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THE USE OF A PSEUDO RANDOM BINARY REACTIVITY INPUT AND THE RESULTING GAMMA RAY FLUCTUATIONS TO DETERMINE THE TRANSFER FUNCTION OF A NUCLEAR REACTOR.

The Ohio State University, Ph.D., 1973
Engineering, nuclear

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THE USE OF A PSEUDO RANDOM BINARY REACTIVITY
INPUT AND THE RESULTING GAMMA RAY FLUCTUATIONS TO
DETERMINE THE TRANSFER FUNCTION OF A NUCLEAR REACTOR

DISSERTATION

Presented in Partial Fulfillment of the Requirements
For the Degree Doctor of Philosophy
In the Graduate School of
The Ohio State University

by

Richard Warman Bailey, B.S.

* * * * *

The Ohio State University

1973

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ACKNOWLEDGMENT

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# VITA

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Partial List of Publications:


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<td>BCD</td>
<td>Binary Coded Decimal</td>
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<td>BSR</td>
<td>Bulk Shielding Reactor</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Delayed Neutron Fraction</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Clock Frequency</td>
</tr>
<tr>
<td>$f_n$</td>
<td>Nyquist Frequency</td>
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<tr>
<td>$\phi_{aa}(\tau)$</td>
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<td>$G(s)$</td>
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<td>Metal Oxide Semiconductor</td>
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<td>NAND</td>
<td>Logical &quot;Not And&quot; Circuit</td>
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<td>NIM</td>
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<tr>
<td>OR</td>
<td>Logical &quot;or&quot; Circuit</td>
</tr>
<tr>
<td>p</td>
<td>Number of Bits in PRBS Sequence</td>
</tr>
<tr>
<td>PAR</td>
<td>Princeton Applied Research, Registered Trademark</td>
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<tr>
<td>PRBS</td>
<td>Pseudo Random Binary Sequence</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Reactivity = ((k-1)/k)</td>
</tr>
<tr>
<td>( \lambda_i )</td>
<td>Decay Constant for Precursor Decay</td>
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<td>( T_m )</td>
<td>Modulator Period</td>
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<td>( T^2L )</td>
<td>Transistor-Transistor Logic Family</td>
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<td>( \tau_n )</td>
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<td>( \omega )</td>
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1.0 INTRODUCTION

1.1 PURPOSE

The purpose of this work is to show that the transfer function of a critical nuclear reactor can be determined by measuring the cross correlation between a PRBS reactivity input and the resulting gamma ray fluctuations.

To make these measurements, considerable effort was expended in designing and building the experimental apparatus. In fact, the primary thrust was directed at constructing machinery and instrumentation that could be used for a variety of reactor noise type experiments.

1.2 PREVIOUS WORK

The transfer function of the CP-2 reactor at Argonne National Laboratory was determined by oscillating a control rod and measuring the fluctuating neutron output \((1)\). Walker \((2)\) in 1967, designed and built a neutron oscillator for use at The Ohio State University Nuclear Reactor. This oscillator was designed to be used in the number one beam port of the reactor. The oscillator causes a fluctuation of the effective reactivity of the reactor in essentially a sine wave fashion. The amplitude of the reactivity driving function is kept small so that the reactor can be considered a linear system. The neutron output of the core is monitored with a gamma compensated neutron ionization chamber. The current due to the steady state power is bucked out and the small oscillating signal is amplified and displayed. The amplitude and phase of this oscillating current is recorded at each frequency.
point of interest. The transfer function then is a plot of the amplitude and phase shift as a function of oscillator frequency. Using this oscillator, about four hours are required to determine the transfer function.

In 1958, M. N. Moore (3) demonstrated theoretically that a nuclear reactor transfer function could be measured by taking the autocorrelation function of the power noise.

In 1960, C. E. Cohn (4) published the first simplified theory of nuclear reactor noise. Cohn's work centered on describing the statistical variations of the neutron population, which occur as natural statistical fluctuations in the rates of neutron adsorption and fission, as a "noise equivalent" neutron source driving the reactor.

In the last 13 years, numerous authors (5,6) have added to the knowledge of nuclear reactor noise.

1.3 GAMMA RAY NOISE ANALYSIS

The initial work on the theoretical aspects of using gamma photon fluctuations to determine dynamic parameters of a nuclear reactor was performed by Gelinas and Osborn (7).

Using the theory of Gelinas and Osborn as motivation, Lehto and Carpenter (8) reported the successful measurement of $\beta/\ell$ by high energy gamma ray detection. A considerable effort was expended in developing a gaseous Cerenkov detector to sense the high energy prompt-fission gamma rays.
The theory developed by Gelinas and Osborn was based on the assumption of a homogenous one speed point reactor model without delayed neutrons and delayed gammas. In 1971, Kostic and Seifritz published a paper (9) on space dependent noise analysis using gamma radiation. It was their conclusion that the spatial correlation range is, in general, greater for gammas than neutrons. This implies that the neutron detector monitors less of the core volume than a gamma detector.

1.4 GAMMA DESCRIPTION OF THIS WORK

It is the work of Kostic and Seifritz that supplies some of the motivation for the investigation described in this dissertation. Their work suggests that it is appropriate to use a neutron driving function and by measuring the resultant photon fluctuations establish a neutron-gamma cross power spectral density function. In particular, the work described below, concentrates on the problem of driving the reactor with a PRBS type of reactivity input and detecting the resultant gamma fluctuations.

The result of this effort has been to design and construct a system that generates a PRBS type of reactivity input and measures the reactor impulse response by cross-correlating the PRBS reactivity input with the fluctuating gamma output. Also, the Fourier transform of the impulse response is taken to yield the reactor transfer function.

To demonstrate the capability of the system, the reactor transfer function is determined and shown to be similar to results predicted by neutron measurements.
This work differs from any performed so far, in that only conventional gamma ionization chambers were used and that a PRBS type of driving function was employed. An additional feature of the described system and techniques, is that it is an on-line system and requires no large digital computers.

1.5 OUTLINE OF THIS RESEARCH

The general theory of a gamma noise experiment along with the basic theory of correlation measurements and the principles of PRBS generation are covered in Chapter 2.

Chapter 3, presents the details of the construction and operation of the experimental apparatus.

Chapter 4, discusses the results of this work and compares it to work performed by other experimenters.

The conclusions and recommendations for future research are covered in the final Chapter, Chapter 5.
2.0 THEORY

2.1 INTRODUCTION

The theory presented in this Chapter is organized into three distinct Sections, each dealing with a particular aspect of the problem of measuring the transfer function of a critical nuclear reactor when a pseudo random reactivity input is employed as a driving function, and when the resulting gamma fluctuations are detected as the reactor output information.

Section 2.2 is concerned with the development of the basic idea of cross-correlation measurement when the input driving function can be made to approximate a Dirac delta function.

Section 2.3 discusses the existing theory and interpretation of the expressions derived in Section 2.2 and how they can be used in a gamma flux measurement.

Finally, Section 2.4 develops the PRBS sequence and examines its properties and how it can be used.

2.2 CORRELATION MEASUREMENTS

Because of the extremely small reactivity variations used (see Appendix A) the reactor is considered a linear system in the first approximation.

In light of this fact, consider a linear system characterized by an impulse response \( h(t) \). For any input \( a(t) \), the corresponding system output \( b(t) \), is given by the convolution integral \( (10, 11, 12) \):

\[
b(t) = \int_{-\infty}^{\infty} h(\lambda) a(t-\lambda) \, d\lambda \quad (2-1)
\]
The cross-correlation function $\phi_{ab}$, between the input $a(t)$ and the output $b(t)$, is defined as (11):

$$\phi_{ab}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} a(t)b(t + \tau) dt$$

(2-2)

by substituting equation (2-1) for $b(t)$ yields:

$$\phi_{ab}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \left\{ \int_{-T}^{T} a(t) \left[ \int_{0}^{\infty} h(\lambda) a(t + \tau - \lambda) d\lambda \right] dt \right\}$$

(2-3)

then by reversing the order of integration, the following results:

$$\phi_{ab}(\tau) = \int_{0}^{\infty} h(\lambda) d\lambda \left\{ \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} a(t) a(t + \tau - \lambda) dt \right\}$$

(2-4)

The expression inside the braces in equation (2-4) above, is defined to be the autocorrelation function (11) of the input $a(t)$:

$$\phi_{aa}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} a(t) a(t + \tau) dt$$

(2-5)

by substituting this equation into equation (2-4), the following results:

$$\phi_{ab}(\tau) = \int_{0}^{\infty} h(\lambda) \phi_{aa}(\tau - \lambda) d\lambda$$

(2-6)

which is interesting, because if the autocorrelation function of the input $a(t)$ is equal to a Dirac delta function, i.e.:

$$\phi_{aa}(\tau) = \delta(\tau)$$

(2-7)
then it follows that equation (2-6) reduces to:

$$\phi_{ob}(\tau) = \int_0^\infty h(\lambda)\delta(\tau) d\lambda$$  \hspace{1cm} (2-8)

which further reduces to:

$$\phi_{ob}(\tau) = h(\tau)$$  \hspace{1cm} (2-9)

by employing the definition of $\delta$ (13).

Two approximations must be made in order to apply the above concepts. Namely, a finite correlation time must be used. The details of the correlation function computer that was employed is presented in Section 3. The second approximation that needs to be made is that of selecting an input function $a(t)$ so that equation (2-7) can be used. Section 2.4 presents the details of choosing such an input. Section 3.5 shows the transfer function the Model 427 current amplifier measured using this technique.

It is noteworthy to mention that the transfer function of a linear system is simply the Fourier transform of the impulse response. Many authors discuss this theory in detail. Lee (11) presents complete discussions and derivations of the frequency response from the time response and this work will not be duplicated here.

Some of the basics of the theory will, however, be discussed in Section 3.6 when the operation of the Fourier Analyzer, used in this research, is described.
2.3 GAMMA PHOTON FLUCTUATIONS

The application of equations (2-1) and (2-2) is straightforward when one is considering the neutron output of the critical reactor that is being driven by a neutron source input.

The applicability of equations (2-1) and (2-2) for the case of a reactivity input has been established (14), and is reflected in the expressions for the "source" and "reactivity" transfer functions. The two transfer functions are related by the constant $n_0/\lambda$, where $\lambda$ is the neutron generation time and where $n_0$ is a constant source neutron density. Namely, if $I(S)$ is the "source" transfer function, then $(n_0/\lambda)I(S)$ is the "reactivity" transfer function.

The assumption that the formalism of Section 2.2 can be extended to include the case of reactivity driving and gamma output cross-correlation is supported by the work of Kenney (6) page 405, and Hetrick (14). Hetrick shows that reactivity transfer function should have the form:

$$G(s) = \frac{n_0}{\lambda} I(s) = \frac{n_0}{\lambda s + \beta - \rho_0 - \sum \frac{\beta_i \lambda_i}{s + \lambda_i}}$$

(2-10)

and Kenney shows by block diagram algebra, that the variation of the gamma intensity as a function of an input reactivity variation can be represented by:

$$\frac{\delta \gamma(s)}{\delta k(s)} = \left(\frac{K_0}{1 + \tau_1 s} + \frac{K_1}{1 + \tau_2 s} + \ldots + \frac{K_n}{1 + \tau_n s}\right)G(s)$$

(2-11)
where $K_p$ is the fraction of prompt gamma radiation and $K_1, K_2, K_n$, are the fractions of delayed gamma radiation.

If no delayed gammas are assumed, then:

$$\frac{\delta \gamma(s)}{\delta k(s)} = K_p G(s) \quad (2-12)$$

and the transfer function measured by gamma fluctuations should have exactly the same shape as that measured using the neutron fluctuations.

The inclusion of delayed gamma radiation should have no appreciable effect on the shape of the transfer function at the high frequency end. This end should be dominated by the prompt gamma radiation.

If this is the case, the upper end of the transfer function should be, according to equation (2-12), identical in shape to the transfer function measured using the neutron fluctuations.

At low frequencies however, the effects of the delayed gamma radiation should be more important.

Measurements made by Kenney, using conventional oscillator techniques (6) support this general interpretation.

The similarity of the gamma and neutron measurements at high frequencies, is clearly shown and is presented in Chapter 4. Because of the problems associated with making a PRBS reactivity modulator of the type described with a large number of bits, the low end of the transfer function where the delayed gammas should be more significant, was not examined.
Theoretical considerations by Gelinas and Osborn (7) and more recently, by Kostic and Seifritz (9) support the assumption that the transfer function of a critical nuclear reactor can be measured by observing the fluctuating gamma field. In fact, Kostic and Seifritz have derived an expression that relates the frequency response of the prompt gamma flux to the frequency response function of the neutron flux. It is their conclusion that it is possible to express the frequency response function of the prompt gamma flux by a product of the frequency response function of the neutron flux and a weighting function. This implies an inseparable link between the information contained in the neutron gamma fluxes.

Of course, this is quite logical, since it is known that prompt gamma rays as well as neutrons, are emitted during fission.

Several authors have succeeded in measuring the critical nuclear reactor's transfer function by making cross-correlation measurements between a reactivity driving function and the resultant neutron fluctuation (5, 6, 12, 15).

Assuming the work of Kostic and Seifritz (9) is correct, and by virtue of the fact the cross-correlation technique indeed works for measuring impulse responses of linear systems, it is logical to perform an experiment to verify that the critical nuclear reactor's impulse response and therefore, its transfer function can be measured by the technique suggested above. Namely, the cross-correlation function between a reactivity driving input and the fluctuating gamma output contains essentially the same information as a reactivity-neutron cross-correlation function.
Section 2.4 below, discusses the details of selecting a suitable reactivity driving function.

2.4 PSEUDO RANDOM BINARY SEQUENCE

In an attempt to find an input that has the desirable feature that its autocorrelation function approximates a Dirac delta function, consider the following sequence of numbers $S(n)$, that has the following properties (16):

$$S(n) = \pm 1 \quad (2-13)$$
$$S(n+p) = S(n) \quad (2-14)$$
$$\sum_{m=1}^{p} S(m) = 1 \quad (2-15)$$
$$\sum_{m=1}^{p} S(m)S(m+k) = \left\{ \begin{array}{ll}
  p & \text{if } k = 0 \\
  -1 & \text{if } k = 1, 2, 3, \ldots, p-1
\end{array} \right. \quad (2-16)$$

For arbitrary $p$, such a sequence may not exist. However, such sequences are known to exist for the values of $p = 2^m-1$. For example, consider the case of $p = 15$, the sequence (000111101011001) where 0 denotes $(-1)$ and 1 denotes $(+1)$ satisfies the above equation.

To show that the above sequence has the properties outlined above, the expressions (2-13) through (2-16) will be evaluated. Equations (2-13) and (2-14) are necessarily met by assumption. Equation (2-15) can be evaluated as follows:
To evaluate all cases of equation (2-16) would be very lengthy and so only two of the possible 15 cases will be calculated. First, the case of \( k = 0 \), i.e., no shift, will be evaluated:

\[
\sum_{m=1}^{15} S(m) = (-1) + (-1) + (-1) + 1 + 1 + 1 + 1 + (-1) + (-1) + 1 = 7(-1) + 8(1) = 1
\]

For the second case, the value \( k = 2 \), will be used:

\[
\sum_{m=1}^{15} S(m)S(m+2) = S(1)S(3) + S(2)S(4) + \ldots + S(15)S(17) \quad (2-18)
\]

\( S(17) \) is interpreted to be \( S(2) \), since setting \( k = 2 \) is equivalent to shifting the sequence by 2 bits and because the sequence is periodic with a period \( p \). Therefore, equation (2-18) can be expressed as:

\[
\sum_{m=1}^{15} S(m)S(m+2) = S(1)S(3) + S(2)S(4) + \ldots + S(14)S(1) + S(15)S(2) \quad (2-19)
\]
which satisfies equation (2-16).

Corresponding to a sequence of this type a periodic function can be defined as follows:

\[ f(t) = S(n) \text{ for } n(\Delta t) \leq t < (n+i) \Delta t \quad (2-20) \]

\[ f(t + p\Delta t) = f(t) \quad (2-21) \]

Figure 2-1 shows such a function where \( p = 15 \). A particular time scale was chosen so that various parameters, displayed later, could be measured in real time.

The average value of the function \( f(t) \) can be evaluated in the following manner:

\[
\bar{f}(t) = \frac{1}{T} \int_{0}^{T} f(t) dt = \frac{1}{p\Delta t} \int_{0}^{p\Delta t} f(t) dt
\]

\[
= \frac{1}{p\Delta t} \left\{ \int_{0}^{\Delta t} + \int_{\Delta t}^{2\Delta t} + \cdots + \int_{(p-1)\Delta t}^{p\Delta t} dt \right\}
\]

\[
= \frac{1}{p\Delta t} \left\{ -\Delta t - \Delta t + \cdots + \Delta t \right\}
\]

\[
= \frac{1}{p\Delta t} \left\{ 8(+\Delta t) + 7(-\Delta t) \right\}
\]

\[
= \frac{\Delta t}{p\Delta t} = \frac{1}{p}
\]
15BIT PSEUDO RANDOM BINARY SEQUENCE

\[ P \times T_{\text{clock}} = \text{SEQUENCE LENGTH} = 15 \times 10 \text{ Ms} = 150 \text{ Ms} \]

CLOCK PULSE TRAIN
CLOCK FREQUENCY ...... 100 Hz

FIGURE 2-1 15 BIT PSEUDO RANDOM BINARY SEQUENCE
The autocorrelation function can be calculated in a similar manner, but in lieu of this calculation, an actual measurement of the autocorrelation function of the above detailed 15 Bit PRBS was done and is presented in Figure 2-2.

Figure 2-2 also displays the autocorrelation function for a 1023 Bit PRBS. The main observable difference is the fact that the baseline returns nearer to zero for the larger bit sequence. This fact implies that the value of the autocorrelation function is inversely proportional to the number of bits. The actual value of the autocorrelation between the peaks can be shown to be equal to \(-\frac{\phi_{aa}(0)}{p}\) and \(\phi_{aa}(0)\) is equal to the peak amplitude of the PRBS squared. The peak amplitude in the example shown is 1 volt so \(\phi_{aa}(0) = 1\) volt\(^2\). The value of the baseline for the 15 Bit case is thus, \(-1/15\) volt\(^2\).

Of additional interest is the Power Density spectrum of the 15 Bit PRBS. Figure 2-3 shows a plot of the envelope of the line spectrum derived by taking the Fourier transform (cosine) of the autocorrelation function.

It is appropriate to present a prescription for generating additional sequences that satisfy the equation (2-13) through (2-16).

The reader is referred to the paper by Roe (16), cited above, for extensive detail and a rather complete reference list, directed to the problem of generating pseudo random binary sequences.

Following the method of Roe, it is necessary to have a generator sequence to construct a complete sequence. Suitable generator sequences for the range of \(p = 3\), to \(p = 1,048,575\), are presented on page
AUTO CORRELATION FUNCTION OF PSEUDO RANDOM BINARY SEQUENCE

INPUT: CLOCK- $10^2$ Hz
AMPLITUDE- $\pm 1$V

CORRELATOR: CH.A; D.C. COUPLED GAIN X1
CH.B; CH.A COUPLED GAIN X1

FIGURE 2-2 AUTOCORRELATION FUNCTION OF PRBS
COSINE TRANSFORM OF AUTO CORRELATION FUNCTION OF PSEUDO RANDOM BINARY SEQUENCE

INPUT: CLOCK - $10^2$Hz
AMPLITUDE - ± IV

CORRELATOR: Ch.A; D.C. COUPLED
GAIN x 1
Ch.B; Ch.A COUPLED
GAIN x 1

FOURIER ANALYZER: INPUT GAIN - 6
OUTPUT GAIN - 5
SCAN - 100 Sec.
HANNING - S.S.

FIGURE 2-3 POWER DENSITY SPECTRUM OF PRBS
No.7 of Roe's paper. With these facts in mind, proceed as follows:

1. Select a $p = 2^{m-1}$ that fits the experiment.
2. Find the generator sequence specified by Roe.
3. Select an arbitrary "starter sequence" of $m$ binary digits. One of the digits must be non-zero.
4. Employ the following recursion relationship to extend the starter sequence.

$$C_n = C_{n-1}a_1 + C_{n-2}a_2 + \cdots + C_{n-m}a_m \quad (2-23)$$

where the arithmetic is modulo 2 and $n > m$.

To illustrate the procedure, part of the 15 Bit sequence will be generated.

From Roe, the generator sequence for $p = 15, m = 4$, is:

$a_0 = 1, a_1 = 1, a_2 = 0, a_3 = 0, a_4 = 1$

As a "starter sequence", the following will be used:

$C_1 = 0, C_2 = 0, C_3 = 0, C_4 = 1$

Two additional elements of the sequence will be calculated as follows, using the recursion relationship (2-12):

$C_5 = C_4a_1 + C_3a_2 + C_2a_3 + C_1a_4$

$= (1.1 + 0.0 + 0.0 + 0.1)_2 = 1$

$C_6 = C_5a_1 + C_4a_2 + C_3a_3 + C_2a_4$

$= (1.1 + 1.0 + 0.0 + 0.1)_2 = 1$

The first six elements of the sequence are thus 000111. The reader can calculate the remaining elements and show the completed sequence is
000111101011001.

By interpreting the 0's as (-1), the sequence generated is the one described above (Figure 2-1), and satisfies equations (2-13) through (2-16).
3.0 EXPERIMENTAL APPARATUS

3.1 REACTOR OSCILLATOR/MODULATOR

The mechanical device used to modulate the reactor is a moving shutter type that was described by Walker (2) in 1967. Walker's oscillator was modified in several areas and a new electronic drive package was built.

The light pipe photocell coupling system for determining the rotor position was re-designed to eliminate the light pipe. The primary reason for this change was that the light pipe suffered radiation damage and became opaque during reactor operation. The new system is a bulb-photocell combination and has proved satisfactory.

The most significant change in the oscillator was to remove the sine wave shaped cadmium parts and subsequently replace them with new cadmium parts shaped to provide a 15 Bit PRBS reactivity modulation. Figure 3-1 shows the shape of these parts. The fabrication of these parts was facilitated by the use of the technique of photomilling. Appendix B, details the procedure that was developed to photomill cadmium.

As can be noted in Figure 3-1, the bit pattern generated by rotating the rotor counter clockwise is the same as was derived in Section 2.4, above.

In addition to the PRBS shaped cadmium parts, a disc was constructed with the same bit pattern and used with the light bulb-photocell combination mentioned above, to generate an electrical output that is
BOTH STATOR AND ROTOR ARE VIEWED FROM CORE END

FIGURE 3-1 15 BIT PSEUDO RANDOM BINARY SEQUENCE REACTIVITY MODULATOR
representative of the reactivity modulation. Figure 3-2 shows this disc. The main problem associated with this construction is that of aligning the light disc and the rotor. This was done by trial and error until suitable alignment was achieved.

3.2 OSCILLATOR/MODULATOR CONTROL ELECTRONICS

Figure 3-3 shows the schematic diagram for the electronics located in the reactor oscillator/modulator. As can be seen from the figure, there are two identical channels. One channel carries the PRBS information and the second channel carries a sync signal that is also generated by the light disc. The sync signal is generated by the outside track of the light disc. The two L14A502 phototransistors, detect the varying light intensity as the light disc rotates. These signals are amplified by the 2N5134 transistors and inverted by the T2L NAND gates. The resulting signal is fed into a Signetics dual line drive that drives the cable connecting the oscillator/modulator to the Model 50 Oscillator control unit.

The Model 50 control unit is shown schematically in Figure 3-4. This control module is a two width NIM (17) module and has the following functions.

The Model 50 is outfitted with the 397A circuit board. The functions of the 397A circuit board are more complex than are required for the PRBS type experiment. It is designed to also be used with the old sine-wave stator and rotor parts and a multichannel pulse height analyzer used as a multichannel voltmeter. In light of this fact, only the functions applicable to the PRBS type of operation will be discussed.
VIEWED FROM CORE END

LOCATION OF PHOTOCCELL ASSEMBLY

BIT PATTERNS

CCW  000111101011001
CW   100110101111000

FIGURE 3-2  15 BIT PRBS LIGHT DISC
FIGURE 3-3 OSCILLATOR INTERNAL ELECTRONICS
First the 397A circuit board contains the regulated power supply for the light bulbs in the modulator. A five volt regulated supply is provided to supply the operating voltages for the remainder of the logic. Another function is that of providing an interlock action with the reactor console. This interlock prohibits the rotation of the oscillator without the cognizance of the reactor operator.

The Model 50 unit is the connecting link between the motor control unit and the oscillator. As such, the Model 50 is fitted with a synchro receiver that shows the position of the modulator rotor and an off-on switch that overrides the motor control.

The electrical outputs of the Model 50 and T²L compatible logic levels and are capable of driving coaxial cables.

3.3 MODEL 59 PRBS AMPLIFIER

The PRBS output of the Model 50 oscillator/modulator control unit serves as the input to the PRBS amplifier.

The PRBS amplifier has nothing to do with PRBS generation. The name was chosen because the amplifier was designed to be used with the PRBS type of experiment. More correctly, a name like "Bi-polar Precision Switch Amplifier", should have been chosen.

In any event, the function of the Model 59 amplifier is to take the single-ended T²L signal from the Model 50, and produce a bi-polar signal of precise amplitude to feed the channel A input of the PAR Correlation Function Computer. The bi-polar nature is chosen to permit the d.c. coupling of the channel A input. By d.c. coupling, no correction for the low frequency roll-off of an a.c. amplifier need be made.
Figure 3-5 shows the Model 59 amplifier. Of particular interest is the manner in which MOS analog switches are employed to switch the input of the 3341/15C amplifier between the adjustable regulated power supply rails. This technique relegates the T2L input to one of supplying time information only. The voltage amplitude is controlled only by the regulated power supplies. Additional exclusive OR and NAND gates are employed to provide set up for amplitude adjustment and also permit 180 degree phase shifting of the output with respect to the input.

3.4 RADIATION DETECTORS AND ELECTRONICS

Three radiation detectors were employed. A Reuter-Stokes RSN-15A gamma compensated neutron ionization chamber with a neutron sensitivity of $4 \times 10^{-14}$ a/nv and a physical size of 3.13 inches in diameter and 22.13 inches long was used in position G-7 of the thermal column. Details of the thermal column are discussed in Section 3.9. The RSN-15A was operated with a polarizing voltage of the +600 volts and a compensating voltage of -70 volts.

A second neutron ionization chamber, a Reuter-Stokes RSN-229A, was used in the dry tube located on the face of the core. The RSN-229A has a physical size of 1 inch in diameter by 9 inches long. The neutron sensitivity is $5 \times 10^{-15}$ a/nv and the gamma sensitivity is $4 \times 10^{-12}$ a/R/hr.

The third detector employed was a RSG-1S-M4 gamma ionization chamber. This detector was also manufactured by Reuter-Stokes. The RSG-1S-M4 was used in the dry tube and has a sensitivity of 1.25 a/R/hr. The physical size of the gamma detector is 2 inches in diameter by 8.13 inches long. +1100 volts was used as the polarizing voltage.
FIGURE 3-5  MODEL 59 PRBS AMPLIFIER
All three of the above detectors were supplied operating voltages from Keithley Model 240A high voltage power supplies.

The output currents of the detectors were amplified and converted to low impedance voltage signals by a Keithley Model 427 current amplifier.

3.5 CURRENT AMPLIFIER

The Keithley Model 427 current amplifier has two features that make it particularly suitable for this application. First the amplifier has a built-in precision bucking or suppression supply, that permits the steady state nominal d.c. current associated with the reactor power to be cancelled. The difference between the bucking current and the reactor power current can then be amplified, usually by 100, to permit expansion of the small fluctuating currents that are a result of the reactivity driving input. The vernier suppression current knob on the Model 427, affords an easy method of correcting for small power drifts and thus permits full use of the maximum gain of the 427.

The second feature of the Model 427 that makes it attractive for this application, is that of wide bandwidth. In fact, the rise time or bandwidth of the 427 is front panel adjustable. For all of the measurements described in this work a rise time of 0.3 milliseconds was employed. Figure 3-6, shows the experimental set-up used to measure the transfer function of the Model 427. Appendix C describes the first instrument in the set-up, namely, the Model 45 X-tal oscillator. In addition, the Model 41 PRBS generator is described in detail in Appendix
FIGURE 3-6  EXPERIMENT TO DETERMINE THE TRANSFER FUNCTION OF THE MODEL 427 CURRENT AMPLIFIER
D.

A current of approximately $10^{-8}$ amps is applied to the input of the 427, and the internal bucking supply is set to balance this steady input current. This d.c. current is modulated by the ±1 volt PRBS signal applied through a $10^3$ ohm resistor. This makes current fluctuations of $±10^{-9}$ amps. The Model 427 amplifier gain is set to $10^{10}$ volts/amp yielding output signals of ±10 volts. These output signals are inverted by Model 42 amplifier and then they are cross-correlated with the ±1 V PRBS input signal to yield the impulse response of the 427 inverting amplifier pair. Figure 3-7, shows the impulse response.

In addition to the impulse response, the transfer function can also be displayed. Figure 3-8, shows the phase and amplitude transfer function as derived by taking the Fourier transform of the impulse response. Section 2, discusses the theory and Section 3.6, discusses the hardware employed for this transform. Of particular interest, is the fact that the response is flat out to 1.2 KHZ (-3db) for the rise time setting of 0.3 milliseconds.

3.6 CORRELATION COMPUTER AND FOURIER ANALYZER

Both the correlation Computer and Fourier Analyzer used in this work are commercial instruments, built by Princeton Applied Research (PAR) Corporation.

The correlation computer, Model 101, is a hybrid computer utilizing both analog and digital techniques, to solve the correlation integral. The Model 101 computes the value of the integral (18):
Figure 3-7 Impulse Response of the Model 427 Current Amplifier
FIGURE 3-8  MODEL 427 CURRENT AMPLIFIER TRANSFER FUNCTION
\[ \phi_{ab}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T} a(t) b(t-\tau) \, dt \] (3-1)

at one hundred incrementally increasing values of the time delay, \( \Delta \tau \).

If \( n \) is the channel number, \( n\Delta \tau = \tau \) is time coordinate of the \( n \)th channel. The \( n \)th channel can be represented as:

\[ \phi_{n}(t) = \frac{1}{RC} \int_{-\infty}^{t} e^{-\frac{t-t'}{RC}} v_a(t')v_b(t'-n\Delta \tau) \, dt' \] (3-2)

where \( R_C \) is the time constant of the averaging circuits and \( t' \) is greater than \( t \) by definition. The time constant employed is 20 sec.

Computation at each point involves three operations, time shifting, multiplication and integration. The rate of time shifting determines the value of \( \Delta \tau \) and therefore, establishes the time base of computation for each channel.

All operations are performed simultaneously in real time so that the length of time required to compute the entire function, depends only on the averaging time constant (20 sec). Each run was greater than 100 sec, so that the error of the correlation function was less than one percent of the true value. As the function is being computed it is stored in a 100 channel analog memory. The stored function can be read out in several ways. In general, an oscilloscope and an x-y plotter were used as serial output devices. Several Figures, like 2-2 and 3-7, are reproductions of the plotted output.
In conjunction with the Model 101 correlation function computer, a PAR Model 102 Fourier Analyzer, was employed to transform the system impulse response into the frequency domain \((19)\).

In Section 2.2, it was shown that the system impulse response could be measured by the technique of cross-correlating an input function with the system output, if the input function had an autocorrelation function approximating a delta function. Lee (11) page 328, shows that the impulse response is related to the system function \(H(\omega)\) by:

\[
h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} \, d\omega \tag{3-3}
\]

this expression is in the form of a Fourier transform. By inverse transformation the following is obtained:

\[
H(\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} \, dt \tag{3-4}
\]

by substituting \(2\pi f\) for \(\omega\),

\[
H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} \, dt \tag{3-5}
\]

is obtained and is the function calculated by the Model 102 Fourier analyzer.

Following the method of Lee, page 341, equation (2-6) can be transformed to yield:

\[
\Phi_{ob}(f) = H(f)/\Phi_{oo}(f) \tag{3-6}
\]
This expression states that the input-output cross-power density spectrum $\phi_{ab}(f)$ of a linear system is the product of the system function and the input power density spectrum.

So by evaluating (3-5) above, the ratio $\phi_{ab}(f)/\phi_{aa}(f)$, is obtained.

Now, to summarize the preceding discussion in words:
The Model 101 correlation function computer, computes the value of the impulse response, $h(t)$, from the cross-correlation of the input $a(t)$ and the output $b(t)$ functions. The Model 102 then computes the system function $H(f)$ by taking the Fourier transform of the impulse response $h(t)$.

The system function is presented at the output of the Model 102 Fourier analyzer as both an amplitude and phase frequency spectrum. In fact, by using the Model 60 dual log amplifier, described in Section 3.8, the Bode plot of the system can be plotted directly.

For completeness, two additional features or characteristics of the Model 101 and Model 102 pair need to be mentioned.

The first of these is the frequency response of the system. Figure 3-9 gives the frequency response in terms of the ratio of $f/f_n$. By nature of this system, $f_n$ (Nyquist frequency) is equal to the full scale frequency output of the Model 102 and can, therefore, be applied directly.

Since the correlation time and thus the maximum frequency transformed can be easily changed, it is seldom necessary to apply the corrections of Figure 3-9.
FIGURE 3-9 FOURIER ANALYZER FREQUENCY RESPONSE
Finally, the Model 102 has switch selectable Hanning lag windows or weighing functions, that can be employed if the impulse response converges too slowly in the correlation interval. This is necessary since the integration limits, equation 3-5, of the Fourier integral extend from $-\infty$ to $+\infty$. The use of the Hanning lag windows is explained by Blackman and Tukey (20). The results of employing Hanning lag windows are illustrated in the Model 102 Instruction Manual (19).

3.7 INTERFACE, MODEL 43

Figure 3-10, show the Model 43 interface unit that was designed and built to facilitate the use of the correlation function computer and the Fourier analyzer. Several useful features are incorporated into the interface. First, and most important, is the ability to disconnect the computer from the analyzer without removing any cables. This is accomplished by placing the Fourier analyzer "In-Out" switch in the "Out" position. This seemingly trivial detail is important, because of the fact that the cable connecting the two units is on the back of the instruments and must be disconnected each time the correlator is used by itself.

Other functions of the Model 43 can be understood by looking at Figure 3-10. For example, by selecting the toggle switch and rotary switch positions, a variety of output conditions are permitted. The usual setup is for the "Scope" section to be set up for observing $c(\tau)$, the correlation function, and the "Record" section set up to plot one of the available spectra selected by the "Spectrum" switch. However, it is also desirable to plot the correlation function, so the "Record" section
FIGURE 3-10 MODEL 43 INTERFACE
can be made to do this by switching the "Record" toggle switch to $c(\tau)$. When the "Record" toggle switch is set to $c(\tau)$, the "Record-x" output is the time axis and the "Record-y" output is the correlation function amplitude. When the "Record" toggle switch is set to FA, the "Record-x" output is the frequency axis and the "Record-y" output is the amplitude of the selected spectrum.

3.8 DUAL LOG AMPLIFIER

Figures 3-11 and 3-12, are the schematic diagram and front panel photograph respectively, for the Model 60 Dual Log Amplifier. This instrument, constructed in a 2-width NIM module, was built to permit Log-Log plotting of the Fourier Analyzer Amplitude Output. The result of this plot, of course, is the familiar Bode plot of the frequency response of the measured amplitude. Both sections of the Model 60 are identical and, as can be seen from the Figures, either can be switched to provide a linear output. This feature is useful when displaying the phase transform as a function of frequency. By selecting log frequency and linear phase, the resulting plot is the Bode phase plot directly.

The function switch permits the selection of output ground, fixed +3.00 volt and fixed +6.00 volt output for recorder calibration, as well as log or linear outputs.

3.9 THE OSU NUCLEAR REACTOR

The OSU Nuclear Reactor is a light water moderated thermal reactor of the BSR configuration. Figure 3-13, is a top view of the
MODEL 60 CONTAINS: 2 CHANNELS
P.C. BOARD LAYOUT No. 440
# DENOTES INTERNAL POT.

FIGURE 3-11 SCHEMATIC DIAGRAM OF MODEL 60 DUAL LOG AMPLIFIER
FIGURE 3-12  FRONT PANEL OF MODEL 60 DUAL LOG AMPLIFIER
FIGURE 3-13  REACTOR CORE SHOWING DRY TUBE

A - CIF  I - BEAM PORT NO. 1
B - SOURCE DRIVE  EXTENSION
C - GRAPHITE ELEMENT  J - FISSION CHAMBER
D - FUEL ELEMENT  DRIVE
E - CONTROL ROD DRIVE  K - THERMAL COLUMN
F - DRY TUBE  EXTENSION
G - RABBIT TUBE  L - IONIZATION CHAMBER
H - BEAM PORT NO. 2  HOUSING
EXTENSION
core, showing the experimental dry tube, as well as the permanent equipment and facilities. The thermal column is shown in Figure 3-14. The particular graphite stringer removed is G-7. This location is on the core centerline and its depth is the full depth of the column. In fact, when the detector is inserted fully, it butts up against a four inch lead gamma shield that is between the end of the graphite and the thermal column liner. This position is approximately 18 inches from the west face if the core.

The number one beam port, with the modulator readied for insertion, is shown in Figure 3-15. The modulator is inserted fully into the beam port and beam port extension (Figure 3-13), and the rotor end is located less than one-half inch from the north core face, and on the core centerline.

All of the experimental locations employed in this study are thus shown in the three Figures: 3-13, 3-14, and 3-15.

3.10 THE OVERALL MEASUREMENT SYSTEM

An operational block diagram of the measurement system is shown in Figure 3-16. All of the individual instruments have been discussed in detail, with the exception of the period meter. Referring to Figure 2-1 and Appendix D, recall the discussion about the sync signal. The period meter monitors the sync signal and thus measures the period of rotation of the modulator. Figure 3-2, shows the light Disc with the outside track used to provide the sync information, and Section 3.2, discusses the electronics.
FIGURE 3-14  THERMAL COLUMN WITH RSN-15A PARTIALLY INSTALLED
FIGURE 3-15    BEAM PORT NO.1 AND REACTIVITY OSCILLATOR/MODULATOR
Figure 3-16 SYSTEM BLOCK DIAGRAM
Shown also in Figure 3-16, is a 1.8 scfm air supply and a console permit. Reference (2), discusses these in detail. The air flow is used to maintain low Argon-41 build-up, and the console permit signal prevents the modulator from being run without the knowledge of the reactor operator.

Finally, for a sense of completeness, Figure 3-17, shows the rack of electronics, the x-y plotter, and the oscilloscope, used throughout this experiment. Each instrument discussed above, can be identified from this photograph.

Appendix E, is included as an illustration of the application of the techniques of Section 2, and the equipment of Section 3, to a standard band pass electronic circuit.
FIGURE 3-17  OVERALL SYSTEM ELECTRONIC EQUIPMENT
4.0 RESULTS

Using the measurement system described in Section 3.10 with the PRBS reactivity modulator discussed in Section 3.1, numerous reactor runs were performed at a power level of 50 watts. The transfer function that was measured during these runs is shown for the three detectors and two detector locations in Figures 4-1 through 4-3. In order to extend the frequency response out beyond 50 Hz, two correlation times were employed. The first time of 200 msec and a corresponding modulator period of 215 msec, was used to determine the transfer function out to 20 Hz. The second correlation time of 100 ms and a modulator period of 107 msec, was used in the frequency region of 20 to 100 Hz. The data from the two runs were normalized at 20 Hz.

The reason for two runs can best be explained by noting that the effective clock frequency of the modulator is the expressed:

\[
\begin{align*}
  f_c &= \frac{1}{T_m/p} \\
  &= \frac{1}{T_m/15}
\end{align*}
\]  \hspace{1cm} (4-1)

\hspace{1cm} (4-2)

where \( f_c \) denotes the clock frequency and \( p \) is the number of bits in the sequence and \( T_m \) is the modulator period. For a modulator period of 215 msec, \( f_c \) equals 69.8 Hertz and the bit width is 14.33 msec. It is generally accepted \((15)\) that data derived from such a sequence is limited to the effective clock frequency divided by two. By adhering to this convention, the data from a 200 msec correlation time run is good to 35 Hz. By employing an additional correlation time and oscillator period of 100 msec, the frequency can be extended to 75 Hz minimum.
FIGURE 4-1 AMPLITUDE TRANSFER FUNCTION BY NEUTRON FLUCTUATIONS
FIGURE 4-2 AMPLITUDE TRANSFER FUNCTION BY NEUTRON/GAMMA FLUCTUATIONS
Figure 4-3 Amplitude Transfer Function by Gamma Fluctuations

Gamma data
RSG-1S-M4 Gamma
Ionization chamber located in dry tube

Frequency in hertz
Amplitude-relative units

14.5 Hz
In Section 5.2, a technique of supplying the PRBS reactivity input is described that will allow the frequency response to be measured with a single cross-correlation measurement.

For both of the detectors (gamma and both gamma and neutron) located in the dry tube, a value of $\ell/\beta$ of 0.0118 sec was measured using the breakpoint shown in Figures 4-2 and 4-3. For the compensated neutron ionization chamber located in the thermal column a $\ell/\beta$ of 0.0109 sec was measured (Figure 4-1).

The lower frequency cut off at about 5 Hz was necessitated by the fact that the natural frequency of PRBS that was used is $f_C/p$ or 4.6 hertz for the 215 msec oscillator period.

Also, the Fourier Analyzer will transform only down to the reciprocal of the correlation time. In this case, this is 1/200 msec, or 5 Hz.

The obvious way to extend the lower frequency cut off and still retain the high frequency end, is to increase $p$. Increasing $p$ is difficult, because of the decreasing angular size of each bit. Perhaps up to 128 bit could ultimately be etched. If this were the case, the lower frequency limit could be extended down to 0.5 Hz with an oscillator period of 2 seconds. The clock frequency would still be comparable at $128/2$ equal to 64 Hz.

Figure 4-4, shows the data measured by Walker (2), page 99, plotted on the same scale as the data presented above. The breakpoint in Walker's data indicates a $\ell/\beta$ of 0.0118 sec, which is consistent with the values measured by cross-correlation of a PRBS reactivity input and the resultant neutron and/or gamma fluctuations.
PLOTTED FROM WALKER'S DATA (2), PAGE NO. 99.

DETECTOR: RSN-15A REUTER-STOKES
LOCATION: POSITION G-7 OF THE THERMAL COLUMN

FIGURE 4-4 AMPLITUDE TRANSFER FUNCTION BY NEUTRON OSCILLATOR
5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

It is the conclusion of this author, that the transfer function of a critical nuclear reactor can indeed, be determined by measuring the cross-correlation between a PRBS reactivity input and the resultant gamma fluctuations. In addition, it is concluded that the measurement can be performed on-line and that the time required to perform the measurement is much less than that required for a conventional oscillator measurement. A further conclusion is, that at least for a relatively clean core, conventional gamma ionization chambers can be employed.

The primary shortcoming of the experiment as performed, is the inability to change the PRBS reactivity driving function during an experiment. In fact, using the modulator as it stands, requires multiple runs (see Section 4.0) for a transfer function measurement.

The experiment could also be significantly extended by acquiring a small compensated ionization chamber, so that neutron and gamma results could be compared at various spatial points.

5.2 RECOMMENDATIONS

It is advisable to design and build a new PRBS reactivity insertion device that can be electronically adjusted by various sequence lengths and clock frequencies. This would enable an experimenter to modify the reactivity insertion while the reactor is at power and study the effects of different PRBS inputs.
One of the more interesting possibilities of the future experimentation lies in spatial studies with both gamma and neutron detectors. It should then be possible to verify the work of Kostic and Seifritz (9). In fact, it should be possible to get a number representative of the weighing function that relates the gamma to the neutron frequency response at several locations.

Also, it would appear that perhaps a ratio of "average number of fission neutrons" to "average number of fission gamma rays" could be determined. This possibly would involve modeling the gamma fission spectrum, perhaps in a manner similar to Maienshein, et al; (21). Information regarding detector efficiencies would need to be studied in detail with respect to gamma and neutron energies.

Additionally, an experiment performed by Kenney and Schultz (22) indicates as does the work of Kostic and Seifritz (9), that the gamma correlation range is much greater than the neutron correlation range. This would suggest the possibility of using multiple detectors located outside the core, to study internal power variations. This is attractive for two reasons. First, the instrumental effect of inserting a detector into the core is minimized. Second, in power reactors with large fluxes, the detector lifetime would be lengthened by being able to have the detector removed from the core.

Finally, it is recommended that a laboratory experiment be refined that would permit students in reactor dynamics courses to familiarize themselves with this type of equipment and measurement technique.
It should be noted, that the bulk of this equipment can be adapted for use with a wide spectrum of physical systems, other than nuclear reactors, simply by varying the input inserting device and the output parameter detector. Typical examples would include: mechanical, fluidic, acoustic, and biological systems.
REFERENCES


REFERENCES


On April 14, 1971, a Hazards Subcommittee of the Advisory Committee on Reactor Operations (ACRO), met at the Nuclear Reactor Laboratory and considered a proposal to modify Walker's Oscillator (2). The proposal reads as follows:

It is my desire to obtain Committee approval to change statement (1) "No modifications of the apparatus within the protective aluminum case shall take place without prior Committee approval", to read: (1) Any modifications made to the apparatus within the protective aluminum case will not differ substantially either structurally or in material composition from the device as approved in September, 1967. In addition, any change made that affects the reactivity worth of the Oscillator will be within the limitations set down under Section 6.2 of the Technical Specifications.

The original statements of restrictions are documented in the minutes of the September 12, 1967, ACRO Meeting. The proposal was accepted by the ACRO Subcommittee on April 14, 1971, and is documented in a letter to ACRO from Dr. W. E. Carey, dated April 14, 1971.

In order to comply with Section 6.2 of the OSU Nuclear Reactor Technical Specifications, and also with the approved version of the experiment, it was necessary to measure the reactivity worth of the oscillator. The worth determination experiment was performed on April 15, 1971, and is documented in the Reactor Logbook on page 1255. The technique used, is described in the Request for Reactor Operations
No. NRL-1004, dated April 15, 1971. The oscillator was loaded into Beam Port No. 1, and the rotor was positioned so that no polyethylene was exposed. An Incremental Control Rod Approach to Critical was performed. The reactor was subsequently leveled at 9.0 watts and the shim rods were banked at 58.0 cm. The regulating rod was positioned at 39.55 cm. By using the regulating rod calibration curve, a total reactivity worth of +0.257% Δk/k over voided beam port No. 1 was measured. A duplicate run with the rotor positioned to expose a 1-Bit poly wedge yielded a differential reactivity worth of .004% Δk/k.

So, as the oscillator is rotated, a PRBS reactivity input of .004% Δk/k modulates the critical reactor.

Referring to Figure 3-1, it can be seen that the exposed area, as a function of rotation, does not have infinite risetime. In fact, the risetime is linearly increasing and decreasing within the single bit space. Figure A-1, shows the shape of the reactivity as a function of the angular position of the rotor.

Figure A-2, shows the autocorrelation functions of both a 15 bit PRBS and the 15 bit reactivity shape of Figure A-1.

Curve B is the autocorrelation function of the funcion shown in Figure A-1. Note the similarity of Curve B and Curve A which is the autocorrelation function of the 15 bit PRBS. The two main differences are that Curve B has about a two percent greater width and about a twenty-five percent lower height.

These differences are not significant when considering the reactivity of Figure A-1 as a driving function.
FIGURE A-1 15 BIT PRBS REACTIVITY VARIATION SHAPE
FIGURE A-2 AUTOCORRELATION FUNCTIONS OF 15 BIT PRBS SHOWING THE EFFECT OF TRANSITION TIME
This fact is demonstrated in Appendix E, where the transition time of the PRBS was deliberately "spoiled" to a shape identical to Figure A-1, and the impulse response of the Band Pass Filter was measured with this "spoiled" function.

Figure E-7, shows the comparison of the impulse response measured using the PRBS and the "spoiled" PRBS driving function.
APPENDIX B  PHOTOMILLING CADMIUM

In order to fabricate the cadmium rotor and stator parts shown in Figure 3-1, a technique was developed to photomill cadmium.

The shape of the part is cut from Ulano "Rubylith" type D3R. A twice-size layout is normally used to make the cutting easier. The completed layout is photographically reduced to the final part size. Two copies of these photographic negatives are employed, one on each side of the cadmium, to minimize undercutting.

The cadmium metal sheet is coated on both sides with Kodak Photoresist "KPR-Type 3". The photoresist is dried per the manufacturer's instructions and the two photographic negatives, described above, are applied to the two sides. Both sides of the cadmium sheet are exposed, through the negatives, to ultraviolet light and the part is subsequently developed with Kodak "Ortho Resist Developer".

The cadmium now has photoresist on the area to be left, and is bare in the area to be removed.

Ammonium Nitrate solution is used as the etchant, and is prepared by adding 300 ml of Ammonium Nitrate to 3.5 liters of water. The operating temperature is about 200° F.

The cadmium is immersed in the etchant and is agitated until all of the material not covered by the photoresist is etched away.

Approximately 20 minutes are required to etch a cadmium thickness of .020 inches. It has been observed that holes of .060 inches in diameter can be etched in .020 inch material.
A crystal controlled pulse generator was designed and constructed to provide a precise frequency source to drive the PRBS generator.

A 1 megahertz, ct cut, crystal was employed as the time base. This clock frequency was divided by a series of 6 BCD decade counters and provided a range of decade frequency steps from 1 hertz to 1 megahertz.

In addition to this divider string, a second string of three presetable BCD counters permit division of the decade frequency steps by another three digit number that ranges from 001 to 999.

The complete range of the Model 45 is from 1/999 hertz on the lower end to 1 megahertz maximum.

Figure C-1, is a photograph of the Model 45 that shows the thumbwheel switches used to set the output frequency.

The outputs of the Model 45 are T2L logic level signals with a pulse width of 200 nanoseconds. Both the normal (0 to +5V) and the complement (+5 to 0) signals are provided.

The mechanical construction conforms to the NIM-2 width module specifications and as such, is usable in any NIM Bin/Power supply system.
FIGURE C-1  MODEL 45 X-TAL OSCILLATOR FRONT PANEL
Following the general principles outlined in Section 2.4 concerning the generation of pseudo random binary sequences, an electronic instrument was designed and built to generate these sequences for $p = 15$ through $p = 1023$. Figure D-1, shows the front panel of the NIM 2-width instrument. The "Clock In" Input is supplied by the Model 45 X-Tal oscillator, described in Appendix C, above. Reference to Figure 2-1, shows the relationship between the clock pulses and the particular PRBS sequence.

In addition to the PRBS sequence, a sync pulse is generated. Figure 2-1, shows the time location of the sync signal with respect to the PRBS sequence. The sync signal occurs in coincidence with the first bit of the PRBS sequence each time the sequence repeats. The value of the sync bit or signal can be seen by again referring to Figure 2-1. In the example shown, a sync signal appears every 150 msec. This is the time period of the sequence, namely 15 bits multiplied by the clock period of 10 msec equals 150 msec. Another use of the sync signal, is to verify that the bit number is correct. By using a clock frequency of one kilohertz, the bit number is displayed directly in milliseconds by measuring the sync output with a period counter.

Both the PRBS and the sync outputs are T$^2$L compatible (0 to +5 volts). The Model 59 PRBS amplifier, described in Section 3.3 above, is excellent for transforming the T$^2$L signal to a bipolar signal of precise amplitude.
FIGURE D-1    MODEL 41 PRBS GENERATOR FRONT PANEL
APPENDIX E  BAND PASS FILTER, AN ILLUSTRATION

An electronic circuit of known response was chosen to illustrate the application of the techniques of Sections 2 and 3.

Figure E-1, is the schematic diagram of the circuit, a band pass filter, (23).

To illustrate that the impulse response of the circuit can be measured by using a PRBS input, refer to Figure E-2. Note the similarity of the two oscilloscope photographs (a, b) shown in Figure E-2. Photograph a, is the output of the circuit shown in Figure E-1 when driven by a +10 volt, 1 millisecond wide impulse. Photograph b, is the cross-correlation function of the circuit's output with a 63 bit, ±1 volt PRBS input. The clock frequency used to make this run was 500 Hz. Figure E-3b, is a block diagram of the circuit used to measure the cross-correlation function. In Figure E-2b, the discreet 100 channel steps are evident. The gain of the measurement apparatus has been chosen to display an amplitude approximately the same as in photograph E-2a, so that a visual comparison can be easily made.

Figure E-4a, shows the impulse response on a 100 msec time scale, with the damping factor envelope sketched in place. Figure E-4b, is a semi-log plot of the damping factor which yields a decay constant (\( \alpha \)) of 78.5 Sec\(^{-1} \). By noting the period of the damped oscillations in Figure E-4a (10 msec), the angular frequency \( \omega = 2\pi f = 2\pi T \), can be determined to be 628\(^{-1} \). From \( \omega \) and \( \alpha \), the damping constant \( \xi = \alpha/\omega = 0.125 \) is determined.
\[ B = \frac{\omega_{CU} - \omega_{CL}}{2 \omega_o} = 0.102 \]
\[ \omega_o = (RC)^{-1} = 628 \text{Sec}^{-1} \]
\[ \xi = 0.125 \]

FIGURE E-1  BAND PASS FILTER
a. IMPULSE INPUT, +10 VOLTS: 1 MS.
HORIZ. 5 MS./ DIV.
VERT. 0.5 V/ DIV.

b. PRBS INPUT, $f_c = 500$ Hz, 63 BITS
HORIZ. 5 MS./ DIV.
VERT. 0.1 V/ DIV.

FIGURE E-2  IMPULSE RESPONSE OF BAND PASS FILTER
a. FREQUENCY SCAN EXPERIMENTAL SET-UP

b. PRBS INPUT EXPERIMENTAL SET-UP

FIGURE E-3 BLOCK DIAGRAM OF EXPERIMENTAL APPARATUS
DAMPING CONSTANT, $\xi = \frac{\alpha}{\omega} = 78.5 \text{ Sec}^{-1} / 628 \text{ Sec}^{-1} = 0.125$

**Figure E-4** IMPULSE RESPONSE, DAMPING FACTOR PLOT

a. IMPULSE RESPONSE OF BAND PASS FILTER

b. SEMI-LOG PLOT OF DAMPING FACTOR

$A = A_0 e^{-\alpha t}$

$A_0 = 0.24$

$\alpha = 78.5 \text{ Sec}^{-1}$
Figure E-5, shows the Bode plots of the phase and amplitude of the band pass filter. Note that in the phase plot, the phase is zero at 100 Hz (628 Sec\(^{-1}\)).

In the amplitude plot, a correction for the frequency response of the correlator and Fourier Analyzer has been applied (see Figure 3-9 for a plot of this correction).

The correlation time of 100 msec was chosen, so that the Fourier Analyzer would transform out to \(50/T = 500\) Hz making this correction necessary for the high frequency side. This illustrates the use of the frequency response correction curve, Figure 3-9.

To further establish the applicability of the techniques applied above, the band pass filter was examined using a variable frequency sine wave. Figure E-3a, is the block diagram of the test set-up. Figure E-6, is the amplitude response curve made by applying a fixed amplitude, variable frequency input, and measuring the resultant output.

The fact that E-5 and E-6 are identical, as well as the fact that E-2a and E-2b are identical, establishes the applicability of the methods described in the body of this work.

For the sake of completeness, the 63 bit PRBS sequence used to measure the impulse response, was modified by "spoiling" the transition time until a waveform similar in shape to Figure A-1 was obtained. This "spoiled" PRBS driving function was then used, as in Figure E-3b, to measure the impulse response.

Figure E-7, shows results of this measurement. Curve A is the impulse response measured using the PRBS, and Curve B is the impulse
FIGURE E-5  BODE MAGNITUDE, PHASE FREQUENCY PLOTS
Figure E-6 Bode magnitude plot by frequency scan.

Amplitude transfer function by using frequency scan set-up.
FIGURE E-7  IMPULSE RESPONSE MEASURED WITH PRBS
AND TRANSITION TIME SPOILED PRBS
response measured using the "spoiled" PRBS. For all practical purposes, these curves are identical. This displays the fact that the driving function need not be of perfect shape, as long as the autocorrelation function still approximates a delta function.