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SITE-DIVERSITY ATTENUATION MEASUREMENTS
AT 28 GHZ BY RADIOMETERS FOR AN EARTH-SPACE PATH

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the Ohio State University

By

Kuan-ting Lin, M.S.

* * * * *

The Ohio State University
1986

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Professor Roger C. Rudduck

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CHAPTER I
INTRODUCTION

The development of satellite communications has tended to increase the operating frequencies up to their limit [1]. The limitation is mainly due to the absorption effect of the Earth atmosphere and of hydrometeors in the forms of clouds, rain, fog, and wet snow, especially rain, which reduce the reliability and performance of space-earth communication links, particularly for those operating above 10 GHz. At higher frequencies, scattering loss as well as absorption loss due to rain becomes significant. Both account for the attenuation of the transmitted signal. Other deleterious effects can also be caused by rain on satellite-earth links, such as cross-polarization and angle-of-arrival fluctuations [2-5]. Here we are interested only in rain attenuation effects and in related methods of improving the reception of satellite-transmitted signals. Since rain is often localized and forms in cells, the utilization of multiple antennas at far separated locations to receive satellite signals will greatly improve the reliability of the communication channels. This method is usually called site diversity. Switching, i.e., instantaneously selecting the signal suffering the least attenuation, was the first type of diversity processing proposed and is easy to visualize conceptually [6]. The linear or nonlinear combination of signals received from
various sites also can be used to improve the system performance. Maximal-ratio combining (simultaneously adjusting each signal channel gain and phase to maximize the signal-to-noise ratio after linear combining) is one of the best among those signal combining methods [7]. Both methods will be considered in Chapter V.

The measurement of the attenuation over a path from a satellite to ground terminal can be used to generate meaningful statistics when a suitable satellite beacon is available over the time span of at least a year, although several years is a preferable interval. Often a beacon is not available for a sufficient time period, and an indirect measurement has been used as an alternative. It consists of measuring the radiometric noise emitted by the propagation medium, which is related to the attenuation over the earth-satellite path. The relationship is not necessarily a simple one and the extraction of attenuation information from radiometric data is discussed in some detail in Chapter III. Radiometers have long been used as a means to measure the noise in order to infer the path attenuation. Several radiometer designs are available [8-10]. They can be divided into two categories. The first is essentially a superheterodyne receiver followed by a square-law detector, a low-pass filter, and an integrator, as shown in Figure 1.1. It measures the total noise power from the antenna plus that from the receiver, and is therefore called a total power radiometer. The second is a receiver of which the input is continuously switched between the antenna and a reference noise source, at a frequency high enough so that the gain does not have time to change
Figure 1.1. Typical total power radiometer block diagram.
during one cycle. Such a radiometer is called a Dicke radiometer. A block diagram of a Dicke radiometer is shown in Figure 1.2. The Dicke radiometer was used here because it is less sensitive to short-term gain variations and does not require as high power handling as a total-power radiometer does.

The problem considered in this dissertation is to investigate the advantages of site diversity reception at 28.6 GHz during rain events for a satellite-earth link. The Comstar D/4 satellite was launched on February 19, 1981, and was stationed in geosynchronous orbit at 127 degrees west. The satellite beacon of Comstar D/4 at 28.6 GHz was only available during the very early stage of the experiment, starting in March 1981, and then was turned off on September 1, 1981. Since there was no suitable satellite beacon available over an entire year, two radiometers were used to measure attenuation indirectly. During an interval when both the beacon and a radiometer were available, the beacon signal was received and used to verify the data taken radiometrically at one site; thus it served as a calibration for the radiometer at one site. An identical radiometer was used at the second site, located at a distance of 9 km. The data taken with the two radiometers over the period from June 1983 to March 1985 constitutes the site-diversity reception data set.

The results are presented in the following manner. Single-site performance is analyzed by computing the cumulative measured attenuation distribution, the distribution of the attenuation by hour of day, and the fade-duration and inter-fade interval distributions. Diversity performance is analyzed by first computing the two-dimensional joint
Figure 1.2. Typical Dicke radiometer block diagram.
attenuation probability and then the system attenuation distribution, its variation with hour of day, and the system fade-duration and inter-fade interval distributions, for both switching and maximal-ratio combining systems.

This dissertation is organized in the following manner. Chapter II gives an account of the experimental hardware and calibration procedures. Chapter III covers the necessary theoretical background material for inferring path attenuation radiometrically and gives the details of the method employed here. Chapter IV contains the details of the data-processing method. The results of the experiment are given in Chapter V. Chapter VI summarizes the conclusions drawn from the results.
CHAPTER II
EXPERIMENT DESCRIPTION

A. INTRODUCTION

This chapter describes the equipment used in the experiment and the calibration procedures for the equipment. First, let us introduce the location of this experiment, and in the following sections, the equipment will be discussed.

The data were collected with radiometers at two sites in the Columbus, Ohio area. One was located at the Satellite Communication Facility of the ElectroScience Laboratory, called the main site in this report; the other site was to the west of Don Scott airport of The Ohio State University and is called the remote site. Both receivers operated at 28.6 GHz. The distance between the two locations is approximately 9 km. A map (Figure 2.1) shows the two sites and also indicates the direction to the satellite COMSTAR D/4 at an azimuth of 236.4° and an elevation of 25.6°. The main-site antenna was pointed in that direction during the entire experiment. The remote-site antenna was inadvertently pointed toward an azimuth of 196.4° and elevation of 25.6° for the period from day 153/1983 to day 40/1984; thereafter the azimuth was corrected to 236.4°. The data will be presented in Chapter V for two yearly periods, i.e. from day 153/83 to day 152/84 and from day 61/84 to
Figure 2.1. Locations of the two diversity sites.
day 60/85. Because of the mispointing, the joint data for the earlier period is not clearly relatable to diversity operation, and this is indicated by a cautionary note wherever it is applicable. The radar was occasionally used to detect the rain rate along the path at the main site, and was not operated on a continuous basis. The rain gauge was used to measure the rain rate at the main site. All the measured data were digitized and recorded on magnetic tapes controlled by an HP2116B computer. The computer is the center of the data-acquisition system and is also located at the main site.

B. RADIOMETERS

The two radiometers are essentially identical. A more detailed description of these radiometers has been given by Pigon [11]. Here we only give a general description of the radiometer system. A block diagram of both radiometers is shown in Figure 2.2. Both antennas are focal point-fed parabolic reflectors with a diameter of 1.6 m and are vertically polarized with a maximum gain of about 50 dB and beamwidth of 0.45°. The RF front end consists of a Dicke switch in the form of a switchable circulator to modulate the RF signal which is to be synchronously detected in the processor, a three-position waveguide switch to aid in system calibration, two waveguide-termination ovens and their control circuits for reference and calibration temperatures, and a solid-state noise source with a waveguide attenuator and a directional
Figure 2.2. 28.6 GHz radiometer block diagram.
coupler, originally intended to provide a positive voltage at the synchronous-detector output (see below), but which is no longer used in this manner. The nominal temperatures for the reference and calibration ovens are 340 K and 440 K respectively.

A double balanced mixer, with a Gunn-diode local oscillator input, beats the modulated RF signal to a 30 MHz Intermediate Frequency (IF). The IF is amplified and filtered by a broadband amplifier and an IF filter centered at 30 MHz with a bandwidth of 8 MHz. The IF signal is then envelope-detected by an amplifier-detector. The frequency after detection is 1951 Hz, the Dicke modulation frequency. This odd number is used because it lies mid-way between 60 Hz harmonics (which may be generated by a power supply unit). The detector is of square-law type, so that the voltage output is linearly related to the temperature difference at the inputs to the Dicke switch, in Kelvins. The signal is further amplified, synchronously detected, and integrated with a integration time of 3 seconds to get a DC voltage output corresponding to a given effective input temperature differential at the Dicke-switch input. The output then is digitized and recorded on magnetic tape together with other temperature information. The remote-site radiometer output is digitized and sent through a telephone data line to the main computer.

The noise source was turned off, in part, because it was becoming unstable (since the noise source was not intended for long-term use) and, in part, to improve the sensitivity of the receiver. The processors had to be redesigned to accept negative outputs from the
synchronous detectors. A detailed description of the rebuilt portion is given in Appendix A. Otherwise, the instruments remained as documented by Pigon [11].

C. RADIOMETER CALIBRATION

The radiometers were calibrated from time to time in order to ensure the system reliability. The calibration adjustments consist of resetting the output level for null input and setting the system gain. The waveguide switch is designed for this purpose; it allows the Dicke-switch port normally connected to the antenna to be switched manually to a matched load in the reference oven or a matched load in the calibration oven. Since the system has a linear input/output characteristic, two known noise temperature inputs are sufficient to determine the system characteristics. The two known noise temperatures are produced by switching the waveguide switch to the reference oven or the calibration oven. The corresponding noise temperatures at the Dicke switch input have to be computed from the oven temperatures, waveguide transmission factors, and waveguide temperatures by the relationship

\[ T_e = T_0 \alpha + (1 - \alpha)T_W, \]  

(2-1)

where

- \( T_e \) is the noise temperature at the Dicke switch input,
- \( T_0 \) is the oven temperature
- (the reference oven or the calibration oven),
$T_W$ is the temperature of the waveguide,
$\alpha$ is the transmission factor of the waveguide
(of the reference oven path or the calibration oven path).
The corresponding outputs are
\[ V_0 = G[T_e - T_{\text{eff}}] \]  \hspace{1cm} (2-2)
where
\[ T_{\text{eff}} = T_k \alpha_k - T_W(1 - \alpha_k) \]  \hspace{1cm} (2-3)
and
$T_k$ is the temperature of the reference oven,
$\alpha_k$ is the waveguide transmission factor for
the reference oven path which directly
connects to the Dicke switch,
$G$ is the system gain in volts/Kelvin.

Table 2.1 shows the waveguide transmission factors of the radiometers as measured prior to the experiment. The temperatures of the reference and calibration oven and the front-end box are measured at each calibration. The calibration procedure is the following. First, the null of the system is set with the waveguide switch switched to the reference-oven position, because the input level is then almost zero. After the radiometer output voltage has been adjusted to agree with the output voltage calculated for the reference oven input, the waveguide switch is switched to the calibration-oven position and the system gain is adjusted until the output agrees with the corresponding precalculated value for the desired gain value, 0.02 volts per Kelvin. The
TABLE 2.1
WAVEGUIDE TRANSMISSION FACTOR OF RADIOMETERS

<table>
<thead>
<tr>
<th>Waveguide path</th>
<th>Losses(dB)/Transmission factor</th>
<th>Main site</th>
<th>Remote site</th>
</tr>
</thead>
<tbody>
<tr>
<td>F to D</td>
<td>.75/.841</td>
<td>.75/.841</td>
<td></td>
</tr>
<tr>
<td>G to D</td>
<td>1.8/.661</td>
<td>1.47/.713</td>
<td></td>
</tr>
<tr>
<td>E to D</td>
<td>1.85/.653</td>
<td>1.08/.780</td>
<td></td>
</tr>
<tr>
<td>A to D</td>
<td>1.55/.700</td>
<td>1.25/.750</td>
<td></td>
</tr>
<tr>
<td>Antenna to D</td>
<td>2.4/.575</td>
<td>2.25/.596</td>
<td></td>
</tr>
</tbody>
</table>
input-output characteristics of the radiometer system is shown in Figure 2.3 and is almost linear. The measured clear-weather antenna temperature is about 65 K during the summer and 40 K during the winter.

D. RAIN GAUGE

The rain gauge used in the experiment is a tipping-bucket rain gauge located at the main site. Each tip of the rain-gauge bucket corresponds to 0.254 mm of rainfall accumulation and is counted cumulatively and recorded on the magnetic tape, with other data, at three-second sample intervals. It is assumed that the rain rate between tips is constant so that the rain rate can be computed from the number of samples between tips by

\[ R = \frac{304.8}{N} \text{[mm/hr]}, \]  

(2-4)

where

- \( R \) is the rain rate between tips,
- \( N \) is the number of samples between tips.

Thus, the rain rate computed from Equation (2-4) has a high limit of 304.8 mm/hr. This rain rate was never exceeded during our experiment. The counter will automatically reset to zero whenever tip counts total 255, the maximum that can be accomodated in the data word. At times the bucket tipped only once throughout an event, then we assume there was no rain. Note that the rain rate was averaged over a non-uniform time period, i.e., the higher the rain rate derived, the shorter was the time used for averaging.
Figure 2.3. The input-output characteristics of the radiometer.

SLOPE $= 19.5 \text{mV/K}$

at the antenna port of Dicke Switch
E. RADAR SYSTEM [12]

The radar located at the main site is a conventional magnetron pulse radar with a 9.1 m Cassegrainian-fed parabolic antenna, operating at 3.064 GHz. The antenna is vertically polarized with a maximum gain of 44 dB and beamwidth of 0.75°. The radar-pulse length is about 1.3 μs, and the pulse is repeated at a rate of 100 Hz, but only for 32 pulses every three seconds. The peak power output from the magnetron is about 3.64 kW. A block diagram of the radar system is shown in Figure 2.4. The receiver IF is centered at 30 MHz with a bandwidth of 10 MHz. The radar interface generates S-FIRE and S-GATE pulses which control the magnetron to fire and the receiver gate to open, respectively. After the radar fires and the receiver gate opens, the returned IF signal is log-amplified and sampled at every 1 μs for 100 samples. The gate initiates the first sample with a delay of 3.82 μs following the firing of a radar pulse. One hundred successive samples are then stored in the integrator shift register. The radar fires again 10 ms later. The process is repeated with the new 100 samples being added digitally to the previous samples on a range-bin by range-bin basis. This procedure is repeated until 32 samples in each range bin have been added, so that the return in dB is effectively integrated over 32 samples. The integrated data are then transferred to the data buffer, ready to be recorded. After the 32 samples, the radar is quiet until the next 3-second interval. The shift register is also set to zero, ready for the next cycle. The whole process is repeated every 3 seconds. Thus,
Figure 2.4. The block diagram of the S-band radar system.
the 100 adjacent range-sorted radar returns are each log-integrated for 32 samples, which takes 320 ms.

F. RADAR CALIBRATION

In order to calibrate the radar receiver, a 3.064 GHz signal generator, see Figure 2.4, with an internal precision step attenuator, was connected to the input of the receiver front end. Then the output of the receiver was recorded for various input levels. The resulting calibration curve is shown in Figure 2.5. Thus, the dynamic range is about 70 dB, with a minimum detectible level of -70 dBm at the calibration port, point J in Figure 2.4. In order to establish the corresponding power levels at the antenna port rather than the calibration port, we must refer to the radar insertion losses, shown in Table 2.2. Thus, the received signal level at the antenna port is 17.85 dB below the calibration level to produce the same radar output.

Due to the logarithmic amplifier used in the radar receiver, an additional correction is required to account for the fluctuation of the received signals. Let us consider the relationship between the log of the average of a set of samples, \( v_0 \), and the average of the log of the individual samples, \( v \), i.e.

\[
v = \frac{1}{N} \sum_{i=1}^{N} 10 \log_{10} p_i,
\]

(2-5)

and

\[
v_0 = 10 \log_{10} \left( \frac{1}{N} \sum_{i=1}^{N} p_i \right),
\]

(2-6)
Figure 2.5. The S-band radar calibration curve.
TABLE 2.2
INSERTION LOSS FOR RADAR

Waveguide path | Losses (dB)
----------------|-----------
A to B          | 3.8       
D to A          | 1.6       
C to A          | 1.85      
A to E          | 3.2       
F to B          | 2.05      
G to H          | 0.5       
G to B          | 2.75      
J to F          | 19.6      

(Refer to Figure 2.4)
or in other words, \( v \) is the value which the radar received and \( v_0 \) is the average power to be inferred. In general, the radar returns are random, so that \( v \) and \( v_0 \) are not equal. If we assume that the amplitude of the radar return is Rayleigh-distributed, according to Marshall and Hitschfeld [13] the relation will be

\[
v_0 = v + 2.51 \text{ dB.}
\]

Thus, the correct difference level between the antenna port and the calibration output port must be reduced to 15.34 dB (17.85 - 2.51 dB) in order for the radar output calibrated in average received power. Thus the minimum detectable level is -85.34 dBm (-70 - 15.34 dBm).

The power monitor system is calibrated by first measuring the peak power of the magnetron and then connecting the magnetron to the input of the monitor and attenuating its power with a precision step attenuator. The calibration curve for the monitor is given in Figure 2.6 [14].

G. DATA ACQUISITION SYSTEM

The data acquisition system is based on an HP2116B computer. A detailed description of the system has been given by Weller [15]. The data are sampled and recorded digitally on 9-track magnetic tapes which can be analyzed by programs running on the VAX 11/780 computer in the ElectroScience Laboratory. The tape format is given in Appendix B. The radiometer data and rain-gauge data were sampled every 3 seconds, but other temperatures were sampled at 60 second intervals.
Figure 2.6. The calibration curve of radar power monitor.
The -5.5 volt output of the receivers and other temperature information data were sampled and converted to a 12-bit two's complement binary number at the main site, and to an 8-bit two's complement binary word at the remote site. The converters were calibrated by injecting with a standard voltage source and the output was recorded on the tape. By utilizing the linear-regression technique, the digital-voltage relations can be found with a maximum error of 20 millivolts for the remote site and 3 millivolts for the main site. The relations are

\[ V = \frac{(D-2048)}{199} \] at the main site,

and

\[ V = \frac{(D-128)}{25} \] at the remote site, \hspace{1cm} (2-8)

where

- \( V \) is the voltage input into the A/D converter,
- \( D \) is the digital value recorded (positive).

H. SUMMARY

The equipment used in obtaining data for the experiment has been discussed in this chapter. With this information, the radiometer output can be converted into the noise temperature at the radiometer antenna terminal. The converting procedure will be shown in Section IV.B. The radar is used in the experiment to monitor the rain structure along the path and then to verify the method of converting sky brightness temperature into path attenuation. This method will be discussed in the
following chapter. The relation between the noise temperature received at the radiometer antenna terminal and the sky brightness temperature incoming at the main beam direction of the radiometer antenna will also be discussed in the next chapter.
A. INTRODUCTION

This chapter describes some of the theoretical relationships involved in determining the attenuation on a satellite-earth path from the radiometer antenna temperature. The specific equations used in our data analysis are derived in Sections III.H and III.J. The preceding sections review radiative transfer theory, the radiative transfer equation, and its relationship to the radiometric formula for an absorbing medium. Those who are not interested in this theory, or who are already familiar with it, can proceed directly to Section III.H. Here we are going to explain first briefly how a radiometer works and then to introduce the radiative transfer theory.

The radiometer has been used as an economic means to derive rain attenuation [16]. It receives noise in the form of electromagnetic fields which originate from any absorbing medium due to spontaneous local electric and magnetic moments arising from the thermally induced random motions of its constituent charges. These randomly generated fields are absorbed and scattered by the surrounding medium, and the absorbed power is converted into thermal motion of the charges of the surrounding medium. And again these thermal motion of charges reradiate fluctuating electromagnetic fields. The radiation energy transfer from one location to others is called radiative transfer. At local
thermodynamic equilibrium, the thermally emitted fields depend on the absorbing property and the physical temperature of the medium, and the radiation energy transport can be formulated as the radiative transfer equation (Equation 3-9). In order to derive the relationship between rain attenuation and the radiative noise one has to solve this equation.

A direct solution to this equation has not been found. Numerical methods are available only for certain special geometries, but with the assumption that the scattering effect can be ignored, the solution can be simplified to a useful formula.

In this section the radiative transfer equation is reproduced and the solutions to the equation are reviewed. The analysis of the equation shows the difficulty of coping with scattering effects. Finally, a method of determining the attenuation on a satellite-earth link using the radiometer antenna temperature is derived for data processing.

This formulation is felt to be an improvement over the more usual ones. Usually, at frequencies below 10 GHz the satellite-earth path attenuation below 10 dB has been computed from the radiometric formula [17] using the radiometer antenna temperature with a guess of the mean absorption temperature, such as setting it equal to the surface temperature or a function of the surface temperature, such as the Wulfsberg formula [18]. As the operating frequencies increase, the above usual way of deriving the mean absorption temperature may lose accuracy because the scattering effect of the radiation becomes
important. Also there is no justification for the assumption of a constant $T_m$. The new method used here determines the mean absorption temperature with due account for its dependence on total attenuation and the absorption and temperature profiles over the path. Aside from being better rooted in theory, it has been proved experimentally to be better than previous methods at 28.6 GHz and at the elevation angle of 25.6°. The method will be described from a theoretical viewpoint in Section III.H and confirmed experimentally in IV.C.

B. RADIATIVE TRANSFER EQUATION

The radiative transfer equation has been reproduced in various textbooks; the treatment given here follows Ishimaru [19]. Radiative energy transferred by electromagnetic radiation is generally treated by a phenomenological theory called radiative transfer theory. The theory does not deal with diffraction and interference effects at all, except that such effects are included in the scattering and absorption characteristics of a single particle or a group of particles. Polarization effects may be included through the use of Stokes' parameters. A basic equation developed by Chandrasekhar [20] is called the radiative transfer equation and is often used as the starting point to solve the radiometer problem.

The central quantity of the transfer theory is the specific spectral intensity, defined as follows: Let $dP$ be the amount of flux or average power flowing within a solid angle $d\Omega$ across an elementary area $da$ pointed in a direction specified by an unit vector $\hat{s}$ in a small
frequency range \((f, f+df)\). To the first order approximation, one can assume \(dP\) is linearly proportional to the projection of \(da\) onto a plane normal to the \(\hat{s}\) direction, as well as to \(d\Omega\) and \(df\):

\[
dP = I(\vec{r}, \hat{s}, f) \cos \theta \; da \; d\Omega \; df, \quad (3-1)^*\]

where \(\theta\) is the angle between \(\hat{s}\) and the normal to \(da\). This defines the specific spectral intensity \(I(\vec{r}, \hat{s}, f)\) which is a function of position, direction and frequency. Let us define the specific intensity \(I(\vec{r}, \hat{s})\) in a narrow frequency band \(B\) over which \(I(\vec{r}, \hat{s}, f)\) does not vary appreciably, with

\[
I(\vec{r}, \hat{s}) = \int_{f-B/2}^{f+B/2} I(\vec{r}, \hat{s}, f_1) \; df_1 = I(\vec{r}, \hat{s}, f) \; B \; [\text{Watts/m}^2/\text{rad}]. \quad (3-2)
\]

Then the total flux passing through a small area within a solid angle \(d\Omega\) pointed in the direction \(\hat{s}\) in the narrow frequency band \(B\) centered at \(f\) is

\[
dF(\vec{r}, \hat{s}) = I(\vec{r}, \hat{s}) \cos \theta \; da \; d\Omega. \quad (3-1a)
\]

Let us consider a specific intensity \(I(\vec{r}, \hat{s})\) incident on a cylindrical volume element with cross section \(da\) and length \(ds\). The volume \(ds \; da\) contains \(n(D, \vec{r}) dD \; ds \; da\) particles with size between \(D\) and \(D+dD\), where \(n(D, \vec{r}) dD\) is the number of particles with size between \(D\) and \(D+dD\) in a unit volume and is called the size distribution. The particles of interest here are rain-drops which are considered as spheres. Each particle with size \(D\) absorbs power \(a_a(D) I(\vec{r}, \hat{s})\) and scatters power \(a_s(D) I(\vec{r}, \hat{s})\), and therefore the decrease of the flux

*the bar in \(\vec{r}\) denotes a vector, the carat in \(\hat{s}\) denotes a unit vector.*

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\[ dI(\hat{r}, \hat{s})d\Omega \] for the volume within a solid angle \( d\Omega \) and across \( da \) is expressed as the sum of power loss due to individual particles and gas absorption within the volume. By taking the ensemble average, the summation can be replaced with integration,

\[
d\Omega \ da \ dI(\hat{r}, \hat{s}) = -d\Omega \ dsda \left[ \int (\sigma_a(D) + \sigma_s(D))n(D, \hat{r})dD + Kg(\hat{r}) \right] I(\hat{r}, \hat{s})
\]

\[
= -K_t(\hat{r})I(\hat{r}, \hat{s}) \ ds \ da \ d\Omega , \quad (3-3)
\]

where \( K_t(\hat{r}) = \int (\sigma_a(D) + \sigma_s(D)) n(D, \hat{r})dD + Kg(\hat{r}) \), \( \sigma_a(D) \) and \( \sigma_s(D) \) are the scattering and absorption cross sections of a particle with size \( D \), respectively, and \( Kg(\hat{r}) \) is the volumetric gas absorption (mainly water vapor and oxygen absorption). The values of \( \sigma_a(D) \), \( \sigma_s(D) \) and \( Kg(\hat{r}) \) are assumed to be almost constant within the frequency band \( B \). This decrease in the specific intensity may be at least partially offset because a portion of the specific intensity \( I(\hat{r}, \hat{s'}) \) incident on this volume from other directions \( \hat{s'} \) is scattered into the direction \( \hat{s} \) and is added to the intensity \( I(\hat{r}, \hat{s}) \). This contribution will now be explored.

Let us consider flux incident from the direction \( \hat{s'} \) on a single particle inside the volume \( dsda \). The incident flux density through a small solid angle \( d\Omega' \) is given by \( S_i = I(\hat{r}, \hat{s'})d\Omega' \). This flux density is incident on the particles in the volume \( dsda \). The power flux density scattered by the particle in the direction \( \hat{s} \) at a distance \( R \) from the particle is then given by \( S_s = \{ |f(\hat{s}, \hat{s'}; D)|^2/R^2 \} S_i \), where \( f(\hat{s}, \hat{s'}; D) \) is the scattering amplitude of the particle with size \( D \), and does not vary
appreciably over the narrow frequency band $B$. Thus, the scattered flux in the direction $\hat{s}$ due to $S_i$ is

$$S_i R^2 d\Omega = |f(\hat{s},\hat{s}';D)|^2 S_i d\Omega = |f(\hat{s},\hat{s}';D)|^2 I(\hat{r},\hat{s}') d\Omega' d\Omega . \quad (3-4)$$

Adding the incident flux from all directions $\hat{s}'$ and the contributions from all the particles inside the volume $dsda$, which is oriented so that the cross-section $da$ is normal to $\hat{s}$, shows that the total flux scattered into a cone of solid-angle extent $d\Omega$ around the direction $\hat{s}$ is

$$dF(\hat{r},\hat{s}) = \int_{\Omega'} d\Omega' dD d\Omega' d\Omega . \quad (3-5)$$

where the integration over all $\Omega'$ is taken to include the contributions from all directions $\hat{s}'$.

If one defines a phase function

$$p(\hat{s},\hat{s}',\hat{r}) = \frac{4\pi}{K_S(\hat{r})} \int_{\Omega'} f(\hat{s},\hat{s}';D)dD dD,$$  

where $K_S(\hat{r})$ is the scattering coefficient

$$K_S(\hat{r}) = \int \sigma_S(D)n(D,\hat{r})dD,$$

the resulting equation is

$$dF(\hat{r},\hat{s}) = \frac{K_S(\hat{r})}{4\pi} dsa \int_{\Omega'} p(\hat{s},\hat{s}',\hat{r})I(\hat{r},\hat{s}') d\Omega' d\Omega . \quad (3-5a)$$

The phase function will be considered in Section III.E for various rain intensities.
The flux also may increase due to emission from within the volume dsda. Denoting by \( e(\vec{r},\hat{s}) \) the power radiation per unit volume per unit solid angle in the direction \( \hat{s} \), the increase of flux is given by

\[
dsda \ e(\vec{r},\hat{s})d\Omega .
\]

Adding all the contributions, then, gives the complete equation governing the energy balance at a position \( \vec{r} \) along the direction \( \hat{s} \) of a pencil ray due to absorption, scattering and emission along the path, as written below:

\[
\frac{dI(\vec{r},\hat{s})}{ds} = -K_t(\vec{r})I(\vec{r},\hat{s}) + \frac{K_s(\vec{r})}{4\pi} \int p(s',\hat{s}',\vec{r})I(\vec{r},\hat{s}')d\Omega' + e(\vec{r},\hat{s}).
\]

(3-7)

For radiometer application, the medium is in local thermodynamic equilibrium at a physical temperature \( T_p(r) \). Kirchhoff's law gives a reasonable approximation for the emission of the radiation intensity,

\[
e(\vec{r},\hat{s}) = K_a(\vec{r})B(2hf^3/c^2)/[\exp(hf/kT_p(\vec{r}))-1]
\]

(3-8)

where

- \( B \) is the narrow frequency band in Equation (3-2),
- \( f \) is the center frequency of the band,
- \( c \) is the velocity of light,
- \( K_a(\vec{r}) \) is the volumetric absorption cross-section [m²/m³], which includes the volumetric gas absorption \( K_g(\vec{r}) \), i.e.,

\[
K_a(\vec{r}) = \int_D \sigma_a(D)n(D,\vec{r})dD + K_g(\vec{r}),
\]

and \( k \) and \( h \) are the Boltzmann and Planck constants, respectively.
Since the physical temperature $T_p(\vec{r})$ for our interest is a normal ambient temperature and frequencies in the millimeter range are being considered, the emission can be approximated by the Rayleigh-Jeans approximation as $K_a(\vec{r})kT_p(\vec{r})/\lambda^2$, where $\lambda$ is the free-space wavelength. The radiative transfer equation for the radiometer may then be rewritten as

$$\frac{dI(\vec{r},s)}{ds} = -K_t(\vec{r})I(\vec{r},\hat{s}) + \frac{K_s(\vec{r})}{4\pi} \int p(\hat{s},\hat{s}',\vec{r})I(\vec{r},\hat{s}')d\Omega' +$$

$$+ K_a(\vec{r}) \frac{kT_p(\vec{r})}{\lambda^2}. \quad (3-9)$$

where $T_p(\vec{r})$ is the actual temperature of the medium. The specific intensity can be changed to an equivalent noise temperature. We define $I(\vec{r},\hat{s})$ as

$$I(\vec{r},\hat{s}) = \frac{k}{\lambda^2} T(\vec{r},\hat{s}) \quad (3-10)$$

and

$$I(\vec{r},\hat{s},f) = \frac{k}{\lambda^2} T(\vec{r},\hat{s},f), \quad (3-10a)$$

where $T(\vec{r},\hat{s},f)$ is the spectral noise temperature.

Thus, the final radiative transfer equation for the radiometer is

$$\frac{dT(\vec{r},\hat{s})}{ds} = -K_t(\vec{r})T(\vec{r},\hat{s}) + \frac{K_s(\vec{r})}{4\pi} \int p(\hat{s},\hat{s}',\vec{r})T(\vec{r},\hat{s}')d\Omega' +$$

$$+ K_a(\vec{r})T_p(\vec{r}). \quad (3-11)$$

Note that $T(\vec{r},\hat{s})$ is the brightness temperature at location $\vec{r}$ pointing in the $\hat{s}$-direction.
C. INTEGRAL EQUATION FOR RADIATIVE TRANSFER

The radiative transfer equation can be recognized as a first-order differential equation with respect to \( s \) and has the form

\[
\frac{dy(s)}{ds} + P(s)y(s) = Q(s). \tag{3-12}
\]

The general solution for this equation is

\[
y(s) = y(s_0)e^{-\tau(s)} + e^{-\tau(s)} \int_{s_0}^{s} Q(s_1)e^{+\tau(s_1)} ds_1, \tag{3-13}
\]

where \( \tau(s) = \int_{s_0}^{s} P(s_1) ds_1 \),

and \( y(s_0) \) is known as a boundary condition. Comparing Equation (3-11) and (3-12) we get

\[
y(s) = T(\hat{r}, \hat{s}),
\]
\[
y(s_0) = T(\hat{r}_0, \hat{s}),
\]
\[
P(s) = K_t(\hat{r}),
\]
\[
Q(s) = \frac{K_s(\hat{r})}{4\pi} \int \Omega' p(\hat{s}, \hat{s}', \hat{r}) T(\hat{r}, \hat{s}') d\Omega' + K_a(\hat{r}) T_p(\hat{r}), \tag{3-14}
\]

where \( s \) is a distance parameter measured along the \( \hat{s} \)-direction.

Then the integral equation for the noise temperature \( T(\hat{r}, \hat{s}) \) is obtained by substituting Equation (3-14) into (3-11), and the result is
\[ T(\hat{r}, \hat{s}) = \int_{\hat{s}_0}^{\hat{s}} \exp[-(\tau(s) - \tau(s_1))] \left[ \frac{K_s(\hat{r}_1)}{4\pi} \int_{\hat{s}_0}^{\hat{s}_1} p(s, s', \hat{r}_1) T(\hat{r}_1, s') \, d\hat{s}' + \right. \\
+ \left. K_a(\hat{r}_1) T_p(\hat{r}_1) \, ds_1 + \exp[-\tau(s)] \, T(\hat{r}_0, \hat{s}) \right] \, ds_1, \quad (3-15) \]

where
\[ \tau(s) = \int_{\hat{s}_0}^{s} K_t(\hat{r}_1) \, ds_1. \]

By using the successive substitution technique, the integral equation can be solved; first choose \( T_1(\hat{r}, \hat{s}) = 0 \) to replace the unknown \( T(\hat{r}, \hat{s}) \) in the integrand and apply the boundary condition at boundary \( \hat{r}_0 \) to obtain
\[ T_2(\hat{r}, \hat{s}) = \int_{\hat{s}_0}^{\hat{s}} \exp[-(\tau(s) - \tau(s_1))] K_a(\hat{r}_1) T_p(\hat{r}_1) \, ds_1 \\
+ \exp[-\tau(s)] \, T(\hat{r}_0, \hat{s}). \quad (3-16) \]

Iterating the above procedure, we obtain the solution to the radiative transfer equation as a sum of exponential integrals (refer to Figure 3.1 for definition of symbols)
\[ T(\hat{r}, \hat{s}) = T_\infty(\hat{r}, \hat{s}) = \int_{\hat{s}_0}^{\hat{s}} \exp[-(\tau - \tau_1)] K_a(\hat{r}_1) T_p(\hat{r}_1) \, ds_1 + \\
\left( \int_{\hat{s}_0}^{\hat{s}} \exp[-(\tau-\tau_1)] K_s(\hat{r}_1) \int_{\hat{s}_0}^{\hat{s}_1} p(s, s', \hat{r}_1) \int_{\hat{s}_0}^{\hat{s}_1} \exp[-(\tau' - \tau'_1)] \right) \times \\
K_a(\hat{r}_1) T_p(\hat{r}_1) \, ds_1 \, d\hat{s}_1 \, d\hat{s}'_1 \, d\hat{s}_1 + \\
\int_{\hat{s}_0}^{\hat{s}} \exp[-(\tau - \tau_1)] K_s(\hat{r}_1) \int_{\hat{s}_0}^{\hat{s}_1} p(s, s', \hat{r}_1) \int_{\hat{s}_0}^{\hat{s}_1} \exp[-(\tau' - \tau'_1)] \right) \times \\
K_a(\hat{r}_1) T_p(\hat{r}_1) \, ds_1 \, d\hat{s}_1 \, d\hat{s}'_1 \, d\hat{s}_1 + \\
\int_{\hat{s}_0}^{\hat{s}} \exp[-(\tau - \tau_1)] K_s(\hat{r}_1) \int_{\hat{s}_0}^{\hat{s}_1} p(s, s', \hat{r}_1) \int_{\hat{s}_0}^{\hat{s}_1} \exp[-(\tau' - \tau'_1)] \right) \times \\
K_a(\hat{r}_1) T_p(\hat{r}_1) \, ds_1 \, d\hat{s}_1 \, d\hat{s}'_1 \, d\hat{s}_1 + \]

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\[
\frac{K_S(s_1)}{4\pi} \int \frac{\Omega''}{p(s',s''_1)} \int \exp[-(\tau''-\tau_1'')] \int \int \int ds_1 d\omega'' ds_1 d\omega' ds_1 \\
+ \ldots
g + \left[\exp(-\tau)\right]T(\hat{r}_0,s) + \\
\int \left[\exp\left(-(\tau-\tau_1)\right)\right] \frac{K_S(s_1)}{4\pi} \int \frac{\Omega'}{p(s',s'_1)} \left[\exp\left(-\tau_1'\right)\right]T(\hat{r}_0',s') \int \int \int ds_1 d\omega' ds_1
+ \ldots \ldots \ldots (3-17)
\]

where
\[
\tau = \tau(s) = \int_{s_0}^{s} K_t(\hat{r}_2) ds, \quad \tau_1 = \tau(s_1),
\]
\[
\tau' = \tau'(s') = \int_{s_0}^{s'} K_t(\hat{r}_2') ds', \quad \tau_1' = \tau'(s_1'),
\]
\[
\tau'' = \tau''(s'') = \int_{s''_0}^{s''} K_t(\hat{r}_2'') ds_2, \quad \tau_1'' = \tau''(s_1''), \text{ etc.}
\]

D. PHASE FUNCTION

Let us compute the phase function for a given rain rate. The rain drop size distribution was assumed to be exponential, given by Marshall and Palmer [21]
Figure 3.1. Multiple-scattering paths for the computation of radiative transfer.

The symbols without primes ($\mathbf{r}_0, \mathbf{r}_1, \mathbf{s}_0, \mathbf{s}_1$ and $s$) are on the direct path. The symbols with one prime ($\mathbf{r}'_0, \mathbf{r}'_1, \mathbf{s}'_0, \mathbf{s}'_1$ and $s'$) are on the first order scattering path. The symbols with double primes ($\mathbf{r}''_0, \mathbf{r}''_1, \mathbf{s}''_0, \mathbf{s}''_1$ and $s''$) are on the second order scattering path. All $s$'s are distance parameters measuring along the path from the boundary ($s_0'$, $s_0''$, and $s_0$) to the scattering points ($s', s''$) or to the observation point ($s_1=s$). All $\mathbf{r}$'s are position vectors relative to a reference point (not shown).
\[ n(D) = N_0 e^{-\Lambda D \text{[drops/(m}^3\text{mm)]}}, \quad (3-18) \]

where
\[ \Lambda = \alpha R^\beta \text{ (mm}^{-1}\text{)}, \]
\[ N_0 = 8000 \text{ [drops/(m}^3\text{mm)]}, \]
\[ \alpha = 4.1, \]
\[ \beta = -0.21, \]

where \( R \) is the rain rate in mm/hr and \( D \) is the droplet diameter in mm. The rain water refractive index was taken to be [22] at 30 GHz and 300 K
\[ m = 5.992584 - j2.808710, \]
\[ \lambda = 1 \text{ cm}. \]

The equation used for the phase function is
\[ p(\theta) = \frac{\lambda^2 \int_0^D \left[ |S_1(\theta, D)|^2 + |S_2(\theta, D)|^2 \right] n(D) dD}{2K_s}, \quad (3-19) \]

where \( \theta \) is the angle between the incident and the scattered directions, \( S_1(\theta, D) \) and \( S_2(\theta, D) \) are the scattering amplitude \( S \) of particle size \( D \) derived from Mie scattering for vertical and horizontal polarizations [23,24], and \( K_s \) is the scattering cross section per unit volume defined below Equation (3-6). The calculated functions due to rain are shown in Figures 3.2, 3.3 and 3.4. The function does not vary significantly with rain rate for rain rates from 1 mm/hr to 25 mm/hr and is recognized to be
Figure 3.2. The computed phase function for 1 mm/hr rain at 30 GHz and 300 K (spherical rain-drops and Marshall and Palmer drop-size distribution).
Figure 3.3. The computed phase function for 10 mm/hr rain at 30 GHz and 300 K (spherical rain-drops and Marshall and Palmer drop-size distribution).
Figure 3.4. The computed phase function for 25 mm/hr rain at 30 GHz and 300 K (spherical rain-drops and Marshall and Palmer drop-size distribution).
dominated by small rain-drops. Although the phase function is simple, the solution to the radiative equation is not easy. Next, we are going to evaluate the volume which contributes most of the radiation and show the difficulty of coping with the scattering effect.

E. ACTIVE SCATTERING VOLUME

The active scattering volume is defined as the volume which contributes most of the radiation received at the observation point \( \hat{r} \) incident in a specific direction \( \hat{s} \). It is analogous to the active rain volume for rain attenuation [25]. For simplicity, let us assume that the rain-filled medium is homogeneous and that no radiation originates from outside the medium. Then from the equations below (3-17) we have

\[
\tau = K_t(s-s_0), \quad \tau' = K_t(s_1-s_0), \quad \tau'' = K_t(s''-s_0)
\]

\[
\tau_1 = K_t(s_1-s_0), \quad \tau'' = K_t(s''-s_0), \quad \tau''_1 = K_t(s''_1-s_0), \ldots
\]

and Equation (3-17) becomes

\[
T(\hat{r}, \hat{s}) = \frac{K_a}{K_t} \int_{0}^{\tau} \exp[-(\tau-\tau_1)] T_p(\hat{r}_1) d\tau_1 +
\]

\[
\frac{K_s}{4\pi K_t} \frac{K_a}{K_t} \int_{0}^{\tau} \exp[-(\tau-\tau_1)] \int_{0}^{\tau'} \int_{0}^{\tau''} \exp[-(\tau'-\tau'_1)] x
\]

\[
T_p(\hat{r}_1) d\tau_1 d\Omega d\tau_1 +
\]

\[
\frac{K_s}{4\pi K_t} \frac{K_s}{4\pi K_t} \frac{K_a}{K_t} \int_{0}^{\tau} \exp[-(\tau-\tau_1)] \int_{0}^{\tau'} \int_{0}^{\tau''} \exp[-(\tau'-\tau'_1)] x
\]

\[
\int \int p(\hat{s}, \hat{s}') \int_{0}^{\tau''} \exp[-(\tau''-\tau''_1)] T_p(\hat{r}_1) d\tau_1 d\Omega d\tau_1 d\Omega
\]

\[
+ \ldots \ldots \quad 42
\]
The first term represents the radiation emitted from the medium without scattering, the second term, radiation that has been scattered once, and others represent energy scattered more than once. If the scattering cross-section per unit volume, $K_s$, is much less than the absorption cross-section per unit volume, $K_a$, the first term will dominate, and it is an integration along the propagation path. In other words, in this case only the medium along the path contributes to the radiation noise received; the other terms in the sum are negligible although positive. When the condition is not satisfied, more terms are needed. Let us consider the second term, first, to find the volume which contributes to the single-scattered radiation noise at $\mathbf{r}$ and in the direction $\hat{s}$. Since there is no hot spot in a uniform medium, and since the exponential factor $\exp[-(\mathbf{r}-\mathbf{r}_1)] = \exp[-K_t(s-s_1)]$ falls as the path length increases, the volume will be confined within the region of small $(s-s_1)$, i.e., near the observation point. Thus, the volume is spherical with the receiver as the center and with a radius $N/K_t$ ($N$ is explained below), and is independent of the receiving direction, as far as calculation of $T$ is concerned. (When the received noise power is calculated, $T$ will be convolved with the antenna pattern, see section J below, and directional considerations will enter at that point.) For multiple scattering terms the active volumes are inside the first-order active scattering volume, since the higher order scattering paths are more likely longer than the first-order scattering path with both the emitted point and received point fixed. The constant $N$ is chosen on the basis that $\exp[-K_t(s-s_1)]$ is set equal to $e^{-N}$ at the active scattering volume boundary;
N determines the magnitude of the contributions that will be neglected when one integrates only over the active scattering volume in determining first-order scattering contributions. For reasonable rain rates, this volume turns out very large. Table 3.1 shows $K_t$ values as a function of rain rate at 28.6 GHz. Even for $N = 1$, the radius turns out over 8 km at 5 mm/hr rain and 0.84 km at 50 mm/hr. It is unreasonable to assume rain to be constant over these distances at these rates. Therefore the active scattering volume concept, which is useful for denser media (larger $K_t$), turns out to be useless for the present purpose; the integrations cannot be limited to a conveniently small volume but must be carried out over most or all of the rain medium.

**TABLE 3.1**

RAIN ATTENUATION $K_t$ IN NEPER/KM

(SPHERICAL RAIN DROPS, MARSHALL-PALMER DROP-SIZE DISTRIBUTION, AT 28.6GHz, 300 K)

<table>
<thead>
<tr>
<th>RAIN RATE (MM/HR)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_t$ (NEPER/KM)</td>
<td>0.1225</td>
<td>0.2519</td>
<td>0.3769</td>
<td>0.4981</td>
<td>0.6171</td>
<td>1.1920</td>
<td>1.7311</td>
<td>2.2353</td>
</tr>
</tbody>
</table>
F. REVIEW OF NUMERICAL SOLUTIONS TO THE RADIATIVE TRANSFER EQUATION

Despite the difficulty of finding solutions to the radiative transfer equation, several researchers have solved the equation numerically for the case of the plane-stratified medium. Oguchi [26] adopted an extension of the spherical harmonics method for the numerical solution of the radiative transfer equation for the plane-stratified random medium. In this method, the specific intensities are expanded in generalized spherical functions, and then the integral term in the transfer equation can be evaluated analytically, thus yielding a set of simultaneous ordinary differential equations which is then solved numerically subject to the boundary conditions. For spherical rain-drops, it is found that the exact expression of Mie scattering can be incorporated into the integral term, allowing the integral to be evaluated without approximation. A numerical solution of the equation is obtainable even for a rather thick rain layer. The solution was intended to describe the incoherent-scattering effect due to rain for millimeter wave propagation, but the technique is applicable to radiometry.

Ishimaru and Cheung [27], following Chandrasekhar, used the Gaussian quadrature formula to replace the integral over $\Omega$ in Equation (3-15) with a sum of $m$ terms, then writing the equation for $m$ directions $S_i$, and thus obtaining $m$ simultaneous ordinary differential equations in $m$ unknown variables $I(\bar{r}, S_i)$, where $i = 1, 2, \ldots, m$. This system can be solved by using the eigenvalues-eigenvectors technique, or by the method of invariant embedding. They showed a result which applies at 30 GHz
and at an elevation angle of 26°: for a 3 km thick, uniform rain, and a uniform rain temperature, \( T = 273 \text{ K} \), the attenuation derived from the sky-brightness temperature underestimated the true attenuation as computed by their method by about 30% for attenuations below 15 dB. In other words, at this frequency and attenuation above 10 dB, the scattering effect cannot be ignored if one wants to determine the path attenuation by measuring radiometric noise.

Tsang et al. [28] used the same numerical method as Ishimaru and Cheung, following Chandrasekhar, and found the angular dependence of the microwave thermal emission. For small optical thickness, brightening occurs at 90° elevation and darkening at small elevation angles. In the case of large optical thickness, darkening occurs at all angles.

Zavody [29] used a method which considers each order of scattering effects separately and which can be derived from the successive substitution method. The solutions converged after computation of a few orders of scattering effects. The analysis showed that multiple scattering in rain cannot be ignored at 37 GHz since the attenuation deduced from brightness temperature with scattering neglected can be too low, by as much as 3 dB for 10 dB true attenuation. The ground emission effect is also shown to be significant by Zavody.

Since rain is seldom uniform in the horizontal plane, conclusions drawn from these numerical results about the relation between path attenuation and antenna temperature for a uniform rain medium must be applied to our measurements with considerable discretion.
G. DERIVATION OF RADIOMETRIC FORMULA FOR ABSORBING MEDIUM

We shall now use the radiative transfer equation to derive the radiometric formula. Scattering effects will be ignored. The derivation will therefore be rigorous only for $K_{t}(\vec{r}) = K_{a}(\vec{r}) \gg K_{s}(\vec{r})$, a condition which is not satisfied at 28.6 GHz for moderate or heavy rains [30]. The justification for using the formula anyway is that it "works", as discussed in IV.C, and that a better one is not available.

When scattering is neglected Equation (3-11) becomes

$$\frac{dT(\vec{r},\hat{s})}{ds} = - K_{t}(\vec{r})T(\vec{r},\hat{s}) + K_{t}(\vec{r})T_{p}(\vec{r}).$$

The solution, Equation (3-15) applies with $T(\vec{r}_{0},\hat{s}) = 0$ to satisfy the boundary condition, and $K_{s}(\vec{r}) = 0$, giving

$$T(\vec{r},\hat{s}) = \int_{S_{0}}^{S} T_{p}(\vec{r}_{1})K_{t}(\vec{r}_{1})\exp\{- \int_{S_{1}}^{S} K_{t}(\vec{r}_{2}) \, ds_{2}\} \, ds_{1}.$$  \hspace{1cm} (3-22)

With constant $T_{p}(\vec{r}) = T_{p}$, the integrand becomes an exact differential

$$T(\vec{r},\hat{s}) = T_{p} \int_{S_{0}}^{S} \exp\{- \int_{S_{1}}^{S} K_{t}(\vec{r}_{2}) \, ds_{2}\} \
= T_{p}[1 - e^{\tau(s)}],$$

where

$$\tau(s) = \int_{S_{0}}^{S} K_{t}(\vec{r}_{1}) \, ds_{1}.$$
In general, $T_p(\vec{r})$ is not a constant. We can then define a mean absorption temperature $T_m$ that satisfies

$$T(\vec{r},s) = T_m \left[ 1 - e^{-\tau(s)} \right], \quad (3-24)$$

as

$$T_m = \frac{1}{1 - e^{-\tau(s)}} \int_{s_0}^{s} T_p(\vec{r}_1) K_t(\vec{r}_1) e^{-\int_{s_1}^{s_2} K_t(\vec{r}_2) ds_2} ds_1. \quad (3-25)$$

Rearrange (3-24) and let $T(\vec{r},s) = T_b$, we obtain

$$\tau(s) = \ln \left( \frac{T_m}{T_m - T_b} \right). \quad (3-25a)$$

This is the "radiometric formula" which will be used to convert measured brightness temperature $T_b$ to optical depth $\tau(s)$ or, equivalently, to path attenuation. To do so, one needs the "mean temperature" $T_m$, defined by Equation (3-25). In practice, this temperature is not known exactly. In the next section we shall derive the method for estimating $T_m$ that will be used in processing the data.

H. ESTIMATION OF THE PARAMETER $T_m$

Historically, the temperature $T_m$ has been chosen, not on the basis of Equation (3-25), but empirically. One approach was simply to choose $T_m$ so that the attenuation derived from data agrees best with attenuation measured independently. This method requires another way of measuring attenuation, and that is not usually available. The second is to calculate $T_m$ from some atmosphere model and assume that this value is approximately good for all weather conditions. Both methods assume a
uniform $T_m$ throughout a rain event. However, if some knowledge about the path is known, one can estimate $T_m$ with much more certainty. Following Leonard and Levis [31] to normalize Equation (3-25) (see Appendix D), we get a formula for the mean absorption temperature $T_m$ independent of the path length $L$,

$$T_m = T_p(r_0) + T_{mn}[T_p(r_a)-T_p(r_0)]$$

and

$$T_{mn} = \frac{\tau(s_a)}{1 - e^{-\tau(s_a)}} \int_0^1 T_n(x_1)K_n(x_1)e^{-\tau(s_a)\nu(x_1)}dx_1,$$  \hspace{1cm} (3-26)

where

$$x_1 = (s_a - s_1)/L,$$

$$\tau(s_a) = \int_{s_0}^{s_a} K_t(r_1)ds_1$$

is the optical depth,

$$\nu(x_1) = \int_0^{x_1} K_n(x_2)dx_2$$

is the optical depth profile along the path,

$$K_n(x_1) = K_t(r_1)/[\int_{s_0}^{s_a} K_t(r_2)ds_2]$$

is the attenuation profile,

$$T_n(x_1) = [T_p(r_1) - T_p(r_0)]/[T_p(r_a) - T_p(r_0)]$$

is the temperature profile,

$K_t(r)$ is the specific attenuation at $r$,

$T_p(r)$ is the physical temperature at $r$,

$T_p(r_0) = 273$ K,

and $T_p(r_a)$ is the surface air temperature.

The parameters $r_a$, $r_1$, $r_0$, $s_a$, $s_1$ and $s$ are shown in Figure 3.5.
Figure 3.5. Path parameters.
Two profiles along the path, those of temperature and of attenuation, must be known or assumed to use the formulation of Equation (3-26) for $T_m$. The temperature profile is relatively uncritical, in part because the temperature range is small compared to its average value and in part because a linear decrease (lapse rate) is generally a good approximation. For the attenuation it is preferable to use radar data when it is available.

The absorption of ice is negligible at 28.6 GHz and the radiation emitted by ice can also be ignored. Thus, the attenuation and the radiation are contributed mostly from the atmosphere up to the 0°C isotherm. If we further assume a uniform rain profile along the path, the equation can be rewritten as,

$$T_m = T_p(R_o) + \left(\frac{1}{1 - e^{-\tau_{sa}}} - \frac{1}{\tau(s_a)}\right)[T_p(R_a) - T_p(R_o)]. \quad (3-26a)$$

The derivation is shown in Appendix D. With minor modifications, Equations (3-25) to (3-26a) were used to analyze the radiometric data of the experiment.
During our experiment radar data was available only for a part of the time since the radar was not engineered for unattended operation. It was found that use of a uniform rain profile, $K_n(x) = 1$, while not as good as use of radar data, gave better agreement than the purely empirical choices when compared with the observed signal attenuation from the Comstar D/4 beacon. This is not unexpected since at least the dependence of $T_m$ on $\tau$ is included correctly, while it is ignored in the case of empirical $T_m$. The experimental evidence will be shown in section IV.C. Despite the theoretical importance of scattering [27, 29] at 30 GHz for medium rain, formula (3-26a) showed good agreement between the radiometer-derived attenuation and that of direct measurement with a satellite beacon, when the latter was available, for less than 15 dB rain attenuation, as is also shown section IV.C. This formula was therefore used in the data analysis for the entire experiment.

J. RADIOMETER ANTENNA EFFECTS

From the above, we know that the attenuation can be inferred by measuring the brightness temperature $T_b$ (Equation 3-25a). But the measured quantity, when using an antenna with finite gain is the convolution of the brightness temperature and the antenna pattern. Here, we will consider the antenna effects on the emission measurement.

The radiation power received by an antenna with receiving cross section $A(\hat{s})$ surrounded by an absorbing medium is

$$ P_r = B \int_{\Omega} I(\vec{f}_{a, \hat{s}}, f)A(\hat{s})d\Omega, $$

(3-27)
where $B$ is the IF bandwidth of the radiometer and $I(\tilde{\mathbf{r}}_a, \hat{s}, f)$ is the incident spectral radiation intensity. From Equation (3-10a), Equation (3-27) reduces to

$$P_r = \frac{kB}{\lambda^2} \int T(\tilde{\mathbf{r}}_a, \hat{s}, f)A(\hat{s})d\Omega.$$  \hspace{1cm} (3-28)

The noise power density available from a resistive load at temperature $T$ is $kTB$. Then, we can define the antenna temperature $T_a$ by $P_r/kB$, and thus

$$T_a = \frac{1}{\lambda^2} \int T(\tilde{\mathbf{r}}_a, \hat{s}, f)A(\hat{s})d\Omega.$$  \hspace{1cm} (3-29)

The antenna gain can be written as

$$G(\hat{s}) = \frac{4\pi}{\lambda^2} A(\hat{s}).$$  \hspace{1cm} (3-30)

Thus,

$$T_a = \frac{1}{4\pi} \int T(\tilde{\mathbf{r}}_a, \hat{s}, f)G(\hat{s})d\Omega.$$  \hspace{1cm} (3-31)

The noise entering into the antenna includes the sky brightness into the main beam of the antenna pattern as well as the contributions from all other lobes. In order to facilitate the computation, the side lobe contributions are defined as $T_{s1}(\hat{s})$, and the main beam sky brightness as $T_{\text{sky}}(\hat{s})$. Letting $\Omega_m$ denote the main beam angle, then the antenna temperature is

$$T_a = \frac{1}{4\pi} \int_{\Omega_m} T_{\text{sky}}(\hat{s})G(\hat{s})d\Omega + \frac{1}{4\pi} \int_{4\pi - \Omega_m} T_{s1}(\hat{s})G(\hat{s})d\Omega.$$  \hspace{1cm} (3-32)
Since the efficiency of the antenna \( e \) is

\[
e = \frac{1}{4\pi} \int_{4\pi} G(\hat{s})d\Omega \equiv 1,
\]

and the beam efficiency of the antenna \( h \) is defined as

\[
h = \frac{1}{4\pi} \int_{\Omega_m} G(\hat{s})d\Omega,
\]

if we let

\[
\hat{\Omega}_{sky} = \frac{1}{4\pi} \int_{\Omega_m} T_{sky}(\hat{s})G(\hat{s})d\Omega
\]

and

\[
\hat{T}_{s1} = \frac{1}{4\pi} \int_{4\pi-\Omega_m} T_{s1}(\hat{s})G(\hat{s})d\Omega
\]

then Equation (3-32) can be simplified as

\[
T_a = h\hat{T}_{sky} + (1-h)\hat{T}_{s1}
\]  (3-35)

The sky brightness temperature \( T_{sky}(\hat{s}) \) varies slowly with the observation angle, and therefore, for a narrow beamwidth antenna, \( T_{sky}(\hat{s}) \) over the main-beam is almost a constant, \( T_b \). Then Equation (3-35) can be rewritten as

\[
T_a = hT_b + (1-h)\hat{T}_{s1}
\]  (3-36)

Rearranging the equation, we obtain

\[
T_b = \frac{T_a - (1-h)\hat{T}_{s1}}{h}
\]  (3-37)

Substituting this into (3-25a) yields
\[
\tau = \ln \frac{h T_m}{h T_m - T_a + (1-h) T_{S1}}
\]  

(3-38)

With no available means to measure \(h\) and \(T_{S1}\), estimates have to be made. Since the receiving antenna was a parabolic reflector with tapered illumination, curved edges, and a low surface tolerance to keep sidelobes low, \(h\) has been assumed to be unity in the data processing. Thus we get

\[
\tau = \ln \frac{T_m}{T_m - T_a}
\]  

(3-39)

This \(h\) value may not be realized for even a well-designed narrow-beamwidth antenna. However, the error introduced by this assumption has been proved to be tolerable. More detailed discussion will be found in Section IV.B.

K. SUMMARY

This chapter has dealt with the theory of inferring path optical depth \(\tau\) (or equivalently path attenuation \(A = 4.34\tau\)) from the measured antenna temperature \(T_a\). An estimate of the "mean" temperature \(T_m\) over the path is needed for the conversion. The method used is based on an extension of a technique proposed by Leonard and Levis, which is not yet readily available in the literature [31]. In this chapter the method was derived, starting with basic radiative transfer theory, to show in detail the assumptions which are implied. They are (1) scattering is negligible, or scattering out of the path is balanced by scattering into
the path (including scattered ground-radiated energy), (2) high aperture efficiency (i.e., the radiation received from the ground via sidelobes can be neglected), and (3) on a statistical basis, $T_m$ can be estimated satisfactorily by assuming a uniform absorption coefficient along the path. Experimental evidence for the validity of these assumptions will be given in the next chapter.
CHAPTER IV
DATA REDUCTION

A. INTRODUCTION

The data reduction process is considered in this chapter. The radiometer data is first reduced to the sky-brightness temperature, and then is converted into the path attenuation by the method described in Section III.H. The proof that this method is appropriate for our data at 28.6 GHz is included in this chapter. With these derived path attenuation data, the attenuation statistics are deduced in two forms: the joint probability density of the attenuations observed at both sites, and the cumulative statistics for the two types of site-diversity processing and for each single site.

B. SKY BRIGHTNESS REDUCTION

In this section, procedures are described for converting the radiometer output into sky brightness. The process is similar to that presented by Pigon [11], but with minor changes since the solid-state noise source was turned off, together with some modification in the signal processor (see section II.B and Appendix A). Next we are going to derive the expression for sky-brightness temperature as a function of the radiometer output and other related information. The radiometer output ranges from -5 to 0 volts and is linearly related to the
difference between the noise temperature at the antenna port and the noise temperature at the reference port of the Dicke Switch (D.S.). The relation, which is similar to Equation (2-2), is given by Equation (4-1),

\[ V_0 = G[T_{aeff} - T_{keff}], \]  

where

\[ V_0 \] is the radiometer output voltage,

\[ G \] is the system gain,

\[ T_{aeff} \] is the noise temperature at the antenna port of the D.S.,

\[ T_{keff} \] is the noise temperature at the reference port of the D.S.

The noise temperature at the output side of a waveguide is related simply to the noise temperature at the input side, the physical temperature of the waveguide and the transmission factor of the waveguide, provided the load is well matched to the guide. In our case, the waveguide connected to the antenna port is partially inside the front-end box, which is at relatively high temperature, and partially outside the box, at a lower temperature. It is therefore appropriate to divide the guide into two sections to compute the output noise temperature. The noise temperatures of the antenna port and the reference oven are therefore given as

\[ T_{keff} = T_k \alpha_k + (1-\alpha_k)T_B, \]

\[ T_{aeff} = [T_a \alpha_o + (1-\alpha_o)T_o] \alpha_B + (1-\alpha_B)T_B, \]  

(4-2)
where

\( T_k \) is the physical temperature of reference oven,

\( T_a \) is the antenna noise temperature,

\( T_0 \) is the physical waveguide temperature outside the box,

\( T_B \) is the physical waveguide temperature inside the box,

\( \alpha_o \) is the transmission factor for the guide outside the box,

\( \alpha_B \) is the transmission factor for the guide inside the box,

\( \alpha_k \) is the transmission factor for the guide from the reference oven.

This equation is similar to Equation (2-3). Substituting Equation (4-2) into Equation (4-1) and simplifying leads to

\[
V_o = G([T_a - T_0] \alpha_a + (T_0 - T_B) \alpha_B + (T_B - T_k) \alpha_k],
\]

(4-3)

where

\( \alpha_a = \alpha_o \alpha_B \) is the total antenna path transmission factor.

Rearranging Equation (4-3) we obtain

\[
T_a = T_0 + \frac{V_o / G - (T_0 - T_B) \alpha_B - (T_B - T_k) \alpha_k}{\alpha_a}.
\]

(4-4)

This is the equation used in data processing to calculate antenna temperatures, from the radiometer output voltages.

All the physical temperatures are sampled every 60 seconds and recorded on magnetic tape together with the radiometer output. The gain
factor of the system is kept fixed and is corrected from time to time, as required, to prevent gain changes, as discussed in Section II.C. The transmission factors were measured prior to the experiment and are shown in Table 2.1. The block diagram, which shows the relative positions of the antenna, waveguides, ovens and the D.S. is shown in Figure 2.2.

Typical "clear day" antenna temperatures measured from our system range from about 40 K during the winter to approximately 65 K during the summer. The calculated sky brightness temperature at 28.6 GHz and 25.6° elevation angle due to gaseous absorption is given in Table 4.1 according to Smith [32], suggesting that on clear days our antenna temperature over-estimates the sky brightness temperature by about 25 K. This could be due to sidelobes, and in retrospect it might have been appropriate to use a value of $h = 0.9$ instead of $h = 1$. However, the error in the attenuation calculated from Equation (3-39) can be shown to be less than 1 dB for the attenuation range 0 to 10 dB, and less than 2 dB for 10 to 15 dB. Thus it has not seemed worthwhile to recompute all the results with what might be a better estimate of $h$.

Due to the long-term continuous operation of the radiometer, the system had to be shut down in certain periods for repair, and sometimes it took several days to repair. Some data loss due to down-time is of course inevitable. Table 5.1 shows the total time that data is available for each month.

Occasionally, a rain drop caught in the horn opening of the antenna, although the antenna and horn were coated with Silibond which should prevent water from adhering to the antenna. The problem was not
<table>
<thead>
<tr>
<th>Water Content g/m³</th>
<th>Air Temperature K</th>
<th>Relative Humidity %</th>
<th>Brightness Temperature K</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>288</td>
<td>17</td>
<td>20</td>
<td>US Atmosphere 40 N</td>
</tr>
<tr>
<td>7.5</td>
<td>288</td>
<td>42</td>
<td>45</td>
<td>US Atmosphere 40 N</td>
</tr>
<tr>
<td>17</td>
<td>303</td>
<td>99</td>
<td>50</td>
<td>Tropical Atmos. 15 N</td>
</tr>
</tbody>
</table>
solved during the operation period and has to be taken into consideration during data processing. Bad data due to the problem was identified as periods during which the reduced sky brightness temperature stayed near the ground temperature for an extended time, sometimes for a period of hours after a rain event. The correct data during such times can not be recovered, and therefore the bad data was removed during the processing. The data for both sites were removed whenever the data of one site were bad. The bad data due to antenna-wetting within the period of rain events detected at the main site by the rain gauge was estimated at about 10% of the time for which the derived attenuation was above 2 dB, and the total bad data due to antenna wetting was about 6% of the total data recorded. Thus, the antenna-horn opening was not often blocked by rain drops hanging in the horn. The statistics shown in Chapter V represent statistics for the specified periods, excluding down time and bad data periods, and they should be a good representation of the true statistics for the period. The derivation of the statistics will be discussed in detail in Section IV.D.

During the initial operation period, from June 1983 to October 1983, the system gain varied as a function of the front-end box temperature. The problem was significant only at the main site, because the IF amplifier and detector unit at only that site were temperature-sensitive. The gain-temperature variation was obtained as follows. First, take the antenna temperature of the remote site during clear days as the true antenna temperature at the main site. Then the
The system gain of the main site was computed by the following expression, obtained from Equation (4-4),

\[ G = \frac{V_o}{[(T_a-T_0)\alpha_a + (T_0-T_B)\alpha_B + (T_B-T_k)\alpha_k]} \]  

(4-5)

The resulting gain was correlated with the box temperature to obtain a relationship, which turned out to be linear. Figure 4.1 shows the gain-temperature variation on a typical day. The scatter plot shows an almost linear relation between gain and box temperature,

\[ G = G_0 + K(T - T_0), \]

where

- \( G_0 \) is the known system gain at a known box temperature \( T_0 \),
- \( K \) is a constant,
- \( G \) is the system gain while the box temperature is at \( T \).

The constant \( K \) changed whenever the gain was readjusted (when a calibration was performed) and there may also have been a variation over a long period of time. Since we are only interested in the attenuation during rain events, the above technique was applied to nearby clear-day data to obtain the system gain-temperature variation during each event. Then, the corrected gain was substituted into Equation (4-4).

Table 4.2 shows the types of temperature sensors used and the corresponding D.C. amplifier gains. The IC sensor output, ranging from -5 to 5 volts, is linearly related to the measuring temperature from -50°C to 50°C. The thermocouple generates a small voltage based on the difference in temperature between two junctions of dissimilar metals. One junction is placed where the temperature is to be measured; the other is an effective junction at the amplifier input. The voltage
Figure 4.1. A typical example of the gain-temperature variation of the main-site radiometer.
### TABLE 4.2

**TEMPERATURE SENSORS AND AMPLIFICATION VALUES**

<table>
<thead>
<tr>
<th>Location of Sensor</th>
<th>Type of Sensor</th>
<th>Amplification Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main-Site PC Board</td>
<td>IC</td>
<td></td>
</tr>
<tr>
<td>Main-Site External Waveguide</td>
<td>IC</td>
<td></td>
</tr>
<tr>
<td>Main-Site Reference Oven</td>
<td>Thermocouple</td>
<td>800</td>
</tr>
<tr>
<td>Main-Site Calibration Oven</td>
<td>Thermocouple</td>
<td>800</td>
</tr>
<tr>
<td>Remote-Site PC Board</td>
<td>IC</td>
<td></td>
</tr>
<tr>
<td>Remote-Site External Waveguide</td>
<td>Thermocouple</td>
<td>670</td>
</tr>
<tr>
<td>Remote-Site Reference Oven</td>
<td>Thermocouple</td>
<td>670</td>
</tr>
<tr>
<td>Remote-Site Calibration Oven</td>
<td>Thermocouple</td>
<td>670</td>
</tr>
</tbody>
</table>
was amplified by a D.C. amplifier, digitized and recorded. A thermocouple table was used to find the temperature difference from the voltage. The thermocouples all were connected to a printed circuit board inside the front-end box. An IC temperature sensor was used to measure the temperature of the PC board. Thus all the temperatures could be determined from the recorded voltages.

After the raw data was processed to yield sky brightness temperature according to the above method, it was reduced into a file-by-file structure. Each file corresponds to a rain period, and contains the values of the sky brightness temperature and of the ambient (ground) temperature. A rain period or event was defined as a time period during which the sky brightness temperature has increased approximately 30 K over its clear-atmosphere value.

C. REDUCTION OF PATH ATTENUATION FROM SKY BRIGHTNESS TEMPERATURE

The attenuation was obtained from the brightness temperature by eq. (3-25a),

\[ \tau = \ln \left( \frac{T_m}{T_m - T_b} \right) = A / 4.34 \]  

(3-25a)

where

- \( \tau \) is the path attenuation in nepers,
- \( A \) is the path attenuation in decibels,
- \( T_m \) is the weighted mean absorption temperature of the medium,
- \( T_b \) is the sky brightness temperature.
As discussed in Section III.G, this equation was originally derived from the radiative transfer equation for a uniform medium temperature $T_m$, but it has been generalized to a non-uniform medium temperature case with a weighted mean absorption temperature $T_m$.

The method used here to determine $T_m$ was first proposed by Leonard and Levis [31], and is summarized in Section III.H. Since radar data was not available for many rain events, we used the uniform rain profile along the path as an approximation. Substituting $T_p(\bar{r}_0) = 273$ and $T_p(\bar{r}) = T_g$ in Equation (3-26a), we obtain

$$T_m = 273 + \frac{1}{1 - e^{-\tau}} - \frac{1}{\tau}(T_g - 273), \quad (4-6)$$

where $T_g$ is the ground temperature.

Therefore the mean absorption temperature is not a function of the path length, but only of the total path attenuation and the ground temperature. $T_m$ also has to satisfy Equation (3-25a). Rearranging this equation, we get

$$T_m = \frac{T_b}{1 - e^{-\tau}} \quad (4-7)$$

Equation (4-6) and Equation (4-7) should be equal,

$$T_m = \frac{T_b}{1 - e^{-\tau}} = 273 + \frac{1}{1 - e^{-\tau}} - \frac{1}{\tau}(T_g - 273), \quad (4-8)$$

Let us define a new function $f(\tau)$,

$$f(\tau) = \frac{T_b}{1 - e^{-\tau}} - 273 - \frac{1}{1 - e^{-\tau}} + \frac{1}{\tau}(T_g - 273), \quad (4-9)$$
which should be zero for correct $T_b$, $T_g$, and $\tau$. With $T_b$ and $T_g$ obtained from measurement, the attenuation $\tau$ was calculated from the zero of this function by Newton's iteration procedure.

From equation (4-6), we know that $T_m$ never exceeds $T_g$, and according to (3-25a) we know $T_m > T_b$. Therefore, if $T_b > T_g$ was encountered during the processing, we were unable to process the data by using the above equations. In this situation, since the sky brightness temperature was very high, we assigned a high attenuation value for that particular sky brightness temperature. We will show shortly that the radiometer-inferred attenuation is valid only up to 15 dB, and therefore the attenuation value of 15 dB was assigned for this high sky-brightness-temperature case.

In the winter time, the ground temperature, $T_g$, may drop below 273 K, and then $T_m$ was chosen to be 273 K. This value is appropriate for freezing rain and wet snow. It is less appropriate for cold, dry snow, but such snow produces very little attenuation at 28.6 GHz and is relatively rare in Columbus, OH. Indeed the data showed low sky brightness whenever $T_g$ fell substantially below 273 K, and the choice of $T_m$ is relatively uncritical in this case, which corresponds to small attenuation.

The above function $f(\tau)$ was also generalized for arbitrary temperature profiles and attenuation profiles. Using Equation (3-26), we obtain

$$f(\tau) = \frac{T_b}{1 - e^{-\tau}} - 273 - T_m(\tau)(T_g - 273),$$

(4-10)
where
\[ T_m(n) = \frac{\tau}{1 - e^{-\tau}} \int_0^1 T_n(x)K_n(x)e^{-\tau v(x)}dx, \quad (4-11) \]

\( T_n(x) \) is the temperature profile along the path,
\( K_n(x) \) is the attenuation profile along the path,
\( v(x) \) is the normalized attenuation factor along the path.

These reduction methods were checked by taking a rain event during the early stage of the experiment (on day 242, 1981), the only major rain event when the satellite beacon was available after the first radiometer became operational. The same period also appears in Pignon's thesis [11]. During that period the radar was used to monitor the rain rate along the path; thus direct transmission, radiometer, and radar data were all available simultaneously. This allowed the path attenuation to be calculated both with \( K_n \) determined by radar and use of (4-10) and with (4-9), which assumes uniform attenuation, \( K_n = 1 \), and the results could be compared with the direct transmission data. Figure 4.2 is the scatter plot of beacon level plotted against radiometrically derived attenuation by using the above method with radar-inferred rain rate along the path and a linear temperature profile. Figure 4.3 is the scatter plot of beacon level plotted against radiometrically derived attenuation calculated using \( T_m \) computed from Wulfsberg formula [33], \( T_m = 1.12 T_g - 50 \). (Since the Wulfsberg formula was proposed for use under the clear-sky conditions [33], its use for a rain event is questionable). Figure 4.4 shows the result with \( T_m \), using the uniform attenuation assumption for the same set of data. Mr. Pignon used a
best-fit constant $T_m$ adjusted empirically to produce a good scatter plot, this is shown in Figure 4.5. Comparison of Figures 4.2 to 4.5 leads to the following conclusions. Use of the rigorous expressions (3-26) for evaluating $T_m$ with the assumption of a linear lapse rate and an attenuation profile determined by radar greatly enhances the accuracy relative to empirical choices of a constant $T_m$. Using the expression with an assumed constant attenuation profile is less satisfactory, but still a very substantial improvement over the empirical approach. The improvement is greatest for large attenuations; it extends the useful range of the radiometric measurement up to 15 dB. The measurement error in the attenuation was within 1 dB up to 10 dB of net attenuation. For higher attenuation ranges, 10 to 15 dB, more error is likely, about 2 dB. Above 15 dB, little accuracy can be expected. Therefore, in this report attenuation results are only shown up to 15 dB. Comparing Figure 4.4 with Figure 4.3, we can easily see the improvement of this method in the 8 - 15 dB attenuation range. The method works because $T_m$ is a function of attenuation, therefore a method which makes use of this relationship should be better than ones that simply ignore it.

The procedure for deriving the radar-inferred rain attenuation along the path at 28.6 GHz was given by Sun [12]. The required equations are summarized in Appendix E.

D. STATISTICS REDUCTION

Since the data is random in nature, the presentation must be in a statistical form. The attenuation derived from the above method, for
Figure 4.2. The scatter plot of the satellite beacon level against the radiometer derived attenuation by using Leonard and Levis' method with the rain attenuation profiles derived from radar data and the assumption of linear temperature profile.
Figure 4.3. The scatter plot of the satellite beacon level against the radiometer derived attenuation by using the Wulfsberg formula.
Figure 4.4. The scatter plot of the satellite beacon level against the radiometer derived attenuation by using Leonard and Levis' method with the assumption of uniform attenuation and linear temperature profiles.
Figure 4.5. The scatter plot of the satellite beacon level against the radiometer derived attenuation by taking a best fit $T_m=280K$. 

5. The scatter plot of the satellite beacon level against the radiometer derived attenuation by taking a best fit $T_m=280K$. 

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values ranging from 0 to 15 dB, was linearly (in dB) digitized in 100 levels, plus a level for attenuation equal to or exceeding 15 dB. Therefore, a total of 101 levels was obtained for each site. The attenuation data was further processed and stored as the number of samples $s(m,n)$ for each rain event, where $m$ and $n$ represent the digitized levels at the main site and the remote site, respectively, and $s(m,n)$ is the number of samples found for which the path attenuations were within the $m$th level observed at the main site and within the $n$th level observed at the remote site. Then the number of samples $s(m,n)$ for a period, i.e., a month, a season or a year, is the sum of the number of samples of events within that period for the corresponding $m$ and $n$ levels. The total number of valid samples, excluding the bad data, for each month was counted during the data processing, and therefore the final probability density $p(m,n)$ was calculated by

$$p(m,n) = \frac{s(m,n)}{S}, \quad (4-12)$$

where $S$ is the total number of valid samples available in the period. Then, the single-site cumulative probabilities are simply,

$$P(M) = \sum_{m=M}^{101} \sum_{n=1}^{101} p(m,n), \quad (4-13)$$

and

$$P(N) = \sum_{n=N}^{101} \sum_{m=1}^{101} p(m,n), \quad (4-14)$$

for the main site and the remote site, respectively. $P(M)$ represents the probability of attenuation exceeding the $M$th level observed at the main site.
The same probability density was used to derive the cumulative statistics for site diversity operations, which can be either switching or maximal-ratio combining. The joint cumulative probability for switching is

\[ P(N) = \sum_{m=N}^{101} \sum_{n=N}^{101} p(m, n). \]  

(4-15)

The probability was summed over all the \((m, n)\) for which the lesser of \(m\) and \(n\) exceeds \(N\), i.e., the switching is assumed to select the channel with least attenuation for each 3-second sample. Under the assumptions of identical receivers and that the front-end noise greatly exceeds external noise, which might be received coherently by the two antennas, maximal-ratio-combining processing (Appendix F) results in an output signal power which is the sum of the signal powers received at both sites,

\[ x = x_1 + x_2, \]

(4-16)

where

\[ x = 10^{-x'/10}, \]

\[ x_1 = 10^{-x_1'/10}, \]

\[ x_2 = 10^{-x_2'/10}, \]

\(x'\) is the output combined signal attenuation in dB,

\(x_1'\) is the attenuation observed at the main site in dB, and

\(x_2'\) is the attenuation observed at the remote site in dB.
For each \((m,n)\) level, we computed the combined signal attenuation

\[
x' = -10 \log_{10} \left( 10 - \frac{0.075 + 0.15(m-1)}{10} + 10 - \frac{0.075 + 0.15(n-1)}{10} \right).
\]

(4-17)

For example, for the \((1,3)\) level, the combined signal attenuation is,

\[
x' = -10 \log_{10} \left( 10 - \frac{0.075}{10} + 10 - \frac{0.375}{10} \right),
\]

and for the \((2,101)\) level, the combined signal attenuation is,

\[
x' = -10 \log_{10} \left( 10 - \frac{0.225}{10} + 10 - \frac{15.075}{10} \right).
\]

Actually, the 101st level included all attenuation values of 15 dB and larger, but since we are only interested in the cumulative statistics, the resulting error in Equation (4-17) would only introduce less smoothness at the large attenuation level. Therefore the joint cumulative probability \(P(x')\) for maximal-ratio combining is the sum of the probabilities of all the \((m,n)\) levels for which the combined signal attenuation is less than or equal to \(x'\).

E. DATA REDUCTION PROGRAMS

Several computer programs were used to read and to write data tapes, to reduce the data, and to make plots of the data. These programs include:

1. A program named NLCRT1 to read the data tapes and to plot directly the voltage outputs of all channels (see Appendix C), excluding
the S-band radar data, against time. This program is intended as an aid in finding rain event occurrences and in identifying problems with the system.

2. A program named ANTEMP to read data tapes, to calculate antenna temperatures based on the recorded and user-given data, and to write results on a new tape on the event basis. The event periods are supplied by the user.

3. A program named TWTP1 to read event data tapes, to calculate the attenuation at both sites, to generate the probability density of the attenuation, and to write the probability density of the attenuation of each event into output data files.

4. A program named F2PLT to read event data files and to make plots of cumulative statistics for each month.

5. A program named F2PLT1 to read event data files and to make plots of yearly cumulative statistics.

6. A program named F2PLT1 to read event data files and to make plots of yearly cumulative statistics on the log-normal scale.

7. A program named TWTP2 to read event data tapes, to calculate the attenuation at both sites, and to store the computed results into another tape, also on an event basis. The computed results are used for computing fade-duration statistics, inter-fade-interval statistics and hour-of-day fade statistics.
8. Programs named ATTENDUL, ATTENDUR, ATTENDUS, and ATTENDUC to read event data tapes (attenuation data) and to compute the starting and ending time when a threshold level was exceeded, the time when the event began and ended for the attenuations observed at the main site and/or at the remote site, and to calculate the joint attenuations for the switching and maximal-ratio combining modes, respectively. The computed results are stored in an output file for each threshold level.

9. A program named ATTENHR to read time data files, to calculate hourly statistics for each month, and to store the results in an output file.

10. A program named ATTENDUI to read time data files, to calculate the durations of events and the intervals between events, and to store the durations and intervals for each month in an output file.

11. Programs named ATTENHRPLOT and ATTENHRPLOTI to read the hourly statistics file and to plot the hour-of-day statistics for each season and for each year, respectively.

12. Programs named ATTENDUPLLOT and ATTENDUPLLOTI to read the duration and interval data file and to plot the duration statistics and the inter-event interval statistics for each season and for each year, respectively.

13. A program named TAPECOPY to read a raw data tape and to copy the radar data to a new data tape together with the radiometer output at
the main site. The time period for this radar data was on day 242/81.

14. A program named CONVR to read the rain event on day 242/81 from the corresponding raw data tape, to calculate the antenna temperature at the main site, rain rate, and the satellite beacon level received, and to store the results in a data file.

15. A program named ATMOD2 to read the radar data tape for day 242/81, to compute the beacon attenuation and radiometer inferred-attenuations by Wulfsberg's method and by the Leonard/Levis method, with the rain rate profile being derived from the radar data, and to plot the scatter plots of beacon attenuation against radiometer-inferred attenuation by these two methods.

16. A program named PPLT to read the data file generated by CONVR, to compute the beacon attenuation and radiometer-inferred attenuations by Wulfsberg's method and by the Leonard/Levis method, with a uniform rain profile along the path, and to plot the relations between beacon attenuation and radiometer-inferred attenuation by these two methods.

17. A program named RAININTENSITY to read the raw data tapes, to compute the rain rate and the statistical distribution of rain rate for each event, and to store the statistical result in a data file. Thus, each file contains the statistics of one event. The time period of the event is supplied by the user.
18. Programs named RAINSTAT and RAINSTAT1 to read the rain-event data files and to plot the monthly and annual rain-rate cumulative statistics, respectively.

19. A program named RAINFILE to read the raw data tapes, to compute the rain rate, and to store the rain rate data with the time information in a data file. The results are used for computing rain-duration statistics, inter-event interval statistics and rain hour-of-day statistics.

20. A program named RAINDU to read the data file generated by RAINFILE, to compute the rain duration and the inter-event intervals and the monthly statistics of both, and to store the statistics in a data file.

21. A program named RAINDUPLOT to read the data file generated by RAINDU and to plot the rain-duration statistics and the inter-event interval statistics.

22. A program named RAINHR to read the data files generated by RAINFILE, to compute the monthly hour-of-day statistics, and to store them in a data file.

23. A program named RAINHRPLOT to read the data file generated by RAINHR and to plot the hour-of-day statistics.
F. SUMMARY

The data reduction process has been discussed in this chapter. The attenuation statistics are processed in two steps: first a joint probability of attenuation is calculated for the two sites, then this is integrated to give cumulative probabilities of attenuation for single-site and two methods of site-diversity reception. Some additional statistics cannot be derived from the joint probability density, such as fade-duration statistics, inter-fade interval statistics, and the hourly cumulative attenuation distributions. These statistics are explained in the next chapter. Their calculation is straightforward, and the programs for their calculation are listed in the list of programs at the end of this chapter.
A. INTRODUCTION

The results of the data reduction discussed previously will now be presented. Table 5.1 lists the data periods in 1983-1985 during which measurement were made. The total data time recorded on magnetic tape is approximately 12,892 hours.

In the problem of millimeter waves propagated over a satellite-earth path, one rarely has sufficient information about the path, and therefore is not interested in the attenuation for a particular instant of time, but in the statistical properties of the path attenuation. Therefore, the data are reduced to show the statistical characteristics. Two kinds of statistical results are obtained from the analysis. The first consists of the cumulative distributions of the attenuation, both for individual sites and for site-diversity operation. The second consists of the time behavior statistics of the attenuation in forms of fade-duration and inter-fade interval distributions, again both for individual sites and site-diversity.
### TABLE 5.1

**READABLE DATA PERIODS**

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<td>x</td>
<td>x²</td>
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<td></td>
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<td></td>
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<tr>
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<td>739</td>
<td></td>
<td></td>
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<td>x⁵</td>
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</table>

1. Starts on June 2.
2. Ends on March 14.
4. Down on August 12.
5. Starts on December 8.
6. The remote-site antenna was repointed on February 9, 1984 (40/84). See section II.A for detailed discussion.
B. WEATHER OF COLUMBUS, OHIO

Since rain is an important factor affecting the wave propagation, we are interested in the rainfall of the Columbus area. According to the climatological summary published by the National Oceanic and Atmospheric Administration (NOAA) (1984) for the Columbus area, the average rainfall is heavier during March through August than for the other months of the year. Thunderstorm weather at Columbus is most prevalent from April to September. Table 5.2 shows the monthly rainfall and number of thunderstorm days for the experiment period and also long-term averages. The monthly data shows large variation from year to year.

C. CUMULATIVE DISTRIBUTIONS OF RAIN RATE

The point rain-rate distribution measured with the rain gauge at the main site from August 1983 to July 1984 is shown in Figure 5.1. (Figures for this chapter will be found at its end, beginning with page 99.) The distribution for each individual month is shown in Figures G.1.1 to G.1.18 in Appendix G. Normally February is the least rainy month, but for the period from August 1983 to July 1984, June happened to be the month with the least rainfall during this year. High rain rates were observed during April, May, and September, and low maximum rain rates during December through March. March is a month with large rainfall but much of it occurred at low rain rates, and September is a month with medium rainfall, but some high rain-rate precipitation occurred. October and November are months with large rainfall and medium-rate rain occurred.
TABLE 5.2

MONTHLY RAINFALLS AND NUMBER OF THUNDERSTORM DAYS FOR
COLUMBUS, OHIO FOR THE PERIOD JUNE 1983 THROUGH MARCH 1985 [34]

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<tr>
<td>April</td>
<td>78.7</td>
<td>81.0</td>
<td>6</td>
<td>4</td>
<td></td>
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<td>May</td>
<td>125.2</td>
<td>94.0</td>
<td>4</td>
<td>6.4</td>
<td></td>
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<td>June</td>
<td>116.6</td>
<td>18.0</td>
<td>94.5</td>
<td>5</td>
<td>5</td>
<td></td>
<td>8.1</td>
<td></td>
</tr>
<tr>
<td>July</td>
<td>71.1</td>
<td>80.0</td>
<td>94.0</td>
<td>10</td>
<td>4</td>
<td></td>
<td>8.1</td>
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<td>Aug.</td>
<td>56.6</td>
<td>75.2</td>
<td>83.0</td>
<td>6</td>
<td>8</td>
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<td>37.6</td>
<td>66.5</td>
<td>3</td>
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<td>Dec.</td>
<td>80.3</td>
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<td>0</td>
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<td>0.3</td>
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</tbody>
</table>

86
May is a month with large rainfall and high-rate rain occurred. It is likely that such monthly distributions may change significantly from year to year.

D. CUMULATIVE DISTRIBUTIONS OF ATTENUATION

As stated in Chapter 4, the attenuation data was first reduced to a joint attenuation probability density \( p(m,n) \) which is shown in Figures 5.2 and 5.3. The reason for this process is that from this data we will be able to compute the attenuation statistics for the individual sites and for switching and maximal-ratio site diversity combining processing (or any other kind of processing). With this technique we separate the purely meteorological weather effects on attenuation statistics from the communication-technique considerations. It is worth noting that the attenuations at both sites are highly correlated at lower attenuation levels and uncorrelated at high levels. The cumulative distribution is shown in Figure 5.4. Table 5.3 also shows this cumulative distribution. The flat top of Figure 5.4 may be due in part to the "event" approach of data processing: if attenuation didn't exceed a threshold it wasn't copied to an "event" tape. Therefore, none of these figures is accurate for very small attenuations.

The radiometer-inferred yearly cumulative attenuation distributions for both sites and for the diversity systems are shown in Figures 5.5 through 5.8. Figures 5.6 and 5.8 are plotted on a log-normal scale. (Since the two antennas were not pointed in the same direction during part of the 6/83 to 5/84 time period, the "J" and unlabelled curves in
## Table 5.3

**Joint Statistics $P(M,N)$**

\[
P(M,N) = \sum_{n=N}^{M} \sum_{m=m}^{n} p(m,n)
\]

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Figures 5.5 and 5.6 should not be taken as indicative of true diversity system performance.) As indicated previously, switching and maximal-ratio combining are the only types of diversity signal processing considered here. The results for both types of processing are shown in these figures. The attenuation is relative to the signal received by a single antenna with no attenuation; thus an "attenuation" of -3dB (i.e., 3 dB of gain) would be attained for maximal-ratio-combining system if there were no signal attenuation on the path. The points for maximal-ratio combining are to the left of the corresponding points for the switching system, and both are to the left of the points for the signals received at the individual sites. This indicates that, relative to the individually received signals, the processed signals have a lower average attenuation level and lower probabilities of deep fades. It is also noted that the improvement for switching processing is greater at the higher attenuation levels and least at the lower attenuation levels. For maximal-ratio-combining processing, the improvement is almost 3 dB at the lower attenuation levels and greater at the higher attenuation levels. The inherent superiority of maximal-ratio combining over switching is obvious; when switching, only the larger signal is used, and when combining, both signals are utilized. Clearly the sum of the larger plus the smaller yields a larger resultant than the larger considered alone. The individual-month performance is shown in Figures G.2.1 through G.2.22. Less fading was observed during the winter months of the year. 

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The improvement due to the site diversity can be quantified by the diversity gain \[6\], which is defined as the difference between the path attenuations associated with the single site and the joint processing for a given percentage of time, and is a function of the single-site path attenuation. Table 5.4 shows the diversity gains for switching and maximal-ratio-combining processing. (Again it should be noted that not all of the 83/84 data represents true diversity operation; but the results certainly are very similar.)

E. CUMULATIVE DISTRIBUTION OVER 24 HOURS

Figure 5.9 shows the cumulative rain-rate statistics for each hour of the day, summed over the period from August 1893 to July 1984 for the rain-rate levels 2, 4, 6, 10 and 30 mm/hr. At the lower rain rates, the distribution is almost uniform over the 24 hours with slightly more minutes during the afternoon period, while for the higher rain rates, the distribution is gathered into two groups, one during the middle of the night and the other during the afternoon, local time. Figures 5.10 through 5.15 are the cumulative fade distributions for each hour of the day, over each year for the attenuation levels of 3, 6, 9, and 15 dBs for each single site and for switching processing. The same kind of distributions for maximal-ratio-combining processing are shown in Figures 5.16 and 5.17, but with the threshold attenuation at 0, 3, 6, and 9 dBs. For path attenuation exceeding 3 dB, the distributions over 24 hours are almost the same for both single-site paths and, like the low rain-rate distributions, they also show slightly more minutes during
### TABLE 5.4

DIVERSITY GAIN FOR SWITCHING AND MAXIMAL-RATIO-COMBINING PROCESSING FOR YEARS 83-84\(^1\) AND 84-85\(^2\)

<table>
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<tr>
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<th>Attenuation (dB)</th>
<th>Year 83-84</th>
<th>Year 84-85</th>
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<td>Switching</td>
<td>Combining</td>
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<td>3.5</td>
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<td>1.6</td>
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<td>1.5</td>
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<td>2.3</td>
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<td>2.4</td>
<td>4.6</td>
<td>2.5</td>
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<td>15</td>
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1. Year 83-84 is the period from June 1983 to May 1984 (153/83 to 152/84).
2. Year 84-85 is the period from March 1984 to February 1985 (61/84 to 60/85).
3. The remote-site antenna was repointed on February 9, 1984 (40/84). See section II.A for detailed discussion.
the afternoon period. For higher threshold values, the distributions also show two groupings of peaks over the 24 hours; one is during the middle of the night and the other is during the afternoon, but there are some differences between the two paths. For each individual season, the distributions are shown in Figures G.3.1 through G.3.28.

The cumulative minutes in each hour-of-day statistics are given in real minutes without any adjustment for the minutes which were excluded due to down-time and bad data periods.

F. PROBABILITY DISTRIBUTIONS OF THE RAIN-LEVEL AND FADE DURATIONS

The time behavior of an event can be characterized by the time duration for which the quantity characterizing the event is above or below a certain level. Thus, the duration of a rain-level occurrence may be defined as the time during which the rain rate remains continuously above a specified level, and the fade duration is defined as the time duration for which the signal is below a given threshold. Figure 5.18 shows an example of a rain event and illustrates a rain duration interval $T_1$ and an inter-rainfall interval $T_2$, both for a threshold of 1 mm/hr. The specific starting and ending times of these periods are not of interest since such times are quite random; it is the time intervals with which we are concerned. A cumulative distribution of rain durations experienced at the main site during the period from August 1983 to July 1984 is shown in Figure 5.19. At the lower rain rates, i.e., 1, 2, 3, and 4 mm/hr, the total number of events over the year was almost the same, but the maximum duration of the rain
occurrences was reduced for the higher of these rates. As the threshold level increases, both the total number of occurrences and the maximum duration of the occurrences decrease. The measured fade durations for the data sorted by years are shown in Figures 5.20 to 5.27, and by seasons in Figures G.4.1 to G.4.28. In these figures, the number of fades per year (or per season) for which the duration equals or exceeds the abscissa has been plotted for rain attenuations exceeding 3, 6, 9, and 15 dB for each single site and for switching processing, while 0, 3, 6, and 9 dB thresholds were used for maximum-ratio-combining processing. As expected, both single-site paths experienced almost the same fade-duration distribution. The usual caution applies regarding the diversity plots for the 6/83 to 5/84 time period.

Based on the measured results, a ratio, $R$, has been computed for each attenuation threshold to study the improvement due to the diversity processing schemes. Let $R$ be defined as

$$R = \frac{\text{Total number of events with } A > x \text{ dB for a single-site path}}{\text{Total number of events with } A > x \text{ dB for diversity processing}}.$$ 

Table 5.5 shows this improvement ratio for switching and maximal-ratio combining at different threshold levels.

G. PROBABILITY DISTRIBUTIONS OF INTER-EVENT INTERVALS

Similarly, an inter-rainfall interval is defined as a time interval for which the rain rate remains continuously less than a specified level, while it is exceeded before and after the interval. An inter-fade interval is the time, between two fade events, during which the attenuation remains less than a given threshold. An example of an
### TABLE 5.5

**IMPROVEMENT RATIO FOR SWITCHING AND MAXIMAL-RATIO-COMBINING PROCESSING FOR YEARS 83-84\(^1,3\) AND 84-85\(^2\)**

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<tr>
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<td>5.34</td>
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1. Year 83-84 means the period from June 1983 to May 1984 (153/83 to 152/84).
2. Year 84-85 means the period from March 1984 to February 1985 (61/84 to 60/85).
3. The remote-site antenna was repointed on February 9, 1984 (40/84). See section II.A for detailed discussion.
4. R means the improvement over the single remote site, and L the single main site.
inter-rainfall interval is shown as $T_2$ in Figure 5.18. Figure 5.28 shows the distribution of inter-rainfall intervals at the main site for the period from August 1983 to July 1984. The measured yearly distributions of inter-fade intervals are shown in Figures 5.29 to 5.36. The seasonal variations are shown in Figures G.5.1 to G.5.28. Again, the distributions of inter-event intervals are almost the same for both single-site paths. From the experiment results, the diversity tends to reduce the number of short intervals at a low threshold level, and to increase the interval and to reduce the number of inter-event intervals at a high threshold level. The latter is expected, since the diversity processing, either switching or maximal-ratio combining, tends to decrease the fade duration and to reduce the number of fades, in other words, to increase the inter-fade interval and to reduce the number of inter-fades.

In the processing of the event-duration and interval-between-events statistics, the data lost due to down time and antenna wetting were replaced with a low attenuation level because we were unable to recover the true data. This will have some effect on the results, but it is the best we can do since it is impossible to just exclude data when dealing with time intervals.

H. SUMMARY

In this chapter, the attenuation statistics results obtained using an experiment with two radiometers separated at a distance of 9 km are presented. The cumulative attenuation statistics and the correlation
between attenuations over these two paths has been found. Both switching and maximal-ratio-combining processing statistics were extracted from the experiment data. The fade-duration statistics, the inter-fade interval statistics and the cumulative attenuation distributions over 24 hours have also been derived from the data for each site-diversity signal processing method and for each single-site operation.
Figure 5.1. Cumulative distribution of rain rate at the main site for the period from August 1983 to July 1984.
Figure 5.2. The joint probability density function of the attenuations at both sites for the period from day 40/1984 to day 40/1985.
Figure 5.3. The same as Figure 5.2 (different view direction).
Figure 5.4. Cumulative two-dimensional distribution of the attenuations at both sites for the period from day 40/1984 to day 40/1985.
Figure 5.5. Cumulative distribution of attenuation for the period from June 1983 to May 1984. 'L' denotes the main site, 'R' the remote site, 'J' switching processing, and no letter maximal-ratio-combining processing.
Figure 5.6. Cumulative distribution of attenuation for the period from June 1983 to May 1984 (log-normal scale). 'L' denotes the main site, 'R' the remote site, 'J' switching processing, and no letter maximal-ratio-combining processing.
Figure 5.7. Cumulative distribution of attenuation for the period from March 1984 to February 1985. 'L' denotes the main site, 'R' the remote site, 'J' switching processing, and no letter maximal-ratio-combining processing.
Figure 5.8. Cumulative distribution of attenuation for the period from March 1984 to February 1985 (log-normal scale). 'L' denotes the main site, 'R' the remote site, 'J' switching processing, and no letter maximal-ratio-combining processing.
Figure 5.9. Cumulative distribution of rain rate over 24 hour day for the period from August 1983 to July 1984. The time shown is GMT. Subtract 5 hours to obtain the local time.
Figure 5.10. Cumulative distribution of the attenuation on satellite-to-main-site path over 24 hour day for the period from June 1983 to May 1984. The time is GMT. Subtract 5 hours to obtain the local time.
Figure 5.11. Cumulative distribution of attenuation on satellite-to-main-site path over 24 hour day for the period from March 1984 to February 1985. The time is GMT. Subtract 5 hours to obtain the local time.
Figure 5.12. Cumulative distribution of attenuation on satellite-to-
remote-site path over 24 hour day for the period from June
1983 to May 1984. The time is GMT. Subtract 5 hours to
obtain the local time.
Figure 5.13. Cumulative distribution of attenuation on satellite-to-remote-site path over 24 hour day for the period from March 1984 to February 1985. The time is GMT. Subtract 5 hours to obtain the local time.
Figure 5.14. Cumulative distribution of attenuation for switching processing over 24 hour day for the period from June 1983 to May 1984. The time is GMT. Subtract 5 hours to obtain the local time.
Figure 5.15. Cumulative distribution of attenuation for switching processing over 24 hour day for the period from March 1984 to February 1985. The time is GMT. Subtract 5 hours to obtain the local time.
Figure 5.16. Cumulative distribution of attenuation for maximal-ratio-combining processing over 24 hour day for the period from June 1983 to May 1984. The time is GMT. Subtract 5 hours to obtain the local time.
Figure 5.17. Cumulative distribution of attenuation for maximal-ratio-combining processing over 24 hour day for the period from March 1984 to February 1985. The time is GMT. Subtract 5 hours to obtain the local time.
Figure 5.18. A typical example of rain duration and inter-rainfall interval.
Figure 5.19. Rain duration distribution for the period from August 1983 to July 1984.
Figure 5.20. Fade duration distribution on the satellite-to-main-site path for the period from June 1983 to May 1984.
Figure 5.21. Fade duration distribution on the satellite-to-main-site path for the period from March 1984 to February 1985.
Figure 5.22. Fade duration distribution on the satellite-to-remote-site path for the period from June 1983 to May 1984.
Figure 5.23. Fade duration distribution on the satellite-to-remote-site path for the period from March 1984 to February 1985.
Before February 9, 1984:

- Antenna pointing
  - Azimuth: 236.4°
  - Elevation: 25.6°

- Antenna pointing
  - Azimuth: 196.4°
  - Elevation: 25.6°

After February 9, 1984:

- Antenna pointing
  - Azimuth: 236.4°
  - Elevation: 25.6°

- Antenna pointing
  - Azimuth: 236.4°
  - Elevation: 25.6°

Figure 5.24. Fade duration distribution for switching processing for the period from June 1983 to May 1984.
Figure 5.25. Fade duration distribution for switching processing for the period from March 1984 to February 1985.
Figure 5.26. Fade duration distribution for maximal-ratio-combining processing for the period from June 1983 to May 1984.
Figure 5.27. Fade duration distribution for maximal-ratio-combining processing for the period from March 1984 to February 1985.
Figure 5.28. Inter-rainfall interval distribution for the period from August 1983 to July 1984.
Figure 5.29. Inter-fade interval distribution on the satellite-to-main-site path for the period from June 1983 to May 1984.
Figure 5.30. Inter-fade interval distribution on the satellite-to-main-site path for the period from March 1984 to February 1985.
Figure 5.31. Inter-fade interval distribution on the satellite-to-
remote-site path for the period from June 1983 to May
1984.
Figure 5.32. Inter-fade interval distribution on the satellite-to-remote-site path for the period from March 1984 to February 1985.
Figure 5.33. Inter-fade interval distribution for switching processing for the period from June 1983 to May 1984.
Figure 5.34. Inter-fade interval distribution for switching processing for the period from March 1984 to February 1985.
Figure 5.35. Inter-fade interval distribution for maximal-ratio-combining processing for the period from June 1983 to May 1984.
Figure 5.36. Inter-fade interval distribution for maximal-ratio-combining processing for the period from March 1984 to February 1985.
CHAPTER VI
CONCLUSIONS

The results of the 28 GHz site diversity propagation measurement made at The Ohio State University, using two radiometers separated by 9 km, have been presented. Two kinds of site-diversity processing, switching and maximal-ratio combining, are considered. The results are presented in the form of statistics. As expected, both site-diversity methods reduce the fade margin considerably and would reduce the system outage time due to fading. Maximal-ratio combining outperforms switching in this respect. The fade-duration statistics also show a reduction in the number of fades for a given fade level and in the fade duration for a given attenuation threshold.

The joint probability density function of attenuation is shown to be a useful starting point for deriving the performance statistics of a variety of systems, e.g. single-site, switching diversity, and maximal-ratio combining diversity systems. It constitutes a meteorologically determined data base from which statistics for diverse type of systems may be derived. It shows that at the lower attenuation values, the attenuations at the two sites were highly correlated during the experiment, while at high attenuation values there was much less correlation.

A newly proposed method of deriving the path attenuation from the measured antenna temperature was utilized, using assumed temperature and
attenuation profiles along the path, and neglecting the scattering effect. The method was shown experimentally to be useful for our measurements at 28.6 GHz and at an elevation of 25.6° for attenuations up to 15 dB, giving substantially better results than the usual approach which assumes a constant, empirical mean temperature. Further research is needed on this subject if one is interested in the radiometrically inferred attenuation for higher attenuation values and higher frequencies, and also to determine a possible dependence on elevation angle.
REFERENCES


[31] Leonard, R. E., "Calculations of Mean Path Temperature Involved in Radiometrically Inferred Attenuation", Thesis for M. S., 1984, The Ohio State University, Department of Electrical Engineering, Columbus, Ohio.


APPENDIX A
THE NEW RADIOMETER PROCESSORS

Figure A.1 shows the circuit diagram for both radiometer processors. The radiometer signal after common-mode rejection (see Figure 2.2) is injected into the 4th pin of a switched capacitor filter, MF10. Integrated circuits CD4046BE and CD4518BE together comprise a digital phase-locked loop. The frequency at its output is 100 times that at the input, the Dicke-switch reference signal, at 1951 Hz. This 195.1 kHz signal becomes the clock input to the switched capacitor filter. The signal output from the filter has almost a sine-wave form with many sampling steps. This signal is then fed into an active filter, a phase-shifter, and a synchronous detector which consists of a single integrated circuit, GAP-01. An active low-pass filter selects the DC component of the resultant. The filter can be controlled from the front panel for the selection of one of three available integration times (1, 3, and 10 seconds).

Figure A.2 shows the radiometer time-base generator, digital phase-shifter and Dicke-switch driver. The frequency generator consists of a crystal oscillator and a ripple counter, CD4060BE. The crystal oscillator oscillates at 31.216 KHz. This frequency is then converted to 1951 Hz by the ripple counter. The signal is fed into both the digital phase-shifter and the Dicke-switch driver.
Figure A.1. Circuit diagram of radiometer synchronous detector and integrator-filter.
Figure A.2. Circuit diagram of radiometer time-base generator, digital phase shifter and ferrite switch driver.
APPENDIX B

TAPE FORMAT

Table B-1

Record Format

<table>
<thead>
<tr>
<th>Word No.</th>
<th>Bit No. (Record Contents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No. of words in record (in binary format)</td>
</tr>
<tr>
<td>1</td>
<td>Clock word 1 (see Table B-2)</td>
</tr>
<tr>
<td>2</td>
<td>Clock word 2 (see Table B-2)</td>
</tr>
<tr>
<td>3</td>
<td>Clock word 3 (see Table B-2)</td>
</tr>
<tr>
<td>4</td>
<td>System Status</td>
</tr>
<tr>
<td>5</td>
<td>Tape number</td>
</tr>
<tr>
<td>6</td>
<td>File number</td>
</tr>
<tr>
<td>7</td>
<td>Record number</td>
</tr>
<tr>
<td>8</td>
<td>Record type</td>
</tr>
<tr>
<td>9</td>
<td>No. of Data sets in record</td>
</tr>
<tr>
<td>10</td>
<td>Seconds sampled</td>
</tr>
<tr>
<td>11</td>
<td>Seconds not sampled</td>
</tr>
<tr>
<td>12</td>
<td>First data set (see Table B-3)</td>
</tr>
<tr>
<td></td>
<td>variable length.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Nth Data set</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>End of record</td>
</tr>
</tbody>
</table>
### Table B-2

**REAL-TIME-CLOCK WORDS**

<table>
<thead>
<tr>
<th>Bit No.</th>
<th>Clock Word 1</th>
<th>Clock Word 2</th>
<th>Clock Word 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2 Minutes, units</td>
<td>2 Days, units</td>
<td>2 Years, units</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2 Minutes, tens</td>
<td>2 Days, tens</td>
<td>2 Years, tens</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Unassigned</td>
<td>Unassigned</td>
<td>Unassigned</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>2 Seconds, units</td>
<td>2 Hours, units</td>
<td>2 Days, hundreds</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>4</td>
<td>Unassigned</td>
</tr>
<tr>
<td>11</td>
<td>8</td>
<td>8</td>
<td>&quot;</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>1</td>
<td>&quot;</td>
</tr>
<tr>
<td>13</td>
<td>2 Seconds, tens</td>
<td>2 Hours, tens</td>
<td>&quot;</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>4</td>
<td>&quot;</td>
</tr>
<tr>
<td>15</td>
<td>Unassigned</td>
<td>Unassigned</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

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Table B-3
DATA-SET FORMAT

<table>
<thead>
<tr>
<th>WORD NO.</th>
<th>BIT NO. (RECORD CONTENTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0</td>
<td></td>
</tr>
</tbody>
</table>

1 NO. OF WORDS IN DATA SET (IN BINARY FORMAT)
2 CHANNEL NUMBER DEVICE NUMBER
3 SECONDS TILL THE FIRST SAMPLE FROM RECORD TIME
4 SECONDS PER SAMPLES IN DATA SET
5 ERROR CODE
6 NUMBER OF SAMPLES IN DATA SET
7 DATA (ONE WORD PER SAMPLE)

•
•
• UNTIL ALL THE SAMPLES ARE STORED.

Each radar data set contains 100 radar range-bin data words.
APPENDIX C

CURRENT CHANNEL ASSIGNMENTS

<table>
<thead>
<tr>
<th>Device Number</th>
<th>Name</th>
<th>Channel Number</th>
<th>Input Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Local A/D Converter (16 channels)</td>
<td>0</td>
<td>S-Band Radar Power Monitor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>Front-End Box Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>Outside Waveguide Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>Reference Oven Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>Calibration Oven Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>28.6 GHz Radiometer Output</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12</td>
<td>Rain Gauge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13</td>
<td>8.5 GHz Radiometer Output</td>
</tr>
<tr>
<td>2</td>
<td>not used</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Remote Site (32 channels)</td>
<td>7</td>
<td>Outside Waveguide Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11</td>
<td>Reference Oven Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>Calibration Oven Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23</td>
<td>Front-End Box Temperature</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31</td>
<td>28.6 GHz Radiometer Output</td>
</tr>
<tr>
<td>4</td>
<td>Radar</td>
<td>0</td>
<td>S-Band Radar (100 radar-return data/sample)</td>
</tr>
</tbody>
</table>

APPENDIX D

DERIVATION OF EQUATIONS (3-26) AND (3-26a)

This appendix shows the derivation of equations (3-26) and (3-26a). The derivation was first given, in somewhat different form, by Leonard [31]. Let us start from equation (3-25),

\[ T_m = \frac{1}{1-e^{-\tau(s)}} \int_{s_0}^{s} T_p(\tilde{r}_1)K_t(\tilde{r}_1) e^{-\int_{s_1}^{s} K_t(s_2) ds_2} ds_1. \]  

(3-25)

First, we let

\[ s = s_a, \quad \tilde{r} = \tilde{r}_a \] (see Figure 3.5),

\[ L = s_a - s_o, \] \hspace{1cm} (D-1)

\[ x_1 = \frac{(s_a - s_1)}{L}, \] \hspace{1cm} (D-2)

\[ K_n(x_1) = K_t(\tilde{r}_1)/\left[\frac{\tau(s_a)}{L}\right] \]

\[ = K_t(\tilde{r}_1)/\left[\frac{1}{L} \int_{s_0}^{s_a} K_t(\tilde{r}_2) ds_2\right], \] \hspace{1cm} (D-3)

and

\[ \nu(x) = \int_{0}^{x} K_n(x_1) dx_1. \] \hspace{1cm} (D-4)

Then rearranging (D-3), we obtain
Here, we replace $s_2$ by $x_2$ with $x_2 = (s_a - s_2)/L$.

From (D-4) and (D-6), we obtain

$$\tau(s_1) = \int_{s_0}^{s_1} \tau(s_a) ds_2,$$

and

$$\tau(s_1) = \int_{s_0}^{s_1} \tau(s_a) ds_2,$$

$$= \frac{1}{L} \int_{s_0}^{s_1} K_n(\frac{s_a-s_2}{L}) \tau(s_a) ds_2,$$

$$= \tau(s_a) \frac{1}{1-e^{-\tau(s_a)\int_{s_0}^{s_1} K_n(x_2) dx_2}},$$

(D-6)

Here, we replace $s_2$ by $x_2$ with $x_2 = (s_a - s_2)/L$.

From (D-4) and (D-6), we obtain

$$\tau(s_1) = \tau(s_a)[1 - \nu(x_1)].$$

(D-7)

Substitute (D-1), (D-5) and (D-7) into (3-25), we get

$$T_m = \frac{1}{1-e^{\tau(s_a)}} \int_{s_0}^{s} T_p(\tilde{r}_1) \frac{K_n(\frac{s_a-s_1}{L}) \tau(s_a)}{L} e^{-\tau(s_a)\int_{s_0}^{s} K_n(x_1) dx_1} ds_1,$$

$$= \frac{\tau(s_a)}{1-e^{\tau(s_a)}} \int_{0}^{s_0} T_p(\tilde{r}_1) K_n(x_1) e^{-\tau(s_a)\int_{0}^{s_1} K_n(x_1) dx_1} dx_1.$$ 

(D-8)
Further normalize the temperature profile by

\[ T_n(x_1) = \frac{T_p(\bar{r}_1) - T_p(\bar{r}_0)}{T_p(\bar{r}_a) - T_p(\bar{r}_0)} \]  \hspace{1cm} (D-9)

and then obtain

\[ T_p(\bar{r}_1) = T_n(x_1)[T_p(\bar{r}_1) - T_p(\bar{r}_0)] + T_p(\bar{r}_0). \]  \hspace{1cm} (D-10)

Substitute (D-10) into (D-8) yields

\begin{align*}
T_m &= \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int T_n(x_1)[T_p(\bar{r}_a) - T_p(\bar{r}_0)] K_n(x_1)e^{\tau(s_a)v(x_1)}dx_1 + \\
&\quad \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int T_p(\bar{r}_0) K_n(x_1)e^{-\tau(s_a)v(x_1)}dx_1.
\end{align*}

But differentiate (D-4), we obtain

\[ \frac{dv(x)}{dx} = K_n(x). \]  \hspace{1cm} (D-12)

Therefore, the second integral in (D-11) is exactly integrable, giving

\begin{align*}
T_m &= \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int T_n(x_1)[T_p(\bar{r}_a) - T_p(\bar{r}_0)] K_n(x_1)e^{-\tau(s_a)v(x_1)}dx_1 + \\
&\quad \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int T_p(\bar{r}_0) K_n(x_1)e^{-\tau(s_a)v(x_1)}dx_1 + \\
&\quad T_p(\bar{r}_0).
\end{align*}

\[ T_p(\bar{r}_0) \],

(D-13)

where \( v(1) = 1 \).
Rearranging (D-13), we then obtain (3-26),

\[ T_{mn} = \frac{T_m - T_p(\bar{r}_0)}{T_p(\bar{r}_a) - T_p(\bar{r}_0)} = \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int_0^1 T_n(x_1)K_n(x_1)e^{-\tau(s_a)v(x_1)}dx_1. \]

(3-26)

With \( K_t(\bar{r}_1) = \text{constant} \) and

\[ T_n(x_1) = \frac{T_p(\bar{r}_1) - T_p(\bar{r}_0)}{T_p(\bar{r}_a) - T_p(\bar{r}_0)} = 1 - x_1, \]

a linear profile, we will be able to derive equation (3-26a), as follows.

From (D-3) and (D-4) and the above assumption, we see

\[ K_n(x) = 1, \]  

(D-14)

and

\[ \nu(x) = x. \]  

(D-15)
Substitute (D-14) and (D-15) into (3-26) to obtain

\[ T_{mn} = \frac{T_m - T_p(\tilde{r}_o)}{T_p(\tilde{r}_a) - T_p(\tilde{r}_o)} = \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int_0^1 (1-x)e^{-\tau(s_a)x} dx \]

\[ = \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int_0^1 e^{-\tau(s_a)x} dx - \frac{\tau(s_a)}{1-e^{-\tau(s_a)}} \int_0^1 xe^{-\tau(s_a)x} dx \]

\[ = \frac{1}{1-e^{-\tau(s_a)}} \left[ -e^{-\tau(s_a)x} \right]_0^1 - \frac{1}{1-e^{-\tau(s_a)}} \left[ xe^{-\tau(s_a)x} \right]_0^1 \]

\[ = \int_0^1 \left[ -e^{-\tau(s_a)x} \right] dx \]

\[ = 1 + \frac{1}{1-e^{-\tau(s_a)}} \left[ e^{-\tau(s_a)x} \right]_0^1 - \frac{1}{1-e^{-\tau(s_a)}} \left[ e^{-\tau(s_a)x} \right]_0^1 \]

\[ = \frac{1}{1-e^{-\tau(s_a)}} - \frac{1}{\tau(s_a)}. \]  

(D-16)

By rearranging (D-16), we get (3-26a),

\[ T_m = T_p(\tilde{r}_o) + \left( \frac{1}{1-e^{-\tau(s_a)}} - \frac{1}{\tau(s_a)} \right) \left[ T_p(\tilde{r}_a) - T_p(\tilde{r}_o) \right]. \]

(3-26a)
Appendix E

DERIVATION OF RAIN RATE AND RAIN ATTENUATION
ALONG THE PATH FROM RADAR DATA

The analysis of radar signals to yield rain rate, given by Sun [12], is summarized below:

The time-averaged received power backscattered from a resolution cell located at a distance of \( r \) meters from the radar antenna is

\[
P_r = \frac{Q}{r^2} Z e^{-2\text{ARD}} \text{ [Watts]}.
\]

(E-1)

The radar calibration constant \( Q \) is given by

\[
Q = \frac{\pi^3 |K|^2 f^2 G T P_T}{2^6 \cdot C},
\]

(E-2)

where

\[
|K|^2 = \left| \frac{m^2 - 1}{m^2 + 1} \right|^2,
\]

\( m \) is the complex refractive index of liquid water,
\( f \) is the radar frequency [Hz],
\( G \) is the maximum antenna gain,
\( \tau \) is the radar pulse length [sec],
\( P_T \) is the peak transmitted power [watts],
\( c \) is the velocity of wave propagation [m/sec],

and
ARD is the one-way attenuation between the radar and the resolution cell.

Except during very heavy rainfall, the attenuation at the radar frequency, 3.064 GHz, can be and was neglected in the data analysis [12]. Thus, Equation (E-1) is rewritten without attenuation factor as

\[ P_r = \frac{Q}{r^2} Z. \]  \hspace{1cm} (E-3)

The radar reflectivity factor \( Z \) is represented by a power law relationship

\[ Z = a Z_{R}^{b Z}, \]  \hspace{1cm} (E-4)

where

\[ a = 296.79 \text{ [mm}^6/\text{m}^3] \]
\[ b = 1.4422 \text{ at } 3.064 \text{ GHz} [35]. \]

The constants for the S-band radar used in this experiment are [12]:

\[ f = 3.064 \text{ GHz}, \]
\[ P_t = 3.64 \text{ kW}, \]
\[ G = 44 \text{ dB}, \]
\[ \tau = 1.3 \mu s, \]
\[ m = 9.001 - j1.398 \text{ [12] at } 0^\circ \text{C}, \]
\[ |K|^2 = 0.9339 \text{ [12]}. \]

Substituting Equation (E-4) into Equation (E-3) and applying the above parameters, we obtain
\[ R = \left( \frac{Z}{a_z} \right)^{1/b_z} = \left( \frac{r^2 p_r}{1.6827 \times 10^{-12} a_z} \right)^{1/b_z} \text{ [mm/hr]} \] (E-5)

\[ = 6.85 \times 10^7 \ (r^2 p_r)^{0.6934} \]

The brightband echoes (supposedly due mostly to wet ice) are very large and, if interpreted as \( R \), would give high rain-rate values. Clearly, the use of the derived rain rate at brightband is inappropriate. But the path attenuation of 28.6 GHz at and above the brightband is negligible due to the low absorption of ice, which is the prevalent form of water above the 0°C isotherm. Thus the derived rain rate along the path only is the true rain rate below the brightband. The brightband is identified by an unusual high reflectivity or apparent rain-rate value along the path which persists throughout the rain event. Figure E.1 shows that for the event of day 242/81 the bottom of the brightband was at about the 40th range bin.

Now, in order to evaluate the specific attenuation of the resolution cell at the beacon frequency, 28.6 GHz, for the rain rate \( R \) at that cell, a power law was used

\[ \alpha = a_\alpha R^{b_\alpha}, \] (E-6)

where

\[ a_\alpha = 0.17494 \text{ [dB/km]}, \]

\[ b_\alpha = 1.0209; \quad \text{at 28.6 GHz [35].} \]
Figure E.1. An example of radar derived rain-rate related to its range and time elapsed.
Since the radar data represents the attenuation at 28.6 GHz contributed by rain only below the brightband, and since the brightband is easily identified, the radar-derived attenuation data used in Figure 4.2 is obtained by summing the specific attenuations for each range bin below the brightband. Similarly, the contours above bin 40 in Figure E.1 should not be read as representing rain rate. Each range bin corresponds to a distance of 150 m (the return signal is sampled every 1 microsecond).
Appendix F

MAXIMAL-RATIO COMBINING

Maximal-ratio combining is a method to combine signals received in different channels linearly to improve the signal-to-noise ratio (SNR) optimally with respect to linear combination. The method can be found in many communication textbooks. Here, we follow Schwartz et al. [7]. Let the total complex envelope at IF in the kth channel be

\[ v_K(t) = g_K(t)u(t) + n_K(t), \quad (F-1) \]

where

- \( g_K(t) \) is the attenuation and phase shift due to the rain effects,
- \( u(t) \) is the signal output without rain effects,
- \( n_K(t) \) is the noise level with or without rain.

The most general linear combining results in an output complex envelope

\[ v(t) = \sum_{K=1}^{N} \alpha_K v_K(t), \quad (F-2) \]

where the \( \alpha_K \) are complex weights which vary slowly in time as the individual channel signals change due to fading, and \( N \) is the total number of channels. The instantaneous combined output has signal and noise,

\[ s(t) = u(t) \sum_{K=1}^{N} \alpha_K g_K, \]
and

\[ n(t) = \sum_{K=1}^{N} \alpha_K n_K(t). \] \hfill (F-3)

Then, the instantaneous output signal power is

\[ P(t) = |u(t)\sum_{K=1}^{N} \alpha_K g_K|^2, \]

and the average output noise is

\[ N = \langle |n(t)|^2 \rangle \]

\[ = \sum_{K=1}^{N} |\alpha_K|^2 |n_K(t)|^2 + \sum_{i,j=1}^{N} \alpha_i^* \alpha_j^* n_i(t)n_j(t) \]

\[ = \sum_{K=1}^{N} |\alpha_K|^2 \langle |n_K(t)|^2 \rangle + \sum_{i,j=1}^{N} \alpha_i^* \alpha_j^* \langle n_i(t)n_j(t) \rangle. \] \hfill (F-5)

Assuming the \( n_K(t) \) are independent of each other, the average output noise power is,

\[ N = \sum_{K=1}^{N} |\alpha_K|^2 N_K, \] \hfill (F-6)

where \( N_K = \langle |n_K(t)|^2 \rangle \). Then the output instantaneous SNR is

\[ \gamma = \frac{\langle |u(t)|^2 \rangle \sum_{K=1}^{N} \alpha_K^2 g_K^2 |^2}{\sum_{K=1}^{N} |\alpha_K|^2 N_K}. \] \hfill (F-7)

By applying the Schwarz Inequality for complex-valued numbers...
we obtain
$$\gamma < \frac{\sum_{i=1}^{N} |u(t)|^2 |g_i|^2}{\sum_{i=1}^{N} N_i} = \sum_{i=1}^{N} \gamma_i$$

The equality holds, and therefore the signal-to-noise ratio is optimized, if and only if
$$\alpha_i = \frac{g_i^*}{N_i}$$
where $K$ is an arbitrary complex constant.

Therefore, in order to maximize the instantaneous SNR, the gain of each channel should be made directly proportional to the signal amplitude level and inversely proportional to the mean square noise in that channel, and it should have a phase canceling out the signal delays so as to give identical phase for the signals from all channels at the summing point. The constant of proportionality $K$ must, of course, be the same for each channel. Thus, the combined output SNR is the sum of the SNR's on the individual channel,
$$\gamma = \sum_{i=1}^{N} \gamma_i$$

If each channel has the same noise output $N_0$ due to identical receivers being used, divide Equation (F-11) by the signal-to-noise ratio in a single channel for a clear weather day, to get
\[
\frac{\gamma}{\gamma_0} = \sum_{i=1}^{N} \frac{\gamma_i}{\gamma_0} = \sum_{i=1}^{N} x_i, \quad (F-12)
\]

where \( \gamma_0 = \frac{|u(t)|^2}{N_0} \) is the signal-to-noise ratio for clear weather in a single channel,

\( x_i = |g_i|^2 \) is the atmospheric power transmission factor for the \( i \)-th channel.

The left-hand side of Equation (F-12) is the output signal-to-noise ratio relative to the maximum (i.e., clear air) signal-to-noise ratio of one channel. Defining a joint transmission factor \( x = \gamma/\gamma_0 \) gives

\[
x = \sum_{i=1}^{N} x_i. \quad (F-13)
\]

The transmission factors \( x_i \) are related to the path attenuations \( x_i' \) (in dB) by

\[
x_i' = -10 \log_{10} x_i \quad (F-14)
\]

and one can define a similar dB measure for \( x \)

\[
x' = -10 \log_{10} x \quad (F-15)
\]

and write

\[
x' = -10 \log_{10} \left( \sum_{i=1}^{N} \frac{x_i'}{10} \right). \quad (F-16)
\]

We need to show now that \( x' \) is the same type of quantity as the individual site attenuations and the joint attenuation for switching, as plotted in Figures 5.5 through 5.8, for example. The quantity plotted there (on a dB basis) is the ratio of rain-attenuated signal power \( S \) to...
the clear weather power $S_0$,

$$y = \frac{s}{s_0} \quad \text{(F-17)}$$

In the present derivation we have assumed identical typical Earth stations (not masers) in which the front-end noise $N_0$ predominates. Then we can write

$$y = \frac{s/N_0}{s_0/N_0} = \frac{\gamma}{\gamma_0} \quad \text{(F-18)}$$

from which it is apparent that the $x$ defined above is precisely the same kind of quantity. To state it directly; when front-end noise predominates, the single-site attenuations and the equivalent "joint attenuation" for switching also can be interpreted as ratios of the actual signal-to-noise ratio to that for clear sky.

For those who prefer an interpretation in terms of signal power, rather than signal-to-noise ratio the following may be helpful. Adaptive combiners normally are active networks and may entail considerable amplification, i.e. $|K| \gg 1$. Clearly the output power of such an amplifier cannot be compared directly to the unamplified output for single-site or switching operation. However, we can conceptually consider such a system as a low-gain adaptive preamplifier followed by an ordinary amplifier. If we adjust the $K$ (i.e., the gains) of the adaptive channels so that their summed output noise power is $N_0$, then the summed signal power will be directly comparable with that for both single-site and switched-diversity operation. Thus $x'$ can be considered the "attenuation", in dB, of the signal at the preamplifier output relative to the clear-weather input signal power at either input.
APPENDIX G
MORE RESULTS

This appendix contains more plots of experiment results which are referenced in Chapter V.

The seasons used in the figures are defined below:

Winter includes December, January and February,
Spring includes March, April and May,
Summer includes June, July and August,
and Fall includes September, October and November.
Figure G.1.1. Monthly cumulative distribution of rain rate (07/83).
Figure G.1.2. Monthly cumulative distribution of rain rate (08/83).
Figure G.1.3. Monthly cumulative distribution of rain rate (09/83).
Figure G.1.4. Monthly cumulative distribution of rain rate (10/83).
Figure G.1.5. Monthly cumulative distribution of rain rate (11/83).
Figure G.1.6. Monthly cumulative distribution of rain rate (12/83).
Figure G.1.7. Monthly cumulative distribution of rain rate (01/84).
Figure G.1.8. Monthly cumulative distribution of rain rate (02/84).
Figure G.1.9. Monthly cumulative distribution of rain rate (03/84).
Figure G.1.10. Monthly cumulative distribution of rain rate (04/84).
Figure G.1.11. Monthly cumulative distribution of rain rate (05/84).
Figure G.1.12. Monthly cumulative distribution of rain rate (06/84).
Figure G.1.13. Monthly cumulative distribution of rain rate (07/84).
Figure G.1.14. Monthly cumulative distribution of rain rate (08/84).
Figure G.1.15. Monthly cumulative distribution of rain rate (12/84).
Figure G.1.16. Monthly cumulative distribution of rain rate (01/85).
Figure G.1.17. Monthly cumulative distribution of rain rate (02/85).
Figure G.1.18. Monthly cumulative distribution of rain rate (03/85).
Figure G.2.1. Monthly distribution of attenuation (06/83).
Figure G.2.2. Monthly distribution of attenuation (07/83).
Figure G.2.3. Monthly distribution of attenuation (08/83).
Figure 6.2.4. Monthly distribution of attenuation (09/83).
Figure 6.2.5. Monthly distribution of attenuation (10/83).
Figure G.2.6. Monthly distribution of attenuation (11/83).
Figure G.2.7. Monthly distribution of attenuation (12/83).
Figure G.2.8. Monthly distribution of attenuation (01/84).
Before February 9, 1984:
Antenna pointing
Azimuth 236.4°
Elevation 25.6°
main site 236.4°
remote site 196.4°

After February 9, 1984:
Antenna pointing
Azimuth 236.4°
Elevation 25.6°
main site 236.4°
remote site 236.4°

Figure 6.2.9. Monthly distribution of attenuation (02/84).
Figure G.2.10. Monthly distribution of attenuation (03/84).
Figure G.2.11. Monthly distribution of attenuation (04/84).
Figure G.2.12. Monthly distribution of attenuation (05/84).
Figure G.2.13. Monthly distribution of attenuation (06/84).
Figure G.2.14. Monthly distribution of attenuation (07/84).
Figure 6.2.15. Monthly distribution of attenuation (08/84).

ngthening the protection

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**Figure 6.2.15.** Monthly distribution of attenuation (08/84).
Figure G.2.16. Monthly distribution of attenuation (09/84).

% OF TIME ABSCISSA EXCEEDED

0.0010 0.0100 0.1000 1.0000 10.0000 100.0000
0.0010 0.0100 0.1000 1.0000 10.0000 100.0000

ATTENUATION (DB)

main site
Antenna pointing 25.6°
remote site
Antenna pointing 25.6°
Figure G.2.17. Monthly distribution of attenuation (10/84).
Figure G.2.18. Monthly distribution of attenuation (11/84).
Figure G.2.19. Monthly distribution of attenuation (12/84).

% OF TIME ABSCISSA EXCEEDED

ATTENUATION (dB)

0.0010 0.0100 0.1000 1.0000 10.0000 100.0000

-0.0010 0.0100 0.1000 1.0000 10.0000 100.0000
Figure G.2.20. Monthly distribution of attenuation (01/85).
Figure G.2.21. Monthly distribution of attenuation (02/85).
Figure G.2.22. Monthly distribution of attenuation (03/85).
Figure G.3.1. Seasonal distribution of attenuation on the satellite-to-main-site path over 24 hours (summer/83).
Figure G.3.2. Seasonal distribution of attenuation on the satellite-to-main-site path over 24 hours (fall/83).
Figure G.3.3. Seasonal distribution of attenuation on the satellite-to-main-site path over 24 hours (winter/84).
Figure G.3.4. Seasonal distribution of attenuation on the satellite-to-main-site path over 24 hours (spring/84).
Figure G.3.5. Seasonal distribution of attenuation on the satellite-main-site path over 24 hours (summer/84).
Figure G.3.6. Seasonal distribution of attenuation on the satellite-to-main-site path over 24 hours (fall/84).
Figure G.3.7. Seasonal distribution of attenuation on the satellite-to-main-site path over 24 hours (winter/85).
Figure G.3.8. Seasonal distribution of attenuation on the satellite-to-remote-site path over 24 hours (summer/83).
Figure 6.3.9. Seasonal distribution of attenuation on the satellite-to-remote-site path over 24 hours (fall/83).
Figure G.3.10. Seasonal distribution of attenuation on the satellite-to-remote-site path over 24 hours (winter/84).
Figure G.3.11. Seasonal distribution of attenuation on the satellite-to-remote-site path over 24 hours (spring/84).
Figure G.3.12. Seasonal distribution of attenuation on the satellite-to-remote-site path over 24 hours (summer/84).
Figure G.3.13. Seasonal distribution of attenuation on the satellite-to-remote-site path over 24 hours (fall/84).
Figure G.3.14. Seasonal distribution of attenuation on the satellite-to-remote-site path over 24 hours (winter/85).
Figure G.3.15. Seasonal distribution of attenuation for switching processing over 24 hours (summer/83).
Figure G.3.16. Seasonal distribution of attenuation for switching processing over 24 hours (fall/83).
Figure G.3.17. Seasonal distribution of attenuation for switching processing over 24 hours (winter/84).
Figure G.3.18. Seasonal distribution of attenuation for switching processing over 24 hours (spring/84).
Figure G.3.19. Seasonal distribution of attenuation for switching processing over 24 hours (summer/84).
Figure G.3.20. Seasonal distribution of attenuation for switching processing over 24 hours (fall/84).
Figure G.3.21. Seasonal distribution of attenuation for switching processing over 24 hours (Winter/85).
Figure G.3.22. Seasonal distribution of attenuation for maximal-ratio-combining processing over 24 hours (summer/83).
Figure G.3.23. Seasonal distribution of attenuation for maximal-ratio-combining processing over 24 hours (fall/83).
Figure 6.3.24. Seasonal distribution of attenuation for maximal-ratio-combining processing over 24 hours (winter/84).
Figure G.3.25. Seasonal distribution of attenuation for maximal-ratio-combining processing over 24 hours (sprng/84).
Figure G.3.26. Seasonal distribution of attenuation for maximal-ratio-combining processing over 24 hours (summer/84).
Figure G.3.27. Seasonal distribution of attenuation for maximal-ratio-combining processing over 24 hours (fall/84).
Figure G.3.28. Seasonal distribution of attenuation for maximal-ratio-combining processing over 24 hours (winter/85).
Figure G.4.1. Seasonal distribution of fade duration on the satellite-main-site path (summer/83).
Figure G.4.2. Seasonal distribution of fade duration on the satellite-main-site path (fall/83).
Figure G.4.3. Seasonal distribution of fade duration on the satellite-main-site path (winter/84).
Figure G.4.4. Seasonal distribution of fade duration on the satellite-main-site path (spring/84).
Figure G.4.5. Seasonal distribution of fade duration on the satellite-main-site path (summer/84).
Figure G.4.6. Seasonal distribution of fade duration on the satellite-main-site path (fall/84).
Figure G.4.7. Seasonal distribution of fade duration on the satellite-main-site path (winter/85).
Figure G.4.8. Seasonal distribution of fade duration on the satellite-remote-site path (summer/83).
Figure G.4.9. Seasonal distribution of fade duration on the satellite-remote-site path (fall/83).
Figure G.4.10. Seasonal distribution of fade duration on the satellite-remote-site path (winter/84).
Figure G.4.11. Seasonal distribution of fade duration on the satellite-remote-site path (spring/84).
Figure G.4.12. Seasonal distribution of fade duration on the satellite-remote-site path (summer/84).
Figure G.4.13. Seasonal distribution of fade duration on the satellite-remote-site path (fall/84).
Figure G.4.14. Seasonal distribution of fade duration on the satellite-remote-site path (winter/85).
Figure G.4.15. Seasonal distribution of fade duration for switching processing (summer/83).
Figure G.4.16. Seasonal distribution of fade duration for switching processing (fall/83).
Figure G.4.17. Seasonal distribution of fade duration for switching processing (winter/84).
Figure G.4.18. Seasonal distribution of fade duration for switching processing (spring/84).
Figure G.4.19. Seasonal distribution of fade duration for switching processing (summer/84).
Figure G.4.20. Seasonal distribution of fade duration for switching processing (fall/84).
Figure G.4.21. Seasonal distribution of fade duration for switching processing (winter/85).
Figure G.4.22. Seasonal distribution of fade duration for maximal-ratio-combining processing (summer/83).
Figure G.4.23. Seasonal distribution of fade duration for maximal-ratio-combining processing (fall/83).
Number of fades duration is exceeded.

0 dB

3 dB

6 dB

Fade duration (min.)

Before February 9, 1984:
Antenna pointing
main site 236.4° 25.6°
remote site 196.4° 25.6°

After February 9, 1984:
Antenna pointing
main site 236.4° 25.6°
remote site 236.4° 25.6°

Figure G.4.24. Seasonal distribution of fade duration for maximal-ratio-combining processing (winter/84).
Figure G.4.25. Seasonal distribution of fade duration for maximal-ratio-combining processing (spring/84).
Figure G.4.26. Seasonal distribution of fade duration for maximal-ratio-combining processing (summer/84).
Figure G.4.27. Seasonal distribution of fade duration for maximal-ratio-combining processing (fall/84).
Figure G.4.28. Seasonal distribution of fade duration for maximal-ratio-combining processing (winter/85).
Figure G.5.1. Seasonal distribution of inter-fade interval on the satellite-to-main-site path (summer/83).
Figure G.5.2. Seasonal distribution of inter-fade interval on the satellite-to-main-site path (fall/83).
Figure G.5.3. Seasonal distribution of inter-fade interval on the satellite-to-main-site path (winter/84).
Figure G.5.4. Seasonal distribution of inter-fade interval on the satellite-to-main-site path (spring/84).
Figure G.5.5. Seasonal distribution of inter-fade interval on the satellite-to-main-site path (summer/84).
Figure G.5.6. Seasonal distribution of inter-fade interval on the satellite-to-main-site path (fall/84).
Figure G.5.7. Seasonal distribution of inter-fade interval on the satellite-to-main-site path (winter/85).
Figure G.5.8. Seasonal distribution of inter-fade interval on the satellite-to-remote-site path (summer/83).
Figure G.5.9. Seasonal distribution of inter-fade interval on the satellite-to-remote-site path (fall/83).
Figure G.5.10. Seasonal distribution of inter-fade interval on the satellite-to-remote-site path (winter/84).
Figure G.5.11. Seasonal distribution of inter-fade interval on the satellite-to-remote-site path (spring/84).
Figure 6.5.12. Seasonal distribution of inter-fade interval on the satellite-to-remote-site path (summer/84).
Figure G.5.13. Seasonal distribution of inter-fade interval on the satellite-to-remote-site path (fall/84).
Figure G.4.14. Seasonal distribution of inter-fade interval on the satellite-to-remote-site path (winter/85).
Figure G.5.15. Seasonal distribution of inter-fade interval for switching processing (summer/83).
Figure G.5.16. Seasonal distribution of inter-fade interval for switching processing (fall/83).
Before February 9, 1984:
Antenna pointing

<table>
<thead>
<tr>
<th>Site</th>
<th>Azimuth</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main site</td>
<td>236.4°</td>
<td>25.6°</td>
</tr>
<tr>
<td>Remote site</td>
<td>196.4°</td>
<td>25.6°</td>
</tr>
</tbody>
</table>

After February 9, 1984:
Antenna pointing

<table>
<thead>
<tr>
<th>Site</th>
<th>Azimuth</th>
<th>Elevation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main site</td>
<td>236.4°</td>
<td>25.6°</td>
</tr>
<tr>
<td>Remote site</td>
<td>236.4°</td>
<td>25.6°</td>
</tr>
</tbody>
</table>

Figure G.5.17. Seasonal distribution of inter-fade interval for switching processing (winter/84).
Figure G.5.18. Seasonal distribution of inter-fade interval for switching processing (spring/84).
Figure G.5.19. Seasonal distribution of inter-fade interval for switching processing (summer/84).
Figure G.5.20. Seasonal distribution of inter-fade interval for switching processing (fall/84).
Figure G.5.21. Seasonal distribution of inter-fade interval for switching processing (winter/85).
Figure G.5.22. Seasonal distribution of inter-fade interval for maximal-ratio-combining processing (summer/83).
Figure G.5.23. Seasonal distribution of inter-fade interval for maximal-ratio-combining processing (fall/83).
Figure G.5.24. Seasonal distribution of inter-fade interval for maximal-ratio-combining processing (winter/84).
Figure G.5.25. Seasonal distribution of inter-fade interval for maximal-ratio-combining processing (spring/84).
Figure G.5.26. Seasonal distribution of inter-fade interval for maximal-ratio-combining processing (summer/84).
Figure G.5.27. Seasonal distribution of inter-fade interval for maximal-ratio-combining processing (fall/84).
Figure G.5.28. Seasonal distribution of inter-fade interval for maximal-ratio-combining processing (winter/85).