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OPTIMAL PUBLIC DEBT POLICY UNDER UNCERTAINTY:
A NEW CLASSICAL APPROACH

DISSERTATION

Presented in Partial Fulfillment of the Requirement for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Tung-Hao Lee, B.A.

* * * * *

The Ohio State University
1986

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Department of Economics
To My Parents, My Wife, 
and My Son
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CHAPTER I
INTRODUCTION

The purpose of this dissertation is to extend Barro’s (1979) approach to the determination of public debt. Between year-end 1980 and year-end 1984 U. S. federal debt held by the domestic non-financial sector increased 621.9 billion dollars in nominal values; on the average it went up 155.5 billion dollars each year. If no deficit-reduction actions are taken, the Office of Management and Budget predicts that the federal debt will increase by over 200 billion dollars each year until 1990. Facing such a rapid rise in the federal debt, Congress passed the Balanced Budget and Emergency Deficit Control Act (a.k.a. Gramm-Rudman-Hollings Act) which requires mandatory spending reduction to balance the federal government budget by fiscal year 1991. On the other hand, economists ask whether there is a theory to determine the optimal level of public debt. If so, can the Gramm-Rudman-Hollings Act be justified by such a theory? Given that government bonds are not wealth (Barro 1974) as a first-order approximation, Barro (1979) proposes to minimize tax distortion cost as a second-order theory for debt determination in a world of perfect foresight. In this dissertation, I extend his case to uncertainty in a barter economy, and then econometrically examine whether his theory is supported by the data.

-1-
Before studying the determination of public debt, it is useful to develop some definitions. One way to define concepts is to explain how to measure them. Unfortunately, techniques for measuring government budget deficits and public debt are still unsettled. Conceptually, budget deficits are the difference between government income and outlay over any period of time. Even this straightforward concept presents many measurement problems. For example, should deficits from the off-budget and federally sponsored agencies be included as part of government deficits? How do we measure the value of different categories of public goods? What are the relevant costs associated with them? How do we value the implicit or explicit contingent promises given by the government, and should they be included in government deficits? Is it necessary to distinguish cyclical, structural, and high-employment deficits, and if so, how? How do we measure endogenous government spending that contributes to the deficit? What measures are relevant for economic analysis? Enormous problems in measuring government deficits and public debt await solutions. This dissertation will not attempt to answer these questions. However, having conceptually tractable government deficit and public debt definitions in a barter economy are necessary for later analyses. Modigliani (1961) defines national debt as all claims against the government, interest bearing or not, held by the private sector of the economy (including bank-held debt and currency), less any claims held by the government against the private sector. Barro (1984b) defines real government deficits as the sum of changes in real value of all claims against the government during a particular period of time. In
this dissertation, I assume that in a barter economy no money circulates; only real deficits and public debt are considered. After such an assumption, I then follow Modigliani's measure of public debt, treat Barro's deficits as full government deficits, and define the excess of government spending over tax revenue as the current government deficits. I also assume that the society treats private bonds and public bonds equivalently. For simplicity, all bonds are assumed to mature in the next period, and only one interest rate prevails in the economy.

The basic questions concerning federal deficits and the level of public debt are whether high federal deficits would suffocate economic recovery or prosperity, and whether public debt would be a burden on future generations. People worry that high federal deficits might absorb too large a share of savings of the economy, leaving it with insufficient investment, and killing the potential growth of the economy—a crowding out effect—and whether our descendants would then have to pay off public debt with a lower level of consumption which represents a burden on them.

In fact, concern about public debt can be traced back to the period of the founding father of economics, Adam Smith, and probably to earlier authors. David Ricardo extensively discusses Britain's public debt problem, and he elaborates the famous Ricardian Equivalence Theorem about public debt. This theorem holds that methods of financing government expenditures have no effect on our economy. However, at that time and later, the major reason for the government issuing debt given by economists was to finance war expenses. With the
emergence of Keynesian theory, views of public debt change significantly. Public debt became a consequence of the government's fiscal policy. The jargon of "crowding out" became familiar in our daily lives, and until recently people still used this line of reasoning in analyzing economic phenomena.

Today, economists analyze public debt problems primarily in one of three ways. The first approach employs Keynesian analysis, by either the IS-LM model or its related Harrod-Domar growth model. Crowding out effects are generally the feature of this analysis. The second approach uses the overlapping generations model based on a pioneering article by Samuelson (1958). This model is useful in determining the equilibrium interest rate and tax effect on different generations with the presence of government debt. Because Wallace (1980) establishes the role of fiat money in the overlapping generations model, the effect of public debt given different monetary policies may also be analyzed in this model. The third approach focuses on a maximizing representative individual whose intertemporal utility function may include money as an argument—a Sidrauski model (1967)—or on minimizing social cost generated by some government policy actions over time. The terminal date of the above optimization problem may be infinite or finite. In general, in a zero-population-growth economy either the terminal date being infinite and the representative individual being assumed to live forever with no probability of death, or he and his descendants being assumed to behave as a single individual, the representative individual would treat public debt as an
implicit future tax. Debt and taxes are equivalent in the intertemporal budget constraint.

Since I intend to analyze optimal tax and debt policy within Barro's framework but extend it to uncertainty, I assume that the government is the only player who live forever, and the terminal date of the optimization problem is infinite. With regard to citizens in the economy, the Ricardian Equivalence Theorem is assumed to hold in principle. Therefore, all policies derived from my model may be considered to have a second-order effect on the economy. Also, because Barro adopts the classical Ricardian Equivalence Theorem as a first-order approximation to public finance, his public-debt-determination theory may be regarded as a new classical approach.

Empirically, numerous works are related to issues of public debt. For example, whether the Ricardian Equivalence Theorem holds or not is discussed in Tanner (1970), Kochin (1974), Yavitz and Meyer (1976), Buiter and Tobin (1979), Feldstein (1982), Kormendi (1983), Aschauer (1985), and Seater and Mariano (1985). If the crowding out effect exists for high government deficits is considered in Feldstein and Eckstein (1970), Plosser (1982), and Evans (1985). If the government is monetizing the debt or not concerns Barro (1978), Hamburger and Zwick (1982), Dwyer (1982), Miller (1983), Meyer (1983), Protopapadakis and Siegel (1984), and King and Plosser (1985). Although these articles are related to public debt issues, they are not directly related to the issue--the optimal public debt policy--that I intend to examine in this dissertation.
Given that the Ricardian Equivalence Theorem holds in the first place, I empirically investigate whether the new classical approach to the determination of public debt is a good approximation to the historical behavior of public debt. In particular, I argue that Barro’s empirical study (1979, 1984a) is logically flawed. His regression method only makes a necessary but not a sufficient condition for accepting the new classical model. Alternatively, I utilize an econometric method which estimates the parameters of the model directly. As a result, I avoid Barro’s weak empirical confirmation of the model and provide a more powerful test for the model. However, results should be interpreted with care because I strongly impose the validity of the Ricardian Equivalence Theorem as a presumption, and accepting or rejecting the public debt determination model is conditional on that presumption. Therefore, my results should not be interpreted as evidence directly supporting the Ricardian Equivalence Theorem per se.

The rest of this dissertation is organized as follows. In Chapter II, I give a selective literature survey on public debt problems in a barter economy, in which the major contributions are examined. In Chapter III, Barro’s new classical approach to the public debt problem is discussed. Because I study the optimal public debt policy in Barro’s framework as a benchmark, differentiating Barro’s work from others is necessary. Chapter IV extends the discussion of Barro’s optimal public debt policy to include uncertainty. Comparative analysis indicates how uncertainty affects the public debt policy. In Chapter V, sources of U.S. and U.K. data sets are given, and descriptive statistics and
simple Granger causality are also investigated. Chapter VI lays out the empirical procedures; from those empirical results, policy implications are investigated. Finally, in Chapter VII, conclusions are drawn and further research directions are proposed.
Notes


3 Although people generally refer Ricardo as the developer of the Equivalence Theorem of public debt, in his own work one may find his own arguments against his Equivalence Theorem. This leaves room for O'Driscoll, Tobin, and others to argue that Ricardo in fact supports the Non-equivalence Theorem of public debt. For details, see the survey in Chapter II.

4 An example of a Keynesian analysis of the U.S. deficit problem may be found in the testimony of Mr. Lyle E. Gramley, a former member of Board of Governors of the Federal Reserve System, before the subcommittee on Domestic Monetary Policy of Committee on Banking, Finance and Urban Affairs, U.S. House of Representatives on June 8, 1984. (Federal Reserve Bulletin, June 1984, pp. 500)

5 Examples can be found in Aiyagari (1984), Sargent and Wallace (1981), Darby (1984), and Miller and Sargent (1984).
CHAPTER II
A LITERATURE SURVEY OF PUBLIC DEBT PROBLEMS IN A BARTER ECONOMY

Much of the literature has already focused on public debt problems. To construct a comprehensive survey is a formidable task. Since I plan to analyze the public debt policy in a barter economy, it would obviously not be appropriate to explicitly review articles related to monetary policy effects on public debt in this survey. Instead, I track a few major theoretical contributions to convey the outlines of how public debt problems have evolved so far.

This survey is organized in chronological order. Classical economists' views on public debt are reviewed first. Keynesian public debt theory is then examined. Because we extend Barro's debt determination model, his new classical approach to macroeconomics is surveyed separately. Finally, some recent developments in public debt theory are reviewed.

I. Classical Economists' View on Public Debt - Adam Smith and David Ricardo

Adam Smith (1776) puts the major reason for the government to issue public debt as the occurrence of war:

The want of parsimony in time of peace, imposes the necessity of containing debt in time of war (1776, pp. 861).
Public reluctance in using new taxes to fund war expense does not necessarily derive from the lack of trust in government behavior, but from the political and economic difficulties inherent in raising new taxes in an already overburdened economy. Adam Smith refutes the opinion that national debt represents additional capital, instead portraying it as a hindrance to the growth of the economy:

The capital which the first creditors of the public advanced to government, was, from the moment in which they advanced it, a certain portion of the annual produce turned away using in the function of a capital, to serve in that of a revenue; from maintaining productive labourers to maintain unproductive ones, and to be spent and wasted,..., without even the hope of any future reproduction (1776, pp. 877).

After the Great War with France (1793-1815), the annual debt charge in Britain became 54% of national expenditures, making public debt a major concern (Shoup, 1960). In his Principles of Political Economy and Taxation, David Ricardo gives his view about the problems attending public debt. Given government purchases, the form in which they are financed is irrelevant:

When, for the expenses of year's war, twenty millions are raised by means of a loan, it is the twenty millions which are withdrawn from the productive capital of the nation. The million per annum which is raised by taxes to pay the interest of this loan, is merely transferred from those who pay it to those who receive it, from the contributor to the taxed, to the national creditor, ..., whether the interest be paid or not be paid, the country will be neither richer nor poorer. Government might at once have required the twenty million in shape of taxes, in which case it would not have been necessary to raise annual taxes to the amount of a million (Works of David Ricardo, ed. by J.R. McCulloch, 1886, pp. 146-147).

This is the Ricardo Equivalence Theorem, named by Buchanan (1976). However, one page later in the same book, Ricardo says:
...,it must not be inferred that I consider the system of borrowing as the best calculated to defray the extraordinary expense of the State. It is a system which tends to make us less thrifty - to blind us to our real situation (op. cit. pp. 148).

This argument is cited as the Ricardian Non-equivalence Theorem. In today's terminology, this proposition posits that public debt illusion exists. In his Funding System, Ricardo finds an effective intergenerational transfer of bequests which has the same result as the Equivalence Theorem is found:

It would be difficult to convince a man possessed of 20,000£ or any other sum, that a perpetual payment of 50£ per annum was burdensome with a single tax of 1,000£. He would have some vague notion that the 50£ per annum would be paid by posterity, and leaves it charged with this perpetual tax, where is the difference whether he leaves him 20,000£ with the tax, or 19,000£ without it? (op. cit. pp.187).

Shoup (1962) thinks this irrelevance proposition is Ricardo's own rational view of public debt, but O'Driscoll (1977), Buiter and Tobin (1979) argue that only public debt illusion is Ricardo's position. Regardless of the position Ricardo actually held, today we generally refer to the irrelevance argument as the Ricardian Equivalence Theorem.

II. Pigou's Public Debt Theory

Only the discrepancy between tax revenue and government expenditures compels governments to resort to public borrowing. In A Study in Public Finance, A.C. Pigou proposes to follow, other things being equal, a stable tax rate policy to avoid the disturbance costs associated with changes in tax codes. Even in the face of variation in
government expenditures, he holds that tax structures should be kept stable.

When the government expenditure is devoted to producing capital equipment, public borrowing is feasible, since the loan may be repaid by its own productivity. For a non-remunerative project like a war, the debt financing ought to be repaid before a new project (a new war) occurs. Pigou agrees that the Ricardian Equivalence Theorem holds in general. If the government follows Pigou's tax and debt policy, the resulting public debt would not be a burden to future generations. However, four points should be noticed. First, in a progressive growing economy, the representative individual may suffer from the illusion (or what Pigou calls a subjective burden) that he should still save as much as he should as if he were in a stationary economy. This illusion may cause him to save too much for his own good. Second, government debt may be classified by its holders. External debt is held by foreigners, while internal debt is held by citizens. Servicing an internal public debt implies transfer payments which generate costs. Third, the choice between debt and taxes is irrelevant only if such a choice won't affect how the real resources are drawn. If an overlarge amount of real resources is withdrawn within a very short period of time, an equivalence result would not hold. Fourth, but not least, tax and debt may have different distribution effects, both because of limited resources of the poor, and because of the diminishing marginal utility of income. With tax financing, the rich may bear more of the burden than under debt financing. In sum, Pigou acknowledges the Ricardian Equivalence Theorem as a first-order approximation, but hints
that the cost associated with different financing methods for government spending should be taken into account in determining the optimal tax and debt policy.

III. Early Keynesian Theory of Public Debt

During 1940s and 1950s, costs and benefits of public debt implied by Keynesian economic policies became a major new issue in debates about short-run stabilization policy. An early interpretation was framed by Lerner (1943). His functional finance doctrine says that the purpose of taxation is to maintain an adequate level of private consumption, one which equilibrates aggregate demand and supply to avoid inflation or unemployment. The difference between government expenditures and taxes is made up by government bonds. Buiter and Tobin (1979) criticize so short-run a focus for the debt policy as inconsistent with its long-run effect on economic growth. According to Buiter and Tobin, Lerner mistakenly assumes that insufficient demand may be expanded by use of public debt without hurting capital accumulation in the long run. However, Lerner's argument is consistent with the Keynesian tradition in assuming that income taxes affect only consumption directly and in the short run. Although public debt may reduce public saving and reduce the capital stock in the long run, according to Keynes, "in the long run, we are all dead".

A second interpretation treats public debt as part of personal, corporate, and government portfolios. Patinkin (1965) shows that a wealth effect or real balance effect is associated with public debt. Explaining why the society treats public debt asymmetrically between
the long and short runs and why public debt forms a part of people's wealth without any reservation for the tax liability in the future are major weaknesses in this analysis. For a new classical economist, these assumptions may not be acceptable.

Domar (1944) studied the national debt problem from another perspective. National debt may cause problems because a permanently increased public debt ultimately has to be serviced by taxpayers. The burden of debt is measured by the income tax rate which must be imposed to finance debt service charges. Because income may grow at high rates, whether the income tax rate for debt service charges will rise is far from evident. Domar considers three peacetime cases, and in all three cases, national debt is assumed to grow as a fraction of national income in each period.

In the first case, national income is constant. This constancy may be due to investment not causing a higher man-hour productivity, which is not incompatible with full employment if the level of national income is sufficiently high. Or it may be that investment causes higher productivity per man-hour, but also decrease the number of man-hour worked, and so results in an ever-shrinking work-week, etc. Domar shows that net income after payment of the tax would eventually be zero, the debt-income ratio would go to infinity, the tax rate would go to 100%, and all taxable income would go to paying the service of the debt.

In the second case, income grows at a constant absolute rate. The nature of the economy may be such that investment fails to raise productivity per man-hour sufficiently to allow national income to grow
faster, Or that an insufficient rise in the number of man-hour worked may result from a diminishing productivity of investment due to the wasteful character of investment expenditures or to a lack of new technological improvements. Or it may be that although the productivity per man-hour rises sufficiently, the number of hours worked, voluntarily or involuntarily, declines continuously. The outcome of this case is that the debt-income ratio would go to infinity, and the tax rate on debt service would go to 100%. However, the net income after tax payment would be a constant, and therefore, non-bond holders would be much better off than in the first case. Still, "it is doubtful whether an economy with an ever-increasing tax rate levied for the sole purpose of paying interest on the debt would be able to escape serious economic and social difficulties which may lead to a repudiation of the national debt".¹

In the third case, national income increases at a constant relative rate. In this case, the debt-income ratio, the tax rate for the service of debt, and the net income of non-bond holders after the payment of taxes would be constant in the limit. In fact, even though a more rapidly rising income would result in a higher level of debt, the greater the income growth rate, the lower would be the tax rate. Domar therefore concludes that the problem of debt burden is essentially a problem of achieving a growing national income. This may be achieved by a rise in government expenditures and actual growth in productivity but without a rise in prices.
IV. The Public Debt Debate since the Late 1950s

Beginning in the late 1950s, economists started to attack the classical Ricardian equivalence argument. Different mechanisms for shifting the burden of debt from present to future generations have been proposed. Contributions to public debt problems during this period have been mostly collected in James Ferguson's *Public Debt and Future Generations*. I only cite three major studies done by Buchanan (1958), Bowen, Davis, and Kopf (1960), and Modigliani (1961), and I also review Diamond's work (1965), which successfully summarizes the studies done by Bowen et al. and Modigliani.

In his *Public Principles of Public Debt*, Buchanan (1958) argues that any involuntary tax payment is a burden. When government expenditure occurs, members in the present generation may freely choose to finance it by either taxes or bonds. Since the choice is based on individual preference, it is reasonably assumed that either tax or debt financing freely chosen is not a burden to the present generation. If bond financing is used, these bonds must be repaid in the next generation. Members of the next generation have no choice but to use taxes to finance bond repayment. As a result, members of the next generation suffer lower levels of utility. Hence, when debt financing is used, the present generation effectively transfers burdens to the next generation.

Tobin (1965) criticizes Buchanan as neglecting the "incidence and effects" of tax financing. Voluntary tax financing decided by a political "majority" may burden the bulk of current taxpayers and may have distortionary effect on the economy. Shoup's (1962) criticism is
that if bond financing occurs at the expense of private consumption, the next generation may enjoy a higher level of capital stock.

Bowen, Davis and Kopf (1960) (BDK) consider the gross burden of public debt as the reduction in lifetime consumption experienced by members of each generation. Even if loan finance reduces not private investment but consumption when government expenditure occurs, they argue that it is still possible through intergenerational bond selling for the present generation to shift a part of the burden to future generations. The mechanism for this transfer assumes the following: when government expenditure occurs, a member of the present generation reduces his consumption by an equal amount in exchange for government bonds, so that the private stock of capital is not hurt by loan financing. But at the end of the life of the first generation, a consumption spree is assumed to occur, which amounts to extracting the resources sacrificed previously from the next generation by selling bonds. As a result, the next generation sacrifices an equivalent level of consumption, and eventually passes the burden to the third generation. This deferment of consumption reduces lifetime consumption and imposes a burden on future generations. BDK conclude that even when capital stock is intact, future generations still shoulder a burden from public debt.

Their argument is criticized by Shoup and Tobin, whose first counterargument emphasizes the similarity of BDK analysis to conventional capital burden analysis. After all, the next generation has to sacrifice consumption to keep its capital intact. Otherwise, it inherits less capital because of the consumption spree undertaken by
the present generation. Second, the burden occurs because of government expenditures. Using taxes to finance these expenditures may result in the same effects as occur in bond financing. A member of the present generation may well increase his consumption at the end of his life and reduce capital stock if he chooses to do so. The burden-causing mechanism proposed by BDK is really not unique to debt financing; tax financing may have the same result.

Following the traditional Keynesian theory, Modigliani (1961) develops an alternative way of evaluating the comparative effects of debt and tax financing. He starts with the assumption that a tax levy decreases disposable income and therefore reduces private consumption and saving as determined by their respective aggregate marginal propensities. On the other hand, government debt financing is assumed to reduce only private saving. Incorporating these asymmetric assumptions into his life-cycle hypothesis, he argues that although the interest payment on debt may approximately measure the burden of debt, the reduced capital stock due to saving reduction causes a burden.

The strongest case occurs when the lifetime propensity to consume is 1 and government spending is not used for productive purposes. Using taxes to finance government spending would cause tax payers to suffer an equivalent loss of their lifetime disposable income. Therefore, they must cut their consumption, spread over their lifetimes. Initially the taxed would cut saving by the marginal propensity to save times government spending. However, in later periods an amount equal to this reduced saving would be matched by a reduced amount of consumption, so that eventually the real capital
stock returns to its original level (or its trend). As long as taxpayers live long enough, the burden of government spending falls principally on them. In contrast, using public debt to finance government spending would not hurt private consumption at the time government spending occurs. But private saving has to be reduced immediately to that equal amount. Because the capital stock is unrecoverably diminished, people suffer a burden. In this way, Modigliani rejects the Ricardian Equivalence Theorem.

Modigliani's assumptions are not without critics. For example, Mishan (1963) does not accept Modigliani's asymmetric assumptions about the effects of debt and taxes. If the capital market is perfect, then only the present value of overall taxes are relevant. It is hard to perceive why tax and debt should be treated differently in determining the level of investment or capital.

Diamond (1965) effectively generalizes the arguments of BDK and Modigliani by focusing on a competitive overlapping-generations equilibrium analyzed within a neoclassical growth model. Capital market equilibrium balances entrepreneurs' capital demand and individual savers' capital supply. Along what is termed the Golden Rule Path, the marginal product of capital equals the rate of population growth. Diamond studies the effects of external and internal debt separately. Using the Golden Rule Path as a benchmark, external debt has two effects in the long run when taxes are employed to finance periodic interest payment. Such taxes would reduce individuals' lifetime consumption possibilities and disposable income. Further, they reduce individuals' saving and capital stock while
raising the interest rate. In addition to the above two effects, internal debt reduces capital stock because of the substitution of government debt for capital stock in individuals' portfolio—a debt swap effect. Because of this additional effect, the substitution of internal for external debt would result in a higher interest rate and lower utility level.

Compared to Diamond, BDK consider only the tax effect of internal debt on capital supply side, while Modigliani discusses only the fall in capital demand by the debt swap effect. Diamond not only considers both of them but adds the effects of tax on capital stock. He therefore summarizes the school of national debt matters in a complete way.

V. Barro’s Public Debt Theory

In recent years, the most influential new classical work on the public debt problem is done by Robert Barro. Barro (1974) assumes that each generation cares about not only its own utility, but also the utility of its descendents. In a perfect competitive capital market, when a representative individual in the current generation maximizes his and his descendant’s utility, it is necessary for him to transfer a part of his wealth to his descendant. It is assumed that this intergenerational bequest transfer is operative. Every individual in the current generation has one child. And even if a 100% inheritance tax is imposed, the representative individual in the current generation may find ways to increase the human capital investment in his descendant. Given such an operative intergenerational bequest transfer
assumption, Barro (1974, 1979, 1981, 1984a) models the role of the public sector in an economy with following features in a perfectly foresighted world:

(A) The Ricardian Equivalence Theorem is a valid first-order approximation with regard to the relationship between tax and public debt in an economy. The operative intergenerational bequest transfer in a perfect market assures that the representative individual will consider his optimal consumption profile over an infinite time horizon. As a result, an individual would treat present public debt as an implicit tax which will be repaid fully in the future. Therefore, given a known government spending stream, using tax or debt as a revenue source really doesn't matter. Since each individual has already taken public debt and tax obligations into his consumption decision, methods of financing government expenditures don't affect by other real economic variables like real interest rate, real wage rate, etc.

(B) The real interest rate and output is affected by government purchases. To be more specific, a transitory increase in government purchases has a larger (but less than one-to-one) positive effect on the real interest rate and output than does a permanent increase in government purchases. The basic reasoning comes from the concept that the net-of-transfer-payment government spending (government spending, for short) has two roles in our economy: it substitutes for private consumption, and it is a factor for private production function. Government spending may be a substitute for private consumption, as for example, if the service of national parks reduces the necessity of
travelling abroad, and the recreational consumption of an average household is thus reduced. The service of law and order may reduce the expense on home security service. Government spending may also be a factor of private production. For example, the service of a highway system may reduce the transportation cost in production processes, and the service of public education may improve productivity of labor.

Any change of government spending would therefore affect the real side of the economy by the substitution effect in consumption demand, and the output effect in the production supply. Assume that the substitution effect of government spending to consumption is less than one but greater than zero of the size of government spending, and the sum of the marginal product of government spending and the substitution effect is less than or equal to one. With permanent government spending fixed, a transitory increase in government spending would increase the excess aggregate demand by approximately one minus the sum of substitution and output effects. As a result, this excess aggregate demand is balanced by a less than one-for-one rise in the real interest rate compared to the rise in the real interest rate when there are no substitution and output effects. In contrast, a permanent rise in government spending by increasing an equal amount of government spending every period thereafter (a "transitory" increase in government spending every period) would decrease the permanent income and therefore the aggregate demand by a factor of about the same magnitude as a transitory increase in government spending but with an opposite sign. In net, these two effects are cancelled out. As a result, permanent government spending has nil effects on real interest rate.
The weakness of the above argument is that the level of government spending itself is presumably determined by some unspecified general dynamic equilibrium framework.

(C) Once we are given values of real economic variables, we may consider the financial side of the public sector. We assume that economy has a simple income tax system. The constraint imposed on the government is a zero present value of national debt in the infinite future (an overall budget constraint). If the government minimizes the present value of deadweight losses due to taxation subject to the overall budget constraint, then it would find that keeping a constant income tax rate is the optimal policy. When it applies such a taxation policy to the economy, the level of public debt is also determined, because any discrepancy between government spending and tax revenue is fulfilled by the creation of public debt. For example, if government spending and income are growing at a same constant rate, public debt would also grow at that rate.

Barro’s approach in fact follows the same way as classical tax-friction proposition approach which is a second-order amendment to the Ricardian Equivalence Theorem. Therefore, Barro’s debt determination model "rediscover" the classical approach of the public debt theory.

VI. Recent Developments in Public Debt Theory

Since 1970s, the ever-growing popularity of applying dynamic programming techniques to economics has revived economists’ interest in research concerning optimal government policies. Kydland and Prescott (1977, 1980) notice that an open-loop optimal policy solved from a
dynamic economic system may not be time-consistent in the sense that if the current decision maker wants to maximize the utility of consumers, he may change policies adopted previously. Setting economic policies is a game against rational economic agents. With no commitment to fulfill previous policy promises, the current policy maker may well invalidate previous policies to optimize the problem he faces. On the other hand, a stationary economic policy may be time-consistent but sub-optimal. For example, in a perfectly foresighted equilibrium context, in Turnovsky and Brock's (1980) model, using the income tax rate as the only policy instrument is not time-consistent.

Lucas and Stokey (1983) discuss the optimal fiscal policy in a dynamic economy without capital. They follow Ramsey's optimal taxation structure (1927) with a temporal instead of a cross-sectional interpretation. In a representative consumer economy, the government purchases goods each period at competitive prices. To finance government purchases, taxes must be raised—assuming a flat-rate income tax. To find the optimal tax rate over time, policy makers try to maximize the expected present value of the representative consumer's utility. In such a framework, they avoid totally the distribution problem caused by taxation and public debt policy. Since time-consistency is a problem in Ramsey's model with capital, Lucas and Stokey consider only an economy with no capital. The output is produced solely by labor with a constant return to scale technology. Government spending follows some exogenously determined stochastic process with a known transition probability distribution. Government spending enters neither the utility function of the representative
consumer nor the production function. How the government disposes of its procured goods is irrelevant to the economy. Only the tax revenue collected by the government from consumers would matter. They also assume that the government fully controls the tax rate and issues new debt at market prices during its administration without incurring any cost, but must honor its predecessors' debt.

Within such a framework, the question is whether fully honored debt commitments are sufficient to induce future governments to continue a tax policy that is optimal initially. If so, the policy is time-consistent. To solve the question, they first derive a competitive market equilibrium containing consumption, leisure, and debt profile. Using the same objective function subject to the competitive market equilibrium constraint, they are able to show that the optimal tax policy is an open-loop one. However, if the government has the additional power to manipulate the maturity structure of public debt, the optimal tax policy may be time-consistent.

Therefore, they conclude that in a barter, capitaless, non-growing economy, if the economy's founding fathers have the power to bind all their descendants' ability to change tax rates, then the Ricardian Equivalence Theorem holds, and debt structure is irrelevant. On the other hand, if their descendents are not so bound, but are merely committed to paying off any debt left to them, then by adjusting the structure of debt for the future, a time-consistent tax policy may still be followed.

Blanchard (1985) studies the public debt policy when the representative individual faces a constant probability of death. Both
individual and aggregate consumption are derived. After deriving the aggregate consumption and studying the dynamic behavior and steady states in open and closed economies, he puts government into the system with the assumptions that government spending has no effect on the marginal utility of private consumption, and that only lump-sum taxes or debt may be used. His concern about optimal debt policy is to eliminate the business cycle of output. In an open economy, he concludes that government debt and foreign assets should follow symmetric but opposite paths. However, no such conclusion can be found in a closed economy. Because the individual is under a constant death threat, Blanchard's model is de facto a finite horizon case. The representative individual doesn't have any "life-cycle" aspects in his model. His model should be treated as an intermediate step toward an infinite horizon case.

Reflecting on this survey, it is not hard to find that theories of public debt in different periods are closely related to the main stream of economic thought. Before the Keynesian revolution in macroeconomics, economists generally agreed that Ricardian Equivalence Theorem holds as a first-order approximation. Minimizing the friction cost of tax collection therefore becomes the objective in determining tax and debt policies. In the Keynesian period, the argument that national debt matters became popular. Public debt belongs to short-run government policies. However, according to Modigliani's and Diamond's models, when policy makers consider public debt as a policy instrument, they should also take its long-run effects into account. With basically conflicting effects in long and short runs, what the optimal
public debt policy should be is unclear. Some social preference assumptions must be made before any optimal public policy is pursued. In contrast, Barro's model has precise policy implications. However, Barro studies only cases in a perfectly foresighted world. In a world of uncertainty, it is interesting to know whether his public debt policy recommendation for smoothing income tax rate still holds or not. Therefore, extending Barro's model to uncertainty may not only complete the model itself, but also provide an evaluation of the validity of previous policy implications in a perfectly foresighted world. This is studied in Chapter IV.
Notes

1 op. cit. pp. 46

2 Barro's affirmation of the Ricardian Equivalence Theorem is echoed by other economists. For example, Carmichael (1982) develops an extended neutrality theorem of public debt such that if the aggregate economy behaves like a composite individual, and either bequest or gift transfer mechanisms are operative, then changes in the steady state level of public debt are neutral regardless of whether the economy's growth rate is greater than the real interest rate as concerned by Feldstein (1976), and Barro (1976). In a representative individual model with a separable utility function in government expenditures, Bryant (1983) shows that effects of transfer payments, government investment, and provisions of public goods on the economy are unaffected by whether they are financed by bonds or taxes, as long as government actions are in a lump sum type. However, we should be cautious that corner solutions, distortions, illusions, and distribution effects can cause non-neutrality of public debt (Carmichael (1983)). Also, nonidentical individuals, or non-lump sum government actions can destroy neutrality (Stiglitz (1983)). Furthermore, distortionary realworld taxes can break down the neutrality of public debt (McCulloch (1985)). Therefore, it is an empirical issue whether the Ricardian Equivalence Theorem is a good approximation of the real world phenomena. In this dissertation, however, I adopt the Ricardain Equivalence Theorem as a first-order approximation, as Barro did in his 1979 paper, and proceed to discuss the optimal debt policy in that framework.


4 Ramsey (1927) concludes that without lump-sum taxes, the optimal excise tax rate should be inversely related to the elasticity of demand or supply of that good. If we treat past capital as a good, then a policy maker always tries to tax it away, but no capital will accumulate in the first place. In a dynamic context, a capital levy becomes time-inconsistent.

5 Some economists try to combine both new classical and Keynesian schools together in this area. For example, Buiter (1983) uses the permanent cost of debt service approach with a comprehensive accounting budget constraint to address the issue of whether given projected real output growth, government spending programs can be financed without raising tax rates or seigniorage. His approach in fact, is an extension of Domar (1944). He considers only objectively the optimal debt policy in terms of sustainability, consistency and credibility, and allows both new classical and Keynesian to work in the model.
CHAPTER III
BARRO'S PUBLIC DEBT MODEL IN A WORLD OF PERFECT FORESIGHT

I. The Model

Assume that we are in a barter economy with a fixed structure of taxes. Given the time profile of government spending and taxable income, the government wants to minimize the present value of deadweight losses of taxation to determine the optimal tax revenue for each period up to infinity, but does not necessarily expect any future government to follow the same policy. A balanced budget is not required for every period. The government may issue one-period bonds to finance its spending. The real interest rate $r$ is treated as constant. Interest returns from government bonds are assumed to be either exempted from taxation or to have already been included in the taxable income in which the real income from production is passively changed to satisfy the time profile assumption of taxable income. Let $b_t$ be the level of public debt at time $t$, $Y_t$ be the economy's taxable income (income for short) at time $t$, $G_t$ be the government spending, and $R_t$ be tax revenue collected at time $t$. The following budget constraint holds at every point of time $t$:

$$b_t = G_t - R_t + e^r b_{t-1}$$

(1)
where interest rate is assumed to be compounded continuously.

I refer to $G_t - R_t + (e^r - 1)b_{t-1}$ as the full government deficit at time $t$ and to $G_t - R_t$ as the current government deficit at time $t$. Equation (1) says that public debt issued at time $t$, $b_t$, is the sum of the current government deficit at time $t$, $G_t - R_t$, plus maturing previous debt with accrued interest, $e^r b_{t-1}$. It is an accounting identity in the absence of money or repudiation of prior debt. Assume that the initial period is time 1, and the government at time 1 inherits $b_0$ from time 0. The sum of the accounting budget constraint (eq. (1)) up to infinity can then be expressed as follows:

$$
\sum_{t=1}^{\infty} G_t e^{-r(t-1)} + e^r b_0 = \sum_{t=1}^{\infty} R_t e^{-r(t-1)} + \lim_{t \to \infty} e^{-r(t-1)} b_t. \tag{2}
$$

Eq. (2) says that the present value of government spending with the initial public debt plus accrued interest should be equal to the present value of tax revenue plus the present value of public debt in the infinite future. Following Barro, we assume that the growth rate of $b_t$ is less than the interest rate $r$ asymptotically, so that from eq. (2) we have eq. (3) as the overall budget constraint:

$$
\sum_{t=1}^{\infty} G_t e^{-r(t-1)} + e^r b_0 = \sum_{t=1}^{\infty} R_t e^{-r(t-1)}. \tag{3}
$$

Eq. (3) is not an accounting identity, but actually constrains the government to live within its means in the long run.

Barro assumes that the tax collection cost is an increasing function of tax revenue and a decreasing function of income. For
analytical convenience, I assume the cost function to have the following particular form:

\[ Z_t = F_t(R,Y) = aR^1 + aY^{-\alpha} \quad a, \alpha > 0. \quad (4) \]

Positive \( a \) and \( \alpha \) guarantee that a larger tax revenue would increase the deadweight loss. The functional form \( F_t(R,Y) \) is assumed invariant over time. Now the policy maker attempts to select \( \{R_t\}_1^\infty \) to minimize the present value of the cost function \( V \) subject to the overall budget constraint, eq. (3), in a world of perfect foresight. His problem becomes:

\[ \text{Min } V = \sum_{t=1}^{\infty} aR_t^{1+\alpha} Y_t^{-\alpha} e^{-r(t-1)} \quad (5) \]

subject to eq. (3).

The first-order condition of eq. (5) is that

\[ a(1+\alpha)R_t^{\alpha-\alpha} Y_t^{-\alpha} e^{-r(t-1)} = \lambda e^{-r(t-1)} \quad t=1,2,3,..., \]

or

\[ (R_t / Y_t) = (\lambda / a (1+\alpha))^{1/\alpha} \quad (6) \]

where \( \lambda \) is the Lagrange multiplier.

If \( t \) is finite, then the second-order condition is satisfied by the fact that the bordered Hessian is positive definite. From equation (6), the optimal income tax rate should be the same for all periods.
Given a constant income tax rate as the optimal tax policy over time, we may solve for the optimal tax revenue from the overall budget constraint eq. (3). Once we get the optimal tax revenue for each period, the public debt policy is then determined from eq. (1) by the accounting budget constraint in each period. Its time profile depends on the time paths of both government spending and income. Barro gives some examples to illustrate such debt policies under different circumstances. In a real barter economy, three cases are considered: (1) both income and government spending are constant over time, (2) income and government spending are constantly growing, and (3) transitory income and government spending occur.

II. Both Income and Government Spending are Constant over Time

In this case, the optimal total tax revenue is also constant over time. From the overall budget constraint, eq. (3), the tax revenue \( R \) is equal to \( G + (e^r - 1)b_0 \). The optimal debt is also constant and is equal to the initial debt, \( b_0 \); it is neither amortized nor built upon and is determined totally independently of \( G, Y, r \), or the form of F-function. No full government deficit or surplus should occur from period 1 on; "(it) is (also) determined independently of the values of \( b_0, G, Y, r \), or the form of the F-function" (Barro 1979, pp. 945). Barro claims such type of result still applies when complications are introduced to the time paths of forcing variables \( Y \) and \( G \) in a world of perfect foresight.
III. Income and Government Spending are Constantly Growing

Assume that income grows continuously at a rate $\eta$ each period, and government spending grows continuously at a rate $\gamma$ each period. In order to have a finite present value of income, $\eta$ is assumed to be less than the interest rate $r$. To prevent government expenditures from exceeding income, $\gamma$ is less than or equal to $\eta$. Therefore, we have $0 \leq \gamma \leq \eta < r$.

Since from eq. (6) the optimal tax policy keeps the income tax rate constant over time, tax revenue also grows at a rate $\eta$ over time. From the overall budget constraint (eq. (3)), and the accounting budget constraint, eq. (1), we have the following general results at time $t$:

$$b_t = e^{\eta} b_{t-1} + G_t \left( \frac{e^{\eta} - e^{\gamma}}{e^r - e^{\gamma}} \right),$$

$$R_t = G_t \left( \frac{e^r - e^{\eta}}{e^r - e^{\gamma}} \right) + (e^r - e^{\eta}) b_{t-1}. \quad (7)$$

It is interesting to note that while tax revenue grows at rate $\eta$, public debt grows but at a rate less than $r$ because the present value of public debt in the infinite future must be zero.

From the debt-income ratio, we may observe the behavior of public debt further:

$$\frac{b_t}{Y_t} = \left( \frac{G_1}{Y_0} \right) \left( \frac{1}{e^r - e^{\gamma}} \right) + \left( \frac{b_0}{Y_0} \right) - \left( \frac{G_1}{Y_0} \right) \left( \frac{1}{e^r - e^{\gamma}} \right) \left( \frac{1}{e^{(\eta - \gamma)t}} \right), \quad (9)$$

and
If \( n = \gamma \), \( R_t = G_t + (e^{\gamma} - e^n) b_{t-1} \). Public debt is growing at a rate \( \eta \), and the debt-income ratio is constant over time. Public debt is determined only by the initial debt and the growth rate of income; it is not determined by government expenditures and the real interest rate at all. If \( \eta > \gamma \), then from the negative coefficient of the last term in eq. (9), the debt-income ratio is increasing and converges to the limit of eq. (10). This implies that public debt is increasing at a declining rate, and in the limit is growing at a rate \( \eta \).

From the above discussion, we may conclude that in a world of perfect foresight, if income and government spending are growing at the same rate, public debt is allowed to grow, but only as fast as income. The interest payment on previous debt should be covered by taxes only to the extent it exceeds the growth rate of the economy.

If the growth rate of income is greater than that of government spending, public debt initially grows faster than \( \eta \) to cover a part of current government spending. Hence, the level of tax collection may be reduced in every period compared to tax collection when income and government spending grow at the same rate. What is important is that no additional taxes would be imposed to redeem the initial public debt. In contrast, public debt may be used to cover a portion of government expenditures governed by the growth rate of the tax base. Also note that the level of debt may not become negative unless the initial level of debt is negative.
IV. Transitory Income and Government Spending

Barro confines the term "transitory" to shocks to the system, e.g. an economic depression is a shock to income, and a war is a shock to government expenditures. Since both depression and war are not expected to last forever, they are transitory to the economy. After an income or government spending shock, income or government spending is assumed to return to its trend, determined by the permanent part of income or government spending, respectively. In a world of perfect foresight, these shocks have already been taken into account by decision makers, and therefore, will not affect the optimal tax policy, which requires a constant income tax rate over time. However, the time profile of tax revenue won't be the same as that in the previous case, because of the occurrence of transitory income and government expenditures. Assume that the trend growth of $G_t$ and $Y_t$ are at the same rate $\eta$. We may observe the following overall budget constraint:

$$\frac{G_t e^r}{e^r - e^\eta} + \sum_{t=1}^{\infty} \frac{\epsilon_t e^{-r(t-1)}}{1} + e^r b_0 = \tau \left[ \frac{Y_t e^r}{e^r - e^\eta} + \sum_{t=1}^{\infty} \mu_t e^{-r(t-1)} \right] \quad (11)$$

where $\epsilon_t$ and $\mu_t$ are perceived by the economy as a government spending shock and income shock at time $t$ respectively. However, they may be zeros. The $\tau$ is equivalent to the flat income tax rate for every period.

For simplicity, Barro assumes that the first $k$ periods exhibit constant positive shocks to government expenditures and the first $n$
periods exhibit constant negative shocks to income. In other words, we have the following assumptions:

\[ \varepsilon = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \ldots = \varepsilon_k > 0 \text{ and } \varepsilon_{k+1} = \varepsilon_{k+2} = \ldots = 0, \]

and

\[ u = u_1 = u_2 = u_3 = \ldots = u_n < 0 \text{ and } u_{n+1} = u_{n+2} = \ldots = 0. \quad (12) \]

The overall budget constraint therefore becomes

\[
\frac{G_1 e^r}{e^r - e^\eta} + \varepsilon \left( \frac{1 - e^{-kr}}{1 - e^{-r}} \right) + e^r b_0 = \tau \left[ \frac{Y_1 e^r}{e^r - e^\eta} + u \left( \frac{1 - e^{-nr}}{1 - e^{-r}} \right) \right] (11')
\]

where \( Y_1 \) and \( G_1 \) are measured from the trend values of \( Y \) and \( G \).

From this constraint, we may derive the value of \( \tau \). Also, some comparative statics may be studied here. In particular:

\[
\frac{d \tau}{d \varepsilon} > 0, \quad \frac{d \tau}{d u} < 0,
\]

and

\[
\frac{d \tau}{d \eta} = \text{sgn}\left[ G_1 u (1 - e^{-nr}) - Y_1 (\varepsilon (1 - e^{-kr}) + e^r b_0) \right] \text{sgn}[G_1 - \tau Y_1] \quad (13)
\]

where \( \text{sgn}[] \) is the "signum" function, which takes on the sign of its argument.

An increase in the magnitude of a positive government expenditure shock would require a higher income tax rate, while an increase in the absolute value of a negative income shock would also need a higher income tax rate. An increase in the trend growth rate of both
government spending and income would reduce the income tax rate if both income and government spending shocks are specified as in eq. (12), and the initial debt is assumed positive. Intuitively, an increase in the growth rate of both government expenditures and income would make transitory shocks relatively less burdensome, and therefore only a lower income tax rate would be required.

With the income tax rate \( \tau \) in hand, both the tax revenue time profile and the public debt time profile can be derived. From the optimal tax policy, tax revenue should have the same time profile as income does. Therefore, in the first \( n \) periods, tax revenue includes proportional income shock and from the \( n+1 \)th period on, it goes back to its trend determined by the permanent part of income, i.e.,

\[
R_t = \tau \left[ Y_1 \ e^{n(t-1)} + u \right] \quad t=1,2,3,...,n,
\]

and

\[
R_t = \tau \left[ Y_1 \ e^{n(t-1)} \right] \quad t=n+1, n+2,...
\]  \hspace{1cm} (14)

Both income shocks and government spending shocks play roles in determining the level of debt. Assume that the number of periods of shocks from government expenditures are less than those from income, i.e., \( k < n \). From the budget constraint (1), the time profile of public debt may be expressed in the following:

\[
b_t = (G - \tau Y_1) \left[ \frac{e^{rt} - e^{nt}}{e^r - e^n} \right] + (e - \tau u) \left[ \frac{1 - e^{rt}}{1 - e^r} \right] + e^{rt} b_0 \quad t=1,2,3...,k,
\]
The public debt time profile has three stages: less than or equal to \( k \), between (including) \( k+1 \) and \( n \), and over \( n \). In these three stages, there are two common factors: the difference between the trend government spending and the trend income tax, and the initial public debt, growing at the interest rate \( r \). Heterogeneous terms are dependent on the magnitude and length of those shocks. In the first stage, \( t < k \), both shocks from income and government spending are accumulated on public debt. In the second stage, \( k < t < n \), new government shocks do not exist; only the old government shock affects public debt by its occurring interest payment. The income shock still has the same effect on public debt as in the first stage. In the third stage, \( t > n \), both old government and income shocks affect public debt via interest payments. Notice that public debt has to absorb all shocks from income and government spending permanently.

In sum, Barro's public debt model is derived from a minimum cost tax policy. Government debt is purely a residual from the current budget constraint after deducting collected tax revenue. The tax policy in his model requires the government to impose a stationary income tax rate over time. As a result, shocks from both government spending and income are absorbed by public debt. A so-called "tax (rate) smoothing theory" of the determination of public debt is therefore established.
Notes

1 If the values of the full government deficit and the current government deficit are negative, we change the term "deficit" to "surplus".

2 Barro assumes that \( z_t = F_t(R, Y) = R_t f(R_t / Y_t) \) is linear homogeneous with respect to \( R_t \) and \( Y_t \) together. Barro requires that \( f' > 0 \). If \( F(R,Y) = aR_tY_t^\alpha \), this implies that \( \alpha > 1 \). I therefore assume eq. (4) as the cost function. See Barro (1979 pp.943).

3 However, the level of employment is assumed unaffected by those shocks.

4 Restrictions on \( u \) and \( e \) must be such that the sum on each side is positive. Otherwise, we have a permanent negative tax rate.
In this chapter I follow Barro’s minimizing cost approach toward public debt determination with non-state-contingent (fully guaranteed) real bonds, but with either uncertain income or government spending. Barro (1979) discusses the determination of public debt with full employment and a fixed program of government expenditure assumptions. However, these assumptions are not good approximations to economic phenomena in the real world. Therefore, I introduce either uncertain income or government spending to relieve from the stringent assumptions imposed by Barro.

With Barro’s public debt determination model in a world of perfect foresight as a benchmark, I extend his model into uncertainty by three different cases. In the first case, I relax the known income time profile assumption with uncertain income. In the second case, I assume that the income time profile is known while government expenditures are uncertain. In the third case, I generalize the first two cases by assuming that both income and government expenditures are uncertain.

Both income and government spending are assumed to have two parts: a deterministic and a stochastic part. The deterministic part of government spending and income is assumed to be known by the decision maker, while for the stochastic part, only the parameters of the
distribution are assumed to be known. Despite the existence of uncertain income and government spending, a constant real interest rate is assumed. This assumption may be a first-order approximation when variations in the growth rate of income and government spending are small and productivity of capital is stationary over time.

I. Uncertain Income with a Fixed Government Expenditure Program

Income uncertainty is the only difference between this case and Barro's model. The budget constraint (eq. (1)), the overall budget constraint (eq. (3)), and the total cost function (eq. (4)) are still the same as before. Assume that income follows a lognormal process; it grows continuously at a constant rate \( \eta \) plus a stochastic term \( \nu_t \) which is unknown at time \( t \). I assume that \( \nu_t \) is identically, independently, and normally distributed (i.i.n.d.) with zero mean and constant variance \( \sigma^2 \). In other words, I specify the stochastic income process:

\[
\ln Y_t - \ln Y_{t-1} = \eta + \nu_t
\]  

(16)

where \( \nu_t \sim N(0, \sigma^2) \) and \( \text{cov}(\nu_t, \nu_{t-1}) = 0 \) for \( i \neq 0 \).

The available information at time \( t \), \( \Omega_t \), includes at least \( b_0, Y_0, r \), and all realized variables before time \( 0 \), all parameters in the cost function and income generating process, and government spending in all periods.\(^2\) The objective is to find taxes in different periods that minimize the expected present value of tax-collection cost subject to
the overall budget constraint (eq. (3)) with the income generating process (eq. (16)):

\[
\text{Min } V = E_1 E \left[ \alpha R_t^{1 + \alpha} Y_{t-1} e^{-r(t-1)} \right] \\
\text{subject to (3), (16)}
\]

where is the expectation operator given information at time 1, i.e., 

\[
E_1 = E( . | G_1) 
\]

I solve eq. (17) by an open-loop solution for \( R_t \). \( R_t \) is therefore non-stochastic. The first-order condition for equation (17) is thus:

\[
e^{r(t-1)} \frac{\partial V}{\partial R_t} = \alpha (1 + \alpha) E_1 [R_t^\alpha Y_t^{-\alpha}] - \lambda = 0 \quad t \geq 1,
\]

or

\[
E_1 [ R_t^\alpha Y_t^{-\alpha} ] = E_1 [ R_1^\alpha Y_1^{-\alpha} ] \quad t \geq 1
\]

where \( \lambda \) is the Lagrange multiplier of eq. (3).

Equation (18) says that if \( \alpha = 1 \), the expected income tax rate in all periods given the information set at time 1 should be the same when income is uncertain.

Since only income is uncertain in the objective function, the expectation operator works only on \( Y_t \). In particular, we have:

\[
E_1 ( Y_{t-1}^{-\alpha} ) = Y_0^{-\alpha} e^{(-\eta \alpha + 0.5 \alpha^2 \sigma_Y^2)} \quad t.
\]
Combining eq. (18) with eq. (19), we may express explicitly the optimal tax policy as:

\[ R_t^\alpha = R_1^\alpha e^{(\eta \alpha - 0.5 \alpha^2 \sigma_v^2)} (t-1), \]

or

\[ R_t = R_1 e^{(\eta - 0.5 \alpha \sigma_v^2)} (t-1). \]  \hspace{1cm} (20)

Equation (20) demonstrates that tax revenue should be growing at the rate \( (\eta - 0.5 \alpha \sigma_v^2) \) each period regardless of the growth rate of government spending. Since \( \alpha > 0 \), the tax revenue with income uncertainty grows more slowly than that under a certain income growth rate \( \eta \), and therefore must start off higher to have the same present value. When income grows randomly, the collection of tax revenue should take this uncertainty factor into account. Therefore, the growth rate of tax revenue should be adjusted accordingly. Also, the higher the \( \alpha \) is, the slower the growth of tax revenue will be. This means that as tax-collection activity becomes more distortionary, the government should reduce the growth rate of tax revenue. If the cost function is quadratic to the tax revenue, i.e., \( \alpha = 1 \), the optimal tax collection would be \( R_t = R_1 e^{(\eta - 0.5 \sigma_v^2)} (t-1) \).

So far only the relation between tax revenue in different periods has been found; it is necessary to derive an explicit solution for tax revenue in any period to establish a complete time profile of the optimal tax policy. As in the certainty case, I use the overall budget
constraint to solve tax revenue at time 1 in terms of government spending. Substituting eq. (20) into eq. (3), we have:

\[ R_1 = \left[ \sum_{t=1}^{\infty} G_t e^{-r(t-1)} + b_0 e^r \right] \left( 1 - e^{(\eta - r - 0.5 \sigma_v^2)} \right) \]  

(21)

where \( r \) is required to be greater than the adjusted growth rate of tax revenue, \( (\eta - 0.5 \sigma_v^2) \), to guarantee that the terms in the overall budget constraint are finite.

Equation (21) says that tax revenue at time 1 should be proportional to the present value of current and future government spending plus the accumulated initial debt up to time 1. If government spending is growing at a constant rate \( \gamma \), where \( \gamma \) is less than \( r \), the optimal tax policy at time 1 becomes

\[ R_1 = \left[ G_t \frac{e^r}{e^r - e^\gamma} + b_0 e^r \right] \left( 1 - e^{(\eta - r - 0.5 \sigma_v^2)} \right). \]  

(22)

Given the optimal tax policy, the debt policy may be derived. Using the budget constraint eq. (1) and the tax policy of eq. (20), we have the following public debt policy under income uncertainty:

\[ b_t = e^{(\eta - 0.5 \sigma_v^2)} b_{t-1} + G_t \left( \frac{e^{(\eta - 0.5 \sigma_v^2)}}{e^r - e^\gamma} - e^\gamma \right). \]  

(23)

Comparing eq. (23) with eq. (7), the debt policy is complicated by the fact that the tax policy itself is more complex because of income uncertainty.
uncertainty. Clearly enough, public debt should grow more slowly when income is uncertain and the level of public debt at time 1 is less than that in a world of perfect foresight.

Furthermore, if $\eta = \gamma$, the coefficient of $G_t$ in eq. (23) is negative. This implies that public debt should be negative after some time point $T$ as long as $G_1 > 0$. Also, because $b_t$ is monotonically decreasing after some time point $T'$ where $T' < T$, a full budget surplus policy should be pursued after $T'$.

While the tax policy of eq. (8) in a perfectly foresighted world implies a constant income tax rate, the tax policy of eq. (20) implies an asymptotically zero income tax rate in the infinite future given the information set at time 1 and $\alpha$ being greater than 0. Note that, given $\Omega_1$,

$$\frac{R_t}{\text{med}(Y_t)} = \left[ \frac{R_1}{Y_0^{\eta}} \right] e^{-0.5\alpha \sigma^2 (t-1)},$$

and

$$\text{S.D.} \left[ \ln \left( \frac{Y_t}{Y_0} \right) \right] = \sqrt{t} \sigma_y, \quad (24)$$

where $\text{med}(Y_t)$ and $\text{S.D.}$ stand for the median of income at time $t$ and standard deviation respectively. From eq. (24), the median income tax rate approaches 0 as time goes to infinity, and the standard deviation of log income tax rate is limited to the order of square root $t$. It can be shown that the probability limit of the income tax rate goes to 0 as time goes to infinity. Therefore, the government at time 1 no longer treats a constant income tax rate as an optimal policy given the
income uncertainty assumption. Rather, a policy that almost surely leads to a declining income tax rate policy should be followed; i.e., rather than leaving debt untended (relative to $Y_t$), the government at time $1$ should pay off debt and even amortize its future expenditures as soon as possible.

II. Uncertain Government Expenditures with a Known Income Time Profile

In this case, I assume that government spending is uncertain but income is known with certainty. Current government spending is determined by previous government spending and a continuously growing factor - a constant drift plus a periodic random draw from a normal distribution. The objective is the same as before. But I substitute public debt for tax revenue as the policy instrument to facilitate the solution. I minimize the present value of a linear homogeneous cost function over the stochastic process $(b_t)_1^\infty$ subject to $b_0$ given and the overall budget constraint.

$G_t$ is assumed to follow a lognormal random walk with drift:

$$\ln G_t - \ln G_{t-1} = \gamma + \zeta_t,$$

and

$$\zeta_t \sim N(0, \sigma^2_{\zeta}) \text{ and } \text{cov}(\zeta_t, \zeta_{t-i}) = 0 \text{ for } i \neq 0. \quad (25)$$

To adapt to the uncertainty case, the overall budget constraint is redefined as

$$\lim_{T \to \infty} e^{-r(T-1)}E_1(b_T) = 0, \quad (3')$$
i.e., the present value of expected debt in the infinite future conditional on information available at time 1 is zero. I also assume that the log expected growth factor for government spending \((\gamma + 0.5\zeta^2)\) is less than the real interest rate plus the constant growth rate of income.

Substituting the current budget constraint (eq. (1)) into the objective function, I solve the process of \(b_t\) such that

\[
\min V = E_1 \sum_{t=1}^{\infty} \left( G_t - b_t + e^r b_{t-1} \right)^{1+\alpha} \gamma^{-\alpha} e^{-r(t-1)}
\]

subject to (3').

To derive the optimum decision rule for the above objective function, I assume that government optimizes only one period at a time. After updating the information set, the next period's optimal policy is then pursued. With such an optimization process, the first-order condition for the objective function at any time \(t\) becomes:

\[
[(G_t - b_t + e^r b_{t-1}) Y_t^{-1}]^\alpha = Y_t^{\alpha} E_t (G_{t+1} - b_{t+1} + e^r b_t)^{\alpha}
\]

where \(G_t\) is assumed known at time \(t\) and \(E_t\) is the expectation operator based on information available at time \(t\). The information set at time \(t\) includes at least the real interest rate \(r\), the time profile of income, the government spending at time \(t\), and all the parameters in both the cost function and government spending generating process.
Alternatively, the first-order condition may be rewritten as:

\[
\left(\frac{R_t}{Y_t}\right)^\alpha = E_t \left( \frac{R_{t+1}^\alpha}{Y_{t+1}} \right).
\]  

(28)

Except for the expected value operators on the power of next period's income tax rate, eq. (28) has the same form as eq. (6) in a world of perfect foresight.

For simplicity, I assume that \( \alpha = 1 \) in the remainder of this section. At time \( t \), eq. (28) then says that the expected next period's income tax rate is equal to the current income tax rate. Equivalently, eq. (27) is transformed:

\[
E_t \left( \frac{Y_{t+1}}{Y_t} \right) = E_t (G_{t+1}) - E_t (b_{t+1}) + e^r b_t,
\]

or

\[
E_t (b_{t+1}) - \left( \frac{Y_{t+1}}{Y_t} + e^r \right) b_t + \left( e^r \frac{Y_{t+1}}{Y_t} \right) b_{t-1} = E_t (G_{t+1}) - \left( \frac{Y_{t+1}}{Y_t} \right) G_t.
\]  

(29)

Eq. (29) is a second-order difference equation with respect to \( b_t \). Two boundary conditions are needed to solve eq. (29) for \( b_t \). One is the information set available at time 1, which includes \( b_0 \). The other in general comes from the transversality condition which takes the limit of the ending period's first-order condition of the objective function eq. (26) with the terminal date being finite. The transversality condition of this problem is that

\[
\lim_{T \to \infty} \frac{\partial V}{\partial b_T} = \lim_{T \to \infty} e^{-r(T-1)} E_T \left[ (G_T - b_T + e^r b_{T-1}) Y_T^{-1} \right] = 0,
\]
or
\[
\lim_{T \to \infty} e^{-r(T-1)} E_t (R_T/Y_T) = 0. \tag{30}
\]

Eq. (30) requires that the present value of the expected income tax rate in the infinite future given the information set available at time 1 should be 0. However, the assumption that the log expected growth factor for government spending is less than the real interest rate plus the constant growth rate of income, and the overall budget constraint (3') are sufficient to satisfy this transversality condition. In fact, aside from the growth factor assumption for government spending, the overall budget constraint (3') imposes a more stringent boundary condition than the transversality condition (eq.(30)). Therefore, given the growth factor assumption for government spending \( r + \eta > \gamma + 0.5\sigma^2 \), the overall budget constraint (3') is sufficient to be one boundary condition.

To solve the stochastic Euler equation of eq. (29), rewrite it lagged one period as

\[
E_{t-1}(b_t) - \left(\frac{Y_t}{Y_{t-1}} + e^r\right)b_{t-1} + \left(e^r \frac{Y_t}{Y_{t-1}}\right)b_{t-2} = E_{t-1}(G_t) - \left(\frac{Y_t}{Y_{t-1}}\right)G_{t-1}. \tag{31}
\]

For simplicity, assume that \( \frac{Y_t}{Y_{t-1}} = e^\eta \), i.e., income is growing continuously at a constant rate \( \eta \) in each period. Also, define \( B \) as a special lag operator such that \( B^{-1}E_{t-1}X_t = E_{t-1}X_{t+1} \). Therefore, \( B \) does not change the content of the information set. Eq. (31) can be rewritten as:
\[ [1 - (e^\eta + e^r)B + e^{r+\eta}B^2] E_{t-1}b_t = E_{t-1}(G_t - e^\eta G_{t-1}). \]  

(32)

There are two roots in eq. (32), \( e^\eta \) and \( e^r \), each of which is greater than one. To satisfy the stochastic Euler equation, I rewrite eq. (32) as:

\[ (1 - e^\eta B) E_{t-1}b_t = \left[ \frac{1}{1 - e^r B} \right] E_{t-1}(G_t - e^\eta G_{t-1}) + c_1e^rt. \]  

(33)

For a lag operator, it is known that \( \frac{1}{1 - \lambda B} = - \sum_{i=1}^{\infty} \lambda^{-i}B^{-i} \) if \( \lambda > 1 \). Also, note that the coefficient of the homogeneous part of the solution, \( c_1 \), is determined by the information set at time 1 and the overall budget constraint, eq. (3). As a result, updating the information set by one period, eq. (33) may be rewritten as

\[ b_t = e^\eta b_{t-1} + \frac{e^x - e^\eta}{e^x - e^r} G_t \]  

(34)

where \( x = \gamma + 0.5 \sigma_t^2 \), the log expected growth factor for government expenditures, and \( c_1 \) is set to 0 to avoid explosive debt.

From eq. (34), the level of public debt depends on the previous period's debt and current government spending. In deriving eq. (34), the interest rate is assumed to be greater than \( x \), the log expected growth factor for government spending (\( r > x \)). For a finite present value of income, \( r \) is required to be greater than \( \eta \). To avoid expected government expenditures exceeding income, I assume that \( \eta > x \). If \( \eta > x \), the coefficient of \( G_t \) is positive. A higher income growth rate
allows public debt to grow positively with government expenditures. If \( t = x \), public debt grows independently of government expenditures. Such a rule for public debt policy is intuitively attractive. Whether government can use public debt to finance its expenditures permanently depends on whether income grows fast enough. This is consistent with Domar's (1944) policy prescription for national debt in a world of certainty.

However, the government at time 1 no longer reads the income tax rate as zero in the infinite future. Rather, the median income tax rate converges to a constant as time goes to infinity, and thus, the uncertainty of government spending does not make the income tax rate collapse at zero. For details, see Case (II) in Appendix B.

III. Uncertain Income and Government Spending

Assume that the tax-collection cost function is still linear homogeneous with respect to taxes and income. Now both government spending and income are stochastic. Specifically, I combine stochastic assumptions in the previous two cases with the following:

\[
\ln Y_t - \ln Y_{t-1} = \eta + \nu_t ,
\]

\[
\ln G_t - \ln G_{t-1} = \gamma + \zeta_t ,
\]

\[
(\nu_t, \zeta_t) \sim \text{BN}(0, \begin{bmatrix} \sigma_{\nu\nu} & \sigma_{\nu\zeta} \\ \sigma_{\nu\zeta} & \sigma_{\zeta\zeta} \end{bmatrix}), \tag{35}
\]

and \( \text{cov}(\nu_t, \nu_{t-i}) = \text{cov}(\zeta_t, \zeta_{t-i}) = \text{cov}(\zeta_t, \nu_{t-i}) = 0 \) for \( i \neq 0 \).
where BN(.) is a bivariate normal distribution and $\sigma_{xx}$ is equal to $\sigma_x^2$.

Equation (35) assumes that both income and government spending follow a lognormal process with $\eta$ and $\gamma$ as the drift parameters respectively. The two stochastic terms are jointly normal and may be correlated contemporaneously. Non-contemporaneous random terms are assumed to be independent either within the same process or across processes.

Using eq. (25) as the objective function, a stochastic process $\{b_t\}_{t=1}^{\infty}$ subject to $b_0$ given and the overall budget constraint (3') can be found. Assume that the log expected growth factor for government spending over income is less than the real interest rate. The first-order stochastic Euler equation is thus:

$$E_t\{[(G_t - b_t + e^r b_{t-1})/Y_t]^\alpha - [(G_{t+1} - b_{t+1} + e^r b_t)/Y_{t+1}]^\alpha\} = 0,$$

or

$$E_t\{ (R_t / Y_t)^\alpha - (R_{t+1} / Y_{t+1})^\alpha \} = 0$$(36)

where both $G_t$ and $Y_t$ are assumed not observable at time $t$.

If $\alpha = 1$, eq. (36) implies a constant expected income tax rate as an optimal income tax policy (compare eq. (6) for a perfect foresight world). Also, eq. (36) becomes:

$$E_t[(-Y_t^{-1}(Y_t^{-1})_t + e^r (Y_{t-1}^{-1})(Y_t^{-1})_t)] = E_t[(-Y_{t+1}^{-1}(Y_{t+1}^{-1})_t + e^r (Y_t^{-1})(Y_{t+1}^{-1})_t)] = 0.$$(37)
For simplicity, we adopt the following notations:

\[
\frac{G_t}{Y_t} = p_t, \quad \frac{b_t}{Y_t} = q_t, \quad \frac{Y_{t-1}}{Y_t} = f_t. \tag{38}
\]

Using these notations in eq. (38), eq. (37) may be rewritten as

\[
E_t(p_t) - E_t(q_t) + e^{t}E_t(q_{t-1})E_t(f_t) = E_t(p_{t+1}) - E_t(q_{t+1}) + e^{t}E_t(q_t)E_t(f_{t+1}),
\]

or

\[
E_t(q_{t+1}) - (1 + e^{t})E_t(f_{t+1})E_t(q_t) + e^{t}E_t(q_t)E_t(q_{t-1}) = E_t(p_{t+1}) - E_t(p_t). \tag{39}
\]

Note that I can separate the expectation of a product into a product of expectations because the decision variable is independent of the unobservable \( Y_t \). In particular, \( q_t \) is a function of \( Y_t \), which is in turn a function of \( \eta \) and the unobservable \( \nu_t \). On the other hand, \( f_{t+1} \) is a function of \( \eta \) and the unobservable \( \nu_{t+1} \). By the assumption that \( \nu_t \) and \( \nu_{t+1} \) are independent, \( q_t \) and \( f_{t+1} \) are therefore independent. In reality, the assumption of the unobservable \( Y_t \) at time \( t \) may be reasonable, since the budget decision process begins much earlier than the realization of \( Y_t \).

The transversality condition of the problem is the same as eq. (30). Following the same reasoning as in the previous case, the overall budget constraint \((3'')\) and the initial information set are two boundary conditions for solving eq. \((39)\).

The expectation of \( f_t \) is constant given information available at time \( t \). In particular,
\[ E_t(f_{t+1}) = e^{-\eta} + 0.5\sigma_v^2 = s = E_t(f_t). \] (40)

Substituting (40) into (39), and reducing the information set by one period, we have

\[ (1 - B)(1 - e^{r_s}B)E_{t-1}(q_t) = E_{t-1}(p_t - p_{t-1}). \] (41)

There are two roots for the above difference equation, 1 and \( e^{r_s} \). From the transversality condition (eq. (30) and eq. (36)), \( r \) is required to be greater than \( \eta - 0.5 \sigma_v^2 \). Therefore, we have \( r > \eta - 0.5 \sigma_v^2 \), i.e., \( e^{r_s} > 1 \), and eq. (41) becomes:

\[ (1 - B)E_{t-1}(q_t) = \left( \frac{1}{1 - e^{r_s}B} \right) E_{t-1}(p_t - p_{t-1}) + c_1(e^{r_s})^t. \] (42)

In a way similar to the previous case, eq. (42) can be solved as:

\[ b_t = b_{t-1}e^{\eta - 0.5\sigma_v^2} - (Y_{t-1}e^{\eta - 0.5\sigma_v^2})^t e^{-(r-0.5\sigma_v^2)i} E_t(p_{t+i} - p_{t+i-1}) \\
+ c_1(e^{r_s})^t s^{-1}Y_{t-1}. \] (43)

Because \( E_1[(e^{r_s})^ts^{-1}Y_{t-1}] = Y_0e^{rt} \), it is necessary to set the coefficient \( c_1 \) of eq. (42) to be 0 to satisfy the overall budget constraint (3').

Again for simplicity, I adopt the following notations:

\[ w = \gamma - \eta + 0.5 \left( \sigma_v^2 - 2\sigma_w^2 + \sigma_\zeta^2 \right), \]
\[ z = \gamma - \sigma_{v\zeta} + 0.5 \sigma_{\zeta}^2. \] (44)

Assume that \( z < 0 \). The \( z + r \) is different from the \( x \) in eq. (34) of the previous case because the covariance between two processes is taken into account. By notations in eq. (44), eq. (43) can then be shown explicitly as

\[
 b_t = b_{t-1} e^{(\eta-0.5\sigma_v^2)} - G_{t-1} e^{(\eta-0.5\sigma_v^2)} (e^\nu - 1) \left( \frac{e^z}{1 - e^z} \right). \] (45)

Note that for a finite \( E_1(G_t/Y_t) \), I assume that \( \nu \leq 0 \). If \( \nu < 0 \), an increase in the previous government spending would increase the current debt level. Intuitively, a sufficiently high income growth rate may make debt policy less burdensome. Therefore, greater public debt is feasible. On the other hand, if \( \nu = 0 \), public debt is growing independently of government expenditures.

It is noted that when \( \nu < 0 \), the time profile of \( b_t \) is not stationary. It has, in fact, an explosive time profile. Only \( q_t \), the ratio of debt to income, has a stationary time profile. However, since \( r > \eta - 0.5 \sigma_v^2 \) and \( z < 0 \), the present value of \( b_t \) may still be zero when \( t \) goes to infinity. The imposed assumptions which satisfy the transversality condition are not in conflict with present assumptions.

Finally, due to the uncertainty of the income process, I arrive at the same result as in the first case; the government at time 1 treats median income tax rate in the infinite future as if it were zero. For details, see Case(III) in Appendix B.
IV. Summary

In this chapter, I consider the new classical optimal debt policy under uncertainty in three cases. In general, intuitive results are derived. When income is uncertain but government spending is growing at a constant rate, tax revenue grows more slowly than under constantly growing income, and therefore must start off higher. Public debt should also grow more slowly when income is uncertain and the level of public debt at time 1 is less than that in a world of certainty. Furthermore, the government at time 1 no longer treats a constant income tax rate as optimal, but rather treats the income tax rate in the infinite future as if it were zero.

When income is constantly growing but government spending is uncertain, whether government can use public debt to finance permanently its expenditures depends on whether income grows fast enough. If the growth rate of income is greater than the log expected growth rate of government spending, public debt can grow positively with government expenditures. On the other hand, if they are equal, the existing public debt grows independently from government expenditures. Furthermore, unlike the first case, the uncertainty of government expenditures does not make the government at time 1 behave as if the median income tax rate in the limit were zero. Rather, a constant median income tax rate in the limit is recognized.

The most general case considered here is when both income and government spending are uncertain. An intuitive result can still be derived: a sufficiently high income growth rate allows public debt to respond positively to government expenditures. Also, due to the
uncertainty of income growth, the government at time 1 behaves as if a zero median income tax rate in the limit were fulfilled.
Notes

1 Lucas and Stokey (1983) discuss a related problem in an Arrow-Debreu state-contingent model with uncertain government expenditures. They show that the optimal structure of government obligations with full information is state contingent. In the real world, this state-contingent character is not a good approximation of the nature of debt or government debt in particular because states are not observed by both parties.

2 Note that $Y_1$ is unknown at time 1.

3 Since $Y_t$ is not in $Q$, in this case the closed-loop solution is the same as the open-loop solution. This can be shown either by comparing eq. (23) to eq. (44), which is a general solution for the first case, or by comparing eq. (20) to the closed-loop solution derived in Appendix A. For simplicity, I solve this problem by the open-loop solution.

4 The open-loop solution for public debt in this case is such that

$$b_{1} = \frac{e^{-rY(t+1)}}{e^{-r} - e^{-r}} G_0 + \frac{c_{1} e^{(\gamma - 0.5 r \sigma^2)t}}{e^{-r} - e^{-r}}$$

where $c_{1} = \frac{1}{e^{-r} - e^{-r}} [b_0 - \frac{e^{-rY(t)}}{e^{-r} - e^{-r}}]$.

5 Depending on $\alpha$ being less than, equal to, or greater than unity, the expected income tax rate as time $t \to \infty$ given $\varphi$ may be infinite positive, constant, or zero, respectively. See Appendix B for the demonstration that income tax rate in limit is zero.

6 The overall budget constraint ($3'$) requires that the present value of $b_0$ to be zero, while the transversality condition ($30$) only involves the $b/Y_t$. Therefore, ($3'$) imposes a more stringent boundary condition than the transversality condition.

7 Such an operator for solving a stochastic Euler equation is employed by Sargent (1979 pp. 336-337).

8 Alternatively, I keep $(1 - e^{-rB})$ instead of $(1 - e^{-\varphi})$ on the left hand side of eq. (33). But such a solution does not satisfy the transversality condition. Therefore, it is ignored in the discussion.
CHAPTER V

DATA

I have collected historical data from both the United States and the United Kingdom. The U.S. data set consists of 96 annual observations of the following 7 key variables from 1889 to 1984: GNP, GNP deflator, the level of public debt, the average annual interest rate paid on government bonds, the level of government revenue and government expenditures, and the estimated total resident population. The data set I constructed uses measures of those variables as close as possible to the ones proposed by Barro (1978, 1979, 1984b).

In the United Kingdom data set, I have collected a total of 129 annual observations of those variables similar to the U.S. data from 1855 to 1983. However, no attempt has been made to adjust the government holdings of public debt.

Once the basic variables had been collected, I adjusted the measures of variables to satisfy the accounting budget constraint, converted the variables into real per capita terms, and derived the ex post real interest rate. Using the converted data sets, I proceed to describe the basic statistical properties of those variables, the stochastic process of variables of interest, and the Granger causality among variables.
I. U.S. Data Collection

The nominal GNP (A) and GNP deflator (B) come from the following sources: 1889-1908 from Historical Statistics of the United States Colonial Times to 1970, Part I, Series F 1-5, pp. 224; 1909-1928, 1929-1975 from National Income and Product Accounts of the United States, 1929-1976 Statistical Tables, Table 1.22, Table 1.1 and Table 7.1, pp. 72, 1-2, and 319 respectively; 1976-1979 from Survey of Current Business, Table 1 and Table 7.1, pp. 22 and 99 respectively in Vol. 62, No. 7 (July, 1982); 1980-1983, ibid., pp. 22 and pp. 81 respectively in Vol. 64, No. 7 (July, 1984); 1984, ibid., Table 1.1 -1.2 and Table 7.22, pp. 8 and pp. 26 respectively in Vol. 65, No. 7 (July, 1985). The GNP deflators between 1889 and 1908 use the 1958 price as the base. They are not consistent with the deflators of other years use that the 1972 price as the base. Accordingly, we have rescaled the deflators between 1889 and 1908 by multiplying an adjusting scale factor of 0.646853, which comes from the comparison between the deflators of 1958 and 1909 of the two series.

The level of public debt (C) includes the total interest bearing debt (D) and federal agency debt (E), but excludes government debt held by federal agencies, trust funds (F), and the Federal Reserve (G). Also, federal agency debts held by the government and the Federal Reserve (H) are excluded. It is assumed that public debt grows smoothly over a year; therefore, the measure of public debt as of June 30 is used as the mean level of the debt during that year. Ideally, federally sponsored agencies’ debt should also be considered, but so far the
related data are insufficient for me to calculate it. The sources of government debt are from the following:


For the government debt held by government agencies and trust funds (F), and by the Federal Reserve (G), 1916-1940, from *Banking & Monetary Statistics*, Section 13, Table No. 149, pp. 512; 1941-1970, from *Banking & Monetary Statistics 1941-1976*, Section 13, Table 13.4, pp. 882-883; 1971-1976, from *Treasury Bulletin*, Table OFS-1, pp. 77, Jan. 1978; for the years 1977-1984, the sources are the same as federal agencies debt issues (E).

For the agency securities held by federal government and the Federal Reserve (H), before and in 1954 the amount is negligible.
Between 1955-1960, the source is the Treasury Bulletin, in Ownership of Federal Securities section, Table 1, pp. 53, Jan. 1962; 1961-1970, ibd., Table OFS-1, pp. 65, Jan. 1971. The years 1971-1984 use same sources as (F) and (G).

Government revenue (I), expenditures (J), and the interest payment on public debt (K) between 1889 and 1976 are from Statistical Appendix to Annual Report of the Secretary of the Treasury on the State of the Finances, Table 2, pp. 4-13, Fiscal 1980. It is noted that before and in 1976 the government fiscal year ended at June 30, but since 1976 it ends at Sept. 30. To get consistent measures of (I), (J) and (K) for 1977-1984 against those before 1976, we use monthly data from Treasury Bulletin, Table FFO-1, and FFO-3, pp. 1 and 6 (FFO-1: pp. 3 for 1983 and 1984); FFO-3: pp. 7 for 1980, 1981 and 1983, and pp. 8 for 1984), Sept. 1977-1983, and Fall Fiscal 1983-1984.

After making the timing adjustment, I adjust the interest payment. First, the total adjusted annual interest payment should include the interest paid by government agencies to the non-federal-government public. Then on the government expenditure side, the total interest payment on public debt should be excluded.

Dividing the adjusted total annual interest payment into the mean level of the public debt before that year, we have the average interest rate of public debt for that year.

II. U.K. Data Collection


For price indices, unlike Barro, who used data from Mitchell's *European Historical Statistics 1750-1970* (Barro, 1984, EHS for short), I inferred them from D & C's data set for 1855-1915 and 1919-1959 by taking the ratio of the National Income per head in the current price and the National Income per head at 1913-1914 prices. In this way, I have avoided the consideration of changing indices over time. For price indices of 1915-1919, I collected them from EHS pp. 741, and adjusted them in the following way: First, I calculated the total inflation rate over the period of 1914-1920 from both EHS and D & C series. Second, I took the ratio of these two inflation rates and derived the rescaling factor 1.315. Third, by using 1914 D & C price index as a base, I re-adjusted the EHS series by rescaling EHS


Nominal public debt is from the following sources. For 1855-1937, it is from "Aggregate Gross Liabilities of the State" in Abstract, pp. 403. For 1938-1965, it is from "Aggregate Liabilities of the State" in 2nd Abstract, pp. 162. In the Abstract, 1938 and 1939 public debt measures are included. The difference between these two sources is that the 2nd Abstract does not include the external debt arising from the World War I years, 1914-1918. The magnitude is about 1 billion sterling. However, I do not know how that discrepancy faded over time; thus for simplicity, I have used the measure in the 2nd Abstract instead of trying to amend the discrepancy. The source for the years

Population measures between 1855 and 1914 and between 1920 and 1959 are computed from D & C, by taking the ratio of National Income in current price and National Income per head in current price. For 1915-1919, the population measures are from the Abstract, pp. 10. However, in the Abstract only civilian males in England and Wales are counted, accordingly, the following adjustment has been made. I have interpolated the male estimate by using 1914 and 1921 measures, and after the interpolation, using the 1914 male estimates in England and Wales as a base, I have added 30,000 each year to the male measure, then totaled estimates in other sectors. For 1960-1983, I have used mid-year estimates from IFS 1985, English edition, pp. 640.

III. Data Manipulation

Given these data, I first adjusted the measure of government expenditures to satisfy the government budget constraint such that \( b_t = G_t - R_t + e^{rt} b_{t-1} \). Note that from the above accounting budget constraint, I use the nominal interest rate for this adjustment. I then transform GNP, \( G_t \), \( R_t \) and \( b_t \) into the real per capita measure. However, the interest rate should be real and ex ante. To satisfy this requirement, either I treat the ex post real interest rate calculated from the budget constraint during per capita transformation as an ex ante rate plus an uncorrelated error with the ex ante real rate per se,
or I make an additional assumption that the price level is perfectly foreseen, assuring that the calculated real rate is consistent with the accounting budget constraint in real terms. Given that the empirical method I am going to apply is basically a generalized instrumental variable method, my treatment for ex post real interest rate should not cause inconsistent estimates as long as errors of selected instruments are uncorrelated with explanatory variables. After such adjustments, I establish the U.S. and U.K. data sets for my empirical study. They are found in Appendix C.

IV. Summary of Descriptive Statistics of the Data Sets

The basic statistics of the data sets are summarized in Table 1. The historical averages of ex post real interest rates in both countries are insignificantly different from zero at the 5% level. The average proportion of government expenditures, revenue, and debt to measured income is higher for the U.K. than for the U.S. However, the difference between the corresponding two-point estimates is insignificantly different from zero because of the large standard deviations of those measures. The same conclusion can be drawn for the historical average growth rate for income, government expenditures, and debt. That is, the U.S. has a higher growth rate, but not significantly greater than that of the U.K.

Figures are also provided to examine patterns of variables. Figure 1 and 2 show the historical proportion of government expenditures, revenue and debt to income for the U.S. and U.K., respectively. Ignoring the scale, the general pattern of these two
figures is very similar. However, some distinction still can be made. For example, after 1981 the debt-to-income ratio for the U.S. rose rapidly compared to the U.K. Although both countries reduced the ratio of government expenditures to income, the revenue-to-income reduction was faster for the U.S. than that for the U.K. As a result, a different debt-to-income pattern arose. Because the growth rates of those variables are major instruments used for my empirical study, Figures 3 to 8 are provided. Their stationarity is a prerequisite for valid estimation in terms of the econometric method I will employ. These figures provide the first clue about the validity of such an assumption. The first impression from these figures is that no obvious time trend exists for these variables; the stationarity assumption may not be violated in the sample period. Also, except for the two World Wars, the similarity of patterns between the two countries is limited.

The limited similarity in patterns for the corresponding variables between the two countries is further confirmed by the Vector Autoregression (VAR). In VAR, the dependent variables are the corresponding current period variables of both countries, while the independent variables are the corresponding variables of both countries lagged by one period. If the corresponding variables under study in the two countries follow a similar process, I would not reject the hypothesis of the equality of the corresponding coefficients across countries. However, the results shown in Table 2 demonstrate that only the income growth and the interest ratio reject the equality restriction. The hypothesis is otherwise rejected. Therefore, these dissimilar processes for the corresponding variables between two
countries make it necessary to study the public debt model for each country separately.

V. Specifying the Stochastic Process of Relevant Variables by the ARIMA Model

In this section, I specify the stochastic processes of the concerned variables by the ARIMA model. Such specifications serve two purposes. First, stochastic specifications of forcing variables in the previous chapter can help to evaluate the appropriateness of those stochastic assumptions according to the collected data sets. Second, since stationarity is an important assumption for future empirical study, I can test that property of the relevant time series from the coefficients of the estimated ARIMA model. Results are summarized in Table 3.

In Chapter IV, the forcing variables Y and G are assumed to follow a lognormal process with a constant drift. When ARIMA is fitted with the log of Y and G individually for both the U.S. and U.K. data sets, such assumptions are shown to be acceptable except that for log Y of the U.S., an additional AR(1) is needed. In general, taking the first order difference of the log of Y and G is sufficient to have white noise error terms in the ARIMA model. Therefore, my stochastic assumptions do not deviate too much from the stochastic property of my collected data.

For future empirical study, I examine the stationary properties of \( Y_t, G_t, R_t, b_t \) and \( r_t \). In growth terms, they are at most AR(2) or MA(1). Since estimated \( |\hat{\phi}_1| < 1 \), if any, they satisfy the stationarity
In this section, I study the Granger causality (1969) relation among variables in the data sets. In Granger's term, x is said to cause y if, given the past values of y's, the past values of x's can predict the current value of y. In my model, I hope to find that income Y and government expenditures G cause either revenue R or public debt b. I use the VAR to examine the existence of such causality. Results are summarized in Table 4.

When variables in level are used, the Granger causality relation is not quite clear. In the U.S. case, government spending G is not exogeneous to the system. Y is caused by G. Revenue R and public debt b interact with each other, and only the real interest rate r is unrelated to the system. In the U.K. case, b is exogeneous to the system; Y causes G; R and Y interact with each other; and r is caused by Y and G.

When variables in growth terms are used (prefix R), the Granger causality relation becomes clearer. In the U.S. case, RY and RG cause RR, and RG causes Rb. Rr is unrelated to the system. In the U.K. case, RY is exogeneous to the system, and it causes RG and Rb. RR is caused by RG; again Rr is unrelated to the system.

In sum, when the Granger causality among variables in level measures is considered, such causality is not the one implied by the model. However, when variables in growth terms are used, the causality
relation becomes clearer: either RY or RG causes RR and Rb. Such a causality relation is therefore consistent with that implied by the model.
Table 1
Descriptive Statistics of the U.S. and U.K. Data Sets

Unit: U.S. $1,000 at 1972 price per capita 
U.K. pound sterling at 1913-14 price per capita

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income^c</td>
<td>3.24049</td>
<td>55.87038</td>
</tr>
<tr>
<td>Gov't Expenditures</td>
<td>0.45726</td>
<td>14.43568</td>
</tr>
<tr>
<td>Revenue</td>
<td>0.42205</td>
<td>13.52982</td>
</tr>
<tr>
<td>Debt</td>
<td>1.00388</td>
<td>62.16232</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-0.00172</td>
<td>-0.00014</td>
</tr>
<tr>
<td>Gov't/Income</td>
<td>0.10691</td>
<td>0.20008</td>
</tr>
<tr>
<td>Revenue/Income</td>
<td>0.09760</td>
<td>0.19359</td>
</tr>
<tr>
<td>Debt/Income</td>
<td>0.27135</td>
<td>1.10768</td>
</tr>
<tr>
<td>Income Growth</td>
<td>0.02342</td>
<td>0.01498</td>
</tr>
<tr>
<td>Gov't Growth</td>
<td>0.11780</td>
<td>0.07663</td>
</tr>
<tr>
<td>Revenue Growth</td>
<td>0.05794</td>
<td>0.02889</td>
</tr>
<tr>
<td>Debt Growth</td>
<td>0.06608</td>
<td>0.01145</td>
</tr>
<tr>
<td>Ex-Post r_t / r_t-1</td>
<td>3.38600</td>
<td>1.88727</td>
</tr>
</tbody>
</table>

Note: a. U.S. annual data are from 1889-1984 and U.K. data are from 1855-1983.
b. The unit is not applicable to ratio or growth variables.
c. U.S. income is measured by GNP. U.K. income is measured by either National Income or Net National Product.
### Table 2

Test of Regression Coefficient Equality Across Equations for Variables between the U.S. and U.K. Data (1889-1983)

\[
V_i(t) = k_i + a_i V_{us}(t-1) + b_i V_{uk}(t-1) + \epsilon_i \quad (i=us \text{ or } uk)
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Country</th>
<th>(a_{us})</th>
<th>(b_{us})</th>
<th>(a_{uk})</th>
<th>(b_{uk})</th>
<th>(R^2)</th>
<th>(H_0: a_{us} = a_{uk}) &amp; (b_{us} = b_{uk})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>U.S.</td>
<td>0.8684*</td>
<td>0.0096*</td>
<td>0.9887</td>
<td>5402.951*</td>
<td>a,b</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>0.3405</td>
<td>0.9883</td>
<td>0.9841</td>
<td>(0.0443)</td>
<td>(0.0029)</td>
<td></td>
</tr>
<tr>
<td>Gov't</td>
<td>U.S.</td>
<td>0.5939*</td>
<td>0.0116*</td>
<td>0.8990</td>
<td>1054.336*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>2.4336</td>
<td>0.9035*</td>
<td>0.9188</td>
<td>(2.6181)</td>
<td>(0.0774)</td>
<td></td>
</tr>
<tr>
<td>Revenue</td>
<td>U.S.</td>
<td>0.7512*</td>
<td>0.0080*</td>
<td>0.9767</td>
<td>8595.446*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>0.3132</td>
<td>1.0039*</td>
<td>0.9892</td>
<td>(1.6186)</td>
<td>(0.0573)</td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>U.S.</td>
<td>0.9459*</td>
<td>0.0006*</td>
<td>0.9434</td>
<td>4352.859*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>-0.6675</td>
<td>0.9930*</td>
<td>0.9783</td>
<td>(1.4772)</td>
<td>(0.1030)</td>
<td></td>
</tr>
<tr>
<td>Ex-Post r</td>
<td>U.S.</td>
<td>0.4296*</td>
<td>0.0052*</td>
<td>0.1705</td>
<td>46.828*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>0.1257</td>
<td>0.5922*</td>
<td>0.3460</td>
<td>(0.1344)</td>
<td>(0.1170)</td>
<td></td>
</tr>
<tr>
<td>RIncome</td>
<td>U.S.</td>
<td>0.1916</td>
<td>0.0528</td>
<td>0.0215</td>
<td>1.161</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>0.1330</td>
<td>-0.0646</td>
<td>0.0073</td>
<td>(0.0805)</td>
<td>(0.1062)</td>
<td></td>
</tr>
<tr>
<td>RGov't</td>
<td>U.S.</td>
<td>0.3147*</td>
<td>0.2737*</td>
<td>0.1767</td>
<td>12.401*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>-0.5268</td>
<td>0.9858</td>
<td>-0.0196</td>
<td>(0.1084)</td>
<td>(0.1039)</td>
<td></td>
</tr>
<tr>
<td>RRevenue</td>
<td>U.S.</td>
<td>0.1697</td>
<td>0.9759*</td>
<td>0.1926</td>
<td>15.423*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>0.0910</td>
<td>0.1225</td>
<td>0.0658</td>
<td>(0.0404)</td>
<td>(0.1036)</td>
<td></td>
</tr>
<tr>
<td>RDebt</td>
<td>U.S.</td>
<td>0.2429*</td>
<td>1.7356*</td>
<td>0.4337</td>
<td>12.401*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>-0.0427</td>
<td>0.7222*</td>
<td>0.4264</td>
<td>(0.0264)</td>
<td>(0.0895)</td>
<td></td>
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<tr>
<td>Rr</td>
<td>U.S.</td>
<td>-0.0271</td>
<td>-0.0363</td>
<td>-0.0210</td>
<td>0.085</td>
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<tr>
<td></td>
<td>U.K.</td>
<td>-0.0008</td>
<td>-0.0204</td>
<td>-0.0218</td>
<td>(0.1097)</td>
<td>(0.3148)</td>
<td></td>
</tr>
</tbody>
</table>

a. The critical test statistic is \(X^2(2, 0.05) = 5.991\)
b. * significant at the 5% level.
c. Prefix R stands for \(x(t)/x(t-1)\).
### Table 3

Time Series in ARIMA(p,d,q) Specifications

\[(1-L)^d Z_t = \mu + (1-\theta L)/(1-\phi_1 L)(1-\phi_2 L) \varepsilon_t\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Country</th>
<th>(p,d,q)</th>
<th>(\mu)</th>
<th>(\phi_1)</th>
<th>(\phi_2)</th>
<th>(\theta)</th>
<th>Variance Est.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Y</td>
<td>U.S.</td>
<td>(1,1,0)</td>
<td>0.01840</td>
<td>0.20790</td>
<td></td>
<td></td>
<td>0.00383</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.00798)</td>
<td>(0.10165)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>(0,1,0)</td>
<td>0.01393</td>
<td></td>
<td></td>
<td></td>
<td>0.00190</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.00385)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log G</td>
<td>U.S.</td>
<td>(0,1,0)</td>
<td>0.04283</td>
<td></td>
<td></td>
<td></td>
<td>0.13397</td>
</tr>
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<td></td>
<td></td>
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<td>(0.03755)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>U.K.</td>
<td>(0,1,0)</td>
<td>0.02895</td>
<td></td>
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<td>0.07078</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.02352)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(Y_t / Y_{t-1})</td>
<td>U.S.</td>
<td>(1,0,0)</td>
<td>1.02053</td>
<td>0.20352</td>
<td></td>
<td></td>
<td>0.00385</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.00797)</td>
<td>(0.10173)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>(0,0,0)</td>
<td>1.01498</td>
<td></td>
<td></td>
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<td></td>
<td>(0.00391)</td>
<td></td>
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<tr>
<td>(G_t / G_{t-1})</td>
<td>U.S.</td>
<td>(2,0,0)</td>
<td>1.11828</td>
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<td>0.20794</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(0.05659)</td>
<td>(0.10110)</td>
<td>(0.10111)</td>
<td></td>
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<tr>
<td></td>
<td>U.K.</td>
<td>(0,0,0)</td>
<td>1.07663</td>
<td></td>
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<td>0.21313</td>
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<tr>
<td></td>
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<td></td>
<td>(0.04080)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_t / R_{t-1})</td>
<td>U.S.</td>
<td>(1,0,0)</td>
<td>1.05780</td>
<td>0.27936</td>
<td></td>
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<td>0.06534</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.03622)</td>
<td>(0.09957)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>(0,0,0)</td>
<td>1.02889</td>
<td></td>
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<td>0.00875</td>
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<td></td>
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<td></td>
<td>(0.00827)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b_t / b_{t-1})</td>
<td>U.S.</td>
<td>(0,0,1)</td>
<td>1.06484</td>
<td></td>
<td></td>
<td>-0.79927</td>
<td>0.08431</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.05881)</td>
<td></td>
<td></td>
<td>(0.06239)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>(1,0,0)</td>
<td>1.01217</td>
<td>0.63723</td>
<td></td>
<td></td>
<td>0.00563</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.01782)</td>
<td>(0.06867)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(r_t / r_{t-1})</td>
<td>U.S.</td>
<td>(0,0,0)</td>
<td>3.38601</td>
<td></td>
<td></td>
<td></td>
<td>381.696</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.00446)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.K.</td>
<td>(0,0,0)</td>
<td>1.88727</td>
<td></td>
<td></td>
<td></td>
<td>84.1125</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.81063)</td>
<td></td>
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</tr>
</tbody>
</table>
### Table 4
The Granger Causality by Observing Results from VAR

<table>
<thead>
<tr>
<th>System</th>
<th>Lags</th>
<th>Country</th>
<th>Dependent Variable</th>
<th>Significant at 5% Explanatory Variable</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y,G,R,b,r</td>
<td>2</td>
<td>U.S.</td>
<td>Y</td>
<td>Y, G</td>
<td>0.99113</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td>G, R, b</td>
<td>0.95315</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R</td>
<td>Y, G, R, b</td>
<td>0.99098</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>b</td>
<td>G, R, b</td>
<td>0.98645</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>r</td>
<td>r</td>
<td>0.28612</td>
</tr>
<tr>
<td>U.K.</td>
<td></td>
<td></td>
<td>Y</td>
<td>Y, R</td>
<td>0.99207</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>G</td>
<td>Y, G</td>
<td>0.95398</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>R</td>
<td>Y, R</td>
<td>0.99506</td>
</tr>
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<td></td>
<td>b</td>
<td>b</td>
<td>0.99089</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>r</td>
<td>Y, G, r</td>
<td>0.56743</td>
</tr>
<tr>
<td>RY,RG,RR,Rb,Rr^a</td>
<td>2</td>
<td>U.S.</td>
<td>RY</td>
<td>----</td>
<td>0.13704</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>RG</td>
<td>----</td>
<td>0.21729</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>RR</td>
<td>RY, RG, RR</td>
<td>0.45475</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rb</td>
<td>RG</td>
<td>0.44997</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rr</td>
<td>----</td>
<td>0.03085</td>
</tr>
<tr>
<td>U.K.</td>
<td></td>
<td></td>
<td>RY</td>
<td>RY</td>
<td>0.12132</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>RG</td>
<td>RY</td>
<td>0.09261</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>RR</td>
<td>RG</td>
<td>0.31316</td>
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<td></td>
<td></td>
<td>Rr</td>
<td>----</td>
<td>0.05645</td>
</tr>
</tbody>
</table>

^a Prefix R represents an operator such that ( $x(t) / x(t-1)$ ).
FIGURE 1
RATIO OF U.S. GOVERNMENTS, REVENUE AND DEBT TO GNP
FIGURE 2
RATIO OF U.K. GOVERNMENTS, REVENUE AND DEBT TO NI
FIGURE 3
U.K. AND U.S. INCOME GROWTH RATE
FIGURE 4
U.K. AND U.S. GOVERNMENT EXPENDITURE GROWTH RATE
FIGURE 5
U.K. AND U.S. GOVERNMENT REVENUE GROWTH RATE
FIGURE 6
U.K. AND U.S. GOVERNMENT DEBT GROWTH RATE
FIGURE 7
U.K. AND U.S. EX-POST REAL INTEREST RATE
FIGURE 8
U.K. AND U.S. EX-POST REAL INTEREST RATE GROWTH RATE
Notes

1 The U.S. data set was jointly collected by Yen Shih and me.


3 I acknowledge Seater's (1985) criticism about the measurement errors of deficits and public debt arising from inflation, changes in interest rates, and the existence of state and local deficits. In this dissertation, I use public debt in real per capita value to avoid the first mentioned error. However, instead of adjusting the value of public debt due to changes of the term structure of interest rates for government securities, I derive "the" nominal interest rate through the ratio of the current year nominal interest payment to the face value of the previous period's public debt by the assumption that the life of all bonds is one period. I also make no attempt to adjust the state and local deficits, for two reasons: first, a data collection problem exists dating back to the 19th century. Second, the nature of state and local deficits and the state constitution restrictive on the state budgeting process make state and local deficits quite a different issue from national debt.

4 Again, Barro's procedure is followed. See Barro (1984b, pp. 369).

5 GNP, GNP deflator and population estimate are collected on December 31 while other variables are collected on June 30 each year.

6 We have insufficient data for both government debt and federally sponsored agencies' debt held by federally sponsored agencies, and federally sponsored agency debt held by federal government agencies and the Federal Reserve.

7 Before and in the 1960s, the available data are called "interest-bearing securities guaranteed by the U.S. government".


9 I use the computer package RATS 4.11 to perform the VAR estimation.

10 I use PROC ARIMA in SAS to perform the estimations.

11 The interest rate in both countries is also stationary with the AR(1) specification.
CHAPTER VI
AN EMPIRICAL STUDY OF THE NEW CLASSICAL APPROACH
TO THE DETERMINATION OF PUBLIC DEBT

I. Introduction

In this chapter I use a structural-type approach to estimate and test Barro's (1979, 1984) "new classical" model of public debt determination. Barro tests his model by regressing the measured debt variable on independent variables which come from relevant comparative studies. Although in general his empirical results match the sign predicted from his comparative studies, it does not necessarily imply that his model should be accepted. Other models may give the same empirical implications.

Alternatively, I adopt the Generalized Method of Moments (GMM) estimator, proposed by Hansen (1982) and Hansen and Singleton (1982), to estimate and test the model directly. Such an econometric method has been applied in different research fields. For example, in asset pricing models, see Hansen and Singleton, Brown and Gibbons (1985) and Rayner (1985). In the term structure of interest rates, see Dunn and Singleton (1984). In consumption models, see Hayashi (1982), and Eichenbaum and Hansen (1984). In general equilibrium macroeconomic models, see Rotemberg (1984), Mankiv, Rotemberg and Summers (1985). In a foreign exchange risk premia model, see Mark (1985).
GMM is a generalized instrumental variable method. One feature of such an estimator is that we can estimate parameters of the model from the stochastic Euler equation without explicitly giving the stochastic processes of the "forcing variables" in the model. As long as those stochastic processes are ergodic stationary, the GMM estimator is strongly consistent and asymptotically normal. Another feature of the GMM estimator is that selected instruments need not be exogenous and a conditional heteroskedastic forecast error is allowed. Note also that if a conditional homoskedastic error is assumed, the GMM estimator reduces to a Nonlinear Two-Stage Least Squares (NL2SLS) estimator asymptotically. However, if the homoskedasticity assumption is violated, the estimated variance-covariance matrix would not then be consistent. Hence, adjusting the variance-covariance matrix of the estimated parameters for consistency is necessary. Finally, GMM (and NL2SLS if the conditional homoskedasticity is assumed) provides a $\chi^2$ test statistic for the over-identifying restriction for the model. Such a test statistic can be used to accept or reject the model directly.

The remainder of this chapter is organized as follows. In section II, I set up the model estimation strategy for GMM and NL2SLS. In section III, I lay out the testing hypotheses to be examined. In section IV, the empirical results are discussed. In section V, I discuss further results when a linear homogeneous restriction is imposed on the model. In section VI, I investigate the robustness of the parameter estimates and policy implication from the empirical study.
II. Model Estimation

The general objective function without any restrictions on parameters (unrestricted model for short) is:

\[
\text{Max } V = \sum_{t=1}^{\infty} \mathbb{E}_t \left( G_t - b_t + e^{r_t} b_{t-1} \right) Y_t e^{1+\alpha - \beta - \delta (t-1)}
\]

(46)

where \( G_t \) is government spending at time \( t \), \( b_t \) is the level of public debt at time \( t \), \( Y_t \) is the income level at time \( t \), \( r_t \) is the interest rate at time \( t \), \( \delta \) is the discount rate applied by the government, and \( \alpha \) and \( \beta \) are cost parameters in the model. Assume that \( \alpha, \beta, \) and \( \delta \) are constant over time. \( \mathbb{E}_t \) is the expectation operator given information available at time \( t \). By the accounting budget constraint, terms in the parentheses of equation (46) are equivalent to \( R_t \), government revenue at time \( t \).

The stochastic Euler equation of the above objective function (46) is

\[
\mathbb{E}_t \left[ \frac{R_t}{Y_t} e^{r_t} - \frac{R_{t+1}}{Y_{t+1}} e^{r_{t+1}} \right] = 0
\]

(47)

where \( \alpha, \beta, \) and \( \delta \) are three parameters unknown to econometricians.

Assume that government policy makers and econometricians do not observe \( Y, R, \) and \( r \) of time \( t+1 \) at time \( t \). Also, both \( (R_{t+1}/ R_t) \), \( (Y_{t+1}/ Y_t) \) and \( r_t \) are assumed to follow some ergodic stationary stochastic processes. As a result, a simplified equation (47) can be shown as:
where $E_t$ is the expectation operator given information available at time $t$.

Assume that $u_{t+1}$ is the disturbance term in the econometric estimation of eq. (48), and $E_t[u_{t+1}] = 0$. Further, assume that $E_t^2$ is nonzero and finite. Let $Z_t$ be the vector of (instrumental) variables with finite second moments that are in the government decision maker's information set. The following function $f_t$ can then be constructed:

$$f_t = [(R_{t+1}/R_t) (Y_t/Y_{t+1}) e^{r_{t+1} - 1}] x Z_t$$

where $x$ is the scalar multiplication operated on a vector. Note that $f_t$ is a function of $R$, $Y$, $r$, $Z$, and parameters $\alpha$, $\beta$, and $\delta$. The dimension of $f_t$ is determined by the dimension of $Z_t$.

From eq. (48) and (49), we have the following orthogonality conditions:

$$E[f_t] = 0$$

where $E$ is the unconditional expectation operator. The number of the orthogonality condition in this case is the same as the dimension of $Z_t$.

The sample counterpart of eq. (50) is constructed according to the method of moments:
where $T$ is the sample size.

Eq. (51) should be close to 0 when the estimated $\alpha$, $\beta$ and $\delta$ are very close to the true parameters $\alpha$, $\beta$, and $\delta$, and when the number of observations $T$ becomes very large. The dimension of $g_T$ in this case is equal to the dimension of $f_t$.

To estimate parameters in eq. (48), an objective function must be used. Following Hansen, and Hansen and Singleton, I choose $\alpha$, $\beta$, and $\delta$ over the parameter space that would minimize function $J_T$ such that

$$J_T(\alpha, \beta, \delta) = g_T^T W_T g_T$$

where $W_T$ is a symmetric positive definite matrix with the order being equal to the number of instruments in this case.

The optimal choice of $W_T$ proposed by Hansen is the following in this model:

$$W_T^{-1} = R_T = \frac{1}{T} \sum_{t=1}^{T} f_t f_t'$$

Note that in order to compute $W_T$, a consistent estimator for $\alpha$, $\beta$, and $\delta$ is needed. Therefore, I apply estimating procedures in the following way. First, a starting value of $\alpha$, $\beta$ and $\delta$ are selected. Second, the suboptimal $W_T$ matrix is formulated according to eq. (53). Third, $\alpha$, $\beta$ and $\delta$ are estimated by minimizing eq. (52) given the suboptimal $W_T$. Fourth, $W_T$ is re-evaluated according to the the estimated $\alpha$, $\beta$ and $\delta$ in
the third step. Fifth, the third and fourth steps are repeated until $J_T$ converges (the convergence criterion is set to be $1.0^{-8}$ in my estimation).^7

The weighting matrix $W_T$ is closely related with the asymptotic variance-covariance matrix of the estimator. Hansen proposes to select $D_T$ as the sample average of the vector of the partial derivative of eq. (49) with respect to these parameters. With such a selected $D_T$, $(D_T' W_T D_T)^{-1}$ approaches the asymptotic variance-covariance matrix as the sample size goes to infinity.

To test the validity of the model, the over-identifying restriction test is used. Such a test follows from the fact that $T$ times the value of $J_T$ has asymptotically a $\chi^2$ distribution with degrees of freedom equal to the number of orthogonality conditions ($\phi$) minus the number of estimated parameters ($k$).

If $u_{t+1}$ is conditional homoskedastic, eq. (48) without the conditional expectation operator but added with $u_{t+1}$ can be estimated by the NL2SLS estimator. In other words, the GMM estimator reduces to the NL2SLS estimator if the error term $u_{t+1}$ is conditional homoskedastic. Also, under the conditional homoskedasticity assumption, the over-identifying restriction $\chi^2$ test statistic can be derived from the number of observations $T$ times the $R^2$ from regressing the NL2SLS estimated residuals on the instrumental variables. However, if the conditional homoskedasticity assumption is not valid, then the estimated variance-covariance matrix would not be consistent and any statistical inference derived from it would not be correct. Therefore,
for consistency and valid statistical inferences, the variance-covariance matrix of the estimated parameters has to be reestimated.\(^9\)

Given such a reestimated variance-covariance matrix, the asymptotic normality of the parameter estimators can be established.\(^{10}\)

Because Barro's constant income tax rate policy is based on his linear homogeneous restriction on the tax collection cost function with respect to tax revenue and income, it is interesting to estimate the parameters of the cost function with such a restriction, i.e., \(\alpha = \beta\) (a restricted model for short). Such an estimation is also performed. However, one should be cautious in interpreting these estimation results if the linear homogeneity hypothesis is statistically rejected in the unrestricted model.

III. Testing the Hypotheses

After estimating the unrestricted model, the following hypotheses are tested. First, whether the model is rejectable or not is tested by testing of the over-identifying restrictions. Second, if the over-identifying restriction test is confirmed, I then test properties of the estimated parameters. In particular, I test the following hypotheses:

\[(A) \quad H_0^A: \alpha = 0 \quad \text{and/or} \quad (A') \quad H_0^A: \alpha = 1 \]
\[H_1: \alpha > 0, \quad H_1: \alpha \neq 1,\]

\[(B) \quad H_0: \beta = 0 \]
\[H_1: \beta > 0,\]
(C) \( H_0: \alpha = \beta \)
\( H_1: \alpha \neq \beta, \)

and

(D) \( H_0: \delta = \bar{r} \)
\( H_1: \delta \neq \bar{r}. \)

Throughout these tests, I also examine the robustness of the GMM and NL2SLS estimation by examining the results of selecting different instruments.

In test (A), I examine whether the tax collection cost is positively related to tax revenue or not. If the model is correct, a higher tax revenue should cause higher costs, i.e., \( \alpha > 0. \) If the null hypothesis of (A) is rejected, a further test (A') needs to be done on whether \( \alpha = 1 \) or not. The problem is greatly simplified by assuming that the objective function is quadratic in terms of tax revenue. Rejecting that \( \alpha \) is equal to 1 can provide a clue as to the appropriateness of such a specification when used in Barro's model.

In test (B), I examine whether the tax collection cost is negatively related to income. A higher income implies a lower burden, given the same level of tax collection activity in the model. Therefore, we should expect that \( \beta > 0. \)

In test (C), I examine the assumption of linear homogeneity with respect to tax revenue and income as proposed by Barro. Such an assumption is crucial for his stable income tax rate prescription in a new classical world. Rejecting such an assumption leaves room for a different pattern of income tax and public debt policy.
The introduction of a government subjective discount rate is a device to make the tax collection cost function be separable in a special way. In test (D), I examine whether the government subjective discount rate is the same as the average real interest rate. From this test, we can therefore conclude the appropriateness of using the average real interest rate as a proxy in our model.

For the restricted model, I test the over-identification restriction and hypotheses in terms of (A), (A') and (D). If in the unrestricted setting we cannot reject the model but do reject that \( \alpha = \beta \), and in the restricted setting we cannot reject the model either, then we should interpret the results from the restricted model with caution. Statistically, such a restriction imposes a risk for specification error. Therefore, reported results only serve as a reference due to Barro's strong specification assumption. On the other hand, if neither model is rejected, and \( \alpha = \beta \) is not rejected either, then the linear homogeneous restricted model can be treated as a consistent further study from the general unrestricted model.

IV. Empirical Estimation and Results

Both restricted and unrestricted models are estimated using the annual U.S. data from 1889 to 1984 and U.K. data from 1855 to 1983. Given the asymptotic normality for GMM and variance-adjusted NL2SLS estimators, I can apply standard test procedures to those testing hypotheses proposed in the previous section.

Different instrument sets are used in the estimation. Since GMM estimates are a complicated nonlinear function of the data, in the
study of small sample properties of GMM estimation for parameters of a constant relative risk aversion utility function, Tauchen (1985) reports that there is a trade-off between variance and bias in terms of the number of lags used in the instrument set. In general, the longer the lags for the instruments, the more concentrated a bias the estimates would have. Therefore, according to his suggestion, the most credence should be placed in the smallest instrument sets. In this study, I use instruments lagged at most by three periods. In fact, I estimate the model twice, using similar instrument sets, of which one lags the other by one period. In particular, in the first round estimation, I select three instruments as a core: a constant, the previous period's income growth rate \( Y_t/Y_{t-1} \), and the previous period's revenue growth rate \( R_t/R_{t-1} \). Adding the previous period's changing rate of the ex post real interest rate, \( r_t/r_{t-1} \) (interest ratio or r ratio, for short), the previous period's national debt growth rate (debt growth, for short), and the previous period's government expenditure growth rate (government growth, for short), I have a six instrument set. When two lags are added to the same set of instruments, I have eleven instruments. Changing instruments in the estimation will check the robustness of the estimation results. In the second round estimation, I lag all the corresponding instruments from the first round by one period, and re-do the estimation.

GMM and NL2SLS results are summarized in Tables 5 and 6 for the first round estimation, and in Tables 21 and 22 for the second round estimation. Note that I use the interest ratio as an instrument in my estimation. In common econometric practice, a researcher probably would
use instead the previous period's ex post real interest rate (interest rate, or r-rate for short) as an instrument. Therefore, I repeat the estimation with the interest rate instrument. Those estimation results are summarized in Tables 7 and 8 for the first round, and in Tables 23 and 24 for the second round. To check the robustness of my estimation results, I repeat the estimation with these different instrument sets, each containing only five instruments, by adding the core instruments with interest rate (or interest ratio) and/or government growth or/and debt growth. Tables 9 through 12 summarize the results for the first round estimation, and Tables 25 through 28 for the second round.

Similar to the above estimation for the unrestricted model, I also estimate the linear homogeneous restricted model. Results are summarized in Tables 13 through 20 for the first round estimation, and in Tables 29 through 36 for the second round.

A. Empirical Results from the General Unrestricted Model

From the J-statistic derived from GMM, changing the composition of the instruments in both rounds of estimation does not in general reject the model. The only exception to rejecting the unrestricted model is the U.K. case when the interest rate variable is used in the eleven instrument set in the second round estimation (Table 23). Such a positive result is distinguishably different from those of Hansen and Singleton, Rotemberg, and Mankiw, Rotemberg and Summers in applying the GMM estimator in different settings.

The inability to reject the model does not imply that the test-statistic J value is weak in power. Rather, the unrestricted model
is flexible enough for the time path of collected variables. Since the model is not rejectable, further investigation of the properties of the estimated parameters is demanded.\(^\text{15}\)

**Hypothesis (A) and (A')**

In the GMM estimation, the first round \(\alpha\) estimates for the U.S. range from \(-0.0960\) to \(0.2399\), while the second round estimates range from \(0.0360\) to \(0.1927\). Except for the cases of six and eleven instruments with the interest rate instrument for both rounds, and two additional cases of five instruments with the interest rate instrument for the first round, they are insignificantly different from zero at the 5% level. On the other hand, the \(\alpha\) estimates for the U.K. range from \(0.2549\) to \(1.1314\) for the first round, and \(0.1761\) to \(0.8771\) for the second round, which are in general higher than in the U.S. cases. Also, except for the case of five instruments without government growth but with the interest ratio instrument for both rounds, and the case of five instruments without debt but with the interest ratio instrument in the first round estimation, values of \(\alpha\) are significantly greater than zero at the 5% level. However, results on whether the estimated \(\alpha\) is significantly different from one are mixed. For those \(\alpha\) significantly greater than zero, all U.S. cases are significantly less than one at the 5% level. On the other hand, four out of seven cases in the first round and two out of six cases in the second round for the U.K. cannot reject \(\alpha = 1\) at the 5% significance level.
In the NL2SLS estimation, the first round $\alpha$ estimates for the U.S. range from 0.1126 to 0.2349, while the second round estimates range from -0.0042 to 0.1595. Except for cases of five, six, and eleven instruments with the interest rate instrument in the first round, all cases are insignificantly different from zero at the 5% level. On the other hand, the first round $\alpha$ estimates for the U.K. range from 0.4474 to 0.9273, while the second round estimates range from 0.2088 to 0.8045, which again are higher than those in the U.S. case. Furthermore, five out of eight U.K. cases in the first round, and five out of seven cases in the second round are significantly greater than zero at the 5% level. For those estimated $\alpha$ values greater than zero in the U.K., three out of eight cases in the first round, but only one out of five cases in the second round, are insignificantly different from one at the 5% level.

It is also interesting to note that except for the case of no debt instrument but with the interest rate instrument in the second round of NL2SLS estimation, in all cases with both methods and both rounds of estimation, the estimated $\alpha$ values for the U.S. are significantly less than those for the U.K. at the 5% level when the interest rate instrument is used. However, none of the corresponding estimates are significantly different from each other at the 5% level when the interest ratio instrument is used.

In sum, for both rounds of estimation, the estimated $\alpha$ values using different instrument sets are in general insignificantly different from zero for the U.S., while they are generally greater than zero for the U.K. However, whether estimated U.S. $\alpha$ values are
significantly less than the corresponding U.K. $\alpha$ values depends on the interest instrument chosen. Furthermore, in GMM and NL2SLS estimation, there are more cases in which the estimated $\alpha$ values for the U.K. are insignificantly different from one than those for the U.S. Therefore, these results imply that revenue has a much more distorted effect on tax collection cost for the U.K. than for the U.S. A quadratic power on revenue distortion cost may not be a bad approximation in some cases for the U.K., but it is not valid for the U.S.

**Hypothesis (B)**

In the GMM estimation, the first round $\beta$ estimates for the U.S. range from $-1.0680$ to $0.3995$, while the second round estimates range from $0.5019$ to $1.0850$. Due to the large estimated standard errors, although five-eighths of $\beta$ estimates are negative, all $\beta$ estimates are still insignificantly different from zero at the 5% level in the first round estimation. However, for the second round estimation, except for the case of six instruments that include the interest rate instrument, they are significantly greater than zero at the 5% level. On the other hand, the first round $\beta$ estimates for the U.K. range from $-0.0834$ to $0.8177$, while the second round estimates range from $-0.8063$ to $0.7734$. Even though some $\beta$ estimates are negative, the hypothesis of $\beta = 0$ cannot be rejected at the 5% significance level for both rounds of estimation.

In the NL2SLS estimation, the first round $\beta$ estimates for the U.S. range from $0.0260$ to $0.7479$, while the second round estimates range from $0.1952$ to $0.9699$. Except for cases without the debt instrument in
the second round estimation, they are insignificantly different from zero at the 5% level. On the other hand, the first round $\beta$ estimates for the U.K. range from -0.3655 to 0.7479, while the second round estimates range from -0.6222 to 0.3109. None are significantly different from zero at the 5% level either.

In sum, the estimated $\beta$ values for both GMM and NL2SLS in the first round estimation are insignificantly different from zero at the 5% level. However, for the second round estimation, the estimated $\beta$ values from GMM with different instrument sets are in general significantly greater than zero for the U.S., although for the U.K. they are insignificantly different from zero. Nevertheless, only two U.S. $\beta$ values from NL2SLS are significantly greater than zero. The reasons for such a difference between GMM and NL2SLS results in the U.S. case, and between U.S. and U.K. results in general in the second round estimation, can be attributed to both the higher estimated standard errors in U.K. cases than the corresponding ones in U.S. cases, and the smaller estimated standard errors of $\beta$ from GMM than those from NL2SLS in the U.S. Furthermore, due to the larger estimated standard errors of $\beta$ than of $\alpha$, except for the GMM case of eleven instruments with the interest ratio instrument in the second round, the estimated $\beta$ values for the U.S. are insignificantly different from those for the U.K. at the 5% significance level in both methods of estimation and in both rounds. Hence, results show only weakly that income is important in reducing the tax collection cost for the U.S. when the second round instruments are used in GMM, but is not at all important for the U.K.
Hypothesis (C)

Given the estimated asymptotic variance-covariance matrix and the asymptotic normality property for GMM and NL2SLS estimates, I test the linear homogeneity hypothesis, i.e., whether $\alpha = \beta$ can be accepted or not. Such a test also determines the relative importance of income and revenue in determining tax collection cost.

In the first round of estimation, all cases cannot reject the linear homogeneity hypothesis. On the other hand, in the second round GMM estimation, except for the case of five instruments including the interest rate instrument without the government growth instrument, all U.S. cases reject the null hypothesis at the 5% significance level. In fact, $\alpha$ is significantly less than $\beta$ at the 5% level. Therefore, there is a strong indication that income has a stronger effect on the tax collection cost than does revenue in the U.S. case for the second round estimation. In contrast, the linear homogeneity hypothesis is not rejectable in the U.K. at the 5% significance level for all cases.

However, the NL2SLS estimation cannot make such a distinction. All cases of the U.S. and U.K. cannot reject the null hypothesis at the 5% significance level. This can attributed to both a smaller difference between, and larger standard errors of, estimated $\alpha$ and $\beta$ from NL2SLS than from GMM in the U.S.

In sum, the linear homogeneity hypothesis cannot be rejected for both countries in general, except that the second round GMM estimation provides evidence against the linear homogeneity hypothesis for the U.S. However, the NL2SLS estimation cannot reject the null hypothesis at all. Therefore, from the second round GMM results, imposing such a
restriction on the tax collection cost function may be a reasonable approximation for the U.K. but may distort the parameter estimates for the U.S. When such a restriction is imposed on the U.S. with the same instrument sets as those in the second round estimation, the results should be interpreted with caution. We, in fact, risk misspecifying the model. Otherwise, the linear homogeneity assumption is a valid restriction.

**Hypothesis (D)**

According to Chapter V, the historical mean (standard deviation) of the ex post real interest rate for the U.S. and U.K. is -0.00172 (0.05764) and -0.00014 (0.06197) per annum, respectively. Assuming that the ex post real interest rate and δ come from two independent normal distributions, I test whether \( \delta = \bar{r} \) or not.

In both the GMM and NL2SLS estimation for both rounds of estimation, I cannot reject the null hypothesis. Therefore, it would not be a bad approximation to substitute the historical mean of real interest rates for the subjective discount rate in practice.

**B Empirical Results from the Linear Homogeneous Restriction Model**

When the restriction that \( \alpha = \beta \) is imposed, only one U.K. case in the second round GMM estimation rejects the model based on the J-statistic at the 5% significance level. As the unrestricted model, the same is the case of the U.K. with eleven instruments using the interest rate instrument. Therefore, as in the unrestricted model, the restricted model is in general not rejectable for both countries. Such
a conclusion is not necessarily consistent with that for the U.S. if the second round instrument sets are used. Results from the second round GMM estimation on the unrestricted U.S. model show that the linear homogeneous assumption is rejectable. Therefore, estimation results from specifying the U.S. tax collection cost function with the linear homogeneity restriction using the second round instrument sets serve only as a reference for comparison.

**Hypothesis (A) and (A') with $\alpha = \beta$**

In the GMM estimation, the first round $\alpha$ estimates for the U.S. range from 0.1378 to 0.2287, while the second round estimates range from 0.0460 to 0.1518. All of the $\alpha$ estimates in the first round are significantly greater than zero but not equal to one at the 5% level. In contrast, none of the second round estimates are significantly different from zero at the 5% level, mainly due to smaller point estimates of $\alpha$. On the other hand, the first round $\alpha$ estimates for the U.K. range from 0.3809 to 0.8671, while the second round estimates range from 0.1933 to 0.7647. All of them are significantly greater than zero at the 5% level. However, except for one-half of all cases in the first round estimation and two cases in the second round estimation, all cases reject the hypothesis that $\alpha = 1$ at the 5% significance level.

In the NL2SLS estimation, the first round $\alpha$ estimates for the U.S. range from 0.1202 to 0.2289, while the second round estimates range from -0.0015 to 0.1409. One-half of the $\alpha$ estimates in the first round are greater than zero, but none of the $\alpha$ estimates in the second round
are significantly greater than zero at the 5% level. Furthermore, for all \( \alpha \) values greater than zero, none of them are insignificantly different from one at the 5% level. On the other hand, the first round \( \alpha \) estimates for the U.K. range from 0.4025 to 0.7821, while the second round estimates range from 0.1733 to 0.5634. While all \( \alpha \) estimates in the first round estimation are greater than zero at the 5% level, only three out of seven cases in the second round estimation are significantly greater than zero at the 5% level. There are three cases in the first round estimation but only one case using the interest rate instrument without debt instrument in the second round estimation that are insignificantly different from one at the 5% level.

In sum, we face the same basic conclusion as in unrestricted model. The \( \alpha \) estimates for the U.K. are higher than those for the U.S. Except for cases of NL2SLS in the second round estimation, such a difference is statistically significant when the interest rate instrument is used. However, imposing linear homogeneous restriction does change the significance of the U.S. \( \alpha \) in the first round estimation when it is compared with the corresponding one in the unrestricted model. Also, different lags of instruments change the significant values of estimated \( \alpha \), which is quite clear from the restricted U.S. \( \alpha \) estimates from both rounds. Furthermore, all estimated \( \alpha \) values from GMM are significantly greater than zero in the first round, while this does not hold for NL2SLS estimates. Therefore, the linear homogeneity restriction, changes in instrument sets by different lags and the change in method of estimation do change the character of \( \alpha \) estimates in the model.
Hypothesis (D) with $\alpha = \beta$

In all cases, the estimated $\delta$ values for the U.S. are indifferent from those for the U.K. at the 5% significance level. Furthermore, none of the estimated $\delta$ values are significantly greater than zero for both rounds of estimation. Also, for both rounds of GMM and NL2SLS estimation in both countries, none of the $\delta$ estimates are significantly different from the historical mean of the real interest rate at the 5% level. Therefore, as with the unrestricted model, we can use the historical mean of the real interest rate in each country as a proxy for its subjective government discount rate.

C. Summary

Empirical results from GMM show that the cost minimization approach to the determination of public debt is in general not rejectable. In the first round estimation with the linear homogeneity restriction, all GMM estimates of $\alpha$ are significantly greater than zero. In the second round estimation, both GMM and NL2SLS result in different patterns of parameter estimates for the U.S. and U.K. In particular, income is a relatively important factor in the U.S. cost function, while revenue is more important to the U.K. Also, for both rounds of GMM estimation, when the interest rate instrument is used, the $\alpha$ estimate for the U.S. is significantly less than the corresponding estimate for the U.K. No such conclusion can be given to cases using the interest ratio instrument. Nor can we say that estimates of $\beta$ are significantly different between the two countries. However, the first round large negative U.S. $\beta$ point estimates seem
beyond the theoretical lower bound although they are still insignificantly different from zero. In the second round GMM estimation, rejecting the linear homogeneity hypothesis for the U.S. cases makes us cautious in interpreting the U.S. estimation results in which such a restriction has been imposed. Less of a risk exists for estimates from the U.K. Although GMM and NL2SLS estimates are similar, at least in testing linear homogeneity hypothesis (C) in the second round estimation, the NL2SLS estimation cannot pick up clearly different results for the U.S. and U.K. In that test, GMM seems to be better. However, the average ex post real interest rate can still be treated as a proxy for $\delta$.

VI. Some Further Results and Implications from the Second Round

Empirical Results

From both restricted and unrestricted models in the second round estimation, the estimated $\alpha$ values for the U.S. sometimes are significantly lower than those for the U.K. Also, in the unrestricted model, estimated $\beta$ values for the U.S. have different patterns from those for the U.K. The following question then arises: What factors could result in such different estimated parameters? Two attempts have been made here to identify the sources of such heterogeneous results. However, due to the nonlinear complexity of timing and patterns among those instrument variables, and between forcing variables and instrumental variables, no conclusive answer can be given. Only some directions are provided.
I first examine how estimates are changed by separating the sample into several subsamples. In the first experiment, I arbitrarily separate the U.S. data set at 1945. Accordingly, the U.K. data set is separated at 1888 and 1945. By doing so, I can examine how estimates vary when one subsample includes majors wars, while the other does not. Furthermore, I can investigate the robustness of GMM estimation in terms of sampling periods. In the second experiment, I delete the 1984 observation from the U.S. data and 1855-1888 observations from the U.K. data to align the U.S. with U.K. data, then separate the sample equally. Each subsample then includes one World War, I examine how estimates differ from those in the first experiment by separating the sample at different points in time. Conclusions here should be tracked with caution, because special events like the Depression may be included in one subsample but not in the other. I use six instruments, with the interest ratio instrument in the second round estimation as my experimental instrumental variable set. Results are summarized in Tables 37 and 38.

A. Results from Separating Sample the at 1945

Comparing the results for the U.S. in the first experiment with the corresponding estimates over the whole sample period in Table 21, the estimated $\alpha$ values increase in both subsamples. The estimated $\beta$ values decrease, but the estimated $\delta$ values increase in the first subsample from 1889 to 1945. Opposite results occur for the second subsample from 1946 to 1984. Furthermore, all estimated standard errors have increased. However, none of these corresponding estimates
in two subsample periods differ significantly from each other at the 5% level.

In contrast to the results of the U.S. estimation, results from the U.K. fall into a more complicated pattern. In the first subsample from 1855 to 1888, while both estimated $\alpha$ and $\beta$ decrease, the estimated $\delta$ increases. For the 1889 to 1945 subsample, all estimated parameters increase. For the 1946-1983 subsample, all estimated parameters decrease. Less than two-thirds of the estimated standard errors for those parameters have higher values.

Furthermore, the estimated $\alpha$ flips its sign to negative in the first and third subsamples. Also, the $\alpha$ estimate in the first subsample is significantly greater than in the second subsample at the 5% level. Additionally, the $\beta$ estimate in the third subsample becomes negative and is significantly different from that in the second subsample at the 5% level. Therefore, estimation results vary widely for the U.K. case.

In sum, except that J-values have increased over all subsample periods, estimation results vary. According to the U.S. results, including the two World Wars in the first subsample does not make the $\alpha$ estimate significantly higher than in the second subsample, although this is true for the U.K. Divergent directions for the estimated $\beta$ in two subsamples for the U.S. and U.K. make it hard to judge the effect of the 1930s Depression. Obviously, separating the sample into subsamples does change the estimated results, but the directions are hard to explain. It is also no longer true that the estimated $\alpha$ ($\beta$) has a higher (lower) value in the U.K. than in the U.S. In this
experiment, I cannot therefore identify sources for divergent results in the unrestricted model.

B. Results from Separating the Sample at 1935

In the second experiment, except for similar comparisons as in the first experiment, I examine changes of estimates due to changes in sampling periods. Again, J-values in both countries and both subsamples have gone up. For both countries in the second subsample from 1936 to 1983, parameter estimates and standard errors are lower compared to the corresponding estimates over the entire sample period, 1889 to 1983. On the other hand, in the first subsample from 1889-1935, all standard error estimates for U.S. parameters increase.

Compared to the corresponding estimates in the previous experiment, the α estimate is lower, while the β estimate is higher for the current experiment in U.S. first subsample. In the second subsample of the U.S., the α estimate is also lower. For the U.K. case, α estimates in both samples are higher for the current experiment, but mixed results occur for β estimates. Changing the sample period and/or including World War II in the sample does change α point estimates, but their directions are different for the U.S. and U.K. Therefore, it is again hard to tell which common factors cause such different results.

In terms of the change of estimates between subsamples, in the U.S. case, the corresponding estimates between two subsamples are not significantly different from each other at the 5% level. Furthermore, the β estimate is no longer significantly greater than zero, and the α
estimate changes its sign to negative in the first subsample. In the U.K. case, both estimates of \( \alpha \) and \( \delta \) in the first subsample are significantly greater than the corresponding estimates in the second subsample at the 5% level. Also, both \( \alpha \) and \( \delta \) estimates change to negative in the second subsample.

In sum, while lower parameter estimates for both countries are found in the second subsample period compared to the corresponding estimates over the whole sample, no such direction is found in the first subsample period. In terms of the pattern of parameter estimates, there is some indication that the divergence of the parameter estimates in two countries diminishes in the second subsample period. In particular, the \( \alpha \) estimates reverse order for these two countries in this subsample. In contrast, no such conclusion holds in the first experiment. However, it is clear that changing the break point for the subsamples could change the estimation results, but including major events like wars does not result in a clear change of directions for these estimates.

C. Policy Implications of Empirical Estimates in a Constant Growing Economy

After searching for reasons for the different patterns of estimates for the U.S. and U.K., I considered what policy implications arise from the empirical results. However, I must emphasize that policy implications for public debt derived in the following occur only in a hypothetical world, although relevant estimates applied come from my empirical results. Such an exercise indicates only how the model
works, and may tell us little about what real public debt policy should be. The following derived debt policy only considers a world of perfect foresight, yet in the real world, the uncertainty of economic forcing variables and the possibility of changes in the political structure make it difficult to formulate a precise debt policy over time.

In a world of perfect foresight, if \( G \) and \( Y \) are growing at a constant rate and \( r \) is constant, the first-order condition of eq. (47) can be transformed into:

\[
R_t = R_{t-1} e^{(\eta \beta + \delta - r)}/\alpha
\]  
(54)

where \( \eta \) is the constant continuous growth rate of income.

By the accounting budget constraint and eq. (54), the optimal level of public debt is determined as:

\[
b_t = G_{t-1} e^\gamma - R_{t-1} e^{(\eta \beta + \delta - r)}/\alpha + e^\gamma b_{t-1}
\]  
(55)

where \( \gamma \) is the constant continuous growth rate of government expenditures.

Since government spending is also assumed to grow at a constant rate \( \gamma \), eq. (55) is a first-order non-stochastic difference equation. Also, because the historical average real interest rate is negative in both country samples, the only root for eq. (55) is less than one and eq. (55) can be solved as:

\[
b_t = G_0 \frac{e^{\gamma(t+1)}}{e^\gamma - e^r} - R_0 \frac{e^{\pi(t+1)}}{e^\pi - e^r}
\]  
(56)
where \( \pi = (\eta \beta + \delta - \rho)/\alpha \), the growth rate of tax revenue.

The coefficient for the homogeneous part of the solution must be set to zero to satisfy the transversality condition. Furthermore, by the transversality condition, I have the following restriction applied in eq. (56):

\[
C_0\left[\frac{e^{(\gamma - r)(t-1)}}{e^\gamma - e^r}\right] - R_0\left[\frac{e^{(\pi - r)(t-1)}}{e^\pi - e^r}\right] \to 0 \text{ as } t \to \infty. (57)
\]

Since in eq. (57) all parameters and initial values are constant except \( t \), unless the terms on two two sides of the minus sign are equal, \((\gamma - r)\) and \((\pi - r)\) must be less than zero to satisfy the above restriction. But because the empirical \( r \) is negative, \( \gamma \) and \( \pi \) must be negative too. This implies that government expenditure and tax revenue growth rates should both be negative. Obviously, from the data I have, such an assumption is not initially fulfilled. Therefore, I have a nonstationary time path for public debt, which can be positive explosive if positive government spending growth rate \( \gamma \) is greater than tax revenue growth rate \( \pi \), or be negative explosive otherwise. For example, using real data in such a hypothetical world, I calculate each country's income growth rate and government expenditure growth rate by fitting initial and ending values of relevant variables in a continuous growth setting. The estimated \( \eta \) and \( \gamma \) are 0.0182 and 0.0428 for the U.S., and 0.0139 and 0.0290 for the U.K. Given those parameter estimates from six instruments in Table 21, the \( \pi \) for U.S. and U.K. is 0.0269 and 0.0220, respectively. Because \( \gamma \) is greater than \( \pi \),
regardless how public debt fluctuates in the interim period, it will be positive explosive in the limit.

E. Summary

In this section, I examined the robustness of the GMM method on parameter estimates and factors possibly causing different estimates for the U.S. and U.K. according to the instruments in the second round estimation. I also investigated the implied public debt policy from my empirical results in a world of certainty.

For the former issue, I find that different sampling periods and/or major events like wars can change GMM point estimates, but the directions of these changes are hard to explain. My investigation of policy implications shows that in a world of a constant growing economy, the negative real interest rate causes a nonstationary public debt policy. Such a policy does not satisfy the transversality condition. Strictly speaking, no optimal debt policy can conform with the actual data. However, such a problem becomes less serious in a world of uncertainty. Although the average real interest rate is negative, the large sample variance of the real interest rate outweighs the negative interest rate such that if there exists a probability distribution with finite second moments for those forcing variables, then the transversality condition in an expectation formed at the initial decision time may still be satisfied.
### Table 5


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>t value of $H_0: \alpha = \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>3.2223</td>
<td>-0.0960</td>
<td>-1.0680</td>
<td>0.02012</td>
<td>0.7281 6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2248)</td>
<td>(1.2892)</td>
<td>(0.02278)</td>
<td></td>
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<tr>
<td>U.K.</td>
<td>1.1673</td>
<td>0.2974*</td>
<td>0.8138</td>
<td>-0.00590</td>
<td>-1.0698 6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1437)</td>
<td>(0.4525)</td>
<td>(0.00906)</td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>4.0890</td>
<td>0.1380</td>
<td>-0.5452</td>
<td>0.01761</td>
<td>1.3671 11 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1056)</td>
<td>(0.5679)</td>
<td>(0.01053)</td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>2.9125</td>
<td>0.3239*</td>
<td>0.6659</td>
<td>-0.00448</td>
<td>-0.8551 6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1091)</td>
<td>(0.3758)</td>
<td>(0.00746)</td>
<td></td>
</tr>
</tbody>
</table>

- \( \chi^2(0.05,3)=7.8147; \chi^2(0.05,8)=15.5073. \)  
- * significant at the 5% level.

### Table 6


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>t value of $H_0: \alpha = \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>4.4252</td>
<td>0.1593</td>
<td>0.3800</td>
<td>-0.00188</td>
<td>-0.4046 6 lag0/lag1</td>
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<tr>
<td></td>
<td></td>
<td>(0.0972)</td>
<td>(0.5752)</td>
<td>(0.01337)</td>
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</tr>
<tr>
<td>U.K.</td>
<td>4.6125</td>
<td>0.5285</td>
<td>0.0523</td>
<td>0.01422</td>
<td>0.4174 6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2928)</td>
<td>(0.9645)</td>
<td>(0.02148)</td>
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</tr>
<tr>
<td>U.S.</td>
<td>7.2864</td>
<td>0.1375</td>
<td>0.3022</td>
<td>-0.00165</td>
<td>-0.2856 11 lag0/lag1</td>
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<tr>
<td></td>
<td></td>
<td>(0.0776)</td>
<td>(0.5875)</td>
<td>(0.01417)</td>
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<tr>
<td>U.K.</td>
<td>6.0375</td>
<td>0.4474*</td>
<td>0.0771</td>
<td>0.01174</td>
<td>0.5011 11 lag0/lag1</td>
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<tr>
<td></td>
<td></td>
<td>(0.1630)</td>
<td>(0.6839)</td>
<td>(0.01489)</td>
<td></td>
</tr>
</tbody>
</table>

- * significant at the 5% level.  
- J-value valid only if conditional homoskedasticity is assumed.
Table 7

<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>t value of $H_0: \alpha = \beta$</th>
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<tr>
<td>U.S.</td>
<td>4.1920</td>
<td>0.1952*</td>
<td>0.3995</td>
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<td>-0.3504</td>
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<td></td>
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<td>(0.0768)</td>
<td>(0.5657)</td>
<td>(0.01199)</td>
<td>6 lag0/lag1</td>
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<tr>
<td>U.K.</td>
<td>3.5236</td>
<td>0.8548*</td>
<td>0.2198</td>
<td>0.01936</td>
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<tr>
<td></td>
<td></td>
<td>(0.1873)</td>
<td>(1.2848)</td>
<td>(0.02083)</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td>U.S.</td>
<td>6.1774</td>
<td>0.2399*</td>
<td>-0.0876</td>
<td>0.01467</td>
<td>0.7610</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0626)</td>
<td>(0.4430)</td>
<td>(0.00975)</td>
<td>11 lag0/lag1, lag1/lag2</td>
</tr>
<tr>
<td>U.K.</td>
<td>8.8280</td>
<td>0.7672*</td>
<td>0.7313</td>
<td>0.01113</td>
<td>0.0598</td>
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<tr>
<td></td>
<td></td>
<td>(0.1648)</td>
<td>(0.5240)</td>
<td>(0.01063)</td>
<td>11 lag0/lag1, lag1/lag2</td>
</tr>
</tbody>
</table>

*a. $\chi^2(0.05,3)=7.8147; \chi^2(0.05,8)=15.5073.$  
*b. * significant at 5% level.

Table 8

<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>t value of $H_0: \alpha = \beta$</th>
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<td>U.S.</td>
<td>3.7076</td>
<td>0.2320*</td>
<td>0.7479</td>
<td>-0.00510</td>
<td>-0.7012</td>
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<td>(0.0870)</td>
<td>(0.7811)</td>
<td>(0.01560)</td>
<td>6 lag0/lag1</td>
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<tr>
<td>U.K.</td>
<td>5.3875</td>
<td>0.8592*</td>
<td>-0.2383</td>
<td>0.02797</td>
<td>0.6422</td>
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<td></td>
<td></td>
<td>(0.2469)</td>
<td>(1.5853)</td>
<td>(0.02832)</td>
<td>6 lag0/lag1</td>
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<td>U.S.</td>
<td>8.8044</td>
<td>0.2068*</td>
<td>0.5292</td>
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<td>(0.5385)</td>
<td>(0.01320)</td>
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<tr>
<td>U.K.</td>
<td>15.6250*</td>
<td>0.6110*</td>
<td>0.5728</td>
<td>0.00993</td>
<td>0.0480</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1580)</td>
<td>(0.7542)</td>
<td>(0.01524)</td>
<td>11 lag0/lag1, lag1/lag2</td>
</tr>
</tbody>
</table>

*a. * significant at the 5% level  
b. J-value valid only if conditional homoskedasticity is assumed.
### Table 9


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>α</th>
<th>β</th>
<th>δ</th>
<th>t value of Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.4292</td>
<td>0.2207*</td>
<td>0.1036</td>
<td>0.00662</td>
<td>0.1719</td>
</tr>
<tr>
<td></td>
<td>(0.0923)</td>
<td>(0.6610)</td>
<td>(0.01353)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>3.4920</td>
<td>0.8572*</td>
<td>0.2271</td>
<td>0.01933</td>
<td>0.4745</td>
</tr>
<tr>
<td></td>
<td>(0.1890)</td>
<td>(1.2928)</td>
<td>(0.02091)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.8248</td>
<td>0.1760</td>
<td>-0.2014</td>
<td>0.01026</td>
<td>0.4974</td>
</tr>
<tr>
<td></td>
<td>(0.1513)</td>
<td>(0.7283)</td>
<td>(0.01340)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>1.1806</td>
<td>0.2549</td>
<td>0.8098</td>
<td>-0.00683</td>
<td>-1.1256</td>
</tr>
<tr>
<td></td>
<td>(0.2295)</td>
<td>(0.4272)</td>
<td>(0.00937)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- a. $\chi^2(0.05,2)=5.99146$.
- b. * significant at the 5% level.

### Table 10


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>α</th>
<th>β</th>
<th>δ</th>
<th>t value of Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.0028</td>
<td>0.2349*</td>
<td>0.3424</td>
<td>0.00160</td>
<td>-0.1496</td>
</tr>
<tr>
<td></td>
<td>(0.1088)</td>
<td>(0.7846)</td>
<td>(0.01578)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.0375</td>
<td>0.9273*</td>
<td>-0.3655</td>
<td>0.03181</td>
<td>0.7232</td>
</tr>
<tr>
<td></td>
<td>(0.2045)</td>
<td>(1.6862)</td>
<td>(0.02933)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.6072</td>
<td>0.1725</td>
<td>0.0260</td>
<td>0.00473</td>
<td>0.2397</td>
</tr>
<tr>
<td></td>
<td>(0.1356)</td>
<td>(0.6964)</td>
<td>(0.01414)</td>
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<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.8000</td>
<td>0.4788</td>
<td>0.0691</td>
<td>0.01262</td>
<td>0.3786</td>
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<tr>
<td></td>
<td>(0.2581)</td>
<td>(0.9161)</td>
<td>(0.02040)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- a. * significant at the 5% level.
- b. J-value valid only if conditional homoskedasticity is assumed.
### Table 11

<table>
<thead>
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<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>t value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.4054</td>
<td>0.2306*</td>
<td>0.1560</td>
<td>0.00576</td>
<td>0.1130</td>
<td>5 lag0/lag1 r rate</td>
</tr>
<tr>
<td></td>
<td>(0.0908)</td>
<td>(0.6405)</td>
<td>(0.01316)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>1.4267</td>
<td>1.1314*</td>
<td>-0.0834</td>
<td>0.03003</td>
<td>0.6598</td>
<td>5 lag0/lag1 r rate</td>
</tr>
<tr>
<td></td>
<td>(0.3259)</td>
<td>(1.7808)</td>
<td>(0.03052)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>1.9968</td>
<td>0.1138</td>
<td>-0.3591</td>
<td>0.00988</td>
<td>0.5569</td>
<td>5 lag0/lag1 r ratio</td>
</tr>
<tr>
<td></td>
<td>(0.1643)</td>
<td>(0.8159)</td>
<td>(0.01486)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>1.1463</td>
<td>0.2982*</td>
<td>0.8177</td>
<td>-0.00586</td>
<td>-1.0715</td>
<td>5 lag0/lag1 r ratio</td>
</tr>
<tr>
<td></td>
<td>(0.1439)</td>
<td>(0.4546)</td>
<td>(0.00909)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. $\chi^2(0.05,2)=5.99146$.
b. * significant at the 5% level.

### Table 12

<table>
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<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>t value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>3.0176</td>
<td>0.2091*</td>
<td>0.5552</td>
<td>-0.00294</td>
<td>-0.4453</td>
<td>5 lag0/lag1 r rate</td>
</tr>
<tr>
<td></td>
<td>(0.0890)</td>
<td>(0.8303)</td>
<td>(0.01611)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>5.1875</td>
<td>0.8782*</td>
<td>-0.3182</td>
<td>0.02969</td>
<td>0.6758</td>
<td>5 lag0/lag1 r rate</td>
</tr>
<tr>
<td></td>
<td>(0.3246)</td>
<td>(1.5962)</td>
<td>(0.02947)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>2.2816</td>
<td>0.1126</td>
<td>0.0546</td>
<td>0.00193</td>
<td>0.0922</td>
<td>5 lag0/lag1 r ratio</td>
</tr>
<tr>
<td></td>
<td>(0.1216)</td>
<td>(0.7024)</td>
<td>(0.01409)</td>
<td></td>
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</tr>
<tr>
<td>U.K.</td>
<td>4.7250</td>
<td>0.4915</td>
<td>0.1236</td>
<td>0.01221</td>
<td>0.3458</td>
<td>5 lag0/lag1 r ratio</td>
</tr>
<tr>
<td></td>
<td>(0.3067)</td>
<td>(0.8785)</td>
<td>(0.02012)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
### Table 13


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>( \alpha )</th>
<th>( \delta )</th>
<th>( H_0: \alpha = \beta )</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>6.0148</td>
<td>0.1967*</td>
<td>0.00328</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0953)</td>
<td>(0.00596)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>2.1615</td>
<td>0.3809*</td>
<td>0.00100</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1544)</td>
<td>(0.00561)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>8.1052</td>
<td>0.1378*</td>
<td>0.00211</td>
<td>Accept</td>
<td>11 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0675)</td>
<td>(0.00518)</td>
<td></td>
<td>lag1/lag2</td>
</tr>
<tr>
<td>U.K.</td>
<td>3.8458</td>
<td>0.3898*</td>
<td>-0.00078</td>
<td>Accept</td>
<td>11 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1091)</td>
<td>(0.00529)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. \( \chi^2(0.05,4)=9.4877; \chi^2(0.05,9)=16.9190. \)
b. * significant at the 5% level.

### Table 14


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>( \alpha )</th>
<th>( \delta )</th>
<th>( H_0: \alpha = \beta )</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>5.3084</td>
<td>0.1309</td>
<td>0.00138</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1044)</td>
<td>(0.00682)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>5.3625</td>
<td>0.4613*</td>
<td>0.00686</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1865)</td>
<td>(0.00772)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>8.2432</td>
<td>0.1215</td>
<td>0.00089</td>
<td>Accept</td>
<td>11 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0918)</td>
<td>(0.00681)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>6.8625</td>
<td>0.4064*</td>
<td>0.00598</td>
<td>Accept</td>
<td>11 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1370)</td>
<td>(0.00664)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
Table 15

<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>4.5957</td>
<td>0.2178*</td>
<td>0.00590</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0718)</td>
<td>(0.00495)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>3.4465</td>
<td>0.7824*</td>
<td>0.01239</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1521)</td>
<td>(0.00740)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>7.6842</td>
<td>0.2048*</td>
<td>0.00797</td>
<td>Accept</td>
<td>11 lag0/lag1 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0543)</td>
<td>(0.00443)</td>
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<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>9.0102</td>
<td>0.7596*</td>
<td>0.01069</td>
<td>Accept</td>
<td>11 lag0/lag1 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1412)</td>
<td>(0.00722)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. $\chi^2(0.05,4)=9.4877; \chi^2(0.05,9)=16.9190.$
b. * significant at the 5% level.

Table 16

<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>7.3968</td>
<td>0.1906*</td>
<td>0.00275</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0859)</td>
<td>(0.00601)</td>
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</tr>
<tr>
<td>U.K.</td>
<td>6.4875</td>
<td>0.7554*</td>
<td>0.01166</td>
<td>Accept</td>
<td>6 lag0/lag1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1733)</td>
<td>(0.00813)</td>
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<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>12.0060</td>
<td>0.1821*</td>
<td>0.00238</td>
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<td>11 lag0/lag1 lag1/lag2</td>
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<tr>
<td></td>
<td></td>
<td>(0.0822)</td>
<td>(0.00619)</td>
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<tr>
<td>U.K.</td>
<td>15.5625</td>
<td>0.6072*</td>
<td>0.00935</td>
<td>Accept</td>
<td>11 lag0/lag1 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1511)</td>
<td>(0.00716)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
Table 17


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.5416</td>
<td>0.2271*</td>
<td>0.00414</td>
<td>Accept 5 lag0/lag1 r rate</td>
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</tr>
<tr>
<td></td>
<td>(0.0735)</td>
<td>(0.0735)</td>
<td>(0.00507)</td>
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<tr>
<td>U.K.</td>
<td>3.3989</td>
<td>0.7822</td>
<td>0.01242</td>
<td>Accept 5 lag0/lag1 r rate</td>
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</tr>
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<td>(0.1521)</td>
<td>(0.0740)</td>
<td>(0.00740)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>1.4482</td>
<td>0.2182*</td>
<td>0.00210</td>
<td>Accept 5 lag0/lag1 r ratio</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0956)</td>
<td>(0.00600)</td>
<td>(0.00579)</td>
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</tr>
<tr>
<td>U.K.</td>
<td>2.1556</td>
<td>0.3821*</td>
<td>0.00103</td>
<td>Accept 5 lag0/lag1 r ratio</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1716)</td>
<td>(0.00579)</td>
<td>(0.00579)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. $\chi^2(0.05,2)=7.81473$.
b. * significant at the 5% level.

Table 18


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.0948</td>
<td>0.2289*</td>
<td>0.00336</td>
<td>Accept 5 lag0/lag1 r rate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0953)</td>
<td>(0.0953)</td>
<td>(0.00599)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>5.9500</td>
<td>0.7821*</td>
<td>0.01210</td>
<td>Accept 5 lag0/lag1 r rate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1816)</td>
<td>(0.00805)</td>
<td>(0.00805)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.7084</td>
<td>0.1869</td>
<td>0.00239</td>
<td>Accept 5 lag0/lag1 r ratio</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1090)</td>
<td>(0.00605)</td>
<td>(0.00605)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>5.4750</td>
<td>0.4025*</td>
<td>0.00605</td>
<td>Accept 5 lag0/lag1 r ratio</td>
<td></td>
</tr>
<tr>
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<td>(0.1746)</td>
<td>(0.00702)</td>
<td>(0.00702)</td>
<td></td>
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</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
Table 19


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>( \alpha )</th>
<th>( \delta )</th>
<th>( H_0: \alpha = \beta )</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.4197</td>
<td>0.2287*</td>
<td>0.00406</td>
<td>Accept 5 lag0/lag1 rate</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0737)</td>
<td>(0.00510)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>2.6783</td>
<td>0.8671*</td>
<td>0.01336</td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2217)</td>
<td>(0.00801)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>2.3509</td>
<td>0.1969*</td>
<td>0.00081</td>
<td>Accept 5 lag0/lag1 ratio</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.0947)</td>
<td>(0.00605)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>2.1660</td>
<td>0.3798*</td>
<td>0.00100</td>
<td>Accept 5 lag0/lag1 ratio</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1654)</td>
<td>(0.00561)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. \( \chi^2(0.05,3)=7.81473 \).
b. * significant at the 5% level.

Table 20


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>( \alpha )</th>
<th>( \delta )</th>
<th>( H_0: \alpha = \beta )</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>4.5632</td>
<td>0.1802*</td>
<td>0.00240</td>
<td>Accept 5 lag0/lag1 rate</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.0808)</td>
<td>(0.00605)</td>
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</tr>
<tr>
<td>U.K.</td>
<td>6.5375</td>
<td>0.7502*</td>
<td>0.01157</td>
<td>Accept 5 lag0/lag1 rate</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.2108)</td>
<td>(0.00790)</td>
<td></td>
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</tr>
<tr>
<td>U.S.</td>
<td>2.2724</td>
<td>0.1202</td>
<td>0.00104</td>
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<tr>
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<td></td>
<td>(0.1028)</td>
<td>(0.00686)</td>
<td></td>
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</tr>
<tr>
<td>U.K.</td>
<td>5.3125</td>
<td>0.4274*</td>
<td>0.00638</td>
<td>Accept 5 lag0/lag1 ratio</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.1807)</td>
<td>(0.00714)</td>
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</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
### Table 21


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>(\beta)</th>
<th>$\delta$</th>
<th>(t) value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.9409</td>
<td>0.1220</td>
<td>1.0236*</td>
<td>-0.01707</td>
<td>-2.1710*</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0.1082)</td>
<td>(0.3392)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>1.1809</td>
<td>0.3246*</td>
<td>0.7734</td>
<td>-0.00377</td>
<td>-0.4355</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1505)</td>
<td>(1.0325)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>4.7504</td>
<td>0.1214</td>
<td>1.0850*</td>
<td>-0.0216*</td>
<td>-4.7582*</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0839)</td>
<td>(0.1509)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>5.9003</td>
<td>0.1761*</td>
<td>0.3542</td>
<td>-0.00334</td>
<td>-0.5156</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0830)</td>
<td>(0.3217)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $a. \chi^2(0.05,3)=7.8147; \chi^2(0.05,8)=15.5073.$
- $b. *$ significant at the 5% level.

### Table 22


<table>
<thead>
<tr>
<th>Country</th>
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<th>$\alpha$</th>
<th>(\beta)</th>
<th>$\delta$</th>
<th>(t) value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.6562</td>
<td>0.1228</td>
<td>0.5598</td>
<td>-0.00665</td>
<td>-0.6640</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1197)</td>
<td>(0.6154)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>0.9300</td>
<td>0.3199</td>
<td>0.3109</td>
<td>0.00479</td>
<td>0.0083</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1568)</td>
<td>(1.1037)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>22.4952*</td>
<td>-0.0042</td>
<td>0.1952</td>
<td>-0.00484</td>
<td>-0.3428</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1035)</td>
<td>(0.6141)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>8.5808</td>
<td>0.2088</td>
<td>0.0260</td>
<td>0.00524</td>
<td>0.4462</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1375)</td>
<td>(0.4360)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $a. *$ significant at the 5% level.
- $b. J$-value valid only if conditional homoskedasticity is assumed.
### Table 23


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$t$ value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.8361</td>
<td>0.1724</td>
<td>0.5019</td>
<td>-0.00292</td>
<td>0.7444</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0988)</td>
<td>(0.4333)</td>
<td>(0.01102)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.0277</td>
<td>0.7371*</td>
<td>-0.1781</td>
<td>0.0220</td>
<td>0.6418</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2178)</td>
<td>(1.2840)</td>
<td>(0.02433)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>7.8700</td>
<td>0.1927*</td>
<td>1.0150*</td>
<td>-0.02328*</td>
<td>-4.0921*</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0747)</td>
<td>(0.1569)</td>
<td>(0.00705)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
<tr>
<td>U.K.</td>
<td>22.6621*</td>
<td>0.6192*</td>
<td>0.4556</td>
<td>0.00872</td>
<td>0.4232</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1259)</td>
<td>(0.3614)</td>
<td>(0.00917)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
</tbody>
</table>

a. $X^2(0.05,3)=7.8147; X^2(0.05,8)=15.5073.$

b. * significant at the 5% level.

### Table 24


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$t$ value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.9110</td>
<td>0.1416</td>
<td>0.5434</td>
<td>-0.00569</td>
<td>-0.6989</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1034)</td>
<td>(0.5810)</td>
<td>(0.01415)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.7368</td>
<td>0.5286*</td>
<td>0.2833</td>
<td>0.01102</td>
<td>0.1794</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1781)</td>
<td>(1.3077)</td>
<td>(0.02246)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>29.8844*</td>
<td>0.0467</td>
<td>0.2061</td>
<td>-0.00296</td>
<td>-0.2912</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0937)</td>
<td>(0.5755)</td>
<td>(0.01372)</td>
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<td>lag2/lag3</td>
</tr>
<tr>
<td>U.K.</td>
<td>21.2136*</td>
<td>0.3476*</td>
<td>0.1815</td>
<td>0.00730</td>
<td>0.3886</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1367)</td>
<td>(0.4656)</td>
<td>(0.00915)</td>
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<td>lag2/lag3</td>
</tr>
</tbody>
</table>

a. * significant at the 5% level

b. J-value valid only if conditional homoskedasticity is assumed.
### Table 25


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>α</th>
<th>β</th>
<th>δ</th>
<th>t value of Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.6960</td>
<td>0.1246</td>
<td>0.5074</td>
<td>-0.00644</td>
<td>-0.8713 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1343)</td>
<td>(0.4220)</td>
<td>(0.01166)</td>
<td></td>
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</tr>
<tr>
<td>U.K.</td>
<td>1.2414</td>
<td>0.8771*</td>
<td>-0.8063</td>
<td>0.03632</td>
<td>1.0181 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.3325)</td>
<td>(1.4471)</td>
<td>(0.03046)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.6140</td>
<td>0.0360</td>
<td>0.9939*</td>
<td>-0.01963</td>
<td>-1.9709 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.2119)</td>
<td>(0.3692)</td>
<td>(0.01291)</td>
<td></td>
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<tr>
<td>U.K.</td>
<td>0.8955</td>
<td>0.5166</td>
<td>0.3230</td>
<td>0.00785</td>
<td>0.1212 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.3290)</td>
<td>(1.3459)</td>
<td>(0.02700)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. $\chi^2(0.05, 2) = 5.99146$.

b. * significant at the 5% level.

### Table 26


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>α</th>
<th>β</th>
<th>δ</th>
<th>t value of Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.4105</td>
<td>0.1071</td>
<td>0.6227</td>
<td>-0.00821</td>
<td>-1.0293 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1525)</td>
<td>(0.5042)</td>
<td>(0.01353)</td>
<td></td>
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<tr>
<td>U.K.</td>
<td>1.3516</td>
<td>0.8045*</td>
<td>-0.6222</td>
<td>0.03226</td>
<td>0.8562 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.2986)</td>
<td>(1.5350)</td>
<td>(0.02997)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.8088</td>
<td>0.0595</td>
<td>0.6906</td>
<td>-0.01092</td>
<td>-0.9582 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1786)</td>
<td>(0.5875)</td>
<td>(0.01676)</td>
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<tr>
<td>U.K.</td>
<td>0.7068</td>
<td>0.4261</td>
<td>0.0365</td>
<td>0.01155</td>
<td>0.2680 5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.3043)</td>
<td>(1.3040)</td>
<td>(0.02522)</td>
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</tr>
</tbody>
</table>

a. * significant at the 5% level.

b. J-value valid only if conditional homoskedasticity is assumed.
### Table 27


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$t$ value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.2732</td>
<td>0.1430</td>
<td>0.9173*</td>
<td>-0.01207</td>
<td>-2.3865*</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.0903)</td>
<td>(0.2886)</td>
<td>(0.00863)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>U.K.</td>
<td>4.3660</td>
<td>0.6042*</td>
<td>-0.1351</td>
<td>0.01907</td>
<td>0.5328</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.2058)</td>
<td>(1.3190)</td>
<td>(0.02288)</td>
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<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.0718</td>
<td>0.1199</td>
<td>1.0148*</td>
<td>-0.01474</td>
<td>-2.1360*</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1081)</td>
<td>(0.3417)</td>
<td>(0.01049)</td>
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</tr>
<tr>
<td>U.K.</td>
<td>0.7211</td>
<td>0.3011</td>
<td>0.2756</td>
<td>0.00358</td>
<td>0.2851</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1554)</td>
<td>(1.1351)</td>
<td>(0.01727)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $\chi^2(0.05, 2)=5.99146$.
- * significant at the 5% level.

### Table 28


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$t$ value of $H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.4095</td>
<td>0.1595</td>
<td>0.9188*</td>
<td>-0.01088</td>
<td>-1.8771</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1074)</td>
<td>(0.3524)</td>
<td>(0.01258)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.6376</td>
<td>0.5279*</td>
<td>0.1206</td>
<td>0.01331</td>
<td>0.2851</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1801)</td>
<td>(1.3928)</td>
<td>(0.02358)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>0.0455</td>
<td>0.1340</td>
<td>0.9699*</td>
<td>-0.01254</td>
<td>-1.6255</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1372)</td>
<td>(0.4142)</td>
<td>(0.01482)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>0.6076</td>
<td>0.3089</td>
<td>0.0401</td>
<td>0.00838</td>
<td>0.2355</td>
<td>5 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td>(0.1662)</td>
<td>(1.1572)</td>
<td>(0.01886)</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

- * significant at the 5% level.
- J-value valid only if conditional homoskedasticity is assumed.
Table 29


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.6876</td>
<td>0.1321</td>
<td>0.00347</td>
<td>Reject</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1121)</td>
<td>(0.00623)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>1.5255</td>
<td>0.3213*</td>
<td>0.00269</td>
<td>Accept</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1566)</td>
<td>(0.00567)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>10.7468</td>
<td>0.0751</td>
<td>0.00805*</td>
<td>Reject</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0661)</td>
<td>(0.00292)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
<tr>
<td>U.K.</td>
<td>6.1523</td>
<td>0.1933*</td>
<td>-0.00107</td>
<td>Accept</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0803)</td>
<td>(0.00458)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
</tbody>
</table>

a. $\chi^2(0.05,4)=9.4877$; $\chi^2(0.05,9)=16.9190$.
b. * significant at the 5% level.

Table 30


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>3.7856</td>
<td>0.1290</td>
<td>0.00089</td>
<td>Accept</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1258)</td>
<td>(0.00708)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>0.9300</td>
<td>0.3187</td>
<td>0.00465</td>
<td>Accept</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2111)</td>
<td>(0.00602)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>25.9441*</td>
<td>-0.0015</td>
<td>-0.00149</td>
<td>Accept</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1065)</td>
<td>(0.00653)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
<tr>
<td>U.K.</td>
<td>8.9776</td>
<td>0.1733</td>
<td>0.00228</td>
<td>Accept</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1598)</td>
<td>(0.00531)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
### Table 31


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.7688</td>
<td>0.1429</td>
<td>0.00430</td>
<td>Accept</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1071)</td>
<td>(0.00568)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.9694</td>
<td>0.6802*</td>
<td>0.00842</td>
<td>Accept</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1785)</td>
<td>(0.00705)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>11.6471</td>
<td>0.0460</td>
<td>0.00157</td>
<td>Reject</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0635)</td>
<td>(0.00422)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
<tr>
<td>U.K.</td>
<td>23.6537*</td>
<td>0.5442*</td>
<td>0.00666</td>
<td>Accept</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1144)</td>
<td>(0.00588)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
</tbody>
</table>

a. $\chi^2(0.05,4)=9.4877$; $\chi^2(0.05,9)=16.9190$.
b. * significant at the 5% level.

### Table 32


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>3.8192</td>
<td>0.1396</td>
<td>0.00112</td>
<td>Accept</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1170)</td>
<td>(0.00665)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.9352</td>
<td>0.4984*</td>
<td>0.00720</td>
<td>Accept</td>
<td>6 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1918)</td>
<td>(0.00619)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>32.9147*</td>
<td>0.0470</td>
<td>-0.00029</td>
<td>Accept</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0954)</td>
<td>(0.00637)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
<tr>
<td>U.K.</td>
<td>26.8460*</td>
<td>0.3160*</td>
<td>0.00463</td>
<td>Accept</td>
<td>11 lag1/lag2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1671)</td>
<td>(0.00541)</td>
<td></td>
<td>lag2/lag3</td>
</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
### Table 33


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.2406</td>
<td>0.1248 (0.1102)</td>
<td>0.00379 (0.00570)</td>
<td>Accept 5 lag1/lag2 r rate</td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>2.3170</td>
<td>0.7647* (0.2148)</td>
<td>0.00963 (0.00770)</td>
<td>Accept 5 lag1/lag2 r rate</td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>1.1794</td>
<td>0.1162 (0.1157)</td>
<td>0.00306 (0.00620)</td>
<td>Accept 5 lag1/lag2 r rate</td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>0.8735</td>
<td>0.4942* (0.2347)</td>
<td>0.00461 (0.00685)</td>
<td>Accept 5 lag1/lag2 r ratio</td>
<td></td>
</tr>
</tbody>
</table>

- $a. \chi^2(0.05,2)=7.81473.$
- $b. *$ significant at the 5% level.

### Table 34


<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>3.8129</td>
<td>0.1379 (0.1337)</td>
<td>0.00108 (0.00629)</td>
<td>Accept 5 lag1/lag2 r rate</td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>3.6704</td>
<td>0.5634* (0.2528)</td>
<td>0.00807 (0.00652)</td>
<td>Accept 5 lag1/lag2 r rate</td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>3.7765</td>
<td>0.1225 (0.1332)</td>
<td>0.00075 (0.00667)</td>
<td>Accept 5 lag1/lag2 r ratio</td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>0.8804</td>
<td>0.3351 (0.3042)</td>
<td>0.00485 (0.00631)</td>
<td>Accept 5 lag1/lag2 r ratio</td>
<td></td>
</tr>
</tbody>
</table>

- $a. *$ significant at the 5% level.
- $b. J$-value valid only if conditional homoskedasticity is assumed.
Table 35

<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1.1926</td>
<td>0.1462</td>
<td>0.00436</td>
<td>Reject</td>
<td>5 lag1/lag2 r rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1073)</td>
<td>(0.00568)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>4.9608</td>
<td>0.6055*</td>
<td>0.00863</td>
<td>Accept</td>
<td>5 lag1/lag2 r rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1876)</td>
<td>(0.00657)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>1.2135</td>
<td>0.1518</td>
<td>0.00445</td>
<td>Reject</td>
<td>5 lag1/lag2 r ratio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1147)</td>
<td>(0.00647)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>0.7150</td>
<td>0.3022*</td>
<td>0.00320</td>
<td>Accept</td>
<td>5 lag1/lag2 r ratio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1537)</td>
<td>(0.00563)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. $\chi^2(0.05,3)=7.81473$.
b. * significant at the 5% level.

Table 36

<table>
<thead>
<tr>
<th>Country</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$H_0: \alpha = \beta$</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>3.7583</td>
<td>0.1409</td>
<td>0.00114</td>
<td>Accept</td>
<td>5 lag1/lag2 r rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1155)</td>
<td>(0.00674)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>5.0096</td>
<td>0.4871*</td>
<td>0.00704</td>
<td>Accept</td>
<td>5 lag1/lag2 r rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1918)</td>
<td>(0.00609)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S.</td>
<td>3.7219</td>
<td>0.1302</td>
<td>0.00091</td>
<td>Accept</td>
<td>5 lag1/lag2 r ratio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1272)</td>
<td>(0.00717)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.K.</td>
<td>0.7688</td>
<td>0.2816</td>
<td>0.00419</td>
<td>Accept</td>
<td>5 lag1/lag2 r ratio</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.2009)</td>
<td>(0.00587)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. * significant at the 5% level.
b. J-value valid only if conditional homoskedasticity is assumed.
<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>1889-1945</td>
<td>2.73262</td>
<td>0.14739</td>
<td>0.77681</td>
<td>-0.00773</td>
</tr>
<tr>
<td></td>
<td>1889-1945</td>
<td></td>
<td>(0.13101)</td>
<td>(0.47187)</td>
<td>(0.02057)</td>
</tr>
<tr>
<td></td>
<td>1946-1984</td>
<td>1.47466</td>
<td>0.23083</td>
<td>1.33866</td>
<td>-0.02288</td>
</tr>
<tr>
<td></td>
<td>1946-1984</td>
<td></td>
<td>(0.30484)</td>
<td>(1.02260)</td>
<td>(0.01640)</td>
</tr>
<tr>
<td>U.K.</td>
<td>1855-1888</td>
<td>2.47565</td>
<td>-0.62365*</td>
<td>0.38971</td>
<td>0.02077</td>
</tr>
<tr>
<td></td>
<td>1855-1888</td>
<td></td>
<td>(0.28778)</td>
<td>(0.44734)</td>
<td>(0.01109)</td>
</tr>
<tr>
<td></td>
<td>1889-1945</td>
<td>3.17358</td>
<td>0.37723*</td>
<td>1.59445</td>
<td>0.01814</td>
</tr>
<tr>
<td></td>
<td>1889-1945</td>
<td></td>
<td>(0.11893)</td>
<td>(0.83193)</td>
<td>(0.01585)</td>
</tr>
<tr>
<td></td>
<td>1946-1983</td>
<td>2.30732</td>
<td>-0.22992</td>
<td>-0.72485</td>
<td>-0.02698</td>
</tr>
<tr>
<td></td>
<td>1946-1983</td>
<td></td>
<td>(0.65308)</td>
<td>(1.33160)</td>
<td>(0.02650)</td>
</tr>
</tbody>
</table>

a. Instruments including the interest ratio.
b. $X^2(0.05,2)=5.99146$.
c. * significant at the 5% level.
### Table 38

**GMM Estimates with Historical U.S. and U.K. Data in the Unrestricted Model in Subsamples Separating at 1935**

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>J-value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S.</strong></td>
<td>1889-</td>
<td>2.34756</td>
<td>-0.03147</td>
<td>1.29784</td>
<td>-0.01216</td>
</tr>
<tr>
<td></td>
<td>1935</td>
<td>(0.52101)</td>
<td>(0.080583)</td>
<td>(0.02206)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1936-</td>
<td>1.72164</td>
<td>0.02888</td>
<td>0.55667*</td>
<td>-0.02442*</td>
</tr>
<tr>
<td></td>
<td>1983</td>
<td>(0.07780)</td>
<td>(0.24887)</td>
<td>(0.00648)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1889-</td>
<td>0.89151</td>
<td>0.11998</td>
<td>1.01593*</td>
<td>-0.01721</td>
</tr>
<tr>
<td></td>
<td>1983</td>
<td>(0.10671)</td>
<td>(0.34030)</td>
<td>(0.01012)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1889-</td>
<td>2.91669</td>
<td>0.76254*</td>
<td>0.20122</td>
<td>0.04872*</td>
</tr>
<tr>
<td></td>
<td>1935</td>
<td>(0.13481)</td>
<td>(0.73266)</td>
<td>(0.01960)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1936-</td>
<td>2.70925</td>
<td>-0.07546</td>
<td>0.34959</td>
<td>-0.03748*</td>
</tr>
<tr>
<td></td>
<td>1983</td>
<td>(0.05953)</td>
<td>(0.32628)</td>
<td>(0.00792)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1889-</td>
<td>1.02787</td>
<td>0.33579*</td>
<td>0.52194</td>
<td>-0.00875</td>
</tr>
<tr>
<td></td>
<td>1983</td>
<td>(0.16523)</td>
<td>(1.01271)</td>
<td>(0.01339)</td>
<td></td>
</tr>
</tbody>
</table>

a. 6 instruments including the interest ratio.
b. $\chi^2(0.05,2)=5.99146$.
c. * significant at the 5% level.
Notes

1 For an interpretation, see Bowden and Turkington (1984), pp. 12-16.

2 However, GMM is not asymptotically efficient. It is well known, however, that if the specification of the stochastic processes of those forcing variables is not adequate, then the MLE estimator is not efficient either.


4 If \( r \) is treated as a constant, the model cannot be estimated by the GMM method. To avoid such a problem and also to match real world phenomenon, \( r \) is treated as a stochastic forcing variable in the estimated model.

5 The weakness of this approach is that the model's transversality condition is ignored. Ideally, it should be imposed during estimation.

6 Indeed, according to my data, they are stationary. See Chapter V.

7 I adopt the GRADX procedure of the GQOPT computer software package (Princeton) to perform the minimization of eq. (52). Also, the subroutine LINV2P of IMSL is used for the matrix inversion. Hansen and Singleton stop the estimation procedure at the fourth step. Also, GRADX gives us the same estimates regardless of the different starting values having been tried (from -3 to +3).

8 Due to the critical role of homoskedasticity in NL2SLS, White's (1980) conditional heteroskedasticity test has been done for the linearized form of eq. (48) under the assumption that forecast errors are uncorrelated with \( \ln(R_{t+1}/R_t) \), \( \ln(Y_t/Y_{t+1}) \) and are homokurtic. For the unrestricted model, the null hypothesis has been rejected strongly for both the U.S. and U.K. For the restricted model, the null hypothesis is rejected in the U.K. at the 5% level, but it is not rejectable in the U.S. However, since the explanatory variables are not in the information set, it is possible that they are correlated with forecast errors. As a result, White's (1980) test is biased (White 1982). Empirically the estimated residuals are weakly correlated with those explanatory variables. Also, due to the strong rejection of the null hypothesis for the unrestricted model, I reestimated the variance covariance matrices of parameters in all cases.

9 I report the corrected standard error from ROBUSTSE option in TSP Version 4.0 (Standford).

10 See White (1982), pp. 488-491.
11 When four lag instruments are used, following Hansen and Singleton’s procedure to stop estimation after first iteration, the results are indeed different from those with two lag instruments.

12 All actual residuals are also tested for normality. According to the Kolmogorov D statistic in SAS, normality is not rejectable at less than the 1% significance level for all cases of the U.S. However, it is not rejectable for the U.K. for at most at the 11.4% significance level. Also, those residuals are regressed against their corresponding instruments. I should expect that coefficients of those instruments are insignificantly different from zero. Results show that for some U.S. and U.K. cases, a few regression coefficients are significantly greater than zero. This leads us to doubt about the specification of the model, even though the J-statistic cannot reject the model. In particular, it is possible that randomness of the preference and/or technology during the sampling period causes the correlation between instruments and forecast errors. See Rotemberg (1984). Furthermore, those residuals show a strong AR(1) process, which according to the model, should be white noise. But my estimates are still consistent, although less efficient. Increasing efficiency can be accomplished by changing the R matrix, but the setup of the model does not allow for randomness of preference and/or technology.

13 NL2SLS J-value results in the first round of estimation are also positive for both the unrestricted and restricted models when the interest ratio instrument is used. A reverse result occurs when the interest rate instrument is used.

14 Tauchen also reports that the overidentifying restriction performs reasonably well in finite sample. It is slightly biased towards accepting the null hypothesis of correct specification. See Tauchen (1985).

15 Since I reject the model for the U.K. using the eleven instrument set with the interest rate instrument in the second round estimation, I don’t report that result in my empirical summary.

16 From White (1980, 1982), this linear hypothesis testing should use a $X_1^2$ test statistic.

17 Although the estimated standard errors of $\omega$s are still higher for the U.K. than for the U.S., I can no longer arrive at such a conclusion for those $\delta$'s.

18 In fact, because of the odd number of observations, the first subsample has one less observation than the second subsample.

19 In all cases, the J-value increases but not to the point of rejecting the model.
CHAPTER VII
CONCLUSIONS

The purpose of this dissertation is to study Barro's (1979) new classical approach to the determination of public debt in a world of uncertainty and to empirically test that model. Barro assumes the Ricardian Equivalence Theorem, which asserts, as a first-order approximation, the irrelevance of the real interest rate and output of methods in financing government expenditures. He then proposes as a societal objective the minimization of tax collection costs. As a result, public debt is used to "smooth" out the income tax rate over time in a world of perfect foresight.

I have extended his model to the uncertainty case. In particular, I have considered three possibilities. In the first, income is assumed to follow a lognormal process with a constant drift. I find that, although the expected income tax rate is the same over time, tax revenue should start off higher and grow slower than in the certainty case. Public debt should also grow slower, and be unaffected by the realized income. The tax policy in this case implies an asymptotically zero income tax rate in the infinite future given the information available at the initial period. Therefore, a constant income tax rate is no longer an optimal policy. Rather, a policy that almost surely leads to a declining income tax rate should be followed.

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In the second case, government spending is assumed to follow a lognormal process with a constant drift. I find that the optimal level of public debt should be positively related to current government expenditures if the certain income growth rate is greater than the log expected growth rate of government spending. On the other hand, if the two growth rates are the same, public debt should grow independently of government expenditures. Therefore, whether a government can use public debt to finance its expenditures permanently depends on whether income grows fast enough or not. Also, it is clear that public debt is not used solely for smoothing out the income tax rate, as Barro predicted. However, the tax policy in this case does not imply an asymptotically zero income tax rate in the infinite future given the available information at the initial period. Uncertainty of government spending does not make the median income tax rate collapse at zero in the limit; rather, a constant median income tax rate is expected.

In the third case, both income and government spending are assumed to follow the lognormal process with different drifts. I again have found that sufficiently high income growth relative to the growth of government spending would make public debt less burdensome, and an increase in the previous government spending would increase the current debt level. Similar to the first case, the government at the initial period reads the median income tax rate in the limit as zero due to the uncertainty of income.

In sum, in a world of uncertainty, public debt can be used to finance a part of government spending, but cannot be necessarily used for smoothing out the income tax rate over time. Whether a government
should use public debt to finance part of its expenditures depends crucially on the relative growth of income to government spending in a certain way. Furthermore, income uncertainty makes it necessary for the government to tax early and to use debt less when its spending increases. In particular, when income is uncertain, the government at the initial period treats at least the median income tax rate zero in the limit. Therefore, the tax smoothing theory for public debt determination does not hold in an uncertain world.

Applying my conclusions to the Gramm-Rudman-Hollings Act, it seems that balancing the federal government budget within a period of time, regardless what the economic growth and the relation between government spending and income would be, is an over-reaction to the current budget "crisis". However, the tendency to propose any alternative policy measures based on this very simple model should be viewed cautiously. For example, both the Senate and House passed different versions of the tax overhaul plan, implying that the existing tax structure is not optimal. Any tax increase recommendation based on this simple model therefore has no social welfare meaning because of possible unfair income redistribution. Furthermore, a potential inflationary finance controlled by the Federal Reserve System complicates the tax structure even more. Unfortunately, all of these critical factors are not considered by this model, but treated rather as assumptions. Therefore, this model serves only for intellectual interest. Any practical policy recommendations should be based on more delicate and more well-thought-out ones.
However, after considering the new classical approach to the optimal public debt policy under uncertainty, the empirical question of whether the historical data support this cost minimization approach in determining public debt in general can still be explored. Barro tests the model by regressing the debt variable on explanatory variables whose coefficients are derived from his comparative static results. Alternatively, I adopt a structural-type approach to provide a more powerful test to the theory. I have directly estimated the parameters of the cost function by using the Generalized Method of Moments, using both the U.S. and U.K. data, and supplemented by the Nonlinear Two-Stage Least Squares method.

My J-statistic results from GMM indicate that the model in general is not rejectable. Also, as shown from the first round estimation, the linear homogeneity hypothesis is not rejectable. Tax revenue is indeed a significant factor in determining the social cost function. This conclusion is accompanied by the fact that when the interest rate instrument is used, estimates related to tax revenue for the U.K. are significantly greater than those for the U.S. However, the linear homogeneity hypothesis on the cost function is rejectable for the U.S. from the GMM estimation, but not from the NL2SLS estimation in the second round estimation where the instruments used are lagged one period more from those in the first round. Although Tauchen's GMM small sample property report does not support using long lag instruments, the instruments used in the second round estimation are lagged at most by three periods. The bias problem should not be serious. Yet, with those instruments in the second round estimation,
income becomes a relatively important factor for the U.S., while tax revenue is important for the U.K.

However, my effort in finding the reasons that cause different estimated patterns of these two countries in the second round estimation has failed because of the nonlinear complexity of the model and between forecast errors and selected instruments. I have tried different sampling periods by including different events like wars in the subsamples. The resulting changes in parameter estimates provide few hints about the direction of the effects of those events on parameter estimates.

Finally, applying those estimates to a model with a constantly growing economy and a negative ex post real interest rate yields a nonstationary non-optimal public debt policy by violating the transversality condition. Although this problem is alleviated a little in a world of uncertainty, it still indicates the weakness of the current GMM technique (as NL2SLS) I have applied. Ideally, I should impose my GMM estimation with the transversality condition restriction. However, due to the still developing stage of GMM, such a necessary effort is temporarily put aside.

Aside from the issue of the transversality restriction in the GMM estimation, future research could take other directions. In terms of the model per se, an interesting but also a more complicated model taking the tax structure and/or inflationary finance into account could be constructed. Taking the tax structure into account would permit the distribution effect of public debt to be studied. On the other hand,
incorporating inflationary finance into the model can aid in discussing the issue of monetizing the debt.

Empirically, GMM still has its merit in the present problem. For example, efficiency of the estimated variance-covariance matrix could be improved by following the procedure suggested by Hansen (1985). Or, an alternative model non-nested from the current one could be proposed, and thus test two models according to the line posited by Singleton (1985). A candidate for this direction would be a model with money playing a role. Therefore, the issue of the new classical approach to the determination of public debt should not stop here. Rather, it is a starting point for future research.
APPENDIX A

Equivalence Solutions between
Open-Loop Form and Closed-Loop Form in Case I of Chapter IV

Given the same assumptions as those in Case I of Chapter IV, the optimal closed-loop solution for tax revenue can be shown exactly the same as the open-loop one. A sketch showing such an equivalence is shown as follows.

The objective function is eq. (17) except that the decision variable now is changed to the level of public debt, $b_t$. The first-order stochastic Euler equation is

$$E_t[\frac{R_t}{Y_t}]^\alpha = E_t[\frac{R_{t+1}}{Y_{t+1}}]^\alpha. \quad (A.1)$$

Define $R_t^\alpha$ as $h_t$. From eq. (A.1) and eq. (19), we have the following relation:

$$E_t(h_{t+1}) = \lambda_1 E_t(h_t) \quad (A.2)$$

where $\lambda_1 = e^{\eta\sigma - 0.5\alpha^2\sigma_v^2}$.

Rewrite eq. (A.2) by lagging one period, we then have

$$E_{t-1}(h_t) = \lambda_1 E_{t-1}(h_{t-1}). \quad (A.3)$$
From the accounting budget constraint (1), components in $h_{t-1}$ are known at time $t-1$. Therefore, eq. (A.3) can be reduced to:

$$E_{t-1}(h_t) = \lambda_1 h_{t-1} = \lambda_1^{t-1} h_1.$$  \hfill (A.4)

Now update the information set by one period and substitute $R^\alpha_t$ back for $h_t$, we have the following solution which is exactly equal to that of eq. (20):

$$R^\alpha_t = R^\alpha_1 e^{(\eta_\alpha - 0.5\sigma^2 v)(t-1)}.$$  \hfill (A.5)

The transversality condition of this problem is satisfied by the assumptions given in Case I of Chapter IV.
APPENDIX B

The Open-Loop Income Tax Rate in the Limit

In the open-loop solution of our problem, we can look into how a government decision maker treats the income tax rate in the infinite future given the available information at the time he makes the public debt policy. Again, I consider three cases as those in Chapter IV.

Case (I): The open-loop solution is the same as the closed one. According to eq. (20), we have that

\[ R_t = R_1 e^{(\eta - 0.5 \sigma^2_v)(t-1)}. \] (B.1)

Also, \( Y_t \) is assumed to follow a lognormal process with a constant drift. In particular, from eq. (6) we have that

\[ \ln Y_t - \ln Y_{t-1} = \eta + \nu_t. \] (B.2)

Given the information available at time 1, \( Y_t \) becomes that

\[ Y_t = Y_0 e^{\eta t + \sum_{i=1}^{t} \nu_i}. \] (B.3)
The median of income given the available information at time 1 is that

$$\text{med}(Y_t) = Y_0 e^{\eta t}. \quad (B.4)$$

From eq. (B.1) and (B.4), the median income tax rate is the following:

$$\frac{R_t}{\text{med}(Y_t)} = \left( \frac{R_1}{Y_0 e^{\eta}} \right) e^{-0.5\sigma^2_v (t-1)}. \quad (B.5)$$

In the limit, the median income tax rate collapses at zero, i.e., one-half of the lognormal distribution collapses at zero. Also, the standard error of \(\ln(R_t/Y_t)\) is the following:

$$\text{S.D.} \left[ \ln \left( \frac{R_t}{Y_t} \right) \right] = \sqrt{t} \sigma_v \quad (B.6)$$

where S.D. stands for standard error.

Taking the logarithm of \(R_t/Y_t\), we have that

$$\ln(R_t/Y_t) = [\ln R_1 - \ln Y_0 - \eta] - 0.5\sigma^2_v (t-1) - \sum_{1}^{t} v_i. \quad (B.7)$$

Since \(\ln (R_t/Y_t)\) is normally distributed and its standard error increases at the rate of square root of \(t\), it can be shown that eq. (B.7) is dominated by \(\sigma^2_v\). Therefore, plim \(\ln(R_t/Y_t)\) is negative.
infinity; i.e., the income tax rate approaches zero in the infinite future given the information at time 1.

**Case (II):** The open-loop public debt policy of this case is the following:

\[ b_t = \frac{G_1 e^{xt}}{e^x - e^r} + c_1 e^{\eta t} \quad (B.8) \]

where \( c_1 = b_0 + \frac{G_1}{e^r - e^x} > 0 \) given the assumptions of Case II in Chapter IV, and \( x \) stands for the expected growth factor of government expenditures.

From the accounting budget constraint (1) and eq. (B.8), \( R_t \) can be solved as a random variable of the following form:

\[ R_t = G_1 e^{y(t-1)} \left[ e^{\frac{c_1}{2}} - e^{0.5\sigma_z^2(t-1)} \right] + c_1 e^{\eta(t-1)} (e^r - e^\eta). \quad (B.9) \]

Again, the median income tax rate is the following:

\[ \frac{\text{med}(R_t)}{\gamma_t} = \frac{G_1}{\gamma_1} e^{(y-\eta)(t-1)} \left[ 1 - e^{0.5\sigma_z^2(t-1)} \right] + \frac{c_1}{\gamma_1} (e^r - e^\eta). \quad (B.10) \]

However, by the assumption that \( \eta > y \), in the limit we have that

\[ \lim_{t \to \infty} \frac{\text{med}(R_t)}{\gamma_t} = \left( \frac{c_1}{\gamma_1} \right) (e^r - e^\eta). \quad (B.11) \]
The median income tax rate does not collapse at zero, i.e., the variable \((R_t/Y_t)\) approaches a stable lognormal distribution with one half the probability lying between 0 and \((c_1/Y_1)(e^r-e^\eta)\). Therefore, the government decision maker cannot treat the income tax rate in the infinite future as zero.

Case (III): The open loop solution for public debt in this case is that

\[ b_t = \left(\frac{e^z}{e^z-1}\right) G_0 e^{(z+r)(t-1)} + c_1 e^{(\eta-0.5\sigma^2) t} \quad (B.12) \]

where \(c_1 = b_0 - \left(\frac{e^{-r}}{e^z-1}\right) G_0\).

The tax revenue at time \(t\) can therefore be derived as the following:

\[ R_t = G_t - G_0 e^{(z+r)(t-1)} + c_1 (e^r - e^{\eta-0.5\sigma^2} e^{(\eta-0.5\sigma^2)(t-1)}) \quad (B.13) \]

where \(z+r = \gamma - \sigma \zeta + 0.5\sigma^2\).

Dividing eq. (B.13) by \(Y_t\), the income tax variable can be shown as the following:

\[ \frac{R_t}{Y_t} = \frac{G_0}{Y_0} e^{(\gamma-\eta)t} e^{\zeta_1 \nu_1} - \frac{G_0}{Y_0} e^{(z+r-\eta)(t-1)} e^{-\Sigma \nu_i} \]
The median income tax rate for time \( t \) at time 1 becomes that

\[
\text{med}(R_t) = (\frac{c_0}{y_0}) e^{(\gamma-\eta)t} \frac{c_0}{y_0} e^{(z+r-\eta)(t-1)-\eta}
\]

\[
+ \frac{c_1(e^{\eta-\eta}e^{-0.5\sigma^2_v})}{y_0} e^{-0.5\sigma^2_v(t-1)} | e^{-\Sigma v_j} |.
\] (B.14)

The median income tax rate for time \( t \) at time 1 becomes that

\[
\frac{\text{med}(R_t)}{y_t} = (\frac{c_0}{y_0}) e^{(\gamma-\eta)t} \frac{c_0}{y_0} e^{(z+r-\eta)(t-1)-\eta}
\]

\[
+ \frac{c_1(e^{\eta-\eta}e^{-0.5\sigma^2_v})}{y_0} e^{-0.5\sigma^2_v(t-1)}.
\] (B.15)

From the assumption that \( \nu < 0 \) in Case III of Chapter IV, we have both that \( \gamma < \eta \) and \( z+r-\eta < 0 \). Taking the limit of eq. (B.15), we have that

\[
\lim_{t \to \infty} \frac{\text{med}(R_t)}{y_t} = 0.
\] (B.16)

As in the first case, the median income tax rate does collapse at zero in the limit given the information available at the initial period. Comparing among eqs. (B.5), (B.10), and (B.15), the cause of such a collapse can be attributed to the income uncertainty factor -- \( 0.5\sigma^2_v \) (t-1).
### APPENDIX C

#### DATA SETS

(A) United States Data Set 1889 - 1984:

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Unit: $1,000 at 1972 price per capita

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<tr>
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<td>0.04971</td>
<td>52.90431</td>
</tr>
<tr>
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<td>0.02129</td>
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<td>2340</td>
<td>117.61937</td>
<td>0.00241</td>
<td>48.10077</td>
</tr>
</tbody>
</table>
APPENDIX D

The GNM Computer Program

The following is the basic GNM program used in Chapter V. Different estimation strategies are variations from it.

// JOB,
// REGION=1024K,TIME=(0,30)
//*JOBPARM V=S,LINES=2500,DISKIO=2000,LINECT=62
//PROCLIB DD DSN=CKA030.PROCLIB,DISP=SHR
// EXEC GQOPT,PARM='GOSTMT,NOMAP',TIME.GO=(0,30),
// REGION.GO=1024K
//FORT.SYSIN DD *

IMPLICIT REAL*8 (A-H,O-Z)
EXTERNAL FUNCT
EXTERNAL GRADX
DIMENSION X(3),BLABEL(1),ALABEL(2),
SZ(6,1),H(1,1),T(6,1),F(6,150),
$R(6,6),WK(90),HH(1,3),TT(6,3),
$D(6,150),D2(6,150),D(6,3),DT(3,6),
$CT(3,6),TTS(3,3),VCM(3,3),GOVG(150),D3(6,150)
COMMON/INDA/GNP(150),GOVT(150),REV(150),
$DEBT(150),BINT(150)
COMMON/INPA/AA(3),INOBS,ORTH
COMMON/WMAT/WT(6,6)
COMMON/BSTACK/AINT(300)
COMMON/BPRINT/IPT,NFILE,NDIG,NPUNCH
COMMON/BSTAK/NQ,NTOP
COMMON/BSTOP/NVAR1,ISTOP(3)
COMMON/BTRAT/ITRFLG
COMMON/SNOBS/NOBS

C THE FOLLOWING COMMON IS FOR NMSIMP ONLY.*****
C COMMON/BNEL/STP1,VAR,KONVGE,NRST
C THE FOLLOWING COMMON BLOCKS ARE FOR GRADX EXCEPT BDFP
C AND BINPUT
COMMON/BOPT/IVER,LT,IFP,ISP,NLOOP,IST,ILoop
COMMON/BFIDIF/FDFRAC,FDMIN
COMMON/BLNSR/STEP1,STPACC,NLNSR
COMMON/BDFP/STPMIN,FOPT
COMMON/BINPUT/INFLG
COMMON/BGRDX/RT,EG,RTMULT
COMMON/BSPD/IPSD
COMMON/BGRDX/ACTI,ACTW,ZFUL,BETH

-150-
```
MAX=2
IQ=300
FDFRAC=0.001
FDMIN=1.0D-08
ISTOP(1)=1
ISTOP(2)=1
C ITRFLG=1
C REAL(5,1050) (ALABEL(J),J=1,NP)
C1050 FORMAT(8(1X,A4))
C CALL LABEL(ALABEL,NP)
C READ(5,1100) ZFMAT
C1100 FORMAT(20A4)
READ(5,*) (AA(I),I=1,3)
READ(5,*) (X(I),I=1,3)
WRITE(6,1270)
1270 FORMAT(' STARTING VALUES ')
WRITE(6,*) (AA(I),I=1,3)
C0NV=1.0D-08
INOBS=96
DN0BS=INOBS*1.0D-3.0D0
PI=3.141592653589793D0
DO 310 I=1,INOBS
READ(8,109) GNP(I),REV(I),DEBT(I),BINT(I),GOVT(I),GOVG(I)
310 CONTINUE
109 FORMAT(6F11.5)
IORTH=6
ALPHA=AA(1)*X(1)
ETA=AA(2)*X(2)
DELTA=AA(3)*X(3)
DO 4000 I=3,INOBS-1
H(I,1)=(((REV(I)/REV(I-1))**ALPHA)*((GNP(I)/GNP(I-1))**ETA)*DEXP(BINT(I-1)-DELTA)-1.0D0)
Z(I,1)=GNP(I-1)/Z(I-2)
Z(2,1)=GOVT(I-1)/GOVT(I-2)
Z(3,1)=REV(I-1)/REV(I-2)
Z(4,1)=DEBT(I-1)/DEBT(I-2)
Z(5,1)=BINT(I-1)/BINT(I-2)
Z(6,1)=1.0D0
CALL PKRON(H,Z,T,1,1,6,1,6,1)
DO 4010 J=1,IORTH
4010 F(J,I-2)=T(J,1)
DO 4000 CONTINUE
4000 CONTINUE
DO 500 J=1,IORTH
DO 501 K=1,IORTH
P(J,K)=0.0D0
IF ( K .GE. J ) THEN
DO 502 I=1,INOBS-3
502 P(J,K)=F(J,I)*F(K,I)/DN0BS+R(J,K)
ELSE
```

R(J,K) = R(K,J)
END IF

501 CONTINUE

500 CONTINUE
CALL LINV2F(R, 6, 6, WT, 5, WK, IER)
C WRITE(6,*), 'WEIGHTING MATRIX WT'
C DO 600 I=1, IORTH
C WRITE(6,601) (WT(I,J), J=1, IORTH)
C WRITE(6,*), 'MATRIX R'
C DO 602 I=1, IORTH
C WRITE(6,601) (R(I,J), J=1, IORTH)
ACC=1.0D-08
IFLAG=0
NP=3
ITERL=100
C IPT=1
CALL OPT(X, NP, FLNL, GRADX, ITERL, MAX, IER, ACC, FUNCT, ALABEL)
C CALL OPTOUT(O)
WRITE(6,*), 'ETA=AA(2)*X(2)
ALPHA=AA(1)*X(1)
DELTA=AA(3)*X(3)
DO 4100 I=3, INOBS-1
H(I,1)=(((REV(I+1)/REV(I))**ALPHA)*((GNP(I)/GNP(I+1))**ETA)*DEXP(BINT(I+1)-DELTA)-1.0D0)
Z(I,1)=GNP(I-1)/GNP(I-2)
Z(I,2)=GOVT(I-1)/GOVT(I-2)
Z(I,3)=REV(I-1)/REV(I-2)
Z(I,4)=DEBT(I-1)/DEBT(I-2)
Z(I,5)=BINT(I-1)/BINT(I-2)
Z(I,6)=1.0D0
CALL PKRON(H,Z,T,1,1,6,1,6,1)
DO 4110 J=1, IORTH
F(J,I-2)=T(J,1)
4110 CONTINUE
DO 4100 J=1, IORTH
DO 511 K=1, IORTH
R(J,K)=0.0D0
IF (K .GE. J) THEN
DO 512 I=1, INOBS-3
R(J,K)=F(J,I)*F(K,I)/DNOBS+R(J,K)
ELSE
R(J,K)=R(K,J)
END IF
511 CONTINUE
510 CONTINUE
CALL LINV2F(R, 6, 6, WT, 5, WK, IER)
PFLNL=0.0D0
C IORTH= # OF ORTHOGONALITY CONDITION
TINC=0
9898 ITNO=ITNO+1
CALL OPT(X,NP,FLNL,GRADX,ITERL,MAX,IER,ACC,FUNCT,ALABEL)
C CALL OPTCUT(0)
WRITE(6,*) "J-STATISTICS =",F20.5,
8004 FORMAT("J-STATISTICS =",F20.5,
ETA=AA(2)*X(2)
ALPHA=AA(1)*X(1)
DELTA=AA(3)*X(3)
DO 4200 I=3,INOBS-1
H(1,1)=(((REV(I+1)/REV(I))**ALPHA)*((GNP(I)/GNP(I+1))**ETA)*DEXP(BINT(I+1)-DELTA)-1.0D0)
DERIV=((REV(I+1)/REV(I))**ALPHA)*((GNP(I)/GNP(I+1))**ETA)*DEXP(BINT(I+1)-DELTA)
HH(1,1)=DLOG(REV(I+1)/REV(I))*DERIV
HH(1,2)=DLOG(GNP(I)/GNP(I+1))*DERIV
HH(1,3)=-1.0D0*DERIV
Z(1,1)=GNP(I-1)/GNP(I-2)
Z(2,1)=G0VT(I-1)/G0VT(I-2)
Z(3,1)=REV(I-1)/REV(I-2)
Z(4,1)=DEBT(I-1)/DEBT(I-2)
Z(5,1)=BINT(I-1)/BINT(I-2)
Z(6,1)=1.0D0
CALL PKRON(H,Z,T,1,1,6,1)
CALL PKRON(HH,Z,TT,1,3,6,3)
DO 4210 J=1,IORTH
F(J,I-2)=T(J,1)
D1(J,I-2)=TT(J,1)
D2(J,I-2)=TT(J,2)
D3(J,I-2)=TT(J,3)
4210 CONTINUE
4200 CONTINUE
DO 520 J=1,IORTH
DO 521 K=1,IORTH
R(J,K)=0.0D0
IF ( K.GE. J) THEN
DO 522 I=1,INOBS-3
522 R(J,K)=F(J,I)*F(K,I)/DNOBS+R(J,K)
ELSE
R(J,K)=R(K,J)
END IF
521 CONTINUE
520 CONTINUE
CALL LINV2F(R,6,6,WT,5,WK,IER)
WRITE(6,*) "WEIGHTING MATRIX WT"
C620 WRITE(6,601) (LT(I,J),J=1,IORTH)
C WRITE(6,*)
C WRITE(6,*), MATRIX R
C DO 622 I=1,IORTH
C622 WRITE(6,601) (R(I,J),J=1,IORTH)
WRITE(6,*)
DO 7000 II=1,IORTH
D(II,1)=0.0D0
D(II,2)=0.0D0
D(II,3)=0.0D0
DO 7001 I2=1,INOBS-3
D(II,1)=D1(II,I2)/DN0BS+D(II,1)
D(II,2)=D2(II,I2)/DN0BS+D(II,2)
D(II,3)=D3(II,I2)/DN0BS+D(II,3)
7001 DT(1,II)=D(II,1)
DT(2,II)=D(II,2)
DT(3,II)=D(II,3)
7000 CONTINUE
WRITE(6,8110) ALPHA,ETA,DELTA
8110 FORMAT('ALPHA=',F16.5,' ETA=',F16.5,
S' DELTA=',F16.5,/) CALL TMAT(DT,WT,CT,3,IORTH,IORTH)
CALL THAT(CT, D, TTS,3,1ORTH,3)
WRITE(6,*)
WRITE(6,*) 'MATRIX D'
WRITE(6,*)
DO 8100 I=1,IORTH
C8100 WRITE(6,601) (D(I, J ) , J=1,3)
WRITE(6,*)
C WRITE(6,*), INVERSE OF VARIANCE-COVARIANCE MATRIX
C DO 8000 I=1,3
C8000 WRITE(6,601) (TTS(I,J),J=1,3)
CALL LINV2F(TTS,3,3,VCM,5,WK*IER)
DO 8011 I1=1,3
DO 8022 JJ=1,3
VCM(I1,JJ)=VCM(I1,JJ)/DN0BS
8022 CONTINUE
8011 CONTINUE
WRITE(6,*) STANDARD ERROR VECTOR
A1=DSORT(VCM(1,1))
A2=DSQRT(VCM(2,2))
A3=DSQRT(VCM(3,3))
WRITE(6,8003) A1,A2,A3
8003 FORMAT(' ',1X,3(F16.5,1X),//) WRITE(6,*) VARIANCE-COVARIANCE MATRIX
DO 8001 I=1,3
8001 WRITE(6,8002) (VCM(I,J),J=1,3)
8002 FORMAT(' ',1X,3(F16.5,1X),//) DFLNL=DABS(FLNL-PFLNL)
IF( DFLNL .GT. CCHV) THEN
PFLNL=FLNL

GO TO 9893
END IF
C
IFLAG=1
C IF (IFLAG .EQ. 1) THEN
C CALL FUNCT(X,NP,FLNL,DUMMY)
C END IF
C WRITE(6,*) ITNO
STOP
END

SUBROUTINE FUNCT(X,NP,FLNL,DUMMY)
IMPLICIT REAL*8 (A-H,O-Z)
DOUBLE PRECISION X(NP),FLNL
DIMENSION H(l,l),Z(6,1),F(6,150),G(6,1),GT(1,6),
ST(6,1),TT(1,6),TTF(1,1)
COMMON/INDA/GNP(150),GOVT(150),REV(150),
$DEBT(150),BINT(150)
COMM0N/INPA/AA(3).INOBS,IORTH
COMM0N/WMAT/WT(6,6)
TD=INOBS*1.0DO
ETA=AA(2)*X(2)
ALPHA=AA(1)*X(1)
DELTA=AA(3)*X(3)
DO 400 I=3,INOBS-1
H(1,1)=(((REV(I+1)/REV(I))**ALPHA)*((GKP(I)/GNP(I+1))
**ETA)*DEXP(BINT(1+1)-DELTA)-1.0DO)
Z(1,1)=GNP(I-1)/GNP(I-2)
Z(2,1)=GOVT(I-1)/GOVT(I-2)
Z(3,1)=REV(I-1)/REV(I-2)
Z(4,1)=DEBT(I-1)/DEBT(I-2)
Z(5,1)=BINT(I-1)/BINT(I-2)
Z(6,1)=1.0DO
CALL PKRON(H,Z,T,1,1,
6,1,6,1) DO 401 J=1,IORTH
401 F(J,I-2)=T(J,1)
DO 402 J=1,IORTH
G(J,1)=0.0DO
DO 403 I=1,INOBS-3
403 G(J,1)=F(J,I)+G(J,1)
G(J,1)=G(J,1)/(TD-3.0DO)
402 GT(1,J)=G(J,1)
CALL TMAT(GT,WT,TT,1,IORTH,IORTH)
CALL TMAT(TT,G,TTF,1,IORTH,1)
FLNL=TTF(1,1)
RETURN
END
SUBROUTINE TMAT(A M, R M, CM, I, M, J)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AM(I, M), BM(M, J), CM(I, J)
DO 10 L1=1, I
  DO 20 L2=1, J
    CM(L1, L2)=0.0D0
  DO 30 L3=1, M
    CM(L1, L2)=AM(L1, L3)*BM(L3, L2)+CM(L1, L2)
10 CONTINUE
RETURN
END

SUBROUTINE PKRON(AM, BM, CM, I, J, K, L, M, N)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AM(I, J), BM(K, L), CM(M, N)
C M=I*K, N=J*L
DO 11 L1=1, I
  DO 21 L2=1, J
    DO 31 L3=1, K
      LR=(L1-1)*K+L3
      LC=(L2-1)*L+L4
      CM(LR, LC)=AM(L1, L2)*BM(L3, L4)
      CONTINUE
21 CONTINUE
11 CONTINUE
RETURN
END

//GO.STEPLIB DD
// DD DSN=SYS1.VFORTLIB,DISP=SHR
//GO.SYSIN DD *
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