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Kim, Young Min

A TWO-DIMENSIONAL CHARGE CONTROL MODEL AND AN ANALYTIC CAD MODEL FOR MODFETS

The Ohio State University  Ph.D.  1986

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A Two-Dimensional Charge Control Model and An Analytic CAD Model for MODFETs

A Dissertation
Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

by

Young Min Kim, B.S.E.E., M.S.E.E.

* * * * *

The Ohio State University
1986

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LIST OF SYMBOLS

$\Delta E_c$: Conduction band discontinuity at the heterointerface

$\phi_m$: Schottky-barrier height

$d_d$: Thickness of doped AlGaAs

$d_i$: Thickness of undoped AlGaAs

$d = d_d + d_i$

$d_c$: 2-DEG channel thickness

$d_s$: 2-DEG channel thickness at the saturated section

$L_s$: Source-to-gate spacing

$L_D$: Drain-to-gate spacing

$L$: Gate length

$\Delta L$: Channel length of the saturated section

$\Delta L'$: Channel length of the transition section

$W$: Gate width

$L'$: Length of GCA section after saturation

$\delta_s$: Length of lateral depletion in AlGaAs due to Schottky-barrier gate at the source side

$\delta_D$: Length of lateral depletion in AlGaAs due to Schottky-barrier gate at the drain side

$K_0$: Dielectric constant in AlGaAs

$K_s$: Dielectric constant in GaAs
<table>
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<tr>
<td>$\epsilon_0$</td>
<td>Permittivity in vacuum</td>
</tr>
<tr>
<td>$E_F$</td>
<td>(Quasi) Fermi-level</td>
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<tr>
<td>$E_{Fi}$</td>
<td>Chemical energy</td>
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<tr>
<td>$V_c$</td>
<td>Channel voltage</td>
</tr>
<tr>
<td>$V_{th}$</td>
<td>Threshold voltage</td>
</tr>
<tr>
<td>$V_{off}$</td>
<td>$V_G - V_{th}$</td>
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<td>$V_G$</td>
<td>Gate voltage</td>
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<td>$\epsilon_c$</td>
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<tr>
<td>$\epsilon$</td>
<td>Parallel electric field</td>
</tr>
<tr>
<td>$\epsilon_{eq}$</td>
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<td>$\epsilon_{cp}$</td>
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<td>$\mu$</td>
<td>Field-dependent mobility</td>
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<td>$\mu_0$</td>
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<tr>
<td>$\mu_0'$</td>
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<tr>
<td>$R_S$</td>
<td>Source resistance</td>
</tr>
<tr>
<td>$R_D$</td>
<td>Drain resistance</td>
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<tr>
<td>$R_c(x)$</td>
<td>Channel resistance from $x = -\delta_e$ to $x$</td>
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\( R_{ct}(x) \): Channel resistance for the gated channel

\( I \): Drain current

\( I_p \): Channel current when \( V_c = V_p \)

\( f \): Transition-section correction factor

\( g \): The factor determining \( \varepsilon_p \)

\( z \): Gate width

\( T \): Room temperature

\( T_0 \): Semiconductor lattice temperature

\( T_e \): Electron temperature

\( v_s \): Electron saturation velocity

\( D \): Diffusion constant

\( D_2 \): 2-DEG density of states

\( \gamma_0 \): Proportionality constant relating the energy level of the first subband of the 2-DEG quantum well to the 2-DEG concentration

\( \gamma_1 \): Proportionality constant relating the energy level of the second subband of the 2-DEG quantum well to the 2-DEG concentration

\( j \): Current density

\( n \): 3-DEG concentration

\( \tau \): Relaxation time

\( k \): Boltzmann constant

\( m^* \): Electron effective mass

\( N_c \): Density of state in AlGaAs

\( E_d \): Donor activation energy

\( N_D \): Donor doping level in AlGaAs

\( N_A \): Unintentional acceptor doping level in GaAs
<table>
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<td>$\varepsilon_2$</td>
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<td>Net electron velocity along the channel</td>
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<tr>
<td>$v_d$</td>
<td>Electron drift velocity</td>
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<td>$v_{d,eq}$</td>
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<td>$n_s$</td>
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<td>$E_0$</td>
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<td>$C'_{GD}$</td>
<td>Intrinsic gate-to-drain capacitance</td>
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CHAPTER I
INTRODUCTION

1.1 Overview

The mobility enhancement of electrons in a modulation-doped multilayered-structure first reported by Dingle et al.[1] has stimulated considerable interest in utilizing such structures to fabricate high-speed low-power devices. Recently Mimura et al. [2] reported a new field-effect transistor fabricated on a modulation-doped \( n - Al_xGaAs_{1-x}/GaAs \) heterojunction. They obtained a value of mobility 30\% higher at room temperature and 5 or 6 times higher at 77 K than that of GaAs MESFET. The transistor I-V characteristics showed 3 times higher transconductance than that of GaAs MESFET. Almost at the same time, modulation-doped heterojunction transistors that have similar electrical performances were reported by N. T. Linh et al. [3] at Thomson CSF Co. and Morkoc at al. at University of Illinois. This device was given various names such as MODFET( Modulation-Doped Field-Effect Transistor), HEMT( High Electron Mobility Transistor), and TEGFET( Two-Dimensional Electron-Gas Field-Effect Transistor).

Continuous improvements were made on device performance by introducing appropriate processing techniques and process-parameter optimization. In single-period selectively-doped or modulation-doped heterostructures with 30\% mole fraction of AlAs in the AlGaAs composition, electron mobility as high as 9,090, 136,000 and 286,000 \( cm^2/Vs \) at 300 K, 78 K and 10 K respectively were obtained by
the University of Illinois group. They showed that multiple-period structures grown at low temperature exhibited low-field mobility comparable to single-period structures, but produced negative differential resistance prematurely at and above electric fields of 1 KV/cm [4]. Single-period structures still appear to be superior to multiple period ones, in terms not only of ease of preparation but also of high-field transport properties [7]. The effect of AlAs mole fraction \(x\) on the mobility was studied by Drummond et al. [5], where it was proved that \(x \approx 0.3\) was the optimum value for all temperature ranges.

Also, a study was performed to determine the performance of MODFETs with inverted structure, where GaAs was grown on top of AlGaAs [6]. Though this device has better saturation characteristics (higher output resistances) and is easier to fabricate than normal MODFETs, where the Schottky gate is formed on GaAs instead of AlGaAs, it shows lower transconductance of about 70 mS/mm than that of the normal structure at room temperature.

Improvement of low-field mobility was obtained by inserting an undoped AlGaAs layer between the n-AlGaAs/GaAs interface to reduce the Coulomb interaction between the parent donors in the n-AlGaAs and the electrons in the GaAs that are transferred from the AlGaAs [8]. An undoped-AlGaAs thickness of 60 Å was found to be optimal at 300 K.

For short-channel FET's, the saturation velocity plays a major role in the I-V characteristics compared to the low-field mobility. In bulk GaAs, the peak electron velocity is about \(2 \times 10^7\) cm/s and occurs at an electric field strength of about \(3.5 \times 10^3\) V/cm at room temperature. However, the effective saturation velocity of a GaAs FET channel is in the range of \(1.3 \times 10^7\) to \(1.7 \times 10^7\) cm/s [9], [10], which implies that the GaAs FET outperforms its Si counterpart by more than a factor of 2, considering that the saturation velocity of Si is about \(6.5 \times 10^6\) cm/s. More
detailed study on two-dimensional electron gas (2-DEG) transport was performed by Drummond et al. [11]. Electron transport in modulation-doped AlGaAs heterostructures agrees well with electron transport in undoped bulk GaAs, according to his Monte Carlo simulation. At 300 K, an electron velocity of $1.7 \times 10^7$ cm/s at electric field of 2 K V/cm was measured, in contrast to $1 \times 10^7$ cm/s in bulk GaAs. Considering the switching time of less than 20 ps, this value is smaller than would be expected. Another study shows that the saturation velocity can be as high as $2 \times 10^7$ cm/s at 300 K and $3 \times 10^7$ cm/s at 77 K. The transconductance obtained from a normally-off MODFET (1 μm gate) prepared by Drummond et al., exceeds 225 mS/mm at 77 K leading to intrinsic transconductances of 305 mS/mm and 565 mS/mm at 300 K and 77 K respectively.

Fabrication methods that are used for quality MODFETs rely heavily on the molecular beam epitaxy (MBE) system. Substrates commonly used are semi-insulating (100) GaAs wafers prepared by etching in $H_2SO_4 : H_2O_2 : H_2O$ solution followed by a $H_2O$ rinse. Then, the resulting surface oxide is removed by in-situ thermal desorption prior to film growth. On top of this substrate, a 1 μm thick undoped p-type GaAs buffer layer is grown and covered by 60 A of undoped $Al_{0.3}Ga_{0.7}As$, and again covered by a 600 A of Si-doped $Al_{0.3}Ga_{0.7}As$ layer. Gate recessing is done by thinning the entire AlGaAs layer to the appropriate thickness needed to control the 2-DEG channel using a solution of $H_2O_2 : NH_4OH : H_2O$. Then, mesa etching for device isolation, ohmic-metal (Ni/AuGe/Ge) deposition and alloying for source and drain ohmic contacts follow. Finally, after etching the AlGaAs for the gate region, Al or Ti/Pt/Au is evaporated onto this region for a Schottky-barrier gate contact.
1.2 Research Objectives

In spite of the fast improvement of HEMT performance, proper models describing the physics of HEMT operation have not been well developed and are essential for further device optimization. The major difficulties that are faced in modeling HEMT devices come from the fact that submicron short-channel effects are involved, which complicate electron transport in the channel and consequently causes hot-electron effects. Since the channel electrons are strongly confined within the quantum well formed by the internal electric field accross the AlGaAs/GaAs interface, quantum-mechanical treatment with degenerate Fermi-Dirac statistics should be used for determination of the electron distribution.

The first 1-dimensional DC model for HEMTs was performed by Delage-beaudeuf et al. [12]. They developed a formulation of the 2-DEG charge-control mechanism and applied it to a 2-piece current-controlled model. In order to calculate the equilibrium 2-DEG concentration in the channel, they introduced a new self-consistent formula for the 2-DEG concentration using Stern's approach [17] which approximates the 2-DEG quantum well as a triangular potential well.

Further improvement of this model was carried out by Lee et al. [13], where the dependence of carrier concentration on Fermi-level was considered. Even though this model produces better estimation of the total current, the resulting I-V curve is piecewise in nature and consequently produces an abrupt current transition near the saturation voltage of the I-V characteristics. A 3-piece mobility-model has also been tried [14] to improve the knee region of the I-V curves, which however still remains as a piecewise-linear I-V model. Smooth transition of I-V curves near the pinch-off region was obtained by Weiler et al.[15], who adopted a continuous field-dependent mobility often used for Si. They adopted a 2-DEG formula without
linearization with respect to the 2-DEG concentration in order to include the near-threshold region of $n_s$ vs $V_G$ curve. Since a strong nonlinearity is present, they had to solve the current equation numerically.

On the other hand, a two-dimensional transient simulation of the HEMT was conducted by Widiger et al. [16], where both hot-electron effects and current conduction outside the quantum-well subband system were treated rigorously using Boltzmann moment equations in the form of Stratton [17]. They introduced a mechanism of coupling between the 2-DEG and 3-dimensional bulk charge that are present in the well together and showed the importance of the 3-dimensional gas neglected by most 1-dimensional models. Even though this simulation shows interesting phenomena, such as the distribution of 2-dimensional and 3-dimensional gas and the electron-velocity distribution along the channel, the doping level for the AlGaAs region used in this simulation was too low to be practical. Consequently, the results could not give significant meaning to the operation of real HEMTs. Charge control by the gate should be strictly limited to the region near the gate so that the electron concentration in the ungated region should return to its near-equilibrium state when the doping level for AlGaAs is set to the value practically used, which is about $10^{18}$ cm$^{-3}$.

Recently a two-dimensional Monte Carlo simulation of the HEMT was performed by a group in France [18], which gives physical insight both into the channel characteristics and the way I-V characteristics are affected by geometrical parameters such as the aspect ratio. The possibility of real-space transfer from the channel to the AlGaAs was studied for cases of both 0.5 $\mu$m and 1 $\mu$m gate lengths. This simulation result gives interesting information about channel profiles, such as normal and parallel electric field components along the channel. They concluded that in 1 $\mu$m devices the saturation mechanism of the I-V curves was due to channel
electron pinch off and that there was no significant real-space transfer of electrons. On the contrary, for the 0.5 μm gate devices real-space transfer of hot electrons could not be neglected when the device is operating in the saturation regime. However, this simulation suffers from using the 3-dimensional gas approximation for the 2-DEG transport, which is especially important in quantum-well devices such as HEMTs. Moreover, they used a low doping level for the AlGaAs region as did Widiger.

Recently a velocity-field relationship good only for bulk GaAs was tried in 2-dimensional HEMT simulation by J. Yoshida et al. [19]. In this work, a velocity-overshoot effect was found to cause a 50% increase of the HEMT's cutoff frequency and that a quasi piecewise-linear velocity-field curve gives reasonable approximation to the results of the Monte Carlo simulation. A drawback of this simulation is the classical treatment of electrons in the electron transport equations. Furthermore, the strict application of velocity overshoot valid only for long channel GaAs devices generates a negative-resistance region on the I-V curves, which has not been observed in short-channel HEMTs.

Recently there has been another study on a 1-dimensional model at OSU extending the 2-DEG concentration versus gate voltage relations into both the subthreshold region and the parasitic MESFET region of I-V curves [20]. This model also uses a quasi piecewise-linear velocity-field relationship and predicts the behavior of HEMTs in the parasitic and subthreshold region of the I-V curves better than previous 1-dimensional models. Moreover, it gives finite output conductance. However, this model also does not produce smooth transitions near the saturation voltage of the I-V curves.

In spite of all these works, there are still basic problems that are not clear and should be solved. One important issue is the determination of the role of the 2-DEG
in HEMT transport and the degree of its contribution to the total current. Even though the effect of the 2-DEG on the I-V characteristics has been incorporated in most 1-dimensional models based on linearization of the 2-DEG concentration formula using the triangular potential-well approximation, the nonlinear portion (the tail of the curves relating the 2-DEG concentration to chemical potential) of the formula is very important for pinch off, saturation and subthreshold regions of the I-V curves. Therefore, this effect should be carefully analyzed.

A second issue is to analyze the mechanism of current saturation. Some of the 1-dimensional models of HEMTs assume that saturation current is directly determined by the velocity saturation of electrons and the onset of saturation is determined by the critical field. Even though this is not a bad approximation, the resulting I-V curves become very sensitive to the critical field which is quite arbitrary in nature. Monte Carlo simulation [18] claims that for 1 \( \mu m \) devices, current saturation is due to the pinch off of electrons at the drain side edge of the gated channel like the case of long channel MOSFET. This argument is obviously wrong because the electric field normal to the heterointerface alone cannot represent the total electron concentration along the channel for nonequilibrium, especially in the saturation region where high parallel electric field is built up. In this region, electron concentration is related to the gradient of the parallel electric field. In other words, the fact that the electric field normal to the heterointerface is zero does not necessarily ensure that the total electron concentration is zero.

A third issue is the diffusion effect and its relation to channel opening. Since most 1-dimensional models assume that there is constant channel opening along the gradual channel approximation (GCA) region and its abrupt transition to the saturation region where the degree of channel opening can not be well defined, the diffusion effect which has a strong relation to the channel opening has not been
properly treated. Also, some 2-dimensional simulations neglecting the quantum features of the channel and treating electrons classically, are in error because the 2-DEG diffusion effects were neglected.

A fourth issue is the hot-electron effect. Since in this study we are dealing with short-channel HEMTs, the feature size of the gate length is on the order of 1 \( \mu \text{m} \). This short-channel device is obviously operating in the hot-electron range, and it is therefore hard to model with a simple form. Since an approach using moments of the Boltzmann transport equation as done by Widiger [16], seems to be too involved to reveal the behavior of the 2-DEG transport clearly, we choose an approach using phenomenological expressions such as the velocity-field relationship. This will allow us to formulate the 2-DEG transport in a simpler way.

The last issue is to extract concepts of HEMT operation and to gain ideas for implementation of future CAD models. Since the 1-dimensional models reported so far have inaccuracies due to usage of many parameters chosen arbitrarily for fitting I-V data, there should be comparisons between various models. In this regard, a proper 1-dimensional CAD model including most of the important device physics is sought while maintaining analytic formulation.
CHAPTER II
HOT-ELECTRON EFFECTS

2.1 A review of Hot-Electron Effects in Bulk Semiconductor

The drift velocity of electrons in a semiconductor is proportional to the applied electric field only when the applied electric field is small and the distribution of electric field inside the material is constant. In practice, at room temperature appreciable degradation of low-field mobility requires a strong field of a few KV/cm for Si or GaAs. The reason for mobility degradation is the increase of the average electron kinetic energy above the zero field, i.e., the thermodynamic equilibrium value $\frac{3}{2}kT_0$. ($k$ is Boltzmann’s constant and $T_0$ is the lattice temperature for non-degenerate semiconductors). Thus, if the field-dependent average kinetic energy is expressed as $\frac{3}{2}kT_e$, then the electron temperature $T_e$ is greater than $T_0$ and tends to $T_0$ in the limit as $\varepsilon$ tends to zero, where $\varepsilon$ denotes the parallel electric field. The difference between $T_e$ and $T_0$ is determined by the condition that in a steady current-carrying state the rate at which energy is supplied by the electric field to the electrons must equal the rate at which energy is transferred from the electrons to the lattice with electron-lattice collisions. The electron mobility will be a function of $\varepsilon$ through its dependence on $T_e$.

Most of the existing experimental and theoretical works are concerned with the hot-electron effect when a uniform field is applied to a homogeneous semiconductor. The current density is then proportional to the density of electrons and their
drift velocity in the applied field. However, in structures which involve potential
barriers, there will be an additional contribution to the current density, namely the
diffusion current, which involves the spatial gradient of the electron density. In the
usual theory where the hot-electron effect is neglected and electron temperature
is assumed to be equal to the lattice temperature (this will be referred to as the
thermal equilibrium approximation or TEA), the one-dimensional current density
$j$ is

$$j = q\mu e + qD \frac{dn}{dx}$$

(2.1)

where $n$ is the electron density, $\mu$ is the mobility, $D$ is the diffusion constant, $q$ is
the charge on the electron, and $j$ is in the positive $x$ direction. In the TEA and
for nondegenerate semiconductors, it can be shown quite generally that $D$ and $\mu
are connected by Einstein’s relation,

$$\frac{D}{\mu} = \frac{kT_0}{q}$$

(2.2)

A question that can be raised is whether the TEA is valid when Eq. (2.1) is
applied to the barrier problem where the built-in electric fields are comparable in
magnitude to the uniform fields when there is an appreciable hot-electron effect.
It has been argued in the past that since $\mu$ is already known as a function of
$\epsilon$ from hot-electron studies, $D$ also has a similar relationship with $\epsilon$. However,
this argument is obviously wrong, since as current $j$ tends to zero the electron
temperature must everywhere equal the lattice temperature and $\mu$ and $D$ must
equal their thermal equilibrium values, i.e., they are independent of the local value
of $\epsilon$. One way of solving this problem is to treat $\mu$ and $D$ as functions of the
electron temperature $T_e$. Then in the barrier region, $T_e$ is not a unique function

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of the local field $\varepsilon$ but is determined by the condition of conservation of energy in such a way that $T_e$ tends to $T_0$ when $j$ tends to zero. This condition can be established by the current density equation given by

$$j = qn\mu(T_e)\varepsilon + q \frac{d[nD(T_e)]}{dx}$$  \hspace{1cm} (2.3)$$

when $\mu(T_e)$ and $D(T_e)$ are known functions of $T_e$, determined from the Boltzmann transport equation for particular electron-scattering mechanisms. This equation also applies for a non-Maxwellian form of the distribution function $f_0$ and in this case $D$ and $\mu$ are defined in terms of integrals involving $f_0$. From Boltzmann's steady-state kinetic equation for the electron distribution function $f(p, x)$ in momentum $p$ and coordinate $x$ space, the Boltzmann equation becomes

$$q\varepsilon \frac{\partial f}{\partial p_x} - \frac{p_x}{m} \frac{\partial f}{\partial x} + \left( \frac{\partial f}{\partial t} \right)_e = 0$$ \hspace{1cm} (2.4)$$

where the first term is due to the field $\varepsilon$, the second is due to diffusion ($m$ is the effective electron mass in the usual parabolic band model), and the third represents the effect of electron-phonon collisions. In this case, other scattering mechanisms such as electron-hole generation and recombination are not considered.

When we expand $f(p)$ in spherical harmonics

$$f(p, x) = \sum_{\nu=0}^{\infty} f_{\nu}(E, x)P_{\nu}(\cos \theta)$$ \hspace{1cm} (2.5)$$

where $\theta$ is the angle between $p$ and the $x$ axis, $P_{\nu}(\cos \theta)$ are the Legendre polynomials, and only the first two terms are considered important. Substitution of Eq.(2.5) into Eq. (2.4) gives

$$\frac{q\varepsilon p}{m} \frac{\partial f_0}{\partial E} - \frac{p}{m} \frac{\partial f_0}{\partial x} - \frac{f_1(E)}{\tau(E)} = 0$$ \hspace{1cm} (2.6)$$
for $P_1(\cos \theta)$ terms. The particular form of the collision term, which involves the relaxation time $\tau(E)$, only occurs for elastic collisions. Substituting $f_1(E)$ into the expression for the current density

$$j = -\left(\frac{q}{m}\right) \int p_x f(p) d^3p = -\frac{q}{3m} \int p f_1(p) d^3p$$

or

$$j = qn \mu(T_e) e + q \frac{d[nD(T_e)]}{dx}$$

where

$$n = \int f_0 d^3p$$

$$\mu = -\frac{2}{3m} \int_0^\infty \frac{\tau \frac{\partial f_0}{\partial E}}{f_0 E^{3/2} dE}$$

$$D = \frac{2}{3m} \int_0^\infty \tau E^{3/2} dE$$

and Einstein’s relation is generalized to

$$\lambda = \frac{qD}{\mu} = \frac{\int_0^\infty f_0 \tau E^{3/2} dE}{\int_0^\infty \tau \frac{\partial f_0}{\partial E} E^{3/2} dE}$$

or

$$\lambda = \frac{2}{3} <\frac{\tau E}{1 + \frac{2}{3} \frac{d \ln \tau}{d \ln E}} >$$

where the angular bracket is defined as

$$<G(E)> = \frac{\int_0^\infty f_0(E) G(E) E^{1/2} dE}{\int_0^\infty f_0(E) E^{1/2} dE}$$

When the Maxwellian distribution function for $f_0$ is used, i.e.
\[ f_0 = n \hbar^3 (2\pi m k T_e)^{-3/2} \exp\left(-\frac{E}{k T_e}\right) \quad (2.15) \]

where \( n \) and \( T_e \) depend on \( x \) but not on \( E \), then

\[ \lambda = \frac{q D}{\mu} = k T_e \quad (2.16) \]

For evaluation of the electron temperature, let \( nB(T_e) \) be the rate at which electrons lose energy to the lattice by electron-phonon collisions and \( S(T_e) \) be the flux of energy in the positive \( x \) direction. Then conservation of energy for electrons requires that

\[ j \varepsilon = nB(T_e) + \frac{dS(T_e)}{dx} \quad (2.17) \]

where

\[ S(T_e) = -\kappa(T_e) \frac{dT_e}{dx} - \frac{j}{q} \delta(T_e) k T_e \quad (2.18) \]

\( \kappa(T_e) \) is the thermal conductivity of electrons and \( \delta(T_e) k T_e \) is the average kinetic energy transported per electron arising from the current. Then Eqs. (2.17) and (2.18) can be combined to give

\[ \frac{j}{q} \delta(T_e) k \frac{dT_e}{dx} + nB(T_e) = j \varepsilon \quad (2.19) \]

where the \( \frac{d^2 T_e}{dx^2} \) term has been neglected.

For materials like Si and Ge, Conwell [26] has shown that the assumption of a Maxwellian distribution at temperature \( T_e \) for the symmetrical part of the electron distribution function yields results in good agreement with experiment. The results for the transport coefficients may be written in the form

\[ \mu(T_e) = \mu_0 \left(\frac{T_0}{T_e}\right)^{\frac{3}{2}} \quad (2.20) \]
Equations (2.20) and (2.21) apply in any region of a semiconductor material or device, not just in homogeneous regions. Values for $\mu_0$ and $v_{sat}$ may be obtained by fitting data to the experimental results, which was done by Haung [27] and is derived below. Using the momentum equation of Stratton [17], i.e., Eq. (2.3) and energy equation, i.e., Eq. (2.19), Poisson's equation can be solved in principle to yield profiles of $n$, $E$, and $T_e$ throughout any device. However, except in the simplest cases numerical methods are required for the solution. When the channel electric field is assumed constant everywhere in the device, a simultaneous solution to Eqs. (2.3) and (2.16) can be obtained. The energy equation must then express conservation of energy for that electron at every point $x$ in space. Thus the current

$$j = nev$$

(2.22)

is replaced by simply $ev$. If we further neglect the contribution to $v$ from gradients in electron temperature for the purposes of the energy equation, i.e., Eq. (2.16), we can write

$$v \approx \mu e$$

(2.23)

Substituting Eqs. (2.22) and (2.23) into Eq. (2.19), we get

$$2\mu(T_e)e\kappa \frac{dT_e}{dx} + B(T_e) = q\mu(T_e)e^2$$

(2.24)

On the other hand, assuming a Maxwellian distribution, Eq. (2.3) can be written using Eq. (2.16) as follows
\[ qnv = qnue - q \left( \frac{d[n\nu kT_e/q]}{q} \right) \]  

(2.25)

If in addition to the "single electron" approximation, \( \mu \) varies slowly with position in those regions where the \( \frac{d}{dx} \) term is important, we have

\[ qv \approx que - \mu \frac{d[kT_e]}{dx} \]  

(2.26)

Since we know that

\[ \frac{1}{2} m v^2 \approx kT_e \]  

(2.27)

substitution of Eq. (2.27) into Eq. (2.26) leads to

\[ mv \frac{dv}{dx} + \frac{qv}{\mu(T_e)} = qe \]  

(2.28)

Setting \( \frac{dv}{dx} = 0 \) in Eq. (2.28), i.e., assuming an infinite semiconductor, the reduced momentum and energy equation reduces to

\[ v = \mu(T_e)e \]  

(2.29)

and using Eq. (2.20)

\[ v = \mu_0 \left( \frac{T_0}{T_e} \right)^{1/2} e \]  

(2.30)

Now, Eq. (2.21) becomes

\[ B(T_e) = q\mu(T_e)e^2 \]  

(2.31)

Inserting Eq. (2.31) into Eq. (2.21) results in
Substituting Eq. (2.32) into Eq. (2.30) yields

\[ v = \frac{\mu_0 \varepsilon}{\sqrt{1 + (\mu_0 \varepsilon/v_{sat})^2}} \]  

(2.33)

Since this expression now contains only \( \varepsilon \), and not \( T_e \), Eq. (2.33) defines the velocity-field characteristics. Then the field-dependent mobility can be generalized as

\[ \mu(\varepsilon) = \frac{\mu_0}{(1 + (\varepsilon/\varepsilon_{cr})^\beta)^{1/\beta}} \]  

(2.34)

where \( \beta = 2 \) and \( \varepsilon_{cr} = v_{sat}/\mu_0 \).

2.2 Hot-Electron Effect in HEMTs

The hot-electron effect for HEMTs has a unique feature which is quite different from that of the bulk semiconductor discussed so far. The consideration that should be given to the hot-electron effects for HEMTs is on the potential barrier regions which form at both edges of the gated channel. When gate voltage is applied, the electron concentration in the gated channel will be decreased and therefore there will be an electron concentration gradient at the edges of the channel. The diffusion current caused by this concentration gradient should be compensated by the drift current originated by the built-in field due to the potential barriers in order to maintain zero drain current. This built-in field might be high enough to cause severe mobility degradation (which will later be shown to be true by the 2-dimensional charge control simulation). However this internal field should not cause electron heating, so that the mobility in this region should not
depart from the zero field value. Therefore, Eq. (2.34) can not be directly applied in the potential barrier region. The solution to this problem can be obtained by defining a new velocity-field relationship such as

\[ \mu = \frac{\mu_0}{1 + \left( \frac{\varepsilon - \varepsilon_{eq}}{\varepsilon_{cr}} \right)^{1/\beta}} \]  

(2.35)

where \( \varepsilon \) is the total parallel electric field, \( \varepsilon_{eq} \) is the equilibrium built-in field and \( \varepsilon_{cr} \) is the critical field. Actually \( \varepsilon_{eq} \) is equivalent to the equilibrium diffusion force at the potential barrier for a given gate voltage. Any electric field applied externally along the channel will be superimposed on the built-in electric field and therefore, will cause mobility degradation in this region. This field-dependent mobility model is valid not only at the potential barrier but also at the normal channel region where the built-in field is zero.

The second important consideration for HEMT transport is the diffusion constant. Due to the strong confinement of electrons in the channel, the electron distribution in the channel becomes degenerate and is no longer Maxwellian. Also due to the 2-dimensional distribution of electrons in space, a 2-dimensional density of states instead of a 3-dimensional density of states should be used together with the degenerate Fermi-Dirac statistics. For a non-Maxwellian distribution-function, the relationship between \( \mu \) and \( D \) deviates from the generalized Einstein’s relationship, i.e., Eq. (2.16). The extension of Fermi-Dirac statistics to nonequilibrium operation can be achieved using the quasi-Fermi level. The quasi-Fermi level can be decomposed into an electrostatic potential which generates the drift component of current and a chemical potential which induces the diffusion current. The chemical potential is related to the nonuniform distribution of carrier concentration along the channel and is very important in the potential-barrier region where
the high concentration-gradient of electrons is present. The idea adopted in this
study is to use the diffusion constant associated with hot-electron effect by setting
$D/\mu$ as a function of electron concentration and lattice temperature. Then the
dependence of the diffusion constant on electron heating has the same form as the
mobility. This formulation ensures that the current equation is valid for both the
potential barrier region and the normal-channel region. Then the total current can
be written as

$$I = zn_3\mu(e') \frac{dE_F}{dx} \quad (2.36)$$

or

$$I = qzn_3\mu(e')[\frac{dV_c}{dx} + \frac{1}{q} \frac{dE_{Fi}}{dx}] \quad (2.37)$$

where $E_{Fi}$ denotes the chemical potential, $n_s$ is the 2-DEG concentration, $z$ is the
gate width, $e'$ is the net electric field ($e - \varepsilon_q$), and $V_c$ is the channel potential.
Expressing Eq. (2.37) in the form

$$I = qz[\mu(e')n_s\varepsilon + D\frac{dn_s}{dx}] \quad (2.38)$$

Diffusion constant can be identified as

$$D \equiv D(e', n_s) = \mu(e')\frac{1}{q} \frac{dE_{Fi}}{d\ln n_s} \quad (2.39)$$

Then the ratio between $D$ and $\mu$ becomes

$$\frac{D}{\mu} = \frac{1}{q} \frac{dE_{Fi}}{d\ln n_s} \quad (2.40)$$
When the 2-level formula for the 2-DEG is used for the chemical potential in the above equation, as \( n_a \) becomes small \( D/\mu \) tends to \( kT/q \), which is Einstein's relationship.
CHAPTER III
TWO-DIMENSIONAL CHARGE-CONTROL MODEL FOR MODFET'S

3.1 Introduction

As mentioned earlier, the transport mechanisms involved in the HEMT structure are quite complicated because both the 2-DEG and 3-dimensional gas are present in the channel. Since most of the HEMT devices are required to be short-channel devices in order to have desired high-speed and high-frequency operation, a strong hot-electron effect coupled with two-dimensional effects should dominate the device operation. The nonuniform distribution of the channel electric-field results in coupling between the 2-DEG and the 3-DEG whose concentrations are governed by different carrier statistics. Even though some 2-dimensional simulations neglecting the 2-DEG gas were developed to explain a general qualitative feature of HEMT transport, these approaches are inconsistent with the strong quantum feature of the HEMT measured by Dingle et al. and will result in appreciable error in the GCA region. Moreover, the unusually low doping concentration assumed in n-AlGaAs results in unrealistic channel characteristics.

Widiger [16] attempted a time-dependent 2-dimensional simulation including both the 2-DEG and the 3-DEG. The treatment of hot-electron effects was well accounted for by a comprehensive use of the Boltzmann transport equation. Also, the coupling mechanism of the 2-DEG and the 3-DEG is based on the reasonable
assumption that the degree of coupling has exponential dependence on electron temperature and the energy difference. However, noting that the subbands in the quantum well should converge to the conduction band edge of the GaAs near the heterointerface when subthreshold condition is reached[30], $E_1$ could not be used as the effective conduction band for the bulk charge. It is more reasonable to assume that the first two subbands, i.e., $E_0$ and $E_1$ remain as the 2-DEG quantum energy levels producing surface charges. The bulk charge should be introduced so that the electron concentrations are determined only by the local potential distribution inside the GaAs bulk. According to Widiger's simulation, the distribution of electrons along the channel shows an unrealistically high velocity peak of more than $1 \times 10^8$ cm/s for the 2-DEG, in conflict with measured data or from Monte Carlo simulation. The peak electron velocity measured is at most $2 \times 10^7$ cm/s at room temperature.

In spite of this complicated nature of HEMT transport in the hot-electron regime, recent Monte Carlo simulation of the 2-DEG transport supports the idea of using a field-dependent mobility instead of the complicated Boltzmann transport equation for hot-electron phenomena of HEMTs. Also, some 1-dimensional models fit measured I-V curves well using a Troffimenkoff type of field-dependent mobility [23], even though the accuracy of the models were not well proven. It is certain that a mobility-degradation effect eventually leading to velocity saturation can adequately be described by the Troffimenkoff type of field-dependent mobility in predicting the linear-to-saturation transition region of the I-V curves. In this regard, the following features are included in the formulation of the 2-dimensional model for HEMTs.

1. General types of velocity-field relationships are implemented to account for the hot-electron effects. Simulation of various kinds of velocity-field relationship
will allow us to examine the effect of velocity-field relationship on the I-V curves and to choose the best one for HEMT devices.

2. A diffusion component of current is included accounting for the variation of the quasi-Fermi level along the channel. Using both the field-dependent mobility and the quasi-Fermi level will ensure a generalized Einstein's relationship, which is applicable up to quite hot-electron velocity range of HEMT operations.

3. A two-dimensional analysis is adopted to obtain the electrostatic-potential solutions inside the AlGaAs region. This approach will allow the model to include the two-dimensional effects of the gate voltage on the channel characteristics and on the I-V curves.

4. Due to the high doping level in the n-AlGaAs region, the electron transport in the ungated regions of the HEMTs is governed by near-equilibrium condition outside the lateral depletion-length of the channel generated by the Schottky-barrier gate. These ungated regions are modeled separately as source and drain resistances.

3.2 Two-Dimensional Charge Control

For gate voltages below which a parasitic MESFET turns on, the n-AlGaAs region under the gate can be assumed to be fully depleted. The side boundaries of the depletion region are expected to extend somewhat beyond the gate region. We assume these boundaries to be simply determined by the lateral depletion length of the Schottky-barrier gate, as shown in Fig. 1. These boundaries actually move as the gate-to-source and gate-to-drain biases are changed, which is in contrast to the boundary conditions adopted by Widiger and Hess [8], where the entire n-AlGaAs region is assumed to be depleted from source to drain. In the depletion region under the gate, the Poisson equation can be written as follows.
The boundary conditions are detailed in Fig. 2. From the band diagram given in Fig. 3, one can verify that the electrostatic potential just inside the n-AlGaAs from the gate is \( V_{GB} = V_G - \phi_m \), where \( V_G \) is the gate potential and \( \phi_m \) is the Schottky-barrier height. The potential on both sides of the gate is then given by the Schottky-barrier potential \( V_{G1}(x) \) and \( V_{G2}(x) \).

\[
V_{G1}(x) = \begin{cases} 
-\frac{qN_D}{2\varepsilon_s}(x + \delta_s)^2 + V_S & ; -\delta_s \leq x \leq 0 \\
0 & ; 0 \leq x \leq L + \delta_d 
\end{cases}
\] (3.2)
Figure 2: Boundary conditions for AlGaAs region under the gate

and

\[ V_{G2}(x) = \begin{cases} 
0 & \text{if } -\delta_s \leq x \leq L \\
-\frac{qN_D}{2\varepsilon_2} (x - L - \delta_d)^2 + V_D & \text{if } L \leq x \leq \delta_d
\end{cases} \]  \hspace{1cm} (3.3)

Here \( \delta_s \) is the lateral depletion width at the source side of the gate, \( \delta_d \) the depletion width of the drain side of the gate, \( \varepsilon_2 \) the dielectric constant at the AlGaAs, \( N_D \) the donor doping-level of the AlGaAs, and \( L \) the gate length. On the channel side, the 2-DEG concentration \( n_s \) directly sets the boundary condition in terms of the normal field to \( E_{nor} = \frac{qn_s}{\varepsilon_2} \). The boundary conditions on the lateral boundaries consist of \( V_S + V_{B3}(y) \) on the source side and \( V_D + V_{B3}(y) \) on the drain side, where \( V_s \) is the source potential and \( V_D \) is the drain potential. The potential \( V_{B3}(y) \) is the depletion potential of the ungated heterojunction in the \( y \) direction.
Figure 3: Band diagram for HEMTs in nonequilibrium condition
\[ V_{B3}(y) = \begin{cases} 
- \frac{qN_D}{2\varepsilon_2} (y - \Delta y)^2 & ; \; d_i \leq y \leq \Delta y \\
- \frac{q}{\varepsilon_2} n_{s0} (d_i - y) - \frac{qN_D}{2\varepsilon_2} (d_i - \Delta y)^2 & ; \; 0 \leq y \leq d_i \\
0 & ; \; \Delta y \leq y 
\end{cases} \] (3.4)

where \( \Delta y \) is the equilibrium depletion-length in the ungated AlGaAs region, which can be calculated from the equilibrium 2-DEG concentration \( n_{s0} \) derived by Lee et al.[6].

\[ \Delta y = \frac{n_{s0}}{N_D} \] (3.5)

and

\[ n_{s0} = \sqrt{\frac{2\varepsilon_2 N_D \Delta E_c - \Delta E_F(T)}{q} + \delta + V(-W_2) + N_D^2 (d_i + d_e)^2 - N_D (d_i + d_e)} \] (3.6)

where

\[ \delta = -\frac{kT}{q} \left[ \ln(1 + g'u) + \frac{4}{N_D} \ln(1 + \frac{u}{4}) \right] \] (3.7)

and

\[ u = \exp\left(\frac{gV(W_2)}{kT}\right) = \frac{-(1 - \frac{N_D'}{4b^2}) + \sqrt{(1 - \frac{N_D'}{4b^2})^2 + 4g'N_D'}}{2g'} \] (3.8)

where

\[ N_D' = \frac{N_D}{N_c} \]

\[ g' = g \exp\left(\frac{E_d}{kT}\right) \]
In the above equations, $N_c$ denotes the density of states in the n-AlGaAs, $g$ is the degeneracy factor and $E_d$ is the donor activation energy. The potentials $V_{B1}(x)$ and $V_{B2}(x)$ are equivalent potentials at corresponding boundaries when we set normal electric fields with 0 values at the channel and gate boundaries of the potential components $V_1$ and $V_5$. These potentials can be written using the depletion approximation as

$$V_{B1}(y) = \begin{cases} 
V_{GB} + V_{p2} - \frac{qN_D}{2\varepsilon_2} & ; d_i \leq y \leq d \\
V_{GB} + V_{p2} & ; 0 \leq y \leq d_i 
\end{cases} \quad (3.9)$$

where

$$V_{p2} = -\frac{qN_D}{2\varepsilon_2}d_d^2$$

$$d = d_d + d_i$$

$$V_{B2}(x) = \frac{V_D - V_S}{L_p}(x + \delta_s) + V_S - V_{GB} \quad (3.10)$$

For the potential component $V_4$, the boundary potential $V_g(x)$ is

$$V_g(x) = V_{G1} + V_{G2}(x) - V_{GB} - V_{B2}(x) \quad (3.11)$$

For the potential component $V_2$, the boundary potential $V_d(y)$ is

$$V_d(y) = V_{B3}(y) - V_{B1}(y) + V_{GB} \quad (3.12)$$

Similarly, the boundary potential $V_S(y)$ for the potential component $V_3$ can be expressed as

$$V_S(y) = V_{B3}(y) - V_{B1}(y) + V_{GB} \quad (3.13)$$
As we wish to obtain an analytic solution to the potential problem of Eq. (3.1), we decompose the complex boundary value problem of Fig. 2 into a set of simple boundary value problems as illustrated in Fig. 4 and Fig. 5.

The boundary conditions for each decomposed potential, \([V_1, \ldots, V_6]\) are chosen to satisfy the superposition principle and minimize truncation errors that might arise from taking a finite number of harmonics in the series representation of each decomposed potential. Truncation error is minimized by avoiding potential discontinuities along the periphery of the rectangle for each boundary value problem. The solutions of these six boundary value problems are in the form of harmonic expansions. These expansions and the related Fourier coefficients can be evaluated as

\[
V_1 = V_{GB} + \frac{qN_D}{2\varepsilon_2} (d^2 - y^2)
\]  
(3.14)

\[
V_2 = \sum_{n=1}^{N} A_n \sin \left(\frac{(2n-1)\pi(y-d)}{2d}\right) \sinh \left(\frac{(2n-1)\pi(x + \delta)}{2d}\right)
\]  
(3.15)

\[
V_3 = \sum_{n=1}^{N} B_n \sin \left(\frac{(2n-1)\pi(y-d)}{2d}\right) \sinh \left(\frac{(2n-1)\pi(x - L - \delta)}{2d}\right)
\]  
(3.16)

\[
V_4 = \sum_{n=1}^{N} C_n \sin \left(\frac{n\pi(x + \delta)}{L_p}\right) \cosh \left(\frac{n\pi y}{L_p}\right)
\]  
(3.17)

\[
V_5 = (V_D - V_S) \frac{x + \delta}{L_p} + (V_S - V_{GB})
\]  
(3.18)

\[
V_6 = \sum_{n=1}^{N} D_n \sin \left(\frac{n\pi(x + \delta)}{L_p}\right) \sinh \left(\frac{n\pi(y-d)}{L_p}\right)
\]  
(3.19)

where

\[
A_n = \left(\frac{\cosh \left(\frac{(2n-1)\pi L_p}{2d}\right)}{2d}\right) \int_0^d V_d(y) \sin \left(\frac{(2n-1)\pi(y-d)}{2d}\right) dy
\]  
(3.20)
Figure 4: Decomposed boundary value problems
Figure 5: Decomposed boundary value problems
\[ B_n = (-\csc(\frac{(2n-1)\pi L_p}{2d}))^2 \int_0^d V_s(y) \sin \left(\frac{(2n-1)\pi(y-d)}{2d}\right) dy \quad (3.21) \]

\[ C_n = (\sech \frac{n\pi d}{L_p})^2 \int_{\delta_s}^{L+\delta_d} V_g(x) \sin \left(\frac{n\pi(x+\delta_s)}{L_p}\right) dx \quad (3.22) \]

\[ D_n = (-\frac{L_p}{n\pi \sech \frac{n\pi d}{L_p}})^2 \int_{-\delta_s}^{L+\delta_d} \frac{qn_s}{\epsilon_2} \sin \left(\frac{n\pi(x+\delta_s)}{L_p}\right) dx \quad (3.23) \]

For large aspect ratio, i.e., \( L_p/d \gg 1 \)

\[ V_0(x,0) = -\sum_{n=1}^{N} \frac{L_p}{n\pi} \tanh \frac{n\pi d}{L_p} \mathcal{F}_x \left[ qn_s \right] \sin \left(\frac{n\pi(x+\delta_s)}{L_p}\right) \quad (3.24) \]

or

\[ V_0(x,0) \approx -\sum_{n=1}^{N} \frac{L_p}{n\pi} \frac{n\pi d}{L_p} \mathcal{F}_x \left[ qn_s \right] \sin \left(\frac{n\pi(x+\delta_s)}{L_p}\right) \approx -\frac{qd}{\epsilon_2} \quad (3.25) \]

and

\[ V_2(x,0) + V_3(x,0) = \sum_{n=1}^{N} (-1)^n \mathcal{F}_y [V_d(y)] U(x) \quad (3.26) \]

where

\[ U(x) = \{ \exp \left(\frac{(2n-1)\pi(x+\delta_s)}{2d}\right) + \exp \left(\frac{(2n-1)\pi(x+\delta_s-L_p)}{2d}\right) \} \quad (3.27) \]

and \( \mathcal{F}_x[\cdot] \) and \( \mathcal{F}_y[\cdot] \) defined as

\[ \mathcal{F}_x[\cdot] = \frac{2}{L_p} \int_{-\delta_s}^{L+\delta_d} \sin \left(\frac{n\pi(x+\delta_s)}{L_p}\right) dx \quad (3.28) \]

\[ \mathcal{F}_y[\cdot] = \frac{2}{d} \int_0^d \sin \left(\frac{(2n-1)\pi(y-d)}{2d}\right) dy \quad (3.29) \]
Then the channel potential can be derived by summing the six electrostatic potentials and evaluating it at \( y = 0 \). The resulting channel potential can be written as

\[
V_c(x) = V_A(x) + V_B(x)
\]  

(3.30)

where

\[
V_A(x) = V_G - \phi_m + V_p - \frac{qd}{\epsilon_2}n_s
\]  

(3.31)

and

\[
V_B(x) = (V_D - V_S) \frac{x + \delta_s}{L_p} + (V_S - V_G + \phi_m)
\]

\[
+ \sum_n \phi_d(y)(-1)^n \left\{ \exp\left[ \frac{(2n - 1)\pi (x + \delta_s)}{2d} \right] + \exp\left[ \frac{(2n - 1)\pi (x + \delta_s - L_p)}{2d} \right] \right\}
\]

\[
+ \sum_n \phi_g(x) \sec \frac{n\pi d}{L_p} \sin \frac{n\pi (x + \delta_s)}{L_p}
\]

Here \( L_p \) represents the total lateral depletion length in the AlGaAs under the gate \((L_p = \delta_s + L + \delta_d)\). As can be seen from this solution, \( V_A(x) \) corresponds to the GCA solution whereas \( V_B(x) \) is the built-in potential at both sides of the channel that accounts for the two-dimensional effect additionally present compared to the 1-dimensional models. This built-in potential is responsible for the 2-dimensional charge-control of the 2-DEG in MODFETs.

### 3.3 Current Control

In formulating current transport, we assume that most of the electrons remain confined within the 2-DEG channel. Indeed, as we shall see later, the channel concentration does not drop appreciably even in the saturation region and the carriers remain tightly confined within the triangular potential well. Obviously, the
heated electrons can still escape from the quantum well and this effect is accounted for by the velocity-field relationship. In order to account for both electron heating and the 2-DEG diffusion effects, the channel current $I$ is expressed using the quasi-Fermi energy $E_F$ by

$$I = -z\mu n_s \frac{dE_F}{dx}$$  \hspace{1cm} (3.32)

where $\mu$ is the field-dependent mobility and $z$ denotes the width of the gate. Integration of Eq. (3.32) from source to $x$ leads to

$$-\frac{E_F(x) - E_F(s)}{q} = I[R_S + R_c(x)]$$  \hspace{1cm} (3.33)

where

$$R_S = \frac{1}{qz} \int_{source}^{-\delta_s} \frac{dx}{\mu n_s}$$  \hspace{1cm} (3.34)

and

$$R_c(x) = \frac{1}{qz} \int_{-\delta_s}^{x} \frac{dx}{\mu n_s}$$  \hspace{1cm} (3.35)

Here $R_S$ represents the source resistance and $R_c(x)$ the channel resistance up to the point $x$ from $-\delta_s$. In fact, the ohmic contact resistance could be included in $R_S$ without disturbing our formulation.

On the other hand, from the works of Delagebeaudeuf [12] the 2-DEG concentration $n_s$ is related to the chemical potential $E_{F_1}$ as shown in the band diagram of Fig. 3.

$$n_s = D_2 kT \ln\{(1 + \exp(\frac{E_{F_1} - E_0}{kT}))\{1 + \exp(\frac{E_{F_1} - E_1}{kT})\}\}$$  \hspace{1cm} (3.36)

Here, $D_2$ denotes the two-dimensional density of states, and $E_0$ and $E_1$ are the energy levels for the first two subbands in the triangular quantum well. Some
manipulation of this equation yields an expression for \( E_{Fi} \) as a function of \( n_s \).

\[
E_{Fi}/q = \frac{kT}{q} \ln \left\{ B(n_s) + [B(n_s)^2 + (\exp \frac{n_s}{D_2kT} - 1) \exp \frac{E_0 + E_1}{kT}]^{1/2} \right\} \tag{3.37}
\]

where

\[
B(n_s) = \frac{1}{2} (\exp \frac{E_0}{kT} + \exp \frac{E_1}{kT}) \tag{3.38}
\]

and \( E_0 \) and \( E_1 \) are related to \( n_s \) by

\[
E_0 = \gamma_0 n_s^{2/3} \]
\[
E_1 = \gamma_1 n_s^{2/3}
\]

Here, \( \gamma_0 \) and \( \gamma_1 \) values are experimental coefficients determined by synchrotron resonance experiments. The values commonly used are \( \gamma_0 = 2.5 \times 10^{-12}(Jm^{4/3}) \), \( \gamma_1 = 3.2 \times 10^{-12}(Jm^{4/3}) \) and \( D_2 = 3.24 \times 10^{17}(m^{-2}eV^{-1}) \) \[22\].

On the other hand, from the band diagram in Fig. 3, we have

\[
-\frac{E_F(x) - E_F(S)}{q} = V_c(x) + \frac{\Delta E_c}{q} - \frac{E_{Fr}}{q} \tag{3.39}
\]

From Eqs. (3.33) and 3.3

\[
V_c(x) + \left[ \frac{\Delta E_c}{q} - \frac{E_{Fr}}{q} \right] = I[R_S + R_c(x)] \tag{3.40}
\]

Obviously, the channel current can be written as

\[
I = \frac{V_D - V_S}{R_S + R_{ct} + R_D} \tag{3.41}
\]

where

\[
R_{ct} = \frac{1}{qz} \int_{-\delta_s}^{L+\delta_s} \frac{dx}{\mu n_s} \tag{3.42}
\]
and

\[ R_D = \frac{1}{qz} \int_{L+\delta_d}^{drain} \frac{dx}{\mu n_s} \]  

(3.43)

Finally, elimination of \( I \) from Eqs. (3.40) and (3.41) results in

\[ G(n_s, x) \equiv V_c(x) + \left[ \frac{\Delta E_c}{q} - \frac{F_F}{q} \right] - \frac{R_S + R_c(x)}{R_S + R_D + R_{ct}} (V_D - V_S) = 0 \]  

(3.44)

Equation (3.44) is an integro-differential equation for \( n_s \). This equation can be solved for \( n_s \) simultaneously with Eqs. (3.30), (3.35), (3.37) and (3.42) using constant \( R_S \) and \( R_D \) based on any desired field-dependent mobility relationship for \( R_{ct}(x) \) and \( R_c(x) \). In this work, the Troffimenkoff type [23] of field-dependent mobility with \( \beta \) ranging from 1 to 2 is used.

\[ \mu = \mu_0 \left[ \frac{1}{1 + (\varepsilon_{cp}/\varepsilon_{cr})^\beta} \right]^{\frac{1}{\beta}} \]  

(3.45)

where

\[ \varepsilon_{cp} = \frac{dV_c}{dx} - \left( \frac{dV_c}{dx} \right)_{equilibrium} \]  

(3.46)

and \( \mu_0 \) denotes the low-field mobility, \( v_s \) the electron saturation velocity, and \( \varepsilon_{cr} = v_s/\mu_0 \) is the critical field. Note that the mobility is defined in terms of a net electric field \( \varepsilon_{cp} \) which vanishes at equilibrium, which accounts for the fact that the electrons are not heated by the large equilibrium built-in field at both sides of the gate. These equilibrium built-in fields are essentially formed by the diffusion of electrons from the ungated channel region to the gated channel region because of the big concentration difference caused by the gate voltage. The procedure for obtaining I-V curves is described below and the algorithm is summarized in the flow chart of Fig. 6.
Figure 6: Flow chart for calculating the I-V curves and channel profiles for 2-dimensional charge-control model
Initially the equilibrium solution for $n_s$ ($V_D = V_G = 0$) at a given gate voltage is calculated using the first two terms of Eq. (3.44). Since this equation is a nonlinear function of $n_s(x)$, two methods such as a trial and error method or a Newton-Raphson method are used for solving this equation sequentially in order to reduce the computation time.

The first step uses the trial and error method to find the approximate location of roots. In this step the constant $n_s(x)$ is scanned in a reasonable range with fixed $n_s(x)$ steps until a sign change occurs. The approximate $n_s(x)$ obtained in this way is used as the initial guess for the subsequent Newton-Raphson procedure to find an accurate solution for $n_s(x)$.

Once the equilibrium solution for a given $V_G$ is obtained, $V_D$ is increased in small steps. Then a new $n_s(x)$ is calculated from the whole integro-differential equation via the Newton-Raphson method using the equilibrium solution as the initial guess. In this iteration loop, the channel electric field is calculated by numerical differentiation of $V_c(x)$ and is used for the mobility calculation. This process for calculation of $n_s(x)$ can be successively applied for the next value of $V_D$, because this time the initial guess can be the $n_s(x)$ of the previous $V_D$ value. Once an accurate value of $n_s(x)$ is obtained for each given $V_G$ and $V_D$, the total current can be evaluated from Eqs. (3.41) and (3.42). Since $V_c(x)$, $E_{Fi}(x)$ and $\varepsilon_c(x)$ are automatically calculated during the process of evaluating $n_s(x)$, these channel profiles may be plotted whenever it is necessary. Then, the drift velocity can be calculated using

$$v_d(x) = \mu(x)\varepsilon(x)$$  \hspace{1cm} (3.47)

Also, the drift and diffusion current can be evaluated with
\[ I_{dr}(x) = qz\mu(x)n_s(x)\varepsilon_{cp}(x) \quad (3.48) \]

and

\[ I_{diff}(x) = -qz\mu(x)n_s(x)\frac{dE_F}{dx} \quad (3.49) \]

### 3.4 Comparison with Experimental Data

In the process of fitting theoretical I-V curves to experimental data, the velocity-field relationship was used as a fitting tool. Most of the measured I-V curves that have been published used AlGaAs alloy composition of \( x = 0.3 \) and were measured at 300 K. According to Mailhot et al. [24], the AlGaAs dielectric constant \( \varepsilon_2 \), the AlGaAs/GaAs conduction-band discontinuity \( \Delta E_c \) and the Schottky-barrier height \( \phi_m \) are

\[
\begin{align*}
\varepsilon_2 &\approx (13.1 - 3x)\varepsilon_0 \approx 12.2\varepsilon_0 \\
\Delta E_c &\approx 1.06 x \approx 0.32 \text{ eV} \\
\phi_m &\approx 0.92 + 0.62x \approx 1.11 \text{ V for Au-AlGaAs contact} \\
\phi_m &\approx \text{between 0.9 and 1.0 V for Al-AlGaAs contact}
\end{align*}
\]

For the experimental I-V curves of Drummond [13] shown in Fig. 7, \( \beta = 1 \) was tried in Eq. (3.45) and gives an excellent fit using their stated parameters

\[
\begin{align*}
N_D &= 10^{18} \text{ cm}^{-3}, \quad d_d = 300 \text{ Å}, \quad d_i = 100 \text{ Å} \\
\mu_0 &= 4300 \text{ cm}^2/\text{V}, \quad v_s = 2 \times 10^7 \text{ cm/s}, \quad z = 145 \mu\text{m}, \quad L = 1 \mu\text{m}
\end{align*}
\]

However, in order to have a better fit, \( \phi_m = 1.22 \text{ V} \) instead of 1.11 V and \( R_S = R_d = 20 \Omega \) instead of 10 \( \Omega \) were used and also a parallel leakage resistance \( R_p \) of 4 k\( \Omega \) was allowed.
Figure 7: I-V comparison for enhancement-mode HEMT. solid line: 2-D charge-control model, broken line: experimental data (Morkoc[13]).

For the data of Fig. 8 from Morkoc et al. [22], good agreement was obtained using their stated parameters, except for the low-field mobility:

\[ N_D = 10^{18} \text{ cm}^{-3}, \quad d_d = 250 \text{ Å}, \quad d_i = 60 \text{ Å} \]
\[ z = 145 \text{ μm}, \quad L = 1 \text{ μm}, \quad R_S = R_D = 10 \text{ Ω}, \quad v_s = 2 \times 10^7 \text{ cm/s} \]

However, the usage of the stated low-field mobility of 6800 cm²/Vs, results in too small a value of drain pinch-off voltage. To obtain better agreement, \( \beta = 2 \) and \( \mu_0 = 4300 \text{ cm}^2/\text{Vs} \) were used instead. For this sample, \( \phi_m = 0.9 \text{ V} \) was used and \( R_p \) of 2.8 kΩ was included to have a relatively large slope in the saturation region.

In order to test the dependence of the pinch-off voltage upon the mobility model, various values of \( \beta \), \( \mu_0 \), and \( v_s \) were tried. The simulation result shows that at low critical field the drain pinch-off point is determined by the critical field itself. However, at high critical field, the pinch-off point becomes insensitive to the
Figure 8: I-V comparison for enhancement-mode HEMT. solid line: 2-D charge-control model, broken line: experimental data (Morkoc [22]).

3.5 Saturation and Diffusion Effects

The diffusion and saturation effects of HEMTs can best be investigated by plotting channel profiles such as the channel voltage, the 2-DEG concentration, the parallel electric field, the electron velocity, and the diffusion and drift current along the channel as illustrated in Figs. 9 through 15.

All parameters used for these curves correspond to the ones used for the I-V curves of Fig. 8. For each of Figs. 9 through 15, a gate voltage of 0.2 V was selected, and the drain voltage was varied from 0 to 0.4 V by increments of 0.08 V.

In Fig. 9, one can easily identify the GCA region as the region where the channel potential increases gradually along the channel. One can see that beyond $V_D = V_{D,sat} = 0.32 V$, the channel potential curves in the GCA region converge to...
Figure 9: Channel potential under the gate, gate voltage=0.2 V

Figure 10: 2-DEG concentration along the channel, gate voltage=0.2 V
Figure 11: Parallel electric field along the channel. gate voltage = 0.2 V

Figure 12: Electron velocity along the channel. gate voltage = 0.2 V
that of the saturation voltage. The excess drain potential \((V_D - V_{D,\text{sat}})\) appears across the potential barrier at the drain side. This value, corresponding to the drain saturation voltage \(V_{D,\text{sat}}\), matches the pinch-off voltage obtained from the I-V curves of Fig. 8 at \(V_G = 0.2\ V\).

A similar phenomenon can be observed in Fig. 10, where the 2-DEG concentration in the GCA region remains unchanged for drain voltage above the pinch-off voltage. A narrow saturation region, where the 2-DEG concentration reaches a minimum, forms at the drain-side edge of the GCA region. Since most of the applied drain potential appears across this narrow region of small conductance, the electric field in this region becomes very large as the drain voltage is increased (see Fig. 11). Also, notice that on the source side of the potential barrier the electric field does not depart from its equilibrium value.

Due to the presence of these large electric fields at the potential barrier on the drain side, the electron velocity quickly saturates as can be seen in Fig. 12, even though a slowly varying \(v - \varepsilon\) relationship (Troffimenkoff type) was used. However, full saturation occurs only for applied drain potentials beyond the pinch-off voltage. As the electron velocity reaches its saturation velocity, the electron concentration cannot decrease further and reaches a minimum value. This minimum value \(n_{s,\text{min}}\) is directly set by the continuity of current along the channel, i.e., \(I = qz_{s,\min}v_s\). The diffusion component of the current can be neglected at this minimum where \(\frac{dn_s}{dx} = 0\). Since the current saturates for the drain potential beyond saturation, the electron concentration also saturates near this minimum value.

From Fig. 12, one can observe that as the drain voltage increases beyond the saturation voltage the saturated-velocity region widens. This phenomenon effectively reduces the length of the GCA region under the gate. Shortening of the GCA region is directly responsible for the finite conductance slope in the saturation
region of the I-V curves.

The above analysis gives evidence of the formation of a saturation region with saturated velocity \( v_s \) and minimum 2-DEG concentration \( n_{s,min} \). This observation is consistent with the two-region saturation picture recently proposed by Rohdin and Roblin [20]. However, the saturation region is established gradually in contrast to the assumption of a sharp transition in their model. This phenomenon is evident in Fig. 9, where a fraction of the applied drain potential occurs across the potential barrier (increasing the barrier) before the drain potential reaches the saturation voltage of 0.32 V.

In order to analyze this gradual saturation further, a comparison was made in Fig. 13 between the I-V curves obtained from the two-region model by Roblin and that from the current 2-dimensional model using an abrupt quasi-linear two-piece velocity-field relationship.
The same device parameters were used in both models. The saturation currents are essentially the same despite a slight overshoot originating from the abrupt two-piece velocity-field model. The 2-dimensional charge-control model yields a smoother I-V knee than the two-region model. This results directly from the gradual development of the saturation region.

The drift current and diffusion currents along the channel are plotted in Figs. 14 and 15.

The small ripples of the current values result from the spectrum truncation in the evaluation of the potential. These ripples have negligible effect on the stable integral formulation used in our model. One can observe that about $\frac{1}{3}$ of the total current is carried by diffusion current for every bias condition. However, the saturation current is not effectively modified (see Fig. 13).
3.6 Two-dimensional Potential Distribution

The electrostatic potential distribution inside the gated AlGaAs region can be solved and plotted using Eqs. (3.14) to (3.19). In order to reduce the computation time, the unknown boundary condition $n_s(x)$ is solved first for a given $V_G$ and $V_D$ using the one-dimensional approach discussed previously. Once all the boundary conditions are fixed, a 2-dimensional potential distribution can be calculated using Eqs. (3.14) to (3.19). Figure 16 shows the equilibrium potential distribution in the AlGaAs region.

The back-side plane corresponds to the gate contact and the front-side plane corresponds to the heterojunction just inside the AlGaAs. The left and right plane corresponds to the depletion edge of the gated region at the source and drain side respectively. Because of the conduction band discontinuity at the heterointerface, the conduction band edge of the GaAs goes below the Fermi level, which corre-
Figure 16: Equilibrium electrostatic-potential distribution in the AlGaAs region under the gate. gate voltage=constant, drain voltage=0 V

sponds to the bottom plane in Fig. 16. It can be noticed from this diagram that normal electric field is decreased when we move from the ungated region to the gated region, which implies that the electron concentration is decreasing due to the gate potential. This electron concentration gradient at both edges of the channel gives rise to diffusion currents, which are compensated by drift currents due to the equilibrium potential barriers there. Figure 17 shows the case where the drain voltage has been applied. It can be noticed that the channel potential increases gradually while the normal electric field decreases when we move from source to drain.
Figure 17: Nonequilibrium electrostatic-potential distribution. gate voltage=0.6 V, drain voltage=0.6 V
CHAPTER IV
AN IMPROVED TWO-SECTION CAD MODEL FOR MODFET'S

4.1 Introduction

From the simulation results of the 2-dimensional charge-control model, the following features are conspicuous and could be further utilized to implement a simple and accurate 1-dimensional model.

1. The channel region is sharply separated into two parts, i.e., the GCA region and the saturation region where an electrostatic potential barrier is formed.

2. Below the drain saturation-voltage $V_{D,sat}$, most of the channel voltage appears across the GCA region.

3. When the drain voltage reaches $V_{D,sat}$, the potential drop across the GCA region remains fixed at the value corresponding to $V_{D,sat}$, and the excess voltage $(V_D - V_{D,sat})$ is applied only to the saturation region, i.e., the potential barrier region.

4. High electric field is confined to the saturation region and velocity saturates quickly as the carrier density becomes small.

5. Even though a sharp transition exists between the GCA and saturation regions, the finite transition region could not be neglected. Actually, in the saturation region instead of using full-saturated velocity distribution, a gradual velocity profile should be used.
4.2 Model Formulation

Based on the previous properties of HEMTs, a two-section model, which is shown in Fig. 18, can be established.

This figure corresponds to the case when the applied drain voltage is well above the saturation voltage. In this figure \(d_c\) and \(d_s\) represent the channel thicknesses of the GCA section and the saturated section respectively. The electron velocity at the saturated section can be assumed to be in full saturation without appreciable error.

4.2.1 Current Equation

Using the charge-sheet model [25], the channel current can be written as

\[
I = z\mu n_s \frac{dE_F}{dx}
\]  

(4.1)

where \(E_F\) represents the quasi-Fermi level, \(\mu\) the field-dependent mobility and \(n_s\) the concentration of the channel electrons. Denoting the chemical potential as \(E_{Fi}\), the diffusion current component can be isolated from the quasi-Fermi level as follows. From Eq. (4.1)

\[
I = qzn_s \mu (-\frac{dV_c}{dx} + \frac{1}{q} \frac{dE_{Fi}}{dx})
\]  

(4.2)

On the other hand, the channel current can be written as

\[
I = qzn_s E + qzD \frac{dn_s}{dx}
\]  

(4.3)

Comparing Eq. (4.1) and Eq. (4.3), the ratio between \(D\) and \(\mu\), i.e., \(D/\mu\) can be identified as \(\frac{1}{q}n_s \frac{dE_{Fi}}{dn_s}\). For a range of gate voltage well above the subthreshold regime, \(n_s\) can be approximated by the 2-DEG. In this case, the expression for the chemical potential \(E_{Fi}\) can be obtained from Eq. (3.37).
Figure 18: Boundary conditions for two-section model
Figure 19: $D/\mu$ vs $n_s$ at room temperature

$$
\frac{D}{\mu} = \frac{1}{q} \frac{dE_F}{dn_s}
$$

(4.4)

Figure 19 shows a plot of $D/\mu$ as a function of $n_s$.

It can be noticed from this plot that above the subthreshold region the $D/\mu$ ratio varies linearly, which gives

$$
\frac{D}{\mu} = b + an_s
$$

(4.5)

where $b = 0.033 \, V$ and $a = 1.2 \times 10^{-17} \, V m^2$. The interesting thing we note here is that when $n_s$ approaches 0, the $D/\mu$ ratio converges to 0.0259 V which is $kT/q$ at room temperature. This means that as the 2-DEG becomes a 3-DEG, the relation between $D$ and $\mu$ reduces to Einstein's relation.
4.2.2 Charge Control

Linear Region

The electron concentration in the GCA section is represented by the commonly used expression

\[ n_x = C(V_{off} - V_c) \]  \hspace{1cm} (4.6)

where

\[ V_{off} = V_G - V_{th} \]  \hspace{1cm} (4.7)

\( V_{th} \) represents the threshold voltage and can be expressed as

\[ V_{th} = \phi_0 - V_p - \frac{\Delta E_c}{q} \]  \hspace{1cm} (4.8)

Also, \( C \) represents the gate capacitance per unit gate width and is

\[ C = \frac{K_o \varepsilon_0}{qd_t} \]  \hspace{1cm} (4.9)

Here, \( d_t = d_d + d_i + d_c \). \( d_c \) accounts for the equilibrium channel opening caused by the effect of Fermi-level variation with respect to the gate voltage. It has been calculated by Lee et al [13]. A general closed form solution for \( d_c \) in terms of the gate voltage is given in Appendix A and enables us to model the subthreshold region more accurately. Differentiation of Eq. (4.6) with respect to \( x \) leads to

\[ \frac{dn_x}{dx} = C \varepsilon \]  \hspace{1cm} (4.10)

Combining Eqs. (4.3), (4.5), (4.6) and (4.10)

\[ I = q z \mu ' C \varepsilon (V_{off} - V_c) \]  \hspace{1cm} (4.11)

where

\[ \mu ' = \mu (Ca + 1) \]  \hspace{1cm} (4.12)
From Eqs. (4.12) and (4.13), we can conclude that the increase of total current by the 2-DEG diffusion effect is equivalent to an increase of the low-field mobility and also an increase of the threshold voltage when the low-field mobility and the threshold voltage are defined as the values for the case when the diffusion current component has been neglected. This phenomenon may explain one of the reasons for the enhanced low-field mobility of 2-DEG compared to that of the 3-DEG. If we consider the Troffimenkoff type of field-dependent mobility with $\beta = 1$

$$v = \begin{cases} 
\mu' \varepsilon = \frac{\mu_0' \varepsilon}{1 + \varepsilon / \varepsilon_c} & ; \varepsilon \leq \varepsilon_p \\
v_s & ; \varepsilon_p \leq \varepsilon 
\end{cases}$$

(4.14)

where $\mu'_0$ denotes the enhanced low-field mobility that could be obtained by Hall measurements and $\varepsilon_c$ is defined as $g v_s / \mu'_0$. Here, $g$ is the fitting parameter that determines the velocity of electrons at the boundary of the GCA section and the saturated section and defined as

$$v_s = \frac{\mu'_0 \varepsilon_p}{1 + \varepsilon_p / \varepsilon_c}$$

(4.15)

Then Eq. (4.11) becomes

$$I = \beta \frac{\varepsilon}{1 + \varepsilon / \varepsilon_c} (V_{off}' - V_c)$$

(4.16)

where

$$\beta = q z \mu'_0 C$$

(4.17)

Solving Eq. (4.16) for $\varepsilon$
\[ I = |\beta (V_{off} - V_c) - \frac{I}{\varepsilon_c}| \]  

Integrating Eq. (4.18) from \( x = 0 \) to \( x = L \)

\[ I = \frac{\beta}{L + V_D/\varepsilon_c} (V_{off}V_D - \frac{1}{2}V_D^2) \]  

Saturation Region

When the operating point is in the saturation region of the I-V curves, a saturated section is already formed in the drain side of the channel and Fig. 18 can be used for the boundary conditions of the channel. Now if we define \( x_p = 0 \) or \( x = L' \) to be the splitting point between the GCA and the saturated section and \( \varepsilon_p \) and \( V_c(L') \) to be the parallel electric field and the channel voltage at that point respectively, the saturation current can be written as follows from Eq. (4.19) assuming \( V_c(L') = V_p \)

\[ I = \frac{\beta}{L' + V_p/\varepsilon_c} (V_{off}V_p - \frac{1}{2}V_p^2) \]  

This equation tells us that as the drain voltage increases beyond the saturation voltage, \( L' \) becomes smaller than \( L \) and accordingly the current will increase. This current increase beyond saturation essentially implies a finite slope in the saturation region of the I-V curves. Then \( V_p \) can be calculated from the linear region of the I-V curves. From Eq. (4.19)

\[ I_p = \frac{\beta}{L + V_p/\varepsilon_c} (V_{off}V_p - \frac{1}{2}V_p^2) \]  

Also from Eq. (4.16)

\[ I_p = \beta \frac{\varepsilon_p}{1 + \varepsilon_p/\varepsilon_c} (V_{off} - V_p) \]
Elimination of $I$ from Eqs. (4.21) and (4.22) and rearrangement of the resulting equation with respect to $V_p$ gives

$$
\left( \frac{1}{\epsilon_c} - \frac{1}{\epsilon_p} \right)V_p^2 + 2 \left( \frac{V_{off}'}{\epsilon_p} + L \right)V_p - 2L V_{off}' = 0
$$

(4.23)

The solution of Eq. (4.23) is

$$
V_p = \frac{\epsilon_c(V_{off}' + \epsilon_p L)}{\epsilon_p - \epsilon_c} \pm \sqrt{\left( \frac{\epsilon_c(V_{off}' + \epsilon_p L)}{\epsilon_p - \epsilon_c} \right)^2 + \frac{2\epsilon_c\epsilon_p LV_{off}'}{\epsilon_p - \epsilon_c}}
$$

(4.24)

When $\epsilon_p = \epsilon_c$, Eq. (4.24) reduces to

$$
V_p = \frac{\epsilon_c LV_{off}'}{V_{off}' + \epsilon_c L}
$$

(4.25)

Then $I_p$ can be obtained from Eqs. (4.22) and (4.24) or (4.25).

On the other hand, Poisson's equation in the GaAs region neglecting the acceptor concentrations is

$$
\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{q}{K_s\epsilon_0}n
$$

(4.26)

where $n$ represents the 3-dimensional electron-gas concentration.

Integrating Eq. (4.26) along the $y$ direction ($y$ direction corresponds to that normal to the heterointerface)

$$
\int_{-\infty}^{0} \frac{\partial^2 V}{\partial y^2} dy + \int_{-\infty}^{0} \frac{\partial^2 V}{\partial z^2} dy = \frac{q}{K_s\epsilon_0} \int_{-\infty}^{0} n dy
$$

(4.27)

which reduces to

$$
\epsilon_\perp + d_c \frac{d^2 V_c}{dz^2} = \frac{q}{K_s\epsilon_0} n_s
$$

(4.28)

where $d_c$ represents the effective channel-thickness in this region. Since in the saturated section $\epsilon_\perp$ can be neglected, Eq. (4.28) can be approximated as
In the saturated section diffusion current can be neglected since \( \frac{dn_s}{dx} \approx 0 \). Then the total current can be represented as

\[
I_D = qzn_s v
\]

where \( v \) denotes the net electron velocity \( (v_{d,tot}-v_{d,equl}) \) along the channel. From Eqs. (4.29) and (4.30)

\[
\frac{d^2V_c}{dx^2} = \frac{I_D}{K_0\varepsilon_d z v}
\]

If we assume full velocity saturation in the saturated section, Eq. (4.31) can be written as

\[
\frac{d^2V_c}{dx^2} = \frac{I_D}{K_0\varepsilon_d z v}
\]

Integrating Eq. (4.32) twice with respect to \( x \) from \( x = L' \) to \( L \) or \( x_p = 0 \) to \( \Delta L \)

\[
V_c = \frac{I_D}{2K_0\varepsilon_d z v} x_p^2 + \varepsilon_p x_p + V_p
\]

When \( x_p = \Delta L \) and \( V_c = V_p \), Eq. (4.33) can be written as

\[
V_D - V_p = K I_D (\Delta L)^2 + \varepsilon_p \Delta L
\]

where

\[
K = \frac{1}{2K_0\varepsilon_d z v}
\]

From Eq. (4.20)

\[
I_D(L' + V_p/\varepsilon_c) = \beta(V_{off}'V_p - \frac{1}{2}V_p^2)
\]
Using $L = L' + \Delta L$, Eq. (4.36) reduces to

$$I_D \Delta L = (L + V_p/\varepsilon_c)(I_D - I_p)$$

(4.37)

Multiplying Eq. (4.34) by $I_D$

$$I_D(V_D - V_p) = K(I_D \Delta L)^2 + \varepsilon_p(I_D \Delta L)$$

(4.38)

From Eqs. (4.37) and (4.38)

$$I_D(V_D - V_p) = K(L + V_p/\varepsilon_c)^2(\varepsilon_p)^2 + \varepsilon_p(L + V_p/\varepsilon_c)(I_D - I_p)$$

(4.39)

Arranging Eq. (4.39) in terms of $(I_D - I_p)$ gives

$$K(L + V_p/\varepsilon_c)^2(I_D - I_p)^2 + \varepsilon_p(L + V_p/\varepsilon_c)(I_D - I_p) - (V_p - V_p)I_p = 0$$

(4.40)

From Eq. (4.40), it can be easily verified that if $V_D = V_p$, then $I_D = I_p$ or vice versa. This result ensures the continuity of the linear region and the saturation region of the I-V curves at that point. Solving Eq. (4.40) for $(I_D - I_p)$,

$$I_D - I_p = B + \sqrt{B^2 + \frac{(V_D - V_p)I_p}{K(L + V_p/\varepsilon_c)^2}}$$

(4.41)

where

$$B = -\frac{\varepsilon_p(L + V_p/\varepsilon_c) - (V_D - V_p)}{2K(L + V_p/\varepsilon_c)^2}$$

(4.42)

Equation (4.41) describes the $I_D$ vs $V_D$ relationship purely analytically in the saturation region of the I-V curves. Once $V_p$ and $I_p$ are calculated from Eqs. (4.24) and (4.22), one can generate I-V curves easily using Eq. (4.41). Therefore, Eqs. (4.19) and (4.41) constitute the complete equations for the I-V curve in the whole operational range.
The asymptotic behavior of the I-V curves in the saturation region can be examined as follows. From Eq. (4.40)

\[ V'_D = \frac{a_1 I''_D + b_1 I'_D}{I'_D + I_p} \]  

(4.43)

where

\[ V_D - V_p = V'_D \]
\[ I_D - I_p = I'_D \]
\[ K(L + V_p/\varepsilon_c)^2 = a_1 \]

and

\[ \varepsilon_p(L + v_p/\varepsilon_c) = b_1 \]

When \( I'_D \approx 0 \), Eq. (4.43) reduces to

\[ V'_D \approx \frac{a_1 I''_D + b_1 I'_D}{I_p} \]  

(4.44)

Equation (4.44) tells us that \( I'_D \) is a square-root function of \( V'_D \), which ensures the smooth knee characteristics of the I-V curves. When \( I'_D \gg 0 \)

\[ V'_D = a_1 I'_D + b_1 \]  

(4.45)

which gives nearly constant output conductance in the saturation region of I-V curves.

### 4.3 A Correction Factor for The Finite Transition Section

As pointed out in the previous section, there exists a transition section between the GCA and saturated sections, where both the normal electric field and parallel electric field components should be included in the Poisson's equation.
Figure 20 shows the boundary conditions in the channel region including the transition section.

In the transition section, Poisson's equation can be written as

$$\varepsilon_\perp + d_c \frac{d^2V_c}{dx^2} = \frac{q}{K_s \varepsilon_0} n_s$$ \hspace{1cm} (4.46)

If the GCA expression is used for $\varepsilon_\perp$

$$n_s = C (V_{off} - V_c) + \frac{K_s \varepsilon_0}{q} d_c \frac{d^2V_c}{dx^2}$$ \hspace{1cm} (4.47)

where $C = \varepsilon_2 / q(d_d + d_i + d_c)$. The second term of this equation represents the electron concentration due to the parallel electric field, which is free from the control of the gate voltage. This term is approximately proportional to the channel potential

$$\frac{K_s \varepsilon_0 d_c}{q} \frac{d^2V_c}{dx^2} = fCV_c$$ \hspace{1cm} (4.48)

where $f$ denotes the proportionality constant. Equation (4.48) implies that the channel potential $V_c$ is the hyperbolic sine function of $x$. This result is consistent with the previous 2-dimensional analysis of the channel potential at the drain side of the potential barrier. Then the total electron concentration is

$$n_s = C (V_{off} - V_c) + fCV_c$$ \hspace{1cm} (4.49)

or

$$n_s = C' (V_{off}'' - V_c)$$ \hspace{1cm} (4.50)

where
Figure 20: Boundary conditions for three-section model
Equation (4.50) tells us that transition section can be included as part of the GCA section by using the reduced capacitance $C'$ and decreased pinch-off voltage $V_{off}'$. This means that the non-GCA section can be extended to the GCA section with reduced total charge-control parameter $C'$. However, electron pinching off is delayed by the increase of electrons accumulated by the parallel electric field in the saturated section. Then, these modified charge-control parameters can be used in the previous two-section model to generate more accurate results.

4.4 I-V Comparison

Figure 21 shows a comparison between the two-section model without $f$ factor and the experimental data [13]. The solid line corresponds to the measured data and the dotted line corresponds to present model. In this comparison, we can see that in the linear region of the I-V curves, good agreement is achieved. However as the saturation region is reached, there are huge differences in the saturation current levels. Most of the 1-dimensional models that incorporate Trofimenkoff type of field-dependent mobility use decreased threshold voltage to align the saturation part of the I-V curves. However, this causes a discrepancy in the linear region of the I-V curves. In order to eliminate these discrepancies, they had to again increase the source resistance values to obtain a reasonable fit for the entire range of I-V curves. The source resistance usually used were almost two times as large as the measured value [15], [21]. This approach is wrong because the source resistance and

\[ C' = C(1 - f) \]

and

\[ V_{off}'' = \frac{V_{off}'}{1 - f} \]

Equation (4.50) tells us that transition section can be included as part of the GCA section by using the reduced capacitance $C'$ and decreased pinch-off voltage $V_{off}'$. This means that the non-GCA section can be extended to the GCA section with reduced total charge-control parameter $C'$. However, electron pinching off is delayed by the increase of electrons accumulated by the parallel electric field in the saturated section. Then, these modified charge-control parameters can be used in the previous two-section model to generate more accurate results.

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threshold parameters cannot be used as fitting parameters. When the \( f \) factor was introduced in the current 2-section model, we could observe excellent agreement with the experimental data as shown in Fig. 22. Table 4.1 and Table 4.2 list the device parameters used in Fig. 21 and Fig. 22 respectively. Actually the I-V fitting without a \( f \) factor cannot avoid the considerable discrepancy near the transition region from the linear-to-saturation of the I-V curves even though overall fitting is reasonable. However, the I-V fitting using a \( f \) factor shows good agreement for every region of the I-V curves.

Figure 23 and Fig. 24 shows another example of the I-V comparisons between the experimental data [22] and the 2-section model with and without a \( f \) factor respectively. Table 4.3 and Table 4.4 list the device parameters associated with these comparisons. Again we observe the same phenomenon as the previous
Figure 22: Comparison of I-V curves for enhancement-mode device. solid line: two-section model with correction factor $f = 0.2$, dotted line: experimental data (Morkoc[13]).

Figure 23: Comparison of I-V curves for enhancement-mode device. solid line: two-section model without correction factor, i.e., $f=0$, dotted line: experimental data (Morkoc[22]).
Table 4.1: Device parameters for I-V comparison for Fig. 21

<table>
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<tr>
<th>Device Parameter</th>
<th>Experimental Data</th>
<th>Model parameter</th>
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<tbody>
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Table 4.2: Device parameters for I-V comparison for Fig. 22

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Table 4.3: Device parameters for I-V comparison for Fig. 23

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Table 4.4: Device parameters for I-V comparison for Fig. 24

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<td>$1 \times 10^{14}$</td>
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<td>$g$</td>
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<td>$R_s$ (Ω)</td>
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<tr>
<td>$d_s$ (Å)</td>
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<td>130</td>
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<tr>
<td>$d_c$ (Å)</td>
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<td>80</td>
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</table>
Figure 24: Comparison of I-V curves for enhancement-mode device. solid line: two-section model with correction factor $f = 0.2$, dotted line: experimental data (Morkoc[22]).

comparisons except that a slightly different value of $f$ was used.

In order to test the importance of using the $f$ factor, the comparison between the 2-dimensional charge-control model described in Chapter 2 and the 2-section model described in this chapter without the $f$ factor was made in Fig. 25. In the linear region of the I-V curves, we observe absolute agreement. However, the 2-section model underestimates the saturation current. Through the repetative I-V simulation process, it has been found that the agreement for the whole range of I-V curves can only be achieved by introducing the $f$ factor.

Figure 26 shows the comparison between the I-V curves of the two-section model with the transition-section correction factor $f$ and the measured data. The dotted lines correspond to the experimental data reported by the group at the University of Illinois [14]. The fitting parameters used in this comparison are
Figure 25: Comparison of I-V curves for enhancement-mode device. dotted line: two-section model without correction factor, i.e., \( f = 0 \), solid line: 2-dimensional charge-control model.

Figure 26: Comparison of I-V curves for enhancement-mode device. solid line: two-section model with correction factor \( f = 0.2 \), dotted line: experimental data (Morkoc[13]).
Figure 27: I-V comparison for depletion-mode device, solid line: two-section model, dotted lines: experimental data (HP sample) illustrated in Table 4.5. In this comparison $f = 0.2$ was used for the best fit.

Figure 27 shows the comparison between the current two-section model and the measured data from the HP sample. Very good agreement between the I-V curves and a smooth transition from the linear to the saturation region of the I-V curves are noticeable. In this fit, a saturation velocity of $1.5 \times 10^5$ m/s was used with a low-field mobility of 0.43 $m^2/Vs$. Table 4.6 illustrates the parameters used for this comparison.

It turns out that the slopes of the saturation regions of the I-V curves are largely dependent on the channel opening in the saturated section and the value of $g$, which determines the magnitude of the parallel electric field at the boundary between the transition section and the saturated section.
Table 4.5: Device parameters for I-V comparison for Fig. 26

<table>
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<th>Device Parameter</th>
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<th>Model parameter</th>
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<td>100</td>
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<td>$N_D$ (cm$^3$)</td>
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<td>g</td>
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<td>Model parameter</td>
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<td>$d_i$ (Å)</td>
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<td>$L_s$ (μ)</td>
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<td>$N_A$ (cm$^3$)</td>
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<td>$1 \times 10^{14}$</td>
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<td>g</td>
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<td>$R_s$ (Ω)</td>
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<td>$d_s$ (Å)</td>
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<td>2500</td>
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<tr>
<td>$d_c$ (Å)</td>
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<td>80</td>
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</tbody>
</table>
Figure 28: Nonlinear equivalent-circuit diagram for HEMTs

4.5 Circuit Parameters

The HEMT can be modeled using the nonlinear circuit parameters as shown in Fig. 28, and the circuit parameters can be evaluated from the I-V relations obtained from the previous section.

4.5.1 Output Conductance

The output conductance of the saturation region of the I-V curves can be easily obtained. From Eq. (4.41)

\[
I_D = I_p - \frac{B - (V_D - V_p)}{2A} + \sqrt{\frac{B - (V_D - V_p)}{2A}}^2 + \frac{V_D - V_E}{A} I_p \quad (4.51)
\]

where

\[
A = K(L + V_p/\epsilon_c)^2
\]
and

\[ B = \varepsilon_p(L + V_p/\varepsilon_c) \]

Then the output conductance at the saturation region can be obtained simply as

\[ g_d' = \frac{\partial I_D}{\partial V_D} = \frac{1}{2A} + \frac{1}{A}(I_p + \frac{V_D - V_p - B}{2A})((\frac{B - V_D - V_p}{2A})^2 + I_p \frac{V_D - V_p}{A})^{\frac{1}{2}} \] (4.52)

For the linear region of I-V curves, from Eq. (4.19)

\[ g_d' = \frac{\partial I_D}{\partial V_D} = \frac{\beta'(LV_{off} - \varepsilon_c LV_D - \frac{1}{2}V_D^2)}{\varepsilon_c(L + V_D/\varepsilon_c)^2} \] (4.53)

4.5.2 Capacitances

From Eq. (4.16)

\[ \frac{1}{\varepsilon} = \frac{\beta'(V_{off}' - V_c) - I/\varepsilon_c}{I} \] (4.54)

Then the total channel-charge can be expressed as

\[ Q_T = qz \int_0^L n_s dx \] (4.55)

or

\[ Q_T = qz \int_{V_S'}^{V_D'} n_s \left( \frac{1}{\varepsilon} \right) dV_c \] (4.56)

where \( V_D' \) and \( V_S' \) represent the intrinsic drain voltage and source voltage respectively. Using Eq. (4.54), we find

\[ Q_T = \frac{\beta'^2}{\mu_0 I} \int_{V_S'}^{V_D'} [(V_{off}' - V_c)^2 - \frac{I}{\varepsilon_c \beta'(V_{off}' - V_c)^2}] dV_c \] (4.57)

where
\[ \beta' = q \mu_0 C' \] (4.58)

which leads to

\[ Q_T = \frac{\beta'^2}{\mu_0} \left[ \frac{1}{2} \frac{1}{\beta'^2 \varepsilon_c} [(V'_{off} - V'_D)^2 - (V'_{off} - V'_S)^2] - \frac{1}{3} [(V'_{off} - V'_D)^3 - (V'_{off} - V'_S)^3] \right] \]

(4.59)

On the other hand, for the linear region

\[ I = \frac{\beta'}{L + V'_{DS/\varepsilon_c}} (V'_{off} V'_L - \frac{1}{2} V'_D^2) \] (4.60)

Using the fact that \( V'_{DS} = V'_D - V'_S \), \( V'_G = V'_G - V'_S \) and \( V'_GD = V'_G - V'_D \), \( Q_T \) can be calculated as

\[ Q_T = \frac{\beta'}{6 \mu_0 \varepsilon_c} \left( 3 \frac{K_1}{K_2} + 2 \frac{K_2}{K_3} K_4 \right) \] (4.61)

where

\[ K_1 = (V''_{off} + V'_{GD})^2 - (V''_{off} + V'_{GS})^2 \]

\[ K_2 = L \varepsilon_c + V'_G - V'_GD \]

\[ K_3 = V''_{off} - \frac{1}{2} (V'_{GS} - V'_{GD}) \]

\[ K_4 = (V''_{off} + V'_{GD})^2 + (V''_{off} + V'_{GD})(V''_{off} + V'_{GS}) + (V''_{off} + V'_{GS})^2 \]

and

\[ V''_{off} = V'_{off} - V_G \]

Then, one finds

\[ c'_GS = \frac{\partial Q_T}{\partial V'_G} \] (4.62)
or

\[ c'_{GS} = \frac{\beta'}{6 \mu_0 \varepsilon} \left[ -6(V'_{off} + V'_{GS}) + 2 \frac{K_3(K_4 + K_2K_5) + \frac{1}{2}K_2K_4}{K_3} \right] \]  \hspace{1cm} (4.63)

where

\[ K_5 = 3V'_{off} + 2V'_{GS} + V'_{GD} \]

Similarly

\[ c'_{GD} = \frac{\beta'}{6 \mu_0 \varepsilon} \left[ 6(V'_{off} + V'_{GS}) + 2 \frac{K_3(K_4 + K_2K_5) - \frac{1}{2}K_2K_4}{K_3} \right] \]  \hspace{1cm} (4.64)

In the current saturation region

\[ c'_{GS,sat} \approx c'_{GS}(V'_{DS} = V_P, V'_{GS}) \]  \hspace{1cm} (4.65)

and

\[ c'_{GD,sat} \approx c'_{GD}(V'_{DS} = V_P, V'_{GS}) \]  \hspace{1cm} (4.66)

4.5.3 Source Resistance

From the diagram of the MODFET structure in Fig. 1, it can be noted that there could be no other current path except the 2-DEG channel along the heterointerface for gate voltages below the parasitic MESFET operation. Also, from Monte Carlo simulation done by Mouis [18], real-space transfer of the 2-DEG into the AlGaAs is almost negligible for a 1 µm gate device even at the drain-side edge of the gate. This fact suggests that the 2-DEG is strictly confined in the channel even though to some extent bulk transport in GaAs may be allowed near
the drain-side edge of the gate region. However, since the ungated channel region enforces a near-equilibrium 2-DEG concentration just outside of the gate, bulk electrons should effectively be collected by the ungated channel. Using the equilibrium 2-DEG concentration formulated by Lee et al. [12]

\[ n_{e0} = \sqrt{\frac{2K_c e_0 N_D}{q} \left[ -V(d_i^-) + V(-W_2) + \delta \right] + N_B^2 d_i^2 - N_D d_i} \quad (4.67) \]

From Fig. 29, with an electron current in the ungated channel, \( V(d_i^-) \) varies along the channel and can be written as

\[ V(d_i^-) = V_c(x) + \frac{\Delta E_c}{q} - \frac{E_{F_i}(x)}{q} \quad (4.68) \]

With the linearized relationship of \( E_{F_i} \) with respect to \( n_s \), Eq. (4.67) can be rewritten using Eq. (4.68) as
\[ n_{s0} = \sqrt{\frac{2K_0\epsilon_0 N_D}{q} \left[ \Delta E_c - \Delta E_{F0}(T) \right] + V(-W_2) + \delta - V_c} + N_d^2 - N_d \] (4.69)

where

\[ N_d = N_D(d_i + d_c) \] (4.70)

Because of the high electron concentration, the channel current in the ungated region can be well represented using the low-field mobility \( \mu'_0 \), where \( \mu'_0 \) denotes the low-field mobility including the diffusion effect.

\[ I = qz\mu'_0 n_{s0} \varepsilon \] (4.71)

Integrating Eq. (4.71) with \( n_{s0} \) replaced by Eq. (4.69) from \( x = 0 \) to \( x = L_S \)

\[ I L_S = qz\mu'_0 \int_0^{V'_S} \left\{ -N_D d'_i + \left( b_2 - a_2 V_c \right)^{\frac{1}{3}} \right\} dV_c \] (4.72)

where

\[ d'_i = d_i + d_c \]

\[ b_2 = N_D^2 d'_i^2 + \frac{2K_0\epsilon_0 N_D}{q} \left[ \Delta E_c - \Delta E_{F0}(T) \right] + V(-W_2) + \delta \]

\[ a_2 = \frac{2K_0\epsilon_0 N_D}{q} \]

Integration of Eq. (4.72) gives

\[ I = \frac{qz\mu'_0}{L_S} \left[ -N_D d'_i V'_S - \frac{2b_2^{\frac{2}{3}}}{3a_2} (1 - \frac{a_2 V'_S^{\frac{2}{3}}}{b_2^{\frac{2}{3}}})^{\frac{3}{2}} - 1 \right] \] (4.73)

As can be seen from Eq. (4.73), source resistance is in general nonlinear. However, for small current or small ungated channel length, this I-V relation can be linearized. Assuming \( \frac{a_2 V'_S}{b_2} \ll 1 \)
Then the source resistance can be expressed from the above equation as

\[
I \approx \frac{qz\mu_0'}{L_S} \left[ -N_D d_1' + b_2' \right] V_S'.
\] (4.74)

For large source to gate spacing, Eq. (4.73) might not be valid since present source resistance model neglects the current conduction in the n-AlGaAs.

\[
R_s = \frac{I}{V_S'} = \frac{qz\mu_0'}{L_S} \left( -N_D d_1' + b_2' \right).
\] (4.75)
CHAPTER V

I-V MEASUREMENTS AND COMPARISON WITH MODELS

5.1 Sample Specification

The samples under test were noninverting single-layered heterojunction MODFETs, which were provided from Hewlett Packard Co. The layers were grown by MBE on a Cr-doped semi-insulating GaAs wafers. As can be seen in Fig. 3.1, an undoped GaAs layer, 40 Å of undoped AlGaAs, and 600 Å of Si-Doped AlGaAs layer are grown sequentially on top of the wafer. The mole fraction of Al used was 0.3 and the doping level of Si was $2 \times 10^{18} \text{cm}^{-3}$. Then the gate recessing was done until the AlGaAs thickness of 440 Å for depletion mode and 240 Å for enhancement mode device were obtained. Aluminium was evaporated on the recessed region to form a Schottky-barrier gate. Figure 30 illustrates the entire test pattern incorporating various test structures, and Fig. 31 represents the identification name for each corresponding subpattern.

5.2 I-V Measurements

The I-V measurements were made at room temperature in a shielded probe-station with Tektronix 577 curve tracer. In order to avoid instability of the measurement, the samples were illuminated while performing I-V measurements.
Figure 30: MODFET test-pattern of the sample
<table>
<thead>
<tr>
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<th>ZIGRING</th>
<th>ZFET2</th>
<th>ZAMRING</th>
<th>ZFET6</th>
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<td>ZFET1</td>
<td>ZAGRING</td>
<td>ZFET4</td>
</tr>
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<td>ZIMRING</td>
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<td>ZCV</td>
<td>ZMICWAVE</td>
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</tbody>
</table>

Figure 31: Identification name of the test pattern for the sample
5.2.1 ZFET3

Measured I-V

ZFET3 is a test pattern that is designed for the analysis of the source-access resistance. It includes three depletion mode HEMTs, i.e., Z3100D3, Z3100DH, and Z3100DX which have gate-to-source spacing of 1 μm, 2 μm, and 4 μm respectively. The measured I-V curves for these devices are shown in Figs. 32, 33 and 34. The comparison of the I-V curves between Z3100D3 and the Z3100DH shows that I-V curves of the Z3100D3 has sharper turn-on characteristics than that of Z3100DH, which is natural considering the smaller source resistance of Z3100D3. However, the current level and the transconductance for Z3100DH is larger than that of Z3100D3. This phenomenon can only be explained by assuming that the low-field mobility under the gate and the saturation velocity should be larger for Z3100DH than that of Z3100D3. The slopes in the linear region and the transconductance
Figure 33: Measured I-V Curves for Z3100DH

Figure 34: Measured I-V Curves for Z3100DX
in saturation region of the I-V curves for Z3100DX are further decreased compared to Z3100DH. The decrease of low-field mobility and the saturation velocity for MODFET with 1 μm source-to-gate spacing seems to be related to the degradation of the channel characteristics under the gate due to improper processing steps. Another common feature that can be noticed here is that the wide range of parasitic-MESFET region is formed at high gate-voltages. This phenomenon is due to the excessive doping level employed in AlGaAs.

Comparison with Theory (2-Section Model)

The measured I-V curves are compared to the 2-section model prediction using device parameters of Z3100D3, Z3100DH and Z3100DX and the results are illustrated in Figs. 35, 36 and 37 with associated device parameters listed in Tables 5.1, 5.2 and 5.3 respectively. In this comparison, the source resistance
Figure 36: I-V comparison between measured data and theory (2-section CAD model) for Z3100DH

Figure 37: I-V comparison between measured data and theory (2-section CAD model) for Z3100DX
Table 5.1: Device parameters for I-V comparison of Fig. 34 (Z3100D3)

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<th>Model parameter</th>
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</tr>
<tr>
<td>$v_s$ (m/s)</td>
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<td>$1 \times 10^5$</td>
</tr>
<tr>
<td>$d_d$ (Å)</td>
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<td>440</td>
</tr>
<tr>
<td>$d_i$ (Å)</td>
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<td>55</td>
</tr>
<tr>
<td>$N_D$ (cm$^{-3}$)</td>
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<td>$2.2 \times 10^{18}$</td>
</tr>
<tr>
<td>$\Delta E_c$ (eV)</td>
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<td>0.32</td>
</tr>
<tr>
<td>$\phi_m$ (eV)</td>
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<td>0.9</td>
</tr>
<tr>
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<td>100</td>
</tr>
<tr>
<td>L (μ)</td>
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<td>1</td>
</tr>
<tr>
<td>$L_s$ (μ)</td>
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<tr>
<td>$N_A$ (cm$^{-3}$)</td>
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<td>$1 \times 10^{14}$</td>
</tr>
<tr>
<td>f</td>
<td>–</td>
<td>0.23</td>
</tr>
<tr>
<td>g</td>
<td>–</td>
<td>0.9</td>
</tr>
<tr>
<td>$R_s$ (Ω)</td>
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</tr>
<tr>
<td>$d_s$ (Å)</td>
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<tr>
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</table>
Table 5.2: Device parameters for I-V comparison of Fig. 35 (Z3100DH)

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<td>$N_D$ (cm$^3$)</td>
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<td>g</td>
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Table 5.3: Device parameters for I-V comparison of Fig. 36 (Z3100DX)

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<tbody>
<tr>
<td>( \mu_0 ) (cm²/V s)</td>
<td>_</td>
<td>2700</td>
</tr>
<tr>
<td>( v_S ) (m/s)</td>
<td>_</td>
<td>1.23x10^6</td>
</tr>
<tr>
<td>( d_d ) (Å)</td>
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<td>440</td>
</tr>
<tr>
<td>( d_i ) (Å)</td>
<td>40</td>
<td>55</td>
</tr>
<tr>
<td>( N_D ) (cm⁻³)</td>
<td>2x10^18</td>
<td>2.2x10^18</td>
</tr>
<tr>
<td>( \Delta E_c ) (eV)</td>
<td>_</td>
<td>0.32</td>
</tr>
<tr>
<td>( \phi_m ) (eV)</td>
<td>_</td>
<td>0.9</td>
</tr>
<tr>
<td>( z_z ) (µ)</td>
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<td>100</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>( L_S ) (µ)</td>
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<td>2.6</td>
</tr>
<tr>
<td>( N_A ) (cm⁻³)</td>
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<td>1x10^14</td>
</tr>
<tr>
<td>( f )</td>
<td>_</td>
<td>0.23</td>
</tr>
<tr>
<td>( g )</td>
<td>_</td>
<td>0.9</td>
</tr>
<tr>
<td>( R_S ) (Ω)</td>
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<td>( d_s ) (Å)</td>
<td>_</td>
<td>300</td>
</tr>
<tr>
<td>( d_c ) (Å)</td>
<td>_</td>
<td>80</td>
</tr>
</tbody>
</table>
model developed in Chapter 4 was incorporated into the two-section model and I-V
curves were generated. Also, in the model different low-field mobilities were used
for the ungated channel and gated channel respectively. The low-field mobility in
the ungated channel was estimated from the mobility profiling measurement. In
the gated region, the mobility and the saturation velocity were used for the fitting
parameters. Much lower low-field mobility in the gated channel was used than in
the ungated channel to fit the data. From the I-V comparison, we can conclude
that the source resistance model gives good results for the I-V curves with source-
to-gate spacing of 1 \( \mu m \) and 2 \( \mu m \). However for the gate-to-source spacing of 4 \( \mu m \),
the source resistance model overestimates the actual source resistance. This result
is due to the invalidity of the linearized formula, i.e., Eq. (4.75). When the spacing
becomes large, the ungated channel acts more likely as an ungated FET and I-V
relation becomes nonlinear. In this case, resistance value should be evaluated us­ing
Eq. (4.74). Since most practical HEMTs should have a source-to-gate spacing
of much less than 2 \( \mu m \), the linearized formula for the source resistance is well
justified. The slight disagreement of the I-V curves at the low gate voltages seems
to be due to the gate leakage current flowing in the doped AlGaAs region from
source to drain in the sample.

5.2.2 ZAGRING

ZAGRING is a test pattern designed for the analysis of the gate-length de­
pendence of the I-V curves. It includes four HEMTs, i.e., ZA1, ZA2, ZA3 and ZA4
which have different gate lengths.

Measured I-V Curves

Measured I-V curves of ZAGRING are shown in Figs. 38, 39, 40 and 41.
A prominent feature that can be observed by comparing the measured I-V
Figure 38: Measured I-V curves for ZA4

Figure 39: Measured I-V curves for ZA2
Figure 40: Measured I-V curves for ZA1

Figure 41: Measured I-V curves for ZA3
curves is that as the gate length increases the transconductance decreases. This phenomenon is due to the fact that the transconductance of the HEMTs is inversely proportional to the gate length, which also is predicted by the theory. For 1 \( \mu \text{m} \) devices, we have maximum external transconductance of about 150 \( \text{mS/mm} \). Also we notice that the output conductances in the saturation region of the I-V curves increases.

Comparison with the Theory

For the I-V comparisons with the theory, ZA2, ZA3, ZA4, which have gate lengths of 1 \( \mu \text{m} \), 2 \( \mu \text{m} \), and 4 \( \mu \text{m} \) respectively are chosen. These devices have identical device geometries to that of Z3100D3 except for the gate lengths. Figs. 42, 43 and 44 shows the I-V comparisons with the theory. In these comparisons, exactly the same device parameters as the ones used in fitting Z3100D3 were used.
Figure 43: I-V comparison between measured data and theory (2-section CAD model) for ZA2

Figure 44: I-V comparison between measured data and theory (2-section CAD model) for ZA3
except the Schottky-barrier potential $\phi_m$ as can be seen in Tables 5.4, 5.5 and 5.6.

This implies that the major fitting parameters $f$, $g$, and $d_s$ are insensitive not only to the gate-to-source spacing but also to the variation of the gate lengths. Actually $g$ and $d_s$ are related to the slopes of the I-V curves in the saturation region. Additionally, it has been found that $g$ and $d_s$ mutually dependent and the parameter $f$ is related to the location of the saturation voltage in the I-V curves.

5.2.3 ZIGRING

ZIGRING is an another test pattern designed for the analysis of the gate-length dependence of the I-V curves. The difference from the ZAGRING pattern is that it has a gate-to-source spacing of 1.5 $\mu$m instead of 1 $\mu$m. It includes four HEMTs, namely ZIG1, ZIG2, ZIG3 and ZIG4, which have different gate lengths respectively.

Measured I-V Curves

The measured I-V curves are shown in Figs. 45, 46, 47 and 48. The gate-length dependence of the I-V curves of the ZIGRING is similar to that of the ZAGRING. However, we note that due to the increased gate-to-source spacing, the slopes in the linear region are smaller than those of ZAGRING.
Table 5.4: Device parameters for I-V comparison of Fig. 41 (ZA4)

<table>
<thead>
<tr>
<th>Device Parameter</th>
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<th>Model parameter</th>
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<tbody>
<tr>
<td>$\mu_0$ (cm$^2$/V s)</td>
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</tr>
<tr>
<td>$v_s$ (m/s)</td>
<td>_</td>
<td>$1 \times 10^5$</td>
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<tr>
<td>$d_d$ (Å)</td>
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<tr>
<td>$d_i$ (Å)</td>
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<td>55</td>
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<tr>
<td>$N_D$ (cm$^3$)</td>
<td>$2 \times 10^{18}$</td>
<td>$2.2 \times 10^{18}$</td>
</tr>
<tr>
<td>$\Delta E_c$ (eV)</td>
<td>_</td>
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</tr>
<tr>
<td>$\phi_m$ (eV)</td>
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</tr>
<tr>
<td>$zz$ (µ)</td>
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<td>1</td>
</tr>
<tr>
<td>$L_s$ (µ)</td>
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<td>1</td>
</tr>
<tr>
<td>$N_A$ (cm$^3$)</td>
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<td>$1 \times 10^{14}$</td>
</tr>
<tr>
<td>$f$</td>
<td>_</td>
<td>0.23</td>
</tr>
<tr>
<td>$g$</td>
<td>_</td>
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</tr>
<tr>
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Table 5.5: Device parameters for I-V comparison of Fig. 42 (ZA2)

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<td>$g$</td>
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<tr>
<td>$R_S$ (Ω)</td>
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<td>$d_s$ (Å)</td>
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<tr>
<td>$d_c$ (Å)</td>
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Table 5.6: Device parameters for I-V comparison of Fig. 43 (ZA3)

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<td>55</td>
</tr>
<tr>
<td>$N_D$ (cm$^3$)</td>
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<td>2.2x10$^{18}$</td>
</tr>
<tr>
<td>$\Delta E_c$ (eV)</td>
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</tr>
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<td>$\phi_m$ (eV)</td>
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<td>4</td>
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<tr>
<td>$L_S$ (μ)</td>
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<td>1</td>
</tr>
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<td>1x10$^{14}$</td>
</tr>
<tr>
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<tr>
<td>$g$</td>
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<tr>
<td>$R_S$ (Ω)</td>
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<td>$d_s$ (Å)</td>
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<tr>
<td>$d_c$ (Å)</td>
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Figure 45: Measured I-V curves for ZIG4

Figure 46: Measured I-V curves for ZIG2
Figure 47: Measured I-V curves for ZIG1

Figure 48: Measured I-V curves for ZIG3
CHAPTER VI

CONCLUSION

A new 2-dimensional simulation-model for HEMTs has been developed which offers fast simulation of the channel characteristics and the 2-dimensional potential distribution in the gated AlGaAs region. The predicted I-V curves are well matched to the experimental data. This model reveals a large amount of diffusion current along the channel. The saturation mechanism of HEMTs is explicitly shown by channel profiles such as the channel potential, carrier concentration, parallel electric field, and electron velocity. These profiles correlate with the I-V curves to show that the saturation voltage of the I-V curves occurs exactly at the point where the formation of the saturated section of the channel begins. This picture supports the idea that for short channel HEMTs the saturation mechanism of the I-V curves is the result of velocity saturation of electrons.

From these results, we observe finite electron concentration at a drain voltage well above the saturation voltage. The argument that complete electron pinch off occurs at the saturated section for 1 μm gate device made by Mouis et.al. [18] seems not to be correct. Another important feature noticeable from the parallel electric field profile and electron velocity profile, is that the high-field region that degrades the mobility substantially is localized in the narrow region, i.e., in the drain-side potential-barrier. The saturation mechanism of the I-V curves is directly related to the formation of this high-field region.
Based on the 2-dimensional charge-control simulation, most of the important device physics is included in the 2-section model. In this model a proper treatment of the diffusion effect in the 2-DEG transport was made via the enhanced mobility and increased threshold voltage due to the gradual channel opening along the 2-DEG channel. By introducing the finite channel opening and an effective channel-length modulation, the slope of the saturation region of the I-V curves was successfully modeled. The smooth transition of the I-V curves from linear-to-saturation region of the I-V curves was possible using the continuous Troffimenkov-type of field-dependent mobility.

Furthermore, a correction factor $f$ was introduced to account for the finite transition section forming between the GCA and the saturated section. This factor removes the large discrepancies in the saturation region of the I-V curves predicted by previous 1-dimensional models. Also, a DC source resistance model was developed which provides reasonable estimation for the channel resistances along the ungated channel. This source resistance model was successfully applied to fit the measured I-V curves. In the 2-section model, only three major fitting parameters were introduced, i.e., $f$, $g$ and $d_s$. Considering the fact that fitting parameters are inevitable for all 1-dimensional models in order to have accurate I-V prediction, the number of parameters associated in our model is reasonable. The fitting parameters chosen in our model were found to be predictable and vary over relatively small ranges of values. Further efforts to reduce these parameters are discussed in Appendices A and B.

Above all, the present 2-section model is completely analytic, which offers great advantage in integrating it into a CAD program such as SPICE. Analytic expressions for nonlinear circuit parameters were extracted from the 2-section model and an equivalent circuit model was formed. This circuit model can be a basis for
large-signal analysis of HEMT circuits.

In order to test this 2-section model, a series of I-V measurements were executed. A source resistance model was tested using measured I-V curves from a set of devices that have various gate-to-source spacings. Also, the gate-length dependence of the I-V curves was examined using devices with different gate lengths. Comparison between models was used to examine the diffusion effect and the importance of the finite transition section. The diffusion effect was best revealed by the comparison between the 2-dimensional charge-control model and the 1-dimensional model of Roblin [20]. The importance of the transition section was demonstrated by the comparison between the experimental data and the 2-section model without a correction factor or between the 2-dimensional charge-control model and the 2-section model without correction factor.
CHAPTER VII
FURTHER STUDY

1. The bulk charge effect in GaAs

Due to the inevitable impurity in the bulk GaAs, the space charge in GaAs will affect the device performance when the device is operating in the subthreshold regime. This phenomenon is covered in neither our 2-dimensional charge-control model nor the 2-section model. Even though no satisfactory study of this problem has been reported so far, this phenomenon might be important in the determination of the threshold voltage of the HEMT.

2. The parasitic-MESFET region of the I-V curves was not incorporated in the current study due to the lack of importance in real device performance. Since the parasitic-MESFET region gives inferior characteristics to the HEMTs, the process parameters are commonly adjusted to avoid this region of operation. However, it will be necessary to include this region for applications such as analog power devices.

3. Complete development of a CAD model is necessary based on the present 2-section model. Further circuit parameters such as the source-to-gate or gate-to-drain resistances or capacitances inside AlGaAs region are also needed for accurate circuit description. The gate-resistance and the Schottky-diode models can be relatively easily implemented using the existing theory. Also further reduction of fitting parameters is desired.
4. The analysis of traps located in AlGaAs or at the heterointerface should be done to provide a stable threshold voltage for HEMTs. High-frequency C-V measurements performed in our laboratory show two dominant trap levels in the subthreshold region of the C-V curves.

5. True AC modeling of HEMTs is desirable to test the present DC models. The comparison of predicted AC circuit parameters and measured scattering parameters could be compared to investigate the physical phenomena which might not be covered by the strict DC models.
REFERENCES


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[22] H. Morkoc, "High speed modulation-doped AlGaAs/GaAs field effect transistors: MODFETs); analysis, fabrication and performance," Internal Reports, Department of Electrical Engineering and Coordinated Science Laboratory, University of Illinois.


APPENDIX A
Calculation of Channel Thickness

The channel thickness is strongly dependent on the channel charge because the potential well confining the channel charge is mainly formed by this charge itself. Here we are neglecting the background bulk-charge, which is reasonable assumption especially for HEMTs having unintentional doping level of only $10^{14} cm^{-3}$. The exact dependence of the channel thickness on the applied gate voltage can be obtained using the 2-level 2-DEG-concentration formula in Eq. (3.36). When there is no current flowing along the channel, the surface charge can be expressed using Lee's notation [13] as

$$n_s = \frac{K_0 \epsilon_0}{qd} (V_G - V_{th} - \frac{E_{F_1}}{q}) \quad \text{(A.1)}$$

Differentiation of Eq. (A.1) with respect to $V_G$ results in

$$\frac{\partial n_s}{\partial V_G} = \frac{k_0 \epsilon_0}{qd} (1 - \frac{1}{q} \frac{dn_s}{dV_G} \frac{dE_{F_1}}{dn_s}) \quad \text{(A.2)}$$

or

$$\frac{dn_s}{dV_G} = \frac{K_0 \epsilon_0}{q(d + \frac{K_0 \epsilon_0}{q^2} \frac{dE_{F_1}}{dn_s})} \quad \text{(A.3)}$$

Then the gate capacitance can be represented as

$$C \equiv q \frac{dn_s}{dV_G} = \frac{K_0 \epsilon_0}{d + d_c} \quad \text{(A.4)}$$
where

\[ d_c = \frac{K_0 \varepsilon_0}{q^2} \frac{dE_{Fi}}{dn_s} \]  

Eq. (A.5) is the general expression for the channel thickness which is related with the dependence of chemical potential upon the channel charge. For the first order approximation for \( n_s \),

\[ n_s = \frac{K_0 \varepsilon_0}{q(d + d_c)} (V_G - V_{th}) \]  

In general, \( d_c \) can be evaluated as a function of \( V_G \) using Eq. (A.5) and Eq. (A.6) iteratively because \( \frac{dE_{Fi}}{dn_s} \) is a function of \( n_s \) only. (the functional dependence of \( E_{Fi} \) upon \( n_s \) is in Eq. (3.37)) However, for CAD purposes this approach is not consistent with the present study. As can be seen in the relation between \( D/\mu \) and \( n_s \) in Fig. 19, we can approximate this curve with two pieces, i.e., one as a linear region and the other as a parabolic region. In the linear region where \( n_s \) is large

\[ \frac{D}{\mu} \equiv \frac{1}{q} n_s \frac{dE_{Fi}}{dn_s} = b' + a'n_s \]  

where

\[ b' = 0.0305 (V) \]

\[ a' = 1.5 \times 10^{-17} (Vm^2) \]

and for the parabolic region where \( n_s \) is small

\[ \frac{D}{\mu} \equiv \frac{1}{q} n_s \frac{dE_{Fi}}{dn_s} = 0.025 + \sqrt{a'n_s} \]
where \( a' = 3.2 \times 10^{-19} \, (V^2 m^{-2}) \). The splitting point between these two regions was chosen so as to have best reproduction of the original curve, and this point turns out to be \( n_s = 3.8 \times 10^{14} \, m^{-2} \).

For the linear region, from Eq. (A.5)

\[
d_c = \frac{K_0 \varepsilon_0 \frac{dE_F}{d E_s}}{q^2} = \frac{b}{n_s} + a
\]

where

\[
b = \frac{K_0 \varepsilon_0 \beta'}{q}
\]

\[
a = \frac{K_0 \varepsilon_0 \alpha'}{q}
\]

Eliminating \( d_c \) from Eqs. (A.6) and (A.9) results in

\[
d_c = \frac{b}{n_s} + a = \frac{K_0 \varepsilon_0}{qn_s} (V_G - V_{th}) - d
\]

The solution for \( n_s \) from Eq. (A.10) is

\[
n_s = \frac{(K_0 \varepsilon_0 / q)(V_G - V_{th}) - b}{a + d}
\]

Then \( d_c \) can be evaluated using Eq. (A.11) and Eq. (A.9) as

\[
d_c = \frac{(a + d)b}{(K_0 \varepsilon_0 / q)(V_G - V_{th}) - b} + a
\]

The gate capacitance associated with the channel thickness for the linear region can be obtained from Eq. (A.4) and Eq. (A.12) as

\[
C = \frac{K_0 \varepsilon_0 / q(V_G - V_{th}) - b}{(a + d)(V_G - V_{th})}
\]

For the parabolic region, from Eq. (A.8)
\[ d_c = \frac{K_0 e_0 }{q^2} \frac{d E_{Ft}}{d n_s} = \frac{\sqrt{a_1}}{\sqrt{n_s}} + \frac{c_1}{n_s} \quad (A.14) \]

where

\[ \sqrt{a_1} = \frac{K_0 e_0 }{q} \sqrt{d'_1} \]

\[ c_1 = 0.025 \frac{K_0 e_0 }{q} \]

Elimination of \( d_c \) combining Eq. (A.6) and Eq. (A.14) results in

\[ \frac{K_0 e_0 }{q n_s} (V_G - V_{th}) - d = \frac{\sqrt{a_1}}{\sqrt{n_s}} + \frac{c_1}{n_s} \quad (A.15) \]

Arranging Eq. (A.15) in terms of \( n_s \) leads to

\[ n_s + \frac{\sqrt{a_1}}{d} \sqrt{n_s} + \frac{c_1}{d} - \frac{K_0 e_0 }{qd} (V_G - V_{th}) = 0 \quad (A.16) \]

The solution of Eq. (A.16) is simply

\[ n_s = \left[ -\frac{\sqrt{a_1}}{2d} + \sqrt{\frac{a_1}{4d^2} + \frac{K_0 e_0 }{qd} (V_G - V_{th}) - \frac{c_1}{d}} \right]^2 \quad (A.17) \]

Then \( d_c \) can be evaluated from Eq. (A.10) and (A.17) as

\[ d_c = \frac{K_0 e_0 (V_G - V_{th})}{q \left[ -\frac{\sqrt{a_1}}{2d} + \sqrt{\frac{a_1}{4d^2} + \frac{K_0 e_0 }{qd} (V_G - V_{th}) - \frac{c_1}{d}} \right]^2} - d \quad (A.18) \]

The gate capacitance associated with this channel opening can be obtained from Eq. (A.4) and Eq. (A.18) as

\[ C = \frac{q \left[ -\frac{\sqrt{a_1}}{2d} + \sqrt{\frac{a_1}{4d^2} + \frac{K_0 e_0 }{qd} (V_G - V_{th}) - \frac{c_1}{d}} \right]^2}{K_0 e_0 (V_G - V_{th})} \quad (A.19) \]
APPENDIX B

List of Computer Programs

B.1. Work Space BUIVNT

- This work space includes programs that generates I-V curves and channel profiles based on 2-dimensional charge control model.

`FNS
ALINEPLOT AUTOFORMAT BARPLOT BITPLOT BNP BOLZ BOX BOX2
CLEAR COSFX CON COSFX CROSS DECOD DEF DFDN
DFDX DFGEN DGDX DGNSF DNX DOTPLOT DPLOTA DRANGE DTGE E
ECGX ENCOD EKNSMEWT EKNSX EXPFX2 EXPFX3 FERMI FF FI
FILTER FNSX FORM2 FORMAT FOURCX FOURCX1 FOURCX11 FOURCY FOURCY1
FOURCY11 FOURST FOURST1 GFRTE GRANGE2 GSAVE GSHOW GTST
HELP HELPDIM IDPLC IDPLT4 IDPLT4D IFOURC IFOURE23
IFOURE23E IFOURS IFOURSH5 IFOURS1 IHYSIL INIT INT INTPL
INTS INTS2 INTS2Y ITERATE IVN IVNS LINEPLOT LPLOT LPLOTA
MUNXT MUNXTT NAMES NS PARA PL1 PL2 PL4 PL4C PL4CD
PL5 PLOT PLL4 PLL5 FRANGE PTG PTG2 R RANGE RANGE2
RX SHAPEPLOT SXFX SXFXS SXFXD SXFXLP SIN SINDX SINFX
SINFY SINXD SMOOTHER SMOOTHPLOT SSINH TANH TESN TESX
TIMEDISPLAY TDST U1 U11 U1T U2 U2N U2NV U2P
U2T U2X U2XIV UARS VCGX VD3PF VG3P VG3PLIN VP VS3PF
VTOT VTOTT VY

*PARA[0]*
[0] Z=PARA
[1] N1=1790000000000 0 NC=1E24 0 NA=1E20 0 ND=1E24
[2] D1=1E-8 0 D3=3E-8 0 DELD=0.9 0 Z2=1.45E-4
[3] L=1E-6 0 LS=1E-6 0 L=1E-6
[4] EPS=0.85E12 0 Q=1.6E19 0 K0=13.1 0 KS=13.1 0 KB=1.8E23 0 T=300
[5] DEC=0.32 0 FIB=1.1 0 DEFOT=0 0 ED=0.042 0 S=0.141 0
[6] Q=2.5E12 0 Q=61.36E12 0 G=2 0 DQ=3.24E17 0 DQ=DQ+Q
[7] VCES=2000000 0 ECR=400000
[8] N1=10 0 N2=10 0 I1=50 0 JJJ=20 0 JUG=6

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*VIVN3D]*
[0] Z*IVN3
[1] *NEWTON METHOD USED
[2] *INCLUDE SOURCE DRAIN RESISTANCE
[3] *V0 STEP SIZE .02V, CONV CRITERION 1E-3
[4] PARA
[5] LDF+ 5 100 \&0
[6] VDF+ 5 100 \&0
[7] L20+VCI+(0.2xJVG)-1
[8] ID=0 \& VDT=0
[9] VDF+(QxNDxD+2)+2xKSxEPS
[10] GFIN+((1.424+2)+(XB*T+x)(NA+NI)
[11] VS=0
[12] VG=VCI-FIB
[13] a
[14] JJ+1
[15] NSX5+NSX4+NSX3+NSXI+NSX2=0
[16] L4+VD=0.02+(JJ-1)
[17] DL5X+((2xNDxEPSx(S-D+V)+QxND)=0.5
[18] DL5X+((2xNDxEPSx(S-D+V)+QxND)=0.5
[19] LP+1+DL5X+DL5X
[20] RS=LS-((2xNDxEPSx(S-D+V)+QxND)=0.5)+2xQxMUNxNS0
[21] ED+(LD-((2xNDxEPSx(S-D+V)+QxND)=0.5)+2xQxMUNxNS0
[22] XT=1+(LP+1)x0, III
[23] a XT=0, ((1.24)xLP+32), ((LPx3+4)+(40)xLP+160), ((LPx7+8)+(14)xLP+32)
[24] a XT=0, ((1.24)xLP+20), ((LPx2)+(12)xLP+40), ((0.8xLP)+(120)xLP+200)
[25] X=XT+DL5X
[26] Y+(0, LJJ)x=JJ
[27] a RX=RX1+(\phi X)=0
[28] a MUNX+DFIOX+DFIX+FI0X+ECX+(\phi X)=0
[29] VUX=UY
[30] VCGX=VCGX
[31] \&JJ=JJ/II
[32] NSX=NSXE=QNSX
[33] a VCGX=VCGX
[34] a VCXE=VCX
[35] a FNSXE=FNSX NSXE
[36] a DFIXVE+DFUX FNSXVE
[37] a ECVEQ=EC2 VCXE
[38] a DFI0X+ECUXQ=DFIXE
[39] a IDEQ+QxZZxMUNxNSXE=DFIXE
[40] XPE=X
[41] JJ=JJ+1
[42] +L4
[43] LII=ILC=0

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vEQNSNEUT(0)
[0] Z+EQNSNEUT NSX
[1] X+0
[2] DLNXT+10000000000
[3] L3:*VCX->VCGXV-(Q*D*NSX+KS*EPS)+Q*D*NSX+KS*EPS
[4] ECX+DFDX VCX
[5] NSXI+NSX-DLNXT
[6] VCXI->VCGXV-(Q*D*NSXI+KS*EPS)+Q*D*NSXI+KS*EPS
[7] ECXI+DFDX VCXI
[8] ??
[9] ??
[10] ??
[12] GNSX+VCX-GNSXI+ECX FNSX NSX
[13] GNSXI+VCXI-GNSXI+ECXI FNSX NSXI
[14] DGNSX+GNSXI=(NSX-NSXI)
[15] NSX+NSX-GNSX*DGSX
[16] ??
[17] ??
[18] +((~I(NSX-NSX)+NSX)~1E-3)=1)/DONEX
[19] K+D+K+I
[20] NSX+D+NSXH
[21] +L3
[22] DONEX:Z+NSX
[23] ??
[24] ??

vEQNSXI(0)
[0] Z+EQNSX;X;GNSX
[1] GNSX+(20,0)*0
[2] X+I
[3] L3:NSX+(rX)*e((1E18)+4*K)
[4] VCX->VCGXV-(Q*D*NSX+KS*EPS)+Q*D*NSX+KS*EPS
[5] ECX+DFDX VCX
[6] ??
[7] ??
[8] ??
[9] GNSXI+VS+DELTA-DECV
[10] GNSXK;J+VCX-GNSXI+ECX FNSX NSX
[12] ??
[13] +(X=10)/DONEX
[14] K+X+1
[15] +L3
[16] DONEX: ??
[17] ??
[18] Z3+NSX GNSX
[19] ??
[20] Z+EQNSNEUT Z3

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\*NS[0]*
[0] Z+MS\ GNSX
[1] Z1+(x)*p0
[2] IIC+1
[3] L9:III+1
[4] *(IIC=\(x\)+1)/DONE9
[5] LB:.*(IIR=E+1)/DONE8
[6] *(1+GNSXII+IIC)/DONE8
[7] IIR+IIR+1
[8] +L8
[9] DONE9:
[10] ZII+IIC+1
[12] DONE9:
[13] Z+Z1

\*FNSX[0]*
[0] Z+ECX\ FNSX\ NSX;BOXT;X1XT;A;B;DDD;E;F
[1] BOXT+G0\*NSX+(2+3)
[2] E1XT+G1\*NSX+(2+3)
[3] NS2x+NSX-KB\*T\*(MA+Q)\*INTS E1CX+NSX\*Q+XS\*EPS
[4] DDD++(BOXT+KB\*T)
[5] E++(E1XT+KB\*T)
[6] F++(NS2x+DQ\*KB\*T)
[7] B++(DDD+K)\*2
[8] A++(B+2)+(F-1)\*DDD\*KE
[9] C++B\*A\*0.5
[10] Z+FNSXV+(KB\*T\*Q)\*C

\*DFDX[0]*
[0] Z+DFDX\ FNSXV
[1] Z1+(x)*p0
[3] Z1(pX)+(FNSXV11-FNSXV(\(x\)-1))+(X\(x\)-X(\(x\)-1))
[4] III+2
[6] *(I1=\(x\)-1)/DONE1
[7] I1+II+1
[8] +L1
[9] DONE1=Z+Z1

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\[ Z = \text{EXPFX2(0)} \]
\[ Z = \text{EXPFX2} \]
\[ Z1 + ((2 \times N2) - 1) \times (X + DLSX - LP) \times 01 + 2 \times D \]
\[ Z2 + 300 \times Z1 \]
\[ Z*((Z2=0) \times -300) + Z1 \times Z2 \]

\[ Z = \text{EXPFX3(0)} \]
\[ Z1 + ((2 \times N2) - 1) \times (X + DLSX) \times 01 + 2 \times D \]
\[ Z2 + 300 \times Z1 \]
\[ Z*((Z2=0) \times -300) + Z2 \times Z1 \]

\[ Z = \text{INTS2(0)} \]
\[ Z = \text{INTS2 FX} \]
\[ Z1 + (\text{FX2}) \times 0 \]
\[ Z112 + (\text{FX12} + \text{FX11}) \times \text{DELX} \times 2 \]
\[ Z1 + 3 \]
\[ L1 : Z111 + Z11 - 2 + (\text{FX11-2} + (4 \times \text{FX111} - 1)) + \text{FX111} \times \text{DELX} \times 3 \]
\[ Z1 + 1 + 1 + 1 \]
\[ Z1 + 1 + 1 \]
\[ Z111 \]
\[ \text{DONE1} : Z2 \times Z1 \]

\[ Z = \text{INTS2 Y(0)} \]
\[ Z = \text{INTS2 FY Z1 DELY} \]
\[ Z1 + (\text{FY12}) \times 0 \]
\[ Z112 + (\text{FY12} + \text{FY11}) \times \text{DELY} \times 2 \]
\[ Z1 + 3 \]
\[ L1 : Z111 + Z11 - 2 + (\text{FY11-2} + (4 \times \text{FY111} - 1)) + \text{FY111} \times \text{DELY} \times 3 \]
\[ Z1 + 1 + 1 + 1 \]
\[ Z111 \]
\[ \text{DONE1} : Z2 \times Z1 \]

\[ Z = \text{FOURS(0)} \]
\[ Z = \text{FOURS AN Z1} \]
\[ Z*((2 \times N1) + N1 \times \text{AN1}(10 Z1) + Z1 + (1 \times N1) \times 01 + 2 \times N1) \times \text{SINFX X} \]

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\begin{verbatim}
  VCGX(DJ)
[0] Z+VCGX
[1] V4N+(6o(\mu M1)x01xD+LP)xFOURCXi1 V4P
[2] Z5+IF0URS V4N
[3] VDS3P+<FP
[4] n V23NN+(\tau 1)N(V22)xFOURCXii VDS3P
[5] n V6N+(Q+XO+EPS)x(\tau 1+x01)x(7o(\tau 1)x01xD+LP)xFOURCXii NS0-NSX
[6] VC1+VG+VP
[7] n VC23+IF0URE23 V23NN
[8] n VC23+(-VFOUR+X+DLSX-LP)x01x2D)+\tau (-x+DLSX)x01x2D)
[9] VCP+VCP
[10] n VC6+IF0URS V6NN
[11] n 0+2
[12] nL1+Z5P+(1)N+(0+1+2)+2
[13] n 0+1+2
[14] n 0+1+1
[15] n 0+2
[16] nD0N1=VC4+Z5
[17] Z+VC1+Z5+VC5

  VG3P(DJ)
[0] Z+VG3P
[1] Z1+((-DLSX)\times X\times 0)<VNG+(Q\times ND\times (X+DLSX)\times 2)+2xKO\times EPS
[2] Z2+((-LX)\times X\times ILP)xV3-VG+(Q\times ND\times (X-L+DLSX)\times 2)+2xKO\times EPS
[3] Z+Z1+Z2+VCP

  VY(DJ)
[0] Z+VY
[1] N6P+NC+ND
[2] GP+X+N(KD+KBX)
[3] DELTA+(-KBX+Q)xX((4xGP+NDP)+(1-NDP+4)x2)x0.5-1-NDP+2xGP
[4] N5O1+NDx1+DEL,
[5] N5O+(2xKO\times EPS\times ND\times (DECV-DEF=D-DELTA)+Q)\times N501\times 2+x0.5-NS01
[6] DELY+NS0+ND
[7] Z1+(-Q\times ND\times (Y-DEL'T))\times 2+2xKO\times EPS
[8] Z+((Y+DEL'T))\times 2

  FOURCXi1(DJ)
[0] Z+FOURCXi1 VX
[1] Z++/(E2)(2\times LP)xINTS2((\mu M1,\rho VX)\times 10(\mu M1)x(DLSX+x01+LP)[;\rho X]

  FOURCXii(DJ)
[0] Z+FOURCXii VY
[1] Z++/(E2)(2\times LP)xINTS2((\mu N2,\rho VY)\times 10(\mu 2\times N2-1)\times (Y-D)x01+2xD)[;\rho Y]

  IF0URE23(DJ)
[0] Z+IF0URE23 AN
[1] Z++/(1)(\mu ((\mu X),N2)\times AN)x(EXPFX2+EXPFX)\times EXPFX3+EXPFX3

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\end{verbatim}
\*INTS[0]*

[0] \*INTS FX; Z1
[1] DELX+X123-X11
[3] Z1+(\*pX)+0
[5] I1+3
[7] +(I1=\*pX)/DONE1
[8] I1+1+1
[9] *L1
[10] DONE1:Z+Z1

\*MUNXT[0]*

[0] \*MUNXT ECV
[1] Z\*MUN+1+MUNX|ECU=VES
[2] \*PR[0]*
[3] \*MUNV R NSV; YK; Z1; KK
[4] Z1+(\*pX)+0
[5] YK++Q=ZZxMUNV:NSV
[6] KK+2
[8] +(\*KK=\*pX)/DONE6
[9] KK+KK+1
[10] *L6

\*FX[0]*

[0] \*MUNX RX NSX
[1] YK++Q=ZZxMUNX:NSX
[2] \*INTS YK

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B.2. Work Space THPHEMT

- This work space includes programs that generates the I-V curves based on analytic CAD model.
\[
\begin{align*}
\text{vITN1[]} & \\
0: & \text{Z} = \text{ITN1}; \text{Z1}; \text{ZP}; \text{Z11}; \text{Z2}; \text{Z3}; \text{Z4}; \text{Z5}; \text{Z6}; \text{Z7}; \text{IDSU}; \text{VDSV}; \text{VD1}; \text{VD2}; \text{V4}; \text{C4}; \text{ITS} \\
1: & \text{ZP} = \text{VD} + 4.67 \times 0 \\
2: & \text{J} = 1 \\
3: & \text{L1} = \text{VG} + 0.2 \times \text{J} \\
4: & \text{(+VG*0.8)/L3} \\
5: & \text{VG} = 0.74 \\
6: & \text{L3} = \text{PARD} \\
7: & \text{VDSV} = \text{VDS1} \\
8: & \text{IDSU} = \text{IDS1} \\
9: & \text{ECR} + \text{ECR} = \text{GN1} + \text{GN} \\
10: & \text{ITL} = \text{VD} + ((\text{VDSV} - 40) \times 0.4) \\
11: & \text{VD2} = \text{VDSV} \times ((2 - \text{VDSV}) \times 50) \times 20, ((2 - \text{VDSV}) \times 10) \times 4 + 10 \\
12: & \text{Z1} = 2 \times \text{XS} \times \text{EPS} \times \text{Z2} \times \text{DCS} \times \text{Z5} \\
13: & \text{Z2} = \text{L} + \text{VDSV} \times \text{ECR} \\
14: & \text{Z4} = 0.5 \times ((\text{ECR} \times \text{Z2}) + \text{VD2} - \text{VDSV}) + \text{Z1} \times \text{Z2} \times 2 \\
15: & \text{Z5} = \text{IDSU} \times \text{VDSV} \times \text{VD} \times \text{Z1} \times \text{Z2} \times 2 \\
16: & \text{ITS} = \text{IDSU} - \text{Z4} - ((\text{Z4} + 2) - 25) \times 0.5 \\
17: & \text{VD[]} = \text{VD} \times \text{VD2} \\
18: & \text{ZP} = \text{J} \times \text{ITL}, \text{ITS} \\
19: & \text{(+J=4)/DONE} \\
20: & \text{J} = \text{J} + 1 \\
21: & \text{L1} \\
22: & \text{DONE} = \text{Z} + \text{ZP} \\
\end{align*}
\]

\[
\begin{align*}
\text{vVDS1[]} & \\
0: & \text{Z} = \text{VDS1}; \text{Z1}; \text{Z2}; \text{Z3} \\
1: & \text{PARD} \\
2: & \text{Z1} = \text{FNC1} \times 0.5 - \text{GN} \\
3: & \text{Z2} = 0.5 \times (((\text{GN} - 1) \times \text{VOFFP}) - \text{GN} \times \text{FNC1} \times \text{X} \times \text{ECR}) + \text{Z1} \\
4: & \text{Z3} = \text{GN} \times \text{ECR} \times \text{X} \times \text{VOFFP} + \text{Z1} \\
5: & \text{Z4} = -2 \times ((\text{Z2} + 2) - \text{Z3}) \times 0.5 \\
\end{align*}
\]

\[
\begin{align*}
\text{vIDS1[]} & \\
0: & \text{Z} = \text{IDS1} \\
1: & \text{Z} = \text{ITL} \times \text{VD} \\
\end{align*}
\]

\[
\begin{align*}
\text{vITL[]} & \\
0: & \text{Z} = \text{ITL} \times \text{VD1}; \text{V2}; \text{Z4}; \text{Z1} \\
1: & \text{PARD} \\
2: & \text{Z1} = ((\text{VOFFP} \times \text{VD}) - 0.5 \times \text{FNC1} \times \text{VD} \times 2 \\
3: & \text{Z} = \text{BETA} \times \text{Z1} \times \text{L} \times \text{VD} \times \text{ECR} \\
\end{align*}
\]

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\( \text{PARD[I]} \)

\[\begin{align*}
\text{0} & : \text{PARD}_I \text{ALPHA} \\
\text{1} & : \text{D} + \text{DI} + \text{DD} \\
\text{2} & : \text{DT} + \text{DLD} \\
\text{3} & : \text{VP2} + (\text{Q} \times \text{MD} + 2 \times \text{K0} \times \text{EPS}) \times \text{DD} + 2 \\
\text{4} & : \text{VOFF} + \text{FIB} + 0.025 - \text{VP2} + \text{ECV} \\
\text{5} & : \text{VOFFP} + \text{VG} - \text{VOFF} \\
\text{6} & : \text{mAP} + (\text{K0} \times \text{EPS} + \text{Q} \times \text{DT}) \times 1 + \text{A} \times \text{API} + \text{K0} \times \text{EPS} + \text{Q} \times \text{DT} \\
\text{7} & : \text{mBP} + (\text{VOFFP} \times \text{API} \times 1 + \text{A} \times \text{API}) + \text{B} \times \text{API} \\
\text{8} & : \text{DT} + \text{DC} \times \text{VG} \\
\text{9} & : \text{ALPHA} + \text{Q} \times \text{ZZ} \times \text{MUN} + \text{L} \\
\text{10} & : \text{CAP} \times \text{K0} \times \text{EPS} + \text{Q} \times \text{DT} \\
\text{11} & : \text{GNC} \times 1 - \text{FNC} \times 2 \\
\text{12} & : \text{BETA} \times \text{Q} \times \text{ZZ} \times \text{MUN} \times \text{CAP} \\
\text{13} & : \text{BETAP} \times \text{BETA} \times \text{GNC} \\
\text{14} & : \text{VOFFPP} + \text{VOFFF} + \text{GNC} \\
\text{15} & : \text{ECR} \times \text{VES} + \text{MUN} \\
\text{16} & : \text{nVGSP} + \text{VG} - \text{VS} \\
\text{17} & : \text{nVGP} + \text{VG} - \text{VD} \\
\text{18} & : \\
\end{align*}\]
B.3. Work Space KIMPOT2

- This work space includes programs that generates the plot of 2-dimensional potential distribution in AlGaAs region under the gate based on the 2-dimensional charge control model.
\[ Z = \text{NSX} \]

\[ \text{VAR} \]

\[ \text{D} = 'V' \times \text{VG} + \text{D} \]

\[ \text{VP} = (Q \times \text{ND} + \text{DD} + 2) \times \text{K0} \times \text{EPS} \]

\[ \text{VS} + \text{VBD} \]

\[ \text{VGB} + \text{VGB} = \text{FB} \]

\[ \text{DLX} = (2 \times \text{K0} \times \text{EPS} \times (\text{VGB} - \text{VGB}) + Q \times \text{ND}) \times 0.5 \]

\[ \text{DSX} = (2 \times \text{K0} \times \text{EPS} \times (\text{VSG} - \text{VGB}) + Q \times \text{ND}) \times 0.5 \]

\[ \text{LP} + \text{DLX} + \text{DLX} \]

\[ \text{III} + \text{SI} \]

\[ \text{JJJ} + \text{SI} \]

\[ \text{XTX} + (\text{LP} + \text{III}) \times 0, \text{III} \]

\[ X + X + \text{XTX} - \text{DSX} \]

\[ Y < 0, \text{JJJ} \times \text{D} + \text{JJJ} \]

\[ \text{XMIN} + (\text{XVX} / \text{XVX}) \times \text{YMIN} + (\text{YVY} / \text{YVY}) \]

\[ 4 + Y \times Y \]

\[ 6 \times \text{VCGX} - \text{VCGX} \]

\[ Z = \text{NSX} \times \text{NSX} \times \text{EQNSX} \]

\[ \text{VP} \]

\[ Z = \text{VXP} + \text{VDP} \]

\[ \text{VP} \]

\[ Z = \text{VXP} \times \text{V2} \times \text{V3} \]

\[ \text{VDP} \]

\[ Z = \text{VXP} \times \text{V2} \times \text{V3} \]

\[ \text{VDP} \]

\[ Z = \text{VXP} \times \text{V4} \times \text{V5} \]

\[ \text{VDP} \]

\[ Z = \text{VNP} + \text{V4N} + \text{V5N} \]

\[ \text{VDP} \]

\[ Z = \text{VXP} \times \text{V5} \times \text{V6} \]

\[ \text{VDP} \]

\[ Z = (\text{VS} - \text{VGB}) \times (\text{VGB} - \text{VRS}) \times (\text{VPS} \times \text{DLSX}) \times \text{LP} \]

\[ \text{VDP} \]

\[ Z = \text{VXP} \times \text{V5} \times \text{V6} \]

\[ \text{VDP} \]

\[ Z = -\text{Q} \times \text{D} \times \text{XP} \times \text{NSX} \times \text{TABLE} \times \text{LOOK} \times \text{XP} \times \text{NS0} \times \text{K0} \times \text{EPS} \]

\[ \text{VDP} \]

\[ Z = \text{VXP} \times \text{V5} \times \text{V6} \]

\[ \text{VDP} \]

\[ \text{XP} + \text{XY} \times (\text{XY} - \text{XY}) \]

\[ \text{VDP} \]

\[ Z = \text{VXP} \times \text{V5} \times \text{V6} \times \text{V7} \times \text{V8} \times \text{V9} \times \text{V10} \times \text{V11} \times \text{V12} \]

\[ \text{VDP} \]

\[ 128 \]
CALCULATION OF CHANNEL CONCENTRATION FOR VD APPLIED

Newton Method Used

Include Source Drain Resistance

VD Step Size .02V, Conv Criterion 1E-3

Para

D+ 'VG' O VG

D+ 'VD' 0 VD=0

VD2+(QxNDxQ+2)+2xKSxEPS

VS=0

VGB=VG-FIB

n

JJ=2

L4: VD=0.05x(JJ-1)

DLDX=(2xKDxEPSx(VD-VGB)+QxND)+0.5

DLSX=(2xKDxEPSx(VS-VGB)+QxND)+0.5

LP=L+DLSx+DLDX

RS=(LS-(2xKSxEPSx(VS-VGB)+QxND)+0.5)+ZZxQxMUNxNSO

RD=(LD-(2xKSxEPSx(VD-VGB)+QxND)+0.5)+ZZxQxMUNxNSO

II=80

JJJ=50

XTA1=((LP=III)xO, III)

X=XTA1-DLSX

Y=(O, JJJ)xO=JJ


VYLY+VY Y

VCXV+VCX

NSX+NSX

LI1:ILC=0

L9: NSXI+NSX+DLNSXI+1E-3xNSX

VCXI+VCXV-(QxDx(NSXI-NSO)+XOxEPS)

ECXI+DFDX VCXI

FNSXV+FNSX NSXI

MUNXI+MINXT ECVI

RXV+MUNX RX NSX

IFI+(VD-VS)+(RS+RD+RXVII+pXI)

GNSI+VCXI-FNSXV+(IFI(RS+RXVI))-DECV

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[37] VCX=VCXV-(Q×D×(NSX-NSO)+KD×EPS)
[38] ECX=DFDX VCX
[39] FNSX=FNNS NSX
[40] MUNX=MUNXT ECU
[41] RXM=MUNRX NSX
[42] IP+(UD-US)+(RS+RD+RXV+O)
[43] GNS+UCX-FNSXV+(IP×RS×RXV)-DECV
[44] DGNX+(GNS-GNS)+DLNSXI
[45] DLNSX+GNS+DGNX
[46] NSXN=NSX-DLNSX
[47] *
[48] *(<((INSX-NSX)+(NSX)×0.01)=1)/DONE9
[49] ILC=0+ILC+1
[50] *(ILC110)/DONE44
[51] *DONE4
[52] DONE44=NSX×0+NSXN
[53] D=UD
[54] *(UD×UDJ)/L41
[55] *DONE4
[56] L41=19
[57] DONE9:
[58] *
[59] MUNX=MUNXT ECU
[60] RX=MUNRX NSX[@X]
[61] ID+1D,(VD-US)+(RS×RD×RV)
[62] *(JJ=101)/DONE4
[63] JJ+JJ+1
[64] XPF×X
[65] *L4
[66] DONE42=Z+NSX
[67] *
[68] *VY(0)}
[0] Z+VY Y1:Z1;Z2
[1] NDP=ND×NC
[2] GP×X×(ED×KB×T)
[3] DELTA=(KB×T×Q)×(((4×GP×NDP)+(1-NDP×4)×0.5)-1-NDP×4)×2×GP
[4] NSO1+ND×DI+DELD
[5] NSO×((2×KO×EPS×ND×(DECV-DEFOT-Delta))×Q)+NSO1×2×0.5)-NSO1
[6] DELTY+DI+NSO+ND
[7] Z1+(Y1×DELTY)×Y1×DI×-Q×ND×(Y-DELTY)×2)+2×KO×EPS
[8] Z2+(Y1×DI)×Y1×O×-Q×NSO×KODEPS)×(DI-Y1+Q×ND×(DI-DELTY)×2)+2×KO×EPS
[9] Z+Z1+Z2

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\text{vvcx(idjv)}

[0] Z*VVCX;V4N;VD3P;V23NN
[1] V4N*+60<((\text{L1})x01xDP)xFOURCX11 V43P
[2] VC4*IFOURS V4N
[3] VDS3P+(VY Y)-(VP Y)-VGB
[4] V23NN+((-1)*((\text{L2}))*FOURCY11 VDS3P
[5] n V6NN+(Q+K0xEPS)x(LP+(\text{L1})x01)\times(70(\text{L1})x01xD=LP)xFOURCX11 NSO-NSX
[6] VC1+VP 0
[7] VC23*IF0URE23 V23NN
[8] n VC23+(-UP1)\times((X+DLSX-LP)x01+2xDP)+(X+DLSX)x01+2xDP)
[9] VC5*V2P
[10] n VC6*IFOURS V6NN
\text{vvg3p(djv)}

[0] Z*VGC3P
[1] Z1+((-DLSX)x01)xX10)xVS-VGB*Q+NDx(X+DLSX)x2)+2xK0xEPS
[2] Z2*(((\text{LX})x1LP)xVD-VGB+(Q+NDx(X-L+DLSX)x2)+2xK0xEPS
[3] Z*Z1+Z2-U2P
\text{vup(djv)}

[0] Z*UP Y;Z1;Z2;Z3
[1] Z1+((Yx0)<Y1D)xVGB-(Q+ND2xK0xEPS)x((Y+DELTY)x2)-(D-DELTY)x2
[2] Z2+((Yx0)<Y1D)xVGB-((Q+NSOxK0xEPS)x(D1-Y))+(Q+ND2xK0xEPS)
[3] Z*Z1+Z2
\text{vup(djv)}

[0] Z*V2P
[1] Z*((V-DxVS)x(X+DLSX)+LP)+VS-VGB

\text{bitplot(djv)}

[0] CH BITPLOT ZZ;A;DATA
[1] LOOP:
[3] LINEPLOT DATA+(2,A)+ZZ
[4] STRING+STRING,(2 1 e3+p5pDATA(1:1),(2 2 e1 -1 0 0),DATA
[5] ZZ+(0,A)+ZZ CH:4+CH
[7] ZZ+(0,A)+ZZ CH:4+CH
[8] +LOOP x0#pCH
\texttt{\textbackslash \textbullet{} FFI(D)\textbackslash \textbullet{}}

[0] \texttt{X FF1 Y1 CH; FF; XX; YY; ZZ; F}
[1] \texttt{Z+(XMIN+(XMAX-XMIN)\times X),(0.1)YMIN+(YMAX-YMIN)\times Y}
[2] \texttt{F+XFMN, ' Z'}
[3] \texttt{F+(F-FMIN)+(FMAX-FMIN) \times Y+1-Y}
[4] \texttt{X+XAB, Y*SC}
[5] \texttt{YY+DAX+(Y\times AS)-DAXF}
[6] \texttt{CH+(x\times A),(A+YY)\times (-A)\times YL \times (A+YY),(A+YY)L(-A)\times YL}
[7] \texttt{ZZ+XX, [0.1]YY}
[8] \texttt{CH BITPLOT ZZ}

\texttt{\textbullet{} TLOOK(D)\textbullet{}}

[0] \texttt{F+TABLE TLOOK X; A; STEP}
[1] \texttt{STEP+=(1+TABLE[1;1])-TABLE[1;1]+1+TABLE[1;1]}
[2] \texttt{A+1+TABLE[1;1]+STEP}
[3] \texttt{TABLE+TABLE, 2 '1 TLOOK}
[4] \texttt{F+TABLE[2; A]+((X-TABLE[1; A]+STEP)\times TABLE[2; A+1]-TABLE[2; A]}
[5]
B.4. Work Space HPPLOT

- This work space includes various subroutines for the generation of terminal graphics and plots in HP plotter.

```
\#HPPLOT(CD)\v
[0] HPPLOT
[1] RESET
[2] HPTEXT TEXT
[3] HPSLICER STRING
\#RESET\v
[0] RESET; B; C
[1] CHARASPECT*1.42 \# HEIGHT TO WIDHT RATIO OF A CHARACTER
[2] SEND 'SRI.36.3t' \# SET THE RELATIVE SIZE OF THE CHARACTERS
[3] A+TEXT 0 WH+AC2) \# WH+AC1) \# WH \# WINDOW WIDTH/HEIGHT IN CHARACTERS
[4] YMAX*9000xWHxCHARASPECT+WH
[5] B+328+(WHxWHxCHARASPECT)xCP)PLOTSIZExYMAX \# C+279+C
[6] Pi1+(Pi1x328),Pi1y+279
[7] SEND 'IP ',(\#PiX),',',(\#PiY),',',(\#B),',',(\#C),',';
[8] Pi2+E, C
[9] U1+(U1xXO), U1y+200
[10] U2+(U2xX400), U2y+0
\#HPTEXTED\v
[0] HPTEXT TX; J; TF
[1] +(\#TEXT=0)\v
[2] J=0
[3] TF+TX(J*4+J);J
[4] +6x(eTF)=0/TF=(eTF)e''
[6] +(\#WH)x3
\#HPWORDED\v
[0] J HPWORD WL; WXY; I
[1] I*FASTER WL \# WL+1+WL
[3] WXY HPLABEL WL
\#FASTER[U]\v
[0] I*FASTER L
[1] I+=/(\#eL)=X \#L
\#SEND[C]\v
[0] SEND X
[1] 1 0 2 2 DARBIN X
```
```
/*HPSLICER[0]*
[0] HPNSLICER X; X
[1] LOOP: XX+(2, X[I+1]; I)*X
[2] X[I+(0, X[I+1]; I)]*X
[3] XX+(0.1+XX*12; 2)*XX
[4] HLINE XX
[5] +LOOPxOweX[I+1]*/
/*HPNSLICER[0]*
[0] HLINE XY; XI; I
[1] XY+HPSCALE XY
[2] X; X
[3] SEND ';' LINETYPE, ': ; PU; PA', (X[I+1; 2]), ': ; PD; PA', (X2+X1), ': ; PU; '
/*HPSCALE[0]*
[0] PXY+HPSCALE DXY; A
[1] LINETYPE=DXYC1; 1) + DXY+ 0, + DXY
[2] LINETYPE= (LINETYPE#1)/$LINETYPE
[3] A+(P2-P1)+U2-U1

/*GRANGE2[0]*
[0] GRANGE2; A
[1] TEXT(<pTEXT>)p'
[2] TEXT22; spA)+A+(XMIN ', $XMIN)
[6] TEXT2; (49-a)+spA)+A+(P = ', P)
[7] TEXT2; (49-a)+spA)+A+(FMIN ', $FMIN)
[8] TEXT2; (49-a)+spA)+A+(FMAX ', $FMAX)
[9] SCREENTYPE TEXT
/*BOX2[0]*
[0] BOX2; PA; PB; PC; PD; PF; PO; PR
[1] OA+100 = POSITION OF MIDDLE HORIZONTAL LINE
[2] SC=400+NN+1 = SC MEASURES THE AMOUNT OF ROTATION
[3] NN=NNX1/NPO+NNX1 = SET THE NUMBER OF POINTS PER LINE TO BE USED
[6] PB=(400-SC), OA = PF+(400-SC), PO
[7] PD+400, 200
[8] LN DLINE STRING=(1, 11; 2)*PA, PC, PD, PB, PA, PO, PR, PD, PB, PP
[9] STRING=(2 3 14 1 "1", 11 2 1, STRING
[10] '0' DWRITE(50-30), 191, 1
[12] 'Y' DWRITE 4,(20+OA), 1
```

\begin{verbatim}
\*PL0TDATA[d]*
[0] PLOTDATA; CP; FA; B; CPMAX; F1; F2; F3; F4; F5; F6; F7; F8; F9
[1] 'GIVE FILENAME' & IUTFilename='i', PLOTLABEL=0
[2] OFUNTI 1 & IUTFilename OFTIE 1
[3] CP+(+DFSIZE 1)[2] & 0
[4] CP+0 & SCALE+ 2 2 =0
[5] & Autoscale RETRIEVE ALL DATA
[7] LOOP: F=0 READ 1, CP+CP+1
[8] & 'F', (CP), '+ F'
[9] S+BRANGE GETDATA F
[10] SCALE(1;2)+SCALE(1;2), S(1;2)
[11] SCALE(1;1)+SCALE(1;1), S(1;1)
[12] SCALE(2;2)+SCALE(2;2), S(2;2)
[13] SCALE(2;1)+SCALE(2;1), S(2;1)
[14] +(CP+CPMAX)/LOOP
[15] OFUNTI 1
[16] & SCALE RECIPE
[17] SCALE(2;1)+SCALE(2;1)-0.1/-SCALE(2;1)
[18] SCALE(2;2)+SCALE(2;2)-0.1/-SCALE(2;2)
[19] PRANGE & PRINT DATA RANGE
[20] 'ENTER GRIDX' & GRIDX=0
[21] 'ENTER GRIDY' & GRIDY=0
[22] TEXT+ 25 50 " "
[23] MODEL+1 & SET LINTYPE
[24] AUTOLINE+1 & MANU(0)/AUTO(1) LINTYPE
[25] HPSCREEN & RESET SCREEN
[26] &
[27] & PLOT CURVES
[28] CP+0
[29] LOOP:
[30] & 'FF', CP+CP+1
[31] SHOWCMD F
[32] GRAPH1 GETDATA F
[33] +(CP+CPMAX)/LOOP1
[34] INIT & SLICER STRING
[35] CURSOR
\end{verbatim}
```
@HPSCREEN{[D3]@}
[0] HPSCREEN;S
[1] " SCALE IS: " @ SCALE @
[2] "LEGEND IS: ",LEGEND " @ STRING
[3] "GRIDX IS: " @ GRIDX @ VECTOR
[4] "GRIDY IS: " @ GRIDY @ VECTOR
[5] @
[6] RESETSCREEN
[7] SCREENTYPE TEXT
[8] MODEL* "1
[9] STRING*GRID 0.04 0.03 @ SET THE GRID ASPECT
[10] @ SET VARIABLE
[12] @
[13] STRING*(24992,(ENCOD 'BOX'), "1 0 0 0 0"),TBX,STRING

@RESETSCREEN{[D3]@}
[0] RESETSCREEN
[1] @ GRAPH PARAMETERS
[2] W+50 @ WINDOW WIDTH
[3] WH+25 @ WINDOW HEIGHT
[4] @
[5] TM+2 @ TOP MARGIN
[6] PH+21 @ PLOT HEIGHT
[7] BM+25-PH+TM @ BOTTOM MARGIN
[8] @
[9] PW+35 @ PLOT WIDTH
[10] LM+8 @ LEFT MARGIN
[11] RM+PW-LM+PW @ RIGHT MARGIN
[12] @
[13] US1*SCALE[1;1],SCALE[2;2]
[14] US2*SCALE[1;2],SCALE[2;1]
[15] PS1+(4,4)+CURSORP(LM),TM
[16] PS2+(4,4)+CURSORP(LM+PW-1),TM+PH-1
@SCREENTYPE{[D4]@}
[0] SCREENTYPE X:A
[1] @ GTYPE O
[2] I+O
[3] @ FASTER XI[1+1;1]
[4] (A+SHORTER XI;J)DGWRITE(CURSORP A,1+"1),1
[5] @*(WH@I)*3

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```