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Smith, Bruce Henry, Ph.D.
The Ohio State University, 1987

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THE THEORETICAL AND EMPIRICAL ANALYSIS OF THE POPULATION DENSITY GRADIENTS OF URBAN AREAS CHARACTERIZED BY COAST LINES PROVIDING AN AMENITY TO CITY RESIDENTS

DISSertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

By

Bruce Henry Smith, B.A., M.A.

The Ohio State University

1987

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To my dear wife Kamala Masihlall Smith
and my Mother Marie Vallet Smith
ACKNOWLEDGEMENTS

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Finally, my wife, Kamala, and my mother deserve special mention as this dissertation is half theirs.
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CHAPTER I

INTRODUCTION

ORIGINS OF THE STUDY

This dissertation focuses on an often neglected topographical feature of urban areas—ocean and lake coast lines. Most urban models are set in a flat featureless plain. The existence of a major water body with a shore line is assumed to be nothing more than a minor nuisance in analyzing the spatial structure of a city. But, a coast line is often thought of as providing some utility-bearing characteristics to residents residing near it. The natural beauty of many water bodies and coast lines provides an esthetically pleasing view to inhabitants. Due to the existence of sea and lake breezes, the water body often ameliorates uncomfortable variations in temperature. "In cities such as Boston, Chicago, and others, people often refer to the sea breeze bringing welcome relief from excessive heat as 'nature's air conditioner.'" [Neuberger and Cahir, 1969, p. 68]. Smaller water bodies such as lakes have similar influences on their coastal cities though to a lesser extent. Likewise, sunlight penetrates deeply into water causing the sun’s energy to be distributed widely and water surface temperature changes to be slow. [Landsberg, 1981, presents an advanced treatment of the climate and cities.] More comfortable temperatures provide residents dwelling along the coast with greater utility than
those residing inland, *ceteris paribus*. Swimming in pristine water requires little or no capital to attain utility from the coast. Other activities associated with the coast require other "goods" to convert the coast or water into utility-bearing "commodities" (Becker, 1965). For example, fishing, boating or sailing require goods to obtain utility from the water (fishing poles and water craft).

In this sense, it is best to think of many coasts as a natural attraction of people rather than a topographical barrier which precludes residential expansion about a CBD. While this may be thought of as a rule-of-thumb, the attractiveness of a coast line will vary widely. For example, an uncrowded beach along the coast of the Sudan may be unattractive owing to its lack of proximity to places to work and shop. Obstructions (seaweed, sand bars, or rocky coasts) to water activity may deter residential location. The existence of commercial congestion or industrial pollution at a site may outweigh the benefits of residential location. In a word, the amenities provided by a coast line will vary geographically.

Over the past twenty years, cities and states have shown an increased interest in the preservation of their coast lines for their natural beauty. Several states have passed laws restricting industrial and commercial development of the shoreline (Hyde, 1975), e.g., New Jersey's Coastal Area Facility Review Act and Delaware's Coastal Zone Act. The changing economic environment has also prompted cities to change their outlook toward the urban waterfront.

Over the past twenty years, waterfronts in older communities throughout the United States have experienced significant shifts
in land use and development potential. Declines in shipping, warehousing, and distribution functions, and the relocation of industrial uses which historically dominated the urban waterfront, have left significant areas of vacant and underutilized land. Although these changes have caused economic dislocations, they have also created redevelopment opportunities, offering the potential to introduce new people-oriented activities—such as recreation, housing, offices, specialty shopping, and a range of entertainment and visitor attractions—on the waterfront. [Waterfront Comprehensive Plan, 1986]

If it is true that people will wish to locate near a coast line providing at least one of the amenities described above, ceteris paribus, then the configuration of a city along an amenity coast should be different from the circular shape predicted by the standard urban model. The Rand-McNally definition of urban areas most closely agrees with the spatial configuration of coastal cities suggested above (Rand-McNally Atlas, 1986). Using the census definition of SMSA's, one may erroneously conclude that some coastal cities which are half oval in shape are neither semicircular nor half oval. The census convention of defining SMSA's by counties often results in the inclusion of large areas of land which are not urbanized. Ranally Metro Areas (RMAs) are based on a restricted definition of metropolitan area:

Under the rules for defining RMAs, each area includes (1) a central city or cities; (2) any adjacent continuously built-up areas; and (3) other communities close by but not connected to the city by continuous built-up territory if a substantial portion of their work force commutes to the central city and its adjacent built-up areas. A place generally meets the commuting requirement for inclusion in an RMA if at least eight percent of its population or twenty percent of its labor force commutes to the central city or its adjacent built-up areas. [Rand McNally, 1986, p.99]
The Rand McNally definition also utilizes a population-density criterion to exclude suburbs with only "scattered" development. Using the Rand-McNally definition of cities, one observes many coastal cities that proliferate along the coast.

The existence of cities, or at least larger cities, stems from economies of scale at a particular location or a site-specific factor such as harbor facilities for the transportation of goods. The existence of pleasant natural living conditions is not usually sufficient for residential location. While the coast may provide residents with greater utility than in its absence, spatial competition between firms and households is still likely to exist. Many firms are naturally attracted to the coast if they provide goods necessary for residents to enjoy the amenity coast, e.g., marinas, surf board vendors, concessionaires, etc. Other firms may wish to locate at or near the coast if by doing so they reduce the costs of manufacture (e.g., dumping of industrial waste) or transportation of freight, or increase sales by locating strategically. A description of this competition is discussed in Chapter III.

Many cities (and private firms) have gone to some length to enhance the attractiveness of coastal areas, especially sites located near major population centers. Zoning and capital investment are two powerful tools jurisdictions use to attract residents of a city to the coast. Residential consumption of coastal attractions implies various amounts of travel to the coast. Because such journeys are costly, residents locating near the coast line will be willing to pay a premium to locate there. The economizing on recreational travel is the focus of
the model developed in Chapter III. The use of zoning at the coast and capital investment to improve coastal appearance increases the chances that a resident will make more visits to the coast per period of time. As will be shown in Chapter V, this increases the property value of land near the coast and hence property tax revenue for coastal jurisdictions. Thus, the urban model developed in Chapter III provides a framework by which local jurisdictions can maximize their property tax revenues by developing their coast lines.

PREVIEW

To recap, Chapter II presents a brief review of the standard (monocentric) urban model which has dominated the urban economics field since Alonso published his *Location and Land Use* in 1964. For contrast with the amenity-coast line model presented in Chapter III, the essential features of the model are developed. A discussion of the rent and population density gradients is presented and, where appropriate, criticized in the context of the model developed in later chapters. The literature in Chapter II serves as a foundation upon which is built a model of a monocentric city with an amenity coast line in Chapter III.

Chapter III limits discussion of the attractiveness of an amenity coast line to only one of the amenities discussed above—recreational activities associated with the water body. Thus, implicitly, the coast line is not esthetically pleasing from the place of residence. The validity of this assumption is discussed in Chapter III. Likewise, residents of the city are assumed exposed to identical climatic
conditions regardless of location. In effect, the urban model presented in Chapter III understates the complete impact of an amenity coast on the location preferences of households. Notwithstanding, it is believed that the model captures much, if not most, of the relevant economic considerations of an amenity coast line.

The urban model with an amenity coast line yields rent and population density functions dependent on a household's distance from the amenity coast line, the CBD and the travel cost per unit distance to each. Chapter IV suggests a test of the urban model developed herein which uses Bureau of Census data on population density by census tract. As explained below, estimation of population density gradients provides a useful summary test of an urban model requiring readily available data not subject to the "vexing" measurement problems of land-value studies.

Chapter V introduces two interesting features of the amenity coast line city. First, the optimal "setback" at the coast, first introduced by Brown and Pollakowski (1978), is examined using the basic urban model of Chapter III. Second, the nature of congestion in the city is examined when commuters have two relevant dimensions to consider: distance to the amenity as well as the CBD.
Since Alonso's (1964) pioneering work the so-called "standard urban model" has come to dominate the field of urban and regional economics. The usual result of these models is a circular city (less pie-slice radians to account for topographical barriers such as a lake or sea shore). Several simplifying assumptions are present in most of these models. Residents of the city are assumed to work in a central place - the central business district (CBD) - which is nonspatial or small relative to the total city. All land outside the CBD is used for residential living or transportation (roads or rail) to commute to the CBD. Residents of the city incur costs (money and time) getting to their workplace (CBD). Thus, residents are only willing to live farther from the CBD if they are compensated for the extra commuting costs incurred, e.g., rents decline as they move to the edge of the city. Several comprehensive reviews of monocentric urban models have been published. Among these are Mills and Mackinnon (1973), Richardson (1976), and Wheaton (1976).

Alonso's urban model begins with the specification of a consumer utility function common to all residents. The arguments include the
consumption of land or space, \( L_2 \), a composite good, \( x \), i.e., all other goods, and perhaps distance from the CBD, \( u \), which accounts for the "nuisance" of commuting. All residents commuting to the CBD are assumed to earn the same nominal income per period, \( W \). Thus, the consumers' problem is to maximize utility,

\[
V(x, L_2, u),
\]

subject to,

\[
W = R(u) L_2 + x + p_3(u).
\]

Lagrangean maximization of (1) subject to (2) yields the first-order condition for spatial equilibrium,

\[
L_2 \frac{dR}{du} = \frac{\delta V/\delta u}{\delta V/\delta x} - \frac{dp_3}{du}.
\]

Where, \( p_3(u) \) expresses the cost of commuting to the CBD depending positively on distance to the CBD. The solution to (3) yields the rent gradient for the city except for the constant of integration. Thus, irrespective of residential location, identical consumers achieve the same level of welfare or utility, \( V^0 \).

Alonso simplified his model by ignoring housing. Muth\( (1969, 1975) \) and Mills\( (1967, 1972) \) developed general equilibrium models of the urban economy which incorporated the production of housing services depending on the use of land and capital inputs. In the Mills-Muth model, consumer utility depends on housing services, \( x_2 \), and the consumption of other goods, \( x \). Housing services are assumed to be produced with capital inputs, \( K_2 \), and land, \( L_2 \). Muth specified a CES production function for housing services while Mills hypothesized the Cobb-Douglas. The unitary elasticity of substitution between land and nonland inputs
found in the Cobb-Douglas production function used by Mills has been criticized as inconsistent with the empirical findings of several researchers (see Clapp, 1979; Koenker, 1972; Muth, 1969; Rydell, 1976; Sirmans, Kau, and Lee, 1979). The price of housing services, \( p_2 \), and in turn the price of land, \( R \), is endogenously determined and dependent on distance \( u \). The price or rent of capital inputs, \( r \), is exogenous.

In both models, consumers maximize utility, \( V \), subject to the budget constraint with money income, \( W \), fixed,

\[
V(x, x_2), \tag{4}
\]

subject to,

\[
W = x + p_2(u) x_2 + p_3(u), \tag{5}
\]

while producers of housing services maximize profits. If the production function for housing services is represented by \( x_{2S} \), profit maximization implies

\[
p_2(u) \frac{dx_{2S}}{dK_2} = r, \tag{6}
\]

\[
p_2(u) \frac{dx_{2S}}{dL_2} = R(u). \tag{7}
\]

That is, inputs land and capital earn the value of their marginal products.

The price of housing in the Mills-Muth model is determined by the well-known locational equilibrium condition,

\[
x_2 \frac{dp_2}{du} + \frac{dp_3}{du} = 0. \tag{8}
\]

Which simply says that if a resident moves a distance away from the CBD and in turn incurs increased commuting costs, his expenditures on housing services must decrease by the same amount. The solution to (8) provides a solution to the price gradient for housing services. Two boundary conditions are necessary to complete the model for a closed
city. Equation (9) ensures that everyone in a city of population $N$ lives somewhere. The constant resulting from the integration of (8) is obtained from (10). Equation (10) is the spatial equilibrium condition which requires rent at the edge of the city, $u_A$, be equal to an exogenously given agricultural (opportunity) price of land, $R_A$.

$$ N = \int_{u}^{u_A} N(u) \, du. \quad (9) $$

$$ R(u_A) = R_A. \quad (10) $$

The rent gradient for the city can be found by making substitutions for $x_2$ and $p_2$ in (8). The negative exponential rent gradient attains when the price elasticity of demand for housing services is minus one and commuting cost per unit distance is constant,

$$ R(u) = R_A \exp\left\{ \frac{-P_3}{\tau} (u - u_A) \right\}. \quad (11) $$

Where, $\tau$ is a constant. Equation (10) serves as a boundary condition in the solution of $R(u)$. In the Mills-Muth model, the existence of an exponential rent function also implies that population density will decline exponentially from the CBD in any direction.

EXTENSIONS OF THE STANDARD URBAN MODEL

The assumption of a monocentric city located on a flat featureless plain has been criticized only infrequently. Pollard (1977) notes that, "Although this assumption leads to simple yet powerful analytical models, it precludes the possibility of explaining the substantial impact of a city's topography on land use patterns." Brown and
Pollakowski (1977) found that the existence of a lake has a significant impact on land values in the Seattle area. Pollard found that the esthetic amenity offered by Lake Michigan has a significant impact on building heights in Chicago.

Beckman (1969) and Montesano (1972) consider a model in which preferences are identical among residents but incomes are different to determine where the more affluent will locate. Haurin (1980) considered the impact of spatial variations of income on the demand for housing and population density. He found that increased income increases the quantity of housing demanded but decreases population density.

In Chapter V, the spatial urban model developed in Chapter III is extended to evaluate how traffic congestion formulated by Vickrey may affect the basic conclusions. Congestion externalities from traffic were considered by several researchers. Vickrey (1965) let congestion be represented by an exponential function of the ratio of the transportation demand to the capacity of the transportation system. Mills (1972), Muth (1975), Oron, Pines, and Sheshinski (1974) all consider models incorporating traffic congestion and its effects on city size, population density, and other urban variables of interest to city planners. Muth assumes that the proportion of land devoted to transportation facilities at a given distance from the CBD increases with distance from the CBD. Mills assumes that the amount of land used for transportation is constant at any distance from the CBD in the city. Fisch (1975), Hochman and Pines (1971), Legey, Ripper, and Varaiya (1973), and Solow (1972,1973) showed that a socially efficient solution entails at least some level of traffic congestion. In the context of
optimal government investment in transportation capacity, Mumy (1980) suggests that, "underpriced congestion externalities lead to neither an overly decentralized spatial structure nor an excessively large population, if the correct investment decisions are made." In all of these studies, the restrictive assumption of one-dimensional travel was postulated. In Chapter V, this assumption is relaxed.

Another externality which has received considerable attention in recent years is the problem of air pollution. In their landmark paper, Ridker and Henning (1967) examined the relationship between air pollution and property values. Ridker's and Henning's work was continued and sometimes criticized by several researchers (see Anderson and Crocker, 1971; Cobb, 1977; Freeman, 1971, 1974a, 1974b; Harrison and Rubinfeld, 1978; Maler, 1977; Polinsky and Shavell, 1975; Small, 1975; and Wieand, 1973). None of these studies have addressed the possibility that some pollution is location-specific and the consequence of the specificity on rent and population density gradients. Chapter III introduces an approach which at least begins to deal with the issue of location-specific pollution.

**POPULATION DENSITY FUNCTIONS**

The familiar negative exponential population density function follows from the model when the rent function is negative exponential, i.e., when the price elasticity of demand for housing services is equal to minus one. Thus, the population density function has the same basic shape as the rent gradient. That is, "Density rises as the CBD is
approached only because rising land values induce developers to substitute away from land in the production of housing and because households substitute away from housing in their consumption bundles. Thus, the steepness of the density function depends on the steepness of the rent function and on the ability of producers and consumers to substitute." [Mills and Hamilton, 1984, p.111] The observed pattern of population density is naturally the combined result of topographical, historical, and cultural as well as economic factors. However, by focusing on economic influences, economists have been able to explain a phenomenon which appears inherently noneconomic in very real economic terms.

The concept of the negative exponential population density function was first used by Bleicher (1892) to describe the distribution of population in Frankfurt, A.M., Germany. Clark (1951) rediscovered this form to estimate the decay of population density from the city center for a number of European and American cities. The population density gradient has since been studied extensively by researchers for many cities. Several researchers have shown the log-linear form of population density follows from the basic assumption of consumer utility maximization (Casetti, 1971; Mills, 1967, 1970, 1972; Papageorgiou, 1971a; Muth, 1969),

\[ D = A e^{-b u} \]  

(12)

Which follows from (11) because the number of people living at \( u \) is expressible as the ratio of the total demand for housing services to individual demand for housing services. Equation (12) is linear if the exponential term is approximated by a Taylor series expansion. For
purposes of estimation, most researchers have used the semi-log form of (12).

The population density gradient has proven to be a very useful tool to economists, geographers, and city planners in not only describing and predicting population patterns but in testing urban spatial models which arise from the assumption of consumer utility maximization. Muth, whose name has become synonymous with pioneering efforts in the modeling and estimation of population density gradients summarizes,

First of all, the spatial pattern of population densities is of great interest for its own sake, and it has important implications for many problems such as the intensity of demand for municipal services in various parts of the city and the design of transport systems. Second,... population density is simply the output of housing per unit of land divided by the per capita consumption of housing. Thus, anything affecting either the output or the consumption of housing in different parts of the city will be reflected in population density. Finally, population data are not subject to the many vexing measurement problems that are inherent in using census housing data. [Muth, 1969, p.139-140]

In order to test the urban model presented in Chapter III, several specifications of the population density functions are presented in Chapter IV. Below is presented a review of the literature dealing with the basis of the appropriate form of the population density function.

A more general functional form of population density described by Kau and Lee (1976) involves the Box and Cox (1964) transformation of population density,

\[ D^\lambda - 1/ \lambda = A + b u \]  

(13)
where, \( \lambda \) is the functional form parameter. Equation (13) assumes the linear form when \( \lambda = 1 \) and the semi-log form when \( \lambda \to 0 \). Equation (13) is approximated by the quadratic in \( u \) form when Taylor's theorem is applied

\[
\ln D = A + b \ u + c \ u^2.
\]

(14)

Where, \( b \) measures the change in density at the center of the city and \( c \) measures the change in density away from the center. Equation (14) has been used by Muth (1969) and Latham and Yeates (1970). This form has been suggested by Latham and Yeates as appropriate when modeling population density for a city in which a spatial CBD exists. Equation (14) also follows when the transportation cost function is quadratic in \( u \), e.g., when describing congestion.

Kau and Lee also considered the Box-Cox transformation on both the dependent and independent variables,

\[
D^{\lambda'} - 1 / \lambda' = A' + b' \left( u^{\lambda'} - 1 \right) / \lambda'.
\]

(15)

Equation (15) assumes the linear form when \( \lambda' = 1 \) and the double logarithmic form when \( \lambda' \to 0 \). Based on a sample of 50 U.S. cities in 1970, they concluded, "the results indicate that the logarithmic specifications dominate the linear specifications. But it is difficult to differentiate the semi-log from the double-log functional forms, since their log likelihood values are similar. Also, there exist some cities in which the optimal functional form is either linear or linear quadratic."

Taylor (1971) considered the general form of exponential density,

\[
D = A \ e^{-b \ f(u)},
\]

(16)
to analyze the specific properties of different density forms. Depending on the specification of \( f(u) \), a wide range of sometimes infrequently used density functions are obtainable. However, Taylor offered no explanation as to why some forms should arise from theory nor if they were likely to explain population density well.

Papageorgiou (1971a) shows that every existing population density gradient can be derived from the maximization of a consumer utility function with arguments congestion at a location and the degree of effort "required to overcome the friction of distance between this location and certain key locations within the region." Differences in the form of the density gradient stem from differences in the assumptions made about transportation costs.

The problem of the assumption of monocentricity has been addressed by several students in the fields of economics and geography. Schroeder and Sjoquist (1972) note, "The use of the negative exponential function implies that density isopleths are circles with the center at the CBD. Such functions, therefore, do not allow for differences in the rate of change in density as one moves in different directions from the CBD or for the possible existence of secondary density centers." In consequence, they use trend surface analysis in which \((z,y)\) coordinates (see Figure 1) are used directly as independent variables on an arbitrarily defined coordinate system to reflect directed distance rather than measured distance to the CBD in a population density regression. Schroeder and Sjoquist introduce an interesting consideration of population densities but in an ad hoc manner. They fail to explain in a rigorous fashion why and how, in terms of
topographical features or urban subcenters, population densities do not fall off uniformly from the CBD. Papageorgiou (1971b) considers a model in which a hierarchy of population centers exist. He shows the conventional negative exponential model represents a general form of population density; so the assumption of a monocentric city will still yield "successful" results because of the averaging of observation points; i.e., because census tracts are not infinitesimal and, therefore, do not reveal some incongruities in population densities.

Estimation of the population density gradient has been the focus of investigation for more than three decades. Muth (1969) estimated \( b \) in (12) for many cities and found that \( b \) usually falls into a range from .2 to .5. He explained differences in density gradients as emanating from differences in i) the transportation costs of getting to the CBD, ii) the spatial distribution of laborers commuting to the CBD and the places they shop, iii) preferences for housing location.

Mills (1970) suggested the two-point estimate of population density. If (12) accurately describes population density, then the number of people locating \( k \) miles from the center of a circular city is expressible as,

\[
N(k) = 2 \pi A / b^2 \left[ 1 - (1 + b k) e^{-b k} \right]. \tag{17}
\]

Letting \( k \) go to infinity yields the total population of the metropolitan area,

\[
N = 2 \pi A / b^2. \tag{18}
\]

The parameters \( A \) and \( b \) can be determined by solving simultaneously (17) and (18) by numerical methods. Mills used information from Census data on the population of the metropolitan area and the central city as well
as an estimate of the radius of the central city to determine values for A and b for several U.S. cities spanning three decades.

In the same paper, Mills hypothesized that densities change over time, but slowly. In consequence, Mills constructed a first order, autoregressive distributed lag model of the process of adaptation of urban density gradients for four time periods. He used his two-point estimates of A and b in a cross-section, time-series regression with independent variables population, median family income of the SMSA's in various time periods, a time trend to capture secular changes in such things as transportation costs, and the lagged dependent variable.

Straszheim (1974) has criticized Mills distributed lag model on the basis that the error term is likely serially correlated for a particular city and that estimates of the coefficients are inefficient and inconsistent. He argues a sample of only four observations "make it virtually impossible to obtain reliable estimates of the nature of the autocorrelation process of the error terms." White (1977) reports results of a Monte Carlo simulation designed to determine the properties of Mills' two-point technique. He suggests that estimates of the density gradient are biased slightly downward, "while the estimate of central density is biased upward or downward depending on the error specification."

In an effort to handle the fact that housing is a durable good, Harrison and Kain (1970) examine a model which "views urban growth as a cumulative process and explains current residential densities as a weighted average of the density of new development over several time periods." They use the percentage of dwelling units built at various
time periods that are single-family detached units as a measure of net residential density for 83 cities. They find that a time trend supposed to measure secular changes in income, tastes, and commuting costs has the significant effect of reducing population density. They also find an increase in city size increases density at a given point in space.

Newling (1969) suggests the quadratic negative exponential density is appropriate since "Within the central area nonresidential use preempts most of the available space, so that the profile displays a central density crater with a run, or crest, of high density bordering the central business district." Newling believes cities can be characterized by stages of maturity depending on the values of b and c in (14). Latham and Yeates (1970) examine the use of the quadratic negative exponential population density function in describing metropolitan Toronto for 1951 to 1963. If b is positive and c is negative, the population density curve is at first increasing and then decreasing as one moves away from the center of the city. Latham and Yeates find that central density for Toronto was declining over time and the concavity of the density function was increasing.

Kemper and Schmenner (1976), Mills (1970, 1972) and Neidercorn (1971) have considered negative exponential employment density functions. Kemper and Schmenner question the usefulness of Mills' two-point technique for employment based on the sensitivity of their parameter estimates. Greene and Barnbrock (1978) criticize Kemper and Schmenner because they did not attribute differences in parameter estimates to a priori imputed differences in error structure of the additive error model in (12) [implying nonlinear least squares] versus
the mutiplicative error structure of (12) [implying log-linear least squares].

Several researchers have explored the econometric implications of the density function. Since census tracts are usually not of equal size, the problem of estimation bias arises. Robinson (1956) notes, "The influence of the chance arrangement of boundaries may be removed by weighting the data by the area of the units to which they refer." Weighting of data is essential because the correlation and regression analysis usually employed is based on evaluation of observations of equal importance. Census tract data, for example, implies many more observations near densely populated areas. Robinson (1961) suggests transforming census tracts of unequal size into hexagonal districts of equal area. Frankena (1978) suggests weighting census tracts in proportion to their area. Griffith (1981) examines several sources of bias: i) model specification error, ii) estimation procedure, iii) unequal census tract area, iv) multiple centers, v) existence of externalities. He offers a general estimation procedure to deal with some of these problems. McDonald and Bowman (1976) suggest that in addition to explanatory of the regression, the density form should be evaluated as to how well it predicts total metropolitan population. They conclude, "that the explanatory power of the negative exponential function can be improved upon in some cases by adding a quadratic distance term, but that population can be predicted more accurately if the quadratic term is omitted."
THE NATURE OF THE PROBLEM

The amenity model usually specified in the literature begins with
the specification of a consumer utility function involving the level of
the amenity, a(u), at a distance u from the CBD as an argument,
\[ V = V( x_2(u), x, a(u) ). \] (19)
That is, utility depends on the consumption of housing services, \( x_2 \), and
a at u as well as a composite good, \( x \) (sometimes \( x \) is replaced by
imported and exported goods). The amenity or disamenity is usually
assumed to dissipate symmetrically about the origin, i.e., the CBD.
This simplifies analysis of the model by preserving the circular shape
of the city.

While the symmetrical approach is useful in analyzing the nature
of many amenities and disamenities surrounding urban areas, it fails to
capture instances in which the amenity does not occur symmetrically.
For example, due to winds aloft, pollution may be carried away from the
CBD such that one part of the city bears a disproportionate share of the
effluent emission. Another example occurs in an urban area which is not
located on a flat featureless plain. A city located on the coast line
of an ocean or lake for which the coast has utility-bearing attributes
to residents will not allow for the typical amenity model. Instead,
residents living close to the coast line will receive the benefits of relatively costless access and perhaps the esthetic view of the amenity if the city is not flat.

To account for the problem considered in the previous paragraph, assume that the city is flat and featureless except that a straight coast line exists as illustrated in Figure 1. In this instance, residents will wish to locate near the coast line represented by the z axis. If all points on the coast provide the resident with equal satisfaction in terms of consuming the amenity, the amenity in the utility function depends on the perpendicular distance to the coast and not on radial distance to the CBD. Thus, (19) becomes,

\[ V = V(x_2(y,u), x, a(y)) \]. (20)

If the resident is able to consume the amenity simply by commuting to the coast in much the same way the worker commutes to the CBD, the budget constraint of the consumer can be written as,

\[ M = p_2(y,u) x_2 + x + p_3 u + s y \]. (21)

Where, \( p_2 \) is the price of housing services for given distances \( y \) and \( u \). The commuting cost per unit distance and time to the CBD is assumed constant and equal to \( p_3 \). The cost of commuting to the coast, \( s \), is likewise assumed constant per mile.

Maximization of (20) subject to (21) yields the indirect utility function for the typical resident and in turn the demand equation for the amenity (Appendix A considers utility maximization subject to both budget and time constraints),

\[ a(y) = f(s, p_3, M, p_2) \]. (22)

For a given distance \( y \) from the amenity, the higher the commuting price,
FIGURE 1
BASIC CONFIGURATION OF THE CITY
s, the fewer units, i.e., trips, of the amenity purchased. Likewise, the farther the residence from the coast the fewer the number of trips that are likely to be made per unit period. For example, if the resident locates far from the amenity, say at \( y_1 \) in Figure 2, he will consume fewer trips per period than a resident locating at distance \( y_0 \).

Assume for simplicity that the quasi income-compensated demand curve in figure 2 is perfectly inelastic. That is, no matter where the resident locates in the city he will always consume the same number of trips per unit time. While this assumption is intuitively not palatable, it greatly simplifies the analysis that follows. In actuality, consumers with preferences toward recreation activities associated with a coast line are most likely to locate near it. When consumers have identical preferences, those locating farther from the coast must be compensated for the increased distance of commuting. The simplification here is similar to the assumption made in the standard urban model in which workers do not increase their consumption of leisure though they may live farther from the CBD. (Henderson (1976) is an exception. He includes a labor/leisure tradeoff in the utility function.). There, as the laborer endures an increased commuting distance, his net daily wage, \( W - p_3 u \), falls with radial distance to his workplace. This assumption is usually justified on the basis that number of days worked per week or year has been traditionally determined institutionally.

Once at the amenity, what prevents a resident from being in a continual state of leisure? An amenity of the type postulated here may be thought of as being consumed with the use of time. It is reasonable
FIGURE 2

DEMAND FOR AMENITY WITH RESPECT TO LOCATION
to suppose that marginal utility diminishes with increased consumption. If the resident values other goods attainable only from earned income, it is safe to suppose he will limit his time at the amenity in a predictable way. That is, the "beach bum" phenomenon is precluded.

Above, it was assumed that residences are located on a flat featureless plain and that at any point, the housing being occupied offered no esthetic view of the amenity coast. Pollard (1975) considered the influence of Lake Michigan on the heights of buildings in Chicago. In that instance, residents were willing to pay a higher rent for a view of the Lake. One problem with this type of model is that if a building is constructed high enough to obscure the view of residents inland, those locations (inland) have less value. Thus, the problem of property rights and appropriable rents is introduced for which a socially efficient outcome may not be forthcoming. Klein, et al. (1978) note that "as assets become more specific and more appropriable quasi rents are created (and therefore the possible gains from opportunistic behavior increases), the costs of contracting will generally increase more than the costs of vertical integration [i.e., internal organization]." If property owner A collects "rents" for a lakeside view from residents in his twenty story apartment building located on property 300 yards from the lake, B who owns the property in front of A's can extract the full value of these esthetic rents by announcing an intent to build a twenty-one story building.

Another instance in which the observed pattern of residential location does not follow that implied by the standard urban model is the existence of a man-made radial sector. This case was first considered
by Hoyt (1939, 1966) who visualized a high-income radial sector. Location near the sector was in consequence valued by city residents. If the axis $z$ in Figure 1 represented a nonspatial avenue at which the residents of the city shopped, the consumer/laborer has an incentive to locate as closely to $z$ as possible in addition to locating closely to the CBD. In this case, the avenue $z$ is a road occupied by the sellers of goods and services consumed by city residents. This argument is useful if the number of workers employed along $z$ is very small relative to the number of workers employed at the CBD. If the consumer can do all of his shopping at any given point along $z$, his commuting route is perpendicular to $z$ as measured by $y$. Thus, once consumers reach the radial sector, they do not have to move east or west to shop or that such movement is costless for a distance before reaching the CBD. Most major cities in the United States have one or more avenues with a rich diversity of often overlapping shopping facilities. Thus, this interpretation of the previous model is not restricted to cities having coast lines used principally for recreational purposes.

The interpretation of the variable $s$ is the cost per unit distance and time of reaching the shopping lane. As in the case where the axis $z$ represents a coast line, residents living farther from the radial shopping route have an incentive to economize on the number of trips they take per period. Residents of the city may not economize on the number of trips they take based on the distance of residence to the radial sector if commuting costs to the shopping facilities are small relative to the transaction and time costs of shopping.
In general, we would not expect the resident to take the same number of trips to the CBD as he does to the amenity for an given period. For example, he may take five trips to the CBD and two trips to the amenity in a week. Thus, in terms of expenditures, for a given period commutes to the amenity are weighted so as to reflect their frequency. This can be expressed by weighting $s$ alone (the appropriate weighting factor is discussed below).

In general, the resident who lives at point A in Figure 3 and commutes to the CBD could economize on the actual combined distance commuting by taking a triangular route, say AOB, with commuting costs equal to $p_3 u + p_3 \sqrt{(u^2 - y^2)} + s y$. This route is always preferred to a route involving separate trips to work and shop which involves total commuting costs equal to $2 p_3 u + 2 s y$. Several factors may prevent this economizing behavior however. Some of these are:

i) the person in the household working at the CBD may not be the person responsible for shopping

ii) physical obstructions may make the triangular route impractical

iii) travel costs per unit distance and time along $z$ may be prohibitive

iv) congestion along $z$ during peak periods (rush hour) may make shopping during off-peak times more economical.

In the discussion above, the assumption of a nonspatial CBD has been retained. In the case of a coast line, it is possible that facilities for shipping CBD products are available at more than one point. For example, in the Los Angeles CSMA, shipping facilities are
FIGURE 3

COMMUTING DISTANCES AT A POINT
available at Wilmington and Long Beach. Radial routes emanating from an existing CBD have been used by many firms as alternatives to locating centrally. White (1976) has considered the case where firms can export their output by using suburban terminals, i.e., major highways bypassing many major American cities. The result is the development of employment subcenters and inability to determine changes in urban land values. In the discussion below, the assumption of a single CBD is retained so as to better analyze the problems considered herein. In the case of a coast line in which residents require auxiliary goods such as surf boards, boats and the like to enjoy the amenity, some firms, in the absence of zoning restrictions, will locate along the coast to sell their goods. Thus, to some degree or another, employment subcenters to cater to recreational desires are likely to arise. The effect of this phenomenon is ignored for the moment since it is believed that it adds little insight to the basic model. The question arises again in Chapter IV when the appropriate econometric expression for the population density function is addressed.

THE URBAN MODEL WITH AN AMENITY COAST LINE

The urban model considered in this chapter most closely resembles the model developed by Mills (1972). The CBD is assumed to be nonspatial. When considering a city characterized by a coast line, the CBD is best thought of as port through which the CBD's output is shipped. In that case, only one relatively small part of the coast line is actually useable as a port. Because the city is located on a lake or
ocean, land is available for urban uses everywhere inland. If we were to use the shopping avenue interpretation discussed above, then no such restriction is implied. In that case, the CBD may be the result of economies of scale in the "key industry or set of industries." [Mills, 1972, p. 98].

The land located outside of the CBD and in the city is used for transportation facilities used by commuters to the CBD or for the construction of housing services for city residents. For the moment, it is assumed all land is used for residential housing.

Following Mills, the production function for housing services is assumed to be Cobb-Douglas. This is not a crucial assumption but it does help to simplify analysis (see Altman, 1982; Altman and DeSalvo, 1985).

\[
X_{2S}(y,u) = A_2 L_2^\alpha_2 K_2^{1-\alpha_2}.
\]

Where, \(X_{2S}(y,u)\) is the supply of housing services at point \((y,u)\). \(L_2\) is land used to produce housing services. \(K_2\) is capital used in producing housing services. \(A_2\) is a scale parameter in the production function. And \(\alpha_2\) \((1-\alpha_2)\) elasticity of housing service production with respect to land (capital) or the share of output accruing to the land (capital) input. Constant returns to scale are implied from the fact that \(\alpha_2 + (1-\alpha_2)\) is unity.

From (23), equilibrium in the competitive input markets for housing services implies that \(L_2\) and \(K_2\) receive the value of their marginal products,

\[
R(y,u) = \alpha_2 p_2(y,u) \frac{X_{2S}(y,u)}{L_2(y,u)},
\]
Where, \( R(y,u) \) is the rental rate of land at \((y,u)\). The rental price of capital, \( r \), is assumed to be given exogenously. The price of housing services is given by \( p_2(y,u) \).

Equations (24) and (25) when substituted in (23) provide a relationship between the price of housing services and the rental rate of land,

\[
p_2(y,u) = C \frac{\alpha_2}{R(y,u)}.
\]  

Where,

\[
C = \frac{1 - \alpha_2}{r} \frac{\alpha_2}{\alpha_2 (1 - \alpha_2)}.
\]

The demand function for housing services of a typical urban resident is also assumed to be Cobb-Douglas,

\[
x_{2D}(y,u) = B_2 p_2(y,u) W^{\theta_1}.
\]

Where, \( W \) is the income per laborer in the household; \( \theta_1 \) is the income elasticity of demand for the typical worker and \( \theta_2 \) is the price elasticity of demand for housing services for the individual. Assume that households are of equal size and have equal incomes. (In Appendix C, we consider the location decisions of households when two income groups exist.) Following Mills, the cost of commuting is excluded from the housing-services demand function to avoid computational inconvenience.
For the fixed number of people, N, in the closed city, total housing service demand is found by multiplying individual housing demand by the total number of workers,

$$X_{2D}(y,u) = x_{2D}(y,u) N(y,u).$$  \hspace{1cm} (29)

Where, \(N(y,u)\) is the number of workers living at point \((y,u)\). Thus, equilibrium in the housing services market implies the equality of (23) and (29),

$$X_{2D}(y,u) = X_{2S}(y,u).$$  \hspace{1cm} (30)

Locational equilibrium requires that a household locating at one point is not made any better or worse off by moving farther or closer to the CBD or by moving farther or closer to the amenity. In the first instance this requires that the increased (decreased) commuting cost of a move away (toward) from the CBD is just offset by a corresponding fall (rise) in housing services expenditures,

$$x_{2D}(y,u) \frac{\partial p_2(y,u)}{\partial u} + p_3 = 0. \hspace{1cm} (31)$$

The second condition requires that at a given distance from the CBD, the increased (decreased) commuting cost of a move away from (toward) the amenity is just offset by a corresponding fall (rise) in housing services expenditures,

$$x_{2D}(y,u) \frac{\partial p_2(y,u)}{\partial y} + s = 0. \hspace{1cm} (32)$$

Where, \(s\) is the commuting cost to the amenity.

Equations (31) and (32) are useful in understanding the spatial configuration of the city. If we assume the consumer expends a fixed proportion of his income on the group of commodities composed of expenditures on housing services, \(p_2(y,u) x_{2D}(y,u)\), expenditures
commuting to the CBD, $p_3 u$, and expenditures on the amenity located at $z$ in Figure 4, $s y$, we can trace out the shape of the city.

Let $y_A$ be the edge of the city associated with agricultural rent, $R_A$, when $z$ is equal to zero. At $y_A$ total commuting expenditures to the CBD (work) and to the amenity (pleasure) will be,

$$p_3 u + s y = p_3 y_A + s y_A = (p_3 + s) y_A.$$  \hspace{1cm} (33)

But for the rent to stay at $R_A$ so as to define the city's boundary it must be the case that total expenditures on commuting, i.e., the sum of $p_3 u + s y$, remains the same,

$$p_3 u + s y = (p_3 + s) y_A.$$ \hspace{1cm} (34)

Thus, for a given $y_A$, there exists a linear relationship between $u$ and $y$ at the city's boundary and for all contours associated with $R > R_A$.

Equation (34) is very useful in understanding the shape of the city. Consider a fixed distance $y^$, $0 < y^ < y_A$. While in the city what are the possible values $u$ may assume when $y = y^$? Clearly, when locating perpendicularly to the coast, commuting distance will be $u = y^$. Any residential location other than $z=0$ implies a longer journey to the workplace. From (34), residential location at $y^$ but at the boundary of the city implies that,

$$u = \frac{p_3 + s}{p_3} y_A - \frac{s}{p_3} y^,$$

when $R = R_A$. In general the bounds on $u$ and $y$ are,

$$y \leq u \leq \frac{p_3 + s}{p_3} y_A - \frac{s}{p_3} y \quad \text{and} \quad 0 \leq y \leq y_A.$$

However, while linear in $u$ and $y$, the shape of the city is not
Where, \( u_A = (p_3 + s)y_A/p_3 \).
linear in y and z, but instead is of the form,

\[ p_3 \sqrt{y^2 + z^2} + s \cdot y = (p_3 + s) y_A. \tag{35} \]

Figure 4 illustrates the configuration of the city. The city is longer along the coast line reflecting consumer preference for the amenity offered by the coast. Longer here of course refers to distance from the CBD. The shortest distance to the CBD occurs when the consumer has to commute an equal distance to both the CBD and the amenity, i.e., when \( z = 0 \). The intuitive reasoning comes from equations (31) and (32). When locating at \( z = 0 \), the commuter paying \( R_A \) in land rent incurs a commuting distance to the CBD which is the shortest commute consistent with that level of rent. Consequently, he also has the longest travel to the amenity of any city resident. On the other hand, a laborer locating at the amenity incurs no commuting cost to enjoy the pleasures it holds. By locating at \( y = 0 \), that laborer must commute distance,

\[ u = \left( \frac{p_3 + s}{p_3} \right) y_A \]

to reach the CBD. In the special case in which \( s \) is equal to \( p_3 \), it so happens that a resident locating at a point on the boundary where \( z = 0 \) and one locating on the boundary where \( y = 0 \) commute the same number of miles per period. This is the case in which the city is twice as wide (i.e. along the coast) as it is long. This does not have to be the case. If \( s \) is constant at any point in the city and \( p_3 \) decreases at distances away from the CBD, the city will stretch out farther along the coast than suggested above.
Until now, the total transportation cost function was assumed to be of the special form,

\[ T(y,u) = p_3 u + s y. \]  \hspace{1cm} (36)

In general, it is expected that transport costs to both the CBD and the amenity will vary at every point in the city. Thus, the general form of transportation costs is representable by,

\[ T(y,u) = p_3(y,u) + s(y,u; \pi). \]  \hspace{1cm} (37)

Where, \( \pi \) is the number of trips a consumer at distance \( y \) from the amenity will make per period of time. The number of trips made would be expected to fall with \( y \), a priori. Unfortunately, without explicit knowledge of the forms \( p_3 \) and \( s \) in (37) it is impossible to make definite statements about the shape of the city as well as some of the topics considered below. A definite form for (37) will be proposed below.

If the consumers' income allocated for expenditure on the three urban goods is \( M \) then the rent associated with a given point in the city is,

\[ R(y,u) = M - p_3 u - s y. \]  \hspace{1cm} (38)

Specifically, at the boundary of the city,

\[ R_A = M - p_3 u - s y. \]  \hspace{1cm} (39)

Thus,

\[ \frac{M - R_A}{p_3 + s} = y_A. \]  \hspace{1cm} (40)
DERIVATION OF THE RENT GRADIENT

The derivation of the rent gradient for this city is actually very similar to that employed by Mills. The principal difference is that the city resident is supposed to commute to two different locations in the city per unit time. Thus, an ordinary differential equation is replaced by a partial differential equation. The two locational equilibrium equations (31) and (32) can be expressed uniquely as one locational equilibrium equation,

\[ \frac{\partial \rho_2(y,u)}{\partial y} + \frac{\partial \rho_2(y,u)}{\partial u} + \rho_3 + s = 0. \quad (41) \]

The relationship between the price of housing services and the rental rate on is given by (26). Taking derivatives of (26) yields,

\[ \frac{\partial \rho_2(y,u)}{\partial y} = \alpha_2 \frac{\partial}{\partial y} R(y,u) \quad (42) \]

\[ \frac{\partial \rho_2(y,u)}{\partial u} = \alpha_2 \frac{\partial}{\partial u} R(y,u) \quad (43) \]

Substituting (28), (42), and (43) into (41), produces the partial differential equation,

\[ \alpha_2 \beta_2 \left( \frac{\partial R(y,u)}{\partial y} + \frac{\partial R(y,u)}{\partial u} \right) + \rho_3 + s = 0. \quad (44) \]

Where, \( \alpha = \alpha_2 \beta_2 + \alpha_2 - 1 \).

Equation (44) is separable if the price elasticity of demand, \( \theta_2 \), is equal to minus one. In general, \( R(y,u) \) can be found by a change of variables approach.

The general solution of (44) when \( \theta_2 = -1 \) is,
\[ R(y,u) = c_1 e^{bu + c_2(y - u)} \]  \hspace{1cm} (45)

Where,
\[ b = -(p_3 + s)/\Gamma, \]
and,
\[ \Gamma = \alpha_2 B_2 W_1. \]

To obtain the constants of integration, \( c_1 \) and \( c_2 \), in (45), recall the structure of the city at the points when \( y=u \) at the agricultural rent, \( R_A \), i.e., when \( y=y_A \), and when \( y=0 \) and \( u=u_A \),
\[ R_A = R(y_A,y_A) = c_1 e^{b y_A} \]  \hspace{1cm} (46)

or,
\[ c_1 = R_A e^{-b y_A}. \]  \hspace{1cm} (47)

And when \( y=0 \) and \( u=u_A \),
\[ R_A = R(0,u_A) = c_1 e^{(b - c_2) u_A}. \]  \hspace{1cm} (48)

So that \( c_2 \) is,
\[ c_2 = \frac{b(u_A - y_A)}{u_A} \]  \hspace{1cm} (49)
\[ = \frac{-s}{\Gamma}. \]

Substituting (47) and (49) into (45) rent at any point \( (y,u) \) in the city is,
\[ R(y,u) = R_A \exp\left\{ \frac{-(p_3+s)}{\Gamma} \left[ \frac{(sy+p_3u)}{p_3+s} - y_A \right] \right\}. \]  \hspace{1cm} (50)
Equation (50) expresses rent in terms of $u$ and $y$ as well as the maximum distance from the amenity the city area reaches, $y_A$. Thus, once $y_A$ is known, $u_A$ is also known.

When is an amenity not an amenity? If the coast line does not offer any utility-bearing characteristics to city residents, no commuting will be done to the coast. In this instance we can treat $s$ as identically equal to zero. In this case, (50) reduces to Mills' equation for the rent gradient,

$$R(u) = R_A \exp\left\{-\frac{p_3}{\Gamma} (u - y_A)\right\}, \quad (51)$$

where $y_A$ is identical to $u_A$. On the other hand, if commuters do not have to commute to work (all home production) for the fixed limits of the city $(y_A, u_A)$, $p_3$ can be taken to be zero. The rent gradient becomes,

$$R(y) = R_A \exp\left\{-\frac{s}{\Gamma} (y - y_A)\right\}. \quad (52)$$

For a coast line in which all points offer the same utility and which is very long relative to the population, $y_A$ will go to zero and only an agricultural level of rent will be paid.

In the case hypothesized above, the boundary of the city was designated by the line $y_Au_A$ as in Figure 5. In this instance, boundary means that all locations in the interior command a rent that exceeds the agricultural rent. As equation (52) suggests, if the amenity coast line is not large relative to the number of farm households, it may be necessary to redefine the population of the city as all residents
commuting to the CBD as well as the amenity. Rent in the exterior of $y_A u_A$ is equal to the opportunity cost of the land if those households locating there do not value the amenity or if the coast beyond $u_A$ has not been developed so as to provide an amenity. In general, neither of these conditions hold. Thus, farm families will still wish to commute to the amenity even though they do not commute to the CBD. Will they ever be observed living in the city? No, they will always be outbid by households that do commute to the CBD. To see this, consider points A and B in Figure 5. If a household that commutes to the CBD is located at B and one that does not is located at A, does a spatial equilibrium exist? No, the household at B must incur a higher commuting cost to the CBD than if located at A, while the one at A incurs no such cost. The household at A incurs the same commuting (to the coast only) whether located at A or B. Thus, the household at B will outbid the the farm household for A since it is willing to pay a higher rent to reduce its transport costs to the CBD (assuming it is costless to bring farm produce to market).

To show that spatial equilibrium in the case of a coast of finite distance is different from the case of a coast of infinite distance, consider the spatial equilibrium condition for agricultural households,

$$\frac{\text{d}R}{R} = -s \, \text{d}y.$$  \hfill (53)

Which yields the rent function for farm households,

$$R_C(y) = R_A e^{-s/\Gamma(y - y_C)}, \quad y \leq y_C.$$  \hfill (54)

Where, from the standpoint of farm households, agricultural rent attains at distance $y_C < y_A$. To show that the part of the city's boundary at
FIGURE 5

CITY BOUNDARY WHEN NONRESIDENTS COMMUTE TO THE AMENITY
y < y_c moves inward along the coast when R^G ≠ R^A, let R(y, u) = R^G(y) when y=0. This implies for u_c depicted in Figure 5,

\[ u_c = u_A - s \frac{y_c}{p_3}, \text{ or } u_c < u_A \text{ for } s, y_c > 0. \]

When (37) attains, the rent function is expressible only in terms of a general form involving p_3(y,u), s(y,u), and y and u,

\[ R(y, u) = c_3 \exp \left\{ -\frac{1}{T} \left( T(y, u) + c_4(u - y) \right) \right\}. \tag{55} \]

Where,

\[ c_3 = R_A \exp \left\{ \frac{1}{T} \left[ T(y_A, y_A) \right] \right\}, \tag{56} \]

\[ c_4 = \left[ p_3(0, u_A) - T(y_A, y_A) \right] / u_A. \tag{57} \]

The solution to (44) when the price elasticity of demand for housing services is not restricted to unity is given by,

\[ R(y, u) = \left\{ c_1 + \frac{b(\alpha + 1)}{2} (u + y) + c_2(y - u) \right\} \frac{1}{\alpha + 1}. \tag{58} \]

Where,

\[ c_1 = R_A^{\alpha + 1} - b(\alpha + 1) y_A, \]

and,

\[ c_2 = \frac{b(\alpha + 1)}{u_A^2} (y_A - \frac{u_A}{2}). \]

**DERIVATION OF THE POPULATION DENSITY GRADIENT**

Mills (1972) has discussed at some length population and employment density derived from the standard urban model. In this section, the population density gradient for a city with an amenity
coastline is considered. The population density gradient for this model is similar to that derived in the standard urban model. It is negative exponential and log-linear in \( u \) and \( y \).

Let the number of people living at the point \((y,u)\) be represented by \( N(y,u) \). This is expressible as the ratio of the total demand for housing services to the individual demand for housing services at \((y,u)\),

\[
N(y,u) = \frac{X_{2D}(y,u)}{x_{2D}(y,u)} .
\]  

(59)

Using the first-order conditions in the input market for housing services, (24) and (25), the supply of housing services is expressible as,

\[
X_{2S}(y,u) = A_2 L_2(y,u) \left\{ \frac{(1-\alpha_2) R(y,u)}{x^{\alpha_2}} \right\} .
\]  

(60)

Since equilibrium in the housing services market requires that \( X_{2S} = X_{2D} \), (60) can be substituted into the numerator of (59). Substituting (27) into (28) and the result into the denominator of (59), yields the result

\[
N(y,u) = \frac{\theta_2 \alpha_2 \beta_2}{B_2} \frac{C A_2 L_2(y,u) R(\tilde{y},\tilde{u})}{\Delta} \ .
\]  

(61)

Where,

\[
\Delta = \frac{(1-\alpha_2) R(y,u)}{x^{\alpha_2}} .
\]

In the special case, when the price elasticity of demand for housing services, \( \theta_2 \), equals minus one, (61) becomes,

\[
N(y,u) = \frac{L_2(y,u) R(y,u)}{\Gamma} .
\]  

(62)
So the population density gradient of the city has the same shape as the rent gradient,

$$\frac{N(y,u)}{L_2(y,u)} = \frac{R_A}{\Gamma} \exp\left\{ -\frac{(p_3+s)}{\Gamma} \left[ \frac{(sy+p_3u)}{p_3 + s} - y_A \right] \right\} \quad (63)$$

Taking the natural logarithm of (63) yields the familiar log-linear population density function,

$$\ln \text{(density)} = \text{constant} - \frac{s y}{\Gamma} - \frac{p_3 u}{\Gamma} . \quad (64)$$

Thus, for a given distance $y$ from the amenity, an increase in commuting distance $u$ to the CBD implies that the population density falls by

$$\frac{p_3}{\Gamma} .$$

Likewise, for a given commuting distance to the CBD, population density falls the farther one is located from the amenity,

$$\frac{s}{\Gamma} .$$

Thus, a city with an amenity coast line has population clustered near the CBD but for a given commuting distance from the CBD population density is heavier along and near the coast as might be expected.

If rent is represented in the more general form (55), the logarithm of density is expressible in the form,

$$\ln \text{(Density)} = A - \frac{p_3(y,u)}{\Gamma} - \frac{s(y,u;\pi)}{\Gamma} + Bu - By . \quad (65)$$
Which, in general, is not expected to be linear in \( y \) and \( u \). Since \( p_3 \) and \( s \) are generally not known, reasonable forms for them must be hypothesized.

**THE AREA OF THE CITY AND COMPARATIVE STATIC RESULTS**

In the standard urban model, the city is circular so that a city of radius \( u \) from the CBD has an area of land available for residential and transportation purposes equal to \( \pi u^2 \). This implies that the total land available at a given distance \( u \) is \( 2\pi u, 0 \leq u \leq u_A \). A city with an amenity coast line is not circular. Thus, the area and circumference of such a city is somewhat different. While the area of the city and comparative static results are interesting issues themselves, the policy issues discussed in Chapter V utilize the results here.

Recall that the relationship between \( y \) and \( z \) was given by equation (35),

\[
p_3(y^2 + z^2) + s \ y = (p_3 + s) \ y_A.
\]

Equation (35) can be solved explicitly for either \( y \) or \( z \). To obtain the area of the city \( z \) as a function of \( y \) is integrated from 0 to \( y_A \) or \( y \) as a function of \( z \) is integrated from 0 to \( \frac{p_3+s}{p_3} y_A \). Three different cases for the area of the city exist depending on whether i) \( p_3 > s \); ii) \( p_3 = s \); or iii) \( s > p_3 \).

**case i.** \( p_3 > s \)
\[ \int_{0}^{z_{A}} f(z)dz = \left[ d_{1} + d_{2} \sin^{-1} \left( \frac{\sqrt{p_{3}^{2} - s^{2}}}{p_{3}} \right) \right] y_{A}^{2} \]

\[ = \Omega y_{A}^{2} . \]

Where,

\[ z_{A} = \frac{p_{3} + s}{p_{3}} y_{A} , \]

\[ d_{2} = \frac{(p_{3} + s)^{2} p_{3}}{2 \left[ \sqrt{(p_{3}^{2} - s^{2})} \right]^{3}} , \]

and,

\[ d_{1} = \frac{-(p_{3} + s)^{2} s}{2 p_{3} (p_{3}^{2} - s^{2})} . \]

Where,

\[ 0 \leq \frac{\sqrt{p_{3}^{2} - s^{2}}}{p_{3}} \leq 1 , \]

and

\[ 0 \leq \sin^{-1} \left( \frac{\sqrt{p_{3}^{2} - s^{2}}}{p_{3}} \right) \leq \frac{\pi}{2} . \]

Or, the area of the city when \( p_{3} > s \) is twice the value of that in (66),

\[ \text{Area} = 2 \Omega y^{2} . \]

The amount of land thus available for residential and transportation at a particular \( y \) is,

\[ L(y,u) = L_{2}(y,u) + L_{3}(y,u) = 4 \Omega y . \]

For simplicity, let \( L_{3}(y,u) \) equal zero.
When the city is circular, i.e., \( s=0 \), equation (66) reduces to the standard result found in most urban models since \( \sin^{-1}(1) \) is equal to \( \pi/2 \), \( d_2 \) reduces to unity and \( d_1 \) reduces to zero,

\[
\text{Area} = \frac{\pi}{4} y^2 .
\] (68)

Equation (68) is thus one quarter the area of a circle. In the city with a coast line not offering an amenity the area of the city is twice that of (68).

**case 2.** \( p_3 = s \)

When \( p_3 = s \), the area of the city does not depend on the absolute values of the cost parameters \( p_3 \) and \( s \),

\[
\text{Area} = \frac{8}{3} y^2 .
\] (69)

And so the area available for residential and transportation purposes at \( y \) is,

\[
L(y) = \frac{16}{3} y .
\] (70)

**case 3.** \( s > p_3 \)

As will be discussed below, the case where \( s > p_3 \) is the least likely case. Here, the area of the city is expressible as,

\[
\text{Area} = (e_1 + e_2 \log\left( \frac{y (p_3+s)}{p_3} \left[ 1 + \frac{s}{\sqrt{s^2 - p_3^2}} \right] \right)) y^2
\] (71)

where,

\[
e_1 = \frac{(p_3+s)^2 s}{(s^2 - p_3^2) p_3} ,
\]

\[
e_2 = \frac{(p_3+s)^2 p_3}{2[s^2 - p_3^2]^{3/2}} .
\]
Clearly, the land available for residential and transportation purposes cannot be expressed linearly from equation (71).

As discussed below, assume that \( p_3 > s \) so that land available at \( y \) is given by (67). Substituting (67) into (63) allows us to express the fixed population of the city, \( N \), as,

\[
N = \frac{4 \Omega R_A}{\Gamma} e^{\frac{y_A}{\Gamma}} \int_0^{y_A} y e^{\frac{y}{\Gamma}} du dy. \tag{72}
\]

Where, the upper limit \( u' \) is taken from (34),

\[
u = \frac{(p_3+s)y_A}{p_3} - \frac{sy}{p_3}.
\]

The solution to (72) yields the outcome,

\[
\frac{p_3 N(p_3+s)^2}{4 \Omega \Gamma^2} = e^{-\frac{(p_3+s)y_A}{\Gamma}} - \frac{(p_3+s)y_A}{\Gamma} - 1 - \frac{(p_3+s)^2 y_A^2}{2 \Gamma^2}.
\]  \tag{73}

The exponential term in (73) can be approximated by the Taylor series expansion,

\[
\frac{(p_3+s) y_A}{e^{\frac{y_A}{\Gamma}}} = 1 + \frac{(p_3+s) y_A}{\Gamma} + \frac{(p_3+s) y_A^2}{2 \Gamma^2} + \frac{(p_3+s) y_A^3}{6 \Gamma^3}.
\]  \tag{74}

Substituting (74) into (73) yields the quite tractable result,

\[
\frac{p_3 N(p_3+s)^2}{4 \Omega \Gamma^2} = \frac{(p_3+s)^3 y_A^3}{6 \Gamma^2}.
\]  \tag{75}

Solving for \( y_A^3 \) in (75) and substituting for \( \Gamma \) yields,

\[
y_A^3 = \frac{3 p_3 N \alpha_2 B_2 W}{2 \Omega (p_3+s) R_A}.
\]  \tag{76}
Most of the comparative static results as to how the size of the city changes can be obtained from (76). These are,

\[
\frac{dy_A}{d\alpha_2} > 0; \quad \frac{dy_A}{dB_2} > 0; \quad \frac{dy_A}{dW} > 0; \quad \frac{dy_A}{d\theta_1} > 0; \quad \frac{dy_A}{d\theta} > 0; \quad \frac{dy_A}{dR}\ > 0; \quad \frac{dy_A}{dR} < 0.(77)
\]

Since in general \( Q \) will be a complicated function of both \( s \) and \( p^3 \), the sign of \( \frac{dy_A}{dp^3} \) and \( \frac{dy_A}{ds} \) will be indeterminate. The logic as to why this will be the case is actually very simple. For example, an increase in \( s \) may result in residents economizing on the cost of transporting to the amenity. Thus, population moves toward the tail of the city, i.e., away from the inland. On the other hand, an increase in \( p^3 \) causes the tail to shrink relatively; but the tendency for the city to shrink in the tail may be counterbalanced by an expansion at \( z=0 \) accommodating the population evacuating the tail of the city. The importance of comparative statics in \( s \) and \( p^3 \) is addressed more fully in Chapter V.

**THE MODEL WITH A SPATIAL CBD**

In this section, the case of a CBD which occupies land is introduced. Thus, firms compete against residents for land. It will be assumed that the existence of the port located at 0 significantly affects the scale parameter of the firms' production function. Below, an explicit form of the goods production, \( X_1 \), is supposed.

For purposes of illustration, assume firms only consume land for production purposes, \( L_1 \). Firm equilibrium is thus,
\[
\frac{\delta R^F(u)}{\delta u} + fQ = 0. \tag{78}
\]

Where, \( R^F(u) \) is the rent function of the firm at distance \( u \); \( Q \) is the total output produced by the firm; and \( f \) is the cost per unit distance of shipping one unit of output to the port. Equation (78) yields a linear rent function, where the rent the firm will offer declines linearly with distance to the port. In this case, the firm finds no value in locating near the amenity, per se. The solution to (78) is expressible as,

\[
R^F(u) = \bar{R} - \frac{fQ}{L_1} u. \tag{79}
\]

Where, \( \bar{R} \) is the maximum amount of rent the firm is willing to pay for \( L_1 \) at the port.

In this special case, it is easy to compare the firms' rent gradient with that of the households. In order for firms to be centrally located, it is necessary that the maximum rent at the port 0 paid by firms, \( \bar{R} \), must exceed that paid by households,

\[
(p_3 + s) y_A
\]

\[
R(0,0) = R_A e^{\frac{\Gamma}{\Gamma}} - s y_0 - p_3 u
\]

Figure 6 illustrates the relative heights of the firms' and households' rent gradients at a given distance \( y \).

At a given distance \( y = y_0 \), the households' rent function is given by,

\[
R(y_0, u) = R_A e^{\frac{\Gamma}{\Gamma}} - s y_0 - p_3 u - \frac{(p_3 + s) y_A}{e^{\frac{\Gamma}{\Gamma}}}
\]

\[
\tag{80}
\]
FIGURE 6
RENT GRADIENTS FOR HOUSEHOLDS AND FIRMS
To find the spatial distance of the CBD at equilibrium, set
\[ R^F(u) = R(y_0, u). \] (81)
The solution to (81) can be obtained by a Taylor's series approximation
of (80) and is given by
\[ u_B(y = y_0) = \frac{\Gamma L_1 \left[ \bar{R} - R_A \exp(\xi) \right]}{f Q \Gamma - R_A \exp(\xi) p_3 L_1}. \] (82)
Where,
\[ \xi = \frac{(p_3 + s) y_A - s y_0}{\Gamma}. \]

What is interesting about (82) is that at the edge of the CBD firms
located at different points do not pay the same rent for land. Those
firms locating along the edge but farther from the port are compensated
for the increased freight charges with lower rent per square foot. This
is in contrast with the traditional circular CBD models.

When \( y_0 = u \) or \( x = 0 \), (82) reduces to
\[ u_B(y = y_0) = \frac{\Gamma L_1 \left[ \bar{R} - R_A \exp(\tau) \right]}{f Q \Gamma - R_A \exp(\tau) [p_3 + s]}. \] (83)
Where,
\[ \tau = \frac{(p_3 + s) y_A}{\Gamma}. \]

Likewise, when \( y_0 = 0 \), the spatial distance along the coast is given by
\[ u_B(y = y_0) = \frac{\Gamma L_1 \left[ \bar{R} - R_A \exp(\tau) \right]}{f Q \Gamma - R_A \exp(\tau) p_3 L_1}. \] (84)

Equation (82) defines the spatial structure of the spatial CBD.

Figure 7 illustrates the shape of the CBD relative to the shape of the
FIGURE 7
SHAPE OF THE CBD AND THE CITY
suburbs. Not unexpectedly, the shape of the CBD contrasts markedly with the shape of the suburbs. The reason is of course is that firms and implicitly their employees do not value the coast line as being a useful tool for production. Again, as has been true several times before, when the coast is not valued, i.e., $s=0$, equation (82) reduces to the familiar case of a CBD which is circular,

$$u_B(s=0) = \frac{\Gamma L_1 \left[ \bar{R} - R_A \exp(\$) \right]}{\int Q \Gamma - R_A \exp(\$) p_3 L_1}.$$  \hspace{1cm} (85)

Where,

$$\$ = \frac{p_3 \gamma A}{\Gamma}.$$

A SPATIAL CBD WITH AN EXPORT GOOD SECTOR

The model developed here involves a spatial CBD in which firms produce an export good with a Cobb-Douglas production function. An export good implies that relative to total production the amount consumed by city residents is negligible. The production of the export good implies the existence of the city. As Mills (1967) notes, "The city may be located where the efficiency parameter in the production function for goods is especially favorable. The production function may have increasing or decreasing returns. If there is no effect of location on the efficiency parameter, we must have increasing returns."

Let the production of the export good, $Q$, be Cobb-Douglas

$$Q(u) = A_1 x_{2D}^\alpha(u) N^\beta_1(u) R_1^\gamma(u), \quad \alpha_1 + \beta_1 + \gamma_1 = H_1 < 0.$$  \hspace{1cm} (86)
Where $Q(u)$ is total output at radial distance $u$ from the port $0$; $x_{2D}$ is the amount of structural (housing) services used in production; $N$ is the input of labor and $K_1$ is the input of capital. If the market for $Q$ is competitive and factor markets are competitive, increasing returns are ruled out. Mills (1967) avoids this problem by assuming that the export goods producer is a monopolist with a very simple demand equation for its product. For simplicity, assume firms take the product price as given. Firms are assumed to maximize profits,

$$\Pi = P \cdot Q - p_2(u) \cdot x_{2D} - WN - rK - fQu.$$  \hfill (87)

Where, $p_2$ is the price of structural services and $W$ and $r$ are the fixed prices of labor and capital. The variable $f$ is the freight cost per unit distance and $fQu$ is the total cost of transporting output from the point of production, $u$, to the port.

If resource markets are competitive, the first-order conditions from maximizing (87) are

$$\frac{\alpha_1 \cdot P \cdot Q}{x_{2D}} = p_2(u),$$ \hfill (88)

$$\frac{\beta_1 \cdot P \cdot Q}{N} = W,$$ \hfill (89)

$$\frac{\gamma_1 \cdot P \cdot Q}{K_1} = r,$$ \hfill (90)

$$\frac{\delta \cdot p_2(u)}{x_{2D}} + fQ = 0.$$ \hfill (91)

Where, (88)- (90) are equilibrium factor market conditions and (91) is the firms' spatial equilibrium condition. The firms' demand for structural services is solved from (88) - (90),
\[
\frac{H_2}{x_{2D}} = \alpha_1 \frac{(PA_1)^{\beta_1/(\beta_1 + \gamma_1 - 1)}}{(r/\gamma_1) p_2(u)}, \tag{92}
\]

and where, \(H_2 = H_1 - 1\). Equation (92) can be written more compactly as,
\[
\frac{1 - \beta_1 - \gamma_1}{H_2} x_{2D} (u) = E_1 p_2. \tag{93}
\]

From the production function for housing services, \(p_2(u)\) is expressible in terms of \(R(u)\),
\[
p_2(u) = C R(u). \tag{94}
\]

Where,
\[
C = \frac{1 - \alpha_2}{r}. \tag{27}
\]

Substituting (94) into (93) yields
\[
(1 - \beta_1 - \gamma_1)/H_2 x_{2D}(u) = E_2 R(u). \tag{95}
\]

Where,
\[
E_2 = E_1 C. \tag{96}
\]

To solve for spatial equilibrium, differentiate (94) with respect to \(u\) to get,
\[
\frac{\delta p_2(u)}{\delta u} = \frac{\alpha_2 - 1}{\alpha_2 C R(u)} \frac{\delta R(u)}{\delta u}. \tag{97}
\]

Substituting (95) and (96) into (91) yields the firms' spatial equilibrium condition in terms of \(R(u)\) and its derivative,
\[
-(H_2 - \alpha_1 \alpha_2)/H_2 E_3 R(u) \frac{\delta R(u)}{\delta u} + \delta fQ = 0. \tag{97}
\]

Where,
If increasing returns export production exist, it is possible for the exponent of R(u) in (97) to be positive or negative. If decreasing returns to scale are present, the exponent will always be negative. The smaller is the product $\alpha_1\alpha_2$ in (97) the closer the exponent comes to minus unity. Let the exponent of R(u) in (97) be $E_4$, (97) reduces to
\[
E_4 \frac{\delta R(u)}{\delta u} + fQ = 0 .
\]

If $R$ is the rent firms are willing to pay for land at the port, the solution to the differential equation (98) is
\[
R^F(u) = \left( \frac{E_4 + 1}{E_3} \right) .
\]

The value of $E_4$ in (99) approaches minus unity if $\alpha_1\alpha_2$ is small. In this case, the firms' rent gradient is approximated by the negative exponential rent gradient,
\[
R^F(u) = \bar{R} e^{E_3} .
\]

To find the edge of the CBD, set the rent function for the firm equal to the rent function for the household,
\[
R^F(u) = R^H(y_0,u) .
\]

For a given distance $y_0$ from the coast, the equilibrium spatial distance of the CBD is
\[
u_B(y_0,u) = \frac{(p^3 + s)y_A}{fQ\Gamma - p^3 E_3} \left\{ \ln \bar{R} - \ln \left( \frac{R_A e^{\Gamma}}{\Gamma} \right) + \frac{sy_0}{\Gamma} \right\} .
\]

Equation (102) implies that
\[ \frac{p_3^+ s}{\Gamma} < \frac{fQ}{E_3}. \]

Figure 8 illustrates the shape of the firms' rent gradient relative to the households'. Along the coast, i.e., when \( y=0 \), the spatial extent of the CBD is

\[ u_B(y_0=0) = \frac{\Gamma E_3}{fQ\Gamma - p_3 E_3} \{ \ln \bar{R} - \ln( R_A e^{\Gamma} ) \}. \quad (104) \]

And when \( y_0=u \), the spatial extent of the CBD is

\[ u_B(y_0=u) = \frac{\Gamma E_3}{fQ\Gamma - (p_3^+ s) E_3} \{ \ln \bar{R} - \ln( R_A e^{\Gamma} ) \}. \quad (105) \]

By taking the ratio of (105) to (104), it is possible to see how much farther inland the CBD extends than along the coast,

\[ \frac{u_B(y_0=u)}{u_B(y_0=0)} = \frac{fQ\Gamma - p_3 E_3}{fQ\Gamma - p_3 E_3 - sE_3} \geq 1. \quad (106) \]

The actual shape of the CBD will depend on many parameters in the system. Equality holds in (106) when \( s=0 \). This is the traditional case of the circular CBD.
FIGURE 8.
RENT GRADIENTS WHEN PRODUCTION OF THE EXPORT GOOD IS INTRODUCED
CHAPTER IV

EMPirical ImplementAtion of the model

introduction

In order to verify the importance of recreational travel on rent and population density functions in coastal cities, it is necessary to test the urban model of Chapter III against the backdrop of empirical evidence. As noted in Chapter II, estimation of population density gradients provides a useful summary measure of an urban model. As shown by Mills, the population density function is uniquely related to the rent function. Thus, anything affecting the rent function for residential land shows up in the population density function. Data on population densities is in general easier to obtain than information on housing values in which it is often difficult to disentangle consumers' implicit purchases of housing structure characteristics and location characteristics. In Chapter V is discussed a hedonic approach originally suggested by Brown and Pollakowski (1978) in dealing with location and land use.

The focus of empirical investigation in this chapter is on population density gradients for a number of coastal and non-coastal cities. Equation (64) implies that the natural logarithm of population density declines linearly with distance to the CBD and the amenity coast. Thus, if the coefficients on y and u are estimated
econometrically, they should be negative and significantly different from zero statistically. This is a necessary but not sufficient condition for acceptance of the model specification of Chapter III. It must also be true that if a circular city located on a flat featureless plain were arbitrarily or randomly bisected, the coefficient in (64) on the measure of perpendicular distance from a given location to the line of bisection would not prove significantly different from zero in all but a few cases.

The data used in this chapter is from two different classes of cities--coastal cities and cities having a topography resembling a flat featureless plain. The latter may be thought of as the control group in testing the urban model of the amenity coast line city. It is expected that the inclusion of the $y$ variable in an econometric specification of (64) will fail to yield a significant coefficient for non-coastal cities. On the other hand, negative, significantly different from zero estimates of the $y$ variable are expected for coastal cities.

**TYPES OF EMPIRICAL MODELS**

When the CBD is nonspatial, Equation (64) provides a simple test the urban model. The estimating equation for the population density function can be written as

$$D = \beta_0 + \beta_1 u + \beta_2 y + \varepsilon .$$

(107)

Where, $D$ is the natural logarithm of population density; $\varepsilon$ is a random disturbance with mean zero and finite variance. The negative exponential model with a multiplicative error term appears appropriate
for describing the distribution of population density based on the conclusion reached by Greene and Barnbrook (1978). In this instance, the econometric specification yields a simple test of the null hypothesis that population density is affected by the existence of an amenity coast line.

Mills' two-point estimate of population density is inappropriate here because it reveals nothing about the decline of population density from the coast. The two-point procedure is dependent crucially on the simple negative exponential model of Chapter II. Instead a more disaggregated approach to population densities is required as suggested below.

Latham and Yeates (1970) suggest that if a CBD is large enough (e.g., Toronto) population density will at first increase and eventually decline from the center of the CBD in a manner consistent with spatial equilibrium of households and firms. That is, because of competition between commercial and residential users of land, population density may not decline in the simple manner predicted by the nonspatial-CBD model of Chapter I. Equation (107) does not hold in Latham and Yeates framework but must be modified in the context of the spatial-CBD model of Chapter III.

When a spatial CBD was introduced in Chapter III, it had the form illustrated by Figure 7. Unlike the standard urban model in which one radius was sufficient to describe the shape of the population density gradient, the analysis here requires careful inspection of several points in the city. The analysis employed in Chapter III illustrated by Figure 7 implies that households will not locate in the CBD. If this
were true, equation (107) would hold unequivocally when observations in the firm-dominated CBD were excluded from analysis. In reality, firms outbid households for central locations only incompletely. Some households will bid more for central locations depending on the relative steepness of the rent gradients. This is the assertion Latham and Yeates have shown for Toronto. Population density will at first increase as one moves away from the center of the CBD but then fall at some point for $y < y_B$. For points $y > y_B$, population density will peak at $z = 0$. For the orthant illustrated in Figure 7, population density will be greatest at $u_B'(y_B') = 0$. Residents locating there need not commute to the coast and need commute only distance $u_B'(y_B')$ to work. Residents locating at $y_B$ must commute distance $y_B$ to both the coast and the CBD.

Figure (9) illustrates the way in which it is hypothesized that population densities peak when a spatial CBD is extant. Population density achieves a global maximum at point A in the orthant where residents locate at the coast and commute $u_B'(y_B') = 0$ to work. Local maximum population densities occur along the boundary of the CBD for $y < y_B$. Figure (9) illustrates the peakedness of density in terms of $y$ and $z$. For $y > y_B$, the peak in density occurs at $z = 0$. For $y < y_B$, as $y$ decreases the peak occurs at larger values of $z$, i.e., farther away from the port.

If figure (9) accurately reflects population density for most coastal cities, an econometric equation which captures the spatial relationships discussed above is required. Equation (107) implies no peaks in the orthant of Figure (9). The approach suggested by Latham and Yeates offers useful insights into the problem. Consider the
FIGURE 9

POPULATION DENSITY STRUCTURE OF THE CITY
specification of the population density function which incorporates a quadratic term in \( u \),

\[
D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 u^2 + \varepsilon .
\]  

(108)

Holding \( y \) constant, the \( u \) at which population density reaches a maximum is found by setting \( \frac{\delta D}{\delta u} = 0 \) and is given by

\[
u = -\frac{\beta_1}{2 \beta_3} .
\]

(109)

Where, either \( \beta_1 < 0 \) or \( \beta_3 < 0 \). To give this substance, express \( u \) in terms of \( y \) and \( z \),

\[
\sqrt{(y^2 + z^2)} = -\frac{\beta_1}{2 \beta_3}
\]

or,

\[
z^2 = \frac{\beta_1^2}{4 \beta_3^2} - y^2 .
\]

(110)

This relationship captures the intuition gleaned from Figure (9). As \( y \) decreases, peaks occur at larger values of \( z \) since \( dz/dy < 0 \). Thus, (108) should reflect the nature of population density reasonably well for coastal cities with firms locating centrally.

Brown and Pollakowski (1978) note that jurisdictions may zone coastal areas so as to impose an optimal setback that maximizes land values. Thus, zoning may proscribe residential location near the coast. Firms catering to residential demand for recreation goods, e.g., marinas, surf board vendors, concessionaires, etc., may outbid households for land near the coast. Both of these features limit household location near the coast. In this instance, population density may not peak at the water's edge but at an inland location. To capture this phenomenon, introduce a quadratic term in the population density equation,

\[
D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 u^2 + \beta_4 y^2 + \varepsilon .
\]

(111)
Thus, holding radial distance to the origin constant, the value of $y$ associated with maximum population density is given by,

$$y = -\beta_4/2 \beta_4.$$  \hspace{1cm} (112)

Which says that the peak from the coast always occurs at this $y$ for a given $u$ in the city. When $u$ is not held constant, the $y$ at which density peaks will vary with $z$. When $z=0$, the $y$ associated with the peak in population density occurs at

$$y(z=0) = -(\beta_1 + \beta_2)/2(\beta_3 + \beta_4).$$  \hspace{1cm} (113)

When $z=z_0 \neq 0$, the $y$ associated with the peak in population density is expressible in the form

$$\beta_2 \sqrt{(y^2 + z_0^2)} + [\beta_1 + 2(\beta_3 + \beta_4) \sqrt{(y^2 + z_0^2)}] y = 0.$$  \hspace{1cm} (114)

A slightly more complicated approach to deal with the relationship between $y$ and $u$ is to introduce an interaction term between $y$ and $u$. In this case the estimating equation is of the form

$$D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 u^2 + \beta_4 y^2 + \beta_5 uy + \varepsilon.$$  \hspace{1cm} (115)

Now, when $y$ is held constant, the value of $z$ associated with the peak in population density is given by

$$z^2 = [(\beta_1 + \beta_5 y)^2 - 4 \beta_3^2 y^2] / 4 \beta_3^2.$$  \hspace{1cm} (116)

The expression, $\beta_5(\beta_1 + \beta_5 y) - 4\beta_3^2 y < 0$, is sufficient to ensure that $dz/dy < 0$. When $u=y$, i.e., when $z=0$, the value of $y$ associated with a maximum in population density is $y = -(\beta_1 + \beta_2)/2(\beta_3 + \beta_4 + \beta_5)$.

Since a change in jurisdictional boundary within a larger metropolitan area may imply changes in zoning laws, property tax rates, provisions of public services (e.g., better schools in the suburbs), or different ethnic composition which are capitalized in housing values, it is necessary to account for these distinctions in a density regression.
This happens because the price of housing services is reflected in population density as noted by Muth (1969). For example, if one jurisdiction has a lower property tax rate, ceteris paribus, the lower rate is reflected in the price of housing services. This induces residential relocation to the lower-rate jurisdiction and hence a higher population density compared with jurisdictions having the same spatial and income characteristics. That is, at the boundary of the lower-rate jurisdiction, a discontinuity in the population density function will occur. Likewise, if a jurisdiction finds it optimal to zone property so that residential property tracts are single-family dwelling units larger than would be observed in the absence of zoning, population density might be lower in that jurisdiction than locales with comparable characteristics that do not zone. In terms of the estimating equation, this implies a change in the intercept term when considering that jurisdiction. The solution is the inclusion of an indicator variable distinguishing jurisdictions in the equation,

\[ D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 \text{DUM} . \]  

(117)

Where, DUM takes a value of one if, say, the observation is in a low tax rate jurisdiction, zero otherwise. Since, as noted, many factors vary with jurisdiction it is impossible to control for all of these parsimoniously. Instead assume that the primary difference in zoning laws, property tax rates, provisions of public goods, etc., occurs between the major city in the SMSA and the surrounding suburbs. Note that some variations between jurisdictions increase population density (e.g., a good bundle of public goods) while others decrease population
density (e.g., zoning laws requiring large lots). As a consequence, no a priori expectation about the sign of $\beta_3$ in (117) can be made.

Brueckner (1986) suggests that switches in population density arise as a result of the aging process of urban structures. Haurin has pointed out that the switches detected by Brueckner could also have arisen as a result of changes in income levels when moving from one part of the city to another as predicted by Hoyt radial sector theory.

An alternative way of dealing with differences in income on population density is to directly employ information on residential income. By doing so it is possible to make statements as to how population density changes with increases in income. Unfortunately, equation (64) describes population density for only one income group. When two income groups exist two different density functions arise but it is easy to attain unbiased econometric parameter estimates by introducing variables income and income interacted with distances. In this case, the gradients and central density can be recovered. When $n$ income groups exist it is necessary to introduce $3n$ variables (three for each group) to capture the desired parameter estimates. As a consequence, the degrees of freedom are quickly exhausted. As a compromise, we introduce an income term and interaction terms into the population density equation,

$$D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 \text{INC} + \beta_4 \text{UINC} + \beta_5 \text{YINC}.$$  \hspace{1cm} (118)

Where, INC is the income at an observation, UINC and YINC are distances $u$ and $y$ interacted with income. The former causes a change in the intercept (central density) at the CBD for each income level whereas the latter produce variations in density gradients depending on income.
Here, \( \frac{dD}{du} = \beta_1 + \beta_4 YINC \) and \( \frac{dD}{dy} = \beta_2 + \beta_5 YINC \) or that the gradients vary with income depending on the sign of \( \beta_4 \) and \( \beta_5 \). Unfortunately, bias exists in the parameter estimates if more than two income groups exist. Estimation of (118) results in parameter estimates which are weighted averages of the true parameters of the various income groups. This happens because of the fitting of several income groups with the use of only the parameters indicated in (118). Nevertheless, this equation functions as a parsimonious method to deal with non-constant income of most cities.\(^5\)

Since the shape of the CBD is not circular in the model, employment density as predicted in conventional models is unlikely to hold in cities characterized by an amenity coast line. Indeed, even if firms received no benefit from the coast, their location decisions (and hence employment density) are affected by the desire of households to be near the water. Likewise, as mentioned above, some firms selling products to residents at the coast, will endeavor to seek coastal locations. The models outlined above provide a unique test of firm location and employment patterns. In this case, employment density is unlikely to fall uniformly from the port. If the CBD were circular, econometric estimation of the models above with the log of employment density as the dependent variable would show that any term involving \( y \) would be irrelevant in explaining employment density patterns.

**ECONOMETRIC ISSUES**

Equation (64) says that for a city with an amenity coast line, the logarithm of population density should decline in \( y \) and \( u \). The usual
approach in estimating population density functions is to estimate the equation

\[ \text{Pop} = \beta_0 + \beta_1 u + \varepsilon^* . \]  

(119)

The theoretical justification for this approach follows from the standard urban model in which the only relevant travel is radially from residence to the CBD. An OLS estimate of \( \beta_1 \) is given by

\[ \hat{\beta}_1 = \beta_1 + \beta_2 \frac{\text{cov}(u,y)}{\text{var}(u)} + \frac{\text{cov}(\varepsilon,u)}{\text{var}(u)} \]  

(120)

where, \( \beta_2 \) refers to the coefficient of \( y \) in (64). Taking expected value of (120), the bias associated with estimating (119) is given by

\[ \text{bias} = \beta_2 \frac{\sum u y}{\sum u^2} \]  

(121)

The value of the second term in (121) is the estimated coefficient of \( \delta_1 \) in the auxiliary equation

\[ y = \delta_0 + \delta_1 u + \text{residual} . \]  

(122)

When the census tract observations are symmetrical about the diagonal in the respective quadrants, the relationship between \( y \) and \( u \) cancels out offdiagonal elements. Thus, the estimated coefficient of \( \delta_1 \) is as when \( y=z \), so that

\[ u = \sqrt{y^2 + z^2} \]

or,

\[ u = \sqrt{2} \cdot y . \]

Thus,
\[ \tilde{\delta}_1 = \frac{1}{\sqrt{2}} \]

In fact, observed symmetry exists when the city is roughly circular. A city with an amenity coast line will likely have more observations located along the coast. Since the model above implies the city is not circular, the estimated value of \( \delta_1 \) will not be equal to \( 1/\sqrt{2} \) but something less. Nevertheless, \( \delta_1 \) will always be positive which implies that the bias in estimating (119) is in the direction of increasing the absolute value of the estimated coefficient \( \tilde{\beta}_1 \). That is, if \( y \) is included in a population density regression of an amenity coast line city, the likely effect is a diminution in the importance of radial distance to the CBD. Since projections of population density are usable by local governments in making planning decisions for public services and by firms in making location decisions, errant estimates of density gradients can result in inefficient allocation decisions.

Since census tracts do not all have the same geographical area, it is necessary to correct for possible bias in estimates of parameters of \( u \) and \( y \) introduced by this distortion. The approach chosen here follows that of Frankena (1978). The form of the bias-corrected estimating equation is given by

\[
D = A_1^5 \ln D = \beta_0 A_1^5 + \beta_1 A_1^5 u + \beta_2 A_1^5 y + A_1^5 \varepsilon
\]

(123)

With an estimating equation of this form, it is necessary to modify the statistical procedure somewhat. In order to obtain efficient parameter estimates, \( \beta \) should be computed as

\[ \hat{\beta} = (X'X)^{-1}X'D \]
i.e., the data should be centered about zero rather than sample means. Likewise, the coefficient of determination of multiple regression should be computed with the variation of the dependent variable about zero,

\[ R^2 = 1 - \frac{\hat{e}'\hat{e}}{D'D}, \text{ where, } 0 \leq 1 - \frac{\hat{e}'\hat{e}}{D'D} \leq 1, \]

instead of its sample mean,

\[ R^2 = 1 - \frac{\hat{e}'\hat{e}}{(D - e\bar{D})'(D - e\bar{D})}. \]

Where, \( \bar{D} \) is the sample mean of \( D \) and \( e \) is the unit vector.

**THE DATA**

The 1980 Census provides information on population in census tracts. Census tracts are relatively small units of observation on population. Tracts vary widely in their geographic size but vary less in the number of inhabitants. The Census does not estimate the geographic area of tracts and these must be estimated with the use of a polar planimeter or obtained from city or metropolitan planning authorities (Although two companies, Geographic Data Technology, Inc. and National Planning Data Corporation, offer researchers a uniform measure of tract areas for the 1980 Census). Likewise, straight-line distances to the CBD and the coast must be ascertained using a ruler.

Population density based on census tracts are the most widely used unit of analysis. They provide a reliable and accurate description of population patterns within a larger metropolitan area. When analyzing population density for coast line cities, census tract data are well-suited for testing the significance the amenity.
Most of the population density studies cited above use tract data which constitutes only a incomplete sampling of tracts within the city studied. In the present study, population density, income, and distance variables were collected for all tracts in the city (where city is defined as that part of the SMSA which appears to reasonably describe the spatial extent of the city) except those which contained institutions such as military bases, state mental hospitals and the like. Central tracts containing few people were also collected and used in the analysis below unless no one was reported living in the tract or no mean income was reported for the tract. Haurin (1981) eliminated central city (CBD) tracts in his analysis of Columbus, Ohio to remove the problem of rising population density away from the CBD until a maximum is reached. This requires that one knows at what point population density is likely to peak. To avoid this, below all tracts are used but the squared and interaction terms of u and y are introduced to capture this effect.

The 100% sampling approach introduces the possibility of spatial autocorrelation discussed by Cliff and Ord (1972, 1982), and Martin (1974). According to this school of thought, disturbance terms of a given tract may be autocorrelated with tracts contiguous to it which implies that OLS yields inefficient estimates. This problem will be ignored in the analysis below because (i) OLS results remain unbiased even in the face of spatial autocorrelation, (ii) the task of constructing the appropriate tests for spatial autocorrelation is not efficient from a cost-benefit standpoint, and (iii) there is nothing
inherent in the model to suggest the presence of spatial autocorrelation.

In terms of census tracts, equation (107) has a unique interpretation. \( D \) is the natural logarithm of population density of a census tract. For a census tract, population density is population of the tract divided by the geographic area of the same tract. The variable \( u \) is the distance from the port (or the center of the CBD) to the center of the census tract. The variable \( y \) is the perpendicular distance from the center of the census tract to the coast line\(^7\). In the case of an arbitrarily defined line of bisection in noncoastal cities, \( y \) is the perpendicular distance from that line to the center of the census tract. The lines of bisection for the noncoastal cities were drawn as follows. Four lines of bisection were plotted on the census maps. These correspond to drawing lines through the CBD going north to south (YNS), east to west (YEW), northeast to southwest (YNE) and northwest to southeast (YNW).

Two types of cities are required to test the hypothesis that a coast line is an amenity which serves as an attraction of residents. Coastal cities with prominent bodies of water are obvious candidates. They should be selected in a random fashion so as not to bias results by selecting cities with observed population patterns predicted by the model. They should also be cities with fairly straight coast lines (except at the port which certainly will not be straight) so as to provide an accurate measure of distance to the coast. For example, this would rule out San Francisco which is situated on an irregular portion of the California coast. Likewise, coastal cities whose CBD is located
on an inland river (e.g., Baltimore or Toledo) complicate the measurement of $u$. Noncoastal cities well-suited for this study are those that are roughly circular, i.e., those located on a flat featureless plain with no important commercial boulevard skewing population in an irregular manner. Table 1 summarizes cities of both classes which were chosen for analysis.

**EVIDENCE**

Tables 2-15 summarize the regression results for the coastal cities in Table 1. Seven models were estimated for each of the coastal cities. Model 1 corresponds to the standard urban model in which radial distance to the CBD is the only RHS argument. Model 2 is the "basic model" of the analysis contained in Chapter III (equation 107). Model 3 is identical to model 2 except that the dummy variable, $DUM$, discussed above, is included. In this model, $DUM$ equals one if the observation is within the corporate limits of the major city of the SMSA, zero otherwise. Model 4 adds the mean of tract income ($INC$) and income interacted with distances $u$ ($UINC$) and $y$ ($YINC$) to model 2 to capture the effect of income on population density. Model 5 adds $DUM$ to model 4. Model 6 is equivalent to model 2 except that higher-order distance terms are included (equation 111). Finally, model 7 includes the variable $DUM$ to model 6. Table 16 summarizes the regression results for the noncoastal cities selected for analysis using the basic model. In table 16, the coefficients and significance levels of the perpendicular
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<th>Coastal SMSAs</th>
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<td>Santa Barbara, California</td>
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<th>Noncoastal SMSAs</th>
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<td>Akron, Ohio</td>
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<td>Oklahoma City, Oklahoma</td>
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<td>Indianapolis, Indiana</td>
<td>Spokane, Washington</td>
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distances to lines of bisection are given for each noncoastal city. These then function as a basis of comparison with their counterparts for the coastal cities in model 2.

The amenity gradient (the coefficient of y) in the basic model varies considerably with the city being studied. In Santa Barbara, California the estimated y gradient is .15 and is clearly the dominant variable explaining the decay of population density. In Bridgeport, Connecticut the y gradient is .16, more than twice the u gradient for that city. In Miami, Florida the y gradient is .23, again considerably larger than the u gradient. However, in Milwaukee, Wisconsin, the y gradient is positive though not significant.

For most cities, the size of the estimated population density gradients for the distance u are lower than reported in earlier studies. In the basic model they range from .009 in Corpus Christi, Texas to .16 in Duluth Minnesota and in model 1 where the amenity distance is excluded the range is from .02 in Daytona Beach, Florida to .21 in Bridgeport, Connecticut. Three reasons appear to account for the differences with earlier studies. First, over time, population density gradients have been observed to be declining due to the supposed decline in transport costs. Here, 1980 census data is used, whereas, for example, Muth (1969) used 1950 census data. Second, by weighting observations by their geographic areas, we remove the upward bias in the u gradient inherent in some earlier studies. Third, in the basic model, the presence of the coast removes some of the explanatory power of radial distance.
For the coastal cities, 93% of the estimated coefficients of the amenity distance term, y, in model 2 were negative and statistically significant at at least the 10% level. Only 16% of the same coefficients for the noncoastal cities were negative and significant at the 10% level. This suggests that while 16% is surprisingly high for a variable which should according to theory be insignificant most of the time, that there does seem to exist a distinct difference in the explanatory power of the variable in coastal versus noncoastal cities.

The likely explanation of why many noncoastal cities experience significant (positive and negative) coefficients of the perpendicular distance to bisection is to be found in the fact that the standard urban model does not capture the richness of the real world. That is, no city studied sits on a flat featureless plain. Instead, many topographical barriers, historical and political factors exist which cause city development to be skewed north or south, east or west and so on. Since the distances YNS, YEW, YNE, and YNW were measured from a tract regardless of where the tract lay, these variables may to some extent reflect differences in development stemming from these factors.

The types and condition of transportation facilities belie the assumption that travel cost per unit distance is everywhere the same. Radial sector theory is but one of many other explanations. Radial-Sector theory would suggest that location decisions depend on transportation routes. For example, Interstate 59 traversing Birmingham, Alabama might be a low-cost transportation route causing residents to locate relatively closely to the northeast-southwest bisection of the city. This could explain why the YNE coefficient is
negative and significant. This interpretation is inadequate in explaining many of the observed results in Table 16. For example, the negative coefficient on YNW in Oklahoma City occurs even though no major road runs in that direction. Even more, the bisection variables were not constructed to correspond to the major road arteries running through the noncoastal cities. It is simply happens to be the case that major roads are built to run north-south, east-west, etc. Thus, the coefficients should not be interpreted as measures of distance to low-cost transportation routes.9

In the basic model, in 64% of the cases for the coastal cities the absolute value of the density gradient for y exceeded that of the traditional density gradient, i.e., the coefficient of u. This suggests that the draw or impact of the coast is considerable for many of the cities studied. It is interesting that in none of the 32 cases studied for noncoastal cities did the y gradient exceed the u gradient in absolute value and was almost always considerably smaller. This is also reflected by the fact that the value of the u density gradient rarely fell appreciably when the y term was introduced into the model.

As would be expected when an additional variable is introduced explaining the decay of population density away from the CBD, the density gradient for u is smaller in absolute value for all cities except Milwaukee after y is introduced into the model. In Cleveland, the density gradient for u falls from .20 to .12 or by .08. For Bridgeport, Connecticut, the coefficient falls by .15, and for Ft. Lauderdale, Florida by .11. This thus suggests that without the inclusion of the variable picking-up the effect of the coastal amenity,
population density gradients for \( u \) are biased upward. Model 2 in the tables then should more accurately measure the true gradient for \( u \).

Usually, when one thinks of a Tiebout world, one has in mind the distinction between the large incorporated central city and the suburbs surrounding it. That is, suburbs surrounding the central city are thought of as relatively homogeneous in their tax structure and provision of public services (as well as in their income level and racial composition). To exploit this distinction, a dummy variable, DUM, was introduced in models 3, 5, and 7 which takes a value one if the tract was in the major city of the SMSA and zero otherwise. As mentioned above, if the tract happened to contain a portion in and a portion out of the central city the tract was split. Thus, it is assumed that the population density gradient does not change as a result of leaving the central city but instead a discontinuity (change in intercept) arises from such a move. This approach by no means represents an adequate test of the Tiebout hypothesis but does at least capture some of the problems addressed by Tiebout.

In 64% of the cases of model 3 containing the variable DUM, the coefficient was significant at at least the 10% level. However, for the larger cities, Chicago, Cleveland, Miami, and San Diego, the coefficient of DUM was not statistically significant. Only in Milwaukee is DUM significant. In eight of nine or 89% of the cases which were statistically significant the variable DUM was positive. In model 5 DUM was significant 71% of the time at the 10% level. In 9 out the 10 times the variable was significant, it was positive. In model 7, DUM was
significant 50% of the time at at least the 10% level. In these seven cases the coefficient was positive.

As noted, for most large cities, the variable DUM does not appear to significantly affect population density. This seems surprising in view of the widely held impression that larger cities offer a less satisfactory bundle of public goods and are more prone to racial problems. The answer may lie in the likelihood that suburbs immediately surrounding larger cities are more like the central city than suburbs surrounding central cities in smaller SMSA's.

The fact that DUM is frequently positive and significant for smaller SMSA's suggests that as one leaves the central city there is a jump in population density. Suburbs tend to zone property so that it may only be used for single-family occupants and to impose larger lot size requirements than the central city. Since the data rejects a fall in density at the city-suburb border it might be inferred that density jumps as a consequence of a more attractive tax-benefit package or quite possibly a different ethnic mix (possibly being reinforced by landlords and real estate agents directing perspective renters and buyers of different ethnicity to "appropriate" areas.

Models 4 and 5 contain an income term (INC, measured as mean of income for the census tract) and income interacted with distances u(UINC) and y(YINC). As noted above, this procedure does not ensure unbiased coefficients but should be thought of as a way to deal with nonconstant income in an SMSA. Unfortunately, the introduction of the interaction terms frequently leads one to conclude that y or u or both should not appear in the model. Keeping in mind that income is measured
in thousands, the magnitude of the estimated parameters is extremely small. The median income of the cities studied varies from $15,351 in Daytona Beach, Florida to $22,830 in Santa Barbara, California. In Daytona Beach, a person with median income will reside at a point where the u gradient is positive .097 and y gradient equal to -.11. In general the values of the coefficients on the interaction terms are not large enough to alter the sign of the gradient. While important in affecting population density, the solution found in model 4 and 5 is inadequate in finding unbiased estimates when multiple income groups exist.

Models 6 and 7 contain the squared terms in u and y. An appealing test of the usefulness of these models was given above. There, population density reaches a maximum at y-u (z=0) when \( y^* = -(-\beta_1 + \beta_2)/2(\beta_3 + \beta_4) \) and \( \beta_3 + \beta_4 < 0 \). Along the coast, when y=0, population density reaches a maximum when \( u^* = -\beta_1/2 \beta_3 \) and \( \beta_3 < 0 \). If these models accurately portray the population patterns for the coastal cities studied, \( y^* \) and \( u^* \) should be rather small in value since the CBD's for all the coastal cities were usually chosen to be within one mile of the ocean or lake front. Interestingly, for most cities population density fell immediately from the CBD both directly inland or along the coast. That is, for most cities \( \beta_3 > 0 \) and \( \beta_3 + \beta_4 > 0 \). In almost all cases the minimum in population occurred at a distance beyond the extent of the city. For example, \( u^* \) occurs at 70 miles along the coast in the city of Miami, 19 miles for Erie, and 22 miles for Chicago. The value for \( y^* \) occurs at 33 miles for San Diego, 20 miles for Daytona Beach, and 30
miles for Milwaukee. Only for the cities of Cleveland, Fort Lauderdale, and Miami do the values of $y^*$ and $u^*$ seem realistic. In only one city (Gary) are the higher-order terms insignificant in both models 6 and 7. This result is interesting in view of Latham and Yeates (1970) findings for Toronto. That is, it is likely that while picking-up variations in population density, the higher-order terms do not, in general, function to locate the points at which land use changes from primarily commercial to primarily residential.

An interesting feature of models 2 and 3 is that they permit one to say something about the predicted spatial extent of a coastal city. Specifically, a coastal city with an amenity coast line is expected to extend along the coast farther than it extends inland. The relative extent can be found by noticing that \((\beta_1+\beta_2)y_A/\beta_1=(p_3+s)y_A/p_3\). For those cities for which $\beta_1, \beta_2<0$, the spatial extent of the city varies considerably. For example, the boundary along the coast is only 1.3 times the boundary directly inland for Duluth but 2.3 times this for Chicago and 4.25 for San Diego and 11 for the city of Corpus Christi. As a general observation, ocean coast cities are more skewed toward the coast than lake cities. The reason is likely due to the larger number of recreational activities associated with an ocean setting (e.g., surfing, deep-sea fishing) and to the tourist trade that entails which may imply more employment opportunities along the coast and thereby attract residents wishing to locate near their place of employment.
TABLE 2.
REGRESSION RESULTS FOR THE CITY OF SANTA BARBARA$^a$

<table>
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<th>$y^2$</th>
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<th>INC</th>
<th>UINC</th>
<th>YINC</th>
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<td>.11</td>
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<td>-7x10^4</td>
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</table>

$^a$A total of 74 census tract observations were used for Santa Barbara. The CBD was defined as the geographical center of tract 9. The city was defined as most of Santa Barbara County.

$^b$The numbers in parentheses are t statistics.
TABLE 3.
REGRESSION RESULTS FOR THE CITY OF CLEVELAND\textsuperscript{a}

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<th>INC</th>
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<th>YINC</th>
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<tr>
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</tbody>
</table>

\textsuperscript{a}A total of 373 census tract observations were used for Cleveland. The CBD was defined as the intersection of St. Clair Ave. and E. 9th Street. The city was defined as all of Cuyahoga County.

\textsuperscript{b}The numbers in parentheses are t statistics.
### TABLE 4.
**REGRESSION RESULTS FOR THE CITY OF GARY, INDIANA**

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<th>y²</th>
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<th>INC</th>
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<th>YINC</th>
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<tr>
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</tbody>
</table>

A total of 101 census tract observations were used for Gary, Indiana. The CBD was defined as the intersection of Madison St. and 15th Avenue. The city was defined as Lake County.

The numbers in parentheses are t statistics.
TABLE 5.
REGRESSION RESULTS FOR THE CITY OF PORTLAND, MAINE

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<td>(1.5)</td>
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\textsuperscript{a}A total of 53 census tract observations were used for Portland, Maine. The CBD was defined as the intersection of Clark St. and Commercial Avenue. The city was defined as the entire SMSA.

\textsuperscript{b}The numbers in parentheses are t statistics.
TABLE 6.
REGRESSION RESULTS FOR THE CITY OF ERIE, PENNSYLVANIA

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<th>y²</th>
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a A total of 58 census tract observations were used for Erie, Pennsylvania. The CBD was defined as the intersection of Holland St. and E. 12th Street. The city was defined as Erie County with some outlying tracts excluded.

b The numbers in parentheses are t statistics.
TABLE 7.
REGRESSION RESULTS FOR THE CITY OF DULUTH, MINNESOTA

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A total of 52 census tract observations were used for Duluth, Minnesota. The CBD was defined as the intersection of Mesaba Ave. and W. 3rd Street. The city was defined as Douglas and Superior Counties with some outlying tracts excluded.

The numbers in parentheses are t statistics.
TABLE 8.
REGRESSION RESULTS FOR THE CITY OF BRIDGEPORT, CONNECTICUT\textsuperscript{a}

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\textsuperscript{a} A total of 105 census tract observations were used for Bridgeport, Connecticut. The CBD was defined as the intersection of Fairfield Ave. and Broad Street. The city was defined as the SMSA.

\textsuperscript{b} The numbers in parentheses are t statistics.
TABLE 9.
REGRESSION RESULTS FOR THE CITY OF CORPUS CHRISTI, TEXAS

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\(^a\) A total of 59 census tract observations were used for Corpus Christi, Texas. The CBD was defined as the intersection of Agnes St. and BrownLee Boulevard. The city was defined as the SMSA.

\(^b\) The numbers in parentheses are t statistics.
TABLE 10.
REGRESSION RESULTS FOR THE CITY OF MILWAUKEE, WISCONSIN

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</table>

a A total of 354 census tract observations were used for Milwaukee. The CBD was defined as the intersection of E. Wisconsin Ave. and N. Van Buren Street. The city was defined as Milwaukee County.

b The numbers in parentheses are t statistics.
TABLE 11.
REGRESSION RESULTS FOR THE CITY OF DAYTONA BEACH, FLORIDA

<table>
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</tbody>
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A total of 53 census tract observations were used for Daytona Beach. The CBD was defined as the intersection of FEC Ry. and 2nd Avenue. The city was defined as Volusia County with some outlying tracts excluded.

The numbers in parentheses are t statistics.
TABLE 12.
REGRESSION RESULTS FOR THE CITY OF FORT LAUDERDALE, FLORIDA$	extsuperscript{a}$

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<th>INC</th>
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<th>YINC</th>
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<td>-.07</td>
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</table>

$	extsuperscript{a}$A total of 157 census tract observations were used for Fort Lauderdale. The CBD was defined as the intersection of Broward Blvd. and Andrews Street. The city was defined as most of Broward County with some outlying tracts excluded.

$	extsuperscript{b}$The numbers in parentheses are t statistics.
### TABLE 13.
REGRESSION RESULTS FOR THE CITY OF MIAMI, FLORIDA

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</tr>
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</table>

*a* A total of 212 census tract observations were used for Miami. The CBD was defined as the intersection of NE 15th Ave. and Biscayne Boulevard. The city was defined as Dade County with the Everglades excluded.

*b* The numbers in parentheses are t statistics.
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<th>INC</th>
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<th>YINC</th>
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<td>(19.90)</td>
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<td>(-15.06)</td>
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</table>

*A total of 363 census tract observations were used for San Diego. The CBD was defined as the intersection of State and Market Streets. The city was defined as San Diego County with some outlying tracts excluded.*

*The numbers in parentheses are t statistics.*
### TABLE 15.
REGRESSION RESULTS FOR THE CITY OF CHICAGO, ILLINOIS

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<td>9x10^4</td>
<td>-1x10^3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(56.7)</td>
<td>(-6.8)</td>
<td>(1.8)</td>
<td>.07</td>
<td>(-5.2)</td>
<td>(6.5)</td>
<td>(-8.6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>10.2</td>
<td>-.21</td>
<td>.07</td>
<td>.005</td>
<td>-.007</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(71.1)</td>
<td>(-9.9)</td>
<td>(3.4)</td>
<td>(7.9)</td>
<td>(-7.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>10.4</td>
<td>-.22</td>
<td>.07</td>
<td>.005</td>
<td>-.007</td>
<td>-.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(47.3)</td>
<td>(-9.4)</td>
<td>(3.1)</td>
<td>(7.9)</td>
<td>(-7.7)</td>
<td>(-1.3)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*a* Total of 1192 census tract observations were used for Chicago. The CBD was defined as the intersection of Madison and State Streets. The city was defined as all of Cook County.

*b* The numbers in parentheses are t statistics.
### Table 16.
The Basic Model for the Noncoastal Cities

<table>
<thead>
<tr>
<th>CITY</th>
<th>u</th>
<th>YNS</th>
<th>YEW</th>
<th>YNE</th>
<th>YNW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akron, Ohio</td>
<td>-.20***</td>
<td>-.04</td>
<td>.01</td>
<td>-.03</td>
<td>.05</td>
</tr>
<tr>
<td></td>
<td>(-11.1)</td>
<td>(-.83)</td>
<td>(.18)</td>
<td>(-.72)</td>
<td>(1.28)</td>
</tr>
<tr>
<td>Birmingham, Alabama</td>
<td>-.21***</td>
<td>-.04</td>
<td>.06***</td>
<td>-.06***</td>
<td>.05***</td>
</tr>
<tr>
<td></td>
<td>(-16.1)</td>
<td>(-.28)</td>
<td>(3.7)</td>
<td>(-3.8)</td>
<td>(2.7)</td>
</tr>
<tr>
<td>Charleston, West VA.</td>
<td>-.21**</td>
<td>.03</td>
<td>.06</td>
<td>.02</td>
<td>-.19</td>
</tr>
<tr>
<td></td>
<td>(-2.3)</td>
<td>(.16)</td>
<td>(.26)</td>
<td>(.06)</td>
<td>(-.87)</td>
</tr>
<tr>
<td>Indianapolis, Indiana</td>
<td>-.25***</td>
<td>-.008</td>
<td>.05</td>
<td>-.06***</td>
<td>.09***</td>
</tr>
<tr>
<td></td>
<td>(-11.6)</td>
<td>(-.33)</td>
<td>(1.8)</td>
<td>(-2.9)</td>
<td>(4.23)</td>
</tr>
<tr>
<td>Knoxville, Tennessee</td>
<td>-.15***</td>
<td>.03***</td>
<td>-.05***</td>
<td>-.02</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>(-16.7)</td>
<td>(2.89)</td>
<td>(-3.63)</td>
<td>(-1.48)</td>
<td>(1.59)</td>
</tr>
<tr>
<td>Lexington, Kentucky</td>
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<td>-.07</td>
<td>-.03</td>
<td>-.12</td>
<td>.23***</td>
</tr>
<tr>
<td></td>
<td>(-4.92)</td>
<td>(-.76)</td>
<td>(-.28)</td>
<td>(-1.65)</td>
<td>(2.89)</td>
</tr>
<tr>
<td>Oklahoma City</td>
<td>-.09***</td>
<td>-.004</td>
<td>.01*</td>
<td>.07***</td>
<td>-.03***</td>
</tr>
<tr>
<td></td>
<td>(-16.6)</td>
<td>(-.67)</td>
<td>(1.74)</td>
<td>(5.19)</td>
<td>(-4.11)</td>
</tr>
<tr>
<td>Spokane, Washington</td>
<td>-.19***</td>
<td>-.07***</td>
<td>.05</td>
<td>-.02</td>
<td>.04**</td>
</tr>
<tr>
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<td>(-11.9)</td>
<td>(-3.88)</td>
<td>(1.57)</td>
<td>(-1.47)</td>
<td>(2.42)</td>
</tr>
</tbody>
</table>

aThe CBD's were defined as intersections of important downtown thoroughfares, Birmingham: 8th Ave. and 17th Street; Charleston, W.V.: Washington St. and Park Avenue; Lexington, Kentucky: W. Main St. and Mill Street; Indianapolis: Delaware and Washington Streets; Akron: Main and Bovery Streets; Spokane: Intersection of N. Howard St. and W. Tent Avenue; Oklahoma City: Main St. and Walker Avenue; Knoxville: Union and Market Streets. The sample size of the cities usually consisted of the major county in the SMSA and sometimes only part of the county if the city was small.

bThe coefficient for u is for model 1 in which the only distance variable included was radial distance to the CBD.

cSignificance levels are as follows: * if significant at the 10% level, ** if significant at the 5% level, and *** if significant at the 1% level.
CHAPTER V
EXTENSIONS

THE OPTIMAL SETBACK

Brown and Pollakowski (1977) note that, "economists have not yet turned their attention to the economic significance of the existence and width of the undeveloped apron offering public use and access to bodies of water in urban areas." They go on to test the significance of the width of the "setback" from the water body on property values in the Seattle area. They hypothesize that property values, V, are related to perpendicular distance to the water body and the width of the setback multiplicatively,

\[ V = A y^b y_s^c \quad b < 0, \quad c > 0 \quad (124) \]

Where, \( y \) is the distance of residence to the water; \( y_s \) is the width of the setback; and \( A \) is a collection of housing attributes and a constant term. They concluded that as the setback width increased, property values increased but at a decreasing rate.

There is some evidence that many coastal cities recognize the advantages of having coastal land preserved for public use. Since 1981, the city of Milwaukee has added 15 acres of park land at the lakeshore, increased public access to the coast, and improved the Summerfest grounds. The city of Erie has acquired Erie Sand and Gravel and the Grain Dock for waterfront development and has embarked on an aggressive marina development program. The city of Chicago has 26 miles of
lakefront with 2,500 acres devoted to parkland. Along its "greenbelt" are located eight recreational harbors and 21 public beaches. Seven major museums are located along the coast.

It is possible in the context of the analysis of Chapter III to incorporate the width of the setback into the residential model. The amenity at the coast may be thought of as providing more utility to residents if the setback width (which before was assumed to be of width \( \varepsilon \)) is increased. Before, the number of trips, \( n \), was assumed independent of the relevant variables of the model. If the number of trips increases as the setback width increases but at a decreasing rate, the setback width can be incorporated into the transportation cost function. The increase in recreational visits might arise because a greater setback width may imply an increase in the number of recreational activities associated with the coast, e.g., biking, walking, picnicking, etc., or easier public access to the water body.

The hypothesis that the number of trips increases at a decreasing rate is justifiable on several grounds. If the commodities the consumer enjoys at the coast are inherently of the same nature and if marginal utility of these goods increases at a decreasing rate as is the case with strictly concave utility functions, "consumption" of trips is likewise likely to increase but at a decreasing rate per period. Since consumers are constrained in the amount of time available per period, they must eventually limit the time spent at the coast. Before, the cost of getting to the amenity in a period of time was of the form \( \pi s y \).
Where, \( \pi \) was constant. Maintain the assumption that the number of trips taken per period is independent of residential location, \( y \), but let the number of trips taken increase with the width of the setback,

\[
\pi(y_S) = \pi_0 + \pi_1(y_S), \quad \pi' > 0, \quad \pi'' < 0.
\]

Where \( \pi_0 \) represents the number of trips taken to the amenity in the time period being considered. Thus, the transportation cost per period to the amenity is

\[
\pi(y_S) s y.
\]

If \( y_s \) is small relative to the entire metropolitan area, the rent function derived in Chapter III is still of the same basic form since \( \pi(y_s) \) does not depend on either \( u \) or \( y \). The residential rent function is reexpressible as

\[
R(y,u) = R_A \exp\left\{ \frac{-\left(p_3 + \pi(y_s)s\right)}{\Gamma} \left[ \frac{p_3u + \pi(y_s)sy}{p_3 + \pi(y_s)s} - y_A \right] \right\}. \tag{125}
\]

Brown and Pollakowski suggest that for a given distance \( y \), \( V_{y_S} > 0 \) and \( V_{y_S} y_S < 0 \). Are these results also true for (125)? To see under what conditions this will be true, consider

\[
R_{y_S} = \frac{R(y,u)}{\Gamma} s \pi' [y_A - y] > 0 \text{ since } \pi' > 0 \text{ and } y_A > y.
\]

Likewise,

\[
R_{y_S y_S} = \frac{s(y_A - y)}{\Gamma} \left[ R(y,u) \pi'' + R_{y_S} \pi' \right] < 0 \text{ if } \frac{R_{y_S}}{R(y,u)} < \frac{-\pi''}{\pi'}.
\]

Note that unlike Brown and Pollakowski's simple estimating form, equation (125) is nonlinear in \( y \) and \( y_S \) making econometric estimation less tractable.
A local government trying to maximize its property tax revenues can do so by choosing the optimal width of the setback. To do so requires knowledge of the way in which setback changes affect residential willingness to pay for location privileges. The relevant criterion a jurisdiction can choose is the difference between the increase in tax revenues as a result of higher rents residents in the city are willing to pay and the opportunity cost in property tax revenues as a result of using more coastal land for the setback.

The choice of the optimal setback varies with the nature of the jurisdiction in question. If the planner is concerned with the larger metropolitan area considered above, his object is to find the total rent in the city residents are willing to pay for a given setback less the amount of rental revenue foregone as a result of not using the setback for residential purposes. This is true if the property tax rate is spatially constant. In this instance, the planner's objective function is given by,

$$
W_0 = \int_{y_s}^{y_A} \int_{u_s}^{u_A} \omega' \cdot R(y,u) \, du \, dy - \int_{y_s}^{y_A} \int_{0}^{u_s} \omega' \cdot R(y,u) \, du \, dy.
$$  

(126)

Where \( u' = \left[ (p_3 + s)y_A - sy \right] / p_3 \),

\[
R(y,u) = R_A \exp\{-b/\Gamma [p_3u + sy]/b - y_A\},
\]

and \( b = p_3 + s \pi(y_s) \).

An interesting feature of this analysis which deserves attention but is ignored below is the possibility of letting commercial developers (or even the city itself) occupy the setback. If private concerns occupy the setback and develop it such that they induce residents to commute to the coast no rental property tax revenue need be foregone. For example,
a commercial establishment may build restaurants, surf and souvenir shops, etc. Dining at the ocean is not the same commodity as dining by city hall; atmosphere is everything.

Equation (126) is expressible in the form

\[ W_0 = \frac{-4\Omega R_A}{p_3} \left[ y_A^2 + \frac{\Gamma}{b} - 2y_s - \frac{2\Gamma}{b} \exp\left(\frac{b}{\Gamma}(y_A - y_s)\right) + \frac{\Gamma}{b} \exp\left(\frac{b y_A}{\Gamma}\right) \right]. \]

Note that \( W_0 \) is equal to the total rent of the city when \( y_s = 0 \) and \( \pi \) is constant and that \( W_0 \) is equal to minus the total rent of the city when \( y_s = y_A \) and \( \pi \) is constant. By using the Taylor series approximations

\[ \exp\left(\frac{b}{\Gamma}(y_A - y_s)\right) = 1 + \frac{b}{\Gamma} (y_A - y_s) + \frac{b^2}{2\Gamma^2} (y_A - y_s)^2 \]

and

\[ \exp\left(\frac{b y_A}{\Gamma}\right) = 1 + \frac{b y_A}{\Gamma} + \frac{b^2}{2\Gamma^2} y_A^2, \]

the objective function can be compactly written as

\[ W_0 = \frac{bR_A}{p_3} \left[ \frac{1}{2} y_A^2 + y_s^2 - 2y_s y_A \right]. \] (127)

To ensure \( W_0 > 0 \), the expression in brackets must be positive, i.e.,

\[ \frac{1}{2} y_A^2 + y_s^2 > 2y_s y_A. \]

For example, if \( y_s > y_A \) then \( W < 0 \) and no land is available residential use.

In general, if \( y_s \) is large relatively to \( y_A \), too much land is being foregone. The approximation of \( W_0 \) given in (127) allows an explicit solution for \( y_s \) when \( W_0 = 0 \). If \( h = y_s/y_A \), then \( h = 2 - \sqrt{2} \) when \( W_0 = 0 \).

To find the optimal setback, the planner need only ascertain the maximal value of (127) and solve for \( y_s \). The expression which implicitly defines the optimal width of the setback is given by,

\[ 0 = 2(p_3 + n s)(y_s - y_A) + sn'[1/2 y_A^2 + y_s^2 - 2y_s y_A]. \] (128)
If $W_0$ has only one maximum value at the optimal setback, $y_s^* > 0$, then $dW_0/dy_s$ evaluated at $y_s = 0$ must be positive or that $\pi y_s^2/2 > 2(p_3 + ns)y_A^*$. If this is not satisfied then the model suggests that the optimal setback is at zero or more accurately, if it were possible, at $y_s < 0$. Likewise, $dW_0/dy_s$ evaluated at $y_s = y_A^*$ must be negative, $-\pi y_A^2/2 < 0$, or that $W_0$ is falling at $y_s = y_A^*$.

It is possible to perform comparative static analysis using (128). The planner can determine the effect on the optimal setback, $y_s^*$, as a result of a change in $p_3$ by appealing to the expression,

$$dy_s^*/dp_3 = 2(y_A^* - y_s^*)/\Delta < 0.$$  \hspace{1cm} (129)

Where, $\Delta < 0$ is an expression which ensures maximization of $W_0$ and

$$\Delta = 4\pi y_s^*(y_s^* - y_A^*) + 2y_s^*(y_A^2/2 + y_s^* - 2y_A^*y_s^*) + 2(p_3 + ns).$$

Likewise, the effect of a change in $s$ on $y_s^*$ is given by,

$$dy_s^*/ds = -\{\pi [1/2 y_A^2 + y_s^2 - 2y_A^*y_s^*] + 2\pi(y_s^* - y_A^*)\}/\Delta.$$  \hspace{1cm} (130)

Which is indeterminate in sign.

The change in the optimal setback when the size of the city, $y_A^*$, changes is also indeterminate using (128). In this case,

$$dy_s^*/dy_A = \{\pi (2y_s^* - y_A^*) + 2(p_3 + ns)y_A^*\}/\Delta.$$  

To derive the comparative static solutions for terms involving $\Gamma$, specifically income, $W$, and the income elasticity of demand for housing services, $\theta$, $W_0$ must be approximated with a higher-order Taylor series approximation. In this case, $W_0$ is written as

$$W_0 = 4\pi R_A b \left[ y_A^2/2 + y_s^2 - 2y_A^*y_s^* + \Gamma \left[ y_A^3/6 - y_A^2y_s^* + y_A^*y_s^2 - y_s^3/3 \right] \right]/p_3$$

$$= 4\pi R_A b P/p_3.$$  

The comparative static solution for $W$ is here given by
\[ \frac{dy_s^*}{dW} = \Theta_1 b \left[ s\pi \varpi_1 - b\varpi_2 \right] / \Delta W . \]

Where \( \Delta < 0 \) is different from the one defined above and
\[ \varpi_1 = \frac{y_A^3}{6} - \frac{y_2^2 y_A}{2} + \frac{y_A y_S^2 - y_S^3}{3} , \]
and
\[ \varpi_2 = (y_A - y_S)^2 , \]
and
\[ \Delta = s\pi \rho + 2s\pi \left[ 2y_S - 2y_A - b \varpi_2 / \Gamma \right] + b[2 + 2b(y_A - y_S) / \Gamma] . \]

Likewise for \( \vartheta_1 \),
\[ \frac{dy_s^*}{d\vartheta_1} = b \ln(W) \left[ s\pi \varpi_1 - b \varpi_2 \right] / \Delta \Gamma . \]

In the case of a jurisdiction which is a small part of the larger metropolitan area, many unique problems arise. An increase in the width of its setback entails not only an increase in the number of visits to the coast by residents living within the jurisdiction but also visits residing beyond. Argumentum ad absurdum, if the jurisdiction is narrowly located along the coast, the optimal setback is of width zero. In general, the planner in such a jurisdiction must carefully evaluate the attendant effects of choosing the setback width. A jurisdiction located along the coast is likely to be irregularly shaped. A simple and useful approximation to capture this irregularity is illustrated in Figure 10. The jurisdiction is bounded east-west by two radians from the CBD of length \( u_0 \) and \( u_1 \) and from the north by line of distance \( y_0 \).

In this case, the objective function is very simple and easily solved,
\[ W_0 = \int_{u_0}^{u_1} \int_{y_S}^{y_0} \Omega \exp \left\{ -b \left[ (p_3 u + sny) / b - y_A \right] \right\} dydu - \int_{u_0}^{u_1} \int_{y_S}^{y_4} \Omega \exp \left\{ -b \left[ (p_3 u + sny) / b - y_A \right] \right\} dydu . \]  

(131)
FIGURE 10

HYPOTHETICAL JURISDICTION WITHIN A METROPOLITAN AREA
Where,
\[ u_0 = \sqrt{y_0^2 + z_0^2}, \]
\[ u_1 = \sqrt{y_1^2 + z_1^2}, \]
and, \( b \) is as defined above.

Equation (131) is solvable in the quite tractable expression given by,
\[
W_0 = \frac{\Gamma c_0}{s\pi} \frac{b}{\Gamma} \frac{y_0}{\Gamma} \left[ 2e^{-s} - e^{1-s} \right].
\] (132)

Where,
\[ c_0 = 4\pi R_A \frac{sny_p^{c_0}}{p_3 > 0}. \]
If \( y_s = 0 \) and \( n \) is constant, then (132) reduces to the total rent of the jurisdiction,
\[
W_0 = \frac{\Gamma c_0}{s\pi} \frac{b}{\Gamma} \frac{y_0}{\Gamma} \left[ 1 - e^{1-s} \right].
\]
If, on the other hand, \( y_s = y_0 \) we get minus this value, or all of the potential rent of the jurisdiction is lost.

Again, the planner requires \( W_0 \) to reach a maximum to solve for the optimal setback, \( y_s^{*} \). Equation (132) is greatly simplified by using a linear Taylor series approximation. In this case, \( W_0 \) is approximated as
\[
W_0 \approx c_0 e^{b/\Gamma} \left[ y_0 - 2y_s \right].
\] (133)
Here, \( W_0 \) equals zero when \( y_0 = 2y_s \). Thus, for \( W_0 \) to be positive we require that \( y_s < y_0/2 \).

The optimal setback width is obtainable numerically from the approximation in (133) and is given by
\[
dW_0/dy_s = c_0 e^{b/\Gamma} \frac{s\pi}{\Gamma} \frac{y_0 - 2y_s}{y_s} = 0.
\] (134)
Again it is possible to say something about the slope of $W_0$ using (134). The optimal $y_s$ will be positive if the term in brackets is positive, otherwise it will be zero. The slope of $W_0$ is definitely negative when $y_s = y_0$ suggesting that indeed $W_0$ is falling at the boundary of the jurisdiction.

Several comparative static solutions follow immediately

$$\frac{dy_s^*}{dy_0} = -\pi'y_A'/\Delta \Gamma > 0 \tag{135}$$
$$\frac{dy_s^*}{dW} = \theta_1\pi' y_A(y_0 - 2y_s)/\Delta W < 0 \text{ (where, } W \text{ refers to income)} \tag{136}$$
$$\frac{dy_s^*}{d\theta_1} = \pi' \ln(W)y_A(y_0 - 2y_s)/\Delta \Gamma < 0 \tag{137}$$
$$\frac{dy_s^*}{ds} = -\pi' y_A(y_0 - 2y_s)/\Delta \Gamma > 0 \tag{138}$$
$$\frac{dy_s^*}{dy_A} = -\pi' (y_0 - 2y_s)/\Delta \Gamma > 0. \tag{139}$$

where, $\Delta = sy_A(\pi''[y_0 - 2y_s] - 2\pi')/\Gamma < 0$ for a maximum in the objective function. This is satisfied as long as $\pi'' < 0$. As noted above, intuitively, $y_s^*$ must decrease when $y_0$ increases. Since the rent function increases as $y_s$ increases, the setback is increased until the point where the marginal increase in rent is equal to that lost as a consequence of using more land for the setback. But, if $y_0$ is increased, that $y_s^*$ is no longer optimal, not because rent is lost as a consequence of the increase in $y_0$, but because rent from the additional part of the jurisdiction is added to the total rent. The signs of the comparative statics on $y_A$ and $s$ are explained by the way in which the rent function of the SMSA changes in response to an increase in these two variables. The rent function of the SMSA increases with both variables suggesting that for the jurisdiction, the optimal setback increases whenever the rent function increases. The intuition goes as follows: if the rent is constant at the opportunity cost of land, the
optimal setback is clearly zero. It is only when the rent gradient becomes negative that a nonzero setback is justified. Since the rent function decreases with both $W$ and $\theta_1$, the optimal setback must fall with $W$ and $\theta_1$.

**THE INTRODUCTION OF CONGESTION**

Mills (1972) assumes that "the only relevant dimension of the urban transportation system is radial distance to the city center." In the model specified above commuters have two relevant dimensions to consider, distance to the amenity as well as the CBD. For simplicity, we shall consider the case where congestion occurs only when traveling to the CBD. This may be the case when, for example, travel to the amenity occurs at different times than travel to the CBD, e.g., on weekends only.

We will assume as does Mills, that the transportation system is made up of roads and that the system's design capacity at each contour of the city designated at $y$ is proportionate to the land devoted to transportation at that $y$,

$$X_{36}(y) = A_3 L_3(y).$$

(140)

Where, $L_3(y)$ is the amount of land devoted to transportation along the contour specifying the shape of the city at distance $y$. The scale parameter in the transportation production function is the value $A_3$ in equation (140).

The demand for transportation leading to the CBD is assumed to be equal to the number of laborers living beyond the contour given at $y$
When congestion occurs, demand for transportation facilities at a given contour about the CBD exceeds the design capacity of the system, given by (140). The approach chosen by Mills to account for road congestion is the same as that specified by Vickrey (1967) and the same chosen here

\[ p_3(y) = \tilde{p}_3 + \rho_1 \left( \frac{X_{3D}(y)}{X_{3S}(y)} \right)^{\rho_2} \]  

(142)

Where, \( \tilde{p}_3 \) is two times the cost of commuting when congestion is absent. This occurs only at the edge of the city in the model because that is the only place where \( X_{3D} \) is zero.

When congestion enters the model (44) is replaced by,

\[ \alpha_2 \, B_2 \, C \, W_1 \, R^\alpha(y,u) \left\{ \frac{\partial R(y,u)}{\partial y} + \frac{\partial R(y,u)}{\partial u} \right\} + p_3 + s = 0 \]  

(143)

Where, \( p_3 \) is given in (142). Equation (143) summarizes all the information contained in the model. Thus, the solution to (143) provides a general equilibrium solution to the urban model with an amenity coast line. In general, no analytic solution exists for (143); so that a numerical solution is required to make comparative statements about the structure of the city specified herein.

Mills also specified a congestion model which had to be solved numerically. Equation (143) is a partial differential equation whose solution is considerably more complex than those found in earlier models. The limits on \( u \) found in (142) depend on \( p_3 \) which is found in the integral argument of (143). The only place where this is known not to be the case is at the edge of the city where congestion is absent.
Thus, for any $y \leq y_A$, the value of $p_3$ must be constantly updated in an iterative fashion.

Mills (1972), Muth (1967, 1975), Altman (1981, 1983), Altman and DeSalvo (1981) consider various specification of the parameters discussed above. In no case, has a city characterized as noncircular been considered. Thus, it is not only important to have values which are current and realistic but parameter values which are representative of cities following the model specified above.

The most obvious difference between a city with an amenity coast line and one without is the existence of commuting to the coast. One obvious approach in dealing with the cost of commuting to the amenity, $s$, is to assume in the absence of congestion that $s$ is some fraction of $p_3$, where the actual cost is the same per unit distance but the fraction reflects the fewer trips made to the amenity by a resident for any given period of time. A choice which may reflect the work and leisure patterns of most residents is a fraction value of two (weekends) as is to five (workdays).
Utility Maximization With Budget and Time Constraints

Let the objective of the consumer be to maximize

\[ V = V(x_2, a(y)) \]  \hspace{1cm} (144)

subject to,

\[ W = p_2(y, u) x_2 + p_3 u + sy \]  \hspace{1cm} (145)

and

\[ T = H + a(y) + t_u u + t_y y. \]  \hspace{1cm} (146)

Where,

- \( T \): total time available per period;
- \( H \): hours worked (spatially-independent);
- \( a(y) \): time spent at the amenity (includes all leisure time);
- \( p_3 \): money costs of travel to the CBD;
- \( s \): money costs of travel to the amenity;
- \( t_u \): time required to travel one mile to the CBD (constant);
- \( t_y \): time required to travel one mile to the amenity (constant).

The lagrangean associated with the maximization process is given by,

\[ L = V(x_2, a(y)) + \lambda_1 [W - p_2(y, u) x_2 - p_3 u - sy] + \lambda_2 [T - a(y) - H - t_u u - t_y y] \]  \hspace{1cm} (147)

Therefore, first-order conditions for utility maximization are:

\[ L_{x_2} = V_{x_2} - \lambda_1 p_2(y, u) = 0 \]  \hspace{1cm} (148)

\[ L_a = V_a - \lambda_2 = 0 \]  \hspace{1cm} (149)

\[ L_y = (V_a - \lambda_2) \frac{\partial a}{\partial y} - \lambda_1 x_2 \frac{\partial p_2(y, u)}{\partial y} - \lambda_1 s - \lambda_2 t_y = 0 \]  \hspace{1cm} (150)

\[ L_u = -\lambda_1 x_2 \frac{\partial p_2(y, u)}{\partial u} - \lambda_1 p_3 - \lambda_2 t_u = 0. \]  \hspace{1cm} (151)

Where,
\( \lambda_1 \): marginal utility of money;
\( \lambda_2 \): marginal utility of time.

Equations (148), (149) and (150) imply

\[
-x_2 \frac{\delta p_2(y,u)}{\delta y} = \frac{p_2(y,u)}{V_a} \frac{V_y}{V_x} t_y + s. 
\]

(152)

That is, the reduction of housing expenditures as a result of a small increase in distance from the amenity must equal the money cost per mile to the amenity plus the valuation of time of travel to the amenity, i.e., \( p_2 \) times the marginal rate of substitution of \( a \) and \( x_2 \) times the amount of time needed to travel one mile. Likewise, (151) can be expressed as

\[
-x_2 \frac{\delta p_2(y,u)}{\delta u} = p_3 + p_2(y,u) t_u \frac{V_a}{V_x}. 
\]

(153)

Which has a similar interpretation as (152).
The Model When The CBD Is Not Centered At Zero

Let the CBD be centered at the point \( \alpha y_A \), \( 0 < \alpha < 1 \). Then the bounds on \( y \) and \( u \) are given by,

\[
0 < y < y_A \quad \text{and} \quad |y - \alpha y_A| < u < \left( \frac{p_3[1-\alpha] + s}{p_3} \right) y_A/p_3 - sy/p_3 .
\] (154)

At the boundary of the city, \( y_A \),

\[
y = y_A \quad \text{and} \quad u = \frac{y_A - \alpha y_A}{(p_3 + s)(1-\alpha) y_A}/p_3 ,
\]

\[
y = 0 \quad \text{and} \quad u = \frac{p_3(1-\alpha) + s}{p_3} .
\]

As Figure 11 suggests, initially, the rent contours (and inconsequence the population density contours) are oblong away from the coast. At what point does \( z_0 = z(y=0) = y_0 \)? At \( y_0 \), \( p_3(y_0 - \alpha y_A) + sy_0 = p_3 u + sy \). The value of \( z \) at the coast is therefore given by

\[
z_0^2 = \left[ p_3(y_0 - \alpha y_A) + sy_0 \right]^2/p_3^2 - (\alpha y_A)^2 .
\]

But, at \( y_0 = z_0 \),

\[
y_0 = \frac{2p_3(p_3 + s)\alpha y_A}{s(2p_3 + s)} .
\]

The procedure for finding the rent gradient for this city is similar to that used in Chapter III except that the boundary conditions are different. The solution to the rent function is given by,

\[
R(y,u) = R_A \exp\left\{ -(p_3 + s)/T \left[ \frac{p_3(u + \alpha y_A) + sy}{p_3 + s} - y_A \right] \right\} . \quad (155)
\]
Where, \( u^2 = (|y - \alpha y_A|)^2 + z^2 \).
Equation (155) implies the rent function is still log-linear in $u$ and $y$. Central density must be evaluated with care. Here, central density occurs where $y = \alpha y_A$ such that $u = \sqrt{([y - \alpha y_A] + z^2)} = 0$.

Where a CBD is located is substantially immutable to a city planner wishing to maximize the total rent payments of a city given by the boundaries above since the physical structure is at any point in time not malleable. However, a priori, the parameter $\alpha$ may be thought of as a choice variable in the planner's calculus. To see if total rent varies with $\alpha$, recall that total rent is expressible as follows,

$$TR = \int_0^{y_A} \int_{|y-\alpha y_A|} 4\Omega R(y,u) \, du \, dy. \quad (156)$$

Where, $R(y,u)$ is defined by (155) and $\Omega$ is the weighting factor defined above which assumes the value $\pi/4$ if $s=0$.

The solution to (155) is given by,

$$TR = \frac{4\Gamma^2 R_A \Omega}{p_3(p_3-s)} \left[ e^{-A/T} - e^{-B/T} \right] - \frac{4\Gamma^2 R_A \Omega}{p_3(p_3+s)} \left[ 1 - e^{-C/T} \right] - \frac{4\Gamma R_A \Omega}{p_3} \quad (157)$$

Where,

$$A = s\alpha y_A - (p_3(1-\alpha) + s)y_A,$$

$$B = p_3\alpha y_A - [p_3(1-\alpha)+s]y_A,$$

$$C = s\alpha y_A - [p_3(1-\alpha)+s]y_A.$$

To check under what circumstances total rent falls when $\alpha$ increases, consider the expression,

$$\frac{dTR}{d\alpha} = \frac{4\Gamma R_A \Omega}{p_3(p_3-s)} \left[ -(p_3+s)e^{-A/T} + 2p_3 e^{-B/T} \right] - \frac{4\Gamma R_A \Omega}{p_3} \left[ e^{-C/T} \right] \quad (158)$$
From (158) $d TR/d\alpha<0$ implies that $p_3>s$, and $d TR/d\alpha>0$ implies that $s>p_3$. This indicates that a planner choosing $\alpha$ will choose the coast if $p_3>s$ and $y_A$ if $s>p_3$. Note that this result hinges on the assumption that all residents commute to the amenity coast on a regular basis.
Location Decisions of Two Income Groups

Where do the rich (one income group) and the poor (the other) live in an amenity-coast city? In the standard urban model, one income group locates centrally and the other in the suburbs depending on parameter values such as the transport costs, income elasticity of demand for housing services, and income of each group. Interestingly enough, the results derived below are identical to that found using the standard urban model when \( s=0 \). Let the incomes of the two groups be \( W_r \) (rich) and \( W_p \) (poor) and the income elasticities of demand for housing services be \( \theta^r_1 \) and \( \theta^p_1 \), and assume the rich have a higher (time) cost of travel, so that \( p^r_3 > p^p_3 \) and \( s^r > s^p \).

The log of the rent function for the two groups is given as,

\[
\ln R^i = \ln R^i_0 - p^i_3 u/ \alpha^i_2 B^i_2 W^i_1 - s^i y/ \alpha^i_2 B^i_2 W^i_1 . \tag{159}
\]

Where, \( i=r,p \) and,

\[
\ln R^i_0 = \ln R^i_A + (p^i_3 + s^i) y^i_A/ \alpha^i_2 B^i_2 W^i_1 . \tag{160}
\]

The poor will locate centrally if they outbid the rich at the CBD or if

\[
(p^p_3 + s^p)/(p^r_3 + s^r) > y^r_A/ y^p_A \tag{161}
\]

Along the coast, when \( y=0 \) and \( z=u \), when do the rich begin to bid more for locations than the poor? That is, when does \( \ln R^p(0,z) = \ln R^r(0,z) \)? The intersection occurs when,

\[
z^* = (\ln R^p_0 - \ln R^r_0)/ [(p^p_3/ \alpha^p_2 B^p_2 W^p_1) - (p^r_3/ \alpha^r_2 B^r_2 W^r_1)] . \tag{162}
\]
In order for \( z^* > 0 \) (and since the numerator is assumed positive), we require that \( p_3^p / \theta_1^p > p_3^r / \theta_1^r \). Or, that the absolute value of the slope of the poor's rent function exceed that of the rich.

When \( z=0 \), \( u=y \), when does \( \ln R_p(y, y) = \ln R_r(y, y) \)? This happens when,

\[
y^* = (\ln R_p^p - \ln R_r^r) / \left[ \frac{((p_3^p + s^p) / \alpha_2 B_2 \theta_1^p) - ((p_3^r + s^r) / \alpha_2 B_2 \theta_1^r)}{\theta_1^p \theta_1^r} \right]. \tag{163}
\]

Again, in order for \( y^* > 0 \) (and since the numerator is assumed positive), we require that \( p_3^p / \theta_1^p > p_3^r / \theta_1^r \).

Finally, is \( y^* < z^* \)? This indeed will be the case as long as \( s^r \theta_1^r < s^p \theta_1^p \). And, note that if \( s^1 = 0 \), \( y^* = z^* \) and the results are the same as what one would find in the standard urban model. As a consequence, the amenity coastline model cannot be used to explain a pattern of location which has the rich locating centrally and in the suburbs with the poor sandwiched in between as in the city of Chicago.

The location decisions of the rich and poor are illustrated in Figure 12 for the present case. As long as (161) holds, the poor outbid the rich for the privilege of locating centrally. When \( s^1 = 0 \), or the standard urban model attains, \( z^* = y^* \), so that the boundary separating the rich and poor is a constant radius from the CBD.
FIGURE 12
THE LOCATION OF THE RICH AND POOR
Introduction of a Disamenity Coastline

The model must be modified if part of the coast is not suitable for use as an amenity described in the text; e.g., effluent emission causes the coast to become a disamenity to city residents or jagged cliffs make recreational consumption of that part of the coast impossible. Assume that a disamenity exists at part of the coast but stops at the waters edge. Let this portion of the coast be the range $z_0$ to $z_1$ in Figure 13.

If the disamenity stops at the coast, residents residing in the range $z_0$ to $z_1$ must commute an additional distance (greater than $y$) to consume the amenity. If they live at $z_0 \leq z \leq (z_0 + z_1)/2$, they will commute to the shortest point from residence suitable as an amenity, i.e., $z_0$. Likewise, if they live at $z_1 \geq z \geq (z_0 + z_1)/2$, they will commute to point $z_1$.

Define

$$y^m = \sqrt{y^2 + (z-z_0)^2} \quad \text{if} \quad z_0 \leq z \leq (z_0 + z_1)/2,$$

and

$$y^l = \sqrt{y^2 + (z_1-z)^2} \quad \text{if} \quad z_1 \geq z \geq (z_0 + z_1)/2.$$

The land rent function must then be redefined as,

$$R(y,u) = R_A \exp\left\{ -\frac{(p_3+s)}{\Gamma} \left[ \frac{(sy+p_3u)}{p_3 + s} - y_A \right] \right\} \quad \text{if} \quad z < z_0, \quad z > z_1 \quad (164)$$

$$R(y^m,u) = R_A \exp\left\{ -\frac{(p_3+s)}{\Gamma} \left[ \frac{(sy^m+p_3u)}{p_3 + s} - y_A \right] \right\} \quad \text{if} \quad z_0 \leq z \leq \frac{z_0 + z_1}{2} \quad (165)$$
FIGURE 13

THE CITY WHEN PART OF THE COAST IS A DISAMENITY
Note that $R(y,u) = R(y^m,u) = R(y^l,u)$ when $z = z_0$ and $z = z_1$. Also note that the rent gradient of $R(y^m,u)$ will be negative. The rent gradient $\delta R(y^l,u)/\delta z$ can be positive under certain circumstances. For a given $y$ the rent gradient in the range $z_1 < z < (z_0 + z_1)/2$ looks as follows,

$$\delta R(y^l,u)/\delta z = R(y^l,u) \left\{ -1/\Gamma \left( p_3 z/u - s[z_1 - z] / y^l \right) \right\} (167)$$

Which is positive when $s[z_1 - z] / y^l > p_3 z / u$. Everywhere else the rent gradient of the city is negative. When $y = 0$, the rent gradient $\delta R(y^l,u)/\delta z > 0$ when $s > p_3$. This follows from the fact that total transport costs increase as one moves away from the CBD at $z > (z_0 + z_1)/2$. The rent gradient in the range $z_0$ to $z_1$ will be below that which would exist in the absence of the disamenity. Figure 14 depicts the rent gradient of the city for a given $y$. The positive slope of the rent function illustrates the possibility of a positive rent gradient noted above.

At no other place in the city is the rent gradient positively sloped. The rent gradient in the range $z_0 < z \leq (z_0 + z_1)/2$ along $z$ for a given $y$ is given by,

$$\delta R(y^m,u)/\delta z = R(y^m,u) \left\{ -1/\Gamma \left( p_3 z/u + s[z_0 - z] / y^m \right) \right\} < 0 \ . \ (168)$$

Likewise, for a given $z$ the rent gradient in the range $z_0 < z \leq z_1$ is given as,

$$\delta R(y^l,u)/\delta y = R(y^l,u) \left\{ -1/\Gamma \left( p_3 y/u + s y / y^l \right) \right\} < 0 \ . \ (169)$$

Where, $i = m, l$. 

\[ R(y^1,u) = R_A \exp\left\{ \frac{-(p_3+s)}{\Gamma} \left[ \frac{(sy^1+p_3u)}{p_3+s} - y^1 \right] \right\} z \geq \frac{z_0+z_1}{2} (166) \]
FIGURE 14
A POSSIBLE RENT GRADIENT OF THE AMENITY-DISAMENITY CITY AT A GIVEN Y
With the introduction of a coastal disamenity, the boundary of the city also changes. For \( z > z_1 \) and \( z < z_0 \), the boundary of the city is still determined by the equation,

\[
p_3u + sy = (p_3 + s)y_A. \tag{170}
\]

But, in the range \( z_0 < z < \left( z_0 + z_1 \right)/2 \), the boundary of the city is given by,

\[
p_3u + s\sqrt{y^2 + [z - z_0]^2} = (p_3 + s)y_A \tag{171}
\]

and in the range \( z_1 > z > \left( z_0 + z_1 \right)/2 \) by

\[
p_3u + s\sqrt{y^2 + [z_1 - z]^2} = (p_3 + s)y_A. \tag{172}
\]

From (172), it is possible now for the city's frontier to be upward sloping. That is,

\[
\frac{dy}{dz} = \frac{(s[z_1 - z]/y^1 - p_3z/u)}{(p_3y/u + sy/y_1)} > 0 \tag{173}
\]

if \( s[z_1 - z]/y^1 > p_3z/u \). Which is the same condition for the existence of a positive rent gradient. This scenario is illustrated in Figure 13 by the segment of the city's boundary given by ab.

Finally, the population density function must be modified to reflect the change in the rent function. Specifically,

\[
N(y, u)/L^2(y, u) = R(y, u)/T \text{ if } z < z_0 \text{ or } z > z_1
\]

and,

\[
N(y^m, u)/L^2(y, u) = R(y^m, u)/T \text{ if } z_0 < z < \left( z_0 + z_1 \right)/2,
\]

and,

\[
N(y^1, u)/L^2(y, u) = R(y^1, u)/T \text{ if } z_1 > z > \left( z_0 + z_1 \right)/2.
\]

The econometric implication for the empirical work in Chapter IV is that when part of the coast is characterized by a disamenity, using straight-line distance from the census tract to the coast, \( y \), is not precisely correct. That is, doing so implies we underestimate the distance that a city resident must travel in order to consume the
amenity. In this sense, it must be supposed that disamenities occur infrequently in order to obtain unbiased parameter estimates. Alternatively, one might have a priori information on coastal sections thought to be a confounding nuisance and adjust the measurements accordingly.
FOOTNOTES

1 Though not invalidating their conclusions, Greene and Barnbrock's equation (9) should read,

\[ E(\varepsilon^2) = D_0 e^{-\beta r}(e^{\sigma^2/2} - 1). \]

2 Technically, \( a(y) \) becomes a new commodity when location \( y \) changes.

3 Personal correspondence with Professor Haurin.

4 Alternatively, if log-density functions for two income groups, say the rich (\( r \)) and the poor (\( p \)) are given as \( D^r = \beta^r_0 + \beta^r_1 u + \beta^r_2 y, \) \( i=r,p \), then by introducing a dummy variable DUMI equal to zero if the observation is a low-income tract and one if it is a high-income tract, and estimating the equation \( D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 \text{DUMI} + \beta_4 u \text{DUMI} + \beta_5 y \text{DUMI}, \) the desired parameters are retrievable. In this case, \( \beta^P_0 = \hat{\beta}_0, \) \( \beta^P_1 = \hat{\beta}_1, \) \( \beta^P_2 = \hat{\beta}_2, \) and \( \beta^P_0 = \hat{\beta}_0 + \hat{\beta}_3, \beta^P_1 = \hat{\beta}_1 + \hat{\beta}_4, \beta^P_2 = \hat{\beta}_2 + \hat{\beta}_5. \)

5 Assume \( n \) income groups exist with density equations \( D^i = \beta^i_0 + \beta^i_1 u + \beta^i_2 y + \beta^i_3 \text{INC}^i + \beta^i_4 \text{UINC}^i + \beta^i_5 \text{YINC}^i, \) \( i=1,n. \) If instead, \( D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 \text{INC} + \beta_4 \text{UINC} + \beta_5 \text{YINC} = X \beta \) was estimated, then \( \hat{\beta} = (X'X)^{-1}X'D. \) However, the true model consists of the partition of \( \beta \) and \( X \) such that \( X \beta = [X_1 \ldots X_n][\beta^1 \ldots \beta^n]. \) Where, the data are partitioned such that \( X_i \) is the partition for income group \( i \) and \( \beta^i \) is the set of true parameters for \( i. \) Then \( \hat{\beta} = (X'X)^{-1}X'[X_1 \beta^1 + \ldots + X_n \beta^n]. \) That is, in general, \( \hat{\beta} \) incorrectly predicts \( \beta^i \) with bias depending on the values of \( \beta^j, i \neq j, \) and the values of the observations. The estimator, \( \hat{\beta}, \) is a weighted average of the true parameters.

6 I wish to thank the following organizations for providing data on census tract areas: Birmingham Regional Planning Commission; Spokane County Planning Department; Office of Research of Economic Development of Oklahoma City; Department of Planning and Land Use for the city of San Diego; Department of Metropolitan Development, Division of Planning for Indianapolis; Regional Planning Commission for Cleveland; Chicago Department of Planning; Planning and Zoning Department of Volusia County, Florida; Department of City Development for the city of Milwaukee; Metropolitan Planning Commission of Knoxville; Community Development Department of Santa Barbara; Development Administration of...
Bridgeport; Office of Economic Development of Lexington; Northeast Ohio Four County Regional Planning and Development Organization and the Department of Planning and Economic Development for Akron; and the Planning Department of Dade County.

7 Some measurement error may arise in estimating distance to the amenity if part of the coast is made a disamenity by, say, pollution. In this instance, a resident residing directly inland from the polluted area must commute a distance greater than y in order to consume the amenity. This possibility is addressed further in Appendix D.

8 In the case of Milwaukee, the presence of multicollinearity between u and y may have resulted in the diminution in the effect of the amenity variable. When y alone was used in the model the coefficient was both negative and highly significant. The answer may lie in a pattern of population peculiar to Milwaukee but unknown to this researcher.

9 Another explanation for the presence of many significant coefficients may lie in the presence of heteroscedasticity or spatial autocorrelation biasing the standard errors of the regression downward. To check for heteroscedasticity, Breusch-Pagan, Goldfeld-Quandt, and Park-Glesjer tests were performed. Indeed, the tests frequently indicated the presence of heteroscedasticity arising from u or y or both. When this was encountered the results of the Park-Glesjer tests were used to "correct" for the heteroscedasticity. Surprisingly, this had no effect on the number of coefficients for which the null hypothesis was rejected.

Anderson (1985) suggested the variance of the disturbance term be proportional to the mean of log-density raised to a constant. Since there is no obvious or esthetically pleasing way to eliminate the effect of u and y on log-density to ascertain the legitimacy of Anderson's assertion, this approach was rejected.

For argument spatial autocorrelation was assumed (no tests for spatial autocorrelation were performed). In order to break-up the autocorrelation each city was randomly sampled at 50% and the model was run again. Again, the results rejected the contention that the standard errors were biased downward.

10 If the density equation is given by \( D = \beta_0 + \beta_1 u + \beta_2 y + \beta_3 \text{UINC} + \beta_4 \text{YINC} \), then \( \frac{dD}{du} = \beta_1 + \beta_3 \text{INC} \) and \( \frac{dD}{dy} = \beta_2 + \beta_4 \text{INC} \). Where, \( \frac{dD}{du} = \beta_1 \) and \( \frac{dD}{dy} = \beta_2 \) if income is equal to zero.

11 The results are virtually the same when the interaction term is included.
REFERENCES


Bleicher, H., Statische Beschreibung der Stadt Frankfurt am Main und ihrer Bevölkerung (Frankfurt am Main, 1892).


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Hoyt, H., One Hundred Years of Land Values in Chicago (Chicago: University of Chicago Press, 1933).


Taylor, P. J., "Distance Transformation and Distance Decay Functions," Geographical Analysis 3 (1971), 221-238.


