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Convective scale selection and the initiation of deep cumulus

Balaji, Venkatramani, Ph.D.
The Ohio State University, 1987
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UMI
CONVECTIVE SCALE SELECTION AND THE
INITIATION OF DEEP CUMULUS

DISSERTATION

Presented in partial fulfilment of the requirements
for the degree Doctor of Philosophy
in the Graduate School of the Ohio State University

by

Venkatramani Balaji, M.Sc.

The Ohio State University
1987

DISSERTATION COMMITTEE:

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Full many a glorious morning I have seen
Flatter the mountain tops with sov'reign eye,
Kissing with golden face the meadows green,
Gilding pale streams with heavenly alchemy.
Anon permit the basest clouds to ride
With ugly rack on his celestial face,
And from the forlorn world his visage hide,
Stealing unseen to west with this disgrace:
Even so my sun one day did shine
with all triumphant splendour on my brow,
But out, alack, he was but one hour mine,
The region cloud hath mask'd him from me now.
Yet him for this my love no whit disdaineth:
Suns of the world may stain, when heaven's sun staineth.

— W. Shakespeare, Sonnet XXXIII
Dedicated to the people of Bhopal
victims of corporate greed
ACKNOWLEDGEMENTS

First of all, I would like to acknowledge my immense gratitude to the Advanced Study Program at the National Centre for Atmospheric Research for providing me with the position that made this research possible. These last two years at NCAR have been the most fruitful of my life. My gratitude also extends to Prof John Shaw, Prof Thomas Seliga and all the others who made the passage from the Physics Department at OSU to the Convective Storms Division at NCAR less rough than it might have been. Support during the early years of graduate study was provided by Tom Seliga through NSF grant ATM-8003376.

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VITA

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FIELDS OF STUDY

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ABSTRACT

Studies have shown that the organized lifting of moist low level air at horizontal scales on the order of tens of kilometres is closely associated with the formation of severe storms. In the absence of large-scale forcing, the organized lifting at the required scale must be produced by local convection. The linear theory of convection in an isolated boundary layer fail to provide the required scales. The studies of Clark et al. (1986) show that a possible explanation may be found in the excitation of gravity waves in the stable tropospheric layer by boundary layer convection in the presence of vertical wind shear. In the present study, it is shown that the reorganization of the planetary boundary layer subsequent to the development of the deep gravity wave mode leads to the selection of scales appropriate to deep convection. Comparisons between models of convection in the linearized and the full dynamical equations suggest that while the onset of the deep mode may be a linear effect, its enhancement and the subsequent suppression of the shallow boundary layer mode may be significantly modulated by non-linear effects.

Several results regarding cloud formation are inferred. The directional shear in the boundary layer appears to a sensitive parameter in selecting spatial organizations of a cloud field. The growth of clouds appears to decouple the boundary layer modes in phase from the stable layer modes, an effect that cyclically enhances and suppresses cloud growth. Finally, the interaction of a moving storm system in its severe stage with the boundary layer modes appear to provide an explanation for the spatial and temporal distribution of new convective cells in a multicellular storm system.
CHAPTER I

CONVECTIVE ORGANIZATION: AN INTRODUCTION

Two of the principal motions seen in the atmosphere are the convective and the wave motion, both of which may play important roles in certain modes of organization of the atmosphere. Of these, convection is the one of the greatest interest to humankind, leading, as it frequently does, to the formation of deep cumulus or cumulonimbus, often giving rise to precipitation or severe thunderstorms. As opposed to seasonal or climatic change, which is the other meteorological phenomenon with great impact on human awareness, deep convection is a phenomenon localized in space and time, and is hence to be studied on vastly different spatial and temporal scales. The study of atmospheric convection can be organized into several different scales, starting from the large or synoptic scale, familiar to all through the daily barrage of broadcast weather information, through the mesoscale (~ 100 to 1000 km), to the small scale (~ 10 km). (Of course, in all this, phenomena at microscale involving the physics of water substance play a central role, and it is in fact the enormous range of scales involved that makes this a compelling subject for intellectual curiosity.) It is the small scale organization of convection that is the subject of study here.

Observations of deep convective phenomena indicate that they often exhibit a distinct horizontal spatial scale associated with them. Isolated thunderstorms generally fall into a relatively narrow band of scales in the range of 10 to 30 km,
and the same holds for individual cells within larger convective systems. These studies have shown that the organized lifting of the moist surface air at horizontal scales on the order of tens of kilometres is closely associated with the formation of severe storms. In the absence of large-scale forcing, the organized lifting at the required scale must be produced by local convection. But when the development of local convection out of a stratified atmosphere is examined, a problem arises, for the small scale convection fails to provide the appropriate scales. The diurnal fair-weather atmosphere generally consists of a boundary layer adiabatically stratified in temperature, so that the adiabatic ascent of a parcel of air occurs under neutrally stable conditions. Above the planetary boundary layer (PBL), the atmosphere up to the tropopause is stable to dry adiabatic ascent. However, once a moist parcel reaches saturation, its ascent follows the moist adiabat. An atmosphere that is stable to dry adiabatic ascent but not to moist, is called conditionally unstable, and is known to be the most suitable to the formation of precipitating systems, if a mechanism to release the instability exists. In the absence of large scale forcing, such a mechanism must be sought in the small scale convection in the shallow boundary layer, which is presumed to continue until the overturning releases the conditional instability by some means and leads to the formation of deep convection. The problem of shallow convection that serves as a description of motions in the PBL is precisely the problem that has historically led to the first theoretical formulations of convection in fluids. Convection arising out of the thermal instability is the classical

1 Reviews of observational and theoretical studies of severe storms may be found in such surveys as Byers and Braham, 1949; Ludlam, 1963; Foote and Knight, 1979; Knight and Squires, 1982.

2 In the mid-latitudes, the PBL is usually 1 to 2 km deep, and the stable tropospheric layer about 10 km deep. In the tropics, the boundary layer is generally much shallower, and the stable layer deeper.
expression. This occurs in flows that are subjected to an unstable (top-heavy) density gradient, or equivalently a temperature gradient opposite to gravity.\(^3\) The inertia of an initially stratified fluid usually renders it stable until a certain critical value of the applied adverse gradient is reached, at which point the fluid becomes convectively unstable. Historically, this critical point is expressed in terms of the non-dimensional Rayleigh number \(Ra\), which expresses the ability of the adverse density gradient to overcome the viscous forces inhibiting vertical motion, depending in general on the fluid, its depth, and the kinematic parameters of the initial flow. Its classic formulation is the Rayleigh-Bénard instability in which the fluid initially at rest and contained between two plates is heated uniformly from below. Since the fluid is unstable for \(Ra\) above the critical value, infinitesimal perturbations tend to grow divergently. The analysis into normal spectral modes (Chandrasekhar, 1961) is the customary approach: the flow is found to be stable to all perturbations except in a few of the spectral modes. This spectral quality of the instability translates into a periodic spatial structure over an infinite domain, with a well-defined scale selection. This scale selection may take the form of convective cells (as in the case of the Bénard instability, which forms hexagonal cells), or other periodic forms, such as rolls or vortices, in other flows with a non-zero velocity profile. Since the normal mode analysis deals only with the onset of convection under an infinitesimal perturbation, it suffices to treat the problem with the Navier-Stokes equations in linear homogeneous form, and this approach is also called a linear stability analysis.

The selection of discrete scales is often observed in different convective phenomena over a wide range of scales from above the mesoscale down to the small

---

\(^3\) As the equation of state negatively correlates density with temperature, an adverse temperature gradient leads to an unstable density gradient for most fluids, except where the layer is deep enough that the density becomes hydrostatically stratified.
scale. Restricting attention for the moment to the PBL, it may be noted that there are modes of boundary layer organization into horizontal roll vortices, and also the related phenomenon of cloud streets, long arrays of small cumuli arranged in rows: these show a horizontal spatial scale of a few km, and can often retain their order over several hundred kilometres. Horizontal roll vortices in the boundary layer are frequently observed by low-flying aircraft, and the allied phenomenon of cloud streets may provide visual evidence (Kuettner, 1959; Malkus and Richl, 1964). Glider pilots often report updraft “streets” even in the absence of clouds, making it possible to travel long distances instead of circling a thermal (Kuettner, 1971). The orientation of the streets was found in these studies to be along the direction of shear for the shallow cases, and the wavelength of these longitudinal modes was between 2 and 6 km. The discrete spectral nature of the phenomenon points to linear analyses to provide an angle of approach: it was hence to the linear stability of shallow convection that attention was first directed. A lot of the initial work on scale selection in convection focused on cloud streets (Kuettner, 1971; Kuo, 1963; Asai, 1970; Asai, 1972; Deardorff, 1972). In its basic form, it is shown that the most unstable mode is longitudinal in convectively unstable plane Couette flow (two-dimensional flow between plates with a constant shear). The preference for longitudinal modes is demonstrated by showing that shear exerts a stabilizing influence (i.e., higher critical Rayleigh number) on perturbations.

4 Radiation terminology, such as wavelength, is used to describe these periodic structures even when they are not wave phenomena.

5 The distinction between deep and shallow convection is one of importance here. Shallow convection is used to signify boundary layer phenomena; deep convection involves the troposphere above as well even when its entirety is not subject to convective overturning. Under stable thermodynamic profiles, deep convection may give rise to shallow clouds. The crucial differences are that variations in density may be neglected in shallow convection, greatly simplifying the equations; and that the non-linear effects of the moisture physics are likely to be small.
in the plane of the flow. This has been extended to consider various types of variable shear profiles. The relative strength of the thermal stratification to the inertial effects of the shear is often described in terms of another non-dimensional quantity, the Richardson number $R_i$. Asai (1970), considering the case of different kinds of variable shear profiles, classifies the cases into different $R_i$ regimes. Where thermal effects dominate, the instability is thermal, as in the case of plane Couette flow; however shear-induced inertial instabilities are also possible. This occurs when the wind profile contains an inflection point, when the shear reverses sign; vorticity normal to the flow plane, which is a conserved quantity under the linearized equations, is equal at points across the inflection point, so that the fluid may overturn while still conserving vorticity (Kuettner, 1971). Preferentially transverse modes may also arise out of the inertial instability.

The spectral nature of the normal modes thus leads to a scale selection mechanism in idealized linear (and laminar) flows. There is no intuitively immediate reason to expect the same behaviour for finite-sized eddies in the terrestrial atmosphere, where the motion is strongly non-linear and often turbulent, where both the Reynolds number and the Rayleigh number typically are asymptotically large; however, models that explicitly include turbulence have also been shown to give rise to modal instabilities. The observations of LeMone (1973) of roll vortices in the boundary layer, suggest that besides the longitudinal thermal modes, the inflection-point instability modes are also observed. She found that certain features of roll vortex morphology (specifically, aspect ratio with respect to convective layer depth, and angle subtended by the mean wind at the roll axis) most closely resembles the models of the inflection point instability of Faller and Kaylor (1960), Lilly (1966), and Brown (1970). These models deal with instabilities in the Ekman
spiral, a term referring to the turning wind profile generally found in the PBL under the influence of the forces arising out of large-scale pressure gradients, the Coriolis force, and the dissipating effect of turbulent viscosity. The presence of an inflection point in the wind profile in the direction transverse to the mean flow produces the instability in these models: since the inflectional instability produces transverse modes, these modes are also longitudinal with respect to the mean wind direction.

Besides the observations of Kuettner and LeMone, Eymard (1985) has related the model of Asai (1972) to multiple Doppler radar observations of the tropical convective boundary layer. Linear stability analyses of parallel shear have also been applied to boundary layer profiler data by Davis and Peltier (1976). Linear theory would thus appear to adequately describe some of the scale selection mechanisms operating in shallow convection. What is common to all these models and observations, and most relevant to the problem of scale selection in deep convection, is that they lead to an aspect ratio of 2 to 4 with respect to the height of the fluid layer. For a PBL even 2 or 3 km deep, an extreme mid-latitude value, the resulting horizontal spatial scales available from boundary layer convection are at best only marginally sufficient to describe the lifting associated with deep convection.

Small scale boundary layer convection thus fails to provide an adequate explanation of the horizontal scales of the lifting associated with deep convection and the development of precipitating systems. Hitherto, this has not been thought a serious problem: often the genesis of new convection is associated with pre-existing synoptic or mesoscale phenomena that give rise to the necessary low-level convergence of moist air to provide the lifting. These may occur in the wake of a frontal passage, or in the moist outflow of a decaying cyclone or other mesoscale convective system (MCS). In the mesoscale modeling of convection, a scale separation between
the mesoscale system and the individual clouds or cells that constitute it is often postulated. Such models treat the convection associated with individual cumulus as a subgrid scale phenomenon, whose mesoscale influence may be parameterized in terms of quantities defined at a point, and which in turn responds to the large scale forcing and may be described thereby. Such a characterization of convection falls into the general class of cumulus parameterizations, and the underlying assumption is that the large scale dynamical influence of a cumulus in its deep, precipitating stage is generally insensitive to the mechanism of instability release involved in its initiation. Such parameterizations include the CISK hypothesis (Ooyama, 1964; Charney and Eliassen, 1964), and the schemes of Arakawa and Schubert (1974) and Fritsch and Chappell (1982). Such schemes have generally chosen to demonstrate that the appropriate conditions, such as the availability of moisture, and a conditionally and convectively unstable thermodynamic stratification, exist, and postulating some mechanism of low-level convergence to release the instability. In the CISK hypothesis, the low-level convergence was caused by frictionally induced differential motions in the moist low-level flow. The hypothesis was extended by Lindzen (1974) to what is called wave-CISK, where the initial convergence is induced by atmospheric waves rather than friction. In the Arakawa-Schubert and Fritsch-Chappell schemes, the low level convergence arises out of large scale forcing associated with synoptic conditions and the aggregate effect of the mesoscale system.

If the scale separation assumed in the studies above are valid, the mesoscale system may indeed be independent of the scales associated with cumulus convec-

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7 The Conditional Instability of the Second Kind, occurring at scales of 100 to 200 km. It is so named to distinguish it from the conventional form of the instability that leads to local convection at much smaller scales.
tion. However, Tripoli et al. (1986) has shown that care is needed in making a proper choice of model resolution for the scale separation assumption to be valid, for the accuracy of the parameterization is strongly dependent upon the choice of scale. Lindzen (1974) also found that convective scales selected by the wave-CISK mechanism are tuned to the wavelength of the applied disturbance. This suggests a sensitivity to scale even in the mesoscale; and there is definitely no reason to suppose that the convective scales associated with individual cumuli are independent of the scale of the forcing. It is apparent that the small scale description of convection, at least, must attempt to reproduce mechanisms of scale selection associated with the initiation of deep cumulus convection. The assumptions involved in cumulus parameterization can only be tested by an investigation of scale selection in the small scale characterization of convection.

In the modeling of individual cumulus, where convection arises out of a blue sky, so to speak, the scale of the forcing must be taken into account. Attempts to model this have generally been done for the case of shallow fair-weather cloud ensembles, such as the models of Hill (1974), Hill (1977), and Yau and Michaud (1982). In the case of deep clouds, however, it is the late and severe stage which has generally been of interest to modelers, and mature clouds have generally been treated as though they were independent of the manner of their initiation. Single clouds have usually been initialized by providing an artificial “bubble”, a localized temperature or moisture anomaly, with its scale chosen to be of the order of 10 or 20 km in order to produce a reasonable simulacrum of the final storm. The underlying assumption of insensitivity to initialization is often justified on account of the quasi-steady circulation that some storms eventually achieve: if the time dependence eventu-

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8 This is discussed in greater detail in section II.7, on model initialization.
ally vanishes, it is a boundary-value rather than an initial value problem, and all the effects of initialization appear only as transients. The quasi-steady assumption however holds only over a limited phase of the thunderstorm life cycle; besides, the periodic regeneration of new cells in a propagating storm system (Browning, 1977; Marwitz, 1972; Chalon et al., 1976) cannot be described so.

It is thus apparent that a mechanism to change the horizontal scales of boundary layer convection to those appropriate to deep convection must be sought. Several studies have looked to non-linear aspects of convection, assuming the low aspect ratio to be an effect of the linearity assumption; a review of these efforts may be found in Busse (1978). Chang and Shirer (1983) have examined the effects of the inclusion of the non-linear advection term in the momentum equation in a numerical model of Rayleigh-Bénard convection: transitions leading to the broadening of cell dimensions, and thus larger aspect ratios, were observed. The authors suggest this mechanism as an explanation of the smooth variation in cloud street spacings observed by Atlas et al. (1983).

One other fact of significance in this regard is the persistence of selected scales over large regions of space, leading to periodic organizations; and it is to periodic phenomena at these scales that attention is now directed. These include propagating cloud bands in mesoscale storm outflows, seen in the satellite pictures in Erickson and Whitney (1973): these are reported to have wavelengths of the order of 10 km. Besides cloud bands, periodic structures are also seen in precipitation systems, called rain bands, in extra-tropical convective systems (Browning, 1974; Harrold and Austin, 1974; Houze et al., 1976; Hobbs and Locatelli, 1978). These

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9 In the case under study, the very severe hailstorm of 1 Aug 1981, the observed duration of the quasi-steady circulation was 45 min in a lifetime of six hours (Tuttle et al., 1987).
are typically several hundred kilometres long, separated by distances of the order of 10 km. Some bands are of small width (~ 2 km), while others are broad bands exhibiting considerable periodic substructure, indicating modulations at more than one wavelength. Tropical squall lines often show a periodic structure consisting of individual cells along the line (Marroquin and Raymond, 1982; Bolton, 1984). Redelsperger and Lafore (1987) reported scales of about 25 km associated with these structures. The presence of fog bands with a spacing of about 30 km has also been noted (Walter and Overland, 1984).

Periodic spacings in the same range are also observed in association with deep cloud streets. Malkus and Riehl (1964) report observations of cloud streets with spacings ranging from 15 to 100 km, with both transverse and longitudinal orientations seen. A correlation was seen between cloud size and street spacing, with the lower end occupied by clouds 5 to 6 km deep, occurring in longitudinal bands of 15 to 25 km wavelength. The longer wavelengths were often associated with transverse bands. The first attempts to explain this phenomenon (Sun, 1978) resorted to an instability associated with a much deeper convective layer; the effects of convection on the large scale circulation were parameterized in CISK fashion, and the linear stability analysis retained the aspect ratios associated with boundary layer roll vortices, but now referenced to a much larger depth. However, the occurrence of spacings in the range 15 to 25 km have been reported even when convection is very shallow (LeMone and Meltin, 1984). The large aspect ratios associated with this case (in the range of 30 to 40 with respect to the 600 m deep subcloud layer) do not admit of an easy explanation based on convective instabilities alone.

An alternate clue to the origin of such structures may be found in the fact that other periodic structures that have their origin in atmospheric waves exist
at commensurate wavelengths. Some manner of interaction between atmospheric waves and convection may be examined as a means of scale selection. Wave motions in fluids generally occur in initially stable stratifications, without the adverse gradients that are the motive for the convective instabilities. The perturbation of a stable flow by any means leads to an oscillatory behaviour that manifests itself in fluid waves. Such waves in the atmosphere are often called gravity waves, since the restoring force is gravity. Gravity waves can be observed at scales ranging from $\sim 10$ km to the gravitational-rotational waves that can be thousands of kilometres in wavelength and have time periods that measure in days, and are significantly mediated by the Coriolis force. The initial perturbation can be orographic or meteorological features serving as obstacles to a stratified flow, such as the waves observed in the lee of mountains and mesoscale convective systems. Since there is almost always a dissipative tendency, some sort of forcing is often necessary to sustain a wave. Shear can serve as forcing for waves, and also serve to transmit perturbations over an entire shear layer, so that waves in a shear layer often tend to scale with the depth of the layer. Gravity waves arising out of the shear instability in the stable tropospheric layer would thus be expected to have wavelengths of the order of the tropospheric depth ($\sim 10$ to $20$ km).

Direct empirical evidence linking convection to propagating waves exists at longer wavelengths than those under consideration here (Eom, 1975; Uccellini, 1975; Matsumoto and Akiyama, 1969; Matsumoto and Tsuneoka, 1969; Matsumoto and Ninomiya, 1969; Stobie et al., 1983): these studies have mainly studied precipitation patterns in the presence of a propagating gravity wave event, and observed pulsations in convective activity with a time period corresponding to the period of the passing wave (wavelength $\sim 500$ km, time period
An explanation for periodic structures at scales of the order of the tropospheric depth was therefore sought in gravity waves at commensurate wavelengths. The studies of Ley and Peltier (1981) related the propagating cloud bands observed in the outflow of a decaying mesoscale convective system (Erickson and Whitney, 1973) to gravity waves launched by the system itself. Chimonas et al. (1980) developed a model of convective onset based on condensation occurring at the crests of a propagating gravity wave event. Various studies also link convection to the mountain lee waves launched in flow over mountains (Klemp and Lilly, 1978; Peltier and Clark, 1979; Clark and Farley, 1984). Gravity wave origins for rain bands have also been suggested (Testud et al., 1980; Wang et al., 1983). These, and the other studies of long gravity wave interactions with convection reported above, have all treated the gravity waves in the far field of a mesoscale source. For the small scale characterization of convection whose significance has been emphasized through this discussion, a near field approach is of greater value; this consists in studying the gravity waves close to the region of their forcing. While the difficulty of accurately describing event sources (such as decaying MCSs or fronts) preclude them as a possible mechanism in the near field, an interesting possibility has been raised in studies of the launching of gravity waves through interactions between the boundary layer and the stable layer above. These interactions, which are eminently suited to near field studies, consist in allowing the impaction of the low aspect ratio boundary layer eddies to deepen and impact upon the lower surface of the stable tropospheric layer to excite gravity waves there. The excitation of gravity waves in this fashion was studied by Townsend (1966) and Townsend (1968) in a model of a sheared stable layer subject to deformations of its lower surface. For the purposes of the present study, these deformations may be
thought to correspond to the random impaction of boundary layer eddies upon it. Gravity waves were launched in the layer with a frequency slightly less than the characteristic frequency of the layer (the Brunt-Väisälä frequency). If the duration of the disturbances was large compared to the time period, the waves took the form of wave packets of small extent. These appear in the atmosphere in the form of patches of clear air turbulence of about 50 m depth, and the study was able to relate these to patches observed by pilots flying low over stratocumuli. In the case of considerable wind shear (Townsend, 1968), a wave packet of small extent is radiated, leading to propagating waves travelling at slightly less than the velocity near the top of the stable layer in the case of positive windshear, and patchy turbulence once again, if the velocity decreases upward.

The converse effect — of boundary layer rolls being modulated by the stable layer flow — is not directly modelled in the Townsend studies: scale selection mechanisms in the boundary layer are not studied. These are central to the present study, and such a modulation may be expected in light of the strong correlations in wavelength between the cloud layer and boundary layer reported in LeMone and Meitin (1984). However, the Townsend study established that gravity waves in the stable layer can be launched by impactions of sufficient magnitude resulting from boundary layer convection.

In direct simulations of interactions between the convective layer and the stable layer (Mason and Sykes, 1980; Mason and Sykes, 1982; Clark et al., 1980), clouds emanating from the convective boundary layer eddies, and even the eddies themselves, have been shown to excite gravity waves through the differential motions in the stable layer over the convective intrusions in the presence of shear. In the Mason and Sykes studies, a small 2-D model domain (4 km x 10 km) was used in
numerical simulations. Using a small heat flux,\textsuperscript{10} they were able to obtain small amplitude gravity waves launched in the stable layer in small shear ($1.4 \times 10^{-3}$ sec$^{-1}$). An absorber was used at the model top to minimize reflection of gravity waves at the lid. Periodic lateral boundary conditions were used. The model contained no moisture, and so did not model the effect of clouds.

The small horizontal extent\textsuperscript{11} suggests that gravity waves at significantly longer wavelengths were not expected. The boundary layer eddy size was 2 km. The gravity waves launched selected this eddy size for their scales and this was explained in terms of the thermally forced boundary layer eddies acting as "small hills" to impede the stratified shear flow of the stable layer. While the coherence in phase between the boundary layer and the stable layer flow is as expected, it is to be noted that the periodic lateral boundary conditions force the system to choose modes that are harmonics of the horizontal extent. The small domain of the Mason and Sykes simulations thus would be unable to disclose a long-wavelength mode arising from gravity wave modulations of the PBL, were such a mode to exist. As noted earlier, the excitation of instabilities in a stably stratified shear layer, either through thermal forcing or by forcing the flow over an obstacle, can yield waves that scale with the depth of the layer; and if the layer is the troposphere above the unstable boundary layer, we may expect waves generated in such an instability to have wavelengths of the order of the tropopause height. It is this possibility that was investigated by Clark \textit{et al.} (1986). As the present study grew out of the

\textsuperscript{10} A heat flux of 30 watts/m$^2$ was used, while the average daytime sensible heat flux from solar radiation is of the order of several hundred watts per m$^2$ over land. Over the ocean, most of the incoming shortwave solar radiation is converted into latent heat through evaporation, and the heat flux used here may be appropriate.

\textsuperscript{11} A domain 4 km across was used in most experiments, with an 8 km domain used in a sensitivity test.
findings reported there, it may be well to examine that investigation in depth.

The Clark et al. study attempts to reproduce aircraft observations of gravity waves on a fair weather day with scattered clouds over the Great Plains. The sounding used corresponds to the environment on 12 June 1984 over eastern Nebraska, where an unstable and strongly sheared \((7 \times 10^{-3} \text{sec}^{-1})\) convective boundary layer 2 km deep lay under an unconditionally stable tropospheric layer extending up to 9.5 km. The lifting condensation level (LCL) was slightly below the top of the convective boundary layer, so that small cumuli would form. Aircraft flying over the clouds on that day reported periodic structures of the updraft velocity \(w\) with a wavelength of about 6 km and amplitudes of about 3 m/sec near the cloud tops and decreasing upward. The cumuli were seen to be placed slightly downwind of the updraft maxima. The Clark et al. study uses the 2-D version of the Clark anelastic model. As the same model is also used in the present study, in 2 and 3 dimensions, it is described in detail in chapter II. Since the model was in 2 dimensions, the obstacle effect of boundary layer eddies was likely to be over-emphasized\(^{12}\) and the exaggerated form drag was compensated, after a fashion, by a reduced surface heat flux \((140 \text{ watts/m}^2)\). Shear was restricted to the lowest 3 km of the atmosphere, in the absence of a deeper sounding. While the full tropospheric shear structure would undoubtedly play a part in the evolution of the atmosphere subsequent to the development of a gravity wave mode, it is likely that the interaction itself is governed mostly by the shear structure of the interaction region — the interface of the layers. The restriction of shear to a small portion of the stable layer is therefore still likely to develop the instability provided the shear spans the interface. Amplitudes of waves generated by the shear instability decay rapidly outside the shear

\(^{12}\) In 3 dimensions the fluid would also be able to flow around the obstacle.
zone (Davis and Peltier, 1976), but the continuous forcing from the PBL eddies may well compensate for this.

The experiments performed in the study attempted, first of all, to demonstrate the launching of gravity waves in the stable layer by convective eddies, and further, to study the influence of shear on the subsequent development. An initially stratified flow was subject to thermal forcing at the ground at all wavelengths (by including a random component into the heat flux), to allow the PBL to choose its appropriate shallow convective mode. The partitioning of energy thus concentrates energy initially into the most unstable shallow mode. As the amplitude of this mode grows at the top of the PBL, the incursions into the stable layer launch the gravity wave, at a longer wavelength. The resulting change in the spectral distribution of energy at the top of the PBL begins to excite a new boundary layer mode. Experiments varying the shear indicated that the shear between the two layers is probably the most sensitive parameter governing the competition within the boundary layer between the initial shallow convective mode, and the late time deep mode. The amplitude of the wave launched varied as the magnitude of shear, and this amplitude determined the efficacy of the spectral redistribution of kinetic energy in the PBL. Comparisons with the isolated boundary layer thus showed that there exists a fundamentally different late time solution for the thermally forced boundary layer when the whole troposphere is permitted to take part in the development. This late time mode that governs the entire troposphere including the PBL is called in this study the dominant forced mode.

In the experiments described, a domain was used that was 15 km deep and 30 km across, with periodic lateral boundary conditions. The wavelength selected by the dominant forced mode, 7.5 km, was the fourth harmonic of the horizontal model.
domain. The forced periodicity over the domain means that solutions of a periodic nature pick out harmonics of the lateral extent, so that the wavelengths tend to jump from one harmonic to the next. The domain is required to be large enough that adjacent harmonics in the neighborhood of the final dominant mode solution (which scales with the stable layer depth) are fairly closely spaced in wavelength, so that the effects of the quantization are not exerting an undue influence. Experiments varying the horizontal extent showed the 30 km domain used to be sufficient to demonstrate the basic physics: increasing the domain to 45 km gave markedly different structures, but the final mode selected at late times was the same in wavelength (but now the sixth harmonic).

Further experiments were performed to investigate the development of clouds by introducing moisture into the model. The results for the wind fields were found to be very similar the earlier dry runs with no appreciable “obstacle” effect from the clouds, as was suggested by Newton (1980). However, deep clouds, not present in this study, may show such an effect. The cloud field itself drew considerable influence from the interaction with the wave field. Clouds generally formed slightly upshear (at their bases) of a thermal eddy updraft, and remained anchored there for their lifetime in the strong shear case. The phase speeds of the wave field at cloud top and the eddy field at the base were different; the influence of the cloud top gravity wave on the cloud was thus time dependent, alternately supporting and suppressing the cloud. A further effect seen is that if an upshear cloud is experiencing upper level support while an adjacent downshear one is undergoing

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13 It may be noted that while the wavelength of this mode closely corresponds to the depth of the stable layer, it is somewhat longer than the waves observed. The model used an energy absorber in the stratosphere to inhibit reflections off the model lid, and the reflection characteristics of the tropopause may have changed as a result, causing the slight discrepancy.
suppression, the downshear cloud gets annexed by the first one. This *dynamic feeder cloud* mechanism is suggested by Clark *et al.* (1986) as a possible explanation for the frequently observed upshear growth of clouds. When the shear was reduced in the moist experiment, the strong coupling between the cloud and the subcloud eddy was no longer present, and the cloud field appeared to move relative to both the boundary layer eddies and the tropospheric gravity wave. The lack of continuous updrafts feeding moist air at the cloud base was seen to be unfavourable to cloud growth, suggesting that shear is an agent crucial to strong cumulus development, which may lead to severe storm outbreak in a conditionally unstable tropospheric layer.

These results appear to be of crucial importance in the quest for the small scale description of deep convection. To summarize the arguments of this chapter: there have hitherto been grave difficulties in describing the growth of deep cumulus convection arising out of fair weather conditions earlier in the day, as is often seen on days characterized by isolated thundershowers, or indeed any precipitating structures not associated with a large scale system. The principal difficulty has been that while the genesis of fair weather convection occurs through the thermal forcing of the PBL at the ground by incoming solar radiation, the instabilities associated with shallow convection fail to provide the appropriate scales of lifting associated with deep cumulus. The Clark *et al.* study suggests that the problem of boundary layer convection yields fundamentally different solutions at late times if the entire troposphere is allowed to participate in the dynamics, through a near field representation of atmospheric gravity waves. These solutions do not appear in a conventional normal mode analysis, which seeks only to find the most unstable mode at \( t = 0 \) in a boundary value problem. This theory also shows that the
genesis and sustenance of gravity waves in a convective situation may occur out of purely dynamical effects, whereas most of the depictions of gravity waves in the far field described earlier have undertaken to use the non-linear effects of moisture in parameterized form as an energy source.

The severe Montana hailstorm of 1 Aug 1981 followed the customary life history of thunderstorm days: fair weather convection → scattered clouds → deep clouds → severe storm. Starting with the mean conditions preceding that storm, the present study attempts to use this treatment of PBL convection and its interaction with gravity waves in the near field to create the conditions suitable for deep convection. The development of a severe storm in 2 and 3 dimensions is demonstrated, and important results derived therefrom on the growth and suppression of deep clouds. An explanation for periodicities in space and time associated with certain types of storms is presented. Linear and non-linear dynamical effects are distinguished through the formulation of a model of deep convection as an initial value problem in the linearized Navier-Stokes equations.

Chapter II outlines the two numerical models used in this study: the non-linear Clark model, and the linear model mentioned above. The results in 2 and 3 dimensions from both models is presented in Chapter III. A discussion of the implications of this research and prospects for further study follows, in Chapter IV.
CHAPTER II
THE MODELS

Two models are used in this study of the initiation of deep convection: the non-linear anelastic Clark model that was also used in the earlier work of Clark et al. (1986), and a linear spectral model developed for this sequence of studies to distinguish linear and non-linear effects in the development of the forced modes. This model was briefly described in Clark and Hauf (1986).

a) THE NON-LINEAR MODEL.

1. Model equations.

The non-linear model is the anelastic model of Clark (1977) the various features of which have been fully described in a series of studies where it was used.\(^1\) In its coupled dynamical-microphysical form, the model uses the parameterized microphysics of Kessler (1969) described below; a detailed microphysical model that runs off the dynamics of the coupled form has been developed by Hall (1980). The coupled dynamical-microphysical model is used in the present study. The model is described only briefly herein. The unsteady state Navier-Stokes equations (including the effects of moisture) are solved in their non-linear form over a finite difference mesh. Stability analyses of the hydrodynamical equations\(^2\) have shown that two

\(^1\) cf. Clark, 1977; Clark, 1979; Clark and Farley, 1984; Klaassen and Clark, 1985; Smolarkiewicz, 1984; Smolarkiewics and Clark, 1986.

\(^2\) cf. Bjerknes et al., 1933; Eliassen and Kleinschmidt, 1956; Eckart, 1960.
types of motions exist at widely separated scales — high frequency, high speed acoustic motions which are of significance in high Mach number flows, and gravity wave scale motions. The Brunt-Väisälä frequency is a frequently used marker dividing the regimes. If one is interested in motions at the time scale of gravity waves (frequencies smaller than the Brunt-Väisälä frequency), as in this study, and in general for cloud scale motions, the acoustic waves pose a problem, for they constrain the computational time step, which scales with the phenomenal time scale, to be so small as to be inefficient for the longer scale motions. One approach of modelers has been to separate the equations into acoustic and longwave terms, solving them on their own chosen time scales, and matching at appropriate times (see, e.g., Cotton and Tripoli, 1978; Klemp and Wilhelmson, 1978). An alternative approach is to filter out the acoustic terms from the equations. The approximation commonly used in fluid dynamical computations the Boussinesq approximation, where variations in the density \( \delta p \) are considered negligibly small compared to the value of \( p \) itself, excepting in such terms as are directly dependent on density gradients, i.e., buoyancy terms. The hydrodynamical equations under the Boussinesq approximation achieve the filtering of acoustic terms; however, this approximation is not suitable for the problem of "deep" convection, as the density varies appreciably over the full depth of the troposphere. It has been shown (Dutton and Fichtl, 1969) that for the Boussinesq assumption to be valid, the height of the model domain must be much smaller than the scale height of the atmosphere, defined as the exponential attenuation length of the hydrostatically stratified density \( \frac{B}{g\rho} \). This is valid only up to an atmospheric depth of about 1 to 2 km.³

³The use earlier in this study of the term "shallow" convection to apply to the isolated boundary layer thus corresponds also to the Boussinesq regime.

The studies of Ogura and Phillips (1962) and Dutton and Fichtl (1969) show
that under alternate approximations, less restrictive on the density and so more suitable to deep convection, a filtering of acoustic terms can be achieved. The density is allowed to have a hydrostatic stratification, and the stratified, horizontally homogeneous density is used in all terms except the buoyancy, where the exact density must be used. This yields the so-called anelastic form of the Navier-Stokes equations. The anelastic form can be derived by assuming that local variations in the density and pressure from their values in an adiabatic atmosphere are small (Batchelor, 1953), or in more straightforward fashion that the local time derivative of density is zero (Ogura and Phillips, 1962). In the latter formulation, a base state is specified with hydrostatic stratification of pressure and density and an adiabatic stratification of temperature. The perturbation terms are represented in a Taylor expansion about this state. In the Dutton and Fichtl (1969) formulation, the restriction on the temperature stratification can be removed in cases when the model depth is comparable to the scale height (a condition that is fulfilled in tropospheric convection). The Clark model uses a formulation similar to the Dutton and Fichtl form: the base state is hydrostatic and has a constant static stability $S$. When $S$ is set to zero, this reduces to the Ogura and Phillips form. The constant stability formulation is preferred as it results in a density stratification that is closer in form to the undisturbed deep atmosphere.

The thermodynamical variables are separated into a mean flow term and a perturbation term, where the undisturbed atmosphere is considered to be in a mean flow corresponding to hydrostatic and geostrophic balance. The mean flow itself is further expanded about the base state described above. The base state

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4 The static stability $S$ is defined by $S = \frac{\partial \ln \theta}{\partial z}$; $S = 0$ corresponds to a dry adiabatic atmosphere, for which $\theta$ is constant. A parcel of dry air under these conditions would everywhere be neutrally stable to vertical perturbations.
is represented by the overbars, and the primed variables are the deviations of the input profiles from the base state. Both the base state and the initial state are assumed to be at hydrostatic equilibrium. The time-dependent perturbation terms are represented by double primes, so that the thermodynamic quantities are written as

\[ T = \overline{T}(z) + T'(z) + T''(x,t) \]  

\[ P = \overline{P}(z) + P'(z) + P''(x,t) \]  

\[ \rho = \overline{\rho}(z) + \rho'(z) + \rho''(x,t) \]  

\[ \theta = \overline{\theta}(z) + \theta'(z) + \theta''(x,t) = \overline{\theta}(z)(1 + \theta^*) \]  

\[ q_u = q'_u(z) + q''_u(x,t) \]

where \( T \) is the absolute temperature, \( \rho \) the density, \( P \) the pressure, \( q_u \) the mixing ratio of water vapour. There is, of course, no \( q_u \) term corresponding to the dry environment (\( \overline{q}_u \equiv 0 \)). \( \theta \) is the potential temperature

\[ \theta = T \left( \frac{P}{P_0} \right)^{-\kappa} \]

where \( \kappa = (c_p - c_v)/c_p \) and \( P_0 \) some reference pressure (usually, 1000 mb). \( \theta \) is conserved in a dry adiabatic atmosphere, which is assumed to be an ideal gas. The mixing ratios of the condensed water categories, \( q_c \) and \( q_r \), exist only as perturbation terms, and will be written without double primes. The Navier-Stokes equations for momentum and mass continuity can be written in their anelastic form

\[ \frac{\mathbf{p}}{\rho} \frac{D\mathbf{u}}{Dt} + 2\rho \mathbf{\Omega} \times \mathbf{u} = -\nabla P - k \rho g + \nabla \cdot \tau - \lambda \rho \mathbf{u} \]  

\[ \nabla \cdot (\overline{\rho} \mathbf{u}) = 0 \]
The second term on the left of (7) is the Coriolis force, and the second term on the right is the buoyancy and precipitation loading term. The third term on the RHS is the divergence of the turbulent stress energy tensor \( \mathbf{\tau} \) i.e., \( \nabla \cdot \mathbf{\tau} \) is shorthand for the vector whose \( i \)th component is \( \frac{\partial r_{ij}}{\partial x_j} \). The last term is a frictional term, the Rayleigh absorber, used near the model lid to inhibit reflection of waves off the top. \( \frac{D}{Dt} \) is the total derivative

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla
\]

The hydrostatic equilibrium of the initial state is written

\[
\frac{\partial}{\partial z} (P + P') = -{(\bar{\rho} + \rho')}g
\]

where \( \rho' \) is understood to be before the inclusion of moisture. The moisture and precipitation loading corrections are applied to the base state density stratification to produce the anelastic equation of state

\[
\rho = \frac{P}{R_d T} - \epsilon_0 \bar{\rho} q_v + \bar{\rho} q_e + \bar{\rho} q_r
\]

where \( \epsilon_0 = R_v / R_d - 1 \).

From (10), the pressure gradient and buoyancy terms cancel except for the perturbation terms. The density perturbation is now linearly expanded in perturbation terms, from (11) and (6):

\[
\frac{\rho''}{\bar{\rho}} = -\frac{\theta''}{\bar{\theta}} + \frac{P''}{\gamma P} - \epsilon_0 q_v + q_e + q_r
\]

where \( \gamma = c_p / c_v \). Using (10) and (11) to simplify the buoyancy term in (7), we rewrite the momentum equations as

\[
\bar{\rho} \frac{Du}{Dt} + 2\bar{\rho} \Omega \times \mathbf{u} = -\nabla P'' + \bar{\rho} kg \left( \frac{\theta''}{\bar{\theta}} + \epsilon_0 q_v - \frac{P''}{\gamma P} - q_e - q_r \right) + \nabla \cdot \mathbf{\tau} - \bar{\rho} \lambda \mathbf{u}
\]

\[
\nabla \cdot (\bar{\rho} \mathbf{u}) = 0
\]
Eqs. (13) and (14) are integrated in the model for momentum.

The Rayleigh absorber in (13) acts only on the perturbation velocities, as we do not want such an artifact to affect the pre-existing flow. The same is true for the Coriolis force. It was mentioned earlier that the initial flow is assumed to be in geostrophic balance: a more precise statement may be in order. There exists in the boundary layer of the real atmosphere a balance between the surface effects, viscosity due to turbulence, the Coriolis force, and cross-domain pressure gradients arising out large scale baroclinicity. Pressure gradients resulting from large-scale or mesoscale temperature gradients, giving rise to the "thermal wind", may also be present. The turning wind profile, known as the *Ekman spiral*, that is usually observed in the boundary layer is the result of the balance between these forces. Frictional and turbulence effects dominate the wind near the surface, while at the top of the Ekman layer, the balance is purely between the pressure gradient (including the thermal wind, if present) and the Coriolis force. The Ekman spiral can only be maintained by explicitly modeling the Ekman layer, which requires the inclusion of horizontal gradients to reproduce the geostrophic balance at the top of the boundary layer, and the balance with friction near the surface. In a model with cyclic boundary conditions, such a balance is difficult to include, though not impossible: while cyclicity precludes explicit cross-domain gradients of temperature, it is possible to model its dynamical effect by specifying such a gradient insofar as it plays a role in the momentum equation, without including it explicitly in the temperature field itself. Surface effects can only be properly modeled by including a high-resolution surface boundary layer model, as was done in parameterized form in Smolarkiewicz and Clark (1985). Thus to determine the Ekman profile, a knowl-

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5 The Ekman layer may be defined as the layer near the ground in which mean surface effects cannot be ignored.
edge of large scale gradients and turbulence profiles is required. It has been thought in the formulation of the experiments testing this model of convective organization that such a detailed invocation of geostrophy is not likely to be of relevance; hence, as a vastly simpler alternative to the independent determination of the Ekman profile, the initial flow is assumed to be in an Ekman balance and this initial balance preserved as best possible. To this end, the Coriolis force is permitted to act only on the perturbation velocities, presuming the term arising out of the mean flow to have contributed to the setting up of Ekman flow. The implications of this assumption are discussed in greater detail during the presentation of the three-dimensional model results.

Under the stipulated conditions, the equations of motion (13) and (14) are invariant under a Galilean transformation, (i.e., a translation of the mean flow in velocity space). Further, the horizontal equations are invariant under a rotation of the mean flow about a vertical axis. While the Coriolis contribution to the vertical momentum equation is not invariant under a horizontal rotation, its effect is small enough to be negligible in comparison to the buoyancy term at convective scale.6

2. Turbulence parameterization.

Since the motion is turbulent in general, energy exists at scales smaller than the size of the finite difference grid. It is this fraction of the energy that cannot be treated directly that was separately included in the $r$ term in (13). Eddies smaller than the mesh are treated in parameterized form, and considered in their bulk effect to act as viscous dissipation to the resolved eddies. At the typical scales of

\[ \text{A typical horizontal speed is } 10 \text{ m/s and } \theta^* \sim 10^{-2} \text{ at convective scale. Then the ratio of the Coriolis term to the buoyancy term } \frac{\Omega V}{\theta^*} \sim 10^{-3}. \text{ Over much larger domains, the buoyancy term will be smaller and the Coriolis force will no longer be negligible (cf. Charney, 1948).} \]
meteorological phenomena, molecular viscosity may be neglected in the model as it is overwhelmed by the effective viscosity of the sub-grid scale eddies.

The non-linear terms of the form $\mathbf{u} \cdot \nabla \mathbf{u}$ are capable of transferring energy between different wave numbers in the energy spectrum. But this does not necessarily mean that there is significant interaction between all pairs of scales: there may be an effective separation of scales, i.e., disjoint non-interacting regions of reciprocal space. (The anelastic form we have chosen for the equations is, in effect, assuming a similar separation in time scales in filtering acoustic terms). In the formulation of sub-grid scale eddies as contributing only an effective eddy viscosity, such a separation of scales is assumed between the large, energy-containing eddies, where spectral energy transfers depend on the resolved-scale physics, and the small energy-dissipating turbulent eddies, where energy cascades into shorter and shorter scales. If the grid size used can be located in a region across which there exists the said scale separation, the model will not be unduly sensitive to the parameterization used for the sub-grid scale kinetic energy in the chosen form of the turbulent stress tensor $\tau$. Proceeding on this understanding, the sensitivity can later be checked by reducing the grid size and seeing if the additional kinetic energy in the resolved scale has a significant impact on the phenomenon under investigation.

The Clark model uses a parameterization of the turbulent stress-energy tensor due to Smagorinsky (1963) and Lilly (1962), in which the loss of kinetic energy from the resolved scale is expected to approximate the turbulent cascade of energy to shorter scales

$$\tau_{ij} = \bar{\rho} K_M D_{ij}$$

where $D_{ij}$ is the deformation tensor

$$D_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k}$$
with the total deformation $Def$ defined by

$$Def = \frac{1}{2} \sum_i \sum_j D_{ij}^2$$  \hspace{1cm} (17)

The eddy mixing coefficient $K_M$ is defined as

$$K_M = \begin{cases} \frac{[C \Delta]^2}{\sqrt{2}} |Def| \left(1 - \frac{K_{hRi}}{K_M} \right)^{1/2}, & Ri \leq 0; \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (18)

where $\Delta$ is a measure of the grid size ($\Delta = (\Delta X, \Delta Y, \Delta Z)^{1/3}$). The Richardson number Ri is a measure of the effectiveness of buoyancy in overcoming inertial forces

$$Ri = \frac{g}{\theta} \frac{\partial}{\partial z} \left( \frac{\partial^2}{\theta} + \varepsilon_0 q_0 - q_e - q_r \right) / Def^2$$  \hspace{1cm} (19)

There is no net energy source available for turbulence for $Ri > 0.25$ (Miles, 1961), and observations have shown that turbulence is unlikely for positive $Ri$. In the face of other gross approximations made in the model, it seems appropriate to use the value $Ri = 0$ to switch between turbulent and non-turbulent regimes — separating the unstable flow of the thermally driven boundary layer from the non-turbulent flow of the stable layer (which may be laminar except in regions of locally generated turbulence near cloud boundaries or under breaking waves). In passing from the neutral boundary layer to the stable layer the value of $Ri$ changes from negative to values greater than 1 very quickly, a further indication that using the value $Ri = 0$ instead of 0.25 is of little significance.

For scales well outside the inertial subrange, a fairly large value of the constant $C$ is suggested. Smagorinsky (1963) and Lilly (1962) used values up to 0.16. Here a value of 0.15 is used. Some authors have used a "background" mixing coefficient in the stable region to insure computational stability. Here $K_M$ is set to zero in stable layer flow.
The turbulence is assumed to be equally effective in diffusing momentum and heat, leading to an "eddy" Prandtl number $\frac{K_H}{K_M} = 1$ (defined analogously to the molecular Prandtl number which is the ratio of the thermal diffusivity and the viscosity). The initial profile used in this model show a neutral (and therefore thermally well-mixed) boundary layer containing shear, and this may lead to the speculation that $\frac{K_H}{K_M} \neq 1$: however, the fact that the wind profile in the boundary layer is maintained by large scale forcing permit no definite conclusion in this regard.

3. Microphysical parameterization.

The remaining equations of the model are those of conservation of heat and water substance

\[
\frac{\partial q}{\partial t} = \frac{\partial}{\partial t} \left( \frac{L}{C_p} \right) (C_d + C_{d_2}) + \nabla \cdot (\bar{\rho} K_M \nabla q) \tag{20}
\]

\[
\frac{\partial q_v}{\partial t} = -\bar{\rho}(C_d + C_{d_2}) + \nabla \cdot (\bar{\rho} K_M \nabla q_v) \tag{21}
\]

\[
\frac{\partial q_c}{\partial t} = \bar{\rho} C_d - S_{ae} - S_c + \nabla \cdot (\bar{\rho} K_M \nabla q_c) \tag{22}
\]

\[
\frac{\partial q_r}{\partial t} + \frac{\partial}{\partial z} (\bar{\rho} \nabla_T q_r) = \bar{\rho} C_{d_2} + S_{ae} + S_c + \nabla \cdot (\bar{\rho} K_M \nabla q_r) \tag{23}
\]

where $\bar{\theta} = \bar{\theta} + \theta'$ and $\bar{T} = \bar{T} + T'$ are the environmental values. The last term in Eqs. (20)-(23) represent turbulent diffusion. $C_d$ represents the rate of condensation of water vapour into cloud droplets (evaporation in subsaturated air when negative). $C_{d_2}$ is the rate at which water vapour is deposited on raindrops (evaporation when negative). $S_{ae}$ is the rate at which cloud droplets collect each other to form raindrops, and $S_c$ is the rate of collection of cloud droplets by raindrops. $L$ is the latent heat of evaporation. There is no ice phase in this version of the model.
The microphysics of water vapour in all its phases in the atmosphere is extremely complex and still in want of a full description, and can only be treated in fairly crude fashion in a model in which the fluid dynamics is given primacy. While severe convection is not merely a fluid dynamical problem, and is strongly affected by the response of available water to the dynamics, and conversely, by the dynamical effects of precipitation loading and the thermal effects of its phase changes, the constraints of practical computing oblige one to underplay the fine detail of the microphysics in a coupled model. Treating the dynamics derived in this fashion as a given, one can later treat the microphysics in much more detail, as in Hall (1980). While the dynamics and the microphysics are still interactive, moisture must be treated in the simple bulk water parameterization. Within the bulk formulation detailed accounts of water and all its phase changes have been attempted by various modelers (see, e.g., Ziegler, 1985). However, not enough is known about the microphysics of ice to allow a reliable formulation even in bulk. In the present study, only the physics of warm rain is included: the parameterization used is similar to that of Kessler (1969). In further studies, the use of the parameterization of Koenig and Murray (1976), which includes a two-component ice phase, is contemplated. In the Kessler physics, water exists in the vapour phase and a two-component liquid phase — cloud droplets (diameter < 100μm) and rain (diameter < 1 cm). $q_v$, $q_c$ and $q_r$ are the mixing ratios in the three fields. Condensation is initiated at saturation or above, and all water vapour in excess of the saturation mixing ratio is condensed out, bringing the air back to full saturation. When the cloud water $q_c$ reaches a threshold value, the rain field is initiated through a parameterization of the stochastic auto-collection of cloud droplets. The parameterization of collection rates has further assumed an exponential distribution of
raindrops $N(D) = N_0 e^{-AD}$ where $N(D)dD$ is the number of drops per unit volume with diameters between $D$ and $D+dD$ (Marshall and Palmer, 1948). The raindrops are assumed to fall with their mass-weighted mean terminal fall velocity

$$
\bar{V}_T = \frac{\int_0^\infty D^3 N(D) V(D) dD}{\int_0^\infty D^3 N(D) dD}
$$

(24)

where $V(D)$ is given a form $V(D) = \alpha_1 - \alpha_2 e^{-\beta D}$ based on a curve fit by Atlas (1973) to the experimental data of Gunn and Kinzer (1949). $\bar{V}_T$ has been used above in (23) in the term for the downward precipitative flux of rainwater.

4. Finite difference methods.

The finite difference form of the equations is written using the staggered grid of Harlow and Welch (1965). In the staggered grid form, all scalars and diagonal tensor terms are written at box centers, vector components are written on box faces, and off-diagonal tensor terms are written at box edges, i.e., if $(i,j,k)$ represents the box centre

$$(i,j,k) \rightarrow \left((i - \frac{3}{2})\Delta X, (j - \frac{3}{2})\Delta Y, (k - \frac{3}{2})\Delta Z\right) \quad i,j,k = 1,2,3,\ldots$$

the scalars are written

$$\phi = \phi(i,j,k)$$

(25)

where $\phi$ represents any of $P, T, \rho, \theta, K_M, De, \tau_{11}, \tau_{22}, \tau_{33}, g_u, g_o, g_r$. Velocity components are written

$$u = u(i \pm \frac{1}{2}, j, k)$$

$$v = v(i, j \pm \frac{1}{2}, k)$$

(26)

$$w = w(i, j, k \pm \frac{1}{2})$$
and the off-diagonal stress tensor terms are written

\[ r_{12} = r_{12}(i \pm \frac{1}{2}, j \pm \frac{1}{2}, k) \]
\[ r_{23} = r_{23}(i, j \pm \frac{1}{2}, k \pm \frac{1}{2}) \]  
\[ r_{13} = r_{13}(i \pm \frac{1}{2}, j, k \pm \frac{1}{2}) \]  

(27)

Terms that occur together in the equations are suitably averaged to be defined at the same space point. Defining the operators

\[ \overline{\phi}^x = \frac{1}{2} \left( \phi \left( x + \frac{\Delta x}{2} \right) + \phi \left( x - \frac{\Delta x}{2} \right) \right) \]  
\[ \delta_x \phi = \frac{\phi \left( x + \frac{\Delta x}{2} \right) - \phi \left( x - \frac{\Delta x}{2} \right)}{\Delta x} \]  

(28)

(29)

where \( x \) represents any of \((x, y, z, t)\), the finite difference form of (13) and (14) may be written in centred-space and "leapfrog" time, following Arakawa (1966). In the "leapfrog" scheme, the tendencies evaluated at time \( t = r \Delta t \) are used to integrate forward from time step \((r - 1)\) to \((r + 1)\). The momentum equations (13) are given by

\[ \delta_t(\overline{\rho^2 u})^t + \text{ADVX}^r = + (\overline{\rho^2 u^2 v} f)^r + \text{PFX}^r + \text{KFX}^{r-1} + \text{RAYX}^r \]  
\[ \delta_t(\overline{\rho^2 v})^t + \text{ADVY}^r = - (\overline{\rho^2 u^2 v} f)^r + \text{PFY}^r + \text{KFY}^{r-1} + \text{RAYY}^r \]  
\[ \delta_t(\overline{\rho^2 w})^t + \text{ADVZ}^r = \text{PFZ}^r + \text{BY}^r + \text{KFZ}^{r-1} + \text{RAYZ}^r \]  

(30)

(31)

(32)
The ADV.. terms are the non-linear advective terms

\[
\text{ADVX} = \delta_x(\bar{\rho}^z u^z u^z) + \delta_y(\bar{\rho}^z u^z v^z) + \delta_z(\bar{\rho}^z w^z u^z)
\]

\[
\text{ADVY} = \delta_x(\bar{\rho}^z u^z v^z) + \delta_y(\bar{\rho}^z v^z v^z) + \delta_z(\bar{\rho}^z w^z v^z)
\]

\[
\text{ADVZ} = \delta_x(\bar{\rho}^z u^z w^z) + \delta_y(\bar{\rho}^z v^z w^z) + \delta_z(\bar{\rho}^z w^z w^z)
\]

(33)

The PF.. terms are the pressure gradient terms

\[
\text{PFX} = -\delta_x P
\]

\[
\text{PFY} = -\delta_y P
\]

\[
\text{PFZ} = -\delta_z P - \frac{g}{\gamma R_d} \frac{\bar{P}^z}{T^z}
\]

(34)

where it may be noted that the pressure perturbation contribution to the buoyancy term in (13) has been included in PFZ for convenience. The KF.. terms are the eddy mixing terms

\[
\text{KFX} = \delta_x (r_{11}) + \delta_y (r_{12}) + \delta_z (r_{13})
\]

\[
\text{KFY} = \delta_x (r_{12}) + \delta_y (r_{22}) + \delta_z (r_{23})
\]

\[
\text{KFZ} = \delta_x (r_{13}) + \delta_y (r_{23}) + \delta_z (r_{33})
\]

(35)

where it may be recalled that the turbulence parameterization led to a symmetric form for the stress-energy tensor. The RAY.. terms are the Rayleigh friction terms

\[
\text{RAYX} = -\bar{\rho}^z \lambda u
\]

\[
\text{RAYY} = -\bar{\rho}^z \lambda v
\]

\[
\text{RAYZ} = -\bar{\rho}^z \lambda w
\]

(36)
The continuity equation (14) is written

$$\delta_x(\bar{\rho} \bar{u}) + \delta_y(\bar{\rho} \bar{v}) + \delta_z(\bar{\rho} \bar{w}) = 0 \quad (37)$$

The buoyancy term $BY^r$ and the equations of heat and moisture conservation are not written out explicitly here, but are obviously deducible. For the advective terms for $\theta$ and the moisture variables, the advective scheme of Smolarkiewicz (1984) and Smolarkiewicz and Clark (1986) is used. The Multi-dimensional Positive Definite Advective Transport Algorithm (MPDATA) of Smolarkiewicz is based on the upstream differencing technique, where space differences are taken "upstream", i.e., to first order against the prevailing flow. This scheme is accurate to first order; the truncation errors give rise to implicit second-order diffusion terms that however serve a purpose in that they act as agencies of computational stability. Based on the upstream scheme, a technique of advection accurate to second order can be derived through the inclusion of anti-diffusive second-order corrections. The Lax-Wendroft schemes attempt to achieve this through an explicit subtraction of second-order truncation errors: positive definiteness is lost as a result, an effect that could be serious in the case of small positive definite quantities such as $q_c$. The MPDATA approach avoids this difficulty: the method used consists in Taylor-expanding the upstream advection scheme to second order in order to estimate the value of diffusive truncation errors. The correction is applied by reversing the sign. The scheme is in general second-order accurate. This process can be applied iteratively to achieve any required limit of overall accuracy. For the flow regimes relevant to dynamic meteorology, one corrective iteration is found to be sufficient (Smolarkiewicz and Clark, 1986).

Since the time derivative in the equations above is taken over non-adjacent time steps in the "leapfrog" scheme, two diverging solution streams may develop.
The development of this computational mode is suppressed with the use of the time filter of Robert (1966) and Asselin (1972). No computational mode is associated with the MPDATA advection scheme, as it is based on forward time differencing. The finite differencing techniques used in the model conserve momentum and kinetic energy to second order in time (Arakawa, 1966).

5. Interactive grid nesting and Spawning.

Two other features of the model deserve mention for their relevance to the model spatial resolution. If the necessary size of the model domain and the required resolution for the physics lead to a large number of grid points, the computation may be impracticable. If the region of the physics of interest is only a portion of the whole domain, the grid nesting approach, described in Clark and Farley (1984), allows the user to nest a fine mesh within a coarser one to achieve increased resolution over a part of the domain. The nesting is interactive: the outer model is first integrated, and the solutions then used to provide boundary conditions for the inner model. After the inner model has been integrated, it in turn is used to correct the coarse mesh solutions at common points. Following this, Klaassen and Clark (1985) developed the spawn feature which allows one to spawn the inner model from the outer by interpolation after the physics of interest has begun to develop. This allows one to integrate only the coarse model until such time as increased resolution is needed. Both these features are used extensively in the studies that follow. Currently in nested runs, the same time step must be used in all models, so that numerical stability requirements are most stringent for the finest grid.

6. Boundary conditions.

At the model ground and top, free-slip boundary conditions are used; the
vertical velocity and vertical gradients of the horizontal velocity are set to zero.

\[ w = \delta_z(p^x u) = \delta_z(p^y v) = \delta_{zz} \theta = 0 \quad \text{at} \quad z = 0, H. \] (38)

As in Clark et al. (1986), cyclic lateral boundary conditions are chosen. The model of convective organization that is being investigated concerns itself with the near field representation of the long time, forced modal response of the system. Under a strong mean flow, a considerable part of a horizontal integration domain of computationally feasible size may be used during the development; the convective eddies may traverse the whole domain before enough time has elapsed for developing the forced, late time, near field response. Computational restrictions on domain size thus render the choice of open boundary conditions unfeasible. The choice of cyclic boundary conditions makes available an infinite domain, though this does not come without its price: in a periodic domain simulation, wave-type solutions are harmonically quantized with respect to the domain size. The cyclicity condition thus comes with a caveat: the domain must be at least large enough to contain a few of the longest wavelength modes, so that adjacent wave numbers are sufficiently close in wavelength so as not to show an excessive effect of the quantization. The grid size must be chosen so as to sufficiently resolve the smallest eddies of significance. Since the eddies scale with the depth of the layer in which they occur, these conditions imply for the case in this study (convective layer depth \( \sim 2 \text{ km} \); tropospheric depth \( \sim 12 \text{ km} \)) that the grid must be no coarser than about 1 km and must have a horizontal extent of 50 to 60 km.\(^7\) The vertical resolution must sufficiently resolve the boundary layer: at least 4 or 5 points within the boundary layer are required (a 500 m resolution). These conditions are marginally met in the 3-D simulations. The

\(^7\) In the simulation of Clark et al. (1986), the effects of harmonic quantization were found to be small in comparisons between domains of 30 km and 45 km.
free slip boundary conditions that are used for the upper and lower model surfaces, are justifiable at the lower surface since the lowest model level is of the order of a grid size (actually, exactly half the grid size) away from the actual surface. The Rayleigh absorber at the model top is found sufficient to model absorption of energy into the upper stratosphere in the case of a deep model such as the present one: for shallow runs where the model top may lie in the tropospheric stable layer, a more accurate radiation condition may have to be used.

A feature of the model that is not used in the current study is still worthy of mention, because of its possible relevance to future studies: the model is capable of handling a non-flat topography of the lower surface through a transformation of the z-coordinate (Gal-Chen and Somerville, 1975). No topography is used in this case, the better to isolate the layer interaction under study. The CCOPE network in the High Plains region of Montana, where the storm used in this study occurred, has gently sloping terrain with no topographic features of appreciable size. Possible slope effects are considered later in the discussion.

7. Model initialization

Numerical models of convective phenomena begin either with empirically ascertained (e.g., Doppler-derived) wind fields, or more generally, with an initially stratified flow that is then suitably disturbed in order to release the convective instability. The initialization of cumulus-scale models is one that is still in want of study in depth and modelers have generally chosen to use such artifacts as bubbles and transient heat fluxes, to disturb the initial stratified flow and set up convection. Such initializations are chosen so as to reproduce the gross features of the observed cloud, to study the subsequent evolution. The underlying assumption, especially in modelling the growth of deep cumuli and severe storms, is that the long-term
solution is thought to be independent of initialization effects. Earlier studies with the Clark model used a Gaussian shaped field containing an applied perturbation to the potential temperature (e.g., Clark, 1979). Klemp and Wilhelmson (1978) also used such a bell-shaped initial temperature perturbation. Schlesinger (1978) uses an initial moist buoyant updraft with a specified radius, containing an updraft profile, potential temperature perturbation, and the associated moisture excess over the environment. Cotton and Tripoli (1978) used a moisture anomaly, in the form of a moist bubble, in whose column the depth and base height of the saturated region, and its radius were all specified based on observations of the actual cloud.

"Bubbles" and transient localized heat fluxes represent particular solutions of the differential equations and may be of relevance only in the modeling of single storms or clouds, as in the studies listed above. The more realistic description of the initiation of deep convection that the present study aims at involves the growth of deep cumuli out of a cloud field subsequent to the development of the appropriate scales for deep convection out of the long term modal response of the boundary layer to thermal forcing from the ground. These forced solutions are indeed the ambition of this study, for the question of horizontal scale choice in the development of severe storms is not addressed in "bubble"-type initiations. While it may be argued that in a flat or bell-shaped applied perturbation all scales are present, albeit at small amplitudes, and even a small perturbation ought to excite an instability at that mode, they cannot exhibit the forced responses, by virtue of their transience, for forced modes can only appear after the decay of transient terms. A continuously applied heat flux, as used in Clark et al. (1986), seems the appropriate way to initialize a model to study the long-time solutions to the forced mode problem. Such a "realistic" initialization has been used earlier by Sommeria (1976) to the problem
of boundary layer forcing, where a temperature difference of 1°K between the actual surface and the lowest model height was used to provide a surface flux of sensible and latent heat. This study was of the isolated unstable boundary layer and does not show the dominant forced mode response. Shallow models of cumulus cloud ensembles (Hill, 1974; Hill, 1977; Yau and Michaud, 1982) have also successfully used a continuously applied heat flux at the surface with a random component. In the current study, a heat flux of 300 watts/m² is used which reproduces the sensible heat flux from shortwave solar radiation corresponding to late afternoon conditions over land in the mid-latitudes. A random 5% white noise is superimposed over the horizontal domain, in order to force all the possible modes of the PBL which will then be able to select its own scale for the eddies.

b) THE LINEAR MODEL.

1. Model equations.

The linear model was developed for studies of wavenumber selection in the forced boundary layer and an early version has been described briefly in Clark and Hauf (1986). The model is used to look at individual wavenumber evolution under thermal forcing of the boundary layer in the absence of spectral energy transfer. The dry anelastic form of the Navier-Stokes equations is used, the Boussinesq approximation again being insufficient for the depth of atmosphere participating in the development, as in the non-linear case. The expansion is made about the environmental profiles, to which the overbar terms now refer. Terms that are of significance only in the lowest portion of the atmosphere (specifically the eddy mixing terms) are simplified under the Boussinesq approximation. The development of the linearized equations begin with the Navier-Stokes (sans Coriolis) and
continuity equations, as in the non-linear case

\[
\rho \frac{\partial u}{\partial t} + \bar{\rho} u \cdot \nabla u = -\nabla P'' + k \bar{\rho} \theta + \nabla \cdot (\bar{\rho} K_M \nabla u) - \lambda \bar{\rho} u \tag{39}
\]

\[
\nabla \cdot (\bar{\rho} u) = 0 \tag{40}
\]

where terms have retained the same meaning as in the non-linear case. \( \theta \) is the fractional deviation of potential temperature that was written as \( \theta^* \) in Eq. (4).\(^8\)

The energy conservation is written

\[
\frac{\partial \theta}{\partial t} + \bar{\rho} u \cdot \nabla \theta = F_H + \nabla \cdot (\bar{\rho} K_M \nabla \theta) - \lambda \bar{\rho} \theta \tag{41}
\]

Once again, separating the variables into a mean flow and a perturbation \( (u = U + u') \), the interaction is linearized by neglecting products of perturbational terms, so that

\[
\begin{align*}
    u \cdot \nabla u &= (U \cdot \nabla u_3) k + u_3 \frac{dU}{dz} \\
    u \cdot \nabla \theta &= U \cdot \nabla \theta + w S
\end{align*} \tag{42, 43}
\]

where \( S = \frac{\partial \ln \bar{\theta}}{\partial z} \) is the stability. The perturbation terms \( u' \) an \( v' \) are not computed and the mean flow is stationary. As a consequence, the explicit treatment of the Reynolds stresses involving transfers of energy between different components of the momentum is prohibited. This will inevitably lead to the unlimited expansion of energy in the perturbation term without affecting the mean flow, clearly an unrealistic result. The results therefore can only be taken seriously up to some value of

\(^8\)This unfortunate change in notation is necessitated by the common use of the asterisk superscript to denote the complex conjugate.
the perturbation velocities beyond which the assumptions of the linear formulation break down. It may be noted that the mean velocity is purely horizontal. Since the perturbation terms are assumed small, not only the mean flow, but also the thermodynamic profiles are assumed stationary and a function of height only \((U=U(z), S=S(z), K_M=K_M(z), \lambda=\lambda(z))\). Under these conditions, the flow equations reduce to a single equation for the variable \(w = \bar{p}^{1/2} u_3\) by the usual process of taking the curl twice

\[
\frac{\partial}{\partial t} \nabla^2 w + U \cdot \nabla_H (\nabla^2 w) - \frac{d^2 U}{dz^2} \cdot \nabla_H w =
\]

\[
g \nabla_H^2 \theta + \nabla \cdot \left( \frac{dK_M}{dz} \nabla \frac{\partial w}{\partial z} \right) + \nabla \cdot (K_M \nabla (\nabla^2 w)) - \lambda \nabla^2 w - \frac{d \lambda \partial w}{dz \partial z}
\]

(44)

The equation of heat conservation gives

\[
\frac{\partial \theta}{\partial t} + U \cdot \nabla_H \theta + w S = F_H + \nabla \cdot (K_M \nabla \theta) - \lambda \theta
\]

(45)

where \(\theta\) and \(F_H\) are normalized by \(\bar{p}^{1/2}\) with respect to the terms in Eq. (40).

Since the equations are linear in \(w\) and \(\theta\), the horizontal spectral decomposition of the fields yield independent solutions for each horizontal wave number. By solving completely the amplitude and phase of each wave number, the convective development including the horizontal motions of the eddies or waves (expressed as the phase velocity of the principal mode) can be reconstructed without explicitly solving for the horizontal perturbation velocities. Assuming cyclicity in the lateral domain, the horizontal mesh chosen only serves in deciding the number of modes and their wavelength, and in fact can be solved for a single mode if the wavelength is already known. Unlike the conventional normal mode analysis, this is a formulation

\(^9\)Note that \(K_M\) is not computed from eq. 18 in the linear model. A representative profile from the non-linear model is prescribed.
of convection as a linear initial value problem; the growth rates are time dependent. The $t = 0$ solution is expected to yield the same shallow boundary layer mode as the normal mode analysis, but when sufficient energy has been transported upward, the deep gravity wave mode is expected. It may then be possible to study the competing growth of the initial shallow mode and the late dominant forced mode in the linear situation.

The remaining development is geared toward solving for the amplitude and phase of each mode through a Fourier transform of Eqs. (44) and (45).

2. Spectral form of the equations

The spectral equations are greatly simplified by invoking a virtual domain which is a mirror image about the ground of the real domain i.e., assuming that $z$ varies from $-H$ to $H$. All the quantities can then be assigned a suitable even-odd symmetry about $z = 0$ to simplify the equations. Assuming free-slip boundary conditions at the surface, the horizontal flow variables $U$ and $V$ must be even about $z = 0$. The stability $S$ has a positive value throughout and is hence even. $w$ and $\theta$ are assigned an odd sense: sinking bubbles and plumes as mirror images of the rising ones in the real domain. With the odd sense of $\theta$ and the even sense of $S$, the physical meaning of warmer air rising and colder air sinking is retained even in the virtual domain. $K_M$ and $\lambda$ are even about $z = 0$ and retain their symmetry. The heat input $G_H$ must then be odd, and the heat flux $F_H$, the vertical gradient of $G_H$, is even. $F_H$ is given the same form as in the non-linear model: a random horizontal variation and a exponential profile in the vertical with a specified attenuation length.
\[ F_H = F_Z(z) \cdot F_{XY}(x,y) \]
\[ F_Z(z) = \frac{\partial}{\partial z} e^{-|z|/\Lambda} \]

The white noise in spectral form for \( F_H \) is generated by creating normal modes of constant amplitude and random phase.

Except for the fields \( F_H, w \) and \( \theta \), all others have no horizontal variation.

The quantities \( w \) and \( \theta \) are then written in spectral form

\[ z(x,t) = \int Z(k,t)e^{ik\cdot x} d^3k \]
\[ Z(k,t) = \frac{1}{(2\pi)^3} \int z(x,t)e^{-ik\cdot x} d^3x \]

where \( z(x,t) \) stands for \( w \) or \( \theta \), and \( Z(k,t) \) for the Fourier-transformed quantities \( W \) or \( \Theta \). The quantities \( S, U, V, \lambda \), and \( K_M \) are all even functions of \( z \) alone and represented in a Fourier expansion as

\[ \phi(x) = \int_{-\infty}^{\infty} \phi(p)e^{ipx} dp = \int_{-\infty}^{\infty} \phi(p) \cos pz dp \]
\[ \phi(p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(x)e^{-ipx} dx \]

where \( \phi(p) \) is the 1-dimensional Fourier transform of \( \phi(z) \). All the quantities \( \phi \) are time-independent. Henceforth in the exposition of the formalism, the time dependence of \( w \) and \( \theta \) will be suppressed.

Since all the fields involved must be real, it follows that all \( \phi(l) \) must be real and even in \( l \) and

\[ Z^*(k) = Z(-k) \]
Since the $z$-dependence of $w$ and $\theta$ is odd, the $k_z$-dependent part of $Z(k)$ is purely imaginary, and we write

$$Z(k_H, k_z) = i Z(k_H) z(k_z)$$  \hspace{1cm} (50)

where $z(k_z)$ is real and odd in $k_z$. Then $Z(k_H)$ also satisfies Eq. (49), as is easily seen from Eqs. (49) and (50).

And finally we have the result for the Fourier transform $c(k)$ of the product of two functions $a(x)$ and $b(x)$, i.e., if $c(x) = a(x)b(x)$,

$$c(k) = \int a(k') b(k - k') d^3 k'$$  \hspace{1cm} (51)

Eq. (44) may now be written in this formalism for a single Fourier component $k = (k_H, k_z)$. Defining $k^2 = |k|^2$ and $k_H^2 = |k_H|^2$, and the temporary variables $k_- = k_z - p$ and $k_+ = k_z + p$, Eq. (44) is written

$$-k^2 \frac{\partial}{\partial t} W(k) = - \int (k_H \cdot U(p)) W(k_H) w(k_-) (k_H^2 + k_-^2 - p^2) dp$$

$$- g k_H^2 \Theta(k) + k^4 W(k) + i \int \lambda(p) W(k_H) w(k_-) (k_H^2 + k_-^2) dp$$

$$+ i \int K_M(p) W(k_H) w(k_-) (p^2 k_-^2 - 2 p k_- (k_H^2 + k_-^2)) dp$$  \hspace{1cm} (52)

For the component $-k$, we have

$$-k^2 \frac{\partial}{\partial t} W(-k) = + \int (-k_H \cdot U(p)) W(-k_H) w(-k_+) (k_H^2 + k_+^2 - p^2) dp$$

$$- g k_H^2 \Theta(-k) + k^4 W(-k) + i \int \lambda(p) W(-k_H) w(-k_+) (k_H^2 + k_+^2) dp$$

$$+ i \int K_M(p) W(-k_H) w(-k_+) (p^2 k_+^2 + 2 p k_+ (k_H^2 + k_+^2)) dp$$  \hspace{1cm} (53)
Making a variable transformation $p \rightarrow -p$ in (53), we obtain for $W(-k)$

\[-k^2 \frac{\partial}{\partial t} W(-k) = - \int_{-\infty}^{\infty} (-k_H \cdot U(p)) W(-k_H) w(k_-)(k_H^2 + k_-^2 - p^2) \, dp\]

\[-gk_H^2 \Theta(-k) + k^4 W(-k) - i \int_{-\infty}^{\infty} \lambda(p) W(-k_H) w(k_-)(k_H^2 + k_-^2) \, dp\]

\[-i \int_{-\infty}^{\infty} K_M(p) W(-k_H) w(k_-) (p^2 k_-^2 - 2pk_- (k_H^2 + k_-^2)) \, dp\]

(54)

From (49), (52) and (54), we get

\[-k^2 \frac{\partial}{\partial t} \Re W(k) = - \int_{-\infty}^{\infty} (k_H \cdot U(p)) \Im W(k_H, k_-)(k_H^2 + k_-^2 - p^2) \, dp\]

\[-gk_H^2 \Re \Theta(k) + k^4 \Re W(k) - \int_{-\infty}^{\infty} \lambda(p) \Re W(k_H, k_-)(k_H^2 + k_-^2) \, dp\]

\[- \int_{-\infty}^{\infty} K_M(p) \Re W(k_H, k_-) (p^2 k_-^2 - 2pk_- (k_H^2 + k_-^2)) \, dp\]

(55)

and

\[-k^2 \frac{\partial}{\partial t} \Im W(k) = + \int_{-\infty}^{\infty} (k_H \cdot U(p)) \Re W(k_H, k_-)(k_H^2 + k_-^2 - p^2) \, dp\]

\[-gk_H^2 \Im \Theta(k) + k^4 \Im W(k) + \int_{-\infty}^{\infty} \lambda(p) \Im W(k_H, k_-)(k_H^2 + k_-^2) \, dp\]

\[+ \int_{-\infty}^{\infty} K_M(p) \Im W(k_H, k_-) (p^2 k_-^2 - 2pk_- (k_H^2 + k_-^2)) \, dp\]

(56)
A similar procedure gives, for $\theta$, the equations

$$
\frac{\partial}{\partial t} \text{Re} \Theta(k) = \int (k_H \cdot U(p)) \text{Im} \Theta(k_H, k_-) \, dp + \int S(p) \text{Re} W(k_H, k_-) \, dp \\
- \text{Re} F_H(k) + \int \lambda(p) \text{Re} \Theta(k_H, k_-) \, dp \\
+ \int K_M(p) \text{Re} \Theta(k_H, k_-) \left( pk_- + (k_H^2 + k_-^2) \right) \, dp
$$

(57)

and

$$
\frac{\partial}{\partial t} \text{Im} \Theta(k) = + \int (k_H \cdot U(p)) \text{Re} \Theta(k_H, k_-) \, dp - \int S(p) \text{Im} W(k_H, k_-) \, dp \\
- \text{Im} F_H(k) - \int \lambda(p) \text{Im} \Theta(k_H, k_-) \, dp \\
- \int K_M(p) \text{Im} \Theta(k_H, k_-) \left( pk_- + (k_H^2 + k_-^2) \right) \, dp
$$

(58)

Eqs. (55) – (58) are integrated in the linear model.

3. Finite difference form of the equations.

It is found convenient to recast the Fourier expansions of $w$ and $\theta$ in sine and cosine terms. For $z = w$ or $\theta$, we have

$$
z(x) = \int Z(k) e^{ik \cdot x} \, d^3 k \\
= \int \left( \text{Re} Z(k) \cos(k \cdot x) - \text{Im} Z(k) \sin(k \cdot x) \right) \, d^3 k \\
+ i \int \left( \text{Im} Z(k) \cos(k \cdot x) + \text{Re} Z(k) \sin(k \cdot x) \right) \, d^3 k
$$

(59)

where the imaginary part is 0 from (49). Now separating $k$ into $k_H$ and $k_z$

$$
z(x) = - \int \left( \text{Im} Z(k_H) z(k_z)(\cos(k_H \cdot x) \cos k_z x - \sin(k_H \cdot x) \sin k_z x) \\
+ \text{Re} Z(k_H) z(k_z)(\sin(k_H \cdot x) \cos k_z x + \cos(k_H \cdot x) \sin k_z x) \right) \, d^2 k_H \, dk_z
$$

(60)
where the coefficient of \( \cos k_z z \) is 0 from (49). Now using the \( z \)-symmetries, we write

\[
z(x) = -2 \int_{k_z > 0} (\Re Z(k) \sin(k_H \cdot x) + \Im Z(k) \cos(k_H \cdot x)) \sin k_z z \, d^2 k_H \, dk_z
\]

\[
= -4 \int_{k_z,k_z > 0} (\Re Z(k) \sin(k_H \cdot x) + \Im Z(k) \cos(k_H \cdot x)) \sin k_z z \, d^2 k_H \, dk_z
\]

(61)

Defining \( k = k_x + l \hat{y} + p \hat{z} \), (61) is rewritten

\[
z(x) = -4 \int_{k,l,p > 0} (\Re Z(k,l,p) \sin(kx + ly) + \Re Z(k,-l,p) \sin(kx - ly)
\]

\[
+ \Im Z(k,l,p) \cos(kx + ly) + \Im Z(k,-l,p) \cos(kx - ly)) \sin pz \, dk \, dl \, dp
\]

\[
= \int_{k,l,p > 0} (A_{klp} \sin kx \cos ly + B_{klp} \cos kx \cos ly
\]

\[
+ C_{klp} \sin kx \sin ly + D_{klp} \cos kx \sin ly) \sin pz \, dk \, dl \, dp
\]

(62)

The equations can thus be written in terms of four coefficients for each spectral mode of \( w \) and \( \theta \). First the domain cyclicity and the finite difference conditions are applied. The cyclicity condition converts the Fourier integral into a sum: if \( f(x) \) is cyclic i.e., \( f(x + X) = f(x) \) then its transform \( f(k) \) given by

\[
f(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-ikx} \, dx
\]

(63)

is 0 unless \( k = \pm \frac{2\pi n}{X}, n = 0,1,2, \ldots \) so that \( f(x) \) is given by a summation

\[
f(x) = \sum_{n=-\infty}^{\infty} f_k e^{ikx} \quad , \quad k = \frac{2\pi n}{X}
\]

(64)

The finite difference condition states that modes of wavelength shorter than \( \Delta x \) cannot be resolved, so that \( k \) has a maximum absolute value \( \frac{2\pi}{\Delta x} \), and \( n \) has a maximum absolute value \( NX = X/\Delta x \), and

\[
f(x) = \sum_{n=-NX}^{NX} f_k e^{ikx} \quad , \quad k = \frac{2\pi n}{X}
\]

(65)
The generalization to 3 dimensions is obvious. \( k \) and the corresponding integer \( n \) are used interchangeably in the text, when it is apparent from the context which is intended.

In converting the infinite integrals of Eqs. (55) - (58) to finite sums of the form of Eq. (65), care must be taken when the argument \((k - l)\) appears in the integrand, to keep them within the summation limits. For functions periodic over the interval \((0, 2\pi)\)

\[
\int_{-\infty}^{\infty} \phi(l)\psi(p - l)\,dl = \int_{-\pi}^{\pi} \phi(l)\psi(p - l)\,dl = |p - l| \leq \pi
\]

\[
+ \int_{-\pi}^{\pi} \phi(l)\psi(p - l - 2\pi)\,dl, \quad p - l > \pi
\]

\[
+ \int_{-\pi}^{\pi} \phi(l)\psi(p - l + 2\pi)\,dl, \quad p - l < -\pi
\]

which yields

\[
\int_{-\infty}^{\infty} \phi(l)\psi(p - l)\,dl \rightarrow \sum_{l=-NZ}^{NZ} \sum_{m=-NZ}^{NZ} \phi_l\psi_m(\delta_{m,p-l} + \delta_{m,p-l-2NZ} + \delta_{m,p-l+2NZ}) \quad (67)
\]

When \( \phi_l = -\phi_{-l} \) and \( \psi_m = \psi_{-m} \), the summation in (67) becomes

\[
\sum_{l=0}^{NZ} \sum_{m=0}^{NZ} \phi_l\psi_m(\delta_{m,p-l} + \delta_{m,l-p} - \delta_{m,p+l} - \delta_{m,2NZ-l-p}) \quad (68)
\]

When \( \phi_l = \phi_{-l} \) and \( \psi_m = -\psi_{-m} \), the summation in (67) becomes

\[
\sum_{l=0}^{NZ} \sum_{m=0}^{NZ} \phi_l\psi_m(\delta_{m,p-l} - \delta_{m,l-p} + \delta_{m,p+l} - \delta_{m,2NZ-l-p}) \quad (69)
\]
The coefficients in (68) and (69) can be elegantly summarized in sine and cosine integrals

\[
SCS_{\text{imp}} = \frac{2}{\pi} \int_{-\pi}^{\pi} \sin lz \cos mz \sin pz \, dz
\]

\[
\rightarrow \frac{1}{2} (\delta_{m,p-1} + \delta_{m,l-p} - \delta_{m,p+1} - \delta_{m,2NZ-l-p})
\]

(70)

\[
CSS_{\text{imp}} = \frac{2}{\pi} \int_{-\pi}^{\pi} \cos lz \sin mz \sin pz \, dz
\]

\[
\rightarrow \frac{1}{2} (\delta_{m,p-1} - \delta_{m,l-p} + \delta_{m,p+1} - \delta_{m,2NZ-l-p})
\]

The formalism developed above is now ready for completely writing out the equations to be integrated for the coefficients A–H where

\[
w(x) = \sum_{k=0}^{NX} \sum_{l=0}^{NY} \sum_{p=0}^{NZ} (A_{klp} \sin kx \cos ly + B_{klp} \cos kx \cos ly + C_{klp} \sin kx \sin ly + D_{klp} \cos kx \sin ly) \sin pz
\]

(71)

\[
\theta(x) = \sum_{k=0}^{NX} \sum_{l=0}^{NY} \sum_{p=0}^{NZ} (E_{klp} \sin kx \cos ly + F_{klp} \cos kx \cos ly + G_{klp} \sin kx \sin ly + H_{klp} \cos kx \sin ly) \sin pz
\]

\[
F_H(x) \text{ is similarly expanded}
\]

\[
F_H(x) = \sum_{k=0}^{NX} \sum_{l=0}^{NY} \sum_{p=0}^{NZ} (F_{klp}^1 \sin kx \cos ly + F_{klp}^2 \cos kx \cos ly + F_{klp}^3 \sin kx \sin ly + F_{klp}^4 \cos kx \sin ly) \sin pz
\]

(72)

Eqs. (55) – (58) now yield (with \( q^2 = k_H^2 + l^2 \)):

\[
k^2 \frac{\partial}{\partial t} A_{k_s k_s p} = \sum_{i=0}^{NZ} \sum_{m=0}^{NZ} \left\{ B_{k_s k_s i} k_x U_m (q^2 - m^2) SCS_{\text{imp}} - C_{k_s k_s i} k_y V_m (q^2 - m^2) SCS_{\text{imp}} - A_{k_s k_s l} K_m \left( (q^4 + m^2 l^2) SCS_{\text{imp}} + 2q^2 mCSS_{\text{imp}} \right) - A_{k_s k_s l} \lambda_m (q^2 SCS_{\text{imp}} - mCSS_{\text{imp}}) \right\}
\]

\[+ k_H^2 g E_{k_s k_s p}\]

(73)
\[ k^2 \frac{\partial}{\partial t} B_{k_x k_y} = \sum_{l=0}^{NZ} \sum_{m=0}^{NZ} \left\{ -A_{k_x k_y l} k_x U_m (q^2 - m^2) \text{SCS}_{imp} \\
- D_{k_x k_y l} k_y V_m (q^2 - m^2) \text{SCS}_{imp} \\
- B_{k_x k_y l} K_m\left((q^4 + m^4)/2\right) \text{SCS}_{imp} + 2q^2 m l \text{CSS}_{imp} \right\} \\
+ k_H^2 \mu F_{k_x k_y} \]  
(74)

\[ k^2 \frac{\partial}{\partial t} C_{k_x k_y} = \sum_{l=0}^{NZ} \sum_{m=0}^{NZ} \left\{ D_{k_x k_y l} k_x U_m (q^2 - m^2) \text{SCS}_{imp} \\
+ A_{k_x k_y l} k_y V_m (q^2 - m^2) \text{SCS}_{imp} \\
- C_{k_x k_y l} K_m\left((q^4 + m^4)/2\right) \text{SCS}_{imp} + 2q^2 m l \text{CSS}_{imp} \right\} \\
- C_{k_x k_y l} \lambda_m (q^2 \text{SCS}_{imp} - m l \text{CSS}_{imp}) \right\} \\
+ k_H^2 \mu G_{k_x k_y} \]  
(75)

\[ k^2 \frac{\partial}{\partial t} D_{k_x k_y} = \sum_{l=0}^{NZ} \sum_{m=0}^{NZ} \left\{ -C_{k_x k_y l} k_x U_m (q^2 - m^2) \text{SCS}_{imp} \\
+ B_{k_x k_y l} k_y V_m (q^2 - m^2) \text{SCS}_{imp} \\
- D_{k_x k_y l} K_m\left((q^4 + m^4)/2\right) \text{SCS}_{imp} + 2q^2 m l \text{CSS}_{imp} \right\} \\
- D_{k_x k_y l} \lambda_m (q^2 \text{SCS}_{imp} - m l \text{CSS}_{imp}) \right\} \\
+ k_H^2 \mu H_{k_x k_y} \]  
(76)

\[ \frac{\partial}{\partial t} E_{k_x k_y} = \sum_{l=0}^{NZ} \sum_{m=0}^{NZ} \left\{ \left( + k_x F_{k_x k_y l} U_m - k_y G_{k_x k_y l} V_m - A_{k_x k_y l} S_m \right) \text{SCS}_{imp} \\
- \left((q^2 K_m + \lambda_m) \text{SCS}_{imp} + l K_m \text{CSS}_{imp} \right) E_{k_x k_y l} \right\} \\
- F_{k_x k_y} \]  
(77)
Eqs. (73) – (80) are integrated in the model for w and θ. As can be seen, each horizontal mode \((k_x, k_y)\) is independent, and can be integrated in isolation. As in the non-linear case, the model is initialized with a random heat flux and is integrated forward in time under the leapfrog scheme i.e., the tendency terms from the RHS of (73)-(80) evaluated at timestep \(r\) is used to generate the coefficient at timestep \((r + 1)\) from the coefficient at \((r - 1)\). The computational mode associated with the leapfrog scheme is filtered with single-step Euler differencing after a number of time steps. Boundary conditions used are the same as in the non-linear case.

4. Phase velocity

A crucial role in determining the efficacy of the interaction is played by the relative motion of eddies with height, providing channels of growth and suppression in a time-dependent manner. It is therefore important to compute the phase velocities associated with the horizontal modes as a function of height. First, the
field decomposition is reconstructed in height by integrating over the vertical wave number, so that (62) takes the form

$$w(x) = \sum_{k=0}^{NX} \sum_{l=0}^{NY} A_{kl}(z) \sin kx \cos ly + B_{kl}(z) \cos kx \cos ly + C_{kl}(z) \sin kx \sin ly + D_{kl}(z) \cos kx \sin ly \quad (81)$$

$$= \sum_{k=0}^{NX} \sum_{l=0}^{NY} \alpha_{kl} \sin(kx + ly + \phi_{kl}) + \beta_{kl} \sin(kx - ly + \psi_{kl})$$

The amplitudes are of no consequence here, and the phases are given by

$$\phi_{kl} = \tan^{-1} \left( \frac{B_{kl} - C_{kl}}{A_{kl} + D_{kl}} \right)$$

$$\psi_{kl} = \tan^{-1} \left( \frac{B_{kl} + C_{kl}}{A_{kl} - D_{kl}} \right) \quad (82)$$

Considering a single mode $(k,l)$ at some height and suppressing the indices for a moment, at time $t + dt$, the fluid element has moved relative to the ground by $(U dt, V dt)$ and the phases $\phi$ and $\psi$ have changed, so that

$$\alpha \sin(kx + ly + \phi) + \beta \sin(kx - ly + \psi)$$

$$\longrightarrow \alpha \sin \left( k(x + U dt) + l(y + V dt) + \phi + \frac{\partial \phi}{\partial t} dt \right)$$

$$+ \beta \sin \left( k(x + U dt) - l(y + V dt) + \psi + \frac{\partial \psi}{\partial t} dt \right)$$

$$= \alpha \sin (k(x - C_x dt) + l(y - C_y dt) + \phi) + \beta \sin (k(x - C_x dt) - l(y - C_y dt) + \psi) \quad (83)$$

where the phase velocity $(C_x, C_y)$ has been defined to mean that the mode looks unchanged except for a relative shift at that velocity. By matching terms in (83), we arrive at

$$k C_x + l C_y = -k U - l V - \frac{\partial \phi}{\partial t}$$

$$k C_x - l C_y = -k U + l V - \frac{\partial \psi}{\partial t} \quad (84)$$

which yields

$$C_x = -U - \frac{1}{2k} \left( \frac{\partial \phi}{\partial t} + \frac{\partial \psi}{\partial t} \right) \quad (85)$$

$$C_y = -V - \frac{1}{2l} \left( \frac{\partial \phi}{\partial t} - \frac{\partial \psi}{\partial t} \right) \quad (86)$$
Eqs. (82), (85) and (86) serve to completely determine the phase velocities.

An alternative method was initially devised to determine the phase velocity in the non-linear runs by maximising the correlation of \( w(x) \) with itself at later times at a given shift in space; i.e., to estimate the phase velocity at a given height at time \( t \), the horizontal spatial cross-correlation of the \( w \) field with itself at a succession of increasing lag times is computed. Evidently, a maximum in the correlation function equal to 1 is found at the origin at lag time 0. This peak is followed in a sequence of plots at increasing lag times: the peak at the origin moves away steadily, decreasing in magnitude as the dynamics evolve. If the displacement of the peak is \( \Delta x \) at lag time \( \Delta t \), the phase speed during the period \( t \) to \( t + \Delta t \) is determined to be \( \frac{\Delta x}{\Delta t} \).

The same was done along the two horizontal axes in the 3-D simulations. It was realised after some comparisons that what this method actually produces is an energy weighted mean phase velocity averaged over all modes. For a narrowly peaked spectrum, both methods yield the same result; when there is some spread in the spectrum, the physical meaning of phase speeds determined by the second method is not obvious. Eqs. (85) and (86) are therefore used to determine phase velocities even in the non-linear case, by generating the appropriate spectral coefficients of \( w(x) \).

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\( ^{10} \) To be precise, the weighting function is the amplitude squared.
CHAPTER III

EXPERIMENTS: The CCOPE storm of 1 Aug 1981

It may be well to begin with a recapitulation of the problem: the scales selected in shallow convection do not appear appropriate for the initiation of deep cumulus. However, to reproduce the usual history of thunderstorm days — fair weather convection → scattered clouds → deep clouds → severe storm — a mechanism to develop the appropriate scales must be found. The Clark et al. study suggests that the problem of boundary layer convection yields fundamentally different solutions at late times if the entire troposphere is allowed to participate in the dynamics, through a near field representation of atmospheric gravity waves. It is important to note that it is the development of deep and severe convection that is being established in this study, not the features of interest specific to a particular storm. Such features will no doubt be largely dependent on the synoptic and mesoscale conditions, the topography of the storm location, and horizontal inhomogeneities in the thermodynamic and wind fields, none of which may appear in the present formulation. The case studied in this series of experiments is one which occurred during the CCOPE\(^1\) experiment in 1981 near Miles City, Montana on Aug 1. It has been used in several studies of microphysical and thermodynamic structure of severe storms.\(^2\) A detailed description is provided

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\(^1\) The Cooperative CONvective Precipitation Experiment.

in Tuttle et al. (1987); a time lapse movie taken from the CP-2 radar site shows several small cumuli developing through the day over the CCOPE network. Most of the clouds subsided before reaching great depth; however, several clouds grew to great depth and one grew fully to its computed level of neutral buoyancy and developed a mesoscale anvil. It is precisely such a storm history that this study hopes to document. The one cumulus that finally grew extremely severe did so rapidly and eventually developed into a pair of storms the left component of which sailed rapidly (≈ 21 m/sec) out of observational range. The right component moved more slowly (≈ 8 m/sec) and developed into a hail-producing supercell that received much attention from the radars and aircraft involved, during its steady state phase, from 1540–1715 MDT. Large hail was collected on the ground, and wind gusts in excess of 30 m/sec were measured by the surface mesonetwork.

1. Environmental conditions.

The environmental input used for the model is a composite of the rawinsonde data taken at 1330 MDT at Miles City, with the low level environment modified with the sounding taken by the Queen Air N306D aircraft during its takeoff phase at 1430 MDT (fig. 1). A well-mixed layer of about 1.5 km depth exists near the surface, with a deeper mixed layer of higher potential temperature aloft, conditions known to be favourable to the development of severe weather. The lifting condensation level (LCL) is at about 2.5 km AGL at a temperature a little above 10°C, and the stability index at 500 mb is above 8°C. The conditional instability above the boundary layer extends up to 9 km AGL, and cloud tops may be estimated from the level of neutral buoyancy to be at about the tropopause at 11 km.

The moisture profile in the rawinsonde data of fig. 1 shows a well-mixed

layer already in existence. The development of the mixed layer indicates the prior presence of convective eddies, as one might expect with a late afternoon sounding. The initialization of the model with stratified, horizontally uniform dynamical and thermodynamical profiles is therefore an idealization. This will be seen to be a point of some significance. The boundary layer wind profile is in nature the result of a balance between horizontal pressure gradients, the Coriolis force and the diffusive effects of turbulent mixing. The idealizations in the model formulation allow the treatment of only the first two forces in the model initialization: this is done by allowing the Coriolis force to act only on the perturbation velocities. The mixing effects of the introduction of turbulence and convection will be seen to tend to destroy shear in the PBL, by diffusing the geostrophic wind $V_g$ downward. Were the model to be initialized with an early morning sounding, prior to the development of a neutrally stable PBL, the creation of the neutral layer and the destruction of the shear layer will proceed apace, since it has been assumed that $K_H/K_M = 1$. Unless the model can be initialized with a prior determination of the vertical turbulence profile, it seems expedient to start with a strongly sheared mixed layer as seen in a mature sounding, such as the one in fig. 1.

An analysis of surface mesonet data by Betts (1984) shows strong gradients of moisture across the network, and moist inflow into the low-level air from the southerly flow. While cross-domain gradients and inflow conditions cannot be introduced in a cyclic model, some experiments in this study attempt to simulate the effect of the moist inflow in terms of a moisture flux from the surface. While this does not reproduce the horizontal distribution of the moisture flux, it provides at least for the increased availability of moisture.

Winds (shown in fig. 2) vary from a weak southerly flow in the convective
boundary layer to strong westerlies aloft, with strong shear in the lowest 5 km
\(\sim 6 \times 10^{-3} \text{ sec}^{-1}\) peaking near the top of the boundary layer, which augurs well
for the development of dominant mode interaction, in which shear is the principal
coupling agent between the boundary layer and the stable layer, as described earlier.
Winds are already veering to westerly near the top the boundary layer, and the shear
in the region crucial to the layer interaction is above \(6 \times 10^{-3} \text{ sec}^{-1}\) and directed
about due WSW. The veer (which would probably be less abrupt in a better resolved
sounding) is acute at 1 km, but the wind profile is more or less constant in shear
direction above and below this height. The initial eddies formed by the thermal
forcing at the surface are expected to be roll vortices in the boundary layer that
align themselves along the axis of the lowest level shear.

2. An overview of the experiments.

The basic nature of the forced mode mechanism of PBL convective scale
selection has already been adequately demonstrated for fair weather convection in
Clark et al. (1986). That study reported preliminary results from 2-D models of
the dominant mode interaction as a mechanism of convective initiation. Here, those
studies are extended to the case of deep cumulus growth in two and three dimen­
sions. Comparisons with the results of Clark et al. (1986) lead to an increased un­
derstanding of the distinction between the response of deep and shallow convection
to the dominant mode interaction, and to this end, the focus is on the mature stage
of storm evolution, and the effects of the initiation mechanism thereon. In view of
this, the generalized sensitivity tests of the model results to numerical aspects such
as model resolution and domain size were not felt to be a logical part of this study,
and will be reported in conjunction with further works in progress. In the 2-D case,
an additional experiment is performed to test the sensitivity of the model results,
in terms of structure of the mature storm, to the increased availability of moisture.  
The 3-D experiments focus on the effect of a turning wind profile, and its implications for convective initiation. In addition, results from the linear model, in two and three dimensions, are presented. These results serve to isolate the purely linear aspects of the dominant mode interaction, by treating the evolution of convection as an initial value problem in the linearized set of equations, as derived earlier.

It is well to clarify at this point some aspects of the terminology used in the discussions that follow: the term shallow mode refers to the modes that develop in the boundary layer when the development is still in its infancy; these are the modes described in the theoretical and experimental studies of the isolated boundary layer. The dominant forced mode, or the dominant mode, is the mode that arises out of the resonance of the boundary layer with the gravity wave launched aloft, and it may occupy a considerable depth of the atmosphere. The phase velocity refers not only to the horizontal phase speed of the gravity wave, but also to the speed of horizontal translation of the convective eddies, and is determined in both cases by an application of Eqs. (85) and (86). The wave number of a mode is the number of wavelengths that represent the horizontal extent of the domain: for instance, a wave number of 5 in a domain 60 km in extent represents a wavelength of 12 km. The wavelength of boundary layer eddies refers to their horizontal spacing. Finally, the aspect ratio of boundary layer eddies is the ratio of their wavelength to the boundary layer depth.

3. Experiments with a 3-dimensional model.

As discussed in Chapter I, linear stability analyses of boundary layer instabilities (including instabilities in the Ekman spiral) suggest that the most unstable and fastest growing modes therein are those represented by longitudinal roll vor-
tices. These obviously will not appear in 2-D simulations. Instabilities that arise in a two-dimensional boundary layer will therefore represent a spurious transverse mode that in reality is overwhelmed by the longitudinal modes. 4 The earliest phase of development in the 2-D simulations, therefore, cannot be taken too seriously. However, there are reasons why this may not be cause to discard 2-D simulations entirely. Gravity waves, at least in the pure and undispersed case, have wavefronts perpendicular to the mean flow, and so can be modelled with some authenticity in two dimensions in the plane of the mean flow. The results of Clark et al. (1986) show that the boundary layer is eventually organized by the gravity wave; if the structure of the gravity wave is determined by the dispersion characteristics of the troposphere, and not by the specific manner of perturbation that excites it, the fact that the initial perturbation in the stable layer is a spurious mode will not seriously affect the final result. 5 What cannot be verified in the 2-D simulation is whether the dominant forced mode is in fact competitive in terms of growth rate with the longitudinal mode in the boundary layer. This question will be addressed in the 3-D simulation. Besides, the 2-D simulation is attractive in ways by virtue of its relative simplicity: it will be seen that phase relations governing the motion of boundary layer eddies and the gravity wave propagation are among the most interesting features of this model of convective organization. These are more readily

4 "Spurious" may seem an overstatement: what the normal mode analysis shows is that the longitudinal mode is more unstable and therefore will grow to greater amplitude than the 2-D mode. In a non-linear sense, however, this may result in a suppression of other modes since the vertical heat flux, represented by the temperature-vertical velocity correlation function \( w' \theta' \), increasingly favours the most unstable mode.

5 The determinants of gravity wave structure are not known with certitude at this point: if our speculation that the wavelength is mainly a function of tropospheric depth is correct, then the presumption above is valid. The results of Ley and Peltier (1981) for a travelling disturbance also support this contention, albeit in the far field.
understood in two dimensions, and facilitate the understanding of the same in the 3-D simulations.

The basic 2-D experiments are run in a box of 60 km horizontal extent and 25 km height. The first experiments were run in a box of only 30 km in the horizontal, but the dominant modes were found to be too low a harmonic of the box size, possibly unduly emphasizing the effects of cyclicity. The domain is larger than the basic one of Clark et al. (1986), as is only to be expected, as the convection is deeper here. Another point of comparison with the studies of Clark et al. (1986) is in the vertical shear structure. There the shear layer was 3 km deep, which was enough to span the boundary layer and the shallow cloud layer. The steering level of the eddies in the boundary layer and the clouds in the free layer stayed fairly close to the region of interaction, facilitating the observation and understanding of the gravity wave support cycle arising out of the relative phase velocities of motions in the two layers. In the Aug 1 case, the shear layer near the ground is deeper, extending up to 5 km; a weakly sheared region above is capped with another region of shear spanning the tropopause. The variation of phase velocity with height under a varying shear profile in the free layer could play a role in the formation of deep growth channels, and deserves scrutiny.

The basic 2-dimensional model V1 is run with a resolution of 0.5 km in both the $y$ and $z$ directions, leading to a grid of 120 by 50 points. Due to the negligible effect of the Coriolis force on vertical cloud scale motions, as described earlier, a rotation of $90^\circ$ is made, to bring the mean winds in the upper troposphere (where they are strongest) to lie along the $v$-axis. The winds are then further translated by 10 m/sec along the $u$-axis to minimize the maximum absolute wind speeds, which

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6 The 2-D Clark model runs in the $y$-$z$ plane.
considerably relaxes the constraint on the timestep based on the Courant-Friedrichs-Lewy (CFL) criterion. The resultant wind profile in the model plane is shown in fig. 3. The timestep used in this and subsequent simulations is varied for computing economy; as the model winds evolve, the timestep is altered to keep the computation stable, as provided for by the CFL criterion. In the experiment V1, a timestep of 20 sec was used for the first two hours of the simulation, and 10 sec subsequently, permitting velocities up to 30 m/sec in either direction. Eddies of small amplitude are seen to fill the boundary layer very quickly, within 20 minutes of initiation (see fig. 4a). The eddies move at a speed of about 10 m/sec\(^7\) corresponding to a steering level of about 1.25 km. Impactions of small magnitude upon the lower surface of the stable layer are already apparent. The horizontal wavelength of the \(w\) field is now about 3 to 4 km, giving an aspect ratio of about 2 with respect to the height of the boundary layer. Thirty minutes later, in fig. 4b, the amplitudes of the eddies have increased considerably, though still small, and incursions into the free layer are present along the entire interface. The eddies in the boundary layer show a pronounced tilt in the direction of the shear. The next sequence of pictures (fig. 5), depict the development of the dominant forced mode. At \(t = 120\) min (fig. 5a) strong incursions into the stable layer exist at several places, though the wavelengths are still resonant with the boundary layer eddies. The shear flow in the stable layer in the presence of these incursions is now beginning to establish the dominant mode. In the next frame \((t = 130\) min; fig. 5b), it is apparent that as the stable layer motions increase in depth and amplitude, they exert a telling influence on the boundary layer flow. The internal gravity wave is now seen to force the boundary layer motion into a commensurate wavelength. The destruction of

\(^7\)All velocities in the 2-D model results are reported in the transformed coordinate system.
smaller scale motions is dramatized at around \( y = 50 \) km, where the gravity wave above is seen to be forcing the flow below. This appears to be accomplished through the downward flux of momentum mixing with and destroying the small scale eddy downdraft, a non-linear Reynolds stress effect.

Hereafter, the boundary layer motions exhibit the late time dominant modes with wavelengths that do not occur in models of the isolated boundary layer. The boundary layer eddies are now seen to move at a speed of about 4.5 m/sec corresponding to a steering level of about 2.5 km, above the top of the boundary layer, showing that the boundary layer motions are now the non-local\(^8\) solutions arising out of the resonant interaction with the stable layer. At this time, smaller scale motions are still present in the boundary layer, but the energy contained therein is rapidly being overtaken by the dominant forced mode, whose growth overwhelms the smaller-scale motions: in fig. 5d \((t = 150 \text{ min})\), where the eddies have filled the entire troposphere, most of the energy is seen to reside in a single mode, with a wavelength of about 10 km. This phenomenon, of the increase in depth and wavelength of eddies proceeding apace, is clearly shown in the spectral decomposition of the field, a sequence of which is seen in fig. 6, spanning the same period as fig. 5. Roll vortices observed in the boundary layer would now exhibit an aspect ratio of 5 or 6. Such anomalous aspect ratios, and even larger ones in the tropics,\(^9\) have frequently been observed (cf. LeMone and Meitin, 1984) and do not admit of a plausible theoretical explanation based on boundary layer analysis alone (LeMone, 1973). These modes are now seen to arise out of the interaction with the stable layer. It is to be noted that these are the dominant modes, and no longer the spurious transverse

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\(^8\) The term non-local is used in the sense that the properties of the boundary layer motion are determined from outside it.

\(^9\) This aspect ratio is believed to arise out of the relative depths of the stable tropospheric layer and the PBL, which is much larger in the tropics.
modes of the two-dimensional boundary layer.

The clouds remain relatively shallow for a while (up to about \( t = 180 \) min). This is linked to the variation with height of the phase speed of the dominant forced mode. This variation implies that a channel for deep growth in the wave modulated convective pattern occurs but seldom, so that a cloud field will exist for a long time before one "statistically fortunate" cloud is permitted deep growth, when the mode arrives in phase over a substantial portion of its depth.\(^{10}\) This corresponds well to the observations of the day (as well as other typical days of summertime convective activity in the Great Plains), where a large field of cumulus was seen to evolve through the day, mostly remaining capped, but culminating in the one cumulonimbus that grew deep and produced a severe storm.

Two shortfalls of the simulation interfere with the deep growth of clouds at this point in the development of model V1. In the absence of shear-supporting gradients, the shear region spanning the two layers is becoming a less efficient coupling agent, the value of the shear in the model plane dropping from \( 6 \times 10^{-3} \text{sec}^{-1} \) to about \( 4.2 \times 10^{-3} \text{sec}^{-1} \) in the period \( t = 150 \) to \( 240 \) min. More significantly, the upward flux of moisture in the deep eddies accompanied by a slight warming near the ground due to the imposed heat flux leads to a progressive drying out of the subcloud layer, so that little precipitation develops, although a substantial cloud develops aloft. The depletion of subcloud water vapour is shown in fig. 7, showing a precipitous decline at the inception of growth of the dominant mode, with some falling off of the rate of depletion at the initiation of storm activity (after three hours of simulation time). It has not been attempted in the 2-D simulations to

\(^{10}\) In fact, it may be possible to compute the time it will take if the difference in phase speed between the LCL and the level of free convection could be determined; however, the resolution of the initial sounding and the model do not permit the distinction.
include a mechanism for shear support; the second problem can however be simply treated by imposing a moisture flux at the surface, a plausible assumption from the mesonet data (cf. Betts, 1984). This moisture flux is imposed in a parallel run after \( t = 130 \text{ min} \) and the subsequent development studied in model V6.

A sequence of plots of the \( w \) field and the cloud water content \( q_c \) is shown for the period \( t = 180 \text{ min} \) to \( t = 230 \text{ min} \) for model V1 in figs. 8a-f and 9a-f. The updraft at the centre of the picture at \( t = 180 \text{ min} \) (fig. 8a) shows the gravity wave and the boundary layer eddy in phase agreement and almost out of phase in fig. 8d, 30 min later. The approximate phase speeds computed during this sequence is about 5 m/sec to the left for the eddies and about 8.5 m/sec to the left for the wave. As noted in Clark et al. (1986), this uncoupling in phase of the dominant forced mode from the gravity wave appears to be an effect of cloud.

Since the principal wave number seems to be 5 (the spectral decomposition of fig. 8a is given in fig. 10), giving a wavelength of 12 km, situations favourable for deep growth at any location\(^{11} \) may be expected to occur about every 57 min, based on a relative phase speed of 3.5 m/sec. In the same sequence, the first turret (fig. 9a) is seen to be almost totally enclosed in the downdraft at \( y = 36 \text{ km} \) resulting from the gravity wave; the moisture is then fed into the cloud just upshear which is supported by an updraft at \( y = 30 \text{ km} \), which is then seen to deepen in fig. 9b. However the updraft at the upshear edge of this cloud is already beginning to part company with its gravity wave support, resulting in suppression. A similar event is seen at the right edge, between \( y = 48 \) and 60 km in the sequence (c-e) of figs. 8 and 9. Note that there is continuity between the left and right edges of the cyclic

\(^{11} \) Since the waves and eddies are not monochromatic, phase relations do not hold over the whole horizontal extent simultaneously; secondary modes (such as the mode of wave number 7 in fig. 10) may render the phase agreement good at one location, and slightly off at another.
model domain.

The recurrence time appears to be an important feature arising out of this model of convective organization: the 57 min period leads to an infrequency of occurrence of channels of deep growth during the diurnal atmospheric cycle where the mixed layer is set up late in the morning and the most severe convective activity occurs in the afternoon, i.e., about 4 to 5 hours of development. Two conditions need to be simultaneously satisfied: the growth cycle, with its characteristic recurrence time, must be at its favourable phase for a given cloud at the same time when a cloud at the downshear edge is undergoing suppression. Several growth cycles may elapse before this occurs. A lot of capped convection resulting in a cumulus field is to be expected before the fortuitous location of feeder clouds supplies moisture at cloud base during a growth phase. If suppressed convection is the rule rather than the exception, the competition for moisture is likely to be subdued and on the few occasions when instability is released through this mechanism, growth is likely to be severe. In the model V1 this does not occur until 4 hours after the inception of the dominant forced mode and 6 hours after the start of the simulation, the convection having been kept alive by the forcing. The surface heat flux by now is surely unrealistic as it is constant, taking no account of reduced incoming radiation due to cloud cover and lower solar elevation.

The next sequence (figs. 11a–d, 12a–d and 13a–d) is taken from the period of maximum storm activity in model V1 from $t = 330$ to $t = 360$ min. The environment is now too dry to support a field of cumulus, and all the available condensed moisture is concentrated in a single large cumulonimbus. This has now grown to its maximum height of 11.5 km as inferred from the sounding in fig. 1. The updraft

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12 See p. xlii.
reaches its maximum of 23 m/sec at \( t = 340 \) min, and while the dryness of the subcloud layer precludes precipitation from reaching the surface, the liquid water content reaches an in-cloud maximum of 2.4 gr/kg at \( t = 350 \) min, which translates to a radar reflectivity of 55 dBz based on the assumed Marshall-Palmer drop size distribution. The model microphysics contains only warm rain processes; in the real atmosphere, such a high radar reflectivity is usually taken to be an indication of the presence of hail. This unusually high liquid water content compares with observations and must be considered remarkable in a model containing no ice water. The updraft strength is somewhat smaller than the observed values which are upward of 35 m/sec (cf. Tuttle et al., 1987) but are of appreciable strength for a 2-D model, where upward motions are subdued with respect to 3-D simulations, since convergences in the transverse direction are prohibited. Horizontal cloud velocity measured at the leading edge of the anvil is about 10 m/sec, slightly less than the mean flow in the model plane at mid-tropospheric levels. As the boundary layer eddies feeding the cloud at its base continue to move to the left at about 4 m/sec, it is seen that at this mature stage, the storm circulation is achieving some degree of independence from the modal structure that gave rise to it. Examination of the storm in the region from \( z = 3 \) to 6 km above ground shows that as the storm moves downstream, new cells form on its downstream edge and old ones are discarded on its upstream side: the débris from a decaying cell is seen at about \( y = 50 \) km at \( t = 340 \) min in fig. 12b, and new growth is seen at the forward end at \( t = 360 \) min in fig. 12d. The cells therefore remain anchored to the boundary layer eddies while the storm moves downstream overhead. The atmosphere being very dry at this stage, the cells dry out once they lose the support of the propagating storm circulation. The spatial and temporal characteristics of the storm compare well with
observations of multicellular storm development. Chalon et al. (1976), in their detailed account of a multicellular storm event in NE Colorado, describe the storm as moving southward along and a little slower than the mean mid-tropospheric winds. New cells appeared at locations approximately 10 km downstream of the preceding developments at intervals of about 13 to 16 min. In the current study, this spatial distribution of new cells follows naturally from the wavelength of the dominant forced mode (~ 12 km). The temporal structure may be deduced from the motion of eddies relative to the mean tropospheric flow guiding the cloud. With a storm motion of 10 m/sec downstream and eddy motion of 4 m/sec to the left, the storm is seen to traverse one complete wavelength in about 14 min, which correspond well to the observations cited.

The fact that the model has generated a linearly aligned multicellular storm rather than the splitting storm leading to supercellular structure as observed is due to the inherently 3-dimensional nature of supercellular evolution, in which the vertical vorticity exerts a telling influence. In supercells as modeled by Marwitz (1972) and Browning (1960), the inflow at cloud base comes in from the right and often normal to the mean upper level flow and anvil direction, while descending cold air curls around the updraft at cloud base producing the characteristic hook echo. These features obviously cannot be modelled in two dimensions.

The degree of apparent order is very much less in the airflow in the vicinity of the storm than further away. One reason for the small scale structure is probably extraneous to the problem under consideration: a region of turbulence around z = 15 km is caused by the breaking of a propagating stratospheric wave of 30 km horizontal wavelength. The long wave is very possibly an artifact of resonance with the horizontal domain extent, and the breaking leads to locally generated turbu-
lence. In the sequence in fig. 11, these small scale motions are seen to have diffused down to cloud top. There is also another region of small scale motions seen somewhat lower down, at mid cloud levels. The spectral decomposition in fig. 14 shows these two regions as distinct. The second region of small scale motions is linked to the effect of the presence of liquid water on the airflow in and around the storm. At \( t = 350 \text{ min} \), when the liquid water content (LWC) reaches its highest value in the simulation, water loading is causing strong velocity gradients near the top of the updraft in the supporting cell and apparently counteracting the gravity wave-induced structure of \( w \) (fig. 11c). This is even more apparent in the next plot at \( t = 360 \text{ min} \), when the in-cloud airflow is governed strongly by the liquid water generation, and updrafts in these regions no longer retain the periodicity of the dominant mode. The weakly sheared mid-level flow also plays a role in enhancing this effect. In a sheared mid-level flow, the tilting of updrafts may cause rain to fall out of the updraft into the adjacent gravity-wave induced downdraft, reinforcing the support cycle and leading to long-lasting circulations.

It is interesting to note that far downstream (or a little upstream, depending on your point of view) of the cloud, the wave-eddy structure is as well-defined as in the earlier sequence shown, at the inception of deep cloud growth. A comparison shows that the boundary layer updraft and gravity wave above at around \( y = 48 \text{ km} \) at \( t = 350 \text{ min} \) (fig. 11c) is very similar to the situation at \( y = 24 \text{ km} \) in fig. 8b, suggesting that while in-cloud motions obscure its existence, the gravity wave still persists even at this mature stage. This tends to support the contention of Ley and Peltier (1981) that gravity wave propagation in the far field is governed by the modal structure of the atmosphere rather than some property of the emitter, i.e., that the dispersion relations of a propagating disturbance follow from the as-
sumption of a polychromatic radiative source transmitting into a medium capable of supporting only one kind of disturbance and attenuating the rest, rather than the assumption of a monochromatic source for the wave. This possibility needs more careful investigation: first, the persistence may be an artifact of harmonic quantization. More importantly, it must be noted that there are very fundamental differences between the upstream-moving standing waves of this near field simulation, and the propagating waves in the far field, in Ley and Peltier (1981). Any comparisons must be considered suspect. However, it is significant that the dominant mode still continues to inform the boundary layer; it is questionable whether this could be so if the gravity wave were totally annihilated.

The extreme dearth of low-level moisture considerably affected the later stages of this simulation, not allowing the "storm" to develop precipitation on the ground. The parallel experiment V6 is identical to V1 until time $t = 130$ min, whereupon a moisture flux corresponding to a latent heat flux of 60 watts/m$^2$ is introduced uniformly across the lower surface. Treating the diffusivity of heat and moisture the same, the same exponential attenuation profile is used as for the forcing heat flux. A comparison of figs. 15a and b show how the subcloud moisture has been stabilized in model V6 as compared to V1.

The main effect of the added moisture flux has been to hasten the periods of storm activity with respect to model V1, with little significant difference in the manner of development. The initial period of deep convective activity occurs at about the same time as in V1, shown in figs. 16a and b for $t = 170$ min. In this early stage of activity, the greater abundance of moisture is showing its effect: unlike in V1, here every updraft is seen to grow a cloud on its downshear side. This correspondence between eddy and cloud is often found in very moist environments,
such as in convection over the tropical ocean (cf. LeMone and Pennell, 1980). Since
the modal character of the environment is not much altered, the phase speeds of the
gravity wave and boundary layer eddies remain close to their values in model V1
at the same stage (∼ 9 m/sec and ∼ 5 m/sec respectively), as also the wavelength
(∼ 12 km). Due to the moister environment, however, strong convective activity
occurs whenever conditions are favourable. The recurrence time corresponding to
the time period of relative phase motion of the gravity wave and the eddies is
rather more evident, and equal to about 50 min here. Three peaks of convective
activity are readily apparent. Beginning with the initial period of deeper convec­
tion at $t = 170-180$ min, two peaks of storm growth are seen at around $t = 240$
and $t = 290$ min, seen in fig. 17 in the form of strong updrafts at mid-cloud levels.
Fig. 18, the time history of the maximum liquid water content, shows peaks cor­
responding to these times. At $t = 230$ min (fig. 19), the cloud has evolved into a
single storm, which retains its identity through these two cycles of enhanced growth.
Rainfall near the ground is still very low, though more widespread than in model
V1, and shown at its most widespread in fig. 20. Note that it is the maximum LWC,
and not the most widespread rainfall that is correlated with the recurrence time of
convective growth. The cessation of strong convection occurs earlier than in model
V1, and no third cycle is seen at $t = 340$ min, when the activity was at its maximum
in model V1. This may be because the consumption of available potential energy
and moisture is spread over a longer period of time, in the first two cycles, during
which time there was little activity in V1. The increased rainout is also a factor.
The peak LWC (2.2 gr/kg) and updraft (10 m/sec) are also smaller, possibly for the
same reason. This may be a result of some significance, pointing to the difference
between the violent, short-lived summer storms in dry environments such as the
Great Plains, and the gentler and more widespread precipitation patterns seen in Europe.

The maximum horizontal extent of cloud is much larger than in model V1, covering at its peak over two-thirds of the model domain with the boundary marking the region with $q_c \geq 0.01$ gr/kg. The gravity wave at mid-cloud levels is hence almost entirely masked by the in-cloud air motions. But the wave, or a reasonable facsimile thereof, can still be seen downstream from the cloud, shown in fig. 21a and b at $t = 320$ min. This persistence was also observed in model V1, and discussed earlier.

4. Experiments with a 3-dimensional model.

In many ways, the analysis of the results of the 3-D model is informed by what has transpired in the 2-D case: the 2-D model indicates to large degree what to look for in 3-D. It is apparent from the 2-D results that the phase relations are of paramount importance, and that will be a principal focus of the current experiment. The surfaces of constant phase in 3-D are not likely to be well-defined, as they are in the 2-D model by default, as it were. The interpretation of phase relations is therefore a matter requiring some care.

It is found that the 3-D model is also subject to the rapid drying out of the low level flow, as in the 2-D case. The basic result of the experiment testing the sensitivity of the storm evolution to an imposed surface moisture flux has no inherently two dimensional features; it is therefore to be expected that similar results will proceed from an imposition of moisture flux in the 3-D case. No analogue of the experiment V6 is attempted in three dimensions.

The most fundamental improvement of the 3-D simulation over the 2-D is that longitudinal modes are now possible; the excitation of the development of the
dominant mode no longer proceeds from a spurious mode. An important question that may now be addressed is whether the dominant mode is able to form the low level flow in competition with a strong instability of the boundary layer. It may be stressed again that this is shown to be a mechanism for the development of suitable scales for the lifting of moisture\textsuperscript{13} above the LCL and the free convection level, and as such, may be a crucial factor in the release of the conditional instability leading to storm formation. The spatial scales involved in the lifting of low level moisture is a matter of some interest in the problem of cumulus parameterization in larger scale models; some attempt may be made to address this question at small scale through the articulation of the dominant mode approach to convective organization.

Another significant feature of the 3-D simulation is the turning with height of the wind profile, a factor of immense significance for the formation of the dominant mode, as will be seen. Convection tends to destroy the spiral; the inability to model mechanisms of shear support directly in the model proves to be a matter of some importance. A second experiment in three dimensions attempts to take this into account, albeit in crude fashion. The basic 3-D model M1 is run in a domain 60 km × 60 km × 15 km height, cyclic in x and y. A resolution of 1 km is used in the horizontal, and 0.5 km in the vertical, leading to a box of 60 × 60 × 30 points. The effects of the reduced horizontal resolution with respect to the 2-D model will not be addressed here. A time step of 20 sec is used for the first two hours of the simulation, and 10 sec subsequently. The wind profile is again translated to minimize the maximum possible wind speed, thus considerably relaxing the constraint on the timestep because of numerical stability requirements. Though a rotation of the

\textsuperscript{13}Fig. 15, for instance, shows a sudden increase in the upward mixing of moisture coincident with the inception of the dominant mode: this is due to the increased efficiency of cumulus growth subsequent to the selection of "deep" scales.
winds is of no consequence here, it is performed in order to retain compatibility with the 2-D simulations. The hodograph in the transformed system is shown in fig. 22; a comparison with fig. 2 shows the nature of the transformation. In the 2-D simulation, the crucial quantity turned out to be the relative phase speed of the gravity wave in the free atmosphere and the boundary layer eddy; this remained the same in the transformed system and was reported thus. Here relative angles are of importance, and velocities will be reported in the original system whenever necessary.\footnote{Unless explicitly stated as being in transformed space, velocities reported have been transformed back to the original system.} The mean wind in the boundary layer (marked in fig. 22) is about $0.5\hat{x} + 9\hat{y}$ m/sec (southerly flow), and the wind at the top of the boundary layer, which in this model is taken to be the geostrophic wind $V_g$, is about $6\hat{x} + 9\hat{y}$ m/sec.

The same forcing heat flux is used as in the 2-D case (300 watts/m$^2$). The first sequence of pictures from model M1 (figs. 23a–d show the $w$ field at $z = 1$ km at 10 min intervals from $t = 55$ to 85 min. Structures parallel to the low level shear fill the entire domain. Their velocity, given by Eqs. (85) and (86) is nearly identical at this stage to the mean boundary layer wind, and hence about 20$^\circ$ to the left of the geostrophic wind. These modes thus closely resemble the longitudinal roll vortices as described by LeMone (1973). The wavelength of these modes is about 6 km. It is to be noted that the major component of the motion of the roll vortices is in a direction parallel to themselves. While this velocity does not appear in theoretical models of a two-dimensional boundary layer, observed rolls are rarely without structure along their axes,\footnote{Individual clouds in streets, for example.} and measurements of the motion of these structures usually yield a velocity at least comparable to, if not larger than, the transverse motion (\textit{cf.} Brümmer, 1985; Kuettner, 1971; LeMone, 1973).
At $t = 85$ min, the convection has very nearly wiped out the low level shear. The hodograph at this time in the model system (fig. 24) shows that there is little or no shear below 1 km. This would seem to be an artifact of the poor resolution of the model near the ground, and the absence of shear-supporting pressure gradients; as discussed earlier, the maintenance of Ekman flow is not properly treated under the assumptions of the model formulation. The data analysed by Smolarkiewicz and Clark (1985) during another CCOPE event suggest that directional shear may indeed vanish as an effect of convection. A sequence of wind profiles over several hours show the low level wind slowly coming into line with the upper level flow. But this process occurs on a time scale of several hours; the rapid decay of directional shear in this simulation is obviously unrealistic. A mechanism of shear maintenance is used in the second 3-D experiment to correct this. The current experiment serves to simulate the development of the dominant mode in the case of weak directional shear, and proves important in comparisons with the results of the second experiment.

The immediate effect of the loss of low level shear is that the lowest 1 km ceases to play a dynamical role. A second longitudinal instability, now in response to the shear in the layer 1 to 2 km, manifests itself, and quickly overcomes the original instability, causing a reorientation of boundary layer rolls on a very short time scale ($\sim 15$ min), a process that probably has no counterpart in the real atmosphere. However, they may still be thought of as rolls or roll vortices, since they are a longitudinal mode. This effect is seen to dramatic effect in the aerial shot of fig. 25. The mean flow is now almost planar. As the roll vortices deepen and begin to intrude upon the stable layer, the profound effect of the reorientation now become apparent. The longitudinal modes are now oriented almost perpendicular to the
preferred orientation of the surfaces of constant phase of the gravity wave. So the gravity wave is not launched uniformly along its length, as in the 2-D case; as the impactions of the boundary layer convection are localized, the waves are launched as though from a distributed array of point sources in the plane of the top of the boundary layer. A sequence of pictures of the $w$ field at $z = 3$ km, near the top of the PBL, illustrate what transpires. In fig. 26a a periodic structure is seen that is probably a manifestation of the interaction of the gravity wave and the rolls: starting near the lower left corner and proceeding along the direction of shear, a wave-like structure is apparent. It is presumed that a gravity wave has been launched at some point along the underlying roll. As the wave develops, it is positively reinforced from below at successive wavecrests. The wavelength is about 13 km in the shear direction, and about 7.5 km in the shear-transverse direction, reflecting the presence of both the gravity wave and the rolls. The points of intersection of the wavecrests and the convective updrafts provide channels for the lifting of moisture past the free convection level. Clouds may be expected to be associated with these; and, indeed, a comparison of fig. 26d with fig. 27d shows sets of small cumuli spaced in consonance with the gravity wave and aligned with the shear. Each of the clouds is supported by an updraft on its downshear edge, corroborating the observation of upshear cumulus development in the 2-D studies, and in Clark et al. (1986).

The suppression caused by the wave troughs at intervals along the roll cause the roll to break up; in fig. 28, the $w$ field at low level ($z = 1$ km) corresponding to fig. 26d still appears to be in a longitudinal mode, but the rolls no longer remain coherent over the whole domain, but instead are broken up at intervals approximately corresponding to the wavelength of the gravity wave. The idealized conception of
the dominant mode in 3-D in a pure speed shear situation is now apparent: short roll vortex couplets exist in the boundary layer aligned along the shear axis, with the updraft anchored at its top to a local wave crest, the corresponding downdraft to an adjacent (in the transverse direction) trough. The length of the short rolls along the shear axis is determined by the wavelength of the gravity wave mode; their spacing, and that of the stable layer modes, in the transverse direction, is determined by the wave number of the boundary layer longitudinal mode. This conceptual model is difficult to verify as boundary layers without directional shear are rarely observed. It is proposed to test this model on cases of very weakly sheared boundary layers over the tropical ocean in subsequent studies.

It was assumed at the outset of the 2-D simulations that the structure of the dominant mode was probably independent of the perturbation that produced gravity waves. The conceptual model just outlined shows that the dominant mode is still structured by the initial longitudinal instability, which appears to contradict the assumption. However, it must be noted that the scale selection imposed in the plane of the shear is still determined by the gravity wave; it is only the spacing in the transverse direction that is informed by the longitudinal mode. The assumption does not appear to have been a grievous one for the 2-D simulation, as the scale selection in the principal flow plane is not affected by the initial perturbation.

The phase velocities are calculated for the period $t = 145$ min to $t = 175$ min. The boundary layer modes are seen to move at approximately the mean velocity within the boundary layer. The gravity wave appears to move at about 6 m/sec upshear with respect to the motion of the dominant modes of the boundary layer and almost wholly along the direction normal to its surfaces of constant phase. Note that this is also true of the boundary layer modes: the roll couplets move transverse
to their major axis. It is apparent that the motion at this stage (30 min after the onset of condensation) is wholly independent in the two layers. This is not to mean that the dominant mode no longer exists: the short rolls are still manifest at low levels 2 hours after the onset of deep and severe convection (fig. 29).

The structure of the storm in its late stages is essentially the same as in the following experiment. Since the main focus is on the development of the dominant mode interaction, a discussion of the various features of the storm is postponed until after the exposition of model M2.

The second simulation in 3-D is run under conditions that preserve the turning boundary layer wind profile against the vertical mixing effect of boundary layer convection. As noted earlier, the spiral is maintained by a balance between the Coriolis force, the horizontal pressure gradient, the mixing of momentum by turbulence, and surface friction. At the top of the Ekman layer, the surface effects are negligible, and the balance between the remaining forces gives rise to the geostrophic wind. Frictional effects of turbulence and the surface induce quite a different balance near the ground, and the analytical model of this results in the so-called Ekman spiral. In the current formulation of the model, horizontal pressure gradients are treated only in implicit fashion, assuming the initial flow to be in Ekman-type balance, which is now simulated by subtracting off the Coriolis component of the initial flow. The top of the Ekman layer is high enough that the boundary layer convection does not seriously affect the geostrophic wind except at the long time scales at which the balance is modified even in the real atmosphere. If the lower end of the spiral is maintained through a simulation of frictional effects, the wind profile can be made to maintain directional shear for realistically long times.

This is achieved for the experiment M2, run over an identical domain as M1,
through a simple parameterization of friction in the form of the drag coefficient $C_D$

The drag coefficient (in either the $x$ or $y$ direction) may be defined as:

$$C_D = \left( \frac{u_*}{u_0} \right)^2$$

(1)

where $u_0$ is the horizontal wind at the lowest grid point of the model, and $u_*$ is a value of the velocity typical for the surface layer.\(^{10}\) Clark and Farley (1984) describe how the drag coefficient is formulated in the model. A value of 1 for $C_D$, for instance, would mean that the surface layer extends up to the height of the first grid point. The height of the surface layer is typically of the order of 10 m; for low resolution models such as these, typical values of $C_D$ are of the order of $10^{-3}$ (Smolarkiewicz and Clark, 1985). Since a rigid maintenance of the directional shear near the ground is the purpose here, and not a realistic parameterization of surface effects, an unrealistically high value of 0.25 was used for $C_D$. The friction was referenced not to zero velocity but to the velocity at the lowest point of the sounding, which may be assumed to lie at the top of the surface layer. The effectiveness of the drag in maintaining the shear profile is illustrated in fig. 30, where the spiral is alive and well at $t = 175$ min, in stark contrast to the rapid vertical mixing in model M1.

The convection that arises at the surface when the fluid is subject to thermal forcing produces eddies with low level convergence acting as production terms for the updraft. The drag at the surface retards the convergence; the effectiveness of

\(^{10}\) At $z = 0$, the horizontal wind must be zero; what is usually considered the "surface wind" refers to a layer of constant momentum fluxes that extends from a height of the order of the surface roughness length away from the ground up to $\sim 10$ m. This is referred to as the surface layer. The $C_D$ parameterization simulates drag by treating a fraction of the vertical flux of horizontal momentum at the lowest grid point of the model to be transmitted through the constant flux layer to the ground.
the heat flux in stimulating convection is therefore reduced. Development in model M2 therefore occurs at a slower rate than in model M1.

As in model M1, the first instability to become apparent is the longitudinal mode parallel to the low level shear. The wavelength is a little over 6 km, and they move at a speed of 6 m/sec along the direction of mean boundary layer flow (somewhat slower than the mean flow itself). This is about 20° to the left of the geostrophic wind at the top of the boundary layer: there is little doubt that these represent good facsimiles of the longitudinal roll vortices of LeMone (1973). They are shown in a set of aerial views in fig. 31 at $t = 65$ min.

The dominant mode interaction begins about an hour later. Spectral decompositions of the $w$ field at $t = 115$ min and $t = 135$ min are shown in fig. 32, in conjunction with the aerial views at these times (figs. 33 and 34). The spectral plots show the growth of the deep mode ($k_x = 2; k_y = 5; \lambda = 11$ km) from the shallow mode ($k_x = 4; k_y = 9; \lambda = 6$ km). The modes are more or less parallel and the ratio of their wavelengths is about 2; here the process by which the dominance of the deep mode is established is similar to model M1, and rather more apparent. The wavecrests are in phase with alternate rolls in the boundary layer; these rolls are supported and the rolls in between are suppressed, as seen in the aerial views. The clarity of the interaction is a fortuitous result of the near-90° veer in the wind at $z = 1$ km: it would appear that the more nearly the wind turns at right angles, the more likely is the appearance of banded convection, as opposed to the distributed convection in the early stages of the dominant mode in model M1. The aerial view in fig. 35 at $t = 155$ min shows that the convection does not remain banded over the whole domain, perhaps because of the slight acuteness of the angle of veer. The bands show a structure along the surfaces of constant phase: wavefronts 15 to
20 km in length are formed. A measurement of phase velocities during the period $t = 150$ min to $t = 180$ min show that both the dominant modes in the boundary layer move in the plane of the low level flow at a speed slightly slower than the mean velocity in the boundary layer; the waves have an additional component of about 2.5 m/sec downstream normal to their surfaces of constant phase. This indicates the progress of the development beyond the phase-locked stage. The cloud field is now becoming of appreciable size, and is probably the agency of phase decoupling. Note that in the 2-D simulations, the wave moved upstream relative to the boundary layer modes, and downstream here. The reason for this difference is not apparent, but it is probably an effect of the dimensionality. It may be noted that in the 3-D case, a cloud that is initially on the downshear edge of an in-phase deep mode will continue to experience enhanced growth as the wave moves over it. Upshear cumulus development is thus enhanced by the sense of relative motion that occurs in the 3-D case.

It was felt that the drag at the surface was unduly inhibiting the low level convergence and the consequent lifting. To accelerate the process of severe growth, the drag was turned off at $t = 180$ min, it having served its purpose in articulating the exact process of dominant mode development under directional shear. The removal of drag does affect the relative motions to some extent: the velocity of the boundary layer modes in the plane of the low level flow measured at $t = 270$ min accelerates to 7.5 m/sec, bringing it closer to, but still below, the mean flow. The wave phase velocity remains more or less unchanged, and the relative motion in the plane of the upper level flow retains its value 2.5 m/sec in the downshear direction.

The recurrence time in this case, corresponding to the wavelength of 11 km, is about 73 min. Such pulsations are difficult to observe directly, unlike the 2-D
case, as the clouds do not move along the coordinate planes of the model. However, a plot of the maximum cloud water content versus time (fig. 36) did show peaks at roughly this interval, at $t = 160, 230,$ and $300$ min, and was showing signs of building up to a fourth peak, when the integration was stopped, at $t = 360$ min. However, an additional peak that does not fit into this schema is also present at $t = 200$ min.

The storm motion is more or less due East (in the original coordinates), and somewhat slower than the midlevel flow ($\sim 12$ m/sec). This is about $18$ m/sec with respect to the gravity waves along the axis of the mean shear. Besides, the advection of the waves in the direction of the mean flow means that the moist inflow comes in from the right rear flank. New cells are formed at the outflow boundary at the forward and are advected toward the right rear. These features are in good accordance with the models of Browning (1977) and several analyses of observations of High Plains storms (e.g., Miller and Fankhauser, 1983). Since new cells appear at intervals corresponding to storm motion with respect to the wave-crests, we may expect the spatial interval to equal the wavelength of the dominant mode, and the interval to correspond to the storm-relative phase velocity of the dominant mode. The appearance of new cells at intervals of $16$ min spaced $11$ km apart may thus be deduced. This corresponds very well with the observations of Chalon et al. (1976), as was also found in the 2-D simulations. When the run was ended, after $6$ hours of model time, the anvil was so large as to completely span the model in the $y$-direction, rendering further integration meaningless (fig. 37). The maximum updraft was about $43$ m/sec and the maximum in-cloud LWC $2.3$ gr/kg. While there is no ice in the model, these values are certainly consistent with the characteristic updraft strengths and LWCs associated with hail-producing storms.
Just as in the 2-D case, an excessive drying out of the subcloud layer in the absence of a moist inflow prevented precipitation from reaching the ground. These features have been discussed in greater detail during the presentation of the 2-D results, and are essentially not different here.

The major three-dimensional feature of the observed storm of 1 Aug 1981 that has not been simulated is the possible splitting vortex. It is probable that the level of vorticity production required to set up the supercellular circulation is largely dependent on the vorticity production in the low level convergent flow by large-scale gradients over the domain, which have not been included in the simulation. It has been noted (Clark, 1979) that the frequent occurrence of splitting vortices in three dimensional numerical models could well be an artifact of the "bubble"-type initialization. The simulation here attempts to simulate general features of the cloud field, showing how storms evolve out of cumulus populations; the fact that a specific pathological feature of the actual storm is not reproduced is not considered a serious shortfall.

5. Experiments with the linear model.

Two experiments were conducted with the linear model, one in two dimensions and one in three. In both of these, the rotated and translated winds of the non-linear model runs are used for purposes of comparison. The thermal profile used is an idealized form of the measured temperature profile, eliminating the small scale structure while retaining the features essential to the dominant mode interaction. This idealized form consists of a neutrally stable boundary layer capped by a tropospheric layer of constant stability. The stability aloft is set equal to the mean stability of the lower tropospheric layer (between 2 and 6 km) in order to reproduce the conditions under which the dominant mode interaction unfolds. The value of
$K_M$ is set equal to a constant representative value of 30 m$^2$ sec$^{-1}$ in the PBL, and $0$ above: this profile was chosen from an inspection of the non-linear runs. The Rayleigh absorber profiles are also set to match those of the non-linear runs.

The purpose of these experiments is to see how well the linear anelastic model reproduces the results from the non-linear model, thereby to delineate non-linear aspects of the dominant mode interaction. Two salient features of the linear model may be recalled at this juncture, as they will be seen to influence profoundly the results that follow. Most important, the mean flow is stationary in time, and initially stratified. All the energy therefore goes into the perturbation velocities, which will in time lead to perturbation magnitudes that conflict with the assumptions of linearity. The equations derived are therefore valid only as long as the amplitude of the motions remain small compared to the magnitude of the mean flow, which is of the order of 10 m/sec.$^{17}$ Under this condition, no horizontal momentum is ever produced or destroyed. Since the domain is subject to continuous vertical forcing at all wavelengths, and the conversion to horizontal momentum from the divergence of the forced vertical motions is forbidden, amplitudes of unstable modes will continue to grow divergently past the point where they may be expected to saturate or decay under the non-linear equations. This may be considered to occur at the point when the growth rates in all the modes stabilize, so that further integration merely increases amplitudes without changing the overall structure. This is seen to occur at amplitudes of $w \sim 1$ mm/sec.

The 2-D model L1 is run over a domain similar to the experiment V1. The domain is of 60 km horizontal extent and 25 km in the vertical. A resolution of 0.5 km in is used in both the $y$ and $z$ directions, leading to a grid of 120 by 50

$^{17}$Only the vertical perturbations are computed, but the perturbed horizontal motions may be considered to scale accordingly.
points. A timestep of 7.5 sec was found to be appropriate. The development is basically similar to the non-linear case, as is seen in the sequence of plots of the $w$ field shown in fig. 38. The earliest picture, at $t = 10$ min, shows the shallow mode at low amplitude; this may be compared to the picture from V1 at $t = 50$ min, in fig. 4b. The principal wavenumber is 0, slightly broader than the shallow mode scales in V1. The development of the deep mode is seen through the sequence. In the last shot, at $t = 50$ min, the shallow mode has grown to an amplitude that begins to strain the limits of the linearity assumption, and may be considered the last stage of useful information from the linear model. The distribution of energy in the deep modes appears in a broad spectrum centred at wavenumber 4 (ranging from 3 to 6).

Two important inferences may be drawn from these results. In the V1 run, the destruction of smaller scale motions was dramatized at around $y = 50$ km in fig. 5b, where the gravity wave above was seen to be forcing the flow below. It was then speculated that this appeared to be accomplished through the downward flux of momentum mixing with and destroying the small scale eddy downdraft, a non-linear Reynolds stress effect. This is borne out by the development in fig. 38d, where the deep modes do not dominate the PBL flow. While the inception of the deep mode is reproduced in the linear formulation, the subsequent suppression of the shallow mode appears to be a non-linear effect. The possibility of investigating this further through a diagnostic treatment of Reynolds stresses in the linear model is discussed in Chapter IV.

The other feature of interest in the linear development is the spread of the spectrum of deep modes. In a non-linear sense, the thermal forcing function may be represented by the temperature-vertical velocity correlation function ($w'\theta'$). The
vertical exponential profile applied to the heat flux in both the non-linear and linear cases represents the turbulent contribution to \( (w'\theta') \). In the resolved scale, this term is forbidden in the linear formulation; while in the non-linear runs, it may have served to more efficiently channel energy into the most unstable mode both in deep and in shallow convection. It would thus appear that correlations arising out of the second-order term omitted in the linearization of \( u \cdot \nabla \theta \) play an important role in enhancing instabilities.

The computation of phase velocities revealed another possible non-linear effect. The phase speed of the principal deep mode of wave number 4 calculated from Eq. (86) was 6 m/sec toward the left, a good correspondence with its value in the PBL in model V1. It remained always constant with height. The decoupling of phase between the gravity wave and the dominant mode is thus seen to be a non-linear effect of moisture. This was already shown in the Clark et al. study from a comparison of dry and moist non-linear runs.

The 3-D model L2 was run over the same grid as the non-linear 3-D runs: a domain 60 km \( \times \) 60 km \( \times \) 15 km height, cyclic in \( x \) and \( y \). A resolution of 1 km is used in the horizontal, and 0.5 km in the vertical, leading to a box of 60 \( \times \) 60 \( \times \) 30 points. No fundamentally new insights into the linearity question are offered by the results from model L2, and the results are presented only briefly. The shallow mode has a wave number 0 in the \( y \)-direction; the deep mode has a principal wave number of 4 and, as in L1, shows a broader spectrum. As in the 2-D run L1, the deep mode fails to suppress the shallow mode. No new inferences are drawn from these results.

Since the mean flow is held stationary, the model M2 is the analogous non-linear run. As in the model M2, the shallow longitudinal modes were parallel to the
wavefronts of the gravity wave launched later; the gravity wave front was therefore launched uniformly along its length. Alternate eddies fail to receive upper level support from the gravity wave: if the non-linear mixing of horizontal momentum were allowed, these convective eddies would be suppressed as they are in model M2. Fig. 39a and b show aerial views of the convection in the boundary layer and the stable layer at the end of the run, at \( t = 60 \) min: it can be seen rather clearly how alternate convective eddies have wavecrests perched on top.
CHAPTER IV
RESULTS AND PROSPECTS

The motivation of this study and its results may now be recapitulated. Deficiencies in the current formulation are pointed out and directions for future research indicated in the later sections of the chapter.

The dominant mode mechanism of convective scale selection.

Studies have shown that the organized lifting of moist low level air at horizontal scales on the order of tens of kilometres is closely associated with the formation of severe storms. In the absence of large-scale forcing, the organized lifting at the required scale must be produced by local convection. But when the development of local convection out of a stratified atmosphere is examined, a problem arises, for the small scale convection fails to provide the appropriate scales. Linear theories of shallow convection reveal that the eddies formed by surface heating yield an aspect ratio of 2 or 3 with respect to the depth of the PBL; and to large extent this is borne out by the observations, though anomalous aspect ratios have been observed as well (LeMone and Meitin, 1984). Non-linear approaches to shallow convection do show transitions leading to cell broadening (Chang and Shirer, 1983); however, the more likely explanation of scale selection leading to deep convection is to be found in treating boundary layer convective scale selection in a non-local manner, allowing it to interact with the stable tropospheric layer. The Clark et al. study was able to simulate the development of a field of fair-weather cumulus, based on a mechanism
which allowed the deepening of PBL convective eddies to launch a gravity wave at longer scales in the stable layer. The response of the boundary layer flow to the gravity wave allowed the development of "deep" scales of convection within the PBL.

In the present study, the same mechanism was studied in relation to the development of deep and severe convection, and the picture of storm initiation based on the dominant mode theory of convective organization that emerges from the simulations appears to be a plausible one. An initially stratified, horizontally homogeneous atmosphere is forced at the ground by an applied heat flux, to initiate convection. The 3-D simulations revealed that the convection in the early stages was well represented by the low aspect ratio boundary layer roll vortex modes that were observed by LeMone (1973). As the convection deepens and fills the boundary layer, a gravity wave is launched in the free atmosphere, which in turn leads to the development of the dominant mode.

Linear and non-linear effects.

The convection model in the linearized time-dependent Navier-Stokes equations did also develop the dominant mode, which do not appear in a conventional normal mode analysis. Following the development of the dominant mode in the stable layer, the non-linear models showed that the shallow modes experience suppression in the boundary layer. Comparison with the linear model, where such a suppression is not observed, assert this to be a non-linear effect, possibly related to the downward diffusion of horizontal momentum from the stable layer. The spectral distribution of kinetic energy also showed an apparent non-linear effect: the spectrum of deep modes was significantly broader in the linear simulations. This may be attributed to the enhancement of unstable modes by the resolved-scale heat
flux in the non-linear simulations. It would thus appear that the basic interaction may be understood in linear terms, but the evolution of the flow is significantly mediated by non-linear effects.

The effect of directional shear.

The 3-D simulations revealed that the angle between the low level wind and the wind at the top of the PBL is a significant parameter. In the first 3-D simulations, the idealizations in the model initialization resulted in an attenuation of low level shear. This was a result of the incorrect treatment of the initial flow to be in an implicit geostrophic balance between the large scale pressure gradient and the Coriolis force. The subsequent addition the diffusive effect of turbulence as well as convection in the resolved scale destroyed low level shear on a short time scale. This resulted in the longitudinal roll vortices suddenly changing their orientation as the low level shear vanished and directional shear was lost. In the subsequent simulation, the low level shear was maintained through a parameterization of surface effects. The loss of directional shear in one case and its retention in the other led in serendipitous manner to the discovery of the profound effect of directional shear. In M1, where directional shear was lost, the axis of the longitudinal roll vortices was perpendicular to the fronts associated with the gravity wave, which are aligned transverse to the shear. While the rolls are at first coherent over the whole domain, the suppression caused by the wave troughs at intervals along the roll cause the roll to break up; Short roll vortex couplets appear in the boundary layer aligned along the shear axis, with the updraft anchored at its top to a local wave crest, the corresponding downdraft to an adjacent (in the transverse direction) trough. The length of the short rolls along the shear axis is determined by the wavelength of the gravity wave mode; their spacing, and that of the stable layer modes, in
the transverse direction, is determined by the wave number of the boundary layer longitudinal mode.

In the simulation M2, the wavefronts and the axes of the longitudinal rolls are parallel: a fortuitous result of the near-90° veer in the wind profile of fig. 2. The waves are launched uniformly along their length; here the process by which the dominance of the deep mode is established is similar to model M1, and rather more apparent. The wavecrests are in phase with alternate rolls in the boundary layer; these rolls are supported and the rolls in between are suppressed, as seen in the aerial views. It would appear that many features of cumulus populations — the appearance of banded or distributed convection, for instance — may to large extent be determined by the directional shear in the boundary layer.

*Storm evolution.*

The picture of storm evolution appears to follow three well-defined stages of convective development, subsequent to the appearance of the deep mode.

*Phase-locked convection.* Just after the inception of the deep mode and the suppression of the shallow mode, when condensation is still minimal, the gravity wave and boundary layer eddies remain coherent over their depth, though there may be tilting in response to the shear. A field of scattered shallow cumuli covers the model domain. This is a period of strong upward mixing of moisture, and in the absence of a moist inflow, considerable drying out of low level air may occur. The development of significant amounts of condensation acts to decouple the wave and the convective eddies in phase, possibly through an “obstacle” effect of the clouds. This effect was also observed in the Clark et al. study. In both the 2-D and the 3-D runs this stage lasts for about an hour after the gravity wave is first launched.

*Phase-decoupled convection.* After the waves and the eddies become decoupled in
phase, they retain well-defined speeds, and the relative motion leads to a stage of
cyclical growth, where convective growth is alternately suppressed and enhanced by
the gravity wave, with a characteristic recurrence time. The recurrence time may
be computed from the wavelength of the gravity wave, and its phase speed with
respect to the clouds, which remain anchored to the downshear edge of boundary
layer eddies. The recurrence time is computed to be 57 min in the 2-D case, and
73 min in the 3-D case. A condition for catastrophic growth may be the fortuitous
concurrence of a growth phase in the cycle, and the favourable position of "feeder"
clouds on the downshear edge; the feeder clouds are of great significance when the
subcloud layer is dry. Several growth cycles may elapse before this occurs. Several
clouds in the field may attempt to grow deep and be suppressed before rising to
their level of neutral buoyancy; a time lapse movie from the CCOPE experiment
shows such a process occurring through the day on 1 Aug 1981. This suggests
an explanation of why boundary layer convection may begin in the morning, but
isolated thunderstorms generally occur in the afternoon. This stage may last several
hours; but given a conditionally unstable sounding and a large enough cumulus field,
severe convection is eventually likely to occur.

**Severe convection.** In the event that deep and severe growth occurs, a mature
storm stage follows. The gravity wave that gave rise to the deep convection may
be obscured, or no longer exist, in the vicinity of the storm. The storm moves at
the velocity of the mid-tropospheric winds; if there is storm motion relative to the
convection in the boundary layer, new cells are formed and old ones discarded as
the storm moves overhead from eddy to eddy, producing structures in time and
space similar to that of the classical multi-cellular storm. Both the 2-D and the
3-D simulations corresponded well with the theoretical models (Browning, 1977;
Marwitz, 1972) and observations (Chalon et al., 1976) of multicellular storms.

Prospects for further investigation.

The results of the present study are only to be considered the first steps toward the establishment of a mechanism for the realistic initiation of scales in deep convection. Several aspects of the study require further corroboration, and the applicability of the mechanism in different meteorological regimes needs to be established. It may be noted that in the mid-latitude case investigated in this study, the depth of the boundary layer (2 km) and troposphere (12 km) led to the development of a shallow mode scale of \( \sim 6 \) km and a deep mode scale of \( \sim 12 \) km whose 2:1 ratio is suspiciously effective in promoting the development, as was seen in the M2 simulation, where alternate rolls were suppressed by the gravity wave. The investigation of the mechanism for tropical cases therefore appears to be an essential corroborative study. It is proposed to test this mechanism on the cloud bands reported by LeMone and Meitin (1984) where aspect ratios were found to be 30 or 40 with respect to a boundary layer depth of 500 m.

The failure of the shallow modes to saturate or decay in the linear model was attributed to the neglect of Reynolds stress terms. The Reynolds stresses transfer energy from the mean flow into the perturbed flow under the non-linear equations. In a linear model terms of the form \( \langle u'w' \rangle \) are disallowed; however, their effect in drawing energy from the mean flow may be simulated in a diagnostic estimation of the horizontal perturbation velocities. In 2-D this may be done quite simply inverting the mass continuity equation after \( w \) is determined in the model; the 3-D treatment requires an extra equation. This may permit a more precise delineation of the non-linear effects involved in the suppression of the shallow mode.

The study revealed the simulation of the Ekman layer to be an important
component in modelling the large scale distribution of convection. The Ekman layer is set up in a balance between the large scale pressure gradients, the Coriolis force, turbulent eddy viscosity, and surface effects. The implicit treatment of the mean initial flow as a geostrophic balance was shown to lead to an early destruction of shear. The proper treatment of shear maintenance in the convective layer is therefore a matter open to question. Reducing the effects of the turbulent diffusion of momentum through an assumption of $K_M \ll K_H$ is a possibility, but has no known physical basis; hence, the only alternative appears to be an independent prediction or measurement of the vertical profile of turbulence in the Ekman layer. One possibility is to determine this profile through an extrapolation of the pressure gradient profile into the Ekman layer based on an examination of the wind profile above it. Attempts are underway.

A final feature of the model requiring further corroboration is the cyclical nature of growth and suppression of clouds prior to the appearance of a severe storm, a feature which as yet has only indirect confirmation. While the discussion above reveals that much still needs to be done in the articulation of this model of convection, it is a model that may be of benefit in refining theories of cumulus parameterization and methods of initializing numerical models of deep cumulus. The deep cloud streets studied by Sun (1978), for instance, were modeled on the basis of a forcing function for the sustenance of the gravity wave that arose out of the latent heat flux from the cloud streets; this flux function was consonant in phase with the gravity wave since the cumulus field was itself modulated by the wave. The present study suggests that the forcing may be provided not by a parametric treatment of the non-linear effects of moisture, but indeed can occur from purely fluid dynamical considerations. The crucial difference in the two approaches appears
in the role of the PBL: in the dominant mode mechanism, the instabilities of the boundary layer play a central part, whereas most parametric approaches omit the PBL altogether. A comparative study of the dominant mode mechanism and a parametric method, such as wave-CISK, should prove interesting, as they are so different in approach.

A further extension of this model to include large scale forcing of low level convergences, ought to be done to further establish the possibility of a realistic initialization of numerical models of deep convection. The idealizations involved in setting up this model revealed themselves in the fact that the severe convection of the model was generic and did not resemble the most severe convection observed on that day. An interesting possibility suggests itself for this case, because of the near-90° veer in the wind. Large scale forcing and the low level moist inflow were omitted because a cyclic lateral boundary conditions were required in order for the fetch of the domain to be sufficient for the development of the deep mode. The 3-D results show that the deep mode development occurs in relation to the mid-level shear: thus cyclicity is only required in this plane. Since the low level flow is normal to this direction\(^1\) it may be possible to formulate a model where open boundary conditions are applied in direction of the low level flow, and cyclic boundary conditions in the direction of the upper level flow. Further study is indicated.

\(^1\) It must be noted that a 90° veer is not commonly observed.
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FIGURES
Figure 1. Temperature and dewpoint curves of the environmental sounding from Miles City, Montana on 1 August 1981. Full wind barbs represent 10-knot increments, half barbs 5 knots; the pennant represents a 50-knot wind.
Figure 2. Wind profile for 1 August 1981. Numbers along hodograph represent heights in km.
Figure 3. Wind profile in the plane of model V1. Winds in fig. 2 have been rotated counterclockwise through 90° and 10 m/sec subtracted from v. (See fig. 22.)
Figure 4. Plots of the vertical velocity \( w \) for model V1. Dashed lines represent negative values. (a) \( t = 20 \) min. Contour intervals are 1 mm/sec. (b) \( t = 50 \) min. Contour intervals are 1 cm/sec.
Figure 5. Plots of the vertical velocity $w$ for model V1 during the period $t = 120$ to $150$ min. Contour intervals are 0.5 m/sec. Dashed lines represent negative values.
Figure 6. Spectral analyses of the vertical velocity $w$ for model V1 during the period $t = 120$ to $150$ min. Contours represent energy units, held constant in all the plots.
Figure 7. Average water vapour mixing ratio $q_v$ in the subcloud layer ($z \leq 2.5 \text{ km}$) in model V1 plotted as a function of time.
Figure 8 (on following page). Plots of the vertical velocity $w$ for model V1 during the period $t = 180$ to 230 min. Contour intervals are 1 m/sec. Dashed lines represent negative values.
Figure 9 (on following page). Plots of the cloud water mixing ratio $q_c$ for model V1 during the period $t = 180$ to 230 min. Contour intervals are 0.25 gr/kg. Dashed line represents cloud boundary ($q_c \geq 0.01$ gr/kg).
Figure 10. Spectral analysis of the vertical velocity $w$ for model V1 at $t = 120$ min.

Contours represent the same energy units as in Fig. 6.
Figure 11. Plots of the vertical velocity $w$ for model V1 during the period $t = 330$ to 360 min. Contour intervals are 1 m/sec. Dashed lines represent negative values.
Figure 12. Plots of the cloud water mixing ratio $q_c$ for model V1 during the period $t = 330$ to $360$ min. Contour intervals are 0.25 gr/kg. Dashed line represents cloud boundary ($q_c \geq 0.01$ gr/kg).
Figure 13. Plots of the rain water mixing ratio $q_r$ for model V1 during the period $t = 330$ to 360 min. Contour intervals are 0.125 gr/kg.
Figure 14. Spectral analysis of the vertical velocity $w$ for model V1 at $t = 340$ min.

Contours represent the same energy units as in Fig. 6.
Figure 15. Horizontally averaged profiles of $q_v$ for (a) model V1 and (b) model V6. Contour intervals are 0.5 gr/kg.
Figure 16. Plots of the vertical velocity and the cloud water mixing ratio for model V6 at \( t = 170 \) min. (a) \( w \) field: contour intervals are 1 m/sec. Dashed lines represent negative values. (b) \( q_c \) field: contour intervals are 0.25 gr/kg. Dashed line represents cloud boundary (\( q_c \geq 0.01 \) gr/kg).
Figure 17. Horizontally averaged profiles of $w$ for model V6. Contour intervals are 1 m/sec.
Figure 18. Maximum LWC plotted as a function of time for model V6.
Figure 19. Plot of the cloud water mixing ratio $q_c$ at $t = 230$ min. Contour intervals are 0.25 gr/kg. Dashed line represents cloud boundary ($q_c \geq 0.01$ gr/kg).
Figure 20. Plot of the rain water mixing ratio $q_r$ at $t = 260$ min. Contour intervals are 0.125 gr/kg.
Figure 21. Plots of the vertical velocity and the cloud water mixing ratio for model V6 at $t = 320$ min. (a) $w$ field: contour intervals are 1 m/sec. Dashed lines represent negative values. (b) $q_c$ field: contour intervals are 0.25 gr/kg. Dashed line represents cloud boundary ($q_c \geq 0.01$ gr/kg).
Figure 22. Transformed wind profile for 1 August 1981. Numbers along hodograph represent heights in km. × represents mean wind in the boundary layer.
Figure 23. Plots of the vertical velocity $w$ for model M1 at height 1 km during the period $t = 55$ to 85 min. Contour intervals are 3.125 cm/sec in (a), 0.125 m/sec in (b), 0.25 m/sec in (c), 0.5 m/sec in (d). Dashed lines represent negative values.
Figure 24. Wind profile at $t = 85$ min for model M1. Numbers along hodograph represent heights in km.
Figure 25. Aerial view of the $w$ field at $t = 96$ min for model M1. Contour encloses regions where $w \geq 1$ m/sec.
Figure 26. Plots of the vertical velocity $w$ for model M1 at height 3 km during the period $t = 105$ to 135 min. Contour intervals are 1 m/sec. Dashed lines represent negative values.
Figure 27. Plots of the cloud water mixing ratio $q_c$ for model M1 at height 3 km during the period $t = 105$ to $135$ min. Contour intervals are 0.125 gr/kg. Dashed line represents cloud boundary ($q_c \geq 0.01$ gr/kg).
Figure 28. Plot of the vertical velocity \( w \) for model M1 at height 1 km at \( t = 135 \) min. Contour intervals are 1 m/sec. Dashed lines represent negative values.
Figure 29. Plot of the vertical velocity $w$ for model M1 at height 1 km at $t = 255$ min. Contour intervals are 1 m/sec. Dashed lines represent negative values.
Figure 30. Wind profile at $t = 175$ min for model M2. Numbers along hodograph represent heights in km.
Figure 31. Aerial view of the \( w \) field at \( t = 65 \) min for model M2. Contour encloses regions where \( w \geq 1 \) mm/sec.
Figure 32. Spectral analyses of the vertical velocity $w$ for model M2 versus $k_x$ and $k_y$ at $t = 115$ and 135 min. Contours represent energy units, held constant in all the plots.
Figure 39. Aerial view of the $w$ field at $t = 115$ min for model M2. Contour encloses regions where $w \geq 1$ m/sec.
Figure 34. Aerial view of the $w$ field at $t = 135$ min for model M2. Contour encloses regions where $w \geq 1$ m/sec.
Figure 85. Aerial view of the $w$ field at $t = 155$ min for model M2. Contour encloses regions where $w \geq 1$ m/sec.
Figure 96. Maximum cloud water mixing ratio plotted as a function of time for model M1.
Figure 37. Aerial view of the $q_c$ field at $t = 345$ min for model M2. Contour encloses regions where $q_c \geq 0.01$ gr/kg.
Figure 38. Plots of the vertical velocity $w$ for model L1 during the period $t = 10$ to 50 min. Contour intervals are 0.01 mm/sec in (a), 0.05 mm/sec in (b), 0.1 mm/sec in (c) and (d). Stipples areas represent negative values.
Figure 99. Aerial view of the $w$ field at $t = 60$ min for model L2. Contour encloses regions where $w \geq 1$ mm/sec. (a) represents a view of the boundary layer, (b), a view of the stable layer.