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Vortex-induced noise and vibration in flow past several flat plates

Kim, Chan Mun, Ph.D.

The Ohio State University, 1989
VORTEX INDUCED NOISE AND VIBRATION
IN FLOW PAST SEVERAL FLAT PLATES

DISSEPTION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Chan Mun Kim, B.S., M.S.

* * * * *

The Ohio State University
1989

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FIELDS OF STUDY

Fluid Mechanics, Acoustics and Flow Induced Vibration
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<td>$b^*$</td>
<td>Cylinder radius</td>
</tr>
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<td>$2c$</td>
<td>Half chord length of the flat plate</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Longitudinal spacing between flat plates (or cylinders)</td>
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<tr>
<td>D</td>
<td>Diameter of the cylinder</td>
</tr>
<tr>
<td>f</td>
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<tr>
<td>$F_{yi}$</td>
<td>Dimensionless y-component of the instantaneous loading on the $i^{th}$ cylinder, $F_{yi} = F_{yi}^<em>/\rho U^2_\infty b^</em>$</td>
</tr>
<tr>
<td>$F_x$</td>
<td>Dimensionless x-component of the instantaneous loading on the flat plate, $F_x = F_x^*/(2\rho U^2_\infty c)$</td>
</tr>
<tr>
<td>$F_y$</td>
<td>Dimensionless y-component of the instantaneous loading on the flat plate, $F_y = F_y^*/(2\rho U^2_\infty c)$</td>
</tr>
<tr>
<td>h</td>
<td>Offset parameter of the splitter plate</td>
</tr>
<tr>
<td>i</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>k</td>
<td>Strength of the nascent vortex</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Dimensionless strength of the $m^{th}$ vortex</td>
</tr>
<tr>
<td>$K(z)$</td>
<td>Cauchy kernel</td>
</tr>
<tr>
<td>$K_\delta$</td>
<td>Desingularized kernel using the smoothing parameter $\delta$</td>
</tr>
<tr>
<td>L</td>
<td>Length of the slat</td>
</tr>
</tbody>
</table>
\( m_{i_s} \) Mass per unit width of the \( i^{th} \) cylinder

\( M \) Mach number

\( N \) Number of elements distributed along the half span of a vortex sheet

\( N_{cyl} \) Number of cylinders (or flat plates)

\( N_{v} \) Number of vortices in free space

\( N_{cyl} \) Number of points distributed along the surface of the cylinder

\( N_{s} \) Number of points distributed along the surface of the flat plate

\( P \) Pressure

\( P_\infty \) Ambient pressure

\( P_a \) Dimensionless acoustic pressure

\( r \) Near field radial coordinate

\( R \) Far field radial coordinate

\( Re \) Reynolds number \((Re = U_\infty L / \nu \text{ or } Re = U_\infty D / \nu)\)

\( Re_c \) Reynolds number based on the scale of the vortex core

\( r_{ij} \) Dimensionless distance between \( i^{th} \) and \( j^{th} \) vortex

\( St \) Strouhal number

\( t \) Dimensionless time

\( t_a \) Dimensionless age of the vortex

\( t_{cj} \) Time of creation of the \( j^{th} \) vortex

\( t_s \) Dimensionless sampling period for power spectrum

\( \Delta t \) Time step

\( u \) x-component velocity

\( u_i \) Dimensionless x-component velocity of the \( i^{th} \) vortex, \( u_i = u_i^* / U_\infty \)

\( U \) Velocity of the elliptically loaded wing
\( U_\infty \) Ambient flow speed
\( U_s \) Velocity at the outer edge of the boundary layer at separation
\( v \) y-component velocity
\( v_i \) Dimensionless y-component velocity of the \( i^{th} \) vortex, \( v_i = v_i^* / U_\infty \)
\( v_\theta \) Azimuthal velocity
\( V_{nj} \) Complex velocity at the \( n^{th} \) vortex due to the \( j^{th} \) vortex
\( V_{ci} \) Dimensionless instantaneous velocity of the \( i^{th} \) cylinder
\( W(z) \) Dimensionless complex potential
\( W_{ij}^F \) Dimensionless complex potential due to image doublets
\( W_{jm}^K \) Dimensionless complex potential due to the circulation around the flat plate
\( W_{jm}^M \) Dimensionless complex potential due to the cylinder motion
\( W_{jm}^V \) Dimensionless complex potential due to the image vortices
\( x \) Dimensionless streamwise coordinate
\( x_i \) Instantaneous streamwise position of the \( i^{th} \) vortex
\( x_i' \) Instantaneous streamwise position of the \( i^{th} \) vortex in centroidal coordinates
\( x_c \) Instantaneous streamwise position of the center of vorticity
\( x_{vi} \) Dimensionless initial streamwise position of the \( i^{th} \) impinging vortex
\( X \) Dimensionless far field streamwise coordinate, \( X = R \cos \theta \)
\( X_i \) Dimensionless streamwise displacement of the \( i^{th} \) cylinder
\( y \) Dimensionless transverse coordinate
\( y_i \) Instantaneous transverse position of the \( i^{th} \) vortex
\( y'_i \)  Instantaneous transverse position of the \( i^{th} \) vortex in centroidal coordinates
\( y_c \)  Instantaneous transverse position of the center of vorticity
\( y_{vi} \)  Dimensionless initial transverse position of the \( i^{th} \) impinging vortex
\( Y \)  Dimensionless far field transverse coordinate, \( Y = R \sin \theta \)
\( Y_i \)  Dimensionless transverse displacement of the \( i^{th} \) cylinder
\( z \)  Complex variable in the physical plane, \( z = x + iy \)
\( z_0 \)  Complex coordinate of the singular corner in the physical plane
\( z_s \)  Complex coordinate of a point on the vortex sheet
\( z_{ci} \)  Instantaneous center of the \( i^{th} \) cylinder
\( z_m \)  Instantaneous position of the \( m^{th} \) vortex
\( z_{vi} \)  Dimensionless initial position of the \( i^{th} \) impinging vortex
\( z_L \)  Complex coordinate of the leading edge in the physical plane
\( z_T \)  Complex coordinate of the trailing edge in the physical plane
\( Z \)  Far field complex coordinate, \( Z = Re^{i\theta} \)
\[ \alpha \quad \text{Angle of attack} \]
\[ \beta \quad \text{Included angle of the singular corner} \]
\[ \delta \quad \text{Artificial smoothing parameter} \]
\[ \phi \quad \text{Velocity potential} \]
\[ \phi_a \quad \text{Dimensionless acoustic potential} \]
\[ \mu_s \quad \text{Mass parameter} \]
\[ \mu_{is} \quad \text{Mass parameter of the } i^{\text{th}} \text{ cylinder, } \mu_{is} = \frac{m_{is}}{\rho b^*} \]
\[ \nu \quad \text{Kinematic viscosity} \]
\[ \theta \quad \text{Angular coordinate} \]
\[ \rho \quad \text{Fluid density} \]
\[ \nu_{is} \quad \text{Dimensionless damping factor of the } i^{\text{th}} \text{ cylinder, } \nu_{is} = \xi_i \omega_i \]
\[ \omega \quad \text{Dimensionless natural frequency} \]
\[ \omega_i \quad \text{Dimensionless natural frequency of the } i^{\text{th}} \text{ cylinder, } \omega_i = \frac{\omega_i^* b^*}{U_{\infty}} \]
\[ \omega_r \quad \text{Dimensionless response frequency} \]
\[ \omega_{re} \quad \text{Reduced frequency} \]
\[ \omega_s \quad \text{Dimensionless sampling frequency for power spectrum} \]
\[ \omega(x) \quad \text{Vorticity distribution} \]
\[ \xi_i \quad \text{Damping ratio of the } i^{\text{th}} \text{ cylinder} \]
\[ \psi \quad \text{Streamfunction} \]
\[ \zeta \quad \text{Transformed variable} \]
\[ \zeta_m \quad \text{Instantaneous position of the } m^{\text{th}} \text{ vortex in the transformed plane} \]
\[ \zeta_L \quad \text{Complex coordinate of the leading edge in the transformed plane} \]
$\zeta_T$ Complex coordinate of the trailing edge in the transformed plane

$\Gamma(\lambda)$ Circulation distribution along the vortex sheet

$\Gamma_m$ Circulation around the $m^{th}$ plate
1.1 Background

The unsteady flow past multiple body geometries at high Reynolds number is an extremely difficult problem; if the bodies are bluff and possibly elastic the complications are even more severe because the flow tends to separate downstream producing a large scale unsteady wake with vortex shedding over a wide range of Reynolds numbers. The proper wake structure for unsteady flow past bluff bodies is at present unknown and in calculations using discrete vortex methods to calculate the evolution of the separating shear layer (or layers), the near wake structure has not been taken into account. Because of these and other factors the complete solution of the flow at high Reynolds number in a complicated geometry such as that depicted on Figure 1.1 is a nearly impossible task and the methodology adopted in this work has been to use modelling techniques to elucidate the salient features of the flow. Of course, the primary goal of the present thesis is to determine what factors are involved in noise production in this flow and which noise-generation mechanisms are most likely to occur in practice. Of particular interest is the relative sound pressure level produced in
Fig. 1.1. Geometry of interest.
(a) Flow past a series of inclined slats embedded in a cavity section.
(b) Flow past multiple inclined slats.
buffeting of a structure by a discrete vortex, by vortex shedding, and by vibration of a part of the system.

The purpose of this thesis is to present the results of an investigation of the unsteady flow past a finite series of physical obstructions. The geometry of the present problem is reminiscent of that covering the ballast tanks of large surface ships; such a geometry may contribute to the overall drag coefficient of the ship and can lead to the involuntary reduced speed of the vessel. In this thesis we consider the flow past one louver assembly (among many) which may cover a single ballast tank. The investigation has centered around the development of self-excited oscillations of the structures as well as vortex shedding from the structures and the consequent production of noise in each case; as mentioned above, complicating the problem is the realization that vortex shedding may occur off one or more of the structures and this aspect of the problem has also been investigated. The principal thrust of this work has been to identify possible noise sources in the problem and to compute the noise field in both the near and far fields; in this regard it should be noted that the geometry of practical interest is highly three-dimensional while the present methods apply to two-dimensional problems; however, it is believed that this restriction will not alter the main conclusion of the present work which is that significant noise may be generated as a result of the development of vibration, vortex shedding or buffeting from upstream or from any combination of the three effects.

The problem of interest is the high Reynolds number flow past a series of (possibly elastic) slats embedded in a cavity as shown on Figure
1.1 (a). The spacing between slats is defined by the parameter \( d_s/L \) and the value of the spacing parameter considered here is assumed to be small such that \( d_s/L < 4 \). The flow is assumed to be inviscid in the main and viscous effects are accounted for in the form of possible vortex shedding from any one or all of the slats. Conformal mapping techniques are employed to map the region depicted into a simpler domain where the complex potential can be determined. The presence of the cavity in the present problem is problematical since the flow inside the cavity may be very complicated; indeed, there is little or no available experimental data on the flow inside ballast tanks and in the present work, the cavity flow has not been considered. In this regard the paper by Buxton and Logan (1987) discuss the performance of the ballast tanks from a system point of view, while Okamura et al. (1988) discuss the emergence of flow induced vibration in enclosed hull cavities without a louver-type cover. There is no other work on the present problem of which the author is aware. The focus has thus been on the flow past a series of inclined plates depicted on Figure 1.1 (b) in which there is no upstream splitter plate. Clearly the flow of Figure 1.1 (b) is still very complicated because of the number of inclined plates and the fact that some form of (perhaps very irregular) vortex shedding phenomenon will probably occur from any or all of the plates. It should be noted that the goal of this work is not to rigorously describe the flow past the set of plates of Figure 1.1; rather it is to deduce the relative unsteady forces and the noise field induced by the physical mechanisms likely to occur in
some way in the actual flow, namely, vortex shedding, vibration, and buffeting.

There are many issues involved in the calculation of a flow of this type and it is useful at this stage to summarize the approach to the problem. For simplicity consider the problem of buffeting of the slats by one (or more) disturbances in the upstream flow, a disturbance which may be modelled by a potential vortex. First a conformal mapping is required to simplify the geometry and the conformal mapping in the present problem is not trivial. Such a mapping requires that each slat be mapped conformally into another body for which the complex potential is known or can easily be calculated. In the present case the region outside each slat is mapped into the region outside a circular cylinder, a region in which the complex potential can be easily determined. This procedure is done iteratively and is complicated by the fact that the transformation is singular at each edge of the slat. Moreover the flow in the transformed plane is not easily calculated, requiring a complicated image procedure involving the placement of additional vortices inside each cylinder in order to satisfy the solid wall boundary condition. The possible vibration of one or more of the slats is also a complicating feature of the work; in general both the flow and the structure position must be calculated simultaneously which requires an iterative procedure at each time step for which the solutions are required. In addition, there is a question of the proper flow field between the slats, as well as downstream of the last slat. Because of the high angle of attack of each slat in the assembly, it is likely that vortex shedding will occur downstream of (at least) the last
slat; the calculation of such a flow is not trivial for the case of a single body and in the present geometry the vortex shedding problem is much more difficult. It is clear from this short discussion that the flow field for this problem is complex and requires technical information from a wide variety of sub-areas within fluid mechanics; at this stage it is useful to discuss the problem of interest in more detail.

1.2 Description of the Problem

The geometry of interest is depicted on Figure 1.1 (a); the flow is characterized by the Reynolds number $Re = \frac{U_\infty L}{v}$, where $L$ is the length of a slat, $U_\infty$ is the freestream velocity upstream of the slat assembly and $v$ is the kinematic viscosity. The Reynolds number is assumed large ($Re \sim 10^5$) and the purpose of this discussion is to describe the possible flow patterns which may occur in this geometry and the nature of the noise which may be generated by unsteady flow processes which may occur in the neighborhood of the slats.

The flow impinges on the first slat having negotiated the trailing edge of the upstream splitter plate. The presence of a cavity-type flow at low velocity will result in a separated shear layer impinging from this upstream splitter plate based on the very rapid adjustment of the velocity from $U_\infty$ above the splitter plate to $\sim 0$ below. The type of flow generated locally in this front section of the louver has been described extensively by Rockwell and Knisely(1979) and the flow depends crucially on the distance between the upstream splitter plate and the downstream slat.
The incorporation of the cavity into the present problem is thus very complicated involving specification of a effectively zero velocity below the splitter plate of Figure 1.1 (a); the lack of experimental data on the cavity flow makes this assumption suspect and in what follows, the mean flow is assumed to be uniform above and below the splitter plate upstream of the first slat and downstream of the last slat.

The flow between the slats is also complicated and because of the complicated geometry and the lack of published experimental data in this configuration, some aspects of the flow which occur here must remain speculative. Thus it is useful to refer to results for the flow past a series of two cylinders for which experimental results exist and to then suggest how the results may apply to the configuration of interest in this investigation.

For the case of two cylinders arranged in a tandem configuration previous work (Zdravkovich (1977)) suggests that several types of steady flow may occur depending on the spacing parameter $d_s/D$ where $d_s$ is the longitudinal spacing between the centers of cylinders and $D$ denotes the diameter of the cylinder. The different regimes of flow are illustrated on Figure 1.2. For $d_s/D < 4$ it appears that vortex shedding from the first cylinder may be suppressed in the case of rigid cylinders and the flow between the cylinders may be either of two types; Figure 1.2 (a) depicts in qualitative terms the inviscid, irrotational flow past two cylinders with vortex shedding only off the second cylinder. In the main, the flow (for rigid cylinders) remains steady upstream of the second cylinder. Figure 1.2 (b) depicts the situation where a Kirchoff
Fig. 1.2. Speculated flow types for two cylinders in a tandem configuration.
(a) Inviscid, irrotational flow with vortex shedding off the second cylinder.
(b) Kirchoff free-streamline flow.
(c) Modified Kirchoff free-streamline flow.
(d) Inviscid, irrotational flow with vortex shedding from both cylinders.
(e) Same as Figure 1.2. (d) except the vortex shedding from the first cylinder is due to the vibration of the cylinder.
Fig. 1.2. (Continued)
freestreamline flow exists between the cylinders. The angle at which the free stream line leaves the cylinder may take on several values depending on the nature of the flow in the viscous boundary layer surrounding the cylinder (Imai(1953)). The region between the cylinders is stagnant in the classical Kirchoff scenario. The third type of flow (Figure 1.2 (c)) which may occur between the cylinders is of the modified Kirchoff type with free streamlines bounding the wake as before but with a recirculatory motion at a pressure $p < p_\infty$ where $p_\infty$ is the pressure outside the wake; $p$ may or may not be constant. Note again that no vortex shedding occurs from the first cylinder due to the rather small value of $d_s/D$. At higher values of the spacing parameter, vortex shedding may occur off the first cylinder and this is depicted on Figure 1.2 (d); in this case the flow in the wake of the first cylinder may or may not be steady. Vortex shedding from the first cylinder may also occur if the cylinders are elastic and this situation is depicted on Figure 1.2 (e). In the geometry of current interest similar flow regimes may occur with the main difference that the separation point of each of the slats is fixed at the sharp edges whereas in the case of the cylinder the separation point may oscillate about a mean value.

On Figure 1.3 (a)-(c) are the postulated flow regimes for the slat assembly corresponding to Figures 1.2 (a), (c), (d). As can be seen the flow is extremely complex and due to the lack of experimental data in this arrangement some speculation regarding the flow about and between the slat assembly must be made. In particular, it is at present unclear whether vortex shedding will occur off each slat or just off the
Fig. 1.3. Associated flow regimes for the slat assembly.
(a) Attached flow with vortex shedding only from last slat.
(b) Modified Kirchoff free-streamline flow.
(c) Full scale flow separation.
last one; because of the proximity of the slats it is likely that significant, organized vortex shedding will only occur off the last slat (for the case where the slats are rigid) and this problem has been investigated. A more detailed discussion for the case where the slats are elastic is given in Section 4.5.

The calculation of the noise field depends upon the nature of the flow in the neighborhood of the slat assembly. For this reason, the calculation of the noise in both the near and far fields is relatively straightforward once the flow field is determined; the critical link between the flow and the noise is a generating function, the form of which depends on the precise geometry and form of unsteadiness in the near field.

Having summarized the critical technical aspects of the problem, previous work on the various aspects of the problem can be reviewed.

1.3 Previous Work

As mentioned above the problem of interest here involves complex technical problems in several diverse areas and it is useful at this stage to summarize previous investigations in each of these areas.

**Flows Past Multiple Body Arrangements**

The steady potential flow past a pair of circular cylinders has been considered analytically by Dalton and Helfinstine (1971) and by
Yamamoto (1976). They obtained the flow field by a rather sophisticated application of the method of images (Milne-Thomson(1968)). Consider the flow past a single cylinder; using simple potential flow theory the solution can be obtained by the superposition of a uniform flow plus a doublet to enforce the no-normal-flow condition on the surface. If another cylinder is placed in its wake there will be a normal velocity induced on the first cylinder which is removed by placing another doublet inside the second cylinder; this second doublet will induce a normal velocity on the second cylinder which is removed by placing a third doublet in the second cylinder; this procedure is continued and theoretically an infinite number of doublets are required. However, because the strength of each doublet decreases rapidly only several doublets in each cylinder are required for sufficient numerical accuracy (in the present work no more than four doublets in each cylinder are required so that the velocity on each cylinder is reduced to less than $10^{-4}$ of the ambient flow speed on each cylinder). The method employed in this work is that described in Dalton and Helfinstine (1971) with the additional complexity that the motion of potential vortices outside the cylinders must be calculated and thus an additional image system in each cylinder is required.

The analytical work described above is deficient in the prediction of the flow past bluff bodies at high Reynolds number because the flows remain attached to the bodies at all times. Experimental work by a number of authors suggest that this is not the case (see the review by Zdravkovich(1977) among many others) and very complex separated flow
patterns are observed over a wide range of Reynolds numbers and for a variety of cylinder configurations. We discuss the particular case of the flow past a tandem arrangement of two cylinders; a useful and comprehensive review of this problem as well as flows past cylinders in other configurations is given by Zdravkovich (1977). The flow past a tandem cylinder arrangement depends on a large number of parameters, the main ones being the Reynolds number and the spacing parameter $d_s/D$. The behavior of the flow as a function of these two parameters is complex, however, some general comments may be made. For $d_s/D < 3-4$ there is an interference effect on both cylinders such that vortex shedding from the first cylinder is suppressed and occurs only off the second cylinder (Figure 1.2 (a)). It is possible that the suppression of significant and coherent vortex shedding off the first cylinder is due to the same mechanism by which vortex shedding is suppressed when a splitter plate is attached to a single circular cylinder as described by Roshko (1955)(see also Zdravkovich (1977)). There is an associated decrease in the combined drag of the two cylinders when compared with the total drag of each of the two cylinders alone. This decrease is presumably due to the absence of vortex shedding from the first cylinder. Additional pressure measurements conducted by Zdravkovich (1977) indicate that the flow between the cylinders consists of a wake-type flow at a spatially constant time-averaged pressure. Although the flow was deemed to be unsteady, only time-averaged data were presented; consequently the precise nature of the unsteady flow between the cylinders could not be determined.
For values of the spacing parameter $d_s/D$ greater than about 4 the flow is observed to undergo a radical change. At $d_s/D = 4$ there is a marked jump in the pressure coefficient and the Strouhal number with vortex shedding occurring from both cylinders at the same shedding frequencies (Ishigai, et al. (1972)). These measurements were taken at a Reynolds number $Re=8000$. Such behavior appears to persist at somewhat higher Reynolds numbers (to about $Re=10^5$) although the critical spacing at which the flow changes character appears to depend on Reynolds number.

It seems clear from the present discussion that the flow between two bodies oriented in a tandem arrangement is complex; although the flow there appears unsteady, very little flow visualization data indicating the precise nature of the unsteadiness and the structure of the wake of each body is available. Nevertheless it seems clear that the main features of the flow between the slats will be qualitatively similar to the flow past the tandem cylinder arrangement. It should be noted that the effect of adding more cylinders to the arrangement has not been addressed; no experimental data which addresses the problem of more than two cylinders in a tandem arrangement appears to be available although there has been a large amount of work on the flow past large-scale tube banks with application to heat exchanger technology (Paidoussis, Mavriplis, and Price (1984) among many others).
Flow Induced Vibration of Bodies

There are several methods by which flow-induced vibration of bodies may be analyzed. Because of the difficulty of calculating complicated flows in this area, experimental data has often been employed to generate fluid force coefficients which are then employed to calculate the vibration characteristics of the body; in the lift-oscillator model the lift coefficient is modelled by a second-order ordinary differential equation in the same way as the vibration is modelled (Landl (1975), Iwan and Blevins (1974), Blevins (1977), Hartlen and Currie (1970)). Unfortunately, much of the detail of the fluid flow is lost in this approach although important results have emerged. An excellent review in this area has been written by Parkinson (1974). In the lift-oscillator approach the form of the fluid force coefficients (i.e. the lift and drag coefficients) are determined by dimensional analysis and may depend in a complicated, perhaps nonlinear way on the motion of the structure. The free parameters which arise in this approach are then determined experimentally. This method has the advantage of being simple since only (in general) a few ordinary (often nonlinear) differential equations are required to solve for the vibration problem. The major disadvantage of this method is that the vibration problem is specific to the set of experimental data used to generate the fluid force coefficients. In some work, vibration of a body has been approached from the point of view of a stability analysis in which harmonic oscillations are assumed and a critical velocity beyond which the oscillations will be
sustained is determined (Lever and Weaver (1982), Price and Paidoussis (1986), Paidoussis, Mavriplis, and Price (1984), Tanaka and Takahara (1981)); experimentally generated fluid force coefficients are often used in this approach as well. It should be pointed out that using experimental data in the vibration problem has often proved necessary because of the extreme complexity of the geometry; an example of this fact is the work of Paidoussis, Mavriplis, and Price (1984) who considered the vibration of a complex tube array using the stability theory referred to above. Much strictly experimental data has been published in this area as well.

In the present work we take a different approach and attempt to model the flow field directly and calculate the vibration along with the flow. Vibration is assumed to be generated by buffeting. The advantage of this approach is that no experimental data is required; a disadvantage is that the analysis becomes relatively complex due to the complexity of the geometry (Figure 1.1).

**Acoustics**

There appear to be several noise sources in the present problem as described earlier; however all the mechanisms for noise generation fall within two categories: vibration of the structure and the generation of vorticity upstream of or near the bodies. The acoustics problem depends on the form of the flow field and solutions will be computed using the method of Crighton (1972). As the Mach number \( M \to 0 \) the acoustic field splits into a near and far field and the method of matched
asymptotic expansions may be employed to calculate the noise field. Crighton's method has been used in similar problems by Conlisk (1985) and by Conlisk, Guezennec and Elliott (1989). In this method the far field noise may be calculated in closed form subject to the determination of a generating function which is characteristic of the near field. Such a method provides a useful alternative to the method of Lighthill (1952) which is difficult to implement in the present problem.

**Vortex Shedding**

Perhaps the most difficult aspect of the present work is the realization that vortex shedding will occur off one or more of the slats. The calculation of the position of the separated shear layer is extremely difficult when only one body is present; the fact that in the present problem there are many renders such a calculation even more difficult. The effect of vortex shedding will be modelled by a set of discrete vortices. The separated flow past an inclined flat plate has been considered using this method by Sarpkaya (1975); he advanced the vortices shed off the two edges by a first order Euler scheme. Two parameters must normally be chosen in these methods and these are the point of introduction of the individual discrete vortices and their associated strengths. Sarpkaya (1975) determined these two parameters by requiring the satisfaction of the Kutta Condition and by the requirement that the amount of vorticity shed from each edge is given by
\frac{dk}{dt} = 5U^2_s \quad (1.1)

where $U_s$ is the velocity at the outer edge of the boundary layer at separation. This formula was first given by Fage and Johansen (1927, 1928) in their seminal experimental work on flows past bluff bodies, mainly single, vertically oriented flat plates and cylinders. This formula also can be shown theoretically to hold although there still remains some doubt whether the constant is really .5 and slightly smaller values have sometimes been used (Sears (1956), Stansby (1981)). A particular problem in these methods in the use of equation (1.1) is the definition of $U_s$ and this is discussed fully in Section 3.4. It should be noted that the imposition of the Kutta condition is associated with the removal of the velocity singularity at the edges of the plate and thus the uniqueness of the solution. Recent development of local interaction (triple-deck) theory has shown that, within restricted parameter ranges, only those 'outer' potential flows that satisfy the Kutta condition are, in general, compatible with an acceptable multilayered 'inner' viscous structure (Daniels (1978), Crighton (1985)). There is some doubt that the standard Kutta Condition holds for unsteady flow (McCroskey (1977), Crighton (1985), Poling and Telionis (1986)). However there is both direct and indirect evidence for the validity of the standard Kutta condition in restricted regions around the edges of the plate (Poling and Telionis (1986)) and in the present problem (at least for a single body) the flow near the (fixed) separation point appears substantially steady (Fage and Johansen (1927, 1928)); thus use of the steady Kutta Condition appears to
be justified. Note that the physical ramifications of imposing the Kutta condition represent the mechanism by which both the fluid loading and the noise field are modified (Crighton(1985)).

Kiya and Arie (1977) repeated the calculation of Sarpkaya (1975) and held the position of introduction of the vortices fixed. A crucial test of the viability of these methods is the prediction of the amplitude of the oscillating drag and lift forces on the plate and in this respect the results of Kiya and Arie (1977) indicate a much larger value than that reported by Lught and Haussling (1974) in their Navier-Stokes calculation for flow past a slender elliptic cylinder placed at incidence. Kiya and Arie (1977) compared their results with those of Lught and Haussling(1974) for the flow past an elliptic cylinder because of their claim that they had no experimental data at their disposal (i.e. such as Fage and Johansen (1927, 1928)). The results of Sarpkaya (1975) for the mean normal force coefficient are also high by about 20-30% and it appears difficult to obtain good agreement with experiment for the forces even though global properties of the wake flow appear to be reproduced. Methods similar to those of Sarpkaya (1975) and Kiya and Arie (1977) have been used by Chein and Chung (1988) who used a very large time step to advance the individual vortices; their claim that their method is cheaper to use in terms of computer time is obviated by the fact that the results they present have not been demonstrated to be independent of time step used (unfortunately this situation is typical of the work in this area.). Indeed, use of these methods has often reflected the fact that there is much art involved in the implementation. Unfortunately, the alternative is to
calculate the solution to the full Navier-Stokes equations, which, for the geometry and Reynolds number of interest would be an extremely difficult, if not impossible, task. Computation and modelling of vortex shedding from multiple bodies in the tandem arrangement of interest here does not appear to have been considered in the open literature.

It should be noted that, as far as the present author can tell, there is no experimental data for the flow past an array of flat plates as is of interest here. This fact makes the analytical and numerical work that much harder and the present work has been guided essentially by the single body results described by Fage and Johansen (1927, 1928) (see also Tyler (1931)).

Slat Assembly Mapping

There are several methods available for mapping multiple cylinders into same number of flat plate airfoils; the flow about the circular cylinders will be known using the imaging procedure described earlier provided the conformal mapping is given. The conformal mapping procedure can be done sequentially as in Ives (1976) or iteratively as in Halsey (1977). However, the method described by Ives (1976) is cumbersome for arbitrary body configurations because a contour, once mapped to a canonical contour, must preserve its shape during mapping of all other contours. The technique described by Halsey (1977) eliminates this problem by iteratively mapping each individual contour.
Garrick (1944) was the first to consider the potential flow past a lattice of airfoils in which an infinite series of airfoils were mapped to a single circle. Ives (1976), as mentioned above has also addressed the problem of interest, however his method becomes cumbersome for more than about two elements. In the present work the method of Halsey (1977) has been applied; details of the present application of the method are provided in Chapter IV.

1.4 Summary and Outline of the Study

As noted in the previous sections, the description of the flow past the slat assembly depicted on Figure 1.1 is extremely complicated involving a wide variety of technical issues, some of which have not been resolved here. The issues which must be addressed in the solution of this problem include determination of a realistic inviscid flow past the slat assembly. This portion of the study involves the determination of the proper conformal mapping for the series of slats and requires the development of a sophisticated imaging procedure in the mapped plane; an additional imaging procedure is required to advance vortices as well. The nature of the flow in the wakes of each of the slats is somewhat speculative because of the lack of experimental data in the geometry of interest here. It appears, however, that based on experimental data for the related problem of flow past a bank of circular cylinders, qualitative conclusions concerning the possible forms of the flow between the slats may be made (Figure 1.3).
The greatest uncertainty in the present problem is the range in spacing parameter between the slats where vortex shedding will take place. In addition, the nature of the flow in the near wake of each of the slats must remain somewhat speculative. Nevertheless, significant results concerning the level and frequency content of this flow field may be obtained if judicious assumptions are made. In this regard some simplifications in computing the flow past the bank of slats of Figure 1.1 have been made based on computations done for the case of two slats, the results of which are discussed extensively in Chapter V.

The noise field of the present problem may be characterized as a dipole resulting from the interaction of the vortex dominated unsteady flow with a possibly elastic structure. It appears that two major sources significantly contribute to the interaction; the upstream flow disturbances and the vortex shedding from the bodies. In addition, the body surface vibration may change the fluid motion and thus provides an additional contribution to the noise generation.

The main objective of the present work is the development and the implementation of a fluid-structure interaction model to investigate the noise field produced in a flow past the multiple slat geometry. The emphasis is made on the modelling of the dynamics of the unsteady flow since the noise field can be directly calculated once the flow field solution is available. A numerical method to conformally map the slats into the circular cylinders is developed and the vortex shedding is modelled by applying the discrete vortex method. Of particular interest is the identification of the significant source of the noise and to this purpose
various characteristics of the noise field are investigated using the method of matched asymptotic expansions.

The plan of the thesis is to consider the geometry of Figure 1.1 through consideration of a series of problems of increasing complexity. In Chapter II we describe the methods for calculating the potential flow past a tandem cylinder arrangement which is the first step in the calculation of the flow past the slat assembly of Figure 1.1. We consider the buffeting of the tandem cylinder arrangement by one or a series of potential vortices and the consequent vibration which may occur if the cylinders are elastic. The acoustic field associated with these flows is also calculated and the frequency content and the sound pressure level for a variety of structure parameters is presented. These potential flow solutions are deficient in the sense that no separation of the fluid from either cylinder is accounted for. To begin to consider vortex shedding from a multiple slat arrangement, the problem of vortex shedding from a single flat plate is discussed in Chapter III. Here the methods to be employed in the calculations of the multiple slats are outlined and the issues involved in approximating vortex sheets by point vortices are summarized. From each of Chapters II and III the noise field is also calculated and analyzed. The conformal mapping of multiple slats into an equal number of circular cylinders is discussed in Chapter IV for a wide variety of configurations. Streamline patterns are plotted and impinging vortex motion and the associated noise field are again calculated. In Chapter V vortex shedding is incorporated into the multiple slat geometry and includes an extensive discussion of the
results for the two-slat case and the necessary simplifications required for the calculation of the flow for the geometry depicted on Figure 1.1. The summary and the conclusions of the present study are given in Chapter VI.
2.1 Introduction

As a first step toward a description of the flow past the multiple slat geometry of Figure 1.3, we deal here with the problem of flow past a tandem cylinder arrangement in which both cylinders may be elastic (in both directions) and may have different structural properties; the geometry is depicted on Figure 2.1. The mechanism of excitation of interest is the case of buffeting by one or more potential vortices. Of particular interest is the interaction between the impinging vortex (or vortices) and the two cylinders and it should be noted that large amplitude vibrations are not precluded here. The results indicate that both the force(s) on the cylinder and the acoustic field so generated exhibit an impulsive character as the vortex (or vortices) passes over the two cylinders followed by a time-periodic component as the vortex moves downstream. It is of interest to establish the frequency of the response (which is not the natural frequency of the structure) as a function of mass parameter (i.e. mass ratio).

The present flow field consists of the steady flow past two cylinders, plus one or more vortices which impinge on the two bodies,
Fig. 2.1. Geometry and flow field. The cylinders may translate in both the x and y directions. The numbers inside each cylinder identify that cylinder for future reference.
plus a component due to cylinder vibration. The steady flow past a pair of circular cylinders has been considered by Dalton and Helfinstine (1971) and Yamamoto (1976); dynamic studies of vibration of tube arrays have been published by a number of authors including Paidoussis, Mavriplis, and Price (1984), Price and Paidoussis (1986), Lever and Weaver (1982) and Tanaka and Takahara (1981) among many others. Bokaian and Geoola (1984 a, b) have investigated very low frequency vibrations in flow past two cylinders experimentally and analytically. In much of this work the cylinder vibration has been investigated from the point of view of a stability theory in which harmonic oscillations are assumed and a critical flow velocity determined. Moreover, at some point, experimental data in the form of fluid force coefficients is usually required to complete the solution. These fluid force coefficient equations are usually obtained using a quasi-static analysis. The modeling of the present problem is fundamentally different; a potential vortex is used to generate the cylinder vibration and no harmonic oscillation is assumed a priori. In addition, the cylinder(s) vibration and vortex motion are computed interactively and so the full solution for the flow and cylinder vibration is calculated. During the time in which the vortex is close to each of the cylinders an extremely non-linear interaction takes place in which the forces and cylinder vibration are far from harmonic.
2.2 Model Formulations

**Fluid Mechanics**

The complex potential for this flow is constructed by the method of images in which, to satisfy the condition of no normal flow into the cylinders, successive images are introduced in the manner of Dalton and Helfinstine (1971). Superimposing NV moving vortices on the steady flow past NC cylinders the complex potential is given by

\[ W(z) = z - \sum_{m=1}^{NV} \frac{ik_m}{2\pi} \ln(z - z_m) + \sum_{m=1}^{NV} \sum_{j=1}^{\infty} W_{jm}^V + \sum_{j=1}^{\infty} (W_j^F + W_j^M) \]  

(2.1)

where the first two terms correspond to uniform flow plus NV vortices in free space, \( W_{jm}^V \) is the image potential for NV vortices outside NC cylinders, and \( W_j^F \) are image doublets corresponding to steady flow past NC fixed cylinders; \( W_j^M \) is the potential corresponding to finite amplitude cylinder motion. The subscript \( j \) denotes the number of the image. In equation (2.1) \( i = \sqrt{-1} \), \( z_m \) is the instantaneous position of the \( m^{th} \) vortex, and \( k_m \) is its strength. In practice only three images of the vortex in each cylinder are necessary for an accurate description of the flow; similarly, only three cylinder image doublets are required since the strength of the image doublets decreases rapidly (Dalton and Helfinstine (1971)); in this regard the error in the normal velocity on the cylinder surfaces using three images is about \( 0(10^{-4}) \). The origin for the coordinate system is fixed at the initial position of the center of the
first cylinder which requires the inclusion of the term $W_j^M$ in equation (2.1) (ie. the normal velocity of the fluid at the cylinder surface is the velocity of the cylinder).

In (2.1) all lengths have been nondimensionalized on cylinder radius $b^*$ and all velocities on the flow speed far upstream, $U_\infty$. The precise form of $W_{jm}^V$, $W_j^F$, and $W_j^M$ are (Dalton and Helfinstine 1971)

\[
W_{jm}^V = \frac{i k_m}{2\pi} C_j e^{i\alpha_j} \left\{ \ln(z - \epsilon_{jm}) - \ln(z - \rho_j) \right\}
\]
\[
W_j^F = A_j \frac{e^{i\beta_j}}{z - \gamma_j}
\]
\[
W_j^M = -D_j \frac{e^{i\beta_j}}{z - \gamma_j}
\]

where

\[
\epsilon_{jm} = z_{cl} + 1/(\delta_{jm} - z_{cl})
\]
\[
C_j = 1
\]

and

\[
\begin{align*}
\theta_j &= \alpha_j = 0 \\
\beta_j &= \tan^{-1}[\text{Im}(V_{cl})/\text{Re}(V_{cl})] \\
\delta_{jm} &= z_m \\
A_j &= 1 \\
D_j &= |V_{cl}| \\
\rho_j &= \gamma_j = z_{cl}
\end{align*}
\]

\{ j \leq NC \}
and

\[
\begin{align*}
\theta_j &= \pi - \theta_k \\
\alpha_j &= \pi + 2\eta_{jk} - \alpha_k \\
\beta_j &= \pi + 2\eta_{jk} - \beta_k \\
\eta_{jk} &= \tan^{-1}[\text{Im}(\sigma_j)/\text{Re}(\sigma_j)] \\
\sigma_j &= r_k - z_c \\
\delta_{jm} &= \varepsilon_{km} \\
A_j &= A_k/|\sigma_j|^2 \\
D_j &= D_k/|\sigma_j|^2 \\
\rho_j &= \gamma_j = z_c + 1/\bar{\sigma}_j
\end{align*}
\]

and

\[A_j = C_j = D_j = 0 \text{ for } i = n, j > NC.\]

The bars denote complex conjugate and here, the subscript indices refer to the \(j^{th}\) image vortex/doublet located in the \(i^{th}\) cylinder which is the image of the \(k^{th}\) image vortex/doublet located in the \(n^{th}\) cylinder. Also \(z_c\) and \(V_c\) represent the nondimensionalized center and instantaneous velocity of the \(i^{th}\) cylinder respectively; \(\text{Im}(V_c)\) and \(\text{Re}(V_c)\) refer to the imaginary and real parts of the velocity \(V_c\) respectively. The notation in this problem is difficult to understand and to elucidate the meaning of the index \(j\), in the present case of two cylinders, there are six images for each of \(W_j^F\) and \(W_j^M\) corresponding to three images in each of the cylinders. Thus, the index, \(j\), in the sum
for $W_j^F$, $W_j^M$ runs from 1 to 6 for this case. The j index for $W_{jm}^V$ runs from 1 to 12 since each vortex induces 2 image vortices in each cylinder.

In order to calculate the acoustic field and vibration characteristics of the tandem cylinders, the vortex path is required and this is obtained by solving the set of equations

$$\frac{dx_j}{dt} = u_j, \frac{dy_j}{dt} = v_j, j = 1,\ldots, NV$$

(2.3)

where $(x_j, y_j)$ is the instantaneous position of the $j^{th}$ vortex and NV is the number of vortices in the flow. Also in equation (2.3), $(u_j, v_j)$ is the velocity at the $j^{th}$ vortex given by

$$u_j - iv_j = (\frac{dW}{dz} + \frac{ik_j}{2\pi} \frac{1}{z - z_j})_{z = z_j}$$

(2.4)

The vortices are advanced using a fourth order Adams-Moulton method; for the present case of buffeting where a comparatively small number of vortices have been used, three-figure accurate results for the vortex path have been produced for a step size $\Delta t = .025$.

**Cylinder Motion**

The structure motion is assumed to occur in the form of a translation of the center of each cylinder. The equations of motion then are(Appendix A)
\[ \ddot{X}_i + 2\nu_{ls} \dot{X}_i + \omega_i^2 X_i = \frac{F_{xi}}{\mu_{ls}}, i = 1, 2 \]  
\[ \ddot{Y}_i + 2\nu_{ls} \dot{Y}_i + \omega_i^2 Y_i = \frac{F_{yi}}{\mu_{ls}}, i = 1, 2 \]  

(2.5)

where \((X_i, Y_i)\) is the position of the center of the \(i^{th}\) cylinder; \(\nu_{ls}\) is the damping factor, \(\omega_i\) the natural frequency and \(\mu_{ls}\) is a mass parameter. These quantities are defined by

\[ \nu_{ls} = \nu_{is} \omega_i b^*/U_\infty, \omega_i = \omega_i b^*/U_\infty, \mu_{ls} = m_{ls}/\rho b^2 \]  

(2.6)

where \(^*\) indicates a dimensional quantity, \(b^*\) is the radius of the cylinder, \(\rho\) is the fluid density and \(m_{ls}\) is the mass per unit width of each cylinder. Also in (2.5) \(F_{xi}\) and \(F_{yi}\) are the dimensionless fluid forces on the \(i^{th}\) cylinder in the \(x\) and \(y\) directions respectively based on \(\rho U_\infty^2 b^*\). The initial positions of the cylinder are given with the initial velocity being equal to zero. The numerical techniques used to solve (2.5) are described in Conlisk (1985). It should be noted that the disturbance causing the structure motion is a vortex (or vortices) placed far upstream at \(t = 0\). The forces \(F_{xi}\) and \(F_{yi}\) are then calculated directly using the Blasius Theorem defined as (Milne-Thomson(1968))

\[ F_x - iF_y = \frac{1}{2} i \oint (\frac{dW}{dz})^2 dz + i \frac{\partial}{\partial t} \oint W d\bar{z} \]  

(2.7)
The motion of the structure is fully coupled with that of the vortex from this equation and no empirical constants have been used in the present problem. Note that the forces in equation (2.7) include the terms corresponding to the fluid damping and the fluid inertial force (Blevins(1977), Sarpkaya(1979)).

The Acoustic Field

It is convenient to calculate the acoustic potential $\phi_a$ which is related to the acoustic pressure as

$$P_a = -\frac{\partial \phi_a}{\partial t}$$

where $P_a$, $\phi_a$ and time are dimensionless. $\phi_a$ satisfies the linearized wave equation in the far field which is defined as $X=XM$, $Y=YM$, where $M$ is the Mach Number and $(x,y)$ are the dimensionless near field coordinates (see Figure 2.1). The far noise field at low Mach number $M$ is generated by the unsteady vortex motion near the two cylinders as well as the structural vibration. To obtain the generating function, the unsteady portion of the complex potential in equation (2.1) must be expanded for $M \to 0$ and then differentiated with respect to time. The algebra is tedious for two cylinders (see Appendix B for details) and the result is (Kim and Conlisk(1988))

$$P_a = -\frac{M}{R} \left\{ H_1 ' \cos \theta + H_2 ' \sin \theta \right\}, \ R \to 0$$

(2.9)
where \( H_1 \) and \( H_2 \) are functions of time (\( H_1' = \frac{dH_1}{dt}, \ H_2' = \frac{dH_2}{dt} \)) and involve both vortex motion and cylinder vibration components, \( R^2 = X^2 + Y^2 \) with \( X = R \cos \theta, \ Y = R \sin \theta \). The solution to the wave equation subject to (2.9) as \( R \to 0 \) and an outgoing wave condition at \( \infty \) is given by (Appendix B)

\[
P_a = -\frac{M}{R} \left[ \cos \theta \int_0^{t-R(t-s)} \frac{H_1''(s)ds}{\sqrt{(t-s)^2 - R^2}} + \sin \theta \int_0^{t-R(t-s)} \frac{H_2''(s)ds}{\sqrt{(t-s)^2 - R^2}} \right] (2.10)
\]

where \( H_1'' \equiv \frac{d^2H_1}{dt^2} \). \( H_1 \) and \( H_2 \) are complicated functions of the vortex path and the structure motion and \( H_1 \) is given by

\[
H_1 = \text{Re} \left[ \sum_{j=1}^{\infty} (A_j e^{i\alpha_j} - D_j e^{i\beta_j}) + \frac{i}{2\pi} \sum_{m=1}^{NV} k_m z_m \right. \\
\left. + \frac{i}{2\pi} \sum_{j=1}^{\infty} C_j e^{i\theta_j} \sum_{m=1}^{NV} k_m \left( \rho_j - \epsilon_{jm} \right) \right] (2.11)
\]

with \( H_2 \) being the corresponding imaginary part. In equation (2.11) the first term is the portion of the noise field due to structural motion, the second term is due to the convection of the NV vortices in the flow and the third term is due to the motion of the image vortices.
2.3 Results and Discussion

Results have been computed for several values of the reduced velocity (i.e. structure natural frequency), mass parameter and for different vortex impingement configurations. Two important numerical parameters in the computation are the time step \( \Delta t \) and the number of points \( N_{\text{cyl}} \) distributed along the surface of each cylinder in the calculation of fluid loading. The numerical accuracy has been investigated for several values of these two parameters for a single vortex of strength \( k_1 = -1 \) and the results are tabulated in Table 2.1. Generally, it was found that the values of \( \Delta t = .025, \ N_{\text{cyl}} = 72 \) gave the extremely accurate solutions and thus have been employed throughout the computation.

To compare with the results for elastic cylinders, Figures 2.2 and 2.3 depict the results for the vortex path, forces, and the acoustic pressure in the near field \( H'_1, H'_2 \) with \( H'_1 = \frac{dH_1}{dt} \) for the fixed-cylinder case where the buffeting is due to a single vortex of strength \( k_1 = -1 \); here \( \Delta x_{\text{cyl}} = 4, \ \Delta y_{\text{cyl}} = 0 \) and the vortex is initially placed at \( x_{v1} = -6, \ y_{v1} = .5 \). Note the purely impulsive character of the motion as the forces and acoustic pressure rapidly decay to their initial static values as the vortex passes downstream. According to equation (2.8), \( H'_1 \) represents the acoustic pressure component involving the streamwise motion of the vortex and two cylinders while \( H'_2 \) involves the transverse motion, and on Figure 2.3, \( H'_2 \) is about an order of magnitude larger than \( H'_1 \). Hence it is likely that the dominant
Table 2.1 Typical numerical results for the solution as a function of time step and the number of points along the cylinder surface. Here $\omega_1 = \omega_2 = 2.23$, $\mu_1 = \mu_2 = 5.0$, $\xi_1 = \xi_2 = 0$, $k_1 = -1$ and the dimensionless times are shown.

(a) Parameters as a function of time step $\Delta t$ ($N_{\text{cyl}} = 72$)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\Delta t$</th>
<th>$x_1$</th>
<th>$y_1$</th>
<th>$H'_1$</th>
<th>$H'_2$</th>
<th>Drag Force</th>
<th>Lift Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Cylinder 1</td>
<td>Cylinder 2</td>
</tr>
<tr>
<td>7</td>
<td>0.025</td>
<td>0.47447</td>
<td>1.18301</td>
<td>-0.00036</td>
<td>0.05047</td>
<td>-0.02688</td>
<td>0.02952</td>
</tr>
<tr>
<td></td>
<td>0.0125</td>
<td>0.47437</td>
<td>1.18305</td>
<td>-0.00037</td>
<td>0.05036</td>
<td>-0.02689</td>
<td>0.02953</td>
</tr>
<tr>
<td>17.5</td>
<td>0.025</td>
<td>9.57612</td>
<td>0.51453</td>
<td>0.00189</td>
<td>0.20969</td>
<td>-0.04544</td>
<td>0.04314</td>
</tr>
<tr>
<td></td>
<td>0.0125</td>
<td>9.57591</td>
<td>0.51444</td>
<td>0.00184</td>
<td>0.20932</td>
<td>-0.04548</td>
<td>0.04308</td>
</tr>
</tbody>
</table>

(b) Parameters as a function of $N_{\text{cyl}}$ at $t = 7.0$ ($\Delta t = 0.025$)

<table>
<thead>
<tr>
<th>$N_{\text{cyl}}$</th>
<th>$x_1$</th>
<th>$y_1$</th>
<th>$H'_1$</th>
<th>$H'_2$</th>
<th>Drag Force</th>
<th>Lift Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Cylinder 1</td>
<td>Cylinder 2</td>
</tr>
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<tr>
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<td>1.18301</td>
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<td>0.02952</td>
</tr>
<tr>
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<td>0.05046</td>
<td>-0.02688</td>
<td>0.02952</td>
</tr>
</tbody>
</table>
Fig. 2.2. (a) Vortex path for a single vortex initially placed at $x_{v1} = 6$, $y_{v1} = .5$, of strength $k_1 = -1$. (b) Drag on the two cylinders as a function of time. (c) Lift on the two cylinders as a function of time. Both cylinders are rigid; --- denotes Cylinder 1, - - - denotes Cylinder 2.
Fig. 2.2. (Continued)
Fig. 2.3. Time variation of the acoustic pressure in the near field as a function of time; $H_1$ and $H_2$ are functions of the vortex path. Both cylinders are rigid. The symbol — is $H'_1 = \frac{dH_1}{dt}$, --- is $H'_2$. 
acoustic source is due to the transverse motion of the vortex and two cylinders.

On Figures 2.4-2.7 are depicted results for the elastic case where the natural frequency for both cylinders is $\omega_1 = 2.23$ with $\mu_{is} = 5.0$ for $i = 1,2$; other parameters are as on Figures 2.2 and 2.3. Note the whirling motion of both cylinders on Figures 2.4 (a), 2.4 (b) which commences soon after the vortex begins its motion. It is also noted that this type of whirling motion is typically observed in wake-induced vibration where the fluid forces couple the motion of the structure through displacement and velocity. The interaction is most intense when the vortex negotiates the cylinders; the vortex path for this case is similar to that depicted on Figure 2.2. On Figure 2.5 are depicted the forces scaled by $\mu_{is}$ (i.e. $F_{Xi}/\mu_{is}$, say) on the two cylinders as a function of time. Note that the motion consists of an impulsive portion similar to that of the fixed case followed by a time periodic component. The response frequency $\omega_r = 1.76$ ($\omega_i = 2.23$). On Figure 2.6 are the results for the near field acoustic pressure $H_1'$ and $H_2'$ as functions of time. The near field acoustic pressure also exhibits an impulsive character followed by a time periodic component at a response frequency also $\omega_r = 1.76$. The effect of damping is depicted on Figure 2.7 where the near field acoustic pressure components $H_1'$ (Fig. 2.7 (a)) and $H_2'$ (Fig. 2.7 (b)) are plotted as a function of time for several values of damping factor. Note the typical decay of the amplitude as $t$ increases.

The case of multiple vortex impingement is depicted on Figures 2.8-11. Here three vortices are placed upstream of the cylinders at
Fig. 2.4 (a) Instantaneous position of Cylinder 1 for a natural frequency of both cylinders of 2.23 with $\mu_i = 5$ for $i = 1, 2$.  
(b) Instantaneous position of cylinder 2. Other parameters are as on Figures 2.2 and 2.3.
Fig. 2.5. (a) $F_{xi} / \mu_{is}$ (i.e. Drag) as a function of time for both cylinders for the parameters of Figure 2.4. (b) $F_{yi} / \mu_{is}$ (i.e. Lift) as a function of time. The symbol — denotes Cylinder 1 and —— denotes Cylinder 2.
Fig. 2.6. The time variation of the near field acoustic pressure as given by $H_1'$ and $H_2'$ for the parameters of Figure 2.4. — is $H_1'$, -- is $H_2'$. 
Fig. 2.7. Effect of damping on the near field acoustic pressure components (a) $H_1'$ and (b) $H_2'$ for the parameters of Figure 2.4.

— corresponds to $\xi_1 = \xi_2 = 0$, - - - corresponds to $\xi_1 = \xi_2 = 0.05$ and the symbol 'x x' corresponds to $\xi_1 = \xi_2 = 0.25$. 
$x_{vi} = -6, -7, -8$ and $y_{vi} = .5$ for $i = 1, 2, 3$. The frequencies of the two cylinders are $\omega_1 = \omega_2 = 2.23$ with $\mu_1 = \mu_2 = 5$. In these results and those to follow, there is no damping. The cylinder motion and the vortex paths are depicted on Figure 2.8 and to facilitate plotting in the x-y plane the cylinders are placed at their position at time $t = 0$ on Figure 2.8 (a). Note again the whirling motion induced by the vortex motion on 2.8 (b). The interesting feature in Figure 2.8 (a) is that the vortices initially placed at the more upstream positions have proceeded further downstream than the other vortex during the same time interval. The interactions among vortices and the possibility of chaotic sound field will be further discussed in Section 2.4. On Figure 2.9 are depicted the drag and lift forces as functions of time. Note again the impulsive nature of the force followed by the periodic behavior as the vortices move downstream. Note that the initial equilibrium drag forces are about of same magnitude but in opposite direction due to the mutual interference effect of two cylinders. The lift forces are depicted on Figure 2.9 (b) and exhibit similar characteristics.

On Figure 2.10 are the results for the time variation of the acoustic pressure $H_1' = dH_1/dt$, and $H_2' = dH_2/dt$. A modulation of the pressure pulse is observed if the natural frequencies (ie. reduced velocities) of the two cylinders are different; on Figure 2.11 are the results for the case where $\omega_2 = 4.46$ with $\omega_1 = 2.23$ as before.

On Figure 2.12 are depicted results for the sound pressure level for the various impingement cases described here. The fast Fourier transform is employed and time series of length 2048 and 4096 points
Fig. 2.8. (a) Vortex paths for three vortices placed upstream of the cylinders; to facilitate plotting, the cylinders are placed at their positions at time $t = 0$ and should be used only for reference. (b) Instantaneous position of Cylinder 1. (c) Instantaneous position of Cylinder 2. Here $\omega_1 = \omega_2 = 2.23$. 
Fig. 2.8. (Continued)
Fig. 2.9. (a) $F_{xi}/\mu_{is}$ (ie. Drag) as a function of time for the three vortex configuration of Figure 2.8. (b) $F_{yi}/\mu_{is}$ (ie. Lift) as a function of time. — denotes Cylinder 1 and --- denotes Cylinder 2.
Fig. 2.10. Time variation of the acoustic pressure $H_1'$ and $H_2'$ for the three vortex configuration of Figure 2.8. --- denotes $H_1'$ while, --- denotes $H_2'$. 
Fig. 2.11. Time variation of the acoustic pressure $H_1'$ and $H_2'$ for the three vortex configuration of Figure 2.8 except that the natural frequency of Cylinder 2 is $\omega_2 = 4.46$. — is $H_1'$ while, --- is $H_2'$. 
were both used and compared; the data was sampled every one, two, or four points. The results of the comparison indicated that the 2048 point run sampled every point gave the best results and are depicted here. For all the sound pressure results (except of course for Figure 2.12 (a)), the transient portion of the time series has been subtracted out. The parameters employed to scale the pressure correspond to flow in water at 

\[ U_\infty = 15.4 \text{ m/sec}, \text{ with cylinder radii of } 0.015 \text{ m}. \]

The acoustic pressure is evaluated in physical space at a location directly above the cylinders at \( \theta = \pi/2 \), at a radial distance of 0.254 meter at a Mach number of \( M = 0.0103 \). For reference, on Figure 2.12 (a) is depicted the rigid-cylinder case; note the rapid decrease in the spectrum to a background level of about 40 dB. On Figure 2.12 (b) is the case of a single vortex impinging on the two cylinders having equal natural frequencies \( \omega_1 = \omega_2 = 2.23 \) (see Figures 2.4-7). Note again the rapid decline in the spectrum, however the background level is considerably increased as a result of the cylinder vibration. The peak at just over 160 dB occurs at 290 Hz (\( \omega_1 = 2.23 \) corresponds to 372 Hz) which is the response frequency of the two cylinders. On Figure 2.12 (c) is the result for the three-vortex configuration of Figure 2.8 for the cylinders having the same natural frequencies. The background is again at just over 100 dB with the peak occurring again at 290 Hz but the sound pressure level has increased to about 181 dB. Finally, on Figure 2.12 (d) is the three vortex case corresponding to cylinder natural frequencies of \( \omega_1 = 2.23 \) and \( \omega_2 = 4.46 \). Two response frequencies emerge corresponding to about 290 Hz and 580 Hz. The background level has also increased as well. Two other
Fig. 2.12. Sound pressure level for two vortex arrays and different structure characteristics. (a) Rigid cylinders, single vortex impingement as on Figure 2.2. (b) Elastic cylinders, single vortex impingement as on Figure 2.4. (c) Elastic cylinders, multiple vortex impingement as on Figure 2.8. (d) Elastic cylinders, multiple vortex impingement as on Figure 2.11.
prominent frequencies also emerge corresponding to about 870 Hz = 3 x 290 Hz and 1160 Hz = 2 x 580 Hz; 290 Hz and 580 Hz may be considered the response frequencies of the two cylinders. Note in figures 2.12 (b), (c), and (d) the activity at higher frequencies due to the continued oscillation of the structures; the vibration is almost but not quite harmonic.

As mentioned previously, it has been of interest to determine the response frequencies as a function of mass parameter. The case depicted on Figure 2.13 is for a single impinging vortex of strength \( k_1 = -1 \) and the figure shows the response frequency as a function of mass parameter for several different natural frequencies; here \( \mu_1 = \mu_2 = \mu_s \) with \( \omega_1 = \omega_2 = \omega \) and \( \omega \) corresponds to the natural frequency measured in a vacuum. It is observed in this figure that the response frequency is always lower than the natural frequency since the fluid inertial force produces an added mass effect on the vibration of the cylinders. This added mass effect is inversely proportional to the mass parameter so that the response frequency asymptotically approaches the natural frequency as the mass parameter increases. Note that the response frequency may not be equal to the natural frequency even at a very large value of mass parameter because of the fluid damping effect; for reference \( \mu_s = 25 \) corresponds to a steel cylinder in water.
Fig. 2.13. Response frequency of the cylinders as a function of $\mu$, for a single vortex impingement with the cylinders having equal natural frequencies. Several natural frequencies are shown.
2.4 Chaotic Motion of an Array of Vortices

Based on the results of Section 2.3, it is clear that the noise field in the present problem depends strongly on the interaction between the vortex motion and the cylinders. This interaction may be characterized by two major parameters; natural frequency of the structure and the initial configuration of the point vortices (i.e., positions and strengths). The dynamic behavior of an array of three, equal strength point vortices has been briefly addressed in Section 2.3 for the initial configuration of $x_{vi} = -6, -7, -8$ and $y_{vi} = .5$ for $i = 1, 2, 3$. While this configuration has not produced a broadband character in the sound spectrum it was observed that the sound pressure level has noticeably increased compared to that of a single vortex configuration. Conlisk, Guezennec and Elliott (1989), in their work on chaotic motion of point vortices, investigated the interaction of an array of vortices above a flat boundary by varying the initial conditions of three point vortices. They found in their study that depending on the initial configuration chaotic motion may or may not occur and the chaotic motion of the vortices results in a broadband, chaotic sound spectrum. A similar approach is taken in this section where the effect of the initial configuration of three and four vortices on the vortex motion and the sound field is studied.

The vortex configuration of interest is depicted on Figure 2.14 where three or four point vortices impinge on the two rigid (or elastic) circular cylinders in a tandem arrangement. The computed results indicate that the presence of two cylinders of finite dimension does not
Fig. 2.14. Initial vortex configuration.
(a) Three-vortex problem.
(b) Four-vortex problem.
play an important role in the vortex motion as vortices convect away downstream and four vortices are generally required for the possibility of chaotic motion and broadband sound.

The complex potential for the present problem has been formulated in equation (2.1) and the position of the individual vortices is computed by solving an initial value problem in equation (2.3). As pointed out in Conlisk, Guezennec and Elliott (1989) the instantaneous position of the individual vortices is conveniently described by introducing centroidal coordinates \((x'_i, y'_i)\) which are defined as

\[
x'_i = x_i - x_c \\
y'_i = y_i - y_c
\]  

where \((x_i, y_i)\) is the vortex position in the original coordinate system and \((x_c, y_c)\) is the center of vorticity defined as

\[
x_c(t) = \frac{\sum_{i=1}^{NV} k_i x_i}{\sum_{i=1}^{NV} k_i} \\
y_c(t) = \frac{\sum_{i=1}^{NV} k_i y_i}{\sum_{i=1}^{NV} k_i}
\]

As noted in Section 2.2, the time step employed in the computation must be small enough such that the vortex trajectories are accurately calculated. Considering that the interaction among vortices would be much more intense in the present problem the time step of \(\Delta t = 0.005\) was chosen in the actual computation. However, this \(\Delta t\) was
considerably too small for the spectral analysis (i.e., the data were oversampled). Hence the data were sampled every 8 points in power spectra to yield an effective sampling period $t_s$ equal to 0.04. All spectral results will be shown in terms of reduced frequencies $\omega_{ro}$ defined as

$$\omega_{ro} = \omega / \omega_s$$  \hspace{1cm} (2.14)

where $\omega_s$ is the sampling frequency defined as

$$\omega_s = \frac{2\pi}{t_s}$$  \hspace{1cm} (2.15)

In applying the fast Fourier transform 4096 points were chosen in the time domain resulting in $\Delta\omega_{ro} = 0.000244$. This high resolution is required to investigate neighboring spectral peaks, particularly in the development of spectral sidebands. As noted by Aref (1983) and Conlisk, Guezennec and Elliott (1989) the qualitative behavior of the motion of all vortices are similar. Hence the spectral results for vortex trajectories were computed for only one coordinate of a vortex, without loss of generality. As shown in Section 2.3 the dominant acoustic source is attributed to the transverse motion of the vortices and two cylinders (if elastic). Several preliminary tests confirmed these observations for the present problem. Hence the sound spectra presented later were computed only for $H_2$ which corresponds to the acoustic pressure component involving the transverse motion (see equation (2.10)).
Results were computed for two different cases; three-vortex problem and four-vortex problem. In the following results the two cylinders are rigid. The initial configuration for the three-vortex problem is depicted on Figure 2.14 (a). To minimize the number of disposable parameters, vortex '1' and vortex '2' always had the same initial position, i.e., $z_{v1}=(-7.0, 0.5)$ and $z_{v2}=(-7.0, 1.0)$ respectively. The remaining vortex '3' was positioned to form an isosceles triangle with the other two, and where the x-coordinate ($x_{v3}$) was varied from an equilateral configuration ($x_{v3} = -6.567$) to an aligned one ($x_{v3} = -7.0$). In all of the following results the strengths of the vortices are same.

Depicted on Figure 2.15 are the vortex trajectories in centroidal coordinates for three initial configurations; (a) $x_{v3} = -6.567$, (b) $x_{v3} = -6.856$, and (c) $x_{v3} = -7.0$. The case for $x_{v3} = -6.567$ (Figure 2.15 (a)) corresponds to the equilateral configuration and the trajectories of vortices are characterized by periodic, quasicircular orbits. These orbits are displaced from one another by a very slow osculation implying that most of the activity in the power spectra would occur in the low frequency range. The characteristics for the case $x_{v3} = -6.856$ (Figure 2.15 (b)) show a similar quasiperiodic behavior but the osculations among vortices are faster than those of previous case. These fast osculations appear to be most active for the aligned configuration ($x_{v3} = -7.0$) in Figure 2.15 (c). It has been observed by Conlisk, Guezennec, and Elliott (1989) that the chaotic motion is characterized by the highly irregular trajectories of the vortices. The three cases illustrated in Figure 2.14 show mostly periodic or quasiperiodic behavior and do not seem to lead to a chaotic motion.
Fig. 2.15. Trajectories of vortices in centroidal coordinates for three-vortex problem. (a) $x_{v3} = -6.567$  (b) $x_{v3} = -6.856$  (c) $x_{v3} = -7$. 
from their initial configurations. Also it is noted in Figure 2.14 that the vortex trajectories never visit the neighborhood of the centroid of vorticity, marked by a small cross.

These observations are confirmed in power spectra depicted on Figures 2.16-2.18. In each figure the first one illustrates the power spectrum of the vortex trajectory in the centroidal coordinates and the second one shows the power spectrum of the sound field. The case corresponding to $x_v = -6.567$ is shown in Figure 2.16. The power spectrum of the vortex trajectory exhibits sharp spectral peaks mostly at the harmonics of the primary frequency. However, these harmonics are confined to the relatively low frequency regime due to the very slow osculation of the orbits. The power spectrum of the sound field is also characterized by sharp spectral peaks. However, the primary frequency and harmonics are not the same as those of the spectrum of the trajectory. It is likely that the two circular cylinders interfere with the pure interaction among vortices. Similar characteristics are observed for the cases corresponding to $x_v = -6.856$ (Figure 2.17) and $x_v = -7.0$ (Figure 2.18). As noted earlier, the sharp spectral peaks at higher frequencies represent the effect of the fast osculation of the orbits. Most of these peaks rise almost more than 50 dB above the background and any noticeable sidebands are not developed in either case.

The initial configurations tested here are limited to three representative ones. Nevertheless, it appears that three vortices do not lead to the chaotic motion in the present problem. It is likely that the presence of two cylinders of finite dimension in the direction of motion of
Fig. 2.16. Power spectra for $x_{v3} = -6.567$.
(a) Centroidal vortex position  (b) Near field sound
Fig. 2.17. Power spectra for $x = -6.856$.
(a) Centroidal vortex position (b) Near field sound
Fig. 2.18. Power spectra for $x_{v3} = -7$.
(a) Centroidal vortex position  (b) Near field sound
vortices does not play a significant role as the vortices convect far downstream and thus for the majority of the time the problem corresponds to the three-vortex problem in free space. Note that the three-vortex problem in free space has been considered by Novikov(1975) and Aref(1979) and they concluded that the vortex motion is regular and periodic.

Based on the results of three-vortex problem, it appears that four vortices are required for the possibility of chaotic motion in the present problem. The initial configuration considered in the four-vortex problem is depicted on Figure 2.14 (b). Similar to the three-vortex problem, vortex '1', '2', and '3' always had the same initial position, i.e., \( z_{v1} = (-7.0, 0.25) \), \( z_{v2} = (-6.5, -0.25) \) and \( z_{v3} = (-6.5, 0.25) \) respectively. Note that these three vortices form an isosceles triangle. The remaining vortex, '4', was positioned in the vicinity of the point which forms the square with the other three vortices. The results have been computed for three initial configurations. The first one corresponds to the square configuration, i.e., \( z_{v4} = (-7.0, 0.25) \). This configuration was slightly perturbed by positioning vortex '4' at \( z_{v4} = (-6.9, 0.15) \) and \( z_{v4} = (-6.9, 0.3) \) respectively in second and third configurations. Again the strengths of all vortices employed in the computation are identical.

Figure 2.19 illustrates the vortex trajectories in centroidal coordinates for the three initial configurations. The trajectory for the square configuration in Figure 2.19 (a) exhibits almost circular and periodic orbits. It should be noted that this configuration represents the inversion symmetry with respect to the centroid of vorticity such that
Fig. 2.19. Trajectories of vortices in centroidal coordinates for four-vortex problem.
(a) \( z_{v4} = (-7.0, 0.25) \)  (b) \( z_{v4} = (-6.9, 0.15) \)  (c) \( z_{v4} = (-6.9, 0.3) \).
$z_{v1} = -z_{v3}, z_{v2} = -z_{v4}$ by translating the origin of the coordinates to the centroid of vorticity. The case corresponding to $z_{v4} = (-6.9, 0.15)$ is shown in Figure 2.19 (b) where the quasicircular orbits exhibit the periodic behavior and are displaced by a very slow osculation from one another. It is also noted that the trajectories never visit the neighborhood of the centroid of vorticity. On the other hand, the trajectories for the case of $z_{v4} = (-6.9, 0.3)$ do not have any particular organization in Figure 2.19 (c) and vary irregularly from orbit to orbit.

As expected from the results of Figure 2.19, the power spectra of the vortex trajectory and sound field for the square configuration show a very sharp peak at the primary frequency in Figure 2.20. The spectrum of the trajectory shown in Figure 2.20 (a) exhibits relatively small second and third peaks at the harmonics of the primary frequency resulting from the nonsinusoidal, yet periodic, character of the trajectory. These sharp spectral peaks are also observed in the high frequency regime in Figure 2.21 which corresponds to the case $z_{v4} = (-6.9, 0.15)$. The osculation through orbital motion again appears to contribute to these peaks at high frequencies. Depicted on Figure 2.22 is the case corresponding to $z_{v4} = (-6.9, 0.3)$. In contrast with the previous cases, dramatic changes are observed here. The spectrum of the vortex trajectory in Figure 2.22 (a) exhibits a fully chaotic behavior with a broadband peak and slow linear decay with frequency. The sound spectrum shown in Figure 2.22 (b) is also broadband and primarily flat over most of its frequency range, except for one modest peak.
Fig. 2.20. Power spectra for \( z_{v4} = (-7.0, 0.25) \).

(a) Centroidal vortex position  (b) Near field sound
Fig. 2.21. Power spectra for $z_v = (-6.9, 0.15)$.
(a) Centroidal vortex position  (b) Near field sound
Fig. 2.22. Power spectra for \( z_4 = (-6.9, 0.3) \).
(a) Centroidal vortex position  (b) Near field sound
Finally, the four-vortex problem has been investigated for the case where the vibration of cylinders is incorporated. Two initial configurations were considered in the computation; (a) $z_{v4} = (-7.0, 0.25)$ and (b) $z_{v4} = (-6.9, 0.3)$. The structural parameters employed in the computation are $\omega_1 = \omega_2 = 2.23$ and $\mu_1 = \mu_2 = 5$. The strengths of vortices are same as those of the rigid cylinder case and there is no damping. The vortex trajectories for the square initial configuration (i.e. $z_{v4} = (-7.0, 0.25)$) and the perturbed one ($z_{v4} = (-6.9, 0.3)$) are depicted on Figure 2.23 (a) and (b) respectively. The vortex trajectories for the square initial configuration are characterized by periodic, quasicircular orbits displaced from one another by a rather moderate osculation while the perturbed one exhibits a quite irregular and chaotic pattern. Compared to Figure 2.19 where the cylinders are rigid, it is likely that the vibration of the cylinders strongly influences the motion of vortices especially when the vortices are close to the cylinders. The effect of cylinder vibration on the vortex motion appears to depend strongly on the strengths of vortices and the structural parameters. Depicted on Figure 2.24 are the power spectra of the vortex trajectory (Figure 2.24 (a)) and the sound field (Figure 2.24 (b)) for the square initial configuration. The power spectrum of the vortex trajectory shows a sharp spectral peak at the primary frequency. This primary frequency corresponds to the response frequency $\omega_r = 1.76$ (note that the power spectrum is based on the reduced frequency, i.e., $\omega_r = \omega / \omega_a$). Note again that several other spectral peaks at higher frequencies are due to the moderate osculation of the vortex. The power spectrum of the sound field shown in Figure
Fig. 2.23. Trajectories of four vortices in centroidal coordinates for the case where the cylinders are elastic.
(a) $z_{v_4} = (-7.0, 0.25)$. (b) $z_{v_4} = (-6.9, 0.3)$. 
Fig. 2.24. Power spectra for $z_{v4} = (-7.0, 0.25)$.
(a) Centroidal vortex position (b) Near field sound
2.24 (b) exhibits a very sharp spectral peak at the primary frequency, but no other peaks emerge noticeably. The primary frequency in this spectrum corresponds to the response frequency $\omega_r = 1.76$. The power spectra for the perturbed initial configuration are depicted on Figure 2.25. The power spectrum of the vortex trajectory on Figure 2.25 (a) exhibits a fully chaotic, broadband character and a slow linear decay with frequency. Compared to the power spectrum of the rigid cylinder case (Figure 2.22 (a)), the slope of the decay is more flat but other essential features are quite similar. In contrast with this similarity in vortex trajectory, the power spectrum of the sound field on Figure 2.25 (b) exhibits a substantially different character from that of the rigid cylinder case (Figure 2.22 (b)). The sharp spectral peak at the primary frequency is about 100 dB higher than that of the rigid cylinder case and other secondary peaks are considerably lower such that the spectrum exhibits a narrowband character. As noted earlier, it appears that the effect of vortex motion is confined to the region where the vortices are close to the cylinders. Hence the sound field is likely to be dominated by the vibration of cylinders once the vortices convect far downstream. It is also likely that the cancellation effect among vortices partly contribute to the narrowband character of the sound spectrum. These speculations are supported by observing the time dependent forces depicted on Figures 2.26 and 2.28. Figures 2.26 (a) and (b) correspond to the drag and lift forces exerted on the downstream cylinder for the square initial configuration. The forces for the upstream cylinder exhibit similar behavior. It is shown in this figure that the chaotic forces occur only
Fig. 2.25. Power spectra for $z_{v4} = (-6.9, 0.3)$.
(a) Centroidal vortex position (b) Near field sound
Fig. 2.26. (a) Drag force and (b) Lift force exerted on the downstream cylinder as a function of time for $z_{v4} = (-7.0, 0.25)$. 
during the period when the interaction between the vortices and the cylinder vibration is intense. The sharp spikes on the figure are not due to numerical error and this is shown on Figure 2.27 which corresponds to the step size $\Delta t = 0.0025$ and compares well with Figure 2.26. The number of points $N_{\text{cyl}}$ distributed along the surface of each cylinder in the calculation of the forces is 72 on Figures 2.26, 2.27 and the results for $N_{\text{cyl}} = 36$ or $N_{\text{cyl}} = 144$ also produce same figures and thus are not presented here. Similar observations are applied on Figure 2.28 (a) and (b) which correspond to the case of the perturbed initial configuration. Figure 2.29 is for the step size $\Delta t = 0.0025$ and again compares well with Figure 2.28.

The set of initial configurations considered in the present computation is far from being complete and any firm conclusions regarding the chaotic motion of vortex arrays should be investigated further for more general initial configurations. Nevertheless, it appears that a certain set of upstream disturbances (represented by an array of point vortices herein) leads to a nonlinear interaction among vortices and thus result in a broadband sound spectrum. Regardless of whether the cylinders are rigid or not, the mutual effect of solid boundaries and the vortex motion seems to diminish rapidly as vortices convect away far downstream and the problem asymptotically converges to an equivalent problem in free space.
Fig. 2.27. Same as Fig. 2.26 except that the step size $\Delta t = 0.0025$.
(a) Drag force. (b) Lift force.
Fig. 2.28. (a) Drag force and (b) Lift force exerted on the downstream cylinder as a function of time for $z_y = (-6.9, 0.3)$. 
Fig. 2.29. Same as Fig. 2.28 except that the step size $\Delta t = 0.0025$. (a) Drag force. (b) Lift force.
The problem of vortex impingement on a pair of tandem cylinders has been examined analytically and numerically. Both the vibration characteristics and the noise field generated have been of primary interest in this study. It has been found that the cylinders respond at a frequency always less than its natural frequency by an amount which depends on the value of its mass parameter. The noise field essentially responds at the cylinder response frequency and the sound pressure level is significantly increased by vibration of the elastic cylinders. In the present problem vortex shedding has not been incorporated into the calculation and the vast majority of the work in this area has been concerned with this mode of excitation. However, the type of oscillation described here is reminiscent of the type of oscillation which sets in during lock-in of the vortex shedding to a frequency close to the natural frequency of the structure (Blevins (1977) and elsewhere). Consequently, it is believed that the vibratory characteristics of the cylinders and the noise field so generated are, at least, qualitatively reminiscent of those which would be observed in excitation of the cylinders by vortex shedding. In this regard, it is to be noted that the program used to compute the vortex path and the associated vibration of the two cylinders is relatively short, much shorter than that necessary for a vortex shedding calculation.

Finally, the possibility of chaotic motion of multiple vortex arrays has been addressed. The calculated results show that the interaction
among vortices and the noise field strongly depend on the initial configuration of the vortices. It appears that in the present problem four vortices are required for chaotic motion to occur.

The results of the present chapter indicate that for a wide range of natural frequencies and mass parameters of the cylinders, rather complex vibration patterns may occur. These vibration patterns are generated by rather moderate strength vortices and may generate a rather significant noise field with sound pressure levels of up to 160 dB being common. Of course, the relevance of the present solution to the actual flow past the cylinders at high Reynolds number is suspect because vortex shedding which is present in the actual flow is not considered. Nevertheless, the results of the present chapter are useful in the sense that they indicate how vibration of the structure can lead to a rather significant noise field under the buffeting action of potential vortices. As mentioned above, vortex shedding will occur in the real flow at high Reynolds number and the modeling of the vortex shedding phenomena is discussed next.
3.1. Introduction

The problem of calculating the large scale separated flow past bluff bodies at high Reynolds numbers is a difficult one and solutions to the full Navier-Stokes equations in this situation are extremely rare. Moreover at the high Reynolds numbers of interest here ($10^4 - 10^5$) the flow is invariably time-dependent, involving the generation of an unsteady shear layer which emanates from the edges of the body. The dynamics of these shear layers are very difficult to calculate because they are thin (of the order of $Re^{-1/2}$) and hence difficult to resolve in a viscous flow calculation using the Navier-Stokes equations. An alternative to this type of calculation is to model the shear layers by a series of point vortices; using this procedure no explicit spatial grid is required and only ordinary differential equations for the vortex paths need to be solved.

In this chapter we discuss the methods employed in the present work by first investigating the large scale separated flow past a flat plate at angle $\alpha$ to the free stream as depicted on Figure 3.1. In addition, the noise field is also calculated and results presented for the sound pressure level in the spectral domain. The procedures discussed here
Fig. 3.1. Geometry of interest for the large scale separated flow past a flat plate. Here \((X,Y)\) corresponds to the far field coordinate for the noise field; \(X = R \cos \theta\), \(Y = R \sin \theta\).
are then carried over to the problem of vortex shedding past the multiple louver assembly which is discussed in Chapter V. Before proceeding with the calculations, it is useful to review the procedure by which a free shear layer may be approximated by a number of point vortices.

It should be pointed out that a number of excellent reviews in this area have been written; a review of the area of vortex interactions in general has been given by Saffman and Baker (1979); a review of the use of the point vortex approximation has been given by Moore (1981) and a general review of the state of the art of vortex-dominated flows has been published by Sarpkaya (1988).

3.2 Approximation of a Free Shear Layer by a Number of Point Vortices

Since Rosenhead (1931) investigated the evolution of a vortex sheet to investigate the Helmholtz instability the discrete vortex method has been a major tool to study a variety of problems in which the motion of the vortex sheet is involved. Among these problems, the rollup of a vortex sheet behind the elliptically loaded wing was first computed by Westwater (1935). Moore (1971) tried to repeat the calculations of Westwater using more vortices and more accurate integration scheme but found no rollup behavior and the motion of individual vortices was extremely irregular; his solution to the problem (Moore (1974)) was to combine vortices in the inner turns of the spiral into a single vortex located at the center of vorticity of the group. Instability of the motion of
the sheet was observed as the number of vortices employed to approximate the sheet was increased. Moore (1974) also compared the fraction of vorticity rolled up as a function of time to the result predicted by the similarity solution found by Kaden (1931) and the agreement was good although a free constant had to be chosen for matching purposes. Fink and Soh (1978) suggested that the difficulty in achieving smooth rollup was the result of the emergence of a logarithmic singularity on the curved portions of the sheet and was removable if the grid points on the sheet were equispaced. Thus, they rediscretized the sheet at selected time intervals and found smooth rollup. It should be noted that the rediscretized field, however, violates the governing equations of motion of the sheet. Baker (1980) indicates that the success of the Fink and Soh rediscretization procedure is a result of the approximation of the inner turns of the vortex spiral by a single large scale vortex. He applied the method of Fink and Soh to the case of a closed vortex curve and found that the sheet crosses itself and the calculation subsequently breaks down. It was thus concluded by Baker (1980) that the method of rediscretization was unreliable. It is useful to note, however, that Moore (1981) suggests that the method of rediscretizing the vortex sheet does work to suppress the Helmholtz instability in some cases. Saffman and Baker (1979) and Moore (1981) suggest that, in fact, the inviscid problem may be ill-posed, in which case, the incorporation of viscosity is necessary to achieve smooth rollup.

Higdon and Pozrikidis (1985) used higher order approximation schemes for the vortex sheet and the circulation along the sheet. They
tested their method on the circular vortex sheet discussed by Baker (1980) using very small time steps in the latter stages of the calculation to achieve smooth rollup. The modified Euler method was employed to calculate the solution and additional points were continually added to the sheet; about four minutes of CPU time on the CDC CYBER175 was required. They also tested their method for the case of an infinite plane vortex sheet subject to a periodic disturbance. Krasny (1986) used a filtering technique to remove spurious sheet behavior near the center of the spiral and indicates the development of round-off error may be an additional source of error in the calculations; he also suggested that his calculations appear to show that the point vortex method converges as the number of vortices NV increases up to the development of a singularity in the sheet profile. In a later paper, Krasny (1987) appears to achieve smooth rollup of the elliptically loaded wing by insertion of an artificial smoothing parameter δ and demonstrates convergence of the scheme as δ → 0, although the details of the tip region depend on the value of δ. Maskew (1977) used a subvortex method in his attempt to accurately calculate the induced velocity near the sheet.

In general, the methods discussed above require either a large amount of computer time or a relatively significant computer coding effort and given the complexity of the present geometry (Figure 1.1) and the need for a relatively extensive conformal mapping procedure, a simpler method of obtaining a smooth rollup of the sheet was sought. The present method is similar to that used by Kuwahara and
Takami(1973)) to approximate the vortex sheet. Their method replaces the vortex sheet evolution equations with

\[
\frac{dx_i}{dt} = -\frac{1}{2\pi} \sum_{j=1}^{NV} k_j \{1 - \exp\left(-\frac{\text{Re} \, r_{ij}^2}{4t_a}\right)\} \frac{y_i - y_j}{r_{ij}^2}, \quad i = 1, \ldots, NV
\]

\[
\frac{dy_i}{dt} = \frac{1}{2\pi} \sum_{j=1}^{NV} k_j \{1 - \exp\left(-\frac{\text{Re} \, r_{ij}^2}{4t_a}\right)\} \frac{x_i - x_j}{r_{ij}^2}, \quad i = 1, \ldots, NV
\]

where \( t_a \) is the dimensionless age of the vortex measured from the time of its creation and \( r_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 \). This procedure has the effect of limiting the induced velocity of neighboring vortices and was applied to the elliptically loaded wing, the vorticity distribution of which is described by equation (3.12) below. Their results are inconclusive, however, because they employed a crude integration scheme with a large time step and a small number of vortices. From their results it is not clear that the incorporation of a viscous-type term in the equations is adequate to achieve smooth rollup.

As mentioned above Krasny (1987) uses a similar procedure in which the vortex sheet is convected with velocity

\[
\frac{\partial \tilde{\omega}}{\partial t} = \int_0^{\pi} K_\delta(z - z')\Gamma'(\tilde{\kappa})d\tilde{\kappa}
\]
where $K_\delta$ is a desingularized kernel defined as

$$K_\delta(z) = K(z) \{ |z|^2 / (|z|^2 + \delta^2) \} \quad (3.3)$$

and

$$K(z) = \frac{iz}{2\pi} \quad (3.4)$$

and $z = z(\Gamma(\lambda), t)$, $z = z(\Gamma(\lambda), t)$ and $\Gamma(\lambda) = \sin \lambda$. Equation (3.2) may be approximated by the trapezoidal rule and the result is

$$\frac{d\bar{z}_j}{dt} = \sum_{k=1}^{2N+1} K_\delta(z_j - z_k) \omega_k \quad (3.5)$$

where $\omega_k$ is the quadrature weight defined by $\omega_k = \Gamma'(\lambda_k) \pi / 2N$ and the vortex sheet has been discretized by subdividing the sheet by '2N' pieces of equidistant elements. The present method of desingularization is somewhat different from Krasny (1987); however, the effect is the same in the sense that the singularity at the center of the point vortex is removed allowing smooth rollup of the vortex sheet.

The validity of the approximation of a continuous vortex sheet by a system of discretized point vortices may be demonstrated as follows. Let $z_\delta(\Gamma_\delta, t)$ denote a point on the sheet. Then the complex velocity induced at $z = z_\delta$ is given by (Rott (1956), Birkhoff (1962))
where \( z \) is written in terms of circulation coordinate (i.e. \( z = z(\Gamma, t) \),
\( z_s = z_s(\Gamma_s, t) \)); the integral in this equation is to be interpreted in the
Cauchy principal sense (Stein and Weiss (1971)). It is clear in this
equation that there exists a singularity at \( z = z_s \) and it is necessary in
any numerical scheme to remove the singularity. To do this we split the
induced velocity into two parts according to

\[
\frac{\partial z}{\partial t} \bigg|_{z=z_s} = -\frac{i}{2\pi} \int \frac{d\Gamma}{z_s - z} - \frac{i}{2\pi} \int \frac{d\Gamma}{z_s - z}
\]

where \([\ ]\) denotes the fact that the singular part of the integral has been
removed. This integral may be approximated by any standard
integration scheme. The second integral is, however, singular and it is
not clear whether the local behavior of the integrand yields a contribution
to the velocity of the sheet.

To analyze the behavior of the integrand around \( z = z_s \) we expand
the integrand using binomial theorem such that

\[
\frac{1}{z_s - z} = \frac{1}{(\Gamma_s - \Gamma) \frac{\partial z}{\partial \Gamma}} + \frac{1}{2} \frac{\partial^2 z}{\partial \Gamma^2} + \Gamma_s - \Gamma
\]

\[(3.8)\]
where \( f \) is an analytic function of \( \Gamma_s - \Gamma \) which vanishes at \( \Gamma = \Gamma_s \) and 's' corresponds to \( z = z_s \). Substituting (3.8) into (3.7), the second integral (say I) in equation (3.7) becomes

\[
I = -\frac{i}{2\pi} \left[ \int_{\Gamma_s - \epsilon}^{\Gamma_s + \epsilon} \frac{d\Gamma}{(\Gamma_s - \Gamma) \frac{\partial z}{\partial \Gamma} |_{s}} + \frac{1}{2} \int_{\Gamma_s - \epsilon}^{\Gamma_s + \epsilon} \frac{\partial^2 z / \partial \Gamma^2 |_{s}}{\partial z / \partial \Gamma} |_{s} \right],
\]

The first integral in (3.9) is zero in the Cauchy principal value sense because the integrand is odd and the second integral (say I_2) has the value

\[
I_2 = -\frac{ie}{2\pi} \frac{\partial^2 z / \partial \Gamma^2 |_{s}}{\partial z / \partial \Gamma} |_{s}.
\]

which tends to zero as \( \epsilon \to 0 \) provided \( \partial z / \partial \Gamma |_{s} \neq 0 \) and the derivative \( \partial^2 z / \partial \Gamma^2 |_{s} \) is bounded. Thus under these conditions the velocity induced at \( z = z_s \) may be written as

\[
\frac{\partial \tilde{z}}{\partial t} \Big|_{z = z_s} = -\frac{i}{2\pi} \int_{\Gamma_s - \epsilon}^{\Gamma_s + \epsilon} \frac{d\Gamma}{z_s - z}.
\]

and the point vortex approximation follows from the discretization of the integral by the trapezoidal rule with suitable consideration of the end points of the sheet. Consequently, the point vortex approximation is a consistent approximation to the continuous sheet (Moore (1981)) and the
truncation error is that due to the integration scheme involved provided the sheet profile remains smooth. Equation (3.7) was first obtained by Van De Vooren (1980). Thus the value of $I$ ($= 0$) is exact for a continuous distribution of vorticity along the vortex sheet; no self-induced motion of the sheet is predicted and the procedure of approximating the vortex sheet by a number of point vortices should converge as the number of vortices increases to infinity.

Unfortunately, this is not the case and may be due to the fact that viscosity is neglected as noted by Moore (1981). In any standard numerical scheme used to calculate the unsteady motion of a vortex sheet, it seems reasonable to identify $\varepsilon$ with the difference in circulation, i.e. $\varepsilon = \varepsilon_j = \Gamma_{j+1} - \Gamma_j$ (Van De Vooren (1980), Moore (1981)). Note that unless $\varepsilon_j$ is constant over the entire sheet the value of $I$ will be non-zero yielding a logarithmic singularity for the self-induced motion of the sheet. This fact is apparently the motivation behind the rediscretization method of Fink and Soh (1978); the fact that the rediscretization method does not remove spurious dynamic behavior in all cases, however, suggests that there is some other mechanism (viscosity) which allows smooth rollup.

The use of a Lamb vortex (Lamb (1945)) to model the viscous core region of a potential vortex has been placed on a firm theoretical foundation by Ting and Tung (1965) who rigorously showed that the singularity at the center of a single two-dimensional point vortex is removed by the incorporation of a viscous core; the sum of the inner and outer solutions (in the sense of matched asymptotic expansions (Van
Dyke (1975)), to leading order, is equivalent to the distribution of a Lamb vortex. The Lamb vortex distribution is not a solution of the Navier-Stokes equations for more than one vortex and consequently, this method of removing spurious fine scale behavior must be viewed as a heuristic alternative to the rediscretization method of Fink and Soh (1978) and other similar methods (cutoff radius, for example (Chorin and Bernard (1973)). The problem of computing an accurate representation of the vortex sheet when vortices come close together is exacerbated by the knowledge that the vortex-merger problem is fully two-dimensional and involves the solution of the Navier-Stokes equations (Ting (1985), Liu et al. (1986)). Clearly such a local calculation imbedded within a larger calculation is not practical. The present method has been tested on the rollup of a vortex sheet defined by an elliptically loaded wing and the results for this problem are described next.

3.3 The Elliptically Loaded Wing

The rollup of a vortex sheet induced by an elliptically loaded wing was first described by Westwater (1935) who employed 20 vortices along a vortex sheet of finite span. Moore (1974) recomputed the solution and found that the representation of the vortex sheet by a finite number of point vortices may not be adequate near the center of the spiral since the spiral has an infinite number of turns. Instead he introduced an amalgamation technique in which the tightly rolled portion of the vortex sheet is represented by a single potential vortex and thus eliminated the
chaotic behavior near the core region. Later, Fink and Soh (1978) made a further contribution to the problem by introducing the redistribution method and Krasny (1987) attempted the desingularization of the Cauchy principal value integral. Recently Bera (1988) has claimed that there exists an exact solution to the Westwater problem which does not show any roll up and this seems to suggest that there may be non-uniqueness of the solution for the evolution of the elliptic sheet.

Based on the discussions in the previous section the Westwater problem has been reconsidered here applying the discrete vortex approximation. The vortex sheet is initially located in the interval \(-1 \leq x \leq 1, \ y = 0\) and has the vorticity distribution

\[
\omega(x) = 2x(1 - x^2)^{-1/2}
\]

(3.12)

where all the lengths and velocities have been nondimensionalized using a length scale 'a' and a velocity scale 'U'; 'a' and 'U' correspond to the half span of the vortex sheet and the constant velocity of the wing respectively. Since the vortex sheet is finite, and \(\omega(0) = \omega(1) = 0\), the approximation of the sheet by a number of point vortices corresponds precisely to discretization of the sheet using the trapezoidal rule. To remove the spurious, fine scale behavior of the solution as the time step is reduced, the singularity in azimuthal velocity has been removed by use of a Lamb vortex; the azimuthal velocity distribution is given by

\[
v_\theta = \frac{k^*}{2\pi r^*} \left\{1 - e^{-r^2/4vt^*}\right\}
\]

(3.13)
where, in equation (3.13), $k^*$, $r^*$, $\nu$ and $t^*$ are the dimensional vortex strength, radial coordinate centered at the location of the vortex, kinematic viscosity and time respectively. Non-dimensionalizing $k^*$, $r^*$, and $t^*$ using a length scale $'a'$, and velocity $U$ the induced velocity on the $n^{th}$ vortex due to the $j^{th}$ vortex, in complex notation, is given by

$$V_{nj} = \frac{ik_j}{2\pi (z_n - z_j)} \left( 1 - e^{-\frac{Re z_n - z_j}{4(t - t_j)}} \right)$$

(3.14)

where $t_{cj}$ is the time at which the $j^{th}$ vortex is created (in this section $t_{cj} = 0$), $k_j$ denotes the strength of the $j^{th}$ vortex and $z_n, z_j$ are the positions of the $n^{th}$ and $j^{th}$ vortices, and

$$Re = \frac{Ua}{\nu}$$

(3.15)

On the scale of the vortex core, the appropriate Reynolds number is

$$Re_c = \frac{k_j^*}{\nu}$$

(3.16)

(Ting and Tung (1965)) and the correction term in $V_{nj}$ in equation (3.14) will only be significant if $|z_n - z_j|^2 Re_c = O(1)$. Let

$$z_n' = z_n \sqrt{Re_c}, \quad z_j' = z_j \sqrt{Re_c}$$

(3.17)
then

\[ V_{nj} = -\frac{ik_j}{2\pi(z_n - z_j)} \left\{ 1 - e^{-\frac{Re \cdot |z_n - z_j|^2}{4(t-t_j)}} \right\} \]  

(3.18)

Note that \( \frac{Re}{Re_c} = \frac{1}{k_j} \). Either (3.14) or (3.18) may be employed in the computation and in what follows (3.14) has been employed.

The point vortex method is applied by dividing the sheet into \( 2N \) pieces of equal-length elements. At the midpoint of each element, a point vortex is placed and the strength of each vortex is computed from (3.12). In the following computed test cases \( N = 60 \) and both a simple first order Euler scheme and a fourth order Adams-Moulton scheme were used to compute the subsequent roll-up. On Figures 3.2-3.8, only half of the vortex sheet is depicted. Note that the time has been nondimensionalized such that \( t = t^* U / a \). On Figure 3.2 are the results for the Euler scheme for two cases corresponding to roll-up using uncorrected potential vortices (Figure 3.2 (a)) and that using equation (3.14) in which the induced velocity is finite (Figure 3.2 (b)). Here \( \Delta t = .002 \) and \( Re = 400 \) and note the existence of spurious and unphysical fine scale behavior of the vortices in Figure 3.2 (a). This fine scale behavior has been removed by use of the formula (3.14) the result of which is depicted on Figure 3.2 (b); these two figures are for a dimensionless time of \( t = .5 \) and compare very well with the results of Moore (1974) who removed the spurious fine scale behavior by coalescing all vortices in the inner spiral into a single vortex with an equivalent strength. Figures 3.2 (a) and 3.2 (b) should be compared with Figure 2(b) of Moore (1974). The results at a later time
Fig. 3.2. Roll-up of a finite vortex sheet using a simple Euler scheme at time $t = .5$, $\Delta t = .002$, $N = 60$, $Re = 400$.
(a) Uncorrected Euler. (b) Corrected Euler.
t = 1.0 are given on Figures 3.3 (a) and (b) and similar comments apply at this time as well (compare with Figure 3 of Moore (1974)). On Figures 3.4 and 3.5 are results corresponding to Figures 3.2 and 3.3 using the Adams-Moulton method which is much more accurate than the Euler scheme. Note that there is substantial agreement between the corrected Euler and Adams-Moulton schemes away from the central portion of the spiral. To insure that the present results are independent of time step, results for Δt = .004 were also produced as well; the results corresponding to the Euler scheme with correction (equation (3.14)) and the Adams-Moulton scheme with correction (equation (3.14)) are given on Figures 3.6 (a) and (b) for t = 1. Note the excellent agreement of Figure 3.6 (b) with that depicted on Figure 3.5 (b) indicating that the numerical results are adequately resolved using the Adams-Moulton scheme. Note, however that the Euler scheme does not do nearly as well and is clearly not adequate to resolve the fine scale motion near the center of the spiral.

Very long time solutions at t = 7 and t = 10 have been computed using the corrected Adams-Moulton scheme and are depicted on Figures 3.7 (a) and (b) for Re=400. The results show a tendency for the turns of the spiral to be elliptical rather than circular as was noted by Moore (1974). The effect of Reynolds number has also been investigated using the corrected Adams-Moulton scheme and the results are depicted on Figure 3.8 for t = 7. While the solution for Re = 400 on Figure 3.7 (a) achieves a smooth rolling-up inside the spiral region, the solution for Re = 1600 (Figure 3.8 (a)) shows irregular behavior near the core region of the spiral and the point vortices cross over one another. This cross-over
Fig. 3.3. Same as Figure 3.2 for $t = 1$.
(a) Uncorrected Euler. (b) Corrected Euler.
Fig. 3.4. Same as Figure 3.2 except the Adams-Moulton scheme is employed to advance the vortices. (a) Uncorrected. (b) Corrected.
Fig. 3.5. Same as Figure 3.4 for $t = 1$. (a) Uncorrected. (b) Corrected.
Fig. 3.6. Roll-up of a finite vortex sheet for time step $\Delta t = .004$ at time $t = 1$, $N = 60$, $Re = 400$. (a) Corrected Euler. (b) Corrected Adams-Moulton.
Fig. 3.7. A very long time solution using the corrected Adams-Moulton scheme for time step $\Delta t = .002$, $N = 60$, $Re = 400$.
(a) $t = 7$. (b) $t = 10$. 
Fig. 3.8. Effect of Reynolds number using the corrected Adams-Moulton scheme at time $t = 7$, $\Delta t = .002$, $N = 60$.
(a) $Re = 1600$. (b) $Re = 3200$. 
phenomena becomes more severe for Re = 3200 in Figure 3.8 (b). It is also observed that the number of spiral turns decreases as the Reynolds number increases.

Depicted on Figure 3.9 is the amount of vorticity rolled up as a function of time on log-log scales for Re = 400. Here \( f \) denotes the fraction of the circulation at time \( t \) contained between the spiral’s center and the outermost point of vertical tangency to the spiral (Moore (1974)). According to Kaden (1931), for \( t << 1 \), Moore (1974) obtained

\[
f = gt^{1/3}
\]  

(3.19)

where \( g \) is a constant. The dashed lines on Figure 3.9 have been obtained by choosing \( g \) such that the values of \( f \) from equation (3.19) are the same as those from Moore’s (1974) and present solutions at \( t = 0.01 \). The slope of \( f \) for \( t \leq 0.1 \) in the present solution is in good agreement with Kaden’s value (= 1/3) and the overall time history of \( f \) compares well with Moore’s (1974).

3.4 Vortex Sheding in Flow Past a Single Fixed Flat Plate at Incidence

Introduction and Formulation of the Problem

In this section we apply the method of the discrete vortex approximation for the flow past an inclined flat plate. The theoretical treatment of this method has been discussed in Section 3.2 along with its
Fig. 3.9. The fraction 'f' of vorticity rolled up as a function of time on log-log scale.
application for the elliptically loaded wing in Section 3.3. The physical geometry of interest is depicted on Figure 3.10 (a); far upstream of the flat plate of the chord length 4c the steady, mean velocity is $U_m$ and the angle of attack is denoted by $\alpha$; $z_L^*$ and $z_T^*$ represent the leading and trailing edge respectively. The domain outside the flat plate may be mapped into the outside of a circular cylinder using a Joukowsky-type transformation which is given by

$$z^* = e^{-i\alpha}(\zeta^* e^{i\alpha} + \frac{c^2}{\zeta^*} e^{-i\alpha}) \quad (3.20)$$

where $i = \sqrt{-1}$, $c$ is the dimensional radius of the cylinder and $*$ denotes a dimensional quantity. The transformation (3.20) is also shown on Figure 3.10 (b) where $\zeta_L^*$, $\zeta_T^*$ denote the transformed coordinates of the leading and trailing edge.

Nondimensionalizing all the lengths and velocities on the half chord length of the plate $2c$ and the mean velocity $U_m$ the complex potential for the flow in the $\zeta$-plane due to a streaming motion past the cylinder and a number of point vortices may be obtained using the circle theorem (Milne-Thomson (1968)) and the result is

$$W(\zeta) = \zeta + \frac{1}{\zeta} - \frac{i}{2\pi} \sum_{j=1}^{NV} k_j \ln(\zeta - \zeta_j) + \frac{i}{2\pi} \sum_{j=1}^{NV} k_j \ln(\zeta - \frac{1}{\zeta_j}) \quad (3.21)$$

where $k_j$ and $\zeta_j$ represent the dimensionless strength and position of the $j^{th}$ vortex, respectively; an overbar denotes the complex conjugate.
Fig. 3.10. The geometry of interest in flow past a fixed inclined flat plate. (a) Physical plane. (b) Transformed plane.
Note that the circle theorem also produces what appears to be an image vortex at the origin for each vortex outside the cylinder which corresponds to having an additional term of \( -\frac{i}{2\pi} \sum_{j=1}^{NV} k_j \ln \zeta \) in equation (3.21). But this term is excluded from equation (3.21) since it violates Kelvin's theorem. To see this consider a closed contour which is large enough to enclose both the flat plate and all the vortices shed from the edges of the plate. Then the total circulation is initially zero and the onset of the flow separation is triggered by generating point vortices near the edges of the plate. This may be interpreted that, in the transformed plane, a point vortex of strength \( k_j \) located outside the cylinder introduces an image vortex of strength \(-k_j\) at the inverse point inside the cylinder and another image vortex of strength \( k_j \) at the center of the cylinder. Hence the total circulation is not zero round the contour violating Kelvin's theorem. Note that no other conditions are violated by excluding the aforementioned additional term from equation (3.21).

The complex velocity may be obtained by direct differentiation of equation (3.21) and the result is valid at a non-vortex position. To advance the vortices the velocity induced at the vortex position is required which is given by

\[
(u - iv)_{z=z_j} = \left( \frac{dW}{d\zeta} + \frac{ik_j}{2\pi} \frac{1}{\zeta - \zeta_j} \right) \zeta = \zeta_j \frac{\zeta_j^2}{\zeta_j^2 - e^{-2i\alpha}} + \frac{ik_j}{2\pi} \frac{\zeta_j e^{-2i\alpha}}{(\zeta_j^2 - e^{-2i\alpha})^2} \tag{3.22}
\]
where the second term is the Routh correction.

The tangential separation of the flow at the edges of the plate may be satisfied by imposing the Kutta condition

\[ \frac{dW}{d\zeta} = 0 \text{ at } \zeta = \zeta_L, \zeta_T \]

where \( \zeta_L, \zeta_T \) are the dimensionless transformed coordinates of the leading and trailing edge respectively. There is some doubt as to whether this standard steady Kutta condition is valid for the unsteady flow of present interest. However there is both direct and indirect evidence for the validity of the standard Kutta condition in restricted regions around the edges of the plate (Crighton(1985), Poling and Telionis(1986)) and it appears that the flow near the separation points is quasisteady (Fage and Johansen (1927, 1928)); thus use of steady Kutta condition appears to be justified.

The fluid loading on the plate can be calculated from the Blasius theorem as presented in Section 2.2

\[ F_x - iF_y = \frac{1}{2} i \oint (\frac{dW}{dz})^2 dz + i \frac{\partial}{\partial t} \oint \overline{W} \, dz \]

where \( F_x, F_y \) respectively represent the dimensionless drag and lift forces based on \( 2\rho U^2 \). On substituting equation (3.21) into equation (2.7) and applying the residue theorem, the forces on the plate may be
analytically calculated as shown in Appendix C. Dividing (2.7) into steady and unsteady part, let

\[ F_{x1} - iF_{y1} = \frac{1}{2} i \oint (\frac{dW}{dz})^2 dz \] (3.24)

\[ F_{x2} + iF_{y2} = -i \frac{\partial}{\partial t} \oint W dz \] (3.25)

then the contour integral in equation (3.24) may be expressed around the cylinder contour 'C' such that

\[ F_{x1} - iF_{y1} = \frac{1}{2} i \oint_c \left( \frac{dW}{d\zeta} \right)^2 \left( \frac{d\zeta}{dz} \right) d\zeta \] (3.26)

and applying the integration by parts on equation (3.25)

\[ F_{x2} + iF_{y2} = -i \frac{\partial}{\partial t} \{ (Wz)_{pc} - \oint z(\frac{dW}{dz}) dz \} \] (3.27)

where \((Wz)_{pc}\) is the difference between the values of 'Wz' at the beginning and the end of the plate contour which is identically equal to zero since there is no jump in the stream function along the contour (Sarpkaya (1975)). Upon substituting (3.20) and (3.21) into (3.26) and applying Lagally's theorem (Milne-Thomson(1968)), the steady part of the fluid force may be obtained as

\[ F_{x1} - iF_{y1} = -i \sum_{j=1}^{NV} k_j (u_j - iv_j) \] (3.28)
where

\[ u_j - iv_j = (u - iv)_{x = x_j} \]  \hfill (3.29)

The unsteady part of the fluid force, represented by (3.27) may be written

\[ F_{x2} + iF_{y2} = i \frac{\partial}{\partial t} \int_C \left( e^{-i\alpha} \left( \frac{e^{-i\alpha}}{\zeta} + \frac{e^{i\alpha}}{\zeta} \right) \right) d\zeta \]  \hfill (3.30)

Inserting (3.21) into (3.30) and carrying out the integration using the residue theorem, we finally obtain

\[ F_{x2} + iF_{y2} = -i \frac{\partial}{\partial t} \left( \sum_{j=1}^{NV} k_j e^{-i\alpha} \left( \frac{e^{-i\alpha}}{\zeta_j} + \frac{e^{i\alpha}}{\zeta_j} \right) \right) \]  \hfill (3.31)

Combining (3.28) and (3.31) the final expression for the fluid force exerted on the plate is obtained as

\[ F_x + iF_y = i \left[ \sum_{j=1}^{NV} k_j (u_j + iv_j) - \frac{\partial}{\partial t} \left( \sum_{j=1}^{NV} k_j e^{-i\alpha} \left( \frac{e^{-i\alpha}}{\zeta_j} + \frac{e^{i\alpha}}{\zeta_j} \right) \right) \right] \]  \hfill (3.32)

which is equivalent to the equation obtained by Sarpkaya(1975).
The Far Acoustic Field

To obtain the acoustic pressure for small Mach number the linearized wave equation is solved using the same treatment as in Section 2.2 except that the generating function changes; the details of the solution procedure are given in Appendix B as noted earlier. To obtain the generating function, equation (3.21) is expanded for \( |\zeta_j / \zeta| << 1 \) using the binomial theorem and the result is

\[
\phi_a = \text{Re}\left\{ \frac{ie^{-i\theta}}{2\pi r} \sum_{j=1}^{NV} (k_j \zeta_j - k_j / \bar{\zeta}_j) \right\} \quad (3.33)
\]

where \( \zeta_j = r_j e^{i\theta_j} \) is the vortex position in the transformed plane. The transformation in equation (3.20) indicates that

\[
z \sim \zeta, \quad |\zeta| \to \infty \quad (3.34)
\]

Substituting (3.34) into (3.33) and letting \( Z = Mz = Re^{i\theta} \), we have

\[
\phi_a = \frac{M}{R} (G_1 \cos \theta + G_2 \sin \theta), \quad R \to 0 \quad (3.35)
\]

where

\[
G_1 = \text{Re}\left\{ \frac{i}{2\pi} \sum_{j=1}^{NV} (k_j \zeta_j - k_j / \bar{\zeta}_j) \right\} \quad (3.36)
\]
with \( G_2 \) its corresponding imaginary part; \( \zeta_j \) corresponds to the position of the \( j^{th} \) vortex in the transformed plane. The solution method for the far field acoustic potential subject to (3.35) and an outgoing wave condition as \( R \to 0 \) is described in Appendix B and then using the relation \( P_a = -\frac{\partial \phi_a}{\partial t} \), the acoustic pressure in the far field may be written

\[
P_a = -\frac{M}{R} \left[ \cos \theta \int_0^{t-R} \frac{(t-s)(d^2 G_1 / ds^2)}{\sqrt{(t-s)^2 - R^2}} ds \right. \\
+ \sin \theta \left. \int_0^{t-R} \frac{(t-s)(d^2 G_2 / ds^2)}{\sqrt{(t-s)^2 - R^2}} ds \right]
\]

(3.37)

Note that the acoustic pressure in this equation is purely a function of vortex motion (i.e., shear layer rollup) since the flat plate has been assumed fixed.

**Numerical Procedure**

Based on the discussion of Section 3.2 the free shear layers separating from the edges of the plate may be represented by a number of discrete vortices which are created in the vicinity of the edges and convect downstream. Two important parameters have to be determined in this process; the initial position of the vortex and its associated strength. With regard to these parameters, much of the previous work may be categorized into two methods; the method of fixed point of
introduction and the method of variable point of introduction. In what follows, the term 'nascent vortex' corresponds to a newly created vortex.

In the method of fixed point of introduction, the position of the nascent vortex is pre-fixed and then its strength is determined using the Kutta condition given by equation (3.23). Because the position of the nascent vortex is a disposable parameter this method is very sensitive to the choice of the initial position. Kiya and Arie (1977) tested several values of this parameter and presented their results for $a_s/2c = 0.0125$ where $a_s$ corresponds to the distance between the separation point and the position of the nascent vortex. While their results produced the typical periodicity in drag and lift forces the mean value of the drag force deviates substantially from the experimental results of Fage and Johansen (1927). Sarpkaya (1975) employed the method of variable point of introduction in which the position of the nascent vortex and its strength are determined by applying the Kutta condition and the formula

$$\frac{dk}{dt} = .5U^2_s$$ \hspace{1cm} (3.38)

where $U_s$ is originally defined as the velocity at the outer edge of the boundary layer at separation. The validity of this formula to evaluate the rate of vorticity shed had first been given and demonstrated by Fage and Johansen (1927, 1928) and a theoretical discussion is given by Sears (1956). To compute the value of $U_s$ in equation (3.38) the boundary layer equations should be solved. However, to avoid this complicating feature, Sarpkaya approximated $U_s$ as the average of the transport velocities of
four most recently introduced vortices in each shear layer. The major advantage of Sarpkaya's method over the method of fixed point of introduction is that the feedback effect of the downstream wake fluctuations can be accounted for in determining the position of the nascent vortex and its strength. The calculated results of Sarpkaya compare better with the available experimental data (Fage and Johansen(1927)) than those of Kiya and Arie (1977) although the mean normal force coefficient is still high by about 20-30%. In the present computation we consider the method of variable point of introduction. The major difference of the present model from the previous work is in the use of the Lamb vortex model to compute the velocity induced at each vortex position as defined in equation (3.14). Several other different numerical features will be explained in detail where they occur.

In the present method of variable point of introduction, we approximate $U_*$ in equation (3.38) as the velocity induced at the position of the nascent vortex. To illustrate the essential features of the numerical scheme, suppose there are 'n' vortices already in the flow field and $(n+1)^{th}$ and $(n+2)^{th}$ vortices are to be introduced near the leading and trailing edges respectively at time 't'. Then differentiating equation (3.21) and equating the result to zero at $\zeta = \zeta_L, \zeta_T$ we obtain

$$\frac{i}{2\pi} \left( \frac{k_{n+1}}{\zeta_L - \zeta_{n+1}} - \frac{k_{n+1}}{\zeta_L - 1/\zeta_{n+1}} + \frac{k_{n+2}}{\zeta_L - \zeta_{n+2}} - \frac{k_{n+2}}{\zeta_L - 1/\zeta_{n+2}} \right)$$

$$= 1 - \frac{1}{\zeta_L^2} - \frac{1}{2\pi} \sum_{j=1}^{n} \frac{k_j}{\zeta_L - \zeta_j} + \frac{i}{2\pi} \sum_{j=1}^{n} \frac{k_j}{\zeta_L - 1/\zeta_j}$$

(3.39)
at the leading edge and

\[
\frac{i}{2\pi} \left( \frac{k_{n+1}}{\zeta_T - \zeta_{n+1}} - \frac{k_{n+1}}{\zeta_T - 1/\zeta_{n+1}} + \frac{k_{n+2}}{\zeta_T - \zeta_{n+2}} - \frac{k_{n+2}}{\zeta_T - 1/\zeta_{n+2}} \right)
= 1 - \frac{1}{\zeta_T^2} - \frac{1}{2\pi} \sum_{j=1}^{n} \frac{k_j}{\zeta_T - \zeta_j} + \frac{i}{2\pi} \sum_{j=1}^{n} \frac{k_j}{\zeta_T - 1/\zeta_j}
\] (3.40)

at the trailing edge; in (3.39) and (3.40), \(\zeta_{n+1}\) and \(\zeta_{n+2}\) correspond to the transformed coordinates of the points of introduction of the nascent vortices in the physical plane (say, \(z_{n+1}\) and \(z_{n+2}\)) and \(k_{n+1}\), \(k_{n+2}\) are the unknown strengths of the nascent vortices. Since the position of the nascent vortices is not known a priori the initial position of the previously introduced vortices is given as an initial guess for \(z_{n+1}\) and \(z_{n+2}\). The strength of the nascent vortex at \(z_{n+1}\) is then computed using equation (3.38) where \(U_s\) corresponds to the velocity induced at \(z_{n+1}\) due to the uniform flow and 'n' vortices already in the flow field. The strength of the other nascent vortex at \(z_{n+2}\) may be calculated similarly using the same equation. After the position and strength of the nascent vortices are determined, these parameters are inserted into equations (3.39) and (3.40) to check whether the Kutta condition is satisfied. If a larger strength is required to satisfy the Kutta condition \(z_{n+1}\) is updated closer to its associated edge by a distance of the half of the distance between the old value of \(z_{n+1}\) and the edge, and vice versa; a similar procedure is applied to update \(z_{n+2}\). These updated \(z_{n+1}\) and \(z_{n+2}\) are again inserted into equation (3.38) to compute the new strength of the nascent vortices and this iteration scheme is repeated until the Kutta condition is
satisfied within the specified tolerance. To determine whether the process converges, the change of the strength of the nascent vortex is checked at each iteration and a relative test is employed with the convergence criterion \( \varepsilon = 10^{-4} \).

The time step \( \Delta t \) employed in the computation is an important parameter and should be chosen small enough to achieve the numerical accuracy within a reasonable computation time. A number of preliminary tests to select the optimum step size showed that \( \Delta t = 0.04 \) gave accurate results with respect to the individual motion of each vortex. Also the nascent vortex was introduced at every \( \delta t = 4 \Delta t \) to save the computation time. Several other values such as \( \Delta t = 0.02, \delta t = 8 \Delta t \) and \( \Delta t = 0.01, \delta t = 16 \Delta t \) have been employed for the comparison and the results will be presented later in this section.

To advance the nascent vortices along with the vortices already in the flow field we solve a set of ordinary differential equations

\[
\frac{dx_j}{dt} = u_j, \quad \frac{dy_j}{dt} = v_j, \quad j = 1, \ldots, NV
\]  

(3.41)

where \((x_j, y_j)\) is the instantaneous position of the \(j^{th}\) vortex in the physical plane and \((u_j, v_j)\) is the velocity at \(z = z_j\) given by equation (3.22). The time integration of equation (3.41) is computed using the Adams-Bashforth two step method such that
\[ x_j(t + \Delta t) = x_j(t) + \frac{\Delta t}{2} \{3u_j(t) - u_j(t - \Delta t)\} \]
\[ y_j(t + \Delta t) = y_j(t) + \frac{\Delta t}{2} \{3v_j(t) - v_j(t - \Delta t)\} \]

The local truncation error of this method is of order \((\Delta t)^3\) and appears to produce accurate results for the range of the time step employed in the present calculation. The accuracy of the solution using equation (3.42) and the comparison with the results using other integration schemes such as Euler scheme and Runge-Kutta scheme will be presented later in this section.

As noted by Fage and Johansen(1927), a part of the vorticity which leaves each edge of the plate appears to be dissipated immediately behind the plate by a mixture of the positive and negative vorticity. Also as the distance behind the plate increases, the vorticity concentrated within a region of small radius tends to spread, break up, and decay. Unfortunately the rigorous computation of the merging and the decay of vortices requires the solution of the Navier-Stokes equations (Ting(1985), Liu et al.(1986), Weston et al.(1986)) which cannot be incorporated into the present model practically. Alternatively, we consider that the core radius of a line vortex is given by (Schaefer and Eskinazi(1959))

\[ r_c^* = \sqrt{5.04vt_{a}^*} \]

where \(r_c^*\) and \(t_{a}^*\) correspond to the dimensional core radius and age of the vortex respectively. In this equation \(r_c^*\) is defined as the radial distance between the center of the line vortex and the point where the
azimuthal velocity $v_\theta$ (equation (3.13)) is a maximum and thus may be obtained by differentiating equation (3.13) with respect to $r^*$. Based on $r_c = r_c^* / 2c$, $Re = 4cU_\infty / v$ and $t_a = U_\infty t_a^* / 2c$, equation (3.43) may be written in a dimensionless form such that

$$r_c = 3.17 \sqrt{t_a / Re}$$

(3.44)

which gives $r_c = 0.04 - 0.05$ for $Re = 40,000 - 50,000$ and $t_a = 10$. Hence, based on this discussion, each individual vortex whose distance to the downstream face of the plate is less than .05 has been removed from the flow field. Also two vortices of opposite strength have been combined into a single vortex when the distance between two vortices is less than .1 such that

$$\begin{align*}
x_j &= \frac{|k_m|x_m + |k_n|x_n}{|k_m| + |k_n|} \\
y_j &= \frac{|k_m|y_m + |k_n|y_n}{|k_m| + |k_n|} \\
k_j &= k_m + k_n
\end{align*}$$

(3.45)

where $(x_j, y_j)$, $k_j$ correspond to the position and the strength of the vortex after combination and $(x_m, y_m)$, $(x_n, y_n)$ denote the position of two vortices before the combination with $k_m$, $k_n$ their individual strength.
Finally a number of schemes have been investigated for solving equation (3.38) for the vorticity shed in a given time interval, the simplest of which is

\[ \Delta k = \frac{\Delta t}{2} U_s^2 \]  

(3.46)

This formula corresponds to a simple forward difference for the derivative in (3.38). Clearly such a formula is not accurate especially when a higher order integration scheme is used for the advancement of vortices. Consequently other schemes have been considered corresponding to integration schemes rather than differentiation schemes. Hence, if vortices are introduced every \( \delta t = 4\Delta t \), the use of Simpson's rule results in

\[
\Delta k = \frac{\Delta t}{6} \{ U_s^2(t) + 4U_s^2(t - \Delta t) + 2U_s^2(t - 2\Delta t) \\
+ 4U_s^2(t - 3\Delta t) + U_s^2(t - 4\Delta t) \} 
\]  

(3.47)

Such a formula has a truncation error of order \( (\Delta t)^5 \) and is much more accurate than (3.46). Hence in the results to follow equation (3.47) has been employed. Notice that (3.47) corresponds to (3.46) if \( U_s = \text{constant} \) and \( \Delta t \) in (3.46) is replaced by \( 4\Delta t \).
Results and Comparison with Experiments

Results were computed for $\alpha = 40^\circ, 50^\circ, 60^\circ$ and $Re = 40,000$. Details of the results and their comparison with the previous work will be confined to the case $\alpha = 60^\circ$ and the results for other angles of attack will be presented whenever necessary. Note that the Reynolds number employed in the present computation is within the range considered in Fage and Johansen (1927).

As mentioned previously, the time step $\Delta t$ should be chosen such that the computed results are not sensitive to the step size. Numerical accuracy has been tested for several values of $\Delta t$ and the results for the wake pattern behind the plate at $t = 10$ for $\alpha = 60^\circ$ are depicted on Figure 3.11. The general features of the wake pattern for each $\Delta t$ are similar except for some slight differences in the fine scale structure. The drag and lift forces for each $\Delta t$ have also been calculated and produced a fair agreement and thus in all of the results which will be presented $\Delta t = 0.04$, $\delta t = 4\Delta t$ has been employed. Note that $\delta t$ in Figure 3.11 corresponds to the time interval between the introduction of vortices and has been chosen in Figures 3.11 (a), (b), (c) such that the total number of vortices in the flow field is same. Another important parameter of physical interest is the location of free vortex layers leaving each edge of the plate. Depicted on Figure 3.12 (a) is the computed position of the free vortices in the wake at time $t = 5$ for $\alpha = 40^\circ$ and $Re = 45,000$. The equivalent plot of the position of free vortices at later times also exhibit similar departure from each edge. This figure may be compared with
Fig. 3.11. The evolution of the wake behind the flat plate of $\alpha = 60^\circ$ for several values of $\Delta t$. (a) $\Delta t = 0.04$, $\delta t = 4 \Delta t$ (b) $\Delta t = 0.02$, $\delta t = 8 \Delta t$ (c) $\Delta t = 0.01$, $\delta t = 16 \Delta t$
Fig. 3.12. The location of free vortex layers leaving each edge of the plate for $\alpha = 40^\circ$, $Re = 45,000$. (a) Present scheme (b) Abernathy's calculated free-streamline (from Abernathy(1962))
Figure 3.12 (b) which corresponds to the calculated location of free vortex layers by Abernathy (1962) using the free streamline theory and the experimental result of Fage and Johansen (1928) for \( \alpha = 40^\circ \). The evolution of free vortices on Figure 3.12 (a) appears to be in a good agreement with that on Figure 3.12 (b). On Figure 3.12 (b) \( K \) corresponds to the constriction ratio which is defined as the ratio of the height of the tunnel test section vs. the chord length of the plate.

Figures 3.13 and 3.14 show the evolution of the wake at \( t = 10 \) for \( \alpha = 60^\circ \) obtained using different numerical schemes or parameters and may be compared with Figure 3.11 (a). Figure 3.13 (a) is the result for the case where the strength of each nascent vortex has been calculated using the simple forward difference formula (equation (3.46)) for \( \text{Re} = 40,000 \). Compared to Figure 3.11 (a) where the vortex strength has been computed using the integration formula (equation (3.47)), it appears that use of the simple forward difference formula produces much more irregular motion of the vortices than does the integration scheme. Depicted on Figure 3.13 (b) is the wake pattern for the case of \( \text{Re} = 400 \). The smooth rolling-up of shear layers in this figure demonstrates the validity of the Lamb vortex model and is in conformity with the results of Section 3.3 in terms of the effect of the Reynolds number. In conjunction with this, the wake pattern depicted on Figure 3.13 (c) has been obtained without using any correction (ie. Lamb vortex model) in calculating the velocity induced at each vortex position; here each vortex strength shed into the wake has been calculated using the integration scheme. As discussed earlier two other integration schemes have been tested to
Fig. 3.13. The evolution of the wake obtained using different parameters or numerical schemes. (a) Same as Fig. 3.11 (a) except that a simple forward difference scheme is used in calculating the strength of each nascent vortex. (b) Same as Fig. 3.11 (a) except Re=400. (c) Same as Fig. 3.11 (a) but no correction in calculating the induced velocity.
check the numerical accuracy in advancing each individual vortex. Figure 3.14 (a) shows the wake pattern obtained using the fourth order Runge-Kutta scheme and compares well with Figure 3.11 (a) where the Adams-Bashforth scheme has been employed. The wake pattern obtained using the first order Euler scheme is shown on Figure 3.14 (b) and appears that the accuracy is not as good as that of the other two schemes.

The evolution of the wake and its subsequent development for $\alpha = 60^\circ$, $\text{Re} = 40,000$ are depicted on Figure 3.15 during a particular time interval. It appears that the rollup of the shear layers for relatively small Reynolds number is much different from that for large Reynolds number due to the increasingly finer scale of the motion as the shear layers roll up (see Figure 3.13 (b)). Results for other angles of attack also produce similar patterns and are omitted. Here the vortices whose dimensionless streamwise coordinate exceeds 25 have been omitted from the picture. At $t = 60$ (Fig. 3.15 (a)) the vortex cluster of larger strength shed from the trailing edge begins to entrain the vorticity of opposite sign by drawing the vortices shed from the leading edge, thereby decreasing in strength. At $t = 62.4$ (Fig. 3.15 (b)) and $t = 64.8$ (Fig 3.15 (c)), the vortices from the trailing edge shifts downstream and the vortices from the leading edge continue to increase in strength until they occupy practically the entire breadth of the wake at $t = 67.2$ (Fig. 3.15 (d)). From $t = 69.6$ (Fig. 3.15 (e)) to $t = 76.8$ (Fig. 3.15 (h)) this process is reversed. During this period the vortex cluster of larger strength from the leading edge draws the vortices of opposite sign from the trailing edge followed by the decrease in
Fig. 3.14. The evolution of the wake obtained using different integration schemes to advance the vortices. Other parameters are same as those on Fig. 3.11 (a).

(a) Fourth order Runge-Kutta scheme.
(b) First order Euler scheme.
Fig. 3.15. The subsequent evolution and development of the wake behind the plate during a particular time interval for $\alpha = 60^\circ$, $Re=40,000$. 
Fig. 3.15. (Continued)
(g) $t = 74.4$

(h) $t = 76.8$

Fig. 3.15. (Continued)
strength. While the vortices from the leading edge shift downstream the vortices from the trailing edge continue to grow until they envelope the most of the wake region and the cycle repeats. Note that the periodic entrainment of the vorticity from the other edge is in conformity with the formation mechanism of vortices behind bluff bodies proposed by Gerrard(1966). The Strouhal number has been calculated based on Figure 3.15 and the result for $\alpha = 60^\circ$ is $St \sim 0.185$. Here the Strouhal number is defined as

$$St = \frac{f^* (4c)}{U_\infty}$$

(3.48)

where $f^*$ corresponds to the dimensional vortex shedding frequency. The Strouhal number for other angles of attack can be computed similarly and Table 3.1 shows the Strouhal number as a function of the angle of attack and the comparison with the experimental measurement of Fage and Johansen(1927). It appears that the results of present study are in fair agreement with those of Fage and Johansen(1927).

Figure 3.16 shows the dimensionless drag and lift forces as a function of time for $\alpha = 60^\circ$. Since the vortices are introduced impulsively into the flow field both the drag and lift forces initially increase rapidly until the periodic vortex shedding takes control. After the initial transient stage, the drag and lift forces exhibit a typical periodic oscillation which has been reported in much of the previous work(Sarpkaya(1975), Kiya and Arie(1977), Chein and Chung(1988)). Based on this figure the time averaged mean drag force $\overline{F}_x$ has been
Table 3.1  Strouhal number (St) and comparison with experimental data.

<table>
<thead>
<tr>
<th>$\alpha$(deg.)</th>
<th>Strouhal number (St)</th>
<th>present study</th>
<th>Fage and Johansen(1927)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.253</td>
<td></td>
<td>0.231</td>
</tr>
<tr>
<td>50</td>
<td>0.214</td>
<td></td>
<td>0.196</td>
</tr>
<tr>
<td>60</td>
<td>0.185</td>
<td></td>
<td>0.173</td>
</tr>
</tbody>
</table>

Table 3.2  Calculated mean drag $F_x$ and comparison with previous work for several values of $\alpha$.

<table>
<thead>
<tr>
<th>$\alpha$(deg.)</th>
<th>$F_x$</th>
<th>present study</th>
<th>Fage and Johansen(1927)</th>
<th>Sarpkaya(1975)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>1.13</td>
<td></td>
<td>1.01</td>
<td>1.12</td>
</tr>
<tr>
<td>50</td>
<td>1.52</td>
<td></td>
<td>1.38</td>
<td>1.53</td>
</tr>
<tr>
<td>60</td>
<td>1.97</td>
<td></td>
<td>1.71</td>
<td>1.95</td>
</tr>
</tbody>
</table>
Fig. 3.16. Fluid loading as a function of time for \( \alpha = 60^\circ \).
(a) Drag force. (b) Lift force.
calculated by taking average of the instantaneous drag forces over a few cycles after the periodic vortex shedding has settled down. Depicted on Table 3.2 are results for $\overline{F_x}$ compared to those of previous work for $\alpha = 40^\circ$, 50°, 60°. Note that the result reported in Fage and Johansen(1927) and Sarpkaya(1975) corresponds to the mean(or asymptotic) normal force $\overline{F_n}$ and has been converted to $F_x$ using the relation $F_x = F_n \sin \alpha$. The larger values of $F_x$ in the present calculation compared to those of Fage and Johansen(1927) appears to be that the numerical scheme is based on the inviscid flow assumption and thus has not incorporated the viscous phenomena of dissipation and diffusion of vorticity. With regard to this speculation Fage and Johansen(1927) observed that only about 60% of the vorticity which leaves the edge travels downstream in the form of individual vortices. It is likely that reducing the strength of vortices would bring about a reduction in the forces and several methods have been proposed to incorporate the reduction with a varying degree of success (see Sarpkaya(1988) for review).

Figure 3.17 shows the rate of vorticity shedding into the wake for $\alpha = 60^\circ$. It is observed in this figure that, following the initial impulsive stage, the vorticity is shed from each edge of the plate almost at the same rate during each cycle. This is in agreement with the measurements of Fage and Johansen(1927). The mean value of the vorticity flux, $\overline{dk/dt}$, measured by Fage and Johansen(1927) is 0.99 for $\alpha = 50^\circ$ and 1.0 for $\alpha = 70^\circ$ and in present calculation $\overline{dk/dt} = 1.1$. 
Fig. 3.17. The rate of vorticity shedding into shear layers from the leading and trailing edges of the plate for $\alpha = 60^\circ$. — corresponds to the leading edge and - - - corresponds to the trailing edge.
The near field acoustic pressure is depicted on Figure 3.18 (a). As discussed in Chapter II, $G_1', G_2'$ represent the acoustic pressure component involving the streamwise and the transverse motion of vortices. The distinctive character observed in this figure is that the magnitudes of both $G_1'$ and $G_2'$ are comparable so that the dominant effect of $G_2'$ over $G_1'$ on the noise field observed in the vortex impingement problem of Chapter II does not hold. Figure 3.18 (b) shows the sound pressure level obtained using the fast Fourier transform. The parameters employed to scale the pressure correspond to flow in water at $U_\infty = 15.4$ m/sec, with plate chord length $4c = .03$ m. The acoustic pressure is evaluated in physical space at a location directly above the flat plate at $\theta = \pi / 2$, at a radial distance of .254 meter at a Mach number of $M = .0103$. The noise field on this figure exhibits a broadband character with peak occurring at about 95 Hz which corresponds to the Strouhal number of $St = 0.185$. Depicted on Figure 3.19 is the sound pressure level for $\alpha = 50^\circ$ (Figure 3.19 (a)) and $\alpha = 40^\circ$ (Figure 3.19 (b)). The essential features of the noise field are about the same as those for $\alpha = 60^\circ$ depicted on Figure 3.18 (b) but the frequency of the peak generally increases as $\alpha$ decreases. The peak occurs at about 110 Hz ($St = 0.214$) for $\alpha = 50^\circ$ and 130 Hz ($St = 0.253$) for $\alpha = 40^\circ$. 


Fig. 3.18. (a) Time variation of the near field acoustic pressure $G_1'$ and $G_2'$ for $\alpha = 60^\circ$; — denotes $G_1'$ and --- denotes $G_2'$.
(b) Sound pressure level.
Fig. 3.19. Sound pressure level for different angles of attack.
(a) $\alpha = 50^\circ$. (b) $\alpha = 40^\circ$. 
3.5 Summary

The problem of vortex shedding in a flow past a single inclined flat plate has been examined using the discrete vortex method. The theoretical background of approximating a free shear layer by a number of point vortices has been discussed and its application to the rollup of a finite vortex sheet behind an elliptically loaded wing has been investigated.

Despite using the inviscid flow assumption the results of present model appear to be in reasonable agreement with those obtained experimentally. The kinematics of the flow field may be characterized by a periodic shedding of vorticity which, in turn, is reflected in the fluid loading as oscillating drag and lift forces exerted on the flat plate. The noise field of the present problem shows that the peak sound pressure level occurs at the vortex shedding frequency and the activity at higher frequencies is also at a comparable level. The present model is directly applicable to the problem of vortex shedding for multiple body geometries which will be considered in Chapter V. For the multiple body geometry it is likely that, depending on the distance between the bodies, the interference effect would alter the vortex motion and thus the noise field. To incorporate the present model into the multiple body problem, a numerical mapping technique is first required and this is discussed next.
4.1 Introduction

In this chapter, a numerical mapping technique is discussed in which a series of circular cylinders are mapped to multiple flat plate airfoils which henceforth are termed slats. The complex potential in the circle plane can be constructed directly by applying the method of images and then the solutions in the physical plane can be obtained using the inverse mapping. Conformal mapping of a single body to another single body can generally be done by a one-step transformation or by a sequence of simple transformations. If the geometry of interest consists of multiple bodies, however, the single body mapping techniques cannot be directly applied since any single simple canonical domain on which all the bodies are mapped does not exist.

Multiple body mapping methods have been developed by a number of investigators. Garrick (1936) developed a method for mapping biplane wing sections of an arbitrary geometry to the annular region between two concentric circles. Conformal mapping of a lattice of airfoils has been considered by Garrick (1944) in which the lattice region is reduced to the region of a single circle at the final stage of the mapping. The basic ideas
of Garrick (1944) have been further improved by Ives (1976) who employed cubic spline fitting and the Fast Fourier Transform. Halsey's (1977) technique expanded the usual two-element mapping methods to a general multi-element mapping methods. His technique has the advantage of mapping the multi-element airfoils to the same number of disjoint circles with no special constraints on holding the other contour shapes.

In what follows Halsey's (1977) ideas are employed to map the multiple slats into same number of circular cylinder sections. In section 4.2 the Karman-Trefftz transformation is discussed in which a single airfoil is transformed into a single near-circle by removing the corner on the airfoil. The application of the Karman-Trefftz mapping to the multiple slat geometry is described in section 4.3 with the detailed mapping procedures and several examples. Described in section 4.4 and 4.5 are examples of flow field solutions using the mapping technique discussed in section 4.3. Streamline patterns are depicted in section 4.4 and in section 4.5, the interaction of an impinging vortex with multiple slats is considered.

4.2 Karman-Trefftz Transformation

The major difficulty in performing the conformal mapping is in the treatment of the sharp corners where singularities usually occur in the form of infinite velocities. Any contour without sharp corners would be more easily mapped to a canonical contour (eg. a circle) where the
solution for the flow field is available. Thus, given a contour with one or more sharp corners, one can think of removing these corners as an intermediate step between mapping a contour in the physical domain to a desired canonical contour. This is illustrated in Figure 4.1, where the inverse Karman-Trefftz mapping has been applied to remove the corner from a single airfoil resulting in a single near circle. In this work the Karman-Trefftz transformation is employed and is defined by (Karamcheti(1966))

\[
\frac{z - z_0}{z - z_0'} = \left(\frac{\zeta - nz_0}{\zeta - nz_0'}\right)^{1/n}
\]

(4.1)

where \(z\) and \(\zeta\) represent the physical and transformed coordinates respectively, \(z_0\) is the corner to be removed and \(z_0'\) denotes the second singular point. Also \(n = 1/(2 - \beta/\pi)\) where \(\beta\) is the included angle of the corner. For the body which has a leading edge corner with the same angle as the trailing edge corner, this transformation can remove both corners at once. If the physical geometry consists of a flat plate, \(z_0\) and \(z_0'\) are located at the leading and trailing edges of the plate and then equation (4.1) may be written as

\[
\frac{z}{z - z_0'} = \left(\frac{\zeta}{\zeta - z_0' / 2}\right)^2
\]

(4.2)
Fig. 4.1. Effect of the corner removing mapping using the inverse Karman-Treffetz transformation (from Halsey(1977)).

Fig. 4.2. Effect of the Karman-Treffetz transformation for the special case of a flat plate geometry.
and the flat plate is mapped to an exact circle as in Figure 4.2. In this equation the coordinate system is translated such that \( z_0 \) is at the origin and \( \beta = 0 \). Solving equation (4.2) for \( \zeta \),

\[
\zeta = \frac{z_0'}{2} \left\{ 1 - \left( \frac{z - z_0'}{z} \right)^{1/2} \right\}^{-1}
\]

and differentiating

\[
\frac{dz}{d\zeta} = \frac{z_0'^2}{\zeta^2} \left( \frac{z - z_0'}{z} \right)^{1/2}
\]

It is noted that, for a flat plate geometry, the Karman-Trefftz mapping maps points on either side of the plate to separate and distinct points on the circle and thus a careful tracking procedure must be employed to avoid mapping points to the wrong Riemann sheet.

4.3 Multiple Body Mapping Method

The ideas discussed in section 4.2 can also be applied for the mapping of the multiple body geometries depicted on Figure 1.1. Given the physical geometry which consists of multiple bodies the inverse Karman-Trefftz transformation can be successively applied on each body until all of the corners on the body are removed. After the completion of the corner removing mapping all of the bodies would be transformed to
nearly circular bodies; if the distance between adjacent bodies in the original physical domain is small, some may still be quite non-circular. However, even these bodies should be much more rounded than they originally were.

Having removed the corners from all the bodies, the next step is to map individual bodies to exact circles. This should not present any difficulty since most bodies would already be nearly circular at this stage. There are now a variety of ways to accomplish the near-circle to circle mapping (Theodorsen (1931), Ives (1976), Halsey (1977)). The basic idea is that one body is first mapped to an exact circle and this would change the rest of the bodies to whatever shape they may take. Then the second body is mapped to an exact circle and all other bodies, including the already-circular first body, are changed again. This procedure is repeated until each body has been mapped to a circle once. At this point all bodies would be more nearly circular than they were immediately after the corner removing mapping. This sequence of mappings can then be repeated iteratively until all the bodies are circular to a specified tolerance. The detailed formulation and individual steps of the near-circle to circle mapping method are given by Halsey (1977).

Listed below are the individual steps in corner removing mapping procedure.

Process the initial input coordinate data of the flat plates in the physical plane. At this point the first body of which corner is to be
removed is at the origin and each flat plate is represented by a number of data points.

Initialize the modulus of the mapping derivative at each input point to unity.

Apply the corner removing mapping to the first body which has a corner. Note that the second singular point \((z_0')\) is also removed in this process. Compute the mapped coordinates corresponding to each input point on each body.

Update the moduli of the mapping derivatives at each point by multiplying the previous values by the value determined from the present mapping.

Translate the corner of the second body on which the corner removing mapping is to be applied to the origin.

Repeat above steps until all bodies are removed of corners.

Depicted on Figure 4.3 are the results of the successive application of the Karman-Trefftz mapping on five parallel flat plates. Initially each flat plate is represented by \(N_a\) number of equally spaced input points in the physical domain. Generally \(N_a = 100\) is sufficiently accurate for our purposes as noted below. Note that the physical coordinates of the upper surface and lower surface are the same but they are mapped to different coordinates in the transformed domain. The element (ie. flat plate) which has had the corner removed is now quasi-circular and all the other elements have been distorted to a degree depending on their proximity to the corner-removing element (Figure 4.3 (b)). This
Fig. 4.3. Corner removing mapping for five parallel flat plates. (a) Initial geometry in the physical plane. (b)-(f) Geometries in the transformed plane after successive application of the Karman-Treffetz transformation in order.
procedure is repeated for each element until all the corners are removed. In this procedure, it is understood that the corners are first translated to the origin according to equation (4.2). In one sweep of this procedure all the elements are nearly circular (Figure 4.3 (f)) and the result is that the area outside the multiple flat plates is mapped to the area outside the same number of disjoint near circles.

When the bodies are sufficiently far apart, removing the corner results in a body which is very nearly circular to engineering accuracy and thus any further mapping may not be necessary for the flow field solution. Figures 4.4 and 4.5 show this case in which multiple flat plates in a tandem arrangement are mapped to the same number of circular cylinders.

To check whether the bodies after the corner removing mapping are close enough to exact circles the radial distance between the transformed input points and the geometric center of the body has been computed for several values of the separation distance between bodies in the physical domain. The geometry in the physical plane (Figure 4.6 (a)) consists of three slats of same chord length in a tandem arrangement separated by a dimensional distance \( d_s \). The tabulated values in Table 4.1 have been nondimensionalized on the chord length \( 4c \) and \( r_{\text{max}}, r_{\text{min}} \) denote the maximum and minimum radial distance computed for each body (Figure 4.6 (b)). Note that the deviation \( (r_{\text{max}} - r_{\text{min}}) \) for each body is kept within \( 10^{-3} \) for \( 3 \leq \frac{d_s}{4c} \) and therefore it may be justified in this range that all of the bodies after the corner removing mapping are
Fig. 4.4. Same as Figure 4.3 for the case of four flat plates in a tandem arrangement. (a) Initial geometry. (b)-(e) Transformed geometries in order.
Fig. 4.5. Same as Figure 4.3 for the case of four inclined flat plates in a tandem arrangement. (a) Initial geometry. (b)-(e) Transformed geometries in order.
Fig. 4.6. The geometry employed to check the accuracy of the corner removing mapping as a function of the separation distance between bodies. (a) Physical geometry. (b) Transformed geometry.

Table 4.1 The deviation of the radial distance as a function of the separation distance between bodies; computed results in the table are nondimensionalized by the chord length of the plate '4c'.

<table>
<thead>
<tr>
<th>$d_s / 4c$</th>
<th>$r_{max} - r_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>body 1</td>
</tr>
<tr>
<td>1</td>
<td>8.5xE-3</td>
</tr>
<tr>
<td>2</td>
<td>2.5xE-3</td>
</tr>
<tr>
<td>2.83</td>
<td>1.0xE-3</td>
</tr>
<tr>
<td>4</td>
<td>3.9xE-4</td>
</tr>
</tbody>
</table>
almost exact circles so that any further near-circle to circle mapping may not be required for the flow field solutions in this case.

In the following section the streamline patterns for several flat plate geometries are obtained using the multiple body mapping techniques discussed in this section. The major objective is to verify the validity of these mapping methods to investigate the flow field of the multiple body geometries depicted on Figure 1.1.

4.4 Streamlines for Flow past Multiple Flat Plates

Using the Karman-Trefftz transformation discussed in the previous section the solutions for the flow field can be obtained in the cylinder plane using the method of images. The complex potential in the presence of the uniform flow and NV vortices outside NC flat plates may be written in transformed variables,

\[
W(z) = W(\zeta) = \zeta - \sum_{m=1}^{NV} \frac{ik_m}{2\pi} \ln(\zeta - \zeta_m) + \sum_{j=1}^{\infty} (W_j^F + W_j^M) + \sum_{m=1}^{NV} \sum_{j=1}^{\infty} W_j^{jm} + \sum_{m=1}^{NC} \sum_{j=1}^{\infty} W_j^{jm} \tag{4.5}
\]

where the first two terms correspond to uniform flow plus NV vortices in free space, \(W_{jm}^V\) is the image potential for NV vortices outside NC cylinders, \(W_j^F\) are image doublets corresponding to steady flow past NC fixed cylinders, and \(W_j^M\) is the potential corresponding to finite amplitude motion of the flat plates; \(W_{jm}^K\) represents the potential due to
the circulation around each body to remove the infinite-velocity singularity at the sharp edge and is associated with the satisfaction of the Kutta condition. The precise form of $W^V_{jm}$, $W^K_{jm}$, $W^F_j$, and $W^M_j$ are

$$W^V_{jm} = \frac{ik_m}{2\pi} C_j e^{i\alpha_j} \{\ln(\xi - \varepsilon_{jm}) - \ln(\xi - \rho_j)\}$$

$$W^K_{jm} = \frac{i\Gamma_m}{2\pi} C_j e^{i\alpha_j} \ln(\xi - \rho_j)$$

$$W^F_j = A \frac{e^{i\varphi_j}}{j\xi - \gamma_j}$$

$$W^M_j = -D \frac{e^{i\varphi_j}}{j\xi - \gamma_j}$$

Equations (4.5) and (4.6) are identical to equations (2.1) and (2.2) in Chapter II except $z$ has been switched to $\xi$ and an additional term, $W^K_{jm}$ has been added and thus the expressions for each term in equation (2.2) are equally applied in equation (4.6) by changing $z$ to $\xi$. Note that all lengths have been nondimensionalized on the half chord length of the plate $2c$, all velocities on the speed of the uniform flow, $U_\infty$ and $k_m = k_m^* / 2cU_\infty$; also $\Gamma_m$, the circulation around the $m^{th}$ plate, is defined by $\Gamma_m = \Gamma_m^* / 2cU_\infty$. Applying equation (4.5) and noting that $W = \phi + i\psi$, the stream function can be computed at each mesh point in the cylinder plane and transferred directly to the physical plane.
In order to assure the finite velocity at the corner, the Kutta condition is imposed at the trailing edge of each plate such that, in the cylinder plane,

\[ \frac{dW}{d\zeta} = 0 \quad \text{at} \quad \zeta = \zeta_{TN}, \quad n = 1, 2, \ldots, NC \quad (4.7) \]

where \( \zeta_{TN} \) represents the transformed coordinate of the \( n^{th} \) trailing edge and \( NC \) denotes the total number of flat plates. Inserting (4.5) into (4.7) a set of linear equations are obtained as

\[
\begin{align*}
\left. \frac{dW}{d\zeta} \right|_{\zeta = \zeta_{T_1}} &= 0 \\
\left. \frac{dW}{d\zeta} \right|_{\zeta = \zeta_{T_2}} &= 0 \\
&\vdots \\
\left. \frac{dW}{d\zeta} \right|_{\zeta = \zeta_{TN}} &= 0
\end{align*}
\quad (4.8)\
\]

which may be written in the form

\[
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
& \ddots & \ddots & \cdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\vdots \\
\Gamma_n
\end{bmatrix} =
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_n
\end{bmatrix}
\quad (4.9)\
\]
where

\[
[a] = \begin{bmatrix}
( \sum_{j=1}^{\infty} W_j / \Gamma_1 )_{\zeta=\zeta_1} & \cdots & ( \sum_{j=1}^{\infty} W_j / \Gamma_n )_{\zeta=\zeta_n} \\
( \sum_{j=1}^{\infty} W_j / \Gamma_1 )_{\zeta=\zeta_2} & \cdots & ( \sum_{j=1}^{\infty} W_j / \Gamma_n )_{\zeta=\zeta_n} \\
\vdots & \ddots & \vdots \\
( \sum_{j=1}^{\infty} W_j / \Gamma_1 )_{\zeta=\zeta_n} & \cdots & ( \sum_{j=1}^{\infty} W_j / \Gamma_n )_{\zeta=\zeta_n}
\end{bmatrix}
\]

and

\[
[b] = \begin{bmatrix}
\frac{d}{d\zeta} \left( \sum_{m=1}^{NC} \sum_{j=1}^{\infty} W_j^K - W(\zeta) \right)_{\zeta=\zeta_1} \\
\vdots \\
\frac{d}{d\zeta} \left( \sum_{m=1}^{NC} \sum_{j=1}^{\infty} W_j^K - W(\zeta) \right)_{\zeta=\zeta_n}
\end{bmatrix}
\]

Figure 4.7 is the case for the single vortex and a uniform flow over a rigid flat plate at incidence. Here the lower edge is at the origin in the physical plane and the vortex position is at \( z_1 = (-2, 0.5) \) with a rather large strength of \( k_1 = -10 \). The circulation around the flat plate has been computed using the Gaussian elimination technique in equation (4.9) and then the value of the stream function at each prescribed mesh point has been calculated in equation (4.5). The same vortex position and
Fig. 4.7. Streamline pattern for a single vortex and a uniform flow over an inclined flat plate.
strength have been applied for the case of two rigid flat plates on Figure 4.8. Note that the stagnation points fall on the edges since Kutta condition was imposed on each plate. Depicted on Figure 4.9 is the case of five rigid flat plates in a tandem arrangement. It is concluded from these streamline patterns that the numerical mapping techniques discussed in Section 4.3 can be directly applied to investigate the flow field solutions for the geometry of multiple flat plates.

4.5 Interaction of an Impinging Vortex with Multiple Slats

The interaction between an impinging vortex (or vortices) and the two elastic cylinders have been discussed in Chapter II. In this section a similar approach is adopted along with the multiple body mapping technique to investigate the flow induced sound on the multiple slat geometry. The mechanism of excitation is again the case of buffeting by a point vortex (or vortices) carried along with the uniform flow of speed $U_\infty$. Of particular interest are the vortex trajectory, fluid loading and radiated sound pressure. Also of interest is whether these solutions bear any qualitative similarity to those of the two cylinder geometry in Chapter II.

A number of previous papers on the vortex-airfoil interaction problem have mostly considered a single airfoil geometry. Huang and Chow (1982) motivated by the work of Saffman and Sheffield (1977) investigated the trapping criteria of a free vortex to enhance the lift
Fig. 4.8. Same as Figure 4.7 for two flat plates at incidence in a tandem arrangement.
Fig. 4.9. Same as Figure 4.7 for five flat plates at incidence in a tandem arrangement.
performance for several airfoil shapes. The noise field due to the blade-vortex interaction has been studied by Hardin and Mason (1985) and later by Hardin and Lamkin (1986). In these papers the Joukowski transformation has been employed and the flow field solutions have been computed using either the complex potential technique or finite difference method.

It does not appear that any previous attempts have been made on the geometry of more than a single airfoil. What is more complicated in the multiple body geometry is that the interference effect changes the vortex motion to a degree depending upon the proximity between bodies. Further complications could arise if one or some of the bodies oscillate as discussed in Chapter II.

The complex potential of the present problem has been given by equation (4.5) in Section 4.4. Each individual point vortex initially placed far upstream of the multiple slats is convected by the velocity induced at the vortex position which is given by

\[
(u - iv)_{z=z_j} = \left( \frac{dW}{d\zeta} + \frac{ik_j}{2\pi} \frac{1}{\zeta - \zeta_j} \right)_{\zeta=\zeta_j} \left( \frac{d\zeta}{dz} \right)_{\zeta=\zeta_j} \frac{ik_j}{4\pi} \frac{(d^2z/d\zeta^2)_{\zeta=\zeta_j}}{(dz/d\zeta)_{\zeta=\zeta_j}^2}
\]

(4.10)

and each vortex is advanced by solving the set of equations

\[
\frac{dz_j}{dt} = (u - iv)_{z=z_j}, \quad j = 1, \ldots, NV
\]

(4.11)
where \((u - iv)_{z = z_j}\) corresponds to the complex velocity induced at the \(j^{th}\) vortex of strength \(k_j\) and \(z_j\) denotes the vortex position in the physical plane with \(\zeta_j\) its transformed coordinate. Note again that the second term in equation (4.10) is the Routh correction appearing as a consequence of the transformation.

The fluid loading exerted on each slat may be calculated from the Blasius theorem given by equation (2.7). Since the complex potential in equation (4.5) includes an infinite number of image terms, an analytical evaluation of the fluid loading appears to be cumbersome. Alternatively, the fluid loading in equation (2.7) may be computed numerically by integrating the pressure distribution on the surface of each slat. To evaluate the steady part of the fluid loading given by equation (3.27), define \(\zeta = ae^{i\theta}\). Then \(dW / d\zeta = d\overline{W} / d\zeta\) on \(|\zeta| = a\) because the stream function \(\Psi\) is constant and thus equation (3.27) may be written as

\[
F_{x1} - iF_{y1} = \frac{i}{2} \int_C \frac{dW}{d\zeta} \overline{\frac{dW}{d\zeta}} \frac{d\zeta}{dz} d\zeta \quad (4.12)
\]

which can be arranged as

\[
F_{x1} - iF_{y1} = \frac{a}{2} \int_0^{2\pi} \left| \frac{dW}{d\zeta} \right|^2 \frac{d\zeta}{dz} \bigg|_{|\zeta| = a} e^{-i\theta} d\theta \quad (4.13)
\]

Similarly, the unsteady part of the fluid loading given by equation (3.26) can be written as
and combining (4.13) and (4.14) the total fluid loading may be obtained as

\[
F_x + iF_y = \frac{a}{2} \int_0^{2\pi} \left[ |dW / d\zeta|^2 (d\zeta / dz) \right]_{\zeta = 0} e^{i\theta} d\theta \\
+ a \int_0^{2\pi} \left\{ \frac{\partial W}{\partial t} \frac{dz}{d\zeta} \right\}_{\zeta = a} e^{i\theta} d\theta
\]  

To evaluate the integrals in this equation a numerical integration scheme using the Simpson's method is employed and the term \( \partial W / \partial t \) in (4.15) is evaluated using the four point backward difference scheme.

The noise field of the present problem is similar to that considered in Chapter II except that the generating function changes to include the term corresponding to the circulation around each slat. Hence following the same treatment of Chapter II the generating function may be obtained as

\[
\phi_a = \frac{M}{R} (F_1 \cos \theta + F_2 \sin \theta), \ R \to 0
\]  

where \( R \) corresponds to the far field radial coordinate in the physical plane and is related to the near field radial coordinate \( r \) by \( R = Mr \) and

\[
F_1 = \text{Re} \left\{ \frac{i}{2\pi} \sum_{m=1}^{NV} k_m \zeta_m + \frac{i}{2\pi} \sum_{j=1}^{\infty} C_j e^{i\theta_j} \sum_{m=1}^{NV} k_m (\rho_j - \epsilon_{jm}) \right. \\
+ \left. \sum_{j=1}^{\infty} (A_j e^{i\alpha_j} - D_j e^{i\beta_j}) + \frac{i}{2\pi} \sum_{j=1}^{\infty} C_j e^{i\beta_j} \sum_{m=1}^{NC} \Gamma_m \rho_j \right\}
\]  

(4.17)
with \( F_2 \) being the corresponding imaginary part.

Results have been computed for three different geometries depicted on Figure 4.10. Placed far upstream of the structure is a point vortex of strength \( k_1 = -1.0 \) at the initial position \( z_{v1} = (-5, .5) \). The point vortex is advanced by solving equation (4.12) using the fourth order Adams-Moulton method.

Case I  The geometry of interest depicted on Figure 4.10 (a) consists of two rigid tandem slats of chord length \( 4c \) and a long splitter plate of length \( 5 \times 4c \). The two slats are placed at an incidence of \( \alpha = 3\pi / 4 \) to the ambient flow with the separation distance \( d_s / 4c = 3 \); the distance between the splitter plate and the most downstream slat is \( 2.5 \times 4c \) and the offset parameter of the splitter plate is \( h = 2c \sin \alpha \). Numerical accuracy has been investigated in terms of time step \( \Delta t \) and the number of points \( N_s \) distributed along each slat in the calculation of fluid loading, and the results are depicted on Table 4.2. Generally, \( \Delta t = .025, N_s = 100 \) produces the extremely accurate solutions and thus have been employed in the actual computation. Depicted on Figures 4.11 (a), (b) and (c) are the vortex path, drag force and lift force respectively. On Figure 4.11 (a) the point vortex initially placed upstream of two slats gradually negotiates with two slats and convects downstream almost parallel to the splitter plate. Both the drag and lift forces exhibit the impulsive character on Figures 4.11 (b) and (c). The drag and lift forces for each slat are of almost the same magnitude but of opposite sign such that the
Fig. 4.10. Geometries considered for the interaction of an impinging vortex with multiple slats. (a) Two slats with a downstream splitter plate. (b) Three slats with a downstream splitter plate. (c) Same as (b) except the middle slat is elastic.
Table 4.2  Numerical accuracy of the solution as a function of time step and the number of points along the slat. Here $k_1 = -1$ and the dimensionless times are shown.

(a) Parameters as a function of time step $\Delta t$ ($N_s = 100$)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\Delta t$</th>
<th>$x_1$</th>
<th>$y_1$</th>
<th>$F'_1$</th>
<th>$F'_2$</th>
<th>Drag Force</th>
<th>Lift Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Slat 1</td>
<td>Slat 2</td>
</tr>
<tr>
<td>2.0</td>
<td>0.025</td>
<td>1.24901</td>
<td>-0.78685</td>
<td>-0.00642</td>
<td>0.22849</td>
<td>1.96681</td>
<td>2.13805</td>
</tr>
<tr>
<td></td>
<td>0.0125</td>
<td>1.24902</td>
<td>-0.78705</td>
<td>-0.00644</td>
<td>0.2285</td>
<td>1.96667</td>
<td>2.13799</td>
</tr>
<tr>
<td>5.0</td>
<td>0.025</td>
<td>9.49349</td>
<td>0.63022</td>
<td>-0.00265</td>
<td>0.01875</td>
<td>1.86752</td>
<td>1.99532</td>
</tr>
<tr>
<td></td>
<td>0.0125</td>
<td>9.48995</td>
<td>0.63015</td>
<td>-0.00271</td>
<td>0.01868</td>
<td>1.86747</td>
<td>1.99525</td>
</tr>
</tbody>
</table>

(b) Parameters as a function of $N_s$ at $t = 2.0$ ($\Delta t = .025$)

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>Drag Force</th>
<th>Lift Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slat 1</td>
<td>Slat 2</td>
</tr>
<tr>
<td>100</td>
<td>1.96681</td>
<td>2.13805</td>
</tr>
<tr>
<td>200</td>
<td>1.96829</td>
<td>2.14121</td>
</tr>
</tbody>
</table>

168
Fig. 4.11. (a) Vortex path for a single vortex placed at $z_{v1} = (-5, .5)$, of strength $k_{v} = -1.0$. (b) Drag on the two slats as a function of time. (c) Lift on the two slats as a function of time. Both slats are rigid; — denotes slat 1, --- denotes slat 2.
total force acts in the direction normal to the slat. The small blip in drag and lift forces is not due to a numerical error and this is shown on Figures 4.12 and 4.13 which correspond to the drag and lift forces calculated for $\Delta t = 0.0125$ (Figure 4.12) and $\Delta t = 0.00625$ (Figure 4.13). It appears that the presence of the splitter plate brings about a feedback effect so that the point vortex is accelerated to negotiate the splitter plate for a short period of time. This feedback mechanism has been shown to be extremely important in producing organized oscillations of flows impinging on downstream bodies resulting in pressure fluctuations felt on the downstream body which is the splitter plate in the present case (Rockwell and Naudascher (1979)). While the present flow is much different from those cases described by Rockwell and Naudascher (1979) it is significant that a feedback effect emerges from the calculation in the present problem. Note that the Kutta condition has not been invoked at the leading edge of the splitter plate.

The acoustic pressure in the near field on Figure 4.14 (a) again exhibits a similar impulsive character. As discussed in Section 2.3, $F_1'$ and $F_2'$ represent acoustic pressure components involving the streamwise and the transverse motion of the vortex respectively and, on Figure 4.14 (a), $F_2'$ is about an order of magnitude larger than $F_1'$ indicating that the qualitative characteristics of the present noise field are similar to those of the tandem cylinder problem considered in Chapter II. On Figure 4.14 (b) is the sound pressure level using the fast Fourier transform. The parameters employed to scale the pressure correspond to flow in water at $U_\infty = 15.4 \text{ m/sec}$, with slat chord length of
Fig. 4.12. (a) Drag force and (b) Lift force for $\Delta t = 0.0125$. Other parameters are same as those of Fig. 4.11.
Fig. 4.13. (a) Drag force and (b) Lift force for $\Delta t = 0.00625$. Other parameters are same as those of Fig. 4.11.
Fig. 4.14. (a) Time variation of the acoustic pressure in the near field as a function of time for the case of Fig. 4.11; $F_1$ and $F_2$ are functions of the vortex path. The symbol — is $F_1' = dF_1 / dt$, --- is $F_2'$. (b) Sound pressure level.
4c = .03 m. Other parameters are same as those employed in Section 2.3. The sound pressure may be characterized by a rapid decrease to a background level of about 60 dB.

<Case 2> The geometry of interest in this case depicted on Figure 4.10 (b) is similar to that of Case 1, but another slat has been added. Other parameters employed in the computation are same as before. Depicted on Figures 4.15 and 4.16 are the vortex path, drag force, lift force, near field acoustic pressure, and the sound pressure level. Most of these properties exhibit quite similar characteristics to those of previous case. Note that the feedback effect of the splitter plate is strongest for the most downstream slat.

<Case 3> The geometry on Figure 4.10 (c) is same as that of Case 2 but in this case the middle slat is allowed to vibrate. For simplicity, the middle slat is assumed to oscillate only in the direction normal to the slat (ie. heaving motion) and the rotating motion of the slat is not considered here. The structural parameters employed for the middle slat correspond to the natural frequency $\omega_2 = 2.23$, $\mu_2 = 5$ and there is no damping. Other parameters are same as those of previous cases. The vortex path depicted on Figure 4.17 (a) is similar to that depicted on Figure 4.15 (a). The drag and lift forces on Figures 4.17 (b) and (c) also exhibit similar characteristics for the upstream and downstream slats. However, the forces on the middle slat are quite different from those of rigid case and it appears that the vortex motion strongly influences the
Fig. 4.15. (a) Vortex path for a single vortex placed at \( z_{v1} = (-5, 5) \), of strength \( k_1 = -1.0 \). (b) Drag on the three slats as a function of time. (c) Lift on the three slats as a function of time. All three slats are rigid; — denotes slat 1, --- denotes slat 2 and \( x \) denotes slat 3.
Fig. 4.16. (a) Time variation of the acoustic pressure in the near field as a function of time for the case of Fig. 4.15. The symbol — is $F_1'$, -- is $F_2'$. (b) Sound pressure level.
Fig. 4.17. (a) Vortex path for a single vortex placed at \( z_{v1} = (-5, 0.5) \), of strength \( k_1 = -1.0 \). (b) Drag on the three slats as a function of time. (c) Lift on the three slats as a function of time. The middle slat is elastic; — denotes slat 1, - - - denotes slat 2 and x x denotes slat 3.
motion of the middle slat. The near field acoustic pressure on Figure 4.18 (a) exhibits an impulsive character followed by a time periodic component at a response frequency $\omega_r = 2.17$. Note that this response frequency is not equal to the natural frequency and it is likely that the motion of the middle slat provides a fluid-damping effect. The sound pressure level is depicted on Figure 4.18 (b) and the peak at about 140 dB occurs at 362 Hz ($\omega_2 = 2.23$ corresponds to 372 Hz) which is the response frequency of the middle slat.

4.6 Summary

The numerical mapping technique presented in this chapter appears to provide a versatile tool in the analysis of a flow over the complicated, multiple body geometries. The streamline patterns depicted using this technique demonstrates that the problem can be directly solved in the transformed plane. The vortex path, fluid loading and the sound field considered in the interaction of an impinging vortex with multiple slats exhibit qualitatively similar characteristics to those considered in Chapter II. It appears that the impulsive nature of the fluid loading and the sound field is typical of the vortex-structure interaction problem due to buffeting. The presence of the splitter plate seems to result in a feedback effect which is felt on the upstream plates as the vortex negotiates the edge of the splitter plate. While this flow is considerably different from those in which feedback has been observed to occur, it is striking that such an effect emerges noticeably in the
Fig. 4.18. (a) Time variation of the acoustic pressure in the near field as a function of time for the case of Fig. 4.17. The symbol — is $F_1'$, --- is $F_2'$. (b) Sound pressure level.
calculations. It is still not clear, however, how this feedback effect plays a role in the sound field for the actual flow field where the vortex shedding is likely to occur and this is further considered in the next chapter.
5.1 Introduction

In previous chapters the modeling of a vortex sheet by a finite number of discrete vortices along with the multiple body mapping technique has established the appropriate methods to investigate the interaction of the unsteady flow with multiple body geometries. While the buffeting mechanism considered in Chapter IV provides the basic understanding of the vortex-induced sound it is deficient in simulating the actual flow field since the flow separation has not been incorporated. It is well known that the unsteady flow past bluff bodies tends to separate producing a large scale wake with vortex shedding over a wide range of Reynolds numbers. The noise generated by the vortex shedding is generally characterized by a broadband spectrum with the peak occurring at the shedding frequency. For the case of multiple bodies it is likely that the interference effect among bodies significantly alters the overall flow field; indeed it has been pointed out in Chapter I that the flow pattern determined experimentally is a strong function of the spacing between bodies and the Reynolds number.
Much of the available information on the unsteady flow past multiple bodies may be derived from experiments on circular cylinders. For the particular case of the flow past two tandem cylinders, much previous work suggests that, for the spacing parameter $d_s/D < 3-4$, the vortex shedding from the upstream cylinder(s) is suppressed and occurs only off the downstream cylinder. This feedback effect of the downstream body appears similar to the mechanism by which vortex shedding is suppressed by the influence of a splitter plate on the flow past a circular cylinder (Roshko(1955)). A comprehensive review of interference effects between two circular cylinders is given by Zdravkovich(1977). Associated with the influence of the downstream body, Unal and Rockwell(1987) investigated the control of vortex formation from a circular cylinder by placing a long plate in its wake. Little work has been reported in flow past multiple bodies other than circular cylinders, but it seems probable that the general form of the flow field is not changed much by the shape of the body (Mair and Maull (1971)). It is not clear at present how the interference effect between bodies influences the sound field and no work has been reported with regard to this point. However, it is likely that the interference effect will alter the fluid motion and its interaction with the bodies and thus leads to a significant effect on the far sound field.

In the present chapter, we focus first on the case of flow past an inclined plate with a splitter plate downstream. In Section 5.2 the formulation for this problem is given in detail and the results are given in Section 5.3. Also described in Section 5.3 is the flow past two inclined
plates. Of course, the flow field of interest is that of Figure 1.1 (b) and in this regard it is to be noted that the inclusion of the vortex shedding process from each plate is enormously complicated; thus, this case is viewed as a heuristic extrapolation from the two situations described above. It is believed that this extrapolation gives a reasonable picture (although by no means rigorous or complete) of the vortex shedding process past a set of multiple plates. Finally the flow depicted on Figure 1.1 is considered by including the vortex shedding only from the last inclined plate and the results are presented.

5.2 Formulation

The problem of interest in this chapter is to investigate the flow-induced sound by the vortex shedding from an inclined flat plate in the presence of other flat plates in its neighborhood. To illustrate the problem Figure 5.1 shows a sample geometry where the physical geometry consists of an inclined flat plate and a long horizontal splitter plate (Figure 5.1 (a)). Using the multiple body mapping technique discussed in Chapter IV the solution for the flow field can be obtained in the cylinder plane on Figure 5.1 (b) using the method of images. In this section we present only the salient features of the formulation specific to the present problem because most of the essential formulations have been developed in previous chapters.
Fig. 5.1. The geometry of interest for the flow past an inclined flat plate in the presence of the downstream splitter plate. (a) Physical plane. (b) Transformed plane.
The complex potential in the presence of the uniform flow and NV vortices outside NC flat plates may be written in transformed variables such that

\[ W(\zeta) = \zeta - \sum_{m=1}^{NV} \frac{ik_m}{2\pi} \ln(\zeta - \zeta_m) + \sum_{j=1}^{\infty} (W_j^F + W_j^M) + \sum_{m=1}^{NV} \sum_{j=1}^\infty W_{jm}^V \]  

(5.1)

The physical meaning of each term in this equation is given in Chapters II and IV and the precise form of \( W_{jm}^V \), \( W_j^F \), and \( W_j^M \) are

\[ W_{jm}^V = \frac{ik_m}{2\pi} C_j e^{i\alpha_j} \ln(\zeta - \zeta_{jm}) \]

\[ W_j^F = A \frac{e^{i\alpha_j}}{j\zeta_j - \gamma_j} \]

\[ W_j^M = -D \frac{e^{i\beta_j}}{j\zeta_j - \gamma_j} \]

(5.2)

The expression for \( W_{jm}^V \) in this equation is different from the one in equation (4.6) because the term corresponding to the image vortices located at the center of each cylinder is excluded from \( W_{jm}^V \) to satisfy the Kelvin's theorem as discussed in Chapter III. Note that the expressions for each righthand side term in equation (5.2) are same as those in equation (2.2) except \( z \) is changed to \( \zeta \). Also in equation (5.2), all lengths have been nondimensionalized on the half chord length \( 2c \) of the plate from which vortex shedding occurs and all velocities have been nondimensionalized on the speed of the uniform flow \( U_\infty \).
The complex velocity induced at each individual vortex is given by equation (4.10) and each vortex is advanced by solving the set of equations similar to equation (4.11). The fluid loading exerted on the plate can be calculated using equation (4.15) and the Kutta condition is given by equation (3.23). Note that the Kutta condition in the present problem is satisfied only at the edges of the plate where vortex shedding occurs. The noise field of the present problem is similar to that considered in Chapter IV except that the generating function changes such that

$$\phi_a = \frac{M}{R}(E_1 \cos \theta + E_2 \sin \theta), \ R \to 0$$

(5.3)

where

$$E_1 = \text{Re}\left\{\frac{i}{2\pi} \sum_{m=1}^{NV} k_m \zeta_m - \frac{i}{2\pi} \sum_{j=1}^\infty C_j e^{i\theta_j} \sum_{m=1}^{NV} k_m \varepsilon_{jm} \right. \right.$$  

$$ \left. + \sum_{j=1}^\infty (A_j e^{i\alpha_j} - D_j e^{i\beta_j}) \right\}$$

(5.4)

with $E_2$ being the corresponding imaginary part. Note that the details of the solution procedure for the far field acoustic pressure are given in Appendix B.
5.3 Results and Discussion

The numerical procedure employed for the present problem is the same as the one described in Section 3.4 and thus will not be repeated here. Because the major interest of this chapter is to investigate the feedback effect of the splitter plate we first illustrate several geometric parameters specifically involved in the present problem. As discussed in Section 3.4 the angle of attack $\alpha$ is again likely to play an important role in terms of determining the pertinent fluid loading and the noise field. More importantly, the streamwise spacing between the inclined flat plate from which the vorticity is shed and the splitter plate appears to be crucial such that the flow development is controlled by the interaction of the wake formed behind the flat plate with the leading edge of the splitter plate. Another important parameter is the offset of the splitter plate which is defined as the transverse position of the splitter plate. To explain this further Figure 5.2 shows two different splitter plate configurations for which the offset parameter $h$ is different. On this figure $h$ is measured from the trailing edge of the inclined flat plate and on Figure 5.2 (b) $h = 0$. It appears that the solution would exhibit a variety of characteristics depending upon the different combination of these parameters. To limit the number of disposable geometric parameters minimum, the angle of attack $\alpha$ and the streamwise spacing have been fixed in the present calculation and the results will be presented for the streamwise spacing of $4c$ and $\alpha = 140^\circ$; the Reynolds number employed is $Re = 50,000$. 
Fig. 5.2. The splitter plate configurations of different offset parameter.
(a) $h = 2c \sin \alpha$.  (b) $h = 0$
To check the numerical accuracy of the scheme in terms of the time step $\Delta t$, Figure 5.3 (a) shows the wake pattern at $t = 16$ for $\Delta t = 0.04$, $\delta t = 4 \Delta t$; note that $\delta t$ corresponds to the time interval between the introduction of the nascent vortex and the offset parameter $h = 2c\sin \alpha$ here (i.e., the transverse position of the splitter plate is at the center of the inclined flat plate). To evaluate the effect of reducing $\Delta t$, the corresponding wake pattern at $t = 16$ for $\Delta t = 0.02$, $\delta t = 8 \Delta t$ is depicted on Figure 5.3 (b) and it appears that the general features of the evolution of the wake are similar on both figures except for the slight differences in the fine scale structure. Hence in all the results what follows $\Delta t = 0.04$, $\delta t = 4 \Delta t$ has been employed.

Depicted on Figure 5.4 (b) is the wake pattern behind the flat plate for the offset parameter $h = 2c\sin \alpha$ at $t = 60$. This figure may be compared with Figure 5.4 (a) which corresponds to the wake pattern at $t = 60$ in the absence of the splitter plate. In contrast with the periodic vortex shedding on Figure 5.4 (a) the wake pattern on Figure 5.4 (b) is significantly altered by the splitter plate so that regular and periodic vortex shedding is strongly suppressed. This feedback effect of the downstream splitter plate on the wake development from the upstream flat plate is in qualitative agreement with the experimental observations by Roshko (1955) and Unal and Rockwell (1988) in which the insertion of a long plate and a wedge respectively in the wake of a cylinder inhibits the regular periodic vortex shedding process. Note that the splitter plate has been placed along the centerline of the wake in both of these experiments and no results with regard to the effect of the offset parameter have been
Fig. 5.3. Numerical accuracy on the evolution of the wake for different time steps. Here $h = 2c \sin \alpha$ and $t = 16$.
(a) $\Delta t = 0.04$, $\delta t = 4\Delta t$. (b) $\Delta t = 0.02$, $\delta t = 8\Delta t$. 
Fig. 5.4. The effect of the offset parameter on the wake development at $t = 60$. (a) A single flat plate without the splitter plate. (b) $h = 2c \sin \alpha$. (c) $h = 0$. 
reported. Figure 5.4 (c) shows the wake pattern at \( t = 60 \) for \( h = 0 \). Compared to the case of \( h = 2csin\alpha \) on Figure 5.4 (b) where the regular vortex shedding process is mostly suppressed, the alternate shedding of vorticity is substantially retained here and the overall flow field is dominantly controlled by the interaction between the vortex clusters of opposite sign although there is a local interaction between the vortices shed from the trailing edge of the flat plate and the leading edge of the splitter plate. Depicted on Figure 5.5 is the subsequent development of the wake for the case of \( h = 0 \). It appears that the periodic entrainment of the vorticity from the other edge is similar to that of the single inclined flat plate discussed in Section 3.4.

The characteristics of the wake development depicted on Figure 5.4 appear to be directly reflected on the fluid loading and the noise field. Depicted on Figures 5.6-5.8 is the dimensionless drag force as a function of time. Figure 5.6, the drag force for the case of the inclined flat plate without the splitter plate exhibits a typical periodic pattern as expected from the results of Section 3.4. The time averaged mean drag \( \bar{F}_x \) is about 1.2 here. In contrast with this, the drag force depicted on Figure 5.7 corresponds to the case with the splitter plate at \( h = 2csin\alpha \) and may be characterized by the irregular ups and downs and at some particular time interval a negative drag is observed. On the average, the magnitude of the drag force here is substantially lower than the one without the splitter plate. Note that the decrease in drag as a result of introducing the splitter plate has been experimentally illustrated by Roshko(1955).
Fig. 5.5. The subsequent development of the wake for the case of $h = 0$. 

(a) $t = 55.2$

(b) $t = 57.6$

(c) $t = 60$
(d) $t = 62.4$

(e) $t = 64.8$

(f) $t = 67.2$

Fig. 5.5. (Continued)
Fig. 5.6. The drag force exerted on the flat plate as a function of time for the case without the splitter plate.
Fig. 5.7. The drag force exerted on the flat plate as a function of time for the case of $h = 2c \sin \alpha$. 
The drag force for the case with the splitter plate at $h = 0$ is depicted on Figure 5.8. The periodic pattern on this figure is similar to that on Figure 5.6 but the time averaged mean drag $\overline{F_x}$ is down to about 70% of the mean drag of Figure 5.6.

The time variation of the near field acoustic pressure is depicted on Figures 5.9-5.11. As discussed in Chapter II, $E_1'$, $E_2'$ on these figures represent the acoustic pressure component involving the streamwise and the transverse motion of the vortices respectively. Compared to Figure 5.9 which corresponds to the case without the splitter plate, the case with the splitter plate at $h = 0$ depicted on Figure 5.10 shows that $E_2'$ is increased by about twice the average magnitude. This may be explained by the fact that the transverse motion of the vortices is strongly influenced by the presence of the splitter plate so that the vortices in the lower side of the splitter plate cannot cross over to the upper side and vice versa. The case with the splitter plate at $h = 2c \sin \alpha$ is depicted on Figure 5.11 and exhibits very erratic behavior. It appears that, in addition to the blocking effect of the splitter plate itself, the interaction of the vortices with the leading edge of the splitter plate significantly influences the motion of each individual vortex.

Figures 5.12-5.14 show the sound pressure level obtained for each of the cases depicted on Figures 5.9-5.11. The parameters employed to scale the pressure are same as those employed in Section 4.5. The noise field corresponding to the case without the splitter plate is depicted on Figure 5.12 and exhibits a broadband character with peak occurring at about 130 Hz which corresponds to the Strouhal number of $St = 0.253$. The
Fig. 5.8. The drag force exerted on the flat plate as a function of time for the case of $h = 0$. 
Fig. 5.9. Time variation of the near field acoustic pressure $E_1'$ and $E_2'$ for the case without the splitter plate. — denotes $E_1'$ and - - - denotes $E_2'$. 
Fig. 5.10. Time variation of the near field acoustic pressure $E_1'$ and $E_2'$ for the case of $h = 0$. — denotes $E_1'$ and --- denotes $E_2'$. 
Fig. 5.11. Time variation of the near field acoustic pressure $E_1'$ and $E_2'$ for the case of $h = 2c \sin \alpha$. $-$ denotes $E_1'$ and $-$ denotes $E_2'$. 
Fig. 5.12. Sound pressure level for the case of Figure 5.9.

Fig. 5.13. Sound pressure level for the case of Figure 5.10.
case with the splitter plate at $h = 0$ is depicted on Figure 5.13. Compared to the case without the splitter plate on Figure 5.12, the noise field on this figure exhibit the peak sound pressure level occurring at the lower frequency of about $105 \text{ Hz}$ which corresponds to the Strouhal number of $St = 0.205$. The magnitude of the peak is about the same as that of Figure 5.12 but the sideband levels are increased by about $10 \text{ dB}$. In contrast with Figures 5.12 and 5.13, the noise field corresponding to the case with splitter plate at $h = 2c \sin \alpha$ on Figure 5.14 does not indicate any peak at a particular frequency since regular vortex shedding is mostly suppressed.

Figure 5.15 shows the wake development at $t = 60$ for the case of streamwise spacing $4 \times 4c$ and $h = 2c \sin \alpha$. Compared to the wake pattern of Figure 5.4 (b) where the streamwise spacing is $4c$, the wake development here is once again characterized by regular vortex shedding in the near wake region followed by a strong interaction between the vortices and the leading edge of the splitter plate. These characteristics of the shedding process produce an oscillating drag force as depicted on Figure 5.16. Compared to the large reduction of the drag in Figure 5.7 which corresponds to the case of streamwise spacing $4c$, the drag force here is not reduced much due to the relatively weaker influence of the splitter plate. The corresponding time variation of the near field acoustic pressure in Figure 5.17 (a) also exhibits a quasi-periodic, oscillating character. The noise spectrum depicted on Figure 5.17 (b) indicates that the peak sound pressure level occurs at about $120 \text{ Hz}$ which corresponds to the Strouhal number of $St = 0.234$. This is in contrast with the noise
Fig. 5.14. Sound pressure level for the case of Figure 5.11.
Fig. 5.15. The wake development at $t = 60$ for larger streamwise spacing of $4 \times 4c$; here the offset parameter $h = 2c \sin \alpha$.

Fig. 5.16. The drag force exerted on the inclined plate as a function of time for the case of Figure 5.15.
Fig. 5.17. The noise field for the case of Figure 5.15.
(a) Time variation of the near field acoustic pressure $E_1'$ and $E_2'$.
— denotes $E_1'$ and --- denotes $E_2'$. (b) The sound pressure level.
field for the case of streamwise spacing 4c where the spectrum does not indicate any particular peak frequency due to the strong suppression of the periodic vortex shedding process.

The effect of another plate at incidence in the wake of the upstream plate is considered next. On Figure 5.18 (a) is the result for vortex shedding from only the downstream plate at t = 60 and the wake pattern in this figure implies that the upstream plate has little influence on the periodic vortex shedding from the downstream plate. This observation may be confirmed by the noise spectrum depicted on Figure 5.18 (b). Compared to the noise spectrum for the single plate case depicted on Figure 5.12, the peak sound pressure level occurs at about the same frequency although the sideband levels are slightly higher. The wake development at t = 60 for the case of vortex shedding only from the upstream plate is depicted on Figure 5.19 (a). Note that the patterns are somewhat irregular as those of Figure 5.4 (b) where the splitter plate is placed at mid-wake. The noise spectrum for this case is depicted on Figure 5.19 (b) and note again that the result is very similar to that of Figure 5.14. These results should be compared with those produced when there is vortex shedding from both plates; the wake development at t = 60 is depicted on Figure 5.20 (a). Here it is observed that the shedding pattern is somewhat irregular though not as irregular as that of Figure 5.19 (a). The noise field is shown on Figure 5.20 (b) and it is seen that there is still some evidence of a single dominant frequency although this primary frequency is somewhat lower than that of Figure 5.18 (b). It should be pointed out that the noise level in each of the three cases just
Fig. 5.18. (a) The wake development at $t = 60$ for the case of vortex shedding only from the downstream inclined plate.
(b) The sound pressure level.
Fig. 5.19. (a) The wake development at $t = 60$ for the case of vortex shedding only from the upstream inclined plate.
(b) The sound pressure level.
Fig. 5.20. (a) The wake development at \( t = 60 \) for the case of vortex shedding from both inclined plates. (b) The sound pressure level.
discussed is about the same even though many details of the vortex
shedding process are very different. This fact is important in that the
noise field in the frequency domain is of great interest and is normally
the output of experimental measurement. Of course this result may not
be surprising since the sound pressure level is computed effectively on a
logarithmic scale.

The results of Figures 5.18-5.20 do have implications for the
modelling of the flow of interest which is depicted on Figure 1.1. At the
present time, the incorporation of vortex shedding from each of the
inclined plates of Figure 1.1 appears to be a very difficult, if not an
impossible task. From the acoustic point of view, however, it appears
from the previous results that the major contributor to the noise field is
the vortex shedding from the last plate. Thus a realistic first step in a
rigorous calculation of the flow depicted on Figure 1.1 is to consider the
case where vortex shedding occurs only from the last plate and the wake
development for this case at $t = 40$ is depicted on Figure 5.21 (a); here the
streamwise spacing between the last inclined plate is $4c$, the offset
parameter $h = 0$, and the spacing between inclined plates is $1.2 \times 4c$. The
wake pattern in this figure is very similar to that of Figure 5.4 (c) in that
the periodic vortex shedding process is largely retained. Note that the
streamwise spacing and the offset parameter are the same in both of
these figures. The noise field depicted on Figure 5.21 (b) indicates a clear
single dominant frequency of about 75 Hz. This peak frequency is
somewhat lower than that of Figure 5.13 which corresponds to the case of
single inclined plate with the downstream splitter plate and it appears
Fig. 5.21. (a) The wake development at $t = 40$ for the flow past four inclined plates with vortex shedding only from the last plate in the presence of the splitter plate. (b) The sound pressure level.
that the other three upstream plates drive the peak lower. Note that the peak sound pressure level here is about the same as that of Figure 5.13.

5.4 Summary

The problem of vortex shedding in flow past two or more flat plates has been examined. Two basic configurations with two plates have been investigated; first, the interaction of the wake flow of an inclined plate with a horizontal splitter plate was considered. Next, the vortex shedding patterns in flow past two plates inclined at the same angle were calculated. In both cases the vortex shedding patterns were significantly altered by the presence of the additional plate. Because of the complexity of the computational procedure when more than two plates are involved, the case of four inclined plates with a downstream splitter plate was considered only heuristically in that vortex shedding was assumed to occur only from the last plate. This flow is believed to embody the basic features in the flow past four plates although the precise details of the results should be viewed as being qualitative and not quantitative in character.

For the case of splitter plate configuration, the effect of the vertical offset parameter, $h$, is very significant and two cases have been considered; $h = 2c \sin \alpha$ and $h = 0$. The wake development for the case $h = 2c \sin \alpha$ is characterized by the strong suppression of regular, periodic vortex shedding and the noise field does not indicate any peak in the spectrum. On the other hand, the results for the case $h = 0$ illustrate
that regular vortex shedding is substantially retained and the interaction between the vortices and the leading edge of the splitter plate appears to be local. The suppression of the regular vortex shedding process when $h = 2c \sin \alpha$ is consistent with the experimental results of Roshko (1955) and Unal and Rockwell (1988). The noise field for this case shows that the vortex shedding frequency is slightly less than that for the case without the splitter plate; the peak sound pressure level occurs at this frequency. In both cases of $h = 0$ and $h = 2c \sin \alpha$, the average drag force is reduced compared to that without the splitter plate and this result is in qualitative agreement with previous experiments in similar geometries.

Next, the case of two plates inclined at the same angle was considered. To determine how the vortex shedding process was affected by the presence of the second plate, three separate cases were considered corresponding to vortex shedding only from the downstream plate, vortex shedding only from the upstream plate and vortex shedding from both plates. The case of vortex shedding from the downstream plate only exhibited many of the characteristics of the single plate results; the noise field for the case of vortex shedding from the upstream plate only was substantially random in the sense that no clear frequency emerged, whereas the case of vortex shedding from both plates indicated a dominant vortex shedding frequency about 30% lower than the single plate results.

Finally, the case of four inclined plates, with a downstream splitter plate was considered for $h = 0$ with vortex shedding only off the last inclined plate. The results indicated a shedding frequency of about 75
Hz; it is likely that incorporation of vortex shedding from all of the plates would either further drive the acoustic response frequency lower or push the acoustic response into the fully broadband range. It is interesting to note that all of the sound pressure level results for all of the multiple plate cases considered indicate a maximum level of about 170 dB.
CHAPTER VI
SUMMARY AND CONCLUSIONS

In the present work, the flow past a series of set of slats at incidence has been calculated; the geometry is reminiscent of the louver assemblies which cover ballast tanks on large surface ships, although only a single louver assembly is considered due to the extreme complexity of the structure. Along with the calculation of the very complicated flow field, the acoustic field has been considered for small Mach number which is indicative of low speed flows in water. Vibration of a component of the structure has also been incorporated where possible. The flow field has been assumed to be two-dimensional and inviscid and complex variable techniques have been employed.

The flow field of practical interest is depicted on Figure 1.1 (a) and includes both upstream and downstream splitter plates which cover a large tank in which the flow velocity is very low. However, because there is very little, if any, experimental data on the flow inside such ballast tanks and because such a flow would be difficult to approximate in practice, the primary focus has been on the flow past a series of inclined plates having no upstream splitter plate and is uniform far upstream. Clearly the flow in such a geometry is still very complicated because of the number of plates and the fact that some form of (perhaps very
irregular) vortex shedding phenomenon will probably occur from any or all of the plates; as such it should be noted that the nature of the near wake flow of each internal plate is still somewhat speculative. Consequently, the calculation of the flow field (and hence the noise field) has been approached by investigation of a number of simpler problems leading finally to the consideration of the geometry of Figure 1.1(b). In this regard, it should be noted here that the goal of this work has not been to rigorously calculate the flow past the series of plates depicted on Figure 1.1; rather we have sought here to elucidate the nature of the unsteady forces and noise field induced by the physical mechanisms likely to occur in some way in the actual flow; namely, vortex shedding, vibration, and buffeting.

As a first step toward a description of the flow in the geometry of Figure 1.1, the potential flow past a tandem cylinder arrangement is calculated in Chapter II. In describing any type of vortex motion in the flow past a single flat plate at incidence, the region outside the flat plate must first be mapped into the region outside a circular cylinder in which the complex potential may easily be found. A similar situation obtains for the case of multiple flat plates and the complex potential for the flow described in Chapter II forms the basis for the calculation of the potential flow past several flat plates.

In Chapter II, the flow past two cylinders buffeted by one or more potential vortices has been investigated. Both of the cylinders are assumed to be elastic in the sense that they may change position (i.e. their centers may change) as a result of the buffeting. The noise field for
this situation has also been calculated and several different vortex configurations and structural parameters have been considered. The noise field, in general, consists of an impulsive portion caused by the impingement of the vortex (or vortices) followed in time by a periodic component caused by the induced vibration of the cylinders. The sound pressure level is increased significantly by the vibration and the acoustic field responds at the response frequency of the cylinders. The response frequency is less than the natural frequency of the cylinders due to the presence of the fluid inertial force which produces an added mass effect. The response frequency approaches the natural frequency of the cylinder for very large values of the mass parameter, however.

Also considered in Chapter II is the possibility that chaotic motion of the vortices may take place. The computed results for both three and four vortex configurations indicate that four vortices apparently are necessary for chaotic motion to occur. This result is somewhat surprising since three vortices above a plane wall have been shown to induce chaotic motion (Conlisk, Guezennec, and Elliott (1989)); however, in the present geometry it appears that the presence of the solid boundaries of finite streamwise dimension has an appreciable effect on the motion of the vortices only for a very small period of time. Consequently the vortices behave as if they were in free space for a majority of the time; it has been shown elsewhere (see Aref (1983) for references) that four vortices in free space are necessary for chaotic motion to occur.
The flow fields calculated in Chapter II are deficient in the sense that vortex shedding from at least the second cylinder, which will occur in practice at high Reynolds number, is not considered. This process in the flow past a single inclined plate is thus considered in Chapter III. First, the basis for the point vortex approximation of a vortex sheet in two dimensions is considered and it is shown how the inviscid rollup process may encounter computational problems. This is apparently because the self-induced motion of the sheet is singular unless the vorticity along the sheet is uniform. Because of this fact, it appears necessary to remove the singularity in the self-induced velocity of the sheet and this has been done here heuristically by using Lamb vortices instead of potential vortices to model the vortex sheet. Justification of this method is provided by the work of Ting and Tung (1965) who rigorously show that the singularity at the center of a single two-dimensional vortex is removed by the incorporation of a viscous core. However, since the Lamb vortex distribution is not a solution of the Navier-Stokes equations for more than one vortex, this method of removing the spurious fine scale behavior of the vortices in the vortex sheet must be viewed as a heuristic alternative to other methods as described in Section 3.2.

This method has been tested on the rollup of the vortex sheet produced by an elliptically loaded wing and the numerical results indicate that there is smooth rollup even for very large times for comparatively low sheet Reynolds number. Extensive numerical results have been produced for Re=400 with 120 vortices employed along the sheet; numerical accuracy has been demonstrated using this method.
and the rollup of the sheet appears to agree very well with the solutions given by Moore (1974) for the same problem. It should be pointed out that the effect of the sheet Reynolds number decreases as the Reynolds number increases and similar results at larger Reynolds numbers still show some irregularity in the vortex motion; this appears to be due to the increasingly finer scale of the motion near the inner spiral of the sheet as the Reynolds number increases although this explanation of the phenomenon must remain somewhat speculative.

Despite the difficulties at higher Reynolds numbers encountered using Lamb vortices, this method has been used in the calculation of the vortex shedding problem in flow past an inclined plate. Better numerical behavior of the solutions as the time step is reduced has been obtained using this method and comparison of the present results with previous experimental and computational results indicate substantial agreement. Another substantial improvement in the present calculation is the use of an integration scheme to calculate the rate of vortex shedding rather than the classical first-order differential schemes employed in the previous work on this problem. At a Reynolds number of 400 the rollup of the vortex sheets shed from each edge of the inclined plate is accomplished with no sign of a singularity as with the elliptically loaded wing problem. The noise field for these solutions indicates a response at the vortex shedding frequency with the frequency of the peak sound pressure level increasing as the angle of attack $\alpha$ decreases.

The physical geometry of interest depicted on Figure 1.1 is extremely complex and requires a special numerical conformal
mapping technique which is the subject of Chapter IV. The method involves the sequential mapping of each plate into a circle; this method is a generalization of the method first employed by Halsey (1977). The method is easily implemented if the flat plates are sufficiently separated; the effect of the separation of the plates is the degree to which the near-cylinder remains circular as each sequential mapping is completed. Once the mapping is completed the method of images employed in Chapter II may be employed to calculate the complex potential in the multiple-cylinder plane. In this chapter only impinging motion of vortices on the array of flat plates is considered.

Of particular interest in this chapter is the case where several inclined flat plates are followed downstream by a horizontal splitter plate. When a vortex impinges on the series of plates and subsequently interacts with the splitter plate, a feedback effect in the form of a small blip in the force on the upstream inclined plates is produced as a result of apparently rapid acceleration of the vortex past the leading edge of the splitter plate. This effect is magnified by vibration of one of the slats as shown on Figure 4.17. A similar type of feedback mechanism has been observed when free shear layers impinge on downstream bodies (Rockwell and Naudascher (1979)) but it is very surprising that a feedback mechanism arises naturally in the present problem. In this chapter, the noise field is also calculated and for each case calculated is similar to that computed in Chapter II.

The methods developed in Chapters II, III, and IV may be employed to calculate the flow field past a series of flat plates and this is
the subject of Chapter V. Several different configurations have been investigated beginning with the flow past a single inclined flat plate having a splitter plate in its wake; for simplicity the splitter plate is of length four times that of the flat plate. A number of different parameters are present in this problem including the angle of attack $\alpha$, the vertical offset $h$, and the distance from the center of the inclined plate to the leading edge of the splitter plate (i.e. the streamwise spacing). To reduce the number of disposable parameters the angle of attack $\alpha$, measured from the upstream horizontal axis, has been taken to be 140 degree and solutions have been computed for different values of $h$ and streamwise spacing.

For the case where the splitter plate is aligned with the top of the plate ($h=0$) the wake is somewhat more irregular than for the case where the splitter plate is absent and the Strouhal number is somewhat lower; it should be noted that in this case there is still a visibly dominant frequency in the sound pressure level. For this first set of results the streamwise spacing is equal to the chord length of the inclined plate $4c$ and the Reynolds number $Re=50,000$. The vortices are advanced using the Adams-Bashforth method; the rest of the computational parameters are as described in Chapter III except where noted. The numerical solutions are substantially resolved for a time step $\Delta t = 0.04$ although some fine scale discrepancies do remain.

The solutions for the case where the splitter plate is placed in the middle of the wake of the plate is considered next. For this case the regular, periodic vortex shedding process seen in the case of a single flat
plate appears to be absent and the wake flow appears very irregular; the results for the sound pressure level exhibit no dominant frequency and the noise field is fully broadband. This result is consistent with the experimental results of Roshko (1955) and Unal and Rockwell (1988) in which the insertion of a long plate and a wedge respectively in the wake of a circular cylinder inhibits the regular and periodic vortex shedding process. It should be noted here that the absence of regular and periodic vortex shedding does not imply that no vorticity is shed into the wake; rather it implies that the vorticity that is shed into the wake does not evolve in a periodic way as is the case for no splitter plate.

Solutions have also been computed for the case where the streamwise spacing is 4x4c and the solutions appear much more regular than those for the smaller spacing since the periodic vortex shedding process is largely retained in the near and far wake region. The noise field also indicates a single dominant frequency which is slightly lower than the one for the case without the splitter plate.

The case of two plates inclined at the same angle has been considered next. To determine how the vortex shedding process is affected by the presence of the second plate, three separate cases are considered corresponding to vortex shedding only from the downstream plate, vortex shedding only from the upstream plate and vortex shedding from both plates. The case of vortex shedding from the downstream plate only exhibits many of the characteristics of the single plate results; the noise field for the case of vortex shedding from the upstream plate only is substantially random in the sense that no clear frequency emerges,
whereas the case of vortex shedding from both plates indicates a dominant vortex shedding frequency about 30% lower than the single plate results.

Finally, the case of four inclined plates with a downstream splitter plate is considered for $h = 0$ with vortex shedding only off the last inclined plate. The results indicates a shedding frequency of about 75 Hz; it appears that incorporation of vortex shedding from all of the plates would either further drive the acoustic response frequency lower or push the acoustic response into the fully broadband range. It is interesting to note that all of the sound pressure level results for all of the multiple plate cases considered indicate a maximum level of about 170 dB.

Several important conclusions may be drawn from the results of the present study. First, the noise field depends crucially upon the nature of the flow in the neighborhood of the structure. Significant noise appears to be generated as a result of the vortex shedding or buffeting from upstream disturbances, or from both, and the vibration of the structure generally increases the sound pressure level. Secondly, the presence of a downstream body in the wake of another body produces a feedback effect upstream which, in turn, has a significant effect on the upstream flow. Finally, in the case of vortex shedding, the presence of a downstream splitter plate in the center of the wake of an inclined plate appears to suppress the regular, periodic vortex shedding process. The addition of more inclined plates appears to reduce the Strouhal frequency which is the frequency at which the noise field responds.
Although the present model incorporates complicated aspects of the fluid dynamics, structure vibration and acoustics in a consistent way, the following areas can be further developed for an improved and more comprehensive model.

i) The boundary layer theory should be incorporated in the calculation to determine the separation point for the vortex shedding problem.

ii) The structure vibration should be considered in the vortex shedding model.

iii) A numerical scheme which includes both the buffeting and vortex shedding in a multiple body geometry should be developed, and

iv) A simplified computational method should be developed by which vortex shedding from four or five bodies can be considered.

Although the extension of the inviscid flow model to incorporate boundary layer theory has been introduced by some investigators, the application is very limited and requires further attention. Theoretical and experimental studies regarding the effects of the structure vibration on the noise generation are relatively unresolved research issues. For a complex multiple body geometry, the incorporation of empirically determined dynamic properties into the model also deserves attention from the practical point of view.

Despite some of the limitations of the present flow model caused primarily by the complexity of the geometry, it is believed that the essential features of the flow field, structural dynamics and the noise field have been illustrated. The present work should be viewed, in part, as a logical first step toward a comprehensive physical understanding of
the vortex shedding process in the flow past multiple bodies. In this regard, an extensive set of measurements in the geometries of interest here would clarify many issues, one of which is the nature of the wake between inclined plates.
APPENDIX A

TWO-DIMENSIONAL STRUCTURAL MODEL
FOR THE TANDEM CYLINDERS

Consider two-degree of freedom structure consisting of the cross section of a circular cylinder of mass 'm' per unit width with springs of stiffness $k_x$ and $k_y$ in the x and y directions respectively. For the simplicity, damping is not considered here. The vibration of the cylinder is assumed to occur in the form of the translation of the center of the cylinder as depicted on Figure A.1. Defining $(x_0, y_0)$ as the equilibrium position of the center and $(x,y)$ as the instantaneous position of the center displaced by the fluid force $\mathbf{F} = (F_x, F_y)$, the equations of motion can be constructed (Meirovitch(1967)) such that

$$m\ddot{x} + k_x \Delta l_x \cos \theta_x + k_y \Delta l_y \sin \theta_y = F_x$$

$$m\ddot{y} + k_x \Delta l_x \sin \theta_x + k_y \Delta l_y \cos \theta_y = F_y$$

where

$$\Delta l_x = l_x - l_{x_0}, \quad \Delta l_y = l_y - l_{y_0}$$

and
Fig. A.1. Two dimensional structural model for the elastically mounted cylinder.
\[
1_x = \left( (1_{x_0} + x)^2 + y^2 \right)^{1/2} \quad 1_y = \left( (1_{y_0} + y)^2 + x^2 \right)^{1/2}
\]

Assuming small amplitude vibration so that \( \theta_x \) and \( \theta_y \) are small, then

\[
\sin \theta_x = y / 1_x, \quad \sin \theta_y = x / 1_y
\]

\[
\cos \theta_x = \cos \theta_y = 1
\]

and equations of motion may be written as

\[
m\ddot{x} + k_x(1_x - 1_{x_0}) + k_y(1_y - 1_{y_0}) \frac{x}{1_y} = F_x
\]

\[
m\ddot{y} + k_x(1_x - 1_{x_0}) \frac{y}{1_x} + k_y(1_y - 1_{y_0}) = F_y
\]

Substituting \( 1_x \) and \( 1_y \) in equation (A.3) into the first equation of (A.5), we obtain

\[
m\ddot{x} + k_x \left[ \left\{ \left( 1_{x_0} + x \right)^2 + y^2 \right\}^{1/2} - 1_{x_0} \right]
+ k_y \left[ 1 - 1_{y_0} / \left\{ \left( 1_{y_0} + y \right)^2 + x^2 \right\}^{1/2} \right] x = F_x
\]

which may be arranged as,

\[
m\ddot{x} + k_x 1_{x_0} \left[ \left\{ \left( 1 + x / 1_{x_0} \right)^2 + \left( y / 1_{x_0} \right)^2 \right\}^{1/2} - 1 \right]
+ k_y \left[ 1 - \frac{1}{\left\{ \left( 1 + y / 1_{y_0} \right)^2 + \left( x / 1_{y_0} \right)^2 \right\}^{1/2}} \right] x = F_x
\]

\( A.6 \)
Applying Taylor series expansion this equation may be linearized as

\[ m\ddot{x} + k_x x_0 \left( \frac{x}{x_0} + \frac{1}{2} \left( \frac{x}{x_0} \right)^2 + \frac{1}{2} \left( \frac{y}{y_0} \right)^2 + \cdots \right) + k_y \left( \frac{y}{y_0} \right)x = F_x \]  (A.8)

Applying similar procedure on the second equation of (A.5), we obtain

\[ m\ddot{y} + k_x y_0 \left( \frac{x}{x_0} + \frac{1}{2} \left( \frac{y}{y_0} \right)^2 + \frac{1}{2} \left( \frac{x}{x_0} \right)^2 + \cdots \right) = F_y \]  (A.9)

Assuming small amplitude vibration the second order terms may be dropped from equations (A.8) and (A.9) and then the equations of motion are decoupled to yield

\[ m\ddot{x} + k_x x = F_x \]  (A.10)
\[ m\ddot{y} + k_y y = F_y \]

These equations may be nondimensionalized based on the cylinder radius \( b^* \) and the free stream velocity \( U_\infty \) to obtain the dimensionless form of the equations of motion corresponding to equation (2.5) in Chapter II.
APPENDIX B

THE FORMULATION OF THE ACOUSTIC PRESSURE IN THE FAR FIELD

We deal here with the sound generated by the interaction of the unsteady flow of low Mach number with the elastic tandem cylinders considered in Chapter II. The method of approach is the method of matched asymptotic expansions employed by Crighton (1972) and note that, in the limit as \( M \to 0 \), only the leading order term is significant in the acoustic pressure. The acoustic potential, to the leading order, satisfies the linear wave equation (Morse and Ingard (1968)),

\[
\nabla^2 \phi_a = M^2 \frac{\partial^2 \phi_a}{\partial t^2} \tag{B.1}
\]

where \( \nabla^2 \) is the Laplacean operator defined by

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \tag{B.2}
\]

In the near field where \( r \) is much smaller than the wave length, equation (B.1) satisfies Laplace equation as \( M \to 0 \), i.e.
\( \nabla^2 \phi_a = 0 \quad (B.3) \)

But in the far field, both terms in equation (B.1) are comparable and defining an outer variable \( R = Mr \), equation (B.1) becomes

\[
\frac{\partial^2 \phi_a}{\partial R^2} + \frac{1}{R} \frac{\partial \phi_a}{\partial R} + \frac{1}{R^2} \frac{\partial^2 \phi_a}{\partial \theta^2} = \frac{\partial^2 \phi_a}{\partial t^2} \quad (B.4)
\]

subject to \( \phi_a = \text{Re}(W_a) \) as \( R \to 0 \), and \( \phi_a \) must behave as an outgoing wave as \( R \to \infty \). In this equation \( W_a \) represents the unsteady part of the complex potential.

Applying the method of matched asymptotic expansion

\[
\phi_a = \phi_0 + O(M) \quad (B.5)
\]

in the near field and

\[
\phi_a = M\tilde{\phi}_0 + O(M^2) \quad (B.6)
\]

in the far field and matching requires, to the leading order,

\[
\lim_{r \to \infty} \phi_0 = \lim_{R \to 0} M\tilde{\phi}_0 \quad (B.7)
\]
In order to find the explicit form of the outer potential \( \bar{\phi}_0 \), the unsteady part of the complex potential is written in terms of wave region variables. The unsteady part of the complex potential may be written

\[
W_a(z) = -\sum_{m=1}^{NV} \frac{ik}{2\pi} \ln(z - z_m) + \sum_{m=1}^{NV} \sum_{j=1}^{\infty} W_{jm}^V + \sum_{j=1}^{\infty} (W_j^F + W_j^M) \quad (B.8)
\]

where

\[
W_{jm}^V = \frac{ik}{2\pi} C_j e^{i\alpha_j} \{\ln(z - \varepsilon_{jm}) - \ln(z - \rho_j)\}
\]

\[
W_j^M = -D \frac{e^{i\alpha_j}}{jZ - \xi_j}
\]

\[
W_j^F = A \frac{e^{i\alpha_j}}{jZ - \xi_j}
\]

In the outer region let \( Z = Mz \), then using the binomial theorem equation (B.8) may be expanded for small \( M \) to yield

\[
W_a(Z) = \sum_{m=1}^{NV} \frac{ik}{2\pi} \frac{M}{Z} z_m + \sum_{j=1}^{\infty} C_j e^{i\alpha_j} \sum_{m=1}^{NV} \frac{ik}{2\pi} \frac{M}{Z} (\varepsilon_{jm} + \rho_j) \quad (B.10)
\]

\[
+ \sum_{j=1}^{\infty} \frac{M}{Z} (A e^{i\alpha_j} - D e^{i\alpha_j}) + O(M^2)
\]
Inserting $Z = \text{Re}^{i\theta}$ into equation (B.10), we obtain

\[
W_a = \frac{M}{R} \left( \cos \theta - i \sin \theta \right) \left\{ \frac{i}{2\pi} \sum_{m=1}^{NV} k_m z_m + \frac{i}{2\pi} \sum_{j=1}^{\infty} C_j e^{i\theta_j} \sum_{m=1}^{NV} k_m \left( \rho_j - \epsilon_{jm} \right) \right. \\
+ \sum_{j=1}^{\infty} \left( A_j e^{i\alpha_j} - D_j e^{i\beta_j} \right) \right\} (B.11)
\]

Defining $H_1$ and $H_2$ as

\[
H_1 + iH_2 = \left( \frac{i}{2\pi} \sum_{m=1}^{NV} k_m z_m + \frac{i}{2\pi} \sum_{j=1}^{\infty} C_j e^{i\theta_j} \sum_{m=1}^{NV} k_m \left( \rho_j - \epsilon_{jm} \right) \right) \\
+ \sum_{j=1}^{\infty} \left( A_j e^{i\alpha_j} - D_j e^{i\beta_j} \right) (B.12)
\]

Then equation (B.11) may be written as

\[
W_a = \frac{M}{R} \left( \cos \theta - i \sin \theta \right) (H_1 + iH_2) (B.13)
\]

and using the relation $\phi_a = \text{Re}(W_a)$

\[
\phi_a = \frac{M}{R} \left( H_1 \cos \theta + H_2 \sin \theta \right), \quad R \to 0 (B.14)
\]

Hence the equation for the outer potential, to the leading order, can be written as

\[
\frac{\partial^2 \phi_0}{\partial R^2} + \frac{1}{R} \frac{\partial \phi_0}{\partial R} + \frac{1}{R^2} \frac{\partial^2 \phi_0}{\partial \theta^2} - \frac{\partial^2 \phi_0}{\partial t^2} = 0 (B.15)
\]
subject to the condition

\[ \Phi_0 = \frac{1}{R} (H_1 \cos \theta + H_2 \sin \theta), \quad R \to 0 \]  \hspace{1cm} (B.16)

and the condition that \( \Phi_0 \) behaves as an outgoing wave at \( \infty \).

To solve equation (B.15) we take the Fourier Transform such that

\[ \frac{\partial^2 \Phi}{\partial R^2} + \frac{1}{R} \frac{\partial \Phi}{\partial R} + \frac{1}{R^2} \frac{\partial^2 \Phi}{\partial \theta^2} + p^2 \Phi = 0 \]  \hspace{1cm} (B.17)

and equation (B.16) yields

\[ \Phi = \frac{1}{R} \left( \hat{H}_1(p) \cos \theta + \hat{H}_2(p) \sin \theta \right), \quad R \to 0 \]  \hspace{1cm} (B.18)

Here the Fourier transform is defined by

\[ \Phi = \int_{-\infty}^{+\infty} e^{-ipt} \Phi_0 \, dt \]  \hspace{1cm} (B.19)

and \( \hat{H}_1, \hat{H}_2 \) are Fourier Transforms of \( H_1, H_2 \). Using the separation of variables, we define

\[ \Phi = \Phi_1(R) \cos \theta + \Phi_2(R) \sin \theta \]  \hspace{1cm} (B.20)
Inserting equation (B.20) into equation (B.17), the solutions are obtained as Hankel function of the first and second kind such that

$$
\phi_1 = c_1 H_1^{(1)}(pR) + c_2 H_1^{(2)}(pR) \\
\phi_2 = c_3 H_1^{(1)}(pR) + c_4 H_1^{(2)}(pR)
$$

(B.21)

Using the property of the Hankel function (Abramowitz and Stegun(1965)), the outgoing wave condition at $\infty$, and the condition

$$
\hat{\phi}_1 = \frac{\hat{H}_1}{R}, \quad \hat{\phi}_2 = \frac{\hat{H}_2}{R} \quad \text{as} \quad R \to 0
$$

(B.22)

the unknown constants can be obtained to yield

$$
\phi_1 = \frac{\pi p}{2i} \hat{H}_1(p)H_1^{(2)}(pR) \\
\phi_2 = \frac{\pi p}{2i} \hat{H}_2(p)H_1^{(2)}(pR)
$$

(B.23)

Applying the standard convolution theorem for Fourier transform (Sneddon(1951)), the inversion results in

$$
\phi_1(R, t) = \frac{1}{R} \int_0^{t-R} \frac{(t-s)(dH_1/ds)}{\sqrt{(t-s)^2 - R^2}} ds \\
\phi_2(R, t) = \frac{1}{R} \int_0^{t-R} \frac{(t-s)(dH_2/ds)}{\sqrt{(t-s)^2 - R^2}} ds
$$

(B.24)
and from equation (B.20)

\[ \bar{\phi}_0(R, t) = \phi_1 \cos \theta + \phi_2 \sin \theta \quad (B.25) \]

Hence using \( P_a = -\frac{\partial \phi_a}{\partial t} \), the acoustic pressure in the far field can be obtained as

\[
P_a = -\frac{M}{R} \cos \theta \int_{0}^{t-R} \frac{(t-s)(d^2H_1/ds^2)}{\sqrt{(t-s)^2 - R^2}} ds \]

\[
+ \sin \theta \int_{0}^{t-R} \frac{(t-s)(d^2H_2/ds^2)}{\sqrt{(t-s)^2 - R^2}} ds \quad (B.26)
\]

which corresponds to equation (2.10) of Section 2.2.

The procedure described here is independent of the precise near field geometry and thus can also be employed to calculate the far field acoustic pressure in other geometries. The only difference is that the generating function is changed.
APPENDIX C

ANALYTICAL EVALUATION OF THE FLUID LOADING ON A FLAT PLATE

The complex potential in the cylinder plane may be written in transformed variables

$$W(\zeta) = \zeta + \frac{1}{\zeta} - \frac{i}{2\pi} \sum_{j=1}^{NV} k_j \ln(\zeta - \zeta_j) + \frac{i}{2\pi} \sum_{j=1}^{NV} k_j \ln(\zeta - \frac{1}{\zeta_j}) \tag{C.1}$$

where $z$-plane and $\zeta$-plane are related by

$$z = e^{-i\alpha}(\zeta e^{i\alpha} + \frac{e^{-i\alpha}}{\zeta}) \tag{C.2}$$

The fluid loading exerted on the plate may be obtained using the Blasius theorem

$$F_x - iF_y = \frac{1}{2} i \oint (\frac{dW}{dz})^2 dz + i \frac{\partial}{\partial t} \oint \overline{W} d\zeta \tag{C.3}$$

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where $F_x, F_y$ correspond to the forces in the streamwise and transverse directions respectively. To evaluate the contour integral in this equation, equation (C.3) may be divided into steady and unsteady parts such that

\[ F_{x1} - iF_{y1} = \frac{1}{2} i \oint (\frac{dW}{dz})^2 dz \quad (C.4) \]

\[ F_{x2} + iF_{y2} = -i \frac{\partial}{\partial t} \oint W dz \quad (C.5) \]

Then the right-hand side of equation (C.4) may be expressed as a contour integral around the circle 'C' such that

\[ F_{x1} - iF_{y1} = \frac{1}{2} i \oint_C (\frac{dW}{d\zeta})^2 (\frac{d\zeta}{dz}) d\zeta \quad (C.6) \]

and applying the integration by parts on equation (C.5)

\[ F_{x2} + iF_{y2} = -i \frac{\partial}{\partial t} (Wz)_{pc} - \oint C (\frac{dW}{dz}) dz \quad (C.7) \]

where \((Wz)_{pc}\) is the difference between the values of \(Wz\) at the beginning and the end of the plate contour which is identically zero since there is no jump in the stream function along the contour (Sarpkaya(1975)).

To evaluate the steady part in equation (C.6) let $S$ be a circle of great radius which includes both the cylinder and all the vortices in free space. Note here that all the image vortices locate inside the cylinder.
Then applying the Lagally's theorem (Milne-Thomson(1968)) the contour integral around the cylinder may be written

\[
\frac{i}{2} \oint_C \left( \frac{dW}{dz} \right)^2 \left( \frac{d\zeta}{dz} \right) d\zeta = \frac{i}{2} \oint_S \left( \frac{dW}{d\zeta} \right)^2 \left( \frac{d\zeta}{dz} \right) d\zeta - \frac{i}{2} \sum_{j=1}^{NV} \oint_{\gamma_j} \left( \frac{dW}{d\zeta} \right)^2 \left( \frac{d\zeta}{dz} \right) d\zeta \quad (C.8)
\]

where \( \gamma_j \) is a small contour drawn round the \( j^{th} \) vortex. Now on the circle \( S \), since \( |\zeta| \) is large, we can expand \( \frac{1}{\zeta - \zeta_j} \) in powers of \( \frac{1}{\zeta} \) such that

\[
\frac{1}{\zeta - \zeta_j} = \frac{1}{\zeta} \left( 1 + \frac{\zeta_j}{\zeta} + \left( \frac{\zeta_j}{\zeta} \right)^2 + \cdots \right) \quad (C.9)
\]

Similarly,

\[
\frac{1}{\zeta - 1/\zeta_j} = \frac{1}{\zeta} \left( 1 + \frac{1}{\zeta} \frac{1}{\zeta_j} + \frac{1}{\zeta} \left( \frac{1}{\zeta_j} \right)^2 + \cdots \right) \quad (C.10)
\]

Inserting (C.9) and (C.10) into the first integrand of equation (C.8) and applying the residue theorem,

\[
\frac{i}{2} \oint_S \left( \frac{dW}{d\zeta} \right)^2 \left( \frac{d\zeta}{dz} \right) d\zeta = 0 \quad (C.11)
\]

To evaluate the second integral in equation (C.8), let
\[
\frac{dW}{d\zeta} = G(\zeta) - \frac{ik_j}{2\pi} \frac{1}{\zeta - \zeta_j} \quad (C.12)
\]

where

\[
G(\zeta) = \frac{d}{d\zeta}\{W(\zeta) + \frac{ik_j}{2\pi} \ln(\zeta - \zeta_j)\} \quad (C.13)
\]

Then

\[
\left(\frac{dW}{d\zeta}\right)^2 = (G(\zeta))^2 - \frac{ik_j}{\pi} \frac{G(\zeta)}{\zeta - \zeta_j} + \left(\frac{ik_j}{2\pi}\right)^2 \frac{1}{(\zeta - \zeta_j)^2} \quad (C.14)
\]

Applying Taylor's theorem

\[
G(\zeta) = G(\zeta_j) + (\zeta - \zeta_j)G'(\zeta_j) + \cdots \quad (C.15)
\]

Hence

\[
\left(\frac{dW}{d\zeta}\right)^2 \left(\frac{d\zeta}{dz}\right) = \left[ (G(\zeta))^2 + \left(\frac{ik_j}{2\pi}\right)^2 \frac{1}{(\zeta - \zeta_j)^2} - \frac{ik_j}{\pi} \frac{G(\zeta_j)}{\zeta - \zeta_j} \right.
\]

\[
+ G'(\zeta_j) + \cdots \right] \frac{\zeta^2}{\zeta^2 - e^{-2i\alpha}} \quad (C.16)
\]

Substituting (C.16) into the second integrand of equation (C.8) and applying the residue theorem
Combining (C.8), (C.11) and (C.17) the steady part of the fluid loading may be obtained as

\[ F_{x_1} - iF_{y_1} = -i \sum_{j=1}^{NV} k_j (u_j - iv_j) \]  

where \((u_j - iv_j)\) corresponds to the velocity induced at \(z = z_j\).

Upon inserting (C.2) into (C.7) the unsteady part of the fluid loading may be written

\[ F_{x_2} + iF_{y_2} = i \frac{\partial}{\partial t} \int_C e^{-i\alpha}(\zeta e^{i\alpha} + \frac{e^{-i\alpha}}{\zeta})(\frac{dW}{d\zeta})d\zeta \]  

Substituting (C.1) into (C.18) and carrying out the integration using the residue theorem, we obtain

\[ F_{x_2} + iF_{y_2} = -i \frac{\partial}{\partial t} \left( \sum_{j=1}^{NV} k_j e^{-i\alpha}(\frac{e^{-i\alpha}}{\zeta_j} + \frac{e^{i\alpha}}{\bar{\zeta}_j}) \right) \]

Hence from (C.18) and (C.20) the fluid loading on the plate may be obtained as

\[ F_x + iF_y = i \left[ \sum_{j=1}^{NV} k_j (u_j + iv_j) - \frac{\partial}{\partial t} \left( \sum_{j=1}^{NV} k_j e^{-i\alpha}(\frac{e^{-i\alpha}}{\zeta_j} + \frac{e^{i\alpha}}{\bar{\zeta}_j}) \right) \right] \]
BIBLIOGRAPHY


