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Modeling and fault detection in electromagnetic devices—applications to synchronous machines and signal conditioning systems

Miri, Seyed-Mehdi, Ph.D.
The Ohio State University, 1987
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MODELING AND FAULT DETECTION IN ELECTROMAGNETIC DEVICES - APPLICATIONS TO SYNCHRONOUS MACHINES AND SIGNAL CONDITIONING SYSTEMS

A Dissertation
Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

by

SEYED-MEHDI MIRI, B.S., M.S.

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CHAPTER I
MOTIVATIONS

Electrical machines are the backbones in a great number of industrial plants. In power plants, large induction motors are used in fan and pump drives. These motors consume about 20% of the power generated by the plant and are essential to the operation of power systems. A variety of electrical motors are used to run the production lines in manufacturing plants. They are commonly used in applications where position or speed control is required.

Due to growing demand and power companies’ desire to keep the cost of electricity down, trends are toward operating power systems near their capacities. This degrades reliability and, in the face of a critical component malfunction, can prove to be very costly and undesirable. When an induction motor which is driving a critical fan or pump fails, the shutdown of a main turbine-generator unit is forced. Such a forced outage is very costly due to the higher cost of replacement fuel burned at a smaller plant which, in addition, is less efficient. The cost involved can best be appreciated by calculating the actual dollar values as shown in Figure 1 [1]. The example highlighted in this figure is the replacement fuel costs for the outage of a 600MW unit. From this figure, the total differential fuel costs, for differential fuel unit cost of 15 mils/KWh, is about $5,400,000, if the repair or replacement of the failed machine takes 25 days. This dollar value does not include the additional costs due to the lower efficiency of the smaller plant and the emergency procedures involved.

Costly motor or generator failures are not limited to power plants. Due to automation and interdependent processes, in an automated manufacturing plant, the cost of a machine failure can be just as staggering.

Recent experiences with catastrophic failures and these high costs have encouraged the development of techniques for the prevention of failures in electric machines. With the availability of more powerful computers, computer-aided failure prophylaxis methodologies have become attractive. In the face of growing demand, existence of such methodologies can maintain the current level of reliability without necessitating system expansion.
Most models of dynamic processes required for systems analysis and automatic control are obtained from frequency or transient response data collected during planned experiments. However, it is often desirable to obtain models from systems' normal operating data. In power systems, synchronous generator models are needed by dynamic and transient stability programs to determine the system stability margins. The parameters of these models are functions of generator's operating conditions, and must be reestimated as the machine moves from one operating condition to another. This can only be done with an on-line model identification procedure which uses normal operating data of the generator.

In the following sections, the motivations for this dissertation are discussed in more detail.

1.1 Power System AC Machines and Detection of Abnormal Operation

In addition to synchronous generators which are the main components of power systems, other ac machines play important roles in providing the consumers with electric power. AC motors are widely used in power systems. Large induction motors (1500HP and up) are the prime movers in power plant fan and pump drives.
They use about 20% of the power generated by the plant which shows the high demand for cooling and pumping. Thus, induction motors are essential for the operation of power systems.

Unloaded over-excited synchronous motors, known as synchronous condensers, are used for power factor correction in power systems substations. They help maintaining load voltages within limits, as well as reducing reactive power flows in transmission lines.

Recent trends toward operating power systems closer to their limits require reliable operations of most power system components. In a power plant with generators operating close to their generation capacities, failure of a critical motor, such as a boiler feed pump drive, will necessitate costly emergency procedures involving load reductions or even shutdown of a main turbine-generator. Failure of a generator would, of course, result in an immediate shutdown of the turbine-generator unit. With transmission lines operating near their transmission capacities, failure of a synchronous condenser may result in overloaded lines and unacceptable load voltages. Unscheduled outages may be necessary to avoid exceeding thermal limits of transmission lines due to overload.

Failures in power systems' ac motors have occurred so frequently that they have become a concern of utility companies. A recent EPRI report [2] is a direct result of this concern. This report is a manual on emergency procedures for bringing, when possible, a critical motor temporarily back on line after a particular failure (stator winding failure) has occurred.

In the face of power systems' growing complexity, current levels of reliability can be increased or at least maintained by one of the following two approaches: 1) constructing new generation and transmission capacities, or 2) developing strategies for prophylaxis of abnormal operation in the critical components of a power system. The first alternative is a very expensive one; this makes the latter far more attractive. Under the second alternative, an abnormal operating condition can be detected, diagnosed and remedied before it leads to a major failure. This in turn allows to lower generation and transmission capacities while maintaining the same level of reliability, or, in the face of load growth, to postpone construction of new capacities.

With the availability of powerful computers, the development of computer-aided methodologies for failure prophylaxis in power systems' critical components is encouraged. The first step in such methodologies is the detection of abnormal conditions (faults) which was one of the motivations for this work.
1.2 Electric Motors in Automated Factories and Detection of Faults

In automated factories, electrical motors are commonly used in applications where position, acceleration, speed or torque control is required. Examples are general machine tool applications, milling machinery, industrial robots, centrifuges, feeders, conveyors and packaging machinery. Depending on the application requirements, dc, induction (squirrel-cage or wound-rotor), synchronous or stepper motors are used.

Due to interdependent processes in an automated factory, the cost of a component failure could be staggering; it will always exceed the component cost and, in many cases, the subsystem cost [3].

Failure prophylaxis in the critical motors of an automated factory will increase reliability and safety, and will reduce production cost.

1.3 Power System Harmonics and Failures Due to Premature Aging

Competitive design of power system components calls for the operating points to be more in the nonlinear regions of the characteristic curves. In the case of transformers and electric machines, this means higher degree of iron-core saturation and larger winding currents. The resulting iron-core and ohmic losses cause the operating temperatures to approach rated values for continuous operation.

The existence of power system harmonics has been realized for many years [4]. With the advent of new technologies and more nonlinear loads, such as arc welders, steel mill furnaces and solid state loads, power system harmonics are certain to increase.

The voltage harmonics, generated by nonlinear load currents, cause harmonic currents to flow in the windings of transformers and electric machines. The additional losses caused by these harmonics result in rise of operating temperatures which are already near their rated values. A component lifetime is greatly effected by this temperature rise [5].

The reduced lifetime expectancy, or the premature aging due to excessive temperature, is unpredictable and thus the components become vulnerable to major failures. Preventive measures should therefore be taken in avoiding catastrophic failures caused by the existence of power system harmonics.
1.4 Modeling of Electromagnetic Devices in Time-Domain

In general, there are many reasons for which we may want to model a physical system. Among these reasons are: 1) to gain intuitive understanding of systems and the way they behave, 2) to simulate scenarios, in the case of system expansion or disturbances, before making expensive prototypes or carrying out field tests, 3) to figure out, through computer simulation of the model, the design modifications which would improve performance or reduce cost, 4) to provide models required in different control schemes such as in optimal or adaptive control schemes, and 5) to use the models as the main tools in early detection and diagnosis of system faults.

Due to magnetic saturation of their cores, most electromagnetic devices are nonlinear and have time-varying parameters. In modeling these devices, one of the two following approaches should be taken: 1) frequency-domain approach, that is to compute the sine and cosine series of each sampled signal and develop a model for each frequency; this approach assumes no interactions among frequencies and, in the case of core model parameters, may result in estimates that are physically meaningless, and 2) time-domain approach, that is to use the real time data collected during an operation of the device and develop a model as to match the input/output relations for that operating condition. In this dissertation, the time-domain approach to modeling electromagnetic devices is chosen.

1.5 Modeling of Synchronous Machines from Operating Data

With power systems complexity and size increasing, with recent trends toward operating the power systems near their capacities, and with power generating plants getting farther from demand sites because of geographical conditions and power demand distributions, it has become imperative that the behavior of power system elements, of which synchronous generators are the most important ones, can be accurately predicted under different operating conditions. The reasons for this necessity are: 1) synchronous generators are allowed to operate near their capacities and in order to design control schemes which permit the generators to operate near their stability margins, accurate models of generator dynamics are needed to avoid loss of synchronism, and 2) in the face of growing demand and longer transmission lines, it is necessary to investigate power system stability in planning and operation of the systems. Different computer programs for the study of power system dynamic and transient stability exist. However, due to lack of models which could mimic the behavior of the system generators, these programs do not yield satisfactory results.

Our objective is to develop models accurately representing the power system
synchronous generators. As the system configuration and/or loading changes, the developed models are to remain valid. Since the parameters of a synchronous generator model vary with its core saturation level as well as with its rotor angle, for a model to remain valid as the system conditions change, its parameters must be reestimated after each change. This implies that a modeling technique which can obtain the parameter estimates by processing the generator operating data on line, is required. In addition, the developed models are to have parameters of physical significance. That is, there must be unique and known relations between model parameters and generator parameters such as inductances and time constants. The need for a synchronous machine modeling technique which satisfies the above requirements was one of the primary motivations for this dissertation.

1.6 Modeling and Stability Enhancement of Power Conditioning Systems

A power conditioning system is a device used to improve the quality of power in order to reduce equipment malfunctions. The only design of a three-phase power conditioning system which has become operational is described in [6]. This design utilizes six ferroresonant transformers to synthesize a set of balanced three-phase sinusoidal output voltages which remain constant over a wide range of input voltage fluctuations. No model for this power conditioning system is known to exist. Our objectives regarding the three-phase power conditioning system are: 1) to develop models that adequately represent this system under steady state and transient conditions, 2) improving its stability by understanding and finding a solution to the problem known as the ferroresonant jump [7], and 3) reducing the transient time, following a sudden load drop, during which the system reaches the no-load steady state.

1.7 Power Transformers and the Need for Time-Domain Models

The existence of power system harmonics has been realized for many years. With the advent of new technologies and more solid state loads, power system harmonics are certain to increase both in amplitude and frequency bandwidth. Signal propagation in power systems depends on transformer impedances. Transformer impedances, on the other hand, are functions of signal frequency as well as input voltage and loading condition. Present power transformer models with single-valued parameters obtained from tests cannot adequately represent the transformer in the presence of harmonics. Accurate modeling of transformers must account for changes in signal frequencies and loading conditions as they occur. This requires algorithms which, for a structurally known transformer model, use time-domain
operating data to continuously update the model parameter estimates, thereby accounting for changes in operating conditions. The development of such an algorithm, which could be applied to three-phase and single-phase n-winding transformers, motivated part of our research.
CHAPTER II
LITERATURE REVIEW

In this chapter, the existing fault detection techniques and various estimation algorithms are reviewed. The existing modeling techniques for synchronous machines and transformers are reviewed in their corresponding chapters.

2.1 A Survey of Fault Detection Techniques

History - In a broad sense, faults in a physical system are defined as unexpected changes in operating conditions of components which tend to degrade overall performance, Chow and Willsky [8]. Note that this need not imply a physical failure; it merely means the existence of undesirable operating conditions which may precede a complete failure of the respective component.

Although more theoretical methods for fault detection have been reported in the literature, most techniques existing in practice rely heavily on the experience of the operating staff with the physical system. The reason for this is the wide gap between general theory and application. No one methodology is suitable for all applications, hence, detailed techniques need to be developed for each particular application.

The majority of fault detection techniques found in the literature are based on continuous or periodic monitoring of at least one of the following process physical variables:

Method I - measurable inputs $U$ and outputs $Y$;
Method II - measurable state variables $Z$;
Method III - process physical parameters;
Method IV - characteristic quantities such as efficiency, fuel and oil consumption, impurities in the cooling stream, and radio frequency noise level.

The emphasis on monitoring physical variables is due to the fact that fault detec-
tion is to be followed by fault diagnosis and remedial measures.

Many references, Isermann [9,10] and others, on fault detection based on Method I are available. The most common technique is the use of a limit check. A fault message is issued as soon as a signal \( y(t) \) has exceeded a maximum value \( y_{\text{max}} \) or has fallen below a minimum value \( y_{\text{min}} \). Other techniques are to formulate the problem of fault detection as a problem of hypothesis testing by regarding the normal operation of the system as the null hypothesis [13,14], or to use signal autocorrelation functions, spectral densities, etc. for vibration analysis [14].

Much less is known about techniques based on Method II [10]. Here attempts are made to estimate the unmeasurable physical variables from the measurable signals. Fault reports are then made using testing methods such as failure sensitive filters [12], a whiteness and a chi-squared test of the residuals of the normal Kalman filter [13], and a generalized likelihood ratio test [11]. All these techniques assume exact knowledge of the system parameters which is a very unrealistic assumption.

Very little is known about fault detection techniques based on estimates of system parameters (Method III). This method is based on combined theoretical modeling and parameter estimation of the continuous-time model [11]. Once the structure of the model has been assumed, changes in the model parameters can be monitored using an on-line parameter estimation algorithm such as a generalized least-squares estimator [18], extended Kalman filter [19], maximum likelihood estimator [20], or instrumental variable technique [21]. Using an extensive database, these changes are used to detect faults.

Fault detection based on characteristic quantities (Method IV) has been applied in actual processes with limited success. Synchronous machine examples are detection of intense corona at 3.8 Mhz and of gap discharges at 8.0 Mhz using electromagnetic interference monitoring, Timperley [22]. This method has not yet been successfully applied to transformers. An example of a power transformer monitoring and fault identification scheme is given in [23]. This method has only a limited range of application since some frequent faults (i.e., winding overload) do not manifest themselves in changes of characteristic quantities.

Examples of power system voltages and currents, which have been distorted by the presence of nonlinear loads, are given in Mahmoud [4]. A typical example is shown below:
It can be seen that this signal contains more than one frequency and is far from sinusoidal.

Fuchs, Roesler, and Kovacs [5] discuss the influence of temperature rise due to harmonics on the lifetime of electrical appliances. They derive a criterion to limit the individual amplitudes of the existing harmonics for a maximum allowable temperature rise. If the amplitudes are higher than those permitted by the criterion, the expected component lifetime will be reduced. No systematic technique for dealing with failures which may result from premature deterioration of transformers and electric machines can be found in the literature.

State of the Art – The most recent survey paper on fault detection methods [10] discusses the possibility of using combined parameter and state estimation techniques. Such techniques are most desirable in applications where some of the process physical variables are not measurable. In those cases, neither state variable techniques nor techniques based on estimates of process physical parameters can be applied. This is because state variable techniques assume an exact knowledge of the process parameters where as physical parameter techniques assume the availability of all the process states.

Isermann [10] suggests the following approach for a process with some unmeasurable states:

1. identify an input/output model of the process,

\[ Y(t) = f(U(t), \theta) \];

2. determine the relationship between the external model parameters \( \theta \) and the physical parameters \( p \): \( \theta = q(p) \); note that \( \theta \) is not uniquely described by this relation;
3. estimate the model parameters $\theta$ from input/output measurements using an on-line estimation algorithm;

4. calculate the process physical parameters $p = g^{-1}(\theta)$ and their changes $\Delta p$; note that $g^{-1}$ may not exist except for simple and well-defined processes;

5. make a fault decision using the existing database (catalogue of faults) which relates process faults to changes in $\Delta p$.

Examples for unmeasurable states are damper winding currents in synchronous machines and rotor currents in induction machines. Except for an electromagnetic interference technique [22], there is no fault detection technique found in the literature suitable for AC machine applications. In fact, there are few cases where a proposed technique has been actually applied to a specific problem. The most common example found is the leak detection in pipelines. An example of a DC motor driven centrifugal pump can be found in [10]. Therefore, combined parameter and state estimation techniques need to be developed for fault detection in systems with unmeasurable states.

2.2 A Survey of Identification Techniques for Dynamic Systems

2.2.1 Definitions of a Dynamic System

There are two definitions of dynamic systems used in the literature by most authors [24]; these are:

**Definition 1**

A dynamic system is a mathematical concept consisting of a time set $T$, which is an ordered subset of real numbers, a state space $X$, a set of instantaneous input values $U$, a set of acceptable input functions $W = \{w : T \to U\}$, a set of instantaneous output values $Y$, and a set of output functions $G = \{g : T \to Y\}$. Furthermore, there exists a state transition function $\phi : T \times T \times X \times W \to X$, and a readout map $\zeta : T \times X \to Y$ which defines the output $y(t) = \zeta(t, x(t))$.

**Definition 2**

In addition to the above properties there is a set $A$ indexing a family of functions $F = \{f_a : T \times W \to Y, a \in A\}$.

**Important Difference**

Consider the class of finite-dimensional linear smooth systems with one input and one output in continuous-time. The definition of this system is given by

1. State space $X = \mathbb{R}^n$, $n \times n$ matrix $A$, $n \times 1$ matrix $B$, $1 \times n$ matrix $C$, and $1 \times 1$ matrix $D$. The input/output transformation is described by the
following system of equations

\[
\frac{dx(t)}{dt} = Ax(t) + Bu(t) \quad x(0) \in X
\]

\[
y(t) = Cx(t) + Du(t)
\]

in the spirit of Definition 1.

2. Impulse characteristic \( h \) and initial response \( y_0 \)

\[
y(t) = y_0 + \int_0^t h(t - \tau)u(\tau)d\tau
\]

in the spirit of Definition 2.

A dynamic system under the first definition has an "interior" consisting of the state set \( X \) and the transition function \( \phi \) mapping the state set into itself depending on the input; the state set \( X \) and the transition function \( \phi \) define the internal structure of the dynamic system. The same dynamic system in the sense of the second definition is specified only by a set of input/output maps; this definition deprives the dynamic system of an a-priori defined internal structure.

2.2.2 Definition and Steps of Identification

Definition of Identification

Identification is the determination of a model within a specified class of models for an existing system, or for a system to be constructed. This model represents the essential aspects of the system in a usable form.

Identification Steps

1. Structural identification; for physical models, normally done by selecting a collection of variables, state variables, and using the fundamental laws governing the system to develop relations among these variables.

2. Parameter identification; the estimation of the parameters of the system which are the coefficients of the dynamic equation(s) derived in Step 1.

3. State estimation; the estimation of the unmeasurable system variables, if needed.
4. Model validation; this is the substantiation that the identified model, in the case of an existing system, adequately represents the system for its intended applications.

2.2.3 Mathematical Models

In our studies, we were interested in the applications of modeling and estimation in fault detection, and in stability enhancement through meaningful sensitivity analysis. The class of models suited for these applications is limited to physical models. Since the structure of a physical model is determined by physical laws and our understanding of the system, no order identification technique is required. Therefore, literature was surveyed only for different techniques of parameter estimation, assuming the order (i.e., structure) of the model is known. Due to assumed digital computer application in estimating the physical model parameters, only the discrete time models were considered.

Using Definition 1 for dynamic systems given in Section 2.2.1, the state space representation of a physical model in discrete time domain can be given by

\[
x(K + 1) = Ax(K) + Bu(K) + w(K + 1) \\
y(K) = Cx(K) + Du(K) + v(K)
\]

(2.1)

where \( x \) is the state vector, \( u \) the input vector, \( y \) the output vector, \( K \) the discrete sampling time, and \( A, B, C, D \) are the matrices of model physical parameters. The components of the noise vectors \( v \) and \( w \) are assumed to be white, mutually independent Gaussian noises for stochastic models. Vectors \( v \) and \( w \) are set to zero for deterministic models.

Stochastic Processes

A set of observations \( y(t) \), \( t \) belonging to \( T \), arranged chronologically is called a time series. When collecting data for a time series (reading and recording of a voltmeter at different times, for example), we can regard them as being one of the many sets of data which might have arisen. In other words, the value read at \( t = K_1 \) is only one of the values that the random variable \( X(K_1) \) could have taken on. Therefore, a time series can be described by an ordered set of random variables \( X(K) (K = \ldots, -1, 0, 1, \ldots) \) for a discrete time series. An ordered set of random variables is called a stochastic process.

White Noise

A sequence \( w(.) \) is by definition called white noise if it is a sequence of independent random variables.
2.2.4 The Techniques of Parameter Estimation

The objective of a parameter estimation algorithm is, by processing records of input/output data, to find those values of the model parameters which result in a model behavior similar, in a pre-defined sense, to the behavior of the system.

**Least Square Estimator (LSE)**

Many estimation algorithms found in the literature can be interpreted as least-squares procedures. LSE arrives at the parameter estimates by minimizing sum of the squares of errors between system outputs and the corresponding model outputs for the same inputs applied to both the system and the model. Rewriting the first equation in (2.1) as

\[ x(K + 1) = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} x(K) \\ u(K) \end{bmatrix} + w(K + 1) \]

and letting subscript \( i \) denote the \( i^{th} \) row of a vector or a matrix, we have

\[ x_i(K + 1) = C_i z(K) + w_i(K + 1) \quad (2.2) \]

where

\[ C_i \triangleq [A_i \ B_i] \]
\[ z(K) \triangleq [z^T(K) \ u^T(K)]^T \]

For the time instants \( K + 1 \) to \( N \), we can write

\[ [x_i(K + 1) \ldots x_i(N)] = [C_i][z(K) \ldots z(N - 1)] + [w_i(K + 1) \ldots w_i(N)] \]

Let

\[ Y_i(N) \triangleq [x_i(K + 1) \ldots x_i(N)] \]
\[ Z(N - 1) \triangleq [z(K) \ldots z(N - 1)] \]
\[ e(N) \triangleq [w_i(K + 1) \ldots w_i(N)] \]
\[ Y_i(N) = C_i Z(N - 1) + e(N) \quad (2.3) \]

The Least Square Estimator finds the estimate of \( C_i \) by minimizing the cost function

\[ J = [e][e]^T \]
with respect to $C_j$, where

$$e \triangleq [Y_i(N) - C_i Z(N - 1)]$$

and $Y_i(N)$ is a set of true measurements of the $i$th state variable $x_i$. Minimizing $J$ with respect to $C_i$ yields the least square estimates of the system parameters in the $i$th rows of matrices $A$ and $B$,

$$\hat{C}_i^T = [Z(N - 1)Z^T(N - 1)]^{-1} Z(N - 1)Y_i^T(N) \quad (2.4)$$

Provided that all the state variables are available for measurements, the least square estimates of the parameter matrices $A$ and $B$ can be obtained row by row.

**Weighted LSE**

Minimizing $J = [e]W[e^T]$ for a symmetric positive-definite matrix $W$, the weighted LSE (WLSE) of $C_i$ can be obtained,

$$\hat{C}_i^T = [Z(N - 1)WZ^T(N - 1)]^{-1} Z(N - 1)WY_i^T(N)$$

The matrix $W$ allows each error term to be weighted differently, we would give heavier weights to the measurements in which we have more confidence. It has been shown [25] that for $W = [E(e^Te)]^{-1}$, we get the minimum variance estimator known as Markov Estimator.

**Recursive LSE**

To avoid the matrix inversion required by Eq. (2.4) each time a new set of data becomes available, the following matrix identity is used to derive a recursive version of LSE,

$$[A + BCD]^{-1} = A^{-1} - A^{-1}B[C^{-1} + DA^{-1}B]DA^{-1}$$

where $A$, $C$, and $A + BCD$ are nonsingular square matrices. For recursive equations, please see Appendix B.

**Generalized LSE (GLSE)**

Substituting for $Y_i(N)$ from Eq. (2.3) in Eq. (2.4), we obtain

$$\hat{C}_i^T = C_i^T + [Z(N - 1)Z^T(N - 1)]^{-1} Z(N - 1)e^T(N) \quad (2.5)$$

and noting that for noisy measurements, the expected value of the second term on the right side of Eq. (2.5) is nonzero, we can show that the LSE of $C_i$ is biased.
GLSE obtains unbiased estimates by modeling the error $e$. Eykhoff [26] and Clarke [27] discuss the details of the GLSE. A recursive version can be found in [28], and a recursive version for multi-output systems is given by Sinha and Kwong in [29].

Maximum Likelihood Estimator (MLE)

MLE minimizes the cost function $J = e(N)e^T(N)$ by maximizing the likelihood function

$$L(e(N)) = \frac{1}{(\sqrt{2\pi}\sigma)^N} \exp\left(-\frac{1}{2\sigma^2}e(N)e^T(N)\right)$$

where $e(N) = [e(1, p), e(2, p), \ldots, e(N, p)]$, $e(K, p) = y(K) - pz(K - 1)$, $p$ is the parameter vector, and $y$ is the output of a single-output system. It is assumed that $e(., p)$ is a sequence of independent, identically and normally distributed random variables with zero mean and variance $\sigma^2$. The likelihood function is maximized with respect to $p$. Due to monotonic property of logarithmic functions, maximization of the likelihood function can be replaced by that of

$$\ln[L(e(N))] = -N \ln(\sigma) - N \ln(\sqrt{2\pi}) - \frac{1}{2\sigma^2}e(N)e^T(N)$$

which results in

$$f(p)|_{p=\hat{p}} = 0$$

$$f(p) = \frac{\partial J(p)}{\partial p}$$

Since $f(p)$ is a nonlinear function of $p$, an iterative approach, such as Newton-Raphson, is used to solve for MLE of $p$, $\hat{p}$. A recursive version of MLE has been developed [30] by making some simplifying assumptions. The equations for the recursive version have turned out to be very similar to those of LSE.

Instrumental Variable Technique

The measured signals obtained from an unknown single-input single-output process are passed through "state variable filters" and then sampled to provide the input data to a digital estimation algorithm. The filters used are either analog [31] or digital [32].

Consider a single-input dynamic system described by a linear differential equation model of the form...
\[ \sum_{n=0}^{N} a_n \frac{d^n x}{dt^n} = u + \sum_{m=1}^{M} b_m \frac{d^m u}{dt^m} \]  \hspace{1cm} (2.6)

where \( x(t) \) is the system output response to the input signal \( u(t) \), and \( a_n \) and \( b_m \) are the unknown parameters of the system. The Instrumental Variable technique obtains an "estimation model" that obeys a similar differential equation as that of the system, but which does not include pure time-derivatives of \( x \) or \( u \). This is accomplished by operating on each of the terms in Eq. (2.6) by a nonunique linear time-invariant filter \( D \) given by

\[
D(s) = \frac{P(s)}{Q(s)}
\]

in the analog case. \( P(s) \) and \( Q(s) \) are constant coefficient polynomials in Laplace operator \( s \) with orders \( p \) and \( q \). It has been shown [33,34] that parameters \( a_n \) and \( b_m \) can be related by the estimation model

\[
\sum_{n=0}^{N} a_n [x_o]_{D_n} = [u_o]_{D_o} + \sum_{m=1}^{M} b_m [u_o]_{D_m} \hspace{1cm} (2.7)
\]

where

\[
[x_o]_{D_n} = s^n D(s)x_o
\]

\[
[u_o]_{D_m} = s^m D(s)u_o
\]

and \( x_o \) and \( u_o \) are the true output and input of a SISO dynamic system. Note that \( x_o \) and \( u_o \) are not known and must be replaced by their observed values \( y \) and \( v \), where \( y = x_o + \omega_o \) and \( v = u_o + n_o \) are contaminated by the measurement noise \( \omega_o \) and \( n_o \). Also note that the filters are physically realizable only if the inequality \( q > p+N-1 \) is satisfied. The Instrumental Variable (IV) estimates of the system parameters \( a_n \) and \( b_m \) are obtained by minimizing the cost function

\[
J = \sum_{K=1}^{L} [Z^T(K)a - w(K)]^2
\]

where

\[
Z^T = [y]_{D_0} \ldots [y]_{D_N} [v]_{D_1} \ldots [v]_{D_M}
\]

\[
w = [v]_{D_o}
\]

\[
a^T = [a_o \ a_1 \ldots a_N - b_1 - b_2\ldots - b_M]
\]
for a set of \( L \) observations. Minimizing \( J \), with respect to \( a \), results in the IV estimate of the parameter vector,

\[
\hat{a} = S^{-1}R
\]

\[
S = \sum_{K=1}^{L} Z(K)Z^T(K)
\]

\[
R = \sum_{K=1}^{L} Z(K)w(K)
\]

To remove the biasedness from the estimates, the solution of \( \hat{a} \) is modified as follows:

\[
\hat{a} = \hat{S}^{-1}\hat{R}
\]

\[
\hat{S} = \sum_{K=1}^{L} \hat{Z}(K)\hat{Z}^T(K)
\]

\[
\hat{R} = \sum_{K=1}^{L} \hat{Z}(K)\hat{w}(K)
\]

where \( \hat{Z} \) is an Instrumental Variable vector with components chosen to be highly correlated with the unmeasurable, noise-free system variables, but totally uncorrelated with the noise that corrupt the signals. A method of generating the IV's is suggested in [31].

**Kalman Filtering Approach**

Kalman filtering can be applied to estimate the coefficients of the ARMA model

\[
y(K) + \sum_{i=1}^{n} a_i y(K - i) = \sum_{i=n+1}^{n+m} a_i u(K + n - i) + v(K)
\]

where the coefficients \( a_i \) are subject to random perturbations,

\[
a_i(K + 1) = a_i(K) + w_i(K)
\]

The following assumptions on the statistical properties of the disturbances and the parameters are made: 1) \( w_i(K) \) belongs to a zero mean, white gaussian random process, independent of \( w_j(K) \) for \( i \neq j \), 2) \( v(K) \) belongs to a zero mean, white gaussian random process, independent of \( w_i(K) \), 3) a variance of \( v(K) \) is known based on our knowledge about the noise introduced by measuring devices, 4) a variance of \( w_i(K) \) is known based on our knowledge about the way \( a_i(K) \) is likely
to vary, 5) a mean and a variance for each parameter are assumed before measurements are taken, and 6) initial value of each parameter $a_i(0)$ belongs to a gaussian random process.

Now, if we define the $(n + m)$-dimensional parameter, perturbation and observation vectors as

\[
p(K) = [a_1(K), \ldots, a_{n+m}(K)]^T
\]

\[
w(K) = [w_1(K), \ldots, w_{n+m}(K)]^T
\]

\[
H^T(K) = [-y(K-1), \ldots, -y(K-n), u(K-1), \ldots, u(K-m)]
\]

then,

\[
y(K) = H^T(K)p(K) + v(K)
\]

\[
p(K + 1) = p(K) + w(K)
\]

and the Kalman filter solution is given by [35]

\[
\hat{p}(K + 1/K) = G(K)y(K) + [I - G(K)H^T(K)]\hat{p}(K/K - 1)
\]

\[
G(K) = C(K/K - 1)H(K) [H^T(K)C(K/K - 1)H(K) + R(K)]^{-1}
\]

\[
C(K/K - 1) = E \left[ (p(K) - \hat{p}(K/K - 1))(p(K) - \hat{p}(K/K - 1))^T / H(K - 1) \right]
\]

\[
C(K + 1/K) = C(K/K - 1) - C(K/K - 1)H(K)[H^T(K)C(K/K - 1)
\]

\[
H(K) + R(K)]^{-1}H^T(K)C(K/K - 1) + Q(K)
\]

\[
R(K) = E \left[ v^2(K) \right]
\]

\[
Q(K) = E \left[ w(K)w^T(K) \right]
\]

**Abstract Realization Theory**

The subject of abstract realization theory is concerned with the construction of a dynamic model in the sense of Definition 1 (Section 2.2.1) using the data provided by Definition 2. That is, abstract realization theory is the theory of modeling the internal structure of dynamic systems from input/output relationship.

Consider a linear time-invariant system in discrete time

\[
X(K + 1) = AX(K) + BU(K)
\]

\[
Y(K) = CX(K) + DU(K)
\]
where $X$ is $n \times 1$, $U$ is $r \times 1$, and $Y$ is $p \times 1$. The objective is to determine an abstract realization $(A, B, C)$ of this system knowing that only the inputs $U$ and the outputs $Y$ are available for measurements.

The Ho-Kalman algorithm [36] can obtain a minimal realization for this system from its input/output measurements assuming the dimension of $X$ is known. The Ho-Kalman algorithm can be summarized as follows:

1) Construct the Hankel matrix

$$H = \begin{bmatrix} H_1 & H_2 & H_3 & \ldots \\ H_2 & H_3 & H_4 & \ldots \\ H_3 & H_4 & H_5 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where each $p \times r$ block $H_K$ is constructed from the data collected during $r$ input/output tests with test $j$ being the test during which the input $j$ is set to a unit input and all other system inputs are set to zero. Each block $H_K$ has the form

$$H_K = \begin{bmatrix} y_{11}(K) & y_{12}(K) & \ldots & y_{1r}(K) \\ y_{21}(K) & y_{22}(K) & \ldots & y_{2r}(K) \\ \vdots & \vdots & \ddots & \vdots \\ y_{p1}(K) & y_{p2}(K) & \ldots & y_{pr}(K) \end{bmatrix}$$

where $y_{ij}(K)$ is the value of the system output $y_i$ measured during the $j$th test at the instant $K$ of time.

2) Select some $N \geq n$ and take the truncated matrix

$$H_{NN} = \begin{bmatrix} H_1 & \ldots & H_N \\ \vdots & \ddots & \vdots \\ H_N & \ldots & H_{2N} \end{bmatrix}$$

to compute its singular valued decomposition [37],

$$H_{NN} = QDR$$

3) Compute the following matrices

$$M = R^TD^+$$
\[ E_{r \times n} = \begin{cases} [I_{r \times r} \ O_{r \times n-r}] & \text{if } r < n \\ [I_{n \times n} \ O_{r-n \times n}]^T & \text{if } r > n \\ [I_{r \times r}] & \text{if } r = n \end{cases} \]

\[ H_{NN1} = \begin{bmatrix} H_2 & \cdots & H_{N+1} \\ \vdots & \ddots & \vdots \\ N_{N+1} & \cdots & H_{2N+1} \end{bmatrix} \]

where superscript + denotes the pseudoinverse, \( I \) is the identity and \( O \) is the zero matrix.

4) Calculate an internal realization of the dynamic system as

\[
A = E_{n \times Np}Q^TH_{NN1}^ME_{N \times n} \\
B = E_{n \times Np}Q^TH_{NN}E_{N \times r} \\
C = E_{p \times Np}H_{NN}^ME_{N \times n}
\]

This realization is unique only up to a similarity transformation.

**Concluding Remarks**

In the absence of noise, most parameter estimation algorithms result in the same noise-free estimates. How well a particular estimator performs in a noisy environment strictly depends on the disturbances acting on the system under identification. Many of the parameter estimation techniques reported in the literature are based on least-squares theory. These techniques are usually recursive, easy to implement, and do not require much computing time. If the noise \( n(K) \) acting on the system can be modeled as

\[ n(K) = \sum_{j=0}^{N} d_j w(K - j) ; \quad d_0 = 1 \]

where \( w \) is normally distributed white noise with zero mean and is statistically independent of the input signals, then it can be shown that unbiased estimates are obtained using least-squares techniques. However, due to presence of non-ideal noise and possible structural errors, the estimates are in general biased. The generalized least-squares estimator attempts to remove this bias by modeling the overall estimation errors. The instrumental variable technique attempts to obtain unbiased estimates by replacing the system variables with Instrumental Variables (IV’s) chosen to be statistically independent from the noise acting on the system.
Our review of modeling and estimation techniques has been broad in this chapter. More focused reviews of literature on techniques applied in modeling synchronous machines and transformers will be presented in chapters devoted respectively to synchronous machines and transformers.
CHAPTER III
PARAMETER TRACKING AND FAULT DETECTION IN SYNCHRONOUS MACHINES

3.1 Introduction

Stability of a large power system is affected by dynamic behavior of generators and thus by their parameter values. Parameter values are in turn affected by magnetic saturation, machine internal temperature, machine aging, etc. Accurate estimate of machine parameters are therefore needed for stability studies. In addition, obtaining trajectories for the machine parameters and states, as the machine moves from one operating condition to another, allows for the detection of developing internal faults at early stages.

Most synchronous machines' parameter estimation procedures found in literature are based on analyzing either transient response data [38,39,40,41] or frequency response data [42,43,44,45,46,47] obtained during planned experiments using special inputs. Other techniques use operating data obtained during transients [48,49,50].

One of the widely accepted procedures for estimating synchronous machine parameters is the derivation of these parameters from the so-called d,q-axis tests [38]. These tests are the load rejection tests, that is, the breakers are opened on a known initial loading condition and oscillograms showing the variations of terminal voltage and field current after the opening of breakers are obtained. Two such tests with different initial loading conditions are performed off-line and oscillograms obtained from each are used to extract the d-axis and the q-axis parameters.

Standstill frequency response tests have been widely used in the estimation of synchronous machine parameters [46,47]. However, the parameters obtained by these tests correspond to standstill conditions and not to operating conditions. A frequency response technique which overcomes this problem is given in [42]. It consists of running the machine at reduced speeds with a line-to-line short circuit between two of the phases. By applying excitation, short circuit current at
fundamental frequency plus harmonics due to saliency is generated. Measurements of this current, of the voltages, along with measurement of rotor angle, are used to obtain operational inductances $L_d(j2\omega)$ and $L_q(j2\omega)$. In order to obtain frequency response characteristics of $L_d(j2\omega)$ and $L_q(j2\omega)$, various off-line tests must be performed with machine running at different $\omega$.

Operating data obtained during system transients are also used to determine the machine parameters under operating conditions \[48,49,50\]. The method proposed in [49] consists of off-line frequency domain analysis for the derivation of the operational impedances from the recorded data of generator voltages, currents, rotor angle and position pulse after a disturbance such as bus-tie switching. Then, the machine parameters are obtained from these operational impedances by a curve fitting technique. Reference [48] uses the recorded data for generator phase currents, line-to-line voltages, field voltage, active and reactive power after major disturbances involving pole slipping conditions to derive the machine parameters. A flux linkage model is used for the machine and the direct and quadrature axis fluxes are approximated using the recorded data. Then, d and q-axis fluxes are also obtained by simulating the machine model with the measured inputs ($i_d$ and $i_q$). Machine parameters are determined by adjusting the “old” parameters in a trial and error process until a close match between the “measured” and simulated values of d and q-axis fluxes is obtained. This method also involves curve smoothing and curve fitting techniques. In Japan [50], a Kalman filter technique which uses the field data in estimating the model parameters has been used. This method estimates the machine and the control units parameters simultaneously.

Finally, a technique that allows the on-line estimation of the parameters of power system controllers and generators is proposed in [51]. This technique uses wide-bandwidth noise signals in an excitation signal.

Adaptive observers which estimate the parameters and the unmeasurable states of linear time-invariant systems have been proposed by many researchers \[52,53,54\]. The problem with adaptive observers is that they estimate the parameters of a certain canonical transformation of the system model; thus, the physical meaning of parameters is lost.
3.2 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$X$</td>
<td>$n$-dimensional state vector</td>
</tr>
<tr>
<td>$Y$</td>
<td>$p$-dimensional output vector</td>
</tr>
<tr>
<td>$Z$</td>
<td>$(n-p)$-dimensional vector of unmeasurable states</td>
</tr>
<tr>
<td>$U$</td>
<td>$r$-dimensional input vector</td>
</tr>
<tr>
<td>$[\ ]^T$</td>
<td>transpose of a matrix</td>
</tr>
<tr>
<td>$Z(.)$</td>
<td>sequence of vector $Z$</td>
</tr>
<tr>
<td>$\hat{Z}$</td>
<td>estimate of vector $Z$</td>
</tr>
<tr>
<td>$i_d$</td>
<td>direct axis current</td>
</tr>
<tr>
<td>$i_q$</td>
<td>quadrature axis current</td>
</tr>
<tr>
<td>$i_{fd}$</td>
<td>field current</td>
</tr>
<tr>
<td>$i_{kd}$</td>
<td>direct axis damper winding current</td>
</tr>
<tr>
<td>$i_{kq1/2}$</td>
<td>quadrature axis damper winding currents</td>
</tr>
<tr>
<td>$\delta$</td>
<td>torque angle</td>
</tr>
<tr>
<td>$\omega$</td>
<td>pu rotor angular velocity</td>
</tr>
<tr>
<td>$E_{fd}$</td>
<td>excitation voltage</td>
</tr>
<tr>
<td>$T_p$</td>
<td>prime mover torque</td>
</tr>
<tr>
<td>$\mathbb{R}^r$</td>
<td>$r$-dimensional real vector space</td>
</tr>
</tbody>
</table>

3.3 Statement of the Problem

3.3.1 Formulation of the Synchronous Machine Modeling Problem

Consider a synchronous machine model with $r$ inputs and $p$ outputs, as shown in Figure 2.

The order of the model of Figure 2 is assumed to be known. Our objective is to estimate the parameters of this given model to fit the input/output relationship.

The discrete dynamic equations for the current model of a round rotor synchronous machine connected to an infinite bus is given by [72]

$$X(k+1) = AX(k) + BU(k)$$

$$X = \begin{bmatrix} i_d & i_{fd} & i_{kd} & i_q & i_{kq1} & i_{kq2} & \delta & \omega \end{bmatrix}^T$$

$$U = \begin{bmatrix} E_{fd} & T_p \end{bmatrix}^T$$

(3.1)
where the components of the state vector $X$ and the input vector $U$ represent deviations from their steady state values. Matrices $A$ and $B$ contain all the information about the actual parameters of the machine [72]. Define:

$$Y = [i_d \ i_{fd} \ i_q \ \delta \ \omega]^T$$

$$Z = [i_{kd} \ i_{kq1} \ i_{kq2}]^T$$

then $Y$ is the output vector and $Z$ is the part of the state vector that cannot be measured.

The identification problem is then the estimation of the matrices $A$ and $B$ from a given record of input/output observations, knowing that the damper winding currents in $Z$ cannot be measured. It is important that the original structure of the matrices $A$ and $B$ be identified so that the actual parameters of the machine can be derived from them.

3.3.2 Formulation of the Incipient Fault Detection Problem

The incipient fault identification problem is the detection of potential machine internal faults through input/output data processing. Some examples of these potential faults are slot discharges, high internal temperature, intense corona, contamination of end windings, loose stator end wedges, and gap discharges [22].

Since the problem of fault detection is not limited to synchronous generators, the discussions of this section will include other electromagnetic systems. It is our intention to lay the groundwork in development of failure prophylaxis schemes for power systems' major components. Detection of internal faults in dynamic systems is the first step in any failure prophylaxis scheme. We shall, therefore, formulate the problem of fault detection based on modeling and estimation in electrical motors,
centrifugal pumps, and power transformers. Since an electromagnetic device is, in general, a nonlinear system whose dynamic behavior, in the absence of high frequency inputs, can be described by a lumped parameter model, we are concerned here with fault detection in dynamic systems modeled by nonlinear state space representation of the form

\[ \frac{dX}{dt} = f[X(t), U(t), t] \]

\[ Y(t) = g[X(t), U(t)] \]

which can be linearized around an operating point

\[ \frac{dx}{dt} = A(t)x(t) + B(t)u(t) \]

\[ y(t) = Cx(t) + Du(t) \]

(3.3)

where \( x = \Delta X, y = \Delta Y, u = \Delta U \) are changes in state, output and input vectors. Coefficients \( A, B, C, \) and \( D \) are matrices of time-varying parameters.

Since detection of faults is, in most cases, to be followed by fault diagnosis (location and cause), it is important to select a set of physical variables (i.e., those variables which we use when we talk about a system) as state variables. With this selection of states and with using physical laws of the process in deriving the system of equations in (3.3), we will obtain a physical model which is suitable for fault identification purposes.

The system of equations in (3.3) describes a normal operation or no fault model. This model is valid only during normal operation and one must be aware of its region of validity.

The problem of failure prophylaxis is concerned with: a) Fault Detection—detection of nonpermitted deviations in any of the time-variables in (3.3) which lead to overall system behavior, as modeled by (3.3), other than the intended one; b) Fault Diagnosis—determination of the location and the cause of the abnormality signaled by the fault detection algorithm; c) Fault Prognosis—determination of the course of the fault and the way it effects the process; and d) Remedial Measures—after the effect of the fault is known, a decision on whether to change or to stop operation must be made.

The process of failure prophylaxis is to be based entirely on the analysis of input/output data obtained at discrete time intervals. Although this has the
advantage of allowing the use of powerful computers in the analysis, it does create new problems. Modeling of a process based on digitized input/output data will result in a discrete time model whose parameters, by themselves, do not have physical meaning, but are transformations of the physical parameters needed for failure prophylaxis. Such a discrete time model can be described by

\[
x(k + 1) = A(k)x(k) + B(k)u(k)
\]

\[
y(k) = C(k)x(k) + D(k)u(k)
\]  \hspace{1cm} (3.4)

where the coefficients A, B, C, and D are now matrices of discrete time parameters.

The problem of failure prophylaxis in a process modeled by (3.4) is summarized in the block diagram shown in Figure 3.

The process inputs and outputs are sampled by sensors which may consist of A/D converters, potential deviders, differential amplifiers, isolation amplifiers, and computer interface. Digitized signals \( u^* = u + s_I \) and \( y^* = y + s_O \) are the sensors' outputs. The bias signals \( s_I \) and \( s_O \) represent, in general, the sensor noise and a class of sensor failures. However, we will not consider the possibilities of sensor
failures here. This issue has been addressed in [55]. We will therefore consider the signals $s_I$ and $s_O$ to be acquisition noise.

The failure prophylaxis problem can now be broken down to the following subproblems as depicted by the block diagram of Figure 3.

Fault Detection: The corrupted vector signals $u^*$ and $y^*$ are to be used to construct the model described by (3.3). This is an unsolved and difficult problem. An intelligent modeling algorithm which can, based on input/output measurements, identify the internal state space model in (3.3) is needed. The algorithm would have to rely on previous knowledge of the process to be able to estimate the unmeasurable states and parameters of the process physical model. Upon arrival of new data, the estimated physical quantities are to be updated so that the faults which manifest themselves as abnormal changes in these quantities can be detected.

Fault Diagnosis: After a fault alarm has been issued, the measurements of fault data are to be processed to enhance the effect of the fault so that it can be recognized. This enhanced fault effect is called the signature of the fault.

Fault Prognosis: The effect of a fault on the process must be evaluated to determine whether the fault can be tolerated and if so, for how long. The fault should be divided into different hazard classes according to their severity, with the most hazardous faults being the ones that cannot be tolerated.

Remedial Measures: Based on fault prognosis results, decisions on changes in operating conditions must be made to avoid the complete failure of the process. Typical decisions should involve reducing the process output such that the fault can be tolerated until the time of scheduled maintenance, or to shutdown the process for immediate preventive maintenance when the fault cannot be tolerated.

Practical Problems: Discrete time modeling based on samples of continuous time signals, required as part of the solution to the failure prophylaxis problem discussed above, raises many questions which need to be answered before any developed methodology can be successfully applied. The aptness of such discrete models should be investigated through the study of the effects of sampling rate, nonsimultaneous sampling of two or more signals (skewing), finite wordlength, acquisition noise, and the use of analog or digital filters in removing the measurement noise. These issues will be investigated in Chapter 5.

To become more specific, we now state the problem of failure prophylaxis in dc, induction, and synchronous motors, as well as in power transformers. In the case of motors, we assume each motor is to drive a centrifugal pump under digital closed loop control. The problem of controller failure detection then becomes the problem of sensor failure detection which, as mentioned earlier, will not be
To formulate the problem of fault detection in a motor/pump system, we need to combine the motor dynamic equations with those of the pump. We do this by first giving the balanced equations for a centrifugal pump, followed by equations for dc, induction and synchronous motors.

**Centrifugal Pump**

Centrifugal pumps are employed in power plants, hydroelectric pumped-storage systems, refineries, chemical processing plants, municipal water supplies, etc. to move fluids from one place to another. The mechanical power output of the motor, $T_m\omega$, is converted to the fluid power, $PQ=$ (Pressure)(Flow Rate), by the pump according to the following balance equations:

\[
T_m = J_p\omega + T_p(Q,\omega)
\]

\[
P = P(Q,\omega)
\]

where

$T_m =$ *motor output torque (shaft torque)*

$J_p =$ *pump moment of inertia in lbf - inch - sec$^2$

$\omega =$ *rotor angular velocity in rad/sec.*
\[ T_p = \text{torque generated by the pump plus friction torque of the pump in lb}_f\text{–inch} \]

\[ P = \text{generated pressure in lb}_f\text{/inch}^2 \text{ (psi)} \]

\[ Q = \text{volume flow rate in inch}^3\text{/sec.} \]

The nonlinear functions \( T_p(Q, \omega) \) and \( P(Q, \omega) \) can be linearized for small perturbations around a steady state operating point with constant values \( T_p^0, Q_0, \omega_0, \) and \( P_0 \) for torque, flow rate, speed, and pressure. Using Taylor series expansion and ignoring the nonlinear terms, we get

\[
T_p(Q, \omega) \approx T_p(Q_0, \omega_0) + \left. \frac{\partial T_p}{\partial Q} \right|_{Q_0, \omega_0} \Delta Q + \left. \frac{\partial T_p}{\partial \omega} \right|_{Q_0, \omega_0} \Delta \omega
\]

\[
P(Q, \omega) \approx P(Q_0, \omega_0) + \left. \frac{\partial P}{\partial Q} \right|_{Q_0, \omega_0} \Delta Q + \left. \frac{\partial P}{\partial \omega} \right|_{Q_0, \omega_0} \Delta \omega
\]

and since in steady state \( T_m = T_p \), for a small change \( \Delta T_m \) in \( T_m \), we obtain the pump linearized dynamic equations as

\[
\Delta T_m = J_p \dot{\omega} + C_{TQ} \Delta Q + C_{Tw} \Delta \omega \tag{3.5}
\]

\[
\Delta P = C_{PQ} \Delta Q + C_{P\omega} \Delta \omega \tag{3.6}
\]

where

\[
C_{TQ} = \left. \frac{\partial T_p}{\partial Q} \right|_{Q_0, \omega_0}, \quad C_{Tw} = \left. \frac{\partial T_p}{\partial \omega} \right|_{Q_0, \omega_0}
\]

\[
C_{PQ} = \left. \frac{\partial P}{\partial Q} \right|_{Q_0, \omega_0}, \quad C_{P\omega} = \left. \frac{\partial P}{\partial \omega} \right|_{Q_0, \omega_0}
\]

and \( J_p \) are the parameters of the pump linearized model. In Eq. (3.5), the term \( C_{TQ} \Delta Q \) represents a load torque presented by the pump, while \( C_{Tw} \Delta \omega \) has the
form of a viscous damping torque. In Eq. (3.6), $C_p \Delta \omega$ is the generated pressure, while $C_{PQ} \Delta Q$ represents a pressure drop due to flow resistance.

Assuming the length of the pipe, through which the pump is to force the fluid at a constant flow rate $Q$, is $L$ and its cross-sectional area is $A$, the pressure drop along the pipe $\delta P_L$ is given by

$$\delta P_L = I_f \frac{dQ}{dt} + R_f Q$$

where

$$I_f = \frac{\rho L}{A} = \text{fluid inerance}$$

$$\rho = \text{fluid mass density, constant along the pipe}$$

$$R_f = \text{fluid resistance in } \frac{\text{psi}}{\text{inch}^3/\text{sec.}}$$

The presence of an orifice in the flow path is modeled by

$$Q = C_d A_o \sqrt{\frac{2 \delta P_o}{\rho}}$$

where

$$A_o = \text{orifice cross-sectional area}$$

$$C_d = \text{orifice discharge coefficient}$$

$$\delta P_o = \text{pressure drop due to orifice}$$

For deviations around steady state, we get the pipe balance equation (for incompressible fluids) as

$$I_f \frac{dQ}{dt} = (C_{PQ} - R_f - C_{PPQ}) \Delta Q + C_p \omega \Delta \omega - \Delta P_{load}$$

(3.7)

where
\[ C_{PPQ} = \frac{\rho Q_0}{(C_d A_o)^2} \]

and \( \Delta P_{load} \) is the increase in pressure at the fluid destination. Eq. (3.5), (3.6) and (3.7) describe the dynamics of the pump and pipe system. These equations must now be combined with motor equations to obtain the sets of dynamic equations suitable for fault detection problems.

**DC Motor**

Assuming the dc motor is under armature voltage speed control with constant field flux, we have

\[ V_t = R_a i_a + L_a \frac{d i_a}{d t} + K_a \Phi_d \omega \]

\[ K_a \Phi_d i_a = T_e = J_m \omega + T_m + T_{mf} \]

where

- \( V_t \) = motor terminal voltage
- \( i_a \) = motor armature current
- \( R_a \) = armature resistance
- \( L_a \) = armature leakage inductance
- \( K_a \) = motor physical constant
- \( \Phi_d \) = field flux per pole (a constant)
- \( T_e \) = electromagnetic torque generated by the motor
- \( J_m \) = rotor moment of inertia
- \( T_m \) = shaft mechanical torque
- \( T_{mf} \approx C_{mf0} + C_{mf1} \omega = \text{bearing friction plus windage torques} \)

Now we consider small deviations around a steady state operating point

\[ \Delta V_t = R_a \Delta i_a + L_a \frac{d i_a}{d t} + K_\Phi \Delta \omega \quad (3.8) \]

\[ K_\Phi \Delta i_a = J_m \dot{\omega} + \Delta T_m + C_{mf1} \Delta \omega \quad (3.9) \]

\[ K_\Phi = K_a \Phi_d \]
The equations for the dynamics of the motor and pump around an operating point can now be combined by eliminating $\Delta T_m$ from Eq. (3.5) and (3.9). In state space form we get

$$\frac{dX(t)}{dt} = AX(t) + BU(t)$$

$$Y(t) = CX(t)$$

where

$$X = [\Delta i_a \ \Delta \omega \ \Delta Q]^T$$

$$U = [\Delta V_t \ \Delta P_{load}]^T$$

$$A = \begin{bmatrix}
-\frac{R_a}{L_a} & -\frac{K_f}{L_a} & 0 \\
\frac{K_f}{J_f} & -\frac{C_f}{J_f} & -\frac{C_{TQ}}{J_f} \\
0 & \frac{C_{pw}}{J_f} & \frac{C_P}{J_f}
\end{bmatrix}$$

$$B = \begin{bmatrix}
\frac{1}{L_a} & 0 \\
0 & 0 \\
0 & -\frac{1}{J_f}
\end{bmatrix}$$

$$C = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & C_{pw} & C_{PQ}
\end{bmatrix}$$

$$J = J_P + J_m$$

$$C_F = C_{mf1} + C_{T\omega}$$
\[ CP = CPQ - Rf - CPPQ \]

and the measurable disturbance \( \Delta P_{load} \) (increase in boiler pressure for example) has been included in the input vector for simplicity.

It has been assumed that all the elements of \( U \) and \( Y \) are measurable in time domain. The problem is the detection of nonpermitted deviations in the physical parameters \( R_a, L_a, K_F, J, C_P, CTQ, C_Pw, CP, If, \) and \( CPQ \) by processing discrete time samples of \( \Delta i_a, \Delta \omega, \Delta Q, \Delta P, \Delta V_t, \) and \( \Delta P_{load} \). These deviations are then used to be related to a particular fault; i.e., motor overload, bearing cracks, pipe leak, etc. This problem becomes much more difficult if any elements of \( Y \) or \( U \) cannot be measured.

**Induction Motor**

Now we combine the dynamic equations of the pump/pipe system with those of a three-phase squirrel cage induction motor.

The transient behavior of a three-phase induction motor can be described in an orthogonal coordinate system (d-q-o) rotating with the angular speed \( \omega_s \) as, Krause and Thomas [56]

\[
\begin{align*}
V_{qs} &= r_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega_s \lambda_{ds} \\
V_{ds} &= r_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega_s \lambda_{qs} \\
V_{os} &= r_s i_{os} + \frac{d\lambda_{os}}{dt} \\
V_{qr} &= r_r i_{qr} + \frac{d\lambda_{qr}}{dt} + (\omega_s - \omega_r)\lambda_{dr} \\
V_{dr} &= r_r i_{dr} + \frac{d\lambda_{dr}}{dt} - (\omega_s - \omega_r)\lambda_{qr} \\
V_{or} &= r_s i_{or} + \frac{d\lambda_{or}}{dt}
\end{align*}
\]

where \( V_{qs}, V_{ds}, \) and \( V_{os} \) represent the applied stator voltages in the d-q-o coordinate system and are algebraically related to these voltages.
For a squirrel cage motor fed from a three-wire system without a neutral, rotor voltages \( V_{qr} \), \( V_{dr} \), \( V_{or} \), and the currents \( i_{qs} \) and \( i_{or} \) are all zero. Thus, in the above equations, these signals are set to zero. The flux linkages appearing in the above equations are given by

\[
\lambda_{qs} = L_{ts}i_{qs} + L_m(i_{qs} + i_{qr})
\]

\[
\lambda_{qr} = L_{tr}i_{qr} + L_m(i_{qs} + i_{qr})
\]

\[
\lambda_{ds} = L_{ts}i_{ds} + L_m(i_{ds} + i_{dr})
\]

\[
\lambda_{dr} = L_{tr}i_{dr} + L_m(i_{ds} + i_{dr})
\]

Substituting for these flux linkages in the voltage equations, we obtain

\[
V_{qs} = r_s i_{qs} + \omega_s(L_{ts} + L_m)i_{ds} + \omega_s L_m i_{dr}
+ (L_{ts} + L_m) \frac{di_{qs}}{dt} + L_m \frac{di_{qr}}{dt}
\]

\[
V_{ds} = r_s i_{ds} - \omega_s(L_{ts} + L_m)i_{qs} - \omega_s L_m i_{qr}
+ (L_{ts} + L_m) \frac{di_{ds}}{dt} + L_m \frac{di_{dr}}{dt}
\]

\[
0 = r_r i_{qr} + (\omega_s - \omega_r)(L_{tr} + L_m)i_{dr} + (\omega_s - \omega_r)L_m i_{ds}
+ (L_{tr} + L_m) \frac{di_{qr}}{dt} + L_m \frac{di_{qs}}{dt}
\]

\[
0 = r_r i_{dr} - (\omega_s - \omega_r)(L_{tr} + L_m)i_{qr} - (\omega_s - \omega_r)L_m i_{qs}
+ (L_{tr} + L_m) \frac{di_{dr}}{dt} + L_m \frac{di_{ds}}{dt}
\]

where

\( r_s = \) stator winding resistance
\( r_r = \) rotor equivalent resistance
\( L_{ts} = \) stator leakage inductance
\( L_{tr} = \) rotor leakage inductance
\( L_m = \) magnetizing inductance
\( \omega_s = \) angular velocity of rotation of \( d - q \) axes
\( \omega_r = \) rotor angular velocity
and all rotor quantities are referred to stator windings by the effective stator/rotor turns ratio.

The instantaneous electromagnetic torque, which is positive for motor, is given by

\[ T_e = \frac{n}{2} \left( \frac{p}{2} \right) \left( \lambda_{qr} i_{dr} - \lambda_{dr} i_{qr} \right) \]

\[ = \frac{n}{2} \left( \frac{p}{2} \right) L_m (i_{qs} i_{dr} - i_{ds} i_{qr}) \]

where \( n = 3 \) is the number of phases and \( p \) is the number of poles. The torque equation is nonlinear and should be linearized around a steady state operating point \((i_{qs0}, i_{dr0}, i_{qr0}, \text{and } i_{ds0})\)

\[ \Delta T_e = 3p/4 L_m (i_{dr0} \Delta i_{qs} + i_{qs0} \Delta i_{dr} - i_{qr0} \Delta i_{ds} - i_{ds0} \Delta i_{qr}) \]

\[ = J_m \dot{\omega}_r + \Delta T_m + C_m f_1 \Delta \omega_r \]  \hspace{1cm} (3.10)

Assuming the d-q coordinate system is rotating with an angular velocity equal to angular frequency of the stator voltages; i.e., \( \omega_s = \omega_c \), we can linearize the nonlinear voltage equations to get

\[ \Delta V_{qs} = r_s \Delta i_{qs} + [(L_{qs} + L_m) i_{ds0} + L_m i_{dr0}] \Delta \omega_e \]

\[ + (L_{qs} + L_m) \omega_{e0} \Delta i_{ds} + L_m \omega_{e0} \Delta i_{dr} \]

\[ + (L_{qs} + L_m) \frac{di_{qs}}{dt} + L_m \frac{di_{qr}}{dt} \]  \hspace{1cm} (3.11)

\[ \Delta V_{ds} = r_s \Delta i_{ds} - [(L_{qs} + L_m) i_{qs0} + L_m i_{qr0}] \Delta \omega_e \]

\[ - (L_{qs} + L_m) \omega_{e0} \Delta i_{qs} + L_m \omega_{e0} \Delta i_{qr} \]

\[ + (L_{qs} + L_m) \frac{di_{ds}}{dt} + L_m \frac{di_{dr}}{dt} \]  \hspace{1cm} (3.12)

\[ 0 = r_r \Delta i_{qr} + [(L_{qr} + L_m) i_{dr0} + L_m i_{ds0}] \Delta \omega_e \]

\[ - [(L_{qr} + L_m) i_{dr0} + L_m i_{ds0}] \Delta \omega_r \]

\[ + (\omega_{e0} - \omega_{r0}) (L_{qr} + L_m) \Delta i_{dr} + (\omega_{e0} - \omega_{r0}) L_m \Delta i_{ds} \]

\[ + (L_{qr} + L_m) \frac{di_{qr}}{dt} + L_m \frac{di_{qs}}{dt} \]  \hspace{1cm} (3.13)

\[ 0 = r_r \Delta i_{dr} + [(L_{qr} + L_m) i_{qr0} + L_m i_{qs0}] \Delta \omega_e \]

\[ + [(L_{qr} + L_m) i_{qr0} + L_m i_{qs0}] \Delta \omega_r \]

\[ - (\omega_{e0} - \omega_{r0}) (L_{qr} + L_m) \Delta i_{qr} - (\omega_{e0} - \omega_{r0}) L_m \Delta i_{qs} \]

\[ + (L_{qr} + L_m) \frac{di_{dr}}{dt} + L_m \frac{di_{ds}}{dt} \]  \hspace{1cm} (3.14)
The equations of the motor and the pump can now be combined by eliminating $\Delta T_m$ from Eqs. (3.5) and (3.10) as shown

$$J\dot{\omega}_r = \frac{3p}{4} L_m (i_{dr0}\Delta i_{qs} + i_{qs0}\Delta i_{dr} - i_{qr0}\Delta i_{ds} - i_{ds0}\Delta i_{qr})$$

$$- C_F \Delta \omega_r - C_T Q \Delta Q$$

(3.15)

with parameters $J$ and $C_F$ as defined earlier. Eqs. (3.7) and (3.11) through (3.15) can be put in state space form with

$$X = [\Delta i_{qs} \Delta i_{ds} \Delta i_{qr} \Delta i_{dr} \Delta \omega_r \Delta Q]^T$$

$$U = [\Delta V_{qs} \Delta V_{ds} \Delta \omega_e \Delta P_{load}]^T$$

$$Y = [\Delta i_{qs} \Delta i_{ds} \Delta \omega_r \Delta Q \Delta P]^T$$

and matrices $A$, $B$, and $C$ as given in Appendix A.

It has been assumed that the motor is to be controlled by varying the magnitudes and the frequency of the applied stator voltages.

The problem of fault detection is now compounded with the facts that the rotor currents $i_{qr}$ and $i_{dr}$ are not measurable, and that the motor parameters are time-varying. The parameter variations are due to saturation for inductances and due to skin effect for resistances, particularly in the rotor where wide variations in $r_r$ is expected.

**Synchronous Motor**

Similar state space formulation can be arrived at for a synchronous motor driving the pump. The state, input and output vectors for a three-phase cylindrical rotor synchronous motor are given by [72]

$$X = [\Delta i_q \Delta i_d \Delta i_{fd} \Delta i_{kd} \Delta i_{kq1} \Delta i_{kq2} \Delta \omega_r \Delta Q]^T$$

$$U = [\Delta V_{qs} \Delta V_{ds} \Delta E_{fd} \Delta \omega_e \Delta P_{load}]^T$$

$$Y = [\Delta i_q \Delta i_d \Delta i_{fd} \Delta \omega_r \Delta Q \Delta P]^T$$

where here, the damper winding currents $i_{kd}$, $i_{kq1}$, and $i_{kq2}$ (used to model the effect of magnetic induction on the body of the cylindrical iron rotor), are not measurable.
Power Transformer

The state space formulation for an n-winding (single or three-phase) transformer dynamics is discussed in Chapter 6. Here, we consider a single-phase two-winding transformer for illustrative purposes (Fig. 5).

\[ X(t) = [i_1(t) \quad i_2(t)]^T \]

\[ U(t) = [v_1(t) \quad v_2(t)]^T \]

\[ Y(t) = X(t) \]

\[ A = \frac{1}{\Delta} \begin{bmatrix} -L_2(R_1 + R_{12}) - L_{12}R_1 & L_{12}R_2 - L_2R_{12} \\ L_{12}R_1 - L_1R_{12} & -L_1(R_2 + R_{12}) - L_{12}R_2 \end{bmatrix} \]

\[ B = \frac{1}{\Delta} \begin{bmatrix} L_2 + L_{12} & -L_{12} \\ -L_{12} & L_1 + L_{12} \end{bmatrix} \]

\[ \Delta = L_1L_2 + L_{12}(L_1 + L_2) \]

where the magnetizing inductance \( L_{12} \) is a nonlinear function of the exciting current \( i_E = i_1 + i_2 \), due to saturation of the transformer core.

Typical hysteresis loops for power transformers indicate that the inductance \( L_{12} \) is generally a fast-varying parameter and therefore difficult to track. This will certainly make the fault detection problem more difficult.
Failures Caused by Power System Harmonics

The problem of prophylaxis of failures caused by power system harmonics is concerned with developing techniques to prevent overheating due to excess losses generated by harmonic signals.

To explain this, consider the one-line diagram shown in Figure 6. This figure shows a typical example where the harmonic currents drawn by nonlinear loads (at Bus 2) generate harmonic voltage drops across the power system's impedances (for example across the impedance of the distribution transformer T shown) which result in amplitude modulations of the power system voltages, and thus an elevated temperature rise in the loads connected to Bus 1. Here, the problem is to prevent the temperatures in the loads at Bus 1, which may be other transformers and electric motors, exceed the rated values.

3.4 Solution Procedures

In this section, we will present an algorithm for tracking of synchronous machine parameters from operating data. This algorithm obtains the combined parameter and unmeasurable state estimates, and thus, can be used as the main tool in the state-of-the-art fault detection procedures.
3.4.1 Synchronous Machine Parameter Tracking

In Reference [72], we give the derivation of a round rotor synchronous generator linearized dynamic equations in state space form,

\[
\frac{dX}{dt} = A^*X(t) + B^*U(t)
\]

\[
X = [i_d \ i_{fd} \ i_{kd} \ i_q \ i_{kq1} \ i_{kq2} \ \delta \ \omega]^T
\]

\[
U = [E_{fd} \ T_p]^T
\]  

(3.16)

with parameter matrices \( A^* \) and \( B^* \) as given in [72]. The components of the state vector \( X \) and of the input vector \( U \) are deviations from their steady state values. The matrix \( A^* \) is a function of the steady state operating point around which the dynamic equations have been linearized.

For sufficiently small time interval \( T = \Delta t \), Eq. (3.16) can be written in discrete time domain as

\[
X((K + 1)T) = A(KT)X(KT) + B(KT)U(KT)
\]

which for simplicity we write as

\[
X(K + 1) = AX(K) + BU(K)
\]  

(3.17)

where the continuous system parameters \( A^* \) and \( B^* \) are related to matrices \( A \) and \( B \) by

\[
A = e^{A^*T}
\]

\[
B = \int_{KT}^{(K+1)T} e^{A^*((K+1)T-\tau)}B^*d\tau
\]

Using Maclaurin series expansion we get

\[
A = \sum_{i=0}^{\infty} \frac{1}{i!}(A^*T)^i
\]

and

\[
B = T \left[ \sum_{i=0}^{\infty} \frac{1}{(i + 1)!}(A^*T)^i \right] B^*
\]  

(3.18)
It is known that the damper winding currents \( i_{kd}, i_{kq1}, \) and \( i_{kq2} \) cannot be measured. Therefore, the system's outputs are

\[
Y = [i_d \ i_f \ i_q \ \delta \ \omega]^T
\]

and the unmeasurable states are

\[
Z = [i_{kd} \ i_{kq1} \ i_{kq2}]^T
\]

The proposed solution for the parameter tracking problem has two steps. First, we estimate the unmeasurable components of the state vector; namely, the damper winding currents in \( Z \). Second, we estimate the machine parameters by processing sequences of measured inputs and outputs, and sequences of estimated damper winding currents.

An observer is designed to solve the damper winding currents estimation problem.

**Development of the Observer**

In an observable linear time-variant system, the unmeasurable states are observable through the outputs of the system. It should therefore be possible to estimate the unmeasurable states of the system by processing sequences of its outputs.

We propose an observer whose inputs are the system outputs and whose outputs are estimates of unmeasurable states of the system. It will be shown that the observer model can be identified using a priori knowledge of the system inputs. With this knowledge, observer model order and parameters which describe how the estimates of the system unmeasurable states can be extracted from the output data are determined by simulation.

Figure 7 shows the block diagram of the proposed scheme. In this figure, subscript "s" denotes simulated and "m" denotes measured quantities. The generator inputs are the step inputs; thus, a priori knowledge about the inputs is a set basis vectors for the input vector space. The response of the generator to each member of the basis set is simulated using latest estimates of the machine parameters. Simulation set \( X_s(.) \) is devided into two sets; output set \( Y_s(.) \) and unmeasurable set \( Z_s(.) \). Cross-correlations between each member of one set and all the members of the other set are calculated. Based on these cross-correlations, order of the observer is assumed. Parameters of the observer are then estimated using the assumed order and an algorithm which will be discussed later. Finally, validity of the assumed order is checked. If valid, the identification process is complete; if not, another order is assumed and the process is repeated.
A Priori Knowledge of Basis Vectors of Input Space

Generator Model
\[ X(K+1) = AX(K) + BU(K) \]

\( X_e(\cdot) \)

\( Y_e(\cdot) \)

\( Z_e(\cdot) \)

Cross-Correlation Between \( Y_e(\cdot) \) and \( Z_e(\cdot) \)

Correlations

Assume an Order for the Observer Based on Correlations Obtained

Observer Order

Estimate Parameters of the Observer \( (A_{obs}) \)

\[ \hat{Z} = (A_{obs})(Y) \]

Check
\[ |Z_e(\cdot) - \hat{Z}(\cdot)| \]

Acceptable
Unacceptable

Done

Figure 7: Block diagram showing the development of the observer
Derivation of the algorithm for estimation of the parameters of the observer will now be given. Recall that in Eq. (3.17), $X$ is the $n$-dimensional state vector, $U$ is the $r$-dimensional input vector, $A$ and $B$ are the $n \times n$ and $n \times r$ matrices of parameters, and $K$ is the instant of time. If we let $Y_{px1}$ denote the output vector and $Z$ denote the unmeasurable part of the state vector, then an observer must be constructed such that

$$
\dot{Y}(K) = [A_{obs}]Y^*(K)
$$

$$
Y^*(K) = \begin{bmatrix}
y_1(K - i_1) & y_1(K - j_1) & \cdots \\
y_1(K - \ell_1) & y_2(K - i_2) & \cdots 
\end{bmatrix}^T
$$

(3.19)

where $y_1, y_2, \ldots$ are the components of $Y$, and $A_{obs}$ is the matrix of the observer parameters with appropriate dimension. The lags $i_1, j_1, \ldots$ are identified based on cross-correlation analysis and extensive simulation. Construction of the observer requires the estimation of the matrix $A_{obs}$ such that Eq. (3.19) gives the best estimate of $Z(K)$ for all $K$. By successive substitutions, Eq. (3.17) for instant $K+1$ can be written as

$$
X(K + 1) = A^{K+1}X(0) + \sum_{i=0}^{K} A^{K-i} BU(i)
$$

(3.20)

If we let

$$
\sum_{i=0}^{K} A^{K-i} = A_K
$$

for a step input and deviations from steady state values, it can be written as

$$
X(K + 1) = A_K BU
$$

(3.21)

Now we partition $A_K$ and $B$ as

$$
A_K = \begin{bmatrix}
A_{K,pp} & A_{K,pq} \\
A_{K,qp} & A_{K,qq}
\end{bmatrix} \text{ and } B = \begin{bmatrix}
B_p \\
B_q
\end{bmatrix}
$$

where $q = n - p$. Then we write Eq. (3.21) as

$$
Y(K + 1) = [A_{K,pp}B_p + A_{K,pq}B_q] U
$$

$$
Z(K + 1) = [A_{K,qp}B_p + A_{K,qq}B_q] U
$$
and we use Eq. (3.19) to write

\[
\begin{bmatrix}
A_{K,qp}B_p + A_{K,qq}B_q
\end{bmatrix} U = [A_{obs}] \begin{bmatrix}
A_{K,pp}^* B_p + A_{K,pq}^* B_q
\end{bmatrix} U
\]

\[
A_{K}^* = \begin{bmatrix}
A_{K,1}^{(1)} & A_{K,1-j_1}^{(1)} & \ldots & A_{K,1-t_1}^{(1)} & \ldots & A_{K,1-i_m}^{(m)} & \ldots
\end{bmatrix}^T
\]  \hspace{1cm} (3.22)

where superscript "m" \((m = 1, 2, \ldots, p)\) denotes the transpose of the \(m\)th row of the given matrix.

The matrix \(A_{obs}\) satisfying Eq. (3.22) would depend on \(U\). If \(U\) belongs to \(R^r\), then for \(A_{obs}\) to satisfy (3.22) for all \(U\) in \(R^r\), it must satisfy (3.22) for a set of basis vectors of \(R^r\). Note that such an \(A_{obs}\) will not generally be unique and that it will be a function of instant \(K\).

In what follows, a least-squares estimation based algorithm for estimation of the matrix \(A_{obs}\) is derived.

Let \(X_{u_i}(.)\) be the system response to the input \(U_i = [0 \ldots 0 1 0 \ldots 0]^T\), where \(U_i\) is an \(r\)-dimensional vector with only one nonzero entry in the \(i\)th row. Then define

\[
y_{u_i}(K) = [y_1(K-i_1) \ y_1(K-j_1) \ \ldots \ y_1(K-\ell_1) \ \ldots \ y_p(K-\ell_p)]^T
\]

and

\[
Z_{u_i}(K) = [Z_1(K) \ Z_2(K) \ \ldots \ Z_{n-p}(K)]^T
\]

where \(y_1, y_2, \ldots, y_p; \ i_1, j_1, \ldots, \ell_p,\) and \(Z_1, Z_2, \ldots, Z_{n-p}\) are defined as in Eq. (3.19). The matrix \(A_{obs}\) must then satisfy the relation

\[
Z_{u_i}(K) = [A_{obs}] y_{u_i}(K) ; \quad i = 1, 2, \ldots, r
\]

for all \(K\). That is, \(A_{obs}\) must be estimated such that at any instant \(K\), the following errors are minimized:

\[
e(1) = Z_{u_i}(1) - [A_{obs}] y_{u_i}(1)
\]

\[
e(2) = Z_{u_i}(2) - [A_{obs}] y_{u_i}(2)
\]

\[\vdots\]

\[
e(K) = Z_{u_i}(K) - [A_{obs}] y_{u_i}(K) ; \quad i = 1, 2, \ldots, r
\]

Define
\[ E_j = [e_j(1) \; e_j(2) \; \ldots \; e_j(K)]_{1 \times K} \]

\[ Z_{ju_i} = [z_{ju_i}(1) \; z_{ju_i}(2) \; \ldots \; z_{ju_i}(K)]_{1 \times K} \]

\[ Y_{ui} = [y_{ui}(1) \; y_{ui}(2) \; \ldots \; y_{ui}(K)]_{p \times K} \]

where subscript "j" denotes the j\textsuperscript{th} row; i.e., \( z_{ju_i}(1) \) denotes the j\textsuperscript{th} entry of the vector \( Z_{ui}(1) \). We can now write

\[ E_j = Z_{ju_i} - A_{jobs}Y_{ui}; \quad i = 1, 2, \ldots, r \]

\[ j = 1, 2, \ldots, n - p \]

where \( A_{jobs} \) denotes the j\textsuperscript{th} row of the matrix \( A_{obs} \). The criterion to be minimized can now be given as

\[ Q_j = \sum_{i=1}^{r} (Z_{ju_i} - A_{jobs}Y_{ui})(Z_{ju_i} - A_{jobs}Y_{ui})^T \tag{3.23} \]

Differentiating \( Q_j \) with respect to \( A_{jobs} \) and equating the result to zero will determine the conditions on the estimate of \( A_{jobs} \) that minimizes our criterion:

\[ \frac{\partial Q_j}{\partial A_{jobs}} \bigg|_{A_{jobs} = \hat{A}_{jobs}} = 0 \]

\[ \hat{A}_{jobs} \sum_{i=1}^{r} Y_{ui}Y_{ui}^T = \sum_{i=1}^{r} Z_{ju_i}Y_{ui}^T \]

and solving for \( \hat{A}_{jobs} \) we get

\[ \hat{A}_{jobs} = \left[ \sum_{i=1}^{r} Z_{ju_i}Y_{ui}^T \right] \left[ \sum_{i=1}^{r} Y_{ui}Y_{ui}^T \right]^{-1} \left[ \sum_{i=1}^{r} Y_{ui}Y_{ui}^T \right] ; \quad j = 1, 2, \ldots, n - p \tag{3.24} \]

Eq. (3.24) gives the best estimate of the j\textsuperscript{th} row of the observer matrix at instant K. At instant K+1, \( A_{jobs} \) can be estimated in a recursive manner. We have derived the recursive algorithm in Reference [72].
A prior knowledge of the system inputs can improve the estimates given by Eq. (3.24). If we have some knowledge about the relative order of the magnitudes of the input vector components, then we would use a set of basis vectors with the same relative order of magnitudes. Selecting the basis vectors in this manner has the effect of minimizing a weighted sum in Eq. (3.23).

The algorithm steps for estimation of the parameters of the observer can now be summarized as follows:

1) With the order of the system model known, use the latest estimates of the system parameters and simulate the response of the system to a set of basis vectors selected based on a priori knowledge of the system inputs,

\[
U_i \rightarrow \text{System Model} \rightarrow X_{U_i} \quad \text{for} \quad i = 1,2,\ldots,r
\]

2) Analyze cross-correlations between \(Y_{U_i}(.)\) and \(Z_{U_i}(.)\), where \(Y_{U_i}\) is the measurable part and \(Z_{U_i}\) is the unmeasurable part of \(X_{U_i}\) obtained in Step (1), to approximate the order of the observer.

3) Define vectors

\[
Y_{U_i}^*(K) = [y_1(K-i_1) \quad y_1(K-j_1) \ldots \quad y_1(K-\ell_1) \quad y_2(K-i_2) \ldots]^T
\]

\[
Z_{U_i}(K) = [Z_1(K) \quad Z_2(K) \ldots]^T \quad \text{for} \quad i = 1,2,\ldots,r
\]

where \(y_1, y_2, \ldots\) are the components of the output vector \(Y\), \(Z_1, Z_2, \ldots\) are the components of the vector \(Z\), and lags \(i_1, j_1, \ldots\) are approximated in Step (2).

4) Let \(\ell_{max}\) be \(\max\{i_1, j_1, \ldots\}\), then define matrices

\[
V_{U_i}(K) = \begin{bmatrix} Y_{U_i}^*(\ell_{max}) & Y_{U_i}^*(\ell_{max}+1) & \ldots & Y_{U_i}^*(K) \end{bmatrix}
\]

\[
W_{U_i}(K) = \begin{bmatrix} Z_{U_i}(\ell_{max}) & Z_{U_i}(\ell_{max}+1) & \ldots & Z_{U_i}(K) \end{bmatrix}
\]

for \(i = 1,2,\ldots,r\) and some integer \(K\).

5) Estimate the observer matrix for instant \(K\) by

\[
\hat{A}_{\text{obs}}(K) = \left[ \sum_{i=1}^{r} W_{U_i}(K)V_{U_i}^T(K) \right] S(K)
\]
where

\[
S(K) = \left[ \sum_{i=1}^{r} V_{u_i}(K)V_{u_i}^T(K) \right]^{-1}
\]

6) At instant \( K+1 \), estimate the observer matrix recursively as

\[
\hat{A}_{obs}(K+1) = \left[ \sum_{i=1}^{r} W_{u_i}(K)V_{u_i}^T(K) + Z_{u_i}(K+1)Y_{u_i}^*(K+1) \right] S(K+1)
\]

where \( S(K+1) \) is recursively calculated by

\[
S_1(K+1) = S(K) - S(K)Y_{u_1}^*(K+1)
\]

\[
\left[ 1 + Y_{u_1}^*(K+1)S(K)Y_{u_1}^*(K+1) \right]^{-1} Y_{u_1}^*(K+1)S(K)
\]

\[
S_j(K+1) = S_{j-1}(K+1) - S_{j-1}(K+1)Y_{u_j}^*(K+1)
\]

\[
\left[ 1 + Y_{u_j}^*(K+1)S_{j-1}(K+1)Y_{u_j}^*(K+1) \right]^{-1} Y_{u_j}^*(K+1)S_{j-1}(K+1)
\]

for \( j = 2, 3, \ldots, r \) and where

\[
S(K+1) = S_r(K+1)
\]

7) Check the validity of the observer model by checking the errors

\[
E(.) = Z_s(.) - \hat{Z}_s(.)
\]

where \( \hat{Z}_s(.) \) is obtained as shown below, and \( Y_s(.) \) and \( Z_s(.) \) are obtained from

\[
Y_s(.) \rightarrow \text{Observer} \rightarrow \hat{Z}_s(.)
\]

simulated response of the system to an arbitrary input different from the members of the basis set. If the errors are unacceptable, go back to Step (2).
8) When the actual system is disturbed with an input $U_m$, use the measurement set $Y_m(.)$ and the observer to estimate the unmeasurable states Z,

$$Y_m(.) \rightarrow \text{Observer} \rightarrow \hat{Z}(\cdot)$$

**Parameter Estimation**

Once the unmeasurable part of the state vector has been estimated by the observer, a generalized least square estimator (Appendix B) is used to estimate the parameters of the generator as shown in Figure 8. Unlike ordinary least square estimator, the generalized version gives unbiased estimates for noisy measurements [29].

![Figure 8: Synchronous machine parameter estimation algorithm](image)

Once the parameter matrices of discrete-time model ($A$ and $B$) have been estimated, the estimates of the continuous-time model matrices ($A^*$ and $B^*$) can be obtained as follows: From Eq. (3.18), solve for $B^*$

$$B^* = [A - I]^{-1} A^* B$$

(3.25)

where $I$ is the identity matrix. The matrix $[A - I]$ is invertible if the matrix $A^*$ has nonzero eigenvalues; therefore, $[A - I]$ is invertible for stable systems. To calculate $A^*$ from $A$, we diagonalize or put $A$ in Jordan form. In any case, there is a transformation matrix $M$ such that

$$\bar{A} = M^{-1} AM = [e^{\lambda_i T}]$$

where $T$ is the period of sampling, $e^{\lambda_i T}$ for $i = 1, 2, \ldots$ are the entries on the diagonal and $\lambda_i$'s are the eigenvalues of $A^*$. Taking natural log of the diagonal
entries in $\tilde{A}$ and dividing by $T$ will give $\lambda_i$'s. Matrix $A^*$ can now be obtained by the inverse transformation

$$A^* = M\tilde{A}^*M^{-1}$$

where $\tilde{A}^*$ is either diagonal or in Jordan form. In both cases, the diagonal entries of $\tilde{A}^*$ are the eigenvalues of $A^*$. In the Jordan case, some entries on the superdiagonal are 1; the exact locations are determined by the structure of the matrix $\tilde{A}$. Once $A^*$ has been calculated, $B^*$ can be obtained using Eq. (3.25).

### 3.4.2 Incipient Fault Detection

In general, incipient faults are manifested by abnormalities in at least one of the following sets of quantities [10]:

1) Measurable and unmeasurable state variables; these abnormalities may be detected using a limit check $X_{\text{min}} < X(K) < X_{\text{max}}$, or a trend check $\Delta X_{\text{min}} < \Delta X(K) < \Delta X_{\text{max}}$, where $X$ is either an output signal or estimate of an unmeasurable state obtainable by the observer. Correlation functions may also be used in detecting state abnormalities. Increase in correlation between any two states, for example, may indicate potential loss of minimality.

2) Process physical parameters; once the structure of the system model has been assumed, changes in the model parameters can be monitored using an on-line parameter estimation algorithm. If the structure of the model is derived from physical laws governing the system, physical parameters such as resistances, inductances, etc. can be tracked. In such a case, fault detection may be followed by fault diagnosis (location and cause). Reference [10] reports that change of 7% in armature resistance of a dc motor has been easily detected using least square estimator. It also reports that a change in armature resistance due to temperature rise after cold start-up of the motor has been detected.

3) Characteristic quantities such as efficiency, fuel and oil consumption, impurities in the cooling stream, and radio frequency noise level; continuous on-line monitoring of these quantities can lead can lead to detection of certain types of incipient faults. Examples are detection of intense corona at 3.8MHz and of gap discharges at 8.0MHz using electromagnetic interference monitoring [22].

It is apparent that all incipient fault detection techniques should rely on an extensive data base. Assuming the required data base is available in the memory of a computer, the occurrence of a fault can be detected, its signature can be recognized, and corrective control laws or preventive maintenance may be scheduled.
The proposed fault identification technique is based on monitoring the abnormalities in the combined sets of quantities listed under (1) and (2) above. The parameter tracking scheme presented in the previous section obtains the estimates of the physical parameters along with the estimates of the unmeasurable states, and thus, can be used as the main tool in our fault detection scheme.

Consider a dynamic process whose linearized state space representation is given by
\[
\frac{dX}{dt} = A(t)X(t) + B(t)U(t)
\]
in continuous-time domain, and by
\[
X(K + 1) = A^*(K)X(K) + B^*(K)U(K)
\]
in discrete-time domain.

The parameter tracking algorithm given in Section 3.4.1 can be used to estimate a trajectory for each parameter of the continuous-time model, by processing records of input/output observations sampled in discrete-time domain, as the system moves from one operating condition to another. These trajectories will have embedded in them, the effect of any other factor which may cause changes in the parameters over time. Contributing factors, apart from changes in operating conditions, are mostly due to aging. Therefore, if the trajectory for a given parameter is compared with earlier trajectories for the same parameter, any deterioration which may cause changes in this parameter can be detected. Once an abnormality has been detected, its signature is recognized using a catalogue of faults, relating abnormal changes in specific parameters to specific faults. Such a catalogue can be developed using the recorded history and extensive simulations of the fault models. Fault models are obtained by altering the normal model for the given faults. After recognizing the signature of the fault, the corresponding fault model can be used to predict the fault state trajectories of the process. Knowing where the process is going, we can determine for how long the present abnormality can be tolerated.

To become more specific, we now consider the applications of the proposed solutions to electric machines and transformers. Again, we will assume that each motor is driving a centrifugal pump and will refer to the models developed in Section 3.3.2.

**DC Motor**

The balanced equations for the dc motor/pump system in state space are given in Section 3.3.2. The state variables were defined to be the changes in armature
current $i_a$, in shaft speed $\omega_r$, and in volume flow rate $Q$. The input variables were the changes in terminal voltage $v_t$ and in pressure at the fluid destination $P_{load}$. All these variables are expected to be measurable. The coefficient matrices $A$ and $B$ contain nine nonzero elements with nine unknown parameters and thus the estimates of the physical parameters are uniquely determined from the estimates of $A$ and $B$. For detection of incipient faults, the following procedure can be used:

a) From the record of input/output observations after a change in the input variables, estimate the discrete-time model coefficients $A^*_1$ and $B^*_1$ using a recursive estimator and initial values $A^*_0$ and $B^*_0$.

b) By use of appropriate transformations (given in Section 3.4.1), calculate the estimates of the continuous-time model coefficients $A_1$ and $B_1$ from $A^*_1$ and $B^*_1$.

c) Obtain the unique estimates of the physical parameters

$$\theta^1 = \begin{bmatrix} R^1_a & L^1_a & J^1 & K^1_F & C^1_F & C^1_T & I^1_f & C^1_p & C^1_p \end{bmatrix}$$

from the estimates of $A_1$ and $B_1$ in an obvious manner.

d) Determine the changes in the physical parameters and states

$$\Delta \theta = \theta^1 - \theta^0 \quad \Delta X = X^1 - X^0$$

e) Using an extensive data base, decide whether these changes are normal. If they are not within normal ranges, activate a fault alarm signal indicating a potential failure somewhere in the process.

f) To pinpoint the developing failure, a catalogue of faults is to be used in relating a given abnormality to a specific failure. For instance, an abnormal change in armature current along with the presence of certain vibrational harmonics may be related to bearing cracks.

**Induction Motor**

Referring to Section 3.3.2, the state vector in the state space representation of induction motor/pump system is $X = [\Delta i_{qs} \ \Delta i_{ds} \ \Delta i_{qr} \ \Delta i_{dr} \ \Delta \omega_r \ \Delta Q]^T$ and the input vector is $U = [\Delta v_{qs} \ \Delta v_{ds} \ \Delta \omega_e \ \Delta P_{load}]^T$.

There are eleven physical parameters to be estimated and with all the states measurable, the procedure for detection of incipient faults in induction motors is the same as that for dc motors. However, in a squirrel cage induction motor, the rotor currents $i_{qr}$ and $i_{dr}$ cannot be measured. The first step in the procedure should now be the estimation of rotor currents from the record of input/output...
observations. This can be accomplished by constructing an intelligent observer such as the one discussed earlier.

A problem with which power system engineers are frequently faced is how to avoid the shutdown of an induction motor after some burnt stator coils have been cut out [2]. The problem is that the phase from which coils have been cut out will draw larger than rated current for rated motor load.

The remedial measure that should be taken is to reduce motor load to allow continuous operation of the motor without overloading any winding. It is therefore necessary to calculate the maximum load that the induction motor can drive after the coils have been cut out. In such cases, the induction motor is no longer a balanced three-phase system and a new model is needed for calculation of the maximum allowable load. An on-line parameter tracking scheme would make the development of a new model possible. The structure of the new model, in this case, will be the same as that for the balanced motor. Once the new model has been identified, the maximum motor load allowable can be calculated from the corresponding maximum stator currents. That is, the thermal capacity of the insulation is used to calculate maximum $I^2R$ losses in the stator. From these losses and estimates of the stator winding resistances, maximum allowable stator currents can be calculated. With the maximum currents and the corresponding losses known, maximum allowable shaft load can be calculated. For the continuous operation of the motor without further failures, the motor load must be reduced to the maximum load calculated above. Estimates of the winding resistances can be used to approximate the winding temperatures, so that we ensure the temperatures remain within thermal capacities.

**Power Transformer**

As mentioned in Section 3.3.2, the state and input vectors in the state space representation of a transformer model are the winding currents and voltages, respectively. The difficulty in tracking the parameters of the transformer is the fast variation of the magnetizing inductance due to magnetic saturation of the core.

For a recursive parameter estimator to track the coefficient matrices A and B closely, the effect of the magnetizing inductance on the entries of these matrices must be removed. One way this can be done is to decouple the state equations by physical reasoning and redefining the state and input vectors as explained in Chapter 5. Another way is to construct an observer to estimate the magnetizing branch voltage as discussed in Chapter 6.

Fault detection based on estimates of physical parameters and states is a powerful technique when applied to transformers. Some years ago in a labor conflict, one of the transformers of a power company was shot. The oil leak went unnoticed for several hours before it was detected by visual inspection. At the time the
leak was detected, windings' temperature had risen dangerously. Cooling system leakages are certain to be detected by the proposed technique long before they can create a catastrophe. For copper conductors, a change in temperature from 20°C to 40°C results in a change of resistance from 0.00172 to 0.00185 which corresponds to a 7.7% change.

**Power System Harmonics**

Referring to Figure 5, to limit the amplitudes of the voltage harmonics at Bus 1, generated by the PCS and the nonlinear loads, a nonparametric model for the voltage signal at Bus 1 can be developed. Based on this model, a low-pass or a band-pass filter can be designed to eliminate or limit the harmonics (or subharmonics) at Bus 1. This should limit the temperature rise caused by the harmonics and thus eliminates potential failures caused by excessive temperature in the electric components connected to this bus. The changes in spectral densities, autocorrelation functions, etc. can be used to adjust the parameters of the filter.

**Round Rotor Synchronous Machine**

For a synchronous motor/pump system, the state vector \( X = [i_q \ i_d \ i_{fd} \ i_{kd} \ i_{kq1} \ i_{kq2} \ \omega \ \delta \ Q]^T \). The damper winding currents \( i_{kd}, i_{kq1}, \) and \( i_{kq2} \) are not measurable. In the next section, it is shown that the proposed observer can obtain accurate estimates of the damper currents. The proposed parameter tracking scheme can be used to monitor changes in the machine physical parameters and states. Estimates of \( i_{kd}, i_{kq1}, i_{kq2}, r_{kd}, r_{kq1}, \) and \( r_{kq2} \) can be used to approximate the internal temperature of the machine by calculating \( i^2 r \) terms. Possible failure due to excessive temperature rise may then be detected.

In the case of corona and gap discharges, the output signals should contain small amplitude high frequency components due to these phenomena. If high frequency signal models are identified and included in the database, changes in the signal model parameters, or in autocorrelation functions, spectral densities, etc. in the case of nonparametric signal models, can be monitored for detection of intense corona, gap discharges, etc.

**3.5 Computer Simulation Results**

In this section, we apply the algorithms discussed in the previous sections to estimate the parameters of a given machine. The machine used is the same as in [38]. Following the proposed procedure in developing an observer, we simulate the response of this machine (with the given parameters in [38]) to the following set of basis vectors for the two-dimensional input space \( R^2 \):
Table 1: Maximum Correlation/Lag

<table>
<thead>
<tr>
<th>(Z/Y)</th>
<th>(i_d(\cdot))</th>
<th>(i_{fd}(\cdot))</th>
<th>(i_q(\cdot))</th>
<th>(\delta(\cdot))</th>
<th>(\omega(\cdot))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i_{kd}(\cdot))</td>
<td>0.27/0</td>
<td>0.22/0</td>
<td>0.78/0</td>
<td>-0.95/30</td>
<td>0.97/9</td>
</tr>
<tr>
<td>(i_{kq1}(\cdot))</td>
<td>0.70/0</td>
<td>0.71/0</td>
<td>0.95/4</td>
<td>0.31/0</td>
<td>0.96/26</td>
</tr>
<tr>
<td>(i_{kq2}(\cdot))</td>
<td>0.29/0</td>
<td>0.25/0</td>
<td>0.81/0</td>
<td>-0.94/29</td>
<td>0.97/10</td>
</tr>
</tbody>
</table>

Table 2: Absolute mean and rms errors for damper currents

<table>
<thead>
<tr>
<th></th>
<th>Absolute Mean Error</th>
<th>RMS Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\frac{1}{10} \sum_{K=1}^{110}</td>
<td>e(K)</td>
</tr>
<tr>
<td>(i_{kd})</td>
<td>1.92E-7</td>
<td>3.09E-7</td>
</tr>
<tr>
<td>(i_{kq1})</td>
<td>2.06E-6</td>
<td>2.47E-6</td>
</tr>
<tr>
<td>(i_{kq2})</td>
<td>5.32E-7</td>
<td>9.52E-7</td>
</tr>
</tbody>
</table>

\[U_1 = [0.1 \ 0]^T, \quad U_2 = [0 \ 0.1]^T\]

with given per-unit steady state values of

<table>
<thead>
<tr>
<th>(I_d)</th>
<th>(I_q)</th>
<th>(I_{fd})</th>
<th>(I_{kd})</th>
<th>(I_{kq1})</th>
<th>(I_{kq2})</th>
<th>(\omega)</th>
<th>(\delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5689</td>
<td>0.4078</td>
<td>1.1161</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
<td>0.7743</td>
</tr>
</tbody>
</table>

and simulation step size of 1.0 ms.

Next, cross-correlations between outputs and unmeasurable states were calculated; the results for maximum correlations are given in Table 1. Based on these correlations, several models for the observer were simulated. The following model resulted in the best estimates of the damper winding currents:

\[
[A_{obs}] Y^*(K) = [i_{kd}(K) \ i_{kq1}(K) \ i_{kq2}(K)]^T
\]

\[
Y^*(K) = [i_d(K) \ i_{fd}(K) \ i_q(K) \ i_q(K-4) \ \delta(K-30) \ \omega(K-10) \ \omega(K-26)]^T
\]

The plots of damper winding currents and their estimates were practically the same. The estimation errors are plotted in Figure 9. Absolute mean and rms errors are shown in Table 2.

The machine was assumed to have been disturbed with the input \(U = [0.08 \ 0.05]^T\).
Figure 9: Damper currents estimation errors
The estimates of the matrix $A_{obs}$ for two different instants of time are found to be

$$A_{obs} = \begin{bmatrix}
1.11 & -0.98 & -0.01 & 0.01 & -0.44 & -16.04 & -8.18 \\
0.222 & -0.199 & -19.0 & 1.98 & -0.405 & -1.87 & -20.33 \\
0.231 & 0.204 & 2.24 & -1.88 & -0.06 & -13.46 & 11.03 \\
\end{bmatrix}$$

$$A_{obs} = \begin{bmatrix}
1.14 & -1.0 & -0.20 & 0.181 & -0.452 & -13.76 & -9.72 \\
0.773 & -0.68 & -4.74 & 5.22 & -0.511 & 41.25 & -49.61 \\
-0.775 & 0.663 & 5.58 & -4.95 & -0.041 & -54.34 & 38.81 \\
\end{bmatrix}$$

It can be seen that the matrix $A_{obs}$ is in fact time-varying.

Having $i_{kd}(\cdot)$, $i_{kq1}(\cdot)$, and $i_{kq2}(\cdot)$ estimated by the observer, we may now estimate the parameters of the machine using GLSE (Appendix B). We use the following initial conditions for the algorithm:

$$S(0) = \left[ \sum_{j=1}^{50} \Phi(j-1)\Phi^T(j-1) \right]^{-1}$$

$$C(0) = \sum_{j=1}^{50} [X(j)\Phi^T(j-1)]S(0)$$

This resulted in an accurate estimate, with maximum error of 2.5% of the machine parameters. The estimates of the machine parameters along with their nominal values and percentage errors are tabulated in Table 3.

### 3.6 Conclusions

In this chapter, we have presented an algorithm for the construction of an observer whose inputs are the operating data of the synchronous machine obtained during transients and its outputs are the estimates of the unmeasurable damper winding currents. It was shown that by using this observer one can obtain accurate estimates of the damper currents $i_{kd}$, $i_{kq1}$, and $i_{kq2}$, and using these estimates obtain accurate estimates for the machine parameters.

The parameter estimation algorithm presented proceeds recursively, allowing for the estimation of trajectories for the parameters, as the machine moves from one steady state condition to another. These trajectories will have embedded in them the effect of any other factor which may cause changes in parameters over time. Contributing factors, apart from saturation, are mostly due to aging. Therefore,
Table 3: Synchronous machine parameter estimation results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
<th>Estimated</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_d$</td>
<td>1.734</td>
<td>1.734</td>
<td>0.0%</td>
</tr>
<tr>
<td>$X'_d$</td>
<td>0.370</td>
<td>0.368</td>
<td>0.54%</td>
</tr>
<tr>
<td>$X''_d$</td>
<td>0.306</td>
<td>0.305</td>
<td>0.33%</td>
</tr>
<tr>
<td>$T''_{do}$</td>
<td>4.842</td>
<td>4.840</td>
<td>0.04%</td>
</tr>
<tr>
<td>$T''_{do}$</td>
<td>0.040</td>
<td>0.039</td>
<td>2.5%</td>
</tr>
<tr>
<td>$X_q$</td>
<td>1.717</td>
<td>1.717</td>
<td>0.0%</td>
</tr>
<tr>
<td>$X'_q$</td>
<td>0.554</td>
<td>0.555</td>
<td>0.18%</td>
</tr>
<tr>
<td>$X''_q$</td>
<td>0.306</td>
<td>0.306</td>
<td>0.0%</td>
</tr>
<tr>
<td>$T''_{qo}$</td>
<td>0.538</td>
<td>0.540</td>
<td>0.37%</td>
</tr>
<tr>
<td>$T''_{qo}$</td>
<td>0.062</td>
<td>0.061</td>
<td>1.61%</td>
</tr>
<tr>
<td>$X_{f}$</td>
<td>0.209</td>
<td>0.210</td>
<td>0.48%</td>
</tr>
<tr>
<td>$r_s$</td>
<td>0.00172</td>
<td>0.00172</td>
<td>0.0%</td>
</tr>
<tr>
<td>$r_{kd}$</td>
<td>10.125</td>
<td>10.160</td>
<td>0.35%</td>
</tr>
<tr>
<td>$r_{ka1}$</td>
<td>3.635</td>
<td>3.619</td>
<td>0.44%</td>
</tr>
<tr>
<td>$r_{ka2}$</td>
<td>7.741</td>
<td>7.836</td>
<td>1.23%</td>
</tr>
</tbody>
</table>

if the trajectory for a given parameter is compared with earlier trajectories for the same parameter, any deterioration which may cause changes in this parameter can be detected. For example, if slot discharges are developed in a machine, an increase in the leakage reactance $X_L$ can be expected. Therefore, when comparing trajectories, increase in $X_L$ could indicate slot discharges. Comparing trajectories of the machine parameters therefore allows for incipient faults (developing internal faults) to be detected at early stages.
CHAPTER IV

NONLINEAR MODELING OF MAGNETIC SATURATION AND HYSTERESIS IN AN ELECTROMAGNETIC DEVICE

4.1 Introduction

The cores of most electromagnetic devices, in their normal operating regions, enter saturation by design or otherwise. Some devices, such as ferroresonant transformers, utilize the saturating properties of their core materials to perform their intended tasks. In other devices, such as power transformers, saturation in an undesirable phenomenon imperative to their economically competitive designs.

A second phenomenon, known as hysteresis, associated with the operation of electromagnetic devices is the non-uniqueness of the core magnetic flux for a given excitation. That is, core flux is a multi-valued function depending on the excitation as well as its own previous states.

Computer simulation of a saturable electromagnetic device requires mathematical models for the device saturation and hysteresis characteristics. A general approach to modeling saturation and hysteresis is proposed in this chapter. A model which is nonlinear in parameter is proposed; however, a linearizing transformation is used to allow estimation of the model parameters by linear estimators.

In simulating the response of an electromagnetic device to an input signal, linear approximations to its nonlinear characteristics are often used. Although linear approximations may be adequate, nonlinear phenomena can best be accounted for by nonlinear models.

In the literature, the nonlinear saturation characteristics are most often modeled by piecewise linear models [57, 58, 59]. In a more recent paper [60], hysteresis is modeled by a controlled current source whose output is a polynomial of odd degree in input voltage. The saturation is modeled by another controlled current source whose output is an odd power of the ratio of exciting current to the currents at which saturation occurs. An algorithm for modeling transformer hysteresis loops, including the minor loops, is given in [61]. The major loop is again modeled by a set of piecewise linear segments. Because this algorithm is capable of modeling
the minor loops, it is suitable for the use in electromagnetic transients program.

In this chapter, we propose two nonlinear models to account for saturation, and a double-valued nonlinear function to model the hysteresis loops. One of these nonlinear models has the attractive property that it can be linearized by means of appropriate transformations. These nonlinear closed-form models allow analytic investigations on the performance of the modeled device be carried out. In addition, the existence of analytic models for an electromagnetic device makes the simulation task easier than when using models with tabular data.

4.2 Statement of the Problem

Consider a nonlinear inductance \( L' \) as shown.

\[
\begin{align*}
\nu &= \frac{d\lambda}{dt} = \frac{d}{dt}(L'i_E) = i_E \frac{dL'}{dt} + L' \frac{di_E}{dt} \\
\end{align*}
\]

For this inductance,

\[
\nu = \frac{\partial \lambda}{\partial i_E} \frac{di_E}{dt} = (L' + i_E \frac{\partial L'}{\partial i_E}) \frac{di_E}{dt}
\]

and using the chain rule of differentiation we can write

\[
\nu = \frac{\partial \lambda}{\partial i_E} \frac{di_E}{dt} = (L' + i_E \frac{\partial L'}{\partial i_E}) \frac{di_E}{dt}
\]

Define \( L = \frac{\partial \lambda}{\partial i_E} = L' + i_E \frac{\partial L'}{\partial i_E} \), then \( \nu = L \frac{di_E}{dt} \). The inductance \( L \) is nonlinear due to magnetic saturation and multi-valued due to hysteresis effect. Our problem can be stated as one of finding a closed form expression for \( L(i_E) \) which would represent both saturation and hysteresis. The inductance \( L \) is known as incremental inductance [62] and it is the slope of the curve which represents the relationships between the flux linkage \( \lambda \) and the exciting current \( i_E \). Thus, our modeling problem becomes that of finding an expression for \( \lambda \) as a function of \( i_E \). The flux linkage \( \lambda(i_E) \) can be found by integrating the inductor voltage.
\[ \lambda(i_E) = \int_{t_0}^{t} v dt + \lambda(t_0) \quad (4.1) \]

The plot of \( \lambda(i_E) \) over a full cycle is known as hysteresis loop. Once an estimator for \( \lambda(i_E) \) is determined, the closed form estimator of \( L(i_E) \) can be found by

\[ \hat{L}(i_E) = \frac{\partial}{\partial i_E} \lambda(i_E) \]

### 4.3 Proposed Solution to the Hysteresis Loop Modeling Problem

To find a suitable closed form estimator for the flux linkage \( \lambda \), we guess the form of the function \( \lambda(i_E) = f(i_E) \) and will then attempt to estimate the parameters of \( f \) numerically. Some of the candidate functions are

\[
\begin{align*}
    f(i_E) &= \frac{e^{a+iE}}{1+e^{a+iE}} \quad \text{(logistic function)} \\
    f(i_E) &= a i^{1/b} \quad (b = \text{an odd integer}) \\
    f(i_E) &= a \arctan(b i_E) + c i_E
\end{align*}
\]

Among these candidates, the logistic function [63] has the attractive property that it can be linearized by means of appropriate transformation; this makes the estimation of the parameters \( a \) and \( b \) by linear estimators, as discussed in the next section, possible.

#### 4.3.1 Use of Logistic Function in Modeling Magnetic Saturation

Consider the logistic function

\[ y(x) = \frac{e^{a+bx}}{1 + e^{a+bx}} \quad (4.2) \]

The plots of this function for \( a = -1, b = 0.4 \) as well as for \( a = -2.5, b = 0.4 \) are shown in Fig. 10. The extreme resemblance of this plot to hysteresis loops makes the logistic function a suitable candidate for modeling \( \lambda(i_E) \). However, note that in Eq. (4.2) with \( b > 0 \)

\[ \lim_{x \to \infty} y(x) = 1 \]
and

\[ \lim_{x \to -\infty} y(x) = 0. \]

Therefore, the logistic function is nonnegative, while \( \lambda(i_E) \) can have positive as well as negative values. Therefore, logistic functions cannot be directly used to model hysteresis loops and suitable transformations of them should be used. A suitable transformation is the one that makes the new \( \lambda(i_E) \) (say, \( \lambda'(i_E) \)) nonnegative, and at the same time satisfies the property given by Eq. (4.3)

\[ \ln\left(\frac{y}{1-y}\right) = a + bx \]

Note that adding a constant to the right side of Eq. (4.2), such that \( y \) can be negative as well as positive, would destroy the property in Eq. (4.3), and unless this property is preserved, the task of estimating \( a \) and \( b \) becomes a formidable one.

The proposed approach to modeling the hysteresis loop is based on finding the transformation \( T \)

\[ T : \lambda(i_E) \longrightarrow \lambda'(i_E) \]

such that \( \lambda'(i_E) \) can be modeled by

\[ \lambda'(i_E) = \frac{e^{a+bi_E}}{1 + e^{a+bi_E}} \]

(4.4)

Figure 10: Typical plots of logistic functions.
since minimum and maximum of $\lambda'(i_E)$ is found by substituting minimum and maximum of $i_E$ in Eq. (4.4), the range of $\lambda'(i_E)$, and thus the transformation $T$ depends on the unknown parameters $a$ and $b$. That is, we seek to transform $\lambda(i_E)$ with the domain $[i_{\min} = \min\{i_E\}, i_{\max} = \max\{i_E\}]$ and the range of $[\lambda_{\min}, \lambda_{\max}]$ to $\lambda'(i_E)$ with the same domain as $\lambda(i_E)$, but with the range of $[e_1, e_2]$ where $e_1 = \lambda'(i_{\min})$ and $e_2 = \lambda'(i_{\max})$. Since the sequence that determines the range of $\lambda'(i_E)$ (i.e., values for $e_1$ and $e_2$) depends on the $a$ and $b$ (i.e. parameters to be estimated), we need to estimate $a$, $b$, $e_1$, and $e_2$ simultaneously.

To estimate these parameters, an iterative approach is proposed. A summary of this approach for the $n$-winding transformer model of Fig. 11 is given.

![N-Winding Transformer Model](image)

**Figure 11: An $n$-winding transformer model.**

In steps 1 through 12 below, the actual data for $\lambda(i_E)$ is used to estimate the parameters $K_1$ and $K_2$ such that the cost function

$$c = \sum_{j=1}^{n} \| \lambda_j - K_1 \left[ \frac{e^{a+b i_E j}}{1 + e^{a+b i_E j}} \right] - K_2 \|^2$$

with

$$K_1 = \frac{\lambda_{\max} - \lambda_{\min}}{e_2 - e_1}$$

$$K_2 = \frac{e_2 \lambda_{\min} - e_1 \lambda_{\max}}{e_2 - e_1}$$

is minimized. Note that the parameters $a$ and $b$ are themselves functions of the unknowns $e_1$ and $e_2$. 63
Step 1: Assuming winding resistances and leakage inductances have already been estimated, calculate the flux linkage $\lambda$ from Eq. (4.1) for one full period of the fundamental frequency; use a value for $\lambda(t_0)$ such that the mean of $\lambda$ becomes zero.

Step 2: Calculate the corresponding exciting current from the measurements of winding currents, $i_E = \sum_{j=1}^{n} i_j$.

Step 3: Normalize $\lambda(t)$ to between 0 and 1

$$\lambda_N(t) = \frac{\lambda(t) - \lambda_{min}}{\lambda_{max} - \lambda_{min}}$$

where

$$\lambda_{min} = \lambda(i_{min})$$
$$\lambda_{max} = \lambda(i_{max})$$

Note that the range $[e_1, e_2]$ is a closed subset of (i.e., within) the range of $\lambda_N(t)$, where $\lambda_N(t)$ has the range of $[0,1]$.

Step 4: Guess the initial values $e_1^0$ and $e_2^0$ such that $0 < e_1^0 << e_2^0 < 1$ and define the transformation $T : \lambda_N \rightarrow \lambda'$ as

$$\lambda'(i_E) = (e_2^0 - e_1^0)\lambda_N(i_E) + e_1^0 \quad (4.5)$$

Note that $e_1^0 \leq \lambda' \leq e_2^0$ and the superscript denotes the iteration number.

Step 5: Make a second transformation $S : \lambda' \rightarrow \lambda''$ defined by

$$\lambda'' = \frac{\lambda'}{1 - \lambda'}$$

Note that for true $e_1$ and $e_2$, this is the same transformation as defined by Eq. (4.3). Now, use Eq. (4.5) to write

$$\lambda''_1 = \frac{(e_2^0 - e_1^0)\lambda_N + e_1^0}{1 - [(e_2^0 - e_1^0)\lambda_N + e_1^0]} \quad (4.6)$$

Note that for a given value of $i_E$, $\lambda''$ is a function of $e_1$ and $e_2$. 64
Step 6: Using Taylor series, expand $\lambda''(e_1, e_2)$, given by Eq. (4.6) around $e_1^0$ and $e_2^0$ for each value of $i_E$

$$\lambda''(e_1, e_2) = \sum_{k=0}^{\infty} \frac{1}{k!} \left( \frac{\partial^k \lambda''(e_1^0, e_2^0)}{\partial e_1} (e_1 - e_1^0)^k + \frac{\partial^k \lambda''(e_1^0, e_2^0)}{\partial e_2} (e_2 - e_2^0)^k \right)$$

$$\approx \frac{\partial^k \lambda''(e_1^0, e_2^0)}{\partial e_1} \Delta e_1 + \frac{\partial^k \lambda''(e_1^0, e_2^0)}{\partial e_2} \Delta e_2 + \lambda''(e_1^0, e_2^0)$$

(4.7)

where

$$\frac{\partial \lambda''(e_1^0, e_2^0)}{\partial e_1} = \frac{(1-\lambda_N)}{[1-(e_2^0-e_1^0)\lambda_N-e_1^0]^2}$$

$$\frac{\partial \lambda''(e_1^0, e_2^0)}{\partial e_2} = \frac{\lambda_N}{[1-(e_2^0-e_1^0)\lambda_N-e_1^0]^2}$$

$$\Delta e_1 = e_1 - e_1^0, \quad \Delta e_2 = e_2 - e_2^0$$

and $\lambda''(e_1^0, e_2^0)$ is given by Eq. (4.6). Note that the truncation of the Taylor series is justified because we are only interested in matching the flux linkage and the incremental inductance with their estimates. Now calculate $\lambda''(e_1^0, e_2^0)$, $\frac{\partial \lambda''}{\partial e_1}(e_1^0, e_2^0)$ and $\frac{\partial \lambda''}{\partial e_2}(e_1^0, e_2^0)$ as given above for all $i_E$.

Step 7: For true $e_1$ and $e_2$, $\lambda''$ will be denoted by $\lambda''_2$ and is given by

$$\lambda''_2(e_1, e_2) = e^{a+biE}$$

(4.8)

where

$$a = \frac{i_{\text{max}}-i_{\text{min}}}{i_{\text{max}}-i_{\text{min}}} \ln \frac{e_1}{1-e_1} - \frac{i_{\text{min}}}{i_{\text{max}}-i_{\text{min}}} \ln \frac{e_2}{1-e_2}$$

$$b = \frac{i_{\text{max}}-i_{\text{min}}}{i_{\text{max}}-i_{\text{min}}} \ln \frac{e_1}{1-e_1} + \frac{1}{i_{\text{max}}-i_{\text{min}}} \ln \frac{e_2}{1-e_2}$$

$$i_{\text{max}} = \max\{i_E\}, \quad i_{\text{min}} = \min\{i_E\}$$

We can expand $\lambda''_2$, as given by Eq. (4.8), around $e_1^0$ and $e_2^0$ using a Taylor series

$$\lambda''_2(e_1^0, e_2^0) \approx \left[ \frac{p_1 + p_3iE}{e_1^0(1-e_1^0)} \Delta e_1 + \frac{p_2 + p_4iE}{e_1^0(1-e_1^0)} \Delta e_2 \right] \lambda''_2(e_1^0, e_2^0)$$

$$+ \lambda''_2(e_1^0, e_2^0)$$

(4.9)

where

$$\lambda''_2(e_1^0, e_2^0) = e^{p_1q_1 + p_2q_2 + (p_3q_1 + p_4q_2)iE}$$

(4.10)
\[ p_1 = \frac{i_{\text{max}} - i_{\text{min}}}{i_{\text{max}} - i_{\text{min}}} \], \quad p_2 = \frac{-i_{\text{min}}}{i_{\text{max}} - i_{\text{min}}} \\
\[ p_3 = \frac{i_{\text{max}} - i_{\text{min}}}{i_{\text{max}} - i_{\text{min}}} \], \quad p_4 = -p_3 \\
\[ q_1 = \ln\left(\frac{e^0_1}{1 - e^0_1}\right), \quad q_2 = \ln\left(\frac{e^0_2}{1 - e^0_2}\right) \]

By equating Eqs. (4.6) and (4.8), we get

\[ \Delta \lambda''(e^0_1, e^0_2) = J_1 \Delta e_1 + J_2 \Delta e_2 \quad (4.11) \]

where \( \Delta \lambda''(e^0_1, e^0_2) = \lambda''(e^0_1, e^0_2) - \lambda''(e^0_1, e^0_2) \) with \( \lambda'' \) given by Eq. (4.6) and \( \lambda'' \) given by Eq. (4.10).

\[ J_1 = \frac{(1 - \lambda N)(1 - (e^0_1 - e^0_2)\lambda N - e^0_1)^2}{[1 - (e^0_1 - e^0_2)\lambda N - e^0_1]^2} - \frac{p_1 + p_3 i_{E}}{e^0_1 (1 - e^0_1)} \lambda''(e^0_1, e^0_2) \]

\[ J_2 = \frac{\lambda N}{[1 - (e^0_1 - e^0_2)\lambda N - e^0_1]^2} - \frac{p_2 + p_4 i_{E}}{e^0_2 (1 - e^0_2)} \lambda''(e^0_1, e^0_2) \]

Now calculate \( \Delta \lambda''(e^0_1, e^0_2), J_1, \) and \( J_2 \) as given above for all values of \( i_{E} \). At each iteration, our objective is to update the values of \( e_1 \) and \( e_2 \) such that the difference between the \( \lambda'' \) (constructed from measurements) and \( \lambda'' \) (constructed based on assumed hysteresis model) is minimized. This can be accomplished using an estimator to estimate \( \Delta e_1 \) and \( \Delta e_2 \) in Eq. (4.11)

Step 8: Using a linear estimator such as Generalized Least-Squares Estimator [15] and calculation results of Step 7, obtain the estimates of \( \Delta e_1 \) and \( \Delta e_2 \) in Eq. (4.11), then update \( e^1_1 \) and \( e^1_2 \) by

\[ e^1_1 = e^0_1 + \Delta e_1 \]
\[ e^1_2 = e^0_2 + \Delta e_2 \]

Step 9: Check the convergence criterion

\[ \sum_{i_{E}} |\Delta \lambda''(e^1_1, e^1_2)| < \epsilon \quad \text{a small positive number.} \]

If this inequality is satisfied, continue with Step 10, otherwise go back to Step 4 using the updated \( e_1 \) and \( e_2 \).

Step 10: Calculate the parameter estimates from
\[ \hat{a} = p_1 \ln\left(\frac{e_1^k}{1-e_1} \right) + p_2 \ln\left(\frac{e_2^k}{1-e_2} \right) \]
\[ \hat{b} = p_3 \ln\left(\frac{e_1^k}{1-e_1} \right) + p_4 \ln\left(\frac{e_2^k}{1-e_2} \right) \]

where it has been assumed that \( e_1 \) and \( e_2 \) have converged to their true values after \( k \) iterations.

**Step 11:** Obtain the expression to be used in modeling the hysteresis loop as follows

\[ \hat{\lambda}'(i_E) = \frac{e^{\hat{a}+bi_E}}{1+e^{\hat{a}+bi_E}} \]
\[ \hat{\lambda}_N(i_E) = \frac{\hat{\lambda}'(i_E) - e_1^k}{e_2^k - e_1^k} \]
\[ \hat{\lambda}(i_E) = (\lambda_{\text{max}} - \lambda_{\text{min}})\hat{\lambda}_N(i_E) + \lambda_{\text{min}} \]

where \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) are defined as before.

**Step 12:** Calculate the mutual inductance \( \hat{L}(i_E) = \frac{\partial\hat{\lambda}(i_E)}{\partial i_E} \),

\[ \hat{L}(i_E) = \frac{(\lambda_{\text{max}} - \lambda_{\text{min}})\hat{b}e^{\hat{a}+bi_E}}{[(e_2^k - e_1^k)(1 + e^{\hat{a}+bi_E})^2]} \] (4.12)

Note that the maximum of \( \hat{L} \) is given by

\[ \hat{L}_{\text{max}} = \frac{d^2(2-d)}{4(e_2^k - e_1^k)} ; \quad d = (\lambda_{\text{max}} - \lambda_{\text{min}})\hat{b} \quad 0 < d < 2 \]

and it occurs at

\[ i_E = \frac{1}{\hat{b}}[\ln\left(\frac{d}{2-d}\right) - \hat{a}] \]

The above estimation algorithm should be carried out for increasing values and decreasing values of \( i_E \) separately.

To get a better fit, a modified version of the proposed algorithm can be used to obtain piecewise models of the saturation, and thus achieve the higher degrees of freedom needed. The required modifications are summarized below:

1. Divide the hysteresis loop for decreasing (or increasing) values of \( i_E \) into \( n \) regions \( R_1, R_2, \ldots, R_n \) as desired. Let \( \lambda_1, \lambda_2, \ldots, \lambda_n \) and \( i_1, i_2, \ldots, i_n \) be the corresponding flux linkages and exciting currents. Include the boundary points in both neighboring regions.
2. Carry out the Steps (1) through (3) as before.

3. Continue with the rest of the algorithm with \( \lambda \) and \( i_E \) replaced by \( \lambda_1 \) and \( i_1 \) and note that \( \min\{\lambda_{1N}\} \neq 0 \) and thus Eq. (4.5) should be modified as follows:

\[
\lambda_1'(i_1) = \frac{(e_i^0 - e_j^0)(\lambda_{1N} - \lambda_{1N,\min})}{(\lambda_{1N,\max} - \lambda_{1N,\min})} + e_j^0
\]

\( \lambda'', \frac{\partial \lambda''}{\partial e_1}, \frac{\partial \lambda''}{\partial e_2}, J_1, \) and \( J_2 \) should also change accordingly.

4. For all other regions an additional condition; namely, the continuity at the corner where two regions meet, must be satisfied. This condition can be satisfied by setting \( \Delta e \)'s corresponding to the corner points between regions \( R_k \) and \( R_{k+1} \) equal zero when estimating the parameters of the region \( R_{k+1} \). Thus, for regions \( R_k \) \((k = 2, 3, \ldots, n)\) the estimation algorithm is similar to that for \( R_1 \) with the exception that \( \Delta e_2 \) is set to zero for decreasing \( i_E \) and \( \Delta e_1 \) is set to zero for increasing \( i_E \).

5. It is important to note that the hysteresis loop at a corner point is smooth if and only if \( \lambda' \)'s in the neighboring regions coming together at that corner have the same parameters, indicating an unnecessary division of the loop at that corner. As a consequence, the mutual inductance \( \hat{L}(i_E) \) is not defined at the corner points. However, the value of \( \hat{L} \) at a corner point is in the range determined by its values around the corner points in the two neighboring regions.

### 4.3.2 Use of Arctan Function in Modeling of Magnetic Saturation

Fig. 12 shows the plot of \( y = a \arctan(bx) \) for \( a = 0.06 \) and \( b = 0.25 \). The similarity of this plot to the typical transformer saturation curves makes the arctan function a potential candidate for modeling magnetic saturation. Since most saturation curves have a definite slope in their deeply saturated regions, a function of the form

\[
y = a \arctan(bx) + cx \quad (4.13)
\]

can model saturation accurately.

The difficulty in using Eq. (4.13) is that no algorithm for the estimation of the parameters \( a, b \) and \( c \) exists. However, a study of Eq. (4.13) reveals the way different characteristics of the function \( y \) can be altered by changing these parameters.
Figure 12: The plot of $y = 0.06 \arctan(0.25x)$.

Figure 13: The plot of $y = 0.06 \arctan(0.25x) + 1.8 \times 10^{-5}$.
Figure 14: The plot of $y = 0.04 \arctan(0.25x) + 1.8 \times 10^{-5}x$.

Figure 15: The plot of $y = 0.04 \arctan(0.1x) + 1.8 \times 10^{-5}x$. 
By plotting Eq. (4.13) for \( a = 0.06, b = 0.25 \) and \( c = 1.8 \times 10^{-5} \) in Fig. 13 and comparing it with the plot of Fig. 12, it becomes clear that the parameter \( c \) controls the slope of \( y(x) \) in its flat regions. Now by plotting \( y(x) \) for \( a = 0.04, b = 0.25 \) and \( c = 1.8 \times 10^{-5} \) in Fig. 14 and comparing it with the plot of Fig. 13, we note that the parameter \( a \) controls the \( y \) value at which the flat regions start. Finally, if we plot \( y(x) \) for \( a = 0.04 \) and \( b = 0.1 \) in Fig. 15 and compare it with Fig. 14, we note that the parameter \( b \) controls the slope of \( y \) in the steep region.

With the above discussions, the use of arctan function for modeling magnetic saturation becomes more attractive. One can intelligently change the parameters \( a, b \) and \( c \) to obtain a saturation model which closely matches the measured saturation curve. The parameters \( a, b \) and \( c \) can be estimated independently. The parameter \( a \) will be adjusted until the knee of the saturation model and the knee of the actual saturation curve both correspond to the same flux level. The parameter \( b \) will be adjusted until the slope of the saturation model in the linear region (below the knee) matches the actual one. Similarly parameter \( c \) will be adjusted until the slope of the saturation model in the saturated region (above the knee) matches the actual one. Finally, fine adjustments of \( a, b \) and \( c \) may be required to get a better match.

### 4.3.3 Modeling of the Hysteresis Effect

The saturation modeling techniques discussed in Sections 4.3.1 and 4.3.2 assume a one-to-one relation between the flux linkage \( \lambda \) and the exciting current \( i_E \). In most electromagnetic devices, however, this relation is not bijective. That is, for a given value of exciting current, the core magnetic flux can take on one of infinitely many values, depending on the value of the exciting current and its own past states. This phenomenon is referred to as the hysteresis effect. The plot of the core magnetic flux versus exciting current over a cycle of the fundamental frequency is known as hysteresis loop.

A hysteresis loop which deeply saturates at both ends (positive and negative saturation) is referred to as the major hysteresis loop. The core magnetic flux can take on values which fall on or inside the major loop. For a model to accurately represent the hysteresis effect, it should allow the core flux to take on all the values limited by the major loop in a calculated manner.

Much simulation time can be saved if we limit the values of the core flux to those on the boundaries of the major loop. The core flux now becomes a double-valued function of the exciting current, depending on whether the flux is increasing or decreasing.

To use logistic function discussed in Section 4.3.1 to model hysteresis effect as
discussed above, we can use the algorithm presented in that section to determine a saturation model for increasing values and another model for decreasing values of the core magnetic flux. The arctan function discussed in Section 4.3.2 can also be used to model hysteresis. In this case, functions of the form

\[ y(x) = a \arctan(bx + d) + cx \]

should be used to fit separate models for increasing and decreasing values of the core flux.

4.4 Conclusions

Algorithms for modeling saturation and hysteresis of an electromagnetic device have been presented. Nonlinear models were used to model saturation. Hysteresis effect was accounted for by modeling saturation curves for increasing and decreasing values of exciting current separately. By using a linearizing transformation, the parameters of the model were estimated by a least-squares estimator. Piecewise modeling of the hysteresis loop was possible using a modified version of the proposed algorithm. The piecewise models were made to be continuous, but not necessarily smooth, at the corners. The incremental inductance, defined as the slope of the hysteresis loop, is not unique at a corner point but has an average value of its two linear segments.
CHAPTER V

PRACTICAL CONSIDERATIONS ON THE ESTIMATION OF PARAMETERS OF PHYSICAL MODELS FROM TIME-DOMAIN DATA

5.1 Introduction

Physical modeling procedures developed for electromagnetic devices in this dissertation require the availability of a data acquisition system. From an application viewpoint, an important consideration arises for implementation with parallel versus serial data acquisition procedures. Factors such as cost, model accuracy, and parameter value convergence dictate the necessity of careful investigations of such issues. This chapter addresses the problem of identification of physical parameters in noisy processes for both discrete and continuous-time models. Simulation results will be given for both cases of parallel and serial data acquisition systems.

Physical modeling of a process is of necessity an internal modeling problem. It is based on selecting a set of physical variables (voltages, currents, positions, velocities, etc.) and mathematically describing the dynamic \( V = \frac{dx}{dt}, V_L = L \frac{di_L}{dt}, \) etc.) and static (Ohm’s law, KVL, KCL, etc.) relations among these variables. This description is generally in the form of a set of differential (difference) equations relating the collection of the physical variables in continuous-time (discrete-time). This step in modeling is referred to as system identification which determines the order of the differential (difference) equations as well as the unknown parameters for the physical process.

Assuming that the order of a given dynamical system has been determined by physical laws governing the system, the next issue is the estimation of the system parameters. These parameters are the coefficients of the dynamical equations.

Here, we compare the parameter estimation results from two different estimation procedures: 1) direct estimation of the parameters of the continuous time model from sampled data; and 2) estimation of the parameters of discrete time model from sampled data and subsequent derivation of the estimates of physical parameters by use of appropriate transformation. The estimation errors resulting from phase shifts between signals introduced by nonsimultaneous sampling are
compared for the two procedures. The estimation results given here are those for the physical parameters (resistances and inductances) of an electrical device (system). This system is a component (subsystem) of an overall larger system. The sample data is collected by a data acquisition system looking into the terminals of the device while operating as a component of the integral system.

5.2 Statement of the Problem

A. Discrete Case

Consider a physical system as shown in Fig. 16.

\[
\begin{align*}
  x(k + 1) &= Ax(k) + Bu(k) \\
  \text{where } u(\cdot) &\text{ is the input and } x(\cdot) \text{ the output, both vector quantities.}
\end{align*}
\]

B. Continuous Case

Under the assumption that system order is known, the continuous time representation is

\[
\frac{dx}{dt} = A^* x(t) + B^* u(t).
\]
Again, the vectors $x(t)$ and $u(t)$ represent the input and output of this system. Assuming that $x(t)$ and $u(t)$ are measurable, the estimation problem involves computation of $A^*$ and $B^*$ from a record of $x(\cdot)$ and $u(\cdot)$. For this case, computation or estimation of the state derivative is necessary.

C. Parameter Estimation

System identification and parameter estimation of physical processes have attracted a great deal of attention in recent years. In this chapter, a generalized least squares estimator is used to estimate the parameters of the system shown in Fig. 17. A recursive version of this algorithm for the multivariable case is given in Appendix B. The derivation of the physical parameters from discrete system parameters is given in Section 3.4.1.

5.3 Study Results

A. Parameter Estimation Errors Caused by Non-Simultaneous Sampling

Consider the system of Fig. 17.

![Figure 17: A two-winding transformer model.](image)

We estimate parameters $R_1, R_2, R_n, L_1, L_2,$ and $L_n$ by sampling the terminal voltages and currents. Four signals (two terminal voltages and two terminal currents) are to be sampled. One can sample these four signals serially, using a serial data acquisition system, or simultaneously using a more expensive parallel data acquisition system. The effect of serial sampling on the accuracy of the parameter estimates is studied here.
To obtain simulation signals, first we assume the secondary terminals are connected to an inductive load and we simulate the response of this device to a given \( v_1 \) with assumed parameters. Then we use simulated \( i_2 \) and the load information to calculate \( v_2 \). The signals \( v_1, v_2 \) and \( i_2 \) are thus generated for \( R_1 = 0.0798 \Omega, R_2 = 0.0798 \Omega, L_1 = 0.325 \text{mH}, L_2 = 0.325 \text{mH}, R_n = 0.55 \Omega, L_n = 16.9 \text{mH} \). The GLS algorithm is employed to estimate the parameters. This is carried out first assuming parallel data acquisition, and then for several experiments assuming serial data acquisition with different skewing time. That is \( v_1(k), v_2(k + \text{lag}), i_1(k + 2\text{lag}), i_2(k + 3\text{lag}) \), for \( k = 1, 2, 3 \ldots, 2000 - 3\text{lag} \) and \( \text{lag} = 0, 1, 2, 3, 4 \) are used in the estimation. Estimation results are tabulated in Table 4, where the sampling frequency is 20 kHz. Note that the frequency of the sampled signal is 60Hz, so that 1 \( \text{lag} = 1/20,000 \text{ second} \) which is 0.3% of the 60Hz period. Included in Table 4 are this percentage lag, true parameters, estimates of the parameters and norm of the error vector defined in the parameter space by

\[
\| E \| = \sqrt{\frac{(R_1 - R_1)^2 + (R_2 - R_2)^2 + (L_1 - L_1)^2 + (L_2 - L_2)^2 + (R_n - R_n)^2 + (L_n - L_n)^2}{(R_1^2 + R_2^2 + L_1^2 + L_2^2 + R_n^2 + L_n^2)}}
\]

Using these estimates, the response of the system in Fig. 17 was simulated for the signals \( v_1 \) and \( v_2 \). The absolute mean and rms errors defined by

\[
E_{abs} = \frac{1}{2000} \sum_{k=1}^{2000} |i_j(k) - \hat{i}_j(k)|
\]

\[
E_{rms} = \sqrt{\frac{1}{2000} \sum_{k=1}^{2000} (i_j(k) - \hat{i}_j(k))^2} \quad \text{for} \quad j = 1, 2
\]

are given in Table 5 for the model simulations in the five separate cases.

B. Estimation of Parameters from Noisy Data

Consider first the case of indirect estimation from the discrete-time model parameters. Random noise from a normal distribution with zero mean and unit variance is added to the signals \( v_1, v_2, i_1, \) and \( i_2 \) of part A. That is \( v_1(k) = v_1(k) + \epsilon_k, v_2(k) = v_2(k) + \epsilon_k, i_1(k) = i_1(k) + \epsilon_k, i_2(k) = i_2(k) + \epsilon_k \), for \( k = 1, 2, \ldots, 2000 \) with \( \epsilon_k \sim N(0,1) \). These signals are then used in the estimation of the parameters in the discrete-time model \( A^* \) and \( B^* \) from

\[
\begin{bmatrix}
i_1(k + 1) \\
i_2(k + 1)
\end{bmatrix} = \begin{bmatrix}
A^* \\
B^*
\end{bmatrix} \begin{bmatrix}
i_1(k) \\
i_2(k)
\end{bmatrix} + \begin{bmatrix}
v_1(k) \\
v_2(k)
\end{bmatrix}
\]
Table 4: GLSE Algorithm Results

| Case | $\text{lag} = \%$ | $R_1 = R_2$ | $L_1 = L_2$ | $R_n$ | $L_n$ | Norm of Error Vector $|E_n|$ |
|------|-----------------|-------------|-------------|-------|-------|----------------|
| True Values | | 0.0798 | 0.325 | 0.55 | 16.9 |
| Case I (parallel acquisition) | 0% | 0.0798 | 0.325 | 0.55 | 16.9 | 0 |
| Case II | .3% | 0.0883 | 0.307 | 0.50 | 16.2 | 0.0916 |
| Case III | .6% | 0.09847 | 0.263 | 0.43 | 15.1 | 0.2184 |
| Case IV | .9% | 0.10916 | 0.225 | 0.38 | 14.2 | 0.3116 |
| Case V | 1.2% | 0.12451 | 0.1722 | 0.35 | 13.3 | 0.3735 |

Table 5: Simulation Results

<table>
<thead>
<tr>
<th>Case</th>
<th>$\text{lag} = %$</th>
<th>$E_{\text{abs}}$</th>
<th>$E_{\text{rms}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t_1$</td>
<td>$t_2$</td>
<td>$t_1$</td>
</tr>
<tr>
<td>Case I (parallel acquisition)</td>
<td>0%</td>
<td>0.0190</td>
<td>0.0189</td>
</tr>
<tr>
<td>Case II</td>
<td>.3%</td>
<td>3.1575</td>
<td>4.4558</td>
</tr>
<tr>
<td>Case III</td>
<td>.6%</td>
<td>7.2307</td>
<td>9.797</td>
</tr>
<tr>
<td>Case IV</td>
<td>.9%</td>
<td>11.224</td>
<td>15.042</td>
</tr>
<tr>
<td>Case V</td>
<td>1.2%</td>
<td>15.804</td>
<td>20.857</td>
</tr>
</tbody>
</table>

77
using GLSE. The parameters of the continuous-time model are derived from $A^*$ and $B^*$ using appropriate transformations as discussed in Section 3.4.1. The estimation results are tabulated in Table 6, accompanied by simulation results, in terms of absolute and rms errors, for the case of parallel data acquisition.

Table 6: Noisy Date Case

<table>
<thead>
<tr>
<th>CASE</th>
<th>$R_1 = R_2$</th>
<th>$L_1 = L_2$</th>
<th>$R_n$</th>
<th>$L_n$</th>
<th>$E_{abs}$</th>
<th>$E_{rms}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrete (Noise-free)</td>
<td>0.0798</td>
<td>0.325</td>
<td>0.55</td>
<td>16.9</td>
<td>0.0190</td>
<td>0.0199</td>
</tr>
<tr>
<td>Continuous (Noise-free)</td>
<td>0.0798</td>
<td>0.325</td>
<td>0.55</td>
<td>16.9</td>
<td>0.0249</td>
<td>0.0247</td>
</tr>
<tr>
<td>Discrete (Noisy)</td>
<td>0.0815</td>
<td>0.323</td>
<td>0.86</td>
<td>16.6</td>
<td>2.04</td>
<td>2.13</td>
</tr>
<tr>
<td>Continuous (Noisy)</td>
<td>0.0801</td>
<td>0.307</td>
<td>0.58</td>
<td>3.35</td>
<td>47.6</td>
<td>47.6</td>
</tr>
</tbody>
</table>

Next consider the direct estimation of continuous-time model parameters. The decoupled equations resulting from analysis of the physical system are: ($r = R_1 = R_2, L = L_1 = L_2$)

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{-r + 2R_n}{L + 2L_n}x_1 + \frac{1}{L + 2L_n}u_1 \\
\frac{dx_2}{dt} &= \frac{-r}{L}x_2 + \frac{1}{L}u_2
\end{align*}
\] (5.1) (5.2)

where

\[
\begin{align*}
x_1 &= i_1 + i_2, \quad u_1 = v_1 + v_2 \\
x_2 &= i_1 - i_2, \quad u_2 = v_1 - v_2
\end{align*}
\]

GLSE is used to estimate the parameters of the model according to

\[ Y_k^j = \beta_1^j x_j + \beta_2^j u_j + \epsilon_k; \quad j = 1, 2 \]

now for the noisy case, where

\[ Y_k^j = \frac{dx_j}{dt} \bigg|_{t=k\Delta T} \]
\[ \Delta T = \frac{1}{20,000} \]

\[ \beta_1^1 = -\frac{r + 2R_n}{L + 2L_n}, \quad \beta_1^2 = -\frac{r}{L} \]

\[ \beta_2^1 = \frac{1}{L + 2L_n}, \quad \beta_2^2 = \frac{l}{L} \]

The estimation results for noisy data, as well as for noise-free data \((\epsilon_k = 0\) for all \(k)\), are tabulated in Table 6, again with simulation results in terms of \(i_1\) and \(i_2\).

The large relative errors in the continuous time case result from the choice of \(T\) in the approximation of the term \(dx/dt\).

Although the estimation error resulting from serial sampling is intuitively expected, its behavior, as a function of the skewing time, is not clear. An effective way of reducing this error is to approximate the skewing times involved, and use a software acquisition system to resample the collected signals in opposite order.

C. Estimation of the Physical Parameters of a Saturating Transformer

The physical system considered in our studies is part of a power conditioning device used at computer installations for the smoothing and regulation of voltage signals. This component is a two-winding saturating transformer which produces one of the pulses used in synthesizing the output voltage. The assumed structure for the saturating transformer model is similar to that shown in Fig. 17 with the primary difference that the magnetizing inductance is now a nonlinear function of the exciting current.

For this device, it is reasonable to assume \(R_1 = R_2\) and \(L_1 = L_2\), and thus simplify the estimation process. This assumption is justified by noting that

\[ L_1 \approx \frac{N_2^2}{\mathcal{R}_1} \quad \text{and} \quad L_2 \approx \frac{N_2^2}{\mathcal{R}_2} \]

where

\[ N_1 = \text{number of primary winding turns} \]
\[ N_2 = \text{number of secondary winding turns} \]
\[ \mathcal{R}_1 = \text{reluctance of the primary leakage flux magnetic path} \]
\[ \mathcal{R}_2 = \text{reluctance of the secondary leakage flux magnetic path} \]

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Inductance $L_2$ referred to the primary side is given by

$$L_2' = \frac{N_1^2}{N_2^2} \frac{N_2^2}{R_2} = \frac{N_1^2}{R_2}$$

It is quite reasonable to assume $R_1 = R_2$ which would imply $L_1 = L_2'$. For the saturating transformer $N_1 = N_2$, implying $L_1 = L_2$. For dc values of $R_1$ and $R_2$ we have:

$$R_1 = \rho_1 \frac{\ell_1}{S_1} \text{ and } R_2 = \rho_2 \frac{\ell_2}{S_2}$$

where

- $\ell_1$ = length of the primary winding
- $S_1$ = cross-sectional area of the primary winding
- $\rho_1$ = primary conductor resistivity
- $\ell_2$ = length of the secondary winding
- $S_2$ = cross-sectional area of the secondary winding
- $\rho_2$ = secondary conductor resistivity

With similar argument, we can conclude that for $N_1 = N_2$, $R_1$ equals to $R_2$. Since $R_1$ and $R_2$ are subjected to the same ac signals (skin effect) and are in nearly the same physical location (proximity effect), the equality of their dc values implies the equality of their ac values.

By replacing the magnetizing inductance $L_n$ with the incremental inductance $L_n(t; i_1 + i_2)$, the decoupled differential equations given by Eqs. (5.1) and (5.2) can be used to describe the dynamics of the saturating transformer.

The GLSE algorithm is used to estimate the parameters of the decoupled model. First, a parallel data acquisition system sampling at a rate of 40 Khz was used to collect the necessary data. Figure 18 shows the two terminal voltages and the two winding currents collected by the parallel acquisition system looking into the terminals of the saturating transformer. Figure 19 shows the states and one of the inputs of the decoupled model to be used in the parameter estimation process.

Figure 20 shows the estimates of the parameters $R, L$, and $L_n$, respectively. It can be seen that the resistance $\hat{R}$ and the inductance $\hat{L}$ have converged to $\hat{R} \approx 0.011 \Omega$ and $\hat{L} \approx 2.38 \mu H$, and that the hysteresis effect on $\hat{L}_n$ is small. The
Figure 18: Signals measured at terminals of saturating transformer.
Figure 19: States and the input of the decoupled model.
resistance \( R_n \) was found to be practically zero.

To check the adequacy of the identified model, its response to the same inputs (terminal voltages of Fig. 18) was simulated. The simulation results along with the actual response for the winding currents are shown in Fig. 21. The absolute mean and rms errors are tabulated in Table 7.

Table 7: Simulation Results (parallel acquisition)

<table>
<thead>
<tr>
<th></th>
<th>( I_1 )</th>
<th>( I_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{abs} )</td>
<td>9.890A</td>
<td>7.02A</td>
</tr>
<tr>
<td>( E_{rms} )</td>
<td>11.06A</td>
<td>8.41A</td>
</tr>
</tbody>
</table>

An attempt was made to estimate the parameters using the data collected by a serial data acquisition system. Figure 22 shows the decoupled model input \( v_1 - v_2 \) calculated from the data of the serial acquisition system. Comparing this figure with that for the parallel acquisition system, shown in Fig. 19, it should not be surprising that the physical parameters of the saturating transformer were not identifiable in this case. In fact, the estimates of the parameters \( R \) and \( L \) did not converge, and \( \hat{R} \) assumed negative values, Figures 23a and 23b.

5.4 Description of the Data Acquisition System Used

Based on the discussions of the previous sections, we can now design a suitable data acquisition system. In this section, the parallel data acquisition system, designed and used in the development of our models, is described.

5.4.1 Data Acquisition System Hardware

Figure 24 shows the block diagram of our data acquisition system hardware. Circuit diagrams for the potential divider and the differential amplifier are shown in Fig. 25. The differential voltage input \( e_1(t) - e_2(t) \) is the instantaneous voltage across a terminal of the device. Individual voltages \( e_1 \) and \( e_2 \) are stepped down by the potential dividers \( R_1 \) and \( R_2 \). This is done to reduce the common mode voltage at the inputs of the amplifier. The voltage \( e_1(t) - e_2(t) \) is normally attenuated by the amplifier. The Zener diodes across \( R_2 \) protect the amplifier against overvoltages by limiting the common mode input voltage to 12V.
Figure 20: Estimation of the Physical Parameters
Figure 21: Comparison of the actual and simulated winding currents.

Figure 22: The decoupled input calculated from the serial data.
Figure 23: Estimates of the parameters obtained by processing serial data.
The output of the amplifier is connected to A/D converter via an analog multiplexer. The Zener diodes at the output are for the protection of multiplexer, A.D, I/O interface, etc. against transient overvoltages. The 5K resistor at the output is to limit the output current in the case of output over-voltages when the Zener diodes turn on.

The 100K potentiometer connected across terminals 2-3 is for the adjustment of the common mode off-set. It is adjusted for minimum output voltage when differential input is zero.

Figure 26 shows the components of the isolation amplifier used for current measurements. A current viewing resistor is placed in the return path of each transformer winding. The voltages developed across these resistors are the inputs of the isolation amplifiers at points A and B of Fig. 26. The combination of resistors and switches provide a range of amplification gain from 1 up to 100.

The built-in isolation transformer provides complete isolations among the input, output, and dc parts of the amplifier. This amplifier provides excellent common mode rejection to the input signals and protection for other devices connected to the outputs of the isolation amplifier at points C and D and beyond. Specifically, A/D is protected against high transient currents, particularly the inrush current.

5.4.2 Data Acquisition System Software

The acquisition of data is under program control. A program reads the binary data from the I/O port for up to 5 channels. The collected data is first stored in the main memory. After the collection of data is complete, the program stores the data on a floppy disk.

In addition to the use of shielded cables, the high frequency noise is filtered by software means. Power spectral density of each recorded sequence of data is computed and plotted. Based on these densities, the cut-off frequency of a digital filter is determined. The digital filter is then used to clean the data. The mathematics of the digital filter is given in Appendix C [64].

The data acquisition system described above has a maximum sampling frequency of about 100 kHz.

Figure 27 shows a 60 Hz sinusoidal signal which has been contaminated by high frequency noise. Figure 28 shows the same signal after passing through the digital filter.
Figure 24: Block diagram of the data acquisition system hardware.

Figure 25: Voltage divider and differential amplifier used for voltage measurements.

Figure 26: Isolation amplifier used for current measurements.
5.5 Conclusions

Effects of parallel and serial data acquisition procedures on the parameter estimation process in physical systems have been investigated. In a generalized least squares setting, discrete-time and continuous-time models were addressed. A comparative analysis has been presented illustrating the relative cause and effect relationships of lag introduced in the serial processing procedures: model simulation results complete the analysis. The techniques used in the simulation analyses were then applied to a real physical process, a saturating transformer, and results were compared to actual observations. With the use of serial data acquisition systems, the necessity of accounting for skewing by software means was clearly dictated by this application.

Based on the results of our studies, a suitable data acquisition system was designed and constructed. The hardware and software of this system were discussed in detail.
Figure 27: Noisy signal

Figure 28: The result of filtering the signal of Fig. 27 using the digital filter.
CHAPTER VI
MODELING OF POWER TRANSFORMERS FROM
TIME-DOMAIN DATA

6.1 Introduction

The use of nonlinear loads such as arc welders, steel mill furnaces, power transistors, and silicon-controlled rectifiers are expected to increase. Furthermore, competitive design of power system components calls for the operating points to be more in the nonlinear regions of the characteristic curves [4]. In the case of power transformers, the hysteresis loop would be extended more into the saturated region. It is due to these harmonic generating loads and components that power system harmonics are expected to play an important role in the modeling of power system components. Accurate models for components of power system are needed to simulate the response of power system to the addition of large nonlinear loads, to study their effects before they are actually added, and to simulate solutions to existing problems.

Signal propagation in power systems depends on transformer impedances. Transformer impedances, on the other hand, are functions of signal frequency as well as input voltage and loading condition. Present power transformer models with single-valued parameters obtained from tests cannot adequately represent the transformer in the presence of harmonics. Accurate modeling of transformers must account for changes in signal frequencies and loading conditions as they occur. The parameters of any assumed transformer model should account for not only nonsinusoidal signals, but for nonlinearities generated by the magnetic saturation of the transformer core as well. Estimates of model parameters derived from test data [57] are valid for only a limited range of operation and frequency. This chapter presents algorithms for estimation of the parameters of a structurally known transformer model. These algorithms use time-domain data and continuously update the estimates, thereby accounting for changes in the operating conditions. The techniques presented here can be applied to three-phase and single-phase n-winding transformers. The results of model identification for a single-phase two-winding transformer will be given. The simulated response of the identified model is compared with the transformer actual response.
6.2 Problem Definition

Consider an n-winding transformer model as shown in Figure 29. The structure of the model of Figure 29 is assumed based on physical reasonings and, in general, it may consist of any combination of resistances, inductances, and capacitances. In particular, if each winding is modeled by an inductance in series with a resistance and each magnetizing branch is modeled by a saturable inductance in series with a resistance, the response of the n-winding transformer can be described by a set of n first order nonlinear differential equations of the form

$$\frac{dX(t)}{dt} = A(X)X(t) + B(X)U(t)$$

which can be written as a set of n first order linear time-variant differential equations of the form

$$\frac{dX(t)}{dt} = A(t)X(t) + B(t)U(t) \quad (6.1)$$

where

*state vector* $X = [i_1, i_2, ..., i_n]^T$

*input vector* $U = [v_1, v_2, ..., v_n]^T$

and $A(t)$ and $B(t)$ are nxn matrices of time-varying parameters. For a given operating condition, the time varying nature of matrices $A$ and $B$ is due to saturation of the transformer core.
By treating the transformer winding as mutually coupled coils, each entry of A(t) and B(t) becomes a 3x3 matrix for three-phase transformers [57].

The problem addressed in this chapter can now be stated as follows: Once a structure has been assumed for the model of a single or three-phase transformer, matrices A(t) and B(t) are to be estimated from a given record of state and input observations. The recorded data may contain harmonic components due to magnetic saturation of the transformer core and harmonics present in the input voltages. The estimation of matrices A and B, and subsequent derivation of the transformer physical parameters must be carried out in the presence of additive noise with reasonable accuracy.

6.3 Solution Procedure

A data acquisition system for sampling and recording the instantaneous values of winding currents and voltages is described in Section 5.4. In this section, we present the mathematical formulation of the approach taken in estimating the parameters of an n-winding transformer under the conditions described in Section 6.2.

Consider an n-winding transformer model in per-unit system as shown in Figure 30. In this model, the mutual inductances $L_{jk}$ and the mutual resistances $R_{jk}$ have been assumed to be the same for all windings j and k. In general, these mutuals do not have to be assumed equal. The matrix representation of the model
of Figure 30 is given by

\[
\begin{bmatrix}
  v_1 \\
v_2 \\
  \vdots \\
v_n
\end{bmatrix} =
\begin{bmatrix}
  R_1 + R_{jk} & R_{jk} & \cdots & R_{jk} \\
  R_{jk} & \ddots & \ddots & \vdots \\
  \vdots & \ddots & \ddots & \vdots \\
  R_{jk} & \cdots & R_{jk} & R_n + R_{jk}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
  \vdots \\
i_n
\end{bmatrix} +
\begin{bmatrix}
  L_1 + L_{jk} & L_{jk} & \cdots & L_{jk} \\
  L_{jk} & L_b + L_{jk} & \cdots & L_{jk} \\
  \vdots & \ddots & \ddots & \vdots \\
  L_{jk} & \cdots & L_{jk} & L_n + L_{jk}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
  \vdots \\
i_n
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix}
i_1 \\
i_2 \\
  \vdots \\
i_n
\end{bmatrix}
\]

For a three-phase transformer, each entry of the parameter matrices becomes a 3x3 matrix and each entry of the state and input vectors become a 3x1 vector; i.e., \( L_1 + L_{jk} \) becomes

\[
\begin{bmatrix}
  L_a + L_{jk} & L_{jk} & L_{jk} \\
  L_{jk} & L_b + L_{jk} & L_{jk} \\
  L_{jk} & L_{jk} & L_c + L_{jk}
\end{bmatrix}
\]

and \( v_1 \) becomes \([v_a \ v_b \ v_c]^T\) where subscripts a, b and c denote corresponding phase quantities and \( L_{jk} \) is the mutual inductance between any of the two phases. The saturable inductance \( L_{jk} \) is a nonlinear function of its current. We treat this inductance as a time-varying parameter whose instantaneous values are to be estimated.

For simplicity consider a two-winding transformer, the matrices \( A(t) \) and \( B(t) \) of Eq. (6.1) are given by

\[
A = \frac{1}{\Delta} \begin{bmatrix}
  -L_2(R_1 + R_{12}) - L_{12}R_1 & L_{12}R_2 - L_2R_{12} \\
  L_{12}R_1 - L_1R_{12} & -L_1(R_2 + R_{12}) - L_{12}R_2
\end{bmatrix}
\]

\[
B = \frac{1}{\Delta} \begin{bmatrix}
  L_2 + L_{12} & -L_{12} \\
  -L_{12} & L_1 + L_{12}
\end{bmatrix}
\]

where

\[
\Delta = L_1L_2 + L_{12}(L_1 + L_2)
\]

\[
X(t) = [i_1(t) \ i_2(t)]^T
\]
\[ U(t) = [v_1(t) \ v_2(t)]^T \]

Our objective is to estimate the parameters \( R_1, R_2, R_{12}, L_1, L_2, \) and \( L_{12} \) from a record of \( i_1(t), i_2(t), v_1(t), \) and \( v_2(t) \) sampled with a known frequency. In discrete-time domain, Eq. (6.1) becomes

\[ X(K + 1) = A^*(K)X(K) + B^*(K)U(K) \]  \hspace{1cm} (6.2)

where \( A^* \) and \( B^* \) are matrices of discrete-time model parameters and \( K \) is the instant of time. Matrices \( A^*(K) \) and \( B^*(K) \) can be estimated from a given observation set \( X(.) \) and \( U(.) \) using Generalized Least Square Estimator (GLSE) given in Appendix B, subject to the condition that they contain only time-invariant or "slowly" varying parameters. The estimates of continuous-time model parameters can be obtained from the estimates of \( A^* \) and \( B^* \), by use of appropriate transformations as described in Section 3.4.1, for a given instant of time.

In the case of transformers, however, the mutual inductances are "fast" varying parameters [62]. For the GLSE to track \( A^*(K) \) and \( B^*(K) \) closely, the effect of this inductance on the entries of these matrices must be removed. Reformulating Eq. (6.1) for the two-winding transformer, we take the input vector to be

\[ U = [v_1 - \hat{v}_{12} \ v_2 - \hat{v}_{12}]^T \]  \hspace{1cm} (6.3)

where \( \hat{v}_{12} \) denotes the estimate of the magnetizing branch voltage \( v_{12} \). With this choice of inputs, matrices \( A \) and \( B \) become

\[
A = \begin{bmatrix}
\frac{-R_1}{L_1} & 0 \\
0 & \frac{-R_2}{L_2}
\end{bmatrix}, \quad B = \begin{bmatrix}
\frac{1}{L_1} & 0 \\
0 & \frac{1}{L_2}
\end{bmatrix}
\]  \hspace{1cm} (6.4)

which do not depend on \( L_{12} \). The identification problem is now the estimation of the matrices \( A \) and \( B \) given by Eq. (6.4), the inductance \( L_{12} \), and the resistance \( R_{12} \) from a given record of winding voltage/current observations, knowing that the voltage \( v_{12} \) cannot be measured.

The proposed solution for the estimation of parameters has two steps. First, from no-load data, we develop an observer for estimating \( v_{12} \) from measurable voltage and current sequences. Second, we estimate the transformer parameters by processing sequences of measured voltages and currents, and the estimated sequence of \( v_{12} \). That is, the input vector defined by Eq. (6.3) is first estimated and then, along with the measured state vector, used for parameter estimation.
Estimation of the Voltage $v_{12}$ by an Observer:

In an observable linear time-variant system, any unmeasurable function of the system inputs and/or states belongs to the functional space spanned by input and state functions. That is, an unmeasurable function $z(t)$ can be expressed as

$$z(t) = a_1(t)x_1(t) + a_2(t)x_2(t) + \ldots + b_1(t)u_1(t) + b_2(t)u_2(t) + \ldots$$

where $x_i$'s are the state and $u_i$'s are the input functions; $a_i$'s and $b_i$'s are time-varying coefficients.

In the case of a transformer, all the states (winding currents) are measurable and thus the system is observable. In what follows, we develop an observer for the estimation of the voltage $v_{12}$. The observer model is identified using no-load data; it is then used to estimate $v_{12}$ under load.

Referring to the equivalent circuit of Figure 30, we can write

$$v_{12} = \frac{1}{2}(v_1 + v_2 - R_1 i_1 - R_2 i_2 - L_1 \frac{di_1}{dt} - L_2 \frac{di_2}{dt})$$

and under the assumptions that $R_1 \approx R_2$ and $L_1 \approx L_2$ we can estimate $v_{12}$ in continuous-time domain by

$$\hat{v}_{12} = f_t \left(v_1 + v_2, i_1 + i_2, \frac{d}{dt}(i_1 + i_2)\right)$$

or in discrete-time domain by

$$\hat{v}_{12} = g_k (v_1(k) + v_2(k), i_1(k) + i_2(k), i_1(k+1) + i_2(k+1))$$

where the linear time-varying functions $f_t$ and $g_k$ are to be estimated. Assuming $g_k$ does not change significantly as the transformer moves from one operating condition to another, it can be estimated from open-circuit data. Since $g_k$ is a linear operator, it can be replaced by a time-varying matrix, $G(k)$. For the unloaded transformer $i_2 = 0$, $v_{12} = v_2$ and we can write

$$\hat{v}_{12}(k) = [G(k)] \begin{bmatrix} i_1(k) \\ i_1(k+1) \\ v_1(k) + v_2(k) \end{bmatrix}$$
When a record of data up to the time instant $k+1$ is collected, the matrix $G(k)$ can be estimated using GLSE. We refer to the matrix $G$ as the observer matrix and to its entries as the observer parameters.

The block diagram of Figure 31 shows the steps in estimating the voltage $v_{12}$ under load.

![Block Diagram](image)

Figure 31: Estimation of magnetizing branch voltage under load

To obtain estimates of $L_{12}$ and $R_{12}$ we proceed as follows. Let the voltage $\hat{v}_{12}$ and the current $i_{12} = i_1 + i_2$ be expressed in terms of their truncated Fourier series expansions

$$i_{12} = \sum_{\ell} (a_{\ell} \sin(\ell \omega t) + b_{\ell} \cos(\ell \omega t))$$

$$\hat{v}_{12} = \sum_{\ell} (c_{\ell} \sin(\ell \omega t) + d_{\ell} \cos(\ell \omega t))$$

Define

$$i^\ell_{12} = (a_{\ell} \sin(\ell \omega t) + b_{\ell} \cos(\ell \omega t))$$

$$\hat{v}^\ell_{12} = (c_{\ell} \sin(\ell \omega t) + d_{\ell} \cos(\ell \omega t))$$
The voltage $v_{12}'$ has two components, one in phase with $i_{12}'$, $v_r'$, and one leading $i_{12}'$ by 90°, $v_L'$. To find these components we can proceed as follows:

1) Write $i_{12}'$ as

$$i_{12}' = (a'_{\ell}^2 + b'_{\ell}^2)^{1/2} \cos(\ell \omega t - \phi_{\ell})$$

$$\phi_{\ell} = \arctan2\left(\frac{a'_{\ell}}{b'_{\ell}}\right)$$

2) Calculate the transformation matrix

$$T_{\ell} = \begin{bmatrix} -\cos\phi_{\ell} & \sin\phi_{\ell} \\ \sin\phi_{\ell} & \cos\phi_{\ell} \end{bmatrix}$$

3) Calculate $v_L'$ and $v_r'$ by

$$v_L' = -c_{\ell}\sin(\ell \omega t - \phi_{\ell})$$

$$v_r' = f_{\ell}\cos(\ell \omega t - \phi_{\ell})$$

where

$$\begin{bmatrix} c_{\ell} \\ f_{\ell} \end{bmatrix} = [T_{\ell}] \begin{bmatrix} c_{\ell} \\ d_{\ell} \end{bmatrix}$$

The inductance $L_{12}$ can now be estimated by

$$\hat{L}_{12} = \frac{\partial}{\partial i_{12}} \left( \int v_L dt \right)$$

$$v_L = \sum_{\ell} v_L'$$

(6.5)

where both the integration and the differentiation are performed numerically for ascending and descending values of $i_{12}$. The resistance $R_{12}$ can be estimated using GLSE (Appendix B) to fit a model of the form

$$v_r(k) = \sum_{\ell} v_r'(k) = \hat{R}_{12}(k)i_{12}(k)$$
6.4 Parameter Estimation Results

Case Study

To test the accuracy of the parameter estimation technique described in this chapter, the algorithm was used to estimate the parameters of a single-phase, two-winding, 5KVA, 110/94V transformer. The transformer was loaded by a resistive load drawing 25A. Primary voltage, primary current, secondary voltage, and secondary current were each sampled and recorded by the data acquisition system, described in Section 5.4, at a rate of 5400 samples per second. The data collected was passed through the digital filter described in Appendix C to remove the high frequency noise. An example of a noisy signal and of the filter output is shown in Figure 32 where the plots of noisy and filtered exciting currents for the loaded transformer are shown.

To estimate the parameters of the observer, another set of data was collected during open circuit. The parameters of the observer were estimated for two periods of the 60Hz signal (180 data points for each parameter) using the open circuit data. These parameters were first used to estimate $v_{12} = v_2$ in the open circuit case. The plots of $v_{12}$ and its estimate were practically the same and thus are not shown. The instantaneous values of $v_{12}$ and its estimate $\hat{v}_{12}$ are tabulated in Table 8 for one period. The absolute mean and rms errors of estimating $v_{12}$ are 1.1975 and 1.3631 volts, or 0.66 and 0.75 percent of the peak value, respectively.

Table 8: Actual and estimated magnetizing branch voltage for one period

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<thead>
<tr>
<th>$V_{12}$</th>
<th>$V_{13}$</th>
<th>$V_{14}$</th>
<th>$V_{23}$</th>
<th>$V_{24}$</th>
<th>$V_{34}$</th>
<th>$V_{42}$</th>
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<tr>
<td>106.98</td>
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99
Figure 32: Noisy and filtered exciting current under load

Figure 33: Harmonics of the input voltage
With observer parameters estimated, we proceed to estimate the parameters of the transformer continuous-time model under loading conditions described above. Figure 33 shows the rms values of the harmonics present in the input voltage $v_1$ as a function of frequency. This figure clearly indicates the presence of other than 60Hz signals in the input voltage.

The observer parameters were used to estimate $v_{12}$ under load (see Figure 31). The plot of the estimated magnetizing inductance which corresponds to ascending and descending sections of the hysteresis loop is shown in Figure 34. The estimates of $R_{12}$, $R_1$, $R_2$, $L_1$, and $L_2$ are tabulated in Table 9.

Table 9: Estimated winding and core loss parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.1309 Ohms</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.1514 Ohms</td>
</tr>
<tr>
<td>$R_{12}$</td>
<td>23.16 Ohms</td>
</tr>
<tr>
<td>$L_1$</td>
<td>0.3026 mH</td>
</tr>
<tr>
<td>$L_2$</td>
<td>0.3415 mH</td>
</tr>
</tbody>
</table>

Finally, these estimated parameters were used to simulate the response of the transformer to the same operating conditions. The simulation result along with the actual response for the exciting current is shown in Figure 35.
6.5 Conclusions

The problem of transformer model identification in the presence of power system harmonics and magnetic saturation of the transformer core was discussed. Due to interactions among harmonics in the magnetizing branch, transformer models are in general nonlinear. A technique for the estimation of the parameters of an n-winding transformer model was presented. The emphasis was on the identification of a physical model using time-domain data. By using an observer to estimate the voltage drop across the magnetizing branch, the n-dimensional estimation problem was decoupled into n + 1 one-dimensional problems, resulting in great numerical advantages. To obtain unbiased estimates in a noisy environment, the generalized least-squares technique was used. In addition, the low-pass digital filter [64] was used to remove the high frequency noise from the contaminated data collected by the data acquisition system. Actual identification results for a single-phase two-winding transformer were given. By comparing the simulated response of the identified model with the actual response of the transformer, the accuracy of the model identified by the proposed technique was demonstrated.
CHAPTER VII

TIME-DOMAIN MODELING AND COMPUTER SIMULATION OF THREE-PHASE SIGNAL CONDITIONING SYSTEMS

7.1 Introduction

The history of digital computer crashes has shown that 80% of the time crashes were due to voltage sags caused by faults occurring on the transmission system in the vicinity of the substation serving the computer complex [66].

Unusual failure rates reported in satellite communications earth stations in some third world countries were due to voltage swings of 100 volts, transients of up to 2000 volts, noise levels of over 60 volts, and harmonic distortions greater than 20% [66].

A power conditioning system is a device used to improve the quality of power in order to reduce equipment malfunctions. In a three-phase power conditioning system which utilizes ferroresonant transformers, the improved quality results from the synthesis of output voltages which remain constant over a wide range of input voltage fluctuations; both sags and surges.

The power conditioning system for which we are to develop the only existing model is a three-phase 60Hz device consisting of ferroresonant circuits. This device is also referred to as a datawave system and thus, in this dissertation, the names "power conditioning" and "datawave" are used interchangeably.

7.2 Analysis of the Output Voltage Waveform Synthesis

To develop a physical model for the power conditioning system and its subsequent use in improving system performance through analysis and simulation, understanding the details of synthesis of balanced three-phase output voltage waveforms is the first step.

Fig. 36 shows the wiring diagram for the three-phase power conditioning system. The six interconnected saturating transformers TX1, TX2, TX3, TX4, TX5,
Figure 36: Wiring diagram of the three-phase power conditioning system
and TX6 generate the building blocks for the synthesized output voltage waveforms. For a set of positive sequence balanced input voltages $V_{AB}$, $V_{BC}$, and $V_{CA}$, the exciting currents of the two-winding transformers TX4, TX5 and TX6 are a set of balanced currents. The exciting currents for the four-winding transformers TX1, TX2 and TX3 are

\[ i_{E1} = i_{E4} - i_{E6} \]
\[ i_{E2} = i_{E5} - i_{E4} \]
\[ i_{E3} = i_{E6} - i_{E5} \]

These six exciting currents can now be shown on a single phasor diagram (Fig. 37). Taking the instance shown as our time reference, the projections of these phasors (as they rotate CCW with an angular velocity of $\omega = 377 \text{ rad/sec}$) along the positive real axis are their instantaneous values in time-domain.

The core of each transformer is designed to saturate at a specific level of its exciting current. A typical exciting current/flux relationship is depicted in Fig. 38. Also shown in this figure is a typical transformer winding voltage $E_{66}$.

The levels of exciting currents at which transformer cores saturate, considering the phase shifts among the exciting currents, are set such that at any given time, five of the cores are in saturation and the sixth is in the linear region putting out a voltage pulse similar to that shown in Fig. 38. The unsaturating sequence can be determined from the phasor diagram of Fig. 37. This sequence is as follows: TX4 in positive direction (i.e., positive slope), TX1 in positive, TX6 in negative, TX3 in negative, TX6 in positive, TX2 in positive direction and so on. The magnitudes of the voltage pulses generated by the two-winding transformers TX4, TX5 and TX6 are $\sqrt{3}$ times larger than those generated by the four-winding transformers TX1, TX2 and TX3.

From Fig. 36 we note that the output phase and line-to-line voltages are

\[ V_{an} = V_4 + V_1 - V_2 \]
\[ V_{bn} = V_5 + V_2 - V_3 \]
\[ V_{cn} = V_6 + V_3 - V_1 \]
\[ V_{ab} = V_{an} - V_{bn} \]
\[ V_{bc} = V_{bn} - V_{cn} \]
\[ V_{ca} = V_{cn} - V_{an} \]

where $V_j \triangleq V_{TXj}$.
Figure 37: Datawave exciting currents.

Figure 38: A saturating transformer current, flux, and voltage relationships.
Figs. 39 through 44 show the instantaneous exciting currents and the corresponding winding voltages for the six saturating transformers. Figs. 45 through 50 show the synthesis of the phase and line-to-line output voltages.

7.3 Statement of the Problem

Consider a component of the power conditioning system as shown in Figure 51. This component is assumed to have 2N terminals. The structure of the model for this component is assumed based on physical reasonings and, in general, it may be any combination of resistances, inductances, and capacitances. The dynamics of this component can be described by a set of N first order nonlinear differential equations of the form

\[
\frac{dx}{dt} = A(x)x(t) + B(x)u(t)
\] (7.1)

which can be written as a set of N first order linear time-variant differential equations of the form

\[
\frac{dx}{dt} = A(t)x(t) + B(t)u(t)
\]

where \( x \) and \( u \) are vectors of currents and voltages, and \( A \) and \( B \) are matrices of time-varying component parameters.

The problem of datawave system modeling from time-domain data can now be stated as follows: For each component of the datawave system while operating as a part of the overall system, matrices \( A \) and \( B \) in Eq. (7.1) are to be estimated from the time-domain records of \( x \) and \( u \) observations. The overall system model is to be obtained by interconnecting the component models.

7.4 Time-Domain Modeling of the Line Chokes from Operating Data

A line choke can be modeled by a nonlinear inductor in series with a resistor as shown in Figure 52.

The dynamics of this choke is described by a differential equation of the form

\[
v(t) = Ri(t) + L(i)\frac{di(t)}{dt}
\] (7.2)
Figure 39: TX4 Exciting current and induced voltage.

Figure 40: TX5 Exciting current and induced voltage.

Figure 41: TX6 Exciting current and induced voltage.
Figure 42: TX1 Exciting current and induced voltage.

Figure 43: TX2 Exciting current and induced voltage.

Figure 44: TX3 Exciting current and induced voltage.
Figure 45: Phase A voltage synthesis.
Figure 46: Phase B voltage synthesis.
Figure 47: Phase C voltage synthesis.
Figure 48: \(ab\) line voltage.

\[ E_{ab} = E_{am} - E_{bm} \]

Figure 49: \(bc\) line voltage.

\[ E_{bc} = E_{bm} - E_{cm} \]

Figure 50: \(ca\) line voltage.

\[ E_{ca} = E_{cm} - E_{am} \]
In discrete-time domain, this equation becomes

\[ i(k + 1) = Ai(k) + Bv(k) \]

where

\[ A = e^{-RT} \]
\[ B = \frac{1}{R}(1 - A) \]
\[ T = \text{sampling time in seconds} \]

Our objective now becomes the estimation of \( A \) and \( B \) from the time-domain records of \( v \) and \( i \).

Another technique for estimating the choke inductances can be developed by noting that

\[ v_L = L(i) \frac{di}{dt} = \frac{d\lambda}{dt} = \frac{\partial \lambda}{\partial i} \frac{di}{dt} \]
and therefore $L(i) = \frac{\partial\lambda}{\partial i}$. This suggests that the estimates of $L(i)$ can be obtained by taking derivative of the estimate of flux linkage as a function of current. That is, estimate of $L(i)$ is the slope of the estimate of hysteresis plot. To obtain the hysteresis plot, we measure the choke voltage $V(t)$ and current $i(t)$ by looking into its terminals when it is operating as a component of the power conditioning system. Then we integrate $v(t) - \hat{R}i(t)$ to obtain the flux linkage $\lambda(t)$

$$\lambda(t) = \int_{t_0}^{t} (V(t) - \hat{R}i(t)) \, dt + \lambda(t_0)$$

The plot of $\lambda$ as a function of $i$ over a full 60 Hz cycle is known as the hysteresis loop. The value of $\lambda(t_0)$ is chosen so that the mean of $\lambda(t)$ over one 60 Hz period becomes zero. The inductance $L(i)$ can be found by

$$L(i) = \frac{\partial}{\partial i} \lambda(i) \quad (7.3)$$

It should be pointed out that the differentiation required by Eq. (7.3), when applied to hysteresis data available in numerical form, does not yield sufficiently accurate results, some sort of curve fitting, followed by analytical differentiation of the fitted function is required.

Choke parameters are estimated for different loading conditions of the power conditioning device. The estimation results for the choke inductance $L_{ch}$ and resistance $R_{ch}$ are tabulated in Table 10 as a function of the loading condition. It is observed that the choke resistance is negligible ($R_{ch} \ll 377 \, L_{ch}$) in all cases. A typical choke voltage and current used to obtain the estimates in Table 10 along with the corresponding hysteresis loop are shown in Figure 53.

Inductances $L_s$, $L_{sm}$, and $L_m$ are defined to be the slopes of a piecewise linear estimation of the major hysteresis loop in saturated, intermediate and unsaturated regions, respectively. From the hysteresis loops, it is estimated that

$$L_{ch} = \begin{cases} 
L_s & \text{for } 60A < \left| i_{ch}(K) \right| \\
L_{sm} & \text{for } 20A \leq \left| i_{ch}(K) \right| \leq 60A \\
L_m & \text{for } \left| i_{ch}(K) \right| < 20A 
\end{cases} \quad (7.4)$$

The classification of the loading conditions in Table 10 is based on Eq. (7.4). It is such that the choke inductance is single-valued for light loads ($L_m$), double-valued for medium loads ($L_m$ and $L_{sm}$), and is triple-valued ($L_m$, $L_{sm}$, and $L_s$) for heavy loads.
Table 10: Estimates of the choke inductance and resistance

<table>
<thead>
<tr>
<th>Loading Condition</th>
<th>Choke Peak Current (A)</th>
<th>Choke Inductance (H)</th>
<th>Choke Resistance (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-Load</td>
<td>10.0</td>
<td>0.00284</td>
<td>0.082</td>
</tr>
<tr>
<td>Light-Load</td>
<td>17.0</td>
<td>0.00280</td>
<td>0.079</td>
</tr>
<tr>
<td>Light-Load</td>
<td>24.0</td>
<td>0.00278</td>
<td>0.076</td>
</tr>
<tr>
<td>Med.-Load</td>
<td>38.0</td>
<td>$L_{sm} = 0.002$</td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_m = 0.00278$</td>
<td></td>
</tr>
<tr>
<td>Med.-Load</td>
<td>55.0</td>
<td>$L_{sm} = 0.0019$</td>
<td>0.0717</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_m = 0.00277$</td>
<td></td>
</tr>
<tr>
<td>Heavy-Load</td>
<td>74.0</td>
<td>$L_s = 0.00149$</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_{sm} = 0.0020$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_m = 0.00275$</td>
<td></td>
</tr>
<tr>
<td>Heavy-Load</td>
<td>95.0</td>
<td>$L_s = 0.00144$</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_{sm} = 0.00206$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_m = 0.00271$</td>
<td></td>
</tr>
<tr>
<td>Heavy-Load</td>
<td>120.0</td>
<td>$L_s = 0.00131$</td>
<td>0.0601</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_{sm} = 0.0020$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_m = 0.0027$</td>
<td></td>
</tr>
<tr>
<td>Heavy-Load</td>
<td>140.0</td>
<td>$L_s = 0.00122$</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_{sm} = 0.0020$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_m = 0.0027$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 53: a) Filtered choke voltage or current.

Figure 53: b) Hysteresis loop for the choke operation.
Figure 54: Closed form estimate of the choke incremental inductance.

We will now fit a curve through the estimated inductance values. This will result in continuities in simulation signals, will facilitate numerical solution, and will allow analytical investigation.

Figure 54 shows the fitted curve. This curve has been found by trial and error and is given by

\[
L(i) = \begin{cases} 
0.00277 & \text{for } |i| \leq 20.25A \\
\frac{0.0125}{\sqrt{|i|}} & \text{for } |i| > 20.25A 
\end{cases}
\]

Based on Table 10, a value of 0.07 ohms is assumed for the choke resistance.

7.5 Time-Domain Modeling of the Four-Winding Transformers

In this section, we present the modeling of the four-winding transformers using a parallel data acquisition system. The modeling procedure consists of two steps. In the first step, the model structure is identified. In the second step, the parameters of the model are estimated from records of input/output data collected by the parallel data acquisition system.

7.5.1 Structural Identification

Consider a four-winding transformer as shown in Figure 55. Neglecting the windings' leakage fluxes, we can write \( V_j = r_j i_j + N_j \frac{d\phi_j}{dt} \); \( j = 1, 2, 3, 4 \) or \( V_j = \)

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Figure 55: A four-winding transformer.

\( r_{ij} + \frac{d\lambda}{dt} \) for \( N_1 = N_2 = N_3 = N_4 = N \), \( r_1 = r_2 = r_3 = r_4 = r \), and \( \lambda = N\phi \). Now using the chain rule we can write

\[
V_j = r_{ij} + \sum_{j=1}^{4} \frac{\partial \lambda}{\partial i_j} \frac{di_j}{dt}
\]  

(7.5)

but \( \frac{\partial \lambda}{\partial i_1} = \frac{\partial \lambda}{\partial i_2} = \frac{\partial \lambda}{\partial i_3} = \frac{\partial \lambda}{\partial i_4} = \frac{\partial \lambda}{\partial i_E} \), where \( i_E = i_1 + i_2 + i_3 + i_4 \) and thus Eq. (7.5) becomes

\[
V_j = r_{ij} + \frac{\partial \lambda}{\partial i_E} \frac{di_E}{dt}
\]

Letting \( L = \frac{\Delta}{\partial i_E} \) we have

\[
V_j = r_{ij} + L \frac{di_E}{dt} ; \quad j = 1, 2, 3, 4
\]  

(7.6)

Eq. (7.6) defines the structure of our model as shown in Fig. 56

7.5.2 Parameter Estimation

To estimate the winding resistance \( r \), we note that by using Eq. (7.6) we can arrive at
Figure 56: Four-winding transformer model structure.

\[ V_j - V_k = r(i_j - i_k) \quad \text{for } j, k = 1, 2, 3, 4 \]

One way of estimating \( r \) from test data is to sample and collect time-domain signals \( V_1, V_2, i_1 \) and \( i_2 \) simultaneously (using a parallel data acquisition system), then calculate the differences \( V_1 - V_2 \) and \( i_1 - i_2 \), and obtain the estimate of \( r \) by the following estimator

\[ \hat{r} = \frac{V_F}{I_F} \]

where \( V_F \) and \( I_F \) are the amplitudes of the fundamental components (60 Hz) of the signals \( V_1 - V_2 \) and \( i_1 - i_2 \), respectively. Figure 5.4 of Chapter 5 shows the typical shapes of \( v_1 - v_2 \) and \( i_1 - i_2 \) for the saturating transformers. For the four-winding transformers it is found that \( V_F = 1.6V, I_F = 240A \), and thus

\[ \hat{r} = \frac{V_F}{I_F} = 0.0067 \text{ ohms} \]

With the winding resistance \( r \) estimated, Eq. (7.6) can be rewritten as

\[ L \frac{di_E}{dt} = V_j - \hat{r}i_j \quad (7.7) \]

Integrating Eq. (7.7), we get
Figure 57: Four-winding transformer hysteresis loop.

\[ L' i_E = \lambda \]  

(7.8)

where \( L' \) is now the apparent inductance and \( \lambda_j = N\phi \) is the magnetic flux linking all four windings. A plot of \( \lambda \) as a function of \( i_E \) (Fig. 57) reveals the nature of the incremental inductance \( L \). The instantaneous value of the inductance \( L \) is equal to the slope of the plot of Fig. 57.

It can be seen that the effect of hysteresis on the inductance \( L \) is very small. With the hysteresis effect neglected, we can find a suitable function of \( i_E \) to model the flux linkage \( \lambda \) and thus the inductance \( L \). Obtaining an analytic expression for the estimate of \( L \), as compared to a tabular form which would be obtained from a least-squares estimator of \( L \), is advantageous in both analysis and simulation of the datawave system. In addition, it is known that most recursive estimators cannot track "fast" varying parameters such as the highly nonlinear inductances of the saturating transformers.

To find a suitable closed form estimator for the flux linkage \( \lambda \), we guess the form of the function \( \lambda(i_E) = f(i_E) \) and will then attempt to estimate the parameters of the function \( f(i_E) \) in the manners described in Chapter 4.

In the first attempt, the logistic function is used to model the magnetic saturation shown in Fig. 57. The parameters of the model are estimated using the algorithm given in Section 4.3.1.

For decreasing values of \( i_E \), the estimator of the flux linkage is found to be
Figure 58: Four-winding transformer actual and estimated saturation curve

\[
\lambda(i_E) = \frac{0.214e^{0.7+0.178i_E}}{1 + e^{0.7+0.178i_E}} - 0.108 
\]

(7.9)

Figure 58 shows the actual (calculated by Eq. (7.10)) and the estimated (given by Eq. (7.9)) saturation curve for decreasing values of \(i_E\).

\[
\lambda(i_E) = \int_{t_0}^{t} (v_1 - \dot{i}_1) dt + \lambda(t_0) 
\]

(7.10)

It can be seen that the estimated curve has smaller slope for all values of \(i_E\) and that the fit is not satisfactory at the knees of saturation.

Next, the Arctan function discussed in Section 4.3.2 is used to model the four-winding transformer core magnetic saturation. The estimator of the flux linkage is now found to be

\[
\hat{\lambda}(i_E) = 0.0635 \arctan (0.1732i_E) + (1.5E - 5)i_E 
\]

(7.11)

Figure 59 shows the actual hysteresis loop along with the normalized estimated saturation curve as modeled by Eq. (7.11). It can be seen that the hysteresis loop can be adequately modeled using the arctan function.

The estimator of the incremental magnetizing inductance can be obtained from Eq. (7.11),
Figure 59: Four-winding transformer actual hysteresis loop and estimated saturation curve

Figure 60: Estimate of the four-winding transformer incremental inductance.
Figure 60 shows the plot of \( \hat{L}(i_E) \) defined by Eq. (7.12). This completes the identification of the four-winding transformer model.

7.6 Time-Domain Modeling of the Two-Winding Transformers

In this section, we present the modeling of the two-winding transformers from records of input/output data collected by a parallel data acquisition system. The identification process is the same as that for the four-winding transformers.

7.6.1 Structural Identification

Referring to Section 7.5.1, for the two-winding transformers we can write

\[
v_j = r_i + L \frac{di_E}{dt}; \quad j = 1, 2
\]  

(7.13)

Eq. (7.13) defines the structure of our model as shown in Fig. 61.

```
\[
\hat{L}(i_E) = \frac{\partial \lambda(i_E)}{\partial i_E} = \frac{0.011}{1 + 0.03i_E^2} + 1.5E - 5
\]  

(7.12)
```

Figure 61: Two-winding transformer model structure.
7.6.2 Parameter Estimation

The resistance $r$ can be estimated using the same estimator given for the four-winding transformers in Section 7.5.2. Using this estimator for the two-winding transformers, we get

$$
\hat{r} = \frac{V_F}{I_F} = 0.011 \Omega
$$

With the winding resistance $r$ estimated, we can numerically integrate Eq. (7.13) to obtain

$$
\lambda = \lambda(t_0) + \int_{t_0}^{t} \left( L \frac{di_E}{dt} \right) + \lambda(t_o)
= \int_{t_0}^{t} (v_1 - \hat{r}i_1)dt + \lambda(t_o)
$$

(7.14)

The plot of $\lambda$ as a function of $i_E$ is shown in Fig. 62. It can be seen that the hysteresis effect on $\lambda$ and thus on the incremental inductance $L$ is small.

With the four-winding transformer experience in mind, we use arctan function to model the saturation. The estimator of the flux linkage is found to be

$$
\hat{\lambda}(i_E) = 0.11 \arctan (0.3i_E) + (4.5E - 5)i_E
$$

(7.15)

Figure 63 shows the actual hysteresis loop along with the estimated saturation.
Figure 63: Two-winding transformer actual hysteresis loop and estimated saturation curve

Figure 64: Estimate of the two-winding transformer incremental inductance
curve. It can be seen that the two-winding transformer hysteresis loop can be adequately modeled using the arctan function.

The estimate of the incremental inductance can be obtained through differentiation of Eq. (7.15)

\[
\dot{L}(i_E) = \frac{\partial \lambda}{\partial i_E} = \frac{0.033}{1 + 0.09 i_E^2} + 4.5 E - 5
\] (7.16)

Figure 64 shows the plot of \( \dot{L}(i_E) \) given by Eq. (7.16).

### 7.7 Summary of the Datawave System Component Models

In this section, the component models developed in the previous sections and those developed in Reference [67] are summarized for quick reference.

**Choke Model**

\[
R_{ch} = 0.007 \Omega
\]

\[
L_{ch} = \begin{cases} 
0.0028 H & \text{for } |i_{ch}| \leq 20 A \\
0.0125 \frac{H}{(|i_{ch}|)^5} & \text{for } |i_{ch}| > 20 A
\end{cases}
\]
Two-Winding Transformer Model

\[ i_E = i_1 + i_2 \]
\[ r = 0.011 \Omega \]
\[ L = \frac{0.033}{1 + 0.09i_E^2} + 4.5E - 5 H \]

Four-Winding Transformer Model (Physical Model)

\[ i_E = i_{E1} + i_{E2} \]
\[ r = 0.0067 \Omega \]
\[ L = \frac{0.011}{1 + 0.03i_E^2} + 1.5E - 5 H \]

Filter Models

i. Second harmonic filter

\[ R_{F2} = 0.68 \Omega \]
\[ L_{F2} = 0.027H \]
\[ C_{F2} = 65\mu F \]

ii. Third harmonic filter

\[ R_{F3} = 0.45\Omega \]
\[ L_{F3} = 0.0142H \]
\[ C_{F3} = 55\mu F \]

**Input/Output Capacitors**

i. Input Capacitors

\[ C_1 = 775\mu F \]
\[ R_{C1} = 1000\Omega \]

ii. Output Capacitors

\[ C_2 = 415\mu F \]
\[ R_{C2} = 1500\Omega \]
**Zig-Zag Transformer Model**

\[ L_{zz} = 0.056 \text{H}, \quad R_{zz} = 0.1 \Omega \]

### 7.8 Datawave System Model

In this section, the datawave system component models are used to derive the overall system model equations. Different formulations of the system model mathematics are presented, and advantages and disadvantages of each are discussed.

The component models given in Section 7.7 can now be synthesized to form the overall datawave system model (Fig. 65). For simplicity, the resistors modeling the input/output capacitor losses are not shown in this figure. The resistor \( r \) models the sum of the winding resistances of one two-winding and two four-winding transformers.

The equations describing the dynamics of the datawave can be written by inspection of Fig. 65. Overall, there are six loop and four node equations describing these dynamics.

#### 7.8.1 Current Model

Referring to Fig. 65 and noting that \( i_A + i_B + i_C = 0 \), \( i_4 + i_5 + i_6 = 0 \), \( v_{a'b'} + v_{b'c'} + v_{c'd'} = 0 \), and that \( r_7 = r_8 = r_9 = r_{ch} \), the primary side loop equations are

\[
v_{AB} = r_{ch}i_A + L_7 \frac{di_A}{dt} + v_{a'b'} - R_{ch}i_B - L_8 \frac{di_B}{dt}
\]

\[
v_{BC} = r_{ch}i_B + L_8 \frac{di_B}{dt} + v_{b'c'} - R_{ch}i_C - L_9 \frac{di_C}{dt}
\]
Figure 65: Datawave System Model
\[ v_{a'b'} = r_i 4 + (L_4 + L_1 + L_2) \frac{di_{E4}}{dt} - (L_3 + L_2 + L_5) \frac{di_{E5}}{dt} - \]
\[ -r_i 5 - L_1 \frac{di_{E6}}{dt} - L_2 \frac{di_{E5}}{dt} + L_3 \frac{di_{E6}}{dt} + L_2 \frac{di_{E4}}{dt} \]
\[ v_{b'c'} = r_i 5 + (L_5 + L_2 + L_3) \frac{di_{E5}}{dt} - (L_1 + L_3 + L_6) \frac{di_{E6}}{dt} - r_i 6 - \]
\[ -L_2 \frac{di_{E4}}{dt} - L_3 \frac{di_{E6}}{dt} + L_1 \frac{di_{E4}}{dt} + L_3 \frac{di_{E5}}{dt} \]

and the node equations are

\[ i_A = i_4 + i_{a'b'} - i_{b'c'} \]
\[ = i_4 + c_1 \frac{dV_{a'b'}}{dt} - c_1 \frac{dV_{b'c'}}{dt} + \frac{1}{R_{c1}} V_{a'b'} - \frac{1}{R_{c1}} V_{b'c'} \]

\[ i_B = i_5 + i_{b'c'} - i_{a'b'} \]
\[ = i_5 + c_1 \frac{dV_{b'c'}}{dt} - c_1 \frac{dV_{a'b'}}{dt} + \frac{1}{R_{c1}} V_{b'c'} - \frac{1}{R_{c1}} V_{a'b'} \]

The secondary side equations are

\[ v_{ab} = r_{i4}' + (L_4 + L_1 + L_2) \frac{di_{E4}'}{dt} - (L_3 + L_2 + L_5) \frac{di_{E5}'}{dt} - \]
\[ -r_{i5}' - L_1 \frac{di_{E6}'}{dt} - L_2 \frac{di_{E5}'}{dt} + L_3 \frac{di_{E6}'}{dt} + L_2 \frac{di_{E4}'}{dt} \]
\[ v_{bc} = r_{i5}' + (L_5 + L_2 + L_3) \frac{di_{E5}'}{dt} - (L_1 + L_3 + L_6) \frac{di_{E6}'}{dt} - r_{i6}' - \]
\[ -L_2 \frac{di_{E4}'}{dt} - L_3 \frac{di_{E6}'}{dt} + L_1 \frac{di_{E4}'}{dt} + L_3 \frac{di_{E5}'}{dt} \]
\[ i'_a + i_a + i_{ab} - i_{ca} + i^2_a + i^3_a + i^{zz}_a = 0 \]

\[ i'_b + i_b + i_{bc} - i_{ab} + i^2_b + i^3_b + i^{zz}_b = 0 \]

where

\[ i_{ab} - i_{ca} = c_2 \frac{dV_{ab}}{dt} - c_2 \frac{dV_{ca}}{dt} + \frac{1}{Rc_2} V_{ab} - \frac{1}{Rc_2} V_{ca} \]

\[ i_{bc} - i_{ab} = c_2 \frac{dV_{bc}}{dt} - c_2 \frac{dV_{ab}}{dt} + \frac{1}{Rc_2} V_{bc} - \frac{1}{Rc_2} V_{ab} \]

for the filters we have

\[ \frac{dV_{ab}}{dt} = \frac{R_{Fj}}{3} \frac{d}{dt}(i^j_a - i^j_b) + \frac{L_{Fj}}{3} \frac{d^2}{dt^2}(i^j_a - i^j_b) + \frac{1}{3C_{Fj}} (i^j_a - i^j_b) \]

\[ \frac{dV_{bc}}{dt} = \frac{R_{Fj}}{3} \frac{d}{dt}(i^j_a + 2i^j_b) + \frac{L_{Fj}}{3} \frac{d^2}{dt^2}(i^j_a + 2i^j_b) + \frac{1}{3C_{Fj}} (i^j_a + 2i^j_b); j = 2, 3 \]

and for the Zig-Zag transformer we have

\[ V_{ab} = 3L_{zz} d/dt(i^{zz}_a - i^{zz}_b) + r_{zz} (i^{zz}_a - i^{zz}_b) \]

\[ V_{bc} = 3L_{zz} d/dt(i^{zz}_b - i^{zz}_c) + r_{zz} (i^{zz}_b - i^{zz}_c) \]

Collecting terms, the equations describing the dynamics of the datawave in matrix form become

\[ \begin{bmatrix} V_{AB} \\ V_{BC} \end{bmatrix} = \begin{bmatrix} r_{ch} & -r_{ch} \\ r_{ch} & 2r_{ch} \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} + \begin{bmatrix} L_7 & -L_8 \\ L_9 & L_8 + L_9 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_A \\ i_B \end{bmatrix} + \begin{bmatrix} v_{a'b'} \\ v_{b'c'} \end{bmatrix} \]

(7.17)

\[ \begin{bmatrix} V_{a'b'} \\ V_{b'c'} \end{bmatrix} = \begin{bmatrix} r & -r \\ r & 2r \end{bmatrix} \begin{bmatrix} i_4 \\ i_5 \end{bmatrix} + \begin{bmatrix} LL_1 & LL_2 \\ LL_3 & LL_4 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{E4} \\ i_{E5} \end{bmatrix} \]

(7.18)

\[ \begin{bmatrix} i_A - i_4 \\ i_B - i_5 \end{bmatrix} = \begin{bmatrix} \frac{2}{R_{e1}} & \frac{1}{R_{e1}} \\ \frac{1}{R_{e1}} & \frac{1}{R_{e1}} \end{bmatrix} \begin{bmatrix} V_{a'b'} \\ V_{b'c'} \end{bmatrix} + \begin{bmatrix} 2C_1 & C_1 \\ -C_1 & C_1 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} V_{a'b'} \\ V_{b'c'} \end{bmatrix} \]

(7.19)
\[
\frac{d}{dt} \begin{bmatrix}
  i_{E4} \\
  i_{E5}
\end{bmatrix} = \frac{1}{2A} \begin{bmatrix}
  LL_3 - 2LL_2 & -LL_2 - LL_3 \\
  LL_1 - LL_3 & 2LL_1 + LL_3
\end{bmatrix} \begin{bmatrix}
  i_{E4} \\
  i_{E5}
\end{bmatrix} + \\
\frac{1}{2A} \begin{bmatrix}
  LL_4 & -LL_2 \\
  -LL_3 & LL_1
\end{bmatrix} \begin{bmatrix}
  V_{ab} + V_{a'b'} \\
  V_{bc} + V_{b'c'}
\end{bmatrix}
\]

with

\[
\frac{d^2}{dt^2} \begin{bmatrix}
  i_a - i_b' \\
  i_a + 2i_b'
\end{bmatrix} = -\frac{R_{Fi}}{L_{Fi}} \frac{d}{dt} \begin{bmatrix}
  i_a - i_b' \\
  i_a + 2i_b'
\end{bmatrix} - \frac{1}{L_{Fi}C_{Fi}} \begin{bmatrix}
  i_a - i_b' \\
  i_a + 2i_b'
\end{bmatrix} + \frac{3}{L_{Fi}} \frac{d}{dt} \begin{bmatrix}
  V_{ab} \\
  V_{bc}
\end{bmatrix}
\]

for the second and third harmonic filters \((j = 2, 3)\), and

\[
\frac{d}{dt} \begin{bmatrix}
  i_{a''} \\
  i_{b''}
\end{bmatrix} = -\frac{R_{zz}}{3L_{zz}} \begin{bmatrix}
  i_{a''} \\
  i_{b''}
\end{bmatrix} + \frac{1}{9L_{zz}} \begin{bmatrix}
  2 & 1 \\
  -1 & 1
\end{bmatrix} \begin{bmatrix}
  V_{ab} \\
  V_{bc}
\end{bmatrix}
\]

for the Zig-Zag transformer.

Where

\[
\begin{align*}
  i_{E4} &= i_4 + i_4' \\
  i_{E5} &= i_5 + i_5' \\
  LL_1 &= 2(L_1 + L_2) - L_3 + L_4 \\
  LL_2 &= L_1 - 2(L_2 + L_3) - L_5 \\
  LL_3 &= 2(L_1 + L_3) - L_2 + L_6 \\
  LL_4 &= L_1 + L_2 + 4L_3 + L_5 + L_6 = LL_3 - LL_2
\end{align*}
\]

The sixteen equations given in Eq. (7.17) through Eq. (7.23) completely describe the dynamics of the datawave system. To put these equations into the form of differential equations, to be numerically solved later, we reformulate them as follows:
Now solving for derivative terms in Eq. (7.17), Eq. (7.19), and Eq. (7.21), we get

\[
\frac{d}{dt}\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \frac{-r_{ch}}{B} \begin{bmatrix} 2L_8 + L_9 & L_8 - L_9 \\ L_7 - L_9 & 2L_7 + L_9 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} + \frac{1}{B} \begin{bmatrix} L_8 + L_9 & L_8 \\ -L_9 & L_7 \end{bmatrix} \begin{bmatrix} V_{AB} - V_{a'b'} \\ V_{BC} - V_{b'c'} \end{bmatrix}
\]  

(7.27)

where \( B = L_7L_8 + L_8L_9 + L_9L_7 \).

\[
\frac{d}{dt}\begin{bmatrix} V_{a'b'} \\ V_{b'c'} \end{bmatrix} = \frac{-1}{R_{c1}c_1} \begin{bmatrix} V_{a'b'} \\ V_{b'c'} \end{bmatrix} + \frac{1}{3C_1} \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} i_A - i_4 \\ i_B - i_5 \end{bmatrix}
\]  

(7.28)

\[
\frac{d}{dt}\begin{bmatrix} V_{ab} \\ V_{bc} \end{bmatrix} = \frac{-1}{R_{c2}c_2} \begin{bmatrix} V_{ab} \\ V_{bc} \end{bmatrix} - \frac{1}{3C_2} \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} i_A - i_4 \\ i_B - i_5 \end{bmatrix}
\]  

(7.29)

To eliminate \( i_4, i_4', i_5, \) and \( i_5' \) from Eq. (7.28) and Eq. (7.29), we note that

\[
i_4 = 1/2[i_{E4} + (i_4 - i_4')]
\]
\[
i_4' = 1/2[i_{E4} - (i_4 - i_4')]
\]
\[
i_5 = 1/2[i_{E5} + (i_5 - i_5')]
\]
\[
i_5' = 1/2[i_{E5} - (i_5 - i_5')]
\]

and that by substituting from Eq. (7.26) we get

\[
i_4 = 1/2i_{E4} + \frac{1}{6r}(2V_{a'b'} + V_{b'c'} - 2V_{ab} - V_{bc})
\]
\[
i_4' = 1/2i_{E4} - \frac{1}{6r}(2V_{a'b'} + V_{b'c'} - 2V_{ab} - V_{bc})
\]
\[
i_5 = 1/2i_{E5} + \frac{1}{6r}(V_{b'c'} - V_{a'b'} + V_{ab} - V_{bc})
\]
\[
i_5' = 1/2i_{E5} - \frac{1}{6r}(V_{b'c'} - V_{a'b'} + V_{ab} - V_{bc})
\]

Now substituting for \( i_4, i_4', i_5 \) and \( i_5' \) in Eq. (7.28) and Eq. (7.29) we obtain

\[
\frac{d}{dt}\begin{bmatrix} V_{a'b'} \\ V_{b'c'} \end{bmatrix} = -\left(\frac{1}{R_{c1}c_1} + \frac{1}{6rc_1}\right) \begin{bmatrix} V_{a'b'} \\ V_{b'c'} \end{bmatrix} + \frac{1}{6rc_1} \begin{bmatrix} V_{ab} \\ V_{bc} \end{bmatrix} + \frac{1}{3c_1} \begin{bmatrix} i_A - i_B + 1/2(i_{E5} - i_{E4}) \\ i_A + 2i_B - 1/2(i_{E4} - i_{E5}) \end{bmatrix}
\]  

(7.30)
Integrating Eq. (7.22), we get

\[
\begin{aligned}
\frac{d}{dt} \left[ \frac{V_{a'b'}}{V_{bc'}} \right] &= -\left( \frac{-1}{R_{c1}c1} + \frac{1}{6rc1} \right) \left[ \frac{V_{ab}}{V_{bc}} \right] - \frac{1}{6rc1} \left[ \frac{V_{a'b'}}{V_{bc'}} \right] - \\
- \frac{1}{3c1} \left[ \begin{array}{c}
- i_a - i_b + i_a^2 - i_b^2 + i_a^3 - i_b^3 + i_a^{xz} - i_b^{xz} + 1/2(i_{E5} - i_{E4}) \\
- i_a + 2i_b + i_a^2 + 2i_b^2 + i_a^3 + 2i_b^3 + i_a^{xz} + 2i_b^{xz} - 1/2(i_{E4} - i_{E5})
\end{array} \right]
\end{aligned}
\]  

(7.31)

The load currents \(i_a\) and \(i_b\) are to be calculated using the load information. For instance, if the load is resistive and is connected as delta, then the equations describing the dynamics of the load are given by

\[
\begin{aligned}
\frac{d}{dt} \left[ \begin{array}{c}
i_a^j - i_b^j \\
i_a^j + 2i_b^j
\end{array} \right] &= -\frac{R_{Fj}}{L_{Fj}} \left[ \begin{array}{c}
i_a^j - i_b^j \\
i_a^j + 2i_b^j
\end{array} \right] - \frac{1}{L_{Fj}C_{Fj}} \int \left[ \begin{array}{c}
i_a^j - i_b^j \\
i_a^j + 2i_b^j
\end{array} \right] dt \\
+ \frac{3}{L_{Fj}} \left[ \begin{array}{c}
V_{ab} \\
V_{bc}
\end{array} \right]; \ j = 2, 3
\end{aligned}
\]  

(7.32)

For given input voltages \(V_{AB}\) and \(V_{BC}\), and for a given load, the nonlinear differential equations in Eq. (7.23), Eq. (7.25), Eq. (7.27), Eq. (7.30), Eq. (7.31), and Eq. (7.32) (along with the load equations) can be solved numerically for \(i_a\), \(i_b\), \(i_{E4}\), \(i_{E5}\), \(V_{a'b'}\), \(V_{b'c'}\), \(V_{ab}\), \(V_{bc}\), zig-zag transformer currents, filter currents, and load currents. Other system variables can be solved for via existing static equations such as those in Eq. (7.26).

### 7.8.2 Flux Linkage Model

A more accurate model for the core of an electromagnetic device can be obtained by including a resistance (modeling the core losses) in parallel with the magnetizing inductance. Such a model for the core of a two-winding transformer is shown in Fig. 66.

Using this parallel combination to model the line chokes, two-winding and four-winding transformers, we can obtain an alternative model for the datawave system. In this model, it is convenient to select the windings' flux linkages as system variables. The current/flux relationship for the transformer \(TX_j\) is given by
\[ \lambda_j = L_j i_{mj} + \lambda_{j,\text{res}} \]  
(7.33)

where \( L_j \) is now the apparent mutual inductance between the windings of \( TX_j \) and \( i_{mj} \) is the magnetizing current. Letting \( G = \frac{1}{R_c} \) in Fig. 66, Eq. (7.33) can be written as

\[ \lambda_j = L_j (i_{Ej} - G j e_j) + \lambda_{j,\text{res}} \]

and if we redefine \( \lambda_j \) as \( \lambda_j = \lambda_j - \lambda_{j,\text{res}} \), we get

\[ \lambda_j = L_j (i_{Ej} - G j e_j); j = 1, 2, ..., 9 \]
(7.34)

where \( \lambda_j \) is now the total flux (including residual flux) linking the windings of \( TX_j \).

Referring to Fig. 65, by KCL we have the following relations among the exciting currents of the two- and four-winding transformers

\[
\begin{align*}
    i_{E1} &= 2i_{E4} + i_{E5} \\
    i_{E2} &= i_{E5} - i_{E4} \\
    i_{E3} &= -i_{E4} - 2i_{E5} \\
    i_{E6} &= -i_{E4} - i_{E5}
\end{align*}
\]
(7.35)

Substituting for \( i_{Ej} = \frac{\lambda_j}{L_j} + G j e_j \), we get

\[
\begin{align*}
    \frac{\lambda_1}{L_1} - 2\frac{\lambda_4}{L_4} - \frac{\lambda_5}{L_5} &= 2G_4 e_4 + G_5 e_5 - G_1 e_1 \\
    \frac{\lambda_2}{L_2} - \frac{\lambda_5}{L_5} + \frac{\lambda_4}{L_4} &= G_5 e_5 - G_4 e_4 - G_2 e_2
\end{align*}
\]

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\[
\frac{\lambda_3}{L_3} + \frac{\lambda_4}{L_4} + 2\frac{\lambda_5}{L_5} = -G_4e_4 - 2G_5e_5 - G_3e_3 \tag{7.36}
\]
\[
\frac{\lambda_6}{L_6} + \frac{\lambda_4}{L_4} + \frac{\lambda_5}{L_5} = -G_4e_4 - G_5e_5 - G_6e_6
\]

For the chokes we have
\[
\frac{\lambda_7}{L_7} + \frac{\lambda_8}{L_8} + \frac{\lambda_9}{L_9} = G_7V_7 - G_8V_8 - G_9V_9 \tag{7.37}
\]

Other loop and node equations are
\[
V_{AB} = V_7 - V_8 + V_{a'b'}
\]
\[
V_{BC} = V_8 - V_9 + V_{b'c'}
\tag{7.38}
\]
\[
\frac{\lambda_7}{L_7} + G_7V_7 - i_4 = c_1 \left( 2\frac{dV_{a'b'}}{dt} + \frac{dV_{b'c'}}{dt} \right)
\]
\[
\frac{\lambda_8}{L_8} + G_8V_8 - i_5 = c_1 \left( 2\frac{dV_{b'c'}}{dt} + \frac{dV_{a'b'}}{dt} \right)
\tag{7.39}
\]
\[
i_a + i_4' + c_2 \left( 2\frac{dV_{ab}}{dt} + \frac{dV_{bc}}{dt} \right) + i_a^2 + i_a^3 + i_a^{zz} = 0
\tag{7.40}
\]
\[
i_b + i_5' + c_2 \left( \frac{dV_{bc}}{dt} - \frac{dV_{ab}}{dt} \right) + i_b^2 + i_b^3 + i_b^{zz} = 0
\]
\[
v_{a'b'} = e_1 - 2e_2 + e_3 + e_4 - e_5 + r(i_4 - i_5)
\tag{7.41}
\]
\[
v_{ab} = e_1 - 2e_2 + e_3 + e_4 - e_5 + r(i_4' - i_5')
\tag{7.42}
\]
\[
v_{b'c'} = e_1 + e_2 - 2e_3 + e_5 - e_6 + r(i_4 + 2i_5)
\tag{7.43}
\]
\[
v_{bc} = e_1 + e_2 - 2e_3 + e_5 - e_6 + r(i_4' + 2i_5')
\tag{7.44}
\]

where \( r = 2r_{4-\omega} + r_{2-\omega} \).
Adding Eq. (7.41) to Eq. (7.42), and Eq. (7.43) to Eq. (7.44), we get
\[ V_{a'b'} + V_{ab} = 2(e_1 - 2e_2 + e_3 + e_4 - e_5) + r \left( \frac{\lambda_4}{L_4} + G_4e_4 - \frac{\lambda_5}{L_5} - G_5e_5 \right) \]
\[ V_{\beta'\zeta} + V_{bc} = 2(e_1 + e_2 - 2e_3 + e_5 - e_6) + r \left( \frac{\lambda_4}{L_4} + G_4e_4 + 2\frac{\lambda_5}{L_5} + 2G_5e_5 \right) \] (7.45)

Now subtracting Eq. (7.42) from Eq. (7.41), and Eq. (7.44) from Eq. (7.43), we get
\[ V_{a'b'} - V_{ab} = r[(i_4 - i_4') - (i_5 - i_5')] \] (7.46)
\[ V_{\beta'\zeta} - V_{bc} = r[(i_4 - i_4') + 2(i_5 - i_5')] \]

Similarly, from Eq. (7.39) and Eq. (7.40) we can get
\[ \frac{\lambda_7}{L_7} + G_7V_7 - \frac{\lambda_4}{L_4} - G_4e_4 - i_a - i_a^2 - i_a^3 - i_a^{i''} = \]
\[ = c_1 \left( \frac{2dV_{a'b'}}{dt} + \frac{dV_{\beta'\zeta}}{dt} \right) + c_2 \left( \frac{2dV_{ab}}{dt} + \frac{dV_{bc}}{dt} \right) \]
\[ \frac{\lambda_8}{L_8} + G_8V_8 - \frac{\lambda_5}{L_5} - G_5e_5 - i_b - i_b^2 - i_b^3 - i_b^{i''} = \]
\[ = c_1 \left( \frac{dV_{\beta'\zeta}}{dt} - 2\frac{dV_{a'b'}}{dt} \right) + c_2 \left( \frac{dV_{bc}}{dt} - \frac{dV_{ab}}{dt} \right) \] (7.47)

\[ \frac{\lambda_7}{L_7} + G_7V_7 - (i_4 - i_4') + i_a + i_a^2 + i_a^3 + i_a^{i''} = \]
\[ = c_1 \left( \frac{2dV_{a'b'}}{dt} + \frac{dV_{\beta'\zeta}}{dt} \right) - c_2 \left( \frac{2dV_{ab}}{dt} + \frac{dV_{bc}}{dt} \right) \]
\[ \frac{\lambda_8}{L_8} + G_8V_8 - (i_5 - i_5') + i_b + i_b^2 + i_b^3 + i_b^{i''} = \]
\[ = c_1 \left( \frac{dV_{\beta'\zeta}}{dt} - \frac{dV_{a'b'}}{dt} \right) + c_2 \left( \frac{dV_{bc}}{dt} - \frac{dV_{ab}}{dt} \right) \] (7.48)

Using Farady's law \( e_j = \frac{d\lambda_j}{dt} \), Eq. (7.36) - Eq. (7.38) and Eq. (7.45) - Eq. (7.48), along with the filters and zig-zag transformer equations given in the
previous section, can be put in the form of matrix differential equations. Except filter, zig-zag and load equations, there are fifteen equations and fifteen variables. These variables are the nine flux linkages \( \lambda \), the four voltages \( V_{d'v'}, V_{y'y'}, V_{ab} \) and \( V_{bc} \), and the two currents \( i_4 - i'_4 \) and \( i_5 - i'_5 \).

The estimate of the apparent inductances used in the flux linkage model can be obtained from the incremental inductances of the previous section; these inductances are

\[
L_{ch} = \begin{cases} 
0.0028H & \text{for } |i_{ch}| \leq 20A \\
0.025 \frac{1}{\sqrt{|i_{ch}|}} - 0.056 \frac{1}{|i_{ch}|} H & \text{for } |i_{ch}| > 20A
\end{cases}
\]

\[
L_{2-W} = 0.110 \frac{\arctan(0.3i_E)}{i_E} + 4.5E - 5
\]

\[
L_{4-W} = 0.064 \frac{\arctan(0.173i_E)}{i_E} + 1.5E - 5
\]

7.9 Datawave System Simulation And Model Validation

The final step in the development of the datawave system model is to check its adequacy for the intended applications. That is, based on some quantitative measures, we must decide whether our model represents the physical system with an acceptable degree of accuracy in its domain of applicability.

To identify those circumstances under which the accuracy of the model is of importance to us, we recall our motivations in modeling the datawave. These motivations were: a) understanding the details of the datawave operation; b) improving its performance, by way of design modifications, such as to eliminate the "oscillation" problem and the problem of output voltage collapse resulting from input voltage notches (due to utility switchings under no load to full load conditions); and c) pretesting proposed systems, such as a system in which the datawave operates in conjunction with an uninterruptable power supply, before building expensive prototypes or carrying out field tests. Model simulation results are to determine the implications and consequences of a proposed course of action.

With these motivations in mind, we test the accuracy of our model by comparing its response to a set of inputs (input voltages and output loads) with the response of the datawave to the same set of inputs. This comparison can be done in several ways: a) a time-domain signal resulting from the computer simulation of the model and the corresponding measured signal can be plotted on the same axis.
and then graphically compared; b) simulated and actual signals can be compared based on error criteria such as absolute mean and rms errors; c) spectral density plots of the actual and simulated signals can be compared; etc.

In this section, we use both graphical and spectral comparisons in our model validation. Graphical comparisons are used to develop a trust in the model; as engineers, we tend to have difficulties trusting a model which has been validated using only some mathematical criterion. With the datawave being nonlinear and a source of harmonics, it is appropriate to use spectral densities to compare the harmonics generated by the system with those generated by the model.

When a signal $f(t)$ reaches steady state and becomes periodic, it can be represented by a Fourier series of the form

$$f(t) = \sum_{K=0}^{\infty} \left[ A_K \cos(K\omega_0 t) + B_K \sin(K\omega_0 t) \right]$$

$$= C_0 + \sum_{K=1}^{\infty} C_K \cos(K\omega_0 t - \theta_K)$$

(7.49)

where

$$C_0 = A_0$$

$$C_K = (A_K^2 + B_K^2)^{1/2}$$

$$\theta_K = \arctan \left( \frac{B_K}{A_K} \right) \quad \text{for } k = 1, 2, ...$$

and $\omega_0$ is the angular frequency of the fundamental component. The plot of $C_K$ as a function of $K$ is referred to as the spectral density plot for the function $f(t)$.

The block diagram of Fig. 67 depicts the model validation procedure, adopted in this section as part of overall model development procedure. To obtain computer simulation results, we need to solve the model equations, given in Section 7.8, numerically. In the next section, the numerical solution of these equations is discussed.

7.9.1 Datawave System Simulation

The nonlinear differential equations describing the dynamics of the datawave, given in Section 7.8.1, can be put in the state space form
\[ \frac{dx}{dt} = A(X)X + B(X)U(t) \] (7.50)

where the state vector \( X \) is given by
\[
X = [i_A \ i_B \ V_{a'd'} \ V_{b'c'} \ V_{ab} \ V_{bc} \ i_{E4} \ i_{E5}]^T
\]
the input vector \( U \) by
\[
U = [V_{AB}(t) \ V_{BC}(t)]^T
\]
and the \( 8 \times 8 \) and \( 8 \times 2 \) coefficient matrices \( A(X) \) and \( B(X) \) contain entries which are functions of the currents in \( X \).

In discrete form with \( T \) as the sampling period, Eq. (7.50) is written as
\[
X((K+1)T) = A^*(X(KT))X(KT) + B^*(X(KT))U(KT) \] (7.51)

with
\[
A^* = e^{AT} \]
\[
B^* = A^{-1}(A^* - I)B \] (7.52)
where it has been assumed that the entries of the matrices $A$ and $B$ remain constant during the intervals between $KT$ and $(K+1)T$.

Knowing the initial conditions $X(0)$, and for given input voltages $V_{AB}$ and $V_{BC}$, we can begin the simulation process by calculating $A^*(X(0))$ and $B^*(X(0))$ from Eq. (7.52) and using them in Eq. (7.51) to find $X(T)$. Knowing $V_{AB}(T)$ and $V_{BC}(T)$, filters, zig-zag transformer and load currents, for the instant $2T$, can be calculated from their corresponding differential equations. Knowing $X(T)$, $A^*(X(T))$ and $B^*(X(T))$ can be calculated and then used to find $X(2T)$, and so on.

It should now be clear that the procedure described above is not attractive. It requires the calculation of a matrix exponent and a matrix inversion at each iteration. With the size of the matrix being $8 \times 8$, this procedure is not practical for most commonly-available computers. To avoid the time-consuming calculations of $e^{AT}$ and $A^{-1}$ at each iteration, a modular approach to the solution of the datawave equations is adopted. In this approach, the time derivative of a state variable $x$ at the $K$th sampling interval is estimated by

$$\frac{dx}{dt} \bigg|_{t=KT} = \frac{x((K+1)T) - x(KT)}{T}$$

from which

$$x((K+1)T) = x(KT) + T \left. \frac{dx}{dt} \right|_{t=KT}$$

This is equivalent to numerical integration of $\frac{dx}{dt}$ using rectangular rule; it is also equivalent to estimating $e^{AT}$ by the first two terms of its Maclaurin series. Here, accuracy is sacrificed for speed. However, for a small sampling interval $T$, the accuracy will not be compromised significantly.

The procedure for the solution of the datawave dynamic equations is given below. For convenience, the sampling interval $T$ is omitted, and $K$ is used to denote $KT$.

**Step 1)** Using the choke loop equations of Section 7.8.1, calculate the choke currents for instant $K + 1$ by

$$\begin{bmatrix} i_A(K+1) \\ i_B(K+1) \end{bmatrix} = -\frac{R_{ch}T}{B} \begin{bmatrix} 2L_8 + L_9 & L_8 - L_9 \\ L_7 - L_9 & 2L_7 + L_9 \end{bmatrix} \begin{bmatrix} i_A(K) \\ i_B(K) \end{bmatrix}$$
\[
\begin{align*}
&+ \left[ \begin{array}{c}
i_A(K) \\
i_B(K)
\end{array} \right] + \frac{T}{B} \left[ \begin{array}{cc}
L_8 + L_9 & L_8 \\
-L_9 & L_7
\end{array} \right]_K \left[ \begin{array}{c}
V_{AB}(K) - V_{a'b'}(K) \\
V_{BC}(K) - V_{b'c'}(K)
\end{array} \right] \\
\text{where } B = (L_7L_8 + L_9L_7 + L_9L_8)_K \text{ and all inductances are calculated from their corresponding estimators and their current values at instant } K.
\end{align*}
\]

**Step 2)** Calculate the primary capacitor voltages at instant \( K + 1 \) by
\[
\left[ \begin{array}{c}
V_{a'b'}(K + 1) \\
V_{b'c'}(K + 1)
\end{array} \right] = \left[ \begin{array}{c}
\frac{T}{R_C} + \frac{T}{6R_c} - 1 \\
\frac{T}{6R_c}
\end{array} \right] \left[ \begin{array}{c}
V_{a'b'}(K) \\
V_{b'c'}(K)
\end{array} \right] + \frac{T}{3c_1} \left[ \begin{array}{c}
i_A - i_B + \frac{1}{2}(i_{E5} - i_{E4}) \\
i_A + 2i_B - \frac{1}{2}i_{E4} - i_{E5}
\end{array} \right]_K.
\]

**Step 3)** Calculate the exciting currents of \( TX4 \) and \( TX5 \) at instant \( K + 1 \) by
\[
\left[ \begin{array}{c}
i_{E4}(K + 1) \\
i_{E5}(K + 1)
\end{array} \right] = \left[ \begin{array}{c}
\frac{rT}{2A} \left[ \begin{array}{ccc}
LL_3 - 2LL_2 & -LL_2 & -LL_3 \\
LL_1 - LL_3 & 2LL_1 + LL_3
\end{array} \right]_K \\
- \left[ \begin{array}{c}
i_{E4}(K) \\
i_{E5}(K)
\end{array} \right] + \frac{T}{2A} \left[ \begin{array}{c}
LL_4 & -LL_2 \\
-LL_3 & LL_1
\end{array} \right]_K \left[ \begin{array}{c}
V_{ab} + V_{a'b'} \\
V_{bc} + V_{b'c'}
\end{array} \right]_K.
\]

**Step 4)** Calculate the load currents from the load information; i.e., for a 3-phase delta connected resistive load
\[
\left[ \begin{array}{c}
i_a(K) \\
i_b(K)
\end{array} \right] = \frac{1}{R_{load}} \left[ \begin{array}{cc}
2 & 1 \\
-1 & 1
\end{array} \right] \left[ \begin{array}{c}
V_{ab}(K) \\
V_{bc}(K)
\end{array} \right]
\]

**Step 5)** Calculate the second and third harmonic filter currents by
\[
\left[ \begin{array}{c}
i_a(K + 1) \\
i_b(K + 1)
\end{array} \right] = \left( \frac{1 - T}{L_{Fj}} \right) \left[ \begin{array}{c}
i_a(K) \\
i_b(K)
\end{array} \right] - \frac{T}{L_{Fj} C_{Fj}} \int_0^K \left[ \begin{array}{c}
i_a \\
i_b
\end{array} \right] dt + \\
\frac{T}{L_{Fj}} \left[ \begin{array}{c}
2V_{ab} + V_{bc} \\
V_{bc} - V_{ab}
\end{array} \right]_K \text{ for } j = 2, 3
\]
where the integral terms are evaluated numerically using Trapezoidal rule; i.e.,
\[
I_a^j(K + 1) = I_a^j(K) + \frac{T}{2} \left[ i_a^j(K + 1) + i_a^j(K) \right]
\]
\[
I_a^j(K) = \int_0^{KT} i_a^j dt
\]

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Step 6) Calculate the zig-zag transformer currents,

\[
\begin{bmatrix}
 i^{zz}(K+1) \\
i^{zz}(K+1)
\end{bmatrix} = \left(1 - T \frac{R_{zz}}{3L_{zz}}\right) \begin{bmatrix}
i^{zz}(K) \\
i^{zz}(K)
\end{bmatrix} + \frac{T}{9L_{zz}} \begin{bmatrix}
2 & 1 \\
-1 & 1
\end{bmatrix} \begin{bmatrix}
V_{ab}(K) \\
V_{bc}(K)
\end{bmatrix}
\]

Step 7) Calculate the output voltages \( V_{ab} \) and \( V_{bc} \),

\[
\begin{bmatrix}
V_{ab}(K+1) \\
V_{bc}(K+1)
\end{bmatrix} = \left(1 - \frac{T}{R_{C2}C_2} + \frac{T}{6rC_2}\right) \begin{bmatrix}
V_{ab}(K) \\
V_{bc}(K)
\end{bmatrix} - \frac{T}{6rC_2} \begin{bmatrix}
V_{d'b'}(K) \\
V_{d'c'}(K)
\end{bmatrix} - \frac{T}{3C_2} \begin{bmatrix}
i_a - i_b + i_a^2 - i_b^2 + i_a^3 - i_b^3 + i^{zz}_a - i^{zz}_b + \frac{1}{2}(i_{E5} - i_{E4}) \\
i_a + 2i_b + i_a^2 + 2i_b^2 + i_a^3 + 2i_b^3 + i^{zz}_a + 2i^{zz}_b - \frac{1}{2}i_{E4} - i_{E5}
\end{bmatrix}
\]

Step 8) Continue with Step 1.

### 7.9.2 System Model Validation

We now have the necessary tools to check the adequacy of the developed model. Based on our motivations and the model validation procedure discussed earlier, the credibility of the datawave system model is checked under different conditions. First, the adequacy of the model under different loading conditions from no-load to full-load is tested. For all cases, the actual and simulated output voltages are compared graphically and the spectral density comparison is used for the choke currents. Occasionally, other signals are compared graphically.

Figs. 68 through 89 show the actual and simulated datawave signals for delta connected resistive loads. The correspondence between these figures and the system loading is shown in Table 11.

<table>
<thead>
<tr>
<th>Load Current in Amps</th>
<th>90</th>
<th>60</th>
<th>40</th>
<th>20</th>
<th>0 (Normal)</th>
<th>0 (Oscillatory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figures</td>
<td>68-75</td>
<td>76-78</td>
<td>79-83</td>
<td>84-85</td>
<td>88-87</td>
<td>88-89</td>
</tr>
</tbody>
</table>

The graphical and spectral comparisons shown indicate that the developed model adequately represents the datawave system under all loading conditions from full-load to no-load.
Since one of our motivations for developing our model was to solve the oscillation problem, the validity of the model in the oscillatory mode of operation must be tested. To do this, we applied the input voltages obtained during the oscillation test to the datawave system model. As it can be seen from Fig. 88-89, the response of the model is also oscillatory, and the model does represent the datawave in the oscillatory mode of operation.
Figure 68: Actual and simulated output voltage for a load of 90A.

Figure 69: Actual and simulated TX4 exciting current for a load of 90A.
Figure 70: Actual and simulated steady state choke current for a load of 90A
Figure 71: Actual and simulated choke current spectral densities for a load of 90A.

Numerical Values for the Plots of Fig. 71

<table>
<thead>
<tr>
<th>FREQ</th>
<th>ACTUAL</th>
<th>SIMULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.45656</td>
<td>1.3589</td>
</tr>
<tr>
<td>60</td>
<td>149.25</td>
<td>141.87</td>
</tr>
<tr>
<td>120</td>
<td>0.50366</td>
<td>4.5146</td>
</tr>
<tr>
<td>180</td>
<td>1.1291</td>
<td>6.1295</td>
</tr>
<tr>
<td>240</td>
<td>0.029451</td>
<td>0.34642</td>
</tr>
<tr>
<td>300</td>
<td>5.9148</td>
<td>4.2525</td>
</tr>
<tr>
<td>360</td>
<td>0.025044</td>
<td>3.1291</td>
</tr>
<tr>
<td>420</td>
<td>1.0793</td>
<td>8.4152</td>
</tr>
<tr>
<td>480</td>
<td>0.029607</td>
<td>0.099022</td>
</tr>
<tr>
<td>540</td>
<td>0.023379</td>
<td>0.068314</td>
</tr>
<tr>
<td>600</td>
<td>0.20631</td>
<td>0.063905</td>
</tr>
<tr>
<td>660</td>
<td>0.36575</td>
<td>0.62835</td>
</tr>
<tr>
<td>720</td>
<td>0.042376</td>
<td>0.085459</td>
</tr>
</tbody>
</table>
Figure 72: Actual and simulated steady state exciting current for a load of 90A.

Figure 73: Actual and simulated steady state output voltage for a load of 90A.
Figure 74: Simulated steady state choke voltage for a load of 90A.

Figure 75: Simulated TX4 voltage for a load of 90A.
Figure 76: Actual and simulated output voltages for a load of 60A.

Figure 77: Actual and simulated choke current for a load of 60A.
Figure 78: Actual and simulated choke current spectral densities for a load of 60A.

**Numerical Values for the Plots of Fig. 78**

<table>
<thead>
<tr>
<th>FREQ</th>
<th>ACTUAL</th>
<th>SIMULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.751</td>
<td>.073043</td>
</tr>
<tr>
<td>60</td>
<td>101.53</td>
<td>93.444</td>
</tr>
<tr>
<td>120</td>
<td>.2508</td>
<td>5.0497</td>
</tr>
<tr>
<td>180</td>
<td>1.2489</td>
<td>.51556</td>
</tr>
<tr>
<td>240</td>
<td>.20236</td>
<td>1.5035</td>
</tr>
<tr>
<td>300</td>
<td>3.3262</td>
<td>1.6221</td>
</tr>
<tr>
<td>360</td>
<td>.082859</td>
<td>.15714</td>
</tr>
<tr>
<td>420</td>
<td>.27522</td>
<td>.23098</td>
</tr>
<tr>
<td>480</td>
<td>.090153</td>
<td>.20017</td>
</tr>
<tr>
<td>540</td>
<td>.19167</td>
<td>.092089</td>
</tr>
<tr>
<td>600</td>
<td>.28516</td>
<td>.13117</td>
</tr>
<tr>
<td>660</td>
<td>.4924</td>
<td>.32172</td>
</tr>
<tr>
<td>720</td>
<td>.12905</td>
<td>.017499</td>
</tr>
</tbody>
</table>
Figure 79: Actual and simulated output voltage for a load of 40A.

Figure 80: Actual and simulated choke current for a load of 40A.
Figure 81: Actual and simulated choke current spectral densities for a load of 40A.

Numerical Values for the Plots of Fig. 81

<table>
<thead>
<tr>
<th>FREQ</th>
<th>ACTUAL</th>
<th>SIMULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.9107</td>
<td>.674</td>
</tr>
<tr>
<td>60</td>
<td>68.157</td>
<td>69.657</td>
</tr>
<tr>
<td>120</td>
<td>1.8011</td>
<td>2.9701</td>
</tr>
<tr>
<td>180</td>
<td>1.0819</td>
<td>.61759</td>
</tr>
<tr>
<td>240</td>
<td>.32313</td>
<td>1.463</td>
</tr>
<tr>
<td>300</td>
<td>2.3325</td>
<td>1.6171</td>
</tr>
<tr>
<td>360</td>
<td>.083548</td>
<td>.46315</td>
</tr>
<tr>
<td>420</td>
<td>.10109</td>
<td>.060082</td>
</tr>
<tr>
<td>480</td>
<td>.089877</td>
<td>.030021</td>
</tr>
<tr>
<td>540</td>
<td>.122</td>
<td>.038386</td>
</tr>
<tr>
<td>600</td>
<td>.11854</td>
<td>.064915</td>
</tr>
<tr>
<td>660</td>
<td>.27232</td>
<td>.19384</td>
</tr>
<tr>
<td>720</td>
<td>.05842</td>
<td>.10748</td>
</tr>
</tbody>
</table>
Figure 82: Simulated TX4 voltage for a load of 40A.

Figure 83: Figure 82 expanded.
Figure 84: Actual and simulated output voltage for a load of 20A.
Figure 85: Actual and simulated choke current spectral densities for a load of 20A.

Numerical Values for the Plots of Fig. 85

<table>
<thead>
<tr>
<th>FREQ</th>
<th>ACTUAL</th>
<th>SIMULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.3201</td>
<td>11.015</td>
</tr>
<tr>
<td>60</td>
<td>40.925</td>
<td>40.224</td>
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<td>.96599</td>
</tr>
<tr>
<td>180</td>
<td>.45104</td>
<td>.75313</td>
</tr>
<tr>
<td>240</td>
<td>.02579</td>
<td>.12201</td>
</tr>
<tr>
<td>300</td>
<td>.32321</td>
<td>.085510</td>
</tr>
<tr>
<td>360</td>
<td>.023651</td>
<td>.13998</td>
</tr>
<tr>
<td>420</td>
<td>.14597</td>
<td>.20506</td>
</tr>
<tr>
<td>480</td>
<td>.019654</td>
<td>.013135</td>
</tr>
<tr>
<td>540</td>
<td>.033595</td>
<td>.038283</td>
</tr>
<tr>
<td>600</td>
<td>.045199</td>
<td>.030802</td>
</tr>
<tr>
<td>660</td>
<td>.36876</td>
<td>.38595</td>
</tr>
<tr>
<td>720</td>
<td>.022627</td>
<td>.052627</td>
</tr>
</tbody>
</table>
Figure 86: Actual and simulated output voltage under no load.
Figure 87: Actual and simulated choke current spectral densities under no load

Numerical Values for the Plots of Fig. 87

<table>
<thead>
<tr>
<th>FREQ</th>
<th>ACTUAL</th>
<th>SIMULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.464</td>
<td>80.622</td>
</tr>
<tr>
<td>60</td>
<td>29.517</td>
<td>34.912</td>
</tr>
<tr>
<td>120</td>
<td>17.448</td>
<td>14.762</td>
</tr>
<tr>
<td>180</td>
<td>2.0008</td>
<td>4.4308</td>
</tr>
<tr>
<td>240</td>
<td>6.2825</td>
<td>3.807</td>
</tr>
<tr>
<td>300</td>
<td>2.7848</td>
<td>2.6395</td>
</tr>
<tr>
<td>360</td>
<td>1.131</td>
<td>1.2511</td>
</tr>
<tr>
<td>420</td>
<td>0.37593</td>
<td>0.5296</td>
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<tr>
<td>540</td>
<td>0.10916</td>
<td>0.081272</td>
</tr>
<tr>
<td>600</td>
<td>0.11069</td>
<td>0.077428</td>
</tr>
<tr>
<td>660</td>
<td>0.046468</td>
<td>0.109</td>
</tr>
<tr>
<td>720</td>
<td>0.0063987</td>
<td>0.14539</td>
</tr>
</tbody>
</table>
Figure 88: Actual and simulated output voltage under oscillatory operation.
Figure 89: Actual and simulated choke current spectral densities under oscillatory operation.

Numerical Values for the Plots of Fig. 89

<table>
<thead>
<tr>
<th>FREQ (Hz)</th>
<th>ACTUAL</th>
<th>SIMULATED</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>160.93</td>
<td>161.24</td>
</tr>
<tr>
<td>60</td>
<td>59.033</td>
<td>69.824</td>
</tr>
<tr>
<td>120</td>
<td>34.895</td>
<td>29.523</td>
</tr>
<tr>
<td>180</td>
<td>4.0016</td>
<td>8.8615</td>
</tr>
<tr>
<td>240</td>
<td>12.565</td>
<td>7.614</td>
</tr>
<tr>
<td>300</td>
<td>5.5768</td>
<td>5.279</td>
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<tr>
<td>360</td>
<td>2.262</td>
<td>2.5022</td>
</tr>
<tr>
<td>420</td>
<td>0.75187</td>
<td>1.0592</td>
</tr>
<tr>
<td>480</td>
<td>0.72759</td>
<td>0.48668</td>
</tr>
<tr>
<td>540</td>
<td>0.21832</td>
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</tr>
<tr>
<td>600</td>
<td>0.22139</td>
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<td>660</td>
<td>0.092937</td>
<td>0.21799</td>
</tr>
<tr>
<td>720</td>
<td>0.012797</td>
<td>0.29078</td>
</tr>
</tbody>
</table>
7.10 Conclusions

Due to the existence of magnetic saturation and harmonic signals, it was necessary to use time-domain operating data to develop models for the line chokes, the four-winding, and the two-winding transformers. Algorithms for modeling magnetic saturation and hysteresis presented in Chapter 4 were applied to derive the component models in this chapter.

Two formulations of the system model mathematics; namely, the current and the flux linkage formulations, were presented. In comparing the two models developed here, we make the following observations:

i. The flux linkage model describes the dynamics of the datawave in more detail; it is a more accurate representation of the physical system.

ii. The number of variables (except the load, zig-zag and filter variables) in the flux linkage model is 1.5 times greater (15 compared to 10); the flux linkage model is a more complex model.

iii. For each iteration, the computer time required to solve for the model variables during simulation is at least 1.5 times greater for the flux linkage model. Based on these observations, a decision regarding the use of an appropriate model in a given application can be made. When the accuracy of the simulation results is of greater importance, the flux linkage model should be used. On the other hand, if the speed of simulation is of greater importance, the first model should be used.

It was shown that the current model adequately represents the power conditioning system for its entire loading range. The adequacy of the model was shown through graphical and spectral comparisons of actual and simulated system responses to the same inputs. To speed up the computer simulation process, a modular approach to the numerical solution of the system dynamic equations was taken. Since one of our goals is to use the model in studying the oscillation problem, as discussed in the next chapter, the validity of the model was tested and verified under oscillatory mode of operation.
CHAPTER VIII

IMPROVING TRANSIENT AND STEADY STATE PERFORMANCE OF SIGNAL CONDITIONING SYSTEMS

8.1 Introduction

A problem known as the "ferroresonant jump" associated with the operation of signal conditioning systems which use ferroresonant circuits has been reported in the literature [7,68,69,70,71]. Some researchers [69,70,71] have studied the occurrence of this phenomenon in single-phase conditioning systems with series or parallel ferroresonant circuits.

Ferroresonant jump is a multimodal operation problem which occurs in both single and three-phase signal conditioning systems. In the case of three-phase systems, this problem has never been studied and has no known solution. An analytical investigation leading to a solution for ferroresonant jump problem in three-phase conditioning systems is carried out in this chapter. The proposed solution has been successfully tested in the laboratory and can be easily implemented for the existing systems in the field.

A second problem addressed in this chapter is that of excessive output voltage "ringing" following a heavy load drop. A modification in system parameters leading to significant reduction in this ringing time is proposed and successfully tested in the laboratory.

8.2 Analytical Investigation of the Ferroresonant Jump Problem

Eq. 7.12 of Chapter 7 gives the estimator of the four-winding transformer magnetizing incremental inductance as

\[
\hat{L}(i_E) = \frac{0.011}{1 + 0.03i_E^2} + 1.5E - 5
\]

For a two-winding transformer, this estimate is given by
The average values of these inductances can be obtained by evaluating their integrals over their corresponding domains determined by the minimum and maximum values of the exciting currents involved ($i_{E,\text{min}}$ and $i_{E,\text{max}}$). These average values (results of the integrations) depend on the amplitudes, as well as the frequencies, of the harmonics of exciting currents.

It is due to magnetizing inductances' nonlinearities that the datawave system exhibits different modes of operation (jump phenomenon), a problem we have called the "oscillation" problem. For a given set of balanced input voltages, there may be several stable waveforms for each exciting current of the six transformers, and thus the system can reach any of the several possible steady states, one desirable and all others undesirable. The values of the different system parameters during the transient, prior to a steady state, will determine the final steady state attained by the system. The motivation for the analysis presented in this section is to develop a mathematical foundation based on which the oscillation problem could be explained.

8.2.1 Datawave Single-Phase Equivalent Circuit

We begin our analysis by synthesizing a single-phase equivalent circuit for the datawave; this is done to reduce our work in further analysis. In determining the equivalent circuit, the following simplifying assumptions are made:

i) All resistances are negligible; this does not have any drawbacks since the equivalent model is being developed for steady state analysis, and the small resistances being ignored provide damping during transience and do not have any significant effects on steady state operations.

ii) Filters and the zig-zag transformer are not included in the equivalent circuit.

iii) The output capacitors are assumed to be on the primary side.

iv) Since the equivalent model is being developed for the study of the oscillation problem, and this problem exists under no-load or light-load conditions, the derivation of the model will be for the no-load condition.

Referring to Fig. 7.30 of Chapter 7 (with the above assumptions), we can now write

\[
L(i_E) = \frac{0.033}{1 + 0.09i_E} + 4.5E - 5
\]
\[ V_{d'v'} = L_4 \frac{d^2 E_4}{dt^2} + L_1 \frac{d^2 E_1}{dt^2} - 2L_2 \frac{d^2 E_2}{dt^2} + L_3 \frac{d^2 E_3}{dt^2} - L_5 \frac{d^2 E_5}{dt^2} \]  

(8.1)

If, for notational simplicity, we let

\[ V_{d'v'} = V_c \]

\[ i_{Ej} = i_j \text{ for } j = 1, 2, ..., 6 \]

and if we use the relations

\[ i_1 = 2i_4 + i_5 \]
\[ i_2 = i_5i_4 \]
\[ i_3 = -i_4 - 2i_5 \]

in Eq. (8.1), we get

\[ V_c = (L_4 + 2L_1 + 2L_2 - L_3) \frac{d^2 i_4}{dt^2} + (L_1 - 2L_2 - 2L_3 - L_5) \frac{d^2 i_5}{dt^2} \]  

(8.2)

In general, the exciting currents \( i_4 \) and \( i_5 \) contain 60 Hz as well as harmonic components,

\[ i_j = \sum_K a_{jk} \cos(K\omega t - \theta_{jk}); j = 4, 5 \]

With this general form, however, the analytical investigation of the oscillation problem becomes a difficult and time-consuming task. As a first attempt, only the fundamental components of \( i_4 \) and \( i_5 \) are included in their expressions.

Under balanced conditions and positive sequence of input voltages, the steady state relation between the currents \( i_4 \) and \( i_5 \) (they are now assumed to be 60 Hz signals) is given by \( \bar{I}_5 = \bar{I}_4 \mid_{-120^\circ} \) which implies

\[ i_5 = -\frac{1}{2} \left( i_4 + \sqrt{3}\omega \frac{d i_4}{dt} \right) \]  

(8.3)
where \( \omega = 377 \text{rad/sec} \) is the angular frequency of the applied voltages.

Substituting for \( i_5 \) from Eq. (8.3) in Eq. (8.2) we get

\[
V_c = (3/2L_1 + 3L_2 + L_4 + 1/2L_5) \frac{d i_4}{dt} - \frac{\sqrt{3}}{2} \omega (-L_1 + 2L_2 + 2L_3 + L_5)i_4
\]

(8.4)

where the relation \( \frac{d^2 i_4}{dt^2} = -\omega^2 i_4 \) has been used.

In the chokes' loop, by KVL we have

\[
V_{AB} = L_7 \frac{d i_A}{dt} + V_c - L_8 \frac{d i_B}{dt}
\]

and by substituting \( I_A \mid -120^\circ \) for \( I_B \), we get

\[
V_{AB} = (L_7 + 1/2L_8) \frac{d i_A}{dt} - \frac{\sqrt{3}}{2} \omega L_8 i_A + V_c
\]

(8.5)

For the purpose of our analysis, we may assume linear chokes and let \( L_7 = L_8 = L_9 = L \). With this assumption, Eq. (8.5) becomes

\[
V = 3/2L \frac{di}{dt} - \frac{\sqrt{3}}{2} \omega Li + V_c
\]

(8.6)

where, again for notational simplicity, \( i \) has been used to denote \( i_A \) and \( V \) to denote \( V_{AB} \).

Eqs. (8.4) and (8.6) can now be used to synthesize our equivalent circuit as shown in Fig. 90. In this circuit, \( i_E \) has replaced \( i_4 \) to remind us that this current is an exciting current.

If we redefine the parameters of the equivalent circuit of Fig. 90 as

\[
L \triangleq \frac{3}{2}L
\]

\[
R \triangleq -\frac{\sqrt{3}}{2} \omega L
\]

\[
L_{eq} \triangleq \frac{3}{2}L_1 + 3L_2 + L_4 + 1/2L_5
\]

\[
R_{eq} \triangleq -\frac{\sqrt{3}}{2} \omega (2L_2 + 2L_3 - L_1 + L_5)
\]

Then we have

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Figure 90: Datawave single-phase equivalent circuit for steady state analysis.

\[
v = L \frac{di}{dt} + R_i + R_{eq}i_E + L_{eq} \frac{d^2i_E}{dt^2}
\]

(8.7)

\[
i_E = i - c \frac{dV}{dt}
\]

(8.8)

\[
V_c = V - Ri - L \frac{di}{dt}
\]

(8.9)

### 8.2.2 Determination of the Number of Steady State Solutions

In this section, we will find the number of steady state solutions of the line current \( i \) for a given input voltage \( V \). This can be done by substituting for \( i_E \) from Eq.’s (8.8) and (8.9) in Eq. (8.7),

\[
i_E = i - c \frac{dV}{dt} + L_c \frac{d^2i}{dt^2} + R_c \frac{di}{dt}
\]

(8.10)

\[
V = L \frac{di}{dt} + R_i + R_{eq} \left( i - c \frac{dV}{dt} + L_c \frac{d^2i}{dt^2} + R_c \frac{di}{dt} \right) + L_{eq} \left( \frac{di}{dt} - c \frac{d^2V}{dt^2} + L \frac{d^3i}{dt^3} + R \frac{d^2i}{dt^2} \right)
\]

Noting that in steady state \( \frac{d^2}{dt^2} = -\omega^2 \) and \( \frac{d^3}{dt^3} = -\omega^2 \frac{d}{dt} \), we get

\[
V = (R + R_{eq} - R_{eq} \omega^2 L_c - L_{eq} \omega^2 R_c)i +
\]
+ (L + \text{Re}_q Rc + L_{eq} - L_{eq} \omega^2 Lc) \frac{di}{dt} +
+ (\omega^2 L_{eq} c) V - \text{Re}_q \frac{dv}{dt}

The estimators of the transformers' magnetizing inductances have the general form of

\[ \hat{L}_j = \frac{a}{1 + bi_j^2} + d \quad \text{for } j = 1, 2, 3 \]

\[ \hat{L}_j = \frac{3a}{1 + 3bi_j^2} + d \quad \text{for } j = 4, 5, 6 \]

where the following steady state relations between \( i_4 \) and other exciting currents exist

\[ i_1 = \frac{3}{2} i_4 - \frac{\sqrt{3} d i_4}{2 \omega \frac{dt}{dt}} \]

\[ i_2 = -\frac{3}{2} i_4 - \frac{\sqrt{3} d i_4}{2 \omega \frac{dt}{dt}} \]

\[ i_3 = \frac{\sqrt{3} d i_4}{\omega \frac{dt}{dt}} \]

\[ i_5 = -\frac{1}{2}(i_4 + \frac{\sqrt{3} d i_4}{\omega \frac{dt}{dt}}) \]

Substituting for

\[ \text{Re}_q = -\frac{\sqrt{3}}{2} \omega \left[ \frac{2a}{1 + bi_2^2} + \frac{2a}{1 + bi_3^2} - \frac{a}{1 + bi_1^2} + \frac{3a}{1 + 3bi_3^2} + 6d \right] \]

\[ L_{eq} = \frac{3a}{2(1 + bi_1^2)} + \frac{3a}{1 + bi_2^2} + \frac{3a}{1 + 3bi_4^2} + \frac{3a}{2(1 + 3bi_5^2)} + 9d \]

in Eq. (8.11) and rearranging, we get

\[ \left[ V - Ri - L \frac{di}{dt} + 3\sqrt{3} \omega d \left( i(1 - \omega^2 Lc) + Rc \frac{di}{dt} - c \frac{dv}{dt} \right) \right] - \frac{169}{169} \]
\[-9d \left( \omega^2 cV - \frac{di}{dt}(1 - \omega^2 Lc) - \omega^2 Rci \right) \right] \Delta =

= -\frac{\sqrt{3}}{2} \omega(1 + 3bi_2^2) \left[ 2(1 + 3bi_1^2)(1 + 3bi_3^2)(2 + bi_2^2 + bi_3^2) + 

+ (1 + bi_2^2)(1 + bi_3^2)(2 + 3bi_1^2 - 3bi_5^2) \right] \left[ i(1 - \omega^2 Lc) + Rce\frac{di}{dt} - c\frac{dV}{dt} \right] + 

+ \frac{3}{2} a(1 + bi_3^2) \left[ 2(1 + bi_1^2)(1 + 3bi_3^2)(2 + bi_2^2 + 3bi_1^2) + 

+ (1 + bi_2^2)(1 + 3bi_1^2)(2 + bi_1^2 + 3bi_3^2) \right] \left[ \omega^2 cV - (1 - \omega^2 Lc) \frac{di}{dt} - \omega^2 Rci \right] (8.13)

where

\[ \Delta = (1 + bi_1^2)(1 + bi_2^2)(1 + bi_3^2)(1 + 3bi_2^2)(1 + 3bi_3^2) \]

For a given input voltage \( V(t) = V_m \cos(\omega t + \theta) \), the fundamental solution of \( i_4 = i_E \) (taken as reference) and that of \( i \) can be assumed to be of the forms

\[ i_E(t) = I_E \cos \omega t \]

\[ i(t) = I_m \cos(\omega t + \phi) \]

Substituting these solutions in Eq. (8.10) we can obtain the following relations

\[
\begin{bmatrix}
I_m \cos \phi \\
I_m \sin \phi
\end{bmatrix} = \frac{1}{D} \begin{bmatrix}
-1(1 + \omega^2 Lc) & -\frac{\sqrt{3}}{2} \omega^2 Lc \\
-\frac{\sqrt{3}}{2} \omega^2 Lc & (1 - \frac{3}{2} \omega^2 Lc)
\end{bmatrix} \begin{bmatrix}
I_E - \omega c V_m \sin \theta \\
-\omega c V_m \cos \theta
\end{bmatrix} (8.14)
\]

where \( D = 3\omega^2 Lc(1 - \omega^2 Lc) - 1 \) and \( L \) is the original choke inductance (not \( 3/2L \)).

From Eq. (8.12) we can obtain the following relations

\[ i_1^2 = \frac{3}{4} I_E^2 (2 + \cos 2\omega t + \sqrt{3} \sin 2\omega t) \]
\[ i_2^2 = \frac{3}{4} I_E^2 (2 + \cos 2\omega t - \sqrt{3} \sin 2\omega t) \]

\[ i_3^2 = \frac{3}{2} I_E^2 (1 - \cos 2\omega t) \]

\[ i_4^2 = \frac{1}{2} I_E^2 (1 + \cos 2\omega t) \]

\[ i_5^2 = \frac{1}{4} I_E^2 (2 - \cos 2\omega t - \sqrt{3} \sin 2\omega t) \]

Substituting these relations along with those given by Eq. (8.14) in Eq. (8.13), the following two equations can be derived after some tedious algebraic manipulations and ignoring higher than fundamental terms

\[ V_m \cos \theta \left[ \frac{1}{D} (3 \omega^4 L^2 c^2 - \frac{3}{2} \omega^2 L c)(J + \frac{3}{8} K b l_E^2) - \right. \]

\[ - \frac{3}{2D} \omega^2 L c (\frac{3}{8} K b l_E^2 + \frac{1}{2} a P \omega^2 c) + \left( J + \frac{3}{8} k b l_E^2 - \frac{3}{2} a P \omega^2 c \right) + \]

\[ + \frac{3}{4D} a \omega^4 L c^2 (1 - \frac{3}{2} \omega^2 L c)(P + Q b l_E^2) + \frac{3}{2D} a P \omega^2 c (\frac{3}{2} L c - 1)^2 \] +

\[ + \sqrt{3} V_m \sin \theta \left[ \frac{1}{2D} \omega^2 L c (J + \frac{3}{8} k b l_E^2) + \frac{1}{D} (3 \omega^4 L^2 c^2 - \frac{3}{2} \omega^2 L c) \right. \]

\[ - \left( \frac{3}{8} K b l_E^2 + \frac{1}{2} a P \omega^2 c \right) + \left( \frac{3}{8} K b l_E^2 + \frac{1}{2} a P \omega^2 c \right) + \]

\[ + \frac{1}{2D} a \omega^2 c (1 - \frac{3}{2} \omega^2 L c)^2 (P + Q b l_E^2) + \frac{3}{4D} a P \omega^4 L c^2 (\frac{3}{2} \omega^2 L c - 1) \right] - \]

\[ - \frac{\sqrt{3}}{D} I_E \left[ \frac{1}{2} \omega L (J + \frac{3}{8} K b l_E^2) + 3 \omega L (\omega^2 L c - \frac{1}{2})(\frac{3}{8} k b l_E^2 + \frac{1}{2} P \omega^2 c) + \right. \]

\[ + \frac{1}{2} a \omega (1 - \frac{3}{2} \omega^2 L c)^2 (P + Q b l_E^2) + \frac{3}{4} a P \omega^3 L c (\frac{3}{2} \omega^2 L c - 1) \] = 0 \hspace{1cm} (8.15)
\[ \sqrt{3V_m \cos \theta} \left[ \frac{1}{2D} \omega^2 Lc (J - \frac{3}{8} K b I_E^2) + \frac{1}{D} \left( \frac{3}{2} \omega^2 Lc - 3\omega^4 L^2 c^2 \right) \right] \]

\[ - \frac{1}{2D} a \omega^2 c (1 - \frac{3}{2} \omega^2 Lc)^2 (Q b I_E^2 - P) + \frac{3}{4D} a \omega^4 Lc^2 P (1 - \frac{3}{2} \omega^2 Lc) \]

\[ + V_m \sin \theta \left[ \frac{3}{2D} (\omega^2 Lc - 3\omega^4 L^2 c^2) (J - \frac{3}{8} K b I_E^2) \right] \]

\[ - \frac{3}{2D} \omega^2 Lc \left( \frac{3}{8} K b I_E^2 - \frac{1}{2} a \omega^2 c \right) + (J - \frac{3}{8} K b I_E^2 - \frac{3}{2} a \omega^2 c) + \]

\[ + \frac{3}{4D} a \omega^4 L c^2 (1 - \frac{3}{2} \omega^2 Lc) (Q b i_E^2 - P) - \frac{3}{2D} a \omega^2 c P (1 - \frac{3}{2} \omega^2 Lc)^2 \]

\[ - \frac{3}{D} I_E \left[ \omega L (\frac{1}{2} - \omega^2 Lc) (J - \frac{3}{8} K b I_E^2) - \frac{1}{2} \omega L \left( \frac{3}{8} K b I_E^2 - \frac{1}{2} a \omega^2 c \right) \right] \]

\[ + \frac{1}{4} a \omega^3 L c (1 - \frac{3}{2} \omega^2 Lc)^2 (Q b I_E^2 - P) - \frac{1}{2} a \omega P (1 - \frac{3}{2} \omega^2 Lc)^2 \]  \( = 0 \)  \( (8.16) \)

where

\[ J \triangleq 1 + \frac{15}{2} b I_E^2 + \frac{81}{4} b^2 I_E^4 + \frac{189}{8} b^3 I_E^6 + \frac{2835}{256} b^4 I_E^8 + \frac{729}{512} b^5 I_E^{10} \]

\[ K \triangleq 1 + 6 b I_E^2 + \frac{189}{16} b^2 I_E^4 + \frac{135}{16} b^3 I_E^6 + \frac{405}{256} b^4 I_E^8 \]

\[ P \triangleq 6 + 36 b I_E^2 + \frac{567}{8} b^2 I_E^4 + \frac{2405}{8} b^3 I_E^6 + \frac{1215}{128} b^4 I_E^8 \]

\[ Q \triangleq (2 + 3 b I_E^2) \left( \frac{9}{2} + \frac{27}{2} b I_E^2 + \frac{81}{16} b^2 I_E^4 \right) \]

Eqs. (8.15) and (8.16) are to be solved (for a given \( V_m \)) simultaneously for \( \theta \) and \( I_E \). These equations contain polynomials of degree eleven in \( I_E \), and sine and cosine terms in \( \theta \). There are eleven solutions for \( I_E \). Among these solutions, only the positive real ones are acceptable; complex solutions are meaningless and for every negative one there will be a positive solution with a \( \theta \) of 180° shifted. The number of positive solutions, for a given \( V_m \), indicates the number of 60 Hz steady state modes of operation of the datawave system.

We have shown in [67] that for a given input voltage, there is only one acceptable solution of \( i_E(t) \) corresponding to a value of the choke inductance. That is, if there were no harmonic components in the line and exciting currents, the problem of oscillation (ferroresonant jump) would not exist.
8.3 Solution to the Ferroresonant Jump Problem

One way to eliminate the multimodal characteristic of the datawave is suggested by the analysis of the previous sections. It was shown in Section 8.2 that, for a given input voltage, there will be only one steady state solution for the transformer current if harmonics and subharmonics are ignored. This implies that, by preventing other than 60 Hz currents from flowing into the windings of the saturating transformers, the datawave system will always be forced into a single steady state, regardless of the prior transients. However, it is not practical to have only 60 Hz currents in the transformers' windings, for some of the harmonics are generated within the transformers themselves. On the other hand, not all harmonics contribute to the oscillation problem. To determine which harmonics cause the datawave to become a multistable system, harmonic components must be included in the analytical investigation of the previous sections. Such an analysis, however, is a formidable task.

Since it is difficult to mathematically identify the harmonic components which are the source of the oscillation problem and, more importantly, since harmonic components found analytically to cause oscillation may not be generated as the result of datawave operation in its environment, we will use the actual oscillation data to identify such components. That is, for a datawave installed in its environment, we create a transient which would lead the system to the oscillatory mode of operation. Then, we sample and collect the steady state choke currents in time domain. The spectral densities of these time-domain signals are then calculated and analyzed to identify the major harmonics.

For the datawave system installed in our laboratory, the necessary transient was created by adding a three-phase wye-connected capacitor bank at the input of the datawave, and energizing the system under no load, as shown in Fig. 91.

The choke currents $i_A$ and $i_B$ were sampled and collected. The current $i_A$ is shown in Fig. 92, its spectral density is plotted in Fig. 93 and tabulated in Table 12. It can be seen that the strongest component of the choke current has a frequency of about 100 Hz. It is therefore reasonable to, first, suspect this component to be the cause of oscillation; the second candidate would of course be the 15 Hz component.

As the first attempt to force the system into the normal (desired) steady state mode of operation, the 100 Hz component must be suppressed. This can be done by providing low impedance paths to divert the flow of this harmonic into the windings of the transformers. An impedance $Z(\omega)$ with the characteristic shown in Fig. 94 will serve our purpose.

One way to realize such an impedance characteristic is to use a series $LC$
Figure 91: Creating the transient leading to oscillatory mode of operation

filter. The impedance magnitude of a series $LC$ filter is given by

$$Z(\omega) = \omega L - \frac{1}{\omega C} = \frac{\omega^2 LC - 1}{\omega C}$$

For $Z(\omega)$ to be zero at $f = 100\text{Hz}$, we must have

$$LC = \frac{1}{\omega^2} = 2.533 \times 10^{-6} \text{ sec}^2$$

We can modify the second harmonic filter to obtain the desired value for the product $LC$. If we let the inductance remain at 0.027$H$, then the capacitance must be changed to

$$C = \frac{1}{\omega^2 L} = 94\mu F$$

The capacitance of the second harmonic filter was changed to 100$\mu F$ and the oscillation test (see Fig. 91) was repeated. The plot of the choke current $i_A$ is shown in Fig. 95, and its spectral density is plotted in Fig. 96 and tabulated in Table 13. It can be seen that the system has reached its normal steady state mode of operation. That is, in this case, the choke current is mainly 60Hz and its no-load magnitude is small as expected.
Figure 92: Choke current — oscillatory operation.

Figure 93: Choke current spectral density — oscillatory operation.
Figure 94: Impedance characteristic for suppression of the 100Hz component.

Table 12: Fig. 93 in Tabular Form

<table>
<thead>
<tr>
<th>Freq (Hz)</th>
<th>Amplitude (Amps)</th>
<th>Freq (Hz)</th>
<th>Amplitude (Amps)</th>
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<td>18.5</td>
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<tr>
<td>15</td>
<td>63.2</td>
<td>105</td>
<td>74.3</td>
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<tr>
<td>30</td>
<td>18.1</td>
<td>120</td>
<td>11.0</td>
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<td>6.6</td>
<td>135</td>
<td>14.5</td>
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<td>27.5</td>
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<tr>
<td>75</td>
<td>12.7</td>
<td>180</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 13: Fig. 96 in Tabular Form

<table>
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<th>Freq (Hz)</th>
<th>Amplitude (Amps)</th>
<th>Freq (Hz)</th>
<th>Amplitude (Amps)</th>
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<td>2.2</td>
<td>180</td>
<td>0.28</td>
</tr>
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</table>
Figure 95: Choke current — normal operation.

Figure 96: Choke current spectral density — normal operation.
It was shown that the problem of oscillation can be remedied by changing the capacitance of the second harmonic filter from 65\(\mu F\) to 94\(\mu F\). The undesirable effect of this change is the increase of the filter 60Hz losses from about 28\(W\) per phase to about 80\(W\) per phase. To minimize this increase in losses when redesigning the filter, it is important to keep the capacitance value as small as possible.

8.4 Analytical Investigation of the Output Voltage Ringing After a Sudden Load Drop

The term ringing is used to describe the transient created in the output voltages by a sudden drop of the datawave load.

Our goal is to reduce the load drop transient time. To study this transient, the laboratory set-up of Fig. 97 is used to capture and record the actual transient.

![Figure 97: Laboratory set-up for load drop transient recording.](image)

The datawave is first loaded up to full load of 80A. Then the load is dropped at once by opening the 3-phase switch S. The rectifier voltage, output voltage, input currents, and input voltages are recorded by the data acquisition system. The collection of data is initiated by the opening of the switch S. Figures 98 through
100 show the collected signals. From the bridge rectifier output, it can be seen that the output voltage ringing lasts for about 0.7 seconds after the load drop. The analysis and simulations of this section are aimed at reducing this transient time.

It is known that the degree by which an output voltage lags the corresponding input voltage is a function of the datawave loading; the heavier the load, larger the lagging angle. At no-load this angle is close to zero. When the switch S is opened, phase angles of voltages at the output of the datawave will move from their full-load steady state values to their no-load steady state values. The time it takes for this transition to take place is the ringing time we want to reduce.

One way to reduce this transition time is to reduce the phase shift between the corresponding input and output voltages when the datawave system is loaded. This causes the output voltages to go through smaller phase angle change after the load is dropped. Before we can find ways to reduce this phase shift, we need to study it closely.

8.4.1 60 Hz Analysis of the Input/output Phase Shift

The power conditioning system's external signals are mainly 60 Hz. These signals are input line-to-line voltages \( V_{AB}, V_{BC} \) and \( V_{CA} \), input or choke currents \( i_A, i_B, \) and \( i_C \), and output line-to-line voltages \( V_{ab}, V_{bc}, \) and \( V_{ca} \). The energy storage capacitors' currents and the exciting currents of the transformers consist mainly of 60 and 660 Hz components. However, their 11th harmonic components are, relative to their fundamental components, small and are ignored in the following analysis.

Noting that the corresponding voltages across the primary and secondary capacitors are practically the same, the difference between the input voltage \( V_{AB} \) and the output voltage \( V_{ab} \) can be written as

\[
V_{AB} - V_{ab} = L_7 \frac{di_A}{dt} - L_8 \frac{di_B}{dt}
\]  

(8.17)

Assuming all energy storage capacitors are on the primary side and labeling the capacitor currents as \( i_{c1a}, i_{c1b} \) and \( i_{c1c} \), we have

\[
i_A = i_{c1a} + i_4
\]  

(8.18)

Now ignoring the small 60 Hz currents which flow in the zig-zag transformer and the harmonic filter, referring to Fig. 7.30 of Chapter 7 we can write
Figure 98: Bridge output during load drop test.

Figure 99: Output voltage during load drop test.

Figure 100: Line current during load drop test.
\[ i_4 = i_{E4} + i_a \]  \hfill (8.19)

and substituting Eq. (8.19) in (8.18) we get

\[ i_A = i_{c1a} + i_{E4} + i_a \]  \hfill (8.20)

The current \( i_{c1a} \) is given by

\[ i_{c1a} = c_1 \left( \frac{dV_{ab}}{dt} - \frac{dV_{ca}}{dt} \right) \]  \hfill (8.21)

Due to parallel resonance between the energy storage capacitor bank and the saturating transformers, the currents \( i_{c1a} \) and \( i_{E4} \) are 180° out of phase. Ideally, \( i_A = i_a \) and \( i_{c1a} = -i_{E4} \).

In frequency-domain, Eq. (8.17) becomes

\[ \bar{V}_{AB} - \bar{V}_{ab} = j\omega L\bar{I}_A - j\omega L\bar{I}_B \]  \hfill (8.22)

and assuming a positive sequence network, the phasor diagram of Fig. 101 depicts the operational principles of the power conditioning system. In this figure, superscripts "res", "ind", and "cap" denote resistive, purely inductive and purely capacitive load currents, respectively. These loads are assumed to be connected as three-phase delta and are determined using the following relations

\[ \bar{I}^\text{res}_a = \frac{1}{R_L} (\bar{V}_{ab} - \bar{V}_{ca}) \]
\[ \bar{I}^\text{ind}_a = \frac{1}{j\omega L} (\bar{V}_{ab} - \bar{V}_{ca}) \]
\[ \bar{I}^\text{cap}_a = j\omega c (\bar{V}_{ab} - \bar{V}_{ca}) \]

For wye connected loads, same results are obtained with different relations

\[ \bar{I}^\text{res}_a = \frac{1}{R_L} \bar{V}_{an} \]
\[ \bar{I}^\text{ind}_a = \frac{1}{j\omega L} \bar{V}_{an} \]
\[ \bar{I}^\text{cap}_a = j\omega \bar{V}_{an} \]

Eq. (8.17) can now be written in frequency-domain as

\[ \bar{D}_{ab} = \bar{V}_{AB} - \bar{V}_{ab} = j\omega L\bar{I}_A - j\omega L\bar{I}_B \]  \hfill (8.23)
To study the phase shifts between input and corresponding output voltages, the phasor diagram of Fig. 102 is constructed using Eq. (8.23) and the phasor diagram of Fig. 101. To minimize $\tilde{D}_{res}$ for a given resistive load, the choke inductances $L_7$ and $L_8$ must be minimized.

So far, it has been assumed that the choke inductances $L_7$, $L_8$, and $L_9$ are linear. However, to achieve higher degree of regulation, it is necessary that the choke inductances be nonlinear. In such a case, Eq. (8.17) becomes

$$d_{ab} = V_{AB} - V_{ab} = L_7(i_A) \frac{di_A}{dt} - L_8(i_B) \frac{di_B}{dt}$$

(8.24)

The choke inductances are designed to decrease as their corresponding currents increase. This reduces the system input impedance allowing the load to draw the additional currents it needs without effecting its voltages. If drawing larger resistive currents from the source causes the source voltages to drop, as it is usually the case, the capacitive components of the choke currents will increase due to the reduction in the choke inductances; see Fig. 103. In such a case, choke and load currents are no longer the same.
Figure 102: Analysis of the phase shift between input and output voltages for different types of loads.

In reducing the phase shift between the input and output voltages, we will first attempt to minimize $d_{ab}$, given by equation Eq. (8.24), via redesigning the nonlinear choke inductances. In redesigning these inductances, we will also attempt to minimize the harmonics generated by their nonlinearities. This reduces chokes’ losses, increases their life times, and results in a more reliable operation of the power conditioning system.

The design of the choke inductances is carried out in two steps. In the first step, a functional form for $L_j(i_j)$ ($j = 7, 8, 9$) is determined such that minimum harmonics will be generated as the result of the chokes operation. In the second step, the coefficients of the functions $L_j(i_j)$ are selected so that $d_{ab}$ in Eq. (8.24) is minimized.

8.5 Output Voltage Ringing and Harmonic Reduction Via Redesign of the Chokes

As mentioned earlier, we will first attempt to find the functional forms of $L_7(i_A)$ and $L_8(i_B)$ such that with $i_A$ and $i_B$ 60 Hz sinusoidal, $d_{ab}$ in Eq. (8.24)
Figure 103: Effect of nonlinearity of the choke inductances on voltage regulation.

contains as little harmonics as possible. Assuming \( i_A = I_m \cos \omega t \) and \( i_B = I_m \cos(\omega t - 120^\circ) \), the inductances \( L_7(i_A) \) and \( L_8(i_B) \) can be represented by their Fourier series as

\[
L_7(i_A) = \sum_{k=0}^{\infty} a_k \cos(k \omega t + \theta_{7k})
\]

\[
L_8(i_B) = \sum_{k=0}^{\infty} b_k \cos(k \omega t + \theta_{8k})
\]

Substituting (8.25), \( i_A \) and \( i_B \) in (8.24), we get

\[
d_{ab} = -\omega I_m \sin \omega t \sum_{k=0}^{\infty} a_k \cos(k \omega t + \theta_{7k}) + \omega I_m \sin(\omega t - 120^\circ) \sum_{k=0}^{\infty} b_k \cos(k \omega t + \theta_{6k})
\]

\[
d_{ab} = \frac{\omega I_m}{2} \left[ \sum_{k=1}^{\infty} b_k (\sin((k + 1) \omega t + \theta_{6k} - 120^\circ) - \sin((k - 1) \omega t + \theta_{6k} + 120^\circ)) - \right.
\]

\[
- a_k (\sin((k + 1) \omega t - \theta_{7k}) - \sin((k - 1) \omega t + \theta_{7k})) \right] -
\]

\[
- \omega I_m a_o \cos \theta_{70} \sin \omega t + \omega I_m b_o \cos \theta_{80} \sin(\omega t - 120^\circ)
\]

For \( d_{ab} \) to contain only fundamental components, we must have

\[
a_k = a_j
\]

\[
b_k = b_j
\]

\[
\theta_{7k} = \theta_{7j}
\]

\[
\theta_{8k} = \theta_{8j} + 240^\circ \text{ for } j = k + 2
\]

(8.26)
In addition, if we truncate this series from the term \( N + 1 \) on, it becomes necessary to have

\[
\begin{align*}
  a_k &= b_k \\
  \theta_{7k} &= \theta_{8k} - 120^\circ \quad \text{for} \quad k = N - 1, N
\end{align*}
\]  

(8.27)

which yields

\[
d_{ab} = \frac{\omega I_m}{2} \left[ a_1 \sin \theta_{71} - b_1 \sin(\theta_{81} + 120^\circ) \right] + \\
\frac{\omega I_m}{2} \left[ a_2 \sin(\omega t + \theta_{72}) - b_2 \sin(\omega t + \theta_{82} - 120^\circ) \right] + \\
\omega I_m \left[ -a_0 \cos \theta_{70} \sin \omega t + b_0 \cos \theta_{80} \sin(\omega t - 120^\circ) \right]
\]

Since \( d_{ab} \) should not have a dc bias, we let \( a_1 \sin \theta_{71} = b_1 \sin(\theta_{81} + 120^\circ) \) to get

\[
d_{ab} = \frac{\omega I_m}{2} \left[ a_2 \sin(\omega t + \theta_{72}) - b_2 \sin(\omega t + 120^\circ + \theta_{82}) - \\
-2a_0 \cos \theta_{70} \sin \omega t + 2b_0 \cos \theta_{80} \sin(\omega t - 120^\circ) \right]
\]  

(8.28)

and

\[
L_7(i_A) = \sum_{k=2}^{N-1} a_{\text{even}} \cos(k \omega t + \theta_{\text{even}}) + \sum_{k=1}^{N} a_{\text{odd}} \cos(k \omega t + \theta_{\text{odd}}) + a_0 \cos \theta_{70}
\]  

(8.29)

\[
L_8(i_B) = \sum_{k=2}^{N-1} b_{\text{even}} \cos(k \omega t + 120^\circ + \theta_{\text{even}}) + \\
\sum_{k=1}^{N} b_{\text{odd}} \cos(k \omega t + 120^\circ + \theta_{\text{odd}}) + b_0 \cos \theta_{80}
\]

Noting that except for the terms with \( k = 0 \) and 2, no other term in \( L_7(i_A) \) and \( L_8(i_B) \) has any effect on \( d_{ab} \), we can simplify Eq. (8.29) by setting these terms equal to zero and get

\[
L_7(i_A) = a_0 \cos \theta_{70} + a_2 \cos(2 \omega t + \theta_{72})
\]  

(8.30)

\[
L_8(i_B) = b_0 \cos \theta_{80} + b_2 \cos(2 \omega t + \theta_{82} - 120^\circ)
\]

For \( L_7 \) and \( L_8 \) to be physically realizable, both \( L_7(i_A) \) and \( L_8(i_B) \) in Eq. (8.30) must be positive for all values of \( i_A \) and \( i_B \) which implies

\[
\begin{align*}
  a_0 \cos \theta_{70} &> |a_2| \\
  b_0 \cos \theta_{80} &> |b_2|
\end{align*}
\]
For datawave to be a balanced system, the average values of \( L_7(i_A) \) and \( L_4(i_B) \) must be equal. That is,

\[
a_0 \cos \theta_{70} = b_0 \cos \theta_{80} \triangleq L'_0 > 0
\]

and also

\[
a_2 = b_2 \triangleq a
\]

With these definitions we have

\[
L_7(i_A) = L'_0 + a \cos(2\omega t + \theta_{72})
\]

\[
L_8(i_B) = L'_0 + a \cos(2\omega t + \theta_{82} - 120^\circ)
\]

Physically, we want for \( L_7 \) (or \( L_8 \)) to decrease as \( i_A \) (or \( i_B \)) increases. This can be accomplished only when \( \theta_{72} = 0 \) and \( \theta_{82} = -120^\circ \) in which case we would have

\[
L_7(i_A) = L'_0 + a \cos 2\omega t
\]

\[
L_8(i_B) = L'_0 + a \cos(2\omega t - 240^\circ)
\]

but

\[
\cos 2\omega t = 2\cos^2 \omega t - 1 = 2(i_A/I_m)^2 - 1
\]

\[
\cos(2\omega t - 120^\circ) = 2\cos^2(\omega t - 120^\circ) - 1 = 2(i_B/I_m)^2 - 1
\]

and we get

\[
L_7(i_A) = L'_0 + \frac{2a}{I_m^2} i_A^2
\]

\[
L_8(i_B) = L'_0 + \frac{2a}{I_m^2} i_B^2
\]

with \( L_0 \triangleq L'_0 - a \) and \( a < 0 \).

The coefficient \( a \) has to be of the form \( a = bI_m^2 \), or else \( L_7 \) and \( L_8 \) become independent from the actual values of \( i_A \) and \( i_B \). Letting

\[
c \triangleq -\frac{2a}{I_m^2}
\]

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we get

\[ L_l(i_A) = L_0 - ci_A^2 \]

\[ L_B(i_B) = L_0 - ci_B^2 \]

\[ L_0 \text{ and } c \text{ constants > 0} \quad (8.31) \]

Substituting Eq. (8.31) in (8.24) we get

\[ d_{ab} = (L_0 - ci_A^2) \frac{di_A}{dt} - (L_0 - ci_B^2) \frac{di_B}{dt} \]

\[ d_{ab} = (L_0 - cI_m^2 \cos^2 \omega t)(-\omega I_m \sin \omega t) + (L_0 - cI_m^2 \cos^2 (\omega t - 120^\circ))(\omega I_m \sin(\omega t - 120^\circ)) \]

which simplifies to

\[ d_{ab} = (L_0 - \frac{c}{4} I_m^2)(-\omega I_m \sin \omega t) - (L_0 - \frac{c}{4} I_m^2)(-\omega I_m \sin(\omega t - 120^\circ)) \]

Note that \( d_{ab} \) contains no harmonics and that it can be written as

\[ d_{ab} = L_{eff}(I_m) \left[ \frac{di_A}{dt} - \frac{di_B}{dt} \right] \]

with

\[ L_{eff}(I_m) = L_0 - \frac{c}{4} I_m^2 > 0 \quad (8.32) \]

That is, for a given operating condition, the nonlinear inductances given in Eq. (8.31) act like linear inductances whose effective values are functions of the chokes peak currents \( I_m \).

Eq. (8.31) gives a functional form for the chokes' inductances which assures no output voltage harmonic is generated as the result of the chokes' operation. We will now attempt to find suitable values for the constants \( L_0 \) and \( c \). In selecting these constants, the following factors should be considered:

a) high degree of nonlinearity is required to achieve good regulation,
b) at no time should the choke inductances get too low to lose their high frequency filtering effects,

c) their effective values should change, as a function of the load current, in a manner that causes as little variation in $d_{ab} = V_{AB} - V_{ab}$ as possible; this would improve system stability and in particular the output voltage ringing.

In mathematical terms these conditions translate to

a) maximizing $(L_{\text{max}} - L_{\text{min}})$ where $L_{\text{max}} = L_0$ and $L_{\text{min}} = L_0 - c_i^2$, $j$, $\text{max},$

b) $L_{\text{min}} \geq k$ where $k$ is a positive constant whose value must be large enough for the chokes to block high frequency transients even when their inductance values are at minimum,

c) minimize variations in $L_{\text{eff}} I_m = L_0 I_m - \frac{c_i}{4} I_M^3$ as a function of $I_m$ for the range $0 \leq I_m \leq I_{m,F, L}$.

The peak value of the choke currents at the full load of 30 KVA, 208V line-to-line is about 120A. Based on experience, $k$ is set to 0.5 mH. Then, the conditions under (a) and (b) dictate that

$$L_0 - c(120)^2 \geq 5E - 4$$  \hspace{1cm} (8.33)

and $c$ as large as possible. In this case, the choke inductances are given by

$$L_j(i_j) = \begin{cases} L_0 - c_i^2 & \text{for } i_j \leq 120A \\ 5E - 4 & \text{for } i_j > 120A \end{cases}$$

The condition under (c) can be best satisfied with $L_{\text{eff}}(I_m)$ having a hyperbolic shape. The plots of $L_{\text{eff}}(I_m)$ for different values of $L_0$ and the corresponding values of $c$ found from Eq. (8.32) are shown in Fig. 104. It can be seen that as the value of $L_0$ increases, the shape of $L_{\text{eff}}(I_m)$ gets closer to its desired hyperbolic shape.

8.6 Reduction of the Output Voltage Ringing Time Via Redesign of the Harmonic Filters.

A second approach in reducing the transient time after a load drop is to reduce the relevant time constants. Intuitively, we can speed up the datawave system transients by lowering the impedances of the second and third harmonic
Figure 104: Choke effective inductance as a function of choke peak current.

filters to 60 Hz. That is by reducing their 60 Hz impedances while maintaining the frequencies at which they are designed to resonate. Since the resonant frequency of the second harmonic filter is closer, than that of the third harmonic filter, to 60 Hz, we will attempt to reduce the ringing time via redesign of this filter.

The impedance of the second harmonic filter is given by

\[
Z_{F2}(\omega) = j \left( \omega L_{F2} - \frac{1}{\omega C_{F2}} \right)
\]

where

\[
L_{F2} C_{F2} = \frac{1}{\omega_0^2}
\]

\[
\omega_0 = 2\pi (2 \times 60) \text{rad/sec.}
\]

The plots of \(Z_{F2}(\omega)\) given by Eq. (8.34) for different values of \(L_{F2}\) and \(C_{F2}\), subject to the constraint given by Eq. (8.35), are shown in Fig. 105. It can be seen that as the inductance is reduced and the capacitance is increased, the 60 Hz impedance of the second harmonic filter becomes smaller. Next, we will study the effect of this lower impedance on the output voltage ringing after a sudden load drop.

Using the developed model, we now simulate a case of full resistive load drop with three sets of second harmonic filter parameters as shown in Table 14. The
Figure 105: Impedance of the second harmonic filter for different values of $L_{F2}$ and $C_{F2}$.

Table 14: Four sets of second harmonic filter parameters selected for system load drop simulations

<table>
<thead>
<tr>
<th>Case</th>
<th>$L_{F2}(H)$</th>
<th>$C_{F2}(F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: Original values (Fig. 106)</td>
<td>0.027</td>
<td>65 E-6</td>
</tr>
<tr>
<td>Case 2: Higher 60 Hz impedance (Fig. 107)</td>
<td>0.05</td>
<td>35 E-6</td>
</tr>
<tr>
<td>Case 3: Lower 60 Hz impedance (Fig. 108)</td>
<td>0.0093</td>
<td>185 E-6</td>
</tr>
<tr>
<td>Case 4: Lower 60 Hz impedance (Fig. 110)</td>
<td>0.005</td>
<td>352 E-6</td>
</tr>
</tbody>
</table>
simulated output voltages and choke currents are shown in Figs. 106, 107, and 108 for Case 1, Case 2, and Case 3, respectively. It can be seen that the transient time after the load drop has been reduced in Case 3 as expected.

To verify the simulation results, the laboratory set up shown in Fig. 97 is used to carry out the load drop test of Case 3. The test results are shown in Fig. 109. Comparing the bridge voltage of Fig. 109a with that of Fig. 98, it can be seen that the redesign of the second harmonic filter in Case 3 has reduced the ringing time significantly. Comparing the results of the three simulation cases, it becomes evident that the output voltage ringing time can be reduced arbitrarily by further lowering $L_{P2}$ and increasing $C_{P2}$. Figure 110 shows the simulated signals for $L_{P2} = 5mH$ and $C_{P2} = 352\mu F$.

8.7 Conclusions

The solution to the ferroresonant jump (oscillation) problem was found through the analysis of the datawave steady state behavior. This analysis showed that the proper control of the harmonics present in the oscillatory mode can lead to the suppression of the oscillation, and hence, force the system into the normal (desirable) mode of operation.

The theoretical solution was successfully tested on the datawave system. That is, an oscillation-causing transient was created through a test. With the system in oscillatory mode, the time-domain samples of one of the input currents were collected and its spectral density was calculated. It was found that the dominating component had a frequency of 100 Hz. Then, a 100 Hz trap was designed and incorporated in the system. Under the same test conditions, no oscillation occurred and the system was forced into the normal mode of operation.

Since the datawave is a highly nonlinear system, it is difficult to accurately classify the transients which cause the oscillation. However, some conclusions can be reached through analysis of the input voltages developed at the terminals of the datawave during the oscillation test. The spectral densities for the first three cycles of the input line-to-line voltages during no-load normal tests are tabulated in Table 15. Comparing these densities, we note that there is no significant difference between the amplitudes except at the zero frequency. That is, the dc component of the oscillation test input during the first three cycles is considerably larger than the corresponding amplitude in the normal test. High dc component during the transient drives the system inductances deep into saturation. With the choke inductances over-saturated, the datawave system becomes capacitive and resonates with the utility lines' inductances. This series resonance draws large input currents, and the cycle repeats.
Figure 106: Simulated choke current and output voltage after a sudden load drop with original second harmonic filter parameters.
Figure 107: Simulated choke current and output voltage after a sudden load drop with higher second harmonic filter 60 Hz impedance.
Figure 108: Simulated choke current and output voltage after a sudden load drop with lower second harmonic filter 60Hz impedance.
Figure 109: Actual bridge voltage, choke current, and output voltage after a sudden load drop in the laboratory with lower second harmonic filter 60Hz impedance.
Figure 110: Simulated choke current and output voltage after a sudden load drop with second harmonic filter 60Hz impedance lower than that in Case 3.
Table 15: Spectral densities for the first three cycles of the input line-to-line voltages during no-load oscillation and no-load normal tests

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Normal Test</th>
<th>Oscillation Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.3</td>
<td>13.3</td>
</tr>
<tr>
<td>15</td>
<td>41.0</td>
<td>48.3</td>
</tr>
<tr>
<td>30</td>
<td>80.5</td>
<td>81.3</td>
</tr>
<tr>
<td>45</td>
<td>101.0</td>
<td>97.2</td>
</tr>
<tr>
<td>60</td>
<td>288.7</td>
<td>290.7</td>
</tr>
<tr>
<td>75</td>
<td>77.2</td>
<td>79.2</td>
</tr>
<tr>
<td>90</td>
<td>51.4</td>
<td>53.7</td>
</tr>
<tr>
<td>105</td>
<td>19.8</td>
<td>15.0</td>
</tr>
<tr>
<td>120</td>
<td>4.4</td>
<td>4.4</td>
</tr>
<tr>
<td>135</td>
<td>10.8</td>
<td>11.4</td>
</tr>
<tr>
<td>150</td>
<td>10.7</td>
<td>13.9</td>
</tr>
<tr>
<td>165</td>
<td>9.1</td>
<td>6.0</td>
</tr>
<tr>
<td>180</td>
<td>1.2</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Therefore, it can be concluded that a class of transients causing oscillation are those which create "large" dc components in the first few cycles of the input voltages. Another solution to the oscillation problem can be obtained by controlling the switching times during start-up and load drops, such that the dc components of the input voltages are minimized.

The problem of output voltage excessive ringing following a sudden load drop was solved by redesigning the second harmonic filter. It was shown, thorough model simulation and laboratory tests, that this transient time can be reduced arbitrarily by lowering the second harmonic filter 60 Hz impedance.
CHAPTER IX
CONTRIBUTIONS AND FURTHER RESEARCH

9.1 Contributions

The contributions made as a result of this work can be summarized as follows:

1) Development of the only existing model for a three-phase signal conditioning system and increasing our understanding of its operational details.

2) Improving steady state performance (solving the problem of ferroresonant jump) and transient behavior (reducing the output voltage ringing following a load drop) of three-phase signal conditioning systems via design modifications.

3) Establishing the practical requirements for successful implementation of an electromagnetic device time-domain modeling scheme.

4) Development of algorithms for nonlinear modeling of magnetic saturation and hysteresis in an electromagnetic device. By obtaining analytic models for saturation, these algorithms make analytical investigations of a given problem, such as the ferroresonant jump, possible. In addition, the use of closed-form models facilitates the computer simulation process.

5) Development of a methodology to obtain accurate models of power transformers in the presence of magnetic saturation and power system harmonics.

6) Development of an algorithm for tracking the physical parameters of synchronous generators needed in power system stability studies.

7) Laying the foundations for developments of fault detection schemes in electric machines, centrifugal pumps, and power transformers. For synchronous machines, an algorithm for the combined estimation of physical parameters and unmeasurable damper currents, needed for development of state of the art fault detection and diagnosis procedures, has been developed.
9.2 Further Research

As a continuation of this work, further research should be directed toward achieving the following goals:

1) Actual implementation of synchronous generator parameter and state tracking algorithms.

2) Development of an extensive data base for detection and diagnosis of faults in synchronous machines through simulation of fault models. This data base should include a catalogue of faults, relating a given abnormality in parameters or states to occurrence of a specific fault. For instance, corona and gap discharges can be related to abnormalities in the parameters of the high frequency output signal.

3) In this dissertation, we have established the design requirements for the non-linear choke of a signal conditioning system which would provide good voltage regulation, would act as a high frequency filter suppressing transients and noise, and would minimize the amount of harmonics generated as a result of its operation. Further work is needed to develop a finite element-based technique which would translate the design equations to detailed constructive specifications.
APPENDIX A

INDUCTION MOTOR / CENTRIFUGAL PUMP / PIPE SYSTEM EQUATIONS

Derivation of coefficient matrices in the state space formulation of the induction motor / centrifugal pump / pipe system is given in this appendix.

Putting Eqs. (3.7) and (3.11-3.15) in matrix form, we get

\[ \mathbf{B}'\mathbf{U} = \mathbf{A}'\mathbf{X} + \mathbf{L} \cdot \frac{d\mathbf{X}}{dt} \]

which can be put in the state space form

\[ \frac{d\mathbf{X}(t)}{dt} = \mathbf{A}\mathbf{X}(t) + \mathbf{B}\mathbf{U}(t) \]

\[ \mathbf{Y}(t) = \mathbf{C}\mathbf{X}(t) \]

where \( \mathbf{X} \), \( \mathbf{U} \), and \( \mathbf{Y} \) are defined as before and

\[ \mathbf{A} = -[\mathbf{L}]^{-1}[\mathbf{A}'] \]

\[ \mathbf{B} = [\mathbf{L}]^{-1}[\mathbf{B}'] \]

\[
\mathbf{B}' = \begin{bmatrix}
1 & 0 & d_1 & 0 \\
0 & 1 & d_2 & 0 \\
0 & 0 & d_3 & 0 \\
0 & 0 & d_4 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
\end{bmatrix}
\]
\[ d_1 = -[(L_{ts} + L_m)i_{dso} + L_m i_{dro}] \]
\[ d_2 = -[(L_{ts} + L_m)i_{qso} + L_m i_{qro}] \]
\[ d_3 = -[(L_{tr} + L_m)i_{dro} + L_m i_{dso}] \]
\[ d_4 = -[(L_{tr} + L_m)i_{qro} + L_m i_{qso}] \]

\[
A' = \begin{bmatrix}
    r_s & (L_{ts} + L_m)\omega_{eo} & 0 & 0 \\
    -(L_{ts} + L_m)\omega_{eo} & r_s & -(L_m \omega_{eo}) & 0 \\
    0 & (\omega_{eo} - \omega_{ro})L_m & 0 & -(\omega_{eo} - \omega_{ro})(L_{tr} + L_m) \\
    -\frac{2P}{L_m}L_m i_{dro} & \frac{2P}{L_m}L_m i_{qro} & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    -\frac{2P}{L_m}L_m i_{qso} & 0 & 0 & 0 \\
    -(\omega_{eo} - \omega_{ro})(L_{tr} + L_m) & d_3 & 0 & 0 \\
    r_r & d_4 & 0 & 0 \\
    -\frac{2P}{L_m}L_m i_{qso} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    \frac{2P}{L_m}L_m i_{qso} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    \frac{2P}{L_m}L_m i_{qso} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
    (L_{ts} + L_m) & 0 & L_m & 0 & 0 & 0 \\
    0 & (L_{ts} + L_m) & 0 & L_m & 0 & 0 \\
    L_m & 0 & (L_{ts} + L_m) & 0 & 0 & 0 \\
    0 & L - m & 0 & (L_{ts} + L_m) & 0 & 0 \\
    0 & 0 & 0 & 0 & J & 0 \\
    0 & 0 & 0 & 0 & 0 & I_f \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
    1 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 1 \\
    0 & 0 & 0 & C_{pw} & C_{pq} \\
\end{bmatrix}
\]

Note that due to changes in \(\omega_e\), the resistances \(r_r\) and \(r_s\), and due to changes in \(v_{qs}\) and \(v_{ds}\), the inductances \(L_m\), \(L_{ts}\), and \(L_{tr}\) are time-varying.
MODIFIED GENERALIZED LEAST-SQUARES ESTIMATOR

A summary of equations for the recursive GLSE is given below:

\[
X^T(K) = [Y_m^T(K) \quad \hat{Z}^T(K)]
\]

\[
\Phi^T(K) = [X^T(K) \quad U_m^T(K)]
\]

\[
\dot{X}(K+1) = \dot{C}(K)\Phi(K)
\]

\[
E(K+1) = X(K+1) - \dot{X}(K+1)
\]

\[
S(K+1) = S(K) - \frac{S(K)\Phi(K)\Phi^T(K)S^T(K)}{1 + \Phi^T(K)S(K)\Phi(K)}
\]

\[
\dot{C}(K+1) = \dot{C}(K) + E(K+1)\Phi^T(K)S(K+1)
\]

\[
\dot{E}(K+1) = \dot{C}_E(K)\Phi(K)
\]

\[
\dot{C}_E(K+1) = \dot{C}_E(K) + [E(K+1) - \dot{E}(K+1)] \Phi^T(K)S(K+1)
\]

\[
\dot{C}(K+1) = \dot{C}(K+1) + C_E(K+1)
\]

where

\[
[\dot{C}] = [\dot{A} \quad \dot{B}]
\]
Initial values are

\[ S = \left[ \sum_{j=1}^{Y} \Phi(j-1)\Phi^T(j-1) \right]^{-1} \]

\[ \hat{C} = \left[ \sum_{j=1}^{K} X(j)\Phi^T(j-1) \right] S \]

\[ \hat{C}_E = \left[ \sum_{j=1}^{K} E(j)\Phi^T(j-1) \right] S \]

for some integer \( k \).
To eliminate the effect of the instrumentation noise from the parameter estimates, an estimation model needs to be developed whose parameters are equivalent to those of the system model. It has been shown [31] that such an estimation model is obtained by filtering the sampled data by a digital filter with the following characteristics:

1. The form of the filter must be such that any initial condition on the system variables have insignificant effect on the filter outputs;

2. The frequency bandwidth of the filter approximately encompasses the frequency band covered by the differential equation model of the system.

A digital filter which meets both of these requirements has been designed in [64]. It meets the first requirement by having all its poles at the origin (well-damped transient response). It meets the second requirement by having an adjustable frequency bandwidth. Knowing the frequency bandwidth of the signal a priori, one can select a proper bandwidth for the filter.

A summary of the equations for the low-pass digital filter used to eliminate the high frequency noise from the experimental data recorded by our computer-controlled data acquisition system is given below:

\[ y_f(N) = \sum_{K=-N_f}^{N_f} b_K y(N - K) \]

\( y(.) \): Filter input

\( y_f(N) \): Filter output
\( N_f \) : Pairs of filter coefficients (weighting factors)

\( b_K \) : Weighting factors

\[
K_f = \begin{cases} 
0.13927(\lambda - 7.95) & \lambda > 21 \\
1.8445 & \lambda < 21
\end{cases}
\]

\( N_f = \) Integer part of \( \{K_f/2\delta + 0.75\} \)

\[
\eta = \begin{cases} 
0.1102(\lambda - 8.7) & 50 < \lambda \\
0.5842(\lambda - 21)^4 + 0.07886(\lambda - 21) & 21 < \lambda < 50 \\
0.0 & \lambda < 21
\end{cases}
\]

\[
\lambda = -20\log_{10}\epsilon
\]

\[
b_K = b_{-K} = \frac{\sin\beta K\pi}{K\pi} \left[ \frac{I_o(\eta\sqrt{1-(K/N_f)^2})}{I_o(\eta)} \right];
\]

\( K = 1, 2, ..., N_f \)

\( I_o(X) = \) modified Bessel function

\[
= 1 + \sum_{K=1}^{\infty} \left[ \frac{(X/2)^K}{K!} \right]^2
\]

\( \epsilon \) : Passband error (deviation from unity)

\( \delta \) : Transition band

\( \beta \) : Desired cut-off frequency

For definitions of \( \epsilon, \delta, \beta \) see Figure 111.
Figure 111: Digital Filter Transfer Function
REFERENCES


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