MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

University Microfilms Inc.
300 N. Zeeb Road, Ann Arbor, MI 48106
INFORMATION TO USERS

This reproduction was made from a copy of a manuscript sent to us for publication and microfilming. While the most advanced technology has been used to photograph and reproduce this manuscript, the quality of the reproduction is heavily dependent upon the quality of the material submitted. Pages in any manuscript may have indistinct print. In all cases the best available copy has been filmed.

The following explanation of techniques is provided to help clarify notations which may appear on this reproduction.

1. Manuscripts may not always be complete. When it is not possible to obtain missing pages, a note appears to indicate this.

2. When copyrighted materials are removed from the manuscript, a note appears to indicate this.

3. Oversize materials (maps, drawings, and charts) are photographed by sectioning the original, beginning at the upper left hand corner and continuing from left to right in equal sections with small overlaps. Each oversize page is also filmed as one exposure and is available, for an additional charge, as a standard 35mm slide or in black and white paper format.*

4. Most photographs reproduce acceptably on positive microfilm or microfiche but lack clarity on xerographic copies made from the microfilm. For an additional charge, all photographs are available in black and white standard 35mm slide format.*

*For more information about black and white slides or enlarged paper reproductions, please contact the Dissertations Customer Services Department.

UMI Dissertation Information Service

University Microfilms International
A Bell & Howell Information Company
300 N Zeeb Road, Ann Arbor, Michigan 48106
Kennedy, Kevin Francis

A METHOD FOR METAL DEFORMATION AND STRESS ANALYSIS IN ROLLING

The Ohio State University

University Microfilms International

300 N. Zeeb Road, Ann Arbor, MI 48106
PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark ✓.

1. Glossy photographs or pages ____
2. Colored illustrations, paper or print ______
3. Photographs with dark background ✓
4. Illustrations are poor copy ______
5. Pages with black marks, not original copy ______
6. Print shows through as there is text on both sides of page ______
7. Indistinct, broken or small print on several pages ______
8. Print exceeds margin requirements ______
9. Tightly bound copy with print lost in spine ______
10. Computer printout pages with indistinct print ______
11. Page(s) ___________ lacking when material received, and not available from school or author.
12. Page(s) __________ seem to be missing in numbering only as text follows.
13. Two pages numbered ______. Text follows.
14. Curling and wrinkled pages ______
15. Dissertation contains pages with print at a slant, filmed as received ______
16. Other _____________________________

_____________________________________
_____________________________________
_____________________________________

University
Microfilms
International
A METHOD FOR METAL DEFORMATION AND
STRESS ANALYSIS IN ROLLING

Dissertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Kevin Francis Kennedy, B.S.M.E., M.S.M.E.

* * * * *

The Ohio State University
1986

Dissertation Committee:
T. Altan
R. A. Miller
A. Bagchi

Approved by

T. Altan
Adviser
Industrial and Systems Engineering
To Bridget, Tracy, and Jacque,

to my Parents, and to the memory of Maureen
ACKNOWLEDGMENTS

The author would like to offer his warmest thanks to Professor T. Altan, his adviser, for taking the author as an apprentice and providing the inspiration for this work. The author would also like to thank Professors R. A. Miller and A. Bagchi for their many helpful suggestions and encouragement. The support of the Lima Technical College, by donating the use of its computer facilities, is also acknowledged. But most of all, my wife Jacque deserves the credit for this dissertation. Only a lifetime of devotion can begin to express my gratitude for her support during the course of this work.
VITA


1977-1980 . . . . . . . . . . . . . . . Research Assistant, Mechanical Engineering Department, University of Massachusetts, Amherst Massachusetts.

1978 . . . . . . . . . . . . . . . . . . . . B. S., Mech. Engin., University of Massachusetts, Amherst, Massachusetts.

1980 . . . . . . . . . . . . . . . . . . . . M. S., Mech. Engin., University of Massachusetts, Amherst, Massachusetts.

1980-1983 . . . . . . . . . . . . . . . Researcher, Engineering and Manufacturing Technology Department, Battelle Memorial Institute, Columbus, Ohio.

1983-1985 . . . . . . . . . . . . . . . Lecturer, Lima Technical College, Lima, Ohio.

1983-1985 . . . . . . . . . . . . . . . Lecturer, The Ohio State University at Lima, Lima, Ohio.

PUBLICATIONS


FIELDS OF STUDY

Analysis and Design of Metalworking Processes, Professor T. Altan, The Ohio State University and Battelle Memorial Institute.

Computer Applications in Manufacturing, Professor R. A. Miller, The Ohio State University.


Lubrication of Drawing and Extrusion Operations by Solid Coatings, Professor W. R. D. Wilson, University of Massachusetts.
# TABLE OF CONTENTS

ACKNOWLEDGMENTS ............................................. iii

VITA. ......................................................... iv

LIST OF TABLES. ............................................... ix

LIST OF FIGURES ............................................... xii

CHAPTER  page

I.  INTRODUCTION. .............................................. 1

I.1  Objectives and Benefits of This Study ........ 5

I.2  Background on Rolling Mill Design .............. 6

II.  PREVIOUS WORK ............................................. 13

II.1  Empirical Methods for Roll Pass Design .... 13

II.2  Computer-Assisted Roll Pass Design .......... 20

II.3  Theoretical Analyses of the Rolling Process. . . . 28

III.  FLAT ROLLING ANALYSIS ................................. 31

III.1  Metal Deformation Analysis ....................... 31

III.2  Admissible Velocity Fields ......................... 34

III.3  Flat Rolling Process Considered in the Analysis. 35

III.4  Velocity Field Equations ............................. 38
<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>III.5</td>
<td>40</td>
</tr>
<tr>
<td>III.6</td>
<td>42</td>
</tr>
<tr>
<td>III.7</td>
<td>45</td>
</tr>
<tr>
<td>III.8</td>
<td>46</td>
</tr>
<tr>
<td>III.9</td>
<td>47</td>
</tr>
<tr>
<td>III.10</td>
<td>49</td>
</tr>
<tr>
<td>III.11</td>
<td>52</td>
</tr>
<tr>
<td>III.12</td>
<td>54</td>
</tr>
<tr>
<td>III.13</td>
<td>57</td>
</tr>
<tr>
<td>III.14</td>
<td>58</td>
</tr>
<tr>
<td>IV.1</td>
<td>62</td>
</tr>
<tr>
<td>IV.2</td>
<td>65</td>
</tr>
<tr>
<td>IV.3</td>
<td>68</td>
</tr>
<tr>
<td>IV.4</td>
<td>86</td>
</tr>
<tr>
<td>IV.5</td>
<td>90</td>
</tr>
<tr>
<td>IV.6</td>
<td>92</td>
</tr>
<tr>
<td>IV.7</td>
<td>97</td>
</tr>
<tr>
<td>IV.8</td>
<td>103</td>
</tr>
</tbody>
</table>

IV. RESULTS FROM THE FLAT ROLLING ANALYSIS. 62

IV.1 Example Flat Rolling Solution. 62
IV.2 Nomenclature for Bulge Profile and Spread Predictions. 65
IV.3 Comparison Between Predicted and Measured Values in Hot Rolling. 68
IV.4 Comparison Between Predicted and Measured Values in Cold Rolling. 86
IV.5 Preliminary Conclusions from the Flat Rolling Analysis. 90
IV.6 Factors Which Influence Percent Bulge and True Spread. 92
IV.7 Factors Which May Lead to Single-Bulge or Double-Bulge Deformation. 97
IV.8 The Flat Bar Test - A Suggested Technique for Estimating the Interface Friction Conditions in Rolling. 103
<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>V. SHAPE ROLLING ANALYSIS.</td>
<td>111</td>
</tr>
<tr>
<td>V.1 Shape Rolling Process Considered in the Analysis.</td>
<td>112</td>
</tr>
<tr>
<td>V.2 Velocity Field Equations.</td>
<td>116</td>
</tr>
<tr>
<td>V.3 Minimization Variables.</td>
<td>118</td>
</tr>
<tr>
<td>V.4 Strain-Rate Equations.</td>
<td>121</td>
</tr>
<tr>
<td>V.5 Integration of Streamline Equations</td>
<td>122</td>
</tr>
<tr>
<td>V.6 Velocity Boundary Conditions.</td>
<td>125</td>
</tr>
<tr>
<td>V.7 Initial Guess Solution.</td>
<td>132</td>
</tr>
<tr>
<td>V.8 Computer Program SHAPES</td>
<td>133</td>
</tr>
<tr>
<td>VI. RESULTS FROM THE SHAPE ROLLING ANALYSIS</td>
<td>134</td>
</tr>
<tr>
<td>VI.1 Shape Rolling Experiments.</td>
<td>134</td>
</tr>
<tr>
<td>VI.2 Simulation of Square-Oval-Round Pass Sequences.</td>
<td>136</td>
</tr>
<tr>
<td>VI.3 Comparison Between Predicted and Measured Values.</td>
<td>159</td>
</tr>
<tr>
<td>VII. CONCLUSIONS</td>
<td>164</td>
</tr>
<tr>
<td>LIST OF REFERENCES</td>
<td>170</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Example track of minimization variables for rolling of square-rectangle pass as simulated by BILLETS computer program.</td>
<td>64</td>
</tr>
<tr>
<td>2. Key input parameters used in the BILLETS program to obtain the example flat rolling solution given in Table 1.</td>
<td>64</td>
</tr>
<tr>
<td>3. Comparison between theoretical true spread predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 72 experiments).</td>
<td>76</td>
</tr>
<tr>
<td>4. Comparison between theoretical maximum spread predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 71 experiments).</td>
<td>76</td>
</tr>
<tr>
<td>5. Comparison between theoretical total roll torque predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 71 experiments).</td>
<td>77</td>
</tr>
<tr>
<td>6. Comparison between theoretical roll separating force predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 71 experiments).</td>
<td>77</td>
</tr>
<tr>
<td>7. Comparison between theoretical exit width predictions made by the BILLETS computer program and experiments for hot rolling of a 0.04 % carbon steel at 1832 F (ref. 23 experiments).</td>
<td>82</td>
</tr>
<tr>
<td>TABLE</td>
<td>Page</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>8. Comparison between theoretical <strong>exit width</strong> predictions made by</td>
<td>82</td>
</tr>
<tr>
<td>the BILLETS computer program and experiments for hot rolling of</td>
<td></td>
</tr>
<tr>
<td>a 0.04 % carbon steel at 1832 F (ref. 23 experiments)</td>
<td></td>
</tr>
<tr>
<td>9. Theoretical <strong>percent bulge</strong> predictions made by the BILLETS</td>
<td>93</td>
</tr>
<tr>
<td>computer program as a function of friction shear factor and thickness</td>
<td></td>
</tr>
<tr>
<td>reduction</td>
<td></td>
</tr>
<tr>
<td>10. Theoretical <strong>percent bulge</strong> predictions made by the BILLETS</td>
<td>93</td>
</tr>
<tr>
<td>computer program as a function of width/thickness ratio and thickness</td>
<td></td>
</tr>
<tr>
<td>reduction</td>
<td></td>
</tr>
<tr>
<td>11. Theoretical <strong>true spread</strong> predictions made by the BILLETS</td>
<td>95</td>
</tr>
<tr>
<td>computer program as a function of friction shear factor and thickness</td>
<td></td>
</tr>
<tr>
<td>reduction</td>
<td></td>
</tr>
<tr>
<td>12. Theoretical <strong>true spread</strong> predictions made by the BILLETS</td>
<td>95</td>
</tr>
<tr>
<td>computer program as a function of roll diameter/thickness ratio and</td>
<td></td>
</tr>
<tr>
<td>width/thickness ratio</td>
<td></td>
</tr>
<tr>
<td>13. Comparison between theoretical <strong>true spread</strong> predictions made by</td>
<td>96</td>
</tr>
<tr>
<td>the BILLETS computer program and the Battelle SHPROL computer program</td>
<td></td>
</tr>
<tr>
<td>(details of the SHPROL program are given in ref. 71)</td>
<td></td>
</tr>
<tr>
<td>14. Example track of minimization variables for rolling of <strong>square-oval</strong></td>
<td>144</td>
</tr>
<tr>
<td>pass as simulated by SHAPES computer program using <strong>no-slip</strong> subsidiary velocity boundary condition</td>
<td></td>
</tr>
<tr>
<td>(ref. 73 experiments)</td>
<td></td>
</tr>
<tr>
<td>15. Example track of minimization variables for rolling of <strong>oval-round</strong></td>
<td>151</td>
</tr>
<tr>
<td>pass as simulated by SHAPES computer program using <strong>constrained corner</strong></td>
<td></td>
</tr>
<tr>
<td>subsidiary velocity boundary condition (ref. 73 experiments)</td>
<td></td>
</tr>
<tr>
<td>16. Example track of minimization variables for rolling of <strong>round-oval</strong></td>
<td>159</td>
</tr>
<tr>
<td>pass as simulated by SHAPES computer program using <strong>constrained corner</strong></td>
<td></td>
</tr>
<tr>
<td>subsidiary velocity boundary condition (ref. 73 experiments)</td>
<td></td>
</tr>
</tbody>
</table>
TABLE

17. Comparison between theoretical exit cross-sectional area predictions and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 73 experiments)\textsuperscript{a}.

18. Comparison between theoretical roll separating force predictions and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 73 experiments).

\textsuperscript{a}
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A selection of commonly used rolled shape cross-sections.</td>
<td>2</td>
</tr>
<tr>
<td>2. An illustration of billet rolling in a primary mill which utilizes the new round-edged-rectangle rolling method(ref. 1).</td>
<td>3</td>
</tr>
<tr>
<td>3. An illustration of the rolling of bars and structural shapes(The American Iron and Steel Institute, with permission)</td>
<td>4</td>
</tr>
<tr>
<td>4. Alternative roll pass schedules used for rolling a round bar from a square billet.</td>
<td>8</td>
</tr>
<tr>
<td>5. An illustration of double-bulge deformation in flat rolling</td>
<td>11</td>
</tr>
<tr>
<td>6. Geometrical designation and approximation of profile of side free surface in diamond-diamond pass(ref. 30)</td>
<td>17</td>
</tr>
<tr>
<td>7. Equivalent rectangle transformation method to determine the effective heights of both the billet material($H_{0m}$) and roll cross-section ($H_{1m}$) in diamond-diamond pass(ref. 30).</td>
<td>17</td>
</tr>
<tr>
<td>8. Different methods for division of I-beams for roll pass design(ref. 47)</td>
<td>19</td>
</tr>
<tr>
<td>9. Various pass shapes considered by the RPDROD computer program.</td>
<td>22</td>
</tr>
<tr>
<td>10. Three-dimensional display of bar cross-sections in the roll-bite for square to oval pass as simulated by the RPDROD program, the arrow denotes the direction of rolling.</td>
<td>24</td>
</tr>
</tbody>
</table>
FIGURE | Page
---|---
11. Three-dimensional display of bar cross-sections in the roll-bite for oval to round pass as simulated by the RPDROD program, the arrow denotes the direction of rolling. | 25
12. Three-dimensional display of the stress distribution for oval to round pass as simulated by the RPDROD program, the arrow denotes the direction of rolling. | 26
13. Predicted elongation distribution across the width of the section for oval to round pass as simulated by the RPDROD program. | 27
14. Schematic illustration of the flat rolling process being considered in the present investigation. | 36
16. Compressive stresses in plane-strain upsetting between inclined plates and with unit depths, divergent outward flow assumed (ref. 71). | 60
17. Schematic illustration of single-bulge profile and definitions of exit width dimensions with $W/D = 1$, $D/H = 10.8$, and 35.4% reduction. | 67
18. Schematic illustration of double-bulge profile and definitions of exit width dimensions with $W/D = 0.5$, $D/H = 5.2$, and 34.4% reduction. | 67
19. Various AISI 1018 steel plate cross-sections produced under hot rolling conditions by rolling a square billet to various thickness reductions ($W/D = 1$, $D/H = 16$). | 70
20. Comparison between predicted and measured values of true spread, $W_f$, in hot rolling of AISI 1018 plate specimens at 1832 F (ref. 72 experiments). | 71
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>Comparison between predicted and measured values of maximum spread, ( W_m ), in hot rolling of AISI 1018 plate specimens at 1832 F (ref. 71 experiments)</td>
<td>73</td>
</tr>
<tr>
<td>22</td>
<td>Comparison between predicted and measured values of roll torque in hot rolling of AISI 1018 plate specimens at 1832 F (ref. 71 experiments)</td>
<td>74</td>
</tr>
<tr>
<td>23</td>
<td>Comparison between predicted and measured values of roll separating force in hot rolling of AISI 1018 plate specimens at 1832 F (ref. 71 experiments)</td>
<td>75</td>
</tr>
<tr>
<td>24</td>
<td>Example flow stress curves for AISI 1045 steel</td>
<td>80</td>
</tr>
<tr>
<td>25</td>
<td>Comparison between predicted and measured values of maximum spread, ( W_m ), in cold rolling of AISI 1018 plate specimens (ref. 71 experiments)</td>
<td>87</td>
</tr>
<tr>
<td>26</td>
<td>Comparison between predicted and measured values of roll torque in cold rolling of AISI 1018 plate specimens (ref. 71 experiments)</td>
<td>89</td>
</tr>
<tr>
<td>27</td>
<td>Comparison between predicted and measured values of true spread, ( W_f ), in cold rolling of lead (ref. 24 experiments)</td>
<td>91</td>
</tr>
<tr>
<td>28</td>
<td>Influence of thickness reduction on bulge profile</td>
<td>99</td>
</tr>
<tr>
<td>29</td>
<td>Influence of billet thickness on bulge profile</td>
<td>100</td>
</tr>
<tr>
<td>30</td>
<td>Bulge profile design chart developed from the theoretical analysis showing the combined influences that width/thickness ratio, roll diameter/thickness ratio, and thickness reduction have on the predicted bulge profile type</td>
<td>102</td>
</tr>
<tr>
<td>31</td>
<td>Bulge profile design chart determined from the results of actual production rolling trials with steel (ref. 16, Courtesy Association of Iron and Steel Engineers)</td>
<td>103</td>
</tr>
<tr>
<td>32</td>
<td>Flat bar test theoretical calibration curves with ( W_b/H_b = 1.0 ) and ( D/H_b = 16.0 )</td>
<td>105</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>33</td>
<td>Flat bar test theoretical calibration curves with $W_b/H_b = 0.5$ and $D/H_b = 16.0$</td>
<td>108</td>
</tr>
<tr>
<td>34</td>
<td>Schematic illustration of the shape rolling process being considered in the present investigation</td>
<td>113</td>
</tr>
<tr>
<td>35</td>
<td>Predicted metal flow distribution and neutral surface for rolling of an airfoil section as predicted using modular upper-bound method</td>
<td>120</td>
</tr>
<tr>
<td>36</td>
<td>Billet and roll surface configurations for the rolling of a rectangle through diamond pass showing where no-slip conditions are imposed</td>
<td>127</td>
</tr>
<tr>
<td>37</td>
<td>Representation of combined convergent and divergent metal flow patterns for an airfoil when the direct thickness reduction begins near the outside corners of the billet entry cross-section</td>
<td>127</td>
</tr>
<tr>
<td>38</td>
<td>Representation of divergent metal flow pattern for an airfoil when the direct thickness reduction begins near the center of the billet entry cross-section</td>
<td>130</td>
</tr>
<tr>
<td>39</td>
<td>Various passes available on the grooved rolls used for validating the shape rolling analysis</td>
<td>135</td>
</tr>
<tr>
<td>40</td>
<td>Schematic illustration of relative locations in the deformation zone for different deforming billet cross-sections</td>
<td>137</td>
</tr>
<tr>
<td>41</td>
<td>Entry billet cross-section for square-oval pass, Pass No. 4 in Fig. 39</td>
<td>138</td>
</tr>
<tr>
<td>42</td>
<td>Predicted deforming billet cross-section at $x/X_d = 0.25$ in square-oval pass</td>
<td>139</td>
</tr>
<tr>
<td>43</td>
<td>Predicted deforming billet cross-section at $x/X_d = 0.50$ in square-oval pass</td>
<td>140</td>
</tr>
<tr>
<td>44</td>
<td>Predicted deforming billet cross-section at $x/X_d = 0.75$ in square-oval pass</td>
<td>141</td>
</tr>
<tr>
<td>FIGURE</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>45. Predicted billet exit cross-section in square-oval pass (entry section superimposed with dashed line)</td>
<td>142</td>
<td></td>
</tr>
<tr>
<td>46. Entry billet cross-section for oval-round pass, Pass No. 5 in Fig. 39.</td>
<td>146</td>
<td></td>
</tr>
<tr>
<td>47. Predicted deforming billet cross-section at $x/X_d = 0.25$ in oval-round pass</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>48. Predicted deforming billet cross-section at $x/X_d = 0.50$ in oval-round pass</td>
<td>148</td>
<td></td>
</tr>
<tr>
<td>49. Predicted deforming billet cross-section at $x/X_d = 0.75$ in oval-round pass</td>
<td>149</td>
<td></td>
</tr>
<tr>
<td>50. Predicted billet exit cross-section in oval-round pass (entry section superimposed with dashed line)</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>51. Predicted billet exit cross-section in oval-round pass using no-slip condition (entry section superimposed with dashed line, billet exit section line segments crossing the roll section were omitted for clarity)</td>
<td>153</td>
<td></td>
</tr>
<tr>
<td>52. Entry billet cross-section for round-oval pass, Pass No. 6 in Fig. 39.</td>
<td>154</td>
<td></td>
</tr>
<tr>
<td>53. Predicted deforming billet cross-section at $x/X_d = 0.25$ in round-oval pass</td>
<td>155</td>
<td></td>
</tr>
<tr>
<td>54. Predicted deforming billet cross-section at $x/X_d = 0.50$ in round-oval pass</td>
<td>156</td>
<td></td>
</tr>
<tr>
<td>55. Predicted deforming billet cross-section at $x/X_d = 0.75$ in round-oval pass</td>
<td>157</td>
<td></td>
</tr>
<tr>
<td>56. Predicted billet exit cross-section in round-oval pass (entry section superimposed with dashed line)</td>
<td>158</td>
<td></td>
</tr>
<tr>
<td>57. Actual rolled shape cross-sections produced in Battelle rod rolling experiments (ref. 73) for Pass Nos. 4, 5, and 6.</td>
<td>162</td>
<td></td>
</tr>
<tr>
<td>58. Square-oval pass which produces pronounced hour-glass-bulge when smaller roll diameter is used (ref. 12)</td>
<td>162</td>
<td></td>
</tr>
</tbody>
</table>
59. Actual deforming billet cross-sections determined at different locations in the deformation zone for oval-round pass (ref. 73)................. 163
CHAPTER I
INTRODUCTION

Rolling is a deformation process in which metal is elongated through several passes into various shapes. Fig. 1 shows a selection of commonly used rolled shape cross-sections. These shapes comprise essentially all the metal feedstock used in basic manufacturing industries. A thorough knowledge of the metal deformation and stresses in rolling is necessary in order to optimize the design of the roll pass schedule, which is the sequence of rolling mill operations required to produce a given shape.

There are many types of rolling mills used in industry, and a specialized terminology has developed to describe them. The term primary mill has gained wide acceptance as a generic term to cover the sizing of semi-finished elongated articles called billets and slabs. Billets are shapes which tend to have square cross-sections, and slabs are shapes which are always rectangular and tend to be relatively wide. An illustration of billet rolling in a primary mill using flat grooveless rolls, a tandem no-twist rolling mill, is shown in Fig. 2. Once produced, billets and slabs are often cooled, stored, and eventually rolled in
Figure 1. A selection of commonly used rolled shape cross-sections.
other mills, or forged. However, it is possible, and often economical, to feed billets and slabs directly into finishing mills. Fig. 3 shows a variety of finishing mills which are used in the rolling of bars and structural shapes. Although there is a wide variety of commonly used rolled shape cross-sections, Fig. 1, the principle of the rolling process for producing different finished shapes is the same.
In rolling the billet is drawn into the roll gap by the friction between the plastically deforming billet and the rotating rolls. The reduction in the pass, or compression of the billet, is accommodated in spread and elongation. Spread is the metal deformation in the lateral direction, and elongation is the metal deformation in the direction of rolling. Thus, complex three-dimensional metal deformation takes place. Although spread may be only a fraction of elongation, the ratio between spread and elongation varies at each pass with the reduction, billet and roll shape geometries, roll diameter, roll speed, roll surface finish,
and the billet material and temperature.

Because the factors influencing the rolling process are so intermixed and complex, the practice of designing the roll pass schedule has traditionally been a purely empirical, intuitive and experienced-based technology. Through decades of experience roll pass schedules have been established for most commonly used shapes, Fig. 1, and from conventional materials. However, for new shapes and new materials, and for high rolling speeds, extensive experimentation is necessary in order to establish a roll pass schedule for a new product. The development of a method for analyzing the complex three-dimensional metal deformation and stresses in rolling, providing information which is important in designing the roll pass schedule, is expected to result in significant productivity increases in rolling process analysis and design.

1.1 Objectives and Benefits of This Study

The initial objective of this study is to develop and validate a method for metal deformation and stress analysis for primary rolling mill operations using flat grooveless rolls. In addition to predicting spread and elongation, the analysis will predict the bulging of the billet free side surfaces. The flat rolling process illustrated in Fig. 2 is one in which there is smooth outward barrelining of
the billet free side surfaces, or single-bulge, a particular kind of metal deformation in rolling. The analysis will assume that the rolled material is rigid perfectly plastic, and the analysis will not consider coupled thermo-mechanical effects. Afterward, the flat rolling analysis will be extended to the analysis of certain grooved roll pass sequences that are used in the production of symmetrical geometric shapes without protrusions, such as: squares, diamonds, ovals, and rounds. The results of this study are expected to:

* increase the fundamental understanding of the rolling process;
* lead to the development of improved rules for roll pass design;
* assist in the rational design of rolling mill controls for modern, automated, high-speed rolling mills; and
* reduce roll pass design costs by reducing the number of in-plant production trials necessary to start-up a new rolling mill installation.

1.2 Background on Rolling Mill Design

Because of the pressing need to achieve major increases in industrial productivity, modern rolling mill design strives to develop flexible-purpose or universal roll sets
which can produce specific shapes, not only for a single material, but for several material grades and over a range of rolling process conditions. Rolling process conditions are factors such as the roll speed, the billet temperature, and the quantity and chemical composition of roll coolant. Thus, accurate estimates of the spread and elongation at each pass are required so that the rolls can be designed in such a way that each billet fills each pass without difficulty. Calculation of the roll separating force, via analysis of the stress distribution within the deforming billet material, and the roll torque are also important in order to be able to compensate for elastic deflection of the rolls and machine members, and to avoid mill overloads.

The reason that the design of the roll pass schedule is so difficult to analyze, however, is that for a given grade of material and final shape there is no unique method for roll pass design. For example, Fig. 4 shows a variety of alternative roll pass schedules which can be used to produce a round starting from a square billet. Establishing the number and the shape of the intermediate passes is a complex problem which depends on the characteristics of the available installation.

Also, there is a variety of rolling mills used for the different stages in basic metals production. For large cross-section billets, the two-high reversing mill with flat rolls, or ingot breakdown mill, is the principal type in use
Figure 4. Alternative roll pass schedules used for rolling a round bar from a square billet.

because of its maximum flexibility in size range of ingot and product. The design of rolling mills for large cross-sections is mainly concerned with the structural design of the mill, to accommodate the tremendous roll separating forces generated with large cross-sections, and with developing rolling sequences which avoid defect formation on the unprotected side surfaces of the billet.

As the size of the billet cross-section decreases, regardless of the overall mill design, grooved roll sets are commonly used because the roll grooves tend to restrict the
billet to known cross-sections, thereby providing better control of the size and shape of the billet as its cross-section approaches that of the finished shape. However, during processing the size, shape, and temperature of the billet must be carefully controlled. Failure to meet this requirement can result in serious defects in the finished shape. Moreover, because grooved roll sets wear faster and less evenly than flat rolls do, traditional rolling mills with grooved roll sets require frequent roll changing and, consequently, have relatively high production costs per ton of product.

In recent years rolling mill builders and engineering design firms have been developing rolling mills which utilize flat grooveless rolling sequences in all but the finishing passes\(^1\). The immediate advantages in grooveless rolling include:

* grooveless rolling mills have a wider product mix and have higher area reductions per pass, which means that fewer roll stands and less roll inventory are required for a given installation;
* the possibility of common grooved rolling defects, such as: overfilling, groove misalignment, and guide misalignment, is reduced; and
* there are also reduced roll costs per ton of product,

\(^1\) Numbers in parentheses refer to entries in the LIST OF REFERENCES.
including both material and labor costs in changing and redressing rolls.

In general, grooveless rolling techniques offer routes to faster and cheaper basic metals production.

The Compact Mill(3,4) is a newly developed grooveless rolling mill which utilizes a patented rolling technique known as the round-edged-rectangle method. The principal of the round-edged-rectangle method, which was illustrated in Fig. 2, is to proportion the billet entry cross-section, roll diameter, and thickness reduction in such a way as to promote smooth outward barreling, or single-bulge, of the billet free side surfaces. This kind of free surface shape is known to decrease the occurrence of rolling defects such as surface cracks and seams which tend to reduce mill yields(5,6). These defects occur most often when the free surface shape is similar to that of double-bulge or hour-glass contour(7). An example of double-bulge deformation in flat rolling is illustrated in Fig. 5.

Another feature of the Compact Mill is that its use of grooveless rolls gives it flexibility and a wide product mix. In addition, the Compact Mill uses the latest forced rolling methods(8) to reduce the roll size required for a given shape, thereby decreasing the roll separating forces and stresses acting on the rolls and their machine members. Smaller roll sizes also promote greater area reductions per pass, thereby further reducing the number of roll stands
Figure 5. An illustration of double-bulge deformation in flat rolling.
required for a given installation. Most of the developing grooveless rolling mills are also equipped with computer-controlled speed regulation systems suited for high-speed operation(9). Because of their flexibility and speed, the new grooveless rolling mills can be linked directly with continuous casting facilities in order to maximize the efficient utilization of the latent heat of the refined metal at the caster and achieve even greater productivity increases(10-15). As another example, a grooveless rolling mill has recently been installed in the roughing and finishing stands of a billet mill of the Mizushima works of the Kawasaki Steel Corporation(16). The use of grooveless rolls has resulted in significant reductions in roll changing, setup, and dressing times which has led to an increase of approximately 5% in mill productivity, a 6% fuel savings for heating, 7% less power consumption for rolling, as well as higher yields.

The development of a method for metal deformation and stress analysis for grooveless rolling, or flat rolling, will facilitate further development of this important technology. The extension of the flat rolling analysis to the analysis of the rolling of symmetrical geometric shapes without protrusions will be a first step towards developing the capability to analyze complete roll pass schedules, Fig. 4, for a much wider variety of finished rolled shape cross-sections, Fig 1.
CHAPTER II
PREVIOUS WORK

II.1 Empirical Methods for Roll Pass Design

A thorough knowledge of the forces and metal deformation in rolling is necessary in order to optimize the design of the roll pass schedule. Many authors (17-60) have proposed methods for roll pass design which are based on the use of empirical design formulae and experimental rolling mill data. Specific details of some of the best known formulae are reviewed in recent papers by Mauk and Kopp (61), and Raghupathi and Altan (62).

Various empirical design formulae have been proposed to predict the roll separating force. These formulae often calculate the roll separating force directly as a function of the billet and roll gap geometries, the billet flow stress, and the friction coefficient in the interface between the billet and roll surfaces. Alternatively, the roll separating force can be calculated indirectly through calculation of the average roll separating pressure. The total roll separating force is then calculated by multiplying the roll separating pressure by the plan contact
areas of the deformation zone. The plan contact areas are calculated with knowledge of the spread and elongation, quantities which have to be determined separately. It has been observed\(^{(62)}\), from experimental results, that the formulae of Jedlicka, Sims, and Orowan work very well for blooming mills, and the formulae of Siebel, Trinks, and Jedlicka work the best for slabbing mills.

By contrast, estimating spread and elongation, by far the most important aspects of the rolling problem, is much more difficult. In flat rolling operations in which the bulge is negligible, such as slabbing mills where the entry cross-section is relatively wide, spread is defined as the width increase for the rolling of a simple rectangular cross-section. However, even in situations in which the basic geometry of the rolling problem is uncomplicated, spread is affected by many factors. These factors include the dimensions of the billet entry cross-section, the roll diameter, and the reduction taken in the pass. In addition, spread is affected by factors related to the rolling process conditions such as the billet and roll temperatures, the billet and roll materials, and the speed of the rolling. Several authors have developed design formulae in which the final dimensions of the billet cross-section are calculated primarily as a function of the geometric aspects of the rolling problem, leaving certain unspecified coefficients in their formulae to be adjusted in
order to fit the specific rolling conditions. These authors include: Siebel(25), Ekelund(26), Wusatowski(27), Hill(28), Sparling(29), Shinokura(30), and others(61,62).

One of the simplest design formulae for spread is the formula proposed by Shinokura(30) in which the final width of the billet cross-section, $B_1$, is expressed as,

$$B_1 = 1 + a \cdot \frac{m L}{B_0 + n H_0} \cdot \frac{F_h}{F_0}$$

(2.1)

where,

$B_0$, $H_0$, $F_0$ : are the width, thickness, and sectional area of the unrolled stock, respectively;

$B_1$, $H_1$, $F_h$ : are the width, thickness, and fractional reduction of the area of the rolled stock, respectively;

$L$ : is the projected length of the roll bite, which can be calculated from the roll diameter, the initial thickness of the stock, and the reduction in the pass;

$a$, $m$, $n$ : are empirical coefficients whose values are determined in order to fit the specific rolling conditions being considered.

Experience has shown that these formulae work well when restricted within the ranges of conditions for which they were empirically determined.
In flat rolling, provided that the bulge is negligible, reliable spread formulae are available, and they may be used directly. However, in shape rolling, where the geometry of the rolling problem is much more complicated, the flat rolling design formulae cannot be used directly. In shape rolling the established approach is to first simplify the billet and roll geometries to comparable flat rolling geometries so that the existing flat rolling design formulae can be used. This may be done, at least for simple geometric shapes, by the equivalent rectangle method\textsuperscript{(17-22)}. Figs. 6 and 7 illustrate how a diamond-diamond pass can be simplified using the equivalent rectangle method. The technique involves reducing the areas of the entering billet and the roll cross-sections (between the points of intersection of the two profiles) to rectangles of equivalent area between the same two points, i.e., the rectangles with heights labeled, respectively, $H_0$ and $H_1$ in Fig. 7. Given the equivalent heights (of the flat rolling problem) the maximum width of the billet exit cross-section, $B_1$, Fig. 6, may be estimated using the flat rolling spread formula, Eqn. (2.1).

Elongation in rolling, or the reduction of the billet cross-sectional area, has also been investigated by many researchers\textsuperscript{(17-22,36-60)}. Most researchers prefer elongation formulae when designing section mills simply because a usable definition of spread for many rolled
Figure 6. Geometrical designation and approximation of profile of side free surface in diamond-diamond pass (ref. 30).

Figure 7. Equivalent rectangle transformation method to determine the effective heights of both the billet material ($H_{0m}$) and roll cross-section ($H_{1m}$) in diamond-diamond pass (ref. 30).
sections can be difficult to establish.

In order to estimate the overall elongation in the design of section mills, the original billet cross-section is usually divided into component areas (17-22). Fig. 8 shows a variety of methods used for dividing I-beams into component areas. During rolling the cross-sections of these component areas change and the elongation is different for each component area, depending on the rolling conditions, and the overall elongation is actually a combined effect of the individual elongations of the component areas. For design purposes, however, the individual elongations are calculated using the method of equivalent rectangles, using an established flat rolling design formula for each component area, without regard for the combined influences of the other portions of the section. The overall elongation is then calculated by taking a weighted average of the individual elongations (20, 21).

An example of an established elongation formula is Wusatowski's formula (27), which is expressed as follows,

\[
\Delta_0 = a \cdot b \cdot c \cdot d \cdot \left( \frac{H_1}{H_0} \right) \quad (2.2)
\]

where,

\[
W = 10 \quad ( - 1.269 \left( \frac{B_0}{H_0} \right) \left( \frac{H_0}{D} \right)^{0.556} )
\]
Figure 8. Different methods for division of I-beams for roll pass design (Ref. 47).
\( B_0, H_0, A_0 \): are the initial width, thickness, and cross-sectional area of the unrolled section, respectively;
\( H_1, A_1 \): are the final thickness, and cross-sectional area of the rolled section, respectively;
\( D \): is the effective roll diameter;
\( a, b, c, d \): are coefficients that vary slightly from unity to allow for variations in steel composition, rolling temperature, rolling speed, and roll material, respectively.

Values of the empirical coefficients \( a, b, c, d \), in Eqn. (2.2) are well known, at least for different grades of steel, and can be obtained in the published literature\(^{(20)}\).

These empirical design formulae give good results when restricted within the ranges of conditions for which they were empirically determined. However, there is no single formula or method that gives good results in all the cases.

**II.2 Computer-Assisted Roll Pass Design**

Because of the complexity of the rolling process, and the fact that there is no unique rolling sequence, Fig. 4, computer-assisted methods for the analysis and design of the rolling process are being developed\(^{(63-74)}\). One such method, developed at Battelle by Akgerman, Lahoti, and
Altan(71), uses a modular upper-bound method of analysis to predict spread and elongation, or established empirical design formulae when they are available, and a slab method of analysis to predict the stresses, the roll separating force, and the roll torque. This approach can predict metal deformation and stresses over a wide range of billet and roll geometries and rolling process conditions, and has been successfully applied to the analysis of the rolling of plates and airfoil shapes.

Recently, using the analysis of equivalent flat rolling processes, Kennedy, Altan, and Lahoti(74) extended the previous Battelle study and developed a computer program named RPDROD for the analysis and design of roll pass schedules for bar and rod rolling. The RPDROD computer program permits the designer to determine an optimum roll pass schedule, interactively, by evaluating a number of alternatives in which individual pass designs are selected from a variety of parametrically described roll cross-section shapes commonly used in bar and rod rolling. The roll cross-section shapes which have been included in the RPDROD program, as possible alternative roll passes, are shown in Fig. 9. The present version of the RPDROD program uses Shinokura’s formula(30), Eqn. (2.1), in order to estimate spread. The elongation is estimated by approximating the profile of the free side surfaces of the billet exit cross-section. The distribution of the billet
Figure 9. Various pass shapes considered by the RPDROD computer program.
cross-sectional area through the roll-bite is then established using another empirical function\(^{(71)}\), from which the stresses, the roll separating force, and the roll torque are calculated using the slab method of analysis that was developed in a previous Battelle study\(^{(71)}\).

The RPDROD computer program has been successfully applied to the analysis and design of roll pass schedules for rod rolling. Laboratory experiments for the hot rolling of mild carbon steel rod were conducted at Battelle\(^{(73)}\) on a two-high rolling mill with grooved rolls which accepted a 1 1/4 in. square billet to produce a 7/8 in. diameter rod. Predicted values of elongation and roll separating force were in good agreement with the results of the laboratory experiments. As an illustration of the capabilities of the RPDROD program, Figs. 10 and 11 show, respectively, the predicted deforming billet cross-sections for square-oval and oval-round type pass combinations. The predicted stress and elongation distributions for the oval-round shape combination are shown, respectively, in Figs. 12 and 13. It is felt that RPDROD program is a very practical design tool because it can easily investigate a wide variety of alternative roll pass schedules.

In addition, this approach is not restricted to the use of empirical design formulae. When an established design formula is unavailable, the RPDROD computer program can use a design formula which has been derived from the results of
Figure 10. Three-dimensional display of bar cross-sections in the roll-bite for square to oval pass as simulated by the PDP-RID program, the arrow denotes the direction of rolling.
Figure 11. Three-dimensional display of bar cross-sections in the roll-bite for oval to round pass as simulated by the RPDROD program, the arrow denotes the direction of rolling.
Figure 12. Three-dimensional display of the stress distribution for oval to round pass as simulated by the RPDROD program, the arrow denotes the direction of rolling.
Figure 13. Predicted elongation distribution across the width of the section for oval to round pass as simulated by the RPDROD program.
a complete three-dimensional analysis of the metal deformation and stresses in rolling for the specific rolled material and rolling process conditions being considered.

II.3 Theoretical Analyses of the Rolling Process

There have been several attempts to analyze the rolling process theoretically. Gokyu, Kihari and Mae (1975) analyzed spread in flat rolling by applying a minimum-work hypothesis to the rolling problem. Kennedy and Wilson (1976, 1977) analyzed spread in flat rolling using an approximate plasticity analysis based on a flat bar forging analogy. Earlier, Hill (1978) developed a general approximation technique for the analysis of metal deformation processes which was based on the virtual work-rate principle for a continuum and outlined its application to steckel rolling, which Lahoti and Kobayashi (1979) completed later. Oh and Kobayashi (1980), and Kummerling and Lippmann (1981) applied similar variational methods for plate rolling. Afterward, Lahoti, Akgerman and Altan (1982) extended the work of Oh and Kobayashi (1980) to develop a modular upper-bound approach for the analysis of the rolling of plates and airfoil shapes. Although these early theoretical analyses were capable of predicting average spread and elongation, the roll separating force, and the roll torque fairly accurately, these studies did not analyze the bulging of the billet free side surfaces, an
important aspect of the developing grooveless rolling techniques (16).

Metal flow models in shape rolling have been developed by El-Nikaily (83), using the visioplasticity technique, and single-bulge deformation in flat rolling, outward barreling only, Fig. 2, has been analyzed by Kato, Murota and Kumagai (84) using the upper-bound method. Single-bulge in flat rolling has also been analyzed using the upper-bound method by Sevenler, Raghupathi, and Altan (85) at Battelle. In this approach the bulge and elongation deformation are analyzed separately, using the analysis by Oh and Kobayashi (80) to predict the elongation and a plane-strain upsetting analysis to predict the bulge. An advantage in this approach, though somewhat approximate in nature, is that the complex metal deformation in rolling can be analyzed using a method, the upper-bound, which has comparatively little computer resource requirements.

Recently, bulge in flat rolling has been studied using the generalized finite-element method of analysis. Kanazawa and Marcal (86), Li and Kobayashi (87, 88), Mori and Osakada (89, 90), have presented rigid-plastic finite-element solutions to the flat rolling problem, and, recently, Kiefer (91) has presented an elastic-plastic finite-element solution. Although these analyses provide very detailed information on the complex three-dimensional metal deformation in rolling, the only disadvantage in using the
finite-element method for rolling analysis is that the computer resource requirements, in terms of both storage and processing time, can be prohibitively high(91). Although computer technology is developing rapidly, in most instances the computer resources that are required for three-dimensional rolling analysis using the finite-element method are much too expensive.

Therefore, with this factor taken into consideration, an upper-bound method of analysis was selected for the present study because this approach can provide useful rolling mill design data for the more significant aspects of the rolling problem(85), and because this approach is less costly than an approach based on the finite-element method of analysis(86-91).
CHAPTER III
FLAT ROLLING ANALYSIS

In terms of the kinematical aspects of the metal deformation problem in rolling, the purpose of the present investigation is to determine the spread, elongation, and bulge profile of the rolled section. The stresses, roll separating force, and roll torque are determined separately, once the actual metal flow distribution is known. The present investigation considers only the purely mechanical aspects of the metal deformation problem in rolling, and only under steady-state rolling conditions. Coupled thermo-mechanical effects and unsteady metal flow phenomena (such as crop formation as the front end of the billet is initially drawn into the roll-bite) are not considered.

III.1 Metal Deformation Analysis

In the analysis of three-dimensional metal deformation problems there are no exact solutions that can be used for practical purposes. Therefore, various approximation methods must be used. The upper-bound method (92-98) is one of many approximation methods that are available.
In the upper-bound method of analysis an admissible velocity field solution (Sec. III.2) is considered. Based on this velocity field, the total energy dissipation rate for the metal deformation process is computed. Since this upper-bound energy dissipation rate necessarily predicts a higher value than that required in the actual metal deformation process, based on limit theorems (92, 93), the lower the energy dissipation rate, the better the prediction. Thus, the metal flow distribution can be determined by minimizing the energy dissipation rate with respect to the velocity field - with the actual minimization performed with respect to certain unknown parameters which are introduced in order to represent the metal flow in the actual process (the spread, elongation, and bulge in the case of the rolling problem). Although increasing the number of unknown parameters in the velocity field will generally improve the solution, the computations become much more complex. Consequently, in the upper-bound method of analysis practical compromises must be made in choosing an admissible velocity field.

In applying the upper-bound method of analysis the following assumptions are usually made:

* the material is isotropic and incompressible;
* the elastic deformations are negligible;
* the inertial forces are negligible;
* the friction shear stress, \( r \), is constant in the
roll-workpiece interface and is given by a constant friction shear factor, \( m \), defined by the relation,
\[ \tau = m \cdot \frac{\bar{\sigma}}{\sqrt{3}} ; \]
* the plastically deforming material flows according to the Levy-Mises flow rule and Mises yield criterion;
and,
* the flow stress of the material, \( \bar{\sigma} \), is constant.

The material flow stress, \( \bar{\sigma} \), and the friction shear factor, \( m \), are experimentally determined quantities which are determined for the specific rolled material and rolling process conditions being considered.

The total energy dissipation rate, \( \dot{E}_{\text{tot}} \), is expressed as the sum of the following components: the plastic deformation energy dissipation rate, \( \dot{E}_p \), the velocity discontinuity energy dissipation rate, \( \dot{E}_d \), and the friction energy dissipation rate, \( \dot{E}_f \), expressed as follows,
\[ \dot{E}_{\text{tot}} = \dot{E}_p + \dot{E}_d + \dot{E}_f , \quad (3.1) \]
where,
\begin{align*}
\dot{E}_p &= \bar{\sigma} \int_V \dot{\varepsilon} \, dV , \quad (3.2) \\
\dot{E}_d &= \bar{\sigma} \int_{S_d} |\Delta v| \, dS , \quad (3.3) \\
\dot{E}_f &= m \cdot \frac{\bar{\sigma}}{\sqrt{3}} \int_{S_f} |\Delta v| \, dS , \quad (3.4)
\end{align*}

where \( V \) is the volume of the plastically deforming body, \( |\Delta v| \) represents the jump in material velocity across the
interfaces, $S_d$, for velocity discontinuities, and, $S_f$, for interface friction, and where the effective strain-rate, $\dot{\epsilon}$, is defined by\(^1\),

$$\dot{\epsilon} = \left[ \begin{array}{l} 2 \dot{\epsilon}_{ij} \\ 3 \dot{\epsilon}_{ij} \end{array} \right]^{0.5} , \quad (3.5)$$

for the strain-rates, $\dot{\epsilon}_{ij}$, defined by,

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) , \quad (3.6)$$

based on the velocity field, $v_j$.

### III.2 Admissible Velocity Fields

The most difficult step in applying the upper-bound method of analysis is to develop a class of admissible velocity fields. The admissible velocity field, $v_j$, is a set of functions which satisfies the incompressibility condition, the essential velocity boundary conditions in the roll-workpiece interface, and continuity across elastic-plastic interfaces within the plastically deforming body.

The incompressibility condition may be expressed as follows,

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 , \quad (3.7)$$

where $v_x$, $v_y$, and $v_z$, are the material velocities in the x, y, and z coordinate directions, respectively. Based on the strain-rates, Eqn. (3.6), the incompressibility condition

\(^1\) Using cartesian coordinates with the summation convention for repeated indices.
can be expressed equivalently as,
\[ \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0. \tag{3.8} \]
The essential velocity boundary conditions, for steady-state metal deformation processes, require that the surface streamlines of the plastically deforming body (those streamlines in contact with the tool surface) have to be perpendicular to the local surface normal vector. This requirement may be expressed analytically as follows,
\[ v_i n_i = 0, \tag{3.9} \]
imposed over the roll-workpiece interface, \( S_f \), and where \( n_i \) are the components of the unit inward normal vector to the roll surface. The continuity condition requires that the material velocity components which are perpendicular to elastic-plastic interfaces, within the plastically deforming body, must be continuous across these interfaces. The admissibility of the velocity field used in the present investigation is established in Sec. III.7.

### III.3 Flat Rolling Process Considered in the Analysis

A schematic illustration of the flat rolling process considered in the present investigation is shown in Fig. 14. It is assumed that the billet entry cross-section has straight edges, Fig. 14, and that a pair of rigid grooveless rolls, with roll diameter \( D \), rotating with tangential roll velocity \( v_r \), draw the billet into the roll-bite, reduce its
Figure 14. Schematic illustration of the flat rolling process being considered in the present investigation.
thickness from $H_b$ to $H_f$ and reduce its cross-sectional area from $A_b$ to $A_f$. It is assumed that billet cross-sections perpendicular to the direction of rolling remain plane during deformation. It is also assumed that plastic deformation begins at the billet cross-section which is located at the initial touching point between the billet and the roll surfaces, called the entry plane to the deformation zone. It is also assumed that the exit plane of the deformation zone, the point at which plastic deformation ceases, is the billet cross-section located within the plane which contains the roll centerlines, Fig. 14. The deformation zone itself is the volume bounded by the entry and exit planes, the roll-workpiece interfaces, and the billet free side surfaces. The projected length of the roll-bite, denoted $X_d$, which is also the distance between the entry and exit planes, is determined from the specified values of $D$, $H_b$, and $H_f$, and can be expressed as,

$$X_d = [r^2 - (r - (H_b-H_f)/2)^2]^{0.5}, \quad (3.10)$$

where, for the flat rolling analysis, $r = (D/2)$. The coordinate axes for the analysis are centered at the centroid of the entry cross-section, Fig. 14, with the x, y, and z coordinate directions corresponding to the directions of elongation, spread, and thickness reduction, respectively. Half the roll separation, $h$, is expressed as,

$$h = r + H_f/2 - [r^2 - (X_d-x)^2]^{0.5}. \quad (3.11)$$
For the flat rolling analysis the function \( h \) also corresponds to half the thickness of the deforming billet cross-sections.

### III.4 Velocity Field Equations

Therefore, based on Hill's velocity field formulation for steady-state rolling without bulge\(^{(78)}\), a velocity field for the flat rolling problem with bulge can be expressed as follows,

\[
\begin{align*}
  v_x &= v_b A_b \cdot \frac{1}{A}, \\
  v_y &= -v_b A_b \cdot c \frac{\partial A}{\partial z}, \\
  v_z &= v_b A_b \cdot c \frac{\partial A}{\partial y} + v_b A_b \cdot \frac{1}{A} \frac{dA}{dx} \tag{3.14}
\end{align*}
\]

where \( v_x, v_y, \) and \( v_z \) are, respectively, the billet material velocities in the \( x, y, \) and \( z \) coordinate directions, the function \( A \) is an unknown function of \( x \) denoting the billet cross-sectional area distribution, \( v_b \) is an unknown parameter denoting the billet \( x \) coordinate direction velocity component at the entry plane of the deformation zone (a parameter that specifies how fast the billet material is entering the roll-bite), and the functions \( c \) and \( B \) are defined by,

\[
\begin{align*}
  c &= \frac{1}{hA} \frac{dh}{dx} - \frac{1}{2} \frac{dA}{dx}, \\
  B &= \frac{1}{hA} \frac{dh}{dx} - \frac{1}{2} \frac{dA}{dx} \tag{3.15}
\end{align*}
\]
\[ B = \left\{ z + \frac{1}{3} B_1 z^3 + \frac{1}{5} B_2 z^5 \right\} b_1 y, \quad (3.16) \]

where \( B_1 \) and \( B_2 \) are additional unknown parameters which are introduced in order to consider nonlinear material velocity distributions through the thickness of the deforming billet cross-sections. The parameter \( b_1 \) is evaluated to satisfy velocity boundary conditions (see Sec. III.7).

For the billet cross-sectional area distribution function, \( A \), it is convenient to assume a cubic polynomial function with coefficients selected to satisfy the following four conditions:

1. \( \frac{dA}{dx} = A' \) at \( x = 0 \),
2. \( A = A_f \) at \( x = X_d \),
3. \( A = A_b \) at \( x = 0 \),
4. \( \frac{dA}{dx} = 0 \) at \( x = X_d \).

With these conditions, the function \( A \) is expressed as follows,

\[ A = a_0 \left( \frac{x}{X_d} \right)^3 + a_1 \left( \frac{x}{X_d} \right)^2 + a_2 \left( \frac{x}{X_d} \right) + a_3, \quad (3.17) \]

where,

\[ a_0 = 2 (A_b - A_f) + X_d A', \]
\[ a_1 = 3 (A_f - A_b) - 2 X_d A', \]
\[ a_2 = X_d A', \]
\[ a_3 = A_b. \]

Conditions 1. and 2. permit different elongation distribution functions to be considered by adjusting the
values of just two parameters: $A'$, the slope of the cross-sectional area distribution function at the entry plane, and $A_f$, the final cross-sectional area of the rolled section. Conditions 3. and 4. are imposed to maintain continuity across the elastic-plastic interfaces (Sec. III.7). Thus, conditions 3. and 4. are fixed, they apply to all elongation distribution functions considered in the metal deformation analysis.

### III.5 Minimization Variables

In the flat rolling analysis there are five unknown parameters in the velocity field equations: $A'$, $A_f$, $B_1$, $B_2$, and $v_b$. Hence, the total energy dissipation rate, Eqn. (3.1), can be expressed as a function of these unknown parameters, as follows,

$$ \dot{E}_{\text{tot}} = \dot{E}_{\text{tot}}(A', A_f, B_1, B_2, v_b) \quad (3.18) $$

Two of the unknown parameters, $A'$ and $A_f$, determine the elongation distribution function, two parameters, $B_1$ and $B_2$, determine the bulge contour, and one parameter, $v_b$, determines the rate at which the billet material enters the roll-bite. However, it is convenient to consider (without changing the actual velocity field equations) an equivalent set of dimensionless minimization variables, as summarized in the following,

* the parameters $A'$ and $A_f$ are replaced by,
respectively, \((A' X_d)/A_b\) and \(A_f/A_b\), in order to minimize \(\dot{E}_{\text{tot}}\) with respect to a dimensionless elongation distribution function.

* the parameters \(B_1\) and \(B_2\) are already dimensionless so they are retained, and

* the parameter \(v_b\) is replaced by \(X_n/X_d\), where \(X_n\) denotes the location of the neutral point, the location where the \(x\) direction component of the surface traction stress, \(\tau\), changes direction; a location which may be determined by the input geometry of the roll pass design, the specified values of \(A'\), \(A_f\), \(B_1\), \(B_2\), and \(v_b\), and the velocity field formulation (Sec. III.4) - when it is assumed that the surface traction stress opposes the relative slip velocity vector between the billet and roll surfaces.

Thus, Eqn. (3.18) can be expressed equivalently as,

\[
\dot{E}_{\text{tot}} = \dot{E}_{\text{tot}}\left( (A' X_d)/A_b, A_f/A_b, B_1, B_2, X_n/X_d \right). \quad (3.19)
\]

The use of dimensionless minimization variables facilitates the development of a practical minimization algorithm (Sec. III.11). However, replacing the parameter \(v_b\) with \(X_n/X_d\) can also be justified on the basis of fundamental considerations because of the physical relationships that exist in rolling between the neutral point location, the overall geometry of the rolling problem, the rolling process conditions, and the force equilibrium of the rolled section.
The relative neutral point location, \( X_n / X_d \), defines the boundary between the region of backward slip (where the roll velocity exceeds the billet velocity and where the friction shear stresses continually draw the billet through the deformation zone) and the region of forward slip (where the billet velocity exceeds the roll velocity and the friction stresses oppose the direction of metal movement). Fig. 15 gives a schematic illustration of strip rolling in which the neutral point location is denoted with the symbol N. The region of backward slip, Fig. 15, is on the entry plane side of the deformation zone, before N, and the region of forward slip is on the exit plane side, after N. The practical significance of the neutral point location is that if there is insufficient roll friction, then the neutral point can shift past the exit plane of the deformation zone (leaving the entire arc of contact in backward slip) and, hence, the billet would begin to skid under the rolls. Since skidding is an inherently nonequilibrium condition, the roll pass design would have to be modified in order to return to an equilibrium condition.

III.6 Strain-Rate Equations

The total energy dissipation rate, \( \dot{E}_{\text{tot}} \), is formulated from the velocity field, \( v_i \), and the strain-rates, \( \dot{\varepsilon}_{ij} \). Based on Eqn. (3.6) and Eqns. (3.12)-(3.14), the strain-rate
Figure 15. Schematic illustration of strip rolling showing neutral point, N, and directions of surface traction stress (From Metal Forming: Fundamentals and Applications. American Society for Metals, 1983, p. 253, With permission).

Equations are expressed as follows,

\[
\dot{\epsilon}_{xx} = - v_b A_b \cdot \frac{1}{A} \frac{dA}{dx}, \quad (3.20)
\]

\[
\dot{\epsilon}_{yy} = - v_b A_b \cdot c \frac{\partial^2 B}{\partial y \partial z}, \quad (3.21)
\]

\[
\dot{\epsilon}_{zz} = v_b A_b \cdot c \frac{\partial^2 B}{\partial y \partial z} + v_b A_b \cdot \frac{1}{A} \frac{dA}{dx}, \quad (3.22)
\]

\[
\dot{\epsilon}_{xy} = - \frac{1}{2} v_b A_b \cdot \left[ \frac{dc}{dx} \frac{\partial B}{\partial y} + c \frac{\partial^2 B}{\partial x \partial z} \right], \quad (3.23)
\]

\[
\dot{\epsilon}_{xz} = \frac{1}{2} v_b A_b \cdot \left[ \frac{dc}{dx} \frac{\partial B}{\partial y} + c \frac{\partial^2 B}{\partial x \partial y} \right]
- \frac{1}{2} v_b A_b \cdot \left[ 2 \frac{dA}{dx} \frac{dA}{dx} - \frac{1}{2} \frac{d^2 A}{dx^2} \right] \frac{z}{A}, \quad (3.24)
\]

\[
\dot{\epsilon}_{yz} = \frac{1}{2} v_b A_b \cdot c \left[ \frac{\partial^2 B}{\partial y^2} - \frac{\partial^2 B}{\partial z^2} \right], \quad (3.25)
\]
where, from Eqn. (3.11),

\[
\frac{dh}{dx} = - \left( \frac{X_d - x}{r^2 - (X_d - x)^2} \right) \frac{0.5}{0.5}, \tag{3.26}
\]

\[
\frac{d^2 h}{dx^2} = 1 \left( \frac{r^2 - (X_d - x)^2}{r^2 - (X_d - x)^2} \right)^{0.5} + \left( \frac{X_d - x}{r^2 - (X_d - x)^2} \right)^{1.5}, \tag{3.27}
\]

from Eqn. (3.15),

\[
\frac{qc}{dx} = \frac{1}{2} \frac{dh}{dx} \frac{dh}{dx} - \frac{1}{2} \frac{d^2 A}{dx^2} \frac{d^2 A}{dx^2} + \frac{1}{2} \frac{d^2 B}{dx^2} \frac{d^2 B}{dx^2}, \tag{3.28}
\]

from Eqn. (3.17),

\[
\frac{dA}{dx} = \left[ 3 a_0 \left( \frac{x}{X_d} \right)^2 + 2 a_1 \left( \frac{x}{X_d} \right) + a_2 \right] / X_d, \tag{3.29}
\]

\[
\frac{d^2 A}{dx^2} = \left[ 6 a_0 \left( \frac{x}{X_d} \right) + 2 a_1 \right] / (X_d)^2, \tag{3.30}
\]

and from Eqn. (3.16),

\[
\frac{dB}{dy} = \left[ z + \frac{1}{3} B_1 h_2 \frac{z^3}{h^2} + \frac{1}{5} B_2 h_4 \frac{z^5}{h^4} \right] b_1, \tag{3.31}
\]

\[
\frac{dB}{dz} = \left[ 1 + B_1 h_2 \frac{z^2}{h^2} + B_2 h_4 \frac{z^4}{h^4} \right] b_1 y, \tag{3.32}
\]

\[
\frac{\partial^2 B}{\partial x \partial y} = - \left[ \frac{1}{3} B_1 h_2 \frac{z^3}{h^2} + \frac{4}{5} B_2 h_4 \frac{z^5}{h^4} \right] b_1 \frac{dh}{dx}, \tag{3.33}
\]

\[
\frac{\partial^2 B}{\partial x \partial z} = - \left[ \frac{1}{3} B_1 h_2 \frac{z^2}{h^2} + \frac{4}{5} B_2 h_4 \frac{z^4}{h^4} \right] b_1 \frac{dh}{dx}, \tag{3.34}
\]

\[
\frac{\partial^2 B}{\partial y \partial z} = \left[ 1 + B_1 h_2 \frac{z^2}{h^2} + B_2 h_4 \frac{z^4}{h^4} \right] b_1, \tag{3.35}
\]

\[
\frac{\partial^2 B}{\partial y^2} = 0, \tag{3.36}
\]

\[
\frac{\partial^2 B}{\partial z^2} = \left[ 2 B_1 h_2 \frac{z^2}{h^2} + 4 B_2 h_4 \frac{z^4}{h^4} \right] b_1 y. \tag{3.37}
\]
III.7 Admissibility of the Chosen Velocity Field

Substituting $\varepsilon_{xx}$, $\varepsilon_{yy}$, and $\varepsilon_{zz}$ from Eqns. (3.20)-(3.22) into the incompressibility condition, Eqn. (3.8), yields the following expression,

$$-v_b A_b \cdot \frac{1}{2} \frac{\partial A}{\partial x} - v_b A_b \cdot c \frac{\partial^2 B}{\partial y \partial z} + v_b A_b \cdot c \frac{\partial^2 B}{\partial z \partial y} + v_b A_b \cdot \frac{1}{2} \frac{\partial A}{\partial x} = 0,$$

(3.38)

which is equal to zero when the polynomial interpolation function, $B$, Eqn. (3.16), has continuous second partial derivatives, which it does. Therefore, incompressibility is satisfied.

For the velocity boundary conditions, in the flat rolling analysis the velocity boundary conditions, Eqn. (3.9), reduce to the following,

$$\left. \frac{v_z}{v_x} \right|_{x=h} = \frac{\partial h}{\partial y}.$$

(3.39)

Substituting $v_x$, from Eqn. (3.12), and $v_z$, from Eqn. (3.14), into the left-side of Eqn. (3.39) gives,

$$\left. \frac{v_z}{v_x} \right|_{z=h} = A c \frac{\partial B}{\partial y} + \frac{1}{2} \frac{\partial A}{\partial y}.$$

(3.40)

Then, substituting Eqn. (3.15) into Eqn. (3.40), and rearranging terms, gives,

$$\left. \frac{v_z}{v_x} \right|_{z=h} = A \frac{\partial B}{\partial y} \left( \frac{1}{h} \frac{\partial h}{\partial y} \right) - \frac{1}{2} \frac{\partial A}{\partial y} \frac{\partial B}{\partial y} + \frac{1}{2} \frac{\partial A}{\partial y}.$$

(3.41)

which shows that,

$$\frac{\partial B}{\partial y} \left|_{z=h} = \frac{\partial h}{\partial y} \right.$$

(3.42)
in order for Eqns. (3.39) and (3.41) to be equivalent, but, from Eqn. (3.31),
\[
\frac{\partial \theta}{\partial y} \mid_{z=h} = h \left[ 1 + \frac{1}{3} B_1 + \frac{1}{5} B_2 \right] b_1 ,
\]
which establishes that,
\[
b_1 = \frac{1}{1 + B_1/3 + B_2/5} .
\]
Thus, with Eqn. (3.44), the velocity boundary conditions in the flat rolling analysis are satisfied.

In the present analysis the continuity condition only requires that $v_x$ is continuous across the elastic-plastic interfaces (the entry and exit planes of the deformation zone). Thus, from Eqns. (3.12) and (3.17),
\[
v_x = v_b , \text{ at } x = 0 ,
\]
\[
v_x = v_b \frac{A_b}{A_f} , \text{ at } x = X_d ,
\]
which establishes that the continuity condition is also satisfied.

III.8 Computer Program BILLETS

A FORTRAN language computer program, named BILLETS, has been developed for the analysis of metal deformation and stresses in flat rolling. The BILLETS program is currently operational on a VAX 11/750 mini-computer, executing in batch-mode using editor-created input data files. Copies of the program source code are available on magnetic tape or floppy disk.
Discussion of the numerical methods that are used in the BILLETS computer program is given in the following sections. Since the flat rolling analysis and the shape rolling analysis are nearly identical, any major differences in their respective numerical methods are discussed here.

III.9 Streamline Equations

An aspect of the present approach is that the spread and bulge of the plastically deforming billet are not explicitly given by the velocity field equations (Sec. III.4). However, since the rolling process is assumed to be steady-state, the spread and bulge can be approximated by calculating the spatial coordinates of the billet surface streamlines - streamlines which emanate from a string of discrete material points which are used to approximate the billet entry cross-section. Once calculated, the billet surface streamlines establish an envelope for the deformation zone - an envelope which completely defines the shape of the plastically deforming body. An example of the calculated billet surface streamlines in flat rolling was illustrated in Fig. 5.

In order to calculate the spatial coordinates of the billet surface streamlines it is necessary to define the streamline equations - equations which actually define the slopes of the pathlines of material points as they flow
through the deformation zone. The streamline equations are expressed in terms of the velocity field (Sec. III.4) as follows,

\[
\frac{\partial Y}{\partial x} = v_y, \quad \frac{\partial Z}{\partial x} = v_z \tag{3.46}
\]

where Y and Z are the spatial coordinates of each streamline which are, individually, functions of the x coordinate variable and, collectively, represent the different deforming billet cross-sections which are considered in the metal deformation analysis. Since each pair of streamline equations, Eqn. (3.46), comprise a set of simultaneous first-order partial differential equations, discrete values of Y and Z can be determined using established numerical methods. In the present flat rolling analysis the Runge-Kutta method is used to integrate the streamline equations. Thus, in this approach the spread and bulge of the side surfaces of the plastically deforming billet are determined from the final position and shape of the billet surface streamlines at the exit plane of the deformation zone.

Once determined, the spatial coordinates of the billet surface streamlines, defined by the functions Y and Z, are also used to calculate the boundaries of the deformation zone (boundaries which include the free side surfaces of the plastically deforming billet). This is accomplished by
monitoring the progress of each streamline, while its spatial coordinates are being integrated, together with use of linear interpolation at each x location between discrete values of Y and Z.

The shape rolling analysis uses essentially the same procedure to integrate the streamline equations and model the free side surfaces of the plastically deforming billet, except that a two-point Newton-Cotes formula(99) is used to integrate the streamlines instead of the Runge-Kutta equations that are used in the flat rolling analysis.

III.10 Energy Dissipation Rate Calculations

The plastic energy dissipation rate, \( \dot{E}_p \), Eqn. (3.2), is integrated over the volume of the deformation zone, Fig. 14, in which the infinitesimal volume element, \( dV \), is given by,

\[
dV = dz \; dy \; dx .
\]

(3.47)

Thus, upon substituting \( dV \), Eqn. (3.47), and \( \dot{\varepsilon} \), Eqn. (3.5), into Eqn. (3.2), the plastic energy dissipation rate, \( \dot{E}_p \), can be expressed equivalently as follows,

\[
\dot{E}_p = \bar{\sigma} \int_x \int_y \int_z \frac{2}{3} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}^{1.5} \; dz \; dy \; dx ,
\]

(3.48)

where the x, y, and z integration limits are determined by:

* the entry plane of the deformation zone,

* the exit plane of the deformation zone,

* the roll-workpiece interfaces, and
Since the flow stress, $\bar{\sigma}$, is given, the integrand in Eqn. (3.48) is solely a function of the strain-rates, $\dot{\varepsilon}_{ij}$, Eqns. (3.20)-(3.25). In the flat rolling analysis $\dot{E}_p$ is integrated analytically in the $y$ coordinate direction using an elementary integration formula that is given elsewhere(100). The remaining integrations, in the $x$ and $z$ coordinate directions, are performed numerically using the three-point Newton-Cotes formula(99). Complete details of the integration procedure are given in the BILLETS computer program source code.

For the energy dissipation rate due to velocity discontinuity, $\dot{E}_d$, Eqn. (3.3), with the present velocity field formulation there is a velocity discontinuity at the entry plane of the deformation zone. Since $v_y$ and $v_z$ are zero before the entry plane (where the billet is rigid), and because $v_x$ is continuous across the entry plane (Sec. III.7), the jump in material velocity, $|\Delta v|$, is given by,

$$|\Delta v| = \left[ v_y^2 + v_z^2 \right]^{0.5},$$

(3.49)

where the values of $v_y$ and $v_z$ in Eqn. (3.49) are taken from Eqns. (3.13) and (3.14), respectively, evaluated at $x = 0$. The infinitesimal surface element, $dS$, at the entry plane is given by,

$$dS = dy \, dz.$$  

(3.50)

Therefore, after substituting Eqns. (3.49) and (3.50) into
Eqn. (3.3), the energy dissipation rate due to velocity discontinuity, $\dot{E}_d$, becomes,

$$\dot{E}_d = \frac{\rho}{\sqrt{3}} \int \int \left[ v'_y^2 + v'_z^2 \right]^{0.5} \, dy \, dz, \quad (3.51)$$

Eqn. (3.51) is integrated analytically in the $y$ coordinate direction (100), followed by a numerical integration, in the $z$ coordinate direction, using the three-point Newton-Cotes formula (99).

For the energy dissipation rate due to friction, $\dot{E}_f$, Eqn. (3.4), which is integrated over the roll-workpiece interface, $S_f$, the friction shear factor, $m$, is given as an input value. The jump in material velocity, $|\Delta v|$, is given by,

$$|\Delta v| = \left[ (v'_t - v'_r)^2 + v'_y^2 \right]^{0.5}, \quad (3.52)$$

where $v'_r$ is the tangential roll velocity, Fig. 15, and where $v'_t$ is given by,

$$v'_t = \left[ v'_x^2 + v'_z^2 \right]^{0.5}. \quad (3.53)$$

For a general roll shape the infinitesimal surface element, $dS$, in the roll-workpiece interface, is given by,

$$dS = \left[ 1 + \frac{\partial z^r}{\partial x^r}^2 + \frac{\partial z^r}{\partial y^r}^2 \right]^{0.5} \, dy \, dx, \quad (3.54)$$

where $z^r$ is the function which represents the three-dimensional roll surface. However, with a solid cylindrical roll,

$$z^r = h, \quad (3.55)$$

and,
\[ \frac{\partial z}{\partial y} = 0 , \]  

(3.56)

where, for the flat rolling analysis, the function \( h \) is given by Eqn. (3.11). Thus, upon substituting Eqns. (3.55) and (3.56) into Eqn. (3.54), the expression for \( dS \) reduces to,

\[ dS = \left[ 1 + \frac{dh^2}{dx} \right]^{0.5} dy \, dx . \]  

(3.57)

Then, substituting Eqns. (3.52) and (3.57) into Eqn. (3.4), gives the energy dissipation rate due to friction, \( \dot{E}_f \), expressed in the following form.

\[ \dot{E}_f = m \bar{\sigma} \sqrt{\frac{2}{3}} \int \int \left[ (v_x - v_r)^2 + v_y^2 \right] \left( 1 + \frac{dh^2}{dx} \right)^{0.5} dy \, dx . \]  

(3.58)

Eqn. (3.58) is integrated analytically with respect to the \( y \) coordinate(100), and numerically, with respect to the \( x \) coordinate, using the three-point Newton-Cotes formula(99).

In the shape rolling analysis, because of the increased complexity of the shape rolling velocity field (CHAPTER V), the three-point Newton-Cotes formula(99) is used to integrate numerically the corresponding expressions for \( \dot{E}_p \), \( \dot{E}_d \), and \( \dot{E}_f \).

III.11 Initial Guess and Minimization Procedures

To begin the minimization procedure it is necessary to determine an initial guess solution for the metal deformation problem by initializing the values of the
minimization variables: \((A' \chi_d / A_b), (A_f / A_b), B_1, B_2,\) and \((X_n / X_d).\) For this purpose, Wusatowski's empirical elongation formula\(^{27}\), Eqn. \((2.2)\), is used to estimate \((A_f / A_b)\) and then, for simplicity, a second-order elongation distribution function is used to characterize the variation of the billet cross-sectional area through the deformation zone, initializing the value of \((A' \chi_d / A_b)\) accordingly. The remaining minimization variables are given consistent initial values, neglecting bulge, and assigning a convenient initial neutral point location - one that is similar to that observed with previous plate rolling analyses\(^{80,72}\), as summarized in the following,

\[
A' \chi_d / A_b = -2 \cdot (1 - A_f / A_b),
\]

\[
A_f / A_b = \text{determined from Eqn. (2.2)},
\]

\[
B_1 = 0.00,
\]

\[
B_2 = 0.00,
\]

\[
X_n / X_d = 0.70.
\]

In the present investigation, for both the flat rolling and shape rolling analyses, minimization of the total energy dissipation rate, \(\dot{E}_{\text{tot}}\), Eqn. \((3.1)\), was performed using the Method of Hookes and Jeeves with Discrete Steps. This method is a multi-dimensional line search technique, similar to the cyclic-coordinate method, but one with an acceleration step. Complete details of the method is given in a book by Bazaraa and Shetty\(^{101}\).
III.12 Estimation of Material Flow Stress

The flow stress, $\bar{\sigma}$, of a given metal alloy is influenced by factors related to the deformation process, such as: the temperature of deformation, the strain, the strain-rate, and by factors unrelated to the deformation process, such as: the chemical composition and microstructure of the alloy, grain size, segregation, prior heat treatment, and deformation history(98). However, the relative importance of these factors can vary greatly with the temperature of deformation.

In hot rolling, defined as rolling temperatures above the recrystallization temperature, for a given metal alloy, prior heat treating, and deformation history, the flow stress can be considered to be primarily a function of temperature and strain-rate. The influence of strain, though present, is not significant. In addition, in a single pass rolling process, especially in a production mill, the temperature and strain-rate do not vary significantly within the plastically deforming body. Thus, in hot rolling the flow stress can be approximated as being a constant value within the plastically deforming body, and, therefore, it is reasonable to use the upper-bound method of analysis (with its assumption of constant flow stress) to analyze the metal deformation. Actually, in practical situations, it is only necessary to estimate the material
flow stress after the metal deformation is determined. By then the flow stress can be estimated from average values of the temperature of deformation, strain, and strain-rate, quantities which are, themselves, already known or determined from the metal deformation analysis in the upper-bound approach.

On the other hand, in cold rolling, at or near room temperature, the flow stress of most metal alloys is primarily a function of strain. Therefore, the application of the upper-bound method of analysis in cold rolling must be approximate because the material flow stress could vary significantly within the plastically deforming body (because of varying degrees of plastic strain throughout the body), and, under these conditions, the assumption of constant flow stress could introduce errors in the metal deformation analysis.

The flow stress data for commonly rolled materials at various temperatures, $T$, effective strains, $\bar{\varepsilon}$, and effective strain-rates, $\bar{\dot{\varepsilon}}$, can be obtained from a variety of sources in the published literature\(\text{(98,102-104)}\). A widely accepted analytical representation of flow stress data in metal forming is in the form of empirical equations. In cold metal forming the flow stress data may be represented as follows,

$$\bar{\sigma} = K \bar{\varepsilon}^n$$

(3.60)
where \( K \) and \( n \) are empirical coefficients which vary slightly with temperature and strain-rate. In hot metal forming the flow stress data may be approximated by,

\[
\bar{\sigma} = C \dot{\epsilon}^m
\]  

(3.61)

where \( C \) and \( m \) are empirical coefficients which vary with temperature and strain.

Since the actual temperature, strain, and strain-rate in practical rolling mill operations will vary somewhat within the plastically deforming body, the average or effective values must be estimated before the flow stress data in the form of Eqns. (3.60) or (3.61) can be used. In the present investigation the average temperature of deformation, \( T \), is taken as the starting temperature of the billet just prior to rolling. The effective strain, \( \bar{\epsilon} \), is taken as the average value of the effective strain through the deformation zone. This average strain is estimated as follows,

\[
\bar{\epsilon} = \frac{1}{3} \int_{X_d}^{X_d} \left[ \frac{2}{3} \left( \epsilon_x^2 + \epsilon_y^2 + \epsilon_z^2 \right) \right]^{0.5} dx , \quad (3.62)
\]

where the integrand in Eqn. (3.62) is evaluated for each deforming billet cross-section, taking the logarithmic strains \( \epsilon_x, \epsilon_y, \) and \( \epsilon_z \) as the principal strains(94). The three-point Newton-Cotes formula(99) is used to integrate Eqn. (3.62) numerically through the deformation zone. The effective strain-rate, \( \dot{\epsilon} \), is estimated from the sum of the
plastic, discontinuity, and friction energy dissipation rates (with unity flow stress assumed initially, i.e., $\bar{\sigma} = 1.0$) as follows,

$$\dot{\varepsilon} = \frac{\dot{E}_p + \dot{E}_d + \dot{E}_f}{V} \quad (3.63)$$

where, the theoretical volume of the deformation zone, $V$, is determined from the billet cross-sectional area distribution function, $A$, as follows,

$$V = \int_0^X A \, dx \quad (3.64)$$

The newly computed values of effective strain, $\bar{\varepsilon}$, and effective strain-rate, $\dot{\varepsilon}$, are then used to revise the flow stress value, $\bar{\sigma}$, and total energy dissipation rate, $\dot{E}_{\text{tot}}$, after minimization (Sec. III.11) is completed. Established empirical flow stress equations (such as Eqn. (3.51) for cold rolling and Eqn. (3.52) for hot rolling) are utilized. The same procedures are used in the shape rolling analysis.

### III.13 Analysis of Roll Torque

With the minimum value of the total energy dissipation rate, $\dot{E}_{\text{tot}} = \dot{E}_p + \dot{E}_d + \dot{E}_f$, given (with the revised flow stress value, Sec. III.12), the roll torque can then be computed by equating the rate of work input required to drive the rolls with the total energy dissipation, $\dot{E}_{\text{tot}}$, rate computed from Eqn. (3.1). Thus, the total roll torque, $M_{\text{tot}}$, is given by,
\[ M_{tot} = \frac{\dot{E}_{tot}}{(\text{rpm} \cdot 2 \cdot 3.14159 / 60)} / 12 \quad (3.65) \]

where Eqn. (3.65) yields \( M_{tot} \) in dimensions of ft-lbf when \( \dot{E}_{tot} \) is given in dimensions of in-lbf/s. The same calculations are used in the shape rolling analysis.

III.14 Analysis of Stresses and the Roll Separating Force

The slab method of analysis\(^{(98,105)}\) is an approximate method for analyzing the stresses in plastic deformation problems. In applying the slab method of analysis the volume of the plastically deforming body is divided into a network of trapezoidal slab-like elements. The stress distribution for individual elements is then determined from elementary stress analysis techniques, permitting the stress distribution for the entire body to be obtained by successive approximation of the stresses for the individual elements. The slab method used the same assumptions that are used in the upper-bound method of analysis (Sec. III.1). But in addition, in applying the slab method of analysis, it is also assumed that:

* plane surfaces in the material remain plane during deformation,
* and, it is assumed that material deforms in plane-strain, i.e., zero strain in one of the principal strain directions.

With these assumptions, the stress equilibrium equations can
be solved using elementary techniques.

The resulting compressive stress distribution across a typical deformation element, an element which is assumed to be forged between flat inclined platens in plane-strain, is illustrated in Fig. 16. Based on this slab method of analysis, it can be shown that the principal stress in the $z$ coordinate direction, $\sigma_z$, across the deformation element, Fig. 16, is given by the following (105),

$$\sigma_z = K_2 \cdot \ln \left[ \frac{h_e}{h_b + K_1 x} \right] + \sigma_{ze} \quad (3.66)$$

where $\sigma_{ze}$ is a known principal stress value, known from analysis of the stress distribution across the adjacent element, and where,

$$K_1 = \tan \alpha + \tan \beta \quad (3.67)$$

and,

$$K_2 = \frac{2 \bar{\sigma}}{\sqrt{3}} K_1 + \frac{\bar{\sigma}}{\sqrt{3}} \left( 2 + \tan^2 \alpha + \tan^2 \beta \right) \quad (3.68)$$

and where the remaining symbols in Eqns. (3.66)-(3.68) are explained in Fig. 16.

In analyzing the stresses in practical situations the stress distribution equations, Eqns. (3.66)-(3.68), are applied across the plastically deforming body until a complete stress distribution is obtained. Integration begins at the perimeter of the body where the stress boundary condition $\sigma_z = -2 \bar{\sigma}/\sqrt{3}$ is imposed. Integration then proceeds from all sides (in discrete steps corresponding to each slab-like element) towards the
Figure 16. Compressive stresses in plane-strain upsetting between inclined plates and with unit depths, divergent outward flow assumed (ref. 71).
interior of the body. Integration proceeds against the direction of the surface shear stress, $\tau$, Fig. 16, always integrating either across the width of the rolled section, or along the directions of the streamlines of the metal flow pattern. As the stress distribution is constructed (since the stresses are integrated in opposite directions along the same streamline) more than one stress value can be predicted at a given interior point. When these conflicts occur they are resolved by taking the lower of the predicted stress values as the actual stress. Consequently, a tent-like stress distribution is obtained.

An example of the application of the slab method of analysis was shown in Fig. 12 - the predicted stress distribution for an oval-round pass shape combination, as simulated by the RPDROD computer program(74). Integration of this tent-like stress distribution (over the plane of contact between the billet and roll surfaces) gives the roll separating force. This technique has been successfully applied to the analysis of the stresses in the rolling of airfoils(71), plates(72), and rods(73,74).
CHAPTER IV
RESULTS FROM THE FLAT ROLLING ANALYSIS

In this section the overall results from the flat rolling analysis are discussed and comparisons are made between predicted and measured values of spread, elongation, bulge, roll separating force, and roll torque for the hot rolling of steel and the cold rolling of steel and lead. These measured values were obtained from various sources in the published literature. Discussion of the factors which may lead to single-bulge or double-bulge deformation in rolling, and a suggested technique for estimating the interface friction conditions in rolling are also included.

IV.1 Example Flat Rolling Solution

In the present investigation the Method of Hookes and Jeeves with Discrete Steps(101) was used to search for the velocity field that minimizes the total energy dissipation rate, $\dot{E}_{\text{tot}}$. The track of the minimization variables (or the path taken in minimizing $\dot{E}_{\text{tot}}$) for an example flat rolling solution is shown in Table 1. Key input values of $W_b$, $H_b$, $H_f$, $D$, rpm, $\bar{\sigma}$, and $m$ for this solution are
summarized in Table 2. Also shown in Table 1 are the total energy dissipation rate calculations, $E_{\text{tot}}$, which were made along this track. The starting values for $(A' X_d/A_b)$, $(A_f/A_b)$, $B_1$, $B_2$, and $(X_n/X_d)$, given in the first row in Table 1, were taken from Eqn. (3.59). The minimization variable tolerances, or the allowable uncertainties in finding the minimizer of the upper-bound functional, are additional input values for the program which were set as follows: 0.002 for $(A' X_d/A_b)$ and $(A_f/A_b)$, 0.02 for $B_1$ and $B_2$, and 0.004 for $(X_n/X_d)$. Internally, the integration step sizes for the Newton-Cotes formulae were adjusted in order to maintain at least six significant digit integration precision. As can be seen in Table 1, by comparing the last two energy dissipation rate values, this precision was necessary in to order clearly identify the minimum of the total energy dissipation rate. The predicted solution to the metal deformation problem, represented by the final values of the minimization variables, is given in the last row in Table 1.

Extensive numerical testing determined that changing the initial guess values for the BILLET analysis does not change the final solution, at least not more than the changes that are allowed by the input tolerances for the minimization variables, only the CPU time was affected. With the initial guess values taken from Eqn. (3.59), the elapsed CPU time for this example solution, Tables 1 and 2,
Table 1. Example track of minimization variables for rolling of square-rectangle pass as simulated by BILLETS computer program.

<table>
<thead>
<tr>
<th>A</th>
<th>X/A</th>
<th>A/Ab</th>
<th>B1</th>
<th>B2</th>
<th>X/Xc</th>
<th>E_tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.363</td>
<td>0.757</td>
<td>0.000</td>
<td>0.000</td>
<td>0.700</td>
<td>14.1713</td>
<td></td>
</tr>
<tr>
<td>-0.395</td>
<td>0.757</td>
<td>-0.320</td>
<td>-0.320</td>
<td>0.700</td>
<td>13.8680</td>
<td></td>
</tr>
<tr>
<td>-0.459</td>
<td>0.725</td>
<td>-0.320</td>
<td>-0.320</td>
<td>0.700</td>
<td>13.8093</td>
<td></td>
</tr>
<tr>
<td>-0.555</td>
<td>0.725</td>
<td>-0.320</td>
<td>-0.320</td>
<td>0.700</td>
<td>13.5128</td>
<td></td>
</tr>
<tr>
<td>-0.683</td>
<td>0.725</td>
<td>-0.320</td>
<td>-0.320</td>
<td>0.700</td>
<td>13.2781</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.725</td>
<td>-0.320</td>
<td>-0.320</td>
<td>0.636</td>
<td>13.2196</td>
<td></td>
</tr>
<tr>
<td>-0.843</td>
<td>0.725</td>
<td>-0.320</td>
<td>-0.320</td>
<td>0.636</td>
<td>13.2467</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.725</td>
<td>-0.160</td>
<td>-0.320</td>
<td>0.668</td>
<td>13.2110</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.725</td>
<td>-0.160</td>
<td>-0.320</td>
<td>0.668</td>
<td>13.2110</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.725</td>
<td>-0.240</td>
<td>-0.320</td>
<td>0.652</td>
<td>13.2078</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.725</td>
<td>-0.240</td>
<td>-0.320</td>
<td>0.652</td>
<td>13.2078</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.725</td>
<td>-0.240</td>
<td>-0.320</td>
<td>0.660</td>
<td>13.2075</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.725</td>
<td>-0.240</td>
<td>-0.320</td>
<td>0.660</td>
<td>13.2075</td>
<td></td>
</tr>
<tr>
<td>-0.779</td>
<td>0.723</td>
<td>-0.220</td>
<td>-0.320</td>
<td>0.660</td>
<td>13.2071</td>
<td></td>
</tr>
<tr>
<td>-0.781</td>
<td>0.723</td>
<td>-0.220</td>
<td>-0.320</td>
<td>0.660</td>
<td>13.2070</td>
<td></td>
</tr>
<tr>
<td>-0.781</td>
<td>0.723</td>
<td>-0.220</td>
<td>-0.320</td>
<td>0.660</td>
<td>13.2070</td>
<td></td>
</tr>
</tbody>
</table>

*One row of entries is given per minimization step - first row is the initial guess - last row is the solution, E_tot, is given in lbf-in/s assuming unity flow stress, and key input parameters used in BILLETS program are given in Table 2.*

Table 2. Key input parameters used in the BILLETS program to obtain the example flat rolling solution given in Table 1.

- \( W = 1.00 \text{ in}, \quad H_D = 1.00 \text{ in}, \quad H_f = 0.60, \)
- \( D = 21.33 \text{ in}, \quad \text{rpm} = 20.10, \)
- \( \bar{\sigma} = 1.00 \text{ psi}, \quad m = 0.75 \)
in which the bulge and elongation were analyzed simultaneously, was approximately 25 min. on the VAX 11/750 mini-computer.

IV.2 Nomenclature for Bulge Profile and Spread Predictions

In the present investigation the predicted exit width profiles for the flat rolling analysis were never truly rectangular, there was always a certain amount of bulge. The specific shape of the predicted bulge profile depended on the rolling conditions. Therefore, in discussing the exit width and bulge profile predictions it was necessary to establish a nomenclature, as outlined in the following:

* referring to the dimensioned exit cross-section shapes shown in Figs. 17 and 18, the single-bulge shape profile, Fig. 17, is defined as a bulge profile for which the exit width at the center or mid-thickness of the rolled section, \( W_c \), Fig. 17, is equal to the maximum exit width of the rolled section, \( W_m \), Fig. 18; however, in the present investigation small dimensional effects are neglected by declaring single-bulge profiles whenever the predicted values of \( W_m \) and \( W_c \) differ by less than 1 percent;

* double-bulge, Fig. 18, is defined as the case in which the maximum width, \( W_m \), exceeds center width, \( W_c \), and
when the exit width at the top of the rolled section, $W_t$, Figs. 17 and 18, is less than the center width, $W_c$; thus, double-bulge corresponds to the case in which $W_m > W_c$ and $W_t < W_c$; and,

* it was convenient to define a third category for the more severe double-bulge profiles, called hour-glass-bulge, for the cases in which the width at the top of the section, $W_t$, was greater than the center width, $W_c$; thus, the term hour-glass-bulge is used when $W_m > W_c$ and $W_t > W_t$ (the hour-glass bulge shape is discussed in detail in following paragraphs).

In addition, in comparing the predicted and measured values it is convenient to define:

\[
\begin{align*}
\text{percent true spread} & = \frac{(W_f - W_b)}{W_b} \times 100, \\
\text{percent maximum spread} & = \frac{(W_m - W_b)}{W_b} \times 100, \\
\text{percent bulge} & = \frac{(W_m - W_t)}{H_b} \times 100, \\
\text{percent thickness reduction} & = \frac{(H_b - H_f)}{H_b} \times 100, \\
\text{fractional thickness reduction} & = \frac{(H_b - H_f)}{H_b}, \\
\text{width/thickness ratio} & = \frac{W_b}{H_b}, \\
\text{roll diameter/thickness ratio} & = \frac{D}{H_b},
\end{align*}
\]

where the parameter $W_f$, defining percent true spread, is the true average width of the exit section, $W_f = A_f/H_f$. 
Figure 17. Schematic illustration of single-bulge profile and definitions of exit width dimensions with $W_D/H_D = 1$, $D/H_D = 10.8$, and 35.4% reduction.

Figure 18. Schematic illustration of double-bulge profile and definitions of exit width dimensions with $W_D/H_D = 0.5$, $D/H_D = 5.2$, and 34.4% reduction.
IV.3 Comparison Between Predicted and Measured Values in Hot Rolling

To evaluate the capabilities of the BILLETTS program, the theoretical results were compared with measured values of true spread, maximum spread, roll separating force, and roll torque which were obtained in previous Battelle studies (71, 72). These studies considered the metal flow in plate rolling under controlled laboratory hot rolling conditions, without lubrication, with AISI 1018 steel specimens. In these studies a range of specimen widths, thicknesses, and thickness reductions were considered. Prior to hot rolling, the specimens were annealed (thermal soak at 1625 F for 4 hr. followed by furnace cool for 8 hr.) and then shot blasted to remove any scale that formed during annealing. After annealing, the specimens were reheated in an electric furnace at 1832 F and soaked for at least 40 min. to ensure uniform temperature distribution. An argon gas atmosphere was used in the furnace to protect the specimens from excessive oxidation, and insulated tongs were used to remove the specimens and quickly introduce them to the rolls. For rolling, the experiments were conducted on a two-high rolling mill with 16 in. diameter steel rolls. The nominal roll temperature was room temperature. The plate specimens were rolled to various thickness reductions at a rolling speed of approximately 100 fpm. All specimen
dimensions, before and after rolling, were measured at room
temperature. Some of the rolled plate specimens were
sectioned to show the spread and bulge profiles. Fig. 19
shows the results from the rolling of 1 in. square plate
specimens to various thickness reductions. All of the
rolled specimens were reported to have single-bulge
profiles.

To estimate the interface friction conditions, ring
compression tests(71) were conducted using unlubricated AISI
1018 steel ring specimens heated to 1832 F, yielding an
estimated friction shear factor, m, equal to 0.75. In the
present investigation the flow stress for rolled alloy, AISI
1018 steel, was estimated using an established empirical
flow stress equation for hot working (Sec. III.12),

\[
\bar{\sigma} = C \dot{\varepsilon}^m,
\]

based on input empirical coefficients (C and m values in the
strain, strain-rate, and temperature range of the hot
rolling experiments) given elsewhere(98).

Comparison between the predicted and measured values of
percent true spread (definition in Sec. IV.2) is shown in
Fig. 20. Experimental values of \(W_f\), Fig. 20, were
estimated from measured values of the lengths of the rolled
specimens before and after rolling, \(L_b\) and \(L_f\), respectively,
using the constant volume condition,

\[
W_f \cdot L_f \cdot H_f = A_b \cdot L_b,
\]
Figure 19. Various AISI 1018 steel plate cross-sections produced under hot rolling conditions by rolling a square billet to various thickness reductions ($W_b/H_b = 1, D/H_b = 16$).
Figure 20. Comparison between predicted and measured values of true spread, $W_e$, in hot rolling of AISI 1018 plate specimens at 1832°F (ref. 72 experiments).
from which,
\[ W_f = \left( \frac{A_b \cdot L_b}{L_f \cdot H_f} \right). \] (4.3)
Theoretical true spread values were obtained using the BILLETS computer program for corresponding roll
diameter/thickness ratio of 16, width/thickness ratios of 1 and 2, thickness reductions ranging up to 40 percent, and for an interface shear friction factor, \( m = 0.75 \), the value which was estimated in the ring compression test(71). The predicted and measured values are given in terms of
dimensionless parameters (roll diameter/thickness ratio, width/thickness, etc.) because this is considered to be a more general presentation. It can be seen that the predicted true spread values, quantities which are directly related to the actual elongation, significantly underestimate the measured values, especially at heavy reductions.

On the other hand, as shown in Fig. 21, the agreement between the predicted and measured values of maximum spread, a quantity which is based on the billet outside width measurement, \( W_m \), is significantly better. The corresponding predicted and measured values of roll torque and roll separating force are shown in Figs. 22 and 23, respectively. The comparisons shown graphically in Figs. 20, 21, 22, and 23 are also provided in numerical form in Tables 3, 4, 5, and 6, respectively.
Figure 21. Comparison between predicted and measured values of maximum spread, W_m, in hot rolling of AISI 1018 plate specimens at 1832 F (ref. 71 experiments).
Figure 22. Comparison between predicted and measured values of roll torque in hot rolling of AISI 1018 plate specimens at 1832 F (ref. 71 experiments).
Figure 23. Comparison between predicted and measured values of roll separating force in hot rolling of AISI 1018 plate specimens at 1832 F (ref. 71 experiments).
Table 3. Comparison between theoretical true spread predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 72 experiments).

\[
\begin{array}{cccc}
W_b/H_b & \% \text{ reduction} & \text{theoretical} & \text{measured} & \text{difference} \\
1 & 10.1 & 3.6 & 4.7 & -1.1 \\
1 & 18.6 & 7.6 & 9.4 & -1.8 \\
1 & 28.7 & 12.5 & 18.1 & -5.6 \\
1 & 38.8 & 16.9 & 29.1 & -12.2 \\
2 & 9.0 & 1.4 & 2.3 & -0.9 \\
2 & 18.6 & 4.0 & 5.2 & -1.2 \\
2 & 27.9 & 6.8 & 8.9 & -2.1 \\
2 & 37.7 & 9.2 & 15.3 & -6.1 \\
\end{array}
\]

Table 4. Comparison between theoretical maximum spread predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 71 experiments).

\[
\begin{array}{cccc}
W_b/H_b & \% \text{ reduction} & \text{theoretical} & \text{measured} & \text{difference} \\
1 & 11.8 & 4.9 & 3.9 & 1.0 \\
1 & 21.3 & 9.7 & 9.1 & 0.5 \\
1 & 31.2 & 15.5 & 17.0 & -1.5 \\
1 & 42.5 & 21.7 & 30.0 & -8.3 \\
2 & 11.1 & 2.2 & 2.4 & -0.2 \\
2 & 20.6 & 5.3 & 4.9 & 0.4 \\
2 & 30.3 & 8.6 & 8.4 & 0.2 \\
2 & 39.9 & 11.4 & 14.3 & -2.9 \\
\end{array}
\]
Table 5. Comparison between theoretical total roll torque predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 71 experiments).

\[
m = 0.75, H_b = 1.0 \text{ in}, D = 16.0 \text{ in},
\]

<table>
<thead>
<tr>
<th>( \frac{W_b}{H_b} )</th>
<th>% reduction</th>
<th>theoretical</th>
<th>measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.8</td>
<td>1.5</td>
<td>1.2</td>
</tr>
<tr>
<td>1</td>
<td>21.3</td>
<td>3.0</td>
<td>2.4</td>
</tr>
<tr>
<td>1</td>
<td>31.2</td>
<td>4.9</td>
<td>3.8</td>
</tr>
<tr>
<td>1</td>
<td>42.5</td>
<td>7.6</td>
<td>5.6</td>
</tr>
<tr>
<td>2</td>
<td>11.1</td>
<td>2.9</td>
<td>2.4</td>
</tr>
<tr>
<td>2</td>
<td>20.6</td>
<td>6.0</td>
<td>4.6</td>
</tr>
<tr>
<td>2</td>
<td>30.3</td>
<td>9.9</td>
<td>7.8</td>
</tr>
<tr>
<td>2</td>
<td>39.9</td>
<td>14.5</td>
<td>10.7</td>
</tr>
</tbody>
</table>

Table 6. Comparison between theoretical roll separating force predictions made by the BILLETS computer program and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 71 experiments).

\[
m = 0.75, H_b = 1.0 \text{ in}, D = 16.0 \text{ in},
\]

<table>
<thead>
<tr>
<th>( \frac{W_b}{H_b} )</th>
<th>% reduction</th>
<th>theoretical</th>
<th>measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.8</td>
<td>6.7</td>
<td>4.0</td>
</tr>
<tr>
<td>1</td>
<td>21.3</td>
<td>10.5</td>
<td>7.5</td>
</tr>
<tr>
<td>1</td>
<td>31.2</td>
<td>14.6</td>
<td>11.5</td>
</tr>
<tr>
<td>1</td>
<td>42.5</td>
<td>19.3</td>
<td>17.0</td>
</tr>
<tr>
<td>2</td>
<td>11.1</td>
<td>12.8</td>
<td>10.5</td>
</tr>
<tr>
<td>2</td>
<td>20.6</td>
<td>20.5</td>
<td>17.5</td>
</tr>
<tr>
<td>2</td>
<td>30.3</td>
<td>28.4</td>
<td>26.0</td>
</tr>
<tr>
<td>2</td>
<td>39.9</td>
<td>36.3</td>
<td>35.0</td>
</tr>
</tbody>
</table>
Overall, except for the true spread measurements, it can be seen that the agreement between the predicted and measured values using the BILLETS program is well within acceptable engineering accuracy. In particular, the maximum spread predictions, Fig. 21 and Table 4, were in good agreement with the measured values. Although the roll torque predictions are somewhat higher than the measured values, Fig. 22 and Table 5, this is not unexpected in an upper-bound analysis because in rolling the roll torque is source of the energy that drives the process, and because the upper-bound functional always predicts a somewhat higher total energy dissipation rate\(^{(92,93)}\). It is felt that the tendency for the predicted roll separating force values to be higher than the measured values, Fig. 23 and Table 6, a tendency which was reported in earlier studies\(^{(71,72)}\), is due to approximations introduced by using the slab method (Sec. III.14) to determine the stress distribution. The tendency for the predicted roll separating force values to be closer to the measured values at heavy reductions, Fig. 23 and Table 6, is attributed to roll chilling which makes the billet flow stress under the rolls somewhat greater than the values calculated assuming an average rolling temperature of 1832 F. To show the extent to which a decrease in temperature (or roll chilling) can increase the value of the billet flow stress, example flow stress data curves for AISI 1045 steel (a representative mild carbon
steel with temperature dependent flow stress behavior similar to that of AISI 1018 steel) are shown in Fig. 24. It can be seen that a drop in temperature of only a few hundred degrees Fahrenheit can increase the billet flow stress appreciably.

To determine the accuracy of the bulge profile predictions, the predicted values were compared with exit width measurements, \( W_c \) and \( W_t \) values, from hot rolling experiments conducted at Aachen University (23). These experiments were very similar to the Battelle hot rolling experiments. The rolling trials were conducted without lubrication with mild carbon steel specimens (a 0.04 % carbon steel) at a rolling temperature of 1832 F and a rolling speed of approximately 100 fpm. However, the specimen geometries considered in these experiments were somewhat different; width/thickness ratios of 1.0 and 0.5 and thickness reductions in the range of 35 percent were considered. Graphical representation of the corresponding entry geometries and reductions for the Aachen University experiments was given in Figs. 17 and 18. However, unlike the Battelle study (71), experimental determination of the interface friction conditions was not considered. Therefore, for comparing the predicted and measured values it was necessary to assume limiting values for the interface friction shear factor based on what can be estimated as the practical range of friction conditions in the hot rolling of
Figure 24. Example flow stress curves for AISI 1045 steel.
steel, specifically, \( m = 0.5 \) and \( m = 1.0 \). The upper limit, \( m = 1.0 \), sticking friction, does occur in the hot rolling of steel, especially in unlubricated hot rolling\(^{(106)}\). The lower limit, \( m = 0.5 \), can be observed in the hot rolling of steel, but only when a roll coolant spray or some other lubricating agent is introduced\(^{(106)}\).

The comparison between the predicted and measured values of \( W_c \) and \( W_t \) for \( W_b/H_b = 1.0 \) are given in Table 7. For the predicted bulge profiles, which were single-bulge type, Fig. 18, it can be seen that \( W_t \) decreases as the friction is increased. It can also be seen that the value of \( W_c \) increases with higher friction. Thus, as expected, the BILLETS analysis predicts greater outward barreling when the friction conditions are more severe (the predicted values of \( W_c \) and \( W_t \) differ by only 1.3 mm. at \( m = 0.5 \), yet they differ by 3.7 mm. at \( m = 1.0 \)). The best agreement between the predicted and measured values, Table 7, is obtained with \( m = 1.0 \), the sticking friction condition, at which point the predicted and measured values of \( W_c \) and \( W_t \) differ by just a few millimeters. As stated earlier, sticking friction, \( m = 1.0 \), is observed in the hot unlubricated rolling of steel.

The comparison between the predicted and measured values of \( W_c \) and \( W_t \) for \( W_b/H_b = 0.5 \) are given in Table 8. Also shown in Table 8 are the predicted values of \( W_m \), the maximum width of the exit cross-section. The predicted
Table 7. Comparison between theoretical exit width predictions made by the BILLETS computer program and experiments for hot rolling of a 0.04 % carbon steel at 1832 F (ref. 23 experiments).

<table>
<thead>
<tr>
<th>m</th>
<th>$W_c$</th>
<th>$W_t$</th>
<th>$W_c$</th>
<th>$W_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>33.3</td>
<td>32.0</td>
<td>38.1</td>
<td>32.3</td>
</tr>
<tr>
<td>1.0</td>
<td>35.6</td>
<td>31.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8. Comparison between theoretical exit width predictions made by the BILLETS computer program and experiments for hot rolling of a 0.04 % carbon steel at 1832 F (ref. 23 experiments).

<table>
<thead>
<tr>
<th>m</th>
<th>$W_m$</th>
<th>$W_c$</th>
<th>$W_t$</th>
<th>$W_c$</th>
<th>$W_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>33.6</td>
<td>32.4</td>
<td>32.4</td>
<td>37.8</td>
<td>29.0</td>
</tr>
<tr>
<td>1.0</td>
<td>35.4</td>
<td>34.6</td>
<td>32.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
values of \( W_m \) are included in Table 8 because the predicted bulge profiles for this situation were double-bulge, Fig. 18, instead of the single-bulge type, Fig. 17, contrary to the single-bulge profile that was observed in the actual experiment\(^{23} \). Although the double-bulge prediction was incorrect, at least the predicted double-bulge profile is not very severe (with \( m = 1.0 \), Table 8, the predicted values of \( W_m \) and \( W_c \) differ by less than 1 mm). The discrepancy in the predicted bulge profile type can be attributed to a certain amount of roll chilling in the actual rolling experiments. Roll chilling would increase the material flow stress (and resistance to plastic deformation) in the billet surface layers, cause the spread to become localized in the billet mid-thickness layers, and, thereby, promote single-bulge profiles\(^6 \). Despite the double-bulge prediction, the BILLETS program still predicts the outside width measurement fairly accurately. With \( m = 1.0 \) the predicted value of \( W_m \) differs from the measured value of \( W_c \) by only 2.4 mm.

In regard to the possibility of folding in these experiments (a factor not considered in the BILLETS analysis), it is felt that folding was not a significant factor. In the Battelle experiments, for example, although the corners of the actual rolled shape cross-sections are somewhat rounded, Fig. 19, they are not significantly rounded. In addition, if folding was significant in the
Aachen University experiments, then the measured values of \( W_t \) would be expected to be much greater than the predicted values, but the measured values are not. In fact, with the very narrow entry section, \( W_b/H_b = 0.5 \), Table 8, the measured value of \( W_t \) is actually less than the predicted value. Also, it is interesting that the measured interface spread with \( W_b/H_b = 0.5 \), Table 8, was actually negative, i.e., the measured value of \( W_t \) was less than \( W_b \).

Although the reported negative spread at the top of the rolled section might be due to experimental error, a negative spread value does suggest that the assumption that plane sections perpendicular to the direction of rolling remain plane during deformation (Sec. III.3) can become untenable under certain circumstances. In reality plane sections do not remain plane in rolling. The actual thickness reduction and elongation is greater in the billet surface layers on the entry plane side of the deformation zone, and greater in the billet mid-thickness layers towards the exit plane side, especially with very tall billets and at heavy reductions(5). Thus, the most plausible explanation for negative spread at the top of the rolled section, Table 8, is that plane sections did not remain plane on the entry plane side of the deformation zone, and then, later on, the billet surface layers were pulled-along by the elongation in the mid-thickness layers, shrinking the top of the rolled section towards the exit plane. The
important issue in analyzing the metal deformation problem in rolling is, however, to determine the practical ranges of rolling conditions for which this assumption can be applied with reasonable results. If the actual spread and metal flow at the very top of the rolled section is the aspect of the metal deformation problem which is of interest, then the Aachen University conditions may be a limiting case. However, in most practical situations the important predicted values would be the maximum spread and bulge profile. The predicted interface spread values are less important.

Concerning the discrepancies between the predicted and measured values of true spread at heavy reductions in the Battelle experiments, Fig. 20, these discrepancies could also be related to the plane sections remain plane assumption because the natural effect of any inappropriate analytical constraint would be to confine the metal flow and, thereby, limit the predicted true spread values. If inappropriate, the application of this assumption in formulating the velocity field (Sec. III.4) could very easily account for the discrepancies at heavy reductions, Fig. 20. However, it is important to consider the possibility that these discrepancies could also be due to the roll chilling effect.
IV.4 Comparison Between Predicted and Measured Values in Cold Rolling

Additional Battelle experiments (71) for the cold rolling of AISI 1018 steel plate specimens are considered. The cold rolling experiments, like the hot rolling experiments, were conducted on Battelle's two-high rolling mill with 16 in. diameter rolls with a rolling speed of approximately 100 fpm. The interface friction conditions were estimated by conducting room temperature ring compression tests. The estimated friction shear factor, m, for the cold rolling of steel, was equal to 0.50. The flow stress of AISI 1018 steel, for cold metal forming conditions, was taken from empirical flow stress data given elsewhere (98).

Comparison between the predicted and measured values of maximum spread for the cold rolling of mild carbon steel are shown in Fig. 25. Though experimental data were available only for low reductions, the agreement between the predicted and measured values is good, comparable to the agreement with the hot rolling data given in Fig. 21. This suggests that the influence of chilling on maximum spread data in the Battelle hot rolling experiments was not significant, at least not significant enough to cause large discrepancies between the predicted and measured values. It is felt that the low thickness reductions taken in the Battelle cold
Figure 25. Comparison between predicted and measured values of maximum spread, $W_m$, in cold rolling of AISI 1018 plate specimens (ref. 71 experiments).
rolling experiments is another contributing factor to the agreement between the predicted and measured values, Fig. 25, because over low reductions any errors due to the influence of strain in the flow stress of AISI 1018, at cold working temperatures, will not be significant. The predicted and measured values of roll torque, Fig. 26, are also in good agreement. Corresponding measured values of true spread and roll separating force were unavailable for comparison.

Concerning the predicted true spread values, the cause of the discrepancies in the hot rolling experiments, Fig. 20, is, as yet, undetermined. These discrepancies could be due to the roll chilling effect, or they could due to the inappropriateness of the plane sections remain plane assumption. In this regard, predicted true spread values were compared with measured values of true spread in the unlubricated cold rolling of lead(24). Since the flow stress of lead is mostly a function of strain-rate at room temperature(98,107), lead in cold working is often used as a model material for the deformation and flow of steel under hot working conditions(107). The friction shear factor, m, for cold rolling of lead, under dry conditions, is assumed to be equal to 0.75. It is felt that this is a very reasonable assumption since measured Coulomic friction coefficients of 1.0 are common for lead-base alloys sliding on steel(108).
Figure 26. Comparison between predicted and measured values of roll torque in cold rolling of AISI 1018 plate specimens (ref. 71 experiments).
Comparison between the predicted and measured values of true spread for the cold rolling of lead are shown in Fig. 27. The agreement between the predicted and measured values is very similar to that which was observed with the hot rolling of steel, Fig. 20, specifically, the predicted true spread values underestimate the measured values throughout, and especially at heavy reductions. This finding suggests that the influence of roll chilling on the true spread values, also, may not be significant. It is felt, therefore, that the tendency for the BILLETS analysis to underestimate the true spread measurements, Figs. 20 and 27, is primarily due to errors introduced by the plane sections remain plane assumption. This assumption, if inappropriate, would certainly confine the metal flow and limit the predicted true spread values. In addition, if predominant, these errors would unquestionably increase at heavy reductions, a trend which is fully consistent with the observed results, Figs. 20 and 27.

IV.5 Preliminary Conclusions from the Flat Rolling Analysis

Based on the comparisons made between the predictions from BILLETS computer program and selected hot and cold rolling experiments, the following conclusions are drawn:

* the BILLETS analysis, using input values of billet flow stress and interface friction conditions
Figure 27. Comparison between predicted and measured values of true spread, $W_f$, in cold rolling of lead (ref. 24 experiments).
determined independently by established experimental techniques, predicts the maximum spread, type of bulge profile, roll torque, and roll separating force fairly accurately;

* however, the predicted true spread and interface spread values are not in good agreement with the measured values, especially at heavy reductions.

Although several factors could contribute to the observed errors in the true spread and interface spread predictions (notably, roll chilling and folding - factors not included in the present flat rolling analysis), it is felt that the predominant errors are due to the plane sections remain plane assumption, an assumption which is a limitation in the velocity field.

IV.6 Factors Which Influence Percent Bulge and True Spread

The factors which influence the percent bulge and true spread predictions in the flat rolling analysis are also considered. The influences of friction, thickness reduction, and width/thickness ratio on percent bulge are shown in Tables 9 and 10. It can be seen that:

* percent bulge increases with increasing thickness reduction and increasing friction; and,
* percent bulge increases with decreasing width/thickness ratio.
Table 9. Theoretical percent bulge predictions made by the BILLETs computer program as a function of friction shear factor and thickness reduction.

\[ \frac{W_b}{H_b} = 1.0, \frac{D}{H_b} = 16.0, \]

<table>
<thead>
<tr>
<th>m</th>
<th>% reduction =</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.89</td>
<td>2.59</td>
<td>4.52</td>
<td>6.66</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>2.71</td>
<td>6.68</td>
<td>12.44</td>
<td>18.47</td>
<td></td>
</tr>
</tbody>
</table>

Table 10. Theoretical percent bulge predictions made by the BILLETs computer program as a function of width/thickness ratio and thickness reduction.

\[ m = 1.0, \frac{D}{H_b} = 16.0, \]

<table>
<thead>
<tr>
<th>( \frac{W_b}{H_b} )</th>
<th>% reduction =</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>3.64</td>
<td>8.80</td>
<td>15.18</td>
<td>23.00</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>2.71</td>
<td>6.68</td>
<td>12.44</td>
<td>18.47</td>
<td></td>
</tr>
</tbody>
</table>
The influences of friction, thickness reduction, and roll diameter on the predicted true spread values are shown in Tables 11 and 12. It can be seen that:

* true spread increases with increasing thickness reduction;
* at a given reduction, true spread increases with increasing friction; and,
* for constant friction and thickness reduction, the predicted true spread values also increase with increasing roll diameter.

Now, in previous work by Sevenler, Raghupathi, and Altan (85) bulge in flat rolling was analyzed using an upper-bound approach in which the bulge and elongation were analyzed separately. The analysis by Lahoti, Akgerman, and Altan (71) was used to predict spread and elongation (without considering the influences of the bulge) and then a plane-strain upsetting analysis was used to calculate bulge once the spread and elongation were given. Therefore, in the present investigation the effects of the simultaneous analysis of spread, elongation, and bulge on the predicted true spread values in flat rolling are also considered. Table 13 shows a comparison between theoretical true spread values predicted by the BILLETs program and corresponding true spread values predicted by the Battelle SHPROL program (71) for the conditions $W_b/H_b = 1$ and 2, $D/H_b = 16$, and $m = 0.75$. SHPROL is the parent computer program for
Table 11. Theoretical true spread predictions made by the BILLETS computer program as a function of friction shear factor and thickness reduction.

\[ \frac{W}{H} = 1.0, \frac{D}{H} = 16.0. \]

<table>
<thead>
<tr>
<th>% reduction</th>
<th>m = 0.5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>3.26</td>
<td>7.25</td>
<td>10.66</td>
<td>12.64</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>3.77</td>
<td>9.22</td>
<td>15.39</td>
<td>21.65</td>
</tr>
</tbody>
</table>

Table 12. Theoretical true spread predictions made by the BILLETS computer program as a function of roll diameter/thickness ratio and width/thickness ratio.

\[ m = 0.75, \% \text{ reduction} = 30.0, \]

<table>
<thead>
<tr>
<th>theoretical % true spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>D/H = 4</td>
</tr>
<tr>
<td>W/H = 0.5</td>
</tr>
<tr>
<td>W/H = 1.0</td>
</tr>
<tr>
<td>8.40</td>
</tr>
<tr>
<td>5.01</td>
</tr>
<tr>
<td>17.12 \text{a}</td>
</tr>
<tr>
<td>13.05</td>
</tr>
</tbody>
</table>

\text{a} This value was determined by linear interpolation between existing solutions having the same geometry but with friction shear factors of 0.5 and 1.0.
Table 13. Comparison between theoretical true spread predictions made by the BILLETS computer program and the Battelle SHPROL computer program (details of the SHPROL program are given in ref. 71).

\[ m = 0.75, \; D/H_B = 16.0. \]

<table>
<thead>
<tr>
<th>( W_b/H_b )</th>
<th>% reduction</th>
<th>BILLETS</th>
<th>SHPROL</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0</td>
<td>8.2</td>
<td>6.8</td>
<td>1.4</td>
</tr>
<tr>
<td>1</td>
<td>40.0</td>
<td>17.4</td>
<td>14.5</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>20.0</td>
<td>4.4</td>
<td>3.2</td>
<td>1.2</td>
</tr>
<tr>
<td>2</td>
<td>40.0</td>
<td>9.8</td>
<td>7.0</td>
<td>2.8</td>
</tr>
</tbody>
</table>

The true spread and elongation analysis that was used by Sevenler, Raghupathi, and Altan(85). It can be seen that analyzing the bulge and elongation simultaneously does make a difference, especially at heavy reductions. The BILLETS computer program consistently predicts a higher true spread value because it uses a more general velocity field, one that is less constrained than the velocity field used in SHPROL.
IV.7 Factors Which May Lead to Single-Bulge or Double-Bulge Deformation

In practical situations the mill power, roll separating force, and maximum thickness reduction (the reduction at which the billet is no longer accepted by the rolls) are often the limiting factors in single-pass rolling mill design. However, the bulge profile is also an important factor because certain bulge profiles, specifically, double-bulge and hour-glass-bulge profiles, tend to increase the occurrence of lapping and edge cracking, rolling defects which decrease mill yields (1, 6, 7, 16).

Based on the results of the BILLETS analysis, it was determined that the type of bulge profile that develops in the pass (whether it be single-bulge, double-bulge, or hour-glass-bulge) is primarily a function of the geometric aspects of the pass design, specifically, the entry billet width and thickness, the roll diameter, and the thickness reduction. Changing the friction shear factor, over the practical range of friction conditions, had little influence on the predicted bulge profile type. However, increasing the friction shear factor does exaggerate the concurrent bulge profile.

The influence of thickness reduction on the bulge profile prediction (for constant width/thickness ratio, roll diameter/thickness ratio, and friction shear factor) is
shown in Fig. 28. It can be seen that an hour-glass-bulge profile develops at a fractional thickness reduction of 0.25, but a single-bulge profile is produced at a fractional reduction of 0.40. In most instances the hour-glass-bulge and double-bulge profiles, as predicted by the BILLET program, can be considered to be the result of insufficient thickness reduction which allows the actual metal flow to be localized in the billet surface layers. Increasing thickness reduction will usually produce a single-bulge shape, Fig. 28, however in practical situations, with a free-standing mill, the maximum thickness reduction may be a limiting factor.

It was also observed that in rolling the predicted bulge profile is strongly dependent on the thickness of the billet entry cross-section. Fig. 29 shows how increasing billet thickness (for constant width, roll diameter, thickness reduction, and friction shear factor) can cause the predicted bulge profile to change from single-bulge, to double-bulge, to hour-glass-bulge. Conversely, Fig. 29 also shows how increasing width/thickness ratio and increasing roll diameter/thickness ratio can promote single-bulge and, thereby, improve the metal flow distribution, since the maximum values of these parameters ($W_b/H_b = 1.0$ and $D/H_b = 10$) correspond to the single-bulge shape, and the minimum values ($W_b/H_b = 0.25$ and $D/H_b = 2.5$) correspond to the hour-glass-bulge shape.
$W/H = 0.5$

$D/H = 4$

$\Delta H/H = 0.25 \quad \Delta H/H = 0.40$

HOUR-GLASS SINGLE-BULGE

BULGE ($W_m > W_c, W_t > W_c$) ($W_m = W_c$)

$m = 0.75$

Figure 28. Influence of thickness reduction on bulge profile.
W = 1 in
D = 10 in
ΔH/H = 0.30
m = 0.75

SINGLE-BULGE
(Wm=Wc)

DOUBLE-BULGE
(Wm>Wc, Wt<Wc)

HOUR-GLASS-BULGE
(Wm>Wc, Wt>Wc)

Figure 29. Influence of billet thickness on bulge profile.
The combined effects of the width/thickness ratio, roll diameter/thickness ratio, and thickness reduction on the bulge profile shape has been treated experimentally(16) by charting the type of bulge profile, at a given reduction, on a base of the width/thickness ratio multiplied by the roll diameter/thickness ratio. Fig. 30 shows the theoretical results from the BILLETS analysis charted in this manner for ranges of $0.25 < \frac{W_d}{H_d} < 1.00$, and $4 < \frac{D}{H_d} < 10$, with $m = 0.75$, using closed markers to denote the more severe hour-glass-bulge shapes. Estimated dividing lines between the different bulge shapes are also shown in Fig. 30. It is felt that Fig. 30 can be a useful design tool by permitting a designer to estimate, for given input values of $\frac{W_d}{H_d}$ and $\frac{D}{H_d}$, how much thickness reduction will be required to avoid hour-glass-bulge and double-bulge profiles, thereby offering the possibility of improved yields. Fig. 31 shows a comparable design chart determined from the results of production rolling trials for steel(16). The theoretically determined hour-glass-bulge dividing line, Fig. 30, and the experimentally determined double-bulge dividing line, Fig. 31, are similar in shape.
Figure 30. Bulge profile design chart developed from the theoretical analysis showing the combined influences that width/thickness ratio, roll diameter/thickness ratio, and thickness reduction have on the predicted bulge profile type.
Figure 31. Bulge profile design chart determined from the results of actual production rolling trials with steel (ref. 16, Courtesy Association of Iron and Steel Engineers).

IV.6 The Flat Bar Test - A Suggested Technique for Estimating the Interface Friction Conditions in Rolling

In previous Battelle studies (71-74) the interface friction conditions in rolling, under controlled laboratory conditions, were estimated using the established ring compression test (98). However, the friction conditions in a production rolling mill, especially a hot rolling mill, can be quite different from the friction conditions in a ring test due to the fact that the metal surface conditions and chilling effects, which strongly influence the friction
conditions, can differ greatly in the different tribological situations\(^{(106)}\). Therefore, because such influences are always included in estimating the friction conditions, errors may be introduced in using the ring test to estimate the friction conditions in rolling.

Among the actual rolling tests which are used to measure the friction conditions, most of the existing measurement techniques are practical only for cold strip rolling\(^{(106)}\). The latest measurement techniques attempt to sense the friction stresses in the roll gap by measuring the elastic deformation of the rolls using strain gages\(^{(109)}\). The flat bar test, on the other hand, is a suggested experimental technique that is considered practical for estimating the interface friction conditions in hot rolling.

It is suggested that the interface friction conditions in hot rolling can be estimated by comparing the predicted and measured values of maximum spread, predicted values coming from the BILLETs analysis, at various reductions for a single-pass flat rolling process. The measured maximum spread values would be compared with theoretical calibration curves. Fig. 32 shows suggested flat bar test calibration curves, predicted maximum spread values versus thickness reduction, for \(W_{b}/H_{b} = 1\), \(D/H_{b} = 16\), and for assumed values of \(m = 0.5\) and \(m = 1.0\), values of the friction shear factor which represent the practical range of friction conditions for the hot rolling of steel. Also shown in Fig. 32 are
Figure 32. Flat bar test theoretical calibration curves with $W/H = 1.0$ and $D/H = 16.0$. 
the corresponding maximum spread measurements for the Battelle hot rolling experiments. It can be seen that the predicted maximum spread values are sensitive to the friction shear factor, m, especially at heavy reductions. The sensitivity of the maximum spread to friction is attributed to the following.

First, as shown in Sec. IV.6, the predicted true spread values increase with increasing friction, and with greater spread there naturally tends to be greater sensitivity of spread to friction. In addition, increasing friction exaggerates the concurrent single-bulge profiles - the type of bulge profile which is prevalent at heavy reductions - which, in turn, increases the maximum spread values even further. Thus, the combination of greater true spread and greater outward barrelling makes the predicted maximum spread values, the suggested flat bar test calibration curves, Fig. 32, surprisingly sensitive to the friction shear factor, m, at least at heavy reductions.

To determine the value of the friction shear factor, m, for a given experimental condition, the measured values of maximum spread versus thickness reduction are placed on the appropriate calibration curve, Fig. 32. From the positions of the experimental points with respect to the theoretical curves, given for various values of m, the value of the friction shear factor that existed in the actual experiment can be determined. From the flat bar test calibration
curves given in Fig. 32, although the friction shear factor tends to increase with increasing reduction - probably due to increased chilling effects(98), a value of \( m = 0.75 \) could be taken as the average value for the friction conditions for the Battelle hot rolling experiments, an average value taken over the full range of thickness reductions. This average value from the flat bar test, \( m = 0.75 \), would be the same value as that determined earlier by the ring compression test(71). In addition to being practical, the flat bar test would be easy to carry out, because it does not require any special measurement equipment, and it only requires the measurement of the outside width of a rolled flat bar cross-section.

The idea of improving the flat bar test sensitivity by decreasing the value of \( W_b/H_b \) was also considered. The new calibration curves, with \( W_b/H_b = 0.5 \), are shown in Fig. 33. It can be seen that (comparing the calibration curves in Figs. 32 and 33) decreasing \( W_b/H_b \) improves the test sensitivity at heavy reductions, but the test sensitivity gets worse at low reductions. Although there may be many contributing factors, it is felt that the influence of the bulge profile is critical because (refering to Fig. 30 using \( W_b/H_b = 0.5 \), \( D/H_b = 16 \), and \( W_b/H_b \times D/H_b = 8 \)) at low reductions (at or below a fractional reduction of 0.15) the hour-glass-bulge shape is important and, as a result, (refering to the hour-glass-bulge profile shown in Fig. 29)
Figure 33. Flat bar test theoretical calibration curves with $W/H = 0.5$ and $D/H = 16.0$. 
the metal flow becomes localized in the billet surface layers. Therefore, the worsening of the flat bar test sensitivity at low reductions is attributed to metal flow localization, caused by the hour-glass-bulge shape, which necessarily limits the effects that changing friction conditions can have on the metal flow of the entire section. Conversely, the improvement in test sensitivity at heavy reductions is attributed to the fact that the single-bulge profiles return at the heavier reductions, at fractional reductions above 0.30 in Fig. 30. The crossing over of the theoretical calibration curves at very low reductions, Fig. 33, is considered an anomaly, attributable to the small, yet nonzero, minimization variable tolerances which are used as input variables to the BILLETS computer program.

A possible solution to the test sensitivity problem would be to increase the value of $D/H_b$ (increase the roll diameter) since this change could shift the hour-glass-bulge transition back to much lower reductions, Fig. 30. However, increasing $D/H_b$ much further, currently $D/H_b = 16$, may not work very well in practical situations (in a free-standing laboratory mill without entry and exit guides) because with large roll diameters and at heavy reductions a very narrow flat bar entry cross-section could fall over, or lean sideways, under the compression of the rolls. This mode of deformation, called the parallelogram defect, would introduce significant errors in using the flat bar test to
estimate the interface friction conditions. Therefore, taking all factors into consideration, it is felt that the flat bar test calibration curves with $W_D/H_D = 0.5$ and $D/H_D = 16$ are very close to the optimum flat bar test geometry.
CHAPTER V
SHAPE ROLLING ANALYSIS

An upper-bound method of analysis for steady-state rolling (Secs. III.1 and III.2) is also developed for grooved roll pass sequences involving the rolling of symmetrical geometric shapes without protrusions, such as: squares, diamonds, ovals, and rounds. The velocity field equations that are used in the shape rolling analysis are adapted from the velocity field equations with bulge that were used in the flat rolling analysis (Sec. III.4).

In the flat rolling analysis the velocity field equations were chosen to consider, through application of the polynomial interpolation function, $B$, Eqn. (3.16), linear velocity distributions in the lateral direction, across the width of the deforming billet cross-sections, and nonlinear velocity distributions through the thickness of the deforming billet cross-sections. The parameters $B_1$ and $B_2$ were used to determine the bulge of the deforming billet cross-sections, and the parameter $b_1$ was evaluated in order to satisfy velocity boundary conditions. In the shape rolling analysis the same basic velocity field formulation is utilized, specifically, Eqns. (3.12)-(3.14). However,
the polynomial interpolation function, \( B \), originally defined in Eqn. (3.16), is modified for the shape rolling analysis to also consider nonlinear material velocity distributions in the lateral direction - lateral velocity distributions which are controlled to satisfy velocity boundary conditions and make the deforming billet cross-sections conform to the specific roll groove shape being considered. Thus, except for the obvious differences in the assumed billet and roll cross-section geometries, and the fact that a more general velocity field formulation is utilized, in most respects the shape rolling analysis is identical to the flat rolling analysis discussed earlier.

V.1 Shape Rolling Process Considered in the Analysis

A schematic illustration of the shape rolling process considered in the present investigation is shown in Fig. 34. In the shape rolling analysis it is assumed that the billet cross-section at entry can be approximated by a string of \( n_p \) discrete cross-section points. It is also assumed that a pair of rigid grooved rolls, with outer diameter \( D \) and outside tangential roll velocity \( v_r \), draw the billet into the roll groove and reduce the billet cross-sectional area from \( A_D \) to \( A_f \). As in the flat rolling analysis, it is assumed that plane sections perpendicular to the direction of rolling remain plane during deformation. It is also
Figure 34. Schematic illustration of the shape rolling process being considered in the present investigation.
assumed that the entry plane of the deformation zone is a plane perpendicular to the direction of rolling located at the initial touching-point between the billet entry cross-section and the roll surfaces. It is assumed that the exit plane of the deformation zone is a plane perpendicular to the direction of rolling located within the plane which contains the roll centerlines. The projected length of the roll gap, $X_d$, the distance between the entry and exit planes, is also determined from the specified billet entry cross-section and the geometric parameters of the groove design. As in the flat rolling analysis, the coordinate axes for the shape rolling analysis are centered at the centroid of the billet entry cross-section with the $x$, $y$, and $z$ directions corresponding to the directions of elongation, spread, and thickness reduction, respectively. The overall setup for the shape rolling analysis is, therefore, the same as that used for the flat rolling analysis, an illustration of which was given in Fig. 14.

In the shape rolling analysis an analytical representation of the three-dimensional roll groove surface is required. This surface is a surface of revolution which can be defined analytically in terms of a function $z_r$, a function of the $x$ and $y$ coordinates, which expresses the $z$ coordinate values of points on the surface. This function, $z_r$, can be expressed as follows,

$$z_r(x,y) = z_c - \left[ (z_c-t_y)^2 - (X_d-x)^2 \right]^{0.5} \quad (5.1)$$
where

\[ z_r(x,y) = \text{the } z \text{ coordinates of the upper roll surface}, \]
\[ z_c = \text{the } z \text{ coordinate of the upper roll axis}, \]
\[ t_y(y) = \text{a function of the } y \text{ coordinate which equals} \]
half the roll separation at the exit plane and
which is, therefore, the function that defines
the roll groove shape.

For different roll groove shapes the function \( t_y \),
Eqn. (5.1), is determined directly from the engineering
drawing of the roll exit cross-section. A variety of roll
cross-section shapes, dimensioned in terms of the fillets
and rounds of the exit sections, was shown in Fig. 9 as the
selection of pass shapes considered by the RPDROD computer
program(74). In each case, Fig. 9, the function \( t_y \),
Eqn. (5.1), would be taken as half the roll separation
across the width of the roll exit cross-section. Given
this representation of the three-dimensional roll surface,
Eqn. (5.1), the slopes and curvature of the surface can be
expressed as,

\[
\frac{\partial z_r}{\partial x} = -\frac{(X_d-x)}{[ (z_c-t_y)^2 - (X_d-x)^2 ]^{0.5}}, \tag{5.2}
\]
\[
\frac{\partial z_r}{\partial y} = \frac{(dt_y/dy)(z_c-t_y)}{[ (z_c-t_y)^2 - (X_d-x)^2 ]^{0.5}}, \tag{5.3}
\]
\[
\frac{\partial^2 z_r}{\partial x^2} = 1/[ (z_c-t_y)^2 - (X_d-x)^2 ]^{0.5}
+ \frac{(X_d-x)^2}{[ (z_c-t_y)^2 - (X_d-x)^2 ]^{1.5}}, \tag{5.4}
\]
and the components of the unit inward normal vector,
\((n_x,n_y,n_z)\), can be expressed as,
\[ n_x = N \left[ \left( X_d - x \right) / \left( z_c - t_y \right) \right], \quad (5.5) \]
\[ n_y = -N \left( d t_y / d y \right), \quad (5.6) \]
\[ n_z = N \left[ \left( z_c - z_r \right) / \left( z_c - t_y \right) \right], \quad (5.7) \]

where
\[ N = \frac{1}{\left( 1 + (d t_y / d y)^2 \right)^{0.5}}, \quad (5.8) \]
defined at a point \((x, y, z_r)\) on the surface, Fig. 34.

### V.2 Velocity Field Equations

The shape rolling velocity field is based on the flat rolling velocity field with bulge, Eqns. (3.12)-(3.14). The parameters \(v_b\) and \(A_b\), the functions \(c\), \(dc/dx\), \(A\), \(dA/dx\), \(d^2A/dx^2\), and the strain-rates, \(\varepsilon_{ij}\), are exactly the same as before. However, the function \(h\), originally defined in Eqn. (3.11), is now taken as half the roll separation along the center of the roll groove, i.e., \(z_r\), Eqn. (5.1), evaluated along \(y = 0\). Consequently, the functions \(dh/dx\), and \(d^2h/dx^2\) are now given by Eqns. (5.2) and (5.4), evaluated along \(y = 0\), replacing Eqns. (3.26) and (3.27), respectively.

For the shape rolling analysis the polynomial interpolation function, \(B\), to replace Eqn. (3.16), is selected to satisfy the following conditions:

* the interpolation function should be continuously differentiable;
* the interpolation function should permit the consideration of nonlinear velocity distributions across the width and through the thickness of the deforming billet cross-sections (to consider folding in the analysis, in addition to bulge); and,

* the interpolation function should give velocity fields which are symmetrical about the y and z coordinate axes (a necessary condition to analyze symmetrical geometric shapes like squares, diamonds, ovals, and rounds).

An interpolation function which satisfies these conditions is expressed in the following,

\[ B = (5.9) \]

\[ \begin{align*}
B &= \left[ z + b_1 \frac{z^3}{h^2} + b_2 \frac{z^5}{h^4} \right] \left[ b_1 y + b_2 \frac{y^3}{h^2} + b_3 \frac{y^5}{h^4} \right],
\end{align*} \]

where \( b_1, b_2, \) and \( b_3 \) are functions of \( x \) which are evaluated to satisfy velocity boundary conditions (see Sec. V.6).

Once again, the parameters \( B_1 \) and \( B_2 \) are additional unknown parameters in the metal deformation analysis, the final values of which determine the bulge of the rolled shape cross-section.

Given that the velocity boundary conditions are satisfied, the shape rolling velocity field is admissible because:

* Eqn. (5.9) has continuous second partial derivatives, satisfying incompressibility (Sec. III.7); and,
with the same cross-sectional area distribution function, $A$, Eqn. (3.17), the continuity conditions are also satisfied.

V.3 Minimization Variables

Thus, for the shape rolling analysis the unknown parameters in the velocity field equations include: $A'$, $A'_f$, $B_1$, $B_2$, and $v_b$, the same unknown parameters that were considered in the flat rolling analysis. Two of the unknown parameters determine the elongation distribution function, $A'$ and $A'_f$, two parameters determine the bulge contour, $B_1$ and $B_2$, and one parameter, $v_b$, determines the rate at which billet material enters the roll-bite. For the purpose of minimizing the total energy dissipation rate, $E_{tot}$, Eqn. (3.1), it is also convenient consider the equivalent set of minimization variables: $(A'X_d)/A_b$, $A'_f/A_b$, $B_1$, $B_2$, and $X_n/X_d$. However, in the shape rolling analysis the parameter $X_n$ is defined as neutral point location only at the center of the roll groove, meaning only at $y = 0$.

In addition, in calculating $v_b$ from the assumed values of $(A'X_d)/A_b$, $A'_f/A_b$, $B_1$, $B_2$, and $X_n/X_d$, complete contact between the billet and roll surfaces is assumed across the width of the rolled section, although in general this assumption may not be realistic. Hence, in the shape rolling analysis some of the physical meaning of $X_n/X_d$ can
be lost if, after minimizing the total energy dissipation rate, $E_{tot}$, Eqn. (3.1), there is incomplete contact between the billet and roll surfaces.

The reason that only the neutral point location at the center of the roll groove is considered as a minimization variable in the shape rolling analysis is that a complete analytical treatment of the neutral point phenomenon in shape rolling would be extremely difficult, especially when incomplete roll contact is considered. To illustrate this point, Fig. 35 shows the predicted neutral surface locations for the rolling of an airfoil section (assuming complete roll contact and divergent flow) as predicted using the modular upper-bound method of analysis for shape rolling developed by Lahoti, Akgerman, and Altan(71). Even in the simpler case (with complete roll contact and divergent flow) the location of the neutral surface varies significantly across the width of the rolled section. This variation, Fig. 35, with forward slip (Sec. III.5) predominant near the center of the pass and backward slip increasing towards the outside edges, is primarily due to slight differences in the tangential roll velocity across the width of the roll groove.
Figure 35. Predicted metal flow distribution and neutral surface for rolling of an airfoil section as predicted using modular upper-bound method (ref. 71).
**V.4 Strain-Rate Equations**

For the strain-rate equations, $\dot{\varepsilon}_{ij}$, Eqns. (3.20)-(3.25), the derivatives of the function $B$, Eqn. (5.9), are expressed as follows,

\[
\frac{\partial \dot{B}}{\partial y} = (5.10) \\
\left[ z + B_1 z_2^3 + B_2 z_4^5 \right] \left[ b_1 + 3 b_2 \frac{y^2}{h^2} + 5 b_3 \frac{y^4}{h^4} \right]
\]

\[
\frac{\partial \dot{B}}{\partial z} = (5.11) \\
\left[ 1 + 3 B_1 \frac{z^2}{h^2} + 5 B_2 \frac{z^4}{h^4} \right] \left[ b_1 y + b_2 \frac{y^3}{h^2} + b_3 \frac{y^5}{h^4} \right]
\]

\[
\frac{\partial^2 \dot{B}}{\partial x \partial y} = (5.12) \\
- \left[ 2 B_1 \frac{z_3^3}{h^3} + 4 B_2 \frac{z_5^5}{h^5} \right] \frac{\partial h}{\partial x} \left[ b_1 + 3 b_2 \frac{y^2}{h^2} + 5 b_3 \frac{y^4}{h^4} \right]
\]

\[
\frac{\partial^2 \dot{B}}{\partial x \partial z} = (5.13) \\
- \left[ 6 B_1 \frac{z_3^3}{h^3} + 20 B_2 \frac{z_5^5}{h^5} \right] \frac{\partial h}{\partial x} \left[ b_1 y + b_2 \frac{y^3}{h^2} + b_3 \frac{y^5}{h^4} \right]
\]

\[
\frac{\partial^2 \dot{B}}{\partial y \partial z} = (5.14) \\
\left[ 3 B_1 \frac{z_2^2}{h^2} + 5 B_2 \frac{z_4^4}{h^4} \right] \left[ b_1 + 3 b_2 \frac{y^2}{h^2} + 5 b_3 \frac{y^4}{h^4} \right]
\]
\[
\frac{\partial^2 B}{\partial y^2} = \left[ z + B_1 \frac{z_2^3}{h} + B_2 \frac{z_4^5}{h^2} \right] \left[ 6 B_2 \frac{y_2^2}{h^2} + 20 B_3 \frac{y_4^3}{h^4} \right].
\]

\[
\frac{\partial^2 B}{\partial z^2} = \left[ 6 B_1 \frac{z_2^2}{h^2} + 20 B_2 \frac{z_4^3}{h^4} \right] \left[ b_1 y + b_2 \frac{y_2^3}{h^2} + b_3 \frac{y_4^5}{h^4} \right].
\]

Since only discrete values of the functions \( b_1, b_2, \) and \( b_3 \) are determined in satisfying velocity boundary conditions (Sec. V.6), in the present investigation the terms \( \frac{db_1}{dx}, \frac{db_2}{dx}, \) and \( \frac{db_3}{dx}, \) Eqns. (5.12) and (5.13), are evaluated using a Newton two-point numerical derivative formula(99).

### V.5 Integration of Streamline Equations

Integration of the streamline equations, Eqn. (3.46), for the \( n_p \) discrete billet cross-section points, determines the deforming billet cross-sections at each location in the roll-bite, and, also determines the approximate boundaries of the deformation zone, information which is not explicitly given by the velocity field equations. This was also the case in the flat rolling analysis. However, unlike the flat rolling analysis, integration of the streamline equations must permit those streamlines which are initially part of the billet free-surface to blend with the roll-workpiece interface. This is accomplished by simply monitoring the position of the free-surface points for each
deforming billet cross-section so that the originally free material points can be placed back on the roll surface (by a movement in the z coordinate direction) whenever the free-points have, in effect, struck the roll surface. Once a free-surface point becomes a contact-point, however, the possibility that the contact-point will pull away from the roll surface in a subsequent integration step must be considered. This is determined by calculating, at the beginning of each integration step, the value of the normal stress at each contact-point, the stress component which is in the direction of the unit normal vector to the roll surface.

The normal stress component may be determined from the definition of the state of stress at a point(110), as follows,

\[ \sigma_n = n_i n_j \sigma_{ij} \]  

(5.17)

where \( \sigma_n \) is the normal stress, \( n_i \) are the components of the unit surface normal vector, Eqns. (5.5)-(5.7), and \( \sigma_{ij} \) are the stresses within the material. In plasticity the stresses, \( \sigma_{ij} \), may be determined from the constitutive equation(94),

\[ \sigma_{ij} = \frac{2}{3} \frac{\tilde{\sigma}}{\epsilon} \varepsilon_{ij} + \sigma_m \delta_{ij}, \]  

(5.18)

where,

\[ \sigma_m = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}), \]  

(5.19)

provided that the hydrostatic stress, \( \sigma_m \), is given.
Unfortunately, in the upper-bound method of analysis the hydrostatic stress, $\sigma_m$, is not given explicitly. However, in the present investigation, to obtain a practical estimate of $\sigma_m$, it is assumed that a contact-point which is about to pull away from the roll surface will be in a condition of plane stress with $\sigma_{yy} = 0$. Then, from Eqn. (5.18), using $\sigma_{yy} = 0$, the hydrostatic stress, $\sigma_m$, is given by,

$$\sigma_m = -\frac{2}{3}\frac{5}{2}\varepsilon_{yy},$$

where $\varepsilon_{yy}$ is determined from Eqn. (3.31). Substituting Eqns. (5.20) and (5.18) into Eqn. (5.17) permits the normal stress, $\sigma_n$, to be calculated directly from the strain-rates, $\varepsilon_{ij}$. If the calculated value of $\sigma_n$ turns out to be positive, or tensile, then the contact-point is permitted to pull away from the roll surface during the next integration step. Otherwise, the material point is assumed to remain in contact with the roll, and is shifted back onto the roll surface after each integration step in order to correct for at least part of the streamline integration error. The combined discretization error, due to the use of a string of discrete points to approximate deforming billet cross-sections, and the streamline integration error usually amounts to no more than 0.5 percent of the theoretical roll-bite volume, $V$, Eqn. (3.64).
V.6. Velocity Boundary Conditions

A major difference between the flat rolling and shape rolling analyses is that there are two different kinds of velocity boundary conditions that are considered in the shape rolling analysis: the first are the essential velocity boundary conditions which were used in the flat rolling analysis, Eqn. (3.9), but the second are quite different, these are subsidiary velocity boundary conditions (to be defined in this section) which are imposed to generate velocity fields which are truly representative of the particular billet and roll groove shape combination being considered. In addition, in the shape rolling analysis the velocity boundary conditions are not satisfied automatically by the chosen velocity field equations, these boundary conditions are imposed continually at each integration step used in determining the streamlines of the deforming billet cross-sections.

For the essential velocity boundary conditions (Sec. III.2), whenever a billet cross-section point is in contact with the roll surface, suppose there are $n_c$ contact-points, then the streamlines through the contact-points must be tangential to the roll surface, a condition which can be expressed analytically as $n_c$ equations of the form,

$$\left[ v_x n_x + v_y n_y + v_z n_z = 0 \right]_i, \text{ for } i = 1, n_c. \quad (5.21)$$

Substitution of Eqns. (3.12)-(3.14) into Eqn. (5.21)
produces a system of linear equations from which discrete values of $b_1$, $b_2$, and $b_3$, Eqn. (5.9), may be determined.

Although the shape rolling velocity field, as formulated using Eqns. (3.12)-(3.14) with Eqns. (5.9) and (5.21), is mathematically correct, preliminary research revealed that additional information was needed to make the shape rolling velocity field equations truly representative of actual metal deformation patterns known to occur in shape rolling (71, 73). For situations in which the initial touching between the billet and roll surfaces occurs at or near the outside corner of the entry cross-section, such as in the rectangle-diamond pass illustrated in Fig. 36, then the actual metal flow distribution will consist of a combination of convergent flow at the center of the cross-section with divergent flow towards the outside corners, instead of just the divergent metal flow which was shown in Fig. 35. The more realistic metal flow pattern for an airfoil is illustrated in Fig. 37. This combined metal flow distribution is possible in shape rolling because, for these configurations, Figs. 36 and 37, the resultant contact force is directed towards the center of the deforming billet cross-sections, and because there is a gap between the billet and roll surfaces which makes the space for the metal to be forced into. It is also important to recognize that the presence of combined convergent and divergent metal flow necessarily produces additional neutral surfaces, Fig. 37.
Figure 36. Billet and roll surface configurations for the rolling of a rectangle through diamond pass showing where no-slip conditions are imposed (ref. 71).

Figure 37. Representation of combined convergent and divergent metal flow patterns for an airfoil when the direct thickness reduction begins near the outside corners of the billet entry cross-section (ref. 71).
However, these neutral surfaces are somewhat different from the neutral surface that discussed earlier, Fig. 35, the one which was produced by changing direction of the billet surface traction stresses - not by changing direction of the actual metal movement. To complicate matters further, one must consider that the neutral surfaces in the lateral flow can exist only when there are gaps between the billet and roll surfaces, and they disappear when the direct thickness reduction reaches the center of the deforming billet cross-sections, after which divergent flow exists throughout.

This kind of complex, three-dimensional, metal flow phenomenon can be extremely difficult to incorporate into analytical models. Therefore, although the exact location of the neutral surfaces along the arc of contact should be established on the basis of equilibrium considerations, it was determined that combined convergent and divergent metal flow patterns can be incorporated into the present analysis, with good analytical results, by imposing the subsidiary velocity boundary condition,

\[ v_y = 0 \], imposed at \((y_1, z_1)\), \( (5.22) \)

where \((y_1, z_1)\) represents the spatial coordinates of the outermost contact-point. Eqn. (5.22) is called the no-slip velocity boundary condition. The no-slip condition is imposed only when the initial touching between the billet entry and roll cross-sections occurs near the outside corner of the billet entry section, and Eqn. (5.22) is dropped once
the direct thickness reduction reaches the center of the roll groove. In regard to the rectangle-diamond pass which was shown in Fig. 36, the no-slip velocity boundary condition would be imposed at the locations which are labeled IDFA(1) and IDFA(4), but not at the locations IDFA(2) and IDFA(3).

On the other hand, when the initial touching begins near the center of the billet entry cross-section, as in the airfoil rolling problem illustrated in Fig. 38, then the metal deformation at the outside corners could be quite different from that at the center of the section because of the absence of direct thickness reduction. In fact, without direct thickness reduction the metal deformation at the outside corners could be nearly uniaxial with the outside corners being pulled-along in order to keep up with the elongation and direct thickness reduction taking place at the center of the billet cross-sections. This mode of deformation is also quite different from that which would normally given by a flat rolling velocity field formulation such as Eqns. (3.12)-(3.14).

Therefore, in the present investigation, when the direct thickness reduction begins near the center of the billet entry cross-section, Fig. 38, a proportional strain-rate programme is imposed at the outside corners of the deforming billet cross-section (a subsidiary condition on the velocity field) which is expressed as follows,
Figure 38. Representation of divergent metal flow pattern for an airfoil when the direct thickness reduction begins near the center of the billet entry cross-section (ref. 71).

\[ \dot{\varepsilon}_{yy} = E_1 \dot{\varepsilon}_{xx}, \text{ imposed at } (w_1, t_1), \]  

(5.23)

where the \((w_1, t_1)\) are the spatial coordinates of the outside corners, \(\dot{\varepsilon}_{yy}\) and \(\dot{\varepsilon}_{xx}\) are the strain-rates in the directions of spread and elongation, Eqs. (3.21) and (3.20), respectively, and where \(E_1\) is a dimensionless parameter (the constant of proportionality) which is treated as a sixth unknown in the velocity field equations, determined along with the values of \(A^r, A^r, B_1, B_2,\) and \(v_B\) via minimization of the total energy dissipation rate, \(\dot{E}_{\text{tot}}\), when Eqn. (5.23) is applied. Eqn. (5.23) is called the constrained corner velocity boundary condition.
As an illustration of the application of the constrained corner condition, with Eqn. (5.23) a condition of uniaxial deformation can be imposed by setting,

\[ E_1 = -0.5, \]

so that Eqn. (5.23) becomes,

\[ \dot{e}_{yy} = -0.5 \dot{e}_{xx}, \text{ imposed at } (w_1,t_1). \]  

Thus, with Eqn. (5.25) the metal deformation at the outside corners of the deforming billet cross-sections would be similar to that which occurs in a tensile yield test with a test specimen elongated in the x coordinate direction. In the present investigation, the constrained corner condition, Eqn. (5.23), is imposed only when the direct thickness reduction begins near the center of the billet entry section. When applied, Eqn. (5.23) is imposed on all deforming billet cross-sections which are considered in the metal deformation analysis.

Therefore, the essential velocity boundary conditions, Eqn. (5.21), and subsidiary conditions, Eqn. (5.22) or Eqn. (5.23), can be expressed as a system of \( n_{bc} \) linear equations, where \( n_{bc} \) denotes the number of boundary conditions, in terms of the unknown parameters \( b_1, b_2, \) and \( b_3 \), for each deforming billet cross-section, expressed as,

\[ C_{ij} b_j + a_i = 0, \quad i = 1, n_{bc}, \quad \text{and } j = 1, 3. \]  

where, in the present investigation, \( n_{bc} \geq 3 \). When \( n_{bc} = 3 \) then \( b_1, b_2, \) and \( b_3 \) are determined by Gauss elimination. When \( n_{bc} > 3 \) then \( b_1, b_2, \) and \( b_3 \) are determined by the
Method of Least Squares, replacing Eqn. (5.26) with the normal equations (111),

\[ C_{ki} C_{kj} b_j + C_{ki} a_k = 0, \quad i, j = 1, 3, k = 1, n_{bc}. \] (5.27)

Eqn. (5.27) is also solved by Gauss elimination.

Preliminary numerical testing for round-oval-round type pass sequences revealed that satisfying velocity boundary conditions using the Method of Least Squares, Eqn. (5.27), with only three \( y \) coordinate terms in the polynomial interpolation function, \( B \), Eqn. (5.9), still permitted the essential velocity boundary conditions, Eqn. (5.21), to be satisfied with great accuracy. Typically, the maximum error in Eqn. (5.21), for the values of \( b_1, b_2, \) and \( b_3 \) determined from Eqn. (5.27), was much less than 0.5 degrees even with complete contact across the width of the deforming billet cross-sections.

### V.7 Initial Guess Solution

The initial guess solution for the shape rolling analysis is essentially the same as that used in the flat rolling analysis, except that Shinokura's formula (30), Eqn. (2.1), instead of Wusatowski's formula (27), Eqn. (2.2), is used to estimate \( \left( \frac{A_f}{A_b} \right) \), and the initial value of \( \left( \frac{A_d X_d}{A_b} \right) \) is set to correspond to an assumed linear elongation distribution function at the entry plane of the deformation zone, an assumption which is supported by
elongation distribution measurements from production rolling trials in shape rolling(112). The remaining minimization variables are assigned consistent initial values, as summarized in the following,

\[ A' \cdot X_d / A_b = \left( 1 - A_f / A_b \right), \]

\[ A_f / A_b = \text{determined from Eqn. (2.1)}, \]

\[ B_1 = 0.00, \]

\[ B_2 = 0.00, \]

\[ X_n / X_d = 0.70, \]

\[ E_1 = -0.50, \text{ when Eqn. (5.23) is applied}. \]

\[ (5.28) \]

V.8 Computer Program SHAPES

A FORTRAN language computer program, named SHAPES, has been developed for the analysis of metal deformations and stresses in shape rolling for symmetrical geometric shapes without protrusions such as: squares, diamonds, ovals, and rounds. The SHAPES program is currently operational on a VAX 11/750 mini-computer, executing in batch-mode using editor-created input data files. Discussion of the numerical methods that are used in the SHAPES computer program was given in Secs. III.9-14. Copies of the program source code are available on either magnetic tape or floppy disk.
CHAPTER VI
RESULTS FROM THE SHAPE ROLLING ANALYSIS

In this section the results from the shape rolling analysis are discussed and comparisons are made between predicted and measured values of elongation and roll separating force in the hot rod rolling of steel.

VI.1 Shape Rolling Experiments

To evaluate the capabilities of the SHAPES program, the theoretical results were compared with results from hot rolling experiments with AISI 1018 steel rod, under controlled laboratory conditions, without lubrication, that were obtained in a previous Battelle study(73). These experiments determined the exit cross-sectional areas, roll separating force, and metal flow (billet material sections before and after each pass and, in selected experiments, at different locations in the deformation zone) for a variety of rolled shapes. The cross-sectional areas and billet sections were determined after allowing the hot rolled test specimens to cool to room temperature. The experiments were carried out on Battelle's two-high rolling mill with
Figure 39. Various passes available on the grooved rolls used for validating the shape rolling analysis (ref. 73).

8 in. diameter steel rolls. Various passes on a pair of grooved rolls were used, Fig. 39. These rolls accept a 1 1/4 in. square billet to produce a 7/8 in. diameter rod in seven passes. Specimens for the experiments were cut from hot rolled AISI 1018 steel stock. As in the Battelle plate rolling experiments (71, 72), prior to hot rolling, the specimens were annealed at 1625 F followed by thermal soak at 1832 F in an electric furnace with an argon gas protective atmosphere. The rolling speed was approximately 50 fpm. In the present investigation, the flow stress data and friction shear factor for the rod rolling experiments were taken as the same values that were used in the hot rolling of AISI 1018 steel plate (71, 72).
VI.2 Simulation of Square-Oval-Round Pass Sequences

The results predicted by the SHAPES program were compared with results from the laboratory experiments for the roll pass schedule illustrated in Fig. 39. In validating the shape rolling analysis the SHAPES program was used to simulate the square-oval-round finisher pass sequence, Pass Nos. 4, 5, and 6, which covered the billet and roll shape combinations that the SHAPES program is designed to handle. The entry cross-section used for Pass No. 4 was taken as the exit cross-section from Pass No. 3, assuming complete fill. The entry cross-section for Pass No. 5 was taken as the predicted exit cross-section from Pass No. 4, after being turned through 90 degrees, as dictated by the roll drawing, Fig. 39. A new entry cross-section was generated for Pass No. 6.

In the present investigation, the deforming billet cross-sections were approximated by a string of 10 material cross-section points, and a total of 50 integration steps were used in solving the streamline equations, Eqn. (3.46). Individual deforming billet cross-sections are denoted by their relative locations in the deformation zone, $x/X_d$, where $x$ is the distance from the entry plane, and $X_d$ is the projected length of the roll-bite. Thus, as shown in Fig. 40, $x/X_d = 0.0$ denotes the entry plane of the deformation zone, and $x/X_d = 1.00$ denotes the exit plane.
Selected deforming billet cross-sections for the square-oval solution, Pass. No. 4, as predicted by the SHAPES program, are shown in Figs. 41-45. Since the initial reduction with the square-oval pass began near the outside corner of the entry section, Fig. 41, the no-slip subsidiary velocity boundary condition, Eqn. (5.22), was used to get the solution. As shown in Figs. 41-45, the predicted deforming billet cross-sections gradually conform to the shape of the roll groove cavity. However, the initial gap between the billet and roll surfaces, Fig. 41.
Figure 41. Entry billet cross-section for square-oval pass, Pass No. 4 in Fig. 39.
Figure 42. Predicted deforming billet cross-section at $x/X_d = 0.25$ in square-oval pass.
SQUARE-oval, \( m = 0.75 \)
\( x/X_l = 0.50 \)
NO SLIP

Figure 43. Predicted deforming billet cross-section at \( x/X_d = 0.50 \) in square-oval pass.
SQUARE-OVAL, \( m = 0.75 \)
\( X/X_L = 0.75 \)
NO SLIP

Figure 44. Predicted deforming billet cross-section at 
\( x/X_q = 0.75 \) in square-oval pass.
Figure 45. Predicted billet exit cross-section in square-oval pass (entry section superimposed with dashed line).
does not close completely by the time the exit plane, Fig. 45, is reached. Hence, there is incomplete roll contact throughout much of the deformation zone. Although the degree of incomplete roll contact varied for different situations, this phenomenon generally seemed to be an artifact of the analysis which was due to the existence, in the chosen velocity field formulation, of specific velocity field solutions which permit the billet surface streamlines to lie just within the roll groove cavity without actually making contact. Given the possibility of solutions with incomplete roll contact, as simulated by the SHAPES program, this kind of metal flow is strongly favored in minimizing the upper-bound functional because by avoiding roll contact, and friction losses, the total energy dissipation rate is reduced. Fortunately, even with incomplete roll contact, the only serious effect on the analytical results was that the SHAPES program tended to underestimate the roll separating force. The predicted elongation values were not adversely affected.

It is interesting that the predicted exit cross-section for this solution, Fig. 45, has an hour-glass-bulge shape. This finding is fairly consistent with what would be predicted from Fig. 44, the theoretically determined bulge profile design chart, because with an average fractional reduction of 0.24 (as simulated by the SHAPES program) and values of $W_d/H_d = 1$, and $D/H_d = 8$, this particular square-
Table 14. Example track of minimization variables for rolling of square-oval pass as simulated by SHAPES computer program using no-slip subsidiary velocity boundary condition (ref. 73 experiments). 

<table>
<thead>
<tr>
<th>A' \cdot \frac{X_d}{A_d}</th>
<th>A_f/A_p</th>
<th>B_1</th>
<th>B_2</th>
<th>\frac{X_n}{X_d}</th>
<th>\dot{E}_{tot}</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.281</td>
<td>0.812</td>
<td>0.000</td>
<td>0.000</td>
<td>0.700</td>
<td>6.50827</td>
</tr>
<tr>
<td>-0.313</td>
<td>0.812</td>
<td>0.320</td>
<td>-0.320</td>
<td>0.700</td>
<td>5.41976</td>
</tr>
<tr>
<td>-0.345</td>
<td>0.780</td>
<td>0.960</td>
<td>-0.960</td>
<td>0.700</td>
<td>5.09286</td>
</tr>
<tr>
<td>-0.345</td>
<td>0.780</td>
<td>1.920</td>
<td>-1.280</td>
<td>0.700</td>
<td>4.77469</td>
</tr>
<tr>
<td>-0.345</td>
<td>0.780</td>
<td>2.880</td>
<td>-1.600</td>
<td>0.700</td>
<td>4.80206</td>
</tr>
<tr>
<td>-0.345</td>
<td>0.780</td>
<td>1.920</td>
<td>-1.280</td>
<td>0.732</td>
<td>4.77392</td>
</tr>
<tr>
<td>-0.345</td>
<td>0.780</td>
<td>1.920</td>
<td>-1.360</td>
<td>0.732</td>
<td>4.77316</td>
</tr>
<tr>
<td>-0.345</td>
<td>0.772</td>
<td>2.000</td>
<td>-1.440</td>
<td>0.732</td>
<td>4.75749</td>
</tr>
<tr>
<td>-0.337</td>
<td>0.772</td>
<td>2.080</td>
<td>-1.440</td>
<td>0.748</td>
<td>4.95505</td>
</tr>
<tr>
<td>-0.341</td>
<td>0.772</td>
<td>2.000</td>
<td>-1.440</td>
<td>0.724</td>
<td>4.75560</td>
</tr>
<tr>
<td>-0.333</td>
<td>0.776</td>
<td>2.000</td>
<td>-1.440</td>
<td>0.708</td>
<td>4.76572</td>
</tr>
<tr>
<td>-0.343</td>
<td>0.772</td>
<td>2.020</td>
<td>-1.460</td>
<td>0.720</td>
<td>4.71546</td>
</tr>
<tr>
<td>-0.343</td>
<td>0.770</td>
<td>2.040</td>
<td>-1.480</td>
<td>0.712</td>
<td>4.68371</td>
</tr>
</tbody>
</table>

a One row of entries is given per minimization step - first row is the initial guess - last row is the solution, and \( \dot{E}_{tot} \), is given in lbf-in/s assuming unity flow stress.

The track of the minimization variables for the square-oval is shown in Table 14. The initial guess values for \( \frac{A' \cdot X_d}{A_b} \), \( A_f/A_p \), \( B_1 \), \( B_2 \), and \( \frac{X_n}{X_d} \) were taken from Eqn. (5.28). The input minimization variable tolerances were taken as the same values that were used with the BILLETS program, specifically: 0.002 for \( \frac{A' \cdot X_d}{A_b} \) and \( A_f/A_p \), 0.02 for \( B_1 \) and \( B_2 \), and 0.004 for \( \frac{X_n}{X_d} \). During numerical testing it was determined that the energy surfaces in the shape rolling problem have severely long and narrow valleys, or rills. Thus, if a poor initial guess is used...
with the SHAPES program, one that is far away from the minimum, then the convergence to the minimum can be extremely slow. However, with the initial guess values taken from Eqn. (5.28) the CPU time for the square-oval was 2 hr. and 14 min. on the VAX 11/750 mini-computer.

The predicted deforming billet cross-sections for the oval-round, Pass. No. 5, as simulated by the SHAPES program, are shown in Figs. 46-50. Since the initial reduction began away from the outside corner, towards the center of the entry section, Fig. 46, the constrained corner subsidiary velocity boundary condition, Eqn. (5.23), was used to get the solution. Comparing the predicted deforming billet cross-sections, Figs. 46-50, reveals how the material conforms to the roll groove cavity while spreading outward to nearly perfect width fill at the exit plane, Fig. 50. Also, it is interesting that the center impression, the surface feature that came from the hour-glass-bulge profile predicted in the Pass No. 4, develops a wrinkle (deepening of the impression) on the entry plane side of the deformation zone, Figs. 46 and 47, which fades away on the exit plane side, Figs. 48, 49, and 50. This wrinkling is attributed to compressive lateral forces that are generated when the billet material presses against the steep groove sidewalls of the round groove shape.
Figure 46. Entry billet cross-section for oval-round pass, Pass No. 5 in Fig. 39.
Figure 47. Predicted deforming billet cross-section at $x/X_d = 0.25$ in oval-round pass.
Figure 48. Predicted deforming billet cross-section at $x/X_d = 0.50$ in oval-round pass.
Figure 49. Predicted deforming billet cross-section at $x/x_d = 0.75$ in oval-round pass.
Figure 50. Predicted billet exit cross-section in oval-round pass (entry section superimposed with dashed line).
Table 15. Example track of minimization variables for rolling of oval-round pass as simulated by SHAPES computer program using constrained corner subsidiary velocity boundary condition (ref. 73 experiments).a

<table>
<thead>
<tr>
<th>A′X/a</th>
<th>A′/A</th>
<th>B1</th>
<th>B2</th>
<th>Xn/Xa</th>
<th>E1</th>
<th>Etot</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.093</td>
<td>0.937</td>
<td>0.000</td>
<td>0.000</td>
<td>0.700</td>
<td>-0.500</td>
<td>4.05020</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.636</td>
<td>-0.500</td>
<td>2.81583</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.640</td>
<td>0.636</td>
<td>-0.500</td>
<td>2.86853</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.604</td>
<td>-0.500</td>
<td>2.81323</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.540</td>
<td>-0.500</td>
<td>2.80835</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.444</td>
<td>-0.500</td>
<td>2.80254</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.316</td>
<td>-0.500</td>
<td>2.79823</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.220</td>
<td>-0.500</td>
<td>2.79730</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.156</td>
<td>-0.500</td>
<td>2.79781</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.236</td>
<td>-0.420</td>
<td>2.79683</td>
</tr>
<tr>
<td>-0.061</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.252</td>
<td>-0.420</td>
<td>2.79692</td>
</tr>
<tr>
<td>-0.057</td>
<td>0.937</td>
<td>0.000</td>
<td>-0.320</td>
<td>0.228</td>
<td>-0.380</td>
<td>2.79295</td>
</tr>
<tr>
<td>-0.053</td>
<td>0.937</td>
<td>-0.040</td>
<td>-0.320</td>
<td>0.228</td>
<td>-0.380</td>
<td>2.83702</td>
</tr>
<tr>
<td>-0.057</td>
<td>0.937</td>
<td>-0.020</td>
<td>-0.320</td>
<td>0.232</td>
<td>-0.400</td>
<td>2.77857</td>
</tr>
<tr>
<td>-0.059</td>
<td>0.937</td>
<td>-0.020</td>
<td>-0.320</td>
<td>0.236</td>
<td>-0.420</td>
<td>2.77178</td>
</tr>
</tbody>
</table>

a One row of entries is given per minimization step - first row is the initial guess - last row is the solution, and Etot is given in lbf-in/s assuming unity flow stress.

The track of the minimization variables for the oval-round solution is shown in Table 15. The appropriate initial guess values were taken from Eqn. (5.28). The minimization variable tolerance for the new parameter, E1, Eqn. (5.23), was taken as 0.02. The other minimization variable tolerances were unchanged. The CPU time for the oval-round solution was 2 hr. and 45 min. on the VAX 11/750 mini-computer.
The oval-round solution was also attempted using the no-slip condition, Eqn. (5.22), instead of the constrained corner condition, Eqn. (5.23). The result, shown in Fig. 51, was that the SHAPES program predicted a fin. The SHAPES program also predicted a fin if no subsidiary condition was imposed, but in this case the energy dissipation rates were much higher than the energy dissipation rates using the no-slip condition. Although the constrained corner condition actually gave the lowest energy dissipation rate, its best attribute, among the alternative solutions, is that it gives a much more realistic solution, since, as shown in Table 15, the final value of the corner constraint, \( E_1 = -0.420 \), closely approximates the uniaxial corner condition, \( E_1 = -0.500 \), Eqn. (5.24). Thus, with the constrained corner condition the shape rolling analysis predicts that the outside corners of the deforming billet cross-sections are being pulled-along by the reduction and commensurate elongation taking place at the center of the rolled section.

For the round-oval solution, Pass No. 6, the predicted deforming billet cross-sections are shown in Figs. 52-56. This solution was also obtained using the constrained corner condition, Eqn. (5.23). The track of the minimization variables for the round-oval is shown in Table 16. The final value of the corner constraint, \( E_1 = -0.600 \), also approximates the uniaxial corner condition, \( E_1 = -0.500 \),
Figure 51. Predicted billet exit cross-section in oval-round pass using no-slip condition (entry section superimposed with dashed line, billet exit section line segments crossing the roll section were omitted for clarity).
ROUND-oval, \( m = 0.75 \)
\( X/XL = 0.00 \)
CONSTRAINED CORNER

Figure 52. Entry billet cross-section for round-oval pass. Pass No. 6 in Fig. 39.
ROUND- OVAL, \( m = 0.75 \)
\( x/X_L = 0.25 \)
CONSTRAINED CORNER

Figure 53. Predicted deforming billet cross-section at \( x/X_a = 0.25 \) in round-oval pass.
Figure 54. Predicted deforming billet cross-section at $x/X_d = 0.50$ in round-oval pass.
Figure 55. Predicted deforming billet cross-section at $x/X_a = 0.75$ in round-oval pass.
Figure 56. Predicted billet exit cross-section in round-oval pass (entry section superimposed with dashed line).
Table 16. Example track of minimization variables for rolling of round-oval pass as simulated by SHAPES computer program using constrained corner subsidiary velocity boundary condition (ref. 73 experiments)*.  

<table>
<thead>
<tr>
<th>( A_1/A_0 )</th>
<th>( A_1/A_2 )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( X_n/X_d )</th>
<th>( E_1 )</th>
<th>( \dot{\varepsilon}_{\text{tot}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.190</td>
<td>0.873</td>
<td>0.000</td>
<td>0.000</td>
<td>0.700</td>
<td>-0.500</td>
<td>2.71788</td>
</tr>
<tr>
<td>-0.190</td>
<td>0.873</td>
<td>0.000</td>
<td>0.000</td>
<td>0.636</td>
<td>-0.500</td>
<td>2.70983</td>
</tr>
<tr>
<td>-0.190</td>
<td>0.873</td>
<td>0.000</td>
<td>0.000</td>
<td>0.572</td>
<td>-0.500</td>
<td>2.70679</td>
</tr>
<tr>
<td>-0.190</td>
<td>0.873</td>
<td>0.000</td>
<td>0.000</td>
<td>0.572</td>
<td>-0.500</td>
<td>2.70679</td>
</tr>
<tr>
<td>-0.190</td>
<td>0.873</td>
<td>0.000</td>
<td>0.000</td>
<td>0.572</td>
<td>-0.500</td>
<td>2.70679</td>
</tr>
<tr>
<td>-0.192</td>
<td>0.873</td>
<td>0.000</td>
<td>0.000</td>
<td>0.572</td>
<td>-0.500</td>
<td>2.70679</td>
</tr>
<tr>
<td>-0.196</td>
<td>0.873</td>
<td>0.000</td>
<td>0.000</td>
<td>0.572</td>
<td>-0.500</td>
<td>2.70679</td>
</tr>
</tbody>
</table>

*One row of entries is given per minimization step - first row is the initial guess - last row is the solution, and \( \dot{\varepsilon}_{\text{tot}} \), is given in lbf-in/s assuming unity flow stress.

Eqn. (5.24). The CPU time for the round-oval solution was only 1 hr. and 49 min. on the VAX 11/750 mini-computer.

The rapid convergence with the round-oval is attributed to the success of the initial guess, Eqn. (5.28), which is based on the use of established empirical design formulae to estimate the elongation(74).

VI.3 Comparison Between Predicted and Measured Values

Comparison between the predicted and measured values of exit cross-sectional area (or total elongation) and roll separating force, for the SHAPES analysis, are given in Tables 17 and 18, respectively. For the exit cross-
Table 17. Comparison between theoretical exit cross-sectional area predictions and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 73 experiments)\(^a\).

<table>
<thead>
<tr>
<th>Pass</th>
<th>Theoretical</th>
<th>Measured</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square-oval (Pass No.4)</td>
<td>0.847</td>
<td>0.856</td>
<td>- 0.009</td>
</tr>
<tr>
<td>Oval-round (Pass No.5)</td>
<td>0.793</td>
<td>0.792</td>
<td>0.001</td>
</tr>
<tr>
<td>Round-oval (Pass No.6)</td>
<td>0.684</td>
<td>0.681</td>
<td>0.003</td>
</tr>
</tbody>
</table>

\(^a\) The actual cross-sectional area measurements given in ref. 73 were for room temperature conditions, the measured values given above are the estimated values for hot rolling conditions, estimated assuming 1.3 percent linear thermal expansion for steel from room temperature to 1832 F.

Table 18. Comparison between theoretical roll separating force predictions and experiments for hot rolling of AISI 1018 steel at 1832 F (ref. 73 experiments).

<table>
<thead>
<tr>
<th>Pass</th>
<th>Theoretical</th>
<th>Measured</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square-oval (Pass No.4)</td>
<td>3.5</td>
<td>8.9</td>
<td>- 5.4</td>
</tr>
<tr>
<td>Oval-round (Pass No.5)</td>
<td>1.7</td>
<td>3.1</td>
<td>- 1.4</td>
</tr>
<tr>
<td>Round-oval (Pass No.6)</td>
<td>3.7</td>
<td>3.8</td>
<td>- 0.1</td>
</tr>
</tbody>
</table>
sectional area, Table 17, there is good agreement between the predicted and measured values. For the roll separating force, Table 18, the agreement is good for the oval-round and round-oval solutions. However, for the square-oval, where the initial reduction began near the outside corner, the SHAPES analysis significantly underestimated the measured value. This discrepancy, with the square-oval, is attributed to the general tendency for the predicted deforming billet cross-sections to avoid roll contact, as discussed earlier, and the fact that the square-oval begins with the largest gap at the entry plane, Fig. 41.

Concerning the predicted exit cross-sections, the experimentally determined exit cross-sections for Passes 4, 5, and 6 are shown in Fig. 57. It can be seen that the actual exit cross-sections are similar in appearance to the predicted exit cross-sections shown in Figs. 45, 50, and 56, respectively. Although the predicted hour-glass-bulge in the square-oval, Fig. 45, is not visible in the actual cross-section, Fig. 57, hour-glass-bulge does occur in square-oval type passes(12), as shown in Fig. 58, when smaller roll diameters are used. Concerning the predicted wrinkling in the oval-round solution, during one of the oval-round tests(73) the laboratory mill was stalled so that the partially rolled shape could be cut at selected locations in the deformation zone in order to provide information on the progressive filling of the roll groove.
Figure 57. Actual rolled shape cross-sections produced in Battelle rod rolling experiments (ref. 73) for Pass Nos. 4, 5, and 6.

Figure 58. Square-oval pass which produces pronounced hourglass-bulge when smaller roll diameter is used (ref. 12).
Figure 59. Actual deforming billet cross-sections determined at different locations in the deformation zone for oval-round pass (ref. 73).

cavity, as shown in Fig. 59. It can be seen that the predicted deforming billet cross-sections for the oval-round solution, Figs. 46-50, are very similar in shape to the actual cross-sections, Fig. 59. However, the predicted wrinkling of the center impression, Figs. 46 and 47, is not visible in the actual experiment.
CHAPTER VII
CONCLUSIONS

A method for metal deformation and stress analysis in rolling was developed. The analysis was based on an upper-bound approach in which an iterative numerical procedure was used to minimize the energy dissipation rate for competing kinematically admissible velocity field solutions for the rolling problem. The major assumptions that were made in the analysis are summarized in the following:

* the billet material is isotropic and incompressible;
* elastic deformations and inertial forces are negligible;
* plane sections perpendicular to the direction of rolling remain plane during deformation;
* the billet material flows according to the Levy-Mises flow rule and Mises yield criterion; and,
* the friction shear factor and the flow stress are constant.

Once the velocity field and the final shape of the plastically deforming body were known, then elementary stress analysis techniques were used to determine the force related aspects of the rolling problem.
The initial application of the theoretical analysis involved an investigation of the metal deformation and stresses in primary rolling mill operations using flat grooveless rolls. Theoretical results for spread, elongation, bulge, roll separating force, and roll torque were obtained as a function of the billet entry cross-section geometry, the roll cross-section geometry, the thickness reduction, and pertinent material data such as the flow stress of the rolled material, and the friction shear factor in the roll-workpiece interface. Afterward, the flat rolling analysis was extended to the analysis of certain grooved roll pass sequences used in the production of symmetrical geometric shapes without protrusions, such as: squares, diamonds, ovals, rounds, and related sections.

In general, the metal deformation and stress analysis showed good agreement with spread, elongation, roll separating force, and roll torque measurements in the hot rolling of mild carbon steel for a variety of workpiece and roll cross-section geometries. But, for the flat rolling analysis:

* although the flat rolling analysis predicted the maximum spread, type of bulge profile, roll torque, and roll separating force fairly accurately,
* the predicted true spread and interface spread values were not in good agreement with the measured values, especially at heavy reductions.
It was concluded that, although several factors could have contributed to the observed errors in the true spread and interface spread predictions (notably, roll chilling and folding - factors not included in the present flat rolling analysis), the predominant errors were due to the plane sections remain plane assumption, an assumption which is a limitation in the velocity field.

The factors which lead to single-bulge or double-bulge deformation in flat rolling were also considered. It was concluded that:

* the type of bulge profile that develops in the pass is primarily a function of the geometric aspects of the pass design, specifically, the entry billet width/thickness ratio, the roll diameter/thickness ratio, and the thickness reduction;
* changes in the friction conditions have little influence on the predicted bulge profile type; although,
* increasing friction will exaggerate the concurrent bulge profile.

The bulge profile predictions were summarized in the form of a bulge profile design chart from which it can be estimated, for specific rolling conditions, the amount of thickness reduction that will be required to avoid hour-glass-bulge or double-bulge profiles and promote single-bulge profiles, thereby offering the possibility of improved yields.
The flat bar test was also suggested as an experimental technique for estimating the interface friction conditions in rolling. The friction conditions are estimated by comparing the predicted and measured values of maximum spread, predicted values coming from the flat rolling analysis, at various reductions for a single-pass flat rolling process. For a given experimental condition, the friction shear factor is estimated by comparing the measured maximum spread values with theoretical calibration curves. It is felt that the flat bar test will be easy to carry out because it does not require any special measurement equipment, and because it only requires the measurement of the outside width of a rolled flat bar cross-section.

The shape rolling analysis utilized essentially the same velocity field formulation that was used in the flat rolling analysis, except that the shape rolling velocity field considered folding of the deforming billet cross-sections in addition to bulge. However, a significant difference between the two analyses was that the shape rolling analysis considered the application of so-called subsidiary velocity boundary conditions in order to generate metal deformation patterns which were truly representative of the specific billet and roll groove shape combination being considered. The no-slip subsidiary condition was assumed for roll passes in which the initial thickness reduction begins at or near the outside corner of the billet.
entry cross-section, as in the square-oval pass. The constrained corner condition was assumed for roll passes in which the initial reduction begins near the center of the billet entry cross-section, as in the round-oval. Thus, in practical situations, square-oval-round type pass sequences can be analyzed by assuming either the no-slip condition or the constrained corner condition, whichever is appropriate for the specific situation.

Based on the comparisons between the predicted and measured values for the shape rolling analysis, it is concluded that:

* the shape rolling analysis predicts the total elongation very accurately; however,
* the shape rolling analysis can underestimate the roll separating force due to a tendency for the predicted deforming billet cross-sections to avoid roll contact whenever possible.

Despite this discrepancy, it was clear that the shape rolling analysis gives a very realistic representation of the complex three-dimensional metal deformation in shape rolling, predicting important metal flow phenomena such as hour-glass-bulge and wrinkling.

Therefore, for relatively simple geometric shapes like squares, ovals, and rounds, the upper-bound method of analysis can be used as a practical design tool. However, despite the advances that were made in the present
investigation, the upper-bound method of analysis cannot be expected to be able to analyze complex geometric shapes with protrusions, Fig. 1, the kind of shapes which are, without question, the most difficult shapes to roll. Consequently, it is concluded that future analytical research should be directed towards developing the generalized finite-element method of analysis (86-91).
LIST OF REFERENCES


42. Wusatowski, Z., "Average Stretching Strains in Irregular Roll-Passes" (in German), Neue Huette 2(1957)1, pp. 24-35.


44. Gulyas, J., "Forming Problems in Rolling I-Profiles" (in German), Neue Huette 21(1976)9, pp. 521-524.

45. Neumann, H., "Division of Profile in Parts in Roll-Pass Design" (in German), Neue Huette 7(1962)8, pp. 480-486.


