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EXPERIMENTAL STUDY OF THE EFFECT OF TWO PROOF FORMATS IN HIGH SCHOOL GEOMETRY ON CRITICAL THINKING AND SELECTED STUDENT ATTITUDES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Sister Carol Harbeck, B.A., M.S.

* * * * * *

The Ohio State University
1972

Approved by

[Signature]
Adviser
College of Education
There are many people to whom I am indebted for their help with the work represented by this paper. Most obvious is my adviser, Professor F. Joe Crosswhite. He has aided me in every way he could. Professor Richard Shumway offered invaluable suggestions from proposal to final copy. Professor James K. Duncan's support and interest was very tangible. The teachers and administrators at Mercy High School in Baltimore, Maryland, made the research not only possible but enjoyable. Sister Mary Joannes Clifford was especially helpful. Encouragement from my family, friends, and religious community, and assistance from many other people at crucial times, enabled me to overcome the inevitable obstacles. To all of these, I am deeply grateful.
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<tr>
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<td>B.A., Mount Saint Agnes College, Baltimore, Maryland</td>
</tr>
<tr>
<td>1963-1967</td>
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CHAPTER I

THE PROBLEM

Introduction and Background of the Problem

This study investigated the effects of two formats for writing deductive proofs in high school geometry on students' logical thinking and on their ability to change from one format to the other. Secondary considerations were given to the interaction of format and texts, attitude toward geometry, and attitude toward the writing of proofs.

Geometry has been the most criticized course in secondary school mathematics. Aims and objectives, content, the several possible ways of development, the focus of emphasis, and teaching methods—all have been disputed over the years and are still vital issues in the field of mathematics education. However, nearly everyone expects that the deductive geometry course improves general thinking abilities.

A review of the relevant literature shows continued effort to obtain scientific support for this assumption. However, not all results are positive. Taken as a whole, the research efforts suggest that the connection between geometry and thinking abilities rests in something more
basic to both of them, namely, in an aspect of cognition hereafter referred to as logical thinking. Other writings suggest that the method of presentation of the logical aspects of geometry is a crucial factor in the actual learning process. The most practical way to acquaint students with logical thinking seemed to most teachers to be the understanding and the construction of deductive proofs. However, students almost always regard proof-making as the most difficult task of the entire geometry course. It was only natural for teachers and theorists both to focus on the teaching of proofs in their attempts to improve logical thinking. Many pedagogical "improvements" for the teaching of proofs have been proposed, but until recently none were reported as having been tested on a wider scale than the classroom experience of the proposers and the relatively few teachers they or their articles influenced.

One of these improvements came to the attention of the present writer during a regional convention for mathematics teachers in 1966, when Frank B. Allen presented a new procedure for writing proofs in geometry which he called "flow diagram proofs." The logical structure of flow-diagrams seemed to offer a method which could overcome some of the problems students experience consistently with deductive proofs. Attempts to introduce the flow-diagram format into her own geometry classes during the next two
years convinced the writer that student could use flow-diagrams with profit. They also demonstrated the need for a comparative study in order to justify the claim that use of flow-diagrams is more beneficial to students than use of the common method of writing proofs. Since there were no indications in the literature available prior to the summer of 1969 that such a study was even envisioned, the present project was designed to deal with some of the major issues involved in the questions arising from classroom experience with both formats.

After the experiment had been designed and partially executed, a study was found which paralleled the present work in one main purpose (to determine the effect of the flow-diagram format and the statement-reason format on critical thinking) but did not intend to have students compare the two formats. Instead Martin (68)¹ concerned himself with teacher reactions to the formats. Several of his recommendations for future research had already been built into the procedures for this study.

**Statement of the Problem**

This study compared the effects of the statement-reason format and the flow-diagram format for writing

---

¹Numbers refer to bibliographical references listed at the end of this study. The first number indicates the reference; if there is a second number following a comma, it indicates page number. Two numbers separated by a semicolon denote two references in the bibliography.
In high school geometry on students' logical thinking and on their ability to change from one proof format to the other. Also considered were interactions of format, text, attitude toward geometry, attitude toward the flow-diagram format, and attitude toward the writing of proofs.

In addition to these primary foci of the research, some of the subjects were used in a subexperiment relating format and achievement.

**Definitions**

The following meanings are assigned to special terms used in this study.

1. **Logical thinking** is the ability to recognize assumptions and to draw valid inferences from a set of statements according to criteria for validity established in formal logic, and to judge when such criteria are not satisfied.

2. A deductive proof consists of a set of initial statements called the hypothesis, a set of statements to be inferred from the hypothesis, and an argument showing that the inference is valid according to the principles of logic. The argument typically consists of a set of intermediary statements or inferences and a set of supporting reasons.
for the validity of each statement or inference.

3. The statement-reason format is a commonly-used method of arranging the intermediary statements and their reasons. In this format, the statements are listed vertically on the left side of the paper and the reasons are listed vertically on the right.

4. The flow-diagram format is a method for horizontally arranging the intermediary statements and their reasons, in which the logical connection between statements is indicated by the implication symbol "→".

5. X denotes either the group of students using the flow-diagram format or the treatment involving the use of the flow-diagram format.

6. G denotes either the group of students using the statement-reason format or the treatment involving the use of the statement-reason format.

7. Flexibility is the ability to change from one proof format to the other.

8. The abbreviation CTA stands for the Watson-Glaser Critical Thinking Appraisal.

   The differences between the two proof formats are most easily understood by studying the same proof written in each of the formats. The statement-reason format will be illustrated first.
**Hyp.:** ABCD is a parallelogram  
**Con.:** \( AB \cong DC \)

**PROOF:**  
<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ABCD is a parall.</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2. ( \overline{AD} \parallel \overline{BC} ) &amp; ( \overline{AB} \parallel \overline{CD} )</td>
<td>2. Def. of a parallelogram</td>
</tr>
<tr>
<td>3. ( \overrightarrow{AC} ) is a transversal</td>
<td>3. Figure &amp; def. of a transversal</td>
</tr>
<tr>
<td>4. ( \angle 4 \cong \angle 3 ) &amp; ( \angle 1 \cong \angle 2 )</td>
<td>4. If 2 parallel lines are cut by a transversal, alt. interior ( \angle )'s are ( \cong ).</td>
</tr>
<tr>
<td>5. ( \overline{AC} \cong \overline{AC} )</td>
<td>5. Identity</td>
</tr>
<tr>
<td>6. ( \triangle ABC \cong \triangle CDA )</td>
<td>6. A.S.A.</td>
</tr>
<tr>
<td>7. ( \overline{AB} \cong \overline{CD} )</td>
<td>7. Corr. parts of ( \cong ) triangles are ( \cong ) (CPCTC)</td>
</tr>
</tbody>
</table>

In the flow-diagram format the proof looks like this:

(1) If two parallel lines are cut by a transversal, alt. interior angles are congruent.
Hypotheses

The hypotheses can be formally stated using the terms defined above.

H₁: There is no significant text-treatment interaction on any of the following criterion variables: January CTA, June CTA, attitude toward geometry, attitude toward proof, attitude toward flow-diagrams, and flexibility.

H₂: There is no significant difference between mean scores of X and G on the January CTA.

H₃: There is no significant difference between mean scores of X and G on the June CTA.

H₄: There is no significant difference between mean scores of X and G on the ability to change to a new proof format, as measured by the "flexibility" subtest of the Format Test.

H₅: There is no significant difference between mean scores of X and G indicating their attitude toward geometry, as measured by the "attitude toward geometry" subtest of the Format Test.

H₆: There is no significant difference between mean scores of X and G indicating their attitude toward proof, as measured by the "attitude toward proof" subtest of the Format Test.
H₇: There is no significant difference between mean scores of X and G indicating their attitude toward flow-diagrams, as measured by the "attitude toward flow-diagrams" subtest of the Format Test.

H₈: There is no significant difference between those classes of X and G which used the text School Mathematics Geometry on achievement in geometry, as measured by items on chapter tests accompanying this text.

H₉: There is no significant difference between those classes of X and G which used the text School Mathematics Geometry on those items of chapter tests which measured certain aspects of the ability to understand and construct deductive proofs.

Research Procedures

Sample

The seven geometry classes at Mercy High School in Baltimore, Maryland, during the 1970-71 school year comprised the sample. Mercy High School is a private Catholic girls' school. Students are grouped homogeneously for mathematics based on national testing percentiles, previous mathematics grades, and recommendations of mathematics teachers. The grouping of freshmen revised in the spring of 1970 resulted in eight groups for sophomore year, of which seven took the geometry course. The homogeneous
grouping affected the choice of text, with the result that the top three classes used *School Mathematics Geometry* (4) and the lowest four classes used *Geometry: A Contemporary Course* (64). The homogeneous grouping is reflected in the labels for the classes used in this paper. C₁ is the top ability group while C₇ ranks lowest.

**Treatment**

The difference in treatments consisted in using different formats for proofs in the texts and in student and teacher exercises. Those students assigned to Treatment X used the flow-diagram format; those assigned to Treatment G used the statement-reason format. In each student's text, the proofs on the pages expected to be covered during the first semester were blackened out and supplements were distributed in which the proofs were written in the appropriate format.

Due to restrictions of faculty and department policy, the researcher was assigned to teach three of the seven geometry classes, and four other teachers were given one each of the remaining classes. Two of these teachers agreed to use the unfamiliar flow-diagram format; the researcher taught the other two classes using the flow-diagram format. The researcher also taught one class using the statement-reason format while two different teachers taught the other
two classes using the statement-reason format.

The assignment of treatment to classes thus had to take into consideration the homogeneous grouping, differences in texts, and differences in teachers. Four classes were assigned to Treatment X and three classes to Treatment G in such a way that of the three top ranked classes which used the same text, the first and third groups were assigned to the same Treatment X, and that of the four lowest ranked classes which used the same text, the fourth and seventh groups were assigned to the same Treatment X. Table 1 on the following page depicts the interplay of text, treatment, grouping, and teacher.

Besides using the designated proof formats, all subjects were administered the Watson-Glaser Critical Thinking Appraisal in September, January, and June, and a specially constructed test (the Format Test) in January. These tests are discussed in detail in the Instrumentation section of Chapter III. Table 2 on page 12 outlines the treatment-testing arrangement.

**Statistical Design**

Multivariate analysis of covariance was employed to test six of the hypotheses. Data for the remaining hypotheses failed to satisfy basic requirements for covariance, and so analysis of variance was employed to test
Table 1.--Assignment of Treatment to Classes

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Students in Study</th>
<th>Teacher&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Text&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Treatment&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>37</td>
<td>T₁</td>
<td>A</td>
<td>X</td>
</tr>
<tr>
<td>C₂</td>
<td>24</td>
<td>R</td>
<td>A</td>
<td>G</td>
</tr>
<tr>
<td>C₃</td>
<td>17</td>
<td>T₂</td>
<td>A</td>
<td>X</td>
</tr>
<tr>
<td>C₄</td>
<td>21</td>
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</tr>
<tr>
<td>C₇</td>
<td>21</td>
<td>R</td>
<td>B</td>
<td>X</td>
</tr>
</tbody>
</table>

Code:

<sup>a</sup>R = Researcher; T₁, T₂, T₃, T₄ = Teachers other than the researcher.

<sup>b</sup>Text A = School Mathematics Geometry.

<sup>c</sup>Treatment X = flow-diagram format
G = statement-reason format
Table 2.—Testing and Treatment Arrangement

<table>
<thead>
<tr>
<th>Class</th>
<th>September Testing</th>
<th>January Testing</th>
<th>Treatment 1st sem.</th>
<th>Treatment 2nd sem.</th>
<th>June Testing</th>
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<tbody>
<tr>
<td>C₁</td>
<td>T₁ᵃ</td>
<td>xᵇ</td>
<td>T₂ᵃ</td>
<td>Fₓᵃ</td>
<td>xᵇ</td>
</tr>
<tr>
<td>C₂</td>
<td>T₁</td>
<td>G</td>
<td>T₂</td>
<td>F₉</td>
<td>G</td>
</tr>
<tr>
<td>C₃</td>
<td>T₁</td>
<td>X</td>
<td>T₂</td>
<td>Fₓ</td>
<td>X</td>
</tr>
<tr>
<td>C₄</td>
<td>T₁</td>
<td>X</td>
<td>T₂</td>
<td>Fₓ</td>
<td>X</td>
</tr>
<tr>
<td>C₅</td>
<td>T₁</td>
<td>G</td>
<td>T₂</td>
<td>F₉</td>
<td>G</td>
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<td>G</td>
</tr>
<tr>
<td>C₇</td>
<td>T₁</td>
<td>X</td>
<td>T₂</td>
<td>Fₓ</td>
<td>X</td>
</tr>
</tbody>
</table>

Code:

- 
  aTest T₁ = Critical Thinking Appraisal Form Zm.
  T₂ = Critical Thinking Appraisal Form Ym.
  Fₓ = Format Test Form X (Flow-diagram Format).
  F₉ = Format Test Form G (Statement-reason Format).

- 
  bTreatment X = Flow-diagram Format.
  G = Statement-reason Format.

these three hypotheses. Covariates were IQ, mathematics NEDT, and the September administration of the CTA. The level of significance for all tests was the .05 level.
Limitations of the Study

The major limitations derived from the practical aspects of the study.

1. The sample was limited to girls in a Catholic high school.

2. Treatment was not randomly assigned to classes, nor were students randomly assigned to classes, due to the policy of homogeneous grouping.

3. Students were compared with students. Ideally, classes would have been compared with classes.

4. There was no satisfactory way to control for teacher bias, teacher differences, and teacher-treatment interactions due to limitations imposed by the school in teacher assignment to classes and to treatment.

5. Finally, better testing materials and data-processing arrangements need to be developed in order to obtain a more complete measure of the developing ability to write deductive proofs for subjects using either text.
CHAPTER II

REVIEW OF THE LITERATURE

The assumption which underlies the present study is that there is a way to improve students' reasoning abilities through instruction in the making of deductive proofs in the high school geometry course. There are several concepts involved in such an assumption, and each of these has its own relatively large body of literature. To try to provide an exhaustive literature search for all of these topics would be unrealistic (there is at least one ongoing research institute dedicated just to the area of critical thinking). Also it would be unnecessary, since much of what has been written on any one of these topics is merely tangential to the major focus of the present study. However, there were certain articles and studies which made the basic issues more meaningful to this researcher, even though they were not directly involved with studying the effect of writing proofs in geometry on students' critical thinking. These will be discussed and their contributions to the background of the research problem will be indicated. Thus this chapter on the relevant literature has six parts, discussing in turn critical thinking, logic, deduction,
proof, the formats for writing proofs which have been proposed by experienced geometry teachers, and finally the research into the connection between proof in geometry and critical thinking.

Critical Thinking

Any list of educational objectives published in the last century has included at least one item dealing with improving the student's ability to think. From time to time, national groups have made even stronger statements, statements such as that of the National Education Association in a 1961 monograph in which development of the ability to think was identified as the central purpose of education in American democracy (29).

In the specific field of mathematics education, development of thinking skills is also a universal objective.

Every report on the teaching of mathematics that has appeared in the last fifty years has included the development of critical thinking among its stated objectives. Perhaps this objective is referred to as logical thinking, problem-solving, scientific thinking, reflective thinking, nature of proof, or clear thinking—but all deal with desirable changes in the student to be effected through the study of mathematics (94, 144).

Many prominent educators have proposed their own phrasing of the objective of improving students' ability to think, but the term which has been most widely used in
recent years is "critical thinking." There seems to be no single definition of what exactly is meant by "critical thinking." In fact, the lack of a universally accepted definition has been deplored by many writers (3; 32; 38; 42). When Allen and Rolt wrote their 1969 report on "The Nature of Critical Thinking" for the Concepts in Verbal Argument Project at the Wisconsin Research and Development Center for Cognitive Learning, they identified three disparate viewpoints of the essence of critical thinking. The three viewpoints consider critical thinking as (1) an act of evaluation, or (2) an act of inquiry, or (3) a pluralistic act involving several essentially different activities including both inquiry and evaluation (3). The last conception of critical thinking agrees most closely with the writings of mathematics educators (86) as well as the present writer's interpretation of current theories of learning. Therefore, for the duration of the present study, critical thinking

is not a general ability but a composite of small group and specific factors, particularly recognition of assumptions and judgments if conclusions follow (38, 1016-A).

The names of Glaser and Ennis stood out among all of the authors reviewed by this writer as most clearly setting forth the various activities involved in critical thinking, developing upon the ideas and positions of a
large group of writers. Glaser explored the difficulties experienced by students required to "think critically" and identified certain critical thinking abilities. His findings became the basis for the development of the *Watson-Glaser Critical Thinking Appraisal*. His original work listed the following critical thinking abilities:

- recognize problems and find workable means for meeting those problems
- gather and marshall pertinent information
- recognize unstated assumptions and values
- comprehend and use language with accuracy, clarity, and discrimination
- interpret data
- appraise evidence and evaluate arguments
- recognize the existence (or non-existence) of logical relationships between propositions
- draw warranted conclusions and generalizations
- put to the test the conclusions and generalizations at which one arrives
- reconstruct one's patterns of beliefs on the basis of wider experience
- render accurate judgments about specific things and qualities in everyday life (41,6).

The manual for the revised (1964) forms of the *Watson-Glaser Critical Thinking Appraisal* defines critical thinking as a composite of
(1) attitudes of inquiry that involve an ability to recognize the existence of problems and an acceptance of the general need for evidence in support of what is asserted to be true;
(2) knowledge of the nature of valid inferences, abstractions, and generalizations in which the weight or accuracy of different kinds of evidence are logically determined; and
(3) skills in employing and applying the above attitudes and knowledge (107,10).

These activities agree essentially with Ennis' list of factors of critical thinking:

(1) Grasping the meaning of a statement
(2) Judging whether there is ambiguity in a line of reasoning
(3) Judging whether certain statements contradict each other.
(4) Judging whether a conclusion follows necessarily.
(5) Judging whether a statement is specific enough.
(6) Judging whether a statement is actually the application of a certain principle.
(7) Judging whether an observation statement is reliable.
(8) Judging whether an inductive conclusion is warranted.
(9) Judging whether the problem has been identified.
(10) Judging whether something is an assumption.
(11) Judging whether a definition is adequate.
(12) Judging whether a statement made by an alleged authority is acceptable (32,84).

A reader of these abilities might point out their lack of the mathematical quality of "completeness." One important ability that is omitted is the judging of value statements. Since there seems to be little agreement among educators and value theorists about how such judging is to
be done, the omission is not too surprising. Nor, for the purposes of this study, is it important; such statements rarely occur as part of the subject matter of the typical geometry course.

Another strong but indirect argument for adopting Ennis' definition of critical thinking comes from Jansson's search for the factors of critical thinking which are most crucial for learning mathematics. He ended his search by taking Ennis' list and adapting them to fit six categories: ambiguity, assumption, deduction, definition, conjecture, and model selection (54).

According to Ennis, deductive logic is fundamental to all aspects of critical thinking (33). Mathematicians and logicians have adequately shown that deductive logic is also fundamental to the nature of proof and hence to the role of proof in geometry.

**Logic**

While the importance of logical thinking in the geometry course is unquestioned, the degree of formality of instruction in logical concepts has been debated for many years. The stand that some type of instruction in logical concepts is beneficial to subsequent mathematical learning is supported by the studies by McAloon and Elder. Sister Mary deLourdes McAloon (71) found that an 8-week unit in
selected logical concepts produced significantly higher scores on a mathematical achievement test and on a reasoning test among 25 classes of third- and sixth-grade students. The scores on the mathematics test were higher for the groups taught logic interwoven with mathematics than for the groups taught logic separate from mathematics; however, those differences did not reach statistical significance. There was no similar trend in reasoning scores. Elder concluded that explicit instruction in some concepts from logic improved college algebra students' ability to verbalize mathematical generalizations (31).

However, studies by Platt and Roy call into question the need for a formal or explicit logic course or unit. Roy (96) constructed a unit on logic and proof for 54 high school Math 12 students in an Advanced Algebra course; the control group (also 54 Math 12 students in Advanced Algebra) received informal instruction involving reasoning and axiomatic systems. The formal instruction in logic had no significant effect on the ability of the Math 12 students to judge the validity of arguments or to use mathematical induction to prove theorems. Platt's experiment purposed to

evaluate the effect of the use of mathematical logic in high school geometry on (1) achievement of students in geometry, (2) achievement of students in reasoning in geometry, (3) critical thinking of
students, and (4) attitudes of students toward logic, deduction, and proof in mathematics (87,6).

Among the conclusions, it is stated that

including instruction in mathematical logic in high school geometry does not result in a course which is significantly superior to the traditional course in its over-all effect upon student achievement in reasoning in geometry, critical thinking ability, or attitude of students toward logic, deductive thinking, and proof in mathematics (87,83).

However, he found evidence that including instruction in mathematical logic did improve reasoning ability in geometry for the high achieving students. His is also one of the few studies found reporting significant differences between boys' and girls' critical thinking abilities.

Morgan claimed that the results of his study "raise serious questions as to the need for explicit teaching of mathematical logic and the logic of proof" (74,63).

However, his criticism was probably aimed at the teaching of mathematical logic and proof which occurs within courses not specifically dealing with logic or proof; for nowhere does he state that part of his treatment was instruction in these topics. Instead, it seemed that he was attempting to evaluate the results of whatever instruction in logic and proof is presently given in the courses a mathematics major would normally have received in college and high school.
Deduction

One of the main reasons proposed for including logical concepts in mathematics courses is to improve students' "powers of deduction." Deduction is both a product and a process of logical thinking and is obviously basic to writing proofs in geometry or any subject area. Balomenos (6) amply documented the nature of deduction and its traditional role in geometry when, in 1961, he presented the growing argument that deduction must be a subject of instruction in secondary school algebra as well as in geometry.

Balomenos found after a 15-week study involving 150 high school students that there were no significant differences between the group studying axiomatic number systems and the group studying geometry on algebra achievement, general mathematical achievement, critical thinking, deduction, and mental ability. The geometry group scored significantly higher than the number systems group on a geometry achievement test, as expected.

Balomenos' experiment is of interest to the present writer for two reasons. First, and less importantly, he used the same instrument to measure reasoning ability of a group of geometry students as the present study, namely the Watson-Glaser Critical Thinking Appraisal. Secondly, the lack of significance between the two groups agrees with
many of the studies to be discussed in the section on geometry and critical thinking, and suggests that there probably is no content area in mathematics that serves as the best vehicle for developing logical thinking. Instead of searching for subject matter which is most appropriate for securing educational objectives that transcend subject matter categories, curriculum developers could include a subject for its own contribution to the student's pool of factual knowledge and methods of approaching problems. Since it would be assumed that no course by itself necessarily improves critical thinking, then educators would be more likely to develop methods of instruction that fundamentally and explicitly involve critical thinking activities.

The studies included in this section on deduction deal more with the development of logical thinking than with any particular subject context. Again it may be noted that there is no attempt here to exhaust the literature on logical thinking. The selections from the literature which follow are included because many of them are quite recent and have contributed a great deal to the implications of the present research and suggestions for future curriculum materials and research which are made in Chapter V.
An annotated bibliography on the development of logical thinking in children summarized some of the literature thus:

According to Piaget, the young child understands classification and relationship on the concrete level, but does not develop the ability to reason logically until the age of eleven or twelve, and then only on the concrete level. Other researchers assert that reasoning ability is more related to natural intelligence than age. Burt (16), Welch and Long (108), Suppes and Binford (102), and many others state that the ability to reason is acquired at least by the age of six or seven, limited only by experience. Furthermore, Hazlitt (95), [Shirley] Hill (50), Moore (73), and others show that young children are capable of abstract reasoning (24,1).

Of special significance for the present study is Hazlitt's conclusion that when an adult has to think about totally unfamiliar material, he makes the same mistakes in reasoning as a child. Thus geometry teachers should be familiar with probable errors and shape their instruction of deduction to make the students aware of those errors and less likely to make them in later situations.

Shirley Hill (51) concluded that students in the primary grades could make valid conclusions and that this ability increased significantly with age. O'Brien and Shapiro (78) found, however, that children of these same ages were less able to determine the logical necessity of conclusions. When made to choose between responses of "yes," "no," or "not enough clues," the children avoided the
"not enough clues" response. The children also tended to treat "if . . . then . . ." statements as if they were "if and only if" statements. This type of error was called "child's logic" in a later study by O'Brien, Shapiro, and Reali. Figure 1 illustrates the difference between "child's logic" and "math logic."

Example A: Given: p → q
q
Conclusion: p?
Child's Logic: yes
Math Logic: not enough clues

Example B: Given: p → q
not p
Conclusion: q?
Child's Logic: no
Math Logic: not enough clues

Figure 1.--"Child's Logic" vs "Math Logic."

In Example A, "Child's logic" would lead the child to conclude the antecedent is "true" if the inference and the conclusion are "true." This is known as the logical fallacy of "affirming the antecedent." It assumes that if the original "if . . . then . . ." statement is true, then its converse must be true. Example B reflects the fallacy of "denying the consequence," and assumes that the truth of the original "if . . . then . . ." statement logically infers the truth of the inverse.

The study by O'Brien, Shapiro, and Reali (79) involved approximately 180 students in each of grades, 4, 6,
and 8, and 120 students in grade 10 in the public school system of a middle class suburb of St. Louis, Missouri. One-third of the subjects at each grade level were randomly assigned to that part of the experiment which involved a replication of the earlier study by O'Brien and Shapiro, measuring the tendency to use "child's logic" over "math logic" with a different sample and a different instrument. This part of the experiment will be discussed here, and the other part will be treated in the section of this chapter which concentrates on the effect of format.

The subjects for the first part of the experiment were divided in half by means of a random procedure. Each half took one of the forms of a specially designed test which included items contrasting "child's logic" and "math logic." Each test was scored twice, once according to "child's logic" and then according to "math logic." Graphs of the results illustrated the predominance of "child's logic" over "math logic" at all levels, as well as the change in use of both logics between grade levels. Data from both forms of the test have been combined in the graphs in Figure 2 below. The second graph, Figure 3, shows the prevalence of "child's logic" as a consistent thought pattern. One obvious implication of this part of the study is that teachers cannot assume at any level that "child's logic" is reserved to children.
Figure 2.—Percentage of Responses Correct According to Child's Logic and Math Logic—Tests I and II Combined (Used by Permission of the authors).

Figure 3.—Percentage of Subjects Consistently Using Child's Logic and Math Logic—Tests I and II Combined (Used by Permission of the authors).
Warren Hill found from his experiment with 60 third grade students that

(1) training on classificatory skills and the meaning of the conditional statement was more effective than training on only classificatory skills for increasing subjects' performance on sentential logic tests;

(2) instruction had no effect upon the recognition of inconclusive inferences; and

(3) the structure of the conditional statement is a relevant variable in the child's understanding of sentential logic (51,5024-B).

At the 48th annual meeting of the National Conference of Teachers of Mathematics, Matulis (70) reported the results of his survey of the understandings of selected concepts of logic by 8- through 18-year old students. He had developed a multiple-choice test which measured the understanding of implication, conjunction, disjunction, and quantifiers of 46 classes of fourth- through twelfth-grades public school students. He found direct relationships between understanding of deductive logic and each of age, intelligence, and socioeconomic status, but no significant relationship between sex and understanding of deductive logic. He also graphed age-related patterns for the growth in the understanding of deductive logic according to intelligence and also to socioeconomic status. Replications of this study may establish such growth patterns as part of the development of logical thinking, and consequently raise
many questions for curriculum revision and further experimental research.

Among the implications of this study for the teaching of deductive proofs in geometry are the following:

(1) The older the group of students, the greater will be the range of abilities to understand logical concepts; hence teachers should be equipped to use a number of different instructional approaches suitable for the different levels of maturation.

(2) A "spiral approach" can be developed for the teaching of deduction similar to the "spiral approach" to the teaching of number.

A number of researchers have investigated the ability of students of various ages to distinguish between valid and invalid patterns of inference.

Paulus set out to determine how well children (grades 5 - 12) can draw conclusions from the premises of deductive arguments and to determine if there are significant empirical differences between this ability and the ability to evaluate conclusions of deductive arguments (82,2101-A).

He found that these are indeed different abilities and not always both present to the same extent in any individual. His data also suggested that the type of content affects
the ability to deduce more than the ability to assess deductions.

Roberge explored the differences in children's abilities to assess the validity of conclusions of deductive arguments when the statements were expressed in "class reasoning" terms (e.g., "At least one of the following is true . . ."); cf. Figure 5, p. 41) or in "conditional reasoning" terms (e.g., "If . . . then . . ."); cf. Figure 4, p. 40). He also considered the influence of other factors such as sex, negation, negation of premises, and item content (concrete-familiar, suggestive, or abstract) on these abilities. The sample consisted of 263 students from the fourth to the tenth grades. He concluded, among other things, that

1) overall the class reasoning test was significantly easier than the conditional reasoning test, but that neither type of reasoning was consistently easier at all grade levels;

2) the developmental patterns of percentages of subjects who had mastered specific principles were remarkably similar in both types of reasoning;

3) no improvement in the percentages of subjects who had mastered the fallacies occurred until the tenth grade; and

4) differences associated with sex and reversal of premises were not significant (92).

An experiment by Martens led her to infer that

... without specific instructions in rules of inference, students do develop certain consistent methods of dealing with inferences, but these include invalid inference patterns as well as valid inference patterns (67,4536-A).
This agrees with the prevalence of "child's logic" already noted, and points to a clearly identifiable objective for the geometry course with suggested devices of measuring success in achieving the objective.

Elmer Miller used 329 tenth-, eleventh-, and twelfth-graders in a comprehensive high school to determine a ranking of logical fallacies according to difficulty. He recommended that formal instruction be given on recognizing the common logical fallacies, and that "... the reasoning process should be studied in the same scientific manner as has been done to establish grade and age norms for spelling, reading, arithmetic, social science concepts, etc."

(72,128).

According to almost every study of critical thinking found by this writer, sex is not a significant factor in the development of logical ability.

Several studies dealt with the effect of attitude or belief on the validity of conclusions made (8;40;48;53). In general, they agreed that people tend to make more invalid conclusions when the invalid conclusion agrees with a prior belief or attitude. Fetzer and Murray (37) concluded from a study involving 190 children (ages 8 to 15) that children younger than 12 years of age tended to judge a syllogism on its empirical validity instead of its logical validity, whereas older children used logical validity as their criterion.
Corley's 1959 study tied investigations into deduction with investigations into readiness for geometry. He divided his experimental work into two parts. In the first part, he taught students in grades six through ten for one week in distinguishing among the uses of intuition, inductive reasoning, and deductive reasoning to reach general conclusions. The students also were taught the structure of the syllogism and recognition of validity and invalidity. An experimenter-made test of achievement in general reasoning was administered at the end of the unit. In the second part of the experiment, Corley replicated the teaching and testing of the unit in general reasoning with one class each of grades six, seven, eight and ten. No ninth grade class was available for participation in this part of the experiment. After the unit on general reasoning, the four classes took part in a four-week instructional unit on geometry which emphasized the logical structure of geometry and used the deductive method for proving theorems. At the end of the unit on geometry, an experimenter-made test measured achievement in (1) knowledge of terms and concepts, (2) understanding of geometric proofs, and (3) ability to use the syllogism for valid reasoning. Analysis of variance techniques along with developmental patterns suggested by the data led Corley to make the following conclusions:
(1) there was no significant interaction between grades and IQ levels with respect to scores on either achievement test;
(2) there is a positive correlation between IQ and achievement on each of the achievement tests;
(3) sixth grade students are quite able to handle the geometric terms and concepts included in the tests used in this experiment; they are moderately prepared to understand the three methods of reaching general conclusions but find the logical structure of geometry and the proof of theorems in geometry too complex for comprehension by all but a very few;
(4) the growth curves, beginning with the sixth grade, differ significantly:
   (a) growth of understanding terms and concepts is linear throughout the grades studied
   (b) growth in general reasoning is rapid between grades six and seven, then progresses more slowly;
   (c) growth in understanding of logical structure and proof seems logarithmic, starting at a low level at grade six, greatly increasing from grades six through eight, then progressing very slowly through grade ten (25,77-83).

Although the above conclusions seem to be in accord with results of other experiments and with the learning theories of Piaget and Ennis, great care must be taken in extending them beyond the 14 classes used in the experiment. They must be considered merely indicative until they are corroborated by other studies and larger and perhaps more representative samples.

The final study to be included in this section dealing with deduction used the making of informal proofs the criterion task for measuring the effects of instruction in deduction. In 1967 Phillips compared the effectiveness of two methods of presenting the rules of deduction. The
methods he employed in his study were to handle the rules of deduction informally in the context of a deductive system of mathematics and to treat the rules of deduction in a formal study. He concluded that

the ability to deal with informal proofs of the type presented [in the study] is independent of a previous knowledge of the formal rules of deduction as these rules were presented in the [quarter] course in deduction (84,104).

Proof

The nature of deductive proof has been explicated by many writers. There are ample discussions of the nature of a deductive proof in books on mathematical logic. Every so often, educational groups include similar treatments of proof in their professional publications; for example, the chapter on proof in the recent yearbook of the National Council of Teachers of Mathematics (34).

One of the better known names connected with logic and proof is Nathan Lazar. In 1934-35 his articles in The Mathematics Teacher (62) explored the essence of deduction through multiple contrapositives, converses, and inverses for the teacher of secondary school mathematics. However profound the impact of his articles on later writers such as Allen (2), they did not initiate a widespread reform in pedagogical procedures. Brockman (11) found in 1962 that there was very little teaching
of even simple contrapositives and inverses, and not much more teaching of simple converses. In the past few years, Frank Allen has spoken about the pedagogical advantages of using contrapositives to groups of mathematics teachers at regional and national conventions. Attendance at and reactions to his talks observed by this writer suggests that the "average math teacher" is quite unfamiliar with the concepts and possible uses of multiple contrapositives in teaching mathematics.

A recent dissertation by Arnsdorf (5) traced the teaching of proof in geometry from antiquity to the present, through the recommendations by groups such as the Committee of Fifteen, and the Commission on Mathematics, and the positions of individuals such as E. H. Moore, Gertrude Hendrix, Frank Allen, and Harold Fawcett. To summarize his findings in one sentence is impossible. However, he documented a trend to pay more explicit attention to the role of proof in the geometry course.

Byham surveyed textbooks in geometry, in logic, in teaching procedures, and in college mathematics, as well as pertinent periodicals in order to identify methods of indirect proof which were logically correct, yet simple enough so that both the methods of indirect proof and the logic underlying these methods would be comprehensible to students of high school geometry (18,2899-A).
He discussed two methods, the method of inconsistency and the method of contraposition, which satisfied the above criterion for recommendation to geometry teachers.

Robinson investigated the ways junior high school students justified their generalizations in mathematics, in order to evaluate the need they felt for proof in a mathematical sense of the word proof. She identified responses by means of five categories:

1. Those characterized by not knowing or by guessing
2. Reasoning by analogy
3. Reasoning by induction
4. Reasoning from false or invalid premises
5. "Proofs" (93,71).

"Proofs" were operationally defined as responses which represented valid deductive reasoning, made explicit reference to mathematically acceptable premises, and showed the generalization as a consequence of these premises (93,58). Seventy-four responses were accepted as proofs; these were offered by 37 of the 48 children (77%). The average number of proofs for the 4 tasks used for analysis of data was 1.55 per child. The large amount of variance in individual children's scores was ascribed to two factors—task and mathematical track. The two factors operated independently, even though the children from the ninth grade accelerated track had completed a year of plane geometry, and 3 of the 4 tasks involved geometric situations.
Regression and analysis of the data suggested that "arithmetic problem solving and reading ability are more predictive of the tendency to offer proof in mathematics than are either computational ability or mental age" (93,83). This conclusion is not too unexpected, for it indicates that children have to be able to express how they grasp a problem situation before they will try to explain it to another person.

The main conclusion from Robinson's research was that if the [sample used] is representative of the junior high school population, then most seventh grade pupils are quite capable of understanding the nature, if not the role, of deductive methods in mathematics (93,98).

In particular, she concluded that

(1) whether or not seventh grade students specifically recognize that induction is inadequate to support generalizations in mathematics, their responses in situations of free choice are characterized by other forms of reasoning, proof being one of them;

(2) most seventh and ninth grade students will, when given free choice, justify a mathematical generalization by deductive reasoning from a set of premises if and only if those premises agree with their intuition;

(3) formal instruction in the use of the strategies of proof does not seem to affect students' understanding of the limits of arguing from a finite number of examples, nor students' willingness to argue from premises contradictory to their intuition;

(4) there is no evidence from this study to suggest that instruction in plane geometry contributes to a student's understanding of the need for proof or to his understanding of the nature of an axiomatic system (93).
The results of Robinson's study are more representative than the number in the sample (48) might superficially indicate, because of the sampling procedures employed.

Robinson also made suggestions for introducing a form of mathematical proof into the seventh grade: definitions and axioms should be intuitively acceptable; avoid use of the term "proof" in order to avoid misconceptions associated with that term; instead use "Is this always the case? Why?"; use familiar contexts to avoid language difficulties.

In King's study (58), a unit of proof was developed for use with sixth-grade students. The development process and consequent use by a teacher other than the researcher revealed that, while some of the kinds of proofs recommended by the Cambridge Report (19) are within the understanding of the students at this level, the teacher at this level may experience great difficulty in coping with students' responses and errors. It would seem that there needs to be training for teachers aimed directly at teaching how to teach proofs.

A computer was used in Jensen's research (55) to help beginning college students to construct simple deductive proofs as they learned deductive reasoning. The object of the study was to determine the comparative
effectiveness of two methods of presenting the axioms. Analysis of the data showed no significant differences.

Deer (27) investigated the effect of learning logic on the ability to prove theorems in geometry, but his sample was too small to support any conclusions.

The means by which 11 teachers of geometry, of algebra, and of general mathematics justified their statements and actions in class were analyzed by Wolfe (111). He identified eight teacher strategies of justification. Deductive proofs comprised 38 per cent of the justifications in geometry, 23 per cent in algebra, and 0 per cent in general mathematics.

A final research dealing explicitly with proof is Bittinger's investigation into the effect of a unit in mathematical proof on later performance in mathematics courses. His results suggested that "teaching a unit in mathematical proof may produce an effect, but not one which is significant" (9,3906-A).

**Format for Geometry Proofs**

All of the literature surveyed so far points to the need of developing a method—or better still, a number of methods—for instructing students in the nature of deductive reasoning. Such methods should incorporate all that the majority of learning theorists as well as logicians
consider crucial to understanding and making valid inferences. While there is no intent to classify the following researchers as learning theorists, their results should, in the opinion of this writer, be taken into consideration when issues dealing with format or mode of presentation are being discussed.

The second part of the research by O'Brien, Shapiro, and Reali (79) investigated the effect of mode of presentation. Two-thirds of the sample were given two deduction tests to determine their use of "child's logic" in contrast to "math logic." The test items were identical in content, but two modes of presenting the implication were employed. Figures 4 and 5 illustrate the two modes of presentation, and hence how Tests I and II differed from Tests IA and IIA.

Figure 4.—Sample Item Illustration the Format of Tests I and II in the Study by O'brien, Shapiro, and Reali (Used by permission of the authors).
Figure 5.—Sample Item Illustrating the Format of Tests IA and IIA in the Study by O'Brien, Shapiro, and Reali (Used by permission of the authors).

It was found, as expected, that the task given in Tests IA and IIA [see Figure 5] was much easier than the logically equivalent task when presented in If-Then language. In general, subjects scored substantially and universally higher on Tests IA and IIA in terms of "Math Logic" than on Tests I and II [see Figure 4] (79,8).

A third test was administered to the remaining third of the sample, again with the two modes of presentation exemplified by Figures 4 and 5. The content of the items of experimental interest was classified as causal in context or class inclusion in context. Analysis of test results indicated definite interaction of format and context. On the tests using the mode of presentation in Figure 5, use of "math logic" over "child's logic" occurred more often for a class inclusion context than for a causal context. However, for the test using the mode of presentation in Figure 4, the "if...then..." mode, there were virtually no
differences between contexts. It was concluded that mode of presentation and context were two significant factors in children's deductions.

The influence of different modes of presentation, or formats, was the primary issue in a study by Earle (28). Three formats were used for self-instruction materials in a college descriptive geometry course. The results showed that some formats are comprehended more readily than others.

Excellent teachers in every generation have known that mode of presentation or format does make a difference, and they have developed their own procedures by constant, and perhaps unconscious, research in their own classrooms. These procedures may have been passed on to individual teachers, but in general remained a local phenomenon. In practice, a procedure does not become widespread until it is incorporated into published materials. Close examination of texts for students or teachers reveal only one major change in the presentation of the writing of proofs. Both Arnsdorf (5) and Martin (68) described the change to the statement-reason format.

Euclid's Elements presented a paragraph form of proof that was taught in European schools and later in American schools. Mathematics educators have long voiced dissatisfaction with the Euclidean approach to proofs. As early as the 1890's, attempts were made to improve Euclid's approach. In the 1920's the two-column, statement-authority form began to be used, as an attempt to lessen the formality of
Euclid. This form of proof is found today in most geometry texts, although "modern" geometry programs of the 1960's re-introduced the paragraph form (5,40).

First signs of the ledger [statement-reason] method were evident in 1895 and the usage of the ledger method in its present form was found in a 1926 geometry textbook. The ledger method soon became quite popular among authors, editors, and publishers. It is now used exclusively in nearly all geometry textbooks.

Teacher dissatisfaction with the ledger method was expressed as early as 1926. However alternative methods were not suggested until the 1950's. Several teachers, working independently, developed methods similar in nature to the flow-proof method (68,28).

Around 1959, there were several alterations to the statement reason format for proofs suggested by experienced teachers.

Schacht (97) added "structure lines" to the statement-reason format in order to highlight the use of syllogistic thinking in the statement-reason format. Berger (7) suggested using cycles of "assumption, given, deduction." Wiseman (110) proposed teaching elementary school children something of the nature of deductive reasoning via chain links: elliptical cutouts on which hypotheses were written at the top of the link and conclusions were written at the bottom. "Chains of reasoning" could then be constructed physically. The outline form was Tenney's alternative (103). Thorsen (104) promoted a structure diagram which superficially resembled a computer flow chart, but the connections
indicating the "logical next step." Klingler (59) adapted
Allen's "flow proof" to geometry to produce the format
basically identical with the one used in the present
study. Marks and Smart (66) added a step between "given"
and "to prove" called "analysis" or "plan" which worked
backwards from the conclusion to the given, via state­
ments of the form "q if p."

While not suggesting other complete formats, several
writers "improved" on parts of the proof. Most common
were replacements for "given ...; prove ..." (17;36;95;
99).

Few of these proposed formats have found their way
into texts. One possible reason could be that only in the
past two or three years have studies been undertaken to
determine whether the proposed formats were really im­
provements. There are two such studies known to this
writer, by Arnsdorf and Martin. Martin's study is very
similar to the present study and will be described in
detail in the next section.

Arnsdorf offered a format called the "analytic
method" as an alternative to the statement-reason format:

[The] analytic method shall be interpreted as
the devising of a strategy and then writing
this strategy. Specifically, "analytic method"
begins as a way of thinking in which reasoning
starts with the conclusion and then proceeds to
determine all of the possible ways to show the
conclusion. The hypothesis is then used to determine which of the ways to show the conclusion applies to that particular proof. The way that may be used to show the conclusion is written and labeled "strategy" (5,6).

Arnsdorf's experiment consisted in an eight-week period of instruction of all subjects in writing proofs in the statement-reason format, then additional instruction in one of the two formats. Measures of general geometry knowledge, constructions, and the writing of proofs were taken along with measures of attitude toward geometry and toward the teacher. No significant differences were found in attitudinal changes nor in gains in general geometry knowledge; the students using the analytic method did acquire significantly more ability in the writing of formal proofs. However, the number of subjects imposes serious limitations on the generalization of any conclusions: there were eleven beginning college students in each treatment group. Also it seems that the treatments may have differed in more than the method of writing proofs. It is not clear what the control group did while the experimental group took part in the group discussions used to teach the analytic proof. It may be that the experimental group in effect had more instruction in proof construction, and it could have been this additional instruction, and not the format, which produced the differences in ability to write proofs.
Geometry and Critical Thinking

The earliest investigations into the relationship between geometry and critical thinking profoundly influenced the whole of mathematics education, as well as the teaching of geometry. Fawcett's study resulted in a National Council of Teachers of Mathematics yearbook, The Nature of Proof (32), which is still a prime reference on the subject for mathematics educators. He, Ulmer, and Gadske "spearheaded the movement to emphasize the use of geometry as a vehicle for the development of logical thinking" (112,103).

Fawcett had built on the results of two earlier works. In 1924, Parker (80) concluded that conscious study of methods of logical reasoning was superior to the current emphasis on mastery and replication. She claimed that such study tended to develop in students the qualities of "resourcefulness, versality, and experimental attitude in geometric and non-geometric situations." The differences reported in her study were too small, however, to achieve statistical significance by today's standards. Perry (83) compared the effects of a traditional approach to teaching demonstrative geometry with an approach which emphasized methods of analysis and generalization in solving "originals."
These enlightened techniques of teaching were markedly helpful to the average students, somewhat beneficial to the superior students, and detrimental to the very superior students in initiating and developing habits of analysis and generalization necessary for successful solutions of originals. In solving non-mathematical problems all groups showed marked improvement with the very superior students showing the greatest gains (98,30).

Kimball criticized Perry's instrument for assessing reasoning abilities, stating that

[it] which actually was a composite test prepared from several standard tests of aptitude and intelligence, [seemed] to bear only a remote relationship to the skill usually labelled as "logical thinking" or "critical thinking." There are indications that the gains in reasoning ability which did result were largely due to practice effects rather than actual improvement in reasoning powers (57,49-50).

Both Parker's and Perry's studies had subjects transfer to geometric situations thinking abilities developed prior to the geometry course. Later studies reversed this, concerning themselves with developing critical thinking abilities within the geometry course (though not necessarily solely on geometric material) and testing for transfer to generalized contexts.

Fawcett developed a course entitled "The Nature of Proof" at The Ohio State University High School during the 1930's. In 1938 he published the results of a two-year experiment with 50 students, from which he concluded that the formal course in demonstrative geometry did not seem
to improve students' logical thinking as it was then taught, whereas participation in a course such as "The Nature of Proof" did involve students' thinking abilities and noticeably improved their logical thinking (34).

Ulmer undertook an extension of Fawcett's work, with more than 1,200 students and 16 teachers participating. He reported his conclusions as follows:

. . . it is possible for high school geometry teachers, under normal classroom conditions, to teach in such a way as to cultivate reflective thinking, . . . this can be done without sacrificing an understanding of geometric relationships, and . . . pupils at all I.Q. levels are capable of profiting from such instruction. The results also indicate that even what is commonly regarded as superior geometry teaching has little effect upon pupils' behavior in the direction of reflective thinking unless definite provisions are made to study methods of thinking as an important end in itself (105,25).

Similar studies were done by Gadske, Henderson, and Cook all within a year or two of each other. Generally, results were similar to those of Fawcett and Ulmer. In addition, Gadske (39) noted improved attitudes toward geometry in the experimental group. Cook (23) included interpreting graphs and deciding on best and poorest solutions to a problem in her measure of critical thinking. Henderson (46) drew the content matter for his study from solid instead of plane geometry.
Stein approached the relationship between critical thinking and geometry from a slightly different angle. The studies already mentioned had been experimental studies. But Stein's 1943 study (100) was more of a correlation of achievement in geometry with twelve variables including general intelligence, spatial relationships, linguistic ability, study habits, logical reasoning, and teachers' estimate of success. He concluded that the intercorrelation of traits such as logical reasoning with general intelligence makes it difficult to gauge their individual effect.

In his purpose and in some aspects of his research design, Stein resembled the earliest studies into the question of geometry and critical thinking, rather than a continuation of the recently finished experimental work of Fawcett and others.

The experimental studies showed the possible contribution of the high school geometry course to the improvement of students' reasoning abilities. At this point, however, some observations seem to be in order. First, in each of the investigations, the investigator had to develop his own way to measure students' reasoning abilities. There was at that time no test which claimed to measure these facets of cognition and which was widely recognized as doing just that. This meant that there was really no common basis of
measurement to facilitate comparison of results from two or more of the experiments. The development and wide acceptance of the Watson-Glaser Critical Thinking Appraisal provided such a means of comparison. All of the studies carried out after 1950 employed the Critical Thinking Appraisal and so shared a common measure of critical thinking, and even more importantly, a common "working definition" of critical thinking.

Secondly, the early studies made use of materials which were atypical of the high school geometry course, and involved a significant portion of non-geometric examples. These studies, especially Fawcett's, made their impact on the teaching of geometry via textbooks which included a few exercises dealing specifically with the reasoning processes outside of geometric contexts. But as for their long-range impact on teaching procedures and materials, Kimball observed in 1957 that

... there was little evidence that any widespread systematic efforts have been made to encourage groups of teachers to organize their geometry instruction so as to emphasize improved thinking ability as its specific objective (57,53-4).

Martin repeated the essence of this statement in 1970 (68).

In 1950 Lewis returned to the question of geometry's effect on critical thinking after a space of almost ten years. Lewis used three classes in his experiment: an
experimental class of 22, a "passing control" class of 35, and a "failing control" class of 21. Both control classes followed ordinary text materials; the experimental group used instead of a text certain materials which somewhat resembled Fawcett's course "The Nature of Proof." Analysis of gains on the Critical Thinking Appraisal over the school year indicated that the experimental group gained significantly more than the control groups, while there was no significant difference between gains of the control groups.

The major contribution of the Lewis study was its very emphatic confirmation of Fawcett's findings. Lewis did go one additional step by analyzing the subtest scores from the critical thinking test (57,59).

Massimiano's 1950 study (69) matched pairs of geometry and non-geometry sophomores in a Pittsfield, Massachusetts, high school on the basis of IQ, pretest scores on the Critical Thinking Appraisal, sex, and age, resulting in three matched sets of paired pupils. Analysis of posttest scores on the Critical Thinking Appraisal administered seven months later revealed that all but one group made gains that were significant at the .001 level. The excepted group consisted of boys not taking geometry; their gain was significant at the .20 level. Included in the groups making significant gains was a group of girls in the commercial curriculum who did not take geometry. In comparing the gains of those who took geometry with the
gains of those who did not take geometry, the differences were not significant at the .05 level. There was a tendency, however, for the geometry groups to increase more than the non-geometry groups on the **Critical Thinking Appraisal**. Correlation procedures did not establish any close association between geometry grades and Critical Thinking Appraisal test scores. Kimball interpreted Massimiano's inconclusive results thus:

... all of the geometry students seem to have received what is usually described as "conventional" geometry instruction. ... If anything, the Massimiano study seems to indicate that if courses are patterned after the Joint Committee Report [of the Mathematics Association of America and the National Council of Teachers of Mathematics] the students will not show any remarkable improvement in critical thinking ability when compared with students who do not study any geometry (57, 60-1).

Kimball developed resource units which "suggested ways to organize the teaching of geometry so as to emphasize improved thinking" (57, 71) while supplementing the content of most geometry texts. Six teachers used the units with their classes, while five teachers and their classes served as the control. Students' critical thinking abilities were assessed by means of three instruments: The **Watson-Glaser Critical Thinking Appraisal**, **A Test of Critical Thinking** (published by the American Council on Education), and a **Critical Thinking Scale** developed by the researcher. The **Critical Thinking Scale** required teachers to rate their
students on certain behaviors considered to exemplify critical thinking skills. The seven-month-long study had the following results:

(1) even with minimal supervisory assistance and possibly minimal enthusiasm for the project on the part of the teachers, the resource units made a significant difference toward improving students' critical thinking;

(2) achievement in geometry was not lessened because time was spent on non-geometric materials in the resource units;

(3) it seemed that standardized geometry tests do not measure "ability to apply general critical thinking skills in non-geometric settings . . . [even though] this objective is commonly professed as a major purpose of geometry instruction" (55, 122);

(4) contrary to Lewis' findings, there were no significant differences on the Critical Thinking Appraisal subscores;

(5) there were no significant differences between the ranking of course objectives by students of both groups at the end of the school year.

The research studies by Corley (25) and Platt (87), discussed earlier, contributed to the literature on critical thinking in geometry, although that was not their major focus.

Platt was primarily concerned with instruction in mathematical logic and used the geometry class as the most suitable place in current curricula. He concluded that formal instruction in logic was not essential to the development of critical thinking ability; however, it was beneficial for high achieving students.
Corley's investigation of the readiness of students of different ages to learn deductive geometry indicated that there were maturation factors to be considered when one attempts to influence critical thinking ability. While most sixth grade students could learn geometric facts very readily, very few were able to deal effectively with the intellectual activities essential to critical thinking.

The most recent study exploring the connection between critical thinking and geometry coincides to some extent with the present research. Martin (68) also was primarily concerned with the effect of different formats for geometry proofs on critical thinking and attitudes. Martin involved 36 teachers and approximately 1,100 students in a year's experiment in which half of the teachers used the flow-diagram format and half used the statement-reason format. Critical thinking was measured in September and May by means of the Watson-Claser Critical Thinking Appraisal. An experimenter-made questionnaire measuring teacher attitudes toward the flow-diagram was supplemented by more subjective evaluation by teachers of experimental classes. There were no significant differences on Critical Thinking Appraisal gain scores, but most teachers using the flow-diagram format considered that format to be a better method of writing proofs. They gave two major advantages of the flow-diagram format over the statement-reason format;
the logical organization of the flow-diagram structure, and increased understanding of the deductive process on the part of the students. The major disadvantage derived from the universal use of the statement-reason format in geometry textbooks now available.

Although Martin's work is obviously similar to the present study in its concern with the flow-diagram format and its use of the Watson-Glaser Critical Thinking Appraisal, procedural details are quite dissimilar.

The present research was limited to one school for girls. The teachers participating in Martin's study were teaching in various public schools in various localities. Hence they represented more varied socioeconomic environments and school structures. As a result, it could be argued that Martin's conclusions can be generalized to other groups of high school geometry students more readily.

Both researchers were well aware of teachers' unfamiliarity with the flow-diagram format and provided different means of overcoming this. Martin developed a manual which explained and exemplified flow-diagrams. Moreover, he consulted with teachers who had questions. The present researcher developed instead materials for the students and instructed teachers in the use of these materials. She was also available for discussion of problems that arose.
Martin focused on teacher attitudes toward the flow-diagram format, while the present study attempted to measure student attitudes. Unlike Martin's treatments, students were at least introduced to the format not used in their classes and were asked to make judgments as to comparative merits of both formats.

In order to maintain "purity" of treatment, Martin stipulated that only one format would be used in all of the classes from a given school which participated in his study. In this way, he intended to make sure that students would not have a chance to see the other format. However, the present writer feels that such is impossible. There is a built-in bias toward the statement-reason format in that it is the format "everyone else" used. It was not clear from Martin's description of the schools and selection of classes that when an experimental class was being taught in School X, there were no non-experimental geometry classes in School X. In fact, there may be hints that the opposite was true; for teachers were allowed to select one of their classes for participation in the experiment if they taught more than one geometry class. Certainly, if there were even one non-experimental geometry class in any such School X, his treatments were not kept pure. In such a case, the only experimental treatment would be disturbed by the presence of another format which
would make the treatments no longer equally pure. Moreover, it is the contention of the present writer that even if all geometry classes in an "experimental" school used the flow-diagram format exclusively, there is still inevitable contact with the statement-reason format. First of all, the statement-reason format is the format found in all texts, and most geometry students rely heavily on the text as a study guide. Also, any recourse to help from parents or older students immediately introduces "impurities" since the traditional format is the statement-reason format. In order to offset this inescapable contact with the statement-reason format by flow-diagram users, the present study employed a somewhat indirect control: all students knew from the first day of school that they were taking part in an experiment although they were never told exactly what was the point of the experiment. Obviously a number of them discovered it for themselves when other classes began to use a different proof format; but by then, the teachers felt that their discovery had no effect on the learning going on and definitely did not matter much to the students by the time they were tested in January. In fact, it is possible that some students having "figured out what was going on" made the measure of attitudes more reliable, in that what were reflected in student responses were not spontaneous reactions but the expressions of evaluations which had been being formulated for some time.
Without knowing it (Martin's work was not available when the present study was designed), the present study incorporated several of Martin's suggestions for future research. Notable was the need for text materials using the flow-diagram format. The blackening out of all proofs for both treatment groups and use of supplements containing the appropriate format approached as closely as practical the ideal of separate texts for each treatment. An added feature of the use of supplements containing the appropriate formats lies in the "instant experience" given to the teacher with regard to the writing of flow-diagrams. It is usually assumed that teachers can learn the material in the text at least quickly enough to "keep a page ahead of the class." Thus, student materials are indirectly instructional materials for teachers also.

Another recommendation by Martin dealt with testing "student attitudes toward geometry, proof, etc" (68, 71). The various parts of the Format Test did exactly that. However, the examination of format effects upon achievement was definitely limited in the present study. Among the recommendations to be made in the final chapter will be more explicit suggestions for developing tests that measure achievement in deductive thinking as well as knowledge of geometric "facts."
Chapter Summary

This chapter began by setting forth Ennis' definition of critical thinking and then explored aspects of the nature of deductive thinking which underlies all of critical thinking. Matulis, Roberge, Corley, Robinson, and O'Brien, Shapiro, and Reali were among those attempting to derive maturation patterns of logical thinking. O'Brien, Shapiro, and Reali showed that "child's logic" predominated over "math logic" in a substantial percentage of the population at each level tested. A few studies indicated that instruction in mathematical logic need not be presented formally. Almost every study showed that sex was not a significant factor in the development of deductive reasoning.

Techniques of making geometry proofs seem to be limited to the use of logically simple connective statements, in effect ignoring efforts by Lazar, Brockman, Allen, and other writers to acquaint teachers and publishers with advanced but usable complex statements.

Also restricted is the format for writing proofs. The statement-reason format replaced Euclid's paragraph format around 1926 and has maintained its almost exclusive dominance in texts since then. Despite a steady expression of teacher dissatisfaction with the statement-reason format since its introduction, alternatives to it were suggested.
only in the past two decades by a number of experienced geometry teachers. Earle's short-term research study indicated that format may be a significant factor in influencing learning. However, Arnsdorf and Martin failed to find support for this position.

The impact of geometry and critical thinking has been investigated within almost every ten year period since the 1920's. Early studies by Parker, Perry, Fawcett, Ulmer, Gadske, Henderson, and Cooke indicated that critical thinking abilities could be improved through materials specifically aimed at doing so. In 1950 Lewis confirmed this conclusion in a study using material similar to the Fawcett and related studies. Use of the resource units developed by Kimball in 1957 likewise produced significant gains. However, an experiment by Massimiano a year earlier than Kimball's showed again that a "conventional" geometry course did not have significant effect on critical thinking.

The last two studies dealing with geometry and critical thinking narrowed their focus to certain aspects of the geometry course. Platt found that formal instruction in logical concepts made no significant improvement in critical thinking. Finally, Martin concluded that proof format also was not influential at the .05 level of significance. Since Martin's study shares almost the same
purpose with the present study, similarities and differences between Martin's procedures and those of the present study were detailed.
CHAPTER III

RESEARCH DESIGN

Sample

The subjects in this experiment were those sophomores taking a geometry course at Mercy High School in Baltimore, Maryland, during the school year 1970-71. Mercy High School is a privately owned but church related (Roman Catholic) four-year high school located in a northern residential area just inside the Baltimore city limits. The basic curriculum at Mercy High School is two-pronged: college preparatory and secretarial. Geometry is a sophomore course, except for a small group of students who take Algebra I over a two-year period. This group then takes geometry as juniors. Students are grouped homogeneously for mathematics courses. This is done initially on the basis of scores on standardized IQ and achievement tests taken as eighth-graders. In the spring of each year, each class is regrouped on the basis of student performance in class.

The revision of freshmen grouping done in the spring of 1970 yielded eight groups for sophomore year. Seven of these took geometry and comprised the sample for the present study. The homogeneous grouping determined prior to
the beginning of the sophomore year was reflected in the labels for the classes used in this study. \( C_1 \) was the top ability group while \( C_7 \) ranked lowest. The top three classes used *School Mathematics Geometry* (4); the lowest four classes used *Geometry: A Contemporary Approach* (64).

Because of the policy of homogeneous grouping, assignment of classes to treatment in no way resembled a random process. Three of the geometry classes were assigned to Treatment G and four classes to Treatment X (treatments will be described later) in such a way as to offset initial differences between classes on the measures to be used as covariates, namely, IQ, mathematics achievement, and critical thinking ability. Mathematics achievement was measured by the mathematics subtest of the *National Educational Development Tests* (NEDT) (14) administered in October of the sophomore year. IQ scores came from administration of the *Lorge-Thorndike Intelligence Tests, Multi-Level Edition* (13) during the eighth grade. The *Watson-Glaser Critical Thinking Appraisal* (CTA1) (15) administered in September of the sophomore year measured critical thinking ability. Table 3 contains the mean scores of the treatment groups on the three covariates. Also included in Table 3, on the next page, are mean scores of the text groups on the covariates. Table 4, on the next page, details the results
TABLE 3--Means and Standard Deviations of Text and Treatment Groups on Covariates

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Group</th>
<th>Number</th>
<th>Mean</th>
<th>S D</th>
</tr>
</thead>
<tbody>
<tr>
<td>IQ Text</td>
<td>A</td>
<td>78</td>
<td>115.615</td>
<td>7.280</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>94</td>
<td>106.255</td>
<td>6.241</td>
</tr>
<tr>
<td>IQ Text</td>
<td>A</td>
<td>78</td>
<td>96.76</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>94</td>
<td>78.94</td>
<td></td>
</tr>
<tr>
<td>NEDT Text</td>
<td>A</td>
<td>78</td>
<td>79.667</td>
<td>15.734</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>94</td>
<td>58.851</td>
<td>17.208</td>
</tr>
<tr>
<td>NEDT Text</td>
<td>A</td>
<td>78</td>
<td>69.802</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>94</td>
<td>66.974</td>
<td></td>
</tr>
<tr>
<td>CTA1 Text</td>
<td>A</td>
<td>78</td>
<td>58.719</td>
<td>8.404</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>94</td>
<td>56.013</td>
<td>7.049</td>
</tr>
<tr>
<td>CTA1 Text</td>
<td>A</td>
<td>78</td>
<td>56.013</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>94</td>
<td>56.013</td>
<td></td>
</tr>
</tbody>
</table>

Table 4--Tests of Significance of Initial Differences Between Treatment and Text Groups on Covariates

<table>
<thead>
<tr>
<th>Effect</th>
<th>Variable</th>
<th>d</th>
<th>f</th>
<th>Mean Square</th>
<th>F</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>IQ, NEDT, CTA1</td>
<td>3, 166</td>
<td>--a</td>
<td>1.428</td>
<td>p &lt; .236</td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>IQ</td>
<td>1, 168</td>
<td></td>
<td>0.036</td>
<td>0.001</td>
<td>p &lt; .978</td>
</tr>
<tr>
<td></td>
<td>NEDT</td>
<td>1, 168</td>
<td></td>
<td>993.277</td>
<td>3.687</td>
<td>p &lt; .057</td>
</tr>
<tr>
<td></td>
<td>CTA1</td>
<td>1, 168</td>
<td></td>
<td>30.465</td>
<td>0.066</td>
<td>p &lt; .798</td>
</tr>
<tr>
<td>Treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>IQ, NEDT, CTA1</td>
<td>3, 166</td>
<td>--a</td>
<td>1.139</td>
<td>p &lt; .335</td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>IQ</td>
<td>1, 168</td>
<td></td>
<td>25.772</td>
<td>0.564</td>
<td>p &lt; .454</td>
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<tr>
<td></td>
<td>NEDT</td>
<td>1, 168</td>
<td></td>
<td>347.897</td>
<td>1.291</td>
<td>p &lt; .257</td>
</tr>
<tr>
<td></td>
<td>CTA1</td>
<td>1, 168</td>
<td></td>
<td>250.596</td>
<td>0.543</td>
<td>p &lt; .462</td>
</tr>
<tr>
<td>Text</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>IQ, NEDT, CTA1</td>
<td>3, 166</td>
<td>--a</td>
<td>35.148</td>
<td>p &lt; .001</td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>IQ</td>
<td>1, 168</td>
<td></td>
<td>3361.975</td>
<td>73.577</td>
<td>p &lt; .001</td>
</tr>
<tr>
<td></td>
<td>NEDT</td>
<td>1, 168</td>
<td></td>
<td>18581.980</td>
<td>68.974</td>
<td>p &lt; .001</td>
</tr>
<tr>
<td></td>
<td>CTA1</td>
<td>1, 168</td>
<td></td>
<td>14985.215</td>
<td>32.497</td>
<td>p &lt; .001</td>
</tr>
</tbody>
</table>

*Not printed by the computer program used.
of tests of significance of differences between the mean scores, computed by means of a multivariate analysis of variance.

The reason for using a multivariate analysis of variance instead of a series of univariate tests is to insure that none of the univariate tests computed as part of the multivariate analysis are spuriously high. Stipulating the .05 level of significance means that the experimenter is willing to risk incorrectly rejecting a hypothesis on an average of one time in twenty. The more tests of significance at the .05 level carried out during the same study, the greater is the probability that a significant difference reported was due to purely random factors. The multivariate tests of significance involve statistical procedures to indicate when the significance of the univariate tests can be explained by random factors rather than by the factors under study. The results of the multivariate tests in Table 4 indicate that there is little likelihood that the results of the univariate tests also in Table 4 are spuriously high due to random factors.

In each geometry class there were a number of students lacking some of the necessary data due to absences. The original numbers and the numbers used in the study are given in Table 5. No scores of such students were included in any data reported in this study, with the exception of
Table 5 in which is listed the original number in each treatment-text cell along with the number actually used in the experiment. It was assumed that these omissions did not seriously bias the sample.

TABLE 5—Comparison of Original Numbers of Students in Treatment-Text Cells with Numbers Participating in the Experiment

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Text</th>
<th>Original Number</th>
<th>Number Used</th>
<th>Per Cent Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>A</td>
<td>59</td>
<td>54</td>
<td>8.5</td>
</tr>
<tr>
<td>X</td>
<td>B</td>
<td>56</td>
<td>42</td>
<td>25.0</td>
</tr>
<tr>
<td>G</td>
<td>A</td>
<td>27</td>
<td>24</td>
<td>12.5</td>
</tr>
<tr>
<td>G</td>
<td>B</td>
<td>59</td>
<td>54</td>
<td>8.5</td>
</tr>
</tbody>
</table>

It should be noted, however, that the relatively large percentages of loss in cells XB and GA correspond to the three classes of a single teacher, the researcher. This may point to some undefined teacher-class interactions. Or it may be simply due to the practice of placing into the lowest-ranked class those transfer students whose incomplete records indicate as "probably college capable," a policy used in the placing of at least four of the ten students "lost" from the single class C7.

Treatment

Due to restrictions in the scheduling of teachers and students, teachers could not be randomly assigned to
classes. Of the five geometry teachers, only three were available to teach experimental groups by decision of the department chairman. The researcher was assigned to teach three classes. Four other teachers taught the remaining four classes. Therefore assignment of treatment to classes was based on teacher considerations and on class rankings. Three of the classes were assigned to group G and four classes to group X. The results of assignments to treatment were depicted in Table 1, p. 11. The third column in Table 1 indicates that classes C₁, C₂, and C₃ used one text entitled School Mathematics Geometry (4) (hereafter referred to as Text A) while C₄ through C₇ used Geometry: A Contemporary Course (64) (hereafter referred to as Text B).

Treatment X consisted basically of using the flow-diagram format for writing proofs in all class work and testing during the first semester (September through December). Those parts of each text to be covered during the first semester which illustrated or explained proofs were blackened out and supplements were distributed in which the proof or explanation was in the flow-diagram format. Treatment G consisted basically of using the statement-reason format for writing proofs in all class work and testing during the same time period as the experimental groups. In order to minimize effects due to difference in
text materials other than the format, relevant portions of each text for students in Treatment G were also blackened out and supplements provided, in this case written in the statement-reason format. Copies of supplements for each text in both formats are included in Appendix A. A few minor changes were also made to insure that the details of each proof differed only in format and not in completeness. There was no need to change the format of chapter tests for either text. Chapter tests were administered whenever the teacher deemed appropriate for her class. Since teachers of C1, C2, and C3 administered exactly the same chapter tests, it was possible to derive a geometry achievement score for each subject in those classes. Similar achievement scores were not available for all of the subjects using Text B due to teacher dissatisfaction with the printed tests. At various times during the year, teachers had adapted test items from the printed tests as well as added original items for their classes. Thus the number of items used by all teachers of Text B was too small to permit analysis of achievement.

Besides using the assigned proof format, subjects took the Critical Thinking Appraisal twice more after September, once in January and again in June. They also responded in January to a specially designed text called the Format Test, to be described in the next section. During the Format Test, each subject was instructed in the alternate format
and could then use either format in classwork and testing from that day on.

**Instrumentation**

Subjects' IQ scores were taken from school records and came from a testing during eighth grade. The test used was the *Lorge-Thorndike Intelligence Tests, Multi-level Edition* (13). The *National Education Development Tests* (14) are given nationwide to high school freshmen and sophomores, usually in October of each school year. Subject scores designated NEDT in this study were the national percentile scores for Grade 10 on the mathematics subtest on the five-test battery administered in October 1970. The mathematics subtest scores were used instead of overall scores so that subjects' backgrounds specifically in mathematics could be used as a covariate along with general ability as measured by IQ scores. The third measure used as a covariate was the total score on the September administration of the *Watson-Glaser Critical Thinking Appraisal*, Form Zm (15).

The *Watson-Glaser Critical Thinking Appraisal* was developed over a period of years (1937-1964) to measure five capabilities collectively described as "Critical Thinking": inference, recognition of assumptions, deduction, interpretation of data, and evaluation of arguments. Five subtests were developed to measure these capabilities, then validated and standardized for various age groups. Hence
the Critical Thinking Appraisal yields five subscores and one combined "total" score. Two forms of the Critical Thinking Appraisal are presently available, Form Zm and Ym. The two forms were made equivalent by means of the procedure of equi-percentile equating. This means, in particular, that the test developers would state that a score of x% on one form is equivalent to a score of x% on the other form. Form Zm was given in September and Form Ym in January and June.

The chapter tests accompanying Text A (89) measured knowledge of geometric facts and concepts. A cumulative score on the tests for Chapters 1 through 6 is referred to hereafter as an achievement score. The tests included items identified by the researcher as measuring some aspects of the ability to write and understand deductive proofs. Tabulations of responses to these items yielded a subscore called achievement-in-proofs.

The Format Test, an untimed test developed by the researcher, has two parts and two forms. Part I Form G introduces the flow-diagram format to subjects who have been using the statement-reason format, then requires the subjects to change two completed proofs from the statement-reason format to the flow-diagram format and to construct two other proofs in the flow-diagram format given only the Hypothesis, Conclusion and Figure. Part I Form X introduces
the statement-reason format to subjects familiar with the 
flow-diagram format and requires completion of two changes 
of format and two complete proofs as in Form G, but this 
time writing the proofs in the statement-reason format. 
Scoring keys were made prior to the actual testing in order 
to insure that credit would be given for partial answers 
and that answers would be scored independent of the format 
used.

Scores obtained from Part I measure subjects' ability 
to adapt to the unfamiliar format. Part II consists of a 
questionnaire of 60 items, 18 measuring subjects' attitude 
toward geometry, 21 measuring subjects' attitude toward 
writing proof, and 21 measuring subjects' attitude toward 
and preference for the flow-diagram format in contrast to 
the statement-reason format. Responses are on a 5-point 
agree-disagree scale. In Form G, the familiar format is the 
flow-diagram format. The Format Test as a whole yields four 
subscores and no total score. The subscores are: attitude 
toward geometry, attitude toward proof, attitude to the 
flow-diagram format, and adaptability to the unfamiliar 
format. Appendix B contains both forms of the complete 
Format Test.

The content validity of the Format Test was estab-
lished by the following procedures. Each item in Part II 
was included only if 8 out of 10 experienced mathematics
teachers judged that it did measure the indicated attitude. From an original pool of 75 items, 60 items satisfied the above criterion for inclusion in the test. The instructions in Part I introducing the new format were critiqued by four experienced geometry teachers before the test was finalized.

As a further check on the validity of individual items, coefficients indicating item test reliability were computed for each of the items on each of the attitude subtests of Part II of the Format Test by means of the Kuder-Richardson Formula 8 for scaled data. For the computation of all such reliability coefficients, the two forms of Part II were combined because they were really the same test. All of the items not identical on the two forms were different only in the phrasing of statements comparing the two proof formats. This involved exactly 16 of the 21 items on the "attitude toward flow-diagrams" subtest. However, the method of scoring these items rendered them equivalent. For example, Item 4 was the statement:

Form G: If I had more experience I would probably find the flow-diagram format easier than the statement-reason format.

Form F: If I had more experience I would probably find the statement-reason format easier than the flow-diagram format.

The scoring of Item 4 gave the same number of points for disagreement with the Form G statement as for the same
degree of agreement with the Form X statement. In general, responses to every item in the subtest measuring "attitude toward flow-diagrams" were scored so that the maximum number of points were assigned to the response most favorable to the flow-diagram format.

Results of the reliability coefficients obtained in the manner just described may be found in Appendix C. The ranges of individual item-test reliability coefficients were from -.060 to .806 for the "attitude toward geometry" subtest, from .303 to .808 for the "attitude toward proof" subtest, and from .414 to .921 for the "attitude toward flow-diagrams" subtest. Only two items (numbers 6 and 12, both included in the "attitude toward geometry" subtest) seemed to the researcher that they may have measured some quality other than the one indicated by the name of the subtest because of their low coefficients.

To determine reliability of the subtests of Part II, Kuder-Richardson 8 (KR8) coefficients were computed for each subtest. This formula, adapted to scaled data from that given for dichotomous data in the primary reference (60), was judged most suitable for it assumes only that the items test the same factor and that the sample is drawn randomly from the population. The professional judgment of the experienced teachers was accepted as evidence that the first
assumption was adequately satisfied. There was no attempt to satisfy the requirement of randomization, since the students were all girls in a private Catholic school.

Reliability coefficients such as the Kuder-Richardson 20 (60), or the Cronbach $\alpha$ (26) are probably more familiar than the Kuder-Richardson 8 to readers of educational research. But the calculations for the KR 20 assume equal inter-item correlations and equal item variance in addition to the assumptions for the KR 8. Cronbach's $\alpha$ is equivalent to KR 20 except that it is explicitly for scaled data whereas all of the original KR formulas deal with dichotomous data and must be adjusted for scaled data. It could not be assumed prior to testing that the Format Test data would satisfy the additional assumptions for these more familiar coefficients. Post facto calculations supported the conclusion that KR 8 coefficients were most valid as measures of the reliability of the subtests of the Format Test.

Calculations yielded coefficients of .904, .877, and .970 for the subtests measuring attitude toward proof, attitude toward geometry, and attitude toward flow-diagrams, respectively. The KR Formula 8 yields a coefficient larger than those obtained by Formula 20 (and the Cronbach equation). So the coefficients listed above are more overestimates than underestimates of the true subtest.
reliabilities. As a means of judging the extent of overestimation, Cronbach $\alpha$ coefficients were computed and are listed in Table 6 along with the KR 8 coefficients. However, it must be kept in mind that these are for comparison purposes exclusively and that only the KR 8 coefficients are valid estimates of reliability in this study. The results in the table indicate that the three subtests are sufficiently reliable for the conclusions made in Chapter V.

Anecdotal data were collected in addition to numerical data. Teachers and students were interviewed by the researcher as to the benefits and limitations of the flow-diagram format.

### Methods of Analysis

In order to determine the effect of format for geometry proofs on students' critical thinking and on several attitudes pertinent to the geometry course, it was necessary to employ methods of analyses that took into consideration not only the difference in format, but the differences deriving from other factors which were not controlled.
because of the sampling procedures used. The homogeneous grouping procedure used for geometry classes at Mercy High School actually did produce groups which were significantly different with respect to the three variables of IQ, mathematics NEDT, and critical thinking. All of these three variables were considered crucial factors influencing performance on each of the criterion measures employed in this experiment. Clearly the differences due to mathematics achievement (NEDT) and IQ needed to be controlled in some way since they determined to a great extent the initial differences in the groups (they provided the major objective basis for the homogeneous grouping procedures).

Using these two measures as covariates was a statistical method of controlling them. Since one of the objectives of the present study was to determine the increase in critical thinking attributable to different formats for geometry proofs, the September Critical Thinking Appraisal served as a pretest for each of the later administrations of the Critical Thinking Appraisal.

Use of the same covariates in the analysis of attitudes seemed appropriate, for a person's attitude toward some activity often correlates highly with the ease with which he can perform the activity. However, the hypotheses dealing with attitudes sought to determine the effect of format and not of the factors deriving from greater ability.
Not only were there initial differences between groups relative to IQ, mathematics achievement, and critical thinking ability, but two different texts were used. A few indications have been noted in the literature survey that differences in physical materials could affect performance in class. Hence the basic procedure used to analyze the data was a $2 \times 2$ multivariate analysis of variance with the three covariates of IQ score, mathematics NEDT score, and September Critical Thinking Appraisal (CTA1) score. There were two factors, text and treatment, and two levels to each factor.

Eight criterion measures had been determined: January CTA, June CTA, attitude toward geometry, attitude toward proof, attitude toward flow-diagrams, flexibility, achievement in geometry, and achievement in proof. The two achievement measures were based on chapter tests accompanying the texts; and, as it has been explained above in Chapter III, there were achievement scores only for the groups using Text A. Analysis of covariance including the treatment factor and omitting the text factor was employed to test the two hypotheses involving achievement.

**Overview of the Next Chapter**

In the next chapter, the results of the analyses are discussed. The over-all tests of significance are detailed first for the set of six criterion variables analyzed by
means of the analysis of covariance, then applied to the first six hypotheses. The change to analysis of variance for the testing of the seventh hypothesis is justified by a more detailed discussion of the originally planned analysis of covariance. Then the results of an analysis of variance are applied to testing the seventh hypothesis. After this are given the results of an analysis of covariance for the achievement data and the tests of the final two hypotheses.

Additional analysis seemed warranted for several of the criterion variables, adding information to the results of the tests of hypotheses. These are presented in the section beginning with a summary of the tests of hypotheses.

Pairwise correlations were computed for the data from all criterion measures available for the complete sample. An appropriate statistical table (43, 580) was used to determine which of these correlations were significant. The fifth section of the next chapter contains the correlations, results of the tests of significance, and some observations based on these figures.

Another point of interest to this experimenter was the effect of text on performance. Although the interest was not formalized into research hypotheses, it was felt that the classes using Text B would do as well on the criterion measures as the classes using Text A once initial
differences on the three covariates were controlled statistically. The sixth section of Chapter IV focuses on the effects of text.

The final section in the next chapter compiles remarks from teachers and students concerning their reactions to the flow-diagram format. There was no attempt to qualify or analyze these comments beyond categorizing and summarizing them.
CHAPTER IV

RESULTS

Tests of the Hypotheses $H_1$ through $H_6$

The first seven hypotheses were tested by means of a $2 \times 2$ multivariate analysis of covariance with six criterion variables and 3 covariate variables. The two treatments were designated by X (using the flow-diagram format) and G (using the statement-reason proof format). The two texts were identified as A (School Mathematics Geometry) (4) and B (Geometry: A Contemporary Approach) (64). The resulting four text-treatment groups were not equivalent with respect to mean scores on the covariate variables of IQ, mathematics NEDT, and September CTA.

The nonequivalence of groups on these three measures was confirmed by means of analysis of variance procedures. Table 3 on page 64 gives the mean score and standard deviation of each group for each covariate, while Table 4 on page 64 details the tests of significance of initial differences. Cell means and standard deviations for text-treatment interactions are listed in Table 7 on the next page. None of the interactions in Table 4 were significant at the .05 level, although the $p$ value for NEDT ($p < .057$)
TABLE 7—Mean Scores of Text-Treatment Cells on Covariates.

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Treatment</th>
<th>Text</th>
<th>Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>A</td>
<td>114.000</td>
<td>15.594</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>106.714</td>
<td>7.562</td>
</tr>
<tr>
<td>IQ</td>
<td>G</td>
<td>A</td>
<td>109.708</td>
<td>20.601</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>105.885</td>
<td>4.973</td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>A</td>
<td>80.574</td>
<td>14.483</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>55.952</td>
<td>17.423</td>
</tr>
<tr>
<td>NEDT</td>
<td>G</td>
<td>A</td>
<td>77.625</td>
<td>18.417</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>62.058</td>
<td>14.940</td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>A</td>
<td>68.667</td>
<td>17.481</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>48.595</td>
<td>24.863</td>
</tr>
<tr>
<td>Sept. CTA</td>
<td>G</td>
<td>A</td>
<td>65.125</td>
<td>21.658</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B</td>
<td>46.827</td>
<td>22.202</td>
</tr>
</tbody>
</table>

approached the stipulated .05 level. Likewise, the treatment groups were not significantly different on the three measures. The same was not true when the students were grouped by text. The students using Text A scored significantly higher than the students using Text B on each of the variables. This result was definitely expected, since Text A was used by the three classes ranked highest on the homogeneous grouping scale. The experimenter concluded from these tests of significance that the covariates should be used in order to control for effects due to these initial differences between text-treatment groups.
Use of the procedure of multivariate analysis of covariance implies that the assumptions underlying the procedure have been satisfied by the data being analyzed. Failure to have normality of criteria factors in the population or to have randomness of selection of the sample from the population is not considered sufficient reason to avoid using the procedure, since results are not usually significantly affected by such a failure. However, there are assumptions which must be satisfied before the results are considered valid. Two assumptions vital to the use of covariates are homogeneity of regression and significance of regression within cells. A test for heterogeneity of regression resulting in a level of significance below the stipulated level indicates that the covariates are having different effects on the criterion variables from cell to cell, and hence their use is invalid. If the result of tests of regression within cells does not fall below the stipulated level of significance, there is no significant adjustment due to the covariates and they need not be used. The need for and interpretation of the multivariate test of significance have already been discussed on page . Calculations aimed at determining whether or not the assumptions are satisfied were part of the computer program for the multivariate analysis of covariance used throughout this study (21) with the test for heterogeneity
of regression identified in the computer program as the effect of equality of regression. Results of these calculations are included in each table of results throughout the remainder of this chapter. Not only does the reporting of these results aid in the correct interpretation of the univariate tests of significance of the criterion variables, but they help to explain the necessity of changing statistical procedures in testing one of the hypotheses.

Data for six of the eight criterion variables had been collected for the entire sample of 172 subjects. Table 8 on the next page contains both unadjusted means with corresponding standard deviations and adjusted means for each text-treatment cell. Henceforth abbreviations will be used to identify these variables according to the following list: January CTA (CTA2), June CTA (CTA3) attitude toward geometry (AttG), attitude toward proof (AttP), attitude toward flow-diagrams (AttFD), and flexibility (Flex). Use of three covariates—IQ, NEDT, and September CTA (CTA1)—had been justified by the significance of initial differences between text groups. Thus multivariate analysis of covariance was employed to test the significance of differences on the criterion measures. Results of the calculations to check the assumptions are given in Table 9. The multivariate test of significance showed that regression occurred equally in all cells for the overall
<table>
<thead>
<tr>
<th>Variable</th>
<th>Text</th>
<th>Treatment</th>
<th>Unadjusted Mean</th>
<th>S.D.</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude toward Geometry</td>
<td>A</td>
<td>X</td>
<td>38.833</td>
<td>13.067</td>
<td>36.625</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>37.917</td>
<td>9.136</td>
<td>36.479</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>X</td>
<td>38.595</td>
<td>12.113</td>
<td>40.457</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>39.288</td>
<td>12.495</td>
<td>40.741</td>
</tr>
<tr>
<td>Attitude toward Proofs</td>
<td>A</td>
<td>X</td>
<td>35.907</td>
<td>16.292</td>
<td>32.885</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>38.667</td>
<td>10.107</td>
<td>36.960</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>X</td>
<td>38.714</td>
<td>14.007</td>
<td>41.170</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>41.038</td>
<td>13.868</td>
<td>42.982</td>
</tr>
<tr>
<td>Attitude toward Flow-Diagrams</td>
<td>A</td>
<td>X</td>
<td>46.093</td>
<td>21.221</td>
<td>44.847</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>20.875</td>
<td>20.066</td>
<td>20.635</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>X</td>
<td>43.976</td>
<td>22.858</td>
<td>44.481</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>19.365</td>
<td>14.666</td>
<td>20.361</td>
</tr>
<tr>
<td>Flexibility</td>
<td>A</td>
<td>X</td>
<td>56.926</td>
<td>19.022</td>
<td>50.771</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>57.125</td>
<td>13.989</td>
<td>53.258</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>X</td>
<td>48.238</td>
<td>16.531</td>
<td>53.115</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>37.519</td>
<td>13.729</td>
<td>41.757</td>
</tr>
<tr>
<td>January CTA</td>
<td>A</td>
<td>X</td>
<td>70.481</td>
<td>7.195</td>
<td>66.243</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>71.500</td>
<td>7.746</td>
<td>69.145</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>X</td>
<td>61.095</td>
<td>10.436</td>
<td>64.233</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>62.385</td>
<td>8.536</td>
<td>65.338</td>
</tr>
<tr>
<td>June CTA</td>
<td>A</td>
<td>X</td>
<td>70.426</td>
<td>9.283</td>
<td>66.753</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>73.583</td>
<td>7.360</td>
<td>71.601</td>
</tr>
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<td></td>
<td>B</td>
<td>X</td>
<td>63.095</td>
<td>9.294</td>
<td>65.607</td>
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<td></td>
<td>G</td>
<td>62.404</td>
<td>8.786</td>
<td>65.104</td>
</tr>
</tbody>
</table>
multivariate analysis of covariance. But univariate tests of significance justified the same assumption for only five of the six criterion variables considered individually, for the slopes of the regression lines were significantly different ($p < .020$) when the variable AttFD was taken alone. A univariate test of within cells regression showed that the covariates were not significant influences ($p < .349$) on AttFD scores. Since the limited exposure of one treatment group to flow-diagrams was expected to create a large differential between mean scores of the groups, it was not surprising that use of the covariates was inappro-
appropriate. The methods used to analyze these scores are de-tailed later, in the discussion of the test of the seventh hypothesis.

A second multivariate analysis of covariance was computed for the five criterion variables for which use of the covariates had been justified. Results of these calculations comprise Table 10 on the following page. The tests of regression in all cells and regression within cells were optional at this point, since results from the first multivariate analysis of covariance indicated that the assumptions were satisfied for the set of five criterion variables. But a test of regression within cells was computed anyway, to serve as a doublecheck of the assumptions as well as an indicator, to some extent, of how the figures might change. Thus, the tests of equality of regression in all cells included in Table 10 are from the first multivariate analysis of covariance while the tests of regression within cells are from the second multivariate analysis of covariance.

Results from the univariate tests of the effect of treatment given in Table 10 were used to test the following hypotheses:

\( H_1 \): There is no significant text-treatment interaction on any of the following criterion variables; January CTA, June CTA, attitude toward geometry, attitude toward proof, attitude toward flow-diagrams, and flexibility.
Table 10.—Tests of Significance of Five Criterion Variables Using Three Covariates (N = 172)

<table>
<thead>
<tr>
<th>Effect</th>
<th>Variable</th>
<th>d</th>
<th>f</th>
<th>Mean Square</th>
<th>F</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equality of Regression</td>
<td>AttFD &amp; all five</td>
<td>54</td>
<td>775</td>
<td>1.062 p &lt; .359</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AttG</td>
<td>9</td>
<td>156</td>
<td>1.120 p &lt; .352</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AttP</td>
<td>9</td>
<td>156</td>
<td>1.086 p &lt; .376</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flex</td>
<td>9</td>
<td>156</td>
<td>1.602 p &lt; .119</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA2</td>
<td>9</td>
<td>156</td>
<td>0.951 p &lt; .483</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA3</td>
<td>9</td>
<td>156</td>
<td>0.720 p &lt; .690</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regression Within Cells</td>
<td>all five</td>
<td>15</td>
<td>445</td>
<td>9.779 p &lt; .001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AttG</td>
<td>3</td>
<td>165</td>
<td>2.702 p &lt; .047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AttP</td>
<td>3</td>
<td>165</td>
<td>3.033 p &lt; .031</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flex</td>
<td>3</td>
<td>165</td>
<td>15.698 p &lt; .001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA2</td>
<td>3</td>
<td>165</td>
<td>29.790 p &lt; .001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA3</td>
<td>3</td>
<td>165</td>
<td>32.127 p &lt; .001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interaction</td>
<td>all five</td>
<td>5</td>
<td>161</td>
<td>2.164 p &lt; .061</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AttG</td>
<td>1</td>
<td>165</td>
<td>0.023 p &lt; .879</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AttP</td>
<td>1</td>
<td>165</td>
<td>0.177 p &lt; .675</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flex</td>
<td>1</td>
<td>165</td>
<td>15.698 p &lt; .001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA2</td>
<td>1</td>
<td>165</td>
<td>29.790 p &lt; .001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA3</td>
<td>1</td>
<td>165</td>
<td>32.127 p &lt; .001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>all five</td>
<td>5</td>
<td>161</td>
<td>3.324 p &lt; .007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AttG</td>
<td>1</td>
<td>165</td>
<td>0.015 p &lt; .904</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AttP</td>
<td>1</td>
<td>165</td>
<td>1.284 p &lt; .259</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flex</td>
<td>1</td>
<td>165</td>
<td>6.064 p &lt; .013</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA2</td>
<td>1</td>
<td>165</td>
<td>3.663 p &lt; .1744</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA3</td>
<td>1</td>
<td>165</td>
<td>3.663 p &lt; .174</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Text</td>
<td>all five</td>
<td>5</td>
<td>161</td>
<td>3.285 p &lt; .007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AttG</td>
<td>1</td>
<td>165</td>
<td>2.534 p &lt; .113</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AttP</td>
<td>1</td>
<td>165</td>
<td>9.381 p &lt; .003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flex</td>
<td>1</td>
<td>165</td>
<td>1.567 p &lt; .212</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA2</td>
<td>1</td>
<td>165</td>
<td>0.992 p &lt; .321</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTA3</td>
<td>1</td>
<td>165</td>
<td>0.112 p &lt; .716</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Not printed by the computer program used.

^Not considered significant because the multivariate test is not significant.
H₂: There is no significant difference between mean scores of X and G on the January CTA.

H₃: There is no significant difference between mean scores of X and G on the June CTA.

H₄: There is no significant difference between mean scores of X and G on the ability to change to a new proof format, as measured by the "flexibility" subtest of the Format Test.

H₅: There is no significant difference between mean scores of X and G indicating their attitude toward geometry, as measured by the "attitude toward geometry" subtest of the Format Test.

H₆: There is no significant difference between mean scores of X and G indicating their attitude toward proof, as measured by the "attitude toward proof" subtest of the Format Test.

Only one of these, H₄, was rejected at the .05 level. The users of the flow-diagram format were significantly better able to change to the unfamiliar format than the users of the statement-reason format. This result was no surprise, due to the experimenter's classroom experience in teaching both formats prior to the experiment.

An unexpected result was the apparent favoring of the statement-reason format on CTA3 scores. Although the
level of significance did not quite reach the stipulated level of .05, it was close enough \( (p < .058) \) to suggest that further investigation is desirable.

**Test of the Hypothesis \( H_7 \)**

\( H_7 \): There is no significant difference between mean scores of \( X \) and \( G \) indicating their attitude toward flow-diagrams, as measured by the "attitude toward flow-diagrams" subtest of the **Format Test**.

When analysis of covariance techniques were used to analyze the data for this hypothesis, a fundamental assumption was not satisfied. The differences in regression in all cells were significant \( (p < .020) \), thus indicating that analysis of covariance was not appropriate. However, the test of regression within cells \( (p < .349) \) showed that the covariates did not significantly influence the scores measuring attitude toward flow-diagrams. These calculations justified the change to analysis of variance. Results are given in Table 11.

**TABLE 11—Tests of Significance of Attitude toward Flow-Diagrams (Unadjusted Scores; \( N = 172 \))**

<table>
<thead>
<tr>
<th>Effect</th>
<th>Mean Square</th>
<th>( F (1, 168) )</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>24635.168</td>
<td>63.167</td>
<td>( p &lt; .001 )</td>
</tr>
<tr>
<td>Text</td>
<td>2708.836</td>
<td>6.946</td>
<td>( p &lt; .009 )</td>
</tr>
<tr>
<td>Interaction</td>
<td>3.566</td>
<td>0.009</td>
<td>( p &lt; .924 )</td>
</tr>
</tbody>
</table>
The outcome of the test of significance of the effect of treatment was definitely expected, for at the end of the year each teacher participating in the study had commented on the preference by students in her class for the proof format they had used. The computed $F$ ratio of 63.167 shows that the students using the flow-diagram format had a much more positive attitude toward flow-diagrams than the students using the statement-reason format.

Tests of the Hypotheses $H_8$ and $H_9$

Since data on achievement—both general achievement in geometry (AchG) and achievement in dealing with proofs (AchP)—were not available for the classes using Geometry: A Contemporary Approach, there was no text factor in the analysis procedures employed to test the last two formal hypotheses:

$H_8$: There is no significant difference between those classes of $X$ and $G$ which used the text School Mathematics Geometry on achievement in geometry, as measured by items on chapter tests accompanying this text.

$H_9$: There is no significant difference between those classes of $X$ and $G$ which used the text School Mathematics Geometry on those items of chapter tests which measured certain aspects of the ability to understand and construct deductive proofs.
It was assumed that a single-factor multivariate analysis of covariance would be appropriate using the same covariates of IQ, NEDT, and CTA1 that were used in the tests of the first six hypotheses. The results are somewhat anomalous (see Table 12 on the next page). Neither of the univariate tests of significance indicated that the treatment groups were different for reasons other than randomness of measurement. However the situation in changed when the measures are considered together. The multivariate test of significance indicates that the treatment groups were significantly different (p < .022) when both achievement in geometry and achievement in proof were considered.

Figure 6 is a scattergram illustrating the ordered pairs (achievement in geometry, achievement in proof) with those for X indicated by solid dots and those for G by open circles. The scattergram suggests a reason why the multivariate test of significance had statistically significant results while neither univariate test had statistically significant results. Approximately 80 per cent of the points for G fall in a rectangular region with "corner" points (170, 46), (170, 38), (190, 38), and (190, 46). No rectangle congruent to this will contain an equivalent percentage of points of X, for they are more spread out over the plane. One of the regions with a large percentage of points of X is that determined by the "corner" points
TABLE 12.—Tests of Significance Between Treatment Groups Involving Two
Achievement Criteria Variables and Three Covariates (N = 78)

<table>
<thead>
<tr>
<th>Effect</th>
<th>Variable</th>
<th>d</th>
<th>f</th>
<th>Mean Square</th>
<th>F</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equality of Regression</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AchG, AchP</td>
<td>6, 138</td>
<td>--a</td>
<td>1.987</td>
<td>p &lt; .072</td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AchG</td>
<td>3, 70</td>
<td>444.338</td>
<td>1.953</td>
<td>p &lt; .129</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AchP</td>
<td>3, 70</td>
<td>32.601</td>
<td>2.134</td>
<td>p &lt; .104</td>
<td></td>
</tr>
<tr>
<td>Regression Within Cells</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AchG, AchP</td>
<td>6, 144</td>
<td>--a</td>
<td>5.044</td>
<td>p &lt; .001</td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AchG</td>
<td>3, 73</td>
<td>2483.530</td>
<td>10.506</td>
<td>p &lt; .001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AchP</td>
<td>3, 73</td>
<td>103.712</td>
<td>6.486</td>
<td>p &lt; .001</td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multivariate Test</td>
<td>AchG, AchP</td>
<td>2, 72</td>
<td>--a</td>
<td>4.041</td>
<td>p &lt; .022</td>
<td></td>
</tr>
<tr>
<td>Univariate Tests</td>
<td>AchG</td>
<td>1, 73</td>
<td>390.480</td>
<td>1.652</td>
<td>p &lt; .203</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AchP</td>
<td>1, 73</td>
<td>7.202</td>
<td>0.450</td>
<td>p &lt; .504</td>
<td></td>
</tr>
</tbody>
</table>

*aNot printed by the computer program used.*
Achievement in Proof

Figure 6. Scattergram of Achievement-in-Geometry and Achievement-in-Proof Scores.
(180, 48), (180, 42), (200, 42), and (200, 48); 23 of the 56 points of X lie in this rectangular region. It would seem that the points for X clustered in a region significantly "higher" than those for G, even though the greater dispersal of points for X made the individual means close enough to the means for G so that differences were not significant.

Summary of the Tests of Significance with Comments

The results of the statistical procedures justified the rejection of two of the nine hypotheses. Table 13 summarizes the hypotheses and the results. It was concluded that the classes using the flow-diagram format were significantly better able to change to the unfamiliar format than those using the statement-reason format, and also had significantly more positive attitudes toward flow-diagrams. While the remaining tests of significance would not justify rejecting hypotheses, comments on a few of these tests are in order.

The failure of treatment to effect significant differences on the measures of critical thinking would have been meaningless if there had been no over-all gains in critical thinking. However, as the figures in Tables 14 show, the sample had significantly improved in critical thinking from September to January, with insignificant improvement occurring from January to June. Since the
### TABLE 13—Summary of the Hypotheses

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Criterion Variable</th>
<th>Direction of Observed Means</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>Text-Treatment Interaction</td>
<td></td>
<td>Multivariate test not significant</td>
</tr>
<tr>
<td></td>
<td>AttG(^a)</td>
<td>GB &gt; XB &gt; XA &gt; GA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AttP(^a)</td>
<td>GB &gt; XB &gt; GA &gt; XA</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AttFD(^b)</td>
<td>XA &gt; XB &gt; GA &gt; GB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Flex(^a)</td>
<td>GA &gt; XB &gt; XA &gt; GB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CTA2(^a)</td>
<td>GA &gt; XA &gt; GB &gt; XB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CTA3(^a)</td>
<td>GA &gt; XA &gt; XB &gt; GA</td>
<td></td>
</tr>
<tr>
<td>H2</td>
<td>CTA2(^a)</td>
<td>G &gt; X</td>
<td>Not significant</td>
</tr>
<tr>
<td>H3</td>
<td>CTA3(^a)</td>
<td>G &gt; X</td>
<td>Not significant</td>
</tr>
<tr>
<td>H4</td>
<td>Flex(^a)</td>
<td>X &gt; G</td>
<td>p &lt; .015</td>
</tr>
<tr>
<td>H5</td>
<td>AttG(^a)</td>
<td>G &gt; X</td>
<td>Not significant</td>
</tr>
<tr>
<td>H6</td>
<td>AttP(^a)</td>
<td>G &gt; X</td>
<td>Not significant</td>
</tr>
<tr>
<td>H7</td>
<td>AttFD(^b)</td>
<td>X &gt; G</td>
<td>p &lt; .001</td>
</tr>
<tr>
<td>H8</td>
<td>AchG(^a)</td>
<td>G &gt; X</td>
<td>Not significant</td>
</tr>
<tr>
<td>H9</td>
<td>AchP(^a)</td>
<td>X &gt; G</td>
<td>Not significant</td>
</tr>
</tbody>
</table>

\(^a\)Scores adjusted for three covariates.

\(^b\)Scores not adjusted for three covariates.
TABLE 14—Tests of Significance of Improvement in Critical Thinking by the Entire Sample (N = 172)

<table>
<thead>
<tr>
<th>Variable 1</th>
<th>Mean</th>
<th>Variable 2</th>
<th>Mean</th>
<th>Student t</th>
<th>Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTA1</td>
<td>57.523</td>
<td>CTA2</td>
<td>65.884</td>
<td>9.818</td>
<td>p &lt; .001</td>
</tr>
<tr>
<td>CTA1</td>
<td>57.523</td>
<td>CTA3</td>
<td>66.651</td>
<td>10.025</td>
<td>p &lt; .001</td>
</tr>
<tr>
<td>CTA2</td>
<td>65.884</td>
<td>CTA3</td>
<td>66.651</td>
<td>1.216</td>
<td>p &lt; .226</td>
</tr>
</tbody>
</table>

First semester of the geometry course was the period of the greatest gains in critical thinking ability and also the period of the treatment in its "pure" form, it was thought that if treatment were to have any impact on critical thinking, it would appear in the CTA2 scores. Thus the lack of significance of difference in these scores is quite significant.

Another variable besides CTA2 and CTA3 deserving additional attention was attitude toward flow-diagrams. It has already been noted that each group exhibited a significant preference for the format it used. Consideration of the theoretical and obtained means of the subtest and of all the items provided more information. If all students were completely indifferent to the format they used for proofs, the expected means would be 42 for the subtest and 2 for each of the test items. If each student were to mildly prefer the format she used, the expected
expected means would be 44.444 for the subtest and 2.116 for each of the test items, since there were more students using the flow-diagram format than the statement-reason format.

The mean for the subtest was computed to be 33.98, which is below both of these theoretical means and thus indicates a more negative than positive overall attitude toward flow-diagrams. This slightly negative attitude is also reflected in the means for individual items (Table 15). Care must be taken in interpreting these means, since 18 of the 21 means fall in the unit interval centered at 2 and so are not that different from 2 statistically speaking. However, the probability that all 18 means in the unit interval from 1.5 to 2.5 would fall in the lower half of the interval is approximately \((1/2)^{18}\) which is quite low indeed. Hence it does not seem unreasonable to conclude that a slight bias toward the negative was operative in the scores obtained. Much more study should be made of the responses to the various items in this subtest of the Format Test, to find out exactly what can be concluded from them. But this is beyond the scope and purpose of the present study.

A final focal point for comments on the tests of the hypotheses is the test of interaction effects. When the multivariate test failed to reach significance at the stipulated .05 level, the univariate tests were disregarded.

<table>
<thead>
<tr>
<th>Item Number</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.51</td>
<td>1.47</td>
</tr>
<tr>
<td>9</td>
<td>1.60</td>
<td>1.47</td>
</tr>
<tr>
<td>10</td>
<td>1.67</td>
<td>1.54</td>
</tr>
<tr>
<td>13</td>
<td>1.79</td>
<td>1.46</td>
</tr>
<tr>
<td>16</td>
<td>1.62</td>
<td>1.50</td>
</tr>
<tr>
<td>18</td>
<td>1.36</td>
<td>1.30</td>
</tr>
<tr>
<td>23</td>
<td>1.54</td>
<td>1.42</td>
</tr>
<tr>
<td>25</td>
<td>1.62</td>
<td>1.40</td>
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<tr>
<td>28</td>
<td>1.63</td>
<td>1.41</td>
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<tr>
<td>30</td>
<td>1.68</td>
<td>1.44</td>
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<tr>
<td>32</td>
<td>1.48</td>
<td>1.39</td>
</tr>
<tr>
<td>34</td>
<td>1.72</td>
<td>1.54</td>
</tr>
<tr>
<td>37</td>
<td>1.71</td>
<td>1.40</td>
</tr>
<tr>
<td>41</td>
<td>1.63</td>
<td>1.29</td>
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<tr>
<td>44</td>
<td>1.45</td>
<td>1.46</td>
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<tr>
<td>48</td>
<td>1.72</td>
<td>1.53</td>
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<td>52</td>
<td>1.56</td>
<td>1.59</td>
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<td>53</td>
<td>1.81</td>
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<tr>
<td>56</td>
<td>1.66</td>
<td>1.28</td>
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<td>59</td>
<td>1.59</td>
<td>1.56</td>
</tr>
<tr>
<td>60</td>
<td>1.63</td>
<td>1.57</td>
</tr>
</tbody>
</table>
However the multivariate test was not far from significance (p < .061), so two of the univariate tests warrant attention.

The data indicated that the interaction effect on the June CTA was due almost exclusively to the scores of one group GA, since the scores of the other groups were nearly the same (75.410 in contrast to 62.488, 64.733, and 65.424). A question comes to mind almost immediately: do text and treatment have relatively the same force in producing this interaction effect? It is not possible to answer this question definitively, but further comments are possible. The univariate test of significance for the effect of text (p < .716) on June CTA scores can be compared with the univariate test of significance for the effect of treatment (p < .058) on June CTA scores (see Table 10). These tests suggest that treatment was more influential than text.

The same conclusion seems to be indicated for the relative effects of text and treatment on flexibility. The univariate tests of significance given in Table 10 suggest that treatment (p < .015) is a more significant factor than text (p < .212) in affecting flexibility scores.

Bracht (10) has developed methods of investigating significant interactions suggested by intersecting graphs such as those in Figures 7 and 8 depicting the interactions on Flex and CTA3 scores. These methods can be used to answer the pedagogical concern: is this treatment more
Figure 7.—Interaction of Text and Treatment on Flexibility Scores.

Fig. 8.—Interaction of Text and Treatment on June CTA Scores.
effective with one subset of the sample than with another? Since the interactions in this study failed to reach significant levels—an assumption of Bracht's tests—further tests were not employed. But future study of the interplay of deduction, proof, critical thinking, and geometry should deal in depth with the questions raised by significant interactions.

**Correlations of Criterion Variables**

There were six criterion variables for which scores were available for all subjects, as well as the three covariates. Three of these variables were CTA measures. Besides a total score, for each administration of the CTA there were five subscores. The group of these 24 measures were correlated in pairs. Partial results, all but correlations involving only CTA scores and subscores, are displayed in Table 16.

With such a large number of correlations being computed from the same pool of data, it is probable that some of these correlation coefficients are spuriously high due to more random factors. Statistical methods have been developed to determine when a coefficient is due to random factors or to actual correlation of the variables. These methods take into consideration both the sample size and the number of variables being correlated. According to interpolation of entries in a statistical table printed in
TABLE 16.—Correlations of Subtests of Format Test, Covariates, and CTA Subscores and Total Scores.

<table>
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<th></th>
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</thead>
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<td>IQ</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>HEDT</td>
<td>.245</td>
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<tr>
<td>Att. Geom.</td>
<td>.054</td>
<td>.132</td>
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</tr>
<tr>
<td>Att. Proof</td>
<td>.130</td>
<td>.105</td>
<td>.701**</td>
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</tr>
<tr>
<td>Att. Flow Diag.</td>
<td>.128</td>
<td>.031</td>
<td>.228</td>
<td>.316</td>
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</tr>
<tr>
<td>Flexibility</td>
<td>.184</td>
<td>.411</td>
<td>.221</td>
<td>.309</td>
<td>.141</td>
<td></td>
</tr>
<tr>
<td>Sept. CTA (tot)</td>
<td>.298</td>
<td>.447*</td>
<td>.166</td>
<td>.127</td>
<td>.102</td>
<td>.526**</td>
</tr>
<tr>
<td>Sub 1a</td>
<td>.318</td>
<td>.285</td>
<td>.049</td>
<td>.025</td>
<td>.102</td>
<td>.296</td>
</tr>
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<td>Sub 2a</td>
<td>.059</td>
<td>.205</td>
<td>.004</td>
<td>.040</td>
<td>.045</td>
<td>.387</td>
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<td>Sub 3a</td>
<td>.187</td>
<td>.200</td>
<td>.166</td>
<td>.089</td>
<td>.044</td>
<td>.233</td>
</tr>
<tr>
<td>Sub 4a</td>
<td>.169</td>
<td>.376</td>
<td>.111</td>
<td>.039</td>
<td>.095</td>
<td>.332</td>
</tr>
<tr>
<td>Sub 5a</td>
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<td>.264</td>
<td>.168</td>
<td>.207</td>
<td>.011</td>
<td>.302</td>
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<tr>
<td>Jan. CTA (tot)</td>
<td>.370</td>
<td>.513**</td>
<td>.095</td>
<td>.135</td>
<td>.075</td>
<td>.469**</td>
</tr>
<tr>
<td>Sub 1a</td>
<td>.293</td>
<td>.381</td>
<td>.084</td>
<td>.083</td>
<td>.021</td>
<td>.286</td>
</tr>
<tr>
<td>Sub 2a</td>
<td>.138</td>
<td>.244</td>
<td>-.010</td>
<td>.008</td>
<td>-.047</td>
<td>.209</td>
</tr>
<tr>
<td>Sub 3a</td>
<td>.287</td>
<td>.399</td>
<td>.036</td>
<td>.074</td>
<td>.097</td>
<td>.407</td>
</tr>
<tr>
<td>Sub 4a</td>
<td>.287</td>
<td>.402</td>
<td>.077</td>
<td>.131</td>
<td>.075</td>
<td>.332</td>
</tr>
<tr>
<td>Sub 5a</td>
<td>.285</td>
<td>.334</td>
<td>.185</td>
<td>.210</td>
<td>.131</td>
<td>.387</td>
</tr>
<tr>
<td>June CTA (tot)</td>
<td>.302</td>
<td>.407</td>
<td>.113</td>
<td>.183</td>
<td>.118</td>
<td>.511**</td>
</tr>
<tr>
<td>Sub 1a</td>
<td>.223</td>
<td>.354</td>
<td>.158</td>
<td>.128</td>
<td>.107</td>
<td>.349</td>
</tr>
<tr>
<td>Sub 2a</td>
<td>.140</td>
<td>.189</td>
<td>.002</td>
<td>.113</td>
<td>-.012</td>
<td>.191</td>
</tr>
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</table>
TABLE 16—Continued

<table>
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<th></th>
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<tbody>
<tr>
<td>June CTA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sub 3\textsuperscript{a}</td>
<td>.226</td>
<td>.332</td>
<td>.125</td>
<td>.125</td>
<td>.101</td>
<td>.528\textsuperscript{**}</td>
</tr>
<tr>
<td>Sub 4\textsuperscript{a}</td>
<td>.161</td>
<td>.246</td>
<td>.052</td>
<td>.133</td>
<td>.141</td>
<td>.345</td>
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<tr>
<td>Sub 5\textsuperscript{a}</td>
<td>.315</td>
<td>.268</td>
<td>.047</td>
<td>.126</td>
<td>.063</td>
<td>.293</td>
</tr>
</tbody>
</table>

\textsuperscript{a} CTA Subscores: 1 = inference; 2 = recognition of assumptions; 3 = deduction; 4 = interpretation; 5 = evaluation of arguments.

*Significant at the .05 level.

**Significant at the .01 level.

Guilford's text on statistics (43,580-1), the coefficient should exceed .424 and .452 in order to be significant at the .05 and .01 levels, respectively, for 25 variables and a sample size of 172. Thus those correlations other than between CTA scores and subscores which were significant at the .05 level were the following: NEDT and September CTA total score (p < .05); NEDT and January CTA total score (p < .01); attitude toward geometry and attitude toward proof (p < .01); flexibility and each of the CTA total scores p < .01 in each case); and flexibility and June CTA deduction subscore (p < .01).

It is not difficult to understand the high correlation between attitude toward geometry and attitude toward proof. Immediately before the students responded to the attitudinal
questionnaire, their attention had been focused on a new format for writing proofs. The proximity in time of this activity involving proof could easily have increased in students' minds the close relationship between geometry and proof-making normal in students' school experiences and traditions.

Somewhat surprising is the fact that the data gave no evidence that IQ correlates significantly with CTA scores. Correlations with the CTA and various intelligence tests reported in the test manual suggested much higher correlations, just as it indirectly suggested that there might be moderately high correlations between NEDT and CTA; for the NEDT is said to have been developed from the Iowa Test of Educational Development (20) and .66 was the correlation coefficient for the CTA and the Interpretation of Literature Subtest of the Iowa Test of Educational Development.

The statistical significance of correlations between flexibility and four scores involving the Critical Thinking Appraisal raises several interesting questions. Obviously, logical thinking as defined and measured by the Critical Thinking Appraisal correlates positively with the ability to adjust to a new format. Is there a causal relationship between these capabilities as well as a correlation? Could practice in adapting to a different proof format
increase the understanding and mastery of logical deductions? Or is it that the ability to adapt to a different proof format calls for a better than average grasp of deduction? The structure of the present research does not permit using the results of the experiment to attempt to answer any of these questions. But procedures and materials need to be developed to deal with the possibilities brought up here.

**Effects Due to Text**

It was found that the differences between text-treatment groups on the three covariates IQ, NEDT, and September CTA were significant (p < .001) when the groups were separated by text and not when the basis of separation was treatment (see Table 4, p. 64). The text groups were also different on each of the six criterion variables, and not all the differences were due to initial differences. Table 10, p. 87 indicates the significance levels of differences remaining after adjustments for initial differences. However the scores measuring attitude toward flow-diagrams were unadjusted for the covariates for the reasons already discussed in the section on testing the hypotheses. The text group means for each of the six criterion variables can be read from Table 17. On the highly correlated attitude variables (attitude toward geometry and attitude toward proof) those students using Text A did not
TABLE 17.—Means of Text Groups on Criterion Variables
(Scores Adjusted for 3 Covariates).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Text</th>
<th>Mean</th>
<th>Direction of Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Att. to Geom.</td>
<td>A</td>
<td>36.780</td>
<td>B &gt; A</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>40.544</td>
<td></td>
</tr>
<tr>
<td>Att. to Proof</td>
<td>A</td>
<td>33.915</td>
<td>B &gt; A*</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>42.358</td>
<td></td>
</tr>
<tr>
<td>Att. to Flow D.\textsuperscript{a}</td>
<td>A</td>
<td>33.484\textsuperscript{a}</td>
<td>A &gt; B*</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>31.671\textsuperscript{a}</td>
<td></td>
</tr>
<tr>
<td>Flexibility</td>
<td>A</td>
<td>50.984</td>
<td>A &gt; B</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>47.396</td>
<td></td>
</tr>
<tr>
<td>Jan. CTA</td>
<td>A</td>
<td>66.163</td>
<td>A &gt; B</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>62.642</td>
<td></td>
</tr>
<tr>
<td>June CTA</td>
<td>A</td>
<td>66.464</td>
<td>A &gt; B</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>65.115</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a}Scores unadjusted for covariates; d f = 1, 168
*Significant at the .05 level.

do as well as those using Text B. In fact, they manifested significantly more negative attitudes toward proof than the other group. Text B does spend more time developing proof-making than Text A, and its exercises are also exclusively proof-oriented. These facts may be reflected to some degree in the differences in attitude toward proof and perhaps to a lesser degree in the differences in attitude toward geometry as a whole.
On all other measures, the mean score of students using Text A was higher than the corresponding mean score of students using Text B. These differences were significant at the .05 level in only one case: attitude toward flow-diagrams. Since the Text A group corresponds to the higher ability classes, their preference for the flow-diagram format may be based in the logical nature of the flow-diagram format. It is interesting to note that attitudes toward the flow-diagram format were significantly different for both the treatment groups \( (p < .001) \) and the text groups \( (p < .009) \) but interaction of text and treatment was not significant \( (p < .924) \). Further research is needed to investigate reasons behind these observed preferences.

**Anecdotal Data**

At the end of the 1970-71 school year, several students and teachers were interviewed, in order to secure their evaluations of the flow-diagram format.

Teacher comments resembled in general those obtained in Martin's study. At least one time during the year, each of the four teachers reacted favorably to the flow-diagram format in a conversation with the fifth teacher, the experimenter. A typical reaction would state that "the flow-diagram format is more logical because you can see more clearly how one statement follows from another."
After the January testing, teachers were asked to let students change formats if they wished. A number of students did change; usually the change was from the flow-diagram format to the statement-reason format. Whether or not a student changed apparently depended to some extent on teacher acceptance of the change, but also on class acceptance of her way of "putting proofs on the board." One class using statement-reason format was taught flow-diagrams during the work on indirect proof, but they did not react favorably to the material nor to the new format. One of the best students in that class tried to continue using flow-diagrams after they finished the chapter on indirect proof, but the class "couldn't understand" her proofs when it was her turn to "put them on the board." So after two weeks, she returned to the format used by the class.

In another class, proofs were discussed in small groups rather than put on the board, and so those who changed formats did not have class opposition to influence them. The teacher of this class noticed some tendency on the part of students to change formats if they were working with other students who had changed.

Some students offered reasons why they or their friends preferred a certain format. The following are typical of their comments.

I changed to the statement-reason format because it is easier to work with. When you look at it after you finish, it's all there; and you don't have to figure out each thing as you go along. It's easier to write each part separately.
I like the flow-diagram better because you can tell where each step comes from and how you go into the next step. And then you can just write "definition" or something next to it.

If you do the statement-reason form you have a whole line of things and then you have to put the "a" (for "angle," in congruence problems) and "s" (for "side") and it gets all confusing. But if you do a flow-diagram, you know exactly what you did to get this, and what you're going to do to get that.

When you have the statement-reason format, you have everything right there. And it's neater.

I changed and then I changed back (after one problem) because I found the statement-reason format harder and more confusing, because if you had it in a flow-diagram you could see exactly step by step. In the statement-reason way, you did it step by step, but it wasn't drawn out step by step.

I feel that the flow-diagram is harder than statement-reason. A flow-diagram is spread out and is more complicated than the statement-reason form. A statement-reason proof is right there.

My friends who use the flow-diagram say it's easier. They've used the statement-reason format for some problems; but they find flow-diagrams much easier because they're simpler to understand.

The statement-reason format is easier to work with because that's the way I was taught. Most of my friends do best the way they were taught.

Apparently, to judge from the above comments and similar observations, the students thought the flow-diagram format showed the thinking process, "step by step," more clearly; and this very characteristic caused most of the
difficulty in using the flow-diagram for both students and teachers. Some students preferred the compactness of the statement-reason format, while others saw it as "confusing." The only statement that everyone agreed to is that most people found it easier to use the format they had been taught. This underlines the responsibility of the teacher and developers of materials, for whatever they offer the student will probably be preferred by the student.
CHAPTER V

SUMMARY AND CONCLUSIONS

Summary of the Study

The present study had as its main purpose to obtain experimental evidence as to the effect of two different formats for writing proofs on students' critical thinking, attitude to the uncommon flow-diagram format, attitude toward geometry and proof, and adaptability to an unfamiliar format. A secondary concern was to relate proof format to students' achievement in general geometry knowledge and also achievement in understanding and constructing a deductive proof in geometry. A selected review of research on critical thinking suggested adopting Ennis' "working definition" of critical thinking and using the Watson-Glaser Critical Thinking Appraisal as an appropriate as well as often used measure of critical thinking congruent with Ennis' definition. Review of the literature relating geometry and critical thinking indicated a continuation of the assumption of some basic causal connection, tested by periodic experimentation with varying degrees of success. Those studies involving materials designed to improve
critical thinking on geometric and non-geometric examples concluded in favor of the new materials, while those studies comparing "conventional" geometry materials with non-geometry courses often found no significant differences.

No research comparing ways to write geometry proofs was found prior to the planning of this experiment. Perhaps the reason was that there are only two formats in current use: the paragraph format and the statement-reason format. Euclid made use of the paragraph format, as did all texts until the 1920's when the statement-reason format became accepted as an improved format. During the "revolution" of the 1960's, certain writers reinstated the paragraph format for some of their proofs. Among alternatives to the statement-reason format suggested over the years, the flow-diagram format attributed to Frank B. Allen and adapted to geometry by Donn L. Klingler seems to reflect best the nature of the logical implication. Prior classroom experience by this researcher convinced her that the flow-diagram could be learned by high school student to their profit.

In order to test out this conviction, an experimental study was designed and executed in a middle-sized (approximately 1,000 total enrollment) private Catholic school for girls. Five teachers and their seven geometry classes were involved in the year-long experiment. Treatments differed in the proof formats required in class activities, homework,
and testing during the first semester. Students were homo-
geneously grouped, so no random assignment to treatment was
possible. Teachers and classes were consciously assigned
to treatments in order to "even out" ability differentials.
Analysis of covariance was also used to control for initial
differences in ability and achievement.

Treatment G consisted in using the assigned text and
the statement-reason format for all proofs during the first
semester. Three classes (76 students) were assigned to
Treatment G. Treatment X consisted in using the assigned
text and the flow-diagram format for all proofs during the
first semester. Four classes (96 students) were assigned
to Treatment X. After the first semester, each treatment
group was introduced to the other format for proofs and
allowed to use the format of their choice. Teachers con-
tinued to use the stipulated format predominately. In
order to minimize the influence of textual use of the state-
ment-reason format, all proofs in the texts of all students
were blackened out on those pages to be covered during the
first semester. Proofs written in the stipulated formats
were dittoed and distributed to each treatment group. Two
different texts were used by part of each treatment group.

Measures of IQ, mathematics achievement, and initial
critical thinking ability were obtained in the first month
of the experiment. The IQ scores from the eighth grade
were taken from school records; mathematics achievement was determined by scores on the mathematics subtest of the National Educational Development Tests taken in early October of the students' sophomore year; Form Zm of the Watson-Glaser Critical Thinking Appraisal was administered in September as a pretest. These three measures served as covariates for most of the analysis of data on criterion measures.

Critical thinking ability was measured twice more, in January and in June. Also in January were the measurements of attitudes toward geometry, proof, and flow-diagrams, and adaptability to the "other" format (flexibility) by means of subtests of an experimenter-made instrument, the Format Test. Multivariate analysis of covariance was used to test the significance of differences among text-treatment groups on the two measures of critical thinking, the measures of attitudes toward geometry and toward proof, and the measure of flexibility. A univariate test of significance was employed to analyze attitude toward flow-diagram scores, after tests of covariance assumptions indicated the unsuitability of covariance procedures. For the students using the text School Mathematics Geometry, achievement-in-geometry and achievement-in-proof scores were analyzed by means of multivariate analysis of covariance.
Nine research hypotheses were considered. They were:

$H_1$: There is no significant text-treatment interaction on any of the following criterion variables: January Critical Thinking Appraisal, June Critical Thinking Appraisal, attitude toward flow-diagrams, attitude toward geometry, attitude toward proof, and flexibility.

$H_2$: There is no significant difference between mean scores of X and G on the January Critical Thinking Appraisal.

$H_3$: There is no significant difference between mean scores of X and G on the June Critical Thinking Appraisal.

$H_4$: There is no significant difference between mean scores of X and G on the ability to change to a new proof format, as measured by the flexibility subtest of the Format Test.

$H_5$: There is no significant difference between mean scores of X and G indicating their attitude toward geometry, as measured by the "attitude toward geometry" subtest of the Format Test.

$H_6$: There is no significant difference between mean scores of X and G indicating their attitude toward proof, as measured by the "attitude toward proof" subtest of the Format Test.

$H_7$: There is no significant difference between mean scores of X and G indicating their attitude toward flow-diagrams, as measured by the "attitude toward flow-diagrams" subtest of the Format Test.
Hₜ: There is no significant difference between those classes of X and G which used the text School Mathematics Geometry on achievement in geometry, as measured by items on chapter tests accompanying this text.

Hₙ: There is no significant difference between those classes of X and G which used the text School Mathematics Geometry on those items of chapter tests which measured certain aspects of the ability to understand and construct deductive proofs.

The results of the tests of significance led to the rejection of H₄ and H₇ at the .05 level of significance. The groups using the flow-diagram format manifested significantly more favorable attitudes toward the flow-diagram format, which was expected. They also found it significantly easier to adapt to the unfamiliar format. For all other criterion variables, the mean score of the students using the statement-reason format was greater than the mean score of the students using the flow-diagram format. Even though none were significant at the .05 level of significance, the differences on the June Critical Thinking Appraisal approached statistical significant, and in favor of the students using the statement-reason format. The multivariate test of significance of interaction of text and treatment slightly exceeded the stipulated .05 level (p < .061), thus casting doubt on the significance of uni-
variate tests for interaction effects on flexibility and June CTA scores. Data on achievement for the students using School Mathematics Geometry suggested that a larger percentage of students using the statement-reason format tended to score lower on both achievement measures than students using the flow-diagram format.

Pairwise correlations of six criterion and three co-variate variables showed that mathematics achievement (NEDT) correlated significantly with the pretest September Critical Thinking Appraisal (p < .05) and with the January Critical Thinking Appraisal (p < .01). Understandably, attitude toward geometry correlated highly with attitude toward proof (r = .701). The highest correlation involving IQ was the total score on January Critical Thinking Appraisal (r = .370); but the chance that it was spuriously high due to random factors exceeded 5 per cent. An interesting feature of the correlations determined from the data is that flexibility correlated significantly with each Critical Thinking Appraisal total score and with one subscore, that of deduction on the June Critical Thinking Appraisal. The probability that any of these correlations were caused by chance was less than .01.

Students using the text Geometry: A Contemporary Approach had significantly more positive attitudes toward
proof. Those using *School Mathematics Geometry* had significantly better attitudes toward flow-diagrams.

Comments by teachers and students indicated that, in general, students tended to prefer the format they were taught first.

**Conclusions and Implications for Further Research**

Because of the evidence of the data obtained from this experiment, the following conclusions seem warranted:

1. It has not been shown that the format for geometry proof has a statistically significant impact on the development of critical thinking abilities, although the data tended to favor the statement-reason format.

2. The format employed for geometry proofs does affect the ability to adapt to another, different format.

3. Students tend to prefer the proof format they are taught first, and

4. The flow-diagram format can be understood and used by geometry students of various levels of general intellectual ability and mathematical achievement.

Because of the general agreement of results from this experiment with those from Martin's experiment which had a more representative sample, at least superficially, it seems that further research into the area of contrasting formats for proofs on their effect on students' critical thinking
ability should take an essentially different approach from that taken by Martin's and this research. In today's educational foment, critical thinking is claimed as a primary course objective of every "modern" and "tradition" curriculum, in every content area. This fact may render it impossible to obtain empirical support for a "significant" contribution by any one course or technique. Future research on the contribution of geometry and related pedagogical procedures should, instead, concentrate on a single aspect of critical thinking—for example, overcoming the predominance of "child's logic."

The final word has not yet been said about the possible impact of the flow-diagram format on forming students' deductive abilities. But first there must be research aimed at development and evaluation of materials which exploit for pedagogical purposes the intrinsic logic in the flow-diagram format. Comparisons of formats should be set aside until there are viable alternatives to the statement-reason format available in texts equally as attractive as the texts using the statement-reason format.

Much more work needs to be done in the area of attitudes to geometry as a whole and to the various activities of the geometry class, including the construction of proofs. Knowledge of general geometric concepts is a kind of achievement distinct from ability to understand what proofs
"do" and why they are necessary; both of these are distinct from being able to write an "original" proof. Instead of relating proof format to a general concept of "achievement," future studies should explore the effect of the use of a particular proof format on each of these three kinds of achievement.

The importance of positive attitudes for effective learning is recognized by educators everywhere. Thus the data gathered from this sample by means of the Format Test raises many questions. Not only was there a negative bias toward flow-diagrams, but the group as a whole was neutral toward geometry and slightly negative toward proofs (compare tests means in Appendix C with expected "neutral" scores of 38 and 42, respectively). Is this typical of tenth grade geometry students? Is it typical for less capable students to have more positive attitudes toward geometry and proofs? How does the stress laid on proof-making by the text affect these same attitudes? How would reliability coefficients obtained from a more representative sample of geometry students compare with those obtained from the present sample? How would students react to the flow-diagram format if there were no comparison with another format? Is a slightly negative attitude typical of students' reactions toward any format for proof? Further research and refinement of tests
such as the Format Test are needed to begin to answer these questions.

It is the conviction of this writer that there are implications of this experiment more important in ways than those given above for the researcher-to-be. These are the implications for the "average math teacher," and for curriculum developers, especially developers of texts.

First, teachers need to become more aware that there are "teachable" alternatives to the statement-reason format. Teacher familiarity with several of the suggested proof formats could improve his students' awareness of the nature of proofs and proof-making even if he decided not to make extensive use of any of them; for knowledge of the formats adds to his store of teaching and re-teaching approaches. Student and teacher reactions to the flow-diagram format during and after the time of the experiment have encouraged the experimenter to continue to seek out those classroom situations in which use of flow-diagrams increases effectiveness of instruction in proofs.

Second, materials for instruction and testing need to be developed in line with the experiences arising from this experiment and the literature read in preparation for it. Students should be required to decide on the validity of a conclusion already made, as well as to be able to make valid conclusions. "True-false" questions are often used to test "ready-made" conclusions. A slight revision would
allow for three possible answers instead of the usual two; for example, "true-false-not necessarily so" or "always-sometimes-never."

Third, student exercises need to provide for the growth of "math logic" as a consistent thought pattern, and tests need to measure this growth. Items should include non-factual contexts as well as factual geometric contexts, in the manner of the O'Brien, Shapiro, and Reali study, so that attention can be focused on the thinking behind the conclusion.

Fourth, teachers should become more conscious of the role construction of proofs plays in their geometry course. The fact that the format had made its significant impact on critical thinking by January suggests that its effect plateaus early in the course, and thus it may be deemphasized later in the course with little loss to the students' over-all learning. On the other hand, learning to prove statements deductively may be more similar to learning to play a musical instrument. Regular practice of constructing proofs without noticeable improvement may be necessary before the student is ready for deeper insights into the nature of proof. Teachers can use the yearly accumulation of experience with various students to develop guidelines for themselves and researchers.
Finally, teachers should be encouraged by the basic agreement of the rather limited study—limited in numbers and in the kinds of students involved—with Martin's study with a more representative sample. It is possible to incorporate many aspects of formal research into classroom attitudes and activities. The students can only profit from well-planned searches into what and how they have learned.
APPENDIX A

TEXT SUPPLEMENTS (Not to be reproduced)

1. School Mathematics Geometry—Flow-Diagram Format
2. School Mathematics Geometry—Statement-Reason Format
1. School Mathematics Geometry--Flow-Diagram Format

We call the "if" part of a theorem, the hypothesis of the given part, and to call the "then" part the conclusion of the given part. Let p represent the hypothesis and q represent the conclusion. Then the "if-then" form can be written symbolically as $p \rightarrow q$. For example, Statement 1 can be written symbolically as $l$ and $l_2$ are different lines and $l$ intersects $l_2 
rightarrow$ the intersection of $l$ and $l_2$ consists of a single point.

Sometimes, when the hypothesis contains more than one clause, each clause is listed separately, then grouped by a brace symbol ({}). Before the --- symbol is written:

$l$ and $l_2$ are different lines

{l intersects $l_2 
rightarrow$ the intersection of $l$ and $l_2$ consists of a single point

The second standard form for a theorem, called the hypothesis-conclusion form, is illustrated by rewriting Statement 1 as follows:

Hypothesis: $l$ and $l_2$ are different lines and $l$ intersects $l_2$

Conclusion: The intersection of $l$ and $l_2$ consists of a single point.

Ex. 1. Write the statements in Exercise 8--14 in "if... then" form, then in the $p \rightarrow q$ form.

E: 112

(aif) $A = B = C$ \(\iff\) (def) $\angle A = \angle B = \angle C$

(aif) $AB$ is not on $\ell_2$ \(\iff\) form a lin. pair \(\rightarrow\) $\angle ABD = \angle DEC$ \(\rightarrow\)

(def) $\ell_1 \cap \ell_2 = \ell_3 \rightarrow m\angle ABD = m\angle DEC$

(hyp) $m\angle ABD = m\angle DEC$

\(\rightarrow\) $\angle ABD = \angle DEC$

(Repetition) $m\angle ABD = m\angle DEC$

\(\rightarrow\) $\angle ABD = \angle DEC$

\(\rightarrow\) $m\angle ABD = 90$

prove supp.

($\rightarrow$) $m\angle ABD = 90$ \(\iff\) $ABD$ is a right angle

$m\angle DEC = 90$ \(\iff\) $ABD$ is a right angle

{1} Supplement Postulate
{1} Substitution
{3} Properties of real numbers
{3} Transitive prop. of equality
The proof of Theorem 5.9 is in flow-diagram format, a form which helps us to organize our proofs carefully with precise statements and correct reasons, and write them down in such a way as to show how they are related to each other. The main part of the diagram is the arrangement of the statements in an adaptation of the \( \rightarrow \) form (p. 87 in this supplementary material). The theorems, postulates, or definitions which justify drawing a conclusion from a statement or group of statements is indicated in parentheses over the \( \rightarrow \) symbol in one of the ways:

(a) If the name of the theorem, postulate, or definition is short enough it is written over the \( \rightarrow \) symbol. (E.g., (def) was used three times in the proof of Th. 5.9. Each time the statement before the \( \rightarrow \) symbol included the word or phrase being defined.)

(b) Otherwise a number code is used; the number goes over the \( \rightarrow \) symbol in the flow diagram and is then repeated with the theorem, etc., at the end of the flow diagram. (E.g., the use of (1), (2), (3), and (4) in the proof of Th. 5.9)

\[\text{Ex. 120}\]

\begin{align*}
&\text{(hyp) } A - B - C \quad \text{(def)} \quad \angle DBC \cong \angle DBA \\
&\text{(hyp) } \text{Def. of straight angle} \quad \text{form a lin. pr.} \quad (1) \quad m\angle BAC + m\angle DBA = 180 \\
&\quad \text{(hyp) } m\angle DBA = 90 \quad \text{(def)} \quad m\angle DBC = 90 \\
\end{align*}

\[\text{Ex. 126}\]

\begin{align*}
&\text{(hyp) } \angle B \text{ in supp. to } \angle A \quad \text{(def)} \quad m\angle A + m\angle B = 180 \quad (1) \quad m\angle B = 180 - m\angle A \\
&\text{(hyp) } \angle Q \text{ in supp. to } \angle P \quad \text{(def)} \quad m\angle Q + m\angle P = 180 \quad (1) \quad m\angle Q = 180 - m\angle P \\
&\quad \text{(hyp) } m\angle A = m\angle P \\
\end{align*}

(1) Property of real numbers
(2) Substitution and property of real numbers
(hyp & def) \( \angle 2 \cong \angle 1 \) (1) \( \angle 1 \) is supp. form a lin. pr. to \( \angle 2 \)

(hyp & def) \( \angle 2 \cong \angle 3 \) (1) \( \angle 3 \) is supp. form a lin. pr. to \( \angle 2 \)

(reflexive) \( \angle 2 \cong \angle 2 \)

(1) Supp. Post.

(2) Supp. of congruent angles are congruent (thm. 5.16)

\[ \frac{\overline{PQ}}{\overline{XY}} \frac{\overline{PR}}{\overline{XZ}} \rightarrow \triangle \overline{PQR} \cong \triangle \overline{XYZ} \]

\[ P = ? \]

A complete proof seldom leaps to mind all at once, so we try to analyze carefully what we know. Essentially we want to find intermediate statements that use what we know to conclude what we want to prove. Our finished proof, a flow diagram, will be some adaptation of the following:

Given data or hypotheses \[ \rightarrow \] your intermediate statement, with reasons \[ \rightarrow \] what we want to prove

In Example 1, for instance, we want to complete this flow diagram:

(hyp) \( \overline{AB} \) bisects \( \overline{CD} \)

(hyp) \( \overline{CD} \) bisects \( \overline{AB} \) \[ \rightarrow \] \[ ? \] \[ \rightarrow \] \( \overline{AC} \cong \overline{BD} \)

What can we use to conclude that two particular segments are congruent? Looking back, we recall that two segments are congruent if they are corresponding sides of congruent triangles. In turn, we can show that two triangles are congruent if we can apply the S.A.S. Postulate (or, of course, the definition of congruent triangles).

At this stage we know only a few facts about congruent segments; hence, there are only a few devices that we can try.

Looking at our drawing we see that \( \triangle \overline{APC} \) and \( \triangle \overline{DBF} \) are triangles with sides \( \overline{AC} \) and \( \overline{BD} \), respectively. Can we find a correspondence between the sets of vertices of those two triangles that will match \( \overline{AC} \) with \( \overline{BD} \)?

Clearly we can. Both \( \triangle \overline{APC} \leftrightarrow \triangle \overline{DPF} \) and \( \triangle \overline{APC} \leftrightarrow \triangle \overline{BPF} \) match \( \overline{AC} \) with \( \overline{BD} \).

Which do we want?

We know that \( \overline{PC} = \overline{PD} \) (since \( P \) bisects \( \overline{CD} \)) and we know that \( \overline{AP} = \overline{BF} \) (since \( P \) bisects \( \overline{AB} \)). Thus \( \overline{CP} \) and \( \overline{BP} \) (and also \( \overline{AP} \) and \( \overline{BF} \)) should be corresponding sides if we hope to get a congruence.

So we write \( \triangle \overline{APC} \leftrightarrow \triangle \overline{BPF} \) and hope that this correspondence will be a congruence. Under this correspondence, \( \overline{AP} = \overline{BF} \) and \( \overline{PC} = \overline{PD} \), which gives us two pairs of congruent sides (also, these are corresponding sides). If we know that \( \triangle \overline{APC} \cong \triangle \overline{BPF} \), then the S.A.S. Postulate would tell us that \( \triangle \overline{APC} \cong \triangle \overline{BPF} \). In turn, that statement would tell us that \( \overline{AC} \cong \overline{BD} \), which is precisely what we want to prove. Now we note that \( \triangle \overline{APC} \) and \( \triangle \overline{BPC} \) are vertical angles, and hence by Th. 5.18, \( \triangle \overline{APC} \cong \triangle \overline{BPC} \).
Having thought through the problem, we can write a proof in flow-diagram form.

**Given:** $AB$ and $CD$ bisect each other at $F$

**PROVE:** $AC = BD$

1. $(\text{hyp})$ $AB$ bisects $CD$ at $F$  \[ CF = DF \]
2. $(\text{hyp})$ $CD$ bisects $AB$ at $F$  \[ AF = BF \]
3. $(\text{SAS})$ $\triangle AFC \cong \triangle BFD$  \[ \angle APC \cong \angle BPD \]
4. $(\text{def})$ $AC = BD$

**(1) Vertical angles are congruent**

We could have changed some words or symbols and still have had an acceptable proof. For instance, we could have begun with:

1. $(\text{hyp})$ $AB$ and $CD$ bisect each other at $F$
2. $(\text{def})$ $\triangle AFC \cong \triangle BFD$  \[ \angle APC \cong \angle BPD \]

The reason for concluding that $AC = BD$ could have been "corresponding sides of congruent triangles" and been written either in abbreviated form over the --- symbol:

$\triangle AFC \cong \triangle BFD$  \[ \text{CPCT} \]  \[ AC = BD \]

or coded as $(2)$:

$\triangle AFC \cong \triangle BFD$  \[ (2) \]  \[ AC = BD \]

with "(2) Corresponding parts of congruent triangles are congruent" written under "(1) Vertical angles are congruent."

There is no definite number of steps that must be used in writing a proof. The number of steps depends on the approach to the problem and the method of organizing the proof.

**P. 158-0 cont'd**

\begin{align*}
\text{(hyp)} \quad & AH \cong PH \\
& \left\{ \begin{array}{l}
(?) \quad \angle AKB \cong \angle FBH \\
(?) \quad \overline{AB} \cong \overline{FB} \\
\end{array} \right. \\
\rightarrow & \angle A \cong \angle F \quad (?)
\end{align*}

**P. 162 Prob. 2**

\begin{align*}
\text{(hyp)} & \quad FG \cong FS \\
\left\{ \begin{array}{l}
(?) \quad \angle QPR \cong \angle SPR \\
(?) \quad \overline{FR} \cong \overline{FR} \\
\end{array} \right. \\
\rightarrow & \quad \triangle QPR \cong \triangle SFR \\
\Rightarrow & \quad \overline{RQ} \cong \overline{RS}
\end{align*}
Theorem 4.4 Let \( l_1 \) and \( l_2 \) be any two intersecting lines. Then there is exactly one plane containing them.

**EXISTENCE:**

\[
\begin{align*}
\text{(hyp)} & : \text{let } A \text{ be a point in the intersection of } l_1 \text{ and } l_2 \quad \left( \text{Th. 4.1} \right) \quad P \not\in l_2 \\
\{1\} & : l_1 \text{ contains a point } P \text{ different from } Q \\
\{\text{Part. 1}\} & : l_1 \subset E \quad \left( \text{def} \right) \quad \text{there exists a plane, namely } E, \text{ containing } l_1 \text{ and } l_2 \\
\{\text{repetition}\} & : l_2 \subset E
\end{align*}
\]
Suppose a plane \( P \) different from \( E \) contains \( l_1 \) and \( l_2 \).

\[
\text{(repetition) } P \cap \mathcal{E} \neq \emptyset \quad \text{(repetition) } P \cap \mathcal{E} = \emptyset
\]

Contradiction to Th. 4.3

(1) Meaning of containment

L.102

(1) \( l \) contains two points \( G \) \& \( R \)

there is an angle, \( \angle QGR \), congruent to \( \angle PRQ \), such that \( \angle QGR \) and \( \angle PRQ \) are in opp.

half-planes with edge 1

in the same

half-plane

(3) there is one pt. \( T \) of \( QR \) such that \( \angle QGT = \angle RGT \), \( T \) \& \( P \) are

in the same

half-plane

with edge 1

L.111

\( TP \) intersects \( l \) at a point \( A \)

(1) Point Post.

(2) Plane Separation Post. \& Angle Const. Post.

(3) Point-Fixing Thm.

(4) Plane Separation Post.

There are not three possibilities for cases) for \( A \):

I. \( A \) is in the interior of \( QR \)

II. \( A = Q \)

III. \( A = Q = R \)

L.111

(\( \text{reflex} \) \( GQ = QA \))

\( \text{(repetition) } \angle QGR \equiv \angle RGT \) \( \rightarrow \) \( \angle QPR \equiv \angle QGR \) \( \rightarrow \) \( \angle QAP \equiv \angle QAT \)

\( \angle QAP \) is a rt. angle \( \rightarrow \) \( \angle 90^\circ \)

\( l \perp \)
Ds. 214

(1) there exists a point $P_1$ of $EF$ such that
$$DP_1 = AC$$

(hyp) $AR \parallel DE$

(hyp) $\angle A \parallel \angle D$

$$ABC \parallel DEF$$

(2) $\implies DEF \sim DEF_1$ (def.)

(hyp) $ABC \parallel DEF$

(repet) $F \in EF$

(repet) $P_1 \in DF$

(hyp) $F = P_1$

(substitution)

(1) Transitivity of Congruence
(2) Angle Construction Post.
(3) The two lines $DP$ and $EF$ cannot intersect in two points.
Dr. 226

(1) Let M be the midpoint, \( \triangle MD = MG \) of BC

(2) Let F be the point on AB such that AF = 2AM

(3) AM = FM

(SAS) \( \triangle AMB \cong \triangle FMC \) (def)

(4) \( m\angle B = m\angle KCP \)

(5) \( F \) and M are in the same half-plane with edge BC

Dr. 227

(1) (lemmas) \( m\angle QRS > m\angle A \)

(2) (def) \( \angle QRS \cong \angle PRT \)

(3) m\angle QRS = m\angle PRT

(repeat) m\angle QRS > m\angle P

(4) Vertical angles are congruent

Dr. 229

(1) Let X be a point of AB such that X = DE

(hyp) \( \triangle ABC \cong \triangle DEF \) (def)

(m\angle A = m\angle B)

(A\triangle) \( \triangle ABC \cong \triangle DEF \) (def)

(2) \( m\angle A = m\angle ASC \)

Suppose \( X = E \)

(3) \( \triangle ABC \cong \triangle ASC \) (def)

(repeat) \( m\angle ABC > m\angle ASC \) CONTRADICTION

(4) Point-Plotting Thm.

(5) Transitivity

(6) Th. 3.4

(7) Exterior Angle Thm.
(1) Let D be the pt. on the ray opp. DC such that DB = AB

\[ \text{(def)} \quad D = C = \text{DB} + \text{BC} \]

\[ \text{(def)} \quad DB = AB \]

\[ \text{(1)} \quad DC = AB + BC \]

(2) DC = AB + BC

\[ \text{(def)} \quad DC = AB + BC \]

\[ \text{(repot)} \quad DB = AB \]

\[ \text{(1)} \quad DC = AB + BC \]

(3) m\angle DAC > m\angle DAB

\[ \text{(repot)} \quad m\angle DAC > m\angle DAB \]

\[ \text{(2)} \quad m\angle DAC > m\angle DAB \]

\[ \text{(repot)} \quad DC = AC \]

\[ \text{(3)} \quad DC = AC \]

\[ \text{(repot)} \quad AB + BC > AC \]

\[ \text{(1)} \quad \text{Point-Plotting Thm.} \]

\[ \text{(2)} \quad \text{Substitution} \]

\[ \text{(3)} \quad \text{Th. 5.2 \& Angle Measure Inequality} \]

\[ \text{(4)} \quad \text{Base angles of isosceles \( \Delta \) are congruent} \]

\[ \text{(5)} \quad \text{Th. 8.5} \]

(2.5) Let X be any point between P and Q. We shall prove that AX = EX

Step I: \( \text{hyp} \) P is equidistant \( \text{def} \) from A and B

\[ \text{(def)} \quad AP = BP \]

\[ \text{(hyp)} \quad Q \text{ is equidistant from A and B} \]

\[ \text{(def)} \quad AQ = BQ \]

\[ \text{(reflex)} \quad PQ = PQ \]

\[ \text{(S.S.)} \quad \triangle APQ \cong \triangle BPQ \]

Step II: \( \text{repot} \) \( \triangle APQ \cong \triangle BPQ \)

\[ \text{(def)} \quad \angle APQ = \angle BPQ \]

\[ \text{(substitution)} \quad \angle APX = \angle BPX \]

Step III: \( \text{repot} \) \( \triangle APX \cong \triangle BPX \)

\[ \text{(reflex)} \quad PX = PX \]

\[ \text{(S.S.)} \quad \triangle APX \cong \triangle BPX \]

\[ \text{(def)} \quad AX = EX \]

(2.61) there is exactly one plane \( E \) containing \( l \) and \( Q \)

(1) there is a line \( l \) in \( E \) perpendicular to \( l \) at \( P \)

(1.1) there is a line \( l \) in \( E \) containing \( l \) and \( Q \)

(1.1) there is a line \( l \) in \( E \) perpendicular to \( l \) at \( P \)

(2.2) \( E \) is perpendicular to \( l \) at \( P \)

(1) Th. 7.1
(Th. 4,4) Let there be a plane \( P \) containing \( l_1 \) and \( l_2 \).

\[ \begin{align*}
\text{(hyp)} & \quad 1 \perp E \quad \text{(def)} \quad 1 \perp E \\
\text{(hyp)} & \quad 1 \perp l_1 \\
\text{(repot)} & \quad l_1 \perp E \\
\text{(repot)} & \quad l_2 \perp E
\end{align*} \]

\[ l \perp l_1 \quad l \perp l_2 \]

(1) Postulate 7
(2) In plane \( P \) there is exactly one line perpendicular to \( l \) at \( P \).
(3) Substitution

---

\[ \begin{align*}
\text{(post)} \quad \angle BDA \perp AC \quad \text{(def)} \quad C \text{ is in same half-plane with edge } AB \\
\text{(post)} \quad \angle BDA \perp AC \quad \text{(def)} \quad C \text{ is in same half-plane with edge } AB
\end{align*} \]

\[ (1) \quad \max_1 + m \angle ABD = 180 \]

(2) \[ \begin{align*}
\text{(post)} \quad \angle BDA \perp AC \quad \text{(def)} \quad m \angle x = m \angle x_1 \\
\text{m} \angle x = m \angle x_1
\end{align*} \]

(1) Angle Addition Theorem
(2) Supplement Postulate
(3) Substitution
(4) Alternate interior angles are congruent

---

\[ \begin{align*}
\text{(Th. 3.7) Let } E \text{ be the midpt. of } BC \quad \text{(def)} \quad EB = EC \\
\text{(1) Let } F \text{ be the point on the ray opp. } E \text{ such that } EF = ED \\
\text{(def)} \quad \angle 2 \text{ and } \angle 3 \text{ are vertical angles}
\end{align*} \]

\[ (2) \quad \angle 2 \cong \angle 3 \]

(Th. 3.7) Let \( D \) be the midpt. of \( AD \)

\[ \begin{align*}
\text{(def)} \quad \angle 4 \cong \angle 5 \quad \text{(def)} \quad \overline{AB} \parallel \overline{CD} \\
\text{ADPC is a parallelogram}
\end{align*} \]

\[ \text{(Th. 19.21)} \quad \overline{AD} = \overline{CP} \]
Ex. 3.27 cont'd

(Repeat) $DE = EF \rightarrow DE = DF$ \(\rightarrow\) $DE = \frac{1}{2} AC$

(Repeat) $AD \parallel CE \rightarrow DF = AC$

1. Point-Plotting Thm.
2. Vertical angles are congruent
3. Thm. 10.5
4. Substitution
5. Prop. of real numbers
6. Thm. 10.16
2. School Mathematics Geometry—Statement-Reason Format

p. 82  We call the IF part of a theorem the **hypothesis** or the **IF part**, and we call the THEN part the **conclusion** or the part to be proved.

The second standard form for a theorem, called the **Hypothesis—Conclusion** form, is illustrated by rewriting statement (1) as follows:

Hypothesis: \( l_1 \) and \( l_2 \) are different lines and \( l_1 \) intersects \( l_2 \)

Conclusion: The intersection of \( l_1 \) and \( l_2 \) consists of a single point

p. 84  Write the statements in Exercises 8—16, in IF...THEN form, then in **Hypothesis—Conclusion** form.

<table>
<thead>
<tr>
<th>PROOF</th>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( A - B = J )</td>
<td>1. Hyp.</td>
<td></td>
</tr>
<tr>
<td>2. ( C ) is not on ( AD )</td>
<td>2. Hyp.</td>
<td></td>
</tr>
<tr>
<td>3. ( \angle ABD ) and ( \angle ADB ) form a linear pair</td>
<td>5. Definition of a linear pair</td>
<td></td>
</tr>
<tr>
<td>4. ( \angle ABD ) and ( \angle ADB ) are suppl.</td>
<td>2. Substitution Postulate</td>
<td></td>
</tr>
<tr>
<td>5. ( m\angle ABD + m\angle ADB = 180 )</td>
<td>6. Def. of supp. angles</td>
<td></td>
</tr>
<tr>
<td>6. ( m\angle ABD = 90 )</td>
<td>7. Substitution in statement 5</td>
<td></td>
</tr>
<tr>
<td>7. ( m\angle ADB = 90 )</td>
<td>8. Properties of real numb</td>
<td></td>
</tr>
<tr>
<td>8. ( 1 + 2 = 180 )</td>
<td>9. Statement 6 and transitivity</td>
<td></td>
</tr>
<tr>
<td>9. ( 1 + 2 = 180 )</td>
<td>10. Def. of rt. angles</td>
<td></td>
</tr>
</tbody>
</table>

The proof of Theorem 5.9 is in **Statement—Reason Format**, a form which helps us to organize our proofs carefully with precise statements and correct reasons, and write them down in an orderly way. The statements are listed on the left and the reasons for the statements are listed on the right.

<table>
<thead>
<tr>
<th>PROOF</th>
<th>STATEMENTS</th>
<th>REASONS</th>
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<tbody>
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<tr>
<td>2. ( C ) is not on ( AD )</td>
<td>2. Hyp.</td>
<td></td>
</tr>
<tr>
<td>3. ( \angle ABD ) and ( \angle ADB ) form a linear pr.</td>
<td>3. Def. of a lin. pr.</td>
<td></td>
</tr>
<tr>
<td>4. ( m\angle ABD + m\angle ADB = 180 )</td>
<td>4. Supplement Post.</td>
<td></td>
</tr>
<tr>
<td>5. ( \angle ABD ) is a rt. angle</td>
<td>5. Hyp.</td>
<td></td>
</tr>
<tr>
<td>6. ( m\angle ABD = 90 )</td>
<td>6. Def. of rt. angle</td>
<td></td>
</tr>
<tr>
<td>7. ( m\angle ADB = 90 )</td>
<td>7. State: ent. 4 and 5 and Subt.</td>
<td></td>
</tr>
<tr>
<td>8. ( \angle ABD ) is a rt. angle</td>
<td>o. Def. of rt. angle</td>
<td></td>
</tr>
</tbody>
</table>

p. 122  Explain what is meant by "the statement-reason format" for a proof. What is the greatest benefit of this type of proof?


1. \( L_1 \) is supp. to \( L_A \)
2. \( uA + uA = 100 \)
3. \( u^2 = 100 - uA \)
4. \( L_4 \) is supp. to \( L_P \)
5. \( uL_4 + uL_P = 100 \)
6. \( uL_4 = 100 - uL_P \)
7. \( L_A = uL_P \)
8. \( uL_4 = uL_P \)

**Reasons**

1. Hyp.
2. Hyp.
5. Same as 2
6. Same as 3
8. Transitive Prop. of real nos.

<table>
<thead>
<tr>
<th>State/Text</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( L_1 ) and ( L_1 ) form a lin. pr.</td>
<td>1. Hyp. and def.</td>
</tr>
<tr>
<td>2. ( L_1 ) is supp. to ( L_2 )</td>
<td>2. Supp. Post.</td>
</tr>
<tr>
<td>3. ( L_2 ) and ( L_3 ) form a lin. pr.</td>
<td>3. Hyp. and def.</td>
</tr>
<tr>
<td>4. ( L_3 ) is supp. to ( L_2 )</td>
<td>4. Same as 2</td>
</tr>
<tr>
<td>5. ( L_2 \parallel L_2 )</td>
<td>5. Reflexive Prop. (Th. 5.13)</td>
</tr>
<tr>
<td>6. ( L_1 \parallel L_1 )</td>
<td>6. Supp. of ( \parallel ) ( L_2 ) are ( \parallel ) (Th. 5.16)</td>
</tr>
</tbody>
</table>

**Statement-Reason Format (p. 119)**

**Re. 131** (Use the statement-reason format for your proof.)

**Re. 157**

\[ \text{if } \overrightarrow{FK} \parallel \overrightarrow{KL} \]
\[ \overrightarrow{FK} \parallel \overrightarrow{KL} \]
\[ \overrightarrow{LP} \parallel \overrightarrow{LX} \]

then \( \triangle PQL \parallel \triangle XLY \)

**Re. 158-9**

Next we divide the page into two columns as follows:

<table>
<thead>
<tr>
<th>Proof</th>
<th>Statement/Text</th>
<th>Reasons</th>
</tr>
</thead>
</table>

Of course, setting up the statement-reason format will be of no use unless we can think of an idea for a proof. A complete proof seldom leaps to mind all at once. We try to analyze carefully what we want to prove in terms of what we know.

We want to prove that two particular segments are congruent. What can we use? Looking back, we recall that two segments are congruent if they are corresponding sides of congruent triangles. In turn we can show that the triangles are congruent if we can apply the SAS Postulate (and, of course, the definition of congruent triangles).
At this stage we know only a few facts about congruent segments; hence there are only a few devices that we can try.

Looking at our drawing we see that \( \triangle APF \) and \( \triangle BPD \) are triangles with sides \( AP \) and \( BD \), respectively. Can we find a correspondence between the sets of vertices of these two triangles which will match \( AP \) with \( BD \)?

Clearly we can. Both \( \triangle APF \rightarrow \triangle BPD \) and \( \triangle APF \rightarrow \triangle BPD \) match \( AP \) with \( BD \).

Which do we want?

We know that \( PF = PD \) (since \( P \) bisects \( \angle JD \)) and we know that \( AF = BF \) (since \( F \) bisects \( \angle JD \)). Thus \( \angle APF \) and \( \angle BPD \) and also \( AF \) and \( BF \) should be corresponding sides if we hope to get a congruence.

So we write \( \triangle APF \rightarrow \triangle BPD \) and hope that this correspondence will be a congruence. Under this correspondence, \( AP = BP \) and \( PF = PD \), which gives us two pairs of corresponding corresponding sides. If we were to have a way to know that \( \angle APF = \angle BPD \), then the \( \text{SAS} \) Postulate would tell us that \( \triangle APF \equiv \triangle BPD \). In turn, that statement would tell us that \( AP = BP \), which is precisely what we want to prove. Now we note that \( \angle APF \) and \( \angle BPD \) are vertical angles, hence by \( \text{Thm. 5,10,} \angle APF \equiv \angle BPD \).

Having thought through the problem, we can write a proof in statement-reason form.

**Given:** \( \angle JD \) and \( \angle JD \) bisect each other at \( P \).

**Prove:** \( AC = DD \)

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle JD ) bisects ( \angle JD ) at ( P )</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2. ( PF = PD )</td>
<td>2. Def. of &quot;bisect&quot;</td>
</tr>
<tr>
<td>3. ( \angle JD ) bisects ( \angle JD ) at ( P )</td>
<td>3. Hyp.</td>
</tr>
<tr>
<td>4. ( AF = BF )</td>
<td>4. Same as 2</td>
</tr>
<tr>
<td>5. ( \angle APF ) and ( \angle BPD ) are vert. ( \angle )s</td>
<td>5. Figure and def. of vert. ( \angle )s</td>
</tr>
<tr>
<td>6. ( \angle APF \equiv \angle BPD )</td>
<td>6. Vert. ( \angle )s are congruent.</td>
</tr>
<tr>
<td>7. ( \triangle APF \equiv \triangle BPD )</td>
<td>7. ( \text{SAS} )</td>
</tr>
<tr>
<td>8. ( AP = BP )</td>
<td>8. Def. of congruent figures</td>
</tr>
</tbody>
</table>

We could have changed some words or symbols and still have had an acceptable proof. For instance we could have been with a single statement combining statements (1) and (3), and another statement combining statements (2) and (4):

1. \( \angle JD \) and \( \angle JD \) bisect each other at \( P \) \( \rightarrow \) 1. Hyp.
2. \( PF = PD \) \( \rightarrow \) 2. Def. of "bisects" |

Reason (6) could have been "corresponding parts of congruent triangles are congruent" and could have been abbreviated as \( \text{CPCTC} \). There is no definite number of steps that must be used in writing a proof. The number of steps depends on the approach to the problem and the methods of organizing the proof.

<table>
<thead>
<tr>
<th>STATEMENTS</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle DL = \angle DL )</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2. ( \angle LAD \equiv \angle LAD )</td>
<td>2. Hyp.</td>
</tr>
</tbody>
</table>
### Problem 2

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $FQ = PQ$</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2. $\angle QFA \cong \angle AFB$</td>
<td>2. Why?</td>
</tr>
<tr>
<td>3. $PA = PA$</td>
<td>3. Why?</td>
</tr>
<tr>
<td>4. $QF \parallel DF$</td>
<td>4. Why?</td>
</tr>
<tr>
<td>5. $RQ = QS$</td>
<td>5. Why?</td>
</tr>
</tbody>
</table>

### Problem 3

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $HI \perp HI$</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2. $\angle HIH$ is a rt. angle.</td>
<td>2. Perp. lines</td>
</tr>
<tr>
<td>4. $\angle HIH$ is a rt. angle.</td>
<td>4. Why?</td>
</tr>
<tr>
<td>5. ?</td>
<td>5. All rt. angles are congruent.</td>
</tr>
<tr>
<td>6. $HR = FR$</td>
<td>6. Why?</td>
</tr>
<tr>
<td>7. $QA = QA$</td>
<td>7. Why?</td>
</tr>
<tr>
<td>8. $\Delta HIH \cong \Delta FRQ$</td>
<td>8. Why?</td>
</tr>
<tr>
<td>9. $NQ = FQ$</td>
<td>9. Why?</td>
</tr>
</tbody>
</table>

### Problem 5

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $HN = dQ$</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2. $HF = HA$</td>
<td>2. Why?</td>
</tr>
<tr>
<td>3. $HI = HI$</td>
<td>3. Why?</td>
</tr>
<tr>
<td>4. $\Delta HNF \cong \Delta QHA$</td>
<td>4. Why?</td>
</tr>
<tr>
<td>5. $PF = AQ$</td>
<td>5. Def. of congruent figures</td>
</tr>
</tbody>
</table>

### Problem 6

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. $CD = DJ$</td>
<td>2. Why?</td>
</tr>
<tr>
<td>4. $AJ = AD$</td>
<td>4. Why?</td>
</tr>
<tr>
<td>5. $\angle BJD \cong \angle BAC$</td>
<td>5. Why?</td>
</tr>
</tbody>
</table>
How we shall give a statement-reason proof for Theorem 4.1.

Let \( l_1 \) and \( l_2 \) be any two lines intersecting at a point. Then there is exactly one plane containing them.

<table>
<thead>
<tr>
<th>PROOF</th>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXISTENCE:</td>
<td>1. Let ( G ) be a point in the intersection of ( l_1 ) and ( l_2 )</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td></td>
<td>2. ( l_1 ) contains a point ( P ) different from ( G )</td>
<td>2. The Ruler Post.</td>
</tr>
<tr>
<td></td>
<td>3. ( F \cap l_2 )</td>
<td>3. Th. 4.1</td>
</tr>
<tr>
<td></td>
<td>4. There is exactly one plane, say ( \mathcal{F} ), containing ( P ) and ( l_2 )</td>
<td>4. Th. 4.3</td>
</tr>
<tr>
<td></td>
<td>5. ( l_1 \subset \mathcal{F} )</td>
<td>5. Post. 5'</td>
</tr>
<tr>
<td></td>
<td>6. ( \mathcal{F} ) is a plane containing ( l_1 ) and ( l_2 )</td>
<td>6. Statements 4 and 5</td>
</tr>
</tbody>
</table>

UNIQUENESS:

1. Suppose that a plane \( \mathcal{P} \) different from \( \mathcal{F} \) contains \( l_1 \) and \( l_2 \)
2. \( P \in \mathcal{P} \)
3. Plane \( \mathcal{P} \) contains \( P \) and \( l_2 \) and \( \mathcal{P} \neq \mathcal{F} \)
4. Contradiction

1. \( l_1 \) contains two points \( Q \) and \( R \)
2. There is an angle, \( \angle QRS \), congruent to \( \angle QRP \), such that \( P \) and \( S \) are in opposite half-planes with edge \( l \)
3. There is one point \( T \) of \( \overrightarrow{ST} \) such that \( T \neq P \)
4. \( T \) and \( P \) are in opposite half-planes with edge \( l \)
5. \( TT \) intersects \( l \) at a point \( A \)

Now there are three possible cases for the position of \( A \):

I. \( A \) is in the interior of \( QR \)
II. \( A = Q \)
III. \( A = R \)
### Case I. \( A \) is in the interior of \( \overrightarrow{EF} \)

6. \( \overrightarrow{EA} = \overrightarrow{EA} \)

7. \( \triangle EAF \cong \triangle AEF \)

8. \( \angle EAF \cong \angle EAF \)

9. \( \angle EAF \) is a right angle

10. \( \overrightarrow{EF} \perp \overrightarrow{EJ} \)

### Reflexive Property

7. \( \triangle \) (statement 2, 3, and 6)

8. \( \) Def. of congruent triangles

9. \( \) Th. 5.9

10. \( \) Def. of perpendicularity

### Proof

**Statements** | **Reasons**
--- | ---
1. There exists a point \( P_1 \) of \( \overrightarrow{EF} \) such that \( \overrightarrow{EP_1} = \overrightarrow{EA} \) | 1. The Point-plotting Theorem
2. \( \overrightarrow{AB} = \overrightarrow{B1} \) (\( \angle A \subseteq \angle B \)) | 2. Hyp.
3. \( \Delta \) (\( \equiv \) \( \Delta \)) | 3. SAS
4. \( \angle ABC \equiv \angle BAC \) | 4. Def. of congruent figures
5. \( \angle ABC \equiv \angle BAC \) | 5. Hyp.
6. \( \angle ABC \equiv \angle BAC \) | 6. Transitive Property
7. \( \angle \) Th. 4.7 (\( \overrightarrow{P} \) and \( \overrightarrow{P_1} \) are in the interior of \( \overrightarrow{EF} \)) | 7. Th. 4.7 (\( \overrightarrow{P} \) and \( \overrightarrow{P_1} \) are in the interior of \( \overrightarrow{EF} \))
8. \( \angle \) Construction Post. | 8. Angle Construction Post
9. \( \angle \) Repetition of statements 1 and 6 | 9. Repetition of statements 1 and 6
10. \( \angle \) Hyp. | 10. Hyp.
11. \( \angle \) The two lines \( \overrightarrow{EF} \) and \( \overrightarrow{EF} \) cannot intersect in 2 points. | 11. The two lines \( \overrightarrow{EF} \) and \( \overrightarrow{EF} \) cannot intersect in 2 points.
12. \( \angle \) Substitution of \( \overrightarrow{P} \) for \( \overrightarrow{P_1} \) in statement 5 | 12. Substitution of \( \overrightarrow{P} \) for \( \overrightarrow{P_1} \) in statement 5

### Proof

**Statements** | **Reasons**
--- | ---
1. Let \( ii \) be the midpoint of \( \overrightarrow{EF} \). Then \( \overrightarrow{ii} = \overrightarrow{KJ} \) | 1. The Midpoint Theorem, Def. of midpoint.
2. Let \( \overrightarrow{P} \) be the point on \( \overrightarrow{MF} \) such that \( \overrightarrow{AP} = 2\overrightarrow{AB} \). Then \( \overrightarrow{AB} = \overrightarrow{AP} \) | 2. The Point-plotting Theorem
3. \( \triangle \) (\( \equiv \) \( \triangle \)) | 3. Vertical angles are congruent.
4. \( \angle \) (\( \angle ABC \equiv \angle BAC \)) | 4. SAS
5. \( \angle \) (\( \equiv \) \( \angle \)) | 5. SSS
6. \( \overrightarrow{P} \) and \( \overrightarrow{ii} \) are in the same half-plane with edge \( \overrightarrow{EF} \) | 6. \( \overrightarrow{P} \) and \( \overrightarrow{ii} \) are points of the interior of \( \overrightarrow{EF} \) and Th. 4.7
7. \( \overrightarrow{P} \) and \( \overrightarrow{D} \) are in the same half-plane with edge \( \overrightarrow{EF} \) | 7. Both \( \overrightarrow{P} \) and \( \overrightarrow{D} \) are in the half-plane opposite to that which contains \( A \)
8. F is a point of the interior of \( \triangle \) OCD
9. \( \angle D F B > \angle M F P \)
10. \( \angle B J D > \angle B \)

### Proof: 237

<table>
<thead>
<tr>
<th>Step</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \angle A J D &gt; \angle A J B )</td>
</tr>
<tr>
<td>2.</td>
<td>( \angle A J D = \angle A J P )</td>
</tr>
<tr>
<td>3.</td>
<td>( \angle A J D &gt; \angle A J B )</td>
</tr>
</tbody>
</table>

1. Let \( X \) be a point of \( \triangle \) such that \( \angle A X B \) is false and so \( X = B \)

### Proof: 238

<table>
<thead>
<tr>
<th>Step</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( A = \triangle B C )</td>
</tr>
<tr>
<td>2.</td>
<td>( \angle A = \angle B )</td>
</tr>
<tr>
<td>3.</td>
<td>( \angle A = \angle B )</td>
</tr>
<tr>
<td>4.</td>
<td>( \angle A = \angle B )</td>
</tr>
<tr>
<td>5.</td>
<td>( \angle A = \angle B )</td>
</tr>
</tbody>
</table>

Suppose \( X \neq B \)

6. Either \( A - X - B \) or \( A - B - X \)

7. \( \angle A X B > \angle A J B \) 8. \( \angle A J B > \angle A X B \)

9. Hence \( X \neq B \) is false and so \( X = B \)

10. \( \angle A X B \neq \angle A J B \)

### Proof: 239

<table>
<thead>
<tr>
<th>Step</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \angle D J B &gt; \angle D J C )</td>
</tr>
<tr>
<td>2.</td>
<td>( \angle D J B &gt; \angle D J C )</td>
</tr>
<tr>
<td>3.</td>
<td>( \angle D J B &gt; \angle D J C )</td>
</tr>
</tbody>
</table>

1. Given that \( P \) and \( Q \) we shall prove that \( \angle A X = B X \)

### Proof: 240

Let \( X \) be any point between \( P \) and \( Q \). We shall prove that \( \angle A X = B X \)

**Step 1:**
1. \( \angle A P = \angle B P; \) \( \angle A Q = \angle B Q \)
Step III:
4. \( \angle APQ \cong \angle BPQ \)
5. Therefore \( \angle APX \cong \angle BPX \)

Step IV:
6. \( AP = BP; \angle APX \cong \angle BPX; PX = PX \)
7. \( \triangle APX \cong \triangle BPX \)

Step V:
8. Therefore \( AX = BX \)
9. \( X \) is equidistant from \( A \) and \( B \)

Post. 4a

1. There is a point \( Q \) not on \( \ell \)
2. There is exactly one plane \( \pi \) containing \( \ell \) and \( Q \)
3. There is a point \( A \) not on \( \pi \)
4. There is a plane \( \pi \) containing \( \ell \) and \( A \)
5. \( \pi \cap \ell = \{Q\} \)
6. In \( \pi \) there is a line \( \ell_1 \) perpendicular to \( \ell \) at \( P \); and in \( \pi \) there is a line \( \ell_2 \) perpendicular to \( \ell \) at \( P \)
7. There is a plane \( \pi \) containing \( \ell_1 \) and \( \ell_2 \)
8. \( \pi \) is perpendicular to \( \ell \) at \( P \)

Post. 3

1. Th. 4.4
2. \( \pi \parallel \ell \)
3. \( \pi \cap \ell = \{P\} \)
4. \( \ell_1 \perp \ell \)
5. \( \ell_2 \perp \ell \)
6. \( \ell_1 = \ell_2 \)
7. \( \ell_1 \subset \pi \)
### PROOF 1

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( BD \parallel \overline{AD} )</td>
<td>1. Post. 15</td>
</tr>
<tr>
<td>2. ( \angle 2 ) and ( \angle 3 ) are in the same half-plane with edge ( \overline{BD} )</td>
<td>2. Def. of half-plane</td>
</tr>
<tr>
<td>3. Let ( B ) be a point on ( I ) and in the half-plane that contains ( C ) and has edge ( \overline{AD} )</td>
<td>3. Post. 3</td>
</tr>
<tr>
<td>4. ( \angle 2 ) and ( \angle 3 ) are in the plane with edge ( \overline{BD} )</td>
<td>4. Def. of half-plane</td>
</tr>
<tr>
<td>5. ( C ) is in the interior of ( \angle ABD )</td>
<td>5. Def. of interior of angle</td>
</tr>
<tr>
<td>6. ( m\angle ABD = \angle D + \angle A )</td>
<td>6. Angle Addition Th.</td>
</tr>
<tr>
<td>7. ( m\angle 1 + m\angle ABD = 180 )</td>
<td>7. Supp. Post.</td>
</tr>
<tr>
<td>8. ( m\angle 1 + m\angle D + m\angle A = 180 )</td>
<td>8. Substitution</td>
</tr>
<tr>
<td>9. ( m\angle 1 = m\angle D ); ( m\angle A = m\angle A )</td>
<td>9. Alternate interior angles of parallel lines are congruent.</td>
</tr>
<tr>
<td>10. ( m\angle 1 + m\angle D + m\angle A = 180 )</td>
<td>10. Substitution</td>
</tr>
</tbody>
</table>

### PROOF 2

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Let ( J ) be the midpoint of ( \overline{BD} )</td>
<td>1. Th. 3.7</td>
</tr>
<tr>
<td>2. ( JS = GJ )</td>
<td>2. Def. of midpoint</td>
</tr>
<tr>
<td>3. Let ( P ) be the point on the ray opposite to ( \overline{BD} ) such that ( \overline{JP} = \overline{DP} )</td>
<td>3. Point-plotting Th.</td>
</tr>
<tr>
<td>4. ( J ) and ( P ) are vertical angles</td>
<td>4. Def.</td>
</tr>
<tr>
<td>5. ( \angle 1 ) and ( \angle 2 ) are vertical angles</td>
<td>5. Vertical angles are congruent.</td>
</tr>
<tr>
<td>6. ( \triangle BDE \equiv \triangle GPF )</td>
<td>6. Th. 3.7</td>
</tr>
<tr>
<td>7. ( DJ = PG )</td>
<td>7. Def. of congruent triangles</td>
</tr>
<tr>
<td>8. Let ( D ) be the midpoint of ( \overline{BD} )</td>
<td>8. Th. 3.7</td>
</tr>
<tr>
<td>9. ( AD = BD )</td>
<td>9. Def. of midpoint</td>
</tr>
<tr>
<td>10. ( AD = JP )</td>
<td>10. Substitution</td>
</tr>
<tr>
<td>11. ( J ) is a parallelogram</td>
<td>11. Def. of congruent triangles</td>
</tr>
<tr>
<td>12. ( \overline{AD} \parallel \overline{JP} )</td>
<td>12. Th. 10.5</td>
</tr>
<tr>
<td>13. ( \angle 1 ) is a parallelogram</td>
<td>13. Th. 10.21</td>
</tr>
<tr>
<td>14. ( \overline{BD} = \overline{AG} )</td>
<td>14. Def. of a parallelogram</td>
</tr>
<tr>
<td>15. ( \angle 1 ) = ( \angle 2 )</td>
<td>15. Th. 10.16</td>
</tr>
<tr>
<td>16. ( \triangle BD \equiv \triangle AGF )</td>
<td>16. Statement 3</td>
</tr>
<tr>
<td>17. ( \overline{BD} = \overline{AG} )</td>
<td>17. Substitution</td>
</tr>
</tbody>
</table>

1.4.1 To illustrate, if \( C \) is the midpoint of \( \overrightarrow{AB} \), then by definition the measure

\[ \overrightarrow{AC} \parallel \overrightarrow{CB} \quad \text{(Fig. 2-5)} \]

of \( \overrightarrow{AC} \) will be equal to the measure of \( \overrightarrow{CB} \). This can be written in mathematical symbols as

\[ C \text{ is the midpoint of } \overrightarrow{AB} \rightarrow \overrightarrow{AC} = \overrightarrow{CB}. \]

Note: "\( \rightarrow \)" is used as "if \( X \) is a true statement then \( Y \) is a true statement."

For example, the statement "If a triangle contains one right angle, the other two angles are acute angles."

A triangle contains one right angle \( \rightarrow \) the other angles are acute.

Another example is the statement "If \( \angle 1 \) and \( \angle 2 \) are congruent and supplementary, then they are right angles."

If the "\( \rightarrow \)" from it is written

\( \angle 1 \) and \( \angle 2 \) are congruent \( \rightarrow \) \( \angle 1 \) and \( \angle 2 \) are right angles.

As a final example, "If \( 2 + 5 = 3x - 2 \), then \( x = 4 \)." becomes \( 2 + 5 = 3x - 2 \rightarrow x = 4. \)

1.4.2 In each of the problems below, place the question mark with a conclusion which can be drawn from the data preceding the symbol "\( \rightarrow \)."

4) \( D \) is the midpoint of \( \overrightarrow{AB} \rightarrow \)?

5) \( E \) is the midpoint of \( \overrightarrow{CD} \rightarrow \)?

6) \( F \) is the midpoint of \( \overrightarrow{AD} \rightarrow \)?

7) \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \\ intersect each other \rightarrow \)

8) \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) intersect each other \( \text{(draw two conclusions.)} \rightarrow \)
In each of the figures below, replace the question mark with a valid conclusion drawn from the given information. Give the reason for your answer.

1. \( \square \rightarrow ? \)
   \[ \begin{array}{c}
   \triangle ABC \\
   A \quad B \quad C
   \end{array} \]

2. \( \triangle \rightarrow ? \)
   \[ \begin{array}{c}
   \triangle ABC \\
   D \\
   \end{array} \]

3. \( \square \rightarrow ? \)
   \[ \begin{array}{c}
   \square ABCD \\
   A \quad B \quad C \quad D
   \end{array} \]

4. \( \triangle \rightarrow ? \)
   \[ \begin{array}{c}
   \triangle ABC \\
   \triangle DEF \\
   \end{array} \]

5. \( \square \rightarrow ? \)
   \[ \begin{array}{c}
   \square ABCD \\
   \square EFGH \\
   \end{array} \]

6. \( \triangle \rightarrow ? \)
   \[ \begin{array}{c}
   \triangle ABC \\
   \triangle XYZ \\
   \end{array} \]

To prove and 8. Think of, write a statement using the given. \( \rightarrow \) giving a conclusion which can be drawn from the given data.

**Note:** Omission of the image in text makes it difficult to provide a precise response. However, based on the given context, it seems the tasks involve geometric proofs and conclusions based on given figures. The responses would typically involve applying geometric theorems and properties to deduce the required conclusions.
Section 2.43

The statements presented in this section will help clarify the concepts presented in the previous section. 

(Example) Consider the statements presented in the previous section. 

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(1) REFLEXIVE PROPERTY FOR SEGMENTS.

(2) SUBTRACTION POSTULATE: IF CONGRUENT SEGMENTS \(\overline{AB} \text{ and } \overline{BC}\) ARE ADDED TO CONGRUENT SEGMENTS \(\overline{DE} \text{ and } \overline{EF}\), THE SUMS WILL BE CONGRUENT SEGMENTS \(\overline{AD} \text{ and } \overline{DF}\).

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\[\overline{AB} + \overline{BC} \rightarrow \overline{AD}\]

\[\overline{DE} + \overline{EF} \rightarrow \overline{DF}\]

(1) REFLEXIVE PROPERTY FOR SEGMENTS.

(2) SUBTRACTION POSTULATE: IF CONGRUENT SEGMENTS \(\overline{AB} \text{ and } \overline{BC}\) ARE ADDED TO CONGRUENT SEGMENTS \(\overline{DE} \text{ and } \overline{EF}\), THE SUMS WILL BE CONGRUENT SEGMENTS \(\overline{AD} \text{ and } \overline{DF}\).

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\[\overline{DE} \text{ and } \overline{EF}\]

\[\overline{AD} \text{ and } \overline{DF}\]

(1) REFLEXIVE PROPERTY FOR ANGLES.

(2) SUBTRACTION POSTULATE: IF CONGRUENT ANGLES \(\angle CAB \text{ and } \angle DAE\) ARE ADDED TO CONGRUENT ANGLES \(\angle ABE \text{ and } \angle AEF\), THE SUMS WILL BE CONGRUENT ANGLES \(\angle ABD \text{ and } \angle AEF\).

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\[\angle CAB + \angle DAE \rightarrow \angle ABD\]

\[\angle ABE + \angle AEF \rightarrow \angle AEF\]

(1) REFLEXIVE PROPERTY FOR ANGLES.

(2) SUBTRACTION POSTULATE: IF CONGRUENT ANGLES \(\angle 1 \text{ and } \angle 2\) ARE SUBTRACTED FROM CONGRUENT ANGLES \(\angle 3 \text{ and } \angle 2\), THE DIFFERENCES WILL BE CONGRUENT ANGLES \(\angle 1 \text{ and } \angle 2\).

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\[\angle 3 - \angle 2 \rightarrow \angle 1\]

\[\angle 3 - \angle 2 \rightarrow \angle 2\]

(1) TRANSITIVE PROPERTY OF CONGRUENCES: IF TWO ANGLES \(\angle 1 \text{ and } \angle 2\) ARE CONGRUENT TO THE SAME ANGLE \(\angle 3\), THEN THEY \(\angle 1 \text{ and } \angle 2\) ARE CONGRUENT TO EACH OTHER.

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UNLESS OTHERWISE STATED, each only one conclusion of the basis of the data given is each of the following programs. Given the \(\rightarrow\) a single put (1). Then below the flow diagram put (1). Next to the postulate use (1). Next to the basis to justify this conclusion.

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Above the \(\rightarrow\) symbols, indicate the postulate that was used to derive the indicated conclusion. Either write an undetermined title for the postulate over the symbol, or put (1) over the symbol and write out the postulate below the flow diagram:

1. \(\angle 1 \equiv \angle 2 \rightarrow \angle 1 \equiv \angle 2\)
2. \(\angle 1 \equiv \angle 2 \rightarrow \angle 1 \equiv \angle 2\)
3. \(\angle 1 \equiv \angle 2 \rightarrow \angle 1 \equiv \angle 2\)
4. \(\angle 1 \equiv \angle 2 \rightarrow \angle 1 \equiv \angle 2\)
**Pair (IV)**

\[ \triangle \text{AEC} \overset{\text{(s)}}{\implies} \triangle \text{AEC} \overset{\text{(iv)}}{\implies} \triangle \text{AEC} \overset{\text{(v)}}{\implies} \triangle \text{AEC} \overset{\text{(vi)}}{\implies} \triangle \text{AEC} \]

\[ \begin{align*}
\text{(i)} & \quad \text{If two angles are congruent to two congruent angles, then they are congruent. (Theorem 8)} \\
\text{(ii)} & \quad \text{If two angles are vertical angles, they are congruent. (Vertical Angle Theorem)} \\
\end{align*} \]

**Pair (V)**

\[ \triangle \text{ABC} \overset{\text{(s)}}{\implies} \triangle \text{ABC} \overset{\text{(iv)}}{\implies} \triangle \text{ABC} \overset{\text{(v)}}{\implies} \triangle \text{ABC} \overset{\text{(vi)}}{\implies} \triangle \text{ABC} \]

\[ \begin{align*}
\text{(i)} & \quad \text{All right angles are congruent.} \\
\text{(ii)} & \quad \text{Transitive Property} \\
\end{align*} \]
(1) CIRCLE \( O \) AND CIRCLE \( P \) INTERSECT AT \( A \) AND \( B \).

(2) \( \triangle \) \( D C \) \( \sim \) \( \triangle \) \( A C \) \( \sim \) \( \triangle \) \( B D \) \( \sim \) \( \triangle \) \( A D \) \( \sim \) \( \triangle \) \( B C \).

(3) ANGLES OF THE SAME CIRCLE ARE EQUAL.

PROOF

1. \( \triangle \) \( D C \) \( \sim \) \( \triangle \) \( A C \) \( \sim \) \( \triangle \) \( B D \) \( \sim \) \( \triangle \) \( A D \) \( \sim \) \( \triangle \) \( B C \).

2. ANGLES OF THE SAME CIRCLE ARE EQUAL.

3. REVERSE OF DEFINITION; \( \angle \) \( D C \) \( \sim \) \( \angle \) \( A C \) \( \sim \) \( \angle \) \( B D \) \( \sim \) \( \angle \) \( A D \) \( \sim \) \( \angle \) \( B C \).

Ex. 11

1. What conclusion can be drawn from the data given in each of the problems below?

(a) \( D \) is the midpoint of \( \overline{AB} \).

(b) \( F \) is the bisector of \( \overline{AD} \).

(c) \( \overline{DE} \) and \( \overline{DF} \) bisect each other.

(d) \( \overline{AE} \) and \( \overline{BF} \) bisect each other.

Ex. 12

Using the information given, name the right angles in each of the figures below:

(a) \( \overline{AB} \bot \overline{AC} \).

(b) \( \overline{AD} \bot \overline{AE} \).

(c) \( \overline{BC} \bot \overline{BD} \).

(d) \( \overline{CD} \bot \overline{CE} \).

(e) \( \overline{AB} \bot \overline{BC} \).

(f) \( \overline{AD} \bot \overline{BD} \).

(g) \( \overline{AE} \bot \overline{CE} \).

(h) \( \overline{AB} \bot \overline{AC} \).
EXERCISES

In Problems 1 through 6 what conclusion can be drawn in terms of the data given?

Ex. 20

In view of these definitions, where formerly we spoke of the equality of the measures of line segments, now we can speak directly to the congruence of these segments, and a similar relation will hold with angles. Hence, the following statements are said to be equivalent:

\[ \overline{AB} = \overline{CD} \] is equivalent to \[ \overrightarrow{AB} \cong \overrightarrow{CD} \]  
\[ \angle XYZ = \angle DEF \] is equivalent to \[ \angle XYZ \cong \angle DEF \]  

Ex. 21

On the basis of the data given, we shall say that \[ \angle EAC \cong \angle EAC \]  
and justify our conclusion by using the revised definition of the direction of an angle.

Ex. 22

Map I to figure 2-30, then given as the midpoint of \( \overline{AB} \), the conclusion drawn would be \[ \overrightarrow{AB} \cong \overrightarrow{AC} \]  
Justification for this conclusion lies in the revised definition of the midpoint of a line segment.

Ex. 46-48

The reader may be questioning this procedure by saying, "How is it possible to justify this conclusion of simple equality to the definition of the direction of an angle other than the direction?" The definition of the direction of an angle can be used as justification of a conclusion only if the data given states that the ray was the direction. In this problem, however, this was not stated. You know only that there were two congruent angles, and from this you infer that two angles are equal. You draw a conclusion in haste; it is justified by using the reverse of the definition. This is so because it is the reverse of the definition that states that the ray that forms congruent angles with the sides of the angle is the direction of the angle.

Ex. 31

Exercises. What conclusion can be drawn on the basis of the data given in each of the problems below? Justify your conclusion by stating the reverse of one of the definitions you have had. In order that your work will follow the pattern that will be used throughout the year, arrange your conclusions and reasons as you are shown below.

ILLUSTRATION

<table>
<thead>
<tr>
<th>CONCLUSION</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle ABC ) is a straight angle.</td>
<td>( \angle ABC ) is the sum of whose measures is the measure of a straight angle and supplementary angles.</td>
</tr>
<tr>
<td>CONCLUSION</td>
<td>REASON</td>
</tr>
<tr>
<td>------------</td>
<td>--------</td>
</tr>
<tr>
<td>LADD and LADD are right angles</td>
<td>Perpendicular lines are two lines that intersect and form right angles. (def.)</td>
</tr>
</tbody>
</table>

**Postulate 5: The Addition Postulate**

If \( A = B \) and \( C = D \), then \( A + C = B + D \)

**Postulate 6: The Subtraction Postulate**

If \( A = B \) and \( C = D \), then \( A - C = B - D \)

**Postulate 7: The Multiplication Postulate**

If \( A = B \) and \( C = D \), then \( AC = BD \)

**Postulate 8: The Division Postulate**

If \( A = B \) and \( C = D \) (where \( C \) & \( D \) are not zero), then \( A \div C = B \div D \)

**Postulate 10: Symmetric Prop. of Equality**

If \( A = B \), then \( B = A \)

Since it follows from the Symmetric Property that \( AB = CD \) then \( CD = AB \)

Therefore, it can be stated that \( AB = CF \) then \( CF = AB \).
Before reading the “Conclusion” and “Reason” that appear for each problem, try to draw your own conclusion and justify it in terms of the postulates.

### Conclusion | Reason
--- | ---
CD = CD | Reflexive Property of congruent segments.
MD == ME | Addition Postulate: If congruent segments (MD & ME) are added to congruent segments (DE & BE), the sums will be congruent segments (MD & ME).

### Conclusion | Reason
TV = TV | Reflexive Property of congruent segments.
TR == TV | Subtraction Postulate: If congruent segments (TV & TR) are subtracted from congruent segments (SD & TV), the differences will be congruent segments (SD & TV).

### Conclusion | Reason
\( \angle CAD \equiv \angle CAD \) | Reflexive Property of congruent angles.
\( \angle BAD \equiv \angle CAD \) | Addition Postulate: If congruent angles (\( \angle CAD \) & \( \angle CAD \)) are added to congruent angles (\( \angle BAC \) & \( \angle DAE \)), the sums will be congruent angles (\( \angle BAC \) & \( \angle DAE \)).

### Conclusion | Reason
\( \angle AED \equiv \angle AED \) | Reflexive Property of congruent angles.
\( \angle ABE \equiv \angle ABE \) | Subtraction Postulate: If congruent angles (\( \angle AED \) & \( \angle AED \)) are subtracted from congruent angles (\( \angle ADE \) & \( \angle ABE \)), the differences will be congruent angles (\( \angle ADE \) & \( \angle ABE \)).

Before reading the conclusion and reason that appears for the problem, try to formulate your own.

### Conclusion | Reason
\( \angle 1 \equiv \angle 3 \) | Transitive Property of congruence: If two angles (\( \angle 2 \) and \( \angle 3 \)) are congruent to the same angle (\( \angle 2 \)), then they are congruent to each other.

It is quite apparent that this method of proof is not only lengthy but also rather tedious. In view of this fact, the “Statement-Reason” format was developed to shorten and simplify the proof of a statement. The proof presented above in the text will now be repeated in its “statement-reason” format.
<table>
<thead>
<tr>
<th>Date</th>
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<th>Action</th>
</tr>
</thead>
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<tr>
<td>17/12/20</td>
<td>07:15</td>
<td>Start</td>
</tr>
<tr>
<td>17/12/20</td>
<td>07:30</td>
<td>Attend meeting</td>
</tr>
<tr>
<td>17/12/20</td>
<td>08:00</td>
<td>File documents</td>
</tr>
<tr>
<td>17/12/20</td>
<td>09:30</td>
<td>Review reports</td>
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<td>17/12/20</td>
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<tr>
<td>17/12/20</td>
<td>14:00</td>
<td>Schedule meetings</td>
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<tr>
<td>17/12/20</td>
<td>16:30</td>
<td>Finish work</td>
</tr>
<tr>
<td>17/12/20</td>
<td>17:00</td>
<td>End day</td>
</tr>
</tbody>
</table>

*Note: The above schedule is subject to change based on unforeseen circumstances.*
<table>
<thead>
<tr>
<th>PROOF</th>
<th>STATEMENTS</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( a = b ) in supplementary to ( a ).</td>
<td>1. Opposite angles are equal in an isosceles triangle.</td>
</tr>
<tr>
<td>2.</td>
<td>( m\angle A = m\angle B ) in supplementary to ( a ).</td>
<td>2. Definition of supplementary angle.</td>
</tr>
<tr>
<td>3.</td>
<td>( m\angle A = m\angle B ) in supplementary to ( a ).</td>
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</tbody>
</table>
4. \( \angle 1 \) is supplementary to \( \angle 2 \).
5. \( \overline{AB} \) and \( \overline{CD} \) are opposite rays.
6. Given is a straight angle.
7. \( \angle 3 \) is supplementary to \( \angle 4 \).
8. \( \angle A \cong \angle 2 \)

**Proof Statements**

1. \( \angle 3 \cong \angle 4 \)
2. \( \angle 1 \cong \angle 2 \)
3. \( \angle 1 \cong \angle 4 \)

**Reasons**

1. Given
2. Given
3. Transitive Property of Congruence
4. Given
5. Given

**Proof Statements**

1. \( \angle 1 \cong \angle 2 \)
2. \( \overline{AB} \parallel \overline{CD} \)
3. \( \overline{AB} \parallel \overline{CD} \)
4. \( \angle 2 \cong \angle 4 \)
5. \( \angle 1 \cong \angle 2 \)

**Reasons**

1. Given
2. Given
3. Given
4. Given
5. Given
6. Transitive Property of Congruence
7. Transitive Property of Congruence
8. Given
9. Given
10. Given
ILL

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4. ∠ACD is a right angle.
5. ∠ACD ≅ ∠ACD (a)
6. ∠ACD ≅ ∠ACD (a)
7. ∠ACD ≅ ∠ACD (a)
8. ∠ACD ≅ ∠ACD
9. ∠ACD ≅ ∠ACD

PROOF | STATEMENTS | REASONS
--- | --- | ---
1. ∠ACD ≅ ∠ACD (a) | 1. Given
2. Let AB be the bisector of ∠ACD. | 2. Every angle has a bisector (Postulate)
3. AC meets CD at some point E. | 3. Phag’s Axiom
4. ∠ACD ≅ ∠ACD (a) | 4. Def. of the bisector of an angle
5. ∠ACD ≅ ∠ACD (a) | 5. Reflexive Property of Congruence
6. ∠ACD ≅ ∠ACD | 6. A.S.A. (Post.)
7. ∠ACD ≅ ∠ACD | 7. Def. of congruent polygons
### Page 110

<table>
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<tr>
<th>PROOF</th>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>ΔABC is isosceles with AB = AC</td>
<td>1. Hyp.</td>
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<tr>
<td>2.</td>
<td>ΔABC ≅ ΔACB (A)</td>
<td>2. If two sides of a triangle are congruent, then the ∠s opposite those sides are ∠s.</td>
</tr>
<tr>
<td>3.</td>
<td>E is median to AB</td>
<td>3. Hyp.</td>
</tr>
<tr>
<td>4.</td>
<td>E is midpoint of AB</td>
<td>4. Def. of median</td>
</tr>
<tr>
<td>5.</td>
<td>E is median to AB</td>
<td>5. Hyp.</td>
</tr>
<tr>
<td>6.</td>
<td>E is median to AB</td>
<td>6. Same as 4</td>
</tr>
<tr>
<td>7.</td>
<td>E is midpoint of AB</td>
<td>7. Halves of congruent line segments are congruent.</td>
</tr>
<tr>
<td>8.</td>
<td>E ≅ E (B)</td>
<td>8. Reflexive property of congruence</td>
</tr>
<tr>
<td>9.</td>
<td>ΔBEC ≅ ΔECB</td>
<td>9. S.A.S.</td>
</tr>
<tr>
<td>10.</td>
<td>E ≅ E</td>
<td>10. Def. of congruent polygons</td>
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</tbody>
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<thead>
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<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>∠EAC is supp. to ∠1</td>
<td>1. Figure and def.</td>
</tr>
<tr>
<td>2.</td>
<td>∠ECA is supp. to ∠2</td>
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<td>3.</td>
<td>∠1 ≅ ∠2</td>
<td>3. Hyp.</td>
</tr>
<tr>
<td>4.</td>
<td>∠ECA ≅ ∠EBA</td>
<td>4. Supp. Angle Th.: If two angles are supp. to two congruent angles, then they are congruent.</td>
</tr>
<tr>
<td>5.</td>
<td>∠1 ≅ ∠2</td>
<td>5. If two angles of a triangle are congruent, then the sides opposite those angles are congruent (Th.).</td>
</tr>
<tr>
<td>6.</td>
<td>ΔABC is isosceles</td>
<td>6. Reverse of def.</td>
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<tr>
<th>PROOF</th>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>ΔABC = ΔDEF</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2.</td>
<td>BE ≅ BF; ∠F ≅ ∠E; BC ≅ EF</td>
<td>2. Def. of congruent polygons</td>
</tr>
<tr>
<td>4.</td>
<td>E is midpoint of AB</td>
<td>4. Same as 2</td>
</tr>
<tr>
<td>5.</td>
<td>EK ≅ EF (B); D ≅ CI (A);</td>
<td>5. Transitive property of congruence</td>
</tr>
<tr>
<td>6.</td>
<td>ΔABC ≅ ΔEKB</td>
<td>6. S.A.S.</td>
</tr>
</tbody>
</table>

### Page 115

<table>
<thead>
<tr>
<th>PROOF</th>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>At point S of line SC there exists an angle cong. to ∠DEF. Let this angle be ∠FBC.</td>
<td>1. Postulate 17</td>
</tr>
<tr>
<td>2.</td>
<td>Extend SC so that RS ≅ EF (A)</td>
<td>2. Postulate</td>
</tr>
<tr>
<td>3.</td>
<td>Let RS be the line through points R and C.</td>
<td>3. Postulate</td>
</tr>
<tr>
<td>4.</td>
<td>Let RS be the line through points R and A.</td>
<td>4. Postulate</td>
</tr>
<tr>
<td>5.</td>
<td>RS ≅ RS (A)</td>
<td>5. Hyp.</td>
</tr>
<tr>
<td>6.</td>
<td>ΔDEF ≅ ΔHBC</td>
<td>6. S.A.S.</td>
</tr>
<tr>
<td>7.</td>
<td>RS ≅ RS</td>
<td>7. Def. of congruent polygons</td>
</tr>
<tr>
<td>9.</td>
<td>RS ≅ RS (A)</td>
<td>9. Transitivity</td>
</tr>
<tr>
<td>10.</td>
<td>ΔCAR ≅ ΔBAC</td>
<td>10. If two sides of a triangle (ΔCAB) are congruent, then the angles opposite them are congruent (Th.).</td>
</tr>
<tr>
<td>13.</td>
<td>Transitivity</td>
<td>13. Transitivity</td>
</tr>
<tr>
<td>14.</td>
<td>Same as 10</td>
<td>14. Same as 10</td>
</tr>
<tr>
<td>15.</td>
<td>ΔCAR ≅ ΔBAC</td>
<td>15. Addition Postulate</td>
</tr>
<tr>
<td>16.</td>
<td>ΔABC ≅ ΔEBC</td>
<td>16. S.A.S.</td>
</tr>
<tr>
<td>17.</td>
<td>ΔABC ≅ ΔEBC</td>
<td>17. Theorem 11</td>
</tr>
</tbody>
</table>
### Page 11-A

<table>
<thead>
<tr>
<th>PROBLEM STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. Let P be the line through H and A</td>
<td>2. Post.</td>
</tr>
<tr>
<td>3. Let Q be the line through H and B</td>
<td>3. Post.</td>
</tr>
<tr>
<td>4. Let R be the line through Q and A</td>
<td>4. Radii of a circle are congruent</td>
</tr>
<tr>
<td>5. Let S be the line through R and B</td>
<td>5. Figures and def.</td>
</tr>
<tr>
<td>6. Let T be the line through H</td>
<td>6. Same as 1</td>
</tr>
<tr>
<td>7. Let U be the line through T and P</td>
<td>7. Reflexivity</td>
</tr>
<tr>
<td>8. ΔAQL = ΔQXI</td>
<td>8. S.S.S.</td>
</tr>
</tbody>
</table>

### Page 166

<table>
<thead>
<tr>
<th>PROBLEM STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. At point A of circle O there exists an angle congruent to ∠FDE. Let this angle be ∠CAR (A)</td>
<td>1. Why?</td>
</tr>
<tr>
<td>2. Extend AF so that AF = FB (S)</td>
<td>2. Why?</td>
</tr>
<tr>
<td>3. Let P be the line through F and A</td>
<td>3. Why?</td>
</tr>
<tr>
<td>4. Let Q be the line through P and B</td>
<td>4. Why?</td>
</tr>
<tr>
<td>5. Let R be the line through Q and A</td>
<td>5. Why?</td>
</tr>
<tr>
<td>6. ΔAPQ = ΔAFO</td>
<td>6. Why?</td>
</tr>
<tr>
<td>7. ∠AFO = ∠F</td>
<td>7. Why?</td>
</tr>
<tr>
<td>8. AB and DE are right angles.</td>
<td>8. Why?</td>
</tr>
<tr>
<td>10. ∠BAC = ∠F (A)</td>
<td>10. Why?</td>
</tr>
<tr>
<td>12. AB = BC (S)</td>
<td>12. Why?</td>
</tr>
<tr>
<td>13. ∠ASP = ∠AFO</td>
<td>13. Why?</td>
</tr>
<tr>
<td>14. ΔAQP = ΔAFO</td>
<td>14. Why?</td>
</tr>
<tr>
<td>15. ∠APQ = ∠AFO</td>
<td>15. Why?</td>
</tr>
<tr>
<td>16. ∠AFO = ∠F</td>
<td>16. Why?</td>
</tr>
<tr>
<td>17. ∠AFO = ∠F</td>
<td>17. Why?</td>
</tr>
</tbody>
</table>

### Page 166

<table>
<thead>
<tr>
<th>PROBLEM STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Circle O and circle H intersect at A and B</td>
<td>1. Hyp. and Figure</td>
</tr>
<tr>
<td>2. Let R be the line through H and A</td>
<td>2. Radii of the same circle are congruent</td>
</tr>
<tr>
<td>3. Let Q be the line through H and B</td>
<td>3. Hyp.</td>
</tr>
<tr>
<td>4. Let S be the line through Q and A</td>
<td>4. Def. of perpendicular lines</td>
</tr>
<tr>
<td>5. Let T be the line through R and B</td>
<td>5. Reflexivity</td>
</tr>
<tr>
<td>6. Let U be the line through T and P</td>
<td>6. M.L.</td>
</tr>
<tr>
<td>7. Let V be the line through U and Q</td>
<td>7. Def.</td>
</tr>
</tbody>
</table>

### Page 166

<table>
<thead>
<tr>
<th>PROBLEM STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. T = T</td>
<td>3. Reflexivity</td>
</tr>
<tr>
<td>4. ΔABC = ΔADC</td>
<td>4. S.S.S.</td>
</tr>
<tr>
<td>5. ΔABC = ΔADC</td>
<td>5. Def.</td>
</tr>
<tr>
<td>7. ΔABC = ΔADC</td>
<td>7. Reflexivity</td>
</tr>
<tr>
<td>8. ΔABC = ΔADC</td>
<td>8. S.S.S.</td>
</tr>
</tbody>
</table>
### Proof 175A

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CD \perp AB</td>
<td>Hyp.</td>
</tr>
<tr>
<td>2. ( \angle BOD ) is a straight angle</td>
<td>Definition</td>
</tr>
<tr>
<td>3. ( \angle 1 ) is sup. to ( \angle 2 )</td>
<td>Hyp.</td>
</tr>
<tr>
<td>4. ( m\angle 1 + m\angle 2 = 180 )</td>
<td>Definition</td>
</tr>
<tr>
<td>5. ( \angle 1 ) \perp ( \angle 2 )</td>
<td>Hyp.</td>
</tr>
<tr>
<td>6. ( m\angle 1 + m\angle 2 = 180 )</td>
<td>Substitution</td>
</tr>
<tr>
<td>7. ( m\angle 1 = 90 )</td>
<td>Prop. of numbers</td>
</tr>
<tr>
<td>8. ( \angle 1 ) \perp ( \angle 2 )</td>
<td>Definition</td>
</tr>
<tr>
<td>9. ( \angle 1 ) \perp ( \angle 2 )</td>
<td>Definition</td>
</tr>
</tbody>
</table>

### Proof 175B

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. CO \perp \triangle ABC</td>
<td>Why?</td>
</tr>
<tr>
<td>2. ( \angle AOC ) \perp ( \angle BOC ) ( (A) )</td>
<td>Why?</td>
</tr>
<tr>
<td>3. O is the center of the circle.</td>
<td>Why?</td>
</tr>
<tr>
<td>4. ( OF \cong OG ) ( (S) )</td>
<td>Why?</td>
</tr>
<tr>
<td>5. ( OC \cong OS ) ( (S) )</td>
<td>Why?</td>
</tr>
<tr>
<td>6. ( \angle AOC \cong \angle EOC )</td>
<td>Why?</td>
</tr>
<tr>
<td>7. ( \angle AOC \cong \angle OCB )</td>
<td>Why?</td>
</tr>
<tr>
<td>8. ( CO \perp AB )</td>
<td>If two lines intersect to form congruent adjacent angles, then the lines are perpendicular.</td>
</tr>
</tbody>
</table>

### Paragraph 160

The sentence that "A is the same distance from B as it is from C" is far too long. It is usually recorded as "A is equidistant from B and C," or "A is equally distant from B and C." Either of these is interpreted as before, \( AB = AC \). For the sake of later reference, we will make the following definition:

**Definitions:** "A is equidistant from B and C" means that \( AB = AC \).

### Paragraph 161

The reason for each statement will be left for you to supply:

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( PA \cong PB )</td>
<td>Hyp.</td>
</tr>
<tr>
<td>2. ( QA \cong QB )</td>
<td>Why?</td>
</tr>
<tr>
<td>3. ( QA \cong QB )</td>
<td>Why?</td>
</tr>
<tr>
<td>4. ( \angle QAD \cong \angle QBD )</td>
<td>Why?</td>
</tr>
<tr>
<td>5. ( \angle QDA \cong \angle QBA ) ( (A) )</td>
<td>Why?</td>
</tr>
<tr>
<td>6. ( QA \cong QB ) ( (S) )</td>
<td>Why?</td>
</tr>
<tr>
<td>7. ( QA \cong QB ) ( (S) )</td>
<td>Why?</td>
</tr>
</tbody>
</table>

### Paragraph 162

The proof is written as follows:

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( GA \cong CD )</td>
<td>Hyp.</td>
</tr>
<tr>
<td>2. ( C ) is equidistant from ( A ) and ( B )</td>
<td>Reverse of def.</td>
</tr>
<tr>
<td>3. ( AB ) and ( CD ) are radii of circles</td>
<td>Fig. and def.</td>
</tr>
<tr>
<td>4. ( OC \cong OD )</td>
<td>Why?</td>
</tr>
<tr>
<td>5. ( C ) is equidistant from ( A ) and ( B )</td>
<td>Reverse of def.</td>
</tr>
<tr>
<td>6. ( AB ) is the bisector of ( \angle C )</td>
<td>If two points ( (O ) and ( C) ) are each equidistant from the endpoints ( (A ) and ( B) ) of a line segment ( (AB) ), then the line joining these two points</td>
</tr>
</tbody>
</table>
### Proof 1: \( P \) is the midpoint of \( AB \)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Let ( P ) be the midpoint of ( AB )</td>
<td>1. Post.</td>
</tr>
<tr>
<td>2. ( PA = PB )</td>
<td>2. Def.</td>
</tr>
<tr>
<td>3. ( PB = PA )</td>
<td>3. Hyp.</td>
</tr>
<tr>
<td>4. ( P ) is the ( \perp ) bis. of ( AB )</td>
<td>4. If two points (( P ) and ( M )) are each equidistant from the endpoints of a line segment (( AB )), then the line joining them (( PM )) is the ( \perp ) bis. of the line segment.</td>
</tr>
</tbody>
</table>

### Proof 2: \( CD \) is the \( \perp \) bis. of \( AB \)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. Let ( CD ) be the line through ( O ) and ( A ). Similarly for ( CB ).</td>
<td>5. Post.</td>
</tr>
<tr>
<td>6. ( CD \parallel AB )</td>
<td>6. Radii of the same circle are congruent.</td>
</tr>
<tr>
<td>7. ( CD ) passes through ( O ).</td>
<td>7. If a point is equidistant from the endpoints of a line segment, then it lies on the perpendicular bisector of the line segment.</td>
</tr>
</tbody>
</table>
APPENDIX B

FORMAT TEST

1. Form X
   Part I
   Part II

2. Form G
   Part I
   Part II
1. Form X

FORMAT TEST
FORM: X. PART I

NAME_________________________ COURSE NO.________

DIRECTIONS: There are several ways to write a proof. You have learned one way called the flow-diagram format. Now you are to learn another way called the statement-reason format. Read the following explanation carefully, then work the problems as directed.

The basic idea of proof-making is to figure out and then write down an argument which shows that the GIVEN CONCLUSION follows logically from the GIVEN DATA. This argument consists of a set of intermediate statements or inferences and a corresponding set of supporting reasons for the validity of each statement or inference. In the statement-reason format, you are to supply a series of statements and reasons so that you start with the GIVEN DATA and end with the CONCLUSION. Example 1 illustrates both formats for the same proof.

Example 1 GIVFN: $\angle 1 \cong \angle 2$
CONCL: $ABC$ is an isosceles triangle

A. Flow-diagram format

\[
\begin{align*}
\text{hyp:} & \angle 1 \cong \angle 2 \\
\text{def:} & \angle BAC \text{ is supp. to } \angle 1 \\
\text{def:} & \angle BCA \text{ is supp. to } \angle 2 \\
\Rightarrow & \angle 2CA \cong \angle 2CA \\
\Rightarrow & \overline{AC} \cong \overline{BC} \\
\Rightarrow & \triangle ABC \text{ is isos.}
\end{align*}
\]
(1) Supplements of congruent angles are congruent.

(2) If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

(3) Reverse of the def. of isosceles triangle.

B. Statement-reason format

<table>
<thead>
<tr>
<th>Proof</th>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 \cong \angle 2 )</td>
<td>1. Hyp.</td>
<td></td>
</tr>
<tr>
<td>2. ( \angle \text{BAC is supp. to } \angle 1 ) and ( \angle \text{BAC is supp. to } \angle 2 )</td>
<td>2. Figure and def. of supp. angles</td>
<td></td>
</tr>
<tr>
<td>3. ( \angle \text{BAC} \cong \angle \text{BAC} )</td>
<td>3. Supplements of congruent angles are congruent.</td>
<td></td>
</tr>
<tr>
<td>4. ( \angle \text{Z} \cong \text{Z} )</td>
<td>4. If two angles of a triangle are congruent, then the sides opposite those angles are congruent.</td>
<td></td>
</tr>
<tr>
<td>5. ( \triangle \text{ABC is isosceles} )</td>
<td>5. Reverse of the def. of isosceles triangle</td>
<td></td>
</tr>
</tbody>
</table>

You will notice that there are several major differences between the two formats. First of all, the flow-diagram format is written from left to right, while the statement-reason format is a vertical listing of sentences. Secondly, the \( \rightarrow \) symbol is used extensively in the flow-diagram format and not at all in the statement-reason format. Thirdly, each reason is written in the right-hand column immediately next to the statement it justifies, in the statement-reason format. Finally, in the statement-reason format, there is no
visible grouping of statements; but no statement can be written down before all the other statements that it logically depends on have been written. For instance, in Example II you can't write that \( \triangle ABC \cong \triangle ABD \) until after you have written that \( AB = AC, \overline{BD} = \overline{DC}, \) and \( \overline{AB} = \overline{AD} \).

Example II  
**Given:** \( AB = AC; D \) is the midpoint of \( BC \)  
**Conclusion:** \( AD \perp BD \)

A. Flow-diagram format

(hyp) \( AB = AC \quad \iff \quad \angle B = \angle C \)

(hyp) \( D \) is midpoint of \( BC \)  
\[ \text{(def)} \quad \overline{BD} = \overline{DC} \]

(hyp) \( \overline{AB} = \overline{AD} \)

\[ \{ \text{def} \} \quad \triangle ABD \cong \triangle ACD \quad \{ \text{def} \} \quad \angle ADB \cong \angle ADB \]

(1) If 2 sides of a \( \Delta \) are \( \cong \), the \( \angle \) opp. them are \( \cong \).

(2) If 2 lines intersect to form \( \parallel \) adj. \( \angle \)s, the lines are \( \perp \).

B. Statement-reason format

<table>
<thead>
<tr>
<th>Proof</th>
<th>Statements</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( AB = AC )</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2.</td>
<td>( \overline{BD} = \overline{DC} )</td>
<td>2. If 2 sides of a triangle are ( \cong ), the angles opposite them are ( \cong ).</td>
</tr>
<tr>
<td>3.</td>
<td>( D ) is the midpoint of ( BC )</td>
<td>3. Hyp.</td>
</tr>
<tr>
<td>4.</td>
<td>( \overline{BD} = \overline{DC} )</td>
<td>4. Def. of midpoint</td>
</tr>
<tr>
<td>5.</td>
<td>( \overline{AB} = \overline{AD} )</td>
<td>5. Hyp.</td>
</tr>
<tr>
<td>6.</td>
<td>( \ldots )</td>
<td>( \Delta \ldots )</td>
</tr>
</tbody>
</table>
7. $\angle ADB \cong \angle ADC$  
8. $\angle ADB$ and $\angle ADB$ are adjacent
9. $AB \perp BD$

7. Def. of congruent polygons
8. Def. of adjacent angles
9. If two lines intersect to form congruent adj. angles, the lines are $\perp$.

The final example illustrates a more complex proof than previous examples.

Example III GIVEN: $\overline{AB} \perp \overline{BD}$; $\overline{AB} \perp \overline{DE}$; $\angle 1 \cong \angle 2$; $\overline{AB} \cong \overline{AD}$

CONCL.: $\angle C \cong \angle E$

A. Flow-diagram format

(hyp) $\overline{AB} \perp \overline{BD}$ (adj. $\angle ABD$ is a rt. $\angle$)
(hyp) $\overline{AB} \perp \overline{DE}$ (def. $\angle ABD$ is a rt. $\angle$)

(add. post. $\angle BAC = \angle 1 + \angle DAB$)
(add. post. $\angle BAC = \angle 1 + \angle DAB$)

(1) All right triangles are congruent.

B. Statement-reason format

<table>
<thead>
<tr>
<th>Proof</th>
<th>Statement/Reason</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AB} \perp \overline{BD}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. \( \triangle ABC \) is a rt. angle
3. \( AB \perp BC \)
4. \( \triangle ADE \) is a rt. angle
5. \( \triangle ABC \cong \triangle ADE \)
6. \( AB = AD \)
7. \( \angle 1 \cong \angle 2 \)
8. \( m\angle BAC = m\angle 1 + m\angle DAC \)
9. \( m\angle DAC = m\angle 2 + m\angle DAC \)
10. \( m\angle BAC = m\angle DAC \)
11. \( \angle BAC \cong \angle DAC \)
12. \( \triangle ABC \cong \triangle ADE \)
13. \( \angle C \cong \angle E \)

Change each of the following proofs from FLDH-DIA-RAK format to STATEMENT-REASON format.

Problem 1. GIVEN: \( AB \perp CD \)

CONCL. \( \angle 1 \cong \angle 2 \)

(hyp) \( AB \perp CD \) (def.) \( \angle 1 \cong \angle 2 \) are rt. \( \angle 1 \perp \angle 2 \)

(1) All rt. angles are congruent.
Problem 2. **GIVEN**: C is the midpoint of \( \overline{AB} \).

**CONCL.**: \( \overline{AB} \equiv \overline{AC} \).

\[
\begin{align*}
\text{(def.) } \angle BAC \; &\text{ and } \angle 1, \\
\text{(def.) } \angle BAC \; &\text{ and } \angle 2, \\
\text{(hyp.) } \angle 2 \; &\equiv \angle 1.
\end{align*}
\]

\[
\begin{align*}
\text{(def.) } \angle ABC \equiv \angle ABD, \\
\text{(def.) } \angle ABC \equiv \angle ABD, \\
\text{(hyp.) } \angle 3 \; &\text{ is the } \text{mid.} \text{ of } \overline{BC} \equiv \overline{CD} \; \text{(a)}.
\end{align*}
\]

\[
\text{AD} \equiv \text{AD}, \quad \text{BC} \equiv \text{BC}.
\]

**CONSTRUCT A PROOF FOR EACH OF THE FOLLOWING PROBLEMS. USE THE STATEMENT-REASON FORMAT.**

**Problem 3.** **GIVEN**: \( \overline{AD} \) is the median to \( \overline{BC} \);

\( \overline{AD} \perp \overline{BC} \); \( \overline{AB} \equiv \overline{AC} \).

**CONCL.**: \( \triangle ABC \) is isosceles \\
that is, \( \angle A \equiv \angle A \).

**Problem 4.** **GIVEN**: \( \overline{AD} \) bisects \( \angle A \) and \( \angle BCD \).

**CONCL.**: \( \overline{AD} \perp \overline{BC} \).
DIRECTIONS: Indicate your agreement or disagreement with each of the following statements on the answer sheet provided.

Mark (A) if you strongly agree
(B) if you mildly agree
(C) if you neither agree nor disagree
(D) if you mildly disagree
(E) if you strongly disagree.

1. Proofs are usually interesting to me.
2. Geometry is interesting to me.
3. Most students dislike writing proofs.
4. If I had more experience I would probably find the statement-reason format easier than the flow-diagram format.
5. I don't understand most of what we're doing in class.
6. The main reason for my taking geometry is because I need it to go to college.
7. Constructing proofs is the hardest part of the geometry course so far.
8. Geometry is the only subject I take which makes me think.
9. I can reason better using the statement-reason format.
10. I can reason better using the flow-diagram format.
11. I can sometimes think of two or more ways to prove a statement.
12. The most important ideas I will remember from my geometry course are the properties or characteristics of the various figures.

13. Even though I haven't really used the statement-reason format, I think it would be more difficult to use constantly than the flow-diagram format.

14. Constructing proofs is easy for most students.

15. I think I like geometry less than most of the other students in the class.

16. The statement-reason format seems easier to understand than the flow-diagram format.

17. Geometry has made me more conscious of assumptions which people make.

18. The statement-reason format appears to be simple to follow and understand.

19. I usually find it easy to write proofs in geometry.

20. I didn't think I'd like geometry, but I really do.

21. I often know what needs to be done in a proof, but I don't know how to write it down.

22. I hate geometry.

23. The statement-reason format seems harder to use than the flow-diagram format.

24. Writing proofs has been easy for me so far.

25. The statement-reason format brings out the reasoning process more clearly than the flow-diagram format.
26. I have no interest in geometry.
27. I almost always know what's going on in my geometry class.
28. Statement-reason proofs may be long but they are easier than flow-diagrams for me to understand.
29. Writing proofs is the most exciting part of geometry.
30. The flow-diagram format is simple to follow and understand.
31. Not too many students in my class really enjoy writing proofs.
32. I can understand the statement-reason format better than the flow-diagram format.
33. I enjoy the study of geometry.
34. I want to see the statement with the reason right next to it.
35. Learning how to construct a proof was hard for me at first but is not so hard now.
36. Most of the students in my class enjoy writing their own proofs for assigned problems.
37. I can figure out the steps in a proof more easily with the flow-diagram format than with the statement-reason format.
38. Most of the students in my class have no trouble constructing proofs.
39. I hate to write proofs.
40. Most of the other students in my class like geometry more than I do.
41. The statement-reason format brings out the reasoning process very clearly.
42. I will never be able to write a proof.
43. Most of the students in my class are having trouble constructing proofs.
44. The statement-reason format confuses me.
45. If I didn't need geometry to pass college entrance tests, I wouldn't be taking it.
46. I usually have no difficulty in writing a proof.
47. I don't see what connection geometry has with my ability to reason.
48. I will probably never use the statement-reason format by choice.
49. The main reason for studying geometry that is important to me is that national tests include items in geometry.
50. There are very few students in my class who like geometry.
51. I have found writing proofs the most difficult part of the geometry course.
52. I would like to continue using the statement-reason format.
53. Assuming that I had more experience with the statement-reason format, I would find it harder to use than the flow-diagram format.
54. Knowing how to prove a statement and writing the proof down are two quite different things.
55. I never think of more than one way to do a proof.
56. If you want to see how clear the reasoning process is,
you have to use the flow-diagram format.

57. Constructing proofs gets easier the more you do it.

58. Of all the subjects I take, geometry is the one which makes me think most.

59. I prefer the statement-reason format.

60. It is much easier for me to write a proof in the statement-reason format than to write a flow diagram.
2. Form G

There are several ways to write a proof. You have learned one way called the statement-reason format. Now you are to learn another way called the flow-diagram format. Read the following explanation carefully, then work the exercises as directed.

The basic idea of proof-writing is to figure out and then write down an argument which shows that the GIVEN CONCLUSION follows logically from the GIVEN DATA. This argument consists of a set of intermediate statements or inferences and a corresponding set of supporting reasons for the validity of each statement or inference. In the flow-diagram format, you are to supply the intermediate statements and the reasons for them so that you end up with a flow-diagram looking like the following:

\[ \text{GIVEN DATA} \rightarrow \begin{cases} \text{STATEMENTS} \\ \text{AND REASONS} \end{cases} \rightarrow \text{GIVEN CONCLUSION} \]

Example I illustrates both formats for the same proof.

x. a) GIVEN: \( L_1 \equiv L_2 \)

CONCLUSION: \( \triangle ABC \) is an isosceles triangle

a. Statement-reason format

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( L_1 \equiv L_2 )</td>
<td>Hypothesis</td>
</tr>
<tr>
<td>2. ( \angle A ) is supp. to ( L_1 ); ( \angle B ) is supp. to ( L_2 )</td>
<td>Figure and def. of supp. angles</td>
</tr>
<tr>
<td>3. ( \angle A \equiv \angle B )</td>
<td>Supp. of congruent angles are congruent</td>
</tr>
<tr>
<td>4. ( \overline{AB} \equiv \overline{BC} )</td>
<td>If 2 angles of a ( \triangle ) are ( \equiv ), then the sides opposite these ( \angle )s are ( \equiv )</td>
</tr>
<tr>
<td>5. ( \triangle ABC ) is isosceles</td>
<td>Reverse of the def. of isos. triangle</td>
</tr>
</tbody>
</table>

b. Flow-diagram format

\[ \begin{array}{c}
\text{Hyp.} \quad L_1 \equiv L_2 \\
\text{Fig./def.} \quad \angle A \equiv \angle B \quad \Rightarrow \quad \overline{AB} \equiv \overline{BC} \quad \text{is isos.}
\end{array} \]

1. Supp. of congruent angles are congruent.
2. If 2 angles of a \( \triangle \) are \( \equiv \), then the sides opp. these are \( \equiv \).
3. Reverse of the def. of isos. triangle
You will notice that there are several major differences between the two formats. First, the flow-diagram format is written from left to right, while the statement-reason format is a vertical listing of sentences. Second, the symbol is used extensively in the flow-diagram format and not at all in the statement-reason format.

"p \Rightarrow q" is read as "If p is a true statement, then q is a true statement." For example, the statement "If a triangle contains one right angle, the other two angles are acute angles" is written:

a triangle contains 1 right angle \Rightarrow the other two angles are acute

Another example of the use of the symbol is the statement "If \( \angle 1 \) and \( \angle 2 \) are congruent and supplementary, then they are right angles." In the "p \Rightarrow q" form, it is written:

\[ \{ \angle 1 \text{ and } \angle 2 \text{ are congruent} \} \Rightarrow \{ \angle 1 \text{ and } \angle 2 \text{ are right angles} \} \]

As a final example of the use of the symbol, "If \( x + 5 = 3x - 2 \), then \( 7 = 2x \)" becomes:

\[ x + 5 = 3x - 2 \Rightarrow 7 = 2x \]

In the next two examples, write a statement using the symbol and giving a conclusion which can be drawn from the given data.

**Example II:** \( \overline{BD} \) is the bisector of \( \angle ADC \)

**Example III:** \( \overline{AD} \perp \overline{BC} \)

You are correct if you wrote:

\( \overline{BD} \) is the bisector of \( \angle ADC \) \( \Rightarrow \angle ADB \neq \angle CDB \) for Example II.

For Example III, there are at least three possible conclusions from your present knowledge of geometry:

**III-A:** \( \overline{AD} \perp \overline{BC} \) \( \Rightarrow \angle ADB \) and \( \angle ADB \) are right angles

**III-B:** \( \overline{AD} \perp \overline{BC} \) \( \Rightarrow \angle ADB \neq \angle CDB \)

**III-C:** \( \overline{AD} \perp \overline{BC} \) \( \Rightarrow \overline{AD} \) is an altitude of \( \triangle ADB \)
Essentially, the arrow symbol indicates that a conclusion is being drawn from the statement of statements preceding the arrow symbol. The statement after the arrow symbol is the conclusion, and a reason must always be given to justify that conclusion. The reason is put over the arrow symbol in abbreviated form, or a code number is written over the arrow symbol and the reason with its code number is written out in full after the flow-diagram is completed. Moreover the same reason appears more than once in a proof, the same code number is used each time, or the same abbreviated form is written over the arrow symbol.

The third main difference between the two formats—perhaps the most important difference for you when you use the flow-diagram format—is the grouping and placement of statements.

Example IV: GIVEN: $\overline{AB} \cong \overline{CD}$; $O$ is the midpoint of $BD$.

CONCLUSION: $\overline{BO} \perp \overline{AD}$

A. Statement-reason format

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{BO} \perp \overline{AB}$</td>
<td>Hyp.</td>
</tr>
<tr>
<td>$\overline{BO} \perp \overline{CD}$</td>
<td>Hyp.</td>
</tr>
<tr>
<td>$O$ is the midpoint of $BD$</td>
<td>Hyp.</td>
</tr>
<tr>
<td>$\Delta AOB \cong \Delta COD$</td>
<td>Def. of midpoint</td>
</tr>
<tr>
<td>$\angle AOB$ and $\angle COD$ are $\cong$</td>
<td>Def. of congruent figures</td>
</tr>
<tr>
<td>$\angle AOB$ and $\angle COD$ are adjacent</td>
<td>Def. of adjacent angles</td>
</tr>
<tr>
<td>$\overline{AO} \parallel \overline{CO}$</td>
<td>If two lines intersect to form $\cong$ adj. angles, the lines are perpendicular.</td>
</tr>
</tbody>
</table>

B. Flow-diagram format

\begin{align*}
\text{(hyp) } & \overline{AB} \cong \overline{CD} \frac{1}{2} \text{ (1) If } \overline{AB} \cong \overline{CD} \text{, the } \cong \text{ sides are } \cong \\
\text{(hyp) } & \text{Def. of midpoint. } \overline{BO} \frac{1}{2} \text{ (2) If two lines intersect to form $\cong$ adj. angles, the lines are perpendicular.} \\
\text{(hyp) } & \overline{AB} \cong \overline{CD} \frac{1}{2} \text{ (def) } \overline{BO} \perp \overline{AD} \text{ and } \overline{BO} \perp \overline{AD} \\
\end{align*}

In Example IV three statements are grouped together by a brace symbol ($\{\}$). These, with reason 1.5, enable one to conclude that triangles $\Delta AOB$ and $\Delta COD$ are congruent, so all three statements must precede the arrow symbol.
symbol for that conclusion. Notice that a part of the GIV. is not written down until it is needed to draw a conclusion.

The final example illustrates more complex groupings and placement than previous examples.

**Example VI:**

**GIVEN:** \( \overline{AB} \perp \overline{BC} \), \( \overline{AD} \perp \overline{BD} \);

**CONCLUSION:** \( \angle 1 \cong \angle 3 \)

**A. Statement-reason format**

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AB} \perp \overline{BC} )</td>
<td>1. Hyp.</td>
</tr>
<tr>
<td>2. ( \angle 1 ) is a right angle</td>
<td>2. Def. of perpendicular lines</td>
</tr>
<tr>
<td>3. ( \overline{AD} \perp \overline{BD} )</td>
<td>3. Hyp.</td>
</tr>
<tr>
<td>4. ( \angle 3 ) is a right angle</td>
<td>4. Same as 2</td>
</tr>
<tr>
<td>5. ( \angle 1 \equiv \angle 3 )</td>
<td>5. Right angles are congruent</td>
</tr>
<tr>
<td>6. ( \overline{AB} \equiv \overline{BD} )</td>
<td>6. Hyp.</td>
</tr>
<tr>
<td>7. ( \angle 1 \equiv \angle 2 )</td>
<td>7. Hyp.</td>
</tr>
<tr>
<td>8. ( \angle 3 \equiv \angle 3 \equiv \angle 3 )</td>
<td>8. Add. Post. for angles</td>
</tr>
<tr>
<td>9. ( \angle 3 \equiv \angle 3 \equiv \angle 3 )</td>
<td>9. Add. Post. for angles</td>
</tr>
<tr>
<td>10. ( \angle 3 \equiv \angle 3 \equiv \angle 3 )</td>
<td>10. Add. Post. for equality</td>
</tr>
<tr>
<td>11. ( \angle 1 \equiv \angle 3 )</td>
<td>11. Reverse of def. of congruent angles</td>
</tr>
<tr>
<td>12. ( \triangle \equiv \triangle )</td>
<td>12. ASA</td>
</tr>
<tr>
<td>13. ( \angle \equiv \angle )</td>
<td>13. Def. of congruent triangles</td>
</tr>
</tbody>
</table>

**B. Flow-diagram format**

\[
\begin{align*}
(\text{hyp}) \quad \overline{AB} & \perp \overline{BC} \\
(\text{def}) \quad \angle 1 \equiv \angle 3 & \text{ is rt. } \angle \\
(\text{hyp}) \quad \overline{AD} & \perp \overline{BD} \\
\end{align*}
\]

\[
\begin{align*}
(\text{add. post. for } \angle) \quad \angle 1 & \equiv \angle 2 + \angle 3 \\
(\text{add. post. for } \angle) \quad \angle 3 & \equiv \angle 4 + \angle 5 \\
(\text{hyp}) \quad \angle 1 & \equiv \angle 2 \\
\end{align*}
\]

\[
\begin{align*}
(\text{def. } \angle \equiv \angle) \quad \angle 1 & \equiv \angle 3 \\
\end{align*}
\]

\(1\) Right angles are congruent.
CHANGE EACH OF THE FOLLOWING PROOFS FROM STATEMENT-REASON FORMAT TO
FLOW-DIAGRAM FORMAT.

Problem 1. GIVEN: $\overline{AB} \perp \overline{BD}$

CONCLUSION: $\angle 1 \cong \angle 2$

PROOF:

STATEMENTS

1. $\overline{AB} \perp \overline{BD}$ 1. Hyp.
2. $\angle 1$ and $\angle 2$ are right angles 2. Def. of perpendicular lines
3. $\angle 1 \cong \angle 2$ 3. Right angles are congruent.

Problem 1. GIVEN: $O$ is the midpoint of $\overline{AB}$

CONCLUSION: $\overline{OC} \cong \overline{OB}$

PROOF:

STATEMENTS

1. $O$ is the midpoint of $\overline{AB}$ 1. Hyp.
2. $\overline{OC} \cong \overline{OB}$ (a) 2. Def. of midpoint.
3. $\angle 1 \cong \angle 2$ 3. Hyp.
4. $\angle CAD$ and $\angle 1$ are vert. $\angle$s 4. Prop 8: def. of vert. $\angle$s
5. $\angle BAO \cong \angle 1$ 5. Vert. $\angle$s are congruent.
6. $\angle 2 \cong \angle 1$ (a) 6. Transitive prop. of congruence
7. $\angle AOB$ and $\angle AOD$ are vert. $\angle$s 7. Same as 4
8. $\angle AOB \cong \angle AOD$ (a) 8. Same as 5
9. $\triangle AOB \cong \triangle AOD$ 9. A.S.A.
10. $\overline{OC} \cong \overline{OB}$ 10. Def. of congruent triangles

CONSTRUCT A PROOF FOR EACH OF THE FOLLOWING PROBLEMS. USE THE FLOW-
DIAGRAM FORMAT.

Problem 3. GIVEN: $D$ is the midpoint of $\overline{BC}$

CONCLUSION: $\overline{BD} \cong \overline{DC}$

Problem 4. GIVEN: $\overline{AJ}$ bisects $\angle BAC$ and $\overline{AJ} \perp \overline{BD}$

CONCLUSION: $\overline{AJ} \perp \overline{BD}$
DIRECTIONS: Indicate your agreement or disagreement with each of the following statements on the answer sheet provided.

Mark: (A) if you strongly agree
      (B) if you mildly agree
      (C) if you neither agree nor disagree
      (D) if you mildly disagree
      (E) if you strongly disagree

1. Proofs are usually interesting to me.
2. Geometry is interesting to me.
3. Most students dislike writing proofs.
4. If I had had more experience I would probably find the flow-diagram easier than the statement-reason format.
5. I don't understand most of what we're doing in class.
6. The main reason for my taking geometry is because I need it to go to college.
7. Constructing proofs is the hardest part of the geometry course so far.
8. Geometry is the only subject I take which makes me think.
9. I can reason better using the statement-reason format.
10. I can reason better using the flow-diagram format.
11. I can sometimes think of two or more ways to prove a statement.
12. The most important idea I will remember from my geometry course are the properties or characteristics of the various figures.

13. Even though I haven't really used the flow-diagram format, I think it would be more difficult to use constantly than the statement-reason format.

14. Constructing proofs is easy for most students.

15. I think I like geometry less than most of the other students in the class.

16. The flow-diagram format appears easier to understand than the statement-reason format.

17. Geometry has made me more conscious of assumptions which people make.

18. The flow-diagram format appears to be simple to follow and understand.

19. I usually find it easy to write proofs in geometry.

20. I didn't think I'd like geometry, but I really do.

21. I often know what needs to be done in a proof, but I don't know how to write it down.

22. I hate geometry.

23. The flow-diagram format seems harder to use than the statement-reason format.

24. Writing proofs has been easy for me so far.

25. The flow-diagram format brings out the reasoning process more clearly than the statement-reason format.
26. I have no interest in geometry.
27. I almost always know what's going on in my geometry class.
28. Statement-reason proofs may be long but they are easier than flow-diagram for me to understand.
29. Writing proofs is the most exciting part of geometry.
30. The statement-reason format is simple to follow and understand.
31. Not too many students in my class really enjoy writing proofs.
32. I can understand the flow-diagram format better than the statement-reason format.
33. I enjoy the study of geometry.
34. The flow-diagram format confuses me. I want to see the statement with the reason right next to it.
35. Learning how to construct a proof was hard for me at first but is not so hard now.
36. Most of the students in my class enjoy writing their own proofs for assigned problems.
37. I can figure out the steps in a proof more easily with the statement-reason format than with the flow-diagram format.
38. Most of the students in my class have no trouble constructing proofs.
39. I hate to write proofs.
40. Most of the other students in my class like geometry more than I do.
41. The flow-diagram format brings out the reasoning process.
very clearly.

42. I will never be able to write a proof.

43. Most of the students in my class are having trouble constructing proofs.

44. The flow-diagram format confuses me.

45. If I didn't need geometry to pass college entrance tests, I wouldn't be taking it.

46. I usually have no difficulty in writing a proof.

47. I don't see what connection geometry has with my ability to reason.

48. I will probably never use the flow-diagram format by choice.

49. The main reason for studying geometry that is important to me is that national tests include items in geometry.

50. There are very few students in my class who like geometry.

51. I have found writing proofs the most difficult part of the geometry course.

52. I would like to continue using the flow-diagram format.

53. Assuming that I had more experience with the flow-diagram format, I would find it harder to use than the statement-reason format.

54. Knowing how to prove a statement and writing the proof down are two quite different things.

55. I never think of more than one way to do a proof.

56. If you want to see how clear the reasoning process is, you have to use the statement-reason format.
57. Constructing proofs gets easier the more you do it.
58. Of all the subjects I take, geometry is the one which makes
me think most.
59. I prefer the statement-reason format.
60. It is much easier for me to write a proof in the flow-diagram format than to write statements and reasons.
APPENDIX C

ITEM AND TEST RELIABILITY COEFFICIENTS FOR THE SUBTESTS OF
PART II OF THE FORMAT TEST (KUDER-RICHARDSON FORMULA 8)
### ATTITUDE TOWARD GEOMETRY SUBTEST

**Mean** = 38.78  
**S.D.** = 12.05  
**Reliability** = .877

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### ATTITUDE TOWARD FLOW-DIAGRAMS SUBTEST

**Mean** = 33.98  
**S.D.** = 23.24  
**Reliability** = .970

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### ATTITUDE TOWARD PROOF SUBTEST

Mean = 38.53  
S.D. = 14.26  
Reliability = .904

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BIBLIOGRAPHY


27. Deer, George Wendell. "The Effects of Teaching an Explicit Unit in Logic on Students' Ability to Prove Theorems in Geometry." Dissertation Abstracts International, XXX, 2284-B.


65. Lloyd, Daniel B. "Teaching a Unit in Logical Reasoning in the Tenth Grade." The Mathematics Teacher, XXXVI (May, 1943), 226-29.


100. Stein, Harry L. "Characteristic Differences in Mathematical Traits of Good, Average, and Poor Achievers in Demonstrative Geometry." The Mathematics Teacher, XXXVI (April, 1943), 164-68.


