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BLOCKING PROBABILITY IN
NON-SYMMETRIC MULTISTAGE NETWORKS

DISSERTATION

Presented in Partial Fulfillment of the Requirements
for the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By

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* * * * *

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1973

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LIST OF SYMBOLS

A = Traffic offered to a group of devices (erlang)
a = Traffic per device (erlang)
c = Average call intensity
h = Average holding time per call
i = Number of A-matrices per line group
j = Number of outputs per A-matrix
    = Number of B-matrices
J = Total number of junctors
k = Number of outputs per B-matrices
    = Number of C-matrices
l₁ = Number of inputs per A-matrix
l₂ = Number of outputs per C-matrix
m = Number of junctors in a subgroup
n = Number of line groups
N = Total number of lines
CHAPTER I

Introduction

Interest in teletraffic theory and switching networks has increased considerably in recent years. This is not only due to the rapid growth of wire communication of classical types (telephony and telegraphy) but also the introduction of processor controlled switching networks and specially designed networks for the transmission of digital information. The introduction of electronic computers -- processors -- as control elements for switching networks has opened the door for network designers to develop multistage networks of a great complexity together with sophisticated route control algorithms fully utilizing the high speed of the electronic processors. The success of Pulse Code Modulation (PCM) for the transmission of information has created a great interest in developing switching networks utilizing Time Division Multiplexing (TDM) principle. The fact that such TDM networks can be analyzed by means of space division analogues [1], [2], [3] has also increased the current interest in teletraffic theory and switching networks.
One of the main parameters of interest in the analysis of a switching network is the probability of success (failure) in establishing a path through the switching network between two idle terminals. The existence of a path depends upon the state of the network which in turn depends upon the traffic intensity, a quantity determined by the call intensity and the duration of a call, and the structure of the network itself. The first systematic mathematical approach to the telephone traffic problem was undertaken by A.K. Erlang [4]. He applied the theory of probability to the trunking problem in a full availability group of a lost call cleared system. Full availability implies that two idle terminals can always be connected through the network. A lost call cleared system discards immediately all calls which can not be completed due to congestion. The number of busy sources, therefore, can not be greater than the number of busy servers in a lost call cleared system. The theory of probability and the concept of statistical equilibrium have been applied to a variety of problems encountered in teletraffic. As probability theory evolved, a more elegant derivation of the Erlang formulation based on a Markov process of the simple birth and death type was given by A. Jensen [5].

Economic considerations associated with the growth of telecommunication necessitated a search for efficient
networks other than single stage networks. The development of multistage networks presented a difficult problem in the analysis of their traffic handling performance. C. Jacobaeus [6], in his doctoral dissertation, presented the classical analysis of multistage switching networks -- Link Systems as he defines them -- under the assumption of stochastic independence between the stages and a priori assumptions regarding the state distribution of links in various stages. Several authors [7]-[10] have since contributed to the theory of link systems. Elldin [11] analyzed a simple two stage link system by developing the equations of state and solving the system of linear homogeneous equations under the assumption of statistical equilibrium. Because of the immense number of possible states of a link system of any practical use, one encounters enormous difficulties in formulating the linear system of equations and especially in obtaining numerical results. By considering certain states with equal occupancy as indistinguishable, Elldin as well as Bashrin [12] have shown that the number of equations can be reduced. However, for a complex network this number is still very high.

Three stage folded networks were considered by Bowers [13] who derived a comprehensive set of equations for blocking probabilities for a variety of network parameters. The 'folded' concept outlined by Bowers assumes
4-wire connection between an input and an output. It requires that all incoming and outgoing trunks appear at both inputs and outputs of the switching network. A connection between an incoming and an outgoing trunk can be established as an input to an output or as an output to an input connection and hence the name folded. Bininda and Daisenberger [14] have developed a recursive formula for the blocking probability for switching networks of almost any description. The method uses combinatorial analysis and assumed or iteratively obtained distribution for the state of the links in various stages and permits calculation of the availability of a path up to a given stage until the last stage is reached.

It assumes that the connection is established from an input to an output and a single attempt to establish a path through the entire network is made when the desired output is pre-selected. A very similar approach was taken by Lotze [15] in his Combined Inlet and Route Blocking (CIRB) method for blocking probability calculations.

A quick and fairly accurate evaluation of switching networks can be obtained easily with Lee's [16] method of probability linear graph. This method gives values for blocking probabilities which are higher than those obtained by more exact methods. However, an expedient measure of relative efficiency of a number of alternative networks can be obtained during initial design of switching networks.
Again, the expression for blocking formula of a complex network is difficult to evaluate. The simulation program NEASIM developed by Grantges and Sinowitz [17] simulates Lee's model for the network. Verma and Durr [18] have presented a method to analyze a non-symmetric network by developing an equivalent Lee's model. The results obtained by this method are approximate. The term non-symmetric as defined by the authors only implies that the number of links between stages is less than one and not the topological structure of the network. There are numerous authors who have contributed to the theory of link systems -- switching networks or connecting networks -- and have given the analysis of the network of a specific design. [19]-[26]

The present study considers a four stage switching network which differs considerably from the conventional four stage networks being used in the industry at the present time, e.g. number 5 crossbar, ESS number 1, 4-A machine, etc. The schematic diagram of a conventional symmetric four stage network is shown in figure 1. By symmetric we mean that the left half of the network is identical to the right half of the network. The configuration of the component matrices of a network does not change the symmetric nature of the network as a whole. The network presented here is a non-symmetric four stage network. The schematic diagram of the network configuration is shown in figure 2. It is seen
Figure 1
A Symmetrical Four Stage Network
Figure 2
A Non-Symmetric Four Stage Network
from figure 2 that in order to provide a path between an input and any output, the lines of the third stage are distributed to all matrices in the fourth stage as opposed to a symmetric network of figure 1 where this distribution takes place in the middle of the network. The control device (processor) follows a definite sequence of steps for path searching and permits multiple attempts.

The analysis presented here is directly related to the control routine. Instead of a stage-by-stage determination of an available path, the present analysis derives blocking probability with respect to blocking events as seen by the processor. This approach is very suitable to observe the effects of configuration changes on total blocking. The changes in path searching routine can also be included with relative ease in the present analysis.

The non-symmetric network presented here is more economical compared to the symmetric network when costs per unit of traffic carried are compared. This is especially true when multiple attempts are permitted. The network can be expanded to any size with less crosspoints than that would be required for a symmetric network. The three stage network is a potential candidate for a non-blocking [19] or a non-blocking in the wide-sense [20] network. This flexibility is not present in a symmetrical four-stage network.
The description of the network, its configuration and the control principle employed for a connection through the network are given in Chapter II. A brief description of a conventional network is also contained in Chapter II. Mathematical analysis for developing the expression for blocking probability is given in Chapter III. The analysis is based on probability theory; combinatorial methods, structure of the network and the path searching routine. Chapter IV contains the results of numerical evaluation. The effects on probability of blocking are shown for various network parameters. Conclusions and recommendations are presented in Chapter V. Those who are not familiar with the terms used in teletraffic theory will find Appendix I containing the glossary of terms very useful. Fundamentals of link systems are explained in Appendix II. Appendix III contains a derivation for the state distribution for a complex combinatorial problem encountered during the development of the final expression for blocking probability.
CHAPTER II

Description of the Switching Network

The basic building block of a switching network is a crosspoint switch. A crosspoint switch with m inputs and n outputs is symbolically denoted as a m x n switch. It has, consequently, m x n crosspoints and any one of m inputs can be connected to any one of n outputs by operating a proper crosspoint. Thus a crosspoint switch is a single stage switching network. The multistage switching networks are constructed by interconnecting -- linking -- a number of crosspoint switches. The interconnections between various stages are called links. Switching networks developed with any number of stages (>1) form a general class of Link Systems. Various symbolic representation of a m x n crosspoint switch are shown in figure 3.

A convention used in step by step exchanges is shown in figure 3(a). The representation shown in figure 3(b) is influenced by the introduction of a crossbar switch signifying the horizontals and verticals of the crossbar. It is also very suggestive of the name crosspoint switch. The link convention shown in figure 3(c) is independent of
Figure 3

Symbolic Representation of a Crosspoint Switch
the hardware used in the building block. It is a very compact and useful representation, especially for representation of a complex multistage network as well as the analysis of the same.

The major portion of the text deals with the concept and analysis of a switching network under study. It is therefore logical to make a few brief remarks at this point about the hardware available for the crosspoint switch. The crossbar switch is a 10 x 20 switch while the code switch is a high density compact 40 x 10 switch. The present state of the art and increasing interest of computer industries in the switching area have made available a variety of electronic crosspoint switches. For example Motorola [27] has developed a 4 x 4 switch for 2-wire application. One can develop the required building block by using FET's or cross-reed relays.

2.1 Switching Network Configuration

The configuration of the switching network -- switching matrix -- is shown in figure 4. The interconnection of junctors to D-stages is shown in figure 5. It is observed that the matrix configuration is non-symmetric due to the fan-out at the third stage as opposed to a fan-out at the second stage normally found in a symmetric four stage network. It may be seen that a connection is made from an input to an input -- line to line -- over eight stages of
Figure 4
Simplified Block Diagram of a Four-Stage Non-Symmetrical Network
Figure 5
D-Stage Junctor Interconnections
selection; however, each input is connected to a common point called a junctor over four stages of selection.

The switching network consists of four component matrices A, B, C and D. They are the basic building blocks -- crosspoint switches -- of different configurations. A group of such matrices are interconnected as shown in figures 3 and 4 to obtain the desired switching network. A-matrices have \( l_1 \times j \) crosspoints; B-matrices have \( i \times k \) crosspoints; C-matrices have \( j \times l_2 \) crosspoints and D-matrices have \( n \times m \) crosspoints. Detailed interconnections of various matrices are described in the following subsections.

2.1.1 Line Group

A line group consists of \( i \) A-matrices and the associated \( j \) B-matrices and \( k \) C-matrices. Each A-matrix has \( l_1 \) inputs where lines are terminated. Thus a line group has \( i \times l_1 \) lines. For a total of \( N \) lines we need \( n \) line groups where

\[
n = \frac{N}{i \times l_1}
\]

A connection to or from a line can be established by utilizing the links from the associated B and C matrices only and any available junctor.

Each of \( j \) outputs of an A-matrix is connected to an input of one B-matrix. Conversely, i inputs of a B-matrix are derived by taking one output from each of \( i \) A-matrices. The
interconnection will be called an A-link. Similarly each output of a B-matrix is connected to only one input of a C-matrix. This interconnection will be denoted as a B-link. With this interconnection scheme, any line in a line group can access any output of C-matrices associated with the given line group.

2.1.2 Interconnection of C and D Matrices

The unique feature of the network presented here is the interconnection of the outputs of C-matrices to the outputs (junctors) of D-matrices. This is where the fan-out occurs which enables any line to be connected to any other line if idle paths to an idle junctor exist from both lines. The number of inputs required for a D-matrix is equal to the number of line groups. The number of subgroups of a D-matrix is equal to the number of outputs from a C-matrix. Each subgroup of a D-matrix contains \( k \) (equal to the number of C-matrices per line group) D-matrices. If we denote the line group number by the first subscript, the C-matrix number by the second subscript and the C-matrix output number by the superscript, then, \( C_{p,q}^r \) indicates the \( r \)th output of the \( q \)th C-matrix in the line group number \( p \). (\( r = 1,2 \ldots, 2; q = 1,2 \ldots, k; p = 1,2 \ldots, n \)) Similarly, we denote the input of a D-matrix by a superscript, the subgroup of a D-matrix by the first subscript and the D-matrix number within a subgroup by the second subscript. Thus \( D_{m,n}^k \) implies the
\( \text{\$th input of the \$th D-matrix in the subgroup \$m. The inter-} \\
\text{connection of C-matrix outputs to D-matrix inputs follow} \\
\text{the rule:} \\
\text{connect} \quad c^r_{p, q} \text{ to } d^p_r, q \\
\text{This interconnection will be referred to as a C-link.} \\
\)

2.1.3 Connection of Junctors to D-Matrices

The junctors are connected to the outputs of D-matrices. There are \( \frac{k}{2} \) junctor groups; each junctor group consists of \( \frac{r}{2} \) junctor subgroups. There are \( m \) junctors in a junctor subgroup. A junctor can be considered as a two-port device; each port is connected to an output from different D-matrices of a given subgroup. The connection scheme is shown in figure 5 for a D-matrix subgroup.

2.2 Switching Network Control

The control element, henceforth called a processor, is a device which searches for a path through the switching network in order to establish a connection between two inputs -- calling and a called line -- after their identities with respect to their locations on A-matrices and the A-matrix numbers are known. Auxiliary devices -- receivers and identifiers -- may be used to provide this information to the processors or the processor may perform this function on its own. Once the respective identities for the two lines involved in a desired connection are obtained, the
processor follows a specific sequence of search routines for an idle path leading from the calling line to the called line. The path searching is broken down into steps to avoid useless repeated searches through the entire network.

There are two major steps in establishing a path from the calling line to the called line:
1. Selection of a path from the calling line to an idle junctor.
2. Selection of a path from the junctor chosen in step 1 to the called line.

It may be seen from figure 5 that only one C-link of a C-matrix can be used to reach a junctor in a given junctor subgroup; e.g. $C_{11}$ is the only C-link in C-matrix number one of the first line group that can reach $m$ junctors connected to the outputs of matrix $D_{11}$. Therefore it is necessary that both C-links corresponding to the C-matrices of the groups involved and having an access to idle junctors of a specific junctor subgroup be idle. Such a pair of C-links is called a coincident idle C-link pair and will be referred to as an idle C-link pair (coincidence implied unless noted otherwise).

A simplified flow diagram for the path searching routine is shown in figure 6. The detailed description of the routine is given in what follows.

1. Search for an idle C-link pair in C-matrix one of the
Figure 6
Flow Diagram for Path Searching
calling line group and C-matrix \( \frac{k}{2} + 1 \) of the called line group. Let \( x \) denote the number of idle C-link pairs, \( (x = 0,1, \ldots, \frac{k}{2}) \). If \( x \) is equal to zero, continue the search in the following ordered pairs of C-matrices -- first matrix is for the calling line group --

\[
C_2, \frac{C_k}{2} + 2; \quad C_3, \frac{C_k}{2} + 3; \quad \ldots \quad \frac{C_k}{2} - 1, \quad C_k.
\]

2. If \( x > 0 \) then search for an idle junctor in the junctor subgroups corresponding to the C-link pairs. If there is no idle junctor in all \( x \) junctor subgroups, go to the next search for an idle C-link pair in 1. If an idle junctor is found, go to 3.

3. From the selected C-link pair search for idle paths to the calling line and the called line. If the search is successful the connection is established by operating the crosspoints involved. If a path from the C-links to either the calling line or the called line can not be established and it is the last permissible search for 1 (i.e. Step 1 has been carried out for \( \frac{C_k}{2} - 1 \)), then return a busy signal to the calling line. If it is not the last permissible search then continue from the next sequence in 1.

The processor abides by certain additional rules in order to obtain uniform utilization of junctors. These rules are (a) non-homing starting point for the search of an idle
C-link pair and junctors. This implies that the search for an idle C-link pair will begin in C-matrices $C_j$, $C_{\frac{k}{2} + j}$ if the search for the C-link pair in the previous attempt ended at $C_{j-1}$ $C_{\frac{k}{2} + j-1}$. A total of $\frac{k}{2}$ passes for the search of an idle C-link pair will always be made. (b) If possible avoid using the idle junctor in a given junctor subgroup if it is the only idle junctor.

2.3 **Example Illustrating the Connection Principle**

Let us assume that an inlet (calling line) in line group number one desires a connection to an inlet (called line) in line group number 5. Also assume that the search for an idle C-link pair can commence from C-matrix one in line group one. The called line is known to be idle. The attempt to establish the connection will follow the steps outlined below.

**Step 1.**

Compare the states of the C-links in matrices $C_{1,1}$ and $C_{5,\frac{k}{2} + 1}$. Suppose that the idle busy states of the C-links is such that no coincident idle pair can be found. Since the search for the idle C-link pair can be continued, compare the states of the C-links in matrices $C_{1,2}$ $C_{5,\frac{k}{2} + 2}$. During this process, for example, assume that three idle
C-link pairs were found: namely, $C_{1,2}^1 - C_{5,k}^{2,2}$, $C_{1,2}^4 - C_{5,k}^{4,2}$; and $C_{1,2}^6 - C_{5,k}^{6,2}$.  

**Step 2.**

The second subscript of the C-links available is two. Hence the search for an idle junctor is restricted to junctor group two. It is obvious that the idle junctors in junctor group one serve no useful purpose. The superscripts of the available C-links indicate the junctor subgroups of the junctor group 2 accessible via the C-links and hence the search for an idle junctor should be restricted only to these subgroups. Assume that all the junctors in junctor subgroups one, four and six of junctor group two are busy.

**Step 3.**

Search for an idle C-link pair in C-matrices $C_{1,3}$ and $C_{5,k}^{2,3}$. Assume that C-links number two, four and seven in both the C-matrices are idle.

**Step 4.**

Search for an idle junctor in junctor subgroups two, four and seven of the junctor group three. Suppose that each subgroup contains at least one idle junctor. Let us say that the junctor in subgroup four is selected.
Step 5.

Test for a path from $C_{1,3}^4$ to the calling line and for a path from $C_{5,k}^{4,\frac{5}{2}+3}$ to the called line. If both paths are idle, close the necessary crosspoints in the appropriate $A$, $B$ and $C$ matrices and the necessary crosspoints in $D$-matrices to establish the total path from the calling line to the called line; i.e. calling line - $A$ - $B$ - $C$ - $D$ - junctor - junctor - $D$ - $C$ - $B$ - $A$ - called line.

If either path tested is busy the entire process starting from the search for an idle C-link pair is repeated.

2.4 Conventional Four Stage Network

As mentioned previously, the conventional four stage network is symmetric and is derived by interconnecting two identical two stage networks back to back. A schematic diagram of the network is shown in figure 7. Note that the non-symmetric network shown in figure 4 is converted to the symmetric network.

The $A$ and $D$ matrices are identical with $\ell_1 \times j$ crosspoints while the $B$ and $C$ matrices are identical with $i \times k$ crosspoints. By combining $i$ $A(D)$ matrices and the associated network an incoming (outgoing) line group is formed. There are $j \times k$ links available at the outlets of $B$-matrices which are connected to $C$-matrix inlets of all the outgoing line groups. The number of $B$-$C$ links thus formed between an
Figure 1
Symmetric Four Stage Network Derived from the Non-Symmetric Network of Figure 4
incoming line group and an outgoing line group is determined by the traffic flow between the two groups. Usually the network can grow to a maximum of k incoming and k outgoing groups.

When a connection is to be established from a terminal at the input of an A-matrix to an idle terminal at the output of a D-matrix the path is tested over the four stages of switching. If a path can not be found, another idle terminal on a D-matrix, preferably requiring different B - C - D matrices is selected and the path search is repeated. Input-to-input connections can be established by having two appearances on different D-matrices for a given output terminal as shown in figure 7. The path search is similar to that in the network we propose, except that the idle junctor is pre-selected and the path search is over four stages.
CHAPTER III

Traffic Analysis of the Switching Network

The traffic analysis of a non-symmetric network is presented in this chapter. By traffic analysis we mean the determination of the probability of failure in establishing a connection between two given inputs (lines). The event that the desired path does not exist (or exists) depends upon the state of the system at the time a request for a connection arrives. The state of the system is described by the number of busy-idle links at the corresponding A matrices, the links in the associated B-C matrices and the number of busy junctors. Figure 8 shows one of many possible states of the system at the instant of a call arrival. Only the matrices involved in the desired connection are shown. The advantage of link symbols for switching network analysis is readily appreciated. The distribution of various states is a function of arrival and departure processes or equivalently of occupancy distribution for various links.

3.1 Assumed State Distributions

Certain assumptions regarding the state distributions
State Distribution of Links at the Instant of a Call Arrival
are necessary in order to obtain the expression for blocking probability as a function of traffic.

1. The arrival process as viewed from all the inputs is Poisson distributed with a constant mean $c$ (i.e., the mean number of arrivals per unit time is $c$).

2. The duration of a call is exponentially distributed with a constant mean $h$. The average number of departures per unit time is $1/h$.

3. The inputs originate calls independent of the state of the system and independent of each other. Since the inputs, as well as the associated links, are occupied when an input originates or receives a call, the average occupancy per input is

$$\frac{2 \times c \times h}{N}$$

This is called traffic per input and is denoted by $a$.

4. The distribution of busy inputs (lines) per $A$-matrix is a binomial distribution.

5. The distributions of busy $B$ and $C$ links also follow a binomial distribution.

6. The states of the links in various stages are stochastically independent.

7. The distribution of busy junctors obeys the Erlang distribution.

8. The search time of the processor is sufficiently small to justify the assumption that the state of the system
does not change during successive retrials in establishing a connection.

Assumptions 1 and 2 are commonly used in teletraffic theory. The measurements taken in working exchanges serving a large number of lines have shown that in most cases assumptions 1 and 2 hold. Assumption 3 implies that each line has the same likelihood of requesting a connection to any other line. This may not be true in general for an individual line. However, by a proper combination of lines of different characteristics, the equivalent effect relative to the network can be obtained on the average. Since the number of lines per A matrix is finite, it is reasonable to assume that assumption 4 applies. Note that Poisson distribution is a limiting distribution achieved from a binomial distribution. Assumption 5 follows from 4. Elldin [27] has shown that for link systems of practical configurations, assumption 6 is reasonable. Assumption of independence between the links in different stages results in a slightly higher probability of blocking than that would be obtained by accounting for the inter-dependence. Hence, it is a safe assumption from the practical viewpoint. The Erlang distribution is assumed for the busy junctors because any input can occupy any junctor (i.e. traffic offered to junctors emanates from a large number of sources). Fast electronic processors justify assumption 8.
3.2 Events Causing Network Blocking

In order to derive a mathematical expression for blocking probability, we must account for all the events contributing to or causing network blocking. The various events which produce blocking are listed in the order the processor will encounter them.

1. The event that there is no coincident idle C-link pair.
2. The event that all junctor subgroups accessible via idle C-links are fully occupied.
3. An idle path from the appropriate C-link to the calling line or an idle path from the appropriate C-link to the called line cannot be found. Note that at this point the processor has found an idle C-link pair and an idle junctor and is searching for a path through the three stage network to both calling and called lines.

3.3 Expressions for the Probabilities of Various Events

In this section we determine the probabilities of the events producing network blocking.

3.3.1 Probability That There is no Coincident Idle Pair of C-links

The problem of calculating the probability that there is no coincident idle pair of C-links or the probability that there are exactly z such pairs is closely related to the problems of matching and guessing [29] or the problems in
psychic research [30]. However, the answers in the realm of traffic control are somewhat easier to verify. Let us assume that there are $x$ busy links in the C-matrix associated with the calling line. Denote this as group 1. Similarly, suppose that there are $y$ busy links in group 2 associated with the called line. There are $\ell_2$ C-links in each C-matrix. We are seeking, first, the probability that none of the $\ell_2 - x$ idle links in group 1 and $\ell_2 - y$ idle links in group 2 have an access to the same junctor subgroup or, simply stated, that they do not match. The problem differs very little from the classical problem of matching cards from a deck in the sense that we are seeking the probability of matching of idle links, a characteristic analogous to color. Secondly, given the states of the links in two groups we wish to determine the probability of exactly $z$ matches for idle links. Note that $z = 0$ is a special case of the latter event.

Let us assume, without the loss of generalities, that $x > y$. Hence the maximum number of idle coincident pairs that can exist is $\ell_2 - x$. If we require that there are exactly $z$ matching idle pairs, then $\ell_2 - x - z$ of $y$ busy links in group 2 must occupy locations corresponding to $\ell_2 - x - z$ of group 1. The remaining busy links of group 2, $y - (\ell_2 - x - z)$ can be distributed among $x$ locations in
\[
\binom{x}{y - (\ell_2 - x - z)} \text{ ways.}
\]
Figure 9 shows number of ways \( y \) busy links can be distributed for a given state of \( x \) busy links in group 1.

'\( z \)' idle locations can be selected in \( \binom{x}{z} \) different ways. Dividing the number of combinations of \( z \) matches by the total number of ways \( y \) links can be arranged, we obtain the desired probability.

\[
P (z \ x, y) = \text{Probability of exactly } z \text{ idle coincident C-links given } x \text{ and } y \text{ busy C-links in the respective groups;} \quad x > y
\]

\[
= \frac{\binom{x}{z} \cdot \binom{y - x - z}{y - z}}{\binom{x}{y}}
\]

\[
= \frac{\binom{x}{z} \cdot \binom{x}{x + y + z - x}}{\binom{y}{y}} \quad (1)
\]

Equation (1) can be written in the following form by observing \( \binom{r}{s} = \binom{r}{r-s} \)

\[
P (z \ x, y) = \frac{\binom{x}{z} \cdot \binom{x}{x + y + z - x}}{\binom{y}{y}}
\]

\[
= \frac{\binom{x}{z} \cdot \binom{x}{x + y + z - x}}{\binom{y}{y}} \quad (2)
\]
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Figure 9

Distribution of y Busy C-links in Group 2 for a Given Distribution of x Busy C-links in Group 1
Note that when $x + y + z < \ell_2$, $z$ idle C-link pairs can always be found. Equation (1) and (2) are true if $x + y + z \geq \ell_2$.

The absolute probability of $z$ idle C-links pairs can be calculated from the above conditional probability by including all combinations of $x$ and $y$ links busy ($x, y = 0, 1, \ldots, \ell_2$)

$$P(z) = \text{Probability of exactly } z \text{ idle C-link pairs}$$

$$= \sum_{x} \sum_{y} P(z \mid x, y) B(x) B(y)$$

$$= \sum_{x=0}^{\ell_2} \sum_{y=\ell_2-x}^{\ell_2} \frac{\binom{\ell_2 - \text{Max}(x, y)}{z} \binom{\text{Max}(x, y)}{x+y+z-\ell_2}}{\binom{\ell_2}{\text{Min}(x, y)}} B(x) B(y)$$

(3)

Where $B(x)$ and $B(y)$ are the probability distribution functions for $x$ and $y$ links busy. Since we have assumed a binomial distribution for busy links

$$B(x) = \binom{\ell_2}{x} p^x (1-p)^{\ell_2-x}$$

(4)

$p$ is the probability of a link being busy and is equal to the average traffic per link.

3.3.2 Probability that there is no idle junctor in the junctor subgroups accessible via $z$ C-links.
This event presents a complex problem in combinatorial analysis. We have a total of \( J \) junctors, where \( J = \frac{k}{2} \times m \times k/2 \). The arrangement of the junctors can be viewed as a rectangular array of \( r = \frac{k}{2} \times k/2 \) rows and \( m \) columns. From a given \( C \)-matrix \( \frac{k}{2} \) rows of \( m \) junctors each are accessible. From a given \( C \)-link, however, only a specific row with \( m \) junctors is accessible. At any instant there may be altogether \( w \) junctors busy occupied by \( 2w \) inputs. These \( w \) junctors are distributed among \( r \) rows each row having some of its junctor busy. We seek the probability that \( z \) specific rows are fully occupied under the condition that \( w \) junctors are busy. This event will produce blocking due to a lack of an accessible idle junctor. The detailed derivation of the probability distribution of \( z \) fully occupies rows is given in Appendix III. The equation in its final form under the assumption of the Erlang distribution for busy junctors is denoted by \( Q_r(z) \) and is given by

\[
Q_r(z) = \binom{r}{z} \sum_{s=0}^{r-z} (-1)^s \left( \begin{array}{c} r-z \cr s \end{array} \right) \frac{E_{ms-mz}(A)}{E_{mr}(A)}
\]

Where

\[
E_N(A) = \frac{A^N}{N!} \sum_{i=0}^{N} \frac{A_i}{i!}
\]
and \( A = \) traffic offered to \( N \) junctors

\[ = c h \]

\[ = \frac{1}{2} \times \text{Number of inputs} \times \text{traffic per input} \]

Since we are interested in a specific row, equation (5) is divided by \( \binom{r}{z} \)

\[ q_r (\text{specific } z) = q_r^z \]

\[ = \frac{q_r(z)}{\binom{r}{z}} \] (7)

3.3.3 Probability that a path from the C-link to the calling (called) line does not exist.

The required probability corresponds to the blocking in a three-stage network employing point-to-point connection. The approach is similar to the one employed in 3.1.1, since this is also a problem in matching. Here we are interested in finding the probability of no match between the idle links, \( z = 0 \) in (3.2). Suppose there are \( a^1 \) lines busy in the A-matrix corresponding to the calling line when the calling line originates a request, and there are \( b^1 \) C-links busy in the appropriate C-matrix when an idle C-link is found. There are \( j \) paths from an inlet to a C-link. At the left side of the B-matrix \( a^1 \) of them are occupied by busy lines and on the right side of the B-matrix \( b^1 \) are occupied by the busy C-links.
If \( j - b_1 \) idle links on the right side of the B-matrix do not have a match with the \( j - a_1 \) idle links on the left side of the B-matrix (or say \( a_1 \) busy on the left of B-matrix spans the specific \( j - b_1 \) on the right of B-matrix) then the path does not exist. Hence

\[
R_1 (a_1, b_1) = \text{Probability that no path exists from the input to the C-link given } a_1, b_1
\]

\[
= \binom{a_1}{j-b_1} \binom{j}{b_1}
\]

The absolute probability of no path is calculated as

\[
R(\text{no path}) = \sum_{a_1} \sum_{B_1} \binom{a_1}{j-b_1} \cdot B (a_1) \cdot B (b_1)
\]

3.4 Blocking Probability with a Single Attempt

Probability of blocking with a single attempt to establish a connection will now be developed by combining the probabilities of various events producing a state of blocking. The single attempt is defined in this section as an attempt where only one C-matrix pair; one junctor group and one set of A - B - C matrices for each line are tested for a complete idle path.

As mentioned at the beginning of this chapter there are three events causing blocking (or failure to establish
a path between two inputs). We have also assumed that occupancy distributions in various stages are independent. Suppose we denote these three events by \( Q_1, Q_2 \) and \( Q_3 \). Let \( q_i \) (\( i=1,2,3 \)) be the probability that event \( Q_i \) occurs and \( p_i \) = 1\(-q_i \) be the probability that event \( Q_i \) does not occur. Then a blocking is encountered if either \( Q_1, Q_2 \) or \( Q_3 \) occurs.

Probability of blocking = \( P_r\{Q_1+Q_2+Q_3\} \)

\[
P_r\{Q_1 + Q_2 + Q_3\} = P_r\{Q_1\} + P_r\{Q_2\} + P_r\{Q_3\}
\]

\[
- P_r\{Q_1Q_2\} - P_r\{Q_2Q_3\} - P_r\{Q_3Q_1\}
\]

\[
+ P_r\{Q_1Q_2Q_3\}
\]

\[
= P_r\{Q_1\} + P_r\{Q_2\} + P_r\{Q_3\} - P_r\{Q_1\} P_r\{Q_2\}
\]

\[
- P_r\{Q_2\} P_r\{Q_3\} - P_r\{Q_3\} P_r\{Q_1\}
\]

\[
+ P_r\{Q_1\} P_r\{Q_2\} P_r\{Q_3\}
\]

\[
= q_1 + q_2 + q_3 - q_1q_2 - q_2q_3 - q_3q_1 + q_1q_2q_3
\]

\[
= q_1 + q_2 (1-q_1) + q_3 (1-q_1) - q_2q_3 (1-q_1)
\]

\[
= q_1 + P_1 (q_2+q_3-q_2q_3)
\]

\[
= q_1 + P_1q_2 + P_1q_3 (1-q_2)
\]

\[
= q_1 + P_1q_2 + P_1P_2q_3
\]

(10)
Equation (10) can be interpreted as
\[
P_r\{Q_1 + Q_2 + Q_3\} = P_r\{Q_1\} + P_r\{Q_2 \mid Q_1\} + P_r\{Q_3 \mid Q_1, Q_2\} \ldots (11)
\]

Substituting the appropriate probabilities derived in section 3.2 we obtain the probability of blocking with a single attempt.

\[P_1 = \text{Probability of blocking with a single attempt}\]

\[
= \sum_{y_1=0}^{\ell_2} \sum_{y_2=\ell_2-y_1}^{\ell_2} P(0, y_1, y_2) \cdot B(y_1) \cdot B(y_2)
\]

\[+ \sum_{y_1=0}^{\ell_2} \sum_{y_2=\ell_2-y}^{\ell_2} \sum_{z=1}^{\ell_2} P(z, y_1, y_2) \cdot B(y_1) \cdot B(y_2) \cdot Q_r(z)
\]

\[+ \sum_{x=0}^{\ell_2-1} B(x) \sum_{y_1=0}^{\ell_2-1} \sum_{y_2=0}^{\ell_2-x} R_1(x, y_1) + \sum_{y_1=0}^{\ell_2-1} \sum_{y_2=0}^{\ell_2-x} R_1(x, y_2) - R_1(x, y_1)
\]

\[R_1(x, y_2) \sum_{z=1}^{\ell_2} P(z, y_1, y_2) \cdot B(y_1) \cdot B(y_2) \cdot (1-Q_r(z))
\]

(12)

3.5 Blocking Probability With Multiple Attempts

It was pointed out in Chapter II that if no idle C-link pair or idle junctor in the corresponding junctor subgroups exist, or if the path to either line from the C-link can not be found at the first attempt, the processor will
make successive attempts until the limit of the retrial capability is reached. The expression for blocking in this case is not obtained by simply raising the expression for blocking with a single attempt to a power equal to the number of retrials. The state distributions involved in the state of blocking are not all different for different attempts. It is seen that the state distributions for the number of lines busy in the A-matrices (calling or called line) do not change for different attempts. Also the same junctor group is accessed via two pairs of C-matrices and hence the same distribution for the busy junctor in the group exists during these two searches for an idle C-link pair. A single attempt is redefined in this section so that the blocking states of the events contributing to total blocking are different for different attempts. This is accomplished by defining a single attempt as an attempt where the processor tests one junctor group for an idle junctor, all the associated C-matrices (two pairs) having an access to the junctor group are tested for the idle C-link pair and the paths from all the usable C-links of the C-matrices (two pairs) to the lines are checked for the connection possibilities. The total number of attempts permissible, $S$, is then equal to the number of junctor groups which equals half the number of C-matrices per line group ($1/2 \ k$).
Blocking will occur if no idle coincident pair of C-links is available or all the junctors are busy, or if when given an idle junctor and the corresponding C-link pair a path to either line (calling or called) can not be established from the appropriate C-link.

The probability that an idle coincident pair of C-links can not be found with S attempts can be obtained from equation (3).

\[
P(0) = \sum_{y_1=0}^{\lambda_2} \sum_{y_2=\lambda_2-y_1}^{\lambda_2} P(0, y_1, y_2) \cdot B(y_1) \cdot B(y_2)
\]  

\[U_1 = \text{Probability that an idle C-link pair can not be found with S attempts} \]

\[= \left[ P(0) \right]^{2S} \]  

The probability, \(U_2\), that all the junctors are busy is given by

\[U_2 = E_{m \cdot r}(A)\]  

Where \(E_{m \cdot r}(A)\) is the Erlang - B formula for the blocking with \(m \cdot r\) devices offered traffic of \(A\) erlangs. In general \(E_N(A)\) is given by

\[E_N(A) = \frac{A^N/N!}{\sum_{i=0}^{N} A^i/i!} \]
A is the traffic offered to junctors.

The probability that exactly \( z, z > 0 \) idle C-link pairs are found during a single attempt is

\[
P_2(z) = P(z) + P(z) - P(z) \cdot P(z)
\]  

(17)

where \( P(z) \) is obtained from equation (3).

Now the probability, \( P_4 \), that during a single attempt an idle C-link and an idle junctor are found is given by

\[
P_4 = \sum_{z=1}^{L_2} P_2(z) \cdot (1 - Q_r(z))
\]  

(18)

Now the paths for two lines from the C-link are tested. Noting that two different C-matrices can be tested we determine the probability, \( u_3 \), that a path from the C-links to a given line can not be established.

\[
u_3 = \sum_{x=0}^{L_1-1} B(x) \left\{ \sum_{y=0}^{L_2-1} \binom{x}{y} \cdot B(y) \right\}^2 \cdot P_4
\]  

(19)

Note that \( u_3 \) gives the probability that a path from a given C-link to a given line does not exist. For a complete connection we need to establish paths to calling and called line from given C-links. Let \( u_4 \) be the probability of blocking in the three-stage network (i.e. either or both paths are blocked)

\[
u_4 = u_3 + u_3 - u_3 \cdot u_3
\]  

(20)
Probability of blocking in establishing a connection between two lines through the three stage network with \( S \) attempts is given by

\[
U_3 = (u_4)^S
\]  

(21)

Now we have three events causing a blocking in establishing the desired connection:

\[
\{U_1\} = \text{no idle C-links}
\]

\[
\{U_2\} = \text{all junctors are busy}
\]

\[
\{U_3\} = \text{no path to through the three-stage network}
\]

We follow the same procedure as outlined in section (3.4) to obtain the final equation for blocking probability.

\[
E = \text{Probability of blocking with } S \text{ attempts}
\]

\[
= P_r\{U_1+U_2+U_3\}
\]

\[
= P_r\{U_1\} + P_r\{U_2\} + P_r\{U_3\} - P_r\{U_1\} P_r\{U_2\}
\]

\[
- P_r\{U_2\} P_r\{U_3\} - P_r\{U_3\} P_r\{U_1\} + P_r\{U_1\} P_r\{U_2\} P_r\{U_3\}
\]

(22)

Appropriate equations for various probabilities are substituted in equation (22) giving the expression for the desired blocking probability.
CHAPTER IV

Results

The results obtained after numerical evaluation of the proposed configuration with various network parameters are given in this chapter. A computer program was written and the results of calculations were obtained with the help of a time-sharing computing system. Before embarking on the presentation of the results, a few preliminary remarks at this point are appropriate.

It is observed from the expression for blocking that any of the three events described before can create a blocking state. It is very possible that one of the events may present a "bottle-neck" and mask the potential improvement obtainable by reducing the probability of occurrence for the other events. This is to stress that the network is at most as good as the weakest link. For example, one can provide an infinite number of junctors with a three-stage network having excessive internal blocking in which case there is a waste of junctors. One may develop an ideal non-blocking three-stage network and still encounter excessive blocking if the number of junctors is too low or there are not enough
links at the outlet of a C-matrix. The results, therefore, are given for various combinations of parameters to illustrate the effects of changes in parameters on blocking probability.

The numerical results are shown in a graphical form in Figures 10 - 22. To determine the effect of number of attempts, \( S \) is varied from 1 to 4; the effect of the three-stage network was derived by varying \( j \) from 8 to 10 in unit steps, the effect of junctors and the number of junctors in a subgroup is obtained by varying \( m \) (4, 6, 8, 10).

4.1 **Effect of Multiple Attempts on Blocking Probability**

Figures 10 - 13 show the variation in blocking probability as a function of number of attempts. It may be seen from these results that the reduction in blocking probability due to an increase in number of attempts follows a rule of diminishing returns. It is also observed that the proportionate improvement is greater if the three-stage network has a significant internal blocking. Figures 12 and 13 indicate that very little is gained by multiple attempts if the number of junctors are inadequate, i.e., a high probability of all junctors busy. This supports the intuitive concept that if probability of all junctors busy is high, multiple attempts do not alter the state of blocking.
Figure 10
Effects of Multiple Attempts on Blocking

\[ \lambda_1 = 8; \lambda_2 = 8; j = 10; J = 256 \]
Figure 11
Effect of Multiple Attempts
\( \lambda_1 = 8; \  \lambda_2 = 8; \ j = 8; \ S = 4; \ J = 256 \)
Figure 12
Effect of Multiple Attempts

$\lambda_1 = 8; \lambda_2 = 8; j = 10; S = 4; J = 128$
Figure 13
Effect of Multiple Attempts
\[ \lambda_1 = 8; \lambda_2 = 8; j = 8; J = 128 \]
4.2 Effect of Improved Three-Stage Network on Blocking Probability

Figures 14 - 17 show the change in blocking probability when internal blocking in the three-stage network is reduced. The improvement is the greatest, as expected, when only one attempt is allowed with adequate number of junctors resulting in a high probability of obtaining an idle junctor. With multiple attempts (S = 4 in Figure 15) the improved three-stage network does not result in the same relative improvement with one attempt. The "bottle-neck" situation is evident in Figures 16 and 17 where the probability of not having an idle junctor substantially masks the effect of the improved three-stage network.

4.3 Effect of Number of Junctors on Blocking Probability

Figures 18 - 21 show the variation in blocking probability as a function of junctors per subgroup. There are two reasons for improvement when the number of junctors per subgroup is increased. First, increasing the number of junctors per subgroup increases the total number of junctors. Consequently the probability of all junctors busy decreases for a given traffic. Second, increased number of junctors per subgroup reduces the probability of a junctor subgroup fully occupied thus increasing the probability of finding at least one idle junctor in a junctor subgroup accessible by a given C-link pair. Again, for an adequately designed three-stage
Figure 14
Effect of Three-Stage Network
\[ \lambda_1 = 8; \lambda_2 = 8; S = 1; J = 256 \]
Figure 15
Effect of Three-Stage Network

$l_1 = 8; \quad l_2 = 8; \quad S = 4; \quad J = 256$
Figure 16
Effect of Three-Stage Network
\( \lambda_1 = 8; \lambda_2 = 8; S = 1; J = 128 \)
Figure 17
Effect of Three-Stage Network
\( \lambda_1 = 8; \lambda_2 = 8; S = 4; J = 128 \)
Figure 18
Effect of Size of Junctor Subgroup
\[ k_1 = 8; \ k_2 = 8; \ j = 10; \ S = 4; \ J = 256 \]
Figure 19
Effect of Size of Junctor Subgroup

\[ \ell_1 = 8; \ell_2 = 8; j = 10; S = 1 \]
Figure 20
Effect of Size of Junctor Subgroup
\( \lambda_1 = 8; \lambda_2 = 8; j = 8; S = 1 \)
Figure 21
Effect of Size of Junctor Subgroup
\[ \lambda_1 = 8; \lambda_2 = 8; j = 8; S = 4 \]
network with multiple attempts the improvement is quite significant. With a single attempt, however, the relative improvement drops rapidly as expected. The relative improvement is further reduced if there is excessive internal blocking in the three-stage network.

4.4 Effect of Increasing the Number of C-Links

If total system performance is ignored, it is easy to implement the apparent improvement measures which could be misleading. Figure 22 shows the results of one such pitfall. It is tempting to deduce that by providing more C-links, blocking probability can be reduced since it is the first place where the processor may encounter blocking if no idle C-link pair can be found. As it stands, the statement by itself appears very sound and logical. However, if the total system is examined, increasing the number of C-links increases the blocking probability through the switching network and overall deterioration in the performance results. One should maintain equal probabilities of blocking through the three-stage network and then there will be a definite improvement in the overall performance when the number of C-links per C-matrix is increased.

4.5 Comparison with a Conventional Four-Stage Network

An approximate analysis of the proposed network and the conventional network is performed utilizing Lee's method.
Figure 22
Effect of an Increase in C-Links

\[ \lambda_1 = 8; j = 10; S = 4; J = 320 \]
Figure 23 shows the probability linear graphs of the symmetrical network shown in figure 7.

Note that two middle links can be used for establishing a connection via a selected junctor. P and q are the busy and idle probabilities for the links shown. r and t are determined by accounting for the fact that either pair is idle. The probability that either pair of middle links is idle is given by

\[ t = 2(1-P)^2 - (1-P)^4 \]
\[ = 2q^2 - q^4 \]  \hspace{1cm} (23)
where $q^2$ is the probability that both links in the pair are idle.

The probability, $P$, that a link is occupied is determined from the traffic offered and the network structure. The blocking expression for the network with a single attempt is

$$E_1 = \left[ 1 - (1-P)^4 \right] \left\{ \frac{2}{3} (1-P)^2 - (1-P)^4 \right\}$$

(24)

The probability linear graph of the proposed network is shown in Figure 24.

![Figure 24](Probability Linear Graph of the Proposed Network)

The occupancy of the C-link must be determined from the network structure. Probability, $t$, of having one idle
C-link pair is determined as before by substituting the occupancy of C-links in place of P in equation 23.

The probability of a success using one connection path is given by

\[ Q = q \cdot q \cdot t \cdot q \cdot q \]  

(25)

and the blocking probability with a single attempt is

\[ E_2 = (1-Q)^j \]  

(26)

Figure 25 shows the result of such comparison. The parameters for both networks are the same except for the topological difference. It is seen that the conventional network performs a little better than the proposed network. However, this fact alone should not be used in evaluating a network.

With multiple attempt capabilities, the difference between two networks may not be significant.

The relative simplicity of the control is another feature in favor of the proposed network. It eliminates the need to search an entire four-stage network every time an idle junctor is selected.

Three-stage networks are easily adaptable for a strictly non-blocking configuration or a non-blocking in wide sense configuration. The control routine in the proposed network is readily suitable for either criteria since it is designed to search for a path via three-stages.
Figure 25
Comparison of a Symmetric and a Non-Symmetric Network
CHAPTER V

Conclusions and Recommendations

5.1 Conclusions

We have presented a new concept in the design of a multistage switching network. Traditionally, four stage switching networks have been designed as symmetric network. A non-symmetric network with four selection stages was designed in this study and its subsequent analysis was presented in the preceding chapters.

The system was described from its topological structure. A path searching routine to establish a connection was formulated. The analysis of the network was based on probabilistic considerations as well as the structure of the network, control mechanism and combinatoric arguments. Few configurations of practical interest were evaluated as to their performance regarding blocking probabilities or grade of service as commonly referred to in teletraffic theory. Several pitfalls, easy to encounter, in designing apparently better networks were pointed out by observing the results of numerical evaluation of the configurations.

The study, we hope, has made a contribution to the
theory and analysis of switching networks employed in telecommunications. There are a large number of problems yet to be solved mathematically in the teletraffic area. Until more is learned about system behavior, interdependence and the characterization of input process, one has to rely on reasonable assumptions to analyze a practical network. We feel we have accomplished this and would like to make certain recommendations for further studies.

5.2 Recommendations

1. Various strategies for the selection of junctors can be employed. We have assumed a random selection implying that every junctor is equally likely to be occupied. One of the many other possible approaches is to maintain at least one idle junctor in every junctor subgroup as far as possible. That is, an idle junctor is not seized if it is the only idle junctor in the subgroup and there are other junctor subgroups accessible via idle C-link pair and having more than one idle junctor. As a result of this selection approach an idle C-link pair will seize an idle junctor with probability one if the number of idle junctors is greater than or equal to the number of junctor subgroups.

2. It will be of a great advantage to know the effect
of making the three stage network non-blocking in wide sense. It would be desirable to develop a rule to make a general three stage network non-blocking in wide sense.

3. The non-symmetric network shown is easily applicable to a five stage matrix. Various possibilities for interconnections exist here.

4. Since the present analysis was made under certain assumptions, it will be desirable to simulate the network (not Lee's model as done in NEASIM) for its performance under different assumptions of state distributions. The simulation will be a useful tool in determining the degree of interdependence between different stages.

5. Determine if there exists a network which is optimum from crosspoints/Erlang criteria for a given blocking probability.

Finally we hope that our study of switching networks stimulates interest among electrical engineering students in telecommunications, which is a field with many challenges and intriguing problems yet to be solved with a sound technical and theoretical approach.
APPENDIX I

Glossary of Teletraffic Terms

The definitions of terms commonly used in teletraffic theory are given in this Appendix. These definitions are derived from the report of the nomenclature committee at the Sixth International Teletraffic Congress, Munich, 1970.

1. **Availability (Accessibility)**
   The number of appropriate outputs in a switching network which can be reached from an input.

2. **Availability, Full**
   Exists when any free input can reach any free output regardless of the state of the system.

3. **Availability, Limited**
   Exists when access is given only to a limited number of outputs of a given group.

4. **Blocking (Congestion)**
   The condition where the immediate establishment of a new connection is impossible.
5. **Blocking, Internal**
In a conditional selection system the condition in which a connection cannot be established between a given input and any suitable free output due to unavailability of paths.

6. **Call Attempt**
Any demand to set up a call.

7. **Call Packing**
A method of ordering the allocation of calls to free paths. The most commonly used method is that which attempts to route new connections via the most heavily loaded part of the switching network.

8. **Congestion**
Same as blocking.

9. **Congestion, Call**
The ratio of the number of call attempts which cannot be served immediately to the number of call attempts offered.

10. **Congestion, Time**
The ratio of the time for which congestion exists to the total time considered. It is an estimate of the probability that an external observer will find the system in a state of congestion.
11. **Connecting Network (Switching Network)**
   An arrangement of switches whose function is to connect inputs to outputs.

12. **Connecting Stage**
   A connection in a homogeneous switching network uses a number of crosspoints in series. If the crosspoints included in a connection are numbered starting with the closest to the network inputs, then all the connecting matrices containing crosspoints that are used as nth crosspoints in such connections are said to belong to the nth connecting stage.

13. **Crosspoint**
   Each connection in a switching network is established by closing one or more crosspoints. A crosspoint comprises a set of contacts that operate together and extend the signal leads of the connection.

14. **Holding Time**
   The total duration of one occupation. There are many classes of holding time, e.g., setting-up, conversation, etc.

15. **Hunting**
   Any rule of selecting an idle output from a number of idle outputs.
16. **Hunting, Sequential**
   A rule of selection among free links or outputs based on their numbering from a starting position which may or may not change.

17. **Hunting, Random**
   A rule of selection among free links or outputs in which there is no preference for one free output over any other.

18. **Link**
   The device in a switching network connecting one output of one of the connecting stages to an input of the next connecting stage.

19. **Link System** (Conditional Selection System)
   A system in which:
   a. There are at least two connecting stages.
   b. A connection is made over one or more links.
   c. The links are chosen in a single logical operation.
   d. Links are seized only if they can be used in a connection.

20. **Loss System**
   A switching system in which call attempts fail when there is no free path for the required connection.
21. **Lost Traffic**
   In a loss system, the portion of traffic offered which cannot be carried because of congestion.

22. **Matrix**
   A simple switching network in which a specified input (matrix row) has access to a specified output (matrix column) via a crosspoint placed at the intersection of the row and column in question.

23. **Occupancy**
   Each use of a device regardless of cause.

24. **Occupancy, Mean** (Load per Device)
   The traffic carried by a group of devices divided by the number of devices in the group.

25. **Rearrangement**
   The disconnection of one or more existing connections in a switching network and their re-establishment via new paths with the objective of obtaining a free path for an additional connection.

26. **Selection Stage** (Switching Stage)
   One or more connecting stages in a switching system which together serve a particular switching function. This function is to be defined in each case.

27. **Seizure**
   The event which is the beginning of an occupation.
28. **Switch**
   A physical assembly of crosspoints.

29. **Traffic Volume**
   The sum of holding time of a certain number of occupations.

30. **Traffic Intensity**
   The traffic volume occurring during a specified period of time divided by the duration of the period, both quantities being expressed in the same time unit. The unit of traffic intensity is Erlang. Usually the specified time is the busy hour. If the period is extremely short, it is recommended that the term "instantaneous traffic intensity" be used. The traffic intensity of traffic carried is also equal to the mean number of seizures during the mean holding time.

31. **Traffic Offered**
   A calculating quantity having the dimension of traffic intensity. It is equal to the mean number of call attempts during the mean holding time.

32. **Traffic Capacity**
   The traffic intensity which can be handled by a given network for a prescribed value of some congestion function. In France, Sweden, United Kingdom and United States traffic capacity is the traffic offered. In
Finland, Germany, Italy and Spain the traffic capacity is the traffic carried.
APPENDIX II

Fundamentals of Switching Networks

We present the basic principles employed in the analysis of switching networks or link systems. With the aid of a simple two stage network, Jacobaeus' theory for the link system is presented first. Basic concepts behind Lee's probability linear graph method are illustrated next using a symmetrical four stage network as an example.

Jacobaeus' Theory:

A link system is a multistage switching network where: (a) a connection between two terminals is performed by one or more links, one in each stage (b) the link or links are seized at the same time as the chosen terminal and (c) only the links which connect the two desired terminals are seized.

An example of a two stage link system using crossbar switches is depicted in Figure 26. Figure 27 shows the symbolic representation, commonly used in the analysis of the same network.

The fundamental principle of the link system analysis is explained by means of the two stage link system symboli-
Figure 26
Two-Stage Link System
Figure 27
Link Diagram of the Two-Stage Network of Figure 26
cally shown in Figure 27. A route is defined as a group of devices leading to a specific destination. An input desiring connection to a given destination, therefore, can utilize any of the devices in a route. Let \( p \) be the number of busy devices in the route at the instance of a call arrival. The probability of this may be written as \( G(p) \). For blocking to occur under this condition it is necessary that \( m-p \) specific links in the B-column which are in the same horizontal rows as the free devices in the route are busy. The probability of this will be written as \( H(m-p) \). The probability of blocking under the condition \( p \) is then \( H(m-p) G(p) \). The total blocking probability will be

\[
E = \sum_{p=0}^{m} H(m-p) G(p)
\]  

(27)

\( H(m-p) \) and \( G(p) \) are deduced from the distribution functions used in teletraffic theory, i.e., Erlang, Engset, Bernoulli (binomial) or Poisson, whichever is appropriate.

The above simple principle can be extended to analyze link systems with more than two stages by properly defining \( G(p) \) and \( H(m-p) \). For a large network of more than four stages, a further breakdown and stage by stage analysis may be necessary.

**Lee's Method (Probability Linear Graph)**

Only the basic concept of Lee's method is presented
Figure 28
Symmetric Network for Lee's Method
here. Interested readers are referred to the original article for details.

A probability linear graph is generated for the switching network under consideration. This graph shows all the paths connecting two nodes, input and output, of the network. Then a generating function yielding the linking probability (or blocking probability) is written down in accordance with the graph. The usual assumption of independence between the various links is made. Note that the linking probability is derived as a point-to-point connection.

The method is illustrated by an example. A four stage switching network is shown in Figure 28. The probability linear graph for such a network is shown in Figure 29.

![Probability Linear Graph for the Network of Figure 28](image)
Let the occupancies of the links, probability of a link being busy be \( p \), \( q \), and \( r \) as shown. The linking probability using links \( X_1^{(1)} \), \( X_2^{(1)} \) and \( X_3^{(1)} \) as a path is 

\[(1-p)-(1-q)(1-r)\] which is the same for all paths. There are \( j \) such paths. The blocking probability is then

\[
\left[1-(1-p)(1-q)(1-r)\right]^j
\]  

(28)

The simplicity of Lee's method is obtained at the expense of accuracy. For example, Figure 30 shows a well known non-blocking network first proposed by Clos. It is too well known, as the name implies, that the blocking probability is identically zero regardless of the link occupancies. Applying Lee's approach for calculating the blocking probabilities, we arrive at the following table of results.

<table>
<thead>
<tr>
<th>( \text{a = occupancy} )</th>
<th>( \text{p = blocking probability} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>( 7.44 \times 10^{-29} )</td>
</tr>
<tr>
<td>0.3</td>
<td>( 3.94 \times 10^{-12} )</td>
</tr>
<tr>
<td>0.5</td>
<td>( 1.34 \times 10^{-5} )</td>
</tr>
<tr>
<td>0.7</td>
<td>( 0.025 )</td>
</tr>
<tr>
<td>0.9</td>
<td>( 0.676 )</td>
</tr>
<tr>
<td>0.95</td>
<td>( 0.907 )</td>
</tr>
<tr>
<td>1.0</td>
<td>( 1.0 )</td>
</tr>
</tbody>
</table>
Figure 30

Non-Blocking Network of Clos
However, Lee's method is a powerful tool during the initial design of switching networks where a relative performance index is to be obtained in order to further study a selected configuration among a vast number of alternatives available. Several other authors have elaborated on the subject in recent years. The NEASIM program simulates Lee's model when the numerical evaluation of the blocking polynomial for a real network presents extreme difficulties.
APPENDIX III

The Distribution of the Number of Entirely Occupied Rows of Junctors When They are Arranged as a Rectangular Array

During the derivation of the expression for the blocking probability it was required to calculate the probability of the event that given j junctor subgroups each contained at least one idle junctor when it was known that there were w junctors busy among mr junctors arranged as an m x r array. The above probability can be derived from the expression for the number of entirely occupied rows of junctors when they are arranged as a rectangular array.

Consider a group of mr devices arranged in r columns and m rows. Occupied devices are placed at random within the group.

Lemma 1. Given that altogether there are w devices occupied, there are

\[ C(p|w) = \binom{m}{p} \sum_{s=0}^{\left\lfloor \frac{w}{r} \right\rfloor} (-1)^s \binom{m-p}{s} \binom{mr-pr-sr}{w-pr-sr} \]

\[ [z] = \text{largest integer } \leq z \]
equally probable combinations in which \( p \) rows are entirely occupied while the remaining \( m-p \) rows have at least one free device each.

**Lemma 2.** Given a probability distribution, \( P(w) \) for the number of occupied devices, the probability \( P_m(p) \) of finding precisely \( p \) rows occupied can be expressed as

\[
P_m(p) = \binom{m}{p} \sum_{s=0}^{m-p} (-1)^s \binom{m-p}{s} H(pr+sr)
\]  

(30)

where

\[
H(x) = \sum_{w-x}^{mr} \frac{\binom{mr-x}{w-x}}{\binom{mr}{w}} P(w)
\]  

(31)

\( H(x) \) is recognized as the \( x \) devices specified in advance are occupied.

**Proof:** Denote by \( v_{r-i} \) the number of rows having \( r-i \) occupied devices and \( i \) free devices \((i=1, 2, \ldots, r-1)\). If rows with the same number of occupations are regarded as equal, we can distinguish.

\[
\binom{m}{p} \binom{m-p}{v_{r-1}} \binom{m-p-v_{r-1}}{v_{r-2}} \ldots \binom{m-p-v_{r-1}-v_{r-2} \ldots -v_2}{v_1}
\]  

(32)

different combinations. The number of combinations of \( r-1 \) occupations with a row is \( \binom{r}{r-1} \). Thus it is readily seen that
where the summation shall be carried out for all \( v_{r-i} \) satisfying the relation

\[
\sum_{i=1}^{r-1} (r-i) v_{r-i} = w - pr
\] (34)

Let us consider the following identity:

\[
\left[ \sum_{i=1}^{r} \left( \begin{array}{c} r \\ r-i \end{array} \right) x^{r-i} \right]^{m-p} = \left[ (1+x)^{r} - x^{r} \right]^{m-p} \] (35)

Binomial expansion of both members will give

Left member = \[
\left[ \left( \begin{array}{c} r \\ r-1 \end{array} \right) x^{r-1} + \sum_{i=2}^{r} \left( \begin{array}{c} r \\ r-i \end{array} \right) x^{r-i} \right]^{m-p}
\]

= \[
\sum_{v_{r-1}=0}^{m-p} \left( v_{r-1} \right)^{(r-1)} v{r-1} \left[ \left( \begin{array}{c} r \\ r-2 \end{array} \right) x^{r-2} + \sum_{i=3}^{r} \left( \begin{array}{c} r \\ r-i \end{array} \right) x^{r-i} \right]^{m-p-v_{r-1}}
\]

= \[
\sum_{v_{r-1}=0}^{m-p-v_{r-1}} \sum_{v_{r-2}=0}^{m-p-v_{r-1}} \sum_{v_{r-1}=0}^{m-p-v_{r-1}} \left( \begin{array}{c} r \\ v_{r-1} \end{array} \right) \left( \begin{array}{c} r \\ r-1 \end{array} \right) x^{r-1}
\]
\[
\left( v_{r-2}^{r-2} \right) \left( v_{r-2}^{r-2} \right) \ldots x \left( v_{r-1}^{m-p-v_{r-1} \ldots v_2} \right) \\
\left( v_{r-2}^{r-2} \right) \left( v_{r-2}^{r-2} \right) \ldots x \left( v_{r-1}^{m-p-v_{r-1} \ldots v_2} \right)
\]

Right member

\[
= \sum_{s=0}^{m-p} \binom{m-p}{s} (-1)^s x^{sr} \left( \frac{1}{1+x} \right)^{m-p-pr-sr} \\
= \sum_{s=0}^{m-p} \binom{m-p}{s} \sum_{j=0}^{m-r-pr-sr} (-1)^s \binom{m-p}{s} \left( \frac{1}{1+x} \right)^{m-r-pr-sr} x^{sr+j}
\]

(37)

The series in (33) appears as the coefficients of \(x^{w-p}\) the left member (35). Using instead the corresponding coefficients in the right member (37) we obtain (29). The assumption of random occupation implies that all occupation patterns with a given value of \(w\) are equally probable. Thus the probability \(P_m(p)\) can be calculated as

\[
P_m(p) = \sum_{w=pr}^{mr} \frac{C(p,w)}{\binom{mr}{w}} p(w)
\]

(38)

Inserting (29) in (38) and changing the order of summation, we obtain

\[
P_m(p) = \binom{m}{p} \sum_{s=0}^{m-p} (-1)^s \binom{m-p}{s} \sum_{w=pr+sr}^{mr} \frac{C(w-pr-sr)}{\binom{mr}{w}} p(w)
\]

(39)
which is identical to (30).

Some special cases:

1. The Erlang Distribution

\[ P(w) = \frac{A^w}{\sum_{i=0}^{\infty} \frac{A^i}{i!}} \]

\[ P(mr) = E_{mr}(A) \quad (40) \]

Substituting \( P(w) \) in the second summation of (39) we have for the terms in this summation, \( B \),

\[ B = \sum_{w=pr+sr}^{mr} \left( \frac{(mr-pr-sr)}{w-pr-sr} \right) \frac{A^w}{\sum_{i=0}^{\infty} \frac{A^i}{i!}} \]

\[ = \sum_{j=0}^{mr-pr-sr} \left( \frac{(mr-pr-sr)}{j} \right) \frac{A^{j+pr+sr}/(j+pr+sr)!}{\sum_{i=0}^{\infty} \frac{A^i}{i!}} \]

\[ = \sum_{j=0}^{mr-pr-sr} \frac{(mr-pr-sr)!}{j!(mr-pr-sr-j)!} \frac{(j+pr+sr)!/(mr-pr-sr-j)!}{(mr)!} \]

\[ \times \frac{A^{mr} A^{-mr=j+pr+sr}}{(j+pr+sr)! \sum_{i=0}^{\infty} \frac{A^i}{i!}} \]
Substituting this in the expression for \( P_m(p) \) we obtain

\[
P_m(p) = \binom{m}{p} \sum_{s=0}^{m-p} (-1)^s \binom{m-p}{s} \frac{E_{mr}(A)}{E_{mr-pr-sr}(A)}
\]

(42)

2. The Bernoulli (Binomial) Distribution

\[
P(w) = \binom{mr}{w} a^w (1-a)^{mr-w}
\]

(43)

Following the method for the Erlang distribution we obtain

\[
P_m(p) = \binom{m}{p} a^{rp} (1-a^r)^{m-p}
\]

(44)
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