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Sharif, Kamaruddin Bin

PENSION FUNDING AND INVESTMENT: A MULTIPLE CRITERIA DECISION MAKING APPROACH

The Ohio State University

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PENSION FUNDING AND INVESTMENT:
A MULTIPLE CRITERIA DECISION MAKING APPROACH

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Kamaruddin Bin Sharif, B.A., M.A.Sc.

* * * * *

The Ohio State University
1985

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DEDICATION

In Memory of my Late Mother
ACKNOWLEDGEMENTS

The writing of a dissertation can often seem to be almost a Sisyphean task. It is made more bearable however, with the providence of Allah Almighty, with the support of families and friends and with the guidance and counsel of the dissertation reading committee.

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Minor Field: Quantitative Methods
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CHAPTER I
INTRODUCTION

INTRODUCTION

Approximately half of all wage earners in the United States today are currently covered by private pension plans. Coverage may be extended to cover all wage earners in the future. Since the late 1940s, private pension systems have shown a phenomenal growth both in terms of funds committed and coverage provided. They have burgeoned into a significant social and financial force. Their dominance in the financial marketplace can be seen from the funds' assets growth over the last four decades. According to the Securities & Exchange Commission (SEC), pension fund assets have risen from a book value of $2.4 billion in 1940 to $57 billion in 1960 and to $872 billion in 1983; an average annual increase of about 8.5 percent over the last 43 years. If the assets of both the public and private pension plans are aggregated their total value will soon be approaching the $1 trillion mark.
Most of the assets in the pension funds are invested in stocks, bonds and other money market instruments with the primary objective of providing sufficient income to cover present and future expenditures. With millions of dollars of contributions flowing into the fund annually, financial executives naturally have been deeply concerned with investment strategies and the right 'mix' of the pension fund portfolio to be used for better investment results.

The 1974 enactment of the Employee Retirement Income Security Act (ERISA) has had a substantial impact on the investment policies of pension funds. Pension costs and contributions have increased considerably since then. At the same time, however, firms have been hard-pressed to meet their pension fund obligations. Before the passage of ERISA, pension liabilities were not regarded as liabilities of the firm; however, such claims are now considered obligatory and are handled as liabilities of the firm. The Pension Benefit Guarantee Corporation (PBGC) - a pension termination insurance plan - was established under ERISA to ensure the protection of employees and retirees should a failing pension plan become terminated. Under the law, the PBGC may rightfully lay claim to a maximum of thirty percent of the firm's net worth for any unfunded benefit obligations. In this way unfunded pension liability can be considered a "full-fledged debt item on corporate balance
sheets" and this has somewhat impaired the ability of the firm to accumulate needed capital (42). In adjusting to the above requirements of ERISA on pension funds, it is imperative for management to improve on their fund's portfolio performance, particularly with regard to decisions on the asset mix.

According to a recent survey conducted by Heidinger Inc. (40), pension plans in many relatively large companies are "not receiving the management attention they deserve". Management tend to focus their attention more towards other human resource programs such as direct compensation and health & welfare plans. In addition, the survey also indicated that many companies "seem to believe there is little they can do to manage their pension contributions or the levels at which their pension liabilities are covered by assets".

In view of the above developments, coupled with the uncertainties of inflation and sluggishness in the economic growth, pension fund managers of today are faced with the dire need to improve the financial management of their pension plans. Efficient management of pension portfolios is of vital importance in optimizing the pension fund in order to offset the increasing pension costs and improve profit and benefit levels. As a prerequisite for better funding and investment strategies, pension managers need to formu-
late better decision-making techniques. It is therefore
the intention of this study to provide decision-makers
with a normative model that can be applied to help allevi­
ate pension fund asset management problems.

OBJECTIVE

The purpose of this study is to demonstrate a multiple
criteria decision-making technique that might be used by
pension investment managers to select optimum funding and
investment strategies for a defined benefit1 trustee pension
plan. An interactive multiple goal programming model
will be developed to assist decision-makers in the selec­
tion of the pension fund portfolio asset mix as well as to
efficiently allocate the appropriate amount of contribution
made periodically to the fund.

In managing pension funds, the vital decision-making
criteria (related particularly to investment and funding)
are: (a) the choice of plan and level to which benefits are
provided; (b) the method by which benefits are financed;
and (c) the investment of pension fund assets (53).

1 the kind of plan in which the benefits are set in advance
by some actuarial formulation based on employee earnings
and length of service; the contributions from the spon­
sors being the dependent variable.
In dealing with the problem of investment and funding of a pension fund, a number of methods, particularly those related to operations research, have been suggested. In general, these methods can be categorized into two different and distinct forms. The first is the construction of simulation models (25), (32), (49), (53), the second is the formulation of mathematical programming models (12), (48). This study will employ the second type. However, unlike the methods that have been suggested previously, the model to be used here will attempt to handle the problem of pension funding and investment on the basis of multiple goals or objectives. In addition, this model is a "systematic procedure for gaining insight into the role of the various conflicts and uncertainties" (47) inherent in pension fund management. Using these insights, management should be able to achieve new solutions as well as relate these solutions to the pension fund.

**DECISION SUPPORT SYSTEM (DSS)**

In dealing with the problems, the use of quantitative techniques with the aid of computers is almost inevitable. Owing to the rapid advancements in computer technology many attempts have been made to merge the use of computers and the decision-making process. Out of this combination of
managerial judgment and computer support (15) grew the concepts and techniques now collectively termed decision support system.

Decision support systems (DSS) which emerged in the late 1960s are computer-based systems enabling analytical inquiry of data on an interactive basis (16). These systems attempt to improve the quality of decision-making rather than improving efficiency. Therefore, instead of attempting to make faster and more accurate decisions, the emphasis is on improving the results of the decisions or making better decisions. As compared with the traditional operations research methods, DSS may include an optimization model but the responsibility of making the final decisions still relies on the judgment of the manager (22). In short, DSS is a system that effectively attempts to exploit the technological capabilities of computers along with that of man and his cognitive skills.

DSS is becoming a highly valued addition to many organization management systems. For instance, in the area of portfolio management systems, DSS has been commonly used by investment managers in handling their clients' portfolios. As Meredith & Turban (33) indicated, DSS now allows subjective judgment by the decision-maker through interactive sessions between the computer, analyst, and manager and vice versa, as opposed to the traditional operations
research approach whereby appropriate investments are selected and optimal allocations of the clients' funds obtained. In so doing, the systems update information from time to time and also consider preferences of clients/managers in accordance with the obtained results.

The model that is to be used here - the interactive multiple goal programming model - is also a type of DSS in that it is computer based and it involves mutual and successive interplay between a decision-maker and an analyst.

METHODOLOGY

(1) Introduction

Traditionally, decision-making models have been based on the single objective of profit maximization. This approach, however, has been seriously probed by many researchers including some early ones such as Chamberlain (6), Hurwicz (19), Cyert & March (8). Bilkey (4), in his article entitled "Empirical Evidence Regarding Business Goals", analyzed the research findings of nine different studies which concluded that the firm in practice does not have a single objective of profit maximization. While it is useful to use cost or profit as a measure of value for most firms, not every feature of the outcome can be measured in these terms. Most firms also have several other
objectives such as customer service, goodwill, safety and other intangibles that cannot be expressed in terms of cost or profit.

In many real life situations multiple and conflicting criteria are often involved. Business establishments, for instance, are made up of different groups of individual: owners, management, employees, customers, etc., and each of these groups pursue their own goals or objectives. For example, the object for assessing the problem of bank portfolio selection (47), are: (a) to maximize stockholder's equity; (b) to maximize cash covering of bank deposits; (c) to maximize loans made; and (d) to minimize the loan-deposit ratio. Pension asset management program objectives may include: (a) reducing the contribution costs; (b) maximizing the returns on investments; (c) reducing the risk level in investments; and (d) immunizing the investment against unexpected changes in the market rate of return.

To solve these multiple-criteria decision problems, various methods have been suggested and utilized. MacCrimmon, (29) classifies the procedures into four main types, namely (1) weighting, (2) sequential elimination, (3) mathematical programming, and (4) spatial proximity methods. The method that is pursued in this study is of the third type, mathematical programming, which includes linear programming, goal programming, and the more recent interactive multi-criteria programming.
(ii) Goal Programming

Goal programming is a management science technique that is a derivative of linear programming which has been perfected in large measure by Charnes & Cooper (7), Ijiri (21), Lee (27), and Ignizio (20). Unlike linear programming a goal programming model does not have single cost-minimizing or utility maximizing objective. Instead, it can handle problems with multiple goals and multiple sub-goals. At the same time it also provides the decision-maker with an opportunity to include, in the problem formulation, objectives which are not reducible to a single target.

Goal programming, in essence, involves the identification of certain specific goals and the definition of new variables (or deviations) that measure the extent to which each goal deviates from what is to be achieved. As a result, a new objective function is developed in terms of the new variables or deviations. This new function is then minimized using similar algorithms as that used in linear programming.

According to Klock & Lee (26), the basic premise of goal programming is that, "when faced with multiple conflicting objectives, a decision-maker will assign priorities to each goal and attempt to minimize the undesirable deviations from the ordered goals rather than attempt to fully achieve
incompatible goals". Unlike other models of multiple objective problems, goal programming requires the decision-maker to designate an objective level they wish to attain for each goal in the model. The general purpose of the model then is to minimize the weighted sum of the deviations from these objective levels.

(iii) Interactive Multiple Goal Programming

Even though goal programming in many ways corresponds closely with decision making in practice, this technique has one important drawback. It requires a fairly detailed amount of a priori information about the decision maker's preferences. It needs to define the aspiration levels, divide the set of goals into priority classes, estimate and attach weights to each priority class. Furthermore, once a ranking has been established among the goals, it will remain unchangeable. In other words, "first order priority goals determine the solution space of goals of less important priority classes" (45).

In an effort to overcome the above problem, management scientists have developed sets of procedures collectively known as interactive methods. The use of interactive techniques for multiple criteria algorithms has been well established and used by many researchers such as Dyer (10), Geoffrion (13), Zionts (56), and others. According to Monarchi (34), "the advantage of the interactive mechanism is
that it embeds the decision maker within the algorithm so it is not necessary for him to express his preference structure analytically prior to the generation of information". Thus, by means of interactive procedures the decision-maker becomes closely involved in the solution-making process while being required to possess relatively little a priori information. Various forms of interactive procedures appear in management science literature. Spronk \(^{(45)}\) has categorized them into three groups, namely:

"(a) methods in which the decision-maker has to determine trade-offs among the goal variables at each iteration, given the goal values in the current solution;

(b) methods in which the decision-maker has to choose the 'best' solution from a limited set of (generally efficient) solutions to each iteration;

(c) methods in which the decision-maker at each iteration has to define minimum and maximum values for one or more of the goal variables which in most methods are translated into restrictions reducing the feasible region."

However, most of the usual interactive approaches lack advantages of traditional goal programming, such as the possibility of including pre-emptive priorities. Therefore, in an attempt to combine the advantages of goal programming with that of interactive procedures (of which he favors group (c)), Spronk \(^{(45)}\) introduced the interactive multiple goal programming (IMGP) method. (see Appendix A)
MODEL FORMULATION

In formulating the model the main considerations will be the choice of pertinent goals that could sufficiently describe the decision-making process of pension funding and investment and the constraints to be encountered.

The pension fund illustrated by Trowbridge (52) is like a large reservoir with various inlets and outlets. The inlets consist of contributions and investment income coming into the fund while the outlets consist of benefit payments and expenses of operations. Since the study is concerned with a defined-benefit pension plan which provide fixed benefits to retirees upon retirement, the outlet or the outflow factor of this model, i.e., the benefits, will be considered as an exogenous variable that is derived outside of the model by actuarial valuation. Therefore, the attention of this study will be focused only on the inlet factor, i.e., contributions and investment income.

In the investment of a pension fund, the main concern will be toward two interrelated factors, the expected rate of return on investments and the risk that the management must take in order to obtain the desired return. The model used in this study adopts a passive buy and hold strategy for an all-bond portfolio. With a portfolio of this kind

\[2\] essentially this means, buying and holding a security or bond to maturity or redemption and then reinvesting cash proceeds in similar securities.
the three goals or objectives to be aimed are: (a) to maximize the expected return on the investment; (b) to minimize any default risk — the risk that the issuer of the bond will default in the contractual payment (principal as well as coupon payments) —; and (c) to minimize the sensitivity of the pension fund to movement in market interest rates, i.e., minimizing any reinvestment risk.

As for the funding, the focus will be on minimizing the contribution cost which in this case is represented by the proportion of fund outflows financed by cash contributions. This factor will determine the type of funding policy, e.g., either full funding, pay-as-you-go policies, or perhaps partial funding, that the management should adopt.

Another factor that is as important as the goals to be considered are the constraints. In formulating an interactive multiple goal programming model, an analyst will encounter two types of constraints: the goal constraints, and the usual constraints. Goal constraints are restrictions imposed on the various goals included in the model. These constraints are usually determined by the decision-maker and can be altered from time to time interactively. The usual constraints are fixed constraints, such as those used in linear programming, which account for the economic and other limitations of the system that the model has to recognize and which will confine the decision variables to values that are attainable.
Given the various goals and constraints and using the IMPG procedures given in Appendix A, a decision-maker would be able to obtain the best attainable values of the goal variables as well as the other decision variables that could determine the best asset mix combination in a given pension fund portfolio.

IMPORTANCE OF STUDY

The model in this study represents an advancement in the state-of-the-art in pension asset management. This model is important especially in view of the fact that previous methods have not been able to explicitly formulate a solution to the multi-dimensional asset management problem involving pluralistic criteria. In problems where the multiple goals can be measured or represented using a common term, a method using linear programming can only formulate it as a single-objective function problem subject to a number of constraints. In other words, if there exists more than one goal in a problem, this form of model tends to regard other goals as constraints. It is important to point out that management goals are not constraints. In problems where the goals are conflicting and not measurable by a common yardstick, a linear programming formulation will produce infeasible solutions. Furthermore, owing to
the interactive nature of the model, subjective judgments made by the decision-makers can be accommodated to influ-
ence the results.

Furthermore, with online interactive computers now easy-
ly accessible and becoming more and more an integral part of the decision making process, the introduction of the interactive multiple goal programming procedure will add another dimension to decision making in pension fund man-
agement.

As Simon (44), in his bounded rationality theory empha-
sized, the human brain's two limitations, namely (a) limit-
ed information-storing capacity and (b) limited retention capacity, will lead managers to look for "good enough" solutions rather than optimal answers. Therefore, IMPG attempts to broaden pension fund manager's limited ration-
ality by adding the computer's abilities to those of the limited human brain. In so doing, the decisions made will be improved and be more efficient.

OUTLINE OF THE STUDY PRESENTATION

This study is organized into five chapters. Chapter I contains the introduction to the study, the objectives, the importance and the rationale underlying the methodology.
Chapter II presents a review of related literature on pension funding and investment in particular those using the simulation and mathematical programming techniques. This chapter will also include a section that explains the interactive multiple goal programming procedures and how they will fit into the evolutionary development process of pension fund management.

In chapter III the IMPG model for pension funding and investment will be developed. The chapter takes into consideration the goals, constraints and the relationships between the goal variables and other decision variables.

Chapter IV presents empirical results from the developed model using a given set of data. An analysis of how the solutions are derived is also offered.

Chapter V summarizes the study, providing possible extensions to the model and also its the limitations.
CHAPTER II
REVIEW OF RELATED LITERATURE & MODELS

This chapter focuses on two different approaches to decision-making in pension fund management that have been used by previous researchers. In the first section various simulation models that are pertinent to this study are reviewed. This is followed by discussion of studies using mathematical programming models. Finally the last section deals specifically with the interactive multiple goal programming model that is being proposed in this study.

SIMULATION STUDIES

In the last few years, there have been a number of studies using the simulation approach (2), (25), (32), (49), (53) to handle pension funding and investment problems. Basically, a simulation technique describes and predicts the characteristics of a given pension fund under different settings or scenarios. It investigates the results or outcomes of numerous alternative strategies as reflected by

- 17 -
the system's performance on key criterion variables. In the case of a pension portfolio, different scenarios of investment strategies involving a different mix of bonds, stocks and other investment instruments are developed. Since these different security mixes have direct impact on either the investment return or pension costs, the 'mix' that produces the highest return or lowest pension costs may be perceived as the "best mix".

In an attempt to present a systematic analytical approach to the setting up of an investment policy for pension funds, Bergstrom & Frashure (2) applied a seven step simulation process. The first three steps projected the future rates of return from the various investment instruments (bonds, equities, etc.), determined the variability of these returns and obtained the correlation between them. Having generated the rate of return, its variability and correlation, a simulation was performed as step four to obtain the different investment outcomes over a period of time. Then, using the simulated results derived in step four, the probability of meeting critical return targets as that set by the actuarial valuation of the fund was determined. In step six, the sensitivity of pension costs to the various asset mixes obtained above was then identified. Bergstrom & Frashure suggested sponsors to prepare a simulation of their liability structure in order to determine
the impact of the various investment outcomes on plan costs. Finally, in step seven, they proposed that a systematic mechanism be developed to translate the above long-term asset mix forecasts into a "specific short-range stock/bond short-term investments mix for each portfolio under management", so as to accurately represent capital market realities.

In his article "Risk Versus Return in Pension Fund Investment", Tepper (49) adopted another simulation strategy in explaining the effect of different combination of asset mixes on pension costs. His pension fund financial planning framework consists of (a) projecting the workforce and their salary increases - so as to determine the liability forecasts (b) projecting the asset values through various investment policies and capital market forecasts and (c) the simulation of the fund's financial status. This technique which permits pension fund managers to measure the impact of alternative investment mix on pension costs can also "assist a sponsor in projecting pension costs, in testing his actuarial model against experience, and in determining the cost of proposed benefit liberalization".

More recently there have been other contributions made on the subject of pension fund management using simulation techniques. Kingsland (25), for instance, uses a pension projection model (PENSIM) which generates liability and
investment results for any given plan on an annual basis. These results are then compared in order to determine the amount of unfunded liabilities and the portion of liabilities accrued for the next fiscal year. Here again, a different set of investment asset mixes are used to match the liabilities.

Winklevoss (53), developed the pension liability and asset simulation model (PLASM). This model consists of four basic parts, namely: (a) deterministic population simulator - which simulates the membership of a pension plan; (b) deterministic liability simulator - which performs actuarial valuations in order to determine the annual contribution to the plan and related liabilities; (c) stochastic inflation and asset simulator - which produces returns from the different asset categories jointly with future anticipated and unanticipated inflation -; and (d) stochastic liability simulator - which integrates the results of the first three parts to predict the firm's payroll, its pension costs and the assets and liabilities of the fund. This model coordinates three vital decision policies in a pension fund namely the benefit policies, funding policies and investment policies which Winklevoss felt were missing in many of the pension plans.

Marshall (32) also proposed the use of a simulation process to handle the problem of pension asset mix decisions
in an inflationary environment. In his model, both the asset and liability side of the pension fund were simulated. On the asset side, three forms of investments (bonds, stocks and money market instruments) were used to determine the different required asset mixes. The liability and asset sides were brought together so as to evaluate the contribution factor. The latter was then compared with the actual cash contribution. The difference of these two entities were termed contribution adjustments which were then used as a foundation for an overall decision criteria. The asset mix which minimizes the expected present value of the contribution adjustments, will then be selected as the best asset-mix for the investment of the fund.

As a decision-making tool, simulation is a powerful and a popular technique which is being increasingly utilized with the advent of the computer. In the management of pension funds, simulation could be used to generate projections for all key elements of a pension plan. However, in estimating these outcomes a great number of iterations or trials would be required because the greater the degree of required precision, the larger the number of settings needed to be simulated. This could prove to be a slow and expensive process.

As opposed to analytical or mathematical models, simulation is a descriptive rather than a normative tool. As a
model it does not have an ordered arrangement or structure of an objective function and constraints, and as a process it will produce, in many instances, suboptimal results. For this reason, a management scientist would first prefer to use an analytical approach. However, there are many complex practical problems that cannot be easily solved using the analytical approach. In these cases then, simulation techniques would be the choice of method.

MATHEMATICAL PROGRAMMING MODELS

Mathematical or analytical models are normative in nature. They are formally structured and are usually comprised of three components (33): (a) result variables - dependent variables that indicate how effective the system is; (b) decision variables - independent variables that can be manipulated and controlled; and (c) uncontrolled variables - factors that are beyond the control of the decision makers. These components are mathematically related and in turn form the main framework of a model consisting of the objective function and the constraints.

Unlike simulation models, a mathematical programming model produces and optimizes any decision of interest within the model itself. It has the advantage of having mathematical characteristics and properties with which one can
study the behavior of any particular system as well as its sensitivities to change. These characteristics are essential elements that enable the model to predict and produce accurate and reliable results. Furthermore, by being able to quantify a problem in a realistic fashion, a computer could be used to program the problem, manipulate its variables so as to test any effect on other variables and to produce a result.

In the area of investment and financial management, many mathematical programming models have been suggested and used including (28), (31), (38), (43). However, within the context of pension funding and investment only a few applications are found. In this section, two contributions namely that of Tepper (48) and Frankfurter & Hill (12) will be discussed.

Tepper, (48) in developing a normative multi-period model to determine an optimal strategy for pension funding and investment, used dynamic stochastic programming as a decision-making technique. Basically, it is a unidimensional model with the single objective of minimizing the multi-period cost of contributions to the fund. Tepper analogized decisions made in a pension fund with regard to contribution and investments to that of consumption and saving decisions made by consumers. Just as the consumer is seen to choose levels of consumption and investment in a
sequential fashion, Tepper deduced that the contributions and investment levels in a pension fund are selected in a similar way.

The recursive nature of the dynamic programming solution process allows the model to produce optimal solutions at every stage of the problem until the end of the period being considered. This can be regarded as an advantage because the model is able to develop some course of action for the future and in this case provides possible market values of the pension fund. The whole operation can be done in one sequence.

However, the model also poses some drawbacks. First, in selecting the investment portfolios the choice was not made explicitly on the basis of risk/return tradeoff but rather, it was based only on the expected returns of different asset mixes during a given period. Also, the model is devoid of any checking mechanisms or constraints to control the risk on investments. Secondly, this form of programming method is usually tailored to a specific problem; each time that a new problem occurs a different formulation has to be developed. Furthermore, as the size of the problem grows - in this case the number of alternative investment choices - the number of computations required will increase tremendously, thus making it difficult to handle.
Another normative approach to pension fund management was offered by Frankfurter & Hill (12). In formulating the problem, they applied a linear programming model with their objective being "to minimize the discounted value of the increments to pension fund value over the planning horizon subject to constraints on investment risk, funding levels prescribed by law and periodic benefit payment obligations". This objective can be seen to be quite similar to that of Tepper's (48) i.e. to minimize the contribution cost to the pension fund.

The model involves a three stage decision-making process: (a) the forecasting of expected benefits using selected plan design and actuarial assumptions which will assist the decision-maker in determining the minimum contribution level; (b) the determining of the level of contribution made to the fund in correspondence to the projected expected benefits; and (c) the determining of the composition of the pension fund portfolio asset mix. As a result, the model is able to produce the annual contribution to the fund over a given period of time, the expected return of the portfolio, other related decision variables as well as the composition of the investment portfolio while at the same time recognizing the impact of risk on the various result obtained.
By Comparison, the linear programming model proposed by Frankfurter & Hill (12) is more easily solved than the stochastic dynamic programming model mentioned earlier. In addition, this model contains an important feature. It includes risk constraints, which are missing in the previous model. These enable a decision-maker to make efficient solutions pertaining to the investment portfolio on the basis of a mean/variance criterion.

The linear programming model introduced by Frankfurter & Hill (12) does provide a useful framework for pension managers to make decisions with regard to the funding and investments of the pension fund. This model not only produces optimal contribution values but also allocates the appropriate investment portfolio mix. However, as a constrained optimization model, this formulation can only handle one objective function at a time, i.e., in this case the minimization of contribution costs, for the optimization criterion.

In a real-world problem such as the managing of a pension fund, simultaneous objectives are often involved. As Zeleny (55) indicated, "no decision making occurs unless two criteria are present. If only one criterion exists, mere measurement and search suffice for making a choice ...theoretically then, if there is only a single criterion

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3 model consisting of an objective function and a number of constraints.
which can be perfectly measured and efficiently searched for, indeed no decision making is involved". Zeleny also pointed out that only in a time-pressured situation, such as in an emergency or a crisis, will a setting with a single criterion be encountered. Mao (30) and many other researchers have also indicated that in practice management seeks a number of functional objectives rather than just a single optimizing criterion.

In many problems where multiple objectives exist, a linear programming model was used as the optimizing tool. In situations such as this, only one of the goals would be chosen to represent the objective function while the other remaining goals would be included as constraints. Perhaps, in cases where the goals are all measured in some common unit, the multiple goals can be reduced into a single objective.

In many other instances, however, these goals could not be condensed into a single criterion or stated in common terms. This could be due to many different reasons such as: (a) the objectives cannot be measured in one common unit; (b) the goals cannot be directly compared; (c) some of the goals are intangibles, thus, they could not be quantified; (d) the goals are on a different ranking basis; or (e) even if the goals can be measured in a common unit, the results obtained would not prove to be useful to the deci-
sion maker. As such, the representation of each of the different types of goals or objectives becomes essential and necessary.

**Interactive Multiple Goal Programming Model**

As explained in the first chapter, various methods are available to handle problems with multiple objectives. Goal programming, for instance, has been cited as one of the better methods that could realistically represent a multi-criteria problem in its formulation. However, its procedures require that the decision-maker's preference on the various goals, i.e., the preemptive priority factors, be adequately and explicitly represented. This creates some difficulties because in practice, these priority factors or aspiration levels cannot easily be obtained and quantified. To remedy this problem, Spronk (45) introduced the interactive multiple goal programming model which is utilized in this study to handle the problem of pension funding and investment.

Using the interactive multiple goal programming (IMP3) technique, for each objective function, the 'best' or 'ideal' and the 'worst' or 'pessimistic' values are computed and placed together as vectors in a matrix called potency matrix by the analyst. The first solution to the prob-
lem is obtained from the various minimum achievable or pessimistic values of each goal. The first solution together with the corresponding potency matrix is presented by the analyst to the decision-maker for their assessment. If the solution was found to be satisfactory or if no potential improvements to the goal variables could be expected, then the operations would be terminated and the first solution would be accepted by the decision-maker. Otherwise, the decision-maker would instruct the analyst to raise the minimum value of one particular goal that they feel should be changed. The decision-maker would specify by how much the goal should be increased. The fact that a certain objective value is required to be altered indirectly indicates that this is an important change that needs to be made at that point in time by the decision-maker. After the change, a new potency matrix would be computed and a new solution derived. This again would be presented to the decision-maker. It should be noted here that the decision-maker at this point has to specify whether the changes made to the pessimistic solution outweigh the changes in the resultant ideal value or not. If it does not, then a new pessimistic value has to be defined. The process will continue until, finally, the decision-maker is satisfied with the solution consisting of the values of the different goals or objectives.
In the process of obtaining the most satisfactory values for the different goals in the problem, the procedure also produces other derived optimal decision variables. For instance, by using this procedure in this study, the pension manager will be able to obtain values of the other decision variables such as the proportions of the fund to be invested in the various investment instruments.

As an interactive device the IMPG procedure provides direct interaction between the analyst and the decision-maker via an online computer facility. It features a dialogue between the decision-maker and the computer and at the same time poses a learning process that will assist the decision-maker in examining the tradeoff or compromise between the different objectives. In addition, the availability of computer-graphics also helps to reduce time and effort in preparing the input data and also enables decision-makers to get feedback instantly.

As mentioned earlier, the IMPG is a hybrid of goal programming and interactive procedures. As such, it has the advantages of both approaches. As a goal programming technique, IMPG has the capability of incorporating multiple conflicting goals that appear in any particular problem. Additionally, as an interactive procedure, IMPG requires only minimum preference information from the decision-maker to solve problems. The decision-maker in this case becomes
more closely involved in the problem-solving process and might benefit greatly from the learning effects that the procedure provides. These learning effects permits the decision-maker to evaluate the sensitivity of his judgment toward any decision made.

Finally, in comparison to other methods, the IMG is a relatively simple technique to understand and to be programmed on the computer. The steps used in the procedure to derive solutions (see Appendix A) occur in a logically sequential and straightforward fashion. In this regard, the IMG can be considered a powerful decision-making tool.

This procedure is particularly appropriate in managing pension funds on an in-house basis. It could either be used to produce short-run solutions or to develop multi-period long-run forecasts. In this study, the focus is on a single-period model that can be used to assist pension managers in making decisions on short-term basis.
CHAPTER III
MODEL FORMULATION

This chapter outlines the formulation for a single period interactive multiple goal programming model for pension funding and investment. First, the various goal variables is discussed. Four relevant goals for an all-bond portfo- lio are proposed and these goals are directly related to the decision-making process of investment and funding. Next, the relationship between the goals are presented and finally followed by the constraints that are imposed on the model.

INTRODUCTION

Mathematical models that have been suggested previously to handle the problem of pension funding and investment are unidimensional in nature. Tepper (48) and Frankfurter & Hill (12) for instance, introduced the dynamic stochastic programming and linear programming technique, respectively.

* the portfolio consists of corporate bonds and U.S. government bonds.
Both of these methods are focused on a single objective, i.e., to minimize the cost of contributions to a pension fund.

However, most real decision situations are characterized by multiple goals or objectives rather than by merely a single objective. In the case of the two methods mentioned above, if there exist more than one goal, e.g. maximizing expected growth, minimizing volatility, and maximizing current dividend, in the selection of an investment portfolio by Lee & Lerro (28), these goals will not be presented in the problem as goals but rather are relegated to being constraints. It is important to make clear that management goals are not constraints. They are goals or targets which do not have to be satisfied at all times. If they are treated as constraints such as those in a linear programming formulation, then, in obtaining a feasible solution, all constraints must be satisfied simultaneously.

FORMULATION

A pension fund may be thought of as a large pool with one or more inlets and outlets (52). Inlets consist of contributions and investments income. Outlets are made up of benefit payments and other supplemental payments - additional normal costs from changes to actuarial assumptions
that increase benefits such as salary growth, workforce growth, and the increase in cost-of-living allowance. The pension fund manager who is responsible for this so-called reservoir of funds has some degree of control over the inflow and/or outflow and the use of his authority is carried out in such a way so as to achieve objectives consistent with the underlying goal of obtaining an optimal portfolio mix for the pension fund assets.

In describing the inflows and outflows of a pension fund the following symbols will be used throughout in the model:

\[ C = \text{Contribution made by the sponsors during year } t; \]
\[ I = \text{Investment income obtained from the fund's portfolio during year } t; \]
\[ B = \text{Benefits paid to retirees in year } t; \]
\[ f_t = \text{The fund size in year } t; \]
\[ A = \text{The rate of growth in } f_t \text{ that accounts for supplementary increases in benefits;} \]

Using the equation of maturity similar to that used by Marshall (32) both the inflows and the outflows can be equated as follows:

\[ C_t + I_t = B_t + \Delta A_t A \quad \text{(3.1)} \]
**Goal Variables**

As earlier mentioned, this study is concerned with two important decision-making areas in pension fund management, namely funding and investment. Therefore, goals to be introduced here will correspond to these two policies.

In investment portfolio analysis, two underlying assumptions are used; namely, (a) any investor would want to maximize the return from his/her investments, and (b) investors are basically risk averse. Investment objectives are usually established based on these assumptions and they often involve the tradeoff between risk and return. Risk and return of an investment are said to be inversely related to each other. In other words, the higher the rate of return one wishes to obtain, the riskier the investment one has to undertake and vice versa. Therefore any evaluation of a portfolio should have goals pertaining to these vital concepts of risk and return.

This model adopts a passive buy and hold strategy for an all-bond portfolio. In a portfolio of this nature, the sources of return will be of three types: (1) coupon payments; (2) appreciation or depreciation of price; and (3) reinvestment returns, also known as the 'interest-on-interest' factor. This means that a portfolio return is being governed by coupon payments and reinvestment proceeds. Thus any decision-making goals involving the
investment of this type of portfolio will be centered around three functional characteristics of the portfolio namely, (1) the maximization of expected returns on the portfolio, (2) the minimization of any default risk or the payment of coupons and principal of bonds in the portfolio, and (3) the minimization of any reinvestment risk owing to changes in the market rate of interest.

Maximizing the Expected Rate of Return

Various measures of return have been suggested in the literature to measure the return on bonds and one of the more widely used measurement is the yield-to-maturity (YTM) or promised yield. This measure can be computed by the following formulation:

\[
YTM = \frac{A + (P - C)/n}{(P + C)/2}
\]

where,

A = Annual Coupon Income;
P = Price of Bond at Maturity;
C = Current Market Price of Bond;
n = Years remaining till Maturity.

which "indicates the fully compounded rate of return available to an investor, assuming the bond is held to maturity" (14). As a measure of return, yield-to-maturity poses sev-
eral problems. Homer & Leibowitz (18) have suggested that another measurement called "realized compound yield" be used. However, due to the fact that most of the data available to be used in this study is based on yield-to-maturity, the former mode of measurement of return will be adopted in this model. If,

\[ X_t = \text{the proportion of the pension assets invested in the } i^{th} \text{ bond in year } t, \]
where \( i = 1, 2, \ldots, n; \)

\[ R_t = \text{the yield-to-maturity of the } i^{th} \text{ bond in year } t \text{ and} \]

\[ R_{pt} = \text{the rate of return on the pension fund portfolio;} \]

then,

\[ R = (X_{R1} + X_{R2} + \ldots + X_{Rn}) \]

\[ pt \quad 1t \quad 1t \quad 2t \quad 2t \quad nt \quad nt \]

If the expected value operator is applied to both sides of the above equation:

\[ E(R) = E(X_{R1} + X_{R2} + \ldots + X_{Rn}) \]

\[ pt \quad 1t \quad 1t \quad 2t \quad 2t \quad nt \quad nt \]

---

5 As pointed by Fong & Fabozzi (11) the problems are: (a) it assumes that the coupon payments can be reinvested at a rate equal to the computed YTM; (b) it also assumes that the whole amount can be reinvested (which is not plausible); and (c) it assumes that the bond will not be redeemed before the maturity date.
the expected rate of return of the portfolio in year \( t \);

\[ E = \text{the expected rate of return of the } \text{ith bond in the portfolio in year } t. \]

The expected rate of return of the various bonds can be estimated using a method adopted by Wagner & Tito (51). Using capital asset pricing techniques\(^6\) to bond portfolios, they constructed a bond market line (see Figure 1) where duration\(^7\) (which in this case is defined as a measure of interest risk on bonds) instead of beta (the market risk of a security) was used as the measure of risk.

Accordingly the equation representing the bond market line will be,

\[ E(R) \text{ or } E = R + E(R - R)D / D \]

where,

\(^6\) technique based on the model that relates the risk of a security measured by beta (market risk of a security) to the level of expected return.

\(^7\) a concept developed by Frederick R. Macauley - Some Theoretical Problems Suggested by the Movement of Interest Rates, Bond Yields and Stock Prices in the United States since 1856 (New York: National Bureau of Economic Research, 1938)
\[ R = \text{the risk-free rate;} \]
\[ E(R_{mt}) = \text{the expected market rate of interest;} \]
\[ D_{mt} = \text{the duration of the market;} \]
\[ D_{it} = \text{the duration of the } i^{th} \text{ bond.} \]

Using the above equation the values of \( E_{it} \) can be estimated. However, as Reilly & Sidhu (41) pointed out, the above bond market line does not take into account the differences in the default of bonds. Therefore in a portfolio consisting of investment grade bonds of different ratings (ranging from Aaa to Baa), a different set of equations must be constructed to represent each particular rating. As demonstrated by Reilly & Sidhu (41), an ideal set of bond market lines are constructed each representing a different bond rating (see Figure 2). In an actual situation, as shown in the next chapter, the bond market lines are usually not parallel to each other.
Figure 1: Bond Market Line
Figure 2: Ideal Bond Market Lines With Different Default Ratings
Once the different values of $E$ are estimated then it

the equation representing $E$ (equation 3.2), i.e., the pt

equation representing $E$ (equation 3.2), i.e., the

expected rate of return of the portfolio, can be formed.

With this equation then one can summarize the objective

of the first goal.

**Minimizing the Default Risk of Investment**

In adopting a passive buy and hold strategy, an all-bond

portfolio is virtually controlled by coupon payments and reinvestment proceeds. This means that the issue of mini-
mizing the risk of default on the coupons and principal

payment is an important goal that an investor or pension fund manager should take into consideration.

Government bonds are considered "riskless" securities.

However, they are subject to interest rate and purchasing

power risks. Other types of bonds such as corporate bonds, considered in this study, are subject not only to the two kinds of risk just mentioned but also to credit or default risks. It can be deduced therefore, that the yield of the "riskless" bonds are influenced by interest rates and pur-

chasing power factors. Furthermore, the yield of corporate bonds is affected by these two factors as well as default or credit factors. If the difference of the yields of these two types of bonds i.e. yield spread or risk premium, are taken into consideration, one sees that the difference
in yields is compensated for the added risk. Under certain simple models of default, Yawitz (54) and Bierman & Hass (3) have shown that the yield spread is directly proportional to the probability of default.

Consider the yields of government bonds (Y) and the yields of the other types of 'risky' bonds (Y). If D is the yield spread for the ith bond then

\[ D_i = (Y_i - Y_{ig}) \]

Each of the 'risky' bonds are rated by the different rating agencies such as Moody's, Standard & Poor's, Fitch Investors Service, Duff & Phelps, and others. Given the ratings as that found in Duff & Phelps, which are given in terms of numbers, one could then regress these ratings against the corresponding yield spread. As shown in Appendix B, the relationship between the two variables was found to be linear. Expressing the two variables in terms of a regression equation,

\[ D_i = \hat{a} + \hat{\beta} R_i \hspace{1cm} (3.3) \]

where,

\[ D_i = \text{yield spread between the ith corporate bond and a government bond} \]

8 in regressing such ordinal measures as ratings, there are two school of thought; one allowing this form of manipulation while the other do not.
\[ R_i = \text{rating of the } i\text{th bond;} \]
\[ \hat{\alpha}, \hat{\beta} = \text{the relevant regression coefficient.} \]

For this second goal, \( D \) (the yield spread) is used as a proxy for the default risk of the \( i\text{th} \) bond. Therefore instead of minimizing the default risk, one could minimize the risk premium \( D_i \).

Minimizing the Sensitivity of the Pension Fund to Movement in Market Interest Rate

Both the assets and the liabilities of a pension fund are affected by changes in market rates of return. For instance, when the market rates of interest increase it will raise the discount rate which in turn will decrease the present value of the pension fund liabilities. This increase will also reduce the value of the fund assets. However, this sensitivity to the movement in interest rates can be reduced by the firm if it can be immunized. By immunizing a portfolio, one is guaranteed a minimum return performance over a specified investment period. This strategy also assists pension managers in securing high yields while simultaneously guarding their portfolios against the erosive impact of inflation. As Keintz & Stickney (23) indicated, "a pension fund is immunized for a
holding period if the value of its net assets at the end of the holding period, regardless of the course of market rates of return during the holding period, is at least as large as it would have been had the market rate of return been constant throughout the holding period.

For a fund with assets $A_t$ and liabilities $L_t$ at time $t$, then for the fund to be accordingly immunized during a period from $t$ to $t+1$, the necessary condition is:

$$\frac{A}{A_t} \cdot D_t = \frac{L}{L_t} \cdot D_t$$

where, $D_t$ and $D_t$ are the measures of duration to the assets $A_t$ and liabilities $L_t$ respectively and by definition,

$$D_t = \frac{\sum_{t=1}^{n} t \cdot CF_t / (1 + r)^t}{\sum_{t=1}^{n} CF_t / (1 + r)^t}$$

where,

$CF_t = $ cash flow to be received or paid;

$n = $ holding period;

$r = $ an appropriate discount rate;
Many of the pension funds observed in practice are either overfunded or underfunded. In fact, statistics on the assets and liabilities of pension funds (those published in the Value Line Investment Survey) indicate that many funds are significantly underfunded. In such a case, the duration weighted values of the assets and the liabilities would not be equal. For an overfunded fund (such as that illustrated in the next chapter) the difference is positive thus,

\[
\frac{A}{L} - \frac{L}{D} > 0
\]

If \( H \) is a non-negative deviation that exists between the values \( \frac{A}{L} \) and \( \frac{L}{D} \) then,

\[
\frac{A}{L} - \frac{L}{D} - H = 0 \quad \text{(3.4)}
\]

In an effort to immunize the fund, one should minimize the value of \( H \), ideally to zero. Thus, in order for the firm to reduce the fund's sensitivity to changes in the market interest rates, the pension manager must minimize the deviation between the duration weighted values of assets and liabilities of the fund.
Minimizing the Contribution Cost

The first three goals described above relate to the investment policy of a pension fund. However, the fourth goal describes the funding policy.

As given earlier, when the inflows and the outflows of a pension fund are equated the following results:

\[ C_t + I_t = B_t + \Delta A_t \]

Using the above relationship, management will determine the proportion of the outflows that is financed by contribution and also determine the proportion that is derived from investment. Using the same notation \( \rho (\text{Rho}) \) as used by Marshall (32) to represent the proportion of the fund outflows provided for by contributions then,

\[ \rho = \frac{C_t}{B_t + \Delta A_t} \quad \text{where,} \quad 0 > \rho > 1 \]

Using \( \rho \), one could determine whether or not an actuarial cost method employed by the fund is conservative. For instance, when \( \rho = 0 \), this implies that no cash contribu-
tions are made. The fund is totally dependent upon investment earnings. When $p = 1$, this signifies that the fund is financed solely by contributions, i.e., a pay-as-you-go method. This strategy is not permissible under ERISA.

So in order to determine the type of funding policy that should be pursued with the investment policy of the fund, the fourth goal is included to minimize the contribution cost. Therefore, for the fourth goal, the objective is to minimize $p$.

**Relationship Between the Goal Variables**

(a) Consider the above relationship,

\[ C_t + I_t = B_t + \Delta A_t \]

and

\[ p = \frac{C_t}{B_t + \Delta A_t} \]

Since $0 > p > 1$ then,

\[ (1 - p) = \frac{I_t}{B_t + \Delta A_t} \]

, the proportion of the fund outflow in year $t$ that is accounted by investment income.
But, \[ I = R \cdot A \] \( t \) \( pt \) \( pt \), where \( R \) is the rate of return of the portfolio in year \( t \).

and if,

\[ R = (X R_{1t} + X R_{2t} + \ldots + X R_{nt}) \]

then,

\[ (1 - \rho) = \frac{R \cdot A}{B + \Delta A} \]

\[ = \frac{(X R_{1t} + \ldots + X R_{nt})A}{(B + \Delta A)} \]

\[ (1 - \rho)(B + \Delta A) = (X R_{1t} + \ldots + X R_{nt}) \]

and,

\[ (X R_{1t} + \ldots + X R_{nt})A + \rho(B + \Delta A) = B + \Delta A \]

(b) As given in the first goal, the expected rate of return on the portfolio \( E \) is related to the various decision variables \( X, X, \ldots, X \) by the equation:

\[ E = (X E_{1t} + X E_{2t} + \ldots + X E_{nt}) \]
(c) By imposing the immunization condition as given in the third goal we have the relationship:

\[ A \cdot D - L \cdot D - H = 0 \]

In this case the \( A \) and \( L \) values are usually known, \( t \) \( t \)

the values of \( D \), \( D \) and \( H \) are to be determined accordingly, \( t \) \( t \)

(d) Considering the pension fund balance sheet at time \( t \), the assets will consist of two main entries, namely the pension assets and the present value of the future normal costs of the fund. The liabilities are the present value of the future benefits. When the assets are equated with the liabilities the following results:

\[ A + \sum_{t=1}^{\infty} C / (1 + r)^i = \sum_{t=1}^{\infty} B / (1 + r)^i \] \hspace{1cm} (3.5)

where,

\[ A = \text{asset of the fund;} \]
\[ \sum_{t=1}^{\infty} C / (1 + r)^i = \text{present value of the future normal costs;} \]
\[ \sum_{t=1}^{\infty} B / (1 + r)^i = \text{present value of future benefits;} \]
\[ r = \text{discount rate of interest;} \]
Assuming that the contribution increases at a rate of $g$ percent per year so that,

$$C_{t+1} = (1 + g)C_t$$

then, considering the left hand side of equation (3.5),

$$A + \sum_{i=1}^{\infty} \frac{C_i}{(1+r)^i} = A + \left( \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots \right)$$

$$= A + \left( \frac{C}{(1+r)} + \frac{C}{(1+r)} \left( \frac{1+g}{1+r} \right) + \ldots \right)$$

$$= A + \frac{C}{(1+r)} \left[ \frac{1}{1 - \frac{1+g}{1+r}} \right]$$

$$= A + \frac{C}{(1+r)} \left[ \frac{1}{1 - \frac{1+g}{1+r}} \right]^{-1}$$

$$= A + \frac{C}{(1+r)}$$

Therefore,

$$A + \sum_{i=1}^{\infty} \frac{C_i}{(1+r)^i} = A + \frac{C}{(1-g)}$$

Next, consider the right hand side of equation (3.5)

$$\sum_{i=1}^{\infty} \frac{B_i}{(1+r)^i}$$

If the annual benefit is assumed to increase annually at a rate of $g$ percent so that,
\[
B_{t+1} = B_t (1 + g)_{2t}
\]

then,
\[
\sum_{i=1}^{\infty} B_i / (1+r)^i = B_t / (1+r) + B_{t+1} / (1+r)^2 + \ldots
\]

\[
= B_t / (1+r) + B_t (1+g) / (1+r)^2 + \ldots
\]

\[
= B_t / (1+r) \left[ 1 + (1+g) / (1+r) + \ldots \right]
\]

\[
= B_t / (1+r) \left[ 1 - (1+g) / (1+r) \right]^{1-1}
\]

Therefore,
\[
\sum_{i=1}^{\infty} B_i / (1+r)^i = B_t / (r-g)_{2t}
\]

Equating the left and the right side of the equation,
\[
A + C_t / (r-g)_{2t} = B_t / (r-g)_{2t}
\]

From equation (3.4),
\[
A_{t}D_{t} - L_{t}D_{t} - H_{t} = 0
\]

Therefore,
\[ A_t = \frac{L_t D_t + H_t}{D_t} \]

Substituting the value of \( A \) in the above equation,

\[
\begin{align*}
  & \frac{L_t D_t + H_t}{D_t} + C_t = B_t \\
  & A_t (r-g_1) = (r-g_2) \\
  \end{align*}
\]

The unknowns in the above equations are, \( C, H, D \) and \( D \).

\( D \) the duration of the asset can be shown to be approximately equal to \( (n+1) \) (see Appendix C) where \( n \) is the holding period.

Thus, the above equation becomes,

\[
\begin{align*}
  & \frac{L_t D_t + H_t}{(n+1)} + C_t = B_t \\
  \end{align*}
\]

(e) As given in equation (3.3), the yield spread of the bonds in the portfolio was found to be linearly related to its respective rankings (see Appendix B).

Thus,
\[ D_t = \hat{\alpha} + \hat{\beta} R^t \]

the values of \( \hat{\alpha} \) and \( \hat{\beta} \) can be obtained from Appendix B.

Since \( D_t \) is the average yield spread of the portfolio in year \( t \), one could substitute for its value with the difference of the expected yield of the portfolio and that of the average yield of government bonds i.e. \( D_t = (E_{pt} - \bar{Y}_{ig}) \), where \( \bar{Y}_{ig} \) is the average yield of the government bond.

Since \( \bar{Y}_{ig} \) is normally a constant, the equation then becomes,

\[ E_{pt} - D_t = \bar{Y}_{ig} \] \hspace{1cm} (3.7)

**Constraints**

As mentioned previously, mathematical programming models consist of an objective function and a number of constraints. The constraints in a problem are restrictions on the values of the variables as well as a description of the various resources required by the model. Constraints divide all possible solutions into two sectors, the feasible and infeasible sectors. Their presence in a problem account for the technological, manpower, time, economic and other limitations of the system which cannot be violated.
In using the IMGP model, two types of constraints are used. The first type consists of the goal constraints and the second consists of the usual constraints which may appear in any linear programming formulation. The goal constraints, as opposed to the more rigid usual constraints, are flexible and can be interactively altered from time to time by the decision-maker.

(a) Goal Constraints

For each of the four goals described above there will be a constraint. For the first goal, the management of the firm would specify a limit on the minimum expected rate of return of the investment portfolio \( E \) that it would be willing to tolerate. In this case a goal constraint such as \( E \geq e \), where \( e \) is some suggested minimum rate of \( pt \) would be used.

If \( d \) is the maximum level of default risk on investment that a firm would be willing to undertake, this would be used as the initial limit to the second goal constraint, i.e., \( D \leq d \). It too could then be changed interactively in the process of solving the funding and investment problem.

With respect to the third goal, it must achieve the necessary equality in value between the duration weighted values of \( A \) and \( (A \cdot D) \) in order for the firm to immunize the fund. Initially a value of \( h \) (where \( h \) is a constant) can be used as the upper limit for \( H \) (the deviation between \( A \cdot D \) and
such that \( H \leq h \). As with the other goals, this value can be reduced interactively from iteration to iteration to decrease it to an ideal value of zero.

For the fourth goal, the value of \( \rho \), the proportion of fund outflows financed by cash contributions, must lie between zero and one. If the firm has a funding policy which indicates that at least \( p \) percent of the contribution to the pension fund outflows should come from investment, the firm is indirectly putting a limit of \( \rho \leq (1-p) \).

It is assumed that the percentage of the contribution to the pension fund outflow will satisfy the minimum funding constraint\(^9\) imposed by ERISA or a defined-benefit plan.

(b) Usual Constraints

This model also contains a number of constraints which are not interactively changed such as:

(i) Non-negativity constraints - the various decision variables \( (X_1, X_2, \ldots, X_n) \) must be non-negative i.e.

\[ X_i > 0 \]; this condition prevents negative investment values to appear in the model.

\(^9\) minimum funding must cover the normal cost plus the amortization of unfunded past service cost over no more than 30 years for new plans and no more than 40 years for existing plans.
(ii) Investment constraints - there are special constraints imposed on the proportion of the portfolio allocated to certain types of bonds. For example, the firm might set a predetermined percentage of government bonds to be held in the portfolio.

(iii) Constraints on the immunization condition factor. This is to ensure that the necessary condition for immunization is maintained.

(iv) Unity constraints. The value of the decision variables (X's) must add to unity. That is, the various proportions invested must amount to 100 percent. This is an upper limit investment constraint.

Summary

Summarizing the model in terms of its' goals, constraints and relationships between them as given above:

Goals:

(a) Max $E_{pt}$ (to maximize the expected rate of return of the portfolio)

(b) Min $D_t$ (to minimize the default risk of investment)
(c) Min $H$  
(to minimize the sensitivity of the pension fund to movement in market interest rate i.e. to minimize reinvestment risk)

(d) Min $\rho$  
(to minimize the contribution cost)

Goal Constraints:

\[ E \geq e \]
\[ D \leq d \quad \text{To be changed interactively from iteration to iteration} \]
\[ H \leq h \]
\[ \rho \leq \rho \]

Other Relationships and Constraints:

\[ (X_R + \ldots + X_R)A + \rho(B + \Delta A) = (B + \Delta A) \]
\[ E = (X_E + \ldots + X_E) \]
\[ A - D - L - D - H = 0 \]
\[ \frac{L}{(n+1)} + \frac{H}{(r-g)} + \frac{C}{1} = \frac{B}{(r-g)^2} \]
\[ D = \hat{A} + \hat{B}R \]
\[ E - D = Y \]
\[ A \cdot D - L \cdot D \geq 0 \]

\[ X_{1} \times X_{2} + X_{3} + \ldots + X_{n} = 1.0 \]

\[ X_{e} + X_{f} + X_{g} + X_{h} = P \]

\[ X_{j} + X_{k} + X_{l} = P \]

\[ X_{i} \geq 0 \quad (i=1, 2, \ldots, n) \]

\[ A \cdot D = n \]

\[ \rho \geq 0 \]

\[ H \geq 0 \]

\[ R \geq 1 \]

\[ R \leq 10 \]
CHAPTER IV
EMPIRICAL APPLICATION

This chapter empirically demonstrates the model developed in chapter three. It begins with the discussion of the data used for the study. This is followed by an evaluation of some preliminary relationships that constitute the framework for the model. Once all the equations and relationships are numerically set, the next step is the implementation of the IMPG procedures. Finally, the results pertaining to the various goals and decision variables are summarized and discussed.

DATA

The data used in this study are obtained from the following sources: (a) BEA Annual Pension Surveys (1); (b) Moody's Manuals (Industrial, Public Utility, Bank & Finance and Transportation) (36); and (c) Moody's Bond Record (35).

As mentioned in earlier chapters, the outlets or the outflows to a pension fund or the benefits, are considered
as an exogenous variable to the model used in this study. As such, the model will not specifically deal with the selection of any actuarial cost methods\(^{10}\), it will however assume that a firm has already chosen a particular cost method to generate its benefit.

**BEA Annual Pension Surveys**

Annually, BEA Associates publish a pension survey of approximately 40 companies in the United States. These companies are selected from ten different industries\(^{11}\). To improvise realism into the model, pension fund data from one of the companies, General Electric, was chosen for the study. Data from three annual pension survey reports for General Electric were extracted from the BEA Pension Surveys as shown in table (4.1).

---

\(^{10}\) the five actuarial cost methods are: (1) entry age normal method; (2) unit credit method; (3) aggregate method; (4) attained age normal method; and (5) individual level premium method.

\(^{11}\) the industries are: (1) automobile industry; (2) chemical industry; (3) drug industry; (4) electrical industry; (5) food processing industry; (6) miscellaneous; (7) office equipment industry; (8) retail industry; (9) steel industry; (10) tire & rubber industry.
### Table 1

**Pension Statistics of General Electric (1981-83)**

<table>
<thead>
<tr>
<th>Year</th>
<th>Pension Expenses (mil.)</th>
<th>Pension Fund Assets (mil.)</th>
<th>Vested Benefits (mil.)</th>
<th>Total Benefits (mil.)</th>
<th>Interest rate assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>549</td>
<td>6801</td>
<td>6203</td>
<td>6543</td>
<td>7.5%</td>
</tr>
<tr>
<td>1982</td>
<td>570</td>
<td>8682</td>
<td>7160</td>
<td>7688</td>
<td>7.5%</td>
</tr>
<tr>
<td>1983</td>
<td>643</td>
<td>10172</td>
<td>7939</td>
<td>8496</td>
<td>7.5%</td>
</tr>
</tbody>
</table>

### Annual Growth of Pension Expense

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1978</td>
<td>13%</td>
</tr>
<tr>
<td>1979</td>
<td>8%</td>
</tr>
<tr>
<td>1980</td>
<td>16%</td>
</tr>
<tr>
<td>1981</td>
<td>15%</td>
</tr>
<tr>
<td>1982</td>
<td>4%</td>
</tr>
<tr>
<td>1983</td>
<td>13%</td>
</tr>
</tbody>
</table>
As can be seen in this table, values of the assets, benefits, interest rate assumption, etc. can be obtained directly. However, other statistics such as the rate of growth of contribution \((g_1)\), the rate of growth of benefits \((g_2)\) must be calculated from the given annual figures. The rate of growth of contribution \((g_1)\) was obtained by averaging the annual growth of pension expense from 1978-1983 and was calculated to be approximately \(11.5\) percent. The rate of growth of benefits \((g_2)\) was found to be approximately \(15\) percent (an average of the growth rate of benefit over the last three years, 1981-83). The above derived figures of \(g_1\) and \(g_2\) are used in this study for the given period \((t)\), and assumes that there are no drastic decision changes with regard to the pension fund parameters to be made in the near future.

**Moody's Reference Manual & Moody's Bond Record**

A representative sample of 200 bonds (see Appendix D) was selected (using stratified sampling) from various Moody's reference manuals, 170 were corporate bonds and the rest, government or treasury bonds. Only investment grade corporate bonds, i.e., those with Aaa, Aa, A and Baa ratings were chosen. The government bonds are categorized by

---

12 According to Moody's or Standard & Poor's the ratings
their terms to maturity, i.e., short-term, intermediate and long-term.

These bonds in turn are categorized into groups of similar ratings and similar types (see Appendix E). In order to include all of these bonds into the model, each group is represented by a respective decision variable \( X \) as noted in table 2.

Both Moody's reference manuals and the bond record data characterize each individual bond by the coupon rate, maturity date, price, and yield obtained (see Appendix D). Using the coupon rate, the yield, and the maturity date, one could estimate the duration value for each individual bond using the "Duration Tables for Bond and Mortgage Portfolio Management" (9). By averaging the yields, as well as the duration of bonds in each of the 19 groups, one could summarize the results in a tabular form (see table 3 and table 4).

have the following designations:
Aaa - High grade investment bonds;
Aa - High grade investment bonds, but the margins of protection are not quite as strong as Aaa;
A  - Medium grade investment bonds;
Baa - Medium grade investment bonds, but may lack certain protective elements against adverse economic conditions.
Table 2

<table>
<thead>
<tr>
<th>X</th>
<th>Consists of Aaa Bank and Finance Corporate Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&quot; &quot; Aa &quot; &quot; &quot; &quot; &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>2</td>
<td>&quot; &quot; A &quot; &quot; &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>3</td>
<td>&quot; &quot; Baa &quot; &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>4</td>
<td>&quot; &quot; Aaa Industrial &quot; &quot;</td>
</tr>
<tr>
<td>5</td>
<td>&quot; &quot; Aa &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>6</td>
<td>&quot; &quot; A &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>7</td>
<td>&quot; &quot; Baa &quot; &quot; &quot;</td>
</tr>
<tr>
<td>8</td>
<td>&quot; &quot; Aaa Public Utilities &quot; &quot;</td>
</tr>
<tr>
<td>9</td>
<td>&quot; &quot; Aa &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>10</td>
<td>&quot; &quot; A &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>11</td>
<td>&quot; &quot; Baa &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>12</td>
<td>&quot; &quot; Aaa Transportation &quot; &quot;</td>
</tr>
<tr>
<td>13</td>
<td>&quot; &quot; Aa &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>14</td>
<td>&quot; &quot; A &quot; &quot; &quot; &quot;</td>
</tr>
<tr>
<td>15</td>
<td>&quot; &quot; Baa &quot; &quot; &quot;</td>
</tr>
<tr>
<td>16</td>
<td>&quot; &quot;</td>
</tr>
</tbody>
</table>

X, X, X are short-term, intermediate and long-term government bonds respectively.
Table 3

Average Yield of the Groups

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>10.43%</td>
<td>13.20%</td>
<td>13.01%</td>
<td>13.31%</td>
<td>11.76%</td>
<td>11.96%</td>
<td>12.53%</td>
<td>13.50%</td>
<td>12.77%</td>
<td>12.75%</td>
</tr>
<tr>
<td>R</td>
<td>12.82%</td>
<td>13.50%</td>
<td>11.98%</td>
<td>12.27%</td>
<td>12.79%</td>
<td>13.33%</td>
<td>10.70%</td>
<td>11.70%</td>
<td>11.83%</td>
<td></td>
</tr>
</tbody>
</table>


Table 4

Average Duration of the Groups

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th></th>
<th></th>
<th>D</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.3867</td>
<td></td>
<td></td>
<td>7.695</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7.546</td>
<td></td>
<td></td>
<td>7.284</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6.998</td>
<td></td>
<td></td>
<td>6.884</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5.222</td>
<td></td>
<td></td>
<td>7.669</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6.004</td>
<td></td>
<td></td>
<td>7.159</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6.957</td>
<td></td>
<td></td>
<td>6.778</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7.065</td>
<td></td>
<td></td>
<td>4.165</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>6.564</td>
<td></td>
<td></td>
<td>6.534</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>7.795</td>
<td></td>
<td></td>
<td>8.327</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>7.878</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(a) Preliminary relationship evaluation

In constructing the bond market line to obtain the expected rate of return (as given in the previous chapter), Reilly & Sidhu (41) suggested that different lines should be constructed to take into account the different ratings and types of bonds under consideration.

Accordingly, the 170 corporate bonds included in the sample were categorized into four respective rating groups (Aaa, Aa, A, Baa) while the government bonds were placed into three groups (short-term, intermediate and long-term). For each group, the individual rate of return was regressed against the respective duration and the results are tabulated in table 5.

By substituting the duration value in table 4 into the respective equations in table 5, it is possible to obtain the expected rate of return of the respective groups (see table 6).
<table>
<thead>
<tr>
<th>Class</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>$E(R_t) = 10.5284 + 0.2087 D_t$</td>
</tr>
<tr>
<td>Aa</td>
<td>$E(R_t) = 11.3671 + 0.1535 D_t$</td>
</tr>
<tr>
<td>A</td>
<td>$E(R_t) = 11.5016 + 0.1801 D_t$</td>
</tr>
<tr>
<td>Baa</td>
<td>$E(R_t) = 11.7823 + 0.2511 D_t$</td>
</tr>
<tr>
<td>Short-term</td>
<td>$E(R_t) = 12.1294 - 0.3432 D_t$</td>
</tr>
<tr>
<td>Intermediate</td>
<td>$E(R_t) = 12.2050 - 0.0774 D_t$</td>
</tr>
<tr>
<td>Long-term</td>
<td>$E(R_t) = 13.4373 - 0.1928 D_t$</td>
</tr>
<tr>
<td></td>
<td>Expected Rate of Return</td>
</tr>
<tr>
<td>---</td>
<td>-------------------------</td>
</tr>
<tr>
<td>1</td>
<td>E = 11.235%</td>
</tr>
<tr>
<td>2</td>
<td>E = 12.525%</td>
</tr>
<tr>
<td>3</td>
<td>E = 12.762%</td>
</tr>
<tr>
<td>4</td>
<td>E = 13.094%</td>
</tr>
<tr>
<td>5</td>
<td>E = 11.781%</td>
</tr>
<tr>
<td>6</td>
<td>E = 12.435%</td>
</tr>
<tr>
<td>7</td>
<td>E = 12.744%</td>
</tr>
<tr>
<td>8</td>
<td>E = 13.431%</td>
</tr>
<tr>
<td>9</td>
<td>E = 12.155%</td>
</tr>
<tr>
<td>10</td>
<td>E = 12.576%</td>
</tr>
</tbody>
</table>
Using the derived expected rate of return figures in table 6, one could obtain equation (3.2) which represents $E_p$, the expected rate of return for the portfolio.

With information from table 1, table 2, and table 3, all the other relationships between the goal variables and other decision variables in chapter three can be obtained.

The next step is to construct the upper and lower limits of the goal constraints. In practice, these values are chosen by decision-makers or by a committee of decision-makers selected by the firm. However, since it is assumed that decisions are being made by an imaginary decision-maker, the limits will be set accordingly.

In the case of the first goal, i.e., maximizing the expected rate of return, management would like the expected rate of return of the portfolio to be in excess of the interest rate assumed in the actuarial valuation of the benefits. In the case of General Electric, the interest rate assumed for 1981-1983 was constant at 7.5 percent (see table 1). Thus, it would be appropriate to set the initial lower limit or the minimum expected rate of return at 7.5 percent, i.e., $E_p \geq 0.075$

With respect to the second goal, the average yield spread (the difference between the yields of individual bonds in the sample and the 11.85 percent average of 'riskless' bonds) was found to be at 0.6135 percent. However, the decision-maker might be conservative by choosing a high initial value which
in this case is 5 percent. Therefore, the goal constraint for the second objective is \( D < 0.05 \).

The ideal value of \( H \), the third goal variable, is zero. This allows \((A - D)\) to equal \((L - D)\), thus achieving the necessary condition for immunization. Any positive arbitrary values can serve as the upper limit of \( H \). For this illustration, the initial value of \( H \) was chosen to be 10,000.

In the fourth goal, the value of \( \rho \) must lie between zero and one. To minimize \( \rho \) a conservative approach is to start with a high initial value of \( \rho \). The value chosen here is \( \rho = 1 \) which implies that the outflows or benefits are being paid by contributions.

Once all the equations and constraints that constitute the framework of the model are obtained (see Appendix F) it is then possible to proceed with the IMGP procedures in order to obtain the best attainable asset-mix allocation, as well as the optimum values of decision variables while satisfying all the goals set by the decision-maker.

**Implementation of the IMGP Procedures**

Using the previously described formulation (see Appendix A) as the basis for the programming of the problem the decision-maker could then proceed with the implementation of the IMGP procedures as described in the flow-chart (Figure 3) below (47).
Calculate the 1st. Potency Matrix and display it to the decision-maker

Is pessimistic solution satisfactory?

Yes → End of Program

No → Change one of the right-hand side values

Calculate the new potency matrix

Is proposed solution acceptable?

Yes → End of Program

No → Change one of the right-hand side values

Figure 3: Flow-chart of the IMPG Procedure
In order to program the problem, Spronk & Nijkamp (39), (46), used the IBM's MPSX/370 package. For this study, the program was handled by the linear programming package LINDO. Using the more advanced MPSX package, a user would be able to get values of the potency matrix in one iteration. The LINDO package requires that a user iterate several times (depending on the number of goals in the problem) in order to obtain a single potency matrix. Regardless of the linear programming package used, both approaches will achieve similar results.

Since the problem consists of solving for four goals at each iteration the interactive procedure produces four solutions i.e. one for each objective. At the same time the other decision variables, such that the value of the X's \( X_1, X_2, \ldots, X_{19} \) are also obtained.

As a result of the first iteration the following values are obtained:

<table>
<thead>
<tr>
<th></th>
<th>( E_{pt} )</th>
<th>( D )</th>
<th>( H )</th>
<th>( \text{Rho}(\rho) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{pt} )</td>
<td>0.12608</td>
<td>0.00758</td>
<td>3136</td>
<td>0.405164</td>
</tr>
<tr>
<td>( D )</td>
<td>0.11850</td>
<td>0.00000</td>
<td>9712</td>
<td>0.439272</td>
</tr>
<tr>
<td>( H )</td>
<td>0.12608</td>
<td>0.00758</td>
<td>3136</td>
<td>0.405164</td>
</tr>
<tr>
<td>( \text{Rho}(\rho) )</td>
<td>0.12608</td>
<td>0.00758</td>
<td>3136</td>
<td>0.405164</td>
</tr>
</tbody>
</table>
From the above matrix, the potency matrix, \( P \), consisting of the best and worst values of each goal variable is constructed. Thus,

\[
P = \begin{bmatrix}
0.12608 & 0.00000 & 3136 & 0.405164 \\
0.11850 & 0.00758 & 9712 & 0.439272
\end{bmatrix}
\]

The pessimistic solution \( S = (0.11850, 0.00758, 9712, 0.439272) \), together with the potency matrix \( P \) is then be presented to the decision-maker.

If they are satisfied with the result they can accept \( S \). However, if the decision-maker is not satisfied with \( S \) and wishes to increase the expected rate of return of investments (since the previous lower limit given in the program is very low, i.e., 7.5 percent) another iteration would be performed.

In the second iteration the lower limit of \( E \) is adjusted. In this case the limit can be changed from \( E \geq 0.075 \) to \( E \geq 0.12229 \) (where the new lower limit is the average of the best and worst values of \( E \), i.e., \( (0.12608 + 0.11850)/2 = 0.12229 \)). The limits of the other goal
variables remain unchanged. The following table shows the values after the second iteration:

<table>
<thead>
<tr>
<th></th>
<th>( E_{pt} )</th>
<th>( D_t )</th>
<th>( H )</th>
<th>( \text{Rho}(\rho) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{pt} )</td>
<td>0.12606</td>
<td>0.00758</td>
<td>3136</td>
<td>0.405164</td>
</tr>
<tr>
<td>( D_t )</td>
<td>0.12229</td>
<td>0.00379</td>
<td>7627</td>
<td>0.428462</td>
</tr>
<tr>
<td>( H )</td>
<td>0.12536</td>
<td>0.00686</td>
<td>3136</td>
<td>0.405164</td>
</tr>
<tr>
<td>( \text{Rho}(\rho) )</td>
<td>0.12536</td>
<td>0.00686</td>
<td>3136</td>
<td>0.405164</td>
</tr>
</tbody>
</table>

From the above values we could construct potency matrix \( P_2 \):

\[
P_2 = \begin{bmatrix}
0.12608 & 0.00379 & 3136 & 0.405164 & \text{Best} \\
0.12229 & 0.00758 & 7627 & 0.428462 & \text{Worst}
\end{bmatrix}
\]

The increase in the lower limit of \( E_{pt} \) has somewhat improved the 'worst' attainable values of \( H \) and \( \rho \) and worsened the 'best' attainable value of \( D_t \). In other words, by increasing the lower limits of the portfolio expected return, the gaps between the 'best' and 'worst' values of the yield spread in the portfolio are reduced.

In addition, the deviation between \( (A_{pt} \cdot D_t) \) and \( (L_t \cdot D_t) \) and the proportion of outflow accounted for by contribution is
also narrowed.

Seeing these improvements, the decision-maker might be encouraged to refine the lower limit of \( E \) further from \( E > 0.12229 \) to \( E > 0.12419 \) (where \( 0.12419 = (0.12608 + 0.12229)/2 \)).

With this change in the lower limit of \( E \), the following third iteration values are obtained.

<table>
<thead>
<tr>
<th></th>
<th>( E ) (_{pt} )</th>
<th>( D ) (_t )</th>
<th>( H )</th>
<th>( \rho(H) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E ) (_{pt} )</td>
<td>0.12608</td>
<td>0.00758</td>
<td>3136</td>
<td>0.405164</td>
</tr>
<tr>
<td>( D ) (_t )</td>
<td>0.12490</td>
<td>0.00569</td>
<td>5376</td>
<td>0.405164</td>
</tr>
<tr>
<td>( H )</td>
<td>0.12536</td>
<td>0.00686</td>
<td>3136</td>
<td>0.405164</td>
</tr>
<tr>
<td>( \rho(H) )</td>
<td>0.12536</td>
<td>0.00686</td>
<td>3136</td>
<td>0.405164</td>
</tr>
</tbody>
</table>

Accordingly, the next potency matrix \( P \) is,

\[
P_3 = \begin{bmatrix}
0.12608 & 0.00569 & 3136 & 0.405164 \\
0.124190 & 0.00758 & 5376 & 0.416782 \\
\end{bmatrix}
\]

Best

Worst

The increase in the lower limit of \( E \) did not seem to improve its 'best' value. However, it had worsened the 'best' values of \( D \) and improved the 'worst' values of \( H \) and \( \rho \). All in all,
the increase in the lower limit of $E$ tends to narrow the differences between the 'best' and 'worst' values of the other goals, i.e., they are converging.

Next, if the decision-maker decides to improve the 'worst' attainable value of $H$, then the upper limit of $H$ must be revalued from $H < 10,000$ to $H < 4256$ (where $4256 = (3136+5376)/2$).

The values obtained in the fourth iteration are:

<table>
<thead>
<tr>
<th></th>
<th>$E_{pt}$</th>
<th>$D_t$</th>
<th>$H$</th>
<th>$\text{Rho}(\rho)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{pt}$</td>
<td>0.12578</td>
<td>0.00728</td>
<td>4256</td>
<td>0.410975</td>
</tr>
<tr>
<td>$D_t$</td>
<td>0.12419</td>
<td>0.00569</td>
<td>4256</td>
<td>0.410975</td>
</tr>
<tr>
<td>$H$</td>
<td>0.12536</td>
<td>0.00686</td>
<td>3136</td>
<td>0.405164</td>
</tr>
<tr>
<td>$\text{Rho}(\rho)$</td>
<td>0.12536</td>
<td>0.00686</td>
<td>3136</td>
<td>0.405164</td>
</tr>
</tbody>
</table>

The value of the next potency matrix $P_4$ will be:

$$P_4 = \begin{bmatrix}
0.12578 & 0.00569 & 3136 & 0.405164 & \text{Best} \\
0.12419 & 0.00728 & 4256 & 0.410975 & \text{Worst}
\end{bmatrix}$$

A change in the $H$'s limit tends to improve the pessimistic values of $D$ and $\rho$. This change did not improve the 'best' value of $H$. In fact, it worsened the ideal value of the expected rate of return of the portfolio $E_{pt}$.
After four iterations one could see that the values of all the four goal variables are converging. The decision-maker may next decide to improve the upper limit of from \( \rho = 1.0 \) (which is unreasonable and not permissible under ERISA) to a more realistic limit of \( \rho \leq 0.40870 \) (where \( 0.40870 = (0.405164+0.410975)/2 \)). As a result of this change the decision-maker will get the following fifth iteration values:

\[
\begin{array}{c|cccc}
 & E & D & H & \text{Rho}(\rho) \\
\hline
E_{pt} & 0.12608 & 0.00758 & 3136 & 0.405164 \\
D_{t} & 0.12419 & 0.00569 & 3817 & 0.408700 \\
H & 0.12608 & 0.00758 & 3136 & 0.405164 \\
\text{Rho}(\rho) & 0.12608 & 0.00758 & 3136 & 0.405164 \\
\end{array}
\]

Constructing the next potency matrix i.e. \( P_5 \)

\[
P_5 = \begin{bmatrix}
0.12608 & 0.00569 & 3136 & 0.405164 & \text{Best} \\
0.12419 & 0.00758 & 3817 & 0.408700 & \text{Worst}
\end{bmatrix}
\]
The improvement in the upper limit of Rho tends to improve the H values further. It also improves the 'best' value of E, yet at the same time worsens the pessimistic value of D.

So far all the goal limits have been altered except the limit on yield spread (D). If the decision-maker decides to change the upper limit of D, accordingly the limit will be altered from $D < 0.05$ to $D < 0.00664$.

The changes will produce the results of the sixth iteration:

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>D</th>
<th>H</th>
<th>Rho(ρ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pt</td>
<td>0.12514</td>
<td>0.00664</td>
<td>3201</td>
<td>0.405502</td>
</tr>
<tr>
<td>t</td>
<td>0.12419</td>
<td>0.00569</td>
<td>3482</td>
<td>0.406962</td>
</tr>
<tr>
<td>t</td>
<td>0.12514</td>
<td>0.00664</td>
<td>3201</td>
<td>0.405502</td>
</tr>
<tr>
<td>Rho(ρ)</td>
<td>0.12514</td>
<td>0.00664</td>
<td>3201</td>
<td>0.405502</td>
</tr>
</tbody>
</table>

Accordingly the sixth potency matrix is as follows:

$$p = \begin{bmatrix} 0.12514 & 0.00569 & 3201 & 0.405502 \\ 0.12419 & 0.00664 & 3482 & 0.406962 \end{bmatrix}$$
After the sixth iteration, one can observe that all the goal variables tend to converge even further, in spite of the fact that the 'best' values of $E$ and $H$ had somewhat worsened.

The decision-maker could terminate the process at this point or could proceed for another iteration of refinement with changes being made to $D_t$.

Assuming that the decision-maker proceeds with another iteration, i.e., by changing $D < 0.00664$ to $D < 0.00617$, the following results will occur:

<table>
<thead>
<tr>
<th></th>
<th>$E_{pt}$</th>
<th>$D_t$</th>
<th>$H$</th>
<th>Rho($\rho$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{pt}$</td>
<td>0.12467</td>
<td>0.00617</td>
<td>3340</td>
<td>0.406224</td>
</tr>
<tr>
<td>$D_t$</td>
<td>0.12419</td>
<td>0.00569</td>
<td>3482</td>
<td>0.406962</td>
</tr>
<tr>
<td>$H$</td>
<td>0.12467</td>
<td>0.00617</td>
<td>3340</td>
<td>0.406224</td>
</tr>
<tr>
<td>Rho($\rho$)</td>
<td>0.12467</td>
<td>0.00617</td>
<td>3340</td>
<td>0.406224</td>
</tr>
</tbody>
</table>

The corresponding potency matrix $P_7$ then is:

$P_7 = \begin{bmatrix}
0.12467 & 0.00569 & 3340 & 0.406224 \\
0.12419 & 0.00617 & 3482 & 0.406962
\end{bmatrix}$

Best

Worst
Since no potential improvements to the four goal variables could be expected, the procedure is terminated at this point.

The solution, \( S = (0.12419, 0.00617, 3482, 0.406962) \) was accepted to be the final solution.

As seen above, the changes made by the decision-maker during the interactive process was made more or less at an intuitive basis. Nevertheless, the fact that a certain goal value is required to be changed indirectly indicates that this is an important change that needs to be made at that point in time by the decision-maker.

(c) Summary of Results

In this section, the results obtained from the above-described IMPG interactive process will be summarized (see table 7) and discussed.

From the final iteration, the solution for the four goals was found to be \( S = (0.12419, 0.00617, 3482, 0.406962) \) which can be translated as:

(a) The maximum expected rate of return to be obtained from the given portfolio of 200 bonds is 12.419 percent.

(b) The default risk, as measured by the yield spread is 0.617 percent. This is the minimum yield spread between a given risky
bond and the average of 'riskless' government bonds. In a portfolio consisting of a mixture of stocks and bonds this figure would be expected to be higher.

(c) The deviation between the duration-weighted values of the assets and the liabilities is found to be 3482 or $3.482 million. This indicates that to some extent the fund is overfunded.

(d) The minimum contribution rate stands at approximately 40.7 percent of pension fund inflow. This implies that 59.3 percent are to come from investments (assuming there is no other form of inflow into the fund).

The final solution obtained above is the result of a number of shifts and compromises made between the four related goals.
Table 7

**Goal Values during the Interactive Process**

<table>
<thead>
<tr>
<th>Goal Variable</th>
<th>Goal Values</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Maximum Expected Return</td>
<td>Pessimistic</td>
<td>0.11850</td>
</tr>
<tr>
<td></td>
<td>Idealistic</td>
<td>0.12608</td>
</tr>
<tr>
<td>Minimum Default Risk</td>
<td>Pessimistic</td>
<td>0.00758</td>
</tr>
<tr>
<td></td>
<td>Idealistic</td>
<td>0.00000</td>
</tr>
<tr>
<td>Minimum Deviation of Durations (H)</td>
<td>Pessimistic</td>
<td>9712</td>
</tr>
<tr>
<td></td>
<td>Idealistic</td>
<td>3136</td>
</tr>
<tr>
<td>Minimum Value of Rho (ρ)</td>
<td>Pessimistic</td>
<td>0.43927</td>
</tr>
<tr>
<td></td>
<td>Idealistic</td>
<td>0.40516</td>
</tr>
</tbody>
</table>
In a pension fund management setting, these goals variables are used to mold the investment and funding policies. They provide management with a set of short-term targets which can be used as a benchmark for the decision-maker. The learning effects provided by the IMGP process also permits the decision-maker to evaluate the sensitivity of his judgment toward investment and funding implications. It also allows the strategies adopted, to be dynamic in nature.

Apart from the goal variable values which are used basically for policy-making indicators, the other decision variables which are used for operational purposes, and in particular for asset mix allocation, are also obtained. After the final iteration (iteration 7), the following (see table 8) are values of the various decision variables:

The composition of the asset-mix can in turn be illustrated in terms of the individual firm that issue the bonds for the respective groups (see table 9).
Table 8  
Values of the Other Decision Variables

Composition of Asset Mix

<table>
<thead>
<tr>
<th>Composition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>X - Aa Finance &amp; Banking Corporate Bonds</td>
<td>7.667%</td>
</tr>
<tr>
<td>X - Baa Industrial Corporate Bond</td>
<td>32.333%</td>
</tr>
<tr>
<td>X - Aaa Public Utilities Bond</td>
<td>20.000%</td>
</tr>
<tr>
<td>X - Long-term Government Bond</td>
<td>40.000%</td>
</tr>
<tr>
<td>Average rank of bonds in the portfolio</td>
<td>1.4537</td>
</tr>
<tr>
<td>Contribution to the fund for year t</td>
<td>881.0995</td>
</tr>
<tr>
<td>Duration weight of liabilities</td>
<td>4.789</td>
</tr>
<tr>
<td>Duration weight of assets</td>
<td>4.000</td>
</tr>
</tbody>
</table>
Table 9
Composition of the Asset-Mix in terms of Bond Issuer

<table>
<thead>
<tr>
<th></th>
<th>Bond Type</th>
<th>Percentage</th>
<th>Issuers</th>
</tr>
</thead>
<tbody>
<tr>
<td>X :</td>
<td>Long-term (15 years or over)</td>
<td>40.0%</td>
<td>Government Bond</td>
</tr>
</tbody>
</table>
CHAPTER V

SUMMARY & CONCLUSION, LIMITATIONS AND EXTENSIONS

SUMMARY & CONCLUSION

The principal aim of this dissertation is to develop and demonstrate a multiple-criteria approach for selecting an optimal funding and investment strategy for a defined-benefit trustee private pension plan. This goal has been accomplished by formulating a model using an interactive multiple goal programming procedure.

Previous mathematical programming techniques that have been proposed (12), (48) for the management of pension funds are single-objective models. These models are constructed with the capability of handling a single objective. The major drawback of these models is that they are not able to handle more than one goal at a time as is the case in most real decision situations. With the presence of conflicting and non-comparable goals in a given problem, the task of aggregating these goals into a single objective
proves to be an almost impossible task under such a prohibitive framework.

The decision-making processes of organizations are now more closely related to the concepts of goals and attainments rather than being based on single-path criterion such as profit maximization or cost minimization. In many cases these goals may include policies, targets, deadlines, etc., which are multi-criteria in nature. Despite the fact that the existence of these goals are recognized, decision-makers tend to use unidimensional models which utilize single criterion as proxy descriptors to such complex situations. By using techniques such as linear programming, managers must reduce the various goals into a single objective. More often than not, these goals tend to be conflicting and incommensurable. Under such circumstances, methods such as linear programming would yield infeasible solutions.

This study explores the use of an interactive multiple goal programming technique to handle the problem. The approach is a hybrid of goal programming and interactive procedures. As a result, a decision-maker will be able to handle a problem with multiple conflicting goals without having to specify any preference functions and representation of any form of ranking weightage on to the objectives.
Four goals were introduced in this study to represent the mechanism of funding and investment in order to portray the risk-return scenario in a typical investment situation as well as to signify the funding aspect of a pension fund. In an all-bond portfolio, the return is basically monitored by the coupon payments and the reinvestment proceeds. The relevant goals corresponding to these sources of expected income, together with the risk factor that is attached to it are: (a) the maximizing of the expected return of the portfolio; (b) the minimizing of the default risk on risky bonds; and (c) the minimizing of reinvestment risk. A fourth goal was also added to minimize the contribution cost to the fund.

In addition to these goals, various relationships between the different objectives are formulated so that the results obtained from this formulation may reflect the presence of these goals. These are the interdependency of contribution and investment decision, which in turn affects the choice of investment alternatives. Also developed are constraints which are imposed on both the goals and the various decision variables.

Once the goals are clearly defined and the constraints developed, the decision maker should be able to use the IMPG procedure to arrive at a decision pertaining to the goals mentioned as well as other related information. In
implementing this procedure, the decision-maker (with the help of the analyst) is required to construct a potency matrix consisting of vectors of the ideal and pessimistic values of each goal to be used in deriving solutions to the problem. In cases where a satisfactory solution is to be determined, a potency matrix will be calculated which the decision-maker can use to indicate his preference as to which goal value ought to be raised or reduced. Executing these changes involves a direct interplay between the analyst and the decision-maker. As a result of the changes made from iteration to iteration, the decision-maker will finally arrive at a satisfactory solution.

As demonstrated in chapter four, the whole procedure used to determine the final satisfactory solution to the four goals took seven iterations during which changes were made interactively by the decision-maker on the different values of the goal variables. Each time a change is made to one of the goals, the decision-maker has to specify "whether the shifts in the ideal solution are outweighed by the shifts in the pessimistic solution" (47). If it is not, the pessimistic solution must be redefined and, correspondingly, a new ideal solution must be computed. The final solution obtained is the result of a number of shifts and compromises between the four related goals. For example, the maximum expected rate of return obtained in the
final solution is not necessarily the highest return that could be achieved from the portfolio. It is, however, the best attainable result considering the other three goals in the model.

The goal variable values obtained (see table 7) for the pension fund are used to shape its investment and funding policies. It provides management with a set of short-term targets that act as a benchmark for the managers. Having procedures like the IMGIP permits the decision-maker to evaluate the sensitivity of his judgment toward investment and funding implications. The periodic reevaluation of the policies as required by ERISA can be readily implemented. This will also allow a process which governs investment and funding strategies such as asset allocation, to be dynamic and sensitive enough to incorporate past management performance and results in order to produce new solutions which are in accordance with the objective.

With a growing number of firms managing in-house pension funds, the use of such a procedure would be more economically feasible especially in light of ERISA's requirements on companies to reassess their actuarial assumptions every three years. Furthermore, by using the IMGIP, which is in line with similar concepts as that in decision support systems, decision-making will be effectively improved.
LIMITATIONS

In conducting this study, a number of limitations with regard to the approach in general, as well as the technical aspects of the model are noted. First, it is assumed that the pension fund and its sponsoring parent company are treated as two separate entities. Legally, they must be separated. However, it has been shown that the fund's performance (especially in the case of a defined-benefit plan) has a strong inverse impact on the firm's contribution level to the fund. As a result of this separation, Hagigi (17) noted that, in general, a defined-benefit pension plan will not be invested in an optimal way. This is due to the fact that in constructing its portfolio, the fund tended to emphasize its own risk rather than the firm's total risk.

Second, the model used in this study is a single-period model and, as such, has a somewhat limited utility. For short-term evaluation purposes this model can be effectively used. However, the model will need adjustments and modifications if intended for long-term planning.

Third, in dealing with some specific aspects of the model, there are a number of limitations as to the choice of variables and measurements used. In constructing the first goal, for instance, yield-to-maturity was used to measure
the expected rate of return. It should be noted that this measurement poses some problems with regard to its underlying assumptions. A more reliable measure in this case would be achieved by using another measurement called realized compound yield.

In the second goal, yield spread was used as a proxy measure for default risk. As indicated by Kharabe (24), yield spread is the sum of the systematic risk premium and the default premium. Therefore, by using a proxy measure the value of the default risk tends to be overstated. On the other hand, by immunizing the portfolio against changes in the market interest rates (which is done through the implementation of the third goal), the systematic risk is somewhat reduced (11).

Finally, it should be noted that all the goals considered in this model are financial in nature. New goals with social considerations could be included, but they must be weighed against ERISA's restrictions which tend to be more favorable toward financial soundness of the fund.

EXTENSIONS

As seen in chapter three, the model used in this study is somewhat simplified. As a single-period model consisting of an all-bond portfolio it assumes no taxes and no
transaction costs on the sale and purchase of investment assets. In this section, several technical changes are recommended to improve the reality of the model.

The portfolio composition of pension funds in practice consists of investments in corporate stocks, corporate bonds, U.S. government securities, and other forms of investment instruments. In 1983, it was indicated that about 50 percent of the total assets of all uninsured private pension plans was invested in corporate stocks, 25 percent was in U.S. government securities, 15 percent in corporate bonds, and the remaining 10 percent in other forms of investment. Over the years the portfolio distribution of trustee pension funds has changed its emphasis from bonds and government securities in the 1940s to stocks in the 1970s. However, the composition of the portfolio basically still consists of these three vital instruments. The combination of bonds and stocks in a pension fund portfolio is essential owing to their contrasting risk-return profiles. An inclusion of stocks investment into the model could probably bring in unnecessary volatility to the portfolio, but as Munnell (37) has indicated the considerable excess of contributions over benefits in a pension fund would "provide a cushion to weather swings in the security price". Furthermore, the higher returns the fund will get

13 Pension Facts, 1984-85, American Council of Life Insurance, Washington D.C.
from the investment of stocks will compensate for the high risk assumed. As a complement, bonds with lower returns can be utilized for the preservation and long-term accumulation of capital which is required by a pension fund.

Black (5), Tepper & Paul (50) however, have suggested that management should invest pension assets entirely into bonds. Bonds, they argued, allow for tax arbitrage. Black recommended that bonds should replace all stocks in the portfolio while at the same time issue new debt of its own. Using the proceeds, management should buy back its own stock. In so doing, he believed the firm will obtain maximum tax deduction. However, as pointed out by Tepper & Paul (50), this all-bond strategy has its drawbacks. Such a strategy would tend to drain corporate liquidity. It also tends to overfund the plan which in turn would lead to demands for more liberal benefits plan. In the case of termination of a plan, this might lead to losses to stockholders.

Another extension that should be included in the model is the tax factor. For a qualified pension plan, employer's contribution to the fund is tax deductible. This exemption does have a direct effect on the cost of contributions to the fund and it could add to a substantial amount. In terms of the model, the tax factor can be visualized as a reducing factor attached to the cost of contri-
butions. In addition to the tax deductible clause, the Internal Revenue Service (IRS) has also stipulated a maximum deduction limit imposed on the amount of deductions allowable from taxable income. This restriction will act as an additional constraint to the model.

The model can be further improved by including the transaction costs of sales and purchases of investment assets and also by extending the model to be a multi-period pension plan. Transaction costs can be included as administrative expenses and they can be tagged to the benefit payments and treated as a cash outflow. An extension of the model into a multi-period plan on the other hand will allow it to be dynamic in nature. In its aggregate form, a multi-period model is not merely a collection of single period models but rather a continuous model in which current results affect future results. A model of this form tends to be complex and computational requirements could prove to be difficult to handle.

Finally, it should be pointed out that this model can be generalized to be as a single goal model. This could be achieved by setting a 0-1 type formulation on to the goals. In so doing, it would have provide more flexibility to the model.
APPENDICES
The interactive multiple goal programming can be explained in terms of the following steps:

Step 1: In this step one first need to identify the various instruments, goals and feasible region of the problem.

Step 2: Once the above variables are known next construct the potency matrix $P$. The potency matrix is a two row matrix comprising of a set of ideal solutions $(g_i^*)$ and a set of pessimistic solutions $(g_i^{min})$ for each goal variables. Given a problem with $m$ objectives or goals the potency matrix will be in the form:
Step 3: The analyst will collect a priori information about the aspiration levels of the decision-maker on each of the goals. Here a new auxiliary vector $\delta_i$ is being introduced where $\delta_i$ by definition

$$
\begin{bmatrix}
* & * & \cdots & * \\
\min & \min & \cdots & \min \\
\min & \min & \cdots & \min \\
\end{bmatrix}
$$

is $\delta_{i} = (g_{i} - g_{i})$ or the difference of the lowest level $g_{i}$ that is being rejected by the decision-maker and the highest level accepted thus far. This vector serves as an adjustment factor to goal levels which sometimes are regarded too high.

Step 4: In this step the analyst will come up with an initial solution $S_1$, where

$$
S_1 = (g_{1}, g_{1}, \ldots, g_{1})
$$

the pessimistic solution. This solution together with the potency matrix $P$ is then presented to the
decision-maker.

Step 5: If the decision-maker finds that the solution is satisfactory, then it will be accepted; otherwise the analyst will proceed to step six.

Step 6: The decision-maker will next decide on which goal variable that should be improved or augmented.

Step 7: Let say the jth goal need to be improved so that 
\[ g_j(s) = g_j(s) + \delta_i^2 \]
then the analyst will get a new trial pessimistic solution \( S_2 \) in which only the value in the jth goal will be changed.

Step 8: Correspondingly, a new potency matrix \( P_2 \) will be constructed.

Step 9: Here the decision-maker will be confronted by the decision that whether the change from \( S_1 \) to \( S_2 \) to be acceptable to justify the change from \( P_1 \) to \( P_2 \). If the decision-maker felt that the change is justified then the new solution \( S_2 \) will be accepted together with 2
the potency matrix $P^2$.

Step 10: If the new solution is not accepted, then another goal variable will be changed. Accordingly, the analyst will go back to step five and repeat the process until a satisfactory solution is obtained.
APPENDIX B

The Relationship between the Yield Spread and Ratings of Corporate Bond

The following are the results obtained from a regression analysis of the above given variables:

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>SUM OF SQUARES</th>
<th>MEAN SQUARE</th>
<th>F VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>1</td>
<td>85.35950509</td>
<td>85.35950509</td>
<td>93.96</td>
</tr>
<tr>
<td>Error</td>
<td>198</td>
<td>179.8810449</td>
<td>0.90849013</td>
<td>PR F</td>
</tr>
<tr>
<td>Corrected</td>
<td>199</td>
<td>265.2405500</td>
<td></td>
<td>0.0001</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>265.2405500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R-SQUARE</th>
<th>C.V.</th>
<th>ROOT MSE</th>
<th>YIELD SPREAD (Yld)MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3211819</td>
<td>155.362</td>
<td>0.95314748</td>
<td>0.613500</td>
</tr>
</tbody>
</table>

PARAMETER ESTIMATE T FOR H0: PR T STD ERRCE OF ESTIMATE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>INTERCEPT</th>
<th>RATING(Rnk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESTIMATE</td>
<td>0.3119213</td>
<td>0.2187757</td>
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Correlation Between Yield Spread (Yld) and Ratings (Rnk)

Covariance Matrix

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Correlation Between Yld and Rnk

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Correlation Coefficients

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APPENDIX C

By definition,

\[ A = \sum_{t=1}^{n} \frac{t \cdot CF_t}{(1+r)^t} = \frac{\sum_{t=1}^{n} CF_t}{(1+r)} \]

where,

\[ CF_t = \text{asset cash flow at time } t; \]

\[ r = \text{the appropriate rate of discount}. \]

Expanding the above expression,

\[ A = \frac{CF_1/(1+r) + 2CF_2/(1+r) + \ldots + nCF_n/(1+r)}{1/(1+r) + CF/(1+r) + \ldots + CF_n/(1+r)} \]

Assuming that the cashflow are increasing at a rate of \( g \) percent per year such that,

\[ CF_{t+1} = (1+g)CF_t \]

Then,
\[
\begin{align*}
A_n &= \frac{CF/(1+r) + 2CF(1+g)/(1+r) + \cdots + nCF(1+g)/(1+r)}{1/(1+r) + CF/(1+r) + \cdots + CF(1+g)/(1+r)} \\
&= \frac{1/(1+r) + 2(1+g)/(1+r) + \cdots + n(1+g)/(1+r)}{1/(1+r) + \cdots + (1+g)/(1+r)} \\

\text{Multiply the numerator and denominator by } (1+g) \\
A_n &= \frac{(1+g)/(1+r) + 2(1+g)/(1+r) + \cdots + n(1+g)/(1+r)}{(1+g)/(1+r) + (1+g)/(1+r) + \cdots + (1+g)/(1+r)} \\

\text{If,} \\
\frac{1}{(1+i)} &= \frac{(1+g)}{(1+r)} = v \\
\text{Therefore,} \\
A_n &= \frac{v + 2v + \cdots + nv}{v + v + \cdots + v} \\

\text{If } a_n \text{ is the present value of an annuity for } n \text{ years where} \\
\frac{2}{n} \\
a_n &= v + v + \cdots + v = (1-v^n)/i
\end{align*}
\]
Therefore,
\[
D = \frac{a + (a - nv)/i}{\frac{a}{n} - \frac{nv}{(ia)n}}
\]

Taking the third term of the right hand side separately,
\[
\frac{n}{nv} = \frac{n}{nv} = \frac{n}{nv} = \frac{n}{nv} = \frac{n(1+i)-n}{1-(1+i)}
\]

\[
= \frac{n}{1 - (1 - ni + n(n+1)/2)i - \ldots}^{\ldots}
\]

\[
= \frac{n - ni + n(n+1)i/2 - \ldots}{ni - n(n+1)i/2 + \ldots}
\]

Ignoring the terms including i and higher powers,
then,
\[
\frac{n}{nv} = \frac{n - ni}{ni} = 1/i - n
\]

Substituting this approximated value in D ,
therefore,

\[
\begin{align*}
A & : 1 + 1/i - 1/i + n \\
D & : 1 + n \\
\end{align*}
\]
## APPENDIX D

Sample of Bond Issuers

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(115) Pacific Bell  A3  4.625  1999  45.500  12.87  8.72
(116) Pacific Corp.  Baa2  10.00  2006  75.000  13.60  7.70
(120) Penny (J.C.) Inc.  A1  8.875  1995  76.125  12.91  6.67
(121) Pfizer Inc.  Aa2  8.500  1999  76.000  11.95  7.82
(123) Phillip Morris Inc.  A2  8.875  2004  72.250  12.71  8.10
(124) Private Export Funding Company  Aaa  7.800  1989  112.000  5.37  4.28
(125) Procter & Gamble Co.  Aaa  7.000  2002  63.250  12.00  8.58
(126) Public Service Electric & Gas Co.  Aa3  7.000  1998  62.750  12.69  7.79
(127) RCA Corporation  Baa2  12.25  2005  95.000  12.94  7.70
(128) Republic Bank Corp.  Aa3  9.375  2001  70.375  13.93  7.39
(129) Revlon Inc.  A1  10.875  2010  83.250  13.15  7.93
(130) Rexnord Inc.  Baa2  8.950  1995  76.000  12.98  6.63
(131) Reynolds (R.J.) Ind. Inc.  Aa1  8.000  2007  68.000  12.16  8.62
(132) RLC Corporation  Baa3  11.625  1990  99.875  11.65  4.48
(134) Santa Fe Natural Resources Inc.  A3  8.350  2002  67.000  13.14  7.90
(135) Sears Roebuck & Co.  Aa2  6.375  1993  73.125  11.09  6.59
(136) Socony Mobil Oil Co.  Aa1  4.250  1993  57.500  11.97  7.01
(137) Sohio Pipeline Co.  A1  8.750  2001  72.625  12.63  7.85
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## APPENDIX E

### Groups of Bond Issuers by Rating & Types

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<td>IBM Credit Corporation</td>
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<td>4</td>
<td>J.P. Morgan &amp; Company Incorporated</td>
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<td>National Westminster Bank DLC.</td>
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<td>General Motor Acceptance Corporation</td>
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<td>Household Finance Corporation</td>
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<td>8</td>
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X : Consist of (Baa) Finance & Banking Corporate Bonds
-4-----------------------------
(1) Centran Corporation
(2) Clark Equipment Credit Corporation
(3) Frueauf Finance Company
(4) Paine Webber Incorporated
(5) Walter E. Heller & Company

X : Consist of (Aaa) Industrial Corporate Bonds

(1) Atlantic Richfield Company
(2) Bristol-Myers Company
(3) Campbell Soup Company
(4) Coca Cola Company
(5) Ekco Products
(6) Exxon Corporation
(7) Federated Department Stores Incorporated
(8) General Electric Company
(9) General Mills Company
(10) International Business Machine
(11) Procter & Gamble Company

X : Consist of (Aa) Industrial Corporate Bonds

(1) Beatrice Food Company
(2) General Foods Corporation
(3) Halliburton Company
(4) International Paper Company
(5) Kraft Incorporated
(6) Monsanto Company
(7) Motorola Incorporated
(8) Pfizer Incorporated
(9) Reynolds (R.J.) Industries Incorporated
(10) Sears Roebuck & Company
(11) Socony Mobil Oil Company
(12) Times Mirror Company
(13) United Technologies Corporation
(14) Upjohn Company

X : Consist of (A) Industrial Corporate Bonds

(1) Aluminium Company America
(2) Burroughs Corporation
(3) Dennison Manufacturing Company
(4) Dow Chemical Company
(5) Ford Motor Company
(6) Mercantile Stores
(7) Owen-Illinois Incorporated
(8) Penny (J.C.) Incorporated
(9) Phillip Morris Incorporated
(10) Revlon Incorporated
(11) Sperry Corporation
(12) Tenneco Company
(13) Union Carbide Company

X : Consist of (Baa) Industrial Corporate Bonds

(1) Amax Incorporated
(2) American Stores Company
(3) Bethlehem Steel Corporation
(4) Carter Hawley Hale Stores Incorporated
(5) Cummins Engine Company Incorporated
(6) Lone Star Industry
(7) Mack Trucks Incorporated
(8) MacMillan Incorporated
(9) Midland-Foss Corporation
(10) RCA Corporation
(11) Rexnord Incorporated
(12) Walter Jim Corporation

X : Consist of (Aaa) Public Utilities Bonds

(1) Dallas Power & Light Company
(2) Northern States Power Company
(3) Northern States Power Company
(4) Texas Electric Service Company
(5) Wisconsin Electric Power Company
(6) Wisconsin Michigan Power Company
(7) Wisconsin Power & Light Company
(8) Wisconsin Public Services Corporation

X : Consist of (Aa) Public Utilities Bonds

(1) American Telephone & Telegraph Company
(2) American Telephone & Telegraph Company
(3) Central Illinois Light Company
(4) Consolidated Natural Gas Company
(5) Duke Power Company
(6) Illinois Bell Telephone Company
(7) Iowa Power & Light Company
(8) Kansas Power & Light Company
(9) Kentucky Utilities Company
(10) Madison Gas & Electric Company
(11) Public Service Electric & Gas Company
(12) Southern California Edison Company
(13) Southwestern Public Service Company
(14) Tampa Electric Company

X : Consist of (A) Public Utilities Bonds

(1) Alabama Power Company
(2) Atlantic City Electric Company
(3) Carolina Power & Light Company
(4) Cleveland Electric Illuminating Company
(5) Commonwealth Edison Company
(6) Florida Power Corporation
(7) Florida Power & Light Company
(8) Niagara Mohawk Power Company
(9) Pacific Bell
(10) Southern Bell Telephone & Telegraph Company
(11) Southern Natural Gas Company

X : Consist of (Baa) Public Utilities Bonds
(1) Appalachian Power Company
(2) Atlanta Gas Light Company
(3) Central Hudson Gas & Electric Company
(4) Georgia Power Company
(5) Hartford Electric Light Company
(6) Michigan Consolidated Gas Company
(7) New York State Electric & Gas Corporation
(8) Ohio Power Company
(9) Pacific Corporation
(10) Philadelphia Electric Company
(11) Union Electric Company

X : Consist of (Aaa) Transportation & Railroad Bonds

-13------------------------------

(1) American President Lines Limited
(2) Boston Carriers Incorporated
(3) Delta Steamship Lines
(4) Exxon Pipeline Company
(5) Falcon Tankers Incorporated
(6) Langfitt Shipping Corporation
(7) Lykes Brothers Steamship Incorporated
(8) Mobil Alaska Pipelines Company
(9) Pacific Far East Line Incorporated

X : Consist of (Aa) Transportation & Railroad Bonds

-14------------------------------

(1) Humble Pipeline Company
(2) Monongahela Railway Company
X  : Consist of (A) Transportation & Railroad Bonds
    - 15

(1) Chesapeake & Ohio Railway Company
(2) Consolidated Freightways Incorporated
(3) Greyhound Corporation
(4) Greyhound Leasing & Financial Corporation
(5) Greyhound Leasing & Financial Corporation
(6) Lakehead Pipeline Company Incorporated
(7) Missouri Pacific Railroad Company
(8) Ryder Systems Incorporated
(9) Santa Fe Natural Resources Incorporated
(10) Sohio Pipeline Company
(11) Southern Pacific Company
(12) Texas & Pacific Railway
(13) Virginia & Southern Railway

X  : Consist of (Baa) Transportation & Railroad Bonds
    - 16

(1) Flexi-Van Corporation
(2) Hertz Corporation
(3) Illinois Central Gulf Railroad Company
(4) Kansas City, St. Louis & Chicago Railway Company

(5) RLC Corporation

X : Consist of Short-term U.S. Government Notes & Bonds 17

X : Consist of Intermediate U.S. Government Notes & Bonds 18

X : Consist of Long-term U.S. Government Bonds 19
APPENDIX F

Framework of the model

Goals:

Max E (Expected Rate of Return)

Min Y (Yield Spread of Bonds)

Min H (Deviation between (A - D) & (L - D))

Min \( \rho(\rho) \) (Proportion of outflow financed by cash contribution)

Initial Goal Constraints:

\( E \geq 0.075 \) pt

\( Y \leq 0.05 \) st

\( H \leq 10,000 \)

\( (\rho)Rho \leq 1.000 \)
Other Relationships & Constraints:

\[ E - 0.1124X - 0.1253X - 0.1276X - 0.1309X \]
\[ \text{pt 1 2 3 4} \]
\[ + 0.1178X + 0.1244X + 0.1227X + 0.1343X + 0.1216X \]
\[ \text{5 6 7 8 9} \]
\[ - 0.1258X - 0.1289X - 0.1361X - 0.1197X - 0.1254X \]
\[ \text{10 11 12 13 14} \]
\[ + 0.1279X + 0.1348X + 0.1070X + 0.1170X + 0.1183X \]
\[ \text{15 16 17 18 19} \]
\[ = 0 \]
\[ 1061X + 1343X + 1323X + 1354X + 1196X + 1217X + \]
\[ \text{1 2 3 4 5 6} \]
\[ + 1275X + 1373X + 1299X + 1297X + 1304X + 1373X + \]
\[ \text{7 8 9 10 11 12} \]
\[ + 1219X + 1248X + 1301X + 1356X + 1088X + 1190X \]
\[ \text{13 14 15 16 17 18} \]
\[ + 1203X + 2169Rho = 2169 \]
\[ \text{19} \]
\[ Yld - 0.2188Rnk - 0.3119 = 0 \]
\[ E - Yld = 0.1185 \]
\[ \text{pt} \]
\[ C - 2169Rho = 0 \]
\[ t \]
\[ 8496DL - 88.89C + H = -34293 \]
\[ t \]
\[ 10172DA - 8496DL \geq 0 \]
\[ X + X + X + \ldots + X = 1.0 \]
\[ 1 2 3 \ldots 19 \]
\[ x + x + x + x = 0.2 \]
\[ 1 \quad 5 \quad 9 \quad 13 \]

\[ x + x + x = 0.4 \]
\[ 17 \quad 18 \quad 19 \]

\[ x \geq 0 \]
\[ 1 \]

\[ x \geq 0 \]
\[ 2 \]

\[ x \geq 0 \]
\[ 3 \]

\[ \cdots \]

\[ x \geq 0 \]
\[ 19 \]

\[ DA = 4.0 \]

\[ \mathbf{Rho} \geq 0 \]

\[ H \geq 0 \]

\[ \mathbf{Rnk} \geq 1 \]

\[ \mathbf{Rnk} \leq 10 \]
Appendix G

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