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Higher order corrections in walking technicolor theories

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The Ohio State University, 1989
HIGHER ORDER CORRECTIONS
IN WALKING TECHNICOLOR THEORIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

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CHAPTER I

INTRODUCTION

The standard model of Weinberg and Salam gives a unified description of two apparently diverse interactions, weak and electromagnetic. The weak interactions are responsible for the slow radioactive decay of nuclei. The electromagnetic interactions on the other hand are relatively stronger and much more familiar. It is this interaction that holds the electron and the proton together inside the Hydrogen atom. The Weinberg Salam model (standard model) is based on a non-abelian gauge group $SU(2)_L \times U(1)_Y$ which is spontaneously broken down to the manifest symmetry $U(1)_Q$ of electromagnetism. A symmetry is said to be spontaneously broken when the Lagrangian of the theory is invariant under transformations of the corresponding symmetry group but the vacuum or ground state of the theory is not. When a symmetry is realized in this mode it will not be manifest in the solutions of the theory.

In the standard model the spontaneous breaking of $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$ is achieved by means of elementary Higgs scalars ($\phi$). However it was realized very soon that a quantum field theory with light elementary scalars is unnatural. A quantum field theory is said to be natural if the radiative corrections to the bare parameters of the theory are not very much greater than their observed values. In
the standard model the radiative corrections to the square of the scalar bare mass diverges quadratically with the cut off. This makes the standard model unnatural as it requires fine-tuning of the bare Higgs mass to extremely high precision to get a renormalized Higgs mass of say 1 TeV. The quadratic divergence of the Higgs self energy can be looked upon as a symptom of the fact that the standard model is really a low energy phenomenological model and must be replaced by a much more complete dynamical theory beyond a certain cut off.

Technicolour was introduced as a dynamical alternative to the elementary Higgs picture to solve the naturalness problem. Technicolour refers to an asymptotically free non-abelian gauge interaction whose running coupling becomes strong near the electroweak scale ($\Lambda_{\text{ew}} \approx 250$ Gev). It is assumed that in the absence of other interactions (colour, electroweak and possibly a hierarchy of others which might be revealed as we go up in energy) the technicolour sector has a global flavor symmetry group $G_F = \prod_r SU(2n_r)_L \approx SU(2n_r)_R$ where $r \in \mathbb{N}$ labels different representations of the technicolour group. (By $SU(2n_r)$ we mean a group of $2n_r \times 2n_r$ unitary matrices of determinant 1. The subscript L(R) means that the symmetry generators (Noether charges) are formed from left-handed $\psi_L$ (right-handed $\psi_R$) fermion fields).

Near the characteristic electroweak scale, technicolour interaction becomes strong and causes a spontaneous breaking of $G_F$ down to the diagonal vectorial subgroup $H = \prod_r SU(2n_r)_V$. (If $Q^a_L$ and $Q^a_R$ are generators of $SU(2n_r)_L$ and $SU(2n_r)_R$ respectively then $Q^a_L = Q^a_R$ are the generators of $SU(2n_r)_V$). The electroweak gauge group $SU(2)_L \times U(1)_Y$ forms a weakly gauged subgroup of $G_F$. As a result the weak gauge
bosons ($W^\pm$ and $Z^0$) that couple to the broken symmetry currents (in this case the axial vector currents $j^\mu_a(x)$, $\mu =$ Lorentz 4 vector index, $a =$ index for SU($N$) generators) pick up masses by eating up the corresponding Goldstone bosons, leaving the photon which couples to the unbroken conserved electromagnetic current as massless. Technicolour therefore can successfully break the full electroweak symmetry group SU(2)$_L$XU(1)$_Y$ down to the manifest symmetry U(1)$_Q$ of electromagnetism and give mass to the weak gauge bosons in the framework of a renormalizable field theory.

In the standard model the Higgs sector gives mass not only to the weak gauge bosons but also to the fermions through Yukawa couplings to fermions of the form $f_{ij} \bar{\psi}_{Li} \phi \psi_{Rj}$ which break the quark lepton chiral symmetries explicitly ($f_{ij}$) are the dimensionless Yukawa coupling constants). Quarks and leptons acquire current masses when $(\phi)$, the vacuum expectation value of the Higgs field acquires a non-zero value ($\approx 250$ Gev). Although technicolour can provide a satisfactory dynamical explanation for the observed mass spectrum of electroweak gauge bosons, it cannot by itself give mass to ordinary fermions (quarks and leptons). Recall that in the standard model it is the spontaneous breaking of the electroweak gauge group SU(2)$_L$XU(1)$_Y$ that gives mass to ordinary fermions. So in order to give mass to ordinary fermions the spontaneous chiral symmetry breaking ($\chi$SB) in the technicolour sector has to be somehow communicated to the ordinary fermion sector. In this way one can use technicolour to explicitly break the chiral symmetry of quarks and leptons and give them mass.
In order to achieve this one has to embed technicolour into a larger gauge group known as extended technicolour. In extended technicolour (ETC) models both ordinary fermions and techifermions sit in the same representation of the extended technicolour gauge group \( G_{ETC} \). At some mass scale \( \Lambda_{ETC} \), \( G_{ETC} \) breaks down to \( G_{TC} \) (technicolour) \( \otimes G_C \) (colour) due to strong interaction effects of some other gauge group \( G_S \) whose flavour symmetry group contains \( G_{ETC} \) as a subgroup. The symmetry currents that mediate transition between ordinary fermions and techifermions will be broken and the gauge bosons that couple to these broken symmetry currents will pick up mass. These gauge bosons can give rise to quark-lepton \( xSB \) masses via self energy like diagram shown in Fig. 1a. In an approximation which treats the ETC gauge boson mediated transition as an effective four Fermi interaction the quark-lepton mass generation mechanism can be depicted as in Fig. 1b. In this approximation we have

\[
\frac{g^2_{ETC}}{m_q, l^2_{ETC} \Lambda_{ETC}} \psi \bar{\psi}
\]

where all matrix indices have been suppressed from \( m_q, l \). \( \langle \psi \bar{\psi} \rangle_{\Lambda_{ETC}} \) is the technifermion condensate renormalized at \( \Lambda_{ETC} \) where \( \Lambda_{ETC} \) is the extended technicolour breaking scale. The renormalization mass scale has been purposely chosen to be \( \Lambda_{ETC} \) because we shall be considering technicolour theories with a slowly running coupling and for such theories the loop shown in Fig. 1b gets the largest contribution from momenta of order \( \Lambda_{ETC} \). So if we renormalize \( \langle \psi \bar{\psi} \rangle \) at \( \Lambda_{ETC} \) we can equate it to the loop shown in Fig. 1b. If on the other hand we renormalize \( \langle \psi \bar{\psi} \rangle \) at \( \Lambda_{ETC} \) (\( xSB \) scale for technicolour sector) we will have to multiply \( \langle \psi \bar{\psi} \rangle_{\Lambda_{ETC}} \) by a large anomalous dimension factor before equating it to Fig. 1b. We have
In a QCD like theory with no small dimensionless parameter we get

\begin{equation}
\langle \bar{\psi}\psi \rangle_{\Lambda_{\text{ETC}}} = \langle \bar{\psi}\psi \rangle_{\Lambda_{\text{TC}}} e^{-\int_{\Lambda}^{\Lambda_{\text{ETC}}} d\mu \frac{\alpha_{\text{TC}}(\mu)}{\mu} \gamma_m(\alpha_{\text{TC}}(\mu))}.
\end{equation}

Using naive scaling arguments and chiral perturbation theory we get

\begin{equation}
\langle \bar{\psi}\psi \rangle_{\Lambda_{\text{ETC}}} = \langle \bar{\psi}\psi \rangle_{\Lambda_{\text{TC}}} e^{-\frac{\beta_1 \alpha_{\text{ETC}}}{\pi} \gamma_{m,1}}.
\end{equation}

where we have used $m_q \approx \frac{1}{2} (m_u + m_d) \approx 7 \text{ MeV}$. For $m=125 \text{ MeV}$ this gives $\frac{\mu_{\text{ETC}}^2}{g_{\text{ETC}}^2(\text{mixing angle factors})} \approx 60 \text{ TeV}$. (1.4)

In the above

\begin{itemize}
  \item $\mu$ = running mass parameter
  \item $\alpha_{\text{TC}}(\mu)$ = running coupling of technicolour sector
  \item $\gamma_m$ = anomalous dimension of $\langle \bar{\psi}\psi \rangle$
  \item $\beta_1$ and $\gamma_{m,1}$ are the coefficients of $\frac{\alpha}{\pi}$ in the perturbative expansions of $\beta(\alpha)$ and $\gamma_m(\alpha)$ respectively. $\beta(\alpha)$ measures the rate of running of the coupling with the renormalization point $\mu$ and is defined by $\frac{d\alpha}{d\ln\mu} = \alpha \beta(\alpha)$. $F_\pi(f_\pi)$ is the technipion (pion) decay constant and $m_\pi$ is the pion mass. $\mu_{\text{ETC}}$ is the mass of ETC gauge boson that gives current mass to strange quark and $g_{\text{ETC}}$ is the coupling constant of extended technicolour.
\end{itemize}
On the other hand in an ETC scenario, when $G_{\text{ETC}}$ breaks down to $G_{\text{TC}} \otimes G_{\text{C}}$, the neutral currents that mediate transition between ordinary fermions of different flavour are also broken. The corresponding gauge bosons therefore become massive. The amplitude of these flavour changing neutral current processes are however severely constrained by experimental data. For example $\Delta m_K$ arising from $K^0 - \bar{K}^0$ mixing is known to be $\Delta m_K \approx 3.5 \times 10^{-6}$ eV. Consider the flavour changing neutral current transition $d_S'^{} L L \rightarrow S'^{} d'_L L$ (Fig. 3) mediated by ETC gauge bosons which can give rise to a $\Delta S = 2$ transition corresponding to $K^0 - \bar{K}^0$ mixing. Using the vacuum saturation approximation, one can estimate the contribution of the process to $\Delta m_K$. The above value for $\Delta m_K$ leads to the following lower bound for the mass scale of flavour changing neutral gauge bosons.

$$\left( \frac{\nu^2_{\text{ETC}}}{\nu^2_{\text{ETC}} \text{(mixing angle factors)}} \right)^{\frac{1}{4}} \geq 1200 \text{ TeV} \quad (1.5)$$

How can the mass scale of the ETC gauge bosons that give mass to quark-leptons be so small compared to the mass scale of flavour changing neutral gauge bosons particularly since both arise from the breaking of $G_{\text{ETC}}$ into $G_{\text{TC}} \otimes G_{\text{C}}$. Normally one would expect the ETC gauge bosons that give mass to quarks and leptons to be at least as heavy as the flavour changing neutral gauge bosons. This is the flavour changing neutral current (FCNC) problem of ETC scenarios. We discuss next a possible dynamical resolution of this problem in which all ETC gauge boson masses responsible for light quark masses are raised to 1000 TeV.
Recently it was proposed by Appelquist, Karabali and Wijewardhara (AKW) that asymptotically-free technicolour theories with a slowly running coupling might be a possible resolution of the long standing FCNC problem of ETC scenarios. By analysing the linearized gap equation (for technifermion propagator) in ladder approximation with leading logs included, and choosing the Landau gauge they showed that if the parameters of the theory are so adjusted that $\beta(\alpha) = 0$ and $\alpha(M) = \alpha_c = \frac{\pi}{3C_2(R)\Lambda}$ for $\Lambda_{\text{ETC}}^2 \ll M^2 \ll \Lambda_{\text{ETC}}^2$ then the dynamical mass $\Sigma(P)$ satisfies a power law solution of the form $\mu(P) \propto P^\epsilon$ over a large momentum range before assuming its rapidly falling asymptotic form ($\Sigma(P) \propto \frac{P^{2\epsilon}}{P^2}$ up to logs). Here $\alpha_c$ is the critical coupling above which chiral symmetry of technicolour sector is spontaneously broken and $\mu$ is the chiral symmetry breaking scale of technicolour sector. The much more slowly falling behaviour of $\Sigma(P)$ over a large momentum range ($\Lambda_{\text{ETC}}^2 \ll P \ll \Lambda_{\text{ETC}}^2$) could be used to raise the mass scale of ETC gauge bosons (that give mass to quarks-leptons) near the mass scale of flavour changing neutral gauge bosons and yet generate sufficiently large fermion masses. This leads to a dynamical resolution of the FCNC problem in ETC scenarios.

The AKW analysis however does raise some serious concerns.

a. The analysis was restricted to ladder approximation whereas the effective expansion parameter in ladder approximation

\[
\frac{3\alpha C_2(R)}{\pi} = \frac{3\alpha_c C_2(R)}{\pi} = 0(1) \text{ with } \alpha = \alpha_c.
\]

Therefore it is difficult to trust the lowest order result and it is important to estimate the effects of $0(\alpha^2)$ corrections to $\Sigma(P)$. We would like to see if, after taking $0(\alpha^2)$ corrections into account we do get a power law solution of the form $\Sigma(P) = \mu(P)^\epsilon$ in the non-running condition (i.e. When the coefficient of the leading log term in the kernel vanishes. In Landau
gauge this condition is satisfied when $B(\alpha)=0$ in some properly chosen
gauge. AKW obtained a power law solution $\Sigma(P) = \mu(P)$ in Landau gauge
under the condition $B(\alpha) = 0$ and $\alpha = \alpha_c$. We would like to see how the
above conditions for getting a power law solution in the fixed point
limit are modified when $O(\alpha^2)$ corrections are included. Of
particular interest is the change of gauge choice (if any) required
for getting a power law solution under the $O(\alpha^2)$ corrections.
Another question that we would like to investigate is if the
coefficient of the $O(\alpha^2)$ term in the exponent $\epsilon$ is indeed small
compared to the coefficient of $O(\alpha)$ term.

b. The power law behaviour of $\Sigma(P)$ in ladder approximation was
obtained by assuming $B(\alpha)=0$ and $\alpha(M)=\alpha_c=\pi/3C_2(R)$. This estimate of
the critical coupling was obtained by Peskin who did a stability
analysis of the CJT (Cornwall, Jackiw and Tomboulis) effective
potential in ladder approximation with a fixed coupling. However
since the effective expansion parameter $\frac{3\alpha C_2(R)}{\pi} \approx O(1)$ in ladder
approximation there are no justified reasons for trusting the lowest
order result for $\alpha_c$. A rigorous determination of $\alpha_c$ by including $O(\alpha^2)$
corrections to the effective potential involves choosing the right
gauge (that would ensure a gauge invariant result for $\alpha_c$) and
carrying out a stability analysis of the effective potential by
including $O(\alpha^2)$ corrections to the 2PI vacuum graphs. Unfortunately
this is a very hard problem. We do not know what is the right gauge
to choose in $O(\alpha^2)$ nor do we know how to do a stability analysis when
$O(\alpha^2)$ effects are included in the effective potential. One can
however make a conjecture about the value of $\alpha_c$ when $O(\alpha^2)$ corrections
are included by setting the expression for $\gamma_m(\alpha)$ to $\mathcal{O}(\alpha^2)$ equal to 1. One is led to make this conjecture by noting that setting $\gamma_{m=1}$ in ladder approximation leads to the result $\alpha_c = \frac{\pi}{3C_2(R)}$ for the critical coupling in ladder approximation. We would like to see if the $\mathcal{O}(\alpha^2)$ correction term in the expression for $\alpha_c$ determined in the above manner is indeed small compared to the $\mathcal{O}(\alpha)$ result.

To answer these questions we started with the integral equation satisfied by $\Sigma(P)$ in terms of the linearized and renormalized Bethe-Salpeter kernel in the chiral symmetric limit. In order to obtain an integral equation for $\Sigma(P)$ that involves only the radial momentum as the integration variable we tried to evaluate the traced and angular averaged kernel for all the diagrams to $\mathcal{O}(\alpha^2)$. However we have not been able to calculate the contribution of a piece of the crossed ladder diagram so far. The calculations were done in an arbitrary covariant gauge to study the question of gauge dependence of dynamical mass in technicolour theories with a slowly running coupling. Loop integrals with two different propagator denominators were done by Feynman parametrization. But loop integrals with more than two different propagator denominators could not be done by Feynman parametrization and so we used the Gegenbauer polynomial method in momentum space to do such integrals. Being unsuccessful in evaluating the last piece of the crossed ladder diagram exactly we decided to study the integral equation by using the end point approximation for the kernel. (The kernel $K(P,k)$ is a function of two variables $p$ and $k$. By end point approximation we mean the limiting form of the kernel when $k \gg p$ or $p \gg k$). The piece of the crossed ladder diagram that we have not been above to evaluate exactly so far does not contribute to the
end point approximation for the kernel. Using the end point approximation for the kernel we solved the integral equation for $\Sigma(P)$ by making a power law ansatz of the form $\Sigma(P) = \mu \left( \frac{p^2}{\mu^2} \right)^{(b+\lambda)}$ where $b = \frac{1}{2} \gamma_m$ and $\lambda = \frac{1}{2} \gamma_2$. (See Chapter II and V for reasons behind this identification). Here $\gamma_2$ is the anomalous dimension associated with the technifermion wavefunction renormalization constant. We find:

a. With the end point approximation for the kernel a scale invariant power law solution of the form $\Sigma(P) = \mu \left( \frac{p^2}{\mu^2} \right)^{(b+\lambda)}$ with $b = \frac{1}{2} \gamma_m$ and $\lambda = \frac{1}{2} \gamma_2$ is obtained in the non-running condition only in Landau gauge ($\xi=0$) and a peculiar gauge ($\xi=-3$). Here $\xi$ is the covariant gauge parameter.

b. If technicolour is an SU(N) gauge theory with large N, then in Landau gauge gauge with technifermions in the first few low lying representations of SU(N) the coefficient of the $0(\alpha^2)$ term in the exponent $b$ is very small compared to the coefficient of the $0(\alpha)$ term. This statement holds even for large $\alpha$. The coefficient of the $0(\alpha^2)$ term in $\lambda$ is also very small but since the $0(\alpha)$ term in $\lambda$ vanishes in Landau gauge we cannot make any useful comparison.

c. If the critical coupling $\alpha_c$ to $0(\alpha^2)$ is determined by $\gamma_m=1$ then the $0(\alpha^2)$ correction to $\alpha_c$ is small compared to the $0(\alpha)$ term (1-18%).

To summarize, within the approximations we have made the $0(\alpha^2)$ and perturbation theory seems reliable.

This thesis is organized as follows. In Chapter II we present a brief discussion of AKW analysis and the major concerns about the results of this analysis. Here we also present the major questions that will be addressed in this thesis. Chapter III contains a proof to justify the use of a linearized kernel for the range of momenta.
(Λ_{TC}^{\alpha P\alpha\Lambda_{ETC}}) we are interested in. In Chapter IV we first present the principles for evaluating the integrals. Then we present the final results of our calculation. As an example of how the method works we have presented the calculations for the contribution of the non-abelian vertex correction diagram to $\Sigma(P)$ in modest detail in Appendix A. In Chapter V we present an approximate solution of the integral equation which is obtained by using the end point approximation for the kernel. Based on this approximate solution we discuss the validity of $0(\alpha)$ results. Finally in Chapter VI we present some questions that remain unanswered in this thesis. These questions will be addressed in near future.
CHAPTER II
Technicolour Theories with Slowly Running Coupling in Ladder Approximation

In this chapter we shall discuss some aspects of $\chi$SB and dynamical mass function $\Sigma(P)$ in technicolour theories with a slowly running coupling in ladder approximation. The chapter is divided into two sections. In Sec. A we present the results of stability analysis of the CJT effective potential due to Peskin. The stability analysis leads to a value for the critical coupling $\alpha_c$. We also present some arguments (due to Banks and Raby) to emphasize the need for working in Landau gauge ($\xi=0$) to get a gauge invariant and reliable result for $\alpha_c$ in $O(\alpha)$ by doing a stability analysis on the CJT effective potential. In Sec. B we review the AKW analysis of the gap equation in ladder approximation. We show how a slowly running coupling can give rise to a $\Sigma(P)$ which behaves as $\mu(P)$ over a large momentum region. We conclude the chapter by noting that this behaviour of $\Sigma(P)$ can be used to solve the long standing FCNC problem of technicolour theories.

A. Stability Analysis

In order to prove that the chiral symmetry underlying a gauge theory is indeed spontaneously broken one must show that the vacuum energy density for the broken phase is lower than that of the
symmetric phase. Cornwall, Jackiw and Tomboulis introduced an
effective potential for a non-local composite operator $\psi(x)\bar{\psi}(y)$ that
can be used to study $\chi$SB in gauge theories. Peskin did a stability
analysis of the CJT effective potential in ladder approximation with a
fixed coupling. He found that an instability to the chiral symmetric
vacuum develops provided $\alpha > \alpha_c = \frac{\pi}{3C_2(R)}$. The analysis was done in
Landau gauge but no reasons were given for this gauge choice. This is
an important issue because the composite operator $\psi(x)\bar{\psi}(y)$ is a gauge
variant operator and if we work with gauge variant operator, we have
no guarantee that its effective potential will satisfy the positivity
requirements needed for the stability criterion. Earlier Banks and
Raby showed that for an abelian gauge theory in Landau gauge, the gauge
variant operator $\psi(x)\bar{\psi}(y)$ coincide with gauge invariant operators.
This result is also true for non-abelian gauge theories to $O(\alpha)$.
Combining Peskin's analysis to that of Banks and Raby we can conclude
that $\alpha_c = \frac{\pi}{3C_2(R)}$ is a gauge invariant and reliable result in $O(\alpha)$.
However there exist some serious concerns about the validity of this
result.

a. The CJT potential is the effective potential for a composite
operator that is non-local in time in Heisenberg picture and therefore
the corresponding Schrödinger picture operator does not have the
interpretation of energy. It is not clear therefore that the CJT
potential should satisfy the positivity requirements for stability.
b. The effective expansion parameter in the single gluon exchange
approximation to the effective potential $\alpha_c C_2(R) \approx O(1)$ and therefore
there is no obvious reason to trust the lowest order result.
We do not have any satisfactory answer to (a) so we shall simply apologize and proceed. However we do have a partial answer to (b) which will be presented in Chapter V.

B. AKW Analysis

Consider massless technifermions with a non-abelian asymptotically-free vector-like gauge interaction. We assume that the theory has an underlying global symmetry \( G_f = \prod_{Y} SU(2n_r)_L \otimes SU(2n_r)_R \). Near the electroweak scale the technicolour interaction becomes strong and the flavour symmetry \( G \) is spontaneously broken to \( \prod_{V} SU(2n_r)_V \). For a QCD-like theory one gets by operator product expansion in Landau gauge

\[
\Sigma(p) = \frac{\Sigma_0}{p^2} \left( \ln \frac{p}{A'} \right)^{-3C_2(R)-1} \quad \text{for } p^2 \to \infty
\]  

Here \( A' \) is the renormalization group invariant mass scale. For a QCD-like theory with no small dimensionless parameter \( \Sigma(p) \) will reach its rapidly falling asymptotic form fairly soon and this in turn leads to the low estimate \((\approx 60 \text{ TeV})\) for the mass of ETC gauge bosons to produce fermion masses of hundred MeV. However if the coupling runs slowly and stays close to the critical coupling i.e. if \( \beta(\alpha) = 0 \) and \( \alpha(M) \approx \alpha = \frac{\pi}{3C_2(R)} \) for a large range of \( M \) \( (\mu_{\chi_{SB}} \Lambda_{ETC}) \) then a careful analysis of the gap equation shows that \( \Sigma(p) \) falls off much slowly (as \( \mu \left( \frac{\Lambda_{ETC}}{p} \right) \)) over this large momentum region. Here \( \Lambda_{ETC} = \) extended technicolour breaking scale and \( \mu = \chi_{SB} \) scale of technicolour sector.

From the axial vector Ward Identity and Goldstone theorem one obtains

\[
\Sigma(p) = \frac{1}{4} \int \frac{d^4k}{(2\pi)^6} \frac{\Sigma(k)}{k^2A^2(k) - \Sigma^2(k)} (\gamma_5)_{cd} K_{ac}d_b(p,k,0;\alpha,\xi)(\gamma_5)_{ba} \]  

(2.2)
where the inverse fermion propagator \( S^{-1}(P) = \mathcal{PA}(P) - \Sigma(P) \) and 

\[ K_{ac,db}(P,k,0;\alpha,\xi) \] is the 2 particle irreducible fermion antifermion scattering kernel. In ladder approximation with leading logs included one gets by working in Landau gauge

\[
\Sigma(P^2) = \frac{3C_2(R)}{4\pi} \int_0^\infty \frac{dk^2}{M^2} \Sigma(k^2) a(M^2) \quad \text{where} \tag{2.3}
\]

\( M = \max(P,k) \). We have actually written the linearized form of the integral equation which is justified for \( P^2 \gg \mu^2 \). Suppose \( B(\alpha) = 0 \) and \( a(M) \approx a_c \) for \( \mu \ll M \ll \Lambda_{ETC} \). Let us further assume that \( \mu^2 \ll P^2 \ll \Lambda_{ETC}^2 \). Taking a power law ansatz for \( \Sigma(P) \) of the form \( \Sigma(P) = \mu \left( \frac{P^2}{\mu^2} \right)^{-b} \). We get by ignoring correction terms of order \( \left( \frac{P^2}{\mu^2} \right)^b \Lambda_{ETC}^2 \)

\[
b_\pm \approx \frac{1}{2} \left[ 1 \pm \sqrt{1 - \frac{3a}{\pi C_2(R)}} \right] = \frac{1}{2} \gamma_m \quad \text{and hence} \quad \Sigma(P) \approx \mu \left( \frac{P}{\mu} \right)^{-b} = \mu \left( \frac{P}{\mu} \right)^{-\gamma_m} \tag{2.4}
\]

The reason for identifying \( b_- \) (the exponent for mechanical mass) with \( \frac{1}{2} \gamma_m \) can be seen by solving the renormalization group equation for the effective mass \( \frac{dm}{m} = -\gamma_m(\alpha)dt \) with a fixed coupling where \( t = \ln \frac{P}{\mu} \). Whereas the reason for identifying \( b_+ \) (the exponent for dynamical mass) also with \( \frac{1}{2} \gamma_m \) can be seen by doing an OPE of the fermion propagator with a fixed coupling. Compare this with the asymptotic form of \( \Sigma(P) \) given in Equation (2.1).

Ordinary fermion masses are given approximately by

\[
m_f \approx \frac{\mu_{ETC}^2}{4\pi^2} \int_0^{\Lambda_{ETC}} dP \Sigma(P) \tag{2.5}
\]
where $N = \text{no. of technifermions}$. The much more slowly falling behaviour $\left(\frac{\mu}{m}\right)$ of $\Sigma(P)$ can be used to raise the mass scale of ETC gauge bosons that give mass to ordinary fermions, roughly near the mass scale of flavour changing neutral gauge bosons and yet generate large enough fermion masses. Thus technicolour theories with a slowly running coupling might be a possible resolution of the long standing FCNC problem of ETC scenarios. An alternative but equivalent way of seeing this is to use OPE. The slowly running behaviour of the coupling near its critical value leads to a large anomalous dimension $\gamma_m(\alpha_c) = 1$ and this enhances the technicolour condensate

$$\langle \bar{\psi} \psi \rangle_{\Lambda_{\text{ETC}}} = \langle \bar{\psi} \psi \rangle_{\Lambda_{\text{TC}}} \sim \Lambda_{\text{ETC}}^2$$

This implies that $m_{q, l} = \frac{g_{\text{ETC}}^2}{\mu_{\text{ETC}}} \langle \bar{\psi} \psi \rangle_{\Lambda_{\text{ETC}}} \approx \frac{g_{\text{ETC}}^2}{\mu_{\text{ETC}}} \frac{\Lambda_{\text{ETC}}^2}{\Lambda_{\text{TC}}} \approx \frac{g_{\text{ETC}}^2}{\mu_{\text{ETC}}} \Lambda_{\text{TC}}^3$ (2.7)

For $m_{q, l} \approx 125 \text{ MeV}$ and $\Lambda_{\text{ETC}} = 10^3 \text{ TeV}$ we get

$$\left(\frac{\mu_{\text{ETC}}^2}{g_{\text{ETC}}^2 \text{(mixing angle factor)}}\right)^{1/2} \sim \left(\frac{\Lambda_{\text{TC}}^2}{m_{q, l}}\right)^{1/2} \approx 1100 \text{ TeV}$$ (2.8)
CHAPTER III
Linearization Analysis of the Gap Equation

In this chapter we shall prove that for $\mu^2 \ll p^2 \ll \Lambda^2_{ETC}$, one can ignore the insertion of dynamical mass in the fermion propagators appearing inside the $O(\alpha^2)$ kernel, after one has renormalized the divergent subgraph in some appropriate renormalization scheme. The proof will be given for Euclidean momenta and the renormalization of divergent subgraphs will be done by MS scheme.\(^{14}\)

A serious concern about the AKW analysis is that they restricted their analysis to ladder approximation whereas the effective expansion parameter in ladder approximation $\alpha C_2(R) \approx 0(1)$. Therefore it is difficult to trust the lowest order result and there is a real need to estimate the effects of higher order corrections to $\Sigma(P)$. Being motivated by this need we made a systematic study of the effects of $O(\alpha^2)$ corrections to $\Sigma(P)$. To $O(\alpha^2)$ the diagrams that contribute to the kernel are shown in Fig. 4. The blob stands for a pseudoscalar vertex. The first problem that one faces while considering $O(\alpha^2)$ corrections to the gap equation is that the gap equation is horribly non-linear in this order. Fermion propagators do not appear inside the $O(\alpha)$ kernel, but they do appear inside the $O(\alpha^2)$ kernel and strictly speaking one must consider them with appropriate dynamical mass. However we shall now prove that for $\mu^2 \ll p^2 \ll \Lambda^2_{ETC}$ one can ignore
the dynamical mass of the fermion propagators appearing inside the kernel i.e. a linearized and renormalized kernel can be used.

The kernel for crossed ladder diagram is ultraviolet finite and its linearization analysis is simplest. We shall concentrate on the linearization analysis of kernels with a divergent subgraph. To $O(\alpha^2)$ the abelian vertex correction diagram, the non-abelian vertex correction diagram and the vacuum polarization diagram are ultraviolet divergent. Among the four vacuum polarization diagrams only the one with a closed fermion loop has a dynamical mass. So we need to consider only the two vertex correction diagrams and the vacuum polarization diagram with a closed fermion loop in the linearization analysis of divergent kernels.

Let $\Gamma(P, k, \alpha, \Sigma)$ denote a generic unrenormalized divergent subgraph with external momenta labelled by $p, k$ and complete fermion propagator with dynamical mass $\Sigma$. One can write

$$\Gamma(p, k, \alpha, \Sigma) = \Gamma(p, k, \alpha, 0)_{1/\epsilon} + \left[\Gamma(p, k, \alpha, 0) - \Gamma(p, k, \alpha, 0)_{1/\epsilon}\right] + \left[\Gamma(p, k, \alpha, \Sigma) - \Gamma(p, k, \alpha, 0)\right] \quad (3.1)$$

where $\Gamma(p, k, \alpha, 0)_{1/\epsilon}$ is the pole term in $\epsilon$ when dimensional regularization is used. The first term $\Gamma(p, k, \alpha, 0)_{1/\epsilon}$ will be subtracted off from $\Gamma(P, k, \alpha, \Sigma)$ in MS scheme (implemented at zero mass). The second term is the renormalized contribution to the subgraph in the limit $\Sigma(P) = 0$. To prove that the linearization of a renormalized subgraph is possible for $\mu \ll \Lambda_{\text{ETC}}$ we should show that the contribution of $[\Gamma(p, k, \alpha, \Sigma) - \Gamma(p, k, \alpha, 0)]$ to the kernel $K_{ac, db}(P, k, 0; \alpha)$ is small compared to the contribution of $[\Gamma(p, k, \alpha, 0) - \Gamma(p, k, \alpha, 0)_{1/\epsilon}]$ for all values of $k$ and $\mu \ll \Lambda_{\text{ETC}}$. Instead of comparing the contributions of $[\Gamma(p, k, \alpha, \Sigma) - \Gamma(p, k, \alpha, 0)]$ and $[\Gamma(p, k, \alpha, 0) - \Gamma(p, k, \alpha, 0)_{1/\epsilon}]$ to $K_{ac, db}(P, k, 0; \alpha)$ we shall compare their
contributions to \( \tilde{K}(P,k,0;\alpha) = (\gamma_5)_{cd}^{ac} K_{ac,db}^{(P,k,0;\alpha)} (\gamma_5)_{db} \) because it is the Lorentz scalar which appears in the Gap equation. The crossed ladder diagram is ultraviolet finite so its linearization analysis consists in showing that \( \tilde{K}(P,k,0;\alpha, \Sigma=0) \) is much greater than \( [\tilde{K}(P,k,0;\alpha, \Sigma)-\tilde{K}(P,k,0;\alpha, \Sigma=0)] \).

Consider the vacuum polarization diagram with a closed fermion loop. Denote it by \( \Pi_{AB}^{\mu \nu}(q=P-k, \Sigma) \) when the fermion lines have a dynamical mass \( \Sigma \). Its contribution to \( K(P,k,0;\alpha) \) in terms of Euclidean momenta is

\[
\tilde{K}(\Sigma) = \frac{32 ig^4 C_2(R) T(R)}{((P-k)^2)^2} \int \frac{d^4 q}{(2\pi)^4} \frac{2 \Sigma(q) \Sigma(q+q) + q \cdot (q+q)}{(q^2 + \Sigma^2(q))(q^2 + \Sigma^2(q+q))}
\]

(3.2)

\[
\tilde{K}(0) = \frac{32 ig^4 C_2(R) T(R)}{((P-k)^2)^2} \int \frac{d^4 q}{(2\pi)^4} \frac{q \cdot (q+q)}{q^2(q+q)^2}
\]

(3.3)

\[
\tilde{K}(\Sigma)-\tilde{K}(0) = \frac{32 ig^4 C_2(R) T(R)}{((P-k)^2)^2} \int \frac{d^4 p}{(2\pi)^4}
\]

\[
\frac{2 \Sigma(q) \Sigma(q+q) - q \cdot (q+q) [2 \Sigma^2(q) + (q+q)^2 \Sigma^2(q)] \Sigma^2(q+q)}{q^2(q+q)^2 [(q^2 + \Sigma^2(q))(q^2 + \Sigma^2(q+q))]} 
\]

(3.4)

The above integral has no ultraviolet (UV) or infrared (IR) divergence. We shall now make an order of magnitude estimate of the contributions to the above integral coming from different regions of \( q \) and \( k \) momenta. We shall consider 3 different momentum scales. \( \mu = \chi_{SB} \) scale for technicolour, \( A_{ETC} \) = some upper cut off beyond which new physics comes in and \( p \) some intermediate momentum such that \( \mu < p < A_{ETC} \). We shall assume that \( \Sigma(p) = \mu(p)^{\alpha} \) with \( 1<\alpha<2 \) for \( \mu < p < A_{ETC} \) and
\[ \Sigma(P) = \mu \left( \frac{H}{p} \right)^2 (\log s) \] for \( P \geq \Lambda_{\text{ETC}} \). In the following analysis we shall leave out the overall multiplicative factor \( \frac{32i\pi C_2(R) \Sigma(R)}{(2\pi)^4} \) because both \( \tilde{K}(\Sigma) - \tilde{K}(0) \) and \( \tilde{K}(0) - \tilde{K}(0)_{1/\epsilon} \) have this multiplicative factor in common.

1: Let \( k \approx P \)

Consider the contribution to \( \tilde{K}(\Sigma) - \tilde{K}(0) \) coming from \( k = \mu \). We have

\[
2 \pi^2 (l+q)^2 \Sigma(l) \Sigma(l+q) \approx 2 \mu^2 p^2 \mu \left( \frac{H}{p} \right)^2 = \mu^4 p^2 \left( \frac{H}{p} \right)^2
\]

\[
\approx \mu p^2 \left( \frac{H}{p} \right)^2 a^2 + p^2 \mu^2 \mu \left( \frac{H}{p} \right)^2 a^2 = \mu^4 p^2 \left( \frac{H}{p} \right)^2 a^2
\]

Hence for the numerator we have

\[
2 \pi^2 (l+q)^2 \Sigma(l) \Sigma(l+q) \approx 2 \mu^2 p^2 \mu \left( \frac{H}{p} \right)^2 = \mu^4 p^2 \left( \frac{H}{p} \right)^2
\]

\[
\approx \mu^3 p^3
\]

For the denominator we have

\[
\pi^2 (l+q)^2 \left[ \pi^2 \Sigma^2(l) \right] \approx \mu^4 p^4
\]

The phase space supplies a factor of \( \mu^4 \) for \( k = \mu \).

\[ \therefore \left[ \tilde{K}(\Sigma) - \tilde{K}(0) \right]_{l = \mu} \approx \frac{1}{p^4} \mu^4 p^3 = \frac{1}{p^2} \mu^3 \]

With \( k = \mu \) consider the contribution to \( \tilde{K}(\Sigma) - \tilde{K}(0) \) coming from \( l = p \).

Proceeding as above we get \( \left[ \tilde{K}(\Sigma) - \tilde{K}(0) \right]_{l = p} \approx \frac{1}{p^2} \left( \frac{H}{p} \right)^2 \). With \( k = \mu \) consider the contribution to \( \tilde{K}(\Sigma) - \tilde{K}(0) \) coming from \( l = \Lambda_{\text{ETC}} \). We get

\[ \left[ \tilde{K}(\Sigma) - \tilde{K}(0) \right]_{l = \Lambda_{\text{ETC}}} \approx \frac{1}{p^2} \left( \frac{H}{p} \right)^2 \left( \frac{H}{p} \right)^{-4} \]. Hence for \( k = \mu \) the leading contribution to the loop integral comes from \( l = \mu \) and its contribution to \( \tilde{K}(\Sigma) - \tilde{K}(0) \) is
\[ [\tilde{K}(\Sigma) - \tilde{K}(0)]_{K=\mu} \approx \frac{1}{p^2} \left( \frac{\mu}{p} \right)^3 . \quad (3.8) \]

2. Let \( k=\mu \). Proceeding as above we find that the leading contribution to the loop integral comes from \( \lambda=\mu \) and its contribution to \( K(\Sigma) - K(0) \) is
\[ [\tilde{K}(\Sigma) - \tilde{K}(0)]_{K=\mu} \approx \frac{1}{p^2} \left( \frac{\mu}{p} \right)^3 . \quad (3.9) \]

3. Let \( k=\Lambda_{ETC} \). Proceeding as above we find that the dominant contribution to the loop integral comes from \( \lambda=\mu \) and its contribution to \( \tilde{K}(\Sigma) - \tilde{K}(0) \) is
\[ [\tilde{K}(\Sigma) - \tilde{K}(0)]_{K=\Lambda_{ETC}} \approx \frac{1}{\Lambda_{ETC}^2} \left( \frac{\mu}{\Lambda_{ETC}} \right)^3 . \quad (3.10) \]

Let \( \tilde{K}_R = \tilde{K}(0) - \tilde{K}(0)_{1/\epsilon} \) be the contribution to \( \tilde{K} \) of the renormalized vacuum polarization diagram with zero dynamical mass for fermions. This can be evaluated exactly (see Chapter IV for details).
\[ \tilde{K}_R = -\frac{16i g^4 C_2(R) T(R)}{(4\pi)^2 (P-k)^2} \left( \frac{5}{3} \frac{\ln (P-k)^2}{\mu^2} + \ln 6\pi - \gamma \right) . \]
\[ (3.11) \]

Leaving out the overall multiplicative factor we get
1. For \( k \approx \mu \), \( \tilde{K}_R \approx \frac{1}{p^2} \)
2. For \( k \approx P \), \( \tilde{K}_R \approx \frac{1}{p^2} \)
3. and for \( k \approx \Lambda_{ETC} \), \( \tilde{K}_R \approx \frac{1}{\Lambda_{ETC}^2} \) . \quad (3.12)

Compare the above estimates with the leading contribution to \( \tilde{K}(\Sigma) - \tilde{K}(0) \) coming from \( k \approx \mu \), \( k \approx P \) and \( k \approx \Lambda_{ETC} \) respectively. We find that
Hence we see that for $v = u, v = p, v = A_{ETC}$ and for all values of $k$, $\frac{\tilde{K}(\Sigma) - \tilde{K}(0)}{K_R} \approx \frac{3}{p^3} \ll 1$.

Instead of going through the details of the linearization analysis for different diagrams separately we summarize the final results of linearization analysis for the kernel in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>$\frac{\tilde{K}(\Sigma) - \tilde{K}(0)}{K_R}$</th>
<th>$k = u$</th>
<th>$k = p$</th>
<th>$k = A_{ETC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>abelian vertex diagram</td>
<td>$\frac{u}{p}$</td>
<td>$\left(\frac{u}{p}\right)^4$</td>
<td>$\left(\frac{u}{A_{ETC}}\right)^3$</td>
</tr>
<tr>
<td>non-abelian vertex diagram</td>
<td>$\frac{u}{p}$</td>
<td>$\left(\frac{u}{p}\right)^3$</td>
<td>$\left(\frac{u}{A_{ETC}}\right)^2$</td>
</tr>
<tr>
<td>vacuum polarization diagram</td>
<td>$\left(\frac{u}{p}\right)^3$</td>
<td>$\left(\frac{u}{p}\right)^3$</td>
<td>$\left(\frac{u}{A_{ETC}}\right)^3$</td>
</tr>
<tr>
<td>crossed ladder diagram</td>
<td>$\frac{u}{p}$</td>
<td>$\left(\frac{u}{p}\right)^4$</td>
<td>$\frac{u}{A_{ETC}}\left(\frac{u}{p}\right)^3$</td>
</tr>
</tbody>
</table>
Hence for all diagrams $K_R \rightarrow K^2(0) - K(0)$ for all values of $k$ if $\mu \ll \Lambda_{ETC}$.

We find, as expected that although the linearization approximation for the kernel holds for $k=\mu$, it is not as good as it is for $k=p$ and $k=\Lambda_{ETC}$.

The integral equation for $\Sigma(P)$ now becomes (in terms of Euclidean momenta)

$$\Sigma(P) = \frac{1}{4} \int \frac{d^2k}{(2\pi)^4} \frac{\Sigma(k)}{k^2 A^2(k) + \Sigma^2(k)} K_R(P,K,0;\alpha,\mu).$$  \hspace{1cm} (3.14)

This is still non-linear due to the $\Sigma^2(k)$ term appearing in the denominator. However we find that

1. The contribution to $\Sigma(P)$ coming from $k=\mu$ is

$$[\Sigma(P)]_{k=\mu} \approx \frac{\mu^2}{p^2} = \mu \left(\frac{\mu}{p}\right)^2$$

2. The contribution to $\Sigma(P)$ coming from $k=p$ is

$$[\Sigma(P)]_{k=p} \approx \mu \left(\frac{\mu}{p}\right)^\alpha \text{ where } 1 < \alpha < 2$$

3. The contribution to $\Sigma(P)$ coming from $k=\Lambda_{ETC}$.

$$[\Sigma(P)]_{k=\Lambda_{ETC}} \approx \mu \left(\frac{\mu}{\Lambda_{ETC}}\right)^2$$  \hspace{1cm} (3.15)

The term $\Sigma^2(k)$ in the denominator is important only for $k \lesssim \mu$. However if $\Sigma(p)$ does indeed behave as $\mu \left(\frac{\mu}{P}\right)^\alpha$ with $\alpha=1$ for $\mu \ll \Lambda_{ETC}$ then $\Sigma(P)$ receives the largest contribution from $k=p$ and one can drop the $\Sigma^2(k)$ term from the denominator. The result is we have a linear and homogeneous integral equation for $\Sigma(P)$. 


CHAPTER IV
Evaluation of Loop and Angular Integrals

In this chapter we present the methodology for doing the loop and angular integrals. Loop integrals which have two different propagators in the denominator will be done by Feynman parametrization and those with three or four different propagators in the denominator will be done by Gegenbauer Polynomial method in momentum space. Ultraviolet divergent subgraphs will be renormalized by \( \text{MS} \) scheme\(^{14} \). In this scheme one subtracts \( \frac{1}{\varepsilon} + \ln\pi\varepsilon^{-\gamma} \) from the UV divergent subgraph where \( D = \) dimensionality of loop integration = 4-\( 2\varepsilon \) and \( \gamma \) is the Euler constant. As an example of how the method works we will present the evaluation of the contribution of the non-abelian vertex correction diagram to \( \Sigma(P) \) in modest detail in Appendix A. At the end of the chapter we present the piece of the crossed ladder diagram that we have not been able to evaluate exactly so far and the reasons why our method does not work for that subintegral. All discussions will refer exclusively to Euclidean momenta.

To \( O(\alpha^2) \) the diagrams that contribute to \( \Sigma(P) \) are shown in Fig. 4. We can write

\[
\Sigma(P) = \Sigma_0(P) + 2\Sigma_1(P) + 2\Sigma_2(P) + \Sigma_3(P) + 2\sigma(P) .
\]  

(4.1)

See Fig. 4 for the meaning of these symbols. One can check by power counting that the integrals we have to evaluate do not have any
infrared (IR) divergence as long as one of the external momentum \( p \) or \( k \) is different from zero. Since we shall be interested in solutions to \( \Sigma(P) \) for \( \mu \equiv \mu_{\pi A} \), we have no intrinsic infrared divergence in the integrals. The two vertex correction diagrams and the vacuum polarization diagram however have a logarithmic ultraviolet (UV) divergence. It turns out that at one loop level these UV divergent integrals can always be reduced to a form so that they have just two different propagator factors (with one or both sometimes squared) in the denominator. Hence they can be done by Feynman parametrization\(^{10,11}\) or other standard techniques. However the integrals which have three or four different propagator factors although finite cannot be done by Feynman parametrization. If one tries to do them in that way then after evaluating the loop integral one obtains such an intractable result that one cannot do the \( dk \) (normalized angular measure in 4-d Euclidean space) integration. Note that the unknown function \( \Sigma(k) \) which appears in the integrand of the integral equation is a function of \( k^2 \) only and so it would be definitely advantageous from the point of view of solving the integral equation if we could find the traced and angular averaged kernel

\[
\kappa(p^2,k^2) = \int dk \ (\gamma_5)^{cd} K_{ac,d} (p,k,0;\alpha,\mu) (\gamma_5)_{ba} \]

(4.2)

\( \Sigma(P) \) then satisfies an integral equation in just the radial momentum variable \( k^2 \) of the form

\[
\Sigma(P) = \frac{1}{32\pi^2} \int \frac{dk \Sigma(k)}{A^2(k)} \kappa(p^2,k^2) \]

(4.3)

Since we wanted to do both the loop and angular \( dk \) integrals and
obtain an integral equation in just the radial momentum variable we
used the Gegenbauer polynomial method in momentum space\textsuperscript{12} to do the
integrals with more than two different propagators in the denominator.
An important point to note about these later integrals is that they are
both IR and UV finite. The Gegenbauer polynomial method in momentum
space however cannot be used directly to evaluate integrals which have
four different propagator factors in the denominator. The reason being
the inverse propagator factor \((\gamma-p-k)^{-2}\) is not at all suitable for
expansion in terms of Gegenbauer polynomials. One encounters integrals
with four different propagator factors in the denominator in connection
with the crossed ladder diagram. To overcome this problem we used the
method of partial fraction decomposition to express an integral which
has four different propagator factors \(\gamma^2, (\gamma-p)^2, (\gamma-k)^2\) and \((\gamma-p-k)^2\)
(in an arbitrary covariant gauge some of them might be squared) in the
denominator as a sum of integrals each of which has only three different
propagator factors \(\gamma^2, (\gamma-p)^2\) and \((\gamma-k)^2\) in the denominator with
coefficients that are simple functions of \(p^2, k^2\) and \(p\cdot k\). Of course
we had to shift the integration variable \(\gamma\) and change the signs of \(p\)
and (or) \(k\) as necessary, in the course of algebraic manipulation. These
techniques enabled us to do all the integrals in an arbitrary covariant
gauge except for a piece of the crossed ladder diagram which is

\[
\int \frac{d\gamma}{(2\pi)^4} \frac{d^4 k}{k^2} \frac{\Sigma(k)}{(2\pi)^4} \frac{\gamma (\gamma + P + k) [\gamma \cdot k P - (\gamma + P) + \gamma P k - (\gamma + k)]}{\gamma^4 (\gamma + P)^2 (\gamma + k)^2 ((\gamma + P + k)^2)^2}
\]

(4.4)

In the integral equation for \(\Sigma(P)\) this piece appears multiplied with a
factor \((1-\xi)^2\) and hence it does not contribute to the integral equation
in Feynman gauge ($\xi=1$). In other words we have a complete and exact
evaluation of $\bar{K}(p^2,k^2)$ to $O(\alpha^2)$ only in Feynman gauge which is the
simplest gauge to work with in higher order perturbative calculations.

We shall now list the basic principles that enabled us to do the integrals.

1. If the loop integral has only two different propagator
denominators evaluate the integral by Feynman parametrization. If the integral has an UV divergence use dimensional regularization to regulate the divergence. After evaluating the integral subtract \[ \frac{1}{\epsilon} + \ln(\pi - \gamma) \] according to MS prescription.

2. If the loop integral has more than two different propagator
denominators in the denominator and is finite, expand all propagator
denominators in Euclidean space in terms of Gegenbauer polynomials by using the formula

\[ \frac{1}{(x-y)^2} = \frac{1}{M^2(x,y)} \sum_n C_n^{1}(\hat{x} \cdot \hat{y}) \, m^{n}(\hat{x},\hat{y}) \]  
\[ (4.5) \]

Here $\hat{x} = \frac{x}{|x|}$ and $\hat{y} = \frac{y}{|y|}$ and $m(x,y) = \min(\frac{x}{y}, \frac{x}{x})$.

Note that the Chebyshev polynomials $C_n(x \cdot y)$ form a complete set of
orthonormal polynomials only on a 3 sphere. This means that Gegenbauer polynomial method in momentum space cannot be applied to dimensionally regularized integrals. It turns out however that at one loop level all the UV divergent integrals have only two different propagators in the denominator and hence they can be trivially done by Feynman parametrization.
3. In an arbitrary covariant gauge one finds integrals with \((\gamma-p)^2\)
or \((\gamma-k)^2\) appearing squared in the denominator. However in these integrals one always finds a factor of \(\gamma.(r-k)\) or \(\gamma.(r-k)\) in the numerator. One can write

\[
\frac{\gamma.(\gamma-p)}{((\gamma-p)^2)} = \frac{1}{(\gamma-p)^2} + \frac{a}{p^2} \frac{1}{\partial^2(\gamma-p)}
\]  

(4.6)

and take \(p \frac{M}{\partial P}\) outside both \(\int d\mathbf{k}\) and \(\int \frac{d^3\gamma}{(2\pi)^6}\) integrations by using integration by parts. The resulting integrations are much simpler.

One encounters integrals of this kind in connection with the contribution of non-abelian vertex correction diagram to \(\Sigma(P)\).

4. Carry out the \(\int d\gamma\) and \(\int d\mathbf{k}\) integrations using the orthonormality condition of Gegenbauer polynomials.

\[
\int d\mathbf{k} \ C_{n_0}^*(\mathbf{a} \cdot \mathbf{k}) \ C_{n_1}^*(\mathbf{b} \cdot \mathbf{k}) = \delta_{n_0 n_1} \frac{1}{n_0 n_1 + 1} \ C_{n_0}^*(\mathbf{a} \cdot \mathbf{b})
\]

(4.7)

5. Next do the radial \(\int_0^\infty d\gamma\) integration by dividing the entire integration range into six different regions. For \(k<p\) the three regions are \(0<\gamma<k\), \(k<\gamma<p\) and \(p<\gamma<\infty\). For \(k>p\) the three regions are \(0<\gamma<p\), \(p<\gamma<k\) and \(k<\gamma<\infty\). Add up the contributions arising from different regions for \(k<p\) and \(k>p\) separately and express the result in a symmetric form.

6. After doing all the integrals one is left with an infinite sum which has to be evaluated in terms of known elementary functions and special functions for \(k<p\) and \(k>p\) separately. The trick is to start with the infinite geometric series and do various mathematical manipulations on it until it reduces to the form one needs.
7. In order to do the integrals without getting into IR divergences sometimes it might be necessary to express the product of two Gegenbauer polynomials with the same argument as a finite sum of Gegenbauer polynomials. The formula one needs to use is

\[ C'_n(x) C'_m(x) = \sum_{k=|n-m|}^{n+m} C'_k(x) \]  

(4.8)

One encounters integrals of this kind in connection with the contribution of abelian vertex correction diagram.

8. Use the method of partial fraction decomposition to express integrals which have four different propagator factors \( \gamma^2, (\gamma-p)^2, (\gamma-k)^2 \) and \( (\gamma-p-k)^2 \) (with some of them squared) in the denominator as a sum of integrals each of which has only three different propagator factors \( \gamma^2, (\gamma-p)^2 \) and \( (\gamma-k)^2 \) in the denominator. This method enabled us to evaluate the contribution of the crossed ladder diagram exactly in Feynman gauge. But the method does not enable us to evaluate the contribution of the crossed ladder diagram exactly in an arbitrary gauge. To make the meaning of partial fraction decomposition clearer and to demonstrate how it works let us consider an example. The contribution of the crossed ladder diagram to \( \Sigma(P) \) in Feynman gauge is

\[ \Sigma_4(P) = -4g^4C_2(R)\left[C_2(R) - \frac{1}{2}C_2(G)\right] \int \frac{d^4k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4\gamma}{(2\pi)^4} \frac{1}{\gamma^2(\gamma+P)^2(\gamma+K)^2(\gamma+P+K)^2} \]  

(4.9)

Here \( C_2(G) \) is the quadratic Casimir for the adjoint representation of the gauge group. Using the identity
Making appropriate shifts the integration variable $\gamma$ and reversing the sign of $p$ and (or) $k$ as required we can write

$$\int \frac{d^4k}{(2\pi)^4} \Sigma(k) \quad \int \frac{d^4\gamma}{(2\pi)^4} \frac{(Y+P)(Y+k)}{Y^2(Y+P)^2(Y+P+k)^2}$$

$$= \int \frac{d^4k}{(2\pi)^4} \Sigma(k) \quad \int \frac{d^4\gamma}{(2\pi)^4} \frac{1}{Y^2(Y-P)^2(Y-k)^2}$$

$$- \int \frac{d^4k}{(2\pi)^4} \Sigma(k) \quad \int \frac{d^4\gamma}{(2\pi)^4} \frac{1}{P+k}$$

$$= \int \frac{d^4k}{(2\pi)^4} \Sigma(k) \quad \int \frac{d^4\gamma}{(2\pi)^4} \frac{1}{Y^2(Y-P)^2(Y+k)^2}$$

$$= \int \frac{d^4k}{(2\pi)^4} \Sigma(k) \quad \int \frac{d^4\gamma}{(2\pi)^4} \frac{1}{P+k}$$

Both the integrals appearing above have only three different propagations in the denominator and can be easily evaluated by Gegenbauer polynomial method (GPM) in momentum space. We shall now present the results of our calculations for the contribution of each diagram separately. They can be readily evaluated by following the principles outlined above.

a. Straight ladder diagram

$$\Sigma_0(P) = (3+\xi)g^2C_2(R) \int \frac{d^4k}{(2\pi)^4} \Sigma(k) \quad \frac{1}{k^2A^2(k) (P-k)^2}$$

$$= \frac{(3+\xi)\alpha}{2\pi} C_2(R) \int \frac{kdk\Sigma(k)}{H^2A^2(k)}$$

$$A(k) = 1 + \frac{\alpha C_2(R)}{4\pi} \xi(1-\ln \frac{k^2}{\mu^2}) \text{ in MS scheme.}$$
b. Abelian Vertex Correction diagram

Consider the contribution of the abelian ($\Gamma_{qqg}$) and non-abelian ($\Gamma_{ggg}$) vertex correction diagrams to $\Sigma(P)$. Both $\Gamma_{qqg}$ and $\Gamma_{ggg}$ are UV divergent and we shall regulate them by dimensional regularization. Strictly speaking the program of dimensional continuation should be applied only to divergent subgraphs $\Gamma_{qqg}$ and $\Gamma_{ggg}$. However the Gegenbauer polynomial method in momentum space which we shall use to evaluate finite integrals can be applied only to Euclidean scalars. To get Euclidean scalars one has to take traces and do some additional $\gamma$ algebra which fall outside the divergent subgraph. These additional algebraic manipulation might lead to spurious factors of $D$ when done in $D$ dimensions. These extra $D$ factors are harmless when they multiply the UV finite piece of $\Gamma_{qqg}$ or $\Gamma_{ggg}$ because $d=4-2\varepsilon$ and ultimately we shall let $\varepsilon$ tend to zero. However they give rise to extra constant factors when multiplying the pole term in $\Gamma_{qqg}$ or $\Gamma_{ggg}$. So we need to multiply the final result by a correction factor if the algebra is done in $D$ dimensions for the entire expression of $\Sigma_1(P)$. The evaluation of this correction factor for $\Gamma_{qqg}$ and $\Gamma_{ggg}$ goes as follows. In a renormalizable field theory we can write by power counting and Lorentz covariance

$$\Gamma_\mu(P,k) = (g_\mu - \varepsilon)^2 \frac{a}{\varepsilon} \gamma_\mu + \text{finite contributions.} \quad (4.13)$$

The pole term is $(g_\mu - \varepsilon)^2 \frac{a}{\varepsilon} (\gamma_\mu)_{ac}$ where $a$ is a constant. Consider the following expression
\[(g_{\mu}-\epsilon)^2 \frac{a}{\epsilon} \left( \gamma_{\mu} \gamma_{\nu} [g_{\mu\nu} - (1-\xi) \frac{(p-k)^{\mu}(p-k)^{\nu}}{(p-k)^2}] \right) \]

\[= \left( g_{\mu}-\epsilon \right)^2 \frac{a}{\epsilon} \sum_{\gamma} \gamma_{\mu} \gamma_{\nu} [g_{\mu\nu} - (1-\xi) \frac{(p-k)^{\mu}(p-k)^{\nu}}{(p-k)^2}] \]

\[= 4( g_{\mu}-\epsilon)^2 \frac{a}{\epsilon} (D-1+\xi) \]

(4.14)

So the lesson is if we work in an arbitrary gauge \( \xi \) and do the algebra in \( D \) dimensions for the entire expression \( \sum \frac{1}{2} \sum (p) \) of \( \sum (p) \) then we must multiply our final result by a correction factor of \( \frac{(3+\xi)}{(D-1+\xi)} = (1+\frac{2\epsilon}{3+\xi}) \).

In \( D=4-2\epsilon \) dimensions the coupling constant \( g \) has dimensions of \( \mu^{\xi} \) where \( \mu \) is some arbitrary mass parameter. To get a dimensionless coupling \( g \) has to multiplied by \( \mu^{-\epsilon} \). Let \( \Sigma_1(p) \) be the contribution of \( \Gamma_{qqg} \) to \( \Sigma(p) \). We get

\[\Sigma_1(p) = g^2 \left( g_{\mu}-\epsilon \right)^2 [C_2(R) - \frac{1}{2} C_2(G)] C_2(R) \int \frac{d^D k}{(2\pi)^D} \frac{\Sigma(k)}{k^2} \mu^{2\epsilon} \frac{(1+2\epsilon)}{(3+\xi)} \int \frac{d^D \gamma}{(2\pi)^D} \]

\[\left[ (D-2)(D+\xi-3)(p-\gamma), (k-\gamma), (1-\xi)(D+\xi-5)(p-\gamma), (k-\gamma) + 2(1-\xi)(D+\xi-3) \right. \]

\[\left. \frac{(p-\gamma)(k-\gamma)(p-k)}{(p-k)^2} - 4(1-\xi) \frac{\gamma(p-\gamma)\gamma(k-\gamma)}{\gamma^2} + 2(1-\xi)^2 \frac{\gamma(p-k)}{\gamma^2(p-k)^2} \right] \times \]

\[\left[ (p-\gamma)(p-k), (k-\gamma), (k-\gamma) + 2(1-\xi)(p-\gamma), (k-\gamma)(p-k), (p-\gamma) \right] \]

\[\frac{1}{(p-k)^2 \gamma^2 (\gamma-p)^2 (\gamma-k)^2} \]

(4.15)
\[
\frac{\alpha}{8\pi^2} C_2(R) \int_0^\infty \frac{kdk\Sigma(k)}{M^2} (C_2(R) - \frac{1}{2} C_2(G)) \left[ \frac{1}{4} ((1-\xi)(7+\xi) - 4(1+\xi)) \right] \left( \frac{\mu^2}{\mu^2} + \frac{k^2}{\mu^2} \right) \\
- \frac{1}{2} (1-\xi)(3-\xi) \ln \frac{M^2}{\mu^2} + \ln \frac{M^2}{\mu^2} (\xi-1)(1+3\xi) - 2 - (1-\xi)(9-\xi) \frac{M^4}{3} \frac{4}{3} (1-\xi) \left[ 1 + (1-\xi) \left( \frac{1}{4} + \frac{m^2}{M^2} \right) \right] \\
+ \frac{1}{4} (1-\xi)^2 \ln \frac{m^2}{M^2} + 2 (1-\xi)(1+\frac{M^2}{m^2}) L_2 \left( \frac{m^2}{M^2} \right) + (1-\xi)(1+\frac{M^2}{m^2}) \ln \frac{m^2}{M^2} \ln \left( \frac{1-m^2}{M^2} \right) \\
- (1-\xi) \ln \frac{M^2}{(M+m)^2} - (1-\xi) \left( \frac{M^2}{m^2} - 1 \right) \ln \left( \frac{1-m^2}{M^2} \right) \\
+ \frac{1}{2} (1-\xi) \ln (1+m)(4 + (1-\xi)(1+\frac{m^2}{M^2}) \ln \left( \frac{1-m^2}{M^2} \right) \\
(4.16)
\]

Here \( m=\min(P,k) \), \( M=\max(P,k) \)

\( L_2(Z) \) is dilogarithm function\(^{16}\) defined by \( L_2(Z) = - \int_0^Z \frac{\ln(1-t)}{t} dt \)

\( F(\frac{3}{2}, 1; 3; \frac{4P}{(P+k)^2}) \) is the Gauss hypergeometric\(^{17}\) function and

\( F' = \left. \frac{\partial F}{\partial a} \right|_{a=1} \)

C. Non-abelian vertex correction diagram. Let \( \Sigma_2(P) \) be the contribution of \( \Gamma_{\text{ggg}} \) to \( \Sigma(P) \). We get

\[
\Sigma_2(P) = \frac{\alpha^2}{16\pi^2} C_2(G) C_2(R) \left( \frac{\gamma - \epsilon}{\mu} \right)^2 \left( \frac{2\epsilon}{1+3+\xi} \right) \int \frac{dk}{(P-k)^2} \int \frac{dY}{(2\pi)^D} \\
\left[ 2(2-D)Y \left( (2\gamma-P-k) - 2(1-\xi)Y \left( (2\gamma-P-k) + (1-\xi) \right) \right) \right] \left( \frac{\gamma-Y}{(\gamma-P)\gamma} \right) \\
\left[ (2-D)(\gamma-P) \cdot (\gamma-k) - D(\gamma-P) \cdot (P-k) \right] + (1-\xi) \left( \frac{\gamma-Y}{(\gamma-k)\gamma} \right) \left[ (2-D)(\gamma-k) \cdot (\gamma-P) \right]
\]
\[ + \frac{\alpha^2}{8\pi^2} \frac{C_2(R) \int \frac{dk \tilde{F}(k)}{M^2}}{\gamma^2} \{ [C_2(G)(5+3(1-\xi)) - 4N_f T(R)] + \ln \frac{(P\cdot k)^2}{\mu^2} \] 

\[ + \frac{C_2(G)(3+3(1-\xi)) - 20}{3} T(R)N_f \} \] 

Here \( T(R) \delta_{AB} = \text{Tr}(T_A T_B) \) where \( T_A \) are the generators of the gauge group in the representation \( R \) and \( N_f \) is the number of technifermion flavours.

In technicolour theories \( N_f \) must be greater than or equal to 2.
\[ \Sigma_4(P) = g^4 C_2(R)[C_2(R) - \frac{1}{2} C_2(G)] \left[ \frac{d^4 k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \frac{d^4 \gamma}{(2\pi)^4} \frac{1}{\gamma^2 (\gamma + P)^2 (\gamma + k)^2 (\gamma + P + k)^2} \right] \times \]

\[ [(1 + \xi)(\xi - 3)(\gamma + P)(\gamma + k) + 4(1 - \xi^2) \gamma (\gamma + k) \gamma (\gamma + P) + \frac{2(1 - \xi)^2 \gamma (\gamma + P + k)}{\gamma^2 (\gamma + P + k)^2} \]

\[- (\gamma + k)(\gamma + P)\gamma (\gamma + P + k) + (\gamma + P)(\gamma + P + k)\gamma (\gamma + k)] \quad (4.19) \]

We have

\[ \Sigma_{4;1}(P) = (q + \xi)(\xi - 3) \int \frac{d^4 k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4 \gamma}{(2\pi)^4} \frac{\gamma (\gamma + P)(\gamma + k)}{\gamma^2 (\gamma + P + k)^2 (\gamma + P + k)^2} \]

\[ = \frac{(1 + \xi)(\xi - 3)}{8\pi^2 (4\pi)^2} \int \frac{k d k \Sigma(k)}{M^2} \left[ (1 + \frac{m^2}{m^2}) [L_2 (-\frac{m^2}{M^2} + \ln (1 + \frac{m^2}{M^2})] + (2 - \ln \frac{m^2}{M^2}) \right] \quad (4.20) \]

\[ \Sigma_{4;2}(P) = 4(q - \xi^2) \int \frac{d^4 k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4 \gamma}{(2\pi)^4} \frac{\gamma (\gamma + P)(\gamma + k)}{\gamma^4 (\gamma + P)^2 (\gamma + k)^2 (\gamma + P + k)^2} \]

\[ = \frac{(1 - \xi^2)}{8\pi^2 (4\pi)^2} \int \frac{k d k \Sigma(k)}{M^2} \left[ 2(1 + \frac{m^2}{m^2}) [L_2 (-\frac{m^2}{M^2} + \ln \frac{m^2}{M^2} + \ln (1 + \frac{m^2}{M^2})] + 3(2 - \ln \frac{m^2}{M^2}) \right.

\[ + (1 - 2 \frac{m^2}{M^2 + m^2}) \ln \frac{m^2}{M^2} \left] \quad (4.21) \right. \]

\[ \Sigma_{4;3}(P) = 2(1 - \xi^2) \int \frac{d^4 k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4 \gamma}{(2\pi)^4} \frac{\gamma (\gamma + P + k)}{\gamma^4 (\gamma + P)^2 (\gamma + k)^2 (\gamma + P + k)^2} \]

\[ [(\gamma + k)(\gamma + P + k)\gamma (\gamma + P) - (\gamma + k)(\gamma + P)\gamma (\gamma + P + k) + (\gamma + P)(\gamma + P + k)\gamma (\gamma + k)] \]

\[ (4.22) \]

\[ = 2(1 - \xi^2) \int \frac{d^4 k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4 \gamma}{(2\pi)^4} \frac{\gamma (\gamma + P + k)}{\gamma^4 (\gamma + P)^2 (\gamma + k)^2 (\gamma + P + k)^2} \]

\[ [\gamma^2 ((\gamma + P + k)^2 - P_k) \]

\[ + \gamma.kP.(\gamma + P) + \gamma.P.k.(\gamma + k)] \]

\[ = \Sigma_{4;3;1} + \Sigma_{4;3;2} + \Sigma_{4;3;3} \quad (4.23) \]
\[ \Sigma_{4;3;1} = 2(1-\xi)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4\gamma}{(2\pi)^4} \frac{\gamma.(\gamma+P+k)}{\gamma^2(\gamma+P)^2(\gamma+k)^2(\gamma+P+k)^2} \]

\[ = \frac{(1-\xi)^2}{4\pi^2(4\pi)^2} \int_0^\infty \frac{kd^k\Sigma(k)}{M^2} \left[ (2-\ln\frac{M^2}{m^2}+1-\frac{M^2}{m^2})\left(\frac{\ln\frac{M^2}{m^2}+\ln(1+\frac{m^2}{M^2})}{M^2}\right) \right] \] (4.24)

\[ \Sigma_{4;3;2} = -2(1-\xi)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4\gamma}{(2\pi)^4} \frac{\gamma.(\gamma+P+k)}{\gamma^2(\gamma+P)^2(\gamma+k)^2(\gamma+P+k)^2} \]

\[ = \frac{(1-\xi)^2}{16\pi^2(4\pi)^2} \int_0^\infty \frac{kd^k\Sigma(k)}{M^2} \left[ (2-\ln\frac{M^2}{m^2}+1-\frac{M^2}{m^2})\left(\frac{\ln\frac{M^2}{m^2}+\ln(1+\frac{m^2}{M^2})}{M^2}\right) \right] \] (4.25)

\[ \Sigma_{4;3;3} = 2(1-\xi)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\Sigma(k)}{k^2} \int \frac{d^4\gamma}{(2\pi)^4} \frac{\gamma.(\gamma+P+k)[\gamma.kP.(\gamma+P)+\gamma.Pk.(\gamma+k)]}{\gamma^4(\gamma+P)^2(\gamma+k)^2(\gamma+P+k)^2} \]

\[ = \frac{(1-\xi)^2}{4\pi^2(4\pi)^2} \int_0^\infty \frac{kd^k\Sigma(k)}{M^2} \left[ (2-\ln\frac{M^2}{m^2}+1-\frac{M^2}{m^2})\left(\frac{\ln\frac{M^2}{m^2}+\ln(1+\frac{m^2}{M^2})}{M^2}\right) \right] \] (4.26)

This is the most difficult integral. We have not been able to do this integral exactly and completely, although we have been able to do parts of it by using Gegenbauer polynomial method (GPM) in momentum space. But since we are not convinced that the entire integral can be evaluated in this approach we will not reproduce the results of those partial evaluations. The problem that one encounters in evaluating this integral is the following: It seems to us that GPM in momentum space is the best bet for doing both the angular and the loop integrations. But the GPM in momentum space cannot be applied directly when one has in the denominator an expression of the form 

\[ \gamma^4(\gamma+P)^2(\gamma+k)^2(\gamma+P+k)^2 \]. If one tries to make (according to our recipé) a partial fraction decomposition of the corresponding
integrands, combined with appropriate variable changed one obtains a
sum of integrands each of which has only a product of $\gamma^2$, $(\gamma-p)^2$ and
$(\gamma-k)^2$ in the denominator. But the coefficients of the loop integrands
are none much more complicated functions of $p^2$, $k^2$ and $p \cdot k$. In fact
after doing the $dy$ integration one encounters angular integrations over
directions of $\hat{k}$ of the form $\int \frac{\hat{d}k}{(\hat{p} \cdot \hat{k})^3} C_n(\hat{p} \cdot \hat{k})$ and $\int \frac{\hat{d}k}{(\hat{p} \cdot \hat{k})^2} C_n(\hat{p} \cdot \hat{k})$.
Note that after done $\int \hat{d}k$ we will be left with one infinite sum over $n$
which has to be done. These angular integrals are individually singular
because the denominator vanishes at $\hat{p} \cdot \hat{k} = \cos \theta$. There is no
doubt that the divergences must cancel each other yielding a finite
result for $\Sigma_{4;3;3}$. But we have not been able to do the individually
divergent angular integrals using some suitable regulator scheme and
show that when we add all the pieces together the terms that have a
singular dependence upon the regulator cancel leaving a finite result
for $\Sigma_{4;3;3}$. We also found that it is not possible to cancel the
singular behaviour of the angular integrands among each other and still
do the $\hat{d}k$ integration and get a tractable closed form result (i.e. no
infinite sums). We tried every conceivable partial fraction decomposi-
tion but in each case we found the same problem. We are almost
convinced that the GPM in momentum space combined with partial fraction
decomposition will work if the loop integrand has $\gamma^2(\gamma-p)^2(\gamma-k)^2(\gamma-p-k)^2$
or $\gamma^4(\gamma-p)^2(\gamma-k)^2(\gamma-p-k)^2$ or $\gamma^2(\gamma-p)^2(\gamma-k)^2((\gamma-p-k)^2)^2$ in the
denominator. But to our utter disappointment it fails if the integrand
has $\gamma^4(\gamma-p)^2(\gamma-k)^2((\gamma-p-k)^2)^2$ in the denominator.

The integral for $\Sigma_{4;3;3}(P)$ however vanishes in the limit
$M \rightarrow 0$. We shall therefore evaluate the entire kernel in this limit and
use this end point approximation for the kernel to obtain an approximate solution for $\Sigma(P)$. The solution, as we shall see, though approximate elucidates some of the interesting features of dynamical mass generation and $\chi$SB in the fixed point limit. We shall derive this approximate solution in the next chapter and discuss its implications.
CHAPTER V

An Approximate Solution of the Integral Equation

In this chapter we present an approximate solution of the integral equation. The approximation consists in using the end point approximation for the kernel instead of the complete and exact kernel. We discuss in which gauge power law solutions to $\Sigma(P^2)$ and $A(P^2)$ can be obtained and the implications of the solution on the validity of lowest order perturbation theory. We also present an estimate of the $O(\alpha^2)$ correction to the critical coupling $\alpha_c$ based on the conjecture that $\alpha_c$ is determined by $\gamma_m(\alpha=1)$ to any order of perturbation theory.

A. The Integral equation in the end point approximation for the kernel.

Let us evaluate the kernel for $M \gg m$ and use it in the integral equation for $\Sigma(P)$. We get

$$\Sigma_0(P^2) = \frac{(3+\xi)\alpha}{4\pi} C_2(R) \int_0^\infty \frac{dk^2}{k^2} \frac{\Sigma(k^2)}{k^2}$$

where $A(k^2) = 1 - \frac{\alpha\xi}{4\pi} C_2(R) [\ln \frac{k^2}{\mu^2} - 1]$. 

(5.1)

$$2\Sigma_1(P^2) = \frac{\alpha^2}{8\pi^2} C_2(R)[C_2(R) - \frac{1}{2} C_2(G)] \int_0^\infty \frac{dk^2}{k^2} \frac{\Sigma(k^2)}{k^2} [\xi(3+\xi) \ln \frac{M^2}{\mu^2} + 2 + (1-\xi)(4+\xi)]$$

(5.2)
The above step needs some explanation.

First the leading log terms in the expression for \( A_1(p^2) \) can always be put into the form \( \ln \frac{M^2}{\mu^2} \) by introducing compensating terms of the form \( \ln \frac{p^2}{k^2} \). Second since the leading log term of the abelian vertex correction diagram to \( 0(\alpha) \) is the same as that of the wave function renormalization it is possible to extract the leading log term proportional to \( (\alpha C_2(R))^2 \) of \( 2A_1(p^2) \) in the form of a prefactor \( \Lambda^2 M^2 \ln \frac{M^2}{\mu^2} \). This explains how one goes from (5.2) to (5.3).

Note however that in (5.3) the leading log term proportional to \( \alpha^2 C_2(R)C_2(G) \) of \( 2A_1(p^2) \) is still explicitly displayed.

\[
\Sigma_3(p^2) = \frac{\alpha^2}{16\pi^2} C_2(R) \int_0^\infty \frac{dk^2 \Sigma(k^2)}{M^2} \left[ \left( \frac{4n_f T(R) - \frac{1}{2}(13 - 9\xi) C_2(G)\ln \frac{M^2}{\mu^2}}{2} \right) + \frac{1}{12} (97 + 18\xi + 9\xi^2) C_2(G) - \frac{20}{3} n_f T(R) \right] \tag{5.5}
\]

\[
\Sigma_4(p^2) = \frac{\alpha^2}{16\pi^2} C_2(R) C_2(G) \int_0^\infty \frac{dk^2 \Sigma(k^2)}{M^2} (3 - 6\xi - \xi^2) \tag{5.6}
\]
Hence \( \Sigma(P^2) = \Sigma_0(P^2) + 2\Sigma_1(P^2) + 2\Sigma_2(P^2) + \Sigma_3(P^2) + \Sigma_4(P^2) \)

\[
\begin{align*}
\Sigma(P^2) & = \frac{\alpha}{4\pi} C_2(R) \int_0^\infty \frac{dk^2 \Sigma(k^2)}{M^2} \frac{A^2(M^2)}{A^2(k^2)} \left[ (3+\xi) - \frac{\alpha}{4\pi} \left( \frac{1}{12} (113+42\xi+9\xi^2)C_2(G) - (3+\xi)^2 C_2(R) - \frac{20}{3} n_F T(R) \right) \right] X(P^2) \\
& + \frac{\alpha}{4\pi} \left( \frac{1}{12} (313+42\xi+9\xi^2)C_2(G) - (3+\xi)^2 C_2(R) - \frac{20}{3} n_F T(R) \right) \ln \frac{M^2}{\mu^2} + \frac{\alpha}{4\pi} \frac{1}{12} (313+42\xi+9\xi^2)C_2(G) - (3+\xi)^2 C_2(R) - \frac{20}{3} n_F T(R) \right] \ln \frac{M^2}{\mu^2} \end{align*}
\]

For small \( \alpha \) one can solve the renormalization group equations for the running coupling \( \alpha(M^2) \) and the running gauge parameter \( \xi(M^2) \) to one loop.\(^{15}\) We obtain

\[
\begin{align*}
\alpha(M^2) & = \alpha [1-\frac{\alpha}{12\pi} \left( 11C_2(G) - 4n_F T(R) \right) \ln \frac{M^2}{\mu^2}] \\
\xi(M^2) & = \xi [1+\frac{\alpha}{24\pi} \left( 13-3\xi \right) C_2(G) - 8n_F T(R) \ln \frac{M^2}{\mu^2}] \end{align*}
\]

Using the above we get

\[
\begin{align*}
\Sigma(P^2) & = \frac{\alpha}{4\pi} C_2(R) \left[ (3+\xi) - \frac{\alpha}{4\pi} \left( \frac{1}{12} (313+42\xi+9\xi^2)C_2(G) - (3+\xi)^2 C_2(R) - \frac{20}{3} n_F T(R) - (3+\xi)^2 C_2(R) \right) \right] \times \\
& \int_0^\infty \frac{dk^2 \Sigma(k^2)}{M^2} \frac{A^2(M^2)}{A^2(k^2)} \left( 3+\xi(M^2) \right) \alpha(M^2) \end{align*}
\]

We will use the above integral equation to obtain a scale invariant power law solution for \( \Sigma(P) \).

B. When can \( \Sigma(P^2) \) and \( A(P^2) \) have scale invariant power law solutions?

Let us assume that in the non-running limit (V.8) admits power law
solutions of the form $\Sigma(P^2) = \mu \frac{p^2}{\mu^2} -(b \pi^2)$ and $A(P^2) = \frac{p^2}{\mu^2} - \lambda$. Here we shall identify $b$ with $\frac{1}{2} \gamma_m$ and $\lambda$ with $\frac{1}{2} \gamma_2$. That $A(P^2)$ should scale as $\frac{p^2}{\mu^2} - \gamma_2$ can be seen most easily by evaluating $A(P^2)$ to $0(\alpha)$ and using the $0(\alpha)$ expression for $\gamma_2$. An equivalent way is to solve the renormalization group (RG) equation for $\gamma_2$ in the non-running limit. That $\Sigma(P^2)$ should scale as $\mu \frac{p^2}{\mu^2} - \gamma_2 (\gamma_m + \gamma_2)$ can be seen by recalling that the effective mechanical mass $m(P^2) = \frac{\Sigma(P^2)}{A(P^2)}$ should scale like $\frac{p^2}{\mu^2} - \gamma_2$ according to the RG equation for $\gamma_m$ in the non-running limit. Equation (V.8) indicates that to achieve a non-running condition we must have

\[(3 + \xi(M^2)) a(M^2) = (3 + \xi) a\]

(5.10)

This leads to $n^2 T(R) = \frac{1}{4} (11 + \frac{3}{2} \xi + \frac{1}{2} \xi^2) C^2_2(G)$

(5.11)

where we used the one loop solutions for $a(M^2)$ and $\xi(M^2)$ given by (5.8). Using the non-running condition (5.10) in the equation for $\Sigma(P^2)$ we get

$$\Sigma(P^2) = (aK_1 + a^2 K_2) \int_0^\infty \frac{dk^2}{M^2} \frac{\Sigma(k^2)}{A^2(k^2)} a^2(M^2)$$

where $aK_1 = \frac{(3 + \xi)a}{4\pi} C_2^2(R)$

(5.12)

and $a^2 K_2 = \frac{a^2}{4\pi} \left[ \frac{1}{12} (313 + 42 \xi + 9 \xi^2) C_2^2(G) - \frac{20}{3} n^2 T(R) - (3 + \xi)^2 C_2^2(R) \right]$

Using the above ansatz for $\Sigma(P^2)$ and $A(P^2)$ in the integral equation we get
\[ b(1-b) = (K_1\alpha + K_2\alpha^2)(1+2\lambda) - \lambda(1+\lambda) + 0(\alpha^3) \]

\[ b_\pm = \frac{1}{2} \left[ 1 \pm \left[ 1+4(\lambda-K_1\alpha)+4\lambda(\lambda-2K_1\alpha)-4K_2\alpha^2 \right]^{1/2} \right] \quad (5.13) \]

\( b_+ \) is the exponent for dynamical mass. This can be seen by examining the small \( \alpha \) limit of \( b_+ \) which corresponds to the much softer asymptotic behaviour \( \frac{1}{p^2} \) up to logs of \( m(p^2) \). \( b_- \) is the exponent for mechanical mass and this can be seen by examining the small \( \alpha \) limit of \( b_- \) which corresponds to a much slow asymptotic fall off for \( m(p^2) \). For \( \alpha \ll \) we get

\[ b_- = -\lambda + K_1\alpha + (K_1^2+K_2)\alpha^2 \]

\[ = \frac{3\alpha}{4\pi} C_2(R) + \left( \frac{\alpha}{4\pi} \right)^2 C_2(R) \left[ \frac{1}{6}(119+9\xi+3\xi^2)C_2(G) - \frac{14}{3} n_f T(R) + \frac{3}{2} C_2(R) \right] \quad (5.14) \]

where we used

\[ \lambda = \frac{1}{2} \gamma_2 \]

\[ = \frac{\alpha K}{4\pi} C_2(R) + \left( \frac{\alpha}{4\pi} \right)^2 C_2(R) \left[ \frac{1}{4}(25+8\xi+\xi^2)C_2(G) - 2n_f T(R) - \frac{3}{2} C_2(R) \right] \quad (5.15) \]

Equating \( b_- \) to \( \frac{1}{2} \gamma_m = \frac{1}{2} \left[ \frac{\alpha}{\pi} Y_1 + \left( \frac{\alpha}{\pi} \right)^2 \gamma_2 \right] \) where

\[ Y_1 = \frac{3}{2} C_2(R) \]

\[ Y_2 = \frac{3}{16} C_2(R) + \frac{97}{48} C_2(R) C_2(G) - \frac{5}{12} C_2(R) T(R) n_f \quad (5.16) \]

we get

\[ n_f T(R) = \frac{1}{8} (22+9\xi+3\xi^2)C_2(G) \quad (5.17) \]
(V.10) and (V.13) agree iff

$$\xi=0 \text{ (Landau gauge)} \text{ and } \xi=-3 \text{ (peculiar gauge).}$$

Hence we conclude that in the end point approximation for the kernel power law solutions of the form $$\sum(p^2) = \mu\left(\frac{p^2}{\pi^2}\right)^{-\frac{2}{3}(\gamma_1+\gamma_2)}$$ and $$A(p^2) = \left(\frac{p^2}{\pi^2}\right)^{-\frac{2}{3}\gamma_2}$$ can be obtained in the non-running limit only in $$\xi=0$$ and $$\xi=-3$$ gauges.

That a scale invariant solution can be obtained in Landau gauge with a slowly running coupling can be reasoned as follows. The scale factor $$\mu$$ appears in the kernel of the integral equation only through the leading log terms. The leading log terms in the kernel can arise either from running coupling or from running gauge parameter. Landau gauge is the fixed point for the gauge parameter to all orders in perturbation theory. Further if the coupling does not run to $$0(\alpha)$$ then the scale factor $$\mu$$ does not appear in the kernel. This makes a scale invariant solution obtainable in Landau gauge if the coupling does not run to $$0(\alpha)$$. It is a little difficult to see why a scale invariant solution can be obtained in $$\xi=-3$$ gauge with a fixed coupling. The contributions of the straight ladder diagram, abelian vertex correction diagram and the non-abelian vertex correction diagram to the kernel vanish individually if $$\xi=-3$$. Hence in $$\xi=-3$$ gauge the leading log term in the kernel arises only from the vacuum polarization diagram. By explicit computation one can show that the coefficient of the leading log term contributed by the vacuum polarization diagram to the kernel is proportional to $$[C(G)(\frac{13}{2}-\frac{\xi}{2})-\frac{2}{3}T(R)_{\overline{f}}]$$ if $$\xi=-3$$ this reduces to $$2\frac{11}{6}C_2(G)\left(\frac{2}{3}T(R)_{\overline{f}}\right)^{2.8}_{-1},$$ which vanishes if the coupling does not run to $$0(\alpha)$$.

C. How important are the $$0(\alpha^2)$$ corrections for large $$\alpha$$?

Let us choose Landau gauge ($$\xi=0$$) and let $$n(T(R)=\frac{11}{4}C_2(G)$$ i.e. $$B(\alpha)=0+0(\alpha^2)$$
The ansatz $\Sigma(p^2) = \mu(p^2)^{-\lambda}$ and $A(p^2) = (\frac{p^2}{\mu^2})^{-\lambda}$ gives

$$b(1-b) = -\lambda(1+\lambda) + (K_1^2 + K_2^2)(1+2\lambda) + O(\alpha^3)$$

(5.19)

Using

$$\lambda = \frac{1}{2} \gamma_2 = \frac{3\alpha}{\pi} C_2(R) \left[ \frac{2 C_2(G) - 2C_2(R)}{192C_2(R)} \right] + O(\alpha^3)$$

(5.20)

We get

$$b(1-b) = \frac{1}{4} \left[ \frac{3\alpha}{\pi} C_2(R) + \left(\frac{3\alpha}{\pi} C_2(R) \right) \frac{2 C_2(G) - 2C_2(R)}{192C_2(R)} \right] + O(\alpha^3)$$

$$1) \quad \frac{1}{2} \gamma_2 \left( \frac{1}{m} + \frac{1}{m} \right)$$

(5.2)

where

$$\frac{1}{2} \gamma_m = \frac{1}{4} \left[ \frac{3\alpha}{\pi} C_2(R) + \left(\frac{3\alpha}{\pi} C_2(R) \right) \frac{2 C_2(G) + 9C_2(R)}{216C_2(R)} \right] + O(\alpha^3)$$

(5.22)

Assume that technicolour is an SU(N) gauge theory with N large. We will now estimate the coefficient of $\left(\frac{3\alpha C_2(R)}{\pi} \right)^2$ in $b(1-b)$ and $\frac{1}{2} \gamma_2$ for large N and $R = \text{fundamental (D), symmetric (} )$ and antisymmetric ( ) representations.
Thus for these low lying representations of SU(N) with N large, the $0(\alpha)$ results in $\xi=0$ gauge $b_\pm = \frac{1}{2}[1\pm \frac{1}{\sqrt{\pi}C_2(R)}]$ and $\lambda = \frac{1}{2} \gamma_2 = 0$ are very nearly true to $0(\alpha^2)$.

In $0(\alpha)$ the critical coupling $\alpha_c$ is determined by $b_\pm = \frac{1}{2}$. If the critical coupling to $0(\alpha_c)$ is also determined by $b_\pm = \frac{1}{2}$ we get from (5.21) $ax^2 + x - 1 = 0$ where $x = \frac{3\alpha_c}{\pi} C_2(R)$ and $a = \frac{14C_2(G)-15C_2(R)}{72C_2(R)}$

$$3 \frac{\alpha_c}{\pi} C_2(R) = \frac{-1+\sqrt{1+4a}}{2a}$$

(5.23)

Assuming $a \ll 1$ we get

$$\alpha_c = \frac{\pi}{3C_2(R)} \left[1-\frac{14C_2(G)-15C_2(R)}{72C_2(R)}\right]$$

$$= \frac{\pi}{3C_2(R)} \left[1\pm .014-.18\right]$$

(5.24)

However this is just a conjecture and probably one should not take it too seriously.
The conclusion is that perturbation theory is valid for

\[ b(l-b) = \frac{1}{4}\left[ \frac{3αc_2(R)}{π} + \left( \frac{3αc_2(R)}{π} \right)^2 \epsilon_\lambda \right], \text{ with } \epsilon_\lambda \ll 1. \]

However we must be concerned with its validity for \( \lambda = \frac{1}{2} \gamma_2 = \left( \frac{3αc_2(R)}{π} \right)^2 \epsilon_\lambda \), with \( \epsilon_\lambda \ll 1 \) since the \( O(\alpha) \) term in the expression for \( \lambda \) vanishes and we have no idea about how large the next order term is.
CHAPTER VI

Conclusions and Outlook for Future

No matter how physically appealing this power law solution is, it is still an approximate solution. Peculiar functions $L_2(z)$, $\frac{F'}{F}$, $\ln(Z)\ln(1+Z)$ appear in the kernel and they might give rise to important contribution for $k=p$ although they are negligible for $M_{\infty m}$. In fact a simple order of magnitude estimate suggests that if

$$\Sigma(P) \approx \mu\left(\frac{\mu}{P}\right)^c$$

with $c$ close to 1 then the most dominant contribution to $\Sigma(P)$ comes from the region $k=p$. The power law solution obtained with the end point approximate kernel is no answer to the solution we expect to obtain with the exact $O(\alpha^2)$ kernel with all its complexities.

We are still trying to evaluate the last integral analytically. However if we cannot do it ultimately we will use numerical methods to do this last integral. When the full integral equation with the exact $O(\alpha^2)$ kernel becomes available we would like to investigate the following questions.

1. We would like to solve the full integral equation for $\Sigma(P)$ in different covariant gauges. We would like to see if the exact integral equation does admit a power law solution for $\Sigma(P)$ with a fixed or slowly running coupling for any value of $\xi$ although the answer to this may turn out to be negative. Only fixed point gauges and some peculiar gauges seem to be the likely candidates for power law
solution. By solving the integral equation in different gauges we would like to answer the following questions.

a. Does the exponent change with the gauge parameter significantly?

b. Are there corrections beyond the power law solution in gauges other than Landau gauge? Are they logarithmic? If not what are they?

2. Does the solution to \( \Sigma(P) \) in different gauges lead to a gauge invariant technicolour condensate \( \langle \bar{\Psi}\Psi \rangle_{A_{ETC}} \)?

3. Try to build realistic ETC models and check if the solution to \( \Sigma(P) \) lead to gauge invariant quark lepton masses to lowest order in \( g_{ETC} \).
APPENDIX A

In this appendix we shall present the evaluation of the contribution of the non-abelian vertex correction diagram to $\Sigma(P)$ in modest detail.

We have

$$\Sigma_2(P) = \frac{g^2}{16\pi^2} C_2(G)C_2(R)(g\mu^{\varepsilon_2})^2 \mu^{2\varepsilon(1+\frac{2\varepsilon}{3+\tilde{\xi}})} \int \frac{dk}{(P-k)^2} \int \frac{d^Dk}{(2\pi)^D}$$

$$2(2-D)(\gamma-P)(2\gamma-P-k)-2(1-\varepsilon)(2\gamma-P-k)+(1-\varepsilon) \frac{\gamma.(\gamma-P)}{(\gamma-P)^2} [(2-D)(\gamma-P)(\gamma-k)$$

$$-D(\gamma-P)(P-k)+(1-\varepsilon) \frac{\gamma.(\gamma-k)}{(\gamma-k)^2} [(2-D)(\gamma-k)(\gamma-P)+D(\gamma-k)(P-k)]$$

$$+(1-\varepsilon)(2-D) \frac{\gamma.(P-k)}{(P-k)^2} [(P-k)(\gamma-P)+(P-k)(\gamma-k)]-(1-\varepsilon)$$

$$2\gamma.(\gamma-P)(\gamma-P)(P-k)(\gamma-k)(P-k) - \gamma.(P-k)(P-k)(\gamma-P) - \gamma.(\gamma-P)(\gamma-P)(\gamma-k)$$

$$(\gamma-P)^2(P-k)^2$$

$$\gamma.(\gamma-P)(\gamma-P)(P-k) - (1-\varepsilon)^2 [2\gamma.(\gamma-k)(\gamma-k)(P-k)(\gamma-P)(P-k)$$

$$\gamma.(P-k)(P-k)(\gamma-k) - \gamma.(\gamma-k)(\gamma-k)(\gamma-k) + \gamma.(\gamma-k)(\gamma-k)(P-k)]$$

$$\frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2}$$

(A.1)
1. \[ \sum_{\gamma=1}^{\infty} \left( P_{-1} \right) - \left( \frac{2\epsilon}{3+\zeta} \right) \int \frac{d\hat{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \gamma^2(\gamma-P)^2(\gamma-k)^2 \] (A.2)

We have

\[ \int \frac{d\hat{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \gamma^2(\gamma-P)^2(\gamma-k)^2 \]

\[ = \int \frac{d\hat{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{1}{(\gamma-P)^2(\gamma-k)^2} + \frac{1}{2} \left( \int \frac{d\hat{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{1}{(\gamma-k)^2} \right) \]

\[ = \frac{1}{2} \left( \frac{P^2+k^2}{(2\pi)^4} \right) \int \frac{d^4\gamma}{(2\pi)^4} \gamma^2(\gamma-P)^2(\gamma-k)^2 \] (A.3)

\[ = \frac{(4\pi)^6}{16\pi^2} \left( \frac{\gamma-P}{2\pi} \right)^2 \int \frac{d\hat{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \gamma^2(\gamma-P)^2(\gamma-k)^2 \]

\[ = \frac{(P^2+k^2)}{16\pi^2} \sum_{n_1=0}^{\infty} \frac{\gamma^2(\gamma-P)^2(\gamma-k)^2}{\sum_{n=0}^{\infty} \frac{M^2(\gamma,P)M^2(\gamma,k)}{\sum_{n_1=0}^{\infty} \frac{M^2(\gamma,P)M^2(\gamma,k)}}} \] (A.4)

We have \[ \int \frac{d\hat{k}}{(P-k)^2} = \frac{1}{M^2(P,k)} \] (A.5)

\[ \int \frac{d\hat{k}}{(P-k)^2} = \frac{1}{M^2(P,k)} \] (A.6)

\[ \sum_{n_1=0}^{\infty} \frac{\gamma^2(\gamma-P)^2(\gamma-k)^2}{\sum_{n_1=0}^{\infty} \frac{M^2(\gamma,P)M^2(\gamma,k)}} \] and

\[ \sum_{n_1=0}^{\infty} \frac{\gamma^2(\gamma-P)^2(\gamma-k)^2}{\sum_{n_1=0}^{\infty} \frac{M^2(\gamma,P)M^2(\gamma,k)}} \] and
Consider first $k<P$

Region 1: $0<y<k$. Here $x = \frac{y^2}{p^2}$

\[- \int_0^k \frac{y^2\ln(1-x)}{M^2(\gamma,P)M^2(\gamma,k)M^2(P,k)x} \quad \text{where} \quad x = m(\gamma)^{P}(\gamma) \frac{m(\gamma)^{k}}{k^{2}P} \quad (A.7)\]

\[- \int_0^k \frac{y^2\ln(1-x)}{M^2(\gamma,P)M^2(\gamma,k)M^2(P,k)x} = - \frac{1}{p^2k^2} \int_0^{k/P} \frac{dt\ln(1-t^2)}{t} = - \frac{1}{2p^2k^2} L_2(\frac{k^2}{p^2}) \quad (A.8)\]

Region 2: $k<y<P$. Here $x = \frac{k^2}{p^2}$

\[- \int_k^P \frac{y^2\ln(1-x)}{M^2(\gamma,P)M^2(\gamma,k)M^2(P,k)x} \quad \text{where} \quad x = m(\gamma)^{P}(\gamma) \frac{m(\gamma)^{k}}{k^{2}P} \quad (A.9)\]

\[- \int_k^P \frac{y^2\ln(1-x)}{M^2(\gamma,P)M^2(\gamma,k)M^2(P,k)x} = - \frac{1}{2p^2k^2} \ln\frac{k^2}{p^2} \quad (A.10)\]

Region 3: $P<y<\infty$. Here $x = \frac{k^2}{y^2}$

\[- \int_P^\infty \frac{y^2\ln(1-x)}{M^2(\gamma,P)M^2(\gamma,k)M^2(P,k)x} \quad \text{where} \quad x = m(\gamma)^{P}(\gamma) \frac{m(\gamma)^{k}}{k^{2}P} \quad (A.10)\]

\[- \int_P^\infty \frac{y^2\ln(1-x)}{M^2(\gamma,P)M^2(\gamma,k)M^2(P,k)x} = - \frac{1}{p^2k^2} \int_0^{k/P} \frac{dt\ln(1-t^2)}{t} = - \frac{1}{2p^2k^2} L_2(\frac{k^2}{p^2}) \quad (A.10)\]

\[\therefore \text{For } k<P\]
Exploiting the symmetry under exchange of $P$ with $k$ we get

$$
\int \frac{dk}{(P-k)^2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-P)^2}
$$

and

$$
\sum_{n_1=0}^{\infty} \int \frac{d\gamma dym_{n_1}(\gamma P) m_{n_1}(\gamma k) \gamma \gamma^{(n_1+1)}}{M^2(\gamma, P) M^2(\gamma, k)} \int \frac{dk}{(P-k)^2} C_{n_1}(\hat{P}, \hat{k})
$$

Using the above results we get

$$
\Sigma_{2,1}(P) = \frac{3}{4\pi^2 M^2} \left[ \left( \frac{2}{3+\xi} + \frac{1}{3+\xi} \right) F' + \frac{1}{2} \gamma \gamma^{(M+m)} - \frac{1}{4} \gamma \gamma^{(M+m)} - \frac{1}{2} (\frac{M}{m^2} + 1) \left( L_2 \frac{m^2}{M^2} + \frac{1}{2} \gamma \gamma^{(1-m^2/2)} \gamma \gamma^{m^2/2} \right) \right] \left( \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \right)
$$

2. In an exactly similar fashion we get

$$
\Sigma_{2,2}(P) = -2(1-\xi) \mu^2 e (1+2\xi) \int \frac{dk}{(P-k)^2} \int \frac{d\gamma dY_{\gamma}(\gamma(2\gamma-P-k))}{(2\pi)^4} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2}
$$

$$
= \frac{-(1-\xi)}{4\pi^2 M^2} \left[ \left( \frac{2}{3+\xi} + \frac{1}{2} F' + \frac{1}{2} \gamma \gamma^{(M+m)^2} + \frac{1}{4} \gamma \gamma^{(M+m)^2} \right) - \frac{1}{2} (1+\frac{M}{m^2}) \left( L_2 \frac{m^2}{M^2} + \frac{1}{2} \gamma \gamma^{(1-m^2/2)} \gamma \gamma^{m^2/2} \right) \right]
$$
\[
\sum_{2,3}(P) = (1-\xi) \mu^{2\epsilon}(1+\frac{2\epsilon}{3+\xi}) \int \frac{dk}{(P-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{\gamma \cdot (y-P)}{\gamma^2((y-P)^2)^2(y-k)^2}
\]

\[
\frac{1}{\gamma^2((y-P)^2)^2(y-k)^2} (2-D)(y-P) \cdot (y-k) - D(y-P) \cdot (P-k) = (1-\xi) \mu^{2\epsilon}(1+\frac{2\epsilon}{3+\xi})
\]

\[
\int \frac{dk}{(P-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{P \cdot (y-P)}{\gamma^2((y-P)^2)^2(y-k)^2} [(2-D)(y-P) \cdot (y-k) - D(y-P) \cdot (P-k)] + (1-\xi) \mu^{2\epsilon}(1+\frac{2\epsilon}{3+\xi})
\]

(A.15)

Consider the first integral

\[
\int \frac{dk}{(P-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{\gamma^2((y-P)^2)^2(y-k)^2} (2-D)(y-P) \cdot (y-k) - D(y-P) \cdot (P-k)
\]

(A.16)

\[
= \int \frac{dk}{(P-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{\gamma^2(y-x)^2} - (D-1) \int \frac{dk}{(P-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{\gamma^2(y-x)^2}
\]

\[
+ (D-1) \int \frac{d^Dy}{(2\pi)^D} \frac{1}{\gamma^2(y-x)^2}
\]

We have \[
\int \frac{d^Dy}{(2\pi)^D} \frac{1}{\gamma^2(y-x)^2} = \frac{(4\pi)^D}{16\pi^2 x^{2\epsilon}} (1+2t) \left( \frac{1}{e} - \gamma \right)
\]

and

\[
\int dk \int \frac{d^4y}{(2\pi)^4} \frac{1}{\gamma^2(y-P)^2(y-k)^2} \quad \text{[using } d^4y = 2\pi^2 y^3 dy d\gamma d\hat{\gamma}] \]

\[
= \frac{1}{8\pi^2} \int dk \int \frac{\gamma d\gamma d\hat{\gamma}}{M^2(y, P)M^2(y, k)} \sum n^1(y, P) n^2(y, k) c^1_n(y, P) c^2_n(y, k)
\]
\[
\frac{1}{8\pi^2} \int d\gamma \int \frac{\gamma d\gamma}{M^2(\gamma, P)M^2(\gamma, k)} \frac{n_1^2(\gamma P)n_2^2(\gamma k)}{n_1^2(\gamma P)n_2^2(\gamma k) + 1} \delta_{n_1, n_2} C'_{\gamma, k}
\]

\[
\frac{1}{8\pi^2} \int \frac{\gamma d\gamma}{M^2(\gamma, P)M^2(\gamma, k)} \frac{n_1^2(\gamma P)n_2^2(\gamma k)}{n_1^2(\gamma P)n_2^2(\gamma k) + 1} \int d\kappa C'_{\gamma, \kappa}
\]

\[
\frac{1}{8\pi^2} \int \frac{\gamma d\gamma}{M^2(\gamma, P)M^2(\gamma, k)} \frac{n_1^2(\gamma P)n_2^2(\gamma k)}{n_1^2(\gamma P)n_2^2(\gamma k) + 1} \int d\kappa C'_{\gamma, \kappa}
\]

where we used \( \int d\kappa C'_{\gamma, \kappa} = \delta_{\gamma, \kappa} \) (A.17)

Let \( k < P \).

Region 1: \( 0 < \gamma < k \)

\[
\frac{1}{2P^2}
\]

(A.18)

Region 2: \( k < \gamma < P \)

\[
\frac{1}{2P^2} \ln \frac{P^2}{k^2}
\]

(A.19)

Region 3: \( p < \gamma < \alpha \)

\[
\frac{1}{2P^2}
\]

(A.20)

For \( k < P \)

\[
\frac{1}{P^2} + \frac{1}{2P^2} \ln \frac{P^2}{k^2} = \frac{1}{M^2} \left[ 1 + \frac{1}{2} \ln \frac{M^2}{m^2} \right]
\]

(A.21)

Exploiting the symmetry under exchange of \( P \) with \( k \) we get

\[
\int d\kappa \int \frac{d^4q}{(2\pi)^4} \frac{1}{\gamma^2(\gamma - P)^2(\gamma - k)^2} = \frac{1}{8\pi^2 M^2} \left[ 1 + \frac{1}{2} \ln \frac{M^2}{m^2} \right]
\]

(A.22)
\[ \therefore (1-\varepsilon) \mu^2 (1+\frac{2\varepsilon}{3+\xi}) \int \frac{d\hat{k}}{(P-k)^2} \int \frac{dP \gamma}{(2\pi)^D} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]

\[(2-D)(\gamma-P).\gamma-k-D(\gamma-P).P-k)]

\[= \frac{(1-\varepsilon)}{16\pi^2 p^2} [-2(\frac{2}{3+\xi}-2)-ln \frac{k^2}{\mu^2}+3ln \frac{p^2}{k^2}] \text{ for } k<P \]

\[= \frac{(1-\varepsilon)}{16\pi^2 p^2} [-2(\frac{2}{3+\xi}-2)-ln \frac{k^2}{\mu^2}+3ln \frac{p^2}{k^2}] \text{ for } k>P \]

Using the identity \[\frac{P.(\gamma-P)}{(\gamma-P)^2} = \frac{1}{2} P^\mu \frac{2}{\partial P^\mu} \frac{1}{(\gamma-P)^2}\] the 2nd integral can be written as

\[\int \frac{d\hat{k}}{(P-k)^2} \int \frac{dP \gamma}{(2\pi)^D} \frac{P.(\gamma-P)}{(\gamma-P)^2(\gamma-k)^2} \frac{1}{((2-D)(\gamma-P).\gamma-k-D(\gamma-P).P-k)]} \]

\[= \frac{1}{2} P^\mu \frac{\partial}{\partial P^\mu} \int \frac{d\hat{k}}{(P-k)^2} \int \frac{dP \gamma}{(2\pi)^D} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \frac{1}{((2-D)(\gamma-P).\gamma-k-D(\gamma-P).P-k)]} \]

\[\int \frac{d\hat{k}}{(P-k)^2} \int \frac{dP \gamma}{(2\pi)^D} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \frac{1}{((2-D)(\gamma-P).\gamma-k-D(\gamma-P).P-k)]} \]

\[= \frac{1}{2} \int \frac{d\hat{k}}{(P-k)^2} \int \frac{dP \gamma}{(2\pi)^D} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \frac{1}{(2-D)P.(\gamma-k)-D-P.(\gamma-P)]} \]

Using (A.4) and the identity \[\frac{P.(\gamma-P)}{(\gamma-P)^2} = \frac{1}{2} P^\mu \frac{\partial}{\partial P^\mu} \frac{1}{(\gamma-P)^2}\] we get

written as
\[
\int \frac{dk}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{P.(y-p)}{(y-p)^2(y-k)^2} [(2-D)(y-p).(y-k)-D(y-p).(p-k)]
\]

\[
= \frac{1}{2} p \mu \int \frac{dk}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{(2-D)(y-p).(y-k)-D(y-p).(p-k)}
\]

\[
- \int \frac{dk}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{(2-D)(y-p).(y-k)-D(y-p).(p-k)}
\]

\[
- \frac{1}{2} \int \frac{dk}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{2(D-1)p.(y-k)-Dp.(y-p)}
\]

(A.24)

Using (A.4) and the identity \( p^\mu \frac{\partial}{\partial p^\mu} = p \frac{\partial}{\partial p} \) we get

\[
\frac{1}{2} p \mu \frac{\partial}{\partial p^\mu} (1-\xi) \mu^2 \left( 1 + \frac{2\epsilon}{3+\xi} \right) \int \frac{dk}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{(2-D)(y-p).(y-k)-D(y-p).(p-k)}
\]

\[
[(2-D)(y-p).(y-k)-D(y-p).(p-k)]
\]

\[
= \frac{(1-\xi)}{8\pi^2 k^2} \left( \left( \frac{2}{3+\xi} + 1 \right) + \frac{1}{2} \ln \frac{k^2}{\mu^2} - \frac{3}{2} \ln \frac{P}{\mu^2} - \frac{3}{2} \ln \frac{P}{k^2} \right) \text{ for } k<P
\]

\[
= 0 \quad \text{for } k>P
\]

(A.25)

We can write after some algebra

\[
\int \frac{d^D(p-k)}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{(2-D)(y-p).(y-k)-D(y-p).(p-k)}
\]

\[
= \frac{1}{2} \int \frac{d^Dk[k^2-p^2-(p-k)^2]}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{y^2(y-k)^2} \frac{2}{(2-D)(y-k)^2} \int \frac{d^Dk[k^2-p^2-(p-k)^2]}{(p-k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{y^2(y-k)^2}
\]

\[
+ \frac{(D-1)}{2} \int \frac{d^Dk[k^2-p^2-(p-k)^2]}{p-k^2} \int \frac{d^Dq}{(2\pi)^D} \frac{1}{y^2(y-k)^2} \quad (A.26)
\]
The first two integrals are trivially done by Feynman parametrization and the last one can be done by GPM in momentum space (see A.12 and A.22). We get

\[ \frac{(1-\xi)}{32\pi^2\mu^2} \left[ \frac{(k^2-p^2)}{M^2(1-m^2(P,K))} - 2 \left( \frac{2}{3+\xi^2} - 1 \right) - \ln \frac{k^2}{\mu^2} + 3\ln \frac{p^2}{\mu^2} \right] \]

\[ + \frac{3(1-\xi)}{16\pi^2M^2} \left[ \frac{(k^2-p^2)}{m^2} \left[ L_{N}^{2} M_{2} + \frac{1}{2} \ln \left( 1 - M_{2}^{2} / M_{2}^{2} \right) \frac{3}{2} \ln \frac{m^2}{M^2} \right] - \frac{1}{2} \ln \frac{m^2}{M^2} \right] \quad (A.27) \]

We can write after some algebra

\[ \int \frac{d^k}{(P-k)^2} \int \frac{d^\gamma Y}{(2\pi)^D} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \left[ 2(D-1)P.(\gamma-k) - DP.(\gamma-P) \right] \]

\[ = \int \frac{d^k}{(P-k)^2} \int \frac{d^\gamma Y}{(2\pi)^D} \left[ \frac{1}{\gamma^2(\gamma-k)^2} - \frac{1}{(\gamma-P)^2(\gamma-k)^2} \right] \]

\[ + (4p^2-3k^2) \int \frac{d^k}{(P-k)^2} \int \frac{d^4\gamma}{(2\pi)^4} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} + 3\int d^k \int \frac{d^4\gamma}{(2\pi)^4} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \quad (A.28) \]

The 1st integral can be done by Feynman parametrization. The 2nd and 3rd integrals can be done by GPM in momentum space (see A.12 and A.22). We get

\[ = \frac{1}{8\pi^2\mu^2} \left[ \frac{(4p^2-3k^2)}{m^2} \left[ L_{N}^{2} M_{2} + \frac{1}{2} \ln \left( 1 - M_{2}^{2} / M_{2}^{2} \right) \frac{3}{2} \ln \frac{m^2}{M^2} \right] + 3 \left[ 1 - \frac{1}{2} \ln \frac{m^2}{M^2} \right] \right] \]

\[ - \frac{1}{2} \left( \ln \frac{k^2}{(P+k)^2} + \frac{p^2}{p} \right) \quad (A.29) \]

Putting all the pieces together we get
\[ \Sigma_{2,3}(P) = \frac{(1-\xi)}{16\pi^2P^2} \left\{ -2 \left( \frac{2}{3+\xi} \right)^2 - \ln \frac{k^2}{\mu^2} + 3 \ln \frac{p^2}{\mu^2} \right\} \]

\[ - \frac{p^2}{k^2} \left( L_2 \left( \frac{k^2}{p^2} \right) + \frac{1}{2} \ln \left( 1 - \frac{k^2}{p^2} \right) \ln \frac{k^2}{p^2} \right) + \frac{1}{2} \ln \frac{k^2}{(P+k)^2} \]

\[ + \frac{1}{2} \left( \frac{p'}{p} \right) \text{ for } k<P \]

\[ = \frac{(1-\xi)}{16\pi^2k^2} \left\{ -2 \left( \frac{2}{3+\xi} \right)^2 - \ln \frac{k^2}{\mu^2} + 3 \ln \frac{p^2}{\mu^2} + 3 \ln \frac{k^2}{p^2} \right\} \]

\[ - \left( L_2 \left( \frac{p^2}{k^2} \right) + \frac{1}{2} \ln \left( 1 - \frac{p^2}{k^2} \right) \ln \frac{p^2}{k^2} \right) + \frac{1}{2} \ln \frac{k^2}{(P+k)^2} + \frac{1}{2} \left( \frac{p'}{p} \right) \text{ for } k>P \] (A.30)

4.

\[ \Sigma_{2,4}(P) = (1-\xi) \mu^2 \left( 1 + \frac{2\xi}{3+\xi} \right) \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d\mathbf{y}}{(2\pi)^D} \frac{\gamma \cdot (y-k)}{\gamma^2 ((y-k)^2)^2 (y-P)^2} \]

\[ = [2-D](\gamma-k) \cdot (\gamma-P) + D(\gamma-k) \cdot (P-k)] \] (A.31)

\( \Sigma_{2,4}(P) \) can be obtained from \( \Sigma_{2,3}(P) \) by interchanging \( P \) with \( k \). We will not write the expression for \( \Sigma_{2,4}(P) \) separately, rather we will add \( \Sigma_{2,4}(P) \) to \( \Sigma_{2,3}(P) \) and express the result in a symmetric form.

\[ \Sigma_{2,3}(P) + \Sigma_{2,4} \]

\[ = \frac{1}{16\pi^2M^2} \left\{ -8(1-\xi) \frac{1}{3+\xi} + 8(1-\xi) + 2(1-\xi) \ln \frac{k^2}{\mu^2} \frac{p^2}{\mu^2} - 3(1-\xi) \ln \frac{m^2}{M^2} \frac{1}{2} (1-\xi) \ln \frac{m^2}{M^2} \right\} \]

\[ + (1-\xi) \left( \frac{p'}{p} \right) - (1-\xi) (1+\frac{M^2}{m^2}) \left( L_2 \left( \frac{m^2}{M^2} \right) + \frac{1}{2} \ln \left( 1 - \frac{m^2}{M^2} \right) \ln \frac{m^2}{M^2} \right) \] (A.32)
\[ \sum_{2,5}(P) = (1-\xi)(2-D)\mu^2(1+2\epsilon) \int \frac{dk}{((P-k)^2)^2} \int \frac{d^Dq}{(2\pi)^D} \]

\[ \frac{\gamma.(P-k)[(P-k).(\gamma-P)+(P-k).(\gamma-k)]}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]  

(A.33)

After doing some simple algebra one can write

\[ \int \frac{dk}{((P-k)^2)^2} \int \frac{d\gamma}{(2\pi)^D} \frac{\gamma.(P-k)[(P-k).(\gamma-P)+(P-k).(\gamma-k)]}{\gamma^2(\gamma-P)^2(\gamma-k)^2} = \int \frac{dk}{((P-k)^2)^2} \left[ \int \frac{d\gamma}{(2\pi)^D} \frac{\gamma.(P-k)}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \right] \]

\[ = \frac{(4\pi)^6(1+2\epsilon)(1-\gamma)}{32\pi^2} \int \frac{dk}{((P-k)^2)^2} \left[ \frac{p.(P-k)}{p^2\epsilon} + \frac{k.(k-P)}{k^2\epsilon} \right] \]  

(A.34)

We have \[ \int \frac{dkP.(P-k)}{((P-k)^2)^2} \]

\[ = -\frac{1}{2} \int dk \frac{\partial p}{\partial \mu} (P-k)^2 \]

\[ = -\frac{1}{2} p \frac{\partial p}{\partial p} \frac{1}{M^2(p,k)} = \frac{1}{p^2} \text{ for } k<P \]

\[ = 0 \text{ for } k>P \]

\[ \therefore \int \frac{dkP.(P-k)}{((P-k)^2)^2} = \frac{1}{2M^2(p,k)} \left[ \frac{(p^2-k^2)}{M^2(p,k)(1-m^2(p,k,\gamma))} + 1 \right] \]  

(A.35)

Similarly \[ \int \frac{dkk.(k-P)}{((P-k)^2)^2} = \frac{1}{2M^2(p,k)} \left[ \frac{(k^2-p^2)}{M^2(p,k)(1-m^2(p,k,\gamma))} + 1 \right] \]  

(A.36)
Using the above results we get

\[ \Sigma_{2,5}(P) = \frac{-(1-\xi)(1+\frac{2}{3+\xi}-\ln \frac{M^2}{\mu})}{16\pi^2M^2} \]  \hspace{1cm} (A.37)

6. \[ \Sigma_{2,6}(P) = -2(1-\xi)^2 \mu^2 \epsilon(1+\frac{2\epsilon}{3+\xi}) \int \frac{d\vec{k}}{((P-k)^2)^2} \frac{d^D\gamma}{(2\pi)^D} \frac{\gamma_i(y-P)(y-P)(P-k)(y-k)(P-k)}{\gamma^2(y-P)^2(y-k)^2} \]

\[ = -2(1-\xi)^2 \mu^2 \epsilon(1+\frac{2\epsilon}{3+\xi}) \int \frac{d\vec{k}}{((P-k)^2)^2} \frac{d^D\gamma}{(2\pi)^D} \frac{(y-P)(P-k)(y-k)(P-k)}{\gamma^2(y-P)^2(y-k)^2} \]

\[ - 2(1-\xi)^2 \mu^2 \epsilon(1+\frac{2\epsilon}{3+\xi}) \int \frac{d\vec{k}}{((P-k)^2)^2} \frac{d^D\gamma}{(2\pi)^D} \frac{P_i(y-P)(y-P)(P-k)(y-k)(P-k)}{\gamma^2((y-P)^2)(y-k)^2} \]  \hspace{1cm} (A.38)

We can write the 1st integral as

\[ \int \frac{d\vec{k}}{((P-k)^2)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{(y-P)(P-k)(y-k)(P-k)}{\gamma^2(y-P)^2(y-k)^2} \]

\[ = \frac{1}{2} \int \frac{d\vec{k}}{((P-k)^2)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{(y-k)(P-k)}{\gamma^2(y-P)^2} - \frac{1}{2} \int \frac{d\vec{k}}{((P-k)^2)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{(y-k)(P-k)}{\gamma^2(y-k)^2} \]

\[ + \frac{1}{2} \int \frac{d\vec{k}}{((P-k)^2)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{(y-k)(k-P)}{\gamma^2(y-P)^2(y-k)^2} \]  \hspace{1cm} (A.39)

We get by using Feynman parametrization

\[ \int \frac{d\vec{k}}{((P-k)^2)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{(y-k)(P-k)}{\gamma^2(y-P)^2} \]

\[ = \frac{(4\pi)^2 \Gamma(\xi)}{32\pi M^2 P^2 \epsilon} \left[ B(2-\epsilon,1-\epsilon) \left( \frac{P^2-k^2}{M^2(1-m^2(P_k+k_p))} +1 \right) + B(1-\epsilon,1-\epsilon) \left( \frac{k^2-P^2}{M^2(1-m^2(P_k+k_p))} +1 \right) \right] \]

\[ = \frac{(4\pi)^2 \Gamma(\xi)}{32\pi M^2 P^2 \epsilon} \left[ B(2-\epsilon,1-\epsilon) \left( \frac{P^2-k^2}{M^2(1-m^2(P_k+k_p))} +1 \right) + B(1-\epsilon,1-\epsilon) \left( \frac{k^2-P^2}{M^2(1-m^2(P_k+k_p))} +1 \right) \right] \]  \hspace{1cm} (A.40)
\[ 62 \]
and \[ \int \frac{dk}{((p-k)^2)^2} \int \frac{d^2Y}{(2\pi)^D} \frac{(y-k)\cdot(p-k)}{y^2(y-k)^2} \]

\[ = \frac{(4\pi)^2 \Gamma(\epsilon) B(2-\epsilon, 1-\epsilon)}{32\pi^2 \alpha^2} \left[ \frac{k^2-p^2}{M^2(1-m^2(\frac{p}{k}, \frac{p}{k}'))} + 1 \right] \]

(A.41)

where \( \Gamma(Z) \) is the Gamma function and \( B(Z, \omega) \) is the Beta function.

Further we can write

\[ \int \frac{dk}{(p-k)^2} \int \frac{d^2Y}{(2\pi)^D} \frac{(y-k)(k-p)}{y^2(y-k)^2(2\pi)^D} \]

\[ = \frac{1}{2} \int \frac{dk}{(p-k)^2} \frac{d^2Y}{(2\pi)^D} \frac{1}{y^2(y-k)^2} \frac{1}{2} \int \frac{dk}{(p-k)^2} \frac{d^2Y}{(2\pi)^D} \frac{1}{y^2(y-k)^2} \frac{1}{2} \int \frac{dk}{(p-k)^2} \frac{d^2Y}{(2\pi)^D} \frac{1}{y^2(y-k)^2} \]

The first two integrals can be done by Feynman parametrization and the 4rd integral can be done by GPM (see A.22). We get

\[ = \frac{1}{32\pi^2 \alpha^2} \ln \frac{p^2}{k^2} - \frac{1}{16\pi^2 \alpha^2} \left[ 1 - \frac{1}{2} \ln \frac{m^2}{\text{M}^2} \right] \]

(A.42)

\[ \therefore \quad \mu^{2\epsilon} \left( 1 + \frac{2\epsilon}{3+\epsilon} \right) \int \frac{dk}{((p-k)^2)^2} \int \frac{d^2Y}{(2\pi)^D} \frac{(y-p)\cdot(p-k)(y-k)\cdot(p-k)}{y^2(y-p)^2(y-k)^2} \]

\[ = \frac{1}{64\pi^2 \alpha^2} \left( \frac{2}{3+\epsilon} - \ln \frac{\text{M}^2}{\mu^2} \right) \]

(A.43)

Using \( \frac{\partial}{\partial p^\mu} \frac{1}{(y-p)^2} = \frac{2P\cdot(y-p)}{((y-p)^2)^2} \) and \( \frac{\partial}{\partial p^\mu} \frac{1}{((p-k)^2)^2} = \frac{4P\cdot(p-k)}{((p-k)^2)^3} \)

We can write
\[ \int \frac{d^k k}{((P-k)^2)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{2P \cdot (y-P) \cdot (y-P) \cdot (y-P) \cdot (y-k) \cdot (p-k)}{\gamma^2((y-P)^2)^2(y-k)^2} \]

\[ = \rho^\mu \frac{\partial}{\partial \rho^\mu} \int \frac{d^k k}{((P-k)^2)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{(y-P) \cdot (p-k) \cdot (y-k) \cdot (p-k)}{\gamma^2((y-P)^2)^2(y-k)^2} \]

\[ - \int \frac{d^k k}{((P-k)^2)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{P \cdot (y-P) \cdot (p-k) \cdot (y-k) \cdot (p-k) \cdot (p-k) \cdot (y-k) \cdot (p-k)}{\gamma^2((y-P)^2)^2(y-k)^2} \]

\[ = -4 \int \frac{d^k k \cdot (k-P)}{(p-k)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{(q-P) \cdot (p-k) \cdot (q-k) \cdot (p-k)}{\gamma^2((y-P)^2)^2(y-k)^2} \]

(A.44)

Using (A.43) we get

\[ \rho^\mu \frac{\partial}{\partial \rho^\mu} \frac{2\epsilon}{1 + 32\pi^2} \int \frac{d^k k}{((P-k)^2)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{(y-P) \cdot (p-k) \cdot (y-k) \cdot (p-k)}{\gamma^2((y-P)^2)^2(y-k)^2} \]

\[ = -\frac{1}{32\pi^2 \rho^2} \left( \frac{2}{3 + \epsilon} + 1 - \ln \frac{\rho^2}{\mu^2} \right) \quad \text{for } k < P \]

(A.45)

\[ = 0 \quad \text{for } k > P \]

After doing some simple algebra one can show that

\[ -\int \frac{d^k k \cdot (k-P)}{(p-k)^2} \int \frac{d^D \gamma}{(2\pi)^D} P \cdot [(y-P) + (k-P)] \int \frac{d^k k}{((p-k)^2)^2} \frac{d^D \gamma}{(2\pi)^D} P \cdot [(k-P) + (y-P)] \]

\[ = -2 \int \frac{d^k k \cdot (k-P)^3}{((p-k)^2)^3} \frac{d^D \gamma}{(2\pi)^D} \frac{(q-P) \cdot (p-k)}{\gamma^2((y-P)^2)^2(y-k)^2} \]

(A.46)
All the integrals written above have only two different propagators in the denominator of the loop integrand and hence they can be done by Feynman parametrization. We get for $k<P$

$$\frac{(4\pi)^{\varepsilon}(1+2\varepsilon)}{64\pi^2p^2(p^2-k^2)} \left[ \frac{2p^2-k^2}{p^2} \frac{k^2}{k^2\varepsilon} \right]$$

and for $k>P$

$$\frac{(4\pi)^{\varepsilon}(1+2\varepsilon)}{64\pi^2k^2(k^2-p^2)} \left[ \frac{1}{p^2\varepsilon} - \frac{1}{k^2\varepsilon} \right] \quad (A.47)$$

We now have all the ingredients for $\Sigma_{2,6}(P)$. Putting them together we get

$$\Sigma_{2,6}(P) = \begin{cases} \frac{(1-\xi)^2}{32\pi^2p^2} \left[ \frac{2}{3+\xi} - \ln \frac{p^2}{\mu^2} + 1 + \frac{1}{2} k^2 \frac{\ln p^2}{(p^2-k^2)} \right] & \text{for } k<P \\ \frac{(1-\xi)^2}{32\pi^2k^2} \left[ \frac{2}{3+\xi} - \ln \frac{k^2}{\mu^2} + \frac{1}{2} \frac{p^2\ln p^2}{(p^2-k^2)} \right] & \text{for } k>P \end{cases} \quad (A.48)$$

7.

$$\Sigma_{2,7}(P) = (1-\xi)^2 \mu^2 e^{\varepsilon} \frac{2\varepsilon}{3+\xi} \int \frac{d\hat{k}}{((P-k)^2)^2 (2\pi)^D} \frac{d\hat{Y}}{\gamma^2(Y-P)^2(Y-k)^2} \frac{y.(P-k)(P-k).(Y-P)}{\gamma^2(Y-P)^2(Y-k)^2} \quad (A.49)$$

After doing some simple algebra we get

$$\int \frac{d\hat{k}}{((P-k)^2)^2} \int \frac{d\hat{Y}}{(2\pi)^D} \frac{y.(P-k)(Y-P).(P-k)}{\gamma^2(Y-P)^2(Y-k)^2}$$

$$= \frac{1}{2} \int \frac{d\hat{k}}{((P-k)^2)^2} \int \frac{d\hat{Y}}{(2\pi)^D} \frac{y.(P-k)}{\gamma^2(Y-P)^2} - \frac{1}{2} \int \frac{d\hat{Y}}{(2\pi)^D} \frac{y.(P-k)}{\gamma^2(Y-k)^2}$$

$$- \frac{1}{2} \int \frac{d\hat{k}}{(P-k)^2} \int \frac{d\hat{Y}}{(2\pi)^D} \frac{y.(P-k)}{\gamma^2(Y-P)^2(Y-k)^2} \quad (A.50)$$
The first two integrals can be done by Feynman parametrization. We have

\[\int \frac{d\mathbf{k}}{((P-k)^2)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{\gamma.(P-k)}{\gamma^2(\gamma-P)^2} = \frac{(4\pi)^\varepsilon(1+2\varepsilon)\Gamma(\varepsilon)}{64\pi^2 M^2 p^2 c^2} \left(\frac{(p^2-k^2)}{M^2(1-M^2/L_k^2)}\right) + 1 \] (A.51)

and

\[\int \frac{d\mathbf{k}}{((P-k)^2)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{\gamma.(k-P)}{\gamma^2(\gamma-k)^2} = \frac{(4\pi)^\varepsilon(1+2\varepsilon)\Gamma(\varepsilon)}{64\pi^2 M^2 k^2 c^2} \left(\frac{(k^2-p^2)}{M^2(1-M^2/L_k^2)}\right) + 1 \] (A.52)

We can also write

\[\int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{\gamma.(P-k)}{\gamma^2(\gamma-P)^2(\gamma-k)^2} = \frac{(p^2-k^2)}{16\pi^2 p^2 k^2} \left[ L_2\left(\frac{m^2}{M^2}\right) + \frac{1}{2} \ln \frac{(1-M^2/L_k^2)}{M^2} \right] \frac{(4\pi)^\varepsilon(1+2\varepsilon)\Gamma(\varepsilon)}{32\pi^2 M^2} \left(\frac{1}{p^2 c^2} - \frac{1}{k^2 c^2}\right) \] (A.54)

We now have all the ingredients for \(\Sigma_{2,7}(P)\). Putting all the pieces together we get

\[
\Sigma_{2,7}(P) = \frac{(1-\xi)^2}{32\pi^2 p^2} \left[ L_2\left(\frac{p^2}{k^2}\right)^2 + \frac{1}{2} \ln \frac{(1-k^2/p^2)^2}{k^2}\right] \left(\frac{1}{p^2 c^2} - \frac{1}{k^2 c^2}\right) \] (A.55)

8.

\[
\Sigma_{2,8}(P) = \frac{(1-\xi)^2}{2c} \left(\frac{1+2\varepsilon}{3+\xi}\right) \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D \gamma}{(2\pi)^D} \frac{\gamma.(P-k)\gamma_y.(\gamma-P)+(\gamma-P).(P-k)}{\gamma^2((\gamma-P)^2)^2} \]

\[
= \frac{(1-\xi)^2}{2c} \left(\frac{1+2\varepsilon}{3+\xi}\right) \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D \gamma}{(2\pi)^D} \left[ \frac{(\gamma-P).(\gamma-k)+(\gamma-P).(P-k)}{\gamma^2((\gamma-P)^2)^2} \right] + \frac{P.(P-k)\gamma_y.(\gamma-k)+(\gamma-P).(P-k)}{\gamma^2((\gamma-P)^2)^2} \] (A.56)
Consider the 1st integral.

Using \((\gamma - P) \cdot (\gamma - k) = \frac{1}{2}[(\gamma - P)^2 + (\gamma - k)^2 - (P - k)^2]\) and \((\gamma - P) \cdot (P - k) = \frac{1}{2}[(\gamma - k)^2 - (\gamma - P)^2 - (P - k)^2]\)

We can write

\[
\int \frac{dk}{(P - k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{((\gamma - P) \cdot (\gamma - k) + (\gamma - P) \cdot (P - k))}{\gamma^2(\gamma - P)^2(\gamma - k)^2}
\]

\[
= \int \frac{dk}{(P - k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{1}{\gamma^2(\gamma - P)^2(\gamma - k)^2} - \int dk \int \frac{d^Dy}{(2\pi)^D} \frac{1}{\gamma^2(\gamma - P)^2(\gamma - k)^2}
\]

The 1st integral appearing above can be done by Feynman parametrization and the 2nd integral can be done by GPM (see A.22). We get

\[
u^2e^{(1 + \epsilon)} \int \frac{dk}{(P - k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{((\gamma - P) \cdot (\gamma - k) + (\gamma - P) \cdot (P - k))}{\gamma^2(\gamma - P)^2(\gamma - k)^2}
\]

\[
= \frac{1}{16\pi^2p^2} \left( \frac{2}{3 + \xi} - \ln \frac{p^2}{\mu^2} + \ln \frac{k^2}{p^2} \right) \quad \text{for } k < P
\]

\[
= \frac{1}{16\pi^2p^2} \left( \frac{2}{3 + \xi} - \ln \frac{k^2}{\mu^2} \right) \quad \text{for } k > P
\]

Using the identity \(\frac{1}{2} \frac{\partial}{\partial p^\mu} p^\mu \frac{1}{(\gamma - P)^2} = \frac{P \cdot (\gamma - P)}{((\gamma - P)^2)^2}\) the 2nd integral can be written as

\[
\int \frac{dk}{(P - k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{P \cdot (\gamma - P)[((\gamma - P) \cdot (\gamma - k) + (\gamma - P) \cdot (P - k))]}{\gamma^2((\gamma - P)^2)^2(\gamma - k)^2}
\]

\[
= \frac{1}{2} \frac{\partial}{\partial p^\mu} \int \frac{dk}{(P - k)^2} \int \frac{d^Dy}{(2\pi)^D} \frac{((\gamma - P) \cdot (\gamma - k) + (\gamma - P) \cdot (P - k))}{\gamma^2(\gamma - P)^2(\gamma - k)^2}
\]
\[ -\frac{1}{2} \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{p^\mu \frac{\partial}{\partial p^\mu} \Sigma((\gamma-P)-(\gamma-k)-(\gamma-P)-(P-k))}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]

\[ -\frac{1}{2} \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{p^\mu \frac{\partial}{\partial p^\mu} \Sigma((\gamma-P)-(\gamma-k)-(\gamma-P)-(P-k))}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]

We have \( p^\mu \frac{\partial}{\partial p^\mu} \frac{1}{(P-k)^2} = \frac{2P.(k-P)}{((P-k)^2)^2} \) \( (A.60) \)

and \( p^\mu \frac{\partial}{\partial p^\mu} \{(\gamma-P)-(\gamma-k)-(\gamma-P)-(P-k)\} = 2P.(k-P) \)

\[ \mu^2 \left(1+\frac{2\epsilon}{3+\xi}\right) \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{p.(P-k)}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]

\[ = \frac{1}{2} \frac{p^\mu}{\mu} \frac{\partial}{\partial p^\mu} \mu^2 \left(1+\frac{2\epsilon}{3+\xi}\right) \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{\{(\gamma-P)-(\gamma-k)-(\gamma-P)-(P-k)\}}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]

\[ - \mu^2 \left(1+\frac{2\epsilon}{3+\xi}\right) \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{P.(k-P)}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]

\[ = \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^4\gamma}{(2\pi)^4} \frac{P.(k-P)}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]  \( (A.61) \)

Using \( (A.17) \) we get for \( k<P \)

\[ \frac{1}{2} \frac{p^\mu}{\mu} \frac{\partial}{\partial p^\mu} \mu^2 \left(1+\frac{2\epsilon}{3+\xi}\right) \int \frac{d\mathbf{k}}{(P-k)^2} \int \frac{d^D\gamma}{(2\pi)^D} \frac{\{(\gamma-P)-(\gamma-k)-(\gamma-P)-(P-k)\}}{\gamma^2(\gamma-P)^2(\gamma-k)^2} \]

\[ = \frac{1}{2} \frac{p}{\mu} \frac{\partial}{\partial p} \frac{1}{16\pi^2p^2} \left(1+\frac{2\epsilon}{3+\xi}\right) - \ln \frac{p^2}{\mu^2} + \ln \frac{k^2}{p^2} \] note that \( \frac{p^\mu}{\mu} \frac{\partial}{\partial p^\mu} = p \frac{\partial}{\partial p} \)

\[ = -\frac{1}{16\pi^2p^2} \left(1+\frac{2\epsilon}{3+\xi}\right) - \ln \frac{p^2}{\mu^2} + \ln \frac{k^2}{p^2} \]
For \( k > P \) we get

\[
\frac{1}{2} \mu^2 \frac{\partial}{\partial \mu} \mu^2 (1+2\varepsilon) \int \frac{dk}{(P-k)^2} \int \frac{d^{4}Y}{(2\pi)^D} \frac{\{(\gamma-P)\cdot(\gamma-k)+(\gamma-P)\cdot(P-k)\}}{\gamma^2(\gamma-P)^2(\gamma-k)^2}
\]

\[
= \frac{1}{2} \mu^2 \frac{\partial}{\partial \mu} \frac{1}{16\pi^2 k^2} \left( \frac{2}{3+\xi} - \ln \frac{k^2}{\mu^2} \right) = 0 \quad (A.62)
\]

Using \( (\gamma-P)\cdot(\gamma-k)+(\gamma-P)\cdot(P-k) = (\gamma-k)^2-(P-k)^2 \) we get

\[
- \mu^2 (1+2\varepsilon) \int \frac{dk}{(P-k)^2} \int \frac{d^{4}Y}{(2\pi)^4} \frac{P\cdot(\gamma-P)}{\gamma^2(\gamma-P)^2(\gamma-k)^2}
\]

\[
= - \mu^2 (1+2\varepsilon) \int \frac{dk}{(P-k)^2} \int \frac{d^{4}Y}{(2\pi)^D} \frac{1}{\gamma^2(\gamma-P)^2(\gamma-k)^2}
\]

\[
= \frac{1}{32\pi^2}(1+2\varepsilon) \frac{1}{\varepsilon} \left[ \frac{(k^2-P^2)}{M^2(1-m^2(k/P)^2)} - 1 \right] \quad (A.63)
\]

\[
\therefore \mu^2 (1+2\varepsilon) \int \frac{dk}{(P-k)^2} \int \frac{d^{4}Y}{(2\pi)^D} \frac{P\cdot(\gamma-P)\{(\gamma-P)\cdot(\gamma-k)+(\gamma-P)\cdot(P-k)\}}{\gamma^2((\gamma-P)^2)^2(\gamma-k)^2}
\]

\[
= - \frac{\ln \frac{k^2}{\mu^2}}{16\pi^2 P^2} \quad \text{for} \ k < P
\]

\[
= 0 \quad \text{for} \ k > P \quad (A.64)
\]

\[
\therefore \sum_{2,8} (P) = \frac{(1-\xi)^2}{16\pi^2 P^2} \left[ \frac{2}{3+\xi} - \ln \frac{P^2}{\mu^2} \right] \quad \text{for} \ k < P
\]

\[
(P) = \frac{(1-\xi)^2}{16\pi^2 k^2} \left[ \frac{2}{3+\xi} - \ln \frac{k^2}{\mu^2} \right] \quad \text{for} \ k > P \quad (A.65)
\]
\[
\begin{align*}
\therefore \Sigma_{2,6}(P) + \Sigma_{2,7}(P) + \Sigma_{2,8}(P) &= \frac{(1-\xi)^2}{16\pi^2p^2} \left[ -\frac{3}{2(3+\xi)} \sum_{k} \frac{\ln \frac{p^2}{\mu^2}}{4} \ln \frac{k^2}{\mu^2} \sum_{k} \frac{1}{2(\frac{p^2}{k^2} - 1)} \left[ L_2\left(\frac{k^2}{p^2}\right) + \frac{1}{2} \ln \frac{k^2}{p^2} \ln \frac{1-k^2}{p^2} \right] \right] \\
&\quad - \frac{1}{4} \frac{k^2 \ln \frac{k^2}{p^2}}{(p^2-k^2)} \quad \text{for } k<P \\
&\quad - \frac{1}{4} \frac{p^2 \ln \frac{p^2}{k^2}}{(k^2-p^2)} \quad \text{for } k>P
\end{align*}
\]

\[\Sigma_{2,9}(P) + \Sigma_{2,10}(P) + \Sigma_{2,11}(P)\] can be obtained from \[\Sigma_{2,6}(P) + \Sigma_{2,7}(P) + \Sigma_{2,8}(P)\] by interchanging \(P\) with \(k\).

\[
\begin{align*}
\Sigma_{2,9}(P) + \Sigma_{2,10}(P) + \Sigma_{2,11}(P) &= \frac{(1-\xi)^2}{16\pi^2k^2} \left[ -\frac{3}{2(3+\xi)} \sum_{p} \frac{\ln \frac{p^2}{\mu^2}}{4} \ln \frac{p^2+k^2}{p^2} \sum_{p} \frac{1}{2(\frac{k^2}{p^2} - 1)} \left[ L_2\left(\frac{k^2}{p^2}\right) + \frac{1}{2} \ln \frac{1-p^2}{k^2} \ln \frac{k^2}{p^2} \right] \right] \\
&\quad - \frac{1}{4} \frac{k^2 \ln \frac{k^2}{p^2}}{(p^2-k^2)} \quad \text{for } k<P \\
&\quad - \frac{1}{4} \frac{p^2 \ln \frac{p^2}{k^2}}{(k^2-p^2)} \quad \text{for } k>P
\end{align*}
\] (A.67)
\[ \frac{3}{3} \left( \frac{3+1}{3+1} \right) \frac{2}{1} + \frac{4}{2} \left( \frac{(3-1)}{3-1} \right) \frac{4}{1} + \frac{2}{1} \left( (3-1) \frac{2}{1} \right) \frac{2}{1} = \frac{2}{1} \left( (3+2) \right) \frac{2}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}{1} \left( (3+1) \right) \frac{4}
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2. By SU(2) \_l we mean that the left-handed fermions transform as the fundamental representation of the weak isospin gauge group, whereas the right-handed fermions transform as singlets. The subscript y refers to weak hypercharge and Q refers to electric charge.


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