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Analysis of deformation-induced heating in tensile testing using a finite element method

Kim, Yong Hwan, Ph.D.
The Ohio State University, 1987
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ANALYSIS OF DEFORMATION-INDUCED HEATING
IN TENSILE TESTING USING A FINITE ELEMENT METHOD

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Yong Hwan Kim, B.S., M.S.

∗ ∗ ∗ ∗ ∗

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To My Wife, Seung Mae
A numerical method for analyzing non-isothermal viscoplastic deformation problems has been developed. The physical problem is cumbersome because the thermal and deformation effects are coupled both ways, i.e. plastic deformation generates heat and the temperature rise affects material flow behavior. As an application of this method, sheet tensile tests conducted in air have been analyzed using a two-dimensional finite element formulation. A modified Bishop's method is used to solve the thermoplasticity problem in decoupled form at each time step. The analysis consists of two main parts: a rigid-viscoplastic finite element method to analyze the deformation, and a transient heat transfer finite element method. Each part is assumed to occur during sufficiently small, consecutive time steps. Using the present method, the various factors affecting the nonisothermal ductility of material and flow characteristics can be investigated. The accuracy of the analysis is confirmed by comparison.
with experimental tensile test data for several engineering materials. Loss of total elongation by adiabatic deformation reaches 6.2% and 35% for I.F. steel and 304 stainless steel, respectively, illustrating the importance of deformation heating. For the intermediate case in air, both uniform and total elongations decrease with testing speed as a result of a drop in heat transfer to the environment. The competing effect of deformation heating and strain-rate sensitivity of AK steel is also examined and the FEM results showed the "near-invariance" of non-isothermal tensile ductility of this alloy. It is observed that the effect of deformation heating becomes more pronounced as necking develops and at higher testing speeds. The development of a temperature gradient is found to have a detrimental effect on ductility as opposed to the stabilizing effect of rate-sensitivity. Consequently, better formability can be achieved by controlling heat transfer conditions during forming. In addition, several numerical techniques, which often arise in finite element analysis of large deformation problems with high strain localization, are examined and an automatic remeshing technique has been developed.
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Computational Mechanics and Metal Forming Theory
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NOMENCLATURE

Latin Alphabetic Symbols

\( A_i, A, A_{i+1} \)  \( \text{domain of analysis at time } t_1, t \text{ and } t_{i+1} \)
\( A_k \)  \( \text{area of element } k \)
\( C \)  \( \text{heat capacity matrix} \)
\( c \)  \( \text{heat capacity} \)
\( E_{ij} \)  \( \text{components of Lagrangian strain tensor} \)
\( \dot{E}_{ij} \)  \( \text{components of Lagrangian strain rate tensor} \)
\( e^u \)  \( \text{uniform engineering strain} \)
\( e_f \)  \( \text{total engineering strain at failure} \)
\( F \)  \( \text{forcing vector in heat balance equation} \)
\( f_k \)  \( \text{measure of efficiency of mesh for remeshing} \)
\( G_{\alpha\beta} \)  \( \text{components of metric tensor } (\alpha, \beta = 1,2) \)
\( e_a \)  \( \text{base vector of convective coordinate system} \)
\( G \)  \( \text{determinant of metric tensor, } G_{\alpha\beta} \)
\( h \)  \( \text{heat convection coefficient} \)
\( K(Au) \)  \( \text{stiffness matrix in deformation part} \)
\( K \)  \( \text{heat conductivity matrix} \)
\( K_T \)  \( \text{tangent matrix} \)
\( k \)  \( \text{thermal conductivity} \)
\( M \)  \( \text{index describing the shape of yield locus} \)
\( \text{(Hill's new theory)} \)
m  strain-rate sensitivity
N  array of element shape function
Nf  fraction of error norm
n  strain hardening exponent
n  normal vector along boundary
Q  activation energy
\dot{q}  prescribed heat flux along boundary \partial S_2
q  heat generation rate during \Delta t
R  universal gas constant
R  correction vector
r  position vector.
\bar{r}  normal anisotropy index
T  temperature of specimen, \dot{T}(x,t)
T  nodal values of temperature
T_i  components of prescribed traction vector on the boundary
T_\infty  temperature of environment
\dot{T}(t)  time derivative of nodal temperature
\hat{T}  prescribed temperature on boundary
t  current time
u, u_i  nodal displacement vector
V  volume of body
v, v_i  velocity vector
\dot{W}  work rate
X  global Cartesian coordinates
Greek Alphabetic Symbols

- $\beta$: thermal expansion coefficient
- $\gamma$: index of strain localization ($= \frac{\Delta \varepsilon_c}{\Delta \varepsilon_g}$)
- $\Lambda_0$, $\Lambda$: initial and current sheet thickness
- $\Delta \varepsilon_{ij}$: components of incremental Lagrangian strain tensor
- $\Delta t$: time increment
- $\Delta u$: incremental displacement during $\Delta t$
- $\Delta u^o$: initial guess of displacement increment
- $\Delta W$: incremental work during $\Delta t$
- $\overline{\varepsilon}$: effective strain
- $\dot{\varepsilon}$: effective strain rate
- $\varepsilon_u$: uniform strain
- $\Delta \varepsilon$: incremental effective strain during $\Delta t$
- $\Delta \varepsilon^o$: initial guess of effective strain
- $\Delta \varepsilon_c$, $\Delta \varepsilon_g$: incremental effective strain at specimen center and gage length
- $\Delta \varepsilon_\alpha$: incremental principal strain ($\alpha = 1, 2$)
- $\eta$: fraction of strain energy converted to heat
- $\Delta \lambda_\alpha$: incremental extension during $\Delta t$
- $\xi^\alpha$: convected coordinates ($\alpha = 1, 2$)
- $\rho$: material density
- $\overline{\sigma}$: effective stress
\[ \sigma_{ij} \] components of Cauchy stress
\[ \partial S_1, \partial S_2 \] boundaries of heat balance equation
\[ \Gamma_{ij} \] symmetric Piola-Kirchhoff stress
\[ \omega \] weight factor
CHAPTER I.
INTRODUCTION

In sheet forming of ductile materials, the forming limit is governed by plastic instability and fracture following strain localization. A number of parameters influence the localization process. Among the most important are strain hardening, strain rate hardening, and anisotropy. For high strain rates, the effect of temperature gradients caused by deformation heating also becomes important. Numerous studies have been performed to study the effect of material properties on sheet metal formability.

Most of the energy expended in plastic deformation is converted to heat. Temperature rises up to 100°C during tensile tests of some engineering materials have been reported by several authors [1-6]. This "natural" temperature increase (i.e. one produced by the deformation itself) can have detrimental effects on the formability of material by promoting strain localization in regions of
high strain rate and temperature. Especially, after necking occurs, the deformation is localized and the temperature rises mainly within the necking region. Since the flow stress decreases with increasing temperature, further deformation is localized preferentially in this zone and this "autocatalytic" process continues until failure occurs. Therefore, the tensile ductility in a non-isothermal test is noticeably less than that in an isothermal case [7].

Control of temperature gradients has led to improvements in several industrial forming operations. Reduction of "natural" temperature gradient promotes enhanced formability, an effect exploited in the ARMCO Cold Forming Process [8,9] and in the usage of flood lubrication ("flood" lubrication means continuous sprays of water-based lubricants during forming) for stamping of stainless steel parts [10]. It is also possible to make use of local "artificial" heating to improve the performance of some forming operations. Line heating [11] and heated die-cooled punch forming of steels [12] and superplastic materials [13] are operations based on controlled temperature gradients to promote formability. The principle on which each of these processes is based is locally changing the flow stress to redistribute strain
toward a more uniform distribution.

It is obvious, therefore, that the temperature rise in a plastic deformation process must be considerable, and the problem of the effect of deformation heating must be studied in conjunction with the flow process. The effect of deformation heating becomes more important in actual application, where higher forming speed is generally involved. In spite of many publications studying temperature effect on deformation, little effort has been expended to model the coupled effects of heat generation, heat flow, and thermal softening during non-isothermal, non-uniform deformation of sheet metal on material ductility.

In the present work, two-dimensional finite element analysis is performed to solve the coupled deformation and heat transfer problem arising in in-plane sheet metal forming problem. The problem can be defined as a general two-dimensional boundary value problem with coupled deformation and heat transfer. The physical problem is cumbersome because the thermal and deformation effects are coupled both ways, i.e. plastic deformation generates heat and the temperature rise affects material flow behavior. A numerical method is proposed for the analysis of the
thermoplasticity problem in a decoupled form at each time step. This model is fully two-dimensional in terms of material and heat flow and therefore avoids errors associated with quasi-one dimensional or "long wavelength" approaches. The model allows ab initio calculations with only material and heat transfer parameters as input data. As an application of finite element formulation, non-isothermal tensile tests of several engineering materials have been analyzed and the effect of deformation heating on tensile ductility of material is discussed. The analytical results are compared with existing experimental results. Also, the combined effect of other material parameters and environmental conditions have been investigated and some numerical problems which arise in large deformation problems are discussed.
CHAPTER II.

LITERATURE SURVEY

This literature survey is directed towards getting a brief overview of existing knowledge in the analysis of sheet metal forming processes. This chapter is divided into three sections. The first section deals with the effect of material properties on sheet metal formability. The second section briefly addresses the historical overview of finite element formulation in large deformation theory. Finally a short review of numerical analysis of thermoplasticity problem will be given.

2.1. Effects of Material Properties on Sheet Metal Formability

As mentioned in the previous chapter, there are numerous factors influencing sheet metal formability. Much effort has been made to predict material response during sheet forming operations. Material behavior under actual forming conditions depends upon a number of
parameters, which include strain hardening, strain rate hardening, anisotropy, microstructure, temperature gradients, strain paths, fracture, and so on. For decades, a considerable amount of research has been done to develop exact material models and to explain material behavior in terms of mechanical properties. In this section, the effect of material properties on sheet metal formability, especially on material response and strain localization process during plastic deformation will be briefly discussed. Since the primary objective of this work is to study the effect of temperature gradient on material behavior during tensile tests, more attention will be paid to the effect of deformation heating. In addition to that, the effect of strain rate hardening will be reviewed because it becomes more important in non-isothermal deformation than in the isothermal case.

2.1.1. Effects of Strain Rate Hardening

The most important material property influencing the formability of sheet metal is the intrinsic ability of the material to harden with deformation. A region in a part undergoing thinning can resist further deformation by virtue of strain hardening and can spread the deformation to its neighboring region, thereby promoting more uniform
thinning [14,15]. Selecting the best measure of strain hardening has been a controversial topic for many years [16-19]. The most popular measure of strain hardening is the strain hardening exponent, n, in the Hollomon's empirical power law $\sigma = K(\varepsilon)^n$ [16]. This simple law has many shortcomings [18,19] and has been shown not to represent strain hardening of materials like brass, copper or stainless steel very well. Another commonly used form of strain hardening law was proposed by Voce [17] with the following relationship:

$$\sigma = \sigma_s - (\sigma_s - \sigma_i) \exp(-\varepsilon/n) , \quad (2.1)$$

where $\sigma_s$ is a saturation flow stress, $\sigma_i$ is the initial yield strength, and n is the strain hardening exponent. The Voce equation has been shown to fit the behavior of brass and copper quite well. In the ideal situation, the hardening exponent n is equal to uniform strain; however, a single value of n does not generally describe the entire stress-strain relationship for most metals.

Positive strain-rate sensitivity of flow stress is another important property that aids strain distribution. This property can be described by an increase in flow stress with increasing strain rate. In deformation
process, gradients in strain and strain rate are always developed by frictional and geometrical constraints, and both strain hardening and strain-rate sensitivity act to reduce the resulting non-uniformity of thinning and to increase the strain at failure.

Strain-rate sensitivity, better known as m value, is commonly expressed as

\[ m = \frac{\partial (\ln \sigma)}{\partial (\ln \dot{\varepsilon})} |_{\varepsilon, T} \]  

(2.2)

where \( \sigma \) is flow stress and \( \dot{\varepsilon} \) is strain rate. The m value of most materials has been found to depend upon strain, strain rate, interstitial contents, temperature, microstructure, and even testing method [2,7,15,20,21].

As is the case for strain hardening, some controversy exists on how best to measure strain-rate sensitivity. One most commonly used form is the power law \( \sigma = K(\varepsilon)^N(\dot{\varepsilon})^m \) [22]. Another common form for rate sensitivity is the overstress description: \( \sigma = \sigma(\varepsilon) + \Delta \sigma(\dot{\varepsilon}) \), in which the rate dependent term is additive [15]. This additive stress can be expressed in the form: \( \Delta \sigma = m' K \ln(\dot{\varepsilon}/\dot{\varepsilon}_0) \), where \( \dot{\varepsilon}_0 \) is reference strain rate.
Wagoner [23-25] found that \( m \) values of zinc and aluminum-killed (AK) steel strongly depend upon strain rate and proposed another form of constitutive equation as follows:

\[
\sigma = \sigma_0 \exp(B \dot{\varepsilon}^A), \quad (2.3)
\]

where \( A \) and \( B \) are constants to be determined from experiments and \( m \) value is expressed as \( m = B' (\dot{\varepsilon})^A \). This expression of rate sensitivity has been used in the analysis of non-isothermal tensile test of AK steel in this work and details will be discussed later.

Rate sensitivity effects show up in the presence of gradients in strain rate and, therefore, in a tensile test, the elongation beyond maximum load is influenced primarily by \( m \). After the onset of strain localization, positive strain-rate sensitivity inhibits further flow localization because additional deformation in areas of high strain rate will require greater stresses. Many studies demonstrate that the amount of post-uniform deformation depends greatly on strain-rate sensitivity [26-29]. Even though the rate sensitivity has a more pronounced influence than the strain hardening, the values of \( m \) for most cold forming materials are rather small
(<0.03). However, the presence of even such a small m can be responsible for 10-20% of post-uniform elongation to failure for some materials. In deformation of superplastic materials, in which m is quite large (>0.3), m is the primary factor which causes elongation up to several hundred percent to occur [30,31].

2.1.2. Effects of Deformation Heating

A number of experimental and analytical studies showing the effect of deformation heating have been reported. Farren and Taylor [32] found that for steels, copper and aluminum, the generated heat represents 86.5, 90.5-92 and 95%, respectively, of plastic work by measuring the plastic work and the temperature rise in a series of tensile experiments. The remainder of the plastic work is stored as internal energy associated with the internal defects. Sachedev and Hunter [2] measured the temperature increase during tensile tests of dual phase steel, high strength low alloy steel (HSLA), and plain carbon steel using an infra-red thermometer. They reported a maximum temperature increase of 76°C for dual phase steel at a strain rate of 1.5x10^{-2}/s. Temperature rises of 58°C and 42°C were measured under similar conditions for HSLA and plain carbon steel, respectively.
Similar experimental results are reported by several authors [3-6,33]. Ayres [3] found that the total elongation dropped from 54% in water (near isothermal case) to 40% in air during tensile test of 1008 aluminium-killed steel and the maximum temperature increase was about 90°C at relatively high strain rates. He found that ductility remained approximately constant for various strain rates in air and concluded that a strain-rate sensitivity which increased with rate and temperature gradients competed in determining ductility at high strain rates. Lin and Wagoner [4-6] measured temperature rises of up to 75°C during tensile testing of I.F. steel and 118°C for 310 stainless steel at a strain rate of 10^-1/s, and a total elongation drop from 45% in water or at low rate in air to 40% in an insulated condition or at high rate in air. For 310 stainless steel, total strain ranged from 60% (isothermal) to 42% (adiabatic) at measured range of rates. Ferron [33] compared tensile ductility of 304 stainless steel using metallic grips and Araldite grips to investigate effect of heat sinks at both ends of the specimen. With the presence of heat sinks, he proposed that temperature gradients resulting from heat flow towards specimen ends played a prominent role in the development of flow localization in different positions along the specimen. Therefore,
necking and failure would occur at the center of the specimen because of both geometric defect and higher thermal effect at this region. By suppressing heat flow to the specimen ends, however, he proposed that the location of the necking site could be made random along the specimen.

Various experimental studies have been made to find the material constitutive relationship including temperature effect. The temperature dependence of flow stress at constant strain and strain rate can be represented by [7]

\[ \sigma = C \exp\left(\frac{Q}{RT}\right) \quad (2.4) \]

where \( Q \) is an activation energy for plastic flow, \( R \) is universal gas constant and \( T \) is temperature. Combined expression of temperature and strain rate dependence of flow stress was first proposed by Zener and Hollomon [34] in the form

\[ \sigma = f(Z) = f(\dot{\varepsilon} \exp(\Delta H/RT)) \quad (2.5) \]

where \( \Delta H \) is an activation energy that is related to the activation energy \( Q \) in Eq. (2.4) by \( Q = m \Delta H \), where \( m \) is
the rate sensitivity. The quantity $Z = e^{\dot{\varepsilon} \exp(\Delta H/RT)}$ is called the Zener-Hollomon parameter and it is referred as a temperature-modified strain rate. Another common form of temperature dependency of flow stress is that flow stress varies linearly with temperature, which was proposed by Hutchison [35] in the form

$$\sigma = \sigma_0 (1 - \beta \Delta T), \quad (2.6)$$

where $\sigma_0$ contains ingredients of flow stress other than its temperature dependency and $\Delta T$ is the temperature difference from the reference state. This type of linear dependency of flow stress was used to develop the constitutive equations with temperature effect by Kleemola and Ranta-Eskola for aluminum-killed steel [36] and by Lin and Wagoner for interstitial free steel [4,5]. Also the exponential dependence of flow stress upon temperature has been used to develop constitutive equations by Wada et. al. for aluminum alloys [37] and Lin and Wagoner for 310 stainless steel [4,5], where flow stress is expressed by

$$\sigma = \sigma_0 \exp(\Delta H/RT), \quad (2.7)$$

where $A$ and $B$ are constants.
Several analytical studies, including effect of heat generation and transfer on material tensile ductility, have been performed recently. Korhonen and Kleemola [38] applied the finite difference method to predict the temperature distribution and the effect of heat transfer during a tensile test of aluminum-killed steel sheet. They concluded that strain rate and deformation heating influenced the flow stress and the uniform strain even at moderate strain rates ($10^{-2} - 10^{-3}$/s). Similarly, rather simple finite difference methods have been used by several others with the same general result: a decrease of ductility due to deformation heating. Wada et al. [37] used a simple one-dimensional analytical form to calculate the temperature distribution of specimen in torsion test of non-work hardening material. They assumed that the strain rate depended on stress and temperature and was independent of strain to include the effect of deformation heating. Similar result was found by Fressengeas and Molinari [39] using a one-dimensional perturbation method. Gao and Wagoner [40,41] solved the transient heat transfer problem using a finite difference method to study the effect of deformation heating for I.F. steel and 310 stainless steel. They investigated the effect of environment and end conditions by changing the heat transfer coefficient and boundary conditions. They used
the empirical equations obtained from experiments for predicting strain and the distribution of generated heat.

In most analytical studies, however, uniform strain has been used as a measure of material ductility and the effect of deformation heating has been investigated only up to the necking point because of the difficulty in analyzing non-uniform deformation after necking. Unfortunately, the effect of a temperature gradient becomes more severe after necking because of the autocatalytic interaction of higher temperatures, reduced flow stress, and strain localization. To include the effect of deformation heating in post-uniform deformation, Raghavan and Wagoner [42-44] used a finite element method, including the effect of specimen geometry. They used empirical equations of the temperature distribution throughout the specimen and compared with two extreme heat flow cases (isothermal and adiabatic), rather than calculating heat transfer directly. They found that the presence of both thermal gradients and geometric notch in the specimen has a harmful effect on tensile ductility of I.F. steel. However, the amount of total elongation drop due to adiabatic deformation is about 15.5% of isothermal total elongation, independent of the size of initial geometric defects. Semiatin et. al. [45] used
one-dimensional finite difference method to solve numerically the coupled deformation-heat transfer problem for both uniform and post-uniform deformation during a uniaxial tensile test of AK steel, showing 12.5 to 14.5% of decrease of total strain in non-isothermal case compared with isothermal condition. They also verified the "near-invariance" for non-isothermal deformation of AK steel by the competing effect of deformation heating and rate sensitivity, which had been found by Ayres' experiment [3].
2.2. Finite Element Analysis of Large Deformation

Finite element analysis has been widely used for analyzing metal forming problems and the literature is expanding at a great rate. Although development of the finite element method started in the early 1950s, real progress in its application to metal forming began in 1960s by Argyris [46], Pope [47], Marcal and King [48], and Yamada et. al. [49]. Stiffness matrices based on the incremental stress-strain relation were derived for small strain problems. Another approach using an "initial stress" computational process was proposed by Zienkiewicz et. al. [50]. However, it was realized that a small deformation formulation is not realistic since forming processes generally involve large plastic strain.

During 1970s, extensive studies on the finite element analysis of plastic forming problems have been performed since Hibbitt et. al. [51] introduced the first complete finite element large strain formulation. They used a total Lagrangian formulation (TLF) in which the reference state is the original undeformed configuration. Later, McMeeking and Rice [52] derived an Eulerian type of finite
element formulation, which could be more properly called updated Lagrangian formulation (ULF) [53], following the principle of virtual work given by Hill [54]. In this formulation, the current configuration at time t is adopted as the reference state for evaluation of deformation in the time interval (t, t+At). Consequently the finite element mesh moves through space with the deforming material. The utilization of the updated finite element mesh as the reference configuration made this approach very appealing to investigators in the area of large deformation analysis, especially for steady state processes, and a number of studies have been performed using this formulation [55-60].

At the same time two major alternatives were successfully introduced for the analysis of metal forming processes. First, rigid-plastic formulation was advocated by Kobayashi and his collaborators [61-64], in which the elastic deformations are assumed to be negligible compared to the large plastic strains. Even though the method fails to predict the stress history wherever elastic loading and unloading is encountered, it has been widely accepted in analyzing the large deformation processes since it is not necessary to check the yield condition during the computational procedure and, therefore, the
computational cost is considerably reduced. Moreover, it seems that this idealization offers excellent solution accuracies because the effects of elastic response at large strain in the actual material are negligible. These advantages have made rigid-plastic formulation more favorable in analyzing large deformation processes.

Another approach, based on a viscoplastic formulation (or flow formulation) was proposed by Zienkiewicz et. al. [65-67] as a generalization of rigid-plastic formulation. This method enables itself to deal with rate-dependent materials by treating the deforming material simply as a non-Newtonian viscous fluid. This method has been modified and used by many researchers [68-72]. Later, elastic strains were included in this formulation by Dawson and Thomson [73].

The number of applications of finite element method to metal forming problems is extensive. Only finite element analysis of sheet metal forming will be reviewed here. Wifi [74] first published a complete FEM solution of the axisymmetric stretch forming and deep drawing problems, using the incremental total Lagrangian formulation for rate-insensitive elastic-plastic material. Later, he used an updated Lagrangian formulation for the
hydrostatic bulging problem [75]. Wang and Budiansky [76] used a TLF to simulate axisymmetric hemispherical punch stretching of elastic-plastic materials. Wang and Wenner [77], Yamada et. al. [55], Derbalian et. al. [59], Nakamachi [60], Iseki et. al. [78,79] and Tang [80] also used elastic-plastic formulation in analyzing various sheet metal forming problems. Meanwhile, use of rigid-plastic formulation in analyzing sheet metal forming has been limited because of nonuniqueness of deformation mode for the quasistatic deformation of a rigid-plastic solid under certain types of boundary conditions and large geometrical change during deformation. Kim and Kobayashi [81,82] proposed a functional, which is adequate to analyze sheet forming problem, for the analysis of axisymmetric bulging and stretching problems. Similar type of rigid-plastic formulation was used by Wang [70-72], Kobayashi and his collaborators [83-85], Gotoh [86-88], and Kim and Yang [89] for solving many sheet forming problems. In most cases, membrane theory neglecting bending effects has been used to simulate the deformation of sheet. Another attempt to include the bending effect was made by Zienkiewicz and his collaborators based on viscous shell theory [90-92] for rigid-viscoplastic material. Also, shell theory has been used for elastic-viscoplastic formulation by Wood et. al.
Rather than using shell theory in order to include bending effect, Stoughton [95] introduced a hybrid model based on the membrane theory by including the effect of bending as an additional degree of freedom.

In the current work, a rigid-viscoplastic finite element formulation using a convected coordinate system developed by Wang [71, 72] has been used to simulate in-plane, non-isothermal sheet forming problem. The functional which had been used in the formulation of sheet forming of rigid-plastic materials [81-85] was introduced and membrane theory was employed to model sheet deformation.
2.3. Numerical Analysis of Thermoplasticity Problem

Including temperature effects in the analysis of metal forming is an important step toward extending the finite element analysis. In thermoplastic analysis, particular attention should be paid to coupling of the deformation analysis calculation. Many efforts have been made to determine the temperature distributions in metal forming processes in recent years. Most efforts have been placed, however, on analyzing bulk deformation at relatively high temperature and few have been reported in sheet metal forming problem.

The first attempt to solve a thermoplasticity problem numerically was made by Bishop [96], who developed a numerical method to compute the temperature distribution in a plane extrusion problem. He assumed the material to be perfectly rigid-plastic and obtained the flow field from a slip line solution. This method made the approximation that the process was separated into two steps, one involving heat generation and transport and the other heat transfer. This approach has been used by Korhonen and Kleomia on simple tension [38], by Altan and
Kobayashi on axisymmetric extrusion [97], and by Lahoti and Altan on compression and torsion [98]. Usually, the temperature calculations are done by finite differences or by finite element, while flow pattern is determined by experiment or by upper bound technique.

Oden et al. [99] proposed the first coupled analysis of deformation and heat transfer for an elastic-plastic materials, although the application was to a rectangular aluminum bar constrained in one direction and heated at a corner. Application of coupled analysis to a real forming problem was done by Zienkiewicz et al. [90,100,101]. They solved steady-state extrusion and rolling problems by solving the heat balance equation and equilibrium equation simultaneously using a flow formulation. They used the same shape function for velocities and temperature and solved directly the stiffness equation containing both velocities and temperature as unknowns. Previously Zienkiewicz et al. [102] had proposed to solve the same problem by separating the equation into flow problem and thermal equation and iterating the results between the two sets of solutions, in which the plasticity values are adjusted according to the temperature field. Other attempts to solve the coupled forming processes using similar coupled analysis have been made by Rebelo and
Kobayashi on compression and upsetting of rigid-viscoplastic material [103,104], by Dawson and Thomson on axisymmetric extrusion and plain strip rolling [105,106], by Chandra and Mukerjee on extrusion of elastic-viscoplastic material [107], and by Argyris and Doltsinis on extrusion and upsetting [108,109] using "natural approach" with infinitesimal tetrahedrons instead of the conventional Cartesian approach of rigid-viscoplastic materials.
CHAPTER III.
FINITE ELEMENT FORMULATION

A finite element method (FEM) has been developed to analyze non-isothermal, in-plane sheet deformation. The formulation is aimed at solving a general, coupled, thermoplasticity boundary value problem and consists of two main parts: one is a two-dimensional rigid-viscoplastic FEM to analyze the deformation and the other is a transient heat transfer problem. Bishop's proposal [96] is adapted to solve the coupled thermoplasticity problem in a stepwise decoupled form. The formulation is from first principles; requiring only material and heat transfer data.

3.1. Governing Field Equations

The governing field equations include the mechanical equilibrium equation and the energy conservation equation.

The mechanical equilibrium equations, neglecting inertia force and body force, are given by
\[ \sigma_{ij,j} = 0 \quad (3.1) \]

with boundary conditions

\[ v_i = v_i \quad \text{or} \quad \sigma_{ij} n_j = T_i \quad . \]

Here \( v_i \), \( T_i \) and \( n_j \) are the prescribed Cartesian components of velocity, traction and the unit outward normal vectors on the boundary, respectively.

Referring to Fig. 3.1, the energy conservation equation in a certain domain in the present configuration has the local form

\[ \rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + q - 2h \frac{\partial}{\partial A} (T - T_\infty) \quad . \quad (3.2) \]

with boundary conditions

\[ T = \hat{T} \quad \text{on} \quad \partial S_1 \quad . \]

\[ k \frac{\partial T}{\partial n} + \frac{h}{A} (T - T_\infty) = -\hat{q} \quad \text{on} \quad \partial S_2 \quad . \]

where \( T \) is temperature, \( \rho \) is the material density, \( c \) is heat capacity, \( k \) is heat conductivity, \( q \) is heat
Figure 3.1. Domain of two-dimensional heat transfer equation.
generation rate, \( h \) is heat convection coefficient, \( \Delta \) is the current sheet thickness, \( \hat{T} \) and \( \hat{q} \) are the prescribed boundary temperature and heat flux over the boundaries \( \partial S_1 \) and \( \partial S_2 \), respectively, and \( \mathbf{n} \) is the outward normal vector over boundary \( \partial S_2 \). Only free convection is assumed herein.

The average heat generation rate \( \dot{q} \) in Eq. (3.2) during the small time interval \( \Delta t \) can be defined by

\[
\dot{q} = \frac{\eta}{\Delta t} \int_{\epsilon_i}^{\epsilon_i + \Delta \epsilon} \frac{\sigma}{\epsilon} \, d\epsilon,
\]

where \( \epsilon_i \) is the effective strain at the beginning of incremental step, time \( t_i \), \( \Delta \epsilon \) is the increment of effective strain during \( \Delta t \) and \( \eta \) is the fraction of strain energy converted to heat. Throughout the study, \( \eta \) is assumed to be 0.9 from experimental data [32].
3.2. Decoupled Analysis - Bishop's Method

The calculation of temperatures in a plastically deforming material is a non-steady state heat transfer problem in a moving incompressible medium with heat sources. Since the problem involves the determination of simultaneous material deformation, heat generation, and heat transfer, a closed-form analytical solution to the two equilibrium equations is impossible to obtain. Bishop [96] first proposed the principles of a numerical method for solving such a problem by separating the problem of heat generation, conduction, and material transport into two parts. In this scheme, the heat generation and material transport are regarded as occurring instantaneously for a small time interval, followed by the same interval in which heat transfer (conduction and convection) occurs as for a stationary medium, even though deformation, heat generation, and heat transfer in fact occur simultaneously. In other words, the method approximates the heat generation and the simultaneous heat transfer as taking place in two consecutive steps. This method has been used and modified by several authors [38,97,98] and its suitability in solving thermoplasticity
problems has been verified as long as a sufficiently small time step is used. The method used in the present study assumes that the heat generation due to deformation takes place at the beginning of a time interval $\Delta t$, associated with a small plastic deformation and then the heat transfer occurs without further deformation, leading to a stepwise decoupled solution procedure. The repetition of these two steps numerically simulates the deformation process and gives the temperature distribution as a function of time.
3.3. Application of the Finite Element Method

In this section, general descriptions of the finite element formulation will be outlined for both the deformation part and heat transfer part. Details can be found in Appendix B.

3-3-1. Rigid-viscoplastic Finite Element Formulation

A rigid-viscoplastic finite element method, based on Wang's variational method [71,72], is used to analyze in-plane plastic deformation. The formulation includes the effect of strain hardening, strain rate hardening, and utilizes Hill's new theory [110,111]. Any type of one-dimensional hardening law can be employed.

The necessary and sufficient condition for satisfying the boundary value problem defined in Eq. (3.1) is given by the virtual work principle

\[ \delta \bar{W} = \int_{V_o} G^{ij} \delta \bar{E}_{ij} \, dV_o, \quad (3.4) \]
are the components of symmetric Piola-Kirchhoff stress and virtual Lagrangian strain rate tensor, respectively, at a certain time, and $\delta \dot{W}$ denotes the external virtual work rate and integration should be carried out on the original configuration. From the equivalent work principle [112], Eq. (3.4) can be written in the form

$$\delta \dot{W} = \int_{V_0} \bar{\sigma} \delta \dot{\varepsilon} \, dV_0 ,$$

(3.5)

where $\bar{\sigma}$ and $\delta \dot{\varepsilon}$ are effective stress and virtual effective strain rate, respectively. The effective stress is assumed to be a function of effective strain ($\bar{\varepsilon}$), effective strain rate ($\dot{\varepsilon}$), and temperature ($T$):

$$\bar{\sigma} = \bar{\sigma}(\bar{\varepsilon}, \dot{\varepsilon}, T).$$

In analyzing the non-steady state deformation by a step-by-step procedure, consider the deformation during one step from $t_i$ to $t_i + \Delta t = t_{i+1}$. Fig. 3.2 shows the deformation of the body which occupies the region $A_i$ at time $t_i$, then afterwards the region $A$ at time $t$ ($t_i < t < t_{i+1}$), and finally the region $A_{i+1}$ at time $t_{i+1} + \Delta t$. In the figure, $i \mathbf{g}_\alpha$, $\mathbf{g}_\alpha$, and $i+1 \mathbf{g}_\alpha$ ($\alpha = 1, 2$) are the base vectors at time $t_i$, $t$, and $t_i + \Delta t$, respectively and motion is described in terms of convected coordinate.
Figure 3.2. Coordinate system in the deformation process.
system, \( \mathbf{\xi}^i \). Then, by integrating Eq. (3.5) during \( \Delta t \), we can define incremental virtual work, \( \Delta(\delta W) = \delta(\Delta W) \), during \( \Delta t \) as

\[
\delta(\Delta W) = \int_{t_1}^{t_1 + \Delta t} \delta \dot{W} \, dt
\]

\[
= \int_{V_0} \int_{t_1}^{t_1 + \Delta t} \bar{\sigma} \, \delta \dot{\varepsilon} \, dt \, dV_o
\]

\[
= \int_{V_0} \int_{\varepsilon_1}^{\varepsilon_1 + \Delta \varepsilon} \bar{\sigma} (d\varepsilon) \, dV_o \quad . 
\]

(3.6)

The assumption involved in Eq. (3.6) is that during a small time interval, the ratio of the principal strain rates does not change significantly. The increment of displacement therefore uniquely determines the incremental strain and the stress state at the end of the deformation step, independent of path. The principal direction is also assumed to be fixed by neglecting the spin during a single step, but it allows change of principal direction from step to step. For the rate-sensitive problem, the deformation rate is also assumed to be constant during each step. In the usual flow theory, this assumption holds only for infinitesimal increments of time or strain.
It implies that the stress-strain relation during each increment in this formulation is that of a total deformation theory of plasticity. However, as time intervals are made smaller and smaller, the flow equations are approximately satisfied.

Both sides of Eq. (3.6) are functions of incremental displacement, $\Delta u$, alone since the choice of incremental strain path allows determination of $\Delta \varepsilon = \Delta \varepsilon(Au)$ and $\sigma = \sigma(\Delta \varepsilon)$. The work principle for a given time step therefore is written from Eq. (3.6):

$$\frac{\partial (\Delta W)}{\partial (\Delta u)} = \int_{V_0} \sigma(t_i+\Delta t) \frac{\partial (\Delta \varepsilon)}{\partial (\Delta u)} dV_0 = f(t_i+\Delta t) \quad , (3.7)$$

or

$$\Phi(\Delta u) = 0 \quad , (3.8)$$

where $f$ and $\Delta u$ are the external load vector and displacement increment, respectively, at time $t_i+\Delta t$ and $\Phi$ is a nonlinear vector function. Effective stress is approximated as $\sigma = \sigma(\varepsilon, \varepsilon_i) \cong \sigma(\varepsilon_i + \Delta \varepsilon_i, \Delta \varepsilon_i)$. An alternative form illustrating the matrix form of Eq. (3.7) may be written:
where $K(\Delta u)$ is the assembled nonlinear stiffness matrix as a function of $\Delta u$. Since the coefficients of the stiffness matrix $K(\Delta u)$ depend on the unknowns $\Delta u$ and their derivatives, a direct solution of Eq. (3.9) is generally impossible. Therefore, the Newton-Raphson method is employed to solve the system of nonlinear equations, (3.9) in an iterative scheme. To apply the Newton-Raphson method, let $\Delta u^0$ be a trial solution. We seek a correction vector $\Delta u'$ defined by

$$\Delta u = \Delta u^0 + \Delta u' .$$  \hfill (3.10)

Corresponding to this displacement, the incremental effective strain can be expressed in the form

$$\Delta \bar{\varepsilon} = \Delta \bar{\varepsilon}^0 + \Delta \bar{\varepsilon}' .$$ \hfill (3.11)

where $\Delta \bar{\varepsilon}'$ denotes the correction term for the trial solution $\Delta \bar{\varepsilon}^0$. Substituting Eqs. (3.10) and (3.11) into (3.9) and retaining the first two terms, the linearized form of equation (3.9) near the trial solution $\Delta u^0$ becomes

$$K_t \cdot \Delta u' = f - R .$$ \hfill (3.12)
where the tangent matrix $K_T = \left[ \frac{\partial^2 \Delta W}{\partial \Delta u_i \partial \Delta u_j} \right]$, and correction vector $R = \left[ \frac{\partial \Delta W}{\partial \Delta u_i} \right]$. Both $K_T$ and $R$ are computed at $\Delta u = \Delta u^0$ and $\Delta \epsilon = \Delta \epsilon^0$. Solving Eq. (3.12) with respect to $\Delta u$, the assumed displacement field is updated by $u + \Delta u$.

Detailed expressions of terms in Eq. (3.12) appear in Appendix B and similar expressions can be found elsewhere [71,72]. Constant strain triangular elements are used to develop the stiffness equations. The information of strain energy and deformed geometry will be used in the transient heat transfer part for estimating the temperature distribution over the specimen.

### 3.3.2. Transient Heat Transfer Equation

Imposing the given boundary conditions on Eq. (3.2) and discretizing into finite elements in Lagrangian spatial coordinates by a Galerkin type procedure [113], the following system of ordinary differential equations is obtained.

$$\mathbf{C} \ddot{\mathbf{T}} + \mathbf{K}_T + \mathbf{F} = \mathbf{0} \quad ,$$

(3.13)
where $K$, $C$, and $F$ are the heat capacity matrix, the conduction matrix, and thermal forcing vector by heat source term, respectively and temperature $T(x,t)$ is approximated by $T(x,t) = N(x) \cdot T(t)$. Here $N(x)$ is the two-dimensional shape function for the linear triangular elements and $T(t)$ is nodal temperature vector as a function of time. The procedure can be found in any textbook [113].

The system of ordinary differential equations, (3.13) is now solved for $\hat{T}$ using finite difference scheme for the time domain, which leads to the following equation:

$$
C \frac{T_{t+\Delta t} - T_t}{\Delta t} + K \frac{T_{t+\Delta t} - T_t}{2} + F = 0 ,
$$

(3.14)

where $C$, $K$, and $F$ are assigned their mid-interval values. This time integration is an adaptation of the Crank-Nicholson method, in which the time derivative is approximated by a central difference referred to the mid interval. The same triangular elements used in the plastic portion of the program are adapted to evaluate the interpolating function. The amount of heat generation during a small time interval by plastic deformation can be estimated from Eq. (3.3). The amount of heat transferred to the surroundings and ends of the specimen during the
same time interval is evaluated and the temperature
distribution is obtained. This temperature in each
element is used to evaluate the effective stress in the
given constitutive equations for the next time step in the
plastic portion of the calculation.
3.4. Hill's New Yield Theory and Hardening Law

In Hill's new yield theory [110,111] for a normal anisotropic material with normal anisotropy parameter \( r \) for the case of plane stress \( (\sigma_3 = 0) \), the effective stress \( \bar{\sigma} \) and its associated effective strain rate \( \dot{\varepsilon} \) are defined in terms of the principal values by

\[
\bar{\sigma} = C_1 \left[ |\sigma_1 + \sigma_2|^M + C_2 \right] |\sigma_1 - \sigma_2|^M \right)^{1/M},
\]

\[
\dot{\varepsilon} = D_1 \left[ |\dot{\varepsilon}_1 + \dot{\varepsilon}_2|^{M-1} + D_2 \right] |\dot{\varepsilon}_1 - \dot{\varepsilon}_2|^{M-1} \right]^{1/M},
\]

where

\[
C_1 = \left[ \frac{2(1+r)}{2(1+r)} \right]^{-1/M}, \quad C_2 = 1 + 2r,
\]

\[
D_1 = \frac{1}{2} \left[ 2(1+r) \right]^{1/M}, \quad D_2 = \left[ 2(1+r) \right]^{-1/(M-1)}.
\]

In Eq. (3.15), \( M \) is the new index describing the shape of the yield locus. If \( M = 2 \), the equation is reduced to Hill's old theory [114], and if \( M = 2 \) and \( r = 1 \), the classical von Mises yield criterion is recovered. From Eq. (3.15), the incremental effective strain during a small time interval \( t_i \) to \( t_i + \Delta t \) will be
\[
\Delta \varepsilon = \int_{t_i}^{t_i+\Delta t} \varepsilon \, dt.
\] (3.16)

and with the condition of a proportional path, the integral can be evaluated as

\[
\Delta \varepsilon = D_1 \left[ \frac{M}{M-1} |\Delta \varepsilon_1 + \Delta \varepsilon_2|^{M-1} + D_2 \frac{M}{M-1} |\Delta \varepsilon_1 - \Delta \varepsilon_2|^{M-1} \right]^{M}.
\] (3.17)

where \( \Delta \varepsilon_1 \) and \( \Delta \varepsilon_2 \) are two principal strain increments between time \( t_i \) and \( t_i + \Delta t \). This integration, in general, cannot be simply evaluated since the effective strain depends not only on the current strain value but on strain path. However, during a small time interval, the ratio of the principal strain rates can be assumed not to vary significantly, so that Eq. (3.17) provides a practical approximation for strain increment \( \Delta \varepsilon \).
3.5. Numerical Consideration

3.5.1. Computational Procedures

A FORTRAN program was developed to solve the problem. Because of its complexity and execution time, the program was run on a CRAY-1/S computer of Boeing Computer Service Company at Seattle, WA, and CRAY-XMP/48 computer of Pittsburgh Supercomputing Center at Pittsburgh, PA. The FORTRAN code was fully vectorized to minimize running time on the CRAY computers. Computational procedures utilizing Bishop’s method are given below and a flow chart of the program appears in Appendix A.

1) Apply the rigid-viscoplastic finite element method to solve for the plastic deformation during a small time interval while holding the temperature constant. Heat generation rates for the incremental displacement solution as well as other mechanical quantities relating deformation are found.

2) The updated geometry and heat generation rates calculated in part 1) are used in the next part to solve the incremental energy balance equation.
3) Solve the transient heat transfer equation and find the temperature distribution. The temperature distribution is used to modify the constitutive relationship for each element in the deformation segment of the calculation at the next time step.

4) Proceed to the next step, repeating steps 1) to 3)

3.5.2. Convergence problem

One of the most important needs of numerical methods is to provide an appropriate convergence criterion. Convergence of solutions in the deformation segment of the calculation is considered to be obtained if the fraction defined by

$$N_f = \frac{\| \Delta u_t \|}{\| \Delta u^0 \|}$$

is less than a prescribed tolerance after solving for $\Delta u$ in Eq. (3.12) for each iteration, where $\| \cdot \|$ denotes $L_2$-norm. For the numerical computation, the tolerance of $10^{-6}$ is used. Use of sufficiently small time increments ensures that the computational errors remain small. The time increment is chosen so that incremental displacement during each step is 0.333 mm at the gage length of the
specimen and the strain increments do not exceed 0.005 for each step. The desired convergence was obtained in about 3-6 iterations for most of steps except the final stages of computation. Since the temperature gradient enhances the flow localization and the localized strain produces sharper temperature gradients ("autocatalytic" process), convergence of the finite element solution is more difficult to achieve than in the analysis of isothermal deformation. Using the following numerical techniques, however, the convergence of the solution can be achieved up to near the physical failure point.

In general, the convergence of the iterative procedure depends heavily on the choice of the trial solution $\Delta u^0$ close to the actual solution. In this study, the solution of the previous step is used as an initial guess for the current step. This method can not be applied for the first step since no previous solution is available. To overcome this difficulty, a corresponding elastic solution is used as the initial guess at the first step. These initial guesses provide convergence of the solution throughout most of deformation until deformation becomes highly non-uniform near the physical failure point of the test. As the deformation localization develops, the difference of deformation distribution between each
time step increases and finally becomes so large that the previous solution is no longer near enough to the current solution to allow convergence [115]. To overcome this difficulty, the step size is reduced as deformation proceeds whenever $N_f$ in Eq. (3.18) tends to increase. With a smaller step size, solutions between the adjacent steps become more similar. This is the most common strategy to improve convergence. In addition, the trial solution is weighed by

$$\Delta u_{i+1}^o = \Delta u_i + (\Delta u_i - \Delta u_{i-1}) \cdot w,$$

(3.19)

if the solution tends to diverge, where $\Delta u_{i+1}^o$ is the trial solution at step $(i+1)$; $\Delta u_i$ and $\Delta u_{i-1}$ are two previous solutions at step $i$ and $(i-1)$, respectively, and $w$ is an arbitrarily chosen weighting factor ranging from 0.0 to 0.5 [115,116].

Furthermore, deformation becomes localized in the center region of the specimen after the maximum load and outside of this region, elastic unloading occurs and spreads from the shoulder toward the center. Since rigid-plastic theory is used here, the problem will be indeterminate during elastic unloading. The unloading zone moves as a nearly rigid body and its
strain-indeterminacy is another source of numerical divergence. The difficulty has been overcome by considering an offset of the effective strain increment, say $\Delta \varepsilon_0$, which is several orders of magnitude smaller than the average strain over the specimen (say, $10^{-2}$ times the average strain increment). The stresses are assumed to vary linearly from zero to the flow stress if the effective strain rate is smaller than this offset strain [68,117,118]. Whenever the incremental strain, $\Delta \overline{\varepsilon}$ becomes less than the offset strain increment, $\Delta \varepsilon_0$ in one element, the constitutive equation is modified as

$$
\overline{\sigma} = K \left( \overline{\varepsilon}_1 + \Delta \varepsilon_0 \right)^n \frac{\Delta \overline{\varepsilon}}{\Delta \varepsilon_0} .
$$

(3.20)

for Hollomon type material, for example, and the element stiffness equations are accordingly modified. Here $\Delta \overline{\varepsilon}$ is the incremental effective strain at the current time step and $\overline{\varepsilon}_1$ is the effective strain at the beginning of the step. Similar modification can be done for other constitutive equations. An alternative way to handle numerical difficulty due to elastic unloading is assuming the boundary of the specimen to undergo rigid-body motion [83,115]. This method was used in the early stage of this study. The rigid region is chosen corresponding to the
elements for which the axial strain rate is smaller than a
certain value (say, $10^{-5}$ times the average strain rate).

The success of convergence in solving Eq. (3.14) for
the transient heat transfer part of the calculation
usually depends on the size of elements and time step
used. Use of the same elements and time step as in the
deformation segment, however, is found to guarantee
convergence of solution in the heat transfer part.
Generally, central difference scheme provides
unconditional stability unless abrupt changes of the
forcing term are involved [113], which is not the case in
this study.

One major advantage of the prescribed decoupled
method is that the transient heat transfer part can be
treated as a piecewise linear problem even though material
parameters are varying with temperature, strain, and
strain rate. The parameters can be regarded as constants
on a certain element during a sufficiently small time
interval. This makes the problem easier to solve and
considerably reduces the computing time as compared with
treating both the deformation and heat transfer aspects
nonlinearly.
3.6. Automatic Remeshing Technique

The accuracy of a finite element solution greatly depends upon how the element discretization is performed. Several methods have been proposed to improve the discretized model in an iterative manner. A natural way of improving the quality of the finite element solution is to increase the number of degrees of freedom. New degrees of freedom can be added by either increasing the order of polynomial approximation inside elements, the so-called p-method [119-122], or by subdividing elements, the h-method [122-124]. Another improvement can be made by optimizing the coordinates of nodal points [125,126], in which nodal points are redistributed after solving the problem without increasing the degrees of freedom.

In a large deformation problem, updating of the nodal quantities on a finite element mesh sometimes leads to unacceptable mesh distortions and in most cases it becomes more important to redefine the mesh after some steps of the process. For a tensile test, the aspect ratio of an element increases as the specimen deforms and may provide unacceptable results. Also, it becomes difficult to
include sufficiently the effect of strain localization because high strain gradients are inhibited by finite element size. Even though there have been many efforts to study the effect of mesh optimization, few publications can be found applying the method to metal forming problems. Most studies [119-125,127,129] have been limited to solving steady-state or elasticity problems with rather simple geometry and to show the improvement of solution by a posteriori error analysis and mesh optimization.

In the mesh optimization process, two key questions are 1) how to define a quantitative measure of the efficiency of a given mesh and 2) how to modify the mesh and interpolate physical quantities from original mesh to new mesh while retaining equilibrium and shape of specimen. The second question is usually encountered for the case of optimization of node positions and transient problem. Numerous attempts have been made for the first question, while few results have been reported for the second question. For any remeshing technique, the most essential thing is to provide a proper criterion to identify the regions of the domain where the finite element solution is poorer. In most cases, these regions are identified by means of element error indicators, which
are defined locally on each element. These error indicators should be related to the size of each element and to local residuals that arise from the force unbalance and obtained during the solution process itself. The most commonly used error indicator is strain energy density (or specific energy difference) given in terms of element size and error norm of the given boundary value problem [121,127]. Other criteria can be used by measuring distortion of element such as Jacobian of element or ability of distorted element to follow the deformation process, especially in large deformation problems [126,128]. Once a criterion of efficiency of mesh is established, a new mesh can be developed by either h-method or p-method or redistribution of nodes. Sometimes completely new mesh can be defined by hand or by an automatic mesh generation procedure. If remeshing is required for computation of the next time step rather than final error analysis, some quantities defined on the old mesh, e.g. strain, stress, and displacement, should be interpolated into the new mesh system. This can be done by using the same shape function used in original finite element approximation and by interpolating quantities at each point in terms of nodal values. More details of this interpolating procedure can be found elsewhere [128]. In this study, the main purpose of remeshing is not analyzing
errors, but reducing error caused by large distortion of elements and continuing computation for the next time steps. Both subdivision of elements and optimization of nodes were reviewed separately for automatic remeshing.

In order to optimize the position of nodes, they are redistributed in such a way that the prescribed element error indicators are minimized on the new mesh system [125], while keeping the previous degrees of freedom unchanged. After solving deformation at each time step, error indicators, $f_k$, which will be explained later, are computed for each element. The nodal points are redistributed in the direction where $f_k$ becomes constant for all elements. Both direction and the magnitude of the change are computed by assigning $f_k$ as a mass-like quantity at the geometric center of each element. This 'mass' is considered to attract the nodes surrounding the element. Displacement, strain and temperature on the old mesh are interpolated into the new mesh by the previously mentioned method. The method is easy to implement and cost-effective because of fixed degree of freedom during computation. However, it is difficult to keep the proper shape of specimen during the change of nodal points and to prevent errors involved in interpolating procedure, especially after some localization of deformation. Even
slight change of shape sometimes leads to violation of force equilibrium during modification of quantities and becomes a source of numerical divergence at the next time step. Therefore, the h-method is found to be more suitable to this study, even though it increases computing time.

In subdivision of elements, the h-method, new degrees of freedom are introduced by selectively subdividing elements where the solution is less accurate. These regions are identified by element error indicators, $f_k$, constructed from the finite element solution. The method used in this study is as follows. After solving deformation and heat transfer parts at each time step, a row of elements are added by evenly subdividing the innermost row of elements if remeshing is necessary. All quantities defined on the old geometry are interpolated into the new nodal points and elements.

Considering the special features of this study, which involves large strain localization within the central region of specimen, the following two criteria are used to determine the efficiency of mesh.
A subdivision of the innermost row of elements, will be done either if 1) error indicator, $f_k$, on a certain element becomes too large compared with other elements, or 2) strain localization indicator, $\gamma_i$, tends to decrease at a certain time step, say step $i$. The error indicator, $f_k$, which is related to strain energy density, is defined on element $k$ as $f_k = A_k \cdot (\Delta \varepsilon_k)^2$ where $A_k$ and $\Delta \varepsilon_k$ are element area and increment of effective strain during a time step $\Delta t$, respectively, on the element. Constant value of $f_k$ over elements provides the most desirable solution [127,129]. In actual computation, remeshing is performed if maximum of $f_k$, which always occurs at the center part of the specimen, exceeds three times its average value. The number was chosen arbitrarily. Sometimes the same order of $f_k$ on each element may be enough to make improvement [125]. Another criterion of remeshing, $\gamma$, which indicates the tendency of strain localization, is measured at each time step by $\gamma = (\Delta \varepsilon_c)/(\Delta \varepsilon_g)$, where $\Delta \varepsilon_c$ and $\Delta \varepsilon_g$ are increment of effective strain at specimen center and gage length during $\Delta t$, respectively. During uniform deformation this ratio will be near unity; while it always increases during post-uniform deformation due to high local strain rate in the necking region. If this number decreases despite further deformation, it implies that deformation solved by the present mesh becomes
uniform because of discontinuity of finite element solution. In most cases, the first criterion is satisfied earlier than the second one.
CHAPTER IV.
RESULTS AND DISCUSSION

4.1. Introduction

The combined deformation and heat transfer analysis described in the previous chapter was performed for several engineering materials. As applications of the formulation, tensile tests of ARMCO interstitial free (I.F.) steel [130], 304 stainless steel and aluminum-killed (AK) steel were analyzed and the results were compared with existing experimental results. Special attention has been paid to the effect of deformation heating on material response and localization process. Effects of test speed and heat transfer condition as well as combined effect with other material parameters were examined. Also, analytical results of tensile test of a superplastic material (SPZ1) and effect of automatic remeshing on FEM results are presented.

The ASTM E-8 standard sheet tensile specimen is chosen for analysis (Fig. 4.1) and the corresponding
finite element mesh is shown in Fig. 4.2. The exact specimen profile corresponds to the template provided by the Tensilkut Corporation [131], and incorporates a 0.6% taper over a distance of 25 mm from the specimen center in order to initiate necking at the specimen center. Because of symmetry, only a quarter of the specimen was analyzed. The finite element mesh contains 245 nodal points and 408 elements.

As mentioned in the previous chapter, the FORTRAN program is vectorized for efficient use on a CRAY supercomputer. About 20% of total CPU time is spent in solving the simultaneous equations. Several solution algorithms were in fact examined. Among them, a vectorized Gaussian elimination algorithm provided the best efficiency with a speed factor of 1.5, as compared with LU decomposition. The program was run on several scalar computers for comparison of execution time, with the following results:

<table>
<thead>
<tr>
<th>Computer</th>
<th>CPU time/CRAY CPU time</th>
</tr>
</thead>
<tbody>
<tr>
<td>CRAY 1/S</td>
<td>1</td>
</tr>
<tr>
<td>IBM 3081/D</td>
<td>14</td>
</tr>
<tr>
<td>VAX 11/780</td>
<td>114</td>
</tr>
</tbody>
</table>
Figure 4.1. Tensile specimen and domain of analysis.

Figure 4.2. FEM mesh geometry (a quarter of specimen).
4.2. Non-isothermal Tensile Tests of I.F. Steel

4.2.1. Material Modeling

To verify the FEM formulation, non-isothermal tensile tests of interstitial free (I.F.) steel by ARMCO Steel Corporation have been analyzed and the results are compared with experiments from other literature. To include the strain hardening and rate hardening effects, the following constitutive equation was used \[4,5]\:

\[
\bar{\sigma} = K (\bar{\varepsilon} + \bar{\varepsilon}_o)^n \left( \dot{\varepsilon} / \dot{\varepsilon}_o \right)^m (1 - \beta \Delta T), \quad (4.1)
\]

where parameters are chosen from other literature as follows \[4,5\]: \(K = 566.0\) MPa, \(n = 0.219\), \(m = 0.018\), \(\bar{\varepsilon}_o = -0.014\), \(\dot{\varepsilon}_o = 0.002/s\), \(\beta = 0.0011/°C\).

Thermal properties of I.F. steel are chosen as follows \[132,133\]: \(\rho = 7.85 \times 10^{-3}\) g/mm\(^3\), \(c = 0.464\) J/g/°C, \(k = 0.054\) J/mm/s/°C. Within the ranges of heat convection coefficients given in the literature \[133\], the following values are chosen to agree with the experimental results by Lin and Wagoner \[4,5\] according to the test conditions:
\[ h = 10^{-5} \text{ J/mm}^2/\text{s}/^\circ\text{C} : \text{natural convection in air}, \]
\[ h = 10^{-2} \text{ J/mm}^2/\text{s}/^\circ\text{C} : \text{natural convection in water}. \]

4.2.2. Material Ductility

The simulated engineering stress-strain curves of I.F. steel are presented for various rates and test conditions in Figs. 4.3a to 4.3d, illustrating the effect of heat transfer conditions on the measured tensile response. Four heat transfer conditions were examined: isothermal, in-water, in-air, and adiabatic cases. Drop of flow stress and total elongation by thermal softening is clearly shown for all range of rates. At the range of strain rates examined, the difference of maximum stress between the isothermal case and the adiabatic case is about 2% of the isothermal one, independent of rate. In intermediate cases such as in-water and in-air, the curves approach the isothermal case for relatively low rates and the adiabatic case for higher rates, since heat transfer increases with decreasing testing speed. Figs. 4.4a to 4.4c show the effect of initial strain rate on the stress-strain relationship for three heat transfer conditions, i.e. isothermal, in-air, and adiabatic cases, respectively. Both for isothermal and adiabatic cases,
Figure 4.3. Simulated engineering stress-strain curves of I.F. steel showing effect of heat transfer conditions at initial rates of a) $10^{-1}$/s, b) $10^{-2}$/s, c) $10^{-3}$/s, and d) $10^{-4}$/s. The curves terminate at the point at which Eq. (4.3) is satisfied.
Figure 4.3. (2/4, continued)
Figure 4.3. (3/4, continued)

initial strain rate $= 10^{-2}/s$  I.F. STEEL

- isothermal
- in water
- in air
- adiabatic
Figure 4.3. (4/4, continued)
Figure 4.4. Effect of the initial strain rates on stress-strain relationship of I.F. steel for a) isothermal, b) in-air, and c) adiabatic cases.
Figure 4.4. (2/3, continued)
Figure 4.4. (3/3, continued)
flow stress always increases with rates because of strain rate hardening. In the in-air case, however, the flow stress at the higher rate falls below the curves at the lower rates because of thermal softening at high local strain rate region, especially after necking.

Simulated engineering stress-strain curves show substantial agreement with experimental ones by Raghavan [43], as shown in Figs. 4.5a and 4.5b. Agreement between measurement and prediction of stress-strain relationship is fairly good in most cases, even though there is some discrepancy during post-uniform deformation. This error is likely to result from uncertainty in parametric values in the constitutive equation, which was originally derived over a range of $0.02 < \bar{\varepsilon} < 0.35$ [4]. Since m-value may vary with strain [134], use of constant m leads to some error in predicting the stress-strain curve. There is some possibility that the m-value is overestimated since the predicted total elongations at high strain rates are 2 to 3% higher than the measurements. Also, error in choosing heat transfer coefficients seems to be another source of discrepancy. Another possibility for discrepancy between analytical results and experiment is that after a certain amount of deformation, simulated strain gradients are inhibited because of the finite
Figure 4.5. Comparison of simulated engineering stress-strain curves of I.F. steel in air with experimental results by Raghavan [43] at rates of a) $10^{-2}$/s and b) $10^{-3}$/s.
Figure 4.5. (2/2, continued)
element size and averaging effect of the FEM solution. This phenomenon can be avoided to some extent by using an optimized finite element mesh and is likely to be negligible in tensile tests of I.F. steel. The principal predictions of the analysis, however, are not significantly affected by the lack of absolute quantitative agreement with experiment. The trends in elongation and stress level with variations in rate and environment are of principal interest.

The predicted uniform and total elongations corresponding to the several rates and heat transfer conditions are shown in Fig. 4.6. Experimental results by Lin [5] are also presented. As mentioned earlier, uniform elongation has mostly been used as a measure of tensile ductility of material, even though the effect of temperature gradient becomes more important after necking because of large strain localization. In this study, both uniform and total elongation are used as measures of ductility. The uniform elongation can be defined as the strain at maximum load. However, total elongation, which is strain at failure point, depends on the choice of failure criterion. Physical failure occurs not only due to strain localization but is also enhanced by the presence of material defects in the specimen. There have
Figure 4.6. Effect of heat transfer conditions on uniform and total elongations of I.F. steel. Marks in the figure denote the experimental results by Lin [5].
been many efforts to express failure strain in terms of material parameters and specimen geometry. An extensive literature survey on this topic can be found elsewhere [135]. In the present work, the failure strain is chosen as the value where the following parameter reaches a certain value, taking into account only the rate of strain localization at the specimen center and in the uniform deformation region [136], which is similar to the strain localization indicator, \( \gamma \), defined in the previous chapter:

\[
\frac{\Delta \varepsilon_c}{\Delta \varepsilon_a} > \text{critical value},
\]

where \( \Delta \varepsilon_c \) and \( \Delta \varepsilon_a \) are incremental strain at center and average incremental strain over the specimen.

In terms of nominal value, neglecting the acceleration rate of strain localization and assuming that the true strain at the center \( (\varepsilon_c) \) is much larger than the strain hardening exponent \( (n) \), the above criterion can be expressed as [136]:

\[
\frac{\Delta \varepsilon_c}{\Delta \varepsilon_a} > \text{critical value}.
\]
\[ \frac{\Delta \bar{e}}{\Delta e} \approx \frac{\Delta F/F}{\Delta e/(1+e)} > 5.0 \quad (4.3) \]

where \( e \) is the engineering strain (50 mm gage length), \( F \) is the corresponding force in the engineering stress-strain curve and \( \Delta \) denotes the change during the time step. The parameter represents the degree of strain localization as well as the slope of logarithmic engineering stress-strain curve. This equation could be used directly in the FEM program to evaluate total elongation. Throughout the study, except in the superplastic material case, the number 5.0 was used with consideration of the limit of convergence of the program and has no direct physical significance. The exact choice of the number, however, has little effect on the value of total elongation unless a very small value is selected, because the number increases very sharply after a certain amount of strain localization.

As shown in Fig. 4.6, both uniform and total elongations are nearly constant in isothermal and adiabatic cases for various rates, which coincides with experimental and analytical results in the literature [4-6, 42-44, 137]. Ghosh [29] derived the criterion of isothermal plastic instability for a rate-sensitive
material, in the form of $\varepsilon_u = n/(1+m)$. Under this criterion, with consideration of prestrain $\varepsilon_o$, the uniform engineering strain, $\varepsilon_u$, of I.F. steel equals 25.7%. The uniform strain from the numerical calculation is about 0.5% greater than this analytical result; the difference presumably arising from two-dimensional effects overlooked in the original criterion. Such a difference has been noted [43,138] for both rate-sensitive and insensitive materials. Agreement between FEM results and experiments by Lin and Wagoner [4,5] for total elongation is fairly good in the isothermal case. Errors in the non-isothermal case arise from inaccuracies in choosing heat transfer coefficients and possible experimental errors.

In adiabatic deformation, the ductility remains nearly constant with respect to rate while the temperature slightly increases with rate. This illustrates the importance of the temperature gradient relative to the absolute value of temperature increase. For simulated tests in air, the lines lie between two extreme cases, i.e. they approach the isothermal case for relatively low rates and the adiabatic case for relatively high rates. Additional calculation shows that the temperature distribution as well as the engineering stress-strain relationship in air are nearly identical with those of the
isothermal case at initial rates less than $10^{-4}$ s and with those of the adiabatic case at rates higher than 1/s.

The predicted differences of uniform and total elongation (i.e. post-uniform elongation) between isothermal and adiabatic cases are about 2.7% and 6.2%, respectively. The uniform elongation is found to drop by 0.1 to 2.7% at moderate rates ($10^{-4} - 10^{-1}$ s), while total elongation decreases by 0.3 to 6.0% testing in air compared to the isothermal case. Contrary to results for AK steel, in which the total elongation in air remains nearly constant for various rates [3], the ductility of I.F. steel decreases with increase of rates. In AK steel, the temperature gradient at high rates is partially compensated for by an $m$-value which increases with rate [24]. This phenomenon will be discussed later.

4.2.3. Effect of Deformation Heating on Strain Localization

Figs. 4.7a to 4.7d show the development of local strain rate at the center and 25 mm away from the center as a function of engineering strain for various initial strain rates. Variations of local strain rates with initial rates for each heat transfer condition are also
Figure 4.7. Variation of local strain rate as a function of engineering strain at the center of specimen and 25 mm away from the center at rates of a) $10^{-1}/s$, b) $10^{-2}/s$, $10^{-3}/s$, and d) $10^{-4}/s$. 
Figure 4.7. (2/4, continued)
Figure 4.7. (3/4, continued)
Figure 4.7. (4/4, continued)
given in Figs. 4.8a to 4.8c. During uniform deformation, strain distribution is almost constant over the specimen except in the shoulder region. In post-uniform deformation, however, strain inside the neck near the center increases sharply while strain outside the neck remains constant. These figures therefore represent the amount of strain localization during deformation and can be used as a measure of material ductility. The ratio of the two strain rates is the strain localization indicator, $\gamma$. This result shows that flow localization in the non-isothermal case begins at an earlier time than in the isothermal case as a result of deformation heating. As in the case of engineering stress-strain curves, curves in air approach the isothermal one at low rate and the adiabatic one at high rate. As shown in Figs. 4.8a and 4.8c, local strain rate is independent of rate for isothermal and adiabatic cases, which verifies the invariance of ductility, i.e. uniform and total elongation with rate despite variation of flow stress. In Fig. 4.8a, a slight decrease in slope of the curves at the final stages of local strain curves at the specimen center implies that localization process tends to decrease contrary to the actual situation and the FEM solution becomes insufficient to represent the actual deformation mode. This insufficiency of the FEM solution can be
Figure 4.8. Variation of local strain rate at specimen center and 25 mm away from the center for different heat transfer conditions: a) isothermal, b) in air, and c) adiabatic cases.
Figure 4.8. (2/3, continued)
Figure 4.8. (3/3, continued)
partially eliminated by remeshing which will be discussed later.

4.2.4. Temperature Distribution

Temperature distributions during testing in air for various rates are given in Figs. 4.9a to 4.9d. The predicted result agrees well with the experiment [43], as shown in Figs. 4.9b and 4.9c. Prior to necking, heat conduction from the center region into the specimen ends establishes a temperature gradient that is significant only outside the gage length, where effect on mechanical strength is not important. After necking occurs, the temperature near the center increases sharply due to flow localization and large local strain increments. At high strain rate ($10^{-1}$/s), the predicted temperature increase reaches about 75°C at the center of specimen, in substantial agreement with experiment [4-6,42-44]. Maximum temperature increase at specimen center at failure point is approximately 48°C at the rate of $10^{-2}$/s, 13°C at $10^{-3}$/s, and 1.5°C at $10^{-4}$/s. Under adiabatic conditions, variation of maximum temperature at specimen center with rate is not as severe as in-air case, showing 70 to 90°C over the range of strain rates examined.
Figure 4.9. Temperature distribution during test in air along specimen tensile axis at rates of a) $10^{-1}$/s, b) $10^{-2}$/s, c) $10^{-3}$/s, and d) $10^{-4}$/s.
Figure 4.9. (2/4, continued)
Figure 4.9. (3/4, continued)
Figure 4.9. (4/4, continued)
Figs. 4.10a to 4.10b show the temperature increase at the center point as a function of engineering strain at four different initial rates, showing sharp increase of temperature after the onset of necking. Also the figures show the effect of heat transfer conditions on temperature variation.

Simulated two-dimensional temperature and effective strain distributions at five steps of the test conducted in air at an initial rate of $10^{-1}/s$ in Figs. 4.11a and 4.11b show the importance of two-dimensional analysis for analyzing the tensile test, a test usually studied by one-dimensional analysis. The figures were drawn by I-DEAS [138]. Temperature and effective strain vary significantly along y-axis, particularly near the failure point. Also, both temperature and effective strain are almost uniform within the gage length before necking. For relatively low rates, temperature variation in the y-direction is less severe than that of effective strain along that axis. This variation is largely from heat convection into the environment.
Figure 4.10. Maximum temperature increase at the center of specimen at initial rates of a) $10^{-1}/s$ and b) $10^{-3}/s$, showing sharp increase of temperature after necking.
Figure 4.10. (2/2, continued)
Figure 4.11. Two-dimensional distributions of a) temperature and b) effective strain over the specimen for the test of I.F. steel in air at rate of $10^{-1}$/s along the process of test.
Figure 4.11. (2/2, continued)
4.2.5. Effects of Heat Conduction at Specimen Ends

The importance of heat conduction into grips at the end of a tensile specimen may be examined by simulations with and without heat sinks at the specimen ends. Figs. 4.12a to 4.12d present temperature-profile data of four initial strain rates ($10^{-1}$ to $10^{-4}$/s). At the higher rate, there is virtually no difference in temperature profile in the gage region throughout the test and only a minor variation at the specimen ends. Little effect on mechanical behavior is therefore anticipated. At the lower rate, there is a noticeable difference in temperature profile between the two cases, but the temperatures and gradients themselves are quite small. At the lower rate, there is little difference between a test in air and an isothermal test, so little difference can be ascribed to the end effect. It therefore appears, contrary to some published results [33], that conduction into specimen ends is unimportant in affecting total elongation, at least for I.F. steel tested in air using standard sheet tensile specimens. Engineering stress-strain curves in Fig. 4.13 show that there is little difference in mechanical response due to the presence of heat sinks at specimen ends. The total elongation increases by about 0.4% due to the presence of
Figure 4.12. Effect of conduction at specimen ends on temperature distribution along specimen at rates of a) $10^{-1}$ /s, b) $10^{-2}$ /s, c) $10^{-3}$ /s, and d) $10^{-4}$ /s.
In Air (strain rate = $10^{-2}$/s)

<table>
<thead>
<tr>
<th>Eng. Strain(%)</th>
<th>w/o heat sink</th>
<th>with heat sink</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>10.1</td>
<td>10.1</td>
</tr>
<tr>
<td>19.7</td>
<td>19.7</td>
<td>19.7</td>
</tr>
<tr>
<td>25.0</td>
<td>25.1</td>
<td>25.1</td>
</tr>
<tr>
<td>29.6</td>
<td>29.7</td>
<td>29.7</td>
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<td>34.4</td>
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<td>34.5</td>
</tr>
<tr>
<td>40.1</td>
<td>40.2</td>
<td>40.2</td>
</tr>
</tbody>
</table>

Figure 4.12. (2/4, continued)
Figure 4.12. (3/4, continued)
Figure 4.12. (4/4, continued)
Figure 4.13. Engineering stress-strain curves for test in air without conduction at specimen ends for various initial rates.
heat sinks at the rates examined.

The slight specimen taper employed in the Tensilkut specimens always causes flow localization at the center of specimen. The position of flow instability (necking) can depend on geometry as well as geometric defects in the specimen, but necking will not generally occur randomly unless a perfectly parallel specimen with no initial internal inhomogeneities is used. If a parallel, uniform specimen is used, the problem will be defined as an eigenvalue problem for abrupt bifurcation rather than a standard deformation problem as in the present study [52]. Therefore, only in special, carefully controlled specimens will the failure position be random and only under these circumstances are end conditions likely to produce any measurable difference in mechanical response.
4.3. Tensile Testing of 304 Stainless Steel

As another application, non-isothermal tensile testing of 304 stainless steel was analyzed. The constitutive equation of this material has the same form as I.F. steel, which was given in Eq. (4.1). The material parameters and thermal properties are chosen as follows [39,132]: $\dot{\varepsilon} = 1450.0$ MPa, $n = 0.52$, $m = 0.02$, $\ddot{\varepsilon}_o = 0.0$, $\ddot{\varepsilon}_o = 0.00166/s$, $\beta = 0.0017/°C$, $\rho = 7.8 \times 10^{-3} \text{ g/mm}^3$, $c = 0.462 \text{ J/g/°C}$, $k = 0.015 \text{ J/mm/s/°C}$. Comparing these parameters with those of I.F. steel, the effect of deformation heating is expected to be larger in this case, because of the large heat source due to higher flow stress and lower thermal conductivity of stainless steel.

In Figs. 4.14a to 4.14d, engineering stress-strain curves of 304 stainless steel are presented, showing the same trend of effect of deformation heating as for I.F. steel. In this material, the drop of flow stress and ductility due to thermal softening is more noticeable than in the case of I.F. steel due to its higher flow stress and higher value of $\beta$. Maximum flow stress in adiabatic condition is about 90% of that in the isothermal case, compared to a 2% drop in the I.F. steel case.
Figure 4.14. Effect of heat transfer conditions on simulated engineering stress-strain curves of 304 stainless steel at rates of a) $10^{-1}$/s, b) $10^{-2}$/s, c) $10^{-3}$/s, and d) $10^{-4}$/s.
Figure 4.14. (2/4, continued)
Figure 4.14. (3/4, continued)
Figure 4.14. (4/4, continued)
Simulated engineering stress-strain curves of stainless steel in air are shown for various initial rates in Fig. 4.15. The effect of heat transfer, which increases with decreasing rate, is clearly shown. At high strain rate, the drop of flow stress due to thermal softening exceeds effect of rate hardening. The variation of uniform and total elongation for various rates and test conditions is presented in Fig. 4.16. As in the case of I.F. steel, tensile ductility in both isothermal and adiabatic cases are almost constant with respect to rates. Total elongations in the isothermal and adiabatic cases are about 105% and 70%, respectively, illustrating that more than 30% of isothermal ductility is lost by temperature effect. For the test in air, uniform elongation varies from 46% to 67% over the range of rates, while total elongation varies 69% to 100% for the same rates. The difference of uniform and total elongation between isothermal and adiabatic cases is about 21% and 35%, respectively.
Figure 4.15. Engineering stress-strain curves for the test in air of 304 stainless steel for different initial rates, showing effect of amount of heat transfer during deformation.
Figure 4.16. Effect of heat transfer conditions on uniform and total elongations of 304 stainless steel for various initial strain rates.
4.4. Effects of Deformation Heating and Rate-sensitivity in Tensile Testing of Aluminum-killed Steel

As mentioned earlier, strain-rate sensitivity tends to raise the flow stress in regions of high strain rate, and therefore has a stabilizing effect on ductility, especially after the onset of necking [139,140]. Ayres [3] found that the tensile ductility of aluminum-killed steel in air remains almost constant for various testing speeds. He ascribed this result to the fact that rate-sensitivity (m-value) of AK steel and the magnitude of deformation heating both increased with test speed. This "near-invariance" of tensile ductility of AK steel for non-isothermal deformation by Ayres is shown in Fig. 4.17 and was verified by Semiatin et. al. [45] using a one-dimensional finite difference method. Wagoner and Wang [23,24] measured an increase in the isothermal tensile ductility of AK steel with test speed and correlated this to a measured increase of m-value with speed. Such effect of deformation heating and strain rate can be evaluated by applying constitutive equations which include the effects of rate on strain-rate sensitivity.

In this section, the competing effect of rate sensitivity and thermal softening of AK steel will be examined using
Figure 4.17. Ayres' experimental results [3] for tensile properties of AK steel measured in air and water at several initial rates, showing "near-invariance" of non-isothermal tensile ductility of this alloy.
finite element analysis.

4.4.1. Constitutive Equation of AK Steel

Wagoner and Wang [23,24] found that the instantaneous (i.e. as measured in a "jump" test) rate-sensitivity of AK steel depends strongly on strain rate at room temperature, while showing little dependency on strain. They proposed the following tensile constitutive equation relating flow stress ($\tilde{\sigma}$) to strain ($\tilde{\varepsilon}$) and strain rate ($\dot{\varepsilon}$), which was given in the same form in Eq. (2.3):

$$\tilde{\sigma} = K \tilde{\varepsilon}^n \exp[b(\dot{\varepsilon}^a - \dot{\varepsilon}_o^a)].$$  \hspace{1cm} (4.4)

The parameters $a$ and $b$ control the variation of rate-sensitivity according to the formula

$$m = \frac{d(\ln\tilde{\sigma})}{d(\ln\tilde{\varepsilon})} = (ab) \dot{\varepsilon}^a.$$  \hspace{1cm} (4.4)

Even $m$-values of other materials such as I.F. steel, high strength low alloy steel (HSLA), hot-rolled steel, dual phase steel were found to slightly vary with strain rate [141]. Three analytical expressions for $m$ used in the present study are identical to those proposed by Wagoner and Wang [24]:
"RHW Law" : \[ m = 0.0318 \varepsilon^{0.179} \]  

"Halfway Law" : \[ m = 0.0265 \varepsilon^{-0.090} \]  

"Power Law" : \[ m = 0.022 \text{ (constant)} \]

They obtained "RHW Law" (\( m_i \) in Ref. [24], where subscript \( i \) denotes "instantaneous" rate sensitivity) through a series of jump tests (i.e. rapid strain rate change tests) in air [23] and "Power Law" (\( m_c \) in the reference, which represents "continuous" rate sensitivity) from the continuous tensile tests in circulating water. The \( m \)-value in "Halfway Law" (\( m_t \) as operant rate sensitivity) simply represents an average of the two other laws, in log \( m \)-log \( \varepsilon \) space. By comparing strain distributions and elongation data from experiments with results of FEM analysis, they concluded that the "Halfway Law" provided the best agreement with experimental results. Variation of \( m \)-value with strain rate for the three laws is shown in Fig. 4.18. The other parameters in Eq. (4.4) are chosen as follows [24]: \( K = 516.0 \text{ MPa, } n = 0.234, \varepsilon_o = 10^{-4} /s, \bar{r} = 1.5 \). For the "Power Law", the following simple constitutive equation is used:

\[
\bar{\sigma} = K \bar{\varepsilon}^n (\bar{\varepsilon}/\varepsilon_o)^m .
\]
Figure 4.18. Variation of strain-rate sensitivity, m-value, of AK steel with strain rate for three constitutive equations examined.
To include the thermal effect on the constitutive relationship, the following equation is used for each isothermal constitutive equation, given in Eqs. (4.4) and (4.8).

\[(\bar{\sigma})_{\text{nonisothermal}} = (\bar{\sigma})_{\text{isothermal}} (1-\beta \Delta T) \quad (4.9)\]

where the term, \((1-\beta \Delta T)\) is introduced to include the effect of temperature change \(\Delta T\) on flow stress. The thermal properties of AK steel are chosen as follows \([45,132,133]\): \(\beta = 0.0015^{\circ}\text{C},\) density \(\rho = 7.87\times10^{-3}\text{ g/mm}^3\), specific heat \(c = 0.464 \text{ J/g/}^{\circ}\text{C},\) thermal conductivity \(k = 0.059 \text{ J/mm/s/}^{\circ}\text{C},\) and heat convection coefficient \(h = 2.0\times10^{-5} \text{ J/mm}^2/\text{s/}^{\circ}\text{C in air.}\)

4.4.2. Material Ductility

The predicted variations of uniform and total elongations with strain rate for the three constitutive equations examined are given in Figs. 4.19a to 4.19c, showing that deformation heating has a considerable influence on both uniform and total strain. In each figure, three heat transfer conditions are modeled: non-isothermal case in air along with the extreme cases of
Figure 4.19. Simulated tensile test results of AK steel illustrating effect of deformation heating on uniform and total elongation for three constitutive relations: a) "Power Law", b) "RHW Law", and c) "Halfway Law".
Figure 4.19. (2/3, continued)
Figure 4.19. (3/3, continued)
isothermality and adiabaticity. When a constant m-value is used ("Power Law"), total elongation as well as uniform elongation remains almost constant for the isothermal and adiabatic cases. This agrees well with previous results for I.F. steel and 304 stainless steel, whose strain-rate sensitivities vary little with strain rate. The differences in uniform and total elongation between the isothermal and adiabatic cases are 3.5% and 7.0%, respectively. For tests conducted in air, elongation decreases with test speed because of progressive deformation heating and differential softening in the neck.

On the other hand, total elongations in isothermal and adiabatic cases vary considerably with test speed when m increases with strain rate. The total isothermal elongation varies from 50.3% at a rate of 0.15/s to 37.9% at 0.0001/s for "RHW Law", while dropping from 48.2% to 39.8% for "Halfway Law" over the same range of rates. The corresponding differences for adiabatic deformation are about 9.8% for "RHW Law" and 6.4% for "Halfway Law". For both laws, the difference of total elongation between the isothermal and adiabatic case increases with speed from about 5.0% to 8.0% at the rates examined, while the drop of uniform elongation remains almost constant with the
value of 3.5% for the three constitutive laws. Little
effect of the choice of constitutive laws is expected
during uniform deformation.

For deformation in air, heating compensates the
effect of increasing rate-sensitivity by decreasing
ductility at higher rates. The amount of compensation
depends on the material parameters chosen. In this case,
the "Halfway Law" provides the "near-invariance" of
ductility and agrees well with the experimental results
(Fig. 4.20). The "Power Law" leads to a decreasing total
elongation with strain rate while the "RHW Law" shows an
increase. The "Halfway Law" therefore appears to be the
best choice in reproducing the non-isothermal results of
Ayres [3] as well as the isothermal data of Wagoner and
Wang [24], for which it was originally proposed.

Figs. 4.21a to 4.21c show the simulated engineering
stress-strain curves for deformation in air at four
initial rates. The flow stress always increases with rate
during uniform deformation. In post-uniform deformation,
however, there is a considerable difference between "Power
Law" and the others. In the "Power Law" case, where a
constant m-value is used, the flow stress at the highest
rate falls below those for the lower rates because of
Figure 4.20. Comparison of uniform and total strain of AK steel for three constitutive laws for tests in air.
Figure 4.21. Simulated engineering stress-strain curves in air at various strain rates for three constitutive equations: a) "Power Law", b) "RHW Law", and c) "Halfway Law".
Figure 4.21. (2/3, continued)
Figure 4.21. (3/3, continued)
thermal softening. In "RHW Law" and "Halfway Law" cases, the flow stresses at higher rates remain higher than those at lower rates because the increased strain rate hardening more than offsets the additional thermal softening.

4.4.3. Temperature Distribution

Temperature distributions along the specimen length are presented in Figs. 4.22a and 4.22b at initial strain rates of 0.15/s and 0.0029/s. The temperature gradients are not very sensitive to the choice of constitutive equation. Prior to necking, the temperature gradient is sizable only outside the gage length, where the effect on mechanical strength is unimportant. After necking occurs, the temperature near the center increases sharply because of flow localization and large local strain increments. At high strain rate (0.15/s), the maximum predicted temperature increase at the specimen center is approximately 70°C, in substantial agreement with experiment [3].
Figure 4.22. Comparison of temperature distributions along tensile axis for "RHW Law" and "Halfway Law" at rates of a) 0.15/s and b) 0.0029/s.
Figure 4.22. (2/2, continued)
4.5. Superplastic Materials

In recent years, many studies have been performed to understand the phenomenon of superplasticity. Numerous authors have measured unusually large values of tensile elongation at high temperatures for a specific class of alloys [30,31,142-145]. The phenomenon of superplasticity usually involves tensile deformation at low stresses to very high strain levels, typically of the order of several hundred percent. Even though superplasticity in metal was first observed in Bi-Sn eutectoid by Pearson [146] in 1934, the phenomenon was regarded mainly as a laboratory curiosity for many years. Only recently, with the growing realization that superplasticity provides new opportunities as a highly versatile tool of fabricating metals on a commercial scale and the fact that it is capable of extending the limitations associated with the more conventional processes, there have been many systematic searches for superplastic alloys and forming processes. The most distinguishable characteristics of superplastic materials are: 1) high strain rate sensitivity of the flow stress [142,143], 2) strong variation of properties with grain size and strain rate [30], and 3) importance of grain boundary sliding as the
Non-isothermal tensile tests of SPZ1 (Zn-Al eutectoid: Zn-22% Al-0.15% Cu) were examined as another example of FEM analysis, partly because of the availability of a constitutive equation in a closed form. The constitutive equation for SPZ1 has the same form given in Eq. (4.1) and the parameters and thermal properties are chosen as follows from other literatures [132,147]:

\[ K = 157.0 \text{ MPa}, \quad n = -0.121, \quad m = 0.420, \quad \epsilon_0 = 0.0, \quad \epsilon_\infty = 1.0, \]

\[ \beta = 0.0025 \degree\text{C}^{-1}, \quad \rho = 7.18 \times 10^{-3} \text{ g/mm}^3, \quad c = 0.40 \text{ J/g\degree C}, \]

\[ k = 0.046 \text{ J/mm/s\degree C}. \]

It should be noted that these quantities are applicable only within a certain range of temperature (about 200-250\degree C) because of their high dependency on temperature [147,148]. At room temperature, SPZ1 does not show the characteristics of superplasticity [146,147]. It becomes more important to include the effect of temperature gradients in analyzing deformation behavior of superplastic material, because a rapid change of temperature may cause the change of microstructure and, eventually, loss of superplastic nature of the material.

Even though many experimental studies have been performed showing the effect of temperature on the flow stress of superplastic material, no closed form of constitutive laws including effect of temperature is available. In this
study, contributions of deformation heating to flow stress and strain localization process will be discussed. Both $n$ and $m$ values in the constitutive relationship of SPZ1 are assumed to be constants in the current study although those values are known to depend strongly upon strain, strain rate, and temperature [142-145,147,148]. Because of the lack of information of constitutive relationship, the results of the current study is limited to examine qualitatively the effect of low flow stress and large $m$ value on material behavior during non-isothermal tensile testing.

Simulated results of engineering stress-strain curves for three heat transfer conditions are presented in Figs. 4.23a to 4.23c, showing effect of strain rate on flow stress. Because of its high $m$-value, the effect of strain rate is considerably larger than for conventional materials. Also, no strain hardening is observed since the material has a negative $n$ value. Maximum engineering stresses in the isothermal case are about 80, 30, 10 MPa for rates of $10^{-1}$, $10^{-2}$, and $10^{-3}$/s, respectively, showing quantitative agreement with the previous experiment [148]. Total elongation of SPZ1 is known to be about 800 [147] to 900% [148]. Little effect of heat transfer conditions is found because of its low flow stress. For the simulated
Figure 4.23. Effect of strain rates on simulated engineering stress-strain curves of SPZ1 for different heat transfer conditions: a) isothermal, b) in air, and c) adiabatic conditions.
Figure 4.23. (2/3, continued)
Figure 4.23. (3/3, continued)
test conducted in air, temperature over the specimen decreases after a certain increase at early stages of deformation, because a larger amount of energy is dissipated by conduction into grips and convection through air (Fig. 4.24). In the adiabatic case, the maximum temperature increase reaches about 50, 20, and 7°C for the rates of $10^{-1}$, $10^{-2}$ and $10^{-3}$/s, respectively, while showing no difference in stress-strain curves. In Figs. 4.25a to 4.25b, the variation of local strain rate at the center of the specimen and 25 mm away from the center is shown as a function of engineering strain. In the isothermal case, local strain rates are not affected by global strain rate, demonstrating the invariance of isothermal ductility [137]. In the non-isothermal case in air, strain localization occurs at an earlier stage as strain rate increases. This is a direct result of temperature gradient. However, deviation of local strain rate between global rates begins after greater than 200% elongation has been reached, where the flow stress has already dropped to less than a quarter of its maximum value. Little effect on global deformation mode is therefore expected.

Deformation-induced heating has little effect on the material behavior of the superplastic material because of
Figure 4.24. Change of maximum temperature increase at the center of specimen at rates of a) 0.1/s and b) 0.01/s.
Figure 4.24. (2/2, continued)
Figure 4.25. Variations of effective strain at specimen center and 25 mm away from the center as functions of engineering strain for a) isothermal and b) in-air cases.
Figure 4.25. (2/2, continued)
its extremely low flow stress and consequent small heat generation. As mentioned earlier, it may be important to include the effect of deformation heating in studying deformation of higher-strength superplastic materials because superplasticity strongly depends upon temperature and strain rate. The importance may increase with new discoveries of superplastic alloys such as low-alloy steels, which involve larger flow stresses. In order to fully understand the deformation behavior of superplastic materials, more attention should be paid to finding constitutive relationships.
4.6. Effect of Element Discretization and Remeshing

It is apparent during finite element analysis of forming problems that both the predicted load and the deformation strongly depend on finite element discretization. The choice of a proper initial finite element mesh becomes a very important aspect. The question of what kind of model provides the best solution is, however, yet unknown. In regards to this question, many studies have been done to investigate the effect of finite element discretization and to develop efficient mesh refinement methods [119-129]. Especially in metal forming problems, which involve large deformation, mesh refinement is very important to avoid possible inaccuracy due to large distortion of the initial mesh system. As mentioned in the previous chapter, subdivision of some elements, the so-called h-method, was examined for three different initial mesh systems (Fig. 4.26) in order to study the effect of element discretization and remeshing. An isothermal tensile test of I.F. steel at an initial strain rate of $10^{-1}/s$ was chosen as a model. Here Mesh A is the same mesh system used in the previous analysis (Fig. 4.2) and Mesh B is a modification of Mesh A so that the length of each element in x-direction has the same
Figure 4.26. Three FEM mesh systems used in analyzing effect of element discretization and remeshing: a) original mesh (Mesh A, Fig. 4.2), which was used in the previous chapters, b) regular mesh (Mesh B), which has the same boundary shape to Mesh A and uniform length of elements in x-direction within the gage length, and c) parallel specimen with uniform mesh (Mesh C).
dimension near the specimen center region. For comparison, a parallel specimen with uniform mesh (Mesh C) was also examined (Fig. 4.26c). The numbers of nodal points in each mesh system are 245, 217 and 245, respectively.

Variations of engineering stress-strain curves for different mesh systems are shown in Figs. 4.27a and 4.27b. The effect of initial mesh system on analytical results is clearly illustrated. As shown in the figures, strain localization tends to be impeded for large elements in high strain regions, while remeshing counteracts this tendency. Also, use of a parallel specimen deters the strain localization process, as observed by other researchers [42,43,151]. Without remeshing, total elongations of each mesh system are 44.7, 51.4, and 55.7%, respectively, showing considerable difference according to the choice of initial element discretization. It is not clear that the original mesh system is optimal, perhaps better solutions can be obtained by optimizing initial mesh system so that more localization can be included.

Figs. 4.28a to 4.28b show the effect of remeshing for original mesh system (Mesh A). In this case, remeshing does not affect stress-strain curves except in the final
Figure 4.27. Engineering stress-strain curves of I.F. steel for three mesh systems showing the effect of initial element discretization and remeshing: a) without remeshing and b) with remeshing. Dots on curves denote failure points defined by Eq. (4.3).
Figure 4.27. (2/2, continued)
Figure 4.28. Effect of remeshing for original mesh system (Mesh A) on a) engineering stress-strain relationship and b) local effective strains at the specimen center and 25 mm away from the center. X's in the figure denote the point where a row of elements was added to the initial mesh system.
Figure 4.28. (2/2, continued)
stages of computation. Considering the criterion of failure strain, which is given in Eq. (4.3), there is little effect of remeshing, which verifies the accuracy obtained using the original mesh. However, effective strain at specimen center increases with remeshing, which implies that more strain localization can be followed by adding more elements at the specimen center. With remeshing, the predicted effective strain at the center increases by 40% at 48% of engineering strain compared to the case without remeshing, even though it is beyond the failure strain, as defined by Eq. (4.3). Thus, it can be said that remeshing is important to studying strain localization processes even though the effect on the global response of material is small by its averaging effect. For more uniform mesh and/or uniform shape, effect of remeshing is more severe than the case for the original mesh. In Figs. 4.29 to 4.30, engineering stress and variation of effective strain at center and 25 mm away from the center are shown as functions of engineering strain for mesh system B and C, respectively. With the same criterion for failure strain, total elongation at failure point with remeshing procedure is about 3% less than that without remeshing for Mesh B. For the uniform specimen in Mesh C, this difference reaches 5%.
Figure 4.29. Effect of remeshing for regular mesh system (Mesh B) on a) engineering stress-strain relationship and b) local effective strains at the specimen center and 25 mm away from the center.
Figure 4.29. (2/2, continued)
Figure 4.30. Effect of remeshing for parallel specimen with uniform mesh (Mesh C) on a) engineering stress-strain relationship and b) local effective strains at the specimen center and 25 mm away from the center.
Figure 4.30. (2/2, continued)
In the analysis of actual forming processes, it is very difficult to predict the strain distribution and the optimal initial mesh system and, therefore, a uniform mesh is often used or modification of mesh is used solely according to experience, which may lead to some unacceptable results. Also, an increase of the number of elements from the beginning is accompanied by an increase of computing time and cost even though it usually provides better results. Therefore, it is highly desirable to use a remeshing technique in analyzing large deformation problems and in optimizing complicated mesh systems.
CHAPTER V.
CONCLUSIONS

Understanding sheet metal formability requires a broad knowledge of material properties and behavior under forming conditions. Many efforts have been made to explain material behavior in terms of material parameters and process parameters. It is, however, difficult to express material behavior in terms of one or several parameters because most parameters influence material behavior during forming operations in combined forms.

One of the important parameters influencing ductility is deformation heating by plastic deformation itself. Even though its importance has been acknowledged for a long time, few analytical studies have been performed to investigate the effect of deformation heating on material ductility during deformation because of the complexity of thermoplasticity problems. Only simplified one-dimensional analysis or "long wavelength" analysis has been reported. A two-dimensional finite element analysis
has been developed to study the effect of deformation heating in the current study. A modified Bishop's method was used to solve the thermoplasticity problem numerically. Using the present method, various factors affecting the material ductility and flow characteristics have been investigated. Based on simulation of sheet tensile tests of several materials, the following conclusions were drawn.

1) A modified version of Bishop's method can be used as a powerful tool for the analysis of thermoplasticity problems. Agreement between analysis and experiment is fairly good for most materials examined.

2) The temperature gradient during tensile testing has a detrimental effect on material ductility opposite to the stabilizing effect of strain-rate sensitivity and the temperature gradient effect becomes more severe after the onset of flow localization (necking). The temperature gradient in air increases with increasing testing speeds and approaches an adiabatic limit while deformation at relatively low strain rates (less than $10^{-1}$/s in most cases) approaches isothermal conditions.
3) For I.F. steel, uniform elongation drops about 0.1 to 2.7% at moderate rates \((10^{-4} \text{ to } 10^{-1}/s)\), while total elongation decreases about 0.3 to 6.0% during tests in air compared to the isothermal case. The predicted difference in uniform and total strain between isothermal and adiabatic cases is about 2.7 to 6.2%, respectively. This shows that the effect of deformation heating becomes more significant after necking occurs and increases with testing speed. These results agree with semi-analytical ones obtained by Gao and Wagoner [40,41] and Raghavan and Wagoner [42-44] and with experimental results by Lin and Wagoner [4-6] for this alloy.

4) The existence of heat sinks at the ends of the specimen does not significantly affect ductility, even though it slightly changes the temperature gradient near the ends, particularly at low rates.

5) Decrease of ductility by deformation heating becomes more significant for materials having large flow stresses. In the simulation of non-isothermal tensile tests of 304 stainless steel, whose flow stress is about three times that of I.F. steel, uniform and total elongation in the adiabatic case drop about 21 and 35%, respectively. On the other hand, the effect of deformation heating is
negligible for a zinc-based superplastic material, whose flow stress is quite small.

6) In order to verify the "near-invariance" of non-isothermal ductility of AK steel [3], the combined effect of deformation heating and strain-rate sensitivity, which depends on strain rate, was examined. Three constitutive equations, differing only in the variation of flow stress with strain rate, were employed to include variation of strain-rate sensitivity with strain rate. The equation ("Halfway Law") found applicable to the isothermal results of Wagoner and Wang [24] was also found to reproduce the non-isothermal results by Ayres [3]. This result demonstrates that the effect of thermal softening and strain-rate sensitivity happen to nearly offset each other for this alloy, resulting in a total elongation independent of strain rate.

7) In order to obtain better convergence of FEM solutions, several numerical techniques were examined. These include the treatment of elastic unloading and an automatic remeshing technique. Using these numerical techniques, solution convergence could be achieved to nearly the physical limit of the problems. The effect of finite element discretization was examined using an automatic
remeshing technique, which shows a strong dependence of solution upon the initial discretization. By employing automatic remeshing, a better solution as well as better numerical efficiency could be achieved.

It is essential to include the effect of deformation heating as well as strain rate in investigating material ductility. In actual metal working processes, where quite high forming speed is usually encountered and, consequently, the effect of deformation is large, better ductility can be obtained by controlling environmental heat transfer conditions or by using low forming speeds. Since there are a number of parameters influencing material ductility during actual processes, more study should be done to understand material properties in terms of their combined effects and application of finite element method would be a powerful tool in studying them.
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APPENDIX A.

FLOW CHART OF PROGRAM SHEET2D

In order to solve a two dimensional thermoplasticity problem by the finite element analysis, a FORTRAN program SHEET2D was developed. A concise presentation of the program is given here to aid discussion of numerical procedure.

The process flow chart is given in Fig. A.1, illustrating the overall computational procedures. Main computation of deformation and temperature is performed in subroutine STIFPL. In Fig. A.2, a modulized flow chart of subroutine STIFPL is given.
START

PREPROCESSOR : read material input data and original
    mesh geometry data.

MAIN PROCESSOR : main part - solve deformation and
    temperature distribution.
    repeat if remeshing is necessary.

ELDATA : compute element data (nodal connectivity,
    degree of freedom, area of element, boundary
    conditions, etc).

STIFIN : compute elastic solution for the initial
    guess of the first time step.

STIFPL : solve deformation and compute temperature.

MODRMD : interpolate quantities from old mesh to new
    mesh if remeshing is necessary.

POSTPROCESSOR : modify data for output display.

STOP

Figure A.1. Simplified flow chart of program SHEET2D.
For output display, commercial packages I-DEAS [138] and
DI-3000 [152] are used in conjunction with program.
START

LOOP OVER TIME STEPS

PLASTIC DEFORMATION

LOOP OVER ITERATION STEPS (Newton-Raphson method)
   compute element stiffness matrix and force vector.
   assemble to global stiffness equation.
   impose boundary conditions.
   solve for displacement increment.
   check convergence of solution.
   if not, update/modify initial guess and repeat the previous steps.

UPDATE GEOMETRY

TEMPERATURE PART
   compute element matrices.
   assemble to global matrices.
   apply time integration.
   apply boundary conditions.
   solve the equations and find temperature.

CHECK IF REMESHING IS NECESSARY
   if not, increase time and repeat the iteration for the next time step.
   if remeshing is necessary, store quantities for interpolation in module MODRMD, and repeat time loop.

RETURN

Figure A.2. Modular flow chart for subroutine STIFPL.
APPENDIX B.

FINITE ELEMENT FORMULATION OF
RIGID-VISCOPLASTIC IN-PLANE SHEET FORMING

B.1. Geometric definition

A body which fills a certain region of space \( A_1 \) at a given time \( t_1 \) is considered (Fig. 3.2). With a passage of time, the point \( P \) of \( A_1 \) undergoes displacement and at a certain time \( t \), the body fills a region \( A \). In order to describe the motion of the body, a convective coordinate system, \( \xi^\alpha \), is introduced which moves with the body in such a way that the coordinates \((\xi^1, \xi^2)\) of any given point \( P \) do not change with time \( t \). Then, the base vector \( g_\alpha \) and the metric tensor \( G_{\alpha\beta} \) at time \( t \) is given by

\[
\begin{align*}
g_\alpha &= \frac{\partial \mathbf{r}}{\partial \xi^\alpha} , \\
G_{\alpha\beta} &= g_\alpha^* g_\beta ,
\end{align*}
\]  

(B.1)
where the position vector \( \mathbf{r} \) denotes the position of the particle at time \( t \) relative to fixed orthogonal Cartesian coordinates \( X^\alpha (\alpha = 1,2) \). In analyzing the non-steady state deformation by a step-by-step procedure, the deformation during one step from \( t_1 \) to \( t_{1+1} (= t_1 + \Delta t) \) is considered. The vector \( \mathbf{r} \) denotes the position of a material point \( P \) at time \( t_1 \) with respect to the fixed Cartesian coordinates \( X^\alpha \). The base vector \( \mathbf{g}_\alpha \) and the metric tensor \( \mathbf{G}_{\alpha\beta} \) at time \( t_1 \) are given by

\[
\mathbf{g}_\alpha = \frac{\partial \mathbf{r}}{\partial X^\alpha}, \quad \mathbf{G}_{\alpha\beta} = \mathbf{g}_\alpha \cdot \mathbf{g}_\beta.
\]  

(B.2)

The displacement \( \mathbf{u} \) from \( t_1 \) to \( t \) is written as

\[
\mathbf{u} = \mathbf{r} - \mathbf{r} = \mathbf{u}_\alpha \mathbf{g}_\alpha.
\]  

(B.3)

The deformation during that time is characterized by Lagrangian strain tensor such as

\[
\mathbf{E}_{\alpha\beta} = \frac{1}{2} (\mathbf{C}_{\alpha\beta} - \mathbf{G}_{\alpha\beta})
\]

\[
= \frac{1}{2} (u_{\alpha,\beta} + u_{\beta,\alpha} + u_{\gamma,\alpha}u_{\gamma,\beta}).
\]  

(B.4)
where the comma signifies the covariant differentiation with respect to the metric tensor \( iG_{\alpha\beta} \) at time \( t_i \). The Lagrangian strain tensor \( i+1E \) at time \( t_i + \Delta t \) is expressed by the metric tensor \( i+1C_{\alpha\beta} \) at time \( t_i + \Delta t \) as

\[
i+1E_{\alpha\beta} = \frac{1}{2} (i+1C_{\alpha\beta} - iG_{\alpha\beta}) \quad . \quad (B.5)
\]

Two principal geometric strains at time \( t \) are defined as

\[
\epsilon_\alpha = \ln \lambda_\alpha \quad (\alpha = 1,2) \quad . \quad (B.6)
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the extension ratios defined by

\[
\lambda_{1,2} = \left[ \frac{1}{2} \left[ iG^{\alpha\beta} - iG_{\alpha\beta} \right] \pm \right]

\sqrt{\left( iG^{\alpha\beta} - iG_{\alpha\beta} \right)^2 - 4 iG / \sqrt{iG}} \right]^{1/2} \quad . \quad (B.7)
\]

where \( G \) and \( iG \) denote the determinants of the metric tensors \( G_{\alpha\beta} \) and \( iG_{\alpha\beta} \), respectively, and \( iG^{\alpha\beta} \) is the inverse of \( iG_{\alpha\beta} \). Similarly, the principal extension ratios between time \( t_i \) and \( t_i + \Delta t \) can be written in the form,
\[ i+1 \lambda_{1,2} = \left[ \frac{1}{2} \left[ iG^{\alpha\beta} \cdot i+1G_{\alpha\beta} \pm \sqrt{(iG^{\alpha\beta} \cdot i+1G_{\alpha\beta})^2 - 4 \frac{i+1G}{iG}} \right] \right]^{1/2} . \quad (B.8) \]

From Eq. (3.15), the incremental effective strain, \( \Delta \varepsilon \) can be expressed in terms of metric tensor as

\[ \Delta \varepsilon = D_1 \left[ (\zeta_1^2)^{2(M-1)} + D_2 (\zeta_2^2)^{2(M-1)} \right]^{M-1} / M . \quad (B.9) \]

where

\[ \zeta_1 = \Delta \varepsilon_1 + \Delta \varepsilon_2 = \frac{1}{2} \ln \phi_2 \]

\[ \zeta_2 = \Delta \varepsilon_1 - \Delta \varepsilon_2 = \frac{1}{2} \ln \left[ \frac{\phi_1 + (\phi_1^2 - 4 \phi_2)^{1/2}}{\phi_1 - (\phi_1^2 - 4 \phi_2)^{1/2}} \right] . \]

and \( \phi_1 = iG^{\alpha\beta} \cdot i+1G_{\alpha\beta} \), \( \phi_2 = i+1G/iG \).
B.2. Finite Element Discretization

Let the domain be discretized into a number of finite elements and \( \Delta u \) be the nodal displacement increment vector from \( t_1 \) to \( t_1 + \Delta t \) at each nodal point. The effective stress \( \bar{\sigma} \) is assumed only to be a given function of the current effective strain \( \bar{\varepsilon} \) and its rate \( \dot{\bar{\varepsilon}} \), say

\[
\bar{\sigma} = \bar{\sigma}(\bar{\varepsilon}, \dot{\bar{\varepsilon}}).
\]  

(B.10)

In fact, effective stress is a function of temperature as well as strain and strain rate. At this stage, however, effect of temperature can be excluded because the temperature problem will be solved in a decoupled form. The effective strain rate \( \dot{\bar{\varepsilon}} \) in Eq. (B.10) permits the dependency of material deformation on rate hardening effect to be included.

When the proper definition of effective strain during deformation, in Eq. (B.9), and the constitutive relationship, in Eq. (B.10), are provided, Eq. (3.12) can be explicitly expressed as follows:
\[ K^T = \sum_{e=1}^{N} K^T(e) \]
\[ f = \sum_{e=1}^{N} f(e) \quad \text{(B.11)} \]
\[ R = \sum_{e=1}^{N} R(e) \]

where \((e)\) denotes element, \(N\) is the number of elements over the domain, and summation symbol implies assembly procedure of element matrix and vectors. And \(K^T(e)\), \(f(e)\), and \(R(e)\) are written in their component form as

\[
K^T(e)_{ij} = A_o \int_{A_o(e)} \left[ \bar{\sigma} \frac{\partial^2 (\Delta \bar{e})}{\partial \Delta u_i \partial \Delta u_j} + \left( \frac{\partial \bar{\sigma}}{\partial \bar{e}} + \frac{1}{\Delta t} \frac{\partial \bar{\sigma}}{\partial \varepsilon} \right) \left( \frac{\partial \Delta \bar{e}}{\partial \Delta u_i} \right) \left( \frac{\partial \Delta \bar{e}}{\partial \Delta u_j} \right) \right] dA_o .
\]
\[
f(e)_{ij} = A_o \int_{A_o(e)} T_j dA_o .
\]
\[
R(e)_{ij} = A_o \int_{A_o(e)} \bar{\sigma} \frac{\partial \Delta \bar{e}}{\partial \Delta u_j} dA_o .
\]

where \(A_o\), \(A_o(e)\) are element thickness and element area in
the original configuration. Here, all quantities are computed at \( \Delta u = \Delta u^0 \) and \( \Delta \varepsilon = \Delta \varepsilon^0 \) and effective stress is approximated as \( \bar{\sigma} = \bar{\sigma}(\bar{\varepsilon}, \bar{\varepsilon}^0) \approx \bar{\sigma}(\bar{\varepsilon}_1 + \Delta \varepsilon, \frac{\Delta \varepsilon}{\Delta t}) \).
B.3. Definition of Deformation in Linear Triangular Elements

Let \((X^1, X^2)\) with base vector \(e_1\) and \((\xi^1, \xi^2)\) with base vector \(g_1\) be the global and local coordinate system, respectively, (Fig. B.1). Then the base vectors of the local coordinate system are chosen as sides of a triangular element, which can be expressed in terms of the global system as

\[
\begin{align*}
g_1 &= R_{12} = (X_2 - X_1) e_1 + (Y_2 - Y_1) e_2, \\
g_2 &= R_{13} = (X_3 - X_1) e_1 + (Y_3 - Y_1) e_2. 
\end{align*}
\]  

(B.13)

where \(R_{12}\) and \(R_{13}\) are position vectors of nodal points, 2 and 3, respectively. Then, the metric tensor at time \(t_i\) will be

\[
{\bar{G}}_{\alpha\beta} = {\bar{g}}_\alpha \cdot {\bar{g}}_\beta
\]

(B.14)

\[
= \begin{bmatrix}
1_3^2 & \frac{1}{2} (1_2^2 + 1_3^2 - 1_1^2) \\
symm & 1_2^2
\end{bmatrix}
\]
Figure B.1. Change of local coordinate system of a triangular element.
where \( l_1 = |R_{23}|, l_2 = |R_{13}|, \) and \( l_3 = |R_{12}| \) at time \( t_1 \).

Similarly, the metric tensor at time \( t_{1+\Delta t} \) can be expressed in terms of lengths \( L_i \) of the sides of the element at time \( t_{1+\Delta t} \) as

\[
\gamma_{a\beta}^{i+1} = \gamma_{\alpha\beta}^{i+1} \cdot \gamma_{\beta \gamma}^{i+1}
\]

\[
= \begin{bmatrix}
L_3^2 & \frac{1}{2} (L_2^2 + L_3^2 - L_1^2) \\
\text{symm} & L_2^2
\end{bmatrix}
\]  

Once the metric tensor is defined as in Eqs. (B.14) and (B.15) for each triangular element at a certain time, the terms in Eq. (B.12) can be easily computed by the previous definition. Similar expressions can be found elsewhere [71,72].