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WORKERS' COMPENSATION AS INCOME INSURANCE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Jong Chul Rhee

The Ohio State University
1986

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Dedicated to
My Father and Mother,
Whose Prayer, Love and Care Provided
Great Inspiration
ACKNOWLEDGEMENTS

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Studies in Econometrics

Studies in Urban Economics
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A. Introduction

Workers' compensation is a compulsory system that requires employers to provide no-fault insurance against work-related injury and disease in return for a limited liability against such events. As a matter of fact, the workers' compensation was introduced as a no-fault substitute for tort liability of employers and is perceived as a benefit which employers are obliged to provide to employees. However, it is important to recognize that employees may bear the entire cost of workers' compensation through decreased wages. Once this is understood, the system can be analysed as an insurance scheme in which benefits to injured employees are paid for by healthy employees with the firm no more than an intermediary in the transaction.

Focusing on the role of workers' compensation as income insurance, this study develops a theoretical model of workers' compensation with particular scrutiny given to informational conditions. The model provides the optimal workers' compensation benefit schemes under differing observational conditions. Alternative workers' compensation benefit schemes under imperfect information are evaluated based on the model.

Workers' compensation was the first form of social insurance to develop in the United States. The states were first stimulated to
enact workers' compensation laws in 1908, when the federal government passed a law covering certain federal employees; the state workers' compensation laws were passed by all but six states between 1911 and 1920, generally authorizing elective coverage and applying mainly to certain highly hazardous occupations.¹ From the beginning, workers' compensation has been a system of individually state-run programs so that there has been considerable variation in the structure of the state programs.

Many reforms in compensation laws were prompted by the recommendations issued in 1972 by the National Commission on State Workmen's Compensation Laws.² Nineteen essential recommendations were proposed, including several on more liberalized benefits and broader coverage. Since then, the states have enacted numerous reforms in their workers' compensation programs in response to the Commission's report. As a consequence, workers' compensation system has expanded significantly with respect both to the breadth of coverage and to the level of benefits provided.

Currently, nearly 90 percent of American workers are covered by the workers' compensation laws of the various states. The total benefit costs of the program to employers have also increased dramatically, from $4.9 billion in 1970 to $22.9 billion in 1980. Among social insurance programs, workers' compensation expenditures are


² For a summary of recommendations, see Chelius(1977), pp.85-97.
currently exceeded in magnitude only by social security disability insurance and unemployment insurance. Table 1 lists the social insurance programs and shows the expenditures on each in selected years.

As the size of program and claim frequency have risen dramatically during the last decade, economists have begun to reevaluate this important form of social insurance. Several theoretical and empirical works on workers' compensation have centered on various reforms of workers' compensation system, or at least attempted to give some direction for a "better" system. However, one of the factors which usually makes difficult an evaluation of the performance of workers' compensation and a search for the optimal system is the lack of agreed-upon objectives of workers' compensation system.

### TABLE 1
**EXPENDITURES ON MAJOR SOCIAL INSURANCE PROGRAMS**

<table>
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<th>Programs</th>
<th>Date Enacted</th>
<th>Public Expenditures (Billions of Current Dollars)</th>
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<tr>
<td>Social Security Disability Insurance</td>
<td>1954</td>
<td>1.6</td>
</tr>
<tr>
<td>Unemployment Insurance</td>
<td>1935</td>
<td>2.2</td>
</tr>
<tr>
<td>Workers' Compensation</td>
<td>1908</td>
<td>1.2</td>
</tr>
<tr>
<td>Railroad Retirement</td>
<td>1937</td>
<td>1.1</td>
</tr>
<tr>
<td>Black Lung</td>
<td>1969</td>
<td>-</td>
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</tbody>
</table>

Source: Social Security Bulletin
In its report, the National Commission articulated five implicit objectives of workers’ compensation system: (1) broad coverage of employees and work-related injuries and diseases; (2) substantial protection against the loss of income (income-protection objective); (3) provision of sufficient medical care and rehabilitation services; (4) encouragement of safety (safety objective); (5) an effective delivery system for benefits and services. This statement of objectives and the accompanying recommendations for their implementation have shaped reform efforts and debate since then.

Several economists emphasized the role of workers’ compensation as income insurance. According to Larson(1973), "...the most important single principle affecting decision on workers’ compensation issues is one branch of 'income insurance', in the same sense as unemployment insurance, social security, and temporary disability insurance." Income insurance is a term describing an essentially simple concept: small and regular contributions are made by the employer, the employee, or both, as a result of which the employee, on the happening of a specified contingency interrupting or terminating his earnings, has a right to be paid him a portion of his lost income. There are several contingencies that may cause this interruption or termination of earnings: economic unemployment; physical disability; old age retirement. "When the matter is put in this way, it is obvious that workers’ compensation is one segment of the total scheme of income

\[\text{^3}\text{For a discussion of objectives, see Darling-Hammond and Kniesner(1980), pp.12-38.}\]

\[\text{^4}\text{Larsons(1973), p.31.}\]
protection, the principal variant being the nature of the contingency insured against." Berkowitz and Burton (1970), and Berkowitz and Johnson (1970), Berkowitz (1973), and Vroman (1973) also emphasized the income-protection objective in workers' compensation.

It is, however, hard to find theoretical work which modeled the workers' compensation as income insurance in the past literature. Much of the literature has focused on the safety objective of workers' compensation, analyzing the safety effects and investigating the optimal form of the program in the light of it. With incomplete experience rating, employers may have a reduced incentive to provide safe workplaces; at the same time employees have less incentive to utilize appropriate care.

This study most closely follows Larson's viewpoint: i.e., workers' compensation system will be regarded as an income insurance system in this work. The primary goal of the compensation authority is to maximize the expected utility of a worker who is faced with random work-related accident. If a work-related accident resulted in a decline in income only and no other typical loss, the insurance decision would be straightforward; the risk-averse worker would fully insure under perfect information if the insurance premium accurately reflected the probability of the occurrence and the amount of loss.

But the disability insurance problem is more complex than this,

---

5 Larson (1973), p.32.

6 This means that no matter what happens he will end up with the same income which would be lower than if he paid no insurance premiums and never had an accident. This is the result of complete insurance with perfect information.
even in the perfect information case. First of all, we may consider the various sources through which health damage induces economic loss. Health problems may decrease wage rate, available hours for work and leisure, and increase work disutility, which, in turn, may force the worker to reduce work hours as well as earnings. Second, labor-leisure choice should be involved in the disability insurance problem, for health damages may affect wage rates as well as work hours. Thus, the appropriate utility structure in the disability insurance problem is at least a bivariate function of income and leisure, not a univariate function of income only, which is the conventional case.\(^7\)

These two points may lead to a different insurance principle from the conventional one even in the perfect information case; full-optimum insurance, for example, may involve more than complete income coverage. This result will be shown in chapter III.

One fundamental problem in the provision of workers' compensation as income insurance is the difficulty of monitoring the true loss of earning capacity due to work-related injury. The workers' compensation program attempts to cope with this problem by using one or another proxy for the true loss of earning capacity. Different benefit schemes, of course, will in general result in different expected utility outcomes and further generate different work disincentives.

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\(^7\)If the implicit price of leisure determined by the wage rate and work disutility is unaffected by health damage, then the optimal insurance with perfect information in this case is identical to that of the case where utility has only one argument, income. For a reference of general discussion on this point, see Arrow(1973). But as long as the wage rate and work disutility are affected by health damage, the analysis is more complicated even with perfect information.
Compared to social security disability insurance which is based on actual earnings losses as well as physical impairment, the current workers' compensation benefit schemes may induce somewhat less work disincentives, for there is no earnings test for most injuries and workers are paid according to "expected" future losses in many cases, although there are some proposals to alter system to create actual earnings loss structure.

The purpose of this study is to develop a theoretical model of workers' compensation and provide the optimal benefit schemes under the differing informational conditions. Finally the performances of alternative benefit schemes under imperfect information are evaluated, based on the model, with particular attention to the work disincentives established.

The paper is organized as follows. In the next section, the current workers' compensation system is briefly outlined. The discussion illustrates the wide range of alternative mechanisms currently in use that we will be considering and evaluating. In chapter II, the past literature is reviewed and the relevance of this work is discussed. Chapter III presents a theoretical model. Full-optimum insurance scheme with perfect information is discussed. In chapter IV, the optimum insurance scheme with imperfect information is discussed. Especially, benefit payment scheme based on the actual earnings loss is analysed. In chapter V, the benefit payment scheme based on the physical impairment is introduced and the performances of alternative benefit schemes are compared.
B. Current Workers’ Compensation System

1. Types of benefits

Workers’ compensation laws provide four benefits: (1) disability income benefits, (2) medical care, (3) death benefits, and (4) rehabilitation services. Among them, disability income benefits are payable after the disabled worker satisfies a waiting period that generally ranges from two or seven days. The weekly amount is based on a percent of the workers’ average weekly wage, typically 66 2/3 percent, and the degree of disability.

Four classifications of disability are generally used to determine the weekly benefit amount: (1) temporary total (2) permanent partial (3) permanent total (4) temporary partial.

(1) Temporary Total: The employee is totally disabled but is expected to recover fully and return to work. The employee is eligible for it after a waiting period of two to seven days. Disability income benefits continue unless either the employee returns to work or Physician makes a written statement that the employee is capable of returning to his former position of employment.

(2) Permanent Partial: In this case, employee is able to work at his regular job or can be trained for other types of work. Two classes of permanent partial disability are used - scheduled and nonscheduled.

Scheduled injuries are listed in the law including the loss of an arm, leg, finger etc. In most states, the amount paid for a scheduled injury is determined by multiplying an appropriate number of weeks by

---

the weekly disability income benefits, subject to some maximum amount.

Nonscheduled injuries are of a more general nature and involve the loss of earning capacity to the body as a whole, such as back or head injury that makes working difficult. The benefits paid for nonscheduled injuries are generally based on some percent of the difference in earnings before and after the injury multiplied by a certain number of weeks.

(3) Permanent Total: The employee is unable to work in gainful employment. Most states pay lifetime benefits.

(4) Temporary Partial: Temporary partial disability means that the disabled worker has returned to work but is earning less than before and still has not reached maximum recovery. The weekly benefit is a percent of the difference in earned wage before and after the injury up to the weekly maximum. Only a few states provided this benefits.

2. Insurance Arrangement

In most states, firms purchase a policy from a private insurer. The policy guarantees payment of the benefits that the employer is legally obligated to pay his disabled workers. In six states, employers must insure in an exclusive state fund. Twelve states permit employers to purchase insurance from either private insurers or competitive state funds.

The workers' compensation premiums paid by individual firms are based on an industrywide (manual rate). When a firm is sufficiently large, its own industry experience is used to modify the industrywide
rate in setting the firm's premium (experience rate). Experience-rated premium is essentially a weighted combination of industrywide and firm-specific insurable losses rises with firm size, the industrywide risk of injury, and the level of workers' compensation benefits in the states. More than 80 percent of all employees worked for firms that were not fully self-rated. Thus, in the typical case, it is clear that workers' compensation premiums only partially reflect a firm's own injury experience.

3. Insurance Benefit Scheme

Burton (1983) has well summarized various benefit schemes currently used in workers' compensation programs. For the permanent partial disability benefits, the schemes currently used can be categorized into five groups.

(1) Benefit Based on Impairments: The schedules for permanent partial benefits found in most statutes usually list injuries for which the extent of the underlying loss (impairment) is the primary but not sole basis for benefits. For example, sixty-five weeks of benefits are provided for the loss of a thumb, with each week's benefit set at two-thirds of the worker's average weekly wage (subject to a statutory maximum). In the extreme cases, for example the statutes in Washington and Oregon, benefits are based solely on the extent of the impairment, all workers with the same impairment receive the same dollar amount.

---

(2) Benefits Based on Disaggregated Functional Limitation: The typical workers' compensation schedule contains, in addition to a list of injuries involving amputations, a list that related benefits to the loss of use of specific body member.

(3) Benefits Based on Aggregated Functional Limitation: The California program has developed work-capacity guidelines to rate the severity of certain types of injuries considered hard to evaluate, such as spinal and heart injuries. These guidelines have eight levels of severity, some of which are stated in terms of aggregated functional limitations like half of his preinjury capacity for performing such activities as bending, stooping, lifting, pushing, etc.

(4) Benefits Based on the Loss of Earnings Capacity: In many jurisdictions nonscheduled permanent partial benefits are based on the loss of earning capacity. This loss of capacity results from the worker's functional limitations interacting with other influences such as age, education, and state of labor market. Most statutes provide little or no guidance about how to evaluate these factors.

(5) Benefits Based on the Actual Loss of Earnings: In some jurisdictions, all or certain types of injuries are assessed in terms of their consequence on the actual earnings of the workers in order to determine the amount of benefits. For an extended period of time after the date of maximum medical improvement, actual earnings are compared with potential earnings to determine if there is wage loss. The 1979 Florida law that adopted a wage-loss approach for permanent partial disabilities and the New York nonscheduled permanent partial
disabilities benefits are the obvious examples for this. Pennsylvania could be said to use this approach.

The five operational approaches to permanent partial benefits can be related to the distinction between scheduled and nonscheduled benefits found in most workers' compensation statues. Each statute contains a list of injuries with a corresponding duration of benefits. The scheduled benefits are based on impairment or disaggregated functional limitations. Each statute also contains a general provision pertaining to injuries that are not rated by use of schedule. Nonscheduled benefits are based on the loss of earning capacity, the fourth approach mentioned above, or the actual loss of earnings, the fifth approach.
CHAPTER II
PAST RESEARCH AND RELEVANCE OF THIS STUDY

Several economists have considered the incentive effects of workers' compensation associated with imperfect information. There are two possible types of imperfect information on which the agents (employee, employer, and workers' compensation authority) actions are based: (1) employees may be incorrect in their estimates of occupational risk, and the employer and the employee each has difficulty monitoring the precautions and safety efforts taken by the other; (2) the workers' compensation authority may not be able to monitor accurately the true loss of earning capacity due to work related injury or disease.

The first type of imperfect information has been considered in several studies regarding safety effect of workers' compensation. Oi(1973), Gregory and Gisser(1973), and Diamond(1977) have argued that employees' underestimation of occupational risk justifies workers' compensation. Their main argument for this conclusion is that the enactment of workers' compensation would compel the employers to pay their share of accident costs more correctly through experience-rating insurance premiums. In turn, the labor costs would more precisely reflect accident costs. This leads to a larger decline in employment in the high risk firm than in the low risk firm.
Chelius(1977) claims that the introduction of workers' compensation does not necessarily improve industrial safety unless "all" the workers underestimate occupational risk because wage premiums associated with work-related risk are determined by the marginal workers. The practical effect of experiencing-rating premium which is applied only to large firm is another factor affecting the safety effect of workers' compensation.

Several studies took into account the safety incentive of workers' compensation as well. Oi(1973), Gregory and Gisser(1973), Diamond(1977), Chelius(1977), and Rea(1981) analyzed the effect of a change in compensation benefits on industrial safety in a theoretical framework. From the firm side, an increase in benefits may encourage the firm to invest more in industrial safety because the firm saves more by preventing injuries through experience-rating premiums. Needless to say, this incentive effect is a function of experience-rating. From the worker's side, increased benefits may induce the worker to be less careful because his private costs of an injury are decreased. These two opposite directions end up with theoretically indetermined effects on the industrial safety of increased compensation benefits. Interestingly enough, Rea(1981) showed that even without the worker's moral hazard problem, increased benefits possibly lead to a decrease in safety level even when the worker underestimates industrial risk.

Chelius(1977), Chelius(1982), Worrall and Appel(1982), Butler and Worrall(1983), and Butler(1983) in their empirical works found a positive relationship between workers' compensation benefits and the
reported injury rate. But the research did not determine whether the increase in reported injuries are the result of a reporting phenomenon from worker side behavior or of more actual injuries.

The second type of imperfect information involves the inability of the workers' compensation authority to monitor the true loss of earnings capacity; this information problem leads directly to insurance problems of a traditional sort. A fundamental problem in the provision of social insurance including workers' compensation is the difficulty of monitoring the true value of loss the disabled suffered. Monitoring the true value of loss may be prohibitively expensive. A natural consequence is that the social insurance authority must use proxies for measuring the true value of loss.

In the case of workers' compensation, two kinds of proxy are used for measuring the true value of earning capacity loss; the physical impairment and the actual earnings loss. Social security disability insurance requires both an actual earnings loss and evidence of disability. Under social security disability insurance, however, benefits are paid only if the worker is totally disabled. In contrast, workers' compensation allows benefits for partial disability. Of course, different kinds of benefit schemes are likely to have different work incentive effects and could lead to quite different levels of expected utilities.

Few theoretical or empirical studies have modeled workers' compensation as income insurance. Earlier works of social insurance with imperfect state verification focused on social security programs. Diamond and Mirrlees(1978) develop a disability model which can be
applied to public provided pensions. They assumed two states of nature; i.e., the worker has either full earnings capacity or none and there is no partial loss of earning capacity. The government is assumed unable to distinguish those who cannot work from those who merely choose not to. No screening is feasible. The basic nature of their model lie in the constraint the potential applicants impose on the level of income maintenance of the disabled. They explore the nature of this unintended-applicant supply constraint both in a static framework and over the life cycle, applying the model to public pension plan.

Parsons (1984) generalizes the static version of the Diamond and Mirrlees model to incorporate a feasible but imperfect disability screen. He applies the model to social security disability program. Professor Parsons especially noticed the social cost of the program due to labor force withdrawing associated with benefit generosity. The fundamental reason why he considered two states of nature is because in the case of disability insurance, benefits are paid only when the disabled worker is presumed to be totally disabled.

In the model I develop, partial loss of earning capacity is allowed as well as total loss. And no screening is feasible; i.e., once the injury of the disabled worker is verified as work-related, he is eligible for disability benefits without any detail.

There are very few empirical studies on the work disincentive effect of workers' compensation benefits. The best known research is Johnson (1983); he estimates that workers' compensation benefits provide
small but statistically significant work disincentives to permanent impaired workers.

In sum, in the past literature, the researchers who dealt with social insurance programs including workers' compensation mainly focused on increased benefit costs and benefit levels, and attempted to predict their indirect effects on safety and various other economic variables. It is, however, expected that the type of benefit scheme itself may be important. More specifically, in the case of workers' compensation, the expected utility the worker could achieved under the system and the labor disincentives of the system may differ under alternative benefit schemes even though the aggregate resources devoted to the program are identical. The form of the compensation scheme may be an important social question.

Taking into account this factor, this work develops a model of workers' compensation as income insurance, and provides a method of evaluating the performances of various benefit schemes based on the model. The approach is a general one which can be applied to other social insurance program.

This study should ultimately be of value to workers' compensation authorities because it (1) provides a theoretical structure for comparing the performances of alternative benefit scheme, and (2) provide some insight into the optimal benefit scheme. The benefit scheme based on the actual earnings loss has received considerable attention in recent years because of the incorporation of a variant of the scheme in the 1979 Florida legislation. The primary force behind the legislation was the belief of Florida employers that before 1979
the law was unduly expensive because the permanent partial benefits were too generous. It is conjectured that the particular version of wage-loss benefits that Florida has adopted will reduce the employers' costs of workers' compensation.\textsuperscript{10} The experience of the social security disability system, which requires both physical impairment and actual earnings loss, suggests that cost reductions may be much less than expected. Individuals may adjust their work efforts to the economic incentives they face under the new system in a way that offsets expected savings.

The methodology of this study can also be applied to other social insurance programs.

\textsuperscript{10} Burton\textsuperscript{(1983)}, pp.41-49.
CHAPTER III
FULL-OPTIMUM DISABILITY INSURANCE UNDER PERFECT INFORMATION

A. Introduction

The basic function of insurance is the reduction in uncertainty; other things being equal, individuals seem to prefer and be willing to pay for a reduction in their risks. Particularly in situations where the individuals face a probability of very large losses, they appear quite interested in insurance. The onset of a substantial work disability is such a situation. Clearly some form of insurance against disability is desirable, whether publicly or privately provided.

In this chapter, I first characterize the ideal structure of disability insurance which achieves the full-optimum with perfect (and costless) information. Especially I scrutinize the shape of the optimal insurance-payoff trajectory as a function of earnings capacity loss in this "first-best" situation. This ideal insurance model serves as a framework for a discussion of the goals of disability insurance, even if it does not serve directly as a guide to policy since any actual insurance program must confront informational difficulties which limit the design of the insurance program.

The analysis of disability insurance requires a more extensive model than the conventional one. In previous insurance analyses, especially when informational issues were not involved, there have usually been two ways to characterize the individual's utility in
uncertain setting. First, utility depends only on income, and is not state-dependent. Second, utility depends only on income, but is state-dependent. Analyses based on the first assumption are simpler than those based on the second.

The first approach amounts to the case where the worker faces the risk of a decline in income directly but no other loss or disability. In this case, the insurance decision is straightforward. If the insurance premium accurately reflects the probability of the occurrence and the amount of loss, the risk-averse worker will completely insure; i.e., the worker will end up with the same income over all health states. Further, utility is also the same over all health states. This conclusion is in accordance with the conventional wisdom on complete insurance under perfect information.\(^\text{11}\)

In the second approach utility is permitted to vary with the state of health, along the lines discussed in the first approach. The optimal insurance of the similar case to this has been described by Arrow(1973). If the insurance premium accurately reflects the probability of the occurrence and the amount of loss, the optimal insurance policy has the form of stating a critical marginal utility of income and paying out an amount sufficient to bring the marginal utility of income to that level; i.e., at optimum, the marginal utility of income should be the same all over health states, but income (or utility) per se is not necessarily constant across health states.

\(^{11}\)In this paper, "complete insurance" is defined as one which completely smoothes the insured's income (or consumption) across states of nature. Some literature, however, defines it as one which fully covers the marginal loss
Previous studies, analyzing the problem of rational insurance purchasing from the point of view of an individual facing a specific risk, have usually taken utility solely as a function of income and derived the optimal insurance coverage as complete insurance under perfect information. However, as Arrow(1973) clarified, employing this kind of utility (a function of income or a composite good only) is appropriate only if all the prices of relevant commodities are unaffected by the state of nature so that all economic effects could be subsumed into changes in income. In the case of disability insurance, suppose we assume that health damage does not affect the (implicit) price of leisure, let alone the prices of other commodities. Then we could take utility as a function of income only and we may end up with complete insurance under perfect information.

Conversely, if we assume that health damage does affect the price of leisure through changing market wage rate and/or work disutility, we must consider a more complex insurance contract. In general we do not end up with complete insurance principle under perfect information. (We have to introduce 'leisure' as another explicit argument in the utility function in order to incorporate the disability effect on the price of leisure.)

As a matter of fact, in the case of disability insurance, the conventional principle of complete insurance under perfect information appears to be a special case. That is to say, the full-optimum disability insurance developed in this chapter shows us that at the optimum, the utility level, total income including insurance benefits, and (effective) leisure are constant over all health states only when
the wage rate and work disutility remain constant across health states. When the wage rate decreases and/or work disutility increases with health damage, the utility level, total income, and (effective) leisure either increase or decrease with health damage even under full-optimum disability insurance depending upon utility structure unless the utility function is an additive one. In the case of additive utility, it is shown that the optimum insurance is a complete one.

In this sense, the disability insurance model which incorporates the effects of health damage on the price of a commodity (leisure) can be regarded as a more general model than the conventional insurance model. From the next section, a disability insurance model is presented and the full-optimum insurance will be characterized.

B. MODEL

In this chapter I focus on a single period model in which a positive probability of health damage exists. Health damage results in utility loss through some loss of earnings as well as leisure. Against such risk, the workers are willing to insure. The question to be answered in this chapter is what are the characteristics of the full-optimum disability insurance under perfect information.

The maintained assumptions and notations are as follows:

1. There exists only one kind of accident, which affects the

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12 Since total income is constant across health states, the full-optimum insurance in this case is a complete insurance in the conventional sense.
worker's health state. The probability of accident is $\theta$. Ex ante $\theta$ is known to both the insurer and the insured. Further, ex post the insurer can verify whether the accident occurs.

(2) Health state affected by injury from accident is a random variable $X$. $X$ takes the range from 0 to 1. The value $x=0$ means no health damage (perfectly healthy) and the value $x=1$, complete health damage (perfectly unhealthy). The function $h(x)$ denotes the conditional probability density function of $X$ given $\theta$.

(3) The loss due to health damage derives from three sources;

(i) the effect on wage rate; health damage may decrease the wage rate as a consequence of a decrease in work productivity. Assume that the wage rate at health state $x$ is $W \cdot f(x)$ where $W$ is the wage rate with no health damage. The function $f(x)$ is decreasing, reflecting the effect of health damage on the wage rate. We will call it the 'disability wage function'; $f(0)=1$, and $f'(x)<0$.

(ii) the effect on work disutility; health damage increases the disutility of work hours. Work disutility at $x$ is denoted by $g(x)$. The function $g(x)$ is increasing in $x$, showing the relationship between the health damage and work disutility. In particular $g(0)=R$, a constant, and $g'(x)>0$. We will call $g(x)$ the 'work disutility function'.

(iii) the effect on available hours; health damage increases the individual's maintenance time (non-work and non-leisure time), thereby reducing the time available for other activities. Total available hours without health damage is denoted by $H_0$. Assume that
the available hours at $x$ are $H_0 \cdot (1-x)$.\(^\text{13}\)

(4) All the workers face the same probability of accident $\theta$ and the same distribution of health damage $h(x)$. There are a large number of workers.

(5) Utility depends on consumption of goods (or alternatively on income including insurance benefits), and leisure. Moreover, the utility function is a strictly concave one, reflecting the risk-averse preferences of all workers. All the workers are assumed to be identical in preferences and wage rates across health states.

Competition in the insurance industry imposes a break-even constraint with premiums equal to payout on average (for simplicity administration costs are assumed to be zero); in our framework

$$(1-\theta) \cdot B_0(z_0) + \theta \cdot \int_0 B(z) \cdot h(x) \, dx = 0$$

where $z$ denotes a vector of arguments in the payoff function $B$ which the insurer monitors in determining his payoff, and subscript $0$ denotes zero health damage. A government program, however, can run at a loss if it is subsidized by taxes; in our framework

$$(1-\theta) \cdot B_0(z_0) + \theta \cdot \int_0 B(z) \cdot h(x) \, dx < 0.$$ 

In this chapter, presenting the optimum insurance scheme, I focus on the competitive insurance market outcome or alternatively government

\(^{13}\)This assumption means that the percentage loss of available hours is used as a standard measure of disability; i.e., we can conveniently say that 10 percent disability indicates 10 percent loss of available hours. The percentage loss of wage rate or the percentage increase of work disutility can play the same role. In any case, three effects are interrelated through health damage variable $X$. 
program with no net subsidy to the worker. The case of positive subsidies introduces no interesting new aspects in complete information case.

The competitive insurance equilibrium can be characterized by assuming the insurer's objective is to maximize the representative worker's expected utility, subject to the break-even constraint and the optimal work decision of the insured. The insured's (the worker's) choice problem in this model lies solely in the labor supply decision; i.e., after the insurance scheme is announced and the health state is realized, the worker will decide his work hours. This labor supply decision of the worker is of course an additional consideration in the insurer's maximization problem.

The relevant uncertainty is parameterized by $\theta$ and by the conditional density function $h$. It affects the wage rate, work disutility and available hours. In the "first-best" world where everyone possesses perfect information, the insurer as well as the insured know the true state of health $x$, work hours $T_x$, in addition to market parameters including the wage rate, and the uncertainty parameters, $\theta$ and $h$.

The insurer's capability of monitoring both $x$ and $T_x$ may induce him to employ an insurance payoff function $B(x,T_x)$. We can, however, easily demonstrate that if the true health state $x$ is known to the insurer, he can force the insured to choose whatever $T_x$ he wants.\(^{14}\) Thus, the additional ability to monitor actual work hours $T_x$ is of no

\(^{14}\) For a discussion of this, see Spence and Zeckhouser(1971), pp. 384-5.
value; i.e., there is no loss of efficiency if the payoff is made solely a function of the insured's health state $x$. Therefore the insurance payoff function is reduced to $B(x)$.

As we assumed, utility is a function of income $M$ and leisure $L$ (more precisely, effective leisure since health damage may affect work disutility). Thus, utility at health state $x$ can be expressed as

$$U = U(M_x, L_x)$$

$$= U(B(x) + Wf(x)T_x, H_o(1-x) - g(x)T_x)$$

where $[H_o(1-x) - g(x)T_x]$ indicates effective leisure.

Given $x$ and payoff function $B$, the worker's optimum work hours is derived from the following first-order condition;

$$\frac{\partial U}{\partial T_x} = 0 \rightarrow U_M \cdot Wf(x) - U_L \cdot g(x) = 0$$

where subscript indicates partial derivative.

With this, the insurer's optimizing problem is to

$$\max_{B(x)} (1-\theta) \cdot U(B(0) + W \cdot T_o, H_o - RT_o)$$

$$B(x) + \theta \cdot \int_0^1 U(B(x) + Wf(x)T_x, H_o(1-x) - g(x)T_x) \cdot h(x) \, dx$$

subject to $(1-\theta) \cdot B(0) + \theta \cdot \int_0^1 B(x) \cdot h(x) \, dx = 0$ (break-even constraint)

and $$U_M \cdot Wf(x) - U_L \cdot g(x) = 0.$$

The Lagrangean function becomes

$$\xi = (1-\theta) \cdot \{U(B(0) + W T_x, H_o - RT_o) - \lambda \cdot B(0)\}$$

$$+ \theta \cdot \int_0^1 \{U(B(x) + Wf(x)T_x, H_o(1-x) - g(x)T_x) - \lambda \cdot B(x)\} \cdot h(x) \, dx$$

Employing the calculus of variation formulae, we can find the two marginal conditions;

$$U_M = \lambda \quad \text{for all } x \text{ including } x=0 \quad (3.1)$$

$$U_L \cdot Wf(x) = U_L \cdot g(x) \quad \text{for all } x \text{ including } x=0. \quad (3.2)$$
The condition (3.1) is the equality of the marginal utilities of income over all health states. The condition (3.2) is basically the marginal condition for labor-leisure choice at each health state. But combined with (3.1), the condition (3.2) is reduced to

$$\lambda \cdot \Psi f(x)/g(x) = U_L.$$ 

This implies that at the optimum, the marginal utility of leisure decreases with health damage when the wage rate decreases and/or work disutility increases with health damage. Otherwise, it is constant. The optimal insurance payoff trajectory $B(x)$ and the work hours $T_x$ are simultaneously determined and basically derived from conditions (3.1) and (3.2) with the break-even constraint.

As a matter of fact, the design of the ideal insurance program reduces here to the insurer's choice of the insurance payoff $B(x)$, namely the payoff that will maximize the expected utility of the insured. It will be useful to separate this into two parts: the magnitude of benefits and the trajectory across health states.

C. The properties of the optimal insurance-payoff trajectory:

Its determinants

It turns out that the two uncertainty factors, $\theta$ and the damage distribution function $h$, affect only the position of $B(x)$, not the curvature of $B(x)$; i.e., a change in the functional form of $h$ or $\theta$ shifts the $B(x)$ curve, but does not affect its curvature. It turns out that the slope of $B(x)$ is always positive and that the curvature of $B(x)$ depends upon the disability wage function $f(x)$, work disutility
function $g(x)$, and the utility structure.  

C.1. The position of $B(x)$

To get more insights into the position of $B(x)$, let us describe how to solve the original optimizing problem in more detail. Rewriting the marginal condition for the optimal labor supply (3.2), we have

$$W_f(x)U_M(B(x)+W_f(x)T_x, H_0(1-x)-g(x)T_x) = g(x)U_L(B(x)+W_f(x)T_x, H_0(1-x)-g(x)T_x).$$

Thus, labor supply function can be expressed as

$$T_x = T(W_f(x), g(x), B(x), x).$$

Substituting this into the marginal condition (3.1), we have

$$U_M(B(x)+W_f(x)T(W_f(x), g(x), B(x), x), H_0(1-x)-g(x)T(W_f(x), g(x), B(x), x)) = \lambda$$

From this we can express $B(x)$ as

$$B(x) = B(W_f(x), g(x), x, \lambda).$$

Here $\lambda$ is a constant, not a function of $x$. That is, $\lambda$ is a shifting factor. Further, $\lambda$ is derived from the following break-even constraint;

$$(1-\theta)\cdot B(0) + \Theta \cdot \int_0^1 B(x) \cdot h(x) \, dx = 0$$

$$\Rightarrow (1-\theta)\cdot B(W, \lambda) + \Theta \cdot \int_0^1 B(W_f(x), g(x), x, \lambda) \cdot h(x) \, dx = 0$$

Thus, $\lambda = \lambda(W, f, g, \theta, h)$, where $f, g, h$ denote functional forms. That is, $\lambda$ is determined by the functional forms of $f, g, h$ and $\theta$.

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15 More precisely, if health damage affects only available hours, $B(x)$ is linear regardless of utility structure. If health damage affects the wage rate or work disutility, the utility structure, especially the income effect on labor supply plays a role in determining the shape of $B(x)$. This result will be shown and discussed in the next section (D).
Therefore, we can express \( B(x) \) as

\[
B(x) = B(Wf(x), g(x), x, \lambda )
= B(Wf(x), g(x), x, \lambda Wf, g, \theta, h))
\]

In general, the optimal \( B(x) \) is a nonlinear function of \( x \) with functional parameters of \( f, g, h \), if specified, and the uncertainty parameter \( \theta \). The functional forms of \( f \) and \( g \) have both a direct effect and an indirect effect through \( \lambda \) on \( B(x) \). The uncertainty factors, \( \theta \) and the functional form of \( h \), have only indirect effects on \( B(x) \) through \( \lambda \) level. Thus, changes in \( \theta \) and \( h \) only shift \( B(x) \).

In order to get more insights into the effect of \( h \) on \( B(x) \), consider the special case: \( h(x) = \gamma - \delta x \). But \( \int_{\delta} h(x)dx = 1 \) since \( h(x) \) is a density function. Thus, \( \gamma = 1 + (1/2)\delta \). That is, \( h(x) = [1 + (1/2)\delta] - \delta x \). The usefulness of this assumption lies in the fact that a change in \( \delta \) implies a change in the distribution of severeness of damage. Particularly, the larger \( \delta \) means that on average the probability of less severe damage is more likely to be higher than that of more severe damage. Here it is easy to see that the larger \( \delta \), the smaller \( \lambda \), which indicate \( B(x) \) shifts downward. The other uncertainty parameter \( \theta \) also shifts \( B(x) \) in the opposite direction; i.e., the larger \( \theta \) leads to the smaller \( \lambda \), which means \( B(x) \) moves downward.

In sum, the shape of the damage distribution \( h(x) \) affects the shape of optimal insurance payoff only indirectly (shifting the curve) under perfect information, while under imperfect information, as we will see in the next chapter, the damage distribution plays a major role in determining the shape of the insurance payoff trajectory.
C.2. The shape of B(x)

To derive an explicit expression for the slope of B(x), it is necessary to take the total differential of condition (3.1) and solve for \( dT_x / dx \). The result is

\[
\dot{T} = \frac{-U_{MM} \cdot \dot{B} + (U_{ML} \cdot \dot{g} - W \cdot U_{MM} \cdot \dot{f}) \cdot T + U_{ML} \cdot H_o}{W \cdot f \cdot U_{MM} - U_{ML} \cdot g}
\]

(3.3)

where \( (\cdot) \) denotes \( d(\cdot)/dx \), \( f = f(x) \), \( g = g(x) \), and \( T = T_x \).

Next, substituting (3.1) into (3.2) gives

\[
W \cdot f \cdot \lambda = g \cdot U_L
\]

(3.4)

Taking the total differential of (3.4) and solving for \( \dot{B} \) yields

\[
\dot{B} = \left[ -W \cdot f + \frac{U_{LL} \cdot g}{U_{ML}} \right] \dot{T} - W \cdot T \cdot \dot{f} + \frac{U_{LL} \cdot T \cdot \dot{g}}{U_{ML}} + \frac{U_{LL} \cdot H_o}{U_{ML}} - \frac{U_L \cdot \dot{g}}{U_{ML} \cdot g} + \frac{\lambda \cdot W \cdot \dot{f}}{U_{ML} \cdot g}
\]

(3.5)

Substituting (3.3) into (3.5), and making some manipulations give

\[
\dot{B} = \left[ \frac{\dot{g}}{g} - \frac{\dot{f}}{f} \right] \cdot W \cdot f \cdot T + \frac{W \cdot f \cdot H_o}{g} - \left[ \frac{\dot{g}}{g} - \frac{\dot{f}}{f} \right] \cdot \frac{U_L \cdot (U_{MM} \cdot W \cdot f - U_{ML} \cdot g)}{(U_{MM} \cdot U_{LL} - U_{ML}^2) \cdot g}
\]

(3.6)

In equation (3.6), recall that \( \dot{g} \geq 0 \), \( \dot{f} \leq 0 \), \( U_L > 0 \), \( U_{MM} < 0 \), and \( U_{LL} < 0 \). Also notice that the parenthesized term \( (U_{MM} \cdot Wf - U_{ML} \cdot g) \) determines the sign of the income effect; i.e., if leisure is a normal good, this term is negative. Further, \( (U_{MM} \cdot U_{LL} - U_{ML}^2) > 0 \) if the utility function is a strictly concave one. Thus, the slope of B(x), which is \( \dot{B} \), is always positive regardless of the utility functional form and the shapes of \( f(x) \) and \( g(x) \) as long as the utility function is a strictly concave one and leisure is a normal good.

The curvature of B(x), which is the second derivative of B(x), depends upon the shapes of the disability wage function f(x), the work disutility function g(x) and the utility structure. As we see in
equation (3.6), the optimal insurance payoff trajectory is in general not linear as long as \( f(x) \) and/or \( g(x) \) are not constant. We can think of two approximations when the optimal payoff trajectory is nonlinear; a linear payoff scheme and a deductible policy.\(^{16}\) If the optimal \( B(x) \) is severely convex over most of its domain, then a deductible policy may be a reasonable approximation to the optimum. Conversely, if the optimal \( B(x) \) is less curved, then linear payoff scheme may be a better approximation to the optimum. Thus, scrutinizing the curvature of \( B(x) \) may shed light on the comparison of the fitnesses of the two alternative insurance schemes to the optimum.

Taking the second differential of \( B(x) \), however, results in an extremely complicated form. We will, therefore, look at two simple cases in the next section (D) which lead to interesting results and will be useful as benchmarks. In the next subsection (D.1), we assume that health damage affects only the available hours, not the wage rate and work disutility (HOURS LOSS MODEL). On the other hand, in the subsection (D.2), we assume that health damage affects only the wage rate (WAGE LOSS MODEL). I do not consider here the case of the work-disutility effect, for it ends up with the similar result to that of the wage-rate effect.

In both cases, I will describe the properties of work hours, utility level, income, and leisure trajectories as well as the curvature of \( B(x) \) under the full-optimum insurance.

\(^{16}\)Deductible policy in this case means that the insurance does not cover small (or partial) loss of health damage, but cover only the large (or total) loss of health damage.
D. Two benchmarks: The hours loss model and the wage rate loss model

D.1. The hours loss model

The case where health damage has an effect only on the available hours is relatively easy to analyze since the rate of exchange between income and leisure does not change with health damage. It turns out that the optimal insurance-payoff trajectory is a linear one; i.e., insurance benefit should increase proportionally with health damage. At the optimum, the corresponding work hours are also derived as a linear function of $x$.

Other interesting things we notice in this case are that at the optimum

(i) total income including insurance benefits is constant across health states

(ii) leisure hours are constant across health states

(iii) utility level is also constant across health states

(iv) work hours decreases with health damage.

Thus, in this case we end up with complete insurance in the conventional sense under perfect information, for the insured worker could maintain the same income.

First of all, it is easy to see from equation (3.6) that $B(x)$ becomes linear when both $f(x)$ and $g(x)$ are constant. Let us suppose that $f(x)=g(x)=1$ to show this in a simple way. It follows that $\dot{B} = WH_0$ in equation (3.6), meaning that $B(x)$ is a linear function of $x$, and has a positive slope.
To obtain \( \dot{T} \), substitute \( \dot{B} = WH_0 \) into equation (3.3) and recall that \( \dot{f} = \dot{g} = 0 \). It immediately follows that \( \dot{T} = -H_0 \), indicating \( T_x \) is also a linear function of \( x \), and has a negative slope.

In order to show that total income \( M \) is constant, let us take the total differential of \( M \) with respect to \( x \). We have

\[
\dot{M} = \dot{B} + W \cdot \dot{T}
\]

\[
= WH_0 - WH_0 = 0
\]

This means that total income is constant across health states.

Leisure hours are also constant, for \( L_x = H_o(1-x)-T_x \) and \( \dot{L} = -H_o - \dot{T} = 0 \).

Since both total income and leisure hours are constant over all health states, utility is also constant across health states.

D.2. The wage rate loss model

In this subsection, we assume that health damage affects only the wage rate and disability is measured by the wage rate. In particular assume that

\[
W_x = W \cdot (1-x), \quad H_x = H_o = \text{constant}, \quad g(x) = \text{a constant}
\]

where \( W_x \) and \( H_x \) are work hours and total available time at \( x \), respectively. This case is more complex to analyze than the previous case since the rate of exchange between income and leisure is affected by health damage. In this case, the curvature of \( B(x) \), which is the second derivative \( B(x) \) in equation (3.6), appears to be a very complicated form. It, however, turns out that the income effect in the conventional labor supply problem plays a major role in determining the curvature of \( B(x) \).
To show this, let us rewrite the expression of $\dot{B}$ of this case from equation (3.6)

$$\dot{B} = W \cdot T - \frac{1}{U_L} \cdot \frac{(U_{MM} \cdot W \cdot (1-x) - U_{ML})}{(1-x)} \cdot \frac{U_{LL} - U_{ML}^2}{U_{ML}} \tag{3.7}$$

Recall that the parenthesized term $(U_{MM} \cdot W \cdot (1-x) - U_{ML})$ in expression (3.7) determines the sign of the income effect in the labor supply problem and it is zero if there is no income effect. Thus, if the income effect is zero, $\dot{B}$ is reduced to

$$\dot{B} = W \cdot T$$

The second derivative of $B(x)$ is

$$\ddot{B} = W \cdot \ddot{T}.$$ 

It indicates that the curvature of $B(x)$ in this case solely depends on the sign of $\ddot{T}$. But it turns out that $\ddot{T}$ is always negative under the full-optimum insurance. Thus, $\ddot{B}$ is negative. All these mean that insurance payoff trajectory is concave if the income effect is zero.

When the income effect is nonzero, the second derivative of $B(x)$ takes a very complicated form and I have been unable to isolate any regularities in the general utility form. Consider, therefore, three specific forms of the utility function which provide us with some insight into the determinants of the shape of $B(x)$. Example 1 is a utility function in which the income effect is zero. Example 2 is the Cobb-Douglas case where work hours is independent of the wage rate when

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$^{17}$It will be shown in the next section (E).
there is no nonearned income such as insurance benefits. Thus, work
hours change with the wage rate (with health damage) only because of
the insurance benefits in this case. Example 3 is an additive utility
function case in which the cross derivative $U_{ML}$ is zero. In the
latter two cases, there is the positive income effect on leisure
demand. We assume in this subsection that the damage distribution is
an uniform one; i.e., $h(x) = 1$. Again this assumption does not alter
the curvature of $B(x)$ since the shape of $B(x)$ does not depend upon the
shape of $h(x)$.

**Example 1.** Let us consider the original problem in the case when
utility is a particular form of separable function, which is
\[
U(M_x, L_x) = U(M_x - V(T_x))
\]
\[
= U(B(x) + W(1-x)T_x - V(T_x))
\]
where $T_x = (H_o - L_x)$ and $V(.)$ is an increasing function describing the
marginal disutility of labor. One of the characteristics of this
utility function is that the income elasticity of leisure demanded
(labor supplied) is zero.

To show explicit relations, let us assume that $V(T_x) = R \cdot T_x^2$,
where $R$ is a constant. This assumption yields
\[
U = U(B(x) + W(1-x)T_x - R \cdot T_x^2).
\]
Then the optimum work hours are
\[
T_x = \frac{W(1-x)}{2R}.
\]
Work hours are unambiguously decreasing in health damage since there is
no income effect on leisure demand.
The insurer's optimizing problem is to

\[ \max_{B(x)} \quad (1-\theta) \cdot U(B(0) + WT_0 - RT_0^2) + \theta \cdot \int_0^1 U(B(x) + W(1-x)T_x - RT_x^2) \, dx \]

subject to \((1-\theta) \cdot B(0) + \theta \cdot \int_0^1 B(x) \, dx = 0\)

and \(T_x = W(1-x)/2R\).

Employing the calculus of variation formulae, we can obtain the optimal \(B(x)\). Taking the total differential of \(B(x)\) yields the expressions for the slope and curvature of \(B(x)\). They are

\[ B(x) = \left\{ \frac{3-2\theta}{6} \right\} \left( \frac{W^2}{2R} \right) - \left( \frac{W^2}{4R} \right) \cdot (1-x)^2 \]

\[ \dot{B} = \frac{W^2 \cdot (1-x)}{2R} \]

\[ \ddot{B} = \frac{-W^2}{2R} . \]

It is easy to see from the second expression that \(\dot{B}\), the slope of \(B(x)\), is always negative. From the third expression, it is clear that \(B(x)\) is concave. This result is due to the fact that the income effect is zero in this separable utility.

It is also interesting to observe utility level, income and leisure trajectories. Substituting the two expressions \(T_x\) and \(B(x)\) into the original utility yields the indirect utility function

\[ U = U\left[ \left( \frac{3-2\theta}{6} \right) \cdot \left( \frac{W^2}{2R} \right) \right] . \]

It indicates that the utility level is constant across health states since the argument in the indirect utility function is independent of \(x\). The slope of the income trajectory \(M_x = B(x) + W(1-x)T_x\) turns out to be

\[ \dot{M} = \left( -\frac{W^2}{R} \right) \cdot (1-x) < 0 . \]

It means that income decreases with health damage. The slope of the leisure trajectory \(L = H_0 - T_x\) is expressed as

\[ \dot{L} = H_0 + \left( \frac{W}{2R} \right) > 0 , \]
indicating that leisure increases with health damage.

**Example 2.** Suppose utility takes a Cobb-Douglas form; i.e.,

\[ U = H^\alpha \cdot L^\beta = (B(x) + W \cdot (1-x) \cdot T)^\alpha \cdot (H_0 - T_x)^\beta \]

where \( 0 < \alpha < 1 \), and \( 0 < \beta < 1 \). The assumption that the utility function is strictly concave means that \((\alpha+\beta-1) < 0\). In this case, work hours and insurance payoff trajectories are derived as

\[
T_x = \frac{(\alpha/\alpha+\beta) \cdot H_0 - (\beta/\alpha+\beta) \cdot B(x) / W(1-x)}{\beta/\alpha+\beta-1} - WH_0 \cdot (1-x)
\]

\[
B(x) = K \cdot (1-x) \cdot \left( \frac{\beta}{\alpha+\beta-1} \right) - WH_0 \cdot (1-x)
\]

where \( K = WH_0 \cdot [1 - (2\beta/3)] / [1 - \theta + ((\alpha+\beta-1) / (\alpha+2\beta-1) \theta)] \) is a constant.

The first and second derivatives of \( B(x) \) with respect to \( x \) are expressed as

\[
\dot{B} = K \cdot (\beta/\alpha+\beta-1) \cdot (1-x) \cdot \left( \frac{\beta}{\alpha+\beta-1} \right) + WH_0
\]

\[
\ddot{B} = K \cdot (\beta/\alpha+\beta-1) \cdot \left( \frac{\beta/\alpha+\beta-1 - 1}{\alpha+2\beta-1} \right) \cdot (1-x)
\]

It is easy to see from the first expression that the slope of \( B(x) \) is negative since \((\alpha+\beta-1) < 0\). The second expression shows us that \( \ddot{B} > 0 \), indicating that the optimum insurance payoff trajectory \( B(x) \) is convex. Notice that the income effect on leisure demand is positive in this utility form.18

Substituting from the two expressions \( T_x \) and \( B(x) \) yields the following trajectories;

\[
U = \left( \frac{\alpha}{\alpha+\beta} \right)^\alpha \cdot \left( \frac{\beta}{\alpha+\beta} \right)^\beta \cdot \left[ (1-x) \right]^{\alpha+\beta} \cdot K \cdot (1-x) \cdot \left( \frac{\beta}{\alpha+\beta-1} \right)
\]

\[
M_x = \left( \frac{\alpha}{\alpha+\beta} \right) \cdot K \cdot (1-x) \cdot \left( \frac{\beta}{\alpha+\beta-1} \right)
\]

\[
L_x = \left( \frac{\beta}{\alpha+\beta} \right) \cdot W \cdot K \cdot (1-x) \cdot \left( \frac{1-\alpha}{\alpha+\beta-1} \right).
\]

---

18 It means that as nonearned income increases, leisure increases and work hours decrease.
where $K$ is the same as before. The sign of the first one is negative
since $K$ is negative, indicating that the utility level increase with
health damage. The second and the third expressions also have the
negative sign. Thus, income as well as leisure increases with health
damage. On the other hand, work hours $T_X$ unambiguously decrease with
health damage since $T_X = H_o - L_X$ and $L_X$ increases with $x$.

Example 3. Finally, suppose that

$$U = M^\beta + L^\beta$$

$$= (B(x) + W \cdot T_X)^\beta + (H_o - T_X)^\beta,$$

where $0 < \beta < 1$. One of the characteristics of this utility function
is that the cross derivative $U_{ML}$ is zero. In this case, work hours and
insurance payoff trajectories are, respectively,

$$T_X = \left[\frac{1}{W(1-x) + (W(1-x))(1/1-\beta)}\right] \cdot \left[\frac{(W(1-x))(1/1-\beta)}{H_o}\right]$$

$$- \left[\frac{1}{W(1-x) + (W(1-x))(1/1-\beta)}\right] \cdot B(x)$$

$$B(x) = \left[\frac{(W(1-x))(\beta/\beta-1) + 1}{}\right] \cdot S - WH_o \cdot (1-x)$$

where $S = \left[\frac{(1-(2/3)) \cdot WH_o}{W((\beta-2)/((\beta-1)) \cdot (1-\theta -(\theta/(\beta-1))) + 1} \right]$. 

Taking the total differential of $B(x)$ subsequently yields

$$\dot{B} = S \cdot W(\beta/\beta-1) \cdot (\beta/1-\beta) \cdot (1-x)^{(1/1-\beta)} \cdot WH_o$$

$$\ddot{B} = S \cdot W(\beta/\beta-1) \cdot (\beta/1-\beta) \cdot (1/1-\beta) \cdot (1-x)^{((\beta-2)/(1-\beta))}.$$ 

It is easy to see that the signs of both expressions are positive since
$\beta < 1$. It indicates that the optimum insurance payoff trajectory is
convex in this case. Notice that the income effect on leisure demand
is positive in this utility form.
The utility level, income, and leisure trajectories are

\[ U = S + S \cdot [W \cdot (1-x)]^{1/\beta-1} \]

\[ M = S \]

\[ L = S \cdot [W \cdot (1-x)]^{1/\beta-1}. \]

where \( S \) is the same as before. It is clear that income \( M \) is constant across health states in this case. The signs of the slopes of the utility level and leisure trajectories are negative since \( \beta < 1 \). It indicates that utility level as well as leisure increases with health damage. On the other hand, work hours decrease with health damage since leisure increases.

The three examples suggest that under the full-optimum insurance, the curvature of the insurance payoff trajectory \( B(x) \) depends critically upon the income effect in the conventional labor supply problem. That is, if the income effect on leisure demand is zero, then \( B(x) \) is concave; if the income effect is positive, \( B(x) \) is convex. Another result we observed is that work hours unambiguously decrease with health damage.

These results are intuitively appealing. Consider first the latter result that the individual works more at the higher wage rate of healthy state and works less at the lower wage rates of states with greater health damage. Recall that in the conventional labor supply theory, income and substitution effects make ambiguous the direction of a change in work hours due to a change in the wage rate. Here, however, there is no ambiguity. With insurance, the worker is constrained by the expectation of income, not by the labor earnings in any particular
state, since he can redistribute consumption by insurance. In this situation, the worker is made better off if he works more when the wage rate is higher (at the less health damage state) and redistributes consumption to low wage state through insurance. In this case, the randomly drawn wage rate induces no income effect because the good luck of the higher wage rates of some states are offset by the bad luck of the lower wage rates of other states. This result will be proved rigorously in the next section (E).

Next, let us return to the first result, that the income effect in the labor supply problem plays a major role in determining the curvature of B(x). Recall that work hours always decrease with health damage under insurance. Consider first the case where there is no income effect so that insurance benefits do not affect work hours. In this situation, the worker’s losses from a decline in the wage rate definitely decrease as health damage x increases. It induces the insurance benefits to increase but at a decreasing rate with x.

Let us turn our attention to the other case where there is the positive income effect on leisure demand so that insurance benefits affect work hours. In this case, work hours decreases with health damage more rapidly than in the case where there is no income effect. Thus, it is expected that the worker’s losses from drop in the wage rate increase as health damage x increases. It induces the insurance benefits to increase at an increasing rate with health damage.

The next section (E) describes the properties of the utility level, income, leisure and work hours trajectories in the general utility functional form.
In order to examine the property of each trajectory, let us move to the two marginal conditions (3.1) and (3.2). The conditions are

\[ U_M(M_x, L_x) = \lambda \]
\[ U_L(M_x, L_x) = Wf(x) \cdot \lambda \]

where \( M_x = B(x) + Wf(x) \cdot T_x \) and \( L_x = H_0(1-x) - g(x) \cdot T_x \). The first equation is equivalent to (3.1). The second equation is obtained by plugging (3.1) into (3.2). Notice that \( \lambda > 0 \).

To derive explicit expressions for \( L \) and \( M \), let us take the total differentials of the two marginal conditions. Differentiating the conditions with respect to \( x \) yields

\[ U_{ML} \cdot \dot{M} + U_{ML} \cdot \dot{L} = 0 \]
\[ U_{ML} \cdot \dot{M} + U_{LL} \cdot \dot{L} = \lambda \cdot Wf(x) \cdot \lambda \]

The solution of \( M \) and \( L \) are, respectively,

\[ \dot{M} = \left( \frac{U_{ML}}{U_{MM} \cdot U_{LL} - U_{ML}^2} \right) \cdot \lambda \cdot \left( \begin{array}{c} Wf \\ g \\ f \end{array} \right) \cdot \left( \begin{array}{c} \dot{g} - \dot{f} \\ g \\ f \end{array} \right) \]  \hspace{1cm} (3.8)

\[ \dot{L} = \left( \frac{-U_{MM}}{U_{MM} \cdot U_{LL} - U_{ML}^2} \right) \cdot \lambda \cdot \left( \begin{array}{c} Wf \\ g \\ f \end{array} \right) \cdot \left( \begin{array}{c} \dot{g} - \dot{f} \\ g \\ f \end{array} \right) \]  \hspace{1cm} (3.9)

It is interesting to notice that some implicit contract literature brings about the similar model and analytical results. See Rosen(1985). In his survey paper he deals with the case where the firm is risk neutral and the production function is linear in labor input. Then the optimizing problem for the firm is similar to the optimizing problem for the insurer in this model. This similarity is basically due to the fact that in the implicit contract theory the firm plays a role as an insurer. Implicit contract theory, however, mainly focuses on the movement of the observed wage rate and employment.
It is easy to see from (3.9) that \( \dot{L} \) is negative as long as \( \dot{f} < 0 \) and/or \( \dot{g} > 0 \) (recall that these are in our maintained assumptions). It also indicates that \( \dot{T} \) is negative, for \( g \cdot \dot{T} = -g \cdot \dot{T} - H_0 - \dot{L} < 0 \).

The intuitive reason why work hours always decreases with health damage has been displayed in section (D).

Equation (3.8) show us that total income is constant when \( U_{ML} = 0 \). We have seen this case in Example 3 in section (D). If \( U_{ML} > 0 \), then total income increases with health damage (Example 2 in section (D)). When \( U_{ML} < 0 \), it decreases with \( x \). Thus, only when \( U_{ML} = 0 \), do we end up with complete insurance. When \( U_{ML} > 0 \), the full-optimum insurance is more than complete one, and vice versa.

The utility level trajectory is derived by differentiating the original utility function and substituting from (3.8) and (3.9). The result is

\[
\dot{U} = U_M \cdot \dot{M} + U_L \cdot \dot{L} \\
= - \left( U_M \cdot U_{ML} - U_L \cdot U_{MM} \right) \cdot \left( \dot{L} / U_{MM} \right)
\]  

(3.10)

The first term \( (U_M \cdot U_{ML} - U_L \cdot U_{MM}) \) in the right hand side of (3.10) is of course the numerator of the income effect on leisure demand, which determines the sign of that effect. That is, if this term is positive (negative), the income effect is positive (negative). If this term is zero, the income effect is zero. Equation (3.10) shows us that utility is constant across health states only when the income effect is

\[20\text{ It shows that total income (or consumption) is completely smoothed only if utility are strongly separable in } M \text{ and } L \text{. This is the same analytical result as Rosen's (1985) in his implicit contract literature.} \]
zero. This case has been shown in Example 1 in section (D.2). It is obvious from (3.10) that the utility level increases with health damage when the income effect on leisure demand is positive since \( U_{MM} < 0 \) and \( \dot{L} > 0 \), which has been shown in Example 2 and 3.

In sum, under the full-optimum disability insurance, labor supply decreases unambiguously as the wage rate drops due to health damage. Leisure increases as the wage rate decreases due to health damage. It is basically the consequence of substitution effects.

Complete insurance is achieved only when the cross derivative \( U_{ML} \) is zero. Notice that in the conventional consumer choice problem in which only the ordinal preferences matter, the sign of \( U_{ML} \) itself does not have any meaning, for some monotonic transformations lead to alternative signs of \( U_{ML} \). But in the multivariate insurance problem such as this disability insurance, the sign of \( U_{ML} \) is meaningful in the sense that whether the full-optimum insurance is complete one depends upon this sign.

F. Insurance payoff versus actual earnings loss

In the current social insurance programs, we can identify two kinds of insurance scheme both of which are based upon the actual earnings loss. Social security disability insurance limits payment to total disability only. (A sort of large deductible policy.) Conversely, some state workers' compensation programs compensate for partial disability as well, based upon the actual earnings loss. The payment schemes of them are linear. Needless to say, the current social insurance systems are designed and implemented under imperfect
information. It may, however, shed light on the comparison of the two alternative insurance schemes to examine the relationship between the optimum insurance payoff and the actual earnings loss under perfect information.

It turns out that when health damage affects only the available hours (HOURS LOSS MODEL), the insurance payoff and the actual earnings loss have a linear relationship regardless of the utility structure. In the case where health damage has an effect on the wage rate (WAGE RATE LOSS MODEL), they have a linear relationship if the cross derivative $U_{ML}$ is zero. Further, if the utility function has a particular form in which the income effect on leisure demand is zero (as in the previous Example 1 in section (D.2)), insurance payoff and the actual earnings loss also have a linear relationship. When utility takes a Cobb-Douglas form (as in Example 2 in section (D.2)), it turns out that the insurance payoff is splitted into two components; a portion compensating for the actual earnings loss and that subtracting the value of leisure gain. Each portion has a linear form. Thus, the insurance payoff and the actual earnings loss also have a linear relationship.

Let us consider first the "hours loss" model. In order to prove that the insurance payoff and the actual earnings loss have a linear relationship, it may suffice to show that $B = \gamma W T$ where $\gamma$ is a constant, since the linear relationship means that

$$B(x) = \gamma (W T_0 - W T_x) + K$$

where $\gamma$, $W$, $T_0$, and $K$ are constants. Recall that one of our previous results in the case of "hours loss" model is that $\dot{M} = 0$, implying that $M_x = B(x) + W T_x = K$, a constant.
Thus, \( B(x) = -WT_X + K \), indicating that the insurance payoff and the actual earnings loss have a linear relationship.

Let us turn our attention to the "wage rate loss" model. In this case, it may suffice to show that \( \dot{B} = -\gamma (\dot{WT} + \dot{WT}) \) in order to display that the insurance payoff and the actual earnings loss have the linear relationship, since linear relationship means that

\[
B(x) = \gamma (WT_0 - WX_TX) + K
\]

where \( W_x \) is the wage rate at \( x \), and \( \gamma, W_T, K \) are constants.

First, consider the case where \( = 0 \). As we derived in the previous section (E), total income is constant across health states in this case; i.e., \( \dot{M} = 0 \). Therefore, \( M_X = B(x) + WX_TX = S \), a constant. Thus,

\[
B(x) = S - WX_TX, \text{ indicating that } B(x) \text{ and the actual earnings loss have a linear relationship.}
\]

Next, let us move to the case of utility structure in which there is no income effect on leisure demand. As we can verify in equation (3.6) in section (C.2), in this case \( \dot{B} \) is reduced to \( \dot{B} = -\dot{WT} \). Thus, if \( \dot{WT} = \gamma (\dot{WT} + \dot{WT}) \), then \( B(x) \) becomes to have a linear relationship with the actual earnings loss. Example 1 in section (D.2) is this case; the utility function is \( U = U(B(x) + WX_TX - RT_X^2) \).

Finally, let us consider the Cobb-Douglas utility form;

\[
U = W^\alpha L^\beta = (B(x) + WX_TX)^\alpha (H_0 - TX)^\beta.
\]

In this case, the relationship between the insurance payoff and the actual earnings loss is derived directly from the labor supply problem. The optimal work hours with insurance is derived as

\[
T_X = (\alpha/\alpha+\beta)H_0 - (\beta/\alpha+\beta)(B(x)/W_x)
\]
Or equivalently,

\[ B(x) = \frac{(\alpha/\beta)}{W_x} \cdot H_x - (\frac{\alpha+\beta}{\beta}) \cdot W_x \cdot T_x \]

\[ = \frac{(\alpha/\beta)}{W_x} \cdot (H_x - T_x) - W_x \cdot T_x \]

\[ = \frac{(\alpha/\beta)}{W_x} \cdot L_x - W_x \cdot T_x \]

where \( L_x \) is leisure hours at \( x \). Thus, the insurance payoff is composed of the portion compensating for the actual earnings loss and that subtracting the value of leisure gain. Each portion has a linear form. Thus, the insurance payoff and the actual earnings loss also have a linear relationship.

G. Summary and policy implication

As we have mentioned before, scrutinizing the curvature of insurance payoff trajectory \( B(x) \) may shed light on evaluating the alternative insurance schemes; linear payoff scheme and deductible policy. If the optimal \( B(x) \) is severely convex over most of its domain, the deductible policy may be a reasonable approximation to the optimum. Conversely, if the optimal \( B(x) \) is linear or less curved, the linear payoff scheme may be a better approximation to the optimum.

It turns out that the type of health damage (type of injury) and the income effect in the conventional labor supply problem are two major factors determining the curvature of \( B(x) \). As we have seen in section (D.1), when the injury affects only the available hours, \( B(x) \) is linear, indicating that the linear payoff scheme is appropriate in this case.

On the other hand, when the injury affects the wage rate, the analysis is more complicated and it turns out that \( B(x) \) is nonlinear.
One definite result is that when there is no income effect on leisure demand, $B(x)$ becomes concave, implying that the linear payoff policy is more plausibly fitted to the optimum than the deductible policy. When the income effect is positive, $B(x)$ appears to be convex, indicating that the deductible policy is possibly more reasonable approximation to the optimum.

We have also examine the relationship between the insurance payoff and the actual earnings loss. It is more practical in the sense that current social insurance programs are based upon the actual earnings loss in some cases. Without limited information problem, it turns out that the insurance benefits and the actual earnings loss have a linear relationship at the optimum in most cases, suggesting that the linear payment scheme which compensates for partial as well as total disability is more fitted to the optimum than the payment scheme which limits to total disability only.
OPTIMAL DISABILITY INSURANCE UNDER IMPERFECT INFORMATION:

INSURANCE BASED ON ACTUAL EARNINGS LOSSES

A. Introduction

The difficulties in designing and maintaining a disability insurance increase dramatically when complete information is not available to all parties. Asymmetric information holding is a particular problem, introducing incentive compatibility problems and moral hazard. The impact of asymmetric information on the ideal disability insurance scheme will be explored in this section. As it happens, income effects on leisure demand are again crucial element in the analysis. Unlike the full information case, however, the distribution of health damage severity is also crucial.

The full-optimum disability insurance is incentive incompatible under asymmetric information about health condition. Suppose that the insurer and the insured share all the information except true health damage. More precisely, both the insurer and the insured know the uncertainty factors (θ and the functional form of h), wage rate and work hours in the market, and the functional forms of f and g. The insurer, however, does not know the true health state of the insured, while the insured knows his own health state.

In this case, the insurer could, if he wished, construct a health insurance scheme that paralleled the ideal scheme, using reported
health rather than actual health. The insurer may require the worker to report his health state, and make the insurance payoff depend upon this reported health. A more complex contract is possible if the insured's work hours can be monitored. Such a contract may require, for example, that the work hours "correspond" with reported health condition. We consider the actual earnings loss model in this section.

Assume that the individual derives no independent utility from honesty and objectivity. In this situation, we can say that the ideal insurance scheme is incentive compatible if the reported health state associated with the actual work hours is the same as the true health state. Under these assumptions, the worker will report health damage higher than the true one if he can obtain higher utility by doing so. Equivalently, the ideal scheme is incentive compatible if the utility the worker can obtain by misreporting health damage is not higher than the utility he can obtain by reporting true health state at each x.

It is easy to show that the ideal scheme is incentive incompatible. Reported health damage is denoted by \( x \) and the true one by \( \hat{x} \). The utility the worker can obtain when he reports the true health damage is expressed as

\[
U = U(B(x)+Wf(x)T_x,H_0(1-x)-g(x)T_x)
\]

And utility he can get when he reports health state as \( \hat{x} \) is

\[
\hat{U} = U(B(\hat{x})+Wf(\hat{x})\hat{T}_x,H_0(1-\hat{x})-g(\hat{x})\hat{T}_x)
\]

where \( B(x) \) and \( T_x \) are insurance payoff and actual work hours respectively associated with reported health \( x \).

If the insurer can observe work hours and if work hours decrease with disability, then in order for the worker to report credibly health
damage higher than the true one, he has to work less hours than he would in the true health state. This implies that the leisure hours associated with reported damage is larger than that associated with the true health damage. More formally,

\[ x > x \rightarrow \hat{T_x} < T_x \]
\[ \rightarrow \hat{L_x} > L_x \]

Thus, if the total income associated with the reported health damage is the same as that associated with the true one, or if the contribution of the increased leisure hours to utility is greater than the loss of utility due to a decrease in total income associated with the reported health damage, then utility increases.

To show that reporting health damage higher than the true one leads the worker to higher utility, consider first the "hours loss" model with wage rate and work disutility constant over all health states. Recall that under the ideal insurance scheme, the worker will obtain a constant total income, regardless of reported health state since income is constant and nonworking hours increase with misreported health damage. Obviously the individual has an incentive to do so.

The incentive incompatibility problem implies that the ideal insurance scheme is not sustainable under asymmetric information because the break-even constraint is not satisfied. Insurers could not sustain such a program based solely on reported health and observed work hours.

Now let us turn our attention briefly to the moral hazard problem. In the previous section (E) in chapter III, one of the major conclusion is that under a full-optimum scheme, utility levels may increase with
health damage as long as the wage rate decreases and/or work disutility increases with health damage. This situation may create a serious moral hazard problem if the insurer cannot monitor the worker's care level (the efforts of the worker to avoid accident). In this model we ignore the care effects. Nonetheless it must be recognized that the worker has an incentive to lower his care level as long as the full-optimum insurance provides him with higher utility with an accident than without one.

Returning to the reporting question, the full-optimum disability insurance is not sustainable due to the insurer's lack of monitoring the true health damage. In the situation where the insurer's monitoring capacity is limited so that he can only monitor some imperfect measure for the unobserved true health damage, he will instead design an insurance scheme as a function of this imperfect measure to maximize the insured's expected utility. As illustrated in the observed work hours model, such imperfect measures will in general not be sufficient to secure ideal coverage.

In social insurance programs in the United States, in fact, two kinds of indirect measures are used for the economic losses of true health damage; they are the actual earnings loss and a measure of physical impairment. When the insurer constructs the insurance payoff as a function of actual earnings loss. Pareto-optimal risk sharing is generally precluded, because it will not induce the proper incentives for making appropriate labor supply decisions. Only a "second-best" solution can be achieved. As a consequence, a program operating with
incomplete information may distort the labor supply decision while achieving only partial insurance. When the insurer takes the physical impairment as a measure for the unobserved true health damage, work hours the worker chooses at each health state may also differ from those under the full-optimum insurance. But in this case, this deviation results from income effects of the insurance benefits.

The analysis of this chapter is restricted to the insurance scheme based upon the actual earnings loss. In this chapter I characterize first the optimal disability insurance based upon the actual earnings loss under imperfect information in section (B). And then the optimal scheme is derived in the case of linear form in section (C). In the last section (D), I compare the performances of two versions of insurance benefit scheme based on actual earnings loss.

B. The optimal disability insurance based on the actual earnings loss

In general, the optimal disability insurance based upon the actual earnings loss is nonlinear when insurer's information is imperfect. In the current social insurance programs, we can identify two kinds of insurance schemes based upon the actual earnings loss. Social security disability insurance limits payment to total disability only (the 'Total only' scheme.) Conversely, workers' compensation programs compensate for partial as well as total disability (the 'Partial also' scheme.) The typical partial payment scheme is linear in health damage, for example, 66 2/3 % of wage loss.

We consider these two insurance schemes as approximations to the optimum. If the optimal disability insurance based upon the actual
earnings loss is severely convex over most of its domain, the 'Total only' scheme may be a reasonable approximation to the optimum. Conversely, if the optimum insurance payoff is linear or less curved, then the 'Partial also' linear scheme may be a better approximation to the optimum. Clearly the shape of the insurance payoff curve and its determinants are crucial to the question of which of these administratively simple schemes best approximates the ideal "second-best" system. In addition to this, I will describe the properties of work hours, utility level, income and leisure in order to characterize the optimum insurance.

It turns out that the shape of health damage distribution \( h(x) \) plays a crucial role in determining the shape of the insurance payoff curve when the insurance payoff depends upon the actual earnings loss. Recall that in the case of the full-optimum insurance, \( h(x) \) does not affect the shape of the insurance payoff trajectory. Another major factor affecting the shape of the benefit trajectory is the income effects. The role of \( h(x) \) and the income effect in determining the shape of insurance payoff curve is shown in this section. We also expect that risk-averseness and the labor supply supply response to the system will play a role in determining the shape of insurance payoff in the moral hazard situation with imperfect information. These factors explicitly appear in the linear compensation scheme case in the next section (C).
In order to show explicitly the role of health damage distribution in determining the shape of the insurance payoff, consider the "hours loss" model in which both the wage rate and the work disutility are constant across health states. Let \( B(WT_0 - WT_x) \) denote the insurance payoff as a function of the actual earnings loss. This function can be simplified as \( B(T_x) \) in this case since \( W \) and \( T_0 \) are constants. Notice that the sign of the first derivatives of the two functions are opposite, but the sign of the second derivatives of them are same. Thus, if the function \( B(T_x) \) is convex, then the function \( \hat{B}(WT_0 - WT_x) \) is convex as well.

To determine this function, the insurer can employ the optimal control formulae with utility \( U \) serving as the state variable, and \( B(T_x) \) as the control variable. Utility function is

\[
U = U(M_x, L_x) = U(B(T_x) + WT_x, H_o(1-x) - T_x).
\]

Given \( x \) and payoff function \( B \), the worker's optimum work hours is derived from the following first order condition;

\[
\frac{\partial U}{\partial T_x} = 0 \rightarrow U_M(B' + W) - U_L = 0 \quad (4.1)
\]

where \( B' = dB(T_x)/dT_x \).

We can also obtain the transition function by taking the differential of utility with respect to \( x \), which is

\[
dU/dx = U_M \cdot (dB/dT) \cdot (dT/dx) + U_M \cdot W \cdot (dT/dx) + U_L \cdot (-H_o) - U_L \cdot (dT/dx) \\
= \{U_M \cdot (B' + W) - U_L\} \cdot (dT/dx) - U_L \cdot H_o.
\]

But from equation (4.1), \( U_M \cdot (B' + W) - U_L = 0 \). Thus,

\[
dU/dx = - U_L \cdot H_o.
\]
This transition function has a negative value, implying that utility level declines with health damage under the optimum insurance with imperfect information.

We have the following break-even constraint;

\[(1-\theta) \cdot B(T_0) + \theta \cdot \int_0^T B(T_x) \cdot h(x) \, dx = 0.\]

Let \(\lambda\) be the multiplier corresponding to the break-even constraint and \(\psi(x) \cdot h(x)\) be the multiplier corresponding to the transition function.

Then the Hamiltonian function becomes

\[H = \{U(B(T_x)+\lambda T_x, H_0(1-x)-T_x) - \lambda \cdot B(T_x) + \psi(x) \cdot (-U_L \cdot H_0)\} \cdot h(x) \quad (4.2)\]

From this we have the following optimizing conditions;

\[\frac{\partial H}{\partial T_x} = 0 \rightarrow \{U_M \cdot (B'+W) - U_L\} - \lambda \cdot B' - \psi(x) \cdot \{U_{LM} \cdot (B'+W) - U_{LL}\} \cdot H_0 = 0 \]

\[\rightarrow \lambda \cdot B' = - \psi(x) \cdot H_0 \cdot \{U_{LM} \cdot (B'+W) - U_{LL}\} \quad (4.3)\]

\[d\psi(x)h(x)/dx = - \frac{\partial H}{\partial U} \rightarrow \psi \dot{h} + \dot{\psi} \cdot h = -h \]

\[\rightarrow \dot{\psi} = - (\dot{h}/h) \cdot \psi - 1 \quad (4.4)\]

where \(\psi = \psi(x), \dot{\psi} = d\psi/dx, h = h(x), \dot{h} = dh/dx\).

And

\[dU/dT_x = 0 \rightarrow U_M \cdot (B'+W) - U_L = 0 \quad (4.5)\]

From equation (4.3)

\[dB(T_x)/dT_x = B' = - \psi(x) \cdot H_0 \cdot \{U_{LM} \cdot (B'+W) - U_{LL}\} / \lambda \]

\[\rightarrow B' = - \psi(x) \cdot H_0 \cdot \{U_{LM} \cdot W - U_{LL}\} / \lambda \]

\[1 + \{\psi(x) \cdot H_0 \cdot U_{LM}/\lambda\} \quad (4.6)\]

Here \(dB(T_x)/dT_x\) is negative as long as consumption increases with income since \(\psi(x) > 0, \lambda > 0, U_{LL} < 0\) and the parenthesized term \(U_{LM} \cdot W - U_{LL}\) > 0 in the case of normal good; i.e., at the optimum, \(B(T_x)\) increases as \(T_x\) decreases.
Let us recall that in the case of the ideal insurance scheme, where \( x \) is observed, \( \frac{dB(x)}{dT_x} = 0 \) given \( x \). Or equivalently, \( \frac{dB(x)}{dL_x} = 0 \) where \( L_x \) is leisure. This implies that when \( x \) can be monitored, the optimal insurance plan will require the worker to pay the full marginal opportunity cost of his spending time on leisure.

In contrast to the above, if only \( T_x \) can be monitored, \( \frac{dB(T_x)}{dT_x} < 0 \) given \( x \). Or equivalently, \( \frac{dB(T_x)}{dL_x} > 0 \) given \( x \). This implies that when only \( T_x \) can be monitored, the optimal insurance plan will not require the worker to pay the full marginal opportunity cost of his spending time on leisure. This, in turn, means that the worker will spend time too much on leisure, or equivalently he will work too less. This is the consequence of moral hazard.

Now let us pay attention to the shape of the utility level, work hours, and income trajectories with respect to health damage \( x \). As we have seen in the transition function (\( \frac{dU}{dx} = -U_L \cdot H_0 \)), the utility level decreases with health damage under the optimal insurance with imperfect information. Work hours also decrease with health damage due to the second-order condition for the insured's optimizing decision. With these two results, it is easy to show that income also decreases with health damage. Taking the total differential of the utility function with respect to \( x \) yields,

\[
\dot{U} = U_M \cdot \dot{M} + U_L \cdot \dot{L}.
\]

But we have

\[
\dot{U} = \frac{dU}{dx} = -U_L \cdot H_0 \quad \text{and} \quad \dot{L} = -H_0 - \dot{T}.
\]

Substituting these yields,
\[-U_L^*H_\alpha = U_M^* \dot{M} - U_L^*H_\alpha - U_L^* \dot{T}\]
\[\Rightarrow U_M^* \dot{M} = U_L^* \dot{T}.
\]
It indicates that \(\dot{M} < 0\) since \(\dot{T} < 0\), \(U_M < 0\) and \(U_L < 0\). Thus, income also decreases with health damage. It, in turn, means that the optimal disability insurance under imperfect information is a partial insurance program.

Now let us turn our attention to the shape of insurance payoff curve. In order to get more insights into the shape of \(B(T_x)\), let us take the case where \(U_{ML} = U_{LM} = 0\). Additively separable utility function satisfies this condition. Then, from equation (4.3), we have

\[
\frac{dB(T_x)}{dT_x} = B' = \psi(x) \cdot H_0 \cdot U_{LL} / \lambda
\]

Let us take the total differential of (4.7) with respect to \(x\) and solving for the \(B''(= \frac{d^2B(T_x)}{dT_x^2})\) to get the curvature of \(B(T_x)\). It yields

\[
B'' \cdot \dot{T} = (\dot{\psi} \cdot H_0 \cdot U_{LL} / \lambda) + (\dot{\psi} \cdot H_0 / \lambda) \cdot (U_{LLM} \cdot (B' \cdot \dot{T} + \dot{W} + U_{LMM} \cdot (-H_0 - \dot{T})
\]

But \(U_{LLM} = 0\) since \(U_{LM} = 0\). Thus,

\[
B'' = (\dot{\psi} \cdot H_0 / \lambda) \cdot (U_{LL} / \dot{T}) + (\dot{\psi} \cdot H_0 / \lambda) \cdot (U_{LLL} / \dot{T} \cdot (-H_0 - \dot{T})
\]

Here \(\dot{T} < 0\) due to the second order condition for the worker's labor supply decision. Furthermore, it may be reasonable to assume that \(dU_{LL}/dx < 0\), indicating that the rate at which diminishing marginal utility of leisure decreases as health damage \(x\) increases. This assumption means that \(U_{LLL} \cdot (-H_0 - \dot{T}) < 0\) in this case since

\[
\frac{dU_{LL}/dx}{dx} = U_{LLM} \cdot (B' + \dot{W}) + U_{LLL} \cdot (-H_0 - \dot{T})
\]

and \(U_{LLM} = 0\). Then the second term of the right hand side of equation (4.8) is positive.
The sign of $B''$, therefore, depends upon the sign and the magnitude of the first term, $(\dot{\psi} \cdot H_0/\lambda) \cdot (U_{LL}/\dot{T})$. But $(U_{LL}/\dot{T}) > 0$ and $(H_0/\lambda) > 0$. Thus, the sign of $B''$ depends on the sign and the magnitude of $\dot{\psi}$. For an instance, if $\dot{\psi} > 0$, then $B'' > 0$, indicating that $B(T_x)$ is convex.

Now recall that $\dot{\psi} = -(\dot{h}/h) - 1$ from equation (4.4), meaning that $\dot{\psi}$ depends upon the distribution of health damage $h(x)$. In order to provide further insight into the determinants of the curvature of $B(T_x)$, let us assume that $h(x)$ is a linear function; $h(x) = \gamma - \delta x$ where $\delta > 0$. Since $h(x)$ is a density function, $\int h(x) dx = \int (\gamma - \delta x) dx = 1$. It indicates that $\gamma = 1 + (1/2) \cdot \delta$. Thus,

$$h(x) = \{1 + (1/2) \delta\} - \delta x.$$  

From this,

$$\frac{\dot{h}}{h} = -\delta$$

and

$$\frac{\partial (\dot{h}/h)}{\partial \delta} = -\frac{1}{\delta \{1 + (1/2) \delta\} - \delta x}.$$  

Now let us back to equation (4.4), which is $\dot{\psi} = -(\dot{h}/h) - 1$. From this and equation (4.9), we have

$$\frac{\partial \dot{\psi}}{\partial \delta} = -\frac{\partial (\dot{h}/h)}{\partial \delta} = -\frac{1}{\delta \{1 + (1/2) \delta - \delta x\}^2} > 0.$$  

Equation (4.10) implies that as $\delta$ increases, $\dot{\psi}$ also increases. This indicates that the larger $\delta$ is, the more it is likely it is that $B''$ will be positive; i.e., as $\delta$ increases, the possibility that $B(T_x)$ becomes convex increases.
Graphically, in figure 1, it implies that as the density function $h(x)$ moves from $C_1$ to $C_2$ to $C_3$, the $B(T_x)$ trajectory moves from $C_1'$ to $C_2'$ to $C_3'$. All these mean that the larger $\delta$ is, it is more plausible that 'Total only' payment scheme is a better approximation to the optimum than 'Partial also' linear payment scheme. This result is intuitively appealing. Suppose that $\delta$ is very large. Recall that $\hat{T} < 0$. Then, there will be a sharp decrease in the frequency with which $T_x$ is chosen as it gets smaller. If this decline in frequency is sufficiently large, the relative importance of furthering risk spreading as opposed to providing appropriate incentives may increase as $T_x$ decreases. This

![Figure 1](image)

**FIGURE 1**

HEALTH DAMAGE DISTRIBUTION AND INSURANCE PAYOFF CURVE
will require that $B(T_x)$ steepen as $T_x$ decreases, implying that $B(T_x)$ is convex (equivalently, $B''(T_x) > 0$).

Another major factor affecting the shape of the optimal insurance payoff with imperfect information turns out to be the income effect. To see this, let us move to the marginal condition for optimum work hours (equation (4.1)). The condition is

$$\frac{\partial U}{\partial T_x} = 0 \Rightarrow U_M \cdot (B'(T_x) + W) - U_L = 0.$$ 

This equation is equivalent to

$$B'(T_x) = \left( \frac{U_L}{U_M} \right) - W$$ 

Taking the total differential of it yields

$$B''(T_x) \cdot \dot{T} = \frac{(U_{LM} \cdot \dot{M} + U_{LL} \cdot \dot{L}) \cdot U_M - U_L \cdot (U_{MM} \cdot \dot{M} + U_{ML} \cdot \dot{L})}{U_M^2}$$

$$= \frac{(U_M \cdot U_{LM} - U_L \cdot U_{MM}) \cdot \dot{M}}{U_M^2} - \frac{(U_L \cdot U_{ML} - U_M \cdot U_{LL}) \cdot \dot{L}}{U_L^2}$$ \hspace{1cm} (4.11)

where $B''(T_x) = \frac{\partial B'(T_x)}{\partial T_x}$, the second derivative of $B(T_x)$.

Notice that in equation (4.11) the first parenthesized term $(U_M \cdot U_{LM} - U_L \cdot U_{MM})$ determines the sign of the income effect in the conventional labor supply problem. If leisure is a normal good, this term is positive. On the other hand, the second parenthesized term $((U_L \cdot U_{ML} - U_M \cdot U_{LL}) \cdot \dot{L})$ determines the sign of the income effect in the consumption decision problem. It is positive if the consumption good is normal one.

Thus, the shape of insurance payoff curve $B(T_x)$ depends on the income effects and the sign of $\dot{T}$, $\dot{M}$, $\dot{L}$. Recall that it has been
verified previously in this chapter that $\dot{T} < 0$ and $\dot{M} < 0$. But the sign of $\dot{L}$ is theoretically ambiguous. Let us consider a simple case first in order to get more insights into it. If the income effect in the consumption decision problem is zero, then $B''(T_x)$ in (4.11) is reduced to

$$B''(T_x) \cdot \dot{T} = \frac{(U_M \cdot U_{LM} - U_L \cdot U_{MM}) \cdot \dot{M}}{U_M^2}$$

It is easy to see from the above equation that $B''(T_x) > 0$ as long as leisure is a normal good, since $\dot{T} < 0$, $\dot{M} < 0$ and $(U_M \cdot U_{LM} - U_L \cdot U_{MM}) > 0$ in the case of normal good. It implies that the insurance payoff curve is convex in this case.

Next, suppose that the income effect in the labor supply problem is zero. Then the $B''(T_x)$ in equation (4.11) is reduced to

$$B''(T_x) \cdot \dot{T} = -\frac{(U_L \cdot U_{ML} - U_M \cdot U_{LM}) \cdot \dot{L}}{U_M^2}$$

This equation itself suggests that the sign of $B''(T_x)$ depends on the sign of $\dot{L}$ since $\dot{T}$ is unambiguously negative and $(U_M \cdot U_{ML} - U_M \cdot U_{LM})$ is positive as long as the consumption good is a normal good. But, in fact, the sign of $B''(T_x)$ and the sign of $\dot{L}$ are simultaneously determined just as $B(T_x)$ and $T_x$ (or $L_x$) are. Again the property of health damage distribution is a crucial factor. Recall that in the previous analysis we assumed the following linear health damage distribution;

$$h(x) = \gamma - \delta x.$$
The analytical result was that if $\delta$ is very large, then $B''(T_x) > 0$, implying that the insurance payoff curve is convex. The intuitive reason was displayed. In this case, the above equation suggests that $\dot{L} > 0$, indicating that leisure increases with health damage at the optimum.

C. Optimal linear insurance scheme based on
the actual earnings loss

The optimal insurance payoff based upon the actual earnings loss can take a convex, linear, or concave form depending upon the shape of health damage distribution $h(x)$, disability wage function $f(x)$, work disutility function $g(x)$, and the utility functional form. The conditions under which $B(T_x)$ takes linear, convex or concave form can be derived, but the form is too complicated to be useful for comparative dynamics analysis. To put the problem in more tractable way, let us deal with linear insurance payoff scheme and derive the optimal form explicitly.

An analysis of the linear structure is appealing for several reasons:\footnote{In research on incentive scheme, similar form of linear incentive scheme is widely used. For examples, see Berhold(1971), and Stiglitz(1975).} (1) As mentioned, it is simple to handle, leading to some fruitful results and (2) under a certain conditions, the optimal insurance scheme based upon the actual earnings loss is in fact linear.

Let us first consider the following simple case:
(1) health damage affects only the wage rate. It decreases proportionately with health damage (that is, \( f(x) = (1-x) \)). Both work disutility and total available time are constant across health states.

(2) the wage rate without health damage is 1.

(3) the insurance payoff is a linear function of the actual earning loss, that is,

\[
B(T_o-(1-x)T_x) = D + \gamma (E_o - E_x) = D + \gamma (T_o - (1-x)T_x)
\]

where \( E_o \) is labor earnings without health damage, \( E_x \) is actual labor earnings at \( x \), \( T_o \) is work hours without health damage, and \( T_x \) is work hours at \( x \). In this scheme \( D \) is a constant component of benefits, paid to all injured workers independent of labor earnings loss. In this simple case it is also independent of the severity of injury. As a matter of fact, \( D \) is a simple form of benefits, based on physical impairment. The term \( \gamma (T_o - (1-x)T_x) \) is another component of benefits, based on actual earnings loss. The parameter \( \gamma \) is compensation ratio.

Now the problem is to find the optimizing \( \gamma \) and \( D \). In this analysis, utility function takes the following general form:

\[
U = U(M_x, T_x)
\]

where \( M_x \) is total income, including insurance benefits, and \( T_x \) is work hours. Total income at \( x \) is

\[
M_x = D + T_o \cdot \gamma + (1-x) \cdot (1-\gamma) \cdot T_x \quad (4.12)
\]

Utility at \( x \) is

\[
U = U(D + T_o \cdot \gamma + (1-x) \cdot (1-\gamma) \cdot T_x, T_x) \quad (4.13)
\]

We have the following break-even constraint;

\[
\int_0^1 \{D + T_o \cdot \gamma - (1-x) \cdot \gamma \cdot T_x\} \, dx = 0
\]
\[ D = -T_0 y + y E((1-x)T_x) \]  \hspace{1cm} (4.14)

where \( E(.) \) is expected value of \( . \) with respect to \( x \).

Substituting (4.14) into (4.12), we have the following income expression reflecting the break-even constraint:

\[ M = (1-x)(1-\gamma)T_x + \gamma E((1-x)T_x) \]  \hspace{1cm} (4.15)

I wish first to derive the indirect utility function based on the utility function (4.13) and the income expression (4.15). In the utility function (4.13), the first-order condition with respect to \( x \) is

\[ U_M(M_x, T_x) = 0 \]  \hspace{1cm} (4.16)

Since \( M_x \) is a function of \( D \) and \( y \) in (4.15), we obtain the following general expression for labor supply from the condition (4.16):

\[ T_x = T(D, y, x). \]

Substituting this into the utility function (4.13), we have the following indirect utility function;

\[ U = U(D + T_0 y + (1-x)(1-\gamma)T(D,y,x), T(D,y,x)) \]  \hspace{1cm} (4.17)

The insurance authority's problem is assumed to be the maximization of the injured worker's expected utility with respect to two policy parameters, \( \gamma \) and \( D \), under a break-even constraint. This maximization problem is reduced to

\[ \max \ EU(D + T_0 y + (1-x)(1-\gamma)T(D,y,x), T(D,y,x)) \]

subject to \( D = -T_0 y + y E((1-x)T(D,y,x)) \) \hspace{1cm} (from (4.14))

One of the marginal conditions for this problem can be expressed as

\[ E\left\{ U_M T_x' \right\} + \gamma E((1-x)T_x') + U_T T_2' = 0 \]  \hspace{1cm} (4.18)

where \( T_2' = \partial T(D,y,x)/\partial \gamma \).
Plugging equation (4.16) into (4.18), we obtain the following reduced expression:

$$ E(U_m - (T_0 - (1-x)T_x)) + \lambda \cdot (-T_0 + E((1-x)T_x) + \gamma \cdot E((1-x)T_2^2)) = 0 \quad (4.19) $$

The other marginal condition is

$$ E(U_m \cdot (1 - T_0 + ((1-x)(1-x)T_x')) + U_T \cdot T_1') + \lambda \cdot (-1 + \gamma \cdot E((1-x)T_1')) = 0 \quad (4.20) $$

Plugging equation (4.16) into (4.20), we can obtain the following reduced expression:

$$ E(U_m) + \lambda \cdot (-1 + \gamma \cdot E((1-x)T_1')) = 0 \quad (4.21) $$

From equation (4.19) and (4.21), we obtain the following expression of optimum condition:

$$ [1 - \gamma \cdot E((1-x)T_1')] \cdot [E(U_m \cdot (T_0 - (1-x)T_x))] \\
= [E(U_m)] \cdot [T_0 - E((1-x)T_x) - \gamma \cdot E((1-x)T_2')] \quad (4.22) $$

Basically, the optimum $\gamma$ should be derived from the optimum condition (4.22). But utilizing the Taylor expansion for approximation, and making further manipulations, we can obtain the following convenient expression for the optimum $\gamma$:

$$ \gamma = 1 + \left[ \eta^* / A \cdot S^2 \cdot E((1-x)T_x^2) \right] \quad (4.23) $$

Here, $\eta^* = E[(d(1-x)T_x/d\gamma) \cdot \gamma \cdot E((1-x)T_x)]$, the expected value of the elasticity of labor earnings with respect to compensation ratio $\gamma$. It reflects the labor response to a change in the compensation ratio. It has a negative sign in general. Thus, if the labor response to compensation ratio is larger, benefit component based on the actual earnings loss should be smaller.

---

$^{22}$Stiglitz (1975) solved the similar kind of problem to this in his incentive scheme literature. I use the same procedure he took in order to obtain the approximation of $\gamma$. 
The parameter \( A = - \frac{U''(x)}{U'(x)} \), a measure absolute risk-averse. Thus, the more risk averse the worker, the larger should be the benefit component based on the actual earnings loss. The parameter \( S^2 \) is the coefficient of variation of labor earnings. The optimum \( D \) is obtained in the following form:

\[
D = \left[ E((1-x)T_x - T_0) \right] \cdot \left[ 1 + \left( \frac{\eta^*}{(E((1-x)T_x) \cdot A \cdot S^2)} \right) \right]
\]

Let us take another simple case where the wage rate and work disutility are constant over all health states, but total available hours decrease proportionately with health damage (that is, the health effect is treated as available time, the "hours loss" model.) The utility function is assumed to take a Cobb-Douglas form. Further, let us suppose that \( B(T_x) \) takes the following linear form:

\[
B(T_x) = K + \delta \cdot T_x
\]

Now the problem is to derive the optimizing \( \delta \) and \( K \). The procedures for deriving the results are similar to the previous case.

The results are

\[
\delta = W \cdot \left[ \frac{\eta}{(R \cdot S^2)} \right] - 1 \quad (4.24)
\]

\[
K = \left( -\delta \right) \cdot \left( \frac{\eta}{(A \cdot S^2)} \right) + \bar{M}_x
\]

where \( \eta = \frac{dT_x}{d\delta}(\delta/T_x) \), which reflects the labor response to a change in compensation rate. (We may notice that \( \frac{dT_x}{d\delta}(\delta/T_x) = \frac{dT_x}{dW}(W/T_x) \), for total income in this case is denoted by \( K + (\delta + W)T_x \). The parameter \( S^2 \) is the coefficient of the variation of work hours, \( R \) is a measure of relative risk-averse, \( A \) is a measure of absolute risk-averse, and \( \bar{M}_x \) and \( \bar{T}_x \) are average income and average work hours, respectively.)
In sum, it is easy to see from the results, (4.23) and (4.24), that two major factors in the determination of each component of benefits are the labor response to the system and the degree of risk aversion. That is, the larger the labor response to the system, the smaller the benefit component based on the actual earnings loss should be. Secondly, the more risk-averse the worker, the larger the benefit component based on actual earnings loss should be.

D. Comparison of performances of 'Total only' payment scheme and 'Partial also' payment scheme: A simulation result

As we discussed above, in social insurance programs in the United States, two kinds of indirect measures are use for the economic costs of true health damage; the actual earnings loss and a measure of physical impairment. The use of indirect measures for the true health damage generally brings about two consequences; labor distortions and insurance payoff (or insurance benefit) distortions.

Labor distortion is defined as the difference between work hours trajectory under the full-optimum insurance and that under the insurance scheme based upon the indirect measure for true health damage. Labor distortion per se has its own meaning; if one scheme generates bigger labor distortion than the other, then we can say that social welfare costs induced by the former one is bigger than the other one. Thus, the smaller the labor distortions, the better the insurance benefit scheme is, other things being equal.
On the other hand, insurance payoff distortion is defined as the difference between the insurance payoff trajectory under the full-optimum insurance and that under the insurance benefit scheme in question. We can say that the loss is 'underinsured' at health state x when benefit level at x under a insurance scheme in question is lower than that under the full-optimum insurance, and vice versa. In general case, under a given benefit scheme, in some range of state x, the loss is underinsured, and in the other range the loss is overinsured. We can expect that this underinsured and overinsured status is the main sources of welfare loss of the worker induced by using indirect measure, instead of using true health damage, in the insurance scheme with imperfect information.

In the appendix, I show the relationship between benefit distortions and expected utility achieved under given benefit scheme. Utilizing that relationship, it is shown that in a certain set of benefit schemes, expected utility and benefit distortions have a direct relations so that we can compare expected utilities only by referring to benefit distortions. In the other set of benefit schemes, it is verified that expected utility achieved under given benefit scheme has the lower bound and the upper bound which can be expressed by certain benefit distortions.

Let us examine the sources of these distortions in alternative insurance benefit schemes. As we verified in section (B) of chapter IV, when the insurer constructs the insurance payoff depending upon the actual earnings loss, the problem of moral hazard occurs. The moral hazard problem arises basically because the worker has control over labor earnings through work hours. In this instance, Pareto-optimal
risk sharing is generally precluded, because it will not induce the proper incentives for making appropriate labor supply decisions. As a consequence, a program operating based on the actual earnings loss under imperfect information distorts the labor supply decision. Insurance payoff in the moral hazard situation is also deviated from the full-optimum one.

Let us turn our attention to the other case where the insurer employs the physical impairment to measure the true health damage which is not observed. When the insurer constructs the insurance payoff function based on physical impairments, he may face both the labor distortions and insurance benefit distortions even though they do not result from moral hazard problem. In this case the labor distortions result from income effects on leisure demand. Let us take an example to describe this. For simplicity, let us assume that wage rate and work disutility are constant across health states, and utility takes a Cobb-Douglas functional form

\[ U = (B(x) + WT_x)\alpha \cdot (H_0 \cdot (1-x) - T_x)^\beta \]

In this instance, the full-optimum insurance payoff trajectory \( B(x) \), and work hours \( T_x \) are, respectively,

\[ B(x) = W \cdot x - (1/2) \cdot \theta \cdot W \]
\[ T_x = (\alpha / \alpha + \beta) \cdot H_0 \cdot (1-x) - (\beta / \alpha + \beta) \cdot (B(x) / W). \]

Now we assume that the insurer cannot monitor the true \( x \), but use an imperfect measure for this. Let this measure be denoted by \( \hat{x} \), which is

\[ \hat{x} = x + \epsilon \]

where \( x \) is a continuous variable representing an abstract form of the physical impairments, \( \epsilon \) is a random variable indicating an error in
measurement. If the insurance structure stayed the same, the insurance payoff trajectory and work hours are described as

\[ B^*(x) = W \cdot \hat{x} - (1/2) \cdot \theta \cdot W \]

\[ T_x = (\alpha/\alpha + \beta) \cdot H_0 \cdot (1 - x) - (\beta/\alpha + \beta) \cdot (B(x)/W). \]

Further, insurance benefit distortion and labor distortion would be, respectively,

\[ B(x) - B^*(x) = W \cdot (x - \hat{x}) \]

\[ T - T = -(\beta/(\alpha + \beta)W) \cdot (B(x) - B^*(x)) = -(\beta/\alpha + \beta) \cdot (x - \hat{x}) \]

Thus, we could recognize the insurance benefit distortions and labor distortions in both cases of the physical impairments and the actual earnings loss measure. These two distortions can be used to evaluate the performances of alternative insurance schemes under imperfect information.

As we discussed above, there are two basic forms of insurance benefit schemes in the workers' compensation programs in the United States; 'impairment' benefit scheme and 'actual earnings loss' payment scheme. In the former case disability benefits are based solely on the fact that the worker suffered work-related injury. In the latter case the benefits are paid on the difference in the disabled worker's earnings before and after the injury. We can identify another version of 'actual earnings loss' payment scheme in social security disability insurance. Social security disability insurance, however, limits payment to total disability only. We will call it 'Total only' case. In that program disability benefits are paid only if the worker has not worked for earnings for five months. On the other hand, workers'
compensation programs compensate for partial as well as total disability. We will call it 'Partial also' case.

In this section, the performances of the two versions of insurance benefit scheme based on actual earnings loss are compared in the light of benefit distortions and labor distortions generated by them. In section (C) of the next chapter V, the performance of benefit schemes based on actual earnings loss is compared with that of benefit scheme based on physical impairments.

Let us describe the full-optimum insurance scheme with a specific utility structure as a first step to compare the performances of the 'Total only' payment scheme and 'Partial also' payment scheme. I assume that the government program runs at a loss (subsidizes the worker). A constant $C$ denotes the amount of subsidy. Utility takes a specific form in which there is no income effect in labor supply problem;

$$U = U(B(x) + Wf(x) \cdot T_x - (R/2) \cdot g(x) \cdot T_x^2).$$

where $f(x)$ is the 'disability wage function' and $g(x)$ is the 'work disutility function' in our maintained assumptions. Let us assume that $f(x) = (1-x)^\alpha$ and $g(x) = (1-x)^{-\beta}$

where $0 < \alpha$ and $0 < \beta$. For simplicity, health damage distribution is assumed to take a uniform distribution.

With these assumptions, the individual's work hours are

$$T_x = (W/R) \cdot (1-x)^{\alpha+\beta} \quad (4.25)$$

The full-optimum insurance benefit scheme is obtained from the following optimizing problem;
The optimizing problem (4.26) can be solved in the same procedure as we did in section (C) of chapter III. Insurance payoff trajectory turns out to be

\[ B(x) = C + \left( \frac{(2x+\beta)(1-\theta)+1}{2(2x+\beta+1)} \right) \left( \frac{W^2}{R} \right) - \left( \frac{W^2}{2R} \right) \cdot (1-x)^{2x+\beta} \]  

(4.27)

Equation (4.27) and (4.25) show us the ideal insurance benefit trajectory and the corresponding work hours respectively.

Benefit schemes under which only total disability benefits are allowed are classified as 'Total only' case, and those under which partial disability benefits are also allowed belong to 'Partial also' case. I consider first labor distortion in both cases. Next, benefit distortions will be derived.

**Labor distortions**

In 'Total only' case, as long as the injured worker earns some labor income, no disability benefits are paid. If he does not work, a fixed amount of benefits are paid. In this situation, when he works, his utility at state \( x \) can be expressed as

\[ U = U(W \cdot (1-x)^x \cdot T_x - (R/2) \cdot (1-x)^{-\beta} \cdot T_x^2) \]

His optimum work hours are

\[ T = (W/R) \cdot (1-x)^{(x+\beta)} \]
Therefore his utility, when working, is

\[ U_h = U(\frac{W^2}{R}) \cdot (1-x)^{2\alpha+\beta} - (\frac{W^2}{2R}) \cdot (1-x)^{2\alpha+\beta} \]

If he does not work, he can obtain a fixed disability benefits \( K \), achieving utility

\[ U_n = U(K) \]

The decision rule states that if \( U_h < U_n \), the worker will choose not to work. It can be more specifically characterized as: leave the work place if \( U((\frac{W^2}{2R}) \cdot (1-x)^{2\alpha+\beta}) < U(K) \).

Let \( x^* \) is a state at which 'working' and 'not working' are indifferent to the injured worker under the benefit scheme; i.e., \( x^* \) satisfies

\[ U((\frac{W^2}{2R}) \cdot (1-x)^{2\alpha+\beta}) = U(K) \]

Since the utility function is monotonically increasing in its argument, \( x^* \) can be obtained from the following equality;

\[ (\frac{W^2}{2R}) \cdot (1-x)^{2\alpha+\beta} = K \quad \text{and} \quad 0 < x < 1 \]

Thus,

\[ x^* = 1 - \left[K \cdot (\frac{2R}{W^2})\right]^{1/(2\alpha+\beta)} \]

Therefore, the decision rule implies that if \( x > x^* \), the injured worker will leave the labor force. Or in this case,

if \( x > 1 - \left[K \cdot (\frac{2R}{W^2})\right]^{1/(2\alpha+\beta)} \), he leaves the labor force.

Now let us turn our attention to the value of \( K \). The break-even constraint in this case is

\[ \theta \cdot \int_{x^*}^{1} K \, dx = C \]

Based on this break-even constraint and the value of \( x^* \), we can obtain the value of \( K \) such as follows:

\[ K = \left[\left(\frac{C}{\theta}\right) \cdot \left(\frac{2R}{W^2}\right)\right]^{-1/(2\alpha+\beta)} \cdot \left[2\alpha+\beta/2\alpha+\beta+1\right]^{2\alpha+\beta/2\alpha+\beta+1} \quad (4.28) \]

Therefore, the decision rule implies that: leave the labor force if \( x > 1 - \left[\left(\frac{C}{\theta}\right) \cdot \left(\frac{2R}{W^2}\right)\right]^{1/(2\alpha+\beta+1)} \)
From the above results, we can obtain work hours trajectory under this scheme such as follows;

\[
T = \frac{W}{R} \quad \text{if } x = 0
\]

\[
= \left(\frac{W}{R}\right) \cdot (1-x)^{a+\beta} \quad \text{if } 0 < x < x^* \quad (4.29)
\]

\[
= 0 \quad \text{if } x^* < x < 1
\]

We can also obtain the probability of labor force withdrawal. We know that if \( x > x^* \), the worker would leave labor force, and we assume that health damage distribution \( h(x) = 1 \) From these, the probability of labor force withdrawal \( P \) is

\[
P = \int_{x^*}^{1} dx = \left[ x \right]_{x^*}^{1} = 1 - x^* = \left(\frac{C}{\theta} \cdot \left(\frac{2R}{W^2}\right)\right)^{1/2a+\beta+1}
\]

This implies that as the government subsidy to the worker in this program \( C \) increases, the probability of non labor participation also increases.

Let us now turn our attention to labor distortions. Labor distortion (denoted by \( LD \)) at each state is defined as the difference between work hours under the full-optimum insurance scheme and those under the benefit scheme in question; i.e., \( LD = T_x^* - T_x \), where \( T_x^* \) is work hours at \( x \) under the full-optimum insurance, and \( T_x \) under the benefit scheme in question. In our case labor distortion \( LD \) can be expressed as

\[
LD = 0 \quad \text{when } 0 < x < x^*
\]

\[
= \left(\frac{W}{R}\right) \cdot (1-x)^{a+\beta} \quad \text{when } x^* < x < 1 \quad (4.30)
\]

where \( x^* = 1 - \left(\frac{C}{\theta} \cdot \left(\frac{2R}{W^2}\right)\right)^{1/2a+\beta+1} \)

Now we define the aggregate labor distortions (denoted by \( ALD \)) as the sum of squared labor distortions; i.e., \( ALD = \int_0^1 (T_x^* - T_x)^2 \, dx \).
In 'Total only' case, the aggregate labor distortions can be expressed as

\[ ALD = [(W/R) \cdot (\alpha + \beta + 1)] \cdot [(C/\theta) \cdot (2R/W^2)]^{[\alpha + \beta + 1/2\alpha + \beta + 1]} \quad (4.31) \]

The aggregate labor distortion reflects the dead weight loss, the social welfare costs associated with the benefit scheme in question; i.e., the larger ALD, the larger social welfare costs on average.

Let us turn our attention to the 'Partial also' case. In this case, benefits are paid based on the difference in the injured worker's earnings before and after injury. In this situation, the disability benefits can be expressed as

\[ B(E) = (W \cdot T_0 - W \cdot (1-x)\alpha \cdot T_x) \cdot \gamma \]

where WT₀ and W(1-x)Tₓ are labor earnings before and after injury, respectively. The parameter γ is the compensation ratio determined by the compensation authority and E denotes the actual earnings loss.

Under this scheme, the worker's utility at each state x is

\[ U = U(\gamma \cdot W \cdot T_0 + (1-\gamma) \cdot W \cdot (1-x)\alpha \cdot T_x - (R/2) \cdot (1-x)\beta \cdot T_x^2) \]

His work hours can be obtained from the following first-order condition;

\[ U'(.) \cdot [(1-\gamma) \cdot W \cdot (1-x)^\alpha - R \cdot (1-x)^{-\beta} \cdot T_x^2)] = 0. \]

Thus, \( T = W/R \) if \( x = 0 \)

\[ = (W/R) \cdot (1-\gamma) \cdot (1-x)^{[\alpha + \beta]} \quad \text{if} \ 0 < x < 1 \quad (4.32) \]

It is easy to see that under this scheme there is no explicit non-labor-force participation.
But work hours at each state under this scheme are lower than those under the full-optimum insurance scheme. From equation (4.25) and (4.32), we can obtain the following labor distortion; at \( x \)

\[ \text{LD} = \gamma \cdot \left( \frac{W}{R} \right) \cdot \left( 1 - x \right)^{\alpha + \beta} \quad \text{where} \quad 0 < x < 1. \tag{4.33} \]

Let us now turn our attention to the value of \( \gamma \). The break-even condition in this case is

\[ \theta \cdot \int_0^1 B(E) \, dx = 0 \quad \Rightarrow \quad \theta \cdot \int_0^1 \left( \frac{W}{R} - W \left( 1 - x \right)^\alpha \cdot T_x \right) \cdot \gamma \, dx = C. \]

Substituting from (4.32) and solving for \( \gamma \) yields

\[ \gamma = \frac{-\left( 2 \alpha + \beta \right) + \sqrt{\left( 2 \alpha + \beta \right)^2 + 4 \left( C/\theta \right) \left( R/W^2 \right) \left( 2 \alpha + \beta + 1 \right)}}{2} \tag{4.34} \]

The aggregate labor distortions in this case are

\[ \text{ALD} = \int_0^1 \left( \gamma \cdot \left( \frac{W}{R} \right) \cdot \left( 1 - x \right)^{\alpha + \beta} \right)^2 \, dx \]
\[ = \gamma^2 \cdot \left( \frac{W}{R} \right)^2 \cdot \frac{1}{2(2 \alpha + 2 \beta + 1)} \tag{4.35} \]

**Benefit distortions**

Benefit trajectory in the 'Total only' case can be easily shown from the previous result. Based on the equation (4.28), we have the following step function representing benefit trajectory in the 'Total only' case;

\[ B = 0 \quad \text{if} \quad 0 < x < x^* \]
\[ = K = \left( \frac{(C/\theta) \cdot (2R/W^2)^{1/2}}{2(2 \alpha + 2 \beta + 1)} \right) \cdot \left( 2 \alpha + \beta / 2 \alpha + \beta + 1 \right) \quad \text{if} \quad x^* < x < 1 \tag{4.36} \]

where \( x^* = 1 - K \).

In the 'Partial also' case, the benefit trajectory takes the following form;

\[ B(E) = \left( \frac{W}{R} - W(1-x)^\alpha \cdot T_x \right) \cdot \gamma \quad \text{if} \quad 0 < x < 1 \tag{4.37} \]

where \( T_0 = \frac{W}{R} \), \( T_x = \frac{W}{R}(1-\gamma)(1-x)^{\alpha + \beta} \), and \( \gamma \) is in equation (4.34).
In the remaining section, using the previous results and assuming plausible values for unknown parameters of the system, benefit and labor trajectory of various schemes are compared. It will show us benefit distortions and labor distortions generated by alternative benefit schemes. Further, total income trajectories, and expected utilities induced by them are compared.

We have five parameters of system; $\alpha$, $\beta$, $R$, $\theta$, and $C$. The following assumptions about them are taking;

(1) $\alpha$ and $\beta$ are assumed to be 1 and 0, respectively.

(2) $R$ is assumed to be 0.1. And $W$ is 1. By doing so $T_0$ is derived as 10, indicating that the worker will work 10 hours and his labor earnings are 10 if he does not suffer from any injury.

(3) $\theta$ is assumed to be 0.2.

(4) $C$ is assumed to be 0.67. From this value of $C$, benefits under the full-optimum insurance are nonnegative for all $x$.

We have two benefit schemes to be compared; 'Total only' payment scheme, and 'Partial also' payment scheme.

Benefit trajectories and aggregate benefit distortions

In the full-optimum insurance, from equation (4.27), benefit trajectory is $B(x) = 5 - 5(1-x)^2$. This benefit trajectory is concave as we expected in subsection (D.2) of chapter III.

In 'Total only' case, from equation (4.29), the worker would leave labor force and receive disability benefits, $K$, when $x > x^*$. In this case, $K = 3.78$ and $x = 0.13$. The benefit trajectory is

$B = 0$ if $x < 0.13$
B = 3.78 if \( x > 0.13 \)

The aggregate benefit distortions (the sum of squared benefit distortions) are

\[ \text{ABD} = 1.23. \]

In 'Partial also' case, from equation (4.34) and (4.37), benefit trajectory is

\[ B(E) = (T_0 - (1-x) \cdot T_x) \cdot \gamma. \]

Thus, in this example, benefit trajectory is

\[ B(E) = 4.16 - 2.43 \cdot (1-x)^2 \text{ and } \gamma = 0.416 \]

The aggregate benefit distortions are

\[ \text{ABD} = 0.56. \]

**Labor trajectory and aggregate labor distortions**

Under the full-optimum insurance, labor trajectory is

\[ T_x = (1-x)/R \]

Thus, in this example, \( T_x = 10 \cdot (1-x) \)

In 'Total only' case, from equation (4.29), labor trajectory is

\[ T = 10 \cdot (1-x) \text{ if } x < 0.13 \]
\[ = 0 \text{ if } x > 0.13 \]

The aggregate labor distortions in this case are, from (4.7)

\[ \text{ALD} = 21.6 \]

In 'Partial also' case, from equation (4.32), \( T_x = (1-x)(1-\gamma)/R. \)

In this case, from equation (4.34),

\[ \gamma = 0.416 \text{ and } T_x = 5.84 \cdot (1-x) \]

The aggregate labor distortions in this example are, from (4.35)

\[ \text{ALD} = 13.1 \]
Total income trajectory and expected utility

Total income is composed of labor earnings and disability benefits. Let $M_x$ denote total income at $x$; i.e., $M_x = B(x) + (1-x)T_x$.

In the full-optimum insurance, from equation (4.27) and (4.25), total income is

$$M_x = 5 + 5 \cdot (1-x)^2.$$

In 'Total only' case, from equation (4.28), total income is

$$M_x = \frac{(1-x)^2}{R} \text{ if } x < x^*$$
$$= K \text{ if } x^* < x.$$

In this case, $M_x = 10 \cdot (1-x)^2$ if $x < 0.13$
$$= 3.78 \text{ if } x > 0.13$

In the 'Partial also' case, from equation (4.32), (4.34) and (4.37), total income is

$$M_x = 4.16 + 3.41(1-x)^2.$$

Expected utility achieved under each scheme is computed by taking the following strictly concave utility function;

$$U = \sqrt{M - \frac{(R/2)T^2}.$$ 

All the above results are summarized in Table 2.

As we expected, expected utility achieved under the full-optimum scheme is higher than those under the other schemes which are based on actual earnings loss. Notice that the loss is 'underinsured' at the higher level of $x$ and 'overinsured' at the lower level of $x$ when one or another benefit scheme is taken, which is mentioned above. This 'underinsured' and 'overinsured' status is the main sources of welfare loss induced by using the imperfect measure.
Aggregate labor distortions are, as we expected, higher in the case of 'Total only' than in the case of 'Partial also'. Aggregate benefit distortions are also higher under the 'Total only' payment scheme. Expected utility is higher in the case of 'Partial also' scheme. Thus, this specific example shows us that the 'Partial also' payment scheme is more desirable than the 'Total only' payment scheme.

TABLE 2
PERFORMANCES OF 'TOTAL ONLY' PAYMENT SCHEME AND 'PARTIAL ALSO' PAYMENT SCHEME

<table>
<thead>
<tr>
<th>Benefit scheme</th>
<th>Benefit Trajectory and Distortions</th>
<th>Labor Trajectory and Distortions</th>
<th>Total Income Trajectory</th>
<th>Expected Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal scheme</td>
<td>( B_x = 5 - 5(1-x)^2 )</td>
<td>( T_x = 10(1-x) )</td>
<td>( M_x = 5 + 5(1-x)^2 )</td>
<td>EU = 2.24</td>
</tr>
<tr>
<td>'Total only' scheme</td>
<td>( B = 0 ) if ( x&lt;0.13 )</td>
<td>( T = 10(l-x) ) if ( x&lt;0.13 )</td>
<td>( M = 10(l-x)^2 ) if ( x&lt;0.13 )</td>
<td>EU = 1.96</td>
</tr>
<tr>
<td></td>
<td>( B = 3.78 ) if ( x&gt;0.13 )</td>
<td>( T = 0 ) if ( x&gt;0.13 )</td>
<td>( M = 3.78 ) if ( x&gt;0.13 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ABD = 1.23</td>
<td>ALD = 21.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>'Partial also' scheme</td>
<td>( B = 4.16 - 2.43(1-x)^2 )</td>
<td>( T = 5.84(1-x) )</td>
<td>( M = 4.16 + 3.41(1-x)^2 )</td>
<td>EU = 2.16</td>
</tr>
<tr>
<td></td>
<td>( 0 &lt; x &lt; 1 )</td>
<td>( 0 &lt; x &lt; 1 )</td>
<td>( 0 &lt; x &lt; 1 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ABD = 0.56</td>
<td>ALD = 5.77</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: ABD = Aggregate Benefit Distortions
ALD = Aggregate Labor Distortions
EU = Expected Utility Calculated from Utility Function \( U \)
\[ U = \sqrt{NM - (R/2)T^2} \]
A. Introduction

In current social insurance programs, including workers' compensation, a commonly used indirect indicator of the economic losses resulting from health damages is a measure of physical impairment. This measure is presumably a proxy for the expected economic losses due to injury. This expected loss approach to benefits may be efficient from the point of view of work incentives; once such a benefit is determined, it is not reduced because of the amount of actual earnings. Monitoring costs are also low. One serious problem of this approach, however, lies in the fact that benefit schedule based on physical impairments may have only a crude relationship to economic losses due to injury; i.e., the benefits may not cover actual, insurable losses.

In this chapter, I construct first a simple 'impairment' model illustrating the essence of an insurance scheme based on physical impairments under imperfect information. In section (B), I describe the insurance scheme based on a measure of 'impairment', comparing and contrasting it to the full-optimum insurance scheme with perfect information. In section (C), I compare the performance of the benefit scheme based on physical impairments with that based on actual earnings loss.
B. Insurance benefit scheme based on physical impairments

Suppose that the insurer cannot observe directly the true health damage \( x \) under imperfect information, but that he can observe some physical impairments or functional limitations due to injury. Further, suppose that he combines these observed impairments and constructs a measure of 'impairment', which is a continuous variable. The insurer uses this 'impairment' measure to obtain 'expected' health damage. For simplicity and without loss of generality, assume that he calculates 'expected' health damage as a linear function of 'impairment' measure.

More formally, the insurer is assumed to calculate the 'expected' health damage \( \hat{x} \) in the following manner:

\[
\hat{x} = \delta + Ky
\]

where \( \hat{x} \) is 'expected' health damage, \( y \) is the 'impairment' measure, and \( \delta \) and \( K \) are constants. Further, true health damage \( x \) and the 'expected' health damage \( \hat{x} \) have the following relationship:

\[
x = \hat{x} + \epsilon
\]

where \( \epsilon \) denotes "measurement" error. For a given measurement technology, \( \epsilon \) is assumed to be a random variable with variance \( \sigma^2 \). Variance \( \sigma^2 \) can presumably be reduced by an improvement in measurement technology.

\[\text{23This approach is similar to the "whole man" approach currently used in workers' compensation benefit schemes of several states. In practice, this approach requires an evaluation of the seriousness of the permanent consequences of the injury compared to a "whole man". The whole man assessment considers impairment and functional limitations in determining rating.}\]
Thus, the relationship between true health damage and 'impairment' measure in this framework is expressed as

\[ x = \delta + Ky + \varepsilon. \]

Other assumptions and notations in chapter III are maintained in this chapter.

Let us describe first the insurance scheme based on this 'impairment' measure. Since the insurer can observe only 'expected' health damage, he must construct the insurance payoff as a function of 'expected' health damage instead of true health damage. Thus, the insurance payoff function is expressed as \( B(\hat{x}) \). In this instance, the (constrained) contract design is for the insurer with limited information first to construct the full-optimum insurance as a function of true health damage \( x \), and then to substitute the 'expected' health damage \( \hat{x} \) for true health damage \( x \), ignoring the random factor \( \varepsilon \) which is unobservable as well as unavoidable for a given technology of measurement. In this case, of course, the break-even constraint in any individual case may not be satisfied because of the randomness of "measurement" error \( \varepsilon \).

Thus, the insurance payoff under the benefit scheme based on the 'impairment' measure takes the same functional form as that under the full-optimum scheme except that the argument is \( \hat{x} \) instead of \( x \). Therefore, when the full-optimum insurance payoff is linear in \( x \), the insurance payoff based on the 'impairment' measure should also be linear in \( \hat{x} \), and vice versa. All the results of analyses on the shape of insurance payoff under the full-optimum scheme can be applied to the
shape of insurance payoff function based on the 'impairment' measure. Particularly, in the "hours loss" model where injury affects only available hours, $B(x)$ should be linear in $x$.

Let us examine some properties of this benefit scheme based on the 'impairment' measure. The insurance payoff in each health state under this scheme deviates from that under the full-optimum scheme whenever "measurement" error $\epsilon$ is nonzero. Further, when a utility structure involves an income effect on leisure demand, work hours at each health state also deviates from those under the full-optimum scheme. The magnitudes of these distortions depend on the technology of measurement and on the utility structure.

To show these more explicitly, let us consider the "hours loss" model. In this instance, as we derived in section (D.1) of chapter III, the insurance payoff under the full-optimum scheme is linear in $x$, expressed as

$$B(x) = K + W H_0 \cdot x$$

where $K$ is a constant. On the other hand, the insurance payoff based on 'impairment' measure should be

$$B(x) = K + W H_0 \cdot x.$$ 

Thus, the benefit distortion is

$$B(x) - B(x) = W H_0 \cdot (x - x)$$

$$= W H_0 \cdot \epsilon.$$ 

The expected value of the sum of squared benefit distortions is

$$E[B(x) - B(x)]^2 = (WH_0)^2 \cdot \sigma^2.$$
Thus, it is obvious that an improvement in the impairment measurement (a smaller $\sigma^2$) can reduce the aggregated benefit distortions on average.

Work hours may also be distorted by the 'impairment' measure through income effects. The post-disability effective wage rate will be unaffected by this system since benefits are based on 'expected', not actual earnings. These distortions cannot be derived from a general utility functional form. It is obvious, however, that if a utility structure involves no income effect on leisure demand, there is no labor distortions. Now let us take a Cobb-Douglas utility structure, in which the income effect on leisure demand exists, in order to show how the "measurement" error $\epsilon$ as well as the utility structure affect labor distortions.

The utility function then is

$$U = H^{a} \cdot L^{\beta}.$$ 

In this case, the ideal insurance payoff and work hour are, respectively,

$$B(x) = (-1/2) \cdot \Theta \cdot WH_0 + WH_0 \cdot x$$

$$T^*_X = (\alpha/\alpha+\beta) \cdot H_0(1-x) - (\beta/\alpha+\beta) \cdot \{B(x)/W\}$$

$$= \left[ \frac{\alpha + (1/2) \cdot \theta \cdot \beta}{\alpha + \beta} \right] \cdot H_0 - H_0 \cdot x.$$ 

On the other hand, under an insurance scheme based on the 'impairment' measure, the payoff and work hours schedules are, respectively,

$$B^{\hat{}}(x) = (-1/2) \cdot \Theta \cdot WH_0 + WH_0 \cdot \hat{x}$$

$$= (-1/2) \cdot \Theta \cdot WH_0 + WH_0 \cdot (x-\epsilon)$$

$$\hat{T}_X = (\alpha/\alpha+\beta) \cdot H_0(1-x) - (\beta/\alpha+\beta) \cdot \{B^{\hat{}}(x)/W\}$$

$$= \left[ \frac{\alpha + (1/2) \cdot \theta \cdot \beta}{\alpha + \beta} \right] \cdot H_0 - H_0 \cdot x + (\beta/\alpha+\beta) \cdot H_0 \cdot \epsilon.$$
From these, benefit distortions (BD) and work hours distortions (LD) are derived as:

\[ \text{BD} = B^*(x) - B(x) = WH_o \cdot \epsilon \]  

\( (5.1) \)

\[ \text{LD} = T^*_x - T_x = (-\beta/\alpha + \beta) \cdot \frac{\{(B(x) - B(x))^2\}}{W} \]

\[ = (-\beta/\alpha + \beta) \cdot H_o \cdot \epsilon. \]  

\( (5.2) \)

The benefit distortion in (5.1) is equivalent to what we have derived in the general utility functional form; obviously, an improvement in the technology of measurement reduces these distortions on average.

The expression for labor distortions in (5.2) suggests that if the insurer overestimates the health damage (negative \( \epsilon \)), then work hours under the scheme based on 'impairment' measure are less than those under the full-optimum scheme, and vice versa. Expected value of aggregate labor distortions with respect to \( \epsilon \) is expressed as:

\[ E(T^*_x - T_x)^2 = \left[ (\beta/\alpha + \beta) \cdot H_o \right]^2 \cdot \sigma^2. \]

Thus, an improvement in measurement technology (a smaller \( \sigma^2 \)) reduces labor distortions on average. The parameter \( (\beta/\alpha + \beta) \) in the utility structure also affects the size of the labor distortions. If the individual values leisure relatively heavily, labor distortions will be larger under this scheme.

Conversely when the 'expected' health damage is lower than the true health damage (positive \( \epsilon \)), total income, leisure and utility level are lower than the full-optimum levels. But as we derived, work hours are greater than those under the full-optimum levels.
C. Comparison of performances of 'impairment' payment scheme and 'actual earnings loss' payment scheme: A simulation result

As we discussed above, in social insurance programs in the United States, two kinds of indirect measures are used for the economic costs of true health damage; the actual earnings loss and 'impairment' measure. The use of indirect measures for the true health damage generally brings about two consequences; labor distortions and insurance benefit distortions. As we defined above, labor distortions are the differences between the work hours trajectory under the full-optimum insurance and that under the insurance scheme based on the indirect measure. Insurance benefit distortions are the differences between the insurance payoff trajectory under the full-optimum scheme and that under the benefit scheme in question.

Let us examine the sources of these distortions in alternative insurance benefit schemes. As we verified in section (B) of chapter IV, when the insurer constructs the insurance payoff depending upon the actual earnings loss, the problem of moral hazard occurs. The moral hazard problem arises basically because the worker has control over labor earnings through work hours. In this instance, Pareto-optimal risk sharing is generally precluded, because it will not induce the proper incentives for making appropriate labor supply decisions. As a consequence, a program based on actual earnings losses under imperfect information distorts the labor supply decision. The insurance payoff in the moral hazard situation also deviates from the full-optimum.
Let us turn our attention to the case in which the insurer employs a physical impairment indicator to measure the unobserved true health damage. Recall that, when the insurer constructs the insurance payoff function based on physical impairments, he may also face both labor distortions and insurance benefit distortions, although they do not result from moral hazard problems. In this case the labor distortions result from income effects of insurance benefits on work behavior.

In this section I compare the performances of the 'impairment' payment scheme with those of 'actual earnings loss' payment scheme (especially 'Partial also' case) based on labor distortions and benefit distortions. A particular point of focus will be the effect of \( \sigma^2 \), the variance of "measurement" error under the 'impairment' payment scheme, on relative performance.

Consider the "hours loss" model with utility taking a Cobb-Douglas form:

\[
U = H^\alpha \cdot L^\beta.
\]

The government is assumed to subsidize the worker by the amount \( C \).

Under these assumptions, the full-optimum benefit trajectory \( B^*_x \) and work hours \( T^*_x \) are, respectively,

\[
B^*_x = C/\theta - (1/2)WH_0 + WH_0 \cdot x
\]

\[
T^*_x = -H_0 \cdot x - (\beta/\alpha+\beta)(1/\theta)(C/\theta) + (\alpha+0.5\beta/\alpha+\beta) \cdot H_0
\]

Under the 'impairment' payment scheme, \( x \) is replaced with \( \hat{x} \):

\[
\hat{x} = x + \varepsilon \quad \Rightarrow \quad \hat{x} = x - \varepsilon
\]

Under this scheme, the benefit trajectory \( B^*_x \) and work hours \( T^*_x \) are, respectively,
\[ B(x) = \left( \frac{C}{9} \right) - \left( \frac{1}{2} \right) WH_0 + WH_0 \cdot x - WH_0 \cdot \varepsilon \quad (5.5) \]

\[ T_x = \left( \frac{\alpha}{\alpha + \beta} \right) \cdot H_0 (1-x) - (\beta/\alpha + \beta)(1/\overline{W}) \cdot B(x) \quad (5.6) \]

Conversely, under the 'actual earnings loss' payment scheme, the benefit function \( B(E) \) is expressed as:

\[ B(E) = (W T_0 - W T_x) \cdot \gamma \]

Under this scheme, benefit trajectory \( B^a(x) \) and work hours \( T^a_x \) are, respectively,

\[ B^a(x) = \gamma \cdot \left[ (\alpha/\alpha + \beta)(\beta/\alpha + \beta)(\gamma/1-\gamma) \cdot WH_0 (1-x) \right] \quad (5.7) \]

\[ T^a_x = \left( \frac{\alpha}{\alpha + \beta} \right) \cdot H_0 (1-x) - (\beta/\alpha + \beta)(\alpha/\alpha + \beta)(\gamma/1-\gamma) \cdot H_0 \quad (5.8) \]

Here the compensation ratio \( \gamma \) satisfies the following budget constraint;

\[ \gamma \cdot \int_0^1 B^a(x) \, dx = C. \]

Thus, \( \gamma \) is derived as:

\[ \gamma = - \left( \frac{1}{2} \right) \cdot \left( \frac{C}{\theta} \right) \cdot \left\{ \left( \frac{\alpha + \beta}{\alpha} \right)^2 / \alpha \beta \right\} \left( 2/H_0 \right) \]

\[ + \left( \frac{1}{2} \right) \cdot \sqrt{\left( \frac{C}{\theta} \right) \cdot \left\{ \left( \frac{\alpha + \beta}{\alpha} \right)^2 / \alpha \beta \right\} \left( 2/H_0 \right) + 4 \cdot \left( \frac{C}{\theta} \right) \cdot \left\{ \left( \frac{\alpha + \beta}{\alpha} \right)^2 / \alpha \beta \right\} \left( 2/H_0 \right)} \]

**Benefit Distortions**

Under the 'impairment' payment scheme, the benefit distortion \( (BD_1) \), derived from (5.3) and (5.5), is expressed as:

\[ BD_1 = B(x) - B^*(x) = WH \cdot \varepsilon \]

The corresponding aggregate benefit distortions \( (ABD_1) \) can be expressed as the sum of squared deviations of benefits from the full-optimum, namely

\[ ABD_1 = \int_0^\infty [B(x) - B^*(x)]^2 \, dx = (WH_0)^2 \cdot \sigma^2 \quad (5.9) \]

On the other hand, under the 'actual earnings loss' payment scheme, benefit distortion \( (BD_a) \) is, from (5.3) and (5.7),
\[ BD_a = B^i(x) - B^o(x) \]
\[ = C/\theta - (1/2)WH_0 + WH_0 \cdot x - \gamma \cdot [(\alpha/\alpha + \beta)(\beta/\alpha + \beta)(\gamma/1 - \gamma) \cdot WH_0(1 - x)] \]

The corresponding aggregate benefit distortions (ABD) are

\[ ABD_a = \int_0^1 (BD_a)^2 \, dx \] (5.10)

Now let us compare the aggregate benefit distortions under the two schemes. In order to do so, we assume reasonable values for unknown parameters of the system. We have six parameters: \( \theta, \alpha, \beta, W, H_0, \) and \( \gamma. \)

1. \( \theta \) is assumed to be 0.2
2. both of \( \alpha \) and \( \beta \) are assumed to be 0.4
3. \( W \) is assumed to be 1
4. \( H_0 \) is assumed to be 20
5. \( \gamma \) is assumed to be 0.7.

From these assumptions, \( C = 0.82. \) Work hours without health damage are 10.

Under these assumptions, aggregate benefit distortions under the 'impairment' payment scheme is, from (5.9),

\[ ABD_i = 400 \cdot \sigma^2 \]

On the other hand, those under the 'actual earnings loss' payment scheme are, from (5.10),

\[ ABD_a = 65.3 \]

Thus, if \( \sigma^2 < 0.16, \) then aggregate benefit distortions under the 'impairment' payment scheme are less than those under the 'actual earnings loss' payment scheme. Or equivalently, if \( \sigma < 0.4, \) then the 'impairment' payment scheme induce less aggregate benefit distortions than the 'actual earnings loss' payment scheme. In order to get more
insights into this result, suppose that \( \epsilon \) has normal distribution and zero mean. Then the result implies that if the "measurement" error is less than 0.4 with the 68\% frequency level, then the 'impairment' payment scheme is better than the 'actual earnings loss' payment scheme as far as benefit distortions are concerned.

As a matter of fact, the critical value of \( \sigma \) is sensitive to the size of subsidy \( C \) (or equivalently, the compensation ratio \( \gamma \)). In the simulation, I consider variations in the compensation ratio \( \gamma \) (and the corresponding \( C \)). The results are in Table 3. As we see in table 3, the critical value of \( \sigma^2 \) increases with \( \gamma \) (or equivalently \( C \)). It implies that, as the size of the subsidy increases, the 'impairment' payment scheme appears to perform better than the 'actual earnings loss' payment scheme as far as benefit distortions are concerned. The intuitive reason is that, as the size of subsidy increases, work hours under the 'actual earnings loss' payment scheme decrease more than under the 'impairment' payment scheme due to the fact that the benefits paid depend on work hours, thereby inducing greater benefit distortions.

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( C )</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.25</td>
<td>0.11</td>
</tr>
<tr>
<td>0.6</td>
<td>0.45</td>
<td>0.13</td>
</tr>
<tr>
<td>0.7</td>
<td>0.82</td>
<td>0.16</td>
</tr>
<tr>
<td>0.8</td>
<td>1.6</td>
<td>0.27</td>
</tr>
<tr>
<td>0.9</td>
<td>4.05</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Labor distortions

Under the 'impairment' payment scheme, the labor distortion ($L_{D_1}$) is expressed as: from (5.4) and (5.6),

$$L_{D_1} = T^*_x - T_x = -(\beta/\alpha+\beta) \cdot H_0 \cdot e$$  \hspace{1cm} (5.11)

The corresponding aggregate labor distortions ($A_{LD_1}$) are

$$A_{LD_1} = \int_{-\infty}^{\infty} [T^*_x - T_x]^2 \, d\epsilon = (\beta/\alpha+\beta)^2 \cdot H_0 \cdot \sigma^2$$  \hspace{1cm} (5.12)

On the other hand, under the 'actual earnings loss' payment scheme, the labor distortion ($L_{D_a}$) is, from (5.4) and (5.8),

$$L_{D_a} = -(\beta/\alpha+\beta)(C/\theta) + (1/2)(\beta/\alpha+\beta) \cdot H_0 \\
+ (\beta/\alpha+\beta)(\alpha/\alpha+\beta)(\gamma/\gamma-\gamma) \cdot H_0 - (\beta/\alpha+\beta) \cdot H_0 \cdot x$$  \hspace{1cm} (5.13)

The corresponding aggregate labor distortions ($A_{LD_a}$) are the sum of squared labor distortions in (5.13). Let us take the same assumptions on parameters as in the benefit distortion analysis;

(1) $\theta = 0.2$  (2) $\alpha = \beta = 0.4$  (3) $W = 1$  (4) $H_0 = 20$  (5) $\gamma = 0.7$.

From these assumptions, $C = 0.82$.

Under these assumptions, aggregate labor distortions under the 'impairment' payment scheme are, from (5.12),

$$A_{LD_1} = 100 \cdot \sigma^2$$

On the other hand, those under the 'actual earnings loss' payment scheme are, from (5.13),

$$A_{LD_a} = 97.6$$

Thus, if $\sigma^2 < 0.98$, then aggregate labor distortions under the 'impairment' payment scheme are less than those under the 'actual earnings loss' payment scheme. Or equivalently, if $\sigma < 0.99$, the 'impairment' payment scheme induces less aggregate labor distortions.

In order to get more insights into the result, suppose that $\epsilon$ is
normally distributed and has zero mean. Then the result implying that if the "measurement" error is less than 0.99 with the 68% frequency level, then the 'impairment' payment scheme is better than the 'actual earnings loss' payment scheme as far as labor distortions are concerned. Recall that the range of $x$ is 1.0. This value of $\sigma$, therefore, is high enough for any crude 'impairment' measure to achieve. As one would expect, payment schemes independent of actual earnings loss are likely to have much lower labor supply disincentives. Notice that this critical value of $\sigma^2$ is much higher than that in the case of benefit distortion analysis. If the actual earnings scheme is to be optimal, it is likely to be based on its better "benefit" performance.

The following simulation results in table 4 also show that the critical value of $\sigma^2$ is sensitive to the size of subsidy $C$ (or equivalently the compensation ratio $\gamma$). The results show us that the critical value of $\sigma^2$ increases with the size of $C$ (or equivalently $\gamma$). This implies, once again, that as the size of subsidy increases, it is plausible that the 'impairment' payment scheme is better one than the 'actual earnings loss' payment scheme.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\gamma = 0.5$</th>
<th>$\gamma = 0.6$</th>
<th>$\gamma = 0.7$</th>
<th>$\gamma = 0.8$</th>
<th>$\gamma = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C = 0.25$</td>
<td>$C = 0.45$</td>
<td>$C = 0.82$</td>
<td>$C = 1.6$</td>
<td>$C = 4.05$</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.27</td>
<td>0.49</td>
<td>0.98</td>
<td>6.84</td>
<td>12.25</td>
</tr>
</tbody>
</table>

TABLE 4
THE CRITICAL VALUE OF $\sigma^2$ IN LABOR DISTORTION ANALYSIS
The critical values of $\sigma^2$ in labor distortion comparison are much higher than those in benefit distortion comparison, given any size of subsidy. This indicates that the 'impairment' payment scheme is relatively more advantageous in labor distortion considerations than in benefit distortion considerations.

For $C$ over 0.82 ($\gamma$ is over 0.7), $\sigma^2$ becomes over 1, implying that labor distortions under the 'impairment' payment scheme are definitely less than those under the 'actual earnings loss' payment scheme since 'expected' health damage cannot be greater than 1.

In sum, simulation results, both in benefit distortions and in labor distortion analyses, show us that

(1) as the size of subsidy increases, it is more plausible that the 'impairment' payment scheme performs better than the 'actual earnings loss' payment scheme, and

(2) the 'impairment' payment scheme is relatively more advantageous when the insurer is concerned about labor distortions than about benefit distortions.
SUMMARY AND CONCLUSION

The main results I have obtained in this study may be summarized by displaying some implications on the design of workers' compensation benefit scheme, derived in this study:

(1) Under limited information on the economic losses of health damage, the optimal benefit scheme based on 'actual earnings loss' is in general nonlinear as we derived in chapter IV. In practice, two approximations to the optimum are currently use in social insurance programs; a linear 'Partial' payment scheme and a 'Total only' payment scheme. I compared in chapter IV how well these two alternative benefit schemes approximated the optimum program, using analytical and simulation methodologies.

In the analytical part, I scrutinized first the curvature of the optimal insurance benefit program as a function of actual earnings losses. If this function is severely convex over most of its domain, then a 'Total only' scheme may be a reasonable approximation to the optimum. Conversely, if the optimal benefit curve is linear or less curved, the linear 'Partial' payment scheme may be a better approximation to the optimum.

We can consider the full-optimum insurance scheme with perfect information as a benchmark for this comparison. In this case, it turns out (in chapter III) that the type of injury is a major factor determining the curvature of the optimal insurance benefit function.
When the injury affects mainly the individual’s available hours, for example, through increased maintenance requirements, the optimal benefit function is linear in health damage. In this case, it is also linear in actual earnings losses. When the injury affects mainly the worker’s productivity (his market wage rate), however, the optimal benefit function is in general nonlinear in health damage. The function is linear in actual earnings losses if either (i) there is no income effect on labor supply or (ii) the individual’s preferences are strongly separable in consumption and leisure.

In the imperfect information case, additional factors affect the ideal structure and in particular the curvature of the optimal insurance benefit function. They are (i) the distribution of health damage (ii) the income effect and (iii) the degree of risk-averseness. An analytical result suggests that if the frequency of severe health damage is 'much' lower than that of lesser damages, then it is plausible that the 'Total only' payment scheme is a better approximation to the optimum. Another analytical result is that, if the income effect on consumption good demand is close to zero, then the optimal benefit function is convex. But it is hard to figure out how severely convex it is. The degree of convexity of the best program under these conditions is difficult to assess in general.

I therefore consider an administratively simple linear benefit scheme (in section (C) of chapter IV) in this environment. It turns out that the insurance benefits associated with actual earnings losses will rapidly decline as labor earnings increase in the optimal program when the degree of risk-averseness is large. This suggests that the
'Total only' payment scheme is plausibly a better approximation to the optimum when the individual is severely risk-averse.

In general, however, in an actual earnings loss program, the linear 'Partial' payment scheme will be a 'better' approximation to the optimum. This is supported by a simulation in section (D) of chapter IV. In that simulation, important aspects of the performance of the two alternative schemes are compared, namely the benefit distortions, labor distortions, and expected utilities achieved under the two schemes. The simulation results reveal that both of benefit and labor distortions are smaller under the linear 'Partial' payment scheme than under the 'Total only' payment scheme under plausible parameter assumptions. Expected utility is also higher under the linear 'Partial' payment scheme than under the 'Total only' scheme over the range of parameters considered.

(2) In 'impairment' based insurance programs, the quality of the performance of the program, of course, depends on the quality of the measurement of the impairment and its relationship to the economic losses of the health damage. Labor supply distortions will not exist in the impairment scheme if income effects on labor supply are zero. Otherwise, it turns out (in section (B) of chapter V) that labor supply is directly related to the direction of "measurement" error. If the insurer overestimates the health damage, then work hours under this benefit scheme are less than the full-optimum level, and vice versa. When the "measurement" error is very large, it is even possible that the labor distortions under the insurance benefit scheme based on
physical impairment are larger than those under the scheme based on actual earnings loss.

In a linear system (section (C) of chapter IV), the analysis suggests that less risk-averse the worker and the more response to the system is his labor supply, the more appropriate the benefit scheme based on physical impairment only.

Simulation results (in section (C) of chapter V), both in benefit distortion and in labor distortion analyses, reveal that: (i) as the size of government subsidy to the worker increases, it is more plausible that the 'impairment' payment scheme performs better than the 'actual earnings loss' payment scheme, and (ii) the 'impairment' payment scheme is relatively more advantageous when the insurer is concerned about labor distortions than about benefit distortions.
APPENDIX

RELATIONSHIP BETWEEN BENEFIT DISTORTIONS AND EXPECTED UTILITY

ACHIEVED UNDER GIVEN BENEFIT SCHEME

In the previous chapter V, we discussed and expected that there exist relationship between benefit distortions and expected utility achieved under benefit schemes in question. In this appendix, that relationship is more explicitly displayed, utilizing approximation method.

For simplicity, the following assumptions are taken; 
\[ \alpha = 1, \beta = 0, \text{ and } W = 1. \]

Also we replace \( (R/2) \) with \( R \) in the utility function. Then we have the following form of utility function under the compensation program:

\[ U = U(B(x) + (1-x)T_x - RT_x^2). \]

Let \( \Phi(x) = (B(x)+(1-x)T_x - RT_x^2) \). Then we have

\[ U = U(\Phi(x)). \]

As we derived in Example 1 in section (D.2) of chapter III, the function \( \Phi(x) \) becomes constant with the full-optimum insurance; i.e.,

\[ \Phi(x) = B(x)^* + (1-x)T_x^* - RT_x^* = k, \] a constant.

where \( B(x)^* \) is benefit level and \( T_x^* \) is work hours at \( x \), respectively under the full-optimum insurance scheme.

What I am going to do first is to obtained an approximation of the following expected utility achieved under any arbitrary insurance
benefit scheme represented by \( B_x \);

\[
EU = \int_0^1 U(B_x + (1-x) \cdot T_x - R \cdot T_x^2) \, dx = EU(\Phi(x)) \tag{6.1}
\]

We have three clues to an appropriate Taylor expansion:

- Form break-even constraint, \( \int_0^1 f'(B) \, dx = \int_0^1 B_x \, dx = C/\theta \tag{6.2} \)
- From definition, \( \Phi(x) = B_x + (1-x) \cdot T_x - R \cdot T_x^2 \tag{6.3} \)
- From the marginal condition, \( B(x)^* + (1-x) \cdot T_x^* - R \cdot T_x^{*2} = k \tag{6.4} \)

where \( C \) is the amount of government subsidy to the worker and \( k \) is a constant.

Utilizing Taylor expansion of the utility function \( U(\Phi(x)) \) at \( k \), and incorporating (6.3) and (6.4), and taking integral, we have the following result;

\[
EU(\Phi(x)) = U(k) + U'(k) \cdot \int_0^1 \left\{ (B - B(x))^* + \left[ (1-x)T_x^*-R \cdot T_x^{*2} - (1-x)T_x^*-R \cdot T_x^{*2} \right] \right\} dx
\]

Substituting from (6.2) yields

\[
EU(\Phi(x)) = U(k) + U'(k) \cdot \int_0^1 \left\{ (1-x)T_x^*-R \cdot T_x^{*2} - (1-x)T_x^*-R \cdot T_x^{*2} \right\} dx
\]

\[
+ \frac{1}{2} \cdot U''(k) \cdot \int_0^1 \left\{ (B - B(x))^* \cdot \left[ (1-x)T_x^*-R \cdot T_x^{*2} - (1-x)T_x^*-R \cdot T_x^{*2} \right] \right\} dx
\]

where \( \{B - B(x)^*\} \) indicates benefit distortion at \( x \).

In equation (6.5), we can find some interesting relationship between benefit distortions and expected utility. Let us define
'aggregate benefit distortions' as the square sum of benefit distortion and denoted by (ABD). More formally,

\[ ABD = \int_0^1 \{B_x - B(x)^*\}^2 \, dx. \]

To show this, let us consider first any two benefit scheme both of which generate no labor distortions. In this case, we can directly compare two different expected utilities achieved under the two schemes only by referring to the aggregate benefit distortions induced by them. This due to the fact that when \( T_x = T_x^* \) for all \( x \) (no labor distortion), from equation (6.5), \( \mathbb{E}U(\Phi(x)) \) is reduced to

\[ \mathbb{E}U(\Phi(x)) = U(k) + \frac{1}{2} \cdot U''(k) \cdot \int_0^1 \{B_x - B(x)^*\}^2 \, dx \quad (6.6) \]

where \( \int_0^1 \{B_x - B(x)^*\}^2 \, dx \) indicates the aggregate benefit distortions. Notice that \( U''(k) \) is negative. Thus, in equation (6.7), it is easy to see that the bigger the aggregate benefit distortions, the smaller the expected utility. This is the simple relationship derived in this case where benefit schemes induce no labor distortions.

The typical case to which this simple rule is applied is the 'impairment' payment case where the insurance benefits are independent of labor earnings loss (or equivalently, labor supply). We can also derived the difference between the expected utility achieved under the full-optimum insurance and that under the benefit scheme based on the physical impairment from equation (6.6). Recall that the expected utility under the full-optimum insurance is \( U(k) \). Thus, the difference between the two expected utilities is, from (6.6)

\[ \Delta \mathbb{E}U = \mathbb{E}U(\Phi(x)) - U(k) \]

\[ = \frac{1}{2} \cdot U''(k) \cdot \int_0^1 \{D - B(x)^*\}^2 \, dx \]
where \( D \) is a constant insurance benefits paid under the 'impairment' payment scheme. Furthermore,

\[
-\Delta \text{EU}/U'(k) = (1/2) \cdot \{ -U''(k)/U'(k) \} \cdot \int_0^\infty \{ D - B(x)^k \}^2 \, dx
\]

\[
= (1/2) \cdot A \cdot (ABD)
\]

where \( A \) is a measure of absolute risk-averse, and \( (ABD) \), aggregate benefit distortions.

Now let us turn our attention to the case of 'actual earnings' payoff scheme where there exist some labor distortions so that \( T_x = T_x^* \). In this case, equation (6.5) cannot be simplified directly so that it is difficult to directly compare different expected utilities achieved under the different benefit schemes only by referring to benefit distortions. But we can find an indirect way by which expected utilities can be compared to some degree based on certain benefit distortions.

Specifically, what I am going to do is to obtain a lower bound and upper bound of expected utility achieved under "actual earnings loss" payment scheme. Each bound is expressed as a function of the aggregate benefit distortions.

Upper bound of expected utility

Let \( \{ B_{ax} \} \) be the benefit trajectory and \( \{ T_a(x) \} \) be the work hours trajectory under the 'actual earnings loss' payment scheme. Of course, \( T_a(x) = (1-x)(1-\gamma)/2R \) in this utility structure. Suppose that the worker is provided with \( B_{ax} \) at each health state \( x \), and he works \( T_x^* \) where \( T_x^* = (1-x)/2R \) which is work hours under the full-optimum
insurance. In this imaginary situation, utility level at \( x \) can be expressed as

\[
U_1(x) = U(B_{ax}, T^*_x) = U(B_{ax} + (1-x)T^*_x - RT^*_x^2).
\]

But under the 'actual earnings loss' payment scheme, the achieved utility level at \( x \) is expressed as

\[
U_a(x) = U(B_{ax}, T_{ax}) = U(B_{ax} + (1-x)T_{ax} - RT_{ax}^2)
\]

where \( T_{ax} \) denotes work hours at \( x \) under the 'actual earnings loss' payment scheme.

According to economic theory, \( U_a(x) < U_1(x) \). We have the following heuristic reasoning for this conclusion: in both cases, the same amount of money \( B_{ax} \) is paid to the worker at each state \( x \). But in the latter case where \( T_{ax} = (1-x)(1-\gamma)/2R \), labor supply is distorted, while the former case, not. Thus, at each state, \( U_a(x) < U_1(x) \). This, in turn, ends up with the following relations; \( EU_a(x) < EU_1(x) \).

But by referring to equation (6.5), we know that

\[
EU_1(x) = U(k) + (1/2)\cdot U''(k) \cdot \int_0^1 (B_{ax} - B(x)^*)^2 \, dx
\]

where \( B_{ax} \) is insurance benefits at \( x \) under the 'actual earnings loss' payment scheme and \( B(x)^* \) those under the full-optimum insurance. Therefore we can say that the upper bound of expected utility achieved under 'actual earnings loss' payment scheme is

\[
U(k) + (1/2)\cdot U''(k) \cdot \int_0^1 (B_{ax} - B(x)^*)^2 \, dx \quad (6.8)
\]

**Lower bound**

Suppose that the individual works \( T^*_x = (1-x)/2R \) at \( x \), and benefits are provided to him based on the 'actual earnings loss' payment
formula. In this imaginary situation, benefits at \( x \) can be expressed as

\[
B_{cx} = (T_0 - (1-x)T_x^*) \cdot \rho
\]

where \( \rho \) satisfies the break-even constraint, \( \int B_{bx} \, dx = C/\theta \).

Further, utility level at \( x \) in this situation can be expressed as

\[
U_c(x) = U(B_{cx}, T_x^*) = U(B_c(x) + (1-x)T_x^* - RT_x^*)^2.
\]

But under the 'actual earnings loss' payment scheme, the worker who made the optimal labor supply decision Achieves the following utility;

\[
U_a(x) = U(B_{ax}, T_x^*) = U(B_{ax} + (1-x)T_{ax}^* - RT_{ax}^*)^2.
\]

According to economic theory, \( EU_a(x) > EU_c(x) \). We have the following reasoning for this: In both cases, the worker faces the same benefit scheme. Under this benefit scheme, \( T_{ax} = (1-x)(1-\gamma)/2R \) is the optimal labor supply of the worker, which allows him the highest utility among all the labor supply decision including \( T_x^* = (1-x)/2R \) under this benefit scheme. Therefore \( EU_a(x) > EU_c(x) \).

But by referring to equation (6.5), we have

\[
EU_c(x) = U(k) + (1/2) \cdot U''(k) \cdot \int B_{cx} - B(x)^* \, dx.
\]

Therefore, we can say that the lower bound of expected utility achieved under 'actual earnings loss' payment scheme is

\[
U(k) + (1/2) \cdot U''(k) \cdot \int B_{cx} - B(x)^* \, dx
\]

(6.9)

where \( B_{cx} \) satisfies that \( B_{cx} = [T_0 - (1-x)/2R] \rho \) and \( \int B_{cx} \, dx = C/\theta \).

Comparing expected utility under 'Impairment' payment scheme and that under 'Actual earnings loss' payment scheme

From equation (6.8) and (6.9), we have the following relationship;

\[
U(k) + (1/2) \cdot U''(k) \cdot \int B_{cx} - B(x)^* \, dx < EU_a(x) <
\]
\[ U(k) + \frac{1}{2} U'(k) \cdot \int_a^x \{ B_{\text{ax}} - B(x)^* \}^2 \, dx \quad (6.10) \]

where \( EU_a(x) \) is expected utility achieved under 'actual earnings loss' payment scheme, \( B_{\text{ax}} \) is the benefits at \( x \) associated with scheme, \( B(x)^* \) is that under the full-optimum insurance, and \( B_{\text{cx}} \) is the same as before.

As we mentioned, expected utility achieved under 'impairment' payment scheme can be expressed as

\[ EU(\Phi(x)) = U(k) + \frac{1}{2} U''(k) \cdot \int_a^x \{ D - B(x)^* \}^2 \, dx \quad (6.11) \]

Referring to equation (6.10) and (6.10), we can find a simple rule which could be applied to comparing two expected utilities achieved under 'impairment' payment scheme and under 'actual earnings loss' payment scheme.

The rule is

1. If \( \int_a^x \{ D - B(x)^* \}^2 \, dx < \int_a^x \{ B_{\text{ax}} - B(x)^* \}^2 \, dx \), then expected utility under 'impairment' payment scheme is definitely higher than that under 'actual earnings loss' payment scheme.

2. If \( \int_a^x \{ D - B(x)^* \}^2 \, dx > \int_a^x \{ B_{\text{cx}} - B(x)^* \}^2 \, dx \), then expected utility under 'impairment' payment scheme is definitely lower than that under 'actual earnings loss' payment scheme.

3. If \( \int_a^x \{ B_{\text{ax}} - B(x)^* \}^2 \, dx < \int_a^x \{ D - B(x)^* \}^2 \, dx < \int_a^x \{ B_{\text{cx}} - B(x)^* \}^2 \, dx \), then we cannot compare them only with lower bound and upper bound.
REFERENCES


