INFORMATION TO USERS

The most advanced technology has been used to photograph and reproduce this manuscript from the microfilm master. UMI films the original text directly from the copy submitted. Thus, some dissertation copies are in typewriter face, while others may be from a computer printer.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyrighted material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each oversize page is available as one exposure on a standard 35 mm slide or as a 17" × 23" black and white photographic print for an additional charge.

Photographs included in the original manuscript have been reproduced xerographically in this copy. 35 mm slides or 6" × 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.
Motion planning for legged locomotion systems on uneven terrain

Vijaykumar, Ramakrishnan, Ph.D.
The Ohio State University, 1987
PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark √.

1. Glossy photographs or pages ______
2. Colored illustrations, paper or print ______
3. Photographs with dark background ______
4. Illustrations are poor copy ______
5. Pages with black marks, not original copy ______
6. Print shows through as there is text on both sides of page ______
7. Indistinct, broken or small print on several pages √
8. Print exceeds margin requirements ______
9. Tightly bound copy with print lost in spine ______
10. Computer printout pages with indistinct print ______
11. Page(s) __________ lacking when material received, and not available from school or author.
12. Page(s) __________ seem to be missing in numbering only as text follows.
13. Two pages numbered ______. Text follows.
14. Curling and wrinkled pages √
15. Dissertation contains pages with print at a slant, filmed as received ______
16. Other ________________________________________________________________

_____________________________________________________________________

U-M-I
MOTION PLANNING FOR LEGGED LOCOMOTION
SYSTEMS ON UNEVEN TERRAIN

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of The Ohio State University

By

R. Vijaykumar, B.Tech., M.Sc.

The Ohio State University
1987

Dissertation Committee:

Dr. K.J. Waldron
Dr. K. Srinivasan
Dr. G.L. Kinzel
Dr. H.R. Busby

Approved by:

Dr. K.J. Waldron - Adviser
Department of Mechanical Engineering
I would like to express my sincere appreciation to Dr. K.J. Waldron for his support and guidance throughout this research. I am especially grateful for the confidence he showed in me and for the freedom and support he gave me to pursue my own interests. I would like to acknowledge Dr. K. Srinivasan, Dr. G.L. Kinzel, Dr. H.R. Busby and Dr. W. Ogden for their criticism and suggestions which helped improve the readability of this thesis. I wish to thank the staff of the Advanced Design Methods Laboratory for providing the much needed computing facilities throughout my doctoral studies. Finally, my deepest thanks go to my family, friends and colleagues whose complete support made this project possible.

The financial support from Defense Advanced Research Projects Agency (DAAE 07-84-K-R001) and the Presidential Fellowship from The Ohio State University is gratefully acknowledged.
VITA

April 12, 1962 Born - Patna, India


March 1985 M.S., Mechanical Engineering, The Ohio State University, Columbus, Ohio.

1983-1987 Graduate Research Associate, The Ohio State University, Columbus, Ohio.

PUBLICATIONS


iii
FIELDS OF STUDY

Major Field: Mechanical Engineering

Studies in Kinematics: Professors K.H. Hunt, K.J. Waldron and G.L. Kinzel

Studies in Mathematics: Professor T. Scheick

Studies in Dynamics and Vibrations: Professors R. Singh, D. Unger

Studies in Controls and System Dynamics: Professors E.O. Doebelin and K. Srinivasan

Studies in Robotics: Professors K.J. Waldron and D.E. Orin
TABLE OF CONTENTS

ACKNOWLEDGEMENTS .............................................. ii
VITA ........................................................................ iii
LIST OF TABLES ...................................................... viii
LIST OF FIGURES ..................................................... ix

Chapter

I. INTRODUCTION .................................................. 1
  1.1 Mobile Robots ............................................... 1
  1.2 Machine Intelligence in Mobile Robots ................. 2
  1.3 The Advantages of Legged Locomotion Systems ...... 3
  1.4 Problem Definition and Scope ............................ 5

II. MOBILE ROBOTICS - A SURVEY OF PAST RESEARCH EFFORTS ..... 14
  2.1 Wheeled Robots ............................................. 14
  2.2 Legged Locomotion Systems .............................. 20
  2.3 The Structure of Machine Intelligence ................. 26
  2.4 The Adaptive Suspension Vehicle ....................... 32

III. GAIT ANALYSIS FOR WALKING MACHINES ................. 37
  3.1 Introduction ................................................ 37
  3.2 History ..................................................... 37
    3.2.1 Natural Systems: .................................... 37
    3.2.2 Theory: ................................................ 39
  3.3 Stability in Locomotion .................................. 41
  3.4 Introduction to Gait Theory: ............................ 43
  3.5 Omnidirectional Walking on Uneven Terrain -
    Definitions .................................................. 52
  3.6 On the Stability and Stability Margins of Gaits ...... 60
    3.6.1 General ................................................. 60
    3.6.2 Longitudinal Stability Margin and Symmetry in
        Gaits: ..................................................... 61
    3.6.3 Wave Gaits: ........................................... 75
    3.6.4 Comparison of the Stability Margin and
        Longitudinal Stability Margin for Crab Wave
        Gaits ..................................................... 85
    3.6.5 Optimization of Crab Gaits: ........................ 88
**Table of Contents (continued)**

3.6.6 Follow-the-Leader Gaits .................................................. 103
3.6.7 Effect of an Arbitrary Load-Wrench on Locomotion on Smooth Terrain .................................................. 106
3.7 Gait Selection and Optimization for a Walking Machine .................................................. 112
   3.7.1 Gait Selection .................................................. 112
   3.7.2 Criteria for Gait Optimization .................................................. 115
3.8 Conclusion .................................................. 120

IV. FORCE DISTRIBUTION IN CLOSED KINEMATIC CHAINS .................................................. 121
   4.1 General .................................................. 122
   4.2 The Force Distribution Problem .................................................. 124
      4.2.1 Governing Equations: .................................................. 124
      4.2.2 Optimization .................................................. 129
      4.2.3 The Pseudo-Inverse: .................................................. 131
   4.3 Decomposition of the Force Field .................................................. 133
      4.3.1 Definitions .................................................. 133
      4.3.2 The Pseudo-Inverse .................................................. 134
      4.3.3 The Equilibrating Force Field .................................................. 139
      4.3.4 The Interaction Force Field .................................................. 150
   4.4 Decomposition of the Load-Wrench .................................................. 163
   4.5 Variable Compliance .................................................. 168
   4.6 Linear Programming .................................................. 173
   4.7 Comparative Study of Different Methods for Walking Machines .................................................. 178
      4.7.1 Introduction .................................................. 178
      4.7.2 Simulation .................................................. 182
      4.7.3 Results .................................................. 183
   4.8 Concluding Remarks .................................................. 198

V. ADAPTIVE GAIT CONTROL AND BODY MOTION REGULATION .................................................. 200
   5.1 Introduction .................................................. 200
   5.2 Kinematics of the Legged System .................................................. 200
   5.3 Control of Gait Parameters .................................................. 203
      5.3.1 Definitions .................................................. 203
      5.3.2 Changing Time Period with a Constant Duty Factor at Constant Velocity .................................................. 209
      5.3.3 Changing Velocity with a Constant Duty Factor .................................................. 209
      5.3.4 Gait Transitions at Constant Velocity with Constant Stroke .................................................. 210
      5.3.5 Maintaining a Constant Time Period and Velocity with a Changing Stroke .................................................. 213
      5.3.6 Controlling the Time Period and Stroke to Maintain a Constant Leg Velocity .................................................. 213
      5.3.7 Change of Gait at a Constant Velocity .................................................. 216

vi
Table of Contents (continued)

5.3.8 Effect of Gait Transitions on Stability ........ 217
5.3.9 Conclusion ........................................ 219
5.4 Foothold Selection ................................... 220
5.5 Automatic Body Motion Regulation ....................... 228
5.6 Conclusion ........................................ 240

VI. A SIMULATION OF THE GUIDANCE MODULE ............. 242
6.1 Introduction ........................................ 242
6.2 Simulation Model .................................... 244
6.3 Foothold Selection .................................. 250
6.4 Gait Control .......................................... 254
6.5 Vehicle Body Dynamics and Control .................... 261
6.6 The Pilot .............................................. 266
6.7 Examples .............................................. 268
6.8 Conclusion ............................................ 302

VII. CONCLUDING REMARKS .................................. 304
7.1 Conclusions .......................................... 304
7.2 Other Research Issues .................................. 305

REFERENCES ............................................... 308

vii
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Placing and Lifting Events in a Locomotion Cycle</td>
<td>63</td>
</tr>
<tr>
<td>3.2</td>
<td>Critical Time Instants in a Locomotion Cycle</td>
<td>65</td>
</tr>
<tr>
<td>4.1</td>
<td>Three Fingerted Grasp for a Hex-Nut</td>
<td>159</td>
</tr>
<tr>
<td>4.2</td>
<td>Computational Time for the Proposed Algorithms</td>
<td>184</td>
</tr>
<tr>
<td>4.3</td>
<td>Force Distribution Generated by the Decomposition of the Load Wrench</td>
<td>185</td>
</tr>
<tr>
<td>5.1</td>
<td>Evaluating Stability of a Foothold</td>
<td>225</td>
</tr>
<tr>
<td>6.1</td>
<td>Dimensions of the Model of the ASV Used in the Simulation</td>
<td>225</td>
</tr>
<tr>
<td>6.2</td>
<td>Constants Used in the Simulation</td>
<td>269</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The Hierarchical Structure of Intelligence in Mobile Robots</td>
<td>7</td>
</tr>
<tr>
<td>1.2</td>
<td>The Guidance Module in Legged Locomotion</td>
<td>10</td>
</tr>
<tr>
<td>2.1</td>
<td>Supervisory Control in Legged Locomotion [75]</td>
<td>23</td>
</tr>
<tr>
<td>2.2</td>
<td>The Adaptive Suspension Vehicle [98]</td>
<td>33</td>
</tr>
<tr>
<td>3.1</td>
<td>Plan View of a Six-Legged Locomotion System</td>
<td>44</td>
</tr>
<tr>
<td>3.2</td>
<td>Measures of Static Stability</td>
<td>47</td>
</tr>
<tr>
<td>3.3</td>
<td>Gait Diagram for Wave Gaits</td>
<td>53</td>
</tr>
<tr>
<td>3.4</td>
<td>Coordinate Systems for a Legged Vehicle</td>
<td>56</td>
</tr>
<tr>
<td>3.5</td>
<td>Quasi-static Stability</td>
<td>59</td>
</tr>
<tr>
<td>3.6</td>
<td>A Support Pattern on the $\sigma$-plane</td>
<td>62</td>
</tr>
<tr>
<td>3.7</td>
<td>A Regular Gait with a Constant Phase Increment on Each Side</td>
<td>69</td>
</tr>
<tr>
<td>3.8</td>
<td>Plan View for a Crab Gait with a Crab Angle, $\alpha$</td>
<td>72</td>
</tr>
<tr>
<td>3.9</td>
<td>Optimal $\phi$ for a Wave Gait</td>
<td>80</td>
</tr>
<tr>
<td>3.10</td>
<td>Isostability Lines for a Wave Gait with Stroke Equal to Pitch</td>
<td>84</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>3.11</td>
<td>Comparison of the Stability Margins for Crab Wave Gaits; (a) Stability Margin (b) Longitudinal Stability Margin</td>
<td>86</td>
</tr>
<tr>
<td>3.12</td>
<td>Stability Contour Plots as a Function of Phase Differences</td>
<td>94</td>
</tr>
<tr>
<td>3.13</td>
<td>Stability of Optimized Crab Wave Gaits for Small Crab Angles; ( \eta = 1.0, \gamma = 1.0 )</td>
<td>102</td>
</tr>
<tr>
<td>3.14</td>
<td>Stability of Optimized Crab Wave Gaits for Small Crab Angles; ( \eta = 1.2, \gamma = 1.0 )</td>
<td>104</td>
</tr>
<tr>
<td>3.15</td>
<td>Motion Along the Longitudinal Axis of the Vehicle Body on Smooth Terrain with an Arbitrary Load Wrench</td>
<td>107</td>
</tr>
<tr>
<td>3.16</td>
<td>Variation of Stability with the Stroke to Pitch Ratio</td>
<td>117</td>
</tr>
<tr>
<td>4.1</td>
<td>The Force Distribution Problem; (a) Legged Vehicle (b) Multifingered Gripper</td>
<td>128</td>
</tr>
<tr>
<td>4.2</td>
<td>Interaction Force Field for a Two Fingered Grasp</td>
<td>152</td>
</tr>
<tr>
<td>4.3</td>
<td>Interaction Force Field for a Three Fingered Grasp</td>
<td>154</td>
</tr>
<tr>
<td>4.4</td>
<td>Interaction Forces for a Three Fingered Grasp (a) Positive Interaction Forces (b) Negative Interaction Forces</td>
<td>156</td>
</tr>
<tr>
<td>Figure</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.5</td>
<td>A Three Fingered Grasp for a Hex-Nut</td>
<td>158</td>
</tr>
<tr>
<td>4.6</td>
<td>Decomposition of the Load-Wrench and the Force Field</td>
<td>164</td>
</tr>
<tr>
<td>4.7</td>
<td>Wire Frame Simulation Model of the ASV and the Rough Terrain</td>
<td>179</td>
</tr>
<tr>
<td>4.8</td>
<td>The Constrained Pseudo Inverse</td>
<td>180</td>
</tr>
<tr>
<td>4.9</td>
<td>Force Distribution for the ASV (Test Case) - Friction Angles and Magnitudes of Forces</td>
<td>189</td>
</tr>
<tr>
<td>5.1</td>
<td>Control of Gait Parameters by the Guidance Module</td>
<td>204</td>
</tr>
<tr>
<td>5.2</td>
<td>Sequence of Leg States</td>
<td>206</td>
</tr>
<tr>
<td>5.3</td>
<td>A Gait Transition in a Perfect Wave Gait</td>
<td>212</td>
</tr>
<tr>
<td>5.4</td>
<td>A Gait Transition in a Wave Gait Involving a Change in Phase Angles</td>
<td>218</td>
</tr>
<tr>
<td>5.5</td>
<td>Predicting Stability for Evaluating Stability Characteristics of Footholds</td>
<td>224</td>
</tr>
<tr>
<td>5.6</td>
<td>Locomotion on Sloping Smooth Terrain</td>
<td>230</td>
</tr>
<tr>
<td>5.7</td>
<td>Body Attitude Control - One Degree of Freedom</td>
<td>231</td>
</tr>
<tr>
<td>5.8</td>
<td>Body Attitude Control - Two Degrees of Freedom</td>
<td>233</td>
</tr>
<tr>
<td>5.9</td>
<td>Yaw Velocity and Crab Angle</td>
<td>236</td>
</tr>
<tr>
<td>5.10</td>
<td>Altitude Control</td>
<td>237</td>
</tr>
</tbody>
</table>
List of Figures (continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>A Detailed Outline of the Guidance Module</td>
<td>243</td>
</tr>
<tr>
<td>6.2</td>
<td>Model of the ASV Used for Simulation</td>
<td>245</td>
</tr>
<tr>
<td>6.3</td>
<td>Workspaces of the Legs for the Model</td>
<td>248</td>
</tr>
<tr>
<td>6.4</td>
<td>Gait Controller in the Pilot Module</td>
<td>255</td>
</tr>
<tr>
<td>6.5</td>
<td>Flowchart for Updating Status of the Legs</td>
<td>260</td>
</tr>
<tr>
<td>6.6</td>
<td>(a) Velocity Control of the Vehicle</td>
<td>262</td>
</tr>
<tr>
<td></td>
<td>(b) Vehicle Body Kinematics and Dynamics</td>
<td>263</td>
</tr>
<tr>
<td>6.7</td>
<td>The Pilot Operation Simulation</td>
<td>265</td>
</tr>
<tr>
<td>6.8</td>
<td>Gait Control on Even Terrain</td>
<td>272</td>
</tr>
<tr>
<td>6.9</td>
<td>Climbing a 3 Foot Step</td>
<td>275</td>
</tr>
<tr>
<td>6.10</td>
<td>Stepping Down a 3 Foot Step</td>
<td>280</td>
</tr>
<tr>
<td>6.11</td>
<td>Crossing a 4 Foot Step</td>
<td>286</td>
</tr>
<tr>
<td>6.12</td>
<td>Rough Terrain Locomotion</td>
<td>291</td>
</tr>
<tr>
<td>6.13</td>
<td>Foothold Selection on Uneven Terrain</td>
<td>297</td>
</tr>
</tbody>
</table>
CHAPTER I
INTRODUCTION

1.1 Mobile Robots

Many robotic applications today require mobility. However, most present day robots are stationary or, have at most, limited motion along guideways in one or two directions. This is because, until recently, it was not economical or practical to have mobile robots. With recent excursions in robotics and allied sciences (especially the development of relatively inexpensive digital computers), mobile robots have become feasible and very attractive prospects. Researchers have realized the enormous potential of having mobility in robots performing repetitive tasks in industrial environments and employing mobile robotic systems for locomotion and transportation in hostile surroundings.

Over the past couple of decades, the focus in mobile robotics has gradually shifted to tasks which require a higher degree of cognitive or reasoning capability for locomotion. Space exploration is one example, in which the robot must exhibit intelligent behavior to adapt to an unknown unstructured environment. Construction and maintenance work in dangerous surroundings presents another application for intelligent mobile robots. The increased emphasis on autonomous mobile systems over the last few decades has made machine intelligence, as
applied to locomotion systems, a very active area of research.

1.2 Machine Intelligence in Mobile Robots

Machine Intelligence is an integral part of any robot and more so for mobile systems. It is the ability of a machine to recognize a set of alternatives in a real world situation and to intelligently choose an alternative without any human interference. The set of choices are a consequence of the redundancy of the robot-world system. Redundancy is used to describe a situation in which the number of input sensory variables (sensory information) is greater than the minimum number of inputs required by the control system or where the machine possesses a greater number of actuated degrees of freedom than is necessary for motion. In the context of locomotion, there is more than one way of synthesizing a desired motion, and there is more than one path to a prescribed goal or destination. An intelligent mobile robot must be able to sense and perceive the environment and must have the intelligence that enables it to cope with constantly emerging new situations even as it moves. The key element in such a system is the ability to resolve the redundancy in each situation by optimizing the operating parameters based on criteria such as energy consumption, efficiency of energetics, time or even suboptimal heuristics derived from an engineer's intuition or experience.
1.3 The Advantages of Legged Locomotion Systems

Over 50% of the land in the world is not negotiable by tracked vehicles. This is because conventional vehicles are woefully ill-equipped to deal with uneven terrain. In fact, even on relatively flat terrain most existing vehicles need a prepared surface for locomotion. This is not to say that the wheeled vehicle is of little use. It has a relatively simple mechanical construction with one moving part and, more important, the pneumatic tire or tracks can adapt automatically to obstacles less than the tire/track thickness. The wheel is, no doubt, an optimum system on prepared roads and is comparable in speed to, if not better than, most legged animals on roads. However, wheeled vehicles are unable to traverse terrains with even small, unimpressive obstacles. Also they have a preferred direction of motion as compared to legged systems which severely constrains their maneuverability.

Legged locomotion systems [2] can step over obstacles and walk on rough terrain as they can judiciously choose support points on the ground --- the locomotion elements (feet) do not have to be in continuous contact with the ground. They are also superior to wheeled or tracked vehicles when going up soft slopes. Legged systems tend to pile the surface material behind their feet, thus decreasing the effective slope and increasing traction. Wheels cause the surface material to be piled up in front of the wheel and this causes the effective slope to increase and impedes motion. On hard surfaces too, all driving elements can interact with the ground (which may not be
possible in wheeled or tracked vehicles) thus maximizing the tractive force in a legged vehicle.

Another advantage of legged vehicles is their efficiency. Soil compaction is the primary source of motion resistance at low speeds. Wheeled or tracked systems have all their support elements in contact with the ground at all times. Walking on the other hand, is a discrete process as the body samples the ground at discrete intervals in space and time which tends to be more efficient. Also the terrain sampling nature of the walking process, and the ability of the legs to extend and contract give the body an active independent suspension. The active adaptive suspension could eventually result in walking machines surpassing wheeled vehicles even on prepared surfaces.

There has, in the past two decades, been a sudden interest in walking machines. This could be attributed to the advent of digital computers and the realization that complex tasks can be performed by digitally controlled mechanisms. The walking machine is vastly more complicated than the conventional vehicle. This is evident from the fact that there are over 40 million gaits (walking sequences) possible for a six legged machine. In addition the problem of static and dynamic balancing of the vehicle is a formidable one --- this problem is virtually non-existent in present day land vehicles. Also, in an autonomous system, the vision system in a walking robot needs to preview terrain in the lateral as well as the longitudinal directions. Further, the required accuracy in the terrain elevation information is
greater, since even small holes could paralyze a legged vehicle. The vision system required for a autonomous wheeled robot is relatively simple and wheeled vehicles have years of research, design and experimentation behind them. However, considerable research in mobile legged systems is now in evidence [27], [64], [73], [75], [81], [82], [98]. Much of this work is concentrated on computer control of the legs, the body attitude, sensors and actuators. Energy efficiency has been emphasized in the design of legs and actuators. Efforts are also concentrated on range sensing and vision, and new light weight, high strength materials. In order to emulate the superior off-road mobility of biological systems it is important to make the walking machine an autonomous and intelligent vehicle in which man’s role as an operator is minimal.

1.4 Problem Definition and Scope

A walking robot is advantageous as a mobility system in many ways. However, as discussed in Section 1.3, without intelligent algorithms to coordinate the legs and control the gait, and the ability to plan paths for the vehicle in unstructured terrain, such an active mobility system is a burden on the human operator. This dissertation explores the possibility of establishing intelligence in the control structure of legged robots with the objective of partially automating the planning process on uneven terrain. An anthropomorphic, hierarchical intelligence structure with four levels has been adopted.
They are the planner, navigator, pilot and the controller [62] and the relationship between these units is shown in Figure 1.1. As we go down the hierarchy, there is a hierarchical decomposition of the task or goal into subgoals and a decrease in the generality and scope of the planning process. The planner is at the highest level and is involved with the task specification and the lowest level or the controller is more of an execution module than a planning module which interacts with the actuators, sensors and the servo loops controlling the actuators. The navigator and pilot represent intermediate and lower levels of intelligence. The navigator is responsible for planning a path for a typical range of ten body lengths with the objective of achieving the goal specified by the planner. The pilot is concerned with the immediate surroundings extending typically to a couple of body lengths. The controller is the only level which is reasonably well understood and extensively reported. The present research effort is aimed at the development of motion planning strategies for the immediately superior level, the pilot, which incorporates the lower level intelligence in locomotion.

The pilot or the guidance module sends commands to and monitors the functions of the controller. The sensory data available to the walking machine is used to construct a geometric model of the surrounding environment and the terrain-vehicle system. The guidance system utilizes this model to plan ahead the motion of the legged robot for 1-2 body lengths. It enables the vehicle to follow the
Figure 1.1: The Hierarchical Structure of Intelligence in Mobile Robots
commands of the immediately superior level, the navigator (or the human operator), as closely as possible without endangering the vehicle. The relative importance of mission completion versus vehicle survival have to be considered by the system.

The objectives of this research endeavour are:

- to identify and understand the functions of the pilot or guidance module for legged locomotion on uneven terrain.
- to develop strategies for motion planning at the guidance level based on a geometric model of the terrain derived from terrain elevation information.
- to study the problem of force distribution in closed kinematic chains and to develop efficient algorithms for computing foot forces required to maintain a walking machine in equilibrium.

The functions at this level include local path selection, selection of footholds in space and time, gait control and optimization, body posture regulation, specification of body accelerations and determination of foot forces required to achieve the specified accelerations. A rough outline of the pilot operation is shown in Figure 1.2. Each of the pilot's functions is discussed briefly below.

As illustrated in Figure 1.1, the range of the model of the environment available to the pilot is lower than that available to the navigator. However, the local map at the guidance level has a greater
resolution than the global map on which the path selection process is based. The local map may be expected to reveal terrain features and even obstacles which cannot be inferred from the lower resolution global map. The local path modification scheme in the pilot operation must be designed to modify, if needed, the planned path in order to circumvent such obstacles.

The guidance module must select appropriate footholds for support. This is especially critical on uneven terrain and unstructured environments. Suitable criteria and algorithms for selecting footholds on rough terrain are described and evaluated here. Irregular distribution of footholds (in space) engender unstructured support patterns and the vehicle must be able to adapt its gait to such deviations from normal conditions. This function is called Adaptive Gait Control and is an essential component of the guidance software. In a situation in which the terrain elevation does not change significantly, thus enabling the vehicle to reach a steady state, it is possible to optimize the gait to suit the operating conditions and the environment. Thus gait optimization is another important function of the guidance module.

Another key issue is the regulation of the body attitude and altitude to suit the terrain, speed and desired goal. The desired longitudinal and lateral velocity of the vehicle may be expected to be specified at the path selection level, but the other four components of velocities must be automatically specified and controlled. This
Figure 1.2: The Guidance Module in Legged Locomotion
function of automatic body posture regulation is also within the scope of the guidance module.

In traditional vehicle systems, coordination of the locomotion elements is done mechanically and is of a passive nature. Digital control and robotics offer the alternative of active coordination which permits optimization of locomotion. In actively coordinated legged systems, the foot forces required to achieve a desired acceleration must be obtained. The problem of computing the desired foot forces is statically indeterminate. Therefore, fast and efficient algorithms are required to obtain an optimal distribution of forces. In uneven terrain, the stability of the vehicle can be inferred from the force distribution. Thus efficient force distribution algorithms must be used to estimate vehicle stability at every planned configuration. Similar algorithms must be used by the controller to specify force set points for the actuators in a computed torque control scheme.

The proposed scheme of control will depend on a geometric description of the terrain-vehicle system and finite-state sequential logic (to describe the stepping sequence) to arrive at suitable decisions for the generation of footholds and body control. This is in opposition to the possibility of symbolic descriptions of the task, vehicle-terrain interaction and control of the task which lends itself to the implementation of precepts based on artificial intelligence. It is believed that numeric rather than symbolic control with the
selection of optimality criteria and good heuristics is more suited to
the lower form of intelligence at this level of motion planning. On
the other hand, the higher levels in the control hierarchy may require
more powerful techniques for interpreting the terrain, navigation and
vehicle control.

The function of path selection must be used at different levels
in the hierarchy on different models of the terrain. The path
selection process at the navigator level requires a low resolution,
long range model and the guidance module needs a high resolution,
short range model which will be able to discover obstacles which were
obscured by the lower resolution at the superior (navigator) level.
Thus path selection at the guidance level is really a local path
modification scheme. Details of path selection schemes can be found
extensively in published literature [7], [8], [12], [18], [23], [36],
[41], [63], [80], [92], [104]. Discussion on local path modification
is confined to a critical review of the existing literature on the
subject.

The organization of this dissertation is briefly outlined below.
The next chapter is a brief review of the research efforts on
intelligent mobility systems. A critical analysis of the published
literature in light of the anthropomorphic intelligence structure is
presented. Of particular interest is the Adaptive Suspension Vehicle
(ASV) [98], a six-legged vehicle being built at The Ohio State
University, which is expected to serve as a test bed for the ideas
proposed in this dissertation. An analysis of gaits which can be used for omnidirectional locomotion on uneven terrain is presented in Chapter 3. The issue of selecting a versatile, efficient and simple gait for this purpose has been discussed. Measures of gait stability have been used as a basis for gait optimization. Chapter 4 addresses the problem of force distribution between the legs of the walking machine. This problem is characteristic of all closed loop kinematic chains, in general, and has been investigated in some detail, digressing in some instances from the subject of legged locomotion. The issues of adaptive gait control and automatic body motion regulation are described in Chapter 5. A computer simulation of the proposed motion planning algorithms has been performed for a simplified model of the Adaptive Suspension Vehicle. The results of the simulation yield some insight into the problem of motion planning. A detailed discussion of these results is presented in Chapter 6. Chapter 7 is a concluding chapter which summarizes this research effort and significant results described in this dissertation. Several other peripheral issues which are outside the scope of this dissertation but merit further research are briefly discussed.
CHAPTER II
MOBILE ROBOTICS - A SURVEY OF PAST RESEARCH EFFORTS

2.1 Wheeled Robots

The first intelligent mobile system dates back to 1968. SHAKEY was an integrated mobile wheeled robotic system built at the Stanford Research Institute. It is considered to be a pioneering effort in Artificial Intelligence (AI). It had a TV camera, an optical range-finder and several proximity sensors. STRIPS was a widely acclaimed decision-making system developed for this project. First order predicate calculus was used to represent the world model and the A* algorithm [23], [69] for path finding using search depth as the cost.

In the early 1970’s, JASON was built at the University of Berkeley [8]. It was a low cost experimental system designed to solve classical AI problems. JASON was equipped with an ultrasonic torch and a proximity sensor for perception but was operated with remote control and hence did not really possess an intelligent locomotion subsystem. An important feature of this work was the use of decision analysis in coping with the uncertainty of sensory information.
The JPL Mars Rover was built (1975-79) with an objective of planetary and space exploration [93]. It had a laser range finder and two parallel solid state cameras. It represented an effort to integrate, for the first time, the subsystems of locomotion, manipulation, vision (sensation) and perception. The software for the locomotion subsystem runs as three concurrent processes: the navigation execution module (NEX), the path planner module (PPM) and the vehicle guidance module (VGM). The NEX invokes the PPM and the VGM and responds to their requests. It communicates with the other subsystems through the robot executive module (REX).

The ANU robot was built at the Australian National University [18]. It had an ultrasonic range finder and a color TV camera but does not seem to have had an intelligent control system. It was built chiefly for vision research.

The effort of the Department of Bioengineering of the Academy of Sciences in USSR is worth mentioning. Unfortunately very little is known about the details of their devices. Three levels of intelligence were identified for locomotion: determining the maneuver, decomposing the maneuver and performance of the elementary actions [62].

The Guide Dog Robot at MITI's Mechanical Engineering Laboratories in Japan is an autonomous device to enhance mobility aids for the blind [92]. It used organized maps and landmarks for navigation. It also had some obstacle detection. It had an excellent speed and direction control. The MEL DEIC was another mid-sized robot with two
on-board cameras and ultrasonic ranging. Object identification based on edge detection and subsequent classification of objects were the basis of decision-making.

The Stanford AI Lab cart was a remotely controlled TV equipped mobile robot [63]. Navigation through cluttered space was demonstrated using the cart. The perception was based on TV camera pictures obtained from a camera mounted on a horizontal slide. Nine views of the same scene provided a redundant database for reliable inferences amidst noise. The path selection was designed to avoid detected obstacles - the path would change as new obstacles were perceived from a new location. The path consisted of a series of tangential segments between cylinders representing obstacles and contacting arcs. The progress of the cart was very slow. Further work on this system was done at the Robotics Institute at Carnegie Mellon University. Currently, research at CMU includes work on REX, an experimental robotic excavator and TERRAGATOR, NEPTUNE, and NAVLAB are autonomous outdoor vehicles [19], [102]. Recent research at Stanford includes the autonomous mobile robot project, which involves research on vehicle navigation in unstructured, indoor environments. The Stanford Mobile Robot [42], a three-wheeled robot, has been used for demonstrating the planning, sensing and locomotion subsystems.

In France, researchers have been very active in this area. In 1981, VESA was built at Laboratoire d'Application des Techniques Avancees in Rennes [18]. It possessed a laser triangulation system to
assist the path selection process. Obstacle avoidance was enhanced by information provided through retractable touch sensors. HILARE was a three wheeled robot which was built in Laas [17]. It had an ultrasonic ranger and a vision system consisting of a video camera and a laser range finder. The decision making system was composed of "special decision modules" (SDM's) each of which was an independent rule-based system with its own hierarchical structure. These SDM's include a general planner, a navigation module and a module for scene analysis (and pattern recognition).

The University of Warwick, in England, has done work on automated trucks which has been briefly described in Reference [15]. They have sonars and carbon impregnated bumpers which change resistance under stress. This is used for tactile sensing to detect collisions for a "groping" strategy in navigation. Accoustic ranging systems for obstacle detection have also been developed at the General Motors Research Laboratories (GMRL) for the GMR Auto Nav-2 [103] and at CMU for the NEPTUNE.

Hermies-II is a self-powered mobile robot built at Oak Ridge National Laboratory with wheels driven by independent D.C. motors with two 5 degree-of-freedom manipulator arms [104]. It is equipped with a ring of five sonar sensing elements which are rotated to scan a 180 degree field of view in seven seconds. It also uses an expert system in real-time for navigation.
In the industry, FMC has developed an autonomous vehicle navigation system capable of road following, route planning and obstacle avoidance for a computer controller M113 armored carrier [43]. ALVIN, the eight wheel drive, all-terrain, Autonomous Land Vehicle at Martin Marietta Denver Aerospace has demonstrated abilities to steer around obstacles during road following using an RGB video camera and a laser range scanner. These programs are representative of navigation research in partially structured environments.

In Japan, research efforts in fork lift systems and operatorless conveyor trucks have proved that it is possible to build intelligent and autonomous mobile systems which function in complex industrial environments. Komatsu, Ltd. and Murata Machinery are two firms to have made inroads into the relatively unknown field of autonomous robots. Barret Electronics' Unicar and Eaton-Kenway's Robocarrier systems are examples of similar systems in the U.S. Typically, systems like these use wire guide paths for communication and guidance of a number of unmanned robot vehicles. Such systems are called autonomous guided vehicles (AGV) [14] and are controlled by a central monitoring processor. Most industrial research work in this area tends to concern Flexible Manufacturing System (FMS) units and the number of such systems is steadily increasing.

A anthropomorphic hierarchical structure of machine intelligence is increasingly being accepted for mobile robots. A three level control hierarchy consisting of the Planner, the Navigator and the
Pilot has been outlined in References [41], [62]. In this system, the Cartographer receives information from the sensors and maintains maps of the world at the three different levels (see Figure 1.1). The issues of map updating (as the vehicle moves) and navigation are also discussed by the authors. The acceleration of the navigation planning with learning or the acquisition of additional sensory information with locomotion has also been studied in Reference [35]. However systems of this kind are still at the proof-of-the-concept stage.

At present, although many autonomous mobile robots exist there are very few intelligent systems. They are either teleoperated or have a preprogrammed, inflexible strategy with little ability for making decisions. Typical examples are the existing autonomous guided vehicles. These machines may seem autonomous but are definitely not intelligent in the sense we are interested in. Another feature about existing (or past) intelligent mobile systems is that they have operated only in two dimensions and in relatively structured environments. Eventually the need for research on interaction with three dimensional surroundings and obstacles has to be recognized. Research has been largely confined to the navigator operation and the path selection problem. More about this is presented in Section 2.3.

The problem of poor processing speed in perception and planning algorithms is apparent. There is also a need for developments in image processing and pattern recognition and especially for fast algorithms which can operate on-line. In addition, the integration of various
subsystems in intelligent mobile machines should be emphasized and understood.

2.2 Legged Locomotion Systems

Walking machines possess an immense potential for rough-terrain locomotion [23], [60] and several articulated legged vehicles have been built in the past two decades to demonstrate this potential [27], [64], [72], [58], [98]. This area of science is relatively new and most of the research has been geared to understanding the mechanics of locomotion [81] and control and coordination of articulated limbs [58]. The study of insect walking [79], [105] has contributed significantly to this understanding. But it should be noted that the high mobility of animals or walking machines is due to their ability to adapt their gaits intelligently to the terrain. The need to incorporate intelligence in motion planning is evident. The following section describes some past and current projects in the areas of legged locomotion systems.

The first studies in legged locomotion were on gaits [66], [67] and they date back to 1872. Almost a century later, Hildebrand carried out a systematic study of quadruped locomotion and associated foothold patterns and sequences [25], [26]. Wilson reported a similar study on insects [105]. Tomovic [94] and McGhee [53] were the first to describe and model gaits mathematically. A "finite state" concept was introduced and a new terminology for gait studies evolved [53]. An
exhaustive study of mechanical linkage systems for walking machines was done by Shigley [87].

The first legged machine completely controlled by computers was built at the University of South California [54], [55]. The Phoney Pony was a four legged machine which had two degrees of freedom in each leg which were coordinated by a computer. McGhee and Frank [55] also carried out a mathematical analysis for quadruped creeping gaits and proposed an optimum crawl gait based on the maximization of longitudinal stability margin of the body. The next walking machine was built in 1968 by General Electric [64]. The GE quadruped was a 3000 lb. machine in which the operator controlled the four legs by his hands and feet through a master-slave type valve controlled hydraulic servo system. This proved to be extremely cumbersome. It served to demonstrate the necessity of automation in the coordination of the legs in a walking machine.

In 1972 an electrically powered six legged machine was built at the University of Rome. About the same time analog computer controlled bipeds were built in Yugoslavia and Japan. At this stage, walking machines required a lot of operator participation but several proofs of the concept now existed.

In the USSR, significant work on locomotion and gaits was done by Bessonov and Umnov [3]. The wave gait was shown to be the most optimal gait for six legged vehicles on level ground using the longitudinal stability margin as a measure of the stability of a gait. A six legged
machine was built at the Moscow Physiotechnical Institute in 1974. Four years later, Okhotsimski et al. developed another walking vehicle [73]. The number of legs reflects a tradeoff between stability and complexity and six was judged to be an optimal number. Although it was powered externally it was under complete computer control and was also equipped with a range scanner. This machine was the first to demonstrate application of AI concepts to legged locomotion systems. The legs were similar to those of insects and had a total of 18 degrees of freedom. The intelligence structure was hierarchical and the decision-making process was organized into a situation-action dictionary. Perception and motion planning algorithms were developed. It seems to have been successful in adapting its gait to the surroundings to some extent. The problem of force distribution between the legs of the machine was also studied.

The OSU Hexapod was a 300 lb. six legged machine built at the Ohio State University [58], [75]. It had insect type legs similar to its Russian counterpart and was driven by electric motors controlled by SCRs. It was powered externally and controlled through a PDP-11/70 computer. The Hexapod was equipped with force sensors at the feet for force control and gyroscopes for attitude control. Work on the design of walking machines was accompanied by research on intelligent control schemes and terrain-adaptive locomotion. The free gait proposed by Kugushev [44] and modified by McGhee [59], was the first gait tailored
Figure 2.1: Supervisory Control in Legged Locomotion [75]
to the need for intelligent gait control on rough terrain. Guided by the walking of animals, the Follow-the-Leader (FTL) gait was developed [76]. In the FTL gait, the four legs behind the two fore legs step on the footprints selected for the fore legs. The operator used a hand-held laser to designate candidate footholds, which were used for the front legs if the computer found them acceptable. Two CID television cameras were used to detect the laser beam. A supervisory control scheme shown in Figure 2.1 was incorporated in the Hexapod [75].

Kessis et al [38] report a four-level control architecture for an autonomous six-legged hexapod built at the University of Paris in 1980. The lowest level (leg level) involves the control of individual legs using a variable leg compliance to adapt to uneven terrain. The second level (gait level) generates gaits according to the commands from level three (plan interpreter). Level four (planner) copes with the perception and modeling of the universe and plans actions according to the encountered situation. The terrain model is built by a rule based production system and path planning uses the A* algorithm in a 3-dimensional environment.

PVII is a quadruped walking machine constructed at the Tokyo Institute of Technology [27]. It weighs only 10 kg. with a 10 watt power consumption. It is powered externally and connected via an umbilical cord to a minicomputer. A hierarchical control system is implemented whose functions include navigation, planning, gait control, posture regulation and generating commands for the
servomechanisms. Obstacle avoidance is accomplished through tactile sensing. More recently, another quadruped, TITAN III has been developed. It is a 320 watt machine which is also equipped with wheels as alternative locomotion in elements. The emphasis in design has been on improving the efficiency of locomotion. Adaptive gait control strategies to negotiate forbidden footholds have also been the subject of active research.

In 1983, in Carnegie Mellon University, Ivan Sutherland built the first six legged machine which was entirely self-contained and had an on-board computer and power supply [81]. This had also a total of 18 degrees of freedom which were hydraulically actuated. The hydraulic circuits were designed to make the legs move "usefully" without being digitally controlled by a microprocessor. The operator controlled the speed, direction and attitude of the body. The questions of optimal parameters for gaits or foothold selection for navigation were not addressed. About the same time, also at CMU, Raibert built a one-legged hopping machine [82]. It was the first successful statically unstable but dynamically stable machine. This was a study in dynamic balancing and height and attitude control through hopping. Similar efforts have led to the design and fabrication of a four-legged machine based on the same principle [82].

The Functionoid ODEX was a six-legged walking machine with an axisymmetric leg configuration built by Odetics Inc. in 1983 [83]. It had an unprecedented strength to weight ratio (5.6 when stationary and
2.3 while walking) and reasonable agility. It had eighteen degrees of freedom too but proved to be very easy to control. However the mode of control was teleoperator-like and the vehicle received commands through a joystick via radio telemetry. The actions of the legs were coordinated by on-board computers but the operator used the camera system to view the surroundings and appropriately direct ODEX. ODEX uses a tripod gait used by many six-legged arthropods.

RECUS is an underwater survey robot [34] built by Komatsu, Ltd. It is an eight-legged robot with telescopic legs designed to operate on uneven ocean floors. Walking of the robot is controlled by a single board computer in the mother ship. It controls the walking sequence control and the attitude but in effect is teleoperated.

Currently, a research effort at the Ohio State University has resulted in the successful design, fabrication and testing of a six-legged vehicle called the Adaptive Suspension Vehicle (ASV) [98]. A brief description of the vehicle can be found in Section 2.4. The ASV is designed to carry an operator but it is hoped that eventually, it will operate as a completely autonomous unmanned system.

2.3 The Structure of Machine Intelligence

In most of the mobile robotic systems developed so far, a common feature has been the recognition for the need of a hierarchical structure of intelligence and hierarchical decomposition of the task (of locomotion) and the model of the perceived world. This is
illustrated in Figure 1.1 in the previous chapter. The higher levels define tasks or subgoals for the lower levels and monitor their status. As we go up the hierarchy, on a time scale there is a decrease in the frequency of updating sensory information and on the length scale, the world taken into consideration is larger but, with fewer details. At each successive level down the hierarchy, there is a decrease in the generality and scope of the search and greater resolution. Four levels may be identified in the hierarchy - the planner or the route layout module, the navigator or the path selection module, the pilot or the guidance module and the controller. The cartographer is concerned with maintaining maps for the path selection and guidance modules at the required resolution and range and with appropriate detail.

Planning: The route layout is done at the highest level. It involves planning a route for the vehicle taking into account the general characteristics of the robot's ability to adapt to different terrains and surmount various obstacles. At this level, the basic element of locomotion (leg or wheel) is not of concern. In a completely autonomous system, it may be the only level which interacts with a human operator. This interaction may be limited to off-line specification of the task. The planner prescribes a set of subgoals for the next lower level. It works on a model of the world which extends typically to maybe a hundred body lengths. Presently, there is
little or no research in evidence in this regime of machine intelligence.

**Path selection**: The navigator is primarily concerned with path selection. It uses the terrain preview data and information from other sensors to chart a "best" course for several "vehicle body lengths" to realize the subgoal command from the upper level. Obstacle avoidance is an integral part of such a process. It in turn prescribes subgoals for the pilot level to meet the requirements of the selected path. It maintains a terrain map and a model of the world confined to a few (typically ten) body lengths which possesses a higher level of resolution than the planner’s model.

Most of the work on autonomous navigation has been at this level. It is specific to wheeled or walking vehicles only to the extent that the characteristics of the vehicle have to be known (dimensions of the body, size of the tire or foot, maximum stride length etc.). However most of the work reported in this field has been with reference to wheeled robots. This problem is similar to the problem of planning manipulator transfer movements without explicit programming of the motion [503]. Given the initial and final location (starting point and subgoal) of the mobile robot, the optimal path circumventing obstacles has to found. The concept of shrinking the robot to a single reference point while expanding the obstacle regions [953] has proved to be useful. Lozano-Perèz has formalized this approach with the concept of a configuration space [493]. This involves approximating the obstacles
by polyhedra. In two dimensions the shortest collision free path is composed of straight lines joining the origin to the destination through a set of vertices of obstacle polygons. A V-graph or visibility graph in which each link represents a straight line between two points which can "see" each other, can be built and a search routine is used to obtain the optimal path (see References [12] and [69]). A similar method was employed to navigate SHAKEY. In 3-dimensions this method gets a little more complicated - Lozano-Peréz uses 'slices' (a projection of any space into a lower dimension space) to overcome this problem.

Thompson [93] describes a path planning module for the JPL rover. This approach avoids the construction of a V-graph for the whole space but builds (expands) the graph as and when needed. Similarly, Koch et al [41] propose the concept of 'sectors' (sectors of a circle containing the current position of the vehicle and the goal with the vehicle at the center) which yields a subset of the visibility graph and avoids the problem of combinatorial explosion.

The free space (space outside the obstacles) can be modeled as a union of generalized cones [5],[6] which eventually leads to a more efficient utilization of space. Brooks uses generalized cones to build a connectivity graph which is the input to the path search routine [5].

HILARE's model of the world was represented by a connectivity graph whose nodes are places or rooms and links are traversible.
boundaries. The rooms are further decomposed into polygonal cells representing free space [17].

The most popular graph/tree search method has been the A* algorithm [23], [69]. Distance, energy consumption, uncertainty in information and many other criteria have been used as heuristics to drive the search process.

**Guidance:** The pilot plans a sequence of elementary acts of motion in space and time to generate a path between the goals prescribed by the path selection level. Minor deviations from the prescribed path may be tolerated to circumvent small obstacles. The level has a short term memory and its perception is confined to one or two body lengths. This level may be virtually absent in wheeled locomotion where the motion is almost fully constrained or specified by the path selection level. However, a legged locomotion system is free to select footholds in space and time, i.e. when and where it "samples" the terrain. Thus optimization of gait parameters, optimal foothold selection, dynamic balancing and attitude control of the vehicle body are tasks which must be performed at this level. The choice of legs as elements of locomotion introduces a new level of intelligence and a new degree of complexity into motion planning for the body. This level has also been described as the guidance module by researchers. The pilot treats the leg as a finite state machine [53] and delegates the lower level task of planning trajectories for the legs and the associated problem of leg collision avoidance to the controller.
Thomson has identified a guidance module in wheeled vehicles which translates the planned path into a set of commands for the actuators and uses feedback to control the vehicle limiting the deviation from the prescribed path to a few evasive movements. However, this description would seem to encompass the function of the next lowest level (controller) too. A real-time system for the control of a nested hierarchy of control modules is described in Reference [4]. The pilot module has been developed and tested on a gas powered dune buggy.

The operation of the pilot has been more actively pursued for legged locomotion. In the four-level hierarchy of the University of Paris hexapod [38], the gait level and the plan interpreter appear to constitute the guidance module. Hirose [27] describes a gait control level partitioned into a local motion-trace generator (which responds to the commands of the navigation planning module) and an adaptive gait controller. For the ASV, the free gait has been used as an automatic heuristic technique to optimize the vehicle walk over unstructured terrain [59] and the guidance algorithm [78] is based on this free gait. This enables automatic selection of footholds and gait parameters but the process is quite inefficient and slow. Current efforts are directed towards supervisory control [75] which is aimed at implementing the "horse-intelligence" in the "rider-horse-system" analogy (see Figure 2.1). This will at least enable supervisory control in rough terrain with man providing the "rider-intelligence".
Controller: The lowest level, the controller, is the only level which interacts directly with the actuators. It represents the "spinal" level associated with control of individual joints in natural systems and involves real-time servo control loops and sensory feedback at the actuator level. This is the only area in this hierarchy which is reasonably well researched and documented and is the foundation for robotic technology. In legged locomotion systems, this also entails leg trajectory planning using proximity sensors, and actuation, and also incorporates "cerebellar" intelligence to a certain extent. The same in wheeled systems controls the actuation of the wheels. This level involves very little intelligence and it is also possible to identify the controller as a "plan execution" module and exclude it from this model of the intelligence structure.

2.4 The Adaptive Suspension Vehicle

The Adaptive Suspension Vehicle (ASV) (see Figure 2.2) is a proof-of-concept prototype of a legged vehicle designed to operate in rough terrain that is not navigable by conventional vehicles [98]. It is 3.3 meters (10.9 feet) high and weighs about 3200 kg (7000 lb.). It presently operates in a supervisory control mode but will eventually operate autonomously. To this end, it possesses over 80 sensors, 17 onboard single board computers and a 900 c.c. motorcycle engine rated at 50 kW (70 hp). It has three actuators on each of the six legs thus
Figure 2.2: The Adaptive Suspension Vehicle [98]
providing a total of 18 degrees of freedom. The 18 degrees of freedom are hydraulically actuated through a hydrostatic configuration. The engine is coupled through a clutch to a 0.25 kW/hr flywheel which smooths out the fluctuating power requirements of the pumps.

The ASV senses over 80 control variables. The most important sensor is an optical scanning rangefinder which is a phase modulated, continuous wave ranging system with a range of approximately 30 feet and a resolution of 6 inches [108]. It has a field of view of 40 degrees on either side of the body longitudinal axis and from 15 to 75 degrees below the horizontal. A inertial sensor package consisting of a vertical gyroscope, rate gyroscopes for the pitch, roll and yaw axes, and three linear accelerometers provide information to determine body velocity and position. Leg position feedback is used from the legs in the support phase for the purpose of correcting for gyro and integration drift in the inertial reference system. Thus, there is considerable scope for sensor cross checking and error detection. The leg control system is based on a force control scheme, when the leg is on the ground, and on a position control scheme, when the leg in transfer phase. Thus the position, velocity and pressure difference across each of the eighteen hydraulic actuators are monitored during operation.

The ASV has six operating modes. The utility mode is a pre-flight check out phase which involves testing and diagnostics in the power up and power down stages. The precision footing mode is the only manual
mode in which the operator has complete control over all the legs and
and the body of the machine. The close maneuvering mode is a low speed
omnidirectional mode of locomotion which employs a free gait. These
three modes are fully operational. The terrain-following mode will be
used in moderately rough terrain where the optical terrain-scanner
[108] will be used to select footholds. The large obstacle mode is
used for crossing large obstacles in which a follow-the-leader gait is
used. The cruise/dash mode is designed for rapid locomotion except
that the cruise mode will emphasize efficiency in the energetics of
locomotion and the dash mode will sacrifice efficiency for speed.

The ASV, unlike its predecessors [27], [64], [75] is completely
computer controlled and independent except for the operator. The
operator performs the functions of path selection and specifies the
linear velocities of the vehicle in the fore-aft and lateral
directions, and the yaw velocity. The roll and pitch rates and the
velocity in the vertical direction are automatically regulated by the
guidance system. In addition to path selection, the operator also
supplies important gait parameters such as stride length and
duty factor, and the mode of control (cruise/dash, terrain-following
or large obstacle mode). Eventually, this could be replaced by an on-
line expert system which performs these tasks.

The guidance module for such a legged system serves to provide
the "horse-level" intelligence as shown in Figure 2.1. Presently, it
restricts the vehicle to states from which it can be safely
decelerated to a stop [78]. However, this implementation does not account for uneven terrain and a geometric model of the terrain-vehicle system. The optical rangefinder will play an important role in this regard. The rest of this dissertation describes strategies for autonomous legged locomotion that may be used for such a legged system.
CHAPTER III
GAIT ANALYSIS FOR WALKING MACHINES

3.1 Introduction

In the past, researchers have worked on modeling and analyzing quantitatively the gaits of natural and artificial legged systems [60]. Most of this work has been confined to idealized gaits for motion along the longitudinal axis of symmetry of the system on even terrain. Though, this work is not very practical for omnidirectional motion on uneven terrain, it provides a foundation for the work described in this chapter. Some of this work is surveyed and a background in gait theory is included in the earlier subsections of this chapter. Some new definitions and concepts are proposed for omnidirectional walking on rough terrain. A new gait, the modified wave gait, is suggested as an alternative to the structured ideal gaits discussed in the literature. The optimization of the modified wave gait as a function of the inertial loading and the direction of motion is also discussed.

3.2 History

3.2.1 Natural Systems:

A vast variety of examples of legged locomotion can be found in
nature and many studies of legged animals have been undertaken with a view to discovering strategies for control and coordination of the legs. Unfortunately, the complexity of the nervous systems of animals has made this problem very difficult and most reports have remained largely descriptive [16], [20], [24].

The earliest studies of gaits date back to 1872 when Muybridge [66] used successive photographs to study the locomotion of animals and later human locomotion. However, Hildebrand [25] was probably the first researcher to analyze gaits quantitatively. Since then much of research has been carried out in this area [13], [26], [77] motivated in some cases, by an interest in legged robots. Of particular interest, in the context of the ASV (as explained in the next section) is the literature on arthropods.

Wilson's report [105] on insect gaits is very informative. He attempted to develop a model for insect gaits by proposing a few simple rules. A wave of protraction (transfer phase) runs from posterior to anterior, and contralateral (on opposite sides of the body) legs of the same segment alternate in phase. The protraction time is constant and the retraction (support phase) time is varied to control the frequency. Further, the intervals between the steps of the hind and middle ipsilateral (on the same side of the body) legs, and those between the middle and the fore ipsilateral legs are constant. Though these rules are not observed in all species [105], [106], most of these rules have been validated qualitatively with a variety of
insects when walking on smooth horizontal surfaces. The classical alternating tripod gait exhibited by fast moving locusts and cockroaches is a good example [13].

More recently, a cinematographic analysis of locusts [79] was undertaken to study locomotion characteristics on rough terrain. Each individual leg was found to have the ability to locate a suitable support site independently without visual control. The use of vision was limited to the detection of terrain changes and obstacles resulting in a change of the general direction and in the movement of a fore leg to the general direction of a support site. No unique gait could be identified on rough terrain though an in-phase coordination of the contralateral legs of the middle segment was frequently observed.

Other studies by Cruse [9], [10] on stick insects show that the posterior legs are placed close to the support sites of the immediately anterior ipsilateral legs. This follow-the-leader behavior may be a general strategy for walking on uneven terrain and has also been observed in domestic (Nubian) goats [77].

3.2.2 Theory:

Tomovic [94] and Hildebrand [25] were the first to study gaits
quantitatively. Hildebrand developed the concepts of a \textit{gait formula}\footnote{The new terms used here will be defined in Section 3.4.} and a \textit{gait diagram} to describe symmetric gaits of horses. McGhee\cite{54} started the development of a general mathematical theory of locomotion based on a finite state concept. A leg was defined as a sequential machine with an output state 1 representing the support phase, where the leg is in contact with the supporting surface, and an output state 0 representing the transfer phase. In the terminology of physiology, the support phase is the period of retraction and the transfer phase is the period of protraction. McGhee and Frank\cite{55} defined the \textit{static stability margin} for gaits as a measure of the static stability of gaits. They identified statically stable gaits for a quadruped as \textit{creeping gaits}, of which the \textit{regular crawl gait} (crawl gaits were defined by Hildebrand\cite{25}) was proved to be the most optimal in terms of the \textit{longitudinal stability margin}. McGhee and Jain\cite{56} attempted to explain the bias shown by animals towards certain gaits through a characteristic called \textit{regular realizability}. The notions of \textit{event sequence} and \textit{gait matrix} were used to describe gaits.

The study of gaits has led to the definition and classification of different gaits. In 1973, Bessenov and Umnov\cite{3} used numerical experimentation for hexapods to demonstrate that the optimal gaits are
regular and symmetric and these gaits were described as "wavy" gaits. This agreed with Wilson's observation of a metachronal pattern of steps in insect walking. More recently, Song [89] has carried out a detailed analysis and classification of gaits, where the wave gait was identified as the optimal gait for hexapods. The follow-the-leader (FTL) gait was studied by Tsai [76] and implemented on the OSU Hexapod. An aperiodic gait called a free gait was proposed by Kugushev and Jaroshevskij [44] and later modified by McGhee and Iswandhi [59] for the OSU Hexapod. This was the first attempt to address the problem of automatic foothold selection in a real world situation. The utility of the free gait was subsequently demonstrated through simulations for the ASV [46]. A more extensive treatment of statically stable gaits can be found in the work reported by Song [89].

3.3 Stability in Locomotion

Broadly speaking, animal locomotion can be classified into two categories. The first type is the one exhibited by insects. Insects are arthropods and have a hard exoskeletal system with jointed limbs. They use their legs as struts and levers and the legs must always support the body during walking, in addition to providing propulsion. In other words, the metachronal or sequential pattern of steps must ensure static stability. The vertical projection of the center of
gravity must therefore always be within the support pattern [55]. This kind of locomotion has been described as crawling [81] and the legs have to provide at least a tripod of support at all times.

Another kind of locomotion may be observed in humans, horses, dogs, cheetahs and kangaroos which have a more flexible structure. These animals require dynamic balance, which is a less stringent restriction on the posture and gait of the animal. The animal may not be in static equilibrium - to the contrary, there may be periods of time when none of the support legs are on the ground, as is observed in trotting horses, running humans and, of course, hopping kangaroos.

Until now, most efforts to build dynamically balanced robots have been confined to bipeds [65] and hopping machines [81], [81]. This is because the complexities of the locomotion system in biological creatures have prevented proper understanding of the involved mechanics and controls. Also, the present state of the art in digital computers has allowed the implementation of only simplified dynamic models for walking machines.

The present generation of walking machines almost exclusively attempt to emulate the mechanism of walking in insects. Control and coordination in such machines are obviously simpler, and are well

2. The term support pattern is defined in Section 3.4, but, for now, it may be assumed to be the two dimensional convex polygon formed by the contact points.
within the reach of modern technology. Also, the statically stable
crawl typified by insects, is better suited to heavy machines with
rigid structures like the ASV. In fact, the ASV incorporates a quasi-
static mode of operation in which the resultant of the inertial forces
and the weight is treated as an effective gravitational force. The
mathematical theory and definitions involved in gait analysis, and the
concept of static stability of legged systems, are discussed in the
next section.

3.4 Introduction to Gait Theory:

A detailed description of terminology and definitions for gait
study can be found in Reference [89]. Some of these past results are
presented here with the idea of providing a foundation for the
presentation of new theorems and ideas. The legs on the left side of
an n-legged machine are numbered 1, 3, ..., n-1, and those on the
right side are designated by even numbers, in accordance with previous
literature [54] (see Figure 3.1). The notions of animal and machine
are used interchangeably here. All the results and derivations are
applicable only to six-legged gaits, although the development of the
theory is more general and can be applied to n-legged gaits. All the
legs are assumed to be identical. It is natural to assume a symmetry
about the sagittal plane as seen in all examples in nature. Also, it
is assumed that the pitch, P (as defined in the figure), between the
legs is constant. That is, there is a fore-aft symmetry in the system
Figure 3.1: Plan View of a Six-Legged Locomotion System
(P = pitch, W = width)
(though this is contrary to the anatomy of some biological systems). Thus, in Figure 3.1, the pitch, $P$, and the width, $W$, or the width to pitch ratio, $\gamma$, determine the geometry of the legged system. Next, a few important terms are defined. Most of these definitions are applicable to the special ideal case, in which the terrain is even, the vehicle velocity is uniform, the longitudinal axis of the body is parallel to the direction of the velocity and the body attitude is parallel to the terrain.

**Definition 1** [94]: A leg is a sequential machine with two output states, 1 and 0. The state 0 represents the support phase of the leg or the state of being in contact with the ground (retraction). The state 1 represents the transfer phase of the leg, in which the leg is above the contacting surface (protraction).

**Definition 2** [54]: The cycle time, $T$, defined for periodic gaits, is the time duration of a complete cycle of motion of a leg, and hence the time duration of a locomotion cycle.

**Definition 3** [54]: The duty factor, $\beta$, is the fraction of time (in one cycle) that a leg spends in the support phase. It is assumed in this analysis that all legs have the same duty-factor as there is no reason to expect one leg to be favored over another (The legs are postulated to be identical).
Definition 4 [25]: The stride length, $\lambda$, of a gait is the distance through which the center of gravity of the system translates during one locomotion cycle.

Definition 5 [89]: The stroke, $R$, is the distance through which the foot translates during a locomotion cycle.

Definition 6 [89]: The pitch, $P$, is the distance between the centers of strokes (mean positions) of two adjacent, ipsilateral legs. For a symmetric machine, the pitch is the same for any two such legs.

Definition 7 [54]: The leg phase, $\phi$, is the time fraction of a cycle time period by which the instant of contact of the leg lags behind a reference time instant. This reference time is selected to be the instant when leg 1 contacts the ground.

Definition 8 [53]: An event of a gait is the placing or lifting of any of the feet during locomotion. For a $n$-legged animal or machine, there are $2n$ events in an event sequence or locomotion cycle.

Definition 9 [55]: A support pattern is the minimum area convex point set in the support plane obtained from the contact points of the feet on the ground at a given time. The support plane on level terrain is the surface on which the contact points of the feet lie, but it is more difficult to define a support plane on uneven terrain. More about this will be discussed later.

Definition 10 [55]: The stability margin (SM) for a support pattern is equal to the shortest distance from the vertical projection of the center of gravity of the machine to any point on the boundary of the
Figure 3.2: Measures of Static Stability

(a) LONGITUDINAL STABILITY MARGIN (LSM)

\[ \text{LSM} = \text{MINIMUM} \left( d_1, d_2 \right) \]

(b) STABILITY MARGIN (SM)

\[ \text{SM} = \text{MINIMUM} \left( r_1, r_2, r_3, r_4 \right) \]
support pattern. This is illustrated in Figure 3.2. The stability margin for a periodic gait is the minimum stability margin over all the support patterns encountered in an entire cycle.

**Definition 11** [55]: The *longitudinal stability margin* (LSM) for a support pattern is the shortest distance along the direction of projection from the center of gravity of the machine to an edge of its support pattern (see Figure 3.2). The longitudinal stability margin for a periodic gait is the minimum LSM over an entire gait cycle. The longitudinal stability margin is positive if and only if the static stability margin is positive and vice versa. To that extent, the LSM, being simpler to compute, may be used as a measure of static stability to permit quantitative comparison of all possible gaits (see Section 3.6).

**Definition 12** [54]: A gait is singular if any two or more events occur simultaneously during a locomotion cycle.

**Definition 13** [54]: A gait formula, $\zeta$, for an $n$-legged machine is defined by

$$\zeta = (\beta_1, \beta_2, \ldots, \beta_n, \phi_2, \phi_3, \ldots, \phi_n).$$

Notice that $\phi_1$ is equal to zero due to the choice of the reference time in Definition 7.

**Definition 14** [54]: Two gaits are identical if and only if the $(2n-1)$ elements of the gait formulae are identical. This leads to a possible total of $(2n-1)!$ gaits. Also, there are $2n$ events in a locomotion
cycle and hence a total of \((2n-1)!\) permutations are possible which agrees with the number obtained earlier.

**Definition 15** [89]: The local phase, \(\phi_L\), of a leg, is defined as the fraction of the locomotion cycle elapsed since the placing event of the leg. In other words, \(\phi_L = 0\) when the leg is placed and \(\phi_L = \beta\) when the leg is lifted.

**Definition 16** [89]: A gait is symmetric if the motion of contralateral legs on the same segment is exactly half a cycle out of phase.

**Definition 17**: A gait is periodic if the motions of each of the legs are periodic. In other words, the time fractions of the support and transfer phases during successive cycles are constant for all legs.

**Definition 18** [94]: A gait of an \(n\)-legged machine is a creeping gait, if and only if every support pattern involves at least \(n-1\) support points.

**Definition 19** [31]: A wave gait is a regular, symmetric gait with the difference in leg phases between any pair of adjacent ipsilateral legs being equal. In other words, if \(\text{mod}\) represents the modula operator,

\[
\beta_1 = \beta_2 = \ldots = \beta_n; \quad \phi_2 = 1/2, \quad \phi_i = (\phi_{i-1} + 1/2) \text{ mod } 1, \quad 2 < i \leq n
\]

and

\[
(\phi_{i+2} - \phi_i) \text{ mod } 1 = (\phi_{j+2} - \phi_j) \text{ mod } 1, \quad 1 \leq i, j \leq n-2.
\]

A perfect wave gait, is a wave gait in which \(\phi_3 = \beta\) and consequently \(\phi_5 = 2\beta \text{ mod } 1, \quad \phi_4 = (\beta + 1/2) \text{ mod } 1\) and so on.
Definition 20 [76]: A follow-the-leader gait (FTL gait) is a gait in which the posterior legs are placed on the support points of the immediately anterior, ipsilateral legs. The advantage of such a gait lies in the simplification of the foothold selection strategy, since, for an FTL gait, the selection process need be made for the front two legs only. Such gaits are well suited to terrain-adaptive locomotion.

FTL gaits can be divided into continuous and discontinuous gaits [89]. A continuous FTL gait is periodic and is a wave gait characterized by a poor LSM (compared to the perfect wave gait). The discontinuous FTL gait is aperiodic and only one leg is moved at a time. It has been implemented on the ASV for crossing obstacles but is very unsuitable for a smooth and uniform motion.

Definition 21 [44]: The motion trace of a vehicle is the desired trajectory for the vertical projection of the vehicle center of gravity.

Definition 22 [59]: The kinematic margin for a designated foothold for a leg is the length of the curve representing the motion trace from the current vertical projection of the CG to the point at which the leg using that foothold reaches its kinematic limits.

The free gait is a sequence of support points in a terrain divided into cells, some of which are classified as forbidden, whose generation is based on the following logic [44], [59]:
(1) The minimum value of the kinematic margin over all supporting legs is maximized by lifting appropriate legs but ensuring always that the vehicle remains stable [44].

(2) If the vehicle is unstable, the leg whose placement can remedy the situation is placed [44].

(3) If, in step (2), there is more than one candidate leg, then the leg with the greatest kinematic margin is placed [59]. In other words, no leg is placed on the ground, unless, by doing so, the minimal kinematic margin over all the support legs is increased.

The computational complexity and the problems associated with a real-time implementation of this algorithm are evident. One way of simplifying the computational burden would be to use a very coarse terrain grid. This suggests an obvious tradeoff.

Gait diagrams [25] are used to describe gaits graphically (see Figure 3.3). They record the duration of the support phase (dark line) or retraction, and the transfer phase (no line) or protraction. Figure 3.3 illustrates an example of a wave gait.

It should be noted that the wave gait has been defined differently in the past. The definition presented here is based on the description of Bessonov and Umnov [3] and that followed by Wilson [105]. Song [89] defines a wave gait by further restricting $\phi_3$ to be equal to $\beta$. This special wave gait has been called the perfect wave gait (Definition 17) in this work to avoid any ambiguity. Gait
diagrams are used to describe gaits in this dissertation although a host of other techniques are available in the literature [89].

3.5 Omnidirectional Walking on Uneven Terrain - Definitions

Most of the existing terminology on gaits is applicable only to walking in a direction parallel to the longitudinal axis of the body on a plane surface with a constant velocity and a body attitude parallel to the surface. Clearly, this is not the real-world situation. It was found necessary to modify existing definitions and propose new ones to describe and study omnidirectional walking on uneven terrain, and the effects of an inertial loading on the vehicle body. In omnidirectional motion, the gait of the machine can be instantaneously described by a crab gait.

Definition 21: A crab gait is defined as a gait in which the direction of progression of the center of gravity of the machine need not coincide with the longitudinal axis of the body. The crab gait is believed to be sufficiently general for this study.

Definition 22: A crab wave gait is a crab gait which is regular and periodic and which has the same phase difference between any pair of ipsilateral, adjacent legs. The sequence of events in a crab wave gait is characterized by three parameters:

(1) The duty factor, \( \beta \)

(2) The longitudinal phase difference, \( \psi \), between any pair of ipsilateral legs
WAVE GAIT

PERFECT WAVE GAIT

Figure 3.3: Gait Diagram for Wave Gaits
(3) The lateral phase difference, $\theta$, or the phase difference between any pair of contralateral legs

In a regular, periodic gait, if the phase difference between any pair of adjacent, ipsilateral legs is a constant, then it follows that the phase difference between any pair of contralateral legs is also a constant. As $\theta$ is not, in general, equal to $1/2$, the crab wave gait need not be symmetric. In a wave gait, $\theta = 1/2$ and in a perfect wave gait, $\psi = \beta$ in addition.

**Definition 23**: The support plane or the $r$-plane is defined as a best-fit plane obtained by doing a linear regression on the points of support at any given state (see Reference [75] for the procedure). On smooth ground, the support plane is the plane of the ground. For uneven terrain, the support plane is not just a function of the terrain but is also a function of the "sampling" of the terrain by the legs.

A system of forces along generally disposed lines of action, and of couples whose directions are quite general, is equivalent to a force along a unique line of action and a couple whose direction is parallel to that line. This force-couple combination is called a wrench and the line is called the wrench-axis or screw-axis [32]. The load-wrench is a term used to denote the wrench which is the resultant of the weight of the vehicle and the inertial forces and moments. The wrench-axis is denoted by the symbol, $\mathfrak{m}$, the intensity of the wrench
is \( f \) and the force and couple associated with the wrench are \( f \) and \( c \) respectively.

The desired support plane, \( \sigma \), is a plane passing through the centroid of the support points, perpendicular to the wrench axis, \( \xi \). This plane is referred to as the \( \sigma \)-plane throughout the rest of the dissertation. It is desired that the support plane, \( \pi \), be coincident with \( \sigma \) and the reasons for this will become apparent in Chapter 5.

The nomenclature used in the rest of this dissertation is described here briefly. A reference frame, \( A \), is denoted by \( \{ A \} \) - the orientation of the coordinate system and the position of its origin expressed in a known reference frame constitute the description of the reference frame. A quantity expressed in a coordinate system attached to a reference frame, \( A \), has a leading superscript, \( A \). The coordinate system is defined by the coordinate axes, \( X_A \), \( Y_A \) and \( Z_A \) or the corresponding unit vectors \( \hat{X}_A \), \( \hat{Y}_A \) and \( \hat{Z}_A \). The position vector and velocity of a point \( P \) expressed in the reference frame \( \{ A \} \) are denoted by \( A \) \( A \) \( \mathbf{r}_P \) and \( A \) \( A \) \( \mathbf{v}_P \) respectively. A \( ^\wedge \) symbol is used to indicate a unit vector.

The reference frame, \( \{ E \} \) is an earth fixed reference frame. \( \{ B \} \) is a reference frame fixed to the vehicle body at the center of gravity, with the \( X \)-axis along the longitudinal axis (in the preferred direction of motion) and the \( Z \)-axis is such that for locomotion on even terrain, if the body is parallel to the plane of the terrain, it points vertically upwards. In other words, the \( X_B \)-\( Z_B \) plane is the
Figure 3.4: Coordinate Systems for a Legged Vehicle
sagittal plane (see Figure 3.4). The origin is chosen to be at \( G \), the center of gravity of the vehicle. It is assumed that the mass of the legs is negligible in comparison with the vehicle weight, and thus the C.G. of the vehicle is fixed with respect to the vehicle and the reference frame \([B]\). \([W]\) is a reference frame with its origin at the centroid of the contact points, \( O \), which obviously lies on the \( \sigma \)-plane and the \( \pi \)-plane. \( \hat{z}_W \) is parallel to the wrench-axis (normal to the \( \sigma \)-plane) and opposite in direction to the force component of \( \mathbf{w} \), \( f \). Let \( C \) be the intersection of the wrench-axis with the \( \sigma \)-plane. If the wrench is considered to be fixed to the body, the instantaneous velocity of point \( C \), \( \mathbf{v}_C \), can be easily computed. \( \hat{\mathbf{x}}_W \) is parallel to that component of the velocity of point \( C \) which is perpendicular to the wrench-axis. In other words, if \( \omega_b \) denotes the angular velocity of the body and \( \mathbf{r}_{C/G} \) the position vector of the point \( C \) with respect to \( G \), then

\[
\mathbf{v}_C = \mathbf{v}_G + \omega_b \times \mathbf{r}_{C/G}
\]

and

\[
\hat{\mathbf{x}}_W = \frac{\mathbf{v}_C - [\mathbf{v}_C \times \hat{z}_W] \mathbf{z}_W}{|\mathbf{v}_C - [\mathbf{v}_C \times \hat{z}_W] \mathbf{z}_W|}
\]

\( \hat{\mathbf{x}}_W \) may be easily obtained by:

\[
\mathbf{v}_W = \mathbf{z}_W \times \mathbf{x}_W
\]
(W) may be described with respect to (B) by using a 4x4 homogeneous transformation matrix, $^W_B$:

$$^W_B = \begin{bmatrix} ^W_B & ^W_B & ^W_B & ^W_B \\ ^W_H & ^W_H & ^W_H & ^W_H \\ 0 & 0 & 0 & 1 \\ -0 & -0 & -0 & 1 \end{bmatrix}$$

In Figure 3.5, let the points of support be projected on to the $\sigma$-plane along a direction parallel to the wrench axis to form a two-dimensional support polygon on the $\sigma$-plane. The quasi-static stability margin is then the stability margin (SM) associated with the support polygon, if the projection of the C.G. were at C. The quasi-static stability margin reduces to the static stability margin for an even terrain with no inertial forces.

If the angular velocity of the vehicle, $\omega_b$, is zero, then the axis $X_H$ (in Figure 3.5) represents the direction of progression of the vehicle. The angle between the projection of the direction of progression along $Z_B$ on the $X_B-Y_B$ plane, and the longitudinal axis of the body measured from the longitudinal axis about the $Z_g$-axis is defined as the crab-angle, $\alpha$.

$$\cos \alpha = \frac{\left( ^B_X_H - \left( ^B_X_H \cdot \hat{k} \right) \hat{k} \right) \cdot \hat{k}}{\mid ^B_X_H - \left( ^B_X_H \cdot \hat{k} \right) \hat{k} \mid}$$  \hspace{1cm} (3.1)
Figure 3.5: Quasi-static Stability
If the angular velocity of the vehicle is nonzero the instantaneous velocity of the point C can still be used to find $B^g_{w}$ and the crab angle.

The quasi-static stability margin is the LSM computed from the two-dimensional support polygon described earlier, assuming the projection of the C.G. to be at C. From now on, the prefix quasi-static will be dropped and the terms SM and LSM will refer to the quasi-static case. These definitions are more general and reduce to the existing definitions for SM and LSM for smooth terrain locomotion under static loading.

Another alternative measure of stability which has been proposed for locomotion on rough terrain is the energy stability margin [61]. The energy stability margin is the energy required to overturn the vehicle and is a measure of the static stability based on the potential energy of the body. However, the SM and the LSM, as defined here, are applicable to a quasi-static situation and are preferred as quantitative measures of stability.

3.6 On the Stability and Stability Margins of Gaits

3.6.1 General

It was seen that the longitudinal stability margin (LSM) and the stability margin (SM) are two measures of static stability and with appropriate modifications, they can also be used for the quasi-static situations. The problem of optimizing gaits based on such measures of
stability has been investigated [3], [55], [89]. This topic has been further investigated using an analytical approach. It is assumed that the metachronal pattern of steps observed for insects by Wilson [105] is optimal. Thus only regular, periodic gaits with the same phase difference between any pair of adjacent, ipsilateral legs, are considered. For the first time, a nonzero crab angle is also considered in such an exercise. The gaits considered here are idealized gaits. That is, the legs are assumed to stroke through their center positions (which define the width and pitch of the vehicle) in a plane perpendicular to the gravity vector. Such an analysis is still relevant to rough terrain locomotion, as the results can be applied to support patterns on the σ-plane.

3.6.2 Longitudinal Stability Margin and Symmetry in Gaits:

Gaits for hexapods can be described as a sequence of 12 events, namely, the lifting and placing events for each of the six legs. The 12 events occur at times given by Table 3.1. The function, F, denotes the modula 1 operator. Figure 3.6 shows a two dimensional support pattern on the σ-plane with the vertical projection of the center of gravity, \( \hat{C} \), parallel to the projection of the instantaneous linear velocity of the vehicle, \( \mathbf{v} \), on the σ-plane. It is assumed for now, that the angular velocity of the body is zero and that the linear velocity of the body, \( \mathbf{v} \), is constant over a cycle. Equivalently, the point C can be assumed to be stationary, with the support plane moving
Figure 3.6: A Support Pattern on the $\sigma$-plane
Table 3.1. Placing and Lifting Events in a Locomotion Cycle

<table>
<thead>
<tr>
<th>EVENTS</th>
<th>t = t/T</th>
<th>LEG NUMBER</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>placing</td>
</tr>
<tr>
<td>2</td>
<td>β</td>
<td>1</td>
<td>lifting</td>
</tr>
<tr>
<td>3</td>
<td>θ</td>
<td>2</td>
<td>placing</td>
</tr>
<tr>
<td>4</td>
<td>F(θ+β)</td>
<td>2</td>
<td>lifting</td>
</tr>
<tr>
<td>5</td>
<td>ψ</td>
<td>3</td>
<td>placing</td>
</tr>
<tr>
<td>6</td>
<td>F(ψ+β)</td>
<td>3</td>
<td>lifting</td>
</tr>
<tr>
<td>7</td>
<td>F(θ+ψ)</td>
<td>4</td>
<td>placing</td>
</tr>
<tr>
<td>8</td>
<td>F(θ+ψ+β)</td>
<td>4</td>
<td>lifting</td>
</tr>
<tr>
<td>9</td>
<td>F(2ψ)</td>
<td>5</td>
<td>placing</td>
</tr>
<tr>
<td>10</td>
<td>F(2ψ+β)</td>
<td>5</td>
<td>lifting</td>
</tr>
<tr>
<td>11</td>
<td>F(θ+2ψ)</td>
<td>6</td>
<td>placing</td>
</tr>
<tr>
<td>12</td>
<td>F(θ+2ψ+β)</td>
<td>6</td>
<td>placing</td>
</tr>
</tbody>
</table>
at a velocity, \(-V\).  

**Theorem 1:** The longitudinal stability margin of the support pattern in a regular, periodic gait is at a minimum just before any one of the six placing events or just after any one of the six lifting events.  

**Proof:**  
If the point C is stationary and the support pattern is considered to be moving at a velocity, \(-V\), the support pattern can not change between events. In Figure 3.6, the distance \(d_1\) decreases and the distance \(d_2\) increases as the support pattern translates. The event of placing a leg has the effect of either increasing \(d_1\) and/or \(d_2\) or of leaving them unchanged. Similarly, the event of lifting a leg has the effect of decreasing \(d_1\) and/or \(d_2\) or leaving them unchanged. The LSM of a support pattern, by definition, is the minimum of \(d_1\) and \(d_2\). The quantity \(d_1\) has to be the smallest just before one of the six events of placing a leg and \(d_2\) the smallest just after one of the six lifting events. As the LSM has to be equal to either \(d_1\) or \(d_2\), it is at a minimum at one of the twelve instants described by the theorem, which completes the proof.  

Although the result of this theorem may be intuitively obvious to some readers, the author is not aware of any formal recognition of this simple conclusion. The computation of the LSM can be based on the calculation of \(d_1\) over six instants and \(d_2\) over another six instants. These "critical time instants" are shown in Table 3.2. Further, analytic expressions can be derived for \(d_1\) and \(d_2\), which simplifies
Table 3.2. Critical Time Instants in a Locomotion Cycle

<table>
<thead>
<tr>
<th>CRITICAL DISTANCE</th>
<th>CRITICAL TIMES</th>
<th>LEG NUMBER</th>
<th>DESCRIPTION OF EVENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>$0^-$</td>
<td>1</td>
<td>before placing</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$\beta^+$</td>
<td>1</td>
<td>after lifting</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$\theta^-$</td>
<td>2</td>
<td>before placing</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$F(\theta+\beta)^+$</td>
<td>2</td>
<td>after lifting</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$\phi^-$</td>
<td>3</td>
<td>before placing</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$F(\psi+\beta)^+$</td>
<td>3</td>
<td>after lifting</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$F(\theta+\psi)^-$</td>
<td>4</td>
<td>before placing</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$F(\theta+\psi+\beta)^+$</td>
<td>4</td>
<td>after lifting</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$F(2\phi)^-$</td>
<td>5</td>
<td>before placing</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$F(2\phi+\beta)^+$</td>
<td>5</td>
<td>after lifting</td>
</tr>
<tr>
<td>$d_1$</td>
<td>$F(\theta+2\phi)^-$</td>
<td>6</td>
<td>before placing</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$F(\theta+2\psi+\beta)^+$</td>
<td>6</td>
<td>after lifting</td>
</tr>
</tbody>
</table>
the calculations considerably.

The idea of optimizing gaits based on the LSM has engaged many researchers [3], [55], [89], but the problem continues to elude completing understanding. The optimization can be formulated as a nonlinear programming problem in seven dimensions for regular, periodic gaits, with a fixed geometry (pitch, P and width, W). If the LSM for a support pattern is described as a function, \( f \), of \( \phi_2, \phi_3, \ldots, \phi_6, \beta, \) and \( \eta \) then

\[
\text{LSM} \triangleq \min \{ f_j(\phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \beta, \eta) \} \quad (3.2)
\]

where \( j \) is an index varying from 1 through 12 denoting the critical time instants in Table 3.2 and \( \eta \) is the stroke to pitch ratio. The problem formulation is as follows:

Minimize \[ -\min \left[ f_j(\phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \beta, \eta) \right] \quad (3.3) \]

Subject to

\[
\begin{align*}
0 & \leq \phi_2 \leq 1 \\
0 & \leq \phi_3 \leq 1 \\
0 & \leq \phi_4 \leq 1 \\
0 & \leq \phi_5 \leq 1 \\
0 & \leq \phi_6 \leq 1 \\
0 & < \eta < \infty \\
0 & \leq \beta \leq 1
\end{align*}
\]
If $\beta$ and $\eta$ are parameters in this problem, then the objective function is nonlinear with piece-wise linearity in the domain of five dimensions. It is intuitively obvious that increasing the ratio of time spent by a leg on the ground will increase the stability, and that the problem can be solved for any prescribed value of $\beta$. The functions $f_j$ are also functions of the vehicle geometry which involve the pitch, $P$, the width, $W$, and the stroke to pitch ratio, $\eta$ and these can be considered as inputs to the problem. For a crab gait, the crab angle, $\alpha$, serves as yet another input. With some ingenuity, for a known stepping sequence, the objective function can be expressed as a linear function and thus enable an ad hoc linear programming formulation. But, in the general case, the optimization problem is quite intractable.

McGhee and Frank have performed this optimization for a quadruped crawl for the case $\eta \leq 1$, $\alpha = 0$ (this simple case yields a solution relatively easily). They proved that the optimum quadruped gait is characterized by

$$\phi_2 = \frac{1}{2}$$

$$\phi_3 = \beta$$

$$\phi_4 = \beta - \frac{1}{2}$$

and the longitudinal stability margin is given by

$$LSM = R(\beta - \frac{3}{4})$$

(3.4)
Considering Definition 22, $a = 0$, $\theta = 1/2$ and $\phi = \beta$, and therefore this gait is a perfect wave gait (as defined in Section 3.5). It was found to be used by horses [25] but with $\eta$ marginally greater than 1, and $\psi$ marginally greater than $\beta$. This is thought to be because of the tendency to use a FTL strategy which requires $\psi$ to be greater than $\beta$ to prevent collision of ipsilateral legs.

Bessonov and Umnov experimented with hexapod gaits numerically to establish the perfect wave gait as the optimal gait in terms of static stability [33]. Song's work [89] sought to formalize this in terms of analytical expressions for the LSM for perfect wave gaits.

A derivation of the LSM for a regular gait with a constant phase difference between adjacent, ipsilateral legs, and a zero crab angle is presented here. Let the phase difference between any pair of contralateral legs be $\Theta$. The phase angles are then given by

\[
\begin{align*}
\phi_1 &= 0 \\
\phi_2 &= \theta \\
\phi_3 &= \psi \\
\phi_4 &= (\theta + \psi) \mod 1 \\
\phi_5 &= (2\psi) \mod 1 \\
\phi_6 &= (2\psi + \Theta) \mod 1 \\
\end{align*}
\]

(3.5)

Let $t$ be a nondimensionalized time such that the time period $T$ equals 1.0. Figure 3.7 shows a gait diagram representation for such a gait. At a time, $t$, the local phases, $\phi_{L,i}$, are:
Figure 3.7: A Regular Gait with a Constant Phase Increment on Each Side

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>θ</th>
<th>ψ</th>
<th>β</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>a = ψ + β - 1</th>
<th>e = θ + ψ + β - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = 2ψ - 1</td>
<td>f = θ + ψ - 1</td>
<td></td>
</tr>
<tr>
<td>c = 2ψ + β - 2</td>
<td>g = θ + 2ψ - 2</td>
<td></td>
</tr>
<tr>
<td>d = θ + β - 1</td>
<td>h = θ + 2ψ + β - 2</td>
<td></td>
</tr>
</tbody>
</table>
\[ \phi_{L,1} = t \]
\[ \phi_{L,2} = (t - \theta) \mod 1 \]
\[ \phi_{L,3} = (t - \psi) \mod 1 \]
\[ \phi_{L,4} = (t - \theta - \psi) \mod 1 \]
\[ \phi_{L,5} = (t - 2\psi) \mod 1 \]
\[ \phi_{L,6} = (t - 2\psi - \theta) \mod 1 \]

If the \( i^{th} \) leg is in support phase then the coordinates \((x^i, y^i)\) in the body fixed reference frame, \([B]\), on the support plane is given by:

\[
x_1 = P + R/2 - R/\beta(\phi_{L,1}) \quad ; \quad y_1 = W/2
\]
\[
x_2 = P + R/2 - R/\beta(\phi_{L,2}) \quad ; \quad y_2 = -W/2
\]
\[
x_3 = R/2 - R/\beta(\phi_{L,3}) \quad ; \quad y_3 = W/2
\]
\[
x_4 = R/2 - R/\beta(\phi_{L,4}) \quad ; \quad y_4 = -W/2
\]
\[
x_5 = -P + R/2 - R/\beta(\phi_{L,5}) \quad ; \quad y_5 = W/2
\]
\[
x_6 = -P + R/2 - R/\beta(\phi_{L,6}) \quad ; \quad y_6 = -W/2
\]

(3.7)

Consider the time instants \( t_1 \) and \( t_2 \) given by

\[ t_1, t_2 \leq 1 \]

and

\[ t_2 = (\beta + 2\psi + \theta - t_1) \mod 1 \]

Then,

\[
x_1(t_1) = -x_6(t_2) \quad ; \quad y_1(t_1) = -y_6(t_2)
\]
\[
x_2(t_1) = -x_5(t_2) \quad ; \quad y_2(t_1) = -y_5(t_2)
\]
\[
x_3(t_1) = -x_4(t_2) \quad ; \quad y_3(t_1) = -y_4(t_2)
\]

Thus, the following theorem can be written.
Theorem 2: For a regular gait (with a duty factor, $\beta$, stroke, $R$, pitch, $P$) with a constant phase difference, $\psi$, between all pairs of ipsilateral adjacent legs, any support pattern has another support pattern in the same cycle symmetric about the origin. If the nondimensionalized times corresponding to the two patterns are $t_1$ and $t_2$, then

$$(t_1 + t_2 - 2\psi - \theta) \mod 1 = \beta$$

where $\theta$ is the phase difference between any pair of adjacent contralateral legs.

Corollary to Theorem 2: The longitudinal stability margin for such a gait is at a minimum just before any one of the six placing events. Similarly, it is at a minimum just after any one of the six lifting events. The corollary is a direct consequence of the symmetry proved in Theorem 2.

Another theorem for regular gaits with a constant phase difference between ipsilateral adjacent legs concerning lateral symmetry can be proved along the same lines (see Reference C893). It is stated here for completeness:

Theorem 3: For a regular gait with a constant phase difference, $\psi$, between any pair of ipsilateral adjacent legs, any support pattern has a mirror symmetric pattern reflected about the frontal plane through the CG. If the two time instants corresponding to the support patterns are $t_1$ and $t_2$, then

$$(t_1 + t_2 - 2\psi) \mod 1 = \beta$$
Figure 3.8: Plan View for a Crab Gait with a Crab Angle, $\alpha$
Both the theorems (2 and 3) also apply to wave gaits as wave gaits only have the added restriction that $\theta$ must be $1/2$. In addition, the support patterns with $\theta$ equal to 0.5 are symmetric about the sagittal plane [89]. This leads to the following theorem (which is rigorously proved in [89]):

**Theorem 4**: The LSM of a wave gait is at a minimum just before any one of the placing events in a half locomotion cycle.

In omnidirectional motion, the crab angle, $\alpha$, modifies equations (3.7). Figure 3.8 shows a plan view of the legged vehicle with a crab angle, $\alpha$. The $X'-$Y' coordinate system is the $X_\mathcal{W}-Y_\mathcal{W}$ system defined in Section 3.5, translated to the vertical projection of G. For now, the load wrench is assumed to be vertical and in addition to being perpendicular to the $X_\mathcal{W}-Y_\mathcal{W}$ plane, i.e., the inertial forces are zero. The positions $(x'_i, y'_i)$ in the $X'-$Y' system are given by (the function, $F$, performs the modula 1 operation):

\[
\begin{align*}
    x'_1 &= P \cos \alpha + \frac{W}{2} \sin \alpha \cos \theta - \frac{R}{2} \sin \theta; \\
    y'_1 &= \frac{W}{2} \cos \alpha - P \sin \alpha; \\
    x'_2 &= P \cos \alpha - \frac{W}{2} \sin \alpha \cos \theta + \frac{R}{2} \sin \theta; \\
    y'_2 &= -\frac{W}{2} \cos \alpha - P \sin \alpha; \\
    x'_3 &= \frac{W}{2} \sin \alpha + \frac{R}{2} - (R/\beta)F(t-\psi); \\
    y'_3 &= \frac{W}{2} \cos \alpha; \\
    x'_4 &= -\frac{W}{2} \sin \alpha + \frac{R}{2} - (R/\beta)F(t-\theta-\psi); \\
    y'_4 &= -\frac{W}{2} \cos \alpha; \\
    x'_5 &= -P \cos \alpha + \frac{W}{2} \sin \alpha \cos \theta - \frac{R}{2} \sin \theta; \\
    y'_5 &= \frac{W}{2} \cos \alpha + P \sin \alpha; \\
    x'_6 &= -P \cos \alpha - \frac{W}{2} \sin \alpha \cos \theta + \frac{R}{2} \sin \theta; \\
    y'_6 &= -\frac{W}{2} \cos \alpha + P \sin \alpha.
\end{align*}
\]

(3.8)

If two instants $t_1$ and $t_2$ are defined so that:
\[(t_1 + t_2) \mod 1 = (\theta + 2\psi + \beta) \mod 1\]

then,
\[
x_1'(t_1) = -x_6'(t_2) ; \quad y_1'(t_1) = -y_6'(t_2)
\]
\[
x_2'(t_1) = -x_5'(t_2) ; \quad y_2'(t_1) = -y_5'(t_2)
\]
\[
x_3'(t_1) = -x_4'(t_2) ; \quad y_3'(t_1) = -y_4'(t_2).
\]

Thus, Theorem 2 also applies to regular, periodic, crab gaits with a constant phase difference between all pairs of ipsilateral, adjacent legs.

**Theorem 5:** In a regular, periodic, crab gait, in which the phase difference between any pair of ipsilateral, adjacent legs is equal to a constant, \(\psi\), any support pattern has another symmetric pattern reflected about the origin and the time instants, \(t_1\) and \(t_2\), are given by:

\[(t_1 + t_2) \mod 1 = (\theta + 2\psi + \beta) \mod 1\]

where \(\theta\) is the lateral phase difference.

**Corollary:** The LSM of a regular crab gait in which the phase difference between any pair of ipsilateral, adjacent legs is equal to a constant, may be computed as the minimum distance, \(d_L\), over the time instants just before each of the six placing events, i.e. at times \(0^-, \theta^-, \psi^-; [(\theta + \psi)^-] \mod 1, (2\psi^-) \mod 1\) and \([(2\psi + \theta)^-] \mod 1.\)
3.6.3 Wave Gaits:

A typical analytical derivation of the expression of the LSM for a wave gait is shown below. Consider Figure 3.7 with Equations (3.7) but with the following restrictions:

(a) \( P > (R/\beta)\psi \) or \( \eta < \frac{1}{\psi/\beta} \)

No leg can be placed in a position ahead of the adjacent, ipsilateral leg preceding it.

(b) \( (1 - \beta) \leq \Theta, \psi \leq \beta \)

This ensures that one of legs 1 and 3 (2 and 4) and one of legs 3 and 5 (4 and 6) are always in support phase.

(c) \( (1-\psi) \leq \Theta \leq \psi \)

Notice that the restrictions (b) and (c) are satisfied by all perfect wave gaits.

At \( t = 0^- \):

\[
x_2 = P + R/2 - R/\beta (1 - \Theta)
\]

\[
x_3 = R/2 - R/\beta (1 - \psi)
\]

and

\[
d_1(0^-) = (x_1 + x_2)/2
\]

\[
= P/2 + R/2 - R/\beta (1 - (\Theta + \psi)/2)
\]

At \( t = \Theta^- \):

\[
x_1 = P + R/2 - (R/\beta)\Theta
\]

\[
x_4 = R/2 - R/\beta(1 - \psi)
\]

and

\[
d_1(\Theta^-) = P/2 + R/2 - R/\beta(1 - \psi + \Theta)/2
\]
Thus $d_1$ can be computed for the other 4 instants also, though without numbers for $\theta$ and $\psi$ this process may be rather difficult. In this case, however, in view of restriction (b), one of legs 1 and 3 is always on the ground and similarly, leg 2 or leg 4 is always in support phase. Also, from restriction (a), the front boundary of the support polygon (which determines $d_1$) has to be formed either by legs 1 and 2 or by legs 1 and 3 or by legs 2 and 3. The worst case times are therefore times at which neither leg 1 or leg 2 is on the ground and the corresponding ipsilateral, succeeding leg is as far back as possible. In other words, the worst case times are those instants at which either leg 1 or leg 2 is not on the ground and is about to descend. Thus, only the times $0^-$ or $\theta^-$ are the critical instants and the LSM is the minimum of $d_1(0^-)$ and $d_1(\theta^-)$. The LSM is maximized when the two expressions for $d_1$ are equated:

$$d_1(0^-) = d_1(\theta^-)$$

or

$$\theta = 1/2$$

This makes the gait symmetric and consequently a wave gait, and the corresponding LSM for this special case is given by:

$$\text{LSM} = d_1(0^-) \bigg|_{\theta=1/2}$$

or
This implies that the LSM of a regular, periodic gait with every pair of ipsilateral, adjacent legs differing by a constant phase angle, \( \psi \), is maximized when the phase difference between any pair of contralateral, adjacent legs (the lateral phase difference) is equal to 1/2. That is, the gait is symmetric and hence a wave gait. This has been shown by numerical experimentation [33] and this analytical exercise serves to confirm it. However, the analytical treatment cannot be called a proof as it is valid only under the three restrictions, (a), (b) and (c) described at the beginning of this subsection. However, Equation (3.9) is always valid. Further, if, \( \psi = \beta \), the gait is a perfect wave gait and from Equation (3.9), the LSM is given by

\[
\text{LSM}_{\text{perfect}} = \frac{P}{2} + R - \frac{3}{4}R/\beta \tag{3.10}
\]

This expression is derived in Reference [89].

Let restriction (a) be replaced by

\[
P > \frac{R \psi}{\beta} \quad \text{for} \quad \psi < \frac{1}{2}
\]

and

\[
P > \frac{R}{\beta} (\psi - \frac{1}{2}) \quad \text{for} \quad \psi \geq \frac{1}{2}
\]
As before, leg $\psi$ have any value between $(1-\beta)$ and $\beta$ and $\theta$ be between $(1-\phi)$ and $\phi$. This precludes the possibility of leg 5 (leg 1) or leg 6 (leg 2) forming the front (rear) edge of the support polygon, but permits leg 3 (leg 4) to be placed ahead of leg 1 (leg 2) and leg 5 (leg 6) to be placed ahead of leg 3 (leg 4). Now, expressions for $d_1(0^-)$ and $d(\theta^-)$ are not sufficient. The following relations can be derived in addition to expressions for $d_1(0^-)$ and $d_1(\theta^-)$.

At $t = \psi$:

\[ x_1 = P + R/2 - R(1-\psi)/\beta \]
\[ x_2 = P + R/2 - R(\psi-\theta)/\beta \]

and

\[ d_1(\psi^-) = P + R/2 - R(\psi - \theta/2)/\beta. \]

At $t = \lceil (\theta + \psi)^{-} \rceil \mod 1$:

\[ x_1 = P + R/2 - R(\theta + \psi - 1)/\beta \]
\[ x_2 = P + R/2 - R\psi/\beta \]

and

\[ d_1(\lceil \theta + \psi \rceil^{-} \mod 1) = \frac{P + R/2 - R/\beta(\theta/2 + \psi - 1/2)}{\beta}. \]

Considering these four expressions for $d_1$, it is evident that $d_1(0^-)$ and $d_1(\psi^-)$ increase with $\psi$ and $d_1(\theta^-)$ and $d_1(\lceil (\theta + \psi)^{-} \rceil)$ decrease with $\psi$. Similarly, $d_1(0^-)$ and $d_1(\psi^-)$ decrease with $\theta$ and $d_1(\psi^-)$ and $d_1(\lceil \theta + \psi \rceil^{-})$ decrease with $\theta$. The gait LSM is equal to the minimum of these four quantities. This suggests optimal phase angles, $\theta^*$ and $\psi^*$ exist.

Equating $d_1(0^-)$ to $d_1(\theta^-)$ gives
Equating $d_1(\psi^-)$ with $d_1(F(\theta+\psi^-))$ gives the same result. On the other hand, equating $d_1(\theta^-)$ with $d_1(\psi^-)$ or $d_1(F(\theta+\psi^-))$ with $d_1(0^-)$ yields

$$\psi^* = \frac{2}{3} \left[ \frac{p}{2(R/\beta)} + 1 \right]$$

This is subject to restriction (b) according to which

$$(1 - \beta) \leq \psi^* \leq \beta$$

The optimal $\psi^*$ for $\theta = 1/2$ can be obtained from a plot of $d_1(0^-)$ and $d_1(\psi^-)$ against $\psi$. Figure 3.9 shows examples in which the distance $d_1$ (normalized by dividing by the pitch) has been plotted for all six instants against $\psi$ for $\theta = 1/2$. For $\psi < 1-\beta$ or for $\psi > \beta$, the stability drops drastically as there are instants at which neither leg 1 (leg 2) nor leg 3 (leg 4) are in the support phase. For large $\beta$, the value of $\psi^*$ is within the prescribed range and for small $\beta$, $\psi^*$ is greater than $\beta$ (Figures 3.9.a and 3.9.b illustrate this fact for two different cases). Thus

$$\psi_{opt} = \text{MIN} (\beta, \psi^*)$$

will obtain the optimum phase difference, $\psi_{opt}$, for wave gaits. It is easy to show that $\psi^*$ is less than $\beta$ only if $\eta$, the stroke to pitch ratio, is greater than or equal to $\beta/(3\beta-2)$ and $\beta$ is greater than 2/3. Thus,
Figure 3.9: Optimal $\psi$ for a Wave Gait

$(1/\eta > 1/2\beta \Gamma(2\phi) \mod 13)$

(a) $\eta = 2.0, \beta = 0.9$
Figure 3.9 (b) $\eta = 1.0$, $\eta = 0.8$
Figure 3.9 (c) $\eta = 3.0$, $\beta = 0.8$
This is in conflict with Reference [89] which describes the perfect wave gait as the optimal gait for straight line locomotion. The wave gait with $\psi$ given by Equation (3.12) would seem to be superior as the LSM obtained by substitution into Equation (3.9) is greater than or equal to the LSM for the perfect wave gait (Equation (3.10)). This is only of academic interest as this conflict arises only for large stroke to pitch ratios and duty factors which will not be employed frequently in walking machines. This is not to say that Equation (3.12) describes the optimal six-legged crab gait. There is no guarantee that if the restriction (a) is removed, Equation (3.12) is the optimal condition. Nevertheless, this argument serves to illustrate a general procedure which can be followed for any given values of $\beta$, $\theta$ and $\eta$. Figure 3.9.b shows a more commonly encountered situation in which $\psi^*$ is greater than $\beta$ and therefore $\psi_{\text{opt}}$ is equal to $\beta$. Figure 3.9.c gives an idea of the variation of the stability for very high strokes.

It has been implicitly assumed in the preceding derivations that the gait is according to the gait diagram shown in Figure 3.7 and the order of stepping events is as described in the figure. Every such analytical derivation has to be based on similar assumptions and, therefore, should be validated numerically. This is a consequence of
Figure 3.10: Isostability Lines for a Wave Gait with Stroke Equal to Pitch (Lines of constant LSM/stroke are plotted for \( \eta = 1.0 \))
the piecewise-linearity of the domain. The philosophy of complementing analytical derivations with numerical experimentation has been adopted in this study.

Figure 3.10 shows the variation of stability \( S_r = \text{LSM/Stroke} \) with \( \psi \) and \( \beta \) for \( \lambda = 1.0 \) (which is a reasonable value). As expected, the stability improves with duty factor but not quite so with \( \psi \). For a given \( \beta \), as \( \lambda \) is unity, Equation (3.12) yields the optimal value of \( \psi \) and the best stability is obtained for \( \psi = \beta \). All the assumptions in this analysis are valid for these values of \( \beta, \psi, \theta \) and \( \lambda \).

3.6.4 Comparison of the Stability Margin and Longitudinal Stability Margin for Crab Wave Gaits

Up to this point the LSM has been the focus of all discussions. This is because, although the SM seems to be a better measure of stability, if the stepping sequence is known analytical expressions for the LSM can be easily found. The relationship between the SM and the LSM is studied here for crab gaits.

For this comparison a typical geometry has been chosen in which the pitch, the width and the stroke have been set to be equal. This geometry is representative of a machine like the ASV. The stability margins are normalized to the pitch. The parameters \( \theta \) and \( \psi \) have been assigned values which are used for perfect wave gaits (i.e. \( \theta = 1/2 \) and \( \psi = \beta \)).
Figure 3.11: Comparison of the Stability Margins for Crab Wave Gaits:
(a) Stability Margin; (b) Longitudinal Stability Margin;
($\psi = \beta$, $\Theta = 0.5$, $\eta = 1$, $\gamma = 1$)
Figure 3.11 shows plots of the variation of the SM and LSM with the crab angle for different duty factors. The angles between 0 and 90 degrees have been considered. The same behavior may be expected for negative crab angles because of the symmetry in the problem.

In Figure 3.11a, there is a transition crab angle below which there is a high stability region and above this value the stability margin drops drastically. The transition value is a function of the dimensions of the vehicle, the stroke and the duty factor. For large duty factors the transition value decreases and for $\beta = 1.0$, it is equal to zero. Within this transition value the SM is not affected significantly.

The variation of the LSM with the crab angle is shown in Figure 3.11b. Again, the presence of a transition value can be observed and the LSM follows similar trends. The LSM drops as the crab angle is increased and the magnitude of the slope increases after the transition value. The transition occurs at $\tan^{-1}(W/2P)$ for $\beta = 1.0$ at which the direction of progression is parallel to the line joining the projection of the CG with the mean position (position at mid-stroke) of leg 1. The transition values for lower duty factors are higher. The exact transition values are not of interest as it is unlikely that the such crab angles will be used for locomotion for any long period of time. If lateral locomotion is required, it is preferable to change the vehicle orientation and decrease the crab angle so that the angle
is reasonable small. A small crab angle (within the transition value) may be tolerated.

It should be noted that the SM is a lower bound on the LSM and that the LSM is a higher bound on the SM. Both measures of stability show similar variations with crab angle and duty factor. If the angular velocity of the vehicle is zero, according to the assumptions on which the plots are based, then the body (and the load-wrench) translates with respect to the support patterns and the LSM can be expected to be a good measure of stability. However, if the body angular velocity is nonzero, the velocity of the points C in Figure 3.5 is very sensitive to the angular velocity and it makes more sense to use the SM as a measure of stability. It should be remembered that both quantities are only measures of stability and use of either measure for motion planning will yield suboptimal results. In such a situation, the considerable reduction in computational complexity and the ease in deriving optimal values of gait parameters favor the use of the LSM for motion planning.

3.6.5 Optimization of Crab Gaits:

A general procedure for deriving optimal values for $\theta$ and $\psi$ in terms of the gait LSM was presented in Section 3.6.3. The same procedure can be applied to crab gaits with reference to Figure 3.8. The edge of the support pattern which determines $d_1$ ($d_2$) is called the front (rear) boundary. $\eta$ is restricted to be small so that leg 5 (1)
or leg 6 (2) cannot form the front (rear) boundary, if the ipsilateral preceding (succeeding) leg is in support phase. Also the restrictions (b) and (c) listed in a previous subsection are assumed to apply. That is, $\psi$ must lie between $(1-\beta)$ and $\beta$, and $\theta$ must be between $\psi$ and $(1-\psi)$. This is not true for large strokes and large crab angles, but it is necessary to make this simplification to analytically derive values for $\psi_{\text{opt}}$ and $\theta_{\text{opt}}$. The results derived from the above assumptions may be valid even when the restrictions are in fact violated. However, there is no guarantee for the validity of the results in such a situation. Also, by Theorem 5, it is necessary to consider $d_1$ only.

A critical crab angle, $\alpha_c$, can be defined as

$$\tan \alpha_c \triangleq \frac{W}{2P}$$

and this can be identified as the transition angle for $\beta = 1.0$ in Figure 3.11(b). It is convenient to define nondimensional parameters to describe the gait:

\[
\begin{align*}
\xi & \triangleq \frac{\tan \alpha}{\tan \alpha_c} = \frac{2P \tan \alpha}{W} \\
\mu & \triangleq 2 \tan \alpha \tan \alpha_c = \frac{H}{P} \tan \alpha = \gamma \tan \alpha \\
\kappa & \triangleq \frac{P \cos \alpha}{R/\beta} = \frac{\beta \cos \alpha}{\eta} \\
\gamma & = \frac{H}{P} \\
\eta & = \frac{R}{P}
\end{align*}
\]

The parameters $\gamma$ and $\eta$ are as defined earlier. The restrictions mentioned earlier can now be expressed analytically as:
\[
\frac{l}{k} < \frac{(1 + uE/2)}{(\psi + \xi /2)} \quad \text{for } \psi < 1/2
\]

and

\[
\frac{l}{k} < \frac{(1 + uE/2)}{((\xi /2 + \psi - 1/2)} \quad \text{for } \psi \geq 1/2
\]

As before, restrictions (b) and (c) apply:

(1-\beta) \leq \psi \leq \beta

and

(1-\psi) \leq \theta \leq \psi

Again, the LSM at times 0^-, \theta^-, \psi^-, (\theta + \psi)^- \mod 1, (2\psi)^- \mod 1 and (\theta + 2\psi)^- \mod 1 can be calculated. In view of the above assumptions, it is sufficient to consider only the first four instants.

The expressions for the foot positions in the [H] frame are given by Equation (3.8). The following expressions may be written for \(d_1\) (the legs forming the front boundary are indicated in parenthesis) for the four time instants:

At \(t = 0^-(\text{legs 2, 3})\):

\[
d_1 = \frac{(W/2) \sin \alpha + R/2}{\cos \alpha + P \sin \alpha} [\sin \alpha + P \cos \alpha - (R/\beta)(\psi - \Theta)]
\]

At \(t = \theta^- (\text{legs 1, 4})\):

\[
d_1 = -\frac{(W/2) \sin \alpha + R/2}{\cos \alpha + P \sin \alpha} [\sin \alpha + P \cos \alpha - (R/\beta)(\psi + \Theta - 1)]
\]
At \( t = \psi \) (legs 1, 2):
\[
d_1 = P \cos \alpha - (W/2) \sin \alpha + R/2 - R/\beta(\psi - \Theta) + \frac{[W/2 \cos \alpha + P \sin \alpha]}{W \cos \alpha}[W \sin \alpha - (R/\beta)\Theta]
\]

At \( t = (\Theta + \psi) \mod 1 \) (legs 1, 2):
\[
d_1 = P \cos \alpha - (W/2) \sin \alpha + R/2 - (R/\beta)\psi + \frac{[W/2 \cos \alpha + P \sin \alpha]}{W \cos \alpha}[W \sin \alpha - (R/\beta)(\Theta - 1)]
\]

Following the same argument as the one presented earlier for the zero crab angle case, the optimal situation is when
\[
d_1(0^-) = d_1(\Theta^-) = d_1(\psi^-) = d_1((\Theta+\psi)^- \mod 1)
\]

All four quantities are functions of \( \Theta \) and \( \psi \) and each equality can be represented as a straight line on the \( \Theta-\psi \) plane. Clearly there are \( ^4\mathbb{C}_2 (=6) \) such lines and it is possible to write equations for them.

\[
d_1(0^-) = d_1(\Theta^-) \text{ yields: } \left[ \frac{1}{2+\xi} + \frac{1}{2-\xi} \right] \Theta + \left[ \frac{1}{2-\xi} - \frac{1}{2+\xi} \right] \psi + \kappa \left[ \mu + \frac{1+\mu}{2+\xi} - \frac{1-\mu}{2-\xi} \right] = \frac{1}{2-\xi} \quad (3.13a)
\]

\[
d_1(0^-) = d_1(\psi^-) \text{ yields: } \Theta(1+\xi) \left[ \frac{1}{2} - \frac{1}{2+\xi} \right] + \left[ 1 + \frac{1+\xi}{2+\xi} \right] \psi = 1 - \kappa(1+\xi) \left[ \frac{(\mu-1)}{(2+\xi)} - \frac{\mu}{2} \right] \quad (3.13b)
\]
Some algebraic manipulation is required before the equations can be written in this form. It is clear that the gait can be described in terms of the three nondimensional parameters $\kappa$, $\xi$, and $\mu$ and the two phase differences, $\theta$ and $\psi$. The optimal gait parameters $\theta_{\text{opt}}$ and $\psi_{\text{opt}}$ are functions of these three parameters.

In general, only three of the six equations are linearly independent and if $\xi$ equals 0, only two are linearly independent. For example, if Equations (3.13a) and (3.13b) are used to determine $\theta$ and $\psi$, Equation (3.13d) is automatically satisfied. The first three equations are linearly independent unless $\xi$ is equal to zero (or the
crab angle is equal to zero). For positive $\xi$ corresponding to positive crab angles, the equations (3.13a) and (3.13b) yield the optimal $\theta$ and $\psi$ and (3.13d) is also satisfied. On the other hand if $\xi$ is negative ($\alpha$ is negative), the equations (3.13a) and (3.13c) yield the optimal $\theta$ and $\psi$ and Equation 3.13e is automatically satisfied. Further, if $\theta$ is replaced by $(1-\theta)$, $\mu$ by $-\mu$ and $\xi$ by $-\xi$ in (3.13b) and (3.13c), Equations (3.13e) and (3.13d) are obtained. For a nonzero $\xi$, Equation (3.13f) is not an optimal condition and need not be considered.

If $\xi$ is equal to zero, all equations are satisfied for the optimal $\theta = 0.5$ and $\psi$. It is easy to see then, that equations (3.13a) and (3.13b) yield $\theta = 0.5$ and equations (3.13b) and (3.13c) yield $\psi = 2/3(1+\kappa/2)$ (which agrees with Equation (3.12)) and that the other three equations are satisfied by these values of $\theta$ and $\psi$.

If the values of $\theta$ and $\psi$ derived from these equations fall outside the limits specified by one of the restrictions, the optimum value may be taken to be the nearest limiting value $(1-\beta)$ or $\beta$. If the restriction on $\kappa$ (and hence $\eta$) is violated, then the whole analysis is invalid and this analytical procedure fails. Again, though the procedure is not valid, the values for $\psi$ and $\theta$ obtained through this process may still yield better results than those obtained by using $\theta = 1/2$ and $\psi = \beta$, but this can not be guaranteed.

The above conclusions may be verified by numerical experimentation. A few examples have been considered to illustrate this point. A good way of understanding the problem is by plotting
Figure 3.12 Stability Contour Plots as a Function of Phase Differences
(a) LSM - $\alpha = 0$ deg., $\eta = 1.2$, $\beta = 0.8$
Figure 3.12 (b) LSM - $\alpha = 25$ deg., $\eta = 1.2$, $\beta = 0.9$
Figure 3.12 (c) LSM - $\alpha = 25$ deg., $\eta = 1.2$, $\beta = 0.9$
Figure 3.12 (p) ISM $\alpha = -25$ deg., $\nu = 1.2$, $B = 0.9$
Figure 3.12 (e) LSM - $\alpha = -25$ deg., $\eta = 1.2$, $\beta = 0.9$
Figure 3.12 (f) SM – α = 25 deg., η = 1.2 , β = 0.9
isostability contour lines for the given nondimensional parameters $\kappa$, $\xi$ and $\mu$ as a function of $\psi$ and $\theta$. This has been done in Figure 3.12 for a few cases. The stability margins have been normalized to the pitch. In Figure 3.12(a), the special case of $\alpha = 0$ degrees has been considered. The optimal $\psi$ is given by Equation (3.12) and $\theta_{opt}$ is equal to 0.5. The symmetry in the contour plot is due to the crab angle being equal to zero and this symmetry was discussed with reference to Theorem 3, earlier in this chapter. This symmetry vanishes when $\alpha$ is nonzero which is the case in Figure 3.12(b). This figure has been zoomed in to get the plot in Figure 3.12(c). The straight lines corresponding to equations (3.13a), (3.13b) and (3.13d) have been plotted on the $\psi-\theta$ plane (see dashed lines in Figure 3.12c) and the point of concurrence, as explained earlier, yields the optimum solution. The plots for $\alpha$ and $-\alpha$ are symmetric about the $\theta=0.5$ line (see Figures 3.12(d)). In Figure 3.12(e), Equations (3.13a), (3.13c) and (3.13e) are shown to yield the optimal solution. Figure 3.12(f) shows a similar plot with SM used as the measure of stability. The optimal value may be obtained numerically with the same ease but it is much more difficult to analytically predict the location of the optima. A comparison of figures 3.12(c) and 3.12(f) illustrates that these two quantities are completely different in behavior though they are both measures of stability.

The location of the optimal solution in the $\psi-\theta$ space may be intuitively understood in the following way. If $\alpha > 0$, the three
 foremost legs are 1, 2 and 3 and the instants before the stepping events for these three legs can be expected to determine $d_1$. Similarly, if $\alpha < 0$, 1, 2 and 4 are the foremost legs and the minimum distance $d_4$ over the three instants before the stepping events for these legs determines the LSM. Hence, equating these distances yields the optimum condition.

The above treatment of the optimization appears to be somewhat ad hoc and, further, it is difficult to prove that the phase differences thus obtained are optimal. However, the plots in Figure 3.13 should serve as evidence of the improvement in the LSM of crab gaits. The analytical derivations are based on the assumption that the legs 5 and 6 never constitute the front boundary and this may not be the case when the crab angles increase beyond $\alpha > \xi > 1$. Thus the above formulation is valid for small crab angles. From the figure it is also evident that the improvement from the optimization process extends to a crab angle for about 40 degrees for the given $\eta$ (stroke to pitch ratio) and $\gamma$ (pitch to width ratio) of unity. The SM (Figure 3.13a) is not significantly improved by this optimization process. The figure shows plots for only positive crab angles since both quantities are even functions of crab angle. A similar variation in the SM and LSM may be seen for $\eta = 1.2$, $\gamma = 1.0$ in Figure 3.14.

As mentioned earlier, a legged vehicle is not likely to operate with a large crab angle for any length of time. On the other hand, the LSM increases (for relatively low crab angles) with an increase in
Figure 3.13 Stability of Optimized Crab Wave Gaits for Small Crab Angles. $\eta = 1.0$, $\gamma = 1.0$. 
crab angle if the above optimization process is used and the optimal $\Theta$ and $\psi$ are used. In addition, in straight line locomotion, the leg interference problem is somewhat alleviated. This is because, in an ideal gait, if the crab angle is zero, the ipsilateral legs must stroke along the same straight line. Hence a small crab angle may even be beneficial. However, the crab angle is likely to be limited typically to within $\alpha_c$. This justifies changing the longitudinal and lateral phase differences ($\psi$ and $\Theta$) based on Equation (3.13) in a steady state condition, depending on the nondimensional parameters $\kappa$, $\mu$ and $\xi$.

3.6.6 Follow-the-Leader Gaits

The discussion on follow-the-leader or FTL gaits is limited to continuous FTL gaits. This is because it is desired to have a continuous uniform motion as far as possible. In the event that it is not possible to navigate using such a scheme a discontinuous gait may have to be chosen. Such a strategy is expected to be employed in a supervisory mode of operation which involves considerable human participation, and is beyond the scope of this dissertation.

Continuous FTL gaits may be thought of as a special class of wave gaits. In the event that these gaits are not symmetric, they may be treated as any other regular, periodic gait with equal phase differences between all pairs of adjacent, ipsilateral legs. The representation and treatment of FTL gaits are exactly the same as
Figure 3.14 Stability of Optimized Crab Wave Gaits for Small Crab Angles. $\eta = 1.2, \gamma = 1.0.$
described for wave gaits in Section 3.6.3 with one major restriction:

\[(\psi/\beta)\eta = 1\]  
(3.14)

This is a direct consequence of the fact that the footprint of a leg must coincide with the footprint of the ipsilateral leg which immediately precedes it [89]. This also means that \(\psi\) must be greater than \(\beta\) to avoid collision between two adjacent, ipsilateral feet and consequently, \(\eta\) is greater than \(\eta (\eta < 1)\). However, the FTL gaits observed in nature are a little approximate in that the footprints of adjacent, ipsilateral legs do not coincide exactly. The succeeding foot is placed beside the other foot in order to circumvent the problem of collision. If this approximate FTL behavior is acceptable then \(\psi\) need no longer be greater than \(\beta\). The problem of collision between legs is automatically avoided in FTL gaits. In an ideal FTL gait the stability is poor, as \(\psi > \beta\). This can be seen in Figures 3.9.b and 3.10 where the LSM falls drastically for values of \(\psi\) greater than \(\beta\). On the other hand, if the approximation can be tolerated then the FTL gaits have good stability as they can have any value of \(\psi\) subject to the restriction in Equation (3.14). The FTL gait may also be considered as a crab gait and the results derived for the crab gait are valid here if \(\kappa\) is made equal to \(\psi\).
3.6.7 Effect of an Arbitrary Load-Wrench on Locomotion on Smooth Terrain

Figure 3.15 shows a plan view of a legged system with a wrench axis intersecting the support plane or the \( \pi \)-plane at \( P_\$ \) or \((x_\$, y_\$)\). The wrench axis is, in general not perpendicular to the \( \pi \)-plane. Any measure of the quasi-static stability as defined in Section 3.5 has to be based on measurements on the \( \sigma \)-plane which is perpendicular to the wrench axis. However, as the terrain is smooth, for a constant load-wrench, the orientation between the \( \sigma \)-plane and the \( \pi \)-plane remains constant for a given load-wrench. It is proposed to derive optimum lateral and longitudinal phase differences using as the measure of stability, the LSM based on the support pattern, which is obtained by projecting the support points on the \( \pi \)-plane. This approximate LSM suffices as it differs from the (quasi-static) LSM by a constant multiple which depends on the orientation between the load-wrench and the \( \pi \)-plane, and any solution generated by an optimization based on the approximate LSM will also be optimal with respect to the quasi-static LSM. For this reason, it is assumed in the rest of this subsection that the \( \sigma \)-plane and the \( \pi \)-plane are the same.

Thus in Figure 3.15, the LSM is given by

\[
\text{LSM} = (x_1 - x_\$) + \frac{(y_\$ - y_1)(x_2 - x_1)}{(y_2 - y_1)}
\]  
(3.15)
Figure 3.15: Motion Along the Longitudinal Axis of the Vehicle Body on Smooth Terrain with an Arbitrary Load Wrench
The crab angle has been set equal to zero to simplify the derivation. Once again the phase differences are restricted by the assumptions listed earlier in Section 3.6.3:

(a) $P > \frac{R\Phi}{\beta}$ for $\Phi < 1/2$

and

$P > \frac{R(\Phi-1/2)}{\beta}$ for $\Phi \geq 1/2$

(b) $(1 - \beta) \leq \theta, \psi \leq \beta$

(c) $(1-\psi) \leq \theta \leq \psi$

This helps simplify the analytical manipulations. Following the procedure described in Section 3.6.2,

$$d_1(0^-) = (R/2 - R/\beta (1-\psi) - x_s) + \frac{(y_s - W/2)(P - R/\beta (\psi-\theta))}{W}$$

$$d_2(\theta^-) = (P + R/2 - (R/\beta)\theta - x_s)^{-1} (W/2 - y_s)(P + R/\beta (1-\psi-\theta))$$

$$d_1(\psi^-) = (P + R/2 - (R/\beta)\psi - x_s) + \frac{(W/2 - y_s)(R \theta/\beta)}{W}$$

$$d_1(F(\theta+\psi)^-) = (P + R/2 - (R/\beta)F(\theta+\psi) - x_s) + \frac{(y_s - W/2)(R/\beta)(F(\theta+\psi) - \psi)}{W}$$

Let $\bar{y}$ be a nondimensional parameter equal to $y_s/W$ and $\kappa$ be defined as earlier, but with the crab angle equal to zero, $\kappa = P/(R/\beta)$.

Equating $d_1(0^-)$ to $d_1(\theta^-)$,
\[ \theta = \frac{1}{2} + \bar{y} \left(2(\kappa - \psi) + 1\right) \] 

Equating \( d_1(\theta^-) \) to \( d_1(\psi^-) \),

\[ \psi = \frac{\kappa (\bar{y} + 1/2) + 1}{(3/2 + y)} \] 

Equating \( d_1(\theta^-) \) to \( d_1((\theta + \psi)^{-\text{mod} 1}) \),

\[ \psi = \frac{\kappa (1/2 - \bar{y}) + 1}{(3/2 - y)} \] 

Equating \( d_1(\theta^-) \) to \( d_1(\psi^-) \)

\[ \theta + \psi (\bar{y} - 3/2) - (\kappa + 1)(\bar{y} - 1/2) = 0 \] 

Equating \( d_1(\theta^-) \) to \( d_1((\psi + \theta)^{-\text{mod} 1}) \)

\[ \theta + \psi (\bar{y} + 3/2) = (\kappa + 2) + (\kappa + 1)(\bar{y} - 1/2) \] 

Equating \( d_1((\theta + \psi)^{-\text{mod} 1}) \) to \( d_1(\psi^-) \)

\[ \theta = \bar{y} + 1/2 \]
Once again these equations are straight lines in the \( \Theta-\psi \) plane and if \( \bar{y} \) is nonzero, three of the six equations are linearly independent. In the special case where \( \bar{y} \) is equal to zero the equations yield the same results as were obtained earlier where \( \Theta_{\text{opt}} \) is equal to 1/2 and \( \psi_{\text{opt}} \) is given by Equation (3.12). Once more, Equation (3.15f) does not represent an optimal condition. Equations (3.15a) and (3.15b) define the optimal parameters when \( \bar{y} \) is greater than zero and Equation (3.15d) is automatically satisfied. When \( \bar{y} \) is less than zero, equations (3.15a) and (3.15c) govern the optimum. This time, Equation (3.15e) is automatically satisfied. These conclusions can be verified by substituting numerical values in these equations. Also, it is evident that Equation (3.15e) can be obtained from Equation (3.15b) and Equation (3.15d) from Equation (3.15c) if \( \bar{y} \) is replaced by \(-\bar{y}\) and \( \Theta \) by \((1-\Theta)\). Clearly, \( \Theta_{\text{opt}} \) is an odd function and \( \psi_{\text{opt}} \) an even function of \( \bar{y} \).

This analysis indicates the need for two nondimensional parameters, \( k \) and \( \bar{y} \). Notice that any projection from the \( \pi \)-plane to the \( \sigma \)-plane leaves these parameters unaffected. Thus the results of such an exercise performed on the \( \pi \)-plane apply to the quasi-static case (referred to the \( \sigma \)-plane). The same procedure may be used to deal with the situation in which the machine is walking on a smooth incline and the weight is the only force acting on it. The load-wrench is no longer perpendicular to the \( \pi \)-plane and the situation is mathematically equivalent to one in which the vehicle is walking on an
even terrain with a load-wrench which does not coincide with the $Z_B$ axis.

The derivation of these expressions is based on assumptions which must now be satisfied if the analysis is to remain valid. If either of $\psi$ or $\theta$ fail to satisfy these constraints then the "optimum" value is the closer of the two limiting values of $((1-\beta)$ or $\beta$ for $\psi$ and $(1-\psi)$ and $\psi$ for $\theta$). An important conclusion, which may be drawn from these expressions, is that they are independent of $x^\parallel$. In other words, the changes in the load-wrench caused by forward accelerations or decelerations do not affect the optimum phase differences. Similarly, a terrain inclined to the $X_B-Y_B$ plane and perpendicular to the $X_B-Z_B$ plane does not affect the optimal characteristics or phase differences.

The same process can be followed for crab gaits on smooth terrain but it is much more cumbersome and will not be attempted here. Unfortunately, no such derivations are possible for rough terrain as the $\pi$-plane is constantly changing. In such a condition, it is meaningless to optimize the gait as it is unlikely that the gait will follow any predetermined pattern as discussed in Chapter 5 with reference to Adaptive Gait Control.
3.7 Gait Selection and Optimization for a Walking Machine:

3.7.1 Gait Selection

The problem of selecting gaits is very complicated and this is evident from observations of biological systems. The criteria for selecting a gait for a quasi-statically balanced machine for operation on uneven terrain are:

1. High stability
2. High mobility
3. Terrain adaptability
4. Versatility
5. Simplicity of finite-state logic and associated computations
6. Energy efficiency

Wave gaits, FTL gaits and the free-gaits are compared based on the above criteria. These gaits are modified for omnidirectional locomotion by the introduction of a crab-angle. There are many other gaits described in the literature [89], and the Large-Obstacle-Gait (LOG) is one such useful gait. Although the LOG can be used for obstacle crossing, its utility in a cruising or a terrain-following mode is limited. The wave gaits, FTL gaits and the free gait are believed to encompass a regime that is general enough for this discussion.

It was shown in Section 3.6.4 that the LSM and SM are both measures of stability and gaits optimized with either of these
measures as a basis are, at best, suboptimal. However, considering the fifth criterion, i.e. simplicity of control computations, it is best to use the LSM as a basis for characterizing stability. On even terrain, the crab wave gait can be optimized for maximum LSM. Although it may not be easy to prove that this optimization produces the optimal gait, the crab wave gait with appropriate values for $\theta$ and $\psi$ (depending on $\alpha$) is believed to be the most stable in terms of the LSM. In terms of speed the wave gait provides greater mobility than the other gaits. The obstacle-crossing abilities of the wave gait may be limited, but at the pilot level of planning the vehicle is not expected to encounter large obstacles. The most attractive feature is the simplicity in the generation of stepping sequences and the symmetry in the stepping patterns.

The free gait optimizes for the best SM and the kinematic margin for every foothold. Though it is difficult to compare it to a periodic, structured gait, its stability is definitely comparable to, if not better than, to the stability of the crab wave gait. It is also versatile as it can navigate through a terrain which offers relatively few support sites for the legged system. However, the two disadvantages lie in the complexity in the associated control computations and its poor mobility in terms of speed. Also the free gait is designed to operate in a two dimensional terrain and is not of much use on uneven terrain.
The FTL continuous gait has poor stability as it is a wave gait with $\psi$ greater than $\beta$ (see Section 3.6.6) but the FTL discontinuous gait has excellent stability. The FTL discontinuous gait suffers from a difficulty in programming a sequence of steps which are general enough to handle variants in terrain relief. It is more suitable to a supervisory mode of control. The FTL continuous gait is attractive in terms of the reduction in complexity of the foothold selection algorithms. However, the restriction on the support-sites of the middle and rear legs do limit its versatility. The approximate FTL gait, as discussed in Section 3.6.6, can be implemented as a wave gait and offers a little more flexibility.

It is proposed to use a gait that combines the simplicity of the wave gait and the versatility of the free gait. This gait is called a modified wave gait. The modified wave gait or (MWG) is a regular periodic gait with the same phase difference between any pair of adjacent ipsilateral legs ($\psi$). The variables $\theta$ and $\psi$ are varied to optimize the gait depending on the crab-angle. The stepping sequence in time is exactly like the wave gait, but the spatial distribution of support sites is not symmetric. The choice of footholds is based on the load-wrench, terrain features and quasi-static stability. Such a gait would maximize suboptimal (to simplify control computations) measures of stability and also permit a wide range of duty factors, stride lengths, pitches, strokes and other gait parameters. In addition, it would enable a maximum utilization of the workspace.
available to each leg. Appropriate control of the lateral difference, $\psi$ would also easily enable an FTL implementation of the modified wave gait.

3.7.2 Criteria for Gait Optimization

So far the optimization of gaits for omnidirectional locomotion has been discussed with $\beta$, $\eta$ and $\alpha$ as input parameters. In the real world situation, for a given hardware configuration, it is possible to vary the pitch, $P$, for a given width (or the width to pitch ratio, $\gamma$), the stroke to pitch ratio, $\eta$, the crab angle $\alpha$, the time period, $T$ and the duty factor, $\beta$ in addition to the phase differences, $\psi$ and $\Theta$. The criteria for the choice of $\psi$ and $\Theta$ has been discussed in Section 3.6.5. It is assumed that the other parameters will be selected by the guidance module in accordance with the objectives outlined by the navigator. In this subsection, the rationale behind the choice of these parameters is discussed.

It is assumed that the navigator specifies the desired average speed of the vehicle, a minimum longitudinal stability margin which the vehicle must maintain, and a desired LSM which the vehicle must strive for. Further, it should be recalled that in rough terrain, the guidance module is expected to select suitable footholds. This will result in irregular support patterns which do not conform to the ideal support patterns which were used as a basis for optimization.
The crab angle $\alpha$ is regulated by the body motion regulation scheme. This regulation scheme is discussed later in greater detail. As was mentioned earlier, a value of $\xi$ which is greater than 1.0 is not desirable, and values of $\xi$ less than 1.0 can be tolerated and may be even desired if the crab gaits are suitably optimized.

The pitch and the stroke are constrained by the hardware of the vehicle and, if the stroke is less than the maximum possible stroke, the centers of stroke may be shifted, thus allowing an increase or a decrease in the pitch [973]. An increase in pitch is advantageous in that it increases the stability of the vehicle. However, the guidance algorithm is expected to use stability considerations for the foothold selection procedure, and in irregular support patterns engendered by such an algorithm, a pitch cannot be defined. Even if an average or an instantaneous pitch is defined, it is not necessary to maximize this quantity as the foothold selection module is expected to select footholds to maximize stability. Thus the pitch can be considered to be fixed at a value which enables the leg to achieve maximum stroke.

The variation of stability with the stroke to pitch ratio has been discussed in detail in Reference [89]. Figure 3.16 shows the variation of the LSM and SM with stroke for the case in which the pitch to width ratio, $\gamma$, is fixed at 1.0, and the crab angle is zero. The LSM and SM are normalize to the pitch in the figure. The plots suggest that, depending on the LSM desired by the navigator, a suitable $\eta$ and $\beta$ may be picked to achieve the desired LSM. The SM
Figure 3.16: Variation of Stability with the Stroke to Pitch Ratio
(a) Stability Margin (SM)  (b) Longitudinal Stability Margin (LSM)  ($\gamma = 1.0$)
shows a similar trend, although there is a saturation effect as the SM can never be less than half the vehicle width. Although only the \( \alpha = 0 \) degrees case is shown in Figure 3.16, similar graphs can be drawn for any value of \( \alpha \).

If the legged system is considered to be a sampled data system relative to the terrain profile, it does not respond to terrain variations with a wavelength less than the stride length, \( \lambda \) [99]. Thus for a given terrain, it is possible to determine a suitable \( \lambda = R/\beta \) and in the absence of terrain information it would seem to be logical to attempt to maximize this value. However, the filtering behavior for the longer wavelength is different. It is better to have shorter steps to negotiate changes in gradients. In addition, on uneven terrain, the irregular support patterns may make it difficult to permit large strokes. Short but sure steps are more likely under such conditions. Finally, the linear velocity of the legs can be related to the gait by:

\[
v = \frac{R}{\beta T}
\]

The minimum time period is restricted by the maximum velocity of the return stroke (\( V_{R,\text{max}} \)) determined by the actuator control system characteristics. Thus,

\[
\frac{R}{(1-\beta)T} \leq V_{R,\text{max}}
\]
These equations constrain $R$, $\beta$ and $T$ for a given vehicle velocity. Another consideration is the budgeting of the load between the feet. The greater the duty factor, the greater the number of feet sharing the load, which is clearly desirable. Thus, there are multiple constraints and obvious trade-offs in selecting the stroke to pitch ratio, or the stride length.

For locomotion on rough terrain, the primary problem is coping with the irregularity of the support patterns engendered by the foothold selection algorithm. The velocity is likely to be low so that time period considerations should not dictate the choice of gait parameters. It is appropriate to select a minimum duty factor which will ensure the desired LSM. It is true that plots like the one in Figure 3.16 are valid only for ideal, symmetric support patterns. Nevertheless, they do provide useful information about the tradeoffs involved and a suitable duty factor may be selected incorporating, if necessary, an appropriate factor of safety.

Though the simulation of the guidance module described in Chapter 6 does not include the gait optimizer, the operation of such a unit is outlined here. In this research, the primary objective is to study aspects of rough terrain locomotion. Therefore, a maximum stroke, $R$, must be selected depending on the terrain gradients ahead of the vehicle. Thus a minimum duty factor, $\beta_{\text{low}}$, and a maximum stroke, $R_{\text{max}}$, are calculated by the gait optimizer. The gait controller attempts to maximize the stride length subject to the constraints on
the stroke and the duty factor. The crab angle is regulated independently by the body motion regulation module. The optimal phase difference \( \Theta_{opt} \) and \( \phi_{opt} \) can be easily computed from the information regarding \( \alpha, R, \beta \) and the load-wrench (\( \mathbf{w} \)).

Optimization of the crab gait is meaningful only if the gait reaches some sort of a steady state. If the terrain is constantly changing, generating optimal values of \( \Theta \) and \( \Theta \) is not beneficial as the adaptive gait controller is likely to change the stroke and the duty factor continuously. In the general case, it is meaningful to configure the optimizer as an expert system which recommends optimal gait parameters to the gait controller using terrain information, past vehicle status and gait history, and the goals set by the navigator to recommend optimal gait parameters to the gait controller.

3.8 Conclusion

This research has been aimed at two aspects of walking machine gaits which have been largely ignored so far. These are omnidirectional locomotion and rough terrain or three dimensional locomotion. A survey, and critical analysis, of the available literature has been presented. Existing definitions and theorems have been modified and generalized to accommodate omnidirectionality and three dimensionality. The crab angle has been incorporated into this study to account for omnidirectionality. The issue of stability is particularly important for walking machines and has been examined in
some detail. However, only static stability or, at best, quasi-static stability has been considered, as most existing walking machines have sought to perfect this mode of operation.

Two measures of stability, the longitudinal stability margin (LSM) and stability margin (SM) have been proposed to measure quasi-static stability. The SM is believed to be a better measure of stability but it is difficult to analytically estimate the SM due to the nonlinearities associated with the computations. Numerical experimentation was used to compare the two measures of stability. A new approach for studying gait stability, based on critical events, has been followed. This approach allows the LSM to be expressed analytically and closed form expressions can be developed for the gait parameters. Analytical optimization of crab gaits based on the LSM is cumbersome but ad hoc treatment of the subject is possible. Such an exercise has been carried out for crab gaits, and the stability characteristics of these gaits have been enhanced for small crab angles. The dependency of the optimal phase differences, based on the LSM, on inertial loading has been established. Such optimization exercises yield analytical expressions for the gait parameters for the modified wave gait which are suboptimal, but easy to compute in real-time environments. The rationale behind the design of such a gait optimizer is discussed. These suboptimal gait parameters serve as inputs to an adaptive gait controller for a legged robot which is described in greater detail in Chapter 5.
CHAPTER IV
FORCE DISTRIBUTION IN CLOSED KINEMATIC CHAINS

4.1 General

The work described in this chapter addresses the problem of the appropriate distribution of forces between the legs of a legged locomotion system. The legs of the walking machine and the terrain form closed kinematic chains. The system is statically indeterminate and an optimal solution is desired for force control [403, 1100]. In addition, as unisense force limitations are imposed on the wrenches acting at the feet (the terrain cannot pull at the feet) it is important to be able to determine if any valid distribution of forces can be found to counteract a given load-wrench (as defined earlier in Section 3.5). If such a valid solution cannot be found, or if it can be shown that such a valid solution does not exist, then the legged system is unstable. This procedure of predicting instability can be used for planning by the guidance module to restrict the vehicle to safe configurations.

Another example of closed kinematic chains can be found in multifingered grippers. The interaction of the legged system with the ground is similar to the interaction between the fingers of a multifingered gripper grasping an object, and the object itself [45]. It is felt that the same analysis can be carried out for the force
distribution in both types of structures with some minor differences. The discussion in this chapter is general enough to be applied to terrain-vehicle systems and gripper-object systems. Any differences in the two problems are highlighted.

Fast and efficient algorithms to compute the contact forces for both these problems have been developed. The trade-off between computational simplicity and optimality makes it necessary to resort to sub-optimal algorithms [100]. The terrain-foot interaction and the finger-object interaction are modeled as point-contacts [52] and this permits 3 degrees of freedom at each support/contact point. The term contact force is used to describe the reaction force and it could be taken to mean the reaction force acting on a leg or the force applied by a finger on the object.

The redundancy in this system compares to the underdetermined system of equations encountered in the inverse kinematics of redundant robot manipulators. The Moore-Penrose Generalized Inverse [70] or the pseudo-inverse has also been suggested as a tool to obtain an artificial synergy for kinematically redundant manipulators [48]. The pseudo-inverse seeks the minimum Euclidean norm solution in an underdetermined system of equations, a least-squares solution in an overconstrained system, and in a general case, a minimum norm, least-squares solution [90]. The kinematic redundancy in robot manipulators has also been resolved through the minimization of the maximum norm (or the box norm) [39], the minimization of quadratic user-defined
Dextrous, multifingered grippers have been the object of considerable research [11], [52], [71]. The kinematics and force control problems engendered by these devices have been analyzed in References [29], [37], [84], [85]. Force control of such a system requires the specification of contact forces between the fingers and the gripped object. The gripper-object system has a high degree of static indeterminacy [37], [84], [85]. In addition to determining the forces required for equilibrium, it is essential to superimpose forces which squeeze the object to ensure that there is no slip at the points of contact [29], [30], [45], [52], [107]. The force distribution problem in walking machines [57], [100] seems to be different to the extent that it is not necessary to squeeze the terrain. This is because the terrain merely supports the machine and the machine need not have the ability to grip the terrain. However, this may not be the case on uneven ground and on gradients. This and the question of predicting instability will be discussed in the following sections.

4.2 The Force Distribution Problem

4.2.1 Governing Equations:

It is assumed that a desired body (object) trajectory is available, i.e. the desired acceleration and the current velocities of the vehicle (object) are available. The legs (fingers) are assumed to
be massless in comparison to the mass of the vehicle body (object). This is a realistic assumption for the Adaptive Suspension Vehicle. The forces \((R_x, R_y, R_z)\) and moments \((M_x, M_y, M_z)\) which must be applied by the feet on the body or by the terrain on the feet (or by the fingers on the object), are related to the given current velocities, \(\omega\) (angular) and \(v\) (linear), and the desired accelerations, \(\alpha\) (angular) and \(a\) (linear), by Euler's equations (all quantities are expressed in the body reference frame (object reference frame), \([B]\)):

\[
\begin{align*}
R &= \frac{d}{dt}(mv) + \omega \times mv - H \quad (4.1) \\
M &= \frac{d}{dt}(H) + \omega \times H \quad (4.2)
\end{align*}
\]

where

\[
H = \frac{R}{E} (-mg) ,
\]

\(\frac{R}{E}\) is the homogeneous transform which describes quantities defined in the earth-fixed reference frame with respect to the body fixed (object fixed) frame and \(H\) is the angular momentum of the body in reference frame \([B]\).

In other words,

\[
\begin{align*}
R_x &= ma_x + m(\omega_y v_z - v_y \omega_z) - \dot{W}_x \\
R_y &= ma_y + m(\omega_z v_x - v_z \omega_x) - \dot{W}_y \\
R_z &= ma_z + m(\omega_x v_y - v_x \omega_y) - \dot{W}_z \quad (4.3)
\end{align*}
\]

and

\[
\begin{align*}
M_x &= I_{xx} \alpha_x + (I_{zz} - I_{yy})\omega_y \omega_z \\
M_y &= I_{yy} \alpha_y + (I_{xx} - I_{zz})\omega_x \omega_z \\
M_z &= I_{zz} \alpha_z + (I_{yy} - I_{xx})\omega_x \omega_y \quad (4.4)
\end{align*}
\]
where $I_{xx}$, $I_{yy}$ and $I_{zz}$ are the principal moments of inertia. It is convenient to adopt the reference frame $\{W\}$ and the coordinate system defined in Chapter 3 with the origin at the centroid of the support points and the $Z$-axis parallel to the wrench axis (see Figure 4.1). If $F_i$ are the contact forces at the contact points and $r_i$ are the position vectors of the contact points, $(x_i, y_i, z_i)$ then, the equations of equilibrium for the vehicle body (or the object) can be written as:

\begin{align*}
\sum_{i=1}^{n} F_i &= R \\
\sum_{i=1}^{n} (r_i \times F_i) &= M
\end{align*}

Equations (4.5) and (4.6) can be represented in matrix notation as:

\[ G \mathbf{q} = \mathbf{w} \]

where,

\[ \mathbf{w} = \begin{bmatrix} R \\ M \end{bmatrix} \]
This represents a 6 by 3n system of equations which, in general, has 3n-6 degrees of freedom. The load-wrench defined in Section 3.5 is now identified with \( \mathbf{w} \). The vector \( \mathbf{w} \) can be written with respect to reference frame \([B]\) as:

\[
\mathbf{w} = \begin{bmatrix}
\mathcal{f} \\
\mathbf{c} + \mathcal{f} \times \mathbf{c}
\end{bmatrix}
\]

where \( \mathcal{f} \) is the force associated with the wrench (equal to \(|\mathbf{R}|\), \( \mathbf{c} \) is
Figure 4.1: The Force Distribution Problem
(a) Legged Vehicle
(b) Multifingered Grippers
the couple parallel to the wrench axis and $r$ is the position vector of
any point on the wrench axis. Each column in the $6 \times 3n$ matrix, $G$, defines a line in Plücker's coordinates [32] and thus represents a wrench of zero pitch. In general, the lines do not all belong to the same linear complex [32] and hence, the columns of $G$ span the six space of all possible wrenches and the equations are consistent.

4.2.2 Optimization

The redundancy in Equation (4.7) can be resolved through optimization of the force field based on suitable criteria. A linear programming approach was suggested by McGhee and Orin [57] for the coordination of legged systems using an energy minimization objective function and similar investigations for gripper-object interactions have been reported [37]. This section discusses the rationale behind such an optimization procedure.

In all such closed loops it is necessary to restrict the contact wrenches so that the normal component at each contact point is positive. In addition, the ratio of the tangential force component to the normal force component should be limited to within a maximum threshold value to prevent the possibility of slipping. The maximum value is the maximum coefficient of static friction available at the interface. In wheeled locomotion systems the specific resistance to locomotion increases drastically with the ratio of the tangential component to the normal component of force [101]. Although similar
data for legged systems is not available it is reasonable to believe that the same trend will prevail for walking machines. Hence, it should be beneficial to minimize the largest friction angle over all the contact points.

It is also desirable to minimize the largest contact force to prevent structural failure and to eliminate the possibility of saturation of the control system. An alternate function to minimize may be the sum of the magnitudes of the contact forces. In the ASV this is meaningful because the leakage losses in the hydraulic circuits driving the legs are proportional to the pressure difference across the actuators and are thus directly related to the contact forces. It is also possible to minimize the maximum actuator pressure difference but such an exercise requires conversions to actuator or joint coordinates and becomes cumbersome and specific to the particular system of interest. This problem is outside the scope of the present effort.

The optimization problem can be summarized as follows (n_i is the normal at the ith contact point, \( \phi_{\text{max}} \) is the maximum allowable friction angle):

**Variables:**

\[ F_{1x}', F_{1y}', F_{1z}', F_{2x}', F_{2y}', F_{2z}', \ldots, F_{nx}', F_{ny}', F_{nz}' \]
Equality Constraints:
- 6 equations of equilibrium

Inequality Constraints:
\[ \frac{E_i}{F_i} \cdot n_i > \cos \alpha_{\max}, \quad i = 1, \ldots, n \]

Objective Function (to be minimized):
- MINIMUM( \[ \frac{E_i}{F_i} \cdot n_i \], \quad i = 1, \ldots, n) 
  OR  MAXIMUM( |E_i|, i=1,\ldots, n) 
  OR  \[ \sum_{i=1}^{n} |E_i| \]

Further, the nonlinear constraints can be approximated by linear functions which makes the use of the linear programming method possible. This is discussed later in Section 4.6. An alternative formulation based on the pseudo-inverse is discussed next.

4.2.3 The Pseudo-Inverse:

The Moore-Penrose Generalized Inverse or the Pseudo-Inverse of \( G \), \( G^+ \), seeks to find the minimum norm, least-squares solution [90] for the force vector, \( g \) in Equation (4.7) and
The Pseudo-Inverse further satisfies the following conditions:

(i) \( GG^+G = G \)

(ii) \((GG^+)^T = GG^+\)

(iii) \(G^+GG^+ = G^+\)

(iv) \((G^+G)^T = G^+G\)

The first two conditions are necessary and sufficient conditions for \( g = G^+w \) to be a least-squares solution of Equation (4.7). The last two conditions are additional necessary and sufficient conditions required to ensure a minimum Euclidean norm solution. In addition, the Moore-Penrose Inverse is unique.

In general, \( w \) belongs to the column space of \( G \) and Equation (4.8) finds the solution vector, \( q \), which lies completely in the row space of \( G \). In other words, if \( g \) is expressed as

\[
g = q_h + q_p,
\]

where \( q_h \) is a solution to the homogeneous system of equations, then

\[
G q_h = 0,
\]

and thus, \( q_h \) belongs to the null space of \( G \). \( q_p \) belongs to the row space of \( G \) and satisfies the non-homogeneous equations:

\[
G q_p = \lambda
\]

Clearly, \( g \), which is the sum of the two components also satisfies the non-homogeneous equations, but, the minimal-norm solution is \( q_p \).

In Equation (4.7), the column space of \( G \) is of dimension 6 and the null space, of dimension \( 3n-6 \). It is also conceivable to employ the
3n-6 degrees of freedom or the null space components of \( q \), to achieve a secondary objective [48]. This possibility is discussed in a later section.

4.3 Decomposition of the Force Field

4.3.1 Definitions

The 3n force components acting at the n contact points constitute the force field of interest. This force field can be decomposed [45] into two force fields:

(a) The Equilibrating Force Field

(b) The Interaction Force Field

The interaction force between two contact points is the component of the vector difference between the contact forces, along the line joining the two contact points. If \( \mathbf{r}_i \) and \( \mathbf{r}_j \) are the position vectors of the ith and jth contact points with contact forces \( \mathbf{F}_i \) and \( \mathbf{F}_j \) respectively, then the interaction force \( \mathbf{F}_{ij} \) is defined by

\[
\mathbf{F}_{ij} = (\mathbf{r}_i - \mathbf{r}_j) \cdot (\mathbf{F}_i - \mathbf{F}_j), \quad i \neq j; \quad i, j = 1, \ldots, n
\]  

These forces are similar to the scalar internal forces which describe the pinch between two fingers in a gripper [11], [85].

The equilibrating forces are the forces required to maintain the body in equilibrium against the load wrench. The term equilibrating forces is used to describe a set of forces which necessarily has a
nonzero resultant force. In particular, the **equilibrating force field** consists of all equilibrating force vectors\(^3\) which do not have any interaction force components. This force field is similar to the velocity field of a rigid body where the difference between the velocities of any two points on the body has no component along the line joining the two points. An interaction force vector has a zero resultant and the **interaction force field** consists of all possible interaction force vectors.

4.3.2 The Pseudo-Inverse

It is possible to correlate the two force fields with the row space and the null space of \(G\) and the pseudo-inverse of \(G\). This is described by the following theorem.

**Theorem:** If a given force field, \(g\), is decomposed into the equilibrating force field and the interaction force field, then all vectors of forces belonging to the row space of the coefficient matrix, \(G\), (where \(G\mathbf{g} = \mathbf{w}\) represents the equilibrium equations) constitute the equilibrating force field and the interaction force field comprises all vectors of forces belonging to the null space of \(G\). In addition, the pseudo-inverse, \(G^+\), yields the solution \(G^+\mathbf{w}\) which belongs to the equilibrating force field.

---

3. The term **vector** is used to describe a column vector of force components and not necessarily a vector force with three components.
Proof:

The pseudo-inverse of $G$ maps the load wrench $w$ into the row space of $G$. This is evident from the properties of the pseudo-inverse and the minimum Euclidean norm solution. If the resulting force field, $q$, belongs to the row space of $G$, then $q$ can be written as a linear combination of the rows in $G$. In other words,

$$ q = G^T c $$

where

$$ c = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 & c_4 \end{bmatrix}^T $$

and $c_0$, $c_1$, $c_2$, $c_3$, $c_4$ and $c_5$ are constants (the superscript $T$ indicates the transpose). Thus,

$$ F_{ix} = c_0 + c_4 z_i - c_5 y_i $$

$$ F_{iy} = c_1 - c_3 z_i + c_5 x_i $$

$$ F_{iz} = c_2 + c_3 y_i - c_4 x_i $$

and if the constants are known then $q$ can be determined. The interaction force between any two contact points $i$ and $j$ is given by Equation (4.9) and substituting the above expressions for $F_i$ and $F_j$ gives:

$$ F_{ij} = (E_i - E_j) \cdot (r_i - r_j) $$

$$ = (F_{ix, i} - F_{ix, j})(x_i - x_j) + (F_{iy, i} - F_{iy, j})(y_i - y_j) $$

$$ + (F_{iz, i} - F_{iz, j})(z_i - z_j) $$

$$ = c_4 (z_i - z_j) - c_5 (y_i - y_j) (x_i - x_j) $$
\[ + [c_3(z_i - z_j) + c_5(x_i - x_j)](y_i - y_j) \\
+ [c_3(y_i - y_j) - c_4(x_i - x_j)](z_i - z_j) \]
\[ = 0 \]

This implies that any vector of forces which is a linear combination of the rows of \( G \) (i.e. a vector which belongs to the row space of \( G \)) does not have any interaction force components. Further if the vector of forces should satisfy the equilibrium equations (which \( G \) does) then the force field to which all such vectors belong, by definition, is the equilibrating force field. Therefore, the minimum norm solution yields a solution vector which lies in the equilibrating force field.

On the other hand, any vector of forces which has a component in the null space has interaction force components. This can be shown in the following way.

In a general case, \( G \) can be reduced by row operations to the form

\[
G = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & -z'_2 & y'_2 & 0 & -z'_3 & y'_3 & 0 \\
0 & 0 & 0 & z'_2 & 0 & -x'_2 & z'_3 & 0 & -x'_3 \\
0 & 0 & 0 & 0 & -y'_3 & x'_3 & Q & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

where

\[ x'_i = (x_i - x_i) \]
The process of elimination can be carried on further but the point of interest is the that any five of the first six columns and the seventh column are, in general, linearly independent. The sixth column is a linear combination of the first five columns as each set of three columns represents three zero pitch wrenches passing through a point of contact and two such sets of wrenches can never be linearly independent – they are reciprocal to a twist about a zero pitch screw along the line joining the two points of contact. These columns can be selected as a basis for the column space of G. Thus the 3n-6 free variables \([903]\) or the undetermined variables are \(F_{2z}’, F_{3y}’, F_{3z}’, F_{4x}’, F_{4y}’), F_{4z}’ \ldots, F_{nx}’, F_{ny}’, F_{nz}’\). The basis for the null space can be found by making all but one of these variables zero (for each of the 3n-6 free variables) and solving the homogeneous equations for the other 6 variables. The first basis vector obtained by constraining \(F_{2z}’\) to be equal to 1 and the other free variables to be equal to zero is:

\[
y_1’ = (y_1 - y_1') \\
z_1’ = (z_1 - z_1') \\
x_3’ = x_3 - \frac{x_2' z_3'}{z_2'} \\
y_3’ = -y_3' + \frac{y_2' z_3'}{z_2'}
\]

and

\[
Q = \frac{x_2' y_3'}{z_2'} - \frac{y_2' x_3'}{z_2'}
\]
Similarly, if $F_{3y}$ is equal to 1 and the other free variables are equal to zero, the second basis vector is obtained:

$$\begin{bmatrix}
-\frac{x_1'}{z_2'} & -\frac{y_1'}{z_2'} & -1 & \frac{x_2'}{z_2'} & \frac{y_2'}{z_2'} & 1 & 0 & 0 & \ldots & 0
\end{bmatrix}^T$$

It can be seen that the first basis vector for the null space involves forces at contact points 1 and 2 and these forces are equal and opposite. Thus, they have interaction force components and in addition, have no equilibrating force components (the resultant of these forces is zero). Hence the basis vector belongs to the interaction force field. The same holds for the second basis vector though it involves forces at contact points 1, 2, and 3 in the x-y plane. The same process may be carried out for all the basis vectors for the null space with similar results. Thus, the null space consists of all possible interaction force vectors which have a zero resultant and these are solutions to the homogeneous system of equations, $G \cdot g = 0$. This completes a proof for the theorem.

In Section 4.2.2, it was stated that an optimal solution might be one which minimizes the maximum contact force. The solution which achieves zero interaction forces minimizes the norm of the solution.
vector and in some ways does limit the contact forces. It means that the feet or fingers do not "fight" each other and thus the isometric work\(^4\) is minimized. Thus the pseudo-inverse is arguably a good technique to resolve the redundancy of the system. The drawback of this method lies in its inability to ensure that the friction angle constraints are met. With reference to walking machines, the variation of the terrain is not taken into account and there is no guarantee that the normal components of the support forces are positive. This suggests a method which superimposes on the equilibrating force solution an appropriate distribution of interaction forces designed to massage the solution so that the friction angle constraints are satisfied.

4.3.3 The Equilibrating Force Field

It was shown that the equilibrating force field comprises all vectors of forces which belong to the row space of the coefficient matrix, G. The screw corresponding to the load-wrench belongs, in general, to the sixth order screw system and hence to the six dimensional space, R\(^6\). If the interaction at each contact is modeled as a point contact, each support provides three components of forces.

\[^4\] A term used frequently in biomechanics to describe work which does not involve contractions of the muscle fibers but instead increases the tone of the fibers.
Thus the terrain or object can resist any zero pitch wrench through the contact point with the restriction that the resultant contact force lies within the friction cone. This restriction is ignored for the present discussion as the definition of equilibrating forces is independent of the terrain geometry.

The set of zero pitch wrenches through a point defines the special third order screw system [32]. Any set of three noncolinear contact points can resist any arbitrary load-wrench. This is because two contact points, each of which can resist wrenches belonging to a special third order screw system, span five degrees of freedom (they can not resist a moment about the line joining the two points), but three such special third order screw systems span the six-dimensional space, \( \mathbb{R}^6 \). Such a condition, which in general, involves more than 3 contact points, is called form closure [52]. In such a condition, there exists a unique vector of forces, which belongs to the equilibrating force field, and can counteract the load-wrench. The equilibrating force vector, \( \mathbf{g} \), is given by

\[
\mathbf{g} = \mathbf{G}^T \mathbf{c}
\]

As

\[
\mathbf{G} \mathbf{g} = \mathbf{w},
\]

\[
\mathbf{G} \mathbf{G}^T \mathbf{c} = \mathbf{w}.
\]

The load-wrench, \( \mathbf{w} \), belongs to \( \mathbb{R}^6 \) and if the number of contact points is \( n \), \( \mathbf{g} \) is a column vector of dimension \( 3n \) and \( \mathbf{G} \) is a \( 6 \times 3n \) matrix. If the \( 3n \) zero pitch wrenches span the six space then the rank of \( \mathbf{G} \) is
six and hence, $GG^T$ is full-rank (equal to 6). Thus if form closure exists, $GG^T$ is invertible and

$g = G^T (G G^T)^{-1} \mathbf{w}$

and the pseudo inverse, $G^+$, is given by:

$G^+ = G^T (G G^T)^{-1}$

(4.11)

The uniqueness of the Moore-Penrose Inverse and the simple, elegant formula for the inverse and the equilibrating vector makes it attractive. However, manipulating $6 \times 3n$ matrices can be cumbersome and is computationally expensive. A sacrifice in the form of a minor deviation from the minimum-norm optimality enables a considerable simplification in the computations. The simplification is achieved by decomposing the equilibrating force field into a force field with forces parallel to the load-wrench axis and another force field with forces on planes perpendicular to the load-wrench axis. In the coordinate system attached to the reference frame [$\mathbf{w}$], in Figure 4.1, this decomposition would result in forces parallel to the x-y plane or the $\sigma$-plane (defined in Chapter 3) and forces parallel to the z-axis. It is desirable that each of these two force fields be equilibrating force fields, though, as seen later, this is not possible. The decomposilion can be performed in two different ways.
Method I

The equilibrium equations involving the forces on the x-y plane can be written in the form:

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & \ldots & 1 & 0 \\
0 & 1 & 0 & 1 & \ldots & 0 & 1 \\
-y_1 & x_1 & -y_2 & x_2 & \ldots & -y_n & x_n
\end{bmatrix}
\begin{bmatrix}
F_{1x} \\
F_{1y} \\
F_{2x} \\
F_{2y} \\
\vdots \\
F_{nx} \\
F_{ny}
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}
\] (4.12)

This is an underdetermined set of three equations with 2n unknowns which can be inconsistent only if the projections of the n contact points on the x-y plane are concurrent. In general and especially for the ASV, this is never the case. The hypothesis that the equilibrating forces cannot have any interaction force components yields nC^2 equality conditions of the form:

\[(E_i - E_j) \cdot (P_i - P_j) = 0\]

or

\[(F_{ix} - F_{jx})(x_i - x_j) + (F_{iy} - F_{jy})(y_i - y_j) = 0; \ i, j = 1, \ldots, n \] (4.13)
143

The 2n zero interaction force restrictions are linearly independent only for the special case in which the number of contact points is equal to 3. In general, only 2n-3 equations are linearly independent and Equations (4.12) and (4.13) form a 2n x 2n system of linearly independent equations. Fortunately, there is no need to resort to numerical inversion. A simple solution is obtained through analytical inversion:

\[ F_{ix} = \frac{c(-y_i)}{n I} \]
\[ F_{iy} = \frac{c(x_i)}{n I} \]

and

\[ I = \frac{\sum (x_i^2 + y_i^2)}{n} \]  \hspace{1cm} (4.14)

It is easily verified that this system of forces satisfies Equation (4.12) and (4.13) and also that the resultant of \( F_{ix} \) and \( F_{iy} \) is related to the position vector, \( p_i \) by:

\[ (F_{ix} + F_{iy}) \cdot p_i = 0 \]  \hspace{1cm} (4.15)

The analogous kinematic system describing a rigid body in rotation would have the angular velocity of the body equal to \( \frac{c}{nI} \) and the velocity of the \( i^{th} \) point on the body would be equal to \( (F_{ix} + F_{iy}) \).
Having found the forces, $F_{ix}$ and $F_{iy}$, the three remaining equations of equilibrium can be written as:

\[
\begin{bmatrix}
1 & 1 & \ldots & 1 \\
(x_1-x_i) & (x_2-x_i) & \ldots & (x_n-x_i) \\
(y_1-y_i) & (y_2-y_i) & \ldots & (y_n-y_i) \\
\end{bmatrix}
\begin{bmatrix}
F_{iz} \\
F_{izz} \\
F_{nz}
\end{bmatrix}
= 
\begin{bmatrix}
f_i \\
\Sigma z_i F_{ix} \\
\Sigma z_i F_{iy}
\end{bmatrix}
\]

(4.16)

The unknown column vector on the right hand side is a function of the components $F_{ix}$ and $F_{iy}$ and hence depends on the equilibrating forces computed earlier. This constitutes a $3 \times n$ system of equations which is underdetermined when $n > 3$ or when $n$ is equal to 3 and the support points are colinear. The later possibility is extremely unlikely and is excluded from this analysis. For this system of equations, the pseudo-inverse solution does not belong to the equilibrating force field. In fact, the zero interaction force hypothesis over-constrains the problem. This hypothesis would imply, that if any two contact points do not lie on the same $z$-plane, the $z$-components of forces are equal. This would constrain all the $z$-components to be equal which would violate the two moment equilibrium relations in Equation (4.16). On the other hand, the pseudo-inverse yields a planar force distribution of the form
\[ F_{12} = A + B \left( x^1 - x^2 \right) + C \left( y^1 - y^2 \right) \] (4.17)

This is because the pseudo-inverse solution has to belong to the row space of the coefficient matrix in Equation (4.16). The coefficients A, B and C can be obtained by Gaussian elimination performed on a 3 x 3 system of equations. Alternatively, analytical expressions can be written for the three constants:

\[
B = \frac{(f_3 + x^2f_1)(y^2 - y_2^2) - (f_3 + y^2f_1)(x'y' - x^2y^2)}{D}
\]

\[
C = \frac{(f_3 + y^2f_1)(x^2 - x_2^2) - (f_2 + x^2f_1)(x'y' - x^2y^2)}{D}
\]

\[
A = f_1 - x^2B - y^2C
\]

where,

\[
D = (x^2 - x_2^2)(y^2 - y_2^2) - (x'y' - x^2y^2)
\]

\[
x'^2 = (x^1 - x^2)
\]

\[
y'^2 = (y^1 - y^2)
\]

\[
x'^2 = \frac{1}{n} \sum_{i=1}^{n} x'^2_i
\]

\[
y'^2 = \frac{1}{n} \sum_{i=1}^{n} y'^2_i
\]
This completes one method of solving for the equilibrating force vector. However, strictly speaking, the solution does not lie in the equilibrium force field. The z-components do not satisfy the zero interaction force hypothesis. This makes the method sub-optimal but its implementation is computationally efficient and fast.

**Method II**

This method decouples the sub-problems of finding forces parallel to the x-y plane and forces parallel to the wrench axis completely. This time, 5 equations are used to solve for $F^x$ and $F^y$.

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & \ldots & 1 & 0 \\
0 & 1 & 0 & 1 & \ldots & 0 & 1 \\
-y_1 & x_1 & -y_2 & x_2 & \ldots & -y_n & x_n \\
 z_1 & 0 & z_2 & 0 & \ldots & z_n & 0 \\
 0 & z_1 & 0 & z_2 & \ldots & 0 & z_n \\
\end{bmatrix}
\begin{bmatrix}
F_{1x} \\
F_{1y} \\
F_{2x} \\
F_{2y} \\
F_{nx} \\
F_{ny} \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
c \\
0 \\
0 \\
\end{bmatrix}
\]

(4.18)

If the pseudo-inverse is used as a tool to minimize the norm of the vector of forces, with some algebraic manipulation, an analytical expression for the forces can be found:
The z-components of the forces are found by considering the three equations in Equation (4.16). The terms $\Sigma z_i F_{ix}$ and $\Sigma z_i F_{iy}$ are zero by Equation (4.18). Again an analytical inversion for Equation (4.16) is possible and the $F_{iz}$ are given by:

$$F_{iz} = (f/D) (1 - A (x_i' - x_1') - B (y_i' - y_1'))$$

(4.20)

where,

$$x_i' = (x_i - x_1')$$
\[ y_i = (y_i - y_j), \]
\[ A = \frac{\Sigma x_i y_i^2}{\Sigma y_i^2} - \frac{\Sigma x_i y_i y_i'}{\Sigma y_i^2} - \frac{(\Sigma x_i y_i')^2}{\Sigma y_i^2}, \]
\[ B = \frac{(\Sigma y_i')}{D} - A(\Sigma x_i y_i'), \]
\[ D = n - \frac{(\Sigma y_i')^2}{\Sigma y_i^2} - A\left(\Sigma x_i' - \frac{\Sigma x_i y_i'}{\Sigma y_i^2}\right). \]

Equations (4.19) and (4.20) do not describe a force field with zero interaction forces (Equation (4.9) does not hold). The interaction forces on the x-y plane can be found to be:

\[ E_{ij} = \frac{c(yz(x_i - x_j) - xz(y_i - y_j))(z_i - z_j)}{n(\Sigma z^2 - yz - xz)} \]  
(4.21)

Unfortunately this quantity is not zero, unless the contact points are all on the same plane perpendicular to the wrench axis or along the same line parallel to the wrench axis. But, on the other hand, the complete decoupling of Equations (4.18) and (4.16) which are required to find the two force fields is an advantage. It is difficult to say which of the two methods is better as in either case, the solution is suboptimal.
Method I involves \(11n+16\) multiplications and \(11n+5\) additions and method II needs \(15n+13\) multiplications and \(16n-3\) additions. If parallel processing is available then method II cuts the time requirement to that needed for \(9n+3\) multiplications and \(7n-4\) additions. Clearly, method II is superior in a parallel processing environment (it takes 15-20% less time) but otherwise, method I is faster.

It was noted that the decomposition of the equilibrating force field, which is needed to simplify the problem, results in a solution which has interaction force components. Thus, the force vector obtained by such a method does not really belong to the equilibrating force field. In addition to this, the decomposition causes singularities in the equations of equilibrium which are briefly discussed here.

In Method I, Equation (4.12) becomes singular (rank of the coefficient matrix is equal to 2) if the projections of all support points on the X-Y plane are coincident. This is an unlikely event for a walking vehicle or a gripper-object system.

Equation (4.16) becomes singular if the projections of the contact points on the x-y plane are colinear. In walking machines, this situation is indicative of a zero or negative stability margin and would not be encountered. However, for multifingered grasps, this may pose a problem. In general, the columns of the coefficient matrix span the subspace consisting of all the lines (forces) perpendicular
to the X-Y plane. In the special configuration in which the projections of all the contact points on the X-Y plane lie along a line, the columns of the matrix span the subspace of all forces which are perpendicular to the X-Y plane and also intersect that line. In such a situation, the gripper cannot resist any external force in the z-direction, unless the line of action of the force intersects the line passing through the projection of the contact points on the X-Y plane. Thus, the decomposition scheme cannot be used to find the equilibrating forces in the special case of a planar grasp when the load-wrench parallel to the plane of the grasp. It is emphasized, that the equilibrium equations in Equation (4.7) and a minimum norm scheme may still be used to find the equilibrating forces.

4.3.4 The Interaction Force Field

The previous subsection discussed methods to find appropriate equilibrating forces. It may be essential to superimpose a set of bias forces or interaction forces (which belong to the interaction force field) which will ensure that the friction angle constraints are met. As explained earlier, the vector of interaction forces always belongs to the null space of the coefficient matrix, G. It is informative to examine the simpler cases in which the number of contact/support points is 2 or even 3, in which it is easier to visualize the interaction force field.
Two Contact Points

As the resultant of the contact forces must be zero, the two forces must act along the line joining the two points and must be equal and opposite. In Figure 4.2, 1 and 2 denote the two contact points, \( \hat{n} \), the unit normal along the line vector drawn from 1 directed towards 2, \( \hat{n}_1 \) and \( \hat{n}_2 \), the two unit normals at the two contact points and \( F_{1E} \) and \( F_{2E} \) the equilibrating force solutions generated as described earlier. \( F_{1I} \) is the interaction force at the \( i \)th contact point and \( F_{1I} = -F_{2I} \). Let \( F_{1t} \) and \( F_{2t} \) be the two tangential components and \( F_{1n} \) and \( F_{2n} \) the two normal components of the net force obtained by superposing the interaction forces on the equilibrating forces.

\[
F_{1n} = F_{1E} \cdot \hat{n}_1 + (F_{1I} \hat{n}) \cdot \hat{n}_1 \\
F_{2n} = F_{2E} \cdot \hat{n}_2 - (F_{1I} \hat{n}) \cdot \hat{n}_2 \\
F_{1t} = | F_{1E} - (F_{1E} \cdot \hat{n}_1) \hat{n}_1 + F_{1I} \hat{n} - F_{1I} (\hat{n} \cdot \hat{n}_1) \hat{n}_1 | \\
F_{2t} = | F_{2E} - (F_{2E} \cdot \hat{n}_2) \hat{n}_2 - (F_{1I} \hat{n} - F_{1I} (\hat{n} \cdot \hat{n}_1) \hat{n}_1 |
\]

The restrictions on the friction angle are:

\[
F_{1t}/F_{1n} \leq \mu
\]

and

\[
F_{2t}/F_{2n} \leq \mu
\]

where \( \mu \) is the maximum coefficient of friction.

There are two nonlinear inequalities relating the interaction forces to the terrain or object geometry. It is possible that an
Figure 4.2: Interaction Force Fields for a Two Fingered Grasp
(a) No valid solution for the interaction forces
(b) Minimum required interaction force
interaction force \( (\mathbf{F}_{1I}) \) which satisfies these inequalities cannot be found (see Figure 4.2(a)). If a solution does exist, it is possible to obtain a minimum required interaction force, \( P \), which will satisfy the inequality constraint for both contact points where \( F_{1I} = P \) and \( F_{2I} = -P \). This is illustrated in Figure 4.2 (b). The interaction force, \( P \), could turn out to be negative depending on the geometry and this would imply that \( F_{1I} \) acts in the opposite direction to \( \mathbf{e} \) and \( F_{2I} \) is along \( \mathbf{e} \).

**Three Contact Points**

The zero resultant condition forces the interaction force field to be planar. This can be verified through elementary statics and geometry. It is not possible for any of the interaction forces to have components perpendicular to the plane defined by the three points, or the \( \pi \)-plane defined in Chapter 3. Also, the interaction forces must all pass through a point. Both these statements can be verified by taking moments about different points on the \( \pi \)-plane. Figure 4.3 shows three contact points on the \( x-y \) plane or the \( \pi \)-plane (support plane) and the \( z \)-axis perpendicular to the plane. If \( (x_o, y_o) \) is the point of concurrence of \( F_{1I} \), \( F_{2I} \) and \( F_{3I} \), then the direction cosines of the \( i^{th} \) interaction force are given by:

\[
\hat{\mathbf{e}}_i = \frac{(x_0 - x_i) \mathbf{i} + (y_0 - y_i) \mathbf{j}}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}}
\]
Figure 4.3 Interaction Forces for a Three Fingered Grasp
The equality constraints are

\[ F_{1I} \hat{e}_1 + F_{2I} \hat{e}_2 + F_{3I} \hat{e}_3 = 0 \]

and the inequalities are of the form

\[ | F_{iE} + F_{iE} \hat{e}_i - (E_{iE} + F_{iE} \hat{e}_i) \cdot \hat{n}_j \hat{n}_j | \leq \mu (E_{iE} + F_{iE} \hat{e}_i) \cdot \hat{n}_i \]

There are five unknowns - \( F_{1I}, F_{2I}, F_{3I}, x_0 \) and \( y_0 \). The three force equilibrium equations are linearly dependent and of rank 2. This is because the three coplanar unit vectors \( \hat{e}_i \) pass through a point and only two of three such vectors can be linearly independent. Thus there are two equality constraints and three inequality constraints.

If the point of concurrence, \((x_0, y_0)\) is fixed, the three equilibrium equations determine the ratios of the magnitudes of the interaction forces and the inequalities can be used to scale the magnitudes of those forces. The smallest scaling factor which ensures that all three inequalities are satisfied is the obvious choice. Again, because of the way the interaction forces are defined, the magnitudes could be negative. This is easier to visualize for multifingered grippers and an example is shown in Figure 4.4. The centroid of the contact points is suggested as a convenient location for the point of concurrence in Reference [45]. The utility of such
Figure 4.4: Interaction Forces for a Three Fingered Grasp
(a) Positive Interaction Forces
(b) Negative Interaction Forces
simplifications is demonstrated by considering an example of a common assembly operation of screwing a nut onto a threaded bolt. Usually such an operation involves a small thrust as well as a moment to thread it in - the weight is assumed to be negligible compared to these forces. With a three-fingered symmetric grip, as described in Figure 4.5, the concurrence of the normals at the centroid of the contact points simplifies the problem enormously. A few numbers for an example case [45] are shown in Table 4.1. The maximum allowable friction angle has been set at \( \tan^{-1}(0.25) \). As a result of the symmetry in this particular case, the friction angles can be made equal for all fingers (see Table 4.1). In this simple example and for other such symmetric grasps, the symmetry assists both, the decomposition of the force field as well as the determination of the interaction forces. Also, the equilibrating forces do belong to the equilibrating force field and do not have interaction components. This is obviously because the contact points are all in the same \( z \)-plane.

If the number of contact points is greater than three it is somewhat more difficult to visualize the interaction force field. It is possible to consider a subset of the entire set of vectors of interaction forces which is composed of the interaction force fields generated by taking all combinations of 2 and 3 contact points at a time. Thus, in a four contact point case we have four possible combinations of 3 contact points \( (4C_3) \), and 6 possible combinations of 2 contact points \( (4C_2) \). Each of these 10 combinations corresponds to
Figure 4.5 A Three Fingered Grasp for a Hex-Nut
Table 4.1 Three Fingered Grasp for a Hex-Nut
(See Figure 4.5)

No. of fingers: 3

Coordinates of contact points:

<table>
<thead>
<tr>
<th>No.</th>
<th>(in meters)</th>
<th>Contact Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.0216, -0.0125, 0.0)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(-0.0216, -0.0125, 0.0)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(0.0000, 0.0250, 0.0)</td>
<td></td>
</tr>
</tbody>
</table>

Load force (N): 10k applied at (0.0, 0.0, 0.0)
Load couple (N-m): 1k

Equilibrating forces:

<table>
<thead>
<tr>
<th>(N)</th>
<th>( F_1 ) = 6.66I + 11.55j + 3.33k</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F_2 ) = 6.66I - 11.55j + 3.33k</td>
</tr>
<tr>
<td></td>
<td>( F_3 ) = -13.33I + 3.33k</td>
</tr>
</tbody>
</table>

Interaction forces:

<table>
<thead>
<tr>
<th>(N)</th>
<th>( F_{11} ) = -47.61I + 27.48j</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( F_{21} ) = 47.61I + 27.48j</td>
</tr>
<tr>
<td></td>
<td>( F_{31} ) = -54.98j</td>
</tr>
</tbody>
</table>

Predicted friction angle: \( \mu_1 = \mu_2 = \mu_3 = 0.25 \)
Actual friction angle: \( \mu_1 = \mu_2 = \mu_3 = 0.25 \)
an interaction force field, and it may be possible to find a suitable vector of interaction forces by superposing one or more of these fields which satisfies the inequality constraints.

Such strategies will work only if the direction of the interaction force, at those contact points at which the equilibrating forces violate the friction angle constraints, makes an angle with the surface normal at the contact point, which is less than the maximum allowed friction angle. Clearly, this was not the case in Figure 4.2 (a). For legged locomotion systems, in general, the interaction forces determined by these schemes will be almost parallel to the horizontal plane, and the normals are likely to point upward. Hence this scheme may not work. Nevertheless, this approach to the problem does lend a good physical insight and is well suited to analyzing the force distribution between the fingers of a gripper.

Screw theory allows a useful interpretation of the interaction force field. If the interaction force field consists of n wrenches (which must have a zero resultant), the screws corresponding to the n wrenches must, in general, belong to a screw system of order n-1. In special cases, they may belong to a screw system of order less than n-1. Further, if the wrenches are pure forces (which is the case for point contacts), the screw system must allow zero pitch screws (wrenches).

If the number of contact points is equal to 3, the interaction force field must correspond to special second order screw systems
[32]. The force field consists of coplanar zero pitch screws, which are all parallel or concurrent. The system of coplanar, zero pitch wrenches is used in Reference [45] (see figures 4.3 - 4.5). If the number of contact points is equal to 4, the zero pitch screws must belong to a third order screw system (including the special-three systems enumerated by Hunt [32]). The axes of the zero pitch wrenches lie along the generators of a hyperboloid of revolution. In a special case, they may all be concurrent [45], and the wrenches constitute the special three-system, which consists of the bundle of lines (axes of zero pitch screws) through a point. For the 5 contact point case, the axes or the zero pitch wrenches are, in general, members of a fourth order screw system. In the general case, only two lines (which may be imaginary) can intersect the axes of the zero pitch wrenches and the axes belong to a linear congruence. If the number of contact points is equal to six, the axes are members of a linear complex and have one reciprocal screw whose axis coincides with the axis of the complex. Finally, if the number of contact points is greater than six, in general, there is no restriction on the axes of the zero pitch wrenches.

The redundancy in the problem may be resolved by determining interaction forces to satisfy a secondary objective function, $\theta$. This involves a trade-off between the minimum norm solution and the optimization of an auxiliary function. The solution, $g$, to Equation (4.7) can be projected into the range space or row space of $G$: 
\[ q_p = G^+Gq \]
where \( q_p \) denotes the particular solution which belongs to the equilibrating force field. Similarly, the homogeneous component belonging to the interaction force field can be obtained by:

\[ q_h = (I_{3n} - G^+G)q \]
where \( I_{3n} \) is a \( 3n \times 3n \) identity matrix.

If \( \theta(q) \) is the auxiliary function to be minimized then \( q \) can be set to be \( -\alpha \nabla \theta \), where \( \alpha \) is a constant gain, and

\[ q_h = \alpha (G^+G - I_{3n}) \nabla \theta(q) \]

The modified inverse would be

\[ q = G^+w + \alpha (G^+G - I_{3n}) \nabla \theta(q) \]  \hspace{1cm} (4.22)
where the first term belongs to the equilibrating force field and the second to the interaction force field. The null space component attempts to minimize the function \( \theta(q) \).

For a given load-wrench, \( w \), the equilibrating force components can be easily found but the interaction force component is not easy to obtain. The process of minimizing \( \theta(q) \) is iterative and time consuming. Moreover, it is difficult to find a tractable function, \( \theta \), which can embody the friction angle restrictions. It is very difficult to find time-efficient techniques to obtain a suitable interaction force field unless the number of legs is less than or equal to three. The idea of superposing a set of bias forces is not very attractive from the point of view of implementation on a computer except for
simple cases, but the decomposition of the force field into an equilibrating force field and an interaction force field does lend a good insight into the problem and the equilibrating force field can be used to good advantage.

4.4 Decomposition of the Load-Wrench

On rough terrain, the normals at the contact points are directed arbitrarily and, in general, the lines are linearly independent. These lines can be represented using Plücker line coordinates [32]. This system of line coordinates is a homogeneous coordinate system in which each line is represented by six components. In a line, $s_1$, denoted by $(L_1, M_1, N_1, P_1, Q_1, R_1)$, $(L_1, M_1, N_1)$ are the direction cosines of a unit vector along the normal and $(P_1, Q_1, R_1)$ is the vector moment of the normal about the origin. The six components are related by the Quadratic Identity,

$$L_1 \cdot P_1 + M_1 \cdot Q_1 + N_1 \cdot R_1 = 0$$

and, in addition,

$$L_1^2 + M_1^2 + N_1^2 = 1$$

This brings the number of independent parameters down to four. The $n$ normals can be arranged as the columns of a $6 \times n$ matrix, $J$. If the $i^{th}$ normal is represented by $s_i$, then

$$J = [s_1 | s_2 | \ldots | s_n]$$
Figure 4.6: Decomposition of the Load-Wrench and the Force Field

$N(J)$, $C(J)$, and $R(J)$ denote the null space, column space, and row space of $J$ respectively.
Consider the system of equations given by

\[ J q = \omega \]  \hspace{1cm} (4.23)

It is interesting to note that this system is mathematically isomorphic to the kinematics of the well-known serial chain manipulator with revolute joints. If the manipulator has \( n \) revolute joints with their axes along the \( n \) normals, then Equation (4.23) represents the instantaneous kinematics of the manipulator - \( \omega \) is the instantaneous twist axis for the end-effector and \( q \) is the vector of unknown joint rates required to produce the desired end-effector rates. The matrix \( J \) is the well-known Jacobian matrix encountered in robotics and the inversion of the Jacobian to obtain the joint rate vector is a common operation.

If the load-wrench (end-effector screw) belongs to the column space of \( J \) or to the subspace spanned by the \( n \) normals, then a vector of forces (joint rates) which satisfy the equations of static equilibrium (kinematics) can be found. The forces obtained by inverting \( J \) have a zero friction angle as they are parallel to the corresponding normals.

In general, the load-wrench will not belong to the column space of \( J \). However, the load-wrench can be projected onto the subspace spanned by the normals, using the orthogonal projection, and onto a subspace orthogonal to the column space of \( J \) which is the null space
of $\mathbf{J}^T$ (see Figure 4.6). It is always possible to find forces acting along the normals which can counteract the projected component of $\mathbf{M}_{\text{proj}}$. The perpendicular or orthogonal component, $\mathbf{M}_{\text{res}}$, is the residual component, which should be small compared to $\mathbf{M}_{\text{proj}}$. This statement is justified in a quasi-static situation where the dominating contributor to $\mathbf{M}_{\text{proj}}$ is the weight of the vehicle body or the object. The gravity vector can be expected to lie in the subspace spanned by the contact normals, since the normals point in the general vertical direction. In any event, the residual component can be expected to be small. Thus, the system of equations can be reduced to:

$$\mathbf{J} \mathbf{g} = \mathbf{M}_{\text{proj}}$$

These equations are consistent and at least one solution exists. However, the normals are not always likely to be linearly independent. This is evident in the special case in which a walking machine encounters level terrain: a maximum of three parallel normals can be linearly independent or in planar grasps: at most three coplanar normals can be linearly independent. Hence, it may be necessary to select a minimum norm solution to the problem which eliminates the null space components. In other words, if $\mathbf{g}$ is decomposed into $\mathbf{g}_h$, the solution to the homogeneous equations and $\mathbf{g}_p$, the particular solution, the homogeneous component is ignored and the equations can be written in the form:

$$\mathbf{J} \mathbf{g}_p = \mathbf{M}_{\text{proj}}$$

(4.24)
This equation has a unique solution which may be also obtained through the pseudo inverse, $\tilde{\eta}^+$:

$$ q = \tilde{\eta}^+ M $$

(4.25)

None of the normal components can be negative and the pseudo-inverse does nothing to ensure this. If one of the components is negative it may be fixed at a small positive value so that the pseudo-inverse may be used to solve for the other components.

The residual component, $M_{res}$, should be small and the force, $q_{res}$, required to maintain equilibrium with $M_{res}$, should not change the friction angles drastically. $q_{res}$ may be selected from the equilibrating force field and the superposition of $q_{res}$ on $q_p$ should not affect the force distribution significantly.

This method relies on the assumption that $M_{res}$ is a small component of $M$. The validity of this assumption is examined in a later section. If this is indeed the case, the dominating component of the load-wrench ($M_{proj}$) is balanced by forces which are normal to the terrain. This method is optimal to the extent that it tends to keep the friction cone angles low. Comparisons of computational time and performance are made in a later section.
4.5 Variable Compliance

Another way of resolving the static indeterminacy is based on the principle of geometric compatibility, which is used for passive structures commonly encountered in structural mechanics or solid mechanics. If the body (object) is assumed to be rigid compared to the legs (fingers) and the legs (fingers) are assumed to be linearly elastic with the $i^{th}$ leg (finger) possessing stiffnesses $k_{ix}, k_{iy},$ and $k_{iz}$, then the force-displacement relations are:

$$F_{ix} = k_{ix} \delta_{ix}, \quad F_{iy} = k_{iy} \delta_{iy} \quad \text{and} \quad F_{iz} = k_{iz} \delta_{iz}$$

where $F_i$ and $\delta_i$ are the contact forces and the displacements respectively at the contact points. If the walking platform (object) has gross linear displacements $\Delta x, \Delta y$ and $\Delta z$ and gross rotations $\Delta \theta_x, \Delta \theta_y$ and $\Delta \theta_z$ about the $x, y$ and $z$ axes, then

$$\delta_{ix} = \Delta x + z_i \Delta \theta_y - y_i \Delta \theta_z$$
$$\delta_{iy} = \Delta y - z_i \Delta \theta_x + x_i \Delta \theta_z$$
$$\delta_{iz} = \Delta z + y_i \Delta \theta_x - x_i \Delta \theta_y$$

These are the geometric compatibility conditions. Combining these conditions with the force-displacement relationships the following equations can be written for each of the $n$ legs (fingers):
This is a very important simplification since the $3n$ unknown force components have been expressed as functions of only six unknown quantities. Further, as the equilibrium equations provide six conditions, in general, a unique solution for the six gross displacements can be obtained by inversion and the $3n$ unknown force components can be obtained in turn by substitution in Equation (4.26). However, the stiffness of the legs (fingers) are not known. The stiffness is a function not only of the structure of the leg (finger) and the drives actuating it but also a function of the simulated electronic compliance.

A further simplification to this problem is obtained by letting the $3n$ stiffness values be identical. If all the $k_{ix}$, $k_{iy}$ and $k_{iz}$ are set equal to $k$, then, equations (4.10) and (4.26) are identical. This can be seen if $c_0$ is substituted by $k\Delta x$, $c_1$ by $k\Delta y$, and so on. Thus the least squares minimum norm solution is also a solution to the equal compliance model. Thus, making all the compliances equal does not yield any new result.
If all the legs (fingers) were isotropic (equal stiffness in all directions) but with different compliances, then:

\[
\begin{bmatrix}
F_{ix} \\
F_{iy} \\
F_{iz}
\end{bmatrix} = k_i \begin{bmatrix}
1 & 0 & 0 & z_i & -y_i \\
0 & 1 & 0 & -z_i & 0 \\
0 & 0 & 1 & y_i & -y_i
\end{bmatrix} \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\Delta \theta_x \\
\Delta \theta_y \\
\Delta \theta_z
\end{bmatrix}
\]

(4.27)

One way to relate the six leg (finger) stiffnesses is by examining the terrain (object) stiffness at the different contact points. To minimize the energy transfer between the vehicle body (end-effector) and the terrain (object) it is advantageous to produce an impedance mismatch between the two subsystems. This is useful if the stiffness of the terrain (object) is known because the compliance of each leg (finger) can be electronically controlled to simulate a desired stiffness. Since the terrain (object) stiffness cannot be estimated with the currently available sensory equipment, this scheme cannot be adopted. However, the local gradients of the terrain be calculated for the walking machine system from the geometric terrain model and this data could be used in selecting a stiffness for each leg.

It is advantageous to select footholds such that the contact normals are parallel to the load-wrench. The stiffness may be selected
based on the orientation of the local normal relative to the load-wrench. Qualitatively, a soft compliant leg is preferred for poor footholds and a stiff leg for good footholds. A quantitative measure (and by no means, the only such measure) for the quality of a foothold is the angle between the wrench axis and the foothold normal. The larger this angle is, the lower should be the desired stiffness.

As the dominant component of the load-wrench is the weight, and hence the force component, it is computationally simpler and more practical to allow variable stiffness in the direction parallel to the wrench axis only. In the x-y plane, all the legs have identical stiffnesses and the expressions presented in Equation (4.14) hold. In the z-direction:

\[
\begin{bmatrix}
  k_1 & k_2 & \cdots & k_n \\
  k_1 y'_1 & k_2 y'_2 & \cdots & k_n y'_n \\
  -k_1 x'_1 & -k_2 x'_2 & \cdots & k_n x'_n \\
\end{bmatrix}
\begin{bmatrix}
  1 & y'_1 - x'_1 & \Delta z \\
  1 & y'_2 - x'_2 & \Delta \theta_x \\
  \vdots & \vdots & \vdots \\
  1 & y'_n - x'_n & \Delta \theta_y \\
\end{bmatrix}
\begin{bmatrix}
  f \\
  f y_s \\
  -f x_s \\
\end{bmatrix}
\]

(4.28)

where

\[
x'_i = \left( x_i - \frac{\Sigma k_i x_i}{\Sigma k_i} \right)
\]
\[
y'_i = \left( y_i - \frac{\Sigma k_i y_i}{\Sigma k_i} \right)
\]
\[
x'_s = \left( x_s - \frac{\Sigma k_s x_s}{\Sigma k_s} \right)
\]
This sums up the variable compliance method. The use of different compliance for different legs (fingers) in the z-direction serves to modify the equilibrating forces obtained through Method I in Section 4.3. It does tend to provide a non-uniform distribution of forces but this is because it redistributes the equilibrating forces so that the legs (fingers) with poorer footholds (contact points) take a smaller
share of the load. It also provides ratios of compliances which can be used in active compliance control schemes [40].

4.6 Linear Programming

Linear programming is one of the most studied techniques in engineering design and excellent canned programs are available for this method. It has proved to be the most successful method for linear systems. It was seen in Section 4.2.2 that the equality constraints are linear but the inequality constraints and the objective function are nonlinear. If a suboptimal, but feasible, solution is adequate as opposed to the optimal solution, compromises may be made in terms of linearizing the objective function and the inequalities.

A foot (finger) coordinate system is defined for each of the contact points with z-axis along the normal and the origin at the contact point. If \( \hat{e}_i \) is the \( i \)th unit normal in the earth-fixed reference frame \( (E) \) and \( F_{R_i}^E \) is the rotation transformation which relates \( (C_i) \), the foot (finger) reference frame, to \( (E) \), then

\[
F_{R_i}^E = \begin{bmatrix}
\hat{e}_i \\
\hat{e}_x \hat{e}_i \\
\hat{e}_y \hat{e}_i \\
\hat{e}_z \hat{e}_i \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
e_z & -e_y & e_x \\
0 & e_x & e_y \\
-e_x & -e_z & e_y \\
\end{bmatrix}
\]
The force components $F_{ix}$, $F_{iy}$ and $F_{iz}$ can be transformed from the body/object fixed reference frame, (E) to (C) to obtain the force vector $f_i$.

$$f_i = \frac{C_i}{E} \frac{E}{B} \frac{B}{F} F_i$$

(4.30)

The linear programming formulation allows only positive variables. $f_{iz}$ must always be positive but $f_{ix}$ and $f_{iy}$ may be either positive or negative. If $\Delta$ is a large constant number whose magnitude exceeds the magnitude of the smallest negative $f_{ix}$ or $f_{iy}$, then, it is convenient to define a set of non-negative variables $f'_i$ given by

$$f'_{ix} = f_{ix} + \Delta$$
$$f'_{iy} = f_{iy} + \Delta$$
$$f'_{iz} = f_{iz} + \Delta$$

If the six components of $w$ are $w_1$, $w_2$, ..., $w_5$ and $w_6$, and $B_i$ is the position vector of the $i$th contact point in (B), then the six equations of equilibrium are

$$\sum_{E}^{B} \sum_{C}^{E} \sum_{R}^{C_i} f'_i = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \sum_{E}^{R} \sum_{C}^{E} \sum_{R}^{C_i} \begin{bmatrix} \Delta \\ \Delta \end{bmatrix}$$
The inequality constraints can be easily linearized by replacing the friction cone with a "friction pyramid" such that the pyramid lies entirely within the cone [57]. If \( \mu \) is the maximum coefficient of friction and \( \mu_{\text{eff}} \) is equal to \( \mu/\sqrt{2} \), each of the four bounding planes of the pyramid can be represented as an inequality constraint:

\[
\begin{align*}
  f_{iz} &\leq \mu_{\text{eff}} f_{iz} \\
  -f_{iz} &\leq \mu_{\text{eff}} f_{iz} \\
  f_{iy} &\leq \mu_{\text{eff}} f_{iz} \\
  -f_{iy} &\leq \mu_{\text{eff}} f_{iz}
\end{align*}
\]

or

\[
\begin{align*}
  f'_{iz} &\leq \mu_{\text{eff}} f'_{iz} + (1 - \mu_{\text{eff}}) \Delta \\
  -f'_{iz} &\leq \mu_{\text{eff}} f'_{iz} - (1 + \mu_{\text{eff}}) \Delta \\
  f'_{iy} &\leq \mu_{\text{eff}} f'_{iz} + (1 - \mu_{\text{eff}}) \Delta \\
  -f'_{iy} &\leq \mu_{\text{eff}} f'_{iz} - (1 + \mu_{\text{eff}}) \Delta
\end{align*}
\]

These inequalities can be reduced to equalities using non-negative slack variables, \( p_{i1}, p_{i2}, p_{i3} \) and \( p_{i4} \):

\[
f'_{iz} - \mu_{\text{eff}} f'_{iz} + p_{i1} - (1 - \mu_{\text{eff}}) \Delta = 0
\]
There are a variety of linearized objective functions which can be used. One such function which can be used is given by:

\[ U = \frac{n}{i=1} f_{iz} \]  

(4.32a)

When the vehicle is on even ground the sum of the contact force components, and hence \( U \), is a constant as the variables are constrained by Equation (4.30). In rough terrain, minimizing \( U \) given by Equation (4.32a), limits the sum of normal components of the contact forces and this does achieve one of the optimal characteristics stated earlier.

Another possible candidate for an objective function is

\[ U = - \left( \frac{n}{i=1} (p_{i1} + p_{i2} + p_{i3} + p_{i4}) \right) \]  

(4.32b)

This function maximizes the sum of the non-negative slack variables which in turn tends to minimize the friction angles.

By introducing more variables the objective functions based on infinity norms can be considered. Let \( d \) be a positive variable. An objective function for minimizing the largest contact force component can be written as follows:
This introduces one additional variable and 6n new inequalities. Note that the inequalities may be written either in body coordinates or in foot (finger) coordinates and minimizing $U$ will minimize the absolute value of the largest force components in the desired coordinate system.

Another alternative for such a minimax type method is to maximize the smallest distance from the side-constraints. This can be done by considering the slack variables of Equation (4.31):

$$
\begin{align*}
    d & \leq p_{i1} \\
    d & \leq p_{i2} \\
    d & \leq p_{i3} \\
    d & \leq p_{i4}
\end{align*}
$$

and

$$
U = -d
$$

(4.34)
A host of canned routines (see Reference [33] for example) are available. They are primarily based on the Simplex Method and the Revised Simplex Method [88]. A comparative study of the different methods presented in earlier sections is reported in the following section.

4.7 Comparative Study of Different Methods for Walking Machines

4.7.1 Introduction

A study of the different algorithms was performed using a simulation of the Adaptive Suspension Vehicle. Details about the simulation can be found in Chapter 6. A wire-frame representation of the rough terrain and the ASV is shown in Figure 4.7. The algorithms are used to evaluate the force distribution at the critical time instants enumerated in Table 3.2. The variation of the forces with time may be assumed to be monotonic between stepping and lifting events. This is true if the variation of the force field with position is linear and the load-wrench does not change drastically between stepping and lifting events. The equilibrating forces do vary linearly between any two such events as the contact points and the support legs remain the same (see Equations 4.10, 4.14, 4.17, 4.19). The distribution produced by the variable compliance method or by the load-wrench decomposition technique also varies linearly with position. The linear programming methods do not ensure such a linear variation. Nevertheless the forces are expected to decrease or
Figure 4.7: Wire Frame Simulation Model of the ASV and the Rough Terrain
Figure 4.8: The Constrained Pseudo Inverse
increase monotonically between the lifting and stepping events. In these circumstances, the force field has local extrema at these critical time instants. The critical instants for stability (which are enumerated in Table 3.2) are therefore adopted as candidate points for evaluating and comparing the algorithms.

The following options were evaluated:

(a) Decomposition of the force field
   - Equilibrating forces by Method I
   - Ignore interaction force field
(b) Decomposition of the load-wrench
(c) Linear Programming
   - Minimizing the largest force component
(d) Linear Programming
   - Maximizing the distance from the side-constraints
(e) Linear Programming
   - Phase I: Simplex Method
(f) Decomposition of the force field
   - Equilibrating forces by Method I
   - Interaction forces by Linear Programming
(g) Variable Compliance Method
4.7.2 Simulation

A 2.97 second time interval of vehicle motion is considered in which the center of gravity of the vehicle starts from rest from the origin (0, 0, 0) and reaches a point (7.45, 2.7, 0.28) (all dimensions are in feet), where the linear velocity is (3.18, 1.17, 0.26) feet/sec². The weight of the vehicle is 7000 lb, and the characteristic length, which is equal to twice the pitch of the vehicle, is 10 feet. More details about the dimensions of the vehicle may be found in Figure 6.2 and Table 6.1. The maximum allowable friction angle was chosen to be 25.6 degrees corresponding to a maximum coefficient of friction of 0.48. The choice of this value is entirely arbitrary and only the linear programming methods, which directly take into account the friction cone angles, are affected by this choice. The gait used for simulation is a modified wave gait as described in Chapter 3.

Figure (4.9) shows the results of these algorithms applied to the test case. The ratio of the tangential to normal components of foot forces, denoted by ε, and the magnitudes of the foot forces have been presented in graphical form. These graphs are not very informative but they do indicate the peak values of the forces and the friction angles. Table 4.2 is a comparison of the computational times for the various strategies. These data are the execution times on the DEC system VAX 8500 and Micro Vax II. They were obtained by averaging the execution time over several runs.
Some of the methods involve the pseudo inverse operation in one form or another. The L-U decomposition scheme [90] is used. The constrained pseudo inverse, described by Figure 4.8 is implemented. In the flow-chart, \( \kappa \) is a small positive value and no component is allowed to fall below it. In this study a value of \((f/3N)\), where \( f \) is the force component associated with the load-wrench, was found to be suitable.

4.7.3 Results

(a) Equilibrating forces

- Method I (see Figure 4.9.1)

The equilibrating force solution shows a reasonable distribution of the load between the legs. The peak load is only 40% of the vehicle weight (on leg 2). However, as discussed earlier, the friction angles may become very high. This is because the algorithm does not account for the terrain geometry. The high friction angles in the beginning for legs 2, 4 and 5 compared to the other legs (as shown in Figure 4.9.1), are undesirable - the largest ratio of tangential to normal force components is almost 0.6. It should be remembered that the equilibrating forces do have some interaction force components irrespective of whether Method I or Method II is used. However, the appearance of these components is incidental. Thus the term equilibrating forces is used loosely to describe this force field.
Table 4.2 Computational Time for Proposed Algorithms

Notes: All figures are in seconds. \( t_a \) and \( t_s \) are the times required for configurations with 4 and 5 support legs respectively. The total time is the time required for the force distribution computations over the 2.97 second interval. The time in parenthesis indicates the average time required for a configuration in the test case.

<table>
<thead>
<tr>
<th>STRATEGY</th>
<th>TURN AROUND TIME</th>
<th></th>
<th>TOTAL TIME (AVERAGE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t_a )</td>
<td>( t_s )</td>
<td>VAX 8500</td>
</tr>
<tr>
<td>(a)</td>
<td>0.016</td>
<td>0.020</td>
<td>0.39(0.014)</td>
</tr>
<tr>
<td>(b)</td>
<td>0.047</td>
<td>0.059</td>
<td>1.31(0.048)</td>
</tr>
<tr>
<td>(c)</td>
<td>0.647</td>
<td>1.130</td>
<td>21.43(0.794)</td>
</tr>
<tr>
<td>(d)</td>
<td>1.773</td>
<td>3.455</td>
<td>60.12(2.227)</td>
</tr>
<tr>
<td>(e)</td>
<td>0.390</td>
<td>0.657</td>
<td>12.47(0.462)</td>
</tr>
<tr>
<td>(f)</td>
<td>3.021</td>
<td>5.332</td>
<td>100.01(3.704)</td>
</tr>
<tr>
<td>(g)</td>
<td>0.028</td>
<td>0.037</td>
<td>0.79(0.029)</td>
</tr>
</tbody>
</table>
Table 4.3 Force Distribution Generated by the Decomposition of the Load Wrench

All forces are expressed as percentage of vehicle weight and moments as percentage of vehicle weight times one fifth the pitch of the vehicle.

Case (a)

Number of support legs: 4

Load wrench, \( \mathbf{M} \)
\[
\begin{bmatrix}
0.0489, & 0.0176, & 1.0018; & 0.0, & 0.0, & 0.0033 \\
0.0146, & -0.0112, & 1.0015; & -0.0112, & -0.1685, & 0.0001 \\
0.0343, & 0.0288, & 0.0003; & 0.0112, & 0.1685, & 0.0032
\end{bmatrix}
\]

Foot forces

<table>
<thead>
<tr>
<th>Leg No.</th>
<th>Contact Normals</th>
<th>Foot Force Magnitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>NOT IN SUPPORT PHASE</td>
<td>0.1228 0.19</td>
</tr>
<tr>
<td>2</td>
<td>(-0.34,0.0,0.94)</td>
<td>0.3428 0.01</td>
</tr>
<tr>
<td>3</td>
<td>(-0.06,0.0,0.99)</td>
<td>0.4988 0.01</td>
</tr>
<tr>
<td>4</td>
<td>(0.0,0.0,1.00)</td>
<td>0.0411 0.38</td>
</tr>
<tr>
<td>5</td>
<td>(0.52,0.0,0.85)</td>
<td>0.0411 0.38</td>
</tr>
<tr>
<td>6</td>
<td>NOT IN SUPPORT PHASE</td>
<td>0.0411 0.38</td>
</tr>
</tbody>
</table>
Case (b)

Number of support legs: 5

Load wrench, $\mathbf{M}^{}$  
\[
\begin{bmatrix}
0.0231, 0.0086, 0.9998; 0.0, 0.0, 0.0015
\end{bmatrix}
\]

$\mathbf{M}_{\text{proj}}$  
\[
\begin{bmatrix}
0.0000, 0.0000, 0.9998; 0.4932, 0.4932, 0.0000
\end{bmatrix}
\]

$\mathbf{M}_{\text{res}}$  
\[
\begin{bmatrix}
0.0231, 0.0087, 0.0000; -0.4932, -0.4932, 0.0000
\end{bmatrix}
\]

Foot forces

<table>
<thead>
<tr>
<th>Leg No.</th>
<th>Contact normals</th>
<th>Foot force magnitudes</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$(-0.34, 0.1, 0.94)$</td>
<td>0.2307</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>$(-0.34, 0.0, 0.94)$</td>
<td>0.2801</td>
<td>0.05</td>
</tr>
<tr>
<td>3</td>
<td>$(-0.06, 0.0, 0.99)$</td>
<td>0.0334</td>
<td>0.71</td>
</tr>
<tr>
<td>4</td>
<td>NOT IN SUPPORT PHASE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$(0.52, 0.0, 0.85)$</td>
<td>0.1945</td>
<td>0.16</td>
</tr>
<tr>
<td>6</td>
<td>$(0.18, 0.0, 0.98)$</td>
<td>0.4000</td>
<td>0.02</td>
</tr>
</tbody>
</table>
(b) Decomposition of the load-wrench

The results obtained by using this method are shown in Table 4.3. In the particular configuration of Case (a), the decomposition of the load-wrench works very successfully. This is clearly because the component of the load-wrench orthogonal to the subspace spanned by the four normals, \( \mathbf{w}_{\text{res}} \), is insignificant compared to the projected component, \( \mathbf{w}_{\text{proj}} \). In general this is not the case as shown in Case (b). The resultant friction angles are very large for this configuration. In some cases it is not possible to obtain a solution in which the normal components of the forces are positive. Hence plots for the force distribution are not shown for this case. Clearly, this is not a very reliable method. The load distribution is more uneven than in method (a) (using equilibrating forces) and is also (2 to 3 times) computationally more expensive.

(c) Linear Programming

- Minimizing the largest component of the contact forces

(see Figure 4.9.2)

This method limits the maximum force component (described in the foot coordinate systems) very effectively, and the maximum foot force is 35% of the vehicle weight (which is 5% less than the same figure for method (a)). There is an obvious tendency for the solutions to lie along the side-constraints (inequalities). At least two legs are at
extremes at each instance and, the friction angles are very close to their allowable maximum. However, the allowable maximum can be suitably selected to obtain a good distribution. The drawback is the heavy computational load - 40 times the load for method (a) and about 15 times that for method (b).

(d) Linear Programming

- Maximizing the distance from the side constraints

  (see Figure 4.9.3)

This problem has a higher number of variables associated with the formulation as explained in the previous section (see Equation 4.34). Consequently this linear programming routine costs about 3 times more than (c) in terms of CPU time. The maximum friction angles may be minimized, and this optimizes the locomotion and also minimizes the possibility of slipping. However, the magnitudes of the forces are much higher and the peak force is about 50% of the vehicle weight. The maximum value for the components may be specified as side-constraints and thus the magnitudes can be reduced if necessary. However, this would result in further increasing the number of inequalities and a consequent increase in the computational load. The trade-off between the two optimal characteristics for locomotion is clearly reflected in the linear programming approach.
Figure 4.9: Force Distribution for the ASV (Test Case) - Friction Angles and Magnitudes of Forces
($
\eta$
 is the ratio of the tangential component to the normal component of the foot force)
Figure 4.9.1: Decomposition of the Force Field
(Equilibrating forces)
Figure 4.9.2: Linear Programming (Method (c))
Figure 4.9.3: Linear Programming (Method (d))
Figure 4.9.4: Linear Programming (Phase I)
Figure 4.9.5: Linear Programming (Method (f))
Figure 4.9.6: Variable Compliance Method
(e) Linear Programming

- Phase I of the Simplex Method (see Figure 4.9.4)

One way of minimizing the time required for a linear programming problem is by restricting the problem to a Phase I problem in the Simplex and Revised Simplex Methods (see Reference [88], pp. 134). The Phase I algorithm generates a feasible solution to the problem without considering the objective function, and the Phase II involves the generation of an optimal solution using the feasible solution as a starting value. By using just the Phase I part of the technique, the computing time may be reduced by almost a factor of 2, but, it still is about 15 times more than that for method (a). The friction angles (or $\varepsilon$ values) are within the maximum acceptable threshold but they, once again, tend to be close to this extreme. The maximum foot forces are very high and go up to 50% of the vehicle weight. The load distribution is very uneven but the advantage of this method (and also the last two methods) lies in the ease with which constraints can be programmed into the method. However, the heavy computational load, although somewhat reduced by relaxing the requirement on the optimality of the solution, remains a major deterrent.
(f) Decomposition of the force field

- Equilibrating forces by method I and interaction forces by linear programming (see Figures 4.9.5)

The problem of finding interaction forces in an time-efficient elegant way proved to be intractable. However the nature of the interaction force field in a real-world problem is of academic interest. The Simplex method is used to solve a linear programming formulation of the problem of determining interaction forces. The maximum interaction force component is used as an objective function for minimization. Comparing Figure 4.9.1 and Figure 4.9.5 it can be seen that the interaction forces are small in magnitude compared to the equilibrating forces (the two sets of plots are almost identical) and only serve to massage the equilibrating force field to yield a desired solution. As shown in Figure 4.9.5, the interaction forces are less than 10% of the vehicle weight and less than 20% of the maximum equilibrating force. This shows that the decomposition of the force field into the equilibrating force field and the interaction force field is very effective. But the lack of a fast and efficient algorithm puts this method at an obvious disadvantage.

(g) Variable compliance method

This method is a derivative of method (a). This is evident from the friction angles which are almost identical for the most part. However the equilibrating force field is modified by the addition of
stiffnesses so that those legs which have a smaller angle between the contact normal and the load-wrench take a larger share of the load. The redistribution of equilibrating forces is evident from Figures 4.9.1 and 4.9.6. For example, the load taken by legs 5 and 2 is reduced because of the larger friction angle and legs 1 and 6 take up a larger share of the load. The computational load is comparable to method (a) (only 30% more). With a computational time of 0.017 seconds for the 4 support legs case and 0.026 seconds for the 5 support legs case (on the VAX 8500), it is possible to use it for real-time computations needed to specify the force setpoints for the controller.

4.8 Concluding Remarks

Different ways of describing and formulating the force distribution problem have been proposed and studied. A comparative study using computer simulation as a tool is described. Algorithms for evaluating and predicting stability in rough terrain for use by the guidance module are studied. Another application of such algorithms is the generation of force set-points for force control algorithms and even the desired compliances in active compliance control schemes.

The concept of decomposing the force field into an equilibrating force field and an interaction force field is particularly useful. Unfortunately, there is no fast simple algorithm to compute interaction forces in real-time. The equilibrating forces can be
efficiently computed if a small deviation from the desired zero-interaction-force condition is permitted. The concept of varying the effective compliance of the leg according to the local terrain geometry can be used to redistribute the equilibrating forces so that legs with better footholds can support a greater part of the load. This method can be easily used for real-time control algorithms judging by its performance on the VAX 8500.

Another useful approach is the well-known technique of Linear Programming. An implementation of the Phase I of the Simplex Method is suggested for verifying if a given configuration is stable. In such an exercise it is not critical to know if an optimal distribution of forces can be found. It is sufficient to know if any valid (feasible) solution can be found to the force distribution problem. The Phase I method is faster than all the other linear programming methods and is guaranteed to find a solution (if one exists) unlike methods which do not involve linear programming. It would be necessary to use this technique six times (before each stepping event) in a locomotion cycle and the average computational time of 0.46 seconds (on the VAX 8500) needs to be reduced at least by a factor of 10 before it may be used for the guidance module. It is believed that it is possible to use distributed processing on stand-alone units to realize this reduction.
CHAPTER V
ADAPTIVE GAIT CONTROL AND BODY MOTION REGULATION

5.1 Introduction

The superior mobility of a legged system (as compared to a wheeled system) on uneven terrain stems from its ability to adapt its gait intelligently to the terrain. In Section 3.7, it was concluded that modified wave gaits (MWG), which incorporate the simplicity of the wave gait and the versatility of the free gait, were the best choice. Another attractive feature is that the MWG lends itself quite easily to the implementation of a follow-the-leader strategy. Terrain-adaptive locomotion involves intelligent foothold selection and the control of gait parameters to produce the desired motion. This demands a departure from the structured, symmetric, and ideal support patterns on which most of gait theory rests. It is proposed to use the MWG to enable a constantly changing velocity and to permit irregular, asymmetric support patterns.

5.2 Kinematics of the Legged System

Consider the earth-fixed reference frame (E) and the body-fixed reference frame (B). Let the transformation from (E) to (B) be denoted by $B^E_t$ and its inverse by $E^B_t$. $v_b$ and $\omega_b$ are the linear and angular
velocities of the vehicle. The position of a foot at point $P$ is indicated by $r_p$ with the velocity given by $v_p$ and the same quantities measured relative to the body fixed frame, $(B)$, are denoted by a trailing subscript $p/b$. A leading superscript denotes the reference frame to which the quantity is referred to.

The following expressions can be written for the legged system:

\[
E_{r_p} = E_{r_b} + E_{r_{p/b}} = E_{r_b} + B_{E_{r_{p/b}}}
\]

and

\[
B_{v_{p/b}} = E_{v_{p/b}} - E_{\omega_b} \times E_{r_{p/b}} \\
E_{v_{p/b}} = E_{v_p} - \dot{E}_{r_p} \tag{5.1}
\]

If the leg is in support phase, then,

\[
E_{v_p} = 0
\]

and

\[
E_{v_{p/b}} = -E_{v_b}
\]

or

\[
B_{v_{p/b}} = -E_{v_b} - \dot{E}_{r_b} \times E_{r_{p/b}} \tag{5.2}
\]

Equations (5.1) and (5.2) describe the relative position and velocity of the foot contact point in terms of the position and velocity of the vehicle. If the leg is not in support phase then $B_{E_p/B}$ and $B_{v_p/B}$ are directly specified by the leg trajectory planner and the kinematics problem is much simpler.
If the angular velocity of the vehicle, $\omega_b$, is zero, the foot contacts of all the legs in support phase have the same velocity with respect to the body. This velocity is equal and opposite to the vehicle velocity. In this simple case, it is easy to relate the foot velocity to the gait parameters, $\beta$ (duty factor), $R$ (stroke) and $T$ (cycle time period):

$$v_b = \frac{R}{(\beta T)} \quad (5.3)$$

Here, $R$ represents the stroke along the relative velocity vector which is, in general, arbitrarily directed. If $\omega_b$ is not zero, $v_{p/b}$ is constantly changing in direction and magnitude and is different for different legs. It is clearly not equal to $v_b$ which makes such a simplistic representation invalid. However, an instantaneous stroke may be defined based on the instantaneous relative velocity ($v_{p/b}$) for each leg. In Equation (5.3), if the $i$th leg is in support phase with an instantaneous stroke $R_i$, and an instantaneous velocity with respect to $[B]$, $v_{p_i/b}$, then $R_i$ may be defined by the following equation:

$$v_{p_i/b} = \frac{R_i}{(\beta T)} \quad (5.4)$$

However, Equation (5.3) may still be used to define an average (effective) stroke, $R_b$. In this work, the latter definition of $R_b$ is
used to calculate an optimum value for \( \eta \), the stroke to pitch ratio, which is in turn required to compute \( \phi_{\text{opt}} \) and \( \theta_{\text{opt}} \), and thus optimize the gait parameters as discussed in Chapter 3. The stroke, \( R_b \), is not used to determine the motion of the vehicle directly. Thus, \( R_b \) need not be determined accurately and it is sufficient to base it on the general direction of the vehicle and not on the instantaneous direction of the velocity of the foot with respect to the body.

5.3 Control of Gait Parameters

5.3.1 Definitions

The modified wave gait is characterized by \( \psi \), the phase difference between any two adjacent, ipsilateral legs, \( \theta \), the phase difference between any pair of adjacent, contralateral legs and \( \beta \), the duty factor. There are three other parameters which do not affect the sequencing of legs, but they do involve the coordination of the legs. They are the stroke, \( R \), the time period, \( T \) and the crab angle, \( \alpha \).

In Figure 5.1, the Gait Optimizer generates optimum values of \( \theta \) and \( \psi \) for a given duty factor, crab angle, stroke and load-wrench (denoted by \( H \) — see Chapter 3 for definition). The other parameters, \( R, T, \beta \) and \( \alpha \) are permitted to change constantly to provide the vehicle its versatility, while \( H \) is determined by vehicle body kinematics and dynamics. It is meaningful to use the Gait Optimizer only if the input variables (\( H, \beta, R \) and \( \alpha \)) are reasonably constant. Thus in a real system, it should be invoked only if \( \beta, R, \alpha \) and \( H \) (the
Figure 5.1: Control of Gait Parameters by the Guidance Module
load-wrench) reach some sort of a steady state, or show a definite trend.

Each leg in support phase is characterized by:

1. $(\ell_{x_i}, \ell_{y_i}, \ell_{z_i})$ - the maximum distance the leg $i$ can be moved in the $x$, $y$ or $z$ directions with the current velocity (given by Equation (5.2)) before the workspace limits restrain the leg.

2. $t_{ri}$ - the maximum time the leg $i$ can be moved with the current velocity, before it reaches the workspace limits.

3. $\phi_{L,i}$ - the local phase of the $i$th leg (see Definition 15 in Chapter 3).

4. $S_i$ or Leg State - a leg $i$ has to be in one of the three states: support ($S_i = 1$), ready ($S_i = 0$) or transfer ($S_i = -1$). The sequence of states is shown in Figure 5.2. The transfer and support states are as defined in the literature and in Section 3.4 but the ready state is introduced as an intermediate state. This is to ensure that the leg is given sufficient time in the transfer state before it can be deployed. In other words, the transfer state is an asynchronous state and each leg spends a definite time interval in this state before it passes on to the ready state (a synchronous state) from which it can be deployed (enter support phase) when required.

A leg in transfer phase is characterized by a deploy time, $t_{di}$.
Figure 5.2: Sequence of Leg States
which is the minimum time needed before that leg can be deployed. In other words, it is the minimum time required before the state of the leg changes to a 0 or a ready state.

If the leg placement is unrestricted, or in other words, if it can be placed anywhere within the workspace, then it is not always possible to keep the time period constant [47]. However, it is desirable to maintain a time period or a cycling frequency which is optimal from the point of view of control of the actuator circuits driving the leg. At this point, it is necessary to define a few new terms to incorporate the concept of a varying and a desired time period.

The instantaneous time period, \( \tau_i \), for any leg, \( i \), is given by:

\[
\tau_i = \frac{t_{ri}}{R \phi_i} = \frac{t_{ri}}{\beta - \phi_{L,i}}
\]

The maximum instantaneous time period, \( \tau \), for the legged system is given by:

\[
\tau = \text{MINIMUM} \{ \tau_1, \tau_2, \tau_3, \ldots, \tau_n \}
\]

where \( \text{MINIMUM} \) is a function which returns the minimum of all the arguments, and the subscripts 1 through \( n \) are the indices for the support legs.

It is also necessary to define a threshold return time, \( t_{r,\text{min}} \), which represents the minimum time required for the duration of the transfer state. This depends on the hardware which actuates the leg or
the specifications of the leg controller. Immediately following a lifting event, the deploy time of the leg which has just been lifted is set equal to the threshold return time. The time period and the duty factor of the gait are restricted by the threshold return time:

$$T > \frac{t_{r,\text{min}}}{(1-\beta)}$$  \hspace{1cm} (5.6)

Also T has to be less than or equal to the maximum instantaneous time period given by Equation (5.5). This restricts T to be in the range specified by:

$$\frac{t_{r,\text{min}}}{(1-\beta)} \leq T \leq \tau$$  \hspace{1cm} (5.7)

There is a minimum duty factor which must be maintained to ensure a minimum required stability margin or LSM. Such a minimum duty factor, $p_{\text{low}}$ would be typically prescribed by an upper level of control, in this case either the navigator or the gait optimizer (see Figure 5.1). The effect of duty factor on stability and vehicle speed has already been discussed (see Chapter 3). The upper limit on the duty factor is always unity. The gait optimizer also specifies an ideal stroke based on the roughness and the gradients of the terrain. A large stroke is not suited to large gradients on rough, uneven terrain. On the other hand, the LSM increases or decreases with the
stroke depending on the duty factor and the pitch as discussed in Chapter 3. Thus an intelligent choice for the stroke has to be made. This ideal stroke is also an upper limit on the stroke and thus the maximum stroke, $R_{max}$, is known. There is no lower limit on the stroke.

5.3.2 Changing Time Period with a Constant Duty Factor at Constant Velocity

If any of the legs face geometric constraints, the time period may fall below the maximum value specified by Equation (5.7), in which case it is necessary to reduce the time period to or below the maximum value. It is always desirable to be able to do this without having to change the velocity of the legs. In Equation (5.4), if $v_{p_1/b}$ has to remain unchanged, the quantity $R_1/(\beta T)$ cannot change. If a reduction in $T$ is matched by a proportional drop in $R_1$, or in other words, if the cycling frequency is increased but the cycling stroke or amplitude is also proportionately decreased, the velocity, $v_{p_1/b}$ remains constant. The duty factor is unchanged and therefore, the stepping sequence and the gait remains unaffected.

5.3.3 Changing Velocity with a Constant Duty Factor

If the velocity of the leg with respect to the body must change with $\beta$ remaining a constant, it is necessary for the $R_1/T$ ratio to change accordingly. This can be done either by changing the stroke or
by varying the time period, or by changing both quantities. In any event, the duty factor remains the same and thus the gait and the stepping sequence do not change.

5.3.4 Gait Transitions at Constant Velocity with Constant Stroke

Let \( t_\text{c} \) be the time at which the change is desired. Let \( R_1, \beta_1, \) and \( T_1 \) indicate the quantities before the change, at time \( t_\text{c}^- \), and \( R_2, \beta_2, \) and \( T_2 \) the quantities after the change, at time \( t_\text{c}^+ \). The quantities \( t_\text{c}^- \) and \( t_\text{c}^+ \) represent time instants just before and after the transition respectively. If \( R_1 \) must equal \( R_2 \) and the velocities are equal, then \( \beta_2 \) must equal \( (\beta_1 T_1)/T_2 \). This obviously involves a change in gaits as the two duty factors are different. One way of ensuring a smooth transition between the two gaits is by ensuring that the local phase of a reference leg, leg \( j \), is the same before and after the change. In other words, \( \phi_{L_{-j}} \) has no discontinuity across the transition. The choice of the reference leg is purely by convenience.

The local phases of the other legs change across this transition as they are assigned new local phases based on \( \beta_2 \) and \( \phi_{L_{-j}} \). Thus, \( \beta_2 \) and the six local phases are still related according to the definition of the MWG. Any time there is a change in the local phases, care should be taken to ensure:

1. No leg with \( S_\text{i} = -1 \) at \( t = t_\text{c}^- \) can have \( \phi_{L_{i}}(t_\text{c}^+) \leq \beta \). In other words, no leg in transfer phase at \( t_\text{c}^- \) can be required to be in support phase at \( t_\text{c}^+ \).
(2) No leg at a geometric limit at \( t = t_t^- \) can be in support phase at \( t = t_t^+ \).

(3) No leg which is required to be in transfer phase at \( t = t_t^+ \) can have a deploy time, \( t_{di} \), which is greater than the time in which it is expected to be deployed. This can be expressed mathematically in terms of known quantities.

If \( \phi_{t_t^+} \) is the new local phase of leg \( i \) and if \( S_i(t_t^+) = -1 \), which in turn implies

\[
\phi_{L,i}(t_t^+) > \beta, \text{ then,}
\]

\[
(1 - \phi_{L,i}(t_t^+)) T_z > t_{di}
\]

Thus any transition of a gait from one duty factor to another must satisfy these three conditions for all the six legs. If the transition is at a stepping event then it is most convenient and natural to choose the reference leg to be the stepping leg. Figure 5.3 shows an example of such a gait transition. The thin lines represent the gait before the time, \( t_t \), and the bold lines indicate the gait after \( t_t \). The transition period is a time interval after \( t_t \) (with a duration of \( \beta z T_z \)), in which the gait is irregular and aperiodic. The transition cycle is the cycle of events following \( t_t \). After \( t_t \), the sequencing of legs is governed by the bold lines (gait 2), although, in the figure, normal and bold lines are shown for the transition cycle.
Figure 5.3: A Gait Transition in a Perfect Wave Gait (constant $\beta T$) (The stepping leg is leg 3 which is also the reference leg. Notice $\phi_{L,3}(t) = \phi_{L,3}(t^*)$.)
5.3.5 Maintaining a Constant Time Period and Velocity with a Changing Stroke

Again, it is assumed that the velocity of the leg should be kept at a constant value. When the leg kinematics and workspace geometry constrain the motion of the leg, a change in stroke with a corresponding change in time period (to maintain a constant velocity) is sufficient. In such an event the duty factor remains the same. But it is possible to combine this change with another change in the time period, which will revert the time period back to its original value with a corresponding change in the duty factor, without changing the stroke. Thus, the effect of changing the stroke without affecting the time period and velocity can be produced by two other changes which are described in Sections 5.3.3 and 5.3.4. The change in the duty factor causes a change in the gait and the conditions described in Section 5.3.4 must be satisfied.

5.3.6 Controlling the Time Period and Stroke to Maintain a Constant Leg Velocity

Equation (5.7) restricts the time period to lie within a certain range. In addition to that the time period has to satisfy Equation (5.4) describing the gait kinematics. The foothold selection procedure may force the stroke to change, and maintaining an optimal time period requires a constant variation of \( \beta \) for a constant leg velocity. Two modes of control are suggested in this subsection. The first mode
optimizes the time period and at the same time, ensures that the constraints on the stroke and duty factor are met. Similarly, the second mode involves the control of the stroke while the time period and the duty factor are kept within the feasible range.

If the instantaneous time period is greater than \( \tau \) (obtained from Equation (5.5)), the time period must be reduced and therefore, it is appropriate to make the time period the controlled variable. If, on the other hand, the instantaneous time period is less than or equal to \( \tau \), the controlled variable is the stroke. This allows the stroke to be increased if it is below \( R_{\text{max}} \). If the stroke is already equal to \( R_{\text{max}} \), the time period may be chosen to be the controlled variable.

**Control of Time Period**

Let \( R_{0}, T_{0} \) and \( \beta_{0} \) represent the stroke, instantaneous time period and duty factor. If \( T_{0} \) is greater than \( \tau \), the time period must be decreased and the new time period, \( T_{1} \) can be at most equal to \( \tau \). If \( T_{1} \) equals \( \tau \) and \( \beta_{1} \) is equal to \( \beta_{0} \), \( R_{1} \) equals \( (R_{0}/T_{0})T_{1} \). This is as discussed earlier in Section 5.3.2. However, \( T_{1} \) may now violate the restriction in Equation (5.6). Even if this is not the case, it is desirable to make the time period equal to \( T_{\text{opt}} \). Thus an increase in time period is now required. Let the new parameters after the change in time period be \( R_{2}, T_{2} \) and \( \beta_{2} \). As \( R_{2} \) cannot be increased above \( R_{1} \) because of geometric constraints, a change in \( \beta \) is essential. \( R_{2} \) should be kept equal to \( R_{1} \) to maintain the maximum possible stroke and
therefore, it is necessary to maintain \( (\beta_1 T_1) \) equal to \( (\beta_2 T_2) \). The limits on the time period, \( T_2 \), can be derived from Equation (5.6) and the lower limit, \( B_{\text{low}} \), on the duty factor, \( \beta_2 \).

\[
T_{\text{max}} = \frac{\beta_1 T_1}{B_{\text{low}}}
\]

\[
T_{\text{min}} = t_{r(\text{min})} + \beta_1 T_1 = t_{r(\text{min})} + \beta_2 T_2
\]

In the absence of information about the hardware, it can be assumed that the optimal time period, \( T_{\text{opt}} \), lies midway between the limits imposed by Equation (5.7). In other words,

\[
T_{\text{opt}} = \frac{1}{2} \left[ T_{\text{max}} + T_{\text{min}} \right] \quad (5.8)
\]

This, arguably, is a good choice as it offers reasonable flexibility for changing the time period on either side.

\( T_2 \) is set equal to \( T_{\text{opt}} \) which yields \( \beta_2 = (\beta_1 T_1)/T_2 \), the new duty factor. Once \( \beta_2 \) is decided, the feasibility of the gait transition from \( \beta_1 \) to \( \beta_2 \) is investigated by seeing if the restrictions in Section 5.3.4 are satisfied. If they are not met, then this suggested change is not possible and a value of \( T_2 \) between \( T_1 \) and \( T_{\text{opt}} \) may be picked for a second trial. This may be repeated for values of \( T_2 \) closer to \( T_1 \) until a value of \( T_2 \) (in the extreme case, equal to \( T_1 \)) is found for which a gait transition is possible. Thus the system always strives to
attain the optimal $T_{\text{opt}}$. If no such value of $T_{2}$ can be found and if $T_{1}$ ($= \tau$) violates the inequality in Equation (5.6), it means $\tau$ is too small. This is usually indicative of a poor choice of foothold for one of the feet. The gait controller cannot deal with such a case and a decrease in velocity is the only alternative.

### Control of Stroke

If the leg kinematics do not restrain the stroke, then it is possible to increase the time period and the stroke. If the instantaneous time period, $T_{0}$, is less than $\tau$, ($R_{0}$ is less than $R_{\text{max}}$) then $R_{0}$ can be increased to achieve a larger stroke. The maximum new time period, $T_{1}$ is equal to $\tau$ and the maximum stroke, $R_{1}$ is equal to $(R_{0}/T_{0})\tau$. The desired stroke $R_{1}$ should lie in the range specified by:

$$\left[\frac{R_{0}^{T_{\text{r(min)}}}}{T_{0}(1 - \beta)}\right] \leq R_{1} \leq \frac{R_{0}^{\tau}}{T_{0}}$$

(5.9)

$R_{1}$ is selected to be equal to $R_{\text{max}}$, or in the event that $R_{\text{max}}$ lies outside this range, equal to the limit closest to $R_{\text{max}}$. The duty factor remains the same and thus, the gait does not change.

5.3.7 Change of Gait at a Constant Velocity

In Figure 5.1, the Gait Optimizer prescribes optimum parameters $R_{\text{low}}, R_{\text{max}}, \phi_{\text{opt}}$ and $\theta_{\text{opt}}$. $R_{\text{low}}$ and $R_{\text{max}}$ are used to determine the ranges for the control variables, time period and stroke, as described
in Section 5.3.6. A change in $\psi_{opt}$ or $\theta_{opt}$ involves a change in the gait and a discontinuity in the variation of the local phases. Such a transition is similar to that described in Section 5.3.4.

Figure 5.4 shows an example of a gait transition between wave gaits. At the transition, $\psi$ is changed from $\beta_1 (=0.6)$ to $\beta_2 (=0.5)$ while $\theta$ is constant at 0.5. The reference leg is selected to be the stepping leg (leg 3) again. The bold lines depict the gait after time, $t^\ast$, at which the transition begins. Though the example in Figure 5.4 is kept relatively simple for illustrative purposes, this procedure can be used to make a transition from any gait to any other provided the three conditions in Section 5.3.4 are satisfied.

The transition period in Figure 5.4 is the period in which the effects of the transition at time $t^\ast$ can be seen. The legs, which were placed using the old gait parameters, are in support phase until at most the end of the transition period. Thus, it is the period of time starting from the transition time, $t^\ast$, with a duration of $\beta_2 T_2$ (see also Figure 5.3). In the transition period, the time period and the stroke are regulated (as explained in Section 5.3.6) to accommodate the irregularities in the vehicle gait.

5.3.8 Effect of Gait Transitions on Stability

In the gait control strategies discussed so far, the spatial constraints (pertaining to workspace limitations) and the temporal constraints (produced by the sequential logic) have been considered.
Figure 5.4 A Gait Transition in a Wave Gait Involving a Change in Phase Angles
However, the question of stability has not been addressed. When a gait transition is considered, it is important to check that the vehicle is stable for at least one cycle of locomotion events after the transition. Such an unstable condition is most likely to occur in the transition period shown in figures 5.3 and 5.4. If the vehicle is not stable for a full cycle of events after the transition, the transition cannot be executed.

5.3.9 Conclusion

In this section, methods of controlling gait parameters to achieve a desired velocity are described. For an accelerated or decelerated motion, the stroke and time period may be controlled to change the velocity. Further, the time period and the duty factor can be adjusted to maintain an optimal time period. If the leg workspaces are not "fully utilized", the stroke can be increased towards the maximum stroke specified by the gait optimizer. It is proposed to monitor, control and specify gait parameters at time instants just before each of the 6 stepping events. Implicit in this idea is the assumption that the state of the vehicle does not change rapidly between the events. Thus, the path of the legs between the events is assumed to be linear. If the angular velocity of the vehicle is nonzero, this will not be the case. However, the discrepancy between the planned and the actual path can be resolved at the controller level in real time.
5.4 Foothold Selection

The rationale for foothold selection is based on the following criteria:

1. Magnitudes of the contact forces
2. Gait stability
3. Workspace constraints
4. Undesirable support sites

Each of these criteria is discussed in this section. The emphasis here is on developing heuristics based on these criteria which will enable easy and efficient implementation of a foothold selection algorithm on a computer.

Contact Forces

The magnitudes of the contact forces should be limited for the reasons mentioned in Chapter 4. The strategy for foothold selection should exploit the advantages of the force distribution algorithm employed for control to produce a synergistic effect which would improve the force distribution characteristics. It is assumed that the equilibrating force solution discussed in Section 4.3.3 is employed for force control. This simplifies the problem as the expressions for contact forces (Equations (4.14) and (4.17)) are compact and simple. This analysis was performed in the [W] coordinate system and it is convenient to continue to refer to the same system.
The forces on the x-y plane or the o-plane are given by Equation (4.14). The magnitude of the contact force projected on the x-y plane at the ith foot, $F_{ixy}$, can expressed in terms of the coordinates of the foot contact as:

$$F_{ixy} = \frac{c p_i}{n I}$$

(5.10)

The moment about the z-axis associated with the contact force at any point is given by:

$$M_{iz} = \frac{c p_i^2}{n I} = \frac{c p_i^2}{n \sum_{i=1}^{n} p_i^2}$$

It is easy to see that the sum of all the moment components in the z-direction is equal to $c$, the external couple. To minimize the largest component on the x-y plane it is clear that all the components must be equal in magnitude. In other words,

$$p_1 = p_2 = p_3 = \ldots = p_n$$

This suggests a symmetrical arrangement of the support sites about the origin (which is also the centroid of the contact points by definition) of the reference frame, $\{H\}$, would be optimal.

The forces in the z-direction are related to the x and y coordinates of the contact points through Equation (4.17). If the weight is assumed to be the dominant contributor to the load-wrench, i.e. $c$ is assumed to be small, then the smallest maximum foot force
will result when $x^* = y^* = 0$. In such a situation, all the $z$-components will be equal. Alternatively, if all the $z$-coordinates are zero, i.e., all the contact points are on the $\sigma$-plane, the resulting effect on the $z$-components is the same as if $c$ were zero. Thus a situation where the $\pi$-plane and $\sigma$-plane coincide is desirable. In addition, the equilibrating force vector will not have any interaction force components in such a situation. This is the reason that the $\sigma$-plane was called the desired support plane in Chapter 3 (Section 3.5).

If the weight is the most significant component of the load-wrench, then the pitch of the load-wrench is small. In this case, the orientation of the contact normal with respect to the wrench axis determines the friction angle at a contact point. The best situation would be one in which the contact normals were all parallel to the load wrench.

In summary, an optimum situation results if:

(a) All the feet are on the $\sigma$-plane or the $\sigma$-plane and the $\pi$-plane coincide.

(b) The feet are symmetrically positioned on the x-y plane about the load-wrench axis and the load-wrench axis passes through the centroid.

(c) The contact normals are parallel to the load-wrench.

Gait Stability

It is assumed that a minimum gait LSM needs to be maintained. The choice of the LSM as a measure of gait stability has already been
discussed in Chapter 3. Every potential foothold is evaluated in terms of the stability it provides. For each foothold, a sequence of events following the foothold selection through one locomotion cycle (into the future) may be considered and the minimum LSM over the cycle may be used to evaluate the stability characteristics of the foothold. Any new footholds which have to be generated in this locomotion cycle can be assumed to be in conformance with the ideal stepping patterns of the wave gait. This method is time consuming. Instead a shorter and simpler method is suggested whereby only two critical events in the succeeding cycle are considered.

Consider the support pattern (on the \( \sigma \)-plane) in Figure 5.5. The \( \{B\} \) coordinate system has been translated so that the origin coincides with that of the \( \{M\} \) coordinate system. Let \( \Gamma_{ij} \) be the segment joining the projection of the support points for feet \( i \) and \( j \) on the \( \sigma \)-plane. The two legs constitute a leg pair. If \( \Gamma_{ij} \) intersects the \( x_\sigma^- \)-axis an intercept, \( \nu_{ij} \) may be found. If the segment \( \Gamma_{ij} \) does not intersect the \( x_\sigma^- \)-axis, \( \nu_{ij} \) does not exist. The distance \( d_1 \) defined in Chapter 3 is quite simply the maximum of all such \( \nu_{ij} \) and \( d_2 \) is the negative of the minimum of the \( \nu_{ij} \). The LSM is obviously equal to the smaller of the two quantities, \( d_1 \) and \( d_2 \). At a critical event, the segment whose intercept is equal in magnitude to the LSM is called the critical segment.

Table 5.1 enumerates the critical events corresponding to the stepping events for each of the six legs, as well as the leg pairs
Figure 5.5: Predicting Stability for Evaluating Stability Characteristics of Footholds
## Table 5.1 Evaluating Stability of a Foothold

<table>
<thead>
<tr>
<th>Leg No.</th>
<th>Critical Distance</th>
<th>Critical Events</th>
<th>Critical Phases</th>
<th>Leg Pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(d_1)</td>
<td>Leg 2 placing</td>
<td>(\theta^-)</td>
<td>1, 4</td>
</tr>
<tr>
<td></td>
<td>(d_1)</td>
<td>Leg 1 lifting</td>
<td>(\beta^-)</td>
<td>1, 2</td>
</tr>
<tr>
<td></td>
<td>(d_1)</td>
<td>Leg 1 placing</td>
<td>(0^- \mod 1)</td>
<td>2, 3</td>
</tr>
<tr>
<td>2</td>
<td>(d_1)</td>
<td>Leg 2 lifting</td>
<td>((\theta+\beta)^- \mod 1)</td>
<td>1, 2</td>
</tr>
<tr>
<td></td>
<td>(d_1)</td>
<td>Leg 1 placing</td>
<td>(0^- \mod 1)</td>
<td>2, 3</td>
</tr>
<tr>
<td></td>
<td>(d_1)</td>
<td>Leg 5 lifting</td>
<td>((2\psi+\beta)^+ \mod 1)</td>
<td>3, 6</td>
</tr>
<tr>
<td>3</td>
<td>(d_1)</td>
<td>Leg 2 placing</td>
<td>(\theta^- \mod 1)</td>
<td>1, 4</td>
</tr>
<tr>
<td></td>
<td>(d_2)</td>
<td>Leg 6 lifting</td>
<td>((\theta+2\psi+\beta)^+ \mod 1)</td>
<td>4, 5</td>
</tr>
<tr>
<td>4</td>
<td>(d_1)</td>
<td>Leg 5 placing</td>
<td>((2\psi)^+ \mod 1)</td>
<td>5, 6</td>
</tr>
<tr>
<td></td>
<td>(d_2)</td>
<td>Leg 6 lifting</td>
<td>((\theta+\beta+2\psi)^+ \mod 1)</td>
<td>4, 5</td>
</tr>
<tr>
<td>5</td>
<td>(d_2)</td>
<td>Leg 6 placing</td>
<td>((\theta+2\psi)^+ \mod 1)</td>
<td>5, 6</td>
</tr>
<tr>
<td>6</td>
<td>(d_2)</td>
<td>Leg 5 lifting</td>
<td>((\beta+2\psi)^+ \mod 1)</td>
<td>3, 6</td>
</tr>
</tbody>
</table>
assumed to form the critical segment. Two critical events are considered for each leg. To evaluate a foothold the intercept, \( v_{ij} \), based on the leg pairs at each of the two instants is computed and the LSM for the locomotion cycle is derived from the two \( v_{ij} \). The velocities at the time of selection of the foothold are assumed to remain constant over the cycle to allow the computation of the positions of the feet at time instants in the future.

For example, if \( t^* \) is the time at which leg 1 has to be placed, to evaluate a potential foothold for leg 1, the support pattern and \( v_{14} \) at \( (t^* + \Theta T)^- \) and the support pattern and \( v_{12} \) at \( (t^* + \Theta T)^- \) are considered. If \( \psi \) is greater than \( (1-\Theta) \) (which has been assumed to be the case in Chapter 3), then the position of leg 4 needed to determine \( d_1 \) at time \( (t^* + \Theta T)^- \) depends on the foothold selected for leg 4 at \( [t^* + ((\Theta + \psi) \mod 1)T] \). In such an event the foothold for leg 4 is assumed to be the ideal foothold which is half a stroke length away from the mean position of the leg.

The assumption that the velocities are constant through one locomotion cycle may not be valid. The critical segment may NOT be the segment formed by the leg pairs in Table 5.1. Also, the selection of footholds after time, \( t^* \) will not be in accordance with the ideal gait and may either improve or degrade the stability. Nevertheless, this method yields a suboptimal measure of the stability and the simplicity of this technique makes it attractive.
Workspace Constraints

The foothold selection procedure must consider only those potential support sites which lie within the leg workspace. In addition, the support sites which permit a complete stroke at the planned velocities and time period are preferred over the footholds which do not. Thus, footholds which permit longer strokes (with the current velocities) are considered more useful and this tends to prevent unnecessary reductions in the stroke leading to a reduction in the time period.

Undesirable Support Sites

The possibility of corruption of data collected by sensors or of uncertainty in the available elevation data cannot be ignored. If the data associated with a site has a poor confidence level, that site must be excluded from the list of potential support sites. In addition, if certain sites are deemed unfit for load-bearing, they should not be considered by the foothold selection routine. A site, which another foot has already occupied, is also excluded. This way, the collision of feet is prevented, but the collision of legs is relegated to a lower level of control, which involves trajectory planning.

These four criteria make conflicting demands on the foothold selection routine. Depending on the mode of operation one or more of these criteria can be emphasized while the others may be de-emphasized. If the four effects can be quantified, a heuristic
function which is comprised of four terms can be constructed. Each of these terms is a constant times a numerical evaluation of one of these effects. The constants are like gains and they can be varied to yield desirable characteristics. For example, in the ASV, the emphasis on larger stroke lengths should be increased in the cruise/dash mode, but the stability should be emphasized for operation in a hostile environment. In rough terrain the force distribution characteristics are more important as the friction angle must kept within a maximum value. In a follow-the-leader mode, the footholds of the anterior legs should be based on the contact normals and workspace constraints but the footholds of the other legs do not have to be selected.

5.5 Automatic Body Motion Regulation

The navigator commands the rates (velocities) in the lateral and longitudinal directions as described in Chapter 2. The other four rates have to be automatically regulated at the guidance level. Work in this area has been reported in References [47] and [98], and this section is a development of the ideas presented in these references. The inertial forces caused by the linear accelerations are ignored to simplify the problem. The foothold selection algorithm discussed in Section 5.4 is capable of making adjustments to accommodate a varying wrench axis.
The important considerations in developing strategies for body motion regulation are:

(a) Vehicle stability
(b) Gradability
(c) Geometric constraints

Waldron et al. [98] propose two types of strategies for walking on inclined slopes. Figure 5.6 shows the vehicle in a desired configuration for two different configurations. In the simple configuration shown in Figure 5.6(a), the angle $\eta$ determines the slope of the terrain and $\eta_D$ the desired body attitude. The two strategies for deciding on a desired attitude are shown in Figure 5.7(a) and Figure 5.7(b). In Figure 5.7(a), the body is commanded to be parallel to the terrain which decreases the vehicle stability [98]. However, a decrease in body height (which is defined in Figure 5.6(a) can compensate for this degradation by improving the stability. The second strategy is to incline the body with respect to the slope - an extreme situation in which the body is parallel to the x-y coordinate system in the frame (E) is shown in Figure 5.6(b). Though this method does not degrade the stability it does adversely affect the gradability because of the typically high ratio of body length to leg lift in walking machines.

Figure 5.6(b) illustrates the case of cross-slope locomotion. In this situation, as the width of the body is comparable to the lift of the legs, the strategy of Figure 5.7(b) does not decrease the
Figure 5.6: Locomotion on Sloping Smooth Terrain
(a) Climbing up an Inclined Surface
(b) Cross-slope Locomotion
Figure 5.7: Body Attitude Control - One Degree of Freedom
(a) Body Attitude Parallel to Incline
(b) Horizontal Body Attitude
(c) Alternative Scheme for Body Attitude Control
gradability and at the same time the stability remains unaffected. In a general case in which the plane of the terrain is arbitrarily oriented with the longitudinal axis of the body, a compromise between the schemes shown in Figure 5.7(a) and 5.7(b) may be desired.

\( \eta_D \) can also be determined by more complicated relationships an example of which can be seen in Figure 5.6(c). This scheme makes the vehicle insensitive to small changes in the support plane orientation which are likely to be encountered on rough terrain. On the other hand, the vehicle responds to larger inclinations and the response decreases as the inclination becomes larger. This is a compromise between the schemes illustrated in Figure 5.7(a) and 5.7(b).

A rough terrain complicates the problem even more as it is no longer a simple plane. One way of approximating the terrain is by using a least-squares formulation to fit a plane through the support points. Such a best fit plane has been defined as the support plane or the \( \pi \)-plane in Chapter 3. \( \hat{n}_S \), the normal to the support plane is, in general, arbitrarily oriented with respect to the axes of the body fixed frame, \( [B] (X_B-Y_B-Z_B) \) or the earth fixed reference frame, \( [E] (X_E-Y_E-Z_E) \). This is shown in Figure 5.8. The body attitude, \( \eta \), is defined to be the angle between the support plane normal, \( \hat{n}_S \) and the Z-axis of the reference frame, \( [E] \). The desired body attitude, \( \eta_D \), is the angle between the desired orientation of \( \hat{Z}_B, \hat{Z}_{BD}, \) and \( \hat{Z}_E \). Therefore, the following equations may be written:
Figure 5.8: Body Attitude Control - Two Degrees of Freedom
\[ \cos \eta = \hat{n}_S \cdot \hat{Z}_E \]  \hspace{1cm} (5.11)

\[ \cos \eta_D = \hat{Z}_{BD} \cdot \hat{Z}_E \]  \hspace{1cm} (5.12)

The desired rates of change of body attitude must be about a vector perpendicular to the plane containing \( \hat{Z}_B \) and \( \hat{Z}_{BD} \). This vector is denoted by \( \hat{m} \) in Figure 5.8 and the desired angle of body rotation, \( \xi \), is given by

\[ \cos \xi = \hat{Z}_B \cdot \hat{Z}_{BD} \]  \hspace{1cm} (5.13)

If a first order system is used to control \( \xi \), the desired body attitude rate is given by

\[ \dot{\xi} = -k_\xi \xi \]  \hspace{1cm} (5.14)

Another way of specifying the desired attitude is by using a different estimate for the support plane. The potential support sites ahead of the vehicle may be used to estimate the support plane. Clearly this is possible only if the vehicle has information about the elevation of the terrain ahead of it. This enables the vehicle to change its attitude according to the terrain features which lie ahead of it before actually encountering those features. This improves the vehicle gradability significantly, and can be used very effectively.
for planning the motion of the body. This is further discussed in the next chapter.

In addition to the regulation of body attitude, a yaw velocity is needed to control the direction of heading. The crab angle, $\alpha$, is shown in Figure 3.5 and the yaw velocity, $\dot{\alpha}$, is the rate about the $Z_B$-axis. The yaw velocity can be related to the crab angle through a number of simple control schemes, some of which are shown in Figure 5.9. In Chapter 3, the LSM was shown to increase with the crab angle within a certain range. Hence, the deviation from the zero crab angle situation may be tolerated and to a certain extent may even be desired. In addition to an increase in LSM, the problem of leg interference is somewhat alleviated. This is because, for symmetric support patterns with a zero crab angle, ipsilateral legs must stroke along the same straight line. Hence the yaw velocity prescribed in Figure 5.9(a) may not be the best solution as this would attempt to maintain a zero crab angle. The other extreme is shown in Figure 5.9(c) in which the yaw velocity is always zero. The situation shown in Figure 5.9(c) may be a suitable compromise. A small crab angle has its advantages and is therefore tolerated, but larger crab angles are resisted. The issue of relating energy efficiency with the crab angle has not been researched and may yield a better optimal value for the crab angle.

So far, the attitude control and the crab angle control have been discussed independently. This treatment is valid only if the angles
Figure 5.9 Yaw Velocity and Crab Angle
Figure 5.10 Altitude Control
involved are small. If this is the case the two rates, \( \dot{\alpha} \) and \( \dot{\xi} \) may be superposed to yield the resultant angular velocity of the vehicle which determines the three components of angular rates.

The fourth velocity component of interest is the rate of change of body height. As mentioned earlier, the height regulation problem is coupled with the problem of attitude control. It is advantageous from the point of view of stability, to minimize the height on an incline when the body is parallel to the incline. The height may be expressed as a function of body attitude in different ways as shown in Figure 5.10. Figure 5.10(c) shows a scheme compatible with the strategy for attitude control illustrated in Figure 5.7(c). Another important consideration in determining the body altitude is the roughness. It is better to increase the body altitude in rough terrain to avoid interference problems associated with the geometry of the vehicle. The rate of change of height can be error driven:

\[
\dot{h} = -K_h (h - h_D)
\]

Thus, if the desired vehicle rates are represented by \( B_{\dot{v}_D} \) (linear) and \( B_{\dot{\omega}_D} \) (angular) in the reference frame \( [B] \),

\[
B_{\dot{v}_D} = \begin{bmatrix} B_{\dot{v}_Dx} \\ B_{\dot{v}_Dy} \\ \dot{h} \end{bmatrix}
\]
where $\mathbf{V}_{Dx}$ and $\mathbf{V}_{Dy}$ are the desired velocities specified at the navigator level, and

$$
\mathbf{B}_D = \mathbf{\dot{u}} + \mathbf{\ddot{u}}
$$  

(5.17)

In Equation (5.17), $\mathbf{\dot{u}}$ is determined in the following way. If $\mathbf{B}_s$ is the normal of the support plane expressed in the $\{E\}$ reference frame, then a simple transformation yields $\mathbf{B}_s$ in the reference frame $\{B\}$:

$$
\mathbf{B}_s = \mathbf{B}_s^E \mathbf{E}
$$  

(5.18)

If $n_{sx}$, $n_{sy}$ and $n_{sz}$ are the direction cosines of $\mathbf{n}_s$ with reference to $\{B\}$, then

$$
\mathbf{\hat{Z}}_{BD} = \left[ \begin{array}{c}
n_{sx} \sin \eta_B \\
n_{sy} \sin \eta_B \\
\cos \eta_B \end{array} \right]
$$  

(5.19)

and

$$
\mathbf{\hat{m}} = \frac{\mathbf{\hat{Z}}_B \times \mathbf{\hat{Z}}_{BD}}{|\mathbf{\hat{Z}}_B \times \mathbf{\hat{Z}}_{BD}|}
$$  

(5.20)

Thus, the three angular rates and the rate of change of altitude are automatically determined by a set of simple control schemes. These
schemes can be used by the control system in real-time. The guidance system uses such strategies to plan the motion of the body. More details on the implementation can be found in Chapter 6.

5.6 Conclusion

It has been shown that the stroke, duty factor and time period can be controlled to achieve a desired velocity and at the same time an optimal value of either the stroke or the time period. Strategies for gait transitions have been discussed. It is possible to incorporate this control scheme into the guidance module. In addition, the phase angles need to be modified to maintain the optimal \( \theta_{opt} \) and \( \phi_{opt} \) output by the Gait Optimizer and this can be done using the same general strategy for gait transitions.

The automatic sequencing of footholds in time is determined by the modified wave gait and the associated gait control scheme. An important issue is the selection of footholds in space. The selection of footholds is based on simple heuristics derived from the kinematics and dynamics of the problem.

The problem of automatic regulation of body motion has been analyzed. The body attitude control depends on an effective support plane which is derived from the support points and potential support sites ahead of the vehicle. The height regulation is dependent on the attitude of the vehicle and the roughness of the terrain. The yaw
velocity is controlled to restrict the angle between the longitudinal axis of the body and the velocity of the body.

The basic framework of the guidance module and the strategies involved have been discussed in this chapter. A computer simulation of these algorithms is presented in the next chapter.
CHAPTER VI
A SIMULATION OF THE GUIDANCE MODULE

6.1 Introduction

The implementation of the guidance module and a computer simulation of the different concepts and strategies discussed in Chapters 3, 4 and 5 are presented in this chapter. The relationship and interface between the different units of the guidance module are also analyzed. Within the guidance module two definite levels can be identified. The upper, or the supervisory level performs the functions of local path modification and gait optimization and the lower level is involved with the foothold selection, gait planning, force distribution and coordination, and body motion planning. The interface between the two levels is completed by a diagnostics and remediation unit which monitors the lower level and suggests changes to either of the two modules at the upper level or the navigator, or transfers control back to the appropriate lower level units.

Figure 6.1 shows the general philosophy of design of a pilot module. The navigator and the controller are shown here to complete the picture, but a detailed discussion of these units is outside the scope of the present research effort. The functions of the different units at the lower level have been discussed in chapters 4 and 5. The
Figure 6.1 A Detailed Outline of the Guidance Module
problem of gait optimization and control was addressed in Chapters 3 and 5. These lower level units have been simulated on a VAX 8500 machine and are discussed in greater detail in the following sections. The broad specifications for the ASV are used here. The geometry of the ASV is chosen for purposes of illustration, but the discussion is general enough to be applied to any six legged robot. The cartographer and the terrain scanner have been simulated by using mathematical functions at a macroscopic level and a random number generator at the microscopic level to create a model of the terrain. The actual terrain-vehicle system has not been simulated and, accordingly, the drift or the discrepancy between the planned and actual paths has not been accounted for. It must be emphasized that the objective of this exercise is merely to demonstrate the feasibility and utility of the control schemes discussed earlier, and not to produce a working control structure for the ASV. The units within the dashed lines in Figure 6.1 represent the scope of this discussion.

6.2 Simulation Model

A brief description of the geometry of the model of the ASV and the leg kinematics is presented here. The geometry of the model is considerably simplified (see Figure 6.2). The exact mechanical system details of the ASV, and analysis of the kinematics of the pantograph leg mechanism can be found elsewhere [89], [97], [98]. A schematic of the leg mechanism is shown in Figure 6.2 (b). The dimensions of the
Figure 6.2 Model of the ASV Used for Simulation
(a) Geometry of the Model
(b) Kinematics of the Pantograph Leg
Table 6.1 Dimensions of the Model of the ASV Used in the Simulation.
(All dimensions are in feet - Figure 6.2)

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Width (TW)</td>
<td>3.5</td>
</tr>
<tr>
<td>Bottom Width (BW)</td>
<td>2.0</td>
</tr>
<tr>
<td>Height (H)</td>
<td>4.0</td>
</tr>
<tr>
<td>Magnification ratio (m)</td>
<td>5</td>
</tr>
<tr>
<td>Offset (r)</td>
<td>1.45</td>
</tr>
<tr>
<td>Dimensions of Pantograph</td>
<td></td>
</tr>
<tr>
<td>- a</td>
<td>4.0</td>
</tr>
<tr>
<td>- b</td>
<td>0.8</td>
</tr>
<tr>
<td>Pitch (P)</td>
<td>5.0</td>
</tr>
<tr>
<td>Length (L)</td>
<td>13.0</td>
</tr>
</tbody>
</table>

Home Positions
- Leg 1                  (5.0, 2.5, -5.5)
- Leg 2                  (5.0, -2.5, -5.5)
- Leg 3                  (0.0, 2.5, -5.5)
- Leg 4                  (0.0, -2.5, -5.5)
- Leg 5                  (-5.0, 2.5, -5.5)
- Leg 6                  (-5.0, -2.5, -5.5)
model can be found in Table 6.1.

In the figure, the point \( K \) is the knee, \( F \) is the foot and \( O \) is the point of intersection of the lift axis with the \( x_B \)-\( y_B \) plane. The variable \( \& \) represents the lift actuator displacement, \( d \) the drive actuator displacement and \( \theta \) the abduction-adduction motion of the leg. In the actual mechanism the abduction-adduction motion is not equivalent to a pure rotation about an axis parallel to the \( x_B \)-axis \cite{89}. Thus, this simple model is only an approximation. However it is sufficient for the purposes of simulation as the leg kinematics is used only for the graphics in the simulation. From Figure 6.2(b),

\[
\begin{align*}
(x_F - x_O) &= md \\
(y_F - y_O) &= (mr + \lambda(m-1)) \sin\theta \\
(z_F - z_O) &= -(mr + \lambda(m-1)) \cos\theta
\end{align*}
\]

where \( m \) is the magnification ratio of the pantograph \((= a/b)\) which is equal to 5. For a given foot position, \((x_F, y_F, z_F)\), the equations for inverse kinematics may be easily written:

\[
\begin{align*}
\theta &= \tan^{-1} [(y_F - y_O)/(z_F)] \\
d &= (x_F - x_O)/m \\
\lambda &= \frac{1}{4} (z_F/\cos\theta - 5r)
\end{align*}
\]

The point \((x_K, y_K, z_K)\) can be determined from simple trigonometry and as the points \( F, K \) and \( O \) are known the position of the leg mechanism
Figure 6.3 Workspaces of the Legs for the Model
(a) Workspace Volume
(b) Partitioning Leg Workspace into Cells
is completely determined. The leg workspace used in the simulation is shown in Figure 6.3 for leg 1. All legs are assumed to have identical workspaces. Again it is only an approximation of the actual ASV leg workspace (which is not a parallelepiped as shown in the figure) and this simplifies the geometric constraint equations. In principle, it is possible to accommodate any leg workspace geometry.

The foot size of the ASV leg is 12" by 6". The workspace may be divided into cells by considering a grid on a plane parallel to the \( x_B - y_B \) plane through the home position of the leg. The home position for a leg is a point in the workspace of the leg which is chosen arbitrarily such that, for an ideal gait, the foot strokes through the home position which is also the mean position. Table 6.1 lists the home positions for all the six legs. A cell size of 18" X 9" (in Figure 6.3) precludes the possibility of interference of feet placed on adjacent sites. Note that this does not eliminate the problem of leg interference which must be dealt with at the lower level of trajectory planning by proximity sensing. The chosen cell size allows 20 candidate support sites for each leg for the assumed leg workspace. The indices \( i \) and \( j \) determine the location, \( B_e \), in body coordinates and \( B_{pfx} \) and \( B_{pfy} \) are simple functions of \( i \) and \( j \) respectively. However, \( B_{pfz} \) depends on the terrain topography. The point of intersection of a line parallel to the \( Z_B \) axis through the center of the cell and the terrain surface is designated to be \( B_e \) and its position vector can be found in the reference frame \( CEJ \). A simple
transformation yields $B_E$. Finding the intersection of a line and a
discretized surface may be cumbersome and an alternative method is
suggested below.

A reference (S) is defined such that $z_S$ is parallel to $z_E$ but $x_S$
and $y_S$ are projections of $x_C$ and $y_C$ along $z_E$ on the $x_E^-$ $y_E$
plane. Thus
the 5 X 4 grid of rectangular cells is projected on to a plane
parallel to the $x_E^- Y_E$ plane. Thus, if the transform $R_S$ describing the
reference frame (S) with respect to (B), is computed, then $S_E$ is
given by:

\[
\begin{align*}
S_{px} &= (i-3)X_{cell} \\
S_{py} &= (j-1)Y_{cell} \\
S_{pz} &= 0 \\
\end{align*}
\]

for legs 1, 3, and 5

\[
= (1-j)Y_{cell} \\
\]

for legs 2, 4, and 6

and $E_E$ can be found quite simply by:

\[
E_E = E_B B_S E_S
\]

The z coordinate of the actual point on the terrain can be determined
from the terrain model as $E_{pz}$ is known as a function $E_{px}$ and $E_{py}$
and $B_E$ can be computed by another simple transformation:

\[
B_E = B_E E_E
\]

6.3 Foothold Selection

The criteria for selecting footholds have been discussed in
Section 5.4. As mentioned earlier, they make conflicting demands and a
compromise between the desired characteristics is sought. It is convenient to use a heuristic function, \( \Omega \), as a linear combination of suitable measures of the different desired effects to evaluate potential support sites:

\[
\Omega = c_s h_s + c_n h_n + c_x h_x + c_y h_y + c_z h_z + c_r h_r
\]  
(6.1)

In this equation, \( c_s, c_n, c_x, c_y, c_z \) and \( c_r \) are constants or gains. The other quantities are functions based on the criteria listed in Section 5.4.

The first function, \( h_s \), is a measure of the stability of the vehicle in the locomotion cycle following the foothold selection for the stepping leg. The procedure of Section 5.4 (see Table 5.1 and Figure 5.5) is followed to estimate the gait LSM by computing the LSM at the two critical events. \( h_s \) is taken to be the LSM obtained by this suboptimal procedure. If the LSM is negative, the possibility of instability exists and a penalty is imposed on the foothold by adding a large negative number to \( h_s \).

\( h_n \) depends on the orientation of the terrain normal at the foothold and is an estimate of the friction angle at that foot for the selected foothold. It is convenient to define \( h_n \) as:

\[
h_n = \hat{W}_n \cdot \hat{k} = \hat{W}_n z
\]
where the leading superscript \( \mathcal{W} \) represents the \( \{ \mathcal{W} \} \) coordinate system and the unit vector \( \hat{n} \) is the normal at the potential foothold. Recall that the \( z \)-axis of the reference frame \( \{ \mathcal{W} \} \) is parallel to the wrench axis. A candidate support-site with a large \( h_n \) is preferred because the ratio of the normal to tangential components of force at that support-site may be expected to be higher.

In Section 5.4, it was concluded that an optimal situation based on force distribution considerations is one in which the feet lie on the \( \sigma \)-plane \( (x_w - y_w \) plane) and the feet are symmetrically placed about the load-wrench axis. In addition the feet are as far away from the axis as possible which also tends to maximize the static stability margin as defined in Chapter 3. If \( E_L \) is the candidate support site for a leg, \( h_z \) is equal to the negative of the absolute value of the \( z \) component of \( W_{E_L} \). The largest value of \( h_z \) is zero in which case the foot lies on the \( x_w - y_w \) plane and, if all the feet lie on this plane, the force vector obtained by the equilibrating forces technique has no interaction force components and does belong to the equilibrating force field (see Chapter 4).

It is difficult to constrain the feet to be symmetrically disposed about the load-wrench. This is because the vehicle is constantly moving while the support pattern is stationary between stepping and lifting events. Also the hardware restricts the legs to lie within their respective workspaces and the arrangement of the legs on the vehicle and the geometric constraints engendered by the
stroking of the legs further restricts the foot placements. However, it is evident that the middle legs must be abducted as much as possible while the front and rear legs must be adducted. Let $\mathbf{r}_C$ be the position vector of point $C$ in Figure 3.5 (the intersection of the wrench axis with the $\alpha$-plane) and $\text{SIGN}$ be equal to $+1$ for odd legs and $-1$ for even legs. For the middle legs $|\mathbf{W}_{P_{fy}} - \mathbf{W}_{R_{Cy}}|$ must be maximized and similarly, for the rear and front legs, the same quantity must be minimized. A similar simplistic analysis can be made for the $x$ direction. Thus $h_x$ and $h_y$ may be defined as:

$$h_x = (\mathbf{W}_{P_{fx}} - \mathbf{W}_{R_{Cx}}) \text{ for legs 1 and 2}$$
$$= -|\mathbf{W}_{P_{fx}} - \mathbf{W}_{R_{Cx}}| \text{ for legs 3 and 4}$$
$$= (\mathbf{W}_{R_{Cx}} - \mathbf{W}_{P_{fx}}) \text{ for legs 5 and 6}$$

and

$$h_y = -\text{SIGN} \times (\mathbf{W}_{P_{fy}} - \mathbf{W}_{R_{Cy}}) \text{ for legs 1, 2, 5, and 6}$$
$$= \text{SIGN} \times (\mathbf{W}_{P_{fy}} - \mathbf{W}_{R_{Cy}}) \text{ for legs 3 and 4}$$

However, if $\mathbf{W}_{P_{fy}} \times \text{SIGN}$ is less than $\mathbf{W}_{R_{Cy}}$ then $h_y$ is set to a large negative number. This is possible when the crab angle is large. This analysis is restricted to the case where the leg phasing is governed by the MWG (modified wave gait) described in Chapter 3. Unfortunately, when the crab angle is large in magnitude, the choice of support sites for legs 1 and 6 (when $\alpha$ is greater than $\alpha_c$) or legs 2 and 5 (when $\alpha$ is less than $-\alpha_c$), is restricted.
$h_r$ takes into account the geometric constraints imposed by the leg kinematics. If placing a leg at a candidate foothold causes a reduction in the current stroke (and consequently a change in the time period), $h_r$ is given a large negative value. This suitably increases the preference for footholds which permit complete stroking. At the same time footholds which cause a reduction in the stroke may be used if better footholds are unavailable.

In different situations, depending on the encountered terrain features, the gains can be tweaked to achieve the desired objective. The function $f_t$ is used to evaluate desirable characteristics for each of the 20 cells shown in Figure 6.3 unless they fall outside the workspace boundary. It is convenient to order them in descending order of $f_t$ in a priority queue and to dispense them to the foothold selection module when required.

6.4 Gait Control

The flowcharts shown in Figure 6.4 capture the essence of the adaptive gait control module. The different modules in this figure are described in some detail. The underlying concepts are discussed in Chapter 3 and Section 5.5.

In Figure 6.4, $\text{LSM}_d$ is a desired minimum LSM which should be maintained and $\text{LSM}_{\text{min}}$ is the minimum allowed LSM. In other words, the system tries to maintain (while planning) a LSM greater than or equal to $\text{LSM}_d$ but at any rate the LSM must not be allowed to fall below
Figure 6.4 Gait Controller in the Pilot Module
The "Foothold_Flag" is a flag which is reset before planning an event and is set when a possible (reachable) foothold is found for the event (only for stepping events). "Update Legs_Status" updates the position, velocity, local phase and state of all the six legs. The function of this unit is described in greater detail later. If the event is a stepping event, the foothold selection routine is invoked and the position of the stepping leg is accordingly determined. The vehicle status before this selection is stored, for reasons that are explained later.

The control of time period and stroke have been discussed earlier in Section 5.3. Following a stepping event, the maximum instantaneous time period, \( \tau \), is computed from the workspace constraints (see Equation (5.5)). The pathological leg is defined as the leg for which the corresponding instantaneous time period is equal to \( \tau \). In other words, the pathological leg is the leg which forces the stroke to be reduced. If \( \tau \) is equal or close in value to the time period there is no need to change the gait parameters. A larger \( \tau \) allows the stroke to be increased and the stroke is the controlled variable (Section 5.3.6). A smaller \( \tau \) complicates the problem considerably as a reduction in stroke is mandatory. Thus the constraint on the stroke to be less than \( r_{\text{max}} \) is less critical and small deviations from this restriction can be easily tolerated. "Control Time Period" varies the gait parameters to reduce the stroke and yet maintain the same leg velocity.
If the time period cannot be reduced below \( \tau \) by "Control Time Period", then, if the pathological leg is not the stepping leg, the problem can be attributed to a leg whose foothold was planned earlier. A simple way to circumvent the problem is by decreasing the velocity of the vehicle. The desired decrease in velocity may be described by a multiplicative factor less than unity. This factor can be computed by dividing \( \tau \) by the current time period. The decrease in velocity cannot be achieved instantaneously. The dynamics of the vehicle must be considered and the planning process must be backed up by a couple of locomotion cycles. This is shown in Figure 6.6. However, if the stepping leg is the pathological leg, the problem is remedied quite easily by selecting another foothold which allows a larger stroke, which is shown in Figure 6.4(b). The vehicle status before the stepping event which is stored in memory is now recovered to try an alternative foothold.

The vehicle stability at the decision symbol is computed by estimating the minimum LSM over the locomotion cycle following the stepping event. The vehicle stability is checked every stepping event. In the event the LSM thus obtained is less than \( \text{LSM}_d \) (which is indicative of potential instability) another foothold is tried. In Figure 6.4(b), if the LSM computed at the decision symbol is greater than the minimum permissible LSM (\( \text{LSM}_{\min} \)), but less than \( \text{LSM}_d \), then the foothold is stored in memory. In the event that a foothold is already in memory, the foothold which produces the greater LSM is
retained in memory. The stored vehicle status is recovered before an alternative foothold is evaluated. This procedure is continued until a foothold with the LSM greater than $\text{LSM}_d$, is found or the foothold priority queue is empty. If the priority queue is empty, and if the foothold_flag is set, then the best available foothold is used. Notice that $\text{LSM}_d$ is reduced for this stepping event and is reinitialized before the next event. If the LSM corresponding to the best available foothold is less than $\text{LSM}_d$, it may not mean that the LSM of the planned motion has fallen below $\text{LSM}_d$. This is because the vehicle stability at the decision symbol is estimated from the LSM over the locomotion cycle following the event and not just from the LSM at or immediately after the event. It is likely that the foothold selection process will remedy the problem in the stepping events which follow.

If no footholds are available and the foothold flag is still reset, it means that a foothold with the LSM $> \text{LSM}_{\text{min}}$ could not be found and an unstable configuration is predicted. This is indicated by the control point, 4, in the figure. The planning process is then aborted and the control is transferred to the diagnostic unit. An example of a corrective action, that may be taken by the remediation unit, is to repeat the planning process with a lower vehicle velocity, so that, a larger number of footholds becomes available to the foothold selection unit.

One of the most important functions of event planning is to implement the 12 event sequential logic and the change of the three
states (see Figure 5.2) associated with each of the six legs. The updating of the status of the legs over the 12 events, or the 24 time instants (for each event the time instant just before and after the event is considered) is outlined in Figure 6.5. The "Event Table" in Figure 6.5 is a list of 24 time instants over a cycle normalized to the time period and sorted in increasing order. Clearly the event table must be reconstructed every time the gait is changed.

The "Event Planner" uses the event_table to calculate the time at which the next event is scheduled to occur. The phase increment is calculated quite simply by dividing the time increment by the time period. The status of all the legs is updated according to these increments. If the new local phase of any leg is greater than the duty factor, the leg must be in the transfer or ready phase. If the previous state is a 1, it is changed to -1 and the deploy time is set to the threshold return time, $t_{R,min}$ (see Section 5.3.1). If the previous state is a 0, it remains at 0. A -1 state is changed to 0 if the time elapsed since the transition from 1 to -1 (lifting of the leg) is greater than the deploy time, and otherwise remains at -1. If the new local phase is less than the duty factor, it indicates that the leg must be in support phase. If the previous state is a 1, it remains at 1 and the foot position and velocity are updated using the leg kinematics equation (Equation 5.2). The maximum displacement, $\bar{Z}$, possible at the current velocity as defined in Section 5.1, is computed from the leg workspace specifications. Clearly, the elements
Figure 6.5 Flowchart for Updating Status of the Legs
of $\&$ must remain positive. The previous state cannot be a -1, as the transition from -1 to 1 is impossible and cannot happen. The 0 state requires the selection of a foothold for the change of state from 0 to 1 and is accompanied by computation of the foot velocity and the maximum possible displacement, $\&$, for the selected foothold.

6.5 Vehicle Body Dynamics and Control

The mass of the legs is assumed to be negligible compared to that of the vehicle. This is a reasonable assumption for the ASV and is easily justified at a planning stage. Thus the dynamics of the vehicle reduces to simple rigid body dynamics which can be described by Euler's equations (as written in Chapter 4) from which the load-wrench may be easily found.

In Figure 6.6(a), the subscript 'd' denotes the desired quantities, 'c' the commanded quantities and 'a' the actual quantities. $\omega_d$ and $v_d$ are the desired angular and linear velocities, and $\alpha_d$ and $a_d$ are the desired angular and linear accelerations. The desired linear velocity in the horizontal plane is specified by the navigator at the path selection level. This linear velocity may be scaled down if, subsequently, the gait controller demands a reduction in the velocity. This is shown by the control point 4, in Figure 6.6(b) (see also Figure 6.5).

The other four components of velocity are determined by the automatic body motion regulation schemes as discussed in Section 5.5
Figure 6.6 (a): Velocity Control for the Vehicle
Figure 6.6.(b): Vehicle Body Kinematics and Dynamics
The body attitude regulation is performed in two different ways depending on the mode of control. Ordinarily, the desired attitude is computed as shown in Figure 5.7(c). The reasons for this choice have already been discussed. If sudden changes in terrain gradients can be expected, the support plane is computed by "looking ahead" to locate potential support sites. This called the "Look Ahead" mode as opposed to the "No Look Ahead" mode. In this mode the scheme illustrated in Figure 5.7(a) is preferred as the control algorithm reacts faster to the changes in the terrain. The yaw velocity and the rate of change of height are regulated as illustrated in figures 5.9 and 5.10 respectively.

Once $u_d$ and $v_d$ are known, an error driven scheme is used to compute the commanded accelerations, $a_c$ and $a_c$. Euler's equations are used to find the commanded force system acting on the vehicle. The force system or the load-wrench, $w$, is the resultant of the inertial forces and torques, and the weight of the vehicle. The force distribution algorithm or the coordination algorithm decomposes the force system into commanded foot forces, $F_4$, which can be used to calculate the actuator forces, $f_4$, for the controller. Thus the control scheme in Figure 6.6(a) is in effect a computed torque scheme. However, for the purposes of planning, it is assumed that the controller and the plant dynamics are ideal and the actual and commanded accelerations are identical. The force distribution algorithm can, in principle, be used to scale the commanded
Figure 6.7 The Pilot Operation Simulation
accelerations back if the desired locomotion characteristics are not present. In this work, however, ceiling values are set for the maximum allowed accelerations. The desired accelerations are scaled back so that all the accelerations are below the maximum allowed values. The actual velocities are obtained by integration as shown in the figure. The vehicle body kinematics and dynamics are represented in Figure 6.6b.

In the event that the gait controller requires the velocity to be reduced, the "Slow_Down_Factor", or the factor by which the vehicle velocity must be reduced by, is computed, as described in Section 6.4 and this is used to scale back the desired velocities. This is shown in Figure 6.6. In such a situation, the whole planning process has to be repeated because the velocities cannot be changed instantaneously. However, it is assumed that discontinuous accelerations can be achieved.

6.6 The Pilot

The important units of the simulation are shown in Figure 6.7. The relationship between these units and the function of the coordination algorithms at the planning level is discussed here.

In Figure 6.7, "At Destination" is a Boolean function which indicates whether the vehicle is within a small distance of the destination. The diagnostics and remediation module monitors the operation of the gait controller, the foothold selection module and
the force distribution algorithm and performs the function of error
detection and recovery. Ideally this module should be configured as an
expert system. In the present framework it can take any of the
following corrective actions:

(1) Slowing down the vehicle
(2) Command the foothold selection unit to select another
foothold
(3) Change gains in the foothold selection heuristics
(4) Change gains in the body motion regulation algorithm
(5) Change mode of computing support plane

The logic on which these discussions are based has been discussed
earlier.

The "Event_Table" is stored in memory and is always available to
the gait controller. A new table must be constructed with every change
in the gait.

It was suggested in Chapter 4 that the Phase I part of the
Revised Simplex method could be adopted for checking for vehicle
stability. The decision symbol below the force distribution unit in
Figure 6.7 indicates this. In the event of instability it is easy to
determine the foot (feet) whose placement caused the instability. This
can be deduced from the friction angles at the feet and the magnitudes
of the support forces. The process may be repeated with a different
foothold(s) for the foot (feet) which engendered the instability. This
corrective action should not be required in general, because the
foothold selection algorithm is based on force distribution considerations, among other factors. In this simulation, the force distribution is merely used as a check for stability.

6.7 Examples

A computer simulation of the guidance module has been used to generate a few examples. The examples demonstrate the feasibility of automation in surmounting typical features in uneven terrain with preview sensing. The computer simulation was done on a Digital VAX 8500 machine using VAX FORTRAN. The VAX FORTRAN language provides PASCAL like enhancements to the FORTRAN 77 language. DI-3000 was used to display a three-dimensional wire frame model of the terrain and the vehicle on a Tektronix 4107 terminal.

The modified wave gait can be recognized to be a powerful and versatile gait which is easy to implement in a gait controller. The ability to automatically regulate body motion by using a few relatively simple control schemes is also demonstrated. The contribution of the foothold selection algorithm in improving the force distribution characteristics is also seen in the examples. The constants, parameters and gains used for these examples are listed in Table 6.2. The figures for the examples can be found in Figures 6.8 - 6.13.
Table 6.2 Constants used in the simulation

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial height above the ground</td>
<td>5.5 feet</td>
</tr>
<tr>
<td>Maximum allowed stroke (R_{max})</td>
<td>4.0 feet</td>
</tr>
<tr>
<td>Desired LSM</td>
<td>1.0 feet</td>
</tr>
<tr>
<td>Minimum allowed LSM</td>
<td>0.1 feet</td>
</tr>
<tr>
<td>Minimum allowed duty factor</td>
<td>0.6</td>
</tr>
<tr>
<td>Minimum return time, t_{R,min}</td>
<td>0.1 sec.</td>
</tr>
<tr>
<td>Maximum allowed accelerations</td>
<td></td>
</tr>
<tr>
<td>Linear accelerations</td>
<td>4.0 feet/sec^2</td>
</tr>
<tr>
<td>Angular accelerations</td>
<td>15.0 degrees/sec^2</td>
</tr>
<tr>
<td>Mass of the vehicle</td>
<td>7000 lb.</td>
</tr>
<tr>
<td>Moments of inertia of the vehicle</td>
<td></td>
</tr>
<tr>
<td>I_{xx}</td>
<td>16880 lb-ft.sec^2</td>
</tr>
<tr>
<td>I_{yy}</td>
<td>128223 lb-ft.sec^2</td>
</tr>
<tr>
<td>I_{zz}</td>
<td>132397 lb-ft.sec^2</td>
</tr>
<tr>
<td>Constants in Figures 5.7 - 5.10</td>
<td></td>
</tr>
<tr>
<td>( \eta_1 )</td>
<td>15.0</td>
</tr>
<tr>
<td>( \eta_2 )</td>
<td>25.0</td>
</tr>
<tr>
<td>( \eta_3 )</td>
<td>45.0</td>
</tr>
<tr>
<td>( \eta_{D1} )</td>
<td>10.0</td>
</tr>
<tr>
<td>( \eta_{D2} )</td>
<td>30.0</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>10.0</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>25.0</td>
</tr>
<tr>
<td>S_{1}</td>
<td>0.067</td>
</tr>
<tr>
<td>S_{2}</td>
<td>0.1</td>
</tr>
<tr>
<td>h_{1}</td>
<td>6.0</td>
</tr>
<tr>
<td>h_{2}</td>
<td>5.8</td>
</tr>
<tr>
<td>h_{3}</td>
<td>5.0</td>
</tr>
<tr>
<td>Gains and Inputs</td>
<td>Examples</td>
</tr>
<tr>
<td>-----------------</td>
<td>----------</td>
</tr>
<tr>
<td>$K_\alpha$</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_\xi$</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_z$</td>
<td>0.2</td>
</tr>
<tr>
<td>$C_x$</td>
<td>1.0</td>
</tr>
<tr>
<td>$C_y$</td>
<td>1.0</td>
</tr>
<tr>
<td>$C_z$</td>
<td>1.0</td>
</tr>
<tr>
<td>$C_n$</td>
<td>2.5</td>
</tr>
<tr>
<td>$C_g$</td>
<td>1.0</td>
</tr>
<tr>
<td>Look Ahead</td>
<td>NO</td>
</tr>
</tbody>
</table>
Example (a) Gait control on even terrain (Figure 6.8)

The efficacy of the gait control strategies proposed in Chapter 5 is demonstrated by this example. A long path of 90 feet (about 6 body lengths) is selected for this demonstration though, in practice, the guidance module would be invoked several times to plan over such large distances. The commanded velocity is 10 feet/sec (6.8 miles/hour). The first order nature of the velocity-time plot in Figure 6.8.1 is due to the simple error driven proportional control law. The system is, in effect, a sampled data system with respect to time as the state of the system is updated only at the locomotion events. The stroke, time period, and duty factor of the gait are changed continuously to maintain this velocity profile. The system reaches a steady state at a time of 8 seconds (and a distance of 56 feet) when the velocity is within 5% of the commanded velocity in Figure 6.8.1. The system tries to increase the stroke to the maximum allowed stroke \( R_{\text{max}} = 4.0 \) feet in Figure 6.8.3. The stroke does overshoot \( R_{\text{max}} \), which happens when the stroke is equal to \( R_{\text{max}} \) and the time period is the control variable. This state continues till the time period is greater than \( \tau \). This happens because the stroke is not treated as a critical variable and as the flow chart indicates a stroke larger than \( R_{\text{max}} \) is tolerated if the time period is at the desired "optimal" value. The steady state values for duty factor, time period and stroke are 0.67, 0.62 and 4.0 respectively. As the terrain is even, all support sites have normals which are parallel to the \( Z_g \) axis, and hence, are equally attractive from force distribution.
Figure 6.8.1
SIMULATION RESULTS

Figure 6.8.2
SIMULATION RESULTS

Figure 6.8: Gait Control on Even Terrain
Figure 6.8.3

SIMULATION RESULTS

Figure 6.8.4

SIMULATION RESULTS

Figure 6.8 (Continued)
considerations. Therefore the foothold selection module attempts to maximize the quasi-static stability. As Figure 6.8.4 indicates, the minimum LSM at steady state is 3.25 and a theoretical, ideal gait with the same duty factor, stroke and a zero crab angle yields a LSM of only 2.21. The improvement of over 60% with the same piece of hardware is an important benefit derived from the foothold selection routine. The SM however, does not improve and both the value derived from the ideal footholds, and that obtained from the simulation results are equal to 1.75. The average computational time for a locomotion cycle is 1.4 seconds on the VAX 8500. It is believed that the time can be reduced by as much as 50% if the opportunities for parallelism (some of which are evident in Figures 6.1 through 6.7) are exploited.

Example (b) Climbing a 3 foot step (Figure 6.9)

Figure 6.9.1 shows the vehicle climbing a 3 foot step. The terrain is level except for the step and the terrain model is thus only two-dimensional. When the vehicle body is directly over the step, all the six legs may not be able to reach the ground in some configurations. The "Look Ahead" mode is essential to "prepare" the vehicle (to avoid such configurations) by changing its attitude before actually encountering the step.

The algorithm is sensitive to values of $K_\phi$ and $K_z$ (gains for the attitude control model, defined in Chapter 5) and 1 or 2 iterations may be required before the correct combination of gains
Figure 6.9: Climbing a 3 Foot Step
Figure 6.9.2

SIMULATION RESULTS

VELOCITY (FT/SEC)

Figure 6.9.3

SIMULATION RESULTS

DUTY FACTOR (.81)

Figure 6.9 (continued)
Figure 6.9.4

SIMULATION RESULTS

Figure 6.9.5

SIMULATION RESULTS

Figure 6.9 (continued)
Figure 6.9.6  
SIMULATION RESULTS  

Figure 6.9.7  
SIMULATION RESULTS  

Figure 6.9 (continued)
is found for the successful completion of the task. The vehicle tries to achieve an average velocity of 3.0 feet/sec (see Figure 6.9.2) and maintain a LSM of at least 1.0 feet (Figure 6.9.5). It is clear that the gait controller is successful in varying the stroke, duty factor and time period to maintain the desired velocity (Figures 6.9.3 and 6.9.4). It is evident, from Figure 6.9.4, that the time period and stroke had to be reduced just before the step. This is because short steps had to be taken by the front legs as the position and orientation of the body relative to the terrain did not permit support sites on top of the step. In Figure 6.9.7, the variation of the body height of the body is plotted against time. The height of the body C.G. in its initial configuration (5.5 feet above the ground as shown in Table 6.2) is taken to be zero. The system overshoots the desired height of 6.0 feet (see Figure 5.10 and Table 6.2 for the values of constants and the gains in the control schemes) above the support plane or 3.5 feet in the figure. At first glance this seems unusual as the control strategy described in Chapter 5 for altitude control is a first order control system. However, the desired rate of change of altitude is prescribed by an error driven proportional controller and the difference between the actual rate of change and the desired rate of change determines the acceleration in the z direction. Hence the system is actually a second order system. \( K_z \), the gain for the altitude control system determines the overshoot. In this example \( K_z \) is set very high which decreases the effective damping of the system.
Figure 6.10: Stepping Down a 3 Foot Step
Figure 6.10.2
SIMULATION RESULTS

Figure 6.10.3
SIMULATION RESULTS

Figure 6.10 (continued)
Figure 6.10.4

Figure 6.10.5

Figure 6.10 (continued)
Figure 6.10.6

Figure 6.10.7

Figure 6.10 (continued)
In other situations, it may be safer to overdamp the system by lowering \( K_z \) so that the overshoot is zero. Figure 6.9.6 shows the body attitude increasing as the vehicle nears the step and then decreasing to come back to an even keel. For small body inclinations, the control system for the body attitude is a second order system. This is not the case here. The LSM in Figure 6.9.5 varies through the cycle but remains well above 1.0 feet. The SM is also above 1.0 feet (though it is not guaranteed to be so) and is the lower bound for the LSM.

**Example (c): Climbing down a 3 foot step (Figure 6.10)**

This is similar to the previous example and the side view of the motion of the vehicle can be seen in Figure 6.10.1. The initial height of the vehicle is set to be 4.5 feet above the ground (-1.0 in the \( \{E\} \) reference frame) so that the vehicle is crouching initially. This initial configuration was chosen to test the altitude regulation subsystem. In Figure 6.10.7, the height increases towards the desired height of 6.0 feet (+0.5 feet in \( \{E\} \)) before the presence of the step is detected and the altitude is reduced. The absolute value of the body attitude angle (\( \eta \)) is plotted in Figure 6.10.6. The attitude angle is actually negative as the front end of the vehicle tilts down. The velocity in Figure 6.10.2 shows an unexpected deviation from the exponential convergence because the desired vehicle velocity is computed in the body fixed reference frame, \( \{B\} \), and the velocity plotted in the figure is in earth fixed coordinates. These velocities
do not include the \( z \) component and therefore their magnitudes differ in the two coordinate systems.

**Example (d) Crossing a 4 foot ditch (Figure 6.11)**

Figure 6.11.1 shows a flat terrain with a 4 foot ditch and a cylindrical hole with a 5 foot diameter. The vehicle is commanded to move to a destination with coordinates \((20.0, 10.0, 0.0)\) in the coordinate system, \( (E) \). This requires a crab gait and the maximum velocity component is restricted to be 3 feet/second in Figure 6.11.2. The crab angle is reduced according to the strategy outlined in Figure 5.9 where \( \alpha = 10 \) degrees and the system is insensitive to crab angles below \( \alpha \) (see Figure 6.11.6). The "Look Ahead" mode is not employed as the estimated support plane would then simulate a slope because of the presence of the ditch. The height increases from 5.5 feet to 6.0 feet (Figure 6.11.7) with a small overshoot. Again the unexpected overshoot is caused by the high gain. The vehicle LSM remains over 1.0 feet but the SM does drop below 1.0 feet once (Figure 6.11.5).

**Example (e) Rough Terrain (Figure 6.12)**

Figure 6.12.1 shows a terrain with a random variation of \( \pm 1 \) feet superposed on a V-feature with an included angle of 120 degrees. The foothold selection routine works to choose suitably oriented support sites and the stroke and time period have to be changed repeatedly to accommodate the resulting irregularity (Figures 6.12.3 and 6.12.4). The body attitude control performs satisfactorily - the absolute value of the body attitude angle is plotted in Figure 6.12.7. The variation
Figure 6.11: Crossing a 4 Foot Ditch
Figure 6.11.2

SIMULATION RESULTS

Figure 6.11.3

SIMULATION RESULTS

Figure 6.11 (continued)
Figure 6.11.4

Figure 6.11.5

Figure 6.11 (continued)
Figure 6.11.6

Figure 6.11.7

Figure 6.11 (continued)
of the altitude is seen in Figure 6.12.6. The height is defined in the body coordinate system as in Chapter 5. The initial increase in height (Figure 6.12.7) is again as a result of the tendency to reach a height of 6.0 feet from the initial 5.5 feet.

The crab angle variation in Figure 6.12.8 comes as a surprise. The wild fluctuation seems to show an instability in the system. The large desired attitude rates prescribed by the attitude control scheme cause large body angular velocities, which affect the velocity of point C (in Figure 3.5 considerably. This velocity, $v_C$, is used to determine the crab angle, and consequently, large spurious crab angles are produced if the attitude rates are high. Thus this technique of determining the crab angles and the quasi-static stability margin is obviously unsuitable for situations with large body attitude changes.

A definition based on the instantaneous screw axis of body motion (instead of the direction of progression or the direction of $v_C$) and the twist about the screw axis of motion (instead of the distance along the direction of $v_C$) may be more suitable. This is suggested for further investigations on this subject.

An instability is produced by large angular rates as the body attitude angle and the crab angle are treated as independent quantities and the two rates are superposed. The principle of superposition for angular displacements is valid only for infinitesimal displacements. It is best to control one of the two orientation angles at a time if large changes in the angles are
Figure 6.12: Rough Terrain Locomotion
Figure 6.12.2

Figure 6.12.3

Figure 6.12 (continued)
Figure 6.12.4

SIMULATION RESULTS

Figure 6.12.5

SIMULATION RESULTS

Figure 6.12 (continued)
Figure 6.12.6

SIMULATION RESULTS

Figure 6.12.7

SIMULATION RESULTS

Figure 6.12 (continued)
Figure 6.12.8

SIMULATION RESULTS

Figure 6.12 (continued)
required. The logical choice would be to let the crab angle remain unchanged. As the studies in Chapter 3 indicate the crab angle is not critical and will not undermine stability greatly. The body attitude, on the other hand, is critical when changes in terrain gradients have to be negotiated.

**Example (f) Force distribution and foothold selection (Figure 6.13)**

Figure 6.13.1 shows a flat terrain with a random fluctuation of a 1 foot magnitude superimposed on it. The effects of the gains in the foothold selection heuristic function ($\Omega$), on the force distribution is investigated here. A destination of $(5, 2, 0)$ and a velocity of 4 feet/sec are specified. The effect of the gains $c_x$, $c_y$ and $c_z$ is not very obvious as the term $c^h \Delta B$ in $\Omega$ tends to improve the stability and thus has much the same effect on the force distribution as $c_x$, $c_y$ and $c_z$ (see Equation 6.1). However, the effect of $c_n$ is clearly seen in Figure 6.13. $c_n$ is set to be zero in Figures 6.13.2 and 6.13.3, whereas in Figures 6.13.4 and 6.13.5 it is equal to 10. It is evident that the friction angles can be reduced significantly, without sacrificing any stability, by appropriately selecting the support sites. Also, the magnitudes of the forces are not changed significantly and the variation of the stroke and time period remains unaffected.
Figure 6.13: Foothold Selection on Uneven Terrain
Figure 6.13.2 (a)

FORCE DISTRIBUTION

Figure 6.13.2 (b)

Figure 6.13 (continued)
Figure 6.13.4 (a)

FORCE DISTRIBUTION

Figure 6.13.4 (b)

Figure 6.13 (continued)
Figure 6.13.5 (a)

Figure 6.13.5 (b)

Figure 6.13 (continued)
6.8 Conclusion

The integration of the functions of foothold selection, gait control, automatic body motion regulation and coordination was presented in this chapter. The implementation of the strategies proposed in Chapters 3 through 5 was discussed. It was shown that preview sensing can be effectively used to automate the functions of the guidance module in legged locomotion on uneven terrain.

To the author's knowledge, this is the first time that the problem of using an adaptive gait control algorithm to negotiate steps and ditches has been addressed. The MHG (modified wave gait) was shown to be a versatile gait. The ability to vary the time period, stroke and the duty factor was found to be particularly useful. Appropriate body motion regulation schemes based on information derived from preview sensing were found to be necessary for surmounting large changes in terrain gradients.

Unfortunately, the presence of large attitude rates poses a problem for control and planning. The motion planning algorithm considers the time instants just before and after locomotion events. This proved to be adequate for most examples, but large attitude angles produce large accelerations which cause a significant change in velocity between events. This contradicts the assumption about constant velocity between "sampling" instants, made earlier in Chapter 4, and may lead to vehicle instability. Also a better analysis is required to deal with situations in which both the
crab angles and the attitude angles are large. It may be beneficial to use screw theory to describe the spatial motion of the vehicle body with respect to the ground.
7. CONCLUDING REMARKS

7.1 Conclusions

The salient research contributions described in this dissertation are listed below.

(1) The functions of the guidance system for a legged robot walking on uneven and unstructured terrain have been identified. They are:

   a. Local path modification
   b. Gait optimization
   c. Foothold selection
   d. Gait control
   e. Body posture regulation
   f. Ensuring vehicle stability

(2) The wave gait, which has been shown (in the literature) to possess good stability characteristics, has been appropriately modified to account for omnidirectionality and irregular foothold patterns. Such a modified wave gait (MWG) has proved to be adequate for robotic applications in terms of versatility, simplicity and ease of implementation. A suboptimal measure of stability, the quasistatic longitudinal stability margin, has been used as a basis for the optimization of the MWG.

(3) A foothold selection scheme, which utilizes a geometric model of the terrain, has been developed. It is based on heuristics, which are derived from kinematic and dynamic considerations, and is geared
towards simplicity and computational efficiency.

(4) A gait controller which intelligently adapts the vehicle gait to the terrain and the irregular, spatial distribution of footholds has been designed. It enables a smooth transition between different duty factors, strokes and cycle frequencies to allow a wide range of speeds.

(5) Body posture regulation schemes, which use information about the terrain elevation and vehicle status to appropriately change the vehicle attitude and height, have been investigated. These schemes have been shown to enhance the mobility of the system.

(6) Methods of resolving the static indeterminacy in the force distribution in closed loop kinematic chains have been studied. It is useful to model each leg as a compliant member and choose the leg compliances according to the available terrain information. It has been shown that such a technique can be easily implemented in a real-time force control scheme. A linear programming implementation (Phase I in the Simplex Method) is believed to be the best alternative, when checking for the stability of a planned vehicle configuration.

7.2 Other Research Issues

The control system used for motion planning at the guidance level was discretized with respect to time, since only critical time instants in the locomotion cycle were considered. Further, the sampling intervals were not the same. This caused problems especially
when the angular velocities of the body were high because of the nonlinearities associated with large angular motions. Another disadvantage is the bandwidth limitation engendered by poor sampling rates. An alternative formulation in terms of screw theory may be more practical, in which the body is assumed to twist about an instantaneous screw axis relative to the ground. This scheme would explicitly account for angular movements and would alleviate problems caused by the discretization of the system.

The simulation of the guidance system reported here is not complete. It is essential to use a simulation of the actual terrain-vehicle system which will account for the vehicle drifting from the planned path. The ability of the guidance system incorporating a local path modification scheme needs to be tested with such a simulation. Also a more in depth study of the diagnostic and remediation unit will be beneficial. As mentioned in Chapter 6, it is best configured as an expert system and it serves as an interface between different units in the guidance system. The integration of these units must be studied in greater detail.

The force distribution problem is not unique to walking machines but it is characteristic of a general class of mechanical systems which involve closed loop kinematic chains. Other examples are multifingered grippers, coordinated multiple robot arm systems, to name a few. The relative emphasis which should be placed on the interaction force field versus the equilibrating force field varies in
different systems. An analytical form of the pseudo inverse solution (which yields the equilibrating force field) for the general three dimensional problem should be pursued further. In addition, the possibility of decomposing the interaction force system into two or more subsystems (each belonging to the null system), in order to simplify the problem, must be investigated.

The analogy (and duality) between statics and kinematics can often lend useful insight to the problem and must be studied further. This is especially useful when both static and kinematic constraints must be satisfied. An example of this kind can be found in an actively coordinated wheeled system described in Reference [1013]. Systems of this kind will eventually play an increasingly important role in unmanned locomotion. This study should contribute significantly towards the development of such systems.


Muybridge, E. 1955. The Human Figure in Motion. Dover. New York.


