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A computer-based investigation of the inter-relationships between the concepts of slope, parallel, perpendicular, and polygons

Whitaker, Paul Harrison, Ph.D.

The Ohio State University, 1988

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A COMPUTER-BASED INVESTIGATION OF THE INTER-RELATIONSHIPS BETWEEN THE CONCEPTS OF SLOPE, PARALLEL, PERPENDICULAR, AND POLYGONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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1988

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Educational Policy and Leadership
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To My Wife, Ellen
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Studies in instructional design for mathematics curriculum with Dr. Suzanne Damarin, Dr. James Leitzel, and Dr. Bert Waits.
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Chapter I
Statement of the Problem

With the advent of the microcomputer much has been written and said concerning the ability or inability of the computer to aid in the teaching of various topics within the mathematics curriculum. Until recently the using of the computer while studying geometric concepts has been quite restricted. Although graphics and computer drawing programs have been available the student has generally been unable to be actively involved in the construction of figures on the computer screen. Most computer software for geometric construction has been directed toward the student making predictions and drawing conclusions from what is seen as a result of what has been preprogrammed to happen within the computer. The student does not manipulate figures directly but rather through the input of numbers or properties (such as the type of quadrilateral or triangle).

Geometric Constructions

Previous to the advent of the computer and, in particular, of interactive graphics and its use in the classroom, geometric constructions involved the use of a compass and a straight edge.

The constructions were tedious, time consuming and for the most part relatively inaccurate. The compass-and-ruler constructions are traditionally
introduced in the plane geometry course. The plane geometry course traditionally follows the first course in algebra.

In the lesson plans for the plane geometry class, the geometric constructions usually follow an understanding of the purpose and methods of geometry, congruence, parallel lines, parallelograms and circles. Students are assumed to have a thorough knowledge of the geometric concepts before attempting the geometric constructions.

The basic geometric constructions in traditional plane geometry give the students the ability to:

a. Construct a line segment congruent to a given line segment.
b. Construct an angle congruent to a given angle.
c. Construct the bisector of a given angle.
d. Construct the perpendicular bisector of a given line segment.
e. Construct a perpendicular to a line segment at a certain point on that line segment.
f. Construct a perpendicular to a given line segment from a point not on that line segment.

After the students have been given practice with each of these constructions they are challenged with more difficult construction problems. All other constructions of plane geometry may be derived from the six listed above.

The construction of parallel and perpendicular lines is easily derived from the above. Perpendicular lines are constructed immediately from e or f.
Parallel lines are constructed by using $b$ along with the characteristic of parallel lines and transversals.

For the most part, teachers avoid spending a lot of time on constructions because of the tediousness of the task involved. Students are easily frustrated because of the time factor and the need for psychomotor accuracy. Both the students and teachers tend to be uncomfortable with this situation.

**Concept of Slope**

The mathematical concept of slope is usually introduced to students in the algebra sequence. It is introduced for the primary purpose of preparing students to find the equation of a line given a point on the line and the slope of the line.

The students are first taught that the slope is the ratio of the increase or decrease of the $y$ variable to the increase or decrease of the $x$ variable, or "Rise over Run." Students are then given exercises to find the equations of lines given two points, one point and the slope, the slope and the $y$-intercept or the $x$ and $y$ intercepts. Students also are expected to consider slope when determining if pairs of lines are parallel or perpendicular.

The concepts of parallel and perpendicular lines are only briefly covered in most texts which do not give the students adequate time for mastery or the concept of slope as related to parallel and perpendicular lines. The exercises traditionally involve the determining the slopes of two or more lines and then considering if the lines are, in fact, parallel or perpendicular. The topic of parallel and perpendicular lines is not usually covered with rigor.
Students learn algorithms involving the slopes ($m_1$ and $m_2$) of pairs of lines. For parallel lines they learn that $m_1 = m_2$. For perpendicular lines they learn that $m_1 = -1/m_2$ or that the product of $m_1$ and $m_2$ is -1. The students have little experience with the meaning or derivation of these rules.

The proof of the relationships of the slopes is left to the higher-level courses. In fact, the usual proofs rely on trigonometry which typically occurs much later in the curriculum. This causes much frustration on the part of the student at higher levels of mathematics. The higher levels of mathematics require a rigorous understanding of slope as it relates to the graphing of combinations of all type of straight lines. This includes parallel lines and perpendicular lines.

**Slope-Stick**

With the introduction of the new technology of the mouse-driven Apple Macintosh computer a new vista for instruction and learning has been revealed. Computer packages, such as Slope-Stick, are becoming available which provide the student the opportunity to generate geometric shapes in a manner which has not been previously available. Slope-Stick, developed for this study, allows the students to perform, easily and quickly, many operations which have not previously been assigned because of the tedious constructions and calculations which might have been necessary.

Slope-Stick provides an environment in which the student may work in a manner similar to the compass and ruler environment of traditional geometry classes but without the demands of time, psychomotor accuracy and drudgery. The primary concepts of Slope-Stick are the slope of a line segment and the length of a line segment.
Slope-Stick gives the user the ability to construct a line segment and, as the line segment is constructed to monitor the slope and the length of the segment appearing (Figure 1). This allows the user to construct accurately a line segment of desired slope and length.

Slope-Stick also gives the user the ability to measure accurately slope and length of a given line segment. Slope is measured to the nearest .05. For instance, the slope might be .65 or .70 with no reading between the two values. Distance is also measured to the nearest .05. Thus, it is possible for a student to construct a line segment 2.50 units in length with slope .45. To bisect the line segment the student would measure a distance 1.25 from one end of the line segment. Slope and length are constantly displayed as the user moves the crosshairs until the mouse button is released.

The distance between two points is also found by selecting the Dist/Pts button from the main menu. This allows the user to pick a point, click the mouse on that point then move the crosshair until the desired point is located (Figure 2). The slope and length are displayed until the mouse button is again clicked signalling the desired point.

Slope-Stick gives the student the ability to perform virtually all of the traditional geometric constructions using the two concepts of slope and distance. The use of slope-stick for such constructions gives the student expertise in constructing parallel and perpendicular lines while forcing the constant use of the relationships of the slopes of parallel and perpendicular lines.
Figure 1 - Constructing a Line Segment - A Slope-Stick Screen

Slope
-0.90 = -3.50 / 4.00

Distance Between Points
5.30
Figure 2 - Measuring Distance Between Two Points - A Slope-Stick Screen

Slope
\[-0.90 = -\frac{2.30}{2.60}\]

Distance Between Points
3.45
In addition, more diverse types of construction are possible with Slope-Stick. As an example, to bisect an angle using Slope-Stick, the user does not follow the tradition methods of copying arcs, etc. The method of construction is:

1. From the vertex, measure equal distances on the two sides of the angle.
2. Using the points marked on the sides of the angles determine the slope of the line segment joining those two points. (Using the Dist/Pts utility).
3. From the vertex of the given angle construct the line segment which is perpendicular to the line segment joining the points on the sides of the angle.

This makes use of theorems from geometry which are not ordinarily emphasized when performing the angle-bisection construction. This construction uses the theorem that the perpendicular from the angle between the two equal sides of an isosceles triangle to the opposite side is the bisector of the angle between the two equal sides.

This is just one example of the diverse types of construction which may be used in Slope-Stick. Not only are the possible types of construction diverse but they are possible in a short amount of time. Because of the time factors teachers would not assign a problem to the students such as the following:

Construct a line segment which is 3 units in length and has slope of 2. At the lower end of the line segment construct another line segment which is 2.35 units in length and and has slope of -2.25.
This type of problem would be unreasonable for the student to attempt but would be easily accomplished using Slope-Stick.

van Hiele Levels of Geometric Thought

In 1957 P.M. van Hiele and D. van Hiele-Geldof developed a theory which suggests that students proceed through five levels of geometric thought. If one level of geometric thought is not mastered before proceeding to work at the next level, the theory predicts that the student will tend to perform in an algorithmic manner rather than with understanding. In 1959 van Hiele further suggested that a student will not perform satisfactorily if the level of instruction is above the level of mastery for the student. Both of these theories put forth by van Hiele and van Hiele-Geldof were supported by research done by Mayberry (1983).

Descriptions of the five van Hiele levels of geometric thought together with examples of tasks that can be performed by students at or above the level are as follows:

The van Hiele Levels of Geometric Learning

Basic Level (Level 0) - (Visual) - Recognition
- Student reasons about basic geometric concepts such as shapes, without regard for specific details of the figure.
  ° Why is that a rectangle? *Because it looks like a table-top.*
- Student names a figure or identifies a particular figure from a field of various shaped figures.
  ° Which of the figures in the picture are rectangles? *Student fails to identify obvious squares as rectangles.*
- Student names figures with regard to orientation on the page.
  ° Which of the figures are squares? *The student fails to realize that all of the figures are squares because*
some are rotated 45° about their centers.

Level One - Analysis
- Student informally analyzes the attributes of a particular figure.
  ° Identify the types of triangles. *The student classifies the triangles into three groups, triangles with three angles equal, two angles equal and no sides equal.*
- Student recognize and name properties of geometric figures.
  ° What is a parallelogram? *The student might list several attributes of the parallelogram without giving the 'true' definition.*
- Student describes properties of a particular figure.
  ° If a triangle has only two equal sides, name it.

Level Two - Abstraction (Informal Deduction) - Ordering
- Student logically orders the concepts of a particular geometric concept.
  ° Starting at the top of the following list and going down the list, how many items on the list are needed to define a square?
    1. Four sides
    2. Two sides parallel
    3. Opposite sides equal
    4. All sides equal
    5. One right angle
    6. All angles equal to 90°.
- Student forms an abstract definition of a geometric concept.
  ° Is it possible for two similar triangles to also be congruent?
- Student determines those properties which are necessary and sufficient for the existence of a particular geometric concept.
  ° Which of the following properties are necessary for the existence of a trapezoid?
  ° Can a right triangle also be an isosceles triangle? *Always? Sometimes? Never?*
  ° Will a right triangle with hypotenuse of 10 inches be
congruent to another right triangle with hypotenuse of 10? Always? Sometimes? Never?

If a triangle has one angle larger than 90°, what can you say about the sum of the other two angles?

Level Three - Deduction (Formal Deduction) -
- Student reasons formally within the concept of a particular mathematical system. This would include, axioms, definitions, and theorems, defined terms and undefined terms.
- If a line segment joins the mid-point of one side of a triangle with the midpoint of an adjacent side, prove the line segment is parallel to the third side.
- If the adjacent mid-points of the sides of a quadrilateral are joined, what is the name of the new figure? Prove it.

Level Four - Rigor
- Student compares geometric systems based on different geometric axioms.
- Cite major differences between Euclidean geometry and hyperbolic geometry.
- Student studies various geometries in the absence of concrete models.
- Using hyperbolic geometry, prove that no quadrilateral is a rectangle.

Mayberry further found that for each geometric concept, a student at level N will correctly answer all questions at a level below N but will not correctly answer all questions above level N. It was also found that seventy percent of the students who had taken high school geometry were below level 3.

Mayberry found that the high school geometry courses, on the whole, assumed a level 2 understanding of geometry to begin geometry learning (at the high school level). Students who enter a geometry class without a mastery at level 3 will only have a 50 percent chance of success in a geometry
course which emphasizes proof. If the geometry instructor begins the instruction at level 3, assuming that all of the students are ready to begin instruction at that level, the students will perform at a low level. Soon after beginning geometry learning the student must be ready for deductive reasoning. This is level 3, one level above most of the students. If they do perform successfully, as stated above, they will be using algorithmic methods rather than understanding the theory behind the studies.

This fact, together with the finding of van Hiele concerning unsatisfactory results of having instruction above the level of mastery of the students, would imply that for at least seventy percent of the high school students who had high school geometry, their instruction was above their heads. Not only was the instruction above their heads but they have no adequate background for further studies of mathematics.

In summary, students who take geometry without first developing level 2 thought processes may not successfully complete a course in formal geometry. This is because their level of geometric knowledge, the level of instruction and the geometric level of the content of the textbook do not coincide.

For this study, only the first three levels (0,1,2) of geometric thought will be considered. The rationale is that the average students who enter high school geometry do not have a level 3 mastery of geometry. This implies that a study involving levels 0, 1, and 2 would be indicated for the group to be used.
**Problem Statement**

Up to the point of using Slope-Stick in the classroom a major portion of this study has been spent in developing the software. Slope-Stick was developed after realizing the lack of understanding of the concept of slope on the part of many students of mathematics. Students who come into the first course in Calculus are found to be lacking in their knowledge and understanding of the mathematical principles of slope. The present version of Slope-Stick is a result of much thought as to how the students might use the concept of slope in a new way. Slope-Stick was developed in the hope that the concept of slope might be easily integrated into other areas of mathematics which might not, normally, consider such a topic.

The purpose of this study will be to investigate whether or not

1. Use of Slope-Stick will improve the student's mastery of geometry.

2. Use of Slope-Stick will enhance students' understanding of the concept of slope and the use of the slope concept in geometric constructions.
CHAPTER II
REVIEW OF THE LITERATURE

The promise of computers for mathematics instruction has been debated and discussed in the literature. Computer use in mathematics first began on larger campuses in the early 1950s when very large main frame computers were used mainly for mathematical calculations. Later in the early 1960s IBM manufactured the first mass produced computers which were available to a wider market and were made available to large corporations, etc. who wished to use their power. All of the execution of the programs was done using batch processing; this meant delays between the definition of a program or potential problem solution and seeing the results. A great breakthrough came with the introduction of interactive computing which allowed for near-immediate results rather than having to wait until later for the output. Since the introduction of the microcomputer in the late 1970s, animated graphics on the computer has become more widely available. Since its inception, the microcomputer has been reduced in size, its usable memory has been increased, its processing speed increased exponentially and its cost greatly reduced. Each month brings new peripheral devices for added usability. Along with graphics came the hope by mathematics teachers and students that the computer would become more usable in the classroom. With the newer microcomputers
and their more detailed graphics capabilities came the desire on the part of mathematics teachers to incorporate the use of the computer into the geometry classroom.

Three questions will be examined in the review of the relevant literature:

1. What does the literature suggest about the promise of computers in learning and teaching of mathematics?
2. What does the literature suggest as a description of the 'average' geometry?
3. What does the literature suggest about the use of computers in the geometry classroom?

**Promise of Computers in Mathematics**

As early as 1967, there was a call for a broader integration of the computer into the mathematics curriculum (Porter, 1988). This was brought about by the availability of the significant computational facilities afforded by computers. There is presently a call for fundamental changes in the mathematics curriculum which will take advantage of these facilities (Porter, 1988; Schwartz, 1986).

Prior to the introduction of interactive computer graphics on affordable machines the uses of the computer were restricted to or reserved for those individuals who understood the inner-workings of the computer to the point that they could 'program' the computer to produce a desired result. Programming in a language 'friendly' to the computer was a prerequisite to any problem solving using the computer.
To use the computer of the 'state of the art' adequately in the mathematics classroom the mathematics community must realize the various modes in which the computer can function. These capabilities lead Schwartz (1986) to assert:

*Students must be given tools that make the making, exploring, and verifying of conjectures easy and engaging to do. Students must be told, both explicitly and implicitly, that such activity is meritorious and exemplary, rather than disruptive.*

When considering the uses of the computer in educational computing there are several paths which could be taken. Taylor (1980) suggests that there are initially three paths. They are: to use the computer as a tool, as a tutor or as a tutee. After choosing one of these paths one must consider which software and hardware configurations are most suitable for the proposed task. In order to determine the appropriate software/hardware configuration one should consider an appropriate model for the evaluation of the products and processes of the instructional design activity (Damarin, 1987).

*Some of the most straightforward and familiar uses of the computer have been outlined by Schoenfield (1988).*

**Drill and Practice** was the first type of programs widely used in the school classrooms. These software packages provide the students with any number of exercises which will drill the student on one or more particular concept(s). Many of these programs exist for use in the elementary school classrooms.
Tools that do the work, while we analyze the results of the work such as Geometric Supposer, Schwartz, Yerushalmy, 1986, a tool enabling the user to input requested characteristics of triangles. Upon specification of a procedure to be performed, the Supposer keeps the description 'in mind' to be retrieved and reproduced at the user's command. This tool gives the user the ability to check conjectures concerning geometric figures quickly.

Gaming environments (Green Globs, Dugdale & Kibbey) - Green Globs presents four games from which the user is able to choose. One game, in particular, displays a portion of the coordinate plane on the screen. At random points on the screen are 'globs'. The object of the game is to name an equation which will pass through the maximum number of 'globs'. The simplest method would be to determine the linear equation of the line through one particular point on the screen. For instance, if a 'glob' was located at the point (a,b) the simplest equation going through that point would be $y = bx/a$. Choosing two lines $x=a$ and $y=b$ would also work. More game points are earned by choosing a second degree equation which would result in the graph being a circle, ellipse, hyperbola, or parabola and going through two or more 'globs' on the screen. The user is thereby encouraged to determine more difficult equations.

Simulations - One of the more popular simulations comes from the field of biology. This piece is named "Frogger". It allows the students to dissect a frog in quite a realistic manner. The expense of laboratory frogs and the students' mistakes with those frogs is not a factor. Simulations have been developed for mathematics which are readily used in the classroom. TABSLAB of The Ohio State University has developed one
particular piece of software which demonstrates to the students the concept of probability. This is the TABS Lottery Disk (Damarin, et al, 1985). In mathematics, the most interesting simulations would mainly be derived from the applications of mathematics to other fields such as chemistry or physics. In discrete mathematics, logic boards are readily adaptable to simulation on the computer. By changing a Boolean expression representing any particular switch the user can readily see the change of the result of the logic system. For example, changing 'a and b' to 'a and not b' would allow the user to see the effect of the change immediately.

**Dynamic representations** - In mathematics these are particularly useful for graphing. Presenting students with an equation such as \( y = Ax^2 + Bx + C \), such software lets them change values of \( A, B, \) and \( C \) and visually determine the effects of the constants on the graph of the function. For instance, what happens when \( C = 0 \)? What will be the \( x \) and \( y \) intercepts of the graph?

A useful example of this is Master Grapher (Waits & Demana). This allows students to work with several types of equations and to see the graphs of those equations. The graphing capabilities which are available to the students are capabilities which would not be available through any other means. Students are able to select from polynomials of any imaginable degree with operations students normally would not be asked to graph. The software is capable of handling equations of conics, polar equations, parametric equations, polynomial equations, rational equations and other equations in two or three variables.
Some non-standard consequences of learning to program. Students being able to work with Boolean operators after programming experience is an example of a worthwhile addition to their mathematical 'repertoires'. Many of the intricate subtleties of mathematics are more easily understood in a programming environment. For instance, when programming for the graph of an equation around the origin, the student will frequently encounter a case where division by zero is a nuisance. A simple way around such a problem is to substitute \( 1 / k \) for the denominator where \( k \) is the largest integer value allowable in the software being used. The computer, being a discrete tool, interprets the very small number as a number other than zero and handles the operation without returning an error message. If students are challenged in this manner, the mathematics will become a more tangible tool.

Intelligent Tutoring Systems (WEST) - In this system the user is challenged to progress through a series of 70 squares. The user is given bonuses and pitfalls which enable the user to move forward or require that the move be in reverse. Each of three spinners exhibits a number. The three numbers are then arranged at the user's discretion. Two operations are inserted with the choices being addition, subtraction, multiplication, or division. The user also has an opportunity to group pairs of the integers which would make use of the distributive property. The computer-based tutor then decides if the decision is a good one. If not, the tutor will make a suggestion. The tutor does not intervene on all choices-only those which seem to be a bit 'far out.'
Computer-Based Microworlds (Logo) - Papert first described microworlds in *Mindstorms* (Papert, 1980). Each person has the capability of having a different microworld. Papert relates the story of a student whose microworld had only one turn on the computer screen. The student only turned right 45 degrees. All constructions were accomplished using only this turn. This student's microworld had a characteristic unlike any other but it belonged to that student. Many learning environments are created such the the student is either doing it right or doing it wrong. There is only one correct way and if the procedure does not use the correct way - then it is wrong. The microworld concept does not chide the user when a task is performed in an unorthodox manner.

These familiar uses of the microcomputer have opened up a wide vista which is available to educators which are interested in enhancing the learning of their students. The responsibility for the selection of this software at the appropriate time in the mental development of the student falls upon educators. Educators must be in a position to evaluate correctly the needs of the students and make wise software selections on the basis of those evaluations.

Developers of instructional software bear the responsibility of developing software which will not only be marketable but will aid in the educational development of the users.

A different approach to the design of educational courseware has been put forth by several educators. Papert (1980) introduced educators to the concept he called "Microworlds." These microworlds when presented within the context of computers and computer software provide the user with
an environment with a well-defined body of primitive operations. The user has the capability of using the defined body in many ways. Two users will not necessarily use the body of material in the same manner. An example of such a microworld is Logo. Within the Logo environment, the user is given relatively elementary rules to follow. It is then possible for the users to explore on their own in a non-threatening environment. One of the finer aspects of this situation is that the user immediately sees the results of the commands given. The computer does all of the calculations and the user is able to make further alterations to the program if so desired.

**Instructional Design and the Computer**  
Gagne and Briggs (1979) set forth a model for instructional design which rests upon a set of assumptions about the nature of the educational activity under consideration. They assume that it is both possible and desirable to perform a detailed analysis of the goals, objectives and priorities of the learner and to list these through the definition of performance objectives. These theories and models have been used for the production of the majority of software using the formats of drill and practice and tutorials.

For software to be developed with the above mentioned (Gagne & Briggs) underlying assumption a developmental model for software should be followed. Damarin (1987) defines such a model. Figure 3 illustrates the model.

The model has seven nodes leading up to the software design. **Content analysis** is the major node. It is at this point that the courseware designer determines just exactly what is to be the content of the proposed courseware. The courseware is developed with the idea in mind that the
computer is going to enable the designer to provide a new meaning or understanding of a concept to the user. The computer allows us to replace verbal explanations with graphical, pictorial and interactive experiences. The computer is able to handle the tedious calculations where feasible and desirable and to release a larger amount of time and effort to the user for discovery. Through the use of the computer it is possible to lower the grade level necessary for successful performance in many areas.
The second and third nodes are considered virtually simultaneously. Both depend upon the defined content and each are separate.

The cognitive learning goal has to do with the user's relationship with the content. Is the user able to perform the mental tasks which are
necessary? Does the user understand the meanings of all of the terms being used? Will the courseware provide the missing meanings? The learning goal has nothing to do with the relationship of the user to the computer or to the courseware per se. The cognitive learning goal has to do with the relationship of the student to the content of the courseware. The cognitive learning goal might be the same for a lesson to be taught in the traditional classroom manner. But, if so, the question, "Is the computer necessary?" must be answered appropriately.

The cognitive interactions pertain to the user's relationship to the computer and to the courseware. Will the user have a new appreciation for the material presented? Will the user be interested in the subtleties of the material? If the computer is able to do the tedious calculations quickly, will the user be encouraged to explore several different ways to accomplish the proposed tasks? Will the user feel comfortable using the peripheral devices necessary for the successful completion of the task? Will the user be encouraged to explore even further with the utilities of the courseware?

The cognitive interactions have to do with the student interaction with the software itself and, as would be imagined, are directly related to and dependent upon the cognitive learning goal. It is imperative that the cognitive interactions reflect the cognitive learning goals. If the cognitive interactions are more difficult to master than the cognitive learning goal, then the design is faulty.

The affective interactions are directly related to the affective learning goal. These interactions reflect and are easier to master than the affective learning goal.
Software and hardware specifications follow after the determination of the goals and interactions. Ideally, the software specifications can be proposed without having to consider available hardware. Practically, the software specifications are many times forced or influenced by the available hardware. For some courseware it may be necessary to restrict the software specifications to fit a particular hardware configuration. For instance, if a particular peripheral device is needed for the successful performance on the software, the available hardware is possibly restricted. If, on the other hand, only one particular hardware configuration is available, the software specifications will have to match this configuration. Thus, the interconnection of the software configurations and hardware configurations in the Figure 3.

After the above steps have been completed the courseware developer is ready to design the courseware. This is not the final step for the courseware has yet to be tested. Most courseware developed for the computer is tested many times. After testing the design may return to the Software Design stage. It may circulate between these two nodes for a time until the developer decides that the courseware is ready for distribution.

Even after distribution and after having received input from the users and others the developer may find that it is necessary to again conduct a content analysis and proceed through the flow chart. Users may suggest further content which might be included in the courseware package. The content analysis will then be expanded and taken through the other six nodes of the design model. It is not a forgone conclusion that revision will have to take place at each node. That possibility should be carefully
examined. The revision of software after such input is done quite frequently when considering the number of version numbers on some courseware. Courseware for the computer is continually being evaluated formatively - rarely summatively. Educators should be wary of courseware which has been produced and marketed with only one version. This would indicate assumed perfection or no further interest in the development of the courseware.

Papert (1980) introduced educators to the concept he called "Microworlds." These microworlds when presented within the context of computers and computer software provide the user with an environment with a definite defined body of instructional material. The user has the capability of using the defined body in many ways. Two users will not necessarily use the body of material in the same manner. An example of such a microworld is Logo. Within the Logo environment, the user is given relatively elementary rules to follow. It is then possible for the user to explore on their own in a non-threatening environment. One of the finer aspects of this situation is that the user immediately sees the results of the commands given. The computer does all of the calculations and the user is able to make further alterations to the program if so desired.

With the innovative technologies that are available to the mathematics community it is going to be exciting to witness the new mathematics which will be forthcoming. For example, the study of fractals has come about as a result of the technology of the computer. The origination of fractals came through the examination of a portion of the complex number plane. The mathematics of fractals has been lost in the beauty of the graphs of those
fractals. Along with fractals comes a new science of "Chaos" (Gleick, 1987).

Some in the mathematics community are voicing the opinion that we should not do too much for the student (Smith, 1988). Others are saying that we should teach the students what to do with mathematics rather than how to do mathematics. As an example of this, many of the statistics programs available for use in the study of statistics allow the software to remain transparent while only the results are used in the final analysis. In analyzing the results the students are able to spend the bulk of their energies on the analysis of the data rather than the arithmetic of their calculations.

**Characteristics of the "Average" Geometry Student**

The geometry knowledge of students at virtually all educational levels is problematic. The 1982 National Assessment determined that fewer than 10% of 13-year-olds could find the measure of the third angle of a triangle given the measures of the other two angles. Only 20% of the same age group could find the length of the hypotenuse of a right triangle when given the other two sides (Carpenter et al, 1983). These findings, among many others point to the dilemma facing educators. That is, a topic is not understood until it is taught. The more difficult concept of the right triangle together with the pythagorean theorem apparently has been taught more frequently or with more "gusto" than the triangle-sum theorem; still only one in five students have mastered it.

The study of proofs in geometry is one of the most important goals of the teacher in the geometry classroom. In a study of 99 high school
geometry classes, 85 of the classes studied proofs long enough for the teachers to be confident that their students had enough experience that a test on proof would be fair at the end of the year (Senk, 1985). None-the-less at the end of the year 28% of the students could not do the easiest triangle congruence proof. Only 31% of the students were judged to have competence with formal proofs. There were high correlations between the simple geometric concepts which the students understood at the beginning of the year and the performance on proof at the end of the year.

Studies have shown that a minimum of 30 percent of geometry students reason at the concrete operational level, with another 30 to 40 percent being classed as transitional reasoners (Farrell and Farmer 1985: McDonald 1982). This would indicate that the teacher would do well to use concrete models and concrete terms along with familiar terms when introducing new material to the student. In this manner the student could be coached into reasoning inductively.

Many geometry students have less than adequate spatial visualization. Geometry demands ability in spatial visualization. Ranucci devoted much of his writing in geometry to activities which would increase the students' expertise in mental manipulation of geometric figures (Ranucci, 1952).

**Geometry Curriculum K-12**

As early as 1902 in an address to the American Mathematical Society E. H. Moore called for an enriched and revitalized approach to the teaching of mathematics in the early grades. Suggested changes covered such
topics as observation, experimentation, reflection and deduction. As vehicles for teaching in a manner to reflect these changes it was suggested that drawing and paper folding in kindergarten could lead to a systematic study of intuitional geometry. Geometry should be closely connected to the numerical and literal arithmetic.

The Final Report of the National Education Association Committee of Fifteen on Geometry which was issued in 1912 declared that it was of the utmost importance that some work in geometry be done in the grade schools. The reasons given were, first, that geometric forms certainly do enter into the lives of every grade school child. Secondly, the ability to control geometric forms is a definite need in the lives of grade school children. An early appreciation of geometric forms is an absolute prerequisite for success in any future work in geometry.

Suggestions for revision of geometry - Allendoerfer

Fifty some years later, in 1969 Allendoerfer acknowledged pressures from reform groups both here and overseas and suggested it was time to 'do something' about geometry instruction. Possible solutions to the problem and also possible curriculum changes which he suggested were very much similar to the suggestions one hears today.

For the elementary school Allendoerfer suggested that informal plane and solid geometry be presented along with some ideas having to do with geometric formations. In the present we have software which is a definite reinforcement to the ideas of geometric transformations. Funky Chicken (Damarin, et al), for example, is a software package which cleverly acquaints the students with the concepts of transformations and in such a
manner as to cause the students to be very much interested.

For the junior high school Allendoerfer proposed more informal geometry along with the use of coordinates in algebra which would lead to graphing concepts and principles. He also suggested that the students be introduced to the elements of deductive proof.

For the tenth grade formal deductive plane geometry was proposed with informal solid geometry together with a possible brief inclusion of analytic geometry. Traditionally, this is the year that most students have taken the 'normal' high school geometry course.

Allendoerfer ventured even further into the high school curriculum to propose a full semester of plane and solid analytic geometry and geometric transformations in preparation for use in calculus. A warning was given that if these topics are not taught in the high school mathematics classes, then students would never get it for this material is rapidly being dropped out of the college mathematics curriculum.

Solutions to the performance dilemma - Usiskin

In 1972 Trafton and LeBlanc reported that by 1970 geometry had gained a permanent place in the elementary school mathematics program. It was thought that the emphasis on informal geometry had grown and developed into a securely held position in the elementary schools. Interestingly enough Usiskin in 1987 called again for a specific unified curriculum in geometry for the elementary schools.

There are two major problems which seem to prompt all writings about school geometry. These are: the poor performance of students and what Usiskin (1987) called an outdated curriculum. As a solution to the
performance dilemma, Usiskin suggests four steps which could be taken in the education community.

First, Usiskin suggested the setting forth of an elementary school geometry curriculum by grade level. This suggestion came in 1987, nearly two and a half decades following the Cambridge Congress on School Mathematics which was held in 1963. As a result of the report of that congress a report was produced which called for geometry to be introduced to and implemented in the elementary grades. This was to begin at the kindergarten level. As a result of the congress and of the report, interest in geometry at the elementary level was reported to be increased. It was felt that the changes which took place in the elementary mathematics curriculum were a result of the congress. Those changes either did not fulfill the expectations of the mathematical community or it was not satisfied with the geometry being taught in the elementary grades at the end of the 1970s.

It may be that changes did take place in the overall philosophy of the elementary grades but the curriculum was very dependent upon the textbooks which were being used. Some of the texts had very little in the students' text but left the bulk of the geometry materials only in the teacher's edition of the book. This would allow each teacher to cover the material thought to be important and enable the teacher to omit topics in geometry if so desired.

If Usiskin's renewed call for a uniform curriculum is heeded a geometry teacher in the high school would know the geometric background of the students entering the geometry classroom for the first time. There is a
definite need for uniformity in geometry education.

Secondly, Usiskin further admonishes educators not to hold back from geometry those students who may be poor in arithmetic or algebra. He states there are more students who are better achievers in geometry than in algebra or even arithmetic. Geometric thinking appeals more to the visually orientated students than the algebra or arithmetic courses. For such learners, a geometric figure might be more easily understood than an algebraic equation. Girls, as well as the boys should be encouraged to take high school geometry. Studies have shown that at the high school level the girls have equal ability to learn geometry as the boys (Senk and Usiskin, 1983).

Thirdly, Usiskin suggests a significant amount of competence in geometry should be required of all students. As a result of starting in the early grades the perceptual and spatial abilities would enhance the adjustment and progress of these students. Studies have shown that students do not enter high school geometry with adequate knowledge concerning geometry to succeed in the geometry classroom. As a result of the poor background in geometry, only about one half of all high school students take geometry while less than 50% of that small percentage achieve satisfactory success in the subject (Usiskin, 1987).

Finally, Usiskin argues that universities and colleges should require that all prospective teachers of mathematics should study geometry at the college level. Many teachers of mathematics have no formal geometry course beyond the high school level. Indeed, many university professors of mathematics have had no formal course in geometry beyond the high
Suggestions for resolution of performance dilemma - Usiskin

As early as 1969, Allendoerfer had suggested the curriculum dilemma stemmed from the fact that there is really no agreement as to what the subject of geometry is all about. Usiskin (1969) set forth three suggestions concerning the resolution of the curriculum dilemma. The suggestions are still pertinent to the present discussion.

First, the semantics used in discussions of geometry should be clarified. Many questions should be answered concerning the semantics. What is Euclidean geometry? What do the words forma, informal, and intuitive mean in relation to the teaching of geometry? What is proof? When is a proof valid? Is there a proof other than a formal proof? Is an informal proof sufficient as a proof or must something be proved formally? What is the purpose of the proof?

Second, the level, quality, and quantity of discourse in the discussions of the geometry curriculum must be refined. There seem to be three levels of geometry which exist. First is the geometry of the real world, second is the geometry mathematicians like to talk about and lastly is the geometry which is actually taught in our high school geometry classrooms. The call is for a unification of the geometry into one package which would be appealing to all three groups - the students, learners, those who apply geometry in the real world, and the mathematicians. Mathematics educators must analyze from the curriculum perspective, the various ways of conceptualizing geometry. Usiskin (1987) suggests four ways.
The first is to present geometry as the study of the visualization, drawing, and construction of figures. Students in the lower grades do not usually have the visualization and drawing abilities which are possible and would be helpful for their understanding of geometric concepts. So often, the teachers in the beginning grades are compelled to teach certain selected topics and thus do not have the time during regular class periods to provide for students the drawing activities and visualization exercises which could enhance their concepts of geometry.

Secondly, geometry should also be presented as the study of the real, physical world. How is geometry used by carpenters? What are some examples of geometric shapes in nature? What are some geometric shapes the students see as they walk or ride to school? How is geometry used to find the volume of the local swimming pool? Or the hexagonal fountain downtown? Examples of geometric shapes and patterns could and should be brought into the classroom for the students to identify, sketch, look at and feel. Examples of geometry in nature provide a direct link between the sometimes abstractness of geometry and the reality of geometry forms.

The presentation of geometry as a vehicle for representing mathematical or other concepts whose origin is not visual or physical is the third necessary approach. Examples of this exist which could easily be used. The cartesian coordinate system for representing algebraic relations geometrically is just one example which leads to examples from calculus having to do with the area under the graph of a polynomial equation from one particular value of x to another value of x. Graphs are actually
geometric expressions of algebraic relations. The graphs intensify the information displayed. Geometric graphs provide useful tools for the comparison of pairs or a group of many curves.

Finally, geometry should be examined as an example of a mathematical system. For many students, this would be their introduction to the concept of a mathematical system. Geometry was the first branch of mathematics to be logically organized. This long history brings with it some ideas that have been slow to be eliminated. In fact, many of them have not been eliminated. This mathematical system should only be the introduction to further mathematical systems. Geometry should be the springboard from which the students are introduced to other mathematical systems.

Recommendations for enhancement of the first course in geometry - Niven

Niven (1987) sets forth recommendations to enhance the beginning course in geometry. Many geometry courses begin in chapter one with the introduction to axioms and postulates and the assignment for the first evening might be to memorize the first eight axioms. According to Niven, beginning geometry should be taught in the same way that beginning algebra and beginning calculus students are taught - without putting extreme emphasis on rigor. Thompson (1910) wrote in his calculus book:

You don't teach the rules of syntax to children until they have already become fluent in the use of speech. It would be equally absurd to require general rigid demonstrations to be expounded to beginners in calculus (and geometry).

A course in geometry without the rigor of 'traditional' geometry would be a fascinating course for the teacher to teach and for the student to experience.
This would allow for an abundance of diagrams and informal proofs which would fascinate and excite the students without having them burdened with the rigors of formal proof.

Niven suggests that teachers should quickly get to the heart of geometry. For example, they should not allow the Pythagorean theorem to be put toward the end of the school year. The students are able to handle a simple algebraic proof of the theorem soon after beginning the geometry course. This would enable the students to have the theorem as a tool in topics and concepts which follow. A linking of geometry to elementary algebra would be a bonus for the students. They would quickly see that the algebra and geometry do have some common bonds.

The techniques of algebra and analytic geometry should be included along with the classical Euclidean models. This would facilitate an integration of geometry with algebra. It would exhibit to the students the connections between algebra and geometry and would allow for proofs of the more classical theorems early in the course. Many geometric proofs are nearly trivial when proved using algebraic or analytic geometry concepts.

Diagrams should be used in all instances - especially when dealing with proofs. Geometry teachers would do well to require figures for satisfactory solutions of any geometry problem. The figures should be correct for the given exercise, although not necessarily to scale, and all possible figures should be included by the student. An example of this would be: Given two sides and the altitude from the first side to the opposite angle - to construct a triangle. An acute and an obtuse triangle should both be presented and be considered as being possible correct solutions.
Geometry should be related to the mainstream of mathematics and to the real physical world. The early Greeks determined the radius of the world to be approximately 3960 miles. This was done using their 'crude' instruments and their knowledge of geometry. As the world has become more highly educated the role of geometry as a topic of study has been assigned the textbook and traditional classroom teaching methods. A geometry teacher would be wise to propose problems from the world of reality. Some problems concerning the theorems related to right triangles which would useful in the building trades would provide the students with a real world example. Students tend to be more eager to learn if they can see an application in 'real' life which would use such information.

Traditionally, geometry has burdened the students with verbose statements, theorems, postulates, etc. The obvious, at times, has been elaborated to great lengths. The theorems could just as easily be related to specific figures: triangle ABC, rectangle ABCD, etc. For example, a theorem as stated by Euclid states:

If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

The same theorem could just as easily have been stated:

Given triangles ABC and DEF: If AB = DE, AC = DF, and angle BAC > angle EDF, then BC > EF.

The proofs of some of the difficult theorems should be postponed or omitted. This would apply, particularly, when a proof would be extremely difficult using the methods of geometry but would be relatively easy in a
later course. The use of the theorem without proof would be of no detriment to the student. The proof of a theorem using more advanced methods which would provide for an easy proof, in no way, takes away from the importance of nor the beauty of a particular theorem. On the other hand, if we use elegant methods to prove a theorem which could be proved just as easily with a more elementary set of theorems then we are causing the theorem to seem more difficult than necessary.

The problems in the textbooks should be numerous and of intermediate difficulty. All students should find problems which they would consider of moderate difficulty. If extra-credit problems are assigned, all students should find problems which would not be considered too difficult or even impossible. All should be given the chance for extra credit.

An example of theorems, theories, etc. in geometry which students hold in awe is the inability of one to construct the trisection of an angle while using an unmarked straight edge and compasses. Students should realize the construction is impossible with those stipulations. The students should also be told and shown that the construction is accomplished using the compass and straight edge and making two marks on the straight edge.

One of the causes for the problem with geometry and how we consider its use in our curriculum is that it is so much unlike algebra. Arithmetic and algebra have for their objectives the development of the properties of the fields of rational, real, and complex numbers. The notion of a field is relatively easy to grasp and the importance of this field is unquestioned. It is all in a neat package and the only room for discussion or innovation is in the pedagogy involved.
On the other hand, geometry is not easily defined. There is not a lot of agreement as to what it is really about. When asked what geometry is, Oswald Veblen said that 'geometry is what geometers do.' Some may think it is the study of the invariants of transformations groups. To others it is just the study of geometric figures. To the average high school geometry student it would be associated closely with a method of proof. The average geometry class consists of proofs of theorems and then proving geometric statements which are based on those theorems. There are many geometries. Some would do away with Euclidean geometry. Others would say we need to study the affine, non-Euclidean, projective, algebraic, and differential geometries. When we decide which geometry we are going to study we then consider if we are going to confine our studies to two dimensions or pursue the study of geometry in three dimensions and even in $n$ dimensions. The problem becomes apparent - which geometry will we agree on as being most important for the elementary and secondary schools? There is no agreement among educators at this point.

**Three approaches to geometry - Allendoerfer**

The geometry of instruction is basically made up of three approaches to geometry. The first is the traditional synthetic method. This is the method which was used by Euclid and is most familiar to mathematics students. The second approach is the analytic method which was used first by Descartes. This provides for the study of geometry in conjunction with a coordinate system which is easily associated with the field of real numbers. This type of geometry is one of the most useful tools used by students of mathematics for the study of the calculus and higher mathematics. It moves
easily into the realm of science and is understood by most students of the sciences. The proofs in analytic geometry are easily understood by the algebra student for after the geometric relations are expressed in algebraic terms, the proofs reduce to algebraic manipulation of symbols. Vector geometry is the third approach. This approach develops its theories through the use of standard vector algebra. This approach is coordinate-free and its methods are closely related to those in physics. This method also relates well to vector spaces which is a concept very important in the study of modern algebra.

Allendoerfer viewed these three approaches as merely tools and each as useful in its own way. The result of this is that students should be conversant with them all. The curriculum problem is congested when one attempts to design a program which would introduce the students to all three geometric methods.

_Turtle Geometry - Papert, et al_

In addition to the three approaches suggested by Allendoerfer consideration should be given to turtle geometry (Papert, 1980; Abelson and diSessa, 1980). Turtle geometry not only provides another approach but is a vehicle whereby geometry is easily adaptable into the mathematics curriculum at all levels. Just as Euclid and Descartes developed their own styles for geometric thinking turtle geometry provides another style of geometry. Turtle geometry is the computational style of geometry while Descartes' style was algebraic and Euclid's was logical.

While the geometries of Euclid and Descartes require a certain amount of mathematical maturity, turtle geometry is easily introduced to the
students in their first educational experience - whether it be in kindergarten or in the first grade. Because of this, it is possible for students to become acquainted with turtle geometry, be comfortable with it, and pursue its ideas to more complex structures within mathematics.

Turtle geometry, for the most part, makes use of the computer. The computer performs the repeated calculations which are necessary for the moving of the turtle. But, as Abelson and diSessa indicated, turtle geometry can be done using only pencil and paper. A comparison of the two technologies, computer and paper-and-pencil, would soon convince the students of the 'niceness' of doing turtle geometry on the computer.

Many conceptualize turtle geometry being a 'nice' programming language for grade school children who might want to do some attractive drawings on the computer screen. Turtle geometry is more than the geometry experienced by grade school children. With the mathematics suggested by Abelson and diSessa (1980), it is possible that turtle geometry could be taken into the high school mathematics classroom and the undergraduate classroom. Many fail to realize the abundance and richness of topics which are used in the programming of the turtle for its movement on the computer screen. Most, if not all, of the data structures contained in Pascal are also available for use in turtle geometry. Would that turtle geometry could follow the students through the middle school and high school years.

**Place of geometry in the mathematics curriculum - Fehr**

In 1973 Fehr suggested geometry should become a part of every other part of the mathematical study - linear algebra, analysis, and applied
mathematics. It should become an integral part of each of these and should become a way of thinking. The study of geometry should be made from many approaches. These approaches coincide closely with those suggested by Allendoerfer.

The first is the physical informal approach. Students would work with drawings, paper folding, and apparatus to gain an intuitive feeling for geometric figures. In the present we could add the computer as a tool for aiding with this approach. In Bulgaria, students use the computer along with the Koala pad to produce drawings on the computer. This would allow the students to draw figures easily, see the results and quickly change the drawing if they wish to make changes or begin again. This approach is easily adapted to the lower grades and will serve to introduce the students to geometric concepts.

Fehr's second suggested approach is the synthetic-axiomatic approach. This would consider the affine plane with a minimum of axioms and would include both the finite and infinite models. To keep the study in two-space would leave the concept at a more elementary level without extended formality.

The coordinate-analytic approach is the one which is used extensively in later mathematics courses. This allows for the assigning of coordinates to the affine plane and subsequently to Euclidean two- and three-space. This allows for the usual algebraic techniques.

Transformations as another approach studies mappings of the plane and three-space and eventually relating the study to group structure and matrix algebra. Vectors as an approach examines the algebra of points in
two- and three- space and introduces students to the concepts of inner product which allows for the study of perpendicular and Euclidean space. Vector spaces and linear algebra follow using the above approaches as a foundation. This entails the building of an axiomatic n-dimensional vector space and its linear algebra.

Throughout the approaches suggested by Fehr (1973) geometry is a definite tool to be used in the more complex disciplines of mathematics. Each of the approaches builds on the previous approach. None is independent of the others mentioned before it. Through these six approaches to mathematics using geometry the serious student not only has an adequate foundation in mathematics principles but also a solid base of geometric concepts to produce further studies.

Curriculum Standards - National Council of Teachers of Mathematics

As recently as 1987, the National Council of Teachers of Mathematics has distributed draft copies of curriculum standards for mathematics in all grades K-12 (Romberg, 1987). This group has divided the grades K-12 into three levels of education. They are: K-4, 5-8, and 9-12, coinciding with primary, middle, and high schools for the most part. There is some irregularities concerning whether it is K-4 or K-5 and 5-8 or 6-8. Each school district may differ from a neighboring school district.

The proposed standards deal with mathematics education for the three levels of education. The commission has suggestions as to which types of geometric instruction should take place for each of the three levels.

For grades K-4 the commission suggests five distinct educational objectives. All of the objectives deal both in two and in three dimensions.
First, students should be able to describe, model, and classify shapes. This gives the students the opportunity for 'hands-on' experience with the various geometric figures. Secondly, the students will investigate and predict the results of combining, subdividing, and changing shapes. This, for example, requires higher order thinking skills beyond just repeating the definition of a triangle. The students will, through prediction and possible rejection of that prediction, soon spend more time in careful consideration of the problem.

The third, and possibly the most important objective is to develop a spatial sense. The teacher in the early grades has the opportunity to challenge those who might possess lower level spatial skills to develop those skills more fully. The teacher also is challenged by those students who have elementary or no spatial skills. This is the chance of the classroom teacher to prepare the student for higher order spatial skills.

The fourth objective is to relate geometric ideas to numeric and measurement ideas. This objective could be correlated with the third objective when using shapes the students can hold in their hands, measure, and feel.

Objective number five is an objective which could be used at all grade levels. That is, to use geometry in solving problems. This provides an excellent opportunity for teachers to relate the lessons from the text books to the everyday shapes and figures with which the students are acquainted.

For the grades five through eight, the commission is suggesting mathematics curriculum should include the exploration of geometry in a variety of situations. This is accomplished through five stated objectives.
This first objective is to identify, describe, and compare geometric figures. This objective is similar to the first objective for grades K-4. Secondly, the students will visualize and represent one-, two-, and three-dimensional geometric shapes. As with the earlier grades, visualization and representation is strengthened through the use of models, manipulatives, and actual items which can be seen, touched, and handled. Contained within this objective would be the students' ability to describe verbally the objects as they are felt, seen, or drawn. The successful teacher will not spend inordinate amounts of time memorizing definitions or computing areas, volumes, or perimeters. The teacher will focus on investigating geometric ideas and relationships. With the middle-school level of social and intellectual development the informal exploration of geometry will be exciting and at the same time productive.

The third objective for middle-school geometry is to understand geometric relationships and their consequences. For instance, a three sided figure is rigid. A four sided figure is not rigid but becomes rigid when a diagonal is inserted. This is excellent for exploration on the part of the students. Objective number four, not surprisingly, deals with problem solving. The students will apply geometric properties and relations in solving problems and in developing other mathematical concepts.

The fifth objective is to recognize and appreciate geometry as a means of describing the physical world. This objective, when successfully accomplished, will provide the student the opportunity to relate this recognition and appreciation not only to geometry but to other topics in mathematics and science.
The geometry objectives for grades nine through twelve become two-pronged. The commission calls for geometry to be taught from a synthetic perspective and from an algebraic perspective. The two perspectives with their objectives for the college-intending students provide a summary. For the synthetic perspective the students will develop an understanding of an axiomatic system through investigating and comparing various geometries. These two go beyond what is traditionally contained in the high school geometry syllabus. The algebraic perspective allows the students to deduce properties of figures using vectors and to apply transformations, coordinates, and vectors in problem solving. Attainment of the objectives for the algebraic and synthetic perspectives will equip the students for further mathematics courses.

In considering the above mentioned sources for the revision of the geometry curriculum at all levels of pre-college education one feels the need to consider what might be the outcome of the suggestions and proposals.

Consideration of the curriculum standards as presented by the NCTM provide an excellent geometric preparation for further study. Several proposed changes in content and emphases for the high school geometry course will provide the students with an excellent pre-college geometry background. Along with the changes in algebra, the changes in geometry will provide for computer-based explorations of two and three dimensional figures.

The proposed change in algebra having to do with the computer has to do with the graphing of equations. The algebra student should be
provided with the opportunity to recognize the graphing of the equations to be the beginning of what has traditionally been called analytic geometry. Analytic geometry, being a subset of affine geometry, provides the perfect opportunity of providing for the joining of geometry and algebra. Theorems from geometry can be proved using elementary algebraic techniques. Teachers should seize this opportunity.

For example, give the students three linear equations, no two of which are parallel. The students will then be asked to find the coordinates of the intersections, find the length of the three sides, find the midpoints of two of the sides, find the distance between those two midpoints, compare the length of the line segment between the midpoints with the length of the third side, compare the slope of the line segment between the midpoints with the slope of the third side of the triangle, and deduce the relationship between the slope and length of the line segment joining the mid-points of two sides of a triangle and the third side. This could be accomplished as a class exercise or an exercise for the students but it is a means of introducing the students to one of the important theorems from geometry. The students will not have proved it but after the teacher confirms the truth of the theorem, the students will recognize its value. Especially, when problems involving the use of this theorem are presented to the students. Many of the theorems of geometry are easily proved using algebraic rather than logical arguments. Teachers should realize the importance of relating geometry to other mathematics topics. Some do not relate geometry to any other subject - "Geometry is only taught after Algebra I and the students will learn geometry in that class. Successive mathematics classes will not do geometry."
The integration of geometry into all pre-college mathematics classes would be a goal to accomplish.

One significant possibility for the geometry curriculum has not been addressed by the NCTM standards. That significant possibility is turtle geometry. Turtle geometry provides for a computational style of geometry. This computational style enhances the inter-relationship between geometry and algebra. This enhancement becomes evident as more advanced problems of turtle geometry are considered. With the acceptance of turtle geometry and thus the acceptance of a computational style the students will easily move into other areas of mathematics which are understood using the computational style. For example, number theory has a definite computational style. In fact, number theory provides examples of several of the program structures and thus the logical structures of computer program. This reality provides for not only a stonger mathematical background but for an introduction to programming logic.

Including turtle geometry with the curriculum of the NCTM would allow the students to transport ideas and concepts from earlier courses into more advanced courses. Recursion in the early grades, as mentioned before, will be an old friend when encountered in more advanced topics of mathematics such as number theory.

Milauskas (1987) contends that students learn problem solving by solving quality problems. Problem solving should be the underlying theme in the mathematics classroom. In light of this, problem solving exercises should be given regularly to students - not just at the end of the course. There are problem solving exercises available which exercise virtually
every problem solving technique. Those would include: recognition, basic drill and algorithmic practice, applications, open applications, true applications, algebra, extensions and open searches. This list provides a wealth of possibilities of problem solving exercises.

In years past competitions have been held in which teams of geometry students were given a group of problems involving geometric construction. These constructions all required problem solving skills for the problems were unlike any problems appearing, at that time, in the geometry textbooks. The problems demanded the students rely on their ability to recall construction procedures and problem solving skills which would allow for successful completion of the problems. This problem solving exercise will improve the student's ability to deal with the more difficult problems encountered in geometry.

**Computers in the Geometry Classroom**

Computers are flexible and powerful drawing tools. Many (most) of the drawings generated on the computer screens are generated from within a program written for the computer. The computer also has the capacity of being a tool for thinking. Considering the fact that geometry involves drawings and thinking it should be a logical conclusion that the computer would be perfectly suited for the teaching and learning of geometry.

There are many variables involved in choosing a path to follow when considering a geometry course which would use the computer heavily. Among the variables to be considered would be which topics in geometry would be well suited for the computer? Which computers would be best utilized in the teaching of those topics? What is the software available or
needed for the teaching of those topics on the chosen computers? If the computer-based software can help to free the mind of the user and allow him to explore the possibly unknown areas of geometry - it serves a very important purpose.

For geometry on the computer, at this time, Logo has been the most widely used piece of software for instruction. Logo is a computer language which must be mastered in order to adequately produce programs which will illustrate various principles of geometry.

Logo is not just a programming language but it allows for learning through discovery, developing problem-solving skills, and it supports the teaching of geometry. Logo is used effectively in the classroom as a tool for the teacher to use in the preparation of materials for students who may be only at the novice level of understanding Logo.

The student who is competent with and comfortable in the use of Logo can be given more challenging work in the form of writing procedures which will produce particular shapes which might be a bit out of the ordinary geometry syllabus. This would provide a classroom atmosphere for the more advanced student which would be stimulating for the student and would add to the overall classroom attitude toward geometry through the realization of the capabilities of Logo as a tool for geometry.

With the coming of Logo-Writer the student is given opportunity to integrate the word-processing capabilities into the construction of geometric figures.

The Geometric Supposer is a program for the microcomputer which allows students to define figures on the screen. This allows for rapid
analysis of figures by the user. The user may then make and test conjectures as to the behavior of further figures and parts of those figures. Swartz and Yerushalmy argue that the student should be encouraged to make conjectures based on the figures and should not be criticized for conjectures which do not prove to be true. The learning experience will be very valuable as the student conjectures and finds that the conjecture is true or false.

With the availability of applications packages on the Macintosh computers the students have available some of the most sophisticated procedures for producing high-quality geometric figures.

MacPaint and MacDraw provide the students with applications packages which are very user friendly. The packages allow for various fonts, shading patterns, line-drawing capabilities, curve sketching, and free-hand paintings and drawings. MacPaint allows for dot-by-dot editing of graphics screens. MacDraw provides the utility of manipulating objects.

SuperPaint is an integrated package allowing for the capabilities of both MacDraw and MacPaint. The user is able to choose the 'layer' in which the work is to be done. The tools for each of the layers vary. It is possible to view each layer while working or to view only the layer that it being used.

The two layers allow the user to place the paint layer underneath the object layer and trace existing detailed dot images with object tools. Or, the user can create objects first, experimenting with their size and shape, then turn them into dots in the paint layer and touch up the fine details.
The application packages provide for full-screen editing, multiple windows for cutting and pasting and three levels of magnification. The view can also be reduced to illustrate how the figure will appear on a printed page. The user is able to produce shapes larger than the screen. In the paint layer it is possible to rotate, distort, and slant.

It is possible to import MacPaint and MacDraw figures for editing and production as SuperPaint files. Also, the user is able to create startup screens in MacPaint. The provides an outlet for the students' creative abilities. LaserBits allows for dot-by-dot editing at 300 dots-per-inch resolution. This provides high quality printing on a laser printer.

The geometric shapes are readily transported to word-processing files allowing for the integration of print and graphics in the same document. This, again, provides a creative outlet for the students' abilities.

What is Slope-Stick?

Slope-Stick is a stand-alone geometry application package for the Macintosh microcomputer. It is a tool for constructing straight lines on the monitor screen.

When beginning Slope-Stick, the user's name is requested, the name is typed in. The main screen with the main menu is displayed giving the user a choice of drawing a line segment, finding the distance between two points, going to the help menu, or renewing the screen.

When choosing to draw a line segment the user finds that the cursor changes to crosshairs. At this point the user will move the cursor to the portion of the screen in which the line segment is to begin. After choosing the point at which the line segment is to be begun the mouse button is
depressed and held in this position until the desired line segment is drawn. The line segment is drawn by moving the cursor away from the point at which the button was engaged. As the student moves the cursor the line segment between the original point and the cursor follows the cursor. As the cursor is moved, the previously displayed line segment disappears and a new line segment is drawn. Also, as the cursor is moved, the length of the existing line segment is displayed in a window to the right of the active window. The slope of the line as it is viewed by the user is also displayed in the display window. The slope is expressed as a ratio of signed numbers and also as a quotient. This allows the user to see the ratio which produces the quotient. This continues until the user releases the mouse button. At this time the cursor changes to a pointer which indicates the user is to move to the menu buttons for another selection.

After releasing the mouse button the screen which the user has just produced is saved to the floppy disk. This allows for later examination by the user or by the teacher, depending on the version of the software.

If the user chooses to find the distance between two points on the active window, that menu button is chosen. The cursor again resembles crosshairs which signals the beginning of the distance-finding activity. The cursor is moved to a point at which the user wishes to begin the measurement. For beginning to find the distance between two points the user clicks the mouse at the desired point. As the user moves the cursor away from this beginning point the distance between the original point and the cursor is being displayed in a window to the right of the active window. The slope of the imaginary line connecting the two points is viewed by the
user in the display window. The slope is expressed as a ratio of signed numbers and also as a quotient actively in the construction window. When the desired distance is measured, the user again clicks the mouse button. At this point the cursor changes to the pointer indicating the need to return to the menu buttons.

After clicking the mouse button the second time the screen which the user has just produced is saved to the floppy disk. This allows for later examination by the user or by the teacher, depending on the version of the software.

If the user chooses to renew the screen that menu button is activated. At that time the construction screen will be completely erased and a clean screen will be visible.

If the user chooses the menu button named 'Help' the screen is completely cleared and a help screen appears. The user has the capability of choosing types of polygons. Those types are square, rectangle, parallelogram, rhombus, kite, and trapezoid. The user also has the opportunity of choosing the main menu which allows for the return to the starting menu. If this option is chosen, the screen reappears exactly as it was when the user chose the help menu.

If the user chooses one of the quadrilaterals for viewing, the viewing screen is filled with types of quadrilaterals which are defined as being the quadrilateral chosen by the user.

Under the 🍎 at the top of the Macintosh screen the user may choose a desktop accessory called calculator. This causes a calculator to appear on the screen for the user's use. The calculator is helpful when the user
wishes to find the negative inverse of a given slope. This provides the slope of the line perpendicular to an existing line or a known slope. It is best that the calculator be used when the help screen is displayed. If the calculator is used while the main menu is visible the calculator will cause everything on the screen and under the calculator to be erased.

The user can choose the main menu button which returns the last screen visible before the help button was chosen. The user then is capable of continuing with the exercises.
Research Design

Subjects
There were four groups of students used for this study. One class of ninth grade Algebra I students who use computers in the classroom and one class of tenth grade Geometry students who use computers in their classroom comprise the experimental group. One class of ninth grade Algebra I students who do not use the computer in the classroom and one class of tenth grade Algebra II students who do not use the computer in the classroom comprise the control group.

All of the students attend an integrated high school in the Columbus, Ohio Public School System. The students in the classes using computers were a part of a larger experimental setting in which computers are used for all classes and were chosen for the ACOT experiment in the following manner. The students' names were drawn from a pool of volunteers which met certain criteria. Those were: they must live within the established school boundaries, they must have performed above the 36th percentile on standardized reading and mathematics tests, were willing to take Algebra I as a required mathematics course, and were able to persuade a parent to become involved. The group was balanced for sex and race to reflect the
racial composition of the school and district.

The students who use the computers in their classwork are located in two rooms on the third floor of West High School. The classrooms are designated as ACOT (Apple Classroom of Tomorrow) classrooms with signs outside the door attesting to this fact. The computers are arranged on tables with two students to a table. Each of the classrooms holds approximately 30 students. The students' computers are networked via the JANET network to two file servers at the back of the room. The file servers also serve as a means of communications with the teachers. There is a bank of printers at the rear of the room for the students print-outs. Each of the students has a Macintosh computer with an external disk drive in the classroom. Each Macintosh has one megabyte of RAM which gives the students the capability to handle the more sophisticated software packages. Each student also has a Macintosh at home for study use.

The control group was composed of the Algebra I and the Algebra II classes which did not use the computers. The classes volunteered to be part of this study. The Algebra I class was taught by the same person who taught the ACOT Algebra I class. The Algebra II class was taught by another teacher not connected with the ACOT classroom. A background study on both classes in the control group was not performed.

**Treatment**

There were five sessions for each group of students in the treatment group and two sessions for each group of students in the control group.
The experimental group of students took the pretest, had three sessions with the Slope-Stick software and then the posttest was given. The control group of students took the pretest on the same day that experimental group took the pretest. The control Algebra I class took the posttest on the same day that the experimental group took the posttest. The control Algebra II class took the posttest on the day following.

Table 1 Schedule of Pretest, Treatment, and Posttest by Groups

<table>
<thead>
<tr>
<th>Date</th>
<th>Activity</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 29</td>
<td>Pretest</td>
<td>All Groups</td>
</tr>
<tr>
<td>May 2</td>
<td>Treatment Part I</td>
<td>Experimental Group</td>
</tr>
<tr>
<td>May 3</td>
<td>Treatment Part II</td>
<td>Experimental Group</td>
</tr>
<tr>
<td>May 4</td>
<td>Treatment Part III</td>
<td>Experimental Group</td>
</tr>
<tr>
<td>May 5</td>
<td>Posttest</td>
<td>All Groups but control Algebra II</td>
</tr>
<tr>
<td>May 6</td>
<td>Posttest</td>
<td>Control Algebra II group</td>
</tr>
</tbody>
</table>

Instruments

The Polygons and Lines test is composed of 31 multiple choice questions. The first 15 have to do with the characteristics of particular polygons; the questions 16-31 test the students on the concept of slope. The Polygons and Lines test was site-tested. The results of the pilot test are in the Appendix A of this document. The site-test used two forms. Only one form of the test was used for the pretest and posttest of this study.

The treatment which was administered to the experimental group is composed of two parts, written construction exercises and the computer software.
Software

Slope-Stick is an application package for the Macintosh written in BASIC and compiled into machine code to add speed to the performance of the user. The software was written by the author for this study and is appropriate for more general use in the mathematics classroom.

Slope-Stick is a tool which allows the students to construct lines and/or measure distances on the screen using the mouse. Upon beginning the program and typing the required name the user then may select to construct a line segment. As a student presses the mouse and holds the button down a line segment is continually drawn from the point at which the button was pressed to the current position of the cursor (Figure 4).

Only one line segment is visible on the screen - the line segment which connects the first point to the present cursor position. As the user moves the mouse over the screen the user is able to read from a window on the screen the slope and length of the line segment being drawn. The slope and length of the line segment are continually changing values as the mouse is moved - until the button is released. The slope is expressed both as a ratio and as a quotient. For example, the slope 1 might have a ratio displayed of -2.5/-2.5.

To find the distance between two points, the user first clicks the mouse and then begins moving the pointer around the window until the desired point is reached (Figure 5).

As the pointer is moved around the slope and length of the present location is continually displayed. The button is again clicked and the distance is displayed on the screen along with the slope.
The software has two other capabilities which the students used.

There is a button to choose called 'Renew' (Figure 6).
Figure 4 - Constructing a Line Segment - A Slope-Stick Screen
Figure 5 - Measuring Distance Between Two Points - A Slope-Stick Screen
Figure 6 - Renewing the Screen - A Slope-Stick Screen
This merely erases the window which the student has been working with. The other button is the help button. After choosing 'help' the student is given a new screen with an opportunity to choose several examples of quadrilaterals (Figure 7).
Figure 7 - Help Menu - A Slope-Slick Screen

Click appropriate button
The choices are square, rhombus, rectangle, parallelogram, kite, trapezoid and quadrilateral. When choosing one of these the student is given a screen full of examples of that particular quadrilateral (Figure 8). After the user has seen the figures he wished to view he can then return to the previous screen. Nothing that was left on the screen before going to 'help' will be lost; the screen will be just as the user left it.
Figure 8 - Demonstration Screen from Help Menu - A Slope-Stick Screen
The version of software used causes the screen to be saved to the floppy disk each time the user completes drawing a line segment or completes finding the distance between two points.

Each student was given a disk with "Slope-Stick" on it. The disk was formatted in such a way that the user went straight into the program after the disk was booted cold. The software does not allow the student to exit the program at any time while the computer is left on. To exit the students typed Command-Shift-1. This allowed all of the screens produced by the users to be saved to the floppy disk.

Each of the three days the students were given a new disk which was a duplicate of the disk received on the first day - but, with nothing else on it. All of the disks for each day were collected and kept in a group.

In addition to the window with the figure being saved to the floppy disk, the time at which the screen was saved was recorded on the disk along with a record of the length and slope of the line segment - slope being expressed as a ratio and a quotient.

**Construction exercises**

The construction exercises are comprised of nine tasks for the students to do using the Macintosh. Each is to a degree more difficult than the preceding task. The tasks are administered in printed form. After each task is a grid on which the students may record lengths, slopes and ratios. There are several questions to be answered as the students progress through the tasks. The questions are meant to prompt the students in their thinking.
All of the students were given the tasks on the first day and given the opportunity to progress at their own pace. The classroom teacher and the author walked around the room answering questions as they occurred. Some of the students finished the tasks on the second day but all finished by the end of the third day.

Statistical Design

The study was conducted using the Nonequivalent Control Group Design (Campbell and Stanley, 1963). This design was chosen because the control group and the experimental group did not have pre-experimental sampling equivalence. The treatment was assigned to a particular group - the 9th and 10th grade students who use the Macintosh computers in the classroom.

A 2 X 2 analysis of variance was used to determine if effects are due to treatment or to group. Regression and interaction between selection and variables such as history, maturation and testing were possible sources of invalidity. Maturation was controlled by means of administering all of the pretests on the same day and all of the postests on or near the same day with the three treatments falling one day after the other. This insured an equal amount of time between the pretest and postest for all groups. Course content in the classroom during this time did not contain any of the information needed for successful performance on either the pretest, treatment or postest. History was controlled in this manner. An advantage of the design was the lack of possible effects from reactive arrangements.
Examination of Student Activity on Slope-Stick Disks

A sampling of the students' disks were examined and data was collected concerning the manner in which the students constructed the line segments.

Method of Examination The students' disks were examined using software created by the author. The students' discs were inserted into the computer and the program was called from the desktop.

Upon beginning, the program requests the name of the student whose software will be examined (Figure 9).
What is the student's name? Joe

Figure 9 - Screen For Entering Student Name - Viewing Student Screens
The student's name is typed on the keyboard and the program begins by displaying the first screen produced by the student using Slope-Stick (Figure 10).
Figure 10 - Initial Viewing Screen - Viewing Student Screens
The students' name is displayed in the rectangle below the rectangle containing the buttons. As well as displaying the first screen, a menu is provided for the user from which to choose the manner in which the software is to be examined. If the software is to be examined screen by screen with no deviation from the order in which the screens are examined, the user will select the "Next" button. If the user wishes to have a grid to assist in the evaluation of the student's work the user will select the "Grid On" button. If this button is selected a grid will overlay the viewing square (Figure 11).
Figure 11 - Screen Showing Grid On - Viewing Student Screens
If the user wishes to remove the grid, the "Grid Off" button will be selected and the screen will appear as in Figure 10. Should the user wish to examine a previous screen the "Previous" button is selected causing the screen just previous to the present screen to be displayed. The occasion might arise on which the user might desire to view a screen significantly numerically higher or lower than the screen presently being viewed. If so, the "Enter No" button is selected at which time the user is to enter the desired number (Figure 12). The user might want to print out some of the screens produced using Slope-Stick. If so, the print screen button is selected causing the entire screen to be sent to the printer for printing.
Figure 12 - Screen for Entering Number of Screen Desired - Viewing Student Screens
CHAPTER IV
STATISTICAL ANALYSIS

Introduction

Statistical analysis of the data from the pretest and posttest using the Slope/Polygon Test was the major consideration. An analysis of what the students did while using the software and an analysis of what was revealed about what they did or did not know was the second consideration.

Research Hypotheses

One of the major tasks was to develop a computer program which would interface with the students and with the concepts of slope, parallel, and perpendicular.

To find the effects of the use of the software in the classroom situation, the following null hypotheses were proposed:

\[ Ho_1 : \text{There is no significant difference between the control group and the experimental group in the increase from pretest to posttest in score after the use of the software.} \]

\[ Ho_2 : \text{There is no significant difference between the control group and the experimental group in the increase from pretest to posttest in the score on problems} \]
having to do with slope.

H₀₃: There is no significant difference between the control group and the experimental group in the increase from pretest to posttest in the score on problems having to do with polygons.

**Findings - Analysis of Pretest and Posttest Data**

The scores on the Slope/Polygon test ranged from 2 to 22, with a possible score of 31. After the students in the experimental group used the software in class for one period each day for three days the Slope/Polygon test was again administered to all subjects. The pretest-posttest scores were compared using single factor Analysis of Variance test. It was found that the groups differed significantly (F = 28.01, df = 3/162, p < .01).

**Comparison of the Control and Experimental groups on pretest**

The Slope/Polygon test was given to the experimental and control groups on the same day. The purpose of the test was to determine the students’ understanding of the content as it related to slope, parallel, and perpendicular.

Examination of pretest means indicated that the groups were not the same at the beginning of the study. The mean score for the control group was 10.632. The mean score of the experimental group was 12.617. Using Scheffe’s Test the F ratio is 2.79 which does indicate a significant difference at the .05 level (F = 2.79, df = 3/82, p > .05). This was not expected considering that the control group consisted of students in the Algebra II and Geometry classes while the experimental group consisted of students in the Algebra I and Geometry classes.
A t test for independent samples was utilized to compare the pretest and posttest achievement of the treatment group. It was found that the two means differed significantly (Table 2).

Table 2  Means of the Pretest Using the Slope-Polygon Test Compared with the Means of the Posttest Using the Slope-Polygon Test

<table>
<thead>
<tr>
<th>Group</th>
<th>M</th>
<th>SD</th>
<th>N</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental Group</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>12.62</td>
<td>3.28</td>
<td>47</td>
<td>2.96</td>
<td>.005</td>
</tr>
<tr>
<td>Posttest</td>
<td>14.92</td>
<td>4.23</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Group</td>
<td></td>
<td></td>
<td></td>
<td>-1.69</td>
<td>&gt; .10</td>
</tr>
<tr>
<td>Pretest</td>
<td>10.63</td>
<td>3.05</td>
<td>38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posttest</td>
<td>9.42</td>
<td>3.11</td>
<td>36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A t test for independent samples was also utilized to compare the pretest and posttest achievement of the control group. It was found that the two means differed significantly but that difference indicated a decrease in the mean score rather than an increase (Table 2).
Table 3  Kuder-Richardson 20 and Standard Error of Measurement Values for Slope-Polygons Test Used as Pretest and Posttest

<table>
<thead>
<tr>
<th>Experimental Group</th>
<th>KR20</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>.43</td>
<td>2.48</td>
</tr>
<tr>
<td>Posttest</td>
<td>.76</td>
<td>2.35</td>
</tr>
<tr>
<td>Control Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>.38</td>
<td>2.45</td>
</tr>
<tr>
<td>Posttest</td>
<td>-.11</td>
<td>2.50</td>
</tr>
</tbody>
</table>

The Kuder-Richardson values along with the Standard Error of Measurement values for each test are given in Table 3.

Point-biserial values changed for both the experimental and control groups. When an item is removed from the list of those whose point biserial scores are negative the indication is that possibly the poorer students may have been guessing on the pretest and could not repeat the guess on the posttest. Or the inference may be that the better students have no idea of the correct answer on the pretest but were able to determine the correct answer on the posttest. Consideration of the pretest and posttest scores for the experimental group and the control group indicates that both of the inferences are possible. A listing of the test items which had negative values is given in Table 4.

Table 4  Items of Slope-Polygon Test Which Provided Negative Point Biserial Values

<table>
<thead>
<tr>
<th>Experimental Pretest</th>
<th>2 5 7 9 14 21 27 29 30 31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental Posttest</td>
<td>3 7 9 17</td>
</tr>
<tr>
<td>Control Pretest</td>
<td>4 6 9 14 21 24 25 27 28 29</td>
</tr>
<tr>
<td>Control Posttest</td>
<td>4 9 14 20 26</td>
</tr>
</tbody>
</table>
The items common to all four group is item 9 which asks the student to identify a triangle with sides 3, 4, and 5. The biserials of the posttests are improved over the biserials of the pretests. The one question which the experimental group had the most difficulty with which would have to do with the concept of slope is item 17. This item gives the students four graphs and the student is to identify the graph which has the negative slope. The biserials on the other questions after item 17 had satisfactory point-biserials. Both groups had difficulty with item 14 on the pretests. This item has to do with the definition of perpendicular bisector and diagonals. The negative point-biserial values indicate that the poorer student tended to correctly answer those items more often than the better students.

Comparison of the Control and Experimental groups on pretest and Posttest - By Class

The data from the Experimental Group was divided into the data from the experimental Geometry and the experimental Algebra I classes while the data from the Control Group was divided into the data from the control Algebra I class and the control Algebra II class (Table 5).
### Table 5 Means of the Pretest Using the Slope-Polygon Test Compared with the Means of the Posttest Using the Slope-Polygon Test - By Classes

<table>
<thead>
<tr>
<th>Group</th>
<th>M</th>
<th>SD</th>
<th>N</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental Algebra I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>12.63</td>
<td>4.04</td>
<td>24</td>
<td>1.69</td>
<td>.10</td>
</tr>
<tr>
<td>Posttest</td>
<td>14.92</td>
<td>4.23</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Algebra I</td>
<td></td>
<td></td>
<td></td>
<td>0.37</td>
<td>&gt;.10</td>
</tr>
<tr>
<td>Pretest</td>
<td>10.05</td>
<td>2.80</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posttest</td>
<td>10.35</td>
<td>2.37</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experimental Geometry</td>
<td></td>
<td></td>
<td></td>
<td>1.77</td>
<td>.025</td>
</tr>
<tr>
<td>Pretest</td>
<td>12.57</td>
<td>2.31</td>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posttest</td>
<td>15.27</td>
<td>4.31</td>
<td>22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Algebra II</td>
<td></td>
<td></td>
<td></td>
<td>-2.509</td>
<td>.025</td>
</tr>
<tr>
<td>Pretest</td>
<td>11.22</td>
<td>3.17</td>
<td>18</td>
<td></td>
<td>No Increase</td>
</tr>
<tr>
<td>Posttest</td>
<td>8.31</td>
<td>3.60</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When comparing means of the classes within the experimental and control groups there is significant increase in the means both the experimental Algebra I and the experimental Geometry.

When comparing means of the classes within the experimental and control groups there is significant increase in the means of the experimental Algebra I while there is a small increase in the means for the control Algebra I class (Figure 13).
When comparing means of the classes within the experimental and control groups there significant increase in the means of the experimental Geometry while there is a sizeable decrease in the means for the control Algebra II class.
When considering the means for the experimental Algebra I and the experimental Geometry are compared, both of the \( t \) scores are significant but the \( t \) score for the experimental Geometry class is more significant than is the score for the experimental Algebra I (Figure 14).

![Comparison of Experimental Algebra I with Experimental Geometry on Pretest and Posttest Means](image)

**Figure 14**  Comparison of Experimental Algebra I with Experimental Geometry on Pretest and Posttest Means

When comparing the means of the control Algebra I and the control Algebra II there is a slight increase in the score for the control Algebra I while there is a significant decrease in the scores for the control Algebra II group.

The pretest/posttest using Slope/Polygons was designed to cover the topic of polygons in exercises 1-15 and the topic of slope in exercises 16-31. The scores were recorded for each of the initial four groups.
according to the scores on exercises dealing with slope and the scores on exercises dealing with polygons. The possible score for polygons was 15 and the possible score for slope was 16. The means comparing slope and polygons are corrected for accuracy. That is, the score made on the slope section was multiplied by 15/16 to equate the two values.

When considering the scores for the questions concerning slope and the scores for the questions concerning polygons it is of interest to investigate the outcomes. For the experimental group when examining the slope questions there is a significant increase in the scores on the posttest as compared to the scores on the pretest. When examining the polygon questions, there is a positive difference between the score on the pretest and the score on the posttest which would indicate a poorer performance on the posttest than on the pretest (Figure 15 and Table 6).
When examining both the slope questions and the polygon questions for the control group, there is a positive difference between the score on the pretest and the score on the posttest which would indicate a poorer performance on the posttest than on the pretest.

Investigation of Table 6 indicates there is no significant differences between pretest scores and posttest scores in either group except for the significant difference between the pretest and posttest scores on the slope questions. The difference between the pretest and posttest scores on the slope questions was significant.
Table 6  Means of the Pretest Using Slope-Polygon Test Compared with the Means of the Posttest Using the Slope-Polygon Test - By Topics

<table>
<thead>
<tr>
<th>Group</th>
<th>M</th>
<th>SD</th>
<th>N</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Experimental Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>6.47</td>
<td>2.6</td>
<td>47</td>
<td>4.22</td>
<td>.005</td>
</tr>
<tr>
<td>Posttest</td>
<td>8.88</td>
<td>2.7</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polygons</td>
<td></td>
<td></td>
<td></td>
<td>-0.23</td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>6.15</td>
<td>2.0</td>
<td>47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posttest</td>
<td>6.04</td>
<td>2.4</td>
<td>48</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Control Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope</td>
<td></td>
<td></td>
<td></td>
<td>-0.88</td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>5.39</td>
<td>2.5</td>
<td>38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posttest</td>
<td>4.86</td>
<td>2.6</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polygons</td>
<td></td>
<td></td>
<td></td>
<td>-1.27</td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>5.24</td>
<td>2.1</td>
<td>38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posttest</td>
<td>4.64</td>
<td>1.8</td>
<td>36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Findings - Analysis of Students' Use of Software

As a result of the design of the software, it was possible to see what the students did while using the software. After the student constructed a line segment or measured the distance between two points the screen was saved to a file on their disk. This allowed for the viewing of what the student did as well as a record of the slope, ratio equivalent of the slope, and the length of the line segment or the length of the segment measured. By using this information it was possible to determine at what point on the screen the student started and at what point the line segment or measurement was
The discs of seven students were examined and two separate sets of data for each student were produced. One of the sets recorded the points at which the students started the construction of a polygon. The second report recorded the points at which the students started the construction of a line segment. These two sets for each student were recorded on a graph similar to Figure 17.

Figure 16 Definition of Sectors of Screen Used by Students
Figure 17 is a sample record of the points at which a student began to construct a polygon.

Figure 17  Sample Record of Points at Which Student Started Polygons
Table 7
Number of Times Each Student Started a Polygon
In the Designated Sector of the Circle

<table>
<thead>
<tr>
<th>Sector of the Circle</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Student 2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 3</td>
<td>2</td>
<td></td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 4</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 5</td>
<td>2</td>
<td></td>
<td>4</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 6</td>
<td></td>
<td></td>
<td>7</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 7</td>
<td></td>
<td>4</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>11</td>
<td>24</td>
<td>21</td>
</tr>
</tbody>
</table>

Sector seven was used most frequently for the beginning of the construction (Table 7). Sectors seven and eight were the two adjacent sectors which were most frequently used. Five of the students began their polygons either in sector seven or in sector eight. One student chose sectors one and eight while another chose sector six and seven. Only one student used sector six and one used sector one in conjunction with sector seven or sector eight. Sectors two, three and five were not used by any of the students for the origination point of a line segment. Sectors seven and eight were used 71% of the time while sectors six and seven were used 55% of the time. Sectors two and three had no points or origination of the line segments.
Table 8  
Number of Times Each Student Started a Line Segment  
In the Designated Sector of the Circle

<table>
<thead>
<tr>
<th>Student</th>
<th>Sector of Circle In Which Polygons Were Begun</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Totals</td>
<td>62</td>
</tr>
</tbody>
</table>

When considering the sectors of the graph in which the students began the construction of a line segment sectors seven and eight are dominant. Sector seven was used 30% of the time while sector eight was used 25% of the time. The next highest percentage of use was sector one. When considering adjacent pairs of sectors, sectors seven and eight are most used. Sector seven or eight was used most frequently by all students except for student number 2. This student used sectors five and six more frequently than any other pair of adjacent sectors. Sectors three, four, and five are least used with sector four have less use than any other sector for the origination of a line segment.

In addition to having the capability to examine the points on the graph which the student uses, the software gives the teacher the capability of determining if the student constructs a line segment or measures the length of a line segment from left to right, right to left, toward the left and upward, toward the left and downward, toward the right and upward, or toward the right and downward. With this capability the graphs which the students
created were examined and the direction in which the line segment was constructed was recorded. Table 9 gives the record of the seven students.

The table lists the directions in which the students construct the line segment. For instance, D/R indicates the line segment was constructed in a direction which was down and to the right, D/L refers to down and to the left, U/R refers to up and to the right, U/L refers to up and to the left. 0/R refers to horizontally and to the right and 0/L refers to horizontally and to the left.
Table 9  Directions In Which Line Segments Were Drawn

<table>
<thead>
<tr>
<th>Student</th>
<th>Sector of Circle In Which Polygons Were Begun</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D/R</td>
<td>D/L</td>
<td>U/R</td>
<td>U/L</td>
<td>0/R</td>
<td>0/L</td>
</tr>
<tr>
<td>1</td>
<td>46</td>
<td>31</td>
<td>11</td>
<td>4</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>20</td>
<td>21</td>
<td>9</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>7</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>44</td>
<td>35</td>
<td>0</td>
<td>1</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>20</td>
<td>1</td>
<td>6</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>38</td>
<td>17</td>
<td>17</td>
<td>1</td>
<td>19</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
<td>17</td>
<td>16</td>
<td>9</td>
<td>19</td>
<td>1</td>
</tr>
<tr>
<td>TOTALS</td>
<td>353</td>
<td>162</td>
<td>81</td>
<td>37</td>
<td>92</td>
<td>22</td>
</tr>
</tbody>
</table>

Considering that sector seven or sector eight was most frequently used by the students for the origination of a line segment or the origination of a polygon it is not surprising to find the line segment constructed most of the time is constructed from one of those sectors down and to the right. This would indicate a line segment with negative slope is most commonly constructed.

When examining the performance of the students as the data was collected concerning the origination of line segments and polygons several difficulties were noticed.

The first obvious difficulty seemed to stem from the lack of understanding of some terms. The students did not understand the phrase "consecutive mid-points". One student did not successfully complete any of the problems dealing with this concept until the second day. On the second day the performance increased. Some students spent more time than others in getting acquainted with the software. One of the better students spent most of the first day just exploring the possibilities of the software.
The second and third day this student was on task a large percentage of the time.

A record was kept as to what percentage of the time the students spent on task, how many screens were used, and how many times they refreshed the screen to begin again. Table 10 lists this information.

Table 10  Sampled Students' Results on Pretest, Posttest, Percentage of Time on Task, and Percentage of Screens Which Were Restarted

<table>
<thead>
<tr>
<th>Student</th>
<th>Pretest</th>
<th>Posttest</th>
<th>Ds</th>
<th>%On Task</th>
<th>%Restart</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>79</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>16</td>
<td>2</td>
<td>81</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>5</td>
<td>-8</td>
<td>36</td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>21</td>
<td>10</td>
<td>93</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>15</td>
<td>4</td>
<td>90</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>11</td>
<td>-2</td>
<td>93</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
<td>20</td>
<td>3</td>
<td>69</td>
<td>24</td>
</tr>
</tbody>
</table>

Students four and five had the largest gain in scores from pretest to posttest and were among the three highest for percentage of time on task. Student three had the highest percentage of restarts, the smallest percentage of time on task, and the lowest score on the posttest.
Chapter V
DISCUSSION AND RECOMMENDATIONS

**Pretest Similarities and Differences**

On the pretest the test results have pointed out several similarities and differences which are of interest.

First, there was no difference on the scores between the experimental pretest for Algebra I and the experimental pretest for Geometry. This is of particular interest since the topics of diagonals, midpoints, etc were covered in the test. This would indicate that the Algebra I students have approximately the same expertise as the Geometry students. It is of interest when one considers the theorems in geometry concerning mid-points, diagonals, parallel and perpendicular.

Secondly, there was no significant difference between the scores of the control Algebra I group and the control Algebra II group. This is significant in that the Algebra II students typically have two more years of mathematical maturity than the Algebra I students.

Next, the experimental Algebra I group scored significantly higher than the control Algebra I group. This is of particular interest when one considers the two classes are taught by the same teacher. It would be of interest to administer further tests to the two classes and to compare past performance records.
The experimental Geometry group scored significantly higher than the control Algebra II class. Again, this was unexpected considering the Algebra II students have had Geometry.

By topics:

There was a significant difference between the experimental group results for the polygon questions and the control polygon group polygon questions. The control group scored lower than the experimental group. That is, the Algebra I/Geometry group scored higher than the Algebra I/Algebra II group on the polygon questions.

There was a significant difference between the experimental group results for the slope questions and the control group slope questions. Again, the Algebra I/Geometry group scored higher than the Algebra I/Algebra II group.

There was no significant difference between the experimental group results for the slope questions and the experimental polygon questions. This would indicate that the classes were equally acquainted with the concepts of slope and polygons as relate to the topics covered.

Posttest Similarities and Differences

There was no significant difference between the experimental Algebra I scores and the experimental Geometry scores. This coincides with no significant difference on the pretest.

There was a significant difference between the control Algebra I scores and the control Algebra II scores. The difference was negative and was not expected. This is discussed further in the section 'Testing Irregularities'
By topics

There was a significant difference between the experimental group results for the slope questions and the experimental polygon questions. Again, the Algebra I/Geometry group scored higher than the Algebra I/Algebra II group.

There was no significant difference between the control group results for the slope questions and the control group results for the polygon questions.

Pretest-Posttest Similarities and Differences

There was a significant difference between the experimental pretest and posttest Algebra I scores. This would indicate that the treatment was instrumental in the increase of scores. This was an expected result.

There was a significant difference between the experimental pretest and posttest Geometry scores. This would indicate that the treatment was instrumental in the increase of scores. This was an expected result.

There was a significant difference between the control pretest and posttest Algebra II scores. The difference was negative and was not expected. The reason for this will be discussed in the 'Testing Irregularities' section.

There was no significant difference between the control Algebra I scores. This was also an expected result.

By topics:

There was a significant difference between the experimental pretest slope questions and the experimental posttest slope questions. This was expected as a result of the treatment.
There was no significant difference between the experimental pretest polygon questions and the experimental posttest polygon questions. This was surprising in that the students had the capability of using the help windows. This would indicate the treatment did not help in further understanding of polygons.

There was no significant difference between the control pretest slope questions and the control posttest slope questions. This was expected since no instruction concerning slope took place in their classrooms in the interim.

There was no significant difference between the control pretest polygon questions and the control posttest polygon questions. Again, this was expected since no instruction concerning polygons took place in their classrooms in the interim.

**Testing Irregularities**

There was a problem with the control Algebra II class. The pretest was given to the students during a regular class period. They were allowed the full class period to complete the test. The posttest was given to the students under different circumstances. After some students were sent out in the hall to make up missed tests, the remaining students were given a lesson in Algebra II at the beginning of the class and were only given the posttest during the last part of the period. The students were allowed only 15 minutes to take the test. This would explain decreases in the scores on the posttest as compared with the scores on the pretest. The students could not have performed normally under these conditions.

There is a problem with research when the control group does not have access to computer and the experimental group is to receive the
treatment using the computer. Classroom teachers of the control group have problems convincing themselves that their students' input is really that necessary or important. If the control group takes the pretest and no new material is taught which is covered on the pretest, then the teachers of the control classroom sometimes find it difficult to justify their students taking the posttest. Further research in the use of computers which uses a non-computer classroom for the control will necessitate a very good understanding and appreciation from the teacher of the control group.

**Recommendations**

Students in geometry should be given more instruction in the simpler concepts concerning polygons such as diagonals, properties of the diagonals, midpoints, and parallel/perpendicular segments.

Students should be given the opportunity to work with a tool such as Slope-Stick in order to improve their knowledge of the concept of slope as it relates to polygons and lines.

Slope-Stick should be used for geometric constructions traditional to the geometry syllabus. All of the geometric constructions except the construction of a triangle given the length of three sides are possible using Slope-Stick.

Slope-Stick is also capable of aiding with the teaching of vector geometry. The addition and subtraction of vectors would be an easy task using Slope-Stick. Also the resolution of vectors into vertical and horizontal components would be possible; making the use of Slope-Stick a valuable asset in elementary physics classrooms.
The computer with the available software should be made available to all mathematics students for use as a tool, tutor or tutee. With the advent of graphing calculators, it may be possible that peripheral devices such as the mouse will become available for the pocket calculators.

**Changes in Slope-Stick suggested by Research Data**

The study has shown that the students use the seventh and eighth sectors of the square along with the first and second. This suggests that the small rectangle reserved for screen data at the top of the screen should be moved to the bottom of the viewing screen. This would not interfere or impede the students' intended moves on the screen.

The time was recorded at which each screen was saved. This time was expressed in hours and minutes only. It is a simple matter to include the time to the nearest hundredth of a second on the screen when the data is recorded on the disk. This would be an addition which would provide data of interest for further research.

The help screen evidently was not used by the students as evidenced by the lack of increase in scores on the polygon questions. The "Help" button will be replaced with a "Print Screen" button to allow students to have a printed copy of their work.

The software used in the study did not allow the users to escape from the program without ejecting the disk and turning off the computer. This will be changed to facilitate classroom use.

**Future Research**

A study of students using Slope-Stick during the duration of a high school Geometry course would be of interest. It would be necessary to
have a classroom equipped with Macintosh computers for the students use similar to the ACOT classrooms. It would be of interest to compare their performance in Algebra II with the performance of students in a regular Geometry course.

With the software's ability to save screens to the disk it would be of interest to further investigate the students' use of Slope-Stick. The manner in which a student constructs a figure would be of interest. How often does the student begin constructing a figure with the initial line in the horizontal position? If the horizontal line is the line first drawn, is the line at the top or bottom of the completed figure? With the slope being displayed as a ratio, one is able to determine if a student has drawn a line from right to left or left to right and top to bottom or bottom to top. With the time being recorded for each screen saved it would also be of interest to consider the time factor. Saving the screens also allows the researcher to examine if the student has stayed on task and if so for what percentage of the time. One could also determine how many times the students erased the screen to redraw an unsatisfactory figure. Comparison of the students' scores as to how carefully they construct the figures would be of interest. One would also be able to answer the question as to which portion of the window does the student use more than any other. The study considering the portions of the screen could follow closely the study done by Vander Embse (1987).

In view of the frequency of using sectors 7 and 8 of the screen, would the same findings be true if the menu was moved to the left hand side of the screen and the construction square to the right hand side of the screen? What would be the difference in results when comparing locating the menu
at the top left or at the bottom left? What difference would be found if the menu was moved to the lower right hand of the screen? In discussing menu locations, would the position of the 'Line' button to the other buttons produce different outcomes? What would be the result if the buttons were put in a horizontal row across the bottom of the screen? If this horizontal row was used, would it make a difference in the students' starting points if the 'Line' button was in the lower middle of the screen, lower right of the screen or lower left of the screen?
APPENDIX A

PILOT OF PRETEST AND POSTTEST
Appendix A

Pilot Test of Pretest/Postest

Two versions, Form A and Form B, of the Pretest/Postest, **Slope/Polygon Test**, were pilot-tested at Mount Vernon Nazarene College using three lower-division mathematics classes. **Slope/Polygon Test** is composed of 31 multiple-choice questions testing subjects' knowledge of the concept of slope, classes of quadrilaterals, classes of triangles, properties of triangles, perpendicular lines, parallel lines, bisectors, perpendicular bisectors, angles, and right triangles. Copies of these tests are included in this appendix.

The two forms were distributed by rows giving alternate forms to alternate subjects.

The 99 subjects included 55 freshmen, 27 sophomores, 14 juniors, and 3 seniors. The mathematical backgrounds indicated by courses taken and when the courses were taken are indicated in Table 11.
Table 11  

Courses Taken by Subjects of Pilot Test

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<tr>
<th></th>
<th>High School</th>
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<th>Never</th>
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<tr>
<td>Algebra I</td>
<td>91</td>
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<td>4</td>
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<tr>
<td>Algebra II</td>
<td>71</td>
<td>3</td>
<td>25</td>
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<td>Analysis</td>
<td>8</td>
<td>2</td>
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<tr>
<td>Precalculus</td>
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<td>1</td>
<td>79</td>
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<tr>
<td>Calculus I</td>
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<tr>
<td>Calculus II</td>
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<td>3</td>
<td>94</td>
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</table>

Mean scores, standard deviations, reliabilities (KR20), and standard errors were computed for the entire test and for each of two subsets. These are reported in tables A.2, A.3 and A.4. Item biserials for both forms of the test were examined to identify any items which functioned poorly. Four questions had negative point-biserial scores.

Form A

| 5, 9, 10 |

Form B

| 6 |

The difficulty on the noted problems seemed to be as follows:

**Form A**  
Problem 5 - "Least numerical value"  
Problem 9 - "triangle with the slope of all sides negative"  
Problem 10- "triangle with the slope of all sides negative"

**Form B**  
Problem 6 - "Least numerical value"
The correlation coefficient between total score on problems 17-31 and a student having taken Plane Geometry previously was found to be .381 which is, with the large N, large enough to make a prediction as to the outcomes using the proposed Research Design.

The difference between the means of the two forms of the test was not found to be statistically significant at the 0.01 level of significance.

Table 12
Results for the complete test - Exercises 1-31

<table>
<thead>
<tr>
<th></th>
<th>Form A</th>
<th>Form B</th>
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<tbody>
<tr>
<td>Mean</td>
<td>14.8040</td>
<td>15.2160</td>
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<tr>
<td>Standard Deviation</td>
<td>3.8510</td>
<td>3.8450</td>
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<tr>
<td>KR20</td>
<td>0.5743</td>
<td>0.5855</td>
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<tr>
<td>Standard Error</td>
<td>2.5125</td>
<td>2.4755</td>
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Table 13
Results for the first section of test - Exercises 1-16:

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<tr>
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<td>KR20</td>
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<td>1.7792</td>
<td>1.7805</td>
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Table 14
Results for the second section of test - Exercises 17-31:

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<tr>
<td>KR20</td>
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<tr>
<td>Standard Error</td>
<td>1.6821</td>
<td>1.6559</td>
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</table>
Slopes and Polygons

1. Which quadrilateral has diagonals which are equal in length but are not perpendicular to each other?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

2. Which quadrilateral has diagonals which are only perpendicular to each other but do not bisect each other and are not equal in length?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

3. Which quadrilateral has diagonals which are equal in length and are perpendicular bisectors of each other?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

4. Which quadrilateral has diagonals which are perpendicular bisectors of each other and are not equal in length?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite
5. Which quadrilateral has diagonals which are bisectors of each other, are not perpendicular, and are not equal in length?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

6. The diagonals of an isosceles trapezoid are:
   A. Perpendicular to each other.
   B. Equal in length.
   C. Bisectors of each other.
   D. None of the above

7. If a triangle is isosceles how many equal sides does it have as a minimum?
   A. One
   B. Two
   C. Three
   D. No equal sides

8. If a triangle is equilateral, how many equal sides does it have?
   A. One
   B. Two
   C. Three
   D. No equal sides

9. What kind of a triangle is a triangle with sides 3, 4, and 5?
   A. Isosceles
   B. Right
   C. Equilateral
   D. Scalene

10. If a triangle has angles of 40 and 55 degrees, what is the number of degrees in the third angle?
    A. 40
    B. 55
    C. 85
11. If a triangle has two angles which are 55 degrees, what kind of a triangle is it?
   A. Isosceles
   B. Right
   C. Equilateral
   D. Obtuse

12. What kind of a triangle has sides 4, 5 and 6?
   A. Isosceles
   B. Right
   C. Equilateral
   D. Scalene

13. If a right triangle has one acute angle of 35 degrees, what is the size of the other acute angle?
   A. 65
   B. 55
   C. 85
   D. 75

14. A triangle has sides of length 3, $3\sqrt{3}$, and 6. What is the size of the angle opposite the side which is 3 units in length?
   A. 30
   B. 45
   C. 60
   D. 90

15. A triangle has sides of length 3, 3, and $3\sqrt{2}$. What is the size of the angle opposite a side which is 3 units in length?
   A. 30
   B. 45
   C. 60
   D. 90
16. Which figure above contains a line with slope = 0?
   A.
   B.
   C.
   D.

17. Which figure above contains a line with negative slope?
   A.
   B.
   C.
   D.

18. Which figure above contains a line with positive slope?
   A.
   B.
   C.
   D.

19. Which figure above contains a line with slope which is undefined?
   A.
   B.
   C.
   D.
20. Which figure contains the line which has the slope of least numerical value?
   A. 
   B. 
   C. 
   D. 

21. Which figure contains the line which has the slope of highest numerical value?
   A. 
   B. 
   C. 
   D. 

22. Which two figures contain lines with positive slope?
   A. A and B 
   B. A and C 
   C. B and C 
   D. B and D 

23. Which two figures contain lines with negative slope?
   A. A and B 
   B. A and C 
   C. B and C 
   D. C and D
24. Is it possible to construct a triangle with the slope of all sides negative?
   A. Yes
   B. No

25. Is it possible to construct a triangle with the slope of all sides positive?
   A. Yes
   B. No

26. What is the slope of a line which is to be constructed perpendicular to a given line with slope = -1?
   A. -1
   B. +1
   C. +1/2
   D. -1/2
   E. Cannot tell without further information

27. What is the slope of a line which is to be constructed parallel to a given line with slope = 2?
   A. 1/2
   B. -2
   C. +2
   D. -1/2
   E. Cannot tell without further information
28. In figure A above, if the slope of the line a is -5, what is the length of the third side of the triangle?
   A. 2.5
   B. 1.5
   C. 5
   D. 1

29. In figure B above, if the slope of the line b is -1, what is the length of the third side of the triangle?
   A. .33
   B. 3
   C. 1
   D. 2

30. In figure C above, if the slope of the line c is 1, what is the length of the third side of the triangle?
   A. 1
   B. 2
   C. 3
   D. 4

31. In figure D above, if the slope of the line d is 3, what is the length of the third side of the triangle?
   A. 3
   B. 4
   C. 9
   D. 6
32. Your present classification is -
   A. Freshman
   B. Sophomore
   C. Junior
   D. Senior

   Indicate when you had the following courses.

33. Algebra I
   A. Before High School
   B. In High School
   C. Never

34. Algebra II
   A. In High School
   C. Never

35. Plane Geometry
   A. In High School
   B. Never
1. Which figure above contains a line with slope which is undefined?
   A. 
   B. 
   C. 
   D. 

2. Which figure above contains a line with negative slope?
   A. 
   B. 
   C. 
   D. 

3. Which figure above contains a line with slope = 0?
   A. 
   B. 
   C. 
   D. 

4. Which figure above contains a line with positive slope?
   A. 
   B. 
   C. 
   D.
5. Which two figures contain lines with positive slope?
   A. A and B
   B. A and C
   C. B and C
   D. C and D

6. Which two figures contain lines with negative slope?
   A. A and B
   B. A and C
   C. B and C
   D. C and D

7. Which figure contains the line which has the slope of least numerical value?
   A.
   B.
   C.
   D.

8. Which figure contains the line which has the slope of highest numerical value?
   A.
   B.
   C.
   D.

9. Is it possible to construct a triangle with the slope of two sides positive?
   A. Yes
   B. No
10. Is it possible to construct a triangle with the slope of two sides negative?
   A. Yes
   B. No

11. What is the slope of a line which is to be constructed perpendicular to a given line with slope = +1?
   A. -1
   B. +1
   C. +1/2
   D. -1/2
   E. Cannot tell without further information

12. What is the slope of a line which is to be constructed parallel to a given line with slope = -2?
   A. 1/2
   B. -2
   C. +2
   D. -1/2
   E. Cannot tell without further information
13. In figure A above, if the slope of the line a is -1, what is the length of the third side of the triangle?
   A. 2.5
   B. 1.5
   C. 5
   D. 1

14. In figure B above, if the slope of the line b is -3, what is the length of the third side of the triangle?
   A. .33
   B. 3
   C. 1
   D. 9

15. In figure C above, if the slope of the line c is 1/2, what is the length of the third side of the triangle?
   A. 1
   B. 2
   C. 4
   D. 8

16. In figure D above, if the slope of the line d is 2, what is the length of the third side of the triangle?
   A. 3
   B. 4
   C. 9
   D. 6
17. Which quadrilateral has diagonals which are only perpendicular to each other but do not bisect each other and are not equal in length?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

18. Which quadrilateral has diagonals which are perpendicular bisectors of each other and are not equal in length?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

19. Which quadrilateral has diagonals which are equal in length but are not perpendicular to each other?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

20. Which quadrilateral has diagonals which are not perpendicular to each other and are not equal in length?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
   E. Kite

21. Which quadrilateral has diagonals which are equal in length and are perpendicular bisectors of each other?
   A. Rectangle
   B. Square
   C. Rhombus
   D. Parallelogram
22. The diagonals of an isosceles trapezoid are:
   A. Perpendicular to each other.
   B. Unequal in length.
   C. Bisectors of each other.
   D. None of the above.

23. If a triangle is equiangular, how many equal angles does it have?
   A. One
   B. Two
   C. Three
   D. No equal sides

24. If a triangle is isosceles how many equal angles does it have as a minimum?
   A. One
   B. Two
   C. Three
   D. No equal sides

25. What kind of a triangle is a triangle with sides 5, 12, and 13?
   A. Isosceles
   B. Right
   C. Equilateral
   D. Scalene

26. If a triangle has angles of 35 and 50 degrees, what is the number of degrees in the third angle?
   A. 40
   B. 55
   C. 85
   D. 95

27. If a triangle has two angles which are 35 degrees, what kind of a triangle is it?
   A. Isosceles
   B. Right
C. Equilateral
D. Scalene
28. What kind of a triangle has sides 4, 6 and 7?
   A. Isosceles
   B. Right
   C. Equilateral
   D. Scalene

29. If a right triangle has one acute angle of 25 degrees, what is the size of
    the other acute angle?
   A. 65
   B. 55
   C. 85
   D. 75

30. A triangle has sides of length 2, $2\sqrt{3}$, and 4. What is the size of
    the angle opposite the side which is 2 units in length?
   A. 30
   B. 45
   C. 60
   D. 90

31. A triangle has sides of length 5, 5, and $5\sqrt{2}$. What is the size
    of the angle opposite a side which is 5 units in length?
   A. 30
   B. 45
   C. 60
   D. 90

32. Your present classification is -
   A. Freshman
   B. Sophomore
   C. Junior
   D. Senior

Indicate when you had the following courses.
33. Algebra I
   A. Before High School
   B. In High School
   C. In College
   D. Never

34. Algebra II
   A. In High School
   B. In College
   C. Never

35. Plane Geometry
   A. In High School
   B. In College
   C. Never

36. Trigonometry
   A. In High School
   B. In College
   C. Never

37. Analysis
   A. In High School
   B. In College
   C. Never

38. Precalculus
   A. In High School
   B. In College
   C. Never

39. Calculus
   A. In High School
   B. In College
   C. Never

40. Calculus II or higher
   A. Previously
41. Are you currently enrolled in a mathematics course?
   A. Yes
   B. No
PLEASE NOTE:

This page not included with original material. Filmed as received.

University Microfilms International
APPENDIX B

TASKS FOR TREATMENT
Constructions

1. Construct a line segment with length 4 and slope 1.
   a. Find its midpoint.
   b. At the midpoint construct a line segment with slope -1 and length 4.
   c. What is the product of the ratios of the two segments? _____

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<tr>
<th>LENGTH</th>
<th>SLOPE</th>
<th>RATIO</th>
<th>LENGTH</th>
<th>SLOPE</th>
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<tr>
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</tr>
</tbody>
</table>

2. Construct a line segment with length 2 and slope -2.
   a. Construct another line segment with length 2 and slope -2.
   b. What is the relationship of the two line segments just constructed?
   c. What is the relationship of the two ratios?

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<th>RATIO</th>
<th>LENGTH</th>
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3. Construct a triangle.
   a. Record the lengths, slopes, and ratios of the three sides.
   b. Locate the midpoints of two sides of the triangle.
   c. Connect the two midpoints which you have found.
   d. How long is this line segment?
   e. What is the slope of the line segment?
   f. What can you say about the line segment you have just constructed and the third side of the triangle?

   How do their lengths compare?

   How do their slopes compare?

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</table>
4. Construct a triangle.
   a. Record the lengths, slopes, and ratios of the three sides.
   b. Find the midpoint of one side of the triangle.
   c. Using the midpoint just found, construct a line from this midpoint parallel to one of the other sides of the triangle.
   d. What is the length of the line just constructed?

   e. How does this length compare with the length of the parallel side?

   f. At the point where the constructed line joins the other side measure the distances to the two endpoints of that side. What is the relationship between the two distances?

| LENGTH | SLOPE | RATIO | LENGTH | SLOPE | RATIO |
5. Construct a parallelogram which is not a rectangle or square.
   a. Construct the diagonals of the parallelogram noting their length, slope, and ratio.
   b. What is the relationship between the lengths of the diagonals?
   c. What do the values of the two slopes tell us about the relationship between the two diagonals?
   d. Measure the distances from the point of intersection of the two diagonals to the four vertices.
      What is true of the distances to the vertices?

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<tr>
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6. Construct a quadrilateral with none of the sides equal. Record the length, slope, and ratio of each side.
   a. Find the midpoint of each of the sides.
   b. Join the midpoints of adjacent pairs of sides making four lines and a new quadrilateral.
   c. What do you notice about the pairs of opposite sides of the quadrilateral?

   d. What is the name of the quadrilateral which was formed?

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7. Construct a rectangle recording the length, slope, and ratio of each side.
   a. Find the midpoint of each of the sides.
   b. Join the midpoints of adjacent pairs of sides making four lines and a
      new quadrilateral.
   c. What is true of the opposite sides of the quadrilateral?

d. What is the name of the quadrilateral which was formed?

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8. Construct a quadrilateral as follows:
   a. Construct a line segment with length = 5 and with slope = 0.
   b. At the left hand end of the segment construct a line with length = 2 and with slope = 2.
   c. At the right hand end of the segment construct a line with length = 2 and slope = -2.
   d. Connect the ends of the last two lines constructed.
   e. What is the slope of the line just constructed?

   f. What is the relationship between this line and the first line constructed?

   g. Locate the mid-points of the four sides of the quadrilateral?
   h. Connect the consecutive mid-points of the four sides of the quadrilateral.
   i. What is the name of the new quadrilateral constructed?

| # | LENGTH | SLOPE | RATIO | # | LENGTH | SLOPE | RATIO |
9. Construct a line with length 3 and slope -2. Using that line for one of the sides, construct a square. Record length, slope, and ratio for each side.
   a. Using the "DIST/PTS" button determine the lengths and slopes of the diagonals of the square noting their slope and length.
   b. What is the relationship between the lengths of the diagonals?

   c. What do the values of the two slopes tell us about the relationship between the two diagonals?

   d. Find the midpoint of each of the sides.
   e. Join the midpoints of adjacent pairs of sides making four lines and a new quadrilateral.
   f. What is true of the opposite sides of the new quadrilateral?

   g. What is the name of the new quadrilateral which was formed?

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Figure 18 - Student #1 - Points at Which Lines Were Begun
Figure 19 - Student #1 - Points at Which Polygons Were Begun
Figure 20 - Student #2 - Points at Which Lines Were Begun
Figure 21 - Student #2 - Points at Which Polygons Were Begun
Figure 22 - Student #3 - Points at Which Lines Were Begun
Figure 23 - Student #3 - Points at Which Polygons Were Begun
Figure 24 - Student #4 - Points at Which Lines Were Begun
Figure 25 - Student #4 - Points at Which Polygons Were Begun
Figure 26 - Student #5 - Points at Which Lines Were Begun
Figure 27 - Student #5 - Points at Which Polygons Were Begun
Figure 28 - Student #6 - Points at Which Lines Were Begun
Figure 29 - Student #6 - Points at Which Polygons Were Begun
Figure 30 - Student #7 - Points at Which Lines Were Begun
Figure 31 - Student #7 - Points at Which Polygons Were Begun
Bibliography


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