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Pressure and position control strategies for pneumatic systems

Lai, Jiing-Yih, Ph.D.

The Ohio State University, 1989

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PRESSURE AND POSITION CONTROL STRATEGIES
FOR PNEUMATIC SYSTEMS

DISSERTATION

Presented in Partial Fulfillment of the Requirement for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Jiing-Yih Lai, M.S.M.E.

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Department of Mechanical Engineering
Dedicated

to the memory of my Father
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LIST OF SYMBOLS

\( A_{nm} \)  
- dual Fourier transform coefficient of PWM

\( A_{p1}, A_{p2} \)  
- actuator areas where 1 and 2 denote left and right chambers respectively

\( A_{v}, A_{v1}, A_{v2} \)  
- orifice areas of valve 1 where 1 and 2 denote charging and discharging ports respectively

\( B_{nm} \)  
- dual Fourier transform coefficient of PWM

\( C_{d1}, C_{d2} \)  
- discharging coefficients of valve 1, where 1 and 2 denote charging and discharging ports respectively

\( C_{a1}, C_{b1} \)  
- coefficients of \( C_{d1} \)

\( C_{a2}, C_{b2} \)  
- coefficients of \( C_{d2} \)

\( C_p \)  
- specific heat at constant pressure

\( C_v \)  
- specific heat at constant volume

\( C_{24} \)  
- function of pressure ratio, see Eq. (2.7d)

\( c_{24p} \)  
- perturbation of \( C_{24} \)

\( D_1, D_2 \)  
- pressure response slopes, see Eqs. (3.20a) and (3.20b)

\( E_0 \)  
- amplitude of sinusoidal wave to PWM

\( E_1, E_2 \)  
- acceleration response slopes, see Eqs. (3.23a) and (3.23b)

\( e_1, e_2 \)  
- coefficients relating \( e_p \) and \( T_p \), see Figure 3-2

\( e \)  
- displacement error between \( x_d \) and \( x_p \), see Eq. (3.32)

\( e_x \)  
- displacement error between \( x_d \) and \( x \), see Eq. (3.33)
\begin{itemize}
\item $e_p$ pressure error
\item $F_c$ Coulomb friction force
\item $F_d$ mechanical friction force
\item $F_f$ feedforward force
\item $F_p$ actuator force
\item $F_s$ stiction force
\item $f_s$ cutoff frequency of the filter
\item $g$ gravitational constant
\item $h$ amplitude of the carrier signal
\item $j$ learning law trial
\item $J_p$ mean square error
\item $k$ number of data points for averaging (Chapter 4)
\item $k_1$ partial derivative of mass flow rate with respect to valve operating, see Eqs. (2.9c)
\item $k_2$ partial derivative of mass flow rate with respect to load pressure, see Eq. (2.9d)
\item $k_3$ coefficient related to compliance
\item $k_4$ no-load flow sensitivity
\item $k_a, k_{a1}, k_{a2}$ valve coefficients where 1 and 2 denote charging and discharging port respectively, see Eqs. (2.11b) and (2.17)
\item $k_b$ valve coefficient, see Eq. (3.15)
\item $k_n$ constant value given by Eq. (2.7d), 0.258 for air
\item $k_o$ system gain of chamber dynamics
\item $k_p$ pressure gain
\item $k_r$ coefficient in mass flow rate, see Eq. (2.5)
\end{itemize}
\begin{itemize}
  \item $k_x$: position gain
  \item $k_v$: velocity gain
  \item $k_w$: PWM gain
  \item $m$: mass of air in the control volume
  \item $m_a$: load inertia
  \item $N_{12}$: ratio of actual to sonic orifice flow
  \item $N_f$: trial of friction test
  \item $n_{12p}$: variation of $N_{12}$
  \item $n$: ratio of specific heat, 1.4 for air
  \item $p$: chamber pressure (Chapter 2)
  \item $P_{12}$: chamber pressure where 1 and 2 denote left and right chambers respectively (Chapter 3)
  \item $P_4$: atmospheric pressure
  \item $P_c$: command pressure
  \item $P_{c0}$: dc component of $P_c$
  \item $P_{ca}$: amplitude of $P_c$ for the first harmonic
  \item $P_f$: feedforward pressure
  \item $P_{s1}, P_{s2}$: supply pressure where 1 and 2 denote left and right chambers respectively (Chapter 3)
  \item $P_s$: supply pressure (Chapter 2)
  \item $\Theta$: heat transferred to the chamber (Chapter 2)
  \item $\Theta_1$: heat transferred to chamber 1 (Chapter 3)
  \item $q$: least square coefficient
  \item $R$: gas constant
  \item $r_c$: critical pressure ratio, 0.528 for air
  \item $s$: Laplace operator
\end{itemize}
T, T₁ temperature where 1 denotes the left chamber

Tₚ pulse period of PWM

Tᵣ pulse width of PWM

Tₛ sampling time

t time

tₖ learning period

tₛ rise time

u valve input signal (Chapter 2)

V chamber volume (Chapter 2)

V₁, V₂ chamber volume (Chapter 3)

vₑ valve input signal (Chapter 3)

x load displacement

xₐ command displacement

xₚ perturbation of x

α duty factor, Tᵣ/Tₚ

αₛ duty factor in the steady state

β scale factor

δ position threshold

ρ density of air in the chamber (Chapter 2)

ρ₁ density of air in chamber 1 (Chapter 3)

τ, τ₁, τ₂ chamber parameters where 1 and 2 denote charging and discharging port, respectively

ω modulating frequency

ωₑ modulated frequency

ωₚ natural frequency
\( \zeta \) damping ratio

\( \Delta x \) displacement ripple

\( \Delta p \) pressure ripple

\( \Gamma_p \) pressure learning gain

\( \Gamma_x \) position learning gain

\( \Gamma_v \) velocity learning gain

**Superscripts**

- \( d \) operating point or time averaged value
- \( \frac{d}{dt} \)
- \( \frac{d^2}{dt^2} \)
- \( \frac{d^3}{dt^3} \)

* Optimal value

\(^\wedge\) estimated quantity

\( \infty \) limiting value as \( t \rightarrow \infty \)

\( i \) step of iterative pressure response

\( s \) steady state

**Subscript**

\( 0 \) initial point
CHAPTER I
INTRODUCTION

1.1 INTRODUCTION

Pneumatic systems for force and motion control are increasingly becoming more popular as new applications range from miniature electropneumatic circuits to complex mechanical, automotive, manufacturing, robotic, biomedical and aerospace systems [Drazan (1976), Thayer(1984) and Liu et al.(1987)]. The potential advantages of using pneumatic actuation systems over competing technologies are lower cost, simpler operational systems, easy maintenance, long term storability and wide temperature capability [Thayer(1984) and Liu et al.(1987)]. However, such advantages may not be realized unless a few fundamental problems are solved first. Some of these problems are as follows: (i) because of the gas compressibility, the time constant associated with the fluid flow process is low compared to that of hydraulic systems, (ii) actuation stiffness changes dramatically for different operating pressures and piston positions, and (iii) the friction force to actuation force ratio is typically high, which results in poor static accuracy. Also, mathematical prediction of the dynamic response is more difficult as the governing processes are highly nonlinear [Thayer(1984), Noritsugu(1987) and Kunt(1988)].
Currently, pneumatic actuation systems for automated manufacturing and pneumatic manipulators are primarily in the open loop control mode. Such systems with simple on-off type control schemes can achieve only gross positioning, but are not suitable for continuous and fine motion control applications. A closed-loop servoactuation system is not generally available because of the above mentioned problems which prohibit its application in precise position control and/or high speed automation. Consequently, present pneumatic actuation systems are limited to applications that require low accuracy and/or can tolerate slow dynamic response. Nevertheless, it is expected that pneumatic actuation integrated with computer-based control and high speed flow control will overcome the above-mentioned difficulties and meet the demand of low cost automation in various applications. Accordingly, the development of novel designs and control strategies is extremely important to advance the state of the art of pneumatic actuation. The main focus of this study is on accurate computer control of pneumatic pressure and load position.

A typical pneumatic servoactuation system block diagram is shown in Figure 1-1 which relates the electric command to the load motion being controlled [Thayer(1984)]. The power source commonly consists of a compressor or an engine bleed and the converter regulates the gas pressure to a level which meets the work requirement regardless of the upstream pressure variation and independent of flow. Power modulation is usually accomplished by means of directional control valves which start or stop pressurized gas
Figure 1-1 General pneumatic servoactuation system
flow and direct it to selected passages. The actuator can be linear or rotary depending on the load motion requirement. The need for accessories such as filters, lubricators and reservoirs is self-evident. A detailed description of various components is given by Yeaple(1984).

1.2 LITERATURE REVIEW

1.2.1 Mathematical Models

The transfer function approach has been used to model the dynamics of a pneumatic double acting actuator and a proportional control valve [Shearer(1956a, 1956b), Halstenberg(1959) and Burrows (1969)]. Similar analyses can be found in the fluid power texts by Blackburn et al.(1960), Andersen(1967), and McCloy(1980). However, such a linearized model is suitable for the mid-stroke position only. Also, it is not suitable for asymmetrical actuators with different cross-sectional areas. In addition, nonlinear mechanical friction is typically neglected as it increases difficulty in analysis. Stability issues of such a linear model have been analyzed by Burrows(1969) using the Routh criterion.

The state space approach has been used to extend the linearized model over several operating positions [Scavarda et al.(1987) and Liu et al.(1987)]. Scavarda et al.(1987) have developed two linearized models for a pneumatic servomechanism under various operating conditions. Liu et al.(1987) have studied the dynamics and control of a one-degree-of-freedom pneumatic manipulator. But, in both studies, the effect of mechanical friction has been neglected.
Doeblin (1958) has examined, using the describing function technique, the frequency response of an on-off valve controlled pneumatic servomechanism using a rotary actuator. A similar analysis has been performed by Araki (1984) who derived the frequency response of a linear pneumatic actuator under proportional flow control by approximating the valve pressure-flow relationship as a third degree polynomial equation. Such a linear pressure-flow model makes it easy to determine the operating pressure.

1.2.2 Control Schemes

Burrows and Webb (1968, 1970) have studied the stabilizing effect of auxiliary tanks in addition to velocity and acceleration feedback for an on-off pneumatic servomechanism. A similar study has been performed by Eun et al. (1982) who conducted extensive experiments to investigate the effects of cylinder orifice which bypasses the air in the actuator, stabilizing tank and velocity feedback on position control accuracy and system stability. Bowns and Ballard (1972) noted the excessive sensitivity of the dynamic response of an on-off pneumatic system to actuator seal friction. The applications of on-off controlled pneumatic actuator to pneumatic manipulators for either position or force control can be found in the publications by Krejnin (1981), Yamafuji (1985), Drazan (1976), Suh et al. (1983) and Krisztinicz (1976).

Pulse-width-modulation (PWM) has been shown to offer considerable advantages in the control of dc motors and hydraulic servos. The control signal to the motor or the valve is given in the
form of pulse-width-modulated wave. The high-frequency switching of this signal, and the resulting dither in the motor shaft or the valve spool, greatly minimizes the effects of such nonlinearities as hysteresis, threshold, stiction, dead-zone and null shift. Such a technique has also been applied to the on-off controlled pneumatic systems by Goldstein et al.(1968), Eun et al.(1984), Thayer(1984), Morita(1985) and Noritsugu(1985,1987). Goldstein et al.(1968) have used the technique of differential pulse-length modulation to modulate the motion of a floating-flapper-disk switching valve. Eun et al.(1984) have experimentally studied the feasibility of a modified on-off controller in which an additional pulse band outside the deadband is added to the reference signal in order to improve the accuracy. They found that increase of the pulseband produces better accuracy but slows the transient response. Thayer(1984) has analyzed the frequency response of a particular electropneumatic actuator system for aerospace applications. Even though the valves are under the PWM control, he did not report the effect of PWM on the system response. Morita(1985) and Noritsugu(1985) have implemented the PWM mode in pneumatic manipulators to control pressure and contact force, respectively; both demonstrate that continuous feedback control can be obtained with this technique without using expensive and sensitive servovalves. Noritsugu(1987) has employed three valves for a particular motion control problem where two valves were under on-off control, and the third valve (automotive injection valve) was under PWM control when operating in the vicinity of the desired position. Linearized analysis was used to study the effect of various parameters,
including friction, on the system response. But this scheme is too complex and expensive. Also, no efficient method can be used to determine some of the parameters in the controller, such as the switch points to change the valve motion and the position feedback gain. Accordingly, this strategy can not be applied in practice.

1.3 SCOPE AND OBJECTIVES

Referring to Figure 1-1, the pneumatic power modulator, actuator, and load blocks will be studied in detail both analytically and experimentally. Nonlinear mathematical models will be developed for the computer simulation studies and linearized models will be formulated to design the control system. Experimental studies will be implemented on a pneumatic robot as shown in Figure 1-2 in which only the extension/retract degree of freedom of the arm is considered. The robot arm will be actuated by a linear displacement cylinder driven by two on-off control valves. This study proposes to operate one valve in the PWM mode control while holding the other valve wide open; specific reasons for this will be discussed later.

Specific objectives for this dissertation are grouped under the main headings of pressure control, position control and learning control as given below. Note that each chapter is written in a journal paper style, hence each chapter is self sufficient. Accordingly, a detailed problem statement is also included in each chapter.
1. Gripper adjustment
2. Retract stop
3. Reach stop
4. Upper stop collars
5. Lower stop collars
6. Cushion adjusting ring
7. Lock nut
8. Rotation stops
9. Speed controls

Figure 1-2 Pneumatic robot used for experiments. Only the extension/retract degree of freedom of the arm is used for position control.
(I) **Pressure control:** Since no analytical study on the effect of PWM mode on the pneumatic system performance has been conducted previously, this study fills this void while proposing a new strategy for pressure control. The valve-chamber system shown in Figure 1-3 is considered for this purpose. A nonlinear computer simulation model will be developed to study the valve-chamber dynamics and a linearized model will be used to analyze the system characteristics in both time and frequency domains. In addition, a new strategy with feedforward pressure compensation will be proposed for pressure control, and the stability of such a closed loop system will be studied using a graphical technique. (Chapter 2)

(II) **Position control:** The actuator system as shown in Figure 1-4 is considered and the control of load motion $x(t)$ is the objective. We establish two major objectives as follows: First, a comprehensive analytical model incorporating pneumatic process nonlinearities and nonlinear mechanical friction will be developed to characterize the actuator dynamics. It will be supported by a computer simulation study. Second, we intend to develop a novel control strategy using the on-off valves, to achieve high position accuracy. A simplified analytical model will be formulated to design the control system. (Chapter 3)
Figure 1-3 Schematic of the valve-chamber system

Figure 1-4 Schematic of the on-off valve controlled pneumatic actuator
(III) Learning control: It is generally difficult to design a fixed-gain controller suitable for all operating points for nonlinear pneumatic systems. To overcome this problem, first, we intend to examine the feasibility of using available learning laws to improve the steady state pressure and position accuracies. Computer simulation and laboratory experiment will be conducted for this study. Next, an optimization scheme based on linearized models will be developed to choose the learning gains. Computer simulation based on nonlinear models will be carried out to verify the feasibility of this optimization scheme. (Chapter 4)
CHAPTER II
PRESSURE CONTROL OF A
PNEUMATIC CHAMBER

2.1 INTRODUCTION

Pulse-width-modulation (PWM) has been found to be very effective in the control of dc motors and hydraulic servos as it reduces the effect of such nonlinearities as hysteresis, threshold and stiction, and improves system reliability and performance. This technique has also been applied to on-off controlled pneumatic actuators to modulate valve performance by Noritsugu (1985, 1987) and Morita et al. (1985). Noritsugu (1987) has employed two valves under on-off control, and a third valve (automotive injection valve) under PWM control when operating in the vicinity of the desired position. However, this scheme is too complex and costly. Morita et al. (1985) and Noritsugu (1985) have also implemented the PWM mode in pneumatic manipulators to control pressure and contact force, respectively. However, their models of mass flow characteristics fail to describe the on-off effect of valve motion. Moreover, the literature does not contain any analytical study on the effect of PWM mode on the pneumatic system performance. This study is presented to fill this void while proposing a
better system model and a control strategy including a successful experimental demonstration.

Various attempts at modeling pneumatic actuators have been made. Shearer (1956), Burrows (1969), and Botting et al. (1970) have used the transfer function approach to model the dynamics of pneumatic actuator systems. However, such a linearized model is suitable only for the mid-stroke position. Recently, Liu (1987) and Scavarda (1987) have used the state space approach to extend the linear model to various operating positions. But, their approach ignores the effect of discharge coefficient which varies with the pressure ratio between downstream and upstream of the valve and the orifice geometry.

A number of techniques for force control of pneumatic manipulators have been proposed. Suh et al. (1983) have carried out experimental work on an on-off controlled robot gripper to examine the effect of various parameters on system performance. Morita's work employing PWM control of the on-off valve has demonstrated the feasibility of using PWM control [Morita et al., 1985]. However, with only proportional feedback implemented, they found that there is deviation between the command pressure and the mean pressure response. Noritsugu (1985) has successfully employed an analog PI control scheme to reduce the steady state error.

In this chapter, a pneumatic valve controlled chamber system shown schematically in Figure 2-1(a) is studied. The on-off valve with supply pressure $p_s$ is operated under the PWM mode to control the chamber pressure $p(t)$. The chamber volume $V$ is fixed, and the valve
orifice area is $A_{v1}$ for charging and $A_{v2}$ for discharging. A periodic pulse signal $u(t)$ with duty pulse width $T_R$ being proportional to the pressure error $e_p$ (pressure difference between actual $p(t)$ and command chamber pressure $p_c(t)$) is input to the solenoid of the valve, as shown in Figure 2-1(b). We extend the literature by developing a flow direction dependent model in which the total pressure term instead of the perturbed one is used and by including a realistic description of the discharge coefficients $c_{d1}$ and $c_{d2}$. Based on this model, analytical studies of the effect of the PWM mode on system performance are carried out in both frequency and time domains. Subsequently, a strategy is proposed to control the chamber pressure $p(t)$.

2.2 FREQUENCY RESPONSE

2.2.1 Chamber dynamics

A lumped parameter dynamic model of the fluid state and flow is developed for the valve-chamber system. The following assumptions are made: (1) fluid properties at a given instant are uniform throughout any one of the control volumes, (2) the ideal gas equation of state is valid, (3) the thermofluid processes is isentropic as the rate of mass transfer with the system is much faster than the rate of heat exchange through the system boundary, and (4) the bandwidth of the valve is larger than the carrier frequency. Note that these assumptions have been made by several investigators [Shearer 1957, Burrows et al. 1967 and Noritsugu 1987]. From the energy equation, the relation among volume $V$, pressure $P$, and temperature $T$ is given by
Figure 2-1 Proposed system schematic. (a) Valve-chamber system. (b) Formulation of the PWM mode.
\[ C_p T \dot{m} - p \frac{d V}{dt} + \frac{d Q}{dt} = \frac{d}{dt}(C_v \rho VT) \]  

(2.1)

where \( C_p \) is specific heat at constant pressure, \( C_v \) is specific heat at constant volume, \( \dot{m} \) is mass flow rate, \( \rho \) is density of air in the chamber, \( \frac{d t}{d} \) is first derivative of time, and \( Q \) is heat transferred to system. From the ideal gas equation of state, \( \rho \) can be expressed as

\[ \rho = \frac{p}{(R T)} \]  

(2.2)

where \( R \) is gas constant. Also, since the thermofluid process is isentropic, the heat transfer rate is zero, i.e.

\[ \frac{d Q}{dt} = 0 \]  

(2.3)

The assumption of isentropic thermofluid process means that the temperature \( T \) is time varying. However, from a practical point of view, \( T \) does not vary appreciably from its initial value \( T_0 \) in most cases. Hence, \( T \) can be regarded as a constant value \( T_0 \). This assumption has been used by several previous investigators [Shearer(1956, 1957), Andersen(1976) and Kawnmnia(1988)]. Also, it seems to be valid in our laboratory experiment. Further, since the chamber volume \( V \) is constant, we have

\[ \frac{d V}{dt} = 0 \]  

(2.4)

Substituting Eqs. (2.2), (2.3) and (2.4) into Eq. (2.1), yields

\[ \dot{p}(t) = k_r \dot{m}(t) \quad , \quad k_r = \frac{n R T_0}{V} \quad , \quad n = \frac{C_p}{C_v} \]  

(2.5)
where \( n \) is ratio of specific heats (1.4 for air). The mass flow rate \( \dot{m} \) across the valve ports is modeled as one dimensional, i.e. fluid properties are uniform across the valve ports. For this purpose a compressible isentropic mass flow rate function can be expressed as follows [Andersen1975, McCloy 1980]:

\[
\dot{m} = \begin{cases} 
C_{d1} A_{v1} P_s k N_{12} / \sqrt{T_0} & , \quad u(t) > 0 \\
-C_{d2} A_{v2} P_4 k C_{24} / \sqrt{T_0} & , \quad u(t) < 0 
\end{cases}
\] (2.6)

where

\[
C_{d1} = C_{a1} - C_{b1} \frac{P(t)}{P_s} , \quad C_{d2} = C_{a2} + C_{b2} \frac{P(t)}{P_4}
\] (2.7a,b)

\[
k = \left[ \frac{ng}{R} \left( \frac{2}{n+1} \right)^{\frac{n+1}{n-1}} \right]^{1/2}
\] (2.7c)

\[
C_{24} = k_n \left\{ \left[ \left( \frac{P(t)}{P_4} \right)^{(n-1)/n} - \left( \frac{P(t)}{P_4} \right)^{(n+1)/n} \right]^{1/2} - \frac{P(t)}{P_s} \right\}^{1/2}
\] (2.7d)

\[
N_{12} = \begin{cases} 
k_n \left[ \left( \frac{P(t)}{P_s} \right)^2 - \left( \frac{P(t)}{P_s} \right)^{(n+1)/n} \right]^{1/2} & , \quad \frac{P(t)}{P_s} > r_c \\
1 & , \quad \frac{P(t)}{P_s} < r_c 
\end{cases}
\] (2.7e)

where \( N_{12} \) and \( C_{24} \) are mass flow parameters, \( P_4 \) is atmospheric pressure, \( k_n \) is a constant (0.258 for air) and \( g \) is the gravitational constant. When the valve input signal \( u(t) > 0 \), air flows through the supply port of the valve to the chamber, and when \( u(t) < 0 \), air from the chamber escapes to the atmosphere. The discharge coefficients \( C_{d1} \) and \( C_{d2} \), defined as the ratio of actual to theoretical steady state mass
flow rate, are empirical factors used in the calculation of mass flow rate through the input and output ports of the valve, respectively. Here $C_{d1}$ and $C_{d2}$ are modelled as a first-degree polynomial function of the chamber pressure as shown in Eqs. (2.7a) and (2.7b) with $C_{a1}=0.53$, $C_{b1}=0.01$, $C_{a2}=0.231$ and $C_{b2}=0.121$. Eq. (2.7e) shows that the mass flow rate $\dot{m}$ under subsonic condition $(p(t)/p_s>r_c)$ is more nonlinear than that in sonic $(p(t)/p_s<r_c)$. In order to study the system characteristics analytically, Eq. (2.6) is linearized as

$$\dot{m} = -k_1 p + k_2 A_v$$

(2.8)

where

$$A_v = A_{v1}, \quad u(t) > 0 \quad ; \quad A_v = -A_{v2}, \quad u(t) < 0$$

(2.9a,b)

$$k_1 = \begin{cases} \frac{p_s k_n}{\sqrt{T_0}} A_{v1} \left[ \frac{C_{b1}}{p_s} (\bar{N}_{12} + n_{12p} p) - C_{a1} n_{12p} \right], & u(t) > 0 \\ \frac{p_s k_n}{\sqrt{T_0}} A_{v2} \left[ \frac{C_{b2}}{p_s} (\bar{C}_{24} + c_{24p} \bar{p}) + C_{a2} c_{24p} \right], & u(t) < 0 \end{cases}$$

(2.9c)

$$k_2 = \begin{cases} \frac{p_s k_n}{\sqrt{T_0}} (C_{b1} n_{12p} \bar{p}^2 / p_s + C_{a1} (\bar{N}_{12} - n_{12p} \bar{p})), & u(t) > 0 \\ \frac{p_s k_n}{\sqrt{T_0}} \left( C_{a2} (\bar{C}_{24} - c_{24p} \bar{p}) - C_{b2} c_{24p} \bar{p}^2 / p_s \right), & u(t) < 0 \end{cases}$$

(2.9d)

Note that due to the on-off effect of valve motion, $k_1$, $k_2$ and $A_v$ depend upon the $u(t)$ sign. Also, the mass flow parameters $N_{12}$ and $C_{24}$ are linearized around the operating pressure $\bar{p}$, which is defined as the average pressure for each cycle of valve motion in the steady state.
Here $N_{12}$ and $C_{24}$ denote the operating values, and $n_{12p}$ and $c_{24p}$ represent the partial differential of $N_{12}$ and $C_{24}$ with respect to $p$, respectively. Substituting Eq. (2.8) into Eq. (2.5), we obtain the governing equation of chamber pressure response around the operating pressure.

$$\dot{p} + \tau p = k_a A_v$$

$$\tau = k_1 k_r, \quad k_a = k_2 k_r$$

Eq. (2.10) shows that the chamber dynamics can be modelled as a first order system with distinct system parameters $k_a$ and $\tau$ depending on the flow direction. Here it is seen that $\tau$ is affected by orifice area, supply pressure, temperature, operating pressure and volume.

### 2.2.2 Transfer Characteristics of PWM

The method of obtaining the transfer characteristics is based on a dual Fourier series expansion in two variables [Black, 1953]. Consider that the input signal $e_p(t)$ to the pulse-width modulator is a sinusoidal wave $E_0 \sin \omega t$, where $E_0$ is the amplitude and $\omega$ is the modulating frequency. The modulated signal $u(\omega t, \omega_c t)$ of carrier frequency $\omega_c$, where $\omega_c$ is defined as $1/T_p$, can be expressed as follows:

$$u(\omega_c t, \omega t) = \frac{1}{2} A_{00} + \sum_{n=1}^{\infty} [A_{n0} \cos n \omega t + B_{n0} \sin n \omega t]$$

$$+ \sum_{m=1}^{\infty} [A_{0m} \cos m \omega_c t + B_{0m} \sin m \omega_c t]$$

$$+ \sum_{m=1}^{\infty} \sum_{n=\pm1}^{\infty} [A_{nm} \cos (m \omega_c t + n \omega t) + B_{nm} \sin (m \omega_c t + n \omega t)]$$

(2.12)

where
\[ A_{nm} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} u(\omega_c t, \omega t) \cos(m\omega_c t + n\omega t) d(\omega_c t) d(\omega t) \]  \hfill (2.13a)

\[ B_{nm} = \frac{1}{2\pi^2} \int_0^{2\pi} \int_0^{2\pi} u(\omega_c t, \omega t) \sin(m\omega_c t + n\omega t) d(\omega_c t) d(\omega t) \]  \hfill (2.13b)

The first term in Eq. (2.12) is the dc component of the pulses. The frequency components of the second term correspond to the frequency components of the modulating wave \( \omega \) and its harmonics. The third term separates frequencies into the fundamental frequency \( \omega_c \) of the pulses and its harmonics. The frequency components of the last term are given by the ensemble of all positive pairs formed by taking the sum and difference of integral multiples of each fundamental.

Note that the modulated signal \( u(\omega_c t, \omega t) \) is a periodic square wave with constant period \( T_p \) and amplitude \( A_{v1} \) when the valve is open, and \( A_{v2} \) when the valve is closed, thus, \( u(\omega_c t, \omega t) \) is given by

\[ u(\omega_c t, \omega t) = \begin{cases} A_{v1} & , \quad 0 \leq \omega_c t < (\pi + \frac{\pi}{h}E_0 \sin \omega t) \\ -A_{v2} & , \quad (\pi + \frac{\pi}{h}E_0 \sin \omega t) \leq \omega_c t < 2\pi \end{cases} \]  \hfill (2.14)

where \( h \) is the amplitude of the carrier signal, as shown in Figure 2-1(b). By substituting Eq. (2.14) into Eq. (2.13), \( A_{nm} \) and \( B_{nm} \) can be evaluated. All cosine coefficients \( A_{nm} \) vanish because the input is a sine function and the modulation function \( u(\omega_c t, \omega t) \) is odd. The first few terms of the coefficients are given below.
21

\[
B_{10} = \frac{E_0}{2\pi h} (A_{V1} + A_{V2}) ; \quad B_{n0} = 0 , \quad n = 2, 3, \ldots ,
\]

\[
B_{01} = \frac{A_{V1} + A_{V2}}{2\pi^2} \left( \int_{0}^{2\pi} \cos(\frac{\pi}{h} E_0 \sin \omega t) \, d\omega t \right),
\]

\[
B_{02} = \frac{A_{V1} + A_{V2}}{4\pi^2} \left( \int_{0}^{2\pi} \left( 1 - \cos(\frac{2\pi}{h} E_0 \sin \omega t) \right) \, d\omega t \right),
\]

\[
B_{11} = \frac{A_{V1} + A_{V2}}{2\pi^2} \left( \int_{0}^{2\pi} \cos \left( \frac{\pi}{h} E_0 \sin \omega t + \omega t \right) \, d\omega t \right), \ldots \quad (2.15)
\]

Figure 2-2 shows that the effect of higher modes \(B_{01}, B_{02}, \ldots\) etc. which induce small amplitude fluctuations in time domain response cannot be neglected. Also, as \(E_0\) is increased, \(B_{10}\) increases more significantly than \(B_{0m}\) and \(B_{1m}\), and hence the effect of the carrier signal is reduced. Substituting the coefficients into Eq. (2.12), obtain

\[
\begin{align*}
\mathbf{u}(\omega_c t, \omega t) = & \frac{1}{2} A_{00} + B_{10} \sin \omega t + \sum_{m=1}^{\infty} B_{0m} \sin m\omega_c t \\
& + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{nm} \sin (m\omega_c t + n\omega t) \quad (2.16)
\end{align*}
\]

It shows that the modulated signal \(u\) is the superposition of the input signal, the carrier signal and the correlated terms between them. If the carrier frequency \(\omega_c\) is larger than system bandwidth, the plant can act as a low-pass filter to attenuate the high frequency component of the modulated signal. As the modulating frequency \(\omega\) is increased, the effect of \(B_{1m}\) becomes more significant, which shows that the effect of the modulator increases.
Figure 2-2  Effect of input amplitude $E_0$ on the Fourier coefficients for the modulated signal.
2.2.3 Computer Simulation Results

To obtain the output frequency spectrum numerically, pressure \( p(t) \) is fed back to form a closed loop system, and the corresponding nonlinear model given by Eqs. (2.5) to (2.7) is used to represent the open loop plant. Various parameters used for the computer simulation are shown in Table 2-1 and the closed loop control block diagram is illustrated in Figure 2-3. The command input \( p_c(t) \) is sinusoidal, defined as \( p_c = p_{c0} + p_{ca} \sin \omega t \), where \( p_{c0} \) is the dc component, \( p_{ca} \) is the magnitude of the sinewave, and \( \omega \) is the modulating frequency. An FFT program was used to transfer the output \( p(t) \) into frequency domain for analysis.

Figure 2-4 compares the normalized magnitude of the output frequency spectra for a number of cases. Figure 2-4(a) shows that the output spectrum contains the input frequency \( \omega \), the carrier frequency of 20 Hz and its higher harmonics which are small. Figure 2-4(b) illustrates that as the input amplitude \( p_{ca} \) is reduced, the effect of the carrier signal is increased. Figure 2-4(c) shows that as the input frequency \( \omega \) is increased, the effect of the side frequencies becomes more significant, and hence the effect of the modulator is increased.

The chamber model given by Eqs. (2.9) to (2.11) is used to study the closed-loop frequency response for a simplified \( u(t) \) expressed as \( E_0 \sin \omega t \). The closed-loop control block diagram is illustrated in Figure 2-3 and the system parametric values used in the computer simulation are shown in Table 2-1. Figure 2-5 indicates that as volume \( V \) is increased, the system bandwidth is reduced. Figure 2-6 illustrates
Table 2-1 System parameters used in computer simulation and experiment

$A_{v1}=2.0 \text{ mm}^2$ (orifice area for the charging port)

$A_{v2}=4.45 \text{ mm}^2$ (orifice area for the discharging port)

$V=538 \text{ mm}^2$ (control volume)

$p_s=0.32 \text{ Mpa}$ (supply pressure)

$T_0=25 \degree \text{C}$ (temperature)

$h=0.035 \text{ Mpa}$ (amplitude of the carrier signal)

$T_s=2 \text{ ms}$ (sampling time)

$T_p=50 \text{ ms}$ (PWM period)

$r_c=0.528$ (critical pressure ratio)

$p_4=0.1 \text{ Mpa}$ (atmosphere pressure)

$C_a_1=0.53$ (coefficient of $C_{d1}$)

$C_a_2=0.231$ (coefficient of $C_{d1}$)

$C_b_1=0.01$ (coefficient of $C_{d2}$)

$C_b_2=0.121$ (coefficient of $C_{d2}$)
Figure 2-3  Proportional feedback
Figure 2-4  Simulation of output frequency spectra. (a) $p_{ca}=0.0345$ Mpa, $\omega = 12.57$ rad/s and $\omega_c = 125.7$ rad/s. (b) $p_{ca}=0.0069$ Mpa, $\omega = 12.57$ rad/s and $\omega_c = 125.7$ rad/s. (c) $p_{ca}=0.0345$ Mpa, $\omega = 31.42$ rad/s and $\omega_c = 125.7$ rad/s.
Figure 2-5  Effect of volume V on system frequency response
Figure 2-6 Effect of the operating pressure $\bar{p}$ on system frequency response
that as the operating pressure \( \bar{p} \) is increased, the system bandwidth is increased as well. For both cases it is expected that the steady state error will appear because the system is a Type 0 system. Therefore, the use of the proportional control alone is not sufficient.

### 2.3 Time Domain Response \( p(t) \)

Since the frequency domain analysis does not provide sufficient information regarding the system performance, an analytical approach is developed here to get time domain behavior. Given the steady state condition as shown in Figure 2-7, where \( p(0) \) represents the initial pressure of each PWM cycle, \( p(\alpha T_p) \) is the maximum pressure when the sign of \( u(t) \) changes, and \( \alpha \) is duty factor, we can expand Eq. (2.10) as

\[
\begin{align*}
\frac{dp}{dt} + \tau_1 p &= k_{a1} A_{v1}, & 0 \leq t < \alpha T_p \\
\frac{dp}{dt'} + \tau_2 p &= -k_{a2} A_{v2}, & 0 \leq t' < (1 - \alpha)T_p
\end{align*}
\]  

(2.17a) (2.17b)

where

\[
\begin{align*}
\tau_1 &= \frac{nRP_s \sqrt{T_0}}{V} A_{v1} \left[ \frac{C_{b1}}{p_s} (\bar{N}_{12} + n_{12p} \bar{p}) - C_{a1} n_{12p} \right] \\
\tau_2 &= \frac{nRP_4 \sqrt{T_0}}{V} A_{v2} \left[ \frac{C_{b2}}{p_s} (\bar{C}_{24} + c_{24p} \bar{p}) + C_{a2} c_{24p} \right] \\
k_{a1} &= \frac{nRP_s \sqrt{T_0}}{V} \left[ \frac{C_{b1}}{p_s} \bar{p}^2 / p_s + C_{a1} (\bar{N}_{12} - n_{12p} \bar{p}) \right]
\end{align*}
\]  

(2.18a) (2.18b) (2.18c)
Figure 2-7  Pressure status in the steady state
\[ k_{a2} = \frac{nRP_4 \sqrt{T_0}}{V} \left[ C_{a2} (\bar{C}_{24} - c_{24p} \bar{p}) - C_{b2} c_{24p} \bar{p}^2 / p_s \right] \]  

(2.18d)

Solve Eqs. (2.17a) and (2.17b) to obtain

\[
p(t) = \frac{k_{a1} A_v}{\tau_1} + \left[ p(0) - \frac{k_{a1} A_v}{\tau_1} \right] e^{-\tau_1 t}, \quad 0 \leq t < \alpha T_p
\]

(2.19a)

\[
p(t') = -\frac{k_{a2} A_v}{\tau_2} + \left[ p(\alpha T_p) + \frac{k_{a2} A_v}{\tau_2} \right] e^{-\tau_2 t'}, 0 \leq t' < (1 - \alpha)T_p
\]

(2.19b)

Since the system is in the steady state, the following conditions can be observed: \( p(t) \big|_{t=0} = p(t') \big|_{t'=(1-\alpha)T_p} \) and \( p(t) \big|_{t=\alpha T_p} = p(t') \big|_{t'=0} \).

Substitute these conditions into Eqs. (2.19a) and (2.19b) to obtain

\[
p(0) = \frac{k_{a2} A_v}{\tau_2} \left( e^{-\tau_1 (1-\alpha)T_p} - 1 \right) + \frac{k_{a1} A_v}{\tau_1} e^{-\tau_2 (1-\alpha)T_p} \left( 1 - e^{-\tau_1 \alpha T_p} \right)
\]

(2.20a)

\[
p(\alpha T_p) = \frac{k_{a2} A_v}{\tau_2} e^{-\tau_1 \alpha T_p} \left( e^{-\tau_2 (1-\alpha)T_p} - 1 \right) + \frac{k_{a1} A_v}{\tau_1} \left( 1 - e^{-\tau_1 \alpha T_p} \right)
\]

(2.20b)

Here, \( p(0) \) and \( p(\alpha T_p) \) are the minimum and maximum pressures, respectively, in the steady state. Accordingly, the pressure ripple \( \Delta p \) can then be expressed as

\[
\Delta p = p(\alpha T_p) - p(0) = \left( \frac{k_{a1} A_v}{\tau_1} + \frac{k_{a2} A_v}{\tau_2} \right) e^{-\tau_2 (1-\alpha)T_p} \left( e^{-\tau_1 \alpha T_p} - 1 \right) \frac{1 - e^{-\tau_1 \alpha T_p} e^{-\tau_2 (1-\alpha)T_p}}{1 - e^{-\tau_1 \alpha T_p} - e^{-\tau_2 (1-\alpha)T_p}}
\]

(2.21)
Also, since the PWM period is very short, the pressure response \( p(t) \) is similar to a triangular wave. Accordingly, the average pressure \( \bar{p} \) can be expressed as

\[
\bar{p} = \frac{p(\alpha T_p) + p(0)}{2}.
\]

\[
\frac{k_{a1} A_{V1}}{\tau_1}(e^{-\tau_2(1-\alpha)T_p} - 1) + \frac{k_{a2} A_{V2}}{\tau_2}(e^{-\tau_1(1-\alpha)T_p} + 1)(1 - e^{-\tau_1\alpha T_p})
\]

Figures 2-8 and 2-9 illustrate the effect of supply pressure \( p_s \), volume \( V \) and carrier frequency \( \omega_c \) on \( \Delta p \) and \( \bar{p} \), respectively. It should be noted that when the system is operated in the PWM mode, pressure ripples \( \Delta p \) always exist. We would like to reduce such ripples to be as small as possible. Figure 2-8 indicates that the effect of \( \omega_c \) on \( \Delta p \) is more significant than the other two parameters. Figure 2-9 shows that \( \bar{p} \) can be affected significantly by varying \( \alpha \). Thus, to achieve zero steady state error, it is necessary to select appropriate \( \alpha \) in the steady state.

### 2.4 CONTROL STRATEGY

It has been shown that proportional control alone is not sufficient because steady state error still exists. The feasibility of implementing a PI control strategy to improve system performance has been studied by Noritsugu(1985). However, such a PI control tends to reduce the system stability. Therefore, we propose a new control strategy as shown in Figure 2-10 in which feedforward
Figure 2-8 Parametric studies for pressure ripple $\Delta p$. (a) Effect of supply pressure $p_s$. (b) Effect of volume $V$. (c) Effect of carrier frequency $\omega_c$. 
Figure 2-9 Parametric studies for average pressure $\bar{p}$. (a) Effect of supply pressure $p_s$. (b) Effect of volume $V$. (c) Effect of carrier frequency $\omega_c$. 
Figure 2-10 Proposed control block diagram
compensation is employed to adjust the duty factor \( \alpha \) and hence reduce the steady state error. Here \( p_f \) is the feedforward pressure. To analyze the closed loop system behavior, we assume that the carrier frequency \( \omega_c \) is sufficiently large such that the effect of the carrier signal is small. Hence the transfer characteristics of the PWM amplifier can be expressed as a pure gain \( k_w \). The chamber pressure \( p(s) \) in \( s \) domain is:

\[
p(s) = \frac{k_o p_c}{s + (k_o + \tau)} + \frac{k_o p_f}{s + (k_o + \tau)}
\]

\[ k_o = k_a k_w \]  \[ k_w = \frac{B_{10}}{E_0} = \frac{A_{v1} + A_{v2}}{2h} \]

Eq. (2.23) shows that the feedforward pressure \( p_f \) does not affect the system stability as it does not appear in the characteristic equation. If the feedforward pressure \( p_f \) is not implemented, the steady state error may exist as the system is a Type zero system. However, if \( p_f \) is employed by the controller as shown in Eq. (2.23), the steady state error can be reduced significantly. The system bandwidth can be increased by decreasing \( h \) and hence increasing \( k_o \) provided that it should not saturate the valve.

### 2.5 EXPERIMENTAL RESULTS

The experimental set-up as shown in Figure 2-11 consists of a pneumatic chamber with adjustable volume \( V \), and a 3-way poppet valve (Parker #T21025) operated by a solenoid which is controlled by
Figure 2-11 Experimental schematic
an electronic relay. A tank is connected to the supply port of the valve to regulate the supply pressure $p_s$. An Intel 86/310 microcomputer with associated A/D (ISBX311) and D/A (ISBX328) converters of resolution 2.44 mv is used as the controller. The pressure $p(t)$ measured with a strain gage type pressure transducer, is sent to a signal conditioner, and then to the A/D. The computer sends the output ($\pm 5$V DC) through D/A to the relay circuit to command the valve motion. The control strategy including the carrier signal is implemented digitally in the FORTRAN language.

Predicted and measured pressure responses during charging and discharging processes are compared shown in Figures 2-12(a) and 2-12(b), respectively. Various parameters used in the computer simulation are given in Table 2-1. Figure 2-12 indicates that the predictions based on the nonlinear model given by Eqs. (2.5) and (2.7) agree with the measurements very well.

Figure 2-13 compares the predicted and measured frequency responses. Measured frequency response is based on sinusoidal excitation technique in which a command pressure $p_c=p_{c_0}+p_{c_1} \sin \omega t$ is input to the system. Here $p_{c_0}$ is the dc component, $p_{c_1}$ is the amplitude of the sinewave and $\omega$ is the input frequency. The pressure response $p(t)$ is then transferred to the frequency domain to obtain the pair of amplitude ratio and phase associated with $\omega$. Such procedures are repeated for different $\omega$ to get the frequency response. Predicted frequency response is based on the linearized model given by Eqs. (2-9) to (2-11). Table 2-1 shows the system parameters used in the computer simulation and $p_{c_0}$ and $p_{c_1}$ used in
Figure 2-12 Comparison between predicted and measured pressure $p(t)$. (a) Charging process. (b) Discharging process
Experimental

Predicted

Figure 2-13 Comparison between predicted and measured frequency response
the experiment are 0.14 Mpa and 0.035 Mpa, respectively. Figure 2-13 indicates that for input frequencies below 4 Hz, the predictions match the measurements. But, if input frequencies are higher than 4 Hz, the measurements have smaller amplitude and more phase lag than the predictions. This is due to system nonlinearity in the measurements. Note that the frequency response is dependent on \( p_{co} \) which affects the operating pressure.

Figure 2-14 illustrates that as the feedforward compensation is implemented, the steady state error in the measured step response is reduced. Figure 2-15 shows that as the pulse period \( T_p \) decreases, the pressure ripple decreases also. However, as \( T_p \) is smaller than 50ms, the pressure response exhibits non-periodic. This is because the PWM frequency exceeds the bandwidth of the valve drive dynamics. Figure 2-15 also indicates that \( T_p \) may affect the steady state error, because the average pressure in the steady state \( \bar{p} \) is dependent on \( T_p \) as shown in Eq. (2.22).

Figure 2-16 indicates that as \( h \) is decreased, the speed of response may be increased slightly. However, if \( h \) is too small as shown in Figure 2-16, the response may become unstable. Figure 2-17 shows that the response is very sensitive to sampling time. In both cases, the pulse period \( T_p = 50 \) ms. We would like \( T_p \) to be as small as possible, but \( T_s \) to be as large as possible. However, due to the limitation of valve bandwidth, \( T_p \) cannot be too small. Also, to insure that for each pulse cycle there is sufficient number of data points for effective pulse-width modulation, \( T_s \) cannot be too large. Therefore, there is tradeoff
Figure 2-14 Effect of the feedforward pressure $p_f$ on measured pressure response
Figure 2-15 Effect of the pulse period $T_p$ on measured pressure response
Figure 2-16 Effect of carrier amplitude h on measured pressure response
Figure 2-17 Effect of sampling period $T_s$ on measured pressure response
between the selection of $T_s$ and $T_p$. The ratio $T_p/T_s$ used in this study is 25.

2.6 CONCLUSION

In this chapter we have presented an analytical method to examine the effect of PWM mode on the performance of a pneumatic valve-chamber system and have proposed a control strategy to control the pressure response. A flow direction dependent first order chamber model incorporating the fundamental frequency component of the PWM mode has been used to study the system frequency response and the governing differential equations have been solved to obtain the steady state behavior. Predictions agree with the measured data extremely well in the frequency domain and in the time domain as well during both charging and discharging processes. Simulation results also indicate that increase in carrier frequency can enhance the system performance significantly. Experimental demonstration of the controller on a pneumatic chamber has been successful in achieving the desired pressure performance. This control strategy can be applied to the pneumatic actuators or manipulators for position or force control.
3.1 INTRODUCTION

Pneumatic actuation is used extensively for numerous position control applications, but only in the open loop control mode where the stroke of the moving part is usually fixed by mechanical stops. Closed loop control is generally not used because of fundamental problems associated with air compressibility, poor damping ability, significant mechanical friction and strong nonlinearities. Nevertheless, it is expected that pneumatic systems under computer control will become more popular to meet the demand of simple and low cost automation. Accordingly, the focus of this chapter is on the development of a new control strategy which will overcome some of the difficulties mentioned above.

The transfer function approach has been used historically to model the dynamics of pneumatically actuated systems [Shearer(1956, 1957), Burrows(1969) and Botting(1970)]. However, such a linearized model is suitable only for the mid-stroke position. Recently, the state space approach has been utilized to extend the linearized model over several operating positions [Scavarda et al.(1987) and Liu et al.(1987)].
But the effect of mechanical friction has been neglected. Since servovalves are very expensive, on-off valve controlled pneumatic systems are typically used in the industry. Using these, full power can be utilized to drive the load, but precise positioning accuracy can not be obtained. Such systems have been studied both analytically [Burrows(1967) and Brown et al.(1972)] and experimentally [Eun et al.(1982, 1984)].

Pulse width modulation offers considerable advantages in the control of dc motors and hydraulic servos as it can reduce the effects of nonlinearities such as hysteresis, threshold, stiction, dead-zone and null shift, and improve system reliability and performance. Such a technique has also been applied to on-off controlled pneumatic systems by Noritsugu(1985, 1987) and Morita(1985). Noritsugu(1987) employed three on-off valves for a particular motion control problem where two valves were under on-off control, and a third valve (automotive injection valve) was under the PWM control when operating in the vicinity of the desired position. Linearized analysis was used to study the effect of various parameters including friction, on the system response. But this scheme is too complex and expensive. Also, he did not report how to determine some of the parameters in the controller such as the selection of switch points to change the valve motion and the position feedback gain. Morita (1985) and Noritsugu (1985) have also implemented the PWM mode in pneumatic manipulators to control pressure and contact force, respectively; both have demonstrated that continuous feedback control
of pressure and grasping force can be obtained with this technique without using any servovalve.

3.2 OBJECTIVES AND SYSTEM DESCRIPTION

Based on the literature review, it is evident that most of the control strategies available for position control are still not satisfactory. Some of the designs are suitable only for low position accuracy; others are so complex that it is difficult to determine control gains or system parameters when applied to a multidegree of freedom system. To overcome such deficiencies typically evident in the literature and to develop accurate pneumatic manipulators, we establish two major objectives. First, a comprehensive nonlinear model is developed to characterize actuator dynamics. It is supported by a computer simulation study. Second, we intend to develop a novel control strategy using on-off valves to achieve high position accuracy. A simplified analytical model is formulated to design the control system. Successful experimental implementation on a single degree of freedom pneumatic robot is demonstrated.

The system, as schematically shown in Figure 3-1, is composed of two on-off valves in conjunction with a linear actuator carrying an external load $m_a$. For control purposes, it is assumed that the displacement $x$, velocity $\dot{x}$, and the chamber pressures $p_1(t)$ and $p_2(t)$ can be measured. To reduce the nonlinearity associated with large pressure variations, we propose to hold the valve 2 open such that $p_2(t)$ remains nearly equal to supply pressure $p_{s2}$. This can be
achieved physically by connecting a large accumulator tank to the air
supply port of valve 2. Valve 1 is under PWM mode control, where a
periodic pulse signal \( v_c(t) \) with duty pulse width \( T_r \) being proportional
to the pressure error \( e_p \) (pressure difference between actual and
command pressure in chamber 1) is input to the solenoid of valve 1.
Figure 3-2(a) illustrates the formulation of the PWM signal, and Figure
3-2(b) shows the design of duty pulse width \( T_r \) vs. \( e_p \). Note that when
\( e_p > e_1 \), valve 1 is open for the whole period, and it is closed when
\( e_p < e_2 \).

3.3 ANALYTICAL MODELS

3.3.1 Nonlinear Model

The following assumptions are made: (i) the ideal gas equation of
state is valid, (ii) the thermofluid processes is isentropic, and (iii) the
bandwidth of the valves is much larger than the carrier frequency. The
last assumption should allow us to determine the dynamics of the
whole system as the combination of chamber dynamics and load
dynamics. Note that the pressure dynamics of chamber 2 is ignored as
\( p_2(t) \approx p_{s2} \). The governing equations for chamber 1 are developed as
follows.

\[
C_p T_1 m_1 - p_1 \frac{d V_1}{dt} + \frac{d Q_1}{dt} = \frac{d}{dt}(C_v \rho_1 V_1 T_1) \tag{3.1}
\]

\[
\rho_1 = p_1 / (R T_1) \tag{3.2}
\]

\[
V_1(t) = V_1(0) + A_{p1} x(t), \quad V_1(0) = V_{10} \tag{3.3}
\]
Figure 3-1 Schematic of the pneumatic system. Here, $p_1$ and $p_2$ are chamber pressures, $A_{p1}$ and $A_{p2}$ are cross-sectional area, $A_{v1}$ and $A_{v2}$ are orifice areas associated with the input and output ports of valve 1, $p_{s1}$ and $p_{s2}$ are supply pressures, $m_a$ is the load and $x$ is the position-actuator displacement.

Figure 3-2 Pulse width modulation (PWM) applied to valve 1. a. Formulation of the PWM signal. b. Duty pulse width $T_r$ vs. pressure error $e_p$. Here $v_e$ is the voltage input to valve 1, $v_c$ is the carrier signal, $h$ is the pulse magnitude of the carrier, $T_p$ is the pulse period, $T_r$ is the pulse width when valve 1 is open, $t$ is time and $e_1$ and $e_2$ are pressure errors.
where $C_p$ is specific heat at constant pressure, $C_v$ is specific heat at constant volume, $V_1$ is volume $1$, $T_1$ is temperature $1$, $\dot{m}_1$ is mass flow rate, $\rho_1$ is density $1$, $R$ is gas constant, $Q_1$ is heat transferred to chamber $1$ and $0$ is initial state. Since the thermofluid is isentropic, we have

$$\frac{d Q_1}{dt} = 0$$  \hspace{1cm} (3.4)

The assumption of isentropic thermofluid process means that the temperature $T_1$ is time varying. However, from a practical point of view, $T_1$ does not vary appreciably from its initial value $T_{10}$ in most cases. Hence, $T_1$ can be regarded as a constant value $T_{10}$. This assumption has been used by several previous investigators [Shearer(1956, 1957), Andersen(1976) and Kawnmnia(1988)]. Also, it seems to be valid in our laboratory experiment. Further, note that the gas constant $R$ can be expressed as

$$R = C_p - C_v$$ \hspace{1cm} (3.5)

Substituting Eqs. (3.2) to (3.5) into Eq. (3.1) yields

$$\dot{p}_1(t)V_{10} + \dot{p}_1(t) A_{p1} x(t) + n p_1(t) A_{p1} \dot{x}(t) - n \dot{m}_1(t) R T_{10} = 0$$ \hspace{1cm} (3.6)

The mass flow rate $\dot{m}_1$ in Eqs. (3.6) can be expressed as follows [Anderson, 1967]:

$$\dot{m}_1 = \begin{cases} C_{d1} A_{v1} p_{s1} N_{12} k / \sqrt{T_{10}} & , \quad v_e(t) > 0 \\ -C_{d2} A_{v2} p_4 C_{24} k / \sqrt{T_{10}} & , \quad v_e(t) < 0 \end{cases}$$ \hspace{1cm} (3.7)
\[
N_{12} = \begin{cases} 
  k_n \left[ \frac{p_1(t)}{p_{sl}} \right]^{2/n} - \left( \frac{p_1(t)}{p_{sl}} \right)^{(n+1)/n} \right]^{1/2}, & p_1 \frac{p_{sl}}{p_{sl}} > r_c \\
  1, & p_1 \frac{p_{sl}}{p_{sl}} < r_c 
\end{cases}, \quad r_c = 0.528 \text{ for air}
\]

\[
C_{24} = k_n \left\{ \left( \frac{p_1(t)}{p_4} \right)^{(n-1)/n} - \left( \frac{p_1(t)}{p_4} \right) \right\}^{1/2}
\]

\[
k = \left( \frac{ng}{R} \right)^{\frac{n+1}{n-1}}
\]

\[
C_{d1} = C_{a1} - C_{b1} \left( \frac{p_1(t)}{p_{sl}} \right), \quad C_{d2} = C_{a2} + C_{b2} \left( \frac{p_1(t)}{p_4} \right)
\]

where \(p_4\) is the atmospheric pressure, \(k_n\) is a constant, 0.2588 for air and \(g\) is the gravitational constant. The discharge coefficients \(C_{d1}\) and \(C_{d2}\) are defined with respect to the input and output ports of valve 1 respectively, and are modeled as a function of pressure ratio \((p_1/p_{sl})\) with coefficients \(C_{a1}, C_{b1}\) and \(C_{a2}, C_{b2}\), respectively. Here \(C_{a1}=0.503, C_{b1}=0.001, C_{a2}=0.231\) and \(C_{b2}=0.121\). It is noted that when \(v_e(t) > 0\), air flows from the supply pressure \(p_{sl}\) port through one orifice of the valve to the actuator; conversely when \(v_e(t)\) is negative, air flows out from the actuator through another orifice of the valve to the atmosphere. The sign of mass flow rate \(m_1\) changes as the valve spool moves from one orifice to another. The actuator-load dynamics is given as follows:

\[
m_a \ddot{x} = F_p(t) - F_d(t) \tag{3.8}
\]

\[
F_p(t) = A_{p1} (p_1(t) - p_4) - A_{p2} (p_2(t) - p_4)
\]
\[
F_d(t) = \begin{cases} 
F_c \text{ sgn}(\dot{x}) & , \dot{x} \neq 0 \\
F_p(t) & , \dot{x} = 0, F_p(t) < F_s \\
F_s \text{ sgn}(F_p(t)) & , \dot{x} = 0, F_p(t) > F_s 
\end{cases} 
\]

where \( F_p(t) \) is the external force acting on the piston, \( F_d(t) \) represents the mechanical friction, \( F_c \) is the Coulomb friction force, and \( F_s \) is the stiction force. This equation shows that the acceleration \( \ddot{x} \) is mainly affected by pressures \( p_1 \) and \( p_2 \). Our approach in keeping \( p_2 \) nearly constant should simplify the control problem.

3.3.2 Simplified Analytical Model

As we can see from Eqs. (3.6) to (3.8), the nonlinear characteristics of this system are primarily due to air compressibility, valve flow rate, and mechanical friction. Here we develop a simplified analytical model to understand the dynamics better and to design the controller. The conventional approach linearizes the system around the equilibrium position assuming that the load displacement \( x(t) \) is small [Shearer 1956, Andersen 1967 and Burrows 1969]. We deviate from this by linearizing the nonlinear equations (3.6) to (3.8) around the command displacement \( x_d \). This should enable us to design the controller such that when the actuator-load is in the vicinity of \( x_d \), the load displacement \( x \) should converge to \( x_d \) quickly. First we examine the characteristics of the PWM mode.
3.3.2.1 Transfer Characteristics of PWM

Our method employs a double Fourier series expansion in two variables [Black, 1957]. Consider that the signal input $v_e(t)$ to the pulse-width modulator is given by $E_0 \sin \omega t$ where $E_0$ is the amplitude and $\omega$ is the modulating frequency. The modulated signal $u(\omega t, \omega_c t)$ of carrier frequency $\omega_c$ can be expressed as follows:

$$u(\omega_c t, \omega t) = \frac{1}{2} A_{00} + \sum_{n=1}^{\infty} [A_{n0} \cos n\omega t + B_{n0} \sin n\omega t]$$

$$+ \sum_{m=1}^{\infty} [A_{0m} \cos m\omega_c t + B_{0m} \sin m\omega_c t]$$

$$+ \sum_{m=1}^{\infty} \sum_{n=\pm 1}^{\infty} [A_{nm} \cos (m\omega_c t + n\omega t) + B_{nm} \sin (m\omega_c t + n\omega t)]$$

(3.9)

Also, since $u(\omega t, \omega_c t)$ is a square wave, it can be given by

$$u(\omega t, \omega_c t) = \begin{cases} 
A_{v1}, & 0 \leq \omega_c t \leq (\pi + \frac{\pi}{h} V_0 \sin \omega t) \\
-A_{v2}, & (\pi + \frac{\pi}{h} V_0 \sin \omega t) \leq \omega_c t \leq 2\pi \end{cases}$$

(3.10)

where $h$ is the amplitude of the carrier, as shown in Figure 3-2(a). When the valve input signal $u(\omega t, \omega_c t)>0$, air flows through the supply port of the valve to the chamber, and when $u(\omega t, \omega_c t)<0$, air from the chamber escapes to the atmosphere. By substituting Eq. (3.9) into Eq. (3.10), all of the coefficients $A_{nm}$ and $B_{nm}$ can be evaluated. All cosine coefficients $A_{nm}$ vanish because the input is a sine function and the modulation function is odd. The dc gain $A_{00}$ and the first few terms of $B_{nm}$ are given below.
\[
B_{10} = \frac{E_0}{2\hbar} (A_{V1} + A_{V2}) \quad B_{n0} = 0, \ n = 2, 3, \ldots , \\
B_{01} = \frac{A_{V1} + A_{V2}}{2\pi^2} \left( \int_0^{2\pi} 1 + \cos \left( \frac{\pi}{\hbar} E_0 \sin \omega t \right) \, d\omega t \right) , \\
B_{02} = \frac{A_{V1} + A_{V2}}{4\pi^2} \left( \int_0^{2\pi} 1 - \cos \left( \frac{2\pi}{\hbar} E_0 \sin \omega t \right) \, d\omega t \right) , \quad (3.11) \\
B_{11} = \frac{A_{V1} + A_{V2}}{2\pi^2} \left( \int_0^{2\pi} \cos \left( \left( \frac{\pi}{\hbar} E_0 \sin \omega t \right) + \omega t \right) \, d\omega t \right) , \ldots
\]

Thus, the modulated signal \( u(\omega t, \omega_c t) \) in Eq. (3.9) is reduced to

\[
u(\omega_c t, \omega t) = \frac{1}{2} A_{00} + B_{10} \sin \omega t + \sum_{m=1}^{\infty} B_{0m} \sin m\omega_c t \\
+ \sum_{m=1}^{\infty} \sum_{n=\pm 1}^{\infty} B_{nm} \sin (m\omega_c t + n\omega t)
\]

(3.12)

The frequency component of the second term corresponds to the frequency component of the input signal. The third term separates into frequencies corresponding to the carrier frequency \( \omega_c \) and to the harmonics of this frequency. The frequency components of the last term correspond to the sum and difference of the modulating frequency \( \omega \) and integral multiples of the carrier frequency \( \omega_c \). Note that as \( \omega_c \) or \( E_0 \) is increased, the effect of the modulation is decreased. For more detail see Section 2.2.2.

### 3.3.2.2 Chamber and Load Dynamics

To simplify the mathematical model, the motion terms \( x, \dot{x} \) and \( \ddot{x} \) are linearized around the command position \( x_d \), but the
pressure $p_1$ is not linearized which should enable us to include the nonlinear mechanical friction. Note that for conventional linearized model, $p_1$ is also linearized. But such a model is valid only if the friction term $F_d(t)$ in Eq. (3.8) can be neglected. Eq. (3.6) is simplified to yield

$$\dot{m}_1 = k_3 \dot{p}_1 + k_4 \dot{x}_p$$

$$k_3 = \frac{V_1}{n R T_{10}}, \quad k_4 = \frac{\bar{p}_1 A_{p1}}{R T_{10}}$$

where $x_p$ represents the perturbation of $x$ around the command position, $V_1$ represents the equilibrium volume, and the operating pressure $\bar{p}_1$ is defined as the average pressure for each cycle of valve motion in the command position. Linearizing Eq. (3.7) for $\dot{m}_1$, we get

$$\dot{m}_1 = -k_1 p_1 + k_2 A_v$$

$$A_v = A_{v1}, \quad v_e(t) > 0 \quad ; \quad A_v = -A_{v2}, \quad v_e(t) < 0$$

$$k_1 = \begin{cases} \frac{p_{s1} k_n}{\sqrt{T_{10}}} A_{v1} \left[ \frac{C_{b1}}{p_{s1}} (\bar{N}_{12} + n_{12p} \bar{p}_1) - C_{a1} n_{12p} \right], & v_e(t) > 0 \\ \frac{p_{s1} k_n}{\sqrt{T_{10}}} A_{v2} \left[ \frac{C_{b2}}{p_{s1}} (\bar{C}_{24} + c_{24p} \bar{p}_1) + C_{a2} c_{24p} \right], & v_e(t) < 0 \end{cases}$$

$$k_2 = \begin{cases} \frac{p_{s1} k_n}{\sqrt{T_{10}}} \left( C_{b1} n_{12p} \bar{p}_1^2 / p_{s1} + C_{a1} (\bar{N}_{12} - n_{12p} \bar{p}_1) \right), & v_e(t) > 0 \\ \frac{p_{s1} k_n}{\sqrt{T_{10}}} \left( C_{a2} (\bar{C}_{24} - c_{24p} \bar{p}_1) - C_{b2} c_{24p} \bar{p}_1^2 / p_{s1} \right), & v_e(t) < 0 \end{cases}$$
Note that the mass flow parameters $N_{12}$ and $C_{24}$ are linearized around the operating pressure $\bar{P}_1$, which is defined as the average pressure for each cycle of valve motion in the steady state. Here $\bar{N}_{12}$ and $\bar{C}_{24}$ denote the operating values, and $n_{12p}$ and $c_{24p}$ represent the partial differential of $N_{12}$ and $C_{24}$ with respect to $p_1$, respectively.

Combining Eqs. (3.13) and (3.14), one has

$$\tau \dot{p}_1 + p_1 = k_a A_{\nu} - k_b \dot{x}_p$$  \hspace{1cm} (3.15)

$$\tau = k_3 / k_1, \quad k_a = k_2 / k_1, \quad k_b = k_4 / k_1$$

Eq. (3.15) represents the pressure dynamics of chamber 1 around the equilibrium position, which is similar to a first order system, but with distinct gains depending on the direction of fluid flow. The equation for load dynamics is given by modifying Eq. (3.8) around the command position $x_d$ as

$$m_a \ddot{x}_p = A_{p1} p_1 - F_t - F_d(t)$$  \hspace{1cm} (3.16)

$$F_t = A_{p1} p_4 + A_{p2} (p_2 - p_4)$$

Eqs. (3.15) and (3.16) represent the simplified model for the system. Note that this model is similar to a linear model except for the friction term $F_d(t)$ and the time-varying gains $k_a(t)$, $k_b(t)$ and $\tau(t)$ which vary according to the sign of $v_c(t)$. However, since the valve is operated in the PWM mode, the high-frequency switching of the pulse-width-modulated wave and the resulting dither effect can
reduce the effect of nonlinear friction and gain variations. To analyze the effect of $k_a(t)$, $k_b(t)$ and $\tau(t)$, first it is necessary to determine $\bar{p}_1$ from steady state considerations.

$$\bar{p}_1 = \left[ A_{p2} p_2 + (A_{p1} - A_{p2}) p_4 + \bar{F}_d \right] / A_{p1}$$

$$\bar{F}_d = \int_t^{t+T_p} F_d(t) \, dt = \int_t^{t+T_p} A_{p1} (p_1(t) - p_4) \, dt - \int_t^{t+T_p} A_{p2} (p_2 - p_4) \, dt$$

where $\bar{F}_d$ represents the average friction acting on the load associated with each cycle of valve motion in the steady state. This equation shows that if $\bar{F}_d$ is a constant, $\bar{p}_1$ is a constant too and does not depend on the command position $x_d$. Since $k_a(t)$, $k_b(t)$ and $\tau(t)$ are very sensitive to the operating pressure $\bar{p}_1$ as evident from Eqs. (3.13) to (3.15), a uniform operating pressure over all equilibrium positions ensures that these parameters do not vary much for different piston positions. But if $\bar{p}_1$ changes with the piston position, then the system parameters and feedback gains would have to be position dependent. For instance consider the case when only one valve is used to control the pressure difference across the chamber. Now $p_2$ would be unsteady and position dependent, and hence $\bar{p}_1 = \bar{P}_1(x)$; also it would be very difficult in this case to determine the operating pressure. Further, note that among the parameters $k_1$, $k_2$, $k_3$ and $k_4$ in Eqs. (3.13) and (3.14), only $k_3$ is affected by the piston location. Accordingly, the major control issue here is how to reduce the effect of $k_3$ and hence of $\tau$ on the pressure dynamics.
3.3.3 Analysis of Displacement Ripple

In the previous chapter, it has been shown that pressure ripple always exists when the valve is operated under the PWM mode. The effect of such a pressure ripple on displacement response is studied in this section using the simplified model of Eqs. (3.15) and (3.16). To simplify the analysis, the friction effect is neglected. Also, the second term on the right hand side of Eq. (3.13) can be neglected as volume variation is small in the steady state. Accordingly, the pressure response $p_1(t)$ in the steady state can be expressed as

$$p_1(t) = \frac{k_{a1}A_v}{\tau_1} + \left[p_1(0) - \frac{k_{a1}A_v}{\tau_1}\right] e^{-\frac{\tau_1}{\tau_1} t}, \quad 0 \leq t < \alpha T_p$$  \hspace{1cm} (3.18a)

$$p_1(t') = -\frac{k_{a2}A_v}{\tau_2} + \left[p_1(\alpha T) + \frac{k_{a2}A_v}{\tau_2}\right] e^{-\frac{\tau_2}{\tau_2} t'}, \quad 0 \leq t' < (1 - \alpha)T_p$$  \hspace{1cm} (3.18b)

where $\alpha_s$ is the duty factor in the steady state and $p_1(0)$ is the initial pressure of any PWM cycle. Since the PWM period $T_p$ is usually very short, Eqs. (3.18a) and (3.18b) can be linearized as

$$p_1(t) = D_1 t + p_1(0) \quad 0 \leq t < \alpha_s T_p$$  \hspace{1cm} (3.19a)

$$p_1(t') = -D_2 t' + p_1(\alpha_s T_p) \quad 0 \leq t' < (1 - \alpha_s)T_p$$  \hspace{1cm} (3.19b)

where

$$D_1 = \left[\frac{k_{a1}A_v}{\tau_1} - p_1(0)\right] \tau_1$$  \hspace{1cm} (3.20a)

$$D_2 = \left[\frac{k_{a2}A_v}{\tau_2} + p_1(\alpha_s T_p)\right] \tau_2$$  \hspace{1cm} (3.20b)
Here \( p_1(0) \) and \( p_1(\alpha_s T_p) \) can be expressed as follows by referring to Eqs. (2.20a) and (2.20b)

\[
p_1(0) = \frac{k_{a2} A_{v2}}{\tau_2} \left( e^{-\tau_2(1-\alpha_s)T_p} - 1 \right) + \frac{k_{a1} A_{v1}}{\tau_1} \left( e^{-\tau_2(1-\alpha_s)T_p} - 1 \right)
\]

\[
p_1(\alpha_s T_p) = \frac{k_{a2} A_{v2}}{\tau_2} \left( e^{-\tau_1 \alpha_s T_p} \right) - \frac{k_{a1} A_{v1}}{\tau_1} \left( e^{-\tau_2(1-\alpha_s)T_p} - 1 \right)
\]

Now, using Eqs. (3.16) and (3.19) to obtain

\[
\dot{x}(t) = E_1 t + \ddot{x}(0) \quad 0 \leq t < \alpha_s T_p
\]

\[
\dot{x}(t') = -E_1 t' + \ddot{x}(\alpha_s T_p) \quad 0 \leq t' < (1 - \alpha_s)T_p
\]

where

\[
E_1 = \frac{A_{p1}}{m_a} D_1
\]

\[
E_2 = \frac{A_{p1}}{m_a} D_2
\]

\[
\ddot{x}(0) = \frac{A_{p1}}{m_a} p_1(0) - \frac{F_t}{m_a}
\]

\[
\ddot{x}(\alpha_s T_p) = \frac{A_{p1}}{m_a} p_1(\alpha_s T_p) - \frac{F_t}{m_a}
\]

Then, integrating Eq. (3.22) to obtain \( \dot{x} \), and then integrating \( \dot{x} \) to obtain \( x \), we have
\[ \dot{x}(t) = \frac{E_1}{2} t^2 + \ddot{x}(0) t + \dot{x}(0) \quad 0 \leq t < \alpha_s T_p \] (3.24a)
\[ \dot{x}(t') = \frac{-E_2}{2} t'^2 + \ddot{x}(\alpha_s T_p) t' + \dot{x}(\alpha_s T_p) \quad 0 \leq t' < (1 - \alpha_s) T_p \] (3.24b)

\[ x(t) = \frac{E_1}{6} t^3 + \ddot{x}(0) \frac{t^2}{2} + \dot{x}(0) t + x(0) \quad 0 \leq t < \alpha_s T_p \] (3.25a)
\[ x(t') = \frac{-E_2}{6} t'^3 + \ddot{x}(\alpha_s T_p) \frac{t'^2}{2} + \ddot{x}(\alpha_s T_p) t' + \ddot{x}(\alpha_s T_p) \quad 0 \leq t' < (1 - \alpha_s) T_p \] (3.25b)

Since the system is in the steady state, the following conditions can be observed:

\[ \int \dddot{x}(t) = 0 \] (3.26a)
\[ \int \ddot{x}(t) = 0 \] (3.26b)
\[ \alpha_s = \frac{D_2}{D_1 + D_2} \] (3.26c)

Using Eqs. (3.24) and (3.25) together with the conditions (3.26a) and (3.26b), we have

\[ \dddot{x}(0) = -\frac{E_1}{2} \left( \frac{E_2}{E_1 + E_2} \right) T_p \] (3.27a)
\[ \dddot{x}(\alpha_s T_p) = \frac{E_1}{2} \left( \frac{E_2}{E_1 + E_2} \right) T_p \] (3.27b)
\[ \dddot{x}(0) = \dddot{x}(\alpha_s T_p) = -\frac{E_1 E_2}{12 \left( E_1 + E_2 \right)} T_p^2 \] (3.27c)
By substituting Eq. (3.27) into Eq. (3.25), we obtain \( x(0) \) and \( x(\alpha_s T_p) \). Thus, the displacement ripple \( \Delta x \), defined as \( x(0) - x(\alpha_s T_p) \), is expressed as

\[
\Delta x = \frac{(E_1 E_2)^2}{12(E_1 + E_2)^3 T_p}
\]

(3.28)

where \( E_1 \) and \( E_2 \) are determined by Eqs. (3.20) and (3.23). However, in Eq. (3.20) the duty factor \( \alpha_s \) is still an unknown. Now \( \alpha_s \) is determined as follows: Using Eqs. (3.20), (3.23) and (3.26c) we have

\[
D_2 = \tau_2 \left( \frac{D_1}{\tau_1} - \frac{k_{a1} A_y}{\tau_1} + \frac{k_{a2} A_y}{\tau_2} + 2 \frac{F_t}{A_{p1}} \right)
\]

(3.29)

Also, using Eqs. (3.20) together with the boundary conditions \( p_1 \big|_{t=\alpha_s T_p} = p_1 \big|_{t'=1-\alpha_s T_p} \), we obtain

\[
D_1 = \left( -\frac{k_{a1} A_y}{\tau_1} - \frac{k_{a2} A_y}{\tau_2} \right) \left/ \left( \alpha_s T_p \frac{\alpha_s}{\tau_2 (1-\alpha_s)} - \frac{1}{\tau_1} \right) \right.
\]

(3.30)

Thus, by solving Eqs. (3.26c), (3.29) and (3.30) numerically, we can obtain \( \alpha_s \).

Eqs. (3.23) and (3.28) clearly show that as the actuator-load parameter \( A_{y1} \) is increased, or \( m_a \) is decreased, the displacement ripple \( \Delta x \) becomes large. The effects of varying the pneumatic parameters \( A_{y1}, P_{s1} \) and \( T_p \) on displacement ripple are shown in Figure 3-3. It indicates that for low frequency operation, \( A_{y1} \) has more
Figure 3-3  Effect of supply pressure $p_{s1}$ and valve orifice areas $A_{v1}$ and $A_{v2}$ on displacement ripple $\Delta p$
effect on $\Delta x$ than $P_{b1}$. Also, to reduce $\Delta x$, the most effective method is to reduce $T_p$.

### 3.4 Proposed Control Strategy

Figure 3-4 illustrates the proposed control block diagram of the system. The closed loop system is composed of two loops: the inner loop involving pressure feedback to control the pressure $p_1$, and the outer loop involving both displacement $x$ and velocity $\dot{x}$ feedback to control $x$. Given the desired displacement $x_d$ and velocity $\dot{x}_d$, the outer loop computes the desired pressure needed to move the load $m_a$ and the inner loop controls the actual pressure response to match the desired one. The inner and outer loops are almost independent except for the $k_b$ term which couples the chamber dynamics and load dynamics. To analyze the closed loop system behavior, we assume that the carrier frequency $\omega_c$ is sufficiently large such that the effect of the carrier signal is small. Hence the transfer characteristic of the pulse-width modulator can be expressed as a pure gain.

#### A. Pressure Control

Figure 3-5 illustrates the inner-loop block diagram. The open-loop dynamics is described as:

$$p_1 = \frac{1}{s + (\tau + k_o)} \left( k_o p_c + k_o p_f - k_b \dot{x}_p \right)$$  \hspace{1cm} (3.31)

where $s$ is the Laplace variable and the expressions for $k_o$ and $k_w$ are shown in Eqs. (2.27) and (2.28), respectively. The first two terms in Eq. (3.28) have been discussed in the previous chapter. The third
Figure 3-4 Proposed control strategy

Figure 3-5 Inner-loop block diagram
term acts as a disturbance, but it does not affect the steady state error if $x_d$ is a step input. Hence, the steady state error can be reduced to zero by selecting an appropriate $p_f$. However, if $x_d$ is a ramp input, a feedforward term proportional to $\dot{x}_d$ has to be added to the controller to compensate the disturbance. We note that as $h$ is decreased, the closed loop time constant can be improved by designing the system such that the response of the inner loop is faster than that of the outer loop; this should ensure that the chamber dynamics does not affect the load dynamics.

b. Position Control: Assume that the inner-loop time constant is much faster than the outer-loop bandwidth, and accordingly the inner-loop dynamics can be neglected. Further, if we let the estimated inertia $m_a'$, as shown in Figure 3-4 to be equal to $m_a$, then the dynamic equation for the error $e(t) = x_d - x_p$ is

$$\ddot{e} + 2\zeta\omega_n\dot{e} + \omega_n^2 e = 0$$

(3.32)

$$k_x = \frac{1}{\sqrt{\omega_n}}, \quad \zeta = \frac{k_v}{2\omega_n}$$

Eq. (3.32) represents the error dynamics for the closed loop system with desired natural frequency $\omega_n$ and damping ratio $\zeta$ for the closed loop system. The poor damping ability in the original open loop system is improved in the closed loop by using the velocity feedback $k_v$.

c. Friction Compensation: Since it is usually difficult to predict the nonlinear friction accurately, a two-stage feedforward force $F_f$ is defined to compensate the friction force as
\[ F_f = \begin{cases} \beta F_c \text{sgn}(e_x) & , |e_x| \geq \delta \\ 0 & , |e_x| < \delta \end{cases} \quad e_x = x_d - x \]  

(3.33)

where \( \beta \) is a scale factor and \( \delta \) is a threshold value. The damping ratio \( \zeta \) is set to 1 (critical value) so that when the actual response \( x \) reaches the command displacement \( x_d \), the velocity \( \dot{x} \) is near zero. Since the feedforward force is switched to zero at this point, the moving parts should stick.

d. Filter Design: The velocity \( \dot{x} \) and pressure \( p_1 \) feedback signals are oscillatory under the PWM mode whenever a periodic voltage is input to the solenoid of valve 1. Since only the average velocity and pressure are to be controlled, digital filters must be used to remove the high frequency components in the data which are primarily due to the periodic spool motion in valve 1. Here, each digital filter is modeled as a first order system with cut-off frequency \( f_s \).

Key features of our proposed control strategy are as follows: (i) since the system is divided into two loops, a better understanding of the process dynamics and control is obtained, (ii) velocity and displacement gains \( k_v \) and \( k_x \) can be used to shift the poles of the load dynamic system to the desired locations, (iii) the amplitude \( h \) of the carrier signal \( u \) can be used to improve the closed loop time constant of the chamber dynamics such that the chamber response \( p_1(t) \) is always faster than the load response \( x(t) \), and (iv) a feedforward force \( F_f \) can be employed to reduce the steady state error \( e \).
3.5 Computer Simulation Results

A computer simulation of the control algorithm described in Section 3.4 is incorporated with the nonlinear model given by Eqs. (3.5) to (3.7) to examine the control strategy. Given the command displacement $x_d$, the controller computes the desired pressure $p_c$ in chamber 1 using the inputs, actual displacement $x$ and velocity $\dot{x}$. The pressure error signal $e_p = p_c - p_1$ is then fed into the PWM mode which drives valve 1. Various parameters and gains used for the computer simulation study are listed below, and the nonlinear mechanical friction $F_d(t)$ is assumed to be given only by the Coulomb friction $F_c$: $m_a=6.5 \text{ kg}$, $p_4=0.101 \text{ Mpa}$, $p_{s1}=0.372 \text{ Mpa}$, $p_{s2}=0.209 \text{ Mpa}$, $A_{p1}=0.0792 \text{ m}^2$, $A_{p2}=0.0596 \text{ m}^2$, $A_{v1}=2.0 \text{ mm}^2$, $A_{v2}=4.45 \text{ mm}^2$, $T_1=25 \text{ °C}$, $h=0.035 \text{ Mpa}$, $p_f=0.01 \text{ Mpa}$, $T_s=2 \text{ ms}$, $\omega_n=10.0 \text{ rad/sec}$, $\zeta=1.0$, and $\delta=2.5 \text{ mm}$.

Figure 3-6 illustrates the effect of $F_c$ on the system response which is similar to that of a second order system except for the appearance of time delay initially, which is due to the fact that at $t=0$, $p_2=p_{s2}$ and $p_{10}=p_4$. Without any compensation force $F_f$, the steady state error $\epsilon$ is large as expected due to the friction. However, as $F_f$ is added to the controller, the steady state error $\epsilon$ is reduced substantially. This plot also indicates that the amplitude of $F_f$ is not necessarily equal to that of $F_c$ as the moving parts may stick at the same position for many values of $F_f$.

3.6 Pneumatic Robot Experiment

A commercial pneumatic robot with five degrees of freedom (Schrader Bellows #MM2) as shown in Figure 3-7 is used for the
Figure 3-6 Effect of feedforward compensation with $F_c=17.78\ N$
1. Gripper adjustment
2. Retract stop
3. Reach stop
4. Upper stop collars
5. Lower stop collars
6. Cushion adjusting ring
7. Lock nut
8. Rotation stops
9. Speed controls

Figure 3-7 Pneumatic robot used for experiments. Only the extension/retract degree of freedom of the arm is used for position control.
experimental study. This robot was originally designed only for the open-loop control mode. Our experimental set-up is shown in Figure 3-8 where only the extension/retract degree of freedom of the robot arm is studied as the example actuator. One port of the arm actuator is connected to a 3-way poppet valve(Parker #T21025) operated by a solenoid which in turn is controlled by an electronic relay. A tank is connected to the other side of the valve to provide the supply pressure $p_{s1}$. The other port of the arm actuator is connected to another tank at pressure $p_{s2}$.

The control routines reside on an Intel 86/310 single board microcomputer which is used with a terminal to control the actuator. A 16 channel A/D module(ISBX311) with a resolution of 2.44 mv is used to get the sensor data from a potentiometer to measure $x$, a velocity pick-up to measure $\dot{x}$ and a strain gage type pressure transducer to measure $p_1$. An output channel from the D/A(ISBX328) converter is used to send the output signal through a relay circuit to command the valve motion. The control strategy including filters and carrier signal algorithms are implemented digitally in the FORTRAN language. Data stored during the operation is transferred to a mainframe computer for further analysis.

Several experiments have been run to demonstrate the feasibility of our proposed control strategy. General gains and parameters are shown below: $m_a=6.5$ kg, $p_{s1}=0.372$ Mpa, $p_{s2}=0.209$ Mpa, $A_{p1}=0.0792$ m$^2$, $A_{p2}=0.0596$ m$^2$, $A_{v1}=2.0$ mm$^2$, $A_{v2}=4.45$ mm$^2$, $T_1=25$ °C, $h=0.035$ Mpa, $p_f=0.01$ Mpa, $T_s=2$ ms, $\omega_n=10.0$ rad/sec, $\zeta=1.0$, $F_f=13.3$ N and $\delta=2.5$ mm.
Figure 3-8. Experimental schematic. The actuator shown here is the extension/retract arm of the pneumatic robot.
Figure 3-9 compares the measured displacement responses $x(t)$ for the conventional on-off control strategy and our approach based on PWM control. The plot shows clearly that the system response is improved significantly under PWM control. Figure 3-10 shows that the steady state error can be reduced substantially by adding feedforward force $F_f$ to the outer-loop of the controller. Note that $F_f$ is tuned manually as it is difficult to determine $F_f$ due to the unmodeled dynamics, variable mechanical friction and system nonlinearity.

Figure 3-11 illustrates that as $T_p$ is reduced, the average mass flow rate of the on-off valve operated under the PWM mode is more similar to that of a servo control, and consequently, the displacement ripple $\Delta x$ is reduced too. On the contrary, if $T_p$ is too large, the system exhibits unstable response. Figure 3-12 shows that for a smaller $k_x$, the system response is slower, but if $k_x$ is too large, significant overshoot is seen.

Figure 3-13 shows that as $k_v$ is reduced, the displacement response $x(t)$ exhibits overshoot. However, if $k_v$ is too large, significant oscillations are noted in the velocity response which results in sluggish $x(t)$. It should be noted that no oscillations are found in $x(t)$ even if $k_v=0$. This should be attributed to the mechanical friction which contributes a certain damping effect to the system. Figure 3-14 illustrates that the position error $e$ is very sensitive to the variation in $h$ as it changes the steady state chamber pressure.

Figure 3-15 compares step response $x(t)$ in dimensionless form for both computer simulation and pneumatic robot experiment. The
Figure 3-9 Comparison of measured position response using two control strategies.
Figure 3-10 Effect of the Feedforward gain $F_f$ on the measured position response
Figure 3-11 Effect of the pulse period $T_p$ on the measured position response
Figure 3-12 Effect of position gain $k_x$ on the measured position response
Figure 3-13 Effect of velocity gain $k_v$ on the measured position response
Figure 3-14 Effect of carrier amplitude $h$ on the measured position response
Figure 3-15 Comparison between measured and predicted $x(t)$ given step command $x_d$. Here $t_s$ is the rise time.
setup time $t_s$ for the measured $x(t)$ is 0.36 s and that for the predicted one is 0.47 s. The discrepancy is because the computer simulation based on the nonlinear model given by Eqs. (3.6) to (3.8) can not predict the actual system precisely. For example, we did not model the valve drive dynamics, also, the friction is not modeled accurately. However, the results shown in Figure 3-15 indicates that the nonlinear model can be used to study the feasibility of the control strategy.

3.7 CONCLUSION

The contributions of this research are as follows: (i) our method of varying only one pressure instead of two to drive the load reduces the system nonlinearity significantly which enables us to design the controller easily, (ii) an improved mathematical model incorporating pneumatic nonlinearities and nonlinear mechanical friction is developed for computer simulation studies and to guide laboratory experiments, and (iii) a method to overcome the nonlinear mechanical friction problem is proposed. Excellent agreement between prediction and measurement demonstrates that our proposed control strategy is indeed useful for an on-off valve controlled pneumatic system as it achieves a high position accuracy without using any mechanical stops. All of components used in the experiment described above are commercially available and hence it is possible to apply our proposed system to industrial automation.
CHAPTER IV
PNEUMATIC PRESSURE AND POSITION
CONTROL USING LEARNING LAW

4.1 INTRODUCTION

It has been shown in Chapter 3 that the proposed control system is effective in achieving the desired end-point position accuracy without using any mechanical stops in the actuator. However, it is difficult to determine the feedforward compensation due to the unmodeled dynamics, variable mechanical friction and system nonlinearity. Also, a fixed-gain controller design is valid only near the operating point. Whenever there is variation in the operating point, a new value of feedforward compensation must be chosen to obtain the same or similar performance. Therefore, the development of the feedforward compensation which is effective for all operating points becomes an important issue and this will be addressed next.

One of the potential methods which may overcome the above mentioned problem is adaptive control. In this scheme a system model with unknown parameters is typically proposed and then such parameters are predicted using an on-line identification algorithm such as recursive least squares [Goodwin et al. (1983), Ljung et al. (1983)]. However, this method is not suitable for our case due to the following reasons even though well known system identification
techniques and adaptive control theories are available [Craig(1984), Harokopos(1986)]. First, it assumes that the unknown parameters change slowly compared with the rate at which they can be estimated. But, in our system the mechanical friction is a nonlinear function of load velocity and possibly load position, and hence its variation may be rapid and large. Second, due to the effect of unmodeled dynamics such as valve mechanism, non-uniform fluid properties at a given instant through the control volume and mechanical friction, it is difficult to design feedforward compensation based on the known, but limited system parameters.

A more suitable method of improving our proposed control strategy is learning control [Togai et al.(1984), Craig(1984) and Harokopos(1986)]. Learning systems have the ability to improve their performance using previous experience provided the system is repetitive. This scheme is very attractive especially when it is difficult to model the system parametrically, which is the case here. Another benefit comes from its ease in implementation especially in small computers. Learning control is applicable only for periodic actuation problems which, however, are typical for most pneumatic systems used in industry. Accordingly, we apply this approach to our system to analyze its feasibility.

4.2 Literature Review

Recently several learning control systems for mechanical manipulators have been proposed. Craig(1984) proposed a learning scheme for the case of repeated trials of a robot and studied its
convergence. The primary benefit of this approach is that it allows compensation for friction and other effects which are difficult to model. Togai et al. (1984, 1986) have proposed discrete learning control algorithms for both time-invariant and time-varying dynamic systems and obtained the conditions for the convergence of system states. An optimization scheme has also been developed to identify the optimal learning gains. Morita et al. (1986) have modified Togai's (1984) algorithm for a sparse data case in which the error signal can be measured only at relatively few points along the path. This work has shown that the position error due to the effects of sparse data can be reduced either by shifting the location of the measurement points slightly in each cycle, or by using forward/backward estimation to predict the points of interest. Subamanian (1988) has studied by computer simulation various learning techniques applied to a two-link manipulator, and compared the performance of various learning strategies under different operating conditions. He observed that drive train resonance may be excited by the learning process even though the learning filter was designed to be stable. Also, by formulating the design of the learning filter as an optimization problem, learning filter coefficients have been derived for reducing position errors.

Arimoto, Kawamura and their colleagues (1984, 1985a, 1985b, 1986) introduced several learning control algorithms which have been successfully applied to industrial robots. They have proposed an iterative learning process which improves the next robot cycle by using the previous cyclic data. Three types of learning laws have been
proposed in which the control inputs were either modified by the acceleration, velocity or position error pattern. The convergence conditions for the motion trajectory for each control law are also given. Further, they have suggested a two-stage learning process for manipulators where the drive train dynamics involve flexible modes. First, the learning process is employed to realize the desired motion for the rigid link subsystem and a second process is then used to control the actuator torques. The convergence of this control scheme to the desired motion trajectory has been assured under some reasonable conditions.

Takagami et al. (1985) have proposed a learning method to isolate vibration from the floor for an air cushion system consisting of an isolated table and an air-spring regulated by a hydraulic actuator. For the periodic vibration isolation case, the control signal to the hydraulic actuator is a combination of several sinusoidal signals in which the phase and amplitude of each harmonic are chosen separately by the learning process. For each trial, the computer converts a pair of phase and amplitude values to a digitized sine wave using a look-up table and then sends it to drive the actuator using a digital to analog converter. The phase and amplitude of each harmonic is modified separately in a step by step manner until the motion of the isolated table is minimal. The corresponding digitized sine wave is stored in computer memory. The optimal control signal is then obtained by synthesizing all of the digitized sine waves over the frequency range of interest. This learning method is promising, but it is not suitable for our position control application as it can only
predict the sinusoidal components of the input, but not the dc component of the input. Also, the proper selection of the number of harmonics involved in the learning process may be a problem. If the number of harmonics is not adequate, position accuracy may be poor, and if too many harmonics are chosen, here the rate of convergence will be too slow.

4.3 PROBLEM FORMULATION

Based on the literature review, it is evident that the application of learning control scheme for pneumatically actuated systems has not been studied. A few learning algorithms are available for mechanical manipulators which do not consider the dynamics of the actuation systems [Craig(1984), Togai et al.(1984, 1986)]; others are suitable only for simple driving systems such as an electric motor modeled as a first order system [Subamanian(1988)]. To fill the void in the literature and to improve pneumatic system performance, we establish two objectives in this area. First, we examine the feasibility of using available learning laws for nonlinear pneumatic system problems. Computer simulation and laboratory experimental studies are included. Second, an optimization scheme based on the linear pneumatic system models is developed to choose the learning gains appropriately. Computer simulation based on the nonlinear mathematical model is carried out to verify the feasibility of this optimization scheme.

An important feature of the learning controller design should be to ensure the complete use of all available knowledge of the system dynamics. We wish to mathematically model a major segment of the
system, and then use the learning process to account for what can not be modeled. Accordingly, our learning control algorithms are composed of a feedback control scheme, a feedforward compensation and a learning law as shown in Figures 4-1 and 4-2 for pressure and position controls, respectively. For both control algorithms, the command pressure $p_c$ and position $x_d$ are periodic. For the pressure control shown in Figure 4-1, the feedback scheme employs a proportional control scheme using pressure feedback. The feedforward pressure $\hat{P}_f$ is adaptively learned by the learning law which specifies how to construct the next feedforward term based on the steady state error $\bar{e}_p$ and the feedforward pressure $\hat{P}_f$ from the last trial. For the position control shown in Figure 4-2, the feedback scheme consists of position, velocity and pressure feedback based on the known structure and parameters. The learning law is employed to adjust the feedforward force $\hat{F}_f$ using the steady state position error $\bar{e}_x$ and $\hat{F}_f$ from the last trial.

In subsequent sections, we will develop the pressure and position learning algorithms for our system. The convergence criterion of these learning algorithms are then outlined based on the linear dynamic models, and the mathematical conditions are established. Also, an optimization scheme is applied to the proposed learning system to identify the optimal learning gains. Finally, computer simulation and experimental results are presented to verify the feasibility of proposed algorithms. Also, a computer simulation study based on the nonlinear model is carried out to evaluate the effectiveness of the optimization scheme.
Feedback Loop

Command Pressure \( P_c \)

Controller

Chamber Dynamics

\( p \)

\( \beta_f \)

Memory

\( e_p \)

Learning Control

Feedback Loop

Command Position \( x_d \)

Controller

Actuator Dynamics

\( x \)

\( \dot{x} \)

\( p_1 \)

Learning Control

Figure 4-1 Pressure learning control scheme

Figure 4-2 Position learning control scheme
4.4 PRESSURE CONTROL

4.4.1 Control Strategy

A pressure iterative rule is proposed which updates the feedforward pressure $\hat{P}_f$ based on the previous (j-1th) cyclic data and reduces the average pressure error $\bar{e}_p$ in the next (jth) steady state operation.

$$\hat{P}_f(j) = \hat{P}_f(j - 1) + \Gamma_p \bar{e}_p (j - 1)$$  \hspace{1cm} (4.1)

$$\bar{e}_p (j - 1) = \frac{1}{k} \left[ \sum_{i=1}^{k} (p_c(i) - p(i)) \right]_{j - 1}$$  \hspace{1cm} (4.2)

where $p_c$ is the command pressure which is a periodic pulse function, $\Gamma_p$ is a pressure learning gain which dominates the rate of convergence of the pressure error $\bar{e}_p$ and $k$ is number of points used for averaging. In our strategy, at the jth pulse period, $\hat{P}_f(j - 1)$ is updated in such a way that the present $\hat{P}_f(j)$ at the jth pulse period is modified by being added a $\Gamma_p$ fraction of the averaged pressure error $\bar{e}_p (j - 1)$. This strategy is similar to the proportional control scheme, but here the pressure error data is available only once per command period.

We wish to design a system in which the feedforward pressure $\hat{P}_f$ converges to a value with minimum steady state error. The linearized model shown in Figure 4-3 is used for the analytical study. Also, it is assumed that the carrier frequency $\omega_c$ is sufficiently large such that the transfer characteristics of the pulse-width modulator can
Figure 4-3  Pressure learning control block diagram
be expressed as a pure gain \( k_w \). Accordingly, from Figure 4-3 we can write the system dynamics in error space \( e_p(s) \) as

\[
e_p(s) = p_c - p_1 = \frac{(s + \tau)p_c - k_o \dot{p}_f}{s + \tau + k_o} \tag{4.3}
\]

Now using the final-value theorem we obtain the steady state error \( \bar{e}_p \) for a step input as

\[
\bar{e}_p = \lim_{s \to 0} s e_p = \frac{\tau p_c - k_o \dot{p}_f}{\tau + k_o} \tag{4.4}
\]

This equation shows that \( \bar{e}_p \to 0 \) if \( \dot{p}_f \to \tau p_c / k_o \). However due to the system nonlinearity, \( k_o \) and \( \tau \) are dependent on the operating point and can not be predicted precisely. Therefore it is impractical to determine \( \dot{p}_f \) using \( k_o \) and \( \tau \). Alternatively, we can express \( \dot{p}_f(j) \) as a function of the pressure error \( \bar{e}_p(k) \), \( k = 1, 2, \ldots, j - 1 \) from Eq. (4.1).

\[
\dot{p}_f(j) = \dot{p}_f(1) + \Gamma_p \sum_{k=1}^{n-1} \bar{e}_p(k) \tag{4.5}
\]

where \( \dot{p}_f(1) \) is the initial guess of \( \dot{p}_f(j) \), which can be taken to be zero. Using Eqs. (4.4) and (4.5) we can express \( \bar{e}_p(k), k = 1, 2, \ldots, j - 1 \) as follows

\[
\bar{e}_p(1) = \frac{\tau p_c - k_o \dot{p}_f(1)}{\tau + k_o} = \frac{\tau p_c}{\tau + k_o} \tag{4.6}
\]

\[
\bar{e}_p(2) = \frac{1}{\tau + k_o} [\tau p_c - k_o \Gamma_p \bar{e}_p(1)] = \frac{\tau p_c}{\tau + k_o} \left( 1 - \frac{k_o}{\tau + k_o} \Gamma_p \right) \tag{4.7}
\]
Eq. (4.9) clearly shows that as \( j \to \infty \), it is required that

\[
\left| 1 - \frac{k_o \Gamma_p}{\tau + k_o} \right| < 1 \quad \text{for} \quad \lim_{j \to \infty} \bar{e}_p(j) \to 0. \tag{4.10}
\]

Eq. (4.10) provides the condition of convergence for the proposed learning algorithm. Hence the learning gain \( \Gamma_p \) in Eq. (4.1) should be bounded as shown below in order to ensure that the feedforward pressure \( \hat{p}_f(j) \) converges to a steady state value with a minimum possible error.

\[
0 < \Gamma_p < \frac{2(\tau + k_0)}{k_0} \tag{4.11}
\]

The next issue is how to choose an optimal learning gain \( \Gamma_p^* \) which optimizes the error response. We define the one-dimensional mean-square steady state error \( J_p \) as

\[
J_p = \sum_{j=1}^{\infty} q(j) \left[ \bar{e}_p(j) \right]^2 \tag{4.12}
\]
where \( q \) is a least square coefficient. If \( q=1 \), \( J_p \) reduces to an integral square-error (ISE) criterion \([\text{Ogata}(1970)]\). Substituting Eq. (4.9) into Eq. (4.12), we obtain

\[
J_p = \sum_{j=1}^{\infty} q(j) \left[ \frac{\tau_p}{\tau+k_o} \right]^2 \left[ 1 - \frac{k_o}{\tau+k_o} \Gamma_p \right]^{2(j-1)}
\]  
(4.13)

To obtain minimum \( J_p \), we let \( \frac{dJ_p}{d\Gamma_p} = 0 \) which yields the optimal learning gain \( \Gamma_p^* \) as

\[
\Gamma_p^* = \frac{\tau+k_o}{k_o}
\]  
(4.14)

This result is valid irrespective of the value of the weighting coefficient \( q \). Also, by substituting Eq. (4.14) in Eqs. (4.6) to (4.9) we find that all steady state errors disappear except for \( \bar{e}_p(1) \). Therefore, if we choose \( \Gamma_p^* \) according to Eq. (4.14), the steady state error will be eliminated from the second (\( j=2 \)) learning cycle. It should however be pointed out that the results developed in this section are strictly based on a linear pneumatic model assumption. Since the actual system is nonlinear, computer simulation must be used to verify such results.

4.4.2 Computer Simulation and Experimental Results

The learning algorithm presented in Section 4.4.1 is used in conjunction with the nonlinear valve-chamber model given by Eqs. (2.5) to (2.7) to study the feasibility of the learning strategy. Various
system parameters used for the computer simulation study are listed below: \( A_v1 = 2.0 \text{ mm}^2 \), \( A_v2 = 4.45 \text{ mm}^2 \), \( V = 538 \text{ mm}^2 \), \( p_s = 0.32 \text{ Mpa} \), \( T_0 = 25 ^\circ \text{C} \), \( h = 0.035 \text{ Mpa} \), \( T_s = 2 \text{ ms} \), \( T_p = 50 \text{ ms} \) and \( t_c = 1 \text{ s} \). The experimental setup as shown in Figure 2-11 and described in Section 2.5 is used for the experimental study.

Figure 4-4 compares predicted and measured time histories of pressure response \( p(t) \) without learning control where \( t_c \) is the learning period. Corresponding pressure ripples \( \Delta p \) are compared in Figure 4-5 where \( i \) is pulse cycle of PWM. For both cases, the feedforward pressures \( \hat{p}_f \) are carefully tuned to minimize \( \bar{e}_p \). It is observed that the predicted \( p(t) \) is more repetitive than the measured \( p(t) \). Also, the predicted \( \Delta p \) is more uniform than the measured one. This could be attributed to the fact that the actual system exhibits more uncertainty than the model in the following manner. First, the nonlinearity associated with the valve mechanism such as partial opening of the valve port and the time delay is significant, but it is neglected in the model. Second, the valve is powered by a relay operated by 60 Hz a.c. voltage which is aliased by the 20 Hz PWM signal in the relay circuits. This is not considered in the model. Due to this aliasing problem, the actual air flow rate is dependent on the PWM cycle even though the pulse width \( T_r \) remains the same, and hence it results in pressure ripple fluctuations. We believe that the pressure response could be improved substantially if a better valve with faster and better dynamic response is used.

Now the learning control is applied. Since the learning control algorithm does not affect the system's response during the first cycle,
Figure 4-4  Comparison of predicted and measured pressure responses $p(t)$ without learning control for $p_c=0.137$ Mpa. (a) Predicted $p(t)$. (b) Measured $p(t)$. 
Figure 4-5  Comparison of predicted and measured pressure ripple $\Delta p$ without learning control for $p_c=0.137$ Mpa.
this cycle will show the system behavior without learning. Figure 4-6
compares the predicted and measured pressure responses p(t) and
the corresponding pressure errors $\tilde{e}_p$ are compared in Figure 4-7,
where j is the learning trial. Figure 4-6 shows that during the first
cycle the pressure response p(t) deviates substantially from the
command $p_c$. The learning controller clearly forces p(t) to converge to
$p_c$ rapidly as early as the second trial. Figure 4-7 illustrates that the
predicted $\tilde{e}_p$ learning curve is smoother than the measured one.
Again, this is because the actual system exhibits more non-periodic
response.

Figures 4-8 depicts the measured pressure responses with
learning control for three operating conditions in which either $p_c$ or
$\bar{x}$ is varied and the corresponding $\tilde{e}_p$ and $\dot{p}_f$ learning curves are
compared in Figure 4-9. These results indicate that the learning
algorithm is effective at all operating conditions without any manual
adjustment of the control gains. Also, Figure 4-9(b) shows that $\dot{p}_f$
converges to a different value as the operating condition changes.
Thus, it is practical to implement learning control as it will be difficult
for any other control algorithms to correctly identify $\dot{p}_f$. Further, note
that for all three cases p(t) converges to $p_c$ almost within two trials.
Therefore, to simplify the learning process, we may use the first trial
to estimate $\dot{p}_f$, then use this $\dot{p}_f$ during the later cycles.

Finally, Figure 4-10 shows the effect of learning gain $\Gamma_p$ using
the computer simulation of the nonlinear mathematical model. The
rate of convergence of $\tilde{e}_p$ and $\dot{p}_f$ is slow at $\Gamma_p$=0.5. However, as $\Gamma_p$
is increased to 2.5, the learning algorithm becomes unstable. It is also
Figure 4-6 Comparison of predicted and measured pressure response $p(t)$ with learning control for $p_c=0.137$ Mpa and $\Gamma_p=1.1$. (a) Predicted $p(t)$, (b) Measured $p(t)$. 
Figure 4-7 Comparison of predicted and measured pressure error $\tilde{e}_p$ with learning control for $p_c=0.137$ Mpa and $\Gamma_p=1.1$. 
Figure 4-8  Measured $p(t)$ with learning control for three operating conditions. (a) $p_c=0.137$ Mpa and $\bar{x}=101.6$ mm. (b) $p_c=0.172$ Mpa and $\bar{x}=101.6$ mm. (c) $p_c=0.137$ Mpa and $\bar{x}=50.8$ mm. Here $\Gamma_p=1.1$ for all three cases.
Figure 4-9 Comparison of measured learning curves for three operating conditions. (a) Learning curve for $\bar{e}_p$. (b) Learning curve for $\rho_f$. Here $\Gamma_p = 1.1$. 
Figure 4-10 Effects of varying $\Gamma_p$ on pressure learning response. (a) Learning curve for $\bar{e}_p$. (b) Learning curve for $\bar{P}_f$. 
observed that if $\Gamma_p$ is chosen appropriately, $\bar{e}_p$ could converge to the desired one within two learning trials. To verify the feasibility of the optimization scheme developed in the previous section, we substitute $\tau$ and $k_o$ into Eqs. (4.11) and (4.14) and find that $\Gamma_p$ should be bounded between $0 < \Gamma_p < 2.35$ for stability and $\Gamma_p^*$ should be 1.17 for the optimal rate of convergence. These results are comparable with those found by the computer simulation. This clearly demonstrates that the convergence criterion Eq. (4.11) and the optimal gain $\Gamma_p^*$ in Eq. (4.14) are indeed useful to determine the system stability and to achieve fast convergence of errors.

4.5 POSITION CONTROL

4.5.1 Control Strategy

We consider the system shown in Figure 4-11 which includes chamber dynamics and load dynamics. We can write the system dynamics in error space $e_x$ as

$$m_a e_x + m_a (\tau + k_o) \dot{e}_x + (k_o m_a k_v + A_{pl} k_b) \ddot{e}_x + k_o m_a k_x e_x = (F_t + F_d) - k_o (F_t + F_f) - A_{pl} k_o p_f$$

(4.15)

The position error $\bar{e}_x$ in the steady state condition given a step input is

$$\bar{e}_x = \frac{(F_t + F_d) - k_o (F_t + F_f) - A_{pl} k_o p_f}{k_o m_a k_x}$$

(4.16)
Figure 4-11 Position learning control block diagram
This equation shows that we can choose either $\hat{F}_f$ or $p_f$ as the learning variable to reduce $\bar{e}_x$. However, since $p_f$ can also affect the pressure response, we prefer to select $\hat{F}_f$ alone as the learning variable and keep $p_f$ as a constant. The position control algorithm as shown in Figure 4-11 is similar to that of Figure 3-3 except that the learning law is now employed to adjust the feedforward force $\hat{F}_f$. The design of the servo loop including pressure $p_1$, velocity $\dot{x}$ and position $x$ feedbacks is not changed at all. The learning law is an iterative rule in which $\hat{F}_f$ at the $j$th pulse period is updated by modifying $\hat{F}_f(j-1)$ through the addition of a fraction $\Gamma_x$ of the steady state position error $e_x(j-1)$.

$$\hat{F}_f(j) = \hat{F}_f(j-1) + \Gamma_x \bar{e}_x(j-1)$$  \hspace{1cm} (4.17)

$$\bar{e}_x(j-1) = \frac{1}{k} \sum_{i=1}^{k} (x_d - x(i)) \big|_{j-1}$$  \hspace{1cm} (4.18)

where $\Gamma_x$ is a position learning gain which dominates the rate of convergence of errors. Instead of $e_x(j-1)$, $\bar{e}_x(j-1)$ is used in Eq. (4.17) to reduce the effect of noise or non-periodic effect. The convergence criterion for this learning algorithm is similar to that given in Section 4.4.1. By comparing Eq. (4.16) and (4.4), we can regard $(F_t + F_d - k_0 F_t - A p_1 k_0 p_f)$ as $\tau p_c$. $\hat{F}_f$ as $\hat{p}_f$ and $k_0 m_a k_x$ as $(\tau + k_0)$, and then use Eq. (4.11) to get the following condition for convergence.

$$0 < \Gamma_x < 2 m_a k_x \quad \text{such that} \quad \lim_{j \to \infty} \bar{e}_x(j) = 0$$  \hspace{1cm} (4.19)
Also, similar to the development of the optimal pressure learning gain $\Gamma_p$ in Section 4.5.1, we obtain the optimal position learning gain $\Gamma_x$ as $\Gamma_x = m_a k_x$ which minimizes the mean square-error criterion $J_x = \sum_{j=1}^{\infty} q(j) [\bar{e}_x(j)]^2$. Again, it should be noted that these results are based on the linearized model description of the pneumatic system.

**4.5.2 Computer Simulation and Experimental Results**

The learning algorithm proposed in Section 4.5.1 is used in conjunction with the nonlinear mathematical model of Section 3.3.1 and the fixed-gain controller of Figure 4-11 to study the feasibility of the learning strategy. Various parameters and gains used for the computer simulation are listed below: $m_a=6.5$ Kg, $p_4=0.101$ Mpa, $p_{s1}=0.372$ Mpa, $p_{s2}=0.209$ Mpa, $A_{p1}=0.0792$ m$^2$, $A_{p2}=0.0596$ m$^2$, $A_{v1}=2.0$ mm$^2$, $A_{v2}=4.45$ mm$^2$, $T_1=25$ °C, $h=0.035$ Mpa, $p_l=0.01$ Mpa, $T_s=2$ ms, $T_p=50$ ms, $t_c=1.5$ s, $\omega_n=10.0$ rad/sec, $\zeta=1.0$, and $\delta=2.5$ mm. The experimental setup is the same as shown in Figure 3-6.

First we compare time histories of the predicted and measured displacement responses $x(t)$ without learning control in Figure 4-12(a). Corresponding position errors $\bar{e}_x$ are compared in Figure 4-12(b). It is observed that the predicted and measured $x(t)$ agree very well, indicating that the simulation model can in fact be used to study the feasibility of the control strategy. Also, the results indicate that $x(t)$ does not display any oscillations. This represents a significant improvement compared with conventional on-off control results of
Figure 4-12 Comparison of predicted and measured position response \( x(t) \) without learning for \( x_d = 101.6 \) mm. (a) Time histories of \( x(t) \) (b) Learning curves for \( \bar{e}_x \).
Chapter 3. But note that the transient response is not quite smooth as the load velocity \( \dot{x}(t) \) oscillates under the PWM mode control. Such oscillations could be reduced if the carrier frequency \( \omega_c \) is increased. Further, from Figure 4-12(b) we note that the experimental repeatability of the position is about 0.7 mm which is much worse than predicted by the model. One of the factors responsible for repeatability error is the nonlinear mechanical friction.

To study the significance of mechanical friction, preliminary experiments were conducted to measure the stiction and Coulomb friction forces associated with the actuator for the following operating conditions: (i) \( \bar{x}=50.8 \) mm and \( p_2=0.0 \) Mpa. (ii) \( \bar{x}=50.8 \) mm and \( p_2=0.172 \) Mpa and (iii) \( \bar{x}=101.6 \) mm and \( p_2=0.0 \) Mpa. Each test condition was repeated 5 times and \( p_1 \) was either increased from a value lower than \( p_2 \) or decreased from a value higher than \( p_2 \). Stiction force was determined by keeping a constant pressure \( p_2 \) in chamber 2 as shown in Figure 3-1 and then slowly increasing or decreasing \( p_1 \) in chamber 1 until the arm starts to move. Coulomb friction force was determined by giving an extra impact force to overcome the stiction while slowly increasing or decreasing \( p_1 \) until the actuator continues to move. Experimental results as illustrated in Figure 4-13 show that stiction is very significant as the stiction force may be up to three times the Coulomb friction force. Also, note that the stiction is larger when \( p_1 \) is increased from a lower level than when it is decreased from a higher level. Further, when the operating pressure is increased, or when the actuator is moved backward, the stiction tends to increase. Accordingly, we conclude that the stiction affects the
Figure 4-13 Measured stiction and Coulomb friction forces without learning control for three operating conditions. (a) $\bar{x} = 50.8$ mm and $p_2 = 0.0$ Mpa. (b) $\bar{x} = 50.8$ mm and $p_2 = 0.172$ Mpa. (c) $\bar{x} = 101.6$ mm and $p_2 = 0.0$ Mpa. Here $N_f$ is trial of friction test.
system repeatability more significantly than the Coulomb dry friction and also varies with the operating conditions.

Other factors such as valve dynamics and the switching condition as given by Eq. (3.32) for the feedforward force $F_f$ may also affect system repeatability. The effect of valve dynamics has already been discussed in Section 4.4.2. The threshold value $\delta$ as shown in Eq. (3.32) is constant for both computer simulation and experiment. However, it may be necessary to design a friction-dependent switching point to improve the system repeatability.

Now we employ learning control and compare predicted and measured $x(t)$ in Figure 4-14(a). Corresponding $\bar{e}_x$ learning curves are compared in Figure 4-14(b). The measured $x(t)$ has slower convergence rate than the predicted $x(t)$ because the stiction effect has not been considered in the simulation model. Also, the repeatability of the measured $x(t)$ was found to be 0.8 mm which is slightly larger than that shown in Figure 4-12 where without any learning algorithm. This indicates that non-periodic dynamics may deteriorate the performance of the learning control slightly.

Figure 4-15 compares three measured $x(t)$ with learning control and the corresponding $\bar{e}_x$ and $\hat{F}_f$ learning curves are compared in Figure 4-16. In general, $\bar{e}_x$ decreases rapidly during the first several trials, but oscillates around the zero while the learning process continues. The non-periodic effect of the system due to mechanical friction and non-uniform pressure response is so significant that system repeatability can not be improved any further even if this learning algorithm were to be implemented. This again demonstrates
Figure 4-14 Comparison of predicted and measured position response $x(t)$ with learning control for $x_d=101.6$ and $\Gamma_x=0.7$. (a) Time histories of $x(t)$. (b) Learning curves for $\bar{e}_x$. 
Figure 4-15 Measured $x(t)$ with learning control for three command $x_d$. (a) $x_d=101.6$ mm. (b) $x_d=76.2$ mm. (c) $x_d=63.5$ mm. Here $\Gamma_x=0.7$ for all cases.
Figure 4-16 Comparison of measured learning curves for three operating conditions (a) Learning curves for $\bar{e}_x$. (b) Learning curves for $\hat{F}_f$. 
that such a learning algorithm can only reduce the errors due to repeated system dynamics, but not the errors associated with non-periodic system dynamics.

The computer simulation model based on the nonlinear mathematical description is now used to study the effect of the following gains and parameters: position learning gain $\Gamma_x$, velocity learning gain $\Gamma_v$, load inertia $m_a$ and pressure disturbance or noise in the actuator. Figure 4-17 shows the effect of varying the position learning gain $\Gamma_x$. As $\Gamma_x$ is increased, the convergence speed of the position error $\tilde{e}_x$ is faster. But, if $\Gamma_x$ is increased too much, say to 1.57, the convergence speed becomes worse due to the significant overshoot in the feedforward force. To verify the feasibility of the optimization scheme based on the mathematical linear model we substitute $k_x$ and $m_a$ into Eq. (4.17) and find that $\Gamma_x$ should be bounded between 0 and 1.47 for stability. This calculation is slightly more conservative than the computer simulation result. The discrepancy may be attributed to the "sticking effect" in the nonlinear simulation model which increases the system stability, but it obviously can not be predicted using the linear model. However the optimal learning gain $\Gamma_x^*$ is computed to be 0.73 which is comparable with the simulation result. Thus the proposed scheme can be used to estimate the learning gain for fast convergence of errors.

Next an additional derivative term $\Gamma_v [\tilde{e}_x(j-1) - \tilde{e}_x(j-2)]$ is added to the learning law Eq. (4.17) to study Arimoto's(1985a) algorithm in which the feedforward compensation is modified by position and velocity patterns. Figure 4-18 shows that without the
Figure 4-17 Effects of varying $\Gamma_x$ on position learning response. (a) Learning curves for $\bar{e}_x$. (b) Learning curves for $\hat{F}_f$. 
Figure 4-18 Effects of varying $\Gamma_v$ on position learning response. (a) Learning curves for $\bar{e}_x$. (b) Learning curves for $\bar{F}_f$. 
derivative term, \( \bar{e}_x \) displays undershoot and oscillations for \( \Gamma_x=1.05 \). However, this situation is not improved at all when the derivative term is implemented. Therefore, there is no need to add a derivative term to the learning law given by Eq. (4.17).

Figure 4-19 illustrates that the learning algorithm performs well for a load inertia \( m_a \) variation of \( \pm 10\% \). This result clearly indicates that no adjustment of position or velocity gain is necessary for a small variation in inertia \( m_a \) as the associated discrepancy can be easily compensated by the learning algorithm. To simulate the effect of pressure disturbance or noise in the actuator, a random function is added to the right hand side of Eq. (3.5). Figure 4-20 indicates that the learning strategy performs well for disturbance force between -2 to 2 N. It should be noted that the load inertia is "stuck" in the steady state. Accordingly, a small force disturbance or noise in pressure signal may not affect the load position status. But if the noise or disturbance is sufficiently larger such that the net force acting on the actuator can overcome the stiction, position response \( x(t) \) will deteriorate.

4.6 CONCLUSION

Two learning control algorithms have been proposed to improve the pressure and position control accuracies of a pneumatic actuator system. Both computer simulation and laboratory experiment exercises have shown that these algorithms are valid as the steady state pressure and position errors for step input commands are reduced significantly without any manual adjustment of the control
Figure 4-19 Effects of varying $m_a$ on position learning response. (a) Learning curves for $\bar{e}_x$. (b) Learning curves for $F_f$. 
Figure 4-20 Effects of noise on position learning response. (a) Learning curves for $\bar{e}_x$. (b) Learnings curve for $\bar{F}_f$. 
gains. The proposed learning law is designed in such a way that it updates the feedforward pressure or force using data from the previous cyclic data and hence reduces the steady error when the next command cycle is performed. Such a learning law reduces the error associated with periodic system response, but is not capable of reducing the error due to non-periodic system dynamics. The associated convergence criterion and optimal learning gains have also been provided based on the linear system theory. But these work very well for the nonlinear system as evident from computer simulation and experimental results.
CHAPTER V
CONCLUSION

5.1 SUMMARY

An experimental and theoretical study of pneumatic systems controlled by on-off valves operated under PWM mode with focus on pressure and position control has been conducted. This pneumatic system consists of a linear, double-sided actuator, an inertia load, two on-off valves, feedback system and a computer system. The emphases have been on the characterization of the pneumatic system dynamics and on the development and real time implementation of new control strategies with focus on the steady state pressure and position accuracies given command step inputs.

An analytical method to examine the effect of PWM mode on the performance of the pneumatic valve-chamber system has been proposed and a pressure control strategy has been developed. A flow direction dependent, first order chamber model incorporating the fundamental frequency component of the PWM mode has been used to study the system frequency response and the governing differential equations have been solved to obtain the steady state behavior. Predictions agree with the measured data extremely well in the
frequency domain and in the time domain as well during both charging and discharging processes. Computer simulation results also indicate that the increase in carrier frequency can enhance the system performance significantly. Experimental demonstration of the controller on a pneumatic chamber has been successful in achieving the desired pressure performance.

A computer simulation model incorporating pneumatic process nonlinearities and nonlinear mechanical friction has been developed to characterize actuator dynamics; then it is used in the simplified form to design the controller. In our control scheme, one valve is held open and the other is operated under the pulse width modulation mode to simulate the proportional control. An inner loop utilizing the pressure control strategy is formed to control the actuator pressure, and an outer loop with displacement and velocity feedbacks and a two staged feedforward force is used to control the load displacement. Experimental results on a single degree of freedom pneumatic robot have indicated that the proposed control system is better than the conventional on-off control strategy as it is effective in achieving the desired actuation performance without using any mechanical stops in the actuator. All of components used in the experiment described above are commercially available and hence it is possible to apply our proposed system to industrial automation.

Two learning control algorithms have been proposed to improve the pressure and position control accuracies of the pneumatic actuator system. Both computer simulation and experimental exercises have shown that these algorithms are valid as the steady state pressure and
position errors for step input command are reduced significantly without any manual adjustment of the control gains. The proposed learning law is designed in such a way that it updates the feedforward pressure or force using the previous cyclic data and hence reduces the steady error when the next command cycle is performed. Such a learning law reduces the error associated with periodic system response, but is not capable of reducing the error due to non-periodic system dynamics. The associated convergence criterion and optimal learning gains have also been provided based on the linear system theory.

5.2 CONTRIBUTIONS

The following contributions are evident of this study:

1. A new pneumatic control scheme to reduce the effect of process nonlinearity has been proposed. This uses one on-off valve under the PWM mode while holding the other valve wide open and without using any mechanical stops.

2. The effect of PWM control mode on the pneumatic chamber dynamics in both time and frequency domains has been identified both experimentally and analytically.

3. Two mathematical models have been developed: (i) a comprehensive nonlinear model is used to characterize the actuator dynamics, and (ii) a simplified linearized model is employed to design the control system.

4. New pressure and position control strategies have been developed. Such strategies have been implemented
successfully and demonstrated experimentally on a pneumatic robot.

5. Two learning algorithms to improve the pressure and position control accuracies have been proposed. Corresponding convergence criterion and optimization scheme have also been given based on the linear control theory.

5.3 FUTURE RESEARCH

Several key areas of future research are suggested here:

1. Include valve drive dynamics, realistic pneumatic process dynamics and improved mechanical friction model in the nonlinear computer simulation.

2. Develop a friction-dependent switching point for the feedforward force \( F_f \).

3. Extend the proposed control strategy to trajectory or path control.

4. Improve the learning control law to accommodate the effect of non-periodic dynamics.

5. Extend the proposed control schemes to a multi-degree-of-freedom pneumatic robot.

A few modifications in the laboratory system hardware are suggested below which should improve the performance of the current system. First, the a.c. poppet valve used for chamber 1 should be replaced as the time lag associated with the valve drive mechanism
is too long and the aliasing between the PWM mode and the a.c. power is significant. Some important features of the new on-off valve should be: d.c. operated, smaller time lag (~10 ms for current valve), poppet type and with a $C_v$ value near 0.9 for both charging and discharging ports. Second, a dynamic flow measurement device should be used to study the valve dynamics. Third, the pipe line between the tank and chamber 2 should be reduced as pressure fluctuations in chamber 2 have been noted which are large enough to affect the fine position accuracy. Finally, the potentiometer should be replaced by a linear encoder as the signal to noise ratio is poor due to the bad connection. Also, an encoder can extend the motion measurement range which is less than 150 mm for the current setup and it can measure velocity simultaneously.
REFERENCES


Miyazaki, F., Kawamura S., Matsumori, M. and Arimoto, S., 1986, "Learning Control Scheme for a Class of Robot Systems with


**Experimental Setup Specification**

**A. Instrumentation**

1. Sensotec pressure transducer (M530)
   - strain gauge type
   - 0-140 psi abs.
   - sensitivity: 0.176 mV/psi

2. Maurey Instrument potentiometer (M1326-8103)
   - stroke: 8 in
   - 10 kΩ ± 5%
   - sensitivity: 0.625 V/in at 5V DC excitation

3. Sanborn LYsyn linear velocity transducer (6LV4N)
   - moving-magnet pickup
   - stroke 95%: 4 in
   - stroke max. usable: 5.56 in.
   - Sensitivity: 264 mV/(in/s)

4. HEISE pressure gauge (H 43527)
   - 0-100 psig
   - Resolution: 0.25 psi

5. Potter & Brumfield relay (JMI 520081)
   - Mercury wetted contact relay
   - 5V DC excitation
   - 120V AC output

6. Conditioners
   - Honeywell Accudata gauge control units (105)
     - strain gauge conditioner
   - Electro Instrument Inc. amplifier (A-20RW)
B. Poppet Valve (Parker T20025D2F01)

3-Way, 2-Position Single Solenoid Direct Operated Valve
1/4 Inch Ports

Description
The T20025 Series Valve is a two (2) position, 2 and 3-way, single solenoid, direct operated, spring returned, ¼” side ported valve.

It may be used as normally closed or normally open, also as a selector valve.

Operation
Valve will operate mounted in any position. See mounting dimensions and port locations.

Selection table shows typical piping connections and maximum pressure differentials for each model number.

"Maximum Pressure Differential" is the maximum allowable difference between pressures recorded at any two working ports of the valve. The highest pressure that may be connected to any port is 150 PSI. For 2-way operation, plugs must be screwed in and sealed bubble tight for valve to work properly.

Specifications
• Operating pressure, vacuum to 150 PSI (10 bar).
• Operating temperature, 0°F to 160°F (—18°C to 71°C).
• Class B solenoid, dual rated 120V/60 Hz., 110V/50 Hz., continuous duty 120V/60 Hz., 7.2 watts, .26 amp inrush, .14 amp holding. Other voltages available.
• U.L. and C.S.A. listed.

Options
• Solenoid Indicator Lights. Insert letter "L" after "T" prefix. Example, T21025D1F01.

• 72 inch electrical leads. Insert "M16" after complete model number. Example, T21025D1F01M16.

Coil Selection

<table>
<thead>
<tr>
<th>Class B</th>
<th>60 Hz.</th>
<th>50 Hz.</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>120VAC</td>
<td>110VAC</td>
<td>6VDC</td>
</tr>
<tr>
<td>02</td>
<td>240VAC</td>
<td>220VAC</td>
<td>12VDC</td>
</tr>
<tr>
<td>03</td>
<td>480VAC</td>
<td>440VAC</td>
<td>24VDC</td>
</tr>
<tr>
<td>04</td>
<td>21</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>22</td>
<td></td>
</tr>
</tbody>
</table>

Dimensions — T200

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.15</td>
<td>3.31</td>
<td>2.25</td>
<td>88</td>
<td>88</td>
<td>.09</td>
<td>.56</td>
</tr>
<tr>
<td>4.01</td>
<td>84</td>
<td>51</td>
<td>22</td>
<td>22</td>
<td>2.4</td>
<td>14</td>
</tr>
</tbody>
</table>

Model Selection Information

1."Maximum Pressure Differential" is the maximum allowable difference between pressures recorded at any two working ports of the valve. The highest pressure that may be connected to any port is 150 P.S.I. Model numbers shown are for 120V/60 Hz. coil. See Coil Selection Chart for other options.

Pneumatic Division
Otsego, Michigan 49078

Parker Fluidpower
MotionMate offers live axes of movement, or "degrees of freedom." They are as follows:

A — Base Rotates, infinitely adjustable up to 180°.
B — Lift of 8".
C — Extend to 18" to 24".
D — Wrist Rotate 90° or 180°.
E — Grasp.
### AXIS OF MOTION

<table>
<thead>
<tr>
<th>AXIS</th>
<th>DESCRIPTION</th>
<th>FUNCTION</th>
<th>STROKE</th>
<th>MOTION/SPEED</th>
<th>AXIS DRIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Base rotate</td>
<td>Horizontal rotation</td>
<td>180°</td>
<td>180° per 1.5 sec. depending on load.</td>
<td>Rack and pinion, air powered, hyd controlled</td>
</tr>
<tr>
<td>B</td>
<td>Lift</td>
<td>Vertical linear</td>
<td>0°</td>
<td>10 to 240 ° depending upon load.</td>
<td>Air cylinder hydraulically dampened</td>
</tr>
<tr>
<td>C</td>
<td>Extend</td>
<td>Horizontal linear</td>
<td>18°</td>
<td>10 to 240 ° depending upon load.</td>
<td>Air cylinder hydraulically dampened</td>
</tr>
<tr>
<td>D</td>
<td>-Wrist</td>
<td>Wrist rotation</td>
<td>90°</td>
<td>½ second</td>
<td>Air cylinder and cam</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>180°</td>
<td>½ second</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Grasp</td>
<td>Open/close Linear or pivot</td>
<td>36° or 180°</td>
<td>½ second</td>
<td>Air cylinder actuated draw bar</td>
</tr>
</tbody>
</table>

### Weight and Load Capacity

- **Weight of robot**: 460 pounds
- **Mass load capacity at wrist**: 10 pounds
- **Repeatability**: ± 0.005
- **Axis drive method**: Pneumatic 40 to 120 psi
- **Power source**: 115V/60Hz, 240V/50Hz optional
- **Axis motion detectors**: Proximity switches on base rotate, lift and extend
- **Adjustable mechanical stops**: On base rotate, lift and extend
- **Operating temperature**: -20°F to +140°F
- **CONTROL**: Electronic microprocessor with remote portable teaching module
- **Memory capacity**: Up to 300 steps
- **Program method**: Push button via portable teach module
- **Maximum time interval between steps**: 10.79 minutes
- **Auxiliary equipment**: 6 outputs and 8 inputs each rated at 115V/60Hz. Other voltages available

### TORQUE DEVELOPED AT WRIST

<table>
<thead>
<tr>
<th></th>
<th>90° WRIST ROTATION</th>
<th>180° WRIST ROTATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSI, INCH LBS, LBS</td>
<td>PSI, INCH LBS, LBS</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>200</td>
</tr>
</tbody>
</table>

### GRIPPER CLOSER FORCES

<table>
<thead>
<tr>
<th></th>
<th>LINEAR FORCE</th>
<th>PIVOT (AT END OF 2&quot; FINGER TOOLING)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSI, INCH LBS, LBS</td>
<td>PSI, INCH LBS, LBS</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• Specifications of the extension/retract actuator
  - piston area (extension stroke): 1.227 in²
  - piston area (retract stroke): 0.921 in²
  - stroke: 18 in
D. Microcomputer (Intel 86/310 Microcomputer)

- iSBC 86/310 single board computer based on the 8 MHz 8086 microprocessor (16 Bits)

- iSBC 304 RAM module and iSBC 337A numeric module (8087) installed on the processor board

- 5.25" flexible disk drive (iSBX 218A flexible diskette controller)

- iSBX 311 analog input multimodule on iSBC 86/35 processor board

- iSBX 328 analog output multimodule board

- Programmable internal timer