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Transient, non-thermal equilibrium, multiphase transport processes in porous media

Sözen, Mehmet Fuat, Ph.D.
The Ohio State University, 1989
TRANSIENT, NON-THERMAL EQUILIBRIUM, MULTI-PHASE TRANSPORT PROCESSES IN POROUS MEDIA

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Mehmet F. Sözen, B.S., M.S.

* * * * *

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<tr>
<td>$a_{sv}$</td>
<td>specific surface area common to solid and vapor phases, m² m⁻³</td>
</tr>
<tr>
<td>$a_{\sigma\beta}$</td>
<td>specific surface area common to $\sigma$ and $\beta$ phases, m² m⁻³</td>
</tr>
<tr>
<td>$a_{\sigma\gamma}$</td>
<td>specific surface area common to $\sigma$ and $\gamma$ phases, m² m⁻³</td>
</tr>
<tr>
<td>$A$</td>
<td>constant in equation (9), 23.4851064</td>
</tr>
<tr>
<td>$B$</td>
<td>constant in equation (9), 2969.2287, K⁻¹</td>
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<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure, J kg⁻¹ K⁻¹</td>
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<tr>
<td>$d_p$</td>
<td>particle diameter, m</td>
</tr>
<tr>
<td>$D_a$</td>
<td>Darcy number, K/H²</td>
</tr>
<tr>
<td>$F$</td>
<td>geometric factor defined in equation (10)</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration, m s⁻²</td>
</tr>
<tr>
<td>$G$</td>
<td>mass velocity, kg m⁻² s⁻¹</td>
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<tr>
<td>$h_{sv}$</td>
<td>fluid-to-particle heat transfer coefficient, W m⁻² K⁻¹</td>
</tr>
<tr>
<td>$h_{\sigma\beta}$</td>
<td>fluid-to-particle heat transfer coefficient between $\sigma$ and $\beta$ phases, W m⁻² K⁻¹</td>
</tr>
<tr>
<td>$h_{\sigma\gamma}$</td>
<td>fluid-to-particle heat transfer coefficient between $\sigma$ and $\gamma$ phases, W m⁻² K⁻¹</td>
</tr>
<tr>
<td>$H$</td>
<td>height of the packed bed, m</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity, W m⁻¹ K⁻¹</td>
</tr>
<tr>
<td>$k_{fb}$</td>
<td>relative permeability for fluid phase</td>
</tr>
<tr>
<td>$k_{\alpha\beta}$</td>
<td>coefficient of capillary pressure gradient with respect to temperature, N m⁻²</td>
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<td>$k_e$</td>
<td>coefficient of capillary pressure with respect to liquid volume fraction, N m⁻²</td>
</tr>
<tr>
<td>$K$</td>
<td>permeability, m²</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the packed bed, m</td>
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<td>LTE</td>
<td>local thermal equilibrium</td>
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\( \dot{m} \) condensation rate, kg m\(^{-3}\) s\(^{-1}\)

P pressure, N m\(^{-2}\)

PCM phase change material

R gas constant for refrigerant-12, J kg\(^{-1}\) K\(^{-1}\)

Re\(_p\) particle Reynolds number, \( \rho_\gamma \nu \cdot d_p / \mu_\gamma \)

s saturation, \( \varepsilon_\beta / \varepsilon \)

S normalized saturation, \( (s - s_{im}) / (1 - s_{im}) \)

t time, s

T temperature, K

\( T_{\text{melt}} \) melting temperature of PCM, K

u velocity component in x-direction, m s\(^{-1}\)

V volume, m\(^{3}\)

\( \mathbf{v} \) velocity vector, m s\(^{-1}\)

\( \alpha \) thermal diffusivity, m\(^{2}\) s\(^{-1}\)

\( \varepsilon \) porosity

\( \varepsilon_\beta \) volume fraction of liquid phase

\( \varepsilon_\gamma \) volume fraction of vapor phase

\( \varepsilon_\sigma \) volume fraction of solid phase

\( \Delta h_{\text{vap}} \) latent heat of vaporization for Refrigerant-12, J kg\(^{-1}\)

\( \Theta \) dimensionless temperature, \( (T - T_0) / (T_{in} - T_0) \)

\( \mu \) absolute viscosity, kg m\(^{-1}\) s\(^{-1}\)

\( \rho \) density, kg m\(^{-3}\)

\( \rho_{\gamma,s} \) saturation vapor density, kg m\(^{-3}\)

\( \tau \) dimensionless time, \( tu^*/L \)
Subscripts

\( f \) fluid (liquid+vapor)
\( \text{feff} \) effective property for fluid
\( \text{in} \) inlet
\( \text{o} \) initial
\( s \) solid
\( v \) vapor
\( \beta \) liquid
\( \gamma \) vapor
\( \sigma \) solid
\( \sigma_{\text{eff}} \) effective property for solid

Superscripts

\( f \) fluid (liquid+vapor)
\( s \) solid
\( v \) vapor
\( \beta \) liquid
\( \gamma \) vapor
\( \sigma \) solid
\( \ast \) reference

Symbols

\(< \) "local volume average" of a quantity
CHAPTER I

INTRODUCTION

This study is concerned with a series of investigations related to the single- and multi-phase transport processes in porous media, and the behavior of sensible and latent heat storage packed beds. The main objective of this work is to analyze different aspects of the applications of packed beds in particular and porous media in general, by considering a set of problems which proceed in the increasing order of complexity. These problems are designed to provide physical insight for a number of outstanding points related to transport phenomena in porous media as well as different packed bed energy storage systems including sensible heat storage and latent heat storage ones.

Packed beds with single-phase flow have been analyzed in applications ranging from chemical catalytic reactors and pebble bed or rock pile heat storage units to fixed-bed nuclear propulsion systems and spacecraft thermal management systems employing packed beds of encapsulated phase change materials. A major part of the studies conducted to date on packed bed applications with single-phase flow concentrates on utilizing incompressible fluids, liquid or gas, as the heat transporting medium. In these investigations a constant mass flow rate is assumed at every cross section of the packed bed and therefore there is no need for solving any continuity or momentum equations. The problem reduces to solving the governing energy equations for the solid and fluid phases. For this category of problems the two commonly employed models are the so-called single-phase conductivity and two-phase Schumann models. The main distinction
between these two models is that, whereas local thermal equilibrium is assumed to prevail between the solid and the fluid phases in the single-phase model, no such assumption is made in the two-phase model. Therefore, the single phase conductivity model reduces to one governing energy equation in which conduction in both phases is taken care of by the use of an effective thermal conductivity, whereas in the two-phase model there are two governing energy equations, each of them possessing a fluid-to-solid convective heat transfer term. The origin of the two-phase models is the classical Schumann model (Schumann, 1929). Riaz (1977) presented an investigation and comparison of both the single-phase conductivity model as well as the two-phase Schumann model. From a theoretical point of view, both of these models are quite incomplete because they incorporate many simplifying assumptions, such as neglecting the transient heat storage and the conduction terms in the fluid phase to name a few.

The second major class of problems dealing with the single-phase flow through porous media studied to date concentrates on the compressible gas flow through a porous medium. In this category of problems too, there is a lack of analysis of the complete transient, non-isothermal and non-thermal equilibrium flow of a compressible gas. One-dimensional, transient, isothermal flow of an ideal gas through a porous medium has been studied analytically by the use of perturbation methods by Kidder and La Habra (1957). Morrison (1972) also obtained analytical solutions by perturbation methods for one-dimensional isothermal and adiabatic flows of an ideal gas. In both of these investigations Darcy's formulation has been used for fluid flow. Nilson (1981), on the other hand, obtained an analytical solution for one-dimensional isothermal flow through a porous medium in a study which utilized Ergun's equation (Ergun, 1952) to account for the inertial effects. In all of these investigations it was assumed that the gas phase was in local thermal equilibrium with the solid phase. Goldstein and Siegel (1971) performed an
analytical investigation on a steady, non-isothermal ideal gas flow in a porous medium. They also utilized the local thermal equilibrium assumption in the governing energy equation, and Darcy's formulation for fluid flow. Most of these studies which incorporated Darcy's formulation for fluid motion have the drawback of not being applicable to high speed flows in which the inertia effects have to be accounted for.

It can be seen that the models used in both categories of problems described above are incomplete in one aspect or another since they concentrate on either the momentum transport in an isothermal or adiabatic flow or the energy transport in an incompressible flow. In addition they incorporate some simplifying assumptions such as local thermal equilibrium between the solid and the fluid phases, neglect of conduction and transient storage terms in the gas phase energy equation, and neglect of inertial effects in the fluid phase momentum equations. Therefore, their application is limited to certain special cases. Thus the need for a more rigorous formulation of the transport processes in a porous medium for establishing a more flexible and reliable model with a wide range of applicability becomes apparent. This is the main motivation for the work that will be undertaken in Chapter II. The main objective of Chapter II is to present a rigorous analysis and investigation for transient transport processes in a compressible gas flow through a packed bed with no assumption of local thermal equilibrium between the solid and fluid phases. Ergun-Forchheimer relation will be employed as the vapor phase momentum equation in order to account for the inertia effects in addition to the viscous effects. This is very essential for non-Darcy regime flows in porous media. More specifically, it is essential in flows with particle Reynolds numbers greater than 0.1 in which Darcy formulation becomes inaccurate. This point has been neglected in many of the previous studies dealing with single- and multi-phase transport phenomena in porous media. Likewise, the majority of the previous works on multi-phase transport in porous
media almost exclusively employ local thermal equilibrium (LTE) between the solid and fluid phases considered. This assumption is usually not satisfactory for the step change problems in which, during the early stages of the transport processes there may be considerable difference between the temperatures of the flowing fluid and solid particles. This is also true even during the later stages of the transport processes in high speed flows or high permeability porous media in which the fluid-to-solid interaction time or interaction surface area respectively may not be large enough for the temperatures of the fluid and solid phases to become close enough for LTE to be a reasonable assumption. The thermal interactions between the vapor and the solid phases, and how the fluid flow as well as the pressure and density fields are affected by these interactions will be analyzed in detail. Another objective of Chapter II is to gain a better understanding of the situations under which the local thermal equilibrium assumption and one- or two-dimensional consideration of certain field variables would be justifiable. In this regard the effect of certain characteristic non-dimensional parameters, such as the particle Reynolds number, the Darcy number and certain thermophysical parameters, on the general qualitative behavior of the transport phenomena in packed beds will be investigated thoroughly.

One further step into these studies brings us to multi-phase flow of a single pure substance in liquid and vapor form, and the accompanying transport processes in packed beds. The flow of a mixture of a vapor and non-condensible gases, such as air and water vapor, through a porous medium, and the accompanying phase change (condensation) and the multi-phase transport processes involved have received considerable attention in a number of investigations related to different applications. These include phase change in building insulation materials (Vafai and Whitaker, 1986; Vafai and Sarkar, 1986; Ogniewicz and Tien, 1981), heat pipe technology (Udell, 1985), drying of different
porous materials (Berger and Pei, 1973; Plumb et al., 1985), and phase change in porous media (Eckert and Pfender, 1980; Motakef and El-Masri, 1986). The condensing flow of a single vapor through a porous medium, on the other hand, received relatively little attention (Nilson and Montoya, 1980). A rigorous model which includes the basic thermodynamics of the condensation process and the concept of non-thermal equilibrium between the solid and the fluid phases under condensing conditions seem to be completely absent. This fact is the main motivation for the investigation in Chapter III, which is aimed at analyzing the forced convective condensing flow of a vapor through a packed bed. The need for analyzing such problems poses an outstanding issue in a number of contemporary applications of the packed beds. Packed beds which have been classically used in chemical industry and for energy storage purposes in the form of pebble bed or rock pile heat storage units, are currently being considered for applications such as a thermal energy storage system for rejection of heat in pulsed space power supplies and some conceptual spacecraft thermal management systems. The operating conditions of the packed beds for such applications may require gas/vapor flow at high speeds as well as high pressures, which in turn will dictate condensation of the working fluid. This may actually be desirable in order to enhance the amount of thermal energy stored in the packed bed. The fundamental study required for analyzing such problems constitutes the main objective of Chapter III.

For analyzing the problem in Chapter III, the model established for single-phase vapor flow in Chapter II will be expanded such that it will accommodate the liquid phase in addition to the vapor phase of the working fluid. This will be carried out by additional conservation equations for the liquid phase as well as accommodations for the phase change terms in the already available conservation equations for the vapor phase.
Moreover, the thermodynamic aspects of the condensation process which will introduce an additional coupling relation will be presented.

The last phase of this study is concerned with the application of an encapsulated phase change material (PCM) as an energy storage medium in packed beds. This problem will be taken up in Chapter IV. The earlier forms of the packed bed energy storage units solely relied on the sensible heat capacity of the solid bed particles for storing thermal energy. This form has been satisfactorily employed for various applications. However, certain applications may impose a limitation on the size and the weight of the packed bed system utilized. For instance, in the case of a heat rejection system in pulsed space power supplies which incorporate packed beds, the reduction of the mass and volume is of utmost importance. In such cases, utilization of only the sensible heat capacity of a certain material for energy storage will most likely prove to be inefficient. The remedy to this can be found in the utilization of latent heat in the process of energy storage. Recently, encapsulated phase change materials (PCM) have received considerable attention as energy storage materials. The use of an encapsulated PCM is very appealing since it makes the utilization of latent heat storage capacity possible. This is achieved by using a PCM which has a melting temperature within the temperature range of operation of the system incorporating the packed bed. The principal advantage of PCMs in packed beds is that the energy storage density of the bed is increased significantly and thus, the size and mass of the storage system required for a particular application is reduced proportionally.

Different PCMs have been considered for use in the packed bed energy storage units in different applications. For applications over 450°C significant consideration was given to salts by Marianowski and Maru (1977). The physical properties of different
PCMs considered for storage of solar energy have been presented by Lane (1986). A number of studies have been carried out on the analysis of latent heat storage packed beds by different researchers. Ananthanarayanan et al. (1987) investigated the dynamic behavior of a packed bed which utilized encapsulated Al-Si shots which have a melting temperature of 577°C. Air was used as the energy transporting fluid in their study. Pitts and Ji (1987) presented another study on transient thermal behavior of a latent heat storage packed bed which utilized an inorganic compound hydrate PCM, namely Na₂HPO₃·12H₂O, as the heat storage medium. Torab and Chang (1988) investigated the use of encapsulated phase change materials for thermal energy storage units in space power systems. They reported an analysis of a latent heat storage packed bed which utilized lithium hydride as the PCM, and lithium as the transport fluid.

In majority of the models employed in the analysis of this category of problems, the superficial velocity of the working fluid is assumed to be constant. This fact reduces the system of governing equations to a set of energy equations for the working fluid and for the PCM respectively. As pointed out before, although this is a satisfactory approach when the working fluid is incompressible, it is not so when the working fluid is a gas or vapor under high pressures. Moreover, when the working fluid itself undergoes phase change (condensation), a rigorous model which consists of the governing energy, mass and momentum balance equations in addition to the relevant coupling thermodynamic relations has to be employed for analyzing any phase change and the corresponding transport phenomena. In Chapter IV we will undertake the analysis of such a problem with a rigorous model which will basically be the expansion of the model developed in Chapter III. The physical aspects of the phase change process in the PCM will be modeled and analyzed.
The main objective of Chapter IV is to analyze the energy storage characteristics of packed beds which utilize phase change materials in the bed particles. Time history of the crucial field variables such as the temperature profiles of the working fluid and the bed particles, and the velocity, density and pressure of the working fluid will be determined. The determination of these quantities is important since the amount of energy flowing into and out of the packed bed, and hence the amount of energy stored in the packed bed as a function of time, can be determined from the time histories of these variables.
CHAPTER II

ANALYSIS OF TRANSPORT PHENOMENA IN A SINGLE-PHASE FLOW OF A GAS IN A PACKED BED

In this chapter, the model developed for analyzing the single-phase fluid flow and transport processes in a packed bed will be presented. The numerical method of solution will be discussed. A benchmarking that was carried out for verification of the results of the numerical code developed in the present work will also be presented. This will be followed by the presentation and a thorough discussion of the case studies performed and the results obtained.

2.1 STATEMENT OF THE PROBLEM

The problem that will be investigated in this chapter is the transfer and storage of heat from a high temperature reservoir into a packed bed which consists of randomly packed spherical particles of uniform size. The schematic diagram of the problem considered is depicted in Figure 2.1. The extent of the packed bed in the z-direction is assumed to be large enough so that the problem will essentially be two-dimensional. Superheated Freon-12 is used as the energy transport medium, i.e., the fluid phase flowing through the packed bed. Freon-12 (known as R-12, or dichloro-difluoromethane) was chosen as the vapor phase because it is a highly inert and stable compound whose critical point is well above the range of temperatures considered in the present
Figure 2.1: Schematic diagram of the problem
study. Moreover, it has a relatively high vapor density making it capable of carrying more thermal energy, and thus requiring smaller volume flow rate for a certain application, than the typical gases such as air. Different materials will be considered for solid particles in order to determine the effect of different thermophysical parameters on the associated transport processes. In this problem, the high temperature vapor in the reservoir is suddenly allowed to flow through the packed bed initially filled by Freon-12 which is also initially at a uniform temperature and pressure throughout the packed bed. The pressure in the reservoir is higher than the initial pressure in the packed bed and it remains unchanged. A uniform pressure equal to the initial pressure in the packed bed is maintained at the exit of the bed so that there is always a pressure difference between the left hand and the right hand sides of the bed, thus sustaining the vapor flow. In essence this is a forced convection flow through a packed bed. Therefore, relatively higher particle Reynolds number flows than the Darcy regime flows are considered. Since the Darcy formulation would fail for these flows, Ergun-Forchheimer relation is employed instead, in order to account for the inertial effects as well as the viscous effects. The problem is analyzed for the case in which the two horizontal walls of the packed bed are maintained at constant temperature.

2.2 FORMULATION

The governing equations for the transport processes in porous media are developed here by using the "local volume averaging" technique. This method enables us to express the governing equations in a localized macroscopic sense for the porous media, which would otherwise require a very complex microscopic analysis that does not seem to be feasible even with the most advanced computational facilities. The fundamentals of the
application of this technique to porous media are outlined by Whitaker (1977) in his work on drying of porous media. In this work the governing conservation equations have been derived for Darcy flow cases. Gray (1975) also presented an extension on the application of the volume averaging technique to convective and diffusive terms in the governing equations. The volume averaging process is performed by associating a small volume $V$ closed by a surface $S$ with every point in the porous medium. The two important averages of a quantity frequently encountered in the governing equations are the so called local volume average and the intrinsic phase average. The local volume average of a quantity $\Phi$ associated with phase $\Psi$ is defined as

$$< \Phi > = \frac{1}{V} \int_{V_{\Psi}} \Phi \, dV$$

(2.1)

while the intrinsic phase average of a quantity $\Phi$ associated with phase $\Psi$ is defined as

$$< \Phi >_{\Psi} = \frac{1}{V_{\Psi}} \int_{V_{\Psi}} \Phi \, dV$$

(2.2)

where $V_{\Psi}$ is the volume associated with phase $\Psi$. With this formulation it becomes reasonable to represent the intrinsic properties such as the temperature and the density of a phase by the intrinsic phase averaged quantities, and to represent the frequently used "superficial velocity" by the local volume averaged quantity.

Considerable information regarding the application of the spatial-averaging theorem for establishing the local volume averaged forms of the conservation equations for the convective and diffusive flows in porous media, and the models formed by using this principle for such problems can be found in the works of Vafai and Whitaker (1986), and Vafai and Tien (1981). The same methodology is used in the present work for
establishing the model for analyzing the energy and momentum transfer in the packed bed with forced convective flow. The major assumptions and simplifications that are employed in this study are:

(1) Freon-12 used as the vapor phase obeys the ideal gas equation of state, and the operation range is chosen such that it is always in the superheated form.

(2) Natural convection effects are negligible. This is a very reasonable assumption based on the fact that the bed is dominated by forced convection.

(3) The solid phase is incompressible, and the packed bed is assumed to have uniform porosity and to be isotropic.

(4) Boundary and variable permeability effects are neglected in the vapor phase momentum equation.

(5) The temperature range used in the analysis of the problem is relatively small (20K), therefore the variation of physical properties, such as thermal conductivity, viscosity and specific heat capacity, with temperature is neglected.

(6) Due to the relatively low temperature range considered in the present study the inter-particle and intra-particle radiation heat transfer is negligible.

With these points taken into consideration the volume-averaged governing equations can be established in the following form by applying the volume averaging technique to the fundamental flow and energy equations in a porous medium:
**Vapor phase continuity equation:**

The volume averaged form of the vapor continuity equation for a compressible fluid takes the following form:

\[
\frac{\partial}{\partial t} (\varepsilon <\rho_v>^v) + \nabla . (\varepsilon <\rho_v>^v \nabla <v_v>) = 0
\]  \hspace{1cm} (2.3)

**Vapor phase momentum equation:**

The vapor phase momentum equation which incorporates the inertia effects as well as viscous effects by the use of Ergun-Forchheimer relation can be written as (Vafai, 1984)

\[
\nabla <P_v>^v = - \frac{<\rho_v>^v}{K_v^{1/2}} \frac{Fe}{[<\nabla v_v>.<\nabla v_v>]} \frac{\nabla v_v}{|\nabla v_v|} - \frac{\mu_v}{K_v} <\nabla v_v>
\]  \hspace{1cm} (2.4)

In this equation, the first term on the right hand side represents the inertia effects or the pressure drop due to form drag resistance of the packed bed, while the second term represents the pressure drop due to viscous effects. Without the first term, the equation reduces to Darcy equation.

When assumption (2) above is incorporated into this equation, the vapor flow essentially becomes one-dimensional and it assumes the following form:

\[
\frac{\partial <P_v>^v}{\partial x} = - \frac{<\rho_v>^v}{K_v^{1/2}} \frac{Fe}{K_v} <u_v>^2 - \frac{\mu_v}{K_v} <u_v>
\]  \hspace{1cm} (2.4a)
Vapor phase energy equation:

\[
\varepsilon < \rho_v >^v c_p v \frac{\partial < T_v >^v}{\partial t} + c_p v < \rho_v >^v < \nabla v >^v \cdot \nabla < T_v >^v = \nabla \{ k_{veff} \nabla < T_v >^v \} + h_{sv} a_{sv} (< T_s >^s - < T_v >^v) \tag{2.5}
\]

This equation accounts for conduction and transient storage of energy in the vapor phase as well as the transport of sensible heat by the vapor motion. Vapor-to-solid heat transfer is modeled by a convective heat transfer term by the use of a fluid-to-solid heat transfer coefficient.

Solid phase energy equation:

\[
(1 - \varepsilon) \rho_s c_p s \frac{\partial < T_s >^s}{\partial t} = \nabla \{ k_{seff} \nabla < T_s >^s \} - h_{sv} a_{sv} (< T_s >^s - < T_v >^v) \tag{2.6}
\]

This equation follows similarly as in the case of the vapor phase energy equation except that there is no sensible energy transport due to motion.

Vapor phase equation of state:

From the ideal gas law, the equation of state can be written as

\[
< P_v >^v = < \rho_v >^v R_v < T_v >^v \tag{2.7}
\]

In the equations presented above, the effective thermal conductivities for the solid and the vapor phases were modeled in the following form:

\[
k_{veff} = \varepsilon k_v
\]

\[
k_{seff} = (1-\varepsilon) k_s \tag{2.8}
\]
The permeability of the packed bed, $K_v$, and the geometric function $F$ in the vapor momentum equation can be obtained from the experimental results of Ergun (Ergun, 1952; Vafai, 1984) in terms of the porosity, $\epsilon$, and the particle diameter, $d_p$, as follows:

$$K_v = \frac{\epsilon^3 d_p^2}{150(1-\epsilon)^2} \quad (2.9)$$

$$F = \frac{1.75}{\sqrt{150 \epsilon^{3/2}}} \quad (2.10)$$

In equations (2.5) and (2.6), $a_{sv}$ represents the specific surface area of the packed bed, i.e., the surface area of the solid particles per unit bulk volume of the packed bed. Based on geometrical considerations, this ratio can be expressed in terms of the particle diameter and porosity as (Dullien, 1979)

$$a_{sv} = \frac{6(1-\epsilon)}{d_p} \quad (2.11)$$

For the fluid-to-particle heat transfer coefficient, $h_{sv}$, it is necessary to choose an empirical correlation from a range of available experimental results. Considerable amount of experimental work has been carried out for determining this quantity for packed beds for different sizes, shapes and packing configurations of solid particles. Typical experimental investigations and some reviews on fluid-to-particle heat transfer coefficients may be found in the works of Baumeister and Bennett (1958), Bhattacharyya and Pei (1975) and Barker (1965). Considering the ranges of the particle diameter, $d_p$, and the particle Reynolds number, $Re_p$, used in this investigation, the empirical correlations established by Gamson et al. (1943) were found to be appropriate for use in the present work. These empirical correlations which were presented in terms of
Colburn-Chilton $j_h$ factors can be expressed in the following form after some manipulation:

\[
h_{sv} = 1.064 \, c_p \, G \left( \frac{c_p \mu}{k} \right)^{-2/3} \left( \frac{d_p G}{\mu} \right)^{-0.41} \quad \text{for} \quad \frac{d_p G}{\mu} \geq 350 \quad \text{(turbulent)} \quad (2.12)
\]

\[
h_{sv} = 18.1 \, c_p \, G \left( \frac{c_p \mu}{k} \right)^{-2/3} \left( \frac{d_p G}{\mu} \right)^{-1} \quad \text{for} \quad \frac{d_p G}{\mu} \leq 40 \quad \text{(laminar)} \quad (2.13)
\]

where $G$ represents the rate of mass flow through a unit surface area perpendicular to the flow direction.

Thus equations (2.3) through (2.13) form the model for analyzing the energy and momentum transport in unsteady forced convection flow of a gas through a packed bed. This is basically a system of five coupled governing equations which must be solved for analyzing the five field variables, namely $\langle \rho_v \rangle^v$, $\langle u_v \rangle$, $\langle T_v \rangle^v$, $\langle T_s \rangle^s$, and $\langle P_v \rangle^v$.

### 2.2.1 Boundary and initial conditions

In the problem under consideration, the packed bed is initially filled with quiescent R-12 vapor at superheated state, which is at a uniform pressure and temperature and in local thermal equilibrium with the solid phase. These physical conditions can be expressed in mathematical form as

\[
P_v(x,y,t=0) = P_0
\]

\[
T_v(x,y,t=0) = T_s(x,y,t=0) = T_0 \quad (2.14)
\]

\[
u_v(x,y,t=0) = 0
\]
Superheated R-12 from a reservoir at higher temperature and pressure than those of the initial conditions is then allowed to flow through the packed bed while the exit pressure is kept constant and equal to the initial bed pressure. Mathematical expression of these boundary conditions is given as

\[ P_v(x=0,y,t) = P_{in}, \quad P_v(x=L,y,t) = P_0 \]
\[ T_v(x=0,y,t) = T_{in} \quad \text{at} \quad t > 0^+ \quad (2.15) \]
\[ T_v(x,L,y,t) = T_v(x,y=H,t) = T_s(x,y=0,t) = T_s(x,y=H,t) = T_0 \]

Constant wall temperature boundary conditions were employed in order to study the two-dimensional effects in the transport phenomena in the packed bed. Also this kind of boundary conditions will more realistically represent the actual physical conditions in applications in which the packed bed is not just utilized as an energy storage medium, but also as an energy release system.

2.2.2 Physical conditions for the numerical runs

To analyze the effect of different parameters on energy and momentum transport in the packed bed, different solid particle sizes and different materials for the solid phase were considered. A complete set of computations were carried out for particle diameter values of 1mm, 2.5mm and 5mm for the set of different materials which were considered for the solid phase. Several runs were also performed for a particle diameter of 1cm. The three different materials considered for the solid phase in this study were lithium-nitrate-trihydrate, sandstone and 1% Carbon-steel. The main criterion in selecting these materials was to obtain a wide range of thermophysical properties and explore their effects on the qualitative behavior of the transport phenomena since the properties of the bed material may differ widely from one application to another. The average porosity
was chosen to be equal to the asymptotic value of 0.39 throughout the packed bed. This is a valid assumption for all cases in which the ratio of the solid particle diameter to the characteristic global dimension of the packed bed is below a certain value as determined by Benanati and Brosilow (1962). The following physical data were used for the boundary conditions and for the properties of the materials considered in the numerical computations:

<table>
<thead>
<tr>
<th>Freon-12</th>
<th>Lithium-nitrate-trihydrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 68.7588 \text{ J/kg.K}$</td>
<td>$c_p = 2090 \text{ J/kg.K}$</td>
</tr>
<tr>
<td>$c_p = 602 \text{ J/kg.K}$</td>
<td>$k = 0.5 \text{ W/m.K}$</td>
</tr>
<tr>
<td>$k = 0.0097 \text{ W/m.K}$</td>
<td>$\rho = 1550 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>$\mu = 12.6 \times 10^{-6} \text{ Pa.s}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sandstone</th>
<th>1% Carbon-steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_p = 710 \text{ J/kg.K}$</td>
<td>$c_p = 473 \text{ J/kg.K}$</td>
</tr>
<tr>
<td>$k = 1.83 \text{ W/m.K}$</td>
<td>$k = 43 \text{ W/m.K}$</td>
</tr>
<tr>
<td>$\rho = 2200 \text{ kg/m}^3$</td>
<td>$\rho = 7800 \text{ kg/m}^3$</td>
</tr>
</tbody>
</table>

$P_0 = 100 \text{ kPa}$  \hspace{1cm} $T_{in} = 300 \text{ K}$  \hspace{1cm} $T_0 = 280 \text{ K}$

The effects of three important parameters have been studied extensively in this investigation. These are the particle Reynolds number, the Darcy number and the ratio of the solid phase thermal diffusivity to liquid phase thermal diffusivity. Different Reynolds numbers were obtained by varying the pressure in the reservoir, i.e., by applying different pressure gradients across the packed bed. Different Darcy numbers resulted from different particle sizes since the permeability of the packed bed varies with the
particle diameter as given in equation (2.9). The ratio of solid to vapor thermal
diffusivity, \(\alpha_s/\alpha_v\), was of a different order of magnitude for each of the three materials
considered for the solid phase.

2.3 SOLUTION PROCEDURE AND THE NUMERICAL CODE

A numerical code was developed for solving the system of governing equations
presented in the previous section. The Beam-Warming type implicit method which is
frequently used in compressible flow problems in regular media was not applicable in this
case because of the nature of the equations. Therefore, it was found necessary to employ
explicit schemes. In developing the finite difference forms of equations (2.3) through
(2.7), central difference approximations were used for most of the spatial derivatives for
the inner grid points. Spatial derivatives for the grid points on the left and right
boundaries were formed by first-order forward and backward difference approximations
respectively. Upwind differencing was implemented for the convective terms in the
vapor energy and continuity equations in order to insure the stability of the numerical
scheme. It was found through numerical experimentation that using central differencing
in the convective term of the continuity equation did not make any appreciable change in
the results.

While the vapor density at each grid point on the left boundary was determined
from the equation of state by using the boundary conditions for temperature and pressure,
the density at each grid point on the right boundary was determined from the equation of
state by using the vapor temperature computed from equation (2.5) and the pressure
given by the right boundary condition. For the rest of the grid points the density was
computed by the vapor continuity equation. The velocity $< u_v >$ for all the grid points except those on the right boundary were determined by the vapor phase momentum equation which is basically a quadratic equation in $< u_v >$. The velocity at the grid points on the right boundary were computed by linear extrapolation from the preceding two grid points in x-direction. Temperatures of the solid and fluid phases were determined by the associated energy equations while the pressure was determined from the equation of state. Each of the governing equations (2.3) through (2.7) was solved in a separate subroutine in the numerical code. The basic sequence of the solution of the field variables was as follows:

1. $< u_v >$ from equation (2.4a)
2. $< \rho_v >^v$ from equation (2.3)
3. $< T_v >^v$ from equation (2.5)
4. $< T_s >^s$ from equation (2.6)
5. $< P_v >^v$ from equation (2.7)

The numerical computations were performed on a CRAY XMP/28. Full vectorization was achieved in all of the subroutines in the numerical code. As a result of vectorization the CPU time required on the CRAY for a certain run using a grid mesh of 21x11 was approximately 30 times smaller than the CPU time required for the same run on a VAX 8550. Due to the nature of the governing equations, small time steps had to be employed for the stability of the numerical scheme. A systematic decrease in the grid size was carried out and the corresponding stable $\Delta t$ was employed until an agreement to within 1% was achieved in the solutions between the consecutive grid sizes, and the effect of reducing the $\Delta t$ further did not make any appreciable change in the solutions.
A number of interesting observations were made on the stability of the numerical code. It was found that in some cases the time step used in the early stage could be increased in the later stage by an order of magnitude without any influence on the accuracy of the results. It was observed that for a fixed particle Reynolds number, \( R_{ep} \), decreasing the Darcy number, \( D_a \), consistently increased the stability of the numerical scheme in both the early and the later stages. Decreasing \( D_a \) at a fixed \( R_{ep} \) requires using smaller particle diameter (hence smaller permeability) and a larger pressure difference across the packed bed. Since decreasing the particle diameter (or permeability) increases the damping forces in the packed bed, the increase in the stability of the numerical scheme could probably be attributed to the increase in the damping forces.

On the other hand, for a fixed \( D_a \), smaller values of \( R_{ep} \) translate into smaller mass flow rates and smaller amounts of thermal energy flowing into the packed bed per unit time. Therefore, for small values of \( R_{ep} \) the CPU time required for simulating the complete charging of the packed bed becomes excessively large. For instance, for the case when lithium-nitrate-trihydrate is the solid phase and \( D_a=1.18\times10^{-8} \), the CRAY CPU time required for simulating the complete charging of the packed bed was of the order of 1 hour for \( R_{ep}=1100 \). For this specific case the \( \Delta t \) employed in the computations was \( 1.7647\times10^{-5} \) sec. for the early stage while it was possible to push it up to \( 1.7647\times10^{-4} \) sec. for the later stage. The CPU time was estimated to increase significantly for small Reynolds numbers. Therefore full simulation of such cases are very costly on CRAY and would be extremely tedious on regular main frames. For this reason no attempt was made to perform the complete simulation for such cases.
2.4 VALIDITY OF THE NUMERICAL CODE

In order to gain confidence in the accuracy of the results of the numerical code developed, it was necessary to perform a benchmark against some analytical solutions, experimental results or other reliable numerical results. As mentioned earlier, no complete analysis of unsteady, non-isothermal and non-equilibrium flow of a gas through a porous medium is available in the literature. Therefore, in order to check the numerical code it was necessary to compare its results with different limiting case analytical solutions dealing with different aspects of energy and momentum transport in porous media. This benchmarking was performed in two main sections. These were namely the comparison of momentum transport (pressure and hence velocity distributions) in an isothermal flow of an ideal gas through a porous medium, and the comparison of energy transport (temperature distribution in solid and fluid phases) of an incompressible fluid flow through a porous medium with no local thermal equilibrium between the solid and vapor phases. The two sources chosen for this benchmarking were the most relevant sources that could be found for possible comparisons.

The benchmarking of the energy transport was carried out by comparing the analytical solution of the simplified Schumann model presented by Riaz (1977) against the results generated by the present numerical code. This was achieved by making the necessary adjustments to our model in order to reduce it to a system which was equivalent to the simplified Schumann model, i.e., by omitting a number of the transport terms. This comparison is depicted in Figure 2.2 for the fluid phase and solid phase temperature distributions respectively in terms of the dimensionless variables that appear in the work of Riaz (1977). As may be seen from these figures the agreement between our numerical results and the analytical solution is excellent.
Figure 2.2: Comparison of the temperature solutions of the present work against the analytical solutions presented by Riaz (1977)
The benchmarking of the momentum transport was carried out by comparing our results with the analytical solution of a one-dimensional isothermal flow of an ideal gas through a semi-infinite porous medium presented by Kidder and La Habra (1957). Although this analytical solution was obtained for a semi-infinite porous medium, it could be safely compared with our numerical results for small times, during which the information from the boundary had not reached the total length of the packed bed under consideration. The comparison of the analytical solution with the results generated by the present numerical code is shown in Figure 2.3 in terms of the dimensional distance and time and the dimensionless pressure which appears in the work of Kidder and La Habra (1957). Again, a very good agreement can be observed between our numerical results and the analytical solution.

2.5 RESULTS AND DISCUSSIONS

For the present analysis, it was found to be appropriate to present the results in a non-dimensionalized form. The reference quantities that are used in non-dimensionalizing the variables are the reference density, reference velocity and reference pressure denoted by \( \rho^* \), \( u^* \) and \( P^* \) respectively. For convenience, the reference pressure, \( P^* \), for each run is taken to be the average of the initial pressure \( P_0 \) and the pressure of the vapor flowing from the reservoir into the packed bed, \( P_{in} \). Likewise, the reference density, \( \rho^* \), is computed from the equation of state by using \( P^* \) and the average value of the initial and the vapor inflow temperatures. Finally the reference velocity, \( u^* \), is computed from equation (2.4a) by using \( \rho^* \) and the global pressure gradient applied across the packed bed. The dimensionless time and temperature chosen for presenting the results are defined as \( \tau = tu^*/L \), and \( \Theta = (T-T_0)/(T_{in}-T_0) \) respectively.
Figure 2.3: Comparison of the pressure solution of the present work with the analytical solution of Kidder and La Habra (1957)
For each of the computational runs two distinct regimes were observed. These two regimes will be referred to as the 'early stage' and the 'later stage'. The early stage lasts for a very short period of time which starts with the application of the high temperature and high pressure boundary conditions on the left, and ends when the pressure distribution across the packed bed becomes almost linear. This process takes place extremely fast, and afterwards the pressure distribution remains in a quasi-steady linear state. During the early stage no appreciable thermal penetration depth develops in the packed bed. The later stage is the period of time following the early stage.

The vapor velocity, density and pressure distributions along the x-direction at the mid-plane of the packed bed during the early stage, for the case where the solid phase material was lithium-nitrate-trihydrate, are depicted in Figure 2.4. It was observed that for this case the variation of different variables in the y-direction for the core region of the packed bed did not exceed 1%. Therefore, it is very reasonable to present the results for this case in one-dimensional form. The variation in the velocity distribution can be explained by the variation of the pressure. Initially when the high pressure boundary condition is applied, the large pressure gradient causes high velocities. Gradually as the pressure propagates in time and space, the large pressure gradients die away, and the velocities follow the same trend. As mentioned earlier, there is no appreciable thermal penetration depth during the early stage. Therefore there is a sharp decrease in the vapor temperature near the entrance of the bed. This drop in temperature is more pronounced than the corresponding pressure drop. The resulting peak value of the vapor density close to the entrance region, therefore, follows from the equation of state. Beyond the peak region the temperature is almost constant while the pressure decreases in the x-direction. Hence, based on the equation of state the vapor density will follow the same trend as the vapor pressure. On the other hand, the temporal increase in the vapor density
Figure 2.4: Variations of different field variable distributions at the mid-plane of the packed bed during the *early stage*
Dimensionless pressure $\frac{\langle P_v \rangle}{P^*}$

$\frac{R_{cp}}{1100} \quad Da = 7.38 \times 10^{-8}$

Figure 2.4 (continued)
during the *early stage* occurs because during this period the mass flux at any point is larger than the mass flux at a point on its right, i.e., there is a continuous net mass flux into the bed. Therefore, the second term on the left hand side of equation (2.3) has a negative value. Thus the density has to increase in time during the *early stage* in order to satisfy the continuity equation of the vapor phase.

The variations of the solid and vapor temperatures, and the density, velocity and pressure of the vapor phase during the *later stage* for the above-mentioned case are shown in Figure 2.5. It should be noted that the *later stage* starts after the pressure distribution in the packed bed becomes almost linear and remains in that quasi-steady state thereafter. During this stage the thermal penetration depth develops as shown in the figure. Since the effect of the transient term in the vapor continuity equation becomes much less pronounced at the end of the *early stage*, an almost steady-state continuity equation prevails in the *later stage*. This requires the existence of an inverse relationship between the vapor density and vapor velocity distributions. This behavior can be seen in the figure as almost a mirror image trend in the variations of the density and the velocity, i.e., as one increases the other one decreases. The vapor pressure distribution is almost linear in the x-direction and remains unchanged. Furthermore, at any time before the packed bed is fully charged, the slope of the temperature profile is much steeper than the slope of the pressure profile within a major portion of the thermal penetration depth. Therefore, during such times the vapor density increases in the x-direction within this zone. Beyond this zone the vapor temperature is almost constant and the vapor pressure decreases linearly, and so the vapor density also decreases linearly. The advancement of the thermal penetration front which translates into a temporal increase in temperature at a given x-location causes a decrease in the vapor density in time at that location. This of course follows from the equation of state. When ultimately a uniform temperature is
Figure 2.5: Variations of different field variable distributions at the mid-plane of the packed bed during the later stage
Figure 2.5 (continued)
reached throughout the packed bed, the density distribution becomes similar to the pressure distribution as dictated by the equation of state. From the variation of the different field variables in Figure 2.5 it becomes evident that the flow field is very much influenced by the temperature field.

A typical example run in which two-dimensional variations are very pronounced is considered next. The solid particle material considered for this case was 1% Carbon-steel. Figures 2.6(a) and 2.6(b) depict the distributions of the solid and vapor phase temperatures, the vapor density and the vapor pressure. As may be seen from this figure, there is considerable heat loss through the horizontal walls of the packed bed due to conduction heat transfer in y-direction and a strongly two-dimensional temperature distribution exists within the packed bed. Apparently, a one-dimensional modeling for such a case will yield erroneous results. In order to better track the two-dimensional behavior of the variables a grid mesh of 21x21 was employed for the cases in which sandstone or steel was considered as solid material.

It was observed from the numerical computations that during the later stage of each run, the value of the vapor-to-solid heat transfer coefficient computed for each grid point at each time step was very stable and did not differ more than 2% from the reference vapor-to-solid heat transfer coefficient which was based on the reference density, \( \rho^* \), and the reference velocity, \( u^* \). This suggests that an assumption that takes this value to be a constant for all the grid points during the later stage would be very reasonable. Considerable CPU time can be saved in this way by eliminating the otherwise required computations from eqns (2.12) and (2.13) at each time step. However, all of the runs for this investigation were performed by using the variable heat transfer coefficient.
Figure 2.6: Temperature, density and pressure distribution in the packed bed for the case when the solid phase is steel, (a) at $\tau = 211.4$, (b) at $\tau = 546.2$
2.5.1 Effect of the Darcy number (Da): 

In order to determine the effect of the Darcy number on energy and momentum transport in the packed bed, different cases with different Darcy numbers were compared. These comparisons were performed for a fixed particle Reynolds number of 1100 using the same solid material. Again lithium-nitrate-trihydrate was used for convenience in presenting the results, since for this material the distribution of the field variables were essentially one-dimensional in all of these cases. Figure 2.7 depicts the temperature distributions in the solid and vapor phases along the packed bed at different time levels for Darcy numbers of $1.18 \times 10^{-8}$, $7.38 \times 10^{-8}$ and $2.95 \times 10^{-8}$. The time levels in this figure were chosen such that they would result in a reasonable presentation of the penetration fronts in each case. It can easily be seen that as the Darcy number decreases, the difference between the solid and vapor phase temperatures decreases. The reason for this is that a decrease in the Darcy number translates into a decrease in the particle diameter (see equation (2.9)). As the particle diameter decreases, the specific surface area, $a_{sv}$, of the packed bed increases, thus increasing the vapor-to-solid heat transfer interaction by offering a larger surface area. Hence the exchange of heat between the solid and vapor phases becomes more efficient. Therefore, the local thermal equilibrium assumption would be more justifiable at low Darcy number flows.

2.5.2 Effect of the particle Reynolds number (Re$_p$): 

The procedure followed for determining the effect of Re$_p$ on transport phenomena was similar to the one carried out in determining the effect of the Darcy number. This time Da was fixed at $7.38 \times 10^{-8}$ for the same solid phase material while Re$_p$ was varied. The temperature distributions for the solid and vapor phases along the x-direction at the
Figure 2.7: Temperature distributions in the vapor and solid phases for different Darcy numbers for a fixed particle Reynolds number.
Figure 2.7 (continued)
horizontal mid-plane of the packed bed are shown in Figure 2.8 for \( \text{Re}_p \) of 400, 1100 and 2126. As can be seen from these figures, the difference between the temperatures of the solid and fluid phases at any point increases with an increase in \( \text{Re}_p \). This increase should be attributed to the velocity of the flow since the specific surface area of the packed bed is the same in all of these cases. As the velocity of the flow increases, i.e. \( \text{Re}_p \) increases, the time for the solid-to-vapor heat interaction decreases. This will cause a decrease in the efficiency of heat exchange between the solid and vapor phases and hence the deviation from the local thermal equilibrium will increase.

The combined effect of increasing both \( \text{Re}_p \) and \( \text{Da} \) was found to have, as expected, a much more pronounced effect on the local thermal equilibrium assumption. It becomes evident that for high \( \text{Re}_p \) and \( \text{Da} \) flows the local thermal equilibrium assumption becomes very erroneous. Similar qualitative behavior was also found to be valid for the cases in which sandstone or steel was used as the solid phase. Lithium-nitrate-trihydrate was chosen just for its convenience in presenting the results.

2.5.3 **Effect of the thermal diffusivities ratio (\( \alpha_s/\alpha_v \)):**

It was found through numerical experimentations that the ratio of the solid phase thermal diffusivity to liquid phase thermal diffusivity, \( \alpha_s/\alpha_v \), was the most appropriate parameter in representing the combined effects of \( k_s/k_v \), \( c_p_s/c_p_v \) and \( \rho_s/\rho_v \) ratios, all of which are determining factors in the overall heat transfer process. Furthermore, from the numerical experimentations it was also found that one-dimensional approach was very satisfactory for all cases in which lithium-nitrate-trihydrate was used as the solid phase and for which \( \alpha_s/\alpha_v \) was of the order of 0.035 to 0.075, which resulted from different vapor density values corresponding to the average operating pressures applied for different runs. Therefore the effect of \( \alpha_s/\alpha_v \) will be shown for cases in which sandstone
Figure 2.8: Temperature distributions in the vapor and solid phases for different particle Reynolds numbers for a fixed Darcy number.
Re_p = 2126

1. $\tau = 17.6$
2. $\tau = 121.3$
3. $\tau = 201.2$
4. $\tau = 320.9$
5. $\tau = 440.6$
6. $\tau = 600.3$

Figure 2.8 (continued)
and steel were used as the solid phase. The temperature distributions in the solid and fluid phases for two cases in which sandstone and steel were used respectively are shown in Figs. 2.9(a) and 2.9(b). In both of these figures the value of $\text{Re}_p$ was 1100 and the Darcy number was chosen to be $1.18 \times 10^{-8}$. In the case of sandstone $\alpha_s/\alpha_v$ was equal to 0.57 whereas in the case of steel it was 5.71. As can be seen from these figures the temperature distributions are strongly two-dimensional in the case of steel and mildly two-dimensional in the case of sandstone for which one-dimensional approach may give fairly good results. It should be noted that the same qualitative behavior was observed for all ranges of $\text{Re}_p$ and $\text{Da}$ considered in this investigation which proves that two-dimensionality effects are not sensitive to the particle Reynolds number and Darcy number.

For a qualitative assessment of the validity of local thermal equilibrium assumption and the strength of the two-dimensional effects in the energy and momentum transport, the findings of the numerical computations will be presented in an integrated form. Figure 2.10 shows this assessment for the local thermal equilibrium assumption for the case of lithium-nitrate-trihydrate, sandstone and steel for the entire ranges of $\text{Re}_p$ and $\text{Da}$ which were considered in this work. It should be noted that for all the runs there is a difference between the solid and vapor phase temperatures at time equal to zero, and therefore the present assessment is based on the later times during which the thermal penetration depth is developed to some appreciable position. The dividing lines in this figure were decided by the maximum difference between the solid and the vapor phase temperatures relative to the overall temperature range (difference between the inlet vapor temperature and the initial temperature, 20K). For the qualitative ratings, the percentage difference relative to the overall temperature range fall into the following categories: very poor, more than 15%; poor, 10-15%; fair, 5-10%; good, 1-5%; very good, less than 1%.
Figure 2.9: Temperature distributions in the solid and vapor phases at the mid-plane of the packed bed for $Re_p = 1100$, $Da = 1.18 \times 10^{-8}$, (a) at $\tau = 103.9$ for the case with sandstone, (b) at $\tau = 568.7$ for the case with steel.
Figure 2.10: Qualitative assessment of the local thermal equilibrium for the cases with lithium-nitrate trihydrate, sandstone and steel as solid phase respectively.
Figure 2.10 (continued)
From Figure 2.10 it can be concluded that Da is the most influential parameter in determining the validity of local thermal equilibrium. The Reynolds number based on particle diameter is also a very important parameter in this regard. The local thermal equilibrium assumption becomes more viable as both $Re_p$ and Da decrease for the reasons which were explained in the sections analyzing the effects of these parameters. This is clearly seen from the lower left corner of each figure. By the same token it is reasonable to observe the opposite behavior as $Re_p$ and Da increase, i.e., approaching the top right corners of these figures. Also it may be seen that the thermophysical parameters have a much less pronounced influence on the local thermal equilibrium condition. Only for the case of steel were the effects of the physical parameters found to be mildly significant. This is because the conduction heat transfer becomes more dominant than the vapor-to-solid convective heat transfer in this case. Therefore, the conduction heat transfer will play an additional role in the case of steel. This was significantly absent in the cases of sandstone and lithium-nitrate-trihydrate.

In a similar manner a qualitative assessment of the strength of the two-dimensional behavior is depicted in Figure 2.11. This figure shows the behavior for a fixed Da of $7.38 \times 10^{-8}$ for the entire range of $Re_p$ used in this investigation, and the behavior for a fixed $Re_p$ equal to 400 for the entire range of Da used in this investigation. This qualitative behavior, for each case in the figure, was found to be exactly the same for the entire ranges of the corresponding Da or $Re_p$ which were considered in this investigation. Therefore, for brevity only these two cases are presented. The above-mentioned fact and Figure 2.11 confirm that the two-dimensionality effects are not sensitive to either $Re_p$ or Da, but very sensitive to the $\alpha_s/\alpha_v$ ratio, i.e., to the thermophysical properties of the materials which are considered for the bed.
Figure 2.11: Qualitative assessment of the two-dimensionality effects
CHAPTER III

ANALYSIS OF MULTI-PHASE TRANSPORT PHENOMENA
IN A CONDENSING FLOW OF A VAPOR IN A PACKED BED

In this chapter, the problem analyzed in Chapter II is extended to cover the multiphase transport phenomena in a packed bed. This basically involves the consideration of the same flow problem at a pressure range in which the vapor (working fluid) density may reach the saturation vapor density causing condensation. A rigorous model for analyzing the multi-phase transport in the packed bed will be developed by considering the basic thermodynamics of the condensation process. Numerical solution procedure will be discussed in detail. This will be followed by the presentation of the results of a set of case studies performed for this problem.

3.1 STATEMENT OF THE PROBLEM

The problem under consideration is the transient condensing flow of a vapor through a fixed bed of regularly sized spherical solid particles packed in a two-dimensional channel. Figure 3.1. depicts the schematic diagram of the problem under consideration. The packed bed is initially filled with the working fluid (Freon-12) at a slightly superheated state and the whole vapor and solid system is at uniform temperature and pressure. R-12 vapor at a higher temperature and pressure from a reservoir is suddenly allowed to flow through the packed bed, thus depositing its thermal energy to
Hot vapor inflow from high temperature reservoir.

Walls of packed bed kept at constant temperature or insulated.

Vapor outflow at uniform pressure.

- L = 0.4m
- H = 0.2m

Figure 3.1: Schematic diagram of the problem
the solid particles of the bed. This basically forms a step change in the temperature and pressure at the inlet boundary. In this problem the pressures are high enough so that at certain times during the thermal charging of the packed bed, the vapor at certain locations becomes compressed enough so that its density reaches the saturation vapor density at the prevailing temperature, thus causing condensation to take place.

3.2 FORMULATION

The volume averaging technique which was used in Chapter II will be employed in this chapter too in developing the governing equations for the present problem. In establishing the model for the analysis of this problem, the following assumptions and simplifications were employed:

(1) The width of the packed bed is significantly larger than the height, and therefore the problem is essentially two-dimensional.

(2) The vapor phase always behaves as an ideal gas.

(3) There is no local thermal equilibrium (LTE) between the solid and fluid phases but there is LTE between the vapor and liquid phases when there is condensate present in the packed bed.

(4) Natural convection effects are negligible compared to the forced convection in the vapor phase. This essentially yields a one-dimensional flow in the vapor phase.

(5) The solid and the liquid phases are incompressible, and the packed bed has uniform porosity and is isotropic.

(6) Boundary and variable porosity effects are neglected.

(7) Inter-particle and intra-particle radiation heat transfer as well as thermal dispersion effects are neglected.
(8) Variations of physical properties such as thermal conductivities, specific heat capacities, viscosity and latent heat of vaporization with temperature are neglected.

With all these assumptions and simplifications taken into account, the volume averaged governing equations can be established in the following form (It should be noted that we have switched to subscripts $\beta, \gamma, \sigma$ and $f$ which denote the liquid, vapor, solid and the combined fluid (liquid+vapor) phases respectively):

**Vapor phase continuity equation:**

$$\frac{\partial}{\partial t}(\varepsilon_{\gamma} <\rho_{\gamma}>^\gamma) + \nabla \cdot (<\rho_{\gamma}>^\gamma <\mathbf{v}_{\gamma}>) = -<\dot{m}>$$  \hspace{1cm} (3.1)

Due to possible phase change (condensation) in the vapor phase, there is an additional source term, $<\dot{m}>$, in this equation as compared to equation (2.3). Moreover, because of phase change, the volume fractions of the vapor and the liquid phases will be functions of time. Therefore, instead of a constant porosity value, $\varepsilon$, in this equation we have a variable for the vapor phase volume fraction, namely $\varepsilon_{\gamma}(t)$.

**Liquid phase continuity equation:**

The volume averaged continuity equation for the non-compressible liquid phase can be established as

$$\frac{\partial \varepsilon_{\beta}}{\partial t} + \nabla \cdot <\mathbf{v}_{\beta} > - \frac{<\dot{m}>}{\rho_{\beta}} = 0$$  \hspace{1cm} (3.2)
Vapor phase equation of motion:

\[
\nabla < P_\gamma > = - \frac{< \rho_\gamma >^{\gamma} F e_\gamma}{K_{\gamma}^{1/2}} \left[ \frac{< \nabla_\gamma >}{< \nabla_\gamma >} \right] - \frac{\mu_\gamma}{K_{\gamma}} < \nabla_\gamma > \tag{3.3}
\]

This equation is similar to the one used in Chapter II except that \( e_\gamma \) is used instead of \( e \), and the definitions of \( F \) and \( K_{\gamma} \) are slightly different as will be discussed later. With assumption (4) above, this equation reduces to the following one-dimensional form:

\[
\frac{\partial}{\partial x} < P_\gamma > = - \frac{< \rho_\gamma >^{\gamma} F e_\gamma}{K_{\gamma}^{1/2}} < u_\gamma >^2 - \frac{\mu_\gamma}{K_{\gamma}} < u_\gamma > \tag{3.3a}
\]

Liquid phase equation of motion:

The liquid phase motion is formulated by Darcy's law since any motion of the liquid will be much smaller than that of the vapor phase.

\[
< \nabla_\beta > = - \frac{k_{\beta} K_{\beta}}{\mu_\beta} \left\{ k_e \nabla e_\beta + k_{< T >} \nabla < T_f >^f + (\rho_\beta - < P_\gamma >^\gamma) \beta \right\}. \tag{3.4}
\]

Fluid phase energy equation:

\[
\left[ e_\beta \rho_\beta (c_p)_\beta + e_\gamma < \rho_\gamma >^\gamma (c_p)_\gamma \right] \frac{\partial < T_f >^f}{\partial t} - < \text{in} > \Delta h_{\text{vap}}
\]

\[
+ \left[ \rho_\beta (c_p)_\beta < \nabla_\beta > + < \rho_\gamma >^\gamma (c_p)_\gamma < \nabla_\gamma > \right] \cdot \nabla < T_f >^f = \nabla \left[ k_{\text{eff}} \nabla < T_f >^f \right]
\]

\[
+ h_{\sigma \rho} a_{\sigma \beta} [ < T_\sigma >^\sigma - < T_f >^f ] + h_{\sigma \gamma} a_{\sigma \gamma} [ < T_\sigma >^\sigma - < T_f >^f ] \tag{3.5}
\]
Comparison of this equation with equation (2.5) shows the additional effect of the liquid phase in the transient storage term, conduction term and the sensible heat transport term due to motion. Moreover, there is an additional term accounting for the heat transfer due to condensation as well as a convection heat transfer term accounting for the energy transport between the solid and the liquid phases.

*Solid phase energy equation:*

\[
\varepsilon_\sigma \rho_\sigma (c_\rho) \sigma \frac{\partial <T_\sigma>_\sigma}{\partial t} = \nabla \left[ k_{\text{eff}} \nabla <T_\sigma>_\sigma \right] \\
- h_{\sigma \beta} a_{\sigma \beta} \left[ <T_\sigma>_\sigma - <T_f>_f \right] - h_{\sigma \gamma} a_{\sigma \gamma} \left[ <T_\sigma>_\sigma - <T_f>_f \right]
\]

(3.6)

In this equation too, the energy transport term between the liquid and the solid phases is modeled by a convective heat transfer term.

*Volume constraint relation:*

Due to the fact that the liquid and vapor volume fractions are variable, the following volume constraint relation provides a coupling equation, namely

\[
\varepsilon_\sigma + \varepsilon_\gamma(t) + \varepsilon_\beta(t) = 1
\]

(3.7)

*Equation of state for vapor phase:*

Following assumption (2), the equation of state for the vapor phase becomes

\[
<P_\gamma>_\gamma = \rho_\gamma <\gamma R_\gamma T_\gamma>_f
\]

(3.8)
Thermodynamic relation for the saturation density of vapor:

\[ \rho_{\gamma s} = \frac{\exp \left( A - \frac{B}{T_f} \right)}{R \gamma T_f} \]

Equation (3.9)

where A and B are constants, \( T_f \) is in degrees Kelvin and \( \rho_{\gamma s} \) is in \( \text{kg/m}^3 \). This relation was obtained by a least squares curve fitting method to a range of data relevant to the present study.

Equations (3.1) through (3.9) yield nine equations in nine unknowns, namely \( \varepsilon_{\beta}(t), \varepsilon_{\gamma}(t), <\rho_{\gamma}>, <\vartheta_{\gamma}>, <\vartheta_{p}>, <\rho_{\gamma}>, <T_f>, <T_s>, <\dot{m}> \).

For the present analysis the effective thermal conductivities for the solid and fluid phases were modeled as

\[ k_{\text{eff}} = \varepsilon_{\sigma} k_{\sigma} \]

\[ k_{\text{eff}} = \varepsilon_{\gamma} k_{\gamma} + \varepsilon_{p} k_{\beta} \] (3.10)

The permeability of the packed bed of spherical particles is given in the following form (Ergun, 1952)

\[ K = \frac{\varepsilon^3 d_p^2}{150 (1 - \varepsilon)^2} \] (3.11)

where \( \varepsilon \) is the porosity and \( d_p \) is the particle diameter. Therefore, the permeability for the vapor phase, \( K_{\gamma} \), and the geometric factor, \( F \), in the vapor phase momentum equation can be expressed as functions of \( d_p \) and \( \varepsilon \), as (Vafai, 1984)
For the relative permeability of the liquid phase, the one suggested by Udell and Fitch (1985) was used. This is modeled in the following form:

\[ k_{r\beta} = S^3 \]  

(3.14)

where \( S = \frac{s - s_{im}}{1 - s_{im}} \) and \( s = \frac{\varepsilon_{\beta}}{\varepsilon} = \frac{\varepsilon_{\beta}}{1 - \varepsilon_{\sigma}} \)

(3.14)

where \( S \) is the normalized saturation, \( s \) is the absolute saturation, and \( s_{im} \) is the "immobile" saturation. The immobile saturation, \( s_{im} \), is the critical value of the absolute saturation. If \( s \) is greater than \( s_{im} \) then the liquid phase in the packed bed becomes mobile. For values of \( s \) lower than \( s_{im} \), any liquid phase present in the porous system will be immobile, or in the so called pendular state. The value of 0.1 used for \( s_{im} \) in the work of Kaviany and Mittal (1987) will be used in the present work because of lack of any better experimental finding. With this value of \( s_{im} \), the critical value of the liquid fraction at which liquid phase becomes mobile, i.e. \( \varepsilon_{\beta\text{crit}} \), was computed from equation (3.14) to be 0.039, with the value of porosity of the packed bed taken to be 0.39, which is the average asymptotic value for packed beds in which the particle diameter to packed bed diameter is below a certain value (Benanati and Brosilow, 1962). For all the cases studied in the present investigation the maximum value of \( \varepsilon_{\beta} \) never reached \( \varepsilon_{\beta\text{crit}} \) and therefore the liquid was always immobile. It should, however, be noted that the liquid was not assumed to be immobile in modeling the problem, but the fact that it turned out to be immobile was the consequence of the governing physical conditions of the problem.
The empirical correlations for the fluid-to-solid heat transfer coefficients that were used in this chapter are the same as those in Chapter II, namely

\[
h_{oj} = 1.064 (c_p)_j \left( \frac{c_p \mu}{k} \right)_j^{-2/3} \left( \frac{d_p G}{\mu} \right)_j^{-0.41} \quad \text{for} \quad \frac{d_p G}{\mu} \geq 350 \quad \text{(turbulent)}
\]

\[
h_{oj} = 18.1 (c_p)_j \left( \frac{c_p \mu}{k} \right)_j^{-2/3} \left( \frac{d_p G}{\mu} \right)_j^{-1} \quad \text{for} \quad \frac{d_p G}{\mu} \leq 40 \quad \text{(laminar)}
\]

where \( G \) represents the mass flow rate through a unit surface area perpendicular to the direction of flow, and the subscript \( j \) denotes \( \beta \) or \( \gamma \) for the liquid or vapor phase respectively.

The specific surface area of the packed bed for the vapor phase is expressed in the following form:

\[
a_{\sigma \gamma} = \frac{6(1 - \varepsilon_\gamma - \varepsilon_\beta)}{d_p}
\]

Strictly speaking, this correlation was originally derived from geometric considerations for a fully saturated packed bed of spherical particles for a single fluid phase, in the form \( a = 6(1 - \varepsilon) / d_p \) (Dullien, 1979). However, since \( \varepsilon_\beta \) is very small (less than 0.01) compared to \( \varepsilon_\gamma \) (approximately 0.38-0.39), equation (3.16) provides a very accurate estimate for \( a_{\sigma \gamma} \).

Also from an analysis of the representative length scales and volume scales of the liquid and vapor phases of the working fluid, one may obtain a relation between the specific surface areas \( a_{\sigma \gamma} \) and \( a_{\sigma \beta} \) as
and hence
\[ a_{\sigma\beta} = a\gamma \left( \frac{e_{\beta}}{e_{\gamma}} \right)^{2/3} \]  \hspace{1cm} (3.17)

\[ a_{\sigma\beta} = \frac{6(1 - e_{\gamma} - e_{\beta})}{d_p} \left( \frac{e_{\beta}}{e_{\gamma}} \right)^{2/3} \]  \hspace{1cm} (3.18)

Since the liquid phase was always in pendular state and therefore \( h_{\sigma\beta} \) was always zero, due to the fact that for the liquid phase the mass flow rate was zero, there was no real need for the above information. However, \( a_{\sigma\beta} \) was modeled as given in equation (3.18) for the sake of complete modeling.

### 3.2.1 Boundary and initial conditions

The physical conditions initially prevailing in the packed bed are the same as those of the previous chapter. They are, therefore, mathematically expressed as follows:

\[ T_f(x,y,t=0) = T_o(x,y,t=0) = T_0 \]
\[ P_{\gamma}(x,y,t=0) = P_D \]
\[ u_{\gamma}(x,y,t=0) = 0 \]  \hspace{1cm} (3.19)

The pressure on the right boundary is kept at a value equal to the initial pressure in the packed bed while vapor at a fixed high temperature and pressure is supplied at the inlet of the packed bed. The mathematical form of the left and right boundary conditions is as follows:

\[ T_f(x=0,y,t) = T_{in} \]
\[ P_{\gamma}(x=0,y,t) = P_{in} \]  \hspace{1cm} at \( t > 0^+ \)  \hspace{1cm} (3.20)
In the present problem, both insulated top and bottom wall boundary conditions as well as constant temperature top and bottom wall boundary conditions will be investigated. For the case with insulated boundary conditions we have

\[ k_{\text{eff}} \left. \frac{\partial T_f}{\partial y} \right|_{y=0, y=H} = 0 \quad \text{and} \quad k_{\text{eff}} \left. \frac{\partial T_o}{\partial y} \right|_{y=0, y=H} = 0 \]  

(3.21)

For the case with constant temperature boundary conditions at the top and bottom walls, the boundary conditions can be expressed as

\[ T_f(x, y=0, t) = T_o(x, y=0, t) = T_{\text{bot}} = T_0 \]  
\[ T_f(x, y=H, t) = T_o(x, y=H, t) = T_{\text{top}} = T_0 \]  

(3.22)

3.2.2 Physical conditions for the numerical runs

In the present investigation too, in order to analyze the effects of different characteristic parameters, such as \( \text{Re}_p \) and \( \text{Da} \), different particle diameters ranging from 1mm to 5mm, and different ranges of operating pressures were considered. Also two different solid materials were considered in the present problem for determining the effect of the thermal properties of the solid phase on transport phenomena. These materials were lead and 1% Carbon-steel. The value of the average porosity of the packed bed was taken to be 0.39 following the arguments in Chapter II. The numerical values of different parameters used as initial and boundary conditions are given as follows:

\[ T_0 = 300 \text{ K} \quad P_0 = 796 \text{ kPa} \quad T_{\text{in}} = 350 \text{ K} \quad P_{\text{out}} = 796 \text{ kPa} \]

and the value of \( P_{\text{in}} \) was different for different cases ranging from 800 kPa to 866 kPa.
The numerical values of the other physical data used in the numerical computations are as follows:

**R-12:**

- \( k_r = 0.0097 \text{W/m.K} \)  
- \( (c_p)_r = 710 \text{J/kg.K} \)  
- \( \mu_r = 12.6 \times 10^{-6} \text{kg/m.s} \)  
- \( R_r = 0.0687588 \text{ J/kg.K} \)  
- \( \Delta h_{\text{vap}} = 111.3 \times 10^3 \text{J/kg} \)  
- \( k_\beta = 0.0545 \text{ W/m.K} \)  
- \( (c_p)_\beta = 1115 \text{J/kg.K} \)  
- \( \mu_\beta = 179.2 \times 10^{-6} \text{kg/m.s} \)  
- \( \rho_\beta = 1190.35 \text{ kg/m}^3 \)

**Lead:**

- \( k_\sigma = 35. \text{ W/m.K} \)  
- \( \rho_\sigma = 11340. \text{ kg/m}^3 \)  
- \( (c_p)_\sigma = 129. \text{ J/kg.K} \)

**1% Carbon steel:**

- \( k_\sigma = 43. \text{ W/m.K} \)  
- \( \rho_\sigma = 7800. \text{ kg/m}^3 \)  
- \( (c_p)_\sigma = 473. \text{ J/kg.K} \)

### 3.3 SOLUTION PROCEDURE

The finite difference techniques utilized in the solution of the problem in Chapter II were also employed in the solution of the present problem. In this problem, however, depending on whether phase change (condensation) takes place in the vapor phase or not, the set of unknown variables and governing equations change. The vapor phase continuity equation plays an important role in determining the solution format of the governing equations. The basic criterion which governs the phase change (condensation) in the working fluid is the attainment of the saturation vapor density. At any point where the vapor density reaches the saturation density corresponding to the temperature at that point, condensation will occur. The solution format of the condensation mode includes a
switching from the vapor phase continuity equation to the thermodynamic relation giving the saturated vapor density for the solution of the vapor density, while the vapor continuity equation is then used for the solution of the condensation rate.

The stability of the numerical scheme was insured by choosing a proper combination of $\Delta x$, $\Delta y$ and $\Delta t$. A systematic decrease in the grid size was employed for obtaining the convergence of the numerical scheme, and the corresponding stable $\Delta t$ was employed. A compromise, however, had to be made between the accuracy and the computer CPU time required for the computational runs. A 41x21 grid configuration (which gives a dimensionless $\Delta x$ of 0.025) was found to yield qualitatively and quantitatively good results for the condensation period and very good results for the problem for the cases with insulated boundary conditions. A 41x41 grid configuration was employed for the cases with constant wall temperature boundaries.

3.4 RESULTS AND DISCUSSION

In this chapter, part of the results that will be presented for the computational runs performed, will be in terms of non-dimensionalized variables. Variables $< P_\gamma >^\gamma$, $< \rho_\gamma >^\gamma$ and $< u_\gamma >$ are non-dimensionalized by using the corresponding reference quantities, namely $P^*$, $\rho^*$, and $v^*$. $P^*$ was chosen to be 100 kPa, $\rho^*$ was calculated from the equation of state by using $P^*$ and the initial temperature, $T_0$. The reference velocity, $v^*$, was computed from the vapor phase momentum equation, (3.3a), using a pressure gradient which was based on the global pressure difference applied across the packed bed and a density which was calculated from the equation of state by using $T_0$ and $P_{av}$ (the average of the inlet and exit pressures). Temperatures of the solid and fluid
phases are non-dimensionalized in the form $\Theta = (T - T_o) / (T_{in} - T_o)$. Time, $t$, is kept in dimensional form for giving an insight of the actual magnitudes of the durations involved. Same thing is done for the condensation rate data and the total condensate variations as well as for the thermal charging data of the packed bed.

As in the case of the solution of the problem in Chapter II, two distinctly noticeable stages were observed in the solution of the problem considered. These were namely the early stage and the later stage. The early stage usually lasts for a very short period of time during which sharp changes in the distribution of certain field variables, such as $<P_\gamma>$, $<\rho_\gamma>$ and $<u_\gamma>$, occur due to the step change boundary conditions which cause very strong transient behavior.

3.4.1 Problem with insulated wall boundary conditions

Figure 3.2 depicts the variations of the density, velocity and pressure of the vapor as well as the variation of the liquid fraction during the early stage at the mid-plane of the packed bed for insulated wall conditions for a case in which lead was used as the solid phase. It was found that, for cases with insulated boundary conditions, one-dimensional formulation would be very accurate since there was no appreciable variation of the field variables in the $y$-direction. Figure 3.2(a) shows that the vapor pressure distribution evolves and becomes almost linear during the early stage. There is no appreciable thermal penetration during this period and the spatial variation of the vapor density follows the same trend as the vapor pressure as dictated by the equation of state, except at the entrance of the packed bed. The temporal increase in the density is a result of the transient effects dictated by the vapor phase continuity equation. At points where the vapor density reaches the saturation vapor density, condensation occurs and the liquid
Figure 3.2: Variations of different field variable distributions during the early stage.
Figure 3.2 (continued)
phase accumulates. Due to the physical conditions of the problem, the period of time during which the vapor phase in the packed bed maintains a high enough density for condensation to occur is relatively short compared to the total thermal charging period of the packed bed. The variation of the liquid fraction is shown in Figure 3.2(d). In Figure 3.2 the *early stage* was somewhat extended to include the time during which more than 99% of the condensation took place for this case.

Beyond the *early stage*, the changes in the field variables are mainly caused by the development of the thermal penetration depth since the pressure distribution remains almost unchanged. Distributions of the field variables of interest by time during the *later stage* are shown in Figure 3.3. In Figure 3.3(a) the solid lines depict the solid phase temperature distribution while the dotted lines depict the fluid phase temperature distribution. During the *early stage*, the effect of the transient term as well as the condensation (source) term in the vapor continuity equation die out and, therefore, during the *later stage* the vapor density variation in time becomes dependent on the convective term in this equation. The mass flow rate in the packed bed becomes constant requiring an inverse relationship between the vapor density and velocity. Figure 3.3(c,d) clearly shows this behavior as a mirror image type of trend in the variations of the vapor density and vapor velocity at any instant. The spatial variation in the vapor density, on the other hand, can be explained from the equation of state. At any instant before the packed bed is fully charged, the slope of the vapor temperature profile will be larger than the slope of the vapor pressure profile for a certain length of the packed bed (meaning a sharper decrease in temperature than in pressure) causing an increase in the vapor density, whereas exactly the opposite behavior can be seen at the locations where the slope of the vapor temperature profile is smaller than that of the vapor pressure profile (meaning a sharper decrease in pressure than in temperature). When the packed bed becomes
Figure 3.3: Variations of different field variable distributions during the later stage
Figure 3.3 (continued)
thermally fully charged, the vapor density variation follows exactly the same trend as that
of the vapor pressure as dictated by the equation of state.

The overall condensation rate in the packed bed was computed by integrating the
individual condensation rates at all the grid points over the associated volumes at each
time step. The average overall condensation rate per unit width of packed bed for the
case for which the early and later stage results have been presented, is depicted in Figure
3.4(a). It can be seen that the overall condensation rate is higher at the beginning when
the transient effects are very strong, and dies away as the sharp changes in the vapor
density variation erode. The accumulative condensate in the packed bed was also
computed by integrating the condensation rates at all grid points over the associated
volumes at each time step and totaling with the previous sum. Figure 3.4(b) shows the
variation in time of the amount of total condensate in the packed bed per unit width of the
bed. As may be seen from this figure, the accumulation is fast at the start due to high
condensation rate, and builds up quickly reaching an asymptotic value in a short time.

The next important analysis carried out was related to the thermal charging of the
packed bed. For this, we needed to track down the variations of the relevant parameters
including heat flow rates and energy storage rate. The variation in the amount of thermal
energy flowing into and out of the packed bed per unit width is depicted as a function of
time in Figure 3.5(a). These were computed by integrating the energy fluxes of the vapor
flowing into and out of the packed bed over the inlet and the exit cross-sections. A very
short section of the time history of the thermal charging process at the beginning was left
out in this figure in order to obtain a better scale that shows the variation of the heat flow
rates clearly for the whole charging duration. During this very short time interval the rate
of heat flow into the packed bed was several times larger due to the presence of large
Figure 3.4:  (a) Variation of average overall condensation rate in the packed bed
(b) Variation of total accumulative condensate in the packed bed
Figure 3.5: (a) Rate of heat flow into and out of the packed bed

(b) Thermal charging of the packed bed (insulated top and bottom walls)
velocities at the onset of the charging process, i.e., during the early stage. Figure 3.5(b) shows the net energy stored per unit width of the packed bed as a function of time. This was computed in the numerical code by integrating the net heat flow rate into the packed bed over time. The value of the total net energy stored in the packed bed was also determined by a thermodynamic balance analysis between the initial and the final (fully charged) states of the packed bed. The result of the numerical computation was found to be in very good agreement with this analytical result.

3.4.2 Problem with constant temperature wall boundary conditions

The same problem was solved for constant temperature top and bottom wall boundary conditions. As expected, strong two-dimensional behavior was found in the variations of many of the field variables. Therefore, rather than presenting the results in the format of the previous section we will resort to two-dimensional contour plots for the field variable distributions. It should be noted that the solution of this problem has early stage and later stage parts just as in the case of insulated boundary conditions case. However, the distribution of three field variables at three time levels in the later stage will be presented here for convenience. These will be sufficient to show the two-dimensional behavior of the problem which becomes apparent during the later stage.

Figure 3.6 depicts the distributions of the fluid temperature, solid temperature and the vapor density in the packed bed at three different time levels. It can be seen from this figure that, although the two-dimensional behavior of the field variables is not very significant at the beginning, it becomes very much pronounced as the thermal penetration in the bed advances. In the core region of the packed bed the advancement of the thermal
Figure 3.6:  Distribution of field variables in the packed bed at different time levels during the later stage
front follows a similar trend as in the case of insulated boundary conditions. Near the top and bottom walls, however, there is a temperature gradient in the y-direction due to heat loss. Since there is no significant variation in the vapor pressure in the y-direction, the vapor density variation in this direction is primarily determined by the fluid temperature variation. Hence, at locations closer to the top and bottom walls where the fluid temperature becomes lower the vapor density becomes higher. In the core region of the packed bed the variation of the vapor density in x-direction depends on the slopes of the temperature and pressure distributions in this direction. At points where the slope of the temperature distribution is sharper than the slope of the pressure distribution, the vapor density will be increasing and vice versa.

The average overall condensation rate per unit width of the packed bed was computed in the same manner described in the previous section. Figure 3.7(a) shows the variation of this quantity in time. Upon comparing Figure 3.7(a) with Figure 3.7(a), one can see that condensation lasts slightly longer in the case of constant wall temperature case. This is reasonable because due to the heat loss from the top and bottom, for the points next to the top and bottom boundaries near the inlet of the packed bed, it takes longer for the vapor phase to reach a temperature at which the saturation vapor density exceeds the vapor density. Hence the duration of condensation at these points is prolonged. This also gives rise to a slightly larger amount of total condensate in the packed bed. This can be seen by comparing Figure 3.7(b) with Figure 3.4(b).

The variation in the amounts of heat flowing into and out of the packed bed per unit width at the inlet and exit are shown in Figure 3.8(a) as functions of time. In this case too, a small time slice at the beginning of the charging process is omitted from this figure in order to obtain a better scale in the figure. Figure 3.8(b) depicts the energy input into
Figure 3.7:  
(a) Variation of average overall condensation rate in the packed bed 
(b) Variation of total accumulate condensate in the packed bed
Figure 3.8:  
(a) Rate of heat flow at the inlet and exit of the packed bed  
(b) Thermal charging of the packed bed (constant temperature walls)
the packed bed, the energy that is lost from the top and bottom walls by conduction, and the net energy stored in the packed bed per unit width of the bed.

3.4.2.1 Effect of particle Reynolds number \( (Re_p) \) on condensation

The effect of \( Re_p \) on condensation was investigated by running three cases with different \( Re_p \)'s while Darcy number (Da) was kept constant by keeping the particle diameter fixed. The solid phase in all these cases was lead. Different \( Re_p \) values were obtained by applying different inlet pressure boundary condition for each case. Figure 3.9(a) depicts the variation of the average overall condensation rate per unit width of the packed bed for the cases in which the nominal \( Re_p \) values were 500, 1000 and 1500 respectively. The corresponding variations of the total condensate in the packed bed per unit width as functions of time are shown in Figure 3.9(b). From these figures it becomes apparent that the higher the \( Re_p \) the higher will be the condensation rate and the total condensate accumulation. On the other hand, higher \( Re_p \) will cause slightly shorter duration of condensation. Higher \( Re_p \) indicates higher mass flow rates as well as higher pressure difference applied across the packed bed (larger compression forces in the vapor phase), thus higher condensation rate. On the other hand, higher \( Re_p \), due to higher pressure gradient applied means potential for faster propagation of the pressure, density and temperature distributions in the vapor phase and thus shorter condensation time.
Figure 3.9: Effect of particle Reynolds number on condensation
3.4.2.2 **Effect of Darcy number (Da) on condensation**

The effect of Darcy number on condensation was also studied by considering three different Da values for a fixed $Re_p$, namely 1500. Different Da values were obtained by changing the particle size. Figure 3.10 shows the variations of the average overall condensation rate and the total condensate accumulation in the packed bed per unit width as a function of time for Da values of $1.49 \times 10^{-8}$, $2.66 \times 10^{-8}$ and $1.067 \times 10^{-7}$. The corresponding $d_p$ values were 0.75, 1.0 and 2.0 mm respectively. As $d_p$ increases and hence Da increases, the specific surface area of the particles decreases. This causes smaller amount of heat transfer between the solid and fluid phases and a faster advancement of the thermal penetration depth of the fluid phase (faster attainment of lower densities than the saturation vapor density at the initial condensation points). Therefore, although condensation zone advances faster the condensation rate and the amount of condensate accumulation remain small. Decreasing the $d_p$ at constant $Re_p$ (by increasing the pressure difference applied across the packed bed) increases the vapor velocities and hence causes larger mass flow rates and larger condensation rates. Also as $d_p$ is reduced the specific surface area of the particles increases. The heat transfer between the solid and fluid phases becomes more vigorous, energy is transferred at a faster rate from the fluid to the solid particles, and hence it takes longer for the fluid phase at the initial condensation points to reach temperatures high enough at which the vapor density becomes less than the saturation vapor density and condensation stops. Hence higher condensation rates are sustained for longer periods of time at these points, resulting in larger condensate accumulation.
Figure 3.10: Effect of Darcy number on condensation
3.4.2.3 Effect of thermal capacity of the solid phase on condensation

In order to analyze whether the thermal capacity of the solid phase utilized had any effect on condensation, a different material, namely 1% Carbon-steel, was employed as the solid phase material for qualitative comparisons. The variations of the average overall condensation rate and the total condensate accumulation per unit width of the packed bed as functions of time are depicted in Figure 3.11, for the case of lead and steel as solid material for three different Re_p values. As may be seen from this figure, although for a fixed Re_p the condensation rate is almost the same at the beginning for both solid materials, high condensation rates are sustained for a longer period of time in the case of steel than in the case of lead. The reason for this is that the thermal capacity of steel per unit volume is approximately 2.5 times that of lead. Due to this fact along with the high heat transfer rate between the solid and fluid phases, the temperature propagation in the fluid phase will be slower in the case of steel than in the case of lead because more of the thermal energy of the fluid is transferred to the solid in the case of steel than in the case of lead. In the case of steel the vapor phase will take a longer time to reach at a high enough temperature at which the vapor density will become lower than the saturation vapor density corresponding to that temperature, and therefore, the condensation durations will be longer.
Figure 3.11: Effect of the thermal capacity of the solid phase of the packed bed on condensation
CHAPTER IV

ANALYSIS OF A LATENT HEAT STORAGE PACKED BED UNDERGOING A CONDENSING FLOW OF A VAPOR

In this chapter, the problem investigated in Chapter III will be extended in complexity by introducing a phase change material (PCM) to replace the sensible heat storage material of the bed particles. Therefore, in addition to phase change in the working fluid, this problem involves phase change in the bed particles. The model used in Chapter III will be modified such that it will accommodate the phase change processes in the packed bed particles for the present problem. The numerical solution algorithm that shows some appreciable differences with that of the previous problem will be discussed. This will be followed by a discussion of the results of a number of case studies performed.

4.1 STATEMENT OF THE PROBLEM

The problem under consideration is the thermal charging of a packed bed of regularly sized spheres of encapsulated phase change material. This form of packed bed is also known as a latent heat storage packed bed. The schematic diagram of the problem considered in the present study is shown in Figure 4.1. As in the previous chapter, the packed bed is initially filled with R-12 vapor at a slightly superheated state at uniform temperature and pressure. Then it is subjected to flow of superheated R-12 from a
Insulated walls

Figure 4.1: Schematic diagram of the problem
reservoir which has higher temperature and pressure than those initially prevailing in the packed bed. The physical conditions for this problem are very similar to those of the previous chapter. In this problem, however, the packed bed incorporates a PCM whose melting temperature falls within the operating temperature range. Therefore, during the course of the thermal charging process of the packed bed there will be phase change (melting) within the encapsulated PCM particles.

4.2 FORMULATION

The volume averaging technique employed previously is also utilized in the present study. All of the eight assumptions and simplifications used in section 3.2 are assumed to be valid here too. In addition, the following assumption is also employed in the analysis of this problem: The temperature within each individual bed particle is assumed to be uniform at any instant, and the encapsulating material is assumed to have the same thermophysical properties as the PCM.

The governing equations for the present problem are similar to those of the previous problem except for the solid phase energy equation. Whereas a similar energy equation can be used when the PCM is not undergoing phase change, this equation becomes inapplicable when phase change is taking place in the PCM. During the process of phase change in the PCM, the PCM temperature is assumed to remain constant at a value equal to its melting temperature. The governing equations for this problem can be summarized as follows:
Vapor phase continuity equation:

\[
\frac{\partial \langle \varepsilon \gamma \rho_{\gamma} \rangle}{\partial t} + \nabla \cdot \left( \langle \rho_{\gamma} \gamma \rangle \nabla \langle \rho_{\gamma} \rangle \right) = -\langle \dot{m} \rangle
\]  (4.1)

Liquid phase continuity equation:

\[
\frac{\partial \varepsilon_{\beta}}{\partial t} + \nabla \cdot \langle \varepsilon_{\beta} \rangle \nabla \langle \tau_{\beta} \rangle = -\frac{\langle \dot{m} \rangle}{\rho_{\beta}} = 0
\]  (4.2)

Vapor phase equation of motion:

\[
\nabla \langle P_{\gamma} \gamma \rangle = -\frac{\langle \rho_{\gamma} \gamma \rangle}{K_{\gamma}^{1/2}} \frac{\langle \nabla_{\gamma} \rangle - \langle \nabla \rangle_{\gamma}}{\langle \nabla \rangle} - \frac{\mu_{\gamma}}{K_{\gamma}} \langle \nabla \rangle_{\gamma}
\]  (4.3)

which takes the following one-dimensional form

\[
\frac{\partial}{\partial x} \langle P_{\gamma} \gamma \rangle = -\frac{\langle \rho_{\gamma} \gamma \rangle}{K_{\gamma}^{1/2}} \langle u_{\gamma} \rangle^{2} - \frac{\mu_{\gamma}}{K_{\gamma}} \langle u_{\gamma} \rangle
\]  (4.3a)

Liquid phase equation of motion:

\[
\langle \nabla_{\beta} \rangle = -\frac{k_{ri} K}{\mu_{\beta}} \left[ k_{\varepsilon} \varepsilon_{\beta} + k_{<T_{f}>} \nabla <T_{f}>^{f} + \langle \rho_{\beta} - \langle \rho_{\gamma} \gamma \rangle \rangle \right]
\]  (4.4)

Fluid phase energy equation:

\[
\left[ \varepsilon_{\beta} \rho_{\beta}(c_{p})_{\beta} + \varepsilon_{\gamma} \rho_{\gamma} \gamma (c_{p})_{\gamma} \right] \frac{\partial <T_{f}>^{f}}{\partial t} - \langle \dot{m} \rangle \Delta h_{\text{vap}} + \left[ \rho_{\beta}(c_{p})_{\beta} \nabla_{\beta} + \langle \rho_{\gamma} \gamma \rangle (c_{p})_{\gamma} \nabla \langle \gamma \rangle \right] \cdot \nabla <T_{f}>^{f} = \nabla \left[ k_{\text{efff}} \nabla <T_{f}>^{f} \right] + h_{\sigma \beta} a_{\sigma \beta} <T_{f}>^{f} - T_{f}^{f} \right] + h_{\sigma \gamma} a_{\sigma \gamma} <T_{f}>^{f} - T_{f}^{f}
\]  (4.5)
Solid phase (PCM) energy equation:

When there is no phase change in the PCM we have

\[
\varepsilon_\sigma \rho_\sigma \sigma \frac{\partial \langle T_\sigma \rangle^\sigma}{\partial t} = \nabla \cdot \left[ k_{\text{eff}} \nabla \langle T_\sigma \rangle^\sigma \right] \\
- h_{\text{op}} a_{\text{op}} \left[ \langle T_\sigma \rangle^\sigma - \langle T_f \rangle^f \right] - h_{\sigma \gamma} a_{\sigma \gamma} \left[ \langle T_\sigma \rangle^\sigma - \langle T_f \rangle^f \right]
\]

(4.6a)

and when the PCM is undergoing phase change we have

\[ \langle T_\sigma \rangle^\sigma = T_{\text{melt}} \]

(4.6b)

Volume constraint relation:

\[ \varepsilon_\sigma + \varepsilon_\gamma(t) + \varepsilon_\beta(t) = 1 \]

(4.7)

Equation of state for vapor phase:

\[ \langle P_\gamma \rangle^\gamma = \langle \rho_\gamma \rangle^\gamma R_\gamma \langle T_f \rangle^f \]

(4.8)

Thermodynamic relation for the saturation density of vapor:

\[
\rho_{\gamma,s} = \frac{\exp \left( \frac{A - \frac{B}{T_f}}{R_\gamma T_f} \right)}{R_\gamma T_f}
\]

(4.9)

where A, and B are known constants, T_f is in degrees Kelvin and \( \rho_{\gamma,s} \) is in kg/m^3.

The modeling of the other parameters including the effective thermal conductivities of the solid (PCM) and the fluid phases, \( k_{\text{eff}} \) and \( k_{\text{eff}} \), the permeability of the packed bed, K, the permeability for the vapor phase, \( K_\gamma \), the geometric factor, F, the relative
permeability of the liquid phase, $k_{rp}$, the empirical correlations for the fluid-to particle heat transfer coefficient, $h_{pj}$, and the specific surface area of the packed bed for the vapor phase and for the liquid phase, $a_{rY}$ and $a_{rB}$, is carried out exactly in the same way as was done in Chapter III in equations (3.10) through (3.18).

4.2.1 **Boundary and Initial Conditions**

Following the arguments in section 3.2.1, the initial conditions for the present problem are mathematically given as

\[
\begin{align*}
T_f(x,y,t=0) &= T_0(x,y,t=0) = T_0 \\
P_y(x,y,t=0) &= P_0 \\
\n_u(x,y,t=0) &= 0
\end{align*}
\]

(4.10)

There is a continuous flow of high temperature vapor into the packed bed from a reservoir while the pressure at the exit of the packed bed is maintained at the initial bed pressure. The following mathematical forms express these boundary conditions:

\[
\begin{align*}
T_f(x=0,y,t) &= T_{in} \\
P_y(x=0,y,t) &= P_{in} \quad \text{at } t > 0^+ \\
P_y(x=L,y,t) &= P_{out} = P_0
\end{align*}
\]

(4.11)

where

\[
T_0 = 300 \text{ K} \quad P_0 = 796\text{kPa} \quad T_{in} = 350 \text{ K} \quad P_{out} = 796\text{kPa} \quad \text{and} \quad P_{in} = 811.2\text{kPa}
\]

Since we were primarily interested in the energy storage characteristics of the packed beds, insulated wall boundary conditions which were the most appropriate ones have been used in this chapter. These are given as
4.2.2 Physical conditions for the numerical runs

In the present problem two different phase change materials (latent heat storage materials) and a sensible heat storage material were considered for qualitative comparisons in the thermal charging behavior of the packed bed. The PCMs selected for this study were myristic acid which has a melting temperature equal to 331 K, and lithium-nitrate-trihydrate which has a melting temperature of 303 K. We will refer to these materials as PCM1 and PCM2 respectively. The sensible heat storage material chosen was 1% Carbon steel.

The physical property values of the materials which were used in the numerical computations are as follows:

R-12:

\[ k = 0.0097 \text{W/m.K} \quad (c_p)_\gamma = 710 \text{J/kg.K} \quad \mu_\gamma = 12.6 \times 10^{-6} \text{kg/m.s} \quad R_\gamma = 0.0687588 \text{J/kg.K} \]
\[ \Delta h_{\text{vap}} = 111300 \text{J/kg} \quad (c_p)_\beta = 1115 \text{J/kg.K} \quad \mu_\beta = 179.2 \times 10^{-6} \text{kg/m.s} \quad \rho_\beta = 1190.35 \text{kg/m}^3 \]
\[ k_\beta = 0.0545 \text{W/m.K} \]

Myristic acid (PCM1):

\[ c_p = 1590 \text{J/kg.K} \quad k = 0.1 \text{W/m.K} \quad \rho = 860 \text{kg/m}^3 \quad \text{for solid phase} \]
\[ c_p = 2260 \text{J/kg.K} \quad k = 0.1 \text{W/m.K} \quad \rho = 860 \text{kg/m}^3 \quad \text{for liquid phase} \]
**Lithium-nitrate-trihydrate (PCM2):**

\[ c_p = 2090 \text{ J/kg.K} \quad k = 0.5 \text{ W/m.K} \quad \rho = 1550 \text{ kg/m}^3 \quad \text{for solid as well as liquid phase} \]

**1% Carbon-steel:**

\[ c_p = 473 \text{ J/kg.K} \quad k = 43 \text{ W/m.K} \quad \rho = 7800 \text{ kg/m}^3 \]

### 4.3 SOLUTION PROCEDURE

The numerical scheme used for the previous problem is used here with some modifications that accommodate the phase change processes in the PCM. These include some flags in the numerical code that perform the checks for differentiating which one of equations (4.6a) and (4.6b) will be used for computing the PCM temperature.

In the solution of this problem, depending on whether phase change is occurring in either the working fluid or the PCM at a given location and instant in the packed bed, the governing equations and the solution format take different forms. It is assumed that the condensation occurs when the density of the vapor becomes equal to the saturation vapor density.

When there is no phase change in either the working fluid or the bed particles, the field variables \( < \rho_\gamma >, \epsilon_\beta, < u_\gamma >, < \nabla >, < T_f >, < T_{\sigma} >, \epsilon_\beta, \) and \( < P_\gamma > \) are determined from equations (4.1) to (4.8) respectively. At the same time \( \rho_{\gamma,s} \) is determined from equation (4.9). This is necessary for determining when to switch to the special solution format used for situations in which condensation takes place in the working fluid. It is carried out in order to determine when to switch from non-condensing mode to condensing mode solution format.
The phase change process in the PCM also needs special consideration. The following physical characteristics of the PCM are built into the numerical code. Once the PCM at a certain location and time reaches its melting temperature during the thermal charging of the packed bed, its temperature remains constant until phase change (melting) is complete in the capsules at that location. During this period, the solid phase (or PCM) energy equation should not be used for determining the PCM temperature. However, the amount of heat that is transferred to the PCM while it is undergoing melting is integrated in time in order to determine when the phase change is completed. Once the phase change is completed in the PCM at a certain location, the procedure of determining the PCM temperature is switched back to solution from the regular PCM energy equation, i.e., equation (4.6a), with appropriate liquid PCM properties incorporated into the numerical code.

At each time step, after the PCM temperature reaches the melting temperature, the convective heat transfer rate from the working fluid to the PCM and conduction heat transfer rates to and from the PCM are summed up for each node and taken into account. Furthermore, during a given time step both the net energy input to the PCM and the net total energy accumulated in the PCM up to the end of that time step are computed on a unit volume basis for each node where the PCM is undergoing phase change. This process is carried out until the net total energy accumulated in the PCM becomes equal to the latent heat of fusion of the PCM. During this period the solid temperature is taken to be equal to \( T_{\text{melt}} \) as given by equation (4.6b). After this period \( < T_{\sigma} >^\circ \) is again computed from equation (4.6a).
Implementation of the stability of the numerical scheme and the accuracy of the solutions were carried out as explained in the previous chapters. A 41x21 grid configuration (which corresponds to a dimensionless $\Delta x$ or $\Delta y$ of 0.025) was found to yield accurate enough solutions.

4.4 RESULTS AND DISCUSSION

It was again found convenient to non-dimensionalize some of the field variables in presenting the numerical solutions, and keep others in dimensional form to have a better understanding of the variations of the important parameters. Variables $<P_\gamma>$, $<\rho_\gamma>$, and $<u_\gamma>$ are non-dimensionalized with respect to the corresponding reference quantities $P^*$, $\rho^*$ and $v^*$. However, the pertinent parameters of the thermal charging of the packed bed and the condensation of the working fluid are presented in dimensional form per unit width of the packed bed as a function of the dimensional time. The value chosen for $P^*$ was 100 kPa, while $\rho^*$ was then computed from the equation of state using $P^*$ and the initial temperature, $T_0$. The calculation of the reference velocity, $v^*$, was based on the vapor phase momentum equation by incorporating the global pressure difference applied across the packed bed and the vapor density as calculated from the equation of state by using the mean value of the inlet and exit pressures.

Due to the physical conditions considered in the present study, the maximum local liquid fraction of the working fluid did not exceed the critical value above which the liquid becomes mobile, i.e., the liquid phase was always in pendular state. This, combined with the fact that only insulated boundary conditions were considered in the present study, made the problem essentially one-dimensional. It was checked through
numerical experimentation that the solution of the one-dimensional form of the governing equations did not have any appreciable difference from the solution to the two-dimensional form. Therefore, the results of the one-dimensional solution will be presented for convenience.

Two distinct stages that were observed in the previous two problems can be easily identified in the solution of the present problem too. The *early stage* with very strong transient effects, i.e., drastic spatial and temporal changes in the field variables, lasts for a very short time. During the *early stage* the pressure distribution across the packed bed evolves and assumes an almost linear form which is maintained afterwards during the rest of the charging process which will again be referred to as the *later stage*.

The first PCM employed for the encapsulated bed particles is myristic acid (or PCM1), which has a melting temperature of 331 K. The results for the case in which this material is used will be presented in detail. Once the high pressure, high temperature vapor is applied at the inlet of the packed bed, the vapor moving through the packed bed becomes compressed. Since it gives most of its excess internal energy to the colder PCM particles, its density reaches the saturation vapor density at certain locations and condensation takes place. Most of this condensation occurs in the *early stage* while the pressure distribution linearizes and the density of the vapor adjusts itself accordingly. Afterwards, the vapor reaches superheated conditions at all locations and no more condensation takes place.

The *early stage* variations of the density, velocity and the pressure of the working fluid, and the liquid fraction are shown in Figure 4.2. The *early stage* is somewhat extended such that it includes the period during which more than 99% of the condensation in the working fluid is completed. The consequence of the step change
Figure 4.2: Variations of different field variable distributions during the early stage
Figure 4.2 (continued)
boundary conditions can easily be seen from the high velocities at the inlet of the packed bed. These high velocities die out as the pressure distribution becomes linear. Except for a short thermal entry region, during the early stage the changes in the field variables are mostly pressure dependent because there is no appreciable thermal penetration. It should be noted that since the liquid fraction never reaches the critical value for becoming mobile, the $\varepsilon_P$ distribution remains the same throughout the later stage. The early stage results for this problem are qualitatively similar to the ones of the problem in Chapter III.

The changes in the field variables during the later stage can be attributed to the development of the thermal penetration depth. As may be seen from Figure 4.3, the pressure distribution remains linear. The variations of the temperatures of the working fluid and the PCM are very smooth until the PCM reaches its melting temperature. After the onset of melting in the PCM, a distinct discontinuity can be observed in the smoothness of the PCM temperature distribution. This is because for a certain length of the packed bed there is no change in the PCM temperature while the PCM is undergoing melting. In this region the working fluid also adjusts itself accordingly. This can be observed in the temperature profiles for time levels 2 through 8 in the Figure 4.3(a). This kind of qualitative behavior can also be seen in the vapor velocity and density variations along the packed bed at the corresponding time levels. This is a consequence of the fact that the vapor density variation is precisely related to the pressure and temperature variations. It can also be noticed that there is an inverse relationship between the vapor density and vapor velocity. This is due to the fact that the transient term in the vapor continuity equation loses its dominance and the convective term dominates during the later stage. Once the packed bed becomes thermally charged, then the vapor density distribution becomes linear similar to the pressure distribution as can be explained by the equation of state.
Figure 4.3: Variations of different field variable distributions during the *later stage*
Figure 4.3 (continued)
The variation of the average overall condensation rate and the accumulative condensate per unit width of the packed bed are shown in Figure 4.4. The overall condensation rate was computed by integrating the condensation rates at the individual grid points over the associated volumes. Integration of this over time yielded the accumulative condensate per unit width of the packed bed. Again the qualitative behavior in these figures is very similar to the ones in the previous chapter.

Of interest to the thermal charging of the packed bed are the rates of heat flowing into and out of the packed bed as a function of time. These were also computed for the given cross-section of the packed bed with a unit width. The variations of these quantities are depicted in Figure 4.5(a) except for a very short time section at the beginning of the charging process, which was left out for obtaining a better scale on the figure. The variation of the net energy stored per unit width of the packed bed as a function of time is also depicted in Figure 4.5(b). At the beginning of the charging process, the vapor flowing out of the packed bed leaves at a low temperature close to the initial temperature and thus, there is a large difference between the heat flow rates at the inlet and at the exit of the packed bed. Hence the rate of energy storage is large. Afterwards, for a major portion of the charging process there is a uniform difference between the rates of heat flowing into and out of the packed bed causing a linear increase in the amount of energy stored. Once the phase change is complete in all particles of the packed bed, both the working fluid temperature and the PCM temperature at the exit of the bed rise rapidly causing a rapid decrease in the gap between the heat flow rates at the inlet and the exit. When the packed bed becomes completely charged thermally, there remains no difference between the amount of heat flowing into and out of the packed bed.
Figure 4.4:
(b) Variation of average overall condensation rate in the packed bed

Accumulative condensate (kg)

Overall condensation rate (kg/sec)

\[ R_0 = 1500 \]

\[ D_A = 2.66 \times 10^{-9} \]
Figure 4.5:  
(a) Rate of heat flow into and out of the packed bed 
(b) Thermal charging of the packed bed (insulated top and bottom walls)
4.4.1 Qualitative comparison of condensation in the working fluid

Besides myristic acid (PCM1), two more materials were considered for the particles of the packed bed. These were lithium-nitrate-trihydrate, or PCM2, and 1% Carbon-steel. Computational runs were made for these cases with the same initial and boundary conditions as in the case of PCM1. Figure 4.6 depicts the overall average condensation rate and condensate accumulation histories for all three cases. Since condensation in the working fluid takes place in a very short span of time at the beginning, with no significant thermal penetration in the packed bed, the difference in the results for the three materials considered can be attributed mainly to their physical properties. An analysis shows that the thermal capacitance per unit volume of PCM2 is only approximately 12% different from that of steel, whereas the thermal capacitance per unit volume of PCM1 is almost three times smaller than that of steel. Due to this fact and the high heat transfer rate from the working fluid to the solid phase, the temperature propagation in both the solid phase as well as the working fluid is slower in the cases with steel and PCM2 than in the case with PCM1. Thus, for the former two cases it takes longer time for condensation to end. This is as a result of the longer time needed for the vapor to reach at a high enough temperature at which the vapor density becomes lower than the saturation vapor density. Consequently, higher condensation rates are sustained for longer periods and larger condensate accumulation takes place in the case of steel and PCM2.

4.4.2 Qualitative comparison of the thermal charging process

The three materials considered for bed particles in this section are the same as those of the previous section. Time histories of the rates of heat flow into and out of the packed bed for the three cases are shown in Figure 4.7, while that of the net energy
Figure 4.6: Comparison of condensation for cases with different bed particle materials
Figure 4.7: Heat flow rate into and out of the packed bed for cases with different bed particle materials.
stored in the packed bed per unit width is shown in Figure 4.8. It can be seen that, although at the beginning of the charging process the energy stored is almost the same for all three cases, it shows a different variation during later times. While that of the case with steel looks like a conventional charging curve which decays exponentially with time, the cases with the PCMs have a linear variation for a major portion of the charging period. These linear portions correspond to the time spans during which the temperature of the working fluid at the exit of the packed bed is fairly constant and approximately equal to the melting temperature of the PCM, since once the PCM reaches its melting temperature its temperature remains constant until phase change is complete in the PCM. During this period the vapor temperature cannot drop below the melting temperature of the PCM and thus we have a fairly constant vapor exit temperature. In order to illustrate this more clearly, the time history of the vapor exit temperature is presented in Figure 4.9 for the three cases. As may be seen from this figure, the vapor exits the packed bed at a lower temperature for a longer period of time in the case of PCM2 than in the case of PCM1. This is partly due to the lower melting temperature of PCM2 and partly due to its larger thermal capacitance. Therefore, the difference between the rates of heat flowing into and out of the bed is larger and lasts longer in this case, thus giving a larger amount of net energy stored.

Figure 4.8 also shows that using a PCM with a certain melting temperature may not always be a better choice over a sensible heat storage material. For instance, for the boundary conditions and the size considered in the present work, steel seems to perform better than PCM1 if we are interested in the energy storage range of up to 2200 kJ per unit depth over approximately 50 seconds with no limitation on the weight of the packed bed.
Figure 4.8: Thermal charging of the packed bed for cases with different bed particle materials
Figure 4.9: Time history of vapor exit temperature for cases with different bed particle materials
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

Each one of the topics investigated in this study was aimed at the fundamental understanding of different aspects of transport processes in porous media. In this respect, the objective of each topic was realized. In this chapter, the significant findings of each of the problems analyzed will be reiterated.

In Chapter II, the forced convective flow of a gas through a porous medium was analyzed by developing a rigorous model which included the thermodynamic aspects of the compressible flow of a gas or vapor by the use of ideal gas model. The main contributions of this study include the facts that it is the first study to use a non-thermal equilibrium analysis and incorporate the Ergun-Forchheimer equation for the vapor phase momentum equation in a non-isothermal and non-adiabatic gas flow in a porous medium. As discussed previously, this makes the model developed applicable to high speed flows and/or non-thermal equilibrium situations. The model developed is more rigorous than the previous models employed in this kind of problem in that, it does not neglect transport terms such as the conduction or the transient storage terms in the fluid phase as most of the previous works do. Moreover, it does not employ the assumption of local thermal equilibrium between the solid and fluid phases contrary to what has been done in a major part of the available works related to transport phenomena in porous media.
Solution of the transient problem with step change boundary conditions showed the existence of two distinct stages, namely the *early stage* and the *later stage*. It was found that the *early stage* is mostly dominated by changes in the pressure distribution, and the time scale for the changes in the distribution of pressure and the dependent variables is very short during this stage. On the other hand, the time scale for changes in the field variables during the *later stage* was found to be relatively larger, and that development of the thermal penetration into the packed bed played an important role in these changes.

Modeling the flow of the vapor/gas by using an equation of state was found to be very useful since this way it was possible to track down the pressure evolution as well as the variations in the velocity and density of the transporting fluid during the *early stage*. These can be crucial information during the start-up procedures of certain applications. It was also shown that although no such assumption has to be made, the pressure distribution in the packed bed assumes a linear form during the *later stage*.

The results obtained in the case studies performed in Chapter II were also helpful in drawing a number of conclusions on the qualitative aspects of transport phenomena in packed beds generally applicable to any porous medium. The first significant outcome of these analyses was related to the local thermal equilibrium (LTE) assumption. It was clearly seen from the results obtained that higher particle Reynolds number resulted in larger differences between the solid and fluid phases, that is larger deviations from local thermal equilibrium. Similar behavior was seen with increase in the Darcy number which translates into an increase in the particle size of the packed bed. On the other hand, the effect of the thermophysical properties on LTE was found to be much less influential. Therefore, it can be concluded that the validity of LTE assumption is mostly dependent on the speed of the flow and the particle size of the porous medium.
The two-dimensional behavior in the transport processes was also examined. It was found that while the two-dimensional effects are very sensitive to the thermophysical properties of the solid and fluid phases considered, particle size or speed of flow do not have significant effect on two-dimensional behavior. The results obtained show that higher the thermal diffusivity of the solid phase relative to that of the working fluid the more pronounced are the two-dimensional effects. This is reasonable since with higher thermal diffusivity in the solid phase the diffusion of heat in the transverse direction can reach the order of magnitude of the convective heat transfer between the solid and the fluid phases.

Numerical experimentations showed that the stability of the numerical code for the later stage simulations was primarily dependent on the magnitudes of the particle Reynolds number, \( \text{Re}_p \), and Darcy number, \( \text{Da} \). The maximum stable time step, \( \Delta t \), was found to increase with an increase in \( \text{Re}_p \) or decrease in \( \text{Da} \). While the reason for increased stability with decrease in \( \text{Da} \) can be attributed to the increased damping forces in the packed bed, the reason for increase in the stability of the numerical code with increase in \( \text{Re}_p \) should be related to the level of constraint that the flowing gas is under, or in other words the magnitude of the pressure difference applied across the packed bed.

It was found out that due to the nature of the governing equations, very small time steps have to be employed in the numerical simulation of the transport processes in compressible flow through porous media. Numerical simulation of the complete thermal charging process of the packed bed for cases with low \( \text{Re}_p \) and/or high \( \text{Da} \) requires excessive CPU time even on vector machines. For such cases, the use of regular main frames becomes highly inefficient.
One significant finding in Chapter II was that the use of a fluid-to-solid heat transfer coefficient based on a representative reference velocity, i.e., a nominal \( \text{Re}_p \) and a reference density throughout the later stage simulations would not cause a significant inaccuracy in the numerical solutions. This is of engineering importance because considerable amount of CPU time can be conserved by introducing this simplified form rather than performing computations for each grid point at each time step.

It should be noted that all of the conclusions of Chapter II were found to be equally valid in the problems in Chapters III and IV.

The main contribution of the investigation in Chapter III is that the development of a model for analyzing condensation in a convective flow of a vapor through a porous medium. This was achieved by incorporating the basic thermodynamic fundamentals of phase change into the model.

From the results of the case studies performed, it can be concluded that the amount of condensation of the working fluid is dependent on several parameters. For a fixed particle size, the higher the pressure difference applied across the packed bed, i.e., higher the nominal \( \text{Re}_p \), larger will be the rate and total amount of condensation, and slightly shorter will be the duration of condensation in the packed bed. This is a consequence of the fact that the working fluid has larger mass flow rates and is under larger compression forces.

The amount of condensation in the packed bed was also found to be influenced by the thermophysical properties of the bed particles. This could be explained by the heat interactions between the working fluid and the solid particles which determine the time taken for the thermal penetration to travel in both the solid and the fluid phases in the
packed bed. Larger thermal capacitance materials for the bed particles will result in maintaining the temperature of the vapor low and the density of the vapor high, resulting in a larger condensation rate.

Constant temperature wall conditions yielded larger amounts of condensation compared to insulated wall conditions, proving the fact that continuous heat removal from the packed bed will result in larger amounts of condensation.

Case studies in Chapter III also established that in the case of insulated boundary conditions with immobile liquid phase in the packed bed, the application of one-dimensional formulation is extremely accurate for the ranges of $Re_p$ and Da considered in this study. On the other hand, for constant temperature boundary conditions two-dimensional formulation is absolutely necessary.

For the type of problem considered in Chapter III, the amount of condensation in the packed bed is very sensitive to the variation of the fluid temperature on which the saturation vapor density is highly dependent. Therefore, it is very crucial to formulate the problem with no local thermal equilibrium assumption between the solid and the fluid phases in order not to introduce significant inaccuracies in tracking the condensation process.

The thermal charging of the packed bed for cases with insulated wall and constant temperature wall boundary conditions showed that the amount of heat loss can be very significant in the case of constant wall temperature conditions. This is a good indication that a packed bed employing solid particles of high thermal conductivity can be used as an energy storage/release system in applications such as spacecraft thermal management systems in which the instantaneous storage and subsequent release of waste heat is a
major issue. That is, during the storage mode the system will be set up with insulated
walls and during the subsequent release mode the insulation will be removed and the
walls will be exposed to a convecting environment.

In Chapter IV, in which the emphasis was given to the study of the behavior of
energy storage packed beds, it was shown that the characteristics of the thermal charging
of the latent heat storage packed beds differed appreciably from those of the sensible heat
storage packed beds. While the thermal charging curve of a sensible heat storage packed
bed looks very much like the conventional charging curve of a capacitor, that of a latent
heat storage packed bed shows different behavior in that its variation is linear in time
during a major part of the charging process. This behavior which is the consequence of a
fairly constant vapor exit temperature during the part of the charging process in which
phase change takes place in the PCM, was found to be dependent on the melting
temperature of the PCM. Case studies performed showed that energy storage in a packed
bed employing a PCM would be more efficient if the melting temperature of the PCM is
close to the lower end of the operating range of the packed bed. Because, this way
optimum use of the storage capacity would be realized, i.e., the least amount of energy
would be able to escape from being stored in the bed particles.

In the investigations in Chapter IV the differences between the solid and fluid
temperatures became even more apparent than in the previous cases in Chapters II and
III, confirming the necessity of modeling such problems with no local thermal
equilibrium assumption.
As established in Chapter III, the influence of the thermophysical properties of the PCM (or the solid phase) on condensation in the working fluid was reconfirmed in Chapter IV. Similarly, one-dimensional formulation was found to be satisfactory for the case with insulated wall boundary conditions.

5.2 RECOMMENDATIONS

For further work related to the present investigations the following points are recommended:

(a) To take the variation of the thermophysical properties with temperature for both the solid phase and the working fluid into account. This is important for applications in which the temperature range is high, e.g. of the order of 100 K.

(b) To investigate the possibilities of implementing a real gas equation of state for the working fluid rather than using the ideal gas model. This way more accurate solutions may be obtained for applications with operating pressures relatively higher than the atmospheric pressure.

(c) To investigate rational ways of implementing convective boundary conditions to the top and bottom walls of the packed bed. This will be helpful in more realistically studying the characteristics of a packed bed as an energy storage/release unit rather than just as a storage system.
LIST OF REFERENCES


