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Triangle concept hierarchy and microcomputers

Shelton, Marilyn Rose, Ph.D.
The Ohio State University, 1989

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Triangle Concept Hierarchy and Microcomputers

DISSERTATION

Presented in Partial Fulfillment of Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Marilyn Rose Shelton, A.G.S., B.S., M.S.

* * * * *

The Ohio State University
1989

Dissertation Committee:
Suzanne Damarin
Cella Genishi
Lorren Stull

Approved by

Adviser
College of Education
dedicated to
the ones I love
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VITA

November 17, 1943. Born - Dallis, Oregon

1973 ............... Associate of General Studies, Linn-Benton Community College, Albany, Oregon

1974 ............... B.S., Oregon College of Education (now Western Oregon State College, WOSC), Monmouth, Oregon

1974-1982 .......... Parent education teacher with kindergarten and preschool labs, Linn-Benton Community College, Scio, Oregon

1980 ............... M.S. in Education - Early Childhood Education, Oregon College of Education (WOSC), Monmouth, Oregon

1983-1987 .......... Graduate Teaching Associate and doctoral student, The Ohio State University, Columbus, Ohio

1987- present ...... Lecturer, Department of Teacher Education, California State University, Fresno, California

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CHAPTER I

INTRODUCTION

Need for the study

The triangle is one of the fundamental shapes which comprise our geometric world. Triangles are of particular importance because of their unique rigid construction, which is used to advantage in the construction of architectural structures like bridges, buildings, and towers. Another important aspect of triangles is their use in calculating the area of all geometric shapes. Thus triangles, and geometry in general, provide a valuable mathematical model for describing and predicting behavior in the biological and physical world (Fey, 1982).

Understanding geometry is therefore important; however, many people don’t develop a complete understanding of the concept of triangle (Vinner/Hershkowitz, 1980). A prior knowledge of the concept of triangles is a necessary prerequisite for understanding higher levels of geometry.

The concept of triangles is an important concept that many children have trouble with (Damarin, 1981), even in the seventh grade and beyond (Vinner/Hershkowitz, 1980). It seems that the triangle concept itself is often not directly taught, and therefore not learned by some children.
It appears to be common practice to teach only a few stereotyped examples of the concept with the expectation that students will generalize appropriately. Common practice in early childhood education is to teach shapes in the following order: circle, square, triangle, oval, rectangle, and diamond. In early childhood settings the concept of triangle is usually introduced to children at the same time and with the same variety of examples and non-examples as are these other shapes. The examples presented include varieties in color and size, which is appropriate for teaching circle.

The fact that triangles can have very different outward appearances usually isn't stressed, even if a few different examples are presented. Ausubel, Novak and Haneslan (1968) described these different outward appearances as those with a "perceptually dissimilar core" (page 100-101). Another problem for children with this common presentation of shapes is that squares are not taught as a subordinate class of rectangles.

Table 1 presents six shapes which range from the less complex circle to the more complex triangle. Each level of change or complexity requires examples and non-examples for that dimension of the concept to be learned. It is because of these different levels of complexity that the rectangle concept needs more examples than the circle, and the
triangle concept needs more examples and non-examples than either the circle or rectangle.

Table 1
Dimensions of Triangle Concept and Other Shapes

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Circle</th>
<th>Oval</th>
<th>Square</th>
<th>Rectangle</th>
<th>Triangle</th>
<th>Diamond</th>
</tr>
</thead>
<tbody>
<tr>
<td>color</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>size</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>ratio of sides</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>angle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Triangles Frequently Presented

Figure 1

Triangles Rarely Presented

Figure 2
It is this researcher's contention that much of the trouble children have with triangles comes from having undergeneralized or prematurely closed the set of examples which they classify as triangles. Many children include only the few examples that they have been shown and have had identified for them as being triangles (Vinner & Hershkowitz, 1980). Microcomputer based research has shown that children as young as two-years-old have demonstrated an ability to acquire an intuitive sense of triangle which includes all orientations and configurations (Shelton, 1985).

The triangles in Figure 1 represent the few examples of triangles that are usually presented and identified for young children. Figure 2 includes examples of triangles that are usually not presented or labeled as triangles when children are being introduced to the concept.

One of the major problems seems to be that triangles are presented and/or taught as content-specific facts (Gillingham & Price, 1987), and not as a concept. Many researchers agree that concepts are one of the foundations of knowledge (Novak, 1977, Sowder, 1980) but concept formation is a complicated process, and not all researchers/scholars agree on how it should be defined or studied (Ginsburg & Opper, 1981, Merrill, & Tennyson, 1977, Reed, 1982, Rosch, 1976, Ross, 1980, Sigel, 1983, Tesmer & Driscoll, 1986). It is commonly hypothesized that children
develop the "full-blown" triangle concept by sequentially discriminating the increasingly dissimilar triangles as all being triangles and/or applying the definition to new examples (Vinner & Hershkowitz, 1980). Wilson (1982) reported in her dissertation literature review that the entire concept of triangle is learned best when instruction includes numerous examples as well as instances that are not examples of the concept.

Even though the concept of triangle has been studied by many researchers, little has been documented about the sequence of specific examples and or orientations which the children include in their concept of triangle. Novak (1977) contended that curriculum planners need to include the hierarchical relationships among concepts in their instructional designs. The present research project was designed to uncover and document the hierarchy for the inclusion of 27 specific triangles in children’s concept.

This dissertation research is comprised of two parts, one is experimental and the other correlational in nature. The experimental part of the study investigates the role microcomputers can play in geometric shape learning. The correlational part of the study constructs and tests a hierarchy of children’s learning the concept of all triangles in different orientations and with different relationships among the sides and angles. The orientations of the triangles include bases which are horizontal,
vertical, or at a 45 degree angle to the bottom edge of the screen. Triangles included represent categories based on angle (equiangular, right, acute, obtuse) and on sides (equilateral, isosceles, scalene). The specific shapes used are shown in Figure 3, where they are arranged by the angle category.
Statement of the Problem

The problem stems from the author/researcher's observation of young children reaching learning plateaus as they acquire the concept of triangles. Other research (Wilson, 1982) has shown that some children's concept of triangle never progress beyond a given plateau. The result is that some children develop premature closure on their concept of triangle.

The research problem has two parts. Part one is the identification of learning plateaus in triangle concept development and related grade levels. Part two experimentally tests the use of appropriate microcomputer software and its relationship to children transcending the plateaus. The research questions follow:

1. Hierarchy
   a. What is the hierarchy of difficulty of instances in the learning of the concept of triangle?
   b. When and where do children have difficulty progressing upward in the hierarchy?

2. Computer
   Can using microcomputer software designed to display instances of triangles facilitate progress through the triangle hierarchy?
Some sample questions asked during the research are the following:

1. Is the classification of triangles effected by spatial orientations with various side and angle configurations?
2. Which orientations of triangles are difficult for children from kindergarten through grade five to recognize as members of a class for which they recognize some members?
3. At which grades between kindergarten and five do children have difficulty classifying triangles that have been rotated either 90 or 45 degrees, from a position with a horizontal base?
4. At what grade do children correctly classify triangles with extremes of variance of side and angle?
5. Are there particular ratios of sides and/or angles of triangle shapes that give children more difficulty than others?

Assumptions

An underlying assumption is that spatial orientation and different configurations of length of sides and/or angles of triangles have a bearing on how readily examples are accepted and included in a child's concept of triangle. While this does not appear to have been verified by
researchers, it is commonly believed by teachers, test developers, and others familiar with the field.

Young children can learn general geometric concepts at an appropriate level (Ashby, 1985, Bruner, 1960, Fey, 1982). A clear intuitive conception of triangles provides the pre-skills which are necessary to understand higher levels of geometry. Geometric concept learning is based more on experience than on development (Ausubel, Novak, and Hanesian, 1968). Different experiences of children account for the discrepancies between children's level of attainment on the hierarchy that includes all examples of triangles.

**Research Hypotheses**

In relationship to the problems associated with how children develop a comprehensive concept of triangles, research is called for which will clarify progression of the developing concept and identify facilitating instructional materials.

1. The order difficulty of items will not vary among groups or from pretest to posttest (although the absolute difficulty may). Analyzing the data for this hypothesis will provide the difficulty of instances of triangles for establishing a hierarchy.
2. Mean pretest computer scores will increase with grade level. If learning the concept of triangles does occur in plateaus, there may not be smooth increases of scores.

3. At each grade level posttest scores will be significantly greater than pretest scores for the treatment group, but not for control group I. This hypothesis will test for the effect of using microcomputer software which was designed to facilitate progress through the triangle hierarchy.

4. No sex differences will be apparent in any of the analysis. Much interest has been expressed in the literature concerning differences between boys' and girls' spatial abilities, therefore the data will be analyzed to provide relevant information on the population.

Note that two of the hypotheses are stated as null hypotheses (1 and 4), and two are not. Each hypothesis was stated in terms of expected outcome rather than forcing them all to be in the same form.
CHAPTER II

LITERATURE REVIEW

Introduction

The present study is defined and shaped by the intersection of: cognitive development, hierarchy of concept learning, geometry, and computers. Therefore, there are four important areas of literature which are relevant for this study. Research on cognitive development is reviewed first to provide a framework for children's development of cognitive understanding and learning. Concept learning is presented in sub-sections based on the following major categories of research: (a) hierarchy of concept development, (b) visual perception, (c) language, (d) misconceptions, naive theories and premature closure, and (e) example, non-example and prototype learning. Research on geometry and on microcomputers in education will be reviewed last.
Cognitive Development and the Role of Experience

"Virtually everyone nowadays agrees with Piaget that we assimilate input to our existing knowledge structures rather than merely copy it - that our learning, comprehension, and other cognitive activities are heavily constructive" (Flavell, 1982, p. 4).

An assumption underlying this dissertation is that young children can learn to include diverse examples into their concept of triangles and that this learning is based on experience. Experience is defined to be opportunities to interact with and construct meaning from concrete and semi-concrete materials. A part of experience is exposure to the appropriate language which provides labels for the items and processes.

Experience with objects in one's environment plays a major role in cognitive development in Piaget's view (Ginsburg & Opper, 1969). In Piaget's view of learning, if an attempt to relate a new experience directly to a preexisting organizational structure is smooth, then the experience is assimilated. If a new experience requires a modification of a preexisting organization, then the experience is accommodated. A state of equilibrium exists when assimilation and accommodation occur simultaneously and are in balance. The equilibrium process leads to the acquisition of new structures or organizational patterns of mental operations. Stable and lasting learning can result
from the equilibration processes as experiences with the environment are encountered.

Another explanation of how experience helps people to learn has been proposed from a physiological point of view (Esler, 1984). Esler contends that the brain develops with use, "It is evident that repeated stimulation of a neuron or cluster of neurons results in the permanent and increased conductivity of the system" (p. 88). The image of neurons being stimulated with electrical impulses may interfere with the acceptance and application of the theory to educational settings. Therefore it is important to note that the stimulation can be provided by appropriate educational experiences. "Multisensory experiences enrich conceptualization and increase the ease of its recall" (p. 88). Esler goes on to say that "both knowledge and rational processes are products of organic changes in the brain tissue that result from activity" (p. 88).

When a particular skill or process is experienced or practiced many times one can expect that physical changes will occur and that the skill or process may become automatic. Skills or processes that can be executed with little or no conscious thought on the part of the doer are said to have reached a level of automaticity. "Some neurologists view automaticity as being similar to conditioned responses although somewhat more complex and continuing for greater time periods" (B.S. Bloom, 1986, p. 76).
Thus, the physiological view that repeated stimulation results in increased efficiency of a system could explain the development of automaticity. Bloom (1986) describes the stages or sequence of acquiring automaticity as: "The first learning in a field typically emphasizes very basic isolated details that are learned to a high level (i.e. repeated stimulation or experience). The second type of learning (which may overlap with the first learning) emphasizes larger units of repeated stimulation or experience composed of the isolated details already learned. The third type of learning emphasizes series of units and processes built out of the previous units in the second type of learning" (p. 75).

Bloom's (1986) description of functions of automaticity have parallels in Piaget's theory of learning. Bloom's "first learning" is comparable to Piaget's description of schema which is an organized pattern of behavior resulting from experience (Hill & Arbib, 1984, Ginsburg & Opper, 1969). The second and third types of learning for automaticity can be viewed as applications of Piaget's processes of assimilation, accommodation and equilibration. It is interesting to note that Bloom did not draw any connections between his work and Piaget's work.

Experience is also important for Ausubel's theory of concept formation, which occurs after repeated encounters with objects and the learning of the language labels (Novak,
Ausubel's meaningful learning is the result of experiences being assimilated into an idiosyncratic experience base of the individual.

Aune and Moskow (1987) describe several axioms and propositions relating content specific learning to information processing theory. The axioms deal with memory, schemata, hierarchy, and the related structure of data in memory. Their first proposition states that new data will be put into a hierarchy based on relevant schema. Proposition two is that analytic and analogic processes are used to analyze incoming data in order to detect structure and place the data within the relevant hierarchy. The third proposition is that an isomorphic fit between new information and an existing structure does not require much in the way of conscious information-processing, whereas dissimilar information requires greater conscious attention, or mediation, in order to be placed into a current conceptual structure. The fourth proposition states that mediated data is treated the same as sensory data, that is structure is imposed if the data does not readily fit an existing cognitive structure. Aune and Moskow's description could also have been explained by assimilation, accommodation, and equilibration. Like Bloom, they made no ties to Piaget's work.

Ashby and Boulton-Lewis (1985) consider cognitive development to be "a function of the child's maturing
capacity to processes information in interaction with the opportunities provided by the environment" (p. 18). That is to say that the quality of experiences provided by the environment are an integral component for cognitive development.

Dienes (1961) discusses the process of forming concepts as a move from experiences with individual elements, to experiences with restricted class to abstraction. This process can be understood by using the concept formation process of triangles as an example. The elements would be specific examples of triangle, like an isosceles triangle with a horizontal base. An example for the restricted class of triangles would be when the child includes both isosceles and right triangles. An abstraction of the triangle concept would be the inclusion of any given example of triangles, which is what many children have difficulty doing.

Teaching and curriculum planning are directly influenced by the perceived capacity of children's ability to understand and learn concepts, that is, their cognitive development. For example, there is a general pattern for early childhood and elementary school teachers to teach only a few examples of the triangle concept. This pattern fits with the notion, once commonly believed, that young children were not capable of understanding complex ideas. The underestimation of young children's cognitive capacities has been discussed by several authors (Donaldson, 1985, Gelman,
1978, Huber, 1985). The growing data base we have about cognitive development and learning is helping to expand on early theories of learning.

Much has been learned about children's learning processes and capabilities which refine Piaget's stage theory. Ross (1980) presented the following explanation for research findings that differ from Piaget's. "... conclusions about children's categorization skills are dependent on the methodology employed, the response required, and the categories presented" (p. 395).

Donaldson (1985) wrote that we have underestimated children's ability to reason and cited research duplicating Piaget's visualization task with a mountain and a doll where the child was to describe the view from the doll's point of view. Piaget's task is difficult for young children, and thus, conclusions were made that young children were egocentric, or unable to see things from another's point of view. In the research Donaldson described, children were asked to "hide the boy so that the policeman can't see him" (p. 158). Donaldson reported that children were successful with hiding the boy and he concluded that tasks which make sense to children are easier for them to master.

Huber (1985) discussed researcher's current understanding of stages of mental growth and the relationship to appropriate computer use. He says that the stages of mental growth are not exactly innate but "are the
result of a particular environment interacting with a particular set of sensory organs connected to a particular brain" (p. 42). Huber gives the example that the concept of randomness is now understood by eight-year-olds whereas it was once believed that the capacity to understand randomness did not develop until age eleven, with formal operations.

Gannon and Ginsburg (1985) also contend that young children's thinking ability is underestimated. They discuss the informal learning and thinking that children bring with them to school, and agree that children know more than they have been given credit for. These authors list reasons for school failure which they see as related to the underestimation of the children's knowledge. The school failure problems are broken down into two areas. The first is on learning problems, which includes: teaching inadequacies, emotional difficulties, stylistic incompatibility, "bugs" or naïve conceptions, and learning incapacity. The second area includes performance problems: cognitive style, boredom, and fatigue. Gannon and Ginsburg recommended that we base our education on the informal abilities possessed by the children.

Research on the experience of the first day of kindergarten and the results for long term memory were reported by Fivush (1984). Beginning with their first day at kindergarten, children were asked to describe a day at school. The "results indicate that children represent an
event as a general spatial-temporal framework based on the first experience with a new routine; this framework becomes more elaborate and the temporal and the hierarchical organization of the representation becomes more complex with increasing experience with the event' (p. 1709). The children formed a script on the first day of school, and it stayed stable over the three month period of the study.

Research findings from the Genevan School reported by Watson and Nida (1986) state that children solve mental problems at their level and at higher levels, by interpreting tasks and solving problems as they see fit, even though they do not understand the task as it was designed. The study involved having young children balance weighted and unweighted blocks on a fulcrum. The children could solve the problem by moving the blocks, but they did not understand their actions or the special properties of the blocks. The children physically "solved a problem which required concrete operational mental structures" (p. 306). Watson and Nida concluded that preoperational children think and act more concretely than previously thought.

In summary, many researchers agree that experience is a necessary component in cognitive development. The views of the exact role of experience in cognitive development vary from researcher to researcher, but the views are all congruent with the idea that experience provides materials for developing cognition. Experiences are necessary for
physiological development of neurological connections, automaticity, and the processes of assimilation, accommodation and equilibrium. Our adult perspective of children's cognitive capabilities is based on current understandings, and we are therefore given the opportunity to accommodate new research findings which demonstrate that we have been underestimating children's thinking abilities.

Concept Learning

"We are all forced to think with concepts and we cannot use those we do not know" (Novak, 1977, p. 10).

Introduction and Definition

The traditional or 'classical' definition of a concept is "a class or category all members of which share a particular combination of critical properties not shared by any other class" (Stanley & Mathews, 1985, p. 58). Gagne' (1968) says that concept-learning is the acquisition of "a common response, often a name, to a class of objects varying in appearance" (p. 14).

Selger-Ehrenberg (1985) offered several definitions of concepts and concluded that lack of understanding of the process of learning and teaching concepts (p. 161) could contribute to student failure to learn. Case studies by Erlwanger (1975) support Selger-Ehrenberg's statement.
Erlwanger found that children "developed a conception which appeared to function as a relatively stable, cohesive system of interrelated ideas, beliefs and views about mathematics" (p. 157). But, the children developed their own concepts in order to make sense out of their work. Some of these contrived conceptions were quite different from formal school mathematics.

An assumption about the function of concepts is that they reduce the level of complexity in one's environment, and thus communication is easier and the cognitive load is reduced (Arnone, 1987, Bruner, Goodnow, & Austin, 1962). Concepts develop slowly, and as a result of a variety of experiences. Arnone lists openness to change as a major factor in facilitating concept development.

Psychologists have studied concept identification using categories which could be distinguished on the basis of simple rules (Reed, 1982), like selecting examples based on color or size. Some literature deals with research studies using logical relations of conjunctive (and), disjunctive (or), and conditional (if, then). Bruner, Goodnow, and Austin (1962) devote an entire chapter to disjunctive concepts and their attainment.

Two approaches to studying concept formation are feature frequency and prototype (Reed, 1982). Rosch, Mervis, Gray, Johnson, and Boyer-Braem (1976) demonstrated a need to use prototypes at the basic level when teaching
concepts. The prototype model uses an example close to the central tendency of the category, and is successfully used for concepts with values that can have a large variance. The feature frequency model classifies patterns by matching features rather than comparing the similarity of the features. Feature frequency categorization is most successful with concepts which have little variability, like color or size.

Rosch, et al. (1976) studied basic objects in natural categories and from their research demonstrated the need to view three levels of categories. At the basic level the items share the most properties and are most easily identified; the example of chair is used. The subordinate level is subcategories, where examples of high-chair and stool fit. The most inclusive level is superordinate, where the chairs fit into the category of furniture.

Mervis and Crisafi's (1982) research on categorization led them to the conclusion that the degree of differentiation within a category is more important than the hierarchical level.

Research on concept learning falls into the following categories: (a) hierarchy of concept development, (b) visual perception, (c) language, (d) misconceptions, naive theories and premature closure, and (e) example, non-example and prototype.
Hierarchy of Concept Development

The hierarchy of concept structure/development begins with the most general elements of a concept and progresses to the more specific elements by increasing the number of attributes used to differentiate the examples (Ausubel, Novak, and Hanesian, 1968, Novak, 1977). Each of the levels or stages "incorporate all earlier learning" (Ashby and Boulton-Lewis, 1985, p. 23) which provides a common framework for integrating cognitive, social, emotional, language, and physical skills (Fischer, 1980). Included in the hierarchy of concept development is the prior knowledge or experience of the learners, and the recommended follow up procedures for teaching concepts.

Young children's development of mathematical concepts has been studied from several different perspectives. Mitchelmore (1984) concluded from his international studies of spatial ability and geometry that young children have a sense of the shapes that are in their environment, but most children are not helped to investigate the shapes nor are they provided with their names. Clements (1984) wrote that young children learn concepts before they can define them. In a book of reports to teachers on research, Driscoll (1980) said that children come to kindergarten with a lot of intuitive mathematical knowledge and that teachers should
consider this prior knowledge before they begin teaching the child.

In the Encyclopedia of Educational Research, Fey (1982) reported that mathematical talent seems to emerge at an early age and that this gives us reason to believe that young children can and do have the mental abilities required for mathematics learning. Pellerey (1984) explained how this early mathematical talent develops, describing the environmental influences of the society on the developing child's perceptions. The child is acculturated into a world of mathematics, in a very subtle way. The young child picks up on the patterns of the people and events he or she watches. The child internalizes and stores the memory of the patterns to be retrieved later.

Research has shown that children as young as two years old can acquire an intuitive understanding of the concept of triangles, including all variations of angles and orientations (Shelton, 1985). Observations and reflections on the teaching of young children lead to speculation that teachers do not present the concept of triangles to young children. What is usually presented are one or two examples of the concept, the first being an isosceles triangle with a horizontal base, and the second a right triangle (also with a horizontal base).

Stice and Alvarey's (1987) research on hierarchical concept mapping with children from kindergarten through
grade five had as its goal to see if "concept maps teach students to represent the hierarchy of propositions" (p. 86) with appropriate concept and label relationships. The conclusion from this research was that even kindergarten children could use the concept mapping skill in a meaningful way. Terminology used in the study was: (1) Event - anything that happens, (2) Object - anything that exists and can be observed, (3) Concept - "sign/symbol pointing to regularities in events or to records of events" (p. 87), (4) "semantic web" - central idea/core concept, and (5) "structured overview" - arrangement of elements of a concept into a hierarchy.

Concept learning is described as a sequential or stage process by the following researchers: Dlenes (1961), Klausmeler (1976), Piaget (1965), and Rosch, et al. (1976). Table 2, Sequential Concept Learning, shows the relationship between Piaget's stages and the sequences outlined by Klausmeler, Dlenes, and Rosch, et al. The stages that children seem to be going through as they acquire concepts appear to follow a pattern (Klausmeler). At a pre-concept level the children have no inkling of the feasibility of the concept. Sigel (1971) said that classification skills are seen as "'preludes' to concept attainment" (p. 170).

At Klausmeler's (1976) concrete level, concept features are discriminated and names are acquired. This description corresponds with Rosch et al.'s explanation that items from
Table 2
Sequential Concept Learning

<table>
<thead>
<tr>
<th>Klausmeler</th>
<th>Rosch</th>
<th>Dienes</th>
<th>Piaget</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete level:</td>
<td>Basic level:</td>
<td>Play:</td>
<td>Pre-operational</td>
</tr>
<tr>
<td>discrimination,</td>
<td>ex., chair</td>
<td>elements,</td>
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<tr>
<td>represent as an</td>
<td></td>
<td>gather facts</td>
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<tr>
<td>image, acquire</td>
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<tr>
<td>and remember</td>
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<td></td>
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<tr>
<td>concept name</td>
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<td></td>
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<td>Identity level:</td>
<td>Abstraction:</td>
<td>restricted</td>
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<td>class</td>
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<td>that two or more</td>
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<td></td>
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<tr>
<td>forms of an item</td>
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<tr>
<td>are the same object</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Classificatory level:</td>
<td>Subordinate:</td>
<td>Class Gener-</td>
<td>Concrete</td>
</tr>
<tr>
<td>generalize that</td>
<td>ex., stool,</td>
<td>alization:</td>
<td>Operations</td>
</tr>
<tr>
<td>two or more examples</td>
<td>rocking</td>
<td>primitive</td>
<td></td>
</tr>
<tr>
<td>are equivalent and</td>
<td>chair</td>
<td>and mathe-</td>
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</tr>
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<td>belong to the same</td>
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<td>matical,</td>
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<td>class of things</td>
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<td>generalized</td>
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<tr>
<td>Formal level:</td>
<td>Super-ordinate:</td>
<td>Logical Analysis</td>
<td>Logical</td>
</tr>
<tr>
<td>define attributes,</td>
<td>ex: chairs</td>
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<td>examples and</td>
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<tr>
<td>nonexamples</td>
<td></td>
<td>are furniture</td>
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the basic category are the first categorizations made during perception of the environment and the earliest sorted and named by children. They describe basic categories as "those which carry the most information, possess the highest category cue validity, and are, thus, the most
differentiated from one another" (p. 382). Diene's corresponding level of concept development includes gathering facts by playing with elements. All of these processes, of acquiring names for images and gathering facts, can and do begin during Piaget's preoperational stage.

At the identity level generalizations are made about the concept. Klausmeler (1976) describes the child who can view the same object from two different angles, and can generalize that the two views are of the same object. Diene's (1961) refers to this process as forming abstractions of restricted classes.

During the classificatory level less obvious attributes are focused on and finer discriminations are made. The shift to this level corresponds to Piaget's concrete operations stage. Klausmeler (1976) says that this level is reached when two or more equivalent examples of a concept are seen as belonging to the same class. This is the subordinate level in Rosch, et al.'s (1976) sequence, where stools and rocking chairs are both seen as belonging to the class of chairs. At this level, Diene's (1961) discusses primitive generalizations which are the realization that one class can belong to or be included in another class.

Generalizations build on previous restricted class concepts and move toward more abstract or generalized concepts. Diene's also discusses the generalization process
for mathematical understanding as being similar to that for primitive generalizations. Piaget's (1965) class-inclusion corresponds with this level of concept development. Class-inclusion is the understanding of relations among different levels of a class (Ginsburg and Opper, 1969).

Klausmeier's (1976) formal level is reached when a definition can be given about the concept that includes the specific attributes of examples and nonexamples. This is the superordinate level in Rosch et al's (1976) sequence, where the class of chairs is seen as a member of the furniture category. Dienes (1961) describes this as the level for logical analysis. This formal level corresponds with Piaget's logical operations stage.

A study using Rosch's levels was conducted by Dickinson (1987), who concluded that, "Certainly by the time they are two-years-old children can categorize the world at what Rosch has called the 'basic object level'" (p. 615). Dickinson also found that, "The changes in responses observed during the school years suggest development in children's concept of material from a perceptually based concept to a more abstract adult-like concept" (p. 624).

Another approach to understanding concept learning is offered by Nantais, Herscovics, and Bergeron (1984); theirs is a constructivist approach which explains that "the acquisition of skills becomes a part of the construction of understanding... the construction of conceptual schemas and
problem solving" (p. 229). The constructivist's model includes: (1) intuitive, or first level understanding, (2) procedural understanding or mathematical "know how" skills, (3) abstraction, and (4) formalization.

Gagne' (1966) described concept development as beginning with concrete experiences and argued that understanding of concepts is enhanced by having definitions. According to Gagne' concepts are learned prior to learning principles and principle - learning is "the combining of concepts into entities variously referred to as 'ideas,' 'facts,' 'principles,' or 'rules'" (p. 14). He also said that the "criterion performance is being able to use the concept" (p. 15) in relationship with other concepts at the principle level.

Arnone (1987) said concept development begins with a personal understanding of a symbol representing facts and that abstractions don't come about until the concrete operations stage. Kingma (1986) studied transfer for short and long term with kindergarten children and concluded that, "seriation is a precondition for comprehending relationships between numbers" (p. 276). Classification and ordination skills develop simultaneously for numbers up to six, but for numbers larger than six, ordination skills develop before cardination (Liddle & Wilkinson, 1987).

Although a substantial amount of literature dealing with the hierarchy of concept development has been reviewed,
only one article outlines how this knowledge may be applied
to the educational settings of schools. Simon (1986) reported the following directions for planning discovery lessons: (1) identify and prioritize what needs to be learned, (2) distinguish between facts, procedures, and concepts, (3) organize concepts hierarchically, (4) divide what is to be learned into appropriate increments, and (5) create or adapt activities that stimulate the development of the desired concept. Simon's plan for implementing the process is: (1) ask questions that require reflection on thought process, (2) provide subtasks where needed, and (3) evaluate students regularly.

In other words, the teacher will need to analyze carefully the hierarchical structure of the concepts to be presented and plan learning activities with concrete experiences for each level. With appropriate activities the students will be able to relate the concepts to their prior knowledge and grasp the concepts at their current level of development.
Visual Perception

"While it is often overlooked as a curriculum goal, spatial ability is of importance to the complete learning of many concepts and skills, especially those in science and mathematics. According to McGee, spatial ability is composed of two main factors: spatial visualization and spatial orientation. Spatial visualization is the ability to rotate objects mentally; spatial orientation is the ability to 'remain unconfused by changing orientations and ... to determine spatial orientation with respect to one's own body' (1979, p. 897)" (Mitchell and Burton, 1984, p. 395).

Spatial orientation is a sub-set of visual perception. Research on various aspects of visualization have been conducted by many researchers. Some important connections between visual attributes and mathematics learning, and geometry learning in particular, are:

1. Spatial visualization is one of the cognitive abilities which is critical for mathematical learning (Fey, 1982).

2. Visual Imagery is one of the processing styles that seems related to performance in mathematics (Fey, 1982).

3. Visualization of the image of the concept helps recall it (Suebsonthi, 1981). "In thinking, almost always the concept image will be evoked.... The person's mental picture of a concept is the set of all pictures that have ever been associated with the concept in the person's mind. ... In order to handle concepts one needs a concept image and not a concept definition" (Vinner and Hershkowitz, 1980, p. 177).

4. Images in one's memory provide a base to relate new experiences to existing knowledge (Pellery, 1984).
5. Visual thinking at a basic level is where people learn to recognize shapes by their appearances. The other levels of mathematical thinking are dependent on this first level (Fuys, 1984; Lunkenbein, 1984).

6. "Spatial ability is the ability to form and manipulate mental images of physical objects. This ability is called into play whenever one does geometry, since geometry is the study of the spatial properties of various figures abstracted from the concrete world of physical objects" (Mitchelmore, 1984, p. 135).

The importance of spatial and visual orientation has been studied for many years by researchers. Ghent published articles on orientation preferences from 1957 to 1964, and then she published more articles as L. Ghent Braine in 1973. Her main contributions were findings that young children prefer orientations of shapes with the focal point in the upper half, and that children have a natural tendency to use a downward scanning eye movement. When shapes have the focal point in the bottom half, then the natural downward scanning is not efficient and the children decide that the shapes are upside down. The judgment of "right-side-upness" is related to the eye scanning and not to the frame around a shape. Antonovsky and Ghent (1964) reported that nursery school children from both America and Iran preferred orientations perceived as being upright.

Frith (1980) reported on Ghent Braine's research and added to it. A developmental hierarchy of orientation perception was suggested with the earliest distinction being
between upright and non-upright (p. 4). The second stage is distinguishing upside down and sideways, the third stage is the differentiation of left and right. Frith also discussed the horizontal and vertical planes, and the findings that the first preferences are for shapes with vertical alignments.

Harris and Schaller (1973) also built on Ghent Braine's research, but with school aged children and the use of letters as the shapes being distinguished. Harris and Schaller found that older students shift to a preference for horizontal orientation and suggested that this is because of a habit derived from reading.

Other researchers have reported that the importance of orientation of the stimulus is that young children have a difficult time in overcoming the seductive perceptual pull of the way things look (Gold, 1984). Wilson studied this phenomenon with respect to triangles and referred to it as stability of base (Damarin, Hinrichs, Wilson & Whiteman, 1985). Fisher (1977) found that even students in the 6th, 9th, and college grades consistently emphasized the upright orientation of geometric figures, even after training. A possible explanation for this predisposition for the upright position is that of the gravitational pull. The gravitational pull tends to make things rest solidly on a base, where they look more stable, and are therefore easier to deal with.
Ghent Braine (1973) studied the stability view, and her description sounds similar to the gravitational pull definition. Ghent Braine's research was with shapes which had the parts considered to be heavier or more stable appearing as often at the top as the bottom. She argued that uprightness was "determined by ... presence of a focal, or salient, feature at the top of the figure, and a vertical orientation of the main lines (or long axis) of the figure" (p. 45). Her data did not support the stability view.

Shelton (1985) observed that young children's recognition of triangles was easier in upright orientations than when the triangles were rotated. Perner, Kohlmann and Wimmer (1984) studied three to seven-year-old children's perception and drawing of vertical and horizontal lines. They found that even though the three-year-olds could pick out pictures with the correct orientation, they still drew their own pictures with chimneys perpendicular to the slanted roofs.

Changes in orientations of figures can come about through rotation around some center or reflection across some line. The ability to visualize the rotation of a shape is referred to as mental rotations. Research on rotations was conducted by Herman (1984) with first through fifth grade children. The ten-year-olds were more accurate than the six-year-olds in perspective-taking but they had equivalent accuracy in mental rotations. In 1985 Somerville
and Bryant reported research findings on rotations of 45 degrees from the horizontal or vertical axis. Their conclusion was that "young children's grasp of Euclidean spatial relationships is more adequate than has often been suggested" (p. 604).

Perceptual development of young children has been studied by investigating projective and Euclidean understanding. Research by Rosser, Horan, Mattson and Mazzeo (1984) on the development of the young (ages three to five) child's understanding and comprehension of Euclidean space support the literature on the perceptual and memory development of young children. In this study children could match shapes equally well either from recall or when the shape was rotated on a single axis. The children had a difficult time matching shapes from memory which had been rotated. "Nonattention to rotational differences is also consistent with many research findings on young children's projective spatial abilities" (Rosser et al., p. 38). These authors observed that the males paid less attention to frame-of-reference cues than did the females.

A second study by Rosser, Ensing, Glider, and Lane (1984) was with four and five-year-old children. The purpose was to see if cues within a visual stimulus or on its outer edge would help the children predict its position after an unobserved rotation or reflection. The results indicated that children needed external edge markers, and
two markers were required for the reflected images. Sex differences favored the males, with the four-year-old females lagging behind in predicting reorientations.

Shapes or images are recalled, or visualized, by children who have visually seen best examples when learning the concept of a regular polygon (Suebsonthi, 1981). She worked with third grade students and compared the use of a list of critical attributes with that of "best examples." The lists were presented either with explanations or with questioning of the child. Suebsonthi reported that the children who received the best examples recalled the concepts as shapes or images, and that the children who received the list recalled parts of the list, but were unable to form an image of the concept.

In summary, visual perception plays an important role in concept formation. The visual images are seen to be a key factor in mathematical thinking, especially in geometry. Orientation of images and how one visually scans items have both been studied by researchers who are trying to learn more about the connection between visualization and learning.
"Meaning making" is what children are doing when they combine language symbols with experiences and can then communicate with other people (Ball & Milson, 1974, p. 58).

Language can give people handles or names or symbols for the concepts they are dealing with. Names for concepts allow people to communicate with each other about the concepts. Knowing what something is called and also having the names of its attributes or characteristics facilitates progressive understanding and use of the concept (Klausmeler, 1976).

Just as concepts are developed from concrete hands-on experiences, language also develops as a result of experience. If children are helped to move from experience to concept to words, their language learning will reflect an understanding of concepts rather than isolated actions and/or experiences. Concept understanding gradually develops from "experience embeded in the social contexts of" daily lives, and words are then associated with the concept (Genishl, 1988, p. 20).

Memory for names, or verbal imagery, is listed as one of the processing styles that seems to be related to performance in mathematics (Fey, 1982). Fey also said that strategies for introducing names for concepts has been one of the more complex topics which is being studied by
researchers. Brown (1965) reported that "...the sequence in which words are acquired is not determined by the cognitive preferences of children so much as by the naming practices of adults" (p. 276). Gold (1984) speculated that some of the findings that Piaget reported may have been due to communicational deficits instead of cognitive deficits on the part of children. Gold states that the children may be responding as they think the adult wants them to, and do not really interpret the questions or tasks as the adult intended.

Smith (1984) in a series of three studies investigated the relationships between young (aged two through six) children's knowledge of attributes, of dimension, and of their linguistic terms. She defined attribute knowledge as isolation of a particular attribute of an item separated from the entire object. The example she used was that of red— which can be found in a variety of objects. Her definition of dimension knowledge was that there are qualitatively distinct kinds of attributes, the example used was that red and blue are attributes of the same kind, and they are of a different kind of attribute than that of big. Smith concluded that (1) Dimension knowledge develops early and is part of the three-year-old's usable conceptual knowledge; (2) children under three can infer rules about attributes; (3) very young children (age not given) possess abstract reasoning skills; (4) two-year-olds can
successfully complete simple analogies; and (5) for the size dimension, children acquire the concepts before they understand the linguistic terms, which is opposite from the order for other attributes.

In a related study, Raven and Gelman (1984) worked with three, four and five-year-old children on their concepts of big and little. The children's meanings for big and little were almost always in error. They usually used big as if it meant greatest height, and they usually used little as if it meant least height. The authors pointed out that the very generality of the concepts big and little may make it difficult for the children to form a clear picture. Compounding this difficulty may be the lack of corrective feedback, misleading input, and perceptual factors; as children develop during this age span, they acquire increasingly consistent use of rules.

Wales, Colman and Pattison (1983) used mothers and their two or four-year-old children to study the effect of contextual cues on the name the mother would assign to an object for her child. Contextual factors were found to have a primary role in determining mothers' selections of category names. The independent variables, taxonomy of words, age of child, and presentation context, were found to have a complex and dynamic effect on the naming. This kind of research looks at the naming of the object as a way of looking at the perception of the concept.
It could be speculated that the findings cited above may also apply to how preschool teachers name or do not name triangles in the classroom. Most preschool teachers do not call the skinny triangle that is a basic part of their unit-block-set a triangle, but use the children's name for it, which is "ramp". Two texts which are widely read by early childhood educators perpetuate the practice of not focusing on geometric concepts by labeling some "triangles" in the unit block set by their category name, but label another triangle as "ramp" (Hirsch, 1984, Maxim's, 1980, 1985 & 1989). The misnaming practice is so wide-spread that this particular triangle is also labeled a ramp by unit block manufacturers. One speculation is that adults rationalize this naming practice to avoid confusing the children with three categories of triangles (but they use four categories of rectangles in the unit-block-set)!
Misconceptions, Naive Theories, and Premature Closure

"Lack of understanding of concept learning/teaching process" could contribute to student failure to learn concepts. Concepts need to be taught differently than facts (Selger-Ehrenberg, 1985, p. 161).

Probably the most widely known research on misconceptions was conducted by Erlwanger (1973) on children's learning of mathematics with Individually Prescribed Instruction. A case study of a boy named Benny showed that he had derived his own rules over a four year period, and that his teachers were unaware that he had developed many misconceptions.

Many studies have been conducted which demonstrate the hierarchical nature of concept learning in the content areas of mathematics. At a most basic level is the need for correct facts before application of them is possible. A reported example is that the effects of imprecise knowledge (incorrect facts) of a geometrical concept leads to specific faults in solving tasks which require the correct application of the concept.

This conclusion was reported by Schmidt (1980) who conducted a study in Germany with fifth graders on the topic of "axial reflection". Schmidt reported on a study of ten-year-olds and the effects of previous knowledge on learning geometry. One of his findings is that "Effects of imprecise knowledge of a geometrical concept ... leads to
specific faults in solving tasks which require the correct application of these concepts" (p. 191).

In the Encyclopedia of Educational Research, Fey (1982) explained the effect of prior knowledge by saying that, "The learning of mathematics seems to be driven so strongly by innate mental abilities, the background of previously acquired knowledge, and the internal structure of the ideas themselves that few short-term or moderate alterations in the teaching approach have any noticeable impact on student achievement" (p. 1178).

There has been much interest in the 1980's in understanding and preventing the development of misconceptions. AERA Division B sponsored a seminar on misconceptions or alternative conceptions and frameworks held by children (Novak, 1987). Untaught children's models are difficult to unlearn and, furthermore, "students defend them strongly, even in light of class demonstrations that show their inaccuracy" (Peterson, Wittrock, and Covington, 1986, p. 62).

Researchers and teachers have noted patterns of difficulty with the learning of concepts. One pattern is that within an age group and subject-matter area, invisible barriers seem to impede concept learning for many students (Peterson, Wittrock, and Covington, 1986, p. 64). These invisible barriers have been described as (1) "brick walls" which limit ability to comprehend, and (2) "roadblocks"
which are "regular discontinuities in a pattern of general class mastery" (p. 65).

Students often develop 'naive' theories before they receive their first instruction and these theories interfere with learning concepts (Resnick, 1983). Resnick's view of learning is: (1) learners construct their own understanding, (2) understanding something is built on knowing relationships, and (3) all learning depends on prior knowledge. The implications for educators that Resnick lists (p. 487) are: (1) it is "never too soon to start", (2) "teaching has to focus on the qualitative aspects of scientific and mathematical problem situations", and (3) "since naive theories are inevitable,... confront them directly." Mintzes' (1984) plan of attack on naive theories is similar to Resnick's.

Recent research in 'cognitive science' (an area that includes the work of cognitive psychologists, computer scientists, linguists, and philosophers) reflect the problems that teachers encounter. Resnick (1983) reports that: (1) students cannot apply laws and formulas, (2) mathematical problem-solving skills lag behind expectations, and (3) students have extensive 'naive' theories" (pp. 382-83).

Hill and Redden (1984) reported that fifth grade Australian students had a common tendency to reach a conclusion before considering all aspects of a situation.
The students used only part of the information which was available and they failed to account for some important factors. Hill and Redden give Wollman credit for calling this phenomenon "premature closure."

**Example, Non-Example, and Prototype**

Research on learning from examples vs non-examples has helped describe how children learn concepts, as well as how using non-examples helps form the concept (Ausubel, Novak and Hanesian, 1968). Shumway and White (1980) reported on a study that they designed to manipulate the frequencies of levels of irrelevant attributes across positive and negative instances of a concept. They suggested that the use of non-examples was more important in mathematical research than in psychological research because of the many compounding irrelevant factors associated with mathematics learning. These compounding irrelevant factors were listed by Driscoll (1980) who offered this explanation: "Until they are about seven or eight ... children are easily swayed by distractions as their minds organize perceptions" (p. 11). Driscoll went on to say that background color, shape, density, and sound can all distract from the intended stimulus.

Cohen and Carpenter (1980) confirmed that examples and non-examples produce better results than examples alone.
They went on to report that the order of presenting examples and non-examples seemed to have no effect on concept formation. The Cohen and Carpenter study was on geometry with older students.

Burts, McKinney, Ford, and Gilmore (1985) reported on the effects of the presentation order of examples and non-examples on first-grade students' acquisition of coordinate concepts. Their findings from several studies indicate that a simultaneous presentation of examples and non-examples is more effective than successive presentations (p. 310). One study compared the effects of presentations which were sensitive to response presentations of examples with insensitive responses (random or determined prior to the lesson). Results of the study indicated that response-sensitive presentation for teaching a single concept was more effective than random presentations.

Tessmer and Driscoll (1986) conducted an empirical study with junior and senior high students. The study used hierarchical concept trees, with words to define the steps. The article referred to three researchers (Markle, Tlemann, & Anderson) who recommend the use of non-examples to test concept understanding.

Flavell (1982) said that, "We lean toward typologies, prototypes, and other complexity-reducing categorizations" (p. 6) in order to reduce the stimulation provided by our environments. In a research project reported by Tennyson,
Youngers and Suebsonthi (1983) the authors described the best example as a prototype and concluded with the recommendation that teachers use a prototype when teaching concepts. Their study was on third-grade students. The implications from their research are for teachers to include prototypes of a variety of triangles when presenting the concept. The "ramp" in the unit block set could be labeled as a triangle as well as a "ramp."

In another study, Tennyson and Cocchiarella (1986) used a concept teaching model which also included the use of example and non-example. Four different categories of concepts were selected, based on an analysis of complexity of content. The items were presented as: expository-only form, interrogatory-only form, or with expository-interrogatory form. The best results were obtained when both the expository and interrogatory form of testing was used.

Overall, the reported research supports the inclusion of non-examples when presenting and/or teaching a concept. The use of non-examples can help children distinguish between relevant and non-relevant features. Presenting the best example as a prototype of the concept is reported to be effective. Simultaneous presentation of examples and non-examples is more effective than successive presentations.
Many researchers and educators agree that young children learn geometry best when they manipulate objects (Choat, 1978, Kamil, 1985). Much of the relevant literature also mentions the importance of informal mathematics in the child's growing knowledge base. Informal mathematical thinking is not generally learned at school, and young children know more than they had been given credit for (Gannon & Ginsburg, 1985). Glenn (1979) presents an ecological view of living in a geometric world, and being aware of the shapes around us, and using one's senses to gather information on how things fit together and the way things move and change position and direction. Glenn says that teachers should build on children's intuitive knowledge of geometry!

Driscoll (1980) wants teachers to build bridges from concrete to abstract (pp. 11-15) and from informal and concrete to more abstract. He says mathematics in kindergarten should be taught at the preoperational level. Young (1982) provides geometry activities for kindergarten through the eighth grade. "One of the roles for geometric activities is to enhance the student's spatial abilities ... ways we perceive are shaped by our experiences, culture, and education" (p. 38).
In agreement with Young's work is that of Choat (1978) who also contends that understanding stems from experiences. "Mathematics originates from the earliest physical, mental, emotional and social activities of children by their seeing and understanding the world around them" (p. 13).

Bellin (1985) wrote about geometry structures and processing strategies in young children, and described two classes of cognitive theories: (1) stage theories including Piaget's theory, and (2) theories based upon processes and functions including information processing theory. Bellin described a study of selected strategies and successful performance. He cautions that heuristics are solution procedures but using them will not guarantee a solution.

Davis and Silver (1982) presented an overview of recent activity in the field of mathematics education, including a shift from focus on product to focus on process. They said that both the process of learning and the child's thought process were important. They also discussed findings that children don't always make steady upward progress, and in fact sometimes lose ground or regress. "This phenomenon of temporary dips (which may last as long as several years) has recently been referred to as 'U-shaped development'" (p 1160).

The work of Piaget in regard to developing the sense of space and developing geometric concepts is so important and broad that one can hardly do it justice by including it with
the other relevant literature. Piaget, Inhelder, & Szeminska, (1960) wrote The Child's Conception of Geometry. This volume focuses on measurement and metrical geometry: conservation and measurement of length, rectangular coordinates, angles and curves, areas and solids. Kamil, (1985) studied and wrote with Piaget and has done more for interpreting his theories into a form understandable and usable by early childhood educators than any other author. Another author who helps teachers understand and use Piaget's theories is Lovell. One of his articles includes six assumptions about Piaget's cognitive development system: (1) general way of knowing, (2) basic operations of uniting, seriating, equalizing, etc., (3) thinking is an action that transforms reality which leads to knowledge, (4) stage concept - successive development, (5) language & thought, and (6) mathematical learning depends on development (Lovell, 1972).
Types of Geometry:

Topology, Projective, Transformational, Euclidean

The hierarchy of understandability of geometry proceeds from primitive topological space to projective space and finally to Euclidean space (Piaget & Inhelder, 1956). Topological understanding is the perception of relations such as proximity, separateness, continuity, open vs closed, but with no conservation of features such as distance, straight lines, angles.

Projective geometry is concerned with the conservation of straight lines and angles, and with relationships where there is "inter-co-ordination of objects separated in space" (Piaget & Inhelder, 1956, p. 154). This co-ordination allows the development of estimation of apparent size and shape when transformations take place. "Transformational geometry is concerned with the movement of geometric figures" (Horak and Horak, 1983, p. 14). The ability to visualize simple movements of a geometric figure will help solve many types of everyday geometric problems. Euclidean geometry is three-dimensional, deals with distances and measurement, proportions, conservation of angles, size constancy, and perceptual coordinates like the concept of parallels.

In agreement with Piaget, Copeland (1984) argues that topological understanding precedes Euclidean geometry. "A
triangle and a square are not congruent, but the three-year-old does not see this difference in Euclidean relationships" (p. 219). "There are many geometries, and in some of these length and straightness are not important [this is]...called topology" (Robinson, 1975, p. 209). The link between Piagetian theory and practice is made by providing activities of a topological nature for young children (Bass, 1975).

Manipulative materials are often recommended for developing basic geometric concepts. Horak and Horak, (1983) gave directions for teachers on making geometry tiles from heavy cardboard or flexible floor tiles. They included activities for developing: (1) topological concepts; open / closed curves, inside / outside, (2) Euclidean concepts; figures, polygon, quadrilateral, congruence, and similarity, and (3) Transformational geometry; slide, rotation, reflection.

Beginning geometry instruction with topological and projective geometry is in keeping with Piagetian research which suggests that Euclidean concepts are slow to develop and build on the other two. Robinson (1975) reported that when geometry is taught in elementary school from a topological perspective, it is not equated with the problems encountered by high-school Euclidean geometry students as they work with: perimeter, area, congruence, straightness of side, and measurement.
Rosser, Horan, Mattson, and Mazzeo, (1984) discussed the comprehension of Euclidean space by young children. They studied the early emergence of understanding and its limits, and reported the following: (1) young children did not pay attention to rotational difference of shapes, and (2) "males were less attentive to display orientation cues than were females" (p. 23).

van Hiele Levels of Geometry

The van Hiele levels of geometric understanding have been widely studied and used with research on children's geometric concepts. Wirszup (1976) credited the van Hiele work as a breakthrough in the psychology of learning and teaching geometry. The van Hieles developed two different reference systems. The first is the conception of learning process and teaching methods which lead to higher levels of thought: (1) information, (2) directed orientation, (3) explanation, (4) free orientation, and (5) integration. Language is an important factor in this process.

The second van Hiele reference system is the more widely known and quoted. This second system is the levels or stages of development of geometric concepts. The van Hiele levels are: (0) recognition of shapes by their appearance, (1) analysis of shape in terms of their parts and properties, (2) logical ordering of properties and informal proofs, (3) deductive reasoning in an axiomatic
geometry system, (4) rigorous study of axiomatic geometries. The order of these van Hiele levels is set and is dependent on education (Fuys, 1984).

Lunkenbein (1984) studied van Hiele level 0 in depth with a look at interior structures of geometrical objects. The study dealt with transformations within the shapes. "Static" elements represented instances of a concept, and "dynamic" elements were called operations or transformations which acted on the states or objects and their relationships to each other. Infrastructural groupings, characterized by the spatio-temporal aspects of states and operations, describe more precisely the underlying visual or spatial structures of van Hiele's first (visual) level.

Wirszup (1976) reported on a study of teaching geometry in Russia, and changes that were made to take advantage of the knowledge gained from the van Hiele research. Another application of the van Hiele levels has been in the development of the VAN HIELE GEOMETRY TEST by Usiskin (1982) for secondary school geometry research.
There is not much literature on the development of the concept of triangles. Some of the research cited already in this chapter included triangles as examples of shapes or as objects to distinguish (Frith, 1980, Ghent Braine, 1973, Klausmeler, 1976, Tennyson & Coccharella, 1986, Wilson, 1982). The van Hiele's used triangles as an object to be distinguished in their research. Clements (1984) used triangles as an example of children knowing a definition but not applying it in a demonstration of the conceptual difficulties young learners experience in mathematics.

One article was found in the literature dealing specifically with triangles, "What makes a triangle?" (Damarin, 1981). The article says that, "Many children are familiar with only a few triangles" (p. 39) and that "Their concepts of triangles do not include" various orientations (p. 39). The article gives activities to help children learn the broader concept.

One older set of materials designed to teach the concept of triangle provides a tie from theory to practice. McMurray, Bernard, and Klausmeler (1974) produced lessons designed to teach the concept of equilateral triangle to fourth grade students. The lessons were based on the following recommendations from concept learning research: use definitions, concept examples, set of examples and
nonexamples, pairing of examples and non-examples, emphasis of relevant attributes, teaching strategy, feedback, and active involvement by student. Two lesson booklets on equilateral triangles were developed for the students to work some pages, then check their own work.

Laurendeau and Pinard (1970) conducted indepth studies on children's concept of space using five of Piaget's tasks. Included with the shapes they used for objects to recognize was an equilateral triangle with a horizontal base. They gathered extensive data for children from preschool to age 12 on the recognition of this triangle; their study included 50 children at each grade level. The data from the Laurendeau and Pinard study will provide a base for comparison of the hierarchy being developed as a part of this dissertation; the equilateral triangle with a horizontal base and the age/grade ranges are the same.

The literature on triangles does not report studies on the development of the concept, with the exception of the pilot study reported by Shelton (1985). This lack of research demonstrates a gap in our knowledge base on the formation of the concept of triangles.
Computers and Concept Learning

"The formation of mathematical concepts is conditional on the learner's active involvement with the subject matter during the process of learning. Theories of learning indicate that the required mental activity on the learner's part can be stimulated by suitably structured (re)presentations of the concepts to be learned. These structures are most easily implemented on a computer using the idea of a microworld: an environment whose objects the learner transforms by his actions" (Dreyfus, 1984, p. 239).

Computers in education have received a lot of publicity, and therefore the body of literature, which includes research reports, is very large. This review will cover some of the computer literature directly related to concept learning. It has been maintained by some that computer graphics, animation, and interactivity can be used to present some visual mathematical concepts in a way that makes the concepts less abstract and therefore more accessible to children than presentations without the computer (Damarin, 1987, Arneson & Kosel, 1981).

One area of computer applications for concept learning has been with microworlds. Computer microworlds present environments with their own definitions and rules. Microworlds can provide appropriate learning for some concepts, as Dreyfus (1984) points out in, "How to use a computer to teach mathematical concepts". Two examples of
dynamic computer microworlds which were designed to teach concepts are Logo and "Rocky's Boots".

Lutus (1985) draws an analogy of having a mathematical partnership with computers as we have a literate partnership with books. Lutus coined a new word "cybermatics" to describe this math/computer partnership. He listed three categories of cybermatic applications. Category-one is the trivial use of the computer, where the solution could be done as easily with pencil and paper. Category-two is using the computer to do something we can do, but at an increased speed, like a "supercalculator." Category-three "provides a new perspective on mathematics. ... Cybermatics is now used to increase the efficiency and productivity of people" (109).

The teaching of cybermatics will present numbers by means of dynamic images that encourage further understanding. Geometry concepts may be presented first, because they can be visually simple and have many connections with the child's world outside the classroom. Concepts will be presented from general to specific and thus children in the future "will have the advantage over today's students of first seeing the whole of which the elements are a part. A typical student will emerge with a grasp of mathematics and practical skills in partnership with the computer that will make the present training system seem like intentional discouragement" (Lutus, 1985, p. 109).
Forman (1985) wrote about the value of kinetic print in computer graphics. Kinetic images on a computer screen are images which move and make transformations. He maintains that by "combining the point-fixing (static) quality of declaratives with the point-moving (dynamic) quality of procedures, children construct a symbol system that helps them to think flexibly about action patterns" (p. 21). He gives the example of the changes which take place when a fist changes into a hand. Where is your fist when your hand is open? Forman asks, "To what extent can children have more powerful thoughts [than] through the use of symbols that move (p. 20)?"

Motion geometry is recommended by Damarin as an appropriate use for computers in education. "Computer graphics promise to be an exciting mode in which to work on 'motion geometry' as well as processes in geometric reasoning" (Damarin, 1984, p. 64). Motion geometry can "present learners with many diverse examples of geometric concepts rather than a few stereotyped examples that textbooks can offer" (p. 64). Motion geometry incorporates the findings from research on concept learning. Damarin listed the following as major areas to consider: concept development, practice with the concept, application, and problem solving.

"Perhaps the use of mixed-mode multiembodiments or the interplay of concrete, manipulative objects and
representational forms such as the computer monitor display could facilitate the development of abstract understandings" (Damarin, Berlin, and White, 1986, p. 127). These authors state that in static conditions, change is not directly observed. Animation involves an observable change which is not necessarily or directly controlled by the learner. Software which allows the learner to manipulate or control observable change (animation) provides a learning environment commensurate with the characteristics of learners and learnings.

Summary

An examination of the literature on cognitive development, hierarchies of concept learning, geometry and computers demonstrates that at their intersection there is little information on how and when the concept of triangle develops.
CHAPTER III

METHOD AND PROCEDURES

Overview

The first research question was to establish a hierarchy of difficulty for recognizing diverse configurations and orientations of triangles. The second research question was to study the effects of appropriate computer software on children's breadth of understanding of the concept of triangle. Other questions regarded gender differences and use of advance organizers. The data collection was done over a seven week period and microcomputers were used as diagnostic and instructional tools in this study. A pilot study using software designed to teach the triangle concept had been conducted in a day care setting.

Pilot Study

The purpose of the pilot study was to see if a computer could help young children expand their preconceived conception of "triangle" when it included only those examples which are equilateral and "pointing straight up."
The sample included twelve children who had this pre-conceived conception of triangles. The youngest child in this sample was two-years-old and the oldest was five-years-old. The hypothesis was that young children could gain an intuitive understanding of the concept if it were presented in an appropriate way (Shelton, 1985).

The pilot instrument was the "Random Triangles" program from an early version of the Technology and Basic Skills (TABS): Geometry, Exploring Triangles Disk (Damarin et al., 1985). The "Random Triangles" program generated three randomly placed points on the monitor; then the points were connected to form a triangle. In the pre-publication version that was used, random placement of the points sometimes resulted in a straight line, in which case non-examples of the concept were displayed.

This early version of the "Random Triangles" program provided interactive semi-concrete activities for the children. When any key was pressed, three random points appeared on the screen. The child could look at them for as long as he or she wished. The children guessed whether there would be a triangle when the points were connected. When another key was pressed, the points were connected with lines. This shape stayed on the screen until a key was pressed, which cleared the screen and produced three new random points. Also, the children could keep any key
depressed and watch the random triangles flash on and off the screen.

A significant finding from the pilot study was that young children can develop an intuitive understanding of the concept of triangles which includes the universe of examples. One boy who was almost five-years-old had been working with the triangle program for about five minutes when he turned to the researcher, with wide-eyes and a look of surprise on his face, and asked, "You mean triangles come in different shapes!?" Eight out of the twelve children who worked with this program could correctly differentiate triangles from non-triangles.

These findings demonstrate that computer software can provide an effective way to teach variations of the triangle concept in appropriate ways for young children. Questions for further research are: (1) Since young children can learn the triangle concept at an intuitive level, why do so many children have problems? (2) What is the extent of the problems of recognition for specific configurations and orientations?

Population

The research population consisted of all the children attending an elementary school in central Ohio. The school had four classes each of kindergarten and grade
one, and three classes for each of grades two through five. There were 460 children attending the school. The school population was predominantly white middle class. Around 5% of the students were from cultural minorities, around 7% were from lower socioeconomic homes, and about 3% were from upper middle class homes.

Research Design

The classrooms at each grade level were randomly assigned to one of three groups. The first group consisted of two kindergarten and two first grade classes, and one class for each grade from two through five. Group one, the treatment group, received the pretest, treatment, and the posttests. The second and third groups each included one class from each grade level, kindergarten through five. Group two, included to control for effect of treatment, received both the pre- and posttests, without the treatment. The third group was used to control for effect of pretest, this group received only the posttest.

Instrumentation

"Shape Families" is the name of the computer assessment tool which was designed for this study. The researcher designed "Shape Families" and gave a flow chart
Table 3
Research Design

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>O₁</th>
<th>X₁</th>
<th>O₂</th>
<th>X₂</th>
<th>O₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R</td>
<td>O₁</td>
<td>X₁</td>
<td>O₂</td>
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<td>O₃</td>
</tr>
<tr>
<td>2</td>
<td>R</td>
<td>O₁</td>
<td>X₁</td>
<td>O₂</td>
<td>X₂</td>
<td>O₃</td>
</tr>
<tr>
<td>3</td>
<td>R</td>
<td>O₂</td>
<td>X₂</td>
<td>O₃</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. = group 1, treatment
2. = group 2, control for effect of treatment
3. = group 3, control for effect of pretest
R = random assignment, intact classroom to groups
O₁ = pretest, Shape Families I
X₁ = treatment, Exploring Triangles
O₂ = posttest I, Shape Families II
X₂ = treatment, advanced organizer mini instruction
O₃ = posttest II, Shape Families III

Table 4
Data Collection Schedule

<table>
<thead>
<tr>
<th>Week</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week one</td>
<td>pretest</td>
</tr>
<tr>
<td>Weeks two-five</td>
<td>treatment, once each week</td>
</tr>
<tr>
<td>Week six</td>
<td>posttest I</td>
</tr>
<tr>
<td>Week seven</td>
<td>mini instruction and posttest II</td>
</tr>
</tbody>
</table>

of it to an experienced programmer who developed the program in an enhanced form of BASIC. After the program was developed, graduate students from the college of education (experts) reviewed the program to judge whether it measured what it was intended to measure.

The triangles used in the program ranged from an equilateral with a horizontal base (classed as regular or
belonging to the basic level) to an obtuse triangle with a base at a 45 degree angle from the bottom of the screen. Non-triangle shapes with straight sides were included as non-example distractors. Three parallel forms of the assessment tool were developed for use as pretest, posttest I, and posttest II. The same sets of shapes were used in each version of the instrument but the order of shapes within the sets was rotated when they appeared on the screen. The specific shapes of examples and non-examples that were used are in Appendix A, Figure 22.

The instrument was designed so that data on the children and their responses would be saved on the program disk. At the beginning of each session the children typed in their names and selected their room number from a menu. The program recorded the student's name, room number, group number, and grade level.

A practice session with six sets of pictures was built into the instrument to insure that the children could manipulate the program. The escape key, return, and arrow keys were used to select answers. Each of the six practice sets contained three shapes. The practice session was based on having children select flowers from the presented shapes, which also included food, numerals, and boxes. A Koala Pad (a computer graphics tablet) had been used to draw colored pictures for the practice session. Screen dumps (pictures) of the six practice screens are in
Appendix A, Figure 25. Screen dumps showing a sequence of selecting a correct example are shown in Appendix A, Figure 26.

The practice session examples reflected most of the variety of choices that were on the triangle portion of the program. Children were provided with experiences in selecting one, two or no examples of the flower concept. Two of the practice sets contained one flower with two non-flowers, two sets contained two flowers with one non-flower, and two sets contained no flowers with three non-flowers.

The instrument was designed to begin at an appropriate level for each grade, kindergarten through fifth. Kindergarten and first grade children began with the most basic level examples, which was screen number seven (Appendix A, Figure 27). Second and third graders who made two or more mistakes during the practice session also started on screen number seven. Second and third graders with one or no mistakes on the practice session began the triangle portion on screen number 15. Fourth and fifth graders also began with screen number 15. At the end of short segments the program evaluated the student’s performance. If more than 40 percent of the examples were missed, data were saved on the disk, the student was thanked for playing and the session ended.
Testing Procedure

Procedures for testing were designed to keep the school schedule and arrangements for computer time as normal as possible. The school had 13 Apple computers, each with one disk drive, on movable tables. The computer lab was in a large and uncrowded section of the school's library.

Normally, one day a week the computers were divided between the kindergarten and first grade classrooms. The computers were rolled into the classrooms for the children. The other four days of the week all of the computers stayed in the computer lab. Each grade level, second through fifth, was assigned a specific day of the week for their classrooms to rotate going to the computer lab. Each of these classes had 90 minutes of computer time per week.

Teachers for grades two through five had two different procedures for their normal computer lab time. The first method was to have children work in pairs at the computers. The second method was to have half of the class work on the computers, one per machine, while the other half of the class spread out at tables or on the floor to do quiet work. The choice for many of the teachers depended on the software being used. An example is that fifth grade students worked in pairs with "Oregon Trail," but worked alone when using a word processor to enter their creative writing.
During the pre- and posttesting weeks the researcher brought in enough extra computers so that children would not have to share computers. All of the computers stayed in the library every day. The kindergarten and first grade children came to the library for their testing. Since these children normally worked with computers in their own classrooms, their schedule experienced the most disruption.

Because of the extra computers that the researcher brought in, all of the children from each classroom could do the "Shape Families" research at one time. Time was still available for regular classroom computer work. The research pre- and posttesting was not very disruptive for grades two through five.

Several college students who were majoring in elementary education and doing field work in the school helped get the computer lab set up and helped get the programs booted before children arrived. The college students also helped the younger students get past the practice session of the program, which required reading, typing, and pressing return.

When children from a class came to the lab for their experience with the "Shape Families" program, the researcher walked them through the first part as a group. This portion of the program involved entering their names, pressing return, selecting their room numbers from the
menu, and the first few screens of the practice session on selecting flowers.

Teachers were asked if they wanted the children sent back to the classroom when they finished or if they wanted the children to all stay together in the computer lab. When the teachers wanted the early finishers to stay, the researcher let the children work on the computer in BASIC.

On the posttests the children could skip the flower practice session if they remembered how to work the program. This option was not offered on the pretest.

Treatment 1

During the treatment phase, only the school's computers were used. The treatment classes worked with the "Exploring Triangles" program as their first activity during regular computer time.

The treatment consisted of computer activities which were designed to teach the broad concept of triangles. Two versions of the TABS: Geometry, Exploring Triangles Disk (Damarin et al., 1985) were used. Kindergarten and first grade children used the same pre-publication version that had been used in the pilot study. This version allowed user control and experimentation with a demonstration portion of the program, it has been explained in detail in the discussion of the pilot study instrument.
The second through fifth graders used the version of "Exploring Triangles" that had been published by Encyclopaedia Britannica Educational Corporation. This version had three different programs: "Random Triangles", "Three Points", and "Constructing Triangles." The level of abstraction increased with each program.

"Random Triangles" on this version had several options for control. The first was "Watch Triangles", which was a demonstration of random triangles being drawn on the screen, one after another. An option was available to make "Watch Triangles" interactive. With this option a generated triangle stayed on the screen until the learner pressed the spacebar to generate another example. Other options controlled sound effects and black or white background.

"Three Points" allowed the children to use game paddles to place three points, or 'dots' as the children called them, where ever they wished on the screen. When the last point was placed, the points were connected to make a triangle (unless the points were all in a straight line).

"Constructing Triangles" involved inputing numbers for the lengths of the legs of a triangle, and then using the game paddles to connect the ends of the legs to form a triangle. This activity was very challenging and helped children learn the principle that the "sum of the lengths
of the two shorter sides must be greater than the length of the third side" (Damarin et al., 1985, p. 17).

Classroom implementation ideas included with the program suggested giving the students paper triangles to use. The children in this study were given two sets of triangle cut outs during weeks three and four of the treatment period. The first set of paper triangles was large; some examples which were traced, photocopied and reduced are shown in Appendix B, Figure 28. The children used these large paper triangles to look for matching examples in the "Random Triangles" program and as models to duplicate with "Three Points." The second set of paper triangles was small and they were used by the older students with the "Constructing Triangles" program. These small triangles were used as aids in estimating and predicting when the children challenged each other to "make one like this". An exact size photocopy of one of the small sets is in Appendix B, Figure 29. The outlines of both sets of triangles have been traced because the triangles were colors that would not show up on the photocopy.

Procedures for the treatment phase were designed to be as easy as possible for the classroom teachers, that is, to require no extra work on their part, and to take no more time from regular computer activities than necessary. The teachers were invited to stay and watch; most of them
stayed just long enough to see what the children were doing. The children worked with the treatment program in pairs, or threes. The program was booted before children arrived, and new activities and materials were demonstrated to the entire class at one time. Teachers decided how to handle the early finishers. Some of the teachers came back to the lab at about the time the first children finished and distributed their regular software. Other teachers wanted the children to work with BASIC, and some had the children return to their classrooms.

Treatment 2

Mini-instruction was given to all children immediately prior to taking posttest II. The mini-instruction was an advanced organizer to help the children focus on triangles. The mini-instruction took only a few minutes. It consisted of some verbal hints or prompts to help children focus on what they knew about triangles having different configurations and orientations. The children contributed their ideas about orientation and three points in a line. For the most part they said they understood that orientation does not matter, as long as the shape stays true to its definition. Many children defined triangles as three dots with three straight sides and "a space in the middle."
Statistical Analyses

For Hypothesis number one on the hierarchy of difficulty of items, grade level percents for correctly identified examples of triangles were used. Posttest percents confirmed the hierarchy which was based on the pretest (correlation). CROSSTABS analyses were used.

Hypothesis number two was that the mean pretest computer scores would increase with grade level. Each correct selection of an example of triangle was given a count of one. The means were generated with BREAKDOWN on SPSSx.

For Hypothesis number three, that the treatment group would have a significantly higher increase of scores over control group one, ANOVA was used.

Hypothesis number four was that there would be no gender differences in any of the above analysis. ANOVA, BREAKDOWN, and CROSSTABS were all run with the sex variable.

Summary

The descriptive correlational design was chosen to help identify specific examples of the triangle concept that are problematic for children at different grade levels. The experimental design was used to examine the
effectiveness of using the computer to teach the triangle concept.

The first research question was to establish a hierarchy of difficulty for recognizing diverse configurations and orientations of triangles. It was thought that some examples of triangles are more frequently excluded as examples of triangles by children than are others and that these troublesome examples can be identified. A hierarchy of learning the triangle concept, including easily learned and troublesome examples was to be constructed.

A computer program was designed to assess the inclusion of 27 specific examples of triangles by children in their concept of triangle. For each triangle the percent of students correctly identifying it was computed. These percents were analyzed for the construction of the hierarchy. Correlational data from CROSSTABS procedure were used to analyze scores by grade levels.

The second research question was to study the effects of appropriate computer software on children's breadth of understanding of the concept of triangle. The major premise was that when young children are presented and/or interact with only a few specific examples of the triangle concept, many of them develop premature closure on the concept of triangle. The reported pilot study demonstrated that young children can acquire an intuitive understanding
of the triangle concept with the help of appropriate computer software.

Classrooms of children were randomly assigned by grade level to either a treatment group or to one of the two control groups. Three parallel versions of the assessment program were developed and were used as pretest, posttest I and posttest II. Treatment consisted of four weeks of using computer software designed to teach the concept of triangles. ANOVA procedures were used to analyze the experimental data. All statistical procedures were run for the gender variable.
CHAPTER IV

FINDINGS

This chapter presents the findings for the two main research questions: what is the hierarchy of examples, and what is the effect of treatment used in the study? The data were collected on the "Shape Families" computer disks as the subjects interacted with the program (the testing instrument). The data were transferred from the program disks and combined into data sets for each of the three tests. The data were then uploaded from the microcomputer to a mainframe computer, and combined into one file for analysis using SPSSx. The .050 level of significance was used for all analyses.

Sample size for the pretest was 290, which included the treatment group and control group 1. The posttests included students from the treatment group and both control groups. Posttest 1 sample size was 378 and posttest 2 sample size was 384.
Analysis for Hierarchy

Establish Identification Codes

Before addressing the research questions related to order of difficulty, identification codes were established for: the specific examples of triangles, screen numbers, screen positions, and sets of data for each triangle.

Twenty seven specific triangles were used, and are referred to numerically as they appeared on the pretest (Appendix A, Figure 22). Triangle examples in figures illustrating identification and/or difficulty order are photo-reductions of screen dumps from the "Shape Families" program.

Screen numbers appeared in the upper left corner of each graphic screen on the "Shape Families" program, providing a built-in reference. The screens were presented in the same numeric sequence for all three versions of "Shape Families." Screens one through six were for the practice sessions with flowers (Appendix A, Figures 25 and 26). Screen number seven was the first screen with triangles (Appendix A, Figure 27).

Three shapes were on each screen and positions of the shapes were identified from left to right as "a", "b", and "c." Screen numbers and screen positions were combined to form a test reference code. The test reference code is the screen number followed by the screen position; example "7b".
Each specific triangle occupied a different screen position on each test and therefore had three different test reference codes. A method of tracking was necessary to ensure a clear alignment of the three sets of data related to each specific triangle. Since each specific instance of triangle had a number, sets of data for each triangle were assigned the same numeral as the specific instance of triangle. Figure 23 in Appendix A contains all of the triangle identification information for each set. Figure 23 includes each specific triangle, test reference codes for all three versions of the instance, descriptions of the triangle (angle, side and orientation of base) and a drawing of the instance of triangle as it appeared on the screen.

An example of set information, for set 1, is presented in Figure 4. The test reference codes for set 1 represent the three presentations of the equilateral triangle with a horizontal base. This triangle appeared on screen number seven on all three versions of "Shape Families." The pretest identification code was 7b, the position in the middle of screen number seven. For posttest 1, the test reference code was 7a, the position on the left of screen number seven. Posttest 2 test reference code was 7c, for screen number seven with the position on the right of the screen.
Set 1: 7b, 7a, 7c
(test reference code: screen number and position)
(pretest, posttest 1, posttest 2)
equilateral (angle)
equal (sides)
horizontal (orientation of base)

Set 1's Identification Codes,
Triangle Description and Illustration

Figure 4

Order Difficulty

With the identification codes established for each triangle, the next step was to determine the order of difficulty for instances of triangle. Results from a CROSSTABS analysis on the pretest were used to establish the table of difficulty which is shown in Table 5. The percent of children at each grade level who had correct choices for each instance of triangle is listed. These percents of correct selection of instances of triangle are the basis for articulating the hierarchy. Data from posttest 1 were used to verify the relative order of difficulty of the specific triangles (see Table 6).
Table 5

Table of Difficulty, Percent of Correct Answers for Choosing Triangles, by Grade Level, on the Pretest

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<th>set number</th>
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<th>3</th>
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<td>98</td>
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Table 6

Table of Difficulty, Percent of Correct Answers for Choosing Triangles, by Grade Level, on Posttest 1

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<th>set number</th>
<th>grades</th>
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The hierarchy of difficulty of instances of triangles which children include in their concept of triangle was developed from these data. The hierarchy is pictured in Figure 5. The specific triangle examples were ranked based on the percent of correct responses for kindergarten children, then data were checked to see if the specific triangle was included as an example of triangle at a similar or a higher percent by children in older grades. Some adjustments were made for triangles which did not have an even progression of percents by grade level. The levels in the hierarchy reflect the overall order of difficulty children have of including specific examples as a member of the set of triangles.

Tables 5 and 6 reveal that eight specific triangles were correctly selected by 98 to 100% of the children from second through fifth grades. Triangle number 6, which had 98 to 100% correct for the upper grades, was near the bottom (level eleven) of the overall hierarchy. This oddity led to a reordering of the hierarchy.

The eight specific triangle examples that had 98 to 100% correct for second through fifth grades were ranked separately from the rest of the specific examples. The resulting split hierarchy of triangles contains two sections and ten levels (Figure 6).
Overall Hierarchy of Triangles, All Grades and Tests

Figure 5
examples with 98-100% correct responses, grades 2-5
1  (91/100%)

2 (74/100%)

3 (62/100%)

4 (57/100%)

5 (47/100%)

tf

examples with < 99% correct responses by all grades
6 (70/98%)

7 (60/93%)

8 (50/93%)

9 (40/91%)

10 (36/80%)

*(kindergarten pretest mean/highest mean all grades)

Split Hierarchy of Triangles

Figure 6
The top section contains examples of triangles which were 98 to 100% correctly selected by the second through fifth grades. The bottom section of the split hierarchy of triangles contains examples with less than 98% correct responses by all grades.

Within each section of the hierarchy, specific triangle examples were rank ordered based on percent of correct responses for kindergarten children's pretest scores. The triangles appear in Figure 6 from left to right on each row in decreasing order of percent correct.

The levels of the split hierarchy are explained and described next, the percents used are those from the pretest (Table 5). Graphs of pretest and posttest 1 percentages of correct selection of instances of triangle for the twenty seven sets of data are Figures 30 through 56 in Appendix C.

Level One, Triangle 1
Figure 7

Level one had the highest overall percentage correct for all grades. The kindergarten pretest percent correct was 91, however the first grade percent correct was only 78. It is important to note that grade two through grade five
had 98 to 100% correct on this triangle. Many 5-year-olds described this triangle as "on top of a house." Triangle 1 is in level one in both the overall hierarchy and the split hierarchy.

Level Two, Triangles: 4, 5, 8

Figure 8

Level two included the second highest percent of correct answers on the pretest; with 74 to 75% for kindergarten and first grade, and 98 to 100% for grades two through five. The examples in level two are either a rotation of Triangle 1, which is in level one, or they are a "skinny" version of Triangle 1, but with the same orientation. The triangles appear in Figure 8 from left to right in decreasing order of percent correct. This arrangement for displaying instances of triangles in decreasing order was used consistently for all levels.

Of special interest is that Triangle 4 is an "upside" down copy of Triangle 1, or a 180 degree rotation of it. Triangle 5 is a 90 degree rotation of Triangle 1. Both level one and level two graphs (Figures 30, 33, 34 and 37
are in Appendix C) show kindergarten and first grade scores close to each other and at the low end of the range of scores, and second through fifth grade scores at the high end.

Triangle 5 is in level two on both the overall hierarchy and on the split hierarchy. Triangles 4 and 8 shifted from level three on the overall hierarchy to level two on the split hierarchy. The move is of minor interest since the overall hierarchy has twelve levels and the split hierarchy has ten levels.

Level Three, Triangles: 2, 3

Figure 9

Level three consists of two instances of right triangles. Both of these right triangles have a horizontal leg as a base, and the orientations are either upright or pointing to the right. The percent of correct responses for kindergarten children on the pretest was in the 60s.

Triangle 2 had a minor shift from level four of the overall hierarchy to level three of the split hierarchy, but Triangle 3 had a shift up from level five.
Level Four, Triangle 7

Figure 10

Level four includes the third highest percent of pretest correct scores, with 57% correct by kindergarten and 100% by at least one upper grade. This level contains only one triangle example.

Triangle 7 is a 90 degree rotation of Triangle 1, this time the figure is pointing to the left. This is the first figure in the hierarchy, which is pointing to the left, to be included by the kindergarten and first grade children. The children in the study identified upright, pointing to the right, and even triangles with a base at a 45 degree angle from the horizontal before including this left orientation. However, the triangles at 45 degree angles were not correctly selected at 100% levels by any of the upper grades, so they occupy a lower (harder) level on the split hierarchy. Triangle 7 is in the fourth level on both the overall hierarchy and on the split hierarchy.
Level Five, Triangle 6

Figure 11

This fifth level has only one example, like levels one and four. The kindergarten pretest percent correct was in the 40s, but the percent correct was at the 98 to 100% level for grades two through five. Triangle 6 is a 180 degree rotation, or an "upside down" example, of Triangle 8 in level two.

Triangle 6 has extremely different positions on the two hierarchies. It moved from level eleven on the overall hierarchy to level five on the split hierarchy. The order difficulty for this specific triangle is extremely different for kindergarten and grade one than it is for the second through fifth grades.

The graph patterns for levels one through five are all similar (Figures 30 through 37 in Appendix C), with kindergarten and first grade low percents being followed by nearly straight lines for 98 to 100% correct for grades two through five. The kindergarten and first grade percents are close to each other and in fact the lines on some of the graphs do cross.
Level Six, Triangles: 26, 12, 24, 27, 17

Figure 12

Level six includes sets which were 70% correct for kindergarten on the pretest, but did not reach 100% correct by any of the other grades. The examples in level six all have their base at a 45 degree angle to the horizontal plane. Another characteristic of these examples is that they are all relatively "fat". A common name given to these triangles by children in the study was "pizza".

Triangles 12, 17, and 26 moved from level three on the first hierarchy to level six on the split hierarchy. Triangles 24 and 27 were on the fifth and sixth levels of the overall hierarchy so there was relatively no change of order difficulty for either of them.

Level Seven, Triangles: 21, 20, 23, 10, 9

Figure 13
Level seven has 60% correct answers for kindergarten pretest scores. The percents for the upper grades are in the 70 to 80% range. Level seven charts show more separation of pretest and posttest scores than was evident in any of the preceding levels.

Triangles 21 and 23 moved from level five of the overall hierarchy to level seven of the split hierarchy, while triangles 10 and 20 moved only one position, from level six. Triangle 9 was on level seven in both hierarchies.

Level Eight, Triangles: 19, 18, 15, 11

Figure 14

Level eight includes triangles with 50% correct for kindergarten children on the pretest. The older children topped out in the high 80s on these triangles. The graphs show definite separation of percents correct for the pretest and for posttest 1, with the posttest 1 percents all being higher.
Triangles 11 and 19 moved one level, from level seven on the overall hierarchy to level eight on the split hierarchy. Triangle 15 moved up to level eight on the split hierarchy from level nine on the overall hierarchy. The greatest move for level eight was Triangle 18's move down from level six.

---

Level Nine, Triangles: 22, 16, 13

Figure 15

Level nine kindergarten percents correct on the pretest were in the 40s and the upper percent correct for the other grades was 84. The triangle examples which fit into level nine are all characterized by relative skinny shapes (angles less than 20 degrees) and bases which are horizontal or at a 45 degree angle to horizontal. Like level eight, the graphs from this level show separation of pretest and posttest 1 percents, and the posttest percents are always higher.

Triangle 22 is on level nine on both hierarchies. Triangle 16 moved down from level eight on the overall hierarchy to level nine on the split hierarchy. The largest
shift on this level is for Triangle 13, which moved up from level twelve.

Level Ten, Triangles: 25, 14

Figure 16

The two triangles represented in level ten are both very "skinny" triangles, with acute angles less than 20 degrees. These triangles were in the lowest percentage correctly identified by all grade levels. On the pretest, the kindergarten children got 30% correct and the fifth graders got 80 to 83% correct. The graphs for these sets also show remarkable increases in mean scores for all grade levels from pretest to posttest.

Triangles 14 and 25 are in level ten on both hierarchies, but because the overall hierarchy has twelve levels, this reflects a relatively more difficult level on the split hierarchy.

Because of the eight triangles that second through fifth grade children correctly selected at 98 to 100%, it appeared that hierarchies should be constructed separately for kindergarten and grade one and for second through fifth
grade. These two additional hierarchies (Figures 17 and 18) were constructed using the percents correct from the pretest data (Table 5). The lowest and highest percents correct for each level are included on the figures. In the case of the kindergarten and first grade hierarchy (Figure 17), which includes only two grades, the lowest percent is given first. On the second through fifth grade hierarchy (Figure 18), the lowest percent from the four grades is given first, and the highest percent is given last.

The most obvious difference between the four hierarchies is the position of Triangle 6, which is an isosceles triangle with a horizontal top and a vertical alignment. Triangle 6 is in the top level of the split hierarchy and in the second level from the bottom on both the overall hierarchy and the kindergarten and first grade hierarchy. Because of the relative difficulty kindergarten and first grade children have with Triangle 6, it comprises the bottom (hardest) level on the top half of the split hierarchy.

Analysis of all four hierarchies show that the "upside-down" (horizontal top) triangles are included for each specific instance after the "upright" (horizontal base) orientation. Triangle 1's 180 degree rotation is Triangle 4. In all cases Triangle 4 was correctly selected by a smaller percent than Triangle 1. Another example with the same pattern is that of Triangle 6, which is a 180
Kindergarten and First Grade Hierarchy
Pretest Percents of Correct Responses

Figure 17
Second Through Fifth Grade Hierarchy
Pretest Percents of Correct Responses

Figure 18
Triangles: 1, 4, 6, & 8, Pretest Means of Percent Correct for Kindergarten and Grade One

Figure 19

<table>
<thead>
<tr>
<th>Triangle</th>
<th>Pretest Means of Percent Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>84.5</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>8</td>
<td>76</td>
</tr>
<tr>
<td>6</td>
<td>56</td>
</tr>
</tbody>
</table>

degree rotation of Triangle 8. Triangles 6 and 8 were separated by seven levels on the overall hierarchy and were two levels apart on the split hierarchy.

When percents correct for Triangles 1, 4, 6, and 8 are compared on Figure 17, it is evident that kindergarten and first grade children selected triangles in the following order: large angles with upward orientation, large angles with downward orientation, small angles with upward orientation, then small angles with downward orientation.

Figure 19 shows these four specific triangles and the means of percent correct on the pretest. The means of percent correct for second through fifth grade are: 98.5% for Triangle 8, 99% for Triangles 1 and 6, and 99.5% for Triangle 4. The second through fifth grade means are all so
close, and on the same level of their hierarchy, that the difficulty is the same.

Hierarchy Discussion

Hypothesis number one is that the order difficulty of items will not vary among groups or from pretest to posttest (although the absolute difficulty may). It was assumed that the data for this hypothesis would provide the difficulty of instances of triangles for establishing a hierarchy. Four hierarchies were determined and they help explain when and where children have difficulty progressing with the inclusion of specific examples of triangle into their own conception of triangles.

Several questions were asked and answered by this research. One of the questions was, "Is children's classification of triangles with various side and angle configurations affected by spatial orientation?" If so, "Which orientations of triangles give children from kindergarten to fifth grade difficulty in recognizing them as belonging to a class for which they recognize some members?"

The answer from this research is that once a triangle was accepted into a student's concept, 90 and 180 degree rotations did not appear to be a problem. Supporting this statement is the fact that kindergarten and first grade
children's second level of difficulty (72/85%) included triangles with 90 and 180 degree rotations of the first level, Triangle 1. The 45 degree rotation of Triangle 1 came in at the 70 to 80% level for kindergarten and first grade children. The pattern for including the rotations is the same for the upper grades.

"At what grade do children correctly classify triangles with short sides and/or angles less than twenty degrees?"

Pretest data for Triangle 25 can be used to illustrate the answer to this question. Triangle 25 has a horizontal base, one small angle, and one very short side. This triangle was correctly identified by 36% of the kindergarten children, 47% of first graders, 37% of second graders, 43% of third graders, 50% of fourth graders and 61% of fifth graders. A wobble of percents correct is visible on the set of data charted for Triangle 25 (Figure 54 in Appendix C). There is not a definitive answer to the above question, however, there is an increasing trend from a little over one-third correct for kindergarten to a little less than two-thirds correct for fifth grade.

Another apparent and important result from the hierarchy analysis is that some of the triangles in the test were not correctly identified by many of the fourth and fifth grade students. Triangle 16, which has a base at a 45 degree angle from horizontal, was correctly selected by just over half (57%) of the fourth graders and just over
three-fourths (77%) of the fifth graders. Triangle 14, which has a right leg at a 45 degree angle from horizontal, was recognized by less than half (48%) of the fourth graders and less than two-thirds (61%) of the fifth graders. Triangle 14 was in each grade level's lowest percent group.

Triangles with angles less than 20 degrees were characterized by drastic changes in percentages correct from pretest to posttest 1 by all grade levels, and had the lowest percents correct. These "skinny" triangles with a horizontal base, or a base at a 45 degree angle form horizontal, were the last to be included. Rotations of triangles seem to be not as much a problem as triangles with angles less than 20 degrees.

In summary: the overall hierarchy reflects the difficulty levels for all triangles by all grades and for all tests. This hierarchy was puzzling because of the relative position of Triangle 6. The split hierarchy provided a way of making sense of the overall hierarchy, but it was still somewhat confusing. The two hierarchies by grade level provide the cleanest view of the order of difficulty for the inclusion of specific triangles.

Hypothesis number one is that the order difficulty of items will not vary among groups or from pretest to posttest (although the absolute difficulty may). Hypothesis number one is accepted based on the data from treatment and control groups and pretest posttest analysis; however there are
Interesting deviations from this hypothesis in relation to grade level.

It is clear for grade level that the order of difficulty is different for some specific examples of triangles between the two hierarchies (Figures 17 and 18). Order difficulty does vary from kindergarten to grade five. Once the specific examples are included at a 98 to 100% level, they stay at this level. Based on the strength of the kindergarten and first grade hierarchy and the second through fifth grade hierarchy, the research hypothesis would be rejected.

Conclusions from analyzing the hierarchies are that triangles with short sides and/or angles less than twenty degrees have more negative impact on triangle recognition than does rotation. Children in the sample population first included large equilateral triangles with a horizontal base and they included smaller obtuse triangles last.
Analysis: Grade Level and Gender Differences

Hypothesis number two was that mean pretest computer scores would increase with grade level. Scores were derived from the children's answers for items on the "Shape Families" computer program. Even though the answers are not scaled, the answers were evaluated and recorded with the following numeric codes:

Table 7
"Shape Families" Answer Code

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>incorrect choosing (selecting a non-example)</td>
</tr>
<tr>
<td>2</td>
<td>incorrect rejecting (not selecting an example)</td>
</tr>
<tr>
<td>3</td>
<td>correctly rejecting (a non-example)</td>
</tr>
<tr>
<td>4</td>
<td>correctly choosing (an example)</td>
</tr>
</tbody>
</table>

Analysis of variance was run on the pretest data by grade using a procedure to count correctly chosen answers, which were number 4 responses. The results are on Table 8, and the ANOVA table (Table 19) is in Appendix C. One can see that the overall trend is for mean scores to increase with grade level. Because of the closeness of mean scores for adjacent grades the overall scores show a staggered increase. These staggered increases in the order of increasing scores for grade levels indicate that learning plateaus may exist.
Table 8
Pretest Means of Correct Choosing by Grade

<table>
<thead>
<tr>
<th>grade</th>
<th>mean</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
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</tr>
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<td>1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19.07</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18.45</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>21.80</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>23.70</td>
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<td>*</td>
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<td></td>
</tr>
</tbody>
</table>

* indicates significant difference in mean (P < .05)

Table 9
Posttest 1 Means of Correct Choosing by Grade

<table>
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<tr>
<th>grade</th>
<th>mean</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>1</td>
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<td>2</td>
<td>22.05</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>21.30</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
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<td>4</td>
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<td>*</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

* indicates significant difference in mean (P < .05)

In order to try to gain a better understanding of these staggered increasing scores for grade level, more procedures were run and analyzed. Mean computer scores were analyzed in several different combinations. Procedures were run on the two posttests and with all three tests combined. Tables
9 and 10 show means for posttest and the analysis of variance tables are Tables 20 – 23 in Appendix C.

When comparing the pretest results in Table 8 with the posttest 1 results which are in Table 9, seven changes are apparent. Undoubtedly the kindergarten drop in mean is a contributing factor for the changes between kindergarten mean and first through third grade means. The first grade mean stayed essentially the same but the second through fifth grade means increased. This relative shift accounts for the posttest significant difference between first grade mean and the second and third grade means. The grade two mean increased more than the grade five mean, thus reducing their difference to a non-significant level. The fourth grade mean increased more than the third grade mean, to a point of significant differences.

Table 10

Posttest 2 Total Score Means by Grade

<table>
<thead>
<tr>
<th>grade</th>
<th>mean</th>
<th>K 1 2 3 4 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>15.55</td>
<td>*</td>
</tr>
<tr>
<td>1</td>
<td>15.92</td>
<td>* *</td>
</tr>
<tr>
<td>2</td>
<td>21.87</td>
<td>* *</td>
</tr>
<tr>
<td>3</td>
<td>23.04</td>
<td>* *</td>
</tr>
<tr>
<td>4</td>
<td>24.78</td>
<td>* * *</td>
</tr>
<tr>
<td>5</td>
<td>24.89</td>
<td>* * *</td>
</tr>
</tbody>
</table>

* indicates significant difference in mean (P < .05)
Results of posttest 2 are shown in Table 10. These data support the hypothesis that the score means will increase with grade level. For posttest 2 each grade level has a higher mean score than all grades lower than itself. There are significant increases for grades two through five as compared to kindergarten and/or grade one.

Table 11 has results of the analysis of total mean scores over all three tests. Again, with the exception of the second-third grade plateau, the mean scores are in the expected order. Grade two scores showed a significant increase over both kindergarten and first grade scores. Grade three scores were also significantly higher than both kindergarten and first grade. Scores from both fourth and fifth grades were significantly higher than scores for

<table>
<thead>
<tr>
<th>grade</th>
<th>mean</th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
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</tr>
<tr>
<td>1</td>
<td>17.51</td>
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<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>2</td>
<td>21.23</td>
<td>*</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>3</td>
<td>21.13</td>
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<td>4</td>
<td>23.48</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>24.29</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

* indicates significant difference in mean (P < .05)
significantly higher than scores for Kindergarten, first, second, and third grades. Grade five scores were higher than grade four, but not significantly.

In all of the preceding tables one can see plateaus in score increases. Analysis of the combined test scores (Table 11) show the grades falling into three cluster groups, as shown in Table 12. The clusters are characterized by no significant differences between their own scores, but each is significantly different from all grades not in the cluster. Kindergarten and grade one formed the first cluster, with the highest internal variance of any of the clusters. The second and third grades formed the next cluster with the smallest internal difference. The last cluster was the fourth and fifth grades.

Table 12

Within Cluster Differences of Grade Mean Scores

<table>
<thead>
<tr>
<th>cluster 1</th>
<th>cluster 2</th>
<th>cluster 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>K. 15.87</td>
<td>2. 21.23</td>
<td>4. 23.48</td>
</tr>
<tr>
<td>1. 17.51</td>
<td>3. 21.13</td>
<td>5. 24.29</td>
</tr>
<tr>
<td>1.64</td>
<td>- .10</td>
<td>.81</td>
</tr>
</tbody>
</table>
Discussion for Increasing Scores With Grade Levels
and Gender Differences

In general, scores do increase with grade level but they do so unevenly. The results of analysis for total combined scores showed significant increases between 12 of the 15 possible combinations. The hypothesis that scores will increase with grade level can be supported on an overall base by looking at the clusters. These data show that second and third graders had significantly higher scores than kindergarten and first graders. The data also showed that the fourth and fifth graders had significantly higher scores than the second and third graders. With logic, and supporting data, this also means that the scores of fourth and fifth graders were significantly higher than the scores for kindergarten and first grade. The other sets of analysis also fit into this pattern. The only grades with scores out of order were within the clusters.

Research hypothesis number four was that there would be no apparent gender differences on any of the analyses. Mean score increases by grade level were examined for differences between males and females. ANOVA was run on pretest treatment group scores by gender and grade. The ANOVA table (Table 24) is in Appendix C. Table 25 in Appendix C shows ANOVA data for gender by pretest and posttest. Many other tests were run seeking gender differences; neither main
effects nor interaction with other variables were determined (Table 26 in Appendix C). Analysis of effects related to gender do support hypothesis number four which states that there will be no gender differences in the analyses.

Analysis for Effect of Treatment

Hypothesis number three states that at each grade level posttest scores will be significantly greater than pretest scores for the treatment group, but not for control group I. A 3-way ANOVA was used for the analysis with total scores by group, by pretest-posttest, and by grade. The results are on ANOVA Table 27 in Appendix C. Other procedures which are not reported here were run on SAS and SPSSx, with the results supporting the ANOVA outcomes. All of the analyses failed to show significant effects for treatment even though there were significant effects between groups (Table 28 in Appendix C).

SPSSx BREAKDOWN procedure was run on the data and the results are displayed on Table 13 and in Figure 20. Expected results were that control group II posttest 1 mean scores would be the same as the pretest mean scores for the treatment group and control group I. The unexpected results are that control group II posttest 1 scores were in many instances higher than the treatment group and/or control group I mean scores.
<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std dev</th>
<th>number</th>
<th>kindergarten</th>
<th>grade one</th>
<th>grade two</th>
<th>grade three</th>
<th>grade four</th>
<th>grade five</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>post 1</td>
<td>post 2</td>
<td>post 1</td>
<td>post 2</td>
<td>post 1</td>
<td>post 2</td>
</tr>
<tr>
<td>treatment group</td>
<td>17.54</td>
<td>7.87</td>
<td>35</td>
<td>16.40</td>
<td>9.16</td>
<td>20</td>
<td>7.65</td>
<td>8.65</td>
<td>38</td>
</tr>
<tr>
<td>control group I</td>
<td>15.78</td>
<td>8.20</td>
<td>18</td>
<td>19.10</td>
<td>20.93</td>
<td>15</td>
<td>22.72</td>
<td>20.94</td>
<td>16</td>
</tr>
<tr>
<td>control group II</td>
<td></td>
<td>-</td>
<td>20</td>
<td>22.13</td>
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<td>20</td>
<td>24.76</td>
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<td></td>
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<td>5.49</td>
<td>23</td>
<td>21.35</td>
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<td>22.13</td>
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<td>20</td>
</tr>
<tr>
<td>treatment group</td>
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<td>6.40</td>
<td>23</td>
<td>6.64</td>
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<td>22.13</td>
<td>22.40</td>
<td>20</td>
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<td>5.38</td>
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<td>20</td>
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<td>control group II</td>
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<td>-</td>
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<td>24.80</td>
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<td>20</td>
<td>24.76</td>
<td>24.74</td>
<td>20</td>
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<tr>
<td></td>
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<td>23.88</td>
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<td>25.17</td>
<td>1.44</td>
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<td>23</td>
<td>3.72</td>
<td>4.42</td>
<td>21</td>
<td>25.17</td>
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<td>control group II</td>
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<td>23</td>
<td>4.33</td>
<td>4.92</td>
<td>21</td>
<td>25.17</td>
<td>1.44</td>
<td>20</td>
</tr>
</tbody>
</table>
Figure 20

Group Means by Grade for Each Test
Discussion of Treatment Effect

Hypothesis number three was that grade level scores for the treatment group would be significantly greater than scores for control group I at each grade level. The hypothesis is supported at the .05 level by the kindergarten and fifth grade, but not by the other grades. The first grade treatment group gain in score is higher than the control group, but not significantly (with the 3-way ANOVA). Grades two, three and four treatment group increases in scores were all less than the increases for control group I.

One question which emerged was: "Why are control group II posttest 1 scores at the same general level or above scores for control group I?" Speculations abounded while trying to answer this question.

Was something wrong with the pretest version of "Shape Families"? Close examination of the program and the data output revealed no malfunctions. Had the children been placed in classrooms based on levels of their background knowledge and/or experience? If this was the case, is it feasible to believe that the "top" classrooms were all randomly selected to be in control group II, and the "low" classrooms randomly selected to be in the treatment group? The odds are against this and the school principal did not mention such a placement of students when discussing random assignment of classrooms to treatment or control groups.
Maturation does not answer the question because of the within cluster reversals of scores, where younger children have higher mean scores than older children.

Contamination in the study may account for some of the unexpected results. All of the classes were in the same school, and children who were not in the treatment group interacted with the treatment group children on the playground, etc. An example of contamination was that in the halls, children from the control groups referred to the researcher as the "Triangle lady!"

At this point, the hypothesis is rejected which states that posttest scores will be significantly greater than pretest scores for the treatment group but not for control group I.

Discussion of Hierarachy Examples and Treatment Effect

Since children had expressed delight in gaining a broader understanding of the concept of triangle while working with the "Exploring Triangles" disk, the lack of significant treatment effects did not make sense. The data were then analyzed in a different way.

The pretest and posttest 1 percentages of correct selections were analyzed separately for the treatment group and for control group I using CROSSTABS procedures. The data for the treatment group and control group I are on
tables 15 through 18 in Appendix C. Results of the CROSSTABS procedure show more increases of score mean for triangle examples from the lower (harder) portion of the split hierarchy for the treatment group than for control group I. The higher increases of percent correct for the treatment group is explained as treatment effect.

Difference between pretest means and posttest 1 means were calculated separately for treatment group and control group I for each instance of triangle from the bottom section of the split hierarchy (Figure 6). For each specific triangle the differences were then calculated between the treatment group and control group I. Figure 21 displays the resulting differences in gain between pretest and posttest 1 for the two groups and the percent of gain the treatment group had over control group I. The highest percent of gain in percent is 22.2% for triangle 14.

Triangle 14 also happens to be in the bottom level on the split hierarchy. These results indicate that, for triangle 14, the effect of treatment was 22% more effective than no treatment. As one looks down the triangle examples in Figure 21, the inverse relationship of order difficulty and effect of treatment becomes evident. The more difficult the triangle is, the greater the effect of treatment.

Figures 22 and 23 contain specific triangles where the treatment group did not have a gain of percent over control group I. There are three specific examples (Figure 22)
where the two groups had the same percent of increase and four specific examples (Figure 23) for which control group I had a higher percent of gain. Triangles which had the larger angles tended to be selected with higher percent gains for the control group than for the treatment group. This higher increase in percent of gain for control group I could be a pretest effect.

The data support a general statement that treatment effect increases along with order difficulty on the hierarchy of specific examples of triangles.
<table>
<thead>
<tr>
<th>set</th>
<th>Treatment group gain</th>
<th>Control group I gain</th>
<th>Treatment group gain over control group I</th>
<th>Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>37.5</td>
<td>15.3</td>
<td>22.2</td>
<td></td>
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<td>29</td>
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<td>13</td>
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Treatment Group Gains Over Control Group I, Pretest To Posttest

Figure 21
CHAPTER V

SUMMARY, DISCUSSION AND CONCLUSIONS, RECOMMENDATIONS

Summary

The main thrust of the dissertation was to investigate children's acquisition of the concept of triangles, as a complete concept rather than as a small set of examples. It has been well documented that many children have difficulty with the concept of triangle, but a hierarchy of triangle by order of difficulty has not been documented.

The literature was reviewed on developmental theory as it applies to geometric concept learning, concept definitions and concept learning, hierarchies, triangles, and microcomputers in relation to concept learning. Concept learning was subdivided into the following sections: development of a concept, visual, linguistics, example and non-example, and prototype.

A pilot study had been conducted in a child care center, with children ranging from two through six years of age. A dynamic and interactive computer program which presented diverse examples of the triangle concept was used by the children. Results of the pilot study indicated that
young children can form an intuitive understanding of the triangle concept regardless of orientation or configuration.

The population for the dissertation research was all of the students attending one elementary school in central Ohio. Classrooms at each grade level were randomly assigned to either the treatment group, or to one of the two control groups. The first control group was used to check for effect of treatment, and the second control group was to check for effect of pretest. Final results included data on almost 400 students, from kindergarten through fifth grade.

The independent variables were: (1) grade, (2) gender, and (3) specific examples of triangles with (a) configuration combinations based on angles and length of sides, and (b) orientation of figures. The dependent variable was the assessment score.

This study was comprised of two major parts. The first part was to develop a hierarchy of specific examples of triangles as children include them in their concept of triangle. The second part of the research was an experimental study investigating the effects of computer software for facilitating progress through the triangle hierarchy.

Three versions of a computer program, named "Shape Families," were developed and used as the testing instrument for the study. The program was designed to collect and save the children's responses on the program disk.
Hypothesis number one was that order difficulty of items would not vary among groups or from pretest to posttest. This hypothesis required the construction of a hierarchy of the order of acquisition of the triangle examples. Four different hierarchies were actually uncovered and interpreted.

An overall hierarchy was established using data collected on the pretest and was confirmed and adjusted with data from both of the posttests. During the construction of the overall hierarchy, it was discovered that the examples of triangles fell into two distinct categories. The first category included "easy" triangles, or those that second through fifth grade children correctly identified 98 to 100% of the time. The second category included "hard" examples, those that were not selected by 98% of the children in any grade. A split hierarchy was constructed which separated the "easy" examples of triangles from the "hard" examples.

Because of the difference in scores for kindergarten/grade one and for the second through fifth grades, hierarchies of order difficulty were established for these two groups of grades.

Hypothesis two was that mean pretest computer scores would increase with grade level. The data were analyzed and it was discovered that in general the scores do increase with grade level, but the increase is in stages or plateaus, whereby the grades fall into clusters. Kindergarten and
grade one scores formed a cluster, the second and third grade scores formed another cluster and the fourth and fifth grade scores comprised the last cluster.

Treatment consisted of four weekly sessions using a computer program, "Exploring Triangles", which was designed to present the concept of triangles. The students worked with "Exploring Triangles" during their regular computer time. The spread of time per session was from 30 minutes for kindergarten children to 10-15 minutes for fifth graders. Treatment effects were greatest for the "hard" examples.

The last hypothesis, that no gender differences would be apparent in any of the analyses, was accepted because no significant effects of gender appeared.

Discussion and Conclusions

The general assumption that children have difficulty with the concept of triangle was found to be valid. The stereotyped examples of triangle that are frequently presented to children appear in the top section of the hierarchies and the examples which are rarely presented are on the bottom sections of the hierarchies. The 19 specific examples of triangle which are on the bottom section of the split hierarchy (Figure 6) are all examples which are not usually presented to children as being examples of the
triangle concept. The relative position of order difficulty remained stable from one hierarchy to another for most of the triangle examples in the assessment.

This finding is consistent with a study reported by Laurendeau and Pinard (1970) in which they tested children's recognition of objects and shapes. The test required children to name the shapes. One of the shapes in their study was an equilateral triangle with a horizontal base.

In the current research, the "easiest" level in all four hierarchies includes the equilateral triangle with a horizontal base. In Table 14, the current findings are compared with the research findings of Laurendeau and Pinard (1970).

<table>
<thead>
<tr>
<th>Table 14</th>
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<td>Triangle 1 Pretest Percents Correct By Grade, Compared With Research by Laurendeau and Pinard</td>
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At the kindergarten level, there is only a three percent difference of correctly selecting this specific triangle between the two research studies. The maximum
The Laurendeau and Pinard results show the same percent correct for kindergarten through second grade, which gives a plateau effect. In contrast, the current research showed a u-shaped (Davis and Silver, 1982) dip for those three grades.

Another difference between results of the two studies is the relative position of grade two. In the current research, the grade two percent is clustered with third through fifth grade but on the Laurendeau and Pinard results, grade two has the same percent as the kindergarten and first grade.

Second grade is when many children have a cognitive change from preoperational to concrete-operational thinking and this difference in the relative position of the second grade is important. The relative position for the second grade may be due to the time of year the children were tested. In the current study, the children were tested in February. If the Laurendeau and Pinard data were collected in the early fall, the time of year may be having an effect on the relative position of the second grade means.

Results of both studies show lower scores for grade five than for the fourth grade. If percentages correct rise again for the untested sixth grade, then this fifth grade dip is part of another u-shaped curve.
Overall analysis of the two studies shows percents of correct selection for the equilateral triangle with a horizontal base increasing with grade levels, but with dips in mean scores at grade one and grade five. In their general aspect these findings of both studies support the hypothesis that recognition of triangles increases with grade level.

Hypothesis number three was that at each grade level posttest scores will be significantly greater than pretest scores for the treatment group, but not for control group I. The overall data did not clearly support this hypothesis. There were significant gains only for kindergarten and fifth grade. In fact, gains in scores for treatment group grades two, three and four were less than those of the control group. One compounding problem was the eight "easy" triangle examples which second through fifth grade children correctly selected at 98 to 100%.

The treatment effects were minimized when all triangle examples were analyzed together. But when each specific triangle example was analyzed separately and patterns of improvement were sought, the treatment effect was evident. The treatment effect was greatest for examples from the bottom of the hierarchies, while the treatment effect decreased as percents of correct selection increased.
Experience Versus Development

The role of experience in geometric concept learning is said by some to be more important than development (Ausubel, Novak, and Hanesian, 1968). The first experiences are strong and lasting (Fivush, 1984) and build the internal structure (Esler, 1984) for ideas which provide the background knowledge for assimilation (Fey, 1982). Ashby and Boulton-Lewis (1985) argue that quality experience is an integral component for cognitive development. The relative importance of development has been questioned as new methodology enables children to comprehend concepts that previously had been presented in ways that were too abstract for their current level of cognition (Huber, 1985, Ross, 1980).

Conclusions from the current research are that maturation and experience are both important in the development of the concept of triangle. It appears that for easily assimilated triangle examples, maturation is most important, but for the hard to assimilate triangles experience/treatment is more important than maturation.

It appears that when many children reached second grade (maturation), they assimilated the eight specific triangles on the top section of the split hierarchy (Figure 6) at the 98 to 100% level. The role of experience within this development can not be addressed by the methodology of this
study. Experience in the form of computer treatment did not help many of the kindergarten and first grade children in the treatment group gain in percent correct over the control group on these eight specific "easy" triangle examples. An important factor these triangle examples have in common is their vertical alignments (Frith, 1980).

Conservation of area (a developmental phenomenon) appears to be less important than orientation in the acquisition of the concept of triangle. If conservation of area was a key factor, then Triangles 24 and 27 (which are rotations of Triangle 1) would have been in the top section of the split hierarchy (Figure 6). Right triangles included at the 98 to 100% level have both an "upward" and right orientation, whereas right triangles with a "downward" or left orientation are in the bottom section of the split hierarchy. These findings support the conclusions of Rosser et al., (1984) who found that three to five year old children pay more attention to external orientation markers than to internal markers.

Experience appears to be more important than maturation for triangle examples which are not easily assimilated, that is the "hard" triangles without large interior space, right orientation, or vertical alignment. Roughly one third of the "youngest" (K-1) children correctly selected triangle examples which are not easily assimilated, which indicates that for them experience played a more important role than
maturation. About one third of the "older" (2-5) children still did not include many of the triangle examples in their concept of triangle, which indicates that maturation alone is not enough for these children to gain a full understanding of the triangle concept.

Implications and Recommendations

Simon (1986) provides steps for teaching mathematical concepts which can be applied to teaching the triangle concept. The plan includes the following five steps, which are followed with appropriate information for teaching the triangle concept.

1. Identify/prioritize what is to be learned
2. Distinguish between facts, procedures, concepts
3. Organize concepts hierarchically
4. Divide what is to be learned into appropriate increments
5. Create/adapt activities to stimulate development of the desired concept

The current research provides guidance for teaching the concept of triangle at each of Simon's levels. High priority needs to be placed on understanding that triangles come in different shapes and that they are more complex than circles or rectangles. The concept of triangle is substantially more complex than the concepts of circle and rectangle (Table 1) because it has more levels in its concept structure, and it varies not only in size and color,
but in general shape. Children at a young age can learn to recognize an infinite variety of triangles, and not form premature closure on their concept.

Teachers should be aware of misconceptions and/or premature closure for concepts that are relatively complex but are often presented to children with no more examples and non-examples than are simple concepts.

The sequential or stage process of learning the triangle concept moves from recognizing specific triangles (as facts) to experiences with collections of triangles (procedures) to generalizations (concepts). Geometry curriculum should include hierarchical relationships (Novak, 1977) comprised of: elements or specific triangle examples, restricted class or a variety of diverse triangle examples, and abstraction (Dienes, 1961) which is the ability to distinguish triangles from non-triangles.

Teaching procedures need to be different for simple and complex concepts. Prototypes, examples and non-examples should be included for complex concepts. The current study can help by providing four different hierarchies of the complex triangle concept (see Figures 5, 6, 17, and 18). The hierarchies by grade levels (Figures 17 and 18) will be the most useful for classroom teachers.

Klausmeier's levels (see Table 2) provide a guide for appropriate increments of instruction for the triangle concept. While it was clear in this study that some fifth
graders did not recognize many "hard" examples of triangles. It was also clear that some kindergarten children did recognize some of the "hard" examples. Recognition of a variety of specific triangles, including "easy" and "hard" examples is appropriate for young children and may prevent premature closure. Although it represents a low level of thinking on Bloom's Taxonomy, recognition is a prerequisite to "having a concept".

Dynamic computer images such as motion geometry can be an integral part of the plan to present the entire triangle concept (Damarin, 1984). Visual perception has been strongly related to geometric concept learning and research results indicate that the triangle concept needs to be taught visually. Computer software can present the concept so that it is transparent to the student, and so that it can be manipulated and experimented with (Damarin, 1987). Teachers can look for computer software and other materials to present a diverse variety of examples and non-examples to help children form a complete concept.

For Further Research And Development

The current research demonstrated that hierarchies of the acquisition of the triangle concept could be developed. It was apparent that there was a major difference between the kindergarten-first grade hierarchy and that of the
second through fifth grades. It was surprising that no gender differences were apparent in the analysis because in other research gender differences do appear. Using data from this research, hierarchies could be developed separately for males and females to see if there are any gender differences for orientation and or order of difficulty.

Other complex geometric and/or mathematical concepts might lend themselves to using hierarchies to help study the relationships between and among specific examples of a concept. Generalizations from triangle concept learning may apply to other areas of geometry, rectangles for example. Would there be a high correlation between concept acquisition and length of sides or orientation? Could teaching or learning the triangle concept transfer to the rectangle concept? Could hierarchies help teach other complex mathematical concepts like fractions and number systems in general?

Hierarchies could be developed separately for males and females to see if there are any gender differences as far as orientation or in order of difficulty. Generalizations from triangle concept may apply to other areas of geometry, to fractions, and to number systems in general.

The hierarchy developed in this research may be used as the basis for an informal assessment tool for children's concept of triangles. An assessment tool could help
teachers help children form a broad understanding of the concept of triangle. Identified triangle examples could be paired with associated shapes in the environment to help bridge the gap from concrete to abstract, from informal knowledge to formal knowledge. Descriptive phrases which children typically give to shapes before they know them as triangles could help teachers assess whether a child is recognizing the shape, under any label. The assessment could include all four of Klausmeier's levels on Table 2 which are: concrete, identity, classificatory and formal.

The current research identifies a learning hierarchy for instances of triangle concept among children. An obvious question is how does this triangle hierarchy relate to stage theory that has been put forth on children's development and learning. For example this study identifies a clear break between examples, separate as easy or hard, and this break in learning corresponds to second grade or seven-year-olds which is the time when Piagetian research indicates a major cognitive change.

The current study cannot answer this question because it included no measures of Piagetian development. It is premature to claim that the triangle hierarchy separates preoperational and concrete operational children, however the question emerges naturally and merits further study.

Papert (1980) and others have claimed that computers can change what can be learned and understood by young
children. The current research's treatment effect on "hard" triangles tends to validate this claim. Applications of teaching other complex concepts with appropriate computer software could also be examined in relation to Papert's position.
APPENDIX A

SHAPE FAMILIES, TESTING INSTRUMENT
Specific Examples and Non-Examples From Shape Families I

Figure 22
set number, screen numbers and positions
(pretest, posttest 1, posttest 2)

angles
sides
orientation of base

set 1, 7b, 7a, 7c
equilateral
equal
horizontal

set 2, 8a, 8c, 8b
right
scalene
horizontal short leg, 90 long leg

set 3, 10b, 10a, 10c
right
scalene
horizontal long leg, 90 short leg

set 4, 11a, 11c, 11b
equilateral
equal
horizontal top (upside down)

set 5, 12a, 12c, 12b
equilateral
equal
90 (pointing right)

set 6, 13a, 13c, 13b
acute
isosceles
horizontal (upside down)

set 7, 13c, 13b, 13a
equilaterial
equal
90 (pointing left)

Triangle Identification, Description and Illustration
Figure 23
Triangle Identification, Description and Illustration

Figure 23
Figure 23, (continued)

set 15, 18c, 18b, 18a
right
scalene
horizontal short leg, 90 long leg

set 16, 19b, 19a, 19c
obtuse
scalene
90 short leg

set 17, 20a, 20c, 20b
acute
isosceles
45 on left

set 18, 21b, 21a, 21c
obtuse
scalene
45 on left

set 19, 21c, 21b, 21a
acute
isosceles
90 pointing left

set 20, 22a, 22c, 22b
acute
isosceles
45 on left

set 21, 22b, 22a, 22c
obtuse
scalene
60 on left, 50 on right

Triangle Identification, Description and Illustration
set 22, 23b, 23a, 23c
obtuse
scalene
90 short on right

set 23, 23c, 23b, 23a
acute
scalene
45 on left

set 24, 24a, 24c, 24b
equilateral
equal
45 on right

set 25, 24b, 24a, 24c
right
scalene
horizontal long leg, 90 very short

set 26, 25a, 25c, 25b
acute
isosceles
90 short of left (pointing right)

set 27, 26a, 26c, 26b
equilateral
equal
45 on left

Triangle Identification, Description and Illustration

Figure 23
(tracings of photoreductions from screen dumps)

Top row, equilateral
Second row, right triangles
Third row, acute angles
Fourth row, obtuse angles

Shape Families I, screen numbers for above figures

- Top row: 7b 11a 12a 13c 24a 26a
- Second row: 8a 10b 16a 18c 24b
- Third row: 13a 14c 17a 20a 21c 22a 23c 25a
- Fourth row: 15a 16b 18a 18b 19b 21b 22b 23b

27 Specific Triangle Configurations and Orientations

Figure 24
Screen Dumps of the Six Practice Screens

Figure 25
Screen Dumps of Sequence of Selecting a Correct Example

Figure 26
To move cursor..........press arrow keys.
When under a Triangle.....press Return.
To erase an answer........press Return.
When finished...............press 'Esc.'
APPENDIX B

EXPLORING TRIANGLES, TREATMENT
Large paper triangles used with
Random Triangles and Three Points
(figure is a photoreduction of triangles used)

Figure 28
Small paper triangles (exact size)
used with Constructing Triangles

Figure 29
APPENDIX C
HIERARCHY
Table 15
Pretest, Group 1 Means For Each Triangle by Grade

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ONEWAY (Analysis of variance)

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ONEWAY (Analysis of variance)

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ONEWAY (Analysis of variance)
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Pretest-Posttest Mean Scores by Grade, ONEWAY

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ONEWAY (Analysis of variance)

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<td></td>
</tr>
<tr>
<td>PP grade</td>
<td>10</td>
<td>110.99</td>
<td>2.55</td>
<td>.005</td>
</tr>
</tbody>
</table>

* PP = Pretest-Posttest
Table 24
Gender by Grade, Pretest Treatment Group
Mean Scores, ANOVA

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>MEAN SQUARES</th>
<th>F</th>
<th>SIG OF F</th>
</tr>
</thead>
<tbody>
<tr>
<td>main effect</td>
<td>6</td>
<td>8034.92</td>
<td>27.25</td>
<td>.000</td>
</tr>
<tr>
<td>gender</td>
<td>1</td>
<td>384.90</td>
<td>1.31</td>
<td>.255</td>
</tr>
<tr>
<td>grade</td>
<td>5</td>
<td>9633.91</td>
<td>32.67</td>
<td>.000</td>
</tr>
<tr>
<td>2-way interactions</td>
<td>5</td>
<td>477.30</td>
<td>1.62</td>
<td>.158</td>
</tr>
<tr>
<td>gender grade</td>
<td>5</td>
<td>477.30</td>
<td>1.62</td>
<td>.158</td>
</tr>
</tbody>
</table>

Table 25
Mean Scores by Gender and Pretest-Posttest, ANOVA

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>MEAN SQUARES</th>
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<th>SIG OF F</th>
</tr>
</thead>
<tbody>
<tr>
<td>main effect</td>
<td>3</td>
<td>147.81</td>
<td>2.79</td>
<td>.039</td>
</tr>
<tr>
<td>gender</td>
<td>1</td>
<td>17.04</td>
<td>.32</td>
<td>.570</td>
</tr>
<tr>
<td>PP*</td>
<td>2</td>
<td>208.99</td>
<td>3.95</td>
<td>.020</td>
</tr>
<tr>
<td>2-way</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>interactions</td>
<td>2</td>
<td>6.94</td>
<td>.13</td>
<td>.877</td>
</tr>
<tr>
<td>gender PP</td>
<td>2</td>
<td>6.94</td>
<td>.13</td>
<td>.877</td>
</tr>
</tbody>
</table>

* PP = Pretest-Posttest
Table 26

Gender, Grade, Pretest-Posttest,
General Linear Models Procedure, SAS

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>TYPE III SS</th>
<th>F</th>
<th>PR &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>gender</td>
<td>1</td>
<td>9.45</td>
<td>0.23</td>
<td>.634</td>
</tr>
<tr>
<td>group (gender)</td>
<td>4</td>
<td>67.72</td>
<td>0.81</td>
<td>.444</td>
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<tr>
<td>grade</td>
<td>5</td>
<td>6704.50</td>
<td>32.17</td>
<td>.000</td>
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<tr>
<td>group (grade)</td>
<td>10</td>
<td>1710.43</td>
<td>4.10</td>
<td>.000</td>
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<tr>
<td>PP*</td>
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<td>140.18</td>
<td>1.68</td>
<td>.187</td>
</tr>
<tr>
<td>PP X gender</td>
<td>2</td>
<td>12.88</td>
<td>0.15</td>
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<tr>
<td>PP X grade</td>
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<td>1441.50</td>
<td>3.46</td>
<td>.000</td>
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<tr>
<td>gender X grade</td>
<td>5</td>
<td>420.51</td>
<td>2.02</td>
<td>.074</td>
</tr>
</tbody>
</table>

* PP = Pretest-Posttest
Table 27
Pretest-Posttest by Group and Grade, ANOVA

<table>
<thead>
<tr>
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<th>MEAN SQUARES</th>
<th>F</th>
<th>SIG OF F</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1738.91</td>
<td>45.92</td>
<td>.000</td>
</tr>
<tr>
<td>group</td>
<td>1</td>
<td>120.27</td>
<td>3.18</td>
<td>.075</td>
</tr>
<tr>
<td>PP*</td>
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<td>4397.75</td>
<td>116.13</td>
<td>.000</td>
</tr>
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<td>5</td>
<td>1026.45</td>
<td>.27.11</td>
<td>.000</td>
</tr>
<tr>
<td>2-way interactions</td>
<td>17</td>
<td>178.46</td>
<td>4.71</td>
<td>.000</td>
</tr>
<tr>
<td>group PP</td>
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<td>32.44</td>
<td>.86</td>
<td>.425</td>
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<tr>
<td>group grade</td>
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<td>4.12</td>
<td>.001</td>
</tr>
<tr>
<td>PP grade</td>
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<td>197.68</td>
<td>5.22</td>
<td>.000</td>
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<tr>
<td>3-way interactions</td>
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<td>1.24</td>
<td>.261</td>
</tr>
</tbody>
</table>

* PP = Pretest-Posttest

Table 28
Pretest-Posttest Mean Scores, ONEWAY

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>MEAN SQUARES</th>
<th>F</th>
<th>SIG OF F</th>
</tr>
</thead>
<tbody>
<tr>
<td>between groups</td>
<td>2</td>
<td>6621.92</td>
<td>148.94</td>
<td>.000</td>
</tr>
<tr>
<td>within groups</td>
<td>1043</td>
<td>44.46</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ONEDAY (Analysis of variance)
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 30
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 31
SET NUMBER 3

Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 32
SET NUMBER 4

O•Pretest  O•Posttest

Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 33
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 34
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 35
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 36
SET NUMBER 8

Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 37
Percent of Students at Grade Level Correctly Selecting Example of Triangle

Figure 38
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 39
SET NUMBER 11

Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 40
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 41
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 42
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 43
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 44
SET NUMBER 16

**Percent of Students at Grade Level Correctly Selecting Example of Triangle**

*Figure 45*
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 46
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 47
SET NUMBER 19

Percent of Students at Grade Level Correctly Selecting Example of Triangle

Figure 48
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 49
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 50
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 51
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 52
Figure 53

Percent of Students at Grade Level
Correctly Selecting Example of Triangle
SET NUMBER 25

Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 54
SET NUMBER 26

% of Correct Answers

Grade K  Grade 1  Grade 2  Grade 3  Grade 4  Grade 5

Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 55
Percent of Students at Grade Level
Correctly Selecting Example of Triangle

Figure 56
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