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Bubble and bubble wake characteristics in a gas-liquid-solid fluidized bed

Kreischer, Bruce Edward, Ph.D.
The Ohio State University, 1989

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BUBBLE AND BUBBLE WAKE CHARACTERISTICS
IN A
GAS-LIQUID-SOLID FLUIDIZED BED

Dissertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Bruce Edward Kreischer, B.S. ChE, M.S. ChE

The Ohio State University
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Bruce Edward Kreischer

1989
To My Grandmothers

Augusta Geis and Idabell Kreischer

The most devoted, caring, and inspirational people

I have ever known.
ACKNOWLEDGEMENTS

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<tr>
<td>$a$</td>
<td>bubble base, cm</td>
</tr>
<tr>
<td>$A$</td>
<td>cross-sectional area, cm$^2$</td>
</tr>
<tr>
<td>$A_0$</td>
<td>constant in Eq. (27)</td>
</tr>
<tr>
<td>$b$</td>
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<tr>
<td>$C$</td>
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</tr>
<tr>
<td>$C_d$</td>
<td>drag coefficient, -</td>
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<td>$d_p$</td>
<td>particle diameter, cm</td>
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<tr>
<td>$E$</td>
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<td>$E_o$</td>
<td>Eotvos number, $g\Delta\rho D_e^1/\sigma$, -</td>
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<tr>
<td>$f_v$</td>
<td>vortex shedding frequency, 1/sec</td>
</tr>
<tr>
<td>$F_d$</td>
<td>drag force, g cm/sec$^1$</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration, cm/sec$^1$</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>pressure head, cm of continuous phase</td>
</tr>
<tr>
<td>$h_{lw}$</td>
<td>liquid wake depth, cm</td>
</tr>
<tr>
<td>$h_w$</td>
<td>wake depth, cm</td>
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$\Delta H$ : distance between pressure taps, cm
$I$ : converted light intensity, in volts
$I_0$ : converted light intensity at $t_s = 0$, in volts
$K$ : ratio of the wake to bubble volume, -
$K_l$ : ratio of the liquid wake to bubble volume, -
$K_s$ : ratio of the solids containing wake to bubble volume, -
$K_w$ : ratio of the spherical vortex volume to bubble volume
$M_o$ : Morton number, $g U^4 \rho / \rho l^4$, -
$n$ : Richardson-Zaki index, -
$n_i$ : number of bubbles, -
$O$ : Opacity, -
$\Delta P / \Delta Z$ : pressure drop, $g/cm^2/sec^1$
$P_i$ : injection pressure, psig
$R$ : radius of curvature for a spherical-cap bubble, cm
$R_{sc}$ : solids circulation rate, $cm^3/cm^2/sec$
$Re_b$ : bubble Reynolds number, $\rho U_b D_b / \mu$, -
$Re_t$ : particle Reynolds number, $\rho U_p d_p / \mu$, -
$Sr$ : Strouhal number, $D_e f / U_b$, -
$Sr_a$ : Strouhal number, $a_f U_b$, dimensionless
$U$ : velocity vector, $cm/sec$
$U_b$ : bubble rise velocity, $cm/sec$
$U_B$ : absolute bubble rise velocity, $cm/sec$
$U_g$ : superficial gas velocity, $cm/sec$
$U_I$ : superficial liquid velocity, $cm/sec$
\( U_{10} \) : minimum fluidization velocity, cm/sec
\( U_t \) : particle terminal velocity, cm/sec
\( V_b \) : bubble volume, cm\(^3\)
\( V_p \) : particle velocity relative to the liquid phase, cm/sec
\( V_v \) : spherical vortex volume, cm\(^3\)
\( V_w \) : cylinder completing wake volume, cm\(^3\)
\( W \) : rate of work, g cm\(^3\)/sec\(^1\)
\( W_s \) : weight of particles within the bed, g
\( X \) : relative wake solids holdup, -
\( z_p \) : distance between the upper and lower probes, cm

Greek
\( \alpha \) : absorbance, 1/cm
\( \delta_g \) : gas phase volume fraction, -
\( \delta_{ls} \) : liquid-solid phase volume fraction, -
\( \delta_w \) : wake phase volume fraction, -
\( \varepsilon_0 \) : void fraction in a liquid-solid fluidized bed, -
\( \varepsilon_g \) : gas holdup, -
\( \varepsilon_l \) : liquid holdup, -
\( \bar{\varepsilon}_s \) : mean solids holdup, -
\( \bar{\varepsilon}_{sc} \) : critical mean solids holdup, -
\( \varepsilon_{sp} \) : solids holdup in the particulate phase, -
\( \varepsilon_{sw} \) : solids holdup in the wake phase, -
\( \theta \) : angular coordinate, -
$\theta$ : included angle of a spherical-cap bubble, deg.

$\kappa$ : viscosity ratio, -

$\nu$ : viscosity, g/cm sec

$\zeta$ : vortex radius, cm

$i$ : measured chord length, cm

$\rho$ : density, g/cm$^3$

$\Delta \rho$ : density difference, $\rho_c - \rho_d$, g/cm$^3$

$\sigma$ : surface tension, dyne/cm

$\tau_1$ : time of contact for bubble, sec

$\tau_2$ : time elapse between bubble striking successive probes, sec

$\psi$ : stream function

$\psi_i$ : stream function inside the spherical vortex

$\psi_o$ : stream function outside the spherical vortex

**Subscripts**

$c$ : continuous phase

d : dispersed phase

e : effective

g : gas

$l$ : liquid

$p$ : particle

$s$ : solid

$t$ : terminal
CHAPTER I

INTRODUCTION

Fundamental studies of gas-liquid-solid fluidization are relevant to industrial processes ranging from fermentation and wastewater technology to the hydrotreating of petroleum resid. Interestingly enough, these widely different applications involve significantly different bubble characteristics and mass and heat transfer behavior. Ultimately, the hydrodynamic behavior within the fluidized bed, in addition to the mass and heat transfer behavior, reflects the bubble size and motion characteristics and their interaction with the liquid-solid medium. Consequently, the study of bubble characteristics provides valuable insight into the physical understanding of the three-phase fluidization process.

This chapter begins with a general description of the hydrodynamic behavior in a gas-liquid-solid fluidized bed; the central theme in this discussion focuses on the relationship between the bubble behavior and characteristics important to reactor design. Then, bubble size, shape, and motion properties are examined in detail with emphasis on the importance of the bubble wake. And finally, the properties of the bubble wake are examined based on their implications to the hydrodynamic behavior.
1.1 General Hydrodynamic Behavior

The overall fluid mechanical, or hydrodynamic, behavior of three phase fluidized beds stems from complex interactions between the individual phases: the most prominent interaction being between the rising gas bubbles and the surrounding liquid-solid medium. The forces and interactions on and between the individual phases vary with axial height because of different predominant factors of the reactor design. A schematic representation of a concurrent fluidized bed is given in Fig. 1. Four distinct zones above the gas-liquid distributor are identifiable based on the prevailing physical phenomena: the gas jet or distributor zone, the bubble formation zone, the bulk fluidized bed zone, and disengagement zone.

The jet zone, as the name implies, covers the region above the gas-liquid distributor yet which is still below the height marking initial bubble formation. The bubble formation zone covers the region from the initial breakup of gas jets into bubbles up to the point the bubbles grow large enough to where the interaction with the solids affects bubble growth characteristics. In the jet region the gas-liquid interfacial area is quite low since no bubbles are yet formed. Upon formation of the bubbles, the gas-liquid interfacial area becomes relatively large; the initially large interfacial area will either remain large or decrease depending on the system properties in the bulk zone. The
Figure 1. Conceptual diagram of a gas-liquid-solid fluidized bed.
hydrodynamic behavior of the jet and bubble formation zones inherently reflect the gas-liquid distributor design.

The bulk fluidization zone includes the main portion of the fluidized bed where the complex interactions of the individual phases occur and the aspect of the reactor, or column, design which will be most important is the diameter. The hydrodynamic behavior in the bulk fluidized bed zone varies drastically over large ranges of operating conditions due to the complex flow behavior involved. Observations of the flow behavior over these large ranges of operating conditions have identified, based on bubble behavior, three basic flow regimes: the dispersed bubble, the coalesced bubble, and the slug bubble regimes. Basically, the differences between the three flow regimes lies in the bubble sizes and size distributions. The dispersed bubble regime has small bubbles of nearly uniform size, the coalesced bubble regime has a broad size distribution of bubbles, and the slugging regime has large bubbles which can approach the size of the column diameter (see, e.g., Matsuura and Fan, 1984). The conditions at which transition between the various regimes occurs reflect the complexity of three-phase fluidized beds.

The dispersed bubble regime with its bubbles of nearly uniform size represents what can be termed homogeneous fluidization (Ermakova et al., 1970). Gas holdup shows a nearly uniform radial profile (see, e.g., Fan et al., 1985), and
consequently, the liquid velocity profile is also nearly flat. Because of the absence of liquid circulation patterns in the dispersed bubble regime, the liquid flow approaches plug flow (the liquid dispersion coefficient approaches zero). Solids, as in all fluidized beds, can be considered completely mixed. In terms of mass transfer, the small bubble sizes provide a large interfacial area and, hence, gas-liquid mass transfer limitations are not usually encountered.

The coalesced bubble regime exhibits the most interesting and diverse behavior. The coalesced bubble regime contains rather large bubbles having a broad size distribution. Due to the complex interaction between the individual phases, the small bubbles found in the bubble formation zone increase in size through bubble coalescence as they rise through the bulk zone. The prevailing bubble growth behavior in the coalesced bubble regime produces interesting results in terms of other hydrodynamic characteristics. Experimental results indicate that gas holdup in the coalesced bubble regime shows a near parabolic radial distribution (e.g., Hu et al., 1986). Accompanying this radial holdup distribution is a forced liquid circulation imposed over the gross upward liquid flow (Morooka et al., 1982). The superimposed liquid circulation promotes significant liquid mixing. The axial liquid Peclet number in the coalesced bubble regime approaches that of a completely mixed state. Gas-liquid
mass transfer, on the other hand, shows a significant reduction because of a lower gas-liquid interfacial area.

The disengagement zone includes the region above the fluidized bed proper. Particles mainly enter this zone within the bubble wake; subsequently, they are shed from the wakes and settle back to the bed surface. Near the bed surface, drift effects play a prominent role and under many conditions the actual bed surface cannot be easily defined.

The relative importance of the three zones to the overall behavior of the fluidized bed is determined mainly from the bed height; small bed heights necessarily emphasize the importance of the jet and bubble formation zones. Column diameter and particle properties will affect the demarcation between the bubble formation zone and the bulk fluidized bed zone.

1.2 Bubble Size and Shape Characteristics

The interaction between a rising gas bubble and the surrounding liquid or liquid-solid medium affects the observed bubble shape and determines the extent of the disturbance in the surrounding flow field. Bubbles in motion are generally classified by shape as spherical, oblate ellipsoidal, or spherical-cap; the exact shape depends upon the predominating forces acting on the bubble.
The size and shape of the rising bubble can be quantified, in general, by specifying the maximum horizontal and vertical lengths (width and height). In two-dimensional beds, based on these two dimensions and assuming an elliptical bubble shape, the bubble area, and consequently an area-equivalent circular bubble diameter \( D_e \), can be reasonably estimated for most bubble size ranges, while in three-dimensional beds, a volume-equivalent spherical bubble diameter can be obtained. The equivalent bubble diameters thus defined are the usual representation of the bubble size. For extremely large bubbles, the bubble has a spherical-cap shape with a flat bubble base and information such as the included angle and the frontal radius of curvature will help estimate the diameter.

In multi-bubble systems, a mean bubble size is usually used to describe the system. When a measurement technique provides individual bubble sizes directly, an averaging technique must be used. In such a case, the mean size is commonly expressed through the Sauter, or volume-surface, mean. The Sauter mean is 

\[
D_{vs} = \frac{\sum n_i D_{bi}^3}{\sum n_i D_{bi}^2}
\]

(1)

where \( n_i \) is the number of bubbles with size \( D_{bi} \). Note that \( D_{bi} \) can be any equivalent diameter (equivalent volume, etc.); the exact choice will naturally affect the magnitude of the Sauter
mean size. Other definitions of the average diameter are also possible.

The shape of a bubble rising in an infinite medium can be generalized in terms of three dimensional variables (Grace, 1973): the Reynolds number

$$\text{Re}_b = \frac{\rho D_e U_b}{\mu}$$

the Eotvos number

$$\text{Eo} = \frac{g \Delta \rho D_e^4}{\sigma}$$

and the Morton number

$$\text{Mo} = \frac{g \mu \Delta \rho}{\rho \sigma}$$

Clearly, the density difference, the surface tension, and equivalent diameter are all quite important; in addition, the bubble rise velocity \(U_b\) also is related to its shape. Table 1 lists the approximate range of values for each of the above dimensionless numbers for each class of bubble shape.

For small bubbles \((D_e < 0.1 \text{ cm})\), the bubble shape resembles a perfect sphere, surface tension forces dominate, and the bubble rises steadily in a rectilinear path. For intermediate size bubbles, both surface tension and liquid inertia effects are important and liquid viscosity and the presence of surface-active contaminants influence the bubble dynamics. The motion of intermediate size bubbles is extremely complex: ellipsoidal bubbles exhibit secondary motion characteristics, such as zig-zag
Table 1. Approximate Conditions for Various Bubble Shapes

<table>
<thead>
<tr>
<th>Shape</th>
<th>Re&lt;sub&gt;b&lt;/sub&gt;</th>
<th>Eo</th>
<th>ln(Mo)</th>
<th>D&lt;sub&gt;e&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td>&lt; 10</td>
<td>10&lt;sup&gt;-1&lt;/sup&gt; ~ 10&lt;sup&gt;1&lt;/sup&gt;</td>
<td>&lt; 1 mm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10 ~ 10&lt;sup&gt;1&lt;/sup&gt;</td>
<td>10&lt;sup&gt;-1&lt;/sup&gt; ~ 10&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-11 ~ 1</td>
<td>&lt; 1 mm</td>
</tr>
<tr>
<td>Ellipsoidal</td>
<td>20 ~ 10&lt;sup&gt;1&lt;/sup&gt;</td>
<td>1 ~ 40</td>
<td>-11 ~ -1</td>
<td>1 ~ 18 mm</td>
</tr>
<tr>
<td>Spherical Cap</td>
<td>15 ~ 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>40 ~ 10&lt;sup&gt;1&lt;/sup&gt;</td>
<td>-14 ~ 6</td>
<td>&gt; 18 mm</td>
</tr>
</tbody>
</table>
or spiral trajectories and shape dilations or oscillations, as they rise. Clift et al. (1978) presented a summary of these complex motions. For large bubbles ($D_e > 1.8$ cm), liquid inertial effects predominate and the shape approaches a spherical-cap with an included angle of about $100'$ and a relatively flat base; the effects of surface tension, viscosity, and presence of impurities in the liquid medium are negligible. These bubbles follow a rectilinear path with some rocking and/or base oscillations.

Differences in the motion of rising bubbles are directly associated with differences in the flow field, or wake, immediately behind the bubble. The flow around small bubbles can be described by the creeping flow approximation. A periodic shedding of vortices from the bubble wake may produce the secondary motion associated with intermediate size bubbles; in fact, Edge and Grant (1971) found that bubble oscillations started with the onset of vortex shedding from the wake.

In multi-bubble systems, instead of a single bubble size, a distribution of sizes are found. At low liquid velocities, Lee et al. (1984) found a normal bubble size distribution; at higher liquid velocities, they found a log-normal bubble size distribution. A log-normal bubble size distribution was also observed by Darton and Harrison (1974a) and Matsuura and Fan (1984). Matsuura and Fan (1984) found similar shape in the bubble size distributions for each flow regime, but the mean size was
largest in the slug flow regime. The interaction between the individual bubble and the surrounding medium still plays a major role in the bubble behavior, but interaction between individual bubbles and other bubbles and their wakes also plays a major role. Such interaction is evident in the rise velocity behavior of bubbles in multi-bubble systems.

From the discussion in Sections 1.1 and 1.2, it is apparent that a synergistic relationship exists between the rising gas bubbles and the surrounding liquid-solid medium. The characteristics of the bubbles, such as size, shape, and motion, depend on the properties of the liquid-solid medium, while the hydrodynamic behavior of the liquid-solid medium depends on the bubble characteristics.

1.3 Bubble Rise Velocity Characteristics

The rise velocity of a single bubble ($U_b$) has been studied extensively. Three famous relationships have been developed to describe $U_b$ in three important size ranges. The first, in Stokes regime, describes $U_b$ for spherical bubbles at low Reynolds numbers ($Re_b < 0.2$):

$$U_b = \frac{gD_e \Delta \rho}{18 \mu}$$

The second, derived through a wave analogy (Marrucci et al., 1970), is
this equation applies to ellipsoidal bubbles rising in pure water (0.1 < D_e < 1.8 cm). For this range the bubble rise velocity is between about 20 to 30 cm/sec. The rise velocity in contaminated systems is lower than that found in pure systems (e.g., 10 < U_b < 30 cm/sec).

For large spherical-cap bubbles (1.8 < D_e < 5 cm), the Davies-Taylor equation (Davies and Taylor, 1950) applies:

\[ U_b = \frac{2}{3} \sqrt{\frac{g R}{\Delta \rho / \rho}} \]  

where R is the radius of curvature of the bubble. Over this range the bubble rise velocity increases from about 25 to 50 cm/sec.

In multi-bubble systems the interaction between bubbles affects the rise velocity. In a case study of successively rising spherical-cap bubbles, Miyahara et al. (1989) found that the trailing bubble rose faster than the leading bubble, and ultimately, depending on the initial separation between the bubbles, could actually catch the leading bubble. Such behavior indicates that average rise velocities in multi-bubble systems should be higher than those for isolated bubbles.

Several studies have been done on the bubble rise velocity in multi-bubble systems: these studies have been done in both 2D and 3D systems. Table 2 gives the proposed correlations presented by the various authors. Note that the reported values of the
Table 2 Bubble Rise Velocities in Multi-Bubble Systems

<table>
<thead>
<tr>
<th>Reference</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Two-Dimensional System</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Kim et al. (1977) | \[ U_b = 18.0 \bar{D}_b^{0.319} \]
| de Lasa et al. (1984) | \[ U_b = 37.92(\bar{D}_b)^{0.439} \]
| | \[ 3 < \bar{D}_b < 8 \text{ cm}; \quad 50 < U_b < 170 \text{ cm/s} \]
| | \[ 1.5 < \bar{I}_b < 7 \text{ cm}; \quad 45 < U_b < 150 \text{ cm/s} \]

| **Three-Dimensional Systems** | |
| Rigby et al. (1970) | \[ [U_b - (U_g + U_i)](\bar{I}_b)^{1.51} = 32.5(\bar{I})^{1.51} \]
| Lee et al. (1984)<sup>a</sup> | \[ U_b = 43.3(\bar{I})^{0.144} \]
| | \[ 0.4 < \bar{I}_b < 4 \text{ cm}; \quad 20 < U_b < 120 \text{ cm/s} \]
| Lee et al. (1984)<sup>a</sup> | \[ U_b = 36.44(\bar{I})^{0.111} \]
| | \[ 0.1 < \bar{I}_b < 1.6 \text{ cm}; \quad 20 < U_b < 45 \text{ cm/s} \]
| Lee and de Lasa (1986)<sup>b</sup> | \[ U_b = 40.59(\bar{I})^{0.193} \]
| | \[ 1.6 < \bar{I}_b < 3 \text{ cm}; \quad 45 < U_b < 120 \text{ cm/s} \]
| Lee and de Lasa (1986)<sup>b</sup> | \[ U_b = 40.48(\bar{D}_b)^{0.511} \]
| | \[ 0.1 < \bar{I}_b < 1.0 \text{ cm}; \quad 20 < U_b < 50 \text{ cm/s} \]
| | \[ 1.0 < \bar{I}_b < 4 \text{ cm}; \quad 50 < U_b < 110 \text{ cm/s} \]

---

<sup>a</sup> \( V_g = 1.214 \text{ cm/s} \)

<sup>b</sup> \( V_g = 1.896 \text{ cm/s} \)
bubble rise velocity in a multi-bubble system exceed those for isolated single bubble by two to three times; in fact rise velocities in multi-bubble systems have been reported to exceed 1 m/sec.

1.4 Hydrodynamic Implications

The wake itself is directly connected with the overall hydrodynamic behavior in a fluidized bed. In 1964, Stewart and Davidson suggested that the formation of a solids-free region behind a bubble could account for reported bed contraction phenomena. They found that such a solid-free region could be a particle depleted wake behind an air bubble or a stabilized water bubble.

In a conceptual model of the hydrodynamics, Ostergaard (1965) postulated that a fluidized bed consisted of a particulate phase, a bubble phase, and a wake phase; for such a situation the combined volume of these regions must equal the total volume, or in terms of volume fractions

\[ \delta_g + \delta_p + \delta_w = 1 \]  

1. Introducing a gas flow into liquid-solid fluidized beds of small (\(<~1 \text{ mm}\)) glass beads caused a reduction in bed height, hence the term bed contraction (for references see Ostergaard and Theisen, 1966; Ostergaard, 1964; and Turner, 1964).

2. A stabilized water bubble has small air bubbles forming a roof over a clear liquid region.
Note the bubble phase volume fraction is equivalent to the mean gas holdup, or

\[ \delta_g = \bar{\xi}_g \]  \hspace{1cm} (9)

Ostergaard (1965) also indicated that the bubble wake travels with a velocity equal to the bubble rise velocity and, contrary to Stewart and Davidson’s perception, has a solids holdup equivalent to the particulate phase. With these assumptions, Ostergaard developed an expression for the bed porosity in terms of the bubble rise velocity and the parameter

\[ K = \frac{\delta_w}{\delta_g} \]  \hspace{1cm} (10)

which is the ratio of the wake volume to bubble volume. Rigby and Capes (1970) provided a similar development, but considered a solid-free wake.

Bhatia and Epstein (1974) suggested that a more general wake model should allow the solids concentration in the wake to vary. They introduced the parameter,

\[ X = \xi_{sw}/\xi_{sp} \]  \hspace{1cm} (11)

to relate the solids volume fraction in the wake to that in the liquid-solid region. With this definition, an overall mass balance on solids within the fluidized bed can relate the mean solids holdup (\( \bar{\xi}_s \)) to the solids holdup in the particulate region (\( \xi_{sp} \)); for example,

\[ \frac{\bar{\xi}_s}{\xi_{sp}} = [1 - \delta_g(1 + K(1-X))] \]  \hspace{1cm} (12)
Eq. (12) can be rearranged to
\[
1 + K(1-X) = \frac{1 - (\bar{e}_s / \bar{e}_{sp})}{\delta g}
\]  
(13)

These two equations express the mean solids holdup of the bed in terms of three variables. Typically, the particulate phase solids holdup, \( \bar{e}_{sp} \), is evaluated based on the Richardson-Zaki (1954) equation and \( K \) and \( X \) describe the properties of the wake phase. Equation (13) indicates, however, a relationship exists between the wake size and solids concentration. If the wake is considered to be solid-free, \( X \) is set to 0 and \( K \) is replaced with \( K_0 \), indicating a liquid wake only.

One distinction can be made between the contrasting pictures of the wake solids content. This distinction comes through evaluation of the solids circulation rate within the bed. Since the net transport of solids in the bed must be zero, the rate of solids circulation within the bed can be characterized by the rate of solids transport within the wake. Based on the wake model concept, this rate of transport is
\[
R_{sc} = U_s \delta_s \bar{e}_sw = U_s \delta_s \bar{e}_{sp} X
\]  
(14)

Obviously, no transport occurs if the wake size or relative solids holdup becomes zero.

Effective utilization of the wake model to predict the hydrodynamic behavior requires a fundamental understanding of the wake properties \( K \) and \( X \). This study concerns the evaluation of \( X \),
hence the remaining discussion in this chapter focuses on the nature of these wake characteristics.

1.5 Bubble Wake Characteristics

As is indicated in part by the motion of individual bubbles, the flow field behind a bubble plays a principal part in the overall observed bubble behavior. The structure of the flow field, or bubble wake, has important fluid dynamic implications and is directly concerned with the separation of the boundary layer from the bubble and the generation of vorticity. Consequently, most of the ensuing discussions include reference to the bubble Reynolds number.

The wake geometry in gas-liquid systems is classified into three types: closed laminar/toroidal wakes, closed turbulent wakes, and open turbulent wakes (Coppus et al., 1977). The closed laminar wake and totally chaotic, turbulent wake are two extreme natures of the bubble wake. A closed laminar wake is hydrodynamically stable, consists of a well-defined boundary with a toroidal vortex ring inside, and exchanges no liquid with the external flow (Coppus et al., 1977; Bhaga and Weber, 1981). Turbulent wakes consist of chaotic flows, and the closed turbulent wake, although visualized by some investigators (e.g., Davies and Taylor, 1950), is not realistic (Maxworthy, 1967; Wegener and Parlange, 1973).
Bubble wakes in gas-liquid-solid fluidized beds may differ somewhat from those in gas-liquid systems due to direct interaction with the particles and because of different external disturbances prevailing in the bulk flow. Some similarities should, however, exist in the wake structure of these two systems, especially in systems involving small particles of low density. Conceptually, in single bubble systems, the bubble wake can be subdivided into a primary and a secondary wake. The primary wake is the region immediately behind and travelling with the bubble: inside the primary wake, broad, although irregular, circulatory flow patterns can exist, but the exact boundary may not always be clearly identifiable. The secondary wake can extend far below the bubble and represents a much less well defined region. Included in the secondary wake can be free shear layers and vortices shed from the primary wake: at lower Reynolds numbers, the secondary wake can be a streaming tail extending along the bubble rise path below the primary wake (Slaughter and Wraith, 1968) or, at higher Reynolds numbers, a vortex street extending far behind the bubble (Coppus et al., 1977). The presence of particles can be expected to affect the behavior in the secondary wake to a much greater extent than in the primary wake; this premise should also extend to the presence of other bubbles in the system. Based on the definition given here, the primary wake represents, to some degree, the wake phase of the wake model.
Wake formation has been described based on the localized fluid dynamic properties of the wake flow field; essentially, this involves the formation and shedding of vortices beneath the bubble. Tsuchiya and Fan (1988a) observed that, in a two-dimensional gas-liquid-solid fluidized bed, wake shedding often occurs asymmetrically and that ellipsoidal and spherical-cap bubbles have fixed separation regions along the edge. As shown in Fig. 2, the separation of the external potential flow at the bubble edge induces vorticity generation at the separation points and conveyance of vorticity along the free shear layer. Due to differences in the characteristic velocities between the outer and inner boundaries of the shear layer, the free shear layer, or vortex sheet, tends to roll up. Eventually, a circular-cross-sectioned vortex is formed which is still attached to the bubble edge via the shear layer: this vortex grows as vorticity is continuously supplied through the shear layer. When the vortex becomes strong enough to draw the external flow and opposite shear layer across the wake, oppositely-signed vorticity of sufficient concentration eliminates further supply of vorticity;

3. The portion of the bubble surface with the sharpest curvature.
Figure 2. A mechanistic description of vortex formation and shedding (from Tsuchiya and Fan, 1988a).
consequently, the vortex ceases to increase in strength and starts detaching from the bubble. At this moment the vortex is said to be shed. One cycle of the vortex formation-shedding process begins with the roll-up of the vortex sheet and ends with the cutoff of the vorticity supply.

Parallel shedding may take place if the above process proceeds almost simultaneously from both edges. Most likely, however, parallel shedding occurs when the vortex sheets become unstable before forming well-established spiral/circulation flow patterns or even before rolling up. This is the case for large circular-cap bubbles.

The vortex shedding process indicates the unsteady nature of the wake. The Strouhal number

$$Sr = \frac{Df_v}{U_b} = \frac{af_v}{U_b}$$

represents the rate of vortex shedding ($f_v$ is the shedding frequency). Typically, the Sr is expressed as a function of the bubble Reynolds number; note, the shedding frequency can be defined based on either the shedding of a single vortex or the shedding of a vortex pair. In three-phase systems, Miyahara et al. (1989) reported the Sr to range from about 0.3 to 0.55. Tsuchiya and Fan (1988) found a shedding frequency between about 3 and 5 per second for bubbles having a base ranging in size from 0.5 to 3.5 cm in length.
In multi-bubble systems, the secondary wake should not be as extensive as in a single bubble system since the passing of subsequent bubbles will tend to disperse the shed vortices much faster. However, the primary wake should be equally as important in both systems.

1.6 Wake Structure

Observations about the wake structure have been based on several different experimental techniques; each method sheds its own particular light on the structure. Visualization of the flow field behind the bubble and observations on the pressure profile and wake solids concentration within the wake have provided invaluable insight into the wake structure.

Tsuchiya and Fan (1988a) identified three main classes of wake structures based on flow visualization methods: a closed wake which occurs prior to onset of vortex shedding or for steady flows at low Reynolds numbers; an open wake resulting from alternating shedding of vortices from opposite edges of the bubble (2D); and an open wake resulting from parallel shedding of vortices. In open wake systems, the primary wake may contain up to two vortices, while the secondary wake contains vortices shed from the primary wake. The open nature of the wakes behind large single bubbles prohibits identification of a clear-cut boundary between the primary and secondary wakes. However, the systematic or
periodic nature of the vortex shedding process allows a logical
definition of the primary wake size.

For steady flows at low Reynolds numbers ($Re_b < 110$), the
primary wake observed behind large bubbles is laminar and closed
and its boundary shape and size are constant. As shown in Fig. 3
(a) and (b), the primary wake contains primarily a well-developed
toroidal vortex ring (three-dimensional) and a stagnant region
about the rear stagnation point. The stagnant region size depends
upon the Reynolds number and/or wake stability.

At higher Reynolds numbers, an open wake structure exists
behind intermediate to large size bubbles. In liquids or liquid
suspensions of low inertia solid particles, overall the wake flow
is unsteady, a static turbulent layer exists directly beneath the
bubble, and a cyclic vortex shedding process occurs; see Fig.
3(c). Well developed vortices, generated in the primary wake
region, are shed into the secondary wake from alternating sides.
In liquid suspensions of larger inertia particles, asymmetric
shedding still occurs and the material shed has large-scale
vortical motion. However, as shown in Fig. 3(d), a chaotic
turbulent region exists about the wake center axis in addition to
the stagnant turbulent layer. The turbulent region may result
from one of several reasons: either high inertia solid particles
deviate from the liquid flow pattern in the vortex and are
expelled to the central wake region or the turbulent shear layers
Figure 3. A systematic description of primary wake boundaries
(from Tsuchiya and Fan, 1988a)
are dragged into the interior.

For very large bubbles rising at high Reynolds numbers the wake flow is unsteady and has parallel wake shedding. The primary wake contains vortex sheets or blobs and a broad region of chaotic turbulence around the wake central axis: the vortex sheets or blobs, in this case, may not have established a well-defined vortical motion. Note that, in this case, the bubble rises in a rectilinear path.

Based on observations of the solids concentration, Kitano and Fan (1988) classified the primary wake into four regions: a stable liquid, a stable solids, a vortex sheet, and a fluctuating solids region. The stable liquid region is located immediately beneath the bubble; note, however, this region does not occur for all flow conditions and may not extend the entire length of the bubble base. The stable solids region follows the stable liquid layer and includes parts of the growing vortices and the turbulent area around the wake centerline. The vortex sheet region has a low solids concentration. The fluctuating solids region includes the main portion of the vortices to be shed; consequently, this region varies drastically in size with time.

A common observation about the wake structure is the existence of a liquid layer directly beneath the bubble. In terms of pressure profiles within the wake, Lazarek and Littman (1974) and Bessler (1984) found pressure minima occurring near the vortex
centers of a toroidal vortex pair and immediately below the bubble base to be separated by a local maximum layer. It is generally accepted that a lower pressure exists in the primary or near wake than in the surrounding medium.

The representative wake structures found in a two-dimensional bed introduced in the previous section provide information on the size of the wake following relatively large spherical cap bubbles. Information about the wake following spherical and ellipsoidal bubbles in two phase systems can also provide important information. Table 3 provides a general description of wake properties reported in the literature for bubbles (or particles) in various systems. Both closed and open wakes have been observed with relative wake sizes reported from near 0 at low Reynolds numbers to as high as 22 in one case.

In two dimensional systems, the visualization of the flow field which led to an understanding of the wake structure also allowed Tsuchiya and Fan (1988) to qualitatively identify the boundaries between the primary and secondary wakes for several representative wake structures. In the case of the closed wake, the boundary is obvious and is shown enclosed by dotted lines and shaded inside in Fig. 3(a) and 3(b). For an open wake with alternating vortex shedding, the primary wake boundary can be considered the cut-off stream which crosses the wake central axis in going from the free shear layer on one side to that on the
<table>
<thead>
<tr>
<th>Shape</th>
<th>Re&lt;sub&gt;b&lt;/sub&gt;</th>
<th>K</th>
<th>Wake Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500 - 1000</td>
<td></td>
<td></td>
<td>Open – alternate</td>
</tr>
<tr>
<td>2000 - 8000</td>
<td></td>
<td></td>
<td>Open – helical&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>8000 - 10000</td>
<td></td>
<td></td>
<td>Open – spirals&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
<tr>
<td>Ellipsoidal</td>
<td>200 - 800</td>
<td>1.5 - 4</td>
<td>Open&lt;sup&gt;1&lt;/sup&gt;</td>
</tr>
<tr>
<td>Spherical Cap</td>
<td>1000 - 10000</td>
<td>22</td>
<td>Open/Turbulent&lt;sup&gt;3&lt;/sup&gt;</td>
</tr>
<tr>
<td>2000 - 15000</td>
<td>1.5 - 4</td>
<td></td>
<td>Open&lt;sup&gt;4&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

1 - liquid-liquid system - stagnant continuous phase
2 - gas-liquid system - stagnant liquid phase
3 - gas-liquid system - moving liquid phase
4 - gas-liquid-solid system - moving liquid-solid medium
5 - solid-liquid system
other. The boundary shape and size vary cyclically due to vortex shedding; the minimum size occurs immediately after vortex shedding, the maximum size occurs just prior to shedding. For an open wake with parallel vortex shedding, the primary wake cannot be distinguished from the adjacent secondary wake region due to the turbulent region, but the boundary can be defined, as shown in Fig. 3(e), by connecting the end point of two vortex sheets hanging from the bubble edges.

1.7 Wake Properties

The studies presented in Sections 1.4 and 1.5 have, in a sense, taken a microscopic view to describe its physical nature. The remaining studies of interest view the wake from a macroscopic view by evaluating both K and X through overall phase holdup data using the wake model concept. The important information available from these macroscopic studies is a quantification of the effect of the operating variables on the wake characteristics.

In order to back calculate K or X from the general wake model, either X or K must be determined independently. Three approaches have been used: (i) assume no solids exist in the wake (Efremov and Vakhrushev, 1970; El-Tamtamy, 1974; Darton and Harrison, 1975; Baker et al., 1977); (ii) assume the relative solids holdup in the wake is 1 (Ostergaard, 1965); or (iii) predefine the wake size (El-Tamtamy and Epstein, 1978). Table 4
Table 4. Correlations of Wake Size

<table>
<thead>
<tr>
<th>Reference</th>
<th>Correlation</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efremov and Vakhrushev (1970)</td>
<td>$K_0 = 5.1 \varepsilon_0 \left[1 - \tanh\left(40 \frac{U_g}{U_{tg}} \varepsilon_0 \right)^{1.1} - 3.32 \varepsilon_0^{1.11}\right]$</td>
<td>$0 &lt; K_0 &lt; 3$</td>
</tr>
<tr>
<td>Bhatia and Epstein (1974)</td>
<td>$K_0 = \left[0.61 + \frac{0.037}{\varepsilon} + 0.013\right] \varepsilon$</td>
<td>$0 &lt; K_0 &lt; 2$</td>
</tr>
<tr>
<td>El-Temtamy (1974)</td>
<td>$K_0 = 0.462 \left(\frac{U_p}{U_g}\right)^{0.11} \sigma_p^{0.44}$</td>
<td>$0 &lt; K_0 &lt; 3$</td>
</tr>
<tr>
<td>Darton and Harrison (1975)</td>
<td>$1 + K_0 = 1.4\left(\frac{U_p}{U_g}\right)^{0.11}$</td>
<td>$0 &lt; K_0 &lt; 3$</td>
</tr>
<tr>
<td>Muroyama (1976)</td>
<td>$K_0 = 1.72 U_g^{-0.44} U_p^{0.44}$</td>
<td>$0 &lt; K_0 &lt; 1$</td>
</tr>
<tr>
<td>Baker et al. (1977)</td>
<td>$K_0 = 1.617 \left(\frac{U_p}{U_g}\right)^{0.11} \sigma^{-0.44}$</td>
<td>$0 &lt; K_0 &lt; 1$</td>
</tr>
<tr>
<td>Chern et al. (1984)</td>
<td>$K_0 = 1.72 U_g^{-0.44} U_p^{0.44}$</td>
<td>$0 &lt; K_0 &lt; 1$</td>
</tr>
<tr>
<td>Darton (1985)</td>
<td>$K_0 = 0.00002D_e^{-1.11}$</td>
<td>$0 &lt; K_0 &lt; 2$</td>
</tr>
</tbody>
</table>

$\varepsilon_0$ is the void fraction of a liquid-solid fluidized bed at the same liquid velocity.
Table 4. Correlations of Wake Size (cont.)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Correlation</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Particle-Containing Wake</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ostoffaard (1965)</td>
<td>( K_i = 0.14 \delta_g^{-1/2} (U_i - U_{i0}) )</td>
<td></td>
</tr>
<tr>
<td><strong>Assumed Wake Size</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>El Tamtamy and Epstein (1978)</td>
<td>( K = K_s \exp(-5.08 \varepsilon_g) )</td>
<td>( 0 &lt; K &lt; 4 )</td>
</tr>
<tr>
<td></td>
<td>( K_s = \frac{8 - \cos(3\theta/2) + 9\cos(\theta/2)}{8 + \cos(3\theta/2) - 9\cos(\theta/2)} )</td>
<td></td>
</tr>
<tr>
<td>Khang et al. (1983)</td>
<td>( K = (1 - \varepsilon_s) \left[ \exp(-1.2 \varepsilon_g) + 2.5 \exp(-32.8 \varepsilon_g) \right] )</td>
<td>( 0 &lt; K &lt; 4 )</td>
</tr>
</tbody>
</table>

\( K_s \) is the volume ratio based on a sphere completing wake for a spherical-cap bubble.

\( U_{i0} \) is the minimum fluidization velocity and \( U_i \) is the superficial liquid velocity.
compiles available correlations for the ratio of the wake volume to bubble volume ratio.

Since, as indicated by Eq. (13), K and X are coupled, making an assumption about one must bias the calculation of the other. Rigby and Capes (1970) found that the ratio of the wake to bubble volume for both wakes with solids holdup equal to that of the particulate region (K,) and solids-free (K) wakes show similar behavior:

(i) K, and K, increase with increasing d_{p}
(ii) K, and K, increase with increasing U_{l}
(iii) K, and K, increase with decreasing U_{g}

K, however, was always larger than K, and the values ranged from 0 to 0.3 at low liquid velocities to as high as 1.8 for 775 μm sand particles at higher liquid velocities. Table 4 provides the approximate ranges of K, values considered by the various correlations. The trends observed by Rigby and Capes (1970) are observable in the correlations presented in Table 4.

El-Tamtamy and Epstein (1978) adopted the approach of defining a wake size then calculating the relative wake solids holdup. They assumed that K can be evaluated by first assuming that a K, for a single bubble can be calculated based on the
assumption of a sphere-completing wake exists behind a spherical-cap bubble, and second, the single bubble value can be multiplied by a second factor to include effects of bubble-bubble interactions: see Table 4. With this method for evaluating $K$, the proposed the following relationship for $X$:

$$X = 1 - 0.877 \frac{U_t}{(U_g/t_g - U_l/t_l)}$$

for $\frac{U_t}{(U_g/t_g - U_l/t_l)} \leq 1.14$

$$X = 0$$

for $\frac{U_t}{(U_g/t_g - U_l/t_l)} > 1.14$

(15a)

Note that, as indicated in Table 4, the values of $K$ calculated by this approach are much larger than those calculated by assuming a particle-free wake.

The importance of the gas and liquid velocities to the wake properties, and ultimately the hydrodynamic behavior, is evident in the above equations. Including the gas holdup in these correlations introduces the bubble size into the equation, but makes the use of the correlation difficult for prediction purposes. Only the equation proposed by Darton (1985) directly relates $K_g$ to the bubble size. Particle properties only directly appear in the correlation of El-Tamtamy. Each of the correlations included in Table 4 lack a fundamental basis.
1.8 Scope of Study

The physical concepts introduced in the wake model can be further refined by fundamentally evaluating the wake parameters. The discussion of the wake characteristics in the previous sections clearly indicates the possibility of identifying the wake boundary based on an understanding of the fluid dynamics involved in the wake structure. Once the wake size is known, the solids holdup in the wake can also be determined.

In general, this study is an examination of the relationship between bubble characteristics and the hydrodynamic behavior in a gas-liquid-solid fluidized bed through an investigation of bubble wake phenomena. In particular, the study of the wake phenomena behind single bubbles in three-dimensional systems provides valuable insight to complement the knowledge gained about single bubbles in two-dimensional beds by Tsuchiya and Fan (1988) and Kitano and Fan (1988). A further look at restricted cases in free-bubbling fluidized beds is instrumental in the proper application of the wake concept to real systems.

Next, a simple model is presented for the wake behind a spherical-cap bubble based on the important physical phenomena identified in this chapter. The model allows the calculation of both the wake size \( K \) and relative solids holdup \( X \) based on pertinent physical properties of the bubble and the particles.
In completing this study, several measurement techniques have been applied to obtain the necessary quantitative and qualitative information. This chapter will describe first the techniques used in determining the overall hydrodynamic characteristics and the equipment involved. Subsequently, the focus will be placed upon measurement of both bubble and bubble wake properties.

2.1 Overall Hydrodynamic Properties

These experiments were conducted in a large laboratory scale gas-liquid-solid fluidized bed; the desired properties to be measured were the overall gas and solid holdups.

2.1.1 Apparatus

A schematic diagram of the experimental annular fluidized bed is shown in Figure 4. The apparatus consists of a 15.2 cm ID cast acrylic outside column 122 cm in height. Nine pressure taps are located at 15.2 cm intervals along the outside column with 2 additional taps located near the distributor at 5.1 cm
Figure 4. Schematic diagram of the 6 in. diameter annular fluidized bed.
intervals. Centered within the outer column is an interchangeable sealed inside column. This study uses inside column diameters of 5.1 cm, 8.3 cm, and 11.4 cm. A wire mesh retaining grid placed on top of the inside column prevented the loss of solid particles into the liquid reservoir. Water manometers allow the monitoring of the local pressure within the bed and the transparent acrylic column permits visual observation of the bed height.

The distributor consists of three sections: the plenum chamber, the gas-liquid distributor, and a fixed bed mixer. The plenum chamber ensures even liquid flow through the gas-liquid distributor. The gas-liquid distributor allows liquid to pass from the plenum chamber to the fixed bed mixer through 6 mm ID acrylic tubes ending in 1.6 mm injection holes. Gas enters the distributor through two ports located on the side walls and exits through 340 injection holes, 1 mm in size, into the mixer region. The fixed bed mixer region improves the gas distribution.

2.1.2 Gas and Solids Holdup Measurements

The solid holdup was determined by measuring the weight of particles within the bed \( W_s \) and by visually identifying and measuring the bed height \( H \). With these two quantities and a known particle density \( \rho_s \) the solid holdup can be calculated with

\[
\varepsilon_s = \frac{W_s/\rho_s}{AH}
\]  

(16)
The following relationships exist between the phase holdups hold (e.g. Muroyama and Fan, 1984):

\[
\frac{\Delta P}{\Delta Z} = (\varepsilon_g \rho_g + \varepsilon_l \rho_l + \varepsilon_s \rho_s)g
\]

(17)

\[
1 = \varepsilon_g + \varepsilon_l + \varepsilon_s
\]

(18)

By determining the pressure profile in the bed, along with knowing the solids holdup, both the gas and liquid holdup can be determined from these two relationships.

The pressure profile is measured using a series of liquid manometers. Two measurements are important when considering the pressure profile: the distance between two successive pressure taps ($\Delta H$) and the difference in liquid height in the manometers corresponding to these pressure taps ($\Delta h$). With these measurements the gas holdup can be expressed in terms of the solids holdup and the physical properties of the components by

\[
\varepsilon_g = \left(\frac{\rho_l}{\rho_g - \rho_l}\right) \varepsilon_s \left(\frac{\rho_s - \rho_l}{\rho_l}\right) - \left(\frac{\Delta h}{\Delta H}\right)
\]

(19)

For no gas holdup, Eq. (16) can be rearranged to give

\[
\varepsilon_s \left(\frac{\rho_s - \rho_l}{\rho_l}\right) = \left(\frac{\Delta h}{\Delta H_0}\right)
\]

(20)

This form of the equation allows the calculation of the gas holdup strictly in terms of manometer readings. Note the subscript 0 indicates a liquid-solid system.
2.2 Bubble Size Measurement

In current practice, bubble sizes are measured by either of two methods: by photographic or probe techniques. Probe techniques are particularly suitable for three-dimensional beds. An impedance or conductivity double probe can be used to infer the bubble size and bubble velocity in three-phase fluidized beds (Rigby et al., 1970, Dorton and Harrison, 1974a, and Fan et al., 1985). Optical fiber techniques for measuring bubble properties in three-phase fluidized beds have been developed that detect either reflected (e.g., Miller and Mitchie, 1970; Calderbank and Pereira, 1977; Abauf et al., 1978; Delhaye, 1981; Ishida and Tanaka, 1982; de Lasa et al., 1984; Hu et al., 1985; Kitano and Fan, 1986; and Frijlink, 1987) or transmitted light (Kitano and Fan, 1986) to measure local bubble properties. Ishida and Tanaka (1982) used a single quartz fiber to detect characteristic signals of reflected light for both bubbles and particles in a three-phase fluidized bed. Hu et al. (1985) used a flat-topped single fiber probe to measure the local behavior of gas bubbles and solids in three-phase fluidized beds. Frijlink (1987) considered several fiber tip shapes and concluded that a rounded tip yielded the highest reflected light intensity. de Lasa et al. (1984), Lee et al. (1984), and Lee and de Lassa (1986) used a silica optical fiber probe to detect the movement of bubbles and measure the vertical length and rising velocity of bubbles. Kitano and Fan
(1986) developed a light transmittance probe to investigate local solids holdup behavior.

2.2.1 Transmittance Probe

An intrusive light transmittance probe assembly located within the three-phase fluidized bed allowed the measurement of both the bubble size and rise velocity. The assembly consisted of two probes separated by a fixed vertical distance. The individual probes had two optical fibers supported by stainless steel tubing and separated by a 1 cm measurement zone: one fiber brings light from a light source, while the other transmits the light to a photomultiplier tube. The intensity of the light sent to the photomultiplier tube depends on both the concentration of the solids within the measurement zone and the presence or lack of presence of a bubble.

To convert the light intensity to a voltage output, a photomultiplier was connected to a voltage divider circuit driven by a variable high voltage (DC) source. The analog voltage output of +10 to -10 V from the photomultiplier was converted by a Micro-Byte Dash-16 A/D board to an integer value between 2048 and -2047 at a fixed sampling rate. A/D sampling rates of 1000 and 200 per second provided high or medium resolution, respectively. The data were then stored in an IBM PC-XT.
Figure 5 gives a schematic diagram of the actual probe. The optical fibers are 1.0 mm in diameter and a distance of 10 cm separates the individual probes. The stainless steel support tubes are 1.0 mm ID and the copper mounting ring is made from a 1/4 in pipe. Appendix A provides a circuit diagram for the photomultiplier - voltage divider circuit.

2.2.2 Procedure and Apparatus

All experiments of this type were conducted in 4 in Plexiglas column systems. The liquid phase was always water. Figure 6 is a schematic diagram of the three-dimensional fluidized bed system. Most experiments involved a single nitrogen bubble injection through a stainless steel injection tube with 1/4 inch tubing. To provide best results, the stainless steel tube was beveled at the injection end. In some experiments, the bubble volume was measured through a glass funnel and graduated cylinder system; the single bubble was trapped in the glass funnel, transported to the graduated cylinder by flowing water, and trapped within an inverted graduated cylinder.

Bubble injection was controlled through a solenoid valve. The bubble volume, and hence bubble diameter, achieved was a function of the injection pressure from the nitrogen cylinder and the length of time the valve remained open. During automatic injection, the length of time between injections had to exceed the
Figure 5. Drawing of the dual optical fiber probe and the copper tubing support ring assembly. The measurement zone for each probe is 1 cm.
Figure 6. Schematic diagram of the 4 in. diameter fluidized bed.
time required for the bed surface to return to a settled state, otherwise flow disturbances within the bed caused the bubble rise path to vary significantly.

In these experiments, the bubble size was controlled to provide sufficient size differences to examine the effect of bubble size on the wake solids holdup, yet to be large enough to minimize both the deflection of the bubble by the probe and the variation in primary wake size. The large bubbles rise in a rectilinear path increasing the probability of striking the probe and have sufficient momentum to resist deflection by the liquid flow field around the probe; in addition, as indicated in Chapter 1, for bubble Reynolds numbers in the range from 3500 to 8000 the primary wake size only varies slightly.

Location of the probe assembly was such that the upper probe was immediately above the fluidized bed surface; consequently, the lower probe was always approximately 9.5 cm below the bed surface. The weight of solids within the bed was adjusted to allow the probe location to be a consistent distance above the liquid distributor plate.

2.2.3 Signal and Analysis

For a single large bubble, typical output signals for the upper and lower probes, in terms of voltage from the photomultiplier, are provided in Fig. 7. The top signal
Figure 7. Voltage output signals from the photomultiplier tube for both the in-bed and above-bed probes in a typical experiment.
represents the in-bed probe, while the bottom signal represents the above-bed probe. Signal analysis, at this stage, requires two steps: (i) identification of the bubble leading and trailing edges and (ii) determination of the bubble rise velocity.

The bubble signal can be identified through very sharp, large changes in the light intensity. Figure 8 gives enlarged views of the bubble signal for the high resolution (1000 points per second) and medium resolution (200 points per second) cases.

At high resolution, the leading edge of the bubble can be identified by a decrease in light transmittance due to reflection of light at the gas-liquid interface followed by a sudden extreme increase in light transmittance (see Fig. 8(a)). Qualitatively this behavior resembles a peak on the signal; the peak height depends in part on the mean solids holdup and also the location of the probe relative to the central axis. For medium resolution, the peak type behavior is not always evident and the leading edge is identified as the point immediately before the sudden increase in light intensity (see Fig. 8(b)).

During the interval the probe penetrates the bubble, the light transmittance stabilizes at a level near that of transmittance through air. The shape of the signal in this region depended on the probe design and alignment in addition to the sampling resolution.
Figure 8. Difference between signals obtained using a) high resolution and b) medium resolution.
Identification of the trailing edge of the bubble is similar to that of the leading edge. At first a sudden large decrease in light intensity marks the point the probe re-emerges from the bubble. In high resolution or if a low solids concentration exists behind the bubble, a second peak like behavior can occur (see Fig. 8(a)); this peak is also due to light reflectance at the gas-liquid interface. If a peak is not evident, the trailing edge of the bubble is taken at the point where a break in the slope of the signal occurs (see Fig. 8(b)). The peak height obtained at the trailing edge of the bubble also depends on the bubble shape and the relative location between the bubble and the probe. The beginning of the bubble wake, as shown in Fig. 8, is taken as the data point immediately following the trailing edge of the bubble.

As shown in Fig. 7, the bubble passes the in-bed probe at about 0.55 seconds. A time, $\tau_1$, elapses while the bubble is in contact with the in-bed probe and after a time, $\tau_2$, the bubble strikes the freeboard probe. The absolute bubble rise velocity can be calculated from the vertical distance between the two probes, $z_p$, and $\tau_2$ by

$$U_B = \frac{z_p}{\tau_2} \quad (21)$$

This approach assumes that both probes in the assembly strike that bubble at the same position; however, such conditions rarely occur thus introducing a small error into the calculation.

A bubble chord length can be calculated from $U_B$ and $\tau_1$ by
\[ I = \gamma_1 \times U_B \] (22)

Note, this chord length does not necessarily represent the bubble diameter or the maximum vertical chord length. Errors in the measurement of \( I \) arise from errors in the velocity measurement and also by any deflection or distortion of the bubble caused by contact with the probe.

2.3 Solids Holdup Measurement

The light transmittance probe can also provide information on the local solids holdup within the fluidized bed. The supposition on which the solids holdup measurements are based is that the amount of light "absorbed" in the measurement zone will depend on the solids holdup in that zone.

By assuming that the amount of light absorbed in a test section is proportional to the light intensity (I), the distance the light travels (x), and both an absorption coefficient and volume fraction for each phase

\[ \frac{dI}{dx} = - (\alpha_s \varepsilon_s + \alpha_f \varepsilon_f)I \] (23)

Integrating Eq. (18) and expressing \( \varepsilon_f \) in terms of \( \varepsilon_s \) yields

\[ I = C \exp \left\{ - (\alpha_s \varepsilon_s + \alpha_f (1 - \varepsilon_s))x \right\} \] (24)

Rearranging Eq. (19) gives

\[ I = C \exp (-\alpha_f x) \times \exp \left\{ -(\alpha_s - \alpha_f)\varepsilon_s x \right\} \] (25)
The first term in Eq (20) represents the intensity of light passing through the liquid medium alone and is defined as $I_0$. Equation (20) can be expressed in terms of the system opacity as

$$0 = \ln \frac{I}{I_0} = \exp \{\alpha_t \chi\}$$

(26)

where $\alpha$ expresses the system opacity ($\alpha_s - \alpha_t$). This equation is equivalent to the Lambert-Beer law with the solids volume fraction replacing the solute concentration.

In a liquid solid medium, unlike in a liquid-solute system, the "absorption" process involves light reflection at the particle surface, and light passing through both liquid and solid phases. At low solids concentrations, the solids may contribute most of the "adsorption" through reflection of the light, while at high solids concentrations, transmittance of the light through the solid phase may become important.

Kitano and Fan (1988) found that the Lambert-Beer law did not extend well to the full range of solids concentrations. Expressing the system behavior in terms of the opacity provides a convenient approach to compare intensity data stemming from different initial light intensities. The lack of any theoretical relationship between the light intensity and the solids holdup requires a calibration approach to make the conversion between light intensity and solids holdup.
2.3.1 Calibration

Calibration involved two basic steps: (i) measure the light intensity of light transmitted through the measurement zone for the case of water only and (ii) insert the probe in a fluidized bed of glass beads at a given liquid velocity; then measure the light transmittance through the measurement zone over a given time interval and calculate the average light intensity (in volts). Step (ii) was repeated many times to include the entire range of solids holdup required.

In a liquid-solid fluidized bed the local solids holdup fluctuates about the mean; these fluctuations depend on the nature of the bed hydrodynamics. The transmittance of light through a liquid-solid fluidized bed also fluctuates about a mean intensity; however, these fluctuations concern the probe and particle dimensions, in addition to the bed hydrodynamics. The basic presumption of the calibration process is that the mean intensity of light transmitted corresponds to the mean solids holdup.

Figure 9 shows the average opacity versus mean solids holdup for liquid-solid fluidized beds of 163 \(\mu\)m, 326 \(\mu\)m, 460 \(\mu\)m, and 760 \(\mu\)m diameter glass beads. As shown, the 163 \(\mu\)m glass beads show the greatest increase in opacity with increasing solids holdup; 326 \(\mu\)m glass beads show a lesser increase, as do the 460 \(\mu\)m and 760 \(\mu\)m glass beads. The behavior indicated by Fig. 9 does show similar behavior for each size particle: at low solids
Figure 9. System opacity as a function of the mean solids holdup and particle size. Note the decrease in probe sensitivity with increasing particle size.
holdups, the opacity increases nearly linearly with mean solids holdup; beyond a break point, the opacity still increases linearly, but with a much greater slope.

Conversion of light intensity data to solids holdup data was done by a Lagrangian interpolation of the calibration data for each individual data point involved. The interpolation procedure is outlined in Appendix B. The basic program used for the interpolation is also presented in Appendix B.

The conversion can also be done using a split interval expression: such as

\[ O = \frac{I_0}{I} = 1 + \alpha_1 \bar{\varepsilon}_s \quad \text{for} \quad \bar{\varepsilon}_s < \bar{\varepsilon}_{sc} \]  
\[ O = \frac{I_0}{I} = A_0 + \alpha_2 \bar{\varepsilon}_s \quad \text{for} \quad \bar{\varepsilon}_s > \bar{\varepsilon}_{sc} \]

where the slope changes values at a break point which, for the data presented in Fig. 9, falls at \( \bar{\varepsilon}_{sc} \approx d_p \) (in mm) for each size particle. The values of \( \alpha(d_p) \) for each size of glass beads are presented in Table 5.

Probe construction plays a significant role in the opacity behavior of a liquid-solid fluidized bed. Figure 10 shows calibration data obtained from two different probes. The probe with a large measurement zone shows a much greater increase in opacity than a similar probe with a smaller measurement zone. Both probes show the same break point behavior mentioned in association with Fig. 9; however, the break point occurs at a lower mean solid holdup for the probe with a larger measurement zone.
Table 5. Opacity calculation parameters

<table>
<thead>
<tr>
<th>Particles</th>
<th>$\alpha_{i}(d_p)$</th>
<th>$\alpha_{q}(d_p)$</th>
<th>$\bar{c}_{sc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>163 μm Glass</td>
<td>4.55</td>
<td>22.7¹</td>
<td>.17</td>
</tr>
<tr>
<td>326 μm Glass</td>
<td>1.88</td>
<td>15.7¹</td>
<td>.30</td>
</tr>
<tr>
<td>460 μm Glass</td>
<td>1.79</td>
<td>-</td>
<td>.42</td>
</tr>
<tr>
<td>760 μm Glass</td>
<td>0.94</td>
<td>-</td>
<td>.48</td>
</tr>
</tbody>
</table>

¹ - $A_0 = -2.39$
⁻¹ - $A_0 = -3.18$
Figure 10. Effect of probe construction on the transmittance behavior in a liquid-solid fluidized bed.
2.3.2 Typical Signals

In the experiments performed in this study, three types of signals are important: a typical example of each of these converted into solids holdup is given in Fig. 11. Fig. 11(a) shows calibration data for a mean solids holdup of 0.43, Fig 11(b) shows a single bubble experiment for a mean solids holdup of 0.43, and Fig. 11(c) shows a multi-bubble experiment for a mean solids holdup of 0.39. In each case, the presence or absence of large bubbles is clearly evident; note, however, that in multi-bubble systems the smaller bubbles present can not be distinguished.

2.3.3 Solids Holdup Distributions

The raw calibration data (intensity vs. time) can also be used to illustrate a second feature of this measurement technique. As shown in Fig. 11(a), the solids holdup in a liquid solid fluidized bed is not a constant single value; that is the calibration method produces a distribution of light intensities. Consequently, for a given set of conditions, a distribution of solids holdups can be constructed (see Fig. 42(b)).
Figure 11. Example solids holdup data taken in a) a multi-bubble experiment, b) a single bubble experiment, and c) a calibration measurement.
2.4 Method Reliability - Intrusive Behavior

Measurement of the bubble properties for numerous bubbles of a given volume provide both an indication of the reliability and an evaluation of the intrusive behavior of the probe. Isolated bubbles injected using a fixed pulse length to the solenoid valve and a constant nitrogen source pressure produce a consistent bubble volume. The volume was measured by trapping the bubble as it entered the exit section of the column and transporting it to an inverted graduated cylinder through Tygon tubing with a water flow.

For 28 single bubbles, which had an average volume of 5.8 cm³, the chord length distribution and the corresponding bubble rise velocities obtained at medium resolution are provided in Figs. 12 (a) and (b). By assuming spherical-cap geometry and a 50° included angle, the maximum bubble height should be 0.91 cm. Comparing this value to the maximum measured value of 1.17 cm, gives a 29% discrepancy; at high resolution, the accuracy is much better. The limited distribution indicates the probe only contacts the bubble near the center. This suggests the high inertial energy of this size bubble minimizes deflection due to the flow disturbance caused by the probe. The measured chord length distribution shows one limitation in the probe: when the probe strikes the bubble near the edge, the bubble peak can become lost in the local holdup fluctuations.
Figure 12. Demonstration of method reliability through experimentation at constant bubble injection conditions: a) bubble rise velocity distribution and b) chord length probability density.
In Fig. 11(b), the error bars indicate the standard deviation of the velocity data for a given chord length. In each case, the standard deviation is less than 1.5 cm/s or about 7% of the mean. Note that, within error, no trend exists between the measured chord length and the bubble rise velocity; this behavior indicates that the measured chord length is not necessarily the maximum chord length.

2.5 Spherical-Cap Bubbles

In all single bubble experiments, large bubbles of spherical-cap shape were involved. Figure 13 shows a diagram of a spherical-cap bubble in which the four important dimensions are labeled: R, the frontal radius of curvature; a, the length of the bubble base; b, the maximum bubble height; and θ, the included angle. The bubble volume can be expressed as

\[ V_b = \frac{\pi}{6} d_e^3 = \left[ \frac{R}{3} (1 - \cos \theta) (2 + \cos \theta) \right] \]  

In this equation, \( d_e \) is the volume equivalent diameter. Recognizing that \( a \) and \( b \) can be expressed in terms of \( R \) and \( \theta \) as

\[ b = R(1 - \cos \theta) \quad (30) \]
\[ a = 2R \sin \theta \quad (31) \]

and rearranging Eq. (26), the volume equivalent diameter becomes

\[ d_e = \left( \frac{3}{4} a + b \right)^{1/3} \]

To calculate \( d_e \), the included angle and \( a \), \( b \), or \( R \) needs to be known.
Figure 13. Diagram of a spherical-cap bubble, which defines the important geometrical parameters.
The position of the probe relative to the central axis can be estimated based on the spherical-cap geometry. The vertical chord length, \( l \), can be related to the bubble height by

\[
\frac{1}{b} = \frac{\cos \theta - \cos \phi}{1 - \cos \theta}
\]

and the radius relative to the central axis is

\[
\frac{r}{R} = \sin \theta
\]

If \( b \) and \( R \) are known, then \( r \) can be calculated from Eqns. (17) and (18).

If the wake is assumed to be a cylinder with a diameter equal to the length of the bubble base (2a) and a length defined as \( h \), the ratio of the bubble to wake volume can be expressed as

\[
\frac{V_w}{V_b} = \frac{6 \sin \theta}{3 \sin \theta + (1 - \cos \theta)} \left( \frac{h_w}{b} \right)
\]

To calculate \( V_w/V_b \), the included angle, \( h_w \), and one of \( a, b, \) or \( R \) needs to be known.

The included angle can be calculated using the following empirical correlation proposed by Clift et al. (1977): for \( E_0 \geq 40 \) and \( R_c > 1.2 \),

\[
\theta = 50 + 190 \exp(-0.62Re^{0.1})
\]

For Reynolds numbers greater than about 250 this reduces to 50°. \( R, a, \) or \( b \) were calculated either based on video recording of the bubble emerging from the bed surface or by trapping the bubble to determine its volume.
2.6 Materials - particle properties

Four sizes of glass beads were used extensively in this study: 163 μm, 326 μm, 460 μm, and 760 μm. Table 6 lists the density, terminal velocity, and the Richardson-Zaki index, n, for each size particle.

Table 6. Pertinent particle properties

<table>
<thead>
<tr>
<th>size</th>
<th>$\rho_p$ (g/cm$^3$)</th>
<th>$U_t$ (cm/sec)</th>
<th>n (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>163 μm Glass</td>
<td>2.52</td>
<td>1.7</td>
<td>4.05</td>
</tr>
<tr>
<td>326 μm Glass</td>
<td>2.52</td>
<td>4.4</td>
<td>3.46</td>
</tr>
<tr>
<td>460 μm Glass</td>
<td>2.50</td>
<td>6.8</td>
<td>3.21</td>
</tr>
<tr>
<td>760 μm Glass</td>
<td>2.50</td>
<td>11.8</td>
<td>2.93</td>
</tr>
</tbody>
</table>
The existence of the wake behind the bubble and its importance towards the hydrodynamic behavior in gas-liquid-solid fluidized bed has been demonstrated in Chapter 1. In this chapter, measurements of the local solids holdup will be used to study the wake behind a single bubble. From these measurements qualitative observations about the wake structure in general and quantitative information about the average solids holdup in the wake can be made.

3.1 Local Solids Holdup Profiles

A series of experiments were conducted in a liquid-solid fluidized bed: in these experiments, single bubbles were injected under the conditions given in Table 7.

The technique for measuring the local solids holdup, which was described in detail in the previous section, provides local solids holdup data prior to, during, and after the passing of the
Table 7. Conditions used in the single-bubble system experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$P_i$ (psig)</th>
<th>$U_i$ (cm/sec)</th>
<th>$\bar{u}_s$ (-)</th>
<th>$Re_b$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>163 µm Glass Beads</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>M23a</td>
<td>7</td>
<td>0.10 - 0.93</td>
<td>0.06 - 0.50</td>
<td>4000 - 5100</td>
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<tr>
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<td>20</td>
<td>0.10 - 0.93</td>
<td>0.06 - 0.50</td>
<td>5700 - 7000</td>
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<tr>
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<td>0.10 - 0.93</td>
<td>0.06 - 0.50</td>
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<tr>
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<tr>
<td>F22a</td>
<td>7</td>
<td>0.23 - 2.06</td>
<td>0.08 - 0.48</td>
<td>4000 - 5100</td>
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<tr>
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<td>0.25 - 0.51</td>
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<tr>
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<td>0.70 - 4.66</td>
<td>0.25 - 0.51</td>
<td>4700 - 5400</td>
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</tbody>
</table>
bubble. In essence, the measured local solids holdup profile extends both upstream and downstream of the bubble. Figure 14 gives a representative local solids holdup profile and identifies the regions and parameters important to the analysis of the data. The signal itself is subdivided into the pre-bubble (prior to passing), the bubble, and the post-bubble (after passing) regions.

The pre-bubble region, typically, shows high frequency fluctuations or noise characteristic of the probe-particle size dimensions. In addition, depending on flow conditions, global variations can be identified which depend on the hydrodynamic conditions within the bed: these global variations also should depend on particle size, but not on the probe dimensions.

The bubble itself can be identified by very sharp, large changes in the light intensity as discussed in Section 2.2.3. In the bubble region, the value of the solids holdup is meaningless. The bubble chord length, usually referred to relative to the bubble rise velocity, is measured from the apex of the initial peak to the apex of the final peak, as shown in Fig. 14. In cases in which the peaks are not evident, the chord length is measured from the point of rapid increase in light transmittance to the point after the stable transmittance through air region in which the rapid extreme changes in light transmittance end.

The post-bubble region begins at the bubble base, defined by the appropriate means described previously, to the end of the
Figure 14. A typical local solids concentration signal subdivided into the pre-bubble, the bubble, and the post bubble regions.
signal. This region consists of the wake (both primary and secondary), and, in some cases, zones of stable liquid-solid fluidization. Generally, the wake has solids holdup variations are larger than found prior to the bubble. The primary wake size, indicated in the figure by the length $h_w$, can not be identified based on the local solids holdup signal in the wake; consequently, $h_w$ was based on results from studies in two-dimensional beds (Tsuchiya and Fan, 1988a and 1988b and Kitano and Fan, 1988).

The local solids holdup variations in the wake can be analyzed both qualitatively and quantitatively to provide insight into the wake structure. Quantitative analysis of the wake solids holdup based on these signals requires an assumption about the boundary between the primary and secondary wake. In single bubble experiments, for the purposes of considering the relative wake solids holdup, the ratio between the pre-bubble average holdup and the average solids holdup in the primary wake is appropriate in calculating the local value of $X$.

3.2 General Wake Behavior - qualitative observations

Several features of the local solids holdup signals need to be examined with reference to the wake behavior. First, does the local solids holdup in the wake depend on the position relative to the bubble central axis; that is does the position of the probe relative to the bubble central axis reflect different behavior.
Second, can observed trends in the wake be repeatable despite the unsteady nature of the wake process and the uncertainty in position relative to the central axis.

Three data sets, each corresponding to a single bubble injected under identical conditions, are presented in Fig. 15: in this case, the bubble Reynolds number was 7200 ($P_i = 30$ psig). The dotted line represents the mean solids holdup of 0.4 and the measured chord length is indicated in the bottom right corner of each figure.

As shown, the measured chord length is not necessarily the same for each repetition: the bubble signal in Fig. 15(a) has the largest measured chord length, Fig. 15(c) the smallest, and Fig. 15(b) falls in between. Since the injection volume should be the same in each case, it follows that the bubble does not necessarily strike the probe at the same location for each repetition and the probe measurements provide data on the local solids holdup within the wake at different locations. For bubbles of a given volume, a larger observed chord length probably indicates the closer to the central axis the probe passed through the bubble. If the maximum chord length of the bubble is known, and by assuming a spherical cap geometry, an approximate radial position relative to the central axis can be determined (see Section 2.5).

Note the similarity between the wake solids concentration profiles in Figs. 15(a) and 15(b): such a similarity suggests a
326 μm Glass Beads

Figure 15. Solids holdup measurements behind single bubbles created under identical conditions (note the sampling rate for this data is 1000 per second).
near uniform solids concentration region near the central axis. But, in Fig. 15(c) the concentration varies significantly over a short distance through the wake and from that found in Figs. 15(a) and 15(b). The behavior pictured in Fig. 15 supports that found by Kitano and Fan (1988) in two dimensional beds in that (i) a stable solids region existed in the wake near the bubble central axis and (ii) closer to the bubble edge a vortex region existed: in the stable solids region the solids concentration variations are relatively small, while in the vortex sheet region the solids concentration variations could be relatively large.

Also, in Fig. 15 several additional important points are evident: the solids holdup variations found prior to the passing of the bubble are small compared to those found in the post-bubble region. The disturbance caused by the passing bubble continues for a significant time afterwards: in each case, at the end of the signal presented, the bubble is over 9 cm, or about 3 1/2 times \( D_e \), above the probe level and the fluidized bed still shows evidence of the disturbance. And, local solids concentration in the wake can exceed the mean solids holdup (this corresponds to \( X \) greater than 1): in the cases shown here this occurred well downstream of the bubble.

1. In this context, as indicated previously, the term variations refers to an overall sense and not the high frequency fluctuations characteristic of the probe-particle dimensions.
Large scale solids concentration disturbances like that shown in Fig. 15(c) may indicate the presence of high vorticity, since the solids concentration should decrease radially towards the vortex center. In some signals, the presence of downstream shed vortices are indicated by several such disturbances. The location of these disturbances can vary from signal to signal due to the unsteady nature of the vortex shedding process and differences in probe location relative to the bubble central axis.

In Section 2.4, the light transmittance probe reliability was examined through bubble velocity and chord length measurements. The reliability towards wake solids measurements can, in part, be examined through testing for repeating trends. The unsteady nature of the wake and the low probability of the probe striking the same position relative to the bubble prevents the complete repeatability of the local solids holdup profile, yet the trends observed in the local solids holdup can be found to recur. Figure 16 shows that two signals taken under identical conditions can indeed show consistent trends ($P_1 = 30$ psig and $U_1 = 0.54$ cm/sec): in both cases the solids concentration in the wake shows a narrow, low solids concentration region immediately below the bubble, followed by a narrow high concentration region, and finally, by approximately sinusoidal variations in solids concentration. Also note that the chord lengths for each signal in Fig. 16 are nearly identical; for bubbles of spherical-cap
Figure 16. Existence of repeatable trends in the local solids holdup behavior behind a single bubble rising in a liquid-solid fluidized bed. (Time scale: 0.01 seconds)
geometry, this indicates a very nearly equal position relative to the bubble central axis.

In each signal in Fig. 16., the spike indicating the bubble base is followed immediately by a low solids concentration region. Such a signal suggests a liquid wake, in which a low or zero solids concentration exists. The length of the liquid wake region can be estimated by determining the time lapsed between the bubble base peak and the occurrence of the high concentration point immediately following the liquid wake: in these cases, \( h_{lw} = 0.2 \) cm and, by assuming a cylindrical shaped liquid wake, this corresponds to a \( K_s \) of about 0.4. Note, however, such a liquid wake or stable liquid region does not appear in all data sets for given conditions.

The most important factor in the development of the wake structure is the relative velocity between the bubble and the surrounding medium. Consequently, the important system parameters to investigate are the bed voidage (liquid velocity), bubble diameter, and particle size. The bed voidage concerns the relative velocity between the particles and liquid, the bubble diameter concerns the relative velocity between the bubble and the liquid medium, and the particle size is concerned in both.

Each of these properties affect the wake behavior and will be examined separately, but ultimately they are interdependent.
Bed Voidage Effects

Bed voidage, or the mean solids holdup, plays an important role in the observed wake behavior. Three ranges of solids holdups are important in the examination of the voidage effect: high solids holdups ($\bar{e}_s < 0.4$), intermediate solids holdup ($0.2 < \bar{e}_s < 0.4$), and low solids concentration ($\bar{e}_s < 0.2$). Figures 17, 18, and 19 show example local solids holdup signals of both 163 μm and 326 μm glass beads exhibiting the differences in behavior at different mean solids holdups.

At high mean solids holdup, the local solids holdup shows only a very small difference in behavior when comparing data prior to the passing of the bubble to that following as shown in Fig. 17(a). The sole effect seems to be a very slight decrease in solids holdup.

At intermediate solids holdup, however, the effects of the wake become clearly evident: as shown in Figs. 17(b), 17(c), 18(a), 18(b), and 18(c). In the post-bubble region, the local solids holdup variations increase with increasing bed voidage, while in the pre-bubble region, the variations change only slightly (see Section 2.3.3). The overall pattern of the local variations resembles a sinusoid with a long wavelength. The variations become more severe and the extent of the disturbance increases with increasing bed voidage; as shown in Fig. 19, the wake itself is evident, but a demarcation between the primary and
Figure 17. Effect of bed voidage on the local solids holdup in the bubble wake (high solids holdup in a 163 \( \mu \)m glass bead system).
Figure 18. Effect of bed voidage on the local solids holdup in the bubble wake (intermediate solids holdup in a 326 μm glass bead system).
Figure 19. Effect of bed voidage on the local solids holdup in the bubble wake (low solids holdup in a 326 μm glass bead system).
secondary wake is not. Note that at times sufficiently long after the bubble passes, the solids holdup behavior returns to that found prior to its passing.

At low mean solids holdup, the variations in local solids holdup begin to decrease in intensity with further decreases in the mean solids holdup. In addition, the tendency for low concentration regions to exist immediately following the bubble base, like those shown in Figs 18(a) and 18(c), increases with decreasing mean solids holdup.

**Bubble Size Effects**

In this study, the equivalent bubble diameter was varied from about 1.7 to 2.8 cm; this corresponds to a bubble Reynolds number between 4500 and 8000. Over this range, the effect of bubble size remains unclear; however, several general observations made by previous researchers can be supported by the observed local solids holdup behavior. Figure 20 shows typical signals for 2.6, 2.3 and 1.9 cm equivalent diameter bubble in a 326 µm glass bead system. In this case, the smaller the bubble size the larger the low solids concentration region immediately behind the bubble; the size of the low concentration regions can be estimated as

2. The liquid layer size is estimated based on the distance from the peak representing the bubble base to the point after the liquid layer where the rapid changes in light intensity diminish.
Figure 20. Effect of bubble size on the local solids holdup in the wake of a bubble rising in a bed of 326 µm glass beads.
0.16, 0.21, and 0.35 for the 2.6, 2.3 and 1.9 cm equivalent diameter bubbles, respectively. An increased in liquid wake size for a decrease in bubble size has been observed or suggested in other studies (e.g.; Rigby and Capes, 1970). Note that the chord length measured for the 2.3 cm equivalent diameter bubble exceeded that measured for the 2.6 cm bubble; this discrepancy reflects a difference in location relative to the bubble central axis.

The behavior demonstrated in Fig. 20 provides a clear example of the differences in behavior found for the different bubble sizes. But, such a clear difference was the exception and not the rule and no general trends in the behavior could be discerned.

Particle Size Effects

Particle size greatly affects the local solids holdup, especially in the in-bed probe. As shown in Fig. 21, the variations in local solids concentration, both in the pre- and post-bubble regions, become larger for increasing particle size. The variations in the pre-bubble region, reflect a greater degree of turbulence in the bed. As a consequence, the in-bed bubble signal is less clear when using the 460 μm and 760 μm glass beads; essentially, the chance of a bubble striking the probe towards its central axis is reduced, thus reducing the effectiveness of the probe. Above-bed signals still provide good information, however.
Figure 21. Effect of particle size on the in-bed local solids concentration signal (note the sampling rate for this data is 200 per second).
Figure 22 gives typical above-bed signals for beds of 163, 326, 460 and 760 μm glass beads.

The intensity of the variations in the wake region and the tendency to find a low solids concentration region increase with increasing particle size. Usually, for a given mean solids holdup, the wake solids holdup is lower in larger particle systems. The change in wake behavior with bed voidage is consistent for all particle sizes considered.

Summary

Several important points are clear from the evidence presented in this section. First, structural phenomena such as vortices and liquid wakes (low solids concentration regions) can be seen in the local solids concentration signals. No clear demarcation between the primary and secondary wakes exists in terms of the solids concentration profile within the wake. The loss of measurement ability inside the bed in large particle systems (460 and 760 μm) results from the increase turbulence in these systems. And, in terms of the wake solids concentration, the particle size, the bubble size and rise velocity, and the bed porosity all are important.
Figure 22. Effect of particle size on the above-bed local solids concentration signal (note the sampling rate for this data is 200 per second).
3.3 Average Wake Solids Holdup

The analysis of the wake solids holdup includes two parts: (i) defining a wake size and (ii) calculating the average solids holdup within the defined wake. Two problems are inherent in this averaging process. First, the wake boundary can not be identified based on the local solids holdup signals obtained in the experimentation. And second, the location of the probe relative to the central axis is not known; thus, considering the radial position in averaging the wake solids holdup is not possible. These difficulties lead to adopting two assumptions about the wake behavior which are based on observed behavior in two-dimensional beds. The assumptions used, however, can be supported, in part, by the observed local solids holdup behavior.

3.3.1 Wake Size - observed behavior

An appropriate definition of the wake in terms of averaging the wake solids holdup, based on Kitano and Fan's (1988) view of the wake, would include only the stable liquid and stable solids region. Over the entire rise of the bubble, these two regions would travel at the bubble rise velocity. Within short distances from the base, large variations in the solids holdup can be expected because of inclusion of both a small low concentration area (stable liquid wake) and the beginning of a higher concentration region (stable solids wake). With increasing
distance the liquid layer becomes less important to the average and approaching a stable value can be expected. But at distances great enough to encounter the fluctuating solids region, the vortex sheet region, and the shed vortices in the secondary wake, large fluctuations in the solids concentration can be expected again. And finally, sufficiently far removed from the bubble base, the wake solids holdup should begin to approach 1. In addition, the probe location within the wake can be expected to introduce different variations because of differences in the importance of the four regions at the given location of the probe relative to the central axis.

Measurement of the solids holdup distribution in the bubble wake can not, by itself, be used to determine the wake size. Consequently, the wake size (K) is assumed to be 2.8 for the calculation of the wake solids concentration (ɛ_s,w). This value, which was based on the findings of Kitano and Fan (1987), was chosen because it falls between the value 2.5 observed by Miyahara et al. (1988) and 4.7 observed by Kojima et al. (1975).

Based on an assumption of K equal to 2.5, and assuming a cylinder completing wake, the ratio of the wake depth to the bubble height becomes

\[
\frac{h_w}{h_b} = 1.498 \quad (37)
\]

However, to avoid over estimating the stable wake size and to facilitate calculations, the measured chord length is substituted
for the bubble height, thus approximating a spherical-cap wake shape. This approach gives the limits on the assumed wake volume as

\[ 1.49 < \frac{V_w}{V_b} < 2.8 \]  

(38)

This actual wake to bubble volume ratio should be near the upper limit since measurements near the bubble edge are obscured due to normal solids fluctuations and as indicated in Section 2.4.

In the constant bubble volume experiment first discussed with reference to repeatability in bubble rise velocity measurements and the bias in chord length measurement, the local solids holdup profiles were averaged over a series of depths behind the bubble base. A relative wake depth, defined as the ratio of the wake depth to the measured chord length, was used to compare the various averaged data; note that the wake depth was calculated assuming the wake travel with the velocity of the bubble.

Figures 23(a) and 23(b) show the mean and the standard deviation of the average local solids holdup profile as a function of the depth into the wake. Several points are important from this figure. First, the scatter becomes smaller as the distance from the bubble base increases; the data scatter is largest near the bubble base. And second, the average values increase towards 1 as the relative wake depth increases; the increase is sharpest near the base. Based on the data given in the figure, the mean of
Figure 23. Average wake solids holdup as a function of the relative wake depth: for this study, $K$ was assumed to be 2.8; this value corresponds to an $h_w/\xi$ of about 1.5.
the local average wake solids holdup shows a minimum scatter at a relative wake depth of about 1.5 and the standard deviation shows a consistent range for a relative wake depth between about 1 and 3. More important, however, is the relative insensitivity of the mean solids holdup over the range of wake sizes in question; this insensitivity indicates that only a small error is introduced by choosing $h_w/l$ equal to 1.5 for the calculations.

3.3.2 Wake Solids Holdup - observed behavior

The local wake solids holdups, derived by averaging the local solids holdup profile from the point immediately following the bubble base to the point where Eqn. 37 is satisfied, for the conditions of this study are presented in Figs. 24 through 33. In Figs. 24 through 29, representing 162 and 326 μm glass bead systems, the data derived from the in-bed probe are compared with those derived from the above-bed probe, while Figs. 30 through 33 only include data from the above-bed probe. Generally, the above-bed probe finds a slightly higher solids holdup than the in-bed probe, yet the agreement is good.

The data presented in Figs. 24 through 33 indicate that both bed voidage and particle size have a clear impact on the wake solids holdup, but the bubble size effect is not obvious. Due to the unsteady nature of the wake formation and shedding process, in addition to the non spatially uniform solids holdup profile, the
Figure 24. The local wake solids holdup behind a single bubble rising in a fluidized bed of 163 μm glass beads ($P_i = 7$ psig).
Figure 25. The local wake solids holdup behind a single bubble rising in a fluidized bed of 163 μm glass beads ($P_i = 20$ psig).
Figure 26. The local wake solids holdup behind a single bubble rising in a fluidized bed of 163 μm glass beads ($P_i = 30$ psig).
Figure 27. The local wake solids holdup behind a single bubble rising in a fluidized bed of 326 μm glass beads (P_i = 7 psig).
Figure 28. The local wake solids holdup behind a single bubble rising in a fluidized bed of 326 μm glass beads ($P_i = 20$ psig).
Figure 29. The local wake solids holdup behind a single bubble rising in a fluidized bed of 326 μm glass beads ($P_i = 30$ psig).
Figure 30. The local wake solids holdup behind a single bubble rising in a fluidized bed of 460 μm glass beads ($P_i = 20$ psig).
Figure 31. The local wake solids holdup behind a single bubble rising in a fluidized bed of 460 \( \mu \text{m} \) glass beads (\( P_i = 30 \text{ psig} \)).
Figure 32. The local wake solids holdup behind a single bubble rising in a fluidized bed of 760 μm glass beads ($P_i = 20$ psig).
Figure 33. The local wake solids holdup behind a single bubble rising in a fluidized bed of 760 μm glass beads ($P_i = 30$ psig).
local wake solids holdup shows considerable scatter; consequently, all data points obtained under given conditions must also be averaged to provide a better estimate of $X$. The data, reduced into terms of the average wake solids holdup and the relative wake solids holdup, will be discussed in terms of the bed voidage, bubble size, and particle size effects. Figures 34 through 37 show the wake solids holdup as a function of mean solids holdup for 163, 326, 460, and 760 μm glass bead systems, respectively. The error bars in the figures indicate the standard deviation of the data.

**Effect of Bed Voidage**

Figures 34 through 37 clearly indicate that the wake solids holdup decreases with decreasing mean solids holdup. In addition, the severity of the decrease depends on the particle size. The 760 μm glass beads show an anomalous trend in that initially the wake solids holdup increases with bed voidage, then falls off rapidly with further decreases in mean solids holdup.

**Effect of Bubble Size**

Both bubble size and particle velocity affect the bubble rise velocity, and hence the wake behavior. The relative solids holdup ($X$) is shown in Fig. 34 for 163 μm glass beads for three different bubble sizes: the error bars indicate the standard
Figure 34. The relative solids holdup as a function of the mean bed voidage in a bed of 163 μm glass beads (7 psig, 1.6 < $D_e$ < 1.8; 20 psig, 1.9 < $D_e$ < 2.2; and 30 psig, 2.2 < $D_e$ < 2.4).
Figure 35. The relative solids holdup as a function of the mean bed voidage in a bed of 326 \mu m glass beads (7 psig, 1.7 < D_e < 2.0; 20 psig, 2.1 < D_e < 2.5; and 30 psig, 2.6 < D_e < 2.7).
Figure 36. The relative solids holdup as a function of the mean bed voidage in a bed of 460 μm glass beads (20 psig, 2.0 < \(d_e\) < 2.4 and 30 psig, 2.3 < \(d_e\) < 2.7).
Figure 37. The relative solids holdup as a function of the mean bed voidage in a bed of 760 μm glass beads (20 psig, 2.2 < \( d_e < 2.4 \) and 30 psig, 2.2 < \( d_e < 2.6 \)).
deviation of the data. At a low bed voidage, X falls near 1 and the data shows little scatter. For increasing bed voidages, X decreases and the variation in the data increases. As can be seen in the figure, bubble size (bubble injection pressure) only slightly affects the wake solids holdup. Bubble Reynolds numbers range from 4000 - 5100 for a 7 psig injection pressure, 5700 to 7000 for a 20 psig injection pressure, and 6400 to 7900 for a 30 psig injection pressure: note that the variation in \( \text{Re}_b \) is not directly related to the variation in bed voidage and \( \text{Re}_b \) has no significant effect on X.

Effect of Particle Size

Particle size has a much greater impact on X than the bubble size over the conditions studied. By comparing the data presented in Figs. 34 through 37, X is seen to decrease as the particle size decreases from 163 \( \mu \text{m} \) to 460 \( \mu \text{m} \) glass beads. However, the 760 \( \mu \text{m} \) glass beads show unique behavior in that at low bed voidages X is very near 1 (in fact X exceeds 1 in some cases); however, at medium bed voidages X begins to fall rapidly with increasing bed voidage. Bubble Reynolds numbers range from 6500 - 7900 for 163 \( \mu \text{m} \) particles, 6700 to 7300 for 326 \( \mu \text{m} \) particles, 5100 to 5900 for 460 \( \mu \text{m} \) particles, and 5000 to 5600 for 760 \( \mu \text{m} \) particles and, once again, no significant direct relationship between the Reynolds number and X is evident: also
note that the larger fluctuations in solids holdup behavior found in large particle systems are indicated through the error bars.

Summary
The measurements and results of this study pertain to spherical cap bubbles with a volume equivalent diameter between 1.5 and 3.0 cm and having bubble Reynolds numbers between 4000 and 7300. The combined data for the four systems considered in this study can be summarized in terms of the bubble size, particle properties, and liquid velocity by the following relationships:

\[ \varepsilon_{sw} = 0.72 \left( \frac{\text{Re}_b}{\text{Re}_t} \right)^{1/8} \left( \bar{v}_s \right)^{5/4} \]  \hspace{1cm} (39)

or

\[ X = 0.72 \left( \frac{\text{Re}_b}{\text{Re}_t} \right)^{1/8} \left( \bar{v}_s \right)^{1/4} \]  \hspace{1cm} (40)

where \( \text{Re}_t \) is the particle Reynolds number at terminal velocity. Both terms of the equation reflect the particle properties, while the bubble Reynolds number term includes bubble properties: note the first term represents the ratio of the bubble momentum to that of the particle momentum.

Figures 38 and 39 show plots of \( \varepsilon_{sw} \) and \( X \) versus \( \left( \frac{\text{Re}_a}{\text{Re}_t} \right)^{1/8} \left( \bar{v}_s \right)^{5/4} \) and \( \left( \frac{\text{Re}_a}{\text{Re}_t} \right)^{1/8} \left( \bar{v}_s \right)^{1/4} \), respectively. The wake solids holdup can be predicted very well by Eqn. (39); however, the data for 760 \( \mu \)m glass beads at a high bed voidage
Figure 38. Correlation of the wake solids holdup ($t_{sw}$): includes data for 163 μm, 326 μm, 460 μm, and 760 μm glass beads.
Figure 39. Correlation of the relative wake solids holdup \( X \): includes data for 163 \( \mu \text{m} \), 326 \( \mu \text{m} \), 460 \( \mu \text{m} \), and 760 \( \mu \text{m} \) glass beads.
shows the greatest deviation. In terms of $X$, the deviation between the predicted and experimental values is larger, but the error in the data is also magnified due to division by $\tilde{e}_s$, a quantity much less than 1.

The relationships in Eqns (39) and (40) could also be expressed directly in terms of the particle properties and liquid velocity as

$$\tilde{e}_{sw} = 0.72 \left( \frac{Re}{Re_t} \right)^{1/8} \left[ 1 - \left( \frac{U}{U_t} \right)^{1/n} \right]^{5/3}$$  \hspace{1cm} (41)

or

$$X = 0.72 \left( \frac{Re}{Re_t} \right)^{1/8} \left[ 1 - \left( \frac{U}{U_t} \right)^{1/n} \right]^{1/3}$$  \hspace{1cm} (42)

In these equations, $n$ is the exponent for the Richardson and Zaki equation.

Under most conditions in this study, the wake solids holdup was found to be lower than the corresponding results found in a two-dimensional system by Kitano and Fan (1987). This discrepancy probably results from the predominance of wall effects in a two-dimensional systems: in particular, the tendency for particles to concentrate and flow down along the wall.

3.4 Conclusions

The local solids holdup signal produced from the light transmittance probe technique proved effective for the measurement of the solids holdup in the wake. Repeatable trends could be
observed in the data, the method produced the most reliable signals near the wake central axis, and data taken immediately above the bed surface agreed quite well with that taken within the bed. The variation in the local solids holdup data increased with increasing particle size. The in-bed data for the 460 and 760 μm particles showed too much variations to accurately identify the bubble; however, the above-bed signal provided good data for these systems.

The local solids holdup data for the wakes behind individual bubbles were averaged to provide a more representative value of the wake solids holdup. Two assumptions were required to calculate the average solids holdup in the wake:

(i) the average solids concentration near the central axis of the wake can be considered a good estimate of the average concentration of the entire wake; and

(ii) a wake depth to bubble height ratio of 1.5 defines an appropriate wake size for the calculation.

Based on these assumptions, wake solids holdup data was obtained under numerous conditions.

Several trends were recognizable in the relative wake solids holdup:

(i) $X$ decreased with increasing bed voidage $(1 - \bar{e}_g)$;

(ii) $X$ decreased with increasing particle terminal velocity;
(iii) over the range considered in these experiments, the effect of bubble size was unclear; and

A correlation of the data was provided to summarize these trends; note, however, that in the correlation the bubble Reynolds number is included, but, over the range considered within these experiments, its value is not significant to the prediction.
CHAPTER IV

SOLIDS HOLDUP IN THE BUBBLE WAKE:

Multi-Bubble Systems

The intent of Chapter 3 was to investigate the solid holdup behavior behind a single bubble rising in a liquid solid fluidized bed. This situation represents a controlled ideal situation. To complete the experimental investigation of the solids holdup behavior in the wake, however, requires an examination of the behavior in a freely bubbling bed: in particular, this examination must focus on determining any differences in behavior and in identifying important phenomena.

In this chapter, measurements of the local solids holdup will be used to study bubble wakes in a freely bubbling bed. From these measurements, important qualitative and quantitative observations can be made.

4.1 General Behavior

In multi-bubble systems, unlike in the ideal case of a single bubble rising in a liquid solid fluidized bed, significant
flow non-uniformities can exist within the bed. Consequently, the mean solids holdup within the bed can not be used to calculate the local relative solids holdup and thus, the particulate phase solids holdup must also be evaluated.

A series of experiments were conducted in a freely bubbling bed to characterize the wake solids holdup behavior under limited conditions. The conditions, as presented in Table 8, are chosen to provide insight into the possible extension of the single bubble data provided in Chapter 3 to actual situations: the Augustus and Octavius series experiments only consider a single location in the bed near its surface, while the Profile series experiments consider 10 locations, both in-bed and above-bed.

In the next two sections a general discussion of the data is presented to introduce the additional measured quantities important to the behavior in a freely bubbling bed.

4.1.1 Local Solids Holdup Profiles

Unlike in single bubble systems, the local solids holdup signal in a multi-bubble system can include several bubbles and, in general, the bubbles are smaller than those studied in the single bubble experiments. Figure 40(a) shows a representative

1. As introduced in Section 1.4, the fluidized bed is considered to consist of the bubble, wake, and particulate phases.
Table 8. Conditions used in the multi-bubble system experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$h_p$ (cm)</th>
<th>$U_i$ (cm/sec)</th>
<th>$U_g$ (cm/sec)</th>
<th>$h_o$ (cm)</th>
<th>$f_s^*$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augustus 44</td>
<td>40.5</td>
<td>0.24</td>
<td>0.73</td>
<td>42.0</td>
<td>0.47</td>
</tr>
<tr>
<td>Augustus 46</td>
<td>40.5</td>
<td>0.40</td>
<td>0.73</td>
<td>42.0</td>
<td>0.44</td>
</tr>
<tr>
<td>Augustus 50</td>
<td>40.5</td>
<td>0.56</td>
<td>0.73</td>
<td>42.0</td>
<td>0.41</td>
</tr>
<tr>
<td>Augustus 54</td>
<td>40.5</td>
<td>0.72</td>
<td>0.73</td>
<td>42.0</td>
<td>0.39</td>
</tr>
<tr>
<td>Octavius 44</td>
<td>40.5</td>
<td>0.24</td>
<td>0.95</td>
<td>42.0</td>
<td>0.46</td>
</tr>
<tr>
<td>Octavius 47</td>
<td>40.5</td>
<td>0.40</td>
<td>0.95</td>
<td>42.0</td>
<td>0.43</td>
</tr>
<tr>
<td>Octavius 50</td>
<td>40.5</td>
<td>0.56</td>
<td>0.95</td>
<td>42.0</td>
<td>0.41</td>
</tr>
<tr>
<td>Octavius 53</td>
<td>40.5</td>
<td>0.72</td>
<td>0.95</td>
<td>42.0</td>
<td>0.38</td>
</tr>
<tr>
<td>Profile 1</td>
<td>22.5 - 70.5</td>
<td>0.20</td>
<td>1.00</td>
<td>41.6</td>
<td>-</td>
</tr>
<tr>
<td>Profile 2</td>
<td>18.5 - 51.3</td>
<td>0.28</td>
<td>1.00</td>
<td>41.6</td>
<td>-</td>
</tr>
<tr>
<td>Profile 3</td>
<td>20.0 - 56.0</td>
<td>0.36</td>
<td>1.00</td>
<td>41.6</td>
<td>-</td>
</tr>
<tr>
<td>Profile 4</td>
<td>20.0 - 56.0</td>
<td>0.44</td>
<td>1.00</td>
<td>41.6</td>
<td>-</td>
</tr>
<tr>
<td>Profile 5</td>
<td>16.0 - 48.0</td>
<td>0.20</td>
<td>1.50</td>
<td>41.6</td>
<td>-</td>
</tr>
<tr>
<td>Profile 6</td>
<td>16.0 - 48.0</td>
<td>0.20</td>
<td>2.00</td>
<td>41.6</td>
<td>-</td>
</tr>
<tr>
<td>Profile 7</td>
<td>16.0 - 48.0</td>
<td>0.20</td>
<td>2.50</td>
<td>41.6</td>
<td>-</td>
</tr>
</tbody>
</table>

* - based on visual identification of the bed surface
Figure 40. Example signals for multi-bubble experiments: a) local solids holdup signals and b) the solids holdup distribution.
local solids holdup profile for a multi-bubble system; this data was acquired by the methods presented in Sections 2.2 and 2.3. In this case, the bubble, as indicated, appears in the left-hand portion of the signal; the pre- and post-bubble regions are also indicated. The difference in the local solids holdup behavior in the pre- and post-bubble regions is not as pronounced in the multi-bubble system as in the isolated bubble case, but the wake region still shows the largest variations.

Identification of all bubbles in the signal presents some difficulties; for example, differentiation between the changes in light intensity caused by the passing of small bubbles and that caused by large solids concentration variations due to vortical fluid motion is difficult. Identifying relatively large bubbles, however, remains straightforward. For these large bubbles, the exact form of the signal resembles that experienced in the single bubble system. Figure 41 provides contrasting bubble signals found in a fluidized bed of 326 μm glass beads: Fig. 41(a) and 41(b) show bubble signals obtained for identical conditions in a multi-bubble system and Fig. 41(c) shows that for a single bubble case corresponding to a $D_e$ of 2.0 cm; note that, in this case, the observed chord lengths for the bubble in the single-bubble case

2. In this context, large refers to the size of a bubble compared to other bubbles in the freely bubbling bed and is not used in a general way.
Figure 41. Typical bubble signals: a) a multi-bubble experiment, b) a multi-bubble experiment, and c) a single bubble experiment.
and those in the multi-bubble case are of the same order. In the multi-bubble case, the bubble signals have a consistent repeatable form, similar to that found in a single-bubble case, but typically the bubbles are smaller than those studied in the single-bubble system.

Two dotted lines are included in Fig. 40(a) to indicate average properties important to the wake model. The first represents the wake solids holdup, $\varepsilon_{s,w}$, and is calculated in the same manner as discussed in Section 3.3. The second represents the particulate phase solids holdup, $\varepsilon_{s,p}$; this property is derived from the solids holdup distribution (see Fig. 40(b)) and will be discussed in detail in Section 4.2. A third important property, $\varepsilon_{s,0}$, also indicated in the figure, is the average solids holdup immediately prior to the passing of the bubble and can be used to calculate a local value of the relative solids holdup, $X$.

The local solids holdup data, like that shown in Fig. 40(a), can also be used to construct a solids holdup distribution. Figure 40(b) gives the solids holdup distribution corresponding to the data given in Fig. 40(a). Recognizing that under the low gas holdup conditions used in this study ($\varepsilon_g < 0.025$), the bulk of the fluidized bed consists of the particulate phase, the solids holdup distribution can then be used as a reasonable description of the particulate phase. For example, if the gas holdup in the bed is
0.025, and assuming the relative wake size is 3, the fraction of the bed consisting of the particulate phase is

$$\delta_p = 1 - \delta_g (K + 1)$$

or 92.5%.

Since the particulate phase includes over 92% of the bed, $\varepsilon_{s,p}$ can be approximated as the average value of the distribution. In addition, the validity of this calculation can be increased by eliminating the bubble and wake portions of the signal.

A second useful calculation based on the solids holdup distribution data is an approximate determination of gas holdup: by recognizing that in relatively high solids holdup systems ($\varepsilon_s > 0.30$) the data found at an $\varepsilon_s$ near 0.10 corresponds actually to the presence of the bubble. Therefore, the ratio of the number of points found corresponding to the bubble to the total number of points provides an estimate of the local gas holdup; essentially this is the percentage of time that gas occupies the measurement zone.

4.1.2 Axial Profiles

Each of the quantities discussed in the previous section can be measured at any location within the fluidized bed. Figure 3. This provides a ball park estimate of the gas holdup in the system to validate that calculated based on the method presented in Chapter 2.
42 relates the measured local wake solids holdup, particulate phase holdup, and estimated gas holdup to the axial location within the bed.

The importance of the three regions introduced in Section 1.1 can be identified in the figure. The distributor region, which includes the formation of the bubble, and hence, the initiation of the wake, is of obvious importance to the wake behavior, and, as shown, at the location nearest to the distributor the difference in solids concentration between the wake and particulate phases is the largest. Within the bulk fluidized zone (roughly between 18 and 52 cm in the figure) the difference in concentration between these two phase steadily decreases. The behavior in the transition between the bulk fluidized region and the freeboard region (between about 50 and 54 cm) and in the freeboard region itself shows that the wake solids holdup can exceed that in the particulate phase.

4.2 Quantitative Solids Holdup Behavior

In multi-bubble systems, both the wake solids holdup and the particulate phase holdup must be determined to evaluate the relative solids holdup, X. In this section these two quantities will be discussed individually at first, then combined to provide important information on X. Note that the wake solids holdup averaging technique used is the same as that presented for the
Figure 42. Axial distribution of local holdups in a multi-bubble system.
single bubble case (see Section 3.3).

4.2.1 Wake Solids Holdup

Figure 43 shows the mean solids holdup in the wake behind a single bubble as a function of the solids holdup found just prior to the passing of the same bubble ($\xi_{s,0}$): the data includes several combinations of the superficial gas and liquid velocities and the solid line in the figure represents a relative solids holdup equal to 1. The open symbols in Fig. 43 represent a higher gas velocity than the corresponding solid symbol. A comparison of similar open and solid symbols indicate that $\xi_{s,0}$ is higher at the larger gas velocity; essentially, by a mass balance (see Eq. (12)), the larger gas holdup requires a higher particulate phase solids holdup.

The local wake solids holdup data, over the range of local mean solids holdups given, reasonably compares with the data found in single bubble systems for the same mean solids holdup; note, however, that the bubble size found in the multi-bubble systems is smaller and $\xi_{s,0}$ does not necessarily correspond to the particulate phase solids holdup.

Figure 44 shows the solids holdup in the wake as a function of axial location for a 0.2 cm/sec superficial liquid velocity and a series of gas velocities. The largest deviation in the data occurs near the bed surface ($\approx 45 - 50$ cm), while the smallest
Figure 43. Variation in wake solids holdup with the local mean solids holdup ($\varepsilon_{\text{so}}$).
Figure 14. Effect of gas velocity on the axial distribution of the wake solids holdup.
occurs near the distributor (18 - 22 cm). At the locations nearest to the distributor (25 cm < ), the solids concentration in the wake shows a consistent value independent of the gas velocity. Above this region, a slight decrease seems to occur, however, in the solids holdup with increasing distance from the gas-liquid distributor; the severity of this decrease increases with an increase in gas velocity.

Figure 45 shows the solids holdup in the wake as a function of axial location for a 1.0 cm/sec superficial gas velocity and a series of liquid velocities. The data clearly shows a decrease in solids holdup in the wake with an increase in liquid velocity. Note, however, the decrease in wake solids concentration with increasing distance from the gas-liquid distributor becomes less evident with increasing liquid velocity.

4.2.2 Particulate Phase Solids Holdup

Figure 46 shows the solids holdup distribution for the Augustus (Aug44 - Aug54) series experiments: indicated in the figure are the mean (s) and particulate phase (s,p) solids holdups. Comparing the distributions for each set of conditions shows that the shape of the distribution remains essentially the same. However, s decreases with increasing gas velocity and the difference between s and s,p also becomes smaller.
Figure 45. Effect of liquid velocity on the axial distribution of the wake solids holdup.
Figure 46. Solids holdup distributions in a multi-bubbling bed.
Data Set: AugSO
$U_g = 0.73 \text{ cm/s}$
$U_l = 0.56 \text{ cm/s}$
$\varepsilon_{s_p} = 0.015$
$\varepsilon_g = 0.015$

Data Set: Aug54
$U_g = 0.73 \text{ cm/s}$
$U_l = 0.72 \text{ cm/s}$
$\varepsilon_{s_p} = 0.010$
$\varepsilon_g = 0.020$
Figure 47 shows the solids holdup for the particulate phase as a function of axial location for a 0.2 cm/sec superficial liquid velocity and a series of gas velocities. At the locations nearest to the distributor, the solids concentration in the particulate phase, like that found in the wake, shows a consistent value independent of the gas velocity. Above this region, the solids holdup decreases with increasing distance from the gas-liquid distributor; the severity of this decrease increases with an increase in gas velocity and is greater than that found for the solids holdup in the wake.

Figure 48 shows the particulate phase solids holdup as a function of axial location for a 1.0 cm/sec superficial gas velocity and a series of liquid velocities: in addition, the particulate phase holdup data presented in Fig. 48 is also shown in the figure. Like found for the wake solids holdup, a decrease in particulate phase solids holdup occurs with an increase in liquid velocity and this decrease becomes less evident with increasing liquid velocity.

4.2.3 Relative Solids Holdup Behavior

The data presented in Figs. 44 and 47 and 45 and 48 can be combined to provide data on the relative solids holdup behavior at various axial locations within the bed. Figure 49 gives these combined results. As shown, the lowest relative solids holdup
Figure 47. Effect of gas velocity on the axial distribution of the particulate phase solids holdup.
Figure 48. Effect of liquid velocity on the axial distribution of the particulate phase solids holdup.
Figure 49. Axial variation in the relative wake solids holdup in a multi-bubble system.
occurs at the location nearest the distributor, and as the
distance from the distributor increases, the relative solids
holdup approaches 1.

The results reported in Fig. 49, and consequently Figs. 44,
45, 47, and 48, indicate an important finding about the wake
phase. First, the distributor region solids holdup conditions
probably determine the solids holdup in the wake. And second, the
relative wake solids holdup in a freely bubbling bed is higher
than expected based solely on single-bubble measurements due to
axial variations in the particulate phase solids holdup.

4.3 Conclusions

The investigation of the local solids holdup in a multi-
bubble system extended the approach used in the single bubble
system to evaluate the wake behavior. In addition, the solids
holdup distribution at a given location was used to evaluate the
particulate phase solids holdup at that location.

The data presented in Figs. 43 through 49 exhibited several
important trends:

(i) the particulate phase solids holdup decreased with an
increase in distance from the distributor: the
magnitude of the decrease increased with an increase
in gas velocity, but the holdup at the lowest
measurement point only depended on the liquid velocity;

(ii) the wake solids holdup appeared to depend on the particulate phase solids holdup near the distributor region; and

(iii) the wake solids holdup, within error, remained nearly constant with increasing distance from the distributor.

Note that the data presented in this chapter considered only 326 \( \mu \)m glass beads.

From these trends several postulations about the difference in behavior expected in multi-bubble systems and the importance of the single bubble data acquired in the previous chapter can be made:

(i) the distributor region, where in these experiments the frequency of bubbles striking the probe was small, is the most important towards the formation of the solids content in the wake;

(ii) as a consequence of (i) the study of the wake solids holdup behind a single bubble rising in a liquid-solid fluidized bed remains particularly relevant; and

(iii) between the distributor and the bed surface the nearly constant wake solids holdup and the decreasing
particulate phase holdup indicates the relative wake solids holdup increases with axial distance from the bubble formation region (this accounts for the local high values of \( X \) near the bed surface exhibited in Fig. 49).

To reiterate, probably the most important result of the qualitative study of the freely bubbling bed is that the single-bubble experiments can provide a solid basis for the determination of the relative solids holdup at the edge of the distributor region. This initial value can then be modified by considering the axial profile of the particulate phase.
CHAPTER V

THEORETICAL HYDRODYNAMICS

The importance of vortical flow to the wake behavior was clearly established in the first chapter. Since, as evidenced by the complex nature of the wake flow field, a rigorous mathematical treatment of the wake is not feasible, a simple model will be proposed based on a simple spherical vortex. From the well known relationships of the spherical vortex, equations will be derived for the wake size and wake solids concentration based on a simple model of the wake. Subsequently, these equations will be used to predict the wake size and relative solids holdup under various conditions. And finally, a multi-bubble system will be considered.

5.1 Spherical (Hill's) Vortex

For incompressible flow, the continuity equation becomes

\[ \nabla \cdot \mathbf{U} = 0 \] \hspace{1cm} (44)

In spherical co-ordinates, and assuming symmetry with respect to \( \phi \), Eq. (44) can be re-expressed as

\[ \frac{1}{r} \frac{\delta (r U_r)}{\delta r} + \frac{1}{r \sin \theta} \frac{\delta (U_r \sin \theta)}{\delta \theta} = 0 \] \hspace{1cm} (45)
The components of the velocity vector in this equation can be defined in terms of a general function, known as the stream function, which satisfies Eq. (45). For example, the radial and tangential velocity components can be defined as

\[ U_r = \frac{1}{r} \frac{1}{\sin \theta} \frac{\partial \psi}{\partial \theta} \]  
(46)

and

\[ U_\theta = -\frac{1}{r} \frac{1}{\sin \theta} \frac{\partial \psi}{\partial r} \]  
(47)

With these definitions, Eq. (44) is automatically satisfied.

Inside Hill's vortex the flow is rotational and can be described by the stream function (e.g., Lamb (1945))

\[ \psi_i = \frac{1}{2} A (\zeta_i^1 - r^1) r \sin \theta \]  
(48)

In this equation, \( A \) is a constant which relates the vorticity to the position and \( \zeta \) is the vortex radius.

The flow external to the vortex, on the other hand, is irrotational, and can be described by the stream function (e.g., Lamb (1945))

\[ \psi_o = \frac{1}{2} U (1 - \frac{r}{\zeta}) r \sin \theta \]  
(49)

where \( U \) is the free stream velocity.

Recognizing that at the boundary of the vortex (\( r = \zeta \)) not only must \( \psi_i \) and \( \psi_o \) be equal, but also \( \partial \psi_i / \partial r \) and \( \partial \psi_o / \partial r \) to insure the continuity of the tangential velocity. Under these conditions, Eq. (48) can be re-expressed as

\[ \psi_i = \frac{3}{4} U (1 - \frac{r}{\zeta}) r \sin \theta \]  
(50)
Thus, Eq. (50) relates the flow field inside the vortex to the free stream velocity outside the vortex.

Since the stream function represents the velocity field inside the vortex, the kinetic energy contained in the vortex can also be directly derived from it. The summation of the kinetic energy for a single vortex can be expressed as

\[ E = \frac{1}{2} \rho \iiint (u^1 + v^1 + w^1) \, dV \]  

where \( u, v, \) and \( w \) are the components of the velocity vector in rectangular coordinates. Redefining the system in spherical coordinates and expressing Eq. (51) in terms of the stream function gives

\[ E = \pi \rho \iiint \psi \omega r \sin \theta \, d\theta dr \]  

where \( \omega \) is the resultant vorticity. Note that the integration with respect to \( \phi \) has already been done. For Hill's vortex, these equations give (e.g., Lamb, 1945)

\[ E = \frac{10}{7} \pi \rho l' U' \]  

Hence, the kinetic energy of the system is directly related to the medium density, the vortex radius, and the free stream velocity.

5.2 Extension to Wake Behavior behind a Spherical-Cap Bubble

Interpreting the wake behind a rising spherical-cap bubble in terms of a spherical vortex can provide the basis for a compact model of the bubble wake. To effectively consider both the size and solids concentration behavior, this mechanistic view will
include what can be termed macroscopic and microscopic views: the macroscopic view considers the wake size can be determined based on average properties of the liquid and solid mixture, but, to analyze the solids holdup behavior in the wake, the microscopic view conceptually resolves the difference in behavior between the liquid and solids subjected to vortical motion. Equations (50) and (53) can ultimately be used for the estimation of both the relative solids holdup in the wake and the wake size.

5.2.1 Estimation of the Wake Volume

Consider a spherical-cap bubble rising in a three-phase fluidized bed at a velocity $U_b$ relative to the surrounding medium. Immediately below the bubble base a spherical vortex develops, grows to a substantial size, and finally is shed to the surrounding medium. During this process, work is being done by the bubble through the drag force which exists between the bubble and the surrounding medium. The work is being done on the surrounding medium: however, it can be assumed to initially be transmitted to kinetic energy in a growing vortex in the wake (primary wake), then subsequently be shed back into the surrounding medium well below the bubble (secondary wake).

The model of the wake already described conceptually envisions the wake growing, on a cyclic basis, from essentially nothing to some maximum size. At any time, the size of the
spherical vortex can be related to the energy contained in the vortex by Eq. (53), thus estimating the wake volume consists of determining the kinetic energy within the wake.

At terminal rise velocity, the loss in potential energy of a rising bubble is converted into work done on the surrounding liquid medium. The rate of work done by the rising bubble can be expressed as

\[ W = F_d \times U_b \]  \hspace{1cm} (54)

The drag force, \( F_d \), can be defined in terms of a drag coefficient, \( C_d \), as

\[ F_d = \frac{1}{2} \rho \left( \frac{\pi D_e}{4} \right) U_b \ C_d \]  \hspace{1cm} (55)

Consequently, the rate of work becomes

\[ W = \frac{1}{2} \rho \left( \frac{\pi D_e}{4} \right) U_b \ C_d \]  \hspace{1cm} (56)

Note that the drag coefficient in this expression is a function of bubble Reynolds number.

Next, consider one basic presumption about the wake: the work done on the surrounding medium by the rising bubble becomes kinetic energy in the wake prior to shedding. Therefore, the energy added to the wake equals the rate of work multiplied by the time elapsed after the last shedding. With these assumptions and Eq. (53) the size of the vortex at any time can be expressed as

\[ E = \frac{10}{7} \pi \rho L \ U_b \ C_d \]  \hspace{1cm} (57)
where $\Delta t$ is the elapsed time. However, the variation in wake size with time is not of particular importance, but determining an average wake size is.

A characteristic wake size can be defined as the size of the spherical vortex obtained by setting $\Delta t$ equal to a fraction of the time interval between the shedding of a vortex ($\tau / f_v$). Substituting $\tau / f_v$ for $\Delta t$, rearranging Eq. (57), and dividing through by $\pi D_e / 8$ gives

$$\frac{\pi \frac{1}{8}}{\pi D_e / 8} = \frac{(4\pi / 3)}{\pi D_e / 6} = \frac{7 U_b}{10 D_e C_d f_v}$$

And finally, introducing the Strouhal number

$$Sr = \frac{D_e f_v}{U_b}$$

yields

$$K_v = \kappa \frac{7 C_d}{10 Sr}$$

a simple relationship involving the ratio of two important dimensionless quantities. Equation (59) suggests that the volume of the vortex relative to the bubble volume depends inversely on the Strouhal number and indirectly on the Reynolds number through the drag coefficient.

To predict the most appropriate characteristic wake size, $\kappa$ must consider the nature of the wake shedding phenomena and the definition of the shedding frequency: in this model, $\kappa$ is set at 1/2 and the shedding frequency is defined based on the shedding of
a vortex pair in two-dimensional systems, on one period of rotation of a bubble rising in a helical path, or the shedding of a single toroidal vortex.

For a liquid solid medium, the overall energy contained in a vortex can be approximated by

\[ E = \frac{10}{7} \pi \left( \rho_l (1 - \varepsilon_s) + \rho_s \varepsilon_s \right) k U_m^1 \]  

(60)

where

\[ U_m = U_b + \frac{\rho_s \varepsilon_s}{(\rho_l (1 - \varepsilon_s) + \rho_s \varepsilon_s)} V_p \]  

(61)

Essentially, Eq. (61) assumes the velocity characteristic of the vortex for the liquid-solid medium is the mass average velocity.

Starting from Eq. (60) and following a derivation analogous to that for the liquid medium leads to

\[ K_v = \kappa \frac{7}{10} \left( \frac{U_b}{U_m} \right)^1 \frac{C_d}{Sr} \]  

(62)

Equation (62) suggests that particle properties can have a significant impact on the predicted wake size: particle properties directly enter through the relative density term and the particle velocity, but also indirectly affect the Strouhal number and the drag coefficient.

5.2.2 Estimation of the Relative Wake Solids Holdup

Since, as indicated in Chapter 1 and as shown in Chapters 3 and 4, the solids holdup varies inside the wake, such behavior must be accounted for. In this regard, consider that, within the
wake, a non-uniform concentration of solids exists due to differences in behavior of particles and a liquid subject to vortical motion. In addition, assume that this non-uniformity can be adequately accounted for by subdividing the wake into two regions: one without solids and another which contains solids with a concentration equal to that of the surrounding liquid-solid medium.

With these assumptions, the average solids holdup in the wake can be expressed as

\[ h_{sw} \frac{\varepsilon}{h_w} = h_{sw}(0) + h_{sw} \frac{\varepsilon}{sp} \]  

(63)

In this equation, the shape of the wake is assumed to be cylindrical. Solving this equation for the relative solids holdup gives

\[ X = \frac{\varepsilon_{sw}}{\varepsilon_{sp}} = \frac{h_{sw}}{h_w} = 1 - \frac{h_{sw}}{h_w} \]  

(64)

hence, the relative solids holdup can be determined from an estimate of the liquid layer height and ultimately from the differences in behavior of the two phases in a vortex.

**Liquid Wake Size**

As considered by Darton and Harrison (1976) both the liquid and particle flow can be envisioned to be in a vortex in the wake. In a Hill's vortex, the free stream velocity characteristic of the liquid vortex is set equal to the bubble rise velocity \( U_b \) and the stream function for the liquid vortex becomes
Similarly, a free stream velocity characteristic of the particle vortex, defined here as $U^*$, gives the stream function for the particle vortex as

$$\psi = \frac{3}{4} U^* (1 - \frac{r}{\zeta_s}) r \sin \theta \tag{65}$$

\(\zeta_s\) and \(\zeta_l\) are the radii of the particle vortex and liquid vortex, respectively.

The difference in size between the different vortices can be evaluated by two methods. Darton and Harrison (1976) evaluated \(\zeta_s/\zeta_l\) based on the particle slip velocity as follows: the radial component of the velocity vector at any point can be evaluated from Eq. (46) and the stream function. For the liquid vortex, the radial component of the velocity vector is

$$U_{r \parallel} = \frac{3}{2} U_b (1 - \frac{r}{\zeta_l}) \cos \theta \tag{67}$$

and for the particle vortex

$$U_{r \parallel s} = \frac{3}{2} U^* (1 - \frac{r}{\zeta_s}) \cos \theta \tag{68}$$

and, recognizing at the point \(r = \zeta_s\) and \(\theta = 0\) the slip velocity is

$$V_p = U_{r \parallel} - U_{r \parallel s} \tag{69}$$

\(\zeta_s/\zeta_l\) can be evaluated by combining Eqs. (67), (68), and (69). This approach yields
Eq. (70) suggests that the particle vortex size is smaller than that of the liquid.

The size of the liquid wake can be estimated as the difference in size between the liquid and particle vortices (Darton and Harrison, 1976); thus

$$ h_{lw} = \zeta_s - \zeta_l = \zeta_l \left(1 - \frac{2V_p}{3U_b}\right)^{1/4} \quad (71) $$

Eq. (71) can be simplified for small values of $2V_p/3U_b$ to

$$ h_{lw} \approx \zeta_l \frac{V_p}{3U_b} \quad (72) $$

Darton and Harrison (1976) found Eq. (72) to satisfactorily predict the liquid wake size for spherical-cap bubbles greater than about 1 cm$^3$ obtained from photographs of bubbles and wakes in two-dimensional beds, but underestimated $h_{lw}$ for small non-cap shaped bubbles.

An alternative approach for estimating the liquid layer height can be derived based on the energy contained in the liquid and particle vortices.

Since, as the bubble rises, a drag force exits between the surrounding liquid-solid medium and the bubble, the energy of the liquid-solid medium is increased. Essential to this transfer of energy is the contact area between the liquid and solid and the bubble. The contact area can be approximated in terms of the
volume fraction by the following dimensional analysis: recognizing that
\[
\frac{\varepsilon_s}{1 - \varepsilon_s} = \frac{V_s/V}{V/L_L} \propto \left(\frac{L_S}{L_L}\right)^4
\]
(73)
where \(L_s\) and \(L_L\) are dimensions of length; in addition
\[
\frac{A_s}{A_L} \propto \left(\frac{L_S}{L_L}\right)^2
\]
(74)
Eqs. (73) and (74) lead to
\[
\frac{A_s}{A_L} \propto \left(\frac{\varepsilon_s}{1 - \varepsilon_s}\right)^{2/7}
\]
(75)
Next, consider the energy per unit volume in each vortex is proportional to the contact area between the individual phases and the bubble: that is
\[
\frac{E_s/\rho_s}{E_L/\rho_L} = \left(\frac{\varepsilon_s}{1 - \varepsilon_s}\right)^{2/7}
\]
(76)
or after substituting in Eq. (53)
\[
\frac{L_s U^*_s}{L_L U_b} = \left(\frac{\varepsilon_s}{1 - \varepsilon_s}\right)^{2/7}
\]
In this case, the characteristic velocity of the particle free stream is assumed to be the velocity of the particle falling relative to the bubble: or
\[
U^*_s = U_b + V_p
\]
(77)
Rearranging Eq. (76) and substituting in Eq. (77) for \(U^*_s\) gives
\[
\frac{L_s}{L_L} = \left[\left(\frac{\varepsilon_s}{1 - \varepsilon_s}\right)^{2/7} \left(\frac{U_b}{U_b + V_p}\right)^{2/7}\right]
\]
(78)
or in terms of the liquid layer height
Equation (79), like Eq. (71), suggests a smaller radius for the particle vortex, but also shows that as the solids holdup decreases to zero the liquid layer approaches the size of the entire vortex.

Relative Solids Holdup

X can be evaluated directly from Eq. (64) by assuming that

\[ \frac{h_{sw}}{h_w} = \frac{l_s}{l_l} \]  \hspace{1cm} (80)

With this assumption and substituting in Eq. (72) gives

\[ X = 1 - \frac{1}{3} \frac{V_p}{U_b} \]  \hspace{1cm} (81)

Equation (81) shows a striking similarity to the correlation for X proposed by El-Tamtamy and Epstein (1979) which appears in Table 4. Equation (81), however, predicts a higher value of X.

If Eqs. (78) and (80) are combined with Eq. (64), a second expression for X is obtained

\[ X = \left( \frac{\epsilon_s}{1 - \epsilon_s} \right)^{1/3} \left( \frac{U_b}{U_b + V_p} \right)^{1/3} \]  \hspace{1cm} (82)

Both Eqs. (81) and (82) predict X to be smaller for larger particle systems, but only Eq. (82) suggests that X goes to zero as the solids holdup goes to zero.
5.2.3 Summary

From this simple picture of the wake, several important physical phenomena can be considered to affect the behavior of the wake, and hence create differences in behavior between the liquid and particle streamlines, in general, or between the wake in a liquid medium or a liquid-solid medium, in particular.

The characteristic wake size reflects the behavior of the drag coefficient, the shedding frequency, and the bubble rise velocity. Each of these phenomena are affected by the presence of particles in the surrounding medium; hence the model incorporates particle effects in these terms in addition to the energy content of the vortex.

The relative solids holdup is explicitly affected by the mean solids holdup in the particulate phase, as indicated in Eq. (82), but it is also indirectly affected by the mean solids holdup through the bubble rise velocity. The best prediction of the relative wake solids holdup must consider the effect of solids holdup on the bubble rise velocity (e.g; see Jean and Fan (1989)).

5.2.4 Model Estimations

The simple spherical vortex model of the wake allowed the derivation of equations to estimate the wake solids concentration (Eqs. (81) and (82)) and also the wake size (Eqs. (59) and (62)). Each of these resulting equations are expressed in terms of
properties of the rising bubble and the particles in the surrounding medium. These properties are discussed briefly below.

**Particle Velocity**

Essentially, the particle velocity, $V_p$, represents the relative velocity between the particle and the liquid phase. For a single particle in an infinite medium, $V_p$ would be the particle terminal velocity. In their development, Darton and Harrison (1976) defined the relative velocity between the particle and liquid through the Richardson-Zaki equation:

$$V_p = U_t (1 - \varepsilon_s)^{n-1}$$  \hspace{1cm} (83)

which, in the limit as the solids concentration approaches zero, approaches the particle terminal velocity. Eq. (83) includes a solids concentration effect into $V_p$.

**Strouhal Number**

Song et al. (1989), based on observations of bubbles rising in a two-dimensional bed, found that the shedding frequency of a vortex pair could be correlated in terms of the bubble and particle properties. Their correlation, modified to express the frequency in terms of a single vortex, is

$$f_v = A \left( \frac{Re_{ba}}{B} \right)^{-0.7} \ln \left[ \frac{2}{1 + \exp \left( -\frac{Re_{ba}}{B} \right) } \right]$$  \hspace{1cm} (84)

The parameters $A$ and $B$ are listed in Table 9 for a water system with no particles or 163, 326, 460, and 760 \textmu m glass beads.
Table 9. Correlation parameters

<table>
<thead>
<tr>
<th>size</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Particles</td>
<td>11.5</td>
<td>5000</td>
<td>22.2</td>
</tr>
<tr>
<td>163 μm Glass</td>
<td>11.5</td>
<td>7000</td>
<td>21.0</td>
</tr>
<tr>
<td>326 μm Glass</td>
<td>11.5</td>
<td>7000</td>
<td>17.6</td>
</tr>
<tr>
<td>460 μm Glass</td>
<td>11.5</td>
<td>9000</td>
<td>15.6</td>
</tr>
<tr>
<td>760 μm Glass</td>
<td>11.0</td>
<td>10000</td>
<td>13.6</td>
</tr>
</tbody>
</table>

* - from Song et al. (1989)
The bubble rise velocity required in the calculation of the Strouhal number can be calculated based on a Davies-Taylor type equation;

\[ U_b = C \sqrt{\frac{gD_e}{\mu}} \]  \hspace{1cm} (85)

In this case the radius of curvature of the bubble is replaced by the equivalent bubble diameter. The constant \( C \) in Eq. (85) was evaluated from the experimental data on the bubble rise velocity and is also listed in Table 9.

In both the correlation for the shedding frequency and the correlation of the bubble rise velocity, solids concentration effects are not included. This behavior reflected the experimental data used in deriving each equation, but it may not reflect a general trend.

**Drag Coefficient**

For spherical-cap bubbles rising in a liquid medium, the drag coefficient can be evaluated for \( Mo > 10 \) and all \( Re \) with \( (Clift \text{ et al.}, \ 1977) \)

\[ C_d = \frac{8}{3} + \frac{8(2 + 3\iota)}{Re_b(1 + \iota)} \]  \hspace{1cm} (86)

where \( \iota = \frac{U_d}{U_c} \). For air rising in water, \( \iota \) is essentially zero.

For bubbles rising in a liquid-solid fluidized bed, Darton and Harrison (1974b) correlated the drag coefficient for high Reynolds numbers by
\[ C_d = 2.7 + \frac{24}{Re_e} \]  

(87)

where

\[ Re_d = \frac{\rho_e U_b D_e}{\mu_e} \]  

(88)

\[ \mu_e = \mu_\text{s} \exp(36.15 \tau_s^{2.5}) \]  

(89)

\[ \rho_e = (1 - \tau_s)\rho_\text{s} + \tau_s \rho_s \]  

(90)

Equation (87) indicates that the higher the solids holdup in the bed, the greater the drag force between the bubble and the surrounding medium. Also, no effect of particle size appears in this relationship. In the limit, Eq. (87) approaches the value given by Eq. (86).

**Wake Size in Particle-Free Systems**

In a water system, for spherical-cap bubbles, the drag coefficient remains constant over bubble Reynolds numbers between 2500 and 7500 (from Eq. (86)), as is shown in Fig. 50(b). Over this same range of Reynolds numbers, the Strouhal number increases initially, reaches a maximum near a Reynolds number of 4500, then decreases with further increases in Reynolds numbers (from Eq. (84)), as is shown in Fig. 50(a): note, however, the relative change in the Strouhal number is expected to be low through this range of Reynolds numbers and the bubble rise velocity was calculated from Eq. (85).

The predicted wake size, using Eq. (59), reflects the
Figure 50. Variation in a) the Strouhal number, b) the drag coefficient, and c) the predicted wake size behind a spherical-cap bubble in a water system.
behavior of the drag coefficient and Strouhal number; consequently, the model predicts the wake size to vary only slightly between Reynolds numbers of 2500 and 7500 as shown in Fig. 50(c). The predicted wake size, initially decreases with Reynolds number, reaches a minimum at a Reynolds number of about 4500, then slightly increases with further increases in Reynolds number. Over the entire range, the predicted wake size falls between 4.0 and 4.5. This value compares favorably with the data presented in Table 3 and the data presented by Tsuchiya and Fan (1988b).

Wake Size in Particle Systems

According to the model, the presence of particles may affect the energy in the vortex by two means. First, the particles require more energy to follow a vortical path thus increasing the energy capacity for a given size vortex (e.g., see Eq. (60)). And second, the particle concentration affects the drag force between the rising bubble and the surrounding medium as indicated by the correlation of Darton and Harrison (1974). These two effects of the particle size are competing and suggest different behavior at low and high solids holdups.

At low solids holdups ($\tilde{\varepsilon}_s = 0.1$), using Eqs. (62), (83), (84), (85), and (87), the model predicts smaller wake sizes in a liquid-solid fluidized bed than in water, and the value gets
progressively smaller with an increase in particle terminal velocity. Between bubble Reynolds numbers of 2500 and 7500, as is shown in Fig. 51, the predicted wake size decreases with increasing Reynolds number. The extent of the decreases becomes progressively smaller over the whole range, but no minimum value is predicted as is the case in a water system. The absolute value of the relative wake size is predicted to vary only slightly over the whole range. In this range, the effect of solids holdup on the drag coefficient (Eq. (87)) is essentially insignificant.

At high solids holdups, once again using Eqs. (62), (83), (84), (85), and (87), the effect of solids holdup on the drag coefficient becomes progressively more important. Figure 52 shows the predicted size of a wake behind a bubble in a liquid-solid fluidized bed of 163 μm glass beads. As shown, the model predicts a significant decrease in size between a particle-free system and that for a solids holdup of 0.1, but further increases in solids holdup do not produce an further significant changes until a solids holdup of near 0.4 is achieved. At high solids holdups, the model predicts the wake size to increase with an increase in solids holdup; this trend is opposite of that predicted at low solids holdups.

1. Such behavior agrees with the assumption used in the calculation of the average wake solids holdup in Chaps. 3 and 4.
Figure 51. Predicted dependence of the wake size on the bubble Reynolds number as a function of particle size ($\bar{r}_s = 0.1$).
Figure 52. Predicted dependence of the wake size on the bubble Reynolds number in a system of 163 μm glass beads as a function of mean solids holdup.
Figure 53 shows a plot similar to Fig. 51, but with $\bar{e}_s = 0.4$. By comparing these two plots an additional point is evident. The effect of solids holdup is predicted to be smaller for the larger size particles.

The magnitude of the predicted wake size can be evaluated by comparing with data obtained in a two-dimensional liquid-solid fluidized bed. Kitano and Fan (1987) presented data on the wake size observed in a two-dimensional bed of 460 μm particles. This data is presented in Fig. 54 along with the corresponding model predictions. Two points are important about this comparison: first, the experimental values and the values predicted by this simple model agree quite well and second, the trend in wake size as a function of solids holdup predicted by the model is also evident in the data.

The reliability of the wake size prediction depends on the relevancy and accuracy of the correlations for the shedding frequency, bubble rise velocity, and drag coefficient. However, the simple model does relate the wake size to important physical properties of the system much easier to identify than the wake size.

Relative Solids Holdup in the Wake

Eqs. (81) and (82) represent the prediction of $X$ based on the spherical vortex model of the wake. As mentioned earlier, Eq.
Figure 53. Predicted dependence of the wake size on the bubble Reynolds number as a function of particle size ($\tilde{\epsilon}_s = 0.4$).
Figure 54. Predicted wake sizes in a bed of 460 µm glass beads compared with the data of Kitano and Fan (1988).
(81) is of the form proposed by El-Tamtamy and Epstein (1979) for correlating X; however, Eq. (81) does predict a much higher X than the correlation of El-Tamtamy and Epstein.

The simple equivalent energy approach leading to Eq. (82) proves to be a much better method for predicting X. If the characteristic particle velocity \( V_p \) is taken as the particle terminal velocity, the predicted variation in X with the mean solids holdup is given in Fig. 55. As shown, X is predicted to decrease with an increase in bed voidage, at first rather moderately, then at a very high bed voidage a very steep decrease is predicted; such behavior is expected for each particle size. Also, as presented in Fig. 55, X is expected to be larger for smaller particles.

For 326 \( \mu \)m glass beads, the predicted difference in X for different bubble sizes is shown in Fig. 56. In fact, Eq. (82) essentially predicts no difference over the range of interest \( 2.0 < D_e < 2.7 \) because of the relatively small change in bubble rise velocity. This trend agrees with that of the data.

Overall the model predicts the trends with respect to particle size and bubble size as reported in the literature (e.g., Rigby and Capes, 1970). The agreement with the data presented in Chapter 3 of this work is also good. In Fig. 57, the data for X in systems using 163, 326, 460, and 760 \( \mu \)m glass beads are shown relative to the model prediction; the prediction is good for the
Figure 55. Predicted dependence of the relative solids holdup on the bed voidage.
Figure 56. Predicted dependence of the relative solids holdup on the bubble size.

326 μm Glass Beads

- $D_s = 2.0$
- $D_s = 2.3$
- $D_s = 2.7$
Figure 57. Comparison of the single bubble system data with the prediction of the simple vortex model.
163, 326, and 460 μm glass beads, but shows significant deviation for the data of 760 μm glass beads at a high solids holdup.

Extension to Ellipsoidal Bubbles

In a pure water system, the wake size prediction can be extended to include the Reynolds numbers corresponding to bubbles of ellipsoidal shape by introducing an extended correlation for the drag coefficient and Marrucci's equation for the bubble rise velocity (see Eq. (6)). A more generalized drag coefficient correlation has been proposed as (Tadaki and Maeda, 1961):

for $ReMo^{0.11} < 6.0$

$$C_d = 1.25(ReMo^{0.11})^{0.11}$$ (91)

for $6 < ReMo^{0.11} < 16.5$

$$C_d = 1.25(ReMo^{0.11})^{0.16}$$ (92)

for $ReMo^{0.11} > 16.5$

$$C_d = 2.6$$ (93)

Equation (93) corresponds to Eq. (85) and Eq. (92) becomes important for the bubbles of ellipsoidal shape.

The behavior of the drag coefficient as the bubbles grow in size and change shape between Reynolds numbers of 750 - 2500, as expressed by Eqs. (91) through (93), is important to the predicted bubble size. Figure 58(b) shows the variation in drag coefficient in a pure water system over the entire range of bubble Reynolds numbers of interest ($500 < Re_b < 7500$): as is shown, the
Figure 58. Variation in a) the Strouhal number, b) the drag coefficient, and c) the predicted wake size in a water system (note figure includes prediction for both spherical-cap and ellipsoidal bubbles).
drag coefficient changes significantly at the lower Reynolds numbers which correspond to the changing shape of the bubbles. The Strouhal number (from Eq. (83)) also shows larger variations in the range corresponding to ellipsoidal bubbles, as shown in Fig. 58(a). Consequently, the predicted wake size in this range also shows a much larger degree of change with increasing Reynolds number: between Reynolds numbers of about 750 through 2000 the predicted wake size increases sharply, reaches a maximum, then decreases sharply; above 2000, however, as indicated previously, the predicted changes in wake size are much smaller. The discontinuity in the predicted wake size results from the discontinuous nature of the drag coefficient correlation.

Extension to Multi-Bubble Systems

Ideally, the model equations for the wake size should apply equally well to the multi-bubble case provided the relationships for both the drag coefficient and the Strouhal number are available in a multi-bubble system. The predicted wake size would be expected to be smaller in a multi-bubble system compared to the single-bubble case because, as suggested by the discussion in Section 1.3, the rise velocity is significantly higher in multi-bubble systems. Essentially, this suggests a lower drag coefficient in the multi-bubble case; also, a lower dependence on
the particle size would be expected due to the higher rise velocity (see Eq. (62)).

In terms of the relative wake holdup, a strict extension of the proposed model would predict an increase in $X$: this results solely from a large $U_b$ and hence, a smaller dependence on the characteristic particle velocity.

5.3 Conclusions

In this chapter, a simple model was proposed for the wake behind a bubble based on Hill's spherical vortex. From the initial assumptions of the model, equations were derived for the prediction of the wake size and relative wake solids holdup. Several conclusions can be made about the model predictions:

(i) provides a very good prediction of the size of a wake behind a large spherical-cap bubble rising in water;

(ii) provides a very good prediction of the size of a wake behind a large spherical-cap bubble rising in a liquid-solid medium;

(iii) correctly predicts observed trends in the size of the wake as a function of solids holdup;

(iv) correctly predicts observed trends in the relative solids holdup in the wake as a function of both solids holdup and bubble size; and
(v) provides a very good prediction of the relative solids holdup in a system of 326 μm glass beads. The model should also apply in freely bubbling beds if information about the Strouhal number and drag coefficient are available.
CHAPTER VI

RECOMMENDATIONS

The results of this investigation suggest two important avenues of further research. Both avenues concern freely bubbling beds.

First, the discovery of the near constant wake solids holdup in a multi-bubble system coupled with a particulate phase solids holdup which decreases axially suggests that the simple wake model, which assumes constant bed properties, may not be adequate. One approach to compensate for the axial variation would be to consider an average value of X based on an appropriate averaging technique: this technique could be based on a model of the axial variation of the particulate phase solids holdup. A second approach, and probably more difficult, would be to modify the wake model itself to include axial variations.

The second avenue should be concerned with determining the rise velocity and, ultimately, the drag coefficient in a freely bubbling bed, to use in predicting K in a freely bubbling bed. In this case, the dual optical fiber probe should be modified to
reduce the distance between the two probes to increase the chances that a bubble passing through the first probe will also pass the second probe, thus allowing the calculation of the rise velocity. In addition, if possible, local liquid velocity calculations should also accompany the local bubble rise velocity calculations to provide the best possible value of the relative rise velocity: this precaution is designed to eliminate values of the rise velocity biased high due to local liquid circulation patterns found in freely bubbling beds.

And last, the ability of the optical fiber probe to measure the local solids concentration suggests a possible method of evaluating the effectiveness of the gas-liquid distributor. Measurements of the local solids concentration in the distributor region under various liquid and gas velocities may identify problem areas, but it will also be essential to the correct application of data gathered in single-bubble systems.
LIST OF REFERENCES


APPENDIX A
PHOTOMULTIPLIER CIRCUIT DIAGRAM
Figure 59. Schematic diagram of the photomultiplier voltage divider circuit.
APPENDIX B
CALIBRATION INTERPOLATION PROCEDURE
An adaptive Lagrangian interpolation routine - the routine adjusts the size of the interpolating polynomial at the end points of the calibration range. The base size, however, represents a compromise between computing speed and maximum accuracy (maximum size)

VARIABLES

\( N^% \) - NUMBER OF CALIBRATION POINTS
\( C^%X \) - LIGHT INTENSITY, CALIBRATION DATA
\( CSH^% \) - SOLIDS HOLDUP, CALIBRATION DATA
\( SH^! \) - INTERPOLATED SOLIDS HOLDUP
\( Y^% \) - LIGHT INTENSITY, EXPERIMENT

FOR \( K^% = 1 \) TO \( N^%-1 \)
  IF \( Y^X > C^%X(K^X) \) AND \( Y^X < C^%X(K^X+1) \) THEN \( NS^%=K^X-1 \):
  \( NF^%=K^X+2 \): GOTO 2090
NEXT \( K^% \)
SH! = 0
IF \( NS^% < 1 \) THEN \( NS^% = 1 \)
FOR \( I^X = NS^% \) TO \( NF^% \)
  \( TERM^! = 1^! \)
  FOR \( M^X = NS^% \) TO \( NF^% \)
    IF \( M^X = I^X \) GOTO 2160
    \( TERM^! = TERM^! \times (Y^X - C^%X(M^X)) / (C^%X(I^X) - C^%X(M^X)) \)
  NEXT \( M^X \)
SH! = \( CSH(I^X) \times TERM^! + SH^! \)
NEXT \( I^X \)
RETURN