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Vagueness, logic and truth

Cohen, Mary Elizabeth, Ph.D.
The Ohio State University, 1987
VAGUENESS, LOGIC AND TRUTH

Dissertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

by

Mary Elizabeth Cohen, B.A., M.A.

* * * * * *

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### FIELDS OF STUDY

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INTRODUCTION

In his *Foundations of Arithmetic*, Frege denies the applicability of classical logic to predicates without sharp boundaries.

A DEFINITION of a concept (of a possible predicate) must be complete; it must unambiguously determine, as regards any object, whether or not it falls under the concept (whether or not the predicate is truly ascribable to it). Thus there must not be any object as regards which the definition leaves in doubt whether it falls under the concept; though for us men, with our defective knowledge, the question may not always be decidable. We may express this metaphorically as follows: the concept must have a sharp boundary. If we represent concepts in extension by areas on a plane, this is admittedly a picture that may be used only with caution, but here it can do us good service. To a concept without sharp boundary there would correspond an area that had not a sharp boundary-line all round, but in places just vaguely faded away into the background. This would not really be an area at all; and likewise a concept that is not sharply defined is wrongly termed a concept. Such quasi-conceptual constructions cannot be recognized as concepts by logic; it is impossible to lay down precise laws for them. The law of excluded middle is really just another form of the requirement that the concept should have a sharp boundary. Any object O that you choose to take either falls under the concept C or does not fall under it; tertium non datur. E.g., would the sentence 'any square root of 9 is odd' have a comprehensible sense at all if square root of 9 were not a concept with a sharp boundary? Has the question 'Are we still Christians?' really got a sense, if it is indeterminate whom the predicate 'Christian' can truly be ascribed to, and who must be refused it?
Clearly, our natural language concepts do not live up to the rigor prescribed for them. Because vague predicates (i.e., those with borderline cases) abound in our language, certain logicians, like Frege, have worried about the applicability of classical logic to natural language. These worries are reinforced by the sorites paradox, which is most often diagnosed as being due to the vagueness of the predicates involved. The classic version of the paradox involves the predicate 'heap':

One grain of sand is not a heap.

For all n, if n grains of sand are not a heap, then n+1 grains of sand are not a heap.

1,000,000,000 grains of sand are not a heap.

The argument is paradoxical because, although the premisses seem obviously true, 1,000,000,000 grains of sand do compose a heap. Sorites arguments can also be given for most other vague predicates.

Because of these worries about vagueness, some logicians have developed and endorsed alternatives to classical logic. On the other hand, rejecting classical logic for natural language is considered by other logicians to be an overreaction to the situation. After all, many of us use and teach classical logic, and we do so most of the time without running into any problems. Those who take this view seek to find a solution to the sorites and an answer to Frege's worries by some means that leaves classical logic intact.

One of the most successful attempts to tackle the problem in this way is illuminated by Kit Fine's application of supervaluation
semantics to natural language. Supervaluation semantics yields classical logic and solves the sorites while correctly reflecting the fact that our language is nonbivalent. Because of the many nice features of this semantics, and because reactions to it have ranged from favorable to critical, this project began as an evaluation of Fine's approach.

In the course of performing this evaluation, however, I noticed that supervaluation semantics is incompatible with the prosentential theory of truth. I found this interesting because it meant that the results of my evaluation could have implications for that theory. I also began to wonder about the theory's compatibility with other semantics for natural language.

At that point, the aim of the dissertation changed. I became primarily interested in whether the prosentential theory of truth could be reconciled with the preferred semantics for natural language, whatever that turned out to be. I continued to focus on supervaluation semantics, however, because it seemed to me to be the best candidate for the semantics of vagueness.

The dissertation is thus structured as follows. In order to evaluate a semantics for natural language, I needed to be clear about its vagueness. In Chapter One, I discuss the nature of vagueness and argue, contra some philosophers, that it is purely linguistic.

In Chapter Two, I attack two arguments. The first is Hilary Putnam's argument that classical logic is not appropriate for natural language because of its vagueness. It was important for me to examine this argument since its soundness would have entailed that
supervaluation semantics was not tenable for natural language. The second is Michael Dummett's argument for the claim that one must be a metaphysical realist in order to justify classical logic. Given one view of what it is to justify a logic, supervaluation semantics justifies classical logic (as opposed to merely "yielding" it). I needed to know whether an evaluation of that semantics would require the evaluation of metaphysical realism as well. Having refuted Dummett's argument, I go on in that chapter to cast doubt upon the more general claim that the justification of logic is a metaphysical matter. I suggest that logics are justified if they reflect linguistic practice.

Chapter Three is a somewhat detailed examination of supervaluation semantics in which I review some of the semantics' known properties and address some criticisms of it. Then, in Chapter Four, I discuss the prosentential theory and show how it is incompatible with supervaluation semantics.

Finally, in Chapter Five, I compare supervaluation semantics with its major competitor, the many-valued semantics of Lakoff and Zadeh (L-Z), which is compatible with the prosentential theory. I show that the adoption of L-Z would require that we change some of our classical reasoning practice and argue that these changes are not sufficiently well-motivated for us to make them when we have a good semantics that yields classical logic. Next, I go on to examine some other many-valued logics, all of which I show to be either incompatible with the prosentential theory or flawed in some way. I conclude that any supporter of the prosentential theory has a choice to make.
He can, of course, give up his theory of truth and accept classical
logic. But if he is too committed to the prosentential theory to do
that, then he must either adopt L-Z in spite of the fact that the
semantics has little to recommend it other than its compatibility
with his theory of truth or else find some other more acceptable
semantics that is compatible with his theory.
NOTES


CHAPTER I

VAGUENESS, WORDS, AND WORLDS

Vague Words

In order to evaluate different semantics for vagueness, it is important to have a good idea of what vagueness is. The term 'vague' comes from the Latin 'vagus,' meaning wandering or undecided. The American Heritage Dictionary currently defines 'vague' as follows:

1. Not clearly expressed or outlined; inexplicit.
2. Lacking definite shape, form, or character.
3. Indistinctly felt, perceived, understood, or recalled.

Thoughts can fail to be expressed clearly; physical objects may lack definite shape, form, or character; and feelings, perceptions, and thoughts can be indistinctly felt, perceived, and understood, respectively. Vagueness, then, is properly predicated of all these things.

There is, however, a more restricted use of 'vague,' time-honored by philosophers since Peirce, in which the term is used
solely to designate a linguistic phenomenon. Loosely described, the phenomenon I have in mind is that certain words—'red,' for example—have borderline cases of application. It is this phenomenon in which I am interested. Hence, in this section, I take 'vague' to be directly applicable only to words or groups of words.

Williams Alston defines 'vague' as follows:

To say that a word is vague is to say that there are cases in which there is no definite answer to whether it applies or not.²

Alston differentiates two species of vagueness—vagueness of application and vagueness of individuation. He further divides each species into two types—degree and combinatory. When Alston says that a word is vague if there are cases in which there is no definite answer to whether it applies or not, he is talking about vagueness of application. On the other hand, a word has vagueness of individuation when there are cases in which the word clearly applies, in some sense, yet it is indeterminate how many times it applies. For example, imagine a mound of rock for which it is indeterminate whether it is one mountain or two. Here, 'mountain' has vagueness of individuation.

I let my characterization of vagueness of individuation remain informal since in what follows I will be interested primarily in vagueness of application. That type of vagueness can be defined as follows.

DEF: A word w has vagueness of application iff there exists some possible entity in the domain of application of w to which w neither determinately applies nor determinately fails to apply.
I include the clause regarding domains of application because one might say there is a sense in which numbers are neither orange nor non-orange. Numbers certainly are not orange, yet perhaps they aren’t non-orange either because 'orange' simply does not apply to numbers. Even if this is true, however, 'orange' is not vague because of this fact, but because there are colored objects which are not clearly orange and not clearly non-orange.

Degree vagueness of application and combinatory vagueness of application may be defined in the following manner:

DEF: A feature f is tolerant with respect to w iff a small change in any possible entity e with respect to f will make no difference to whether w applies to e.3

DEF: A word w has degree vagueness of application iff w has vagueness of application and whether or not w applies to e depends upon the degree to which e possesses some feature which is tolerant with respect to w.

DEF: A word w has combinatory vagueness of application iff w has vagueness of application and whether or not w applies to e depends upon whether or not e possesses some combination of features.

'Red,' then, has degree vagueness of application because whether or not 'red' applies to an entity e depends upon the degree to which e reflects wavelengths of light. The reflection of light wavelengths is tolerant with respect to 'red' because a small change in that feature makes no difference with respect to the application of the predicate. 'Religion,' on the other hand, has combinatory vagueness of application because whether or not 'religion' applies to an entity depends upon whether it possesses some combination of features, such
as belief in supernatural beings, to holding of certain objects as sacred, etc.

These two types of applicational vagueness are not mutually exclusive. Consider a predicate P whose application is dependent upon 200 features which are each worth roughly .5 percent in that regard. In other words, we can think of an entity with 199 of these features as instantiating P to around degree .995, one with 198 as instantiating P to around degree .99, etc. Then P is certainly compositionally vague. But P is also degree vague because whether or not P applies to any e depends upon the number of relevant features e has. This feature (the number of relevant features e has) is itself tolerant with respect to P because a small change in the number of relevant features e has makes no difference to P's application.

There are some important differences between degree vagueness and combinatorial vagueness. (Hereafter, I shall be discussing only vagueness of application.) The most important of these is the fact that words with degree vagueness generate sorites paradoxes, while words with only combinatorial vagueness do not. This may seem surprising. Yet in order for a combinatorial vague predicate to produce a sorites paradox, that predicate must also be degree vague. For consider a combinatorial vague predicate P which is not also degree vague. Then there is no feature f, upon which P's application depends, which is tolerant with respect to P. But the truth of the sorites' inductive premise depends upon there being such a feature. So, predicates with only combinatorial vagueness cannot produce the sorites paradox.
Nor have I been able to think of any analogous paradox which words with combinatory vagueness only generate. For this reason, I call degree vagueness 'problematic vagueness' and combinatory vagueness 'unproblematic vagueness.' I am more interested in problematic vagueness for the obvious reasons.

My definition of vagueness differs from Alston's in at least one important way. His definition is of what Fine would call 'extensional vagueness.' In other words, Alston's definition makes a word's vagueness depend upon the actual world alone. My definition, on the other hand, is of intensional vagueness. According to it, a word's vagueness is dependent upon the possibility of there being cases for which there is no definite answer to whether or not the word applies. This feature I take from Peirce's definition of 'vague' which occurred in Baldwin's 1901 edition of The Dictionary of Philosophy and Psychology.

A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition. [Emphasis is mine.]

According to Alston's definition, it is unclear whether a word such as 'apple' is vague. This is because it is hard to know if there are actual cases for which there is no definite answer to the question whether 'apple' applies to them. According to Peirce's definition and mine, however, 'apple' is vague. In fact, 'apple' has degree vagueness, according to my definition.
To see this, consider the following example. Imagine a possible world which has, in addition to apple trees, trees which grow fruits that are not quite as round as apples, not quite as red as apples, and taste just a bit different than apples. In fact, think of an apple and then think of changing the apple's shape, color, and taste so slightly that one cannot perceive the difference in the original fruit and the changed fruit without some scientific instrument, and so that if one continued changing the fruit in the same way eventually the fruit would be a pear. Now change the fruit again along the same lines. Keep changing it until you have a pear. Imagine that for each change you thought of making in the apple there are trees in the possible world described which grow fruit corresponding to that change.

In such a world, there are entities for which there is no determinate answer to the question whether 'apple' is applicable to them. This makes 'apple' vague by my definition. Also, in such a world, whether 'apple' applies to an entity e is dependent upon the degree to which e possesses a feature (the combined feature of roundness, redness, and taste) which is tolerant with respect to 'apple.' So, 'apple' has degree vagueness and generates a sorites in the following manner.

Imagine a series of fruits $F_1, \ldots, F_n$ in the possible world described where $F_1$ is an apple, $F_n$ is a pear, and any fruit $F_i$ is just a bit less apple-like than $F_{i-1}$ and a bit more pear-like than $F_{i-1}$. Then the possible world described is one in which the premises
of the following valid argument appear to be true, yet the conclusion is false.

(1) \( F_1 \) is an apple.

(2) \((\text{if } F_i \text{ is an apple, then so is } F_{i+1})\).

(3) \( F_n \) is an apple.

'Apple,' then, does generate a sorites paradox. The fact that 'apple' has degree vagueness of application may seem counterintuitive. If so, it is only because in this world we don't often stumble upon cases for which it is indeterminate whether 'apple' applies to them.

Of course, one might argue that the second premiss of the sorites above is false and that 'apple' is precise on the grounds that it is a natural kind term. Consider the natural kind term 'water.' Kripke and Putnam have argued that common nouns such as 'water' are rigid designators, i.e., refer to the same stuff in all possible worlds. According to Kripke and Putnam, 'water' refers to \( H_2O \) in all possible worlds. But, the argument would continue, if any term is precise, then \( H_2O \) is. Hence, 'water' is precise. Since any natural kind term refers, on this view, to a kind that will eventually be picked out by some scientific term (like '\( H_2O \)'), and since scientific terminology is ideally precise, one can conclude that all natural kind terms are precise. 'Apple,' then, refers to the kind that science will eventually identify in terms of some genetic property, and so the second premiss of the 'apple' sorites is actually false in any possible world.
This argument, however, depends upon the assumption that we are correct in thinking that apples form a natural kind. Perhaps applehood is more like redness. We may, for instance, eventually discover that there is no way to define 'apple' precisely in terms of our scientific terminology. Maybe being an apple is a matter of having a genetic property which is a member of some fuzzy set of genetic properties. I do not know if we have enough evidence to rule out this possibility yet. If this is the case, then whether or not certain "natural kind terms" like 'apple' are vague is an empirical question for which we have no answer at this time.

Vagueness should be distinguished from some other phenomena with which it is sometimes confused. First, vagueness, as I have defined it, is different from the phenomenon which people often call 'vagueness' when they say, for example, "He was very vague concerning the details of the contract." Here 'vague' is being used in accordance with the first entry falling under the dictionary definition cited at the outset. In this case, to say that he was vague is just to say that he was inexplicit; he did not give the details. But what I call 'vagueness' has nothing to do with the absence of details that could have been given. Vagueness is a type of semantic indeterminacy.

The indeterminacy associated with vagueness should not be confused with epistemic indeterminacy. Sometimes we do not know whether to apply a word to an object because we simply have no way of knowing whether the object is in the word's extension, even though the meaning of the word may be precise. Our inability to say whether the
word applies to the object would, in this case, be due to nothing more than an epistemic limitation. It would not be due to any indeterminacy associated with the meaning of the word. Alston is careful to distinguish vagueness from epistemic indeterminacy. However, the dictionary definition quoted at the beginning of this section includes a species of that phenomenon. The definition allows that something (a word or expression) is vague if it is not distinctively understood.

Bertrand Russell gave the name 'vagueness' to a type of epistemic indeterminacy. Russell described "the vagueness of the knowledge derived from the senses" as being due to the "fact that things which our senses do not distinguish produce different effects." As an example of this, he cites a case in which an observer cannot tell, merely by looking, the difference between a glass of pure water and one which is contaminated with typhoid bacilli.

Since Russell was talking about the vagueness of a type of knowledge, and I use the term only to apply to words, our two notions of vagueness are to be differentiated. Russell, however, goes on to say that "the vagueness of the knowledge of the senses infects all words in the definition of which there is a sensible element." Therefore, he does connect this vagueness with the vagueness of words. But the phenomenon which Russell described should not properly be termed a species of vagueness at all. Russell has described a type of epistemic indeterminacy. The case he cites is one in which we are unable to see a particular difference obtaining between two objects. Yet the difference clearly obtains.
Is it really so clear that vagueness is not a form of epistemic indeterminacy? Consider the following case. Harold goes to the carpet store, having been directed by Martha to purchase some red carpet for the dining room. Harold sees some carpet which he likes very much but which he considers to be a borderline case of red. He cannot decide whether the carpet is red, and so he cannot decide whether purchasing it will be in accordance with Martha’s directive. Harold solves his problem by calling her on the phone and asking whether she considers Pepsi cans red, for he has decided that his carpet is very close in color to the color of Pepsi cans. Martha says that she does think that Pepsi cans are red. Harold buys the carpet, and they both live happily ever after.

Why should cases like this one not convince us that the indeterminacy connected with ‘red’ is epistemic after all? Harold was initially missing some information regarding the context. Once he possessed this information, he was able to classify the carpet as red. Perhaps it is true of any context that it contains information that determines the extension of vague predicates. But then the extension of ‘red’ is just context dependent rather than indeterminate. Any indeterminacy associated with vague terms would be merely epistemic.

Yet there do seem to be contexts that simply do not contain the information relevant to determining the extension of a vague term like ‘red.’ It seems sensible to ask of some purported borderline case of red, whether or not, apart from any particular context, it is
red. Of course, we may just decide to treat 'red' as an indexical. In cases where there is not enough contextual information to determine the referent of an indexical like 'that,' we simply say that the word is being used incorrectly, that the use is nonsensical, rather than that the referent of the term is indeterminate.

But this is just where vague terms differ from indexicals. Suppose you are asked the following question. "Is that a correct mathematical equation?" in a context in which there is no obvious referent for 'that.' You are likely, in this case, to say that the question is not well-formed or that it does not make sense because 'that' fails to refer in this context. Now suppose you are asked whether x is red, where x is a borderline case of red and you are given no further contextual information (and, in fact, there is no further relevant contextual information to be given). In this case, you are not likely to say that the question does not make sense or that 'red' fails to refer. Some things still get into the extension of 'red' and some into the anti-extension. 'Red' still refers, it just refers incompletely. This incompleteness is the source of the indeterminacy associated with 'red.' And that indeterminacy is semantic, not epistemic.

Generality, or what Alston calls 'lack of specificity,' should also be distinguished from vagueness. Russell is of some help in making this distinction. He points out that the difference between vagueness and generality is that a word is general when it applies to many things rather than to a single thing, whereas a word is vague if
there are certain things for which its application is indeterminate. Hence, 'mammal' is general because there are many mammals, but vague because there are some things for which it is indeterminate whether they are mammals.

Russell's formal definition of vagueness, however, differs greatly from mine. What Russell called 'vagueness' (and I shall call 'R-vagueness') applies to more than words and so is a broader phenomenon than that which I call 'vagueness.' 'R-vagueness' is applicable to any representation. Photographs, for example, may be R-vague.

Russell said that:

A representation is vague when the relation of the representing system to the represented system is not one-one, but one-many. 10

He also defined 'accurate' as follows:

One system of terms related in various ways is an accurate representation of another system of terms related in various other ways if there is a one-one relation of the terms of the one to the terms of the other, and likewise a one-one relation of the relations of the one to the relations of the other, such that, when two or more terms in the one system have a relation belonging to that system, the corresponding terms of the other system have the corresponding relation belonging to the other system. 11

It is fairly clear from the context of Russell's paper that he meant for a representation to be vague if and only if it is not accurate. Therefore, I attribute to him a definition of 'vague' which is richer than that quoted above. What Russell seems to have had in mind is
that a representation is vague just in case it is not isomorphic to what it represents.

R-vagueness has very little to do with vagueness as I have defined it. First, R-vagueness is a property of representational systems rather than of pieces of such systems. 'R-vague,' then, would be directly applicable to languages, whereas 'vague' is directly applicable only to words. (Of course, it is indirectly applicable to languages. A language is 'vague' if and only if it contains vague words.)

Second, it is not the case that a language is R-vague if and only if it contains words which are vague according to my definition. A language could easily fail to be isomorphic to the world even though the language contained no vague words. R-vagueness and vagueness are distinct phenomena.

Vague Worlds

In the current literature, philosophers sometimes speak of the vague world and sometimes of vague non-linguistic objects. This use of 'vague' is not in accordance with mine. To what phenomenon or phenomena is the word being applied in these instances? What interesting connections, if any, are there between "world-vagueness" and linguistic-vagueness?

There are at least six different ways philosophers have construed the thesis that the world is vague.
(WV₁) The world is such that any true description of it must be vague.¹²

(WV₂) Reality cannot be fully described by stating hard facts alone.¹³

(WV₃) Some vague expressions have more than merely superficial vagueness.¹⁴

(WV₄) There is some simple property P, the presence of which in an object is a matter of degree, such that the relation of being more P than is not a total ordering.¹⁵

(WV₅) Some possible non-linguistic objects are such that it is indeterminate whether or not they have P, for some non-linguistic property P.¹⁶

(WV₆) Some objects have, as a matter of fact, fuzzy boundaries.¹⁷

The types of vagueness listed above stand in need of some clarification. WV₁ is ambiguous. It may mean that any individual sentence that truly describes the world must be vague, i.e., cannot have its vagueness eliminated and still describe the same part of the world truly, (call this 'strong WV₁') or it could mean that any true total description of the world must employ some vague expressions (call this 'weak WV₁').

WV₂ is due to Michael Dummett. Dummett does not define 'hard facts' directly, but he defines 'soft facts' as ones stated by forms of sentence the condition for whose truth—or for a justified assertion of them—is either not fully determinate or else depends in part upon facts about ourselves, e.g., about what we know and do not know, that do not enter explicitly into the statement of the fact.¹⁸
WV₂, then, asserts that a full description of reality must involve some sentence the condition for whose truth (or justified assertion) is either not fully determinate or depends in part upon facts about us.

WV₃ claims that some vague expressions have more than merely superficial vagueness. This notion of world-vagueness is due to Christopher Peacocke. He defines 'superficial vagueness' in the following way.

The vagueness of a vague expression E is superficial if for any language L whose sole vague expression is E, there is some language L' containing only sharp expressions, and such that ... if two situations agree in all respects describable using the language L', then they agree in all respects describable using the language L.¹⁹

There is a problem with Peacocke's definition of superficial vagueness. It is not clear that any language could contain only one vague expression. But if there cannot be such a language, then, according to his definition, every expression has superficial vagueness. The definition will be vacuously satisfied for any expression.

Also, notice that Peacocke defines 'superficial vagueness' for expressions, yet it is unclear how much he wanted that term to cover. Perhaps he intended it to range over words and groups of words such as 'the red door,' which are grammatical but are not sentences. On the other hand, he may have wanted it to include sentences and groups of sentences.

I prefer to drop talk of superficially vague expressions and talk about the superficial vagueness of words instead. A word has
superficial vagueness just in case its vagueness can be eliminated in an interesting way. There are, however, at least two ways one could do this. One of these is to precisify the word, where:

**DEF:** W is precisifiable iff there is a precise word w* applying only to possible entities in w's domain of application, and for any possible entity e, if w determinately applies to e, then so does w*, and if w determinately does not apply to e, then so does w*.

The other way would be to drop the word entirely from the vocabulary of the language in question and replace it with other precise words which in some sense "do the same job as w."

Thus, suppose one wanted to eliminate the vagueness of 'red' in a language L. One could do so by retaining 'red' in L's vocabulary and precisifying it or by omitting 'red' from the vocabulary of L and replacing it, say, with light wavelength terminology.

Notice that the two methods of vagueness elimination are related. In order to precisify a vague word such as 'red,' there has to be some parameter, like light wavelength, upon which redness supervenes, in terms of which the precisification is couched. So, in order to eliminate the vagueness of 'red' by precisification, L must contain or have added to it the same terminology which one adds to L in eliminating the vagueness the other way. This suggests that the two methods of vagueness elimination may not be importantly different. It may be that 'red' can be precisified if and only if there is some feature or features upon which redness supervenes which we can talk about instead of talking about redness.
This seems to be true for 'red.' In fact, it appears that for any predicate $R$ with degree vagueness, $R$ is precisifiable if and only if there is some feature or group of features upon which $R$-ness supervenes which we can talk about instead of $R$-ness. But this fact does not hold in general for predicates with combinatorial vagueness. Consider 'community.' There are features we can talk about upon which something's being a community supervenes, such as number of residents, spatial continuity, political organization, etc. Suppose all of these notions could be precisified. It is by no means clear that 'community' could also be precisified. There may be no way of redefining 'community' so that everything which currently gets called a community falls under the extension of the precisification and everything which currently is not a community falls outside that extension. It may be possible to devise a precisification of 'community' by taking every possible situation one would describe using that word (let all such situations be enumerated $s_1, s_2, \ldots, s_n$), describing those situations with the new precise terminology and then disjoining all those descriptions to form necessary and sufficient conditions for being a community. However, this possibility presupposes that there are only a finite number of situations. That this is actually the case is very unlikely, however.

I wanted to say that a word has superficial vagueness just in case its vagueness can be eliminated in either of the two ways discussed above. But if a word can be precisified, then its vagueness can be eliminated in the second way also. For that reason, it will suffice to define 'superficial vagueness' as follows.
DEF: R is superficially vague in L iff R is vague and either L contains precise vocabulary for a feature or features upon which R-ness supervenes or L could have such vocabulary added to it.

I shall say that WV₁ entails that some predicate of our language has more than merely superficial vagueness.

I turn now to WV₅. Hilary Putnam has said that objects are vague because they are mind-dependent. He claims that

'objects' do not exist independently of conceptual schemes. We cut up the world into objects when we introduce one or another scheme of description.

The mind-dependency of objects establishes their vagueness in the following manner. For Putnam, language is the conceptual tool with which we "cut up" the world. Objects inherit their vagueness from the vagueness of the language used to describe (constitute) them. For example, consider some borderline case of a chair. The sentence 'x is a chair' (where 'x' refers to this borderline case) is neither determinately true nor determinately false. A metaphysical realist would say that this indeterminacy is due to some failure on the part of our language to describe a determinate world. Putnam, on the other hand, claims that the piece of the world being described is itself indeterminate, i.e., the object being described is neither determinately a chair nor determinately not one. Rather than the indeterminacy of language being due to that of the world, it works the
other way around. According to Putnam, our use of vague language constitutes an indeterminate world.

WV$_4$ is just a special case of WV$_5$. To say that there is a simple property $P$, whose presence in an object is a matter of degree, such that the relation of being more $P$ than does not determine a total ordering, is just to say that there are two objects, $o_1$ and $o_2$, which have $P$ to different degrees, such that it is indeterminate which has more $P$. But this is just a special case of it being indeterminate whether some object has some property.

WV$_6$ is due to Gareth Evans. In "Can There Be Vague Objects?," Evans writes that:

It is sometimes said that the world might itself be vague. Rather than vagueness being a deficiency in our mode of describing the world, it would then be a necessary feature of any true description of it. It is also said that amongst the statements which may not have a determinate truth value as a result of their vagueness are identity statements. Combining these two views we would arrive at the idea that the world might contain certain objects about which it is a fact that they have fuzzy boundaries. But is this idea coherent?  

Evans gives an argument which he takes to be a proof that there can be no objects which as a matter of fact have fuzzy boundaries. His argument involves showing that vague identity statements are incoherent. Evans, however, gives no explication of the notion of an object as a matter of fact having fuzzy boundaries. Peter van Inwagen has tried to make Evans' notion of a vague object more explicit. Van Inwagen presents an identity statement, indeterminate in truth-value, the terms of which are rigid designators. He argues that the
terms' rigidity ensures that the indeterminacy of the statement cannot be due to the vagueness of the language employed, but must be due to the "fuzziness" of the objects named. Van Inwagen concludes that there can be an object for which it is indeterminate whether it is identical with another (?) object.

Van Inwagen has failed to notice that the indeterminacy of the statement could be due to the vagueness of 'is identical with.' We have no reason to think that this predicate is not vague. In fact, cases of the type van Inwagen raises themselves suggest that it is vague. For this reason, I think that van Inwagen has merely presented us with a case of linguistic vagueness. However, I am going to count WV6 as another special case of WV5. For, if van Inwagen has isolated a case of world-vagueness rather than a linguistic one, then it is just a special case of it being indeterminate whether an object has a certain property.

Although it may not be obvious, two of the types of world-vagueness I have considered, weak WV1 and WV5, are equivalent. The following diagram shows this equivalence and the other connections between the types of world-vagueness I have examined.
WV₁ is the strongest type of world-vagueness, while WV₂ and WV₃ are the weakest. That strong WV₁ entails weak WV₁ is trivial given the assumption that the world is not empty. The entailment from weak WV₁ to WV₂ is also trivial. That strong WV₁ entails WV₃ can be seen fairly easily. For, suppose any individual sentence that truly describes the world must be vague. Now, consider any sentence S in any language L that truly describes the world. S must be vague. Suppose, for reductio that all words have merely superficial vagueness. Then any vague word in S can be eliminated. But the resulting sentence will be a precise true description of the world.

Establishing the equivalence of weak WV₁ and WV₅ requires some argumentation. Suppose that for every possible object o and any non-linguistic property P, o either determinately has P or determinately lacks P. Let L' contain a name for every possible object and every property so that a predicate P of L' applies to a possible object o just in case o has the property P names. L' is precise. Since L' can describe every object in the world truly, L' provides a precise true total description of the world. So, weak WV₁ implies WV₅.

On the other hand, suppose that some possible object o is neither determinately P nor determinately not P, for some property P. Then a true total description of the world must say that o is neither determinately P nor determinately not P. Such a description must contain a name for P. But any name for P will be vague, from which it follows that any true total description of the world must be vague. So, WV₅ implies weak WV₁.
It is fairly obvious that WV\textsubscript{3} does not entail strong WV\textsubscript{1}. Also, WV\textsubscript{3} does not entail weak WV\textsubscript{1} because the vague word referred to in WV\textsubscript{3} may not be needed in a true total description of the world. The entailment from WV\textsubscript{3} to WV\textsubscript{2} fails for similar reasons. Weak WV does not entail WV\textsubscript{3} because it might be that any true total description of the world must be vague, even though all words have merely superficial vagueness. Eliminating the vague words may not leave one with a true total description of the world.

The other entailments and entailment failures are fairly obvious. There are, then, six different notions of world-vagueness. There is apparently no consensus regarding what it could mean to say that the world is vague. 'World-vagueness,' for the most part, is used to name any phenomena which has something to do with linguistic vagueness and the world.

Both strong and weak WV\textsubscript{1} have as much to do with the nature of descriptions as they do with the world. The same can be said of WV\textsubscript{2}. WV\textsubscript{3} is a linguistic phenomenon. The description of WV\textsubscript{5} is not stated in such a way that it appears to be a linguistic phenomenon, yet it may be that the best way to make sense of what it could mean for some object o to be indeterminately P, for some property P, is to say that the sentence 'o is P' is not determinately true and not determinately false. For, as Russell has said,

Apart from representation, whether cognitive or mechanical, there can be no such thing as vagueness or precision; things are what they are, and there is an end of it. Nothing is more or less what it is, or to a certain extent possessed of the properties which it possesses.\textsuperscript{24}
I think that Russell is right about this. There is no way of making sense of what it could be for an object to be vague without somehow appealing to properties of representations. There is no purely metaphysical vagueness.

In short, I do not have to investigate linguistic vagueness and metaphysical vagueness. Some of the phenomena which have been called 'work-vagueness' have something to do with linguistic vagueness, and it may be that an investigation of linguistic vagueness will prompt some discussion of them. But there is no metaphysical phenomenon which requires my immediate attention, since the phenomena discussed above can all be defined in terms of linguistic vagueness.

A piece of information unearthed in the present section has interesting consequences for the next chapter. That is the fact that weak $WV_1$ and $WV_5$ are equivalent notions. I.e., any true total description of the world must employ some vague expressions if and only if there is some possible object $o$ which neither determinately has $P$ nor determinately lacks $P$, for some non-linguistic property $P$. This fact is interesting because the characterization that Putnam gives of metaphysical realism (which I cite in the next chapter) requires that metaphysical realists reject $WV_5$. But then they must also reject weak $WV_1$. In other words, a metaphysical realist's views entail that there can be a precise true total description of the world. The relevance of this fact for the next chapter will quickly become apparent.
NOTES


3. This notion of tolerance is basically Wright's. See his definition in Crispin Wright, "On the Coherence of Vague Predicates," Synthese, 30 (1975), p. 334.


7. Ibid., p. 87.

8. Ibid.

9. Ibid., p. 88.

10. Ibid., p. 89.

11. Ibid.


15. Ibid.


VAGUENESS, REALISM AND LOGIC

Vagueness and Realism

In "Vagueness and Alternative Logic," Putnam gives the following argument for the claim that the presence of vagueness in the world requires that we adopt some alternative to classical logic.

(P1) The presence of vague linguistic items is inconsistent with the truth of metaphysical realism.

(P2) The truth of metaphysical realism is required in order to justify classical logic.\(^1\)

(C) The presence of vague linguistic items is inconsistent with the justification of classical logic.

In this section, I refute (P1).

Putnam takes his characterization of metaphysical realism from Dummett, who describes the view as one which:

(MR1) assumes a world consisting of a definite totality of discourse independent objects;
(MR2) assumes "strong bivalence"—that is, that an object either determinately has or determinately lacks any property $P$ which may significantly be predicated of that object; and

(MR3) assumes the correspondence theory of truth in a strong realist sense of "correspondence": i.e., a predicate corresponds to a unique set of objects, and a statement corresponds to a unique state of affairs, involving the properties and objects mentioned in (MR1), and is true if that state obtains and false if it does not obtain.²

Putnam's argument that metaphysical realism is inconsistent with vagueness follows.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(1)</td>
<td>Our language is vague.</td>
</tr>
<tr>
<td>(2)</td>
<td>The truth-status of some statements in natural language is indeterminate (where this means that it is indeterminate what truth-status some statements have rather than that some statements have a value called 'indeterminate').</td>
</tr>
<tr>
<td>(3)</td>
<td>Every object $o$ is such that for any property $P$, either $o$ determinately has $P$ or $o$ determinately lacks $P$. (MR2)</td>
</tr>
<tr>
<td>(4)</td>
<td>Every statement $S$ of natural language uniquely corresponds to some state of affairs $X$ and it is determinate that $S$ corresponds to $X$. (MR3), (3)</td>
</tr>
<tr>
<td>(5)</td>
<td>Every state of affairs either determinately obtains or determinately fails to obtain. (3)</td>
</tr>
<tr>
<td>(6)</td>
<td>A statement $S$ is true iff there exists a state of affairs $X$, $S$ corresponds to $X$ and $X$ obtains. (MR3)</td>
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</tbody>
</table>
Every statement of natural language is either determinately true or determinately false.

Every statement of natural language has determinate truth-status.

(2) and (8) are contradictory. So, Putnam concludes that at least one of the tenets of metaphysical realism must be false. The argument is not simply that vagueness forces us to reject the principle of bivalence, whereas metaphysical realists accept it. Putnam concedes that a realist could give up bivalence for sentences—the principle that every sentence is either true or false—although he cannot reject strong bivalence, that is, (MR2).

A metaphysical realist who rejected bivalence (for sentences) could say that some sentences correspond to more than one state of affairs. He might, in effect, treat vagueness as a form of ambiguity. Such a realist could claim, then, that a sentence is neither true nor false if it corresponds to at least two states of affairs, one of which obtains and the other which does not. In this case, (MR3) would have to be revised.

Notice, however, that even a metaphysical realist who rejected the principle of bivalence in this way would be committed to the claim that every sentence has a determinate truth-status. Every sentence will be determinately true, determinately false, or determinately truth-valueless. Yet vagueness, according to Putnam, forces us to reject that claim and thus to reject metaphysical realism.
Putnam does not give any argument for (2). He simply takes it for granted that (1) entails (2). I want to suggest a way that one might argue for (2) on the basis of (1), but I have to clarify (2) somewhat first. What does it mean to say that the truth-status of some statement S is indeterminate? How is this different from saying that S has a value called 'indeterminate'? To say that S's truth-status is indeterminate is to say that it is indeterminate what truth-value S has. We could claim that S has the truth-value called 'indeterminate' without thereby claiming that it is indeterminate what truth-value S has. S could have the value indeterminate determinately.

Now perhaps we can begin to see how one might argue for (2). Suppose that although some predicates have borderline cases, there were never any borderline cases of borderline cases of these predicates. Then it would be determinate what borderline cases of predicates there were. For example, if there were never any borderline cases of borderline cases of red, then we could label a representation of a certain piece of the color continuum in the following fashion.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<tbody>
<tr>
<td>NONRED</td>
<td>BORDERLINE</td>
<td>RED</td>
<td>BORDERLINE</td>
<td>NONRED</td>
</tr>
</tbody>
</table>

Notice that each of A, B, C, D, and E has precise boundaries. In this case one could argue that it is always determinate what truth-status 'X is red' has. If X's color lies in Sections A or E,
then 'X is red' is false; if in C, then 'X is red' is true; if in B or D, then 'X is red' has some third value (call it what you will).

Even if borderline cases of borderline cases of red existed, if there were then no borderline cases of these, it would again be tempting to say that the value of 'X is red' for any X would be determinate. In this case the diagram above would have more areas (those corresponding to borderline cases of borderline cases of red), but the boundaries between areas would again be precise. On the other hand, if there are always borderline cases of borderline cases of any predicate, then it seems that there will be some sentences for which it is only indeterminate what value they have.

So, if one wanted to argue for (2), one could try arguing that there are always borderline cases of borderline cases. Perhaps an argument based on this strategy would be a good one, and perhaps not. We will be better equipped to evaluate such an argument if we make its structure clear.

First, we need the following definitions.

DEF: A word w is vague to degree 0 iff w is vague.

DEF: A word w is vague to degree n+1 iff w is vague to degree n and 'is a borderline case of ' is vague, where the blank is to be replaced by 'w' proceeded by 'a borderline case of' n times.

The argument itself would look something like this:

(9) There is at least one vague word w such that, for all n, w is vague to degree n.
(10) For any vague word \(w\), if every sentence containing \(w\) is to have a determinate truth-status, then there must be some \(n\) such that \(w\) is not vague to degree \(m\) for any \(m > n\).

Therefore,

(11) There exists at least one vague word \(w\) such that some sentence containing \(w\) lacks determinate truth-status.

We have reason to believe that (9) is true because, for any \(n\), the predicate produced by replacing the blank in 'is a borderline case of _______ ' with 'w' preceded by 'a borderline case of' \(n\) times can be used plausibly to generate a sorites paradox. One might argue for (10) in the following way. Suppose every sentence containing some vague word \(w\) has a determinate truth-value. Let truth-values be assigned to sentences of the form

\[ x \text{ is } w, \]

where \(x\) is any term, so that 'x is w' is true if and only if \(x\) is w, false if and only if \(x\) is non-w, and has some third value if and only if \(x\) is a borderline case of w. Suppose, for reductio, that \(w\) is vague to degree \(n\) for all \(n\). Then, we know \(w\) is vague to degree 1, so some possible object o is neither determinately a borderline case of \(w\) nor determinately not a borderline case of \(w\). Hence, given the truth-conditions above, 'o is w' has no determinate truth-status.

Suppose we add to those truth-conditions so that 'x is w' has some fourth value just in case x is a borderline case of a borderline
case of w. Then, since w is vague to degree 2, there will again be
some sentence whose truth-status is indeterminate. Clearly we could
keep adding clauses to the truth-conditions above that specify new
values, but since w is vague to degree n for all n, some sentence
will be indeterminate in truth-value. This, of course, contradicts
our assumption for reductio.

Suppose, though, that truth-values for sentences of the form 'x
is w' are assigned so that 'x is w' is true if and only if x is w;
false if and only if x is not w; and truth-valueless if and only if x
is a borderline case of w, or x is a borderline case of a borderline
case of x, or x is a borderline case of a borderline case of a
borderline case of w, etc. Then, the truth-status of 'x is w' would
always be determinate. But, this strategy gives rise to another
problem. Consider, for example, any clear case of red. 'This is
red,' where 'this' refers to the clear case just mentioned is true
according to this way of assigning truth-conditions. Notice that this
clear case of red may also be a borderline case for some predicate
which can be constructed by preceding 'a borderline case of red'
with 'is a borderline case of' n times, for some n. But in that
case, the truth-conditions discussed above assigned truth-valueless
to 'This is red.' In short, these truth-conditions do not yield
unique truth-status for sentences. So, one cannot take this option.

Perhaps, then, a metaphysical realist would have to accept (2).
At any rate, let us suppose he would so that Putnam's argument is
allowed to get off the ground. The other premisses of the argument
are either taken from Putnam’s characterization of metaphysical realism or else follow easily from some preceding premises. The argument is valid and so, granted that (2) is true, shows that some tenet of metaphysical realism is false. There is a way, however, of escaping Putnam’s argument; it involves rejecting the Putnam-Dummett characterization of metaphysical realism.

Recall that in Chapter One I showed that the claim that no true total description of the world can be precise is equivalent to the claim that some possible object is neither determinately P nor determinately not-P, for some property P. Metaphysical realists reject the latter claim and hence must reject the former. Thus, if a realist is going to account for the vagueness of our language, he must claim that that vagueness can be eliminated.

A metaphysical realist who made this claim would probably want to deny that each predicate corresponds to a unique determinate set of possible objects. Such a realist might argue that the vagueness of our language is due to the fact that many of our predicates do not correspond to properties. On this view, only an ideal language, one in which every predicate corresponds to a property, could be precise.

This line of defense requires that the metaphysical realist deny (MR3) as it is now stated. The denial involves holding that (MR3) is true only with respect to predicates and sentences of an ideal language. But in that case, lines (4), (6), (7), and (8) would have to be amended so that ‘natural’ was replaced with ‘ideal’ in
them. Call these amended lines (4'), (6'), (7'), and (8'), respectively. (8') would then read:

(8') Every statement of an ideal language has a determinate truth-status.

But (8') does not contradict (2) as (8) did.

Putnam calls this response to his argument the "What, Me Worry?" Response and claims that it is ultimately problematic. Since a good deal of our language is vague, a metaphysical realist who takes this line must claim that most of the predicates in our language don't correspond to properties. So the realist cannot accept the correspondence theory of truth for natural language. It follows that he must either deny that natural language sentences have any truth-status at all or else adopt a non-correspondence theory of truth for them.

The realist's first option is not acceptable. Certainly, sentences of natural language are proper candidates for truth-values. Consider 'X is red,' when 'X' refers to a clear case of red. Certainly such a sentence is true. The metaphysical realist must, then, adopt some theory of truth for natural language which does not involve direct correspondence with reality.

Putnam suggests that a realist might claim that sentences of vague natural language get their truth-values only relative to their translation into an ideal language. The realist can hold that a vague sentence is true if and only if its correct translation in an
ideal language is true. But, Putnam claims, this line of defense has its own problems.

The problem concerns whether the expression 'correct translation into an ideal language' is an expression of natural language or an ideal language. It seems to be an expression of vague natural language. But then Putnam points out that "we have answered the challenge, 'in what sense can a sentence which employs vague notions be true?' with an answer that uses a vague notion." But this is not necessarily the case. Putnam presumably thinks that it is because he understands his metaphysical realist to be saying that any sentence in natural language is true if and only if its correct translation into an ideal language is. But the realist need not think that every sentence in natural language needs translating. He may hold that some parts of natural language are already precise and are able to function "as is" in the ideal language. The important question, then, is whether the metaphysical realist could argue successfully that 'correct translation into an ideal language' is a precise expression. I grant Putnam that this is doubtful given all the problems that arise with the notion of 'correct translation.'

The problem appears to be a serious one. A vague sentence S of natural language will be true if and only if there exists a sentence S* which is a correct translation of S into an ideal language and S* is true. But then T, 'S* is a correct translation of S,' will be true if and only if there exists a sentence S** which is a correct
translation of \( T \) into an ideal language and \( S^* \) is true. '\( S^* \) is a correct translation of \( T \)' will be true if and only if..., and so on.

The claim that a vague sentence is true if and only if its correct translation into an ideal language is true may not be the only option open to a metaphysical realist. It may be helpful, then, to consider an actual realist account of the truth conditions for vague sentences. Such an account is expressed by Kenton Machina in "Vague Predicates."\(^5\) Machina's primary purpose in this paper is not to defend metaphysical realism, but is, rather, to provide a non-bivalent formal semantics for vague predicates. But, as I shall show, Machina does say some things that place him in the camp of "metaphysical realism according to Dummett and Putnam." His view can, therefore, be seen as an attempt to deal with vagueness on realist grounds.

The following considerations lead me to believe that Machina is appropriately characterized as a metaphysical realist. He describes what he calls the 'received view' of model-theoretic semantics. Although Machina rejects parts of this view, he accepts that:

(i) The properties of a thing are language-independent.

(ii) Any given particular either has or does not have any given property.

These are basically the first two tenets of metaphysical-realism-according-to-Dummett.

Machina also explains vagueness just as we might expect that a metaphysical realist would. In particular, he rejects the claim that
all predicates of natural language correspond to properties. His ex-
planation of the vagueness of 'red' is as follows.

For example, I think that the problem about "red" arose because there really is no one (determinate or determinable) property of being red, which could serve as the ultimate factual basis for grounding the truth or falsity of the sentence S. Rather, there are many closely related color properties which from our point of view arrange themselves into a kind of natural sequence along the various dimensions of hue, saturation, and brightness. These color properties are each completely determinate, and any given object either has one of them (at a particular time, on a particular part of its surface) or it doesn't. But the word "red" does not signify some precisely bounded set of these properties.

It looks as if Machina does explain vagueness from a realist standpoint using Putnam's "What, Me Worry?" response. That response, recall, involved claiming that vague predicates do not correspond to properties and this is clearly what Machina is claiming. How, then, does he think the truth-status of vague sentences of natural language is determined?

Machina proposes a semantics for vague languages based on a formal system of Zadeh and Goguen. The system includes a set F of interpretation functions. Each member of F maps n-place predicate letters followed by n individual parameters onto one of three values: in, out, and borderline. A sentence formed by preceding n individual parameters with an n-place predicate letter is true if and only if at least one member of F assigns the value in to the sentence and no member of F assigns out to the sentence. One could, I think, presume
that all expressions in the specification of truth-status given above are precise.

Of course, the problem here will arise in formulating the set $F$. If one really wants to say that the truth-status of vague sentences is determined in this way, then $F$ will have to be the correct or, at least, a reasonable interpretation of the language. But here the same old problem arises. If 'reasonable interpretation' is an expression of natural language, as it appears to be, then it is probably a vague one.

And so the regress is allowed to begin again because, unlike the case of the ideal language where the world "makes a claim true" and stops the regress, there is no mind-independent reality corresponding to vague sentences. A vague claim is true iff another vague claim is true. That claim is true iff still another vague claim is true, and so on.

Notice, though, that this problem, which Putnam raises for metaphysical realism, is actually a very general problem that must be addressed by anyone wanting to give a semantics for natural language. It is impossible to give precise necessary and sufficient truth conditions for a vague sentence $S$. For suppose one could give such conditions. Then, because the conditions are precise, $S$ will have a determinate truth value. But we have already seen that vague sentences do not always have determinate values.

It follows that Putnam has failed to unearth a problem unique to metaphysical realism. Rather, he has shown that the semantics for
a vague language must be given in a metalanguage which is vague as well. Eliminating vagueness need not be a matter of translating it away. The realist’s claim that vagueness can be eliminated is merely the claim that a precise description of the world is possible. Moreover, we shall see in the next section that the regress we noticed here does not occur for certain sentences, namely, the classical tautologies. While some vague sentences are genuinely indeterminate in truth-value, others receive determinate values. But this is what we should expect. So, not only has Putnam failed to uncover a problem which is specific to metaphysical realism, it turns out that he has failed to uncover any problem at all. The world is safe for metaphysical realism and vagueness, too.

Realism and Classical Logic

In the last section I attacked Putnam’s argument for (P1). (P2), however, is of independent interest for this project. If the justification of classical logic is dependent upon the truth of metaphysical realism, then in order to evaluate supervaluation semantics, which preserves classical logic (in some sense), and Lakoff-Zadeh semantics, which does not, I need to do metaphysics.

Putnam points out that history suggests logic has always been conditioned by metaphysics. According to Joseph Owens, Aristotle justifies the law of noncontradiction on metaphysical grounds. For Aristotle, the character of an accident, like whiteness, depends upon the substance in which it inheres (e.g., there is "white-for-a-swan"
and "white-for-a-dog"), whereas the character of a substance depends upon nothing other than itself. Now consider the following pair of statements: (1) The swan is white; (2) The swan is not white. (1) and (2) cannot both be true since they are implicitly relativized as follows: (1') The swan is white (for a swan); (2') The swan is not not white (for a swan). Of course, if swans were relative to something in the same way that white is relative to swans, then (1) and (2) might both be true since 'swan' might be relativized to one thing in (1) and another in (2). But, on this Aristotelian view, the term 'swan' inherits nonrelativity from the substance it names. This nonrelativity guarantees that (1) and (2) are contraries. It is in this way that Aristotelian metaphysics is supposed to have conditioned and justified Aristotelian logic.

Putnam continues that "in time the picture of substances and their predicates became the standard metaphysical picture of a world of fully determinate particulars characterized by their fully determinate properties." He attributes this "standard picture" to metaphysical realism. But what happens, Putnam asks, when that view becomes dubious? He answers that "if the metaphysical picture that grew up with and conditioned classical logic is wrong, then some of the 'tautologies' of classical logic may have to be given up." What evidence can be given in support of Putnam's suggestion that the justification of logic is a metaphysical matter? Certainly an historical argument would not be enough. Even if logic has been conditioned by metaphysics in the past, it does not follow that it
should continue to be so conditioned. Is there, then, any convincing argument for this claim? If there is, it is likely to involve the notion of truth. Truth is the best candidate for the alleged connection between logic and metaphysics. Logics are justified by semantics, which usually involve the notion of truth, and it has been argued that different conceptions of truth yield different metaphysical views.

In fact, Dummett does have an argument, in which truth plays a central role, that the rejection of metaphysical realism entails that classical logic cannot be justified. His argument is, roughly, that

(3) the justification of classical logic for a class C of statements requires a notion of truth which is valent (i.e., according to which each statement has a determinate value)

and

(4) if one accepts valence for a class C of statements, then one must be a realist regarding the subject matter of C.

Leaving (3) aside for the moment, what reason do we have to accept (4)? The justification of (4) involves, as Dummett points out, the principle

(T) For any statement which has a truth-value, there must be something in virtue of which it has that value.

(T) seems obviously true. Statements just do not have their truth-values arbitrarily. There must be something in virtue of which
statements have the values they do. (T), however, does not in and of itself justify (4). One could, for example, believe that every statement of C receives one of the values true or false by some conventional means. Ordinarily, this type of view would not be considered realistic regarding C's subject matter.

Suppose, however, that C contains some statements which are undecidable by any means we currently have for determining their truth-value. Then someone who wanted to maintain valence and (T) for C would be stuck with the task of explaining in virtue of what the undecidable statements get their truth-values. It is not clear that one would be able to do this from a non-realist perspective.

As an example of the difficulties that might arise in attempting to give such an explanation, consider mathematical statements. Suppose one holds bivalence (which, of course, is a particular type of valence) for mathematical statements while being an anti-realist about sets. Consider Cantor's Generalized Continuum Hypothesis, which is formally undecidable (independent of ZF), and assume there are no considerations that lead us to regard it as either true or false. If the hypothesis is bivalent, then it must be, "in reality," either true or false, even though we cannot determine its value. The undecidability of the hypothesis is, therefore, the result of nothing more than epistemic indeterminacy. But then there must be something we do not know about which makes the hypothesis have the value it does, yet it is hard to see what type of anti-realist about mathematics one could be and still maintain this.
More specifically, the problem for the mathematical case involves Gödel's incompleteness results. Because of Gödel's work, we know that any consistent theory in which arithmetic can be modeled is not recursively axiomatizable. So a mathematical anti-realist cannot claim that true mathematical statements are those that can be derived from the ideal mathematical theory, where such a theory would be complete, consistent and recursively axiomatizable. The only way a mathematical anti-realist could hold that every statement of mathematics is determinately true or false in virtue of either being derivable or refutable in an ideal theory, is if the theory in question is either inconsistent or not recursively axiomatizable. But in that case it is unclear what grounds one could possibly have for maintaining that the theory is ideal.

Of course, I am not here ultimately concerned with the mathematical case. Consider the instance of Dummett's argument in which I am particularly interested.

(3') The justification of classical logic for natural language requires a valent notion of truth for statements of natural language.

(4') If one accepts valence for a class C of statements that includes some undecidable statements and for which T is true, then one must be a realist regarding the subject matter of C.

(5') The class of statements of natural language includes some undecidable statements (i.e., some statements whose truth-values, if determinate, are not knowable by us).

(6) T is true for statements of natural language.

(7) One must be a realist regarding the subject matter of natural language in order to justify classical logic.
(6) is unproblematic because (T) seems obviously true for statements of any class. (5) is also true; vagueness guarantees that some statements of natural language are undecidable. But what about (4')?

We have just seen that if the class of natural language statements in question contains all mathematical statements, then (4') is at least plausible. But what happens if we restrict the class so that it excludes all statements of mathematics? Then one might think that the truth of (4') becomes dubious.

In this case it seems that one could plausibly maintain that an ideal theory would assign a determinate truth-value to every sentence of the class in question. One could even say that true sentences are just those that would be assigned that value by some group of ideal decision-makers who are constrained to assign a determinate value to every sentence, thereby preserving both anti-realism and valence. In fact, even undecidable statements of mathematics could receive a value in this way. So it turns out that the restriction to non-mathematical statements is not necessary after all.

This "valent anti-realism" seems to be coherent, although it is hard to see what might motivate an anti-realist to go to such pains to preserve valence. In fact, it may be that all actual adherents of valence are also realists, even though they could coherently be anti-realists. But this fact is not enough to get Dummett the conclusion he wants. The 'must' in (4') is essential to his argument's validity.

Dummett might try changing (4') so that it says that a well-motivated theorist who accepts valence must also be a realist. After
all, further consideration might prompt us to say that the view above is silly, since the ideal decision-makers would have no non-arbitrary grounds for assigning truth-values to certain sentences. But rather than argue that this response on behalf of Dummett is not a good one, I simply claim that the truth of (4') is at least disputable. As we shall see, Dummett's argument fails even if (4') is true.

The truth of (3') is heavily dependent upon what is meant here by 'justified.' Given one reading of that term, classical logic can be justified without appeal to any notion of truth whatsoever. On this view, logics are justified if they reflect actual linguistic practice, if they mirror the way we reason, when we reason correctly. The job of a semantics, then, is not to justify logic (i.e., not to tell us that our reasoning is "okay") but, among other things, to give us extra apparatus for determining the validity or invalidity of arguments for which our intuitions are unclear. For example, a sound logic can be used to give semantic counterexamples to certain arguments. Given this reading of 'justified,' (3') is certainly false. It is clear, however, that Dummett has a much different reading of 'justified' in mind here. He has argued that a correct semantics for natural language may show that parts of our current linguistic practice are unwarranted and should, therefore, be abandoned. According to Dummett, to justify a logic is to give a semantics for it. So, (3') is meant to be the claim that no semantics for classical logic can be given which does not appeal to a valent notion of truth.
Putnam gives one counterexample to (3'). Supervaluation semantics provides another. According to that semantics, a statement is true if every admissible way of precisifying it would make it true, false if every admissible way of precisifying it would make it false, and neither true nor false otherwise. Now consider a vague statement of natural language such as 'This is red,' where 'this' refers to a borderline case of red. Some admissible ways of precisifying the statement will make it true, while some will make it false. Hence, the sentence will be neither true nor false. In fact, we could say that the truth-value of the statement is indeterminate. So the notion of truth for statements of natural language appealed to here is not valent, yet supervaluation semantics does yield classical logic.

Of course, it may be objected that the notion of truth appealed to above is actually valent. The objection would be that it is not important whether we call a statement like the one presented above indeterminate in truth-value or having a third value. This is just a matter of labeling. What is important is whether the truth-status of every sentence is determinate, and, the objection continues, since on supervaluation semantics any statement will either be made true by every admissible way of precisifying it, made false by every admissible way of precisifying it, or made true by some ways and false by some others, the truth-status of every statement of natural language will be determinate.
This objection is unfounded because 'admissible way of precisifying statement S' is itself a vague expression. It will, therefore, sometimes be indeterminate what truth-status a statement has because it is indeterminate what counts as an admissible precisification of it. Of course, this feature of supervaluation semantics has itself been called problematic on the grounds that the semantics may not yield classical logic if the expression 'admissible precisification' is not precise.\(^{17}\)

Take, for example, the sentence S, 'This is red or this is not red.' If the set of admissible precisifications of S is not itself precise, then it appears that the condition that every admissible precisification of S be true will not be determinately satisfied. But this line of thinking is mistaken. Notice that every way of precisifying S that does not allow the extension and anti-extension of 'red' to overlap will make the resulting sentence true. And since every admissible precisification of S is one that will not allow the extension and anti-extension of 'red' to overlap,\(^{18}\) then every admissible precisification of S will be determinately true. Similar reasoning will show that other classical tautologies are determinately true under supervaluation semantics. Hence, classical logic is preserved.

Dummett's argument is unsuccessful. First, I showed that an anti-realist might be able to uphold valence for natural language. He could claim that sentences get their truth-values in virtue of the decisions that would be made by an ideal group of decision-makers constrained to give every sentence a determinate truth-value. Of
course, while it seems that an anti-realist could take this stance, it is not at all clear that any would want to take it. For this reason I would not want my attack on Dummett's argument to end here. Fortunately, my case against Dummett's second premiss is much stronger. One can justify classical logic without using a valent notion of truth using supervaluation semantics.

Apart from Dummett's argument, however, there are other reasons for thinking that metaphysics determines logic. Suppose, as Putnam would have us do, that metaphysical realism is false. Then, according to Putnam's version of metaphysical anti-realism, there are "vague objects" in the world, i.e., there is an object o and a property P such that o neither determinately has nor lacks P. Hence, it seems that the statement 'O is P or o is not P' should be false, and so the classical law of excluded middle is not justified. This argument appears to be very straightforward. A logic which includes 'P or not P' as a tautology might lead us astray in cases of reasoning involving objects which neither determinately have nor determinately lack some of their properties.

If the argument above is a good one, then the rejection of metaphysical realism does require that classical logic be rejected also. The argument fails, however, because to reject metaphysical realism is not necessarily to believe that some sentences of the form 'O is P,' where 'O' corresponds to an object and 'P' to a property, are neither determinately true nor determinately false. An anti-realist can believe the negation of that claim, as does the realist.
To do so, of course, the anti-realist must believe that some predicates correspond to properties, while some do not. But he can do that. The realist believes that only precise predicates correspond to properties. There is no reason the anti-realist cannot believe that claim as well. The anti-realist can even say that a predicate $P$ corresponds to a property just in case $P$ is precise. Once the anti-realist defines $P$'s corresponding to a property in this way, it follows that on his view all sentences of the form '0 is $P$' where '0' corresponds to an object and '$P$' to a property will be either determinately true or determinately false. So, rejecting metaphysical realism does not force one to reject that claim as well. Hence, it has yet to be shown that the rejection of metaphysical realism requires that we give up classical logic.

**Metaphysical Realism Reconsidered**

So far, I have examined two arguments for the claim that classical logic is justified only if metaphysical realism is true, neither of which has proved to be successful. It is interesting to note that both arguments break down, at least in part, because of a failure to correctly characterize metaphysical realism. I showed that Dummett's attempt to equate being a realist with believing that our language is valent is not entirely successful, since an anti-realist could believe in the valence of natural language. Then, I showed that the belief that objects either determinately have or determinately lack properties is not characteristic only of metaphysical realists. There is nothing to stop an anti-realist from
believing that claim also. But if metaphysical realism cannot be adequately characterized and distinguished from anti-realism in either of the preceding ways, how is this task ultimately to be accomplished.

Simon Blackburn argues that the task cannot be achieved because nothing substantive distinguishes realism about a subject matter from anti-realism about that subject. He claims that any principle, acceptance of which is purported to be the mark of the realist, can be shown, with a little ingenuity, to be acceptable to an anti-realist also. If Blackburn's contention is correct, then certainly the issue whether or not one should adopt classical logic cannot justifiably depend upon whether one accepts metaphysical realism.

Blackburn shows that neither recognition of the regulative/constitutitive distinction which Kant makes nor the adoption of an explanatory view towards the subject matter in question is enough to brand one as a realist. There are, however, other options which have yet to be discussed.

Dummett proposes to give the name 'metaphysical realism' to the view which consists of the following three tenets: (MR1) The world consists of a definite totality of discourse-independent objects and properties; (MR2) An object either determinately has or determinately lacks any property which may significantly be predicated of it; and (MR3) Truth is correspondence with reality.

I have already shown that (MR2) by itself cannot play the role of metaphysical realist-identifier (although whether it will serve
that purpose when taken in conjunction with (MR1) and (MR3) remains to be seen). But if (MR2) cannot serve as the mark of a realist, it is easily shown that (MR3), taken in isolation, cannot either. I have shown how an anti-realist can mimic realist "correspondence-talk." But once an anti-realist can talk about correspondence with reality, there is nothing to stop him from holding a correspondence theory of truth as well. It should also be obvious at this point that adherence to both (MR2) and (MR3) will not work to set off the realist from the anti-realist, since an anti-realist can hold both of these principles as easily as he can hold one or the other of them.

(MR1), on the other hand, holds more promise as a candidate for realist-identifier. But any attempt to use (MR1) to differentiate metaphysical realists from anti-realists requires that some sense be made of the expression 'discourse-independent' which will not allow anti-realists to accept the principle. Blackburn argues that it is not enough to use metaphors like 'capable of being investigated like a piece of geography' to explain the phrase. Some literal sense must be made of it.

Geoffrey Hellman has addressed the analogous problem of finding a reading of 'mind-independent' which allows him to use the following principle,

\[(SRI) \quad \text{Much of science investigates a mind-independent material world.}\]

which is relevantly similar to (MR1), to distinguish scientific realists from both instrumentalists and constructive empiricists.\(^{22}\)
Hellaan initially finds that there are several intuitively appealing ways of making sense of 'mind-independent' which, unfortunately, make (SRI) acceptable to constructive empiricists. He does, however, eventually arrive upon a way of reading 'mind-independent' which he thinks makes (SRI) acceptable only to scientific realists. He suggests that we say that objects of scientific discourse are mind-independent iff

the phenomenal facts do not suffice to determine uniquely the physical facts ... [i.e.,] there is at least a pair of models (meeting whatever standardness requirements you like) of the totality of scientific laws (known or not but formulable in our overall scientific language) which agree on the truth-values of all sentences in phenomenal vocabulary but which differ in the truth-values of some sentences in physical vocabulary.²³

Can we use a reading of 'discourse-independent' which is analogous to Hellman's reading of 'mind-independent' to successfully distinguish metaphysical realism from Putnam-type anti-realism? Perhaps we can. A metaphysical realist must believe that facts about our discourse are not enough to determine ontological facts, but perhaps a Putnam-type anti-realist cannot believe this.

But what must we mean here by 'facts about our discourse do not determine ontological facts'? Hellman explicates 'determination of physical facts by phenomenal facts' in terms of models. But it is not so clear that this can be done in this case, since in order to set up a model one must already be clear about one's ontology.

It can't be enough to say that metaphysical realists believe that there are at least two models of the same discourse with
different domains, while anti-realists don't, because this would make anti-realism too easily refutable. Another of Hellman's suggestions may be useful at this point. He notes that a scientific realist will believe that "even if all sentience were to disappear, there would still remain galaxies, messenger RNA, and so forth," although an anti-realist would at least have a lot of trouble believing this claim. Perhaps, then, metaphysical realism can be characterized by the belief that a world without discourse would still be a world of objects.

A little thought will show that this suggestion is not a workable one either because a metaphysical anti-realist can hold the same belief. The anti-realist will say that 'objects' in the claim above refers to the objects that our current discourse defines. Certainly those things would still exist if we were not around to talk about them. Thus, the claim above is not problematic for the anti-realist. He can accept it as easily as the realist can. Hellman's suggestions, although they may work to distinguish scientific realists from scientific anti-realists, will not distinguish metaphysical realists from metaphysical anti-realists.

But there are still other possible ways of making the distinction we seek. Putnam suggests at one point that what is definitive about metaphysical realists is their willingness to accept modal claims such as:

(A) Venus might not have carbon dioxide in its atmosphere even though it follows from our theory that Venus has carbon dioxide in its atmosphere,
and

(B) A statement can be false even though it follows from our theory (or from our theory plus the set of true observation sentences).25

Blackburn points out that an anti-realist can justifiably subscribe to (A) and (B). He can do this because he can admit that there may be a standpoint or theory which is different from his current one, yet which is admirable from the standpoint of his current theory, and from which some statement which his current theory says is false may be derived. This much I think is straightforward. But what about the following claims?

(C) Venus might not have had carbon dioxide in its atmosphere even though it followed from an ideal theory that Venus has carbon dioxide in its atmosphere.

(D) A statement could be false even though it followed from an ideal theory.

Could an anti-realist accept them, too? C and D require that there be some gap between epistemology and truth. One might think that it is such a gap that anti-realists want, above all, to deny. But even this is not so clear. Sophisticated anti-realists will want to accept that such a gap exists. They will refuse to equate truth with any epistemological notion. For example, consider an anti-realist who believes in the redundancy theory of truth. Such a theorist believes that to say a sentence S is true is to do nothing more than to assert S. But there is no reason that this theorist must also equate
S with some sentence that attributes some epistemological property to S.

If not even (C) and (D) can be used to set metaphysical realists apart from metaphysical anti-realists, then there is no reason to think that the issue whether we should accept or reject classical logic has anything to do with whether we accept or reject metaphysical realism. But if one's stand on the metaphysical realism/anti-realism dispute is not important for the justification of logic, then what is? Perhaps the justification of logic does have something to do with metaphysics, even though it does not depend on the realism/anti-realism dispute. For although claims such as the following—that the world consists only of objects which either determinately have or determinately lack any property which can significantly be attributed to them—cannot be used to differentiate metaphysical realists from anti-realists, they are still claims about the world and can be opposed to other views about the world. So, perhaps, the claim above can be used as a premiss in an argument for the justification of classical logic.

The argument I have in mind is that because the claim above is true, the classical law of excluded middle is justified. Such an argument is appealing, yet it is ultimately unsuccessful. This is because what is important for the justification of logic is not how the world is, but how the world is represented by us. The world might consist only of objects which either determinately have or determinately lack any property which can be significantly attributed
to them, but if we represent the world as if this is not the case, then the classical law of excluded middle may not be justified after all.

For example, suppose that the world consists of objects which either determinately have or determinately lack any light wavelength property which can be significantly attributed to them. Suppose also that 'red' does not correspond to a real property in the world. In that case, it could be said that although the world is determinate, we represent it as if it is not using 'red' and other vague color words. Then, because there are sentences in our language such as 'X is red' which are sometimes neither determinately true nor determinately false, we may want to reject the classical law of excluded middle, even though all objects and properties in the world are determinate.

It is the way we represent the world that is important to the justification of logic. In other words, logics are justified if they correctly reflect our reasoning practice. Obviously this is a deep issue and I shall have more to say about it in Chapter Five. For now I shall just say that a good logic for our natural language should make arguments valid which we intuitively take to be valid and should make statements tautologies which play the role in our reasoning of tautologous statements. But if this is what the justification of logic rests upon, then the door is open for an argument for the rejection of classical logic after all, although the argument will be
based solely upon the fact that our language is vague, rather than on any metaphysical grounds.

The fact that our language is vague ensures that it contains statements which are neither determinately true nor determinately false. Perhaps this fact alone, rather than any metaphysical fact which may or may not underlie it, is enough to make us reject classical logic. Of course, this depends on what one means by 'reject classical logic.' If one means reject classical truth conditions, then vagueness will force us to reject classical logic. The indeterminacy in truth-value of some statements that vagueness causes will force us to give up bivalence and, hence, the classical truth conditions. On the other hand, if one means by 'reject classical logic' 'reject classical tautologies and theory of deduction,' then vagueness will not force us to give up classical logic, since truth-conditions can be given which preserve these things in spite of vagueness. So, there is a sense in which vagueness does force us to give up classical logic and a sense in which it does not. But it is now clear that vagueness does force us to give something up—namely, classical truth-conditions.
NOTES

1. My wording of (P2) is actually stronger than Putnam's. In Hilary Putnam, "Vagueness and Alternative Logic," Erkenntnis, 19 (1983), p. 299, he only makes the following cautious suggestion: "if the metaphysical picture that grew up with an conditioned classical logic is wrong, then some of the tautologies of classical logic may have to be given up."


3. I talk frequently in this chapter about the metaphysical realist's conception of a predicate corresponding to a property. For the metaphysical realist this is often a metaphorical notion that never gets cashed out. The realist's picture is of predicates reaching out and latching onto properties. I grant that this metaphorical notion is not satisfactory.

I refrain, however, from making an attempt to cash out the metaphorical notion of a predicate corresponding to a property. My aim in this section is solely to refute the argument of Putnam's which is given on pp. 32-33 of this chapter and I can do that without addressing this issue. I am in no way attempting to endorse the realist's view. I only want to show that the realist view is compatible with the presence of vagueness. Those readers who are unhappy with pictures will please try to bear with me.


6. Ibid.

7. In other words, S-V preserves classical theorems and theory of deduction, even though it does not preserve bivalence.


11. Ibid., p. 299.


15. Supervaluation semantics was first developed by Bas van Fraassen to deal with sentences containing non-denoting singular terms. See Bas van Fraassen, "Singular Terms, Truth-Value Gaps, and Free Logic," Journal of Philosophy, 63 (1966), pp. 481-495. For more on S-V semantics in application to vague
languages, see Kit Fine, "Vagueness, Truth and Logic," *Synthese*, 30 (1975), pp. 265-300. Also, see Chapter Three of this volume.

16. See Chapter Three of this volume.


18. See Chapter Three of this volume.


23. Ibid., p. 236.

24. Ibid.

CHAPTER III

SUPERVALUATION SEMANTICS

The Semantics and Some of its Properties

Supervaluation semantics (S-V) was developed by van Fraassen to handle sentences containing non-denoting singular terms.\(^1\) Fine has subsequently applied the semantics to vague languages.\(^2\) S-V is an appealing semantics for vagueness because it reflects the nonbivalence of such languages, yet still yields classical logic. In this section, I lay out S-V following Fine.\(^3\) Then, in Part Two, I go on to defend S-V against some objections. I conclude that, while many objections have been raised against the semantics, none pose any serious threat to it. In fact, most of the criticisms that have been leveled at S-V arise because of a failure to perceive correctly some feature of it which is actually quite desirable.

First, an S-V model is an ordered triple \((D, T, P)\), where (1) \(D\) is a nonempty set; (2) \(T\) assigns denotations in \(D\) to all terms and a domain \(T_1(F) \subseteq D^n\) and (disjoint) anti-domain \(T_2(F) \subseteq D^n\) to each n-ary
predicate; and (3) $P$ is a nonempty set of classical models all with
domain $D$.

Next, truth and falsity in an S-V model are defined as follows.

**DEF:** A is true in $(D, T, P)$ iff $A$ is true in all models
in $P$.

**DEF:** A is false in $(D, T, P)$ iff $A$ is false in all mo­
dels in $P$.

$A$ is undefined in $(D, T, P)$ otherwise.

Finally, we define S-V validity and S-V implication (where '$\models$'
is read 'S-V implies').

**DEF:** A is S-V valid iff $A$ is true in all S-V models.

**DEF:** $\Gamma \models A$ iff in every S-V model in which all members
of $\Gamma$ are true, $A$ is true.

Informally, the semantics can be thought of as working in the
following manner. The domain and anti-domain that are assigned to a
predicate $F$ by $T$ constitute the extension and anti-extension, respec­
tively, of $F$. $T_1(F)$ will be the set of things of which $F$ determin­
ately holds. Similarly, $T_2(F)$ will be the set of things of which $F$
determinately fails to hold. We have imposed the formal requirement
that $T_1(F) \cap T_2(F) = \emptyset$, since $F$ cannot both determinately hold and
fail to hold of the same things. Also, if $n$-ary $F$ is vague, then
$T_1(F) \cup T_2(F) \neq D^n$. $D^n - (T_1(F) \cup T_2(F))$ will be the set of border­
line cases of $F$.

The set $P$ in an S-V model can be thought of as the set of ad­
missible precisifications for vague predicates in that model. This
The notion of admissibility is primitive for Fine. The idea is roughly that a precisification \( M \) of a predicate \( F \) is \textit{admissible} in an S-V model just in case \( M \) is in accordance with \( F \)'s meaning as "fixed" by the model. For example, in our "intended" model a precisification of 'bald' which made a man with 10,000 hairs bald, a man with 10,001 hairs nonbald, and a man with 10,002 hairs bald would not be admissible. Contained in the meaning of 'bald' (as fixed by the "intended" model) is the following constraint: once the number \( n \) gets large enough so that a man with \( n \) hairs is not bald, then any many with \( m \) hairs, where \( m > n \), is not bald as well. Fine also argues that constraints concerning logical relations are contained in the meaning of terms. For example, according to Fine, it is part of the meanings of 'red' and 'orange' that an object cannot be both red and orange. Fine refers to the possibility that logical relations hold among vague predicates as \textit{penumbral connection}. One can think of the meaning of a vague predicate as containing constraints for admissible precisifications of that predicate in accordance with penumbral connection.

One constraint on admissibility is clear. Admissible precisification of predicates should leave clear cases and clear non-cases alone. For example, what is a clear case of red before precisification should remain so after precisification. Accordingly, one would want to impose at least the following formal constraint on \( P \).

\[
(4) \text{ If } (D, T, P) \text{ is an S-V model and } (D, \psi) \in P, \text{ then } T_1(F) \subseteq \psi(F) \text{ and } T_2(F) \subseteq D^P - \psi(F). \]
Notice that this constraint poses only a necessary condition for admissibility, thus leaving admissibility primitive.

(4) cannot function as more than a necessary condition for admissibility because it imposes none of the constraints required by penumbral connection. Were (4) to be a sufficient condition for admissibility, a precisification that let a blob be both red and orange could be admissible. But this is what penumbral connection will not allow. Precisifications of predicates must ebb and flow together.

For Fine,

if language is like a tree, then penumbral connection is the seed from which the tree grows. For it provides an initial repository of truths that are to be retained throughout all growth. Some of the connections are internal. They concern the different borderline cases of a given predicate: if Herbert is to be bald, then so is the man with fewer hairs on his head. But many other of the connections are external. They concern the common borderline cases of different predicates: if the blob is to be red, it is not to be pink; if ceremonies are to be games, then so are rituals; if sociology is to be a science, then so is psychology. Thus penumbral connection results in a web that stretches across the whole of language. The language itself must grow like a balloon, with the expansion of each part pulling the other parts into shape.

The fact that admissibility is a primitive notion is not problematic because our intuitions regarding which precisifications are admissible and which are not are quite clear. (I shall have more to say about this in the next section.) Nor is the use of a primitive notion in a semantics anything new or unusual. For example, the similarity relation in Lewis' semantics for counterfactuals is primitive.
Given that formally we are imposing only a necessary condition for admissibility, however, the question of what S-V models there are does arise. Fortunately, we can give at least a partial answer to this question. Classical models are (in effect) S-V models. Classical models are precise (allow for no borderline cases), and precise languages are just limiting cases of vague ones. A precise language is a vague language with no vague predicates. Accordingly, a classical model M combined with the set of admissible precisifications for it (which will, of course, contain only M) will be an S-V model. To make this precise, let $M = (D, T)$ be a classical model. The model induced by M is the triple $(D, T^*, \{M\})$, where $T^*$ is the function that assigns the same denotations to terms as does $T$ and assigns both $T(F)$ and $D^n - T(F)$ to n-ary F.

Now our partial answer to the question of what S-V models there are can be formally stated:

If M is a classical model, then the model induced by M is an S-V model.

While this partial answer is extremely modest, it is also very important. Notice that if no S-V models existed, then, vacuously, every sentence would be S-V valid, thereby robbing S-V of its interest.

Finally, we are equipped to see the intuitive idea behind the formal conditions for truth and falsity in an S-V model. A is true in an S-V model just in case every precisification of A's predicates compatible with the meaning of the predicates (as fixed in the model)
would result in a true sentence, false just in case every such precisification would result in a false sentence, and truth-valueless whenever some precisifications result in true sentences and some in false.

Before I show that S-V yields classical logic and the classical theory of deducibility, I want to point out a nice feature of the semantics. Notice that we defined S-V implication in the standard way. An intuitive way of defining that notion, however, is to say that S-V implies $A$ just in case in each S-V model, every way of precisifying the predicates in the members of $\Gamma \cup \{A\}$ that makes all members of $\Gamma$ true also makes $A$ true. We can make this intuitive idea precise with the following definition. Let '$\vdash *'$ be read 'S-V implies *'.

**DEF:** $\Gamma \vdash * A$ iff for every S-V model $(D, T, P)$ and every $M \in P$, if $M$ satisfies $\Gamma$, then $A$ is true in $M$.

It turns out that it makes no difference whether we use $\vdash$ or $\vdash *$ for S-V implication. This is because $\Gamma \vdash A$ if and only if $\Gamma \vdash * A$.

**CLAIM:** $\Gamma \vdash A$ iff $\Gamma \vdash * A$.

**PROOF:** Suppose $\Gamma \vdash * A$. Now consider any S-V model $(D, T, P)$ in which all members of $\Gamma$ are true. Then all members of $\Gamma$ are true in every $M \in P$. Then, since $\Gamma \vdash * A$, $A$ is true in every $M \in P$. But then $A$ is true in $(D, T, P)$.

On the other hand, suppose $\Gamma \vdash * A$. Then there is a classical $M = (D, T)$ that satisfies $\Gamma$ but makes $A$ false. But then the model induced by $M$ is an S-V model that satisfies $\Gamma$ but makes $A$ false.
The equivalence of these two notions is a nice feature of S-V, since it means that our intuitive notion of S-V-implication and the standard notion come to the same thing.\(^7\)

I shall now show that S-V yields classical logic and the classical theory of deducibility.

CLAIM: A is S-V valid if and only if A is classically valid.

PROOF: Suppose A is not classically valid. Then A is false in some classical model M. But then the model induced by M is an S-V model in which A is also false. So A is not S-V valid.

On the other hand, suppose A is classically valid. Then A is true in all classical models. Consider any S-V model \((D, T, P)\). A is true in all models in P, so A is true in \((D, T, P)\). Hence, A is true in all S-V models, i.e., A is S-V valid.

CLAIM: \(\Gamma \vdash \ast A\) iff \(\Gamma\) classically implies A.

PROOF: Suppose \(\Gamma\) does not classically imply A. Then there is some classical \(M = (D, T)\) which satisfies \(\Gamma\) but in which A is false. But then, since the model induced by M is an S-V model, \(\Gamma \vdash \ast A\).

On the other hand, suppose \(\Gamma\) classically implies A. Then in all classical models M which satisfy \(\Gamma\), A is true. Consider any S-V model \((D, T, P)\). In every \(M \in P\) which satisfies \(\Gamma\), A is true. So \(\Gamma \vdash \ast A\).

Some Criticisms of S-V

Van Fraassen and Putnam have both raised the same important problem for the supervaluation approach.\(^8\) Notice that the semantics requires that we give up bivalence while accepting the law of
excluded middle. Yet there is good reason to think that this cannot be done. From

\(P \text{ or not-}P\)

and

\(P \text{ iff } 'P' \text{ is true}\)

it follows that

\('P' \text{ is true or 'not-}P' \text{ is true.}\)

More specifically, the argument can be put like this:

\begin{align*}
(1) \quad & P \lor \neg P & \text{S-V semantics} \\
(2) \quad & P \rightarrow 'P' \text{ is true} & \text{T-schema} \\
(3) \quad & \neg P \rightarrow 'P' \text{ is true} & \text{T-schema} \\
(4) \quad & P & A \\
(5) \quad & 'P' \text{ is true} & 2,4 \text{ MP} \\
(6) \quad & 'P' \text{ is true } \lor 'P' \text{ is true} & 5 \text{ vI} \\
(7) \quad & \neg P & A \\
(8) \quad & 'P' \text{ is true} & 3,7 \text{ MP} \\
(9) \quad & 'P' \text{ is true } \lor 'P' \text{ is true} & 8 \text{ vI} \\
(10) \quad & 'P' \text{ is true } \lor 'P' \text{ is true} & 1,4,6,7,9 \text{ vE}
\end{align*}

This version of the argument is Putnam's. Van Fraassen, who developed S-V semantics as a way of dealing with sentences that contain non-referring singular terms, formulated two other versions. They are:

\begin{align*}
(11) \quad & P \lor \neg P & \text{S-V semantics} \\
(12) \quad & P \leftrightarrow 'P' \text{ is true} & \text{T-schema} \\
(13) \quad & 'P' \text{ is true } \lor \neg P & 1,2 \text{ sub. equiv.} \\
(14) \quad & \neg P \leftrightarrow 'P' \text{ is true} & \text{S-V semantics} \\
(15) \quad & 'P' \text{ is true } \lor 'P' \text{ is true} & 13,14 \text{ sub. equiv.}
\end{align*}

and
(16) $P \rightarrow 'P' \text{ is true} \quad \text{T-schema}

(17) -('P' \text{ is true}) \rightarrow -P \quad 16 \text{ contrapos.}

(18) -P \rightarrow '-P' \text{ is true} \quad \text{T-schema}

(19) -('P' \text{ is true}) \rightarrow '-P' \text{ is true} \quad 17,18 \text{ transit.}

(19) is not literally the principle of bivalence, but, as van Fraassen remarks, if (19) is true, then so is bivalence.

There is a way of responding to these arguments which is, unfortunately, very ad hoc. One could define an argument to be valid if and only if it is impossible for its premisses to be true and its conclusion false. Accordingly, it would be possible for a valid argument to have all true premisses and a non-true conclusion. One could claim, then, that the conclusions of the three versions of the argument are neither true nor false.

This solution is problematic because there is no good reason to think that (10), (15), and (19) are not false. Intuitively, we want to say of those schemas that they are false for choices of 'P' like 'X is red' where 'X' refers to a borderline case of red. There are no ground for claiming that (10), (15), and (19) are merely not true rather than false.

Another response is to claim that whereas each version of the argument relies on classical inference rules, those rules are not applicable to metalinguistic arguments. After all, if the metalanguage contains the object language, or can otherwise translate it, then the metalanguage will have truth-value gaps. But there is no reason to
think that classical inference rules will hold for nonbivalent languages.

Consider, for example, v-Elimination, which is used in the first version of the argument. To infer C, in a nonbivalent language, given P v ¬P, it should not be enough to derive C from P and from ¬P, because neither of those may be true. It looks, then, as if v-Elimination is not justified in this case. Perhaps there are similar problems with some of the other classical inference rules used to derive bivalence in the arguments above.

This reasoning, though it may seem convincing initially, is specious. There are nonbivalent languages for which the classical inference rules are valid. For example, if one applies supervaluation semantics to object-language English, as Fine advocates, the result is just such a language. Although the language will be non-bivalent, classical inference rules will be valid, provided that a valid inference is defined as one such that if its premisses are true in some S-V model, then its conclusion is also true in that model. The fact that the metalanguage is nonbivalent is not enough, in and of itself, to show that some classical inference rules are invalid. Whether or not they are invalid will depend upon how we choose to interpret the metalanguage.

Van Fraassen offers a solution to the problem. He asks us to notice that on one way of interpreting the metalanguage for a nonbivalent object language the two sides of a T-sentence sometimes differ in truth-value. Consider, for example,
(20) 'X is red' is true iff X is red.

Let 'X' refer to a borderline case of red. Then the right side of the biconditional is truth-valueless and it is reasonable to think that the left side will be false.

Van Fraassen suggests that, although we do not want to say of nonbivalent languages that

(21) 'P' is true iff P,

where 'iff' is construed as a material biconditional (i.e., is read as implying that both sides of the biconditional will always have the same truth-value), we do want to say both

(22) A ⊨ 'A' is true, and

(23) 'A' is true ⊨ A.\(^{11}\)

But (22) and (23) are not enough to let any of the three versions of the argument go through.

The first version is blocked because lines (2) and (3) are replaced by (22) and (23). But then the inferences at lines (5) and (8) cannot be made. Consider line (5). One cannot conclude ''P' is true' from line (2) and the assumption that 'P' is true, which occurs at line (4), although one may conclude ''P' is true' from line (2) and the fact that P is true.\(^{12}\) The same thing can be said of the inference at line (8).
The second version of the argument is blocked because line (12) has to be replaced by (22) and (23). But then the operation Sub. Equiv., performed at lines (13) and (15), will not be justified. It is clear that we should not be able to substitute 'P* is true* anywhere for 'P,' given that the two sentences may differ in truth value. It is also easy to see that the third version of the argument will be blocked by van Fraassen's suggestion. Line (16) will be replaced by line (22). But then the inference at line (17) will not be justified.

Of course, there is also some intuitive appeal to the thought that the metalanguage should be interpreted so that 'P* is true* is truth-valueless when 'P' is. But if we interpret the metalanguage in that way, then the arguments above are all sound. Van Fraassen rejects this way of interpreting the metalanguage on grounds that it would make the metalanguage nonbivalent. He writes:

And we are so used to discussing the structure of languages ourselves in the usual language of ordinary (that is, classical) logic that this [non-bivalence] may appear as a drawback. That is, if the metalanguage is not bivalent, then it does not seem to be a correct model of the part of natural language that we actually use to discuss the object language.13

Van Fraassen, however, was using supervaluations to interpret sentences with non-denoting singular terms rather than to interpret vague language. For this reason, he had no need of appealing to admissible precisifications. Our use of supervaluation semantics requires that we have the vague term 'admissible precisification' in
the metalanguage and hence that the metalanguage be nonbivalent. We can, therefore, not justify our way of interpreting the metalanguage—so that "P is true" is false when 'P' is truth-valueless—as van Fraassen does.

Our way of interpreting the metalanguage is nevertheless correct. Suppose that an object language sentence 'P' is truth-valueless according to supervaluation semantics. Then some admissible precisification of 'P' results in a classically false sentence. But 'P' is true iff every admissible precisification makes 'P' classically true, so "P is true" must be false. So, when 'P' is truth-valueless, "P is true" is false.

Dummett gives another criticism of S-V. He argues that the semantic's rescue of classical logic is gained at the cost of not really taking vague predicates seriously, as if they were vague only because we had not troubled to make them precise. A satisfactory account of vagueness ought to explain two contrary feelings we have: that expressed by Frege that the presence of vague expressions in a language invests it with an intrinsic incoherence; and the opposite point of view contended for by Wittgenstein, that vagueness is an essential feature of language. The account just given, on the other hand, makes a language containing vague expressions appear perfectly in order, but at the cost of making vagueness easily eliminable. But we feel that certain concepts are ineradicably vague. Not, of course, that we could not sharpen them if we wished to; but, rather, that, by sharpening them, we should destroy their whole point.14

Let us put blame here where blame is due. The problem is not that S-V fails to take vagueness seriously, but rather that Dummett
has failed to take S-V seriously. S-V semantics, besides being compatible with an explanation of the two feelings mentioned by Dummett, functions in an account of why we are generally so successful with our use of vague predicates. Although Dummett did not call our attention to that success, he must be aware of it. Why else would he consider the feeling that vague language is incoherent to be contrary to the feeling that vagueness is essential to our language? I take it that he finds these feelings contrary to one another because: (1) their joint truth entails that our language is essentially incoherent; and (2) this essential incoherence is mysterious in light of the great success of our language use.

Our feeling that vague language is intrinsically incoherent can be explained by appeal to the sorites paradox. Pretheoretically, the paradox is confusing to us and seems to point to some incoherence on the part of vague predicates. The fact that S-V provides a solution to the sorites does not make our previous thought that a solution to the sorites does not make our previous thought that vague predicates might be incoherent strange. The paradox gave us good reason to suspect incoherence; S-V gives us good reason to reject it.

On the other hand, we feel that vagueness is an essential part of our language simply because it is. First, we are aware that to precisify 'red' once and for all would be to change its meaning. Second, Crispin Wright has shown that this change in meaning will sometimes be an essential one. To permanently precisify vague terms such as 'red' is to "pull language apart from experience." 'Red' is
an observational predicate, i.e., we determine whether a thing is red simply by looking at it. Suppose we were to precisify 'red' by picking a point on the color continuum where red stopped and non-red began. Then the shades immediately to either side of that point would be observationally indistinguishable from one another. There would be two observationally indistinguishable shades of color, one red and the other not. It follows that we would no longer be able to judge an object's color simply by looking at it. 'Red' loses its observationality upon precisification.

Alston points to another way in which vagueness is essential to the expressive capacity of our language. He says:

In many areas our situation is such that we cannot make maximally precise statements without going far beyond the evidence. Thus, we have reason to think that city life imposes more psychological strain on people than country life. In formulating this principle we have used a vague word like "city." Of course, we could remove the vagueness in question, which has to do with the minimum number of inhabitants required, by stipulating that any community containing at least 50,000 inhabitants will be called a city. But having done so, we can no longer make the statement with any assurance. There is no precise population cutoff point above which there is a marked increase in the psychological strain imposed by the community.16

Dummett's suggestion that S-V semantics treats vagueness as easily eliminable from our language is mistaken. A special semantics for vague language would be beside the point, if vagueness were so easily eliminable. S-V is a semantics for classical logic which does "take vagueness seriously," which does, in other words, reflect the indeterminacy in truth-value characteristic of vague sentences. The
statement of the semantics itself contains the vague term 'admis-
sible.' This, to repeat a point, is the beauty of the semantics; it
justifies classical logic while fully accepting the failure of bi-
valence due to vagueness. It does one thing which classical semantics
does not, that is—take vagueness seriously.

Because we appeal to precisifications in assigning truth-values
to sentences containing vague predicates does not mean that we see
vagueness as being easily eliminable. There is nothing contradictory
in our doing both the following: (a) seeing vagueness as an essential
phenomenon that contributes to the communicative power of our
language, and (b) appealing to precisifications in an account of the
semantics of that language.

Another criticism of S-V is raised by David Sanford and upheld
by La Verne Shelton. Sanford writes:

Suppose that if Jones reads any of the books on his
shelf he will know something about the history of type
design. From this it does not follow that he will know
something about the history of type design even if he
reads none of the books. Nor does it follow that Smith
has done his fair share of the work simply because he
has done his fair share of the work if he has done any
one of the designated chores. For he may have done none
of the designated chores. Grant then that a certain
statement is true if its predicates are made completely
precise in any appropriate way. Why should the state-
ment thereby be regarded as true if its predicates are
not made precise in any of these appropriate ways?17

Shelton refers to Sanford's criticism and continues:

Fine could go in two directions here. He could say that
in fact the sentence will be precisified in one of the
potential ways; or, he can say that this need not be the
case, but that in some sense the sentence already has all of these precise referents. The first alternative is clearly wrong. It is almost never the case that ordinary concepts are relieved of their borderline cases.... We have both scientific and legal expressions that have exact reference, but these are not in common use: "Out of the office" people tend to use expressions like 'adult' or 'fruit' with their old inexact reference.

If Fine opts for the second alternative, he is claiming that precisifications exist in whatever sense we once thought referents existed. We could take him to be postulating them as theoretical entities. If this is the case, a minimum criterion for accepting them is they save the phenomenon in at least as natural a way as alternatives. But though they do solve the puzzles we have had with inexact reference—exemplified most graphically by the sorites paradox—they do so at the cost of putting a structure on semantics that seems directly orthogonal to the structure it has. Reference varies through contexts. It is not various at a given context.18

Sanford's analogies need not trouble us. Sanford points out that it does not follow logically from the fact that Jones will know some history if he reads his books that Jones will know some history. But Fine never meant to deduce that 'S' is true from the fact that 'S' is true on any precisification. Still, Sanford's point can be put more charitably—why should we even think that a sentence is true just in case all of its admissible precisifications are?

Shelton is wrong to think that Sanford's question allows for only two possible answers. Fine can agree that vagueness is here to stay while still avoiding any commitment to abstract precisifications. S-V semantics does not require that reference be various at a given context. 'Red' does not refer to many different sets on one occasion. Rather, it refers to one set in every context—the set of
things that would be red, given every way we could admissible precisify 'red.' Notice that no appeal to abstract precisifications is needed here. (We can talk about what would be the case if we were to precisify without committing ourselves to the existence of any abstract entities.) But now Sanford and Shelton will ask why we should think that an object o falls under the extension of a word w just in case it does so given every admissible precisification of w. We avoid appeal to abstract precisification, but Sanford's question remains unanswered.

Several things can be said here on behalf of S-V. First, Fine points to our tendency to regard an ambiguous sentence as true when either of its disambiguations are. Consider

(S) John went to the bank,

when John has been both to the river's edge and a financial institution. In such a case, we are happy to accept the suggestion that (S) is true. So why not accept that a vague sentence is true when all its admissible precisifications are?

Next, Fine appeals to his claim that the ways of precisifying a word are contained in its meaning. But, then it should not surprise us that the truth-conditions for vague sentences appeal to precisifications. Shelton rejects Fine's claim in favor of the thesis that the meaning of a word must change upon change of reference. She thinks that to precisify 'red' so that its reference changes is to change the meaning of 'red.' Meaning, according to Shelton, cannot remain
stable through precisification. But Shelton offers little defense of her thesis and to simply accept that thesis, while denying Fine's, is to beg the question.

Another reason for accepting the truth-conditions afforded vague sentences by S-V semantics is that by doing so we get a semantics for classical logic which takes vagueness seriously and validates our intuition that 'red' is univocal across contexts. The semantics is justified by the fact that it does the required work.

Another criticism of S-V semantics is made by Roy Sorensen. In "Vague Entailment and Nonstandard Sorites Arguments," Sorensen calls our attention to the previously ignored fact that vague predicates can lack even possible cases of clear application and non-application. William P. Alston has said:

To say that a word is vague is to say that there are cases in which there is no definite answer to whether it applies to something.

This is a definition of extensional vagueness. Intensional vagueness can be defined roughly by inserting 'possible' before 'cases' in the definition above. Whereas neither of these definitions requires that a vague predicate have even possible cases of clear application or non-application, Alston goes on to say:

Thus "middle-aged" is vague, for it is not clear whether a person aged 40 or a person aged 59 is middle-aged. Of course, there are uncontroversial areas of application and non-application. At age 5 or 80 one is clearly not middle-aged, and at age 45 one clearly is. But on either side of the area of clear application there are indefinitely bounded areas of uncertainty.
This case is typical. Even where definitions of vagueness do not require the possibility of clear cases of application and non-application, theorists have assumed that such cases exist. Sorensen's arguments to the effect that vague predicates can lack these clear areas of application are convincing. And even in the absence of such convincing arguments, theorists have been unwise simply to ignore the possibility of "purely vague predicates." Sorensen's focus on these predicates is thus very welcome. His attempts to draw consequences from their existence for popular logics of vagueness, however, are not convincing.

Sorensen argues that singly anchored vague predicates (those with clear cases of application but lacking clear noncases, or vice versa) and unanchored vague predicates (those with neither clear cases nor clear noncases) pose problems for S-V, but not for many-valued semantics. Whereas he is correct to think that a many-valued semantics such as that of Lakoff and Zadeh is adequate for a treatment of these predicates, he is wrong to think S-V is inadequate for this purpose.

Sorensen's first criticism of S-V with regard to these predicates follows. He writes:

First, supervaluationists must revise their rules for an admissible precisification. They cannot require that new F's always be preceded by old F's. For the absence of old F's would only permit a single precisification; the one making all borderline cases non-F's. Thus, the predicate will have the same status as contradictory predicates. Parallel reasoning ensures that vague predicates having only clear positives and borderline cases would acquire the status of tautologous predicates.
Worse, predicates having only borderline cases would resist precisification altogether.\textsuperscript{24}

Recall from Part One of this chapter that, for Fine, admissibility is a primitive notion and that, loosely speaking, a precisification is admissible just in case it is consistent with the meaning of the term being precisified. Technically, then, the notion of admissibility will require no revisions in order to accommodate singly anchored and unanchored vague predicates. An admissible precisification of such a predicate is simply one consistent with the meaning of that predicate.

From a practical standpoint, however, one may want to formulate rules of thumb regarding the notion of admissibility. One such rule is that upon precisification old F's remain F's and old non-F's remain non-F's. The rule of thumb Sorensen proposes—that new F's be preceded on the continuum by old F's and new non-F's be followed by old non-F's (which, by the way, is not a rule Fine adopts)—is not only inadequate for singly-anchored and unanchored vague predicates; it is inadequate for some doubly-anchored vague predicates as well.

Consider the following diagram of a piece of the color continuum.

\textbf{Diagram 1}

\begin{center}
\begin{tabular}{ccc}
\textsc{nonred} & \textsc{fuzzy area} & \textsc{red} \\
& & \textsc{fuzzy area} & \textsc{nonred} \\
\end{tabular}
\end{center}
A precisification of 'red' which makes the color at point A a case of red is one in which a new case of red is not preceded by an old case. This precisification does not conform to Sorensen's rule, yet it is clearly admissible. Sorensen has failed to notice that some doubly anchored degree vague predicates have clear cases of non-application at either end of the continuum. The rule of thumb he suggests is adequate only for doubly anchored predicates like 'bald' which have clear cases of non-application at only one end of the continuum.

Are there general rules of thumb which apply across the board to all degree vague predicates? The rule that old F's remain F's and old non-F's remain non-F's is such a rule. The fact that conformity to this rule will not guarantee admissibility is not problematic, since that notion is, after all, primitive.

The fact that we may not be able to give a set of rules which constitute necessary and sufficient conditions for an admissible precisification is not a problem for supervaluation semantics. We have intuitions regarding which precisifications are admissible and which are not, and these intuitions are no less clear in the case of singly-anchored and unanchored vague predicates than they are in other cases.

Just as it is clear that the precisification in Diagram 1 is admissible, it is equally clear that the precisifications in Diagrams 2 and 3 are inadmissible.
Diagram 2

FUZZY AREA

NON-TINY VATS
(Tiny containers which are clearly non-vats.)

NON-TINY VATS
(Vats which are clearly non-tiny.)

B C D E
TV NTV TV NTV

Diagram 3

(Let J be some unanchored predicate.)

FUZZY AREA

F G H I
J NJ J NJ

In fact, another general rule of thumb seems to be that new F's and new non-F's cannot be alternated within old fuzzy areas.

Sorensen's main point regarding the problems which these predicates create for S-V, however, has little to do with the discussion above. For, he argues, any acceptable notion of admissibility will necessarily include in its extension the precisification of a singly anchored predicate in which all former borderline cases are counted as clear F's (or non-F's, as the case may be). And this he finds problematic. He writes:

For example, given that 'tiny vat' has no clear positives, it will have one "contradictory" precisification in which it acquires the status of a contradictory predicate. Now suppose 'tiny vat' is embedded in a negative sorites:
(C) 1. Something with a capacity of one million liters is not a tiny vat.

2. If a container with a capacity of n milliliters is not a tiny vat, then a container with a capacity of n-1 milliliters is not a tiny vat.

3. A container with a capacity of 1 milliliter is not a tiny vat.

Under "consistent" precisifications where we allow some borderline cases of 'tiny vat' to be resolved as clear positive cases, the second premise is false. But under the "contradictory" precisification the premise is true. We thus have an example of a sorites whose induction step does not come out false under all precisifications. Hence the supervaluationist cannot diagnose all sorites as unsound because of a false induction step.25

Sorensen is wrong to think that (C) constitutes any paradox at all. Notice that (C3) is true because a container with a capacity of 1 milliliter is not a vat at all. But more importantly, substitute any numeral for '1' in (C3). Then (C3) will not be determinately false, for, by hypothesis, there can be no possible clear case of a tiny vat. It follows that singly-anchored vague predicates are incapable of producing sorites-type arguments of the most troublesome kind—those which, besides being valid, appear to have all true premises and a determinately false conclusion.

This is not surprising when we consider the structure of sorites-type paradoxes. The first premise of a positive sorites affirms that some object is a clear case of some predicate P. Hence, as Sorensen noticed, the paradox requires at least a singly-anchored predicate. The inductive premise then allows us to infer that some other object is also a P. Notice that, in order for the argument to
produce the robust version of the paradox, the object which we infer to be $P$ must, in fact, be a clear non-$P$. Hence, true positive sorites (and, as is easily shown, true negative sorites) can only be generated by doubly-anchored vague predicates.

I want to suggest that this is one reason that unanchored and singly-anchored predicates have previously been ignored. While unanchored vague predicates produce no paradox at all, singly-anchored vague predicates are incapable of producing the robust version of the paradox. Most of the attention in the literature has been devoted to vague predicates which produce this paradox.

On the other hand, singly-anchored vague predicates are not entirely unproblematic. Using them, we can still produce a valid inference which appears to have all true premisses and a conclusion which is neither true nor false. But these apparently problematic inferences are readily diagnosed by supervaluation semantics. As Sorensen himself points out, according to $S-V$, the inductive premiss of these arguments is actually neither true nor false.

This is a virtue of the supervaluation approach, not a vice. Sorensen criticized the approach because he thought that a supervaluationist would be unable to diagnose all sorites as unsound because of a false induction step. But, in fact, $S-V$ semantics can diagnose all true sorites in this way. More importantly, as I show above, the approach also provides an illuminating diagnosis of the problematic inferences which can be produced by singly-anchored vague predicates.
NOTES


3. While Fine never explicitly states this, it is clear that he intends his treatment to cover a language in which vagueness is confined to predicates only and for which vagueness is the only type of semantic deficiency. The following version of Fine's approach is similarly limited.


5. Ibid., pp. 275-276.

6. Nor should it bother us that 'admissible' is itself vague. See "Realism and Classical Logic" in Chapter Two, this volume.

7. Stewart Shapiro pointed out to me that the two notions of S-V validity will come apart when they are defined for the meta-language. The proof will fail at the very last step where we infer from the fact that there is a classical model $M = (D, \tau)$ that satisfies $\Gamma$ but makes $A$ false that the model induced by $M$ does likewise. The truth in a precisification of a meta-language sentence like "'A' is true" depends upon the other precisifications in the S-V model. This is not true of object language sentences. So, if the members of $\Gamma$ and $A$ are in the
metalanguage, there will be no guarantee that they retain the
truth-values they had in M in the S-V model induced by M. The
fact that there may be different precisifications in M and in
the S-V model induced by M may make a difference to the truth-
values of members $\Gamma$ and A.

8. See Hilary Putnam, "Vagueness and Alternative Logic,"
Erkenntnis, 19 (1983), pp. 310-311; van Fraassen, "Singular
Terms," pp. 494-495; and Bas van Fraassen, Formal Semantics and

9. Kneale and Kneale also use this argument as a criticism of
Aristotle in W. Kneale and M. Kneale, The Development of Logic

10. See "The Semantics and Some of its Properties" in this chapter.

11. Van Fraassen's solution can be found in van Fraassen, Formal
Semantics and Logic, p. 166.

12. In classical two-valued semantics, the distinction between the
assumption that 'A' is true and the fact that A is unimportant.
For suppose Q follows from the fact that P. Now suppose one
assumed P when P is in fact false. If one then infers Q from
P, one is in no worse shape than one was when one assumed P,
for Q can only be true or false.

In our case, however, the difference is crucial. Suppose
again that Q follows from the fact that P. Now suppose one as-
sumes P when P is truth-valueless and then infers Q. One can
now be worse off than when one assumed P because Q may be false.
In the S-V object language, the distinction collapses again. There, if Q follows from the fact that P, then when P is truth-valueless, Q cannot be false. This has to do with the fact that the two notions of S-V validity come to the same thing with respect to the object language. Suppose Q follows from the fact that P, i.e., P |- Q. Then P |- * Q. Now suppose P is truth-valueless. Then some way of precisifying the predicates in P and Q makes P classically true. But then some way of precisifying those predicates makes Q classically true. So Q cannot be false.


23. Ibid.


25. Ibid.
The Theory and its Incompatibility With S-V

In their 1975 paper, "A Prosentential Theory of Truth," Grover, Camp, and Belnap showed that the grammatical role of constructions such as 'That is true' is easily viewed as that of an anaphoric prosentence. Anaphoric prosentences are similar to anaphoric pronouns. Consider (1), (2), and (3) below.

(1) If Sally is tired, she will fall asleep easily.
(2) Paul: The water is cold.
(3) Sue: That's true.

Whereas 'she' in (1) is a pronoun that inherits its content from the noun 'Sally,' according to the prosentential theory, 'That's true,' in (3), is a prosentence that inherits its content from the declarative sentence in (2). Anaphoric pronouns stand in for and inherit their content from nouns; anaphoric prosentences stand in for and inherit their content from declarative sentences.
The theory can be generalized, as shown by Grover, Camp, and Belnap, to cover what appears to be almost every English use of 'true.' In particular, according to the theory,

(4) 'Snow is white' is true,

is a prosentence which has the propositional content of 'Snow is white.' We can now see why any theory which does not license the T-sentences in their traditional form will be incompatible with the prosentential theory. According to that theory, 'P' is true' has the same propositional content as 'P,' so the T-sentences in their traditional form will be trivially true. I showed in the last chapter that S-V does not license the T-sentences in their traditional form. Hence, the prosentential theory and S-V are incompatible.

Much more needs to be said about all this. First of all, there are other ways in which the prosentential theory and the supertruth theory come apart. According to the prosentential theory, the law of excluded middle and the principle of bivalence cannot be separated. The theory has as a consequence that (5) and (6) below have the same propositional content.

(5) For all p, p or not p.

(6) For all p, p is true or not-p is true.

But (5) expresses the law of the excluded middle, whereas (6) expresses the principle of bivalence. On the other hand, using S-V makes (5) and (6) false. So the theories diverge on this point.
What about the ontological status that the theories accord to truth? The prosentential theory, strictly speaking, is simply a theory about the grammatical role which 'true' plays in our ordinary discourse. Yet the theory does have important implications concerning the ontological status of truth. Grover, Camp, and Belnap point out that the traditional way of analyzing the grammatical role of 'true' has led us to believe that truth is a property. Traditionally, the construction 'Snow is white' is true' has been accorded subject-predicate form. 'True,' then, was seen as a predicate which characterized a subject. This made it very natural to think that truth was a property.

But the grammatical analysis given by the prosentential theory erases any reasons for thinking of truth in that way. As we have just seen, according to that analysis, 'true' is not a predicate but part of a complex prosentence. As Grover, Camp, and Belnap put it,

if the prosentence theory is right, semantical reflection on truth talk should not cause us to think that there are sentences or statements which exemplify a property of 'truth.' Perhaps there are language-world relations of various kinds; perhaps 'Snow is white' does somehow picture the fact of snow being white, but on our account it is just a confusion to suppose that this has anything to do with some truth property.

...we precisely do not know that there is a property of truth that some sentences have. We may believe that the subject-predicate form of 'That is true' and its kin speaks property ascription rather than (as the prosentence account would have it) just grammatical convenience.
So the prosentential theory gives us reason to eschew the traditional view on which truth is thought to be a property. But if we view truth as it is necessary to do in order to utilize S-V, then the grammatical analysis given by the prosentential theory cannot be accepted. That would, most naturally, return us to the old subject-predicate analysis. But then we are back to viewing truth as a property. So, whereas the use of supervaluation semantics does not directly require any particular stance towards truth's ontological status, it would be very easy for a supervaluationist to think that truth is a property.

This difference between the two theories of truth is important. If it could be shown, independently of considerations raised by the prosentential theory itself, that truth is not a property, then the supervaluationist would be forced either to admit defeat or to develop a new grammatical analysis of 'is true' which was both compatible with supervaluation semantics and made it natural to withhold property status from truth.

Dorothy Grover argues that truth is not a property on grounds "that properties should 'partition' domains that are theory recognized categories or sorts." She then tries to show that truth does not do this. She writes:

for a given predicate P, let the set of objects satisfying 'x is P' be the extension of P, and the set of objects satisfying 'x is not P' be the anti-extension of P. P will be said to partition a domain just in case the set containing the extension and the anti-extension (completely) partitions the domain. Similarly, if a
predicate is said to be property ascribing, the candidate property $P$ partitions a domain just in case its extension and anti-extension partition the domain.\(^3\)

Grover attempts to justify the requirement that property ascribing predicates must partition domains in two ways. First, she claims that if a primary reason for talking about properties within a theory is to individuate and make comparisons between objects, then the question whether or not an object has a certain property $P$ should always be significant. If it were not, Grover claims, then in some cases we would be unable to make the wanted comparison between objects.

Suppose there is an object which neither has nor fails to have some property. In fact, consider the case which Grover ultimately has in mind, the case of the liar sentence: 'This is false.' That sentence is neither true nor false. It can't be true, for if it were, it would also be false; likewise, it cannot be false. According to Grover, since the liar sentence cannot be said to have either truth or falsity, we cannot compare it with other sentences in that respect. But, she argues, if a primary reason for talking about properties within a theory is in order to be able to make such comparisons, then, since truth will not allow us to do this in every case, its property status is dubious.

The argument just cited fails to justify the requirement Grover wants to impose. If a property does not partition a domain, it does not necessarily become useless when it comes to comparing and
individuating objects in that domain. Take the case of the liar sentence. It is neither true nor false. But 'Snow is white,' for example, if true, and no contradiction arises in supposing it is true. So, the obvious follows: 'This is false' is not the same sentence as 'Snow is white.' Of course, we knew that all along. The example does show, however, that truth can be used to individuate objects even when some of the objects in question are neither true nor false.

Grover also attempts to justify the requirement that properties partition domains by calling our attention to the fact "that theory recognized categories of sorts are—in the case of fairly well developed theories—sets of objects that are identified by a theory for the articulation of lawlike generalizations." She claims that properties should partition appropriate categories because each instance of a lawlike generalization should be determinately true or false.

It is not clear exactly what instances of lawlike generalizations Grover is worried about. One would assume that she is concerned with generalizations of the form 'If x then y,' where 'x' is neither true nor false. Yet her example of a sentence which is neither true nor false, the example upon which she bases her argument, is the liar sentence. But if that sentence is substituted for 'x' in the schema above, then the antecedent of the resulting sentence no longer retains the propositional content of the original liar. Due to the indexical use of 'this,' the liar sentence will now assert
that the entire conditional—'if x then y'—is false. So the liar cannot function as the antecedent in a conditional and retain its identity. Of course, the resulting sentence—'If this is false, then y'—is such that the truth of its antecedent will be dependent upon the truth of the entire sentence, yet the truth of the whole sentence will depend upon the truth of its antecedent. It is not clear what to say about the truth-value sentence as a whole. Perhaps it will be indeterminate in truth-value or else be truth-valueless. This may be what Grover is worried about, although the troublesome truth-status of the sentence in question is not a direct result of the lack of truth-value accorded to the liar sentence.

The example does indicate, however, that because 'true' fails to partition the domain of sentences, some lawlike generalizations will not be determinately true or false. Also, there are examples of sentences other than the liar which are neither true nor false, e.g., 'X is red,' when ''X' refers to a borderline case of red.' These sentences might be enough to make Grover's argument go.

Yet, Grover gives no reason for thinking that it should be determinate, for every instance of a lawlike generalization, whether it is true or false. Certainly, it would be nice if things worked out that way, but I am not sure that the fact that they do not, given some conception of what counts as a property, is enough to make us reject that conception. In other words, the fact that counting truth as a property gives one some instances of lawlike generalizations
which are not determinately true or false, should not be enough, in and of itself, to make us take away truth's property status.

A stronger argument needs to be made here. If it can be shown that we have no other reason for counting truth as a property, then perhaps Grover's worry would move us to take away the property status usually accorded to truth. But the real strength of such an argument would rest upon the claim that we don't need the truth property for any interesting reason. Grover goes on to argue that a prosentential 'true' will do any job that we think we need a truth property to do. This is where the real interest of Grover's paper lies.

Grover gives one more argument for the claim that 'true' is not property-ascribing which needs to be considered before moving on to her claims regarding the utility of a truth property. The argument assumes that if 'true' and 'false' ascribe properties, they do so in every case. But, Grover claims, paradox arises if we allow this property ascription to be so unrestricted. In particular, the argument once again concerns the use of 'false' in the liar sentence. Suppose 'false' is property ascribing in this case. Then, as we know, problems arise, for if the liar is false, then it must be true, and if true, it must be false.

Grover contends that the problems vanish when 'true' is seen as part of a prosentence rather than as ascribing a property. Prosentences are expressions which gain their content from some declarative sentence which already has content. ''Snow is white' is true' gets its content from 'Snow is white.' But the prosentence 'This is
false' is constructed so as to receive its content from itself. This shows that the prosentence is not well-formed. Since it has no independent content, it cannot possibly gain any such content from itself! In order to be successful, prosentences must be constructed to receive content from sentences which have some content independently.

The prosentential theory does provide an illuminating account of the paradoxical liar sentence. But a similar account can be given if one accepts the subject-predicate analysis of sentences containing 'true' and 'false.' Suppose, for example, that one is a correspondence theorist. Then 'This is false' says that the state of affairs which corresponds with a certain sentence does not obtain. In the case of 'Snow is white' is false,' this type of analysis presents no problems because the sentence 'Snow is white' already represents a state of affairs independently of the ascription of the property of being false to it. But in the case of the liar, the sentence which is having falsity attributed to it has no content independently of that attribution. In other words, one can only attribute truth or falsity to a sentence that corresponds to a state of affairs independent of that very attribution.

Do We Need Truth?

Grover's contention that truth is not a property needs to be backed up by her claim that we do not need a truth property for any interesting tasks. In this regard, she argues that the prosentential 'true' can be used by meaning theorists and logicians to do
anything we might think a truth property should do. She describes a theory of meaning as one that

must contribute to an account of the function of the various expressions of the language in linguistic activities; furthermore, insofar as a theory of meaning provides an account of what a speaker must know, it must capture something of the speaker's facility with language.6

She then argues that the prosentential theory illuminates the claim that a theory of meaning can be given in terms of truth conditions. Her contention is that the prosentential 'true' makes it plausible that the meaning of a sentence is captured (at least in part) by a statement of its truth conditions. Consider the following statement which might be included in a theory of meaning.

   (1) In conditions C1, 'The milk has curdled' is true.

According to the prosentential theory, (1) has the same propositional content as

   (2) In conditions C1, the milk has curdled.

'The milk has curdled' is true,' then, functions in two ways in (1). First, it is a prosentence standing for 'The milk has curdled.' In addition, it calls attention to that sentence. (1) enables us to say something non-linguistic—namely, what (2) says—while calling attention to a linguistic item at the same time. The utility of the prosentential 'true' for a theory of meaning is, according to Grover, that it lets one say what a speaker of the language must know in
order to use a sentence while simultaneously calling attention to the function of that sentence.

Dummett has argued that a theory of meaning in terms of truth conditions is incompatible with any redundancy-type theory of truth. His concern is that the compatibility of these two types of theories would entail that a Tarski-style truth definition could be used to give a theory of meaning while simultaneously providing an explication of the concept of truth. Dummett claims that truth must be explicated before it can be used to give a theory of meaning. Grover shows that the prosentential 'true' allows the Tarski-style truth definition to do both jobs at once. She claims that

for any language in which we are presenting a theory of meaning, we must assume that it comes equipped with connectives and other "logical" items like quantifiers and pronouns. A prosententialist will, in the same spirit, assume that prosentences are available. The use of 'true' and 'false' in prosentential constructions, can therefore be assumed understood when they are used in providing an account ... along the lines of Tarski of the meaning of sentences that belong to an object language. Now, once 'true' has been used ... to "give meaning," we can think of a sentence like 'Snow is white' as providing the condition on the application of 'true' to the sentence 'Snow is white....' The statement of truth conditions is thereby viewed in two ways— as a statement of truth conditions, and as a so-called truth definition.8

Grover also argues that logicians need only the prosentential 'true.' But I have already shown that any logician who uses super­valuation semantics needs more. Supervaluation semantics has been put to many uses; it has been offered as a semantics for non-denoting singular terms, as a way of dealing with semantic paradoxes and as a
semantics for vagueness. Still, it is worth asking whether even classical logicians require only the prosentential 'true.'

Grover points out that the prosentential 'true' allows the classical logician to provide a formal system of inference, while also making the appropriate connections with our linguistic practice. For example, she claims that

'true' and 'false' make it possible to have certain of the moves in the model parallel moves in inference, since assertion of membership or non-membership in the extensions of 'true' and 'false' carries out extra-model-theoretic (inheritor) reading.

Consider a model in which sentences get assigned 1 or 0. To assign a sentence 1 is, according to Grover, to use the prosentential 'true.' So, in assigning 1 to S, the logician both asserts S and gives to S a value which functions in the formal system. Thus, the formal system gets connected in the appropriate way with linguistic practice.

An antiredundancy theorist, however, would argue that there is something the prosententialist logician is missing. Logicians explain that what all valid arguments have in common is their ability to preserve truth. What would it be for a prosententialist to describe an inference as truth-preserving? A prosententialist such as Grover would have to say, of course, that there is no property of being truth-preserving. She could explain "truth-preservation talk" in much the same way that she explains the function of constructions such as

(3) Everything Tom says is true.
According to the prosentential theory, constructions like (3) provide us with a shorthand way of agreeing with everything Tom says. Without prosentences and certain other linguistic devices, the propositional content of (3) could only be conveyed by repeating everything Tom says. The prosentential 'true,' therefore, gives us a way of making certain general claims. In like manner, "truth-preservation talk" could be explained by the prosententialist as providing us with a shorthand way of saying something about many different arguments. On the prosentential theory, to say of a particular argument—if p then q, p, therefore q, for example—that it is truth-preserving would be to say that whenever if p then q, and p, then q. To say of all valid arguments that they are truth-preserving would just be to say that

- whenever if p then q, and p, then q;
- whenever p or q, and not q, then p;
- whenever p iff r, and p, then r;

and so on, for every valid argument. The prosententialist logician is forced to say that there is no property shared by all valid inferences which constitutes their validity. Of course, it is not at all clear that he should be bothered by this.

It is natural to think that some vague sentences are neither true nor false. For when X is a borderline case of red, we are uncomfortable with the supposition that 'X is red' is true and also
with the supposition that 'X is red' is false. But, according to the prosentential theory,

(4) 'Bertha is red' is neither true nor false,

has the same propositional content as

(5) Bertha is not red and Bertha is red.

But whereas we would like to say that (4) is true, (5) is a contradiction. Grover gives some attention to this general problem, although she does not specifically address lack of truth-value due to vagueness. She does discuss the use of 'neither true nor false' to characterize linguistic items that fail to be truth claims. She notes that we often define truth claims in the following way.

(6) Something is a truth claim iff it is either true or false.

This moves us to say, in addition, that

(7) Something fails to be a truth claim iff it is neither true nor false.

So, for example, if we maintain that

(8) The present King of France is wise,

is not a truth claim, then we will also say that it is neither true nor false. But then, according to the prosentential theory,

(9) The present King of France is wise and not wise,
is true. But this seems wrong.

Grover addresses this problem by arguing that the move from (6) to (7) is unjustified. Her contention is that

(10) 'Runs' is a truth claim iff 'Runs' is true or false,

is nonsense and that (6) is implicitly qualified as follows.

(6') Providing an expression is a truth claim: it is a truth claim if it is either true or false.

But one is not entitled to infer (7) from (6'). Grover concludes:

Thus, while (on present theories at least) we do seem to want the truth claim/nontruth claim distinction, and we do want to say of a sentence like 'The queen of England is wise' that it is either true or false, we do not need to say of a sentence like 'The King of France is wise' that it is neither true nor false.11

Hence, Grover claims that the problem is avoided because we do not need to say of the sentences in question that they are neither true nor false. I am not convinced that (10) is nonsense; but even if we do not need to say of nontruth claims that they are neither true nor false, we should not be convinced that the expression 'neither true nor false' is useless. We may want to maintain that certain sentences containing vague expressions lack truth-value. And notice that supervaluation semantics allows us to say both that

(11) 'S' is neither true nor false,
(12) 'S and not S' is false, because no matter what truth-status S has, every way of precisifying 'S and not S' will result in a classically false sentence.

The problem for the prosentential theory here is the fact that it does not allow for the expression of lack of truth and falsity to a sentence. As was noted above, this is because, according to the prosentential theory, an expression of that type is equivalent in propositional content to expressing the conjunction of a sentence and its negation. Grover tries to avoid the problem by claiming that the expressibility in question is not needed. But it remains to be determined whether that expressibility is needed in order to give an adequate account of vagueness. If it is needed, then besides being incompatible with supervaluation semantics, the prosentential theory will be inadequate for the task of giving an account of vagueness.

The claim that the prosentential theory is incompatible with supervaluation semantics needs to be qualified. The theory is only incompatible with the semantics when supertruth is accorded the status of ordinary truth. For a prosententialist might respond to the arguments in this chapter by claiming that supertruth is distinct from truth. Consider the explication of supervaluation semantics given above.

A vague sentence is true iff every way of precisifying it makes it classically true.
This, the prosententialist can claim, is not a condition for attributing ordinary truth to vague sentences. It is, in fact, a definition of another "type" of truth, that is, of supertruth. Actually, the prosententialist would probably want to claim that there is no property of supertruth either. But the theory that "explains away" supertruth would have to be developed. Of course, the 'true' in 'classically true' in the definition above will still be subject to the prosentential theory. So, for the prosentential theorist the definition above would have the same content as the following one: a vague sentence 'R' is true just in case given every admissible way of precisifying 'R,' R. At any rate, the point of the response would be that the prosentential theory is only supposed to be a theory of truth, which it can be without being a theory of supertruth as well.

If supertruth is distinguished from truth in this way, then any incompatibility of the prosentential theory with supervaluation semantics disappears. One can now claim that 'p' and ''p' is true' do have the same propositional content—just as the prosentential theory says; it is only 'p' and ''p' is supertrue' that differ in content. In other words, the prosententialist can claim that truth is bivalent. So, 'p' and ''p' is true' always have the same truth-value. Supertruth, on the other hand, is nonbivalent. When 'p' is truth-valueless, then ''p' is supertrue' is false.

The observation what whereas truth is bivalent, supertruth is not, also erases any apparent incompatibility due to the fact that supervaluation semantics must pull apart bivalence and excluded
middle, whereas the prosentential theory equates them. The prosen
tentialist can now claim that the two only come apart when they are
stated with respect to supertruth.

This response on behalf of the prosententialist is problematic.
There is no motivation for distinguishing truth from supertruth inde­
pendent of that which stems from wanting to save the prosentential
theory. When Fine applied supervaluation semantics to natural
language, he saw himself as giving the truth-conditions for vague
sentences. Is there any reason to think that this is not what he is
doing?

In their original paper, Grover, Camp, and Belnap take a stand
that is similar to the line of defense I am here proposing for them.
They state:

The preceding sections indicate some connections between
the prosentential theory and certain technical or
theoretical uses of 'true' by philosophers, but we wish
to emphasize that we have by no means tried to explain
all such uses. In particular, the prosentential theory
highlights the predicate's ordinary and hence nonmeta-
linguistic uses, whereas many technical philosophers
think of themselves as using the predicate metalinguis-
tically. Can the prosentential theory be extended to
cover these technical uses?

We don't know much about this. Sometimes theory pre­
cedes usage: those who have been brought up on a
particular theory, say Tarski 1936, or who have figured
one out for themselves, may actually decide to use the
truth predicate in accordance with that theory, no mat­
ter what happens in fluent English. (After all, it is
hardly surprising that one should, when provided with a
predicate, fine some characterizing use for it.) And if
a certain technical use originates from a theory, there
is no reason a priori why the prosentential theory
should cover it. But we leave open, or at least for
another occasion, the question of just what such a
theoretically based truth property could be, and how it
might be related to the ordinary -- i.e., prosentential -- uses of 'true.' We do, however, hope that those which are in the spirit of ordinary usage will be accommodated in the prosentential theory. 12

In his paper, Fine states that he is attempting to answer the question, "What are the correct truth-conditions for a vague language?" 13 I take it that, in answering this question, Fine has no special sense of 'true' in mind. He wants to know -- just as I might want to know whether it is really true that philosophers never make any money -- when certain sentences in vague natural language are true. I assume, then, that his use of 'true' is "in the spirit of ordinary usage" and that any time we seek the truth-conditions for sentences of natural language our use of 'true' is ordinary. The prosentential theory, if correct, should cover this use.

Proponents of the prosentential theory who point to various uses of 'true' which their theory does not accommodate and quickly brand them as "technical" (hence, not within the scope of their theory) are attempting to have their cake and eat it too. We should not allow them to do this. The prosententialist must find some means of delimiting the scope of his theory which is independent of whether or not the theory is adequate for the contexts in question. If he wants to claim that some use of the truth predicate is "technical" and, hence, differs from ordinary usage, then he must somehow account for this "technical" use. He must explain how it is different and take some stand on whether or not it is property ascribing.
At any rate, to claim that truth and supertruth are distinct would force one to say that whereas a sentence like 'Bertha is red' (when Bertha is a borderline case of red) is truth-valueless with respect to supertruth, it is also either true or false. Besides giving an account of what this could possibly mean, one who took this line would still be faced with accounting for our intuitions that 'Bertha' is red is neither determinately true nor determinately false.

The better line of defense for the prosententialist is to agree that classical semantics are not preferable to supervaluation semantics for vague language and then to argue that some other nonclassical logic which is compatible with the prosentential theory is preferable. As we shall see in the next chapter, a good candidate for such a nonclassical logic is the many-valued logic of Lakoff and Zadeh.
NOTES


2. Ibid., p. 119.


4. Ibid.

5. The argument I attribute to Grover here is actually derived from two different arguments she gives. The first occurs in Grover, "Truth: Do We Need It?," p. 88; the second, in D. Grover, "'This is False' on the Prosentential Theory," Analysis, 30 (1976), pp. 80-83.


7. Dummett's argument is cited by Grover in Grover, "Truth: Do We Need It?," pp. 83-84.

8. Grover, "Truth: Do We Need It?," p. 84.

9. I have already mentioned that van Fraassen applies S-V semantics to sentences with non-denoting singular terms in Bas van Fraassen, "Singular Terms, Truth-Value Gaps, and Free Logic," Journal of Philosophy, 63 (1966), pp. 481-495. For an example of the application of S-V to the semantic paradoxes, see Bas van Fraassen, "Truth and Paradoxical Consequences"; Brian Skyrms, "Notes on Quantification and Self-Reference"; and

10. Grover, "Truth: Do We Need It?," p. 87.

11. Ibid., p. 94.


CHAPTER V

S—V VS. MANY—VALUED SEMANTICS

The Justification of Semantics

It seems clear that the preferred semantics for natural language should do at least two things: (a) yield the appropriate logic, and (b) yield the appropriate truth-values for sentences. In order to evaluate S—V and the many—valued semantics, we first need to know what the criteria are for selecting the "appropriate" logic.

In Chapter Two, I said that logics should, in general, be justified by appeal to how well they reflect linguistic practice. Several questions about this claim were left unanswered. First, what is it exactly for a logic to reflect linguistic practice? If logic is to be a normative enterprise, then only certain parts of our practice should be mirrored. Those are the parts that the experts would take to be correct after considerable thought. Of course, the development and choice of a "correct" logic is itself a factor which affects our reasoning practice. We begin with some intuitions about
which patterns of inference are good ones. The logic we develop then gives us some principles of correct reasoning. These principles may extend to cases for which our prior intuitions were unclear or even mistaken. In this way, the logic not only reflects our practice, but guides it as well.¹

Next, how large a role is the requirement that a logic reflect linguistic practice supposed to play? The following quote from Rescher may be helpful here.

In giving a central place to the idea of logic as the systematization of a presystematic practice of reasoning, normatively regarded, we must note the role of various regulative principles in the construction of such a "systematization." I have in mind here such conceptions as those of precision and exactness, of economy and simplicity, and of coherence and consistency. Above all, one must here stress the regulative ideal of by-and-large conformity to the key features of the presystematic practice, of "saving the phenomena" that are involved in the presystematic practice.... Early in the game, one abstracts certain of the key regulative features of the logical enterprise from an informal examination of the presystematic practice of reasoning. Thereupon one injects the requirement of conformity to the regulative features into the consideration of systematizations, as well as "pragmatic" considerations relating to the specific area of application at issue. In taking this stance, the logician ... veers away from primary emphasis upon the empirical features of the presystematic practice with which he deals: his insistence upon the regulative principles limits the extent to which he is satisfied with an empirical survey of inferential practice.²

Notice that, in addition to the regulative requirements such as precision, economy, and consistency, Rescher mentions "'pragmatic' considerations relating to the specific area of application at issue." These pragmatic considerations will vary greatly depending
upon the nature of the task at hand. For example, programming a
computer to do reasoning in natural language requires finding a logic
with special "programmable" features. In such a case, the require-
ment that a logic have those features may be primary, whereas the
requirement to reflect human practice may have little importance. The
choice of a "correct" logic is purpose relative. There is not just
one correct logic for a language prior to the imposition of some
pragmatic and purpose relative constraints. In our case, however, we
are not interested in which logics can be programmed into computers;
we are interested in which logic is justified for our ordinary every-
day use.

We can now restate the first requirement that the preferred
semantics for natural language must meet. Such a semantics should
yield the logic that reflects the parts of our practice the experts
would take to be correct upon reflection. If several logics reflect
our practice adequately, then constraints regarding simplicity and
elegance will come into play. The second requirement for the pre-
ferred semantics for natural language is that it should assign the
appropriate truth-values to sentences. It is fairly clear that we
should mean by 'correct truth-values' nothing other than 'those the
experts would take to be correct upon reflection.' Let us say that a
semantics that meets both these requirements reflects linguistic
practice.

There is another requirement we should impose upon our prefer-
red semantics. Just as that semantics should yield a logic that
respects considerations of simplicity and elegance, the semantics itself should respect these considerations. This requirement will not weigh as heavily as the two above, but it should play some role in the justificatory process.  

Competition for S-V

It is fairly clear that because of vagueness we require a semantics that does not yield bivalence. Such a semantics can yield some truth-vagueless sentences, as does S-V, or else can assign degrees of truth to some sentences. I have yet to compare S-V with any semantics that assigns degrees of truth. Indeed, S-V's major competitors in the field of vagueness semantics are the many-valued ones. In particular, the many-valued semantics of Lakoff and Zadeh (L-Z) holds much promise in this regard.

In L-Z, 0 = falsity, 1 = truth, and all other values fall between 0 and 1. A valuation v is thus a function that assigns to each atomic sentence a value in the internal [0,1]. The truth-value V(A) of A under v is defined as follows.

\[ V(A) = v(A), \text{ for } A \text{ atomic.} \]
\[ V(\neg A) = 1 - V(A). \]
\[ V(A \land B) = \min\{V(A),V(B)\}. \]
\[ V(A \lor B) = \max\{V(A),V(B)\}. \]
\[ V(A \rightarrow B) = 1 \text{ if } V(A) \leq V(B); \ 0 \text{ otherwise.} \]
\[ V(\forall x \ (A x/\psi)) = \text{GLB}\left\{ \frac{V_{d/\psi}(A)}{d \in D} \right\}. \]
\[ V(\exists x \ (A x/\psi)) = \text{LUB}\left\{ \frac{V_{d/\psi}(A)}{d \in D} \right\}. \]
Lakoff and Zadeh do not give a criterion for argument validity. Two suggestions for defining L-Z valid inferences follows.

(i) An inference is L-Z valid iff the value of the conclusion cannot be less than the GLB of the values of all the premisses.

(ii) An inference is L-Z valid iff the value of the conclusion cannot be less than the GLB of the values of all the premisses if that GLB is greater than .5.

An interesting property of L-Z for our purposes is that it would seem to be compatible with the prosentential theory of truth. Since every sentence receives some degree of truth under L-Z, we are not required to say of any sentence that it is neither true nor false. Recall from Chapter Four that the prosentential theory will not allow us to say that. In addition, whereas S-V is incompatible with the prosentential theory because on S-V the Tarski T-sentences are not true in their traditional form, L-Z seems to be compatible with the traditional version of those sentences. On L-Z, it is reasonable to think that when 'The chair is red' is .8 true, then 'The chair is red' is true' will be .8 true as well. So the two sides of a T-sentence will always take the same truth-value.

The prosentential theory will have to be developed a bit to handle the use of 'true' in L-Z, but there is no reason to think that this development will be problematic. The expanded theory will have to include something to the effect that to say that 'Bertha is green' is .8 true is just to say that Bertha is .8 green. The details would have to be worked out, but the idea seems reasonable enough.
The differences in truth-value assignments that sentences receive under S-V and L-Z are fairly obvious. While a sentence such as 'Hector is red' will be truth-valueless according to S-V when Hector is a clear borderline case of red, the sentence will receive a value around .5 according to L-Z. And 'Hector is red or Hector is not red' will be true according to S-V, while receiving a value around .5 according to L-Z.

With respect to the validity of arguments, the two semantics differ regardless of which way we define L-Z validity. Suppose we define L-Z validity as in (i). Then the two semantics yield wildly different results. Along with many other classically valid arguments, the classical disjunctive syllogism:

\[
\begin{align*}
(P_1) & \quad R \lor Q \\
(P_2) & \quad \neg R \\
(C) & \quad Q
\end{align*}
\]

fails to be L-Z valid. Let \(V(R) = .5\) and \(V(Q) = .1\). Then \(V(P_1) = .5\) and \(V(P_2) = .5\), yet \(V(C) = .1\).

Suppose we take (ii) to be the definition of L-Z validity. Then the classical disjunctive syllogism is L-Z valid. For suppose that \(\min(V(P_1), V(P_2)) > .5\). Suppose, for reductio, that \(V(C) < \min(V(P_1), V(P_2))\). Then \(V(C) < V(P_1)\). So, \(V(Q) < V(P_1)\) and hence \(V(Q) < V(R)\). But then \(V(P_1) = V(R)\), and so \(V(R) > .5\). But then \(V(P_2) = V(\neg R) < .5\). This contradicts our original assumption.
Not all classically valid inferences are L-Z valid when (ii) defines L-Z validity. The following classically valid argument, for example, is not L-Z valid according to (ii).

\[ \begin{align*}
(P_1) & \quad P \\
(C) & \quad Q \rightarrow P
\end{align*} \]

Let \( V(P) = .7 \) and \( V(Q) = .8 \). Then \( V(P_1) = .7 \) (which is greater than .5), but \( V(C) = 0 \) (which is less than the value of the only premiss). When (ii) defines L-Z validity, however, the disagreement between the two semantics with respect to valid arguments is much smaller.

Which semantics, L-Z or S-V, best reflects our linguistic practice? Recall that to reflect our practice correctly a semantics must yield the logic the experts would take to be correct upon reflection and assign truth-values the way the experts would. At this point in time, the experts simply do not agree about these matters. Some logicians favor classical logic, while others adopt the logic yielded by L-Z. Some think bivalence should be denied in virtue of degrees of truth, others in virtue of truth gaps. Given that the experts disagree about these matters, there is room for more discussion regarding which semantics is best. We go on to explore this question in the next two sections.

**Sorites, Compositional Vagueness, and Hedges**

The "correct" semantics for vagueness must provide some solution to the sorites paradox. I use 'solution' very loosely here. It is open for a theorist to count the paradox as genuine on the grounds
that vague predicates are "incoherent" as along as the purported incoherency is explained and argued for and some account is given of the broad range of the predicates’ successful use.

Both L-Z and S-V provide intuitively plausible solutions to the paradox. Consider this version:

\[
B_0 \\
(n)(B_n \rightarrow B_{n+1}) \\
B_1,000,000,000
\]

where 'Bn' is read 'A man with n hairs on his head is bald.' Because L-Z allows for infinitely many degrees of truth, the second premiss of the paradox is intuitively false. Along with infinitely many degrees of truth go infinitely many degrees of baldness. If a man with n hairs is bald to the kth degree, then it does not follow that a man with n+1 hairs will be bald to that same degree. A man with n+1 hairs will be bald to a degree slightly less than k. On L-Z, \(V((n)(B_n \rightarrow B_{n+1})) = 0\) because the value of each instance of the universally quantified statement is 0. A man with n+1 hairs is ever so slightly less bald than a man with n hairs.

On S-V, the second premiss of the paradox is false, also, Notice that every way of precisifying the negation of the premiss, \((\exists n)(B_n \& \neg B_{n+1})\), makes it true. The negation of the second premiss is true and so the second premiss itself is false. So both semantics solve the paradox by denying the truth of the inductive premiss.

Up to this point, I have only compared S-V and L-Z with respect to how well they handle predicates with degree vagueness. In Chapter
One, two types of vague predicates, those with degree vagueness and those with combinatorial vagueness, were distinguished roughly from one another. It remains to be seen how well each semantics handles combinatorial vagueness.

L-Z semantics treats combinatorial vague predicates adequately. For example, suppose an analysis of the term 'religion' gives us reason to weight in the following way the factors 1-4 which are relevant to the application of that term.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Belief in some Supreme Being</td>
<td>Rituals</td>
<td>Organized Meetings</td>
<td>Place of Worship</td>
</tr>
<tr>
<td>W1</td>
<td>40%</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Suppose one now wants to evaluate 'X is a religion.' Each of the factors is itself named by a vague expression. These expressions can be given functions which will assign a member in [0,1] to any practice depending upon the degree to which that practice involves the factor the expression names. Then, the value of 'X is a religion' will just be

\[(W_1 \times V_{x1}) + (W_2 \times V_{x2}) + (W_3 \times V_{x3}) + (W_4 \times V_{x4}),\]

where \(W_i\) is the weight assigned to factor \(i\) and \(V_{x_i}\) is the value assigned to \(x\) by the function for factor \(i\). According to this method, 'Catholicism is a religion' will have value 1 since it is reasonable
to think that the practice of Catholicism involves each factor to degree 1. On the other hand, 'Unitarianism is a religion' would receive .52 on the following assignment of values:

\[
V_u(1) = 0.5 \quad \text{(read 'the degree to which the practice of Unitarianism involves factor 1 is 0.5')},
\]

\[
V_u(2) = 0.3,
V_u(3) = 1,
V_u(4) = 0.3.
\]

S-V semantics can also function in an account of the truth conditions for combinatory vague predicates. In fact, the semantics will function in this case just as it does otherwise. A sentence S containing a combinatory vague predicate will be true just in case every admissible way of precisifying S results in a classically true sentence. So, 'Catholicism is a religion' will be true according to S-V semantics because it is reasonable to think that given every admissible way of precisifying 'Catholicism' and 'religion,' Catholicism will fall under the extension of 'religion.' On the other hand, 'Unitarianism is a religion' will be neither true nor false since under some precisifications Unitarianism will count as a religion and under some it will not.

The weights assigned to the various composing factors of a compositionally vague term can be utilized by S-V semantics as well as by L-Z. In this case, the weights serve as a guide to which precisifications of a term count as admissible. A factor assigned a heavy weight will probably be required in every admissible precisification of the term, whereas a lightly weighted factor will be required in
some admissible precisifications and not in others. For example, given the relatively large weight assigned to belief in a supreme being, one would expect every admissible precisification of 'religion' to require that a religion be a practice which involves this belief. On the other hand, only some admissible precisifications will involve the requirement that a religion have a specific place of worship.

More specifically, the admissible precisifications for 'religion' might conform, according to the weights assigned to the application factors for that term above, to the following list.

LIST 1
1 2
1 3
1 4
1 2 3
1 2 4
1 3 4
1 2 3 4

In other words, the first precisification on the list is one where X is a religion iff X involves belief in a supreme being and also involves rituals. According to the second precisification, X is a religion iff X involves belief in a supreme being and organized meetings, and so on for the other precisifications on the list. Then, in order to tell whether or not X is a religion according to the first admissible precisification, for example, one will have to appeal to all admissible precisifications of 'belief in a supreme being.' The treatment parallels that given by L-Z semantics in an intuitive way.
Lakoff has also applied L-Z semantics in a treatment of linguistic hedges in English. Hedges are words and expressions such as 'very,' 'sort of,' and 'loosely speaking' that allow us to avoid precision. Lakoff claims that as far as he knows these hedges cannot "be handled by any logic developed to date [1973]" other than the many-valued logic he proposes. DeWitt has subsequently shown, however, that S-V semantics is also adequate for a treatment of these hedges.

The connection between the hedges Lakoff treats and vagueness has not been explored. In fact, they are intimately connected with vagueness. Hedges operate on vague expressions to create new ones. For example, consider the hedge 'very' and the vague term 'tall.' Modifying 'tall' with 'very' creates a new vague expression: 'very tall.' Hedges such as 'technically' can also be thought of as creating new vague expressions, e.g., 'things that are technically birds.'

A rough intuitive distinction can be made between hedges like 'very' and those like 'technically.' Hedges like 'very' (call these 'degree hedges') operate on degree vague predicates such as 'tall,' 'cold,' and 'bald.' Hedges that are similar to 'technically' (compositional hedges) operate on compositionally vague predicates such as 'religion,' 'fish,' and 'bird.' Lakoff gives different technical treatments for the two types of hedges even though the distinction between them is itself a fuzzy one and the types are not mutually exclusive. For example, 'sort of' can modify both degree and compositionally vague predicates, as (a) and (b) show.
(a) Woody is a sort of bird.

(b) Woody is sort of red.

The informal treatment of compositional hedges given by Lakoff requires that compositionally vague terms like 'fish' have (at least) four sets of application criteria: definitional, primary, secondary, and characteristic though incidental. For example, the sets for 'fish' might be structured somewhat as follows.

Definitional: cold-blooded, totally aquatic.

Primary: gills, scales.

Secondary: fins.

Characteristic though incidental: likes water, good swimmer.

According to Lakoff, 'Technically, X is a fish' will be true just in case X satisfies all the definitional criteria and fails to satisfy at least one primary criterion. Other hedges are treated similarly.

'Strictly speaking, X is a Y' is true iff X satisfies all definitional and primary criteria for Y.

'Loosely speaking, X is a Y' is true iff X satisfies all secondary criteria for Y, but fails to satisfy some definitional criterion and some primary criterion for Y.

'X is a regular Y' is true iff X satisfies only all characteristic though incidental criteria for Y.

If John is a person who loves the water and swims well, then 'John is a regular fish' will be true, whereas 'John is a fish' preceded by
either 'technically,' 'strictly speaking,' or 'loosely speaking' will be false.

Lakoff's formal treatment is a bit more complicated, since it must accommodate degrees of truth. First, as above, the membership function for a combinatory vague predicate can be thought of as a function of the membership functions of the criteria for that predicate's application. So, the value of 'X is a fish' will be a function of the degree to which X is cold-blooded, is aquatic, has gills, scales, etc. Now Lakoff defines four more functions: def, prim, sec, and char. Def(X_{fish}), i.e., the function def applied to X with respect to 'fish,' = the average of the value of 'X is cold-blooded' and the value of 'X is totally aquatic.' Prim(X_{fish}) = the average of the value of 'X has gills' and the value of 'X has scales,' and so on for sec and char.

Now the truth conditions for hedged sentences can be given.

V(technically, X is a Y) = AV{def(X_y), Neg(prim(X_y)),
where NEG(F(X_y)) = 1 - f(X_y).

V(strictly speaking, X is a Y) = AV{def(X_y), prim(X_y)}.

V(loosely speaking, X is a Y) = AV{sec(X_y), Neg(def(X_y)),
prim(X_y)}.

V(X is a regular Y) = Av{char(X_y), Neg(def(X_y)),
prim(X_y), sec(X_y)}.

A similar treatment of compositional hedges can be given in terms of S-V semantics. First, the value of 'X is cold-blooded,' 'X is aquatic,' 'X has gills,' etc., will be true, false, or neither,
depending upon the values each sentence gets with respect to its admissible precisifications. Then, where 'd' ranges over definitional criteria for Y, 'p' over primary criteria for Y, 's' over secondary criteria for Y, and 'c' over characteristic though incidental criteria for Y,

'Technically, X is a Y' is

true if for each d, 'X is a d' is true and for some p, 'X is a p' is false,
false if for some d, 'X is a d' is false or for each p, 'X is a p' is true,
undefined otherwise.

'Strictly speaking, X is a Y' is

true if for each d, 'X is a d' is true and for each p, 'X is a p' is true,
false if for some d, 'X is a d' is false or for each p, 'X is a p' is false,
undefined otherwise.

'Loosely speaking, X is a Y' is

true if for each s, 'X is a s' is true and for some d, 'X is a d' is false and for each p, 'X is a p' is true,
false if for some s, 'X is a s' is false or for all d, 'X is a d' is true or for some p, 'X is a p' is false,
undefined otherwise.

'X is a regular Y' is

true if for each c, 'X is a c' is true and for some d, 'X is a d' is false and for each p, 'X is a p' is true and for each s, 'X is a s' is true,
false if for some c, 'X is a c' is false or for each d, 'X is a d' is true or for some p, 'X is a p' is false or for some s, 'X is a s' is false,

undefined otherwise.

Actually, the treatment will be quite a bit more complicated when we take into consideration that the definitional criteria will themselves be weighted, as will the primary, secondary, and characteristic though incidental criteria. We will ignore these complications, however, as they do not affect the present discussion.

Lakoff's treatment of degree hedges can also be mimicked by S-V semantics. Basically, on Lakoff's account, the value of 'X is a d-Y,' where d is a degree hedge modifying Y will be a function of the value of 'X is a Y.' For example, \( V(X \text{ is very } Y) = V(X \text{ is } Y)^2 \). On the S-V account, whether or not a precisification of d-Y is admissible will be a function of what precisifications of Y are admissible. For example, if the admissible precisifications for 'tall' fall along the continuum of heights as follows,

\begin{center}
\begin{tabular}{c|c}
\text{NON-TALL} & \text{TALL} \\
\hline
\text{admissible precisifications} & \\
\end{tabular}
\end{center}

then the admissible precisifications for 'very tall' will be shifted over some distance on the continuum to the right.

S-V semantics handles combinatory vague predicates and hedges just as well as L-Z does.
Truth-values and Inferences

Since L-Z and S-V differ with respect to the truth-values they assign sentences, perhaps we can find grounds for preferring one semantics over the other on this basis. First, L-Z has a certain intuitive appeal here. After all, there is something seemingly right about assigning a truth-value of .5 to 'X is red' when 'X' refers to a borderline case of red. We may want to say of such a borderline case that it is "sort of red and sort of not red." Hence, the assignment of .5 seems appropriate. Also, it is obvious that baldness comes in degrees. But then it seems that the value of 'Fred is bald' should not always be either 1 or 0. When Fred is bald to some degree, then 'Fred is bald' should be true to some degree as well. It may seem mysterious to talk about degrees of truth, but this should be no more mysterious than talk of degrees of baldness.

An S-V semanticist would say that a semantics need not assign degrees of truth to sentences like 'Fred is bald' even though baldness comes in degrees. While Fred may come closer to a paradigm case of a bald man than Joe does, Fred and Joe may both be determinately bald, so a correct semantics will assign 'Fred is bald' and 'Joe is bald' 'true' according to this view. The supervaluationist will give a similar argument regarding borderline cases and negative cases of baldness.

L-Z and S-V also differ with respect to the values they assign some existence claims. For example, imagine a series of 10,000 cards, the first bunch being determinately non-red, the last bunch
determinately red, and each one is a little more red than the one before it. Now consider the following claim: there exists a first red card in the series. Represent that claim as

\[(\exists x)(Rx \land \forall y (y < x \rightarrow \neg Ry))\]

and call it ‘\((\exists x)A x/\circ\).’ Notice that \((\exists x)A x/\circ\) will be true according to S-V because given any acceptable precisification of ‘red’ there will exist a first red card in the series. L-Z, on the other hand, yields a different result. On that semantics, \(V(\exists x)A x/\circ\) = \(\text{LUB}\{V d/\circ(A)\}\) now, since \(y < u\) is precise and will thus get only 1 or 0, for any \(d \in D\), \(V d/\circ(\forall y (y < u \rightarrow \neg Ry))\) will be 1 if for every card \(y\) before \(d\), \(V(\neg Ry) = 1\) and will be 0 otherwise. Now suppose that for some card \(y\) before \(d\), \(V(\neg Ry) < 1\). Then, since the value of a conjunction is the minimum of the value of its conjuncts, \(V d/\circ(\neg Ry) = V d/\circ(\forall y (y < u \rightarrow \neg Ry)) = 0\). On the other hand, suppose that for every card \(y\) before \(d\), \(V(\neg Ry) = 1\). Then \(V d/\circ(A) = V d/\circ(\neg Ry)\), and either \(V d/\circ(\neg Ry) = 0\) or \(V d/\circ(\neg Ry) = \text{some very small nonzero value}\). So, \(V d/\circ(A) = 0\) or \(V d/\circ(A) = \text{some very small nonzero value}\). So \(\text{LUB}\{V d/\circ(A)\} = V((\exists x)A x/\circ) = \text{some very small nonzero value}\).

Intuition might lead one to think that \((\exists x)A x/\circ\) should be false, or at least mostly false. After all, the vagueness of ‘red’ seems to ensure that there will be no first red card. If this is how one’s intuitions do, then L-Z will seem to have the upper hand here.
The theory behind S-V, however, may cause us to reconsider our intuitions. The defender of S-V will point out that even though there is no card in the series that will count as the first red card in every situation, in any situation in which we precisify 'red' there will be some first red card. Vague terms are set up to be flexible. Sometimes our purposes require that one card be the first red card, sometimes another. But the meaning of 'red' constrains us to precisify so that some card is the first red card. That card will vary across contexts, however.

What about tautologies? A few classical tautologies which are not L-Z tautologies are listed below.

\[
\begin{align*}
\text{LIST 2} \\
P \lor \neg P \\
\neg (P \land \neg P) \\
P \rightarrow (Q \rightarrow P) \\
\neg P \rightarrow (P \rightarrow Q) \\
((P \land Q) \rightarrow R) \iff (P \rightarrow (Q \rightarrow R)) \\
(P \rightarrow (Q \land \neg Q)) \rightarrow \neg P \\
(P \land \neg P) \rightarrow Q \\
Q \rightarrow (P \lor \neg P)
\end{align*}
\]

Lakoff justifies L-Z's failure to validate some of these classical logical truths. He writes:

Incidentally, I consider it a virtue of this system that \('\neg P \lor P' is not a tautology. Suppose 'P' is 'This wall is red.' Suppose the wall is pretty red, say to degree 0.6, according to the given semantics. This seems to me within the range of plausibility. Certainly one would not want to say that the sentence was true in such a situation. Similarly, 'P \& \neg P' is not a contradiction in the above system. And similarly, the sentence 'This wall is red and this wall is not red' in the situation given where the wall is red to some extent seems to me not to be false, but rather to have a degree of truth.
The status accorded 'P v ~P' and 'P & ~P' by L-Z is somewhat intuitive. Lakoff makes no attempt to justify the failure of L-Z to validate the other classical tautologies in List 2. It may be that L-Z's treatment of these classical tautologies is not so plausible. Consider 'P → (Q → P).' Suppose we have color patch X which is .3 blue and Y which is .5 red. Intuitively, 'If X is blue, then if Y is red, X is blue' should be determinately true regardless of the fact that the component sentences are not determinately true. But the value assigned by L-Z to the sentence in this situation is 0. My intuitions are that the sentence is determinately true, but at any rate, it is certainly counterintuitive to suppose that it is determinately false.

We arrive now at the task of comparing the theories of deduction yielded by the two semantics. Recall that in Part One of this chapter I made two suggestions regarding how to define L-Z validity. Notice that, when (ii) is taken as the criterion for L-Z validity, it is not true that an inference is L-Z valid just in case its corresponding conditional is L-Z tautologous. (Let an inference's corresponding conditional be that conditional having the conjunction of all the argument's premises as antecedent and the argument's consequent as conclusion.) \( \forall((((R v Q) & ~R) \rightarrow Q), \) for example, is 0 when \( V(R) = .5 \) and \( V(Q) = .1. \) Yet, as I showed in Part One, the corresponding inference is L-Z valid according to (ii). A semantics that tells us it is always permissible to infer Q from \( (R v Q) & ~R, \) but that also tells us that \( (((R v Q) & ~R) \rightarrow Q \) is sometimes
determinately false, is counterintuitive. Such a semantics would be particularly offensive to a prosententialist, since on that theory to say that \((R \lor Q) \land \neg R, \text{ therefore } Q\)' is a valid argument is just to say that whenever \((R \lor Q) \land \neg R\) then \(Q\).

If we are looking for a semantics which is compatible with the prosentential theory, then L-Z as supplemented with (ii) as the definition for validity will not do. For this reason, I shall consider only the deductive theory yielded by L-Z when (i) is taken as the definition of an L-Z valid argument.

Recall from Part Two that, according to (i), classical disjunctive syllogism is not a valid argument form. Since most of us reason successfully by means of such arguments all the time, one wonders why L-Z would count it invalid. Consider the argument again.

\[
\begin{align*}
(P_1) & \quad R \lor Q \\
(P_2) & \quad \neg R \\
(C) & \quad Q
\end{align*}
\]

When \(V(R) = .5\) and \(V(Q) = .1\), the argument is L-Z invalid, for \(V(P_1) = .5\) and \(V(P_2) = .5\), but \(V(C) = .1\). (Remember that, according to (i), an argument is L-Z valid iff the value of the conclusion cannot be greater than the GLB of the values of all the premises.)

The defender of L-Z will say at this point that disjunctive syllogism is not valid because, according to L-Z, there are ways of assigning truth-values to the premises and conclusion of the argument so that the conclusion has a value less than the minimum of the premises' values. So, disjunctive syllogism, according to L-Z, does
not preserve truth, i.e., it allows the conclusion to be less true than the premises are.

Of course, in a situation such as this—when we have an argument that ordinarily seems intuitively valid, yet the semantics we are considering counts the argument as invalid—we have several options available to us. We can: (a) accept the result of the semantics and the fact that our intuitions are just wrong; (b) reject the semantics in order to preserve our intuitions; or, in this case, (c) accept another definition of validity which counts the argument as valid.

Were we not in search of an alternative to S-V that is compatible with the presentential theory of truth, I would recommend picking option (c) in this case. Opting for (c) would allow us to retain all of L-Z semantics except for the definition of validity. When we have an argument that is intuitively valid, and one definition of validity counts it as valid, while the other does not, then the former definition would seem to have some evidence in its favor. Also, in this case, the definition that makes disjunctive syllogism valid is intuitively plausible in its own right. An argument should be counted valid if, whenever its premises are true (or, in this case, whenever its premises have values on the "true side of the scale"), then its conclusion is at least as true as the minimum of the values of its premises. In classical reasoning, normally we are not concerned with what happens when an argument has at least one false premise, and so it seems appropriate that in this case we should not be concerned
with what happens when an argument has at least one "falseish" pre-
miss.

Of course, since we are in search of an alternative to S-V that
is compatible with the prosentential theory, and since the definition
of L-Z validity that makes disjunctive syllogism valid also makes L-Z
incompatible with that theory, option (c) is not available to us.
Concerning option (a), I am loath to reject the validity of disjunctive
syllogism because a semantics that may very well have a faulty
notion of validity deems the argument invalid. Of course, a prosen-
tentialist would probably be more willing to bite the bullet and ac-
cept (a). I shall discuss this issue further shortly.

Another classically valid inference which fails to be L-Z valid
under (i) is the following one.

\[ \begin{align*}
  (P_1) \ P \land \lnot Q \\
  (C) \ lnot (P \rightarrow Q)
\end{align*} \]

Let \( V(P) = .5 \) and \( V(Q) = .6 \). Then \( V(P_1) = .4 \), while \( V(C) = .0 \). I
think our intuitions support the classical treatment in this case,
so, it is up to the defender of L-Z to motivate the failure of the
above inference. I do not see what this motivation would be.

Notice that adopting L-Z in lieu of S-V will require that we
make changes in our classical practice. Accepting L-Z would mean, at
the very least, that we cannot condone disjunctive syllogism or the
other inference given above. Perhaps not everyone engages primarily
in classical reasoning now, but classical logic is the most
prevalent. After all, it is the logic that we teach beginning logic students.

Altering our classical practice might be worth the trouble if we had serious theoretical reasons for doing so. Were we to discover that no nonbivalent semantics yields classical logic, and that our reasoning is often faulty because of this, then we might want to go to the trouble of altering our current practice. But we are not in that desperate situation. We have a semantics that yields classical logic in spite of the failure of bivalence. In the absence of any compelling defense of L-Z, then, we are justified in leaving our classical practice alone.

On the other hand, a prosententialist who has good theoretical reasons for holding his theory of truth may find that those reasons alone are enough to make him advocate the adoption of L-Z, which is compatible with his theory, in lieu of S-V, which is not. He must, however, accept the fact that unless he comes up with a semantics for classical logic that is compatible with his theory of truth, his views about truth will not come cheaply; he must advocate that we give up parts of our current reasoning practice.

More Many-valued Semantics

A prosententialist who is happy with our classical practice would be pleased to find a semantics that does justice to the non-bivalence of natural language, while yielding classical logic, and is compatible with his theory of truth. While L-Z does not yield
classical logic, perhaps some other many-valued logic does. A good candidate for a semantics that does what the classical prosentential-ist wants is that which I shall call 'DSV' (for 'degree supervaluation semantics'). DSV combines features of both L-Z and S-V. While yielding classical logic, DSV assigns degrees of truth to sentences which S-V made truth-valueless. Under DSV,

\[ V(S) = 1 \text{ iff every admissible precisification of 'S' is classically true.} \]

\[ V(S) = 0.5 \text{ iff approximately half of the admissible precisifications of 'S' are classically true and half are classically false.} \]

\[ V(S) = 0 \text{ iff every admissible precisification of 'S' is classically false.} \]

The dots represent similar clauses for values between 1 and 0.5 and between 0.5 and 0. One can think of sentences receiving values in the interval \([1, 0.5]\) as being true to some degree and those receiving values in the interval \([0.5, 0]\) as being false to some degree.\(^\text{12}\)

Admittedly, DSV is not a precise semantics. First, the statement of truth-conditions contains words like 'approximately.' Second, one obviously cannot count admissible precisifications and then calculate what percentage of them makes 'S' true and what percentage makes 'S' false. In fact, since there are an infinite number of
admissible precisifications, talk about 1/2, 3/4, etc., of them makes no sense. But, in any given context, we will only need a finite number of admissible precisifications, so that problem can be avoided. Also, in most cases, one can get a reasonably good idea of what value a sentence takes just by eyeballing the situation. For example, consider the following diagram.

Notice that almost every admissible way of precisifying 'red' makes 'A is red' true and makes 'B is red' false. Approximately half of the admissible precisifications make 'C is red' true and half make it false. Apparently, the values given to these sentences by DSV are just the values that L-Z would assign to them.

DSV turns out to be a very interesting semantics. As I mentioned above, it yields classical logic and yet makes use of the notion of degrees of truth. In addition, DSV appears at first glance to be compatible with the prosentential theory of truth.

DSV appears to be compatible with the prosentential theory because of its similarity in certain respects to L-Z, which is compatible with that theory. We no longer make use of the notion of a sentence which is neither true nor false. According to DSV, whereas some sentences are neither determinately true nor determinately false, all sentences are true or false to some degree. Recall that,
on the prosentential theory, to say that 'S' is neither true nor false is just to say that S and not S. But S-V allowed us to say that 'S' can be neither true nor false, even though 'S and not S' is false. The prosentential theory would not allow us to say what we needed to say. The use of DSV, however, only requires that we say that 'S' can be neither determinately true nor determinately false, while 'S and not S' is false. But this is coherent on the prosentential theory, since 'S is neither determinately true nor determinately false' is not equivalent in content to 'S and not S,' according to the theory.

Another way in which the prosentential theory is incompatible with S-V is that S-V requires that 'P' and 'P is true' differ in truth-status when 'P' has no truth-value, while on the prosentential theory, 'P' and 'P is true' have the same propositional content and so can never differ in truth-value. The prosentential theory is compatible with DSV on this accord provided that we give a semantics for the metalanguage such that 'P' and 'P is true' always take the same value. It is reasonable to think that just as when Fred is .8 bald, 'Fred is bald' is .8 true, when 'S' is .8 true, ''S is true' will be .8 true. So when DSV is supplemented with the appropriate semantics for the metalanguage, the Tarski T-sentences will be true in their traditional form.

Now we run into problems. Because the Tarski T-sentences are true in their traditional form on this account, Putnam's inference of Chapter Three, Part Two, goes through. From 'P ∨ ¬P' and the
T-schema, which are not both determinately true, one can derive in the metalanguage for each P, 'P' is true or '¬P' is true.' But this is not what we want to say in the metalanguage. For according to DSV, for each P, 'P' is true to some degree or '¬P' is true to some degree.

In addition, a closer look at DSV reveals that it is actually incompatible with the prosentential theory. Notice that the way the semantics is now set up, a sentence S can have some degree of truth yet be not at all false. On the prosentential theory, this amounts to saying "sort of S but not at all non-S." For example, according to the prosentential theory, to say that 'X is red' is .7 true but not at all false is to say that X is sort of red but not at all non-red. But this is not coherent because 'X is sort of Y' implies 'X is at least a bit non-Y.'

DSV can be changed so that a sentence assigned value n is considered n true and 1-n false. This is in line with the prosentential theory. Suppose 'X is red' is .7 true and so .3 false. This amounts to saying, on the theory, that X is non-red exactly to the extent that x fails to be red.

But now a different problem arises. Notice that since DSV validates all classical tautologies,

(1) (P) ¬(P & ¬P),

will be determinately true. And, as we saw above, the Tarski T-sentences must also be determinately true. But then it will be
determinately true that

(2) (P)¬ (‘P’ is true & ‘¬P’ is true).

But, on this way of construing DSV, when ‘S’ is n true, ‘¬S’ will be 1−n true. So any sentence which is not determinately true will be such that it and its negation are both true to some degree. But then we should want (2) to have some degree of truth, rather than being determinately true. Call this problem the 'Putnam problem-two.'

In fact, a more general argument can be given to the effect that no workable semantics which both validates all classical tautologies and assigns degrees of truth is compatible with the prosentential theory. For consider any semantics R which has both these properties. A sentence S which is assigned a degree of truth n by R must either be construed as having, in addition, (i) no degree of falsity, or, (ii) some degree of falsity. If (i), then R is not compatible with the prosentential theory. If (ii), then either R yields the traditional version of the T-sentences or not. If it does yield them, then R is subject to the Putnam problem-two. If it does not, then it is incompatible with the prosentential theory. So, R cannot both be workable and compatible with the prosentential theory.

A semantics that never assigned any classical tautology a value less than .5 would not necessarily be subject to the above criticism. Such a semantics might yield the traditional version of the T-sentences, yet escape the Putnam problem-two because it need not
assign \(-(P \& \neg P)\) the value 1. And depending upon the theory of deduction for such a semantics, the logic yielded in this case might be classical enough for our purposes. The many-valued semantics which is just like L-Z except that the truth conditions for conditional statements are:

\[ V(A \rightarrow B) = \max\{1 - V(A), V(B)\} \]

(call this semantics 'N') will never assign any classical tautology a value less than .5. This claim follows easily from the theorem below.

**THEOREM**: For any N-valuation V, let P[v] be the classical valuation such that for atomic A,

\[ P[v](A) = \begin{cases} 
1 & \text{if } V(A) \geq .5, \\
0 & \text{if } V(A) < .5.
\end{cases} \]

Then, for any N-valuation V and any formula A, if V(A) > .5, P[v](A) = 1, and if V(A) < .5, P[v](A) = 0.

**PROOF**: We proceed by induction on the complexity of A.

**Basic Step** - Trivial.

**Inductive Hypothesis** -

For any N-valuation V and any formula B less complex than A, if V(B) > .5, then P[v](B) = 1, and if V(B) < .5, then P[v](B) = 0.

**Inductive Step** -

**Case 1.** A is \(\neg B\). Suppose \(V(A) > .5\). Then \(V(B) < .5\). By I.H., \(P[v](B) = 0\). Then \(P[v](A) = 1\). Suppose \(V(A) < .5\). Then \(V(B) > .5\). By I.H., \(P[v](B) = 1\). Then \(P[v](A) = 0\).

**Case 2.** A is \(B \lor C\). Suppose \(V(A) > .5\). Then \(V(B) > .5\) or \(V(C) > .5\). By I.H., \(P[v](B) = 1\) or \(P[v](C) = 1\). Then \(P[v](A) = 1\), and similarly for when \(V(A) < .5\).
Case 3. A is B & C. Suppose \( V(A) > 0.5 \). Then \( V(B) > 0.5 \) and \( V(C) > 0.5 \). By I.H., \( P[v](B) = 1 \) and \( P[v](C) = 1 \). But then \( P[v](A) = 1 \), and similarly for when \( V(A) < 0.5 \).

Case 4. A is \( B \rightarrow C \). Suppose \( V(A) > 0.5 \). Then either \( V(B) < 0.5 \) or \( V(C) > 0.5 \). By I.H., \( P[v](B) = 0 \) or \( P[v](C) = 1 \). But then \( P[v](A) = 0 \), and similarly for when \( V(A) < 0.5 \).

Case 5. A is \( (\forall x)B x^D \). Suppose \( V(A) > 0.5 \). Then \( V d^D(B) > 0.5 \) for all \( d \in D \). By I.H., \( P[v]d^D(B) = 1 \) for all \( d \in D \). But then \( P[v](A) = 1 \), and similarly for when \( V(A) < 0.5 \).

Case 6. A is \( (\exists x)B x^D \). Suppose \( V(A) > 0.5 \). Then \( V d^D(B) > 0.5 \) for some \( d \in D \). By I.H., \( P[v]d^D(B) = 1 \) for some \( d \in D \). But then \( P[v](A) = 1 \) and similarly for when \( V(A) < 0.5 \).

\( N \) is not subject to either Putnam problem. Recall that the original Putnam problem arises when a semantics yields both that 'P v ~P' is tautologous and that the T-sentences are determinately true. Putnam problem-two arises when a semantics yields both that '~(P & ~P)' is tautologous and that the T-sentences are determinately true. But, according to \( N \), neither 'P v ~P' nor '~(P & ~P)' is a tautology. So both problems are avoided.

From the standpoint of the prosentential theory, however, \( N \) is not without problems. Notice that, under \( N \), conditional sentences will sometimes be assigned an intermediate degree of truth. (Under L-Z, conditionals were only assigned 1 or 0.) But what is it, on the prosentential theory, to say that a conditional such as 'If X is red, then X is not blue' is true to some degree? When I put on my prosententialist hat, I understand what it is to say that 'X is red' is 0.8
true. That is just to say that X is .8 red or that X is very red. But I do not know what it could possibly be to say that a conditional statement is .8 true.

According to the prosentential theory, when we appear to be attributing the property of truth to a sentence, we are actually saying something about the non-linguistic world. But if 'If X is red, then X is not blue' is .8 true' is a prosentence, then it is unclear from what sentence about the world it receives its content. Because of this problem, N will not be acceptable to the prosententialist.

The semantics (call it L-Z*) that is just like L-Z except that the truth-conditions for its conditional are

\[ V(A \rightarrow B) = \begin{cases} 1 \text{ if } V(A) \leq 0.5 \text{ or } V(B) \geq 0.5, \\ 0, \text{ otherwise,} \end{cases} \]

assigns only 1 or 0 to conditionals, so it is compatible with the prosentential theory on that accord. L-Z* also escapes both Putnam problems because neither 'P v ~P' nor '(P & ~P)' is a tautology under this semantics. In addition, L-Z* is fairly classical. First, no L-Z* valuation will assign any classical tautology a value less than .5. (The proof of this claim is identical to the proof of the similar claim about the logic N, except for Inductive Step Case 4, which is fairly trivial.)

Also, if we define argument validity as follows,

An argument is valid iff it is impossible for the value of each premiss to be greater than .5 while the value of the conclusion is less than .5,
then an argument will be L-Z* valid just in case its corresponding conditional is an L-Z* tautology. But since arguments are classically valid just in case their corresponding conditionals receive a value greater than or equal to .5 under every L-Z* valuation, and L-Z* assigns only 1 or 0 to conditional statements, it follows that arguments are classically valid just in case their corresponding conditionals are L-Z* tautologies. But then arguments are L-Z* valid just in case they are classically valid. So far, L-Z* seems to be a good semantics for the classical prosententialist.

L-Z*, however, fails to provide a solution to some sorites arguments. For example, consider a series of 10,000 cards. Let the first card be determinately red, the last card be determinately nonred, and some other card n be such that L-Z* would assign the sentence 'N is red' the value .5. Also, for any n, let card n+1 be barely less red than card n. Now consider the following sorites argument.

\[
\begin{align*}
R_1 \\
(n)(R_n &\rightarrow R_{n+1}) \\
R_{10,000}
\end{align*}
\]

where 'Rn' is read 'Card n is red.' Since L-Z*'s theory of deduction is classical, the argument is L-Z* valid. The only way the semantics can solve this paradox is to deny the truth of one of the premisses. The truth of the first premiss, however, is noncontroversial. And, as it turns out, the second premiss is determinately true under L-Z* also.
For any card \( n \), \( L-Z^* \) must assign \( R_n \) a value either less than or equal to .5 or greater than .5. In either case, the value of \( R_n \rightarrow R_{n+1} \) will be 1. (We know this because we stipulated above that for some card \( n \), the value of \( R_n \) will be .5. But that card will be card \( m+1 \) for some \( m \). Hence, there are no cards \( n \) and \( n+1 \) such that the value of \( R_n \) is greater than .5 while the value of \( R_{n+1} \) is less than .5.) Since every instance of the universally quantified second premise receives the value 1, so does the premise itself. Thus, \( L-Z^* \) provides no solution to this sorites. We have yet to find a logic that will please the classical prosententialist.

**Conclusion**

S-V emerges from this investigation as my choice for a semantics for natural language. S-V takes vagueness seriously. Unlike classical semantics, S-V respects the failure of bivalence due to the vagueness of natural language. In addition, S-V yields classical logic and the classical theory of deducibility. I take this to be a desirable feature, since had we not found a plausible semantics with this feature, we would have had good cause to worry about our classical practice. Of course, I also found some many-valued semantics that preserve almost all of classical logic. \( N \), for example, never assigns a classical tautology a value less than .5.

I showed, however, that neither S-V nor any acceptable "near-classical" many-valued semantics is compatible with the prosentential theory of truth. In fact, the only viable semantics I examined that
is compatible with that theory is the many-valued semantics L-Z. L-Z does not yield classical logic. Thus, were we to adopt L-Z in lieu of S-V, we would need to make changes in our classical practice. I showed that these changes have little to recommend them other than the fact that they are required by a semantics that is compatible with that theory.

A prosententialist faced with adopting a semantics for natural language has several choices. He can give up his theory of truth and adopt S-V. It is unlikely that many prosentential theorists will be willing to do that. On the other hand, he can continue to defend the prosentential theory and adopt L-Z, reasoning that the semantics' compatibility with his theory of truth is enough to justify his choice. Or he may be able to find an acceptable semantics that is compatible with his theory while yielding classical logic. But these are matters on which the prosententialist must take a stand. He must realize that accepting the prosentential theory narrows one's options when it comes to giving a semantics for natural language.
NOTES


4. I have made no mention of the psychological reality of the semantics we choose. There is a debate in the literature as to whether semantics should be viewed as mathematics, psychology or a combination of both. I hold (as do Dowty, Thomason, Cresswell, Resnik, and others) that semantics is separate from psychology. The general idea is that semantics is normative while psychology is descriptive. For a sophisticated defense of this view, see David Dowty, *Word Meaning and Montague Grammar* (Amsterdam: D. Reidel, 1980), Chapter 8; Richmond Thomason, in the introduction to Richard Montague, *Formal Philosophy: Selected Papers of Richard Montague* (New Haven, CT: Yale University Press, 1974); Max Cresswell, "The Autonomy of Semantics," in S. Peters and E. Saarinen (eds.), *Processes,*
Beliefs and Questions (Amsterdam: D. Reidel, 1982), pp. 69-86; and Resnik, "Logic: Normative or Descriptive?," pp. 221-238.

For the opposing view, see Partee, Barbara Hall, "Belief-Sentences and the Limits of Semantics," in S. Peters and E. Saarinen, Processes, Beliefs and Questions, pp. 87-106.


8. Ibid., p. 478.

9. Richard DeWitt, Ph.D. Thesis (in progress), The Ohio State University, Columbus, Ohio.


11. In particular, Belnap, who in addition to being a founder of the prosentential theory is also a relevance logician, will have no trouble doing without disjunctive syllogism. The relevance logicians vehemently deny the validity of that argument form. See Anderson and Belnap, Entailment, pp. 165-167.
12. The reason I divide things up this way, rather than saying that a sentence which gets .5 is sort of true and sort of false, will become clear in the discussion that follows.

13. Actually, what I have just said is a bit too bold and requires additional support. While it is clear that we will only need only a finite number of admissible precisifications when we are dealing with a finite number of objects, it is not immediately clear that we only need finitely many precisifications in other instances. But this is, in fact, the case.

Consider the following example. Suppose we are concerned with a hypothetical case where there are uncountably many cards. Some cards are determinately red, some are determinately nonred, and some are inbetween, but each is a different shade of color. Suppose we want to know the truth-value of the following sentence according to DSV.

There is one red card such that every card less red than it is determinately nonred.

Now consider the following diagram of the color continuum.

Suppose we use only the admissible precisifications marked above. Now suppose we added 10 more. Suppose we added 10 more after that. The truth-value of the sentence above will be the
same in each case. As long as we have a certain number of admissible precisifications and as long as they are arranged correctly (we want them to be spaced evenly along the continuum), we should be able to make do with a finite number of precisifications. I think that, in most cases, one will find that using some finite number of admissible precisifications to evaluate a sentence will yield an intuitively appropriate value. Imagine what value would be assigned to other sentences stating claims about the cards in the situation described above.

14. The proof of this is just like the proof that S-V yields classical logic. See "The Semantics and Some of its Properties" in Chapter Three of this volume.


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