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Preservice teachers' understanding of division as assessed by concept mapping

Merrill, William Lord, Ph.D.
The Ohio State University, 1987
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PRESERVICE TEACHERS' UNDERSTANDING OF DIVISION
AS ASSESSED BY CONCEPT MAPPING

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By
William Lord Merrill, B.S., M.Ed.

* * * * *

The Ohio State University
1987

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To Kathy
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Instructional Design and Technology
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CHAPTER I
BACKGROUND TO STUDY

This first chapter is a general overview of the dissertation topic. The need and value of the study will be discussed, a statement of the problem will be presented, assumptions, variables, and limitations will be discussed, relevant vocabulary will be defined, and the research hypotheses will be presented.

Need for the Study

Through three separate national assessments of mathematical ability (Carpenter, Coburn, Reys, & Wilson, 1978; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Third National Mathematics Assessment: Results, Trends, and Issues, 1983) it has become increasingly clear that today's schools are teaching topics and skills most easily learned in a rote manner as demonstrated by the gains made by nine-, thirteen-, and seventeen-year-old students. United States students have shown modest gains from one testing to the next in the areas of routine computational and measurement skills as well as on problems which require simple recall of memorized information. Areas not included in this rote learning are more real-life, non-routine problems which require the solver to use higher level thinking skills such as comprehension and application (Bloom et al., 1956). None of the three age groups tested showed significant gains from the second to the third national assessments in solving the nonroutine problems (Third National Mathematics Assessment: Results, Trends, and Issues, 1983).
The United States fared equally poorly on the Second International Mathematics Study (SIMS), an international study comparing the mathematical abilities of eighth and twelfth grade students from various developed countries. Both eighth and twelfth grade students ranked dismally low in comparison with other nations of the world. United States eighth grade students, when compared with eighth graders from 19 other developed countries, ranked eighth in statistics, tenth in arithmetic, twelfth in algebra, sixteenth in geometry, and eighteenth in measurement. United States twelfth grade students, when compared with twelfth graders from 14 other developed countries, ranked tenth in sets and relations, twelfth in number systems, geometry, probability and statistics, and elementary functions/calculus, and fourteenth in algebra (Second Study of Mathematics: Detailed National Report, United States, 1985).

Due to the repeated call in the education community for meaningful learning, a concerted effort needs to be made to determine whether today's students are learning mathematical material meaningfully. This effort appears especially difficult with more states, school districts, and schools moving toward standardized testing to assess the progress of children and the knowledge of the teachers. Many areas are using the results of the standardized tests to evaluate the mental abilities of both children and teachers. However, standardized tests generally do not measure higher-level cognitive skills and understanding of concepts. Questions which measure these skills usually do not fit into the multiple-choice format and most concepts, those which are truly understood, develop over many months or years. Piaget (1952) noted that children
develop mental images or patterns of actions which represent experiences. Children interact with their environment and construct mental models of the world and how it operates. These images are then filed into a network of concepts which are continuously modified as new learnings are integrated with old ideas. Since standardized tests generally measure lower level knowledge and facts, teachers tend to teach the lower level material which will help their students perform well on these tests.

If educators wish children to learn meaningfully and our current system of evaluation fails to assess meaningful knowledge, then there is a need to determine what mathematical knowledge is being learned in a meaningful manner by the students and what mathematical knowledge is being learned rotely by rule and algorithm, leaving students unsure as to how the memorized material relates to their own cognitive structure. To this end, this study attempts to determine the level of understanding of the division concept in preservice elementary teachers by using concept mapping. Preservice elementary teachers are employed since they will be the future teachers and it is desirable, if one expects a teacher to provide meaningful instruction, for the teacher to have a meaningful understanding of the concept which will be taught.

Problems have been noted in the literature pertaining to the level of expertise of elementary teachers in the mathematics area. In a review of the research, Suydam and Riedesel (1970) noted that the background of the teachers is related to the achievement of their pupils. Burton and Bussell (1978) found that many elementary teachers suffer from some degree of mathematics anxiety, "an unpleasant condition characterized by
sweating palms, nausea, shortness of breath and pale cheeks" (p. 23), so they "remain uncomfortable when they must use or teach mathematics" (p. 23) and often pass the anxiety on to their own students. Wright and Miller (1981), after studying mathematics anxiety, concluded that "persons with high math anxiety perceive their math skills as less proficient than their skills in other academic areas and generally will not like math or like to teach math" (p. 72). Thus, it can be inferred that some teachers are instilling negative feelings and are developing a poor knowledge base in the children they teach because they themselves have mathematics anxiety and an inferior knowledge base.

Division was selected since it still occupies a good share of the elementary mathematics curriculum in the intermediate grades (Agenda for Action, 1980), it is the most difficult of the four operations to teach (Slesnick, 1982), and it is one of the areas that most people tend not to understand (Hazekamp, 1978).

Concept mapping (Ault, 1985; Malone & Dekkers, 1984; Moreira, 1979; Novak, 1981b; Novak & Gowin, 1984; Novak, Gowin, & Johansen, 1983) is simply a pictorial means of showing the cognitive structure of a learner. This procedure is based upon the learning theory of Ausubel (1968).

Statement of the Problem

This study deals with two distinct but interrelated problems. First, the study addresses the level of understanding of preservice elementary teachers concerning the concept of division and, second, the study explores the relationship between preservice elementary teachers' mathematics achievement and their understanding of the division concept.
Assumptions

The major assumption of this study is that concept mapping is a valid representation of the learner's understanding of a topic and the interrelationships which occur between concepts. It is recognized here, and by the developers of concept mapping, that there are other factors which could interfere with the final product of the concept map. Novak writes:

while the criteria for the assessment of student-constructed maps consider structure as well as content, the maps are an externalization of the student's cognitive structure. While we believe that the concept map reflects, in some degree, the cognitive structure of the individuals, we do not know what systematic errors are introduced in this representation....However, in the two years of the project's work, we were unable to devise an instrument that would consider all three of the elements that impinge on the construction of concept maps: ability to construct the map, the content correctness as represented on the maps, and the map as a reflection of the cognitive structure. (1981b, p. 36)

Variables

The variables which will be used in the study include:

1) an achievement variable which includes:
   (a) a mathematics achievement test score;
   (b) grades for university courses Mathematics for Elementary Teachers 1 and 2;
   (c) overall college grade point average;

2) an attitude toward mathematics variable which includes:
   (a) an attitude assessment score;

3) an understanding of division variable which includes:
   (a) the total concept map score;
   (b) subscale mapping scores—concept recognition, grouping, hierarchy, branching, and propositions.
Limitations to the Study

The major limitation to this study was one of time. The subjects were selected, taught to map, and tested in one eleven-week period, which is the length of the university quarter in which the data were gathered. This limitation was in effect due to the fact that the classes would disband and the subjects would move on to other endeavors and be unavailable to complete the study.

Definition of Terms

The following is a list of the major terms and their definitions:

A concept is "regularities in objects or events designated by a sign or symbol" (Novak, 1980b, p. 58). For example, the objects which generally have four legs or a platform, support a mattress, and are used to rest or sleep on by one or more persons show the regularity we designate with the symbol bed.

A concept map is defined as "the identification of concepts...and the organization of those concepts into a [two-dimensional] hierarchical arrangement from the most general, most inclusive to the least general, most specific concept" (Novak, 1981b, p. 3).

A logical connective is "a term which serves to link a phrase, clause, or sentence to another clause, or sentence" (Gardner, 1980, p. 224). In concept mapping logical connectives are used to connect two expressed concepts in the map. Examples of logical connectives would be "are related," "has," or "used in."

A proposition is the combination of two or more concepts with logical connectives in order to form an idea which means more than the sum of the individual concepts used in making the proposition (Ausubel,
Novak, & Hanesian, 1978). A very simple proposition would be "The sky is blue." As a concept map, this proposition would be:

![Concept Map]

Meaningful learning occurs when the learner consciously tries to associate new meaningful knowledge to existing cognitive structures in the brain (Novak, 1977).

**Hypotheses**

There are three major hypotheses which will be examined in this study. They are:

Number One: None of the subjects in the high or the low mathematics achieving groups will be able to attain an overall score of 75% accuracy on the total division concept map and on each of the four sub-areas of the map.

Number Two: High mathematics achieving preservice elementary teachers will demonstrate a greater understanding of division, as assessed by concept mapping, than low mathematics achieving preservice elementary teachers.

Number Three: There will be a subset of component variables from the understanding of division composite variable which will predict mathematics achievement of subjects in both the high and the low mathematics achieving groups.

**Overview of the Following Chapters**

The first chapter has given a broad overview of the research surrounding meaningful learning and the learning theory of Ausubel, the basics of concept mapping, the importance of the study, assumptions,
variables, limitations, definition of important terms, and the hypotheses to be tested.

The second chapter describes, in more detail, the meaningful approach to learning, concept mapping, the research currently being conducted using this new technique, and the general research concerning the lack of understanding of division by children and adults. The third chapter details the methods and procedures including the study's design, instrumentation, treatments, and statistical analysis. The fourth chapter is the study itself: reporting hypothesized and unhypothesized findings and a general discussion of the observations obtained from the maps and interviews. The fifth chapter contains conclusions and recommendations derived from the study.
CHAPTER II
REVIEW OF THE LITERATURE

The second chapter will review and discuss in some detail the general mathematics and Ausubelian meaningful learning theories, the process of concept mapping, and the concept of division in order to develop the foundation for measuring division conceptions with concept mapping.

Meaningful Learning

Meaningful Learning in Mathematics

In 1935 Brownell became the spokesperson for meaningful learning with his chapter in the Tenth Yearbook of the National Council of Teachers of Mathematics Yearbook. Brownell (1935) outlined three different theories of teaching arithmetic. He began with the drill theory which, for two reasons, was very popular in the 1920's and 1930's. First, the drill theory relied heavily on a stimulus-response philosophy of learning which created bonds between certain numbers, such as the stimulus 6 and 3 and a specific response, 9. It was thought that arithmetic could be reduced to this bonding procedure. Second, the belief by adults who use numbers daily, that mathematics was easy to do. Therefore, students should be able to learn arithmetic procedures quickly and easily by simply memorizing the facts and procedures. Of course, the adults had forgotten how difficult it was to learn arithmetic in the first place.

Brownell (1935) discussed a second theory, the incidental learning theory of arithmetic learning, which was popular in the mid- to late
1930s. The practitioners of this theory believed that children would learn as much arithmetic as they needed and would learn it better if they were not systematically taught the subject. The premise was founded on the belief that for children to learn, they must be interested in the subject. Therefore, children would learn when they were ready to learn. Arithmetic was not taught except in conjunction with other subjects as needed. This theory became popular in education once again in the 1960's and 1970's with the rise of certain open and free education movements.

Finally, Brownell (1935) set forth the meaning theory of arithmetic instruction. Brownell spent much time and effort promoting the meaning theory since he believed that it was the method which should be employed in teaching children. The meaning theory incorporated aspects from both the drill and incidental theories recognizing important contributions from both. "The 'meaning' theory conceives of arithmetic as a closely knit system of understandable ideas, principles, and processes" (p. 19). The meaning theory encourages understanding by learning in a meaningful manner, and placing a strong emphasis upon relationships which exist.

Once Brownell began the emphasis on meaningful learning with his yearbook article, others started to share the enthusiasm of teaching and learning meaningfully. Morton (1938) described the National Council of Teachers of Mathematics' committee report on arithmetic and its description of the meaning theory. The committee's position was that the social and mathematical phases of arithmetic instruction were equally important. The drill theory ignored both phases and the incidental theory ignored the mathematical phase. The committee, therefore, supported the meaning theory and advocated practice, not merely mindless drill, and
the advantageous use of incidental number experiences in a meaningful manner.

In 1949, Van Engen, in a two-part article, proposed a general theory of meaning. He summarized his theory by writing "in any meaningful situation there are three elements. (1) There is an event, an object, or an action. In general terms, there is a referent. (2) There is a symbol for the referent. (3) There is an individual to interpret the symbol as somehow referring to the referent" (p. 323). Van Engen (1949) was quick to point out that true understanding developed from an operation. The term operation referred in a broad sense to an action, event, or object. Therefore, according to Van Engen (1949), "a knowledge of arithmetic implies...that the individual becomes aware of a correspondence between a set of symbols and a set of operations" (p. 325). Since the operation was an action, event, or object, and the symbol always represented something other than itself, the key to this theory of meaning was the ability to link mentally the operation and the symbol.

The National Council of Teachers of Mathematics in 1959 reaffirmed its support of the meaningful approach (Jones, 1959) by adopting two basic principles. First, "the best learning is that in which the learned facts and processes are meaningful to and understood by the learner..." and second, "understanding and meaningfulness are barely, if ever, 'all or none' insights in either the sense of being achieved instantaneously or in the sense of embracing the whole of a concept and its implications at any one time..." (Jones, 1959, p. 1)

Additional, more recent references (e.g. Agenda for Action, 1980; Driscoll, 1981), have also exhorted teachers to structure more
meaningful instructional activities to help children see the relationships which occur between what they know and what they are learning. If taught meaningfully, students should be able to use the information in more varied situations, remember it longer, and reconstruct the knowledge more easily if it is forgotten.

Ausubel's Meaningful Learning Theory

Most teachers today see the need for meaningful learning to take place in the classroom but may be ill-prepared pedagogically to accomplish meaningful learning with their children. The problem may lie in the notion that teachers do not know what constitutes meaningful learning since most instruction in the classrooms of today encourages rote learning. Ausubel, Novak, and Hanesian (1978) detail meaningful learning as new learning which is symbolically presented in a "substantive" and "nonarbitrary" manner. New learning must be substantive or nonverbatim so the learner can recognize and relate the new learning in many different forms and situations. In other words, a learner should recognize that "a method of combining objects two at a time is associative if the result of the combination of the three objects (order being preserved) does not depend upon the way in which the objects are grouped" (Mathematics Dictionary, 1959, p. 21), or

\[( a + b ) + c = a + ( b + c )\]

as both being valid representations of the associative law. Secondly, new material needs to be nonarbitrary since it must follow some rules in order to be related to the pre-existing knowledge of the learner. In order for a student to learn the properties of rhombus meaningfully, a nonarbitrary relationship would need to be made between the new
material, the concept of rhombus, and the general class of the rhombus, the parallelogram, which should already be in the cognitive structure of the student.

While the material must be substantive and nonarbitrary, this alone does not guarantee that meaningful learning will occur. Ausubel and Robinson (1969) state that material with these two attributes is invested with "logical meaningfulness." In order for the learning to be "psychologically meaningful," or meaningful to the learner, the learner must possess the necessary cognitive structures which make the new learning meaningful.

If a particular learner does possess ideas in his cognitive structure to which the new learning material can be related in a substantive and nonarbitrary fashion, then we say that the material is potentially meaningful to him or that it possesses potential meaningfulness. (Ausubel & Robinson, 1969, p. 53)

The third factor to assure meaningful learning is the intent of the learner to relate the material in a substantive and nonarbitrary manner. Even if the material is logically and psychologically meaningful, the learner must want to relate the material to previously learned material. Any material can be learned in a rote fashion if the learner is so predisposed. Ausubel and Robinson (1969) refer to this intent to learn meaningfully as the learner having a "meaningful learning set."

Ausubel, Novak, and Hanesian (1979) outline three basic types of meaningful learning: concept learning, representational learning, and propositional learning. Concept learning involves the learner identifying and distinguishing the critical attributes of unitary generic or categorical ideas. The child learning the concept "four" will examine and manipulate varying examples of sets which embody "fourness." In
doing so, the child is conceptually developing an understanding of the number four.

Representational learning is the meaningful learning type which most closely resembles rote learning. During representational learning, the learner associates a single word with the concept which it represents. As a child learns the concept four, as in the example above, the word four is also being learned as representing the concept. This type of learning is rote in so much as any word could be given for the concept as long as everyone refers to the same concept with the same word. The word only assumes meaning when it is associated with the meaningful concept.

Propositional learning involves learning the meaning of groups of concepts expressed in propositional form (usually a sentence or phrase). However, the meaning derived from propositional learning is greater than the sum of the meanings of the individual concepts. Additional meaning is developed by the manner in which the concepts are connected.

The more the learning process described above occurs, the stronger the relationships become between the new knowledge and pre-existing knowledge. The pre-existing psychological entities in the learner's brain are termed "subsuming concepts" or more simply "subsumers" by Ausubel (1968). Therefore, meaningful learning occurs when the new knowledge results in growth or modification of existing subsumers.

The size and complexity of the subsumers of the learner depend upon the experiential background of the learner. Therefore, subsumers could be complex and well developed or relatively limited in size and scope (Novak, 1977). These subsuming concepts are not adhesive spots in the
brain to which knowledge is stuck, but rather they act as interactive ports which provide linkages for new information which becomes part of or a modification of the cognitive structure.

We do know that information is stored in localized regions of the brain and that many brain cells are involved in the storage of knowledge units. New learning results in further changes in the brain cells, but some cells affected during meaningful learning are the same cells that already store information similar to the new information being acquired. In other words, the neural cells or cell assemblies active in storage during meaningful learning are undergoing further modifications and are probably forming synapses or some functional association with new neurons. (Novak, 1977, p. 74)

Ausubel (1968) describes the subsumption process as a learner encountering some new information, t, which is related to and assimilated by an existing concept, T, in the cognitive structure. Finally, the interaction of t and T produces a modified subsumer, T't'. As additional information is related to and assimilated by an existing concept, the existing concept is further modified to accommodate the new information. Ausubel (1968) refers to this process as "progressive differentiation."

Progressive differentiation is the process by which the existing cognitive structure is modified and made more specific by new information. A young child acquires concepts through concept formation. As a toddler continually sees examples of dogs, and has these encounters labeled by adults or older children, his or her concept of "dog" slowly develops. After the child has the concept of dog in his or her cognitive structure, additional encounters with dogs tend to progressively differentiate the subsumer. The child may next divide the subsumer, dog, into adults and puppies, then into big, medium, and small dogs, and finally into different breeds of dogs such as collie, German shepherd, and others. The process, if represented pictorially, could resemble the branch
of a tree as it grows, sprouting buds and twigs from the main branch. All new material which grows from the branch is still related to the original branch.

Progressive differentiation also implies a general hierarchical structure for the subsumers. If new information progressively differentiates existing subsumers, as discussed above, then the new information must be more specific than the existing subsumer it is differentiating. Therefore, according to Ausubel, the structure of the subsumers is from general to specific.

In a study which supports the notion that new information is continually incorporated into the cognitive structure modifying existing subsumers, Spiro (1977) asked a group of subjects to read a story. During the reading, the subjects were given some additional information which related to the story and was familiar to the subjects. The subjects were then asked to recount the story three to six weeks later, making sure only to relate the ideas from the story. Generally, in as little as six weeks, the subjects could not relate the story accurately. Spiro (1977) notes that the readers had incorporated the additional information as fact, had inferred much about the story read, and had transferred these inferences into the retelling of the story. Ausubel would contend that the readers' subsumers were modified by the story and the additional information, changing the cognitive structure of what the reader "knew." This modified subsumer could then be used to retell the story, accounting for the inaccurate recounting.

Ausubel, Novak, and Hanesian (1978) propose that meaningful learning could happen by subordinate, superordinate, or combinatorial
learning. All three are considered part of the assimilation theory in as much as new meaningful learning results in preexisting information being altered by the new learning.

Subordinate learning happens when more specific new material is linked to an existing more general idea as either additional cases, extensions, modifications, or qualifications of the existing idea. For example, a child who understands division as sharing of a set between a group of people, experiences subordinate learning after learning a second meaning of division, namely, division as repeated subtraction. The child's understanding of the division concept has been altered by the new information.

Superordinate learning happens when a new more general idea is learned which encompasses previously learned more specific concepts. For example, a child may understand the concepts square, rectangle, rhombus, and parallelogram. When this child learns the general class for all the four-sided plane geometric figures, quadrilateral, superordinate learning has occurred since a more general concept has been learned under which the preexisting more specific concepts may be organized.

Combinatorial learning happens when a new idea is developed which has critical attributes in common with previously learned information but is neither more general nor more specific than the ideas with which it combines. Combinatorial learning is a type of consolidation of existing knowledge. For example, when a child learns the relationships between length, mass, and volume in the metric system, he or she has experienced combinatorial learning. All of the existing concepts (length, mass, and volume) and the new idea were equally general, but a new conceptual relationship was formed between these ideas.
Ausubel contends that even though meaningful learning is maintained and useable much longer than rote learning, it can be forgotten. Ausubel, Novak, and Hanesian (1978) refer to this "meaningful forgetting" as "obliterative subsumption." The authors differentiated meaningful forgetting from the forgetting of rote material by asserting that when the former is forgotten, it does not result in proactive interference of similar new material which is currently being learned, as does the latter. The residual concepts which remained, after obliterative subsumption occurred, served to facilitate new relevant meaningful learning. For example, a child might forget the specific formula for finding the area of a triangle but remember that there exists a relationship between area, length of the base, and the length of the height of the triangle. "Specific details have been obliteratively subsumed, but a concept's usefulness for learning new things...remains a positively functional element in the cognitive structure" (Novak, 1977, p. 457).

Rote Learning

Students learn in a rote fashion, especially when they feel inadequate, because it is easy to memorize a few terms and definitions and even appear to be using the terms and definitions correctly during classroom discussions. Students are forced to do this when the material to be learned lacks logical meaningfulness, the learner lacks the cognitive structure needed for psychological meaningfulness, and/or the learner lacks a meaningful learning set. Teachers also encourage this type of behavior by teaching simple knowledge material, which best lends itself to rote learning, and by giving only full credit to test answers which are regurgitated verbatim (Ausubel & Robinson, 1969; Bloom, Madaus, & Hastings, 1981).
Many students find the subject of mathematics very difficult. They do not truly understand the needed concepts because, more than likely, they were taught by teachers who did not truly understand the needed concepts (Fearnley, 1972). Burton and Bussell (1978) note that many teachers suffer from mathematics anxiety, a debilitating condition which impairs mathematical ability. Research confirms that elementary teachers generally do not establish meaningful learning situations in their classrooms by using concrete referent objects with the children. Milgram (1969) found that the vast majority of the intermediate grade mathematics teachers' time, 51%, is spent with oral or written drill, and 25% is spent going over previous assignments. However, researchers found that children learn skills better when less time is spent drilling and more time is spent on developmental activities (e.g., Schuster & Pigge, 1965; Shipp & Deer, 1960; Zahn, 1966). Many teachers use few or no manipulative materials during mathematics instruction and once the child leaves the primary grades, evidence shows that even less are used in the intermediate and middle school grades (Danforth, 1978; Merrill, 1984).

Probably no learning, beyond infancy, is totally rote in nature. Rather, learning occurs somewhere along a rote/meaningful continuum and the important notion is not whether something is learned rote, but rather to what extent the new material is meaningful to the learner. Phone numbers are often referred to as being rote learning; however, every adult user of the phone system has a subsuming structure which tells him or her that all United States phone numbers have a three digit area code followed by a seven digit number. The adult may also know the three digit exchanges for a particular city and must now only memorize
the last four digits of the phone number. This adult would be said to have a meaningful learning set since only part of the information will be stored arbitrarily in the cognitive structure (Novak, 1977).

Conversely, a person can also learn phone numbers, definitions, descriptions, in fact anything at all, without ever attempting to relate these new data to existing subsumers. To the extent that there is no conscious effort to link new knowledge to old, rote learning has occurred.

Novak (1977) writes:

> the extent to which learning is rote or meaningful is partly a function of the learner's predisposition toward the learning task...; it is also a function of the degree to which relevant concepts in cognitive structure have been developed,...and the potential range of linkages between new information elements and existing cognitive structure, (p. 81)

Ausubel, Novak, and Hanesian (1978) further outline the problems associated with rote learning (verbatim and arbitrary). The authors present two important points for consideration: first, most humans have great difficulty remembering arbitrary and verbatim information and said information can only be remembered for a short period of time unless overlearned; and second, arbitrary and verbatim learning is highly vulnerable to previously learned concepts and new material of a similar nature. Miller (1956) discusses the human's capacity for information processing by noting that most people can only remember seven random numbers or bits of information. He continues by explaining the "chunking" ability of people which allows humans to combine information into new bits of information to be memorized. However, this process is truly rote since no attempt is made to relate this material in a meaningful fashion to any of the cognitive structures of the brain.
Concept Mapping

Concept Mapping Theory

Novak (1977), upon examination of our schools, asked the question which many educators have pondered, "Why do so many students learn so little?" (p. 9). In thinking about this question and trying to help students learn, he developed a mapping system which incorporates Ausubelian learning theory of how people learn in a meaningful manner. This system, which is designed to allow the learner to organize concepts in a meaningful manner, is called concept mapping. Since Ausubel (1968) believes that information in the mind is organized in a top-down fashion, concept maps would also be constructed in a general to specific manner. Figure 1 shows a simplified model for concept mapping. Note that the most general inclusive concept is listed at the top of the map and successively more specific concepts are subsumed below. A concept map will demonstrate visually how the concepts are related hierarchically (vertically) and how concepts of approximately equal generality are related (horizontally). "A concept map depicts hierarchy and relationships among concepts. It demands clarity of meaning and integration of details" (Ault, 1985, p. 38).

Stewart, Van Kirk, and Rowell (1979) outline the steps used in making a concept map. The first step is to identify the major concepts to be included in the map. These concepts are then organized in a superordinate-to-subordinate order that is in keeping with the structure of the content of the subject. Once the concepts are ordered, they are arranged in a map to show how the map-maker believes the concepts relate. Connecting lines are drawn and logical connectives are inserted to
Most General,  
Inclusive Concepts

Subordinate,  
Intermediary  
Concepts

Most Specific, Least  
Inclusive Concepts

Figure 1. Simplified Model for Concept Mapping
demonstrate the exact relationships which exist between the connected concepts. The concept map is then a series of interrelated propositions which show "the structure of the discipline" as viewed by the map-maker.

"The concept map...is a device for representing the conceptual structure of a discipline, or segment of a discipline, in two dimensions" (Stewart, Van Kirk, & Rowell, 1979). Concept maps have an advantage over more traditional outlining because of the added dimension which allows the map-maker to show the interrelationships which exist between concepts of approximately equal generality. The mapping procedure also requires the map-maker to think in multiple directions at one time, a task not easily accomplished unless the map-maker has a deep understanding of the concepts being mapped. This procedure would be totally impossible unless the material being mapped was learned in a meaningful manner. As rote material is learned in a verbatim and arbitrary manner, no connections are made in the brain of the learner to make the associations called for in the mapping procedure.

In contrast to rote learning methods, concept mapping permits variation in right answers. There are many possible patterns capable of connecting concepts. Maps permit comparison of student understanding with expert knowledge. They improve understanding by searching personal meaning for misconceptions or incorrect relationships among concepts. (Ault, 1985, p. 40)

According to Stewart, Van Kirk, and Rowell (1979), concept maps can be used for three basic reasons: 1) as curricular tools; 2) as instructional tools; and 3) as evaluation tools. As a curricular tool, concept maps can be used to organize the information in the general curriculum. Concept maps could organize the cognitions, cognitive competencies, and skills which need to be taught during instruction in order to help assure meaningful material is logically and potentially meaningful.
Material so organized has the necessary attributes to be learned meaningfully and only requires the learner to have a meaningful learning set.

Closely tied to using the maps as curricular tools, are using them as instructional tools. Once the material is meaningfully organized, the instructor needs to determine which examples from the map are most appropriate to illustrate the chosen concepts. After a unit of instruction, a teacher would expect a child to answer a series of questions pertaining to a number of concepts but might not be sure that the child understands the underlying concepts which govern the examples being tested. A teacher could use maps to organize the sequence in which the concepts are to be taught, or children could construct maps before instruction began in order to help the teacher visualize what concepts were understood and what concepts needed further development.

As evaluative tools, concept mapping might be used after a unit is taught to see if a person has organized the material in a meaningful manner. If evaluation is viewed as an assessment of the person's knowledge, then concept maps are one method of assessing what the person truly understood by the concepts listed, the connections which are made, and the logical connectives used to describe the relationships between the concepts.

This study is utilizing concept maps as an evaluative tool to attempt, in part, to ascertain what parts of the division concept are truly understood by preservice teachers.
Research Using Concept Mapping

Moreira (1979) found that students who were taught to organize information using the Ausubelian learning theory were more successful in organizing information in map form than were students taught using the content organization found in the textbook. Two groups of students were taught a unit on electromagnetism, each group using a different method. All the students were finally asked to map the 19 major concepts using the general rules for developing concept maps. While neither of the groups had practiced the concept mapping process, the students taught to organize material using Ausubel's learning theory could follow the experimenter's directions better, complete maps more fully, and organize the information in a general to specific fashion more successfully than the traditional group.

The process of concept mapping has been taught to students by Novak, Gowin, and Johansen (1983). The authors wanted to determine if junior high school science students could learn to use concept mapping and epistemological Vee mapping as tools to encourage meaningful learning. The authors concluded that both types of mappings were valuable to students' understanding of the new material because these students scored higher on novel problem solving tests which measured higher-level cognitive processes than students who were taught the more traditional outlining process for organizing information. It was hypothesized that students were meaningfully learning (in Ausubelian terms) and hierarchically organizing the new knowledge in their cognitive structure. The belief of the authors was that since students who were proficient at concept mapping scored better on higher-level cognitive questions, then
the process of concept mapping facilitates and reflects meaningful learning.

Lehman, Carter, and Kahle (1985) built upon the work of Novak, Gowin, and Johansen by designing a study which measured the conceptual understanding of black high school students in science using concept/Vee mapping and outlining treatment groups. Although this study failed to gain statistical significance, the authors were still convinced that these two heuristics were important tools in the teaching and learning of science. The authors found that "in virtually every case, student-constructed review activities using the experimental heuristics were more concise than the corresponding outlines" (p. 672).

Sinatra, Stahl-Gemake, and Berg (1984) used concept mapping with reading disabled children to determine if the visual mapping procedure produced higher vocabulary scores and greater comprehension than the traditional verbal method of learning. Overall, 70% of the students scored higher on vocabulary and comprehension tests when taught the mapping procedure as a method of organizing material than when taught the verbal method of learning the same material.

Additionally, researchers have been working at refining the concept mapping procedure (Ault, 1985; Gardner, 1980; Novak, 1977; Novak, 1979; Novak, 1980a; Novak, 1981a; Novak, 1981b; Novak, 1986; Novak & Gowin, 1984; Stewart, 1982; Stewart, Van Kirk, & Rowell, 1979). These authors have altered the mapping procedure from being one in which the major concern was the order in which the concepts were listed to one in which the connections and the logical connectives are of utmost importance since these connections indicate whether the map-maker has an
understanding of the stated concepts. Concept mapping is also being taught to students of different age levels to see if there are ages which cannot comprehend the mapping procedure. To this point, all age levels from second grade to college which have been used are able to learn the mapping procedure given ample time.

Division

Division Concept

Division is an operation which is most difficult for students (Hazekamp, 1978). On the third national assessment, 13- and 17-year-old students scored lowest on computational problems involving division. While scores were respectable for simple one-digit divisor problems, scores drastically fell as the problems became more difficult (Third National Mathematical Assessment: Results, Trends, and Issues, 1983). Lindquist, Carpenter, Silver, and Matthews (1983) note that "low results in [long] division raise some serious questions whether the time and effort spent drilling on computation might be devoted more productively to other topics" (p. 16). The division algorithm is not potentially meaningful since the learners probably lack the prerequisite skills needed to be successful. In Ausubelian terms, the learners lack relevant subsumers in their cognitive structure to understand division.

Even though students may lack computational abilities it is vital for them to know when to divide and how to interpret the results since problem situations requiring division solutions occur daily. Two problems presented to 13-year-old students on the third and second national assessments, respectively, indicate the need for students' understanding of the concepts associated with division.
An army bus holds 36 soldiers. If 1,128 soldiers are being bussed to their training site, how many buses are needed? (Third National Mathematics Assessment: Results, Trends, and Issues, 1983, p. 48)

While 70% of the students recognized the problem as requiring division and calculated correctly, only a "small percentage" gave the correct answer, 32 buses. Of the rest, 29% gave the exact quotient as the answer and 18% ignored the remainder. Most of the students demonstrated their failure to understand the problem and what was being asked.

A man has 1310 baseballs to pack in boxes which hold 24 balls each. How many baseballs will be left over after the man has filled as many boxes as he can? (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981, p. 27)

Once again, while approximately two-thirds of the students recognized that this situation required division, only about one-fourth of them correctly answered the question.

It is clear that while most of these students could recognize the division situations and do the physical calculations, the majority did not understand the division questions being asked and, therefore, were not capable of reporting the correct answers. In order to secure the correct answer, students must not only understand the concept of division but also "must know when remainders occur as well as how they can be determined, and they must interpret the results" (Driscoll, 1981, p. 95).

The Agenda for Action (1980) calls for students to solve most complex division problems with a calculator. While this is an excellent recommendation, using a calculator without understanding the division concept, can result in more wrong than right answers. While it is true that performance increased 36% for 13-year-olds and 41% for 17-year-olds using calculators to solve computation problems such as \( \frac{28}{3052} \), it
is equally true that performance fell for 13-year-olds from 29% to 6% on the baseball problem when they were allowed to use a calculator. And, although the 17-year-olds had been exposed to division for nine or ten years, only 19% of them could correctly solve the baseball problem with a calculator. While a calculator is an invaluable tool, the user must still understand the concept and be able to interpret the results of the work.

Similar difficulties were encountered when children were asked to complete division problems using decimal values. Bell, Fischbein, and Greer (1984) asked 12- and 13-year-old students to write the mathematical sentence which could be used to solve certain division word problems (they did not actually solve the problems). The authors note two major difficulties which appear to occur continuously. First, the children had trouble writing expressions which involved division by a decimal value of less than one. An example would be:

A convict is digging his way out of a prison cell. After the first day he had only dug a tunnel of length 0.174 miles. At this rate, how long will it take him to reach a forest 3 miles away?" (p. 133)

The researchers concluded that thirteen of the 30 students tested wrote a multiplication sentence for this problem because they intuitively believed the answer needed to be larger than the two numbers. The major misconception which interfered with the correct sentence was the notion that multiplication always results in a larger answer and division a smaller one. Similar results occurred for other problem situations which required division by a decimal value of less than one.

Second, children reversed numbers in division problems when asked to divide a smaller number by a larger one. An example would be:
25 men do the football pools together. This week they have won $22. How much will they receive each?" (p. 133)

Eight of the 30 students reversed the two numbers in the division sentence since they felt that division involves only large numbers divided by smaller ones.

The work of Fischbein, Deri, Nello, and Marino (1985) supported the observations made by Bell, Fischbein, and Greer (1984) that there are difficulties that children encounter when dealing with certain types of division problems. Fischbein, Deri, Nello, and Marino (1985) attribute the learners' difficulties to tacit models of the operations in the learners' mind. The authors found that "the initial didactical models seem to become so deeply rooted in the learner's mind that they continue to exert unconscious control over mental behavior even after the learner has acquired formal mathematical notions that are solid and correct" (p. 16).

**Division Work with Young Children**

Work has been done on ways to help children understand the concept of division. Nelson and Kirkpatrick (1975) completed studies with young children using problem solving and partitive and measurement division situations. The authors found that with manipulation of real objects, children as young as three and four could begin to develop the concept of partitive and measurement division and could solve simple verbal problems. Izzo (1960) and Lovell (1971) support the work of Nelson and Kirkpatrick (1975) when they each conclude that children were ready to conceptually understand division at about the same time as they traditionally learn addition fundamentals. Copeland (1974) agrees but stipulates that the children must have reversibility of thought so as to be
able to see the relationship between division and multiplication before "truly understanding" the division concept.

Bechtel and Weaver (1976) interviewed second graders to ascertain the methods used by children to manipulate objects in order to solve partitive and measurement situations before formal instruction in division began. The authors found that conceptually these two types of division are different to the children and recommend that instruction should be designed to take this finding into consideration. They also found that problems with zero remainders are no more difficult than those with non-zero remainders for the children and, therefore, "no sharp dichotomy should be made between such instances when providing pre-division experiences in an instructional program" (p. 5).

Schunk (1981) found that: 1) modeling division situations for and with children; 2) providing problem solving principles; and 3) corrective feedback, produce greater gains with children who had "experienced profound failure" in mathematics than similar children taught with didactic instruction. The children exposed to cognitive modeling exhibited greater understanding of the division concept and had greater success with the operation.

This work on the basic understanding of the division concept is important since it supports the general call by educators for more meaningful instruction. Children can learn the division concept meaningfully if the proper instructional activities are used. Dawson and Ruddell (1955) summarized studies across the mathematics curriculum and concluded that "meaningful teaching generally leads to greater retention, greater transfer, and increased ability to solve problems independently" (p. 398).
Long Division

There are two basic long division algorithms taught in mathematics classrooms today:

Distributive

\[
\begin{array}{c}
21)569 \\
-42 \\
149 \\
-147 \\
2
\end{array}
\]

Subtractive

\[
\begin{array}{c}
21)569 \\
-210 \\
359 \\
-210 \\
149 \\
-147 \\
2
\end{array}
\]

In a study of third grade students, Scott (1963) taught the distributive algorithm as being associated with partitive division situations and the repeated subtraction algorithm as being associated with measurement division situations [Van Engen & Gibb's (1956) work with children supported this arrangement of algorithm with situation]. The author notes that most text series selected only one of the two algorithms to teach to children, which meant that one of the two division situations was not being well represented in the text or the children's minds. Scott found, after teaching the two algorithms to children, that the children were: 1) not confused by the two algorithms; 2) performed at least as well on problems involving the division algorithm; 3) solved problem situations as well and often better; and 4) conceptualized division better, than those students who learned only one algorithm. While some would argue that it is unimportant for students to classify problems as being partitive or measurement situations, "some clear conception of the types of problems appropriately solved through division would seem to be a minimum requirement for successful problem solving performance" (p. 751). Scott concluded that a program of systematic
attention to the understanding of division was more successful than one which teaches simple mechanical skills.

Slesnick (1982) found that understanding the division algorithm requires a greater cognitive demand on children than understanding the concept of division or rotey solving the division algorithm. The author contends that children are not ready to understand the division algorithm until they are formal thinkers. For this reason, children might be forced into learning the algorithm before they are cognitively ready, thus, guaranteeing failure and future blockages in this area.

From the information presented above on division, it is clear that students generally do not have a complete or proper conception of the division concept, even though it is possible for them to develop the conception meaningfully. It has been shown that children begin school (and before) able to divide using manipulative materials. However, as teachers use ineffective methods during division instruction, these same children who experience early success with the concept, are unable to perform simple division problems which require an understanding of the meaning of division. Teachers are expected by administrators and parents to teach these children the division algorithm and the meaning of, and methods for, solving problem situations requiring the use of the division algorithm. However, these teachers might be unsuccessful due to a general feeling of mathematics anxiety stemming from not being taught meaningfully during their own mathematical education.

Summary

The second chapter outlined: 1) the meaning theory of mathematics in an attempt to demonstrate the general concensus of mathematics
educators that mathematics instruction should be developed meaningfully; 2) the Ausubelian learning theory which dictates the framework of the concept mapping process; 3) the general mechanics of concept mapping and the related research which has been completed using this experimental heuristic; and 4) the related research pertaining to division in order to show that students have the most difficulties with division and that they generally do not understand the division concept even though they could, if taught meaningfully. This study was developed on the premise that preservice elementary teachers should understand the concepts they will be expected to teach so they will be able to teach children in a meaningful manner. In order to ascertain the conceptual understanding of this important concept, concept mapping is being used since it is designed to show the meaningful conceptual representation of any concept in the learner's cognitive structure.
CHAPTER III

PROCEDURES

The first two chapters dealt with the value of and need for the study and the literature review. This chapter will detail the specific design, instrumentation, treatments, statistical analysis, and procedures used in designing this study to determine how well preservice elementary teachers conceptualize the division concept and if there are differences between high and low mathematics achieving preservice elementary teachers' conceptualization of division.

The Pilot Study

A pilot study was completed with a small group of preservice elementary teachers prior to the start of the current study in order to accomplish the following: 1) assure that preservice elementary teachers could learn to concept map; 2) develop and test the materials and pilot the instruments used in the study; 3) develop a time line for the study; 4) develop the protocol used to teach the mapping procedure; 5) allow the researcher to practice teaching the mapping process; 6) allow the researcher to practice scoring maps; and 7) identify, through a series of Pearson correlations, the relevant variables.

Design

In order to ascertain the concept development of preservice teachers in division, a group of students was taught to concept map and then asked to individually construct a map of the division concept.
used for purposes of the study because of the intent to determine if high achieving students understand the division concept better than the low achieving students.

Once the high and low mathematics achieving groups were identified, the subjects were labeled as Category 1 for low achieving or Category 2 for high achieving mathematics students. This categorical label and the achievement variable were submitted to discriminant analysis in order to determine if the identified parts of the variable (mathematics achievement test score, course grades for Mathematics for Elementary Teachers 1 and 2, and overall grade point average) and the categorical label were strongly related to each other.

The subjects in the study were gathered in small groups twice for the purpose of receiving the treatment. During the first session the subjects were taught to concept map using the strategies developed by Novak and Gowin (1984). Once mastery of the mapping process was attained, the students were assembled for the second session and asked to map the division concept. All maps from both sessions were evaluated blindly using the scoring procedure presented in Malone and Dekkers (1984).

Instrumentation

The mathematics achievement test used in the sample selection was developed by a committee of five education professors (four of whom have a major background in educational mathematics), one mathematics professor, and one graduate student whose major emphasis is in elementary mathematics. The test was developed as a screening instrument to be used with students seeking admission to the college of education and is
therefore secure (See Appendix B for sample questions). The test was piloted and Table 1 is a summary of the test statistics as analyzed by the Statpack ItemA program. The test has construct validity since it consists of relevant concepts from the university's mathematics preparation program for elementary teachers and items which were drawn from the general kindergarten through twelfth grade curriculum.

Before the instructional sequence in concept mapping began, an attitude-toward-mathematics assessment was used with all the students included in the sample (See Appendix C for a copy of the Likert attitude scale). Four semantic differential attitude scales (attitude-toward-mathematics, -teaching mathematics, -teaching elementary school children, and -elementary school children) were developed for a previous study designed to compare alternative programs for preparing elementary mathematics teachers (Leitzel, Schultz, & White, 1979). However, the entire scale was not used since it dealt with more than the subject's attitude toward mathematics. The scales were developed and semantic differential scoring weights were assigned by the project staff consisting of elementary education, mathematics education, and mathematics professors. The attitude scale developed by the project staff was designed to be used with the same type of population as was this study so the decision was made to use part of the scale. The attitude score was used to contribute to the profile of the high and low mathematics achieving students. The results obtained by use of the attitude scale in the pilot study were analyzed using the Fortap program. The Hoyt Reliability for the instrument was 0.86.
Table 1.
Test Statistics for Pilot Data for the Mathematics Achievement Test

<table>
<thead>
<tr>
<th>Type of Statistic</th>
<th>Pilot Statistics</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>Range</td>
<td>9 - 38</td>
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<tr>
<td>Mean</td>
<td>25.04</td>
</tr>
<tr>
<td>Median</td>
<td>24</td>
</tr>
<tr>
<td>Mode</td>
<td>20, 21, 25, 26 (10)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.58</td>
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<tr>
<td>Skewness</td>
<td>+ 0.01</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>- 0.71</td>
</tr>
<tr>
<td>Reliability KR(20)</td>
<td>0.82</td>
</tr>
<tr>
<td>Number of Items</td>
<td>44</td>
</tr>
</tbody>
</table>
Treatments

All students in the high and low achieving groups, as noted above, were taught to concept map. The major thrust of the study was to determine the conceptual formation of the preservice teachers in regard to the division concept. The overall division map was sub-divided into four separate groupings (measurement division, partitive division, three laws which apply to division, and the division/multiplication relationship) in order to see if students had any particular difficulties mapping any of the sub-concepts of the division concept. The total maps and the four sub-areas of the maps (see figures 2 - 5) were scored to see if the students were connecting related concepts in a meaningful manner or if little understanding was demonstrated by the ways in which the concepts were or were not connected. It was hypothesized that no subjects in the high or the low achieving groups would be able to map the division concept at a 75% criterion level. The types of conceptions and misconceptions made by members of the two achievement groups were examined to determine the recommendations for curricular and/or methodological change so as to improve the current curriculum and to allow students to grasp more fully the division concept.

A second purpose of the study was to determine if higher achievers in mathematics were able to concept map division more completely and meaningfully than were low achievers.

The third purpose of the study was to determine if there was a sub-scale of understanding of division scores which could be used to predict achievement in mathematics for preservice elementary teachers.
BASIC OPERATIONS

is

DIVISION

is

PARTITIVE

has indicated by

LARGE SET

shared in

SMALLER SETS

gives

LABEL

has

ANSWER

with

without

REMAINDER

which are

DIVIDED EQUALLY

gives

REMAINDER

example of

27 COOKIES GIVEN TO 6 CHILDREN, HOW MANY COOKIES PER CHILD?

24 COOKIES GIVEN TO 6 CHILDREN, HOW MANY COOKIES PER CHILD?

Figure 2. The Partitive Division Sub-Area Concept Map
Figure 3. The Measurement Division Sub-Area Concept Map
Figure 4. Relationship Between Division and the Three Laws of Division
Sub-Area Concept Map
Figure 5. Relationship Between Division and Multiplication Sub-Area Concept Map
Statistical Analyses

The achievement test which was used, in part, to sort the students into high and low achieving groups, was analyzed using the Statpack ItemA program. The division item scores were correlated with the remaining item scores to see if the subjects' total achievement test score reflected the scores on the division items.

The attitude survey was analyzed using the Fortap program.

The concept maps were analyzed using the scoring procedures in Malone and Dekkers (1984).

Once all data were gathered, the following statistics were computed:

1) For hypothesis number one, the descriptive statistics of the high and low group means and the individual highest and lowest map scores were graphed for the total concept map, and each of the four sub-areas of the division map. t-tests were computed between the total group means and the criterion level of 75%. A Bonferoni correction was applied to the t-test significance values in order to account for the inflated significance level obtained when computing multiple t-tests.

2) For hypothesis number two, the variables achievement and division were submitted to discriminant analysis yielding a canonical correlation and Wilks' Lambda. A Hotellings $T^2$ was computed to discriminate the differences between the high and low groups for the total division concept map, the five subscale scores for the division map, and the four sub-area total map scores. Univariate F-tests, adjusted by a Bonferoni correction, were also computed to
examine the differences between the high and low groups on each of
the ten scores listed above.

3) For hypothesis number three, the division variable was submitted
to discriminant analysis Wilks' stepwise selection process.

4) The variables achievement and attitude were submitted to dis­
criminant analysis yielding a canonical correlation and Wilks'
Lambda.

Procedures

Students currently enrolled in the college of education and partic­
ipating in specific teacher education programs were administered a math­
ematics achievement test which, in conjunction with the other achieve­
ment scores, was used to determine the high and low mathematics achiev­
ing students. The achievement test consisted of 44 questions and took 75
minutes to complete. The use of calculators was permitted on the test.
Once the students in the two groups were identified, the selected sub­
jects were given the attitude-toward-mathematics survey and then taught

Students were given one two-hour training session in concept map­
ing during which they were taught to identify concept words and con­
struct a concept map. Concept words invoke a memory in the mind of the
person hearing the word. When a person hears the word car, the person
might think of his or her first car, a car he or she might want to own,
or a futuristic car. It was important that the students knew that not
all concept words were concept words for them. If the students had never
had experiences with a certain concept, then the word which represented
that concept had no meaning in their cognitive structure. Concept words
can fall into two separate categories: object words and event words. Object words are concept words which represent actual objects which could be seen or touched such as car, cloud, or red. Event words are concept words which represent things which happen during a person's life such as thunder, playing, or birthday party.

Next, the subjects were taught to identify and use logical connectives which are used to show the relationships which exist between two concept words on a concept map. Logical connectives, such as had, are, or contained in, are important since they show whether the student understands how two concepts are interrelated.

Students were given a general explanation of Ausubel's learning theory and how meaningful information is organized in a general to specific manner in the cognitive structure. (See Chapter 2 for a complete discussion of Ausubelian learning theory.) To highlight this theory and its relationship to concept mapping, examples of general to specific concepts, such as person and Robert Davis, were presented to demonstrate the general to specific component of the map. Students were shown the method by which these two concepts would be mapped. The concept William Merrill was added to the same line as Robert Davis on the map to show that concepts of equal generality were placed on the horizontal axis of the map (See Figure 6 for the example used in the training session).

A series of concepts surrounding plants were presented to the students and a concept map was developed. The students were taught the mapping procedure using the steps outlined in Ault (1985). The relevant concepts were identified and then rank-ordered from most general and inclusive to the most specific and exclusive. In the initial mapping
Figure 6. Mapping Example of the Structure of Concept Mapping
activity, the concepts were listed for the subjects but in subsequent mapping activities, the concepts were identified by the subjects from a set of paragraphs. The concepts in the list were clustered to show concepts of equal generality and then arranged in map form on the paper. Finally, the connecting lines and the logical connectives were inserted to complete the map. The development of the "plant" concept map (See Appendix D) was strongly influenced by this author since it was the first attempt at the mapping procedure by the subjects.

The second attempt at mapping used the information contained in several paragraphs about animals' homes. The students used the steps described above and, as a group decided how to complete the map. The complications encountered when creating a group map was that no two people totally conceptualize any topic in exactly the same way, so a compromise method of developing the map must be used. However, it was stressed that each student should think strongly about the manner in which he or she would adapt the map to his or her own way of thinking about the concepts. (See Appendix E for a complete set of concept mapping training materials.)

Following the training session, the students were asked to complete a final concept map on their own (See Appendix F). Each student received the paragraphs to read and was expected to construct the map completely without additional clues or help. Time was given to complete this final map as needed by the students. These concept maps were scored blindly using the scoring procedures outlined in Malone and Dekkers (1984). This pretest was used before the final division mapping in order to see if the subjects could accomplish the task of concept mapping. This was
necessary in order to assure that the results from the division maps were not contaminated by the subject's inability to construct a map. The resulting total pretest map score and subscale scores (concept recognition, hierarchy, grouping, branching, and propositions) all needed to be at least 75% or better correct if the student was then to be permitted to complete the division concept map (see Appendix G for scoring procedures). If a student failed to master all the subscales of the mapping procedure, additional instruction was provided before retesting. All training and scoring of the maps was completed by the researcher.

In a separate session the students were asked to map the division concept from a given set of paragraphs (Appendix H). Once again, the students were given only the paragraphs and were then expected to complete the map without further help or clues. The students were supplied with their folders containing the training materials which were used to teach them initially. Subjects were permitted the use of their training material for review purposes during the final mapping session and to help assure that forgetting part of the mapping procedure did not contaminate the final mapping. Students were given a choice of constructing the division concept map by using their pencils to write on a single piece of paper, or by first writing the concepts on small pieces of paper, then arranging them on a larger piece of paper before taping them down in the desired order. After the design of the map was complete, the subjects added connecting lines and logical connectives. The students' maps were scored blindly. Each subject had: 1) a total division score; 2) five subscale scores (concept recognition, grouping, hierarchy, branching, and propositions) for the total division map; and 3) four
sub-area scores from the total map (partitive division, measurement division, three laws of division, and multiplication/division relationship) from the division map. Subscale scores for the four sub-areas were not included in the analysis due to the relatively small raw score values involved. Large changes in the percentages occurred with only one or two points difference in the raw scores. A scoring map (Appendix I) was used to judge the students' maps.

Half of the subjects in each group was selected at random to participate in a post-mapping interview (Appendix J) in an attempt to identify areas of the mapping procedures which they perceived as being easy or difficult. Careful attention was paid to the students' perceptions of the division concept map and to their perceived success or failure. (See Appendix K for a complete time table of the study.)

Finally, the students' maps were analyzed for concept formation and misconceptions in the structure of the map. All the variables were submitted to the statistical analysis described above. The subscale and sub-area scores from the concept maps were analyzed as a composite, linear combination, and as separate variables. The information gathered from the analysis of the data and the conceptual formation of the students as demonstrated by their maps is reported in Chapter 4, and is used to draw the conclusions and make the recommendations reported in Chapter 5.
CHAPTER IV
FINDINGS

In the previous three chapters, the value of and need for the study and general rationale, the literature review, and the procedures were presented. In Chapter 4, the results of the statistical and non-statistical procedures will be presented as they relate to the hypotheses.

Subject Selection

Students were selected to be in this study based upon the scatter plots of: 1) the mathematics achievement test score with the course grade earned in Mathematics 1; 2) the mathematics achievement test score with the course grade earned in Mathematics 2; and 3) the mathematics achievement test score with the overall college grade point average (See Figures 7-9 for scatter plots). Each scatter plot was divided into nine cells. High mathematics achieving students were identified as being located in the upper right quadrant in at least two of the three scatter plots. Of the 30 high achieving students identified, 87% were located in this upper right quadrant on all three plots. Low mathematics achieving students were identified as being located in the lower left quadrant in at least two of the three scatter plots. Of the 32 low achieving students identified, 91% were located in this lower left quadrant on all three plots. Original cell sizes were unequal due to the need to make cuts in the data so that all of equal scores were either included in or excluded from the analysis.
Figure 7 Scatter Plot of Achievement Test Score and the Course Grade for Mathematics 1
Figure 8 Scatter Plot of Achievement Test Score and the Course Grade for Mathematics 2
Figure 9 Scatter Plot of Achievement Test Score and the Overall Grade Point Average
Pearson Correlations were computed between all of the achievement measures to see if these various measures were related to each other (See Table 2).

Table 2
**Pearson Correlations Between all Achievement Measures**

<table>
<thead>
<tr>
<th>Variables</th>
<th>r value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achievement Test Score with:</td>
<td></td>
</tr>
<tr>
<td>Course Grade for Mathematics 1</td>
<td>0.62*</td>
</tr>
<tr>
<td>Course Grade for Mathematics 2</td>
<td>0.82*</td>
</tr>
<tr>
<td>Grade Point Average</td>
<td>0.36*</td>
</tr>
<tr>
<td>Course Grade for Mathematics 1 with:</td>
<td></td>
</tr>
<tr>
<td>Course Grade for Mathematics 2</td>
<td>0.58*</td>
</tr>
<tr>
<td>Grade Point Average</td>
<td>0.40*</td>
</tr>
<tr>
<td>Course Grade for Mathematics 2 with:</td>
<td></td>
</tr>
<tr>
<td>Grade Point Average</td>
<td>0.63*</td>
</tr>
</tbody>
</table>

* p < .01

All of the measures correlated significantly at the .01 alpha level so it appeared that these measures were properly identified.

To see if the achievement test score, course grades for mathematics 1 and 2, and grade point average were, in combination, good predictors of achievement of mathematics for preservice elementary teachers, the data were submitted to discriminant analysis. For the purposes of the analysis, the high achieving mathematics students were assigned to category 2 and low achieving mathematics students were assigned to category 1. The analysis was completed and the results are summarized in Table 3.
This analysis resulted in a canonical correlation of 0.96 and a Wilks' Lambda of 0.11. These statistics showed that the achievement variable which consisted of the mathematics achievement test score, course grades for Mathematics 1 and 2, and grade point average, was very highly correlated with the high/low categorical labels. The small Wilks' Lambda meant that there was much between group variability and little within group variability, which indicated a good discriminant function. Therefore, the analysis demonstrated that the data used to define the two groups was generally a good measure of mathematics achievement.

Table 3
Discriminant Analysis of High/low Category and Achievement Variable

<table>
<thead>
<tr>
<th>High/Low Category With</th>
<th>Canonical Correlation</th>
<th>Wilks' Lambda</th>
<th>Chi-Squared</th>
<th>df</th>
<th>Significance</th>
</tr>
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<tbody>
<tr>
<td>Achievement Variable</td>
<td>0.96</td>
<td>0.11</td>
<td>95.09</td>
<td>4</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

*p < .01

The study had a mortality rate of 16%. Due to the time constraints, four of the high achieving subjects and six of the low achieving subjects chose not to participate in the study, leaving 26 high and 26 low mathematics achieving students who completed the study.

Statistical Results for the Instrumentation

Mathematics Achievement Test

The achievement test was given to 114 subjects at the beginning of the study. The results were analyzed using the Statpack ItemA program and the results are summarized in Table 4. The results are similar to the statistics attained during the piloting of the test and reported in Chapter 3.
Table 4
Test Statistics for Dissertation Data for the Mathematics Achievement Test.

<table>
<thead>
<tr>
<th>Type of Statistic</th>
<th>Dissertation Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>114</td>
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<tr>
<td>Range</td>
<td>10 - 38</td>
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<tr>
<td>Mean</td>
<td>23.76</td>
</tr>
<tr>
<td>Median</td>
<td>24</td>
</tr>
<tr>
<td>Mode</td>
<td>29 (14)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.01</td>
</tr>
<tr>
<td>Skewness</td>
<td>- 0.00</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>- 0.40</td>
</tr>
<tr>
<td>Reliability KR(20)</td>
<td>0.78</td>
</tr>
<tr>
<td>Number of Items</td>
<td>44</td>
</tr>
</tbody>
</table>
In an attempt to discern if the total achievement test scores were reflective of the subjects' ability to correctly answer items which involved division or the division concept, the subset of division items was correlated with the remainder of the test. Of the 44 questions on the test, 14 were identified either as directly relating to the division concept or as items in which division was an integral part of finding the solution (KR 20 = 0.59). As the Pearson Correlation coefficient was $r = 0.78 (p < .01)$ it seems likely that the achievement score reflects achievement of the preservice elementary teachers on the division items (See Table 5 for division item statistics).

**Attitude Scale**

The students included in the sample were administered a semantic differential attitude-toward-mathematics assessment (See Appendix C). Data gathered were subjected to the Fortap program, resulting in a Hoyt Reliability of 0.88.

**Pretest Mapping Activity**

Subjects were required to complete a pretest map at or above a 75% criterion level in order to determine if the subjects grasped the mapping procedure. Figure 10 is a graphical display of the pretest concept map scores. All the mapping scores were well above the 75% criterion level; thus no additional mapping instruction was necessary. Figure 11 is an example of a pretest map constructed by one of the subjects and generally represents the types of maps produced.

**Hypothesis Number One**

None of the subjects in the high or the low mathematics achieving groups will be able to attain an overall score of 75% accuracy on
Table 5
Test Statistics for Dissertation Data for the Division Items on the Mathematics Achievement Test

<table>
<thead>
<tr>
<th>Type of Statistic</th>
<th>Dissertation Statistics</th>
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</thead>
<tbody>
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<td>N</td>
<td>114</td>
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<tr>
<td>Range</td>
<td>1 - 13</td>
</tr>
<tr>
<td>Mean</td>
<td>7.76</td>
</tr>
<tr>
<td>Median</td>
<td>8</td>
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<tr>
<td>Mode</td>
<td>7 (22)</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.44</td>
</tr>
<tr>
<td>Skewness</td>
<td>- 0.27</td>
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<tr>
<td>Kurtosis</td>
<td>- 0.04</td>
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<tr>
<td>Reliability KR(20)</td>
<td>0.59</td>
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<tr>
<td>Number of Items</td>
<td>14</td>
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</table>
Figure 10. Pretest Concept Map Results
Figure 11. Example of Pretest Concept Map
Figure 12. Total Division Concept Map Results for High and Low Groups
Table 6
Total Division Concept Map and Subscale Scores for All Subjects

<table>
<thead>
<tr>
<th>Subject ID</th>
<th>Total Concept Map</th>
<th>Recognition</th>
<th>Grouping Hierarchy</th>
<th>Branching Propositions</th>
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Low Mathematics Achieving Group

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<thead>
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<th>Subject ID</th>
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<th>Grouping Hierarchy</th>
<th>Branching Propositions</th>
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* p < .01
Figure 13. Partitive Sub-Area Division Concept Map Results for High and Low Achieving Groups
Figure 14. Measurement Sub-Area Division Concept Map Results for High and Low Achieving Groups
Figure 15. The Laws of Division Sub-Area Concept Map Results for High and Low Achieving Groups
Figure 16. Division/Multiplication Relationship Sub-Area Concept Map Results for High and Low Achieving Groups
this level was the relationships between division and the three laws of division. Four subjects performed exactly at the 75% correct level on this sub-area map. While individual scores on the sub-scale scores occasionally surpassed the desired level of performance, the vast majority of the scores was well below the level chosen as demonstrating proficient understanding (See Table 8).

The total map score, the subscale scores (concept recognition, grouping, hierarchy, branching, and propositions), and the sub-area scores (partitive division, measurement division, three laws of division, and division/multiplication relationship) were submitted to Fortap in order to attain an estimate of reliability for each variable and to multiple regression analysis in order to obtain means and standard deviations. Means, standard deviations, and Hoyt reliability for these ten variables are reported in Table 9.

t-tests were computed to compare each of the sub-area map means and the desired criterion. Like the total division map, each of the sub-area means significantly differed from 75% ($p < .01$) (See Table 7). A Bonferroni correction was applied to the t-test significant values reported in Table 7 to account for the inflated significance level obtained when computing multiple t-tests.

**Decision on the Hypothesis**

Based upon the distribution of scores and the t-test values, it appears very likely that preservice teachers do not understand: 1) the concept of division as a total concept; 2) the measurement sub-area; 3) the partitive sub-area; 4) the three laws of division sub-area; and 5) the relationship between multiplication and division sub-area.
Table 8
Division Sub-Area Concept Map Scores for All Subjects

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<th>Subject ID</th>
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<th>3 Laws Map</th>
<th>Mult./Division Map</th>
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<th>3 Laws Map</th>
<th>Mult./Division Map</th>
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Table 9
Means, Standard Deviations, and Hoyt Reliabilities for Total Division Map, the Five Subscale Scores for the Division Map, and the Four Sub-Area Total Map Scores

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<th>Variable</th>
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<th>Hoyt Reliability</th>
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<td>0.78</td>
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<td>Branching</td>
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<td>0.76</td>
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<td>0.74</td>
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Hypothesis Number Two

High mathematics achieving preservice elementary teachers will demonstrate a greater understanding of division as assessed by concept mapping, than low mathematics achieving preservice elementary teachers.

Statistics

A composite, linear combination of the total concept map score and the sub-scale scores of concept recognition, grouping, hierarchy, branching, and propositions representing the understanding of division, was compared with the achievement variable, to yield a canonical correlation of 0.57 and a Wilks' Lambda of 0.68 (See Table 10). The Wilks' Lambda showed significant differences for the composite, linear combination across the components of the entire division variable between the high and low achieving groups ($p < .01$).

<table>
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<tr>
<th>Achievement Variable with Division Variable</th>
<th>Canonical Correlation</th>
<th>Wilks' Lambda</th>
<th>Chi-Squared</th>
<th>df</th>
<th>Significance</th>
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* $p < .01$

Ten of the scores (the total division map score, the five subscale scores for the division map, and the four sub-area total map scores) obtained by each subject were analyzed using MANOVA, yielding a Hotellings $T^2$ of 0.85 ($p < .01$) and univariate $F$-tests for each of the ten scores. The Hotellings $T^2$ treats all ten scores together and showed that the high achieving mathematics group scored significantly higher on the
ten scores than did the low mathematics achieving group (See Table 11). The univariate F-tests show the significant differences between high and low achieving groups on each of the ten scores. However, the significant levels with one or no stars have not been adjusted for multiple comparison so no strong conclusions can be drawn from this data. After a Bonferroni correction was applied to the significant values, the values associated with two stars, total division map, hierarchy subscale score of the total division map, and the three laws of division sub-area map, were all still highly significant (See Table 12).

Table 11
Hotellings $T^2$ Multivariate Test of Significance for Ten Concept Mapping Scores

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Value</th>
<th>Exact F</th>
<th>Hypoth. df</th>
<th>Error df</th>
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* $p < .01$

A correlation matrix was computed for the variables submitted to MANOVA and the relative size of the correlations was important to inspect. The largest correlations of the individual variables with the high/low grouping occur with the three laws sub-area map and the hierarchy subscale score. Very high correlations also are found between the partitive and measurement sub-area map scores and the concept recognition and grouping subscale scores (See Table 13).

Decision on the Hypothesis

Based upon the significant differences between the division and the achievement variables, it appears very likely that high mathematics
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Table 13
Correlation Matrix for Division Concept Map Variables

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<td>0.61</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Partitive Sub-Area</td>
<td>0.39</td>
<td>0.69</td>
<td>0.71</td>
<td>0.76</td>
<td>0.37</td>
<td>0.53</td>
<td>0.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement Sub-Area</td>
<td>0.36</td>
<td>0.68</td>
<td>0.71</td>
<td>0.75</td>
<td>0.37</td>
<td>0.57</td>
<td>0.71</td>
<td>0.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three Laws Sub-Area</td>
<td>0.54</td>
<td>0.79</td>
<td>0.60</td>
<td>0.56</td>
<td>0.67</td>
<td>0.38</td>
<td>0.48</td>
<td>0.48</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division/Mult. Sub-Area</td>
<td>0.35</td>
<td>0.67</td>
<td>0.54</td>
<td>0.49</td>
<td>0.48</td>
<td>0.32</td>
<td>0.31</td>
<td>0.52</td>
<td>0.51</td>
<td>0.52</td>
<td></td>
</tr>
</tbody>
</table>
achieving preservice elementary teachers demonstrated a greater understanding of division than the low mathematics achieving preservice teachers.

Hypothesis Number Three:

There will be a subset of component variables from the understanding of division composite variable which will predict achievement of subjects in both the high and the low mathematics achieving groups.

Statistics

The component variables of the division composite variable were submitted to the Wilks stepwise selection procedure of discriminant analysis which identified potentially useful variables for predicting achievement in preservice teachers.

Four variables entered the analysis in the following order: number one was the hierarchy subscale score; number two was the grouping subscale score; number three was the concept recognition subscale score; and number four was the branching subscale score. (See Table 14).

Decision on the Hypothesis

Based upon the analysis the subset of scores of the division variable, hierarchy subscale score, grouping subscale score, concept recognition subscale score, and branching subscale score can be used to predict mathematics achievement in preservice elementary teachers.

Unhypothesised Statistical Results

Attitude data were used to help establish a profile of the high and the low mathematics achieving preservice teacher. The data, which were analyzed using the Fortap program, yielded scores which were submitted
Table 14
Discriminant Prediction of Division Sub-set Variables Which Predict Mathematics Achievement in Preservice Teachers

<table>
<thead>
<tr>
<th>Entered Value</th>
<th>Wilks' Lambda</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hierarchy Subscale Score</td>
<td>0.82</td>
<td>0.00*</td>
</tr>
<tr>
<td>Grouping Subscale Score</td>
<td>0.74</td>
<td>0.00*</td>
</tr>
<tr>
<td>Concept Recognition Subscale Score</td>
<td>0.72</td>
<td>0.00*</td>
</tr>
<tr>
<td>Branching Subscale Score</td>
<td>0.70</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

* p < .01
to discriminant analysis (See Table 15). The canonical correlation between the high and low achievers was 0.51 and the Wilks' Lambda was 0.74 (p < .01). This result showed that, based upon the attitude survey used in this study, high mathematics achieving preservice teachers had significantly better attitudes than the low mathematics achieving preservice teachers.

Table 15

<table>
<thead>
<tr>
<th>Achievement Variable with Attitude Variable</th>
<th>Canonical Correlation</th>
<th>Wilks' Lambda</th>
<th>Chi-Squared</th>
<th>df</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude Variable</td>
<td>0.51</td>
<td>0.74</td>
<td>14.76</td>
<td>1</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

* p < .01

Unhypothesised, Non-statistical Results

Concept Map Data

Analysis of the concept maps indicated that the majority of the top scorers on the division activity were high achieving students and the majority of the bottom scorers were from the low achieving group. In fact, three out of four of the top 30% of the total scores on concept maps were obtained by high achieving mathematics students. Conversely, three out of four of the bottom 30% of the total scores belonged to low achieving mathematics students, with all of the lowest 10 scores being from the low achieving group. Table 16 has a complete ordered listing of the raw scores, overall percentage, and group (high or low) to which each subject belongs.

Since none of the maps approached the 75% criterion for successful completion of the activity, all of the maps used as examples in the
Table 16
Rank Order of Total Concept Mapping Scores

<table>
<thead>
<tr>
<th>Number</th>
<th>Raw Score</th>
<th>Percent</th>
<th>High/Low</th>
<th>Number</th>
<th>Raw Score</th>
<th>Percent</th>
<th>High/Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>177.5</td>
<td>65</td>
<td>H</td>
<td>27</td>
<td>103.5</td>
<td>38</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>170</td>
<td>62</td>
<td>H</td>
<td>28</td>
<td>103</td>
<td>37</td>
<td>H</td>
</tr>
<tr>
<td>3</td>
<td>167.5</td>
<td>61</td>
<td>L</td>
<td>29</td>
<td>102</td>
<td>37</td>
<td>L</td>
</tr>
<tr>
<td>4</td>
<td>160.5</td>
<td>58</td>
<td>H</td>
<td>30</td>
<td>101.5</td>
<td>37</td>
<td>L</td>
</tr>
<tr>
<td>5</td>
<td>156</td>
<td>57</td>
<td>H</td>
<td>31</td>
<td>100</td>
<td>36</td>
<td>L</td>
</tr>
<tr>
<td>6</td>
<td>153.5</td>
<td>56</td>
<td>H</td>
<td>32</td>
<td>88.5</td>
<td>32</td>
<td>L</td>
</tr>
<tr>
<td>7</td>
<td>152</td>
<td>55</td>
<td>L</td>
<td>33</td>
<td>85.5</td>
<td>31</td>
<td>H</td>
</tr>
<tr>
<td>8</td>
<td>149</td>
<td>54</td>
<td>H</td>
<td>34</td>
<td>85</td>
<td>31</td>
<td>H</td>
</tr>
<tr>
<td>9</td>
<td>145.5</td>
<td>53</td>
<td>H</td>
<td>35</td>
<td>84</td>
<td>31</td>
<td>L</td>
</tr>
<tr>
<td>10</td>
<td>144</td>
<td>52</td>
<td>L</td>
<td>36</td>
<td>83</td>
<td>30</td>
<td>L</td>
</tr>
<tr>
<td>11</td>
<td>142.5</td>
<td>52</td>
<td>H</td>
<td>37</td>
<td>82</td>
<td>30</td>
<td>L</td>
</tr>
<tr>
<td>12</td>
<td>140.5</td>
<td>51</td>
<td>H</td>
<td>38</td>
<td>81</td>
<td>29</td>
<td>H</td>
</tr>
<tr>
<td>13</td>
<td>139.5</td>
<td>51</td>
<td>L</td>
<td>39</td>
<td>81</td>
<td>29</td>
<td>L</td>
</tr>
<tr>
<td>14</td>
<td>136</td>
<td>49</td>
<td>H</td>
<td>40</td>
<td>79.5</td>
<td>29</td>
<td>H</td>
</tr>
<tr>
<td>15</td>
<td>135.5</td>
<td>49</td>
<td>H</td>
<td>41</td>
<td>78.5</td>
<td>29</td>
<td>H</td>
</tr>
<tr>
<td>16</td>
<td>135.5</td>
<td>49</td>
<td>H</td>
<td>42</td>
<td>76</td>
<td>28</td>
<td>H</td>
</tr>
<tr>
<td>17</td>
<td>134</td>
<td>49</td>
<td>H</td>
<td>43</td>
<td>76</td>
<td>28</td>
<td>L</td>
</tr>
<tr>
<td>18</td>
<td>133</td>
<td>48</td>
<td>H</td>
<td>44</td>
<td>69</td>
<td>25</td>
<td>L</td>
</tr>
<tr>
<td>19</td>
<td>123.5</td>
<td>45</td>
<td>L</td>
<td>45</td>
<td>67</td>
<td>24</td>
<td>L</td>
</tr>
<tr>
<td>20</td>
<td>118</td>
<td>43</td>
<td>L</td>
<td>46</td>
<td>65.5</td>
<td>24</td>
<td>L</td>
</tr>
<tr>
<td>21</td>
<td>115</td>
<td>42</td>
<td>H</td>
<td>47</td>
<td>65</td>
<td>24</td>
<td>L</td>
</tr>
<tr>
<td>22</td>
<td>113</td>
<td>41</td>
<td>H</td>
<td>48</td>
<td>63.5</td>
<td>23</td>
<td>L</td>
</tr>
<tr>
<td>23</td>
<td>112</td>
<td>41</td>
<td>H</td>
<td>49</td>
<td>61</td>
<td>22</td>
<td>L</td>
</tr>
<tr>
<td>24</td>
<td>111.5</td>
<td>41</td>
<td>L</td>
<td>50</td>
<td>61</td>
<td>22</td>
<td>L</td>
</tr>
<tr>
<td>25</td>
<td>105</td>
<td>38</td>
<td>H</td>
<td>51</td>
<td>56</td>
<td>20</td>
<td>L</td>
</tr>
<tr>
<td>26</td>
<td>104.5</td>
<td>38</td>
<td>L</td>
<td>52</td>
<td>45.5</td>
<td>17</td>
<td>L</td>
</tr>
</tbody>
</table>
discussion below, including the top scoring maps, will be flawed. Only certain aspects of each map will be high-lighted for the purpose of the discussion. The maps in this chapter have been reproduced for easier examination. (See Appendix L for the actual student produced maps.)

The maps were scrutinized to see if differences existed in the construction of the maps between the top and bottom scorers on the map. The hierarchical structure of the maps was one area where a distinctive difference was found. Figure 17 is an example of a high scoring map. Note the attempt by the map-maker to rank more general concepts toward the top of the map while progressively less general concepts are arranged in a vertical fashion down the map. Figure 18 is an example of a low scoring map. While there was a vertical component to the map, the major sub-concepts were listed horizontally under the division concept with no attempt made to distinguish between the generality of the listed concepts.

Top scoring maps tended to identify more relevant concepts from the paragraphs than the low scoring maps. Figures 19 and 20 are dichotomous examples to demonstrate the differences in the complexity of the high and low scoring maps. Figure 19, while not perfect in the concept listing and connecting lines, demonstrated an attempt to use more concepts in explaining the division concept while Figure 20 showed a relatively simplistic view of the concept.

As shown in Figure 21, high scorers tended to list relevant inter-relationships between concepts. This action resulted in higher scores since closed groupings resulted in more points as defined by the scoring schemata. Low scoring maps, like Figure 22, tended to list everything in
Figure 17. High Scoring Map Demonstrating the Use of Hierarchical Levels
Figure 18. Low Scoring Map Demonstrating the Lack of Use of Hierarchical Levels
Figure 19. High Scoring Map Demonstrating the Large Number of Relevant Concepts Identified
Figure 20. Low Scoring Map Demonstrating the Relative Simplistic Map with few Relevant Concepts Identified
Figure 21. High Scoring Map which Demonstrates the Complex Groupings Used in Higher Scoring Maps
Figure 22. Low Scoring Map which Demonstrates the Simple Groupings Used in Lower Scoring Maps
point or open groupings, which results in fewer points. Top scorers also tended to have more branching points due to the fact that they included more concepts and had more complex grouping schemata which resulted in more concepts having two or more branching points.

The logical connectives used to make the propositions tended to be equally simple on all the maps regardless of the score on the map. Instead of mathematical jargon, very simple vocabulary was used to explain how the two concepts were related. Top scores simply had more propositions because they had more relevant concepts identified to interrelate. On 29% of the maps, including maps from all scoring levels, the map-maker omitted connectors needed to describe one or more of the relationships. Figure 23 is an example of a map where a relationship was believed to exist but no logical connectors were supplied to explain the relationship.

Also, there were five observations which were found to transcend all the scores and ability levels. First, 52% of the maps demonstrated a general lack of understanding of what the vocabulary of division is, how it related to the division concept, and how the multiplication vocabulary is interrelated to the division concept. Figure 24 shows an example of a subject who seems to believe that factors, quotient, dividend, and divisor are all division vocabulary and the general structure of the map suggests a lack of understanding of how the vocabulary in division is structured.

Second, 40% of the map-makers tended to list the major sub-concepts: laws, types of division, and signs or forms, all on the same level on the maps. Also, 63% of the maps showed no attempt to interrelate
DIVISION

has 3 governing
laws which are

is the

has two types

DIVISOR-QUOTIENT

which states
that

IF c/a=b
THEN c/b=a

CONTINUOUS

which states
that

(a+b)/c=(a/c)+b

DISTRIBUTIVE

which states
that

(a+b)/c=(a/c)+(b/c)

INVERSE OF
MULTIPLICATION

and has these
3 pairs to
each problem

MEASUREMENT

has

PARTITIVE

has

NO REMAINDER

DIVIDEND

which is the

NUMBER BEING
DIVIDED

DIVISOR

which is the

NUMBER USED
TO DIVIDE

QUOTIENT

which is the

ANSWER

No Logical Connectives

Figure 23. Example of Connecting Lines Being Used Without Logical Connectives
Figure 24. Example of Part of a Map Which Shows a Lack of Understanding of the Division Vocabulary
these major sub-concepts. Figure 25 demonstrates this problem: concepts all on the same level and no attempt to interrelate the concepts other than to link them all with the division concept.

Third, 29% of the maps were classified as being generally "confused." A confused map was one in which major problems were found in the way in which concepts were related, where concepts were placed, or a combination of the two. Figure 26 is one example of a map in which major problems exist in the placement of concepts and in the relationships shown between concepts. This map-maker has associated the law of distribution with measurement division, the law of continuous division with partitive division, and the divisor/quotient law with division in general.

Fourth, subjects had trouble relating division to multiplication. While 25% of the maps listed multiplication as a related concept for division, and 27% of the maps operationally defined division as the inverse of multiplication, only 10% made the actual connection on their map to explicitly show the relationship which exists.

Fifth, 13% of the maps left one or more concepts "floating." A floating concept is one which is not connected to another concept with a logical connective. Therefore, no proposition is formed and the concept is not part of the map. In Figure 27, the map-maker probably felt that the examples at the bottom of the map should be included in the map, but was unsure of where each fit in the overall schema of the division concept. The map-maker in Figure 28, however, probably omitted connecting lines and logical connectives purely by oversight, as judged by the one proposition which is formed between the "division laws" concept and the "continuous division" concept.
Figure 25. Example of a Horizontal Placement of Major Division Sub-Concepts
Figure 26. Example of a Confused Concept Map
Figure 27. Example 1 of Floating Concepts
Interview Data

One-half of each of the high and low groups was selected randomly and was interviewed upon completion of the division mapping activity to determine how they felt about the difficulty level of the concept mapping activity. All the students interviewed were asked if the mapping was difficult, of moderate difficulty, or easy before and after the division mapping activity. Table 17 shows the total responses from the high and low achieving groups. Clearly, the low group felt that concept mapping was "easy" to "moderately difficult" before the division mapping occurred, with a pronounced shift to "difficult" after the division mapping. The high group also showed a shift, but it occurred from "easy" before division mapping to "moderately difficult" or "difficult" following division.

Table 17
Classification of Concept Mapping by subjects as Easy, Moderately Difficult, or Difficult Before and After the Division Concept Map

<table>
<thead>
<tr>
<th>Classification</th>
<th>Before Group (n=13)</th>
<th>Before Group (n=13)</th>
<th>After Group (n=13)</th>
<th>After Group (n=13)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Easy</td>
<td>9 (69%)</td>
<td>5 (38%)</td>
<td>2 (15%)</td>
<td>1 (8%)</td>
</tr>
<tr>
<td>Moderately Difficult</td>
<td>4 (31%)</td>
<td>7 (54%)</td>
<td>7 (54%)</td>
<td>2 (15%)</td>
</tr>
<tr>
<td>Difficult</td>
<td>0 (0%)</td>
<td>1 (8%)</td>
<td>4 (31%)</td>
<td>10 (77%)</td>
</tr>
</tbody>
</table>

More interestingly, no one, high or low achievers, felt that concept mapping became easier when comparing their responses from before and after the division mapping. In fact, 69% of all the subjects interviewed in both groups felt that the task became harder after the
division task. There were, however, twice as many low achievers than high achievers who felt that the task was harder after they had experienced the division concept mapping task. (See Table 18 for complete results.)

Table 18
Subjects' Perceived Change in the Concept Mapping Task From Before to After the Division Concept Mapping Task

<table>
<thead>
<tr>
<th>Change</th>
<th>High Group (n=13)</th>
<th>Low Group (n=13)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy to Difficult</td>
<td>2 (15.5%)</td>
<td>4 (31%)</td>
</tr>
<tr>
<td>Easy to Moderately Difficult</td>
<td>5 (38%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Moderately Difficult to Difficult</td>
<td>2 (15.5%)</td>
<td>5 (38%)</td>
</tr>
<tr>
<td>Easy (Both Times)</td>
<td>2 (15.5%)</td>
<td>1 (8%)</td>
</tr>
<tr>
<td>Moderately Difficult (Both Times)</td>
<td>2 (15.5%)</td>
<td>2 (15.5%)</td>
</tr>
<tr>
<td>Difficult (Both Times)</td>
<td>0 (0%)</td>
<td>1 (8%)</td>
</tr>
</tbody>
</table>

Subjects were also asked to explain why they felt the mapping activities were easy, moderately difficult, or difficult before and after the division mapping. Similar responses made by the subjects were classified in categories in an attempt to see the general patterns. With few exceptions, responses from the low achieving subjects could be classified into four basic categories before the division mapping: 1) the activity was logical and organized; 2) it required a lot of thought; 3) it seemed easy; and 4) they felt they knew a lot about the topics mapped. After the division mapping, the low group believed that: 1) the mapping was hard to understand, 2) it was hard to organize and coordinate the concepts, and 3) they couldn't identify key concepts.
The high group had views about the mapping procedure similar to those of the low achievers before and after the division mapping, but the high achievers included a more varied set of responses for their views after division. Division was viewed as being abstract by three of the high achievers and, therefore, was more difficult than the training mappings. One of the subjects thought that the number of concepts had a direct relationship with the difficulty of the mapping activity. Since the division paragraphs had more concepts, it was therefore more difficult. Two of the high group subjects saw the division mapping as easy and offered the following two reasons: 1) one simply understood what the division paragraphs were saying; and 2) the other thought that the use of slips of paper to write concepts on prior to arranging them on the map, made the mapping simpler. (See Appendix M for complete interview data.)

Summary

This chapter presented the statistical and non-statistical findings of the study. In summary, the seven major findings of this study were:

Statistical Findings
1) None of the subjects, high or low achieving, could complete the division map, or any of the four sub-area maps, at the desired 75% criterion level. ($p < .01$)

2) Even though no one mapped the division concept completely, the high achieving subjects constructed better maps than the low achievers, therefore demonstrating better understanding of division. ($p < .01$)
3) A subset of division scores consisting of the hierarchy, grouping, concept recognition, and branching subscale scores can be used to predict achievement for this group of subjects. \((p < .01)\)

4) High achieving subjects had a better attitude toward mathematics than low achieving subjects. \((p < .01)\)

5) High scoring maps have a more complex hierarchical structure, more relevant concepts recognized, more complex groupings. \((p < .001)\)

**Non-statistical Findings**

6) All subjects tended to have trouble with the division and multiplication vocabulary as well as with the manner in which the vocabulary related to the total map.

7) Low achieving subjects perceived the mapping of the division concept as being harder than the high achieving subjects.
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Summary of Previous Chapters

This study was conducted in order to assess preservice elementary teachers' conceptual understanding of division. Preservice elementary teachers were administered an achievement test, and the results, in conjunction with course grades for Mathematics for Elementary Teachers 1 and 2 and the overall grade point average, were used to classify the students into high and low achieving mathematics groups.

Concept mapping, an experimental heuristic developed by Novak and his colleagues and based upon the learning theory of Ausubel, was the procedure taught and used in the study to assess the conceptual understanding of the subjects. Research shows that students taught to organize material by concept mapping score higher on higher-level cognitive questions than do students taught to organize material by the traditional outline. Ausubel believes that if information is to be learned meaningfully, it must be learned in a substantive and nonarbitrary manner by a person who has the necessary prerequisite knowledge and who is predisposed to learn the material in a meaningful manner. Meaningful knowledge is hypothesized to be organized in a general to specific manner in the cognitive structure of the learner's mind; consequently, a concept map is developed similarly.
Both the high and low achieving groups were administered an attitude-toward-mathematics scale and taught to concept map during the first of two treatment sessions. The first session ended with the administration of a pretest concept mapping activity. Performance at the 75% level of efficiency on the pretest was required, and was achieved by all subjects before admission to the second session. During the second session, all the subjects were expected to map the division concept. Finally, after the second treatment session half of each of the high and low achieving groups was randomly selected to be interviewed about the relative difficulty of the mapping procedure.

The division maps were analyzed and it was found that: 1) none of the students could map the division concept at the 75% criterion level; 2) the subjects in the high achieving group could map the division concept significantly better than the subjects in the low achieving group; and 3) the hierarchy, grouping, concept recognition, and branching subscale scores can be used to predict the mathematics achievement of elementary preservice teachers. Also, scoring of the attitude scale revealed that the subjects in the high achieving group had significantly better attitudes toward mathematics than the subjects in the low achieving group.

Examination of the division maps showed that subjects who had high scoring maps constructed more complex maps than low scoring subjects, as demonstrated by the subscale scores: concept recognition, grouping, hierarchy, branching, and propositions. Additionally, high scorers on the map construction had a better command of the relationship of the various sections of the division map than did the low scorers.
High achieving subjects reported the concept mapping activity to be easier both before and after mapping the division concept than did the low achieving subjects.

Hypothesized Interpretations and Conclusions

It was hypothesized that none of the subjects in the study could map the total division concept or the four sub-area concepts at a 75% criterion level. In order to assure that the difficulty encountered in construction of a division concept map would not be attributable to inability to construct concept maps in general, the subjects were trained to concept map at a level better than 75%, as assessed by a pretest mapping activity.

None of the subjects in the high or the low achieving groups could map the division concept at the prescribed level. It therefore appears that the preservice elementary teachers employed in this study encountered genuine difficulty with the understanding of the division concept. Even subjects who scored very well on items from the achievement test, which required the use of division, and who had high grades in the required mathematics courses, could not show mastery of this concept through concept mapping. These subjects obviously became very good at recognizing division situations and were adept at using the division algorithm, but they could not demonstrate an understanding of the concept and the complex interrelationships found in the concept. Therefore, there appears to be a need to better train the future teachers as to the meaning and interrelationships of this complex topic. Division occupies a great deal of the mathematics curriculum from the third to the eighth grades, and teachers need to conceptualize the division topic if there
is any hope that they will teach this important topic in a meaningful manner.

Surprisingly, there was a parallelism between the highest and lowest scores on the total division map. This parallel aspect suggests that all of the students, high and low achieving, were successful on certain subscales (concept recognition and propositions), moderately successful on other subscales (grouping and branching), and very unsuccessful on the last subscale (hierarchy). Therefore, the achievement level of the students did not influence their relative ability to accomplish the subscales.

Subjects in the high achieving group scored almost identically on the partitive and measurement sub-area maps while subjects in the low achieving group more often mapped the two sub-areas differently, especially subjects who were at the lower end of the scoring range. These two maps received the same number of points on the five subscales and, therefore, on the total map. That these sub-areas were mapped differently may suggest that subjects who mapped the division concept very poorly see these two forms of division as being conceptually different.

While it was believed, and later held true, that none of the subjects could adequately map the division concept, it was also believed that the high achieving subjects could map the concept better than the low achieving subjects. This belief was also supported by the statistical analysis. Overall, the higher achieving students scored significantly better on the total map and the four sub-areas of the division concept than did the lower achieving students. Also, the higher achieving students identified more concepts, had a more complex hierarchy, grouped
concepts in more complex configurations, identified more branching points, and identified more propositions on the total division map than did the lower achieving students.

Therefore, the higher achieving students, while experiencing difficulty mapping the division concept, demonstrated greater understanding of the concept and would need less meaningful instruction to gain a true conceptualization of division, than would be required by the lower achieving students.

It was also believed that there would be a subset of the division variable which would predict mathematics achievement in preservice elementary teachers. Subjects who listed more hierarchical levels, had more complex groupings, identified more relevant concepts, and had more branching points were higher achievers in mathematics as assessed by the achievement variable used in this study. Therefore, for this population of subjects, these scores predicted mathematical achievement.

**Unhypothesized Interpretations and Conclusions**

The high achieving subjects tended to have higher scores on the division mapping activity while the low achieving subjects tended to have lower scores. There were, however, a few low achievers who produced a high scoring map and several high achievers who produced a low scoring map.

The reason for finding low achieving subjects who score high on division concept mapping is hard to explain based upon the statistical findings reported earlier. One possible explanation could be that certain subjects, regardless of their mathematical achievement ability, exert maximum effort to complete any activity. These subjects are
juniors and seniors in college and, therefore, are basically intelligent persons. The concept mapping heuristics may have come quite close to the manner in which they organize material in other subject areas. It is conceivable that this ability to organize material was transferred to the division mapping exercise. A second possible explanation could be that some of the low achieving students are poor test takers. Since the concept mapping activity is not a conventional test, the students may have felt more at ease and therefore, may have demonstrated what they truly knew.

Finding high achieving subjects who score low on the division concept map is somewhat easier to understand. Many high achievers in mathematics are very good at memorizing material and the related algorithms associated with various operations. These subjects tend to have high grades and score well on tests of mathematical ability. However, they may or may not understand the concepts for which they have memorized algorithms. These subjects would not necessarily do well on a concept mapping activity which is designed to determine conceptual understanding.

Generally, most of the subjects felt that the mapping activity was fairly easy before they were asked to map the division concept. However, following the division mapping activity, there was a pronounced shift by the high group toward assessing the division mapping as moderately difficult to difficult and the low group assessing the division mapping as difficult. These perceptions made by the subjects support the notions that: 1) the division mapping was more difficult than the pre-test mapping activity, causing lower scores on the maps for everyone;
and 2) the high group, who did better on the mapping than did the low group, perceived the division concept as being less difficult than did the low group.

**Programmatic Recommendations**

The programmatic recommendations derived from this study have been divided into two separate groups: 1) understanding of division; and 2) future uses of concept mapping.

**Understanding of Division**

There appears to be a general lack of understanding of the division concept by preservice elementary teachers. Therefore, specific steps need to be taken in order to assure that the future teachers have a proper conceptual understanding of the topics they will be expected to teach. The following recommendations are made:

1) Since none of the preservice elementary teachers could demonstrate a thorough understanding of the division concept by concept mapping, the division concept needs to be taught in a manner which will enhance understanding.

2) In order to assist preservice elementary teachers' understanding of the division concept, work needs to be encouraged at a more concrete level.

3) Based upon the fact that the abilities to list concepts in a hierarchical structure and group concepts accordingly are the best predictors of mathematics achievement in preservice teachers, more emphasis should be place upon development of classification skills during mathematics instruction.
4) Since high mathematics achieving preservice elementary teachers have a better attitude toward mathematics than do low mathematics achievers, concerted efforts to develop instructional programs which will affect positive attitudinal changes must be encouraged.

**Future Uses of Concept Mapping**

In various studies which use concept mapping, including this one, the mapping heuristic has been shown to be an effective tool to use for the assessment of conceptual understanding, and it is a relatively easy heuristic to teach, learn, and score. Overall, highly successful preservice elementary teachers in mathematics score significantly better on the division concept mapping activity, which is used to assess understanding, than do less successful preservice elementary teachers. Therefore, the following recommendations are made:

1) A concerted effort should be made to instruct students in the concept mapping process and it should be used, instead of recall tests, to assess their understanding of concepts in mathematics.

2) Students should be encouraged to construct maps which organize their class notes and text information so as to facilitate study habits and build their understanding of the topics.

**Recommendations for Further Study**

This study has developed a foundation for using concept mapping in a subject area other than science and reading, the curricular areas focused upon in prior studies. The recommendations for further research on concept mapping and mathematics are:

1) Replicate this study with a different population and larger sample of preservice teachers to see if the results remain constant.
2) Develop additional concept maps covering other mathematical topics and test each to see if the results are similar to those found with the division topic.

3) Conduct an experimental study which teaches concept mapping as one of two methods used for studying and organizing mathematical content before testing to determine if the mapping group could score better on a test of higher-level questions than the group taught a more traditional method of studying mathematics.

4) Conduct a study to determine the extent to which logical and formal reasoning abilities influence concept mapping of mathematical topics.

5) Explore whether concept mapping can be taught to, and used by, students at all levels (elementary, middle school, high school, and college).
APPENDIX A

Course Material for Mathematics for Elementary Teachers 1 and 2
Course Material for Mathematics for Elementary Teachers 1 and 2

1. Problem Solving and Strategies for Problem Solving
2. Place Value and Numeration
3. Operations with Whole Numbers
   - Addition
   - Subtraction
   - Multiplication
   - Division
4. Number Theory
   - Divisibility
   - Prime Numbers
   - GCD and LCM
5. Fractions
   - Definition of Fractions
   - Operations with Fractions
6. Ratio and Proportion
7. Decimals and Percents
8. Integers
   - Definition of Integers
   - Operations with Integers
9. Geometry
   - Geometry of Shapes
   - Congruence
   - Relationships of Geometry
   - Constructions
   - Symmetry and Transformations
10. Measurement
    - Length
    - Perimeter
    - Angle Measure
    - Area
    - Units of Measure
11. Probability
12. Statistics
13. Graphing
14. Functions
15. Logic and Proofs

Text for both courses:
APPENDIX B

Sample Questions for Mathematics Achievement Test
Sample Questions for Achievement Test

Which numeral names the number represented below?

5 thousands + 23 hundreds + 9 tens + 13 ones

(A) 523,913
(B) 8,321
(C) 7,423
(D) 7,403
(E) 5,333

If p and q are primes, which of the following can never be true?

(A) p \times q is prime
(B) p - q is prime
(C) p - q is a whole number
(D) p + q is odd
(E) p \times q is odd

If a map has a scale of 1 inch equals 40 miles, how many miles apart are two towns separated on the map by $5 \frac{3}{4}$ inches?

(A) 230
(B) 150
(C) 200
(D) 275
(E) 30
APPENDIX C

Semantic Differential

Attitude-Toward-Mathematics Scale
<table>
<thead>
<tr>
<th>Attitude Instrument</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tr>
<td>CONFIDENT</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>THINKING</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>SYMBOLS</td>
<td>X</td>
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<td>X</td>
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<tr>
<td>UNCOMFORTABLE</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>EASY</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>PRACTICAL</td>
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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>BORING</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>RIGID</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>ONE WAY</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>
APPENDIX D

"Plant" Master Map Used During Training
PLANTS have FLOWERS, STEM, and ROOTS. FLOWERS have petals, nectar, and stems. NECTAR is collected by bees to make honey. Some flowers are colored to attract bees. Examples are orange, yellow, red, and green.
APPENDIX E

Complete Set of Concept Mapping Training Materials
Training Material

Page 1

CAR RAINING
DOG PLAYING
CHAIR WASHING
TREE THINKING
CLOUD THUNDER
BOOK BIRTHDAY PARTY

Page 2

ROBERT DAVIS / PERSON
COLUMBUS, OHIO / CITY AND STATE
THE MEMORIAL / A GOLF TOURNAMENT
BURGER KING / RESTAURANT

Page 3

ARE
WHERE
THE
IS
THEN
WITH
HAVE
LIKE
MAKE
BEES
HONEY
COLORED
PETALS
FLOWERS
PLANTS
ROOTS
STEMS
GREEN
LEAVES
RED
YELLOW
NECTAR
BULBS
ORANGE
Page 5

The world's waters are all either fresh water or salt water. All rivers and most lakes are fresh water. These rivers and lakes are home to many creatures. Fish, like the bass, and perch, live in the rivers and lakes, while other animals, like the otter and beaver, spend part of their time in the water and part out, and most other animals, like the deer and bear, live near the lakes and rivers to assure a supply of fresh drinking water and sometimes food.

All the oceans and a few of the lakes are salt water. Land animals do not drink salt water but some animals do live in the salt water with the other fish and crustaceans. The animals that live in the salt water include the whale and dolphin. Fish which live in salt water include the shark and sunfish and examples of the crustaceans which inhabit the salt waters are the lobster and crab.
APPENDIX F

Pretest Paragraph and Master Concept Map
Figure 30. Pretest Master Concept Map
APPENDIX G

Concept Map Scoring Procedures
Table 19 **Scoring Procedures for Concept Mapping**

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Definition</th>
<th>Scoring Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept Recognition</td>
<td>Concepts are objects, events, situations or properties of things that are designated by a label or symbol.</td>
<td>Count all concepts that are connected to other concepts by propositions. Score one point for each concept.</td>
</tr>
<tr>
<td>Grouping</td>
<td>Groupings are the ways concepts can be linked or joined together. Three types of groupings are:</td>
<td>Scoring of groupings</td>
</tr>
<tr>
<td></td>
<td>Point Grouping: a number of single concepts emanate from one concept.</td>
<td>Point Grouping: 1 point for each concept in the group.</td>
</tr>
<tr>
<td></td>
<td>Open Grouping: Three or more concepts are linked in a single chain.</td>
<td>Open Grouping: 2 points for each concept in the group.</td>
</tr>
<tr>
<td></td>
<td>Closed Grouping: Concepts form a closed system.</td>
<td>Closed Grouping: 3 points for each concept in the group.</td>
</tr>
<tr>
<td>Hierarchy</td>
<td>Concepts on a map can be represented as a hierarchical structure in which the more general, more inclusive concepts are at the top of the map; the specific and exclusive concepts are at the lower end of the map.</td>
<td>Only First Column concepts are scored for the degree of hierarchy in the map. This is based upon the extent concepts are present in assigned levels. Four points are given to each concept correctly assigned to a level; 2 points for each concept one level removed from an assigned level; no score for concepts that are two levels removed.</td>
</tr>
<tr>
<td>Branching</td>
<td>Branching of concepts refers to the level differentiation among concepts, that is, the extent the more specific concepts are connected to more general concepts</td>
<td>Score one point for each branching point which has at least two statement lines.</td>
</tr>
<tr>
<td>Proposition</td>
<td>Concepts acquire meaning through the relationship between concepts. The relationships are represented by connecting word(s) phrases written on the line joining any two concepts.</td>
<td>Score one point for each word or phrase; give half a point for repeated use of proposition.</td>
</tr>
</tbody>
</table>

(Malone & Dekkers, 1984, pp. 227-228)
APPENDIX H

Division Paragraphs
Division, one of the four basic operations, has been defined as the operation of finding either of two factors when their product and the other factor is given. In this sense it is evidently the inverse of multiplication. In generalized form, a divided by b is a number c such that \( b \times c = a \).

Division may be indicated by the sign "\( \div \)" or by using the fraction form. When the sign is used, the dividend comes before it and the divisor after it. When the fraction form is used, the dividend is above the line and the divisor below. Thus "six divided by two" may be written as \( 6 \div 2 \) or \( \frac{6}{2} \). The answer in a division problem is the quotient. The usual operational form is \( 2 \sqrt[ ]{6} \), in which 2 is the divisor and 6 the dividend.

Measurement or repeated subtraction division is one of two types of division. If a student has 24 cookies and wants to give 6 cookies to as many children as possible, the student will count out 6 cookies at a time until all the cookies are gone. The student would ask, "How many 6's are there in 24?" If the same student has 27 cookies and wants to give 6 at a time, the student would ask the same type question; however, there would be 4 groups of 6 with 3 remaining and since another group cannot be measured out, the division is complete leaving a remainder of 3 cookies. The answer in a measurement division problem requires no label since the answer is a simple count of how many times one number is found in another.

Partitive or sharing division is the other type of division. If a student has 24 cookies and wants to give equal amounts to 6 friends, the
student will share the cookies by giving them out one at a time. The student would ask, "If I have 24 cookies and want to give them equally to 6 friends, how many cookies will each get?" If the same student has 27 cookies and wants to share them equally with six friends there would be 6 groups with 4 in each group and 3 remaining. The remainder of 3 cookies could be divided equally between the six friends by giving each another half of a cookie. The answer in a partitive division problem requires a label since a number of things are being divided into groups.

There are three basic laws of division which are: 1) the divisor-quotient law which states that if \( c \div a = b \) then \( c \div b = a \); 2) the law of continuous division which states that \( (a \div b) \div c = (a \div c) \div b \); and 3) the law of distribution which states \( (a + b) \div c = (a \div c) + (b \div c) \). (Buckingham, 1953)
APPENDIX I

Master Division Concept Map
Figure 31. Master Division Concept Map
APPENDIX J

Questions for Post-Mapping Interview
Questions for the Interview

1) Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?

2) Explain your answer. What in particular made it [easy, hard, in-between]?

3) Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?

4) Explain your answer. What in particular made it [easy, hard, in-between]?
APPENDIX K

Time Table for Study
Time-Table for Completion of the Study

1) During the testing session, the subjects are tested using the achievement test (Appendix B). (75 minutes)

2) During the first session, subjects are:
   (a) given an attitude-toward-mathematics assessment (Appendix C);
   (b) trained using materials outlined in Novak and Gowin (1984) (Appendix E);
   (c) given the Pretest for Concept Mapping paragraph (Appendix F) and required to produce a map from scratch. Master scoring map, also included in Appendix F, will not be supplied to the subjects. (Approximately two hours)

3) During the second session, subjects will be given the division paragraphs (Appendix H) and required to produce a map from scratch. The master scoring map (Appendix I) will not be supplied to the subjects. (Approximately one hour)

4) During the last session, subjects will be individually interviewed to determine the subjects' perceived success during the mapping procedure (Appendix M). (Approximately 15 minutes)
APPENDIX L

Student Produced Concept Maps
Open Grouping

2 ways

- "c" of things
- sum of things

Fraction form

Two Types

- Unique Grouping (most to least)
- Two Types

Laws

Identity Law

A + 0 = A

Law of Addition

A + B = B + A

Distributive Law

A + (B + C) = (A + B) + C

Point Grouping

Power

A^2 = A

Exponent

A^n = A * A * ... * A (n times)

Symbol for square

A^2

Symbol for cube

A^3

Distributed Power

A + b : c = a + (c * b)

Distributed Multiplication

A + b : c = a + b : c
No Logical Connectives
Division LAWS

\[
\begin{align*}
\text{divisor} & \rightarrow \text{quotient} \\
\downarrow \text{states} & \quad \downarrow \text{are} \\
\frac{c}{a} = b & \quad \frac{(a \div b)}{c} = \frac{(a \div c) \neq b}{(a + b) \div c} \\
\text{then} & \quad \downarrow \text{states} \\
\frac{c}{b} = a & \quad \frac{(a \div c) + (b \div c)}{}
\end{align*}
\]
APPENDIX M

Interview Data
Interview responses: Low achieving group

# 710
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
   I really couldn't tell you cause I had no preconceived notion of what to expect or what came of it.
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   It really took a lot of thought...I wouldn't say it was hard.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
   Very hard.
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   The division concept was hard to understand...nothing seemed to fall into place like it did on the earlier maps.

# 428
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
   In-between
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   Concept mapping seemed to follow a type of logical, organization pattern which was relatively easy to follow.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
   Hard.
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   It was hard because the division concept seemed to have so many different offshoots which had to be organized and coordinated.

# 620
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
   Easy.
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   The concepts were in more isolated form and not interrelated. I also seemed to know more about the topics we concept mapped before the division topic.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
   Hard.
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   The things that I thought I understood kept changing and the whole concept kept expanding and moving in terms of their order and relationships.
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
In-between.

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
It was in-between because I believed I was getting some of the concepts in the right order. I felt like I was going from general to specific and I felt like everything was falling into place.

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
Hard

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
I was lost. I did not know how to integrate the numbers or identify key concepts that need to be placed in the map so I became very frustrated.

Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
I thought it was fairly easy until the division concept...Whew.

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
The other maps were of more familiar concepts with limited ways you could go.

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
Much harder

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
Division encompasses a large range of many interacting ways and different paths through its concepts.

Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
In-between

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
Mapping is not my favorite thing to do, however, after it was explained it did not seem so difficult.

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
In-between...some of the mapping seemed to be confusing for me in some ways.

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
Some of the mapping was tricky, like trying to figure out what went on each level, while other parts seemed in-between.
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept? I felt that concept mapping was in-between because at that point, it was easier than I thought it was going to be. 

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]? I think I felt it was in-between because the passages had terms that I felt were thoroughly defined. 

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept? 

I felt it was some what hard. 

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]? I think it was hard because I wanted to be extra careful...it was so time consuming cause I didn't know where anything went.

Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept? My first thoughts were in-between...I could relate it to webbing. 

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]? I considered it to be a great deal like webbing and the maps used in the groups seemed to be fairly easy...I knew something about the topics being mapped. 

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept? 

Hard 

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]? I felt it was hard because I couldn't relate or tie together things as easily as the first maps.

Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept? Easy 

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]? This technique seemed to be a very logical process to use. I'm a very logical note taker and this technique seemed to fit with my note taking style and study techniques. 

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept? Easy 

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]? Same as for the second question.
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept? In-between

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   I thought it was in-between because I seemed to understand it better when we were in a whole class situation...maybe with more class time to spend on it, I would have probably had a better grasp of it.

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept? In-between

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   If I was given more instruction and examples I probably would have gotten the hang of it...I liked the idea but maybe more time would be needed if people are to understand it better. Otherwise I really enjoyed it.

Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept? Easy

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   I didn't have any trouble figuring out the concepts and how they went together...I mean...I just seemed to know something about the concepts.

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept? Hard

Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   It wasn't like the first things we did...I didn't seem to know as much about division as I thought I did...I couldn't figure out what went where.

Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept? Hard

Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   Even though I could do it...it was hard because I had to think about where every little thing went...I never knew learning was so tough.

Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept? Very hard
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
Division was highly organized and seemed to run altogether...I didn't know what to put together and what not to.

# 211
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
Easy
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
Cause that process was similar to the way that I organize the things I learn...it just seemed to be easy.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
Very hard
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
Division was harder than the other maps I made because I thought I knew how to divide and that would...ah...let me make the map easier...but it didn't.

# 116
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
In-between
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
Ahh...I don't know...it was not easy and not hard.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
Hard
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
I thought it would be easy since I knew how to divide...but...it didn't seem to help when I was faced with all those words.

Interview responses: High achieving group

# 434
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
I thought it sounded pretty easy before hand.
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
When we were told that first graders could be trained to do the task, I decided in my own mind that it must be fairly simple and basic and it seemed that way during the initial training...things just seemed to fall into place because I knew what I was doing.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
In-between
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   I had a difficult time putting them in a general to specific order...but I didn't seem to have trouble picking out the concepts themselves.

#460
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?  
   In-between
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   I wasn't use to thinking this way so specifically...we all classify without realizing it. but not to actually map concepts on a paper.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?  
   The same
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   We had no feedback about whether or not we were on the right track, so I'm not sure.

# 390
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?  
   In-between
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   I realized that you have to really discern the lower levels and consistent revision...restructuring is needed...but I enjoyed this so it wasn't overwhelming but it was a challenge.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?  
   Hard
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
   Because I am a perfectionist I worried about how to pictorially cover all the complexities...that's why I took so long...I wanted to get it perfect.

# 154
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?  
   Easy
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
   Cause there were written steps for each part of the thinking process between the paragraph and the final map.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?  
   In-between
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
The topic was not about objects...the objects were cookies and such...the concept was division, not plants...I guess I didn't know as much about division as I thought I did.

# 511
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
In-between
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
The first one we did was very easy because it did not involve a lot of concepts but the second one was hard because of the number of concepts...I guess it depends on the number of concepts involved as to how difficult the mapping is.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
In-between
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
Again, I think the difficulty depends on the number of concepts involved in the particular map.

# 102
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
In-between
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
It wasn't easy but instead required a lot of thinking...I thought it was in-between because I could do it but it took some thinking.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
Hard
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
I couldn't get my thoughts together...I was not sure which way to attack it nor which came first, last, or in-between.

# 030
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?
Easy
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
The process for concept mapping was easy to understand and could be done without too much difficulty.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?
Easy
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
    I found that putting each concept on a separate slip of paper enabled me to rearrange and more efficiently organize my concepts.

# 922
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?  
    Easy
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
    The concepts were simple and in an organized manner.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?  
    Hard
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
    The division information was complexly organized and seemed to run altogether...I didn't know what to put together and what not to.

# 211
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?  
    Easy
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
    Cause that process was similar to the way that I organize the things I learn...it just seemed to be easy.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?  
    It was still easy.
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
    I think I did OK on the division map cause I knew what the paragraphs were talking about...and I knew where everything went.

# 332
Question 1: Did you feel that concept mapping was easy, hard, or somewhere in-between before you were asked to map the division concept?  
    Easy
Question 2: Explain your answer. What in particular made it [easy, hard, or in-between]?
    I could...see the picture of what the map should look like in my mind...it seemed very concrete.
Question 3: Did you feel that concept mapping was easy, hard, or somewhere in-between after you were asked to map the division concept?  
    In-between
Question 4: Explain your answer. What in particular made it [easy, hard, or in-between]?
    It was harder to see the picture of the map in my head...I wasn't sure where everything went.
List of References


