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Relativistic analyses of inelastic nucleon-nucleus scattering

Shim, Sugie, Ph.D.
The Ohio State University, 1989
RELATIVISTIC ANALYSES OF INELASTIC NUCLEON-NUCLEUS SCATTERING

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of the Ohio State University

by

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1.1 Introduction

Recently, there has been considerable interest in relativistic descriptions of nuclear phenomena. A relativistic approach based on the Dirac equation as the relevant wave equation has been remarkably successful in treating nuclear structure and nuclear reactions. While complete relativistic descriptions of nuclear matter and finite nuclei using a relativistic quantum field theory are absent, considerable progress has been made by using approximate descriptions, see Refs. 1-27. In this work, we extend the Dirac approach to nucleon-nucleus inelastic scattering problems. Although a relativistic treatment of inelastic scattering is not yet fully developed, one can make considerable progress using various extensions of nonrelativistic approaches to nuclear reactions as in Refs. 11-16.

In this chapter we briefly review the organization of this thesis. In chapter 2, we will present the development of Dirac phenomenology\textsuperscript{17,18,19,20} and a brief review of some of the relevant relativistic approaches used in treating
nuclear structure and reactions. In both cases the potentials consist mainly of a
large attractive Lorentz scalar and a large repulsive Lorentz four-vector (time­
like component only) terms. Some of the elastic optical potentials described
in chapter 2 are used in the relativistic coupled channel (CC) calculation,
described in chapter 4.

In the first use of the Dirac equation in analyzing proton-nucleus elastic
scattering, it was found that either Lorentz scalar or Lorentz four vector optical
potentials could be used to fit the experimental data. The situation changed
when $p - ^4He$ elastic cross section and analyzing power $(A_y)$ data became
available. This additional observable placed a crucial constraint on the Dirac
approach. To reproduce the large spin observables, some new features of the
model was necessary. As it is obvious when the second order Dirac equation
is calculated, there are two ways to proceed. One, which resembles the
nonrelativistic approach, involves introducing a tensor optical potential, which,
in the second order Dirac equation, produces a spin-orbit term. The tensor,
in combination with either a Lorentz scalar or a Lorentz vector potential,
comprise the optical model. The second approach, which was motivated by
meson exchange models of the nucleon-nucleon force, was to use large canceling
Lorentz scalar and time-like vector potentials to obtain the required spin-orbit
enhancement. This is the approach taken by B.C. Clark and her collaborators,
which is now called Dirac phenomenology. They assume the shapes of the scalar and vector potentials resemble the shape of the charge density and used Woods-Saxon form. With this model they could find a good fit to the $p \rightarrow {}^4\text{He}$ cross section data and analyzing power data. The development of the model is discussed further in chapter 2.

The theoretical basis for the Dirac approach can be found in a relativistic, many body quantum field theory. The current prototype is the Walecka model, although similar models were developed in the 1950's, which was developed to calculate the properties of cold, condensed stellar objects such as neutron stars. In this model, which is discussed in chapter 2, the renormalizable Lagrangian density contains baryons and neutral scalar and vector mesons. Approximate solutions are obtained by replacing the meson field operators and baryon sources by their classical expectation values. In infinite nuclear matter, the classical meson fields are uniform and constant. A number of investigations have attempted to use these ideas to treat finite nuclei. In chapter 2, we discuss the approach of Horowitz and Serot. They use an extended Lagrangian which includes the interactions of pions, $\rho$-mesons and photons. The coupled nonlinear differential equations obtained from their extended Lagrangian is solved self consistently in the Hartree approximation. Using fixed experimental values for the mass of proton, the masses of vector
mesons, \( m_v \equiv m_w, m_p \), and \( \alpha \), the coupling constants and mass of scalar meson are searched to obtain the bulk properties of nuclear matter. The results lead to an accurate description of charge densities, rms radii, and single particle nuclear energy levels for doubly magic nuclei.

For nucleon-nucleus scattering, the relativistic impulse approximation (RIA) calculations provide a theoretical, parameter-free description of the nuclear optical model appropriate for use at intermediate energies.\(^\text{23}\) This approach was motivated by the success of the Dirac phenomenological SV model. McNeil et al.\(^\text{2}\) and Clark et al.\(^\text{3}\) developed the relativistic formalism for folding the free nucleon-nucleon amplitudes in Lorentz covariant form\(^\text{27}\) with the nuclear density distributions to obtain a relativistic form for the nuclear optical potential. The resulting nuclear potentials, calculated without free parameters, are, at energies above 400 MeV, essentially the same as those found in Dirac phenomenology. More detailed discussion of the RIA calculation will be given in chapter 2.

In chapter 3 we employ collective models, which are patterned after the conventional nonrelativistic collective models,\(^\text{28,29,30,31}\) to describe the nuclear collective states. The most easily recognized collective motions are the shape vibrations of spherically symmetric nuclei and the rotations of permanently deformed nuclei. For the vibrations of spherical nuclei, we assume that we can
use a vibrational model which is patterned after the nonrelativistic classical oscillation model. For the static rotational collective motions, we assume that we can use a generalization of the nonrelativistic rotational model where the form of the transition potentials are obtained by taking the derivatives of the elastic optical potentials with adjustable deformation parameters. The detailed procedure for obtaining transition potentials will be described in chapter 3. The forms of these transition potentials will be used in the Dirac coupled channel calculations described in chapter 4. The deformation parameters, which are assumed to be proportional to the strength of the coupling between channels, are varied to reproduce the experimental data.

Finally in chapter 4, we describe the coupled channel Dirac formalism and discuss the results of relativistic CC calculations for proton-nucleus inelastic scattering from the several spin zero nuclei at intermediate energies from 360 MeV up to 1 GeV.
CHAPTER I REFERENCES


CHAPTER II

In this chapter, we review the treatment of nucleon-nucleus elastic scattering as it has been described using either Dirac phenomenology or the relativistic impulse approximation (RIA). In section 2.1, we discuss several approaches which have been used for the phenomenological calculations. A review of the usual Dirac phenomenology, where in general, conventional Woods-Saxon shapes are used for the Lorentz scalar and four-vector potentials, is given first. Characteristic features of this approach are discussed in terms of the second order Dirac equation. The standard model of Dirac phenomenology consists of a local, spherically symmetric optical potential which has Lorentz scalar and four-vector terms (time-like component only). The SV model is compared with models which contain other Lorentz character optical potentials and resulting ambiguities are discussed. We review some of the applications of the SV model to analysis of nucleon-nucleus elastic scattering data and give some comparisons with other approaches.

Next, we summarize the results of an approach in which the Dirac optical potentials are constrained by relativistic Hartree considerations. It was
demonstrated that Dirac elastic optical potentials whose real parts are constrained by relativistic Hartree theory were capable of producing good agreement with the elastic cross section and spin observables.\textsuperscript{14}

In recent work, global potential parameters have been obtained for the SV model. These potentials were determined by considering a number of elastic scattering data sets over wide range of energy.\textsuperscript{6,7} This work is continuing and being expanded to include both $E$ (energy) and $A$ (atomic number) dependences.\textsuperscript{8}

In the section 2.2, the parameter-free RIA calculation for the elastic scattering is reviewed. Here, free NN amplitudes in Lorentz covariant form\textsuperscript{12} are folded with the nuclear densities to give the RIA optical potentials. Relativistic Hartree calculations\textsuperscript{13} provide the nuclear densities.

In this thesis we are interested in the description of inelastic nucleon-nucleus scattering. Coupled channel (CC) Dirac equations\textsuperscript{14} have previously been used for inelastic electron scattering,\textsuperscript{15} as well as for the treatment of the inelastic nucleon-nucleus scattering.\textsuperscript{16,17,18,19} In much of the work here, local Dirac SV optical potentials, determined by fitting elastic nucleon-nucleus data with Woods-Saxon form factors, are used to provide the initial diagonal optical potentials for what we term the purely phenomenological approaches described in chapter 4. In the other approach used in this thesis elastic RIA
optical potentials are used as the diagonal potentials for a collective model calculation with the RIA potential used to obtain the transition potentials. This is described in the section 3.4. The results of the Dirac CC analyses are given in chapter 4.

2.1 Relativistic phenomenology

In this section, the SV model of Dirac phenomenology is reviewed. First, the development of the Dirac phenomenology is described and several characteristic features of the Dirac approach are discussed. Next, an approach where the real scalar and vector optical potentials are constrained by relativistic Hartree theory is described and the results are comparing to experimental data. Finally, a global fitting approach is examined by searching energy dependence considering wide ranges of energies above 150 MeV up to 1 GeV.

2.1.1 Review of Dirac phenomenology

As discussed by Miller\textsuperscript{20} the most general local, time independent Dirac equation contains five tensorial types; Lorentz scalar, pseudoscalar, four-vector, axial vector and tensor. In this case the Dirac equation is
\[
\{\vec{\alpha} \cdot \vec{p} + \beta(m + U_S(\vec{r})) + \gamma^\mu U_{\gamma\mu}(\vec{r}) + \gamma^5 U_{PS}(\vec{r})
+ \gamma^\mu \gamma^5 U_{A\mu}(\vec{r}) + \sigma^{\mu\nu} U_{T\mu\nu}(\vec{r})\} \psi(\vec{r}) = E\psi(\vec{r}).
\] (2.1)

If one has parity conserving, spherically symmetric potentials, Eq. (2.1) simplifies to

\[
\{\vec{\alpha} \cdot \vec{p} + \beta(m + U_S) - (E - U_\gamma^p) - \beta \vec{\gamma} \cdot \vec{U}_T + i\vec{\alpha} \cdot \vec{U}_T\} \psi(\vec{r}) = 0, \quad (2.2)
\]

where

\[U_S(\vec{r}) = U_S(r),\]
\[\gamma^\mu U_{\gamma\mu}(\vec{r}) = \gamma^0 U_\gamma^0(r) - \vec{\gamma} \cdot \vec{U}_T(r),\]
\[\sigma^{\mu\nu} U_{T\mu\nu}(\vec{r}) = -\gamma^0 \vec{\gamma} \cdot \vec{U}_T(r) = \beta i\vec{\alpha} \cdot \vec{U}_T(r).\]

The notation of Bjorken and Drell\textsuperscript{21} is used for the \(\gamma\)-matrices.

Equation (2.2) may be rewritten as two coupled equations for the upper \((\psi_u)\) and lower \((\psi_l)\) components of \(\psi(\vec{r})\). Solving for \(\psi_l\) in terms of \(\psi_u\) in the usual way gives for \(\psi_u\)\textsuperscript{1,2,22}

\[
[(E - U_\gamma^p)^2 - (m + U_S)^2 - Q(r)]\psi_u(\vec{r}) = 0, \quad (2.3)
\]

where

\[
Q(r) = A(E + m)(\vec{\sigma} \cdot \vec{p} - \vec{\sigma} \cdot \vec{U}_T - i(\vec{\sigma} \cdot \vec{\gamma})U_T) \frac{1}{(E + m)A} \\
x(\vec{\sigma} \cdot \vec{p} - \vec{\sigma} \cdot \vec{U}_T - i(\vec{\sigma} \cdot \vec{\gamma})U_T), \quad (2.4)
\]
and

$$A = \frac{(m + U_S + E - U_V^0)}{(m + E)}.$$  \hfill (2.5)

Algebra gives the second order Dirac equation for $\psi_u$ as

$$\{\nabla^2 + (E - U_o - V_c)^2 - (m + U_S)^2 - U_V^2 - T^2\}
+ \left(\frac{1}{rA} \frac{\partial A}{\partial r} - \frac{T}{r}\right) \vec{\sigma} \cdot \vec{L} - \frac{2}{r} (iU_V + T) + \frac{1}{A} \frac{\partial A}{\partial r} (iU_V + T)
+ \frac{i}{r} [(\vec{r} \cdot \vec{p}) (U_V - iT)] - \left[\frac{i}{rA} \frac{\partial A}{\partial r} - \frac{2U_V^r}{r}\right] (\vec{r} \cdot \vec{p}) \psi_u(\vec{r}) = 0,$$  \hfill (2.6)

where

$$U_0 = U_V^0 - V_c,$$  \hfill (2.7)

for a static Coulomb potential $V_c$, nuclear potential $U_0$ and

$$T = U_T + U_{AM},$$  \hfill (2.8)

where $U_{AM}$ is the potential due to the interaction of the anomalous moment of the projectile with the Coulomb field of the nucleus, and $U_T$ is the nuclear tensor. To remove the first derivative terms from Eq. (2.6), we let

$$\psi_u(\vec{r}) = K(r) \phi(\vec{r}),$$  \hfill (2.9)

where $K(r) \to 1$ as $r \to \infty$. Direct substitution of Eq. (2.9) into Eq. (2.6) gives

$$\frac{\partial}{\partial r} K(r) = \frac{1}{2} \left[ \frac{1}{A} \frac{\partial A}{\partial r} + 2iU_V^r \right] K(r),$$  \hfill (2.10)
or

\[ K(r) = A^{1/2} \exp\left( \int iU_T(r)dr \right). \quad (2.11) \]

Under this wave function transformation, Eq. (2.6) becomes,

\[ \{ \nabla^2 + (E - U_o - V_c)^2 - (m + U_S)^2 - T^2 + \frac{T}{A} \frac{\partial A}{\partial r} - 2T \frac{\partial T}{\partial r} \]

\[ - \frac{3}{4A^2} \left( \frac{\partial A}{\partial r} \right)^2 + \frac{1}{2r^2A} \frac{\partial}{\partial r} (r^2 \frac{\partial A}{\partial r}) + \left( \frac{1}{rA} \frac{\partial A}{\partial r} - 2 \frac{T}{r} (\hat{\sigma} \cdot \hat{L}) \right) \phi(r) = 0. \quad (2.12) \]

Notice that the three vector part of the Lorentz four-vector potential does not appear in Eq. (2.12). The tensor potential contributes in a complicated way to the central potential and also contributes to the spin-orbit term. In addition, there are cross terms between \( U_T \) and derivatives of \( U_0, V_c, \) and \( U_S \). One may define what are called Schrödinger equivalent or effective central, spin-orbit and Darwin potentials. They are

\[ U_{\text{cent}} = \frac{1}{2E} \left[ 2EU_0 + 2mU_S - U_0^2 - U_S^2 - 2V_cU_0 + U_T^2 \right] + 2U_TU_{AM} - \frac{U_T}{A} \left( \frac{\partial A}{\partial r} \right) + \frac{2U_T}{r} + 2EU_{\text{Darwin}}, \]

\[ U_{\text{Darwin}} = \frac{1}{2E} \left[ - \frac{1}{2r^2A} \frac{\partial}{\partial r} (r^2 \frac{\partial A}{\partial r}) + \frac{3}{4A^2} (\frac{\partial A}{\partial r})^2 \right], \]

\[ U_{so} = \frac{1}{2E} \left[ - \frac{1}{rA^2} \left( \frac{\partial A}{\partial r} \right) + \frac{2}{r} (U_T + U_{AM}) \right], \quad (2.13) \]

for a Schrödinger equivalent equation given by

\[ [\vec{p}^2 + 2E(U_{\text{cent}} + U_{so} \hat{\sigma} \cdot \hat{L})] \phi(r) \]

\[ = \left[ (E - V_c)^2 - m^2 - \frac{2U_{AM}}{r} - \frac{\partial U_{AM}}{\partial r} - U_{AM}^2 \right] \phi(r). \quad (2.14) \]
Thus, both central and spin-orbit potentials depend on $U_S$, $U_\psi$, and $T$ while the Darwin term depends on $U_S$ and $U_\psi$ only.

It can be shown that in order to obtain agreement with the large proton-nucleus scattering spin observables at least two of the three potentials in Eq. (2.12) must be included. In fact, a transformation of the form,

$$\phi(\vec{r}) = e^{i\gamma F(\vec{r})}\psi(\vec{r}),$$

(2.15)

where $F(r) \to 1$ as $r \to \infty$, yields for $\phi(\vec{r})$,

$$[(E - U_\psi)\gamma^0\cos F - (m + U_S)\cos2F + i(E - U_\psi)\sin2F$$

$$-i(m + U_S)\gamma^0\sin2F + i\gamma^0\vec{q} \cdot \vec{r}T + \gamma^0\vec{q} \cdot \frac{\partial F}{\partial \vec{r}} - \vec{q} \cdot \vec{r}\phi(\vec{r}) = 0.$$

(2.16)

Hence, by proper choice of $F(r)$, one can change from a SV model to an equivalent ST or VT or SVT model. The transformation of Eq. (2.15) has been used to obtain equivalent ST and VT potentials from fits obtained using a SV model. Thus, there exist a set of equivalent potentials and an ambiguity in their Lorentz character. At this time it does not appear that the same type of ambiguity exists for inelastic scattering although this is still under investigation. There is, however, strong theoretical evidence from relativistic nuclear structure and RIA calculations favoring the SV model. That it is a preferable phenomenological choice is apparent.
The applications of the SV model to nucleon-nucleus elastic scattering have, in many cases, used two parameter Fermi functions (Woods-Saxon shapes) for the geometries. The optical potentials are given by

\[
U_S = V_S f_s(r) + iW_S g_s(r),
\]

\[
U_0 = V_0 f_0(r) + iW_0 g_0(r),
\]

with

\[
f_i, g_i(r) = [1 + \exp(r - R_i)/a_i],
\]

where \( R \) is the nuclear radius parameter defined as \( r_0 A^{1/3} \) and \( a \) represents the diffusiveness. Therefore, Dirac model contains 12 free parameters, as does the standard phenomenological Schrödinger optical model. At low energy, excellent agreement with experiment is obtained, even at angles where nonrelativistic analyses usually require an additional term in the optical potential. Similar situation also holds for neutron scattering. At higher energies it was found that the SV model is superior to the usual Schrödinger approach in reproducing experiments, especially with regard to the spin observables. The classic example is given by \( p^{-}Ca \) at 497.5 MeV, where the complete set of measurements, cross section, analyzing power and spin rotation function were available. A comparison of Schrödinger and Dirac optical model calculations, where both used simple Fermi shape for the geometries, clearly showed the
ability of the Dirac model to reproduce the spin data while Schrödinger calculation failed. It should be recalled that the spin-orbit potential arises naturally in the second order Dirac equation. Another success of Dirac phenomenology was the natural appearance of the unorthodox ‘wine-bottle-bottom’ shape of the real effective central potential which is required by the data in the transition energy region (around 200 MeV). It was not possible to reproduce the unorthodox shape in the usual nonrelativistic calculation which contained only Woods-Saxon shape or derivatives of the Woods-Saxon shape. Applications to other targets have shown that relativistic optical model is capable of fitting the data quite well.

From extensive phenomenological studies various systematic features of the SV model have emerged. They are:

1. The phenomenological scalar and vector potentials have large strengths and are opposite in sign.

2. The strengths $V_0$ and $V_S$ are energy dependent and the ratio, $R_R$, of the volume integrals of scalar and vector potentials varies linearly with energy.

3. The geometries of the real scalar and vector potentials resemble the nuclear densities.
4. Spin observables are better represented in Dirac analyses than in Schrödinger analyses when both have the same number of parameters.

5. The scalar and vector potentials exhibit a smooth energy variation as do the central and spin-orbit potentials obtained from them.

6. The unorthodox 'wine-bottle-bottom' shape of the real effective central potential arises naturally.

7. While there exist ambiguities in the Lorentz character of the relativistic optical model, both theory and experiment favor the SV model.

2.1.2 Dirac optical potentials constrained by a Dirac-Hartree theory

In the work of Refs. 1 and 4, the real parts of the elastic optical potentials were constrained by relativistic Hartree calculations. The real part of the nucleon-nucleus optical potential was calculated using the relativistic Hartree approach of Ref. 13. Here we summarize briefly the results as they pertain to the present calculation. The Dirac-Hartree equations for a finite nucleus may be derived from a model relativistic Lagrangian density. As a prototype,
consider the model of Walecka\textsuperscript{29}, for which

$$\mathcal{L} = \mathcal{L}_0 + g_s \bar{\psi} \psi \phi - g_s \bar{\psi} \gamma^\mu \psi V_\mu, \quad (2.19)$$

where $\mathcal{L}_0$ is the noninteracting Lagrangian density for baryon ($\psi$), scalar meson ($\phi$), and vector meson ($V_\mu$) fields. The results presented in Refs. 1 and 4 used an extended model that included additional boson degrees of freedom.\textsuperscript{30}

In the Walecka model one begins with the field equations derived from Eq. (2.19) and replaces the meson field operators with static, classical fields. The baryon densities, which serve as source terms in the meson field equations, are evaluated approximately as sums over single-particle wave functions. These techniques lead to a set of coupled, nonlinear differential equations for the classical meson fields and single particle wave functions. If we write the Dirac wave functions as\textsuperscript{31}

$$\psi(\mathbf{r}) \equiv \psi_{nkmt}(\mathbf{r}) = \begin{bmatrix} \frac{G_{nk}(r)}{r} \Phi_{km} \eta_t \\ -\frac{F_{nk}(r)}{r} \Phi_{-km} \eta_t \end{bmatrix}, \quad (2.20)$$

where $\Phi_{km}$ are spin-$\frac{1}{2}$ spherical harmonics labeled by $k$ and $m$, and $\eta_t$ are isospinors labeled by $t$, one finds

$$\frac{d}{dr} G_\alpha(r) + \frac{k}{r} G_\alpha(r) - [E_\alpha - U_0(r) - V_c(r) + m_p + U_s(r)] F_\alpha(r) = 0, \quad (2.21)$$

$$\frac{d}{dr} F_\alpha(r) - \frac{k}{r} F_\alpha(r) + [E_\alpha - U_0(r) - V_c(r) - m_p - U_s(r)] G_\alpha(r) = 0. \quad (2.22)$$
Here $U_0$, $V_c$ and $U_S$ are the fourth component of the Lorentz vector, Coulomb and scalar potentials respectively, and we have assumed spherical symmetry, as appropriate for a doubly magic nucleus. The meson potentials are determined self-consistently by solving additional differential equations driven by the baryon densities. The scalar potential, $U_S$ satisfies

$$\frac{d^2}{dr^2} U_S(r) + \frac{2}{r} \frac{d}{dr} U_S(r) - m_s^2 U_S(r) = g_s^2 \rho_s(r) = g_s^2 \sum_{\text{occupied}}^\alpha \left[ \frac{2j_\alpha + 1}{4\pi r^2} \right] (|G_\alpha(r)|^2 - |F_\alpha(r)|^2),$$

(2.23)

where $m_s$ is the scalar meson mass. The vector potential $U_V$ contains contributions from both isoscalar ($\omega$) and isovector ($\rho$) mesons, as determined by similar equations driven by the baryon density which is proportional to $\rho$. As discussed in Ref. 13, these field equations determine the meson potentials and single-particle nucleon wave functions in this model, and depend on four parameters: $g_s, g_\omega, g_\rho$ (the $\rho$ meson coupling to nucleons), and $m_s$. They considered the parameters $m_\rho = 939\text{MeV}$, $m_\omega \equiv m_\omega = 783\text{MeV}$, $m_\rho = 770\text{MeV}$, and $\alpha = e^2 / 4\pi \simeq \frac{1}{137}$ to be fixed at their experimental values. The remaining four free parameters were normalized as far as possible to the bulk properties of nuclear matter. As discussed in Ref. 13, they took the values $g_s^2 = 109.63$, $g_\omega^2 = 190.43$, $g_\rho^2 = 65.226$, and $m_s = 520\text{MeV}$. These values lead
to an accurate description of charge densities, neutron densities, rms radii and single-nucleon energy levels for doubly magic nuclei throughout the periodic table.

To utilize the preceding results in the relativistic optical model formalism, Refs. 1 and 4 assumed that the geometries of the real parts of the nucleon-nucleus optical potential could be obtained from the bound state Dirac-Hartree solutions $U_0$, $U_S$, and $V_c$. To account for the finite-size effects of the nucleons themselves (which are also neglected in the Hartree approximation), they determined the optical potentials by folding over a suitable single-nucleon form factor. The scalar optical potential $U^\text{opt}_S$ generated from this folding procedure was,

$$U^\text{opt}_S(r) = \int d\vec{r}'d\vec{r}_1d\vec{r}_2 \rho_b(|\vec{r} - \vec{r}'|)\rho_\nu(|\vec{r}_1 - \vec{r}_1|)\rho_\nu(|\vec{r}_1 - \vec{r}_2|)\rho^\text{Hartree}_s(r_2). \quad (2.24)$$

Here $\rho_b$ and $\rho_\nu$ are single nucleon form factors, $v_s(\vec{r})$ is the static Yukawa meson propagator, and $\rho^\text{Hartree}_s(r)$ is the scalar meson density generated by the tadpole loop or, equivalently, the sum on single-nucleon wave functions in Eq. (2.23). Relations analogous to Eq. (2.24) define the Lorentz vector optical potential $U^\text{opt}_L$ and, of course, these lowest order potentials are summed to all orders by solving the Dirac equation for the four-component scattering-state wave function.
In principle, the form factors $\rho_b$ and $\rho_{b'}$ should be used in the static calculation of the Hartree potentials for bound nucleons. Instead, since the Hartree potentials are calculated self-consistently with no form factors, they set $\rho_{b'}(r) = \delta^{(3)}(r)$. The scalar and vector optical potentials were then written as

$$U_{S,0}^{opt}(r) \approx \int d\tilde{r} d\tilde{r}_1 \rho_b(|\tilde{r} - \tilde{r}_1|) \rho_{b'}(|\tilde{r}_1 - \tilde{r}'|) \rho_{S,0}^{Hartree}(\tilde{r}_1)$$

$$= \int d\tilde{r}' \rho_b(|\tilde{r} - \tilde{r}'|) U_{S,0}(r'),$$

(2.25)

where the last line follows from the solution to Eq. (2.22). The nucleon-nucleus optical potential was determined by folding the calculated Hartree potentials with a suitable form factor for the projectile nucleon.

To include possible sources of the energy dependence in the vector and scalar potentials, the real optical potentials of Eq. (2.25) were scaled by factors $C_0$ and $C_S$. The real optical potentials contained only these two parameters; the radial dependence was fixed by the relativistic Hartree calculation. The scalar and vector potentials were given by

$$V_s^{opt}(r) = C_S(r) U_s^{opt}(r) + iW_s f_s(r)$$

(2.26)

$$V_0^{opt}(r) = C_0(r) U_0^{opt}(r) + iW_0 f_0(r),$$

(2.27)

where $f(r)$ is a Woods-Saxon shape. The model contained a total of eight parameters which were varied to obtain fits to experimental data for a number
of spin-zero, closed-shell targets appropriate for the constraint.

This work provides a link between Dirac phenomenology and relativistic mean field theory calculations. The relativistic Hartree densities are used as input to the RIA calculations discussed in section 2.2.1.

2.1.3 Global fitting approaches

In this section the current status of the treatment of proton-nucleus elastic scattering using global optical model potentials is reviewed. Although work has been done for lower energy global potentials\(^7\) we review here work which considers energies between 150 MeV and 1000 MeV\(^8\) because the inelastic data we consider is in this energy region. The optical potentials used in Ref. 6 have the form given in Eq. (2.17), but \(f(E,r)\) and \(g(E,r)\) are now chosen to be symmetrized Woods-Saxon shapes,

\[
f(E,r), g(E,r) = [1 + \exp\left(\frac{r - r_0(E)A^{1/3}}{a(E)}\right)]^{-1}[1 + \exp\left(-\frac{r + r_0(E)A^{1/3}}{a(E)}\right)]^{-1}.
\]  

(2.28)

Several different parametrizations have been used. In one parametrization they restricted the geometries of the real potentials to shapes shown to produce high quality fits at each energy individually. These constraints were based on the meson theory of the NN force. Two different ways of determining the
parameters for the form factors of Eq. (2.28) were considered. For case 1, we considered a simple folding model, described in Ref. 28, based on two-body Yukawa potentials for the exchange of scalar ($\sigma$) or vector ($\omega$) mesons. A large body of elastic scattering data at energies above 150 MeV for $p + ^{40}Ca$ case has been considered for this specific case. For case 2, two-parameter fits to relativistic Hartree potentials described in the previous section were used.\textsuperscript{1,2,4} The geometries of the real potentials are fixed by these constraints and are energy independent. For both cases, all of the energy dependence of the scalar and vector potentials were in the strengths and in the imaginary geometries. The real scalar and vector strength parameters and the imaginary scalar and vector strengths and geometry parameters were taken to have parabolic energy dependence of the form,

\begin{align*}
V_0(E) &= 300(a_1 + b_1 E + c_1 E^2), \\
W_0(E) &= -100(a_2 + b_2 E + c_2 E^2), \\
V_s(E) &= -400(a_3 + b_3 E + c_3 E^2), \\
W_s(E) &= 100(a_4 + b_4 E + c_4 E^2),
\end{align*}

where $E = (T_p - 400)/400$ with $T_p$ the projectile kinetic energy in the laboratory frame. The imaginary geometry parameters are also parabolic, for example, $r_0 = (a + bE + cE^2)$. For each case a total of 24 parameters are
varied. Considering that the data set had 1600 data points, this number of parameters was not excessive.

Next a more restrictive scenario was investigated in which the parametrized geometries were assumed to be energy independent. All energy dependence was embedded in the strength parameters. This does not imply that the second-order Dirac equation central and spin-orbit (Schrödinger equivalent) optical potential geometries are energy independent, as neither is linear in $S$ and $V$. To improve the systematic behavior of the imaginary strengths, the imaginary scalar potential was taken to be energy independent. This constrained the ratio of vector to scalar strengths and puts all of the energy dependence in the imaginary vector potential. This also insured a reasonable behavior of the imaginary potential strengths.

Two assumptions for the energy dependence of the real scalar and vector and imaginary vector strengths were made. First, the energy dependence was taken to be parabolic (case 3) as in Eq. (2.29), and second (case 4), it was assumed to be cubic with the addition of a term $dE^3$ to Eq. (2.29). The number of parameters is 18 in the parabolic case and 21 in the cubic case. In every case the fits to the $p + ^{40}Ca$ data at the individual energies of 161, 181.3, 200, 300, 362, 497.5, 613, 797.5, and 1040 MeV were very good.
The predictive power of these global fits was tested by predicting observables for the 400 MeV data not included in the fitting. The predicted fit for case 1 is shown in Fig. 2.1, which is quite acceptable. All cases predict the 400 MeV observables essentially as well; the calculated observables are almost indistinguishable on a graph with the scale shown in Fig. 2.1. Currently work is underway to determine both the E and A dependence of global optical models. 

2.2 Elastic nucleon-nucleus scattering using RIA

In this section the construction of a Lorentz invariant nucleon-nucleon (NN) amplitude using the c.m. frame NN amplitude in the Pauli representation is described first, and RIA optical potential for proton-nucleus elastic scattering is defined using an optimal impulse approximation. This approach provides a parameter-free description for proton-nucleus scattering.

2.2.1 The Relativistic Impulse Approximation

In proton-nucleus elastic scattering, we assume that momentum transfer between projectile and target nucleus occurs through the single two-body free
The predicted fit to the $^{40}$Ca at 400 MeV elastic cross section and spin data.

Figure 2.1:
The single scattering approximation to the proton-nucleus scattering amplitude in the proton-nucleus c. m. frame is given by,

$$ S_{fi} = -\frac{k}{2\pi v_l} \sum_j (2\pi)^{-3} \int d^3 p \psi_j^*(\vec{p} + \frac{1}{2} \vec{q} - \frac{1}{A} \vec{k}_a) $$

$$ \times (\vec{k}_a - \frac{1}{2} \vec{q}; \vec{p} + \frac{1}{2} \vec{q} - \frac{1}{A} \vec{k}_a | t(T - H) | \vec{k}_a + \frac{1}{2} \vec{q}; \vec{p} - \frac{1}{2} \vec{q} - \frac{1}{A} \vec{k}_a) $$

$$ \times \psi_j(\vec{p} - \frac{1}{2} \vec{q} - \frac{1}{A} \vec{k}_a), $$

(2.30)

where $\vec{k}_a = (\vec{k} + \vec{k}')/2$ and $\vec{q} = \vec{k} - \vec{k}'$ are the average of initial ($\vec{k}$) and final ($\vec{k}'$) proton momentum and the momentum transfer, respectively. For elastic scattering, $\vec{k}_a \cdot \vec{q} = 0$. In Eq. (2.30), $H$ is the nuclear hamiltonian, $T$ is the proton energy in the laboratory frame and $v_l$ is the laboratory velocity of the projectile. If we take the NN $t$ matrix to be on shell, which is termed as the optimal impulse approximation, we can factor out the $t$ matrix at its value at $\vec{p} = 0$. This $\vec{p} = 0$ situation corresponds to Breit frame. The procedure for calculating the Breit frame NN amplitudes via the invariant NN amplitudes from the c. m. frame NN amplitudes are given in Refs. 12, 34 and 35. Assuming isospin, parity, time-reversal conservation, the strong interaction scattering matrix can be expressed in terms of five complex amplitudes for pp and five more for pn. The parameterization of the c. m. NN amplitudes in
Figure 2.2:

Illustration of impulse approximation for nucleon-nucleus scattering.
The terms of the Pauli matrices is

\[ \frac{f_c}{k_c} = A + B\sigma_{1n}\sigma_{2n} + C(\sigma_{1n} + \sigma_{2n}) + D\sigma_{1q}\sigma_{2q} + E\sigma_{1k}\sigma_{2k}, \quad (2.31) \]

where \( \hat{k} \) is a unit vector parallel to average momentum, \( \hat{q} \) is a unit vector along the momentum transfer, and \( \hat{n} \) is a unit vector perpendicular to the scattering plane, \( \hat{n} = \hat{q} \times \hat{k} \). The complex amplitudes \( A, B, C, D \) and \( E \) are calculated from the experimental NN phase shifts.\(^{38,39,40} \) Note that \( f_c/k_c \) is a Lorentz invariant, but five individual terms in Eq. (2.31) are not. Now, introduce a Lorentz covariant representation involving Dirac matrices \( \gamma^\mu \) as follows:

\[ \hat{F} = F_s + F_p\gamma_1^5\gamma_2^5 + F_v\gamma_1^\mu\gamma_2^\mu + F_a\gamma_1^5\gamma_2^5\gamma_2^\mu + F_t\sigma^{\mu\nu}\sigma_{2\mu}, \quad (2.32) \]

with the scalar \( (F_s) \), pseudoscalar \( (F_p) \), vector \( (F_v) \), axial vector \( (F_a) \) and tensor \( (F_t) \) components. These Dirac amplitudes are complex functions of the Lorentz invariants \( s \) and \( t = -q^2 \). We can relate \( \{F_s, F_p, F_v, F_a, F_t\} \) to \( \{A, B, C, D, E\} \) by requiring

\[ \bar{u}_{s_1}(\tilde{k}_c)\bar{u}_{s_2}(-\tilde{k}_c)\hat{F}\bar{u}_{s_1}(\tilde{k}_c)\bar{u}_{s_2}(-\tilde{k}_c) = \chi_{s_1}^\dagger\chi_{s_2}^\dagger \left( \frac{f_c}{k_c} \right) \chi_{s_1}\chi_{s_2}, \quad (2.33) \]

where \( \bar{u}_s(\tilde{k}) \) are positive energy Dirac spinors,\(^{21} \) and \( \chi_s \) are two component Pauli spinors. One can obtain a set of the Lorentz covariant NN amplitudes from the Breit frame NN amplitudes in similar fashion. Defining the Breit
frame NN amplitudes to be

\[
\frac{f_B}{k_B} = A + B\sigma_{1n}\sigma_{2n} + C_1\sigma_{1n} + C_2\sigma_{2n} + D\sigma_{1q}\sigma_{2q} + E\sigma_{1k}\sigma_{2k}, \tag{2.34}
\]

and requiring

\[
\bar{u}_{s_1}(\vec{k})\bar{u}_{s_2}(\frac{1}{2}\vec{q} - \frac{1}{A}\vec{k}_a)\hat{F}\bar{u}_{s_1}(\vec{k})\bar{u}_{s_2}(-\frac{1}{2}\vec{q} - \frac{1}{A}\vec{k}_a) = \chi_{s_1}^{+}\chi_{s_2}^{+}\left(\frac{f_B}{k_B}\right)\chi_{s_1}\chi_{s_2}, \tag{2.35}
\]

we obtain a matrix equation between the Lorentz covariant NN and the Breit frame NN amplitudes. This relation permits the assignment of values for each component of \(\hat{F}\) in terms of the Pauli representation of the NN scattering amplitude. This matrix equation is given by\(^{12}\)

\[
\begin{bmatrix}
A \\
B \\
C_1 \\
C_2 \\
D' \\
E
\end{bmatrix}_{ij} = B_1
\begin{bmatrix}
F_S \\
F_P \\
F_V \\
F_A \\
F_T
\end{bmatrix}_{s,t} \tag{2.36}
\]

where explicit values for the \(B_1\) matrix are provided in Tables I and II of Ref. 12. Because of time reversal invariance, \(C_1 = C_2\) in the c. m. frame. However, \(C_1 \neq C_2\) in the Breit frame because the momenta in Fig. 2.3 do not transform into one another under time reversal transformation.
Figure 2.3:

Illustration of Breit frame nucleon-nucleon scattering.
2.2.2 The RIA optical potential for nucleon-nucleus scattering

In this section, the RIA optical potential is described following the approach of Refs. 10 and 22. The elastic scattering of a proton by a spin-zero nucleus is described by the fixed energy Dirac equation;

\[(\gamma - m - U_{opt}(E))\psi_{k,a}^{(+)}(\vec{r}) = 0,\]  

\[\text{(2.37)}\]

where \(U_{opt}(E)\) is the elastic scattering optical potential and the constant energy \(E\) is related to the asymptotic momentum \(k\) by \(E = (k^2 + m^2)^{1/2}\). The solution can be written as

\[\psi_{k,a}^{(+)}(\vec{r}) = \psi_{k,a}(\vec{k})e^{i\vec{k}\cdot\vec{r}} + \int d^3\vec{r}'\psi_{k,a}(\vec{k})\langle \vec{r}'|U_{opt}|\psi_{k,a}^{(+)}(\vec{r})\rangle.\]  

\[\text{(2.38)}\]

The asymptotic solution at \(r \to \infty\) becomes

\[\psi_{k,a}^{(+)}(\vec{r}) \to \psi_{k,a}(\vec{k})e^{i\vec{k}\cdot\vec{r}} + \frac{e^{i\vec{k}\cdot\vec{r}}}{r} \left[ -\frac{E\gamma^0 - \vec{k}\cdot\vec{r} + m}{4\pi} \int d^3\vec{r}'e^{i\vec{k}'\cdot\vec{r}'}\langle \vec{r}'|U_{opt}|\psi_{k,a}^{(+)}(\vec{r})\rangle \right].\]  

\[\text{(2.39)}\]

where the term inside of the bracket corresponds to the scattering amplitude, and \(\vec{k}' = \vec{k}^f\) represents the outgoing wave momentum. Using the identity

\[E\gamma^0 - \vec{k}^0 + m = 2m \sum_{a'} u_{a'}(\vec{k})\bar{u}_{a'}(\vec{k}),\]  

\[\text{(2.40)}\]

the scattering amplitude can be written as

\[S_{a'a}(E) = -\frac{m}{2\pi} u_{a'}(\vec{k})\bar{u}_{a'}(\vec{k}') \int d^3\vec{r}e^{i\vec{k}'\cdot\vec{r}'}\langle \vec{r}'|U_{opt}|\psi_{k,a}^{(+)}(\vec{r})\rangle.\]  

\[\text{(2.41)}\]
Now introduce a Dirac $T$ matrix for elastic proton-nucleus scattering by

the definition

$$U_{opt}|\psi_{E_{nucl}}^{(+)}\rangle = T_{00}|\bar{k}\rangle u_s(\bar{k}), \quad (2.42)$$

where $|\bar{k}\rangle u_s(k)$ is the incoming plane wave state. Hence the scattering amplitude can be written in terms of $T_{00}$,

$$S_{\nu'\nu}(E) = -\frac{m}{2\pi} \bar{u}_{\nu'}(\bar{k}') \langle \bar{k}'|T_{00}|\bar{k}\rangle u_s(\bar{k}). \quad (2.43)$$

Now by analogy to the nonrelativistic impulse approximation to multiple scattering theory, we write the Dirac $T$ matrix as a sum of two body NN amplitude ($t_i$) in the nuclear ground state, $|\psi_0\rangle$, as

$$\langle \bar{k}'|T_{00}|\bar{k}\rangle = \langle \psi_0|\sum\limits_{i=1}^{A} (\bar{k}'|t_i(\bar{k}', \bar{k})|\bar{k})|\psi_0\rangle. \quad (2.44)$$

In the RIA, the NN $t$ matrix elements are equated to the Lorentz covariant NN amplitude, $\hat{F}$, by the relation

$$-\frac{m}{2\pi} \langle \bar{k}'|t_i|\bar{k}\rangle = k_\nu \hat{F}(q^2, s) e^{i\nu_\nu'}. \quad (2.45)$$

so that

$$\langle \bar{k}'|T_{00}|\bar{k}\rangle = -\frac{2\pi k_\nu}{m} \langle \psi_0|\sum\limits_{i=1}^{A} \hat{F}(q^2, s) e^{i\nu_\nu'}|\psi_0\rangle. \quad (2.46)$$

The nuclear ground state may be expressed by a Slater determinant of the four-component Dirac single particle wavefunctions

$$|\psi_0\rangle = \frac{1}{\sqrt{A!}} det \Pi_{i=1}^{A} \psi_{n_i m_i}, \quad (2.47)$$
where isospin dependence is omitted. Therefore

\[
\langle \psi_0 | \sum_{i=1}^{A} \hat{F}(q^2, s) e^{i\pi \cdot r_i} | \psi_0 \rangle = \sum_{\nu} \int d^3 r e^{i \pi \cdot r} F_{\nu} \sum_{nlm}^{occ} \left( \frac{-G_{n l} r^+}{r} \phi_{lm}^+, \frac{F_{nl} r^-}{r} \phi_{-lm}^- \right) \times O_{\nu} \left( \begin{array}{c} G_{nl} r^+ \phi_{lm}^+ \\ F_{nl} r^- \phi_{-lm}^- \end{array} \right),
\]

(2.48)

where \(\nu\) represents five types of Lorentz tensor and \(O_{\nu}\) are \(1, \gamma_1^5, \gamma_2^5, \gamma_1^\mu \gamma_2^\mu, \gamma_3^\mu \gamma_3^\mu\) and \(\sigma_1^{\mu \nu} \sigma_2^{\mu \nu}\) respectively. For spherical nuclei, parity implies only the \(S, V,\) and \(T\) parts of \(\hat{F}\) contribute, so that

\[
U_{os}(r) = U_S(r) + \gamma^0 U_0(r) - 2i \gamma^0 \gamma^7 U_T(r),
\]

(2.49)

where

\[
U_S(r) = -\frac{k}{4\pi^2 m} \sum_{i=p,n} \int d^3 q e^{-i \pi \cdot q} F_s(0)(q) \tilde{\rho}_s(0)(q),
\]

(2.50)

\[
U_0(r) = -\frac{k}{4\pi^2 m} \sum_{i=p,n} \int d^3 q e^{-i \pi \cdot q} F_0(0)(q) \tilde{\rho}_0(0)(q),
\]

(2.51)

\[
U_T(r) = -\frac{k}{4\pi^2 m} \sum_{i=p,n} \int d^3 q e^{-i \pi \cdot q} F_t(0)(q) \tilde{\rho}_t(0)(q).
\]

(2.52)

Here \(\tilde{\rho}\) indicates density form factors, which are given by

\[
\tilde{\rho}_s(0)(q) = \int d^3 x e^{-i \pi \cdot x} \sum_{nl}^{occ} \frac{2j + 1}{4\pi r^2} (|G_{nl}|^2 - |F_{nl}|^2),
\]

(2.53)

\[
\tilde{\rho}_0(0)(q) = \int d^3 x e^{-i \pi \cdot x} \sum_{nl}^{occ} \frac{2j + 1}{4\pi r^2} (|G_{nl}|^2 + |F_{nl}|^2),
\]

(2.54)

\[
\tilde{\rho}_t(0)(q) = \int d^3 x e^{-i \pi \cdot x} \sum_{nl}^{occ} \frac{2j + 1}{4\pi r^2} |G_{nl}| F_{nl}.
\]

(2.55)
In the actual calculation, the relativistic Hartree approximation is used to calculate the scalar, vector and tensor densities\(^{13}\). The resulting RIA scalar and vector optical potentials are large (100 ~ 200 MeV) and opposite in sign with the scalar attractive and the vector repulsive. These results are in agreement with Dirac phenomenology\(^{1,2}\) and with relativistic nuclear structure calculations.\(^{15,24,25,42,43}\) The tensor contribution is small\(^{44}\) and has not, in general, been included in this work. However we do consider its effects.

The results for \(p + ^{40}Ca\) and \(p + ^{208}Pb\) at 500 MeV and 800 MeV are shown in Figs. 2.4 through 2.7 and the agreement with experiment is quite remarkable. The RIA calculations are indicated by solid curves and the non-relativistic KMT-IA\(^{45}\) results by dashed curves.

Medium effects such as Pauli blocking and exchange effects are not included in the calculation although they can be approximately taken into account.\(^{46}\) The agreement with experimental data is obviously much better than nonrelativistic IA calculation, especially for the spin observables (see Appendix A for the definitions of observables). However, the RIA calculation does not reproduce cross section data at very large angles.\(^{47}\) This discrepancy will be discussed in chapter 4, regarding the coupled channel effects.
Figure 2.4:

The calculated elastic observables for $p + {}^{40}Ca$ at 497.5 MeV using RIA (solid lines) and KMT-IA (dashed lines).
Figure 2.5:

The calculated elastic observables for $p+^{40}Ca$ at 797.5 MeV using RIA (solid lines) and KMT-IA (dashed lines).
Figure 2.6:

The calculated elastic observables for $p + ^{208}Pb$ at 497.5 MeV using RIA (solid lines) and KMT-IA (dashed lines).
Figure 2.7:

The calculated elastic observables for $p+^{208}Pb$ at 797.5 MeV using RIA (solid lines) and KMT-IA (dashed lines).
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CHAPTER III

3.1 Introduction

In this chapter several collective models are used to construct transition potentials for the nucleon-nucleus inelastic scattering. Collective excitations in nuclei mean long range correlated motions that are slow compared to the underlying motions of individual nucleons. Such collective excitations can be classified into two categories. One case is the vibrational excitation of the nuclear shape of spherically symmetric nuclei, while the other case is the rotational excitation of deformed nuclei.\textsuperscript{1,2,3} In the work here we take a simple ansatz, one which has been used in nonrelativistic collective models, that when the matter distribution in the nucleus is deformed by the collective motion of the excited nucleons, the optical potentials are deformed in the same way.\textsuperscript{4,6,6} The motivation for this treatment can be found in the usual folding model for the optical potential\textsuperscript{7,8,9} under the assumption of a zero range nucleon-nucleon interaction. In such a case the optical potentials are directly proportional to the nuclear densities. Of course this is a gross oversimplification; however, it
is one commonly used in nonrelativistic analyses and we follow it here. Thus, in describing collective excited states of the target, Dirac optical potentials are treated in the same way as the central potential in the nonrelativistic Schrödinger equation. The relativistic approach assumes that one may deform the Dirac optical potentials to obtain the transition potentials just as the central potentials are deformed in nonrelativistic models. This procedure is used for the phenomenological optical potentials described in Chapter II (section 2.1.1) as will be discussed in section 3.3.

We have considered other approaches as well. In particular, transition potentials may be constructed using the relativistic impulse approximation (RIA) discussed in Chapter II, section 2.2. In section 3.4, we discuss two different methods of obtaining the transition potentials using the RIA optical potentials.\textsuperscript{10,11,12,13,14,15} This treatment is motivated by the success of the RIA in describing elastic proton scattering observables for spin-zero targets above 400 MeV.\textsuperscript{10,11} For one case the transition potentials are assumed to be proportional to the derivatives of the RIA elastic potentials themselves. For the other case the transition potentials are obtained from folding appropriate invariant nucleon-nucleon amplitudes with the derivatives of the input nuclear densities.
Not surprisingly RIA calculations do not give good agreement with the elastic data at high momentum transfer.\textsuperscript{16,17} This deficiency will be investigated using a Dirac coupled channel approach. The feedback to the elastic channel due to channel coupling is critical, and the importance of channel coupling in obtaining a better representation of the experimental data at large angles is clear.\textsuperscript{18}

3.2 Collective nuclear motions

In this section we give some typical examples of collective nuclear motion, as discussed in Ref. 19. There are four important types of collective motion, they are, surface vibrations of nuclear shape, nuclear rotations, nuclear fission and giant resonance motion. While the first three collective motions take place near the nuclear surface, the last case is related to the collective behavior of nuclear interior. Even though our interest is in vibrational and rotational collective motions, which correspond to low energy excitations, we present here illustrations of all four cases to complete the picture.

Surface vibrations of the nuclear shape are shown in Fig. 3.1(a). The arrows indicate the collective ordered motion which transfer nucleons from one region of the nucleus into another one. They are also shown in Fig. 3.1(b), where the nuclear shape periodically oscillates about an equilibrium radius.
This case is known as a compression mode and the nuclear density also increases and decreases periodically.

The rotations of deformed nuclei shown in Fig. 3.2(a) are another example of coherent, in phase motion of nuclear matter. A sort of tidal wave of nucleons rotate around a nuclear core, and the interior spherical core may be viewed as being mainly at rest during the rotations. Only the bumps outside of the core may be considered to move. The third case, the process of nuclear fission is shown in Fig. 3.2(b), indicating the various stages of this process from one spherical nucleus to two spherical nuclei. The arrows represent the collective flow of nuclear matter. The final case is photonuclear giant resonance motion, which is related to collective behavior involving the nuclear interior. Since the electric field acts on protons only, the neutrons must move in an opposite direction to that of protons in order that the center of mass remains at rest, as shown in Fig. 3.2(c). The nuclear surface is fixed during this process.

3.3 Collective models

In this section we discuss how the transition potentials are obtained. We assume, as is done in nonrelativistic treatment, that the collective motion of excited nucleons in nuclei can be described by shape vibrations or rotations and that the changes in shape are concentrated near the nuclear surface. If we
Figure 3.1:
Schematic figure of surface vibrations.
Figure 3.2:

Schematic figure of (a) the rotations of a deformed nucleus, (b) nuclear fission and (c) giant dipole motion in nuclei.
consider only the first order multipole transitions and a very simplified model in which the transition potentials are proportional to the first derivatives of the elastic potentials, then the dynamical deformation parameters, \( \alpha_{\lambda \mu} \), discussed below, will be the dynamical variables of the model. A model patterned after the nonrelativistic vibrational model can be introduced to describe the vibrational collective excitations of the nuclear densities of spherically symmetric nuclei, and a rotational model can be employed to describe the rotational collective excitation in deformed nuclei. In both cases the relativistic transition potentials are assumed to be generalizable from the nonrelativistic models.

3.3.1 Nuclear deformations

The simplest model for describing the nuclear deformation assumes an incompressible nucleus with a constant density in the interior and a sharp edge at \( r = R_0 \). The deformations can be described by a set of multipole deformation parameters \( \alpha_{\lambda \mu} \) obtained by expanding the surface in spherical harmonics,

\[
R(\theta, \phi) = R_0 (1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y_{\lambda \mu}^*(\theta, \phi)),
\]

(3.1)

where \( R_0 \) is chosen to insure volume conservation, and \( \theta \) and \( \phi \) are polar angles with respect to an arbitrary space-fixed axis. For \( R \) to be real, \( \alpha_{\lambda \mu} \) must
have the same transformation properties as $Y_{\lambda \mu}$, that is,

$$\alpha_{\lambda \mu}^+ = (-)^{\mu} \alpha_{\lambda - \mu}.$$  \hspace{1cm} (3.2)

The requirement that the volume of the nucleus is a constant gives us

$$\frac{4\pi R_0^3}{3} = \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \int_0^{R(\theta, \phi)} r^2 dr$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \frac{1}{3} R_0^3 \left[ 1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y_{\lambda \mu}^*(\theta, \phi) \right]^3$$

$$= \frac{1}{3} R_0^3 \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \left[ 1 + 3 \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y_{\lambda \mu}^*(\theta, \phi) \right]$$

$$+ \frac{3 \cdot 2}{2!} \left( \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y_{\lambda \mu}^*(\theta, \phi) \right)^2 + \cdots .$$  \hspace{1cm} (3.3)

Here, we can use the orthonormality of the spherical harmonics, that is,

$$\int_0^{\pi} \int_0^{2\pi} Y_{\lambda m}(\theta, \phi)^* Y_{\lambda m}(\theta, \phi) \sin \theta d\theta d\phi = \delta_{\lambda \mu} \delta_{mm'},$$  \hspace{1cm} (3.4)

so that we obtain

$$\frac{4\pi R_0^3}{3} = \frac{4\pi}{3} R_0^3 \left[ 1 + \frac{3 \alpha_{00}}{\sqrt{4\pi}} + \frac{3}{4\pi} \sum_{\lambda \mu} |\alpha_{\lambda \mu}|^2 + 0(\alpha^3) + \cdots \right],$$  \hspace{1cm} (3.5)

where the higher-order terms are neglected, assuming the deformation to be small. The term in Eq. (3.1) with $\lambda = 0$ corresponds a compression without changing the shape and will cause a change of the nuclear volume, thus it is discarded. The terms with $\lambda = 1$ are associated with a displacement of the center of mass, which will disappear if we constrain the center of mass at
origin. Hence we can write

\[ R_0^2 = R_\alpha^2 [1 + \frac{3}{4\pi} \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} |\alpha_{\lambda\mu}|^2 + 0(\alpha^3) + \cdots], \]  

(3.6)

which gives,

\[ R_\alpha \approx \frac{R_0}{\left(1 + \frac{1}{4\pi} \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} |\alpha_{\lambda\mu}|^2 + \cdots\right)}. \]  

(3.7)

Now we have

\[ R(\theta, \phi) \approx \frac{R_0}{\xi}(1 + \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi)), \]  

(3.8)

where \( \xi \) is the scale factor required by volume conservation, and is given by

\[ \xi \approx 1 + \frac{1}{4\pi} \sum_{\lambda=2}^{\infty} \sum_{\mu} |\alpha_{\lambda\mu}|^2. \]  

(3.9)

If we ignore the volume changes to the second order in \( \alpha \), then \( \xi \) is unity, and we obtain,

\[ R(\theta, \phi) \approx R_0(1 + \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi)). \]  

(3.10)

This is the expression which is standardly used for the deformation of the nuclear shape. Of course, such a model is not realistic as a nucleus does not have sharp surface and constant density, but more realistic nuclear densities may be accommodated in the above manner if \( R_0 \) is a parameter in the definition of the density as, for example, in a Woods-Saxon shape. When we consider a nucleus whose density is not constant, the volume is not conserved and it is possible for the nucleus to have a \( \lambda = 0 \) mode. However, this monopole
mode will cost too much deformation energy and is not usually considered in the treatment of low lying excited states. Although $0^+ \rightarrow 0^+$ transitions have been observed, these breathing modes are generally weak, and presumably the associated transition densities have little to do with the collective ones discussed here.\textsuperscript{21,23}

The deformation of the nuclear matter density may be given in terms of $\alpha_{\lambda\mu}$ using a Taylor series expansion,

$$\delta \rho = \rho(r, R(\theta, \phi)) - \rho(r, R_0)$$

$$= \sum_{n=1}^{\infty} \frac{(\delta R)^n}{n!} \frac{d^n \rho(r, R_0)}{dR_0^n},$$

where,

$$\delta R = R_0 \sum_{\lambda\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta, \phi).$$

We assume that we can write the deformation of the optical potential in the same way so that

$$\delta U = U(r, R(\theta, \phi)) - U(r, R_0).$$

As mentioned earlier, the motivation for this procedure is the usual folding model\textsuperscript{7,8,8} with a zero range nucleon-nucleon interaction. In the Dirac approach, the scalar and vector potentials are deformed, instead of the central potential.\textsuperscript{5} Furthermore, in Dirac phenomenology we generally use
Woods-Saxon's for the scalar and vector potentials, and since the Woods-Saxon shape is parametrized in $R$, the deformations of the Dirac optical potentials may be written in terms of $\delta R$. However, it should be noted that each of the four optical potentials (real scalar, imaginary scalar, real vector and imaginary vector) will have its own $R$ parameter since now $R$ is one of the free parameters as discussed in Chapter 2. Under these assumptions, the first order transition potentials can be written as

$$U_{i,s}^{(1)} = R_i \sum_{\lambda\mu} \alpha_{\lambda\mu,i} x_{\lambda\mu}(\theta_i, \phi_i) \frac{dU_i(r, R_i)}{dR_i},$$

(3.14)

where $i$ represents one of the four Dirac optical potentials, scalar real and imaginary, and vector real and imaginary. This is the form of the transition potential which is used in conjunction with the phenomenological approach.

3.3.2 The vibrational modes of spherical nuclei

In this section, we describe a simple vibrational model for obtaining transition potentials for the use in a Dirac coupled channel treatment of nucleon-nucleus inelastic scattering. We follow the nonrelativistic harmonic oscillator collective model. In this model the nuclear single particle states are expressed as a sum of annihilation and creation operators.
A spherical nucleus may be assumed to be a liquid drop, its excitation modes arising from small oscillations about the spherical equilibrium shape. Although, as mentioned in section 3.3.1, this is not too realistic, this model introduces the concept of phonon excitations in an elegant manner. Consider a spherical nucleus of radius $R_0$. The deformed surface can be conveniently described by the expression of Eq.(3.10) where we now write the time dependence explicitly as,

$$R(\theta, \phi; t) = R_0(1 + \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t)Y_{\lambda\mu}^\ast(\theta, \phi)),$$

with

$$R_0 = r_0A^{1/3}. \tag{3.16}$$

Of course, the nucleus does not have a sharp surface like that of a liquid drop, but appropriate modifications can be made for the case of a diffuse surface.

Let us assume that the various nuclear densities, neutron and proton scalar and vector as discussed in Chapter II can be expressed as Woods-Saxon's. The next assumption is a critical one. It is commonly assumed in nonrelativistic calculations that the deformed potential can be obtained by expanding the central optical potential in a Taylor series expansion. The form of the deformed potential is taken to be a Woods-Saxon, motivated by the belief that the optical potential should be closely related to the nuclear
density. If we follow the same assumptions, then one would express the scalar and vector Dirac optical potentials in terms of Woods-Saxon's (the explicit form is not important, just the fact that the distribution is similar to what one would expect for a nuclear density),

\[ U_i(r, R_i(\theta_i, \phi_i), a_i) = \frac{V_i}{(1 + \exp((r - R_i)/a_i))}, \]  

where \( R_i \) is described by Eq. (3.15) with different equilibrium radius \( R_{0,i} \) for each potential, scalar real and imaginary, and vector real and imaginary and \( V_i \) represents the potential strength. Assuming oscillations of the scalar and vector potentials are of small amplitude compared to the radius parameter \( R_{0,i} \), we can expand the deformed potential, Eq. (3.17), in a Taylor series to the first order,

\[ U_i(r, R_i, a_i) \approx \frac{V_i}{1 + e^{(r - R_{0,i})/a_i}} + R_{0,i} V_i \frac{d}{dR_{0,i}} \left[ \frac{1}{1 + e^{(r - R_{0,i})/a_i}} \right] \sum_{\lambda \mu} \alpha^i_{\lambda \mu} Y^*_{\lambda \mu}(\Omega_i). \]  

The first term is termed the direct (elastic) potential and the second term given by,

\[ U_{t,i} = R_{0,i} V_i \frac{d}{dR_{0,i}} \left[ \frac{1}{1 + e^{(r - R_{0,i})/a_i}} \right] \sum_{\lambda \mu} \alpha^i_{\lambda \mu} Y^*_{\lambda \mu}(\Omega_i) \]  

\[ \equiv \hat{V}_{t,i}(r) \sum_{\lambda \mu} \alpha^i_{\lambda \mu} Y^*_{\lambda \mu}(\Omega_i), \]  

is used in the construction of a transition potential.
Next, we describe the classical harmonic oscillator model for the oscillation of nuclear matter,\textsuperscript{24} from which the usual nonrelativistic vibrational potential model has traditionally been obtained. First we consider the restoring forces associated with the oscillations of the nuclear matter density which are related to the potential energy of the system. Because of the deviation from sphericity, the potential energy changes and the surface energy increases as a result of an increase in the surface area of the nucleus. This change in the potential from its original value due to the restoring force can be obtained starting from the Hooke's law, \( F_\lambda = -k_\lambda \delta R_\lambda \) for each multipole excitation. Then the potential energy change which is denoted by \( V \) is, in the quadratic approximation,

\[
V = \frac{1}{2} \int r^2 dr \int d\Omega \sum_{i=2}^{\infty} \sum_{\lambda=2}^{\infty} k_\lambda^i |R^i_\lambda - R_0|^2 \\
= \frac{1}{2} \int r^2 dr \int d\Omega \sum_{i=2}^{\infty} \sum_{\lambda=2}^{\infty} k_\lambda^i |\delta R^i_\lambda|^2 \\
= \frac{1}{2} \int r^2 dr \int d\Omega \sum_{i=2}^{\infty} \sum_{\lambda=2}^{\infty} k_\lambda^i R_0^2 |\sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y^*_{\lambda \mu}(\theta, \phi)|^2 \\
= \frac{1}{2} \sum_{i,\lambda \mu} C^i_i |\alpha_{\lambda \mu}|^2 , \tag{3.21}
\]

where \( k_\lambda \) is related to the restoring force. Here \( \int d\Omega Y^*_{\lambda \mu}(\theta, \phi)Y_{\lambda' \mu'}(\theta, \phi) = \delta_{\lambda \lambda'}\delta_{\mu \mu'} \) is used. Also there must be a kinetic energy associated with the
oscillation, which can be written as

\[ T = \frac{1}{2} \int r^2 dr \int d\Omega \sum_{\lambda=2}^{\infty} \rho_\lambda \left| \delta R_\lambda \right|^2 \]

\[ = \frac{1}{2} \int r^2 dr \int d\Omega \sum_{\lambda=2}^{\infty} \rho_\lambda R_0^2 \sum_{\mu=-\lambda}^{\lambda} |\hat{\alpha}_{\mu\mu} Y_{\lambda\mu}|^2 \]

\[ = \frac{1}{2} \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} B_\lambda \left| \hat{\alpha}_{\mu\mu} \right|^2, \]  \hspace{1cm} (3.22)

where \( dr \) ranges only over the surface area where the vibrations occur. \( \rho_\lambda \) is the nuclear density which is assumed to be uniform. These equations satisfy the requirement of rotational and time reversal invariance.

It should be noted that here we are describing the conventional vibrational model simply to introduce the concept of phonons. Since we are going to search on the deformation parameters, which contain all the information from \( B_\lambda \) and \( C_\lambda \), we are not interested in the detailed form of those constants.

We have used this nonrelativistic form of the kinetic energy in Eq. (3.22) since we are considering low lying exited states with excitation energies, in general, less than 5 MeV. As the excitation energy is much smaller than the nucleon mass, the use of nonrelativistic kinematics is reasonable. If one were doing a consistent relativistic treatment, the target should be treated in a relativistic framework. This would add considerable complexity to the argument. We present here a discussion of the usual nonrelativistic vibrational model just as a motivation for the calculations we present in chapter 4. We
are not considering a relativistic harmonic oscillator model of the nucleus in this work. The classical Hamiltonian for the vibrating nuclear system can now be written as,

\[ H_{\text{osc}} = \sum_i \sum_{\lambda \mu} \frac{1}{2} B_{\lambda}^i |\dot{\alpha}_{\lambda \mu}^i|^2 + \frac{1}{2} C_{\lambda}^i |\alpha_{\lambda \mu}^i|^2. \]  

(3.23)

In order to quantize the collective excitation modes of the nucleus and to introduce the concept of phonon excitations, define the momentum conjugate to the variable \( \alpha_{\lambda \mu}^i \) as,

\[ \pi_{\lambda \mu}^i = \frac{\partial T}{\partial \dot{\alpha}_{\lambda \mu}^i} = B_{\lambda}^i \dot{\alpha}_{\lambda \mu}^i. \]  

(3.24)

The Hamiltonian then becomes,

\[ H_{\text{osc}} = \sum_i \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \left( \frac{1}{2B_{\lambda}^i} |\pi_{\lambda \mu}^i|^2 + \frac{1}{2} C_{\lambda}^i |\alpha_{\lambda \mu}^i|^2 \right). \]  

(3.25)

In the quantization procedure, the dependence of the inertial parameter \( B \) on the parameter \( \alpha_{\lambda \mu} \) is ignored, and the variables \( \alpha_{\lambda \mu} \) and \( \pi_{\lambda \mu} \) satisfy the commutator, \([\alpha_{\lambda \mu}, \pi_{\lambda \mu'}] = i\hbar \delta_{\lambda \lambda'}\delta_{\mu \mu'}\). As usual, we define

\[ \alpha_{\lambda \mu}^i = \sqrt{\frac{\hbar}{2B_{\lambda}^i \omega_{\lambda}^i}} (b_{\lambda \mu}^i + (-)^\mu b_{\lambda \mu}^{\dagger i}), \]  

(3.26)

\[ \pi_{\lambda \mu}^i = i\sqrt{\frac{\hbar B_{\lambda}^i \omega_{\lambda}^i}{2}} (b_{\lambda \mu}^{\dagger i} - (-)^\mu b_{\lambda \mu}^i), \]  

(3.27)

so that by substituting Eq. (3.26) and (3.27) into Eq. (3.25), we have

\[ H_{\text{osc}} = \sum_i \sum_{\lambda \mu} \hbar \omega_{\lambda}^i (b_{\lambda \mu}^{\dagger i} b_{\lambda \mu}^i + \frac{1}{2}), \]  

(3.28)
The operator $b_{\lambda \mu}$ and its Hermitian conjugate $b_{\lambda \mu}^\dagger$ are the phonon annihilation and creation operators with the commutator $[b_{\lambda \mu}, b_{\lambda' \mu'}^\dagger] = \delta_{\lambda \lambda'} \delta_{\mu \mu'}$. The ground state is a state with no phonons present, while the first excited state has only one phonon excitation and is $(2\lambda + 1)$-fold degenerate and, therefore, presumably has angular momentum $\lambda$. Since $H_{osc}$ describes the excitation of the nucleus, of course, the expectation value of $H_{osc}$ will be the excitation energy of the nucleus, hence it satisfies

$$H_{osc}\psi(\vec{r}) = \sum_{\lambda} \varepsilon_\lambda \psi(\vec{r}),$$

(3.30)

where $\varepsilon_\lambda (= \hbar \omega_\lambda)$ is the excitation energy of the target nucleus, which can be obtained experimentally. $\psi$ represents a wavefunction for the nuclear system.

We assume that a vibrational model of this type can be used in the relativistic case as well so that Eq. (3.26) can be used for the $\alpha_{\lambda \mu}$ in Eq. (3.19) for the transition potentials. Then $U_{t;i}$ may be expressed in terms of the annihilation and creation operators,

$$U_{t;i} = \hat{V}_{t;i}(r) \sum_{\lambda \mu} \alpha_{\lambda \mu}^i \chi_{\lambda \mu}(\Omega_i)$$

(3.31)

$$\equiv \hat{V}_{t;i}(r) \sum_{\lambda \mu} \beta_{\lambda \mu}^i (b_{\lambda \mu}^{\dagger} + (-)^\mu b_{\lambda \mu}^{\dagger \mu}) \chi_{\lambda \mu}^*(\Omega_i),$$

(3.32)
where \( \beta_\lambda \) is called the deformation parameter for multiploarity \( \lambda \). In this model, the diagonal matrix elements of the transition potentials vanish because \( \delta_{\lambda \mu} \) or \( \delta^\dagger_{\lambda \mu} \) can only connect states which differ by one in the number of phonons.\(^{25}\) In fact, it is this feature which is taken over to the relativistic case. In order to illustrate this, let's consider the form of the coupled channel Dirac equations appropriate for nucleon-nucleus scattering where the nucleus is represented by Eq. (3.28).

We assume that the total Hamiltonian for the projectile-target system is given by

\[
H = H_p + H_I + H_{osc},
\]

where

\[
H_p = \bar{\alpha} \cdot \vec{p} + \beta m, \tag{3.34}
\]

\[
H_I = \beta U_S + U_0 + \beta U_{t,s} + U_{t,0} + V_c, \tag{3.35}
\]

\[
H_{osc} = \sum_i \sum_{\lambda \mu} \hbar \omega_i (\delta^\dagger_{\lambda \mu} \delta_{\lambda \mu} + \frac{1}{2}), \tag{3.36}
\]

where \( V_c \) is the Coulomb potential. We follow Ref. 26 for the definitions of \( \beta \) and \( \alpha \) matrices. The total wave function \( \Psi \) satisfies

\[
H \Psi(\vec{r}) = E \Psi(\vec{r}), \tag{3.37}
\]

where \( E \) is the total energy of the system. \( H_p \) is the Hamiltonian for the projectile, and \( H_{osc} \) describes the excitations of the nucleus. \( H_I \) represents the
interaction between the projectile and target nucleus in this model. $U_{t,i}$ is defined in Eq. (3.19) and $U_S$ and $U_0$ are defined as the first term in Eq. (3.18). The subscripts $S$ and $0$ represent scalar and time-like vector potential parts respectively.

Substituting Eq. (3.33) in Eq. (3.37) gives

$$
(H_p + H_T + H_{osc})\Psi(\vec{r}) = E\Psi(\vec{r}),
$$

(3.38)

Hence the Dirac equation for this system becomes,

$$
[i \cdot \vec{p} + \beta(m + U_S + U_{t,0}) + V_c + \epsilon_\lambda]\Psi(\vec{r}) = E\Psi(\vec{r}).
$$

(3.39)

As shown in the first section of chapter 4, when Eq. (3.39) is multiplied by the final state from the left, one obtains the radial CC equations given by Eqs. (4.20) and (4.21). The transition potentials will appear between the initial and final states to form the matrix elements,

$$
\langle f[U_{t,i}]i \rangle = \langle f[V_{t,i}] \sum_{\lambda,\mu} \beta_\lambda(b_{\lambda,\mu}^* + (-)^\mu b_{\lambda,\mu})Y_{\lambda,\mu}^*(\Omega)i \rangle,
$$

(3.40)

where $f$ represents a final state, and $i$ represents an initial state of the total wavefunction. Since $b_{\lambda,\mu}$ or $b_{\lambda,\mu}^\dagger$ operate in target space and only connect the target states which differ by one in the number of phonons, the diagonal matrix elements of transition potentials vanish and only off-diagonal terms survive in this form of the vibrational model.
3.3.3 The rotational modes of deformed nuclei

In this section we outline the nonrelativistic rotational model approach which is commonly used in the treatment of deformed nuclei. In this case the stable nuclear shape is nonspherical, and this deformed nucleus slowly rotates\(^3\) (also refer to section 3.2, Fig. 3.2(a)). The deformation considered in this model is static. The deformation is usually assumed to be axially symmetric so that

\[
\alpha_{\lambda\mu} = \beta_\lambda \delta_{\mu\lambda},
\]

if the body fixed z axis is directed along the symmetry axis and \(\beta_\lambda\) is termed the deformation parameter. Thus,

\[
R = R_0 (1 + \sum_{\lambda} \beta_\lambda Y_{\lambda 0}^* (\theta')) ,
\]

where \(\theta'\) refers to the body fixed system, and

\[
Y_{\lambda 0}^* (\theta') = \sum_{\mu} D_{\lambda 0}^{\ast} (\theta_i) Y_{\lambda \mu} (\theta, \phi),
\]

where \(D\) is a rotational matrix which transforms the space fixed coordinates to the body-fixed coordinates\(^3\) and \(\theta_i\) stands for the Euler angle between the body fixed and space fixed coordinates as shown in Fig. 3.3.

We use only Lorentz scalar and time-like vector optical potentials for spherically symmetric nuclei, as it has been shown for elastic scattering
Figure 3.3:

Schematic figure of the rotation of axes from the lab to the intrinsic frame of the reference.
calculations that the space part of the Lorentz four-vector potential may be removed by a wavefunction transformation.\textsuperscript{28} However, if the potentials are not spherically symmetric, the space parts of vector cannot be transformed away; thus one should include them for deformed nuclei. However, for simplicity, we have not included the space vector terms in the calculations presented in chapter 4. We assume that replacing $\alpha_{\lambda\mu}Y_{\lambda\mu}^*$ in Eq. (3.19) by $\beta_{\lambda}D_{\mu0}^\lambda Y_{\lambda\mu}^*$ we have an appropriate expression for the coupling potential patterned after the nonrelativistic rotational model. The main difference between the rotational model and the vibrational model in this work is that the diagonal parts of the transition potentials survive in the rotational model while they vanish in the vibrational model.\textsuperscript{25}

### 3.4 RIA based collective model

In this section we use two different assumptions to obtain the collective model transition potentials from the RIA optical model potentials described in Chapter 2. In case 1, we deform the optical RIA potentials themselves to obtain transition potentials. In the second case we deform the target densities and then fold them with the invariant NN amplitudes to obtain the transition
potentials. The diagonal scalar and vector optical potentials were obtained as in Refs. 10 and 11 and contain no free parameters. The scalar and vector target densities were obtained from the relativistic Hartree (RH) calculations of Horowitz and Serot\textsuperscript{29} and the free NN amplitudes\textsuperscript{30} expressed in a Lorentz invariant form as in Ref. 31. In general, we have omitted the tensor term that also appears in the RIA.\textsuperscript{32}

The first order, impulse approximation optical potentials do not account for nuclear collectivity.\textsuperscript{33} Inclusion of this important nuclear dynamics through channel coupling via first order, impulse approximation diagonal and transition potentials is well justified within standard, nonrelativistic multiple scattering theory.\textsuperscript{16,17} For the case 1, the parameters of the model are the scalar, $\delta_s$, and vector, $\delta_v$, deformation lengths. These $\delta$'s may be compared to the deformation parameters $\beta$'s introduced in the previous sections by defining the deformation length in that case as $\delta = \beta R$, where $R$ is the nuclear radius parameter. As mentioned above, the small tensor RIA contribution was not included for most of the cases. However, for heavier nuclei, tensor potential as well as the contribution from the interaction of the anomalous magnetic moment of the projectile with the charge distribution of the nucleus are included in the calculation. In some cases, as discussed in chapter 4, these terms have an appreciable effect on elastic spin observables at small angles. We have not
included deformed nuclear tensor potentials in this work. Nor, in general, have we included a deformed Coulomb potential. In chapter 4, we show that the effect of including the deformed Coulomb potential is small.

As described in chapter II, the RIA optical potentials are given by

\[ U_s(r) = -\frac{k}{4\pi^2 m} \sum_{i=p,n} J^i \int d^3 q e^{-i\vec{q}\cdot\vec{r}} \rho_s^{(i)}(q) \int d^3 r' e^{i\vec{q}\cdot\vec{r}'} \rho_s^{(i)}(r'), \]

\[ U_v(r) = -\frac{k}{4\pi^2 m} \sum_{i=p,n} \int d^3 q e^{-i\vec{q}\cdot\vec{r}} \rho_v^{(i)}(q) \int d^3 r' e^{i\vec{q}\cdot\vec{r}'} \rho_v^{(i)}(r'), \]

\[ U_t(r) = -\frac{2k}{4\pi^2 m} \sum_{i=p,n} \int d^3 q e^{-i\vec{q}\cdot\vec{r}} \rho_t^{(i)}(q) \int d^3 r' e^{i\vec{q}\cdot\vec{r}'} \frac{1}{r'} \rho_t^{(i)}(r'), \]

where the subscripts s, v, t refer to Lorentz scalar, vector (time-like) and tensor quantities, the superscripts, \( i = n \) or \( p \) refers to neutrons and protons and \( k \) is the c.m. projectile wave number.

One may define the transition form factors as the radial parts of the transition potentials by writing

\[ U_i^\lambda = \sum_\mu f_i^\lambda Y_\mu^*. \]  

(3.45)

For case 1 we write

\[ f_i^\lambda = \delta_i^\lambda \frac{\partial}{\partial r} U_i, \]  

(3.46)

where \( \lambda \) is the multipolarity of the excited state and \( i \) represents scalar, vector and tensor optical potentials, and \( \delta \) is the deformation length. This expression for the transition potential has the same form as Eq. (3.14) except that \( \frac{\partial}{\partial R} \) is changed to \( \frac{\partial}{\partial r} \).
For case 2, the transition potential is obtained by folding the derivatives of the input RH densities with the Lorentz covariant NN amplitudes, $F(q)$. Hence the transition form factors for this case are,

$$
\begin{align*}
 f_s^\lambda(r) &= \delta^\lambda_s \left( -\frac{k}{4\pi^2m} \right) \sum_{i=p,n} \int d^3q e^{-i\vec{q}\cdot\vec{r}} F_s^{(1)}(q) \int d^3r' e^{i\vec{r}'\cdot\vec{r}} \frac{d\rho_s^{(1)}(r')}{dr'}, \\
 f_v^\lambda(r) &= \delta^\lambda_v \left( -\frac{k}{4\pi^2m} \right) \sum_{i=p,n} \int d^3q e^{-i\vec{q}\cdot\vec{r}} F_v^{(1)}(q) \int d^3r' e^{i\vec{r}'\cdot\vec{r}} \frac{d\rho_v^{(1)}(r')}{dr'}, \\
 f_t^\lambda(r) &= \delta^\lambda_t \left( -\frac{2k}{4\pi^2m} \right) \sum_{i=p,n} \int d^3q e^{-i\vec{q}\cdot\vec{r}} F_t^{(1)}(q) \int d^3r' e^{i\vec{r}'\cdot\vec{r}} \frac{1}{r'} \frac{d\rho_t^{(1)}(r')}{dr'}.
\end{align*}
$$

For both cases, we have scalar, vector and, if desired, tensor deformation lengths which are determined by fitting the inelastic observables. One could also take the protons and neutrons to have different $\delta^\lambda$'s although we have not done this.

For both cases we consider only coupling between the $0^+$ ground state and the excited state of interest, and pattern the calculation after nonrelativistic vibrational model calculations as discussed in section 3.3.2. In the calculations described in Chapter 4, we have taken two free parameters, $\delta_S$ and $\delta_V$, for each excited state considered. The $\delta$'s for real and imaginary transition potentials are taken equal. The $\delta$'s corresponding to protons and neutrons for case 2 were also taken equal in order to reduce the number of parameters.
The difference between these two cases is,

\[ f_1^\lambda - f_2^\lambda = 2\delta^\lambda \frac{k}{4\pi^2m} \sum_{i=p,n} \int e^{-iqr} F^{(i)}(q) \int \frac{\rho^{(i)}(r')}{r'} e^{iqr'} d\tau' d\tilde{q}, \]  

(3.48)

considering either the scalar or the vector parts of the optical potential. Because of the \(1/r\) factor, the difference is apparent only at small \(r\). In Fig. 3.4 the transition potentials for case 1 (solid line) and case 2 (dashed line) are compared for the case of proton - \(^{40}\)Ca scattering. The projectile energy is 497.5 MeV. The significant difference between two calculations occurs only inside the nucleus, at \(r\) less than 2\(fm\) (the rms radius of \(^{40}\)Ca is 3.51\(fm\)).

Hence, it is expected that the \(\delta\)'s obtained from either case should not be significantly different, and this is true in general, as will be discussed in the next chapter.
Figure 3.4:
Comparison of the transition potentials of case 1 and 2 for $^{40}Ca$ at 500 MeV.
CHAPTER III REFERENCES


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CHAPTER IV

In this chapter we present Dirac coupled channel equations for nucleon-nucleus inelastic scattering and discuss the results of coupled channel (CC) calculations for this process. We consider both spherically symmetric nuclei and deformed nuclei as targets. The simple collective models described in the previous chapter are used to obtain the transition potentials for the use in the Dirac coupled channel calculations. For most of the cases, we consider the Lorentz scalar and time-like vector potentials only and deform them to describe the inelastic scattering to the collective states. Tensor potentials are included in the calculation for several cases in order to examine the effect on the analysis. However, we did not include the deformation of the tensor potentials in this work. For the Coulomb potential, the Woods-Saxon parameters given in Ref. 1 are used for the charge distribution and the potential is obtained by solving Poisson's equation.
4.1 Dirac coupled channel equations

We start with the total Hamiltonian of the projectile-target system, which consists of the Hamiltonian of the projectile, $H_p$, the interaction between the target nucleus and the projectile, $H_I$, and the nuclear Hamiltonian, $H_N$. Then, we can write the total Hamiltonian, $H$, as

$$H = H_p + H_I + H_N,$$

where

$$H_p = \bar{\alpha} \cdot \vec{p} + \beta m,$$

$$H_I = \beta U_S + U_0 + i\beta \bar{\alpha} \cdot \vec{r} U_T + V_C + U_{AM},$$

where the complex potentials consist of a Lorentz scalar potential $U_S$, a time-like component of a four vector potential $U_0$ and a tensor potential $U_T$. And $V_C$ represent the static Coulomb potential, and $U_{AM}$ is for the anomalous magnetic moment term. The potentials $U_S$ and $U_0$ are now deformed and include transition potentials, $U_t$ of the form given in Eq. (3.14). $V_C$ is the static Coulomb potential. The total wave function $\Psi(\vec{r})$ satisfies

$$H \Psi(\vec{r}) \equiv (H_p + H_I + H_N) \Psi(\vec{r}) = E \Psi(\vec{r}),$$
where $E$ is the total energy of the system. The nuclear Hamiltonian $H_N$ satisfies

$$H_N \psi_n(\vec{r}) = \epsilon_n \psi_n(\vec{r}), \quad (4.5)$$

where $\psi_n$ is a nuclear eigenstate and $\epsilon_n$ is its energy eigenvalue. Hence by inserting Eq. (4.2), Eq. (4.3) and Eq. (4.5) into Eq. (4.4), we have

$$[\bar{\alpha} \cdot \vec{\sigma} + \beta(U_S(\vec{r}) + m) + U_0(\vec{r}) + V_C + U_{AM} + i\beta \bar{\alpha} \cdot \vec{r}U_T(\vec{r}) + \epsilon_n] \Psi(\vec{r}) = E \Psi(\vec{r}). \quad (4.6)$$

The Dirac matrices $\alpha$ and $\beta$ are 4 by 4 matrices operating on the components of $\Psi(\vec{r})$. They are defined by

$$\bar{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad (4.7)$$

where $I$ is the 2 by 2 unit matrix and the components of $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli spin matrices. Of course, all the five possible Lorentz covariant potentials could be included in the Dirac equation when we consider inelastic scattering. It should be noted that only scalar, vector and tensor potentials survived for elastic scattering from spin zero targets, because of parity conservation. However, for inelastic scattering, the other types of potentials, such as pseudoscalar and axial vector terms may also survive. However, in the collective model approach used here, we assume we can obtain appropriate
transition potentials for treating collective excitations of the nucleus by deforming the elastic scalar and time-like vector potentials without adding the other types of potentials. The tensor potentials are included in the optical potentials for several cases to see their effects on spin calculation for several cases. However, only the scalar and time-like vector potentials are deformed, the tensor potentials are not deformed in this work.

To obtain the explicit form of the total Dirac wavefunction, we write the target and projectile wavefunctions following the notation of Ref. 4 and 5. The target nucleus can be described by a finite number $N$ of degenerate-energy eigenstates $|nM\rangle$, each an eigenstate of spin $I_n$, $z$ component of spin $M_n$, and parity $p_n$, where $1 \leq n \leq N$. Eigenstates of the orbital angular momentum and spin of the (projectile) proton, $|LM_L\rangle$ and $|\frac{1}{2}M_S\rangle$, are combined into eigenstates of the total angular momentum of the proton in the usual manner:

$$|J\mu\rangle = \sum_{m, M_L} \langle LM_L \frac{1}{2} m_S | J \mu \rangle |LM_L\rangle |\frac{1}{2} m_S\rangle,$$

(4.8)

where $J$ is the total angular momentum of the projectile, and $\mu$ is the third component of $J$. To each value of $J$, there correspond two values of $L$: $L = J + 1/2$ and $L = J - 1/2$. Consequently, $J$ and $L$ can be combined into a single quantum number $\chi$ as follows:

$$J = |\chi| - 1/2,$$

(4.9)
where \( \text{sgn} \chi \) represents the positive or negative sign of \( \chi \). With \( \vec{F} = \vec{I} + \vec{J} \), the total angular momentum of the system, the total wavefunction can be expanded in eigenfunctions of \( \vec{F}^2 \), \( \vec{F}_3 \), and \( P \), the total parity operator for the system. One such set of eigenfunctions is

\[
| f m_f \chi n \rangle = \sum_{\mu M} \langle J \mu I_n M | f m_f \rangle | \chi \mu \rangle | n M \rangle,
\]

(4.11)

for which \( p = n(-1)^L \) and \( f \) is the eigenvalue of the operator \( F \). In general, to a pair of eigenvalues \( f \) and \( p \), there correspond several possible pairs \( (\chi, n) \).

Let \( \mathcal{N}(f,p) \) be the set of such pairs, and let \( N(f,p) \) be its cardinality. It is clear that if \( (\chi, n) \in \mathcal{N}(f,p) \), then \( (-\chi, n) \in \mathcal{N}(f,p) \). As a notational convenience, we introduce a single quantum number \( j \), such that as \( j \) ranges from 1 to \( N(f,p) \), \( (\chi_j, n_j) \) ranges over all the pairs in \( \mathcal{N}(f,p) \). Finally, if \( j \) is the quantum number corresponding to the pair \( (\chi, n) \) in \( \mathcal{N}(f,p) \), let \( j_- \) be that corresponding to the pair \( (-\chi, n) \) in \( \mathcal{N}(f,p) \). Now we may expand the total wavefunction as,

\[
\Psi(\vec{r}) = \sum_{f m_f p i} a_{f m_f p} \frac{1}{r} \begin{pmatrix} g_j^p(\vec{r})| f m_f p \rangle \\ -\text{sgn} \chi_j f_j^p(\vec{r})| f m_f - p \rangle \end{pmatrix},
\]

(4.12)

where the coefficients \( a_{f m_f p} \) are constants which must be chosen to satisfy the scattering boundary conditions.
To obtain the coupled equations for the radial upper component, $g_j$, and lower component $f_j$ of the Dirac spinors, first substitute Eq. (4.12) into the Dirac equation, Eq. (4.6), then we have,

$$ \sum_{fmfpj} \left\{ (m + U_S + U_0 + V_c + \epsilon_j - E)^{-1} g_j f^p(r)|fmfpj\rangle + |\tilde{\sigma} \cdot \tilde{p} + i\tilde{\sigma} \cdot \tilde{r} U_T \right. 
- \left. \frac{\nu}{2m} i\tilde{\sigma} \cdot (\nabla V_c)^{-1} (sgn\chi_j) f_j f^p(r)|fmfpj\rangle \right\} = 0 \quad (4.13) $$

$$ \sum_{fmfpj} \left\{ |\tilde{\sigma} \cdot \tilde{p} - i\tilde{\sigma} \cdot \tilde{r} U_T + \frac{\nu}{2m} i\tilde{\sigma} \cdot (\nabla V_c)^{-1} g_j f^p(r)|fmfpj\rangle + (-m - U_S + U_0 + V_c + \epsilon_j - E)^{-1} (sgn\chi_j) f_j f^p(r)|fmfpj - pj\rangle \right\} = 0. \quad (4.14) $$

where $U_{AM} = \frac{\nu c e}{2mc^2} \hat{\gamma} \cdot \hat{\nabla} V_c(r)$ is written explicitly, which is the potential due to the interaction of the anomalous magnetic moment of the projectile with the Coulomb field of the target nucleus. The coefficient $\nu$ is the anomalous magnetic moment of the projectile. ($\nu = 1.79$ for protons and $\nu = -1.91$ for neutrons.)

Following Refs. 4 and 5, we have the relationships,

$$ |fmfpj\rangle = -i(sgn\chi_j)(\tilde{\sigma} \cdot \tilde{r})|fmfpj - p\rangle, \quad (4.15) $$

$$ |fmfpj - p\rangle = i(sgn\chi_j)(\tilde{\sigma} \cdot \tilde{r})|fmfpj\rangle. \quad (4.16) $$

For convenience, let's simplify the notation by dropping the quantum numbers $f, m_f$ and $p$, that is,

$$ |j\rangle \equiv |fmfpj\rangle, \quad (4.17) $$
If we consider $\vec{\sigma} \cdot \vec{p}$ term in Eq. (4.14), it gives\(^4\)

\[
(\vec{\sigma} \cdot \vec{p}) g_j(r) |j\rangle = \left( \frac{\vec{\sigma} \cdot \vec{r}}{r^2} \right) (\vec{\sigma} \cdot \vec{r}) g_j(r) |j\rangle,
\]

\[
= \left( \frac{\vec{\sigma} \cdot \vec{r}}{r^2} \right) \left[ (\vec{r} \cdot \vec{r}) + i\vec{\sigma} \cdot (\vec{r} \times \vec{p}) \right] g_j(r) |j\rangle,
\]

\[
= \left[ -i \frac{d}{dr} g_j(r) - i(1 + \chi_j) \frac{g_j(r)}{r^2} \right] (\vec{\sigma} \cdot \vec{r}) |j\rangle,
\]

\[
= \left( -\frac{1}{r} \frac{d}{dr} g_j(r) + \frac{1}{r^2} g_j(r) - \frac{1}{r^2} g_j(r) - \chi_j \frac{g_j(r)}{r} \right) \frac{1}{\text{sgn} \chi_j} |j\rangle,
\]

\[
= -\frac{1}{r} \left( \frac{d}{dr} + \frac{\chi_j}{r} \right) g_j(r) \frac{1}{\text{sgn} \chi_j} |j\rangle,
\]

(4.17)

where $(\vec{\sigma} \cdot \vec{r})^2 = 1$ and $(\vec{\sigma} \cdot \vec{L}) |j\rangle = (-1 - \chi_j) |j\rangle$ are used. The proof of the latter relation is the following:

\[
(\vec{\sigma} \cdot \vec{L}) |j\rangle = 2 \vec{\sigma} \cdot \vec{L} |j\rangle
\]

\[
= 2 \times \frac{1}{2} \left[ (\vec{\sigma} + \vec{L})^2 - \vec{\sigma}^2 - \vec{L}^2 \right] |j\rangle
\]

\[
= [J(J + 1) - s(s + 1) - L(L + 1)] |j\rangle
\]

\[
= [(|\chi_j| - 1/2)(|\chi_j| + 1/2) - 3/4 - (|\chi_j| - 1/2 + 1/2)(|\chi_j| + 1)] |j\rangle
\]

\[
= (-1 - |\chi_j|) |j\rangle
\]

\[
= (-1 - \chi_j) |j\rangle,
\]

(4.18)

where the definitions of Eqs. (4.9) and (4.10) are used. Similarly, $\vec{\sigma} \cdot \vec{p}$ term in Eq. (4.13) will become

\[
(\vec{\sigma} \cdot \vec{p}) \frac{f_j(r)}{r} (-\text{sgn} \chi_j) |j\rangle = -\frac{1}{r} \left( \frac{d}{dr} - \frac{\chi_j}{r} \right) f_j(r) |j\rangle.
\]

(4.19)
If we multiply \(-r\langle j |\) on the left of Eq. (4.13) and use the \(\vec{\sigma} \cdot \vec{p}\) term calculation given above, we have

\[
\left( \frac{d}{dr} - \frac{\chi_j}{r} \right) f_j - (m + \epsilon_j - E)g_j = \sum_{j'} [(j|U_S + U_0 + V_c|j')g_{j'} - \langle j|\vec{\sigma} \cdot \vec{p}_T - \frac{\nu}{2m}i\sigma \cdot (\vec{\nabla}V_c)|j'_j)sgn\chi_j\rangle f_{j'}].
\]  
(4.20)

For the second equation, multiply by \(-r(\text{sgn}\chi_j)(j_-|\) on the left of Eq. (4.14) and use the results of Eq. (4.17), then we obtain

\[
\left( \frac{d}{dr} + \frac{\chi_j}{r} \right) g_j + (-m + \epsilon_j - E)f_j = \sum_{j'} [(j_-| - i\vec{\sigma} \cdot \vec{p}_T + \frac{\nu}{2m}i\sigma \cdot (\vec{\nabla}V_c) \\
\times (\text{sgn}\chi_j)|j'_j)g_{j'} - \langle j_-| - U_S + U_0 + V_c(\text{sgn}\chi_j)(\text{sgn}\chi_j)|j'_j)g_{j'}].
\]  
(4.21)

Here, we separate the elastic optical potentials, \(U^0_T\), the scalar and time-like vector potentials (and Coulomb potentials), from transition potentials, \(U^\lambda_T\), and rewrite

\[
\left[ \frac{d}{dr} + \frac{\chi_j}{r} \right] g_j = \left[ E - \epsilon_j + m + U^0_S - U_0 - V_c \right] f_j + \left( U^0_T - \frac{\nu}{2m} \frac{\partial V_c}{\partial r} \right) f_j \\
= \sum_{\lambda \neq j} P_{j_1}^j [(U^\lambda_S - U^\lambda_0)(\text{sgn}\chi_j)(\text{sgn}\chi_j')f_{j'} - U^\lambda_T f_{j'}] 
\]  
(4.22)

\[
\left[ \frac{d}{dr} - \frac{\chi_j}{r} \right] f_j = \left[ E - \epsilon_j - m - U^0_S - U_0 - V_c \right] g_j - \left( U^0_T - \frac{\nu}{2m} \frac{\partial V_c}{\partial r} \right) f_j \\
= \sum_{\lambda \neq j} P_{j_2}^j [(U^\lambda_S + U^\lambda_0)g_{j'} + U^\lambda_T f_{j'}].
\]  
(4.23)

The subscript \(\lambda\) refers to the multipole order of the particular transition potential. The detailed form of \(P_{j_2}^j\) is given in Appendix B.
These complicated coupled equations are solved numerically using the sequential iteration method in the computer code, ECIS87\textsuperscript{7,8} written by J. Raynal. The results are checked using the computer code, CENITH\textsuperscript{4,9} written by R.L. Mercer, which solves the exact coupled channel equations. ECIS87 is considerably faster than CENITH. For most cases we find the results of these two codes are almost the same with differences occurring at very large angles. An example of the comparison of the results from two programs is shown in Fig. 4.1.

4.2 General search procedure

In this section we describe the various procedures we use in the analysis of inelastic proton-nucleus scattering data. The computer code, ECIS87\textsuperscript{7}, originally written for the nonrelativistic CC calculation and recently, modified to admit some features of the Dirac approaches\textsuperscript{8} is used in the analyses. The vibrational and rotational models, described in chapter 3, are used in ECIS87 with the input changed to allow scalar, vector and tensor potentials rather than central and spin-orbit potentials. We use a model based on the
Figure 4.1:

Comparison between calculated p+$^{40}$Ca elastic observables of 500 MeV from ECIS87 (solid lines) and CENITH (dashed lines). In these calculations the $3^-$ state is coupled using the $\beta$'s from 2 parameter fit discussed in section 4.2.
nonrelativistic vibrational model for spherically symmetric nuclear targets such as $^{40}$Ca, $^{48}$Ca, $^{90}$Zr and $^{208}$Pb, and in the same spirit, both rotational and vibrational models for deformed nuclear targets such as $^{12}$C and $^{154}$Sm.

In the search, we include both elastic and inelastic data together, and minimize the total chi-square defined by

$$\chi^2 = \sum \chi^2_{el,i} + \chi^2_{inel,i}, \quad (4.24)$$

by searching adjustable parameters. For each observable, $\chi^2$ is defined as

$$\chi^2_i = \sum_\theta \frac{|x_i^{th}(\theta) - x_i^{ex}(\theta)|^2}{(\Delta x_i^{ex}(\theta))^2}, \quad (4.25)$$

where $x_i^{th}$ denotes the theoretical value, $x_i^{ex}$ denotes the experimental value and $\Delta x_i^{ex}$ denotes the experimental error. For some cases, the elastic data set has very much smaller experimental error than the inelastic data set (especially inelastic $A_y$ data) and it was not possible to obtain the desired quality of the fit. In these cases, we attempted to improve the fit to the inelastic $A_y$ data by increasing its importance in the overall fit by multiplying $\chi^2_{inel,A_y}$ by a factor of $W$. In that case, we are minimizing the total $\chi^2$ given as

$$\chi^2_{T} = \sum \chi^2_{el,i} + \chi^2_{inel,cs} + W \times \chi^2_{inel,A_y}. \quad (4.26)$$

We call this procedure a 'weighting' procedure. Clearly weighting can be applied to any observable in the data set. In some cases both elastic and inelastic
data sets have been weighted. The results are discussed in the following sections.

Diagonal potentials are obtained using either the phenomenological potentials or RIA potentials as discussed in chapter 3. In the first case, which we call the 'phenomenological case', the potential strength parameters are readjusted when the inelastic data are included in the calculation. In the RIA calculation, this is not done, for the reasons discussed below.

The procedures for the phenomenological cases are discussed first. For this case, Woods-Saxon forms are used in the potentials which describe the elastic scattering observables and, as described in chapter 2, the model has a total 12 parameters. The optical potentials for inelastic scattering, which are deformed, are assumed to have the form

\[
U_i(r, R_i(\theta_i, \phi_i), a_i) = \frac{V_i}{1 + \exp((r - R_i)/a_i)},
\]

(4.27)

where \(R_i\) is the radius parameter, \(a_i\) is the diffusiveness and \(V_i\) is the potential strength for each potential \(i\), scalar real and imaginary, vector real and imaginary. The transition potentials are given by,

\[
U_{t, i} = R_{0,i} V_i \frac{d}{dR_{0,i}} \left[ \frac{1}{1 + e^{(r - R_{0,i})/a_i}} \right] \sum_{\lambda \mu} \beta_{\lambda}^i Y_{\lambda \mu}^*(\Omega_i),
\]

(4.28)

where the deformation parameter, \(\beta_{\lambda}^i\) is treated as a free parameter which is searched to fit the inelastic data. The procedures followed are described next.
First, we try a 2 parameter search with deformation parameters $\beta_5$ for the scalar and $\beta_V$ for the vector for each inelastic state. We assume the real and imaginary parts are the same and consider one excited state at a time.

Next, since elastic potential parameters are determined by fitting elastic data, with all the reaction effects in them, and now we treat the coupling to the one excited state explicitly, it is clear that the elastic potential parameters should be changed to accommodate this new situation. Hence, we add the four potential strengths to the search, increasing the number of parameters to 6.

Buck\textsuperscript{10} concluded that the geometry parameters were not changed very much by the inclusion of the inelastic state in the calculation. Since his calculation was nonrelativistic, we cannot assume that the same thing is true in the relativistic calculation. In order to test this, the geometry parameters were included in the search starting from the results of the 6 parameter search, making a 14 parameter search. This allows us to check the stability of this procedure.

We also compare the CC calculation results with distorted wave Born approximation (DWBA) method using the same $\beta$'s, obtained in the 2 parameter search using the CC analyses. Both calculations are done in Dirac formalism.

Next, the procedures for the RIA based calculations are described. Here the diagonal scalar and vector potentials were obtained using RIA as described
in Ref. 11 and contain no free parameters (refer to section 2.2). The input consists of the scalar and vector target densities obtained from relativistic Hartree (RH) calculations of Horowitz and Serot\textsuperscript{12} and free NN amplitudes given in Lorentz covariant form as in Ref. 13. The Sp82 or the Sp88 solutions\textsuperscript{14} were used for \textsuperscript{40}Ca at 500 MeV case, and we found essentially no difference between the results obtained although, in the least-squares sense, the Sp88 gave slightly better agreement with both elastic and inelastic data. Hence, in the calculations here the Sp88 amplitudes are used.

As described in the section 3.3.4 of chapter 3, two different models are employed to obtain the transition potentials. In case 1, the transition potentials were assumed to be proportional to the derivatives of the RIA potentials themselves. In case 2, they are obtained by folding the derivatives of the input RH densities with the appropriate invariant NN amplitudes. Since the first order RIA optical potentials do not account for nuclear collectivity,\textsuperscript{15} the diagonal optical potentials should remain unchanged. Hence, here we use the deformation lengths (\( \delta \)'s) for the adjustable parameters. The deformation length is related to the deformation parameter (\( \beta \)) obtained in the phenomenological approach by \( \delta = \beta R \), where \( R \) is the appropriate nuclear radius parameter. There are two free parameters for each state, \( \delta_s \) and \( \delta_v \); the real and imaginary \( \delta \)'s are taken equal in these calculations. The form of the transition potential
for case 1 is given as

\[ U_i^\lambda = \sum_\mu \delta_i^\lambda \frac{\partial U_i^{RIA}}{\partial r} Y_{\lambda\mu}^* \]  

(4.29)

where \( i \) represents the real and imaginary, scalar and vector potentials. The deformation of the small tensor RIA contribution is not included here.

The RIA calculation including only scalar and vector potentials gives very good agreement with both elastic and inelastic data for most of the cases considered. However, we found that the RIA calculations give less structure in \( A_y \) than observed in the small angle elastic data. It had been found\(^{16} \) that the inclusion of the RIA tensor potential improved this situation. We find that the effects on the extracted \( \delta \)'s by the inclusion of the tensor potentials to be small. These results will be discussed in section 4.3.4.

For \(^{208}\text{Pb} \) case, we found some instability in the observables at large angles. To improve this, we increased both the number of partial waves and the size of \( R_{\text{max}} \), which is the radius at which the solutions are matched to the point Coulomb solutions. The results were not very sensitive to the number of partial waves as long as it is not too small. A typical number for \( \text{Pb} \) at 800 MeV is 150. But the results were sensitive to the value of \( R_{\text{max}} \). The stability tests shown in Fig. 4.2, in which we change \( R_{\text{max}} \) from 10 fm to 15 fm, to 20 fm, show that it is necessary to use \( R_{\text{max}} \) of at least 15 fm to obtain stable behavior out to 40 degrees, if the calculation is extended beyond the point
20 fm must be used. For \(^{40}\text{Ca}\), \(R_{\text{max}} = 10\) fm is adequate for the cases we consider.

A third procedure based on the RIA potentials is followed for \(^{40}\text{Ca}\) at 362 MeV and \(^{12}\text{C}\) at 800 MeV where the RIA potentials give rather poor agreement with the elastic observables. Since the collective model calculation is very sensitive to the quality of the fit to the elastic observables, it is not useful to proceed using these RIA potentials. In view of this we multiplied four RIA potentials by scale parameters which were then searched on along with the \(\delta\)'s. and found much better agreement with both elastic and inelastic data.

For deformed nuclei, both rotational and vibrational models are used. The results from both calculations were about the same. For \(^{12}\text{C}\), a number of search procedures were tried. However, it was not possible to find the same quality of fits to \(^{12}\text{C}\) as for the other targets. The various search procedures for \(^{12}\text{C}\) case are discussed in the section 4.4.1. For \(^{154}\text{Sm}\), where the low lying excited states are very closely leveled, first we considered the lowest excited state in the calculation, and next we included two excited states at once to find the coupling effect between those two excited states. These results are given in section 4.4.2.
Figure 4.2:
The differences between the results using different values of $R_{\text{max}}$, 10 fm (dotted lines), 15 fm (dashed lines) and 20 fm (solid lines) for p+ $^{208}$Pb elastic observables at 800 MeV. RIA potentials are used and the data are from Ref. 42.
4.3 Results of the Vibrational Model Analysis

In this section we present the results of a relativistic coupled channel analysis of proton inelastic scattering from the spherically symmetric nuclei $^{40}\text{Ca}$, $^{48}\text{Ca}$, $^{90}\text{Zr}$ and $^{208}\text{Pb}$. The procedures described in the previous section are followed in the analysis. Much attention will be focused on $^{40}\text{Ca}$ as the most extensive data set exists for this target. The low lying excited states of these spherical nuclei are assumed to be collective vibrational states of the type discussed in chapter 3, and a simple first order vibrational model, patterned after the nonrelativistic vibrational model, is used to obtain the transition potentials. Each excited state is assumed to be a one phonon state. We discuss the results of the analysis of each target in a separate subsection, beginning with $^{40}\text{Ca}$, and present observations regarding the analysis in general, such as the effects of the target mass, in the conclusion of this chapter.

4.3.1 Results for $^{40}\text{Ca}$

In this subsection we discuss the results of the analysis of inelastic $p + ^{40}\text{Ca}$ at incident proton energies 362 MeV, 500 MeV and 800 MeV. The 362 MeV elastic and inelastic data are from Ref. 17, the 500 MeV elastic data are from Refs. 18, 19 and 20, and the inelastic data are from Refs. 21 and 22; the elastic data at 800 MeV are from Refs. 23, 24 and 15, and the inelastic data are
given in Ref. 25. The first three low lying excited states are considered. They are the 3$^-$ with excitation energy 3.74 MeV, the 2$^+$ with excitation energy 3.90 MeV and the 5$^-$ with excitation energy 4.49 MeV. We will discuss first the results of the phenomenological analysis and then give the results of an analysis based on optical potentials calculated using the RIA.

**Phenomenological Analysis**

Here the diagonal parts of optical potentials are obtained by fitting elastic observables using the approach described in chapter 2. We consider two cases, one in which the optical potential parameters are determined by fitting data at each energy, and another which uses the global optical model parameters given in Ref. 26. A two parameter search on the deformation parameters using either of these optical potential produced essentially the same results with respect to the quality of the fit to the inelastic observables and the extracted deformation parameters. As global parameters are not yet available for the other targets we use single energy fits to the elastic data to determine the direct potentials for use as the starting point of the analysis. We consider each energy seperately, the effects of energy dependence are discussed after the analyses of all three energies are completed. The results presented in this section have not been obtained using weighting method described in section 4.2.
We begin with a discussion of the $^{40}\text{Ca}$ data as we have the most complete data set at this energy. The 500 MeV data set extends to quite large angles and one must consider an appropriate procedure for treating the data during the search. One expects the effects of channel coupling to be more pronounced at larger angles, thus, an appropriate diagonal optical potential would be one determined by fitting the data over a restricted angular region. In this case the maximum angle included the elastic data set when the elastic potential is determined, was $30^\circ$ in the c. m. system. When the search includes both elastic and inelastic data all of the inelastic data set is included but the elastic data is cut at $30^\circ$. Thus, we predict the large angle elastic observables as well as the effects channel coupling on these observables. One caveat is required, in any such calculation there is model dependence. The apparent importance of channel coupling is affected by the precise form of the optical potential.

As was outlined in the section on general search procedures, we try first a two parameter search on $\beta_5$ and $\beta_V$ treating each inelastic state independently and then proceed to a 6 parameter search, again treating each state independently. For the 500 MeV data we have also considered a 14 parameter search, in which one varies the geometrical parameters as well as the strengths.

The results of these searches are given in Tables 4.1 and 4.2 and the calculated observables are compared with experiment in Figs. 4.3 and 4.4.
Table 4.1. The changes in $\beta$ and $\chi^2$ for the 2 and 6 parameter searches for $^{40}\text{Ca}$ at 500 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$\chi^2/N(N^*)$</th>
<th>$\chi^2_{incl}/N(N)$</th>
<th>$\beta_s$</th>
<th>$\beta_V$</th>
<th>potential strengths (MeV)</th>
<th>$\sigma_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>El. only</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^2p$ : $CS$</td>
<td>6.9(154)</td>
<td></td>
<td></td>
<td></td>
<td>-302.4</td>
<td>551</td>
</tr>
<tr>
<td>$Ay$</td>
<td>17.9(134)</td>
<td></td>
<td></td>
<td></td>
<td>118.7</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>2.3(32)</td>
<td></td>
<td></td>
<td></td>
<td>198.3</td>
<td></td>
</tr>
<tr>
<td>$3^-$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-117.9</td>
<td></td>
</tr>
<tr>
<td>$2p : CS$</td>
<td>16.6</td>
<td>42.8(68)</td>
<td>.353</td>
<td>.369</td>
<td></td>
<td>549</td>
</tr>
<tr>
<td>$Ay$</td>
<td>4.3</td>
<td>19.2(67)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>3.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6p : CS$</td>
<td>12.3</td>
<td>42.1</td>
<td>.343</td>
<td>.361</td>
<td>-304.2</td>
<td>550</td>
</tr>
<tr>
<td>$Ay$</td>
<td>3.1</td>
<td>20.1</td>
<td></td>
<td></td>
<td>118.8</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td>200.5</td>
<td></td>
</tr>
<tr>
<td>$2^+$</td>
<td>$2p : CS$</td>
<td>7.4</td>
<td>16.1(42)</td>
<td>.111</td>
<td>.112</td>
<td>551</td>
</tr>
<tr>
<td>$Ay$</td>
<td>14.2</td>
<td>2.6(41)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6p : CS$</td>
<td>6.9</td>
<td>16.4</td>
<td>.104</td>
<td>.107</td>
<td>-300.7</td>
<td>549</td>
</tr>
<tr>
<td>$Ay$</td>
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<td>1.2</td>
<td></td>
<td></td>
<td>103.2</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>5.9</td>
<td></td>
<td></td>
<td></td>
<td>198.7</td>
<td></td>
</tr>
<tr>
<td>$5^-$</td>
<td>$2p : CS$</td>
<td>8.0</td>
<td>294(70)</td>
<td>.172</td>
<td>.197</td>
<td>551</td>
</tr>
<tr>
<td>$Ay$</td>
<td>14.6</td>
<td>60.4(68)</td>
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</tr>
<tr>
<td>$Q$</td>
<td>2.3</td>
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<td></td>
</tr>
<tr>
<td>$6p : CS$</td>
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<td>193</td>
<td>.165</td>
<td>.191</td>
<td>-316.2</td>
<td>543</td>
</tr>
<tr>
<td>$Ay$</td>
<td>6.6</td>
<td>56.9</td>
<td></td>
<td></td>
<td>99.5</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>8.8</td>
<td></td>
<td></td>
<td></td>
<td>210.4</td>
<td></td>
</tr>
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</table>

*: $N$ is the number of the data points.
Table 4.2. The $p+^{40}Ca$ 500 MeV potential parameters for the elastic fit and the 14 parameter searches with the $2^+$, $3^-$ and $5^-$ separately included.

<table>
<thead>
<tr>
<th></th>
<th>Elastic</th>
<th>$2^+$ cpd.</th>
<th>$3^-$ cpd.</th>
<th>$5^-$ cpd.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SR$ (MeV)</td>
<td>-302.4</td>
<td>-306.9</td>
<td>-297.6</td>
<td>-303.1</td>
</tr>
<tr>
<td>$R_{SR}$ (fm)</td>
<td>1.029</td>
<td>1.027</td>
<td>1.052</td>
<td>1.036</td>
</tr>
<tr>
<td>$a_{SR}$ (fm)</td>
<td>.736</td>
<td>.710</td>
<td>.690</td>
<td>.699</td>
</tr>
<tr>
<td>$SI$ (MeV)</td>
<td>118.7</td>
<td>134.5</td>
<td>56.9</td>
<td>55.4</td>
</tr>
<tr>
<td>$R_{SI}$ (fm)</td>
<td>1.065</td>
<td>1.029</td>
<td>1.112</td>
<td>1.046</td>
</tr>
<tr>
<td>$a_{SI}$ (fm)</td>
<td>.487</td>
<td>.559</td>
<td>.459</td>
<td>.531</td>
</tr>
<tr>
<td>$VR$ (MeV)</td>
<td>198.3</td>
<td>201.8</td>
<td>201.0</td>
<td>207.5</td>
</tr>
<tr>
<td>$R_{VR}$ (fm)</td>
<td>1.024</td>
<td>1.026</td>
<td>1.048</td>
<td>1.028</td>
</tr>
<tr>
<td>$a_{VR}$ (fm)</td>
<td>.722</td>
<td>.687</td>
<td>.664</td>
<td>.667</td>
</tr>
<tr>
<td>$VI$ (MeV)</td>
<td>-117.9</td>
<td>-128.0</td>
<td>-79.0</td>
<td>-84.1</td>
</tr>
<tr>
<td>$R_{VI}$ (fm)</td>
<td>1.063</td>
<td>1.038</td>
<td>1.084</td>
<td>1.034</td>
</tr>
<tr>
<td>$a_{VI}$ (fm)</td>
<td>.507</td>
<td>.560</td>
<td>.509</td>
<td>.560</td>
</tr>
<tr>
<td>$\beta_S$</td>
<td>.101</td>
<td>.345</td>
<td>.179</td>
<td></td>
</tr>
<tr>
<td>$\beta_V$</td>
<td>.105</td>
<td>.364</td>
<td>.205</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2_{el,CS}/N^*(N)$</th>
<th>$\chi^2_{el,A_1}/N(N)$</th>
<th>$2.3(32)$</th>
<th>$\chi^2_{el,Q}/N(N)$</th>
<th>$\chi^2_{mel,CS}/N(N)$</th>
<th>$\chi^2_{mel,A_2}/N(N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.9(154)</td>
<td>17.9(134)</td>
<td>2.3(32)</td>
<td>9.7(68)</td>
<td>70.5(42)</td>
<td>169(70)</td>
</tr>
<tr>
<td></td>
<td>8.3</td>
<td>3.5</td>
<td>4.1</td>
<td>7.8</td>
<td>18.2(41)</td>
<td>41(68)</td>
</tr>
</tbody>
</table>

*: N is the number of the data points.
Figure 4.3:

Elastic $p+{^{40}Ca}$ observables at 500 MeV along with data. Case 1 (dotted line) has no channel coupling, case 2 (dashed line) contains coupling to $3^-$ state with 2 parameter $\beta$'s, and case 3 (solid line), the 6 parameter search.
Figure 4.4:

Inelastic observables for p+\(^{40}\)Ca at 500 MeV. Solid and dashed lines are defined as in Fig. 4.3.
The quality of the fits is quite good for every case except for the $5^- A_y$ observable, and even in this case the agreement is reasonably good out to 20 degrees. A number of procedures were tried in order to improve the agreement with this data, including weighting the observables but the fit was essentially unchanged. From Fig. 4.4 it seems likely that the geometry of the transition potential is probably incorrect as is signaled by the shifted dip structure. Changing this geometry without changing the direct potential is not consistent with the simple collective model. We agree with the authors of Ref. 21 that this state is not strongly collective and that a more general model would be required to better describe it. Let us now return to consideration of the results in more detail.

The 2 and 6 parameters searches, including the values for the strengths of the optical potentials given in the order scalar, real and imaginary and vector real and imaginary, are given in Table 4.1 and the results for the 14 parameter search including the values of all optical model parameters, are given in Table 4.2. There are a number of observation that can be made regarding the results presented. First, it is very interesting to note that the inclusion of the inelastic $A_y$ observable succeeds in reducing the chi-square for the elastic $A_y$ by almost a factor of 4 in the case of the $3^-$ state, and by factors of 2 or more in the other states. One can conclude from this that the spin observables
for the excited states contain information and influence the phenomenology in a positive manner. In comparing the 2 parameter and 6 parameter fits one notices that there is little change in the extracted $\beta$'s. This is true for each state (the maximum change is 6.3\%) even though the total $\chi^2$, as well as the $\chi^2$ for both elastic and inelastic observables considered separately, is lowered, in some cases by a quite significant amount. We further note that the changes in the strengths produced in the 6 parameter search is not large, the maximum change is less than 17\% in the imaginary scalar strength. Turning now to the 14 parameter fit we see again a reduction in $\chi^2$, as one would expect when the number of parameters is increased, again the reduction is most pronounced for the elastic $A_y$ and the inelastic $A_y$ fits are also improved a little. The case where one would expect the most change in the parameters as the fit to the data is the worst is the 5$^-$ state. In this case we observe rather large change in imaginary potential strength parameter as given in Table 4.2, even though the change in geometry parameters is not substantial. The fit to the 5$^-$ state has not been improved very much. For the 3$^-$ state, we find a large reduction in imaginary potential strengths, however, the extracted $\beta$'s are changed only slightly from the values obtained in the 2 and 6 parameter searches. For the 2$^+$ state, the changes between the results of 6 parameter and 14 parameter search are also small. The maximum difference in geometry parameters is less than 15
% in the diffusiveness of the imaginary scalar. A comparison of the scalar and vector potentials determined from the 6 and 14 parameter searches is given in Fig. 4.5 showing that how little difference exhibits between the potentials. But more importantly the values of the extracted $\beta$’s are almost unchanged. This implies that it is not necessary to do more than a 6 parameter search to obtain results at the 5% level, for this target. We assume that this is in fact true for the other targets considered in this chapter and we will not further consider 14 parameter searches.

Table 4.3 gives the deformation parameters for the inelastic states for both the 2 and 6 parameter searches at 362 MeV, 500 MeV and 800 MeV. Table 4.4 gives the optical model parameters for 362 MeV and 800 MeV. In all three cases the $3^-$ state has the largest $\beta$’s. This is one of the characteristics of spherically symmetric nuclei,27 while the $2^+$ state is generally the strongest state for deformed nuclei. We note that the values of the $\beta$’s at the other energies are within 16% of their values at 500 MeV so that while there is energy dependence, it is not strong. We show in Fig. 4.6 a comparison of the results of this analysis with the other analysis of the same data, and with the corresponding electromagnetic transition discussed in Appendix C. The deformation parameters from these analysis and references for them are given in Table 4.5. In Table 4.6 we give a comparison with the results of analyses
Figure 4.5:

The changes in the potentials by geometry search. The solid lines represent before geometry search and the dashed lines represent after geometry search.
Table 4.3. The deformation parameters in 2p and 6p search for $^{40}\text{Ca}$ at 362, 500 and 800 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>362</th>
<th>500</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_s$</td>
<td>$\beta_v$</td>
<td>$\beta_s$</td>
</tr>
<tr>
<td>$3^-$</td>
<td>2p</td>
<td>.343</td>
<td>.347</td>
</tr>
<tr>
<td></td>
<td>6p</td>
<td>.340</td>
<td>.344</td>
</tr>
<tr>
<td>$2^+$</td>
<td>2p</td>
<td>.142</td>
<td>.140</td>
</tr>
<tr>
<td></td>
<td>6p</td>
<td>.148</td>
<td>.135</td>
</tr>
<tr>
<td>$5^-$</td>
<td>2p</td>
<td>.196</td>
<td>.207</td>
</tr>
</tbody>
</table>
Table 4.4. The optical potential parameters in 6 p searches for $^{40}Ca$ at 500 MeV when the $3^-$ state is included.

<table>
<thead>
<tr>
<th>case</th>
<th>362 (MeV)</th>
<th>800 (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic SR only</td>
<td>-419.9</td>
<td>-284.1</td>
</tr>
<tr>
<td>R</td>
<td>.999</td>
<td>1.03</td>
</tr>
<tr>
<td>a</td>
<td>.668</td>
<td>.690</td>
</tr>
<tr>
<td>SI</td>
<td>99.0</td>
<td>114.5</td>
</tr>
<tr>
<td>R</td>
<td>1.12</td>
<td>.998</td>
</tr>
<tr>
<td>a</td>
<td>.538</td>
<td>.749</td>
</tr>
<tr>
<td>VR</td>
<td>294.6</td>
<td>165.6</td>
</tr>
<tr>
<td>R</td>
<td>1.01</td>
<td>1.04</td>
</tr>
<tr>
<td>a</td>
<td>.641</td>
<td>.647</td>
</tr>
<tr>
<td>VI</td>
<td>-97.0</td>
<td>-115.4</td>
</tr>
<tr>
<td>R</td>
<td>1.12</td>
<td>.991</td>
</tr>
<tr>
<td>a</td>
<td>.552</td>
<td>.665</td>
</tr>
</tbody>
</table>

$3^-_{cpd.}$ (6p)  
-431.1 | -280.4
95.0 | 113.5
302.2 | 163.5
-94.4 | -116.0
Figure 4.6:
Comparison of 6 parameter search \( \beta \)'s with the results of the other calculations.
of electron, neutron and alpha particle inelastic scattering. It is clear that our results are reasonably consistent with the work of others, and, as we produce better fits to the inelastic and elastic observables, represent an improvement over previous work. We note that when comparing with nonrelativistic analyses, it is, considering the second order Dirac equation, approximately correct to simply average $E/m$ times $\beta_S$ and $\beta_V$ to compare with $\beta_{NR}$, which is the deformation parameter in nonrelativistic calculation.

The quality of the fits to the inelastic states for 362 MeV, 500 MeV and 800 MeV are shown in Fig. 4.7, Fig. 4.4, and Fig. 4.8, respectively. For 500 MeV and 800 MeV, the $3^-$ and $2^+$ observables are quite well reproduced; however, $5^- \ A_y$ data are not. We believe that this is because the collective nature of the $5^-$ state is not strong enough to be described by this collective model calculation. For the 362 MeV data as shown in Fig. 4.7 the calculation shows more pronounced structure than the experimental data for the $2^+$ state. A similar discrepancy for the $2^+$ state has been found in the nonrelativistic collective model calculation. Just as at 500 MeV and 800 MeV the same kind of discrepancy occurs for the $5^- \ A_y$ observable. As mentioned above we attribute this to the lack of collectivity in this state.

We have also tried to fit the existing inelastic data for the higher excited states, the $1^-$ state at 5.90 MeV and the second $3^-_2$ state at 6.29 MeV. It was
Table 4.5. Deformation parameters for $^{40}\text{Ca}$ at 500 MeV.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E$ (MeV)</th>
<th>$\beta_S$</th>
<th>$\beta_V$</th>
<th>$\beta_{NR}$</th>
<th>$\beta_{RDWBA}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3$^-$</td>
<td>3.74</td>
<td>.343</td>
<td>.361</td>
<td>.41$^a$, .372$^b$</td>
<td>0.39$^c$</td>
</tr>
<tr>
<td>2$^+$</td>
<td>3.90</td>
<td>.104</td>
<td>.107</td>
<td>.14$^a$</td>
<td>.15$^c$</td>
</tr>
<tr>
<td>5$^-$</td>
<td>4.49</td>
<td>.165</td>
<td>.191</td>
<td>.25$^a$, .17$^b$</td>
<td></td>
</tr>
</tbody>
</table>


RDWBA means the relativistic DWBA calculation.

Table 4.6. Comparison of $\beta$'s with other probes for $^{40}\text{Ca}$ at 500 MeV.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E$ (MeV)</th>
<th>$\beta_S$</th>
<th>$\beta_V$</th>
<th>$\beta_{ao}$</th>
<th>$\beta_{mn}$</th>
<th>$B(E1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3$^-$</td>
<td>3.74</td>
<td>.343</td>
<td>.361</td>
<td>.28</td>
<td>.359</td>
<td>0.40</td>
</tr>
<tr>
<td>2$^+$</td>
<td>3.90</td>
<td>.104</td>
<td>.107</td>
<td>.096</td>
<td>.14</td>
<td></td>
</tr>
<tr>
<td>5$^-$</td>
<td>4.49</td>
<td>.165</td>
<td>.191</td>
<td>.12</td>
<td>.260</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.7:

Cross sections and analyzing powers of the inelastic states for $^{40}\text{Ca}$ at 362 MeV. The solid lines represent the collective model calculation with phenomenological diagonal optical potentials.
Figure 4.8:

Cross sections and analyzing powers of the inelastic states for $^{40}$Ca at 800 MeV. The solid lines represent the collective model calculation with phenomenological diagonal optical potentials.
not possible to get an adequate fit to these high lying states with our simple collective model, even though we tried several different initial values for the parameters. Hence, like the $5^-$ state, we assume that these are not collective states and that it would be necessary to use a different model to explain them.

As has been discussed by several groups,$^{28,29}$ coupled channel effects are particularly important at large angles. We found that the inclusion of channel coupling reduces the large angle $p + ^{40}Ca$ cross sections at 500 MeV, producing better agreement with the data.$^{30}$ This is also true for $p + ^{40}Ca$ at 800 MeV. For $p + ^{40}Ca$ at 362 MeV, we included all the elastic data out to around $67^0$ in the calculation when we search for the potential parameters. As a result, we have a very good fit to the elastic data even at large angles. Hence inclusion of channel coupling reduced the calculated cross sections further down from the experimental data. In Fig. 4.9 and 4.10, we show the channel coupling effects at large angles for those two cases for each excited state. When $3^-$ state is included the effects of channel coupling are largest, and inclusion of either the $2^+$ or the $5^-$ state does not appreciably alter the result of elastic calculation. However, as discussed above, these results depend upon the form of the potentials. It has been shown that it was possible to fit even the large angle data using more general form for the potentials$^{31}$ without channel coupling. Although we point out that a consistent phenomenology which attempts to
Figure 4.9:

Comparison of the CC effects for $^{40}$Ca at 362 MeV at large angles when each excited state is coupled. CC effects are the difference between solid line (with coupling) and dashed line (without coupling).
Figure 4.10:

Comparison of the CC effects $^{40}$Ca at 800 MeV at large angles when each excited state is coupled. The dashed lines are from elastic calculation (without coupling).
treat such large momentum transfer data ($q^2$ larger than $3 \text{ fm}^{-1}$) should be done in a coupled channel framework.

Next we turn to a discussion of the transition potentials for the three energies considered. Of course, at a given energy they differ only in strength from state to state. In Fig. 4.11 we show the results from the analysis of the $3^-$ states using a 6 parameter searches. It is interesting to note that the real scalar and vector transition potentials vary smoothly with energy, exhibiting almost no change in shape, while the imaginary potentials have rather different geometries at each energy. In the search the real and imaginary $\beta$'s are the same so this behaviour may be a result of this constraint although it is more likely due to the energy dependence of the imaginary geometries.

To date almost all analyses of this inelastic proton-nucleus inelastic scattering data have employed the DWBA approximation. Because of this, it is of interest to compare a CC calculation with a DWBA calculation, using the same deformation parameters. In general, about same quality fit to the inelastic observables are obtained; however, the CC calculations are able to produce a superior fit to the elastic data, at large angles. Figure 4.12 shows one example of this comparison for $^{40}\text{Ca}$ at 500 MeV and the effects of the channel coupling are clear.
The energy dependence of the transition potentials for $3^-$ state of $^{40}$Ca. The solid lines are from 362 MeV, the dashed lines are from 500 MeV and the dotted lines are from 800 MeV calculation.
Figure 4.12:

Comparison of CC calculation (solid lines) with DWBA calculation (dashed lines) in the $3^-$ coupled case for $^{40}\text{Ca}$ at 500 MeV, with the same $\beta$'s. Dotted lines are from elastic only calculation.
Finally, we investigated the effect of deforming the Coulomb potential. We consider the $3^-$ state for the 800 MeV data. We found no substantial change in the fit, the largest change in $\chi^2$ (15.6 %) occurred in the inelastic $3^- A_y$ data. The coulomb deformation parameter was taken to be the same as that of the nuclear vector potential. The results for the $3^-$ state with and without Coulomb deformation, shown in Fig. 4.13, differ slightly at angles beyond $35^\circ$. Because of this result we have not included the deformed Coulomb in our calculations.

The RIA Based Calculation

Here the diagonal scalar and vector optical potentials are obtained using the RIA as described in Ref. 11. We employed two different models to obtain the transition potentials, as described in the chapter 3. In case 1 the transition potentials are proportional to the derivatives of the RIA potentials, in case 2 the derivatives of the scalar and vector densities are folded with the invariant NN amplitudes. The two free parameters for each state; the real and imaginary $\delta$'s were taken equal in these calculations. For both models the deformation lengths were adjusted to give good fits to the inelastic observables. The results of using these two procedures produced essentially the same $\delta$'s as is shown in Table 4.7 for the 500 and 800 MeV analyses, the analysis of the 362 MeV
Figure 4.13:

The changes in the $3^-$ state of $^{40}\text{Ca}$ at 800 MeV by including the Coulomb deformation. The solid and dashed lines are with and without Coulomb deformation, respectively.
data will be discussed separately. The deformation lengths determined by fitting the data are in reasonable agreement with previous work\textsuperscript{21,32} as is shown in Table 4.8. The calculated cross sections and analyzing powers for these three states using the transition potentials of case 1 are compared with the experimental data in Figs. 4.14 and 4.15. Naturally the phenomenological approach is capable of producing somewhat better fits, but these RIA based calculations are quite impressive.

Next we investigate the effects of channel coupling for the RIA based calculation. In this case, as discussed earlier, the direct potential is not changed during the search. The effects of channel coupling are again most pronounced for the $3^-$ state. Including the $2^+$ and $5^-$ states does not appreciably alter the results. Figure 4.16 shows the result of coupling the $3^-$ state on the 500 MeV elastic observables. The impact of the channel coupling on the elastic observables, especially at angles beyond $35^\circ$ is pronounced, and causes a marked improvement in the agreement with experiment. This is in disagreement with the results of Ref. 19 which were based on a non-relativistic coupled channel approach using the Schrödinger equation where no spin-orbit coupling was included.

For 362 MeV the agreement with the elastic observables when RIA potentials are used is not very good. The calculated elastic cross section
Table 4.7. The deformation lengths for case 1 and case 2 RIA calculation for $^{40}$Ca at 500 MeV and 800 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$E_x$ (MeV)</th>
<th>500(MeV)</th>
<th>800(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\delta_S$</td>
<td>$\delta_V$</td>
</tr>
<tr>
<td>$3^-$</td>
<td>3.74</td>
<td>1.210</td>
<td>1.261</td>
</tr>
<tr>
<td></td>
<td>case 1</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>1.173</td>
<td>1.231</td>
</tr>
<tr>
<td>$2^+$</td>
<td>3.90</td>
<td>.375</td>
<td>.383</td>
</tr>
<tr>
<td></td>
<td>case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>.362</td>
<td>.373</td>
</tr>
<tr>
<td>$5^-$</td>
<td>4.49</td>
<td>.603</td>
<td>.683</td>
</tr>
<tr>
<td></td>
<td>case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>.583</td>
<td>.666</td>
</tr>
</tbody>
</table>

Table 4.8. Deformation lengths for $^{40}$Ca at 500 MeV.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E$ (MeV)</th>
<th>$\delta_S$ (fm)</th>
<th>$\delta_V$ (fm)</th>
<th>$\delta_{NR}$ (fm)</th>
<th>$B(El)^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^-$</td>
<td>3.74</td>
<td>1.21</td>
<td>1.26</td>
<td>$1.41^{a,b,c}$</td>
<td>1.40</td>
</tr>
<tr>
<td>$2^+$</td>
<td>3.90</td>
<td>.375</td>
<td>.383</td>
<td>$.53^a$</td>
<td>.48</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>$.52^b, .49^c</td>
<td>.74</td>
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<tr>
<td>$5^-$</td>
<td>4.49</td>
<td>.603</td>
<td>.683</td>
<td>$.84^a, .86^c</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.14:

Cross sections and analyzing powers of the inelastic states for $^{40}$Ca at 500 MeV. The solid lines represent the collective model calculation with RIA diagonal optical potentials.
Figure 4.15:

Cross sections and analyzing powers of the inelastic states for $^{40}\text{Ca}$ at 800 MeV.

The solid lines represent the collective model calculation with RIA diagonal optical potentials.
Figure 4.16:

The elastic cross section for $^{40}\text{Ca}$ at 500 MeV. The solid and dashed lines represent with and without coupling to the $3^{-}$ state, in RIA, case 1 calculation.
overestimates the data at angles larger than 10°. As discussed in section 4.2 this is not unexpected for energies below 400 MeV. In order to improve the elastic fit we adjusted the four potential strengths and obtained much better agreement with elastic data, as is shown in Fig. 4.17. Of course, this procedure is not consistent. However, it is well known that the medium and exchange effects, which are not included in these RIA potentials, are not negligible below 400 MeV. The changes in RIA potential strengths are given in Table 4.9, along with the δs for each state. Next, we used RIA potentials which included both medium and exchange effects in an approximate manner. The comparison between the RIA based phenomenological results (those with adjusted potential strengths), the unadjusted RIA potentials and the results from the RIA potentials of Ref. 33 (RIA-MH) is given in Fig. 4.17. The adjusted RIA potentials and the RIA-MH potentials are in rather close agreement, thus, we can conclude that medium and exchange effects are mocked up by scaling of the RIA. The results for the inelastic observables, with and without medium and exchange effects, are shown in Fig. 4.18 for 3 state, and the extracted δ's are given in Table 4.9. We observe the agreement with inelastic data is also improved. We find the 2+ and 5- Aγ data are not well reproduced by any of the three types of calculations, just as in phenomenological case discussed in the last section.
Figure 4.17:

The elastic observables for $^{40}$Ca at 362 MeV, at large angles. The dashed and dotted lines are with adjusted RIA potentials and original RIA potentials respectively. Solid lines use the RIA-MH potentials.
Table 4.9. The deformation lengths for various searches in RIA calculation for $^{40}Ca$ at 362 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$E_x$ (MeV)</th>
<th>$\delta_s$ (fm)</th>
<th>$\delta_v$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3$^-$</td>
<td>3.74</td>
<td>a: 1.12</td>
<td>a: 1.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b: 1.04</td>
<td>b: 1.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c: 1.24</td>
<td>c: 1.23</td>
</tr>
<tr>
<td>2$^+$</td>
<td>3.90</td>
<td>a: 1.428</td>
<td>a: 1.441</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>c: 1.487</td>
<td>c: 1.463</td>
</tr>
<tr>
<td>5$^-$</td>
<td>4.49</td>
<td>a: 1.216</td>
<td>a: 1.315</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b: 1.420</td>
<td>b: 1.485</td>
</tr>
<tr>
<td></td>
<td></td>
<td>c: 1.389</td>
<td>c: 1.490</td>
</tr>
</tbody>
</table>

a; Results of RIA based calculation.
b; Results with adjusted RIA potentials.
c; Results of RIA potential with medium effects, described in the text.
Figure 4.18:

The $3^-$ state for $^{40}\text{Ca}$ at 362 MeV. The solid and dashed lines are RIA based calculation with and without medium effects respectively.
Next we consider the energy dependence of the RIA transition potentials. The results are shown in Fig. 4.19. Just as in the phenomenological cases we see that the real parts vary in a smooth way with energy; however, in this case, for the unaltered RIA, the imaginary parts all seem to be essentially independent of energy. The real RIA transition potentials agree fairly well with the phenomenological ones, except at the origin where shell effects in the input densities are apparent. The RIA potentials are shown in Fig. 4.20. This is not the case for the imaginary potentials. We plan to investigate these effects in more detail. Perhaps removing the constraint on the search on the deformation lengths will produce some insight into this behaviour.

Finally we include a brief discussion of the comparison between the RIA and nonrelativistic KMT\textsuperscript{34} approaches when applied to inelastic scattering. In both cases the simple collective model of Chapter 3 will be used and the deformation lengths will be searched to in an attempt to obtain agreement with the inelastic observables. We will consider only the 500 MeV data and restrict our attention to the 3\textsuperscript{-} state. In both cases two parameters are varied. For the KMT these are the real and imaginary central potential deformation lengths. We are unable to obtain agreement with the inelastic, or elastic spin observables when the KMT potentials are used. This is expected because the deformation of spin-orbit potential is not included in the KMT calculation.
Figure 4.19:

The energy dependence of the RIA transition potentials for $3^-$ state of $^{40}\text{Ca}$.
The solid lines are from 362 MeV, the dashed lines are from 500 MeV and the
dotted lines are from 800 MeV calculation.
Figure 4.20:

The RIA potentials for $p+^{40}Ca$. The solid lines are for 362, the dashed lines are for 500 and the dotted lines are for 800 MeV calculation.
However, we believe that when the deformation of the spin-orbit potential is included, which we plan to do, we expect the agreement with the spin data will not be improved drastically because the elastic spin observables are poorly represented. As is shown in Fig. 4.21 the RIA is capable of producing agreement with the data for the 3- state while the KMT is not.

4.3.2 Results for $^{208}Pb$

Here we discuss the results of the analysis of inelastic $p + ^{208}Pb$ at incident proton energies 500 MeV and 800 MeV. The 500 MeV elastic and inelastic data are from Refs. 15 and 22, and the 800 MeV data are from Refs. 35, 36 and 37. The first seven low lying excited states are considered. They are the 3- with excitation energy 2.61 MeV, the 5- at 3.20 MeV, the 5+ at 3.73 MeV, the 2+ at 4.08 MeV, the 4+ at 4.32 MeV, the 6+ at 4.42 MeV and the 8+ at 4.61 MeV. While both inelastic cross section and $A_{\gamma}$ data exist at 500 MeV, only inelastic cross section data are available at 800 MeV. The results of the phenomenological analysis are discussed first, and the results of an analysis based on optical potentials calculated using RIA are given next.
Figure 4.21: Comparison of the RIA calculation for $p+^{40}Ca$ with KMT calculation. The solid lines are from RIA, the dashed lines are from KMT calculation.
Phenomenological analysis

Here the diagonal parts of optical potentials are obtained by fitting elastic data. We begin with a discussion of 800 MeV data as the large angle data (out to 40°) exists at this energy. In this case we used all of the elastic data, including the large angle data, in the search. This procedure is different from the procedure taken for the large angle data of $^{40}$Ca at 500 MeV. However, after running several test cases, we found that the inclusion of the large angle data did not alter the results. The results presented in this phenomenological analysis have not been obtained using the weighting method described in section 4.2.

As discussed in general search procedure section, we search the elastic potential parameters as well as the deformation parameters. Here we begin with the 6 parameter search, treating each inelastic state independently. The results of these searches at 500 MeV and 800 MeV are given in Table 4.10 and the calculated observables are compared with experimental data in Figs. 4.22, 4.23 and 4.24.

In table 4.11, we also give the results from other analyses at 800 MeV. Again, our results are consistent with both the nonrelativistic calculation and a relativistic DWBA calculation. As shown in Figs. 4.22 through 4.24, all the inelastic states are very well reproduced, except at very large angles. At 500 MeV, Barlett et al. have obtained very good fits to the lowest $3^-$ and
Table 4.10. The deformation parameters in 6p search for $^{208}$Pb at 500 and 800 MeV.

<table>
<thead>
<tr>
<th>State</th>
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<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_s$</td>
<td>$\beta_v$</td>
</tr>
<tr>
<td>$3^-$</td>
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<td>.114</td>
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<tr>
<td>$5^-$</td>
<td>.049</td>
<td>.050</td>
</tr>
<tr>
<td>$5^-_2$</td>
<td>.037</td>
<td>.039</td>
</tr>
<tr>
<td>$2^+$</td>
<td>.054</td>
<td>.059</td>
</tr>
<tr>
<td>$4^+$</td>
<td>.074</td>
<td>.078</td>
</tr>
<tr>
<td>$6^+$</td>
<td>.065</td>
<td>.069</td>
</tr>
<tr>
<td>$8^+$</td>
<td>.043</td>
<td>.047</td>
</tr>
</tbody>
</table>
Figure 4.22:

Inelastic cross sections for $^{208}Pb$ at 500 MeV. Solid lines are from the phenomenological 6 p calculation.
Figure 4.23:

Same as Fig. 4.22 for inelastic $A_y$. 
Figure 4.24:

Inelastic cross sections for $^{208}$Pb at 800 MeV. Solid lines are from the phenomenological $6p$ calculation.
Table 4.11. Deformation parameters for $^{208}$Pb at 800 MeV.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E$ (MeV)</th>
<th>$\beta_s$</th>
<th>$\beta_v$</th>
<th>$\beta_{NR}$</th>
<th>$B(EL)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^-$</td>
<td>2.61</td>
<td>.128</td>
<td>.126</td>
<td>.133</td>
<td>.128</td>
</tr>
<tr>
<td>$5^-$</td>
<td>3.20</td>
<td>.0450</td>
<td>.0510</td>
<td>.065</td>
<td>.064</td>
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<tr>
<td>$5^-$</td>
<td>3.73</td>
<td>.0263</td>
<td>.0338</td>
<td>.046</td>
<td>.047</td>
</tr>
<tr>
<td>$2^+$</td>
<td>4.08</td>
<td>.0528</td>
<td>.0608</td>
<td>.075</td>
<td>.066</td>
</tr>
<tr>
<td>$4^+$</td>
<td>4.32</td>
<td>.0755</td>
<td>.0805</td>
<td>.088</td>
<td>.088</td>
</tr>
<tr>
<td>$6^+$</td>
<td>4.42</td>
<td>.0689</td>
<td>.0731</td>
<td>.083</td>
<td>.10</td>
</tr>
<tr>
<td>$8^+$</td>
<td>4.61</td>
<td>.0476</td>
<td>.0503</td>
<td>.058</td>
<td>.048</td>
</tr>
</tbody>
</table>

5\textsuperscript{−} states using a DWBA model with a nonrelativistic effective interaction. Our results give comparable agreement with the inelastic cross section data and much better agreement with the inelastic $A_y$ data.

Next we discuss the effects of channel coupling on the large angle calculations at 800 MeV. The large angle data have been analyzed within a nonrelativistic framework by two groups\textsuperscript{28,29}. Amado and Sparrow\textsuperscript{28} found in their eikonal approximation calculations that coupled channel effects were large and their inclusion improves the agreement with large angle elastic cross section data. However, Ray and Hoffmann\textsuperscript{29} using either Schrödinger phenomenology or KMT-IA,\textsuperscript{34} concluded that the effects were much smaller than those reported by Amado and Sparrow. Here we give the results of the relativistic calculations based on Dirac phenomenology. In Figs. 4.25 which show the results of the phenomenological (6 parameter search) calculation, the solid lines are with coupled channel effects and the dotted lines are without coupling effects. Only the lowest 3\textsuperscript{−} state, which is the strongest coupled state, is included. We found our results were closer to the results of Amado and Sparrow, and that the CC effects caused the calculation to agree much better with the large angle cross section data. The results for the elastic spin observables at 800 MeV are given in Figs. 4.26. The transition potentials at 500 MeV and 800 MeV for the 3\textsuperscript{−} state are shown in Fig. 4.27. We observe some slight
Figure 4.25:
The elastic observables for $^{208}$Pb at 800 MeV. The solid and dashed lines are for $3^-$ coupled case and elastic only case in the phenomenological 6 p calculation respectively.
Figure 4.26:

Same as Fig. 4.25 for elastic $A_y$ and $Q$. 
energy dependence in the geometries.

**The RIA based calculation**

Here the diagonal optical potentials are obtained using RIA as was done for $^{40}\text{Ca}$. Again we employ two different models, case 1 and case 2, to obtain the transition potentials. In both cases, the deformation lengths, $\delta_S$ and $\delta_V$ were adjusted to give good fits to the inelastic observables. The two cases give essentially identical results as is shown in Fig. 4.28, for the $3^-$ state. The results of using these two procedures are even closer than those obtained for $^{40}\text{Ca}$. The extracted deformation lengths are given in Table 4.12, and as shown in Table 4.13, the deformation lengths determined by fitting the inelastic data are in reasonable agreement with previous work.

The results for the seven inelastic states at 500 MeV and 800 MeV using the transition potentials of case 1 are compared with inelastic data in Figs. 4.29, 4.30 and 4.31. Most of the inelastic states are remarkably well reproduced, considering that there are only two free parameters. By deforming scalar and vector potentials, the deformation of the 'spin-orbit' potential is automatically included. This is an advantage of relativistic approach, which is especially useful for representing spin data. However, at small angle elastic $A_y$, RIA calculations have much less structure than the experimental data.
Figure 4.27:

The energy dependence of the phenomenological transition potentials for $3^-$ state of $^{208}\text{Pb}$. The solid lines are from 500 MeV, the dashed lines are from 800 MeV calculation.
Figure 4.28:

Comparison of case 1 calculation (solid lines) with case 2 calculation (dashed lines) in the $3^-$ state for $^{208}\text{Pb}$ at 800 MeV.
Table 4.12. The deformation lengths for case 1 and case 2 RIA calculation for $^{208}Pb$ at 800 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$E_x$ (MeV)</th>
<th>Scalar $\delta_S$</th>
<th>Vector $\delta_V$</th>
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<tr>
<td>$3^-$</td>
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<td>.664</td>
<td>.727</td>
</tr>
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<td></td>
<td>case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>.653</td>
<td>.719</td>
</tr>
<tr>
<td>$5^-$</td>
<td>3.20</td>
<td>.178</td>
<td>.262</td>
</tr>
<tr>
<td></td>
<td>case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>.176</td>
<td>.260</td>
</tr>
<tr>
<td>$5_2^-$</td>
<td>3.73</td>
<td>.130</td>
<td>.197</td>
</tr>
<tr>
<td></td>
<td>case 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>.132</td>
<td>.197</td>
</tr>
<tr>
<td>$2^+$</td>
<td>4.08</td>
<td>.395</td>
<td>.449</td>
</tr>
<tr>
<td></td>
<td>case 1</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>case 2</td>
<td>.390</td>
<td>.446</td>
</tr>
<tr>
<td>$4^+$</td>
<td>4.32</td>
<td>.372</td>
<td>.464</td>
</tr>
<tr>
<td></td>
<td>case 1</td>
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<tr>
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<td>case 2</td>
<td>.366</td>
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<td></td>
<td>case 2</td>
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<td>$8^+$</td>
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<td></td>
<td>case 2</td>
<td>.169</td>
<td>.246</td>
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</table>
Table 4.13. Deformation lengths for $^{208}$Pb at 800 MeV.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E$ (MeV)</th>
<th>$\delta_\Sigma$</th>
<th>$\delta_\nu$</th>
<th>$\delta_{NR}$</th>
<th>$B(E\lambda)^a$</th>
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<tr>
<td>3$^-$</td>
<td>2.61</td>
<td>0.664</td>
<td>0.727</td>
<td>0.825</td>
<td>0.798</td>
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<tr>
<td>5$^-$</td>
<td>3.20</td>
<td>0.178</td>
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<td>0.401</td>
<td>0.395</td>
</tr>
<tr>
<td>5$^-$</td>
<td>3.73</td>
<td>0.130</td>
<td>0.197</td>
<td>0.283</td>
<td>0.290</td>
</tr>
<tr>
<td>2$^+$</td>
<td>4.08</td>
<td>0.394</td>
<td>0.449</td>
<td>0.466</td>
<td>0.409</td>
</tr>
<tr>
<td>4$^+$</td>
<td>4.32</td>
<td>0.372</td>
<td>0.464</td>
<td>0.546</td>
<td>0.546</td>
</tr>
<tr>
<td>6$^+$</td>
<td>4.42</td>
<td>0.263</td>
<td>0.365</td>
<td>0.514</td>
<td>0.635</td>
</tr>
<tr>
<td>8$^+$</td>
<td>4.61</td>
<td>0.170</td>
<td>0.247</td>
<td>0.359</td>
<td>0.298</td>
</tr>
</tbody>
</table>

This discrepancy will be discussed in section 4.3.4 regarding the effects of the RIA tensor potentials.

We find that the agreement with the inelastic $A_y$ data at 500 MeV are not as good as the other cases, and the data set has large error bars. In order to improve the fit to the inelastic $A_y$ data, we tried the weighting procedure described in section 4.2. We multiplied the inelastic $A_y \chi^2$ by 10 ($W = 10$ in Eq. 4.26). The factor of 10 is chosen arbitrarily. This procedure resulted in a better fit to the inelastic $A_y$ data, as shown in Fig 4.32 for the $3^-$ state. Of course, the $\chi^2$'s of the other observables are raised, but the other observables are changed very slightly. Table 4.14 gives the changes in the $\delta$'s when this weighting is done and no dramatic changes are observed.

For the 800 MeV case, where only the cross section data exist for inelastic states, we are able to reproduce all the inelastic data, without weighting. Our results appear to agree somewhat better with experiment than more sophisticated nonrelativistic calculations given in Refs. 37 and 38.

Next we discuss the effects of channel coupling in the calculation at 800 MeV. We use the case 1 transition potentials. We find essentially the same results as those obtained in the phenomenological calculation given in the previous subsection. The channel coupling to the lowest $3^-$ state improved the agreement with the large angle cross section data substantially, as shown
Figure 4.29:
Inelastic cross sections for $^{208}$Pb at 500 MeV. Solid lines are from the RIA-case 1 calculation.
Figure 4.30:

Same as Fig. 4.29 for inelastic $A_y$. 
Figure 4.31:
Inelastic cross sections for $^{208}$Pb at 800 MeV. Solid lines are from the RIA-case 1 calculation.
Figure 4.32:

Inelastic $A_y$ for $^{208}\text{Pb}$ at 500 MeV. Solid and dashed lines are from with and without weighting in the RIA- case 1 calculation.
Table 4.14. The changes in $\delta$'s by weighting procedure ($W$) for $^{208}\text{Pb}$ at 500 MeV, in RIA calculation.

<table>
<thead>
<tr>
<th>State</th>
<th>$E_x$ (MeV)</th>
<th>$\delta_S$</th>
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<tr>
<td>$3^-$</td>
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<td>$W$</td>
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<td>.254</td>
<td>.275</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>.274</td>
<td>.290</td>
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<td>$5^+_2$</td>
<td>3.71</td>
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</tr>
<tr>
<td></td>
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<td>.356</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>.371</td>
<td>.395</td>
</tr>
<tr>
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<td>4.32</td>
<td>.372</td>
<td>.420</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
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<td>.480</td>
</tr>
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<td>.398</td>
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<tr>
<td></td>
<td>$W$</td>
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<td>.412</td>
</tr>
<tr>
<td>$8^+$</td>
<td>4.61</td>
<td>.245</td>
<td>.277</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>.266</td>
<td>.292</td>
</tr>
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</table>
in Fig. 4.33. The results for the spin observables are shown in Fig 4.34. This result is different from the result of the KMT-IA calculation done by Ray and Hoffmann,\(^{15}\) where they found the channel coupling effects were small.

The energy dependence of the transition potentials is shown in Fig. 4.35. We observe a similar energy dependence of real parts as in the transition potentials for \(^{40}\text{Ca}\), that is, the strengths of real transition potentials are reduced smoothly as the incident energy increases at the surface area. Also we find that imaginary parts increase as the incident energy increase from 500 to 800 MeV. Also we note that these RIA potentials are smaller than the phenomenological potentials shown in Fig. 4.27.

### 4.3.3 Results for \(^{48}\text{Ca}\) and \(^{90}\text{Zr}\)

In this subsection the results of 500 MeV and 800 MeV proton inelastic scattering from either \(^{48}\text{Ca}\) or \(^{90}\text{Zr}\) are analyzed. The 500 MeV elastic experimental data for \(^{48}\text{Ca}\) are taken from Ref. 21 and the inelastic data are from 22. The 800 MeV data for \(^{48}\text{Ca}\) are from Ref. 39 and 25. For \(^{90}\text{Zr}\), the 500 MeV data are from Ref. 18 and Ref. 22, the 800 MeV data are from Ref. 37. For \(^{48}\text{Ca}\), the first four low lying excited states, the \(^2+\) at excitation energy 3.83 MeV, the \(^3-\) at 4.51 MeV, the \(^5-\) at 5.73 MeV and the \(^4+\) at 6.34 MeV, are considered at 500 MeV, and first \(^2+, 3-, 3^-\) (at 5.38 MeV) and \(^5-\)
Figure 4.33:

The elastic observables for $^{208}\text{Pb}$ at 800 MeV. The solid and dashed lines are for $3^-$ coupled case and elastic only case in the RIA calculation respectively.
Figure 4.34:

Same as Fig. 4.33 for elastic $A_y$ and $Q$. 

Figure 4.35:
The energy dependence of the RIA transition potentials for $3^{-}$ state of $^{208}$Pb.
The solid lines are from 500 MeV, the dashed lines are from 800 MeV calculation.
are considered at 800 MeV. For $^{90}$Zr, first seven excited states are analyzed. They are the $2^+$ state at 2.19 MeV, the $5^-$ at 2.32 MeV, the $3^-$ at 2.75 MeV, the $4^+$ at 3.08 MeV, the $2^+_2$ at 3.31 MeV, the $6^+$ at 3.45 MeV and the $8^+$ at 3.59 MeV. While both cross section and $A_y$ data are available for the excited states at 500 MeV, only cross section data for the excited states are available at 800 MeV.

The results of the phenomenological calculations are discussed first. Here, only the six parameter searches are done, for the reason discussed in the previous sections. The results of these searches are given in Table 4.15 for $^{48}$Ca, and in Table 4.16 for $^{90}$Zr. The deformation parameters are in reasonable agreement with the results of other analyses. Even though the $2^+$ state has the lowest energy, the $3^-$ state has the largest $\beta$'s as is usual spherically symmetric nuclei. The calculated observables for both $p^+^{48}$Ca and $p^+^{90}$Zr are compared with experimental data in Figs. 4.36 through 4.40. The inelastic states are very well reproduced overall, including inelastic $A_y$ data. However, in our analyses of the $^{48}$Ca data at 500 MeV, we were not able to obtain quality fits to the inelastic $A_y$ data. However, our results are slightly better than the nonrelativistic phenomenological calculation done by Seth et al.\textsuperscript{21} We did the analysis again including the weighting procedure for the inelastic analyzing power. Again a weight factor of 10 was used. We did this
Table 4.15. Deformation parameters for $^{48}$Ca at 500 and 800 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$E$ (MeV)</th>
<th>$500$(MeV)</th>
<th>$800$(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_s$</td>
<td>$\beta_v$</td>
<td>$\beta_s$</td>
</tr>
<tr>
<td>$2^+$</td>
<td>3.83</td>
<td>.104</td>
<td>.118</td>
</tr>
<tr>
<td>$3^-$</td>
<td>4.52</td>
<td>.189</td>
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<td>$3^-$</td>
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<td></td>
</tr>
<tr>
<td>$5^-$</td>
<td>5.73</td>
<td>.102</td>
<td>.110</td>
</tr>
<tr>
<td>$4^+$</td>
<td>6.34</td>
<td>.049</td>
<td>.058</td>
</tr>
</tbody>
</table>

Table 4.16. Deformation parameters for $^{90}Zr$ at 500 MeV and 800 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$E$ (MeV)</th>
<th>500(MeV)</th>
<th>800(MeV)</th>
<th>$B(E^\lambda)$a</th>
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</thead>
<tbody>
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<td>$\beta_v$</td>
<td>$\beta_s$</td>
<td>$\beta_v$</td>
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<td>.085</td>
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<td>.082</td>
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<tr>
<td>$5^-$</td>
<td>2.32</td>
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<td>.078</td>
<td>.061</td>
</tr>
<tr>
<td>$3^-$</td>
<td>2.75</td>
<td>.168</td>
<td>.172</td>
<td>.162</td>
</tr>
<tr>
<td>$4^+$</td>
<td>3.08</td>
<td>.047</td>
<td>.051</td>
<td>.026</td>
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<td>$2_2^+$</td>
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<td>.037</td>
<td>.035</td>
</tr>
<tr>
<td>$6^+$</td>
<td>3.45</td>
<td>.025</td>
<td>.028</td>
<td>.018</td>
</tr>
<tr>
<td>$8^+$</td>
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<td>.0083</td>
</tr>
<tr>
<td>$2_1^+$</td>
<td>3.84</td>
<td></td>
<td></td>
<td>.050</td>
</tr>
</tbody>
</table>

a ; M.M. Gazzaly et al. and references in there.
Figure 4.36:

Cross sections and analyzing powers of the inelastic states for $^{48}\text{Ca}$ at 500 MeV.
The solid lines represent the collective model calculation with phenomenological diagonal optical potentials.
Figure 4.37:

Cross sections of the inelastic states for $^{48}Ca$ at 800 MeV. The solid lines represent the collective model calculation with phenomenological diagonal optical potentials.
Figure 4.38:
Cross sections of the inelastic states for $^{90}$Zr at 500 MeV. The solid lines represent the collective model calculation with phenomenological diagonal optical potentials.
Figure 4.39:

Analyzing powers of the inelastic states for $^{90}$Zr at 500 MeV. The solid lines represent the collective model calculation with phenomenological diagonal optical potentials.
Figure 4.40:

Cross sections of the inelastic states for $^{90}$Zr at 800 MeV. The solid lines represent the collective model calculation with phenomenological diagonal optical potentials.
for both $^{48}Ca$ and $^{90}Zr$. We were able to obtain better fits to the inelastic $A_y$ data as shown in Figs. 4.41 and 4.42. As expected, we find only a small improvement in the $^{90}Zr$ $A_y$ where the agreement was already quite good without weighting. On the other hand, the improvement in the $^{48}Ca$ $A_y$ representation is substantial as shown in Fig. 4.41. The deformation parameters are given in Tables 4.17 and 4.18. Of course, the changes in $\beta$'s are larger for $^{48}Ca$ with average change of 14.0 %, compared to an average change 6.7 % for $^{90}Zr$.

At 800 MeV, only inelastic cross section data are available for both targets. The agreement with experimental data is very good, for almost every excited state, as shown in Figs. 4.37 and 4.40. Our results are remarkably better than the nonrelativistic DWBA calculations done by Adams et al.\textsuperscript{29} for $^{48}Ca$, and done by Gazzaly et al.\textsuperscript{27} for $^{90}Zr$.

In Fig. 4.43, we show the effects of channel coupling at large angles for $^{90}Zr$ at 500 MeV when the strong $3^-$ state is included. Unfortunately, large angle elastic cross section data do not exist for $^{48}Ca$ and $^{90}Zr$, but we present our prediction for the cross section and $A_y$ at large angles for these targets. Over the range of existing elastic data, there is almost no change in the elastic observables when inelastic states were included; however, the effects of channel coupling are apparent beyond 30°.

Finally the energy dependence of transition potentials are examined for
Figure 4.41:

Inelastic observables for $^{48}$Ca at 500 MeV. Solid and dashed lines are from with and without weighting in the phenomenological calculation.
Figure 4.42:

Inelastic observables for $^{90}\text{Zr}$ at 500 MeV. Solid and dashed lines are from with and without weighting in the phenomenological calculation.
Table 4.17 The changes in $\beta$'s by weighting procedure ($W$) for $^{48}\text{Ca}$ at 500 MeV, in phenomenological calculation.

<table>
<thead>
<tr>
<th>State</th>
<th>$E_a$ (MeV)</th>
<th>$\delta_S$</th>
<th>$\delta_V$</th>
</tr>
</thead>
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<tr>
<td>$2^+$</td>
<td>2.19</td>
<td>.104</td>
<td>.118</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>.128</td>
<td>.137</td>
</tr>
<tr>
<td>$3^-$</td>
<td>4.51</td>
<td>.189</td>
<td>.198</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>.202</td>
<td>.207</td>
</tr>
<tr>
<td>$5^-$</td>
<td>5.73</td>
<td>.102</td>
<td>.110</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>.109</td>
<td>.115</td>
</tr>
<tr>
<td>$4^+$</td>
<td>6.34</td>
<td>.048</td>
<td>.058</td>
</tr>
<tr>
<td></td>
<td>$W$</td>
<td>.063</td>
<td>.069</td>
</tr>
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</table>
Table 4.18. The changes in $\beta$'s by weighting procedure (W) for $^{90}\text{Zr}$ at 500 MeV, in phenomenological calculation.

<table>
<thead>
<tr>
<th>State</th>
<th>$E_x$ (MeV)</th>
<th>Scalar $\delta_S$</th>
<th>Vector $\delta_V$</th>
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</thead>
<tbody>
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<td>$2^+$</td>
<td>2.19</td>
<td>.085</td>
<td>.089</td>
</tr>
<tr>
<td></td>
<td>W</td>
<td>.093</td>
<td>.094</td>
</tr>
<tr>
<td>$5^-$</td>
<td>2.32</td>
<td>.073</td>
<td>.078</td>
</tr>
<tr>
<td></td>
<td>W</td>
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<td>.080</td>
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<td>$3^-$</td>
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<td>.172</td>
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<td></td>
<td>W</td>
<td>.169</td>
<td>.173</td>
</tr>
<tr>
<td>$4^+$</td>
<td>3.08</td>
<td>.047</td>
<td>.051</td>
</tr>
<tr>
<td></td>
<td>W</td>
<td>.050</td>
<td>.053</td>
</tr>
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<td>$2^+$</td>
<td>3.31</td>
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<td>.037</td>
</tr>
<tr>
<td></td>
<td>W</td>
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<td>$6^+$</td>
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<td>W</td>
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<td>.015</td>
</tr>
<tr>
<td></td>
<td>W</td>
<td>.011</td>
<td>.014</td>
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</table>
Figure 4.43:

Large angle elastic observable predictions for $^{90}$Zr at 500 MeV when $3^-$ state is coupled. The solid and dashed lines represent with and without channel coupling.
these nuclei. Figures 4.44 and 4.45 shows the phenomenological transition potentials for $3^-$ state in $^{48}$Ca and $^{90}$Zr. The real scalar and vector transition potentials vary smoothly with energy, with almost no change in shape, while the imaginary parts have slightly different geometries.

Next, the results of the RIA based calculations using the transition potentials of case 1 are presented. We find essentially the same results as in the phenomenological calculation. The resulting deformation lengths are given in Table 4.19 and 4.20, along with the results of the analyses using other probes. The agreement with experimental data are shown in Figs. 4.46, 4.47 and 4.48. Because of the large error bars of inelastic $A_\gamma$ data, again we find the inelastic $A_\gamma$ data are not reproduced as well as the other data. Hence, we employed the weighting procedure to improve this situation. Figures 4.49 and 4.50 show the improvement in the agreement with inelastic data for $3^-$ states of $^{48}$Ca and $^{90}$Zr by performing the weighting procedure, where the $\chi^2$ of the inelastic $A_\gamma$ data was multiplied by a factor of 10 ($W=10$ in Eq. 4.26).

The RIA based calculation results for the 800 MeV observables, shown in Figs. 4.51 and 4.52, were not as good as the phenomenological ones showing deeper minimums and more structure. In order to reproduce the 800 MeV inelastic data better, we employed the same weighting procedure as discussed previously with a weighting factor of 10, this time for the inelastic cross section.
Figure 4.44:

The energy dependence of the transition potentials for $3^{-}$ state of $^{48}\text{Ca}$. The solid lines are from 500 MeV, the dashed lines are from 800 MeV calculation.
Figure 4.45:
The energy dependence of the transition potentials for 3\(^-\) state of \(^{90}\)Zr. The solid lines are from 500 MeV, the dashed lines are from 800 MeV calculation.
Table 4.19. Deformation lengths for $^{48}$Ca at 500 and 800 MeV.

<table>
<thead>
<tr>
<th>State</th>
<th>$E$ (MeV)</th>
<th>$500$ (MeV) $\delta_S$</th>
<th>$\delta_V$</th>
<th>$\delta_{a,b}$</th>
<th>$800$ (MeV) $\delta_S$</th>
<th>$\delta_V$</th>
<th>$\delta_c$</th>
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<td>.61, .71</td>
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<td>.64</td>
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<td>$3^-$</td>
<td>4.52</td>
<td>.669</td>
<td>.717</td>
<td>.84, .88</td>
<td>.908</td>
<td>.874</td>
<td>.87</td>
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<td>$3^+_2$</td>
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<td></td>
<td></td>
<td>.410</td>
<td>.400</td>
<td>.41</td>
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<td>.390</td>
<td>.417</td>
<td>.38</td>
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<td>$4^+$</td>
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<td>.205</td>
<td>.234</td>
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</tr>
</tbody>
</table>

Table 4.20. Deformation lengths for $^{90}Zr$ at 500 and 800 MeV.

<table>
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<tr>
<th>State</th>
<th>$E$ (MeV)</th>
<th>$\delta_s$</th>
<th>$\delta_v$</th>
<th>$\delta_s$</th>
<th>$\delta_v$</th>
<th>$\delta^a$</th>
<th>$B(E\lambda)^a$</th>
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<tr>
<td>$2^+$</td>
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<td>$5^-$</td>
<td>2.32</td>
<td>.320</td>
<td>.347</td>
<td>.480</td>
<td>.460</td>
<td>.408</td>
<td>.98</td>
</tr>
<tr>
<td>$3^-$</td>
<td>2.75</td>
<td>.725</td>
<td>.762</td>
<td>1.24</td>
<td>1.10</td>
<td>.889</td>
<td>.510</td>
</tr>
<tr>
<td>$4^+$</td>
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<td>.203</td>
<td>.226</td>
<td>.492</td>
<td>.368</td>
<td>.275</td>
<td>.35</td>
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<tr>
<td>$2_2^+$</td>
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<td>.354</td>
<td>.301</td>
<td>.216</td>
<td>.171</td>
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<td>$6^+$</td>
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<td>.124</td>
<td>.211</td>
<td>.177</td>
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<tr>
<td>$8^+$</td>
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<td>.090</td>
<td>.095</td>
<td>.088</td>
<td>.13</td>
</tr>
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<td></td>
<td>.524</td>
<td>.423</td>
<td>.308</td>
<td>.30</td>
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</tbody>
</table>

a; M.M. Gazzaly et al. and references in there.
Figure 4.46:

Cross sections and analyzing powers of the inelastic states for $^{40}\text{Ca}$ at 500 MeV.
The solid lines represent the collective model calculation with RIA diagonal optical potentials.
Figure 4.47:

Cross sections of the inelastic states for $^{80}$Zr at 500 MeV. The solid lines represent the collective model calculation with RIA diagonal optical potentials.
Figure 4.48:

Analyzing powers of the inelastic states for $^{90}Zr$ at 500 MeV. The solid lines represent the collective model calculation with RIA diagonal optical potentials.
Figure 4.49:

Inelastic observables for $^{40}\text{Ca}$ at 500 MeV. Solid and dashed lines are from with and without weighting in the RIA based calculation.
Inelastic analyzing powers for $^{90}$Zr at 500 MeV. Solid and dashed lines are from with and without weighting in the RIA based calculation.
Figure 4.51:

Cross sections of the inelastic states for $^{48}\text{Ca}$ at 800 MeV. The solid lines represent the collective model calculation with RIA diagonal optical potentials.
Figure 4.52:

Cross sections of the inelastic states for $^{90}$Zr at 800 MeV. The solid lines represent the collective model calculation with RIA diagonal optical potentials.
Some improvement was found as shown in Figs. 4.53 and 4.54 for $3^-$ state.

We observe a large difference in the $A_y$ prediction, as shown in Fig. 4.53. The changes in $\delta$'s are large this case, for example, $\delta_S$ and $\delta_V$ are 1.24 and 1.10 without weighting, and they are 0.772 and 0.834 with weighting, for $^{90}$Zr $3^-$ state. This ambiguity may be reduced when the inelastic spin data for these cases become available.

The energy dependences of transition potentials for $^{48}$Ca and $^{90}$Zr is shown in Figs. 4.55 and 4.56. The shapes of the transition potentials inside of the nucleus are rather irregular, as found in the RIA transition potentials of other nuclei in the previous sections. This behavior is most likely due to shell effects in the input densities. We find that the shapes near surface are almost the same and the strengths change smoothly with energy.

4.3.4 Effects of tensor potentials in RIA calculation

In the RIA based calculation, we find a pattern of less structure in elastic spin observables at small angles than in the experimental data. This discrepancy at small angles occurs at 800 MeV for all of the targets considered. At 500 MeV, the discrepancy shows up for the heavier nuclei, such as $^{90}$Zr and $^{208}$Pb. Hoffmann et al.\(^1\) reported similar effects at small angles in their non-relativistic $KMT - IA$\(^{34,33}\) calculation for elastic $A_y$ data from $^{40,48}$Ca, $^{90}$Zr
Figure 4.53:

Inelastic observables for $^{48}$Ca at 800 MeV. Solid and dashed lines are from with and without weighting in the RIA based calculation.
Figure 4.54:

Inelastic observables for $^{90}$Zr at 800 MeV. Solid and dashed lines are from with and without weighting in the RIA based calculation.
Figure 4.55:

The energy dependence of the RIA transition potentials for $3^-$ state of $^{48}$Ca. The solid lines are from 500 MeV, the dashed lines are from 800 MeV calculation.
Figure 4.56:

The energy dependence of the RIA transition potentials for 3\textsuperscript{−} state of \textsuperscript{90}Zr. The solid lines are from 500 MeV, the dashed lines are from 800 MeV calculation.
and $^{208}Pb$ at 500 MeV and 800 MeV. They concluded that the nuclear medium effects such as Pauli blocking should be included in IA calculation to describe the elastic spin data at small angles for 500 and 800 MeV cases. Ottenstein et al.\textsuperscript{40} include exchange effects, vacuum polarization and Pauli blocking corrections in their RIA calculations and were able to improve the agreement at small angles. However, they used rather phenomenological adjustments to include vacuum polarization effects.

Here, we added the tensor potential parts to the RIA calculation and found the situation at small angles improves substantially. This procedure is motivated by the fact that even though the tensor contribution is known to be small,\textsuperscript{16} its influence on the spin observables is not negligible.\textsuperscript{41,18} As described in Ref. 16, actually three tensor terms contribute for even-even targets. One is due to the interaction of the projectile proton’s anomalous magnetic moment with the Coulomb field of the nucleus, the second arises from the relativistic invariant nucleon-nucleon tensor amplitude, while the third contribution is due to the interaction of the magnetic moments of the target nucleons with the Coulomb field of the projectile. The last term is already included in the calculation since empirical charge distribution is used to obtain the projectile-target Coulomb potential. The first term is expressed as

$$U_{AM} = \frac{ve\hbar c}{2me^2} \vec{i} \cdot \nabla \nu, \quad (4.30)$$
where $\nu$ is the anomalous magnetic moment of the proton (1.79). The results are shown in Figs. 4.57 through 4.62. As shown Fig. 4.57, the effect of including the anomalous magnetic moment term amounts to about half of the total effect. The contribution from the anomalous magnetic moment term is as significant as that of the RIA tensor potential.

The changes in $\delta$'s by including the tensor potentials in the diagonal potential are given in Table 4.21. Here, only the $3^-$ state, which is the strongest coupled state, coupled cases is considered. No substantial change is noticed in deformation parameters, the average change is 1.52%. The change in the inelastic observables is negligible. For this reason, as well as to reduce the number of parameters, tensor potential was generally not included in this work, nor did we consider the deformed tensor potentials.

### 4.4 Results of the Rotational Model Analyses

Here we investigate inelastic proton scattering from two deformed nuclei, $^{12}$C and $^{154}$Sm, using both the vibrational and rotational models described in chapter 3. In the rotational model of ECIS87, the deformation of the nuclear surface is written (refer to section 3.3.3),

\[
R(\theta) = R_0(1 + \beta_2 Y_{20} + \beta_4 Y_{40} + \cdots),
\]  

(4.31)
Figure 4.57:

Elastic spin observables for $^{40}$Ca at 800 MeV. The solid and dashed lines represent the calculation with and without inclusion of tensor terms.
Figure 4.58:

Elastic spin observables for $^{48}$Ca at 800 MeV. The solid and dashed lines represent the calculation with and without inclusion of tensor terms.
Figure 4.59:

Elastic spin observables for $^{90}$Zr at 500 MeV. The solid and dashed lines represent the calculation with and without inclusion of tensor terms.
Figure 4.60:

Elastic spin observables for $^{90}$Zr at 800 MeV. The solid and dashed lines represent the calculation with and without inclusion of tensor terms.
Figure 4.61:

Elastic spin observables for $^{208}$Pb at 500 MeV. The solid and dashed lines represent the calculation with and without inclusion of tensor terms.
Figure 4.62:

Elastic spin observables for $^{208}Pb$ at 800 MeV. The solid and dashed lines represent the calculation with and without inclusion of tensor terms.
Table 4.21 The changes in δ’s by adding tensor terms in RIA calculation, when 3⁻ state is coupled.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>E (MeV)</th>
<th>Scalar δₛ change(%)</th>
<th>Vector δᵥ change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>δₛ</td>
<td>δᵥ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+Tensor</td>
<td></td>
</tr>
<tr>
<td>⁴⁰Ca</td>
<td>797.5</td>
<td>1.288</td>
<td>0.599</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.339</td>
<td>1.357</td>
</tr>
<tr>
<td>⁴⁰Ca</td>
<td>797.5</td>
<td>0.908</td>
<td>0.874</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.898</td>
<td>0.866</td>
</tr>
<tr>
<td>⁹⁰Zr</td>
<td>497.5</td>
<td>0.725</td>
<td>0.762</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.749</td>
<td>0.779</td>
</tr>
<tr>
<td>⁹⁰Zr</td>
<td>797.5</td>
<td>1.242</td>
<td>1.104</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.265</td>
<td>1.115</td>
</tr>
<tr>
<td>²⁰⁸Pb</td>
<td>497.5</td>
<td>0.558</td>
<td>0.599</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.560</td>
<td>0.600</td>
</tr>
<tr>
<td>²⁰⁸Pb</td>
<td>797.5</td>
<td>0.664</td>
<td>0.727</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.661</td>
<td>0.725</td>
</tr>
</tbody>
</table>
as the quadrupole excitation band is dominant in the deformed nuclei. For \( p^+^{12}C \) we consider inelastic data for the \( 2^+ \) state at 4.43 MeV and \( 3^- \) at 9.6 MeV for 800 MeV incident protons, and the \( 2^+ \) data is considered for 1040 MeV incident proton. For \( p^+^{154}Sm \), the low lying \( 2^+ \) at 0.082 MeV and the \( 4^+ \) state at 0.267 MeV are considered and the coupling between the two states are examined. It is sufficient to consider only up to \( Y_{40} \) in Eq. (4.31).

Also in this section, an analysis of antiproton-\( ^{12}C \) scattering at 179.5 MeV is considered. Two different optical potential parameter sets which give equivalent fits to the elastic data are examined, as well as a RIA based calculation. It should be noted that for spherically symmetric nuclei, the space vector part can be transformed away,\(^{41}\) but for deformed nuclei the space vector should be included in the calculation. We used simple scalar form of time-like vector potential here to simplify the calculation.

4.4.1 Results for \( ^{12}C \)

Here we will discuss the results of an analysis of inelastic \( p^+^{12}C \) scattering at incident proton energies of 800 MeV and 1040 MeV. The 800 MeV elastic and inelastic data are taken from Refs. 42 and 43, the 1040 MeV data are from Ref. 45. At 800 MeV, while both cross section and \( A_y \) data are available for the \( 2^+ \) state, only cross section data are available for the \( 3^- \) state. At
We discuss the results of the phenomenological calculation first. Since it was hard to find parameters which produce good agreement with the $^{12}C$ data, various approaches were tried. Some of them are not described in the section on the general search procedure because they are used only in this subsection. We discuss the results for the analysis of the 800 MeV data first. For the $2^+$ state, both rotational and vibrational model calculations are examined, but for $3^-$ state, which could not be described using the rotational model, only the vibrational model calculation is done. We start with the usual 12-parameter Woods-Saxon fit to the elastic scattering data. We found a good fit to the experimental data, even though the calculated large angle $A_y$ showed an abnormal shape, see Fig 4.63. For the $2^+$ state, first a rotational model calculation is performed and then compared with the results of a vibrational model calculation. Six parameters, four potential strengths and two deformation parameters ($\beta_S$ and $\beta_V$), were searched for both cases. As shown in Fig. 4.64, the results from these two different model calculations give about the same quality of fit to the inelastic data. We found that the $2^+$ $A_y$ data were not very well represented in either case. A nonrelativistic phenomenological calculations\textsuperscript{43} in which 9 parameters were used in a potential of Woods-Saxon form, also failed to reproduce the $2^+$ $A_y$ data. Figure 4.65 shows the results of the
Figure 4.63:

The elastic observables for p+^{12}C at 800 MeV. The solid lines represent the 12 parameter fit to the elastic data using Woods-Saxon shape.
vibrational model calculation for the $3^-$ state, the agreement with experiment is quite reasonable.

Since the 12 parameter fit to the elastic data gives an abnormal shape in the large angle $A_y$ and the fact that the $2^+$ state $A_y$ was not well reproduced, we decided to use a 3-parameter Fermi shape for the direct potential geometry. In this case, there are 16 parameters for the diagonal optical potentials. The 3-parameter Fermi shape is given by

$$U_i = \frac{V_i(1 + w_i R_i^2)}{1 + \exp \left( \frac{r - R_i}{a_i} \right)}, \quad (4.32)$$

where $V_i$ is the potential strength, $R_i$ is the nuclear radius parameter, $a_i$ is the diffusiveness parameter and $w_i$ is a parameter which effects the interior for each potential $i$. With this model, we found very good fits to the elastic data, as is shown in Fig. 4.66. We then did both a two parameter search and a six parameter search for the $2^+$ inelastic data set. The results are given in Table 4.22, along with the results of Refs. 42 and 44. In Fig. 4.67, we show the comparison between the two and six parameter searches for the $2^+$ state. In Table 4.22 we give $\delta$'s instead of $\beta$'s because we use the 3-parameter Fermi shape in a tabulated form as a input for ECIS87. Our values for the $\delta$'s are in reasonable agreement with the results of previous nonrelativistic calculations where they use the Woods-Saxon form for the potentials.\textsuperscript{42} However, even
Figure 4.64:
The $2^+$ state for $p+^{12}C$ scattering at 800 MeV. The solid and dashed lines represent the results of the rotational and vibrational model calculation respectively.
Figure 4.65:

The $3^-$ state for $p^{+12}C$ scattering at 800 MeV. The solid lines represent the results of the vibrational model calculation.
Figure 4.66:

The elastic observables for $p+^{12}C$ at 800 MeV. The solid lines represent the 16 parameter fit to the elastic data using 3 parameter Fermi shape.
with the additional parameters the fit to the $2^+$ $A_y$ data was still not very good.

In order to try to find a better representation of both elastic and inelastic data, we returned to the Woods-Saxon shape and searched on the geometry parameters, $R_i$ and $a_i$. A total 8 geometry parameters were searched. Starting from the result of the 6 parameter search, the 8 geometry parameters and 2 $\beta$'s, making total 10 parameters, were varied. The results for the $2^+$ state are shown in Fig. 4.68. Also, we tried various weighting procedures and even deformed the Coulomb potential. However, none of these procedures improved the quality of fit to the $2^+$ $A_y$ data.

Therefore, we conclude that neither Woods-Saxon nor 3-parameter Fermi shapes are the correct form factors to use for this deformed target $^{12}\text{C}$. Of course, it should be mentioned that the space-vector optical potential parts should not be ignored in the treatment of deformed targets. Also, there is a possibility that the multistep process may play an important role in describing the excited states of $^{12}\text{C}$, as discussed by the authors of Ref. 43.

Next we investigate the effects of the channel coupling on the calculated large angle elastic cross sections. For example, in Fig. 4.69, we show the results when the strong $2^+$ state is included. In these calculation, we used the 3-parameter Fermi shape for the diagonal potentials and did a 6 parameter
Table 4.22 Deformation lengths in the phenomenological calculation for $^{12}\text{C}$ at 800 MeV.

<table>
<thead>
<tr>
<th>$J^z$</th>
<th>$E$ (MeV)</th>
<th>$\delta_S$ (fm)</th>
<th>$\delta_V$ (fm)</th>
<th>$\delta_{NR}$ (fm)</th>
<th>Potential scale</th>
</tr>
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<tbody>
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<td>1.68</td>
<td>1.66</td>
<td>1.91$^a$, 1.58$^b$</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.35</td>
<td>1.38</td>
<td></td>
<td>.603</td>
</tr>
<tr>
<td></td>
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<td>$3^-$</td>
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<td>1.12</td>
<td>1.03</td>
<td>1.21$^a$</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>.490</td>
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<td>.841</td>
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<td>.865</td>
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</table>

Figure 4.67:
The $2^+$ state for $p + ^{12}C$ scattering at 800 MeV. The solid and dashed lines represent the results of 2 p and 6 p searches in the vibrational model calculation.
Figure 4.68:
The $2^+$ state for $p^{+12C}$ scattering at 800 MeV. The solid lines represent the results of 10 p searches described in the text.
search. The channel coupling effects are apparent at large angles, as well as at small angles where we observe that cross sections at the first minimum no longer agree with the data.

Finally, we give the results of the analysis of $p + ^{12}C$ at 1040 MeV. Both rotational and vibrational model calculations are done for this case. The results are given in Table 4.23, and the observables are shown in Fig. 4.70. Six parameter searches were done for both model calculations. In Table 4.23 we find large changes in potential strengths, however the analysis is hampered by the lack of spin data. The $\beta$'s are in reasonable agreement with the results of Ref. 43, where nonrelativistic calculation using the potentials of Woods-Saxon form is done. We found reasonable agreement with the data. In Fig 4.71 we show the effects of the coupling to $2^+$ state.

The phenomenological transition potentials, shown in Fig. 4.72 were determined from 6 parameter searches using Woods-Saxon shapes for the diagonal potentials. They show a smooth dependence on energy. For the real transition potentials, the strength of potentials decrease as energy increase, while the imaginary transition potential strengths increase as the energy increase. Also it is observed that the values of the radius where potential reaches the maximum value, is larger for the imaginary transition potentials than for the real ones.
Figure 4.69:

The elastic observables for $p+^{12}C$ scattering at 800 MeV. The solid lines represent the results of a search when the $2^+$ state is included.
Table 4.23 Deformation parameters in the phenomenological calculation for $p^+^{12}C$ scattering at 1040 MeV.

<table>
<thead>
<tr>
<th>$J^*$</th>
<th>$E$ (MeV)</th>
<th>$\beta_s$</th>
<th>$\beta_v$</th>
<th>$\beta_{NR}$</th>
<th>potential (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic only</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^+$</td>
<td>4.44 vib.</td>
<td>.691</td>
<td>.657</td>
<td>.81</td>
<td>-309.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>479.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>206.5</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>-287.1</td>
</tr>
</tbody>
</table>

Figure 4.70:
The $2^+$ state observables for $p + ^{12}C$ scattering at 1040 MeV. The solid and dashed lines represent the results of 6 parameter search from rotational and vibrational model calculation.
Figure 4.71:

The elastic observables for $p+^{12}C$ scattering at 1040 MeV when $2^+$ state is included, using either vibrational (solid line) or rotational model (dashed line). The dotted line is from elastic calculation.
Figure 4.72:

The energy dependence of the transition potentials for $p+^{12}C$ scattering in the phenomenological calculation.
Next, we present an RIA based calculation for $p+^{12}C$ using the case 1 transition potentials discussed in chapter 3. First, we review the result of the RIA calculation when only elastic data are considered. As shown in Fig. 4.73, in dashed lines, the RIA calculation does not satisfactorily reproduce the elastic data. Even though the positions of the diffraction minima and maxima are correct, the cross sections are too small beyond 10 degrees and the analyzing power shows too much structure. To improve this situation, we adjusted the four strengths of the RIA potentials (scalar real and imaginary, vector real and imaginary), as we did for the $p+^{40}Ca$ scattering at 362 MeV. This improved the agreement with the experimental data remarkably, as shown by the solid lines in Fig. 4.73. The changes in potential strengths and $\chi^2$s are given in Table. 4.24.

For the inelastic scattering calculation, we used the adjusted RIA potentials for the direct potentials and did a two parameter search on $\beta_S$ and $\beta_V$. The results from these calculations are given in Figs. 4.74 and 4.75. We also examined several different search procedures such as an 8 parameter search, 4 strengths and 4 different $\delta$'s one for each of the real and imaginary scalar and vector terms, and found that the agreement with $2^+$ cross section data was improved, but the $2^+ A_y$ data was still not well reproduced.
Figure 4.73:

The elastic observables for $p+^{12}C$ scattering at 800 MeV. The solid and dashed lines represent the results of the adjusted RIA and original RIA calculations respectively.
Table 4.24 The changes in potential strengths and $\chi^2$ in adjusted RIA potentials for $^{12}$C at 800 MeV.

<table>
<thead>
<tr>
<th>potential scale</th>
<th>$\chi^2/N$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>original</strong></td>
<td></td>
</tr>
<tr>
<td>$2p$</td>
<td></td>
</tr>
<tr>
<td>.752</td>
<td>$CS$ 552</td>
</tr>
<tr>
<td>.752</td>
<td>$A_y$ 70</td>
</tr>
<tr>
<td>.920</td>
<td></td>
</tr>
<tr>
<td>.920</td>
<td></td>
</tr>
<tr>
<td>$4p$</td>
<td></td>
</tr>
<tr>
<td>1.137</td>
<td>$CS$ 271</td>
</tr>
<tr>
<td>1.858</td>
<td>$A_y$ 32</td>
</tr>
<tr>
<td>1.218</td>
<td></td>
</tr>
<tr>
<td>1.339</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.74:
The $2^+$ state observables for $p + ^{12}C$ scattering at 800 MeV. The solid lines represent the results of the 2 parameter search with adjusted RIA potentials.
Figure 4.75:

The $3^-$ state observables for $p^{+12}C$ scattering at 800 MeV. The solid lines represent the results of the 2 parameter search with adjusted RIA potentials.
As shown in Table 4.25, it is not easy to find any definite pattern in
the extracted $\delta$'s and potential strengths, although it is clear that $2^+$ state is
the strongest coupled state in this deformed target. From these results, we
conclude that better models, perhaps including the space parts of the Lorentz
four-vector potential, are required to improve the results of RIA calculation
for $^{12}C$.

4.4.2 Results for $^{154}Sm$

Inelastic proton scattering from another deformed target, $^{154}Sm$, is in­
vestigated in this section. The experimental data are taken from Ref. 46.
Only the phenomenological calculation is done for this case. First, a six pa­
rameter search is performed for the lowest $2^+$ state at 0.082 MeV using both
vibrational and rotational models. As shown in Fig. 4.76, they give almost the
same results. Since the next excited state, the $4^+$ at 0.267 MeV, is so close,
the coupling between these two excited states is expected to be very strong.46
Thus, we tried an eight parameter search using the rotational model, includ­
ing both states in the calculation. The results are compared with the previous
calculation which included only the $2^+$ state. The results are given in Table.
4.26, and the calculated observables are shown in Fig.4.77. Also the results of
analyses of inelastic data obtained using other probes are given in Table 4.26.
Table 4.25 Deformation lengths in the phenomenological calculation for p+^{12}C scattering at 800 MeV.

<table>
<thead>
<tr>
<th>J^π</th>
<th>E (MeV)</th>
<th>δ_s,real (fm)</th>
<th>δ_s,imag. (fm)</th>
<th>δ_V,real (fm)</th>
<th>δ_V,imag. (fm)</th>
<th>potential scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>2^+</td>
<td>4.44</td>
<td>1.79</td>
<td>1.79</td>
<td>1.89</td>
<td>1.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2p^a</td>
<td>2.34</td>
<td>-2.67</td>
<td>2.59</td>
<td>-0.67</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8p</td>
<td>2.14</td>
<td>.421</td>
<td>2.31</td>
<td>.858</td>
<td>1.21</td>
</tr>
<tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.36</td>
</tr>
<tr>
<td>3^-</td>
<td>9.60</td>
<td>-1.30</td>
<td>-1.30</td>
<td>-.829</td>
<td>-.829</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2p^a</td>
<td>1.17</td>
<td>-3.16</td>
<td>.868</td>
<td>-.659</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8p</td>
<td>1.43</td>
<td>-.982</td>
<td>1.032</td>
<td>.053</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>1.17</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>1.20</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>1.26</td>
</tr>
</tbody>
</table>

a; These calculations are done using the adjusted RIA potentials to fit the elastic data with the scale factors given in Table 4.24, 4p search.
Figure 4.76:

The $2^+$ state observables for $p+^{154}$Sm scattering at 800 MeV. The solid and dashed lines represent the results of the rotational and vibrational model calculation respectively.
Figure 4.77:
The $2^+$ state observables for $p+^{154}\text{Sm}$ scattering at 800 MeV. The solid and dashed lines represent the results of the 8 p and 6 p searches, described in the text, respectively.
Table 4.26 Deformation parameters in the phenomenological calculation for p+\(^{154}\text{Sm}\) scattering at 800 MeV.

<table>
<thead>
<tr>
<th></th>
<th>(E) (MeV)</th>
<th>(\chi^2/N)</th>
<th>(\chi^2/N)</th>
<th>(\beta_s)</th>
<th>(\beta_V)</th>
<th>(\beta_{NR})</th>
<th>potential (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VIB. Elastic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^+) cpd.</td>
<td>235</td>
<td>0.082</td>
<td>468</td>
<td>630</td>
<td>0.236</td>
<td>0.259</td>
<td>-237.7</td>
</tr>
<tr>
<td>(2^+, 4^+) cpd.</td>
<td></td>
<td>0.082</td>
<td>411</td>
<td>304</td>
<td>0.241</td>
<td>0.260</td>
<td>0.301</td>
</tr>
<tr>
<td>(2^+, 4^+) cpd.</td>
<td></td>
<td>0.272</td>
<td>199</td>
<td>0.068</td>
<td>0.076</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td><strong>ROT. Elastic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2^+) cpd.</td>
<td>247</td>
<td>0.082</td>
<td>469</td>
<td>711</td>
<td>0.245</td>
<td>0.261</td>
<td>-429.0</td>
</tr>
<tr>
<td>(2^+, 4^+) cpd.</td>
<td></td>
<td>0.082</td>
<td>441</td>
<td>330</td>
<td>0.261</td>
<td>0.269</td>
<td>0.301</td>
</tr>
</tbody>
</table>
We observe that the $\chi^2$ for the $2^+$ state is improved by a factor of 2, when the $4^+$ state is included. In Fig. 4.77, the case where two inelastic states are included (solid line) shows better agreement with $2^+$ data than the case where only the $2^+$ state was included (dashed line). The fit to the $4^+$ state was fairly good, as is shown in Fig. 4.78. The small discrepancy could be explained by the fact that there exist several more excited states with excitation energies less than 2 MeV, which are probably also strongly coupled to $2^+$ and $4^+$ states. It is expected that the coupling between the $4^+$ state and next higher inelastic state, the $6^+$ state, is not negligible, and thus should be included in the calculation to describe $4^+$ state better. However, unfortunately, the existing computer program takes too much computer time to do a search with all of these states coupled. Even for the case with two excited states coupled, it took more than 100 hours on the SUN 4. Plans to use the CRAY computer are being considered. Also, of course, we need more complete model if we consider the coupling between several states. The effects of the coupling to $2^+$ state on the calculated large angle cross section is similar to the other cases considered. When both the $2^+$ and $4^+$ states are included, as shown in Fig. 4.79, the calculated cross section show the coupling effects clearly.
Figure 4.78:

The 4+ state observables for $p+^{154}Sm$ scattering at 800 MeV. The solid lines represent the results of the 8 p search, described in the text.
Figure 4.79:

The elastic observables for $p + ^{144}Sm$ scattering at 800 MeV. The solid and dashed lines represent the results of the 6 p and 8 p searches respectively, described in the text.
4.4.3 Results for Antiproton+\(^{12}\)C

In this section, we consider 180 MeV antiproton-\(^{12}\)C inelastic scattering to several low lying states, the \(2^{+}\) at 4.43 MeV, the \(0^{+}\) at 7.6 MeV and the \(3^{-}\) at 9.6 MeV.\(^{47}\) Unfortunately, only cross section data are available, even for the ground state.\(^{48}\) We present phenomenological calculations for antiproton-\(^{12}\)C scattering using two different sets of optical potential parameters for the direct channel potentials. Also a RIA based calculation will be investigated.

In the phenomenological calculation, we found equally good fits to the elastic data using two different sets of potential parameters.\(^{49}\) One set in which the real scalar and vector potentials are large and attractive, and one in which the real vector is large and repulsive and the real scalar is large and attractive. The parameters are given in Table 4.27, where \(r_0\) is related to the nuclear radius parameter as \(R = r_0 \ast A^{1/3}\). The equivalent central and spin-orbit potentials are shown in Fig. 4.80. It should be noticed that in the latter case the real scalar and vector cancel just as the RIA potentials do.\(^{50}\) Certainly, lack of the data for spin observables is one of the reasons for the ambiguity in the elastic potentials and it was hoped that the inclusion of the inelastic data in the analyses might help to resolve the ambiguity. We performed both 2 and 6 parameter searches beginning with both parameter sets. The change in the \(\chi^2\)'s between the two fitting procedures and the resulting potential
Table 4.27. The optical potential parameters of the two different potential sets for antiproton-$^{12}$C scattering at 180 MeV.

<table>
<thead>
<tr>
<th></th>
<th>strength (MeV)</th>
<th>$R$ (fm)</th>
<th>$a$ (fm)</th>
<th>$\chi^2$</th>
<th>$\chi^2/(d.o.f.)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cancelling</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>potential</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>scalar Real</em></td>
<td>$-284.2$</td>
<td>.865</td>
<td>.696</td>
<td>47</td>
<td>1.27</td>
</tr>
<tr>
<td>Scalar Imag.</td>
<td>$117.2$</td>
<td>1.01</td>
<td>.702</td>
<td></td>
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</tr>
<tr>
<td>Vector Real</td>
<td>$123.8$</td>
<td>.966</td>
<td>.773</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vector Imag.</td>
<td>$-196.5$</td>
<td>1.02</td>
<td>.636</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Adding</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>potential</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scalar Real</td>
<td>$-238.7$</td>
<td>.582</td>
<td>.578</td>
<td>48.5</td>
<td>1.31</td>
</tr>
<tr>
<td>Scalar Imag.</td>
<td>$16.8$</td>
<td>1.27</td>
<td>.239</td>
<td></td>
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</tr>
<tr>
<td>Vector Real</td>
<td>$-439.9$</td>
<td>.363</td>
<td>.100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vector Imag.</td>
<td>$-118.3$</td>
<td>1.08</td>
<td>.510</td>
<td></td>
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</tr>
</tbody>
</table>
Figure 4.80:

The real and imaginary effective central and spin-orbit potentials for antiproton+^{12}C scattering calculated from either cancelling (solid lines) or adding potential (dashed lines) set.
strengths and the $\beta$'s are given in Table 4.28. We observe that the fit to the elastic data is destroyed in the two parameter search. The $\chi^2$ for the elastic cross section is increased by a factor of 6 when the strong $2^+$ state is included. Apparently this is because of the double counting of the reaction effects. In the six parameter search, the $\chi^2$ is reduced back to the original value. The change in the potential parameters from their original values are quite large. It is clear that we must search on the direct potential parameters when analyzing antiproton-nucleus inelastic scattering, especially when the coupling to the low lying excited states are strong. The change in the elastic $\chi^2$ is larger when the $2^+$ state is included than when the $3^-$ state is included, as is expected from the values of the deformation parameters. For the $0^+$ state, the changes in potential strengths produced in the six parameter searches are unreasonably large. The values are given in Table 4.28. We believe this happens because the $0^+$ state is not well described by a first order collective model.

For the cancelling potential parameter set, a maximum change of 43 % in the potential parameters (considering only $2^+$ and $3^-$ states) occurred in imaginary scalar potential when $3^-$ state was included. For the adding potential set, the maximum change was 33 %, also found in imaginary scalar potential, but in this case when the $2^+$ state was coupled. The changes in the potentials when the $2^+$ and $3^-$ states are coupled are shown in Figs. 4.81 and
Table 4.28. The changes in $\beta$ and $\chi^2$ in the 2 and 6 parameter searches for antiproton-$^{12}$C scattering at 180 MeV.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\chi^2_N$</th>
<th>$\chi^2_{set}$</th>
<th>$\beta_S$</th>
<th>$\beta_V$</th>
<th>Potential strengths (MeV)</th>
<th>$\sigma_R$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cancelling</strong></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>2$^+$ cpd.</td>
<td>2p</td>
<td>6.5</td>
<td>3.3</td>
<td>.583</td>
<td>.593</td>
<td>480</td>
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<tr>
<td></td>
<td>6p</td>
<td>1.1</td>
<td>1.8</td>
<td>.688</td>
<td>.624</td>
<td>-372.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td>128.8</td>
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<td></td>
<td>164.5</td>
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<td></td>
<td></td>
<td></td>
<td>-209.4</td>
</tr>
<tr>
<td>3$^-$ cpd.</td>
<td>2p</td>
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<td>2.4</td>
<td>.115</td>
<td>.276</td>
<td>485</td>
</tr>
<tr>
<td></td>
<td>6p</td>
<td>1.0</td>
<td>2.2</td>
<td>.035</td>
<td>.201</td>
<td>-227.3</td>
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<tr>
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<td>167.6</td>
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<td></td>
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<td>92.3</td>
</tr>
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<td>-246.9</td>
</tr>
<tr>
<td>0$^+$ cpd.</td>
<td>2p</td>
<td>0.9</td>
<td>47</td>
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<td>.024</td>
<td>488</td>
</tr>
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<td>6p</td>
<td>2.7</td>
<td>3.6</td>
<td>-0.014</td>
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</tr>
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<td>-8.7</td>
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</tr>
<tr>
<td>2$^+$ cpd.</td>
<td>2p</td>
<td>7.7</td>
<td>4.1</td>
<td>.629</td>
<td>.619</td>
<td>483</td>
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<tr>
<td></td>
<td>6p</td>
<td>1.3</td>
<td>2.3</td>
<td>.027</td>
<td>.547</td>
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</tr>
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<td></td>
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<td>11.2</td>
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<tr>
<td></td>
<td></td>
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<td></td>
<td>-122.8</td>
</tr>
<tr>
<td>3$^-$ cpd.</td>
<td>2p</td>
<td>1.5</td>
<td>3.0</td>
<td>1.147</td>
<td>.487</td>
<td>490</td>
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<tr>
<td></td>
<td>6p</td>
<td>1.0</td>
<td>2.8</td>
<td>.877</td>
<td>.500</td>
<td>-261.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-391.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-121.2</td>
</tr>
<tr>
<td>0$^+$ cpd.</td>
<td>2p</td>
<td>0.9</td>
<td>96</td>
<td>.612</td>
<td>.042</td>
<td>492</td>
</tr>
</tbody>
</table>
We find that the deformation parameters ($\beta$'s) with the cancelling potential set are more reasonable, in comparison to nonrelativistic analysis. The values are given in Table 4.29. It seems that the case where the real scalar and vector potentials have the opposite sign is preferred.

We found that both potential sets could give very good fits to the elastic and inelastic data, with chi-square per-degree-of-freedom ($\chi^2/(d.o.f.)$) around 1. Figures 4.84 and 4.85 show the results of a 6 parameter search using the vibrational model for the $2^+$ and $3^-$ states using either the cancelling or the adding potentials. Also we performed rotational model calculations for the $2^+$ state. The vibrational model reproduce the inelastic data a little better, as shown in Fig. 4.83, for $2^+$ state.

For the $0^+$ state, the 6 parameter search using an adding potential set did not converge. We were able to find a good fit to the $0^+$ state using the cancelling potentials, as shown in Fig. 4.86, however, we believe the deformation parameters obtained using a first order collective model are not reliable, since the number of data points is very small. It is also clear that the collective nature of this state is weak.

In a recent nonrelativistic phenomenological calculation, Choudhury and Guo found some disagreement between their calculated values and the experimental data for the $3^-$ state. They claimed the that underlying cause for
Figure 4.81:

The changes in the effective central and spin-orbit potentials when the $2^+$ state is included, in a 6 parameter search.
Figure 4.82:

The changes in the effective central and spin-orbit potentials when the $3^-$ state is included, in a 6 parameter search.
Table 4.29 Deformation parameters for antiproton-$^{12}C$ at 180 MeV.

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$E$ (MeV)</th>
<th>$\beta_s$</th>
<th>$\beta_V$</th>
<th>$\beta_{NR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+$</td>
<td>4.44</td>
<td>CP</td>
<td>.688</td>
<td>.624</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AP</td>
<td>.027</td>
<td>.547</td>
</tr>
<tr>
<td>$3^-$</td>
<td>9.60</td>
<td>CP</td>
<td>.035</td>
<td>.201</td>
</tr>
<tr>
<td></td>
<td></td>
<td>AP</td>
<td>.877</td>
<td>.500</td>
</tr>
</tbody>
</table>

a; Cancelling potential set is used.
b; Adding potential set is used.
Figure 4.83:

$2^+$ state observables from a 6 parameter search using vibrational (solid lines) and rotational (dashed lines) model calculations. The cancelling potential set is used.
Figure 4.84:

$2^+$ state observables in 6 parameter search using cancelling potentials (solid lines) and adding potentials (dashed lines) in a vibrational model calculation.
Figure 4.85:

3− state observables in 6 parameter search using cancelling potentials (solid lines) and adding potentials (dashed lines) in a vibrational model calculation.
Figure 4.86:

$0^+$ state observables in a 6 parameter search using the cancelling potential set in vibrational model calculation.
this discrepancy could be that the collective structure of the $3^-$ state in this light nucleus is much more complicated than could be expected to be described by a simple collective model approach. However, in the relativistic approach here, we were able to find very good agreement with $3^-$ data as well as $2^+$ data and $0^+$ data.

Elastic cross sections are shown in Fig. 4.87, showing the effects of the coupling to the $2^+$ state. We find that inclusion of the strong inelastic state cause the calculated cross sections at large angles to increase, which is opposite in direction from proton scattering. This point is under investigation.

Next, we give the analysis of the RIA based calculation. Even though the incident energy 180 MeV is smaller than the valid energy range of the RIA calculation for proton scattering, it has been shown that for antiproton-$^{12}\text{C}$ scattering, even at very low energies such as 46.8 MeV, the RIA calculation reproduces the experimental data very well.\(^5^0\) This is due to the dominance of the absorptive part of the potential.

The RIA calculation for the elastic observables behave in a manner similar to p-$^1\text{H}$ case, that is, the calculated cross sections gave smaller values than the data. However, when the inelastic states are included, the situation is improved. Again the elastic cross sections at large angles are increased by the inclusion of the inelastic states, resulting in better agreement with the
Figure 4.87:

Elastic observables in a 6 parameter search when the $2^+$ state is included using cancelling potentials in a vibrational model calculation.
experimental data. The results are shown in Fig. 4.88 for elastic observables, and in Figs. 4.89 and 4.90 for inelastic observables. The deformation lengths are given in Table 4.30.

4.5 **Summary and conclusion**

In this work we have considered the extension of relativistic treatments of nucleon-nucleus elastic scattering to the description of certain types of inelastic nucleon-nucleus scattering. There are two major aspects of this work. One is the computational. We obtain the scattering observables by performing a coupled channel calculation which uses the Dirac equation as the relevant wave equation for the scattered proton. The other aspect has to do with the models chosen for the relativistic potentials used in the coupled channel equations. A true relativistic nuclear many-body theory of nuclear structure or reactions does not exist. Further, its development, although the work in the field to date has been impressive, is not eminent. Because of this we rely on extensions of the nonrelativistic approaches to obtain the model transition potentials which describe the coupling between the various states considered. The two models are generalizations of the simplest nonrelativistic models which have been used to describe low lying collective states. They both take the transition potentials to be proportional to the derivative of the direct potentials. This,
Figure 4.88:
Elastic observables in 2 parameter search when $2^+$ state included in RIA based calculation.
Figure 4.89:

$2^+$ state observables for antiproton-$^{12}$C scattering in 2 parameter search in RIA based calculation.
Figure 4.90:

$^3$- state observables for antiproton-$^{12}$C scattering in 2 parameter search in RIA based calculation.
Table 4.30. The deformation lengths for antiproton-$^{12}$C scattering at 180 MeV from the RIA based calculation.

<table>
<thead>
<tr>
<th>State</th>
<th>$\chi_{el}^2/N$</th>
<th>$\chi_{inel}^2/N$</th>
<th>$\delta_S$</th>
<th>$\delta_V$</th>
<th>$\sigma_R$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+$</td>
<td>27.4</td>
<td>11.8</td>
<td>.189</td>
<td>.739</td>
<td>489</td>
</tr>
<tr>
<td>$3^-$</td>
<td>47.2</td>
<td>7.6</td>
<td>-.039</td>
<td>.380</td>
<td>485</td>
</tr>
</tbody>
</table>
of course, introduces an important constraint on the calculations, namely the transition potentials have the same Lorentz character as the direct potentials. The constants of proportionality, termed deformation parameters or deformation lengths in analogy with the nonrelativistic treatments, are the relevant physical observables obtained in the analysis of the elastic and inelastic data.

One model for the transition potentials, called the vibrational model, seems to be appropriate for the description of the low lying collective states of spherical nuclei, especially doubly magic nuclei, the other, called the rotational model, may be more appropriate to the description of permanently deformed nuclei. The determination of the direct potential proceeded in two ways. In one case purely phenomenological potentials were determined by fitting elastic and inelastic data together in the coupled channel environment. In the second the direct potentials were calculated using the relativistic impulse approximation, RIA. This approach makes the connection between empirical nucleon-nucleon free scattering amplitudes, i.e., the two-body problem, and the nuclear many-body problem via the ideas of multiple scattering theory, and represents the method of choice for the treatment of nuclear reactions within a relativistic framework.

Next we summarize the salient features of our analyses.

1. For spherically symmetric nuclei the scalar-vector model of Dirac
phenomenology can be used successfully as a basis for the analysis of inelastic nucleon-nucleus data.

2. Although nucleon-nucleus inelastic scattering data has generally been analysed using the nonrelativistic or relativistic distorted wave Born approximation there is clear evidence that the Dirac coupled channel approach can improve the description of large angle elastic nucleon-nucleus or antinucleon-nucleus cross section observables.

3. A simple generalization of the non-relativistic collective vibrational model has proved successful in describing most of the inelastic data included in the analysis of spherically symmetric nuclei. When the model failed it was due to the lack of collectivity in the state under consideration.

4. In general excellent representations of the inelastic spin observables have been obtained for those states where the collective model was appropriate. In every case the agreement with data is superior to non-relativistic analyses, especially for spin observables.

5. Analyses based on the RIA potentials were successful where the RIA was successful in describing the elastic observables. The two RIA based models for the transition potentials produced essentially the same results.

6. There is clear evidence for the superiority of the RIA based approach over the KMT approach, again especially in the ability to reproduce the
empirical spin observables.

7. For deformed nuclei the generalized Dirac collective vibrational or rotational models met with mixed success. In some cases the agreement with experiment was poor. We attribute these results mainly to two deficiencies in the models used. One the lack of proper inclusion of multi-step processes. The other due to the inappropriateness of the Dirac SV model as a basis for describing permanently deformed nuclei.

8. In our analysis of anti-proton-nucleus scattering we find evidence that the treatment of the inelastic data within a coupled channel framework may remove the ambiguities in the optical potential.

In the future we plan to investigate other targets, including deformed nuclei. We have evidence from the work presented here that the relativistic approach is capable of producing high quality fits to experimental data, especially spin observables, and thus has promise as a superior basis for phenomenology. However, the most important task is the development of better models for the transition potentials.
CHAPTER IV REFERENCES


1.1 The Calculations of the Cross Sections and Spin Observables

We can start from the scattering amplitude $f_{m_s}(\theta, \phi)$, given by

$$f_{m_s}(\theta, \phi) = \sum \chi M_\rho m'_s \sqrt{4\pi (2L + 1) M_x} C^{L_l/2J}_{0m_s m_s}$$

$$\times \left( (\frac{1+i}{2})^{1/2} C^{L_l/2J}_{L_l m_l m_s}, Y_{L_l m_l} (\Omega) |1/2m'_s) \right)$$

where $L_\rho = L (-\chi)$ and $M_x = \frac{e^{i\chi x}}{2ik}$. In terms of the scattering amplitude, the differential cross section is given by

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{m_s} = f^1_{m_s}(\theta, \phi) f_{m_s}(\theta, \phi).$$

The spherical harmonics $Y_{lm}(\theta, \phi)$ bear the following relationship to the associated Legendre polynomials $P^l_m(\theta)$:

$$Y_{lm}(\theta, \phi) = (-)^m (Y_{l-m}(\theta, \phi))^*$$

$$= (-)^m \sqrt{\frac{2l + 1 (l - m)!}{4\pi (l + m)!}} e^{i\phi} P^l_m (\theta), \quad m \geq 0.$$
If we use several well known recursion relations for the Legendre polynomials
and let

\[ f_+ (\theta) = \sum_{L=0}^{\infty} P_0^L (LM_L + (L + 1)M_{-L-1}) \]  
\[ f_- (\theta) = \sum_{L=0}^{\infty} P_0^L (-\cot \theta (LM_L + (L + 1)M_{-L-1})) \]

\[ + \frac{1}{\sin \theta} (LM_{-L} + (L + 1)M_{L+1}), \]  
(A.5)

then the differential cross section is

\[ \frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{m_\sigma} \frac{d\sigma}{d\Omega} m_\sigma \]
\[ = |f_+ (\theta)|^2 + |f_- (\theta)|^2. \]  
(A.6)

The polarization, which is equal with the analyzing power \( A_y \) for the elastic scattering, is given by (see Ref. 4 in chapter 4)

\[ P(\theta) = \frac{2 \text{Im}(f_+^*(\theta)f_- (\theta))}{|f_+ (\theta)|^2 + |f_- (\theta)|^2}. \]  
(A.7)

Finally, the spin rotation function, \( Q(\theta) \) is given by

\[ Q(\theta) = \frac{2 \text{Re}(f_+^*(\theta)f_- (\theta))}{|f_+ (\theta)|^2 + |f_- (\theta)|^2}. \]  
(A.8)
APPENDIX B

2.1 The Evaluation of the Matrix Element \( P_{jj'}^{\lambda} \)

Here we start by considering more general case for the matrix elements evaluation. The result for the specific case we considered will be discussed at the end. Expand the matrix elements of a scalar operator in a multipole series,

\[
A(r) = \sum_m A_{lm}(r) Y^*_m(\Omega) \quad (B.1)
\]

then,

\[
\langle IM| A(r) |I'M' \rangle = \langle IM | \sum_m A_{lm}(r) Y^*_m(\Omega) |I'M' \rangle \quad (B.2)
\]

\[
= \sum_m (-)^m \langle IM | A_{l-m}(r) |I'M' \rangle Y^*_m(\Omega) \quad (B.3)
\]

\[
= \sum_m C_{Ml-m}^{II} \langle I | |A_l||I' \rangle Y_{lm}(\Omega) \quad (B.4)
\]

\[
= \sum_m C_{Ml-m}^{II} \frac{i^l}{(2l+1)^{1/2}} a^{II'}_l(r) Y_{lm}(\Omega). \quad (B.5)
\]

If we consider the evaluation of \( \langle(fm_{jj'}) A(r) | fm_{jj'} \rangle \), we have
\[ \langle fm_j | A(r) | fm_j' \rangle = \sum_{j \ell} \sum_{M_S M_L M_J M_I} C^{L_1/2, J}_{M_L M_S M_J} C^{J'f}_{M_I M_I'} C^{L_1/2, J'}_{M_J' M_J' M_I' M_I'} \]
\[ \times \frac{(-1)^{m+M_J}}{2L+1} \frac{x^L L' L' - L}{(2L + 1)^{1/2}} C_{M_S - M_S}^{1/21/2} \]
\[ \times a_{2j}^{m'n'} \int d\Omega Y_{LM} (\Omega) Y_{L'M}' (\Omega) Y_{LM} (\Omega). \] (B.6)

Since

\[ \int d\Omega Y_{LM} (\Omega) Y_{L'M}' (\Omega) Y_{LM} (\Omega) = \]
\[ \frac{(2L + 1)(2l + 1)(2L' + 1)}{4\pi} \left( \begin{array}{c} L L' \\ 000 \end{array} \right) \left( \begin{array}{c} L L' \\ -M_L M'_L \end{array} \right), \] (B.7)

we have

\[ \langle fm_j | A(r) | fm_j' \rangle = \sum_{j \ell} P_{jj'}^{II'} a_{2j}^{m'n'} (r), \] (B.8)

where

\[ P_{jj'}^{II'} = \sum_{M_S M_L M_J M_I} C^{L_1/2, J}_{M_L M_S M_J} C^{J'f}_{M_I M_I'} C^{L_1/2, J'}_{M_J' M_J' M_I' M_I'} \]
\[ \times \frac{(-1)^{m+M_J}}{2L+1} \frac{x^L L' L' - L}{(2L + 1)^{1/2}} C_{M_S - M_S}^{1/21/2} \]
\[ \times \frac{(2L + 1)(2l + 1)(2L' + 1)}{4\pi} \left( \begin{array}{c} L L' \\ 000 \end{array} \right) \left( \begin{array}{c} L L' \\ -M_L M'_L \end{array} \right). \] (B.9)
We can evaluate this sum by changing all the Clebsch-Gordon coefficients into Wigner 3j symbols, and carrying out the sum over $M_L, M_S, M'_L, M'_S, \mu,$ and $m'$. The result is

$$P_{jj'} = (-)^{I+J+I'+J'} \left\{ \begin{array}{ccc} 2J + 1 & 2J' + 1 & 2L + 1 & 2L' + 1 & 2I + 1 & 2I' + 1 \\ 2 & 2 & 2 & 2 & 2 & 2 \end{array} \right\}^{1/2}$$

$$\times \left( \begin{array}{c} L \ll L' \\ 000 \end{array} \right) \{ J \ll J' \} \left\{ \begin{array}{c} 1/2 \\ \frac{1}{2} \end{array} \right\} \{ L \ll L' \} \{ J \ll J' \}.$$  \hspace{1cm} (B.10)

When we wish to indicate explicitly the dependence of $P_{jj'}$ on $L, J, I, L', J', I'$, and $f$, we may write $P_{jj'}^I(LJIL'J'I'f)$. The $\lambda$ in $P_{jj'}^\lambda$ in Eq. (4.23) corresponds to $I' \Theta I$ in this notation. In this work since we consider only the ground state for the target initial state, $I$ is always 0. Therefore, $I'$ in Eq. (A.10) corresponds to the $\lambda$ in Eq. (4.23).
APPENDIX C

3.1 The Relation between $B(E\lambda)$ and $\beta_\lambda$

Since the results of hadronically induced excitations are often compared to electromagnetic transition rates, we give here expressions for the transition probabilities $B(E\lambda)$ for electric multipoles. The electric multipole operator is $eP_{\lambda\mu}$, where

$$P_{\lambda\mu} = \sum_{i=1}^{Z} r_i^\lambda Y_{\lambda\mu}(\hat{r}_i),$$

and for the transition $I_n \rightarrow I_{n'}$

$$B(E\lambda) = \sum_{M'^{\prime}n'^{\prime}} |\langle I_n^\prime M_n^\prime | eP_{\lambda\mu} | I_n M_n \rangle|^2$$

$$= e^2 |\langle I_n^\prime || P_{\lambda} || I_n \rangle|^2 (2I_n^\prime + 1)/(2I_n + 1).$$

For a $\lambda$–pole excitation of an even nucleus ($I_n = 0$, $I_n^\prime = \lambda$), we have the first order in the proton deformation parameter $\beta_\lambda$:

$$\langle \lambda || P_{\lambda} || 0 \rangle = \hat{\lambda}^{-1} \beta_\lambda \int f_\lambda(r)r^{\lambda+3} dr,$$

where $f_\lambda(r)$ is the form factor of the deformed density, and

$$B(E\lambda) = |\beta_\lambda \int f_\lambda(r)r^{\lambda+3} dr|^2 e^2.$$
If we use the standard prescription,

\[ f_\lambda = R_0[d\rho(r, R_0)/dR_0], \]  

(C.6)

for the deformation of the density and assume that \( \rho(r, R_0) \) depends only on \((r - R_0)\), after integration by parts, \( B(E\lambda) \) reduces to

\[ B(E\lambda) = |(\lambda + 2)\beta_\lambda Z R_0(r^{\lambda-1})/4\pi|^2 e^2. \]  

(C.7)

This expression gives the relation between \( B(E\lambda) \) of electric transition and \( \beta_\lambda \) of a transition induced by hadronic scattering (see Ref. 21 in chapter 3.).
LIST OF REFERENCES

CHAPTER I


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CHAPTER IV


4 Atomic data and Nuclear data tables, 14, 479-508 (1974).


