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AN ANALYTICAL INVESTIGATION OF THE COMBINED EFFECT OF GEOMETRIC DEFECTS AND THERMAL GRADIENTS ON TENSILE DUCTILITY

The Ohio State University

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An Analytical Investigation of the Combined Effect of Geometric Defects and Thermal Gradients on Tensile Ductility

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

By

Kavesary S. Raghavan, B.S., M.S.

* * * * *

The Ohio State University
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To My Late Father

Kavesary N. Seshan
Acknowledgements

I would like to express my sincere appreciation to Dr. R. H. Wagoner for his invaluable guidance, advice and encouragement throughout the course of this research. Financial support for this work was provided by the National Science Foundation through a Presidential Young Investigator’s award (DMR-8351486) to Dr. Wagoner and is gratefully acknowledged.

I would like to thank Dr. Kwansoo Chung and Mr. Yong Kim for many helpful discussions and Dr. Muh-Ren Lin for providing the constitutive equation and help with some of my experimental work. Special thanks are also due to all my friends here at Columbus, especially Mr. Vijay Madhav Parthasarathy who helped me with getting this dissertation in good order.
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\( \alpha \) — \((\partial \ln \varepsilon / \partial \ln e)\) parameter indicating test condition (constant load, constant crosshead etc.)

\( \beta \) — constant in constitutive relation denoting magnitude of temperature effect.

\( \gamma \) — \((1/\sigma [\partial \sigma / \partial \varepsilon])\) a measure of strain hardening.

\( \Delta \varepsilon, \Delta \varepsilon^P, \Delta \varepsilon^P \) — true incremental plastic strain.

\( \Delta \bar{\varepsilon}, \Delta \bar{\varepsilon}^P \) — incremental effective strain.

\( \Delta U \) — correct solution vector at the end of each time step.

\( \Delta U^w \) — guess solution at each time step (correct solution at the end of previous time step.)

\( \Delta T \) — temperature rise above the ambient value assumed 26°C.

\( \Delta W \) — incremental work at each step.

\( \sigma \) — true stress.

\( \sigma_1 \) — true stress in the axial direction.

\( \bar{\sigma} \) — effective stress.

\( \sigma_{ij} \) — flow stress tensor.

\( \sigma_0 \) — prestress term in Ludwik law.

\( \varepsilon \) — true strain.

\( \varepsilon_1 \) — true axial strain.

\( \bar{\varepsilon} \) — effective strain.

\( \varepsilon_{ij} \) — plastic strain tensor.
\( e_u \) — uniform strain.

\( \varepsilon_d \) — swift diffuse instability strain.

\( \rho, \rho^* \) — proportional straining parameter \((\varepsilon_1/\varepsilon_2)\).

\( \rho_d \) — density.

\( \eta \) — fraction of plastic work converted to heat.

\( \theta, T \) — temperature.

\( A \) — cross-sectional area.

\( c_p \) — specific heat.

\( e \) — engineering strain \((e=1-\exp(\varepsilon))\)

\( e_f \) — engineering failure strain (measure of ductility)

\( f_0 \) — parameter indicating size of initial geometric defect.

\( \{f\}_e \) — element load vector.

\( \{F\} \) — global force vector.

\( K \) — strength coefficient in constitutive relation.

\( k \) — boltzman constant.

\( [k]_e \) — element stiffness matrix.

\( [K] \) — global stiffness matrix.

\( m \) — \((\partial \ln \sigma / \partial \ln \dot{\varepsilon})\), strain rate sensitivity parameter.

\( M \) — Hill's new parameter controlling shape of yield surface.

\( n \) — \((\partial \ln \sigma / \partial \ln \varepsilon)\), strain hardening parameter.

\( Q \) — heat generation due to plastic work.

\( r \) — \((d\varepsilon_2/d\varepsilon_3)\), plastic anisotropy parameter.
s — engineering stress \( (s=\sigma/(1+e)) \).

\( t \) — time.

\( u \) — correction displacement vector at each iteration.

\( \{u\} \) — element displacement vector.

\( \{U\} \) — global displacement vector.

\( V \) — volume.

\( X \) — \( (\sigma_1/\sigma_2) \), ratio of principal stresses.
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CHAPTER I.
INTRODUCTION.

The ability to plastically deform metal to useful shapes requires an intimate understanding of the process of flow localization[1,2]. The flow localization process is complicated, not only by the large number of parameters influencing it, but also by the number of forms in which it is manifested. The primary parameters affecting flow localization include material behavior, state of stress, lubrication conditions, strain path, press speed and a host of other less known parameters.

Different forms of flow localization have been observed depending on the forming process and process conditions. During bulk forming common forms of flow localization are barreling, which is usually associated with axisymmetric forming modes[3], and shear bands produced by plane strain modes[4,5]. Causes for flow localization are very much dependent on processing conditions. During cold working, flow localization causes may be either friction related[6] or due to flow softening following an exhaustion of strain hardening capacity. This loss of strain hardening may occur due to a number of physical effects such as the achievement of stable deformation textures, dissolution of precipitates or thermal softening[7-9]. Common causes for flow
localization during hot working are chilling at the tool workpiece interface[10] and thermal softening[3]. In secondary forming operations common forms of flow localization include wrinkling of sheet metal during stamping due to compression instability and tearing at the bottom in deep drawing.

The perplexing number of parameters controlling the localization of plastic flow and the myriad forms in which it is manifested have up to now inhibited the use of a comprehensive plasticity approach to solve practical strain localization problems. A number of die tryouts with varying die designs, lubrication conditions and sometimes change of material is often called for to form individual stampings. In recent years, the rising costs associated with such a trial and error based die tryout approach has spawned a lot of research directed at developing a systematic plasticity-based approach to analyze forming operations. The primary objectives in such an approach include an ability to:

a) Predict metal flow during a forming operation.

b) Establish limits of formability, i.e. determine whether it is possible to perform a specific forming operation without causing any surface or internal failures in the deforming material.

c) Predict stresses, forces and energy necessary to carry out the forming operation. This information is
needed for tool design and for selection of appropriate equipment and process conditions.

Two significant advances have been made in the last couple of decades in the analysis of metal forming operations. Forming limit diagrams (FLDs) have been developed and are increasingly used in the press shop to predict the amount of useful deformation that can be imposed on the workpiece prior to onset of localization. A number of experimental methods [11-20] have evolved to construct FLDs, and attempts to predict FLDs from theoretical considerations have also found reasonable success [21-46]. The second significant advance has been the development of numerical techniques to simulate forming operations, in particular the use of finite element methods [47-60]. The major advantage of the finite element method (FEM) is its applicability to a wide class of boundary value problems with little restriction on workpiece geometry.

A number of stumbling blocks do exist today in any predictive analysis of forming operations, the principal hurdles being:

a) Limited knowledge of friction conditions at the tool metal interface.

b) Absence of material models valid over large strain ranges, strain rates and temperatures.

c) Numerical difficulties in solution of nonlinear
equations which generally arise in plasticity problems due to the large strains involved and also due to complicated material models.

This dissertation primarily addresses the question of development of good material models. More specifically, finite element analysis of nonisothermal tensile tests has been performed to quantitatively establish the combined effect of geometric defects and deformation heating on tensile ductility. This study was prompted by an increasing awareness of the detrimental role of temperature gradients on formability[109-128]. In fact, by control of temperature gradients enhanced formability has been obtained in industrial forming operations. Examples of commercial exploitation of this effect include usage of flood lubrication (i.e., continuous spraying of water-based lubricants during forming) for stamping of stainless steel parts[125], line heating[126] and heated die-cooled punch superplastic forming[127,128] and in the development of the ARMCO cold forming process[114].

The results from this work are expected to lead to a better understanding of the flow localization process in tension and to improved material constitutive relations including temperature effects. An offshoot of the present study may lead to improvements in existing tensile test standards. Since the ambient heat transfer conditions affect the eventual ductility at commonly used test rates by
changing thermal gradients in the specimen, consistent tensile test results could be obtained by enforcing either adiabatic or isothermal conditions. These two extreme heat transfer conditions can be achieved by testing at either very high rates or at very low rates, or by altering the surrounding medium to affect heat transfer rates.
CHAPTER II.
LITERATURE SURVEY.

This literature search was directed toward getting an overview of existing knowledge in the analysis of sheet forming operations. This chapter is divided into two sections, the first addressing the various techniques hitherto employed to assess sheet metal formability. Since the major thrust in this work has been the study of the combined effect of geometric defects and thermal gradients on tensile behavior with a view to improving existing constitutive equations, the second section deals with the development of material models. More specifically, the role of various physical effects on onset of instability and flow localization has been reviewed.

1.1 Determination of Forming Limit Diagrams (FLD's)

1.1.1 Experimental Methods

As early as 1963, a study of failure in biaxially stretched sheets by Keeler and Backofen[11] spurred the development of what is known variously today as the Keeler-Goodwin diagram[12,13] in the context of low carbon steels, the Forming Limit Curve or most commonly as the Forming Limit Diagram (FLD). The forming limit diagram is
the loci of the onset of failure of sheet material (as defined by the strain measured in standard grid circles near the failure site) for various ratios of in-plane principal engineering strains \( e_1 / e_2 \) and is finding increasing use in the press shop to assess the sheet formability limit. A typical forming limit diagram taken from the work of Keeler[1] is shown in Fig.(1). A number of experimental testing methods have evolved since the mid sixties to construct FLD's. The common features involved in all these methods include electrochemical etching of circular grid patterns, varying principal strain ratios by lubrication or punch size, and measurement of principal strains near the site of localized thinning.
FIGURE 1. Typical Forming Limit diagram[1]
Mechanical Dome Testing

The mechanical dome testing, also known as rigid-punch stretching[14-16,139], is shown in Fig.(2a). A circular gridded sheet specimen is firmly clamped between two die-plates and then stretched over a rigid hemispherical punch. By varying lubrication conditions principal strain ratios($\rho = e_1/e_2$) ranging from plane strain to biaxial tension can be achieved. FLD's constructed from this test are however dependent on the size of punch used for the stretching experiments[139]. Other problems with this method include curvature of the deformed sheet making measurement of strains difficult and variation of friction conditions at the tool metal interface, making repeatability of tests difficult.

In-Plane Stretching

A schematic of the set up developed by Azrin and Backofen[19] to perform in plane stretching experiments is shown in Fig.(2b). An elongated patch of reduced thickness is produced at the center of the specimen and a circular spacer of polyethylene is placed so as to encircle the patch. This assembly is stretched over a punch as in the mechanical dome test. By varying the patch length to width ratio $\rho$ can be varied from plane strain to biaxial
conditions. The advantage in this technique is that the patch section remains flat during the test allowing filming of the grid section to determine strains accurately and proportional straining conditions can be achieved. Machining, however, may change material properties and surface roughness variations from specimen to specimen may influence the forming limits established by this technique.

Hydraulic Bulge Testing

In this method, a sheet metal membrane is securely clamped between two dies and is plastically deformed by means of oil pressure (Fig. (2c)). Using elliptical dies with increasing aspect ratios the right hand side of the forming limit diagram ($\rho > 0$) can be constructed [17, 140-142]. Hydraulic bulge tests have the characteristics of both mechanical dome and in-plane stretching tests. Curvature and strain gradients are present as in rigid punch tests, though less severe. Interface friction is absent and balanced biaxial stress states are easily achieved.

Marciniak Double Blank Method

The working principle of this technique is illustrated in Fig. (2d). Two blanks, securely clamped along their edges are used in each experiment. The top blank, known as the driving blank has a central hole which facilitates the stretching of the central part of the second blank over the
punch. Elliptical punches with varying aspect ratios are used to construct the right hand side of the forming limit diagram[23,32]. The principal advantages of this method are:

a) The specimen blank is not in contact with any other bodies in the central area. Friction effects are thus eliminated.

b) Sheet remains flat during stretching enabling accurate determination of strains.

c) Proportional straining can be achieved.

Besides the above methods tensile tests can be used to construct the left hand side ($\rho < 0$) of the forming limit diagram by using specimens with different notches.

Forming limit diagrams constructed experimentally may vary depending on forming process, strain measurement practice, and the criterion used to determine the limit strains[20]. Ghosh and Hecker[18] showed that the FLDs obtained from in-plane stretching and mechanical dome tests differ significantly because of different instability considerations. For in-plane stretching deformation is uniform and local necking is not permitted under biaxial tension unless local inhomogeneities are admitted[23]. In mechanical dome tests strain gradients develop immediately as soon as the punch contacts the sheet. In spite of this strain gradient, stability during rigid punch stretching is
prolonged because of geometric constraints delaying the attainment of localized plane strain required for localized necking. FLDs determined from in-plane stretching are therefore much lower than those constructed from mechanical dome testing. Strain measurements are made from electrochemically gridded circles on the test specimen. However, the strain gradient near the broken specimen is usually large. Consequently the size of the grid circle used influences the forming limit. Further if the deforming process is such that the failure site is a curved surface (as in mechanical dome or hydraulic bulge tests) considerable error is introduced in the strain measurement. Two commonly used techniques to determine limit strain are: a) Principal strain measurements from the nearest circle to fracture which has not been affected by localized necking and b) the strain at the onset of a visible neck. Filming of the specimen during forming allows determination of limit strain from an appropriate frame in the second method[86]. However, in both these cases multiple neck growth, groove formation or sub surface void formation may lead to error in limit strain determination.
FIGURE 2a.
Mechanical Dome Test Setup[14]

FIGURE 2b.
In-Plane Stretching[19]

FIGURE 2c.
Hydraulic Bulge Testing[17]

FIGURE 2d.
Marciniak Double Blank Method[23]
1.1.2 Theoretical Prediction of FLDs

Flow localization in sheets occurs as a two stage necking process involving initially a gradual and mild thinning of the sheet, the extent of which is many times the sheet thickness and therefore referred to as a diffuse neck, followed by a rapid localized thinning, leading to eventual failure.

In 1952, Swift\cite{21} proposed that diffuse instability is initiated when the increment in applied effective stress due to geometric softening exceeds that which can be produced by strain hardening. The concept of an effective stress can be easily understood from a reference to the flow theory of plasticity. The effective stress is the parameter describing magnitude of current yield surface. In other words, it represents all possible combinations of the stress tensor for which yielding occurs. For a material conforming to Von Mises yield criterion\cite{73} and assuming plane stress conditions Swift's criterion reduces to the following expression:

\[
\frac{d\overline{\sigma}}{d\overline{\varepsilon}} = \frac{\overline{\sigma}}{Z_d} \tag{1}
\]

where:

\[\overline{\sigma} = \left( \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 \right)^{1/2}\] is the Von Mises effective stress.

\[d\overline{\varepsilon} = \frac{2}{3} \left( d\varepsilon_1^2 + d\varepsilon_1 d\varepsilon_2 + d\varepsilon_2^2 \right)^{1/2}\] is the effective strain.

and \(Z_d\) is a function of the applied stress ratio.
Since substantial amounts of deformation are often obtainable beyond onset of diffuse instability, Swift's criterion is used only to provide a gross approximation of forming limits for materials with minimal rate sensitivity. Theoretical predictions of FLDs are often based on the onset of a localized neck. Hill[22] proposed that the necking mode will shift from diffuse to local when the rate of strain hardening associated with the larger, in-plane, principal strain is just balanced by the rate of geometric softening along a thickness direction. This criterion is valid only when one of the principal in plane strains is negative, providing a plane of zero extension for a local neck to form along the thickness direction. With assumptions of proportional straining, \( \frac{d\varepsilon_1}{d\varepsilon_2} = \frac{\varepsilon_1}{\varepsilon_2} = \rho^* = \text{constant} \), Hill's local instability criterion reduces to:

\[
\frac{d\sigma_1}{d\varepsilon_1} = \sigma_1 (1+\rho^*)
\]  

(2)

where:

- \( \sigma_1, \varepsilon_1 \) are the true stress and strain corresponding to the larger in plane strain direction and \( \rho^* \) is the ratio of the in plane principal strains and is constant under conditions of proportional straining.

Many actual sheet forming operations involve biaxial stretching with both in plane strains positive. Though localized necking prior to ultimate failure is observed during biaxial stretching, no plane of zero extension exists
and Hill's criterion for onset of localized necking is not applicable. Two independent approaches have been taken to solve this problem, both of them retaining the proportional straining assumption.

A. Marciniak Kuczynski Analysis (M-K model)

In 1967, Marciniak and Kuczynski[23] proposed a model, referred commonly today as the M-K model. The main features of the M-K model are that it presumes that the local loss of stability occurs through a continuous process and includes the concept of an initial thickness inhomogeneity to allow a local neck to form. While the model can handle initial defects, both material-related and geometric, it is easiest to visualize the presence of the latter type of defect.

Consider the specimen with an initial thickness groove shown in Fig. (3a) consisting of two distinct regions A of thickness \( t_A \) and a region B of lower thickness \( t_B \). Assuming proportional straining conditions in region A \( (\dot{\varepsilon}_1/\dot{\varepsilon}_2 = \rho^m) \) through the entire loading process, and prescribing an initial strain \( \varepsilon_{1A} \) the principal steps involved in the M-K analysis can be listed as follows:

1. From the initial value of \( \varepsilon_{1A}, \rho^m \) and \( f = t_A/t_B \) the effective strain in region A \( (\ddot{\varepsilon}_A) \) can be determined once a choice of plasticity theory is made. Assuming a constitutive equation to describe material behavior the effective stress in region A \( (\ddot{\sigma}_A) \) can be calculated
from the effective strain($\varepsilon_A^e$). The ratio of principal stresses in region A can be expressed in terms of $\rho^e$ and from a knowledge of $\bar{\sigma}$ the principal stresses $\sigma_{1A}$ and $\sigma_{2A}$ can be uniquely determined.

2. From force equilibrium considerations along the 1 direction $\sigma_{1B}$ is calculated from $\sigma_{1A}$ as follows:

$$\sigma_{1A} t_A = \sigma_{1B} t_B$$  \hspace{1cm} (3a)

3. Kinematical restrictions force $d\varepsilon_{2A}$ to be equal to $d\varepsilon_{2B}$. To determine the state of strain in region B the M-K approach normally involves providing an initial guess for the principal stress ratio($X_B = \sigma_{1B}/\sigma_{2B}$).

Since $\sigma_{1B}$ is known from step 2, guess estimates of $\sigma_{2B}$ and the effective stress $\bar{\sigma}_B$ can be made at this stage. Using the constitutive equation the effective strain $d\varepsilon_B^e$ is computed. The principal strains $d\varepsilon_{1B}$ and $d\varepsilon_{2B}$ can now be determined from known values of the effective strain and the principal strain ratio $\rho^e$ in region B which is related to $X_B$. The principal strain $d\varepsilon_{2B}$ is compared with $d\varepsilon_{2B}$ and iteration is continued by updating $X_B$ till these are nearly equal. When this strain compatibility requirement is satisfied the thickness strain $d\varepsilon_{3B}$ is determined from $d\varepsilon_{1B}$ and $d\varepsilon_{2B}$ assuming the plastic incompressibility criterion.

4. The entire strain field in region B due to an imposed incremental strain $d\varepsilon_{1A}$, an initial defect size
f and assumed strain ratio $\rho_A^*$ in region A is now completely solved. At the end of each incremental step the defect size is updated and this increment is given by:

$$df = d(t_B/t_A) = f(\Delta e_{3B} - \Delta e_{3A})$$  \hspace{1cm} (3b)

Incremental straining by updating $\Delta e_{1A}$ is continued till the strain state in region B is one of plane strain:

$$\rho_B = \Delta e_{1B}/d_{2B} = 0$$  \hspace{1cm} (3c)

When this condition is achieved principal strains in region A ($\epsilon_{1A}$ and $\epsilon_{2A}$) are taken as the limit strains. By varying the initial loading path different points on the forming limit diagram can be established for positive minor strains.

Sowerby and Duncan[24] have provided a very useful geometric interpretation of the M-K analysis using the plane stress yield surface highlighting the main consequences of choice of yield function and size of initial defect on the forming limit(Fig.(3b-d)). Consider the loading of a sheet element with distinct regions A of thickness $t_A$ and B of thickness $t_B$ as in the M-K model. Initially both regions will lie on the same yield surface with B lying below A because $\sigma_{1B}$ is greater than $\sigma_{1A}$ from force equilibrium considerations. During loading region B will reach the yield locus first because of higher stress(Fig.(3b)).
However because $\text{de}^{2B}$ is kinematically constrained to be the same as $\text{de}^{1B}$ point B will move around the yield surface till point A reaches the yield locus along a predefined proportional strain path. When this condition is achieved both A and B will plastically deform on subsequent loading. Since $\text{de}^{2B} = \text{de}^{1B}$ the strain $\text{de}^{1B}$ will be greater than $\text{de}^{1A}$ since B lies below A on the yield surface (Fig. (3c)). The larger strain on B will lead to higher strain hardening and for subsequent loading A and B will be on different yield surfaces. Point B will move along this new yield locus till A reaches this point. Eventually assuming no fracture occurs in the B region it will reach a position on the yield surface corresponding to plane strain (Fig. (3d)). From normality considerations plastic deformation along $\text{de}^{2B} = 0$ and this would imply $\text{de}^{2A}$ to be equal to zero. For a constant strain path ($p^*$ is constant) a zero $\text{de}^{2A}$ would force all deformation to cease in the A region and therefore limit strains can be determined from the uniform strain region A when the grooved region attains a state of plane strain.

The geometrical interpretation illustrates the significance of shape of yield function, the size of initial defect and accuracy of the constitutive equation used on the forming limit obtained from the M-K model.
FIGURE 3a.
Marciniak-Kuczynski Model with thickness defect in region B[23]

FIGURE 3b.
Region A remains elastic when region B is just plastic[24]

FIGURE 3c.
Kinematical restrictions force
\[ \Delta e_{1B} > \Delta e_{1A} \][24]

FIGURE 3d.
Final stage indicating onset of localized necking when region B is in plane strain[24]
The original M-K analysis was developed assuming a Von-Mises yield function and the material constitutive equation took into account only the influence of strain hardening. A body of literature has developed revealing shortcomings associated with the original M-K model when compared with experimental data and modifications have been made assuming different yield functions and improved constitutive equations\[25-46\]. Extension of the M-K model to include alternative yield functions have been made by Parmar and Mellor\[25\] and others\[26-28\], different types of imperfections\[29,30\], effect of rate sensitivity\[31,32\], effects of plastic anisotropy and yield surface shape\[33-35\].

B. Storen and Rice Model:

The second approach due to Storen and Rice\[36\] invokes no imperfection in the thickness direction to cause localized necking. Instead it assumes the development of a vertex at some corner of the yield surface, a process allowed by crystal plasticity considerations of multiple slip in polycrystalline materials\[37,38\]. The normality condition would allow a bifurcation to form in the presence of a vertex and permit a localized neck to be formed.

Theoretical and experimental FLDs discussed up to this point are determined for proportional strain paths, i.e. the
ratio of major to minor strain is maintained constant at any stage of deformation. Industrial stampings of complex shape often involve multi-stage forming operations and the linear strain path assumption is no longer true. Muschenborn and Sonne[39] have proposed two empirical methods to determine FLDs for complex strain paths assuming that instability occurs at critical values of the effective strain(or thickness strain) determined by the strain increment ratio in the final stage of deformation. In recent years interest has been generated in this area in view of the significance of strain paths on ultimate formability[40-46].

1.2 Process Modeling of Metal Forming Processes:

Numerical modeling of forming operations has gained increasing significance in recent years primarily due to the advent of the finite element method[47-54]. It is indeed foreseeable that in the next few years this approach will be used for:

a) Practical design of metal forming dies.

b) Optimization of preform shapes for cold and hot forging.

c) Prediction of local microstructures and properties of formed parts from the calculated distributions of strain strain rate and temperature.

d) Optimization of metal forming process sequences for desired metal flow and product microstructures.

A number of approximate methods such as 'Slip Line
Field Analysis', 'Slab Analysis', 'Upper and Lower Bound Analysis' have been used in the past to analyze forming processes. These have been useful in predicting forming loads, overall geometry changes of the workpiece, qualitative modes of metal flow and determining optimum process conditions. The advantage of the finite element method over these approximate methods is its versatility in the range and complexity of boundary value problems that can be solved and higher accuracy in determining quantitatively the effects of various parameters involved in the process of metal flow.

The essence of the finite element method can be broken down into five stages\[47,48\]:

Stage 1. Formulation:

Description of the problem in terms of a mathematical model. This results in a set of governing equations which the solution should attempt to satisfy, namely the equilibrium, compatibility and material constitutive behavior. These equations are usually given as a combined statement employing a virtual work principle, energy principle or a weighted residual method.

Stage 2. Discretization

The domain over which the problem is defined is broken up into a finite number of elements to obtain a finite element mesh. The process of discretization is done to reduce the infinite degrees of freedom for the field
variable to a finite number of unknowns, the nodal values. Appropriate interpolation functions are then used to determine the approximate field variable at any point within an element from nodal values.

Stage 3. Element Characteristics:
The variational principle is employed in turn with each element to obtain element stiffness and force. The resulting equations take the following form:

\[ [K]_e \{u\} = \{f\}_e \]  \hspace{1cm} (4a)

where:

\[ [K]_e \] is the element stiffness which contains the constitutive behavior.

\[ \{u\} \] is the unknown displacements at the nodal points.

\[ \{f\}_e \] is the element force.

Stage 4. Assembly:
The element stiffness and force are globally assembled over the whole domain using the element connectivity to give the global stiffness and force.

\[ [K]\{U\} = \{F\} \]  \hspace{1cm} (4b)

Stage 5. Solution:
Boundary conditions are now imposed and an appropriate solution method is used to obtain the nodal displacements. Once these are known secondary quantities such as stresses and force at specified points in the domain can be determined.

The use of finite elements in plasticity problems has
been limited because of the problems involved:
1. Large digital computation times and large storage capacities are required for implementation of large non-linear problems.
2. Magnitude of strains involved is large leading to geometric nonlinearities.
3. Friction characteristics at the tool metal interface are usually not well known.
4. Material models valid for large strains are generally not available.

Early finite element approaches in plasticity employed an infinitesimal definition of strain and the Cauchy stress[49]. More refined finite plasticity definitions of strain have since been developed by Lee[50], Hibitt, Marcal and Rice[51] and Mcmeeking and Rice[52]. Currently, a number of formulations based on Lagrangian, updated Lagrangian or convected coordinate framework have been developed using material models based on either elasto-plasticity, rigid-plasticity or rigid-viscoplasticity theories[53-60]. In the current work a rigid viscoplastic finite element formulation using a convected coordinate framework developed by Wang[53] has been used to simulate the non-isothermal tensile test. More details regarding this formulation will be provided in a subsequent section.
2. Development of Material Models:

The accurate prediction of material response during a forming operation requires reliable constitutive relations. Material behavior under actual processing conditions depends upon a number of factors which include strain hardening, strain rate hardening, temperature gradients, strain paths, response to abrupt changes in strain path, structural changes such as phase transformations and recrystallization. In recent years, a considerable amount of research has been directed to develop good material models[61-127]. Most of the work has been on the development of phenomenological models, though some attempts have been made to extend crystal plasticity to explain macroscopic behavior[61-69].

In a phenomenological approach two requirements are necessary to develop good material models.

a) A reliable constitutive relation relating the effective stress to the effective strain, usually obtained from tests involving simple stress states.

b) A proper form of the flow theory of plasticity is necessary to extend test results from simple stress states to other more complex states[71,72]. The primary function of the flow theory is to enable reduction of the generalized stress tensor to a single effective stress scalar through the yield function and by invoking the concept of
equivalence of plastic work determine a single effective strain scalar to describe the strain response. A number of yield criteria have been proposed [73-77] over the years, but the technologically important ones are those due to Von-Mises [73] and Hill [73,74].

A well developed material constitutive relation must embody all the primary physical effects involved in determining the flow response of the material to an applied stress. Since tensile tests are commonly used to assess material behavior, considerable attention has been directed toward understanding the flow localization process in tension in an effort to capture all the physical effects involved in the constitutive relation. In a general sense, the constitutive response of any material depends on:

$$\bar{\sigma} = f(\bar{\varepsilon}, \dot{\varepsilon}, T, s, h)$$  \hspace{1cm} (5)

where:

- $\bar{\sigma}$ effective stress
- $\bar{\varepsilon}, \dot{\varepsilon}$ effective strain and strain rate
- $T$ Temperature
- $s$ Microstructure (grain size, metallurgical defects....)
- $h$ Prior history (texture, transient loading....)

In this section the role of different physical effects on onset of instability and on flow localization in tension will be examined.
2.1 Role of Strain hardening:

The primary role of strain hardening is on the onset of tensile instability, with a smaller impact on post uniform elongation.

A. Representation of Strain Hardening in Constitutive Law

Different forms of strain hardening law have been found to be valid for different materials. Common representations of strain hardening behavior include:

\[ \sigma = K e^n \] (Hollomon Power Law[79]) \hspace{1cm} (6a)

\[ \sigma = A(e+e_0)^b \] (Swift Law[21]) \hspace{1cm} (6b)

\[ \sigma = \sigma_0 + \delta e^d \] (Ludwik Law[80]) \hspace{1cm} (6c)

\[ \sigma = F(1-G\exp(he)) \] (Voce saturation Law[81]) \hspace{1cm} (6d)

B. Influence of Strain Hardening on Onset of Instability:

The criterion describing onset of instability in uniaxial tension for a material which is only influenced by strain hardening is due to Considere[78], according to which instability occurs at load maximum. For a material in which flow stress depends only on strain, this criterion may be written as follows:

\[ \frac{d\sigma}{d\varepsilon} = \sigma \] \hspace{1cm} (7a)

This criterion states that an instability occurs when the load increase due to strain hardening is just equal to the load decrease due to geometric softening. A simple geometrical construction of the Considere criterion to determine the engineering strain at the onset of instability is shown in Fig.(4).
FIGURE 4. Geometric method to determine uniform strain using Considere's criterion[78]
For a Hollomon type power law hardening material[79] the uniform strain($\varepsilon_u$) corresponds to onset of instability as given by the following equation:

$$\varepsilon_u = n$$

(7b)

C. Influence of Strain Hardening on Flow Localization:

Strain hardening does not have a significant impact on post uniform behavior. For rate insensitive materials however, the onset of instability determines the eventual ductility.

D. Parameters Influencing Strain Hardening Coefficient:

The strain hardening coefficient has been found to vary with strain path, and with abrupt changes in strain path and temperature, the extent being dependent on the material tested. Kleemola et al.[82] tested specimens of steel, copper and brass in uniaxial and biaxial tension. Steel showed the same rate of strain hardening in both uniaxial and biaxial tension but for copper and brass the work hardening rates were significantly different in the two stress states. Wagoner[83-85,87] also found significant differences in plane strain and tensile work hardening rates for aluminum and brass. Modification of Hill's flow theory[75] was found necessary to allow extension of tensile test results to other stress states[86].
2.2 Role of Strain Rate Sensitivity:

The primary effect of strain rate sensitivity is to inhibit the flow localization process and lead to enhanced ductility, though it also influences the onset of instability slightly.

A. Representation of Strain Rate Effect in Constitutive Law

The strain rate sensitivity ($m = d\ln \sigma / d\ln \dot{e}$) is commonly incorporated in constitutive relations in either a multiplicative or additive manner:

$$\sigma = K f(\dot{e}) (\dot{e})^m \quad \text{(Multiplicative Law)} \tag{8a}$$

$$\sigma = K [f(\epsilon) + m \ln(\dot{e})] \quad \text{(Additive Law)} \tag{8b}$$

B. Influence of Strain Rate on Onset of Instability:

Extension of the idea of instability occurrence at the load maximum for materials exhibiting strain and strain rate hardening was originally made by Hart[88]. The onset of instability in this case is given by:

$$\gamma < 1 - m \alpha \quad \text{(9a)}$$

where:

$$\gamma = (1/\sigma) (\partial \sigma / \partial \epsilon)$$

is the strain hardening coefficient.

$$m = (\partial \ln \sigma / \partial \ln \dot{e})$$

is the strain rate sensitivity

$$\alpha = d\ln \dot{e} / d\epsilon.$$ 

For instability occurrence when the rate of crosssection reduction at an imperfection site begins to decrease at a faster rate than the rest of the specimen the parameter $\alpha = 1$ and Hart's criterion reduces to:

$$\gamma < 1 - m \quad \text{(9b)}$$

Ghosh[89] has since argued that this criterion is valid only
for defects which are material-related and not for geometric defects. His criterion can be expressed as:

\[ \gamma < \frac{1-m}{(1-\delta \ln \alpha_0/\delta \ln \alpha)} \] (9c)

In the presence of initial geometric defects, Ghosh's criterion predicts deformation to be unstable from the very beginning. Simplified one dimensional neck analysis in the presence of geometric defects and material-related defects support Ghosh's contention that instability occurs from the very beginning in the presence of a geometric defect.

Other workers have developed instability criteria including strain and strain rate hardening effects, but instead of using area reduction at the imperfection site at a faster rate than in the rest of the material as a basis to determine onset of instability, they have used other choices such as strain gradients or gradients in dislocation density or differences in local length[90-94].

C. Influence of Strain Rate on Flow Localization

A number of simplified neck growth analyses have been developed which take into account both strain hardening and strain rate sensitivity effects. Most of these assume the length of the neck in the axial direction is much greater than the diameter of the bar (usually referred to as the long wavelength approximation). Two distinct types of defects (geometric/deformation) have been identified[95] and flow localization studies have been performed assuming either the presence of an initial geometric defect or an
initial deformation defect.

A geometric defect is essentially a reduced section induced by machining and necking analyses in the presence of initial geometric defects are based on axial force equilibrium considerations in both the homogeneous uniform region and the inhomogeneous defect site[89,92,97].

\[ \sigma_h A_h = \sigma_i A_i \]  
\[ (10) \]

where:

\( \sigma_h \) and \( A_h \) are the axial stress and crosssection area in the homogeneous region and \( \sigma_i, A_i \) are the corresponding variables in the inhomogeneous defect site. For a power law strain and strain rate hardening material the force equilibrium relation can be expressed in terms of \( \varepsilon_h, \varepsilon_i \) and using numerical integration with limit of \( \varepsilon_i \) approaching infinity Ghosh[89] determined the failure strain from the eventual value of the homogeneous strain

\[ e_f = (1-f_0^{1/m})^{-m/(1-\gamma)} - 1 \]  
\[ (11a) \]

where:

\( e_f \) — Engineering failure strain

\( f_0 = (A_{oh} - \delta A_{oh})/A_{oh} \) is a measure of initial defect size.

The deformation defect on the other hand can be thought of as a reduction in crosssection at a defect site caused by a prestrain in the region. For a constant load creep test using a material which exhibits no strain hardening after onset of neck, Hart[88] obtained an expression for the difference in crosssection in the defect site and the
homogeneous region for any stage in the deformation process in terms of initial conditions. A more general expression for failure strain has since been obtained\[91,96]\n
\[ e_f = (1-f_0)^{-m/(1-\gamma)} - 1 \]

Nichols\[96\] has compared the failure strains obtained from both geometric defect analysis and deformation defect analysis with experimental data from Woodford\[98\] and concluded that a geometric defect analysis considering defect sizes of 1-2% led to the best agreement with experimental results. The long wavelength analyses discussed so far neglect triaxial state of stress in the neck region. Nevertheless, work by Hutchinson et al.\[99\], have shown using a Bridgeman correction to account for the triaxial stress state that neck growth assuming long wavelength approximation was faster and provided a safer estimate for the failure strain.

The above analyses are applicable strictly only for round bar tensile specimens. For sheet tensile tests the diffuse neck is initiated when the effective stress hardening rate is just balanced by geometric softening. The biaxial stress state in sheet tensile tests must be considered to predict instability strains. Ghosh\[100,101\] has numerically studied the process of flow localization in materials exhibiting both strain and strain rate hardening using a finite difference approach to solve the one-dimensional axial force equilibrium problem. Biaxial
effects were included by relating trajectory stresses and strains to the corresponding axial quantities. He showed that rate sensitivity influences post uniform elongation, strain to maximum load and also ductility increased with increasing $n$ for rate insensitive materials and that a geometric defect grows from the very beginning, unlike deformation defects which grow only after the Hart[88] instability strain is reached. More recently, Semiatin et al.[102] have modified the one-dimensional finite difference formulation of Ghosh by using a correction factor to account for biaxiality rather than converting trajectory stresses and strains to axial quantities. The resulting formulation is claimed to be much simpler, while maintaining the same accuracy in the results. Chung and Wagoner[103] have since analysed the effects of strain hardening and strain rate hardening by solving the actual two dimensional equilibrium problem using a rigid visco-plastic finite element formulation and have provided quantitative relations for the ductility for various combinations of $n$ and $m$.

D. Parameters Influencing Strain Rate Sensitivity

The rate sensitivity($m$) is commonly determined from continuous tensile tests, jump tests or stress relaxation tests. Because of the different characteristics specific to each test method, there is considerable variation in measured $m$ values[104]. Wagoner[105] has developed an analytical method to measure $m$ from a single tensile test by
mathematically deconvoluting strain hardening and strain rate hardening effects. A number of difficulties have arisen in the past in understanding the rate sensitivity effect because $m$ varies with strain, strain rate, strain path, and temperature; the extent being dependent on material[108,97,107]. Some of these have been resolved by development of new laws relating rate sensitivity to strain rate[106]. However, the importance of strain path in establishing the effective rate sensitivity has still not been identified and should this be significant, further refinements in the flow theory of plasticity may be necessary in order to extend tensile test results to commercially important stress states.

2.3 Temperature Gradient Effects:

Relatively little work has been done to determine the influence of temperature gradients on the onset of instability and flow localization. A number of experimental studies have been made in recent years reporting significant rise of temperature during tensile testing and in actual forming operations leading to a considerable lowering of ductility[112-114,116,118,119,121]. Sachdev and Hunter[112] conducted tensile tests at $3 \times 10^{-3}$/s and $1.5 \times 10^{-2}$/s to determine the temperature effect in plain carbon steels, high strength low alloy steels (HSLA) and dual phase steels. Dual phase steel exhibited the maximum temperature effect among the three steels tested and temperature rises of $21^\circ C$
at uniform elongation and around 76°C at fracture were measured at the higher testing rate. In this section previous work done on understanding the influence of temperature gradients on onset of instability and flow localization will be reviewed.

A. Representation of Temperature Effect in Constitutive Law

Zener and Hollomon[109] first recognized the effect of thermal softening and introduced a parameter \( Z \) to reflect the exponential dependence of flow stress on temperature.

\[
\sigma = f(Z) = \dot{\varepsilon} \exp(-\Delta H/kT) \quad (12a)
\]

Other representations include those due to Lin and Wagoner[113,120]. For interstitial free steel[I.F.] they found that the temperature dependence of flow stress took the following form, as did Kleemola et al.[115]:

\[
\sigma = f(\varepsilon, \dot{\varepsilon})(1-\beta(T-T_0)) \quad (12b)
\]

while stainless steel exhibited a temperature dependence of the form:

\[
\sigma = K(1-A\exp(B\varepsilon))(\dot{\varepsilon}/\dot{\varepsilon}_0)^{aT} + b(T/T_0)^\beta \quad (12c)
\]

For aluminum killed steels, Ayres[119] found the temperature dependence of flow stress could be included in the strain and strain rate effects.

\[
\sigma = K(\varepsilon^n(T))(\dot{\varepsilon}^m(T)) \quad (12d)
\]

B. Influence of Thermal Gradients on Onset of Instability:

Kleemola and Ranta Eskola[115] measured stress strain curves of steel and 70:30 brass at various temperatures and strain rates. They found that uniform strains decreased
from 0.20 to 0.187 when comparing isothermal tests to results in air at a rate of $6 \times 10^{-2}/s$ for steel and from 0.412 to 0.408 for brass. At lower rates the effect of deformation heating was less pronounced and beyond $10^{-4}/s$ both steel and brass exhibited the same uniform elongation for both isothermal and tests in air. Korhonen and Kleemola[116] numerically analyzed sheet steel tensile tests using a constitutive law developed earlier[115]. Considering the onset of diffuse necking to be the limit of uniform strain, they modified Swift's criterion[21] to include temperature effects. For tests conducted at constant crosshead speeds their criterion for onset of diffuse instability is given by:

$$
\tilde{\varepsilon}_d = \frac{n}{R(1+m)+\beta(d\theta/d\varepsilon)} \frac{1}{1-\beta\theta}
$$

where:

$\tilde{\varepsilon}_d$ is the uniform strain

$n$ strain hardening parameter.

$m$ strain rate sensitivity.

$R$ normal anisotropy parameter.

$\beta$ constant obtained from constitutive relation.

$\theta$ temperature.

Expressions for rise in temperature in terms of effective stress, effective strain, and thermal properties can be obtained from solutions of the heat conduction equation for different heat transfer conditions in combination with the
material constitutive relation. This gives rise to a transcendental equation from which the uniform strain can be obtained approximately. The calculated values of uniform strain were found to be close to those obtained experimentally by Kleemola et al.[115]. These authors also calculated temperature profiles and predicted stress-strain curves in the uniform strain region. Ferron et al.[117] have developed instability criteria incorporating temperature effects both in the presence and absence of heat sinks. Their analysis is based on the load maximum criterion and is applicable strictly for round bar tensile specimens. In the presence of initial geometric defects their instability criterion can be written as:

\[
\frac{\delta A}{\dot{A}} = \frac{1}{m} (\gamma-1+m)\delta (\ln A) + \frac{mQ}{kT} \delta (\ln T) - \delta (\ln A_o)
\]  \hspace{1cm} (13b)

Their principal contention is that in the presence of heat sinks at the ends of the specimen, a number of active necks may be formed and failure occurrence will be at the place offering the most favorable balance between the original size of defect and proximity to the center of the specimen. In the absence of sinks failure occurrence will be at the site corresponding to largest initial size of defect.

More recently, Gao and Wagoner[118] have developed instability criteria incorporating temperature effects using constitutive relations for interstitial free steel [I.F. Steel] and stainless steel[S.S 310] which
simultaneously incorporate strain, strain rate and temperature effects[113,120]. Their analysis is an extension of Hart's[88] approach for instability in round bar tensile specimens and the expression for uniform strain is given as:

\[ \varepsilon_u = n/(1+m + \frac{\beta}{(1-\beta T)} \frac{\partial T}{\partial \varepsilon}) \]  

(13c)

As in the work of Korhonen et al.[116], these authors find good correlation between theoretical predictions and experimental measurements of uniform strain for both I.F steel and stainless steel[113,120].

C. Influence of Thermal gradients on Flow Localization:

The principal role of temperature gradients in accelerating the flow localization process and thereby lowering eventual ductility has been recognized in recent years. Ferron et al.[117] found during tensile testing of stainless steel in air, that the total elongation dropped from about 28% to about 22% over a strain rate variation from $8 \times 10^{-3}$ to $8 \times 10^{-1}$ s. The same steel tested in water exhibited an ultimate elongation of 29%, independent of the strain rate. These results reveal the detrimental effect of temperature gradients on tensile ductility. In air, at higher rates little time is available for heat generated during deformation to be conducted or convected, leading to the development of pronounced temperature gradients. This explains the lowering of ductility with increasing rate in air. In water, however, thermal gradients are largely
suppressed and consequently ductility is independent of rate and improved relative to results in air. Ayres[119] found during tensile testing of 1008 A.K (aluminum-killed) steel that the ductility dropped from about 54% in water to about 40% in air. Surprisingly, A.K steel deformed in air exhibited no differences in ductility over a strain rate variation from $10^{-1}/s$ to $10^{-3}/s$ as one would have expected due to thermal gradient effect. This behavior was ascribed to a rising rate sensitivity with rate, which increases post uniform elongation as suggested earlier[107] and offsets the thermal softening effect appropriately. Lin and Wagoner[113,120] have experimentally investigated the thermal effect on interstitial free steel[I.F.] and stainless steel(S.S. 310). They measured a temperature rise of over $120^\circ C$ for stainless steel which has relatively poor thermal properties and about $75^\circ C$ for I.F. steel at strain rates of $10^{-1}/s$ in air and found significant influence of temperature gradients on flow localization.

Very little analytical work has been done to analyze the effect of temperature gradients on post uniform elongation. Gao and Wagoner[118] have been able to predict temperature distributions from measured strain distributions for I.F. steel and stainless steel during necking. Analytical studies of temperature and strain distributions have been made by Wada and Nakamura[121]. By assuming a simple relation for strain rate as a function of applied
stress and temperature, ignoring strain hardening effects, these authors have studied the plastic deformation of aluminum under constant average strain rate conditions. Their calculations showed that decreasing the average strain rate, increasing the ambient temperature or using shorter specimens led to higher uniformity in the deformation. Fressengeas and Molinari[122] have studied the influence of inertia and thermal effects on the localization of plastic flow using a one dimensional model of the tensile test assuming the long wavelength approximation. They estimate significant reductions in ductility with higher imposed thermal gradients but do not show any experimental comparison. Semiatin et al.[123] have performed a one-dimensional finite difference analysis of non isothermal sheet tensile tests incorporating biaxial stresses using a correction factor. To solve the coupled deformation and heat conduction problem, Bishop's approach[110] involving solution of equilibrium deformation problem and the heat equation in separate time steps was employed. These authors have obtained reasonable correlations with experimental work done by Ayres[119] for A.K. steel and suggest that the effect of deformation heating primarily influences post uniform necking. In the present dissertation the role of deformation heating has been analyzed for I.F. steel, incorporating temperature effects within a two dimensional rigid viscoplastic finite element formulation[53,56] using
measured temperature distributions. More recently Kim and Wagoner[124] have solved the two dimensional coupled deformation and heat transfer problem numerically. Their work confirms the semi-empirical results obtained in this work.
CHAPTER III
THEORETICAL BACKGROUND

In this chapter, the theoretical background of the finite element formulation used in the present study along with a description of the material model employed is presented.

1. FINITE ELEMENT FORMULATION:

A rigid viscoplastic finite element formulation employing an incremental work principle was used to simulate the tensile test.

1.1 INCREMENTAL WORK PRINCIPLE:

The incremental work done at each step of incremental straining can be expressed in terms of effective stress and incremental effective strain as:

$$
\Delta W = \int_V \bar{\sigma} \Delta \bar{\varepsilon} \, dV
$$

(14a)

Stiffness equations are derived from this work principle to give:

$$
\frac{\partial \Delta W}{\partial \Delta \bar{u}} = \int_V \bar{\sigma} \frac{\Delta \bar{\varepsilon}}{\Delta \bar{u}} \, dV = F(t_0 + \Delta t)
$$

(14b)

where:

- $\bar{\sigma}$ — Effective true stress
- $\Delta \bar{\varepsilon}$ — Incremental effective true strain
- $\Delta \bar{u}$ — Incremental displacement vector from time $t_0$ to $t_0 + \Delta t$
The principal assumption made in the formulation is that during a small time increment the ratio of the principal strain rates is constant. This assumption allows a unique determination of incremental effective strain $\Delta \varepsilon$ from incremental displacement vector $\Delta u$ at the end of each time step. It should be noted that the above assumption does not hold in the context of the usual flow theory of plasticity since the effective strain depends on strain path. The formulation developed above can be considered as being based on an incremental deformation theory of plasticity, briefly discussed below.

1.2 INCREMENTAL DEFORMATION THEORY OF PLASTICITY

Let $A_0$ represent the initial sheet domain at time $t=0$ and let $A_1$ and $A_2$ correspond to the deformed shape of the sheet at time $t$ and time $t_0 + \Delta t$ as shown in Fig.(5). Let $g_\alpha$, $G_\alpha$, $G^\alpha$ for $\alpha=1,2$ be the convected base vectors at time $t=0$, $t=t$ and $t=t+\Delta t$ respectively. The incremental principal stretch from time $t=t$ to time $t+\Delta t$ is determined from a consideration of the convected metric tensors at time $t$ and $t+\Delta t$ respectively.

$$\Delta \lambda_{1,2} = \left[(G^\alpha_{\beta \gamma} G_{\alpha \beta \gamma}) + \left((G^\alpha_{\beta \gamma} G_{\alpha \beta \gamma})^2 - 4 \frac{G^\alpha}{G}\right)^{1/2}\right]^{1/2}$$

(15a)

where $G_{\alpha \beta}$ and $G$ are the convected metric tensor and its determinant respectively at time $t=t$ and $G_{\alpha \beta}$ is the inverse
of $G_{\alpha\beta}$.

Once the principal stretches $\Delta \lambda_{1,2}$ are known the corresponding incremental principal strains are determined from:

$$\Delta \varepsilon_{1,2} = \ln(\Delta \lambda_{1,2}) \quad (15b)$$

With the assumption stated earlier, the incremental effective strains are determined from the incremental principal strains by reference to the yield theory and stresses can be determined once the constitutive law is known. The choice of yield theory and constitutive law used in this study are discussed in detail in a later section of this chapter.

1.3 NUMERICAL PROCEDURE:

The system of equations in (3) are nonlinear and a full Newton Raphson scheme is utilized to determine the solution $\Delta y$ at current step from the known solution at the previous step $\Delta u^*$. At the very first step the corresponding elastic solution is used as the initial guess.

$$\Delta y = \Delta u^* + u \quad (16a)$$

Iterations are carried out within each time step to determine correction vector $u$ until the fraction norm $N_f$ is less than a prescribed tolerance:

$$N_f = \frac{||u||}{||\Delta u^*||} < 10^{-6} \quad (16b)$$

A number of numerical schemes are employed to increase effectiveness of the iterative procedure. These include reduction of step size, use of weighted trial solutions and
modification of boundary conditions when $N_f$ exceeds a prescribed number.

More complete details regarding the formulation and convergence scheme used has been presented elsewhere by Wang[53] and Chung & Wagoner[56].
FIGURE 5. Sheet domain at different times in the deformation showing the convected base vectors[53]
2. MATERIAL MODELING:

2.1 CONSTITUTIVE RELATION

Commercial grade interstitial free steel (I.F. steel) produced by ARMCO Steel Corporation was the material used in this study. The composition, mechanical and thermal properties of I.F. steel are provided in table 1. This material is a nearly carbon-free steel to which small amounts of titanium and niobium are added to tie up residual carbon and nitrogen as carbides and nitrides. The resulting microstructure is a ferrite matrix which has virtually no interstitial atoms. The grain size of the material studied was determined to be approximately 20-25 microns.
Table 1—Properties of Armco I.F. steel

<table>
<thead>
<tr>
<th>Chemical composition</th>
<th>Mechanical/Thermal Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-0.008</td>
<td>Yield strength: 140 Mpa</td>
</tr>
<tr>
<td>Mn-0.29</td>
<td>Tensile strength: 326 Mpa</td>
</tr>
<tr>
<td>P-0.008</td>
<td>Elongation (2 inch gage): 45%</td>
</tr>
<tr>
<td>S-0.012</td>
<td>Density: 7.86 gm/cm³</td>
</tr>
<tr>
<td>Si-0.011</td>
<td>Specific Heat: 0.108 cal/gm/°C</td>
</tr>
<tr>
<td>Al-0.049</td>
<td></td>
</tr>
<tr>
<td>N-0.0066</td>
<td></td>
</tr>
<tr>
<td>Nb-0.055</td>
<td></td>
</tr>
<tr>
<td>Ti-0.13</td>
<td></td>
</tr>
</tbody>
</table>
LIN & WAGONER[113], based on a series of isothermal tensile tests over a range of strain rates, derived the following constitutive equation:

$$\sigma = K(\varepsilon + \varepsilon_0)^n(\dot{\varepsilon}/\dot{\varepsilon}_0)^m(1-\beta \Delta T)$$  \hspace{1cm} (17)

where:

$$K = 566.3 \text{ MPa} \quad \dot{\varepsilon}_0 = 0.002$$
$$\varepsilon_0 = -0.014 \quad \beta = 0.0012$$
$$n = 0.219 \quad \text{(Strain Hardening Parameter)}$$
$$m = 0.018 \quad \text{(Strain Rate Sensitivity factor)}$$
$$\sigma \quad \text{- True Stress}$$
$$\varepsilon, \dot{\varepsilon} \quad \text{- True plastic strain and strain rate}$$
$$\Delta T \quad \text{- Temperature rise above ambient temperature (26°C)}$$

The overall standard error of fit was found to be 3.7 MPa over a domain of 273 K < T < 358 K, 0.02 < \varepsilon < 0.35 and 10^{-5}/s < \dot{\varepsilon} < 10^{-1}/s.

2.2 CHOICE OF YIELD FUNCTION:

Expressions for definitions of effective stress and effective strain are obtained from Hill's[75] new theory.

$$\bar{\sigma} = \left[\frac{1}{2(1+r)}\right]^{\frac{1}{M}} \left[|\sigma_1+\sigma_2|^M+(1+2r)|\sigma_1-\sigma_2|^M\right]^{\frac{1}{M}}$$  \hspace{1cm} (18a)

$$\Delta \varepsilon^e = C_3 \left[(\Delta \varepsilon^e_1+\Delta \varepsilon^e_2)^{\frac{M}{(M-1)}} + C_4 (\Delta \varepsilon^e_1-\Delta \varepsilon^e_2)^{\frac{M}{(M-1)}} \right]^\frac{(M-1)}{M}$$  \hspace{1cm} (18b)

where
\[ C_3 = \frac{1}{2} \left[ 2(1+r) \right]^\frac{1}{\bar{M}} \quad \text{and} \quad C_4 = (1+2r)^{-\frac{1}{(M-1)}} \quad (18c) \]

In Eqns. 18(a-c), \( r \) is the normal anisotropy coefficient and \( M \) is HILL'S[75] new index describing the shape of the yield surface.

For I.F steel an \( M \) value of 2, and an \( r \) value of 1.5 was used as measured in previous studies by Wagoner[84].

For the case of uniaxial tension the effective stress reduces to the axial stress and the material constitutive relation(Eqn.(17)) obtained from uniaxial tensile tests can be substituted for \( \overline{\sigma} \) in Eqn. 14(a & b).
CHAPTER IV

EXPERIMENTAL AND ANALYTICAL PROCEDURES

In this chapter, the experimental and analytical procedures followed in the present study to establish the combined influence of geometric defects and thermal gradients on instability and localization of plastic flow in sheet tensile specimens are presented. This chapter is organized in two sections. The first section presents the procedures used obtained in the experimental verification of the process model. In the second section the analytical procedures used to study the combined influence of geometric defects and thermal gradients on tensile ductility are provided.

1. EXPERIMENTAL VERIFICATION OF PROCESS MODEL

Experimental verification of the material model and finite element formulation was first done by performing a series of tensile tests and comparing these results to FEM predictions. Care was taken to ensure that the FEM mesh geometry used closely matched the experimental tensile test specimen. The effect of deformation heating was incorporated into the constitutive equation using experimentally measured temperature distributions. In addition, two extreme cases were treatable without temperature measurement: isothermal and adiabatic heat
transfer conditions. In the adiabatic case, the temperature rise in each element is derived from the total plastic work done on that element. By comparison of modeling results using measured temperature distributions with experimental results, the accuracy of FEM predictions was ascertained. Comparison of FEM results from isothermal, adiabatic, and measured temperature conditions allowed determination of the critical strain rate range to be avoided for consistent tensile test results. In this section the detailed procedures used to verify finite element simulation results is discussed.

1.1 EXPERIMENTAL PROCEDURE

All tests were conducted on an INSTRON test frame using specimens conforming to ASTM E-8 specifications (50.8mm by 12.77mm nominal gage section) machined from 0.9mm thick annealed I.F. steel sheets along the rolling direction. A TensilKut machine was used for this machining and a taper of about 1% over the 50mm gage section was obtained. This taper ensured that all tensile specimens failed near the center. The deformation produced at the edges by this machining was relatively shallow and no anisotropy effects due to this are expected. No heat treatment was done after machining. A series of thermocouples was spot welded on to the specimen within the 50mm gage-length to monitor the temperature rise during deformation. Fig.(6) shows the specimen geometry and the
relative location of the thermocouples.

Tensile tests were conducted in air at four crosshead speeds: 0.033 mm/s, 0.083 mm/s, 0.83 mm/s and 3.33 mm/s. These correspond to nominal engineering strain rates (\(\dot{\varepsilon}\)) of \(4.2 \times 10^{-4}/s\), \(10^{-3}/s\), \(10^{-2}/s\) and \(4.2 \times 10^{-2}/s\), respectively. Temperature information was collected using a MASSCOMP high speed data acquisition system[136] and the load versus elongation profile was recorded using a strip chart recorder. An LVDT type extensometer (50 mm gage length) was mounted on the tensile specimen to monitor the engineering strain. Details regarding the actual temperature measurement procedure is provided in the next section.
FIGURE 6. Tensile specimen showing the relative positions of the thermocouples.
1.1.1 TEMPERATURE MEASUREMENT PROCEDURE

Chromel/alumel thermocouples (K type) approximately 0.1mm in diameter were used to measure temperature distributions during the tensile test.

A. CAPACITANCE DISCHARGE SPOT WELDER

A spot welder employing a capacitance discharge principle was used to weld the thermocouples on to the specimen surface (Fig. (7)). By controlling the charging voltage, thermocouples were welded perpendicularly to the specimen without damage to the surface. The optimum charging voltage was determined through trial and error to be approximately 23 volts for I.F. steel. Care was taken to ensure that the weld was strong enough to withstand the conditions of the test.
FIGURE 7. Circuit diagram of capacitance discharge welder
B. MASSCOMP DATA ACQUISITION SYSTEM

A data acquisition system manufactured by MASSCOMP CORP.[136] was used to collect temperature data from the series of eight thermocouples welded to the specimen. A photograph of the set up used is shown in Fig. (8). The MASSCOMP data acquisition system, driven by UNIX/C, has a capability to transfer data from different STD devices including a 12 channel A/D converter, 8 channel D/A converter, 15 channel programmable clock, parallel digital I/O port, GPIB(IEEE 488) interface controller and 16 channel thermocouple port at speeds of up to $10^6$ Hz. and store data in an 80 Mb hard disk. Temperature data were collected in this study using a scan rate (channel-to-channel sampling speed) of $10^4$ Hz. and a sweep rate(sampling speed between sets of data from the 8 channels) ranging from $10^0$-$10^2$ Hz. The sweep rate and scan rate is set up using the main and auxiliary clocks as illustrated in Fig.(9). Due to the weak signals from the thermocouples (in mV range), excessive noise problems were encountered in the early stages, but these were alleviated by using adequate shielding and grounding methods. In order to suppress noise, shielded cables and a global ground were used. To obtain a global ground, all thermocouple boards that were not used were grounded together with other boards on the Masscomp. This system was grounded globally along with the
Instron chart recorder to the ground of the power source.
The error involved in the temperature measurements was estimated to be +/- 2.5°C.
FIGURE 8. Photograph of Setup used for data acquisition
FIGURE 9. Trigger Frequency of Data Acquisition System
1.2 ANALYTICAL PROCEDURE

A. DEVELOPMENT OF EMPIRICAL TEMPERATURE DISTRIBUTION FUNCTION.

The temperature data were fitted using a non-linear least square fitting routine\[137\] employing a simplex algorithm:

a) The temperature rise at a distance of 25.4mm from the specimen center was first fit to a two-stage function of engineering strain(e). Fig.(10):

\[
T_b = K_1 e + K_2 \quad (e < e^*) \\
T_b = K_3 \quad (e > e^*)
\]  

(19a)  

(19b)

The cut off parameter e*, represents the strain level beyond which the temperature at 25.4mm is nearly constant, Fig.(10). This value varies with rate and was obtained by minimizing the error of fit with Eqs.(19a & 19b).

b) The temperature at the specimen center, T₀, is expected to equal Tₐ in the early deformation stages and to rise sharply after necking is initiated. Strain gradients, and therefore temperature gradients can develop rapidly as necking proceeds. An exponential relationship between center temperature and engineering strain was found appropriate, Fig.(11). The discontinuity of T₀ at e=e* occurs because of its dependence on Tₐ, which itself is a two stage function of strain:

\[
T₀ = Tₐ + \exp(K_4 e + K_5)
\]

(19c)
c) The temperature data was globally fit to the final spatial gaussian form:
\[ T(x, e) = (T_o - T_b) \exp\left(-K_g x^2 e\right) + T_b \]  
(19d)
where:
\[ T_o = T_o(e) \] is the peak temperature corresponding to the center of the specimen, Eqn.(19c).
\[ T_b = T_b(e) \] is the temperature at a distance of 25.4mm away from the center, Eqs.(19a & 19b).
\[ x \] = distance from center of specimen.

For this fit a total of six parameters were used. These fit parameters and the cut-off parameter \( e^x \) are provided in table 2 with the associated standard error of fit and normalized error for the four testing rates. The standard error of this fit ranged from 0.5°C to 2.25°C, the higher value corresponding to highest testing rate and the lower value to the lowest testing rate used. The normalized error of fit \( \frac{\text{std.error}}{(\text{max AT})} \) was consistently near 0.05, except at the lowest rate. A representative fit is shown for a nominal strain rate of \( 4.2 \times 10^{-2}/s \) in Fig.(12).
FIGURE 10. Sample fit corresponding to eqs. (19a & 19b) for the highest test rate ($4.2 \times 10^{-2}/s$)
FIGURE 11. Sample fit corresponding to eqn. 19c for the highest test rate ($4.2 \times 10^{-2}$/s)
FIGURE 12. Sample fit of measured temperature data to empirical distributions for the highest test rate ($4.2 \times 10^{-2}/s$) eqn. 19d
### Table 2—Fit Parameters for Temperature Distribution

<table>
<thead>
<tr>
<th>Strain Rate</th>
<th>$4.2 \times 10^{-2}$/s</th>
<th>$10^{-2}$/s</th>
<th>$10^{-3}$/s</th>
<th>$4.2 \times 10^{-4}$/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_0^*$</td>
<td>0.30</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>$K_1$</td>
<td>71.13</td>
<td>66.24</td>
<td>8.256</td>
<td>2.63</td>
</tr>
<tr>
<td>$K_2$</td>
<td>23.94</td>
<td>23.73</td>
<td>26.26</td>
<td>26.01</td>
</tr>
<tr>
<td>$K_3$</td>
<td>45.28</td>
<td>37.0</td>
<td>28.73</td>
<td>27.06</td>
</tr>
<tr>
<td>$K_4$</td>
<td>10.10</td>
<td>15.57</td>
<td>3.884</td>
<td>1.459</td>
</tr>
<tr>
<td>$K_5$</td>
<td>-0.708</td>
<td>-2.288</td>
<td>-0.244</td>
<td>0.0115</td>
</tr>
<tr>
<td>$K_6$</td>
<td>-0.02</td>
<td>-0.018</td>
<td>-0.019</td>
<td>0.013</td>
</tr>
</tbody>
</table>

| Std. Error ($^\circ$C)  | 2.25 | 1.72 | 0.55 | 0.52 |
| Std. Error / max($\Delta T$) | 0.056 | 0.049 | 0.042 | 0.13 |
It should be noted that the above form of the temperature distribution function is not necessarily the best from either physical considerations or the lowest error of fit. For our purposes however, i.e. to simulate the existing heat transfer conditions in air over a strain rate range from $10^{-2}/s - 10^{-4}/s$, this form was found to be sufficiently accurate, as typically illustrated in Figs. (10-12).

B. SIMULATION OF NON-ISOTHERMAL TENSILE TESTS:

In the present study modifications were made to the finite element program to incorporate the influence of deformation heating. The temperature rise at any given deformation step was assumed to influence material behavior only at the next step. The temperature effect was included in the finite element calculations through the constitutive relation (Eqn. (17)), which simultaneously reflects the influence of plastic strain, strain rate and temperature on material flow stress. To simulate isothermal conditions, $\Delta T$ was prescribed a constant value of zero in the constitutive equation.

Classical thermodynamics has been assumed here in order to derive the expression for the adiabatic temperature rise, even though some details of its application to plasticity have been questioned [131, 132].
From the first law of thermodynamics:

\[ dU = \delta Q + \delta W \]  \hspace{1cm} (20a)

where \( dU \) = internal energy, \( \delta Q \) = heat transferred into the closed system (i.e. material element), and \( \delta W \) = external work done on the system.

We restrict ourselves to mechanical work on a unit volume of incompressible (i.e. plastic) material element via external forces and write as follows:

\[ \frac{\delta W}{V} = \sigma_{ij} \frac{\delta e_p^{ij}}{V} = \sigma \delta e^p \]  \hspace{1cm} (20b)

where \( \sigma_{ij} \) and \( \delta e_p^{ij} \) are work-conjugate measures of stress and plastic strain increment with work expressed per unit volume of original or deformed material; the distinction being lost in incompressible flow. \( \sigma \) and \( \delta e^p \), representing effective stress and plastic strain increments, are also work conjugate quantities.

Under adiabatic conditions (\( \delta Q = 0 \)), the internal energy change of a unit volume of material is equal to the work done on the element:

\[ \frac{dU}{V} = \frac{\delta W}{V} = \sigma \delta e^p \]  \hspace{1cm} (Adiabatic)  \hspace{1cm} (20c)

The internal energy change may be somewhat arbitrarily partitioned into a term corresponding to increased temperature and a term corresponding to other storage processes:

\[ \frac{dU}{V} = (\frac{\delta U}{\delta T})_v \delta T + \delta U_{\text{other}} \]  \hspace{1cm} (20d)

\[ = c_v \delta T + \delta U_{\text{other}} \]

The "other" storage processes may include kinetic energy
imparted by external loading and changes in structure of the material. For a quasistatic deformation, $\delta U_{\text{other}}$ will represent only changes in the material state from the beginning to the end of the deformation increment. Examples of material state changes may include changes in grain boundaries, dislocations, point defects, internal cracks, and other metallurgical imperfections.

If the ratio of $c_v dT$ to $dU/V$ is defined to be an efficiency factor, $\eta$, where the "other" storage mechanisms account for $(1-\eta)$ of the internal energy change, it follows from Eqn.(20d) that, in the limit, the temperature change for plastic, quasi-static, adiabatic deformation may be expressed as:

$$dT = -\frac{\eta}{\rho_d c_v} \dot{\sigma} \dot{\varepsilon}^P$$

(20e)

where $\rho_d$ = density and $c_v$ = heat capacity. Although $\eta$ depends on material and deformation temperature (approaching 1 at high temperature), room temperature values for steel have been measured to be about 0.87[111]. An upper bound for $\eta$ of 1 has been used throughout this work, as has often appeared in the literature[123,119].

Empirical temperature forms, Eqs.(19(a-d)), were used to simulate the actual heat transfer conditions in air. These were incorporated into the FEM calculations through the constitutive relation (Eqn.(17)), which is temperature dependent.

Care was taken to match the shape of the FEM mesh to
the actual shape of tensile specimen, in particular the taper, in order to compare modeling stress-strain curves to those experimentally obtained. The mesh is shown in Fig. (13).
FIGURE 13. Geometry of FEM mesh used to verify process model.
2. COMBINED INFLUENCE OF GEOMETRIC DEFECTS AND THERMAL GRADIENTS ON TENSILE DUCTILITY

Once the accuracy of the material model and FEM formulation was verified, a series of FEM simulations were carried out by varying mesh geometry to model different size and shape of imposed taper. Comparison of simulation results assuming isothermal and adiabatic conditions allowed a study of the relative influence of geometric defects and thermal gradients on the flow localization process. Details regarding the simulation process to assess the combined role of geometric defects and thermal gradients on tensile ductility is presented in this section.

2.1 ANALYTICAL PROCEDURE

A. DEVELOPMENT OF TENSILE SPECIMENS/GEOMETRIES

To study the effect of size and shape of imposed geometric defects a series of tensile specimens was developed in the following manner:

1. An initially uniform specimen 6.35mm wide and 12.7mm fillet radius was taken as the 0% taper "reference" specimen.

2. Different specimens varying in magnitude and shape of geometric defect were produced using a cosine function of the following form:

\[ Y(x) = (Y_0 - \Delta y/2.) - \Delta y/2. \cos(\pi x / \lambda) \]  

(21a)

where:
Y(x)—The width of the specimen at position x along the axial direction.

Y₀—The width of reference specimen at x=λ.

ΔY—The difference in width at position x=λ and x=0, the parameter controlling size of geometric defect.

λ—Half wavelength of the cosine function, the parameter controlling shape of geometric defect.

This choice of cosine function form was made to ensure a smooth specimen configuration at either end of the tapered section. Eqn.(21a) was used only to determine the width of the specimen in the tapered section, defined by the parameter λ. Beyond x=λ, the specimen width is retained the same as in the reference specimen. The relative specimen taper is defined to be:

\[ \text{Taper} = \frac{ΔY}{Y₀} \times 100. \quad (21b) \]

In all cases, the specimens were modifications of the uniform width section of the 0% reference specimen and resulted in a constant value of \( Y₀ = 6.35 \text{mm} \). Typical specimens with varying size and shape of initial geometric defect are shown in Fig.(14).

B. Simulation of Tensile tests

Modeling of the tensile test with varying size and shape of geometric defect was achieved by varying the node geometry data but the same mesh was used in each case. This
mesh is shown in Figure (15).

To isolate the effects of a geometric notch on tensile ductility simulation was done by setting the value of $\Delta T$ in the constitutive relation (Eqn. (17)) to be identically zero, resulting in flow stress dependence on strain and strain rate alone.

To simulate adiabatic conditions complete conversion of plastic work to heat was assumed [eqn. 20e, $\eta=1$] even though a fraction of the plastic work is known to be converted to stored internal energy associated with metallurgical defects [111].

At each time step Eqn. (7) is utilized to determine $\Delta T$ and this in turn affects the strain distribution at the next time step due to thermal softening.
TAPER GENERATING FUNCTION

\[ Y(X) = (Y_0 - \Delta Y/2) - \Delta Y/2 \cos \left( \pi X / \lambda \right) \]

FIGURE 14. Schematic of tensile specimen geometries with varying magnitudes of imposed taper.
FIGURE 15. Sample geometry of FEM mesh used for a specimen with 0% initial taper.
CHAPTER V
RESULTS

Two sets of simulations were carried out to analyze the combined influence of geometric defects and thermal gradients on tensile stress strain behavior. For experimental verification, a series of simulations of the non isothermal tensile test was performed using mesh geometries with the same 1% taper as in the actual test specimens. Once the model was verified, another series of simulations was performed by varying mesh geometry to study the combined effect of geometric notches and thermal gradients on instability and flow localization. This chapter is therefore organized as two sections: the first dealing with results from the experimental verification of process model and the second dealing with results from a study of the combined influence of geometric notches and thermal gradients.

A. Experimental Verification of Model

Stress-strain curves corresponding to the four rates and three heat transfer conditions (isothermal, adiabatic and empirical) are shown in Figs.(16-19). The experimentally obtained stress-strain curves are also shown for comparison. The scatter in the experimental ductility values from the mean had a standard deviation of 1.5%. Thus,
there could be some shift in the absolute values in the predictions made here in this study but these are not expected to alter the trend in the results. The experimental data reported here are not generally the average of a large number of tests performed under the same conditions, but two or three tests were made at each rate to check for the overall uncertainty of the experimental data reported. Although the experimental curves consistently show substantially lower elongations than the FEM-generated ones, the relative elongations show the correct trend and the amount of shift due to the heat transfer condition. The predicted difference in total elongation between isothermal and adiabatic cases is 7% while the experimental difference between the highest$(4.2 \times 10^{-2}/s)$ and the lowest$(4.2 \times 10^{-4}/s)$ test rates is 5%. This result suggests that the strain rates examined nearly span the extreme heat transfer conditions, particularly in view of the assumption of total plastic work conversion into heat, which overestimates adiabatic heating. In the finite element formulation used in the present work rigid plasticity was employed. This is justifiable, since plastic strain increments are much larger than elastic strain increments after yielding occurs. Engineering stress strain curves are presented in these figures for comparison with experimentally obtained curves. These relate to true stress and strain measures as follows:
\[ \sigma_t = \sigma_e (1+e) \quad \text{and} \quad \epsilon_t = \ln(1+e) \]  

(22)

where \( \sigma_t, \epsilon_t \) are true stress and true strain and \( \sigma_e, \epsilon \) are engineering stress and strain respectively. The same results are rearranged in Figs.(20-23) to show the influence of strain rate on stress-strain behavior under different heat transfer conditions.

Temperature gradients for the adiabatic condition are presented for the highest and lowest rates \((4.2 \times 10^{-2}/s \text{ and } 4.2 \times 10^{-4}/s)\) in Figs.(24 & 25) along with the measured experimental data. At the low rate Fig.(25) the temperature rise is approximately 2-3 °C; considerably lower than the adiabatic rise. At the high rate Fig.(24), observed temperature rises are nearly equal to the adiabatic values. It must be noted, however, that the adiabatic calculation will always overestimate the heat generation terms because only a part of the deformation work actually becomes heat.

The development of true strain gradients under adiabatic and existing heat transfer conditions are compared for \(4.2 \times 10^{-2}/s\) and \(4.2 \times 10^{-4}/s\) in Figs. (26 & 27). Because of the influence of deformation heating, the production of strain gradients is accelerated at the higher rate (Fig.(26)), while at the lower rate (Fig.(27)) strain gradients develop relatively slowly. At the low rates necking reflects the difference in cross-section between the center and the regions away from it, rather than any
temperature gradient. Thus, the effect of deformation heating is expected to be most pronounced in initially uniform specimens. In this study specimens with a pronounced taper of 1% were used, and even in this case the effect of deformation heating in enhancing strain gradients is apparent (Figs. (26 & 27)).

Typical plots of the development of local strain rate gradients for $4.2 \times 10^{-2}$/s are presented in Figs. (28 & 29) for the two extreme heat transfer conditions (isothermal and adiabatic). The divergence of true strain rate at 0.17mm and 6.33mm away from the center occurs at a lower engineering strain level for the adiabatic condition relative to the isothermal condition. This plot is indicative of enhanced flow localization due to deformation heating.

B. Combined Influence of Geometric Notches and Thermal Gradients on Tensile Ductility

The effect of geometric defects of varying amplitude on tensile ductility, in the absence of any deformation heating, is presented in Fig. (30). This condition is achieved by imposing $\Delta T$ to be zero in Eqn. (17). Tensile ductility decreases from 55.45% for a specimen with no taper to 11.7% for a specimen with 50% taper. Figs. (31–33) compare the influence of notch wavelengths for fixed amplitudes on tensile ductility. The ultimate elongation is lowered uniformly by decreasing the axial extent (period) of
the tapered section for a given magnitude of taper. The simultaneous influence of geometric defects and deformation heating can be seen in Fig. (34). For the same magnitude and shape of taper the ductility is lowered due to the detrimental influence of deformation heating. The amount by which the ductility is lowered is absolutely greater for a specimen which has a smaller initial geometric defect (Fig. 35), but the reduction relative to original elongation is roughly constant (Fig. 36).
FIGURE 16. Comparison of FEM generated engineering stress-strain curves with experiment for the strain rate \(4.2 \times 10^{-2} / \text{s}\).
FIGURE 17. Comparison of FEM generated engineering stress-strain curves with experiment for the strain rate ($10^{-2}/s$)
FIGURE 18. Comparison of FEM generated engineering stress-strain curves with experiment for the strain rate($10^{-3}$/s)
FIGURE 19. Comparison of FEM generated engineering stress-strain curves with experiment for the strain rate \(4.2 \times 10^{-4}/s\)
FIGURE 20. Effect of strain rate on stress-strain behavior under forced isothermal conditions.
FIGURE 21. Effect of strain rate on stress-strain behavior under forced adiabatic conditions.
FIGURE 22. Effect of strain rate on stress-strain behavior from a simulation of heat transfer conditions in air using measured temperature distributions.
FIGURE 23. Effect of strain rate on stress-strain behavior in air from experiment
FIGURE 24. Comparison of adiabatic temperature rise to measured values for the strain rate ($4.2 \times 10^{-2}/s$)
FIGURE 25. Comparison of adiabatic temperature rise to measured values for the strain rate ($4.2 \times 10^{-4} \text{/s}$)
FIGURE 26. Comparison of development of strain gradients under adiabatic and measured temperature conditions for the strain rate ($4.2 \times 10^{-2}$/s)
FIGURE 27. Comparison of development of strain gradients under adiabatic and measured temperature conditions for the strain rate($4.2 \times 10^{-4}/s$)
FIGURE 28. Sample plot showing the increase of local strain rate at the center relative to other regions after onset of necking for the highest rate ($4.2 \times 10^{-2}/s$) under isothermal conditions.
FIGURE 29. Sample plot showing the increase of local strain rate at the center relative to other regions after onset of necking for the highest rate (4.2x10^-2/s) under adiabatic conditions.
FIGURE 30. Effect of magnitude of geometric defect on tensile ductility assuming isothermal conditions.
FIGURE 31. Effect of geometric notch wavelength on tensile ductility assuming isothermal conditions (Taper=0.1%)
FIGURE 32. Effect of geometric notch wavelength on tensile ductility assuming isothermal conditions (Taper=1.0%)
FIGURE 33. Effect of geometric notch wavelength on tensile ductility assuming isothermal conditions (Taper=10.0%)
FIGURE 34. Combined effect of geometric defects and adiabatic thermal gradients on tensile ductility.
FIGURE 35. Absolute effect of thermal gradients on tensile ductility for various magnitudes of taper.
**FIGURE 36.** Proportional effect of thermal gradients on tensile ductility for various magnitudes of taper
CHAPTER VI
DISCUSSION:

The onset of instability and subsequent flow localization is a complicated phenomenon involving a large number of interrelated physical effects. These include strain hardening, strain rate hardening, thermal gradients set up by deformation induced heating, defects of mechanical or metallurgical nature and other history-related effects such as texture and transient loading. A convenient means of understanding the flow localization problem is to determine the role of various physical effects on the development of strain gradients and to visualize failure as occurring when these strain gradients reach a critical value.

In a qualitative sense, the effect of a number of physical effects on flow localization is understood. It has been recognized that the primary role of strain hardening is to prolong the onset of instability, while the strain rate sensitivity \( m \) tends to stabilize the neck formed by suppressing strain gradients and thereby delaying the occurrence of eventual failure. These two effects are relatively independent of each other for small values of parameters \( n \) and \( m \).[103]. Defects serve as preexisting instability sites and therefore promote development of
strain gradients leading to early failure.

The major role of deformation heating in promoting instability in the alloy tested here is mainly by setting up thermal gradients in the specimen. These thermal gradients preferentially soften the material in the neck and enhance the existing strain and strain rate gradients, thereby leading to lower ductility. While the effect of thermal gradients strongly resemble strain rate sensitivity by primarily influencing post-uniform elongation, the uniform temperature rise is also important in establishing total ductility in materials like 310 stainless steel which exhibits temperature dependent strain hardening[113,120]. Abrupt changes in strain path by varying loading conditions produce transient behavior involving changes in strain hardening characteristics. Materials such as mild steel exhibit transients involving increased work hardening leading to increased ductilities, while brass exhibits the opposite effect[46,129].

In the present study finite element modeling of non-isothermal tensile tests offered a unique perspective by allowing a close look at the quantitative role of thermal gradients and geometric notches on flow localization, keeping the influence of other parameters constant.

The magnitude of thermal gradients was found to have an important bearing on ultimate ductility Figs.(16-19). At each of the four rates examined, the adiabatic condition
(corresponding to largest thermal gradients) exhibited the lowest ductility, while isothermal conditions (absence of any thermal gradients) exhibited highest ductility values. The primary role of strain rate seems to be the way in which heat transfer conditions are altered. Experimental and simulated results for I.F. steel show that adiabatic conditions are achieved beyond $10^{-2}$/s while isothermal conditions are achieved below $10^{-4}$/s (Figs. (18 & 19)). The exact range depends strongly on the thermal and mechanical properties of the material and for materials with low thermal diffusivities, adiabatic conditions can be achieved at relatively high rates leading to significant lowering of ductility due to deformation heating[120].

Once the extreme heat transfer conditions are achieved, the ductility becomes independent of rate as shown in Figs. (20 & 21). Fig. (20) shows that for the isothermal condition there is no rate effect on ductility, which is constant at 47.5%. The only effect of strain rate on stress-strain behavior in the isothermal case is to increase the stress level as expected from the constitutive relation (Eqn. (1)). This result is in agreement with Ferron's[117] observations in stainless steel and Ayres'[119] work with A.K. (aluminum-killed) steel in isothermal tests, and has been proven mathematically for power law materials by Chung & Wagoner[130]. It must be noted however, that A.K. steel and I.F. steel behave differently in air. For A.K. steel
Ayres[119] found that the rate sensitivity parameter (m) in (Eqn.(1)) varied strongly with rate, while for I.F. steel this variation is small[113]. For this reason the ductility decreases with increasing rate for I.F. steel, but is roughly independent of rate for A.K. steel in air over the proper range of rates.

For the adiabatic case(Fig.(21)) there is again almost no influence of rate on ductility, which remains constant at 40.5%, even though the absolute difference in the adiabatic temperature rise for 4.2x10^{-2}/s and 4.2x10^{-4}/s is nearly 10°C. In this case the effect of strain rate is more involved. At a higher strain rate, the stress level at a particular strain level is higher. This in turn increases the net heat generation within a material element leading to higher temperature rise prompting one to expect significant lowering of ductility with rate for the adiabatic condition.

The result from Fig.(21) emphasizes the importance of temperature gradients relative to uniform temperature increases.

Fig.(22), which is obtained from FEM by substituting in the empirical temperature distribution (Eqs.19(a-d)) for the rise in temperature in the constitutive relation (Eqn.(17)), is indicative of actual heat transfer conditions at different test rates. The results from Fig.(22) are therefore expected to reflect the same trends as the experimental results from Fig.(23). The scatter in the
experimental ductilities was estimated to about 1.5% from the mean and the predicted values between experiment and modeling presented in this study could vary by that amount.

In the present study, modeling resulted in higher elongations than that exhibited by the actual specimens (Figs. (22 & 23)). This difference may arise from a combination of factors: a) The constitutive equation used is known only over a strain range from 0.02-0.35 and therefore may not adequately represent material behavior during the latter stages of necking. b) Production of defects such as voids in the test specimen at high strains could result in lower elongations during testing. c) Our FEM formulation makes use of constant strain triangles with no remeshing. It is possible that strain gradients are inhibited because of the finite element size. The principal predictions, however are not significantly affected by the lack of absolute quantitative agreement with FEM. Fig. (22) simulates existing heat transfer conditions in air and shows essentially the same results as obtained from experiment (Fig. (23)). In both cases above $10^{-2}/s$ the ductility is almost independent of rate, suggesting adiabatic conditions while at rates below $10^{-4}/s$, the ductility is again independent of rate suggesting isothermal conditions. The higher elongation shown for $4.2 \times 10^{-2}/s$ over $10^{-2}/s$ may result from experimental error or from nearly adiabatic conditions and an enhanced rate sensitivity as observed in
the work of Ayres[119]. The empirical curves of Fig.(22) contain this relationship without any change of rate sensitivity with strain rate, but this seems to be an artifact of the empirical temperature distribution function.

The deleterious effect of deformation heating for I.F. steel is manifested mainly during post uniform stress-strain behavior because the temperature rise is nearly uniform (in the gage section) in the early stages of deformation and work hardening and rate sensitivity are roughly independent of temperature for this material. After necking is initiated, most of the deformation takes place in the neck and strain and strain rate gradients develop. This, in turn leads to development of temperature gradients and as a consequence the ultimate ductility is lowered. The accelerated development of strain gradients due to the thermal gradients set up by deformation heating is shown in Figs.(26 & 27), which compare the adiabatic and ambient heat transfer conditions for $4.2 \times 10^{-2}$/s and $4.2 \times 10^{-4}$/s. This result, i.e that deformation heating primarily affects post uniform behavior, concurs with Semiatin et al.[123] who used a one-dimensional finite difference method to solve for the coupled deformation and heat transfer problem.

The FEM procedure presented here has the capability of defining an "envelope" of possible tensile elongations based on the extreme heat transfer conditions: isothermal and adiabatic. The adiabatic limit is approximate because it
will vary slightly with strain rate, as discussed earlier. These extreme cases require a constitutive equation for calculation, but no experimentally determined temperature distributions. Heat transfer conditions inside the "envelope" can be investigated using FEM techniques in conjunction with measured temperature distributions or by solving coupled heat-flow/deformation boundary value problems. The results from these semi-empirical calculations suggest that tensile testing for I.F. steel in air is nearly isothermal at strain rates lower than $10^{-4}$/s. Strain rates greater than $10^{-2}$/s are nearly adiabatic. In this range of strain rates the measured total elongation shift is approximately the same as the theoretical envelope range although there is a discrepancy in absolute magnitude, the reason for which is unknown.

The results discussed thus far relate to a 1% taper specimen, bringing up the question of the relative effect of magnitude of geometric notch and thermal gradients on flow localization. Modeling tensile tests with varying notch size and shape under assumed isothermal and adiabatic conditions at the rate of $10^{-2}$/s provided the relevant answers.

The tensile ductilities were significantly reduced with increasing size of geometric notch, though shape effects were relatively small (Figs. (30, 31-33)). These results were compared to those obtained from a two element 1-D necking
analysis based on a consideration of axial force equilibrium[89,92,97]. Tensile ductilities obtained from one dimensional geometric defect analysis from the expression

$$e_f = \exp\left[n-m \ln\left(1-f_0^{1/m}\right)\right] - 1.$$

(23)

are much lower than the experimental results. Eqn.(23) can be easily derived from Eqn.(11a) by substituting $\tau = n/\varepsilon$ and $\varepsilon = \ln(1+e)$ to relate true strain and engineering strain in the second expression. The principal assumptions involved in deriving Eqn.(11a) include:

a) 1-D analysis using two elements, ignoring biaxial effects.

b) Strain hardening influences material behavior only up to uniform strain($\varepsilon_u$).

c) Strain rate only influences post uniform material behavior.

Simulation in this study predicts ductility values similar to test results and other predictions which specifically include biaxial effects[101,102]. In a sheet tensile test gripping at either end of the specimen induces biaxial stresses, with degree of biaxiality increasing as necking progresses and contributing to increased ductilities[138]. Table.3 compares tensile ductility results from one dimensional analysis, Eqn.(23), to results from the present work for different size of geometric notch. The same results are presented in graphical form in
Fig. (37). The much lower ductilities in the analytical model are not all due to ignoring biaxial stresses effects, but are primarily due to the poor assumptions involved. The effect of strain hardening has been neglected during post uniform flow, while strain rate effects have been ignored prior to occurrence of instability. Results from Chung and Wagoner[103] corresponding to n and m values used in this study reveal that the cross-effect between n & m neglected in the 1-D analysis constitutes a significant decrease in ductility. The defect size gradually increases along the gage length in the tensile specimen. In the analytical model, only two extreme elements were used leading to an over-estimation of the actual defect size.

The present results agree with trends shown in the work of Ghosh[101] and by Semiatin[102] who modified one dimensional finite difference formulations to incorporate biaxial stress effects.

In any simulation, the choice of failure criterion used influences the results obtained. The actual physical failure occurs not only due to strain localization but is also enhanced by presence of voids and other defects in the specimen. A failure criterion taking into account only the rate of strain localization at the center and in the uniform region has been developed[138].

\[
\frac{\Delta e_c}{\Delta e_{av}} > \text{critical value} \quad (24a)
\]
In terms of engineering values, with the assumption that the true strain in center($\varepsilon_c$) is much larger than the strain hardening exponent($n$) the above criterion can be expressed as:

$$\frac{d\varepsilon_c}{d\varepsilon_{av}} = \frac{dF/F}{d\varepsilon/(1+\varepsilon)} \quad (24b)$$

In the present work the slope of stress strain curve was in most cases nearly infinite when the program stopped due to numerical difficulties. The engineering strain corresponding to the last step was taken as the failure strain in this study. The resulting error involved in failure strain determination is approximately estimated as 0.5%, corresponding to the step size used in the calculations. In all cases failure strains from FEM predictions were higher than experimental results, which is to be expected.

Thermal effects as a proportion of total elongation, have the same magnitude of effect on flow localization, independent of size of geometric defect[Fig.22]. From an absolute perspective though, thermal effects are most significant for specimens with small initial geometric defects[Fig. 21].
Table 3
Comparison of one dimensional analysis to present work

<table>
<thead>
<tr>
<th>Taper (%)</th>
<th>Fractional size of defect ( f_0 )</th>
<th>1-D Analysis ( e_f ) [Eqn. 23]</th>
<th>Present Work ( e_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.000</td>
<td>----</td>
<td>55.47</td>
</tr>
<tr>
<td>0.1</td>
<td>0.999</td>
<td>31.19</td>
<td>53.13</td>
</tr>
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<td>0.5</td>
<td>0.995</td>
<td>27.69</td>
<td>47.76</td>
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<td>1.0</td>
<td>0.99</td>
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<td>5.0</td>
<td>0.95</td>
<td>24.61</td>
<td>33.69</td>
</tr>
<tr>
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<td>0.90</td>
<td>24.48</td>
<td>28.33</td>
</tr>
<tr>
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<td>0.75</td>
<td>24.48</td>
<td>19.12</td>
</tr>
<tr>
<td>50.0</td>
<td>0.50</td>
<td>24.48</td>
<td>11.66</td>
</tr>
</tbody>
</table>
FIGURE 37. Comparison of simplified one dimensional necking analysis to present 2-D FEM analysis.
CHAPTER VII

CONCLUSIONS

A rigid viscoplastic finite element program has been modified to allow incorporation of non-isothermal effects during tensile deformation. The following conclusions apply to sheet tensile tests of I.F. steel:

For I.F. steel sheet tensile specimens with 1% geometric taper the following results hold:
1. Heat transfer conditions in air approximate isothermality for engineering strain rates less than $10^{-4}$/s and approximate adiabatic conditions at rates greater than $10^{-2}$/s.
2. Over this strain rate range, a theoretical loss of total elongation with increasing rate of up to 7% is expected. This agrees with the results of others and with experiments presented here.
3. Although relative shifts in total elongations are predicted well by FEM, absolute elongations are consistently predicted to be higher than those observed.

Further, a study of the combined influence of geometric notches and thermal gradients revealed that:
4. The ductility is drastically decreased with increasing size of geometric notch, though shape effects are relatively
5. Assumptions involved in 1-D, two element analysis lead to poor agreement with FEM results. The neglect of the cross-effect of strain hardening parameter(n) and rate sensitivity(m) may be one of the principal causes for reduced ductilities predicted by 1-D analysis.

6. In a proportional sense, the effect of thermal gradients in lowering tensile ductility is independent of magnitude of taper and is estimated as about 15.5% for I.F. steel.

7. In an absolute sense however, thermal gradients appear to be most significant for specimens with small initial geometric defect.
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APPENDIX A.
SIMPLEX CURVE FITTING ALGORITHM[137]

The Simplex algorithm is a powerful and efficient tool for fitting curves to data, especially when dealing with nonlinear equations. To initiate the computation, a reasonable starting value of the unknown parameters is provided. The algorithm generates the appropriate "Simplex," an n-dimensional space polygon whose vertices lie on the response surface. A 2-D example is shown in Fig. 38, where a and b are the fitting parameters and SDV is the error of fit. A generated "Simplex" is shown if Fig. 39. The algorithm works by moving the "Simplex" sequentially downhill, vertex-by-vertex, until the minimum sum of squared residuals is found. The possible operations used in moving the "Simplex" downhill include reflection(R), expansion(E), contraction(C), or shrinkage(S) as illustrated in Fig. 40.

A good choice of starting values may be necessary to ensure convergence to the correct root, especially for highly nonlinear fits. In general, this algorithm worked very satisfactorily for obtaining fits to measured data in the present work.
FIGURE 38. Example of response surface for 2-D Simplex
FIGURE 39. 2-D Simplex in response surface
FIGURE 40. Mechanisms of moving the Simplex