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ON THE SCAN IMPEDANCE OF AN ARRAY OF V-DIPOLES AND THE EFFECT OF THE FEEDLINES

The Ohio State University Ph.D. 1985

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ON THE SCAN IMPEDANCE OF AN ARRAY OF V-DIPOLES
AND THE EFFECT OF THE FEEDLINES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Sun James Lin, B.S., M.S.E.E.

*****

The Ohio State University
1985

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ACKNOWLEDGMENT

The author would like to express his sincere gratitude to his academic adviser, Professor Benedikt A. Munk, for his encouragement, assistance and guidance in the research and preparation of this dissertation. Also, appreciation is extended to Professors Edward K. Damon and Prabhakar H. Pathak for their review of this manuscript. Finally, special thanks goes to the editorial and drafting staffs at the ElectroScience Laboratory, specifically Becky Thornton, James Gibson and Robert Davis, who made this task much less painful.
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CHAPTER I
INTRODUCTION

Straight dipoles have been widely used in the phased array design due to their low cost and easy construction. However, the straight dipole array has an inherent drawback, the scan impedance changing very fast with the scan angle, which makes impedance matching very difficult. In order to achieve a decent match of impedance between the array element and its feed network for all scan angles, we have to minimize the impedance variation to some extent. To achieve this goal, the V-shaped dipole array was developed [1] and has shown a very stable scan impedance over a very large scan range. The V-shaped dipole is made by bending the two legs of a dipole downward in a "V" shape. Since the V-shaped dipole array is newly developed, very little information about its performance is available in the open literature. In this dissertation, we will study the impedance behavior of the V-shaped dipole array in more detail. From the results presented in this dissertation, one can gain an insight into this array.

The blind spots occurring in the direction close to broadside for the straight dipole array also have bothered antenna engineers for many years. These blind spots are caused by the strong coupling from the unbalanced mode current in the array. When the blind spot is induced,
the array is totally blind, and no power can be received or transmitted by the array. This problem should be avoided in the phased array design. In this dissertation, we will investigate this coupling in an analytical way. Hopefully, through this method, the mechanism which causes the blind spots can be clearly understood.

The model we used to evaluate the coupling between the balanced and unbalanced mode currents can be described as follows. Figure 1.1 shows a dipole array fed with two-wire feedlines. The balanced and unbalanced mode currents on the dipole and feedline are illustrated in Figure 1.2(a). When the two wires of the feedline are closely spaced, the net radiation from the feedline due to the balanced mode current can be neglected. In this case, the feedline can be modelled by a cylindrical wire [2] carrying only the unbalanced mode current as shown in Figure 1.2(b). The cylindrical wire, with radius $r_f$, has the same scattering property as that of the original feedline. Using this model, the array in Figure 1.1 can be represented by Figure 1.3.

The balanced and unbalanced mode currents may be separated hypothetically as illustrated in Figure 1.4. Array "b" which carries the balanced mode current can be considered as an antenna array, whereas Array "u", which carries the unbalanced mode current, may be viewed as a scatterer array. The coupling between these two arrays can be described by using a circuit representation as shown in Figure 1.5. The terminal $R(u)$ and $R'(u)$ represent the terminals of the reference element for Array "u" at point $R(u)$; likewise, $R(b)$ and $R'(b)$ represent the terminals at the point $R(b)$. A voltage source $V_b$, connected between
Figure 1.1 An infinite dipole array fed by two-wire feedlines.
Figure 1.2 Two-wire feedline is modelled by a cylindrical wire with radius $r_f$. 
Figure 1.3 An infinite dipole array with modelled feedlines.
Figure 1.4 Using Array "b" and Array "u" to hypothetically separate the balanced and unbalanced mode currents.
Figure 1.5 Circuit equivalence illustrating the coupling between the Array "b" and Array "u" shown in Figure 1.4.

R(b) and R'(b) is used to excite the balanced mode current. In this dissertation, we assume the unbalanced mode current is induced by the field radiated by the balanced mode current. Therefore, no voltage source is connected between R(u) and R'(u). Zbb is the self-impedance of Array "b", while Zuu is the self-impedance of Array "u"; Zub and Zbu are the mutual impedances between the two arrays.

By using Kirchoff's voltage law at the terminal R(b) and then at R(u), the set of equations describing the coupling between the balanced and unbalanced mode currents is obtained.
\[ z^{bb}I_b + z^{bu}I_u = V_b \quad (1.1) \]

\[ z^{ub}I_b + z^{uu}I_u = 0 \quad (1.2) \]

From Equation (1.2), we know

\[ I_u = -\frac{z^{ub}}{z^{uu}} I_b \quad (1.3) \]

Inserting Equation (1.3) into Equation (1.1), we have

\[ \left[ z^{bb} - \frac{z^{bu}z^{ub}}{z^{uu}} \right] I_b = V_b \quad (1.4) \]

Now we define

\[ Z_d \triangleq \frac{V_b}{I_b} = z^{bb} - \Delta Z_f \quad (1.5) \]

where

\[ \Delta Z_f = \frac{z^{bu}z^{ub}}{z^{uu}} \quad (1.6) \]

\( Z_d \) is the scan impedance of the dipole array when the effect of the unbalanced mode current is taken into account. Since the unbalanced mode current is supported by the feedline, the effect caused by this current is sometimes called the "feedline effect". \( \Delta Z_f \) is the quantity used to evaluate the strength of the feedline effect. Since \( z^{bb}, z^{uu}, \)
Zub and Zbu are functions of frequency and scan direction, Zd and ΔZf will change when the frequency and/or scan direction are varied. When ΔZf = 0, Equation (1.5) reduces to

\[ Z_d = Z_{bb} \]  

(1.7)

which is the self-impedance of Array "b". In this case, the feedline effect is absent. However, if the real part of ΔZf happens to be the same as that of Zbb, the real part of Zd will be zero. Under this situation, no power can be received or transmitted by the array; therefore, the blind spot is created.

To investigate the impedance performance of a V-shaped dipole array and to evaluate the feedline effect, the impedance quantities Zbb, Zuu, Zub and Zbu should be calculated.

In this dissertation, we consider every dipole array as an infinitely large periodic structure. Also, the array elements are fed with Floquet's type currents. Therefore, the Plane Wave Expansion Method [3] will be used to derive the field radiated from the array. In Chapter II, we will briefly review this method. We begin by introducing the field radiated from an infinite array of Hertzian dipoles located in a homogeneous medium. Next, the field radiated from the array of Hertzian dipoles in the presence of ground plane and dielectric slabs will be considered. Then, the results will be extended for a nonplanar linear array with arbitrary element current distribution. Finally, the field radiated from the infinite array of dipoles and feedlines with current distributions defined in Figure 2.6 will be derived.
Using the field expressions obtained in Chapter II, the impedances $Z_{bb}$, $Z_{uu}$, $Z_{ub}$ and $Z_{bu}$ will be formulated in Chapter III. The ring summation scheme described at the end of that chapter is used to evaluate all the impedance quantities.

In Chapter IV, first, we will examine the impedance $Z_{bb}$ for V-shaped dipole arrays under various situations. The cases to be studied are summarized in Table 4.1. Next, the feedline effect for a straight dipole array will be investigated. By examining the impedance behaviors of $Z_{ub}$, $Z_{bu}$ and $Z_{uu}$, several phenomena caused by the feedline effect can be realized. From this study, we also found an easy way to eliminate the feedline effect in the desired frequency range. The method and some numerical results will be given at the end of this chapter. The summary and conclusions are presented in Chapter V.
CHAPTER II

FIELDS RADIATED FROM
INFINITE ARRAY OF DIPOLES AND FEEDLINES

A. INTRODUCTION

In order to analyze the impedance performance of a V-shaped dipole array and to investigate the effect of the feedline, four important quantities, namely $Z_{bb}$, $Z_{uu}$, $Z_{ub}$ and $Z_{bu}$, are needed to be calculated. To calculate these quantities, the fields radiated from the infinite array of dipoles and feedlines should be known. The reason will be clear when we proceed to the next chapter.

The phased array used for scanning purposes is designed such that the scan beam can be shifted to any direction without changing the position of the array elements. This kind of phased array can be achieved by using an infinite number of elements arranged in a periodic structure and fed with currents satisfying the Floquet's theorem. The Floquet's theorem requires that the magnitude of the currents on all elements to be identical and the phase varies linearly along the array. By properly adjusting the phase difference between adjacent elements, the scan beam of the phased array can be switched to any desired direction. The field radiated from the phased array of this type can be
obtained by using the Plane Wave Expansion Method developed by Munk and his associates [3]. As the name suggests, the field radiated from the phased array of Floquet type elements can be expressed as an infinite sum of discrete plane inhomogeneous waves. In the following discussion, all the arrays are assumed to be infinitely large and fed with Floquet type currents.

In this chapter, we will review the techniques developed by Munk et al [3] for the analysis of infinite periodic array. We begin by introducing the expression for the field radiated from an infinite array of Hertzian dipoles located in a homogeneous medium. Next, the field radiated from the array of Hertzian dipoles in the presence of the ground plane and dielectric slabs will be discussed. Then, the results will be extended for a nonplanar linear array with arbitrary element current distribution. The fields radiated from the infinite arrays of dipoles and feedlines with specific current distributions will be derived in the last section.

B. FIELD RADIATED FROM AN INFINITE ARRAY OF HERTZIAN DIPOLES LOCATED IN A HOMOGENEOUS MEDIUM

Figure 2.1 shows an infinite array of Hertzian dipoles located in a homogeneous medium with characteristic impedance \( Z_m \). The dipoles in the array are \( d \) long and oriented in the \( p \)-direction. The grid structure of the array is specified by the parameters \( D_x, D_z, \) and \( \Delta z \) as defined in Figure 2.2. The dipole currents obey the Floquet relationship and the
Figure 2.1 Infinite array of Hertzian dipoles imbedded in a homogeneous medium.
Figure 2.2 Grid structure of \( \hat{p} \)-directed Hertzian dipoles, showing the parameter \( D_x \), \( D_z \) and \( \Delta z \).
current along each dipole is assumed to be uniform because of its infinitesimal length.

If the reference point of the array is positioned at \( R' \) and the field point \( R \) is to the right of the array, the field is given by

\[
dE(R) = i\frac{d}{Dx}z \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m (R-R') \cdot r_m} e_{m+}^*,
\]

\( y \cdot R' < y \cdot R \). \( (2.1) \)

When the field point \( R \) is to the left of the array, the field is given by

\[
dE(R) = i\frac{d}{Dx}z \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m (R-R') \cdot r_m} e_{m-}^*,
\]

\( y \cdot R < y \cdot R' \). \( (2.2) \)

The pertinent quantities in Equations (2.1) and (2.2) are:

\[
^m \hat{r} = x^m r_{mx} \pm y^m r_{my} \pm z^m r_{mz}, \quad (2.3)
\]

\[
r_{mx} = s_{mx} + k \frac{\lambda_m}{Dz} - \frac{n \Delta \lambda_m}{Dx Dz}, \quad (2.4)
\]

\[
r_{mz} = s_{mz} + n \frac{\lambda_m}{Dz}, \quad (2.5)
\]

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It is obvious from the expressions in Equations (2.1) and (2.2) that the field radiated from the array consists of an infinite number of plane waves. Their directions of propagation are determined by the unit vectors \( \mathbf{r}_{m} \) of Equation (2.3). The unit vector \( \mathbf{r}_{m+} \) is pointing in the +y direction, while its counterpart \( \mathbf{r}_{m-} \) is pointing in the -y direction. Note that for \( k=n=0 \), \( \mathbf{r}_{m} \) are identical to \( \mathbf{s}_{m} \) which are the scan directions of the array (see Figure 2.1). In the lossless medium, \( r_{mx} \) and \( r_{mz} \) are always real, but \( r_{my} \) is either real or imaginary. When \( r_{my} \) is real, the plane waves will propagate away from the array with no attenuation. However, when \( r_{my} \) becomes imaginary, the plane waves degenerate into evanescent waves which will be attenuated when one moves away from the array. Thus, the field expression given in Equation (2.1) or Equation (2.2) consists of a finite number of propagating waves and an infinite number of evanescent waves. The wave vectors \( \mathbf{e}_{m} \) given by Equation (2.8) indicate the polarizations of the E-field associated with each plane wave propagating in the directions \( \mathbf{r}_{m} \).
When the Hertzian dipole array is placed in the stratified medium bounded between a ground plane and a set of dielectric slabs, as shown in Figure 2.3, the waves radiated from the array will bounce back and forth between ground plane at \( y=0 \) and dielectric interface at \( y=b_m \). Some electric field will be transmitted through the dielectric slabs and radiated into the free space. Since only the field information inside the medium \( m \) between ground plane and the dielectric slabs is important in the following discussions, we will concentrate on the fields in this bounded region.

To describe the field reflected from a set of dielectric slabs, the effective reflection coefficient for the dielectric interface should be used. Since the effective reflection coefficient in general depends on the polarization of the incident wave on the interface, two kinds of effective reflective coefficients will be used: one is used for the field with polarization parallel to the plane of incidence denoted by \( \Gamma_{m\parallel} \), another one is used for the field with polarization perpendicular to the plane of incidence, denoted by \( \Gamma_{m\perp} \). The subscript \( m \) is used to indicate that the interface is at the boundary of medium \( m \). The (+) sign is attached when the dielectric interface is on the right side of the medium, while the (-) sign will be used when the dielectric interface is on the left side of the medium. Some details about the effective reflection coefficient will be presented in Appendix B.
Figure 2.3 Infinite array of Hertzian dipoles is placed between a perfectly conducting ground plane and a set of dielectric slabs.
The plane of incidence is defined as the plane containing the vector normal to the dielectric interface and the direction of propagation. Pictorial definition of the plane of incidence for a wave travelling from medium \( m \) to medium \( m+1 \), in the \( +y \) direction, is shown in Figure 2.4. Two unit vectors \( \hat{n}_{m+} \) and \( \hat{n}_{m+} \) which are parallel and perpendicular respectively to the plane of incidence are also shown in the figure. These two unit vectors are mathematically given by

\[
\hat{n}_{m+} = \frac{\hat{n}_D \times \hat{r}_{m+}}{|\hat{n}_D \times \hat{r}_{m+}|} \quad \hat{n}_{m+} = \frac{\hat{x}r_{mx} + \hat{z}r_{mx}}{\sqrt{r_{mx}^2 + r_{my}^2}}
\]

(2.9)

\[
\hat{n}_{m+} = \hat{n}_{m+} \times \hat{r}_{m+}
\]

\[
\hat{n}_{m+} = \frac{1}{\sqrt{r_{mx}^2 + r_{my}^2}} \left[ -\hat{x}r_{mx} + \hat{y}(r_{mx}^2 + r_{my}^2) -\hat{z}r_{my}r_{mx} \right]
\]

(2.10)

The unit vectors \( \hat{n}_{m-} \) and \( \hat{n}_{m-} \) for the wave travelling in the \(-y\) direction can be obtained by using the relationships

\[
\hat{n}_{m-} = \hat{n}_{m+},
\]

(2.11)

and

\[
\hat{n}_{m-} = -\hat{n}_{m-} \times \hat{r}_{m-}.
\]

(2.12)
Figure 2.4 Illustration of a plane wave propagating in the direction \( \hat{r}_{m+} \) incident on dielectric interface \( \varepsilon_m, \varepsilon_{m+1} \). The wave is decomposed along the orthogonal and parallel unit vectors \( \hat{n}_{m+} \) and \( \hat{n}_{m+}^\parallel \), respectively.
The unit vectors $\hat{m}_{m-}$ and $\hat{n}_{m-}$ are then given by

$$\hat{m}_{m-} = \frac{-xr_mz + zr_mx}{\sqrt{r^2_{mx} + r^2_{mz}}},$$  \hspace{1cm} (2.13)$$

and

$$\hat{n}_{m-} = \frac{1}{\sqrt{r^2_{mx} + r^2_{mz}}} \left[ -\hat{x} r_{mx} r_{my} - \hat{y} (r^2_{mx} + r^2_{mz}) - \hat{z} r_{my} r_{mz} \right].$$  \hspace{1cm} (2.14)$$

The orientations of these four unit vectors are shown in Figure 2.5.

In order to calculate the field reflected from a dielectric interface by using the effective reflection coefficients described above, it will be necessary to decompose the incident plane wave into two components: one in the $\hat{m}_{m-}$ direction, another one in the $\hat{n}_{m-}$ direction. This can be done by simply expressing the wave vectors, $\mathbf{e}_{m\pm}$ as defined by Equation (2.8), in terms of the unit vectors just defined, i.e.

$$\mathbf{e}_{m\pm} = \hat{m}_{m\pm} (\mathbf{e}_{m\pm} \cdot \hat{m}_{m\pm}) + \hat{n}_{m\pm} (\mathbf{e}_{m\pm} \cdot \hat{n}_{m\pm}).$$  \hspace{1cm} (2.15)$$

Since $\mathbf{e}_{m\pm}$, $\hat{m}_{m\pm}$, and $\hat{n}_{m\pm}$ are all transverse to the direction of propagation $r_{m\pm}$, the wave vector $\mathbf{e}_{m\pm}$ can be written in an alternate form

$$\mathbf{e}_{m\pm} = -\hat{m}_{m\pm} (\mathbf{p} \cdot \hat{m}_{m\pm}) - \hat{n}_{m\pm} (\mathbf{p} \cdot \hat{n}_{m\pm}).$$  \hspace{1cm} (2.16)$$

The detailed derivation will be given in Appendix A.
Figure 2.5 Relative orientation of unit vectors $\mathbf{r}_{m\pm}$, $\mathbf{n}_{m\pm}$ and $\mathbf{\eta}_{m\pm}$. 
It has been demonstrated in [3] that the total field at a field point inside a bounded dielectric medium is composed of four plane wave modes. But for convenience sake, these four plane wave modes will be further decomposed into six plane wave modes as illustrated in Figures 2.6-2.11. Mode 1 and Mode 2 are single-bounce modes. Mode 3 and Mode 4 are double-bounce modes. Mode 5 and Mode 6 are direct modes. It is noticed from Figures 2.6-2.9 that every bounce mode wave is characterized by an initial bounce and an infinite number of subsequent bounces. In those figures, the initial bounce is represented by a wider line and the subsequent bounces are represented by a series of broken lines.

In order to understand the field phenomenon inside the bounded medium, the field comprising these four bounce mode waves will be introduced in some detail. In the following discussion, we assume that the array is located somewhere inside the bounded medium between \( y=0 \) and \( y=b_m \) and its reference element is at \( R' \). The field point \( R \) can be anywhere inside the same region, either on the left or on the right of the array.

For Mode 1, the left-going plane waves emanating from the array are bouncing back and forth between the ground plane and the dielectric slabs. All the bounced waves are arriving at the field point from the left as indicated by the arrows shown in Figure 2.6. After the initial bounce, an infinite number of subsequent bounces will arrive at the field point with a phase delay \( e^{-j2\pi b_m y} \) for every complete bounce.
Figure 2.6 Illustration of "Mode 1" (a) when \( \hat{\gamma} \cdot \hat{R} < \hat{\gamma}^* \cdot \hat{R}^* \), (b) when \( \hat{\gamma} \cdot \hat{R} > \hat{\gamma}^* \cdot \hat{R}^* \).
Figure 2.7: Illustration of "Mode 2" (a) when \( \hat{g}_R > \hat{g}_R' \) and (b) when \( \hat{g}_R < \hat{g}_R' \).
Figure 2.8 Illustration of mode 3

(a) \( \delta < \delta \text{ max} \)

(b) \( \delta > \delta \text{ max} \)
Figure 2.9 Illustration of "Mode 4" (a) when \( \gamma R < \gamma^* R^* \), (b) when \( \gamma R > \gamma^* R^* \)
Figure 2.10 Illustration of "Mode 5."
Figure 2.11 Illustration of "mode 6."
The field contribution from Mode 1 is given by

\[
\begin{align*}
\vec{dE}_1(R) &= \text{Idz}' \frac{Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{-j\varepsilon_m(\vec{R}-\vec{R}') \cdot \vec{r}_{m+} - j2\varepsilon_m(\vec{y} \times \vec{R}')} {\vec{r}_{my}} e^{j2\varepsilon_m(b_{m+} \cdot \vec{y} + \vec{R}')} \\
&= \left[ \hat{I}_{m+} (\vec{e}_{m+} \cdot \hat{n}_{m-}) \hat{I}_{m-} \hat{I}_m + \hat{n}_{m+} (\vec{e}_{m+} \cdot \hat{n}_{m-}) \hat{I}_{m-} \hat{I}_m \right].
\end{align*}
\]

(2.17)

where

\[
\hat{I}_m = \frac{1}{1 - \frac{1}{\hat{I}_{m-} \hat{I}_{m+} e^{j2\varepsilon_m(b_{m+} \cdot \vec{y} + \vec{R}')}}}.
\]

(2.18)

The factors \(\hat{I}_m\) account for the total contribution from the initial bounce and an infinite number of subsequent bounces. When \(\hat{I}_m\) in Equation 2.17 are replaced by 1's, the result will represent the field contribution from the initial bounce only.

For Mode 2, the wave emanating from the array is moving toward the right and all the bounced waves arriving at the field point are coming from the right. The field contribution from Mode 2 is given by

\[
\begin{align*}
\vec{dE}_2(R) &= \text{Idz}' \frac{Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{-j\varepsilon_m(\vec{R}-\vec{R}') \cdot \vec{r}_{m-} - j2\varepsilon_m(b_{m-} \cdot \vec{y} + \vec{R}')} {\vec{r}_{my}} e^{j2\varepsilon_m(b_{m-} \cdot \vec{y} + \vec{R}')} \\
&= \left[ \hat{n}_{m-} (\vec{e}_{m-} \cdot \hat{n}_{m+}) \hat{I}_{m+} \hat{I}_m + \hat{n}_{m-} (\vec{e}_{m-} \cdot \hat{n}_{m+}) \hat{I}_{m+} \hat{I}_m \right].
\end{align*}
\]

(2.19)
For Mode 3, the wave emanating from the array is moving toward the left and all the bounced waves arriving at the field point are coming from the right. The field contribution from Mode 3 is given by

\[
d\vec{E}_3(\vec{R}) = \text{Id} \Delta \delta \frac{Z_m}{20\lambda} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(\vec{R}-\vec{R}')} \cdot r_{m-} e^{-j2\beta_m b_m r_{my}} \\
\left[ \hat{n}_{m-}(\hat{e}_{m-} \cdot \hat{n}_{m-}) \hat{r}_{m-} \hat{r}_{m+} T_m + \hat{n}_{m-}(\hat{e}_{m-} \cdot \hat{n}_{m-}) \hat{r}_{m-} \hat{r}_{m+} T_m \right].
\]

(2.20)

For Mode 4, the wave emanating from the array is moving toward the right and all the bounced waves arriving at the field point are coming from the left. The field contribution from Mode 4 is given by

\[
d\vec{E}_4(\vec{R}) = \text{Id} \Delta \delta \frac{Z_m}{20\lambda} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(\vec{R}-\vec{R}')} \cdot r_{m+} e^{-j2\beta_m b_m r_{my}} \\
\left[ \hat{n}_{m+}(\hat{e}_{m+} \cdot \hat{n}_{m+}) \hat{r}_{m-} \hat{r}_{m+} T_m + \hat{n}_{m+}(\hat{e}_{m+} \cdot \hat{n}_{m+}) \hat{r}_{m-} \hat{r}_{m+} T_m \right].
\]

(2.21)

Note that Equations (2.17)-(2.21) are always true for the field point which is located anywhere in the bounded medium \( m \). In other words, the validity of these four equations is independent of the relative position between the field point and the array; the field point \( \vec{R} \) can be either on the left or on the right of the array.
Mode 1 and Mode 4 can be added up to form one bounce mode plane wave spectrum \( dE_{B+} \), arriving at the field point \( \tilde{R} \) in the direction \( \hat{r}_{m+} \)

\[
dE_{B+}(\tilde{R}) = dE_1(\tilde{R}) + dE_4(\tilde{R})
\]

\[
= \frac{Z_m}{2D_D^2} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m (\tilde{R} - \tilde{R}')} \frac{\hat{r}_{m+}}{r_{my}}
\]

\[
\left\{ e^{-j2\beta_m (y \cdot \hat{R}')} r_{my} \left[ \hat{e}_{m+} \cdot \hat{r}_{m+} \right] \left[ \hat{e}_{m-} \cdot \hat{r}_{m-} \right] \right. \\
\left. \hat{r}_{m+} \left( \hat{e}_{m+} \cdot \hat{r}_{m+} \right) \hat{r}_{m-} \hat{r}_{m-} \right\} 
\]

\[
\left\{ e^{-j2\beta_m \hat{b}_m} r_{my} \left[ \hat{e}_{m+} \cdot \hat{r}_{m+} \right] \left( \hat{e}_{m-} \cdot \hat{r}_{m-} \right) \right. \\
\left. \hat{r}_{m+} \left( \hat{e}_{m+} \cdot \hat{r}_{m+} \right) \hat{r}_{m-} \hat{r}_{m-} \right\} 
\]

Similarly, Mode 2 and Mode 3 can be added up to form one bounce mode plane wave spectrum \( dE_{B-} \), arriving at the field point \( \tilde{R} \) in the direction \( \hat{r}_{m-} \).
\[ dE_{\perp}(R) = dE_2(R) + dE_3(R) \]

\[ = \frac{Z_m}{\lambda x} 2D_{Dz} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(R-R')} \cdot r_m \cdot \sum_{m} e^{-j2\beta_m h_m r_{my}} \]

\[ \left\{ e^{j2\beta_m(y-R')} r_{my} \left[ \hat{e}_{n_m-}(e_m+\hat{m}_{n_m+}) \perp r_m + \perp T_m \right] + \right\} \]

\[ \left[ \hat{e}_{n_m-}(e_m-\hat{m}_{n_m-}) \perp r_m - \perp r_m + \perp T_m \right] \]

\[ \left[ \hat{e}_{n_m-}(e_m-\hat{m}_{n_m-}) \perp r_m - \perp r_m + \perp T_m \right] \]

\[ \left(2.23\right) \]

Now, we will consider two direct modes, Mode 5 and Mode 6. The waves of these two modes are arriving at the field point directly from the array. To obtain the field expressions for these two modes, one can imagine that the ground plane and dielectric slabs were removed and the entire array were immersed in a homogeneous medium \( m \). This situation is exactly the same as the one mentioned in the last section. The equations obtained there can, therefore, be applied here to describe the
fields for Mode 5 and Mode 6. The only difference is that, in the present case, the field point \( \mathbf{R} \) is limited to the region between \( y=0 \) and \( y=b_m \). The field of Mode 5 exists only at the field point which is on the left of the array, i.e.

\[
\hat{dE}_5(R) = \hat{dE}_5(R) = \frac{Z_m}{\sqrt{2}\pi D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(R-R') \cdot \mathbf{r}_m} e_{m-},
\]

\[
0 < \hat{y} \cdot \mathbf{R} < \hat{y} \cdot \mathbf{R}', \tag{2.24}
\]

whereas the field of Mode 6 exists only at the field point which is on the right of the array, i.e.

\[
\hat{dE}_6(R) = \hat{dE}_6(R) = \frac{Z_m}{\sqrt{2}\pi D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(R-R') \cdot \mathbf{r}_m} e_{m+},
\]

\[
\hat{y} \cdot \mathbf{R}' < \hat{y} \cdot \mathbf{R} < b_m. \tag{2.25}
\]

The total field at the field point \( \mathbf{R} \) is the vector sum of \( \hat{dE}_B^+(R) \), \( \hat{dE}_B^-(R) \) and one of the two direct mode fields just mentioned. The choice of the direct mode is dependent upon the relative position
between the field point and the array. When the field point is on the left of the array, the total field is given by

\[ d\vec{E}(\vec{R}) = d\vec{E}_B^+(\vec{R}) + d\vec{E}_B^-(\vec{R}) + d\vec{E}_S(\vec{R}) \]

\[ 0 < y \cdot \vec{R} < y \cdot \vec{R}' \]  \hspace{1cm} (2.26)

When the field point is on the right of the array, the field is given by

\[ d\vec{E}(\vec{R}) = d\vec{E}_B^+(\vec{R}) + d\vec{E}_B^-(\vec{R}) + d\vec{E}_S(\vec{R}) \]

\[ y \cdot \vec{R}' < y \cdot \vec{R} < b_m \]  \hspace{1cm} (2.27)

D. FIELD RADIATED FROM AN INFINITE ARRAY OF NONPLANAR LINEAR ELEMENTS IN THE PRESENCE OF GROUNDPLANE AND DIELECTRIC SLABS

In this section, the results obtained in the previous section will be generalized further to describe the field radiated from a nonplanar linear array with an arbitrary current distribution. Consider a linear array located inside a bounded dielectric medium as shown in Figure 2.12, where the array elements are oriented in the \( \vec{p}^{(1)} \) -direction and the current distribution \( I(\vec{\xi}') \) along the element is a function of
Figure 2.12 An infinite array of linear elements located in a medium bounded by a perfectly conducting ground plane and a set of dielectric slabs.
position. Such an array can be envisioned as being composed of infinitely many arrays of Hertzian dipoles. The field radiated from this linear array can be obtained by adding up all the contributions from these "subarrays."

The field contributions from the bounce mode waves generated by each of these subarrays are given by Equations (2.22) and (2.23). Therefore, the field contributions from the bounce mode waves generated by the linear array with an arbitrary current distribution can be obtained simply by integration of Equations (2.22) and (2.23) over the reference element. The field contribution from the bounce mode waves arriving at the field point \( \mathbf{R} \) in the direction \( \hat{r}_{m+} \) is given by

\[
\mathbf{E}_{B+}(\mathbf{R}) = \frac{Z_m}{2D} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{j\beta_m \mathbf{R} \cdot \hat{r}_{m+}} \mathbf{I}_m \cdot \mathbf{D}_n \cdot \mathbf{I}_{m-1} \mathbf{T}_m ^+ \\
\left\{ \left[ \mathbf{I}_m \cdot \mathbf{D}_n \mathbf{I}_{m-1} \mathbf{T}_m ^+ \right] \mathbf{I}(\mathbf{R}'_m) e^{j\beta_m (\mathbf{R}' \cdot \hat{r}_{m+} + 2r_{my})} \right\} \\
+ e^{-j2\beta_m b_{my}} \left[ \mathbf{I}_m \cdot \mathbf{D}_n \mathbf{I}_{m-1} \mathbf{T}_m ^+ \right] \mathbf{I}(\mathbf{R}'_m) e^{j\beta_m \mathbf{R} \cdot \hat{r}_{m+}} \right\}.
\]

Equation (2.28)
If an arbitrary point \( \mathbf{R}^{(1)} \) on the reference element is chosen as the reference point, the source point \( \mathbf{R}' \) can be expressed as

\[
\mathbf{R}' = \mathbf{R}^{(1)} + \mathbf{p}(1)z',
\]

where \( z_a < z' < z_b \) ,

\[(2.29)\]

where \( z_a \) and \( z_b \) represent the two endpoints of the reference element.

Substituting Equation (2.29) into Equation (2.28), we have

\[
\mathbf{E}_{B^+}(\mathbf{R}) = \frac{I(\mathbf{R}^{(1)}Z_m)}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j2\pi m(R-R^{(1)}\cdot \mathbf{r}_{m+}}
\]

\[
\left\{ e^{-j2\pi m(y\cdot \mathbf{R}^{(1)})} \mathbf{r}_{my} \left[ \mathbf{\hat{n}}_{m+}(\mathbf{e}_{m+} \cdot \mathbf{\hat{n}}_{m+}) \mathbf{p}_{m-} \mathbf{r}_{m-} \mathbf{T}_{m} \right]
\]

\[
\mathbf{\hat{n}}_{m+}(\mathbf{e}_{m+} \cdot \mathbf{\hat{n}}_{m+}) \mathbf{p}_{m-} \mathbf{r}_{m-} \mathbf{T}_{m}
\]

\[
+ e^{-j2\pi m^b m^\tau y \mathbf{r}_{my}} \left[ \mathbf{\hat{n}}_{m+}(\mathbf{e}_{m+} \cdot \mathbf{\hat{n}}_{m+}) \mathbf{p}_{m+} \mathbf{r}_{m-} \mathbf{r}_{m+} \mathbf{T}_{m} \right]
\]

\[
\mathbf{\hat{n}}_{m+}(\mathbf{e}_{m+} \cdot \mathbf{\hat{n}}_{m+}) \mathbf{p}_{m+} \mathbf{r}_{m-} \mathbf{r}_{m+} \mathbf{T}_{m}
\]

\[(2.30)\]
where

\[
P_{m\pm} = \frac{1}{I(R(1))} \int_{z_a}^{z_b} I(z') e^{j \beta_m p \cdot r_{m\pm}} dz
\]  \hspace{1cm} (2.31)

Note that \( P_{m\pm} \) are merely the far field radiation pattern factors of the linear array element oriented in the \( p^{(1)} \) direction with current distribution \( I(z') \).

Following a similar procedure, the field contributions from the bounce mode waves arriving at the field point \( \bar{R} \) in the direction \( \hat{r}_m \) can be obtained as:

\[
\bar{E}_{B-}(\bar{R}) = \frac{I(R(1)) Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j \beta_m (\bar{R} - R')} \hat{r}_m \cdot \hat{r}_m e^{-j \beta_b m \gamma_{my}}
\]

\[
e^{j 2 \beta_m (y \cdot \bar{R}(1)) \gamma_{my}} \left[ \hat{n}_{m-} (\hat{e}_{m+} \cdot \hat{n}_{m+}) P_{m+1 m+1 T_m} \right. \\
+ \left. \hat{n}_{m-} (\hat{e}_{m-} \cdot \hat{n}_{m-}) P_{m-1 m-1 T_m} \right] \\
+ \hat{n}_{m-} (\hat{e}_{m-} \cdot \hat{n}_{m-}) P_{m-1 m+1 T_m} + \\
\left. \hat{n}_{m-} (\hat{e}_{m-} \cdot \hat{n}_{m-}) P_{m+1 m-1 T_m} \right) \\
\]  \hspace{1cm} (2.32)
As indicated in Equation (2.16), we know

$$\hat{e}_{m^\pm} \cdot \hat{n}_{m^\pm} = -p^{(1)} \cdot \hat{n}_{m^\pm} \quad (2.33)$$

Now we define the parallel and orthogonal components of $P_{m^\pm}$

$$\hat{p}_{m^\pm} = [p^{(1)} \cdot \hat{n}_{m^\pm}] P_{m^\pm} \quad (2.34)$$

Making use of Equations (2.33) and (2.34), the field contributions from the bounce mode waves can be expressed as:

$$E_{B^+}(R) = \frac{I(R^{(1)}) Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j \beta_m (R-R^{(1)}) \cdot r_{m^\pm}} r_{m^\pm}$$

$$\left\{ \begin{array}{l}
e^{-j2\beta_m (y \cdot R^{(1)}) r_{m^\pm}} \hat{n}_{m^\pm} P_{m^\pm} r_{m^\pm} \hat{T}_m \\
+ e^{-j2\beta_m b_{m^\pm} r_{m^\pm}} \hat{n}_{m^\pm} P_{m^\pm} r_{m^\pm} \hat{T}_m \end{array} \right\}$$

$$\hat{n}_{m^\pm} P_{m^\pm} r_{m^\pm} \hat{T}_m \quad (2.35)$$
Finally, the fields \( \vec{E}_B(\vec{R}) \) and \( \vec{E}_B(\vec{R}) \) can be added up to form a total bounce mode field \( \vec{E}_B(\vec{R}) \) which is the total field contribution from all the bounce mode waves generated by the entire linear array, i.e.

\[
\vec{E}_B(\vec{R}) = \vec{E}_{B+}(\vec{R}) + \vec{E}_{B-}(\vec{R})
\]
\[ I(R(1)) \frac{Z_m}{2D_x D_z} \left\{ \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \right\} \]

\[ \frac{e^{-j \beta_m (R-h) \cdot r_m \cdot r_{my}}}{r_{my}} \left[ \hat{\eta}_{m+1} B_{m+} + \hat{\eta}_{m+1} B_{m+} \right] \]

\[ + \frac{e^{-j \beta_m (R-h) \cdot r_m \cdot r_{my}}}{r_{my}} \left[ \hat{\eta}_{m-1} B_{m-} + \hat{\eta}_{m-1} B_{m-} \right] \]  

where

\[ B_{m+} = \tau_m T_m e^{-j 2 \beta_m (y-R(1)) r_{my}} \]

\[ + \tau_m T_{m+1} e^{-j 2 \beta_m b_{m+} r_{my}} \]  

(2.38)

\[ B_{m-} = \tau_m T_m e^{-j 2 \beta_m (b_{m-y} \cdot R(1)) r_{my}} \]

\[ + \tau_m T_{m+1} e^{-j 2 \beta_m b_{m+} r_{my}} \]  

(2.39)

Note that the factors \( B_{m+} \) and \( B_{m-} \) defined above are associated with the linear array only and are independent of the location of the field point \( R \).
Now, we will consider the field contributions from the direct mode waves generated by the linear array. In this case, it is convenient to divide the bounded medium into three regions as illustrated in Figure 2.13. The boundaries between the regions are defined by the tips of the elements. If the two endpoints of the reference element are located at \( \bar{R}_a \) and \( \bar{R}_b \) respectively, (see Figure 2.13) the domains of these three regions can, then, be defined as:

(a). Region I : \( 0 < \hat{y} \cdot \bar{R} < \hat{y} \cdot \bar{R}_b \)
(b). Region II : \( \hat{y} \cdot \bar{R}_b < \hat{y} \cdot \bar{R} < \hat{y} \cdot \bar{R}_a \)
(c). Region III : \( \hat{y} \cdot \bar{R}_a < \hat{y} \cdot \bar{R} < b_m \)

where \( \bar{R} \) is the location of the field point. Note that when the array is planar, Region II does not exist. In that case, the field point can always be considered in either Region I or Region III.

Region I is on the left of the array. In this region, the direct mode contribution is coming from the Mode 5, left-going waves, generated by the entire linear array; the field at an arbitrary field point \( \bar{R} \) is obtained by integration of Equation (2.24) over the domain of the reference element

\[
\tilde{E}_{D1}(\bar{R}) = \frac{I(\bar{R}(1)) Z_m}{2 D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j \beta_m (\bar{R}-\bar{R}(1)) \cdot \mathbf{r}_m - e^{m-p_m}}}{r_{my}}, \quad 0 < \hat{y} \cdot \bar{R} < \hat{y} \cdot \bar{R}_b .
\]

(2.40)
Figure 2.13 Three regions defined inside the bounded medium.
For nonplanar arrays, Region II always exists. Region II covers the entire domain of the array. At an arbitrary point in this region, the observer will receive two direct mode waves, Mode 5 and Mode 6, simultaneously; Mode 5 is generated by the subarrays which are on the right of the field point, Mode 6 is generated by the subarrays which are on the left of the field point. For illustration purposes, we draw the reference element of the linear array as shown in Figure 2.14, where a field point $\mathbf{R}$ is located in Region II. The direct mode field at $\mathbf{R}$ can be obtained by integrating Equation (2.24) over the source points from $\xi_a$ to $\xi_0$ and Equation (2.25) over the remaining source points from $\xi_0$ to $\xi_b$. The point $\xi_0$ is defined as the point on the reference element having the same $y$-coordinate as the point of the field point:

$$y \cdot \mathbf{R} = y \cdot \mathbf{R'} \bigg|_{\xi' = \xi_0} \quad (2.41)$$

Substituting Equation (2.29) into Equation (2.41), with $\xi' = \xi_0$, we have

$$\xi_0 = \frac{\hat{y} \cdot (\mathbf{R} - \mathbf{R}^{(1)})}{\hat{y} \cdot \hat{p}^{(1)}} = \frac{\hat{y} \cdot (\mathbf{R} - \mathbf{R}^{(1)})}{\hat{p}^{(1)}} \cdot \frac{p^{(1)}}{y} \quad (2.42)$$

where

$$\hat{p}^{(1)} = p^{(1)} x + p_y^{(1)} y + p_z^{(1)} z \quad (2.43)$$
Figure 2.14 Reference element of a linear array with a field point in Region II.
Therefore, for a nonplanar linear array, the direct mode field in
Region II is given by

\[
\mathbf{E}_{D2}(\mathbf{R}) = \frac{Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right. \\
\left. \begin{array}{c}
\frac{e^{-j\beta_m (R-R(1)) \cdot r_m} - e^{-j\beta_m (R-R(1)) \cdot r_m}}{r_{my}} \\
\frac{e^{-j\beta_m (R-R(1)) \cdot r_m} - e^{-j\beta_m (R-R(1)) \cdot r_m}}{r_{my}} \\
\end{array} \right. \\
\frac{\mathbf{I}(\mathbf{z}')e^{j\beta_m (R-R(1)) \cdot r_m}}{\mathbf{I}(\mathbf{z}')e^{j\beta_m (R-R(1)) \cdot r_m}} \\
\frac{\mathbf{I}(\mathbf{z}')e^{j\beta_m (R-R(1)) \cdot r_m}}{\mathbf{I}(\mathbf{z}')e^{j\beta_m (R-R(1)) \cdot r_m}} \\
\left. \right. \\
\hat{y} \cdot \mathbf{R}_b < \hat{y} \cdot \mathbf{R} < \hat{y} \cdot \mathbf{R}_a \\
\right.
\]

Region III is to the right of the array. In this region, the
direct mode contribution is coming from the Mode 6 generated by the
entire linear array. The direct mode field at an arbitrary field point
\( \mathbf{R} \) in this region can be obtained by integration of Equation (2.25) over
the domain of the reference element
\[ E_{D3}(\vec{R}) = \left( \frac{I(R^{(1)})Z_m}{2D \frac{D}{x \cdot z}} \right) \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(\vec{R}-\vec{R}^{(1)}) \cdot \vec{r}_{m+}} \frac{\vec{e}_{m+}}{r_{my}} \]  
\[ \hat{y} \cdot \vec{R}_a < \hat{y} \cdot \vec{R} < b_m \]

\[ (2.45) \]

The total field at the field point \( \vec{R} \) in the bounded medium is the vector sum of the total bounce mode field and the direct mode field generated by the linear array, i.e.

\[
E(\vec{R}) = \begin{cases} 
E_B(\vec{R}) + E_{D1}(\vec{R}) ; & 0 < \hat{y} \cdot \vec{R} < \hat{y} \cdot \vec{R}_b \\
E_B(\vec{R}) + E_{D2}(\vec{R}) ; & \hat{y} \cdot \vec{R}_b < \hat{y} \cdot \vec{R} < \hat{y} \cdot \vec{R}_a \\
E_B(\vec{R}) + E_{D3}(\vec{R}) ; & \hat{y} \cdot \vec{R}_a < \hat{y} \cdot \vec{R} < b_m 
\end{cases}
\]

\[ (2.46) \]

where \( E_B(\vec{R}) \) is the total bounce mode field, given by Equation (2.37); \( E_{D1}(\vec{R}), E_{D2}(\vec{R}) \) and \( E_{D3}(\vec{R}) \) are the direct mode fields, given by Equations (2.40), (2.44) and (2.45), respectively.
E. FIELDS RADIATED FROM AN INFINITE ARRAYS OF FEEDLINES AND DIPOLES IN
THE PRESENCE OF GROUND PLANE AND DIELECTRIC SLABS

At this point, we are ready to derive the fields radiated from
the infinite arrays of feedlines and dipoles. Consider a V-shaped
dipole array with feedlines located in a bounded medium as shown in
Figure 2.15. The feedlines are perpendicular to the ground plane and
the two legs of each dipole are bent downward making the same angle, \( \theta_b \),
with respect to their own feedline. The angle \( \theta_b \) is defined as the
"bend angle" of the dipole. The feedlines and the dipole legs are
oriented in the directions \( \hat{p}^{(3)} \), \( \hat{p}^{(1)} \) and \( \hat{p}^{(2)} \), respectively, as
indicated in Figure 2.15. Without loss of generality, we will assume
that all the dipoles are parallel to the yz-plane in the following
discussion. Due to the special geometry of the feedlines and dipoles
and the above assumption, the unit vectors, \( \hat{p}^{(1)} \), \( \hat{p}^{(2)} \) and \( \hat{p}^{(3)} \) can be
expressed as:

\[
\hat{p}^{(1)} = p_y y + p_z z, \quad (2.47)
\]

\[
\hat{p}^{(2)} = p_y y + p_z z, \quad (2.48)
\]

\[
\hat{p}^{(3)} = p_y y, \quad (2.49)
\]

where
Figure 2.15 An infinite dipole array with feedlines situated in a medium bounded by a perfectly conducting ground plane and a set of dielectric slabs.
\[ p_{y}^{(1)} = p_{y}^{(2)} = -\cos(\theta_b) , \]  

\[ p_{z}^{(1)} = p_{z}^{(2)} = \sin(\theta_b) , \]  

\[ p_{y}^{(3)} = \]  

\[ p_{y} = 1 \]  

The balanced and unbalanced mode current distributions along the dipole and feedline are shown in Figure 2.16. \( \lambda_e \) is the effective length of the leg of the dipole and is always greater than \( \lambda_d \), the physical length of the leg. \( \beta_d \) is the effective propagation constant of the current along the dipole. It depends upon the \( \beta_m \) of the medium in which the dipole is located. The parameters \( U_a, U_b, U_c, U_d \) and \( U_e \) are used to generate the possible unbalanced mode current patterns along the feedline. These parameters can be determined by satisfying some boundary conditions for the unbalanced mode currents on the feedline and dipole, the details of which will be discussed in Appendix C.

The array shown in Figure 2.15 can be decomposed into two arrays, namely feedline array and dipole array as shown in Figure 2.17 and Figure 2.19, respectively. These two arrays have the same grid structure and are both infinite in extent. The feedline array can be considered as a linear array with elements of length \( \lambda_f \) and oriented in the \( \hat{p}^{(3)} \) direction. The dipole array consists of two linear arrays with elements of equal length \( \lambda_d \) and oriented in the directions \( \hat{p}^{(1)} \) and \( \hat{p}^{(2)} \), respectively.
Figure 2.16 Balanced and unbalanced current distributions.
Figure 2.17 Three regions defined for dipole array inside the bounded medium.
The reference point, \( R^{(d)} \), of the dipole array is chosen at the feed point of the reference dipole. For the feedline array, the reference point, \( R^{(f)} \), is located close to the open end of the reference feedline, see Figure 2.16.

1. Bounce Mode Fields

The bounce mode field generated by the balanced or unbalanced mode current on the \( p \) directed legs of the dipole array can be obtained, by using Equation (2.37), expressed here in a generalized form as:

\[
E_{B}^{(i)M}(R) = - \frac{I^{(1)M} \sin(\beta_{d} e_{y})Z_{m}}{2D_{x}D_{z}} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right.
\]
\[
\frac{e^{-j\beta_{m}(R-R^{(d)}) \cdot r_{m+}}}{r_{my}} [\hat{B}_{m+}^{(i)} + i\hat{B}_{m+}^{(i)}] + \frac{e^{-j\beta_{m}(R-R^{(d)}) \cdot r_{m-}}}{r_{my}} [\hat{B}_{m-}^{(i)} + i\hat{B}_{m-}^{(i)}] \}
\]

\[
i = 1, 2
\]

(2.53)
where

\[
I(i)M = \begin{cases} 
(-1)^i I_{bd} & \text{when } M = b \\
I_{ud} & \text{when } M = u 
\end{cases},
\]

(2.54)

\[
\int_{m+}^B(i) = \int_{m-}^r m m \left[ \int_{m-}^p(i) e^{-j2\delta_m(y-R(d))r_{my}} 
+ \int_{m-}^P(i) \int_{m-}^r m m \right],
\]

(2.55)

\[
\int_{m-}^B(i) = \int_{m+}^r m m \left[ \int_{m+}^p(i) e^{-j2\delta_m(b_m-y-R(d))r_{my}} 
+ \int_{m-}^P(i) \int_{m-}^r m m \right],
\]

(2.56)

\[
\int_{m\pm}^p(i) = \int_{m\pm}^r m m \int_{m\pm}^p(i),
\]

(2.57)

\[
p(i)_{m\pm} = \frac{1}{\sin(\beta_d\theta_e)} \int_{\theta_d}^{\theta_d} \frac{\sin(\beta_d\theta_e') \int_{m\pm}^r m m \int_{m\pm}^p(i)}{d\theta_e'},
\]

(2.58)
The superscript $M$ is used to denote the current mode on the $p^{(i)}$-directed legs which generated the field; $M$ may be "b", balanced mode, or "u", unbalanced mode. $I_{bd}$ and $I_{ud}$ are the magnitudes of the balanced and unbalanced mode currents on the dipole, as indicated in Figure 2.16. The factors $p^{(i)}$ defined by Equation (2.58) will be evaluated in Appendix D.

Following a similar procedure, the bounce mode field generated by the feedline array can be expressed as:

$$E_{B}^{(3)}(R) = -\frac{I^{(3)}(0)z_{m}}{2D_{x}D_{z}} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{e^{-j\pi n(R-R_{f})}r_{m+}}{r_{my}} \left[ \hat{I}_{m+}B_{m+}^{(3)} + \hat{I}_{m+}B_{m+}^{(3)} \right] \right.$$ 

$$+ e^{-j\pi n(R-R_{f})}r_{m-} \left[ \hat{I}_{m-}B_{m-}^{(3)} + \hat{I}_{m-}B_{m-}^{(3)} \right] \right\} ,$$

$$\hfill (2.59)$$
\[
\begin{align*}
\hat{P}_m^{(3)} &= I_m^{r-m} T_m \left[ I_m^{p(3)} e^{-j 2 \hat{a}_m (y - \hat{R} (f)) r_m} ight. \\
& \quad \left. + I_m^{p(3)} e^{-j 2 \hat{a}_m b_m r_m} \right]
\end{align*}
\] (2.60)

\[
\begin{align*}
\hat{P}_m^{-} &= I_m^{r-m} T_m \left[ I_m^{p(3)} e^{-j 2 \hat{a}_m (b_m - y + \hat{R} (f)) r_m} ight. \\
& \quad \left. + I_m^{p(3)} e^{-j 2 \hat{a}_m b_m r_m} \right]
\end{align*}
\] (2.61)

\[
\begin{align*}
\hat{P}_m^{(3)} &= \hat{P}_m^{(3)} \hat{P}_m^{(3)} \\
&= \left[ \hat{P}_m^{(3)} \hat{P}_m^{(3)} \right] \\
&= \left[ \hat{P}_m^{(3)} \hat{P}_m^{(3)} \right]
\end{align*}
\] (2.62)

\[
\begin{align*}
\hat{P}_m^{(3)} &= \frac{1}{I(3)(0)} \int_{-x_u}^{x_u} I(3)(z') e^{-j \hat{a}_m \hat{p}(3) \hat{r}_m \hat{z}'} dx' \\
&= \frac{1}{I(3)(0)} \int_{-x_u}^{x_u} I(3)(z') e^{-j \hat{a}_m \hat{p}(3) \hat{r}_m \hat{z}'} dx'.
\end{align*}
\] (3.63)
The current distribution of \( I^{(3)}(l') \) is defined in Figure 2.16. The factor \( P_{m\pm}^{(3)} \) defined by Equation (2.63) will also be evaluated in Appendix D. Since the feedlines are \( y \)-directed (i.e. \( p^{(3)} = y \)), upon the application of Equation (2.9), we have

\[
\hat{P}^{(3)} \cdot \hat{n}_{m\pm} = 0
\]

which will lead to the following results

\[
\hat{P}^{(3)} = 0
\]

and

\[
\hat{B}^{(3)} = 0
\]

Consequently, Equation (2.59) reduces to

\[
\frac{E_{B}^{(3)}(R)}{I^{(3)}(0)Z_{m}} = -\frac{1}{2D \omega_{X}} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{e^{-j \beta_{m}(R-R(f)) \cdot r_{m}^{+} \cdot \hat{r}^{(3)}}}{r_{my}^{+}} \hat{n}_{m}^{+} \hat{B}^{(3)} + \frac{e^{-j \beta_{m}(R-R(f)) \cdot r_{m}^{-} \cdot \hat{r}^{(3)}}}{r_{my}^{-}} \hat{n}_{m}^{-} \hat{B}^{(3)} \right\}
\]

\[
(2.67)
\]

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2. Direct Mode Fields

As we consider the direct mode field generated by the V-shaped dipole array, we have to divide the bounded medium into three regions as shown in Figure 2.17. The domains of these three regions can be defined as:

Region I_d : \( 0 < \hat{y} \cdot \vec{R} < \hat{y} \cdot \vec{R}(d) - \varepsilon_d \cos \theta_b \)

Region II_d : \( \hat{y} \cdot \vec{R}(d) - \varepsilon_d \cos \theta_b < \hat{y} \cdot \vec{R} < \hat{y} \cdot \vec{R}(d) \)

Region III_d : \( \hat{y} \cdot \vec{R}(d) < \hat{y} \cdot \vec{R} < b_m \)

(2.68)

Employing Equations (2.40), (2.44) and (2.45), we obtain the direct mode fields generated by the balanced or unbalanced mode currents on the \( \hat{p}^{(i)} \)-directed legs of the dipole array

Region I_d :

\[
E_{D1}^{(i)}(R) = \frac{(i)M \sin(\theta_d e)Z_m}{\sqrt{2D \times D}} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(R-R(d)) \cdot e_{m-}^{(i)}} e_{m-}^{(i)} \cdot e_{m-}^{(i)},
\]

\( i = 1,2 \)

(2.69)
Region II_d:

\[
E_{D2}^{(i)M}(R) = \frac{I_{(i)M}Z_m}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{c}
ed^{-j\beta_m(R-R^{(d)})r_{m-}^{(i)}} e^{j\beta_m^p(i)r_{m-}^{(i)}} \int_{z_0}^{z_d} \sin\beta_d(z_e-z') e^{-j\beta_m^p(i)r_{m-}^{(i)}} dz' \\
ed^{-j\beta_m(R-R^{(d)})r_{m+}^{(i)}} e^{j\beta_m^p(i)r_{m+}^{(i)}} \int_{z_0}^{z_d} \sin\beta_d(z_e-z') e^{-j\beta_m^p(i)r_{m+}^{(i)}} dz'
\end{array} \right. \\
i = 1,2
\]

(2.70)

Region III_d:

\[
E_{D3}^{(i)M}(R) = \frac{I_{(i)M}\sin(\beta_d z_e)Z_m}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{-j\beta_m(R-R^{(d)})r_{m+}^{(i)}} e^{-j\beta_m^p(i)r_{m+}^{(i)}}}{r_{my}^{m+ m+}} \\
i = 1,2
\]

(2.71)
where

\[ l_0 = \frac{\hat{y} \cdot (\bar{R} - \bar{R}(d))}{p_y^{(i)}} = \frac{\hat{y} \cdot (\bar{R} - \bar{R}(d))}{-\cos \theta_d} \]

(2.72)

The superscript \(M\) and the current \(I^{(i)M}\) have the same meanings as those described earlier. The pattern factors \(p_m^{(i)}\) are defined by Equation (2.58). Figure 2.18 shows the structure of the reference dipole and the domain of Region II_d.

As demonstrated in Appendix E, Equation (2.70) can be simplified as

\[
E^{(i)M}_{D2}(R) = \frac{I^{(i)M}_{z_m}}{2Dx z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{e^{-j \beta_m (R - R_d) \cdot r_m} - (i) (i) e_{m+} c_{d+}}{r_{my}} \right\}
\]

\[ + \frac{e^{-j \beta_m (R - R_d) \cdot r_0}}{2r_{my}} \]

\[ + \frac{e^{-j \beta_m (R - R_d) \cdot r_0}}{2r_{my}} \]

\[ \beta_{d+} - \omega_{+}^{(i)} \]

\[ \beta_{d+} - \omega_{+}^{(i)} \]

\[ \beta_{d+} - \omega_{+}^{(i)} \]

\[ \beta_{d+} - \omega_{+}^{(i)} \]

\[ \beta_{d+} - \omega_{+}^{(i)} \]
Figure 2.18 Reference element of dipole array with a field point in Region $\text{II}_d$. 

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\[ + \sum_{i=1}^{2} \frac{\lambda_i}{h^{m_i}} \left( \frac{\lambda_i}{h} \right) \left[ \frac{\lambda_i}{h^{m_i+1}} \right] \]

\[+ y \Delta (i) \delta(dx + kD_x) \delta(dz + nD_z) \sum_{i=1}^{2}, \]

where

\[ (2.73) \]

The other quantities in Equation (2.73) are given in Appendix E.
The Dirac delta functions in Equation (2.73) will vanish when the field point \( \bar{R} \) in Region II\(_d\) is not located at the array element. The elimination of the Dirac delta functions from the y-component of the field expression is a very important step for obtaining a convergent series representation for the coupling between the array and an external element situated in Region II\(_d\).

By adding up the contributions from the bounce mode field and the direct mode field, we obtain the total field generated by the balanced or unbalanced mode current on the \( \hat{(i)} \)-oriented legs of the dipole array.

Region \( I_d \):

\[
E_{1M}(\bar{R}) = \frac{I^{(i)}M \sin \zeta e}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{e^{-j\beta_m (\bar{R}-\bar{R}) \cdot r_{m+}^{(i)}}}{\gamma_{my} \Lambda_{1+}} + \right. \\
\left. \frac{e^{-j\beta_m (\bar{R}-\bar{R}) \cdot r_{m-}^{(i)}}}{\gamma_{my} \Lambda_{1-}} \right\},
\]

\( i = 1, 2 \) \hspace{1cm} (2.75)
Region II\(_d\):

\[
\mathbf{E}^{(i)M}_{\bar{d}}(\mathbf{R}) = \frac{I^{(i)M}\sin(\beta_d z_0)Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{c}
-\frac{e^{-j\beta_m (R-R_x)\hat{r}_m^+} - (i)}{r_{my}} + \frac{e^{-j\beta_m (R-R_x)\hat{r}_m^-} - (i)}{r_{my}} \\
+ \frac{e^{-j\beta_m (R-R_y)\hat{r}_m^+} - (i)}{2r_{my}\sin(\beta_d z_0)} + \frac{e^{-j\beta_m (R-R_y)\hat{r}_m^-} - (i)}{2r_{my}\sin(\beta_d z_0)} \\
+ \frac{e^{-j\beta_d (z_e - z_0)} - (i)}{r_{hy}} + \frac{e^{-j\beta_d (z_e - z_0)} - (i)}{r_{hy}} \end{array} \right\} ,
\]

\[i = 1, 2\] .

(2.76)

Region III\(_d\):

\[
\mathbf{E}^{(i)M}_{\bar{d}}(\mathbf{R}) = \frac{I^{(i)M}\sin(\beta_d z_0)Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{c}
-\frac{e^{-j\beta_m (R-R_x)\hat{r}_m^+} - (i)}{r_{my}} + \frac{e^{-j\beta_m (R-R_x)\hat{r}_m^-} - (i)}{r_{my}} \\
+ \frac{e^{-j\beta_m (R-R_y)\hat{r}_m^+} - (i)}{2r_{my}\sin(\beta_d z_0)} + \frac{e^{-j\beta_m (R-R_y)\hat{r}_m^-} - (i)}{2r_{my}\sin(\beta_d z_0)} \\
+ \frac{e^{-j\beta_d (z_e - z_0)} - (i)}{r_{hy}} + \frac{e^{-j\beta_d (z_e - z_0)} - (i)}{r_{hy}} \end{array} \right\} ,
\]

\[i = 1, 2\] .

(2.77)
where

\[
\Lambda_{1+} = \hat{n}_{m+} B(i) + \hat{n}_{m+} B(i) \quad , \quad (2.78)
\]

\[
\Lambda_{1-} = \hat{n}_{m-} B(i) + \hat{n}_{m-} B(i) - e_{m-} p(i) \quad , \quad (2.79)
\]

\[
\Lambda_{2+} = \hat{n}_{m+} B(i) + \hat{n}_{m+} B(i) - e_{m+} \frac{c_d(i)}{\sin(\beta_d \gamma e)} \quad , \quad (2.80)
\]

\[
\Lambda_{2-} = \hat{n}_{m-} B(i) + \hat{n}_{m-} B(i) - e_{m-} \frac{c_d(i)}{\sin(\beta_d \gamma e)} \quad , \quad (2.81)
\]

\[
\Lambda_{g} = \frac{-g_{m-}}{\beta_d - \omega_-} - \frac{-g_{m+}}{\beta_d - \omega_+} \quad , \quad (2.82)
\]

\[
\Lambda_{h} = \frac{-h_{m-}}{\beta_d + \omega_-} - \frac{-h_{m+}}{\beta_d + \omega_+} \quad , \quad (2.83)
\]

\[
\Lambda_{3+} = \hat{n}_{m+} B(i) + \hat{n}_{m+} B(i) - e_{m+} p(i) \quad , \quad (2.84)
\]

and

\[
\Lambda_{3-} = \hat{n}_{m-} B(i) + \hat{n}_{m-} B(i) \quad . \quad (2.85)
\]
For the feedline array, we only need to divide the bounded medium into two regions, i.e. Region II\(_f\) and Region III\(_f\), as illustrated in Figure 2.19. The domains of these two regions are defined as:

\[
\text{Region II}_f : \quad 0 < y \cdot R < \zeta_f \\
\text{Region III}_f : \quad \zeta_f < y \cdot R < b_m
\]

(2.86)

The direct mode field generated by the feedline array in Region III\(_f\) is given by

\[
\begin{align*}
\mathbf{E}^{(3)}_{D3}(R) &= \frac{I^{(3)}(0)Z_m}{2\pi D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\epsilon_m (R - R^{(f)}) \cdot r_m} e_m^+ p_m^+ e^{j\epsilon_m+ r_m^+} \\
&= \frac{I^{(3)}(0)Z_m}{2\pi D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\epsilon_m (R - R^{(f)}) \cdot r_m} e_m^+ p_m^+ e^{j\epsilon_m+ r_m^+}
\end{align*}
\]

(2.87)

Since the current distributions on the left and on the right of \(R^{(f)}\) are represented by two different functions, when we discuss the direct mode field in Region II\(_f\) generated by the feedline array, the field points located on the left and on the right of \(R^{(f)}\) should be considered separately, see Figure 2.20. Employing Equation (2.44), the direct mode fields in these two domains can be formulated.
Figure 2.19 Two regions defined for feedline array inside the bounded medium.
Figure 2.20 Reference element of feedline array with a field point (a) when $0 < y < l_u$ (b) when $l_u < y < l_f$. 
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\[
E^{(3)}_{D2R}(\bar{R}) = \frac{Z_m}{2D_x D_y} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right.
\]
\[
e^{-j\beta_m (\bar{R}-\bar{R}(f)) \cdot \bar{r}_m + \beta_m^0 \cdot \bar{r}_m + \beta_{\beta_m}^0 \cdot \bar{r}_m + \beta_{\gamma_m}^0 \cdot \bar{r}_m} e^{-j\beta_m (\bar{R}-\bar{R}(f)) \cdot \bar{r}_m - \beta_m^0 \cdot \bar{r}_m - \beta_{\beta_m}^0 \cdot \bar{r}_m - \beta_{\gamma_m}^0 \cdot \bar{r}_m} I_L^{(3)}(\bar{z}^\prime) e^{\beta_{\beta_m}^0 \cdot \bar{r}_m + \beta_{\gamma_m}^0 \cdot \bar{r}_m} d\bar{z}'
\]
\[
+ \int_{\bar{z}_o'}^{
\hat{y} \cdot \bar{R}(f)} I_R^{(3)}(\bar{z}^\prime) e^{\beta_{\beta_m}^0 \cdot \bar{r}_m + \beta_{\gamma_m}^0 \cdot \bar{r}_m} d\bar{z}' \]
\[
+ e^{-j\beta_m (\bar{R}-\bar{R}(f)) \cdot \bar{r}_m - \beta_m^0 \cdot \bar{r}_m - \beta_{\beta_m}^0 \cdot \bar{r}_m - \beta_{\gamma_m}^0 \cdot \bar{r}_m} I_L^{(3)}(\bar{z}^\prime) e^{\beta_{\beta_m}^0 \cdot \bar{r}_m + \beta_{\gamma_m}^0 \cdot \bar{r}_m} d\bar{z}'
\]
\[
\left. \right \}
\]
\[
y \cdot \bar{R}(f) < \hat{y} \cdot \bar{R} < \bar{z}_f
\]
\[
(2.89)
\]
\[
\hat{z}_o' = \frac{\hat{y} \cdot (\bar{R}-\bar{R}(f))}{p_y} = \hat{y} \cdot (\bar{R}-\bar{R}(f))
\]
\[
(2.90)
\]

and \(I_L^{(3)}(\bar{z}^\prime)\) and \(I_R^{(3)}(\bar{z}^\prime)\) are given by Equations (C.29) and (C.29), respectively.
As demonstrated in Appendix F, Equations (2.88) and (2.89) can be written in the following forms

\[ \mathbf{E}_{\text{D2L}}^{(3)}(\vec{R}) = \frac{Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right. \]

\[ \quad \frac{e^{-j\beta_m (R-R_x)^2 p_{mn}^{(3)}} e_{m-} c_{L-}}{r_{my}} \]

\[ + \frac{e^{-j\beta_m (R-R_x)^2 p_{mn}^{(3)}} e_{m+} c_{L+}}{r_{my}} \]

\[ + \frac{e^{-j\beta_m (R-R_x)^2 p_{mn}^{(3)}}} {r_{my}} \left[ \frac{e_{m+} - e_{m-}}{(j\beta_m r_{my})^3} \right] \]

\[ \quad \frac{e^{(3)} e_{m+} - e_{m-}}{(2U_b y) - \frac{2 U_b y^2}{j\beta_m r_{my}}} \]

\[ \quad \frac{\delta(\vec{d}_x + kD_x) \delta(\vec{d}_z + nD_z)}{L_0} \left\{ \right. \]

\[ 0 < y < R < y^{(f)} \]

(2.91)
\[ E^{(3)}_{\text{D2R}}(R) = \frac{Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right. \]

\[ e^{-j \beta_m (R-R') \cdot r_m^-} \cdot r_m^- \cdot e_{m^-} C_{R^-} \]

\[ + e^{-j \beta_m (R-R') \cdot r_m^+} \cdot r_m^+ \cdot e_{m^+} C_{R^+} \]

\[ + \frac{e^{-j \beta_m (R-R') \cdot r_{\text{rot}}} \cdot r_{\text{rot}} \cdot e_{m^+} e_{m^-}}{(j \beta_m r_{\text{my}})^3} \cdot (2U_e) \]

\[ - \frac{e_{m^+} - e_{m^-}}{(j \beta_m r_{\text{my}})^2} (U_d + 2U_e y) \]

\[ - \frac{2}{j \beta_m r_{\text{my}}} (U_c + U_d y + U_e y^2) \]

\[ + y \Delta_{R_0} \delta(d_x + kD_x) \delta(d_z + nD_z) \]

\[ y \cdot R < y \cdot R < \varepsilon_f \]

\[ \left( \right) \]

(2.92)
where

\[ \vec{r}_{ot} = r\hat{x} + r\hat{z} \quad , \quad (F.13) \]

\[ \hat{d}x = x - x(f) \quad , \quad (F.20) \]

\[ \hat{d}z = (z - z(f)) - (x - x(f)) \frac{\Delta z}{\Delta x} \quad , \quad (F.21) \]

and

\[ \vec{R}(f) = x(f)\hat{x} + y(f)\hat{y} + z(f)\hat{z} \quad . \quad (2.93) \]

The other quantities in Equations (2.91) and (2.92) are given in Appendix F.

Similarly, in Region II, if the field point \( \vec{R} \) is not located at the array element (i.e. feedline), the Dirac delta functions in Equations (2.91) and (2.92) will all vanish. The purpose of the elimination of the Dirac delta functions from the field expressions for the feedline array is the same as described earlier for the dipole array.

By adding up the contributions from the bounce mode field and the direct mode field, we obtain the total field generated by the feedline array.
Region III$_f$:

\[
E_3^-(\vec{R}) = -\frac{I^{(3)}(0)Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right.
\]

\[
e^{-j\beta_m(R-R') \cdot \vec{r}_{m+}^{(f)}} \frac{\Lambda^{(3)}_{m}}{r_{my}} + e^{-j\beta_m(R-R') \cdot \vec{r}_{m-}^{(f)}} \frac{\Lambda^{(3)}_{m}}{r_{my}} \left\} ,
\]

Region II$_f$:

\[
E_{2L}^-(\vec{R}) = -\frac{I^{(3)}(0)Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right.
\]

\[
e^{-j\beta_m(R-R') \cdot \vec{r}_{m+}^{(f)}} \frac{\Lambda^{(3)}_{m}}{r_{my}} + e^{-j\beta_m(R-R') \cdot \vec{r}_{m-}^{(f)}} \frac{\Lambda^{(3)}_{m}}{r_{my}} \left\} ,
\]

\[
+ e^{-j\beta_m(R-R') \cdot \vec{r}_{ot}} \frac{\Lambda^{(3)}_{m}}{I^{(3)}(0)r_{my}} + e^{-j\beta_m(R-R') \cdot \vec{r}_{ot}} \frac{\Lambda^{(3)}_{m}}{(j\beta_m r_{my})^3}
\]

\[
\left\{ \begin{array}{l}
\frac{-e^{-j\beta_m(R-R') \cdot \vec{r}_{m+}^{(f)}}}{(2U_b y) - \frac{\Lambda^{(3)}_{m}}{j\beta_m r_{my}}} - \frac{\Lambda^{(3)}_{m}}{j\beta_m r_{my}} \\
\end{array} \right. \right) ,
\]

\[
0 < y \cdot \vec{R} < y \cdot \vec{R}^{(f)},
\]

(2.95)
\[
\begin{aligned}
E^{(3)}_2(R) &= - \frac{I^{(3)}(0)Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \right.
\end{aligned}
\]

\[
\begin{aligned}
&\frac{e^{-j\beta_m(R-R)^2} \hat{r}_{m+}}{r_{my}} \Lambda^{(3)}_{R+} + e^{-j\beta_m(R-R)^2} \hat{r}_{m-} \Lambda^{(3)}_{R-} \\
&+ e^{-j\beta_m(R-R)^2} \hat{r}_{rot} \left[ \frac{e^{(3)}_{m+} + e^{(3)}_{m-}}{I^{(3)}(0)r_{my}} \right] \frac{(2U_e)}{(j\beta_m r_{my})^3} \\
&= \frac{e^{(3)}_{m+} - e^{(3)}_{m-}}{(j\beta_m r_{my})^2} \left( U_d - 2U_e y \right) - y \frac{2}{j\beta_m r_{my}} \left( U_c + U_d y + U_e y^2 \right) \left\{ \right. \\
&\left. \hat{r}_{R^{(f)}} < \hat{r} < \hat{r}_f \right. \\
\end{aligned}
\]

where

\[
\Lambda^{(3)}_{3+} = p_{m+}^{(3)} - e_{m+}^{(3)} p_{m+} \\
\Lambda^{(3)}_{3-} = p_{m-}^{(3)} - e_{m-}^{(3)} p_{m-} \\
\Lambda^{(3)}_{L+} = p_{m+}^{(3)} - e_{m+}^{(3)} \frac{c_{L+}^{(3)}}{I^{(3)}(0)}
\]

(2.96)
\[
\lambda_{L^-} = \hat{n}_{m^-} B_{m^-} - e_{m^-} \frac{C_{L^-}}{I^{(3)}(0)}, \quad (2.100)
\]

\[
\lambda_{R^+} = \hat{n}_{m^+} B_{m^+} - e_{m^+} \frac{C_{R^+}}{I^{(3)}(0)}, \quad (2.101)
\]

and

\[
\lambda_{R^-} = \hat{n}_{m^-} B_{m^-} - e_{m^-} \frac{C_{R^-}}{I^{(3)}(0)}. \quad (2.102)
\]
A. INTRODUCTION

In this chapter, the expressions of four impedances, \( Z_{bb}, Z_{uu}, Z_{ub} \) and \( Z_{bu} \) will be formulated. At the beginning, we will briefly introduce the mutual impedance concept. Based on this concept, all the impedances can be defined and derived.

First, consider two different arrays, Array A and Array B inside a bounded medium as shown in Figure 3.1, where the reference points of these two arrays are positioned at \( R^{(A)} \) and \( R^{(B)} \) respectively. Assuming Array A is radiating and Array B is under transmitting conditions, according to Schellkunoff [4], the voltage induced at \( R^{(B)} \) due to the field radiated by the entire Array A is given by

\[
v_{BA}(R^{(B)}) = \frac{1}{I^{(B)}(R^{(B)})} \int_{\text{ref. element of Array B}} \hat{E}^{(A)}(s)I^{(B)}(s)ds
\]
Figure 3.1 Array A and Array B located between a ground plane and a set of dielectric slabs.
where $I^{(B)t}(x)$ is the current distribution on the reference element of Array B under transmitting condition and $I^{(B)t}(x)$ denotes the current at terminal $R^{(B)}$. $E^{(A)}(x)$ is the field radiated by Array A. The unit vector $\hat{z}$ follows the direction of current $I^{(B)t}(x)$; in the section from $x_a$ to 0, $\hat{z} = -\hat{z}(B2)$, while in the section from 0 to $x_b$, $\hat{z} = \hat{z}(B1)$, see Figure 3.1.

The mutual impedance at Array B due to the coupling from Array A can be defined as

$$Z^{BA} = -\frac{\Phi^{BA}(R^{(B)})}{I^{(A)}(R^{(A)})} \int_{\hat{z}(x)} E^{(A)}(x) I^{(B)t}(x) dx$$

where $I^{(A)}(R^{(A)})$ is the terminal current of Array A at $R^{(A)}$. The mutual impedance defined by Equation (3.2) can be described as the voltage induced in Array B caused by a unit current in Array A.

If Array B happens to be the same as Array A and is located one wire radius away from Array A, then the result of Equation (3.2) will represent the self-impedance of Array A. When considering the self-impedance of Array A, it is convenient to utilize a test element being parallel to and one wire radius away from the reference element of Array A as shown in Figure 3.2. The test element has the same shape,
Figure 3.2 A test element located at a specific position on the test contour for self-impedance evaluation.
length and orientation as the array element in Array A. In this case, the reference element and the test element are assumed to be infinitesimally thin; the reference element is located at the center and the test element is on the outer surface of the wire. Assuming the test element is under transmitting condition, the self-impedance of Array A is, then, given by

\[ Z_s^{AA} = -\frac{1}{I^{(A)}(R(A))I^{(A)t}(R_s^{(A')})} \int z \cdot E_s(z)I^{(A)t}(z)dz \, \text{test element} \]  

(3.3)

where \( I^{(A)t}(z) \) is the current distribution on the test element and \( I^{(A)t}(R_s^{(A')}) \) denotes the current at \( R_s^{(A')} \), the reference terminal of the test element. The subscript "s" is used to specify the position of the test element.

Note that the self-impedance given by Equation (3.3) is a function of the location of the test element. Therefore, it may be expressed as

\[ Z_s^{AA} = Z_s^{AA} \]  

(3.4)

where \( \phi_s \) specifies the position of the test element.

According to Schellkunoff [4], the self-impedance should be obtained by allowing the test element to move around a circular contour as shown in Figure 3.2, yielding an average value of coupling.
Therefore, the self-impedance of Array A, denoted by $Z^{AA}$, should be expressed in a more accurate form as

$$Z^{AA} = \frac{1}{2\pi} \int_0^{2\pi} Z^{AA}(\phi_s) d\phi_s$$

$$= -\frac{1}{2\pi I^{(A)}(\vec{R}^{(A)})} \int_0^{2\pi} \frac{1}{I^{(A)}t(\vec{R}^{(A')})} \left[ \int \hat{z} \cdot E_{\phi_s}^{(A)}(z) I^{(A)}t(\xi) dz \right] d\phi_s$$

(3.5)

However, for most nonplanar arrays, the integral shown in Equation (3.5) is very difficult to evaluate exactly. The difficulty is due to the complexity of the field expression appearing in the integrand of Equation (3.5); usually, there is no unified expression can be used to describe the field at the test element located at all positions around the test contour. An alternate way to obtain an approximate result of Equation (3.5) is by putting many test elements at some discrete positions around the test contour and evaluating the average coupling. Therefore, Equation (3.5) may approximately be represented by

$$Z^{AA} \approx \frac{1}{N_t} \sum_{n=1}^{N_t} Z^{AA}(\phi_n)$$

$$= -\frac{1}{N_t I^{(A)}(\vec{R}^{(A)})} \sum_{n=1}^{N_t} \frac{1}{I^{(A)}t(\vec{R}^{(A')})} \int \hat{z} \cdot E_{\phi_n}^{(A)}(z) I^{(A)}t(\xi) dz$$

(3.6)
where $N_t$ denotes the number of test elements used, $\phi_n$ indicates the position of the test element around the circular test contour.

Theoretically, to obtain more accurate results, more test elements should be used. However, having done some investigation, we found that this condition is not always necessary; using a small number of test elements at some selected positions, a similar accurate result may also be obtained. A short survey on the locations and number of test elements used in obtaining the accurate self-impedance results for the dipole and feedline arrays will be given in Appendix G.

B. OUR SITUATION

As mentioned earlier, the current in the "dipole + feedline" array is comprised of two current modes, balanced mode and unbalanced mode. For easy understanding, we have created a fictitious dipole array, called Array "b", to carry solely the balanced mode current as shown in Figure 3.3. The Array "u", shown in the same figure, is carrying the unbalanced mode current only. The spacing between the balanced and unbalanced dipole array is assumed to be zero.

Using two arrays, Array "b" and Array "u", to hypothetically isolate the balanced and unbalanced currents makes it easier to describe the coupling between these two current modes. $Z_{bb}$ is the self-impedance of Array "b". $Z_{bu}$ is the mutual impedance at Array "b" caused by the coupling from Array "u". $Z_{uu}$ and $Z_{ub}$ can be described in a similar manner. The expressions of these four impedances will be formulated
Figure 3.3 Using Array "b" and Array "u" to hypothetically separate the balanced and unbalanced mode currents.
employing the concept set forth in the previous section. In our case, under transmitting condition, the current distribution on Array "b" and Array "u" are assumed to be the same as those shown in Figure 2.16. Therefore, the superscript "t" used to denote the current under transmitting condition becomes unnecessary and will be omitted in all expressions.

When evaluating the coupling between the dipoles of the same current mode, the test dipole shall be moving around the test contour and obtaining an average coupling. If a small number of test dipoles are used, the positions for these test dipoles should be correctly chosen. A similar procedure is necessary for evaluating the coupling between feedlines. However, when considering the coupling between feedline and dipole, these two elements will be located at their original positions in the array. To evaluate the coupling between the dipoles of different current modes, usually, one test position is enough.

C. SELF-IMPEDEANCE Zbb

In this section, the self-impedance of Array "b", i.e. Zbb, will be considered. The dipoles in the array can be straight or V-shaped. We start by examining a straight dipole as shown in Figure 3.4. For simplicity, the figure only shows the reference dipole and some test dipoles. We assume that the field arriving at the test dipoles is radiated by the entire dipole array in the presence of ground plane and dielectric slabs is understood. The same understanding should be
Figure 3.4 Locus of test dipoles for straight dipole.

\[ d x = r_d \sin \phi_d \]
\[ d y = r_d \cos \phi_d \]
applied to the other cases encountered in the following discussions.

For a straight dipole array, the test dipoles are located at a circular contour as illustrated in Figure 3.4. This circular contour is centered at the reference dipole with a radius $r_d$ equivalent to the wire radius of the dipole element. In this case, the test dipole can be considered to be in Region $I_d$ or Region $III_d$.

For a V-shaped dipole array, the test dipoles are placed on a contour as shown in Figure 3.5. The reference terminals of the test dipoles are on an ellipse centered at the reference terminal of the reference dipole. In this case, part of a test dipole may be situated in Region $II_d$.

To evaluate the self-impedance $Z_{bb}$, the field on the test dipole, due to the radiation from the balanced mode current on the entire dipole array, should be known explicitly. Consider a V-shaped dipole as shown in Figure 3.5. If the reference terminal of a test dipole is positioned at $R$, the field point on the $p^\wedge(j)$-directed leg of the test dipole can then be expressed as:

$$R = R^\wedge(dt) + \lambda p^\wedge(j)$$  \hspace{1cm} (3.7)

where $\lambda p$ is the length measured from $R^\wedge(dt)$ along the $p^\wedge(j)$-directed leg of the test dipole.

Substituting Equation (3.7) into Equations (2.75) and (2.77), we obtain the fields on the $p^\wedge(j)$-directed leg of a test dipole in Region $I_d$.
Figure 3.5 Locus of test dipoles for V-shaped dipole.
and Region \(III_d\) due to the radiation from the balanced mode current on the \(p^{(i)}\)-directed legs of the dipole array

\[
E_{(j)}(z_D) = -\frac{(-1)^{i+1}I_{bd}\sin(\beta_{d}z)e^{z}}{2Dx} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ \begin{array}{c}
\frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m+2D}}{D_{d1+}} + \frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m-2D}}{D_{d1-}} \\
\frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m+2D}}{D_{d3+}} + \frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m-2D}}{D_{d3-}} \end{array} \right\} \]

\(i = 1, 2\) and \(j = 1, 2\) (3.8)

\[
E_{(j)}(z_D) = -\frac{(-1)^{i+1}I_{bd}\sin(\beta_{d}z)e^{z}}{2Dx} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ \begin{array}{c}
\frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m+2D}}{D_{d1+}} + \frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m-2D}}{D_{d1-}} \\
\frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m+2D}}{D_{d3+}} + \frac{-j \beta_{m} \hat{d}_{j} \cdot \hat{r}_{m-2D}}{D_{d3-}} \end{array} \right\} \]

\(i = 1, 2\) and \(j = 1, 2\) (3.9)

where

\[
D_{d1+} = \psi_{\Lambda} \hat{d}_{1+}
\]

(3.10)

\[
D_{d1-} = \psi_{\Lambda} \hat{d}_{1-}
\]

(3.11)

\[
D_{d3+} = \psi_{\Lambda} \hat{d}_{3+}
\]

(3.12)
\[ \hat{d}(i) = \psi d3 \]  
\[ d3_\perp = - \Lambda_3 \]  
\[ (3.13) \]

with

\[ \psi_\pm = \frac{e^{-j \beta_m (\hat{R}^\perp - \hat{R}^{(d)}) \cdot \hat{r}_{my}}}{r_{my}} \]  
\[ (3.14) \]

When the field point \( \hat{R} \) on the \( \hat{p}^{(j)} \)-directed leg of a test dipole is situated in Region \( II_d \), \( \xi_0 \) given by Equation (2.72) can be written as

\[ \xi_0 = \frac{\hat{y} \cdot (\hat{R}^{(dt)} - \hat{R}^{(d)})}{p_y^{(i)}} + \xi_D \frac{p_y^{(j)}}{p_y^{(i)}} \]  
\[ (3.15) \]

Since \( p_y^{(i)} = p_y^{(j)} = -\cos \theta_b \) (for \( i=1,2 \) and \( j=1,2 \)), as indicated in Equation (2.50), \( \xi_0 \) can be simplified to

\[ \xi_0 = \xi_{Dd} + \xi_D \]  
\[ (3.16) \]

where

\[ \xi_{Dd} = - \frac{\hat{y} \cdot (\hat{R}^{(dt)} - \hat{R}^{(d)})}{\cos \theta_b} \]  
\[ (3.17) \]
Substituting Equations (3.7) and (3.16) into Equation (2.76), we have

in Region $II_d$

$$E^{(i)b} (z_D) = - \frac{(-1)^{i+1} L_b \sin(\beta_d z_e) z_m}{2 D D x z} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ 
\begin{align*}
- & (i) e^{-j \beta_m (j) r_{m+Z_D}} + (i) e^{-j \beta_m (j) r_{m-Z_D}} \\
+ & (i) e^{-j \beta_m (p \cdot r_0 - \frac{\beta_d}{\beta_m}) Z_D} \\
+ & (i) e^{-j \beta_m (p \cdot r_0 + \frac{\beta_d}{\beta_m}) Z_D}
\end{align*}
\right\}$$

for $i = 1, 2$ and $j = 1, 2$ (3.18)

where

$$D^{(i)} = \frac{dd}{d2+} \frac{- (i)}{\Lambda_{2+}} \quad (3.19)$$

$$D^{(i)} = \frac{dd}{d2-} \frac{- (i)}{\Lambda_{2-}} \quad (3.20)$$
\[
\begin{align*}
D_{(i)g}^{-} &= - \psi_{(i)g} \cdot \frac{j \beta_d (\xi_e - \xi_d)}{2 \sin(\beta_d \xi_e)} \quad (3.21) \\
D_{(i)h}^{-} &= - \psi_{(i)h} \cdot \frac{j \beta_d (\xi_e - \xi_d)}{2 \sin(\beta_d \xi_e)} \quad (3.22)
\end{align*}
\]

with
\[
\psi_{(i)} = e^{-j \beta_{m}(R'_{(d)} - R'_{(t)}) - r_0} \\
\psi_{(i)} = \frac{r_{(d)}}{r_{my}} \quad (3.23)
\]

and $\psi_{(i)}$ are defined by Equation (3.14).

It should be pointed out that the points $R'$ and $R''$ are always lying on the same plane (xy-plane as illustrated in Figures 3.4 and 3.5). These two points can be related by
\[
R'_{(d)} - R'_{(t)} = r_{d} \sin \phi_{d} \hat{x} + \frac{r_{d}}{\sin \phi_{d}} \cos \phi_{d} \hat{y} \quad (3.24)
\]

where $\phi_{d}$ is the position of the test dipole on the test contour as indicated in the figure.

Due to the fact of Equation (3.24), the fields $\tilde{E}_{(j)b}$, $\tilde{E}_{(j)b}$ and $\tilde{E}_{(j)3}$ given above are also functions of $\phi_{d}$ and may be expressed as:
Employing the concept set forth in the previous sections, the self-impedance $Z_{bb}$ can be formulated as

$$Z_{bb} = \frac{1}{N_{td}} \sum_{m=1}^{N_{td}} Z_{bb}^{(\phi_{dm})}$$

(3.26)

where

$$Z_{bb}^{(\phi_d)} = -\frac{1}{I_{bd}(\sin \phi_d e)} \sum_{i=1}^{2} \sum_{j=1}^{2} (-1)^{j+1} g_{(i)b (j)}^{(i)b (\phi_d)}$$

(3.27)

with

$$g_{(i)b (j)}^{(i)b (\phi_d)} = \int_{0}^{2 \pi} p^{(j)} \cdot \mathbf{E}_{(j)}^{(i)b (\phi_d, \omega)} I_{bd} \sin \phi_d (\omega_e - \omega) d \omega$$

(3.28)
In Equation (3.25) $N_{td}$ denotes the number of test dipoles used in evaluating $Z_{bb}$, $\phi_{dm}$ indicates the location of the m-th test dipole on the test contour. These two quantities will be determined in Appendix G.

Now, we will consider $G^{(i)b}_{(j)}(\phi_d)$ defined by Equation (3.28). From Equation (3.25), we notice that the field expressions in Region I$_d$, Region II$_d$ and Regions III$_d$ are different. Therefore, when we perform the integration of Equation (3.25), the field $E^{(i)b}_{(j)}(\phi_d)$ should be correctly chosen.

For a straight dipole array, we always considered the test dipole is situated in either Region I$_d$ or Region III$_d$. When $0^\circ < \phi_d < 90^\circ$ or $270^\circ < \phi_d < 360^\circ$, the entire test dipole is situated in Region III$_d$

$$G^{(i)b}_{(j)}(\phi_d) = \int_0^{2\pi} E^{(i)b}_{(j)}(\phi_d, \phi_d) I_b \sin \phi_d (\phi_d - \phi_d) d\phi_d (3.29)$$

When $90^\circ < \phi_d < 270^\circ$, the entire test dipole is situated in Region I$_d$

$$G^{(i)b}_{(j)}(\phi_d) = \int_0^{2\pi} E^{(i)b}_{(j)}(\phi_d, \phi_d) I_b \sin \phi_d (\phi_d - \phi_d) d\phi_d (3.30)$$

However, for the V-shaped dipole array, the situation is more complicated; a test dipole may be entirely situated in one region (Region I$_d$ or Region II$_d$ or Region III$_d$) or in two adjacent regions (Region I$_d$ and Region II$_d$ or Region II$_d$ and Region III$_d$), depending on $\theta_b$ (bend angle) and $\phi_d$ (position of the test dipole on the test contour). To determine the region(s) in which a test dipole is
situated, we should project the entire test dipole on the yz-plane containing the reference dipole as illustrated in Figures 3.6 and 3.7. Also, we will define two variables with which the proportion of a test dipole in two different regions can be exactly determined.

First, let's examine a situation shown in Figure 3.6, where a test dipole placed on the right hand side of the reference dipole (i.e. when $0^\circ < \phi_d < 90^\circ$ or $270^\circ < \phi_d < 360^\circ$), is projected on the yz-plane. Here, we define a variable as

$$\zeta_{III} = \frac{\Delta y}{\cos \theta_d}$$  

(3.31)

where

$$\Delta y = \frac{r_d}{\sin \theta_d} |\cos \phi_d|$$  

(3.32)

is the absolute value of $dy$ defined in Figure 3.5. The variable $\zeta_{III}$ is used to detect which portion of the test dipole will be situated in Region III$_d$ when $\theta_d$ and $\phi_d$ are varied.

When $\zeta_{III}$ is less than $\zeta_d$, for the $p^-$-directed leg of the test dipole, the portion from $\zeta_D = 0$ to $\zeta_D = \zeta_{III}$, is situated in Region III$_d$, while the rest portion (from $\zeta_D = \zeta_{III}$ to $\zeta_D = \zeta_d$) is in Region II$_d$. In this case, $G_{(j)}^{(i)b}(\phi_d)$ should be expressed as
Figure 3.6 A test dipole projected on the yz-plane containing the reference dipole, when $0^\circ < \phi_d < 90^\circ$ or $270^\circ < \phi_d < 360^\circ$. 

\[ l_\text{III} = \frac{\Delta y}{\cos \theta_b} \]
Figure 3.7 A test dipole projected on the yz-plane containing the reference dipole, when $90^\circ < \phi_d < 270^\circ$. 

\[ l_{II} = l_d - \frac{\Delta y}{\cos \theta_b} \]
\[ G^{(i)b}(\phi_d) = \int_{0}^{\pi} x^{(j)} \cdot \begin{array}{c} \phi_d \\ 2 \end{array} I_{bd} \sin \beta_d (\varnothing_e - \varnothing_D) d\varnothing_D \]

\[ + \int_{\pi}^{2\pi} x^{(j)} \cdot \begin{array}{c} \phi_d \\ 2 \end{array} I_{bd} \sin \beta_d (\varnothing_e - \varnothing_D) d\varnothing_D \]

(3.33)

If \( \beta_{III} > \chi_d \), the test dipole is totally situated in Region III; therefore, \( G^{(i)b}(\phi_d) \) can be represented by Equation (4.29).

When \( \beta_{III} = 0 \) (i.e., \( \phi_d = 90^\circ \) or \( 270^\circ \)), the entire test dipole is situated in Region II; \( G^{(i)b}(\phi_d) \) will be

\[ G^{(i)b}(\phi_d) = \int_{0}^{\pi} x^{(j)} \cdot \begin{array}{c} \phi_d \\ 2 \end{array} I_{bd} \sin \beta_d (\varnothing_e - \varnothing_D) d\varnothing_D \]

(4.34)

Now, if the test dipole is placed in a position when its reference terminal is located at \( 90^\circ < \phi_d < 270^\circ \), we have a situation shown in Figure 3.7. Here, we shall define another variable
\[ \xi_{II} = \xi_d - \cos \theta_b \]  

(3.35)

where \( \Delta y \) is defined in Equation (3.32). The variable \( \xi_{II} \) is used to detect which portion (measured from terminal \( R_\text{nt} \)) of the test dipole will be situated in Region II_d, when \( \theta_b \) and \( \phi_b \) are varied (see Figure 3.7).

When \( 0 > \xi_{II} \), the entire test dipole is situated in Region I_d; in this case, \( G^{(i)b}_{(j)}(\phi_d) \) can be represented by Equation (3.30). However, when \( 0 < \xi_{II} < \xi_d \), the test dipole is existing in two regions, with \( 0 < \xi_d < \xi_{II} \) in Region II_d and \( \xi_{II} < \xi_d < \xi_d \) Region I_d. Therefore, \( G^{(i)b}_{(j)}(\phi_d) \) should be expressed as

\[
G^{(i)b}_{(j)}(\phi_d) = \int_0^{\xi_{II}} p^{(i)b}_{(j)} \cdot E^{(i)b}_{(j)2}(\phi_d, \xi_d) I_b \sin \beta_d (\xi_e - \xi_d) d\xi_d + \int_{\xi_{II}}^{\xi_d} p^{(i)b}_{(j)} \cdot E^{(i)b}_{(j)1}(\phi_d, \xi_d) I_b \sin \beta_d (\xi_e - \xi_d) d\xi_d
\]

(3.36)

D. SELF-IMPEDANCE \( Z_{uu} \)

\( Z_{uu} \) is the self-impedance of Array "u", an array carrying unbalanced
mode current as shown in Figure 3.3. When evaluating the self-impedance $Z_{uu}$, there are four kinds of coupling needed to be considered. The first one is the coupling from the dipole array on the test dipoles(s). The second one is the coupling from the feedline array on the test feedline(s). The third one is the coupling from the dipole array on the feedline. The last one is the coupling from the feedline array on the dipole. Therefore, we may consider the self-impedance $Z_{uu}$ is made up of four components

$$Z_{uu} = Z_{uu}^{dd} + Z_{uu}^{ff} + Z_{uu}^{fd} + Z_{uu}^{df}$$  

(3.37)

where $Z_{uu}^{dd}$, $Z_{uu}^{ff}$, $Z_{uu}^{fd}$ and $Z_{uu}^{df}$ are the impedances associated with the four couplings mentioned above, they will be formulated in order.

1. Impedance $Z_{uu}^{dd}$

Since the geometric structure of the dipoles in Array "u" and Array "b" are identical, the impedance $Z_{uu}^{dd}$ can be formulated in a similar fashion as for $Z_{bb}^{dd}$ described above

$$Z_{uu}^{dd} = \frac{1}{N_{td}} \sum_{m=1}^{N_{td}} Z_{dd}^{uu}(\phi_{dm})$$  

(3.38)
where

\[ Z_{-dd}(\phi_d) = -\frac{1}{[I(3)(0)]^2} \sum_{m=1}^{N_{td}} \sum_{i=1}^{2} \sum_{j=1}^{2} G(i)^u_{(j)}(\phi_d) \]  

(3.39)

with

\[ G(i)^u_{(j)}(\phi_d) = \int_0^{2\pi} \int_0^\pi E(j)^u(\phi_d, \zeta_d) I_{ud} \sin \theta_d (\zeta_e - \zeta_d) d\zeta_d d\zeta_e \]  

(3.40)

and \( I(3)(0) \) is the current at \( R(f) \) (the reference point of Array "u"), \( E(j)^u(\phi_d, \zeta_d) \) is the field on the \( \hat{d} \)-directed leg of a test dipole due to the radiation from the unbalanced mode currents on the \( \hat{d} \)-directed legs of the dipole array. \( N_{td} \) (number of test dipoles) and \( \phi_{dm} \) (positions of \( m \)-th test dipole on the test contour) are exactly the same as those chosen for \( Z \) in Equation (3.26). Note that when we perform the integration of Equation (3.40), the procedure described in the previous section for \( G(j)^b_{(i)} \) should be followed.

The field \( E(j)^u(\phi_d, \zeta_d) \) in Region I_d, Region II_d and Region III_d can be obtained simply by replacing \((-1)^{j+1}I_{bd}\) with \( I_{ud} \) in Equations (3.8), (3.18) and (3.9) in Region I_d

\[ E(j)^u(\phi_d, \zeta_d) = -\frac{I_{ud} \sin(\theta_d \zeta_e)}{2\pi D z} \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left\{ D \left( i \right) e^{-j \beta_m r_{m+\zeta_d}} + D \left( i \right) e^{-j \beta_m r_{m-\zeta_d}} \right\} \]

(3.41)
2. Impedance $Z_{ff}$

When we evaluate the impedance $Z_{ff}$, the test feedlines are placed at a circular contour centered at the reference feedline as illustrated in Figure 3.8. The radius of the contour, $r_f$, is equivalent to the radius of the cylindrical wire modelled for the two-wire feedline as described in Chapter I. The reference terminal of the test feedline is chosen at $R^R$, having the same $y$-component as $R^f$. From Figure 3.8, we know

$$R^R - R^f = y \cos \phi_f \hat{x} + y \sin \phi_f \hat{z}$$

(3.44)

where $\phi_f$ is the location of the test feedline on the circular test contour indicated in the figure.

In Figure 3.8, we also noticed that the test feedline occupies the entire Region II$_f$. To locate the field point on a test feedline, we should write $R$ as

$$R = R^f + \zeta_p^f(3)$$

(3.45)
Figure 3.8 Locus of test feedlines.

\[ dx = r_f \cos \phi_f \]
\[ dz = r_f \sin \phi_f \]
where

$$\hat{p}(3) = \hat{y}$$  \hspace{1cm} (3.46)$$

is the orientation of the feedlines, $p_F$ is the length measured from $R^{(ft)}$ along the test feedline in the $\hat{p}(3)$-direction. The $y$-component of $R$ is

$$y = y^{(ft)} + p_F$$  \hspace{1cm} (3.47)$$

Substituting Equations (3.45) and (3.47) into Equation (2.95), we obtain the field on the test feedline in the section which is on the left hand side of $R^{(f)}$, due to the radiation from the feedline array.

$$E^{(3)} \left( \Phi_f, z_F \right) = -\frac{R^{(3)}}{2D} \frac{Z_m}{2} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ \begin{array}{c} f \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ \right. \\
-k \left. \right. \end{array} \right.$$

$$+ \left[ \tilde{F}_{f\omega} + \tilde{F}_{f\omega F} + \tilde{F}_{f\omega 2F} \right] e^{-j\beta mp^{(3)} \cdot \hat{r}_{ot} \cdot \hat{F}} \right\}$$

$$\hspace{1cm} (3.48)$$
where

\[ F_{fL^+} = \psi^+ \Lambda_{L^+} \tag{3.49} \]

\[ F_{fL^-} = \psi^- \Lambda_{L^-} \tag{3.50} \]

\[ F_{fL^0} = \frac{\psi^f}{I^{(3)}(0)} \left[ - \frac{\bar{e}(3) + \bar{e}(3)}{m^+} - \frac{\bar{e}(3) - \bar{e}(3)}{m^-} \right] \frac{(2U_b)}{(j \beta m_{my})^3} + \frac{(2U_B \xi_u)}{(j \beta m_{my})^2} \]

\[ - y \frac{2}{j \beta m_{my}} (U_a + U_b \xi_u^2) \]

\[ F_{fL^1} = \frac{\psi^f}{I^{(3)}(0)} \left[ - \frac{\bar{e}(3) - \bar{e}(3)}{m^+} \right] \frac{(2U_b)}{(j \beta m_{my})^2} - y \frac{4U_b \xi_u}{j \beta m_{my}} \]  

\[ F_{fL^2} = - y \frac{\psi^f}{I^{(3)}(0)} \frac{2U_b}{j \beta m_{my}} \]  

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Similarly, substituting Equations (3.45) and (3.47) into Equation (2.96), we obtain the field on the test feedline in the section which is on the right hand side of $R^f$, due to the radiation from the feedline array.

\[
E^{(3)}_{(3)2R}(\phi_f, z_f) = -i^{(3)_m}Z_m \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ \right.
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where

\[
F_{fR^+} = \frac{\psi^f}{\Lambda^R^+} \quad (3.57)
\]

\[
F_{fR^-} = \frac{\psi^o}{\Lambda^R^-} \quad (3.58)
\]

\[
F_{fR^0} = \frac{\psi^o}{I(3)(0)} \left[ \frac{e(3) + e(3)}{m^+ m^-} (2U_e) - \frac{e(3) - e(3)}{m^+ m^-} (U_d - 2U_e \varepsilon_u) \right]
\]

\[
- \hat{y} \frac{2}{j \beta_m \gamma_y} \left( U_c + U_d + U_e \varepsilon_u \right) \quad (3.59)
\]

\[
F_{fR^1} = - \frac{\psi^o}{I(3)(0)} \left[ \frac{e(3) - e(3)}{m^+ m^-} (2U_e) - \hat{y} \frac{2}{j \beta_m \gamma_y} (U_d + 2U_e \varepsilon_u) \right]
\]

\[
F_{fR^2} = - \hat{y} \frac{\psi^o}{I(3)(0)} \cdot \frac{2U_e}{j \beta_m \gamma_y} \quad (3.61)
\]
Knowing the field on the test feedline, the impedance $Z_{ff}^{uu}$ may readily be formulated as

$$Z_{ff}^{uu} = \frac{1}{N_{tf}} \sum_{m=1}^{N_{tf}} Z_{ff}^{uu}(\phi_{fm})$$

(3.62)

where $N_{tf}$ is the number of the test feedlines used in evaluating $Z_{ff}^{uu}$, $\phi_{fm}$ is the location of the $m$-th test feedline on the circular test contour (see Figure 3.8). These two quantities will be determined in Appendix G. The current distributions $I_{L}^{(3)}(z_f)$ and $I_{R}^{(3)}(z_f)$ are given by Equations (C.28) and (C.29), respectively.

3. Impedance $Z_{fd}^{uu}$

The impedance $Z_{fd}^{uu}$, associated with the coupling from the dipole
array on the feedline may tentatively be given by

\[
Z_{fd}^{uu} = -\frac{1}{[i^{(3)}(0)]^2} \sum_{i=1}^{2} \int_{\text{feedline}}^{p} E^{(3)}(x) I^{(3)}(x) dx
\]

where the \( E^{(3)}(x) \) is the field on the test feedline due to the radiation from the unbalanced mode currents on the \( p^{(i)} \)-directed legs of the dipole array, \( I^{(3)}(x) \) is the current distribution along the feedline.

Figure 3.9 shows a feedline and a V-shaped dipole, the feedline is situated in Region \( I_d \) and Region \( II_d \). When the field point \( \tilde{R} \) is located on the feedline, we have

\[
\tilde{R} = R^{(f)} + \pi F P^{(3)}
\]

where \( R^{(f)} \) is the reference point of the feedline, \( \pi F \) is the length measured from \( R^{(f)} \) in the \( p^{(3)} \)-direction. Note that \( p^{(3)} = \hat{y} \).

Substituting Equation (3.65) into Equation (2.75), we obtain the field on the feedline in the section which is situated in Region \( I_d \) due to the radiation from the unbalanced mode currents on the \( p^{(i)} \)-directed legs of the dipole array.
Figure 3.9 Relative position between dipole and feedline.
\[ E_{(3)}(z_F) = -\frac{\text{I}_{ud}\sin(\delta_{de})Z_m}{2\pi D_z} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left\{ D_{\pm} e^{-j\beta_m(3)\cdot r_{m\pm z_F}} + D_{\mp} e^{-j\beta_m(3)\cdot r_{m-z_F}} \right\} \] (3.66)

where

\[ D_{\pm} = \psi_{\pm} \] (3.67)

\[ D_{\mp} = \psi_{\mp} \] (3.68)

with

\[ \psi_{\pm} = e^{\frac{-jR_m(R(f) - R(d))\cdot r_{m\pm}}{r_{mn}}} \] (3.69)

Using Equation (3.65), \( \varepsilon_0 \) defined by Equation (2.72) can be written as:

\[ \varepsilon_0 = \varepsilon_0 f - \frac{\varepsilon_f}{\cos\theta_b} \] (3.70)
where
\[
\xi_{Df} = -\frac{y(f)-y(d)}{\cos \theta_b} = \frac{\xi_f - \xi_u}{\cos \theta_b}
\]  

(3.71)

Substituting Equations (3.65) and (3.70) into Equation (2.76), we obtain the field on the feedline in the section which is situated in Region II due to the radiation from the unbalanced mode currents on the \(^{(i)}\) \(p\)-directed legs of the dipole array.

\[
E(x, z) = -\frac{I_d \sin(\beta_d z)}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{c}
\xi_{(i)p} \left( \xi^{(3)} \right)^{-} \left( m + \xi_F \right) + \xi_{(i)p} \left( \xi^{(3)} \right)^{+} \left( m - \xi_F \right) \\
\frac{\beta}{\beta_m \cos \theta_b} \left( \frac{\beta_d}{\beta_m \cos \theta_b} \right) \xi_F \\
+ \xi_{(i)p} \left( \xi^{(3)} \right)^{-} \left( r_o(i) - \frac{\beta}{\beta_m \cos \theta_b} \right) \xi_F \\
+ \xi_{(i)p} \left( \xi^{(3)} \right)^{-} \left( r_o(i) + \frac{\beta d}{\beta_m \cos \theta_b} \right) \xi_F
\end{array} \right\}
\]

\[i = 1, 2\]

(3.72)
where

\[ D_{2+} = \psi + A_{2+} \quad (3.73) \]

\[ D_{2-} = \psi - A_{2-} \quad (3.74) \]

\[ D_{fg} = -\psi (i) \Lambda_{g} \cdot \frac{e^{jB_{m}(\xi_{e}-\xi_{D_f})}}{2\sin(\beta_{d_e})} \quad (3.75) \]

and

\[ D_{fh} = -\psi (i) \Lambda_{h} \cdot \frac{e^{-jB_{m}(\xi_{e}-\xi_{D_f})}}{2\sin(\beta_{d_e})} \quad (3.76) \]

with

\[ \psi (i) = e^{-jB_{m}(R(f)-R(d))\cdot r_{o}} \cdot \frac{1}{r_{my}} \quad (3.77) \]

When we perform the integration of Equation (3.64), several situations need to be considered. Again, referring to Figure 3.9, when the dipole is straight, the entire feedline is situated in Region \( I_{d} \). In this case, \( Z_{fd} \) is given by
When the dipole is bent downward in a "V" shape, a part of the feedline will be situated in Region II<sub>d</sub>, see Figure 3.9. Here, we define an angle

\[ \theta_c = \cos^{-1} \left( \frac{\lambda_f - \lambda_u}{\lambda_d} \right) \]  

and a position variable

\[ z_c = (\lambda_f - \lambda_d \cos \theta_b) - z_u \]  

where

\[ \lambda_f = \text{length of feedline} \]

\[ z_u = y \cdot R \]

\( \theta_c \) and \( z_c \) are also shown in Figure 3.9.
If $\Theta_c < \Theta_b < 90^\circ$, only a part of the feedline section which is on the right hand side of $R^{(f)}$ will be situated in Region $II_d$. When the dipole is bent further, the domain of Region $II_d$ becomes larger; therefore, a larger portion of the test feedline will be situated in Region $II_d$. When $\Theta_b < \Theta_c$, the feedline which is on the left hand side of $R^{(f)}$ will also fall in Region $II_d$. Following the previous analysis, we have the following results

when $\Theta_c < \Theta_b < 90^\circ$

\[
\begin{align*}
Z_{fd} &= -\frac{1}{[I(3)(0)]^2} \sum_{i=1}^{2} \int_{-\epsilon_u}^{\epsilon_u} (3)^{(i)u} (z_F) I^{(3)}_F (z_F) dF \\
&+ \int_{0}^{\epsilon_c} (3)^{(i)u} (z_F) I^{(3)}_R (z_F) dF \\
&+ \int_{\epsilon_c}^{\epsilon_f - \epsilon_u} (3)^{(i)u} (z_F) I^{(3)}_R (z_F) dF \\
&+ (3.81)
\end{align*}
\]
when $\theta_b < \theta_c$

\[
Z_{uu}^{\text{fd}} = -\frac{1}{[I^{(3)}(0)]^2} \sum_{i=1}^{3} E^{(i)}_{L}^{(3)} \int_{-\xi_u}^{\xi_c} p^{(3)} \cdot E^{(i)}_{L}^{(3)} (\xi_F) d\xi_F \\
+ \int_{\xi_c}^{o} p^{(3)} \cdot E^{(i)}_{L}^{(3)} (\xi_F) d\xi_F \\
+ \int_{0}^{\xi_F-\xi_u} p^{(3)} \cdot E^{(i)}_{R}^{(3)} (\xi_F) d\xi_F
\]

(3.82)

It is obvious that when $\theta_b = 90^\circ$, Equation (3.81) reduces to Equation (3.78).
4. Impedance $Z_{df}^{uu}$

The impedance $Z_{df}^{uu}$ is associated with the coupling from the feedlines of Array "u" on the dipole of the same array. It can be formulated as

$$Z_{df}^{uu} = -\frac{1}{[I(3)(0)]^2} \sum_{j=1}^{2} \frac{x_d^{\hat{p}(j)} E^{(3)}(\epsilon_+) \sin\theta_d(\epsilon_+ - \epsilon)dx}{\int_0^{x_d^{\hat{p}(j)}}}$$

(3.83)

where $E^{(3)}(\epsilon)$ is the field on the $\hat{p}$-directed leg of the test dipole due to the radiation from the feedlines.

Since the reference point of the test dipole is located at $\hat{R}^{(d)}$, the field point $\hat{R}$ on the $\hat{p}$-directed leg of the test dipole can be represented by

$$\hat{R} = \hat{R}^{(d)} + x_D^{\hat{p}} \hat{r}(j)$$

(3.84)

where $x_D^{\hat{p}}$ is the length measured from $\hat{R}^{(d)}$ along the dipole leg oriented in the $\hat{p}$-direction.

The $y$-component of $\hat{R}$ is

$$y = y^{(d)} + x_D^{\hat{p}} y^{(j)}$$

$$= x_f - x_D \cdot \cos \theta_b$$

(3.85)

$$j = 1, 2$$
where \( y(d) \) is the \( y \)-component of \( \vec{R}(d) \), which is equivalent to \( \ell_f \) (length of the feedline).

Substituting Equation (3.84) into Equation (2.94), we obtain the field on the \( p \)-directed leg of the test dipole which is situated in Region III due to the radiation from the feedline array.

\[
E^{(3)}_{(j)}(Z_D) = \frac{i^{(3)}(0)Z_m}{2D\xi D_z} \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{c}
F_{d+} e^{-jk\beta_{mp}} \cdot r_{m+Z_D} - F_{d-} e^{-jk\beta_{mp}} \cdot r_{m-Z_D} \\
\end{array} \right\}
\]

\( j = 1, 2 \)

(3.86)

where

\[
F_{d+} = \psi_{3+} \quad (3.87)
\]

\[
F_{d-} = \psi_{3-} \quad (3.88)
\]

with

\[
\psi_{\pm} = e^{-j\beta_m(R(d)-R(f)) \cdot r_{my}} \quad (3.89)
\]

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Substituting Equations (3.84) and (3.85) into Equations (2.95) and (2.96), we obtain the field on the \( \hat{e}^{(j)} \)-directed leg of the dipole which is situated in Region \( \mathrm{II}_f \) due to the radiation from the feedline from the feedline array.

\[
E^{(3)}_{(j)2L_D} = -\frac{I^{(3)}(0)Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{c}
-F_d \cdot e^{-j \beta_m p \cdot r_m + \omega D} + F_d \cdot e^{-j \beta_m p \cdot r_m - \omega D} + \\
\frac{F_{dL} e^{-j \beta_m p \cdot r_{m+D}} dL + F_{dL} e^{-j \beta_m p \cdot r_{m-D}} dL}{dL} + \frac{2 F_{dL} e^{-2j \beta_m p \cdot r_{\omega D}}}{dL dD} + \frac{2 F_{dL} e^{-2j \beta_m p \cdot r_{\omega D}}}{dL dD} + \frac{2 F_{dL} e^{-2j \beta_m p \cdot r_{\omega D}}}{dL dD} \end{array} \right\}
\]

\[
j = 1, 2
\]

\( (3.90) \)
on the right of $R(f)$

\[
E_{(j)2R}(\varepsilon_D) = - \frac{1^{(3)}(0)Z_m}{2Dx_Dz} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ 
\begin{array}{c}
- j\beta_{mp} \cdot r_{m+2D} + j\beta_{mp} \cdot r_{m-2D} \\
F_{dR^+} - j\beta_{mp} \cdot r_{m+2D} + F_{dR^-} - j\beta_{mp} \cdot r_{m-2D} \\
+ \left[ F_{dR^0} + F_{dR^1D} + F_{dR^2D} \right] e^{-j\beta_{mp} \cdot r_{rot2D}} 
\end{array} \right. 
\]

\[
j = 1, 2 
\]

(3.91)

where

\[
F_dL^+ = \psi \Lambda^+ 
\]

(3.92)

\[
F_dL^- = \psi \Lambda^- 
\]

(3.93)
\[
F_{dL0} = \frac{d\phi}{\psi_0} \left[ \frac{(3) + (3)}{e_{m^+} - e_{m^-}} \cdot \frac{(2U_b)}{(j\beta r_{my})^2} - \frac{(3) - (3)}{e_{m^+} - e_{m^-}} \cdot \frac{(2U_b \ell_f)}{(j\beta r_{my})^2} \right]
\]
\[
- y \cdot \frac{2}{j\beta r_{my}} \left( U_a + U_b \ell_f \right)
\]

(3.94)

\[
F_{dL1} = \frac{d\phi}{\psi_0} \left[ \frac{(3) - (3)}{e_{m^+} - e_{m^-}} \cdot \frac{(2U_b \cos \Theta_b)}{(j\beta r_{my})^2} \right]
\]
\[
+ y \cdot \frac{\cos \Theta_b}{j\beta r_{my}} \left( 4U_b \ell_f \right)
\]

(3.95)

\[
F_{dL2} = - y \cdot \frac{d\phi}{\psi_0} \left[ \frac{(3) - 2U_b}{(j\beta r_{my}) \cdot \cos \Theta_b} \right]
\]

(3.96)

\[
F_{dR+} = \psi_+ \Lambda_{R+}
\]

(3.97)

\[
F_{dR-} = \psi_- \Lambda_{R-}
\]

(3.98)
\[ (\omega^2) \cdot \frac{\Lambda_\omega m t}{\mu^2} \cdot \frac{(0)(\varepsilon)^I}{\frac{\alpha}{\mu}} = PR_R \]

with

\[ (\omega_2) \cdot \frac{\Lambda_\omega m t}{\mu^2} \cdot \frac{(0)(\varepsilon)^I}{\frac{\alpha}{\mu}} = PR_R \]

and

\[ [ \left( \frac{\Lambda_\omega m t}{\mu^2} \cdot \frac{(0)(\varepsilon)^I}{\frac{\alpha}{\mu}} \right) \cdot \frac{\Lambda_\omega m t}{\mu^2} \cdot \frac{(0)(\varepsilon)^I}{\frac{\alpha}{\mu}} = PR_R \]

(69.3)
For a straight dipole (i.e. \( \theta_b = 90 \)), the entire dipole is situated in Region III, \( Z_{df}^{uu} \) can be written as

\[
Z_{df}^{uu} = -\frac{1}{[I(3)(0)]^2} \sum_{j=1}^{2} \int_{0}^{\ell_d} p \cdot E_{(3)} \text{I}_d \text{d} \sin \theta_d (\ell_e - \ell_D) d\ell_D \]

(3.103)

Referring to Figure 3.9, when \( \theta_c < \theta_b < 90^\circ \), the entire dipole is situated in Region II and on the right of \( R^- \), \( Z_{df}^{uu} \) will be

\[
Z_{df}^{uu} = -\frac{1}{[I(3)(0)]^2} \sum_{j=1}^{2} \int_{0}^{\ell_d} p \cdot E_{(3)} \text{I}_d \text{d} \sin \theta_d (\ell_e - \ell_D) d\ell_D
\]

(3.104)

However, when \( \theta_b < \theta_c \), a part of the dipole will be extending into the region on the left of \( R^- \). In this case, \( Z_{df}^{uu} \) should be

\[
Z_{df}^{uu} = -\frac{1}{[I(3)(0)]^2} \sum_{j=1}^{2} \left[ \int_{0}^{\ell_{2R}} p \cdot E_{(3)} \text{I}_d \text{d} \sin \theta_d (\ell_e - \ell_D) d\ell_D \right]
\]

\[
+ \int_{\ell_{2R}}^{\ell_d} p \cdot E_{(3)} \text{I}_d \text{d} \sin \theta_d (\ell_e - \ell_D) d\ell_D \]

(3.105)
where

\[ Z_{2R} = \frac{z_f - z_u}{\cos \theta_b} \quad (3.106) \]

denotes the length of the \((j)\) directed leg of the dipole existing on the right of \(R\) (see Figure 3.9).

E. MUTUAL IMPEDANCE \(Z_{ub}\)

\(Z_{ub}\) is the mutual impedance at Array "u" due to the coupling from Array "b". When we evaluate \(Z_{ub}\), the couplings from Array "b" on the feedline and dipole of Array "u" will be considered separately. Therefore, we may consider \(Z_{ub}\) is made up of two components

\[ Z_{ub} = Z_{ub}^{fd} + Z_{ub}^{dd} \quad (3.107) \]

where \(Z_{ub}^{fd}\) and \(Z_{ub}^{dd}\) are associated with the two couplings mentioned above. They will be formulated in the following.

1. Impedance \(Z_{ub}^{fd}\)

The impedance \(Z_{fd}^{ud}\) may be expressed as.
where \( E^{(i)b}(z) \) is the fields on the feedline due to the radiation from the balanced mode currents on the \( p \)-directed legs of the dipole array.

The field \( E^{(i)b}(3) \) in Region \( I_d \) and Region \( II_d \) can be obtained simply by replacing \( I_{ud} \) with \((-1)^i I_b \) in Equation (3.66) and (3.72), i.e.

in Region \( I_d \)

\[
E^{(i)b}(3)(z_F) = \left(-1\right)^i \frac{I_b \sin(\beta_d z_F \epsilon)}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ D^{(i)} e^{-j \beta_m^p (3)^p r_{m+2F}} + D^{(i)-} e^{-j \beta_m^p (3)^p r_{m-2F}} \right\}
\]

\[
E^{(i)b}(3)(z_F) = \left(-1\right)^i \frac{I_b \sin(\beta_d z_F \epsilon)}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ D^{(i)} e^{-j \beta_m^p (3)^p r_{m+2F}} + D^{(i)-} e^{-j \beta_m^p (3)^p r_{m-2F}} \right\}
\]

\[
i = 1, 2
\]

(3.109)
in Region \( II_d \)

\[
E_{(3)2}^{(i)b}(z_F) = (-1)^{i+1} \frac{I_{bd} \sin(\beta_d z_F) Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ 
\begin{align*}
-&(i) e^{-j\beta_m \hat{p}(3) \cdot \hat{r}_m + \hat{z}_F} + (i) e^{-j\beta_m \hat{p}(3) \cdot \hat{r}_m - \hat{z}_F} \\
+ &D_{fg} e^{-j\beta_m [\hat{p}(3) \cdot \hat{r}_o + \frac{\beta_d}{\beta_m \cos \theta_b}] \hat{z}_F} \\
+ &D_{fh} e^{-j\beta_m [\hat{p}(3) \cdot \hat{r}_o - \frac{\beta_d}{\beta_m \cos \theta_b}] \hat{z}_F}
\end{align*}
\right\}
\]

\( i = 1, 2 \) \quad (3.110)

where \( I_{bd} \) is the magnitude of the balanced mode current on the dipole element of Array "b", \( z_F \) is the length measured from \( R \) in the \( y \)-direction.
Following a similar procedure as for $Z_{\text{uu}}$ described in the previous section, $Z_{\text{ub}}$ can be formulated as:

When $\theta_c < \theta_b < 90^\circ$

$$
Z_{\text{ub}} = -\frac{1}{I(3)^{(0)}I_{bd}\sin(\theta_d\theta_e)} \sum_{i=1}^{2} \left[ \int_{-\theta_u}^{\theta_c + i\theta_b} (i)b_{(L_F)}I_{L(3)}^{'(3)}(i)E_{(3)}^1 d\theta_F - \int_{-\theta_u}^{\theta_u} (i)b_{(L_F)}I_{R(3)}^{'(3)}(i)E_{(3)}^1 d\theta_F \right] + \int_{\theta_c}^{\theta_u} (i)b_{(L_F)}I_{R(3)}^{'(3)}(i)E_{(3)}^2 d\theta_F
$$

(3.111)

When $\theta_b < \theta_c$

$$
Z_{\text{ub}} = -\frac{1}{I(3)^{(0)}I_{bd}\sin(\theta_d\theta_e)} \sum_{i=1}^{2} \left[ \int_{-\theta_u}^{\theta_c + i\theta_b} (i)b_{(L_F)}I_{L(3)}^{'(3)}(i)E_{(3)}^1 d\theta_F - \int_{-\theta_u}^{\theta_u} (i)b_{(L_F)}I_{R(3)}^{'(3)}(i)E_{(3)}^1 d\theta_F \right] + \int_{\theta_c}^{\theta_u} (i)b_{(L_F)}I_{R(3)}^{'(3)}(i)E_{(3)}^2 d\theta_F
$$

(3.112)

where $\theta_c, \theta_b$ are given by Equations (3.79) and (3.80), respectively.
2. Impedance $Z_{dd}^{ub}$

When we evaluate $Z_{dd}^{ub}$, the test dipole carrying unbalanced mode current is placed at a position with its reference terminal at $\phi_d = 90^\circ - \epsilon$, where $\epsilon$ is a very small angle. Therefore, for a straight dipole array, the test dipole is completely situated in Region III$_d$ and $Z_{dd}^{ub}$ can be formulated as

$$Z_{dd}^{ub} = -\frac{1}{I^{(3)}(0)I_{bd}\sin(\beta_d\phi_e)} \sum_{i=1}^{2} \sum_{j=1}^{2} \left[ \int_{0}^{\phi_d} \hat{E}_{d}^{(j)} \cdot \frac{\tau^{(j)}b}{(j)3} (\phi_{D})I_{ud}\sin\beta_{d}(\phi_e-\phi_{D})d\phi_{D} \right]$$

(3.113)

where $\hat{E}_{d}^{(j)}$ is given by Equation (3.9).

When the dipoles are V-shaped, we set $\epsilon = 0$. In this case, the entire test dipole is situated in Region II$_d$ (i.e., $\phi_d = 90^\circ$). $Z_{dd}^{ub}$ will be formulated as

$$Z_{dd}^{ub} = -\frac{1}{I^{(3)}(0)I_{bd}\sin(\beta_d\phi_e)} \sum_{i=1}^{2} \sum_{j=1}^{2} \left[ \int_{0}^{\phi_d} \hat{E}_{d}^{(j)} \cdot \frac{\tau^{(j)}b}{(j)2} (\phi_{D})I_{ud}\sin\beta_{d}(\phi_e-\phi_{D})d\phi_{D} \right]$$

(3.114)
where \( \frac{E^{(i)b}}{E^{(j)2}} (Z_D) \) is given by Equation (3.18).

F. MUTUAL IMPEDANCE \( Z_{bu} \)

Comparing with \( Z_{ub} \), the situation for \( Z_{bu} \) is just the other way around; \( Z_{bu} \) is the mutual impedance at Array "b" due to the coupling from Array "u". Similarly, when evaluating \( Z_{bu} \), the couplings from the dipoles and feedlines of Array "u" on Array "b" will be considered separately. Therefore, we may consider \( Z_{bu} \) is made up of two components

\[
Z_{bu} = Z_{bu}^{df} + Z_{bu}^{dd}
\]  

(3.115)

where \( Z_{bu}^{df} \) and \( Z_{bu}^{dd} \) are associated with the two couplings mentioned above. They will be formulated in the following.

1. Impedance \( Z_{bu}^{df} \)

\( Z_{bu}^{df} \) is the impedance component associated with the coupling from the feedlines of Array "u" on the dipoles of Array "b".

Again, referring to Figure 3.9 and following a similar procedure as for \( Z_{uu}^{df} \) described in the earlier section, \( Z_{bu}^{df} \) can be formulated as

when \( \theta_b = 90 \)
\[ Z_{bf}^{bu} = - \frac{1}{I(3)(0)I_{bd}\sin(\beta_{de})} \sum_{j=1}^{2} \left[ (-1)^{j+1} \int_{\lambda_{D}}^{\lambda_{D}} p^{(j)} \cdot E^{(3)}(\lambda_{D})I_{bd}\sin\beta_{d}(\lambda_{e}-\lambda_{D})d\lambda_{D} \right] , \]

when \( \theta_{c} < \theta_{b} < 90^\circ \)

\[ Z_{df}^{bu} = - \frac{1}{I(3)(0)I_{bd}\sin(\beta_{de})} \sum_{j=1}^{2} \left[ (-1)^{j+1} \int_{0}^{\lambda_{D}} p^{(j)} \cdot E^{(3)}(\lambda_{D})I_{bd}\sin\beta_{d}(\lambda_{e}-\lambda_{D})d\lambda_{D} \right] , \]

when \( \theta_{b} < \theta_{c} \)

\[ Z_{df}^{bu} = - \frac{1}{I(3)(0)I_{bd}\sin(\beta_{de})} \sum_{j=1}^{2} (-1)^{j+1} \left[ \int_{\lambda_{2R}}^{\lambda_{2R}} p^{(j)} \cdot E^{(3)}(\lambda_{D})I_{bd}\sin\beta_{d}(\lambda_{e}-\lambda_{D})d\lambda_{D} \right. \]
\[ + \int_{\lambda_{2R}}^{\lambda_{2R}} p^{(j)} \cdot E^{(3)}(\lambda_{D})I_{bd}\sin\beta_{d}(\lambda_{e}-\lambda_{D})d\lambda_{D} \left. \right] , \]

(3.118)
where \( \theta_c \) and \( \varepsilon_{2R} \) are defined by Equations (3.79) and (3.106), the fields 
\( E(3) \), \( E(3) \), \( E(3) \), 
\( E(j)3 \), \( E(j)2L \) and \( E(j)2R \) are given in Equations (3.86), (3.90) and 
(3.91), respectively.

2. Impedance \( Z_{dd} \)

When evaluating \( Z_{dd} \), the test dipole carrying balanced mode 
current will be placed at a similar location as for \( Z_{dd} \). \( Z_{dd} \) can be 
formulated in a similar fashion

when \( \theta_b = 90^\circ \)

\[
Z_{dd}^{bu} = - \frac{1}{I(3)(0)I_{bd} \sin(\theta_d \varepsilon_e)} \sum_{i=1}^{2} \sum_{j=1}^{2} \left[ (-1)^{j+1} \int_{0}^{2\pi} E(3)_{u} \cdot E(i)_{u} (d, \varepsilon) \left( I_{bd} \sin(\theta_{d} \varepsilon_{d} - \theta_{d} \varepsilon) \right) d \theta_{d} \right]
\]
\( \phi_d = 90^\circ - \varepsilon \)

(3.119)

when \( \theta_b < 90^\circ \)

\[
Z_{dd}^{bu} = - \frac{1}{I(3)(0)I_{bd} \sin(\theta_d \varepsilon_e)} \sum_{i=1}^{2} \sum_{j=1}^{2} \left[ (-1)^{j+1} \int_{0}^{2\pi} E(j)_{u} \cdot E(i)_{u} (d, \varepsilon) \left( I_{bd} \sin(\theta_{d} \varepsilon_{d} - \theta_{d} \varepsilon) \right) d \theta_{d} \right]
\]
\( \phi_d = 90^\circ \)

(3.120)

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where \( E(j)2(\xi_d, \eta_d) \) and \( E(j)3(\xi_d, \eta_d) \) are given in Equations (3.42) and (3.43), respectively. Note that \( \varepsilon \) is any small angle.

G. THE SUMMATION SCHEME

Every self or mutual impedance term described in the previous sections can be expressed in a doubly infinite summation form as

\[
Z = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} z(k,n)
\]

(3.121)

It means that the impedance \( Z \) is the total sum of the contribution, i.e. \( z(k,n) \), from all integer pairs \((k,n)\) in the \( kn \)-space defined in Figure 3.10. This doubly infinite sum may approximately be replaced by a finite sum of rings, i.e.

\[
\sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} z(k,n) \sim \sum_{m=0}^{N_\tau} \text{RING}_m
\]

(3.122)

where \( \text{RING}_m \) is the sum of \( z(k,n) \) over the \( m \)-th square ring shown in Figure 3.10. Note that \( \text{RING}_0 \) is equivalent to the \( k=n=0 \) term. \( N_\tau \) is the number at which we truncate the summation. The way to determine \( N_\tau \) can be described as follows. Let the sum of the first \( N \) rings be denoted by \( S_N \), which is a complex number. The absolute value of \( S_N \) can be multiplied by a ratio \( r \) to generate a comparison number \( C_0 \). Next,
Figure 3.10 Ring summation scheme used to evaluate the doubly infinite series in the impedance computation.
calculate the contribution from the terms in the N-th ring, denoted the sum by $R_{RING_N}$. Then, compare the absolute value of $R_{RING_N}$ with $C_p$. Usually, the comparison starts at $N=3$. The summation in Equation (3.122) will be truncated when the absolute value of $R_{RING_N}$ is smaller than or equal to the corresponding $C_p$ for $N_c$ consecutive times. Both $r$ and $N_c$ can be varied. Normally, more rings will be summed for a smaller $r$ or larger $N_c$. In this dissertation, $r$ is 0.01 and $N_c$ is 3, typically.
A. INTRODUCTION

The major objectives of this chapter are (1) to examine the impedance performance \( Z^{bb} \) of V-shaped dipole arrays under various situations, (2) to investigate the effect of the unbalanced mode current (so-called feedline effect) on the scan impedance of a dipole array by using Equation (1.5). It will be shown that without dielectric compensation the straight dipole array has a very large impedance variation with scan angle. This large variation in the scan impedance makes impedance matching more difficult. Stable scan impedance can be achieved by either bending the two legs of every dipole element in a "V" shape or placing the entire array in a stratified medium. It will also be shown that the feedline effect tends to be strong in the E-plane and in the scan direction close to broadside. Strong feedline effect might cause the real part of the scan impedance to be zero at some frequency, creating a so-called blind spot. To eliminate the possible blind spots occurring in the desired frequency range, we may install a proper high impedance load at every feedline in the array.
Based on the information given in the previous chapters, a computer program has been developed to perform the calculation of impedances $Z^{bb}$, $Z^{uu}$, $Z^{ub}$, and $Z^{bu}$ which are the necessary datas for processing the analysis to be presented in this chapter. This program is very general; it can compute all the mentioned impedance quantities for a V-shaped dipole array embedded in an arbitrary number of stratified medium.

In the following investigations, we assume every array is arranged in a rectangular grid, i.e. $\Delta z=0$ in Figure 2.2. For reference purpose, some important quantities associated with the array structure and dimensions of array element are defined and summarized in Figure 4.1. These definitions will be applied throughout this chapter.

The scan direction of an array always refers to the free space and its direction $\hat{S}_{0+}$ is specified by the angle pair $(\alpha, \eta)$ as illustrated in Figure 4.2. Since all the dipoles in the array are parallel to the $yz$-plane, we might consider $\alpha=0^\circ$ as the H-plane (of the dipole array) and $\alpha=90^\circ$ as the E-plane.

The current distributions along the dipole and feedline are shown in Figure 2.16 and Figure C.2. The locations of the test elements for impedance evaluations have been described in detail in Chapter III and Appendix G. Ring summation scheme explained in the previous chapter is used to compute all the impedance quantities.

**B. IMPEDANCE $Z^{bb}$**

In this section, we will present the calculated impedances $Z^{bb}$ for V-shaped dipole arrays under various situations. The data cases to be
Figure 4.1  Geometry of a V-shaped dipole and grid structure of a dipole array.
Figure 4.2 The specification of scan direction $s_{0+}$ by the angle pair $\alpha$ and $\eta$.

$s_{ox} = \sin \eta \cos \alpha$
$s_{oy} = \cos \eta$
$s_{oz} = \sin \eta \sin \alpha$
presented are summarized in Table 4.1. Although our major interest is in the cases when the arrays are located in free space with the absence of dielectric layer (Cases 1-20), some cases (Cases 21-23) showing the effect of dielectric layer(s) on the scan impedance of a dipole array are also included for illustration purpose.

The impedance variation from broadside to \( \eta=80^\circ \) in two major scan planes, \( \alpha=0^\circ \) and \( \alpha=90^\circ \), will be examined. All the investigations are limited to the frequency range within which the grating lobe does not exist, i.e. only the \( k=n=0 \) term of the plane wave spectrum is propagating. In this section, the frequency range of interest is within 4 -7.5 GHz.

1. Cases 1-6

Cases 1-6 investigate the impedance improvement of a V-shaped dipole array when its bend angle \( \theta_b \) is varied from \( 90^\circ \) to \( 40^\circ \) with \( 10^\circ \) increment. In this study, \( \theta_b \) is the only variable and all the other array parameters will be kept constant. Calculated results for these six cases are presented in Figures 4.3-4.8. Each figure contains five impedance results for frequencies of 4, 5, 6, 7 and 7.5 GHz.

We start by examining the scan impedance of a straight dipole array (\( \theta_b = 90^\circ \)) as shown in Figure 4.3. From there, we notice three trends:

(T1) Scan impedance varies very fast with scan angle in both the E-plane and the H-plane.

(T2) The real part of the scan impedance decreases monotonically with increasing scan angle, and its value drops to zero at \( \alpha = 90^\circ \), \( \eta = 80^\circ \).
### TABLE 4.1

**SUMMARY OF CASE DATA FOR THE STUDY OF $Z^{bb}$**

<table>
<thead>
<tr>
<th>CASE NUMBER</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$d_1$ (cm)</th>
<th>$d_2$ (cm)</th>
<th>$d_x$ (cm)</th>
<th>$d_z$ (cm)</th>
<th>$d_f$ (cm)</th>
<th>$d_d$ (cm)</th>
<th>$d_r$ (cm)</th>
<th>$d_0$ (deg)</th>
<th>FREQUENCY (GHz)</th>
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<td>0</td>
<td>2.0</td>
<td>2.0</td>
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<td>90</td>
<td>4, 5, 6, 7, 7.5</td>
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<td>0</td>
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<td>2.0</td>
<td>1.0</td>
<td>0.98</td>
<td>0.1</td>
<td>80</td>
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<td>0.1</td>
<td>70</td>
<td>4, 5, 6, 7, 7.5</td>
</tr>
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<td>0</td>
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<td>0.1</td>
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<td>0.1</td>
<td>90</td>
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</tr>
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<td>0.1</td>
<td>80</td>
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<td>0.1</td>
<td>50</td>
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<td>0</td>
<td>0</td>
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<td>2.0</td>
<td>0.8, 1.0, 1.25</td>
<td>0.98</td>
<td>0.1</td>
<td>40</td>
<td>4, 7</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>0.98</td>
<td>0.1</td>
<td>90, 40</td>
<td>7</td>
</tr>
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<td>0</td>
<td>1.0</td>
<td>1.3</td>
<td>1.0</td>
<td>0.98</td>
<td>0.1</td>
<td>40</td>
<td>7</td>
</tr>
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<td>1.414</td>
<td>0.707</td>
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<td>0.0707</td>
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<td>7</td>
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<td>0.08</td>
<td>1.414</td>
<td>1.414</td>
<td>0.707</td>
<td>0.693</td>
<td>0.0707</td>
<td>90, 40</td>
<td>7</td>
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<td>1.0</td>
<td>0.5</td>
<td>0.49</td>
<td>0.05</td>
<td>90, 40</td>
<td>7</td>
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</tbody>
</table>
Figure 4.3 Scan impedance $Z_{bb}$ for straight dipole array at $F = 4, 5, 6, 7$ and $7.5$ GHz.

$D_x = 2.0$ cm
$D_z = 2.0$ cm
$\delta_d = 0.98$ cm
$l_f = 1.0$ cm
$r_d = 0.1$ cm
Figure 4.3 (continued)
Figure 4.3 (continued)
Figure 4.4 Scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 80^\circ$ at $F = 4, 5, 6, 7$ and $7.5$ GHz.
Figure 4.4 (continued)
Figure 4.4 (continued)
Figure 4.5 Scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_d = 70^\circ$

at $f = 4, 5, 6, 7$ and $7.5$ GHz.

$D_x = 2.0 \text{ cm} \\
D_z = 2.0 \text{ cm} \\
\theta_d = 0.98 \text{ cm} \\
\ell_1 = 1.0 \text{ cm} \\
r_d = 0.1 \text{ cm}$
Figure 4.5 (continued)
Figure 4.5 (continued)
Figure 4.6 Scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 60^\circ$

at $F = 4, 5, 6, 7$ and $7.5$ GHz.

- $D_x = 2.0$ cm
- $D_z = 2.0$ cm
- $\beta_d = 0.98$ cm
- $l_f = 1.0$ cm
- $r_d = 0.1$ cm

$F = 4, 0$ GHz
- $\Theta - E$-PLANE
- $\Box - H$-PLANE

- REF (OHM)
- $\cdots$

- REAL (OHM)
- $\cdots$

- IMAGINARY (OHM)
- $\cdots$
Figure 4.6 (continued)
Figure 4.6 (continued)
Figure 4.7 Scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 50^\circ$

at $F = 4, 5, 6, 7$ and $7.5$ GHz.
Figure 4.7 (continued)
Figure 4.7 (continued)
Figure 4.8, Scanned impedance $Z_{mb}$ for Y-shaped dipole array with $\phi_b = 40^\circ$

$D_x = 2.0$ cm

$\phi_d = 0.98$ cm

$D_z = 2.0$ cm

$\phi_d = 0.1$ cm
Figure 4.8 (continued)
Figure 4.8 (continued)
Scan impedance varies rapidly with frequency. At higher frequency, the real part of the scan impedance tends to be larger.

When the real part of the scan impedance becomes zero, phased array will be "blind", no power can be transmitted/received by the array in that direction. This will limit the scanability of the array. Since trend (T3) is common to the phased arrays without dielectric compensation and is expected to exist in dipole arrays with \( \theta_b < 90^\circ \), this trend will not be considered again in this section.

Figures 4.4-4.8 show the scan impedances for the dipole arrays with smaller bend angles \((80^\circ < \theta_b < 40^\circ)\). From these figures, we notice two facts:

1. The scan impedance becomes more stable when \( \theta_b \) is decreased.
2. The real part of the scan impedance becomes smaller when \( \theta_b \) is decreased.

Since the impedance behavior of a phased array is strongly dependent upon the radiation pattern of its array element, the two facts mentioned above can be realized from the radiation pattern of the dipole element in the array. Figure 4.9 shows the radiation patterns of a single dipole element with various bend angle \( \theta_b \) at frequency 7.5 GHz. For comparison purposes, all the radiation patterns shown in Figure 4.9 are normalized to the same level. In this figure, we observe that the radiation pattern becomes more uniform, especially in the E-plane, when \( \theta_b \) is decreased. In the meantime, we also notice that the power.
Figure 4.9 Radiation patterns of a dipole element for $\theta_b = 90^\circ, 80^\circ, 70^\circ, 60^\circ, 50^\circ, 40^\circ$. 

$F = 7.5 \text{ GHz}$

* $\delta_d = 0.98 \text{ cm}$
* $\ell_d = 1.0 \text{ cm}$
* $r_d = 0.1 \text{ cm}$
Figure 4.9 (continued)
Figure 4.9 (continued)
Figure 4.9 (continued)
Figure 4.9 (continued)
Figure 4.9 (continued)
radiated from a dipole decreases when $\theta_b$ becomes smaller. Usually, a phased array comprised of elements having uniform radiation pattern will have more stable scan impedance. Therefore, one can expect the scan impedance will be more stable for the dipole array with smaller $\theta_b$. Since the real part of the scan impedance is responsible for the power radiated from an array, weaker power radiated from an array element will cause the real part of the scan impedance to be smaller.

2. Cases 7-12

Cases 7-12 investigate the effect of $\lambda_f$, distance between the dipole feed point and the ground plane, on the scan impedance of a dipole array. Three distances, $\lambda_f=0.8$, 1.0 and 1.25 cm, are used. $\theta_b$ is varied from 90° to 40° as before. All the other array parameters are remained the same. In this study, only the impedance variations at frequencies of 4 and 7 GHz will be considered.

Calculated results for cases 7-12 are presented in Figures 4.10-4.15 for frequency 4 GHz. Similar results for frequency 7 GHz are given in Figures 4.16-4.21. Examining the impedance variations shown in these figures, we notice that the scan impedance tends to be unstable when the distance $\lambda_f$ is increased. We also observe that the real part of the scan impedance is smaller for smaller $\lambda_f$. Scan impedance is still more stable for the dipole array with smaller $\theta_b$ for all $\lambda_f$. 

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Figure 4.10 Comparison of scan impedance $Z_{bb}$ for straight dipole array when $l_f = 0.8, 1.0, 1.25 \text{ cm}$ for $F = 4.0 \text{ GHz}$. 

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Figure 4.11 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 80^\circ$, when $\ell_f = 0.8$, 1.0, 1.25 cm for $F = 4.0$ GHz.
Figure 4.12 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 70^\circ$, when $l_f = 0.8, 1.0, 1.25$ cm for $F = 4.0$ GHz.
Figure 4.13 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 60^\circ$, when $z_f = 0.8, 1.0, 1.25 \text{ cm}$ for $F = 4.0$ GHz.
Figure 4.14 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 50^\circ$, when $\ell_f = 0.8, 1.0, 1.25$ cm for $F = 4.0$ GHz.
Figure 4.15 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 40^\circ$, when $\xi_f = 0.8, 1.0, 1.25$ cm for $f = 4.0$ GHz.
Figure 4.16 Comparison of scan impedance $Z_{bb}$ for straight dipole array when $l_f = 0.8, 1.0, 1.25$ cm for $F = 7.0$ GHz.
Figure 4.17 Comparison of scan impedance effect for V-shaped dipoles when $\phi = 0.8$, $1.0$, $1.25$ cm for $F = 7.0$ GHz.
Figure 4.18 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 70^\circ$, when $Z_f = 0.8, 1.0, 1.25$ cm for $f = 7.0$ GHz.
Figure 4.19 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_d = 60^\circ$, when $L_f = 0.8, 1.0, 1.25$ cm for $F = 7.0$ GHz.
Figure 4.20 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 50^\circ$, when $\ell_f = 0.8, 1.0, 1.25$ cm for $F = 7.0$ GHz.
Figure 4.21 Comparison of scan impedance $Z_{bb}$ for V-shaped dipole array with $\theta_b = 40^\circ$, when $l_f = 0.8, 1.0, 1.25$ cm for $f = 7.0$ GHz.
3. Cases 13-14

Cases 13-14 examine the effect of the interelement spacings, $D_x$ and $D_z$, on the scan impedance of a dipole array. In the present cases, we only consider the scan impedance at frequency 7 GHz. Figure 4.22 shows the scan impedances for the dipole arrays with $\theta_0=90^\circ$, $40^\circ$ when $D_x=1.0$ cm and $D_z=2.0$ cm. Comparing Figure 4.22(a) with Figure 4.3 and Figure 4.22(b) with Figure 4.8, we notice that the real part of the scan impedance is doubled when $D_x$ is reduced to one-half of its original value. Impedance patterns are similar for the arrays with different $D_x$.

Figure 4.23 presents the scan impedance of dipole array with $\theta_0=40^\circ$ when $D_x=1.0$ cm and $D_z=1.3$ cm. Comparing Figure 4.23 with Figure 4.22(b), it is apparent that the scan impedance becomes unstable when $D_z$ is decreased.

4. Cases 15-17

Cases 15-17 deal with the situations when the dipole arrays are embedded in a dielectric layer with dielectric constant $\varepsilon_1$ and thickness $d_1$ as shown in Figure 4.2. When an array is completely inside the dielectric layer, all the array parameters should be scaled to $1/\sqrt{\varepsilon_1}$ of their normal values in free space. In cases 16 and 17, another piece of dielectric slab with dielectric constant $\varepsilon_2$ and thickness $d_2$ is placed in front of the array.

Figure 4.24 presents the calculated results for case 15. The results for cases 16 and 17 are shown in Figures 4.25 and 4.26, respectively. It is noted that the presence of dielectric layer(s) will
Figure 4.22 Scan impedances $Z^{bb}$ for dipole array with $\theta_d = 90^\circ$, $40^\circ$ when $D_x = 1.0$ cm, $D_z = 2.0$ cm (Case 13).
Figure 4.23 Scan impedance $Z_{bb}$ for dipole array with $\theta_b = 40^o$ when $D_x = 1.0 \text{ cm}, D_z = 1.3 \text{ cm}$ (Case 14).
Figure 4.24  Scan impedance $Z_{bb}^b$ for Case 15 (a) $\theta_b = 90^\circ$, (b) $\theta_b = 40^\circ$. 

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Figure 4.24 (continued)
Figure 4.25 Scan impedance $Z^{bb}$ for Case 16 (a) $\theta_b = 90^\circ$, (b) $\theta_b = 40^\circ$. 
Figure 4.25 (continued).
Figure 4.26 Scan impedance $Z^{bb}$ for Case 17 (a) $\theta_b = 90^\circ$, (b) $\theta_b = 40^\circ$. 
**Figure 4.25 (continued).**
substantially improve the scan impedance for the straight dipole (see Figure 26 for Case 17). We also notice that the scan impedance is slightly improved for the arrays with $\theta_d = 40^\circ$.

B. SCAN IMPEDANCE $Z_d$

The scan impedances ($z^{bb}$) presented in the previous section are for the dipole arrays in the absence of feedline scatterers. In this section, we will investigate the effect of feedline scatterers (i.e., feedline effect) on the impedance performance of a dipole array. It was shown in Chapter I that when the feedline effect is taken into account, the scan impedance of a dipole array, denoted by $Z_d$, might be expressed as

$$Z_d = z^{bb} - \Delta Z_f$$  \hspace{1cm} (4.1)

where

$$\Delta Z_f = \frac{Z_{bu} z_{ub}}{Z_{uu}}$$ \hspace{1cm} (4.2)

$Z_{bu}$, $Z_{ub}$ and $Z_{uu}$ are defined in Chapter I and formulated in Chapter III. $\Delta Z_f$ defined by Equation (4.2) is a quantity used to measure the strength of the feedline effect.

In this section, all the impedance results will be presented as functions of frequency. The frequency range of interest is from 4 to 10 GHz. Since the effect of feedline scatterers on the impedance behavior of a dipole array is our major concern, we should restrict our
investigations to the arrays located in free space (i.e. \( \varepsilon_1 = \varepsilon_2 = 1 \) in Figure 4.1). In the following study, the quantities listed below will be kept constant:

\[
\begin{align*}
D_x &= 2.0 \text{ cm} \\
D_z &= 2.0 \text{ cm} \\
Z_d &= 0.98 \text{ cm} \\
Z_f &= 1.0 \text{ cm} \\
Z_u &= 0.8 \text{ cm} \\
r_d &= 0.1 \text{ cm} \\
r_f &= 0.1 \text{ cm} \\
\xi &= 0.104
\end{align*}
\]

where \( \xi \) is the current ratio as defined in Equation (C.22).

Table 4.2 is a summary of the data cases for the results to be presented in this section. In each figure, the impedances \( Z_{bb} \) and \( Z_d \) will be shown on the same page. By comparing these two impedances, one can easily observe the strength of the feedline effect existing at various frequencies.

Figures 4.27-4.34 show the impedances \( Z_{bb} \) and \( Z_d \) at various scan planes (\( \alpha = 20^\circ, 45^\circ \) and \( 90^\circ \)) and scan angles (\( \eta = 15^\circ, 40^\circ \) and \( 60^\circ \)) for a straight dipole array. In these figures, we notice that the feedline effect is very strong in the E-plane and in the direction close to broadside. The strength of the feedline effect becomes weaker when \( \eta \) is larger and/or \( \alpha \) is smaller. It will be shown in the next section that the feedline effect is negligibly small in the H-plane (\( \alpha = 0^\circ \)). Closer
### TABLE 4.2

**SUMMARY OF CASE DATA FOR \( Z_{bb} \) AND \( Z_d \)**

**PRESENTED IN FIGURES 4.27 THROUGH 4.36**

<table>
<thead>
<tr>
<th>FIGURE NUMBER</th>
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<th>( \alpha ) (deg)</th>
<th>( \eta ) (deg)</th>
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<td>4.36</td>
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</tr>
</tbody>
</table>
Figure 4.27 Impedances $Z_{bb}$ and $Z_d$ as functions of frequency at $\alpha = 90^\circ$, $n = 15^\circ$ for a straight dipole array.
Figure 4.28 Impedances $Z_{bb}$ and $Z_d$ as functions of frequency at $\alpha = 90^\circ$, $\eta = 40^\circ$ for a straight dipole array.
Figure 4.29 Impedances $Z_{bb}$ and $Z_d$ as functions of frequency at $\alpha = 90^\circ$, $\eta = 60^\circ$ for a straight dipole array.
Figure 4.30 Impedances $Z^{bb}$ and $Z_d$ as functions of frequency at $\alpha = 45^\circ$, $n = 15^\circ$ for a straight dipole array.
Figure 4.31 Impedances $Z_{bb}$ and $Z_d$ as functions of frequency at $\alpha = 45^\circ$, $\eta = 40^\circ$ for a straight dipole array.
Figure 4.32 Impedances $Z_{bb}$ and $Z_d$ as functions of frequency at $\alpha = 45^\circ$, $\eta = 60^\circ$ for a straight dipole array.
Figure 4.33 Impedances $Z_{bb}$ and $Z_d$ as functions of frequency at $\alpha = 20^\circ$, $\eta = 15^\circ$ for a straight dipole array.
Figure 4.34 Impedances $Z^{bb}$ and $Z_d$ as functions of frequency at $\alpha = 20^\circ$, $\eta = 40^\circ$ for a straight dipole array.
examination of Figure 4.27-4.34, we found that the feedline effect tends
to be stronger at frequencies between 7 and 8 GHz (reasons will be given
in Section E). When the feedline effect is strong, we will observe a
very large impedance variation with frequency. In some cases, this
large impedance variation may cause the real part of the scan impedance
to be zero at some frequency, creating a so-called "blind spot". In
Figure 4.27, we notice a blind spot occurs at frequency 7.68 GHz.

The impedances $Z_{bb}$ and $Z_d$ in the scan direction $\alpha=90^\circ$, $\eta=15^\circ$ for
the dipole arrays with $\Theta_b=70^\circ$ and $50^\circ$ are shown in Figures 4.35 and
4.36, respectively. In these figures, we observe that the feedline
effects are still very strong at frequencies between 7 and 8 GHz. It is
evident that the feedline effect can not be avoided for any dipole array
without loading the feedlines.

D. QUANTITY $\Delta Z_f$ AND FEEDLINE EFFECT

The impedance $Z_d$'s presented in the previous section were
calculated using Equation (4.1). The quantity $\Delta Z_f$ defined by Equation
(4.2) represents the strength of the feedline effect which perturbs the
impedance (i.e. $Z_{bb}$) of a balanced dipole array. The relationship
between $Z_{bb}$, $Z_d$ and $\Delta Z_f$ for a straight dipole array is illustrated in
Figure 4.37.

In this section, we will investigate the impedance behaviors of
$Z_{ub}$, $Z_{bu}$ and $Z_{uu}$ which define the quantity $\Delta Z_f$. Based on these
informations, some phenomenons observed in the previous section can be
Figure 4.35 Impedances $Z_{bb}$ and $Z_d$ as functions of frequency at $\alpha = 90^\circ$, $n = 15^\circ$ for a V-shaped dipole array with $\theta_b = 70^\circ$. 
Figure 4.36 Impedances $Z^{bb}$ and $Z_d$ as functions of frequency at $\alpha = 90^\circ$, $\eta = 15^\circ$ for a V-shaped dipole array with $\theta_b = 50^\circ$. 

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**Figure Caption:**

Impedances $Z^{bb}$ and $Z_d$ as functions of frequency at $\alpha = 90^\circ$, $\eta = 15^\circ$ for a V-shaped dipole array with $\theta_b = 50^\circ$. 

The graphs show the real and imaginary parts of the impedances for different frequencies, illustrating how these quantities vary with frequency. The data points are marked at specific frequencies, indicating the impedance values at those points.

---

**Graph Details:**

- **$Z^{bb}$ Graph:**
  - Real axis (0 to 300 Ohms) and Imaginary axis (0 to 300 Ohms).
  - Data points are marked for various frequencies, showing the trend of impedance change.

- **$Z_d$ Graph:**
  - Similar setup as $Z^{bb}$ but with different data points.
  - The graphs are overlaid on a grid background for clarity.

---

**Analysis:**

The graphs provide a visual representation of how the impedances $Z^{bb}$ and $Z_d$ evolve with frequency. The specific data points at different frequencies help in understanding the behavior of these impedances in the context of the V-shaped dipole array.

**Implications:**

Understanding the impedances is crucial for designing and optimizing the performance of dipole arrays in various applications, such as in antenna systems or in signal processing.

---

**Mathematical Notes:**

- Impedance is a measure of a circuit's resistance to alternating current.
- The real part represents the power consumed or delivered by the circuit, while the imaginary part indicates the power absorbed or stored.
\[ Z_d = Z^{bb} - \Delta Z_f \]

\[ \Delta Z_f \triangleq \frac{Z_{ub}Z_{bu}}{Z_{uu}} \]

Figure 4.37 The relationship between \( Z^{bb} \), \( Z_d \) and \( \Delta Z_f \) for a straight dipole array.
explained. From Equation (4.2), we know that the strength of the feedline effect is determined by the magnitudes of $Z_{ub}$, $Z_{bu}$, and $Z_{uu}$. The feedline effect always exists when $Z_{ub}$ or $Z_{bu}$ is not zero, and its strength becomes stronger when $|Z_{ub}|$ and $|Z_{bu}|$ are larger and/or $|Z_{uu}|$ is smaller. In this section, only the impedance behaviors of $Z_{ub}$, $Z_{bu}$ and $Z_{uu}$ for the straight dipole array will be considered.

Figures 4.38-4.40 show the impedances $Z_{ub}$ and $Z_{bu}$ for $\alpha = 20^\circ$, $45^\circ$ and $90^\circ$ and $\eta = 15^\circ$, $40^\circ$ and $60^\circ$. In these figures, we notice that, for a fixed $\eta$, the magnitudes of $Z_{ub}$ and $Z_{bu}$ decrease with decreasing $\alpha$. Two plots showing the variations of $Z_{ub}$ and $Z_{bu}$ as functions of $\alpha$ for $\eta = 15^\circ$, $\nu = 7.0$ GHz are presented in Figure 4.41. It is shown that both $Z_{ub}$ and $Z_{bu}$ vary as functions of $\sin(\alpha)$. When $\alpha = 0^\circ$ (i.e. H-plane), $Z_{ub}$ and $Z_{bu}$ are both zero, forcing $\Delta Z_f$ to be zero. Therefore, feedline effect does not exist in the H-plane.

Now, we shall examine the impedance variations of $Z_{uu}$ at $\alpha = 20^\circ$, $45^\circ$ and $90^\circ$ and $\eta = 15^\circ$, $45^\circ$ and $60^\circ$ depicted in Figures 4.42-4.44. It is noted that, for a fixed $\alpha$ (i.e. same scan plane), the magnitude of impedance $Z_{uu}$ increases very fast with increasing scan angle ($\eta$). However, for a fixed $\eta$ (i.e. same scan angle), impedance $Z_{uu}$ does not change too much with $\alpha$ at the frequencies below 7 GHz.

From the analysis given above, we may conclude that: (1) feedline effect becomes weaker for larger $\eta$ is due to the large increase in $|Z_{uu}|$. (2) feedline effect decreases with decreasing $\alpha$ is because both $|Z_{ub}|$ and $|Z_{bu}|$ are varied as functions of $\sin(\alpha)$. 

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Figure 4.38 Mutual impedances $Z_{ub}$ and $Z_{bu}$ as functions of frequency and $\alpha$ for $\eta = 15^\circ$, $\theta_0 = 90^\circ$ (straight dipole array).
Figure 4.39 Mutual impedances $Z_{ub}$ and $Z_{bu}$ as functions of frequency and $\alpha$ for $n = 40^\circ$, $\theta_b = 90^\circ$ (straight dipole array).
Figure 4.40 Mutual impedances $Z^{ub}$ and $Z^{bu}$ as functions of frequency and $\alpha$ for $\eta = 60^\circ$, $\theta_b = 90^\circ$ (straight dipole array).
Figure 4.41 Variations of mutual impedance $Z_{ub}$ and $Z_{bu}$ as functions of $\alpha$ for $\eta = 15^\circ$, $\theta_b = 90^\circ$, $f = 7.5$ GHz.
Figure 4.42 Impedance $Z_{uu}$ as a function of frequency and $\eta$ at $\alpha = 90^\circ$ for a straight dipole array.
Figure 4.43 Impedance $Z_{uu}$ as a function of frequency and $\eta$ at $\alpha = 45^\circ$ for a straight dipole array.
Figure 4.44 Impedance $Z_{uu}$ as a function of frequency and $\eta$ at $\alpha = 20^\circ$ for a straight dipole array.
E. ELIMINATION OF FEEDLINE EFFECT

In Section C of this chapter, we noticed that whenever the feedline effect exists its strength tends to be strong at frequencies between 7 and 8 GHz. To understand the reason behind it, we shall examine the impedance behaviors of \( Z_{uu} \) shown in Figures 4.38-4.40. In these figures, we observe that at frequencies between 7 and 8 GHz the feedlines are very close to resonance. Therefore, the unbalanced mode current becomes very large, causing the feedline effect to be strong. It is obvious that to eliminate this feedline effect, we should prevent the feedlines from being close to resonance.

Based on the foregoing analysis, we will propose two methods, shown in Figure 4.45, which may be used to eliminate the feedline effect in the desired frequency range. We can install an inductor at every feedline or place a circular ferrite pipe around the feedline as indicated in this figure. The presence of the inductor and ferrite pipe will not affect the balanced mode current on the dipole element, but it will increase the impedance of Array "u". Due to the existence of the inductor/ferrite pipe, the quantity \( \Delta Z_f \) defined in Equation (4.2) should be modified as

\[
\Delta Z_f = \frac{z_{ub}z_{bu}}{z_{uu} + Z_L}
\]  

(4.3)

where \( Z_L \) is the impedance of the inductor/ferrite pipe. From this equation, it is apparent that the strength of the feedline effect can be reduced by increasing the magnitude of impedance \( Z_L \). For an inductor,
\[ Z_L = j \omega L, \] where \( \omega \) is angular frequency and \( L \) is inductance of the coil. The impedance \( Z_L \) of a ferrite pipe is controlled by its length and thickness.

In the following, we will present some results showing the effect of the loaded inductor on the scan impedance of an array. Figures 4.46 and 4.47 show the scan impedance \( Z_d \) of a straight dipole array for \( L=10 \) and 30 nH, respectively. In these figures, we observe that the feedline effect shifts to different frequency range and its strength becomes smaller when \( L \) is varied. Increasing the magnitude of \( L \) will force the strong feedline effect to exist at lower frequencies. Figure 4.48 shows the impedances \( Z_{bb} \) and \( Z_d \) when \( L=100 \) nH. Comparing these two impedance curves, we note that the feedline effect becomes negligibly small at all frequencies.
Figure 4.45 Loading the feedline with an inductor (top) or a circular ferrite pipe (bottom).
Figure 4.46 Scan impedance $Z_d$ for a straight dipole array with $L = 10$ nH inductor installed at every feedline.
Figure 4.47 Scan impedance $Z_d$ for a straight dipole array with $L = 30$ nH inductor installed at every feedline.
Figure 4.48 Comparison of Scan impedances $Z_{bb}$ with $Z_d$ when $L = 100$ nH.
CHAPTER V  
SUMMARY AND CONCLUSIONS

In this dissertation, Plane Wave Expansion Method [1] has been applied to (1). investigating the scan impedance behaviors of V-shaped dipole arrays under various situations, (2). analyzing the effect of feedline scatterers on the impedance performance of a dipole array. Although the theory developed in Chapters II and III are for the dipole arrays embedded in a general stratified medium, in order to obtain clear pictures about how the bend angle (\( \theta_b \)) and feedline effect will affect the scan impedance of a dipole array, our investigations were focused on the cases when the arrays were located in free space, i.e. in the absence of dielectric layer(s).

Without dielectric compensation, a straight dipole array (\( \theta_b=90^\circ \)) has very large impedance variation with the scan angle in both E-plane and H-plane, which makes impedance matching more difficult. An easy way to stabilize the scan impedance (\( Z_{bb} \)) of a dipole array is to bend the two legs of every dipole element in a "V" shape. It was shown that the scan impedance became more stable when \( \theta_b \) was smaller, which is due to the uniform radiation pattern of the array element.

The spacing between the ground plane and dipole feed point has some effect on the scan impedance; the scan impedance tends to be unstable when the spacing is increased. The interelement spacings \( D_x \) and \( D_z \) also...
affect the scan impedance in different ways. Decreasing $D_x$ will increase the real part of the scan impedance without affecting the overall impedance pattern. Decreasing $D_z$ (for array with smaller $\theta_b$) will degrade the stability of the scan impedance (compare Figure 4.23 with Figure 4.22).

The feedline effect which perturbed the impedance performance of a balanced dipole array was shown very strong in the E-plane and in the direction close to broadside. The strength of the feedline effect became weaker when $\eta$ was larger and/or $\alpha$ was smaller. For a fixed $\eta$, the strength of the feedline effect varied as a function of $\sin^2(\alpha)$ (see Figure 4.1 for definitions of $\alpha$ and $\eta$). Therefore, the feedline effect was absent in the H-plane ($\alpha=0^\circ$). The feedline effects were strong when the feedlines were close to resonance. Strong feedline effect might cause the real part of the scan impedance to be zero at some frequency creating a so-called blind spot. Our analysis also showed that the feedline effect could not be avoided for the dipole arrays with smaller $\theta_b$, which is contrary to the conclusion by Schuman, et al [5]. A possible way to eliminate the feedline effect in the desired frequency range is to install a proper high impedance on every feedline to prevent the feedline from being close to resonance.
APPENDIX A

DIRECTION OF PROPAGATION AND WAVE VECTORS

It was mentioned in Chapter II that when the currents on the elements of an infinite periodic array satisfy the Floquet's theorem, the field radiated from the array may be expressed as an infinite sum of discrete plane waves. Each plane wave is propagating in a different direction specified by a pair of indices $k$ and $n$. The propagation direction for the $(k,n)^{th}$ plane wave is given by

$$\hat{r}_{mz} = x\hat{r}_{mx} \pm y\hat{r}_{my} + z\hat{r}_{mz}$$  \hspace{1cm} (A.1)$$

where

$$r_{mx} = s_{mx} + k \frac{\lambda_m}{D_x} - \frac{n \Delta z \lambda_m}{D_x D_z}$$  \hspace{1cm} (A.2)$$

$$r_{mz} = s_{mz} + n \frac{\lambda_m}{D_z}$$  \hspace{1cm} (A.3)$$
and $D_x$, $D_z$ and $\Delta z$ are the parameters defining the grid structure of the array, see Figure 2.2. The subscript $m$ refers to the dielectric medium where $r_{m_+}$ and $s_{m_\pm}$ are evaluated.

The unit vector $\hat{r}_{m_+}$ is pointing to the right of the array, while the unit vector $\hat{r}_{m_-}$ is pointing to the left of the array, see Figure A.1(a). Note that when $k = n = 0$, $\hat{r}_{m_\pm} = \hat{s}_{m_\pm}$ which is the propagation direction of the scan beam (dominant propagating wave) in the medium $m$.

As indicated in Equation (A.4), when $r_{mx}^2 + r_{mz}^2 > 1$, we choose the $-j$ root. This corresponds to an evanescent wave which will decay exponentially when moving away from the array. If $r_{my}$ is real when $(k,n) \neq (0,0)$, we will have another propagating wave (so-called grating lobe). Some energy will be carried away from the array in this undesired direction. That is the situation needed to be avoided in the phased array design. In order to hold off the onset of the grating lobes, we should maintain the interelement spacings $D_x$ and $D_z$ as small as possible.

\[
\begin{align*}
    r_{my} &= \begin{cases} 
    \sqrt{1 - (r_{mx}^2 + r_{mz}^2)^2} & 1 > r_{mx}^2 + r_{mz}^2 \\
    -j \sqrt{(r_{mx}^2 + r_{mz}^2)^2 - 1} & 1 < r_{mx}^2 + r_{mz}^2
    \end{cases} \\
    \hat{s}_{m_\pm} &= \hat{x} s_{mx} \pm \hat{y} s_{my} \pm \hat{z} s_{mz} \quad (A.5)
\end{align*}
\]
When a plane wave is transmitted from one medium into another, its propagation direction will change. The propagation directions of the \((k,n)\)th plane wave in two adjacent mediums can be related by:

\[
\begin{align*}
    r_{(m-1)x} &= \frac{\beta_m}{\beta_{m-1}} r_{mx} \quad \text{(A.6)} \\
    r_{(m-1)z} &= \frac{\beta_m}{\beta_{m-1}} r_{mz} \quad \text{(A.7)}
\end{align*}
\]

and

\[
    r_{(m-1)y} = \sqrt{1 - r_{(m-1)x}^2 - r_{(m-1)z}^2} \quad \text{(A.8)}
\]

Consider a situation shown in Figure A.1(b) where a phased array located in medium \(m\) is isolated from the free space by a set of dielectric slabs. Assuming the scan beam (plane wave with \(k=n=0\)) radiated from the array is propagating in the direction \(\hat{s}_m^+\), when this scan beam is transmitted through the dielectric slabs, its direction will change to \(\hat{s}_o^+\) which may not be the same as that of \(\hat{s}_m^+\). The directions of \(\hat{s}_o^+\) and \(\hat{s}_m^+\) can be related by repeatedly using Equations (A.6) and (A.7)

\[
\begin{align*}
    s_{ox} &= \frac{\beta_m}{\beta_o} s_{mx} \quad \text{(A.9)} \\
    s_{oz} &= \frac{\beta_m}{\beta_o} s_{mz} \quad \text{(A.10)}
\end{align*}
\]
When dealing with a wave being reflected from or transmitted through a dielectric interface, it is necessary to express the incident field in terms of two unit vectors, along unit vectors \( \hat{n}_{m\pm} \) and \( \perp n_{m\pm} \) which are defined in Chapter II. The relative orientations between \( \hat{r}_{m\pm} \), \( \hat{n}_{m\pm} \) and \( \perp n_{m\pm} \) are illustrated in Figure 2.5.

The wave vector for a \( \hat{p} \)-directed element is defined as

\[
\bar{\varepsilon}_{m\pm} = (\hat{p} \times \hat{r}_{m\pm}) \times \hat{r}_{m\pm}
\]  

(A.12)

which specifies the polarization of the electric field radiated from the element. By application of the vector identity

\[
(A \times B) \times C = (A \cdot C)B - (B \cdot C)A
\]

(A.13)

Equation (A.12) can be written in an alternate form

\[
\bar{\varepsilon}_{m\pm} = (\hat{p} \cdot \hat{r}_{m\pm}) \hat{r}_{m\pm} - \hat{p}
\]

(A.14)
Expressing wave vector $\hat{e}_{m\pm}$ in terms of $\hat{n}_{m\pm}$ and $\perp(n_{m\pm})$, we have

$$\hat{e}_{m\pm} = (\hat{e}_{m\pm} \cdot \perp(n_{m\pm})) \hat{n}_{m\pm} + (\hat{e}_{m\pm} \cdot |n_{m\pm}|) \perp(n_{m\pm})$$  \hspace{1cm} (A.15)$$

where

$$\hat{n}_{m\pm} = \frac{1}{\sqrt{r_{mx}^2 + r_{mz}^2}} \left[ -x r_{mx} r_{my} \hat{y} (r_{mx}^2 + r_{mz}^2) - z r_{my} r_{mz}^2 \right]$$  \hspace{1cm} (A.16)$$

and

$$\perp(n_{m\pm}) = \frac{-x r_{mx} + z r_{mz}}{\sqrt{r_{mx}^2 + r_{mz}^2}}$$  \hspace{1cm} (A.17)$$

Noting that

$$\hat{r}_{m\pm} \cdot \hat{n}_{m\pm} = 0$$  \hspace{1cm} (A.18)$$

and

$$\hat{r}_{m\pm} \cdot \perp(n_{m\pm}) = 0$$  \hspace{1cm} (A.19)$$

Equation (A.15) may be written as

$$\hat{e}_{m\pm} = -(\hat{p} \cdot \hat{n}_{m\pm}) \hat{n}_{m\pm} - (\hat{p} \cdot \perp(n_{m\pm})) \perp(n_{m\pm})$$  \hspace{1cm} (A.20)$$
(a) Plane wave directions;

(b) Scan directions $\hat{s}_{0+}$ for the array in the presence of dielectric slabs.

Figure A.1
APPENDIX B
EFFECTIVE REFLECTION COEFFICIENTS $I^m_{m+}$

The purpose of this appendix is to present a method for evaluating the effective reflection coefficients $I^m_{m+}$. Consider a situation shown in Figure B.1, where a set of dielectric slabs placed between medium $m$ and the free space is illuminated by an incident wave $E^i$ propagating in the $+y$-direction. In this figure, $E_f$ represents the total reflected field resulting from an infinite number of bounces inside the dielectric slabs. If the effective reflection coefficient for the interface $(m,m-1)$, is defined as the ratio of $E_f$ to $E^i$ at that interface, we have [6]

$$
I^m_{m+} = \frac{I^r_{m,m-1} + I^r_{m-1,m} e^{-j2\beta_{m-1}d_{m-1}}I^r_{m-1,m}y}{1 - I^r_{m-1,m} + I^r_{m,m-1} e^{-j2\beta_{m-1}d_{m-1}}I^r_{m-1,m}y}
$$

(B.1)

where $I^r_{m,m-1}$ are the regular Fresnel reflection coefficients at the interface between two semi-infinite mediums $m$ and $m-1$. They are given by

$$
I^r_{m,m-1} = \frac{Z_{m-1}I^r_{m}(m-1)y - Z^r_{m}r_{my}}{Z_{m-1}I^r_{m}(m-1)y + Z^r_{m}r_{my}}
$$

(B.2)
\[ I_{m,m-1}^{1} = \frac{Z_{m-1}r_{my} - Z_{m}r_{(m-1)y}}{Z_{m-1}r_{my} + Z_{m}r_{(m-1)y}} \]  \hspace{1cm} (B.3)

where \( Z_{m} \) and \( Z_{m-1} \) are the characteristic impedances of the mediums \( m \) and \( m-1 \) respectively. \( r_{my} \) and \( r_{(m-1)y} \) are given in Appendix A. From Equations (B.2) and (B.3), it is obvious that

\[ I_{m-1,m}^{1} = -I_{m,m-1}^{1} \]  \hspace{1cm} (B.4)

Note that \( I_{m+}^{1} \) are dependent upon \( I_{(m-1)+}^{1} \), which is the effective reflection coefficients at the interface \((m-1,m-2)\). To obtain \( I_{(m-1)+}^{1} \), we should use a reverse approach. First, realizing that \( I_{1+}^{1} = I_{1,0} \) and employing Equation (B.1), we can have \( I_{2+}^{1} \). Then, applying Equation (B.1) interatively, \( I_{(m-1)+}^{1} \) will finally be obtained.
Figure B.1  Multiple reflections in a stratified medium.
APPENDIX C
UNBALANCED MODE CURRENT ALONG FEEDLINE

In this appendix, the unbalanced mode current along a feedline will be simulated. Figure C.1(a) schematically shows the unbalanced mode currents on a two-wire feedline and a dipole. When the two wires of the feedline are closely spaced, the feedline can be modelled by a single wire carrying the total unbalanced mode current on the feedline as shown in Figure C.1(b).

The approach we used to simulate the unbalanced mode current on the feedline can be described as follows. We start assuming that the unbalanced mode current distribution along the dipole is sinusoidal as illustrated in Figure 2.16. The expressions for the currents on the two legs of the dipole are given by

\[ I^{(1)}(z) = I_{ud} \sin \beta_d (e - z_1) \]  
(C.1)

and

\[ I^{(2)}(z) = I_{ud} \sin \beta_d (e - z_2) \]  
(C.2)

where

- \( I_{ud} \) = magnitude of the unbalanced mode current on dipole,
- \( \beta_d \) = effective propagation constant along dipole,
- \( e \) = effective length of one dipole leg,
Figure C.1 Schematic diagrams showing the unbalanced mode currents only on dipole and feedline.
and $L_1$ and $L_2$ denote the lengths measured from point H along with legs in the directions as indicated in Figure C.1(b). Next, a point S on the feedline is chosen in such a way that the points S and R (reference point of the feedline array) are at the same distance away from the ground plane. We next represent the current on the feedline by:

$$I_{L}^{(3)}(y) = A + By + Cy^2 \quad 0 < y < L_u$$  \hspace{1cm} (C.3)

$$I_{R}^{(3)}(y) = D + Ey + Fy^2 \quad L_u < y < L_f$$  \hspace{1cm} (C.4)

where $A$, $B$, $C$, $D$, $E$ and $F$ are the parameters to be determined. These parameters can be obtained by satisfying the boundary conditions for the unbalanced mode currents at points G, S and H(H') simultaneously. Note that the coordinate origin is chosen at the ground plane and $L_f$ is the length of the feedline.

A. Boundary Condition at Point G

Since the feedline is connected to the ground plane (at point G), the derivative of $I_{L}^{(3)}(y)$ at $y = 0$ should be zero, i.e.

$$\left. \frac{d}{dy} I_{L}^{(3)}(y) \right|_{y=0} = 0$$  \hspace{1cm} (C.5)
This will lead to

\[ B = 0 \]  \hspace{1cm} (C.6)

Thus, \( I^{(3)}_L(y) \) becomes

\[ I^{(3)}_L(y) = A + Cy^2 \]  \hspace{1cm} (C.7)

B. Boundary Condition at Point S

Now consider the current at point S, where \( y = z_u \). Due to the continuity property of the current, we know

\[ I^{(3)}_L(z_u) = I^{(3)}_R(z_u) \]  \hspace{1cm} (C.8)

Since no voltage is applied at point S, the derivatives of \( I^{(3)}_L(y) \) and \( I^{(3)}_R(y) \) should be equal at \( y = z_u \)

\[ \left. \frac{d}{dy} I^{(3)}_L(y) \right|_{y=z_u} = \left. \frac{d}{dy} I^{(3)}_R(y) \right|_{y=z_u} \]  \hspace{1cm} (C.9)

Inserting Equations (C.4) and (C.7) into Equations (C.8) and (C.9), we have

\[ A + Cz_u^2 - D - Ez_u - Fz_u^2 = 0 \]  \hspace{1cm} (C.10)

and

\[ 2Cz_u - E - 2Fz_u = 0 \]  \hspace{1cm} (C.11)
C. Boundary Condition at Point H(H')

Applying Kirchoff's current law at point H (where \( y = \xi_f, \xi_1 = 0 \) and \( \xi_2 = 0 \)), see Figure C.1(b), we get

\[
I_R^{(3)}(\xi_f) = I^{(1)}(0) + I^{(2)}(0)
\]  
(C.12)

Now, referring to Figure C.1(a), since there is no unbalanced voltage source connected at points H and H'

\[
\frac{d}{dy} I_R^{(3)}(y) \bigg|_{y = \xi_f} = \frac{d}{d\xi_1} I^{(1)}(\xi_1) \bigg|_{\xi_1=0} = \frac{d}{d\xi_2} I^{(2)}(\xi_2) \bigg|_{\xi_2=0}
\]  
(C.13)

Substitution of Equations (C.4) and (C.7) into Equations (C.12) and (C.13) results in

\[
D + E\xi_f + F\xi_f^2 = 2I_{ud}\sin(\theta_d\xi_e)
\]  
(C.14)

and

\[
E + 2\xi_u F = -2\beta_d I_{ud}\cos(\theta_d\xi_e)
\]  
(C.15)

Finally, we use \( I_{uf} \) to denote the unbalanced mode current at \( y = \xi_u \)

\[
I_{uf} = A + C\xi_u^2
\]  
(C.16)
Solving Equations (C.10), (C.11), (C.14), (C.15) and (C.16) simultaneously, we have

\[ A = I_{uf} \left( \frac{\ell_f}{\ell_f - \ell_u} \right) - I_{ud} \left[ -\frac{2\ell_u}{\ell_f - \ell_u} \sin(\beta_d\ell_e) + \ell_u \beta_d \cos(\beta_d\ell_e) \right], \]  

(C.17)

\[ C = I_{uf} \left[ \frac{-1}{\ell_u(\ell_f - \ell_u)} \right] + I_{ud} \left[ -\frac{2}{\ell_u(\ell_f - \ell_u)} \sin(\beta_d\ell_e) + \frac{\beta_d}{\ell_u} \cos(\beta_d\ell_e) \right], \]  

(C.18)

\[ D = I_{uf} \left( \frac{\ell_f}{\ell_f - \ell_u} \right)^2 - I_{ud} \left\{ 2 \left[ -\frac{\ell_f}{\ell_f - \ell_u} \right]^2 - 1 \right\} \sin(\beta_d\ell_e) + \frac{2\ell_f \ell_u \beta_d}{\ell_f - \ell_u} \cos(\beta_d\ell_e), \]  

(C.19)

\[ E = I_{uf} \left[ -\frac{-2\ell_f}{(\ell_f - \ell_u)^2} \right] + I_{ud} \left[ -\frac{4\ell_f}{(\ell_f - \ell_u)^2} \sin(\beta_d\ell_e) + 2 \left( \frac{\ell_f + \ell_u}{\ell_f - \ell_u} \right) \beta_d \cos(\beta_d\ell_e) \right]. \]  

(C.20)
and

\[
F = I_{uf} \left[ -\frac{1}{(z_f - z_u)^2} \right] - I_{ud} \left[ -\frac{2}{(z_f - z_u)^2} \sin(\theta_{de}) \right] + \frac{2\theta_d}{z_f - z_u} \cos(\theta_{de}) \right]
\] 

(C.21)

Note that the parameters given in the above are proportional to the unknowns $I_{ud}$ and $I_{uf}$.

Here, we assume that the ratio of $I_{ud}$ and $I_{uf}$ is known and defined as

\[
\xi = \frac{I_{ud}}{I_{uf}}
\] 

(C.22)

Applying Equation (C.22) and making the following changes in notation

\[
\begin{align*}
A & \rightarrow U_a \\
C & \rightarrow U_b \\
D & \rightarrow U_c \\
E & \rightarrow U_d \\
F & \rightarrow U_e
\end{align*}
\]

we have

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\[ U_a = I_u \left[ \frac{\xi_f}{\xi_f - \xi_u} \left( \frac{2\xi_u}{\xi_f - \xi_u} \right) \xi\sin(\beta_d \xi_e) - \xi_u \beta_d \xi\cos(\beta_d \xi_e) \right] \] (C.23)

\[ U_b = I_u \left[ -\frac{1}{\xi_u (\xi_f - \xi_u)} + \frac{2}{\xi_u (\xi_f - \xi_u)} \xi\sin(\beta_d \xi_e) + \frac{\beta_d}{\xi_u} \xi\cos(\beta_d \xi_e) \right] \] (C.24)

\[ U_c = I_u \left[ -\frac{\xi_f}{\xi_f - \xi_u} \right]^2 - 2 \left[ \left( \frac{\xi_f}{\xi_f - \xi_u} \right)^2 - 1 \right] \xi\sin(\beta_d \xi_e) \left( \frac{2\xi_f \beta_d}{\xi_f - \xi_u} \right) \xi\cos(\beta_d \xi_e) \] (C.25)

\[ U_d = I_u \left[ -\frac{2\xi_f}{(\xi_f - \xi_u)^2} + \frac{4\xi_f}{(\xi_f - \xi_u)^2} \xi\sin(\beta_d \xi_e) \right] + 2\left( \frac{\beta_d}{\xi_f - \xi_u} \right) \xi\cos(\beta_d \xi_e) \] (C.26)
Now, all the constants presented in the above is proportional to one unknown \( I_{uf} \) (the current at point S) defined by Equation (C.16).

Using the parameters given by Equations (C.23) through (C.27), the unbalanced mode current along the feedline can be represented by

\[
I^{(3)}(\xi) = \begin{cases} 
I_L^{(3)}(\xi) = U_a(\xi + \xi_u) + U_b(\xi + \xi_u)^2 & -\xi_u < \xi < 0 \\
I_R^{(3)}(\xi) = U_c(\xi + \xi_u) + U_d(\xi + \xi_u) + U_e(\xi + \xi_u)^2 & 0 < \xi < \xi_f - \xi_u 
\end{cases} 
\]

where \( \xi \) denotes the length measured from point S in the y-direction.

For illustration purposes, two sets of current distributions are simulated using Equations (C.28) and (C.29) and presented in Figure C.2. Note that all the current distributions shown in this figure are normalized to \( I_{uf} \), the current at \( \xi = 0 \) (i.e., point S).
Figure C.2 Simulation of unbalanced mode current along feedline.
The pattern factor is the pattern of the radiated field from an element, obtained by integrating the field expression over the whole domain of the source points. The purpose of this appendix is to evaluate the pattern factors for the \( p(i) \)-directed leg of a dipole and a \( y \)-directed feedline, with given current distributions, in a homogeneous medium.

A. The Pattern Factor of the \( \hat{p}(i) \)-directed Leg of a Dipole

When dealing with a V-shaped dipole, it is convenient to treat its two differently oriented legs as two independent elements and consider their effects separately. The pattern factor of the \( \hat{p}(i) \)-directed leg of a dipole is given by

\[
p_{m \pm}^{(i)} = \frac{1}{I(i)(0)} \int_{0}^{\beta_d} I(i)(\xi) e^{j \beta \hat{p}(i) \cdot \hat{r}_{m \pm} \xi} \, d\xi
\]

\[i = 1, 2\]

(D.1)

where \( I(i)(\xi) \) is the current distribution along the \( \hat{p}(i) \)-directed leg, \( I(i)(0) \) denotes the current at the reference terminal located at the feed point of the dipole (see Figure 2.17), \( \beta_d \) is the physical length of
the leg. The current $I^{(i)}(\xi)$ is a sinusoidal distribution

$$I^{(i)}(\xi) = I_m \sin \beta_d (\xi_e - \xi)$$

$i = 1, 2$

where

$\beta_d = \text{effective propagation constant along the leg},$

$\xi_e = \text{effective length of the leg}.$

Inserting Equation (D.2) into Equation (D.1), we have

$$p^{(i)}(m) = \frac{1}{\sin(\beta_d \xi_e)} \int_0^{\xi_d} \sin\beta_d (\xi_e - \xi) e^{j\beta_d (m) \xi} \hat{r}^{(i)}_{m=\xi} d\xi$$

$i = 1, 2$

(D.3)

It is obvious that Equation (D.3) is independent of $I_m$ (magnitude of the current on the $p^i$-directed leg).
From the integration table, we can have

\[
\int_a^b \sin(b \theta - \xi) e^{-jX \xi} d\xi = \left\{ \frac{1}{2} \left[ \frac{e^{-j\beta_d(\xi_e - \xi)}}{\beta_d - X} + \frac{e^{-j\beta_d(\xi_e - \xi)}}{\beta_d + X} \right] \right\}_{a}^{b} e^{jX \xi}
\]

Making use of Equation (D.4), with \(a = 0, b = \xi_d\) and \(X = \beta_d \hat{p}(i) \hat{r}_{m\pm}\), Equation (D.3) becomes

\[
p^{(i)}_{m\pm} = \frac{1}{2} \left[ \frac{e^{-j\beta_d(\xi_e - \xi_d)}}{\beta_d - \omega_{\pm}^{(i)}} + \frac{e^{-j\beta_d(\xi_e - \xi_d)}}{\beta_d + \omega_{\pm}^{(i)}} \right] e^{j\omega_{\pm}^{(i)} \xi_d}
\]

\[
- \frac{1}{2} \left[ \frac{e^{-j\beta_d \xi_e}}{\beta_d - \omega_{\pm}^{(i)}} + \frac{e^{-j\beta_d \xi_e}}{\beta_d + \omega_{\pm}^{(i)}} \right]
\]

\[
i = 1, 2
\]

where

\[
\omega_{\pm}^{(i)} = \beta_d \hat{p}(i) \hat{r}_{m\pm}
\]

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B. The Pattern Factor of the y-directed Feedline

The pattern factor of the y-directed feedline is given by

\[ p_m(3) = \frac{1}{I(3)(0)} \left[ \int_{-z_u}^{z_f-z_u} I(3)(z) e^{j \beta_d z} \cdot r_{m \pm z} d\zeta \right] \]  

(D.7)

where \( I(3)(z) \) is the current distribution along the feedline, \( z \) denotes the length measured from the reference point in the y-direction (see Figure 2.16), \( z_u \) indicates the distance between the reference point and the ground plane, \( z_f \) is the length of the feedline. Note that \( \hat{p}(3) \) denotes orientation of the feedline, i.e. \( \hat{p}(3) = \hat{y} \).

Since the current distributions on the left and right of the reference point are represented by two different functions (see details given in Appendix C), Equation (D.7) may be written as

\[ p_m(3) = \frac{1}{I(3)(0)} \left[ \int_{-z_u}^{0} I_L(3)(z) e^{j \beta_d z} \cdot r_{m \pm z} d\zeta \right] \]

\[ + \int_{0}^{z_f-z_u} I_R(3)(z) e^{j \beta_d z} \cdot r_{m \pm z} d\zeta \]  

(D.8)
where

\[ I^{(3)}_L (\ell) = U_a + U_b (\ell + \ell_u)^2 \quad -\ell_u < \ell < 0 \]  
(C.28)

and

\[ I^{(3)}_R (\ell) = U_c (\ell + \ell_u) + U_d (\ell + \ell_u) + U_e (\ell + \ell_u)^2 \]
\[ 0 < \ell < \ell_f - \ell_u \]  
(C.29)

The parameters \( U_a, U_b, U_c, U_d \) and \( U_e \) are given by Equations (C.23), (C.24), (C.25), (C.26) and (C.27), respectively. As noticed in Appendix C, those parameters are proportional to one unknown \( I_{uf} \) (i.e. \( I^{(3)}(0) \)), the current at the reference point.

Carrying out the two integrals in Equation (D.8), we get

\[
\rho^{(3)}_{m\pm} = \frac{1}{I^{(3)}(0)} \left\{ \frac{1}{j\omega_{\pm}} \right\} \left[ \begin{array}{c}
- \frac{U_a + U_b \ell_u^2}{j\omega_{\pm}} - \frac{2U_b \ell_u}{(\omega_{\pm})^2} \\
- \frac{2U_b}{(\omega_{\pm})^2}
\end{array} \right]
\]

\[
- \frac{-j\omega_{\pm} \ell_u}{j\omega_{\pm}} \left[ \begin{array}{c}
U_a - \frac{2U_b}{(\omega_{\pm})^2}
\end{array} \right]
\]
where

\[ \omega_{\pm}^{(3)} = \beta_d \hat{\varphi}_{\pm}^{(3)} \cdot \hat{r}_{m \pm} \]  \hspace{5cm} (D.10)

From the current boundary conditions given in Equations (C.8) and (C.9), we know

\[ U_a + U_b \hat{\varphi}_a^2 = U_c + U_d \hat{\varphi}_d^2 + U_e \hat{\varphi}_e^2 \]  \hspace{5cm} (D.11)

and

\[ 2U_b \hat{\varphi}_b = U_d + 2U_e \hat{\varphi}_e \]  \hspace{5cm} (D.12)
Making use of Equations (D.11) and (D.12), Equation (D.9) reduces to

\[
P_{m \pm}^{(3)} = \frac{1}{I^{(3)}(0)} \left( \frac{1}{j\omega^{(3)}} \right) \left\{ \frac{2(U_b - U_e)}{(\omega^{(3)})^2} - \right.
\]
\[-e^{-j\omega^{(3)} x_u} \left[ U_a - \frac{2U_b}{(\omega^{(3)})^2} \right]
\]
\[+ e^{j\omega^{(3)} (\delta_f - x_u)} \left[ U_c + U_d \delta_f + U_e \delta_f^2 - \frac{U_d + 2U_e \delta_f}{j\omega^{(3)}} \right]
\]
\[- \frac{2U_e}{(\omega^{(3)})^2} \right\}
\]

(D.13)

Since \( U_a, U_b, U_c, U_d \) and \( U_e \) are proportional to \( I^{(3)}(0) \), Equation (D.13) is independent of \( I^{(3)}(0) \). In other words, the pattern factors \( P_{m \pm}^{(3)} \) depend only on the shape of the current distribution on the feedline.
APPENDIX E

DIRECT MODE FIELD RADIATED FROM DIPOLE ARRAY IN REGION II_d

The Region II_d of a V-shaped dipole array was defined in Chapter II. The direct mode field radiated from the dipole array in Region II_d will be derived in this appendix.

Consider a reference dipole and a field point R located in Region II_d as illustrated in Figure 2.18. In Region II_d, the direct mode field \( \hat{X}_{ii} \) radiated from the \( p \)-directed legs of the dipole array due to the balanced or unbalanced mode current is given by Equation (2.70), i.e.

\[
\hat{E}^{(i)}_{\text{D2}}(R) = \frac{I^{(i)}_{Z_m}}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left[ \begin{array}{l}
e^{-j\beta_m (R-R') \cdot \hat{r}_m} \cdot e^{-(i)_{m-}} \int_{\xi=0}^{\xi=\xi'} \sin\beta_d (\xi_{e'-\xi'}) e^{j\beta_{mD} (i)_{m-e'}} d\xi' \\
+ e^{-j\beta_m (R-R') \cdot \hat{r}_m} \cdot e^{-(i)_{m+}} \int_{\xi=0}^{\xi=\xi'} \sin\beta_d (\xi_{e'-\xi'}) e^{j\beta_{mD} (i)_{m+e'}} d\xi'
\end{array} \right],
\]

\[i = 1, 2 \quad (2.70)\]
where
\[ i^{(i)M} = \begin{cases} \frac{(-1)^{i+1}i_{bd}}{M} & \text{when } M = b \\ i_{ud} & \text{when } M = u \end{cases}, \]  
(2.54)

\[ \varepsilon^{(i)}_{m\pm} = (p^{(i)} \times \hat{r}_{m\pm}) \times \hat{r}_{m\pm} \] 
(E.1)

and
\[ \ell_0 = \frac{y \cdot (R - R(d))}{p^{(i)}} y. \]  
(2.72)

The superscript M denotes the current mode on the p^\text{i} directed legs which generates the field. In Equation (2.54), "b" refers to the balanced mode current, whereas "u" refers to the unbalanced mode current.

Using Equation (D.4), the two integrals in Equation (2.70) can be evaluated

\[ \int_{\ell_0} \sin \beta_m (\varepsilon_{e-\ell}) e^{j \beta_m \hat{r}_{m\pm} \ell' d\ell}, \]

\( \ell_0 \)

\[ = \frac{1}{2} \left[ \frac{e^{-j \beta_d (\varepsilon_{e-\ell_0})}}{\beta_d - \omega} + \frac{e^{j \beta_d (\varepsilon_{e-\ell_0})}}{\beta_d + \omega} \right] j \omega (i) \ell_0. \]
\[ -\frac{1}{2} \left[ \frac{e^{j\beta_d \xi_e}}{\beta_d - \omega_+^{(i)}} + \frac{e^{-j\beta_d \xi_e}}{\beta_d + \omega_+^{(i)}} \right] \]

and

\[ \int_{\lambda_0}^{\lambda_d} \sin \beta_d (\xi_e - \xi') e^{j\beta_m P \cdot r_{m+1}} d\xi' \]

\[ \frac{1}{2} \left[ \frac{e^{j\beta_d (\xi_e - \xi_0)}}{\beta_d - \omega_+^{(i)}} + \frac{e^{-j\beta_d (\xi_e - \xi_0)}}{\beta_d + \omega_+^{(i)}} \right] e^{j\omega_+^{(i)} \xi_d} \]

\[ -\frac{1}{2} \left[ \frac{e^{-j\beta_d (\xi_e - \xi_0)}}{\beta_d - \omega_+^{(i)}} + \frac{e^{j\beta_d (\xi_e - \xi_0)}}{\beta_d + \omega_+^{(i)}} \right] \]

where

\[ \omega_+^{(i)} = \beta_m P \cdot r_{m+1} \]

Thus, \( E_{d2} (R) \) given in Equation (2.70) can be written as

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\[
E_{D2}^{(i)}(R) = \frac{I_{D2}^{(i)}}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{e^{-j\beta_m (R-R^m)^{\cdot} - (d)^{\cdot} r_m^{\cdot}}}{r_{m+}} e^{(i)} c_{m+} \right\}
\]

\[
e^{-j\beta_m (R-R^m)^{\cdot} - (d)^{\cdot} r_m^{\cdot}} i^{(i)} (i) + \frac{e^{(i)} m-}{r_{m+}}
\]

\[
e^{-j\beta_d (d_e-d_0)} \left[ \frac{e^{(i)} m-}{\beta_d - \omega(i)} + \frac{e^{-j\beta_d (d_e-d_0)}}{\beta_d + \omega(i)} \right] e^{-j\beta_m k_{d-}^{(i)}}
\]

\[
e^{(i)} m+ \frac{e^{(i)} m+}{r_{m+}} \left[ \frac{e^{(i)} m+}{\beta_d - \omega(i)} + \frac{e^{-j\beta_d (d_e-d_0)}}{\beta_d + \omega(i)} \right] e^{-j\beta_m k_{d+}^{(i)}}
\]

\[
i = 1, 2
\]

(E.5)

where

\[
C_{d+}^{(i)} = \frac{1}{2} \left[ \frac{e^{(i)} m+ d_e-d_0}{\beta_d - \omega(i)} + \frac{e^{-j\beta_d (d_e-d_0)}}{\beta_d + \omega(i)} \right] e^{j\omega(i) z_d}
\]

(E.6)
Here, we will consider the factors $K_{d+}$ and $K_{d-}$ defined above. Substituting Equation (2.72) into Equations (E.8) and (E.9) and doing some manipulation leads to

$$K^{(i)}_{d+} = (R - R^{(d)}) \cdot r_{m+} - \frac{\omega^{(i)}}{\beta_{m}} z_{o}$$  \hspace{1cm} (E.8)

$$K^{(i)}_{d-} = (R - R^{(d)}) \cdot r_{m-} - \frac{\omega^{(i)}}{\beta_{m}} z_{o}$$  \hspace{1cm} (E.9)

$$K^{(i)}_{d+} = K^{(i)}_{d-}$$

$$= (R - R^{(d)}) \cdot [r_{mx} \hat{x} - \left( \frac{p^{(i)}_{y}}{z_{y}} \right) r_{my} \hat{y} + r_{mz} \hat{z}]$$  \hspace{1cm} (E.10)
Due to the fact of Equation (E.10), Equation (E.5) can be expressed as

\[
E^{(i)M}_{D2}(R) = \frac{1^{(i)M}z_m^{(i)M}}{2D_xD_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{e^{-j\beta_m(R-R_{(d)})r_m}e^{-j\beta_n(R-R_{(d)})r_n}}{r_{my}} e^{-(i)p(i)} \right\}
\]

\[
+ \frac{e^{-j\beta_m(R-R_{(d)})r_m}}{r_{my}} e^{-(i)p(i)}
\]

\[
+ \frac{e^{-j\beta_m(R-R_{(d)})r_o}}{2r_{my}} \left[ e^{j\beta_d(z_e-z_0)} \left[ \frac{e^{-(i)}}{e^{m-}} - \frac{e^{-(i)}}{e^{m+}} \right] \right]
\]

\[
+ e^{-j\beta_d(z_e-z_0)} \left[ \left[ \frac{e^{-(i)}}{e^{m-}} - \frac{e^{-(i)}}{e^{m+}} \right] \right]
\]

\[
i = 1, 2
\]

(E.11)
where

\[
\mathbf{r}_0 = \mathbf{r}_{mx} \times \left( \frac{\mathbf{p}(i)}{\mathbf{p}_y} \right) \mathbf{r}_{mz} y + \mathbf{r}_{mz} z
\]  

(E.12)

Making use of Equation (A.14), \( e_{m+} \) defined by Equation (E.1) may be written as

\[
e_{m+} = (\mathbf{p} \cdot \mathbf{r}_{m+})\mathbf{r}_{m+} - \mathbf{p}
\]  

(E.13)

or expressed explicitly as

\[
e_{m+} = (\mathbf{p} \cdot \mathbf{r}_{m+})\mathbf{r}_{mx} x + (\mathbf{p} \cdot \mathbf{r}_{m+})\mathbf{r}_{my} y + (\mathbf{p} \cdot \mathbf{r}_{m+})\mathbf{r}_{mz} z - \mathbf{p}
\]  

(E.14)

Rearranging the y-component term, the wave vector \( e_{m+} \) can be represented by

\[
e_{m+} = \left( \frac{\mathbf{r}_{my}}{\mathbf{p}_m} \right) \left( \beta_d - \omega(i) \right) y
\]  

(E.15)

or

\[
e_{m+} = \left( \frac{\mathbf{r}_{my}}{\mathbf{p}_m} \right) \left( \beta_d + \omega(i) \right) y
\]  

(E.16)
where

\[ g_{m^+} = (\hat{p} - r_{m^+}) r_{mx} \hat{x} + \left( \frac{\beta_d}{\beta_m} \right) r_{my} \hat{y} + (\hat{p} - r_{m^+}) r_{mz} \hat{z} - p \]  
(E.17)

\[ h_{m^+} = (\hat{p} - r_{m^+}) r_{mx} \hat{x} - \left( \frac{\beta_d}{\beta_m} \right) r_{my} \hat{y} + (\hat{p} - r_{m^+}) r_{mz} \hat{z} - p \]  
(E.18)

Similarly, we may express \( e_{m^-} \) as

\[ e_{m^-} = g_{m^-} + \left( \frac{\beta_d}{\beta_m} \right) (\hat{p} - \omega_{m^-}) \hat{y} \]  
(E.19)

or

\[ e_{m^-} = h_{m^-} - \left( \frac{\beta_d}{\beta_m} \right) (\hat{p} + \omega_{m^-}) \hat{y} \]  
(E.20)

where

\[ g_{m^-} = (\hat{p} - r_{m^-}) r_{mx} \hat{x} - \left( \frac{\beta_d}{\beta_m} \right) r_{my} \hat{y} + (\hat{p} - r_{m^-}) r_{mz} \hat{z} - p \]  
(E.21)

\[ h_{m^-} = (\hat{p} - r_{m^-}) r_{mx} \hat{x} + \left( \frac{\beta_d}{\beta_m} \right) r_{my} \hat{y} + (\hat{p} - r_{m^-}) r_{mz} \hat{z} - p \]  
(E.22)
Substitution of Equations (E.15), (E.16), (E.19) and (E.20) into Equation (E.11) results in

\[ E_D^{(i)}(R) = \frac{I^{(i)}M^m_z}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ e^{-j \beta_m (R-R^d) \cdot r_m+} e^{-\gamma^{(i)}(i)} \right\} \]

\[ + \frac{e^{-j \beta_m (R-R^d) \cdot r_m+} e^{-\gamma^{(i)}(i)}}{r_{my}} \cdot \frac{e^{-j \beta_m (R-R^d) \cdot r_m-} e^{-\gamma^{(i)}(i)}}{r_{my}} \]

\[ + \frac{e^{-j \beta_m (R-R^d) \cdot r_0}}{2r_{my}} \left[ e^{j \beta_d (z_e-z_0)} \left\{ e^{-\gamma^{(i)}(i)} \frac{g_m-}{\beta_d - \omega_-} - \frac{g_m+}{\beta_d - \omega_+} \right\} \right] \]

\[ + e^{-j \beta_d (z_e-z_0)} \left[ \frac{h^{(i)} m-}{\beta_d + \omega_-} - \frac{h^{(i)} m+}{\beta_d + \omega_+} \right] \]

\[ + \frac{I^{(i)}M^m_z}{2D_x D_z} \Delta_d \ y \]

\[ i = 1, 2 \]

(E.23)
where
\[ \Delta_d = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m(R-R') \cdot [r_{mx} \hat{x} + \frac{p_z(i)}{p_y} r_{mz} \hat{y} + r_{mz} \hat{z}]} \] (E.24)

and
\[ \chi_d = \frac{2j}{\beta_m} \sin \beta_d (z_e - z_0) \] (E.25)

Employing Equations (2.4) and (2.5) and defining two constants
\[ \omega_x = \frac{2\pi}{D_x} \] (E.26)

and
\[ \omega_z = \frac{2\pi}{D_z} \] (E.27)

Equation (E.24) can be written as
\[ \Delta_d = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\omega_x}^k d^x \hat{x} e^{-j\omega_z}^n \] (E.28)

where
\[ \Delta_d = \chi_d e^{-j\beta_m(R-R')} \cdot [s_{mx} \hat{x} - \frac{p_z(i)}{p_y} s_{mz} \hat{y} + s_{mz} \hat{z}] \] (E.29)
\[ \Delta x = x - x^{(d)} \] (E.30)

and

\[ \Delta z = (z - z^{(d)}) - (y - y^{(d)}) \frac{p_z^{(i)}}{p_y^{(i)}} - (x - x^{(d)}) \frac{\Delta z}{\Delta x} \] (E.31)

Using Poisson's Sum Formula [7], Equation (E.28) can be transformed to

\[ \Delta = \Delta_0 \sum_{k=-\infty}^{\infty} \delta(dx + kDx) \sum_{n=-\infty}^{\infty} \delta(dz + nDz) \] (E.32)

It is obvious that:

(i) if \( dx \neq kDx \) for all \( k \)

\[ \sum_{k=-\infty}^{\infty} \delta(dx + kDx) = 0 \] (E.33)

and

(ii) if \( dz \neq nDz \) for all \( n \)

\[ \sum_{n=-\infty}^{\infty} \delta(dz + nDz) = 0 \] (E.34)
Therefore, if the field point is not located at the array element (i.e. \( \Delta x_i \neq k \Delta x \) and \( \Delta z_i \neq n \Delta z \) for all \( k \) and \( n \)), \( \Delta_d^{(i)} \) given by Equation (E.32) will vanish. The purpose of eliminating \( \Delta_d^{(i)} \) from the field expression is to improve the converging series for evaluating the coupling when the test element is situated in Region II_d.
APPENDIX F

DIRECT MODE FIELD RADIATED FROM FEEDLINE ARRAY IN REGION II_{f}

When considering the direct mode field radiated from the feedline array in Region II_{f}, the field point R located in the two domains shown in Figure 2.20 should be considered separately. The feedline is \( z_{f} \) long, its reference point is chosen at \( R' \), where \( y \cdot R' = \varepsilon_{u} \).

\[ 0 < y \cdot R < z_{u} \]

In this region, the direct mode field radiated from the feedline array is given by Equation (2.88). Carrying out the integrals in Equation (2.88), we obtain

\[
\begin{align*}
E^{(3)}(R) &= \frac{Z_{m}}{2D_{x}D_{z}} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ e^{-j\beta_{m}(R-R') \cdot r_{m}^{\perp}} \frac{E_{c}^{(3)}(3)}{r_{my}} + \frac{e^{-j\beta_{m}(R-R') \cdot r_{m+}^{\perp}} E_{c}^{(3)}(3)}{r_{my}} \right\} \\
&+ \frac{1}{r_{my}} \left[ U_{a} + U_{b}(\varepsilon' + \varepsilon_{u})^{2} \right] + \frac{2U_{b}}{(\varepsilon_{+}^{(3)})^{2}} (\varepsilon' + \varepsilon_{u})
\end{align*}
\]
\[
\begin{align*}
&- \frac{2U_b}{j(\omega_+)^3} \left[ e^{-j\beta_m L_+} \right] \\
&- \frac{e^{-m-}}{r_{my}} \left[ -\frac{1}{j\omega_-} \left[ U_a + U_b (s_0^i + s_u^i)^2 \right] + \frac{2U_b}{(\omega_-)^2} (s_0^i + s_u^i) \right]
\end{align*}
\]

where
\[
\omega_+ = \hat{s}^{(3)}_{m+} \cdot \hat{r}_m^+ 
\]

\[
G^{(3)}_{L+} = (R-R \cdot) \cdot \hat{r}_{m+} - \frac{\omega_+}{\beta_m} \hat{s}_0^i 
\]

\[
G^{(3)}_{L-} = (R-R \cdot) \cdot \hat{r}_{m-} - \frac{\omega_-}{\beta_m} \hat{s}_0^i 
\]

\[
C^{(3)}_{L+} = \frac{e^{-j\omega_+ s_u^i}}{j\omega_+} \left[ U_a - \frac{2U_b}{(\omega_+)^2} \right] 
\]

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\[ c_{L-}^{(3)} = \frac{1}{j\omega_-} (u_a + u_b z_f^2) + \frac{2u_b z_f}{(\omega_-)^2} - \frac{2U_b}{j(\omega_-)^3} \]

\[ + \left[ \frac{1}{j\omega_-} (u_c + u_d z_f + u_e z_f^2) + \frac{1}{(\omega_-)^2} (u_d + 2u_e z_f) \right] \]

\[ \frac{2U_e}{j(\omega_-)^3} \]

\[ - \frac{j\omega_- (z_f - z_d)}{j(\omega_-)^3} \]

\[ + \left[ \frac{1}{j\omega_-} (u_c + u_d z_u + u_e z_u^2) - \frac{1}{(\omega_-)^2} (u_d + 2u_e z_u) \right] \]

\[ \frac{2U_e}{j(\omega_-)^3} \]  \hspace{1cm} (F.6)

and \( \xi_0 \) is given by Equation (2.90). Since \( \hat{p}^{(3)} = \hat{y} \) and the field point \( \vec{R} \) and the reference point \( \vec{R}^{(f)} \) are

\[ \vec{R} = x \hat{x} + y \hat{y} + z \hat{z} \]  \hspace{1cm} (F.7)

and

\[ \vec{R}^{(f)} = x^{(f)} \hat{x} + y^{(f)} \hat{y} + z^{(f)} \hat{z} \]  \hspace{1cm} (F.8)

\( \xi_0 \) can be expressed as

\[ \xi_0 = y - y^{(f)} \]  \hspace{1cm} (F.9)
Note that \( y'(f) = z_u \) (see Figure 2.20), thus, \( z_o \) may also be written in an alternate form as

\[
z_o = y - z_u \tag{F.10}
\]

Employing Equations (D.11) and (D.12), \( c_L \) given in Equation (F.6) can be simplified to

\[
c_L^{(3)} = \left[ \frac{1}{j\omega_\infty^{(3)}} \right] \left( U_c + U_d z_f + U_e z_f^2 \right) + \frac{1}{(\omega_\infty^{(3)})^2} \left( U_d + 2U_e z_f \right)
\]

\[
- \frac{2U_e}{j(\omega_\infty^{(3)})^3} \left( z_f - z_u \right) + \frac{2}{j(\omega_\infty^{(3)})^3} \left( U_e - U_b \right) \tag{F.11}
\]

Now, we will consider the factors \( G_{L+}^{(3)} \) and \( G_{L-}^{(3)} \) defined by Equations (F.3) and (F.4), respectively. Substituting Equations (F.2), (F.7), (F.8) and (F.9) into Equations (F.3) and (F.4), we found that

\[
G_{L+}^{(3)} = G_{L-}^{(3)} = (x-x'(f))r_{mx} + (z-z'(f))r_{mz} \tag{F.12}
\]

Making use of the face given above, Equation (F.1) becomes

\[
E_{D2L}^{(3)}(R) = \frac{z_m}{2\omega_\infty^{(3)} d_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ \frac{-j \omega_m (R-R'(f)) \cdot r_{m-}^{(3)} e_{m-}^{(3)} c_{L-}^{(3)}}{r_{my}} \right. \\
\left. + \frac{-j \omega_m (R-R'(f)) \cdot r_{m+}^{(3)} e_{m+}^{(3)} c_{L+}^{(3)}}{r_{my}} \right\}
\]

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where

\[ \mathbf{r}_{ot} = r_{mx} \hat{x} + r_{mz} \hat{z} \]  \hspace{1cm} \text{(F.14)}

Note that the vector sum of \( \mathbf{e}_{m^+} \) and \( \mathbf{e}_{m^-} \) is

\[ \mathbf{e}_{m^+} + \mathbf{e}_{m^-} = -2(1-r_{my}^2) \hat{y} \]  \hspace{1cm} \text{(F.15)}

Therefore, using Equation (F.15), we may write Equation (F.11) in a slightly different form as

\[
\begin{align*}
E_{D2L}(R) &= \frac{Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ e^{-j \beta_m (R - R')} \frac{r_{m^-}}{r_{my}} e_{m^+} (3) \right. \\
&\quad + \frac{e^{-j \beta_m (R - R')} \frac{r_{m^+}}{r_{my}}}{r_{my}} e_{m^-} (3) \left. \right\} \\
&\quad e^{j \beta_m (R - R')} \left( e_{m^+} (3) + e_{m^-} (3) \right)
\end{align*}
\]
\[ e^{-j\beta_m (R-R(f)) r_{ot}} \frac{e^{-\epsilon (3)} + e^{\epsilon (3)}}{r_{my}} \left[ \frac{2 U_b}{(j\beta_m r_{my})^3} \right] \]

\[ + \frac{e^{-\epsilon (3)} - e^{\epsilon (3)}}{m^+ - m^-} \left( \frac{2 U_b y}{(j\beta_m r_{my})^2} \right) \left( U_a + U_b y^2 \right) \]  

\[ + \frac{Z_m}{2D_x D_z} \Delta_L^{(3)} y \]  

where

\[ \Delta_L^{(3)} = \chi_{fL} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m [(x - x(f)) r_{mx} + (z - z(f)) r_{mz}]} \]  

\[ \chi_{fL} = \frac{2}{j\beta_m} (U_a + U_b y^2) \]  

Applying Poisson's Sum Formulas [7] and following a similar procedure as described in Appendix E for the dipole array, \( \Delta_L^{(3)} \) can be transformed to

\[ \Delta_L^{(3)} = \Delta_L^{(3)} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(d x + k D_x) \sum_{n=-\infty}^{\infty} \delta(d z + n D_z) \]  

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where

\[ \Delta_{L0}^{(3)} = X_{fL} e^{-j \beta_m[(x - x(f))] \alpha_{mx} + (z - z(f)) \alpha_{mz}] } \]  \hspace{1cm} (F.20)

\[ \delta x = x - x(f) \]  \hspace{1cm} (F.21)

and

\[ \delta z = (z - z(f)) - (x - x(f)) \frac{\Delta z}{D_x} \]  \hspace{1cm} (F.22)

It is noticed that \( \Delta_{L0}^{(3)} \) will be zero when

(i). \( \delta x \neq kDx \) for all \( k \) \hspace{1cm} (F.23)

or/and

(ii). \( \delta z \neq nDz \) for all \( n \) \hspace{1cm} (F.24)

In other words, \( \Delta_{L0}^{(3)} \) will vanish as long as the field point is not located at the array elements (i.e., feedlines).

(B) \( \bar{\lambda}_u < \bar{y} < R < \bar{\lambda}_f \)

In this region, the direct mode field radiated by the feedline array is represented by Equation (2.89). Following a similar procedure given above, we have

\[ E_{D2R}^{(3)}(R) = \frac{Z_m}{2D_x D_z} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left\{ e^{-j \beta_m(R-R')} r_m e^{-j \frac{(f)}{(3)}} \right\} \]
\[
\frac{e^{-j\beta_m(R-R^{(f)})} \cdot r_{m+}}{r_{my}} e_{m+}^{-(3)} (3) + \frac{e^{-j\beta_m(R-R^{(f)})} \cdot r_{rot}}{r_{my}} e_{m+}^{-(3)} e_{m-}^{-(3)} \left[ \frac{1}{(j \beta_m r_{my})^3} (2U_e) \right]
\]

\[
\frac{e^{-3} e^{-3}}{m+} - \frac{e^{-3} e^{-3}}{m-} (U_d + 2U_e y)
\]

\[
\left[ \frac{2}{j \beta_m r_{my}} (U_c + U_d y + U_e y^2) \right]
\]

\[
\frac{Z_m}{2D_x D_z} \Delta_R^{(3)} y
\]

\text{(F.25)}

where

\[
C_{R+} = \frac{2}{j \omega^{(3)}_+} (U_e - U_b) - \frac{e^{-j\omega^{(3)}_+} \eta_u}{j \omega^{(3)}_+} \left[ U_a - \frac{2U_b}{(\omega^{(3)}_+)^2} \right]
\]

\text{(F.26)}

\[
C_{R-} = \left[ \frac{1}{j \omega^{(3)}_-} (U_c + U_d \eta_f + U_e \eta_f^2) + \frac{1}{(\omega^{(3)}_-)^2} (U_d + 2U_e \eta_f) \right]
\]

\[
\frac{2U_e}{j \omega^{(3)}_-} \left[ \frac{e^{-j\omega^{(3)}_-} (\eta_f - \eta_u)}{j \omega^{(3)}_-} \right]
\]

\text{(F.27)}
\[ \Delta_R^{(3)} = x_{fR} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{-j\beta_m[(x-x(f))r_{mx} + (z-z(f))r_{mz}]} \]  

(F.28)

\[ x_{fR} = \frac{2}{j \beta_m} \left( U_c + U_d y + U_e y^2 \right) \]  

(F.29)

Applying Poisson's Sum Formula [7], Equation (F.26) can be transformed to

\[ \Delta_R^{(3)} = \frac{\Delta_{R_0}^{(3)}}{r_0} \sum_{k=-\infty}^{\infty} \delta(\bar{d}x + kDx) \sum_{n=-\infty}^{\infty} \delta(\bar{d}z + nDz) \]  

(F.30)

where

\[ \Delta_{R_0}^{(3)} = x_{fR} e^{-j\beta_m[(x-x(f))s_{mx} + (z-z(f))s_{mz}]} \]  

(F.31)

\( \bar{d}x \) and \( \bar{d}z \) are defined by Equations (F.21) and (F.22), respectively.

Since the expression of Equation (F.28) is similar to that of Equation (F.17), when the field point \( R \) is not located at the feedlines, \( \Delta_R^{(3)} \) will also vanish.
APPENDIX G

LOCATIONS OF TEST ELEMENTS FOR EVALUATING $Z_{bb}$ AND $Z_{ff}$

It was mentioned in Chapter III that when we evaluate the self-impedance $Z_{bb}$ or $Z_{ff}$, an appropriate number of test elements should be placed at some discrete positions on the test contour to obtain an average coupling. Theoretically, more test elements will lead to better results. However, for the sake of computation economy, the number of test elements should be as few as possible. In this appendix, it will be demonstrated that when the positions of the test elements are correctly chosen, the same accurate result may be obtained by using a small number of test elements.

In the following investigation, we assume that every array is located in free space and backed by a perfectly conducting ground plane. Also, the variables given below will be held constant:

- $F = 7.0$ GHz, operation frequency
- $D_x = D_z = 2.0$ cm, element spacings
- $\Delta z = 0$ cm, offset
- $L_d = 0.98$ cm, length of one dipole leg
- $L_f = 1.0$ cm, length of feedline
- $r_d = 0.1$ cm, wire radius of dipole
- $r_f = 0.1$ cm, wire radius of feedline
- $\xi = 0.104$ current ratio defined in Equation (C.22)
The angle pair \( \alpha \) and \( \eta \), defined in Figure 4.2, will be used to specify the scan direction \( \hat{s}_{o+} \) of the arrays.

A. SELF-IMPEDANCE \( Z_{bb} \)

The self-impedance of a dipole array with one test dipole located at \( \phi_d \), denoted by \( Z_{bb}(\phi_d) \), is defined in Equation (3.27). The angle \( \phi_d \) is defined in Figure 3.4 for a straight dipole or in Figure 3.5 for a V-shaped dipole. The average self-impedance of the dipole array is defined as

\[
Z_{bb} = \frac{1}{N_{td}} \sum_{m=1}^{N_{td}} Z_{bb}(\phi_{dm}) \tag{G.1}
\]

where \( N_{td} \) is the number of test dipoles used.

Using Equations (3.27) and (G.1), a set of calculated results are obtained and presented in Tables G.1 through G.9 for a straight dipole array and in Tables G.10 through G.18 for a V-shaped dipole array with bend angle \( \theta_b = 70^\circ \).

In each table, we have listed the calculated \( Z_{bb}(\phi_d) \)'s for 24 test dipoles which are uniformly distributed on the test contour. Also shown are the average impedance \( Z_{bb} \) obtained by using two different schemes: (1) by averaging the \( Z_{bb}(\phi_d) \)'s of all 24 test dipoles, (2) by averaging the \( Z_{bb}(\phi_d) \)'s for the test dipoles located at the positions marked by "\( \Rightarrow \)" (i.e. at \( \phi_d = 45^\circ, 135^\circ, 225^\circ \) and \( 315^\circ \) when \( \alpha \neq 90^\circ \), and at \( \phi_d = 45^\circ \) and \( 135^\circ \) when \( \alpha = 90^\circ \), regardless of the shape of the dipoles).
The average impedance $Z_{bb}$ obtained by scheme (1) should be very close to the true self-impedance of the dipole array. Comparing the results obtained by schemes (1) and (2), we notice that their real parts are exactly the same and the imaginary parts are slightly different, with error less than 1.5% in most cases.

B. SELF-IMPEDANCE $Z_{ff}$

The test contour for a feedline is shown in Figure 3.8. The self-impedance of a feedline array with one test feedline located at $\phi_f$, denoted by $Z_{ff}(\phi_f)$, is given in Equation (3.63). The angle $\phi_f$ is defined in Figure 3.8. The average self-impedance $Z_{ff}$ of the feedline array is defined as

$$Z_{ff} = \frac{1}{N_{tf}} \sum_{m=1}^{N_{tf}} Z_{ff}(\phi_{fm})$$  \hspace{1cm} (G.2)

where $N_{tf}$ is the number of test feedlines used.

Using Equation (3.63), a set of calculated results are obtained and presented in Tables G.19 through G.27. Similarly, in each table, we have listed the calculated $Z_{ff}(\phi_f)$ for 24 test feedlines which are uniformly distributed on the test contour. The average of these 24 calculated results is shown at the end of each table. This average result should be close to the true self-impedance of the feedline array.

From these tables, it is interesting to notice that this average impedance can be approximately represented by the $Z_{ff}(\phi_f)$ evaluated at
the position marked by "=>" as shown in each table. This specific position $\phi_f$ can be determined by

$$\phi_f = \alpha + 90^\circ$$  \hfill (6.3)

where $\alpha$ is the scan plane of the array.
Table G.1 Calculated self-impedance $Z_{bb}^{\phi_d}$ for a dipole array with $\theta_b = 90^\circ$ when $\alpha = 0^\circ$ and $\eta = 15^\circ$.

<table>
<thead>
<tr>
<th>$\phi_d$ (DEG)</th>
<th>$Z_{bb}^{\phi_d}$ (OHM)</th>
<th>REAL(OHM)</th>
<th>IMAG(OHM)</th>
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<tbody>
<tr>
<td>0</td>
<td>0.200113E+03</td>
<td>0.647402E+02</td>
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</tr>
<tr>
<td>15</td>
<td>0.199083E+03</td>
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<td>0.198084E+03</td>
<td>0.639782E+02</td>
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</tr>
<tr>
<td>45</td>
<td>0.197142E+03</td>
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</tr>
<tr>
<td>105</td>
<td>0.193390E+03</td>
<td>0.624530E+02</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0.192360E+03</td>
<td>0.835108E+02</td>
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</tr>
<tr>
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<td>0.191526E+03</td>
<td>0.853440E+02</td>
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<td>0.874939E+02</td>
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<tr>
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<td>0.896835E+02</td>
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<td>180</td>
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<td>0.917140E+02</td>
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<td>195</td>
<td>0.192413E+03</td>
<td>0.934405E+02</td>
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<td>0.947972E+02</td>
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<td>0.957359E+02</td>
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<td>0.963366E+02</td>
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AVERAGE $Z_{bb}^{\phi_d}$

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<tr>
<th>NUMBER OF TEST DIPOLES</th>
<th>AVERAGE OF $Z_{bb}^{\phi_d}$</th>
<th>REAL(OHM)</th>
<th>IMAG(OHM)</th>
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<td>0.196679E+03</td>
<td>0.809903E+02</td>
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</table>
Table G.2 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta = 90^\circ$ when $\alpha = 0^\circ$ and $\eta = 40^\circ$.

<table>
<thead>
<tr>
<th>$\phi_d$ (DEG)</th>
<th>$Z_{bb}^{\phi_d}$ (OHM)</th>
<th>REA(OHM)</th>
<th>IMAG(OHM)</th>
</tr>
</thead>
<tbody>
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<td>0.217397E+03</td>
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<tr>
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<td>0.155150E+03</td>
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<td>0.214350E+03</td>
<td>0.149420E+03</td>
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<tr>
<td>45</td>
<td>0.212147E+03</td>
<td>0.147237E+03</td>
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<td>0.207199E+03</td>
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<td>0.205905E+03</td>
<td>0.146171E+03</td>
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<td>0.195845E+03</td>
<td>0.163914E+03</td>
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<td>0.195069E+03</td>
<td>0.162035E+03</td>
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<td>0.195055E+03</td>
<td>0.173025E+03</td>
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<td>195</td>
<td>0.195071E+03</td>
<td>0.177830E+03</td>
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<td>0.182460E+03</td>
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<td>0.202493E+03</td>
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<td>0.211727E+03</td>
<td>0.190792E+03</td>
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<td>345</td>
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AVERAGE OF $Z_{bb}$

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<th>NUMBER OF TEST DIOLES</th>
<th>AVERAGE OF $Z_{bb}$</th>
</tr>
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<tr>
<td>4</td>
<td>0.206422E+03</td>
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Table G.3 Calculated self-impedance $Z^{bb}$ for a dipole array with $\theta_b = 90^\circ$ when $\alpha = 0^\circ$ and $\eta = 60^\circ$.

<table>
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<th>$\phi_d$ (DEG)</th>
<th>$Z^{bb} (\phi_d)$</th>
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<th></th>
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</thead>
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<td>IMAG(OHM)</td>
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<td>0.261724E+03</td>
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</tr>
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<td>30</td>
<td>0.190477E+03</td>
<td>0.276553E+03</td>
<td></td>
</tr>
<tr>
<td>$\Rightarrow$ 45</td>
<td>0.189551E+03</td>
<td>0.272746E+03</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.187676E+03</td>
<td>0.270761E+03</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>0.185006E+03</td>
<td>0.271042E+03</td>
<td></td>
</tr>
<tr>
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<td>0.182310E+03</td>
<td>0.273830E+03</td>
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</tr>
<tr>
<td>105</td>
<td>0.177794E+03</td>
<td>0.270472E+03</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0.173563E+03</td>
<td>0.269473E+03</td>
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</tr>
<tr>
<td>$\Rightarrow$ 135</td>
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<tr>
<td>150</td>
<td>0.169500E+03</td>
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<td>210</td>
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<td>0.293763E+03</td>
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<td>$\Rightarrow$ 225</td>
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<td>240</td>
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<td>285</td>
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<tr>
<td>$\Rightarrow$ 300</td>
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<tr>
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<td>345</td>
<td>0.186537E+03</td>
<td>0.348111E+03</td>
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<tr>
<th>NUMBER OF TEST Dipoles</th>
<th>AVERAGE OF $Z^{bb}$</th>
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</thead>
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<td>REAL(OHM)</td>
<td>IMAG(OHM)</td>
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<td>0.268307E+03</td>
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<tr>
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<td>0.267313E+03</td>
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</table>
Table G.4 Calculated self-impedance $Z_{bb}^{\phi_d}$ for a dipole array with $\phi_b = 90^\circ$ when $\alpha = 45^\circ$ and $\eta = 15^\circ$.

<table>
<thead>
<tr>
<th>$\phi_d$ (DEG)</th>
<th>$Z_{bb}^{\phi_d}$ (PH)</th>
<th>REAL (OHM)</th>
<th>IMAG (OHM)</th>
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<tbody>
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<tr>
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<td>0.751072e+02</td>
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<td>0.691316e+02</td>
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<td>0.191740e+03</td>
<td>0.638492e+02</td>
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<tr>
<td>330</td>
<td>0.191469e+03</td>
<td>0.594552e+02</td>
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<tr>
<td>345</td>
<td>0.190940e+03</td>
<td>0.561621e+02</td>
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AVERAGE OF $Z_{bb}^{\phi_d}$

<table>
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<tr>
<th>NUMBER OF TEST DIPLOES</th>
<th>AVERAGE OF $Z_{bb}^{\phi_d}$ (PH)</th>
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Table G.5 Calculated self-impedance $Z_{bb}$ for a dipole array with $\phi = 90^\circ$
when $\alpha = 45^\circ$ and $\eta = 40^\circ$.

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<th>$\phi_d$ (DEG)</th>
<th>$Z_{bb} (\phi_d)$ REAL(OHM)</th>
<th>IMAG(OHM)</th>
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<tr>
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<td>0.682971E+02</td>
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<th>IMAG(OHM)</th>
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Table G.6 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta_b = 90^\circ$ when $\alpha = 45^\circ$ and $\eta = 60^\circ$.

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<th>IMAG (OHM)</th>
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<th>IMAG (OHM)</th>
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Table G.7 Calculated self-impedance $\bar{Z}_{bb}$ for a dipole array with $\theta_b = 90^\circ$
when $\alpha = 90^\circ$ and $\eta = 15^\circ$.

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<th>IMAG (OHM)</th>
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**AVERAGE OF $ar{Z}_{bb}$**

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<th>IMAG (OHM)</th>
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Table G.8 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta = 90^\circ$ when $\alpha = 90^\circ$ and $\eta = 40^\circ$.

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**AVERAGE OF $Z_{bb}$**

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<th>IMAG(OHM)</th>
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Table G.9 Calculated self-impedance $Z^{bb}$ for a dipole array with $\theta_b = 90^\circ$ when $\alpha = 90^\circ$ and $\eta = 60^\circ$.

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<th>AVERAGE OF $Z^{bb}$</th>
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<th>$\text{IMAG}(\Omega)$</th>
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Table G.10 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta_b = 70^\circ$ when $\alpha = 0^\circ$ and $\eta = 15^\circ$.

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<th>IMAG(OHM)</th>
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**NUMBER OF TEST DIPOLES**

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Table 6.11 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta_b = 70^\circ$
when $\alpha = 0^\circ$ and $\eta = 40^\circ$.

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AVERAGE OF $Z_{bb}$

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Diagram showing the scan plane and the dipole array configuration.
Table G.12 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta_b = 70^\circ$ when $\alpha = 0^\circ$ and $\eta = 60^\circ$.

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Average of $Z_{bb}$

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Table G.13 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta_b = 70^\circ$
when $\alpha = 45^\circ$ and $\eta = 15^\circ$.

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Table G.14 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta_b = 70^\circ$ when $\alpha = 45^\circ$ and $\eta = 40^\circ$.

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AVERAGE OF $Z_{bb}$:

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Table G.15 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta = 70^\circ$ when $\alpha = 45^\circ$ and $n = 60^\circ$.  

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<thead>
<tr>
<th>$\phi_d$ (DEG)</th>
<th>$Z_{bb}(\phi_d)$ REAL (OHM)</th>
<th>$Z_{bb}(\phi_d)$ IMAG (OHM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.054110E+02</td>
<td>0.018030E+02</td>
</tr>
<tr>
<td>15</td>
<td>0.059836E+02</td>
<td>0.079235E+02</td>
</tr>
<tr>
<td>30</td>
<td>0.060048E+02</td>
<td>0.077432E+02</td>
</tr>
<tr>
<td>$\Rightarrow$</td>
<td>45</td>
<td>0.054099E+02</td>
</tr>
<tr>
<td>60</td>
<td>0.045067E+02</td>
<td>0.072896E+02</td>
</tr>
<tr>
<td>75</td>
<td>0.039103E+02</td>
<td>0.071393E+02</td>
</tr>
<tr>
<td>90</td>
<td>0.021002E+02</td>
<td>0.068007E+02</td>
</tr>
<tr>
<td>105</td>
<td>0.045677E+02</td>
<td>0.071307E+02</td>
</tr>
<tr>
<td>120</td>
<td>0.074661E+02</td>
<td>0.072615E+02</td>
</tr>
<tr>
<td>$\Rightarrow$</td>
<td>155</td>
<td>0.075654E+02</td>
</tr>
<tr>
<td>150</td>
<td>0.073817E+02</td>
<td>0.076520E+02</td>
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<tr>
<td>165</td>
<td>0.072463E+02</td>
<td>0.078145E+02</td>
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<tr>
<td>180</td>
<td>0.071439E+02</td>
<td>0.080200E+02</td>
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<tr>
<td>195</td>
<td>0.070093E+02</td>
<td>0.081845E+02</td>
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<tr>
<td>210</td>
<td>0.070003E+02</td>
<td>0.083023E+02</td>
</tr>
<tr>
<td>$\Rightarrow$</td>
<td>225</td>
<td>0.071339E+02</td>
</tr>
<tr>
<td>240</td>
<td>0.072270E+02</td>
<td>0.084409E+02</td>
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<tr>
<td>255</td>
<td>0.073651E+02</td>
<td>0.084693E+02</td>
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<td>270</td>
<td>0.074795E+02</td>
<td>0.082932E+02</td>
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<tr>
<td>295</td>
<td>0.077210E+02</td>
<td>0.085330E+02</td>
</tr>
<tr>
<td>300</td>
<td>0.079201E+02</td>
<td>0.085915E+02</td>
</tr>
<tr>
<td>$\Rightarrow$</td>
<td>315</td>
<td>0.081125E+02</td>
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<tr>
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<td>0.082960E+02</td>
<td>0.085021E+02</td>
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<td>345</td>
<td>0.084340E+02</td>
<td>0.083783E+02</td>
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AVERAGE OF $Z_{bb}$ 

<table>
<thead>
<tr>
<th>NUMBER OF TEST DIPOLES</th>
<th>AVERAGE OF $Z_{bb}$ REAL (OHM)</th>
<th>AVERAGE OF $Z_{bb}$ IMAG (OHM)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.794920E+02</td>
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<tr>
<td>4</td>
<td>0.703027E+02</td>
<td>0.797610E+02</td>
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Table G.16 Calculated self-impedance $Z_{bb}$ for a dipole array with $\theta_b = 70^\circ$ when $\alpha = 90^\circ$ and $\eta = 15^\circ$.

<table>
<thead>
<tr>
<th>$\phi_d$ (DEG)</th>
<th>$Z_{bb}$ ($\phi_d$)</th>
<th>REAL (OHM)</th>
<th>IMAG (OHM)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.148056E+03</td>
<td>0.466203E+02</td>
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</tr>
<tr>
<td>15</td>
<td>0.148714E+03</td>
<td>0.470362E+02</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.148274E+03</td>
<td>0.480292E+02</td>
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</tr>
<tr>
<td>45</td>
<td>0.147506E+03</td>
<td>0.481596E+02</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.146379E+03</td>
<td>0.494097E+02</td>
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</tr>
<tr>
<td>75</td>
<td>0.144086E+03</td>
<td>0.510677E+02</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.143071E+03</td>
<td>0.619097E+02</td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>0.141038E+03</td>
<td>0.560928E+02</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0.139915E+03</td>
<td>0.590326E+02</td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>0.137012E+03</td>
<td>0.617541E+02</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>0.135434E+03</td>
<td>0.649253E+02</td>
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</tr>
<tr>
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<td>0.655728E+02</td>
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</tr>
<tr>
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<td>0.134402E+03</td>
<td>0.659528E+02</td>
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<tr>
<td>210</td>
<td>0.134344E+03</td>
<td>0.649253E+02</td>
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</tr>
<tr>
<td>225</td>
<td>0.137012E+03</td>
<td>0.617541E+02</td>
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<tr>
<td>240</td>
<td>0.139915E+03</td>
<td>0.590326E+02</td>
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<tr>
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<td>0.560928E+02</td>
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</tr>
<tr>
<td>270</td>
<td>0.143071E+03</td>
<td>0.619097E+02</td>
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<tr>
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<td>0.510677E+02</td>
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</tr>
<tr>
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<td>0.146379E+03</td>
<td>0.494097E+02</td>
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<tr>
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<td>0.147506E+03</td>
<td>0.481596E+02</td>
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<td>0.148274E+03</td>
<td>0.480292E+02</td>
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</tr>
<tr>
<td>345</td>
<td>0.148056E+03</td>
<td>0.466203E+02</td>
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</tr>
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</table>

NUMBER OF TEST DIPOLES | AVERAGE OF $Z_{bb}$ | REAL (OHM) | IMAG (OHM) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.142260E+03</td>
<td>0.557923E+02</td>
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<tr>
<td>2</td>
<td>0.142259E+03</td>
<td>0.549569E+02</td>
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</table>

Diagram showing the orientation of the dipoles and the scan plane.
Table G.1 Calculated self-impedance $Z^{bb}$ for a dipole array with $\theta = 70^\circ$ when $\alpha = 90^\circ$ and $n = 40^\circ$.

<table>
<thead>
<tr>
<th>$\phi_d$ (DEG)</th>
<th>$Z^{bb}(\phi_d)$</th>
<th>REAL (OHM)</th>
<th>IMAG (OHM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.935265E+02</td>
<td>0.230321E+02</td>
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</tr>
<tr>
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<td>0.933810E+02</td>
<td>0.239110E+02</td>
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</tr>
<tr>
<td>30</td>
<td>0.929478E+02</td>
<td>0.241599E+02</td>
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</tr>
<tr>
<td>$\Rightarrow$ 45</td>
<td>0.92232E+02</td>
<td>0.243301E+02</td>
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</tr>
<tr>
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<td>0.912400E+02</td>
<td>0.246513E+02</td>
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</tr>
<tr>
<td>75</td>
<td>0.900330E+02</td>
<td>0.250940E+02</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.896530E+02</td>
<td>0.252331E+02</td>
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</tr>
<tr>
<td>105</td>
<td>0.871726E+02</td>
<td>0.273566E+02</td>
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</tr>
<tr>
<td>120</td>
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<td>0.295672E+02</td>
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</tr>
<tr>
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<td>0.310005E+02</td>
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<tr>
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<td>0.322010E+02</td>
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<td>0.827206E+02</td>
<td>0.330252E+02</td>
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<td>0.332682E+02</td>
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<td>0.827206E+02</td>
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<td>0.833930E+02</td>
<td>0.322010E+02</td>
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<tr>
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<td>0.844235E+02</td>
<td>0.310005E+02</td>
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<tr>
<td>240</td>
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<td>0.295672E+02</td>
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<tr>
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<td>0.886453E+02</td>
<td>0.241231E+02</td>
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<tr>
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<td>0.250940E+02</td>
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<tr>
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<td>0.912400E+02</td>
<td>0.246513E+02</td>
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<td>0.92232E+02</td>
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<td>0.929478E+02</td>
<td>0.241599E+02</td>
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<tr>
<td>345</td>
<td>0.935265E+02</td>
<td>0.230321E+02</td>
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AVERAGE OF $Z^{bb}$

<table>
<thead>
<tr>
<th>NUMBER OF TEST DIPOLES</th>
<th>AVERAGE OF $Z^{bb}$</th>
<th>REAL (OHM)</th>
<th>IMAG (OHM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
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Table G.18 Calculated self-impedance $Z_{bb}^{fd}$ for a dipole array with $\phi = 70^\circ$

when $\alpha = 90^\circ$ and $\eta = 60^\circ$.

<table>
<thead>
<tr>
<th>$\phi_d$ (DEG)</th>
<th>$Z_{bb}^{fd}$ (REAL(OHH))</th>
<th>$Z_{bb}^{fd}$ (IMAG(OHH))</th>
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<tbody>
<tr>
<td>0</td>
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<td>0.103965E+02</td>
</tr>
<tr>
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<td>0.103074E+02</td>
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<tr>
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<td>0.489114E+02</td>
<td>0.102339E+02</td>
</tr>
<tr>
<td>45</td>
<td>0.486533E+02</td>
<td>0.098439E+01</td>
</tr>
<tr>
<td>60</td>
<td>0.480359E+02</td>
<td>0.095826E+01</td>
</tr>
<tr>
<td>75</td>
<td>0.474342E+02</td>
<td>0.102472E+02</td>
</tr>
<tr>
<td>90</td>
<td>0.467720E+02</td>
<td>0.078472E+01</td>
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<tr>
<td>105</td>
<td>0.460906E+02</td>
<td>0.114337E+02</td>
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<tr>
<td>120</td>
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<td>0.118066E+02</td>
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<tr>
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<td>0.131074E+02</td>
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<tr>
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<td>0.145328E+02</td>
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<td>0.148028E+02</td>
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<tr>
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<td>0.142460E+02</td>
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<td>0.131074E+02</td>
</tr>
<tr>
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<td>0.454389E+02</td>
<td>0.118069E+02</td>
</tr>
<tr>
<td>255</td>
<td>0.460906E+02</td>
<td>0.114337E+02</td>
</tr>
<tr>
<td>270</td>
<td>0.467720E+02</td>
<td>0.078472E+01</td>
</tr>
<tr>
<td>285</td>
<td>0.474342E+02</td>
<td>0.102472E+02</td>
</tr>
<tr>
<td>300</td>
<td>0.480359E+02</td>
<td>0.095826E+01</td>
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<td>0.098439E+01</td>
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<td>330</td>
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<td>0.103965E+02</td>
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AVERAGE OF $Z_{bb}^{fd}$

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<tr>
<th>NUMBER OF TEST DIPOLES</th>
<th>REAL(OHH)</th>
<th>IMAG(OHH)</th>
</tr>
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<tbody>
<tr>
<td>24</td>
<td>0.467008E+02</td>
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<td>0.467009E+02</td>
<td>0.114749E+02</td>
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Table G.19 Calculated self-impedance \( Z_{ff}^{uv} \)
when \( \alpha = 0^\circ \) and \( \eta = 15^\circ \).

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<th>( \phi_1 ) (DEG)</th>
<th>( Z_{ff}^{uv} (\phi_1) )</th>
<th>REAL(OHM)</th>
<th>IMAG(OHM)</th>
</tr>
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<tbody>
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</tr>
<tr>
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<td>-0.510113E+03</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.555991E+01</td>
<td>-0.512643E+03</td>
<td></td>
</tr>
<tr>
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<td>0.100184E+02</td>
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</tr>
<tr>
<td>60</td>
<td>0.159496E+02</td>
<td>-0.512333E+03</td>
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</tr>
<tr>
<td>75</td>
<td>0.229017E+02</td>
<td>-0.509493E+03</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.303525E+02</td>
<td>-0.507910E+03</td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>0.377900E+02</td>
<td>-0.500879E+03</td>
<td></td>
</tr>
<tr>
<td>120</td>
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<td>-0.511181E+03</td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>0.506647E+02</td>
<td>-0.512198E+03</td>
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</tr>
<tr>
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<tr>
<td>195</td>
<td>0.570225E+02</td>
<td>-0.507809E+03</td>
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<tr>
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<td>-0.510649E+03</td>
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<tr>
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<td>-0.512198E+03</td>
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<tr>
<td>240</td>
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<td>-0.511181E+03</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
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<tr>
<td>285</td>
<td>0.229017E+02</td>
<td>-0.509493E+03</td>
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<tr>
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<td>0.159496E+02</td>
<td>-0.512333E+03</td>
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<tr>
<td>315</td>
<td>0.100184E+02</td>
<td>-0.513627E+03</td>
<td></td>
</tr>
<tr>
<td>330</td>
<td>0.555991E+01</td>
<td>-0.512643E+03</td>
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</tr>
<tr>
<td>345</td>
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<td>-0.508846E+03</td>
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</table>

**Number of Test Feedlines**

<table>
<thead>
<tr>
<th>Number of Test Feedlines</th>
<th>Average of ( Z_{ff}^{uv} )</th>
<th>REAL(OHM)</th>
<th>IMAG(OHM)</th>
</tr>
</thead>
<tbody>
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Table G.20 Calculated self-impedance $Z_{ff}^{uu}$ when $\alpha = 0^\circ$ and $n = 40^\circ$.

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<th>$Z_{ff}^{uu}$ ($\phi_f$)</th>
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<th>IMAG (OHM)</th>
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<td>0.270025E+03 - 0.573629E+03</td>
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<tr>
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<td>345</td>
<td>0.180125E+03 - 0.607226E+03</td>
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<tr>
<th>Number of Test Feedlines</th>
<th>AVERAGE OF $Z_{ff}^{uu}$</th>
<th>REAL (OHM)</th>
<th>IMAG (OHM)</th>
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Table G.21 Calculated self-impedance $Z_{ff}^{uu}$ when $\alpha = 0^\circ$ and $\eta = 60^\circ$.

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<tr>
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<td>30</td>
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<tr>
<td>45</td>
<td>0.744238E+03</td>
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<tr>
<td>60</td>
<td>0.770405E+03</td>
</tr>
<tr>
<td>75</td>
<td>0.81630E+03</td>
</tr>
<tr>
<td>90</td>
<td>0.86089E+03</td>
</tr>
<tr>
<td>105</td>
<td>0.899177E+03</td>
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<tr>
<td>120</td>
<td>0.935072E+03</td>
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<tr>
<td>135</td>
<td>0.966500E+03</td>
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<tr>
<td>150</td>
<td>0.999464E+03</td>
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<tr>
<td>165</td>
<td>0.100349E+04</td>
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<tr>
<td>180</td>
<td>0.100814E+04</td>
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<tr>
<td>195</td>
<td>0.100345E+04</td>
</tr>
<tr>
<td>210</td>
<td>0.999464E+03</td>
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<tr>
<td>225</td>
<td>0.966500E+03</td>
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<tr>
<td>240</td>
<td>0.935072E+03</td>
</tr>
<tr>
<td>255</td>
<td>0.899177E+03</td>
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<td>270</td>
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<tr>
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<td>0.744237E+03</td>
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<td>330</td>
<td>0.717896E+03</td>
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<td>0.695765E+03</td>
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<table>
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<th>NUMBER OF TEST FEEDLINES</th>
<th>AVERAGE OF $Z_{ff}^{uu}$</th>
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Table G.22 Calculated self-impedance $Z_{ff}^{uu}$
when $\alpha = 45^\circ$ and $\eta = 15^\circ$.

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<td>0.600906E+01</td>
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<td>0.501158E+02</td>
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<td>0.546660E+02</td>
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<td>0.586357E+02</td>
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<td>270</td>
<td>0.501158E+02</td>
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<td>285</td>
<td>0.443047E+02</td>
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<td>0.375993E+02</td>
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<td>0.303513E+02</td>
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<tr>
<td>330</td>
<td>0.251172E+02</td>
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<td>0.163079E+02</td>
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<th>AVERAGE OF $Z_{ff}^{uu}$</th>
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Table G.23 Calculated self-impedance $Z_{ff}$ when $\alpha = 45^\circ$ and $\eta = 40^\circ$.

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<th>IMAG(Ohm)</th>
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<td>-0.539505E+03</td>
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<td>-0.458905E+03</td>
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<tr>
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<table>
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<tr>
<th>NUMBER OF TEST FEEDLINES</th>
<th>AVERAGE OF $Z_{ff}^{uu}$</th>
<th>REAL(Ohm)</th>
<th>IMAG(Ohm)</th>
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Table G.24 Calculated self-impedance $Z_{ff}^{uu}$ when $\alpha = 45^\circ$ and $\eta = 60^\circ$.

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<th>$Z_{ff}^{uu}$ ($\phi_f$)</th>
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<th>IMAG(ΩH)</th>
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<td>-0.706605E+03</td>
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<tr>
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<td>-0.632769E+03</td>
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<tr>
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Average of $Z_{ff}^{uu}$

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<th>IMAG(ΩH)</th>
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Diagram showing the relationship between $\phi_f$, $\alpha$, and $\eta$. The scan plane is indicated with $\alpha = 45^\circ$. The ground plane is denoted with $x_f$. The feedline is shown as a circle with points at various $\phi_f$ values.
Table G.25 Calculated self-impedance $Z_{ff}^{uu}$ when $\alpha = 90^\circ$ and $\eta = 15^\circ$.

<table>
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<th>IMAG (OHM)</th>
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<th>AVERAGE OF $Z_{ff}^{uu}$</th>
<th>REAL (OHM)</th>
<th>IMAG (OHM)</th>
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Table G.26 Calculated self-impedance $Z_{ff}$ when $\alpha = 90^\circ$ and $\eta = 40^\circ$.

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<tr>
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<th>REAL(OHMS)</th>
<th>IMAG(OHMS)</th>
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Diagram showing the scan plane and ground plane with scan angle $\alpha = 90^\circ$. The vector $\phi_i$ is shown in the plane of the feedlines.
Table G.27 Calculated self-impedance \( Z_{ff} \)
when \( \alpha = 90° \) and \( \eta = 60° \).

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APPENDIX H

SCAN IMPEDANCE $Z_{bb}$ OF A DIPOLE ARRAY WITH ELEMENT BEND ANGLE GREATER THAN 90° (Y-DIPOLE)

In this appendix, the scan impedance $Z_{bb}$ of a dipole array with element bend angle ($\theta_b$) greater than 90° will be examined (y-dipole). It will be shown that when the bend angle of the dipole elements in the array is greater than 90° a blind spot always exists in the E-plane scan. This blind spot is caused by the pattern null of the array element.

In this study the array is located in free space and backed by a perfectly conducting ground plane as illustrated in Figure H.1. For simplicity, the quantities $D_x$, $D_z$, $\xi_d$ and $r_d$ are kept constant. The only variables are the bend angle $\theta_b$ and $\xi_f$, spacing between dipole feedpoint and ground plane.

In the following, we will present the scan impedances ($Z_{bb}$) (both E- and H-planes) for the arrays with $\theta_b=90°$ (straight dipole), 110° and 140°. First, we will examine the calculated $Z_{bb}$ for spacing $\xi_f=0.6$ cm at frequency 7.5 GHz as shown in Figure H.2. In this figure, we notice that the real part of scan impedance drops to zero at 80° in the E-plane for straight dipole array. We also observe that there is a blind spot in the E-plane at 65° for $\theta_b=110°$ and at 60° for $\theta_b=140°$. When the
spacing $\lambda_f$ is reduced to 0.1 cm, the calculated results are shown in Figure H.3 where we see that the blind spot still exists in every E-plane scan. From the above observation, it seems that the blind spot can not be avoided for the dipole array with element bend angle greater than 90°.

It is known that the impedance performance of a phased array is strongly dependent upon the radiation pattern of the array element. Therefore, to understand the cause of the blind spot, we shall examine the radiation pattern of the array element.

The radiation patterns of a dipole element at frequency 7.5 GHz for $\theta_e=90^\circ$, 110° and 140° are shown in Figure H.4 for $\lambda_f=0.6$ cm and in Figure H.5 for $\lambda_f=0.1$ cm. In these figures, we observe that there is a deep null in every E-plane pattern, regardless of $\theta_e$ and $\lambda_f$. Comparing Figure H.2 with Figure H.4 and Figure H.3 with Figure H.5, it is easy to notice that the blind spots exist exactly in the same directions as that of the pattern nulls. From the above investigation, we realize that the blind spot is created by the null in the element pattern. The radiation patterns of a dipole element at frequency 4.0 GHz are shown in Figure H.6 for $\lambda_f=0.6$ cm and in Figure H.7 for $\lambda_f=0.1$ cm. In those figures, we found that a null still exists in every E-plane pattern. It is obvious that the pattern null will exist at the frequencies between 4.0 GHz and 7.5 GHz.

From the foregoing analysis, it is apparent that dipoles with bend angles greater than 90° are not suitable for phased array design, in general.
Figure H.1 Geometry of a dipole element and dimensions of the array.
Figure H.2 Scan Impedances $\mathbf{Z}_{bb}$ for a dipole array with $\theta_b=90^\circ$, $110^\circ$ and $140^\circ$ when $f=7.5$ GHz, $\ell_f=0.6$ cm.
and 140° when $f = 7.5$ GHz, $z_r = 0.1$ cm.

Figure H.3: Scan impedances zdp for a dipole array with $\theta = 90^\circ$, $110^\circ$.
Figure H.4 Radiation patterns of a dipole element for $\theta_b=90^\circ$, 110$^\circ$ and 140$^\circ$ when $f=7.5\ GHz$, $\ell_f=0.6\ cm$. 

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Figure H.5 Radiation patterns of a dipole element for $\theta_0 = 90^\circ$, $110^\circ$ and $140^\circ$ when $f = 7.5$ GHz, $\delta_f = 0.1$ cm.
Figure H.6 Radiation patterns of a dipole element for $\theta_b=90^\circ$, $110^\circ$ and $140^\circ$ when $f=4.0\,\text{GHz}$, $d_f=0.6\,\text{cm}$.
Figure H.7 Radiation patterns of a dipole element for $\theta_b=90^\circ$, $110^\circ$ and $140^\circ$ when $f=4.0$ GHz, $\lambda_f=0.1$ cm.
REFERENCES


