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MODELING AND CONTROL OF WELDING DISTORTION IN TUBULAR FRAME STRUCTURES

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MODELING AND CONTROL OF WELDING DISTORTION
IN TUBULAR FRAME STRUCTURES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Chien-ann Hou, B.S., M.S.

*****
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1986

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Finally, all the glories to Almighty God !!
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PUBLICATIONS


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Chapter 1

INTRODUCTION

1.1 THE NEED FOR STUDY

Before World War II, most ships and structures were riveted; today most of them are fabricated by welding. In fact, many of the structures presently being built, e.g. the space shuttle, submarines, and heavy containment vessels for nuclear reactors, could not have been constructed without the proper application of welding technology.

However, fabrication by welding also creates problems such as defects and distortion. Due to local heating during welding, complex thermal stresses occur; and residual stresses, and distortion result. Thermal stresses, residual stresses, and distortion can cause cracking and mismatching. In addition high tensile residual stresses in the area near the weld may cause fracture under certain conditions.

Welding distortion of structural tubes can occur in the form of longitudinal bowing about the tube axis or as local geometric deformations. The longitudinal bowing causes problems such as alignment, while local geometric
deformation causes other problems such as lack of flatness, parallelism, straightness, and squareness.

The analysis of welding distortion is a very difficult task, because it involves many principles in many different fields such as heat transfer, mechanics of materials, elasticity, plasticity, and transient, nonlinear analysis. Hence, there are not many publications on this subject. Most engineers solve the problem utilizing experience and post-welding heat treatment. However, after robots were introduced in industry, the automation of welding became popular. This automation makes prediction of welding distortion highly desirable to industry for efficient operation.

In this dissertation, square tubes jointed by fillet welds were selected and the angular distortions due to welding analyzed. This weldment and distortion is shown in Figure 1. The reason for this selection is that the Tee-joint is the simplest and most basic joint type among many different joint configurations, and angular distortion is the most serious distortion problem which occurs in these joints.
Figure 1: Examples of Angular Distortion of Fillet Welded Tee-Joint
1.2 LITERATURE REVIEW

The heat supplied by a welding arc produces a complex thermal cycle in the weldment and this, in turn, causes changes in the microstructure of the heat-affected zone, transient thermal stresses, metal movement, and eventually results in the creation of residual stresses and distortion in the finished structure. Analysis of the thermal stresses and metal movement, in general, involves the following steps:

1. heat flow analysis,
2. transient thermal and stress analysis,
3. and post welding residual-stress analysis.

An exact analytical solution for heat flow from a moving point heat source was first obtained by Rosenthal in 1935 and modified in 1941 [38, 39]. Experiments were conducted by Rosenthal and Schmerber in 1938 to validate the two dimensional solution [40]. In their computations, they used constant thermal conductivity and diffusivity with adiabatic boundary conditions. Under these assumptions, they reported that the computed temperature distributions for a distance far away from the arc was satisfactory and in agreement with those measured. However, in the neighborhood of the arc, the computed isotherms were farther away from the origin of the heat source than the measured values.
This disagreement was due to the fact that the arc heat source is not concentrated at a point, but rather is distributed non-uniformly over a finite area.

After the point heat source theory was developed, many researchers such as Mahla, Wells, Roberts and Adams worked to modify this theory [25, 51, 37, 1]. In 1965, Christensen and co-workers compared experimental results with the idealized Rosenthal's solution and concluded that it was not possible to use the analytical approach alone to determine the temperature distributions very accurately because the heat transfer from the arc to the base metal is so complicated [9]. They also concluded that boundary heat losses are not significant when calculating the cooling history during welding in air.

In 1969 Pavelic and his co-workers introduced a semi-empirical approach to determine a two-dimensional temperature distribution in the base metal provided that the shape and the size of the molten pool was given [36]. The boundary contour of the molten pool was obtained by experiment. They developed a mathematical model for the heat flow problem by considering 1) a finite heat source with an assumed Gaussian heat distribution, 2) boundary heat losses by convection and radiation, and 3) temperature dependent thermal properties. After comparing the
calculated values with experimental data during gas tungsten arc welding of thin, mild steel plate, they reported accurate results.

In 1983, Nunes extended the point source theory to include the effects of phase changes and circulation in the weld pool [33]. In the same year, Tsai modified this theory and combined a Gaussian heat distribution for the welding arc to permit a study of the pulsed current gas tungsten arc welding process [32]. Hou and Tsai developed a general three dimensional closed-form solution which is capable of analyzing the thermal response of a weldment either at its transient or quasi-stationary state during welding [19].

Recently, numerical analyses using finite difference or finite element techniques have become increasingly popular. Hibbitt and Marcal introduced a finite element heat flow model for the prediction of temperature distributions during welding for the quasi-stationary state in 1973 [18]. During 1974 through 1976, Friedman followed by using the finite element method with consideration of both phase change and latent heat effects to conduct a thermo-mechanical analysis [12, 13, 14]. He reported that a more reliable temperature solution than those obtained from either the point or line source theory was obtained by using the finite element method incorporated with additional numerical techniques.
He also summarized and concluded that this analysis technique was also applicable to planar or axisymmetric welds with arbitrary cross-sectional geometries under the quasi-stationary condition [15].

In 1977, Tsai extended the finite difference technique to analyze a three-dimensional model in an investigation of underwater welding [48]. Kou and his co-workers in 1982 reported that the results from a finite difference method agreed very well with experiments of a two-dimensional heat flow analysis of welding thin aluminum alloy plate [22]. He also concluded that his model could be used to study both solidification and solid-state phase transformation during thin plate welding.

However, numerical methods which require a large computer memory storage are quite costly. The advantage of calculational accuracy is sometimes diminished by the correspondingly high cost.

Studies of residual stresses and distortion in weldments were undertaken as early as the 1930's. In 1936, Boulton and Lance-Martin discussed transient thermal stresses during welding along the edge of a plate [8]. Spraragen and associates in 1937, 1944 and 1950 prepared a series of comprehensive reviews on distortion and shrinkage in welds [42, 43, 44].
During the 1950's, several Japanese investigators such as Kihara, Watanabe, and Masubuchi conducted extensive studies on residual stresses and distortion in welded plates [20, 21, 50]. At the same time, Russian researchers such as Okerblom and Vinokurov wrote several books which contained considerable information on the analysis of weldment distortion [34, 49].

However, most early studies considered residual stresses and distortion after welding are based on experimental data. Only limited studies were made on transient thermal stresses and metal movement. This is due to the complexity of the problem and the computations required for analyzing the transient state.

The first significant attempt to use a computer in the analysis of thermal stresses during welding was done by Tall in 1961 [45, 46]. He developed a simple program for thermal stresses induced during bead-on-plate welding along the centerline of a strip. The temperature distribution was treated as a two dimensional problem; however, in analyzing stresses it was assumed that the longitudinal stress $\sigma_x$ is only a function of the lateral distance $y$, while $\sigma_y$ and $\tau_{xy}$ are 0. Such an analysis is referred to as a one-dimensional analysis.

In the 1970's there was a considerable increase in
computer analysis of transient thermal stresses and distortion during welding. Investigators at M.I.T. led by Masubuchi and some laboratories outside the United States are currently working on this subject. In 1975, Masubuchi, Muraki, Nishida etc. developed one-dimensional and two-dimensional finite element programs for the simulation of thermal stresses and metal movement during butt welding. They also did extensive experimentation and comparison with the calculated results. They claimed that the results were in agreement but they also suggested that the model still needed improvement to obtain better accuracy and to handle more complex problems which occur in actual fabrications [26, 28, 35, 27]. However, this was the beginning of systematic research in welding distortion. It also provided valuable information on welding distortion. In 1980, White and Leggatt conducted several experiments comparing a parametric study on weld shrinkage and obtained an empirical relationship between welding distortion and welding conditions [53]. The same year, Friedman developed a two dimensional finite element model for quasi-stationary state arc welding of bead-on-plate weldments [12, 13, 14]. Through-the-thickness temperature variation and completely restrained motion in the welding direction were assumed.

In 1982, numerical analyses of welding became popular.
Lau and Dave used the finite element method to analyze a welded truss connections [24]. They used two dimensional triangular elements for elastic-plastic structural analysis and obtained some empirical relationships for design procedures. Argyris, Szimmat, and Willam also worked on welding stress analysis using finite elements of a butt weld on a plate [31]. Papazoglow and Mashbuchi considered phase transformation effects for a butt weld [11]. They treated the problem as two dimensional and used the finite element program ADINA to calculate the temperature distribution and thermal stresses. Finally, they noted that the analysis was expensive because it required significant amounts of computer memory and CPU time.

In studying the thermal stresses that occur during welding, it is important to compare any analytical predictions with experiment results. Table 1 lists the experimental studies conducted recently [52]. The experiments were conducted with low carbon steel, high strength steel, or aluminum alloys with bead-on-plate and butt welds.

During a recent literature survey there were only a few published papers found considering distortion due to angular changes in fillet welds. In 1954, Kumose and associates conducted experiments on the angular distortion caused by
<table>
<thead>
<tr>
<th>Principal Personnel</th>
<th>Sponsor</th>
<th>Material, Joint Type, etc.</th>
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<td>1. Arita</td>
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<td>Welding along the longitudinal edge of rectangular plates in low-carbon steel, quenched and tempered steels, stainless steel, and a titanium alloy</td>
</tr>
</tbody>
</table>

Table 1: Recent Experimental Studies on Thermal Stress [52]
one-pass fillet welding in T-joint weldments of mild steel plate [23]. They reported that angular distortion appears to increase as the welding speed is increasing, and a maximum value of angular distortion is found in weldments when the plate thickness is between 0.4 to 0.5 in. In 1972 and 1974, Taniguchi and Henry reported on experimental results of angular distortions produced by fillet welds on aluminum plate [47, 17].

Unfortunately, no papers were found which discussed the angular change produced by fillet welds on square tubing.

1.3 THESIS OBJECTIVE AND SCOPE

During heating and cooling in the welding cycle, thermal strains occur in the weld metal and base metal regions near the weld. The strains produced during heating are accompanied by plastic upsetting. The stresses resulting from these strains combine and react to produce internal forces that cause bending, buckling, and rotation. The result of such metal movement is called distortion.

The research involved in this dissertation is part of a project called "Optimization of Robot Arc Welding Procedures for Minimization and Control of Distortion in Tubular Frame Structures", sponsored by IBM.

The goal of this project is to investigate and minimize
the welding distortion of several kinds of joints used in frame structures and to optimize the procedures for robotic welding. Robotic welding is part of an automatic process which can reduce the thermal disturbance, welding distortion and maintain the quality of joints. However, to optimize such a process, a welding engineering analysis that considers distortion and robotic welding procedures is needed.

The main objective of the research described in this dissertation is the modeling and analysis of the welding distortion and thermal stresses in a fillet welded T-joint fabricated from square tubing. The model will consider the out-of-plane distortion caused by angular changes along the fillet weld because this is the most serious problem for a fillet welded Tee joint. If a simple Tee joint with two fillet welds can be analyzed, then a general tubular frame structure can also be analyzed using the same principles with some modifications. The whole tubular frame structure can be constructed by the finite element technique using beam elements. The individual joint analyses can provide information on local effects such as plastic strain around the welded zone and displacement at different locations. With this information on local effects, some modification can be made in the finite element beam structure analysis
which predicts deformations in the whole structure. This metal movement information can be feed back to the joint analysis which predicts the plastic strain in the next time step. With this procedure, the distortion analysis of a general tubular frame structure would be possible. Therefore, this study of the modeling and analysis of a Tee joint can be the foundation for welding distortion and distortion control in the actual welding fabrication of tubular structures.

This dissertation consists of three parts. First, a new and simple model was developed which includes a finite source theory in the heat flow analysis and an elastic-plastic analysis using a strain hardening material. This model was developed to predict the angular distortion for industrial applications. Second, a finite element package named ANSYS was used to check the results. In addition, an experimental set up using robotic arc welding was used to verify the analytical model. The logic of this dissertation is shown in Figure 2. Third, a parametric study using the analytical model was conducted to investigate several welding factors such as heat input, welding speed, external constraint effects, and time breaks between each weld. This study provides valuable information, and gives as a guideline for minimizing and controlling welding distortion for Tee-joints in tubular structures.
Parametric Study
An application to predict and control welding distortion of a tee joint for tubular structure.

Develop a relatively simple computer model to predict and control welding distortion for industrial application.

Finite Element Analysis (Math. Tool)
Experimental Analysis (Mesh. Tool)

Figure 2: Methodology for Angular Distortion Analysis
Chapter 2
MODELING OF WELDING DISTORTION
- THERMAL ANALYSIS

2.1 INTRODUCTION

The thermal response of a material to a welding heat source sometimes causes metallurgical and mechanical problems. The heat supplied by a welding arc produces complex thermal cycles in the weldment, and these in turn cause changes in the microstructure of the heat-affected zone, transient thermal stress, and metal movement. This eventually results in the creation of residual stresses and distortion in the finished structure. In order to solve these problems, an analysis of welding heat flow is necessary.

Many investigators have studied heat flow during arc welding, both analytically and experimentally. Analytical methods already developed are capable of computing temperature distributions with reasonable accuracy in simple weldments such as bead-on-plate welds. Today, with the advancement of computer technology and such numerical
methods as the finite element method, it is quite possible to significantly improve the accuracy of a weldment heat flow analysis. However, there have been relatively few papers published concerning the analysis of welding-heat flow in thin square tubes.

In this chapter, a relatively simple model will be developed. First, background on welding heat flow will be discussed. Then, the governing equation and solution will be derived. Finally, a computer program will be discussed which carries out the numerical calculation.

2.2 BACKGROUND

Perhaps the most critical data required for a welding thermal analysis are the parameters necessary to describe the heat input from the arc to the weldment. To avoid the extremely complex detail of the energy transfer from the arc, the magnitude of the heat input to the work piece is expressed simply as the product of the arc power and an efficiency factor $\eta$:

$$\dot{Q} = \eta \cdot E \cdot I$$

(2.1)

where $E$ is the arc voltage and $I$ is the welding current. The arc efficiency is a rather nebulous factor intended to account for all of the power losses from the arc.
In welding practice, the heat input per unit length of weld (heat intensity) which is supplied by the moving arc is commonly used as a parameter for cooling rates. It is defined by:

\[ Q' = \frac{\dot{Q}}{S} = \frac{\eta \cdot E \cdot I}{S} \]  

(2.2)

where \( S \) is the traveling speed of the arc. The heat intensity parameter not only considers the total heat input rate but also the welding speed. Thus, it also reflects the influence of the traveling speed of the arc.

The heat loss to the air environment from the part of the molten pool surface outside the heat input circle during welding is basically due to radiation and some natural convection. Figure 3 shows the calculated heat loss coefficient and the measured data which was obtained by Tsai [48]. At temperatures higher than 1000°C, radiation is the dominant phenomenon and at temperatures lower than 200 °C, natural convection becomes important but the magnitude is small.

The exact analytical solution for heat flow from a moving point heat source was first obtained by Rosenthal in 1935 and modified in 1941 [38, 39]. In this theory, a constant point or line source moving with constant speed was introduced on a Cartesian coordinate system. Rosenthal
Figure 3: Boundary Heat Losses for Welding in Air [48]
obtained the solutions for the quasi-stationary state condition for one-, two-, and three-dimensional heat flow problems based on the following assumptions 1) constant thermal properties, 2) adiabatic boundary conditions on the plate surface, 3) the base metal being of infinite extent, and 4) joule heating effects being negligible. The welding system is shown in Figure 4, and \((w,y,z)\) is the moving coordinate system which is fixed at the heat source. After the mathematical derivation, the quasi-stationary state solution for an infinitely thick plate (three dimensional problem) using a moving point heat source was obtained as

\[
T - T_0 = \frac{\dot{Q}}{2\pi\lambda R} \exp\left(-\frac{S}{2\kappa W}\right) \exp\left(-\frac{S}{2\kappa R}\right)
\]  \hspace{1cm} (2.3)

for which

\(T\) is the temperature at the point \((w,y,z)\),

\(T_0\) is the reference temperature,

\(\dot{Q}\) is the heat input rate,

\(\lambda\) is the thermal conductivity of the base metal,

\(\kappa\) is the thermal diffusivity of the base metal and,

\(S\) is the welding speed,

\(R\) is the distance from the point to the origin of the coordinate \((w,y,z)\).

For a two dimensional problem (thin plate) and a moving line heat source, Carslaw's instantaneous point heat source
Figure 4: Arc Welding Coordinate System
equation was extended [41], and the quasi-stationary state solution was

\[ T - T_0 = \frac{\dot{Q}}{2\pi k} \exp \left( -\frac{SW}{2k} \right) K_0 \left( \frac{SR}{2k} \right) \]

(2.4)

where \( \dot{Q} \) is the heat input rate,

\( K_0 \) is the modified Bessel function of the zeroth order and,

\( R \) is the distance from the point to the origin of the coordinate \((w,y)\),

\( h \) is the thickness of the plate.

\((w,y)\) is the moving coordinates.

2.3 THERMAL MODEL AND DERIVATION

The model of heat flow through a thin wall tube for a Tee joint was developed, as shown in Figure 5. The weld is short and welding speed fast enough that the heat is assumed instantaneously deposited into the tubular joints. Therefore, the temperature distribution will be symmetric about \( y = 0 \). Also the heat loss from the structural surface is neglected i.e. the surface boundary is insulated. It is then possible to assume that the thin square tube can be cut from the bottom surface and expanded as a plate; therefore, the problem can be reduced from a three dimensional analysis to a two dimensional analysis.
Figure 5: Welding Thermal Model
In order to avoid mathematical complexity in solving a nonlinear equation, almost all analytical studies of heat flow during welding assume that thermal properties do not change with temperature, i.e. the thermal conductivity and diffusivity are constants. Figure 6 shows the conductivity and diffusivity of a mild steel, and indicates that this assumption is reasonable [52].

In general the heat conduction equation in a solid can be derived in the form:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{\rho c}{\kappa} \frac{\partial T}{\partial t}
\]  

(2.5)

However, a dimensionless heat conduction equation also can be written in the following form in order to generalize the problem:

\[
\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} + \frac{\partial^2 \Theta}{\partial Z^2} = \frac{\partial \Theta}{\partial \tau}
\]  

(2.6)

where

\[
\Theta = \frac{(T - T_0)}{(T_m - T_0)}
\]

\[
\tau = \frac{\kappa t}{h^2}
\]

\[
X = \frac{x}{h}
\]
Figure 6: Thermal Conductivity and Diffusivity of A Mild Steel [52]
and $T_m$: melting temperature,

$h$: plate thickness (i.e. thickness of tube),

$t$: time,

$(x,y,z)$: Cartesian coordinate

The solution for an instantaneous point source with a heat input $Q$ at $(x,y,z)$ and an adiabatic boundary condition on the surface of a semi-infinite body is developed by Carslaw and Jaeger and is given as [41]:

$$
\Theta = \frac{Q}{4(\pi \tau)^{1.5}} \exp \left( -\frac{R^2}{4\tau} \right)
$$

(2.7)

where $R^2 = [(X - X')^2 + (Y - Y')^2 + (Z - Z')^2]$.

$$
\bar{Q} = \frac{Q \kappa}{\lambda h^3 (T_m - T_0)}
$$

For a continuous heat source where heat is released from time $t=0$ ($\tau=0$) to $t=t_s$ ($\tau=t_s$), the solution can be integrated with respect to time and to give
\[ \Theta = \int_{0}^{T_s} \frac{\dot{Q}}{4\pi^{1.5}(\tau - \tau')^{1.5}} \exp \left( \frac{-R^2}{4(\tau - \tau')} \right) d\tau' \quad (2.8) \]

where \( R^2 = [(X - X')^2 + (Y - Y')^2 + (Z - Z')^2] \)

\[ \frac{\dot{Q}}{Q_k} = \frac{\dot{Q} \kappa}{\lambda h^3(T_m - T_0)} \]

where \( t_s \) (or \( \tau_s \)) is the time when the arc is extinguished. Assuming the heat flux is constant and is discharged into a small area, which is the weld leg size times the weld length, then a finite heat source can be applied [19]. The solution becomes

\[ \Theta = \int_{X_0}^{X_1} \int_{Y_0}^{Y_1} \int_{T_0}^{T_s} \frac{c' \dot{q}'}{(\tau - \tau')^{1.5}} \exp \left( \frac{-R^2}{4(\tau - \tau')} \right) d\tau' dY' dX' \quad (2.9) \]

where

\[ c' = \frac{1}{4 \pi^{1.5}} \]

\[ \dot{q}' = \frac{\dot{Q} \cdot h}{\lambda \cdot L \cdot a \cdot (T_m - T_0)} \]
and \( a \) is the weld length, and \( L \) is the weld leg size.

After integration with respect to time, the solution becomes

\[
\Theta(X,Y,Z,\tau) = c \int \int \int_{X_0}^{X_1} \int_{Y_0}^{Y_1} \frac{G_1 - G_2}{R} \, dY' \, dX' \quad (2.10)
\]

where

\[
G_1 = \text{erf}\left[ \frac{1}{2} \sqrt{\frac{R^2}{(\tau - \tau')}} \right]
\]

\[
G_2 = \text{erf}\left[ \frac{1}{2} \sqrt{\frac{R^2}{\tau}} \right]
\]

\[
c = \frac{1}{2 \pi}
\]

and

\[
\tau' = \begin{cases} 
\tau_s & \text{if } \tau > \tau_s \\
\tau & \text{if } \tau \leq \tau_s
\end{cases}
\]

For a plate with finite thickness and width in the \( Z \) and \( Y \) directions, respectively, the analysis can be conducted by an approximate method called the image method. This method assumes the system is linear and the superposition principle holds. The image method is used very often in linear elasticity, especially in the theory of
Figure 7: Illustration of Image Method in One Direction

\[ \Phi_n(w, y, 2nH + z) \quad n = 1, 2, 3 \ldots \]

Image No. 2 \[ \Phi_1(w, y, 2H + z) \]

Image No. 1 \[ \Phi_2(w, y, 2H - z) \]

Image No. 3 \[ \Phi_3(w, y, 4H - z) \]

\[ \Phi_{\text{max}}(w, y, 2nH - z) \quad n = 1, 2, 3 \ldots \]
plates and shells [10]. If the welding heat flow problem with a semi-infinite body has been solved, the solution for a welding heat flow problem associated with a plate with finite dimensions and adiabatic surface conditions can be obtained by applying the image method. Figure 7 shows how the image method works. Assuming $\Phi_0(x,y,z)$ is the solution for the welding heat flow problem with a semi-infinite body, it will satisfy the differential equation but not the boundary conditions at $z = h$, where $h$ is the thickness of the plate. Now imagine the plate extended in both the positive and negative Z directions and loaded with a series of identical heat sources at their respective locations. By superimposing $\Phi_0(x,y,z)$ and $\Phi_1(w,y,2h-z)$, the solution $\Phi_0 + \Phi_1$ will satisfy the differential equation and boundary condition at $z = h$ now but not at $z = 0$. However, continuing this process, the error due to not satisfying the boundary condition exactly will be reduced with each iteration. After summing all the possible solutions, the final solution will satisfy the differential equation and approach the right boundary conditions at $z = 0$ and $z = h$.

Therefore, after applying the image method in the Y and Z directions for a finite thickness and width, the solution becomes
\[ \Theta_{\text{final}}(X,Y,Z,\tau) = \Theta(X,Y,Z,\tau) + \]
\[ \sum_{m=1}^{\infty} \left[ \Theta(X,Y,Z^m,\tau) + \Theta(X,Y,Z^m,\tau) \right] + \]
\[ \sum_{n=1}^{\infty} \left[ \Theta(X,Y,Z^n,\tau) + \Theta(X,Y,Z^n,\tau) \right] + \]
\[ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ \Theta(X,Y,Z^m,Z^n,\tau) + \Theta(X,Y,Z^m,Z^n,\tau) \right] \]

\[ y' = 2m \frac{2L}{h} + y \quad , \quad y'' = 2m \frac{2L}{h} - y \]
\[ z' = 2n + Z \quad , \quad z'' = 2n - Z \]

2.4 COMPUTER PROGRAM DEVELOPMENT

A computer program was developed to carry out the numerical calculation of equation (2.11). In Figure 8, a simple computer implementation flow chart is shown.

The program is able to simulate the welding process to calculate the temperature distribution in a finite plate. Due to the plate thinness, the temperature calculated in the midplane will be representative of the average temperature at the top and bottom surfaces.

The image method was applied on both the thickness and width directions to satisfy the adiabatic boundary conditions. In addition, the error function was expanded as
Figure 8: Flowchart for Welding Thermal Analysis

- START
- CALCULATE SOME CONSTANTS
- SET TIME $t = 1$
- POSITION LOOP $I = 1, J = 1$
- SOLUTION WITH DOUBLE INTEGRAL (FINITE HEAT SOURCE THEORY)
- APPLY IMAGE METHOD IN WIDTH AND THICKNESS DIRECTIONS
- CHECK FOR POSITION LOOP
- $t = t + \Delta t$
- INCREASE DATA POINTS USING LINEAR INTERPOLATION
- CHECK FOR TIME
- END
a series developed by C. Hastings, Jr., in 1955 [16]. The error of this approximation is less than $10^{-4}$.

Gaussian quadrature formulas were applied to carry out the integration. This procedure can be applied only when the function $F(x)$ is explicitly known, so that it can be evaluated at any desired value of $x$. It also requires that the integrals be first transformed to integrals with limits $(-1,1)$. In the computer program, there are double integrals to be calculated, therefore double (or area) Gaussian quadrature formulas are used. They can be presented in the following form:

$$\int_{c}^{d} \int_{a}^{b} f(x,y) \, dx \, dy = \int_{c}^{d} \int_{-1}^{1} f \left[ \frac{(b-a)\zeta + b + a}{2} \right] d\zeta \, dy$$

$$= \frac{(d-c) (b-a)}{4} \int_{-1}^{1} \int_{-1}^{1} f \left[ \frac{\zeta + b + a}{2}, \frac{\psi + d + c}{2} \right] d\zeta \, d\psi$$  \hspace{1cm} (2.12)

The program uses 4 point Gaussian quadrature formulas, which are valid for up to a 7th order polynomial at the points which are distant from the welding arc. The region close to the welding arc experiences large temperature gradients. This necessitates use of a 7 point Gaussian quadrature formula in this area. This is valid for up to a 13th order polynomial.
Many iterations are required to carry out the integration, so computer run times are fairly long depending on the accuracy. However, this program does not require a large computer memory.

Thermal conductivity and diffusivity are $0.0005891 \text{ Btu/ln-sec-F}$, and $0.01853 \text{ in}^2/\text{sec}$ respectively for the mild steel in the computer model. Figure 9 shows temperature history on the top surface at a point 0.5 inch away from the weld and with a heat input rate of $1.71 \text{ Btu/sec}$ and different thermal property values. The differences in peak temperature were less than a hundred degrees F. Even though there is a time shift of a few seconds among the peak values of the curves, the assumptions of constant thermal properties are still reasonable.
Figure 9: Comparison of a Welding Temperature History for a Tubular Structure with Different Thermal Properties
3.1 INTRODUCTION

Due to local heating by the welding arc, complex thermal stresses are produced in regions near the welding arc. Figure 10 shows schematically the change of temperature and resulting stresses that occur during welding. A bead-on-plate weld is represented along the X axis to illustrate the thermal and residual stresses. The welding arc, which is moving at a speed S, is presently located at the origin O as shown in Figure 10. The figure also shows the temperature distributions and normal stresses along several cross sections.

Along section A-A, which is ahead of the welding arc, the temperature change and the thermal stress is almost nonexistent. Along section B-B, which crosses the welding arc, the temperature change is extremely rapid, the distribution is very uneven, and stresses in regions near the arc are compressive. The expansion of these areas is restrained by
Figure 10: Schematic Diagram for Change in Temperature and Stress during Welding [52]
the surrounding metal which is at a lower temperature. It should be mentioned that stresses in these regions are as high as the yield strength of the material at the corresponding temperature. Inelastic strain is produced since the yielding stress decreases as the temperature increases. Along section C-C, which is some distance behind the welding arc, the temperature change due to welding again diminishes. Because the base metal regions near the weld have cooled, they try to contract causing tensile stresses in the regions close to the weld. After the weld has cooled, high tensile stresses are produced in regions near the weld, while compressive stresses are produced in the regions away from the weld. Also, the inelastic strain will cause some metal movement and produce distortion. Several types of weld distribution are shown in Figure 11.

During welding, complex strains in the weld metal and in the adjacent base metal regions are caused by a non-uniform heating and cooling cycle. This thermal-mechanism will produce internal forces which induce stresses usually accompanied by plastic upsetting in some instances. Because of the difficulty in analyzing plastic deformation, especially at elevated temperatures, mathematical analyses were limited to very simple cases such as spot welding.

In the previous chapter, the thermal analysis was
Figure 11: Various Types of Weld Distortions [52]
conducted and temperature distribution on the square tube was calculated during and after welding. In this chapter, a distortion model will be developed in conjunction with previous thermal model to calculate stress and strain. First, thermal elasticity and elastic-plastic behaviors will be discussed. A simple one dimensional model and yielding criteria will be presented. Then, the distortion model will be derived. Finally, a complete computer program, which combines thermal and distortion analyses will be discussed.

3.2 BACKGROUND

Broadly speaking, thermal stresses may arise in a heated body either because of a non-uniform temperature distribution or external constraints. However, the non-uniform temperature distribution is the most important cause. The formulation employed here rests on three assumptions: 1) the temperature distributions can be determined independently of the deformation of the body, therefore decoupling of the heat conduction equation and material behavior is possible, 2) the deformations are small, so no distinction is needed between the coordinate of a particle before and after deformation, 3) the material behaves elastically at all times if thermal elasticity is assumed. [7].
The stress-strain relationship for thermal elasticity in cartesian coordinates \((X,Y,Z)\) is

\[
\varepsilon_{ij} = \sigma_{ij}/2G - \delta_{ij}(\mu\Theta/E - \alpha T)
\]

i.e.

\[
\varepsilon_x = \sigma_x/2G - \mu\Theta/E + \alpha T
\]
\[
\varepsilon_y = \sigma_y/2G - \mu\Theta/E + \alpha T
\]
\[
\varepsilon_z = \sigma_z/2G - \mu\Theta/E + \alpha T
\]
\[
\varepsilon_{xy} = \tau_{xy}/2G
\]
\[
\varepsilon_{xz} = \tau_{xz}/2G
\]
\[
\varepsilon_{yz} = \sigma_{yz}/2G
\]

Where

\(E\) is the elasticity modulus,
\(\mu\) is the Poisson's ratio,
\(\alpha\) is the linear thermal expansion coefficient,
\(T\) is the temperature,
\(G\) is the shear modulus, i.e.

\[
G = E/2(1+\mu)
\]

\(\Theta\) is the first invariant of the stress tensor, i.e.

\[
\Theta = \Sigma \sigma_{ii} = \sigma_x + \sigma_y + \sigma_z
\]

\(\delta_{ij}\) is Kronecker delta function defined as

\[
\delta_{ij} = \begin{cases} 
0 & i \neq j \\
1 & i = j 
\end{cases}
\]

The relationship between the first invariant of the stress and strain tensors is
\[ \theta = (1 - 2\mu)\Theta/E + 3\alpha T \]  

where \( \theta \) is first invariant of the strain tensor, i.e.

\[ \theta = \sum \varepsilon_{ii} = \varepsilon_x + \varepsilon_y + \varepsilon_z \]  

The total strains at each point in a heated body are made up of two parts. The first part comprises the strains required to maintain the continuity of the body as well as those arising because of external forces. The second part is a uniform expansion proportional to the temperature rise in the three principal directions. In other words, the total strain can be expressed as

\[ \varepsilon_{ij} = \varepsilon_{ij}' + \varepsilon_{ij}'' \]  

Where \( \varepsilon_{ij}' \) is the component of the elastic strain which follows Hooke's law and \( \varepsilon_{ij}'' \) is the component of the thermal strain.

\[ \varepsilon_{ij}' = \sigma_{ij}/2G - \delta_{ij}(\mu\Theta/E) \]  

\[ \varepsilon_{ij}'' = \delta_{ij}\alpha T \]  

At high temperatures and for high stress levels, the behavior of the material will be plastic and the thermal elastic model will not be valid [29]. In general, the rate of strain is

\[ \dot{\varepsilon}_{ij} = \varepsilon_{ij}' + \varepsilon_{ij}'' \]
Here superscripts, (e) and (p) refer to elastic and plastic strains, respectively.

If the Prandtl-Reuss relations are valid, then

\[ \varepsilon_{ij} = S_{ij} \, d\Delta \]  

(3.8)

where \( d\Delta \) is a constant and \( S_{ij} \) is the stress deviator tensor i.e.

\[ S_{ij} = \sigma_{ij} - \Theta \delta_{ij}/3 \]  

(3.9)

There are two more assumptions used [29]. First, a loading function exists, therefore at each stage of the plastic deformation, there exists a yield function. Second, the relationship between an infinitesimal stress and plastic strain is linear so that the superposition principle may be applied to the stress and strain increments. The rate of plastic strain is assumed to be

\[ \varepsilon_{ij}^{(p)} = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \]  

(3.10)

Where \( \Lambda \) is a proportionality constant,

\( f \) is a yield function.

The yield function is assumed to be a function of the stresses, plastic strains, strain hardening and temperature; i.e.,
where \( w \) is the parameter related to the strain hardening of the material.

Differentiating Equation (3.11) and using the result with Equation (3.10), the following relationship is obtained [26].

\[
\dot{\varepsilon}_{ij}^{(p)} = \hat{G} \left[ \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}} \sigma_{kl} + \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial T} T \right]
\]

(3.12)

where

\[
\hat{G} = \frac{1}{\frac{\partial f}{\partial \varepsilon_{ij}}^{(p)} + \frac{\partial f}{\partial \omega \varepsilon_{ij}}^{(p)} + \frac{\partial f}{\partial \sigma_{ij}}}.
\]

If the von Mises' yield criterion is adopted, then the temperature dependency can be considered

\[
f = \bar{\sigma} - c(\varepsilon_{ij}^{(p)}, T)
\]

(3.13)

where \( c \) is the parameter related to the strain hardening of the material, and \( \bar{\sigma} \) is the equivalent stress defined as

\[
\bar{\sigma} = \sqrt[3]{2} S_{ij}^{\frac{1}{2}} s_{ij}
\]

(3.14)
Using Equation (3.13), Equation (3.12) becomes

$$
\dot{G} = - \frac{1}{\dot{\sigma} \cdot \sigma} \left[ \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{ij}} \right] \tag{3.15}
$$

If we define the rate of equivalent strain as

$$
\dot{\varepsilon}(p) = \sqrt{\frac{2}{3}} \sqrt{\varepsilon_{ij} \cdot \varepsilon_{ij}} \tag{3.16}
$$

and then, using Equation (3.10) and (3.14), the result is

$$
1 = \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial \varepsilon(p)}{\partial \sigma_{ij}} \frac{\partial \varepsilon(p)}{\partial \varepsilon(p)} \tag{3.17}
$$

so the Equation (3.15) becomes

$$
\dot{G} = - \frac{1}{\dot{\varepsilon}(p) \cdot \dot{\varepsilon}(p)} \left[ \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial \varepsilon(p)}{\partial \sigma_{ij}} \right] \tag{3.18}
$$

$$
H' = - \frac{\partial f}{\partial \varepsilon(p)}
$$

Finally, using Equation (3.13), (3.14), and (3.18), the rate of plastic strain becomes
Equation (3.19) shows that if the temperature dependency of the yield function is taken into account in the form of Equation (3.13), the effect of temperature is given as the additional term in the well-known relationship between strain and stress rates.

For the elastic part, we have:

\[
\dot{\varepsilon}_{ij} = \frac{1}{H^2} \left\{ \frac{3S_{ij}}{2G} \dot{\sigma} + \frac{3S_{ij}}{2G} \frac{\partial f}{\partial T} T \right\} \quad (3.19)
\]

or

\[
\dot{\varepsilon}_{ij} = \frac{1}{H^2} \left\{ \frac{3S_{ij}}{2G} \frac{3S_{kl}}{2G} \dot{\sigma}_{kl} + \frac{3S_{ij}}{2G} \frac{\partial f}{\partial T} T \right\} \quad (3.20)
\]

\[
\dot{\varepsilon}_{ij} = \frac{1-2\nu}{E} \dot{\sigma}_{ij} + \frac{S_{ij}}{2G} - \frac{1-2\nu}{E^2} \dot{E} \sigma_{ij}
\]

\[
- \frac{1}{2G^2} \dot{G} S_{ij} + \dot{\varepsilon}^\theta_{ij}
\]

where \( \sigma \) and \( \varepsilon \) are the average hydrostatic stress and strain respectively, and \( \varepsilon^\theta \) denotes the thermal strain caused by the temperature distribution.

Substituting Equation (3.19) and (3.21) into Equation (3.7), the total strain rate becomes [35]
The inverse relation of Equation (3.22) becomes

\[ \dot{\sigma}_{ij} = \frac{E}{1-2v} \dot{\varepsilon}_{ij} + 2G \dot{\varepsilon}_{ij} - \frac{3G \delta_{ij} S_{kl} E_{kl}}{e^2 (H' + 1)} + \dot{\sigma}^\theta_{ij} \quad (3.23) \]

where \( \dot{\sigma}^\theta_{ij} \) consists of the terms related to the rate of thermal strain and the temperature dependency of the material properties and the yield criterion.

\[ \dot{\sigma}^\theta_{ij} = -\frac{E}{1-2v} \dot{\varepsilon}_{ij} + \frac{G}{E} \dot{\varepsilon}_{ij} + \frac{\sigma_{ij}}{G} \left\{ 1 - \frac{e}{H' (3G + 1)} \right\} \dot{\gamma} \quad (3.24) \]

\[ -\frac{3G S_{ij} \frac{df}{dT}}{\sigma H'} \left\{ 1 - \frac{e}{H' (3G + 1)} \right\} \dot{T} \]

For a strain hardening material, if an element is in the plastic region, the element takes one of the following states at the next time increment:
\[ f = 0 \quad (\text{unloading}) \]  
\[ f = 0 \quad (\text{neutral}) \]  
\[ f = 0 \quad (\text{loading}) \]

where

\[ f' = \frac{\partial f}{\partial \sigma_{ij}} \sigma_{ij} + \frac{\partial f}{\partial T} \dot{T} \]  

If the yield function does not move and does not change in size, loading in the element does not occur. This means that the unloading or neutral state is possible.

### 3.3 MATERIAL PROPERTIES

In thermal stress and distortion analyses, the temperature is elevated from room temperature to the melting temperature, then cooled back to the room temperature again. The material properties are not constants but vary with respect to temperature. Figure 12 shows two of the material properties for mild steel [31].

If the thermal properties of the material are assumed temperature dependent in the heat flow analysis, the heat conduction equation becomes nonlinear. To avoid the mathematical complexity involved in solving nonlinear
Figure 12: Mechanical Properties of Mild Steel [31]

Elasticity Modulus (x1000 Ksi)

Yielding Stress (Ksi)
equations, almost all of the previous analyses of heat flow during welding (even those that have employed computers), have used constant properties. Masubuchi used both constant thermal properties and iteration techniques when varying thermal properties were assigned, depending on the temperature of the locations being studied [28].

However, the change of mechanical properties with temperature is much greater than that of thermal properties. The yielding stress and elastic modulus are strongly dependent on temperature, such that they can not be treated as constants. Figure 13 shows the stress strain relationship at different temperatures for mild steel.

3.4 THERMAL DISTORTION MODEL

After the temperature distribution at a given time is obtained, the calculation of the welding distortion is possible. The model assumes that the distortion was caused by two contributions. The first contribution is from inelastic strains which exist due to a non-uniform temperature distribution in the tubes. Second, the shrinkage in the weld nugget zone is due to the contraction of weld metal and its subsequent cooling.

The model of the inelastic strain system divides the flange into many sections as shown in Figure 14, and each
Figure 13: Stress-Strain Curves as a Function of Temperature for Mild Steel
Figure 14: The Model for Welding Inelastic Strain Analysis
section has its own temperature field at any given time. During tube heating, the local temperature of the tube is rising which causes suppressed deformation and therefore thermal stresses. If the temperature is high enough, yielding will occur. Therefore, after cooling, the inelastic strains which cause distortion and residual stresses will still exist. After the summation of all of the inelastic strains in each section, the final results will be obtained. In this model, a one dimensional stress ($\sigma_x$) is considered, namely, that normal to the weld. A narrow band of the ring element was cut from the base member, and the plane of the cross-section of the base member is assumed to remain a plane during the thermal loading process (simple beam theory assumption). Also the thermal loading is assumed monotonic during each time step. The stress-strain relationship can then be written as [30]:

\[
\begin{align*}
\varepsilon_x &= S/H + \tau + \varepsilon_p \\
\varepsilon &= \varepsilon_x - \tau \\
\varepsilon_p &= \Sigma \Delta \varepsilon_p + \Delta \varepsilon_p
\end{align*}
\]

(3.27) \hspace{1cm} (3.28) \hspace{1cm} (3.29)

where $\varepsilon_x \text{ is the dimensionless total strain,}$

$\left(\varepsilon = \varepsilon_x / \varepsilon_o\right)$

$\tau \text{ is the dimensionless thermal strain,}$

$\left(\tau = \alpha T / \varepsilon_o\right)$

$\varepsilon \text{ is the dimensionless engineering strain,}$

$\left(\varepsilon = \varepsilon / \varepsilon_o\right)$
\( \varepsilon_p \) is the dimensionless plastic strain,
\[
(\varepsilon_p = \varepsilon_p / \varepsilon_0)
\]
\( \Sigma \Delta \varepsilon_p \) is the dimensionless sum of previous plastic strain,
\( \Delta \varepsilon_p \) is the dimensionless current plastic strain,
\( S \) is the dimensionless stress,
\[
(S = \sigma_x / \sigma_0)
\]
\( H \) is the dimensionless elastic modulus,
\[
(H = E / E_0)
\]
\( \eta \) is the dimensionless coordinate, starting from the midpoint of the top surface and moving along surface to the midpoint of the bottom surface.
\( \sigma_x \) is the normal stress in the x direction, and
\[
\sigma_0, \varepsilon_0, E_0 \text{ is the yielding stress, yielding strain and elastic modulus, at room temperature.}
\]

Then, the stress \( S \) is a function of \( \eta \) only in each element. By assuming \( \sigma_y = \sigma_z = \tau_{xy} = 0 \) the compatibility equation becomes
\[
\delta^2 \varepsilon_x / \delta \eta^2 = 0
\]
which results in
\[
\varepsilon_x = m + 2(m - n)U(\eta - 3/4)(\eta - 3/4) - U(\eta - 1/4)(\eta - 1/4)\\)
\]
(3.31)
where \( m \) and \( n \) are constants, and \( U(\eta) \) is the step function.

The equilibrium equation in the X and Y directions for the structure are:
\[
\Sigma F_x = P' \quad (3.32)
\]
\[
\Sigma M_y = M' \quad (3.33)
\]
where \( P' \) and \( M' \) are an externally applied force and moment, respectively. For the case where no external forces are present, then \( P' = M' = 0 \).

The expansion of the equilibrium equation for \( F_x \) is

\[
\int_0^1 S \, d\eta = P^* \tag{3.34}
\]

where \( P^* \) is the dimensionless force, and

\[
P^* = P' / 4a^4 \sigma_0 \tag{3.35}
\]

Due to the stress symmetry about the Z axis, the moment with respect to the Z axis is automatically zero. Only one moment component, \( M_y \), will exist and

\[
\int_0^1 S \, g(\eta) \, d\eta = M^* \tag{3.36}
\]

\( M^* \) is the dimensionless moment where

\[
M^* = M' / 8a^4 h^4 \sigma_0 \tag{3.37}
\]

and \( g(\eta) \) is defined as

\[
g(\eta) = \begin{cases} 
1 & 0 \leq \eta < 0.25 \\
2 - 4\eta & 0.25 \leq \eta < 0.75 \\
-1 & 0.75 \leq \eta \leq 1
\end{cases}
\]

Substituting Equation (3.27) into Equations (3.34) and (3.36) and solving for the constants \( m \) and \( n \), yields
\[ e = -\tau + D_1(\tau) + D_2(\Sigma \Delta \varepsilon_p) + D_3(\Delta \varepsilon_p) + D_4(P^*) + D_5(M^*) \quad (3.38) \]

where

\[ D_1(\tau) = B_1 \int_0^1 \tau H \, d\eta - B_2 \int_0^1 \tau H \, g(\eta) \, d\eta \]

\[ D_2(\Sigma \Delta \varepsilon_p) = B_1 \int_0^1 \Sigma \Delta \varepsilon_p \, H \, d\eta - B_2 \int_0^1 \Sigma \Delta \varepsilon_p \, H \, g(\eta) \, d\eta \]

\[ D_3(\Delta \varepsilon_p) = B_1 \int_0^1 \Delta \varepsilon_p \, H \, d\eta - B_2 \int_0^1 \Delta \varepsilon_p \, H \, g(\eta) \, d\eta \]

\[ D_4(P^*) = B_1 P^* \]

\[ D_5(M^*) = B_2 M^* \]

\[ B_1 = \frac{A_4 + 2(A_4 + A_3) \, P(\eta)}{A_1 \, A_4 - A_2 \, A_3} \]

\[ B_2 = \frac{A_2 + 2(A_1 + A_2) \, P(\eta)}{A_1 \, A_4 - A_2 \, A_3} \]

\[ A_1 = \int_0^{.25} H \, d\eta + 1.5 \int_{.25}^{.75} H \, d\eta - 2 \int_{.25}^{.75} H \, \eta \, d\eta \]

\[ A_2 = \int_{.75}^1 H \, d\eta - .5 \int_{.25}^{.75} H \, d\eta + 2 \int_{.25}^{.75} H \, \eta \, d\eta \]

\[ A_3 = \int_{.25}^{.75} H \, d\eta + 3 \int_{.25}^{.75} H \, d\eta - 10 \int_{.25}^{.75} H \, \eta \, d\eta + 8 \int_{.25}^{.75} \eta^2 \, H \, d\eta \]

\[ A_4 = -\int_{.25}^{.75} H \, d\eta + 6 \int_{.25}^{.75} H \, \eta \, d\eta - 8 \int_{.25}^{.75} \eta^2 \, H \, d\eta \]

\[ P(\eta) = U(\eta - .75)(\eta - .75) - U(\eta - .25)(\eta - .25) \]

For the case with constraints, it is assumed that the
centerline of the vertical member is a reference point and is fixed with a rigidity coefficient, $f_r$. Therefore, the displacement of the reference point is zero, and it also carries reaction forces and moments. The rigidity coefficient $f_r$ is an empirical coefficient and must be determined from experiment. The rigidity coefficient represents the rigidity of the joint. If $f_r$ equals 1, then the boundary condition will be the same as a fixed support while $f_r$ equals to 0, represents the simple support condition.

When constraint cases are applied, it is assumed that there is a reaction force $F_r$ acting on the point where a jig or fixture is applied to make the displacement 0. The boundary condition on the constraint point of a fixture is assumed simply supported and therefore carries the reaction force but no moment. However, the distributed moment produced from this effect will be treated as an externally applied moment which satisfies equilibrium equations. The model is shown in Figure 15.

The effect of shrinkage of weld metal was modeled by springs. This is shown in Figure 16. For a weld leg size $L$, the hypotenuse is $1.4 \times L$, and the contraction cooling from the melting temperature $T_m$ through $T$ will be

$$
\Delta = \alpha (T_m - T) \times 1.4 \times L
$$

(3.39)
Welding distortion without constraints.

Applied reaction forces on the restraint points make the displacements at the restraint points zero.

Welding distortion with constraints.

Figure 15: Model of Constraints for Tubular Structures
Figure 16: Model for Weld Metal Shrinkage in a Tee-joint
The shrinkage produces a force $F_s$ directed at an angle of 45 degrees as shown in Figure 16. The magnitude of force $F_s$ will be

$$F_s = K \cdot \Delta$$  \hspace{1cm} (3.40)

where $K$ is the temperature dependent spring constant for weld metal.

If we consider the force component in the $Z$ direction, then Equation (3.40) will be

$$F = \alpha \cdot E \cdot (T_m - T)$$  \hspace{1cm} (3.41)

This force coupled with the rigidity coefficient, will produce the moment. This moment will then add to the moment which was produced by the constraints and thus satisfies the equilibrium equations.

3.5 COMPUTER PROGRAM DEVELOPMENT

A computer program was developed to calculate the thermal distortion of a Tee-joint. The program has the capability of reading the temperature solution as a thermal input from external files or calculating the temperature solution internally if thermal properties are included. After that, the program will calculate some basic constants. Then, it will carry out the thermal distortion analysis section by section for the specific time. If there are
constraints applied, then an iteration method will be used, and the solution must satisfy all boundary conditions before progressing to the next time step. During calculation of the metal movement on each section, the program will check the elastic-plastic status of that section for every point. Figure 17 is a simple flowchart for the welding distortion analysis program.

When the temperature gradient is not large and the thermal stress has not exceeded the yielding stress, the plastic strain is zero, and that the solution is in the elastic range. If the solution is in the elastic range, then the thermal stress and strain can be determined from the temperature distribution using Hook's Law alone.

During a welding heating cycle, the temperature rises very fast in a small area close to the weld. The temperature gradient is steep, and the yield phenomenon will occur with plastic strains being formed. When the stress is beyond yield, the method of successive approximations must be applied. The loading path is divided into a number of increments, and the iteration is as follows:

1. For the first increment of load, \( \varepsilon_p \) and \( \Delta \varepsilon_p \) are assumed zero everywhere.
2. Calculate the engineering strain from the solution at every point.
3. For each value of engineering strain, compute \( \Delta \varepsilon_p \).
Figure 17: Flowchart for Welding Distortion Analysis
Figure 17, continued
from the stress strain curve.

4. After a plastic strain $\Delta e$ is obtained, use Equations (3.27) and the solution to calculate better approximations of engineering strain.

5. Repeat steps 2 to 4 until convergence is reached, and the solution is obtained.

Figure 18 shows a block diagram for computing the plastic strain increment. During the cooling cycle, the thermal unloading process occurs. The material will follow the new stress-strain curve with the same slope as that of the original curve as shown as Figure 19. At this stage, the value of the plastic strain remains constant until reverse yielding occurs.

When calculating the effect of constraints on a Tee-joint, the following iteration method is used:

1. At first, assume a reaction force and recalculate the distortion of the whole structure.

2. Find and check the constraint-point displacement which should be small enough to be within acceptable tolerances.

3. Use linear interpolation to obtain new reaction forces.

4. Repeat step 1 to calculate the distortion of the whole structure until the displacement of the constraint point satisfies the boundary condition.

The program was also designed for graphics presentations. After the results are obtained, the user can employ a graphics terminal to see an animation of the
Figure 18: Block Diagram for Computing Plastic Strains
Figure 19: Typical Dimensionless Stress-Strain Curve of Metallic Material
Figure 20: The Animation of Welding Distortion in a Tee-Joint
distortion as shown in Figure 20. This program was written in FORTRAN and with the Tektronix PLOT10 graphics package.
Chapter 4

FINITE ELEMENT ANALYSIS

4.1 INTRODUCTION

The finite element method is a very popular method used in analyzing engineering problems. It has been applied to structural analysis, heat transfer, and fluid mechanics with very good results. However, the application of finite elements in welding engineering is still relatively rare.

After 1980, there were several publications concerning thermal stresses due to welding, welding heat flow, and strength of welded joints. However, almost all the analyses are two dimensional and can only be used in the quasi-stationary state or are only valid for bead-on-plate butt welds.

There are several commercial finite element programs such as ANSYS, IDEAS, and ABAQUS, but many programs are not suitable for welding analysis applications. Some programs can only be used for structural analyses, and some programs can not perform plastic analyses. However, ANSYS is a general program and was chosen for the analysis of welding.
The ANSYS analysis capabilities include static and dynamic, elastic, plastic creep, swelling, buckling, small and large deflections, steady and transient heat transfer, fluid and current flow [2]. ANSYS is a general purpose program which can be used to calculate temperature distributions in a structure and then convert to the results to a structural type analysis allowing calculation of thermal stresses using nonlinear material properties.

In this chapter, the background for a three dimensional finite element method will be discussed. The background section written is based on the ANSYS Theoretical Manual [3]. In addition, the procedures for modeling of a 1 X 1" square tube will be discussed. Finally, an example will be given to show the results.

4.2 BACKGROUND

For a finite element approach to heat transfer and stress-strain problems, the principle of virtual work is most convenient. The principle of virtual work says that a small virtual change of internal strain energy must be offset by an identical change in external work due to the applied loads or,

\[ \delta U - \delta V = 0 \] (4.1)
where

$\delta U$ is virtual strain energy,

$\delta V$ is virtual external work.

In general, the virtual work principle written in a form for heat transfer is [4]

$$
\int (\nabla T) \cdot K (\nabla T) \, d(vol) + \int h_f (T - T_b) \, d(area) \quad (4.2)
$$

$$
+ \int \rho c_p \frac{\partial T}{\partial t} \, d(vol) - \int \overline{q} \, d(vol) - \Sigma Q_e = 0
$$

where

$K$ is the thermal conductivity,

$T$ is the temperature fields,

$T_b$ is the buck temperature of environment,

$h_f$ is convection coefficient,

$c_p$ is specific heat,

$\rho$ is material density,

$Q_e$ is heat flux acting on the nodes,

$\overline{q}$ is heat generation per volume.

and in the finite element form
\[
\left( \int [B]^T[D][B] \, d(\text{vol}) + h_f \int \{N|_s\}\{N|_s\}^T \, d(\text{area}) \right) \{T_e\} \\
+ \rho c_p \int \{N\}\{N\}^T \, d(\text{vol}) \{\mathbf{i}_e\} = -q \int \{N\} \, d(\text{vol})
\]

(4.3)

\[-h_f T_B \int \{N|_s\} \, d(\text{area}) = \{Q_e\} = 0\]

where

\{N\} is the shape function vector

\{N|_s\} is the shape function vector on the convection surface

[D] is the conductivity matrix

and [B] is the matrix of shape function derivative.

Due to the transient nature of heat transfer problems, a time integration method is used [3]. For the basic diffusion equation

\[
[C] \{\dot{T} \} + [K] \{T\} - \{Q\} = 0
\]

(4.4)

The integration scheme is based on a quadratic iteration. Then

\[
\left( \frac{2\Delta t_0 + \Delta t_1}{\Delta t_0} \cdot \frac{1}{\Delta t_0} \right) \{C\} \{T_n\} = \{Q(t)\} + \\
[C] \left( \frac{\Delta t_0}{\Delta t_0 \Delta t_1} \{T_{n-1}\} - \frac{\Delta t_0}{\Delta t_0 \Delta t_1} \{T_{n-2}\} \right)
\]

(4.5)
where \( \Delta t_0 = t_n - t_{n-1} \)  
\( \Delta t_1 = t_{n-1} - t_{n-2} \)  
\( \Delta t_{01} = t_n - t_{n-2} \)

\( t_n \) is present time  
\( t_{n-1} \) is previous time  
\( t_{n-2} \) is time at second previous time  
\( T_n \) is temperature at this time step (to be calculated)  
\( T_{n-1} \) is temperature at previous time step (known)  
\( T_{n-2} \) is temperature at second-previous time step

After the temperature field has been calculated at any time step, the solution becomes a thermal input to a stress-strain analysis which calculates the thermal stresses and displacements. As for a thermal analysis, the virtual work principle for stress-strain problems is [5]

\[
\int (\delta \varepsilon) \cdot \sigma \, d(vol) - \int R_i \delta u_i \, d(vol) - \int P_i \delta u_i \, d(area) - \Sigma F_e = 0
\]

where \( R_i \) is the body force, \( P_i \) is the prescribed surface force on the boundary surface \( S \), and \( F_e \) is the external force acting on the nodes. The stress is related to the strains by

\[
[\sigma] = [D] \left( [\varepsilon] - [\varepsilon^{th}] \right)
\]
where \( \{ \sigma \} \) is the stress vector, 

\[
[D] \text{ is the material properties matrix (stress-strain matrix),}
\]

\( \{ \epsilon \} \) is the mechanical strain vector, 

\( \{ \epsilon^{th} \} \) is the thermal strain vector.

After solving for the mechanical strain and expanding in three dimensions

\[
\{ \epsilon \} = \{ \epsilon^{th} \} + [D]^{-1} \{ \sigma \}
\] (4.8)

or

\[
\begin{bmatrix}
\epsilon_x \\
\epsilon_y \\
\epsilon_z \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix}
= 
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{xz}
\end{bmatrix}
\begin{bmatrix}
a_{x} dT \\
a_{y} dT \\
a_{z} dT \\
0 \\
0 \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
1 & \nu_{xy} & \nu_{xz} \\
\nu_{yx} & 1 & \nu_{yz} \\
\nu_{zx} & \nu_{zy} & 1
\end{bmatrix}
\begin{bmatrix}
\frac{1}{E_x} & 0 & 0 \\
0 & \frac{1}{E_y} & 0 \\
0 & 0 & \frac{1}{E_z}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\] (4.9)

Therefore, for the finite element form

\[
\int [\mathbf{B}^T [D] [\mathbf{B}] (\mathbf{u}) \, d(\text{vol}) \quad (u)^T - \int [\mathbf{B}^T [D] (\epsilon^{th}) \, d(\text{vol})
\]

\[
- \rho \int [\mathbf{N}^T [\mathbf{N}] \, d(\text{vol}) - \int [\mathbf{N}]^T (\mathbf{P}) \, d(\text{area}) = 0
\] (4.10)
\{ \mathbf{B} \} \text{ is the matrix of shape function derivative (strain-displacement matrix).}

Since the thermal strain is very large on some portions of the structure, the material will go plastic and cause permanent deformations.

Plasticity is both nonlinear and non-conservative. The nonlinear nature of the problem dictates that a number of iterations are required. The non-conservative nature of the problem dictates that the final results of the analysis are path-dependent. That is, even a monotonically increasing load should be applied gradually rather than in one step, as a sharp increase in load might cause an improper distribution of internal forces. Therefore, the following procedure is usually recommended: On the first load step, apply a load level such that the most highly stressed element is about to yield. Then increase the load stepwise by $\Delta P$ and let the program converge to a solution. Then, continue to apply the load in this manner until the final desired load level is reached. This is called initial stress method.

In ANSYS, plasticity is based on the Von Mises yield function and the Prandtl-Reuss flow equations. The material is assumed to be isotropic and the basic technique is the initial stress method. The static equilibrium equation (neglecting unrelated effects) is \([3]\):

\begin{align*}
\end{align*}
\[
[K] \{u\} = \{ F^{\text{nd}} \} + \{ F^{\text{pl}} \}
\]  \hspace{1cm} (4.11)

where 
- \([K]\) is the total stiffness matrix
- \(\{u\}\) is the nodal displacement vector
- \(\{ F^{\text{nd}} \}\) is the applied nodal load vector
- \(\{ F^{\text{pl}} \} = \sum_{m=1}^{N} \{ F_{m}^{\text{pl}} \}\) is the plastic strain element load vector

\[
\{ F_{m}^{\text{pl}} \} = \int_{\text{vol}} [B]^T [D] \{ \varepsilon_{m}^{\text{pl}} \} \text{d(vol)}
\]

where 
- \(N\) is the number of elements
- \(\{ \varepsilon_{m}^{\text{pl}} \}\) is the plastic strain vector computed in the previous iteration

The overall flow chart is given in Figure 21. For numerically integrated elements, the plasticity calculation are done at each of the integration points.

Strains are used throughout the entire development with the only exception being the equivalent stress. Equivalent stress and equivalent strain define the current location on the stress-strain curve.

The Prandtl-Reuss equations for three dimensions are:

\[
\Delta \varepsilon_{x}^{\text{pl}} = \frac{\Delta \varepsilon_{x}^{\text{pl}}}{3 \varepsilon_{t}} (2 \varepsilon_{x}^{c} - \varepsilon_{y}^{c} - \varepsilon_{z}^{c})
\]

\[
\Delta \varepsilon_{y}^{\text{pl}} = \frac{\Delta \varepsilon_{y}^{\text{pl}}}{3 \varepsilon_{t}} (2 \varepsilon_{y}^{c} - \varepsilon_{z}^{c} - \varepsilon_{x}^{c})
\]

\[
\Delta \varepsilon_{z}^{\text{pl}} = - \Delta \varepsilon_{x}^{\text{pl}} - \Delta \varepsilon_{y}^{\text{pl}}
\]
Start

Initialization
Set all strains and stresses to zero. Set $v^e$ to $v$.

Generate stiffness matrix

Re-use Matrix?

YES
Compute and print stresses based on $e^e$ if desired.

NO

Element Stress Pass

Compute $e^t$ (modified total strains) from nodal displacements and then subtract thermal and previous nonlinear material effects (equation 4.1.9).

NO
Solve for displacements

Plasticity Calculations

Compute $e^e$ and $\sigma^e$.
Compute new values of $\varepsilon_{pl}$, $\varepsilon$, $v$, etc. based on their old values and the current values of $e^t$.

Finished iterations?

YES

Finished load steps?

YES

NO

F inish o"r

Figure 21: Procedures for Plasticity Calculation [3]
\[
\Delta \varepsilon_{xy}^p = \frac{\Delta \varepsilon_{x y}^p}{\varepsilon_{e t}} \varepsilon_{xy}^e \\
\Delta \varepsilon_{yz}^p = \frac{\Delta \varepsilon_{y z}^p}{\varepsilon_{e t}} \varepsilon_{yz}^e \\
\Delta \varepsilon_{zx}^p = \frac{\Delta \varepsilon_{z x}^p}{\varepsilon_{e t}} \varepsilon_{zx}^e
\]

which \( \Delta \varepsilon_{ij}^p \) are new plastic strain increments, \( \Delta \varepsilon_{ij}^p \) is the equivalent plastic strain, \( \varepsilon_{ij}^e \) are the engineering strains, and \( \varepsilon_{e t} \) is the equivalent engineering strain defined by the equation

\[
\varepsilon_{e t} = \frac{e^{ij}}{3} \left[ (\varepsilon_x^e - \varepsilon_y^e)^2 + (\varepsilon_y^e - \varepsilon_z^e)^2 + (\varepsilon_z^e - \varepsilon_x^e)^2 + 6(\varepsilon_{xy}^e)^2 + 6(\varepsilon_{yz}^e)^2 + 6(\varepsilon_{zx}^e)^2 \right]^{1/2}
\]

(4.13)

The change in plastic strain is computed by Equation (4.12):

\[
\Delta \varepsilon^p = \frac{\varepsilon_{et} - \frac{2(1+\nu)}{3E} \varepsilon_{n-1}}{1 + \frac{2(1+\nu)}{3E} \left( \frac{d\sigma_e}{d\varepsilon_e} \right)_{n-1}}
\]

(4.14)

where
\( \sigma_{e,n-1} \) is the equivalent stress at previous iteration.

\( \frac{d \sigma_e}{d \varepsilon^{pl}} \) is the rate of change of equivalent stress with respect to the plastic part of the equivalent strain.

Strain quantities with no iteration subscript are at the current iteration (n) for the rest of this section. When \( \Delta \varepsilon^{pl} \) is less than or equal to zero, it is assumed that no plastic straining is occurring, and a transfer is made to the unloading part of the logic.

4.3 FINITE ELEMENT MODEL

The finite element model of a tee-joint is shown in Figures 22 and 23. The isoparametric thermal solid element for thermal analysis and the three dimensional isoparametric solid element for stress-strain analysis are used. This requires 8 nodes for each element. Figure 24 shows the symbolic diagram and shape function for these two types of elements.

The model consists of 552 nodes and 264 elements. Each node has one degree of freedom (temperature) for thermal elements and 3 degree degrees of freedom (displacements in x, y, and z directions) for structural elements.

During the thermal analysis, the thermal properties
For other layers, the node number starts \((n-1)\times 24+1\) and ends \(n\times 24\) with same order as first layer.

For other layers, the element number starts \((n-1)\times 12+1\) and ends \(n\times 12\) with same order as first layer.

where \(n\) is layer number.

Figure 22: Finite Element Model: Front View
The vertical member is drawn for display purpose only.

Figure 23: Finite Element Model: Oblique View
\[
T = \frac{1}{8} (T_I (1-s)(1-t)(1-r) + T_J (1+s)(1-t)(1-r) + T_K (1+s)(1+t)(1-r) + T_L (1-s)(1+t)(1-r) + T_M (1-s)(1-t)(1+r) + T_N (1+s)(1-t)(1+r) + T_O (1+s)(1+t)(1+r) + T_P (1-s)(1+t)(1+r))
\]

**Figure 24: Three-Dimensional Solid Element** [2]
such as conductivity and specific heat are assumed constants. In addition to this, it is assumed that a small amount of heat convection occurs on the whole surface of the structure. The welding arc was simulated by a moving heat flux which was deposited on the nodes of the elements on one side of tubular structures.

After the temperature field solution at any given time step was obtained, conversion is needed from a thermal type of element to a structural type of element. These two models look alike, and ANSYS directly converts the temperature solution to applied forces for a stress-strain analysis.

The temperature solution in general is not monotonic, but for a very small time step, it can be treated as a monotonic loading and the Prandtl-Reuss principle is valid. During the stress-strain analysis, the variations in material properties such as the elastic modulus and the yielding stress are assumed nonlinear. The values of these properties are chosen as the data for the simple analytical model. As a boundary condition, the middle point of the bottom surface at z=0, i.e. node 276, is considered fixed. Also it is assumed that the points on the top surface at z=0 are constrained in the axial direction (z direction) to prevent the whole structure from rotating.

Although the model has been established, a loading
table must be generated to tell the computer the time step and the maximum number of iterations required in each calculation.

ANSYS also provides a post analysis capability. POST1 is a general postprocessor which can plot results such as distortion, stress contours, temperature contours, etc. POST26 is a postprocessor which can handle transient results which consider time as a coordinate.

4.4 INPUT AND OUTPUT

As an example, it was assumed that only one weld was made. The welding current and voltage are 180 amperes and 23.5 volts, and it was also assumed that the welding efficiency is 0.6. The convection coefficient is 0.000004 Btu/sec-in$^2$-F. The welding speed is 12 ipm (5 seconds for 1 inch of weld). Therefore, the heat flux is applied for one second each from nodes 220 to 224 to simulate a moving welding arc.

Figures 25 and 26 show the temperature history for different points. Figure 26 shows three points. One is under the welding arc (heat flux), here the temperature rises very fast, reaches 8000°F, and then cools very fast. The reason for this is because the latent heat and liquid phase is not considered in this analysis due to the
Figure 25: Temperature History at x=0 Inch
Figure 26: Temperature History at Points on Top Surface
simplified model. The other point is 1 inch from the weldment and the peak temperature reaches only about 700° F. The third point is 5 inches away the weldment, and the thermal disturbance is negligible. Figure 25 shows two points which are at the same distance from the weldment, but one is on the top surface while the other is on the bottom.

Figure 27 shows the results for the distortion history which was measured at different points. This result was taken as an average of points which are located on two sides at the same distance from the reference point. Figure 28 shows the stress history. It shows that the magnitudes of $\sigma_x$ and $\sigma_y$ are very small as compared to $\sigma_z$ after welding. In fact, it showed that the simple analytical model which considers only one stress component $\sigma_z$ is valid if only the angular distortions are of interest. Figures 29, 30, 31 and 32 show the metal movement and stress contours during welding. Figures 33 and 34 show the final distortion and residual stress contour for the z direction.
Figure 27: Distortion History for Different Locations
Figure 28: Stress History on Element # 77, (0.25 Inch from Weld)
Figure 29: Distortion for a Tubular Joint at Time 3.5 Sec
Figure 30: Stress Contour $\sigma_x$ for a Tubular Joint at Time 3.5 Sec
Figure 31: Stress Contour $\sigma_y$ for a Tubular Joint at Time 3.5 Sec
Figure 32: Stress Contour $\sigma_z$ for a Tubular Joint at Time 3.5 Sec
Figure 33: Final Distortion for a Tubular Joint
Figure 34: Residual Stress Contour $\sigma_z$ for a Tubular Joint
5.1 INTRODUCTION

Due to the assumptions involved in the analytical model and the finite element model, the results of the computations will be approximate. Therefore, using experimental data as a mechanical tool to check and verify the models is very important.

In this study, the Hobart Motoman welding robot which is electrically powered and microprocessor controlled was used to conduct the experiments. The welding process used was the GMAW (Gas Metal-Arc Welding) process with C-25 shielding gas. The fixture of this experiment was designed to measure the angular distortion of a tee joint. In addition, the component which directly read the displacement was an LVDT. These LVDTs were connected to a data acquisition unit and a personal computer.

In this chapter, the set up of the equipment for experiment will be presented. The experiment was conducted by Loyyd. The more detail information can be obtained from
his M.S. Thesis [6]. Also the comparisons between experimental data and results from the analytical and finite element models will be discussed.

5.2 EQUIPMENT SET-UP AND RESULTS

The first step towards conducting the experiment was to build a fixture which was sufficiently rigid to securely hold the parts being welded. The fixtures of this experiment were designed to be easily moved and fixed on a slide so that they could be used for various constraint distances [6].

LVDT’s (Linear Variable Differential Transformers) were chosen to be the measurement devices since the accuracy and speed necessary are obtainable with these transducers. A total of five transducers mounted on a slide, and these were used to measure the displacements at different points. This is shown in Figure 35. Transducer number 3, always fixed to the centerline of the vertical member, was treated as a reference point.

The data acquisition unit employed was the Hewlett Packard 3497A data logger. This unit is capable of scanning all five transducers in less than one tenth of a second. This system was connected to a HP 85A scientific computer which served as a controller.
Figure 35: Configuration for Experimental Set Up
The welding for these experiments was performed using the GMAW process, a Hobart power supply, and a Hobart welding torch. This is shown in Figure 36. The wire used was 0.035 E70S-1 welding wire. The shielding gas used was C-25, which is a mixture of 75 percent argon and 25 percent carbon dioxide. A complete list of welding parameters, including the equipment used, is given in Table 2.

The data taken directly from the distortion measuring system is shown in Figure 37. However, these raw data needed to be transformed by subtracting the displacement of the reference point (point 3 in Figure 35). Figure 38 shows the results obtained by averaging the displacements on both sides of the weldment. Distortions were measured at points 1.65 inches away from the reference point, and welding currents of 130 A and 98 A were used. Welding voltage of 23.0 and 19.8 Volts, respectively, were used. There was no constraint. The tests were done twice for each welding condition, and the results indicated very good repeatability.
S-axis: Rotation
L-axis: Lower Arm
U-axis: Upper Arm
T-axis: Wrist Turning
B-axis: Wrist Bending

Figure 36: Welding Robot (Hobat Automan)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welding Robot</td>
<td>Hobart Motoman L-10 5-axis Robot</td>
</tr>
<tr>
<td>Power Supply</td>
<td>Hobart Mega-Mig 450 RV</td>
</tr>
<tr>
<td>Robot Controller</td>
<td>Hobart Mega-Con 114-X</td>
</tr>
<tr>
<td>Welding torch</td>
<td>Hobart welding torch which was provided especially for the Hobart welding robot</td>
</tr>
<tr>
<td>Welding wire</td>
<td>AWS classification E70S-1</td>
</tr>
<tr>
<td>Wire feed rates</td>
<td>114 to 295 inches per minute</td>
</tr>
<tr>
<td>Welding currents</td>
<td>70 to 140 amps</td>
</tr>
<tr>
<td>Arc voltages</td>
<td>19 to 23.1 volts</td>
</tr>
<tr>
<td>Shielding gas</td>
<td>C-25 (75% Ar / 25% CO2)</td>
</tr>
<tr>
<td>Wire diameter</td>
<td>.035 inches</td>
</tr>
<tr>
<td>Travel speeds</td>
<td>18.2 to 64.3 cm per minute</td>
</tr>
<tr>
<td>Base metal</td>
<td>ASTM 1010 cold-drawn, high-frequency welded tubing</td>
</tr>
</tbody>
</table>

Table 2: Welding Parameters Used for Welding Robot [6]
Figure 37: Distortion Data Taken Directly from Measuring System
Figure 38: Distortion History from Experimental Weld Test
5.3 COMPARISON OF RESULTS

An analytical model or finite element model is not useful if there is no verification of the model's accuracy. In this research, the experimental work will serve as a verification tool to validate the models.

Figure 39 shows a comparison between thermal contours for surfaces which reached 400°F during and after welding. A thermal paint was brushed on the surface of the tubes. This thermal paint melts if the temperature exceeds a specific temperature. A weld was made on one side of the weldment with 130 A, 23 V and 12 ipm. The finite element model and analytical model were constructed for the same welding conditions. The figure shows that the thermal contours from these three sources are in agreement. The only exception is for the tube side. The thermal contours from experiment were larger than any others here. The reason for this is because during welding initiation or extinction, the arc was on the corner between the top and side surfaces, and there was some heat applied directly to the side surface.

Figure 40 shows a comparison of the distortion measured at 1.65 inches away from the reference point for the same welding conditions. The results show that the final values and the peak values of the three models are matched very
Figure 39: Comparison of Thermal Contour for Three Models at 400 F
Figure 40: Comparison of Distortion in One Weld of a Tee-Joint from FEM, Analytical Model, and Experiment
well. The difference is that the peak time for the experimental result is shifted to the right slightly. This means the reaction time for metal movement in the experiment is slower than that predicted by the analyses.

Figure 41 shows a comparison for the thermal stress which is parallel to the tube axis. The element was chosen on the center of the top surface and was 1.5 inches away from the reference point. The results are all small positive values at the final state. This means the residual stress at this point is negligible. The results show a comparison of FEM and analytical solutions and indicate that they are in agreement. Figure 42 shows the thermal stress histories which were calculated at the point 0.75 inch away from reference point. The results show a very high tensile residual stress, (close to the yield stress), occurring at this point and making the joint weaker than its base metal.

Figure 43 compares the final distortion for these three models. Because the joint has one weld, the distortion on the two sides of the reference point is different. It should be mentioned that the FEM solution did not consider the contraction of weld metal, so that the results are very different from the other models. However, if the average distortion is considered, the results are in agreement.

There was another case studied which used the same
Figure 41: Comparison of Stress Histories in Z Direction at z=1.5 Inch

Calculated at 1.5 inches away from reference point
Midpoint of top surface

E = 23 V
I = 130 A
S = 12 ipm
Figure 42: Comparison of Stress Histories in Z direction at x=0.75 Inch

Calculated at 0.75 inches away from reference point
Midpoint of top surface

E = 23 V
I = 130 A
S = 12 lpm

Analytical Result
FEM Result
Figure 43: Comparison of Final Distortion for One Weld Joint from FEM, Analytical Model, and Experiment

E = 23 V
I = 130 A
S = 12 lpm
welding conditions but made two welds with a 1 second lapse time between welds. Figure 44 shows the distortion history measured or calculated at a distance of 1.65 inches away from the reference point on two sides. Angle 1 was defined as the distortion at the first weld side, and angle 2 was at the other side. It was found that the results were fairly close for each single displacement. The FEM solution was not included in the figure because the weld metal contraction was not considered in the model.

The last cases which were conducted for comparison were for the constraint cases. Figure 45 shows the results for welding condition of 23 V, 130 A and 12 ipm and applied constraints on both sides 5.5 inches away from the reference point. The constraints were released after steady state conditions were reached. The data taken from the experimental measuring system were observed to have a cyclic behavior with frequency between 0.8 to 2.5 cycles per minute. This observation was made only in the cases having applied constraints [6]. The reason for this is not clear. Therefore, a continuous distortion history from the experiment was not possible, and the final distortion was defined as an average value of the last 5 data points. The results indicated the comparison between experimental data and analytical solutions was not very good although still
Figure 44: Comparison of Distortion in Joint with Two Welds Based on Analytical and Experimental Model
Figure 45: Comparison of Distortion for Constraint Case 1

- Experimental Result
- Analytical Result

Points measured

- Measured at 1.65 inches away from reference point
- Constrained at 5.5 inches away from reference point
- Constraints released after cooled down

Parameters:
- E = 23 V
- I = 130 A
- S = 12 lpm

Time (sec)

- After constraints released
Figure 46: Comparison of Distortion for Constraint Case 2
reasonable. Figure 46 shows another case with different welding conditions, and similar results were obtained.

There are several sources of error in the analytical model. The first source is the original data errors. Constant thermal properties were used in the thermal analysis. This caused some errors in the temperature distributions for the structure. Later on, these temperature distributions were used as a thermal loading to the stress-strain analysis which introduced some error in the distortion calculations. The mechanical properties such as yield stress and elastic modulus were modeled as bilinear functions of temperature. The use of bilinear distributions will improve the results but some errors are still introduced because accurate temperature dependent mechanical data are very hard to obtain. The second source of error is computational errors. In the computer program, only a few iterations and a generous convergence criteria were used. During the calculation of plastic strain, the convergence criteria was chosen as 0.001 times the current plastic strain. This means the relative accuracy of the plastic strain will be within a thousandth of the current plastic strain. For the constrained cases, an iteration method also was used to calculate the reaction forces on the constraint points and to make the displacements at the constraint
points approach 0. The displacement convergence criteria was chosen as $10^{-6}$ inch. In addition, the calculation of thermal strains also involved several numerical integrations. Simpson's rule was used to calculate Equation (3.38). This rule will have an global error of order $O(h^4)$ where $h$ is the interval between two integration points. Although it is difficult to quantify the exact error associated with each source, in general the error caused by material properties is probably the most significant. This is based on the results of several trials when testing the computer program.

Overall, for a free Tee-joint, the results matched very well. The analytical model will produce less than a 10 percent error in final distortion. However, if the constrained cases are considered, the results are not as accurate as those of the free joint. The analytical model gave up to 50 percent error for some cases, and it always underestimated the distortion in the final stage. The probable reason for this is the reference point not being 100 percent fixed. This yields a rigidity coefficient value of less than 1.0.

With these comparisons, the analytical model was considered verified. Although results for the constraint cases were not as good as those for free joints, the model
is accurate enough in most cases to predict the distortion of a tee joint in a square tubular structure.
Chapter 6
PARAMETRIC STUDY

6.1 INTRODUCTION

A simple analytical model for tubular tee-joints was developed and verified by the finite element results and experiments as discussed in previous chapters. Also a computer program was written to carry out the numerical calculations.

Welding distortion is a serious problem in the manufacture industry. Due to highly non-uniform temperature distribution in welded members, welding distortion is impossible to avoid. Some engineers correct welding distortion problems by post welding processes such as postheating, and stretching; however, the best way to control distortion is in the design stage before making welds. Then, the correction process can be omitted, saving time and money. Therefore, the effects of each welding parameter and constraint parameter should be evaluated, and a design guideline for reducing welding distortion constructed.
In this chapter, a parametric study of some important factors which cause welding distortion is described. Then a design guideline for reducing welding distortion in a tee-joint are discussed.

6.2 PARAMETER SELECTION

In a welding process, the welding heat input always plays an important role. For arc welding, the heat input is the product of welding current and welding voltage. The heat input is also a parameter used in controlling the size of a weldment and the strength of welded joint. Therefore, the heat input range chosen is the one necessary to ensure adequate joint strength.

The welding speed will affect the temperature distribution in welded members during and after welding. However, the welding speed usually is not considered an independent parameter because it will affect the heat input per length of weldment. In fact, another parameter, namely heat intensity, is used more often than traveling speed itself. The restrictions on welding speed are dictated by maintaining a stable arc and by the mobility of humans or robots. In addition, a tee-joint has two welds and the breaking time between each weld is also a parameter.

Jigging conditions are another set of parameters which
are used to fix structural members temporarily during the welding process. It would be interesting to investigate the distortion effect for different constraint positions. Eventually the constraints have to be removed after welding, but the optimum time to remove the constraints is also an interesting subject. Therefore, constraint time and constraint position are also parameters to be investigated.

6.3 RESULTS

Twenty one arc welding simulations of Tee-joints were made. The parameters used for these simulations are listed in Table 3. The welding conditions were simulated by making 2 fillet welds on a Tee-joint made of 1 x 1" square tubes with thickness 0.125". The physical properties of mild steel are found in Appendix B. The arc efficiency was assumed to be 0.6 and rigidity coefficient $f_r$ to be 1 (clamped condition).

The test H1 is a standard test for the other simulations. Others involved varying one parameter at a time with respect to the standard test H1 depending on which parameters were of interest. The results from all simulations are listed in Table 4.
<table>
<thead>
<tr>
<th>TEST NO.</th>
<th>WELDING VOLTAGE (V)</th>
<th>WELDING CURRENT (A)</th>
<th>WELDING SPEED (IPM)</th>
<th>BREAK TIME (SEC)</th>
<th>CONSTRAINT TIME (SEC)</th>
<th>CONSTRAINT LENGTH (IN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>1.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H2</td>
<td>19.5</td>
<td>85.</td>
<td>12.</td>
<td>1.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H3</td>
<td>25.</td>
<td>150.</td>
<td>12.</td>
<td>1.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H4</td>
<td>15.8</td>
<td>82.</td>
<td>12.</td>
<td>1.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H5</td>
<td>20.</td>
<td>178.75</td>
<td>20.</td>
<td>1.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H6</td>
<td>20.</td>
<td>107.25</td>
<td>8.57</td>
<td>1.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H7</td>
<td>35.</td>
<td>429.28</td>
<td>60.</td>
<td>1.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H8</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>1.</td>
<td>240.</td>
<td>1.6</td>
</tr>
<tr>
<td>H9</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>1.</td>
<td>240.</td>
<td>2.5</td>
</tr>
<tr>
<td>H10</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>1.</td>
<td>240.</td>
<td>5.5</td>
</tr>
<tr>
<td>H11</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>1.</td>
<td>12.</td>
<td>5.5</td>
</tr>
<tr>
<td>H12</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>1.</td>
<td>30.</td>
<td>5.5</td>
</tr>
<tr>
<td>H13</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>1.</td>
<td>100.</td>
<td>5.5</td>
</tr>
<tr>
<td>H14</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>3.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H15</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>7.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H16</td>
<td>23.</td>
<td>130.</td>
<td>12.</td>
<td>0.</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>H17</td>
<td>19.5</td>
<td>85.</td>
<td>12.</td>
<td>1.</td>
<td>240.</td>
<td>1.6</td>
</tr>
<tr>
<td>H18</td>
<td>19.5</td>
<td>85.</td>
<td>12.</td>
<td>1.</td>
<td>240.</td>
<td>5.5</td>
</tr>
<tr>
<td>H19</td>
<td>19.5</td>
<td>85.</td>
<td>12.</td>
<td>1.</td>
<td>12.</td>
<td>5.5</td>
</tr>
<tr>
<td>H20</td>
<td>19.5</td>
<td>85.</td>
<td>12.</td>
<td>1.</td>
<td>100.</td>
<td>5.5</td>
</tr>
<tr>
<td>H21</td>
<td>20.</td>
<td>107.25</td>
<td>8.57</td>
<td>1.</td>
<td>16.</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Table 3: Parameters of Welding Simulations
<table>
<thead>
<tr>
<th>TEST NO.</th>
<th>DISPLACEMENT 1 (in)</th>
<th>DISPLACEMENT 2 (in)</th>
<th>AVERAGE DISPLACEMENT (in)</th>
<th>AVERAGE ANGLE (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>0.1144</td>
<td>0.1083</td>
<td>0.1114</td>
<td>1.1598</td>
</tr>
<tr>
<td>H2</td>
<td>0.01082</td>
<td>-0.006211</td>
<td>0.002305</td>
<td>0.0240</td>
</tr>
<tr>
<td>H3</td>
<td>0.2136</td>
<td>0.3363</td>
<td>0.2750</td>
<td>2.8619</td>
</tr>
<tr>
<td>H4</td>
<td>0.009028</td>
<td>-0.006957</td>
<td>0.001036</td>
<td>0.0108</td>
</tr>
<tr>
<td>H5</td>
<td>0.1100</td>
<td>0.3555</td>
<td>0.2328</td>
<td>2.1232</td>
</tr>
<tr>
<td>H6</td>
<td>0.01773</td>
<td>0.008122</td>
<td>0.01293</td>
<td>0.1347</td>
</tr>
<tr>
<td>H7</td>
<td>0.09860</td>
<td>0.3134</td>
<td>0.2060</td>
<td>2.1450</td>
</tr>
<tr>
<td>H8</td>
<td>0.007921</td>
<td>0.01129</td>
<td>0.009606</td>
<td>0.1001</td>
</tr>
<tr>
<td>H9</td>
<td>0.006968</td>
<td>0.01061</td>
<td>0.008789</td>
<td>0.0916</td>
</tr>
<tr>
<td>H10</td>
<td>0.005264</td>
<td>0.007792</td>
<td>0.006528</td>
<td>0.0680</td>
</tr>
<tr>
<td>H11</td>
<td>0.005183</td>
<td>0.007282</td>
<td>0.006233</td>
<td>0.0649</td>
</tr>
<tr>
<td>H12</td>
<td>0.005172</td>
<td>0.007406</td>
<td>0.006289</td>
<td>0.0656</td>
</tr>
<tr>
<td>H13</td>
<td>0.005211</td>
<td>0.007636</td>
<td>0.006424</td>
<td>0.0669</td>
</tr>
<tr>
<td>H14</td>
<td>0.1192</td>
<td>0.1592</td>
<td>0.1392</td>
<td>1.4367</td>
</tr>
<tr>
<td>H15</td>
<td>0.1422</td>
<td>0.1624</td>
<td>0.1523</td>
<td>1.5718</td>
</tr>
<tr>
<td>H16</td>
<td>0.1132</td>
<td>0.7251</td>
<td>0.09285</td>
<td>0.987</td>
</tr>
<tr>
<td>H17</td>
<td>0.002339</td>
<td>0.005185</td>
<td>0.003762</td>
<td>0.0388</td>
</tr>
<tr>
<td>H18</td>
<td>0.001030</td>
<td>0.001517</td>
<td>0.001274</td>
<td>0.0133</td>
</tr>
<tr>
<td>H19</td>
<td>0.001024</td>
<td>0.001435</td>
<td>0.001230</td>
<td>0.0128</td>
</tr>
<tr>
<td>H20</td>
<td>0.001024</td>
<td>0.001517</td>
<td>0.001271</td>
<td>0.0131</td>
</tr>
<tr>
<td>H21</td>
<td>0.002656</td>
<td>0.004055</td>
<td>0.003356</td>
<td>0.0350</td>
</tr>
</tbody>
</table>

The displacements were calculated at 5.55 inches away from reference point.

Table 4: Results of Parametric Study
6.4 ANALYSIS AND DISCUSSION

As expected, the angular distortion increased with increasing heat input rate. Figure 47 shows the angular distortion of angle 1, angle 2, and the average angle for the point calculated at 5.5 inches away from the reference point. With low heat input such as 1,000 to 2,000 watts, the distortion is very small and the distortion for the two sides of the welds are similar. However, when heat input increases to more than 3,000 watts, the results show a sharp increase in angular distortion, and angle 2 becomes much larger than angle 1 due to the contraction of weld metal on weld two which caused the horizontal member to bend.

Figure 48 shows the effect of speed on welding distortion with constant total heat input or constant heat intensity. It shows that there is a critical speed which will make the welding distortion a maximum. With speeds higher than this, the heat input rate is increased but the thermal disturbance is shortened and the angular distortion is reduced. When the speed is slower than the critical speed, the heat input rate is reduced, and even though it takes a longer time for welding, the temperature distribution is much more uniform and the angular distortion is reduced. However, lower speed welding still has its limitations in that the heat input rate has to be large
Figure 47: Effects of Distortion for Different Heat Inputs.
Figure 48: Effects of Distortion for Different Welding Speeds
enough to initiate the arc and make it stable.

Figure 49 shows a significant difference in distortion with varying break time. It shows that a shorter break time will reduce the distortion significantly. The reason for this is a minimization of the period of thermal disturbance and this will reduce distortion. With a longer break time, the first weld will cool down almost completely, then the second weld starts and makes another thermal disturbance. Therefore, the second weld should be made as soon as possible after the first.

Applying constraints is the method which reduces the welding distortion significantly. The magnitude of distortion for the constrained case was much smaller than that of the free joints. The explanation for this is that constraints will produce reaction forces and moments which will reduce the plastic strain formation. Figure 50 shows the effects of angular distortion for different constraint distance from the reference point. It shows that the constraints should be placed far away from the reference point. Figure 51 shows the effect of distortion as a function of the time after welding when the constraints are released. It is interesting to note that the constraints should be released as soon as possible to reduce the distortion, but the time is not very critical. It has been
Figure 49: Effects of Distortion for Different Break Times
Figure 50: Effects of Distortion for Different Constraint Lengths
Figure 51: Effects of Distortion for Different Constraint Times
very carefully examined and experiments have been done to prove this point. The reason for this is still not clear, but probably because the formation of inelastic strain is occurring in the early stage of welding, a short time constraint is long enough to reduce the angular distortion.

After this parametric study, a simple guideline for welding square tubular structures can be made. It is obvious that decreasing heat input rate and using small fillet size will reduce the angular distortion. However, a small weld leg size will also reduce the strength of a joint. Therefore, heat input rate should be chosen as small as possible but still such that the required strength is obtained. Then, use slower welding speeds and keep the same total heat input energy making sure the power is high enough to melt the materials. If two welds are required, make the break time as short as possible depending on the maneuverability of the welding system. The advantage of using jigging constraints is to reduce distortion. These constraints should be placed as far from the weld as possible to get maximum benefit. The jigging constraints should be released as early as possible after welding. Test H21 is made based on the above conditions. It is observed that the minimum distortion was obtained when compared to tests H1 and H7.
Chapter 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

Following the derivation of the relatively simple analytical model for angular welding distortion and the parametric study using this analytical model, several results can be concluded:

1. A relatively simple analytical model for prediction of welding angular distortion in square tubular structures was developed. It can be used to analyze and predict the angular distortion during either the transient or final states.

2. The solution derived has two parts. The first one is a thermal analysis which was based on Carslaw's point heat source theory but modified for a finite heat source theory to improve the accuracy in the region which is very close to the origin of the heat source. The second part is a thermal stress analysis which was based on the elastic-plastic theory.

3. A computer program was written, based on this
analytical model, to carry out the numerical solution. The program required little memory but needed a long time for iteration. Therefore, this program can be used not only in a main frame computer but also can be transferred to a microcomputer in the future.

4. Comparing analytical solutions with experimental data, it was shown that these results were in agreement. The errors were less than 20 percent on each case.

5. A finite element model for the same analysis was also developed. This model contains thermal heat transfer analysis and stress-strain analysis. The interface between the two parts were established. The results from the FEM were in agreement with these from the analytical model.

6. The finite element solution has some advantages which the analytical model does not have. The finite element solution provides more information, not only for angular distortion but also squareness of the tubes, distortion of other axis, and also all the stresses components. The analytical solution only provides angular distortion solutions and one component stress. However, the finite element solution required a large memory (approximately 1,000 times as much as the analytical model), and a long calculation time (approximately 25 times as much as for the analytical model). Therefore, if angular distortions were
the only subject of interest, the analytical model is the best program to choose.

7. According to the parametric study, the angular distortion is increased with the heat input rate. Also a slower welding speed but the same amount of heat input will reduce the angular distortion. The reason for this is because the weld is short and the transverse shrinkage is not significant. However, slower speed will create other types of distortion problems such as rotational distortion.

8. From the parametric study, it was found that making the second weld as soon as finishing the first weld can reduce angular distortion due to minimizing thermal disturbances.

9. Constraints applied to a member during welding and released thereafter can reduce the angular distortion significantly. The best way is by placing the constraints as far away from welds as possible. Also release the constraints immediately after welding.

10. This research provides an analytical procedure and valuable information for automatic robotic welding. As an advanced production method, it is evident that both control and minimization of distortion are improved.

11. This research is a primary contribution in theoretical analysis of welding distortion on tubular
structures. The literature survey did not yield any information on this topic. The most common cases discussed in the literature are on the distortion of plates.

7.2 RECOMMENDATIONS

This study has suggested several areas where additional research would be desirable. These areas include:

1. Continuing development of the analytical model and modifying the program to be more flexible and user friendly. Also integrate all the programs into one package.

2. Doing more tests on the properties of materials, especially for the temperature dependent material properties. This data will affect the solution significantly.

3. Relaxing some of the assumptions used in the analytical model. Thus, for example, slower welding speeds could be considered. In addition, more stress components could be considered.

4. Extending the analytical model, so that the model can be used not only for a tee joint but also for a lap joint or corner joint. Although the tee joint is the most basic joint type, a real structure usually has more than one joint type.

5. Combining the joint analysis program with a
structural analysis program. The analysis of the frame structural distortion is not as simple as that of a single joint. The frame structure contains more than one joint and constraint. The interaction among joints will affect the distortion of the whole structure. It is worth trying to combine these two types of analyses to analyze and control the distortion of a general three dimensional structure.
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APPENDIX A

PROGRAM LISTING
MAIN PROGRAM

THE PURPOSE OF THIS PROGRAM IS TO PREDICT THE ANGULAR DISTORTION OF A TEE OR LAP JOINT OF SQUARE TUBULAR STRUCTURES. THIS PROGRAM CONSISTS OF TWO PARTS: FIRST, A THERMAL ANALYSIS WHICH CALCULATES THE TEMPERATURE FIELDS DURING AND AFTER WELDING USING FINITE HEAT SOURCE THEORY; SECOND, A THERMAL STRESS ANALYSIS WHICH ANALYZES THE THERMAL STRESS, STRAINS DURING WELDING TO OBTAIN THE RESIDUAL STRESSES AND FINAL DISTORTION AFTER SUBSEQUENT COOLING.

PROGRAMMED BY C. A. HOU
DATE: AUG 19 1984
REVISED: MAY 15 1985

SUBROUTINES CALLED: THERM, INTPL, MECHAN, DISTO, METCON

INPUT VARIABLES:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOLT</td>
<td>Welding Voltage (V)</td>
</tr>
<tr>
<td>CURR</td>
<td>Welding Current (A)</td>
</tr>
<tr>
<td>EMET</td>
<td>Welding Efficiency</td>
</tr>
<tr>
<td>MLSIZE</td>
<td>Weld Leg Size (IN)</td>
</tr>
<tr>
<td>TUBDIM</td>
<td>Square Tube Dimension (IN)</td>
</tr>
<tr>
<td>TWC</td>
<td>Tube Wall Thickness (IN)</td>
</tr>
<tr>
<td>TMM</td>
<td>Material Melting Temperature (°F)</td>
</tr>
<tr>
<td>TEMF</td>
<td>Reference Temperature (Room Temp) (°F)</td>
</tr>
<tr>
<td>INCON</td>
<td>Constraint Indicator (0-Free, 1-Constrained)</td>
</tr>
<tr>
<td>DISCON</td>
<td>Distance from Weld to Constraint (IN)</td>
</tr>
<tr>
<td>CONTIM</td>
<td>Constrained Time (SEC)</td>
</tr>
<tr>
<td>COND</td>
<td>Material Thermal Conductivity (BTU/IN-SEC-F)</td>
</tr>
<tr>
<td>DIFC</td>
<td>Material Diffusivity (IN-IN/SEC)</td>
</tr>
<tr>
<td>YER</td>
<td>Reference Yielding Stress (KSI)</td>
</tr>
<tr>
<td>ELR</td>
<td>Reference Elastic Modulus (KSI)</td>
</tr>
<tr>
<td>SMH</td>
<td>Strain Hardening Coefficient</td>
</tr>
<tr>
<td>ALF</td>
<td>Temperature Dependent Thermal Expansion Coefficient (IN/IN°F)</td>
</tr>
<tr>
<td>ELM</td>
<td>Temperature Dependent Elastic Modulus (KSI)</td>
</tr>
<tr>
<td>YED</td>
<td>Temperature Dependent Yielding Stress (KSI)</td>
</tr>
<tr>
<td>TWP</td>
<td>Welding Time for Each Weld (SEC)</td>
</tr>
<tr>
<td>STTIM</td>
<td>Time Start for Calculation (SEC)</td>
</tr>
<tr>
<td>CNTIM</td>
<td>Time Increment for Calculation (SEC)</td>
</tr>
<tr>
<td>ENTIM</td>
<td>Time Stop for Calculation (SEC)</td>
</tr>
<tr>
<td>DEXX</td>
<td>Thickness of Each Ring Element (IN)</td>
</tr>
<tr>
<td>NAL</td>
<td>Number of Elements, Maximum is 20 (It will automatically increase double when thermal stress analysis)</td>
</tr>
<tr>
<td>LB</td>
<td>Number of Points for Each Element, Maximum is 16 (It will increase three times when thermal stress analysis)</td>
</tr>
<tr>
<td>KP1</td>
<td>NonDimensional Temperature Printout Indictor</td>
</tr>
<tr>
<td>KP2</td>
<td>Dimension Temperature Printout Indictor</td>
</tr>
<tr>
<td>KP3</td>
<td>Stress-Strain Printout Indictor</td>
</tr>
<tr>
<td>LWED</td>
<td>No. of Weld (1 = One Weld, 2 = Two Welds)</td>
</tr>
<tr>
<td>WRED</td>
<td>Indicator for Temperature Reading (0 - Calculated from this program, 1 - Reading from other files)</td>
</tr>
<tr>
<td>WME1</td>
<td>Time to Start Weld 1 (SEC)</td>
</tr>
<tr>
<td>WME2</td>
<td>Time to Start Weld 2 (SEC)</td>
</tr>
<tr>
<td>WNPA</td>
<td>Which Part to Look At (1 - First Weld Side, or 2 - Second Weld Side)</td>
</tr>
<tr>
<td>MCT</td>
<td>Consideration of Weld Metal Contraction (0 - No</td>
</tr>
</tbody>
</table>
C RIGIDITY COEFFICIENT

BYTE INP(20), OUT(20), DAT(20), MHH(20)
COMM /DI/TISP, LWSIZE, TUBDIM, TH
COMM /PRO/CNDF, DIFC, TEMP, TREF
COMM /H5/DX, LA, LB, LA1, LB1
COMM /FU/DY, TY, DTE, DIF, IER1
COMM /REC/VER, ER, BTH, STRAND, MALF, MHH, MVED, ALF(10, 2),
  ELM(10, 2), YED(10, 2)
COMM /AC/TENT(50), INTT, EAT, H(50), N1, N2
COMM /CC/TS, TS
COMM /DC/ENGSTR(60, 50), TOTSTR(50), SURP(60, 50), TSP(60, 50), IN(60, 50)
COMM /EC/ME1, ME2, NPA, TISP2
DIMENSION TEM(30, 16), ACT(60, 50, 2), DTO(60, 2)
DIMENSION SLOP(60, 50, 2), IF(60, 50), STRB(50)
DIMENSION INC(60, 50), SURFC(60, 50), IPC(60), ENGSR(60, 50)
DIMENSION T0PP(7), TEP(7)

C OPEN INPUT AND OUTPUT FILES
C
C READ(3, 1) LK, INP
C FORMAT(0,20A1)
C INP(LK+1)=0
C OPEN(UNIT=1, NAME=INP, TYPE=‘OLD’)
C READ(3, 1) LK, OUT
C OUT(LK+1)=0
C OPEN(UNIT=5, NAME=OUT, TYPE=‘NEW’)

C READ INPUT DATA AND ECHO
C
C READ(1, *) VOL, CURT, EMET
C READ(1, *) LWSIZE, TUBDIM, THC, TEMP, TREF
C READ(1, *) CNDF, DIFC, YER, ELR, BTH, MALF, MHH, MVED
C DO 500 I=1, MALF
C 500 READ(1, *) (ALF(I, J), J=1, 2)
C DO 501 I=1, MHH
C 501 READ(1, *) (ELM(I, J), J=1, 2)
C DO 502 I=1, MVED
C 502 READ(1, *) (MLM(I, J), J=1, 2)
C READ(1, *) INCON, LMD, YED
C IF(CON, EQ, 1) READ(1, *) DISCON, CONTI, RIG
C READ(1, *) TISP, BTIM, CNTIM, ENTIM
C READ(1, *) DEXX, LA, LB, MCT
C IF(LMD, EQ, 2) READ(1, *) W1, W2, NPA
C READ(1, *) KPI, KPE, KPS
C IF(RIG, EQ, 1) THEN
C READ(3, 1) LK, DAT
C DAT(LK+1)=0
C OPEN(UNIT=2, NAME=DAT, TYPE=‘OLD’)
C ELSE
C END IF
C IF(MCT, EQ, 1) THEN
C READ(3, 1) LK, MHH
C MHH(LK+1)=0
C OPEN(UNIT=4, NAME=MHH, TYPE=‘OLD’)
C ELSE
C END IF
C
WRITE(49, 199)
WRITE(49, 200) VOLT, CURT, EMET, WLSIZE, TISP
WRITE(49, 213) TEND, LMEW
IF(MCT. EQ. 1) WRITE(49, 214)
IF(MCT. EQ. 0) WRITE(49, 215)
IF(LMEW. EQ. 2) WRITE(49, 212) W1, W2, NPA
IF(INCON. EQ. 0) WRITE(49, 210)
IF(INCON. EQ. 1) WRITE(49, 211) DISCON, CONTIN, RI9.
WRITE(49, 201) TUBIDM, THC
WRITE(49, 202) BTIM, CONTM, ENTIM
WRITE(49, 203) DEX, LA, LB, JRED
WRITE(49, 204) TEMP, COND, DIFC
WRITE(49, 209) VEN, ELR, STH
WRITE(49, 206)
DO 503 I=1, MHALF
503 WRITE(49, 205) ALF(I, J), J=1,2)
WRITE (49, 207)
DO 504 I=1, MELM
504 WRITE(49, 205) ELM(I, J), J=1,2)
WRITE(49, 208)
DO 505 I=1, MRED
505 WRITE(49, 205) YED(I, J), J=1,2)
199 FORMAT(//5X, '******** MELDING ANGULAR DISTORTION', 'ANALYSIS OF SQUARE TUBULAR STRUCTURE **********'//)
200 FORMAT(3X, 'MELDING CONDITIONS'//7X, 'MELDING VOLTAGE(V)' ,3X, F10.2 
2 /7X, 'WELDING CURRENT(A)' ,5X, F10.2/7X, 
3 'MELDING EFFICIENCY', 10X, F5.2/7X. 
7X, F7.2/7X, 'EACH MELDING TIME(SEC)' ,3X, F7.2)
213 FORMAT(7X, 'REFERENCE TEMPERATURE, 5X, F7.3/7X, 12, 'WELD(8)''//)
214 FORMAT(7X, 'CONSIDERATION OF METAL CONTRACTION')
215 FORMAT(7X, 'NO CONSIDERATION OF MELD METAL CONTRACTION')
201 FORMAT(3X, 'DIMENSIONS OF SQUARE TUBE'//7X, 'TUBE DIMENSION(IN)' ,1 8X, F7.2/7X, 'WALL THICKNESS(IN)' ,8X, F7.3/7X)
202 FORMAT(3X, 'CALCULATION INFORMATION'//7X, 'STARTING TIME(SEC)' ,1 11X, F7.1/7X, 'TIME INCREMENT(SEC)' ,10X, F7.1/7X, 'STOP TIME(SEC)' ,2 15X, F7.1)
203 FORMAT(7X, 'THICKNESS OF ELEMENT(IN)' ,5X, F7.3/7X, 'NO. OF ELEMENT' ,1 1X,15X/7X, 'NO. OF POINT IN EACH ELEMENT' ,3X, 15X/7X, 'JRED', 
2 7X, 15/7X)
204 FORMAT(3X, 'MATERIAL PROPERTIES'//7X, 'MELTING TEMPERATURE(F)' ,12X. 
1 F7.1/7X, 'CONDUCTIVITY(STU/IN-SEC-F)' ,5X, E10.4/7X, 
1 'DIFFUSIVITY(IN-SEC)' ,9X, E10.4)
209 FORMAT(7X, 'REF. YIELDING STRESS(KSI)' ,6X, E10.3/7X, 
2 'REF. ELASTIC MODULUS(KSI)' ,6X, E10.3/7X, 
3 'STRAIN HARDENING COL' ,15X, F5.3)
205 FORMAT(10X, F7.1, 10X, E10.4)
206 FORMAT//7X, 'TEMPERATURE DEPENDENT THERMAL EXPANSION COEFFICIENT'// 
1 10X, 'TE(1) (F)' ,5X, 'THEM. EPN. COE. (IN/IN-F)'//)
207 FORMAT(7X, 'TEMPERATURE DEPENDENT ELASTIC MODULUS'// 1 10X, 'TE(2) (F)' ,5X, 'ELASTIC MODULUS (KSI)'//)
208 FORMAT(7X, 'TEMPERATURE DEPENDENT YIELDING STRESS'// 1 10X, 'TE(3) (F)' ,5X, 'YIELDING STRESS (KSI)'//)
210 FORMAT(5X, 'NO CONSTRAINT')
211 FORMAT(5X, 'CONSTRAINTS—DISTANCE FROM MELD TO CONSTRAINT IS ',F8.2 
1 /5X, 'CONSTRAINED TIME (SEC)' ,5X, F8.2/5X, 
1 'RIGIDITY COEFFICIENT ',5X, F9.3)
212 FORMAT(5X, 'TIME TO START MELD 1 ',5X, F8.1/5X, 
1 'TIME TO START MELD 2 ',5X, F8.1/5X, 'WHICH PART TO LOOK AT',5X, 15)
***INITIALIZATION***

```fortran
C  HEATC=VOLT*CURT*EMET/(WSIZE*TBDDM)/1094.39
KINC=0
LA1=LA+1
LB1=LB+1
DX1=DEXX
DY1=2.*TBDDM/LB
INTA=2*LA+1
INTT=3*LB+1
DX2=DX1/2.
DY2=DY1/3.
N1=(INTT-1)/4+1
N2=(INTT-1)*3/4+1
STRND=YEY/ELR
EAT=1./INTT
KX=INTA
TIME=STIM
JCT=(DISCON/DEXX)*2+1
E1=2.*ELR*(TBDDM**4-(TBDDM-2.*THC)**4)
TISP2=TISP+4*E2
DO 550 LP=1,INTA
IP(LP)=0
DO 550 LG=1,INTT
SURF(LP,LG)=0.
MC=INT(TBDDM/2./DEXX+2.)*2+2
550 ACT(LP,LG)=0.
C  THERMAL ANALYSIS
C  **********************
10 IF(JRED.EQ.0) THEN
  READ(2,998) TIME
  WRITE(49,101) TIME
  WRITE(49,102) (TEM(K,J),K=1,LA1)
  ELSE
    CALL THERM(HEATC,TEM,TIME,LUED)
    END IF
998 FORMAT(14X,F10.2)
500 WRITE(49,102) (TEM(K,J),K=1,LA1)
100 FORMAT(1X,'ORIGINAL DIMENSIONLESS TEMPERATURE FIELD',/7X,'DX= ',F6.3,' IN',/7X,'DY= ',F6.2,' IN'/)
101 FORMAT(1X,'TIME'=,F10.2,' SEC')
102 FORMAT(7X,10F4.4)/9X,10F4.4)/9X,10F4.4)
20 CONTINUE
1010 CONTINUE
C  PREPARATION FOR STRESS ANALYSIS
C  ************************************

C  DO 560 LP=1,INTA
  DO 560 LG=1,INTT
560 ACT(LP,LG)=ACT(LP,LG,2)
  CALL INTPL(TEM,ACT)
  DO 50 KC=1,INTT
  DO 40 KB=1,INTA
40 ACT(KB,KC,2)=ACT(KB,KC,2)*(TEM-MEM)
```
IF(KP2.EQ.0) GO TO 50
WRITE(49,49) DX2, DY2
WRITE(49,31) (ACT(KB,KC,2),KB=1,INTA)
49 FORMAT(///9X 'TEMPERATURE FIELD (DEGREE F)',
         /7X,'DX=', 'F6.2', ' IN', 5X,'DY=', 'F6.2', ' IN')
51 CONTINUE
C
********** THERMAL STRESS ANALYSIS
C
********** THERMAL STRESS ANALYSIS
C
IF(MC.T.EQ.1) READ(4,4) CONB
KCC=0
CONF=0.
DO 52 I=1,7
TEPP(I)=(ACT(MC, I, 2)+ACT(MC, I, 1))/2.
52 CONTINUE
C
DO 600 I=1,INTA
IPC(I)=IP(I)
DO 600 J=1,INTT
INC(IJ)=INC(IJ)
SURP(JJ)=SURP(JJ)
ENOSRC(IJ)=ENOSRC(IJ)
600 CONTINUE
C
DO 291 I=1,INTT
291 STRESS(I)=(ENOSTR(KK, IQ)-SURP(KK, IQ))#H(IQ)#YER
DO 291 IQ=1,INTT
291 CONTINUE
C
FOR EACH SECTION AT PARTICULAR TIME
KCC=KCC+1
DO 30 KK=1,INTA
30 CONTINUE
C
IF(KP3.EQ.1) THEN
ELSE
ENDIF
C
FORMAT(///5X, 'THERMAL STRESS AND STRAIN ANALYSIS'/
1 7X, 'TENT: TEMPERATURE (F)', 1AX, 'TBN: NODIMENSION TOTAL STRAIN'
2 /7X, 'BNB: NODIMENSION MECHANICAL STRAIN', 8X
3 'SPB: NODIMENSION PLASTIC STRAIN/7X,
4 'STS: THERMAL STRESS(KSI)'/
220 FORMAT(5X,'SECTION',15.7X,'ITERATION',15)
221 FORMAT(2X,'TEM ',F8.2,3X,'TN ',E10.3,3X,'EHN ',
1 E10.3,3X,'SPS ',E10.3,3X,'STS ',E10.3)
30 CONTINUE
C
C ********** DISTORTION ANALYSIS
C
CALL DISTO(DTG,DX2,INTA,SLOPE,DISS)
IF (INCON.EQ.1) THEN
IF (CONTIN.LE.TIME) GO TO 68
IF (ABS(DISS(JCT,2)).LE.1.E-6) GO TO 68
IF (KCC .GE. 20) GO TO 68
IF (KCC.EQ.1) THEN
BMONT=RIG+DISS(JCT,2)*1.0*E13/DISCN**2/(S.*VER*TUBDIM**3*THC)
ELSE IF (DISS(JCT,2).GT.0.) UPP=BMONT
IF (DISS(JCT,2).LE.0.) BOT=BMONT
BMONT=RIG+(UPP-BOT)/2.
ENDIF
GO TO 250
ELSE ENDIF
68 WRITE(49,71) BMONT,CONF,KCC,JCT
71 FORMAT(10X,'MONT ',F10.3,'CONF ',E10.3I1,'KCC',19,'JCT',19)
WRITE(49,AA)
AA FORMAT(7///9X,'SLOPE AND DISTORTION'
7X,'SLOPE NO UNIT. DISTORTION IN INCHES')
AA FORMAT(9X,'X* ',E10.4,3X,'SLOPE- ',E10.4,3X,'DI8T- ',E10.4)

C
C ***** CALCULATE EFFECT LENGTH
C
C
IF (TIME.GT.TISP2 .AND. TEN(1,1).LE.1.E-4) THEN
DO 570 KX=KMC,INTA
IF (IP(KX).EQ.0) GO TO 580
570 CONTINUE
580 ELENQ=(KX-1)*DX2
ELSE END IF
C
C
C ***** CHECK FOR STOP
C
C
IF (JRED.NE.1) THEN
IF (KINC.EQ.1) GO TO 230
IF (TIME.LE.INTIN) THEN
IF (TIME.LE.40.) TIME=TIME+INTIN
IF (TIME.GT.40. .AND. TIME.LE.120.) TIME=TIME+4.
IF (TIME.GT.120. .AND. TIME.LE.200.) TIME=TIME+20.
IF (TIME.GT.200.) TIME=TIME+50.
GO TO 10
ELSE KINC=1
DO 234 ITT=1,LA1
DO 234 ITB=1,LAB1
234 TEM=ITT,ITB)=0.
GO TO 1020
END IF
ELSE
    GO TO 10
END IF
C
C ********** THE END
C
C
230 WRITE(45,225) ELENO
225 FORMAT(///3X,'EFFECT LENGTH IS ','F10.4,' IN')
CLOSE(UNIT=1)
CLOSE(UNIT=49)
IF(JRED.EQ.1) CLOSE(UNIT=2)
IF(MCT.EQ.1) CLOSE(UNIT=4)
STOP
END
C
C
C SUBROUTINE METCON(MC,TEPP,TEPC,CONF)
C
C THIS SUBROUTINE IS CALCULATING WELD METAL CONTRACTION
C
C PROGRAMMED BY C. A. NGO
C
C DATE: MAY 1989
C
C SUBROUTINE USED:
C
C PROGRAM CALLED: MAIN
C
C COMMON /D1/TISP,WL8IZE,TUBDIM,THC
C COMMON /MEC/YER,ELR,8TH,STRNG,MALF,MELN,HYED,ALF(10,2),
1
C DIMENSION CON(7),TEPP(7),TEPC(7),AFC(7),E(7)
C
C FIND YOUNGS MODULES
C
C DO 10 I=1,7
IF(TEPC(I) .LE. ELM(I,1)) THEN
  E(I)=ELM(I,1)
ELSE IF(TEPC(I) .GE. ELM(MELH,1)) THEN
  E(I)=ELM(MELH,1)
ELSE
  DO 11 KC=1, (MELH-1)
    KD=KC+1
    IF(TEPC(I),GT.ELM(KC,1) .AND. TEPC(I),GT.ELM(KD,1)) GO TO 11
    E(I)=(ELM(KC,2)+(ELM(KD,2)-ELM(KC,2)*TEPC(I)-ELM(KC,1))/
1  (ELM(KD,1)-ELM(KC,1)))
  11 CONTINUE
GO TO 100
END IF
C
C TEMPERATURE DEPENDENT THERMAL EXPANSION COEFFICIENT
C
100 IF(TEPC(I) .LE. ALF(1,1)) THEN
    AFC(I)=ALF(1,1)
ELSE IF(TEPC(I) .GE. ALF(MALF,1)) THEN
    AFC(I)=ALF(MALF,2)
ELSE
    DO 20 KC=1, (MALF-1)
      KD=KC+1
      IF(TEPC(I),GT.ALF(KC,1) .AND. TEPC(I),GT.ALF(KD,1)) GO TO 20
      AFC(I)=(ALF(KC,2)+(ALF(KD,2)-ALF(KC,2)*TEPC(I)-ALF(KC,1))/
1  (ALF(KD,1)-ALF(KC,1)))
GO TO 10
CONTINUE
END IF
10 CONTINUE
END IF
CON(1)=TUBDIM*WLSIZE*E(1)+AFC(I1)*TEP(1)

CALCULATING AVERAGE FORCE AND/OR MOMENT

CONF=(CON(1)+CON(7)+2.0*(CON(2)+CON(3)+CON(4)+CON(5)+CON(6)))+
O.6/12./@YER*TUBDIM**2*THC
RETURN

END

SUBROUTINE THERMAL<HEATC.TEN.TIME.WELD)

THIS SUBROUTINE IS TO DO THERMAL ANALYSIS WHICH CALCULATES THE TEMPERATURE FIELDS DURING AND AFTER WELDING USING FINITE HEAT SOURCE THEORY.

PROGRAMMED BY C. A. HOU
DATE: AUG. 18 1984

SUBROUTINES USED: FUNC
PROGRAMS CALLED: MAIN

COMMON /DX/T18P,WLSZ,WE.TUBDIM,THC
COMMON /PRO/CON, DIFC, TEM, TEMF
COMMON /DFF/DFF, LAL, LB, LAI, LB1
COMMON /EC/H1, WE2, NPA, TIP2
DIMENSION WET(7), COO(7), WET1(4), COO1(4)
DIMENSION TEM(30,16), DIZ(2,4), DIB(2,4)

DATA WET1/0.3478548451, 0.6521451548, 0.6521451548, 0.3478548451/
DATA COO/-0.611363116, -0.3399810436, 0.3399810436, 0.611363116/
DATA WET/0.1274848451, 0.2797039214, 0.3818300039, 0.4179591836/
1 0.3915000059, 0.2777039214, 0.1274848451/
DATA COO/0.4941079123, -0.7415311852, -0.4058451313, 0.0000000000/
1 0.4058451313, 0.7415311852, 0.4941079123/

GAUSSIAN QUADRATURE INTEGRATION METHOD FITS THE POLYNOMIAL OF DEGREE 13 DURING MELDING OR DEGREE 7 AFTER MELDING IN X AND Y DIRECTION EACH.

CALCULATE SOME CONSTANTS

PI=ATAN(1)
GDOT=HEATC/THC/(COND*(TEM-TEMF))
CC=GDOT/2./PI
I=O.9

INITIALIZATION

DO 200 J=1, LB1
DO 200 N=1, LAI
200 TEM(K, J)=O.
DX=DFF/THC
DY=TUBDIM*2./LB/THC
XD=WLSIZE/2./THC
YD=TUBDIM/2./THC
IER1=0
IF(TIME.LE. TISP) IER1=1
DIFT=DIFF*(TIME-TISP)/(THC*THC)
DTME=DIFF*(TIME/(THC*THC))
KKK=7
IF(TIME.GT. TISP) KKK=4
IER2=0
IF(TIME.LE. TISP2) IER2=1
DIFT2=DIFF*(TIME-WE2-TISP)/(THC*THC)
DTME2=DIFF*(TIME-WE2)/(THC*THC)
KKK2=7
IF(TIME.GT. TISP2) KKK2=4

C
C POSITION LOOP
C
C DO 80 J=1, LS1
Y=DEY*(J-1)
C DO 800 K=1, LA1
C
60 IF(LMED, EQ. 2) THEN
C****** TWO WELDS
C IF(TIME-WE2).GE. 0.001) THEN
C****** STARTING SECOND WELD
C
KCK=KCK+1
IF(NPA, EQ. 1) THEN
IF(NPA, EQ. 1) X=DEX*(K-1)-(XD+YD)
IF(NPA, EQ. 2) X=DEX*(K-1)+(XD+YD)
IER1=IER11
KKK=KKK1
DTME=DTME1
DIFT=DIFT1
TISPP=TISP
ELSE
IF(NPA, EQ. 1) X=DEX*(K-1)+(XD+YD)
IF(NPA, EQ. 2) X=DEX*(K-1)-(XD+YD)
IER1=IER12
KKK=KKK2
DTME=DTME2
DIFT=DIFT2
TISPP=TISP2
END IF
ELSE
C****** STARTING FIRST WELD
C
IF(NPA, EQ. 1) X=DEX*(K-1)-(XD+YD)
IF(NPA, EQ. 2) X=DEX*(K-1)+(XD+YD)
IER1=IER11
KKK=KKK1
DTME=DTME1
DIFT=DIFT1
TISPP=TISP
END IF
ELSE
C****** ONE WELD
C
X=DEX*(K-1)
IER1=IER11
KKK=KKK1
DTME=DTME1
DIFT=DIFT1
TISPP=TISP
END IF
RESULT=0.

INTEGRATION RESPECT TO X AND Y

DO 100 JJ=1, NKK
COX=COO(JJ)
WEI=NET(JJ)
IF(TIME.GT.TSPP) COX=COO1(JJ)
IF(TIME.GT.TSPP) WEI=NET1(JJ)
DX=COX*ID

DO 100 LI=1, NKK
COY=CO01(LI)
WEY=NET1(LI)
IF(TIME.GT.TSPP) COY=CO011(LI)
IF(TIME.GT.TSPP) WEY=NET11(LI)
DY=COY*YD
CALL FUNC(X, Y, Z, AIMAOE)
KEI=-3

IMAGE METHOD

KE1=KE1+4
KE2=KE1+3
OLDA1M=AIMAGE
DO 30 M=1, 4
DISZ(1, M)=2.*((KE1+(M-1)))*Z
DISZ(2, M)=2.*((KE1+(M-1)))*Z
DISY(1, M)=2.*((KE1+(M-1)))*2.*TUBDIM/THC+Y
DISY(2, M)=2.*((KE1+(M-1)))*2.*TUBDIM/THC-Y
CALL FUNC(X, Y, DISZ(1, M), ZREB1)
CALL FUNC(X, Y, DISZ(2, M), ZREB2)
CALL FUNC(X, DISY(1, M), Z, YREB1)
CALL FUNC(X, DISY(2, M), Z, YREB2)

30 AIMA0E=AIMAGE+ZREB1+ZREB2+YREB1+YREB2
DO 40 M=1, 4
DO 40 N=1, 4
CALL FUNC(X, DISY(1, N), DISZ(1, M), A1)
CALL FUNC(X, DISY(1, N), DISZ(2, M), A2)
CALL FUNC(X, DISY(2, N), DISZ(1, M), A3)
CALL FUNC(X, DISY(2, N), DISZ(2, M), A4)

40 AIMAGE=AIMAGE+A1+A2+A3+A4

C IF(AIMAGE.OLDA1M).LT. 1.E-4) GO TO 100
C IF((AIMAGE.OLDA1M).GT. 1.E-01) GO TO 5

C ERROR OF IMAGE METHOD IS WITHIN 1% OF TOTAL

100 RESULT=RESULT+WEI*WEY*AIMAGE*ID*YD

C TEM(K, J)=CC*RESULT+TEM(K, J)
IF(KCH, EQ, 1) GO TO 40
IF(TEM(K, J).LE. 1.E-4) GO TO 20

800 CONTINUE
20 CONTINUE
RETURN
END

C SUBROUTINE ERROR(ARG, RESULT)
C
C THIS SUBROUTINE IS TO CALCULATE THE ERROR FUNCTION.
ERROR FUNCTION APPROXIMATION FROM C. HASTINGS, JR.  
"APPROXIMATION FOR DIGITAL COMPUTER", PRINCETON UNIVERSITY PRESS, 1955  
ACCURACY IS ABS(10^-9)  

PROGRAMMED BY C. A. HOU  
DATE: AUG 12 1984  

PROGRAMS CALLED: FUNC

THIS SUBROUTINE CALCULATES THE VALUES OF THE FUNCTION FROM TEMPERATURE SOLUTION.  

PROGRAMMED BY C. A. HOU  
DATE: AUG 12 1984  

SUBROUTINES USED: ERROR  
PROGRAMS CALLED: THERM  

SUBROUTINE INTPL(PT, ACT)  

THIS SUBROUTINE IS TO PREPARE THE DATA FOR THERMAL STRESS ANALYSIS USING INTERPOLATION METHOD TO EXPAND THE SOLUTION OF TEMPERATURE FIELD.  

PROGRAMMED BY C. A. HOU
PROGRAMS CALLED: MAIN

COMMON /MIS/EXX, LA, LB, LA1, LB1
DIMENSION PT(30,18), AT(60,90,2), AN(2,2), AT(2,2)

DO 20 J=1, LB
   ID=ID+1
   DO 20 I=1, LA
      IC=IC+1
      IC1=2*(IC-1)
      AT(1,1)-PT(I,J)
      AT(1,2)-PT(I+1,J)
      AT(2,1)-PT(I,J+1)
      AT(2,2)-PT(I+1,J+1)
   20 CONTINUE

DO 400 IA=1,3
   IF (I.EQ.1 .AND. IA.EQ.1) GO TO 400
   X=0.5*(IA-1)
   IF (J.EQ.1 .AND. JA.EQ.1) GO TO 400
   Y=(JA-1.)/3
   IM=IC1+IA
   IN=ID1+JA

DO 10 N=1,2
   RESULT=RESULT+AN(IM, IN)*AT(M,N)
   10 CONTINUE

RESULT=RESULT+AN(M, N)*AT(M, N)
1400 ACT(M, IN,2)=RESULT
20 CONTINUE
1000 RETURN
END

SUBROUTINE MECHAN(KR, ACT, DTO, IP, J, BM, CONNF)

THIS SUBROUTINE IS TO CALCULATE THE THERMAL STRESS-STRAIN DURING AND AFTER WELDING FOR EACH SECTION. TOTAL MECHANICAL STRAINS WILL BE CALCULATED.

PROGRAMMED BY C. A. HOU
DATE: AUG 18 1984

SUBROUTINES USED: CONST, FM, ELAS, PLAS, HATPER
**Subroutine Program Description**

**Program Call:** MAIN

**Common Blocks:**
- D1/TISP, M:size, TB, DB, TMB, TMBF
- PRO/COND, DIFC, TEMF
- M/DE/EX, L.A, L.B, L.A1, L.B1

**Additional Common Blocks:**
- SEC/VER, ELR, STH, STR, MAL, MELM, MVED, ALF(10,2),
  - ELH(10,2), YED(10,2)
- AC/ENT(50), INTT, EAT, H(50), N1, N2
- CC/TEA, TRB
- DC/ENSTR(60,50), TOTSTR(90), SURF(60,50), TSP(60,50), IN(60,50)

**Dimension Blocks:**
- B1(50), B2(50), TOE(50), STR, STR(50), ETO, ENOYE(50)
- DIMENSION ACT(AO, 90, 8), DTO(AO, 90, 8), THPN(90), IP(AO)

**Temperature Interval:**

KTI=3
DO 99 LC=1, KTI
DO 101 I=1, INTT
TENT(I)=ACT(KK, I, 1)+(ACT(KK, I, 2)-ACT(KK, I, 1))*LC/KTI

**Pick Up Temperature Dependent Material Properties:**

CALL MATPER(STHARD, ENOYE, THPN, I)

**Calculate Thermal Stress-Strain:**

TOE(I)=TENT(I)*THPN(I)/8TRN0
CALL CONST(81, 82)
CALL F(TOE, TRSA, TR88)
IF(IP(KK), EQ. 0) CALL ELA8(B1, BS, TOE, ENOYE, IP, KK, BNN, CONFF)
IF(IP(KK), EQ. 1) CALL PLA8(B1, BS, TOE, ENOYE, STHARD, J, KK, BNN, CONFF)

**Store Metal Movement Information:**

DO 30 NI=1, 2
DTO(KK, 1)=TOTSTR(1)
30
DTO(KK, 2)=TOTSTR(INTT)

**RETURN**

END

**Subroutine Substitution:**

**Subroutine SUBROUTINE:** CONST(A, B)

**This Subroutine is to Calculate Some Integration Constants.**

**Programmed by C. A. KOU**

**Date:** Aug 19 1984

**Subroutines Used:** SIMPSON

**Program Called:** MECHAN

**Common Blocks:**
- AC/ENT(50), INTT, EAT, H(50), N1, N2
- DIMENSION H0(50), H1(50), N1, N2, A(INTT), B(INTT)

**Constants:**
- M1=N1
- M2=N2-(N1-1)
- DO 10 I=1, INFT
- H0(I)=H(I)
HI(I) = H(I) + EAT*(I - 1)

CALL SIMPSON(H0(I), N1, EAT, H0A)
CALL SIMPSON(H0(N1), N2, EAT, H0B)
CALL SIMPSON(H0(N2), N1, EAT, H0C)
CALL SIMPSON(H1(N1), N2, EAT, H1B)
CALL SIMPSON(H2(N1), N2, EAT, H2B)

A1 = H0A + 3. * H0B / 2. - 2. * H1B
A2 = H0C - H0B / 2. + 2. * H1B
A4 = -(H0C + H0B) + 6. * H1B - 8. * H2B

DE0 = A1 * A4 - A8 * A3

DO 20 J = 1, INTT
  IF (J . LE. N1) P = 0.
  IF (J . GT. N1 . AND. J . LE. N3) P = EAT *(J - 1) + 0.25
  IF (J . GT. N2) P = 0.5
  A(J) = (A4 + 2. * (A4 + A3) * P) / DE0
20 CONTINUE

RETURN
END

SUBROUTINE FMCF_A, B

THIS SUBROUTINE IS TO CALCULATE INTEGRATION CONSTANTS FOR FORCE AND
MOMENT EQUILIBRIUM.

PROGRAMMED BY C. A. HOU
DATE: AUG 1984

SUBROUTINES USED: SIMPSON
PROGRAMS CALLED: MECHAN, PLAS

COMMON /AC/ TEMT(50), INTT, EAT, H(50), N1, N2
DIMENSION F(INTT), AA(50), BB(50)

DO 20 I = 1, INTT
  IF (I . LE. N1) ARM = 1.
  IF (I . GT. N1 . AND. I . LE. N2) ARM = 2. - 4. *(I - 1) * EAT
  IF (I . GT. N2) ARM = -1.
  AA(I) = F(I) + H(I)
  BB(I) = F(I) + H(I) * ARM
20 CONTINUE

CALL SIMPSON(AA, INTT, EAT, A)
CALL SIMPSON(BB, INTT, EAT, B)
RETURN
END

SUBROUTINE ELAS(A, B, E, I, K, P, T, M)

THIS SUBROUTINE IS TO CALCULATE THE STRAINS IN ELASTIC RANGE.

PROGRAMMED BY C. A. HOU
DATE: AUG 1984
PROGRAMS CALLED: MECHAN

COMMON /AC/TEST(50), INTT, EAT, H(50), N1, N2
COMMON /CC/TEST, RE9
COMMON /DC/ENSTK(50), TOTSTR(50), SURF(50, 50), TSP(50, 50), IN(50, 50)
DIMENSION BI(INTT), B2(INTT), TOE(INTT), EN8YE(INTT), IP(60)

DO 30 I=1, INTT
    ENSTK(KK, I)=B1(I)*RE9-92(I)*(TEST+BMH+COFF)-TOE(I)
    TOTSTR(I)=ENSTK(KK, I)+TOE(I)
    TSP(KK, I)=ENSTK(KK, I)
    IF(ENSTK(KK, I) .GE. EN8YE(I)) THEN
        IN(KK, I)=1
        IF(ENSTK(KK, I) .LE. EN8YE(I)) IN(KK, I)=2
    ELSE
        IN(KK, I)=-1
        IF(A8S(IN(KK, I)) .LE. (-EN8YE(I))) IN(KK, I)=2
    END IF
    IF(IN(KK, I)) .GT. 1 .AND. IP(KK) .EQ. 0) IP(KK)=1
30 CONTINUE
RETURN
END

SUBROUTINE PLA8(B1, B2, TOE, EN8YE, STHARD, J, KK, BMH, COFF)

THIS SUBROUTINE IS TO CALCULATE THE STRAIN IN THE PLASTIC RANGE AND THERMAL UNLOADING PROCESS.

PROGRAMMED BY C.A. HOU
DATE: AUG 16 1984

SUBROUTINES USED: FM

PROGRAMS CALLED: MECHAN

COMMON /AC/TEST(50), INTT, EAT, H(50), N1, N2
COMMON /CC/TEST, RE9
COMMON /DC/ENSTK(50), TOTSTR(50), SURF(50, 50), TSP(50, 50), IN(50, 50)
DIMENSION BI(INTT), B2(INTT), TOE(INTT), CURP(SO), D8(50)
DIMENSION EN8YE(INTT), STHARD(INTT), ENSUR(50)

INITIALIZATION

CP0=0.
CP1=0.
J=0
40 J=J+1
CP0R=CP0
CP1R=CP1

FIND CURRENT PLASTIC STRAIN WITH STRESS-STRAIN CURVE
1- LOADING IN ELASTIC, 2- LOADING IN PLASTIC RANGE
3- UNLOADING WITH CONSTANT PLASTIC STRAIN, 4- REVERSE YIELDING

DO 10 I=1, INTT
    IF(IN(KK, I)) .EQ. 1) THEN
        CURP(I)=0.
        IF(ENSTK(KK, I) .LE. 0.) IN(KK, I)=-1
    ELSE
        IF(ENSTK(KK, I) .LE. EN8YE(I)) IN(KK, I)=-1
    END IF
10 CONTINUE

RETURN
END
IF(ENGSTR(KK, I) . GT. ENOYE(I)) IN(KK, I)=2
ELSE IF(IN(KK, I) . EQ. 2) THEN
  CURP(I)=1-(STHARD(I))*(ENGSTR(KK, I)-ENOYE(I))-SURP(KK, I)
IF(ENGSTR(KK, I) . LT. TSP(KK, I)) IN(KK, I)=3
ELSE IF(IN(KK, I) . EQ. 3) THEN
  CURP(I)=0.
ELSE IF(IN(KK, I) . EQ. 4) THEN
  CURP(I)=1-(STHARD(I))*(ENGSTR(KK, I)-SURP(KK, I)+ENOYE(I))
IF(ENGSTR(KK, I) . LT. T8P(KK, I)) IN(KK, I)=-3
ELSE IF(IN(KK, I) . EQ. -3) THEN
  CURP(I)=0.
ELSE IF(IN(KK, I) . GE. 0.) IN(KK, I)=1
ELSE IF(IN(KK, I) . LT. -ENOYE(I)) IN(KK, I)=-2
ELSE IF(IN(KK, I) . EQ. -2) THEN
  CURP(I)=(1-(STHARD(I))*(ENGSTR(KK, I)+ENOYE(I)))-SURP(KK, I)
ELSE IF(IN(KK, I) . EQ. -3) THEN
  CURP(I)=0.
ELSE IF(IN(KK, I) . EQ. -4) THEN
  CURP(I)=(1-(STHARD(I))*(ENGSTR(KK, I)-SURP(KK, I)-ENOYE(I))
END IF
CONTINUE

CC CALCULATE MECHANICAL STRAIN
******************************
DO 100 I=1, INTT
100 ONSR(I)=SURP(KK, I)
CALL FM(ONSR, SPO, SPI)
DO 30 I=1, INTT
30 ENOSTR(KK, I)=N(I)*(TRSA+SPO+SP1)+a(N(I))*(TR88+SPO+CP1)+BMM1+COf#F>-TOE(I)

CC CHECK CONVERGENCE
******************************
IF(J . LE. 9) GO TO 40
IF(AbBS(CP0). LE. 1.E-7 . OR. ABS(CP1). LE. 1.E-7) GO TO 50
IF(J . EQ. 50) GO TO 50
IF(AbBS(CPOR). LE. 1.E-10 . OR. ABS(CP1R). LE. 1.E-10) GO TO 40
IF(AbbBS(CP0-CPOR)/ABBS(CPOR) . ST. 1.E-3 . OR.
1 ABBBS(CP1-CPOR)/ABBS(CPOR) . ST. 1.E-3) GO TO 40

CC FIND TOTAL STRAINS
******************************
DO 60 I=1, INTT
60 TOTSTR(I)=ENGSTR(KK, I)+TOE(I)
SURP(KK, I)=SURP(KK, I)+CURP(I)
TSP(KK, I)=ENGSTR(KK, I)
CONTINUE
RETURN
END

SUBROUTINE SIMPSON(F, N, H, RESULT)
******************************
THIS ROUTINE PERFORMS SIMPSON'S RULE INTEGRATION OF
A FUNCTION DEFINED BY A TABLE OF EQUISPACED VALUES.
PARAMETERS ARE -

F    ARRAY OF VALUES OF THE FUNCTION
N    NUMBER OF POINTS
H    THE UNIFORM SPACING BETWEEN X VALUES
RESULT ESTIMATE OF THE INTEGRAL THAT IS RETURNED
TO CALLER

MODIFIED FROM GERALD, R. D. "APPLIED NUMERICAL ANALYSIS"
DATE: AUG 18 1984

PROGRAMS CALLED: CONST.FM.PLAN

*** CHECK IF THE NO. OF POINTS IS LESS THAN 3.
IF(N.EQ.2) RESULT=H/2. *(F(1)+F(2))
IF(N.EQ.2) RETURN
IF(N.EQ.3) RESULT=H/2. *(F(1)+2.*F(2)+F(3))
RESULT=H/3. *(F(1)+F(2)+F(3))
IF(N.EQ.3) RETURN

*** CHECK TO SEE IF NUMBER OF PANELS IS EVEN.
*** NUMBER OF PANELS IS N-1
NPANEL=N-1
NHALF=NPANEL/2
NBEGIN=1
RESULT=0.
IF(NBEGIN=NHALF) GO TO 9

*** NUMBER OF PANELS IS ODD. USE 3/8 RULE ON FIRST THREE
*** PANELS. 1/3 RULE ON REST OF THEM.
RESULT=3.*H/8. *(F(1)+3.*F(2)+3.*F(3)+F(4))
IF(N.EQ.4) RETURN
NBEGIN=NBEGIN+1
SAME APPLY 1/3 RULE - ADD IN FIRST, SECOND, LAST VALUES.
RESULT=RESULT+H/3. *(F(NBEGIN)+F(NBEGIN+4)+F(NBEGIN+6))
NBEGIN=NBEGIN+3
IF(NBEGIN.EQ. N) RETURN

*** THE PATTERN AFTER NBEGIN+2 IS REPEETITIVE.
NBEGIN=NBEGIN+2
DO 10 I=NBEGIN, NEND, 2
RESULT=RESULT+H/3. *(2.*F(I)+4.*F(I+1))
10 RETURN

** SUBROUTINE MECHANICAL PROPERTIES SUCH AS STRENGTH
** HARDENING COEFFICIENT, YIELDING STRESS, ELASTIC MODULUS ACCORDING
** TO THE TEMPERATURE.

PROGRAMMED BY C. A. HOU
DATE: AUG 18 1984

PROGRAMS CALLED: MECHAN

** COMMON /MEC/YER, ELR, STM, STRND, HALF, MYED, ALF(10.2).
1 COMMON /AC/TENT(50), INTT, YST(50), N1, N2
DIMENSION STR(INTT), YST(INTT), TPM(INTT)
TEMPERATURE DEPENDENT ELASTIC MODULUS

ARAMEM(I.I).LE.ELM(I.I))THEN
H(I)=ELM(I.I)/ELR
ELSIF(TEM(T(I).GE.ELM(HELM.1))THEN
H(I)=ELM(HELM.2)/ELR
ELSE
DO10KC=1,HELM-1
KD=KC+1
IF(TEM(T(I).GT.ELM(KC.1).AND.TEM(T(I).GT.ELM(KD.1))GO TO10
H(I)=(ELM(KC.2)+ELM(KD.2)-ELM(KC.1))/(TEM(T(I)-ELM(KC.1))
1
END IF
10 CONTINUE

TEMPERATURE DEPENDENT THERMAL EXPANSION COEFFICIENT

100 IF(TEM(T(I).LE.ELM(I,1))THEN
TPN(I)=ALF(I,1)
ELSE IF(TEM(T(I).GE.ALF(HALF.1)))THEN
TPN(I)=ALF(HALF.2)
ELSE
DO20KC=1,HALF-1
KD=KC+1
IF(TEMP(I).GT.ALF(KC.1).AND.TEMP(I).GT.ALF(KD.1))GO TO20
TPN(I)=(ALF(KC.2)+ALF(KD.2)-ALF(KC.1))/(TEMP(I)-ALF(KC.1))
20 CONTINUE

TEMPERATURE DEPENDENT YIELDING STRESS AND STRAIN HARDENING COEFFICIENT

200 IF(H(I).LE.1.E-3)THEN
STR(I)=O.
YST(I)=O.
ELSE
STR(I)=8TH
IF(TEMP(I).LE.YED(I,1))THEN
ACD=YED(I,2)/YER
YST(I)=ACD/H(I)
ELSE IF(TEMP(I).GE.YED(HYED.1))THEN
ACD=YED(HYED.2)/YER
YST(I)=ACD/H(I)
ELSE
DO30KC=1,HYED-1
KD=KC+1
IF(TEMP(I).GT.YED(KC,1).AND.TEMP(I).GT.YED(KD.1))GO TO30
ACD=(YED(KC.2)+YED(KD.2)-YED(KC.1))/(TEMP(I)-YED(KC.1))
30 CONTINUE
END IF
300 RETURN
END
SUBROUTINE DISTO(DTO, DX2, INTA, SLOPE, DISS)

THIS SUBROUTINE IS TO DO DISTORTION ANALYSIS.

PROGRAMMED BY C. A. HOUC
DATE: AUG 18 1984

SUBROUTINE USED: SIMPSON
PROGRAMS CALLED: MAIN

COMMON /DI/TISP, WLS, TUBDIM, TMC
COMMON /MEC/YER, ELR, STRM, MELM, MYED, ALP(10, 2),
   ELM(10, 2), YED(10, 2)
DIMENSION DTO(60, 2), ANG(60), SLOPE(60), DISS(60, 2)

CALCULATE SLOPE OF EACH SECTION

DO 10 I=1, INTA
   SCC=(DTO(I, 1)-DTO(I-1, 1))*STRM*DX2
   IF(I .EQ. 1) THEN
      SLOPE(I)=ATAN(STRNO, SCC)
   ELSE
      SLOPE(I)=ATAN(SCC, TUBDIM)+SLOPE(I-1)
   END IF
   CONTINUE

DO 20 I=1, INTA
   IF(I .EQ. 1) DISS(I, 2)=0.
   IF(I .NE. 1) CALL SIMPSON(SLOPE, I, DX2, DISS(I, 2))
   DISS(I, 1)=DISS(I-1)
   CONTINUE

RETURN
END
APPENDIX B

MATERIAL PROPERTIES
### THERMAL PROPERTIES USED IN THE PROGRAM

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal Conductivity (Btu/sec-in-°F)</td>
<td>0.0005891</td>
</tr>
<tr>
<td>Thermal Diffusivity (in²/sec)</td>
<td>0.01853</td>
</tr>
<tr>
<td>Melting Temperature (°F)</td>
<td>2800</td>
</tr>
<tr>
<td>Thermal Expansion Coefficient (in/in-°F)</td>
<td>0.000006</td>
</tr>
</tbody>
</table>

### MECHANICAL PROPERTIES USED IN THE PROGRAM

<table>
<thead>
<tr>
<th>Properties</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yielding Stress (Ksi)</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>1400</td>
</tr>
<tr>
<td>Elastic Modulus (Ksi)</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>1100</td>
</tr>
<tr>
<td></td>
<td>1400</td>
</tr>
<tr>
<td>Strain Hardening Coefficient</td>
<td>0.07</td>
</tr>
</tbody>
</table>