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Analysis of flow, heat and mass transfer in porous insulations

Tien, Hwa-Chong, Ph.D.
The Ohio State University, 1989
ANALYSIS OF FLOW, HEAT AND
MASS TRANSFER IN POROUS INSULATIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

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* * * * *

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To My Parents, Wife and My Beloved Children
ACKNOWLEDGMENTS

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A = aspect ratio, $\bar{H} / \bar{L}$
B = Biot number referring to heat transfer, $\bar{h} \bar{L} / \bar{k}_{\text{eff}, o}$
B* = Biot number referring to mass transfer, $\bar{h}^* \bar{L} / \bar{\alpha}_{\text{eff}, o}$
Bv = Biot number referring to vapor transport, $\bar{h}_v \bar{L} / \bar{\alpha}_{\text{eff}, o}$
$\bar{C}_p$ = average heat capacity, W-s/kg-K
$\bar{c}_i$ = dimensional heat capacity for the ith phase at constant pressure, W s/kg-K
$\bar{c}_o$ = reference heat capacity, W-s/kg-K
$\bar{D}_{\text{v, eff}}$ = effective vapor diffusivity coefficient, m$^2$/s
$\bar{g}$ = dimensionless gravity vector
$\bar{h}$ = heat transfer coefficient, W/m$^2$-K
$\bar{h}^*$ = mass transfer coefficient, m/s
\( h_v \) = species transfer coefficient, m/s

\( \Delta h_{\text{vap}} \) = enthalpy of vaporization per unit mass, J/kg

\( \bar{H} \) = height of the porous insulation, m

\( H' \) = length of the opening of the porous insulation, m

\( k_i \) = dimensional thermal conductivity for phase i, W/m-K

\( k(T) \) = \(- \partial (p_c) / \partial (T)\), N/m²-K

\( k_e \) = \(- \partial (p_c) / \partial \varepsilon \), N/m²

\( k_{\text{eff}} \) = dimensional effective thermal conductivity, W/m-K

\( \bar{K} \) = permeability, m²

\( \bar{K}_\beta \) = effective liquid permeability, m²

\( \bar{K}_\gamma \) = effective gas permeability, m²

\( K_{r\beta} \) = relative permeability for the liquid phase

\( K_{r\gamma} \) = relative permeability for the gas phase

\( \bar{L} \) = thickness of the insulation, m

\( Le \) = Lewis number = \( \bar{\alpha}_{\text{eff},o} / \bar{D}_v, \text{eff} \)

\( \dot{m} \) = dimensional condensation rate, kg/m³-s
Nuh = Nusselt number as defined separately in equations (4.9), (5.5) and (6.6)

pa = dimensionless air pressure, \( \bar{P}_a / \bar{P}_{a,0} \)

\( \bar{P}_c \) = capillary pressure = \( \bar{P}_\gamma - \bar{P}_\beta \), N/m²

pv = dimensionless vapor pressure, \( \bar{P}_v / \bar{P}_{v,0} \)

py = dimensionless gas phase pressure, \( \bar{P}_\gamma / \bar{P}_{\gamma,0} \)

Pe = Peclet number, \( \bar{v}_\gamma,0 \bar{L} / \bar{\alpha}_{\text{eff},0} \)

\( \bar{r} \) = characteristic length of the porous matrix, m

\( \bar{R}_a \) = air gas constant, N-m/kg-K

\( \bar{R}_v \) = vapor gas constant, N-m/kg-K

s = scaled fractional liquid saturation = \( (s - s_{\text{pp}}) / (1 - s_{\text{pp}}) \)

sβ = fractional liquid saturation = \( e / (e + e_\gamma) \)

sβp = the saturation for the immobile liquid

t = dimensionless time = \( \bar{t} / (\bar{L}^2 / \bar{\alpha}_{\text{eff},0}) \)

T = dimensionless temperature = \( \bar{T} / \Delta\bar{T} \)

\( \bar{T}_h \) = reference temperature for the hot side of the insulation, K
\( \bar{T}_c \) = reference temperature for the cold side of the insulation, K

\( \bar{T}_{\infty,h} \) = hot-side ambient temperature, K

\( \bar{T}_{\infty,c} \) = cool-side ambient temperature, K

\( y_o \) = reference position in the vertical direction for pressure

**Greek symbols**

\( \bar{\alpha}_{\text{eff},o} \) = reference effective thermal diffusivity = \( \bar{k}_{\text{eff},o} / (\bar{\rho}_o \bar{c}_o) \), m\(^2\)/s

\( \varepsilon \) = non-dimensional pressure difference across the insulation matrix, \( \Delta\bar{p} / \bar{p}_{\gamma,o} \)

\( \varepsilon_{\sigma} \) = volume fraction for the solid phase

\( \varepsilon_{\beta} \) = volume fraction for the liquid phase

\( \varepsilon_{\gamma} \) = volume fraction for the gas phase

\( \eta \) = opening size ratio, \( \bar{H}' / \bar{H} \)

\( \bar{\mu}_\beta \) = liquid dynamic viscosity, kg/m-s

\( \bar{\mu}_\gamma \) = gas dynamic viscosity, kg/m-s

\( \bar{\rho} \) = dimensional total density defined in equation (2.17), kg/m\(^3\)

\( \rho_v \) = dimensionless vapor density, \( \bar{\rho}_v / \bar{\rho}_{v,o} \)
\( \rho_i \) = dimensionless density for phase i, \( \bar{\rho}_i / \bar{\rho}_{i,o} \)

\( \sigma_{\beta y} \) = surface tension at the gas and liquid interface, N/m

\( \omega \) = relative humidity

\( \omega_{\infty, h} \) = hot-side ambient relative humidity

\( \omega_{\infty, c} \) = cool-side ambient relative humidity

**Subscripts**

- \( a \) = denotes the air phase
- \( c \) = denotes the cool side of the insulation
- \( \text{eff} \) = denotes the effective properties
- \( h \) = denotes the hot side of the insulation
- \( i \) = denotes the ith phase
- \( o \) = denotes the reference quantities
- \( s \) = refers to the saturation quantities
- \( v \) = denotes the vapor phase
- \( x \) = denotes the component in x direction
- \( y \) = denotes the component in y direction
$\beta$ = denotes the liquid phase

$\gamma$ = denotes the gas phase

$\sigma$ = denotes the solid matrix

$\infty$ = denotes the ambient quantities in the surroundings

**Superscripts**

$-$ = refers to dimensional quantities

**Other**

$<>$ = denotes the "local volume average" of the quantity.
CHAPTER 1
INTRODUCTION

There are many studies in the heat transfer literature which fall within the general area of heat and mass transfer in porous media with or without phase change. This is because a lot of applications in thermal engineering require extensive knowledge in this area. Such applications include but are not limited to building insulations, heat exchangers, grain storage, energy conservation, drying technology, oil extraction and geothermal systems, etc. Some of the studies in this general area, such as Phillip and Devries [1], Devries [2], Luikov [3], and Whitaker [4] presented theoretical formulations for these types of problems. Among these formulations, the model proposed by Whitaker [4] is very general and rigorous. The resulting transport equations can be applied to a class of problems in heat and mass transfer in porous materials. Most of the studies in this area are related to more specific applications. In the latter category, the subject of drying in a porous material has received considerable attention. For example, Ceaglske and Hougen [5] investigated the drying of granular porous media and found that the drying rate is determined not by diffusion but by capillary forces; Berger and Pei [6] proposed a mathematical model of drying, taking both vapor diffusion and capillary liquid flow as well as heat transfer through solids into account; Whitaker and Chou [7] constructed a simplified theory for drying granular porous media, and Kaviany and
Mittal [8] investigated the drying of a non-hygroscopic porous slab initially saturated with liquid up to the critical time. There are also a number of studies which are related to other applications. Reddy et al. [9] analyzed the heat and mass transfer in soils with heat sources. Udell and co-workers [10,11] investigated the heat transfer in porous media with phase change and capillarity - the heat pipe effect. Eckert and Faghri [12], and Dinulescu and Eckert [13] studied the one-dimensional moisture migration caused by temperature gradients in a porous medium. Both studies by Eckert and Faghri [12], and Dinulescu and Eckert [13] are related to the problems such as the moisture redistribution in soil under the influence of solar heat. The above-mentioned examples clearly indicate the continuous interest in the subject of simultaneous heat and mass transfer in porous media.

An important topic in the area of energy conservation and building insulation design relates to the influence of condensation on the thermal performance of a porous insulation matrix. Water vapor condensation can take place anywhere in a porous insulation when the vapor density is greater than the saturation vapor density which corresponds to the local temperature at that point. The condensation phenomenon has been observed in a porous wall insulation especially when the insulating material is exposed to relatively moderate temperature differences and/or relatively high humidity environments. As the condensation occurs, the latent heat of vaporization is released acting as a heat source in the heat transfer process. Furthermore, the liquid phase resulting from condensation will cause an increase in the energy transfer across the insulation and hence it affects the
thermal performance of the insulation. Equally as important, the condensate deteriorates the quality of the porous materials.

Infiltration is also one of the important subjects in the field of building insulation and energy conservation. Usually a porous material is introduced in an enclosure in order to reduce the convective and radiative heat transfer. However, it has been found that at room temperature the radiation effects on the overall energy transfer are small, as shown in the studies by Verschoor and Greebler [14], Mumaw [15], and Lopez [16]. Therefore, convection and conduction are the major modes of heat transfer in porous insulation materials. Since the fibrous materials are frequently used as building insulations, any improvement in the thermal performance will result in a great reduction in the overall energy consumption. Apparently, it is of applied and fundamental importance to investigate the problem of heat and mass transfer in porous insulation materials accounting for both infiltration and condensation effects.

The heat and moisture transport in a porous insulation is a multi-dimensional problem with multi-component (air-vapor mixture) flow. In general, a wet porous insulation consists of three phases. These are the solid matrix, the liquid water, and a binary gas phase composed of air and vapor. The transport mechanisms involved in the process are quite complicated. In the gas phase, there is vapor diffusion due to the vapor concentration gradients, bulk convection due to the density variation induced by temperature gradients, and air infiltration due to the small difference in gas pressure across the insulation. There is heat conduction in all three phases and heat convection in the gas phase. There will also be convection
in the liquid phase if it is mobile. In addition, there is heat transfer caused by phase change at the gas-liquid interface. The above-mentioned phenomena present complex interactions between heat and mass transport mechanisms.

There are several experimental studies on the subject of condensation such as Tye and Spinney [17], Stewart [18] and Langlais [19]. There are also several analytical and numerical studies in the literature. A one-dimensional quasi-steady work was done by Ogniewicz and Tien [20]. In [20], the condensation process was assumed to be characterized by three distinct regimes while the solution obtained was for the second (quasi-steady) regime only. Also, the variations in gas phase properties were assumed to be constant. Vafai and Sarkar [21] studied one-dimensional transient as well as steady state heat and mass transport in the insulation materials. Most of the assumptions made in [20] were resolved such as the three-stage condensation process assumption and the constant gas phase property assumption. It is noted that both studies mentioned above accounted for uniform infiltration. Vafai and Whitaker [22] conducted a transient two-dimensional numerical study with simplifying assumptions. In addition, there are also two investigations by Burns et al. [23] and Burns and Tien [24] dealing with steady state forced convection (infiltration) and free convection. However in both studies [23,24], neither the transient effects nor the condensation and the multi-component effects are included.

The objective of this work is to perform a systematic study on transient two-dimensional heat and mass transfer in porous materials with condensation and infiltration effects. The present work is structured in the following way. In
Chapter 2, the equations which govern the multiphase transport process are first presented in a dimensional form and then in non-dimensional form. The physical meanings for the non-dimensional parameters are explained in some detail. In addition to the governing equations, the convective type of boundary conditions are also formulated. In Chapter 3, different numerical schemes were discussed and an appropriate one was chosen. A two-phase format routine is incorporated into this numerical scheme in order to account for phase change. Chapter 4 deals with a fundamental problem of condensation in porous insulations. The vertical boundaries of the two-dimensional matrix are totally permeable while the top and bottom boundaries are impermeable and insulated. Basic step-change boundary conditions are imposed at the left boundary. This chapter serves as a fundamental investigation for a variety of condensation problems. In Chapter 5, a more realistic porous system is considered. The vertical boundaries are partially permeable to simulate the holes and cracks in the walls. Convective temperature and vapor density conditions are applied at the vertical boundaries. Both infiltration and condensation effects are included. Four typical cases which correspond to four representative opening locations are investigated. Chapter 6 constitutes a parallel study with Chapter 5, in which hydrostatic pressure conditions are imposed on the vertical boundaries. This will simulate the real situations even more closely. Finally, the general conclusions and recommendations are given in Chapter 7.
CHAPTER II

PHYSICAL MODEL AND FORMULATION

2.1 Governing Equations

Since this study aims at fully simulating condensation and infiltration in porous materials, a rigorous and generalized model is needed to describe such a complicated transport process of multiphase heat and mass transfer in porous media with or without phase change. The transport phenomena are first modelled by a system of governing conservation of mass, momentum and energy equations separately for the solid, liquid and the gas phases. These equations are based on a microscopic (pore) scale. Next, the boundary conditions at the three phase interfaces, i.e., solid-liquid, liquid-gas, and solid-gas, are formulated to relate these microscopic governing equations to each other. However in addition to the complexity of these governing equations, it is extremely difficult to exactly specify the pore geometry and formulate the appropriate boundary conditions. Therefore, volume-averaging technique is utilized to work out a system of macroscopic equations to rationally tackle the problem.
The derivation of these governing equations requires significant amount of analysis. After some algebraic manipulations on the work of Whitaker [4], the governing equations are found to be

Thermal energy equation:

\[
\bar{p} \bar{C}_p \frac{\partial \langle \bar{T} \rangle}{\partial t} + (\bar{p} \bar{e}_p \langle \bar{V} \rangle + \langle \bar{p} \gamma \rangle \langle \bar{V} \rangle) \cdot \nabla \langle \bar{T} \rangle \\
+ \Delta h_{vap} \langle \bar{m} \rangle = \nabla \cdot (\bar{k}_{ef} \nabla \langle \bar{T} \rangle) \quad (2.1)
\]

Liquid phase equation of motion:

\[
\langle \bar{V} \rangle = - (\bar{K}_e / \bar{p}_e) [\bar{k}_e \nabla e_e + \bar{k}_T \nabla \langle \bar{T} \rangle - (\bar{p}_e - \bar{p}_y) \bar{g}] \quad (2.2)
\]

Liquid phase continuity equation:

\[
\frac{\partial \bar{e}_e}{\partial t} + \nabla \cdot \langle \bar{V} \rangle + \langle \bar{m} \rangle / \bar{p}_e = 0 \quad (2.3)
\]

Gas phase equation of motion:

\[
\langle \bar{V} \gamma \rangle = - (\bar{K}_\gamma / \bar{p}_\gamma) (\nabla \langle \bar{p} \gamma \rangle - \langle \bar{p} \gamma \rangle \bar{g} / \bar{g}) \quad (2.4)
\]
Gas phase continuity equation:

$$\frac{\partial}{\partial t}(\varepsilon_\gamma \langle \tilde{\rho}_\gamma \rangle \gamma) + \nabla \cdot (\langle \tilde{\rho}_\gamma \rangle \gamma \langle \tilde{\nu}_\gamma \rangle) - \langle \tilde{m} \rangle = 0$$ (2.5)

Gas phase diffusion equation:

$$\frac{\partial}{\partial t}(\varepsilon_\gamma \langle \tilde{\rho}_v \rangle \gamma) + \nabla \cdot (\langle \tilde{\rho}_v \rangle \gamma \langle \tilde{\nu}_\gamma \rangle) - \langle \tilde{m} \rangle = \nabla \cdot [\langle \tilde{\rho}_\gamma \rangle \gamma \tilde{D}_{v, \text{eff}} \langle \tilde{\rho}_v \gamma \rangle \langle \tilde{\rho}_\gamma \rangle \gamma]$$ (2.6)

Volume constraint:

$$\varepsilon_\sigma + \varepsilon_\beta + \varepsilon_\gamma = 1$$ (2.7)

Thermodynamic relations:

$$\langle \tilde{\rho}_v \rangle \gamma = \langle \tilde{\rho}_v \rangle \gamma \tilde{R}_v \langle \tilde{T} \rangle$$ (2.8)

$$\langle \tilde{\rho}_a \rangle \gamma = \langle \tilde{\rho}_a \rangle \gamma \tilde{R}_a \langle \tilde{T} \rangle$$ (2.9)

$$\langle \tilde{\rho}_\gamma \rangle \gamma = \langle \tilde{\rho}_v \rangle \gamma + \langle \tilde{\rho}_a \rangle \gamma$$ (2.10)
In the above equations, the variables with a bar on top refer to dimensional quantities. There are three averaging quantities which appear in the volume averaged governing equations. The spatial average for a quantity $\Psi$ is defined as

$$\langle \Psi \rangle = \frac{1}{V} \int_{V} \Psi \, dV \quad (2.13)$$

where $V$ is an averaging volume which is bounded by a closed surface in the porous medium. The averaging volume $V$ is composed of three phases, the solid phase $V_\alpha$, the liquid phase $V_\beta(t)$ and the gas phase $V_\gamma(t)$. The phase average of a quantity $\Psi_\alpha$ has the following definition:

$$\langle \Psi_\alpha \rangle = \frac{1}{V} \int_{V} \Psi_\alpha \, dV \quad (2.14)$$

where $\Psi_\alpha$ is a quantity associated with an $\alpha$ phase which could be the solid phase, liquid phase or the gas phase. Another average quantity of interest is the intrinsic phase average which is defined by

$$\langle \Psi_\alpha \rangle^\alpha = \frac{1}{V_\alpha(t)} \int_{V_\alpha(t)} \Psi_\alpha \, dV \quad (2.15)$$
The variable properties in the porous insulation are

\[
\bar{k}_{\text{eff}} = \varepsilon_\sigma \bar{k}_\sigma + \varepsilon_\beta \bar{k}_\beta + \varepsilon_\gamma \frac{(\bar{k}_v (\bar{\rho}_v)^\gamma + \bar{k}_a (\bar{\rho}_a)^\gamma)}{(\bar{\rho}_v)^\gamma + (\bar{\rho}_a)^\gamma)}
\]  
\hspace{3cm} (2.16)

\[
\bar{\rho} = \varepsilon_\sigma \bar{\rho}_\sigma + \varepsilon_\beta \bar{\rho}_\beta + \varepsilon_\gamma (\bar{\rho}_v)^\gamma + (\bar{\rho}_a)^\gamma
\]  
\hspace{3cm} (2.17)

\[
\langle \bar{c}_p \rangle^\gamma = \left(\langle \bar{\rho}_v \rangle^\gamma \bar{c}_v + \langle \bar{\rho}_a \rangle^\gamma \bar{c}_a / \langle \bar{\rho}_\gamma \rangle^\gamma
\right)
\]  
\hspace{3cm} (2.18)

\[
\bar{C}_p = \frac{\varepsilon_\sigma \bar{\rho}_\sigma \bar{c}_\sigma + \varepsilon_\beta \bar{\rho}_\beta \bar{c}_\beta + \varepsilon_\gamma \left(\langle \bar{\rho}_v \rangle^\gamma \bar{c}_v + \langle \bar{\rho}_a \rangle^\gamma \bar{c}_a \right)}{\bar{\rho}}
\]  
\hspace{3cm} (2.19)

\[
\bar{\alpha}_{\text{eff}} = \frac{\bar{k}_{\text{eff}}}{\bar{\rho} \bar{C}_p}
\]  
\hspace{3cm} (2.20)

It should be mentioned that there are 12 unknowns appearing in equations (2.1) - (2.12). These unknowns are:

\[\varepsilon_\beta, \varepsilon_\gamma, \langle \bar{T} \rangle, \langle \bar{v} \rangle, \langle \bar{v}_\beta \rangle, \langle \bar{v}_\gamma \rangle, \langle \bar{m} \rangle, \langle \bar{\rho} \rangle^\gamma, \langle \bar{\rho}_\gamma \rangle^\gamma, \langle \bar{\rho}_v \rangle^\gamma, \langle \bar{\rho}_a \rangle^\gamma, \text{ and } (\bar{\rho}_a)^\gamma.\]

In arriving at the governing equations, only three major assumptions were made. These are: (1) the porous insulation material is homogeneous and isotropic;
(2) the three phases in the porous system were assumed to be in local thermodynamic equilibrium; and (3) Darcy's flow model is valid in describing the motion of the gas phase and the liquid phase, i.e. non-Darcian (boundary, inertia and dispersion) effects are assumed negligible. The first two assumptions are the common simplifying procedure in dealing with problems of multiphase heat and mass transport in porous materials. The third assumption is justified due to the following reasons. Vasseur et al. [25] used the results of Vafai and Tien [26] to examine the validity of Darcy's law. Based on their results, two conditions which respectively characterize the inertia and the boundary effects, should be satisfied if the results obtained from Darcy's law are to be within a 10% error band. These two conditions are

\[
\bar{U} < \frac{6 \times 10^{-3} \bar{v}}{(1 - \epsilon_\sigma) F \sqrt{K}} \quad \text{and} \quad \bar{L} > \frac{Pr}{\sqrt{K / (1 - \epsilon_\sigma)}}
\]

where \( \bar{U} \) is the characteristic fluid velocity, \( \bar{v} \) the kinematic viscosity, \( (1 - \epsilon_\sigma) \) the porosity, \( F \) a coefficient related to the inertia parameter, \( K \) the permeability, \( \bar{L} \) the characteristic length of the porous material, and \( Pr \) is the Prandtl number. Based on the physical data used in this work, these two conditions are apparently satisfied. These data as displayed in Table 2.1 denote the typical thermophysical properties for porous insulation materials. All the results presented in this study will be based on the data in Table 2.1. The dispersion effects are not significant in porous insulations. Aside from these assumptions, the governing equations are very
Table 2.1: Physical data.

(a) Reference quantities

<table>
<thead>
<tr>
<th>$\bar{\rho}_0$</th>
<th>$\bar{c}_0$</th>
<th>$\bar{\rho}_v$</th>
<th>$\bar{\rho}_s$</th>
<th>$\bar{\rho}_Y$</th>
<th>$\bar{T}_0$</th>
<th>$\bar{P}_Y$</th>
<th>$\bar{k}_{\text{eff},0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{(kgm}^{-3}\text{)}$</td>
<td>$\text{(Jkg}^{-1}\text{K}^{-1}\text{)}$</td>
<td>$\text{(kgm}^{-3}\text{)}$</td>
<td>$\text{(kgm}^{-3}\text{)}$</td>
<td>$\text{(kgm}^{-3}\text{)}$</td>
<td>$\text{(K)}$</td>
<td>$\text{(N m}^{-2}\text{)}$</td>
<td>$\text{(Wm}^{-1}\text{K}^{-1}\text{)}$</td>
</tr>
<tr>
<td>76.89</td>
<td>842</td>
<td>0.03966</td>
<td>1.08216</td>
<td>1.12182</td>
<td>308</td>
<td>1.013x10$^{-5}$</td>
<td>0.026</td>
</tr>
</tbody>
</table>

(b) Solid phase

<table>
<thead>
<tr>
<th>$\varepsilon_{\sigma}$</th>
<th>$\bar{L}$</th>
<th>$\bar{\rho}_{\sigma}$</th>
<th>$\bar{c}_{\sigma}$</th>
<th>$\bar{k}_{\sigma}$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>(m)</td>
<td>(kgm$^{-3}\text{)}$</td>
<td>(Jkg$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(Wm$^{-1}\text{K}^{-1}\text{)}$</td>
</tr>
<tr>
<td>0.03</td>
<td>0.12</td>
<td>2563</td>
<td>835</td>
<td>0.043</td>
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</table>

(c) Liquid Phase

<table>
<thead>
<tr>
<th>$\bar{\rho}_\beta$</th>
<th>$\bar{c}_\beta$</th>
<th>$\bar{k}_\beta$</th>
<th>$\bar{\mu}_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(kgm$^{-3}\text{)}$</td>
<td>(Jkg$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(Wm$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(kgm$^{-1}\text{S}^{-1}\text{)}$</td>
</tr>
<tr>
<td>1000</td>
<td>4182</td>
<td>0.603</td>
<td>0.8 \times 10$^{-3}$</td>
</tr>
</tbody>
</table>

(d) Gas phase

<table>
<thead>
<tr>
<th>$\bar{c}_v$</th>
<th>$\bar{c}_a$</th>
<th>$\bar{k}_v$</th>
<th>$\bar{k}_a$</th>
<th>$\bar{R}_v$</th>
<th>$\bar{R}_a$</th>
<th>$\bar{\mu}_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Jkg$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(Jkg$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(Wm$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(Wm$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(Jkg$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(Jkg$^{-1}\text{K}^{-1}\text{)}$</td>
<td>(kgm$^{-1}\text{S}^{-1}\text{)}$</td>
</tr>
<tr>
<td>1866</td>
<td>1000</td>
<td>0.0191</td>
<td>0.0262</td>
<td>462</td>
<td>287</td>
<td>1.846 \times 10$^{-5}$</td>
</tr>
</tbody>
</table>

(e) Other quantities

<table>
<thead>
<tr>
<th>$\bar{K}$</th>
<th>$\bar{D}_{v,\text{eff}}$</th>
<th>$\Delta \bar{T}$</th>
<th>$\Delta \bar{h}_{\text{vap}}$</th>
<th>$\bar{\sigma}_{\beta Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m$^2\text{)}$</td>
<td>(m$^2\text{s}^{-1}\text{)}$</td>
<td>(K)</td>
<td>(Jkg$^{-1}\text{)}$</td>
<td>(kg$^{-2}\text{)}$</td>
</tr>
<tr>
<td>7.25 \times 10$^{-10}$</td>
<td>2.8 \times 10$^{-5}$</td>
<td>20</td>
<td>2.4425 \times 10$^{6}$</td>
<td>0.07</td>
</tr>
</tbody>
</table>
general and the results obtained can be applied to a general class of problems for heat and mass transfer in porous materials.

It is noted that the effective gas permeability $\bar{K}_g$ and the effective liquid permeability for partially liquid saturated media $\bar{K}_\beta$ can be expressed in terms of the permeability $K$ and the relative permeabilities, $K_{r\beta}$ and $K_{r\gamma}$, as

$$\bar{K}_g = K_{r\gamma} \bar{K}$$
$$\bar{K}_\beta = K_{r\beta} \bar{K}$$

Based on the relative permeability model suggested by Wyllie [27], which agrees well with the data in the work of Fatt and Klikoff [28] and is also used by Udell [10], the relative permeabilities are taken to have the following forms:

$$K_{r\beta} = s^3$$
$$K_{r\gamma} = (1 - s)^3$$

where

$$s = \frac{s_\beta - s_{\beta p}}{1 - s_{\beta p}}, \quad \text{and} \quad s_{\beta} = \frac{\varepsilon_\beta}{\varepsilon_\beta + \varepsilon_\gamma}$$

The variable $s_{\beta p}$ is the saturation of the liquid at pendular state in the porous medium. Below this saturation, the liquid is essentially immobile due to no inter-pore connections. The liquid is considered immobile for $\varepsilon_\beta < \varepsilon_{\beta p}$, where $\varepsilon_{\beta p}$ is the
liquid fraction corresponding to the saturation value of $s_{\beta_p}$. In this study, the value for $s_{\beta_p}$ is taken to be 0.1. This value was also used by Kaviany and Mittal [8].

2.2 Non-dimensional Governing Equations

In order to cast the dimensional governing equations into a non-dimensional form, a group of dimensionless variables are defined, and as a result, a number of non-dimensional parameters are generated. The dimensionless variables are defined as

$$x = \frac{x}{L}, \quad y = \frac{y}{L}, \quad t = \frac{\alpha_{\text{eff}, o} t}{L^2}$$

(2.25)

$$\tilde{v}_\beta = \frac{\tilde{v}_\beta L}{K_{\text{eff}}/\tilde{\mu}_\beta}, \quad \tilde{v}_\gamma = \frac{\tilde{v}_\gamma}{\tilde{v}_{\gamma, o}}$$

(2.26)

$$T = \frac{T}{\Delta T}, \quad \tilde{m} = \frac{\tilde{m} L^2 \Delta \tilde{h}_{\text{vap}}}{K_{\text{eff}, o} \Delta T}$$

(2.27)

$$\rho_v = \frac{\tilde{\rho}_v}{\tilde{\rho}_{v, o}}, \quad \rho_a = \frac{\tilde{\rho}_a}{\tilde{\rho}_{a, o}}, \quad \rho_\gamma = \frac{\tilde{\rho}_\gamma}{\tilde{\rho}_{\gamma, o}}$$

(2.28)

$$p_v = \frac{\tilde{p}_v}{\tilde{p}_{v, o}}, \quad p_a = \frac{\tilde{p}_a}{\tilde{p}_{a, o}}, \quad p_\gamma = \frac{\tilde{p}_\gamma}{\tilde{p}_{\gamma, o}}$$

(2.29)

$$\tilde{g} = \frac{\tilde{g}}{\tilde{g}_o}$$

(2.30)
and the dimensionless parameters have the following expressions:

\[
P_1 = \frac{\bar{p}_\beta}{p_o}, \quad P_2 = \frac{\bar{c}_\beta}{c_o}, \quad P_3 = \left(\frac{\bar{c}_\gamma}{c_o}\right)^\gamma
\]

\[P_4 = \frac{\bar{p}_{\gamma, o}}{p_o}, \quad P_5 = \frac{\bar{p}_\gamma \bar{g}_o \bar{L}}{p_{\gamma, o}}, \quad P_6 = \frac{\Delta \bar{h}_{\text{vap}}}{c_o \Delta T}
\]

\[P_9 = \frac{\Delta T}{T_o}, \quad P_{11} = \frac{\bar{p}_v, o}{p_{\gamma, o}}, \quad P_{12} = \frac{\bar{p}_a, o}{p_{\gamma, o}}
\]

\[P_{13} = \frac{\bar{p}_v, o}{p_{\gamma, o}}, \quad P_{14} = \frac{\bar{p}_a, o}{p_{\gamma, o}}, \quad P_{15} = \frac{2\bar{\sigma}_{\beta \gamma}}{\bar{r}^p \bar{R}_v \Delta T}
\]

\[P_{16} = \frac{\Delta \bar{h}_{\text{vap}}}{R_v \Delta T}, \quad P_{18} = \frac{\bar{\alpha}_{\text{eff}}}{\bar{\alpha}_{\text{eff}, o}}, \quad P_{19} = \frac{\bar{k}_{\text{eff}}}{k_{\text{eff}, o}}
\]

\[P_{20} = \frac{Kp_{\gamma, o}}{\bar{\mu}_\gamma \bar{v}_\gamma, o}, \quad \text{Pe} = \frac{\bar{v}_\gamma, o \bar{L}}{\bar{\alpha}_{\text{eff}, o}}, \quad \text{Le} = \frac{\bar{\alpha}_{\text{eff}, o}}{D_{v, \text{eff}}}
\]

\[\psi_T = \frac{k_{(1)} \Delta T}{k_{\epsilon}}, \quad \psi_g = \frac{(\bar{p}_\beta - \bar{p}_\gamma) \bar{g}_o \bar{L}}{k_{\epsilon}}, \quad \psi_\epsilon = \frac{k_{k_{\epsilon}}}{\bar{\mu}_\beta \bar{\alpha}_{\text{eff}, o}}
\]
After non-dimensionalization, the dimensionless governing equations were found to be

**Thermal energy equation:**

\[
\frac{\partial (n)}{\partial t} + \frac{P_1 P_2 P_3 P_{18}}{P_{19}} \nabla \cdot \nabla (T) + \frac{P_3 P_4 P_{18} P_{\gamma}}{P_{19}} (\rho_\gamma) \gamma \nabla (T) = \frac{P_{18}}{P_{19}} \nabla^2 (T) + \frac{P_{18}}{P_{19}} \nabla \cdot \nabla (T)
\]

**Liquid phase equation of motion:**

\[
\langle \nabla (\dot{\gamma}) \rangle = - K_{18} (\nabla \epsilon_\beta + \nabla T) - \psi \nabla \langle \dot{m} \rangle
\]

**Liquid phase continuity equation:**

\[
\frac{\partial \epsilon_\beta}{\partial t} + \psi \nabla \cdot \langle \dot{\gamma} \rangle + \frac{1}{P_{18}} \langle \dot{m} \rangle = 0
\]

**Gas phase equation of motion:**

\[
\langle \nabla (\dot{\gamma}) \rangle = P_{20} K_{17} (\nabla (\rho_\gamma) \gamma + P_5 (\rho_\gamma) \gamma \nabla \langle \dot{m} \rangle)
\]

**Gas phase continuity equation:**

\[
\frac{\partial (\epsilon_\gamma (\rho_\gamma) \gamma)}{\partial t} + \psi \nabla \cdot (\langle \rho_\gamma \gamma \rangle \nabla (\dot{\gamma})) - \frac{1}{P_{18} P_6} \langle \dot{m} \rangle = 0
\]
Gas phase diffusion equation:

\[
\frac{\partial}{\partial t} \rho_{\gamma} \frac{\nabla}{\rho_{\gamma}} + \text{Pe} \nabla \cdot \left( \rho_{\gamma} \left( \frac{\nabla}{\rho_{\gamma}} \right) \right) - \frac{1}{\text{Pe}_{\gamma}} \frac{\nabla}{\rho_{\gamma}} = \frac{1}{\text{Le}} \nabla \cdot \left( \rho_{\gamma} \left( \frac{\nabla}{\rho_{\gamma}} \right) \right)
\]

(2.43)

Volume constraint:

\[\varepsilon_\sigma + \varepsilon_\beta + \varepsilon_\gamma = 1\]  

(2.44)

Thermodynamic relations:

\[\langle p_v \rangle_{\gamma} = P_{\rho_v} \langle \rho_v \rangle_{\gamma} \langle T \rangle\]  

(2.45)

\[\langle p_a \rangle_{\gamma} = P_{\rho_a} \langle \rho_a \rangle_{\gamma} \langle T \rangle\]  

(2.46)

\[\langle \rho_{\gamma} \rangle_{\gamma} = P_{\rho_{\gamma}} \langle \rho_{\gamma} \rangle_{\gamma} + P_{\rho_{a}} \langle \rho_{a} \rangle_{\gamma}\]  

(2.47)

\[\langle p_{\gamma} \rangle_{\gamma} = P_{\rho_{\gamma}} \langle p_{\gamma} \rangle_{\gamma} + P_{\rho_{a}} \langle p_{a} \rangle_{\gamma}\]  

(2.48)

\[\langle \rho_{v, s} \rangle_{\gamma} = \frac{1}{P_{\rho_{v}} \langle T \rangle} \exp \left( - \frac{P_{\rho_{15}} + P_{\rho_{16}}}{P_{\rho_{16}} \langle T \rangle} + \frac{P_{\rho_{16}}}{\langle T \rangle} \right)\]  

(2.49)
In the above equations, the main variables of interest are the temperature \(T\), the liquid volume fraction \(\varepsilon_b\), the vapor density \(\langle \rho_v \rangle^\gamma\), the gas density \(\langle \rho_g \rangle^\gamma\), the gas volume fraction \(\varepsilon_g\), and the condensation rate \(\langle \dot{m} \rangle\).

2.3 Physical Meanings of the Non-dimensional Parameters

The physical meanings of the non-dimensional controlling parameters are explained in this section. The controlling parameters \(P_1, P_2, P_4, P_6, P_9, \) and \(P_{11}\) through \(P_{14}\), which are defined in equations (2.31) - (2.34), are constants and are fixed by the choice of the reference quantities. \(P_3, P_{18}\) and \(P_{19}\) are the parameters which vary with the variations of the properties. \(P_{15}\) and \(P_{16}\) are the non-dimensional parameters which appear in the Clausius-Clapeyron equation. \(P_{15}\) accounts for the Kelvin effect and is far less dominant than \(P_{16}\) in determining the saturation vapor density. \(P_5, P_e\) and \(P_{20}\) have direct influence on the gas phase convective terms. \(P_5\) accounts for the body force, and \(P_e\) and \(P_{20}\) affect the numerical stability. \(\psi_g\) and \(\psi_T\) describe the relative importance of gravity and thermal gradients in transporting the liquid phase. Finally \(Le\) and \(\psi_e\) are the controlling parameters which characterize the importance of the vapor diffusion and liquid transport relative to the energy transport.
2.4 Boundary Conditions

2.4.1 Dimensional Convective Boundary Conditions

In addition to the specified-value type of boundary conditions, another type of the boundary conditions is also frequently encountered, namely, convective type of boundary conditions. The convective boundary conditions at the porous medium-surroundings gas interface are formulated in a way similar to deriving the governing equations. The dimensional boundary conditions for the mass, energy and species balance are found to be

Mass balance:

\[ \langle \bar{p} \rangle \langle \bar{v} \rangle + \langle \bar{p}_\gamma \rangle \langle \bar{v}_\gamma \rangle \cdot \bar{n} = \bar{n}^* \left( \langle \bar{p}_\gamma \rangle - \bar{p}_\infty \right) \]  \hspace{1cm} (2.50)

Energy balance:

\[ \bar{p} \langle \bar{v} \rangle \Delta h_{vap} \cdot \bar{n} + k_{eff} \nabla T \cdot \bar{n} = \bar{n}(T_\infty - \langle \bar{T} \rangle) \]  \hspace{1cm} (2.51)

Species balance:

\[ [\bar{p} \langle \bar{v} \rangle + \langle \bar{p}_v \rangle \langle \bar{v}_\gamma \rangle - \langle \bar{p}_\gamma \rangle \bar{D}_v, \text{eff} \nabla \langle \bar{p}_v \rangle \langle \bar{p}_\gamma \rangle] \cdot \bar{n} = \bar{n}_v (\langle \bar{p}_v \rangle^\gamma - \bar{p}_v, \infty) \]  \hspace{1cm} (2.52)
where \( \mathbf{n} \) is the unit normal vector which points out from the porous medium into the surrounding gas phase, and \( h^*, \ h^\gamma \) and \( h_v \) are the mass transfer, heat transfer, and species (vapor) transfer coefficients, respectively.

### 2.4.2 Non-dimensional Convective Boundary Conditions

The non-dimensional convective boundary conditions were obtained by using the same non-dimensionalization procedure as in arriving at the dimensionless governing equations. They are now given as below.

**Mass balance:**

\[
(P_1 \psi e \langle \vec{v}_\beta \rangle + Pe P_4 \langle \rho \gamma \rangle \langle \vec{v}_\gamma \rangle) \cdot \mathbf{n} = B^* P_4 \langle \rho \gamma \rangle^\gamma - \rho_\infty
\]  
(2.53)

**Energy balance:**

\[
P_1 P_\delta \psi e \langle \vec{v}_\beta \rangle \cdot \mathbf{n} + P_1 \gamma \nabla T \cdot \mathbf{n} = B (T_\infty - \langle T \rangle)
\]  
(2.54)

**Species balance:**

\[
[P_1 \psi e \langle \vec{v}_\beta \rangle + Pe P_4 P_{11} \langle \rho \gamma \rangle^\gamma \langle \vec{v}_\gamma \rangle - \frac{P_4 P_{11}}{Le} \langle \rho \gamma \rangle^\gamma \nabla (\frac{\langle \rho \gamma \rangle^\gamma}{\langle \rho \gamma \rangle})] \cdot \mathbf{n} = B_v P_4 P_{11} \langle \rho \gamma \rangle^\gamma - \rho_{v, \infty}
\]  
(2.55)
where $B^*$, $B$ and $B_\nu$ are the Biot numbers referring to, the mass transfer, heat transfer, and the species transport, respectively. In particular for impermeable and insulated boundaries, all of the three Biot numbers reduce to zero, and therefore the right-hand sides of equations (2.53)-(2.55) drop out. The non-dimensional governing equations, equations (2.38)-(2.49), and the boundary conditions, equations (2.53)-(2.55), presented above constitute a complete system of working equations which will be invoked in the latter chapters to solve the problems of condensation and air infiltration in porous insulations.
CHAPTER III
SOLUTION METHOD AND
COMPUTATIONAL PROCEDURES

3.1 Solution Scheme

An explicit finite difference scheme was employed to solve the complete
governing equations associated with the boundary conditions. In this scheme, the
spatial derivatives are discretized by the central differencing except for most of the
convective terms which are approximated by an upwind differencing scheme.
Since the physical phenomena for this problem are very complicated, the required
time step size was found to be relatively small. This numerical scheme is composed
of two different formats in time and space to account for phase change. Based on
the experimental work of Langlais et al. [19], the liquid content was considered to
be part of the adsorbed water for \( \varepsilon_B < 10^{-5} \). Therefore, no bulk condensation was
allowed for \( \varepsilon_B < 10^{-5} \) and hence the condensation rate was set to be zero. For \( \varepsilon_B > 10^{-5} \), the liquid was considered to be part of the condensate. This two-phase format
routine which was originated by Vafai and Whitaker [22] now is described as
follows with some modifications.
Format I: This format is for the situation with no phase change, i.e. when $\epsilon_{B} < 10^{-5}$.

1. Since the initial values for $\epsilon_{B}, \langle T \rangle, \langle p_{\gamma} \rangle_{\gamma}, \langle \dot{m} \rangle$ and $\langle p_{\gamma} \rangle_{\gamma}$ are known, the values for $\langle p_{\gamma} \rangle_{\gamma}, \langle p_{a} \rangle_{\gamma}, \langle \rho_{a} \rangle_{\gamma}, \langle p_{\gamma} \rangle_{\gamma}$ and $\epsilon_{\gamma}$ can be obtained by solving equations (2.45), (2.48), (2.46), (2.47) and (2.44). It is noted that step one is applied only initially.

2. $\langle \dot{m} \rangle$ is set equal to zero due to reasons mentioned above.

3. Equations (2.40), (2.44), (2.42), (2.38) and (2.43) are solved for $\epsilon_{B}, \epsilon_{\gamma}, \langle \rho_{\gamma} \rangle_{\gamma}, \langle T \rangle$ and $\langle \rho_{\gamma} \rangle_{\gamma}$ respectively.

4. The values obtained in step three are used to update $\langle \rho_{\gamma} \rangle_{\gamma}, \langle \rho_{a} \rangle_{\gamma}, \langle p_{a} \rangle_{\gamma}$ and $\langle p_{\gamma} \rangle_{\gamma}$ from equations (2.45), (2.47), (2.46) and (2.48).

5. Calculate $\langle \rho_{\gamma}, s \rangle_{\gamma}$ from the Clausius-Clapeyron equation, equation (2.49).

6. At any location, if the $\langle \rho_{\gamma} \rangle_{\gamma}$ obtained in step three is greater than or equal to $\langle \rho_{\gamma}, s \rangle_{\gamma}$ in step five, Format II will be employed for the next time step at that location.
Format II: This format is for the situation with phase change, i.e. when $\varepsilon_B > 10^{-5}$.

1. At the beginning of this format the known values of $\varepsilon_B, \langle T \rangle, \langle P_\gamma \rangle, \langle \dot{m} \rangle$ are used to obtain $\langle \rho_v \rangle_\gamma, \langle p_v \rangle_\gamma, \langle p_a \rangle_\gamma, \langle \rho_a \rangle_\gamma, \langle \rho_\gamma \rangle_\gamma$ and $\varepsilon_\gamma$ by solving equations (2.49), (2.45), (2.48), (2.46), (2.47) and (2.44). It is noted that step one is applied only initially. Also note that $\langle \rho_v \rangle_\gamma$ is set equal to $\langle \rho_v, s \rangle_\gamma$.

2. $\varepsilon_B, \langle \rho_\gamma \rangle_\gamma, \langle T \rangle$ and $\varepsilon_\gamma$ are updated by solving equations (2.40), (2.42), (2.38) and (2.44).

3. The new values obtained in step two are used to update $\langle \rho_v \rangle_\gamma, \langle p_v \rangle_\gamma, \langle p_a \rangle_\gamma, \langle \rho_a \rangle_\gamma$ and $\langle p_\gamma \rangle_\gamma$ from equations (2.49), (2.45), (2.47), (2.46) and (2.48).

4. The condensation rate $\langle \dot{m} \rangle$ is found from equation (2.43).
3.2 Numerical Experimentation

Numerical experimentation was conducted for different versions of upwind differencing methods such as first order upwind, third order upwind, and third order upwind plus fourth order artificial viscosity etc., to determine the accuracy and numerical stability of each scheme. After extensive numerical experimentation was performed on all of these different forms of the upwind differencing, the first order upwind difference scheme was chosen to approximate the convective terms, except in the gas continuity equation, due to its numerical stability. This numerical scheme was further compared with several semi-implicit schemes (i.e., the implicit schemes were used in solving some of the transport equations which have strong convective terms). It was found that the total computational time could not be reduced by using the semi-implicit schemes. As a side product of these extensive comparisons it was found that the results from all of the above-mentioned schemes (explicit or semi-implicit) were in very good agreement with each other.

It was found that the higher the gas permeability is, the stronger the convection terms will be, and hence a smaller time step size is required. This is because the gas phase permeability directly affects the advection terms and hence it has very significant effect in determining the time step size. This fact can be easily seen after examining the expression for $\mathcal{P}_{20}$ in equation (2.36). For this reason, the value of permeability used in this work was chosen to represent a typically high porosity insulation material so that, in addition to corresponding to a very important application, it will also highlight some of the pertinent features of the analysis. For
such a highly porous material, the required time step size is relatively small. Of course the time step size is also affected by the number of grids. The required time step size becomes much smaller as we refine the grids. The accuracy of the numerical results will be discussed in detail in the latter chapters.
CHAPTER IV

A FUNDAMENTAL INVESTIGATION OF
PHASE CHANGE EFFECTS IN POROUS MATERIALS

4.1 Introduction

Transient two-dimensional heat and mass transport accounting for phase change in a porous slab is investigated in this chapter. The variations and the inter-coupling effects of the important field variables such as temperature, vapor density, condensation rate and liquid content are presented. The transient heat transfer rate through the insulation is quantified and the validity of using a one dimensional or constant pressure assumption is investigated. The location and regions of high liquid accumulation are analyzed. Also, the interesting effects of variations of the aspect ratios and humidity levels on the condensation rate, liquid accumulation and the energy transfer will also be discussed in detail.
4.2 Problem Statement

To investigate two-dimensional transient heat and moisture transport accounting for phase change in a porous insulation, a systematic study has been performed on a case which is important from a fundamental point of view as well as an application side. The schematic diagram is shown in Fig. 4.1. The top and bottom boundaries are insulated and impermeable, while the left and right boundaries are permeable and exposed to two different environments, a hot and humid environment on the left-hand side and a cooler environment on the right-hand side. The boundary conditions on the temperature, relative humidity, liquid content and gas phase pressure for the left and right boundaries are specified as

\begin{align*}
T_h &= T(x = 0, y, t) = 15.4 \\
T_c &= T(x = 1, y, t) = 14.65 \\
0 &< \omega_h = \omega(x = 0, y, t) \leq 1 \\
\omega_c &= \omega(x = 1, y, t) = 1 \\
\varepsilon_{\beta}(x = 0, y, t) &= 5.0 \times 10^{-5} \\
\varepsilon_{\beta}(x = 1, y, t) &= 0 \\
p_{\gamma}(x = 0, y, t) &= 1 \\
p_{\gamma}(x = 1, y, t) &= 1
\end{align*}
Fig. 4.1: Schematic diagram of a two-dimensional porous matrix.
It should be noted that the above non-dimensional temperatures translate into \( T_h = 308 \text{ K} \) and \( T_c = 293 \text{ K} \) which are based on physical grounds. Since the top and bottom boundaries are subjected to insulated and impermeable boundary conditions, the convective boundary conditions, equations (2.53)-(2.55) in Chapter 2, will be used with the Biot numbers, \( B^* \), \( B \) and \( B_v \), set equal to zero. Therefore, the boundary conditions for the top and bottom boundaries become

\[
(P_1 \psi \langle \tilde{v}_\beta \rangle + PeP_4 \langle \rho \gamma \rangle^\gamma \langle \tilde{v}_\gamma \rangle) \cdot \mathbf{n} = 0
\]  

(4.5)

\[
P_1 P_6 \psi \langle \tilde{v}_\beta \rangle \cdot \mathbf{n} + P_{19} \nabla T \cdot \mathbf{n} = 0
\]  

(4.6)

\[
[P_1 \psi \langle \tilde{v}_\beta \rangle + PeP_4 P_{11} \langle \rho \gamma \rangle^\gamma \langle \tilde{v}_\gamma \rangle - \frac{P_4 P_{11}}{Le} \langle \rho \gamma \rangle^\gamma \nabla \langle \frac{\rho \gamma}{\langle \rho \gamma \rangle} \rangle] \cdot \mathbf{n} = 0
\]  

(4.7)

The initial conditions are specified as

\[
T(x, y, t = 0) = 14.65
\]

\[
\omega(x, y, t = 0) = 1
\]  

(4.8)

\[
\varepsilon_\beta(x, y, t = 0) = 0
\]

\[
p_\gamma(x, y, t = 0) = 1
\]
To evaluate the heat transfer across the porous insulation matrix, the non-dimensional heat transfer rate at the hot surface, $\text{Nu}_h$, which includes both the heat as well as the mass transfer is given as

$$\text{Nu}_h = \frac{\int_{0}^{A} \left( -P_{19} \frac{dT}{dx} + P_{3}P_{4}Pe\gamma_{\gamma}T_{x} + P_{1}P_{2}\gamma\varepsilon\beta_{x}T \right) \ dy}{AP_{19}(T_{h} - T_{c})} \bigg|_{x=0} (4.9)$$

The Nusselt number, as defined in equation (4.9), accounts for the contribution of heat conduction, infiltration and bulk convection. The physical data used in this investigation, which were based on a typical fibrous insulation, are listed in Table 2.1.

### 4.3 Results and Discussion

As mentioned earlier, this work is aimed at a fundamental investigation of the thermal behavior of the porous material and the dynamic response and the interaction between the field variables such as temperature, liquid content, vapor density, and the condensation rate in a two-dimensional porous medium. It is also aimed at investigating the effects of the aspect ratios, humidity levels and some other pertinent physical parameters. It should be noted that even though the Kelvin effect has been taken into account in equation (2.49), as shown in Chapter 2, a surface tension of 0.07 kg/m² translates into $P_{15} << P_{16}$ if $O(\bar{r}) > 10^{-9}$ m. This
fact was also given in the work of Whitaker and Chou [7] where it was mentioned that surface tension presented very little effect on the saturation vapor pressure.

4.3.1 Transient Variations of the Field Variables

Figure 4.2 shows the temperature distributions inside a porous slab at four different times. As shown in Fig. 4.2, the interior temperature of the insulation material is found to increase with time when the temperature is suddenly increased at the left boundary. The temperature rise results from simultaneous heat conduction and heat convection along with condensation which acts as a local heat source. The increase in temperature starts from the region which is close to the external boundary which is at a higher temperature and then gradually moves inwards into the porous slab. This wave-like propagation was also observed for the vapor density, condensation rate and the liquid content as it can be seen in Figs. 4.3-4.5. For brevity, the contours for the gas phase density are not presented here. However the same type of wave-like propagation was observed for $\langle \rho \gamma \rangle^\gamma$. It should be mentioned that in all of the three dimensional plots for the condensation rate, which are presented in this chapter as well as in Chapters 5 and 6, a positive $\langle \tilde{m} \rangle$ corresponds to condensation whereas a negative $\langle \tilde{m} \rangle$ corresponds to evaporation. The times $t_1, t_2, t_3$ and $t_4$ in figures 4.2 through 4.5 were chosen so as to demonstrate the significant regimes and variations of the field variables. A very important result which becomes apparent from this investigation is that the common assumption of setting $\langle \rho \gamma \rangle^\gamma = \text{constant}$ is not valid at all. This of course,
Fig. 4.2: Spatial variation of temperature inside the porous material for $A = 1$ and $\omega_0 = 1.0$, at four different times: $t_1 = 0.0005, t_2 = 0.0015, t_3 = 0.005$ and $t_4 = 0.01$. 
Fig. 4.3: Vapor density distributions for $A = 1$ and $\omega_h = 1.0$, at four different times corresponding to Fig. 4.2.
Fig. 4.4: Three-dimensional condensation rate plots for $A = 1$ and $\omega_h = 1.0$, at four different times corresponding to Fig. 4.2.
Fig. 4.5: Liquid content distributions at four different times, corresponding to Fig. 4.2, for $A = 1$ and $\omega_h = 1.0$. 
is to be expected as this assumption does not satisfy the continuity equation and hence it does not even yield convergent solutions. The validity of this assumption was also discussed in detail in Vafai and Sarkar's work [21].

The Lewis number, a measure of the relative importance of heat transport to the vapor transport, affects the relative movement of the temperature wave front compared to the vapor density wave front. For the case under investigation with Lewis number less than one, as it can be seen in Figs. 4.2 and 4.3 the vapor density wave front moves faster than the temperature wave front as expected. For Lewis number greater than one, the opposite effect was observed. Another important result which can be observed in Fig. 4.5 is that the liquid accumulates in the region which is next to the hot and humid environment much more than the rest of the porous slab.

4.3.2 Velocity Fields

Figure 4.6 depicts the velocity field distributions for the gas phase. As it can be seen the flow starts from both right and left sides and then moves toward the interior region of the porous insulation since the top and bottom walls are impermeable. Due to the effect of gravity, the fluid moves downward and finally flows out of the porous matrix. This is the only flow configuration that satisfies the gas continuity as well as the gravity requirements. Fig. 4.6 is presented for times $t_1$ and $t_4$ only since the velocity distributions at $t_2$ and $t_3$ are similar to the velocity distributions at $t_1$ and $t_4$. The contours in Figs. 4.2, 4.3 and 4.5 clearly indicate that the physically pertinent variables are dependent on both dimensions of the
Fig. 4.6: Gas phase velocity plots for $A = 1$ and $\omega_h = 1.0$, at two different times:

$t_1 = 0.0005$ and $t_4 = 0.01$. 
porous matrix especially in the mid-region. Therefore a one dimensional analysis would lead to errors especially in the mid-region of the insulation. The two-dimensional behavior results mainly from the fluid motion in the porous matrix. It should be noted that the presented results are for moderate temperature differences imposed across the porous matrix. For larger temperature differences, these two dimensional distortions will become more pronounced. It should also be noted that higher pressure gradients significantly reduce these two dimensional distortions. This is because the vertical boundaries are permeable and therefore the pressure gradients imposed across the insulation cause strong infiltration through the insulation.

4.3.3 Curvilinear Shape of the Field Variable Distributions

The peculiar signature of the contours, with respect to their curvatures, for the temperature, vapor density and the liquid content can be explained as follows. For the early times such as $t_1$, $t_2$ and $t_3$, the wave fronts of the contours are essentially in the left half of the spatial domain. Due to the fluid motion, the propagation of the upper half of the wave fronts is enhanced whereas the propagation of the lower half of the wave fronts is depressed. Therefore the upper parts of the wave fronts are moving faster than the lower parts. Furthermore the top and bottom walls are impermeable and insulated, making the contours perpendicular to the top and bottom boundaries. The combination of these two effects results in the curvatures which are observed in Figs. 4.2, 4.3 and 4.5. For later times such as $t_4$, the wave fronts of the contours are in the right half of the
porous insulation. The fluid motion hinders the diffusion phenomenon in the upper right region while assists the diffusion in the lower right region. However the diffusion in the left domain is still affected by the fluid motion from the left. Therefore even though the curvilinear shape of the contours experience a gradual change, their overall characteristics are still maintained. It should be noted that the gradual change for the shape of the contours, such as temperature and vapor density contours, might not be so apparent, when the wave fronts move into the right half domain. But at the later times $t_5$ and $t_6$, which will be discussed later on, changes in the signature of these contours become more apparent. As result of this type of curvature at the same $x$ locations in the insulation, the values of the temperature and vapor density in the upper region are higher than the corresponding values in the lower region. It should be also noted that one might expect that the liquid content values in the lower region should be larger than the values in the upper region as a result of the gravitational force. That this is not so is due to the small amounts of liquid which are formed inside the porous slab, i.e. when $e_\beta < e_{\beta_p}$ where $e_{\beta_p}$ denotes the liquid content below which the liquid is immobile, the liquid is essentially trapped in the pores. Therefore the gravity will not have a significant role on $e_\beta$ and the liquid cannot pile up in the bottom region of the porous material. However, the current solution scheme did account for and incorporated the possibility of the liquid mobility.
4.3.4 Effects of the Aspect Ratio

Figures 4.2 through 4.6 are based on the case with an aspect ratio of one and the relative humidity of one at the left boundary. For the case with aspect ratio of two, the flow field was found to be qualitatively similar to that of aspect ratio of one. The distributions of the important variables such as the temperature, vapor density, condensation rate, and the liquid content are also qualitatively similar to the case for aspect ratio of one, and hence they are not presented here. In Fig. 4.7, the transient non-dimensional heat transfer rates across the hot wall for \( A = 1, \omega_h = 1.0 \) and \( A = 2, \omega_h = 1.0 \) are depicted. As it can be seen, there is almost no difference between the two cases. This is due to the similarity between the two configurations, and the identity of the boundary conditions in both cases. A close examination of Fig. 4.7 reveals the presence of very small amplitude oscillations, around \( t = 0.0014 \), in the Nusselt number. This type of oscillating phenomenon was also reported by Patterson and Imberger [29], Penot [30], and Staehle and Hahne [31] in studying transient natural convection flows.

4.3.5 Effects of the Humidity Levels

All of the results which were discussed so far were for \( \omega_h = 1.0 \). To examine the effect of different humidity levels, the results for the vapor density distributions and condensation rate are presented for \( \omega_h = 0.8 \) and \( A = 1 \) in Figs. 8 and 9. As expected, the humidity levels at the exterior boundaries have a direct influence on the vapor transport, condensation rate, and hence the liquid content. By comparing Figs. 4.8 and 4.9 with Figs. 4.3 and 4.4 respectively, it can be
Fig. 4.7: Effects of the aspect ratio on the transient Nusselt number distribution.
Fig. 4.8: Vapor density distributions for $A = 1$ and $\omega_n = 0.8$, at four different times corresponding to Fig. 4.2.
Fig. 4.9: Three-dimensional condensation and evaporation rate plots for $A = 1$ and $\omega_h = 0.8$, at four different times corresponding to Fig. 4.2.
Fig. 4.10: Temperature distributions for $A = 1$ and $\omega_h = 0.8$, at four different times corresponding to Fig. 4.2.
found that decreasing the humidity level depresses the vapor transport, the condensation rate and the liquid content. It is also quite interesting to examine the effect of changing the humidity levels on the temperature field while the boundary conditions for the temperature are kept unchanged. If we carefully compare the temperature distributions in Fig. 4.10 which is for $\omega_h = 0.8$ and $A = 1$ with the temperature distributions in Fig. 4.2 which is for $\omega_h = 1.0$ and $A = 1$, we find that the temperature contours are altered due to the change of humidity boundary conditions. For example, the temperature wave front for $\omega_h = 1.0$ in Fig. 2 is moving faster than the temperature wave front for $\omega_h = 0.8$. This is because increasing the humidity level enhances the vapor transport, and the enhancement in vapor transfer will also cause an increase in thermal penetration. This again indicates another aspect of the complex interaction between the temperature and moisture fields.

4.3.6 Verification of the Results

Since there were no available analytical or experimental results which could be used for direct comparison, the numerical scheme was benchmarked through various physically pertinent alternatives. First, runs were made for a case dealing with a square porous slab which is subjected to step-change boundary conditions from all four sides of the slab. Physically, at the steady state, we would expect that the condensation rate would go to zero everywhere in the slab and that the variations for all of the field variables would die out and approach the field values at the periphery of the slab. These expectations were completely verified by our
numerical results. It was also found that for this case, the common constant pressure approximation can yield results which are quite close to the ones without such approximation. In fact the constant pressure approximation for this case reduced the CPU time by several orders of magnitude. This was because the magnitude of the gas phase convection terms, which were only gravity driven, in the governing equations was significantly reduced. However this simplification was found not to be valid for the present case under investigation since the gas phase velocity field was totally different from the velocity field which was obtained without employing the constant pressure approximation. Secondly for the limiting case with pure conduction where there is no convection and condensation, it was found that the numerical results agreed very well with the analytical conduction results. Next, the approach toward steady state at longer times was analyzed. Figure 4.11 shows the temperature and the vapor density contours at later times $t_g$ and $t_5$ for the case $A=1$ and $\omega_H=1$. The approach toward the steady state for the temperature and vapor density distributions can be clearly observed in these figures. The three-dimensional plots for the condensation rate in Fig. 4.12 further confirm the above argument as it can be seen that the condensation rate is significantly diminished as time passes by. Furthermore, as mentioned earlier the results for all of the different explicit and semi-implicit schemes which were investigated in this work, were in very good agreement with each other.

Finally, the accuracy of the numerical results was checked by decreasing the time step size and increasing the number of grids. The results obtained by using different time step sizes while keeping the number of grids constant were found to
Fig. 4.11: Temperature and vapor density distributions for $A = 1$ and $\omega_h = 1.0$, at later times: $t_5 = 0.02$ and $t_6 = 0.03$. 
Fig. 4.12: Three-dimensional condensation rate plots for $A = 1$ and $\omega_h = 1.0$, at later times corresponding to Fig. 4.11.
be in good agreement, both quantitatively and qualitatively. When the number of grids was increased, say from 11x11 to 15x15 and then 21x21, the required time step size decreased drastically even though per time step, computational time did not increase that much since the computation routine was highly vectorized. Although the quality of contours was improved by increasing the number of grids, the primary features of contours were not changed. Therefore most runs were done based on a grid size of 15x15 for A=1 and 15x29 for A=2 except for the results in the Figures 4.11 and 4.12, which are based on grids of 11x11. It should be noted that the numerical computations for these types of problems are extremely intensive. For example after full optimization and strong vectorization of the computation routine, it took about 5.1 hours of Cray X-MP/28 to generate a single curve in Fig. 4.7. To the author's knowledge, this type of analysis which fully simulates the multi-phase transport process with phase change is presented for the first time.

4.4 Conclusions

The phase change process in a porous material was thoroughly analyzed in the present investigation. The problem deals with multiphase heat and mass transfer accompanied by phase change in porous media. The problem was analyzed without making any significant simplifications. In what follows some of the significant conclusions are summarized.

1. The wave-like propagation phenomenon was observed for all of the important field variables such as the temperature, liquid content, vapor density and the condensation rate.
2. The liquid accumulation was found to be heavily concentrated in the region which was adjacent to the hot and humid environment compared to the remainder of the porous insulation.

3. The aspect ratio had an insignificant effect on the Nusselt number.

4. The humidity levels had a direct affect on the vapor transport and condensation process. Increasing the humidity level enhanced the vapor transport, condensation rate, liquid content as well as the thermal penetration.

5. The Lewis number was found to be a very good yardstick for characterizing the relative movement of the temperature wave-front compared to the vapor density wave-front. For Lewis numbers less than one, the vapor density wave-front moves faster than the temperature wave-front. For Lewis numbers greater than one, the reverse trend was observed.

6. The one dimensional model is not valid for a number of situations. This is especially true when the porosity is high and the pressure gradient is very small or zero. However, higher pressure gradients significantly reduce the two dimensional distortions.

7. The constant pressure simplification was found to be a good approximation for a case dealing with a porous slab subjected to step-change boundary conditions on all four sides. Making such simplification can reduce the CPU time drastically. However, this simplification is not at all valid for the general case which is considered in this work. This common assumption should be employed with extreme caution for such type of phase change problems.
CHAPTER V

INFILTRATION AND CONDENSATION EFFECTS
ON AN INSULATION MATRIX

5.1 Introduction

In this chapter, air infiltration and phase change effects on heat and mass transfer in a porous insulation are investigated. The vertical boundaries of the porous system are partially permeable in order to simulate the holes or cracks in the building insulations. The interactions of the field variables and the influence of the Biot numbers and the opening size are investigated. The effects of the opening locations on the Nusselt number are researched through investigating four representative cases. This study can serve as a significant step towards full simulation of condensation problems with infiltration in porous insulations.
5.2 Problem Statement

In chapter 4, a fundamental study has been performed on condensation in porous insulations where basic step-change boundary conditions were imposed at the left boundary as driving forces. In this chapter, however, attention is given to a more realistic physical system as shown in Fig. 5.1. The vertical boundaries of the porous insulation are partially permeable to simulate the holes or cracks in walls, which are subjected to the external pressure forces (infiltration). The horizontal boundaries are assumed to be impermeable and insulated. The impermeable parts of the vertical boundaries are also assumed to be insulated. The temperature and vapor density boundary conditions for the permeable parts are not specified; rather, convective boundary conditions are applied.

The convective boundary conditions, equations (2.53)-(2.55), presented in Chapter 2 are used in this chapter. For the permeable portions of the vertical boundaries, $B$ and $B_v$ are non-zero. However for the impermeable portions of the vertical boundaries as well as the horizontal boundaries, $B$, $B_v$ and $B^*$ are both set to be zero. The pressures are specified at the permeable parts of the vertical boundaries as

$$p_y(x = 0, y, t) = 1 + \varepsilon$$

$$p_y(x = 1, y, t) = 1$$

(5.1)

where $\varepsilon$ denotes the non-dimensional pressure difference across the insulation. In this study $\varepsilon$ is taken as $10^{-5}$ to simulate the real situation for which the pressure
Fig. 5.1: Schematic diagram of a two-dimensional porous matrix with partially permeable boundaries for four representative opening locations.
differences are very small. It is this pressure gradient which causes air infiltration through the insulation. The ambient conditions are

\[ T_{\infty, h} = 15.4 \]
\[ T_{\infty, c} = 14.65 \]  (5.2)

and

\[ \omega_{\infty, h} = 1 \]
\[ \omega_{\infty, c} = 1 \]  (5.3)

The initial conditions are specified as

\[ T(x, y, t = 0) = 14.65 \]
\[ \omega(x, y, t = 0) = 1 \]
\[ \varepsilon_{\beta}(x, y, t = 0) = 0 \]
\[ p_{\gamma}(x, y, t = 0) = 1 \]  (5.4)

The overall heat transfer across the porous insulation, which includes both the heat and mass transfer, is best characterized by the average Nusselt number for each permeable portion of the walls. The Nusselt number at the hot side, \( \text{Nu}_h \), is defined as

\[
\text{Nu}_h = \frac{1}{\eta A} \left[ \frac{\int_{y^*}^{y^* + \eta A} \left( -P_{19} \frac{\partial T}{\partial x} + P_3 \rho \gamma \gamma_x T + P_1 P_2 \Psi \varepsilon \beta_x T \right) \, dy}{P_{19}(T_{\infty, h} - T_{\infty, c}) + P_3 P_4 \rho^* \gamma v^* \gamma_x T_{\infty, h}} \right]_{x = 0}
\]  (5.5)
The Nusselt number, as defined in equation (5.5), accounts for the contribution of heat conduction, infiltration and bulk convection. In the above expression, the numerator accounts for the actual heat transfer and the denominator represents a suitable chosen reference heat transfer. It should be noted that the integration in the numerator is done over the permeable area only. Since, as it will be seen later, the major contribution to the overall heat transfer is due to infiltration, it is then reasonable to include the infiltration effect in the reference heat flux which is used to form the Nusselt number. For this reason, the second term in the denominator on the right-hand side of equation (5.5) accounts for a one-dimensional infiltration. Also, \( \eta \) is the ratio of the opening height to the total height of the vertical boundary.

### 5.3 Results and Discussion

Two checks were made in order to investigate the accuracy of the numerical results. First, the effect of the time step size was checked by fixing the grid size while varying the time step size. It was found that decreasing the time step size beyond a certain value which was used in the numerical experiments did not have an effect on the heat transfer results and the distributions of the field variables. Next, a comparison of the Nusselt number results was made using three different grid systems, given by 11x11, 15x15 and 21x21. It was found that there is a good agreement between 15x15 and 21x21 results except at the very initial stage. It should be mentioned that the CPU time becomes very excessive for the 21x21 grid system. The intensive CPU time which is required for this problem is due to the intercoupled and complicated nature of the problem, which dictates the use of a very
small time step size. In order to conserve the CPU time, the 15x15 grid system is used for most of the runs.

As mentioned earlier, the objective of this study is to investigate the dynamic response and thermal behavior of porous materials with partially permeable boundaries which are subjected to specified pressure gradient and convective boundary conditions. The interactions between the field variables are demonstrated. In addition, the effects of the Biot numbers, B's and B_y's, the area and the locations of the openings are investigated. Four fundamental cases are analyzed in this chapter. These four configurations are represented in Fig. 1. The results, shown in Figs. 5.2-5.14, are based on the case with the openings on the upper left-hand side and lower right-hand side of boundaries, respectively, case (1) in Fig. 5.1.

Figures 5.2 to 5.5 illustrate the spatial and temporal distributions for the temperature, vapor density, condensation rate and the liquid content. As it can be seen in Fig. 5.2, when the boundaries of the porous system are suddenly exposed to the ambient conditions, the left boundary which is next to the hot and humid environment experiences a sudden increase in temperature. This temperature increase then propagates into the interior region of the porous material at later times. The heat transfer process results from heat conduction, convection, and condensation which acts as a local heat source. The wave-like propagating behavior seen in Fig. 5.2 was also observed for the other variables, such as the vapor density, condensation rate and the liquid content shown in Figs. 5.3-5.5. This wave-like propagating behavior has been reported in Chapter 4 in studying the
Fig. 5.2: Spatial variation of temperature inside the porous material for $B=1000$, $B_v=5\times10^4$ and an opening size ratio of $1/2$ at four different times:

$t_1 = 0.0005$, $t_2 = 0.0015$, $t_3 = 0.005$ and $t_4 = 0.01$. 
Fig. 5.3: Vapor density distributions for $B=1000$, $B_v=5\times10^4$ and an opening size ratio of $1/2$, at four different times corresponding to Fig. 5.2.
Fig. 5.4: Three-dimensional condensation rate plots for $B=1000$, $B_v=5\times10^4$ and an opening size ratio of 1/2 at four different times corresponding to Fig. 5.2.
Fig. 5.5: Liquid content distributions at four different times, corresponding to Fig. 5.2, for $B=1000$, $B_v=5\times10^4$ and an opening size ratio of $1/2$. 
condensation problem with step-change boundary conditions. It should be mentioned that the selected four different times, $t_1$, $t_2$, $t_3$, and $t_4$ in Figs. 5.2-5.5 were chosen so as to characterize the significant variations of the field variables. It should be noted that, as seen in Fig. 5.5, there is a larger concentration of the liquid at the left boundary which is exposed to the hot and humid environment. The velocity field distribution for this case is depicted in Fig. 5.14(a). The velocity distribution is presented at time $t_3$ only since the velocity distributions at other times are quite similar to each other. As it can be seen from this figure, the fluid flow starts from the upper left opening, moves towards the interior region and finally flows out of the porous system through the lower right opening. No circulation is observed inside the enclosure. This is because the gas phase velocity is only of the order of $10^{-4}$-$10^{-3}$m/s. Although the average infiltration velocity is quite small, as it will be shown later, it still has a dominant effect on the overall heat transfer.

5.3.1 Effects of the Convective Heat Transfer Boundary Conditions

To examine the effects of the convective heat transfer boundary conditions on the distributions for the field variables as well as on the Nusselt number, the effects of changes in the Biot number, $B$, were investigated. This was done for $B=10$, 100 and 1000 while keeping other parameters fixed. Figs. 5.6-5.9 depict the distributions for the temperature, vapor density, condensation rate and the liquid content for $B=10$. For brevity, the results for $B=100$ are not presented here. Comparing Figs. 5.6-5.9 with Figs. 5.2-5.5 shows that an increase in the Biot number, $B$, causes an increase in the energy transfer and as a result, enhances the
Fig. 5.6: Spatial variation of temperature inside the porous material for $B=10$, $B_v=5\times10^4$ and an opening size ratio of 1/2 at four different time corresponding to Fig. 5.2.
Fig. 5.7: Vapor density distributions for $B=10$, $B_v=5\times10^4$ and an opening size ratio of $1/2$, at four different times corresponding to Fig. 5.2.
Fig. 5.8: Three-dimensional condensation rate plots for $B=10$, $B_\gamma=5\times10^4$ and an opening size ratio of $1/2$, at four different times corresponding to Fig. 5.2.
Fig. 5.9: Liquid content distributions at four different times, corresponding to Fig. 5.2, for $B=10$, $B_y=5\times10^4$ and an opening size ratio of $1/2$. 
temperature penetration. However the vapor density distributions are not altered significantly since $B_v$ is kept constant. As a result, the condensation rate is decreased with an increase in the Biot number since the saturation vapor density is mainly dependent on the temperature. Subsequently, the trend for the liquid content follows that of the condensation rate.

The effects of the Biot number on heat transfer results are illustrated in Fig. 5.10. As expected, as the Biot number increases from 10 to 100 and then 1000, the Nusselt number keeps on increasing. It should be noted that for two orders of magnitude increase in the Biot number, the Nusselt number increases only about 2%. This is because the infiltration is the dominant mode in the overall heat transfer. Therefore, changes in Biot number do not present a significant effect on the Nusselt number.

5.3.2 Effects of the Convective Species Transfer Boundary Conditions

To examine the effects of the convective species transfer boundary conditions, the effects of changes in the species transfer Biot number, $B_y$, were examined. Fig. 5.11 presents the Nusselt number results for two different $B_y$'s; $B_y=5\times10^4$ and $B_y=5\times10^3$. It is observed that at the initial stage, the Nusselt number for the case with higher $B_y$ is greater than the Nusselt number for the case with lower $B_y$, while the reverse trend is true at the later stage. This is because initially, the condensation rates close to the left boundary are significantly higher for $B_y=5\times10^4$ than for $B_y=5\times10^3$. However, at later times the temperature next to the
Fig. 5.10 Effects of the heat transfer Biot number, $B$, on the Nusselt number for $B_v=5\times10^4$ and an opening size ratio of 1/2.
Fig. 5.11: Effects of the mass transfer Biot number, $B_v$, on the Nusselt number for $B=1000$ and an opening size ratio of 1/2.
hot side increases more rapidly for $B_v=5\times10^4$ than for $B_v=5\times10^3$. Therefore, the temperature gradient at the hot side for the case with larger $B_v$ becomes smaller than the case with smaller $B_v$ at the later stage. As for the effect of $B_v$ on the other field variables, it is found that increasing $B_v$ will enhance the vapor transport, and the liquid content. For brevity, only the condensation rates for the case of $B_v=5\times10^3$ and $B=1000$ is presented in Fig. 5.12. The above-mentioned results clearly indicate the complex inter-coupled nature of heat transfer and vapor transport in these type of problems.

5.3.3 Effects of the Permeable Region

A number of interesting results are obtained through examining the effects of the opening area on the field variables and the Nusselt number. Fig. 5.13 illustrates the Nusselt number results for two different opening area ratios, 1/4 and 1/2, with $B=1000$ and $B_v=5\times10^4$. The Nusselt number is significantly higher for the case with smaller opening than the case with larger opening. This is because the case with smaller opening will produce a larger average flow velocity than the case with larger opening, as shown in Fig. 5.14. It should be noted that the Nusselt number defined in this work is based on the averaged heat transfer along the permeable area of the boundary. So even though the "averaged" heat transfer rate decreases as the opening increases, the total heat transfer rate will still increase since the heat and mass transfer area increases. This was rigorously checked from our numerical results. The distributions for the field variables are qualitatively similar to the previous results and hence they are not presented here.
Fig. 5.12: Three-dimensional condensation rate plots for $B=1000$, $B_v=5 \times 10^3$ and an opening size ratio of $1/2$, at four different times corresponding to Fig. 5.2.
Fig. 5.13: Effects of the opening size on the Nusselt number for $B=1000$ and $B_v=5 \times 10^4$. 
Fig. 5.14: Effects of the opening size on the gas velocity distributions at $t_3$ for $B=1000$ and $B_v=5 \times 10^4$.

(a) $\eta(\text{opening}) = \frac{1}{2}$

(b) $\eta(\text{opening}) = \frac{1}{4}$
5.3.4 Effects of the Opening Locations, Cases 1-4

Fig. 5.1 illustrates the four representative configurations which were discussed earlier. Even though these four cases do not represent all possible combinations, they do present the basic combinations that can exist with respect to the opening locations. An investigation of these different opening locations will therefore provide valuable qualitative information in the design of building insulation configurations.

Fig. 5.15 depicts the Nusselt number results for the aforementioned four configurations with the other parameters, i.e. $B=1000$, $B_v=5 \times 10^4$ and the opening size $=1/4$, being fixed. Clearly, case 1 gives the highest Nusselt number while case 4 gives the lowest. The Nusselt numbers for cases 2 and 3 are in between and almost equal. The difference in the Nusselt numbers for the four cases can be best explained from the gas velocity distributions which are given in Fig. 5.16, since the dominant mode of the overall heat transfer is due to infiltration. In case 1, the gravitational force enhances the flow while in case 4 the reverse effect is true. Cases 2 and 3 are in between. In particular, it should be noted that the flow field for case 4 in Fig. 5.16 is not only quantitatively but qualitatively different from the other cases. This is due to the relatively weak pressure gradient which interacts with the gravitational force and depresses the fluid flow. This kind of flow field is similar to that reported in Chapter 4 in which no pressure gradient was imposed across the insulation. As a result, the Nusselt number for case 4 as seen in Fig. 5.15, is less than one. However it should be noted that the Nusselt number in this work is not defined in the usual way. The effect of one-dimensional uniform
Fig. 5.15: The Nusselt numbers for the four cases corresponding to Fig. 5.1 for $B=1000$, $B_v=5 \times 10^4$ and an opening size ratio of 1/4.
Fig. 5.16: Velocity distributions at $t_3$ for the four cases shown in Fig. 5.1 for $B=1000$, $B_v=5 \times 10^4$ and an opening size ratio of 1/4.
infiltration has been included in the reference heat flux which is used in defining the Nusselt number. If the Nusselt number were defined in the usual way, the Nusselt number value for case 4 would be greater than one (with one corresponding to pure conduction). The effects of the opening locations on the field variables follow a trend similar to that of Nusselt numbers. The above results make it amply clear that the crack or hole locations in a building have a very significant effect on the heat transfer results as well as the generated liquid condensation.

5.4 Conclusions

The air infiltration and condensation effects on heat transfer rates and the generated liquid condensate has been investigated in this work. The important field variables, their subsequent interactions and the effects of the heat and mass transfer Biot numbers were investigated. Furthermore, the influence of the permeable region and the opening locations was systematically researched. In what follows some of the key conclusions are listed.

1. The liquid accumulates much more in the region which is next to the hot and humid environment.

2. Infiltration is the dominant mode of the overall heat transfer even in the presence of very small pressure gradients across the insulation slab.

3. An increase in the heat transfer Biot number causes a subsequent increase in the Nusselt number.
4. An increase in the mass transfer Biot number causes a subsequent increase in the Nusselt number at the initial stage. However, the reverse trend was observed at an intermediate stage.

5. An increase in the opening area decreases the Nusselt number.

6. The effects of the opening locations on the Nusselt number, velocity field, and the other field variables were found to be very significant.
CHAPTER VI

PRESSURE STRATIFICATION EFFECTS
ON AN INSULATION MATRIX

6.1 Introduction

In this chapter, thermal stratification effects on heat and mass transfer in a porous insulation are analyzed. As in Chapter 5, the vertical boundaries of the porous system are partially permeable for simulating the holes or cracks in walls. However, in this work hydrostatic pressure variations are considered on the vertical boundaries. The dependence of the Nusselt number and the field variables on several important parameters is investigated systematically. The numerical results can be utilized for estimating the heat loss due to holes or cracks in the building insulations and other related applications.
6.2 Problem Statement

Fig. 1 shows the physical system considered in this chapter. The vertical boundaries of the porous matrix are partially permeable to simulate the cracks or holes in the walls as in Chapter 5. The pressures are not uniform; rather, hydrostatic pressure variations are imposed on the vertical boundaries. The reference zone is set at the mid-height of the porous system. Due to the pressure difference across the insulation, the fluid is forced to flow through the porous insulation. The horizontal boundaries are taken to be insulated and impermeable. The impermeable parts of the vertical boundaries are also taken to be adiabatic. The permeable parts of the vertical boundaries are exposed to two different environments, a hot and humid environment on the left-hand side and a cooler environment on the right-hand side. Convective temperature and vapor density boundary conditions which were presented in Chapter 2 will be used in this chapter.

For the horizontal boundaries and the impermeable portions of the vertical boundaries, $B$, $B^*$ and $B_y$ are zero and therefore equations (4.5)-(4.7) which appear in Chapter 4 are employed here. However for the permeable portions of the vertical boundaries, $B$ and $B_y$ are non-zero. It should be mentioned that to investigate the effects of the variations of $B$ and $B_y$ is one of the main tasks in the study. The pressures at the permeable portions of the vertical boundaries are assumed to be hydrostatic, i.e.,
Fig. 6.1: Schematic diagram of a two-dimensional porous matrix with partially permeable boundaries for four representative opening locations.
\[ p_\gamma(x, y, t) - p_\gamma(x, y_0, t) = p_5 g_y \int_{y_0}^{y} \rho \gamma \, dy \quad \text{for } x = 0, 1 \] 

where \( g_y \) denotes the dimensionless \( y \) component of the gravity vector, and \( y_0 \) is the reference position in the vertical direction, which in this work is taken at the mid-height of the enclosure. Also the pressures at the two reference positions at the left and right vertical boundaries, respectively, are

\[
\begin{align*}
p_\gamma(x = 0, y_0, t) &= 1 + \varepsilon \\
p_\gamma(x = 1, y_0, t) &= 1
\end{align*}
\]

Therefore, it is this pressure difference which will induce the infiltration through the porous system. In the present work, \( \varepsilon \) is again taken as \( 10^{-5} \). This value corresponds to a very small pressure difference which in turn relates to what exists in a realistic situation. It should be noted that since \( \langle \rho \gamma \rangle \) varies along the vertical boundaries, iterations are needed to make \( \langle \rho \gamma \rangle \), \( \langle \rho \gamma \rangle \), as well as \( \langle \rho \gamma \rangle \), \( \langle \rho \gamma \rangle \), \( \langle \rho \gamma \rangle \), and \( \langle \rho \gamma \rangle \) satisfy equations (2.45)-(2.48), (2.55) and (6.1) simultaneously. The temperature and humidity conditions at the ambient environments are specified as

\[
\begin{align*}
T_{\infty, h} &= 15.4 \\
T_{\infty, c} &= 14.65
\end{align*}
\]
The initial conditions are specified as
\[
T(x, y, t = 0) = 14.65
\]
\[
\omega(x, y, t = 0) = 1
\]
\[
\varepsilon_\beta(x, y, t = 0) = 0
\]
\[
\rho_\gamma(x, y, t = 0) = 0
\]
(6.5)

It is noted that the initial pressure condition is also taken to be hydrostatic.

As in Chapter 5, the overall energy transfer across the porous insulation can be expressed by the averaged Nusselt number for each permeable portion of the wall, accounting for the contribution from both the heat and mass transfer. The Nusselt number at the hot side, \( \text{Nu}_h \), is defined as

\[
\text{Nu}_h = \frac{1}{\eta A} \frac{1}{\int_{y^*}^{y} \left( -P_{19} \frac{\partial T}{\partial x} + P_3 P_4 \rho_\gamma \gamma_x T + P_1 P_2 \varepsilon_\beta \gamma_x T \right) dy \bigg|_{x = 0}} \]
\[
p_{y^*}(x, y, t = 0) - p_{y^*}(x = 1, y_0, t = 0) = P_3 \varepsilon_\beta \int_{y_0}^{y} \rho_\gamma dy
\]

where \( \eta \) is the ratio of the opening height to the total height. The Nusselt number defined in the above equation includes the effects of heat conduction, infiltration and bulk convection. The numerator denotes the actual energy transfer and the denominator represents a reference heat transfer with infiltration effect included.
since the dominant mode of the overall energy transfer is due to infiltration (forced convection). This fact will be shown clearly in the later sections. It should be noted that the integration in the numerator is done over the permeable portions only.

6.3 Results and Discussion

In this work, the transient thermal behavior of the porous materials as well as the interactions of the field variables are studied. The effects of the opening size, heat transfer Biot number, $B$, and the vapor transfer Biot number, $B_v$, are also investigated. In addition, four cases which correspond to the four representative opening locations are thoroughly investigated.

6.3.1 General Behavior of the Field Variables - 'Wave-like' Propagation

The distributions for the temperature, vapor density, condensation rate, and liquid content at four different times are illustrated in Figs. 6.2-6.5. These results are based on case 1 as shown in Fig. 6.1 with $B=1000$, $B_v=5\times10^6$ and an opening size ratio of $1/2$. The four different times, $t_1$, $t_2$, $t_3$ and $t_4$ are chosen to clearly demonstrate the temporal variations of the field variables. As it can be seen from Fig. 6.2, since the left boundary of the porous matrix is suddenly exposed to the ambient conditions at higher temperature, the left boundary experiences a sudden increase in temperature. The temperature increase will then gradually penetrate into the interior region of the porous system. This 'wave-like' propagating behavior was also observed for the other variables such as the vapor density, condensation
Fig. 6.2: Spatial variation of temperature inside the porous material for $B=1000$, $B_v=5\times 10^4$ and an opening size ratio of $1/2$ at four different times:

$t_1 = 0.0005$, $t_2 = 0.0015$, $t_3 = 0.005$ and $t_4 = 0.01$. 
Fig. 6.3: Vapor density distributions for B=1000, \( B_v = 5 \times 10^4 \) and an opening size ratio of 1/2, at four different times corresponding to Fig. 6.2.
Fig. 6.4: Three-dimensional condensation rate plots for $B=1000$, $B_v=5 \times 10^4$ and an opening size ratio of $1/2$ at four different times corresponding to Fig. 6.2.
Fig. 6.5: Liquid content distributions at four different times, corresponding to Fig. 6.2, for $B=1000$, $B_v=5\times10^4$ and an opening size ratio of 1/2.
rate, and the liquid content shown in Figs. 6.3-6.5. It should be mentioned that for brevity, the gas phase density distributions, which showed a similar 'wave-like' behavior are not presented here. The 'wave-like' propagation phenomena were observed in all of the results presented in Chapters 4, 5 and the present chapter. As expected a larger amount of condensate was formed at the left boundary since it is exposed to a hot and humid environment. This finding was found to be valid for all the cases discussed in the present work. The gas phase velocity distributions for this case are depicted in Fig. 6.6(a). As it can be seen from the figure, due to the pressure difference across the insulation, the fluid is introduced from the upper left opening then flows through the porous matrix, and it is finally injected out of the right-hand side opening. The infiltration speeds were found to be approximately between $10^{-4}$ to $10^{-3}$ m/s. Due to these relatively small infiltration speeds, circulation was not observed in the porous system. However as mentioned in Chapter 5, although the average gas speed is very small, it still presents a significant effect on the overall energy transfer. Also note that only the velocity distributions at time $t_3$ are presented in this work since the quality of the velocity fields does not change significantly with respect to time.

6.3.2 Effects of the Heat Transfer Biot Number, B

The influence of the heat transfer Biot number, B, on the Nusselt number and the distributions for the field variables is studied for a selected configuration, namely case 1, while the other parameters are unchanged, i.e., $B_v=5\times10^4$ and an opening size ratio of 1/2. Fig. 6.7 demonstrates the comparison between the
Fig. 6.6: Effects of the opening size on the gas velocity distributions at $t_3$ for $B=1000$ and $B_v=5\times10^4$. 

(a) $\eta$(opening) = $\frac{1}{2}$

(b) $\eta$(opening) = $\frac{1}{4}$
Nusselt number results for \( B = 10 \) and \( B = 1000 \). As expected, the Nusselt number for \( B = 1000 \) is greater than the Nusselt number for \( B = 10 \). However, it should be noted that although the Biot number has been increased from 10 to 1000, i.e., two orders of magnitude, the increase in the Nusselt number is not proportional. This is because the energy transfer due to infiltration plays a very important role in determining the overall heat transfer. As a result, the change in the Biot number does not have much influence on the Nusselt number. In Fig. 6.7, both curves present some small amplitude oscillations, between \( t = 0.001 \) to \( t = 0.0015 \). This kind of oscillating phenomenon was also reported in the work of Patterson and Imberger [29], Penot [30], and Staehle and Hahne [31] on transient natural convection flows.

It is of equal importance to examine the effects of the heat transfer Biot number on the field variables. Figs. 6.8-6.10 illustrate the distributions for the temperature, the condensation rate and the liquid content for case 1 with \( B = 10 \), \( B_v = 5 \times 10^4 \) and an opening size ratio of 1/2. By comparing Fig. 6.8 with Fig. 6.2, it can be seen that a decrease in the Biot number causes a decrease in the energy transfer rate and as a result the thermal penetration length decreases. However, the vapor density contours were found almost unchanged since the vapor transfer Biot number, \( B_v \), was the same in both situations. Since the saturation vapor density is mainly dependent on the temperature, the condensation rate for \( B = 10 \) is higher than that of \( B = 1000 \), as shown in Figs. 6.4 and 6.9. This is directly related to the thermal penetration effects. Slower thermal penetration will translate into higher condensation rates. This trend was also observed for the liquid content results as
Fig. 6.7: Effects of the heat transfer Biot number, $B$, on the Nusselt number for $B_{\text{v}}=5\times10^4$ and an opening size ratio of $1/2$. 
Fig. 6.8. Spatial variation of temperature inside the porous material for $B=10$, $B_v=5\times10^4$ and an opening size ratio of 1/2 at four different times corresponding to Fig. 6.2.
Fig. 6.9. Three-dimensional condensation rate plots for $B=10$, $B_v=5 \times 10^4$ and an opening size ratio of 1/2, at four different times corresponding to Fig. 6.2.
Fig. 6.10: Liquid content distributions at four different times, corresponding to Fig. 6.2, for $B=10$, $B_v=5 \times 10^4$ and an opening size ratio of 1/2.
shown in Fig. 6.10. This is because the liquid content is mainly related to the time integration of the condensation rate.

6.3.3 Effects of the Vapor Transport Biot Number, $\text{B}_v$

To examine the effects of the convective vapor transfer boundary conditions, the effects of changes in the vapor transfer Biot number, $\text{B}_v$, were examined. Fig. 6.11 presents the Nusselt number results for two different $\text{B}_v$'s; $\text{B}_v=5\times10^4$ and $\text{B}_v=5\times10^3$. It is observed that at the initial stage, the Nusselt number for the case with higher $\text{B}_v$ is greater than the Nusselt number for the case with lower $\text{B}_v$, while the reverse trend is true at the later stage. This is because initially, the condensation rates close to the left boundary are significantly higher for $\text{B}_v=5\times10^4$, due to higher rate of vapor transport, than for $\text{B}_v=5\times10^3$. However, at later times the temperature next to the hot side increases more rapidly for $\text{B}_v=5\times10^4$, as a result of that extra condensation, than for $\text{B}_v=5\times10^3$. Therefore, the temperature gradient at the hot side for the case with larger $\text{B}_v$ becomes smaller than the case with smaller $\text{B}_v$ at the later stage. The effects of $\text{B}_v$ on the other field variables were also investigated. It was found that an increase in $\text{B}_v$ will result in an enhancement in vapor transport, condensation rate and the liquid content. The temperature, vapor density and the condensation rates for the case of $\text{B}_v=5\times10^3$ and $\text{B}=1000$ are presented in Figs. 6.12 and 6.14. The above-mentioned results clearly display the complex inter-coupled nature of heat transfer and vapor transport in these type of problems.
Fig. 6.11: Effects of the mass transfer Biot number, $B_v$, on the Nusselt number for $B=1000$ and an opening size ratio of 1/2.
Fig. 6.12: Spatial variation of temperature inside the porous material for $B=1000$, $B_v=5\times10^3$ and an opening size ratio of 1/2 at four different times corresponding to Fig. 6.2.
Fig. 6.13: Vapor density distributions for $B=1000$, $B_v=5 \times 10^3$ and an opening size ratio of $1/2$, at four different times corresponding to Fig. 6.2.
Fig. 6.14: Three-dimensional condensation rate plots for $B=1000$, $B_v=5\times10^3$ and an opening size ratio of $1/2$, at four different times corresponding to Fig. 6.2.
6.3.4 Effects of the Size of the Permeable Area

The effects of the size of the open region on the Nusselt number and the field variables were investigated for all cases; however due to similarities between all of the cases only the results for case 1 are presented here. The Nusselt number results for two different opening areas, for case 1, are illustrated in Fig. 6.15. As shown clearly in Fig. 6.15, the Nusselt number for a larger opening is less than the Nusselt number for a smaller opening. This is because the mean air infiltration velocity for the case with a larger opening is lower than that with a smaller opening. This can be clearly seen from the corresponding gas velocity distributions which are shown in Fig. 6.6. Therefore, as seen from the definition of the Nusselt number in equation (6.6), the Nusselt number for the smaller opening size will become higher than that of the larger opening size. However it should be clarified that the Nusselt number defined in this work is based on an averaged heat transfer rate over the permeable area. Therefore, the total heat transfer rate for the larger opening is still greater than that for the smaller opening since the total heat transfer rate is directly dependent on the heat transfer area. This fact can be clearly checked by multiplying the Nusselt number for $\eta=1/2$ in Fig. 6.15 by two. The profiles of the field variables for opening=1/4 are quite similar to the results for opening=1/2 and hence they are not presented here.
Fig. 6.15: Effects of the opening size on the Nusselt number for $B=1000$ and $B_v=5 \times 10^4$. 

- $\eta_{\text{opening}} = \frac{1}{2}$
- $\eta_{\text{opening}} = \frac{1}{4}$
6.3.5 Effects of the Opening Locations, Cases 1-4

The effects of the opening locations on the transient thermal performance of the porous materials were also investigated in some detail in this work. The Nusselt number results for the four cases illustrated in Fig. 6.1 are shown in Fig. 6.16. Other than the opening locations, all other parameters are fixed for the results presented in Fig. 6.16. That is $B=1000$, $B_v=5\times10^4$ and the opening size ratio of 1/4. It can be seen that case 2 generates the highest Nusselt number, cases 3 and 1 are next, and case 4 produces the lowest Nusselt number values. The interesting interrelationship between the Nusselt number distributions presented in Fig. 6.16 can be explained as follows. First, it should be noted that even though the hydrostatic pressure differences across the porous insulation for the four cases are the same, the gradients across the insulation for cases 2 and 3 are higher than that for cases 1 and 4 due to major differences in the flow paths for these cases as it can be clearly seen in Fig. 6.17. Next, note that the buoyant effect diminishes the through flow for case 3 as compared to case 2. This of course will result in a lower Nusselt number for case 3 compared to case 2. Finally, the reason for the Nusselt number for case 1 being higher than case 4 can also be explained in terms of the flow field. In case 1 the flow at the permeable portion of the boundary on the left-hand side essentially enters parallel to the flow field which exists inside the cavity over that portion of the boundary. However in case 4, the flow at the permeable portion of the boundary enters almost perpendicular to the existing flow field in the cavity. Therefore this causes the Nusselt number for case 1 to become higher than
Fig. 6.16: The Nusselt numbers for the four cases corresponding to Fig. 6.1 for $B=1000$, $B_v=5\times10^4$ and an opening size ratio of 1/4.
Fig. 6.17: Velocity distributions at $t_3$ for the four cases shown in Fig. 6.1 for $B=1000$, $B_v=5\times10^4$ and an opening size ratio of $1/4$. 
case 4. The effects of the opening locations on the field variables follow a trend similar to that of Nusselt numbers. The above results can be utilized for evaluating the heat loss due to cracks and holes in the building insulations.

The accuracy of the numerical results was examined rigorously in three different ways. First, for a chosen grid system the time step size was systematically decreased until a time step size was found where any further decrease would not have any significant effect on the field variables or the Nusselt number results. In parallel with the above-mentioned series of tests, the number of grid points was also systematically increased so as to find the minimum number of grid points which are required for achieving a reasonable level (with respect to the required CPU time) of accuracy. This was done by increasing the number of grids from 11x11 to 15x15 and ultimately to 21x21. Table 6.1 illustrates the Nusselt number results for these three different grid systems. It can be seen that the results which are based on the 15x15 and 21x21 grid systems are in good agreement. Since the CPU time became quite excessive for the 21x21 grid system, and because the results for 15x15 grid system were in good agreement with the 21x21 grid system, most of our computations were then based on the 15x15 grid system. Finally the good agreement between our explicit and implicit results, as discussed in Chapter 2, constituted the third examination of the accuracy of our results.
Table 6.1: Effect of the grid size on the Nusselt number

<table>
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<tr>
<th>t</th>
<th>( \text{Nu}_h ) (11x11)</th>
<th>relative difference between (11x11) and (21x21) grid systems</th>
<th>( \text{Nu}_h ) (15x15)</th>
<th>relative difference between (15x15) and (21x21) grid systems</th>
<th>( \text{Nu}_h ) (21x21)</th>
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</thead>
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<td>1.00%</td>
<td>1.8083</td>
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<td>1.6670</td>
<td>0.77%</td>
<td>1.6800</td>
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<tr>
<td>0.0015</td>
<td>1.5917</td>
<td>2.31%</td>
<td>1.6238</td>
<td>0.34%</td>
<td>1.6294</td>
</tr>
<tr>
<td>0.002</td>
<td>1.5555</td>
<td>2.87%</td>
<td>1.5881</td>
<td>0.84%</td>
<td>1.6015</td>
</tr>
<tr>
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</tr>
<tr>
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<td>1.5487</td>
<td>0.80%</td>
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</tr>
<tr>
<td>0.0035</td>
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<td>2.81%</td>
<td>1.5368</td>
<td>0.74%</td>
<td>1.5482</td>
</tr>
<tr>
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<td>3.00%</td>
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</tr>
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<td>2.90%</td>
<td>1.5192</td>
<td>0.71%</td>
<td>1.5300</td>
</tr>
<tr>
<td>0.005</td>
<td>1.4783</td>
<td>2.90%</td>
<td>1.5111</td>
<td>0.74%</td>
<td>1.5224</td>
</tr>
</tbody>
</table>

Note:

\[
\text{relative difference} \equiv \left( \frac{\text{\( \text{Nu}_h \), 21x21} - \text{\( \text{Nu}_h \), 11x11 or 15x15}}{\text{\( \text{Nu}_h \), 21x21}} \right) \times 100\%
\]
6.4 Conclusions

Pressure stratification effects on heat and mass transfer in porous insulation materials has been investigated in this paper. Appropriate pressure, temperature and vapor density boundary conditions were employed to fully simulate the problem of stratified air infiltration in building insulations. The thermal performance of the insulation and the interactions of the field variables were investigated systematically through examining the effects of the Biot numbers, $B$ and $B_v$, opening area, and the opening locations. Several important results are concluded as follows.

1. Infiltration was found to have a major impact on the overall heat transfer even when the pressure gradients were small.

2. An increase in the heat transfer Biot number results in an increase in the Nusselt number. However, only very large increase in the heat transfer Biot number will result in a significant increase in the Nusselt number.

3. Variations in vapor transfer Biot number clearly reveal the complex interaction between the heat and mass transfer fields. The vapor transfer Biot number has an augmenting effect on the Nusselt number during the initial transient heat transfer process; however the trend is reversed at the later times.

4. The Nusselt number decreases with an increase in the size of the permeable area. However, the total heat transfer rate for the larger opening area is still greater than that for the smaller opening area.
5. In all cases, the liquid was found to accumulate more in the region next to the hot and humid environment as compared to the other regions in the porous insulation.

6. The opening locations had a profound effect on the heat transfer rate as well as the other pertinent field variables.
CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

In this research, a systematic study of heat and mass transfer with phase change in porous insulation materials has been performed. This was done by investigating three inter-related problems. These problems are modelled by a set of transient inter-coupled governing equations along with appropriate boundary conditions. The solution algorithm allows full simulation without any significant simplifications. The solution scheme consists of a two-phase format routine which has the capability of accounting for phase change. This type of the full simulation and analysis of these problems were done for the first time in the literature.

In the first problem, which was discussed in detail in Chapter 4, a fundamental investigation was conducted on a two dimensional porous insulation. The top and bottom boundaries are insulated and impermeable while the left and the right boundaries are permeable and exposed to two different environments, a hot and humid environment on the left-hand side and a cooler environment on the right-hand side. The second and the third problems deal with air infiltration and condensation effects on heat and mass transfer in a porous insulation matrix. The vertical boundaries of the porous system are partially impermeable for simulating
the holes or cracks in walls of the buildings. Convective boundary conditions are imposed on the vertical boundaries. In particular for the third problem, hydrostatic pressure variations are considered on the permeable portions of the vertical boundaries.

Several important results obtained in this study are concluded as below.

1. For all the cases studied in this research, the 'wave-like' propagation behavior was observed for all of the important field variables such as the temperature, vapor density, gas density, condensation rate and the liquid content.

2. All the results revealed that the liquid accumulated much more in the region close to the hot and humid environment than the other regions.

3. Either the humidity levels, $\omega_h$, or the vapor transport Biot number, $B_v$, had a direct influence on the vapor transport and condensation process. Increasing $\omega_h$ of $B_v$ enhanced the vapor transfer, condensation rate, liquid content as well as thermal penetration.

4. The relative movement of the energy transfer relative to the vapor transport is well characterized by the Lewis number. It was found that the vapor density wave front moved faster than the temperature wave front for Lewis number less than one. For the Lewis number greater than one, the reversed trend is true.
5. Only a large increase in the heat transfer Biot number, B, will cause a significant increase in the Nusselt number since infiltration is a major mode in the overall energy transfer.

6. An increase in the opening size will decrease the Nusselt number.

7. The opening locations have significant effects on the Nusselt number and field variables in both problems studied in this work. The information extracted in this study can be utilized for estimating the energy loss due to cracks or holes in the building insulations and other related applications.

7.2 Recommendations for Further Research

Several possible modifications for the present study are recommended. These are:

1. Non-Darcian effects can be included when the gas infiltration speed becomes higher due to larger pressure gradients.

2. It would be of interest and importance to study the dynamic response of the porous insulation materials to the cyclic or transient boundary conditions.

3. A three dimensional analysis will be even more closely simulating the real building insulations. However, it should be noted that such an analysis will be quite CPU intensive and will require the use of even faster supercomputers.
LIST OF REFERENCES


