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Experimental study of electron scattering mechanisms in superconductors using nonequilibrium techniques

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The Ohio State University, 1987
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Experimental Study of Electron Scattering Mechanisms in Superconductors Using Nonequilibrium Techniques

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of the Ohio State University

By

Yeouchung Yen, B.S.

** * * * *

The Ohio State University
1987

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Experimental Study of Electron Scattering Mechanisms in Superconductors Using Nonequilibrium Techniques

By

Yeouchung Yen, Ph.D.

The Ohio State University, 1987

Professor Thomas R. Lemberger, Adviser

We have studied the low-voltage resistance of low-resistance and high-resistance Superconductor/Insulator/Normal-Metal proximity-effect tunnel junctions as functions of temperature and transport supercurrent both theoretically and experimentally. The experimental results are compared to numerical calculations based on a Boltzmann equation appropriate for a steady state, spatially homogeneous nonequilibrium charge-imbalance. The data are in excellent agreement with the theoretical calculations from $-0.6 \ T_c$ to $-0.95 \ T_c$ for low-resistance junctions and from $-0.33 \ T_c$ to $-0.85 \ T_c$ for a high-resistance junction. The electron-phonon rate \( r_{\text{e-ph}}(T_c)^{-1} \) we obtained for Sn, \((2.8 \pm 0.3) \times 10^9 \ s^{-1}\), is in agreement with theoretical calculation of \(3.65 \times 10^9 \ s^{-1}\). The unaveraged intrinsic mean square gap anisotropy parameter \(<a^2>_0\) we obtained, \(0.0053 \pm 0.0004\), is smaller than theoretical calculation of 0.02 but is reproducible among all samples. Our results verify the theory of low-voltage resistance of SIN tunnel junctions. Our technique of extracting information of electron scattering mechanisms is particularly useful in study of novel materials in which the pair-breaking or pairing mechanisms are not well understood.
TO MY PARENTS AND MY WIFE
ACKNOWLEDGEMENTS

I would like to express my full-hearted appreciation to my advisor Dr. Thomas R. Lemberger not only for his guidance and insight throughout the research but also for his friendship over the years. He has made my graduate study truly exciting and fruitful. My thanks also extend to my colleagues and all the technical and supporting staff. To my wife Yuh-Guin, who stayed up with me when I took data late in the evening, I sincerely express my gratitude for her encouragement and support.

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CHAPTER I

INTRODUCTION

This dissertation deals with the development of a new technique (R1) in nonequilibrium superconductivity (R2) developed for extracting information on electron scattering mechanisms in superconductors. The work was originally motivated by the first theory (R3) of low-resistance superconductor/insulator/normal-metal tunnel junctions, a theory which is valid only near $T_c$, the superconducting transition temperature. In the early stage of this research we verified (R4,5) quantitatively the existence of the novel charge-imbalance (R1-29) relaxation process predicted by the theory (R3) through measurements on extremely low-resistance junctions based on Al films. (A copy each of the published results (R4,5) are in Appendices A and B.)

Later, we extended the theory to lower temperature and to include the effects of supercurrents on the low-voltage resistance of low-resistance SIN tunnel junctions (R1).

In the final stage of this research we made detailed measurements on both low and high-resistance (R29) Sn and SnIn based junctions to demonstrate the validity of the theory (R1) and the feasibility of applying the technique to study superconducting films other than the
commonly used Al films.

In short, there was only a very simplified theory for the low-voltage resistance of low-resistance SIN junctions with no experimental validity at the beginning of this research. At the present stage, there is a relatively complete theory, with appropriate computer programs, and a great deal of verifying data.

The field of charge-imbalance started more than a dozen years ago when Clarke(*)R6) discovered a nonequilibrium state(*)R2) in a superconductor resulting from quasiparticle injection through a superconductor/insulator/normal-metal tunnel junction. He found that the quasiparticle chemical potential in the nonequilibrium region was different from that of the condensate. This difference in the chemical potential could readily be measured across a second tunnel junction which shared the superconducting electrode with the injector junction. A similar phenomenon had not been observed in a tunnel junction with all electrodes consisting of normal metals.

The origin of this nonequilibrium state is understood as due to charge-imbalance(*R1-29) in the superconductor. Charge-imbalance is a nonequilibrium state in which the density of electron-like \( k > k_f \) and the hole-like \( k < k_f \) excitations are unequal. (Here \( k \) is the electron wave vector and \( k_f \) is the Fermi wave vector.) In a normal metal the density of the electron-like and the hole-like excitations must be equal so that the metal stays electrically neutral. In superconductors, however, the density of the electron-like and the hole-like excitations can be unequal. To maintain charge neutrality,
the chemical potential of the condensate shifts from its equilibrium value by an amount determined by the degree of imbalance.

The imbalance in the electron-like and hole-like excitations is relaxed by intrinsic and/or applied pair-breaking scattering mechanisms. For example, a quasiparticle can be scattered inelastically via electron-phonon scattering from $k > k_F$ to $k < k_F$, thereby reducing the degree of imbalance. In steady state, the shift in chemical potential is proportional to the degree of imbalance which in turn is determined by the injection rate and the relaxation rate. By measuring the injection rate and measuring the chemical potential shift associated with the imbalance, one can obtain information on electron scattering mechanisms in superconductors.

This information is important to understanding of superconductivity because scattering mechanisms govern the behavior of the superconductor under study.

Much work, both theoretical (*R1,3,17-28) and experimental (*R4-14), has been done on the subject of charge-imbalance after the pioneering work of Clarke. A theory was first developed by Tinkham and Clarke(*R17) and later elaborated by Tinkham(*R18) to calculate the charge-imbalance relaxation rate due to electron-phonon processes and elastic intrinsic gap anisotropy process. Their theory was later extended by several authors(*R19,21,22,25) to more rigorous treatment and to include other relaxation processes. From this theory the total charge-imbalance relaxation rate $\Gamma_0^{x-1}$ is shown to be $(\pi A/k_B T) \times [r_{in}^{-1} (r_{in}^{-1} + 2r_{el}^{-1} + 2r_s^{-1})]^{1/2}$ (for $T = T_C$) where
\( r^{-1}_{\text{in}} \) is the inelastic rate, \( r^{-1}_{\text{el}} \) the elastic scattering rate due to intrinsic gap anisotropy, \( r^{-1}_{s} \) the elastic scattering rate due to magnetic field, magnetic impurities, or transport supercurrent, and \( \Delta \) is the order parameter.

Clark and Paterson (\*R7) extended Clarke's original measurements to Sn, SnIn and Pb based junctions. In addition to obtaining an electron-phonon rate for these materials, they showed qualitatively that intrinsic gap anisotropy (\*R30) could also contribute to relax charge-imbalance. Subsequently, Chi and Clarke (\*R9) studied the effect of intrinsic gap anisotropy effect on charge-imbalance relaxation for both clean and disordered Al. From the measured \( r_{Q*} \) vs. \( \Delta/k_B T \) curves they were able to determine a value for the intrinsic gap anisotropy parameter \( <a^2>_0 \) for Al. \( <a^2>_0 \) is the mean square intrinsic gap anisotropy averaged over the entire Fermi surface. This is the quantity used in the literature to characterize the degree of anisotropy.) In both of these studies, the temperature dependence of the data agree with the theory only near \( T_C \), hence the electron-phonon rate at \( T_C \) was determined only to about \( \pm 50\% \) and \( <a^2>_0 \) showed a scatter of \( \pm a \) factor of two.

The effect of magnetic impurities and transport supercurrent on charge-imbalance relaxation was studied by Lemberger and Clarke (\*R11,12). Once again, the measured \( r_{Q*}^{-1} \) was in good agreement with the theory only for data very close to \( T_C \).

All the experimental work mentioned above used a three-film, two-junction geometry first used by Clarke. This particular geometry
has the advantages of having separate injector and detector junctions and of measuring directly the shift in the chemical potential. Nevertheless, to obtain sizable signal, this technique requires very high injection voltage which almost always causes heating and other complications. Moreover, besides having to grow two types of oxide for insulators, the sample preparation requires two well aligned junctions which limits the choice of certain materials as electrodes or prevents in situ sample preparation.

Hsiang and Clarke (HC) (R10) studied the boundary resistance for a variety of SN interfaces. The boundary resistance measured in this type of experiment arises from a region in the superconductor in which the charge-imbalance exists. Unfortunately, the theory of charge-imbalance for SN interfaces that HC used is not very transparent and it holds only for temperatures extremely close to $T_c$. Although HC obtained an electron-phonon rate that is in good agreement with the theoretical calculation, it is not certain how large the uncertainty is for the rate obtained since the temperature range in which the data are valid is very small.

In their work on three-film, two-junction geometry samples, almost all the authors observed a minimum in the low-voltage resistance of the detector junction where none was expected. The major difference between these (SIN) detector junctions and those used for injector junctions was the magnitude of the intrinsic junction resistance. (The intrinsic resistance of the junction is defined as the resistance of the junction at $T_c$.) For the detector
junctio̅ns used the intrinsic junction resistance was usually very small, typically of the order of mΩ or smaller.

To explain this resistance minimum, Lemberger(*R3) developed a theory that included nonequilibrium charge-imbalance effects in the S film. A novel prediction was a charge-imbalance relaxation mechanism associated with the proximity(*R31,32) of the superconducting electrode to the normal metal electrode. The characteristic rate associated with this mechanism is inversely proportional to the intrinsic resistance of the junction. Basically, this process allows nonequilibrium quasiparticles to tunnel through the oxide barrier into the normal metal and relaxes the charge-imbalance. This mechanism was verified experimentally by Lemberger and Yen(*R4) and later more systematically by Yen and Lemberger(*R5). (See Appendices A and B) In the course of this project, we built on the theory and made extensive measurements. The theory has been published and is in Appendix C. The measurements are described in the following pages. The outcome of the analysis is information on the electron scattering processes.

The electron-phonon rate for the superconductor under study can be obtained from the resistance vs. temperature data. Qualitatively speaking, the shape of the resistance vs. temperature curve depends on the ratio of the proximity effect induced tunneling rate and the electron-phonon rate. By calculating the tunneling rate and determining the ratio from the data one can determine the electron-phonon rate of the superconductor under study.
Unfortunately, this technique suffers from a similar drawback as the boundary resistance measurement (R10). Namely, the technique is most useful only for relatively high temperatures. At lower temperatures, the less-than-ideal quality of most real junctions greatly affects the measured junction resistance and the data are no longer useful in determining the scattering rate. Moreover, for pair-breaking scattering other than the electron-phonon and the tunneling process, the temperature range within which the data are useful is too narrow to conclude convincingly the existence of any other pair-breaker.

A way to extend the technique to lower temperatures is to modulate the junction resistance with a supercurrent in the superconducting film. It is well known that a supercurrent acts in many ways like the pair-breaking effect of magnetic impurities (R33-35). This equivalence as charge-imbalance relaxation mechanisms was demonstrated by Lemberger and Clarke (R11,12). Varying the applied supercurrent has the advantage of varying the applied pair-breaking rate without making a lot of samples with different concentrations of magnetic impurities. Therefore, it is useful to apply or induce a supercurrent as a pair-breaker to adjust the degree of nonequilibrium charge-imbalance. A theory (R1) was developed to study the possibility of extracting information on electron scattering from measurements of low-voltage junction resistance as a function of supercurrent. A copy of this paper is in Appendix C.
The goal of this thesis is to verify this theory. Sn was chosen as the superconductor because the intrinsic pair-breaking rates, electron-phonon scattering and elastic scattering with an intrinsically anisotropic order parameter, are reasonably well understood. Also, the electron mean free path, and hence the magnitude of the intrinsic gap anisotropy could be varied by adding a few percent of In.

We (*R15,29) performed measurements of the low-voltage resistance of both low-resistance and high-resistance Sn/Sn-oxide/Cu and SnIn/SnIn-oxide/Cu tunnel junctions as functions of temperature and transport supercurrent. For the temperature range $0.6 \leq T/T_C \leq 0.95$, our supercurrent effect data were in excellent agreement with numerical calculations based on a Boltzmann Equation appropriate for a steady state, spatially homogeneous charge-imbalance. The electron-phonon rates and the intrinsic gap anisotropy parameters we extracted were reproducible among similar samples and were in good agreement with existing values in the literature. Our work not only establishes the validity of the technique but also provides the best measurement of the electron-phonon rate and intrinsic gap anisotropy parameter.

Our new technique is especially interesting in studies of the electron pair-breaking scattering of novel materials. The possibility of applying this technique to studies of novel processes and/or novel materials is the motivation of further work.
The flow of this Dissertation is as follows. In Chapter II we
describe the theoretical background necessary to extract information
on electron scattering in a superconductor. In particular, we
discuss the effect of intrinsic gap anisotropy on the relaxation of
charge-imbalance and on the reduction of the low-voltage resistance
of SIN tunnel junction in the presence of transport supercurrent.
Chapter III gives the relevant experimental details. Chapter IV
contains the experimental results and discussions. Chapter V gives a
concluding summary.
CHAPTER II
THEORETICAL BACKGROUND

In this chapter, we discuss the theoretical background necessary for extracting information on electron scattering in superconductors from measurements of the low-voltage resistance $R_j$ of Superconductor/Insulator/Normal-metal (SIN) tunnel junctions as functions of temperature and transport supercurrent. Since the Sn used for the S electrode has a non-negligible intrinsic gap anisotropy, we discuss in particular how intrinsic gap anisotropy affects the reduction of junction resistance in the presence of a transport supercurrent.

A. Formalism of $R_j$

The theoretical background for extracting information on electron scattering mechanisms from measurements of the low-voltage resistance of SIN tunnel junction has been discussed in detail in Ref.(*R1). (A copy of Ref.(*R1) is in Appendix C.) For clarity and simplicity, Ref.(*R1) left out the effect of intrinsic gap anisotropy in the relaxation of charge-imbalance. However, the gap in a real metal is seldom isotropic (*R30). Intrinsic gap anisotropy allows the quasiparticles to cross the Fermi surface and relax charge-imbalance (*R18) via elastic scattering. In this section we summarize the
salient points of Ref. (*R1) to provide the background information needed for this study. The effect of intrinsic gap anisotropy is discussed in section B.

One important concept discussed in Ref. (*R1) is that there is a proximity effect (*R31,32) induced charge-imbalance generation/relaxation mechanism associated with a SIN tunnel junction (*R3-5). This relaxation mechanism allows quasiparticles to tunnel back from the S electrode into the N electrode. The existence of this surprising mechanism was verified quantitatively in the initial stage of this dissertation. The published results are reproduced in appendices A and B. As a result of this backflow of quasiparticles, the total bias current flowing through the junction should be thought of as the difference between a forward 'equilibrium' current proportional to the electrostatic potential \[ V - Q^*/2N(0)|e| \] and a reverse 'nonequilibrium' current proportional to the chemical potential drop \[ Q^*/2N(0)|e| \] across the junction due to existence of charge-imbalance. Here \( 2N(0) \) is the normal state density of states, \( e \) is the electronic charge and \( Q^* \) is the charge-imbalance per unit volume. The physical meaning of this separation of current is that the electrostatic potential is the applied perturbation to the system and the chemical potential shift is the response of the system to the perturbation.

In analogy to separation of the total current, one should also regard the total quasiparticle injection rate as the difference between the equilibrium and nonequilibrium components corresponding
to the applied perturbation and the system response. There is a
difference between the total current $I$ and the total quasiparticle
injection rate. Basically, only $F^*$ of the equilibrium current goes
into the superconductor as quasiparticle charge, the rest of the
current goes directly into the condensate. $F^*$ starts with unity at
$T_c$ and decreases towards lower temperatures ($R24$).

The resistance of the junction is obtained by expressing $Q^*$ in
terms of $F^*$, $g_{NS}$, $r_{tun}$, and $r_{Q^*}$ in the expression for the total
current $I$. From Equations (27) through (29) of Ref.$(R1)$ we have

$$R_j(T) = R_{eq}(T) + R_{Q^*}(T), \quad (E1)$$

$$R_{eq}(T) = R_N / g_{NS}(T), \quad (E2)$$

$$R_{Q^*}(T) = R_{eq}(T) / [r_{tun}/F^*r_{Q^*} - (1-g_{NS})]. \quad (E3)$$

Equation $(E1)$ shows that the total junction resistance consists of
the equilibrium part $R_{eq}(T)$ and the nonequilibrium part $R_{Q^*}(T)$.
$g_{NS}(T)$ is the conductance of the junction normalized to its value at
$T_c$. It has been tabulated by Bermon $(R36)$. $R_N$ is the intrinsic
resistance of the junction so that $R_{eq}(T)$ is just the junction
resistance when there is no charge-imbalance. As expected, the
nonequilibrium part of the junction resistance depends on $r_{tun}^{-1}$ and
$r_{Q^*}^{-1}$. Here, $r_{tun}^{-1}$ is the proximity effect induced tunneling rate
defined by $(R3,28)$

$$r_{tun}^{-1} = (2N(0) e^2 \Omega R_N)^{-1} \quad (E4)$$

where $\Omega$ is the volume of the nonequilibrium region.

The charge-imbalance relaxation rate $r_{Q^*}^{-1}$ is determined by all
the scattering mechanisms in the superconductor under study. For the
Sn and SnIn films we report in this paper the important scattering mechanisms are inelastic electron-phonon scattering, proximity effect induced tunneling process, and elastic scattering due to intrinsic gap anisotropy. The first two processes have been discussed extensively by several authors (*R17-23,25-27) and will not be repeated here. Charge-imbalance relaxation in the presence of intrinsic gap anisotropy has been discussed for a three-film, two-junction geometry (*R7,9). We give in the following details of gap anisotropy effect on the resistance of a SIN junction.

B. Effect of Intrinsic gap anisotropy

The intrinsic gap anisotropy affects the transition temperature of the superconductor since the effective phonon-mediated electron-electron interaction depends on the relative direction of electron momentum and the crystal axes (*R39). The superconductor takes advantage of the existence of gap anisotropy in forming pairs and obtaining highest $T_c$ possible. Below $T_c$, however, elastic scattering from disorder combined with the intrinsic gap anisotropy results in an elastic pair-breaking rate (*R3,9,28).

The scattering rate for intrinsic gap anisotropy that enters the Boltzmann Equation has the following form (*R7,9,18,28):

$$ G_{el}(E) = r_1^{-1} C_{cf} N_1(E) [2\delta f_E]. $$

(*E5)

$r_1^{-1}$ is the elastic scattering rate in the normal state. $C_{cf}$ is the coherence factor for this process. $N_1(E)$ is the superconducting tunneling density of states. $\delta f_E$ is the deviation of quasiparticle distribution function from its equilibrium value.
Charge-imbalance relaxation due to elastic scattering in the presence of an isotropic gap is prohibited since the coherence factor $(uu' - vv')^2$ is zero for an isotropic gap (*R38), i.e., $u = v'$ and $u' = v$ since the initial and final states have the same magnitude of effective charge, $|q| = |q'|$. However, the coherence factor is finite when the gap is anisotropic (*R18) because $|q| = |q'|$ if the directions of momentum is different for the initial and final states. By using the definitions of $u$, $v$ (*R38), and writing the effective charge $q$ in terms of anisotropy parameter the coherence factor for this process can be expressed as (*R39):

$$C_{cf} = (uu' - vv')^2$$

$$= \frac{E^2 (\partial q/\partial E)^2}{2 (1 - q^2)} \langle a^2 \rangle$$

(*E6)

where

$$\langle a^2 \rangle = \frac{\langle (\Delta - \langle \Delta \rangle)^2 \rangle}{\langle \Delta \rangle^2}$$

(*E7)

is the average gap anisotropy (*R37). Combining Equations (*E5) and (*E6) we have

$$G_{el}(E) = \frac{\langle a^2 \rangle}{2 r_1} \frac{E^2 (\partial q/\partial E)^2}{(1 - q^2)} N_1(E) \left[ 2 \delta f_E \right]$$

(*E8)

where $\langle a^2 \rangle/2r_1$ is the characteristic rate for intrinsic gap anisotropy scattering. This is the scattering rate involved in determining the critical pair-breaking rate. The rate is zero at $T_c$ since $\Delta$ is zero at $T_c$. Note that in Equation (*E6) the coherence factor is proportional to the gap anisotropy parameter $\langle a^2 \rangle$ so that
the coherence factor is zero and the elastic scattering is prohibited when the gap is isotropic. For simplicity, we shall replace $\langle \Delta \rangle$ with $\Delta$ through out the rest of this paper.

In most superconductors, gap anisotropy is reduced by the Anderson(*) averaging effect when the electron mean-free-path is shortened by the addition of nonmagnetic impurities or by the existence of boundaries in thin films. The basic idea behind this is that the elastic scattering rate is increased by the existence of additional nonmagnetic scattering centers or by the boundaries and electrons are scattered more frequently to all directions and positions on the Fermi surface. As the electrons are scattered they carry information of the Fermi surface from one place to the other. The end result is that the gap anisotropy is, so called, "averaged out". The characteristic rates to enter this averaging are the elastic scattering rate $r_1^{-1}$ and the gap rate $\Delta/\hbar$.

The degree of reduction in gap anisotropy due to this averaging is unclear from Anderson's original theory(*R40). From the work of Markowitz and Kadanoff(*R37) on the behavior of $\Delta$ near $T_c$, Tinkham(*R18,41) assumed the averaged gap anisotropy parameter (we shall refer to this averaging as Anderson averaging) to be

$$<a^2> = <a^2>_0/[1+(\hbar/2r_1\Delta)^2]$$

(*E9)

where $<a^2>_0$ is the intrinsic gap anisotropy parameter commonly used in the literature for a bulk, clean superconductor. Note that $<a^2>$ increases as temperature decreases since $\Delta$ increases while $r_1^{-1}$ is constant. The elastic scattering rate $r_1^{-1}$ is determined by $v_F/\ell$ if
the scattering process is mean-free-path limited or by $v_F/d$ if the process is boundary limited. The factor $(\hbar /2\tau_1\Delta)^2$ is usually very large for dirty films, typically of the order of $10^2-10^3$ at 0.6 $T_c$, so that one can rewrite Equation (*E9) as

$$<a^2> = 4 <a^2>_0 \tau_1^2 \Delta^2 / \hbar^2.$$  \hspace{1cm} (*E10)

Combining Equation (*E5), (*E6), and (*E10), for $(\hbar^2/2\tau_1\Delta)^2 \gg 1$, we have

$$G_{el}(E) = \frac{4 <a^2>_0 \tau_1 \Delta^2}{\hbar^2} \frac{E^2 (\delta q/\delta E)^2}{2 (1 - q^2)} \frac{N_1(E) [2f_E]}{E} \hspace{1cm} (*E11)$$

$$= r_{qe10}^{-1} \frac{\Delta^2}{(k_BT_c)^2} \frac{E^2 (\delta q/\delta E)^2}{(1 - q^2)} \frac{N_1(E) [2f_E]}{E} \hspace{1cm} (*E12)$$

where

$$r_{qe10}^{-1} = \frac{2 <a^2>_0 \tau_1 (k_BT_c)^2}{\hbar^2} \hspace{1cm} (*E13)$$

is the characteristic rate for intrinsic gap anisotropy scattering in the dirty limit used by CC (*R9). This characteristic rate is temperature independent and is proportional to the elastic scattering time $\tau_1$ instead of the elastic scattering rate $\tau_1^{-1}$ because of the strong elastic scattering tending to make the order parameter $\Delta$ isotropic. The rate is also proportional to $<a^2>_0$ as expected. Since our pure Sn films do not satisfy $(\hbar^2/2\tau_1\Delta)^2 \gg 1$ at the temperature range studied we should deal with the quantity $<a^2>/2\tau_1$ instead of $r_{qe10}^{-1}$.

We now focus our attention on the energy dependence of the scattering rate. From Fig.(*F6) of Ref.(*R1) and Equation (*E8) the
coherence factor for gap anisotropy scattering is largest for energies near \( \Delta \) and decreases towards higher energies. The tunneling density of states also peaks near \( \Delta \) and decreases towards higher and lower energies. The product of the coherence factor and the tunneling density of states determines the energy dependence of the effective scattering rate. The intrinsic gap anisotropy scattering rate is apparently largest for quasiparticles near the gap edge. That is, low-energy quasiparticles are more likely to scatter elastically across the Fermi surface. High-energy quasiparticles have to lower their energies by inelastic scattering before they are scattered elastically.

The temperature dependence of the scattering rate comes from that of the product of \( C_{CF} \) and \( N_1(E) \) too. To see this better, we assume that all quasiparticles contributing to the scattering rate have an energy of \( \Delta + k_B T \). This is very reasonable because quasiparticles are thermally excited and are mostly populated within energy \( k_B T \) of \( \Delta \).

Fig. (**F1**) shows the temperature dependence of the pair-breaking rate due to intrinsic gap anisotropy calculated using parameters appropriate for sample D30. At high temperatures, \( \Delta \) is small and the product of \( C_{CF} N_1(E) \) is small for most of the quasiparticle states. At low temperatures, \( \Delta \) is large and so is the product of \( C_{CF} \) and \( N_1(E) \). For example, the gap anisotropy rate may start with a value 30 times smaller than the electron-phonon rate at, say, 0.95 \( T_C \) and turn out to be the dominating relaxation rate at, say, 0.6 \( T_C \) because the gap anisotropy rate is enhanced while the inelastic
Fig. 1. The elastic scattering rate \([r_1^{-1} C_{cf} N_1(E)]\) vs. \(T/T_C\). This is calculated assuming that all the quasiparticles have energy \(\Delta + k_B T\) using \(r_1^{-1} = 3.47 \times 10^{12} \text{ s}^{-1}\), as appropriate for sample D30 (pure Sn sample).
electron-phonon rate is frozen out. We emphasize that although the rate is largest at low temperatures, the strongest temperature dependence of this rate occurs at high temperatures. As we will see later in our data analysis, this is very advantageous in our supercurrent effect study since we can then determine the pair-breaking scattering rate due to intrinsic gap anisotropy using data taken at relative high temperatures.

C. Effect of a transport supercurrent

The pair-breaking rate $r_s^{-1}$ caused by a transport supercurrent has been discussed by several authors (*R1,15,33,42,43). A supercurrent induces a gap anisotropy in the superconductor thereby allowing elastic scattering to relax a charge-imbalance. From Equation (3)-(6) of Ref.(*R15), in the dirty limit, $r_s^{-1}$ can be written in terms of measurable quantities and the density of superconducting electrons $n_s$: 

$$r_s^{-1} = \frac{0.1669k_B T_C}{\hbar} \frac{n_s(0,0)^2}{n_s(T,r_s^{-1})^2} \frac{I_s^2}{I_c(0)^2}$$

(*E14)

where

$$I_c(0)^2 = \frac{2.562 \times 2N(0) d^2 w^2 (k_B T_C)^3}{\hbar \rho}$$

(*E15)

is the zero temperature critical current (*R44,45). $d$ and $w$ are the thickness and the width of the superconducting film respectively. $\rho$ is the residual resistivity of the $S$ film.

We point out that the entire temperature dependence of $r_s^{-1}$ is in that of $n_s(T,r_s^{-1})$. $n_s(T,r_s^{-1}=0)$ increases from zero at $T_C$ to
\( \frac{mc^2}{\pi\lambda(0)^2e^2} \) at \( T=0 \) (*R38). \( \lambda(0) \) is the zero temperature magnetic penetration depth. The elastic pair-breaking rate due to a given supercurrent is largest at high temperatures because there are fewer superconducting electrons so they must move faster, and the pair-breaking rate is proportional to the velocity squared.

It is time to pause and discuss our numerical calculations. The computer programs used in this study are the same as the ones used in Ref.(*R1) except that the effects of intrinsic gap anisotropy are included in the present study. Since the pair-breaking rate due to the intrinsic gap anisotropy depends on the order parameter itself we iterated the calculation of \( \Delta \) until a self-consistent set of gap anisotropy pair-breaking rate and \( \Delta \) was found. Checks on the program can be found in the appendix of Ref.(*R1).

D. \( R_j(T,I_a) \)

We now discuss low-voltage resistance as a function of temperature and transport supercurrent. The solid curve in Fig.(*F2) of Ref.(*R1) (Appendix C) shows the theoretical calculation of \( R_j/R_N \) vs. \( T/Tc \) using parameters appropriate for typical Sn junction. The resistance of the junction is composed of the 'equilibrium' part, \( R_{eq}(T) \), and the 'nonequilibrium' part, \( R_Q^*(T) \). The equilibrium part is shown in Fig.(*F2) of the same reference as a dashed curve. The resistance of the junction diverges towards \( T_c \) since \( R_Q^* \) diverges at \( T_c \). \( R_{eq}(T) \) diverges towards low temperature since quasiparticles are thermally excited and therefore the junction resistance depends exponentially on \( \Delta/k_BT \) at low temperature. Both \( R_{eq} \) and \( R_Q^* \) change
in the presence of applied transport supercurrent.

The normalized reduction in the equilibrium part of the junction resistance, \(-\delta R_{eq}(T,I_s)/R_{eq}(T,I_s=0)\), in the presence of transport supercurrents was investigated both theoretically(*R1) and experimentally (*R15) as part of this dissertation. (A less rigorous study was made by Paterno et al (*R46) with different junction geometry.) Basically, the initial reduction in junction resistance is linear in \(I_s^2\) since for small pair-breaking rate the rate is proportional to current squared. For large pair-breaking rate, the rate increases faster than current squared because the density of superconducting electrons decreases as current increases. That is, a transport supercurrent is more effective in producing a pair-breaking rate when the density of superconducting electrons is small as can be seen from Equation (*E14).

For wide films (*R47), it is usually difficult to realize the theoretical critical current (*R44,45). For our Sn and SnIn films we obtained only about 20% of the theoretical critical current, and hence about 4% of the theoretical maximum value of \(I_s^{-1}\). Therefore, we will confine our discussion of supercurrent effect below to only small currents. Fig. (*F2) shows the calculated results of normalized equilibrium resistance of a SIN tunnel junction \(R_{eq}(T,I_s)/R_N(T,I_s=0)\), as a function of supercurrent squared normalized to the zero temperature critical current squared, \(I_s^2/I_c(0)^2\), using parameters appropriate for sample J17. At high temperatures, the decreasing slope of the initial reductions towards
FIG. 2. Calculated normalized low-voltage resistance $R_j(T,I_s)/R_N$ vs. $I_s(T)^2/I_c(0)^2$ for a high-resistance SIN junction. The dashed line indicates the experimentally accessible range of supercurrents in the present study.
lower temperatures reflects the increasing density of superconducting electrons towards lower temperatures. The slope starts to increase, however, below \( -0.65 T_c \) because of the exponential dependence of junction resistance on \( \Delta/k_B T \).

The reduction in the nonequilibrium part of the junction resistance is due to an increase in the charge-imbalance relaxation rate although the reduction in the order parameter slightly decreases the relaxation rate from gap anisotropy. The effect is illustrated in Fig. (*F3) which shows the normalized reduction in the nonequilibrium part of the junction resistance \( -\delta R_q(T,I_s)/R_j(T,I_s=0) \) vs. normalized supercurrent squared \( I_s(T)^2/I_c(0)^2 \) for two different gap anisotropy rates at three different normalized temperatures. One can see that, at the same normalized temperature, the sizes of the effect are quite different for different gap anisotropy rates. The size of the effect also depends strongly on the temperature given same amount of supercurrent and gap anisotropy rate. Thus, we can obtain \( \langle a^2 \rangle_0 \) very accurately from measurements of \( \delta R_j \) vs. \( I_s^2 \).

The resistance of the junction that one measures is the total junction resistance \( R_j(T,I_s) \). The slope of the initial reduction in the junction resistance depends on the relative importance of changes in the equilibrium and nonequilibrium components. Near \( T_c \), the charge-imbalance component dominates the change since application of supercurrent greatly increases the charge-imbalance relaxation rate. The change in density of states does not affect the junction resistance a great deal because most of the change in the density of
Fig. 3. Calculated reduction of the charge imbalance part of the junction resistance (normalized to the junction resistance when there is no supercurrent), $-\delta R_q(T,I_S)/R_J(T,I_S)$, vs. the normalized supercurrent squared, $I_S(T)^2/I_c(0)^2$, for $\langle a^2 \rangle_0$ of 0.0053 and 0.00265 at 0.9, 0.8 and 0.6 $T_c$. At same normalized temperature, gap anisotropy has an important effect on the size of the reduction. Also, given same $\langle a^2 \rangle_0$, the effect is strongly temperature dependent.
states occurs near the gap edge while quasiparticles populate states of higher energies. At lower temperatures, the reduction in 
'equilibrium' component becomes more and more important while the 
'nonequilibrium' component dies out due to the existence of intrinsic 
gap anisotropy.

In actual data analysis, the temperature dependence of the 
junction resistance at high temperatures determines the 
electron-phonon rate, the tunneling rate, and the intrinsic junction 
resistance, R_N. The slope for the initial reduction in the 
supercurrent effect serves as a check for the correct choice of 
scattering rates involved.
CHAPTER III
EXPERIMENTAL DETAILS

We discuss in this chapter the sample preparation procedures, cryogenic techniques, and the measurement techniques that we used during the course of this research. The goal is to make superconductor/insulator/normal-metal tunnel junctions and measure their low-voltage resistances as functions of temperature and/or transport supercurrent below 4.2 K.

A. Sample preparation

Al and Sn have been used as the superconducting electrodes in our study. Al was used in the early stage of the research since it is easier to handle during sample preparation and is commonly used in the literature. Sn was chosen later since we wished to establish the validity of our new technique in materials other than Al. As we shall see in Chapter IV, the physical processes that are involved in the relaxation of nonequilibrium charge-imbalance are quite different in Al and Sn so that it is not simply changing the spices in the recipe. The sample preparation procedures for Al based junctions have been published elsewhere (*R4,5) and will not be repeated here. A copy each of our previous papers are in Appendices A and B. We shall discuss in the rest of this section the sample preparation
procedures for Sn based SIN tunnel junctions.

The samples were Sn/Sn-oxide/Cu or SnIn/SnIn-oxide/Cu tunnel junctions. All films were deposited from resistively heated sources through one eighth inch thick mechanical masks onto glass substrates. The masks were end-mill machined to have the desired pattern of opening and care was taken to reduce the shadow effect created by the thickness of the masks to minimum. Heavy duty Al foil was used to define the narrowest part of the film which was about 300 μm. To improve the uniformity of current distribution, samples were always built on substrates with a ~5000Å thick Nb ground plane. These ground planes were anodized (*R48) and then coated with ~2000Å of SiO for insulation before use.

Fig. (*F4) shows the sample geometry. The Sn and SnIn films were normally 800Å thick, the Copper-Aluminum-Iron film were 2000 - 3000Å, while the Lead overlayer were usually 2000Å. SiO (~1500 Å) was used to define the area of the junctions to be ~300μm x 300 μm.

For simplicity, we shall discuss the procedures for Sn/Sn-oxide/Cu samples first and then state the changes for SnIn/SnIn-oxide/Cu junctions. First, 99.999% pure Sn was deposited onto a previously cleaned glass substrate through an aperture mask at a rate of 20 - 30 Å/sec. in typical pressure of less than 10^-6 torr. The substrate was held at -65 to -80 °C during evaporation using a cold finger. It was found that this substrate temperature range yielded the highest quality Sn and SnIn films. It was also found that the SiO underlayer, used for insulating the Nb plane and the Sn
FIG. 4. Sample configuration: plan(top), side(bottom). I and V are the current and voltage in the junction, Iₘ is the supercurrent in the superconducting strip. For simplicity the Nb ground plane is not shown here. For the Sn sample a ~0.2 Ω wire was connected in parallel with the narrow part of the S film so that most current flows through the 0.2 Ω short when the critical current is exceeded.
film, is important in obtaining high quality films. The substrate was warmed up to room temperature before the film was oxidized. Several methods have been developed for oxidation of films. Each is suitable for different strengths of oxide barriers. We found for low resistance junctions that oxidation in reduced oxygen atmosphere (-2-10 mmHg) for -2 hours gave junctions of reasonably low resistance and high quality. For high resistance junctions, an oxygen glow discharge yielded the highest quality junctions.

After oxidizing the base film, the area of the junctions was defined by depositing SiO to mask off all but 300 μm long of the narrow part of the base films. The substrate was then cooled to about -10 to -20 °C before counter-electrode deposition. Mixture of Cu, Al(3 Wt. %), and Fe(3 Wt. %) was deposited in 2x10⁻⁴ torr Oxygen partial pressure to form the tunnel junctions. Addition of Al and Fe as well as evaporation in an Oxygen atmosphere shortened the electron mean free path and the normal state coherence length in Copper thus preventing a supercurrent from flowing from Sn to the Lead overlayer. It was essential to keep the substrate cold during deposition of the copper mixture to avoid wetting of copper by Sn (*R49). The Pb overlayer was deposited to reduce the series resistance of the normal metal stripe. This was of great importance since the junction resistance could be as low as 40μΩ and any series resistance in the circuit would impair the resolution of the voltage measurements. The samples were then covered with SiO to protect films from recrystallization due to exposure to moisture in the air.
In the case of SnIn/SnIn-oxide/Cu junctions the initial procedure was slightly different. Instead of depositing 800Å of Sn at one time we deposited in sequence 360Å of Sn (@ -20 - 30Å/sec.), 40Å of In (@ 4 Å/sec.), and 400Å of Sn at the same temperature range mentioned above. The completed film stayed in high vacuum (10^{-6} torr) for at least one hour after it was warmed up to room temperature so that interdiffusion between Sn and In is completed. The rest of the sample preparation procedures were the same as those for Sn/Sn-oxide/Cu junctions. It is well known that Sn and In interdiffuse quickly at room temperature provided proper high vacuum is maintained (*R50,51). The resistivity of the completed SnIn alloy films remained unchanged after it was left at room temperature for more than one day suggesting completion of interdiffusion. The apparent difference in the electron mean free path of Sn and SnIn gives additional support for complete interdiffusion.

B. Cryogenic and measurement techniques

We attached wires to the samples using In pads to enable us to perform four-terminal measurements on $R_{\text{stripe}}$, $R_j$, and $R_j$ vs. $I_s$ as functions of temperatures. The In pads were far away from the narrow part of the stripe so that no unwanted alloying effect occurred. This is supported by the film resistivity measurements. Fig. (*F4) shows the wiring diagram. We connected a manganin wire of about 0.2 Ω in parallel with the Sn or SnIn film so that most of the current went through this low-resistance path when the critical current of the superconducting film was exceeded. This is very important since
the normal state resistance of the Sn or SnIn stripe was about 1-3 Ω while the experimental critical current could be more than half an Ampere. As a result, Sn films which didn’t have a normal low-resistance short in parallel burnt out once the critical current was exceeded.

The samples were mounted on a copper plate inside the inner can of a double-can probe with both cans coated with superconducting solder. A superconducting Lead bag was used to further reduce external magnetic fluctuations. Usually it took about 3 hours for the samples to reach 77 Kelvin starting from room temperature. The probe was immersed in liquid Helium with the inner-can jacket evacuated to reduce thermal conduction. The temperature of the bath could be lowered to -1.2 K with a mechanical pump and measured with a Germanium resistor. A pressure regulator and/or an electronic temperature regulator was used to regulate the temperature to better than 1 mK above the λ point and -100 μK below.

Low-Voltage measurements were done using a rf Superconducting Quantum Interference Device (SQUID) in external feedback mode with a voltage resolution of -10 pV in a 10 Hz band-width. The performance of the circuit is limited by the Johnson noise due to the resistor $R_{\text{std}}$ (-7 mΩ) in our feedback circuit. Typical signal voltage across the tunnel junction were less than -5 μV.

For junction resistance measurements we simply measured the low-voltage current-voltage characteristic at different temperatures. In our supercurrent effect measurements we fixed the current bias
through the junction and measured the reduction in voltage across the junction. The reduction in voltage was then converted into reduction in junction resistance. For the samples reported here, within experimental error, no voltage change was detected across the junction in the absence of a junction bias current with supercurrents up to the critical current.

We have found that a thin coating of SiO on thin film Sn decreased the resistivity of the film even at 4.2 K. To avoid this problem we always measured stripe properties with a bare Sn film which was deposited at the same run the base films for the junctions were deposited. Properties of Sn films in the junctions were not affected by SiO since they were only in contact with either a thicker SiO layer or the normal metal counter-electrode. It was experimentally verified that thick layer of SiO did not affect the properties of Sn films.

We expect that the supercurrent density to be relatively uniform through the thickness of the film since the penetration depth was comparable to or larger than the thickness of the film. The coherence length ($\xi$) ($\approx 2300\text{Å}$) was also larger than the film thickness. The current distribution across the width of the films was improved by the presence of the Nb ground plane.
In this chapter, we report experimental results on both high-resistance and low-resistance junctions in which we have observed a reduction in junction resistance with increasing supercurrent. By high resistance junction we mean that the characteristic pair-breaking rate associated with the proximity effect induced tunneling process is negligible compared to the characteristic electron-phonon rate.

This chapter consists of four parts. In section A, we discuss the behavior of the junctions as a function of temperature. Section B contains results on the supercurrent effect. We discuss the electron-phonon rate and the intrinsic gap anisotropy parameter in sections C and D respectively.

Tables 1 and 2 list the relevant parameters of the samples. $T_c$ is the transition temperature of the superconductor. It is very close to that of the bulk of Sn, 3.74K. $\rho_{4.2K}$ and $l$ are the resistivity and the mean-free-path of the films at 4.2K. We used $\rho_{4.2K} l = 1.05 \times 10^{-15}$ $\Omega$-m$^2$ (R7,52) to calculate $l$. $w$ and $d$ are the width and thickness of the films. $w_{S10}$ is the length of the junction defined by S10. $R_N$ is the intrinsic resistance of the junction used.
Table 1. Material and sample parameters. Samples discussed in the text have higher quality than the rest of the samples that are listed here.

<table>
<thead>
<tr>
<th>S film</th>
<th>material</th>
<th>Tc</th>
<th>$\rho_{4.2}$ (nΩ-m)</th>
<th>$I^{(a)}$ (μm)</th>
<th>$w$ (μm)</th>
<th>$w_{SiO}$ (μm)</th>
<th>d(b) (nm)</th>
<th>$I_c(0)^{(c)}$ (Amp.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J17</td>
<td>Sn</td>
<td>3.71</td>
<td>7.45</td>
<td>141</td>
<td>315</td>
<td>350</td>
<td>80</td>
<td>6.47</td>
</tr>
<tr>
<td>D20</td>
<td>Sn</td>
<td>3.72</td>
<td>5.63</td>
<td>187</td>
<td>254</td>
<td>385</td>
<td>80</td>
<td>6.06</td>
</tr>
<tr>
<td>D26</td>
<td>Sn</td>
<td>3.72</td>
<td>6.58</td>
<td>160</td>
<td>254</td>
<td>346</td>
<td>80</td>
<td>5.98</td>
</tr>
<tr>
<td>D30</td>
<td>Sn</td>
<td>3.72</td>
<td>5.73</td>
<td>183</td>
<td>244</td>
<td>385</td>
<td>80</td>
<td>6.16</td>
</tr>
<tr>
<td>A13</td>
<td>SnIn</td>
<td>3.69</td>
<td>28.0</td>
<td>37.5</td>
<td>280</td>
<td>323</td>
<td>80</td>
<td>3.48</td>
</tr>
<tr>
<td>S28</td>
<td>SnIn</td>
<td>3.68</td>
<td>29.3</td>
<td>35.8</td>
<td>297</td>
<td>330</td>
<td>80</td>
<td>3.44</td>
</tr>
</tbody>
</table>

(a) Determined by using $\rho_{4.2} = 1.05 \times 10^{-15}$ Ω-m$^2$ for Sn.
(b) Reading from a crystal thickness monitor.
(c) This is the critical current density used in the fit.
Table 2. Material and sample parameters continued

<table>
<thead>
<tr>
<th>Sample</th>
<th>$R_N$ (μΩ)</th>
<th>$R_N'(a)$ (μΩ)</th>
<th>$\tau_{\text{un}}^{-1}$ $(10^8 s^{-1})$</th>
<th>$r_{\text{e-ph}}(T_c)^{-1}$ $(10^9 s^{-1})$</th>
<th>$\langle a^2 \rangle_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>J17</td>
<td>12500</td>
<td>12500</td>
<td>0.02</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>D20</td>
<td>26.0</td>
<td>27.7</td>
<td>11.8</td>
<td>2.78</td>
<td>0.0056</td>
</tr>
<tr>
<td>D26</td>
<td>40.4</td>
<td>40.4</td>
<td>7.90</td>
<td>2.80</td>
<td>0.0057</td>
</tr>
<tr>
<td>D30</td>
<td>39.4</td>
<td>39.4</td>
<td>7.60</td>
<td>2.86</td>
<td>0.0053</td>
</tr>
<tr>
<td>A13</td>
<td>47.0</td>
<td>47.0</td>
<td>6.00</td>
<td>3.00</td>
<td>0.0046</td>
</tr>
<tr>
<td>S28</td>
<td>85.5</td>
<td>81.1</td>
<td>3.17</td>
<td>2.47</td>
<td>0.0053</td>
</tr>
</tbody>
</table>

(a) Determined from Eq.(*E4) and measured sample dimension.
(b) $4.0 \times 10^9$ s$^{-1}$ used in the numerical calculations.
(c) Not available for this sample.
to normalize the junction resistance data in our best fit. $R_N'$ is determined from Eq. (*E4) using measured sample dimension and literature value of $2N(0)$. That these quantities are the same, within uncertainties, demonstrate the self-consistency of the fit. $I_C(0)$ is the zero temperature critical current calculated using measured dimension and material parameters. The normal state density of states $2N(0)$ used was $2.78 \times 10^{28} \text{ eV}^{-1} \text{m}^{-3}$ (*R52).

$r_{\text{tun}}^{-1}$ is the characteristic proximity effect induced tunneling rate. $r_{\text{e-ph}}(T_C)^{-1}$ is the electron-phonon scattering rate at the Fermi energy at $T_C$. $\langle a^2 \rangle_0$ is the unaveraged intrinsic mean square gap anisotropy parameter defined in Equation. (*E7).

A. Resistance as a function of temperature

Fig. (*F5) shows the normalized low-voltage resistance $R_j(T)/R_N$, vs. normalized temperatures, $T/T_C$, together with calculated results (solid curves) for sample J17. For this sample $r_{\text{tun}}^{-1}$ is negligible compared to $r_{\text{e-ph}}(T_C)^{-1}$ so that the nonequilibrium charge-imbalance is not important. Near $T_C$, $R_j(T)/R_N$ is only linear in $T$ because $k_B T$ is comparable to or larger than $\Delta$, so the gap in the density of states blocks the tunneling of relatively few quasiparticles. At low temperatures, however, the resistance rises exponentially as the number of quasiparticles sufficiently energetic to tunnel into the superconductor vanishes exponentially.

The junction reported above have slightly larger resistances at low temperature than predicted by BCS (*R53) theory. The difference can be explained by an enhancement of $\Delta$ of about 10% due to strong
Fig. 5. The normalized junction resistance $R_j(T)/R_N$ vs. $T/T_c$ for sample J17 (high resistance Sn junction). The circles are the data and the solid curve is calculated. The junction has a slightly larger resistance at low temperature than predicted by BCS theory. The difference can be explained by an enhancement of $\Delta$ of 10% due to strong electron-phonon coupling. Strong coupling effects are not included in our data analysis.
electron-phonon coupling (*R54,55). Strong coupling effects are not included in the analysis of $R_j(T,I_s) vs I_s$ below.

Fig. (*F6) and (*F7) show the normalized low-voltage resistance $R_j(T)/R_N$ vs. normalized temperatures, $T/T_c$, together with calculated results (solid curves) for sample A13 and D30. Also shown on the figures is the equilibrium part of the normalized junction resistance, $R_{eq}(T)/R_N$ (dashed curve). These junctions have much lower resistances than sample J17 so that the proximity effect induced tunneling rate is important. The junction resistance diverges towards $T_c$ since the charge-imbalance relaxation rate vanishes at $T_c$ (*R3). The junction resistance also diverges towards lower temperatures as discussed above. Generally speaking, the shapes of these curves depend on $r_{tun}^{-1}$ and $r_{e-ph}(T_c)^{-1}$. The minimum in the curve locates further away from $T_c$ as $r_{tun}^{-1}$ becomes larger, and it serves as a convenient indicator in initial data analysis.

In Figs. (*F6) and (*F7) we have included gap anisotropy rates in our fits. The intrinsic gap anisotropy relaxes the charge-imbalance and therefore reduces $R_{eq}(T)$. Since the gap anisotropy rate becomes more and more important towards low temperatures the net effect of gap anisotropy on the resistance vs. temperature curve is to make it closer to the $R_{eq}(T)/R_N$ vs. $T/T_c$ curve at low temperatures. In practice one can obtain a reasonable fit to junction resistance vs. temperature data using the ratio $r_{tun}^{-1}/r_{e-ph}(T_c)^{-1}$ as the sole fitting parameter. The electron-phonon rates we obtain with and without incorporating intrinsic gap anisotropy into our data analysis
Fig. 6. The normalized junction resistance $R_j(T)/R_N$ vs. $T/T_C$ for sample A13 (low resistance SnIn junction). The squares are the data and the solid curve is calculated. The resistance diverges toward $T_C$ since the charge imbalance relaxation rate vanishes towards $T_C$. Also shown is the theoretical equilibrium part of the junction resistance curve (dashed curve). The difference between the solid curve and the dashed curve is the charge imbalance part of the junction resistance.
Fig. 7. The normalized junction resistance $R_j(T)/R_N$ vs. $T/T_C$ for sample D30 (low resistance Sn junction). The squares are the data and the solid curve is calculated. The dashed curve is the equilibrium part of the junction. The difference between the solid curve and the dashed curve is the charge imbalance part of the junction resistance.
Fig. 8. Low temperature behavior of sample A13 (low resistance SnIn junction). The squares are the data and the solid curve is calculated. The data lie slightly below the theory curve. At 0.4 $T_c$, the measured normalized resistance is about 10% smaller than the theoretical calculation.
Sn/Sn-oxide/Cu
$T_c = 3.720 \text{K}$
$R_N = 39.4 \mu \Omega$
$\tau_{\text{fun}} = 7.60 \times 10^5 \text{s}^{-1}$
$\tau_{e-ph}(T_c)^{-1} = 2.86 \times 10^5 \text{s}^{-1}$
$\langle \alpha^2 \rangle_0 = 0.0053$

Fig. 9. Low temperature behavior of sample D30 (low resistance Sn junction). The squares are the data and the solid curve is calculated. At the lowest temperature the data lie significantly below the theory curve. At 0.4 $T_c$, the measured normalized resistance is about 23% smaller than the theoretical calculation.
of $R_j/R_N$ vs. $T/T_c$ usually differ by less than 20% depending on the magnitudes of gap anisotropy rate and tunneling rate.

The resistance of the low-resistance junctions deviates from the theory at lower temperatures presumably due to defects in the junction oxide barrier or due to non-tunneling behavior. Fig.(*F8) and (*F9) show the low temperature behavior of the junction resistance for sample A13 and D30. Evidently, the low temperature resistance vs. temperature data are not reliable in determining the scattering rates. Our results on supercurrent effect to be discussed below show that the quality of the junction does not affect the scattering rates we extract from our data analysis of supercurrent effects if the temperature dependence of the normalized junction resistance follows the theory curve down to $-0.6\ T_c$. A simple model (*R56) which assumes some areas of the junction to have unity normalized conductance for all temperatures accounts for the temperature dependence of the junction resistance extremely well but fails when applied to the analysis of supercurrent effect.

B. Resistance as a function of supercurrent

Fig. (*F10) shows the measured reduction in junction resistance (normalized to the junction resistance when there is no supercurrent), $-\delta R_j(T,I_s)/R_j(T,I_s=0)$, vs. normalized supercurrent squared, $I_s^2/I_C(0)^2$, together with numerically calculated curves (solid curve) for sample J17, a high-resistance junction. The data and the theory are in perfect agreement from $-0.33\ T_c$ to $-0.85\ T_c$. The value obtained for the only fitting parameter, the critical
FIG. 10. Reduction in junction resistance normalized to the junction resistance at zero supercurrent, $-\delta R_j(T, I_s)/R_j(T, I_s=0)$, as a function of normalized supercurrent squared, $I_s(T)/I_c^2(0)$, for sample J17 (high resistance Sn junction). The solid lines are the theoretical results. The temperature dependence of the data is in excellent agreement with theory over the temperature range $-0.33 T_c$ to $-0.85 T_c$. 

Sn-SnO$_x$-Cu

$T_c = 3.71 K$

$R_N = 12.5 m\Omega$

$V_{bias} \leq 5 \mu V$
current of the Sn film at T=0, was $I_C(0)=6.47$ Amp., in good agreement with the value of 6.90 Amp. expected from the measured sample properties and Eq.(*E15). For higher temperatures the effect was too small to measure before we exceeded the experimental critical current.

Note that, at high temperatures, the slope of the initial linear depression in $-\delta R_j(T,I_s)/R_j(T,I_s=0)$ decreases as T decreases. However, the slope increases again for lower temperatures. Near $T_C$, the different slopes associated with curves for different temperatures reflect the temperature dependence of $n_s$. That is, the pair-breaking rate caused by a given current is largest near $T_C$ where $n_s$ is small and $v_s$ is therefore large. At low temperatures, $n_s$ is nearly independent of T, so the pair-breaking rate for given current is independent of T. The slope of $-\delta R_j(T,I_s)/R_j(T,I_s=0)$ vs. $I_s^2/I_C(0)^2$ is nearly constant at low T.

Fig. (*F11) and (*F12) show the measured reduction in junction resistance (normalized to the junction resistance when there is no supercurrent), $-\delta R_j(T,I_s)/R_j(T,I_s=0)$, vs. normalized supercurrent squared, $I_s^2/I_C(0)^2$, together with numerically calculated curves (solid curve) for samples A13 and D30, low-resistance junctions. A comparison between Fig. (*F10) and Fig.(*F11) shows that the reduction in the equilibrium part of the junction resistance is less than 10% of the total reduction. The data and the theory are in perfect agreement from $-0.95 T_C$ to $-0.6 T_C$ in both the magnitude and the temperature dependence. We point out in particular that, at the
FIG. 11(a). Reduction in junction resistance normalized to the junction resistance at zero supercurrent, $-\delta R_j(T,I_s)/R_j(T,I_s=0)$, as a function of normalized supercurrent squared, $I_s(T)^2/I_c(0)^2$, for sample A13 (low-resistance SnIn junction). The solid lines are the theoretical results. The temperature dependence of the data is in excellent agreement with theory over the temperature range $-0.55T_c$ to $-0.95T_c$. 

Sample A13

$T_c = 3.690K$

$R_N = 47.0 \mu\Omega$

$\tau_{mun}^{-1} = 6.0 \times 10^8 s^{-1}$

$\tau_{ph}(T_c)^{-1} = 3.0 \times 10^8 s^{-1}$

$\langle \alpha^2 \rangle_0 = 0.0046$
FIG. 11(b). Same as in Fig. 11(a), but for $T$ near $T_c$ only. Note changes in scales on both axes.
FIG. 12(a). Reduction in junction resistance normalized to the junction resistance at zero supercurrent, $-\delta R_j(T, I_s)/R_j(T, I_s=0)$, as a function of normalized supercurrent squared, $I_s(T)^2/I_c(0)^2$, for sample D30 (low-resistance Sn junction). The solid lines are the theoretical results. The temperature dependence of the data is in excellent agreement with theory over the temperature range $-0.55 T_c$ to $-0.95 T_c$. 
FIG. 12(b). Same as Fig. 12(a), but for T near to $T_c$. Note changes in scales on both axes.
same normalized temperature, the gap anisotropy rates $<a^2>/2r_1$ for sample A13 and S28 differ by a factor of $-5$ while the mean-free-paths of electrons in the two samples also differ by a factor of $-5$. This agrees with the theoretical prediction that the gap anisotropy rate is proportional to the mean-free-path for dirty superconductors.

For lower temperatures, the measured resistance drops below the divergent calculated value, presumably due to defects in the insulator between the S and N films. For sample A13, the temperature dependence of the junction resistance deviates from theory only slightly and the supercurrent effect is only $-10\%$ smaller than the expected value at $T = 0.4\ T_C$. For sample D30, however, the temperature dependence of the junction resistance data is off by $-23\%$ at $T = 0.4\ T_C$ and the supercurrent effect data deviates further from the theoretical prediction.

For all samples discussed here, the $\delta R_j(T,I_s)$ vs $I_s$ curves we measured were usually symmetric with respect to the $I_s = 0$ point. The current bias through the junction was usually negligible compared to the applied supercurrent. However, the point of symmetry occasionally shifted to a different value which was larger than half of the bias current through the junction. We showed data for which supercurrent was measured with respect to the symmetry point. The uncertainty created in doing this is about $5$ to $10\%$. Generally speaking, the error bar in the supercurrent effect measurements is about $5\%$ for, say, $2\%$ supercurrent effect, and about $10\%$ for, say,
0.2% supercurrent effect.

Our results suggest that the charge-imbalance component of the junction resistance does depend on the quality of the junction. This complicates the data analysis since we don't know then down to what temperature are the theory applies. Systematic studies of the supercurrent effect for junctions of different qualities reveal that the electron-phonon rate and the intrinsic gap anisotropy rates extracted from the data analysis are consistent among similar samples as long as the temperature dependence of the junction resistance follows the theory curve down to about 0.6 $T_c$.

There are basically two reasons that make the above statement realistic. First, the reduction in the equilibrium part of the junction resistance, $-\delta R_{eq}(T, I_s)$, is usually small compared to that of the nonequilibrium part, $-\delta R_{q^*}(T, I_s)$, for temperatures above $-0.6T_c$ so that the quality of the junction, which affects $R_{eq}(T)$ most, is not important. For lower temperatures, the exponential dependence of $R_{eq}(T)$ on $\Delta/k_BT$ greatly increases the reduction in $R_{eq}$ upon application of a supercurrent.

The second reason relates to the temperature dependencies of the intrinsic gap anisotropy rate and the electron-phonon rate. Fig. (*F1) shows the effective gap anisotropy rate as a function of temperature calculated assuming that all the quasiparticles have energy $k_BT$. The effective gap anisotropy rate increases by a factor of about 40 from 0.95 $T_c$ to 0.6 $T_c$. The electron-phonon rate, which is the dominating rate at high temperatures, on the other hand, is
frozen out because phonons are thermally excited. In our Sn and SnIn films at ~0.6 $T_c$, the pair-breaking rate due to intrinsic gap anisotropy is either larger than or comparable to that of electron-phonon scattering. Consequently, supercurrent effect data taken above 0.6 $T_c$ is sufficient to extract the gap anisotropy rate since it has a deciding effect on the size of $R_Q^*(T,I_b)/R_N$ and therefore $R_j(T,I_b)/R_N$ for the temperature range considered.

Note that the electron-phonon scattering is inelastic so that the pair-breaking rate is only half of the scattering rate. For samples discussed here, at 0.95 $T_c$, the pair-breaking rate due to intrinsic gap anisotropy is about one twentieth to one hundredth of the pair-breaking rate due to electron-phonon scattering. At 0.6 $T_c$, the gap anisotropy rate become ~35-40 times larger while the electron-phonon rate becomes $(0.6)^3=0.22$ times smaller. For a smaller intrinsic gap anisotropy rate one needs to deal with lower temperature data so that the intrinsic gap anisotropy rate can be more important in the problem. To extend studies of gap anisotropy to lower temperatures, it would also help to make junctions of even lower resistance so that the reduction of the charge-imbalance component dominates the reduction in junction resistance even at below ~0.6 $T_c$.

The transition temperature of the superconductor was determined from stripe resistance vs. temperature data for a bare film deposited at the same run the base film was deposited. The $T_c$ we used for junction resistance vs temperature data was within the uncertainty of
the stripe $T_c$ measurement so that $T_c$ is not regarded as a fitting parameter. The depression in transition temperature due to the proximity-effect coupling (*R4,5,28) between the normal metal Cu and the superconducting Sn is less than 2 mK so that $T_c$ of a bare film is the same as that in a junction to within experimental error.

There are four fitting parameters for data analysis of low-resistance junctions: $R_N$, $r_{tun}^{-1}$, $r_{e-ph}(T_c)^{-1}$, and $<a^2>_0$. As noted above, the magnitude and the temperature dependence of the junction resistance data were used to determine $R_N$, $r_{tun}^{-1}$, and $r_{e-ph}(T_c)^{-1}$. The supercurrent effect data was used to determine $<a^2>_0$ and check the correctness of the scattering rates chosen.

The work on high resistance junction suggests that inevitable variation in the film thickness and supercurrent density, as are usual for wide films, have negligible effect for this type of measurement. Therefore, we can use the measured film thickness $d^{mass}$ in the initial data analysis of low-resistance junctions to see how well $r_{tun}^{-1}$ is described by Eq. (*E4).

Table 2 lists $R_N$ used in normalizing the junction resistance data and $R_N'$ that we obtained with Eq. (*E4) using measured sample dimensions and literature value of $2N(0) = 2.78 \times 10^{28}$ eV$^{-1}$m$^{-3}$. The excellent agreement between the two $R_N$s demonstrates that the tunneling rate is accurately described by Eq. (*E4). In practical data analysis, the procedure was iterated until a self-consistent set of $r_{tun}^{-1}$, $R_N$, and $d$ is found for both $R_j$ vs. $T$ and $R_j$ vs. $I_s$ data. The final choice of $d$ was always within 10% of the measured $d$. 
The uncertainties in the scattering rates extracted for sample D30 are about 10% for \( r_{e-ph}(T_c) \) and 20% for \( \langle a^2 \rangle_0 \). The electron-phonon rate is determined more accurately than the gap anisotropy parameter \( \langle a^2 \rangle_0 \) because both \( R_j \) vs. \( T \) and \( R_j \) vs. \( I_a \) data are used to determine \( r_{e-ph}(T_c) \). The uncertainty in the rates extracted are larger for junctions of lower quality because the temperature range in which the data agrees with the theory is narrower and uncertain. The uncertainty in \( \langle a^2 \rangle_0 \) also becomes larger when the change in \( R_q \) vs. \( T \) becomes less important, i.e. when \( R_q \) is large.

C. Electron-phonon scattering rate

In this section we discuss the electron-phonon rate obtained from our data analysis of low-resistance junctions and those in the literature. The electron-phonon rate we obtain is the inelastic electron-phonon scattering rate at the Fermi surface at \( T = T_c \), \( r_{e-ph}(T_c) \). This is obtained assuming the phonon density of states to have an \( \omega^2 \) dependence on energy \( \omega \) (SR19,20) in our calculation so that the electron-phonon rate has a \( T^3 \) temperature dependence above \( T_c \).

Various experimental methods have been used to determine the electron-phonon rate in Sn. Table 3 lists our result of \( r_{e-ph}(T_c) \) together with those in the literature. One thing common to all these techniques is that they all involve nonequilibrium superconductivity. That is, nonequilibrium quasiparticles are generated and then allowed to relax with the help of inelastic scattering.
Table 3. Measurements of the electron-phonon scattering rate in Sn.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Quantities measured</th>
<th>$r_{e-ph}(T_0)^{-1} \times 10^9$ s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>57</td>
<td>Nonlinearity in flux flow</td>
<td>2.0</td>
</tr>
<tr>
<td>58</td>
<td>Nonlinearity in flux flow</td>
<td>2.0</td>
</tr>
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<td>59</td>
<td>Quasiparticle recombination lifetime</td>
<td>$\leq 0.1$ (a)</td>
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<td>60</td>
<td>$\delta\mu_p(x)$ in Phase-slip-center</td>
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<tr>
<td>61-63</td>
<td>$\delta\mu_p(x)$ in Phase-slip-center</td>
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<tr>
<td>7</td>
<td>$\delta\mu_a$ due to $Q^*$</td>
<td>7.14</td>
</tr>
<tr>
<td>10</td>
<td>SNS interface resistance</td>
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<tr>
<td>20</td>
<td>(theory)</td>
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<tr>
<td>present</td>
<td>$R_j$ vs. $T$ and $R_j$ vs. $I_g$ for work</td>
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</tr>
<tr>
<td>work</td>
<td>Sn/Sn-oxide/Cu junctions</td>
<td></td>
</tr>
</tbody>
</table>

(a) Deduced from low temperature recombination time data.
The electron-phonon rate we obtain for Sn is $(2.8 \pm 0.3) \times 10^9 \text{ s}^{-1}$. This is in reasonable agreement with the theoretical calculation (*R20) of $3.65 \times 10^9 \text{ s}^{-1}$ and with other experimental results (*R7,10,57-63).

Musienko *et al* (*R57) and subsequently Klein *et al* (*R58) studied the energy relaxation phenomenon in dynamic mixed states of superconducting Sn and obtained $r_E^{-1}$ of $2.0 \times 10^9 \text{ s}^{-1}$. Parker and William (*R59) studied the energy gap relaxation under phonon irradiation. The electron-phonon rate deduced from low temperature recombination lifetime data was smaller than $1.0 \times 10^8 \text{ s}^{-1}$ for pure Sn. For both of these experiments, large uncertainties arise in estimation of the effective thermal coupling between the samples and the substrate and/or the liquid Helium bath. Both of these experiments give only a phenomenological energy relaxation rate $r_E^{-1}$ which is assumed to be the electron-phonon rate for the case of Sn.

Several experiments on phase-slip centers (*R60-63) have also been used to extract the inelastic scattering rate in Sn. The measured resistance of quasi one dimensional films are attributed to normal regions which have a characteristic charge-imbalance decaying length $\Lambda$. This length is necessary for the normal current to convert into supercurrent or vice versa since inelastic processes are necessary. The inelastic rates that these authors found for Sn range from $1.2 \times 10^9$ to $8 \times 10^9 \text{ s}^{-1}$. This type of experiment is complicated by the large uncertainty in estimation of effective thermal coupling strength between the sample and the environment and by joule heating.
in the normal region.

Compared to the techniques discussed above for studies of electron-phonon rate, our charge-imbalance technique is apparently superior in the sense that heating is negligible. Therefore, the phonons are always in thermal equilibrium. This allows a relatively precise theory to be formulated for charge-imbalance effects in SIN junctions. This is quite different from just using a phenomenological energy relaxation rate $r_E^{-1}$ to account for all the details of interactions and relaxation mechanism.

The main difference between our results and those of the traditional three-film, two-junction charge-imbalance technique (R7) is in the reproducibility. Heating complicated the data analysis in the traditional technique because of the high injection currents used. Also, the (SIN) detector junctions were of generally poor quality. Our technique, although complicated by the occasional poor quality of the junctions, gives consistent results in the electron-phonon rate and the gap anisotropy rate for similar samples.

Measurements on the interface resistance of an SNS sandwich also provides information about electron scattering rate. However, it is very difficult to make sure that the interface is free from oxidation and/or contamination. Furthermore, the theory of SNS interfaces is not as transparent as that of a SIN tunnel junction and applies only for an extremely small temperature range near $T_c$. 
D. Gap anisotropy parameter

In the data analysis of \(-\delta R_j(T,I_0)/R_j\) vs. \(I_s^2\) for low-resistance junctions, we adjust the intrinsic gap anisotropy parameter \(<a^2>_0\) to obtain the best fit in Figs. (*F11) and (*F12). In doing this, we have used a mean-free-path limited elastic scattering rate \(v_f/l\) for all samples. We assume the resistivity and the gap anisotropy in a thin film are determined by same type of elastic scattering. Boundary scattering measurement (*R64) suggested that for Sn the ratio for transport and elastic scattering rates is about 0.9, that is, very close to 1. This is very reasonable since most of the electrons that are involved in the transport process do not move in the direction perpendicular to the film surface so that the electron mean-free-path could be larger than the film thickness. That is, the elastic scattering rate in the film should be the mean-free-path limited rate \(v_f/l\) where \(l\) is determined from the resistivity \(\rho\). This is supported by the film resistivity measurement which shows the mean-free-path of the pure Sn film to be \(-1800\text{Å}\), which is much larger than the film thickness of 800Å. The fact that the gap anisotropy rate is proportional to the mean-free-path in our films is consistent with this picture.

Note that the unaveraged gap anisotropy parameter \(<a^2>_0\) is about the same for Sn and SnIn films although the 'Anderson-averaged' value of \(<a^2>\) are different by a factor of five. This together with the fact that \(<a^2>/2r_1\) is proportional to the mean-free-path suggests that impurity effect on gap anisotropy is described well
qualitatively by the theory.

It is important to understand how gap anisotropy is measured experimentally in order to compare results from two different experiments (*R30). Roughly speaking, measurements on gap anisotropy can be classified into two categories. In the first category, an equator on the Fermi surface or a specific crystalline direction is probed. Ultrasonic attenuation (*R65) and tunneling measurements on a single crystal (*R66) belong to this category. These techniques are useful in studies of detailed Fermi surface change in the presence of impurities. The second category, which our technique belongs to, probes the averaged gap anisotropy of the entire superconductor. Therefore, one has to be very careful in comparing results from different experiments.

Our values of \(<a^2>_0\) are smaller than theoretical estimation (*R37) of 0.02 as well as those of most measurements on clean, bulk samples [ \(<a^2>_0 \geq 0.01\) for most measurements (*R30)]. We considered the possibility that this discrepancy is related to the fact that our samples are evaporated thin films. It is known that any less than ideal situation that exist in the superconductor will reduce the intrinsic gap anisotropy significantly. For example, any stress that might have developed in the superconducting films during sample preparation would have reduced the intrinsic gap anisotropy. However, consistency among \(<a^2>_0\) from different samples make this unlikely the cause. Another possible cause is the granular structure in our films. Existence of grains directly results in reduction in
the electron mean-free-path and therefore $<a^2>$. On the other hand, the grain size should have had a deciding effect on the electron mean-free-path if it has any effect on the size of intrinsic gap anisotropy. In both cases, we would have concluded a larger instead of a smaller $<a^2>_0$ from the data analysis.

It is possible that the theory of Anderson averaging is off by a factor of four for large reductions in $<a^2>$. The data suggests that perhaps the factor $A/2\tau_1\Delta$ in Eq. (E9) should be $A/\tau_1\Delta$ instead.

It is amazing that Anderson averaging applies so well even for such a large reduction in the gap anisotropy. The temperature dependent gap anisotropy parameter $<a^2>$ in our dirty films is $10^2$-$10^3$ times smaller than the intrinsic gap anisotropy parameter $<a^2>_0$ at high temperatures. Small gap anisotropy is usually difficult to measure with methods other than nonequilibrium technique not to speak of reproducibility or verifying the correct energy and temperature dependence.

It is important to note the relative length scales involved. The charge-imbalance diffusion length, which is typically a few microns, is much larger than the grain size ($\sim$800-4000Å), the coherence length (2300Å), the electron mean-free-path, and the thickness of the films. This ensure the charge-imbalance to be quite uniform across the thickness of the film. The perfect agreement we have between data and theory suggests that the inevitable variation in the film thickness and supercurrent density, as are usual in wide films, have negligible effects on this type of measurement.
CHAPTER V
CONCLUDING SUMMARY

We have studied the low-voltage resistance of low-resistance and high-resistance Sn/Sn-oxide/Cu and SnIn/SnIn-oxide/Cu tunnel junctions as functions of temperature and transport supercurrent both theoretically and experimentally. The experimental results are compared to numerical calculations based on a Boltzmann equation appropriate for a steady state, spatially homogeneous charge-imbalance. The data are in excellent agreement with the theoretical calculations from -0.95 \( T_c \) to -0.6 \( T_c \) for low-resistance junctions and from -0.33 \( T_c \) to 0.85 \( T_c \) for a high resistance junction.

Our major results are the electron-phonon rate at the Fermi energy at \( T = T_c \), \( r_{e-ph}(T_c)^{-1} \), and the intrinsic gap anisotropy parameter \( <a^2>_{0} \). The electron-phonon rate we obtain for Sn is \( (2.8 \pm 0.3) \times 10^9 \) s\(^{-1}\). This value is in reasonable agreement with theoretical calculation of 3.85\( \times 10^9 \) s\(^{-1}\) and agrees with other measurements on Sn films.

We find that the mean square anisotropy of the gap, \( <a^2> \), is proportional to \( l \), in agreement with Anderson's theory of averaging of anisotropy effect. The magnitude of \( <a^2> \) is smaller than expected.
from theory and other measurements. Specifically, the value of the intrinsic mean square gap anisotropy \(<a^2>_0\) for pure Sn that we obtain from the measured \(<a^2>\) and the Anderson's theory is \(<a^2>_0 = 0.0053 \pm 0.0004\) for all samples, whereas literature values run from 0.01 to 0.04.

The \(<a^2>_0\) we obtain relies on the accuracy of the Anderson averaging theory. Nevertheless, we feel our technique is one of the most reliable ones since the values of \(<a^2>_0\) are consistent with each other even when the systems studied are a factor of 5 different in the mean-free-path. We conclude that the Anderson's theory of averaging of anisotropy effect describes effect of electron mean-free-path on the gap anisotropy qualitatively.

Our data suggest that inevitable variations in the film thickness and supercurrent density, as are usual in wide films, have negligible effects on the type of measurements reported here.

In summary, our initial work on Al based SIN tunnel junctions verified the existence of the novel charge-imbalance relaxation process thereby established the validity of the theory of low-resistance SIN tunnel junctions. The results reported in this dissertation validate the theory of the effect of supercurrents on the resistance of high and low-resistance SIN tunnel junctions and demonstrate the feasibility of using such measurements to study electron scattering processes in superconductors.
LIST OF REFERENCES


2. For several reviews and further references, see Non-equilibrium Superconductivity. Phonons and Kapitza Boundaries (Proceedings of the NATO Advanced Study Institute), K.E. Gray, ed. (Plenum, N.Y., 1981).


29. Y. Yen and T.R. Lemberger, present work.


41. In Tinkham's original paper the square bracket factor appeared squared. CP and subsequently CC proved that this factor should not be squared.


48. The solution used for anodization is made up of 56 grams of Ammonium Pentaborate, 400 ml of Ethylene Glycol, and 270 ml of water. We apply ≤ 1 ma/cm² of current and 60 Volt DC across the electrodes.

49. We thank Dale van Harlingen for pointing out the fact that Sn wets Cu.


56. Y. Yen and T.R. Lemberger, unpublished.


APPENDIX A

Initial measurement of the low-voltage resistance of low-resistance Al/Al-oxide/Cu tunnel junctions is reproduced in this Appendix. The existence of the novel tunneling relaxation process predicted by theory was verified quantitatively by the depression of $T_C$ due to this pair-breaking process and by the magnitude and temperature dependence of the diverging junction resistance towards the transition temperature. This work, together with the work in Appendix B, supported the theory of SIN junctions and formed the foundation of the technique discussed in the text.
Observation of a novel, inelastic, charge-imbalance relaxation process in superconductor-insulator-normal-metal tunnel junctions

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(Received 13 February 1964)

A recent experiment in liquid helium A superconducting quantum interference device (SQUID) was used to null the feedback mode to measure the Al voltages across the junction. The sample was immersed in liquid helium. A superconducting quantum interference device (SQUID) was used in a nulling feedback mode to measure the voltage across the junction. The junction resistance R(T) was determined from the conductance of the junction. The junction resistance R(T) was determined from the slope of the P-V curve, and Ic was defined as the bias current at which the P-V curve broke sharply from linearity.

Data for sample I are shown in Fig. 2. Because R(T) diverges as 1/(Tc - T1/2) for T near Tc, it is convenient to plot the square of the conductance of the junction G = I/R. Then G2 extrapolates linearly to zero at a temperature that we define to be the transition temperature Tc of the Al in the junction. This definition of Tc is supported by the observation that the critical current Ic, plotted as open circles in Fig. 2, also goes to zero at Tc. We determined the critical current Ic of the Al not in the junction from the current at which the P-V characteristic of the strip switched from zero to a finite voltage. The transition temperature Tc of the Al not in the junction was determined by extrapolating Ic(T) to zero by eye. Ic is plotted as solid circles in Fig. 2. The dashed curves in Fig. 2 are the theoretical maximum values for Ic and Ic, calculated by using measured parameters and by assuming a uniform 2000 Å ground plane centered on the junction and isolated from the sample by a 2000-Å-thick SiO film. The film thicknesses were determined from a thickness monitor that uses a vibrating quartz-crystal sensor.

The sample configuration is illustrated in Fig. 1. The shape of each film was defined with an aperture mask. The Al (200 Å) film was evaporated onto a clean, room-temperature glass substrate. Its narrow section was about 2 mm long and 320 μm wide. Silicon monoxide (500 Å) defined the junction width of about 320 μm. The normal counterelectrode was 2000-3000 Å thick, and it was made by evaporating a mixture of roughly 98% Cu, 1% Al, and 1% Fe to completion from a W boat. The Fe was necessary to prevent a supercurrent from flowing from the Al film into the Pb (1000 Å) overlayer on the counterelectrode. From visual observations of the color of the counterelectrode during its deposition, we believe that the Fe evaporated after the Cu, as expected from the respective vapor pressures of Fe and Cu at the evaporation temperature of about 1000°C. The circles in Fig. 1 represent a 6-mm-diam Pb (2000 Å) ground plane centered on the junction and isolated from the sample by a 2000-Å-thick SiO film. The film thicknesses were determined from a thickness monitor that uses a vibrating quartz-crystal sensor.

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The parameters for sample 1 (2) (3) were as follows: electron mean free path $^{19} l = 4 \times 10^{-9} \text{m}$/μ (1 K) - 130 nm (10 K) (63 nm); film thickness $d = 20$ nm (20 nm); $T_c = 1.07 \text{K}$ (1.07 K) (1.21 K); $T_a = 1.33 \text{K}$ (1.48 K) (1.38 K); $R(T,2) = 4.50 \text{m}$ (9.00 m); $T = 0.0 \text{K}$ (52 m); $T_a = 0.0 \text{K}$ (52 m). From Eq. 1, the current density $j$ was limited by the Fermi surface and the Ohm law. The measured current density $j$ is shown at a lower temperature $T < 1$.$^{12}$

The theory of low-voltage measurement of SIN tunnel junctions is based on the Boltzmann equation approach of Pethick and Smith. Near $T_c$, the physical picture is especially clear. The bias current generates a large potential drop in the insulator and it generates a nonequilibrium state characterized by a charge imbalance in the $S$ electrode. The presence of a charge imbalance implies an excess of quasiparticles on one side of the Fermi surface and a deficit on the other side. Therefore, there is a nonzero density of quasiparticle charge. To maintain local charge neutrality, the density of the condensate charge must change by the opposite amount by adjusting its chemical potential. This results in a chemical potential drop across the junction, in addition to the electric potential drop.

Because of $R_j$'s nonlinearity, the bias current causes a finite value linearly as $T / T_c$, and a resistance $R_j(T)$ associated with the charge imbalance, that diverges as $1/T$ at $T_c$. The divergence of $R_j$ can be understood physically by imagining the bias current to be fixed at some small value while the temperature is increased towards $T_c$. The size of the charge imbalance, and also the chemical potential drop, generated by the current is proportional to the charge-imbalance relaxation time $\tau_0$. As $T \to T_c$, $\tau_0$ continuously approaches its normal-state value, which is infinite because a charge imbalance can never exist nor not exist in the normal state. Hence, the measured voltage diverges.

To compare our results with theory, we numerically solved the Boltzmann equation describing a SIN junction biased at a small voltage $V$ by including electron-phonon scattering and the nonequilibrium parts of the quasiparticle injection term. The total electric current was calculated from the resulting quasiparticle distribution function by using Eq. (11) of Pethick and Smith. There are two important physical parameters in the theory. One is the ratio $\tau_0 / \tau_{eq}$ of the characteristic rate associated with quasiparticle tunneling.

$$\frac{\tau_0}{\tau_{eq}} = \frac{1}{1 + \frac{4}{\pi} \frac{D_T}{\nu_T}} \frac{1}{1 + \frac{4}{\pi} \frac{D_T}{\nu_T}}$$

This ratio is larger than about 0.2, then pair breaking effects on the superconducting density of states and order parameter must be included in the calculation. For the three samples reported here, $\tau_0 / \tau_{eq} < 0.05$, so that the BCS density of states and order parameter were used in the calculation.

The end result of the calculation is the normalized junction conductance

$$g(T,2) = \frac{I R(T,2)}{R_j(T,2)}$$

as a function of the normalized temperature $T / T_c$. Near $T_c$, the calculated values of $g(T,2)$ agree with the analytic result:

$$g(T,2) \sim \frac{1}{1 + \frac{4}{\pi} \frac{D_T}{\nu_T}} \frac{1}{1 + \frac{4}{\pi} \frac{D_T}{\nu_T}}$$

A graph of $g(T,2)$ in the limit $\tau_0 / \tau_{eq} \gg 1$ is shown as a dashed curve in Fig. 3, the solid line is discussed below. Experimental values of $g(T,2)$ are also shown in Fig. 3, where $R(T,2)$ is a fitting parameter determined from the slope of $g(T,2)$ near $T_c$, by using Eq. (4). Clearly, the measured values of $g(T,2)$ have the predicted temperature dependence for $T > 0.05 T_c$, but they deviate from theory at lower temperatures.

Before discussing the deviations, we emphasize that it is very important to check the theory quantitatively by comparing the measured value of $\tau_0 / \tau_{eq}$ with the value calculated from Eqs. (1) and (2) by using measured sample parameters from the experimental values for $T_c$, $\alpha$, $\nu_T$, $R(T,2)$, and $1/\tau_0$ listed above, and the literature value.

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The resistance of the junction is the sum of two parts, corresponding to the electronic and the chemical parts of the electrochemical potential the resistance the insulator $R(T,2)$, that approaches a finite value linearly as $T / T_c$, and a resistance $R_{eq}(T)$ associated with the charge imbalance, that diverges as $1/T$ at $T_c$. The divergence of $R_{eq}$ can be understood physically by imagining the bias current to be fixed at some small value while the temperature is increased towards $T_c$. The size of the charge imbalance, and also the chemical potential drop, generated by the current is proportional to the charge-imbalance relaxation time $\tau_0$. As $T \to T_c$, $\tau_0$ continuously approaches its normal-state value, which is infinite because a charge imbalance can never exist nor not exist in the normal state. Hence, the measured voltage diverges.

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THOMAS E. LEMBERGER AND YBOUCHUNG YEN

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2N(0) = 3.48 × 10^{14} \text{eV m}^2, \text{for sample } 1, 2) (3) we find
T_c/T_F = 0.056 (0.030) (0.035). These values agree very
well with the directly measured values 0.050 (0.009)
(0.048). The measurements near T_c, therefore, confirm
quantitatively the existence of an inelastic charge-imbalance
relaxation process associated with the proximity of a normal
metal to a superconductor.

The solid lines in Fig. 3 represent a modified junction
conductance:

\[ g_j' = R(t) + R(0) - R(t) + R(0) \]  

In effect, \( g_j' \) differs from \( g_j \) in that the temperature-
dependent BCS\(^{1,2}\) resistance of the insulator \( R(t) \) has
been replaced by its value at \( T_c, R(0) \), in the calculated
value of the total junction resistance, \( R(t) \). We have no
explanation for why \( g_j' \) fits the data better than \( g_j \). We can
note only that measurements\(^2\) on junctions with a specific
resistance \( [R(T_c)] \) 100 times larger than those presented
here also were fitted better by assuming that \( R(t) \) was
independent of \( T \).

In conclusion, data on three SIN tunnel junctions clearly
demonstrate an inelastic charge-imbalance relaxation process
associated with the proximity of a normal metal to a super-
conductor. Furthermore, they quantitatively link this pro-
cess to the pair-breaking nature of the proximity effect.
This process may be important in any measurement of
inelastic scattering rates involving superconductors in proximity
to normal metal.

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us (T.R.L.) acknowledges support from the Alfred P. Sloan
Foundation.

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FIG. 3. Measured values of the square of the normalized junction conductance of samples 1(0), 2(0), and 3(1). The dashed curve represents \( g_j' \), the square of the normalized junction conductance calculated numerically. The solid line represents \( g_j \), a modified conductance discussed in the text. Note that the data and the theory curves for samples 1 and 2 are displaced vertically by 0.04 and 0.07 units, respectively, for clarity.

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APPENDIX B

Systematic study of the low-voltage resistance of low-resistance
Al/Al-oxide/Cu tunnel junctions with different intrinsic junction
resistances $R_N$ is reproduced in this Appendix. The (modified) theory
accurately describes the low-voltage resistance of the SIN junctions
for $T/T_C \geq 0.85$, and for $1 \leq r_E/r_{Cu} \leq 200$, thereby confirming the
basic validity of the theory of resistance of SIN junction. $r_E^{-1}$ is
the same as $r_{e-ph}(T_C)^{-1}$. The junctions presented in this
dissertation are well fitted by the unmodified theory. We do not
know why these early measurements deviated systematically from the
theory.
CHARGE-IMBALANCE IN SUPERCONDUCTOR-INSULATOR-NORMAL METAL TUNNEL JUNCTIONS

Tenchung YEH and Thomas R. LEMHOFER**

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The low-voltage dc resistance of a Superconductor-Insulator-Normal metal (SNH) tunnel junction contains contributions from the insulating layer and from the charge-imbalance generated in the S electrode by the measurement current. Measurements on very low resistance junctions are in excellent qualitative agreement with a modified theory that includes the postulate that the resistance of the insulating layer is independent of temperature. In this paper, we report measurements on junctions with resistances that are up to two hundred times larger than those reported earlier. The data agree with the modified theory over the entire resistance range studied.

1. INTRODUCTION

A recent theory (1) predicts that the charge imbalance that is generated in the superconducting electrode of an SNH tunnel junction can relax through a novel process involving tunneling into the normal electrode, in addition to bulk scattering processes. The tunneling process is inelastic and is characterized by a rate:

\[ \frac{1}{\tau_{\text{tun}}} = \frac{1}{2\pi e} R_i(T_f) \]

where \(2\pi e\) is the density of electron states per unit volume, \(e\) is the electron charge, \(i\) is the area of the junction times the thickness \(d\) of the S electrode, and \(R_i(T_f)\) is the resistance of the insulating layer at temperature \(T_f\).

An analytic result of the theory, valid for \(T_f << T_c\), shows how the tunneling process affects the low-voltage dc resistance of the junction, \(R_i(T_f)\):

\[ R_i(T_f) = R_i(0) \left[ \frac{1}{\tau_{\text{tun}}} \right] \]

where \(\tau_{\text{tun}}\) is the electron-phonon scattering rate and \(A\) is the order parameter. From Eq. (2), \(R_i(0)\) diverges as \((T_f-T)^{-1/4}\) with a magnitude that is determined by \(1/\tau_{\text{tun}}\) thus, \(R_i(T_f)\) should reflect the existence of the tunneling process if \(1/\tau_{\text{tun}}\) is significant compared with \(1/g\), i.e., \(1/\tau_{\text{tun}} < 1/g\).

Measurements have been reported (2) on junctions in which \(R_i(T_f)\) was so small that \(30\% > 1/\tau_{\text{tun}}\). For \(T_f < 0.9T_c\), the data were in qualitative agreement with Eqs. (1) and (2). For lower temperatures, 0.85 \(T_f / T_c\approx 0.95\), \(R_i(T_f)\) was smaller than expected from a numerical extension of the theory. However, the theory fitted the data within about 30\% when the theory was modified with the postulate that the resistance of the insulator, \(R_i(T_f)\), was independent of \(T_f\). The modification was made by subtracting the BC's result for \(R_i(T_f)\) from the calculated value of \(R_i(T_f)\) and then adding \(R_i(T_f)\) to \(R_i(T_f)\).

In the paper, we report measurements of the low-voltage dc resistance of junctions for which \(1/\tau_{\text{tun}} > 10^7\). The purpose of the measurements is to test the range of resistances over which the modified theory is valid.

2. DATA

The sample configuration is shown in Fig. 1. Each sample was made by evaporation films onto a room temperature glass substrate. The order of evaporation and the film thicknesses were: Al(100Å), Sn(0.5-500Å), Cu(0-100Å), Pb(0-1000Å), Sn(1500Å), Pb(2000Å). The Cu alloy

![Sample configuration](image)

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**Alfred P. Sloan Foundation fellow
was -0.8% Cu, 33% and 13.8%. The Fe was necessary to prevent a supercurrent from flowing between the Al film and the Pb film on top of the Cu alloy film. The area of the junction was 240, as is 330. Film thicknesses were determined by a thickness monitor with a vibrating mirror sensor. The important physical parameters for our samples are listed in Table I. Values of $1/\tau_2$ were estimated from the approximate relation (3), $1/\tau_2 = (T_c/T)^2/2$), where $T_c$ is the mean-free-path was determined from (4) $\rho = 4 \times 10^{-5} \mu m$.

Table I. Sample parameters.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$t$ (A)</th>
<th>$d$ (A)</th>
<th>$T_c$ (K)</th>
<th>$R_2$</th>
<th>$1/\tau_2$ ($10^7 s^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>105</td>
<td>200</td>
<td>1.405</td>
<td>770</td>
<td>34</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>200</td>
<td>1.404</td>
<td>337</td>
<td>13.3</td>
</tr>
<tr>
<td>3</td>
<td>63</td>
<td>150</td>
<td>1.432</td>
<td>63.5x10^2</td>
<td>1.47</td>
</tr>
<tr>
<td>4</td>
<td>103</td>
<td>200</td>
<td>1.321</td>
<td>3.5x10^2</td>
<td>1.11</td>
</tr>
</tbody>
</table>

The junction resistance $R_j$ was measured with a SQUID used in a feedback mode. For voltages less than 50% the current-voltage characteristics were linear. The critical current of the Al film was measured, but will not be discussed here. Because $R_j$ diverges at $(T_c-T)^2$ near $T_c$ it is convenient to present the results in terms of the conductance, $G_j = 1/R_j$, squared since $G_j$ extrapolates linearly to zero at $T_c$.

Figure 2 shows measured values of $G_j(R_j(T_c)/R_j(T))$, where $R_j(T_c)$ and $T_c$ were chosen for each sample for a best fit to the modified theory (solid curves). The temperature dependence of $R_j$ is quite sensitive to the value of $T_c$ for $0.01 \leq T_c/T \leq 0.1$, so that the values of $R_j(T_c)$ and $T_c$ determined for each sample in this way were accurate to 2%. To check the theory qualitatively, the ratio $G_j(T)/G_j(T_c)$ was estimated from Eq. (1) and from the sample parameters in Table I. The 'best fit' values and the estimated values are given in Fig. 2, and they are in excellent agreement.

3. CONCLUSION

The modified theory accurately describes the low-voltage dc resistance of SIN tunnel junctions for $T/T_c > 0.85$, and for $T/\rho(T)/\rho(T_c) > 0.03$. It seems unlikely that such a large correction could be made with such high accuracy by such a simple modification for all values of $T/T_c$.

REFERENCES

APPENDIX C

The theory of the effect of supercurrents on the low-voltage resistance of SIN tunnel junctions is reproduced in this Appendix. This dissertation verifies this theory and establishes the feasibility of using this technique in studies of electron scattering mechanisms in novel materials.
A great deal of work, both theoretical and experimental, has focused on charge-imbalance phenomena in tunnel junctions involving superconducting films. Experimental results near the superconducting transition temperature have been generally in good agreement with theoretical expectations. Data at lower temperatures often differ from theory. Despite the discrepancies, it is clear that in addition to their intrinsic interest, charge-imbalance phenomena offer the possibility to measure quantities such as the electron-phonon scattering rate, quantities that are difficult to measure in other ways. This possibility is the motivation for further work.

Several experimental studies have dealt with a three-film, two-junction geometry in which the superconducting film to be studied is sandwiched between two other films. One junction is biased at a relatively high voltage to generate a charge imbalance in the superconducting film. The other junction detects the imbalance. This geometry has the favorable feature that the charge imbalance is generated with one junction and detected with another. However, it would be difficult to use with superconductors that either do not grow on another metal film or do not form good tunnel barriers. The problem of junction alignment would make measurements on quench-condensed films, for example, very hard.

More recent work has demonstrated that charge-imbalance effects can be observed in a single superconductor-insulator-normal-metal (SIN) tunnel junction if the resistance of the junction is small enough. The single junction serves as both generator and detector. Bias voltages are very small to avoid heating. This is a convenient geometry for study of diverse materials. We presently are using this geometry to study superconducting films. We have observed that the resistance \( R_J \) of a SIN junction is reduced by supercurrents, either applied directly with a current supply or induced with a magnetic field applied parallel to the superconducting film. Such measurements should yield information similar to that obtained from other nonequilibrium techniques like NMR and ultrasonic attenuation.

I. INTRODUCTION

The basic idea is how charge imbalance affects the resistance of tunnel junctions. In the junction, a small bias voltage generates a current, which in turn generates a quasiparticle charge imbalance whose amplitude is linearly proportional to the voltage applied across the junction. The quasiparticle charge associated with the imbalance leaks back into the normal-metal film, reducing the total current across the junction. Experimentally, this is interpreted as an additional "nonequilibrium" resistance \( R_{\text{eq}} \), which is very nearly proportional to \( R_{\text{eq}} \), and the familiar "equilibrium" resistance \( R_0 \) of the junction is observed. If \( R_{\text{eq}} \) is small enough, then \( R_{\text{eq}} \) is observed in addition to \( R_0 \). Thus, the relaxation time \( \tau_{\text{eq}} \), which depends on the interesting coherence factors and scattering rates, can be studied via the junction resistance \( R_J(T) \).
if they were measured data. Electron-phonon scattering in the S film is included in one junction and neglected in the other for comparison. The intrinsic resistance $R_I$ of the latter junction is chosen to be smaller than in the first, so that the total inelastic-electron-scattering rate at $T_c$, which includes the proximity-effect coupling between $S$ and $N$, is the same for both. Treating the calculated curves as data, we show how the general features of $R_I$ yield information about the relevant scattering rates, coherence factors, and density of superconducting electrons. To include a realistic, albeit small, amount of pair breaking in calculating quantities such as the superconducting density of states, the parameters for $S$ were chosen to correspond to Sn.

The results of this paper are based on the Green's-function theory of charge imbalance in tunnel junctions, rather than the simpler Boltzmann-equation theory, to include the effects of pair breaking on the order parameter, density of states, and coherence factors in the superconductor. Some approximations are made to simplify calculations, as discussed below. In regard to the effects of pair breaking, we note that pair-breaking effects due to intrinsic processes such as electron-phonon scattering are typically small, so the two theories give nearly the same results. In such cases, pair-breaking effects become important only when the pair-breaking rate due to applied supercurrents and magnetic fields is large. In other cases—for example, superconductors containing magnetic impurities—pair-breaking effects due to the intrinsic rate may be important.

We restrict the discussion to include the effects of electron-phonon scattering, the proximity of the normal-metal film to the superconductor, and applied currents and magnetic fields. Relaxation associated with intrinsic anisotropy in the order parameter can be significant, but is omitted here for simplicity. Electron-electron scattering, scattering from magnetic impurities, and the pair-breaking effect of supercurrent fluctuations may also be important under some circumstances, but they are similarly omitted.

To clarify the experimental geometry and some of the assumptions inherent in the theory, the discussion begins with some considerations associated with choosing experimental parameters.

II. EXPERIMENTAL CONSIDERATIONS

A simplified experimental configuration is shown in Fig. 1. The superconducting film is deposited on a glass substrate, oxidized lightly, and masked with insulating SiO films. Then the normal-metal electrode is deposited to complete the junction. The normal-metal film is excited with a superconducting film with a relatively high $T_c$, such as Pb. The normal-metal film is thick and/or dirty enough that any superconductivity induced by the Pb film decays before reaching the junction insulator. The Pb film and the superconducting film form equipotentials so that the current density through the junction is as uniform as possible.

Convenient parameter values that we have used are junction area $300 \times 300 \, \mu \text{m}^2$, superconducting film width $300 \, \mu \text{m}$, superconducting film thickness $300-800 \, \AA$, normal-metal thickness $4000 \, \AA$, SiO thickness $500 \, \AA$, and intrinsic junction resistances from $4 \, \mu \Omega$ to $4 \, m\Omega$. Considerations that go into choosing these values are as follows:

1) The intrinsic resistance $R_I$ of the junction should be chosen so that the proximity-effect tunneling rate $1/\tau_{pe}$ is about one-tenth of the rate of interest.
2) The length of the superconducting strip actually in the junction should be much larger than the distance $\lambda_D = \sqrt{D \tau_{pe}}$, that a quasiparticle diffuses in a charge-imbalance relaxation time $\tau_{pe}$. $D$ is the electron-diffusion constant. This condition ensures that the charge imbalance is essentially uniform across the entire junction area, decay to zero in a length $\lambda_D$ near the edge of the junction.
3) The superconducting film should be much thinner than both $\lambda_D$ and the Ginzburg-Landau coherence length, so that the charge imbalance and the order parameter $\Delta$ are uniform through the thickness of the film.
4) The normal metal must be thick enough that a supercurrent cannot flow through it from the Pb film on top to the S film. However, it should be thin enough that its resistance is much less than the junction resistance.
5) The resistance of the junction should be measured with the smallest, easily measured, voltage, typically a few hundred nanovolts. Over this range, current $I$ is proportional to voltage $V$, so the resistance $R(T) = V/I$ is well defined. Heating is avoided.

III. NUMERICAL RESULTS AND GENERAL ANALYSIS

It is useful to discuss in general terms how useful information can be obtained from $R_I$ as a function of temperature and of applied current or field before developing the microscopic model. For this purpose, we will consider calculated results for two hypothetical S/N junctions. The difference between the two junctions is that electron-phonon scattering in the S film is omitted for one but included in the other. To include a realistic amount of pair breaking due to inelastic processes, calculations are done using typical values for Sn for the S film. Intrinsic gap anisotropy that exists in real Sn films is omitted. For all situations illustrated in the figures, modifications due to inelastic pair breaking are a few percent or less.
For the junction that includes electron-phonon scattering, the parameters are $S$-film thickness $d = 600 \, \text{Å}$, width $w = 300 \, \text{µm}$, and resistivity $\rho = 3 \, \text{µΩ cm}$, a junction area of $(1500 \, \text{µm}^2)$, and an intrinsic junction resistance $R_H = 100 \, \text{µΩ}$. The electron-phonon scattering rate in the $S$ film for an electron at the Fermi surface at $T = T_F = 3.715 \, \text{K}$ is about $4 \times 10^3 \, \text{s}^{-1}$, and is proportional to $T^5$ in the normal state. The proximity (tunneling) rate $1/r_{\text{vm}}$ that characterizes the strength of coupling between the $S$ and $N$ films is inversely proportional to $R_H$ and independent of $T$.

\[
1/r_{\text{vm}} = 1/2N(0) \kappa^2 R_H = 4 \times 10^4 \, \text{s}^{-1},
\]

for the present junction, with a density of states $N(0) = 2.9 \times 10^{16} \, \text{eV}^{-1} \text{cm}^{-3}$, and an injected volume $N = 1000 \, \text{µm}^2 \times 600 \, \text{Å}$. Note that the tunneling process acts like an inelastic scattering process in parallel with electron-phonon scattering so that the total inelastic scattering rate is their sum. For this example junction, $1/r_{\text{vm}}(T_F)$ is about 10 times larger than $1/r_{\text{vm}}$, and will dominate the charge-imbalance relaxation process down to $T/T_F = 0.3$, where $1/r_{\text{vm}}(T_F) = 0.3/r_{\text{vm}}(T_F) = 1/2N(0) \kappa^2 R_H$.

For the junction that does not include electron-phonon scattering in the $S$ film, the parameters are the same, except that $R_H$ is 11 times smaller, so that the tunneling rate is $1/r_{\text{vm}} = 4.4 \times 10^3 \, \text{s}^{-1}$. Thus, the total inelastic scattering rate at $T = 3.715 \, \text{K}$ is $4.4 \times 10^3 \, \text{s}^{-1}$ in both junctions.

The calculated resistances $R_J(T)/R_H$ for these junctions is shown in Fig. 2. The "equilibrium" resistance $R_m(T)/R_H$ is the resistance the junction would have if nonequilibrium effects were negligible. Although it nominally was calculated for $1/r_{\text{vm}} = 4.4 \times 10^3 \, \text{s}^{-1}$ and $T_F = 3.715 \, \text{K}$, i.e., $4/3 k_B T_F/(\hbar) = 0.009$, the curve labeled $1/r_{\text{vm}} = 0$ is a limiting curve that is independent of $1/r_{\text{vm}}$ as long as $\Delta < k_B T_F$. Values of $R_J(T)/R_H$ for junctions with $0 < 1/r_{\text{vm}}(T_F)$ lie between the two curves in Fig. 2. Close inspection of Fig. 2 reveals that the two $R_J(T)/R_H$ curves are merging as $T$ decreases and electron-phonon scattering freezes out.

To see in more detail how the scattering rates influence $R_J(T)$, consider the calculated curves as data. From $R_J(T)$ near $T_F$, we can obtain $R_H$ and the rate $1/r_{\text{vm}}$ and $1/r_{\text{vm}}(T_F)$. Near $T_F$, $R_m(T) = R_H$ is nearly constant compared with $R_m(0)$, which diverges as $k_B T_F/\Delta$, where $\Delta$ is the order parameter. By fitting $R_J(T)$ near $T_F$ with the form

\[
R_J(T) = R_H + \frac{4k_B T_F}{\Delta} \frac{1}{1/r_{\text{vm}} + 1/r_{\text{vm}}(T_F)},
\]

we obtain both $R_H$ and the rate

\[
1/r_{\text{vm}} = 1/r_{\text{vm}} + 1/r_{\text{vm}}(T_F).
\]

From $R_H$ and measured sample parameters, $1/r_{\text{vm}}$ is calculated, then subtracted from $1/r_{\text{vm}}$ to give $1/r_{\text{vm}}(T_F)$. This procedure has been used successfully even in very low resistance junctions. The factor $4k_B T_F/\Delta$ is essentially an average over energy of the coherence factor for charge-imbalance relaxation due to inelastic electron scattering near $T_F$. If an elastic process were dominant, then the coherence factor would be proportional to $(\Delta/k_B T_F)^3$.

Away from $T_F$, it is difficult to extract information from $R_J(T)$ since both $R_m(T)$ and $R_m(0)$ depend on $T$. Measurements of $R_J$ as a function of supercurrent along the $S$ strip of or magnetic field parallel to the $S$ strip, are very useful. In the present examples, we will see that these measurements allow determination of the electron-phonon scattering rate and the density of superconducting electrons as functions of $T$ below $T_F$. The procedure works because the nonequilibrium resistance $R_m(0)$ is much more sensitive than $R_m(T)$ to external pair-breaking perturbations such as a supercurrent $I_s$ along the superconducting strip or a magnetic field $B_z$ parallel to the strip, so measurements at low fields and currents probe $R_m(0)$ almost exclusively.

The experimental arrangement for measuring $R_J(T,I_s)$ is illustrated in Fig. 1. The current $I_s$ through the junction is fixed, and the voltage $V$ across the junction is measured while the current $I_s$ along the superconducting film is increased. The effect of the supercurrent is to shorten the charge-imbalance relaxation time $r_{\text{vm}}$, and thereby to reduce $R_m(0)$. The equilibrium resistance is also reduced. Measurements of $R_J(T,B_z)$ for a function of magnetic field $B_z$ are made in a similar fashion, but the external current passes through a solenoid around the sample rather than along the superconducting film.

Figure 3 shows calculated values of $R_J(T,B_z)/R_H$, and $R_m(T)/R_H$ versus magnetic field for several temperatures. The divergence in $R_J/R_H$ evident in Fig. 4, occurs as
the field approaches the critical field, and the order parameter vanishes. In other words, the resistance diverges as \( T = T_c \), just as it diverges when \( T = T_c \) in zero field, as shown in Fig. 2. This divergence has not been observed due to the large fields required, but would be interesting to study because it would give a more accurate value for the critical field than determinations based on resistance measurements.

In practice, the low-field portion of the curves in Fig. 3 are measured. If they were data on an ideal BCS superconductor, then careful numerical fits to the theory presented below would yield all the information desired about electron scattering rates and coherence factors. Since the most interesting applications of this technique will be to materials in which one is unsure of the correct microscopic model, e.g., disordered superconductors in which electron-electron scattering may be important, it is useful to dissect the calculated curves to see how their general features can be used to infer the microscopic physics. We have found that this sort of crude analysis of data offers a starting point for more detailed fitting.

As seen in Fig. 3, for small fields \( R_m/R_s \) is nearly constant. The dominant effect of the field is to reduce \( R_{2s}/R_s \). The inflection point in \( R_{2s}/R_s \) is marked with an arrow because its location allows a convenient parametrization of the entire low-field curve. As one might guess, the inflection point occurs when the pair-breaking rate \( 1/\tau_s \) due to the field reaches a characteristic value, which turns out to be roughly half of the intrinsic pair-breaking rate. This relationship can be demonstrated near \( T_c \) by the following argument.

Consider the case in which the intrinsic pair breaking is due entirely to inelastic processes so that the intrinsic pair-breaking rate is one-half of the total inelastic scattering rate. The argument is easily generalized to include intrinsic elastic pair-breaking processes. Schmid and Schönholtz showed that, near \( T_c \), \( 1/\tau^* \) depends on the geometric mean of the total pair-breaking rate \( 1/2 \tau^* + 1/\tau^* \) and the inelastic pair-breaking rate \( 1/2 \tau^* \).

\[
\frac{1}{\tau^*} = \frac{\Delta}{4 \pi T} \left[ \frac{1}{2 \tau_m + \tau_s} \right]^{1/2}
\]

(3)

(This result is modified extremely close to \( T_c \), where the dynamic charge-imbalance relaxation rate, in large square brackets in Eq. (3), becomes larger than the frequency \( \Delta/\hbar \). We are not concerned with this range here.) In the present case, the inelastic rate is \( 1/\tau_m = 1/4 \tau_m(1 + 1/\tau_{ph}) \) and the elastic pair-breaking rate \( 1/\tau_s \) is\(^4\)

\[
1/\tau_s = B_0^{d2}/12\pi N(0)M^2
\]

(4)

Not too close to \( T_c \), the dependence of \( \Delta \) on \( 1/\tau_s \) is small, and \( R_m(B_s) \) has an inflection point at \( 1/\tau_s = 1/4 \tau_m \), i.e.,

\[
B_0^{d2}/12\pi N(0)M^2 = \left[ 1/\tau_m + 1/\tau_{ph} \right]^{1/2}
\]

(5)

To the extent that \( R_m(T) \) is proportional to \( \tau_m \) and \( R_m(T) \) is unaffected by the relevant fields, the inflection point in \( R_m/R_s \) coincides with that in \( \tau_m \), and serves as a crude indicator of the inelastic scattering rate.

Figure 4 shows normalized values of \( 1/\tau_s \), the low-field inflection point in \( R_m/R_s \) versus \( B_s \) for example junctions with and without electron-phonon scattering [Plots of \( R_m/R_s \) versus \( B_s/B_s(0) \) for the junction with no electron-phonon scattering are shown below]. For the junction including electron-phonon scattering, \( \tau_m^{-1} \) is
The normalised elastic pair-breaking rate \( \tau_{\text{el}}^{-1} \) at the low-field inflection point in \( R_{\text{el}}(I/1) \) vs \( I/1 \) for the two example junctions. Curves are shown for junctions with and without electron-phonon scattering and with the same inelastic scattering rate at \( T_i, 0.4 \times 10^{-6} \). Including electron-phonon scattering results in a strong \( T \) dependence roughly parallel to the \( T \) dependence of the electron-phonon scattering rate in the normal state.

The density of superconducting electrons \( n_s(T) \) can be obtained from \( R_{\text{el}}(I/1)/R_R \) versus supercurrent \( I_1 \). Calculated curves for \( R_{\text{el}}(I/1)/R_R \) are shown in Fig. 5 for the SIN junction including electron-phonon scattering. The inflection point at low current has a very strong temperature dependence, in contrast to the field measurements in Fig. 3. The reason is that the pair-breaking rate for a current \( I_1 \) depends on \( n_s(T) \), whereas the pair-breaking rate due to an applied supercurrent is valid in the dirty limit and for \( I_1 \) much less than the critical current \( I_{c1} \). Since the rate on the right-hand side (rhs) of Eq. 6 is determined from the field measurements, the density \( n_s(T)/n_0(0) \) is determined from

\[
\frac{n_s(T)}{n_0(0)} = \left( \frac{2\sqrt{3}\pi a^2}{\pi n_s(T)} \right)^{\frac{1}{2}} \left( \frac{1}{R_{\text{el}}(I/1)/R_R} \right)
\]

where \( \pi/a = 1.765 \). This expression for the pair-breaking rate due to an applied supercurrent is valid in the dirty limit and for \( I_1 \) much less than the critical current \( I_{c1} \). The density of states has cancelled out, leaving only measurable parameters and constants.

IV. THEORY OF THE RESISTANCE OF SIN TUNNEL JUNCTIONS

The above discussion has shown how the general features of \( R_{\text{el}} \) can be used to obtain useful information. Of course, with real data, detailed numerical fits will yield more precise values for the various rates. The basic concepts needed for such calculations are outlined in this section, which adapts the theory of charge imbalance to SIN junctions. The principal contribution to the literature of this formulation is that it includes the proximity-effect coupling between the \( S \) and \( N \) films.

A. Background and assumptions

We consider a SIN junction in which the superconducting film is sufficiently thin, and the area dimensions of the junction sufficiently large, that quasiparticle diffusion
ensures a uniform nonequilibrium quasiparticle distribution throughout the volume of superconductor adjacent to the insulator. The relevant diffusion length is 

\[ L_D = \sqrt{D \tau_0} \]

where \( D \) is the charge-imbalance relaxation rate, \( \tau_0 = \mu / \eta \) is the diffusion constant, \( \mu \) is the Fermi velocity, and \( I \) is the electron mean-free path that is measured by the normal-state residual resistivity.

The discussion is restricted to junctions biased at very low voltages, \( |V| \ll \eta \mu D \) and \( \Delta(T) \). Heating can be neglected, since it is proportional to \( V^2 \). The nonequilibrium quasiparticle distribution in the superconducting film is a charge imbalance; that is, quasiparticles are added on one side of the Fermi surface and removed from the other side such that the total number of quasiparticles remains constant. In this case, the magnitude of \( \Delta \) is unaffected by the quasiparticle disequilibrium; only the phase of \( \Delta \) changes.

We basically follow the dirty-limit, Green's function formulation of the problem, adapting the theory to SIN junctions to properly include the proximity effect coupling between the \( S \) and \( N \) films through the insulator. Equations are cast in a form that emphasizes their relationship to the Boltzmann-equation pictures developed by Pethick and Smith. \(^{3,4} \)

The order parameter \( \Delta(T, 1/\tau_0, 1/\tau_m) \) is calculated following Makl. \(^{3} \) Since we consider only cases in which the inelastic pair-breaking rate is small compared with \( \tau_m \), we treat the inelastic rate like an elastic pair breaker in calculating \( \Delta \). For large inelastic rates, the full Eliashberg equations would be required. \(^{10} \) The density of superconducting electrons \( \rho(T, 1/\tau_0, 1/\tau_m) \) also is calculated according to Makl. \(^{3} \) with inelastic pair-breaking treated the same as elastic. The superconducting density of states \( N(E) \) and the other functions \( N_s, R_s, \) and \( R_1 \) that are needed to calculate the effective quasiparticle charge \( q(E) \) and coherence factors are calculated from Eqs. (17), (19), and (34) of Ref. 9. Checks on our computer programs are described in the Appendix.

B. Formal calculation of \( R_s(T) \)

The main purposes of this section are to show that \( R_s \) is the sum of the usual equilibrium resistance \( R_{eq}(T) \) and a nonequilibrium resistance \( R_{neq}(T) \), and to relate \( R_{eq}(T) \) to the charge-imbalance relaxation time \( \tau_0 \). We consider only the dirty limit, in which the momentum distribution of quasiparticles is isotropic, so that quasiparticle states can be labeled by energy \( E \). As usual, \(^{3,4,10,40} \) the tunnel junction is described by the tunneling Hamiltonian with a tunneling probability that is independent of energy.

The quasiparticle distribution function in the superconductor is written

\[ f_s = f_s(E) + \delta f_s, \]

where \( f_s(E) \) is the Fermi function with argument \( E/k_B T \), and \( \delta f_s \) is the nonequilibrium part of the distribution function. The quasiparticles in the normal-metal electrode are assumed to be in equilibrium. \( E \) is measured relative to the actual, nonequilibrium, chemical potential \( \mu \) in the superconductor.

There is a difference in the meaning of \( \delta f_s \) in the Green's-function and Boltzmann-equation theories, as discussed in Appendix A of Ref. 9. In the former, \( \delta f_s \) evolves into the probability of finding an electron as \( T \) increases past \( T_c \), whereas in the latter \( \delta f_s \) evolves into the probability of finding a normal excitation, i.e., an electron above the Fermi surface and a hole below. For a charge-imbalance disequilibrium, in the Green's-function theory \( \delta f_s \) is symmetric about the Fermi surface, while in the Boltzmann-equation theory \( \delta f_s \) is antisymmetric about the Fermi surface. To avoid confusion in the interpretation of relations given below, we note that the important quantity here is the charge associated with nonequilibrium excitations, i.e., the product of \( \delta f_s \) and the effective quasiparticle charge \( q(E) \). The product \( q \delta f_s \) for a charge-imbalance disequilibrium is symmetric about the Fermi surface in both theories. \( Q^* \) is the number density of electrons associated with excitations. \( Q^* \) is zero in thermal equilibrium, where \( f_s = f_s(E) \). If a disequilibrium exists, then

\[ Q^* = 4N(t) \int_0^\infty dE N_s(E) q(E) \delta f_s. \]

Extension of the limit of integration to infinity assumes that the width of the conduction band is much larger than \( k_B T_c \), since \( \delta f_s \) is substantial only within \( k_B T_c \) of the Fermi surface.

Since \( q(E) \) and \( N_s(E) \) play important roles, it is worthwhile to pause here to examine how they depend on energy and pair-breaking rates. Following the notation of Refs. 9 and 10, we define normalized elastic and inelastic pair-breaking rates \( \Gamma_r = \eta / \tau_0 A \) and \( \Gamma_m = \eta / 2\tau_m A \). The functions \( N_s, R_s, R_1, R_2 \) are calculated from Eqs. (17), (19), and (34) of Ref. 9, with the approximations \( \Gamma(t) = \Gamma_r A \) and \( \phi = \Delta \), thus neglecting strong-coupling effects on \( \Delta \) while retaining the broadening of excitation energy levels due to inelastic scattering. In the same notation, \( q(E) = (N_s - N) / N_s \), as seen from a comparison of Eq. (4.3) of Ref. 9 and Eq. (9) above. In the BCS limit of small pair breaking, \( \Gamma + \Gamma_r, \epsilon \ll 1 \), and, for \( E > 0 \),

\[ q(E) = (1 - \Delta^1)^{1/2} E, \]

\[ N_s(E) = E (1 - \Delta^1)^{1/2} E, \]

so that as \( E \to 0 \), \( q \to 0 \) and \( N_s \to \infty \) such that \( N_s(E) \approx 1 \).

Figure 6 shows \( q(E) \) and \( N_s(E) \) for different amounts of elastic pair breaking. \( E \) is normalized to the value of \( \Delta \) in the absence of pair breaking. Values of \( \Gamma_r \) and \( \Gamma_m \) are for the example junctions at \( T/T_c = 0.9 \), and for \( 1/\tau_m = 0 \) and \( 1/\tau_m = 0.9 \). Note that the gap edge is lowered by increased elastic pair breaking. For both cases illustrated, \( q(E) \) rises from zero at the gap edge in 1 at high energies, as the nature of excitations evolves from an equal mixture of electron and hole, to pure electron.

The junction resistance \( R_s(T) \) is obtained by calculating the current \( I \) that flows in the presence of a bias voltage \( V \). The sign convention used here is that when \( V \) is positive, electrons flow from \( S \) to \( N \), so that \( Q^* \) is positive. From Eqs. (4.5) and (4.6) in Ref. 10, we find

\[ I = \delta f_s(T)/R_s \to Q^* / 2N(t) e / R_s. \]
FIG. 6. Normalized density of states $N(E)$, effective quasiparticle charge $q(E)$, and the coherence factor $C_{in}(E)$ for elastic charge-imbalance relaxation processes vs $E/\Delta(0K)T_i, 1/r_i=0$. The solid curves are calculated for an external elastic pair-breaking rate of $\Delta = 0.5$, and for an internal inelastic pair-breaking rate appropriate to $\Delta = 0.7K$. They are very close to the BCS results, Eqs. (10) and (11), because the internal rate is small compared with $\Delta$. The dashed curves are calculated for an elastic pair-breaking rate $8$ times larger than the internal inelastic pair-breaking rate. The depression in $\Delta$ is calculated for $\Delta = 0.9K$.

$$n_{in}(T) = \frac{1}{2} \int_{0}^{\infty} dE N(E) \left| \frac{-\partial^2 N(E)}{\partial E^2} \right|,$$  \footnote{1}

where $R_{en}$ is the intrinsic resistance of the junction. From Eq. (12), $R_{en}/R_{th}(T)$ is the equilibrium resistance $R_{en}(T)$ of the junction, i.e., the resistance when $Q^*$ is negligible. As shown in Fig. 7 for the case of a small pair-breaking rate, $\Delta S$ is a linear function of $T$, and decreases monotonically as $T$ decreases. Equation (12) shows that the charge imbalance $Q^*$ reduces the total current. This leads ultimately to the nonequilibrium resistance $R_{en}$.

Before calculating $R_{en}(T)$, we pause to consider what voltage the voltmeter across the junction reads. Keeping track of electric and chemical potentials is tricky in detail, but the end result is simple: the voltmeter measures the electrochemical potential difference between the electrons in $N$ and the superconducting electrons in $S$.

There is a chemical-potential shift in the injected region of the superconductor because bulk charge neutrality requires that when the quasiparticle electron density increases from $0$ to $Q^*$, the condensate density decreases by the same amount. This requires a shift $\Delta \mu_1$ in the condensate chemical potential such that

$$2N(0)\Delta \mu_1 + Q^* = 0.$$  \footnote{2}

This is consistent with the voltmeter measuring the line integral of the electric field because there is an electric field in the $S$ film at the edges of the junction where the charge imbalance decays to zero. The electric field in the film is required to keep the condensate electrochemical potential constant throughout the $S$ film, as it is in steady state. The end result is that the voltmeter reads the sum of the two electric potential drops, which equals the electrochemical potential drop across the junction. This potential is larger in magnitude than the electric potential alone.

Now we can define the charge-imbalance relaxation rate $1/r_q$, and calculate $R_{en}(T)$. The total current $I$ in Eq. (15) should be thought of as the difference between a forward "equilibrium" current proportional to the electrostatic potential $\Delta \mu = Q^*/2N(0)|e|$ (with magnitude less than $|F|$, which includes the two electric potential drops across the junction, and a reverse "non-equilibrium" current proportional to the chemical potential drop $Q^*/2N(0)|e|$ across the junction, i.e.:

$$I = I_{eq} - I_{rev}.$$  \footnote{3}

$$I_{eq} = \frac{V - Q^*/2N(0)|e|}{R_{en}/R_{th}}.$$  \footnote{4}

$$I_{rev} = \frac{Q^*/2N(0)|e|}{R_{en}/R_{th}}.$$  \footnote{5}

The reverse current $I_{rev}$ represents tunneling of quasiparticle charge from $S$ to $N$ through the tunnel barrier. The physical meaning of the separation of $I_{rev}$ into two parts is that the electrostatic potential across the junction is the applied perturbation, and the chemical-potential shift, or charge imbalance, is the response of the junction to the perturbation.

Now consider the total rate $G_{en}(E)$ at which quasiparticles are generated at each energy $E$, i.e., the rate at which $N(E)|e|/\Delta^2$, is replenished by tunneling electrons. From Eq. (15), in Ref. 10, in the limit $|eV| < k_BT_i$,

$$G_{en}(E) = \left| \frac{-\partial^2 N(E)|e|}{\partial E^2} \right| e |V|/\tau_{en}.$$  \footnote{6}

where $N(E)$ is the density of states.

FIG. 7. Calculated, normalized charge-imbalance relaxation time $T_q^{-1}/\tau_{en}$ vs $T/T_i$, for a SIN junction with no electron-phonon scattering in the $S$ film. Also shown are $P^*$ and the normalized conductance $G_{ns}$. Pair-breaking effects are small for the temperature range shown.
Analogously to \( \Gamma(T) \) (Ref. RM-24), we divide \( G_{\text{eq}}(E) \) into equilibrium and nonequilibrium parts proportional to the electrostatic and chemical potentials:

\[
G_{\text{eq}}(E) = G_{\text{eq},\text{eq}}(E) - G_{\text{neq}}(E), \tag{19}
\]

The notation used here differs slightly from Ref. 13. Generation and relaxation rates here are larger than those in Ref. 13 by a factor of \( N_f \) since Ref. 13 deals with an equation for \( \delta n_f \) rather than \( N_f \).

We define the charge-imbalance generation rate, \( \frac{dQ^n}{dt} \) |\( \Gamma_{\text{eq}} \), to be the rate at which quasiparticles are generated by only the equilibrium part of \( G_{\text{eq}} \). The fraction \( \gamma \) of the "equilibrium" current of electron charge to enter the superconductor at quasiparticle charge is commonly denoted \( \gamma \), and from Eqs. (13), (16), (20), and (22),

\[
rac{dQ^n}{dt} |\Gamma_{\text{eq}} = 4N(0)| e | V - Q^n / 2N(0)| F^* / \Gamma_{\text{eq}}. \tag{22}
\]

The fraction of the "equilibrium" current of electron charge to enter the superconductor as quasiparticles is commonly denoted \( F^* \), and from Eqs. (13), (16), (20), and (22),

\[
F^* = \int_0^\infty dE N_f(E) \left| \frac{-\partial f^n}{\partial E} \right| \int_0^\infty dE N_f(E) \left| \frac{-\partial f^n}{\partial E} \right|. \tag{23}
\]

The rest of the current goes directly into the condensate \( F^* \) lies between 0 and 1, decreasing monotonically from 1 as \( T \) decreases from \( T_c \) as shown in Fig. 7. Both \( F^* \) and \( \nu_{\text{eq}} \) have been discussed previously for the case of no pair breaking. It is convenient to write the charge-imbalance generation rate in terms of \( F^* \) and \( \nu_{\text{eq}} \),

\[
\frac{dQ^n}{dt} |\Gamma_{\text{eq}} = 2N(0)| e | V - Q^n / 2N(0)| F^* \nu_{\text{eq}} / \Gamma_{\text{eq}}. \tag{24}
\]

The charge-imbalance relaxation rate \( r_{Q^n} \) is the generation rate divided by the steady-state value of \( Q^n \).

\[
r_{Q^n} = \frac{dQ^n}{dt} |\Gamma_{\text{eq}} / Q^n \tag{25}
\]

\[
r_{Q^n} = 2N(0)| e | V / Q^n | F^* \nu_{\text{eq}} / \Gamma_{\text{eq}}. \tag{26}
\]

For low voltages, \( Q^n = V \), so \( 1/r_{Q^n} \) is independent of \( V \). Finally, to relate \( \Gamma(T) \) and \( 1/r_{Q^n} \), solve Eqs. (24) and (25) for \( Q^n \) in terms of \( r_{Q^n} \), and use the result in Eq. (12).
depends on \( I_g \) for \( I_a \), close to the critical current \( I_c \). We calculate the coherence factor \( G(E) \) for the elastic charge-imbalance relaxation following Eq. (14) in Ref. 10. In the BCS limit of small pair-breaking, 2-3
\[
G(E) = \Delta^2 E^3, \tag{35}
\]
so it decreases monotonically from unity at the gap edge. Figure 6 shows that with or without pair breaking the coherence factor is unity at the gap edge and decreases smoothly as energy increases.

Simple elastic scattering contributes to charge-imbalance relaxation when a supercurrent flows because the supercurrent suppresses the equality between equal-energy excitations above and below the Fermi surface by bending the order parameter \( \Delta \) different for different directions of momentum. This makes the coherence factor nonzero, and allows simple elastic scattering to scatter excitations across the Fermi surface. In the dirty limit, the pair-breaking rate is proportional to the elastic scattering time, not the rate, because of the strong tendency of elastic scattering to make the order parameter isotropic in spite of the current.

The term \( G(E) \) represents inelastic scattering of quasiparticles from phonons. There are eight basic terms. A quasiparticle can (1) absorb a phonon and stay on the same side of the Fermi surface, or (2) cross the Fermi surface; (3) emit a phonon and stay on the same side of the Fermi surface, or (4) cross the Fermi surface; (5) recombine with a quasiparticle on the same side of the Fermi surface, or (6) with one on the other side. Also, two quasiparticles can be created by absorption of a phonon, with quasiparticles created (7) on the same side of the Fermi surface, or (8) on both sides. Processes (2), (4), (5), and (7) are especially effective in relaxing a charge imbalance, but in any of these processes, the net quasiparticle charge generally changes, depending on the initial and final quasiparticle states. For the calculations presented here, the electron-phonon coupling function, \( \alpha^2 F(\omega) \), is assumed to be proportional to \( \omega^3 \). This results from an energy-independent matrix element and a Debye density of phonon states proportional to \( \omega^3 \). These terms have been discussed in detail by several authors, and are not discussed further here.

The characteristic rate associated with electron-phonon scattering, \( 1/\tau_{\text{el-ph}}(T) \), is the electron-phonon scattering rate of a quasiparticle at the Fermi surface, \( \delta = 0 \), evaluated at \( T = T_c \).
\[
1/\tau_{\text{el-ph}}(T_c) = 1.4 \times 10^{-8} \text{sec}^2 (k_b T_c/\hbar)^3 A_R \text{g} T_c/\hbar = T_c^4, \tag{36}
\]
where \( \Gamma(1.202) \) is the Riemann zeta function. At temperatures below \( T_c \), a characteristic electron-phonon rate can be estimated from the value at \( T_c \) by using \( T_c^4 \) as a rough guess for its dependence on \( T \). In the superconducting state, there is no standard definition for the rate. A detailed numerical analysis of measurements of \( \tau_g \) will yield \( \alpha^2 F(\omega) \), which can then be compared with measurements in the dirty state.

The strategy for calculating \( R\), \( T/R \), numerically is to solve Eq. (11) for \( N, \delta_{ph}/V \), calculate \( Q^*/V \) from Eq. (9), and finally calculate \( R \) from Eq. (12).

\( R_{\text{eff}}(T)/R_N = 1/\delta_{\text{Teff}} \) is calculated by using the definition, Eq. (13), and the density of states calculated with pair-breaking effects included.

### D. Proximity effect only

In this subsection we discuss a junction in which the dominant internal charge-imbalance relaxation mechanism is the tunneling process. This limit can be realized in very low resistance junctions in which \( 1/\tau_{\text{elas}} \) is much larger than the electron-phonon scattering rate at all relevant energies and temperatures. The equation for \( \delta_{\text{T}} \) is no longer an integral equation:
\[
G_{\text{T}}(E) = G(E) = 0. \tag{37}
\]
It is worthwhile to explore this limit in detail because it is easy to solve and therefore provides a particularly clear example illustrating the basic concepts. Some results for this limit were shown in Figs. 2 and 4.

First, consider the case where there is no supercurrent. The solution for \( N, \delta_{\text{T}} \) is
\[
N, \delta_{\text{T}} = N_{\text{T}}(E) \left[ -\frac{3\omega^3}{\delta E} \right] |e|/V, \tag{38}
\]
\[
N_{\text{T}}(E) = |V|/V(k_b T)^{1/2} \text{cos}^4(E/2k_b T). \tag{39}
\]
With the definition of \( Q^* \) and the above solution for \( \delta_{\text{T}} \), we can solve for \( Q^* \) and \( 1/\tau_{\text{ph}} \) in terms of the functions \( F^* \) and \( \delta_{\text{T}} \):
\[
Q^* = 2N(0)F^* \delta_{\text{T}} |e|/V, \tag{40}
\]
\[
1/\tau_{\text{ph}} = (1/\tau_{\text{elas}}) (1 - F^* \delta_{\text{T}}). \tag{41}
\]
Figure 7 shows \( \tau_{\text{ph}}(T)/\tau_{\text{elas}} \), \( F^*(T) \), and \( \delta_{\text{T}}(T) \) in this limit where only the proximity effect contributes to charge-imbalance relaxation. At low temperatures, \( F^* \) and \( \delta_{\text{T}} \) vanish, so \( 1/\tau_{\text{ph}} \approx 1/\tau_{\text{elas}} \). Near \( T_c \), \( F^* \approx 1 - \Delta/4k_b T \) and \( \delta_{\text{T}} \approx 1 \), so \( 1/\tau_{\text{ph}} \approx 1 \approx \delta_{\text{T}} \), as expected for an inelastic pair breakage. 2-3

By combining the general result (37)–(29) for \( R_g(T) \) with Eq. (40) for \( 1/\tau_{\text{ph}} \), we find
\[
R_g(T)/R_N = 1/\delta_{\text{T}} - F^* \delta_{\text{T}} \tag{42}
\]
This result is plotted in Fig. 2. It appears to be independent of \( 1/\tau_{\text{elas}} \), but it is not because \( F^* \) and \( \delta_{\text{T}} \) depend implicitly on the pair-breaking rate. For a large pair-breaking rate, i.e., \( \delta_{\text{T}}/\tau_{\text{elas}} \) comparable to \( k_b T \) or \( \Delta \), pair-breaking effects on the density of states \( \Delta \) will change \( R_g(T)/R_N \) significantly from the values shown in Fig. 2.

Now we examine the effect of externally applied supercurrents and magnetic fields. In the presence of these elastic pair-breaking perturbations, the solution for \( N, \delta_{\text{T}} \) is simply the ratio of the generation rate and the relaxation rate:...
Figure 8 shows calculated normalized values of $N_f E F$ versus energy $E$, normalized to $A T / T_c = 0.9, 1/r_t = 0.1$, for a SIN junction with $1/r_{hp} = 0$. As $1/r_t$ increases, $N_f E F$ decreases. The largest depression is at low energies where the coherence factor and density of states are largest. The gap edge also decreases as $1/r_t$ increases.

Figure 9 shows how $R_f / R_N$ and $R_m / R_N$ are reduced by a magnetic field for the example junction with $1/r_{hp} = 0$. A comparison with Fig. 3 for a junction with a finite electron-phonon scattering rate shows that $R_f + R_T / R_N$ is much larger here, but the low-field inflection point is at about the same field for $T$ near $T_c$, where the inelastic scattering rates are about the same.

A number of effects contribute to the shapes of these curves, as emphasized by the following few figures calculated with parameters appropriate for Sn. As the field is increased, the pair-breaking rate increases, and the charge-imbalance relaxation time $r_{hp}$ decreases, Fig. 10. The order parameter $A$ decreases, Fig. 11. The fraction $F$ of current to charge the superconductor as quasiparticle charge increases, Fig. 12. At low fields, the dominant effect of the field is to reduce $r_{hp}$, so that the low-field inflection point in $R_f$ is near the inflection point in $r_{hp}$. At higher fields, the effects of pair breaking on $A, F^*$, and $r_{hp}$ become dominant, with the ultimate rise in $R_f + R_T / R_N$ near the critical field occurring as $A \rightarrow 0$.

Figure 4 plots the normalized pair-breaking rate $r_{hp}^{-1} | E (T_c) |$ at the inflection point in $A(1/rT)$ versus $r_{hp}$ as a function of $T / T_c$. The value near $T_c$ is smaller than for the junction including electron-phonon scattering because, in this limit, $R_{hp}$ is not quite proportional to $r_{hp}$. Note the very weak dependence of $r_{hp}^{-1} | E (T_c) |$ on $T$, reflecting the $T$-independent rate $1/r_{hp}$.

E. Electron-phonon scattering included

With the inclusion of electron-phonon scattering, the equation for $6_f$ becomes an integral equation. Numeri-
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FIG. 10. $r_{e1}^* / r_{em}$ vs $B_f / B_{t1}(0)$ for the SIN junction in which electron-phonon scattering is neglected. Note the similarity between the behavior of $r_{e1}^* / r_{em}$ and $R_j / R_0$ (Fig. 9), especially at low fields.

cal results were presented in Figs. 2-4 above. Here, we wish only to add a discussion of the approximations used to calculate the curves and to illustrate approximately how the effective electron-phonon scattering rate for charge-imbalance relaxation depends on energy and $T$.

In our calculation of $Q^*$ when electron-phonon scattering is included, we neglect the effects of pair breaking on the coherence factors and density of states in the electron-phonon scattering integral, but include the reduction in $\Delta$. This is valid for small external pair-breaking rates $1/\tau_s, S1/\tau_{em}$ because then the total rate is also small. For larger external rates, neglecting pair-breaking effects introduces an error in $Q^*$ and hence $R_j$. However, for large $1/\tau_s, Q^*$ is so much reduced that errors in its calculation are relatively unimportant in a calculation of $R_j / R_0$. Pair-breaking effects on $R_{em}(T)/R_0$ are important and always included.

We obtain an effective rate $1/\tau_{em}(E, T)$ by fitting Eq. (43) to values of $\delta_f$ calculated with $1/\tau_s=0$. In the fitting, the relaxation rate $r_{e1}^*$ in the denominator of Eq. (43) is replaced by

$$r_{e1}^* + (\pi \hbar / 4 k_B T) \gamma_{e1}(E, T).$$

The factor of $\hbar \pi / 4 k_B T$, represents the effective coherence factor for charge-imbalance relaxation due to electron-phonon scattering. Strictly speaking, the coherence factor should always be less than unity, but for this rough analysis, we relax that restriction a little. Figure 13 shows the effective rate as a function of $E/k_B T$ for several temperatures. The effective rate at the lowest energy, $\Delta(T)$, is roughly proportional to $T^4$. It depends slightly on energy, increasing by about 10% over the relevant energy range from $\Delta$ to $\Delta + k_B T$.

Unfortunately, when elastic pair breaking is added this simple model breaks down, and the effective electron-phonon scattering rate determined in this simple way has a complicated energy dependence. Nevertheless, it is pleasing to see the similarity between the effective rate $1/\tau_{em}(\Delta, T)$ shown in Fig. 13 and the pair-breaking rate at the inflection point in $R_j$ versus $B_f$, shown in Fig. 4.

V. CONCLUSIONS

Our main result is that the dc resistance of low-resistance SIN tunnel junctions contains information about pair-breaking electron-scattering rates, coherence factors, and the density of superconducting electrons in
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FIG. 13. Normalized effective electron-phonon scattering rate, \( r_{\text{e-ph}}(E, T)/r_{\text{e-ph}}(0, T) \), vs \( E/\Delta T \) for several temperatures. The value at the minimum energy, \( E_{\text{min}} = 0.17T \), scales roughly as \( T^{-1} \). The rate depends somewhat on energy, increasing by about 10% over the relevant energy range between \( \Delta \) and \( \Delta + 4\Delta T \).

There is a need for theoretical work in these areas as well. In disordered superconductors, the relative importance of pair breaking due to inelastic electron-electron scattering and elastic scattering in the presence of supercurrent fluctuations is unresolved. In heavy disordered systems, it is unclear how the disorder affects the nature and scattering rates of excitations. Model calculations of the resistivity of SIN junctions for the heavy-fermion systems are needed, since it would be very interesting to compare measurements on SIN junctions made with these materials with other nonequilibrium measurements.

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APPENDIX

Various parts of the computer program were checked against analytic or numerical results available in the literature.

The program used to calculate \( IQ^2 \) including electron-phonon scattering and elastic pair breaking was basically the same program used in Refs. 18 and 19. The program accurately reproduces all of the results discussed in those papers. Those results have been reproduced independently.

The program to calculate the order parameter \( \Delta \) in the presence of pair breaking was based on Ref. 32, and it was checked against plotted results in that paper. The program to calculate the density of superconducting electrons \( n_{\text{e}} \) in the dirty limit in the presence of pair breaking was an integral part of the program to calculate \( \Delta \). The results were checked by calculating the critical current density \( J_c(T) \) from the maximum value of \( J_c = n_{\text{e}}(T, 1/r) \), vs as a function of \( 1/r = Dp^2/2k^2 \), where \( p^2 = 2m^* \). The results were checked against the usual Ginzburg-Landau result near \( T_c \), and against numerical results in Ref. 36 at lower temperatures.

The program to calculate the density of states \( N_s(E) \), the effective quasiparticle charge \( g(E) \), and the coherence factor \( C_s(E) \) for elastic charge-imbalance relaxation, was based on Ref. 10. The functions \( N_s(E) \) and \( N_s(E) \) were checked against plotted results in Ref. 9. The functions \( N_s, N_2, R_3 \), and \( R_5 \) all reduced to the appropriate analytic forms given in Ref. 10 in the limit of zero pair breaking. In addition, calculated values of \( N_s \) for elastic pair breaking only were checked against plotted results in Ref. 32.

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15) For several reviews and further references, see Non-equilibrium Superconductivity, Phonons and Kapitza Boundaries, Proceedings of the NATO Advanced Study Institute, edited by K. E. Gray (Plenum, New York, 1981).


