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Experimental investigation of stress transients in interstitial-free steel and 70/30 brass

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The Ohio State University, 1987
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Experimental Investigation of Stress Transients in Interstitial-Free Steel and 70/30 Brass

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Anne E. Browning, B.S., M.S.

The Ohio State University

1987

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R H Wagoner
Advisor
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To Glen

and

To My Family
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Studies in Solidification
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Studies in Electron Microscopy and Interface Structure
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CHAPTER I
INTRODUCTION

In the mathematical modeling of sheet forming operations, one critical piece of input data is the material response to the imposed forces or displacements. Various strain states are encountered within a sheet during a forming operation and much research has been conducted to characterize the material behavior in important and common strain states, such as plane strain\(^1-3\), balanced biaxial\(^4\), and uniaxial tension\(^1,5-7\). In cases when multi-stage forming operations are required to obtain the final part shape, abrupt changes in strain state can occur. The material response to such changes in strain state must also be input into the modeling programs for successful prediction of part geometry, possible failure, and die design.

In order to model the abrupt changes in strain state which can occur between successive forming operations, several kinds of two-stage strain path experiments have been devised. After a prestrain in one strain state, the
material is reloaded in another strain state and strained to failure. Experimental work has been conducted in the balanced biaxial/uniaxial tension mode, the tensile/tensile mode, and the plane strain/uniaxial tension mode. Unusual flow behavior has been observed during the second stage of the experiments and the results generally follow the same trend:

**POSITIVE TRANSIENT:** High subsequent yield stress followed by reduced work-hardening rate and premature load instability (ferritic type steels), or

**NEGATIVE TRANSIENT:** Lowered subsequent yield stress followed by increased work-hardening and enhanced or unaffected load instability strain (brass type). Aluminum alloys and a dual phase steel generally showed little transient behavior.

The micromechanistic origin of such stress transients has been suggested by several authors. The evolution of dislocation substructures during two-stage experiments has been studied using transmission electron microscopy by Rao et al. and Ronde-Oustau et al. The observed changes in substructure and the concepts of cell wall "polarization" and "memory" have been used to
rationalize the transient behavior of prestrained aluminum-killed (AK) steel\textsuperscript{17}. Hutchinson et al.\textsuperscript{8} claim that dislocation theory can only explain the small strain transient period, not the long term change in work hardening rate caused by the abrupt change in strain state which they observed in brass, steel, an aluminum alloy, and copper.

It has been suggested\textsuperscript{19} that this transient phenomenon is a manifestation of static strain aging or other interstitial effects rather than an artifact of changing dislocation configurations. Strain aging involves the thermally activated diffusion of interstitial solutes in the matrix\textsuperscript{20}. After prestraining and during aging, interstitial solutes (primarily carbon and nitrogen) migrate to dislocations and form solute atmospheres\textsuperscript{21-24} which pin the dislocations against glide. A typical strain aging experiment using a mild steel results in a sharp increase in yield stress on reloading, an increase in flow stress throughout the test, and a decrease in total elongation. This is similar in appearance to stress transients observed in AK steel, however this can not explain the negative transients seen in brass.
The first objective of this work was to determine whether strain aging is a contributing factor to the appearance of stress transients in steels. This was done by performing two-stage strain path experiments on a material which is not susceptible to strain aging. Interstitial-free steel (IF steel) was chosen as the test material because it is similar in structure to steels which exhibit positive transients yet it is widely known that IF steel does not strain age because of its low interstitial content.

The development of deformation textures during straining has been the subject of much research. A preferred crystallographic orientation (texture) different from the texture developed during sheet rolling can form during prestraining. This microscopic texture is manifested macroscopically as a change in $r$, the plastic anisotropy ratio. The parameter $r$ is a measure of the strain ratio anisotropy measured from tensile tests in various directions and is presumed to be related to the relative deformation resistances of in-plane directions to through-thickness directions. This ratio has been correlated $^{25,26}$ with a material's formability in deep drawing and other forming operations which require stretching of the sheet without thinning.
The change in or development of deformation texture by prestraining is believed to result in different work hardening rates in different strain states and therefore may explain the long term effects of the abrupt change in strain state, as seen by Hutchinson, et al.

The second objective of this work was to study, on a macroscopic scale, the role of dislocation rearrangements during the transient. The motion of dislocations during plastic flow is seen macroscopically as plastic strain. Measurement of local plastic strains in the axial and transverse directions during the second stage experiments was performed in order to obtain a better understanding of the transient phenomenon from the view of strain ratios rather than flow stress alone.

In order to model material behavior under complex load histories, yield theories are used for the prediction of yielding and deformation behavior. In general, a yield theory defines a surface in stress space (Fig. 1) inside which elastic behavior occurs. When the stress tensor intersects the yield surface (point A) and an outward stress increment is applied, yielding occurs and plastic flow begins. At this point, the yield surface expands or
Figure 1. Schematic two-dimensional yield surface.
moves so that the current stress always lies on the yield surface. In such a formulation, the outside of the yield surface has no meaning.

The yield theory must define the initial shape of the yield surface as well as the manner in which it moves or expands (hardens) as plastic deformation takes place. The initial yielding may be isotropic (yielding is independent of sheet direction) or anisotropic (material behavior is a function of direction).

Classical plasticity relies on three principal ideas of hardening. First, isotropic hardening, in which the yield surface shape and center are constant but expansion occurs equally in all directions; second, kinematic hardening, in which the yield surface does not expand but moves as a rigid body in either the direction of the stress vector or the normal to the surface at the loading point. The third category is a combination of the first two in which the ratio of isotropic/kinematic hardening can be determined from experimental results. In addition, other, more general, hardening models have been proposed.

Numerous yield theories have been proposed in the last thirty years to account for various experimental
observations such as the Bauschinger effect\textsuperscript{28,29} (early yield on reversal of load from tension to compression), creep\textsuperscript{30} (high temperature deformation), cyclic hardening and softening\textsuperscript{31}, and transient behavior during complex loading histories\textsuperscript{32}. In the present work, several theories designed to predict material behavior in complex loading histories are discussed and qualitatively evaluated in reference to the experimental results. A simple extension of a commonly used anisotropic yield theory with isotropic hardening is presented to provide a comparison to and modeling of the experimental results.

The work presented in this thesis can be divided into three sections. First, the monotonic behavior of the two materials studied, IF steel and 70/30 brass, is investigated in both plane strain and uniaxial tension. The best form of Hill's plasticity theory\textsuperscript{33,34} which adequately describes the flow behavior in these two strain states is determined for the two materials. Second, the transient behavior following an abrupt change in strain state from plane strain to uniaxial tension is investigated. Both the stress transients and transients in strain ratios are measured for the two materials. Lastly, a model is proposed which can describe the transient behavior observed after an abrupt change in strain state.
The model is based on Hill's plasticity theory and incorporates a rapidly changing $r$ value (plastic anisotropy ratio) during the transient.
CHAPTER II
LITERATURE SURVEY

1. EVIDENCE OF TRANSIENT BEHAVIOR

It has long been known that a change in strain path may be accompanied by inhomogeneous deformation and unusual stress-strain curves\(^{35-37}\). As early as 1952, Polakowski\(^{38}\) studied the changes in yield point and elongation in subsequent tensile and compressive testing following various predeformation modes. Such transient behavior is not limited to activation by changes in strain path; abrupt changes in strain rate\(^{39-44}\) as well as in strain amplitude\(^ {45}\) can also produce transient behavior.

Nieh and Nix\(^ {46}\) have observed a transient in some aluminum alloys simply by unloading and reloading in the same strain state. They term this the "unloading yield effect" and explain it in terms of strain softening by shearing of coherent precipitates and their subsequent healing by local diffusion. Transient behavior has also been observed on reversal of loading. In addition to the Bauschinger effect (decrease of yield stress on load
reversal). Christodoulou et al.\textsuperscript{47} have observed a transient in the work hardening behavior after the reversal of loading in polycrystalline copper. They correlate the transient with transmission electron microscopy (TEM) observations of the partial dissolution of cell walls and changes of dislocation density inside the cells.

Two-stage tests have been used to model abrupt changes in strain path, such as those that occur between successive forming operations. Experimental work has been conducted in the balanced biaxial/uniaxial tension mode by Ghosh and Backofen\textsuperscript{4} and the tensile/tensile mode by several authors\textsuperscript{8-12}. Wagoner\textsuperscript{1,13}, and Wagoner and Laukonis\textsuperscript{14,15} performed two-stage strain path experiments in the plane strain/uniaxial tension mode using an inplane plane-strain tension test\textsuperscript{2,48}. Generally the results follow the same trend:

**POSITIVE TRANSIENT:** High subsequent yield stress followed by reduced work-hardening rate and premature load instability (ferritic type-steels\textsuperscript{1,4,8,10,11,14}), or

**NEGATIVE TRANSIENT:** Lowered subsequent yield stress followed by increased work-hardening and enhanced or unaffected load instability strain (brass type\textsuperscript{8,13}).
Aluminum alloys\textsuperscript{8,9} and a dual phase steel\textsuperscript{15} generally showed little transient behavior.

Two-stage strain path tests have also been used to investigate the work-hardening characteristics of several materials in various proportional, non-tensile strain states\textsuperscript{4}. By prestraining in the strain state of interest and subsequent testing in uniaxial tension, the flow curve of the first strain state can be inferred. Ranta-Eskola\textsuperscript{16} has criticized this procedure, recognizing that two-stage tests do not necessarily reproduce proportional path work-hardening curves.

Ranta-Eskola\textsuperscript{16} claimed the different localization behaviors of steel and brass were determined by the sign of the stress change following the abrupt change in strain path. In contrast, Wagoner and Laukonis\textsuperscript{14} proposed that the change in work-hardening rate was the important feature in the transient. Reduced work-hardening rate (positive transient) can cause premature strain localization and lead to early failure, whereas increased work-hardening rate (negative transient) may reduce existing inhomogenieties. This latter proposal was verified by the use of finite element modeling in conjunction with transients of varying magnitude and sign\textsuperscript{49}. Positive stress transients (reduced
work-hardening) promote premature strain localization and failure relative to monotonic hardening. Negative stress transients (increased work-hardening) promote homogeneous, uniform deformation relative to monotonic hardening.

2. ORIGIN OF TRANSIENTS

2.1 DISLOCATION SUBSTRUCTURES

The micromechanistic origin of such stress transients has been suggested by several authors\textsuperscript{8,9,11,16-18} and investigated using transmission electron microscopy by Rao and Laukonis\textsuperscript{17}. They found that a plane strain tension prestrain creates an elongated, well-defined, "polarized" substructure. On reloading in a direction $90^\circ$ to the original tensile axis, the cell structure is no longer stable and dissolves, leading to early plastic instability. This cell dissolution is attributed to the strain reversal that occurs along the prestrain tensile axis.

Ronde-Oustau and Baudelet\textsuperscript{18} investigated the effects on formability of a change in strain path between biaxial and uniaxial tension by observing the differences in the cell structure generated by the individual strain paths. In equibiaxial stretching, the cells are equiaxed and thick-walled. Uniaxial tension produces rectangular dislocation cells with thin, straight long-side walls and
thick, curved short-side walls. These rectangular cells rotate during straining until they are oriented with the long-side walls parallel to the tensile axis at strains $\approx 30\%$. The tangled dislocation walls in the former cause the mean free path of the mobile dislocations to be decreased in subsequent tensile deformation, thus not allowing large tensile strains to be reached.

Schmitt and Baudelet\textsuperscript{50} studied the effect of dislocation substructures on the yielding of prestrained low carbon steel samples by considering the slip systems which become active upon reloading in a different strain state. During prestraining, dislocations move on particular slip systems dependent on the crystal structure. On changing the strain state, for example from plane strain to uniaxial tension, a number of new slip systems become active. The dislocations moving on these slip systems see obstacles formed during the prestrain, such as pileups or forest dislocations, and therefore fewer mobile dislocations are left to produce plastic flow\textsuperscript{51}. This hardening effect is commonly termed latent hardening. In latent hardening, the critical resolved shear stress (stress required for shear on a particular slip system) is increased and therefore an increase in yield stress on reloading is observed. Schmitt and Baudelet\textsuperscript{50} also state
that after reloading, a transient part of the plastic behavior exists which corresponds to a rapid evolution of the substructure. They also note that strain softening and early plastic instability may occur when the number of new active slip systems (ones not activated during prestrain) increases.

Basinski and Jackson\textsuperscript{52} studied the effect of extraneous deformation on second stage flow curves in copper single crystals. They noted that the extraneous deformation (twisting or tension) increased the density of forest dislocations. On reloading along another strain path, obstacles to dislocation motion created by the previously developed dislocation network caused an increase in flow stress. This network created in the first deformation mode was unstable to slip on the newly activated slip systems and resulted in a lowered work hardening rate.

In a companion paper, Basinski and Jackson\textsuperscript{53} extended these ideas to include crystals oriented for multiple glide and polycrystals. They consider a broad forest dislocation network produced on one slip plane and the reorientation of the crystal such that the activated slip plane is slightly inclined to the original glide plane. In this case, the
forest network will intersect the new glide plane along a thin line and will not act as a strong barrier to dislocation motion, thus not causing hardening.

Hutchinson, et al.⁸ claim that dislocation theory can only explain the small strain transient period but not the long term change in work hardening rate caused by the abrupt change in strain state which they observed in prestrained copper, brass, an aluminum alloy, and Aₙ steel.

2.2 STRAIN AGING

It has been suggested¹⁹ that the transient phenomenon is a manifestation of static strain aging or other interstitial effects rather than an artifact of dislocation configurations. When a mild steel sample is prestrained, aged at an elevated temperature (100-200°C) and reloaded, the following may be observed: a return of the sharp yield point, an increase in yield strength, and a decrease in ductility. This phenomenon is commonly termed static strain aging; and is generally accepted as being caused by the presence of interstitial solutes in the matrix²⁰. During aging, interstitial solutes (primarily carbon and nitrogen) migrate to dislocations and form solute atmospheres²¹-²⁴ which pin the dislocations against glide.
A schematic strain aging load-elongation curve is shown in Fig. 2. The increase in yield stress on reloading is similar in appearance to the positive stress transients found by Wagoner for AK steel (See Reference 14, Fig. 2f); however, in that work the second stage curves nearly rejoined the monotonic stress-strain curve so that $\Delta U$, as defined in Fig. 2, would be zero or perhaps slightly negative.

Gawne investigated the static strain aging characteristics of several AK steels with carbon (C) contents in the range 0.003-0.05%. Steels containing 0.003-0.004% C exhibited no strain aging at 50°C because of insufficient interstitial solutes to cause pinning of the dislocations. Steels containing 0.01% C exhibited aging at 50 and 160°C, those containing 0.035% C aged at 160°C but not at 50°C, and those with 0.05% C did not age at either temperature. Gawne explained these seemingly anomalous results by discussing the morphology of the carbides contained in the different steels. Only primary carbides were observed in the 0.05% C steel, whereas the 0.035% C steel contained primary and finer secondary carbides, and the 0.01% C steel contained coarse primary, finer secondary, and extremely fine tertiary carbides. Gawne contends that it is the dissolution of the very fine
Figure 2. Schematic load-elongation curve for a strain aging experiment.
carbide particles which causes strain aging. The size and spacing of the carbides is a function of several process variables, including composition, aging temperature, aging time, and cooling rate.

Very low carbon levels are required in order to reliably avoid strain aging, such as the levels found in an "interstitial-free" (IF) steel. IF steel is often used as a baseline reference material in strain aging work because the lack of interstitial solutes avoids aging effects after aging at relatively low temperatures (100-200 °C).

Static strain aging in IF steel, however, is possible by substitutional solute trapping and has been studied by several authors. Solute concentrations of 3% Ni, 3% Si, or 0.7% Ti were found to produce strain aging effects after aging at 300-450 °C. Purified IF steel (not containing significant substitutional solutes) was found not to strain age at these temperatures. Note that the aging temperature required for strain aging by substitutional solutes is much higher than that required by interstitial solutes.
2.3 PLASTIC ANISOTROPY

As a result of the fabrication processes used in producing a sheet material, large crystallographic reorientations of the grains into an overall preferred orientation, or texture, may occur. During annealing of a cold rolled sheet, recrystallization takes place by the migration of high-angle grain boundaries. Therefore recrystallization is always accompanied by large changes in orientation. During subsequent grain growth, a gradual evolution of the texture occurs. It is widely understood that the properties of grains are a function of orientation and therefore favorable properties in polycrystalline materials can be achieved by obtaining certain preferred orientations.

During straining of a sheet material, a similar change in the crystallographic orientations of the grains occurs, commonly termed "deformation texture". It has been shown, by Stout and Staudhammer, that the texture developed in the sheet depends on the strain or stress state. The flow curves of 70/30 brass subjected to several biaxial deformation modes exhibited widely different work hardening rates. They contend that this is a direct result of the development of deformation textures during the straining operation.
In 1984, Tome, et al.\textsuperscript{59} presented a microscopic hardening law to account for experimental results from copper tested in torsion, compression, and tension. The work hardening rates for the three strain states were widely different when compared on the basis of von Mises effective stress-strain values. Approximately half of the discrepancy between the stress-strain curves of the three strain states was eliminated with the use of the microscopic hardening law which takes into account grain reorientation during straining (deformation texture development). Although the inclusion of texture development did not account for all the differences in work hardening rates, a significant improvement was made over the von Mises definition and, like Stout and Staudhammer\textsuperscript{27}, they conclude that the deformation texture does contribute to the widely different work hardening rates observed in various strain states.

In 1958, Burns and Heyer\textsuperscript{60} established the relationship between the plastic anisotropy ratio, $r$, and texture. The ratio $r$ is defined in a uniaxial sheet tensile test as the ratio of the strain in the width direction to the strain in the thickness direction, as shown below:

$$r = \frac{\varepsilon_w}{\varepsilon_t} \tag{1}$$
where \( w = \text{width} \) and \( t = \text{thickness directions} \). It is difficult to measure \( \varepsilon_t \) in a sheet, therefore the constancy of volume criterion \( (\Sigma \varepsilon_i = 0) \) of the plastic regime is imposed to obtain a usable form of Eq. 1:

\[
r = \frac{1}{-\varepsilon_a / \varepsilon_w - 1}
\]

(2)

where \( a = \text{axial direction} \). As can be seen from Eq. 1, a high \( r \) value is a result of a small strain in the thickness direction compared to the width direction, thus \( r \) is a measure of the strength in the thickness direction (resistance to thinning) to the strength in the planar directions.

The plastic anisotropy ratio has been correlated with a material's formability in deep drawing and other forming operations which require stretching of the sheet without thinning. Whiteley\textsuperscript{25} presented the first conclusive evidence that the deep drawability of a sheet is strongly influenced by the plastic anisotropy ratio, \( r \). A material which has a high \( r \) can support a greater punch load in the wall of the cup and therefore, a greater diameter blank can be drawn without failure in the cup wall.

Several researchers have suggested a relationship between \( r \) and the stress transient which occurs after a
change in strain state. Laukonis and Ghosh found that after a prestrain in biaxial tension, the second stage uniaxial specimens showed a decrease in r value with increasing prestrain in AK steel. An aluminum alloy which showed a small negative transient exhibited no change in r with respect to prestrain.

Dabrowski et al. found that deep drawing and balanced biaxial stretching decreased the plastic anisotropy (decreased r) in sheet steels. They concluded that this decrease in r would be detrimental for subsequent deformations which require high plastic anisotropy, such as deep drawing. They also suggest subsequent deformation in uniaxial tension will be degraded by a prestrain which decreases the plastic anisotropy ratio.

Charpentier and Piehler performed two stage experiments using a high strength low alloy (HSLA) steel in which the first stage was an in-plane compression (used to simulate drawing) and the second stage was uniaxial tension or compression. A significant decrease in yield stress was observed in the second stage when the strain state was uniaxial tension in a direction parallel to the initial compression. The authors attributed this phenomenon to a Bauschinger-type effect as load reversal occurred from
compression to tension in the direction of straining. They calculated the effect of the change in deformation texture during the prestrain and found that it gave a small change in the yield stress in the opposite sense to the Bauschinger effect, and therefore was not a contributing factor to the observed decrease in yield stress.

In a series of papers, Parker et al. investigated the behavior of alpha brass subjected to various loading paths including two-stage strain paths. They derived a possible yield surface and experimentally measured the slope of the normal to the yield surface at various points along the loading path. The normal is inversely related to the plastic anisotropy ratio, \( r \). The second stage normals were not constant as would be expected for an isotropically hardening material but generally tended towards the value that would be predicted by the isotropic hardening theory.

3. MODELING OF COMPLEX LOADING HISTORIES

One of the most commonly used plasticity theories for simple predictive applications is the von Mises yield criterion proposed in 1913. As an isotropic theory with isotropic hardening, the theory is not accurate for the modeling of many engineering materials. Hill proposed a theory based on the same quadratic function used by von
Mises but including initial anisotropy in the sheet. If two of the three sheet symmetry axes (the two in the plane of the sheet) are equivalent, the theory exhibits normal anisotropy and the fitting of only one anisotropy parameter, $\bar{r}$, is required. A newer version of this theory incorporates another parameter, $M$, to adjust the shape of the yield surface.

The von Mises and Hill yield theories include isotropic hardening, i.e., the yield surface expands uniformly upon an increase in the plastic strain. Drucker recognized in 1949 that experimental observations of the Bauschinger effect (early yielding on reversal of loading from tension to compression) indicate that isotropic hardening does not provide a satisfactory model if stress reversal is to be encountered.

In order to model the Bauschinger effect, Prager introduced kinematic hardening; a theory in which the yield surface moves rigidly while staying in contact with the loading point. This theory allows for a different yielding point after a stress reversal than for the initial straining, however experimental results often show that expansion of the yield surface occurs along with translation. Therefore, theories which
combine both isotropic and kinematic hardening have been developed to account for such experimental observations. In general, yield theories have been developed in order to explain a particular observation or set of observations and only yield theories which address complex strain histories will be discussed in this survey.

In 1951, Edelman and Drucker\textsuperscript{73} presented a mathematical investigation of several yield criteria, from the simplest isotropic theory to a theory which incorporates both isotropic and kinematic hardening and material anisotropy. The theories are presented in order of complexity and the user is advised to choose the simplest theory which would capture the desired effects. This is the approach most often taken today. For a particular material and set of expected loading paths, a yield theory is chosen which will capture the desired properties without the added complexity of modeling effects which would not be encountered during the proposed loading histories.

In order to model behavior which varies with complex loading histories, the required yield theory will be inherently complicated, as the theory must be matched to the material behavior in several combinations of loading
paths. Many theories have been proposed in order to model particular experimental observations. For example, Nicholson\textsuperscript{74} proposed a theory in 1975 to account for several phenomena at once, including movement of the yield surface until it no longer includes the origin, contraction of the yield surface, and decrease of the tangent modulus during loading. This theory would serve to model materials which experience plastic strain on unloading, the Bauschinger effect, and a stress/strain curve with a decreasing slope.

As another example, Besseling\textsuperscript{30} presented the "mechanical sublayer" model in 1958 to model creep deformation. Stress-strain relations are presented for an initially isotropic material which is macroscopically homogeneous but microscopically inhomogeneous. This volume is composed of subelements which may have different properties but which all exhibit isotropic hardening and secondary creep. By enforcing a set of boundary conditions, the predicted stress-strain curve demonstrates anisotropic hardening, creep recovery, and primary and secondary creep. Analytical results are presented, however no comparison is made to experimental results.
Several experimental observations have prompted the development of multiple surface yield theories. In usual one-surface models, small strain behavior is considered the same as large strain behavior, however, it has been shown that the behavior in these two strain regimes can be quite different. For example, the hardening modulus immediately after yield is quite different than the asymptotic value reached at large strains. Also, loading, unloading, and re-loading along the same or different strain paths can result in yielding behavior much different than that observed for the initial loading path. Cyclic deformation often cannot be adequately described by one surface theories due to cyclic softening or hardening.

In order to model complex loading histories such as cyclic loading, Mroz presented a conceptual model in 1967 based on multi-loading surfaces and a "field of work-hardening moduli". The theory consists of an inner yield surface and an infinite set of loading surfaces, each with a different modulus. Yielding occurs when the loading point reaches the yield surface. With further plastic deformation, this surface moves to come into contact with the next surface, and so on. Because each surface has a different work hardening modulus, a stress-strain curve can be generated which has a high modulus immediately after
yielding and a continually decreasing modulus (slope) until an 'asymptotic' value is reached.

In 1971, Eisenberg and Phillips\textsuperscript{68} presented a theory involving non-coincident yield and loading surfaces. Yielding occurs when the loading point reaches the yield surface and the two surfaces can then move and/or expand with the loading surface always enclosing the yield surface. The loading point is always in contact with the loading surface. On unloading, the loading surface contracts isotropically until the loading point comes into contact with the yield surface. On reloading, yielding occurs when the loading point reaches the yield surface. This enables yielding at a lower stress than was attained before unloading in the initial path. The stress-strain curve would then asymptotically approach the monotonic curve. This theory was able to accurately predict small strain transient behavior observed on loading, unloading, and reloading in the same or different strain states\textsuperscript{68}. The formulation is presented in a general manner and detailed derivations and examples are included only for an isotropically hardening case. It is implied that more complicated hardening laws could be included, however no evidence of this is provided.
In order to model material behavior during cyclic loadings, Dafalias and Popov\textsuperscript{31} also proposed a model including two surfaces: loading and bounding surfaces. Unlike the previous two-surface models, the outer, bounding surface can expand and move before the loading point reaches that surface. This surface expands or translates at a slower rate than the loading surface so that eventually the two surfaces join and move together. Qualitatively the theory should be able to model cyclic hardening and softening, however it is not compared to experimental results and therefore no quantitative indication of the usefulness of the theory is presented.

Using the foundation laid by Mroz\textsuperscript{76}, and independently of Dafalias and Popov\textsuperscript{31}, Krieg\textsuperscript{29} presented a two surface theory in 1975 which is capable of predicting a gentle transition from elastic to fully plastic behavior and a realistic Bauschinger effect. The theory consists of a loading surface which defines yielding and can isotropically grow and move, and a limit surface which hardens independently of the loading surface. The plastic stiffness, or hardening modulus, is a function of the distance between the two surfaces. When the loading point (on the loading surface) contacts the limit surface, the asymptotic value of the modulus has been reached. This
theory was used qualitatively by Wagoner\textsuperscript{13} to model the transient behavior observed in 70/30 brass on an abrupt change in strain state from plane strain tension to uniaxial tension.

In the late 70's, Lee and Zaverl presented a series of papers\textsuperscript{32,69,70} developing a generalized strain rate dependent anisotropic theory, considering history dependence in Ref. 32. In concept, the theory is similar to the two-surface yield theories discussed above, however the outer, bounding surface is not described explicitly. A set of primary material parameters are used to describe the yield surface distortion, translation, and size. A set of secondary material parameters describe the behavior at large strains by specifying how the primary parameters change along the strain path until asymptotic values are reached. In this way the limit surface is defined implicitly. The model is able to simulate cyclic loading paths and the Bauschinger effect with reasonable accuracy. Because the hardening rate is defined by the distance between the two surfaces, the hardening rate is the same for tension and compression on load reversal, however an increased work hardening rate is normally associated with the early yield characteristic of the Bauschinger effect.
Therefore a more complicated hardening rule must be used to incorporate this effect.

In 1984, Krieg and Key\textsuperscript{71} presented a single-surface yield theory to model many of the same effects as Krieg's two-surface theory\textsuperscript{29}. Yielding is defined by the loading surface as before but the hardening modulus is defined by an exponential function decreasing to the asymptotic value at large strains instead of a function of the distance between the yield and limit surfaces. The theory can predict a gentle knee in the stress/strain curve and a realistic Bauschinger effect. However, as with the Lee and Zaverl model\textsuperscript{32} the hardening rate after load reversal is the same as in the first loading path, leading to an unrealistic model of the "transient" region.

In order to model the smooth elastic-plastic transition observed in elastoplastic materials such as granular media, Hashiguchi\textsuperscript{72} presented a theory introducing a subyield state. The materials in question can both harden and soften upon plastic flow and Hashiguchi noted that although the Mroz\textsuperscript{76,77}, Krieg\textsuperscript{29}, and Dafalias and Popov\textsuperscript{31} models could predict the smooth elastic-plastic transition (through changes in hardening modulus with
plastic strain), they could not predict the softening phenomenon. In the previous theories, on loading into the plastic regime, the yield surface contacts the outer limit surface. After partial unloading (not enough to cause translation of the yield surface away from the limit surface), and reloading, the two surface theories would predict an abrupt change from elastic to plastic states, an effect not observed in many materials.  

Hashiguchi considers a loading surface inside the yield surface and terms this the subyield surface. When the loading path reaches this subyield surface, plastic strains as well as elastic strains occur. The ratio of plastic/elastic strain increases until the fully plastic state is reached at the yield surface. This concept is extended into the framework of the two-surface yield theories so that three surfaces are considered. This allows some elastic behavior on reloading after a partial unloading as well as a smooth transition from the elastic to fully plastic regimes.
CHAPTER III

EXPERIMENTAL APPARATUS AND PROCEDURES

1. MATERIALS

Two materials which exhibit opposite responses to abrupt changes in strain state were chosen for this study: the first is IF steel, a body centered cubic (bcc) material, and the second is 70/30 brass, a face centered cubic (fcc) material. 70/30 brass sheet was shown to exhibit negative stress transients in earlier work\textsuperscript{13}, and plain-carbon steels show positive stress transients\textsuperscript{14}.

Interstitial-free steel (IF steel) was chosen from among many low-carbon steels because of its lack of susceptibility to the complicating effect of strain aging. IF steel, produced by Armco Corporation\textsuperscript{79}, is a nearly carbon-free steel to which small amounts of columbium and titanium are added to tie up residual carbon and nitrogen, producing a ferrite matrix virtually free of interstitial atoms. The chemical composition, as shown in Table 1, results in a non-strain aging material at room temperature.
<table>
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<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Al</th>
<th>N</th>
<th>Nb</th>
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2. SPECIMEN PREPARATION

2.1 PLANE STRAIN SHEET SAMPLES

In order to obtain plane strain conditions in a sheet material, the sample must be wide enough to prevent general contraction of the sheet in the transverse direction. The specimens used, as shown in Fig. 3, have been shown to provide a large region of plane strain in the center of the sample. The samples were machined from IF steel and brass sheets with the tensile axis along the rolling direction (RD) using numerically controlled equipment.

2.2 UNIAXIAL TENSION SPECIMENS

Standard sheet tensile specimens (ASTM E-8) were used in the monotonic experiments (Figs. 4 a,b). The specimens were machined using a Tensilkut machine to a nominal width of 12.7 mm and a nominal gauge length of 50.8 mm. Half-size tensile specimens were used in the second stage experiments and were machined in the same way to a nominal width of 6.4 mm and a gauge length of 50.8 mm. For second stage specimens which would later be used with strain gages, the nominal width was increased to 7.0 mm to allow better bonding of strain gages.
Figure 3. Plane strain sheet specimen.
Figure 4. Uniaxial tension sheet specimens (ASTM E-80 standard) a) full size, and b) half size.
The Tensilkut machine has the capability of machining up to 8 specimens at a time, however, at most 5 specimens were machined at once in order to increase heat transfer from the center specimens. The effort was made to minimize temperature rises in the specimens in order to eliminate the possibility of the occurrence of thermally activated processes or any other changes in the original material structure and properties. During machining, a 0.6% taper was automatically introduced along the specimen width. After machining, the width and thickness of the specimens were measured along the gauge length using a micrometer.

2.3 PHOTOGRIdding TECHNIQUES

In order to monitor strain distributions in subsequent tests, the plane strain samples were photogridded with a grid of 7.5 mm circles, and the second stage samples were gridded with 2.5 mm circles. Photogridding is a photofabrication method in which a grid is placed on the surface of the sample which will deform as the sample is strained.

Photogrids were made using Kodak\textsuperscript{82} photofabrication techniques. The specimen is first immersed into Kodak photosensitive negative resist solution in a Kepro Bench-Top Coater machine\textsuperscript{83} to apply a thin coating
to the surface. After drying, the specimen is covered with a negative gridded film and exposed to ultraviolet light in a Kepro UV Exposure Frame. The areas which are exposed become sensitized and are not washed away, however the unexposed areas are washed away in the next step. The exposed specimen is then immersed in Kodak Photo Resist Developer, dried, and immersed in Kodak Photo Resist Dye in order to make the grids clearly visible. The specimen is then rinsed in methanol to wash away the excess dye.

The two variables in the process are the thickness of the Photo Resist layer (controlled by the withdrawal rate from the Bench Top Coater) and the exposure time. Before each set of specimens were photogridded, a series of trials were performed for different combinations of the variables. The optimum set of variables changes with time because of the degradation of the chemicals and also with the size of grid used. Generally, a withdrawal rate of 3.5 (a unitless value specified by a dial on the machine) and an exposure time of 45 seconds was satisfactory.

3. PLANE STRAIN TESTING APPARATUS

Plane strain testing was performed using a hydraulic Instron testing machine Model #1332 with a 50,000 lb. load cell. Because the plane strain tension test used in
this work is not a standard or common test. special grips had to be machined. These are shown schematically in Fig. 5, and are designed similar to ones used in earlier research. The samples are wide and even gripping had been difficult to achieve in the earlier work. Therefore, special wedge shaped teeth were used to grip the metal better as shown in the inset in Fig. 5. The inner grip plates were made of oil-hardenable tool steel and were heat treated to Rockwell "C" hardness 58-60. The remaining parts of the assembly were made of 4130 steel and were heat treated to Rockwell "C" hardness 34-36. Care was taken in the heat treatment to avoid warping of the steel parts, however a small amount of warping did occur.

For testing, the sample is placed between the two halves of the upper grip and bolts are inserted to keep them together. This assembly is then placed in the testing machine and the bottom grip attached in a similar way. Careful alignment of the grips is critical if symmetrical loading is to be achieved. Equal distances along the grip width between top and bottom grips reproducibly provided symmetric strain patterns. After alignment, the bolts are tightened down to a torque of 90 foot pounds for steel and 75 foot pounds for brass. When the load is applied to the sample, the inner grip plates slide down and the grip teeth
Figure 5. Plane strain testing grips.
dig into the sample. Because of the small amount of warping of the grips which occurred during heat treating, the grips did not hold the sample exactly evenly. This effect was most severe in the steel as it is harder and resists the digging of the grip teeth into the surface. To offset this resistance, the samples were compressed to a load of 20,000 pounds between the grips before loading. This allowed the teeth to dig into the sample surface and also form the sample to accommodate the small amount of warping in the grips.

4. RESISTANCE STRAIN GAGES

4.1 STRAIN GAGE APPLICATION

Resistance strain gages (CEA-06-062WT-350) supplied by Micromeriteasurements Corp. were used to measure the axial and transverse strains of the second stage specimens. The stacked strain gage is shown schematically in Fig. 6. Strain gages are normally used in small strain applications, for example monitoring of elastic strains. For large strain applications, special care must be taken to insure good bond strength of the adhesive between the gage and specimen surface. Under ideal conditions, the gage/adhesive combination used would allow elongations up to 6-8%.
Figure 6. Stacked strain gage for strain measurement in two perpendicular directions.
The strain gage application procedure suggested by Micromeasurements was used and involved the following steps:

1. The sample surface is degreased using a chlorinated solvent, Chlorothene SM.
2. The sample is dry abraded using 220 grit silicon-carbide paper to remove surface scale.
3. The sample is abraded with 320 grit silicon-carbide paper while keeping the surface wet with M-Prep Conditioner A.
4. The sample is scrubbed with cotton-tipped applicators again while wet with the conditioner.
5. The surface is neutralized by scrubbing with a cotton-tipped applicator while applying M-Prep Neutralizer 5.
6. For high elongation studies the surface must be further altered. The sample is abraded in a direction 45° to the original axis, and then abraded in a direction 90° to that.
7. The sample is again degreased, scrubbed with conditioner and then scrubbed with neutralizer.
8. The gage and bonded terminals are arranged on a glass plate and covered with cellophane tape.
9. The gage assembly is positioned on the sample
and the tape pulled back from one end taking the gage with it.

11. The adhesive (M-Bond AE-10) is prepared by mixing the resin and curing agent.

12. The adhesive is then applied to the sample and gage surfaces and the gage put in place.

13. Pressure is applied to the sample for 24-48 hours at \( \approx 30^\circ \) C

14. Thin wires are soldered on the lead tabs and connected to the bonded terminals.

4.2 STRAIN MEASUREMENT

The strain gage was connected to a Signal Conditioner and Amplifier System (Micrometrieasurements Model #2100)\(^{85}\) through twisted multiconductor wires soldered to the lead tabs external to the gage. The quarter bridge arrangement with an internal dummy was used. The signal conditioner allows for balancing of the signals before the tests by zeroing the error, and during the test measures the change in resistance of the gage and the imbalance of the gage with the Wheatstone bridge circuit. The change in resistance is converted to a voltage and amplified. The output can be monitored through a voltmeter or, as in this work, recorded on an X-Y plotter. The sensitivity of the signal was limited by the resolution obtained from the plotter. The strains could be read to \(+/-\ 6 \times 10^{-7}\).
The output from the strain gages is voltage. The Wheatstone bridge analysis presented by Dally and Riley was used to convert the output voltage (E) into strain ($\epsilon_a$). There are four arms in the Wheatstone bridge: three have fixed resistance ($R_2 = R_3 = R_4 = 350 \, \Omega$) and the forth is the strain gage resistance. At the start of the test, the bridge is balanced so that the initial resistance of the gage ($R_o$) is also 350 $\Omega$. During the test, the resistance of the gage (R) changes and the imbalance is converted into a voltage (E). The following equations were used in the analysis:

$$E = \frac{RR_3 - R_2 R_4}{(R+R_2)(R_3+R_4)} V$$ \hspace{1cm} (1)

or, rearranging,

$$R = \frac{R_2(1+2E/V)}{(1-2E/V)}$$ \hspace{1cm} (2)

$$R = R_o + R_o S_g \epsilon_a$$ \hspace{1cm} (3)

where $V = 4 \, V$ (input excitation voltage), and $S_g = 2.08$ (property of strain gage used).

5. PLANE STRAIN TESTING

5.1 EXPERIMENTS

A series of plane strain specimens were strained at a crosshead speed of 0.0106 mm/s (initial uniform strain rate $\approx 1.8 \times 10^{-4} /s$). Photographs were taken at 60 s intervals with a Nikon 35 mm camera and the load recorded. A
A photograph of the experimental set up is presented in Fig. 7. Two external flashes were used and the camera was positioned as close to the sample as possible while still allowing the complete width of the sample to be viewed. Quarter power flashes were used with the camera set at 1/60 s and F8. Specimens to be used for comparison with monotonic uniaxial tension data were strained to failure, others were prestrained to a desired amount and later used for second stage experiments.

5.2 DATA COLLECTION

The diameters of the photogrid circles were measured from the photographs using a Nikon Measurescope\textsuperscript{87}. One row of circles was used for the transverse measurements (along the specimen width); the two center rows were used for the axial measurements (in the direction of straining) in order to get a more averaged value for strain. This provided less scatter in the axial strain measurements.

During measurement of the circles, the data were directly input into an IBM\textsuperscript{88} personal computer through an IEEE interface connecting the digital readout and the computer. The data were then edited to achieve the proper form for input into the data analysis program described in Chapter IV.
Figure 7. Experimental setup for plane strain tension testing.
6. UNIAXIAL TENSION TESTING

All uniaxial tension tests were performed in an electric Instron\textsuperscript{84} machine, Model #1362. This machine was equipped with a 20,000 lb. load cell.

6.1 MONOTONIC TENSILE TESTS

Virgin uniaxial tension samples were strained to failure to provide baseline data for comparison with plane strain and second stage data. The standard ASTM E-8\textsuperscript{80} uniaxial tensile samples were cut both with the tensile axis along the rolling direction (same as plane strain samples) and the transverse direction (same as second stage tensile samples). The samples were strained at an initial average strain rate of $1.8 \times 10^{-4}$/s, and a 0.5" extensometer was used to measure the engineering strain. The load and strain were recorded using an X-Y plotter. The load is converted into engineering stress in the following way:

$$\sigma_{\text{engr}} = \frac{\text{load}}{\text{initial area}}$$  \hfill (4)

True stress and strain can then be calculated:

$$\sigma = \sigma_{\text{engr}} \left( 1 + \varepsilon_{\text{engr}} \right)$$ \hfill (5)

$$\varepsilon = \ln \left( \varepsilon_{\text{engr}} + 1 \right)$$ \hfill (6)
6.2 STRAIN AGING EXPERIMENT

A simple strain aging experiment using IF steel was also conducted to verify that this material does not strain age. A tensile sample was prestrained 8%, followed by aging at 100°C for 2 hours, after which it was strained to failure.

6.3 MEASUREMENT OF STRESS TRANSIENTS

After the desired prestrain, the plane strain sample was unloaded and removed from the assembly. Four half-size tensile samples were then machined from the center area of the prestrained sheet as shown schematically in Fig. 8. The tensile axis of each of the specimens was parallel to the direction of zero initial extension (transverse sheet direction). Two specimens from each prestrain were then strained to failure at an initial average strain rate of $1.8 \times 10^{-4}$/s, and a 0.5" extensometer was used for strain measurement.

6.4 MEASUREMENT OF STRAIN RATIOS

The two remaining half-size second-stage tensile specimens of each prestrain were photogridded with 2.5 mm circles and resistance strain gages were applied to the other side. These were strained to failure at an initial average strain rate of $1.8 \times 10^{-4}$/s. Load vs. time
Figure 8. Schematic of location of second stage specimens in prestrained plane strain blank.
was recorded on a strip chart recorder, and load vs. axial strain and transverse strain vs axial strain were recorded on an X-Y-Y' recorder, where the strain values are output from the strain gage conditioner. Pictures of the deforming grids were taken at approximately 30 s intervals. A photograph of the experimental setup is presented in Fig. 9. The same camera settings were used as in the plane strain test procedure (Section 5.1).
Figure 9. Experimental setup for second stage uniaxial tension testing a) front view showing photogridded specimen, b) rear view showing specimen with strain gage.
1. ANALYTICAL PROCEDURES FOR PLANE STRAIN DATA ANALYSIS

1.1 USE OF HILL'S THEORY

The analysis of the plane strain experiment is not as straightforward as a simple uniaxial tension test. The condition for plane strain (one principal strain is zero) requires no contraction in the width direction of a plane strain sheet specimen. This is not possible in the complete width of the sample and the strain state ranges from uniaxial tension on the edge to plane strain in the center area. Therefore, a data analysis program must be used which subtracts the effect of the edge regions so that the flow curve of the plane strain region alone is calculated.

Wagoner\textsuperscript{48} developed a plane strain data analysis program which incorporated Hill's "old" theory\textsuperscript{33}. This theory is represented by the following equation (with $M = 2.0$) for normal anistropy in two-dimensions:

$$2(1+r)\sigma^M = (1+2r) |\sigma_1 - \sigma_2|^M + |\sigma_1 + \sigma_2|^M$$

\[(1)\]
where $M$ is Hill's parameter and $r$ is the plastic anisotropy ratio. The effect of these parameters on the shape of the yield surface will be discussed later.

A yield theory provides a set of equations which combine the experimentally observed principal stresses ($\sigma_1$) into one effective stress ($\bar{\sigma}$) for comparison with other strain states. The yield theory is also a compact means of storing data from different strain states for use in analytical modeling techniques. Therefore, it is of interest to determine the proper form of the yield theory which describes the material behavior in the various strain states. By definition, the uniaxial tension curve is invariant with respect to yield theory. The proper yield theory would define a surface which would pass through the plane strain and uniaxial tension points in stress space.

Although the original program employed Hill's "old" theory in the analysis, this theory does not satisfactorily describe the plastic behavior of many materials. Hill's "new" theory$^{34}$ (Eq. 1 with $M \neq 2.0$) incorporates other constant values of $M$ to provide a better description of material behavior.
The parameter \( M \) affects the shape of the yield surface by defining the relative values of the yield function at different stress states. For \( M < 2.0 \), the ellipse is elongated along the \( \sigma_1 = \sigma_2 \) line and for \( M > 2.0 \), it is shortened along this direction, while the slope at \( \sigma_2 = 0 \) remains the same (i.e., \( r \) is kept constant) as shown in Fig. 10. This parameter \( M \) does not, however, allow for different work-hardening rates in the various strain states. To allow for this added complication, \( M \) would have to vary with strain, as has been proposed for aluminum\(^{14}\) and brass sheet\(^{13}\). For materials in which the work-hardening rates in the strain states of interest are approximately the same, i.e., the effective stress-strain curves (based on \( M = 2.0 \)) are separated by a constant multiple, a constant value of \( M \) would transform the data from a non-uniaxial tension strain state into the tensile data.

The plastic anisotropy ratio, \( r \), fixes the slope of the yield surface at the intersection of the surface with the stress axes. By differentiating the yield function given by Eq. 1, the following relation can be found (for \( M = 2.0 \)):

\[
\frac{d\sigma_2}{d\sigma_1} = \frac{1 + r}{r} \quad \text{at} \quad \sigma_2 = 0.
\]  

Therefore, as \( r \) increases, the slope decreases, as shown in
Figure 10. Effect of parameter $M$ of yield surface shape.
Fig. 11. From the normality condition (the incremental strain vector is normal to the yield surface):

$$\frac{d\epsilon_2}{d\epsilon_1} = -\frac{r}{1 + r} \quad \text{at} \quad \sigma_2 = 0$$

(3)

where 1 = axial and 2 = transverse directions.

The plastic anisotropy ratio was originally defined by the width-to-thickness strain ratio for a sheet tensile test, as shown below:

$$r = \frac{\epsilon_x/\epsilon_z}{1/(-\epsilon_y/\epsilon_x - 1)}$$

(4)

or, incrementally:

$$r = \frac{d\epsilon_x/d\epsilon_z}{1/(-d\epsilon_y/d\epsilon_x - 1)}$$

(5)

where y = axial, x = transverse, and z = thickness directions for a sheet tensile test and the final relationship is obtained by considering constancy of volume in the plastic regime. For a proportional tensile path, Eq. 4 is identical to Eq. 3, illustrating the physical relation between the yield surface form and measured material properties.

For materials which exhibit planar anisotropy, r varies with the orientation of the tensile sample in the sheet. In this case, an average r,

$$\bar{r} = \frac{r_{90} + 2r_{45} + r_0}{4}$$

(6)

is used in Hill's theory. The subscripts refer to the angle between the rolling direction of the sheet and the
Figure 11. Effect of parameter $r$ on yield surface shape.
tensile axis of the sample. In the following discussions, the r value used corresponds to the particular direction of the test.

Wagoner\textsuperscript{13} has presented a discussion of Hill's "new" theory in terms of its use for the analysis of different strain states. If we consider only plane strain (in which \(\sigma_1 = 2\sigma_2\)) and working in two dimensions, Hill's theory can be reduced to equations relating the effective stress (or strain) to the axial stress (or strain). The set of coefficients in these equations are grouped together and termed \(f_{\sigma}\) (or \(f_{\varepsilon}\)) and are defined as:

\[
f_{\sigma}^{(M)} = \frac{\sigma^{(M)}}{\sigma_1} = \left[ \frac{2\kappa}{(1+\kappa)(1+r)^{1/(M-1)}} \right]^{(M-1)/M}
\]  \hspace{1cm} (7)

\[
f_{\sigma}^{(2)} = \frac{\sigma^{(2)}}{\sigma_1} = \frac{\sqrt{1+2r}}{1 + r}
\]  \hspace{1cm} (8)

\[
f_{\varepsilon}^{(M)} = \frac{\varepsilon^{(M)}}{\varepsilon_1} = \left[ \frac{2\kappa}{(1+\kappa)(1+r)^{1/(M-1)}} \right]^{M/(M-1)}
\]  \hspace{1cm} (9)

\[
f_{\varepsilon}^{(2)} = \frac{\varepsilon^{(2)}}{\varepsilon_1} = \frac{1 + r}{\sqrt{1+2r}}
\]  \hspace{1cm} (10)

where \(\sigma^{(M)}\), \(\varepsilon^{(M)}\) = effective stress and strain based on Hill's "new" theory with user specified M value, \(\sigma^{(2)}\), \(\varepsilon^{(2)}\) = effective stress and strain based on
Hill's "old" theory with $M = 2.0$, 

$\sigma_1, \varepsilon_1 =$ stress and strain in axial direction, 

$r =$ plastic anisotropy ratio, 

$\kappa = (1 + 2r)^{1/(M-1)}$, and 

$M =$ Hill's new parameter. 

Rearranging Eqs. 7-10:

\[
\frac{\sigma(M)}{f(M)} = \frac{\sigma(2)}{f(2)} = \sigma_1 \tag{11}
\]

\[
\frac{\varepsilon(M)}{f(M)} = \frac{\varepsilon(2)}{f(2)} = \varepsilon_1 . \tag{12}
\]

These equations provide a convenient way to calculate the effective stress or strain from experimentally observed axial values, and will be used in the following analyses.

The original data analysis program developed by Wagoner allows only for the use of Hill's "old" theory. Thus it does not provide a self-consistent stress-strain curve for materials which do not obey this theory. For such materials, Wagoner presented a method to determine the dependence of $M$ on effective strain. Using the plane strain stress-strain curve obtained from the data analysis program with Hill's "old" theory, $M$ can be found which will translate the plane strain curve to coincide with the
invariant uniaxial tension curve. This procedure will be discussed in more detail in Section 1.3.

A modified version of Wagoner's plane strain data analysis program is presented in this work. The variation of \( M \) with effective strain is found as presented in Ref. 48. This function is input into the modified program to provide a self-consistent calculation of the plane strain flow curve. As a simplification of the program, \( M \) can be taken as constant. A method is presented to estimate an initial guess for the constant \( M \) value. Using this initial guess, the modified program will calculate the standard deviation between the uniaxial tension constitutive equation and the calculated plane strain flow curve. \( M \) is then iterated to find the lowest standard deviation.

1.2 MODIFIED PLANE STRAIN DATA ANALYSIS PROGRAM

The original and modified programs are described in flow chart form in Figs. 12 and 13. In both cases the input data are the grid circle dimensions and load for each picture which was taken during the experiment. The output is the plane strain effective stress-strain curve based on the yield theory used in the analysis. The procedure used to determine the variation of \( M \) with effective strain is
Figure 12. Flow chart of Wagoner's original plane strain data analysis program.
Input $X_i$, $Y_i$, $L_i$

Calculate spline functions of $\xi$, $\eta$

Divide sample into edge (TS) and center (PS) regions

**TS $\xi$ data**

Calculate $L_e$ using $M = f(\bar{\varepsilon})$ with $M$ different at each point

**Center**

$L_c = L_i - L_e$

$M = f(\bar{\varepsilon})$

Calculate $\bar{\sigma}_1(M)$ vs $\bar{\varepsilon}_1(M)$ for PS region using $M = f(\bar{\varepsilon})$ where $\bar{\varepsilon}$ is defined as $\bar{\varepsilon}$ in center of PS region

If $M$ = constant compute SDY between TS and PS curves iterate $M$

If $M$ = $f(\bar{\varepsilon})$ quit

**Figure 13. Flow chart of modified plane strain data analysis program.**
presented in Section 1.3. The basic steps of the program are presented below:

1. **Generate cubic splines:** For each set of grid readings, the axial and transverse strains are calculated from the current and initial pictures. These strains are used to generate the axial and transverse strain profiles by fitting the strains to least squares cubic splines. The splines are forced to be symmetric about the center of the specimen.

2. **Divide specimen into regions:** The width of the specimen is divided into three regions: the center section in which the strain state is close to plane strain tension, and two edge sections which vary in strain state between plane strain and uniaxial tension. The edge regions are defined by the areas in which

\[
-\frac{\varepsilon_x}{\varepsilon_y} \geq \frac{r}{2(1+r)}.
\]  

3. **Calculate load supported by edge regions:** The effective strain is calculated in increments along the width of the specimen, using the proper M value corresponding to the local effective strain. The stress in the edge regions is calculated from these effective strains. Using this stress and the edge
area, the load supported by the edge regions is calculated.

4. **Calculate effective stress in plane strain:** The load supported by the edge regions is subtracted from the total axial load to give the load supported by the center region. This load is divided by the area of the center region to get the axial stress. The average effective stress of the center section is then calculated from the axial stress using Hill's theory with $M(\bar{\varepsilon})$ and assuming this region is in plane strain tension.

5. **Calculate effective strain in plane strain:** The average effective strain in the center section is calculated as that effective strain which, when assumed to occur uniformly over the entire center section width, would produce the center axial load predicted using Hill's theory with $M(\bar{\varepsilon})$. The strains used to calculate the effective strain for the determination of $M(\bar{\varepsilon})$ are the axial and transverse strains at the center of the plane strain region.

In Steps 3-5, the effective strain must be calculated from the axial and transverse strains using the yield theory with $M(\bar{\varepsilon})$ and the following equation:
\[
\begin{align*}
\dot{\varepsilon} &= \frac{2(1+r)\frac{1}{M}}{2} \left[ \frac{1}{(1+2r)^{1/(M-1)}} (\varepsilon_y - \varepsilon_x)^{M/(M-1)} \right. \\
&\quad \left. + (\varepsilon_y + \varepsilon_x)^{M/(M-1)} \right]^{(M-1)/M} 
\end{align*}
\] (14)

In this case \( M \) is a function of effective strain, and therefore, an iterative procedure must be used. For this purpose, a subroutine was written which would accept as input the axial and transverse strains \( (\varepsilon_x, \varepsilon_y) \) and iterate to determine the corresponding \( M \) value and effective strain. Specifically, a trial \( M \) value, \( M_0 \), is assumed, and the effective strain is calculated from Eq. 14 using \( \varepsilon_x \), \( \varepsilon_y \), and \( M_0 \). This calculated effective strain is then used to find the \( M \) value from the known fit of \( M \) vs. \( \dot{\varepsilon} \). \( M \) is now used as the trial \( M_0 \), and the procedure repeated until the error between \( M_0 \) and \( M \) is less than \( 10^{-4} \).

1.3 VARIATION OF M WITH EFFECTIVE STRAIN

Following the work of Wagoner\(^4\), the variation of \( M \) with effective strain can be calculated in two ways, using a consistent scheme or a simplified scheme. In both methods, the fit equations for the plane strain and uniaxial tension data are required. In the consistent scheme, the problem is solved by moving along the plane strain flow curve (which is calculated based on \( M = 2.0 \)) in increments. Because \( M \) is changing with strain, the stress state is not following a proportional path. In this case, the total \( \dot{\varepsilon} \) cannot be evaluated based on the current \( M \)
value alone because the total $\bar{\varepsilon}$ is comprised of increments for which different $M$ values are used. Therefore the strain increments must be used in the evaluation of the effective strain and Eq. 9 is no longer valid. From Eq. 11:

$$\frac{\bar{\sigma}(M)}{\sigma} = \frac{f(M)}{\sigma} \bar{\sigma}(2)$$

(15)

and using the incremental form of Eq. 12:

$$d\bar{\varepsilon}(M) = \frac{f(M)}{\varepsilon} d\bar{\varepsilon}(2)$$

(16)

where

$$F^M_{\sigma} = \frac{f(M)}{f(2)}$$

and $F^M_{\varepsilon} = \frac{f(M)}{f(2)}$

To find the $M$ value which translates the plane strain effective stress ($\bar{\sigma}^P(2)$) into the uniaxial tension stress ($\sigma^t$), the following equality is used:

$$\sigma^t = \bar{\sigma}(M) = F^P_{\sigma} \bar{\sigma}(M).$$

(17)

If both the tensile and plane strain curves can be expressed as Voce\textsuperscript{89} curves ($\bar{\sigma} = S (1 - A e^{\beta \bar{\varepsilon}})$), then

$$S_t (1 - A_t \exp(\beta_t \bar{\varepsilon}_t)) = F^P_{\sigma} S^P (1 - A^P \exp(\beta^P \frac{1}{F^P_{\varepsilon}} \int_0^{\varepsilon^m} d\bar{\varepsilon}(M))).$$

(18)
To solve Eq. 18, the strain path is followed in increments, and during each strain increment, the following steps are taken:

1. An initial $M$ value is assumed.
2. The effective strain increment is calculated based on this $M$ value and added to the total effective strain which occurred before the increment in order to obtain the current total effective strain.
3. The right and left hand sides of Eq. 18 are calculated and compared.
4. $M$ is iterated and steps 2-4 repeated until the difference between the right and left hand sides of Eq. 18 is less than $10^{-5}$.

The procedure can be greatly simplified if the effective strain is calculated assuming a constant $M$ value up to the current strain. Eq. 18 is now replaced with the following:

$$S_t (1 - A_t \exp(\beta_t \bar{\varepsilon}_t)) = F_{\sigma} S_p (1 - A_p \exp(\beta_p \frac{1}{F_{\varepsilon}} \bar{\varepsilon}(M)))$$

Again the problem is solved using strain increments, however, in this case the total strain is translated using the $M$ value guess in Step 2 above. $M$ is again iterated.
until the error between the plane strain and the uniaxial
flow curves is less than \(10^{-5}\).

The plane strain data analysis program calculates the
effective strain corresponding to a picture taken during
the experiment based on the grid readings of that picture
compared to the grid readings of the first picture (at time
= 0.0) and the current load. No strain increments are
considered in this approach. In this way, a proportional
path is assumed with the hardening law constant up to the
current strain. This is, in concept, consistent with the
simplified scheme for the calculation of \(M\) discussed above.
Therefore, the determination of \(M\) by the simplified scheme
was used in the plane strain data analysis.

1.4 CONSTANT \(M\) VALUE

As a special case of the analysis program including a
variable \(M\) value, \(M\) can be taken as constant. The program
follows the same flow chart, however the determination of \(\dot{\varepsilon}\)
is straightforward since \(\varepsilon_x\), \(\varepsilon_y\), and \(M\) are known. In order
to find the proper \(M\) value, an iteration scheme is used.
After the plane strain flow curve is calculated using a
trial \(M\) value, the standard deviation in stress (Eq. 20) is
calculated between the effective stress-strain data
generated from uniaxial and plane strain experiments. The
The standard deviation is given by:

\[
\text{Std. error} = \left[ \frac{\sum_{i=1}^{N} \left( \bar{\sigma}^{\text{PS}}_i - \bar{\sigma}^{\text{TS}}_i \left( \bar{\varepsilon}^{\text{PS}}_i \right) \right)^2}{N-1} \right]^{1/2}
\]

(20)

where \((\bar{\varepsilon}^{\text{PS}}_i, \bar{\sigma}^{\text{PS}}_i)\) is an effective stress-strain point generated using the current M value throughout the analysis and \(\bar{\sigma}^{\text{TS}}_i\) is the calculated stress at the strain \(\bar{\varepsilon}^{\text{PS}}_i\) using a constitutive equation based on the uniaxial data.

The M value is automatically incremented, the plane strain effective stress-strain data re-calculated based on the new M value, and the new standard deviation calculated. M continues to be incremented to find the minimum standard deviation.

The success of the iteration procedure is dependent on the initial M value chosen. In order to determine an accurate first guess, the calculation with M = 2.0 was used. The data analysis procedure outlined in Ref. 48 was used to estimate a constant M that nearly superimposes the plane strain and uniaxial effective stress-strain curves. This translation procedure is equivalent to using Hill's "new" theory in Steps 4 and 5 in Section 1.2 but using Hill's "old" theory in the calculation of the loads supported by the edge regions (Step 3). Because the edge
regions are near a strain state representing uniaxial tension and because the uniaxial tension flow curve is invariant with respect to yield theory, this approximation is not expected to produce large errors\textsuperscript{13}.

The equations presented in Section 1.1 are used to find an initial guess for $M$. Given $\dot{\sigma}^{(2)}$ and $\dot{\varepsilon}^{(2)}$ from the plane strain data analysis program with $M = 2.0$, and specifying an $M$ value, Eqs. 11 and 12 are used to find $\dot{\sigma}^{(M)}$ and $\dot{\varepsilon}^{(M)}$. $M$ is incremented in order to find the value which gives the lowest total standard deviation as defined in Eq. 20.

The $M$ value found using the translation scheme is then used to re-examine the raw plane strain data and a new effective stress-strain curve is generated. After each effective curve is generated, it is compared with the effective curve from uniaxial data and the standard deviation calculated. $M$ is varied in this way until the error is minimized. The $M$ value at this condition is denoted the self-consistent value.
Yield theories have been used to model a wide variety of physical material phenomena, such as cyclic hardening and softening, the Bauschinger effect, and creep. Wagoner used the two surface yield theory of Krieg to qualitatively model the negative stress transient behavior observed after an abrupt change in strain state from plane strain to uniaxial tension in brass. This theory, however, has several shortcoming. For example, the theory would predict a transient during reloading after a prestrain in the same strain state. The theory by Lee and Zaverl overcomes this problem by allowing the inner yield surface to elongate in the direction of the prestrain rather than just translate. Both theories are limited in use when one considers the modeling of positive stress transients. The inner, transient, yield surface would have to move outside the limiting surface on reloading. As loading continued, the transient surface would move toward the origin until it reached and was enclosed by the limit surface. Equivalently, single-yield surface models would interpret the positive transient as a positive loading with a stress increment toward the elastic region.

The movement of the yield surfaces in the ways discussed above are contrary to the ideas of classical
plasticity. Therefore a yield theory is desired which could model both positive and negative transients in a novel way which would not violate classical plasticity. Since Hill's "new" theory has been used in this work to describe the behavior of both IF steel and brass in plane strain tension, it is desirable to describe the transient behavior using the same yield theory.

In Section 1, the determination of $M$ using plane strain and uniaxial tension monotonic data with a fixed $r$ was discussed. After prestraining in plane strain tension, unloading, and reloading in uniaxial tension, yielding does not occur at the stress predicted using Hill's theory with the $M$ and $r$ values obtained from monotonic tests. $M$ is determined by the fitting of monotonic plane strain and uniaxial tension curves and defines the position of the yield surface in the direction of plane strain tension. The ratio $r$ is a constant material parameter obtained from uniaxial tension tests, however, it will be shown in Chapter V that $r$ changes with strain in the second stage experiments.

A variable $r$ value was used in conjunction with the equations defining Hill's "new" theory to translate the plane strain monotonic curve (with $M$ determined using monotonic data) to coincide with the second stage curve.
Because \( r \) will change with strain, the incremental form of Eq. 12 must be used in order to correctly determine the effective strain. Thus a procedure similar to the consistent scheme for finding \( M = f(\varepsilon) \) was used. The second stage stress-strain data was reduced to \( \Delta \sigma \) vs. \( \Delta \varepsilon \), where \( \Delta \varepsilon \) is the strain after reloading (or \( \varepsilon_{\text{total}} - \varepsilon_{\text{pre}} \)) and \( \Delta \sigma \) refers to the difference between the monotonic and second stage stresses at \( \Delta \varepsilon \). The analysis presented in Section 1.1 is followed once again. In this case the relations, \( f_\cdot \), are now functions of \( r \) and \( M \), and \( M \) is a constant obtained from monotonic results. Eq. 11 becomes

\[
\frac{-\sigma(r, M)}{\sigma(r, M)} = \frac{-\sigma(r_0, M)}{\sigma(r_0, M)} = \sigma_1
\]

and Eq. 12 becomes

\[
\frac{-\varepsilon(r, M)}{\varepsilon(r, M)} = \frac{-\varepsilon(r_0, M)}{\varepsilon(r_0, M)} = \varepsilon_1
\]

where \( r_0 \) is the initial \( r \) value found from monotonic tension tests. Eq. 15 becomes:

\[
-\sigma(r, M) = \frac{F(r, M)}{\sigma} - \sigma(r_0, M)
\]

and using the incremental form of Eq. 16:

\[
d\varepsilon(r, M) = \frac{F(r, M)}{\varepsilon} d\varepsilon(r_0, M)
\]

where \( F(r, M) = \frac{f(r, M)}{\sigma} \)}{f(r_0, M)} \]
To find the $r$ value which translates the plane strain effective stress ($\sigma(r,M)\sigma$) onto the second stage uniaxial tension stress ($\sigma_t$), the following equality is used:

$$\sigma_t = \sigma(r,M)$$

$$= \frac{F(r,M)}{\sigma} \sigma(r_0,M)$$

(25)

The second stage data is expressed in terms of $\Delta \sigma - \Delta \varepsilon$ as discussed above. If the plane strain curve can be expressed as a Voce curve, then

$$-\sigma(r_0,M) + \Delta \sigma = \frac{\sigma}{\sigma} S_p \left(1 - A_p \exp(\beta_p \int_0^{\varepsilon} \frac{1}{F(r,M)} d\varepsilon(r,M)}\right)$$

(26)

where $\Delta \sigma$ is positive for a positive transient. For the case in which the plane strain curve can be represented by a Hollomon curve (See Section 3), Eq. 26 becomes

$$-\sigma(r_0,M) + \Delta \sigma = \frac{\sigma}{\sigma} K_p \left(\int_0^{\varepsilon} \frac{1}{F(r,M)} d\varepsilon(r,M) - \varepsilon_o\right).$$

(27)

To solve Eq. 26 or 27, the strain path is followed in increments, and during each strain increment, the following steps are taken:
1. The proper $M$ value is determined for the current effective strain level.
2. The plane strain effective stress is calculated from the fit equation (assuming a constant monotonic $r_0$).
3. A transient $r$ value is assumed.
4. The effective strain during the increment is calculated based on this $M$ value and added to the total strain which occurred before the increment.
5. The right and left hand sides of Eq. 26 or 27 are solved and the error between them calculated.
6. The $r$ value is iterated and steps Steps 4-6 are repeated until the error is less than $10^{-4}$.

3. CURVE FITTING PROCEDURE

In order to develop constitutive equations and fit other experimental data, a non-linear least squares analysis was used. The data were fit to four equations and were also fit to a cubic spline function. The equations are:

1. Hollomon$^{90}$: $\sigma = K \epsilon^n$ \hspace{1cm} (28)
2. Swift$^{91}$: $\sigma = K (\epsilon + \epsilon_o)^n$ \hspace{1cm} (29)
3. Voce$^{89}$: $\sigma = \sigma_o (1 - A e^{-\beta \epsilon})$ \hspace{1cm} (30)
4. Ludwik$^{92}$: $\sigma = \sigma_o + K \epsilon^n$ \hspace{1cm} (31)
The cubic spline fit is used to determine the incremental K and n values for a Hollomon-type equation.

The computer program carries out a variational procedure which minimizes the standard deviation of the fit. The fit is considered optimized when the incremental change in each fit parameter is less than $10^{-4}$ of its value. The information provided are the coefficients and total deviation in stress for all four equations.

The second stage uniaxial tension data were converted to the form $\Delta \sigma - \Delta \varepsilon$ and fit to an equation which is described later. The data appears to follow an exponential curve, however a fit was desired which had an intercept to the $\Delta \sigma$ axis and which decreased exponentially to zero. The experimental data includes another effect which was to be neglected. At larger strains the data did not decrease to zero, but instead, reached a minimum and began rising in the case of brass or decreased continually in the case of IF steel.

The equation used to describe the second stage data is presented by the following:

$$
\Delta \sigma = \frac{1}{\sqrt{2} \pi R} \frac{\Delta \varepsilon - \varepsilon_0}{V_0} e^{-\left(\frac{\Delta \varepsilon - \varepsilon_0}{V_0}\right)}
$$

(32)
where $R$, $V_o$, and $\epsilon_o$ are constants. $R$ mainly affects the magnitude of $\Delta \sigma$, and $V_o$ especially affects the range over which $\Delta \sigma$ is non-zero (decay constant). The constant $\epsilon_o$ causes a translation of the curve along the $\Delta \varepsilon$ axis. The deviation of the fit to experimental data was not calculated since the important feature was the form of the equation. The constants $R$, $V_o$, and $\epsilon_o$ were adjusted manually until a usable form was determined.
CHAPTER V
RESULTS & DISCUSSION

1. INTERSTITIAL-FREE STEEL

1.1 UNIAXIAL TENSION MONOTONIC EXPERIMENTS

The constitutive equation for the same batch of IF steel as used in this study was determined by Lin and Wagoner. This equation incorporates strain, strain rate, and temperature as shown in Eq. 1.

\[ \sigma = K (\varepsilon + \varepsilon_0)^n (\dot{\varepsilon} / \dot{\varepsilon}_0)^m (1 + \beta (T - T_0)) \]  

(1)

where \( K = 566 \text{ MPa}, \ \varepsilon_0 = -0.014, \ n = 0.219, \ m = 0.018, \ \dot{\varepsilon}_0 = 0.002/\text{s}, \ \beta = -0.0012, \ \text{and} \ T_0 = 298^\circ \text{K}. \) The overall standard error of fit is 3.7 MPa. All of the experiments in this study were conducted at room temperature and therefore the temperature term in Eq. 1 could be neglected.

This equation is based on data from specimens cut along the rolling direction of the IF steel. The second stage tensile specimens of this work, however, were cut along the transverse sheet direction, so uniaxial tensile tests were performed using virgin sheet specimens cut from both the transverse (TD) and rolling (RD) directions. The
results. Fig. 14, show that the flow curves in the two directions are nearly the same. Therefore, Eq. 1, as generated from RD data, can be used for subsequent comparisons. This usage is for convenience since the strain rate can easily be changed in the equation, thus providing quick comparison of uniaxial tensile tests to experiments run at various strain rates.

1.2 STRAIN AGING EXPERIMENT

Fig. 15 shows the results of the strain aging experiment described in Chapter III. After prestraining and subsequent aging at 100°C for 2 hours, the specimen exhibited no additional hardening upon reloading. During the experimental procedure and machining of the half size specimens, care was taken not to expose the samples to elevated temperatures and certainly 100°C was not reached. Therefore it is satisfactorily shown, in combination with the evidence in the literature\textsuperscript{23,24,54}, that this material does not strain age at the operating conditions encountered in the two-stage experiments.

1.3. PLANE STRAIN MONOTONIC EXPERIMENTS

The plane strain effective stress-strain curve is calculated from grid circle measurements and the recorded load. The grid circle measurements are converted into true
Figure 14. True stress-strain curve of IF steel for specimens cut in the RD and TD directions.
Figure 15. Results of strain aging experiment using IF steel.
strain by

\[ \epsilon = \ln \left( \frac{L}{L_0} \right) \]  

(2)

where \( L \) = current diameter of circle and \( L_0 \) = original diameter of circle. The axial and transverse strain profiles for the final picture of a monotonically experiment are plotted in Fig. 16. The cubic spline fit is also shown. The boundaries between the edge and center regions are plotted as vertical lines and are determined using the following criterion for edge regions:

\[ -\frac{\epsilon_x}{\epsilon_y} \geq \frac{r}{2(1+r)} \]  

(3)

The original data analysis program using Hill's "old" theory with \( M = 2.0 \), was used to generate the plane strain curve shown in Fig. 17. The plane strain data is compared to the uniaxial tension constitutive equation at a strain rate of \( 1.8 \times 10^{-4} /s \), the same as the initial uniform strain rate of the plane strain test. The variation in effective stress between tests was less than 5% and data from a representative test are presented.

If Hill's "old" theory adequately described the plastic behavior of IF steel, then these two curves would coincide. This is not the case, although the two curves appear to be separated by a constant multiple. Because the work-hardening rates of the plane strain and uniaxial
Figure 16. The final transverse and axial strain profiles for a monotonic plane strain experiment using IF steel. The cubic spline fits are shown by dotted lines.
Figure 17. Plane strain effective stress-strain data for IF steel using $M = 2.0$ in the analysis compared to the uniaxial tension constitutive equation.
curves appear the same, a constant M value (different from 2.0) was sought which would transform the plane strain curve into the tensile curve. By definition, the tensile curve is invariant with respect to yield theory.

Using the translation procedure described in Chapter IV, the value $M \approx 3.1$ was found to best transform the plane strain curve into the uniaxial curve, as shown in Fig. 18. This fit is sufficiently accurate that the added complexity of a variable $M$ fit was not justified. Translating the data in such a manner is equivalent to replacing Hill's "old" theory with the correct yield theory in representing the plane strain stress-strain data in effective stress-strain format. However, Hill's "old" theory is still used in the experimental analysis of the edge regions, and determination of the load carried by the center region.

The M value found from the translation scheme was used as a starting point for the consistent calculation and iteration scheme, in which the plane strain data is analyzed using Hill's "new" theory and M is automatically iterated to find the lowest standard deviation between the plane strain and uniaxial tension curves. Using this consistent calculation, the value $M = 2.92$ was found to
Figure 18. Plane strain effective stress-strain data for IF steel using $M=3.1$ in the translation scheme compared to the uniaxial tension constitutive equation.
give the minimum error of 8.20 MPa, as shown in Fig. 19. Values for \( M \) between 2.8 and 3.1 give standard errors in the range 8.2 - 10.5 MPa. Although 2.92 yields the lowest standard deviation, values of \( M = 2.9 \pm 0.1 \) are equally acceptable given the scatter in the experimental data. This range of acceptable \( M \) values also illustrates that the yield surface of Hill's theory becomes flattened in the \( \sigma_1 = \sigma_2 \) direction as \( M \) becomes larger, making the plane strain flow curve insensitive to small changes in \( M \).

The consistent calculation does not provide any added accuracy in this case. This insensitivity arises because the edge regions are nearly in uniaxial tension and the choice of yield theory in the analysis of these regions is not critical (uniaxial tension flow curve is invariant with respect to yield theory). Therefore Wagoner's earlier estimate\(^{13}\) that the error in using Hill's "old" theory for the analysis of the edge regions is of the order of the experimental scatter is verified.

Originally a constant \( M \) value was sought because the work hardening rate or curvatures, of the monotonic plane strain and uniaxial tension curves appeared the same. Fig. 20 shows the variation of work-hardening rate, \( n \) \((\delta \ln \sigma / \delta \ln \varepsilon)\), with effective strain for the plane strain
Figure 19. Plane strain effective stress-strain data for IF steel using $M = 2.92$ in the consistent scheme compared to the uniaxial tension constitutive equation.
Figure 20. Logarithmic work hardening as a function of effective strain for the IF steel plane strain data (evaluated with $M = 2.0$).
data and the uniaxial tension constitutive equation. The work hardening rate was simply calculated as the slope in \( \ln \sigma - \ln \varepsilon \) space. As shown, the work hardening in plane strain is not significantly different from that in uniaxial tension, therefore justifying the use of a constant value of \( M \).

### 1.4 MEASUREMENT OF STRESS TRANSIENTS

The results from the two stage, plane strain/uniaxial tensile tests are presented in Fig. 21. Although two half-size tensile specimens from each prestrain were available for testing, only the results from the specimens cut from the centermost location of the prestrain blanks are reported. The other specimens also exhibited positive stress transients but the magnitudes were generally smaller. The highest axial strains occur in the center of the plane strain sample. Therefore, samples cut outside this region were subjected to lower strains and the hardening would be less. Because of this uncertainty in the actual strain reached by these samples, the stress-strain curve cannot be accurately placed on the two-stage curve and therefore the data are not included.

The definitions for effective stress and strain used in constructing Fig. 21 are based on \( M = 2.9 \), which is
Figure 21. Two-stage plane strain/uniaxial tension experimental results for IF steel. The definitions of effective stress and strain are based on $\dot{M} = 2.9$. 
consistent with the monotonic curves. As can be seen, large positive stress transients are observed upon reloading the prestrained specimens.

The shape of transients observed in IF steel is similar to that found by Wagoner and Laukonis\textsuperscript{14} for two-stage, plane strain/uniaxial tension tests of AK steel, in which the uniaxial tensile axis was perpendicular to the plane strain tensile axis. From Ref. 14, Fig. 2d, the following prestrains, $\bar{\varepsilon}_\text{pre}$, and corresponding increases in yield stress over the monotonic uniaxial curve, $\Delta\sigma$, are obtained.

\begin{align*}
\bar{\varepsilon}_\text{pre} &= 0.091 & \Delta\sigma &\approx 35 \text{ MPa} \\
\bar{\varepsilon}_\text{pre} &= 0.187 & \Delta\sigma &\approx 40 \text{ MPa} \\
\bar{\varepsilon}_\text{pre} &= 0.268 & \Delta\sigma &\approx 55 \text{ MPa} \\
\bar{\varepsilon}_\text{ave} &= 0.182 & \Delta\sigma_\text{ave} &= 43 \text{ MPa} \pm/ -10 \text{ MPa}
\end{align*}

These prestrain values were obtained using the assumption of Hill's "old" theory ($M = 2.0$). Because the plane strain curve lies slightly above the uniaxial curve for AK steel, a translation of the plane strain curve to coincide with the uniaxial curve would yield slightly higher prestrain values. Therefore the corrected curve would move the second stage curves to the right and the $\Delta\sigma$ values would be slightly smaller, so the average $\Delta\sigma$ of 43
MPa may slightly overstate the consistent $\Delta \sigma$. Such an error is small in this case because $M = 2.0$ agrees closely with the data for AK steel.

The $\Delta \sigma$ values obtained from the present work on IF steel are presented below. They are taken from Fig. 21 with the prestrain values based on a consistent calculation with $M = 2.9$.

\begin{align*}
\dot{\varepsilon}_{\text{pre}} &= 0.083 \quad \Delta \sigma \approx 68 \text{ MPa} \\
\dot{\varepsilon}_{\text{pre}} &= 0.126 \quad \Delta \sigma \approx 64 \text{ MPa} \\
\dot{\varepsilon}_{\text{pre}} &= 0.161 \quad \Delta \sigma \approx 87 \text{ MPa} \\
\dot{\varepsilon}_{\text{pre}} &= 0.238 \quad \Delta \sigma \approx 87 \text{ MPa} \\
\dot{\varepsilon}_{\text{ave}} &= 0.152 \quad \Delta \sigma_{\text{ave}} = 77 \text{ MPa } \pm/\pm 15 \text{ MPa}
\end{align*}

Comparison of the transient magnitudes reveals that IF steel exhibits larger transients than AK steel, but perhaps not significantly larger. Therefore, no significant effect of interstitials or strain aging can be ascribed to the kind of transient phenomenon reported here.

1.5 STRAIN RATIO MEASUREMENTS

Axial and transverse strains were obtained from the second stage experiments using biaxial strain gages. These data are shown in Fig. 22 with the negative of the transverse (compressive) strain plotted vs. the axial
Figure 22. Transverse (x) vs. axial (y) strains as measured using resistance strain gages for IF steel.
strain. Again, only data from second stage specimens cut from the center of the plane strain blank are presented.

Strain localization occurred before 1% strain in prestrained IF steel. Although the gages were placed in the center of the samples, localization never occurred at gage locations. It is possible that the gage and adhesive strengthened the sample enough to offset the taper in the gage section. The instability of localization could be triggered by any slight geometric defect. Because localization did not occur at the gage, the gage experienced elastic unloading and axial strains in excess of 0.9% were never reached at the gage locations.

As seen in Fig. 22, an increase in prestrain did not change the initial portion of the curve, the elastic slope. The monotonic curve exhibits a sharp apparent transition from elastic to plastic behavior based on the changing slopes. An increase in prestrain, however, causes a more gradual apparent elastic-plastic transition. The slope of this curve \((-\Delta \epsilon_x/\Delta \epsilon_y)\) is related to the incremental value of the plastic anisotropy in the following manner:

\[
r = \frac{\Delta \epsilon_x^P}{\Delta \epsilon_y^P} = \frac{1}{(-\Delta \epsilon_y^P / \Delta \epsilon_x^P - 1)}
\]

where \(y = \) axial, \(x = \) transverse, and \(z = \) thickness directions and \(p \) denotes plastic strains. The final
relationship is obtained by considering constancy of volume in the plastic regime.

The curves in Fig. 22 include both elastic and plastic strains. In the present work, only the plastic effects are of interest and therefore it is desirable to eliminate the elastic strains. The axial elastic strains \((\varepsilon_y^e)\) can be estimated by

\[
\varepsilon_y^e = \frac{\sigma}{E'} \tag{5}
\]

where \(E'\) = apparent Young's modulus (i.e. the asymptotic slope of \(\sigma\) vs \(\varepsilon_y^t\) as \(\varepsilon_y^t \to 0\)). The total strain \((\varepsilon_y^t)\) is defined as

\[
\varepsilon_y^t = \varepsilon_y^e + \varepsilon_y^p \tag{6}
\]

Since both axial and transverse strains are used in the analysis, both must be corrected for elastic strains. They are related by

\[
- \frac{d\varepsilon_x}{d\varepsilon_y} = \nu' \tag{7}
\]

where \(\nu'\) = apparent Poisson's ratio (i.e. the asymptotic slope of \(\varepsilon_x^t\) vs. \(\varepsilon_y^t\) as \(\varepsilon_y^t \to 0\)). To find the plastic strain, Eqs. 5-7 are combined to give:

\[
\varepsilon_y^p = \varepsilon_y^t - \varepsilon_y^e
= \varepsilon_y^t - \frac{\sigma}{E'} \tag{8}
\]

\[
\varepsilon_x^p = \varepsilon_x^t - \varepsilon_x^e
= \varepsilon_x^t + \nu'\frac{\sigma}{E'} \tag{9}
\]
Elastic unloading of the sample occurred before failure of the gage, therefore, the apparent Young’s modulus, $E'$, was calculated from the slope of the unloading curve of load vs. $\epsilon_y$, and the apparent Poisson’s ratio, $\nu'$, was calculated as the slope of the unloading curve of $\epsilon_x$ vs. $\epsilon_y$. The apparent Young’s moduli and Poisson’s ratios were also calculated using the loading curves and the values are presented in Table 2. The average loading values of $E' = 170279$ MPa and $\nu = .361$ were used to evaluate the elastic strains. This value of $E'$ yielded higher elastic strains than the value obtained from the unloading curves and resulted in a monotonic plastic strain curve with a more constant slope.

The transverse vs. axial strain and stress vs. axial strain curves are plotted in Figs. 23 a-d for both total and plastic strains. The curve corresponding to a prestrain of 0.167 is not presented as the load (and therefore, stress) data were not available from the experiment. The incremental slopes of the transverse vs. axial strain curves in Figs. 23 a-d are presented in Figs. 24 a-d. Slopes from both the total strain and plastic strain data are presented for comparison. Subtracting the elastic strains eliminates this complicating factor and the true transient response can be seen. The slope of the
TABLE 2

APPARENT YOUNG’S MODULI AND POISSON’S RATIOS
FOR IF STEEL SECOND STAGE SAMPLES

<table>
<thead>
<tr>
<th>prestrain</th>
<th>$E'$ unloading / loading</th>
<th>$\nu'$ unloading / loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>167068. / 204546.</td>
<td>-- / .304</td>
</tr>
<tr>
<td>0.082</td>
<td>--- / 244809.</td>
<td>-- / .360</td>
</tr>
<tr>
<td>0.126</td>
<td>169546. / 260866.</td>
<td>.380 / .354</td>
</tr>
<tr>
<td>0.167</td>
<td>-- / 244809.</td>
<td>.360 / .367</td>
</tr>
<tr>
<td>0.238</td>
<td>174224. / 212754.</td>
<td>.343 / .360</td>
</tr>
<tr>
<td>AVERAGE:</td>
<td>179279. / 230744.</td>
<td>.361 / .360 (without low value)</td>
</tr>
</tbody>
</table>
Figure 23. Transverse vs. axial strain and stress vs. axial strain plotted using both total and plastic strains for IF steel with a) 0.0 prestrain, b) 0.082 prestrain, c) 0.126 prestrain, and d) 0.238 prestrain.
Figure 23. (2/4)
Figure 23 (3/4)
Figure 23. (4/4)
Figure 24. \((-\frac{\partial e_x}{\partial y})\) vs. axial strain calculated from total strain and plastic strain data for IF steel a) 0.0 prestrain, b) 0.082 prestrain, c) 0.126 prestrain, and d) 0.238 prestrain.
Figure 24. (2/4)
Figure 24. (3/4)

- PRESTRAIN = 0.126
  - DOTTED TOTAL STRAINS
  - SOLID PLASTIC STRAINS
Figure 24. (4/4)
monotonic curve (0% prestrain) is now a constant, as expected (constant plastic anisotropy ratio). An increase in prestrain creates a transient region in Figs. 24 b-d in which the slope decreases to a constant value at very small strains ($\approx 0.002$).

From Figs. 24 a-d, it can be seen that the asymptotic strain ratio is the same when calculated using plastic or total strains. This is expected since the elastic strains which are subtracted are very small and are a minor effect at large total strains. Therefore the asymptotic strain ratio was determined from the total strain data in Fig. 22. Each curve was fit to a straight line in the largest strain region (axial strains $\geq 0.1$) and the plastic anisotropy ratio, $r$, was calculated from the slope of this line. The values are listed below:

$$\bar{\varepsilon}_{\text{pre}} = 0.0 \quad r = 2.267$$

<table>
<thead>
<tr>
<th>$\bar{\varepsilon}_{\text{pre}}$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.082</td>
<td>2.088</td>
</tr>
<tr>
<td>0.126</td>
<td>2.105</td>
</tr>
<tr>
<td>0.161</td>
<td>2.041</td>
</tr>
<tr>
<td>0.238</td>
<td>1.945</td>
</tr>
</tbody>
</table>

The value of $r$ calculated in this way decreases with increasing prestrain, an observation which is consistent with those of Laukonis and Ghosh$^{61}$ on AK steel using $r$ calculations from grid circles and of Dabrowski et al$^{57}$. 
Burns and Heyer\textsuperscript{60} have established that the plastic anisotropy ratio, $r$, is a measure of the texture, or preferred crystallographic orientation of the grains. The decrease in $r$ after a prestrain indicates that the prestrain operation may have produced a deformation texture different from that produced during processing (rolling) of the sheet.

The reverse side of each gaged second stage sample was photogridded with circles and pictures were taken during the test. However, the premature strain localization allowed no additional information to be obtained from the pictures.

1.6 NUMERICAL ANALYSIS OF STRESS TRANSIENTS

The variation of $r$ value which would allow the plane strain curve to coincide with the second stage uniaxial tension curve during the transient period was calculated using the analysis scheme described in Chapter IV, Section 2. The second stage stress-strain data were converted to $\Delta\sigma-\Delta\varepsilon$, where $\Delta\varepsilon$ is the measure of strain after reloading or $\varepsilon_{\text{total}} - \varepsilon_{\text{pre}}$, and $\Delta\sigma$ is the difference in stress between the monotonic and second stage data at a given $\Delta\varepsilon$. An equation was desired for the data fit which exhibited a y
intercept and decreased exponentially to zero. The actual experimental data does not asymptotically approach zero but continues to decrease. This is caused by premature strain localization and is a complicating effect which is not required in this analysis. Thus, an idealized positive stress transient is used in the analysis which rejoins the monotonic stress-strain curve after a small transient region.

The fitting equation is given by the following:

$$\Delta \sigma = \frac{1}{4} \frac{\Delta e - \epsilon_o}{R} e^{-\frac{\Delta e - \epsilon_o}{V_o}}$$

(10)

where $R$, $V_o$, and $\epsilon_o$ are parameters to adjust the shape of the function. The parameters listed below were used to fit the second stage IF steel data which is presented in the form $\Delta \sigma - \Delta e$ in Figs. 25 a-d. The fit equation is also shown in these figures.

<table>
<thead>
<tr>
<th>prestrain</th>
<th>$R$</th>
<th>$V_o$</th>
<th>$\epsilon_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.082</td>
<td>$9.8 \times 10^{-6}$</td>
<td>0.0145</td>
<td>0.0145</td>
</tr>
<tr>
<td>0.126</td>
<td>$1.05 \times 10^{-5}$</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>0.161</td>
<td>$4.6 \times 10^{-6}$</td>
<td>0.0082</td>
<td>0.0093</td>
</tr>
<tr>
<td>0.238</td>
<td>$4.8 \times 10^{-6}$</td>
<td>0.0052</td>
<td>0.0042</td>
</tr>
</tbody>
</table>

The variation of $r$ with effective strain calculated using the analysis scheme presented in Chapter IV, Section 2 is plotted in Figs. 26 a-d, and as can be seen $r$
Figure 25. $\Delta \sigma$ vs. $\Delta \epsilon$ for IF steel second stage experimental data compared with fit equation for a) 0.082 prestrain, b) 0.126 prestrain, c) 0.161 prestrain, and d) 0.238 prestrain.
PRESTRAIN = 0.126
+ EXPERIMENTAL DATA
--- FIT EQUATION

Figure 25. (2/4)
Figure 25. (3/4)
Figure 25. (4/4)

PRESTRAIN = 0.238
+ EXPERIMENTAL DATA
--- FIT EQUATION

d)

Figure 25. (4/4)
Figure 26. Calculated variation of $r$ value with effective strain ($\Delta e$) for IF steel second stage stress transients with a) 0.082 prestrain, b) 0.126 prestrain, c) 0.161 prestrain, and d) 0.238 prestrain.
Figure 26. (2/4)

b)
Figure 26. (3/4)
Figure 26 (4/4)
increases with effective strain until a constant value is reached. Using the relation in Eq. 4, the data are converted to \( (-\frac{d\epsilon_x}{d\epsilon_y}) \) for comparison with experimental data where available and are plotted in Figs. 27 a-d. In each case, the calculated and experimental data appear to approach the same asymptotic value but the transient region is not properly predicted by the numerical analysis. The experimental results show the slope decreasing to the asymptotic value whereas the computed results show the slope increasing to the asymptotic value. For the two highest prestrains the computer program does not converge for the first strain increments and a value of \( r \) cannot be found which allows the yield surface to go through both the plane strain and the second stage uniaxial tension points.

One major difficulty in modeling the positive transient with conventional yield theories has its origin in the definition of the yield surface. The yield surface defines the point of yielding and the surface expands as plastic loading continues. There is no definition for the outside of the surface. To model this behavior with a two-surface theory, the inner yield or transient surface would have to travel outside the outer, limit surface. As the loading continued, the transient surface would move toward the origin until it reached and was enclosed by the
Figure 27. \((-\frac{d\varepsilon_x}{d\varepsilon_y})\) vs. effective strain \((\Delta\varepsilon)\) calculated from variation of \(r\) value and compared to experimental data for IF steel where available for a) 0.082 prestrain, b) 0.126 prestrain, c) 0.161 prestrain, and d) 0.238 prestrain.
PRESTRAIN = 0.126
+ CALCULATED VALUES
- EXPERIMENTAL DATA

Figure 27. (2/4)
Figure 27 (3/4)
Figure 27. (4/4)
limit surface. The material behavior would then be described by this limit surface. This idea is conceptually appealing, however the motion of a surface outside the limit surface does not fit into the conventional idea of yield surfaces.

The simple model introduced in this work employs a well known and relatively simple theory (Hill's "new" theory) in a new way. The parameter which is usually taken as a material constant, \( r \), is now used merely as a fitting parameter. For use in a program to model complex loading paths, such as finite element analysis, the \( r \) value would have to be stored as a function of loading path and prestrain. Further research would be required to determine if a general form of the \( r \) variation could be used to account for different prestrain paths.

Physically, for the yield surface to pass through both the plane strain and second stage uniaxial tension stress states with the slope at \( \sigma_2 = 0 \) prescribed by the experimental second stage \( r \) values, there are two possibilities. The first requires that an unusual yield surface shape develop, for example a bulge or corner may form between the two stress stages. The other possibility is that normality is not obeyed, i.e., the strain vector is
no longer normal to the yield surface at the uniaxial tension stress state. In this case, the experimentally determined $r$ values would not determine the yield surface shape.

2. 70/30 BRASS

2.1 UNIAXIAL TENSION MONOTONIC EXPERIMENTS

The constitutive equation reported by Wagoner for 70/30 brass is shown in Eq. 11.

$$\bar{\sigma} = \sigma_0 \left( 1 - A e^{B\bar{e}} \right)$$  \hspace{1cm} (11)

where $\sigma_0 = 840$ MPa, $A = 0.86$, and $B = -1.81$. This equation was not determined for the same batch of material as used in this study, therefore uniaxial tension tests were performed using specimens cut along the RD and TD sheet directions, with initial uniform strain rate $= 3 \times 10^{-4}$. The RD and TD curves are compared to the constitutive equation in Fig. 28. As shown, the TD and RD data do not coincide and the constitutive equation by Wagoner$^{13}$ does not fit the data properly. The form of Eq. 11, however, seems correct. The data were fit to Eq. 11 and several other constitutive equations described in Chapter IV, Section 3. The Voce equation$^{89}$ (Eq. 11) gave the best fit with the following constants:

RD: $\sigma_0 = 942.4$, $A = 0.875$, $B = -1.414$

TD: $\sigma_0 = 800.5$, $A = 0.883$, $B = -1.907$
Figure 28. Comparison of RD and TD uniaxial tension data for brass with Wagoner's constitutive equation.
where the standard deviation of fit was 1.59 MPa and 2.17 MPa, respectively.

The TD uniaxial data are used for subsequent comparisons with results from second stage specimens which were also cut in the TD sheet direction. The RD data are used in comparisons with results of monotonic plane strain tests for which the samples were cut in the RD direction.

2.2 PLANE STRAIN MONOTONIC EXPERIMENTS

The grid circle measurements were converted to true strain as discussed in Section 1.3 and are presented in Fig. 29. The cubic spline fits and center/edge region boundaries are also shown. The plane strain monotonic effective stress-strain curve using Hill's "old" theory is shown in Fig. 30. The deviation in stress between experiments was less than 5% and therefore only a representative curve is shown. The plane strain curve is compared to the uniaxial tension constitutive equation for RD data. The two curves do not have the same work hardening rates (slopes) as was the case for IF steel and they tend to diverge. Fig. 31 presents the work hardening rate \( \delta \ln \bar{\sigma} / \delta \ln \bar{\varepsilon} \) vs. effective strain for both the plane strain data and uniaxial tension curve from Fig. 30. The plane strain work hardening rate is lower that that of
Figure 29. The final transverse and axial strain profiles for a monotonic plane strain experiment using brass. The cubic spline fits are shown by dotted lines.
Figure 30. Plane strain effective stress-strain data for brass using $M = 2.0$ in the analysis compared to the uniaxial tension constitutive equation.
Figure 31. Logarithmic work hardening as a function of effective strain for the brass plane strain data (evaluated with $M = 2.0$).
uniaxial tension at effective strains greater than 0.2. Hill's "new" theory with a constant \( M \) value cannot transform the plane strain data into the uniaxial curve, however. Wagoner\(^{13}\) has shown that Hill's "new" theory with an \( M \) value which is a function of effective strain can be used.

The dependence of \( M \) on effective strain was calculated using the simplified and consistent schemes discussed in Chapter IV. As shown in Figs. 32 a and b, \( M \) increases rapidly at high strain values for both schemes. The simplified scheme was chosen to fit \( M \) vs. \( \bar{e} \) because it is conceptually consistent with the plane strain data analysis program. The data analysis program allows \( M \) to vary, but for each picture a constant \( M \) is assumed up to the current strain and the total strain is used. In this way a proportional path is assumed with the hardening law constant up to the current strain.

The simplified \( M \) curve was fit to a cubic spline function in four sections. The cubic spline fit guarantees a smooth connection of the sections through a continuous slope. The fit equation was incorporated in the plane strain data analysis program using a variable \( M \) as described in Chapter IV. Fig. 33 was obtained using this
Figure 32. Variation of M value with effective strain calculated using a) consistent scheme, and b) simplified scheme.
Figure 32. (2/2)
Figure 33. Plane strain effective stress-strain data for brass using a variable $M$ in the consistent scheme compared to the uniaxial tension data.
program to analyze the raw plane strain data. The plane strain curve coincides with the uniaxial tension curve, illustrating the effectiveness of this procedure.

2.3 MEASUREMENT OF STRESS TRANSIENTS

The results from the two-stage, plane strain/uniaxial tensile tests are presented in Fig. 34. Although two specimens from each prestrain were allocated for this experiment, the data from the specimens cut from the centermost portion of the prestrained blanks are shown here. As explained in Section 1.4, the actual prestrain is not as accurately known for the other specimens.

The definitions for effective stress and strain used in constructing Fig. 34 are based on a variable $M$ as described in the previous section. As shown in this figure, the flow stress in uniaxial tension is reduced after a prestrain in plane strain tension. The difference in second stage yield stress and the monotonic flow stress at the effective strain ($\Delta \sigma$) for each prestrain is presented below. The yield stress in the second stage is defined as the stress at 0.5% strain after reloading.

\[
\bar{\varepsilon}_{\text{pre}} = 0.054 \quad \Delta \sigma \approx -6.7 \text{ MPa}
\]

\[
\bar{\varepsilon}_{\text{pre}} = 0.075 \quad \Delta \sigma \approx -7.8 \text{ MPa}
\]
Figure 34. Two-stage plane strain/uniaxial tension experimental results for brass. The definitions of effective stress and strain are based on a variable $M$. 

EFFECTIVE STRESS (MPa) versus EFFECTIVE STRAIN
The second stage stress-strain curves do not rejoin the monotonic tensile curve (or equivalently, the monotonic plane strain curve with variable $M$) but remain lower than it. This phenomenon was also observed by Wagoner. The second stage curves also exhibit reduced work hardening rates and slight increases in final elongation.

### 2.4 STRAIN RATIO MEASUREMENTS

The axial and transverse strains obtained from the gaged second stage samples are shown in Fig. 35. Again, only the data from specimens cut from the centermost portion of the prestrained blanks are shown here. The prestrain increases the work-hardening rate and thereby decreases the effect of any discontinuities or defects to cause localized flow in brass and therefore premature localization does not occur. In the case of brass, the termination of the curves in Fig. 35 is caused by failure of the bond between the gage and metal and not elastic unloading. Although the gage and bonding agent used are
Figure 35. Transverse (x) vs. axial (y) strains as measured using resistance strain gages for brass.
rated to remain intact and in working condition for strains up to 4-6%, these strains were never attained in these experiments. It is thought that the narrow width of the specimen caused early weakening of the bond between the gage and metal.

The sharp transition from elastic to plastic behavior observed in IF steel is not seen in brass. Distinct elastic and plastic slopes cannot be determined from the prestrain curves, instead, the plastic loading regime seems to be characterized by an initially high slope followed by a lower slope to give an S shaped curve. Thus, it is especially helpful to eliminate the elastic effects by subtracting the elastic strains as discussed in Section 1.5. The values obtained from loading curves for the apparent Young's moduli and Poisson's ratios are given in Table 3. The gages did not undergo elastic unloading and therefore unloading values are not available.

The transverse vs. axial strain and stress vs. axial strain curves are plotted using total and plastic strains in Figs. 36 a-e for prestrains in which load (and, therefore stress) data is available. The slopes of the axial vs. transverse strain curves in Figs. 36 a-e are plotted in Figs. 37 a-e for both the total and plastic
TABLE 3

APPARENT YOUNG'S MODULI AND POISSON'S RATIOS
FOR 70/30 BRASS SECOND STAGE SAMPLES

<table>
<thead>
<tr>
<th>prestrain</th>
<th>$E'$</th>
<th>$\nu'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>106501.</td>
<td>.316</td>
</tr>
<tr>
<td>0.075</td>
<td>105373.</td>
<td>.317</td>
</tr>
<tr>
<td>0.097</td>
<td>84947.</td>
<td>.300</td>
</tr>
<tr>
<td>0.141</td>
<td>111637.</td>
<td>.300</td>
</tr>
<tr>
<td>0.167</td>
<td>150904.</td>
<td>.300</td>
</tr>
<tr>
<td>0.193</td>
<td>104482.</td>
<td>.289</td>
</tr>
<tr>
<td>0.254</td>
<td>103179.</td>
<td>.286</td>
</tr>
</tbody>
</table>

AVERAGE: 106234. .301
(not including high and low values)
Figure 36. Transverse vs. axial strain and stress vs. axial strain plotted using both total and plastic strain for brass with a) 0.0 prestrain, b) 0.076 prestrain, c) 0.097 prestrain, d) 0.141 prestrain, and e) 0.264 prestrain.
Figure 36. (2/5)
PRESTRAIN = 0.141
TOTAL STRAINS
+ PLASTIC STRAINS

Figure 36. (4/5)
Figure 37. $(-\frac{\partial e_x}{\partial e_y})$ vs. axial strain calculated from total strain and plastic strain data for brass a) 0.0 prestrain, b) 0.075 prestrain, c) 0.097 prestrain, d) 0.141 prestrain, and e) 0.253 prestrain.
Figure 37. (2/5)
Figure 37. (3/5)
Figure 37. (4/5)
Figure 37. (5/5)
strains. This correction eliminates the complicating effect of elastic strains and reveals the transient response in greater detail. The monotonic (0% prestrain) data in Fig. 37a exhibits a nearly constant slope. For the prestrain data, the slope decreases with increasing axial strain and appears to approach an asymptotic value.

Pictures were taken of the deforming grid on the reverse side of each sample. The data from both the strain gages (Fig. 34) and the pictures were used to calculate the plastic anisotropy ratio, r. It was difficult to choose the proper slope from the strain gage data because of the rapidly changing slope. Despite this difficulty, the slope was calculated for the approximately linear region at higher strains (> 0.005).

The picture and strain gage data are presented in Figs. 38 a-f. The picture data was fit using a least squares linear regression in which the best fit line was forced to pass through (0,0). The standard deviation of the slope of this line was calculated. The confidence limits drawn in Figs. 38 a-f are calculated by adding (or subtracting) the deviation in slope to the best fit slope and plotting the line based on that value. The r value was calculated from this slope and is presented below along
Figure 38. Transverse vs. axial strain plotted using strain gage data and picture data for brass with a) 0.0 prestrain, b) 0.075 prestrain, c) 0.097 prestrain, d) 0.141 prestrain, e) 0.193 prestrain, and f) 0.254 prestrain.
Figure 38. (2/6)
Figure 38. (3/6)
Figure 38. (4/6)

BRASS PRESTRAIN = 0.141

- STRAIN GAGE DATA
- PICTURE DATA
- STRAIGHT LINE FIT TO PICTURE DATA
- CONFIDENCE LIMITS

X STRAIN

Y STRAIN
Figure 38. (5/6)
Figure 38. (6/6)
with the best fit from the strain gage data.

\[ r_{\text{from gage}} \quad r_{\text{from pictures}} \]

\[
\tilde{\varepsilon}_{\text{pre}} = 0.0 \quad \begin{array}{l}
0.075 & 0.638 & 0.814 \\
0.097 & 0.625 & 0.666 \\
0.141 & 0.644 & 0.660 \\
0.193 & 0.634 & 0.731 \\
0.254 & 0.642 & 0.754 \\
\end{array}
\]

The strain gage data imply that the prestrain has little or no effect on the asymptotic plastic anisotropy ratio except at the highest prestrain. These values are from very low strains and are subject to interpretation as to what region to use for the determination of the slope. Because the slope is still changing at the time of gage breakage, there is no guarantee that this initial behavior is representative of the higher strain behavior and the data obtained from the pictures will be used in the case of brass. The values from the pictures have much more scatter in the data as the resolution is not as good as for the gages, however, the range of strain over which pictures were taken is very large. It is expected that the \( r \) values from the pictures give a better indication of the large strain anisotropy.
In the case of the large strain data obtained from the pictures, \( r \) appears to decrease as a result of the prestrain. This change in \( r \) is believed to be linked to the deformation texture produced during the prestrain. Stout and Staudhammer contend that the widely different work-hardening rates of stress-strain curves for various strain states is a result of the texture developed during the deformation. This provides an explanation for the difference in work-hardening rates between the monotonic and second stage curves (Fig. 34).

2.5 NUMERICAL ANALYSIS OF STRESS TRANSIENTS

The variation in \( r \) value which would superimpose the plane strain curve on the second stage uniaxial tension curve was calculated using the analysis scheme presented in Chapter IV, Section 2. The second stage stress-strain data were converted to \( \Delta \sigma - \Delta \epsilon \) as discussed in Section 1.6. The experimental data (\( -\Delta \sigma \)) and fit equation chosen to describe the data are shown in Figs. 39 a-f. The experimental data does not decrease exponentially to zero as in the equation because of the large-strain change in work hardening rate exhibited by the second stage brass samples. The fit equation was used to describe the experimental data so that only the transient behavior is analyzed, not other complicating effects. Thus, an idealized negative
Figure 39. $\Delta \sigma$ vs. $\Delta \epsilon$ for brass second stage experimental data compared with fit equation for a) 0.054 prestrain, b) 0.075 prestrain, c) 0.141 prestrain, d) 0.193 prestrain, and f) 0.254 prestrain.
PRESTRAIN = 0.075

EXPERIMENTAL DATA

FIT EQUATION

Figure 39. (2/6)
PRESTRAIN = 0.097
+ EXPERIMENTAL DATA
--- FIT EQUATION

c)

Figure 39. (3/6)
Figure 39. (4/6)
Figure 39. (5/6)
PRESTRAIN = 0.254

+ EXPERIMENTAL DATA

--- FIT EQUATION

**Figure 39** (6/6)
transient in which the second stage stress-strain data rejoin the monotonic curve has been analyzed. The parametric values used in the fit equation (Eq. 10) are given below:

<table>
<thead>
<tr>
<th>prestrain</th>
<th>R</th>
<th>V₀</th>
<th>ε₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.054</td>
<td>6.0 x 10⁻⁵</td>
<td>0.0020</td>
<td>0.0020</td>
</tr>
<tr>
<td>0.075</td>
<td>3.3 x 10⁻⁵</td>
<td>0.0012</td>
<td>0.0012</td>
</tr>
<tr>
<td>0.097</td>
<td>1.0 x 10⁻⁵</td>
<td>0.0027</td>
<td>0.0027</td>
</tr>
<tr>
<td>0.141</td>
<td>1.6 x 10⁻⁵</td>
<td>0.0033</td>
<td>0.0033</td>
</tr>
<tr>
<td>0.193</td>
<td>4.0 x 10⁻⁶</td>
<td>0.0025</td>
<td>0.0025</td>
</tr>
<tr>
<td>0.254</td>
<td>1.5 x 10⁻⁶</td>
<td>0.0030</td>
<td>0.0030</td>
</tr>
</tbody>
</table>

The variation of r with effective strain required for the translation of the plane strain curve onto the second stage data is shown in Figs. 40 a-f. As can be seen, r decreases with strain until an asymptotic value is reached. Eq. 4 is used to convert r into (−dεₓ/dεᵧ) for direct comparison with experimental data where available. The comparisons are shown in Figs. 41 a-f. Like the experimental data in Figs. 37 a-e, the slope decreases to an asymptotic value, however that value is different for the two cases.

Unlike the positive transient, the modeling of the negative transient poses no problems with conventional
Figure 40. Calculated variation of r value with effective strain ($\Delta \varepsilon$) for brass second stage stress transients with a) 0.054 prestrain, b) 0.075 prestrain, c) 0.097 prestrain, d) 0.141 prestrain, e) 0.193 prestrain, and f) 0.254 prestrain.
Figure 40. (2/6)
Figure 40. (3/6)

+ PRESTRAIN = 0.097

Effective Strain vs. Reduction in Area
Figure 40 (4/6)

[Graph showing effective strain with a specified prestrain value.]
Figure 40. (5/6)
Figure 40.

f)
Figure 41. \( \left( -\frac{d\varepsilon_x}{d\varepsilon_y} \right) \) vs. effective strain (\( \Delta\varepsilon \)) calculated from the variation of r value and compared to experimental data for brass where available for a) 0.054 prestrain, b) 0.075 prestrain, c) 0.097 prestrain, d) 0.141 prestrain, e) 0.193 prestrain, and f) 0.254 prestrain.
Figure 41. (3/6)
PRESTRAIN = 0.141
+ CALCULATED VALUES

- dex/dey

EFFECTIVE STRAIN

d)

Figure 41. (4/6)
Figure 41. (5/6)
Figure 41.
plasticity theories. Wagoner\textsuperscript{13} used Krieg's\textsuperscript{29} two-surface plasticity model to qualitatively model a prestrain in plane strain tension followed by straining in uniaxial tension. In this model the inner yield surface describes the transient, early yield and the distance between the yield and limit surfaces defines the work hardening rate. The model does have an undesirable feature in that a prestrain in plane strain followed by reloading in plane strain would also result in a transient behavior. This effect can easily be taken care of by redefining the shape and hardening behavior of the yield surface\textsuperscript{32}. By allowing it to elongate in the original loading direction while keeping the same width in the tensile direction, this inconsistency can be eliminated.

Most two-surface yield theories\textsuperscript{71,74} prescribe a constant, underlying work-hardening rate characteristic of the limit surface. These theories can possibly model idealized negative stress transients, however they cannot easily account for the change in work-hardening rate observed in brass second stage samples. Hart\textsuperscript{44} has developed a set of state variable-type constitutive equations in which deformation history is included in the evolution of the state variables. In this type of theory, the total accumulated strain is not an important quantity.
The values and rates of change of the state variables describe the deformation history. The large strain change in work-hardening rate observed in brass could be modelled by changes in state variables.

The use of a variable $r$ value in Hill's "new" theory provides another method to model the negative transient behavior observed in brass, while providing a way in which to model the positive transient also.

3. COMPARISON OF IF STEEL AND BRASS RESULTS

In 1973, Ghosh and Backofen\textsuperscript{4} discussed the different patterns of failure limits of several types of metals and alloys. They divided the materials into two groups, the brass-type in which the failure limit strain is insensitive to strain state and the ferritic-type in which limit strain is a function of strain state. Since that time, more differences in the plastic behavior of brass and steel have been recorded\textsuperscript{1,4,8,10,11,13,14}. The differences in brass and IF steel are fundamental, perhaps stemming from their different crystal structures; brass is face centered cubic (fcc) and IF steel is body centered cubic (bcc).

The onset of plastic flow signals the start of dislocation motion and interaction\textsuperscript{93}. The slip systems
(sets of planes and directions on which dislocation motion can occur) are a function of the crystal class; fcc crystals have only 12 available slip systems whereas bcc crystals have 48. The particular system which is activated depends on the orientation of the grain with respect to the tensile axis.

Several researchers\textsuperscript{17, 18, 50} have studied the behavior of dislocations during the second stage of two stage strain path experiments. Their work has concentrated on ferritic materials (bcc), in particular aluminum-killed (AK) steel. Schmitt and Baudelet\textsuperscript{50} rationalize the positive stress transient in the following way. On changing the strain state, a number of new slip systems are activated. Dislocations moving on these new systems see obstacles formed during the prestrain, such as pileups or forest dislocations which can not move on the new slip systems. This hardening (often called latent hardening) may account for the positive stress transients. Using transmission electron microscopy, Rao and Laukonis\textsuperscript{17} observed dissolution of dislocation cells in AK steel after reloading a prestrained sample. They propose that this dissolution of cells leads to early plastic instability; a phenomenon observed in this work.
Basinski and Jackson\textsuperscript{52} studied the effect of extraneous deformation on second stage flow curves in copper single crystals (fcc). They found an increase in flow stress with predeformation and attributed this to an increase in the density of forest dislocations. On reloading, obstacles to dislocation motion created by the previously developed dislocation network caused the increase in flow stress (latent hardening). They extended these ideas to polycrystalline materials\textsuperscript{53} and postulated that a decrease in flow stress would be observed if the dislocation network developed in the first strain state was reoriented so that the network intersected the new glide plane in a thin line. This dislocation network would no longer be a strong barrier to glide and the new glide plane would not be hardened.

No formal microscopy study of the type performed by Rao and Laukonis\textsuperscript{17} or Schmitt and Baudelet\textsuperscript{50} for AK steel prestrained in plane strain and subsequently strained in uniaxial tension has been performed for similar two-stage experiments using brass. There is no doubt that the transient behavior in brass also has its origin in the motion of the dislocations, however the question of the actual mechanism of softening and the slightly increased ductility of prestrained brass has yet to be answered.
Although the two materials exhibit opposite stress transient responses, the experimentally observed variation of \( r \) or \( -\frac{\partial \varepsilon_x}{\partial \varepsilon_y} \), the ratio of the strain increments, during the second stage has the same trend in both materials: \( -\frac{\partial \varepsilon_x}{\partial \varepsilon_y} \) decreases with increasing axial strain \( (\varepsilon_y) \). In both cases, an asymptotic value is reached at very low strains (\(< 0.005\)). The data obtained from pictures of the deforming grids on second stage brass samples and the data obtained from fitting the larger strain region of the strain gage data for both brass and IF steel exhibit a decrease in plastic anisotropy ratio, \( r \), with increasing prestrain. Thus, evidence of a deformation texture developed during prestraining is provided for brass and IF steel\(^{57,61}\). The change in work-hardening rate observed in brass second stage samples may be a result of the development of deformation texture\(^{27}\).

The transient in strain ratios, however, cannot be explained by texture, as the change in strain ratios (or \( r \)) occurs at strains too small for deformation texture to be a contributing factor. Jonas et al.\(^{94}\) have studied the effects of crystallography and texture on the flow curves of face centered cubic materials (for example, brass) subjected to various stress states. If the crystallographic nature of slip is taken into account, the
difference in flow curves between, for example, torsion and uniaxial tension can be rationalized up to strains $\approx 30\%$ (based on von Mises effective strain definition). At strains higher than 30%, the development of deformation texture must be considered in order to rationalize the difference in flow curves between various stress states. Thus deformation texture is thought to be important only at high strains $^{59,62,94}$ and cannot be a significant factor in the strain ratio transients discussed above.
The transient behavior of interstitial-free steel (IF) steel and 70/30 brass which results from an abrupt change in strain state has been investigated experimentally and modeled analytically. After a plane strain prestrain, reloading in uniaxial tension results in a negative transient for brass and a positive transient for IF steel.

The monotonic behavior of brass and IF steel was analyzed in both plane strain tension and uniaxial tension. The plane strain data analysis program developed by Wagoner was modified to incorporate Hill's "new" theory, and also to utilize a proposed formulation by Wagoner incorporating non-isotropic hardening. The flow behavior of IF steel in plane strain tension was well characterized by Hill's "new" theory with \( M = 2.9 \). It was found that the consistent calculation (plane strain data analysis using the proper yield theory) did not provide a significant improvement in accuracy over a simple, translation scheme. This verifies Wagoner's estimate that the error in using
Hill's "old" theory in the analysis of the plane strain specimen edge regions in of the order of the experimental error. The formulation proposed by Wagoner\textsuperscript{13} which incorporates non-isotropic hardening (variable M value) into Hill's "new" theory was used to describe the behavior of brass in plane strain tension.

After prestraining in plane strain, second stage uniaxial tension specimens were cut from the prestrained blank. Half of these specimens were used to measure the second stage stress-strain curve and the other half were used to measure the strain ratios during the second stage loading. Brass exhibited a negative transient response after the abrupt change in strain state, whereas IF steel exhibited a positive transient response. In both cases the severity of the transient (absolute magnitude of the change in yield stress on reloading) increased with increasing prestrain.

It had been suggested\textsuperscript{19} that the positive stress transient was a result of strain aging, a thermally activated process involving interstitial solutes. It was demonstrated that IF steel does not strain age at the operating temperatures encountered in this work, however, the positive transient was still observed. Therefore it has
been shown that strain aging is not a contributing factor in this phenomenon.

Resistance strain gages were used to monitor the local axial and transverse strains during the second stage experiments. These strains were corrected in order to include only plastic strains by subtracting the elastic part. The negative of the ratio of strain increments \((-\frac{\varepsilon_x}{\varepsilon_y})\) was calculated from the plastic strain data. The monotonic data exhibited a constant strain ratio, however, the prestrain data exhibited a transient strain ratio response. The strain ratios decreased with increasing axial strain \(\varepsilon_y\) for both IF steel and brass prestrain data. This change in \(-\frac{\varepsilon_x}{\varepsilon_y}\), or equivalently the change in \(r\), occurs at too small of strains for deformation texture to be a contributing factor. Thus it is believed that the motion of dislocations after the change in strain path is the cause of the unusual behavior of the strain ratios and not a rapidly changing texture.

The asymptotic plastic strain ratios for IF steel second stage samples calculated from strain gage data decreased with increasing prestrain. The asymptotic strain ratios for brass showed little change with prestrain. During the second stage experiments, pictures were taken of
the deforming photogrids on the reverse side of the specimens. In the case of IF steel, premature strain localization did not allow any additional information to be obtained from the pictures, but for brass the plastic anisotropy ratio, $r$, was calculated from the strains measured from the pictures. This data indicated that $r$ decreased with increasing prestrain, implying that a deformation texture is developed during the prestrain and affects the large strain behavior of the second stage specimens. The texture developed during deformation is thought to be the cause of the widely different work-hardening rates observed in experiments conducted in different strain states. The change in texture may thus be the cause of the difference in work-hardening rates observed in brass second stage and monotonic experiments.

The transient behavior of these two materials has been modeled analytically by incorporating a variable $r$ value into the equations defining Hill's "new" theory. The $r$ value required to translate the plane strain monotonic curve onto the second stage stress-strain curve was calculated for brass and IF steel. This $r$ value was converted to $(-d\varepsilon_x/d\varepsilon_y)$ and compared to experimental data. The calculated and experimental curves for brass second stage samples have the same trend; the negative of the strain
ratios decreases until an asymptotic value is reached. The asymptotic r values however are not the same. The calculated and experimental strain ratios for IF steel reach the same asymptotic value, however, the experimental curve shows the strain ratios decreasing to that value and the computed curve shows the strain ratios increasing to that value. In order for the yield surface to pass through the plane strain and uniaxial tension stress states, the surface may develop an unusual shape between these two points. It is also possible that normality is not obeyed during the second stage experiments. A simple analytical model has been proposed which can handle the stress transient phenomenon, however, the variation of the fitting parameter, r, does not have a direct correlation with experimentally obtained r values in the case of IF steel.
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