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QUADRUPOLE MOMENT EFFECTS IN LITHIUM-7 ELASTIC SCATTERING

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QUADRUPOLE MOMENT EFFECTS
IN 7LI ELASTIC SCATTERING

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

by

John William Kerns, B.A.

* * * * *

The Ohio State University
1985

Reading Committee:
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Approved by

William D. Ploughe
Adviser
Department of Physics
To my Mother

Patricia Kerns

whose love, guidance, and patience
made everything possible.
ACKNOWLEDGEMENTS

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PUBLICATIONS AND COMMUNICATIONS


FIELD OF STUDY

Major Field: Physics
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Chapter I
INTRODUCTION

1.1 Motivation For Doing The Experiment

In the past several years measurements of the elastic and inelastic scattering of lithium ions have been made for a wide range of target nuclei and bombarding energies. The investigation into the scattering produced by these ions is of interest as one attempts to understand in more detail the reactions induced by them, since these ions lie in the mass range 4\( \leq A \leq 12 \) whose elastic scattering is said to exhibit a transition between the scattering characteristic of light and heavy ions (DEVR77). Light ions (\( A \leq 4 \)) tend to exhibit diffraction structures in their differential cross sections, and they contain both discrete and continuous ambiguities in the potential strengths occurring in optical model analyses of the data; i.e. the scattering data may be fit equally well with a number of potentials of similar shape differing in central strength by 50 MeV or more, as well as potentials characterized by parameter correlations of a continuous character. These ambiguities can sometimes be resolved for data at higher energies and with an angular range sufficient to observe the refractive nuclear rainbow effect. On the
other hand, heavy ion elastic scattering ($A>12$) is usually characterized by strong absorption which prohibits nuclear rainbow effects. Provided that the potential strength in the vicinity of the strong absorption radius is similar, many parameter sets are capable of providing reasonable optical model fits to heavy ion data.

There are five stable ions in this transition region that are available to use as projectiles, and these nuclei have a variety of projectile spins. Any projectile with a ground state spin greater than zero may contain non-zero multipole moments, and those projectiles with a spin $I>1$ may have a substantial quadrupole moment. Of these five transition nuclei, only $^7$Li, $^9$Be, and $^{10}$B have sizable ground state quadrupole moments ($Q>4\,\text{e-fm}^2$), where $e$ is the elementary charge. In order to understand the elastic scattering of these ions, one must understand the contribution of their electric quadrupole moment to the scattering process.

Many investigations have been performed using projectiles belonging to this transition region (POLI72, SCHU73, GLOV80, PARK80, FULM82) covering an energy range between 4.5 MeV and 79 MeV, with the majority of the experiments performed at energies below 50 MeV. In all these papers, the data have been analyzed with a phenomenological optical model potential and/or a microscopic double-folding model potential with various
degrees of success. In almost all cases where the optical model was employed, a reasonable fit to the data for several different parameter sets was obtained, even though these models ignored any quadrupole moment effects. One notable exception was found by Vineyard et al. (VINE84), in which a satisfactory fit could be obtained using an optical model analysis on the scattering of 34 MeV $^7$Li projectiles from a $^{12}$C target when only the angles $\theta < 80^0$ were considered.

Fits to data (PERT77, GLOV80, HNIZ81, VINE84) using a microscopic double-folding model description for the real part of the interaction potential usually produced equally good results, but the real part of the potential required a normalization of approximately one-half for $^6$Li, $^7$Li, and $^9$Be ions in order to reproduce the data. Since the real part of the double-folding potential adequately reproduces the nucleus-nucleus potential for both light and heavy ions, the anomalous behavior of these transition projectiles needs to be investigated. One suggestion (SATC79a) for this behavior is that it can be attributed to the very small breakup energies of these nuclei, while another explanation (HNIZ81) is that additional terms must be included in the potential to account for the large static quadrupole moments of the $^7$Li and $^9$Be nuclei. It was first suggested by Blair (BLAI59) that ground state quadrupole moments could be important in the elastic scattering of heavy ions, and preliminary work (PARK77) has shown that there can be
important quadrupole terms in the optical potential for the elastic scattering of $^{10}$B ions.

1.2 Previous Investigations of the Quadrupole Moment Effect

The effects of the projectile's static quadrupole moment can be investigated by bombarding identical targets with different isotopes of the same element at the same energy. For the case of lithium ions, $^7$Li has a quadrupole moment that is approximately fifty times that of $^6$Li. On the other hand, $^{10}$B has only twice the static quadrupole moment of $^{11}$B. Investigations of the systematics and ambiguities of optical parameters for lithium and boron elastic scattering on various targets have been performed by various groups. Poling et al. (POLI72) found that similar optical model potential parameters could fit both $^6$Li and $^7$Li on $^{12}$C for each energy in a range between 4.5 MeV and 13 MeV, while Schumacher et al. (SCHU73) needed different sets of optical model parameters for his 36 MeV $^7$Li data as compared to the parameters found for his 34 MeV $^6$Li data on the same targets. Vineyard et al. (VINE84), repeating the 34 MeV run on carbon over a larger angular range, found that not only did the $^7$Li data exhibit a completely different angular distribution (especially over the 60-90° range), but that the optical model was incapable of providing a satisfactory fit to the $^7$Li data when it included angles greater than 60°.
Kemper et al. (KEMP79) and Cutler et al. (CUTL77) found that in general a different parameter set was required for the $^7\text{Li}$ data at the same center-of-mass energy as the $^6\text{Li}$ data for the same target in an energy range between 34 and 50 MeV, although there were some exceptions for targets in the medium mass range ($A=50$). In searching for a "global" optical model potential for both lithium projectiles, Cook (COOK82a) found that for targets with $A>24$ the average potential parameters for the two isotopes were very similar, with the major difference residing in the volume integral per interacting nucleus. This difference suggested that $^7\text{Li}$ is less strongly absorbed than $^6\text{Li}$.

Double-folding analyses on the same data, when performed, usually resulted in equally good fits provided that the real part of the potential was normalized by a factor of approximately 0.5.

Investigations using boron ions on light targets yielded results comparable to that of the lithium isotopes. Two studies by Parks et al. (PARK77) and (PARK80) on respective targets of $^{27}\text{Al}$ and $^{16}\text{O}$ at 50 MeV both showed that the same potential parameters used to fit the $^{11}\text{B}$ data failed to provide adequate fits to the $^{10}\text{B}$ data. The differences in the two distributions were quite apparent for both targets for angles greater than $60^\circ$, where the ratio of the experimental elastic cross section to the Rutherford cross section fell below $10^{-2}$. The quadrupole moment
contribution of the $^{10}$B projectile was shown to be significant when a double-folding model for the interaction potential was used in conjunction with either a distorted-wave Born approximation (DWBA) calculation or a coupled-channels (CC) calculation. The latter study also showed that coupled-channels calculations were required to accurately account for the quadrupole moment contribution to the elastic scattering, since this contribution produces interference effects between the various L-value folded potentials.

Another method used to investigate the quadrupole moment effect in elastic scattering was to use aligned or polarized beams. Several investigations have taken place using aligned (spin and quadrupole moment) lithium ions at low bombarding energies. Dreves et al. (DREV78) scattered aligned $^6$Li and $^7$Li ions on $^{12}$C at 12 MeV and found that when polarized beams were used, $^6$Li and $^7$Li scattering behaved quite differently. On the other hand, the two angular distributions were impossible to distinguish when the alignment of the beam projectiles was turned off. One possible explanation for the above effect would be contributions due to spin dependance, but Petrovich and Stanley (PETR77) have shown that these contributions should be small. Either the theoretical treatment is incorrect or other effects are involved.
1.3 Reasons For Choosing These Reactions

Since ambiguities in the potential parameters used in optical model calculations tend to be resolved for data at higher energies with large angular ranges, it is desirable to perform this investigation at the highest possible energy consistent with angular resolution and measurable cross sections. In order to investigate quadrupole moment effects in elastic scattering, the use of lithium ions would appear to be the preferred choice since there exists a substantial difference in the ground state quadrupole moment of the two stable isotopes. This investigation will therefore consist of the elastic and inelastic scattering of \(^7\text{Li}\) ions at an energy near 100 MeV on targets of \(^{12}\text{C}\) and \(^{58}\text{Ni}\). The reason for this particular choice of energy and targets was determined by the fact that similar data existed for the same energy and targets using \(^6\text{Li}\) ions (SCHW81), and therefore a comparison could be made between the results of this investigation and those of Schwandt et al. Another reason for the choice of these particular targets is that target quadrupole moment contributions to the elastic scattering exist only for odd-A targets, and so they will not be present in this data.

The angular distributions produced by the elastic scattering of \(^7\text{Li}\) from the two targets will be fit using an optical model calculation, and the parameters obtained will be compared with those of \(^6\text{Li}\) scattering. Fits using a
microscopic double-folding model will also be attempted. Since the double-folding model is based on a realistic nucleon-nucleon interaction, the potential generated by this method should presumably include any quadrupole moment contributions. Finally, using the double-folding model to generate interaction potentials for both the quadrupole moment transition and the transition to the first excited state of the $^7\text{Li}$ nucleus, the contributions to the elastic scattering from both transitions will be explored by means of a coupled-channels calculation.

The projectile excitation will be considered since a low-lying collective state can have an influence on the elastic scattering, and since it has been suggested (SATC74) that the ground state quadrupole moment of the projectile produces contributions to the elastic cross section that are typically as large as those belonging to the inelastic cross section for a low-lying quadrupole (E2) transition. Based on the above suggestion, the quadrupole contributions should add incoherently to the elastic scattering cross sections because inelastic cross sections tend to be out of phase with the elastic distribution. When Petrovich and Stanley (PETR77) subtracted the experimental differential cross sections to the first excited state of the $^7\text{Li}$ projectile, a new optical model potential was found that produced a slightly better fit to the elastic data. To see if the failure of the double-folding model is due to
quadrupole moment effects and/or low-lying collective effects, the inclusion of these effects will be treated explicitly in a coupled-channels calculation. Hnizdo et al. (HNIZ81) were able to fit his 34 MeV data for $^7$Li on targets of $^{40}$Ca and $^{54}$Fe without having to renormalize the real double-folded potential when both the quadrupole moment and projectile excitation effects were treated explicitly in the coupled-channels formalism, but Cook et al. (COOK82b) were unable to obtain a suitable fit for $^7$Li on $^{12}$C and $^{24}$Mg at energies between 63 and 88 MeV with the normalization constant to the real double-folded potential $N_R=1$ when he included the same effects. Such effects and procedures will be explored in this investigation.
Chapter II
THEORY

2.1 Introduction

The accounting for the processes that take place when two nuclei collide is the goal in analyzing any nuclear reaction. Among the information to be gained is the type of reaction products along with their relative frequencies and angular distributions. The information is then used to test models for the structure of nuclei. By comparing the predictions of the various models to the experimental data, additional knowledge can be gained about nuclear structure and about the usefulness of any given model.

Nuclear reactions are usually classified under two separate categories. Reactions in which the outgoing particles are emitted in a time that is comparable to the transit time of the incident particles across the nucleus are known as direct reactions. In direct reactions, few collisions occur, and the way that the reaction takes place is important. During the collision, the projectile makes a glancing blow with the target and can either be elastically or inelastically scattered. Direct reactions also include transfer reactions, where the projectile picks up or
deposits one or more nucleons from the target; and knock-out reactions, where the projectile knocks out a nucleon or a cluster of nucleons from the target. The other class of reactions are known as compound nuclear reactions. The projectile is absorbed by the target to form an intermediate state in this reaction, with the incident particle sharing its energy with all the nucleons of the compound system. The compound nucleus is an excited state, and after a long time compared to the travel time of the particle across the nuclear diameter, the compound system emits particles and/or $\gamma$-rays until it returns to its ground state. The compound-nuclear state loses any information about its formation due to the large number of collisions between individual nucleons, so the reaction channels available for its decay are determined by the "excess" energy of the compound system. The compound-nuclear reactions can therefore also contribute to the reaction channels used by direct reaction processes, and so the compound nuclear contribution to the cross-section for a direct reaction would also have to be accounted for before a comparison is made with the experimental data. Fortunately, both processes compete with comparable magnitudes only at low energies, usually less than 10 MeV (HODG63).

At energies low compared with the Coulomb barrier, the projectile interacts only with the Coulomb field of the target and exhibits Rutherford scattering. As the
projectile energy approaches the energy associated with the Coulomb barrier height, the projectile starts to interact with the nuclear field and may be absorbed by the target to form a compound nucleus. There are very few reaction channels that are energetically allowed into which this compound-nuclear state can decay, and so there is a large probability that the projectile will be emitted with the same incident energy. This process is known as compound elastic scattering, and its probability of occurrence decreases as the projectile energy increases since a greater number of reaction channels open up. At the projectile energy of this investigation, 100 MeV, the projectile has sufficient energy above the Coulomb barrier of the two targets, 4.9 and 16.6 MeV for $^{12}$C and $^{58}$Ni respectively, that a large number of reaction channels are indeed available to the compound nucleus. This results in a high improbability that the compound elastic process will occur, and so its contribution will be ignored in the theoretical development of the various models used to describe the data.

The simplest and most popular model used to describe elastic scattering is the optical model. This model was first proposed by Serber in 1947 and is based on an analogy between the scattering and absorption of particles by a nucleus and the scattering and absorption of light by a cloudy crystal ball. This model reduces the complex
many-bodied process to a one-body problem with the scattering described by a simple potential depending only upon the relative separation of the two nuclei. An alternative to the optical model is the double-folding model, which will next be investigated. This model relates the potential for the scattering of the two nuclei to the underlying nucleon-nucleon interaction and the properties of the colliding nuclei by integrating over the matter distributions of the two nuclei, somewhat analogous to determining the Coulomb potential between two charged bodies. Lastly, the formalism associated with the coupled-channels theory of elastic and inelastic scattering will be discussed. This model is required whenever there exists a strong coupling between the elastic channel and a particular inelastic channel or channels.

2.2 Elastic Scattering

The solution of the Schrödinger equation for the reaction under consideration is the basic task of any nuclear reaction theory. Since this equation describes the motion for all interacting particles, its solution becomes increasingly complicated as the number of nucleons involved in the interaction increases. To overcome these complications, simplifications must be introduced. The most common simplification is to represent the interaction between the projectile and the target by a simple one-body
potential that is only dependent on the relative separation between the projectile and the potential. The solution for this simplified situation is still quite complicated, and so the calculations for a neutral particle with no spin from a spherical local potential will first be considered. This treatment follows that presented by Hodges (HODG71).

In the asymptotic region, the incoming particle is in a definite state of momentum, and hence can be represented by a plane wave. For a particle traveling in the \( z \) direction, the incoming wave function has the form

\[
\psi_i = Ae^{ikz},
\]

where \( k = (2mE)^{1/2}/\pi \) is the wave number of the incoming particle. Since \( \psi \) must describe the motion of the particle for all regions of space even after scattering, the total wave function will be defined as

\[
\psi = \psi_i + \psi_s,
\]

where \( \psi_s \) represents all the situations not described by \( \psi_i \). If the interaction is spherically symmetric, it can be shown (PARK74) that the outgoing scattered wave in the asymptotic region can be represented by an outgoing spherical wave that is axially symmetric about the \( z \) axis. At large distances from the scattering center, the total wave function may be
written as

\[ \psi = \psi_i + \psi_s = A e^{ikz} + A f(\theta) e^{ikr} \]  

(3)

where \( f(\theta) \) is defined to be the scattering amplitude for a given scattering potential \( V(r) \). From the definition of the differential cross section (PARK74), we have

\[ \frac{d\sigma}{d\Omega} = \frac{1}{J_i} \frac{dN(\theta)}{d\Omega} \]  

(4)

where \( dN(\theta)/d\Omega \) is the number of events per second per unit solid angle scattered by angle \( \theta \), and \( J_i \) is the incident flux of the projectiles. Since the projectile density can be expressed as

\[ \rho = |\psi|^2 , \]  

(5)

where \( \psi \) is a solution to the Schrödinger equation for a free particle

\[ i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi ; \]  

(6)

and \( dN(\theta) \) can be expressed by
\[ dN(\theta) = j_s dA, \] (7)

where \( j_s \) is the scattered flux and \( dA \) is the area into which the particles are scattered, expressions for the particle fluxes \( j_1 \) and \( j_s \) need to be found. Starting with the equation of continuity

\[ \frac{\partial \rho}{\partial t} = - \nabla \cdot j, \] (8)

the left hand side can be re-expressed as

\[ \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t} (\psi^* \psi) = \frac{\hbar}{2m} (-\psi^* \nabla^2 \psi + \psi \nabla^2 \psi^*), \] (9)

which reduces to

\[ \frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m} \nabla^* (\psi^* \nabla \psi - \psi \nabla \psi^*). \] (10)

Therefore, the flux can be expressed as

\[ j = (-i\hbar/2m)(\psi^* \nabla \psi - \psi \nabla \psi^*). \] (11)

Now, upon substituting equation 1 into equation 11, the incident flux may be written as

\[ j_1 = 2i\kappa A^2 (-i\hbar/2m) = A^2 \kappa k/m. \] (12)
Similarly, by substituting $\psi_s$ from equation 3 into equation 11, one obtains

$$j_s = \frac{A^2 \hbar k}{m} \frac{f(r)^2}{r^2}.$$  \hspace{1cm} (13)

Finally, using the expressions for the incident and scattered fluxes in equation 4, the differential scattering cross section is given by

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2.$$  \hspace{1cm} (14)

The Schrödinger equation appropriate to elastic scattering of a particle of energy $E$ by a real spherically symmetric potential $V(r)$ is

$$\nabla^2 \psi + \left(\frac{2m}{\hbar^2}\right)(E-V(r))\psi = 0.$$  \hspace{1cm} (15)

In order to solve this equation, $\psi$ must be expanded into a product of radial and angular parts

$$\psi = \sum_L \frac{u_L(r)}{r} P_L(\cos(\theta)).$$  \hspace{1cm} (16)

where $P_L$ is a Legendre polynomial associated with the angular momentum quantum number $L$. Now, by substituting equation 16 into equation 15 and making use of the
orthonormality of the Legendre polynomials, we arrive at the radial wave equation for $u = r\psi$

$$\frac{d^2 u}{dr^2} + \left[ \frac{2m}{\hbar^2} (E-V(r)) - \frac{L(L+1)}{r^2} \right] u_L = 0. \quad (17)$$

For the case with $L=0$, this equation reduces to

$$\frac{d^2 u_o}{dr^2} + \frac{2m}{\hbar^2} (E-V(r)) u_o = 0, \quad (18)$$

which in the asymptotic region becomes

$$\frac{d^2 u_o}{dr^2} + k^2 u_o = 0. \quad (19)$$

This is a standard second-order differential equation and has a general solution

$$u_o = B\sin(kr) + C\cos(kr) = B\sin(kr + \delta). \quad (20)$$

Equating the general solution to the asymptotic form of the total wave function yields

$$B\sin(kr + \delta) = r[Ae^{ikz} + \frac{Af(\theta)e^{ikr}}{r}]. \quad (21)$$

The $L=0$ component of $e^{ikz}$ is found from the expansion
\[ e^{ikz} = \sum_{L} (2L+1) i^{L} j_{L}(kr) P_{L}(\cos(\theta)), \quad (22) \]

and results in

\[ e^{ikz}(L=0) = j_{0}(kr) P_{0}(\cos(\theta)) = \frac{\sin(kr)}{kr}. \quad (23) \]

Next, expressing all the terms of equation 21 in exponential form with \( L=0 \),

\[ \frac{Be^{-i\delta}}{2i} (e^{2i\delta e^{ikr}} - e^{-ikr}) = \]

\[ \frac{rA}{2ikr} (e^{ikr} - e^{-ikr}) + \frac{rA f(\theta) e^{ikr}}{r}, \quad (24) \]

leads to

\[ [f(\theta) \left( \frac{A e^{ikr}}{r} \right)] r = \]

\[ \frac{Be^{-i\delta}}{2i} (e^{2i\delta e^{ikr}} - e^{-ikr}) + \frac{A}{2ik} (e^{ikr} - e^{-ikr}). \quad (25) \]

Since the left-hand side of the above expression just represents the outgoing wave function, all terms in \( e^{-ikr} \) must vanish. This will happen if

\[ \frac{Be^{-i\delta}}{2i} = \frac{A}{2ik}. \quad (26) \]

Then
\[ f(\theta)Ae^{ikr} = \frac{A}{2ik}(e^{2i\delta}e^{ikr} - e^{ikr}), \]  

(27)

which results in the following form for the \( L=0 \) solution for \( f(\theta) \):

\[ f_0(\theta) = \frac{1}{2ik}(e^{2i\delta} - 1). \]  

(28)

The solution to equation 17 for \( L \neq 0 \) may be obtained by following a similar procedure, thus yielding the total scattering amplitude for all partial waves given by (HODG63)

\[ f(\theta) = \frac{1}{2ik} \sum_L (2L + 1)(e^{2i\delta_L} - 1)P_L(\cos(\theta)). \]  

(29)

Upon examination of equation 29, it can be seen that all the physical information from the potential to the cross section resides in the \( \delta_L \)'s. This information is determined by examining how the wave function is affected, or shifted in phase, in the asymptotic region by the presence of the potential. In practice, these phase shifts, \( \delta_L \), are usually used to define the scattering matrix element

\[ S_L = e^{2i\delta_L}, \]  

(30)

where the matrix elements are the amplitudes of the outgoing waves of different angular momenta for unit incoming waves.

In the scattering of charged particles, Coulomb
scattering also has to be taken into account. For a projectile with charge \(ze\) incident on a target of charge \(Ze\), the Coulomb interaction can be expressed as

\[
V_C(r) = \begin{cases} 
Zze^2 \left(3 - \frac{r^2}{R_c^2}\right)/2R_c & \text{for } r < R_c \\
\frac{Zze^2}{r} & \text{for } r > R_c
\end{cases}
\] (31a)

The total scattering amplitude is then the algebraic sum of a Coulomb term \(f_C(\theta)\) and a nuclear term \(f_N(\theta)\). In terms of partial wave expansions (HODG63), the scattering amplitudes in the presence of the Coulomb field are given by

\[
f(\theta) = f_C(\theta) + \frac{1}{2\pi k} \sum L(2L+1)(S_L-1)e^{2i\delta_L} P_L(\cos(\theta)),
\] (32)

where the Coulomb scattering amplitude is

\[
f_C(\theta) = \frac{-\eta}{2k} (\sin(\theta/2))^{-2}\exp[2i\delta_\theta - 2i\eta \ln(\sin(\theta/2))],
\] (33)

with the Coulomb parameter \(\eta\) for the reduced mass of the system \(\mu\) given by

\[
\eta = \frac{\mu Ze^2}{k\hbar}.
\] (34)

Finally, if the incoming charged particle has a spin component \(m\), the partial wave expansion for the scattering
amplitude for an outgoing spin component $m'$ is given by

\[(SCHW67)\]

\[f_{m'm}(\theta) = f_c(\theta)\delta_{mm'} + (1/2i\alpha) \sum_L [4\pi(2L+1)]^{1/2} \times\]

\[e^{2i\delta_L} (S_L^{J-1})(LS0m|Jm)(LSm-m'|Jm)Y_L^{m-m'}(\theta,0), \quad (35)\]

where the last three quantities in equation 35 are the usual Clebsch-Gordon coefficients and the spherical harmonics.

The scattering matrix elements $S_L$ used to calculate the differential cross sections given by equation 14 are found by solving the Schrödinger equation, equation 15, for particles with energy $E$ scattered by a potential $V$. For simplicity, it is assumed that the potential is local and spherical, and has no explicit momentum or isospin dependance. One of the simplest forms for the potential is the Woods-Saxon potential to be discussed in the following section.

2.3 Optical Model

The optical model is based on an analogy between the scattering of nucleons by a potential with the scattering of light by a cloudy crystal ball. To account for the removal of flux from the incident beam due to non-elastic processes, the spherically symmetric potential includes an imaginary part and is of the form
The optical potential \( V(r) \) between the projectile and the target nucleus is the sum of all the interactions between the nucleons in the projectile and the nucleons in the target. This form for the interaction leads to severe complications when it is used in conjunction with the scattering formulas of the previous section, and so it necessitates an approximation for the form of the potential.

On investigation of the scattering of low energy nucleons by a range of nuclei, Barschall (BARS52) concluded that the scattering process is essentially determined by the properties of nuclear matter in bulk, and that the individual structure of the particles enters only as a second order effect. This suggested that the target nucleus acts on the projectile like a potential well, with the formation of compound-nuclear states taken into account by allowing \( V \) to be complex. By refining the potential well on physical grounds to contain a surface diffuseness, the potential can be expressed as

\[
V(r) = (U + iW)f(r,R,a), \tag{37}
\]

where \( f(r,R,a) \) is a form factor characterized by a radius parameter \( R \) and a surface diffuseness parameter \( a \) that measures the distance over which the potential falls from
its maximum value to zero. The exact shape for the form factor is somewhat arbitrary, with the most commonly used shape being that of a Woods-Saxon form (WOOD54)

\[ f(r,R,a) = [1 + \exp((r-R)/a)]^{-1}. \]  

(38)

The optical model form for the potential represents the total averaged effect of all the nucleon-nucleon interactions between the projectile and the target, and hence gives only the gross structure of the scattering cross-sections.

The Woods-Saxon form is usually taken as the shape for the form factor, since Fernbach et al. (FERN55) showed that to first order the real part of the optical potential is proportional to the nuclear density, which is adequately represented by a Fermi distribution. Since the imaginary part of the potential takes account of all non-elastic processes, it is hard to determine a standard shape for it. There is no physical reason to use the Woods-Saxon form for the imaginary part, but it is commonly used for computational convenience. Calculations (GREE68) have shown that the imaginary potential is peaked at the nuclear surface at low energies and spreads through the nuclear volume as the energy increases. Since the scattering of nucleons by nuclei is more sensitive to the absorption in the nuclear surface than to that in the interior, the shape
of the form factor should have a similar surface behavior. Besides the standard Woods-Saxon shape, a surface peaked potential of the form

$$g(r) = -4a \frac{d(f(r, R, a))}{dr}$$

(39)

is also commonly used for the imaginary part of the optical potential, where the factor $-4a$ is included in order to make the maximum value of $g(r)$ equal to unity. Fits (PERE63) have been found to show no preference for either form of the imaginary part as long as the form factor parameters for the imaginary potentials are allowed to differ from those for the real part of the potential.

For projectiles with spin, an additional term is added to the optical potential. It is usually of the form (HODG63)

$$V_s(r) = (U_{so} + iW_{so}) \left( \frac{n}{mc} \right)^2 \frac{1}{r} \frac{d}{dr} f_s(r, R, a) \mathbf{l} \cdot \mathbf{s},$$

(40)

where $f_s$ is the form factor for the spin-dependant term. In practice it is found that the cross-sections are substantially determined by the central term of the potential, while the spin-orbit term determines the polarizations. The inclusion of the spin-orbit term usually produces an improvement to the cross-section fit, but the inclusion of polarization data is required to attribute any
physical significance to the spin-orbit potential. Without the polarization data, the improvement to the cross section fit is usually due to an increased flexibility that covers up deficiencies in the optical model. Therefore, the spin-orbit contribution will be ignored since polarization data was not available for this analysis.

The optical model potential used for this investigation is of the form

\[ V(r) = V_c(r) - \tilde{V}_o(f_o(r))^{a} - iW_s f_w(r) + 4iW_D \frac{d(f_w(r))}{dr}, \quad (41) \]

where \( V_c(r) \) is the standard Coulomb potential given by equation 31, and \( f_o \) and \( f_w \) are the real and imaginary form factors and are assumed to have the Woods-Saxon shape with \( R_{o,w} = r_{o,w}A^{1/3} \). Modifications to the standard Woods-Saxon form for the real well form factor will also be investigated by altering the value of the exponent \( a \), where \( a \) is usually equal 1. Since the elastic scattering is usually insensitive to the exact shape of the imaginary potential, both the volume absorption potential \( W_v \) and the surface absorption potential \( W_d \) will be used; but as customary practice, not both at the same time.

The appropriate optical potential can not be deduced directly from the experimental data. The method used to generate an acceptable potential was to begin with form factor parameters that produced a suitable potential for a
similar situation, and then search or iterate on the numerical values of its parameters by some prescribed method in order to optimize the fit to the data. The search is carried out by calculation at each step of the quantity

\[ \chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{\sigma_{\text{exp}}(\theta_i) - \sigma_{\text{th}}(\theta_i)}{\Delta\sigma_{\text{exp}}(\theta_i)} \right]^2, \]  

(42)

where \( N \) is the number of data points, \( \sigma_{\text{exp}}(\theta_i) \) and \( \sigma_{\text{th}}(\theta_i) \) are the respective experimental and theoretical cross sections at angle \( \theta_i \), and \( \Delta\sigma_{\text{exp}} \) is the uncertainty associated with \( \sigma_{\text{exp}} \), until a minimum value for \( \chi^2 \) is obtained. The quantity \( \chi^2 \) is a measure of the discrepancy between the experimental data and the theoretical values calculated with the assumed optical potential, with the best fit occurring at the minimized value of \( \chi^2 \). The theoretical cross-sections, equation 14, using the scattering amplitude from equation 35, were calculated using the \( S \) matrix obtained by solving the Schrödinger equation with the interaction potential given by equation 41. The calculations were performed on The Ohio State University's Amdahl 470 computer at the Instructional and Research Computer Center using the optical model code SNOOPY6 (SCHW77).
2.4 Double-Folding Model

To better understand the elastic and inelastic scattering processes, Satchler and Love (SATC79b) suggested that a microscopic formalism should be used that is based on the underlying nucleon-nucleon interaction. The double-folding model is an attempt to calculate the first-order term of the elastic scattering optical potential for the colliding ions in a microscopic way. In this model, the optical potential is obtained by averaging an appropriate nucleon-nucleon interaction over the matter distributions for the two colliding nuclei. This potential can be expressed as

\[ U_F(r) = \int dr_1 \int dr_2 \rho_1(r_1) \rho_2(r_2) V(r_{12} = R + r_2 - r_1), \quad (43) \]

where \( \rho_i(r_i) \) is the density of the \( i^{th} \) nucleus and \( V \) is the effective nucleon-nucleon interaction. In order to simplify the calculation, the nucleon distributions are assumed to be spherical and static. This presentation of the double-folding model theory follows that presented by Satchler and Love (SATC79b).

The Hamiltonian for the elastic and inelastic scattering of the two nuclei \( A + A \) can be expressed as

\[ H = H_a + H_A + T_R + V, \quad (44) \]
where $H_a$ and $H_A$ are the internal Hamiltonians for the two separate nuclei, $T_R$ is the kinetic energy operator of relative motion, and $V$ is a potential that describes the interaction between the two nuclei. Associated with $H_a$ and $H_A$ are complete sets of orthonormal states denoted by $\psi_{a_i}$ and $\psi_{A_j}$ that describe the nuclei in states $i$ and $j$ with energies $\varepsilon_{a_i}$ and $\varepsilon_{A_j}$ respectively. If the total wave function $\psi$, which is a solution to the Schrödinger equation $H\psi = E\psi$, is expanded in terms of the internal eigenstates of the two nuclei

$$\psi = \sum_{i,j} \psi_{a_i}\psi_{A_j} x_{ij}, \quad (45)$$

where $x_{ij}$ describes the relative motion of the two nuclei in states $i$ and $j$ respectively, then $x_{00}$ gives the elastic scattering provided that both nuclei are in their ground states ($i=j=0$). Use of the projection operator formalism of Feschbach (FESC62) allows one to project out the elastic term. This operation results in the Schrödinger equation

$$(T_R + U_o)x_{00} = E x_{00}, \quad (46)$$

with the potential $U_o$ given by (SATC79b)

$$U_o = U_F + \Delta U; \quad (47)$$
where

\[ U_F = \langle \psi_{a0} \psi_{A0} | V | \psi_{a0} \psi_{A0} \rangle \]  

(48)

and

\[ \Delta U = \langle \psi_{a0} \psi_{A0} | V Q \frac{1}{E + i \eta - QH} QV | \psi_{a0} \psi_{A0} \rangle \]  

(49)

The operator Q in the above expression projects out the ground state.

In an optical model analysis, \( U_o \) is approximated by a local, complex potential; but in the double-folding analysis, \( U_F \) is assumed to approximate the real part of \( U_o \) while an additional phenomenological imaginary potential is used to account for the effects of coupling to all non-elastic channels contained in \( \Delta U \). For this approach to be accurate \( U_F \) must be real, and the real part of \( \Delta U \) must be negligible. The first part of \( U_o \) (\( U_F \)) is real if the nucleon-nucleon interaction \( V \) is real, while the second part of the potential (\( \Delta U \)) has been shown by Love et al. (LOVE77) to be predominately imaginary in the important surface region and so it can be neglected since the contribution from its real part is small.

The bare nucleon-nucleon interaction is too strong to be used for the optical potential \( V \) because of the strong,
short-ranged nature of the nucleon-nucleon force. Instead, an effective interaction is usually used which is based upon a realistic nucleon-nucleon force. Satchler and Love (SATC79b) proposed that this interaction should be similar to the G-matrix for two nucleons bound near the Fermi surface. The G-matrix that is most commonly used is that of Bertsch et al. (BERT79), which consists of a sum of three Yukawa terms and is commonly referred to as the M3Y interaction. The first term ensures that the effective interaction has the correct one pion exchange potential (OPEP) behavior in the tail region, the second term simulates multiple pion exchange, while the third term is used to improve the fit to the nucleon-nucleon interaction. The G-matrix elements of the interaction were then calculated in a harmonic oscillator basis, while adjusting the strengths of the latter two Yukawa terms in order to obtain the best fit to the G-matrix elements of the nucleon-nucleon force.

The central part of the effective interaction may be written as (GLOV80)

$$v_{12}(s) = v_{00}(s) + v_{01}(s)\hat{\tau}_1 \cdot \hat{\tau}_2 + v_{10}(s)\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$+ v_{11}(s)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\hat{\tau}_1 \cdot \hat{\tau}_2),$$

where all the contributions due to spin-orbit and tensor
interactions have been ignored. The terms with spin $S=1$ and isospin $T=1$ only contribute to the potential when both nuclei have non-zero spin and isospin respectively, and they are relatively unimportant compared to the $S=0$ and $T=0$ terms even when they do contribute to the interaction. For the last reason, and since both targets under investigation have zero spin, only the $v_{00}$ term will be retained. The explicit form for the central component of the M3Y interaction is

$$v_{00}(r) = \frac{7999 e^{-r/0.25}}{(r/0.25)} - \frac{2134 e^{-r/0.4}}{(r/0.4)}$$

where the energies are in MeV and the radii are in fm. Because the first Yukawa term represents the OPEP, which has $\Delta T=\Delta S=1$, it does not explicitly contribute to the nucleon-nucleon interaction and is not included in the above expression for the interaction.

Although the individual wave functions $\psi_a$ and $\psi_A$ for the two nuclei are assumed to be antisymmetric, the expression for the potential in equation 48 does not explicitly account for antisymmetrization effects between the nucleons in different ions. The largest correction to this potential near the strong absorption radius (SAR) is likely to be a single nucleon exchange between the colliding ions, which for nucleon-nucleus scattering is known as single-nucleon knock-on exchange (SNKE). This correction can be well approximated by the pseudopotential (GOLI76).
$J_{ST}(E)\delta(r_{12})$, where the elastic component for a projectile with incident energy $E$ is given by

$$J_{00}(E) = -421(1 - 0.017E/A_p) \text{ MeV fm}. \quad (52)$$

The full effective interaction potential for this $^7\text{Li}$ elastic scattering analysis at 99 MeV is then given by

$$v_{00}(r) = \frac{7999e(-r/.25)}{(r/.25)} - \frac{2134e(-r/.4)}{(r/.4)} - 3208(r). \quad (53)$$

The matter distributions of the projectile and the target are the other essential parts in the folding model. The most direct measure of the nuclear densities comes from electron scattering, and this source primarily provides information about the charge density distribution of the proton. Information about the density distribution of the neutron is much more difficult to come by, although it is common practice to assume that the two distributions are nearly identical when $N=Z$. Since the scattering of ions, and especially heavy ions, is mainly sensitive to the tail of the nucleon-nucleon potential, this region is especially important. The main contribution to the folded potential near the SAR comes from the tail and surface region of the nuclear densities, and this results in the condition that the densities must have the correct root-mean-square (rms) radii (COOK84a).
Various prescriptions exist for the construction of nuclear densities. The most common shapes used for the matter distributions are a Gaussian distribution

\[ \rho(r) = (A + Br^2)\exp(-\alpha^2 r^2) , \]  

(54)

where \( \alpha \) is adjusted to reproduce the correct rms radius, and a Fermi distribution

\[ \rho(r) = \rho_0 \left[ 1 + \exp\left(\frac{r - R_m}{a_m}\right) \right]^{-1}. \]  

(55)

For both of the above distributions, the parameters \( A, B, \) and \( \rho_0 \) are adjusted they so that they fulfill the normalization condition

\[ \int \rho(r) r^2 dr = A/4\pi. \]  

(56)

Since the M3Y interaction is real, the double folded potentials calculated using this interaction are also real. To account for all non-elastic processes, an imaginary part must be added to the double-folded potential. This part is generally treated phenomenologically since better fits are usually obtained with a Woods-Saxon shape than the same shape as the real folded potential, with both the surface form and the the volume form being acceptable for the imaginary potential. The double-folded potential, \( U_F \) in
equation 43, is calculated by the computer code DFPOT (COOK82c) for appropriate projectile and target density distributions. The folded potential is then used as input to the optical model program SNOOPY6 in which the parameters of the expression

\[ U(r) = N_F U_F(r) - iW_s f(r, R, a) \]  \hspace{1cm} (57)

are searched on in the same manner as discussed in the previous section, where \( N_F \) is a normalization parameter which may be adjusted in order to produce a better fit to the data. If the double-folded model correctly predicts the interaction potential for the two colliding ions, the value of \( N_F \) should be equal to 1.

2.5 Coupled-Channels Model

\(^7\text{Li}\) is the lightest of all projectiles that possesses a low-lying bound collective state, with this state occurring at an energy of only 0.478 MeV. If there exists a strong coupling between the elastic and inelastic channels in the lithium nucleus, this coupling may have an effect on the elastic scattering process and would necessitate that a coupled-channels calculation be performed in order to correctly describe the elastic scattering. Furthermore, the ground state may be coupled to itself during the scattering via the ground-state quadrupole re-orientation matrix.
element since the $^7\text{Li}$ nucleus is non-spherical and has a ground-state spin of 3/2. This effect can also be accounted for in a coupled-channels calculation.

Inelastic scattering is the simplest type of the non-elastic reactions. In the usual case, the incident nucleus interacts with the target and usually gives the target a sufficient amount of energy to raise it to an excited state before proceeding on unchanged except for a reduction in energy. The cross section for this process is usually calculated as a perturbation to the elastic scattering theory; but this method is inadequate when the coupling between the elastic and inelastic channels is strong. It is then necessary to solve the appropriate coupled-channels equations for the wave functions in the elastic and inelastic states, which will be described below for a neutral, spinless projectile. This treatment follows that given by Hodgson (HODG71).

If $T$ represents the kinetic energy operator of the incident particle and $V$ represents the interaction between the incident particle and the target nucleus, the Schrödinger equation for the whole system can be expressed as

$$[T - V(\vec{r},\xi) + H(\xi)]\psi(\vec{r},\xi) = E\psi(\vec{r},\xi),$$  \hspace{1cm} (58)$$

where $\xi$ represents all the nuclear coordinates of the target.
nucleus and $H$ is the nuclear Hamiltonian for the nuclear states $\chi_\alpha(\xi)$ which are defined by

$$H(\xi)\chi_\alpha(\xi) = \epsilon_\alpha \chi_\alpha(\xi). \quad (59)$$

By expanding the total wave function $\psi$ in terms of the target and projectile states,

$$\psi(\vec{r}, \xi) = \sum_\alpha \psi_\alpha(\vec{r}) \chi_\alpha(\xi), \quad (60)$$
equation 58 can be rewritten as

$$[T - \mathcal{E} + \epsilon_\alpha] \psi_\alpha(\vec{r}) = \sum_\alpha' V_{\alpha\alpha'}(\vec{r}) \psi_{\alpha'}(\vec{r}) \quad (61)$$

where

$$V_{\alpha\alpha'}(\vec{r}) = \int \chi^{*}_\alpha(\xi) V(\vec{r}, \xi) \chi_{\alpha'}(\xi) d\xi. \quad (62)$$

The angular coordinates can then be removed through the partial-wave expansion

$$\psi(\vec{r}) = \sum_{L} \frac{(u(r))}{r} Y^{M}_{L}(\theta, \phi) \quad (63)$$
to yield the radial wave functions for the elastic and inelastic channels by the coupled equations (HODG71)
\[
\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + k_\alpha^2 - W_{\alpha\alpha'}(r)u_{\alpha}(r) = \sum_{\alpha'} W_{\alpha\alpha'}(r)u_{\alpha'}(r), \tag{64}
\]

where

\[
W_{\alpha\alpha'}(r) = (2m/\hbar^2) \sum_{L\Lambda} Y_L^{M*}(\theta,\phi)V_{\alpha\alpha'}(r)Y_L^M(\theta,\phi) d\Omega. \tag{65}
\]

The above set of equations are solved by integrating them numerically and then matching them to known asymptotic forms of the wave functions in the asymptotic region.

The exact solution to the set of coupled equations becomes impractical due to time considerations, and so a suitable approximations are normally employed. One approximation involves truncating the set of equations to a small number of channels, and then letting the effects of all the other channels be accounted for by allowing the interaction to be complex. The accuracy of the results obtained with this simplification will depend on the number of channels that are actually used and the strengths of the couplings between them. If a non-negligible coupling strength exists between the ground state and any state that is not explicitly included in the coupling scheme, the calculations will not yield accurate results.

In order to use the expression in equation 64 for a particular reaction, the matrix element \( V_{\alpha\alpha'} \) must be determined. This will involve choosing suitable forms for
both the interaction potential $V(\mathbf{r}, \xi)$ and the nuclear wave functions $\chi_a(\xi)$. The model used to describe the excited states of a nucleus depends on the character of the nucleus concerned. The simplest model to use is a collective model for the wave functions for nuclei that display collective or rotational motions. For a rotational motion, the wave function can be expressed as

$$\psi_I^k = ((2I+1)/8\pi^2)^{1/2} D_{k0} I(S),$$

(66)

where the $D$'s are elements of a rotation matrix.

A simple model for the interaction potential involves using a deformed optical potential. If a rotational model is considered for the wave functions, the nucleus is considered to be permanently deformed with its radius given by

$$R(\theta, \phi) = R_0 [1 + \beta Y_2^0(\theta, \phi)],$$

(67)

where $R_0$ is the radius of the spherical nucleus and $\beta$ is the deformation parameter. By assuming that the nuclear potential depends on the distance from the nuclear surface, the potential can be expanded to first order in $\beta$ about the surface to yield the deformed potential

$$V(\mathbf{r}, \xi) = V(r-R_0) - \beta R_0 Y_2^0(\theta, \phi) \frac{dV}{dr}.$$  

(68)
The first term in the above expression is the usual optical potential, while the second term accounts for the coupling between the elastic and rotational inelastic channels. A similar expression involving the deformation parameter $\beta$, expressed in terms of the dynamic distortion parameters $a_m$, can be derived for a nucleus that undergoes vibrational motions.

When using either of the two collective models, the only additional parameter that is introduced into the calculations is the deformation parameter $\beta$. The differential cross sections for the elastic and inelastic scattering can be calculated by using an appropriate value for $\beta$, which can be determined from other phenomena or experiments, along with the undeformed optical potentials. Even if the value for $\beta$ is unknown, calculations using different values of $\beta$ could be performed until a reasonable fit is obtained for the data under investigation. In the case of strong coupling between the elastic and inelastic states ($\beta>0.3$) (MARM70), the perturbation of the elastic channel by its inelastic companions complicates the determination of the potential parameters since the scattering calculated with the coupled-channels approximation is not the same as that obtained with a simple optical model calculation. For this condition, a systematic variation of the optical model parameters along with the deformation parameter in the coupled-channels calculations
must be initiated in order to obtain the best simultaneous fit to the elastic and inelastic scattering differential cross sections. In fact, Nishioka et al. (NISH84) found that the inclusion of a low-lying inelastic state in a coupled-channels calculation yielded an improved fit to the elastic scattering data.

The simplest inelastic scattering process that can be analysed by the strong coupling formalism is that leading to the excitation of the lowest $2^+$ state in even-even nuclei. In performing these calculations, an approximation is usually made in which the set of equations is truncated, with the effects of all the channels that are not explicitly included taken into account by letting the potential $V$ in equation 58 be complex. For this process, it is usually acceptable to neglect any spin-orbit contributions as long as the data is confined to differential cross sections (HODG71). The effects due to Coulomb excitation are also negligible for light projectiles at high energies (COOK82b). It is observed for this type of excitation that the rotational and vibrational models yield almost identical results (HODG71). A first approximation to the optical potential may be obtained by a standard optical model analysis in which the parameters are adjusted in order to produce a fit to the experimental elastic data. If the coupling to the inelastic channels is weak ($\beta$ small), the elastic cross section will be unchanged and the inelastic
cross section will be proportional to $\beta^2$ when applying this strong coupling model. On the other hand, both the deformation parameter and the optical model potential parameters will have to be adjusted when the coupling is strong.

If the projectile has a low-lying collective state, then coupling to its excited state must also be taken into account. The calculation of inelastic scattering involving excitation of the projectile is, in principle, no different to that involving excitation of the target, except for the fact that the angular momentum of the target does not change while that of the projectile does (COOK82b). While target excitation involves angular momentum transfer, projectile excitation involves spin transfer. A coupling diagram for the inelastic scattering to the first excited states of two even-even nuclei is shown in Figure 1a. In this figure, the horizontal lines represent specific states of the nuclei while the vertical lines represent couplings between the scattered states. Each scattering state is labeled by the quantum number for the angular momentum of the target $I$ and the angular momentum or spin of the projectiles, and the matrix element couplings are labeled by the amount of angular momentum $l$, spin $s$, and total angular momentum $j$ transferred for the excitation respectively. The circular arrows connecting the states to themselves are the reorientation matrix elements, which are allowed for states
Figure 1: Coupling diagrams

(a) two even-even nuclei
(b) 7Li (spin 3/2) and even-even nucleus
with angular momentum greater than one-half.

When a nucleus has spin $I > 0$, it may also possess $2^\lambda$-pole moments with $\lambda < 2I$. If the interaction also possesses a $2^\lambda$-pole moment, only the even values of $\lambda$ are allowed to contribute to elastic scattering (COOK84a). This $2^\lambda$-pole term gives rise to angular momentum transfer $l = \lambda$ and contributes incoherently to the elastic cross section, with the quadrupole term $\lambda = 2$ having the largest contribution. This contribution is sometimes referred to as a reorientation effect since it may reorientate the projection of the nuclear spin.

The coupling diagram for the nuclei involved in this experiment is shown in Figure 1b. The $^7\text{Li}$ projectile has a ground state spin-parity of $3/2^-$ and a first excited state spin-parity of $1/2^-$. Both the $^{12}\text{C}$ and the $^{58}\text{Ni}$ nuclei are even-even nuclei with spin-parity states of $0^+$ and $2^+$ for their respective ground states and first excited states. For this coupling scheme, the ground state of the lithium projectile has an allowed reorientation matrix element and this may have a significant effect on the elastic scattering. Since the amount of computation increases non-linearly with the number of couplings, simplifications should be investigated. The simplification used in this investigation will be that the coupling to the first excited state of the target nucleus will be ignored. The coupled-channels calculations will be performed with a
modified version of the computer code CHUCK3 (COMF81). This version is capable of using either phenomenological optical model potentials or folding-model potentials for the interaction. The use of folding model potentials is discussed below.

The failure of the M3Y interaction in the double-folding model for 7Li projectiles may also be due to coupled-channels effects (HNIZ81), in particular, the ground-state re-orientation matrix element. To see if coupled-channels effects are responsible for the required renormalization of 0.5, the folding model potential will be used for the interaction potential in equation 68.

Following the works of Cook et al. (COOK83), the ground-state or monopole part of the folded potential will be the same as the one discussed in the previous section while the transition density for the quadrupole term in the 7Li ground state will be calculated using a macroscopic transition density of derivative form given by

$$\rho_2(r) = \delta_2 \frac{d}{dr} \rho_0(r), \tag{69}$$

where \(\rho_0(r)\) is the ground-state density and \(\delta_2\) is the deformation length. The quadrupole deformation length is determined by normalization to the intrinsic quadrupole moment \(Q_{20}\) of 7Li.
Assuming a rotational model for $^7$Li, the intrinsic quadrupole moment $Q_{20}$ is related to the experimental value of the static moment $Q_2 = -4.5$ e-fm$^2$ (NIFT71) by $Q_{20} = -5Q_2$. The real part of the quadrupole potential is calculated by folding the effective interaction of equation 53 with the quadrupole density of the projectile and the spherical density of the target. The real potential is finally multiplied by the same renormalization factor $N_R$ as the real monopole potential. For the imaginary quadrupole potential, a Woods-Saxon derivative form is used along with a variable deformation length $\delta_I$. The phenomenological parameter values in the imaginary potential are usually taken to be the same as those found for the elastic monopole case. If the coupling to the ground-state quadrupole moment is responsible for the required renormalization of the folded potentials, the inclusion of this coupling should enable the real folded potentials used for describing the elastic scattering to be used with a "successful" normalization of $N_R = 1$.

In a multipole expansion of the effective interaction, only the $L=0$ term contributes to the elastic scattering. The terms of the interaction with $L>0$ apply to other processes. For an inelastic transition of multipolarity $L$, the $2^L$ component of the interaction is selected, with the
main contribution for a single transition to a collective state coming from the $S=T=0$ part of the effective interaction (C00K84a). In computing the transition density for the excited nucleus (either target or projectile), the transition density used should reproduce the charge form factors for the transition obtained by electron scattering. The most important quantity to obtain correctly is the $B(EL)$ value (C00K84a). The transition density for the excitation is also given by equation 69, with its deformation length determined by the normalization to the $B(EL)$ value,

$$B(EL; J_1+J_f) = \frac{e^2}{2J_1+1} \left( \frac{Z}{A} \int \rho_L(r) r^L dr \right)^2$$  \hspace{1cm} (71)

The real folded form factor is obtained by folding the ground-state density of the unexcited nucleus and the effective interaction $v_{oo}$ with the transition density for the excited nucleus. A Woods-Saxon derivative form multiplied by a variable deformation length $\delta_I$ is also used for the imaginary potential. As with the case for the quadrupole potentials, the same values are used for the normalization factor $N_R$ and the Woods-Saxon imaginary form factor parameters for both the ground-state and excited-state potentials. If the coupling to the low-lying collective state of the $^7$Li projectile is responsible for the required renormalization of the folded potentials, the inclusion of the coupling to this projectile excitation
state should allow the use of $N_R=1$ without destroying the fit to the elastic cross sections.
3.1 Introduction

The $^7$Li scattering data was obtained at the Indiana University Cyclotron Facility. The accelerator consists of three stages shown schematically in Figure 2 and is comprised of a preinjector or ion source, an injector cyclotron, and a main cyclotron capable of producing a beam of $^7$Li$^{3+}$ ions at the desired energy of 100 MeV. Details of the accelerator design may be found in an IUCF internal report (IUCF77).

The facility at IUCF was chosen because it was able to produce the required $^7$Li beam. A schematic diagram of the experimental area is shown in Figure 3. This experiment made use of the magnetic spectrograph, whose characteristics are listed in another report (IUCF73). The spectrograph consists of an entrance quadrupole, a split dipole, and an exit multipole magnet. A set of kinematic correction and trim coils are located in the middle of the spectrograph at a vertical cross-over point in order to maintain a constant focal plane, thus allowing a focal-plane detector to remain a fixed distance behind the exit multipole. Field clamps
Figure 2: Arrangement of IUCF accelerator stages
Figure 3: General layout of IUCF experimental area
Figure 4: Schematic of QDDM magnetic spectrograph
are also used in order to reduce fringe field effects. The spectrograph vacuum system is attached by a metal sliding-seal band to a stationary 24" diameter target chamber. This band allows vacuum to be maintained while the spectrometer is rotated around the chamber. Located at the other end of the spectrometer at the focal plane is a position sensitive helical wire gas proportional chamber. A schematic of the spectrograph arrangement is shown in Figure 4.

The target chamber is 24" in diameter, 14" deep, and includes a target ladder capable of holding 4 separate targets that are held in standard target frames. The target ladder is externally controlled with the ability to choose both the target to be bombarded and the angle of the target relative to the beam direction. Two different Faraday cups are available for beam integration. A Faraday cup located well outside of the scattering chamber in a shielded beam dump and connected to the chamber by standard beam pipe was available; but since the experiment required small scattering angles as well as a few negative angles in order to check the absolute angle of the spectrometer, a split Faraday cup placed in the rear of the scattering chamber was utilized. This split internal Faraday cup was located 9 7/8" behind the target ladder, and its 0.8" diameter surface presented a 5.15 m sr solid angle for beam integration. The split in the Faraday cup provided a method
for keeping the beam positioned on the same area of the target. This was accomplished by initially steering the beam until the integration counts from the left side of the Faraday cup were comparable to the counts from the right side of the cup, which was insulated from the left side.

The targets used in this experiment were a 2.08 mg/cm\(^2\) natural carbon target and a 8 mg/cm\(^2\) isotopically enriched \(^{58}\)Ni target. A 28.5 mg/cm\(^2\) \(^{58}\)Ni target was also placed on the target ladder for possible use at large angles where the count rates are small, and for use as a thickness check for the 8 mg/cm\(^2\) nickel target. The bottom position on the ladder was occupied by a scintillator, which was used to aid the machine operator in aligning the beam. The angle that the targets made relative to the beam was varied during the experimental data runs between 0 and 30 degrees, with an angle of 0 degrees being defined as the beam being parallel to the normal to the target surface. This rotation was utilized in order to reduce kinematic broadening of the projectile's image on the focal plane, as well as prevent the projectiles and scattered particles from striking the target ladder holder at larger scattering angles. This rotation can also be used to increase the effective thickness, and hence the number of scattering centers, of the targets in order to increase the count rates at large angles.
The focal plane of the spectrograph makes an angle of 44° with respect to the emerging beam. The detection system used for the spectrometer was a helical wire gas proportional chamber. This system was chosen by IUCF because of its high position resolution, near unit efficiency, and its ability to accommodate reasonably high count rates (BENT73).

The cathode wire planes which form the helix consist of a 0.05 mm diameter copper wire wound with a 0.5 mm pitch. The wire is wrapped around insulating supports in which are located eight stainless steel anode wires. The system is enclosed by a thin Kapton window and filled with a mixture of argon and carbon dioxide gases. The choice of a thin wire for the helix resulted in good position resolution, but made particle identification difficult (BENT73). Since ⁷Li has a small break-up energy of 2.47 MeV into an alpha particle and a triton, the possibility of other particles entering the spectrometer is enhanced. If the background particles meet the momentum and charge requirements set for the elastically scattered ⁷Li projectiles in the spectrograph, they will also be transported through the spectrometer and will create signals in the helical wire detector. To distinguish between the desired ⁷Li signals and the background signals, a ΔE-E particle identification detector system was used. This consisted of a plastic scintillator paddle of sufficient area that was located
Figure 5: Plastic scintillator/light guide schematic
behind the helical detector and employed a single light pipe unit which coupled to a single 3" photomultiplier tube as shown in Figure 5. The thickness of the scintillator used in this experiment was 1/4", which was sufficient to stop the $^7\text{Li}$ particles of interest. The anode of the proportional counter supplied the $\Delta E$ information with the scintillator paddle supplying the $E$ information. The combination of the $dE/dX$ and the $E$ signals from the helix anode and the plastic scintillator respectively, along with the spectrograph's momentum analysis, permitted sufficient particle identification for the experiment. Shielding to prevent stray background radiation from entering the detectors was obtained with suitably placed bricks around the detectors. Monitoring of the background was accomplished with a second plastic scintillator, which was located directly behind the first scintillator paddle.

3.2 Electronics

The electronics used in the experiment are shown in Figure 6. When a charged particle passes through the helical wire chamber, it ionizes the gas mixture inside the helical detector. Since the cathode and anode wires of the helix are held at a potential difference of 2100 V, the gas ionizes and sets up pulses whose amplitudes are independent of the location of the incident particle in both the helix cathode and the anode. The anode pulse is used in
Figure 6: Magnetic spectrograph electronics diagram
conjunction with the signal from the scintillator to isolate the desired events. The positive pulse set up in the helix cathode travels outward towards both ends of the coil. These two signals from the helix are then routed to a time-to-amplitude converter (TAC) with the signal from the right side of the helix acting as the start signal while the delayed left output acts as the stop signal. The position of the particle along the helix can be determined by the time difference between the two signals arriving at the TAC. The delay in the pulse from the left or lower end of the helix effectively shifts the TAC signal corresponding to the center of the helix to one end of the helix so that the start pulse will always arrive at the TAC before the stop pulse, regardless of the location of the incident particle. The TAC is gated by requiring that the discriminator signals from both the first or forward scintillator and the anode of the helix arrive at the fast/slow coincidence module (FSC) simultaneously. This coincidence requirement also reduces the neutron and gamma background. The TAC output is started by a separate delayed output from the FSC, with the amount of delay set so that the TAC can receive and process both the start and stop signals. To help eliminate false TAC signals produced by simultaneous events on the helix, the TAC output is fed to a linear gate and stretcher (LGS) operating in the gated mode. The gate signal to the LGS is produced by demanding a triple coincidence between the
linear outputs of the first scintillator and the helix anode with the logic pulse produced by the FSC. Finally, the real gated TAC signals are sent to the 1st analog-to-digital converter (ADC) of the Tennelec PACE (pulse analog-to-digital converter and encoder) system, where the PACE system transfers the digital information serially to a computer through a CAMAC interface.

In order to reduce the background to the $^7\text{Li}$ scattering spectra, particle identification was employed using the modules in the lower portion of Figure 6. The linear output signal from the helix anode and the photomultiplier base of the 1/4" scintillator (#1) are amplified and shaped in the delay line amplifiers and passed through linear gate modules, which are gated by the FSC module used to gate the TAC. The signals then encounter timing single channel analyzers (TSCA) that ensure that the signals fall within the proper amplitude window. Their outputs are then routed to a multi-purpose coincidence module (MPC) which requires a coincidence with a delayed FSC signal. A dc-coupled anticoincidence input signal of +5 V is also supplied to the the MPC to inhibit false output pulses created by either a dc voltage or an input pulse that overlaps the period of coincidence of the input pulses. The output of the MPC is used to gate the linear signals from the anode and the first scintillator to the ADC's of the PACE system, with the linear signal from the helix anode being fed into ADC 3 and
the linear signal from the forward scintillator being fed into ADC 2. The information supplied to these two ADCs is then received by the data-acquisition program DERIVE operating in the spectrograph mode, which was programmed to run on a Harris minicomputer. The program produces a two-dimensional particle identifier array used to gate data into other arrays. A sample identification plot is shown in Figure 7. By setting a window around the group of events that correspond to $^7\text{Li}$, the acquisition program routes only event signals that fall within the window into a separate array. A comparison between the raw spectrum and the spectrum employing the window is shown in Figures 8 and 9 for the same run. This run has poor statistics, and as can be seen in the two figures, identification of the peaks of interest would be for all practical purposes impossible without the computer gate.

The appropriate delay times required for the experiment in order for the real events to pass through the electronic arrangement were set by use of a tail pulser. The pulser is also used during the acquisition runs to help determine the dead time of the acquisition system. By externally triggering the pulser with one of the two integrator signals, a pulser signal can be introduced into the system along with the discriminator signals from the detectors. The pulse height is set such that the pulser events will not be confused with the real events as seen in Figure 7.
99 MeV \(^7\text{Li} + 58\text{Ni}\)

- **Pulser events**
- **\(^7\text{Li}\) events**
- **Energy**
- **Non-identified events** (most likely produced by \(^8\text{Li}\) ions)

**Figure 7: Particle identification spectrum**
$^{58}\text{Ni}(^7\text{Li},^7\text{Li})^{58}\text{Ni}$

99 MeV

$\theta_{\text{lab}}=40^\circ$

Figure 8: Ungated spectrum
\(^{58}\text{Ni}(^{7}\text{Li},^{7}\text{Li})^{58}\text{Ni}\)

99 MeV

\(\theta_{\text{lab}}=40^\circ\)

Figure 9: Computer-gated spectrum.
A separate window is then drawn around the pulser peaks so that the pulser events will be sent into its own array. The system dead time is found by comparing the number of events in the pulser array to the number of trigger pulses fed into the system, which were counted on a scaler.

3.3 Data Acquisition

Three separate data runs on the $^{58}$Ni target and one run on the natural carbon target were taken. For each run a beam of $^{7}$Li$^{3+}$ ions at approximately 100 MeV was delivered on target by the machine operator. The plastic scintillator on the target ladder was used for initial alignment, and then the beam's position on target was monitored during the data runs by examining the integrator currents on each of the two halves of the split Faraday cup. Approximate current values for the quadrupole and multipole magnets were calculated and then set according to beam energy and target nucleus, while exact values for both the dipole magnets and the kinematic correction coils were used for each scattering angle. For each run, the center of the helix was chosen to correspond to an excitation energy of 1.5 MeV so that the first excited state of the target would also be included in the spectrum. The last step before the actual data acquisition involved taking a sample spectrum in order to set the digital windows around the $^{7}$Li and pulser events.
During the data runs, three separate parameters were varied in order to achieve a count rate which neither caused a large dead time in the computer processing system nor required a lengthy counting time. For low counting rates, the entrance slits to the spectrograph were opened from their normal horizontal setting of 18 mrad, which produces an entrance solid angle of 1.08 msr when combined with a vertical window of 60 mrad, to a horizontal opening of 30 mrad that produced a new solid angle of 1.56 msr. The target angle was also increased which improved the count rate. A thicker target was also tried, but it degraded the resolution so badly that the peak belonging to the first excited state of the projectile was buried under the elastic peak. For high count rates, the entrance slits were both closed down to yield a new entrance solid angle of only 0.36 msr and the beam was reduced by the machine operator so that there existed a smaller beam current on the target.

Data was gathered for each angle with the requirements that the counting statistics be less than 3% (preferably 1%) in the elastic peak. For angles with low count rates, generally occurring for laboratory angles greater than 40°, time was usually the major consideration in terminating any run. For all angle runs, the system dead time was kept below 10% and usually was no greater than 2%. After each run, the contents of each ADC array was recorded onto a 9-track dump tape to be analyzed later on an Eclipse $$/130
computer system located at the OSU Van de Graaff Laboratory.

Three separate data runs were taken on the 8 mg/cm$^2$ $^{58}$Ni target, with the angular region under investigation depending on the intensity and stability of the lithium beam on target. These three separate runs provided an angular range in laboratory angle from $6^\circ$ to $57.2^\circ$ in mainly $1^\circ$ intervals. Only one data run was taken on the 2.08 mg/cm$^2$ natural carbon target, but since the beam intensity was adequate, a range of lab angles from $6.5^\circ$ to $53^\circ$ was obtained at 1 to $1\frac{1}{2}^\circ$ intervals. Angles were set manually by physically rotating the spectrometer around the scattering chamber and reading the angular position of the spectrometer from a digital readout fed by shaft encoders that displayed the angle to two decimal places. A zero angle offset from the read out value was checked for by assuming isotropy in scattering angle about the true 0 degree angle. Several spectra were taken at angles from -8 to -15 degrees, and a modified elastic peak cross section for these angles were compared to the cross sections for the same angles on the positive or standard angle side. Upon examination of plots made from the above two angular regions, it was noted that they almost precisely overlapped when the negative angle values were shifted approximately $2/3$ of a degree towards the positive values. This meant that there was a $1/3^\circ$ offset from the readout values towards increasing angle values. This procedure was repeated for
each data run with similar results.

Monitoring the condition of the detectors and the electronics was done with the use of scalers. Scaler values were obtained for the two scintillators, the helix anode, and the number of fast/slow coincidences; as well as running time, integrator counts, and pulser counts.

3.4 Errors

Two current integrators with a stated precision of 0.02% for all scales were used to provide the measurement of beam charge on target. Uncertainties in the integrated charge also arise from imprecision in the extent to which the integrator's zeroing feature offsets leakage currents in the Faraday cup, as well as drifts in the zero setting during a run. This uncertainty is estimated to be less than 1%. Errors associated with counting statistics for the integrator will be included with the uncertainty for each angle.

The errors due to target thickness were hard to determine since an additional calibration experiment was not attempted. The two targets were standard targets used and kept at IUCF. As mentioned earlier, the stated thickness of the targets were 2.08 mg/cm² and 8 mg/cm² for the carbon and nickel targets respectively. Since other experimenters have used these targets and have had no real difficulty with their stated values, the thickness are assumed to be correct
within a small error. A rough check of the thickness for the nickel target was attempted by comparing the differential cross sections at a few selected angles to the values obtained with another nickel target of the same isotope with a stated uncertainty of 4%, but no conclusive results were obtained. Errors in the angular position of the targets with respect to the beam axis introduce a further uncertainty in the effective thickness. The target ladder is attached to the scattering chamber by means of a rotating holder that is located on top of the chamber. This holder can be externally rotated, and is marked in half degree intervals. The target angle is then read off this scale and can be set to within an uncertainty of a half degree. This uncertainty results in a very small error even at a target angle of 30°.

The efficiency of the helical wire gas proportional chamber is another area for possible error. An assumed 100% efficiency will be used since that was one of the reasons that this particular detection system was chosen. The actual efficiency of the detector was not determined, but the assumption of 100% efficiency in the detection system is standard procedure. Therefore, the uncertainty in the efficiency is taken to be small.

The entrance solid angle to the spectrometer from the scattering chamber also has to be considered. Since the position of each slit is calibrated to three decimal places
and set using a micrometer, the associated error should be small and equal to the inherent error associated with all geometrical measurements. Also, since the QDDM spectrograph system is used by many experimenters, a substantial error in the slits would quickly be noticed.

Other areas for error are fluctuations in the beam axis position about the reference axis, non-uniformity in target thickness, and deterioration of the target during the run. The non-uniformity in target thickness only has an effect if the beam strikes the target at different positions during the run, but this possibility was minimized by keeping the beam centered on the split Faraday cup. Monitors could have been installed to check for target deterioration; but since the intensity of the beam on target was almost always less than 100 nA, this error source should not have been a problem.

Most of the errors or uncertainties were estimated to be small, and so an overall or absolute error of 5% for the carbon data and 8% for the nickel data was assigned to each run. This value was meant to include all of the above mentioned sources of error except for the statistical uncertainties in the counting rates for both the focal plane detector and the charge integrators. The 5% uncertainty estimate agrees with that obtained for the 99 MeV $^6$Li runs also taken at IUCF (SCHW81).
4.1 Introduction

Now that the raw data have been acquired, the task of reducing the spectra to differential cross sections must take place. This will be accomplished by using the spectra array containing the computer gated $^7\text{Li}$ events and numerically integrating the area under the elastic peak. Possible contributions to the elastic peak due to background noise, the 0.478 MeV excited state of $^7\text{Li}$, and any target contaminations will be subtracted out. Once the peak area, and hence the number of elastically scattered events, has been determined, it is a simple procedure to transform the peak values into differential cross sections. Since multiple spectra exist for many angles, a method for averaging and normalizing the multiple values will also be considered. This will especially be true for the $^{58}\text{Ni}$ angles since three separate data runs were taken on this target in order to cover the entire angular distribution.

After the data have been reduced to differential cross sections, complete with associated errors, the data will be analyzed with six-and-nine parameter optical model
potentials. Additional fitting attempts will be tried with a double-folding model for generating the real part of the optical potential, and a coupled channels analysis that will include the first excited state of the $^7$Li projectile. This chapter will describe the data analysis methods in detail along with the results.

4.2 Data Reduction

Reduction of the raw data was performed using a modified version of the program GAUFIT (KONC82). GAUFIT is a peak fitting program that can fit a spectrum with up to five Gaussian peaks on a variety of backgrounds. The fit is accomplished by employing the chi-square minimizing routine STEPIT (CHANOO). GAUFIT returns the number of counts in the peak (peak area), the full width at half maximum (FWHM) of the peak, and the position of the peak's centroid.

For each spectrum, an initial background, which was usually taken to be quadratic in shape, was calculated before the fitting procedure was attempted. This was done since the routine varies the peak parameters before the background parameters, thus causing wide peaks usually on top of a negative background. After an initial reasonable background was determined for the region of interest, the region of the spectrum to be fit was defined. The initial position and widths of all peaks in the region of interest were input to the program by on-line cursor selection before
the actual fitting searches began. Once an acceptable fit to the dominant peak, usually the elastic peak, had been obtained, the widths of all the other peaks were set equal to that of the dominant peak and kept fixed by the use of a mask. The fitting then continued until no appreciable improvement in chi-squared is found. The fitting region for all nickel spectra contained four peaks as shown in Figure 10, consisting of the elastic and first excited state of the target along with an excited $^7\text{Li}$ projectile state for each of the two $^{58}\text{Ni}$ states. For the natural carbon target, the fitting region usually consisted of only the $^{12}\text{C}$ elastic peak and the $^7\text{Li}$ inelastic peak, except at forward angles ($\theta_\text{L}<30^\circ$) where the scattering kinematics brought peaks from $^{13}\text{C}$, along with peaks from possible target contaminants, into the upper, or higher energy, part of the spectrum. A sample carbon spectrum that contains a contaminant peak is presented in Figure 11.

At large angles, the above procedure ran into problems because of poor counting statistics. Various measures were taken in trying to overcome the fitting problems created by small non-distinct peaks and are explained in Appendix A.

The actual number of particles scattered into each peak was determined by the following method. After a suitable fit was obtained for each spectrum, the summing option of GAUFIT was utilized. In the modified version of this option, the total number of counts in any portion of the
Figure 10: Sample GAUFIT-analyzed nickel spectrum

The above figure shows the peaks involved in the fitting region for a typical spectrum produced by scattering from the nickel target.
The above figure shows the peaks involved in the fitting region for a typical spectrum produced by scattering from the carbon target.
fitted region could be determined by denoting the portion of
the region of interest with cursors and summing all the
counts that lie between the cursor marks. The number of
counts belonging to the background are also obtained by
integrating the background function determined during the
fitting option between the chosen cursor limits. The number
of real events will just be the difference between the above
two values. Finally, the number of events in any given peak
is

\[ P_i = (S - B)A_i\left(\sum_{j=1}^{N} A_j\right)^{-1} \]  

(72)

where \( S \) = the total number of counts in the region, \( A_i \) = computed area of the peak of interest, \( B \) = the number of
background counts in the region, and \( A_j \) = is the computed
area of the \( j^{th} \) peak. The uncertainty in each peak was
taken to be a quadrature combination of the statistical
uncertainties in the peak and background and is computed by

\[ \Delta P_i = \sqrt{\left(\sqrt{P_i}\right)^2 + (-B)^2} = \sqrt{P_i} + B. \]  

(73)

This probably slightly underestimates the actual statistical
error.

The last task in reducing the data is to transform the
peak values into differential cross sections. The
differential cross section in the lab is given by the
following expression:

\[ \sigma_L(\theta) = \frac{d\sigma}{d\Omega} = \frac{NeZ_pC}{Qn'\varepsilon d\Omega} \quad (74) \]

with \( N \) = number of particles scattered by the target at angle \( \theta \), \( e \) = electronic charge constant \( 1.602 \times 10^{-19} \) Coulombs, \( Z_p \) = charge of the projectile, \( C \) = percent of target containing the isotope of interest, \( Q \) = integrated beam charge, \( n' \) = number of target nuclei per unit volume, \( \varepsilon \) = efficiency of the detector, and \( d\Omega \) = the entrance solid angle into the detector. The integrated beam charge is determined by

\[ Q = N_Q S F \text{ Coulombs}, \quad (75) \]

where \( N_Q \) = integrator counts, \( S \) = full-scale reading of the integrator, and \( F \) = pre-scaling integrator factor (=10\(^{-3}\)).

Now, by re-expressing equation 74 in terms of experimental parameters for \(^7\text{Li}\) we get

\[ \sigma_L(\theta) = k \frac{N\cos\theta D}{N_Q S d\Omega} \text{ mb/sr}, \quad (76) \]

with \( \theta_T \) = the target angle and \( D \) = dead time correction.

The value of \( k \) is a target-dependent constant and is equal to 4604.18 for \(^{12}\text{C}\) and 5780.73 for \(^{58}\text{Ni}\), providing that \( S \) is
expressed in nanoamps and dσ in millisteradians.

Transformation of the cross sections to center-of-mass values is achieved by multiplying the value calculated in equation 76 by an angle dependant factor calculated by the program KINMAT (RINC82). The program also computes the center of mass angle for each lab angle.

The system dead time was determined by comparing the number of integrator-triggered pulser pulses to the number of these pulses stored in its own separate spectrum as discussed in the previous chapter. The summing routine of GAUFIT was adequate for determining the number of counts in the pulser peak since it was in a region with little or no background and was the only peak in the selected region of the spectrum. Measured dead times, equal to the number of triggered pulses recorded by a scaler divided by the number of pulses in the pulser peak, were usually maintained below 10% and ranged from 1% up to 12%.

4.3 Data Averaging and Normalization

Now that the data have been reduced to differential cross sections for each data angle run, multiple cross section values that exist for any angle must be "averaged". These multiple values may exist within any particular data run as well as between the data runs. The first step in the averaging process was to average all values corresponding to the same angle within each data run. For the three nickel
runs, the angular distributions were then compared to each other at common angles to check for any consistent discrepancies between the three runs. It was found that the cross section values for the first data run at angles common to either of the other two data runs were larger, on average, by 25%. Primarily since the last set of nickel data was taken with the most intense and stable $^7$Li beam that resulted in data with the best statistics, and secondarily since the cross section values from this run compared favorably with those belonging to the second nickel run, it was assumed that the cross sections for the third run are the most accurate. Therefore, all the differential cross sections belonging to the first data run were reduced by 25%.

The next step consisted of reducing the three nickel runs into a single angular distribution. Now that the three data runs are normalized between each other, a simple average differential cross section value was computed for each angle using all values that existed for that angle, with each existing value for a particular angle given equal weight. Also calculated during this step was a point-to-point uncertainty in the data. This was done by calculating the variance or uncertainty in the mean ($\mu$) of each angle ($x_i$), where the variance is defined by

$$
\sigma_\mu = \frac{s}{\sqrt{N}},
$$

(77)
80

\[
s = \left[ \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu)^2 \right]^{1/2}.
\]

(78)

A fractional variance was then computed by

\[
f = \frac{\sigma \mu}{\mu}.
\]

(79)

When all of the \( f \) values were averaged, a mean value for \( f \) was found to be approximately 0.08. This average, when multiplied by the average differential cross section and divided by the square root of the number of data values at each angle, will be taken as the point-to-point uncertainty for each angle within the final nickel angular distribution, i.e.

\[
\Delta \sigma_{pp} = (f_{ave})(\sigma_{ave})/\sqrt{N}.
\]

(80)

The angular distribution for \(^{12}\)C was obtained in the same manner. Since only one data run was taken on the carbon target, a run-to-run normalization was not required. An average cross section value, along with its associated peak-to-peak uncertainty with \( f \) equal to 0.03 was then calculated for each separate angle.
The errors or uncertainties associated with each point in the final angular distributions must also be calculated. Errors associated with uncertainties in the experimental quantities have been discussed in Chapter 3. For each angle in the distribution, a statistical uncertainty due to counting is determined for both the detector and the integrator, with an average uncertainty to be used if more than one run exists at that angle. An absolute error of 8% was assigned to the nickel distribution, which covers the errors discussed in Chapter 3. The absolute error in the carbon target was taken to be 5% since the target thickness was known to a higher precision. The final uncertainties associated with each differential cross section was then computed by adding in quadrature the above two errors with the peak-to-peak errors discussed in the previous paragraphs.

Absolute normalization presented its own complications in the final data reduction. The standard method of normalizing angular distributions is to assume that at forward angles the scattering resembles Rutherford (Coulomb) scattering; therefore, the ratio of the experimental differential cross section to the Rutherford cross section at each forward angle should be equal to 1. The cross sections are then multiplied by some factor so that the condition is true. Rutherford scattering does occur at forward angles for low projectile energies and for targets of
heavy nuclei, but as the projectile energy increases this phenomenon occurs at progressively smaller angles (COOK82a). Since this experiment deals with neither low energies nor heavy targets, and since the limiting forward angle was 6°, this method of normalization can not be employed. Therefore, the absolute normalization to the data for each target will be taken to be 1. The differential cross sections and their associated uncertainties (errors) for the 12C and 58Ni targets are given in Appendix C.

4.4 Optical Model Analysis

Initial attempts to reproduce the 7Li data were made with a six-parameter phenomenological optical model potential using the optical model search code SNOOPY (SCHW77). The form of the optical potential used is given by

\[ U(r) = U_c(r) - V[f_0(r)]^\alpha - iW_s f_w(r), \]  

where the Woods-Saxon form factors

\[ f_x(r) = \left(1 + \exp[(r - r_x A_T^{1/3})/a_x]\right)^{-1}. \]

\[ U_c(r) \] has the form given by equation 31 with \( r_c \) set equal to 1.3 fm. The value of \( \alpha \) is equal to 1 for the standard Woods-Saxon shape and will maintain this value during the
discussion of this section unless otherwise noted. One of the criteria used in judging the quality of the fit for a given set of parameters was the value of chi-squared. This quantity is calculated by the expression

$$
\chi^2 = \sum_{i=1}^{N} \left( \frac{\sigma_{i \, \text{exp}}(\theta) - \sigma_{i \, \text{calc}}(\theta)}{\Delta \sigma_{i \, \text{exp}}(\theta)} \right)^2,
$$

(83)

where \( \sigma_{\text{calc}} \) is the calculated value, \( \sigma_{\text{exp}} \) is the experimental value, and \( \Delta \sigma_{\text{exp}} \) is the experimental uncertainty in the value of the differential cross section at a given angle for the N data angles. During the parameter searching routines, the values of the parameters that are searched on are varied such as to produce a reduction in the value of chi-squared for the entire distribution. A reduced chi-squared value is obtained by dividing the result obtained in equation 83 by the number of data points, or degrees of freedom. It is these values that are listed in the last column of Tables 1 and 2.

Calculations using the 99 MeV \(^6\text{Li}\) potential parameters (Set I in Table 1) of Schwandt et al. (SCHW81) were first performed since a comparison between those results and the results of this present experiment is a major objective of this study. As one can see by the solid curve in Figure 12,
the above calculations after changing only the mass and center of mass energy of the projectile are in good agreement with the \(^7\)Li data on this target. Improvements on the fit to the data were also looked for by utilizing the parameter searching options of SNOOPY. Starting at the \(^6\)Li parameter values, a search on the six potential parameters in various combinations produced a slight improvement in the fit (Set II). This improvement results from a better agreement between the theory and the data in the phase of the oscillations beyond 40° as shown by the dashed line in Figure 12. An additional fitting attempt was made using the the global \(^7\)Li potential parameters determined by Cook (COOK82a). Calculations using these parameters resulted in a slightly poorer fit at angles greater than 30° due to phase differences between the data and the theory, but the differences in phase were generally eliminated when searches on these parameters were carried out. The resulting fit with these parameters (Set III) is represented by the dotted curve in Figure 12.

**TABLE 1**

Optical model parameters for fits to \(^7\)Li + \(^{58}\)Ni

<table>
<thead>
<tr>
<th>Set</th>
<th>(V) (MeV)</th>
<th>(r_o) (fm)</th>
<th>(a_o) (fm)</th>
<th>(W_s) (MeV)</th>
<th>(r_w) (fm)</th>
<th>(a_w) (fm)</th>
<th>(\chi^2/N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>94.0</td>
<td>1.304</td>
<td>0.865</td>
<td>21.00</td>
<td>1.732</td>
<td>0.773</td>
<td>8.6</td>
</tr>
<tr>
<td>II</td>
<td>99.5</td>
<td>1.304</td>
<td>0.845</td>
<td>22.33</td>
<td>1.732</td>
<td>0.791</td>
<td>5.4</td>
</tr>
<tr>
<td>III</td>
<td>114.2</td>
<td>1.286</td>
<td>0.820</td>
<td>28.99</td>
<td>1.637</td>
<td>0.850</td>
<td>6.0</td>
</tr>
</tbody>
</table>
Figure 12: Optical model fits to the nickel data

$^7$Li + $^{58}$Ni angular distribution of the differential cross sections at 99 MeV.
The solid curve is the calculations using the 99 MeV $^6$Li parameters (Set I of Table 1). The dotted curve is the prediction using the Set III parameters. The dotted curve is the best optical model fit to the data (Set III).
Upon examination of the three fits shown in Figure 12, a common discrepancy between the data and the calculations is the prediction of very deep minima occurring at angles between 40° and 50°. These differences can sometimes be explained in part by the fact that the data was gathered with a non-zero angular resolution. In this angular region, the angular resolution was slightly over 1 degree. Attempts to correct for the 1° angular resolution did produce more shallow minima with improvements in the total chi-squared value on the order of 10%, but they failed to substantially improve the fit in this region.

\(^{12}\text{C}\)

The calculations using the shallow potential set of Schwandt et al. (Set I of Table 2) for \(^6\text{Li}\) on \(^{12}\text{C}\) fail to reproduce the \(^7\text{Li}\) data after adjustment for the new projectile, unlike the good agreement obtained for the fit to the nickel data using the appropriate potential set from the same source. As shown by the solid curve in Figure 13, the region of agreement extents out to approximately 30°, and then only if the data is renormalized. Besides the calculations becoming progressively out of phase with the data for increasing angles, they seriously overpredict the differential cross sections for angles beyond 50°. Since optical model calculations usually reproduce forward angle data without much difficulty, several additional attempts to
reproduce the data besides those discussed during the fitting of the nickel data will be investigated with their resulting parameters listed in Table 2.

**TABLE 2**

Optical model parameters for fits to $^7\text{Li} + ^{12}\text{C}$

<table>
<thead>
<tr>
<th>Set</th>
<th>V (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W_S$ (MeV)</th>
<th>$r_W$ (fm)</th>
<th>$a_W$ (fm)</th>
<th>$\chi^2/N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>105.0</td>
<td>1.278</td>
<td>0.800</td>
<td>44.70</td>
<td>1.259</td>
<td>1.117</td>
<td>1.38E4</td>
</tr>
<tr>
<td>II</td>
<td>97.0</td>
<td>1.278</td>
<td>0.834</td>
<td>67.12</td>
<td>1.280</td>
<td>1.077</td>
<td>1.65E2</td>
</tr>
<tr>
<td>III</td>
<td>114.2</td>
<td>1.286</td>
<td>0.853</td>
<td>36.17</td>
<td>1.739</td>
<td>0.809</td>
<td>8.79E3</td>
</tr>
<tr>
<td>IV</td>
<td>60.0</td>
<td>1.278</td>
<td>1.045</td>
<td>24.92</td>
<td>1.271</td>
<td>1.335</td>
<td>3.45E1</td>
</tr>
<tr>
<td>V</td>
<td>105.0</td>
<td>1.278</td>
<td>0.907</td>
<td>39.79</td>
<td>1.765</td>
<td>0.725</td>
<td>4.64E3</td>
</tr>
<tr>
<td>VI</td>
<td>105.0</td>
<td>1.278</td>
<td>0.800</td>
<td>33.34</td>
<td>1.913</td>
<td>0.665</td>
<td>4.66E2</td>
</tr>
<tr>
<td>VII*</td>
<td>140.0</td>
<td>1.278</td>
<td>0.932</td>
<td>44.70</td>
<td>1.265</td>
<td>1.094</td>
<td>3.75E1</td>
</tr>
<tr>
<td>VIII*</td>
<td>152.5</td>
<td>1.286</td>
<td>0.868</td>
<td>25.00</td>
<td>1.820</td>
<td>0.793</td>
<td>1.46E1</td>
</tr>
</tbody>
</table>

* $\alpha=2$

A search on the Set I parameters of Table 2 in various combinations yielded parameter Set II, whose results are illustrated by the dashed curve in Figure 13. Although the fit is still very poor, the calculations now have the proper phase and no longer greatly overestimate the differential cross sections at large angles. A fit to the data was also attempted using the global potentials of Cook adjusted for the $^{12}\text{C}$ target. These calculations (Set III) yielded rather surprising results as they matched the phase of the oscillations out to $50^0$, as shown by the dotted curve in
Figure 13: Optical model fits to the carbon data

$^7\text{Li} + ^{12}\text{C}$ angular distribution of the differential cross sections at a bombarding energy of 99 MeV.

The solid curve is the calculations using the 99 MeV $^6\text{Li}$ parameters (Set I of Table 2). The dashed curve is the predictions using the Set II parameters. The dotted curve is the fit produced by the global optical model parameters (Set III).
Figure 13. Although the curve is still too high for angles beyond 50°, it has a theoretical value that is one order of magnitude closer to the data point at 81° than the \(^6\)Li parameters prediction. The calculations also require a renormalization of the data by a factor \(N=0.822\) in order to reproduce the data at angles forward of 30°, whereas the data needs to be renormalized by a factor of \(N=0.593\) in order for the Set I parameters to provide a good fit to the forward angle data. This fit is surprisingly good considering that the parameters of Set III were derived using data for targets with \(Z > 12\), since Cook was concerned that the structure of the light nuclei targets might unreasonably influence the global parameter values (COOK82a). Searches on the Cook potential parameters were also carried out, and although they did produce improvements in chi-squared by up to two orders of magnitude, the fits were not greatly improved since they exhibited oscillations with deep minima that were generally out of phase with the data and were not present in the angular distribution.

The next attempt at fitting the data entailed searching for a separate potential family that might produce a better fit. This attempt seemed justified since several potential sets, due to discrete ambiguities in the potential strength, were found for the 99 MeV \(^6\)Li data (SCHW81). Calculations using the two deeper potential sets of Schwandt et al., even after searching on the parameters of the two sets, failed to
produce significant improvements in the fit. A parameter grid on $V$, the strength of the real potential, between 40 and 200 MeV at 5 MeV intervals produced a minima in a chi-squared vs $V$ plot near $V=60$ MeV. Further searches on the parameters in this region resulted in the parameters of Set IV. The fit obtained with these parameters is shown by the dashed line in Figure 14. Although never differing from the data by any significant amount at any given data point, the parameters produce a fit with oscillations too small in amplitude to properly fit the data.

A common failure among all the carbon fits so far is their underprediction of the correct amplitudes for the first three oscillations in the angular distribution. This failure may imply that a renormalization to the data might be required. To check for this possible requirement, an attempt was made to fit only the data involved in the first three oscillations ($\theta<35^0$). Parameter searches starting at the values of Set I from Table 2 were preformed, and an acceptable fit was found with the parameters of Set V. The calculations for this new set of values still underpredict the amplitude of the first oscillation centered near $15^0$, but the differences are now small. Unfortunately, when the calculations are extended out to $80^0$, they also fail to reproduce the data at the larger angles as shown by the solid curve in Figure 14. This result is not surprising since the forward scattering angles can usually be reproduced
Figure 14: Optical model search fits to the carbon data

The solid curve is the fit produced by fitting only the data at forward angles - $0<35^\circ$ (Set V of Table 2). The dashed curve is the resulting fit obtained after gridding on the strength of the real potential $V$ (Set IV).
by many different potential families, with the larger angles required to uniquely define a parameter set (GOLD72).

Since the above attempt did not eliminate the question as to whether the data needed to be renormalized, attempts to fit a renormalized data set were then made. The appropriate renormalization factor was determined in the program SNOOPY by performing calculations on the first twelve data points, which define the first two oscillations in the angular distribution. Renormalization factors of 0.595 and 0.789 were suggested for parameter Sets I and III of Table 2 respectively. Searches were then conducted on the parameters from these two sets, but none of the searches produced a decent fit to the renormalized data. The solid curve in Figure 15 shows the results of the renormalization fit to the Set I parameters, while the dashed curve in the same figure presents the best fit (Set VI from Table 2) obtained through searching on the parameters of Sets I or III. Definitive conclusions about the need for the data to be renormalized can not be made since an acceptable fit was also not obtained with the renormalized angular distribution, although the fit to the forward angle data was vastly improved without requiring adjustments to the Set I parameter values.

The failure of the standard Woods-Saxon shape for the optical potential to properly fit the data in a region that exhibits exponential-like falloff characteristic of nuclear
Figure 15: Normalized optical model fits to the carbon data

The solid curve is the fit to the data using the Set I parameters of Table 2 after the data was renormalized by a factor of 0.595, while the dashed curve is the best fit to the renormalized data (Set VI).
rainbow scattering has been observed elsewhere for composite projectile scattering (GOLD74). This falloff region for the carbon data begins at data angles beyond 40°, with the small oscillations in the exponential-like falloff resulting from a superposition of absorptive diffraction scattering with refractive rainbow scattering. It has been shown (GOLD75) that the parameters which fit the forward angle scattering data, by optimizing the tail of the real potential, are different from the parameters needed to fit the data lying in the refractive region at larger angles, which are affected by the surface of the nuclear potential. A successful fit to the entire angular distribution that contains both diffractive forward angle and refractive large angle scattering results from a compromise in parameter values needed for both regions. Goldberg suggests that the square of the Woods-Saxon form to the real part of the potential, given by

$$U(r) = U_C(r) - V[f(r)]^2 - iW_s f(r),$$  \hfill (84)

better represents the correct shape of the potential, especially in both the surface and tail regions of the potential. The dashed curve in Figure 16 shows the effect of using this new potential shape with the Set I parameters of Table 2. For comparison, the calculations using the
Figure 16: Woods-Saxon shape fit comparison

The solid curve is the fit produced by the standard Woods-Saxon shape ($\alpha=1$) for the real form factor, while the dashed curve represents the predictions using a Woods-Saxon squared ($\alpha=2$) real form factor. Both curves use the Set I parameters of Table 2 in their calculations.
standard Woods-Saxon shape for the same parameters are illustrated by the solid curve in the same figure. Comparing the two results, you can see that the Woods-Saxon squared (WS2) result has a much larger slope along with the slight oscillations that are also present in the data in the falloff region, although this shape for the potential severely underpredicts the data over the entire angular range.

Using α=2, extensive searches on the Set I parameters were performed after initial searches failed to provide a good fit. These searches were done by incrementing the values of the real potential depth V and the imaginary potential depth $W_s$ from 80 to 180 MeV and 30 to 65 MeV in 5 MeV steps respectively, while searching on the geometrical parameters at each grid point. The resulting best fit parameters are listed in Table 2 as Set VII, while the calculations for the parameters are shown by the dashed line in Figure 17. This fit closely resembles the fit provided by the Set IV parameters (dashed line in Figure 14) and is unacceptable for the same reasons.

As a final attempt to fit the data, the above procedure was repeated starting with the Set III global parameters of Cook since they yielded a better initial fit. The results of this search are illustrated by the solid curve in Figure 17, which was generated with the Set VIII parameters of Table 2. This was by far the best fit to the carbon data.
Figure 17: Woods-Saxon squared optical model fits

The dashed curve is the fit (Set VII of Table 2) found from searching on the Set I parameters using a Woods-Saxon squared shape for the real form factor, while the solid curve is the fit (Set VIII) obtained by searching on the Set III parameters. Similar grids on both V and W were made in obtaining the final parameters that resulted in these two curves.
using the phenomenological optical model, and it was the only fit which exhibited the proper behavior for the data angles beyond 40°. Renormalization requirements were also investigated using the same procedure discussed earlier in this section for both α=2 fitting attempts since the first two peaks were still underpredicted by at least 25%. Renormalization factors of 0.449 and 0.931 were determined for Sets VII and VIII respectively, but searches on both parameter sets failed to provide vast improvements in the fits to the renormalized data.

As a few final comments on the optical model fit to carbon, none of the methods employed with α=1 produced a reasonable fit. Any set of parameters that provided a good fit to the data at angles of 35° or less, resulted in a poor fit to the data in the falloff or refractive region. Only after modifying the shape of the optical potential as suggested by Goldberg did a reasonable fit occur, unlike the case for the nickel data. Fits including a spin-orbit term in the optical potential were not attempted since they had little effect on the ⁶Li data of Schwandt et al. and were found to be of little use for ⁷Li scattering scattering at lower energies (POLI76). Also, since the ground state of ⁷Li has a spin of 3/2 while the optical model program SNOOPY was written to handle projectile spins only up to 1, any physical significance to the fit would have to be questioned.
4.5 Double-Folding Analysis

In recent years, the double-folding model has been used to describe the elastic scattering of composite particles. Unlike the optical model, which reduces the interaction of one nucleus with another to a one-body problem in which the potential describing the scattering depends only on the relative separation of the two nuclei, the folding model attempts to approximate the scattering potential in a microscopic manner. The folded potentials are therefore considered to be more fundamental since they are based on realistic density distributions obtained from high energy electron scattering and on an effective interaction determined by nucleon-nucleon scattering. Because of these reasons, folding model fits to both the carbon and nickel angular distributions were attempted in the hope of obtaining improved fits.

For this double-folding model analysis, the real part of the nuclear scattering potential that was used had the form given by equation 43 in Chapter 2. The final form of the scattering potential is given by

\[ U(r) = N_R U_F(r) - i W_S [1 + \exp((r-R_i)/a_i)]^{-1}, \quad (85) \]

where \( N_R \) is a renormalization constant. The renormalization constant and the three imaginary potential parameters were then varied using the program SNOOPY in order to produce a
suitable fit to the data.

The real folded potential $U_F(r)$ was calculated using the program DFPOT (COOK82c). DFPOT computed the density distribution for each nucleus upon input of the type of distribution and its associated parameters. After the two densities were determined, DFPOT then computed the double-folded potential for the chosen effective interaction. The output from DFPOT consists of a table of potentials at discrete radius intervals, and it was used as input to the optical model program SNOOPY. The form of the density distribution selected for each nucleus involved will be discussed in the following subsections for each of the two different targets.

$^{12}_C$

In order to generate the potential of equation 43 suitable choices for the two matter distributions must be made. Most knowledge of matter densities comes from high energy electron scattering, and this only provides information about the proton's charge distribution. Many prescriptions are available for determining a reasonable matter distribution (PETR77), (SATC79b), etc.; with the result that the fit to the cross-section data is not very sensitive to the form of the density used provided that the densities have the correct root-mean-square radii (COOK84a). This lack of sensitivity to the exact shape of the
distributions exists because the surface and tail regions of the densities provide the main contribution to the folded potential near the strong absorption radius. The importance of the strong absorption radius is discussed in Appendix B.

The density distribution chosen for the $^7$Li projectile was the one used by Cook et al. (COOK83). It has a harmonic oscillator form given by

$$\rho(r) = (A + Br^2)\exp(-\alpha^2r^2), \quad (86)$$

with $A = 0.1647$, $B = 0.03086$, and an adjustable value of $\alpha = 0.6$, which was adjusted in order to reproduce the experimental rms radius.

The insensitivity of the folding model to the exact form of the density distributions was explored by choosing three different distributions, each obtained through different means, for the $^{12}$C density. The first distribution is of the form given by equation 86, while the other two carbon densities are given by

$$\rho(r) = A(1 + Br^2)[1 + \exp((r - R_R)/A_R)]^{-1}. \quad (87)$$

The values for the parameters $A$, $B$, $R_R$, and $A_R$ for each density are given in Table 3. The set for the first carbon density (H0) are from Cook (COOK84b), the second set are the 3pF set from DeJager et al. (DEJA74), and the third set (LD)
was calculated using the liquid drop model (MYER70). The latter two distributions were normalized such that their volume integrals \( J \),

\[
J = 4\pi \int_0^\infty \rho(r) r^2 dr,
\]

yielded the same result as the first distribution, with \( A \) being the normalization parameter in equation 87. The table also lists the rms radius for each distribution, which should be about 2.314fm (SATC79b) if the distribution is to yield the correct potential strength near the strong absorption radius.

**TABLE 3**

Ground state density parameters for \(^{12}\)C

<table>
<thead>
<tr>
<th>Type</th>
<th>( A )</th>
<th>( B )</th>
<th>( A_R ) or ( \alpha )</th>
<th>( R_R )</th>
<th>( r_{rms} )</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HO</td>
<td>.1843</td>
<td>.09919</td>
<td>.635</td>
<td>-0-</td>
<td>2.318</td>
<td>COOK84b</td>
</tr>
<tr>
<td>3pF</td>
<td>.1829</td>
<td>-.02687</td>
<td>.5225</td>
<td>2.353</td>
<td>2.445</td>
<td>DEJA74</td>
</tr>
<tr>
<td>LD</td>
<td>.1846</td>
<td>-0-</td>
<td>.550</td>
<td>2.101</td>
<td>2.612</td>
<td>MYER70</td>
</tr>
</tbody>
</table>

The folding model potentials produced by each of the three different carbon distributions when folded with the lithium distribution and the effective interaction are shown in Figure 18. As seen in the figure, the three real potentials are almost identical out to the strong absorption radius (SAR = 6.3 fm for 99 MeV \(^7\)Li on \(^{12}\)C) and are within
one order of magnitude of each other out to 10 fm, where the potential has dropped off by five orders of magnitude from its central value and three orders of magnitude from its value at the SAR.

Since the parameters for the harmonic oscillator form (HO) most closely reproduce the "correct" $^{12}\text{C}$ rms radius, this distribution was used in eq. 43 to determine the folding model real potential. A phenomenological Woods-Saxon form was used for the imaginary potential. The optical model search program SNOOPY was used for the calculations in which the normalization to the real potential along with the three imaginary potential parameters could be varied in order to obtain a fit to the data. The fitting procedure involved a two-parameter grid on the normalization parameter $N$ and the imaginary volume potential strength $W_s$ while searching on the remaining two parameters at every grid point. The resulting best fit is shown by the solid line in Figure 19 and the parameter values are given in Table 4. Although the fit is not quite as good as the one obtained with the phenomenological Woods-Saxon squared shape in the previous section, it is a vast improvement over the fit obtained with the standard Woods-Saxon shape. The dashed curve and the dotted curve in Figure 19 show the respective results of the calculations using the 3pF and the W-S (LD) shapes for the carbon density distributions in the real folded potential with the search
Figure 18: Real double-folded potentials for the carbon data

The solid curve shows the real double-folded potential obtained with a harmonic oscillator form for the carbon density distribution, while the dashed and dotted curves correspond to the potentials obtained when a 3-parameter Fermi shape and a Woods-Saxon shape using a liquid drop model are used to describe the carbon density distributions respectively. The arrow indicates the location of the strong-absorption radius.
Figure 19: Double-folding model fits to the carbon data

The solid, dashed, and dotted curves show the fits to the carbon data using the harmonic oscillator, 3 parameter Fermi, and Woods-Saxon (LD) shapes for the carbon density distribution in the double-folding model respectively. The three fits are very similar since their potentials have almost the same strength at the strong-absorption radius.
parameters found for the HO distribution. Almost identical results are obtained regardless of the form of the density distribution, even with a fairly significant deviation from the "correct" rms radius value. This illustrates the fact that the double-folding model is fairly insensitive to the shape of the density distributions, at least for this target nucleus.

**TABLE 4**

Double-folding Model parameters for fits to $^7$Li data

<table>
<thead>
<tr>
<th>Target Nucleus</th>
<th>Density Type</th>
<th>$N_R$</th>
<th>$W_s$ (MeV)</th>
<th>$r_w$ (fm)</th>
<th>$a_w$ (fm)</th>
<th>$\chi^2/N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}$C</td>
<td>HO</td>
<td>.535</td>
<td>23.983</td>
<td>2.034</td>
<td>0.669</td>
<td>58.6</td>
</tr>
<tr>
<td>&quot;</td>
<td>3pF</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>47.9</td>
</tr>
<tr>
<td>&quot;</td>
<td>LD</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td>50.4</td>
</tr>
<tr>
<td>$^{58}$Ni</td>
<td>WS</td>
<td>.480</td>
<td>37.500</td>
<td>1.500</td>
<td>0.938</td>
<td>5.78</td>
</tr>
</tbody>
</table>

$^{58}$Ni

The procedure used to fit the nickel data was nearly identical to that for the carbon data. The same lithium density distribution was used in developing the folded nickel potential. The density used for the $^{58}$Ni target was determined by Glover et al. (GLOV80) and consists
of the sum of two Woods-Saxon shapes. The form of the
distribution is given by

\[ \rho(r) = A[1 + \exp\left(\frac{r-B}{C}\right)]^{-1} + D[1 + \exp\left(\frac{r-E}{F}\right)]^{-1}, \]  (89)

with the respective values for the constants given in
Table 5. A similar search procedure using SNOOPY on the
four free parameters yielded the fit shown in Figure 20 for
the parameter values given in Table 4. The double-folding
model provided a fit that was just as good as the optical
model fit for this case.

<table>
<thead>
<tr>
<th>Type</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>( r_{\text{rms}} )</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>WS</td>
<td>0.08152</td>
<td>4.1156</td>
<td>.55</td>
<td>0.08558</td>
<td>4.1468</td>
<td>.55</td>
<td>3.798</td>
<td>GLOV80</td>
</tr>
</tbody>
</table>

As seen in Table 4, renormalizations to the real
potential of .54 for the carbon data and .48 for the nickel
data were required to obtain a proper fit. This implies
that the double-folding model overpredicts the strength of
the real potential by nearly a factor of two. This behavior
has been observed for other intermediate ions with
\( 4 < A < 12 \), and has been postulated that this behavior could
Figure 20: Double-folding model fit to the nickel data
be connected with the small breakup energies (2.45 MeV for $^7\text{Li}$ into $t + \alpha$) for the anomalous nuclei (SATC79c). Since very little energy is needed for the breakup of these projectiles, Glover suggested that the static approximation does not remain valid for the angular ranges sampled by most elastic scattering data (GLOV80). In his paper, Glover found that the forward angle data with $\sigma/\sigma_\text{R} > 10^{-1}$ could be fit without any renormalization ($N=1$). This implies that the scattering for these projectiles is sensitive to the potential well inside the strong absorption radius where the static double-folding model is probably not valid. Other explanations for the required renormalizations for the $^7\text{Li}$ and $^9\text{Be}$ projectiles involve their large static quadrupole moments or a strong coupling to a low-lying excited state in the projectile (HNIZ81). These last two possible explanations will be examined in the next section.

4.6 Coupled-Channels Analysis: Quadrupole Effects

In the double-folding model analysis, the density of the ground state for each of the two interacting nuclei was taken to be spherical and static. This assumption resulted in a potential that was too strong by approximately a factor of two. Since $^7\text{Li}$ has a large deformation, or quadrupole moment, it would not be unreasonable to expect the double-folding model to fail when $^7\text{Li}$ projectiles are involved. In fact, Blair (BLAI59) stated that shape
dependance was important in heavy-ion scattering and that it should have a contribution in the elastic scattering process. A significantly improved fit to the data for $^7\text{Li} + ^{40}\text{Ca}$ at 34 MeV and $^7\text{Li} + ^{54}\text{Fe}$ at 48 MeV (HNI81) was obtained when a shape-dependant contribution to the elastic scattering was included. The contribution from the large ground-state quadrupole moment was included in a coupled-channels calculation by Hnizdo et al., and it produced a fit without requiring a renormalization of the double-folded potential. This possible solution to the renormalization requirement for the double-folding will be investigated in this section.

The quadrupole moment contribution from the ground state of the lithium projectile is known as a reorientation term since the interaction may reorientate the spin of the projectile. The real form factor for the reorientation contribution was obtained by folding the effective nucleon-nucleon interaction with the quadrupole density of the lithium projectile and the ground state density of the target of interest. Following the procedure used by Hnizdo et al., the quadrupole density was taken to have the derivative form

$$\rho_2^{ij}(r) = \delta_2^{ij} \frac{d(\rho_o(r))}{dr}, \quad (90)$$

where $\delta_2^{00}$ is the deformation length for the reorientation
and $\rho_0(r)$ is the $^7\text{Li}$ ground-state density. The deformation length was determined by normalization to the intrinsic quadrupole moment $Q_{20}$. Taking the $3/2^-$ ground state of $^7\text{Li}$ as belonging to the $K=1/2^-$ rotational band,

$$Q_{20} = \left(\frac{16\pi}{5}\right)^{1/2} \frac{(7e)}{A} \int \rho_{20}(r) r^4 dr.$$ \hspace{1cm} (91)

Given these spin and rotational states, $Q_{20}$ was found from the experimental value of the static quadrupole moment $Q_2 = -4.5$ e-fm$^2$ (NIFT71) by $Q_{20} = -5Q_2$, which resulted in a value for the quadrupole-transition deformation length $\delta_{200}$ of 3.38 fm. As in the case for the monopole potential, this reorientation potential was multiplied by a renormalizing factor $N$, which was assigned the same value as that for the monopole term. For the imaginary part of the quadrupole potential, a form factor with a derivative Woods-Saxon form,

$$W_2(r) = \delta_{2'} W d(f_1(r))/dr,$$ \hspace{1cm} (92)

was used with the imaginary deformation length $\delta_{2'}$ taken to be equal to $\delta_{200}$. The coupled-channels calculations were performed using a modified version of the computer code CHUCK3 (COMF81), which itself is a modified version of the original code by Kunz (KUNZ73).
If the double-folding model gives the correct potential strength, then the renormalization parameter $N$ should be close to unity. Calculations including quadrupole effects, but leaving the value of the normalization parameter $N$ set at 0.54, are shown by the dashed line in Figure 21. The oscillations are now damped to a reasonable magnitude as compared to the fit produced by the spherical term (solid line in the same figure), but the oscillations become progressively out of phase with the data for angles beyond $30^\circ$. The quadrupole fit also has a smaller slope than that exhibited by the data. Since the renormalized potential with the quadrupole scattering contributions did not give a good description of the data, calculations with $N=1$ were attempted. The results of these calculations are given by the dotted curve in Figure 21. The deep oscillations associated with the $N=.54$ case are now almost completely damped out for angles beyond $35^\circ$, and the slope is much too small to fit the data. The potential parameters and deformations lengths that were used to generate the two curves are given in Table 6.

$^{12}_C$

The double-folding model after a renormalization of $N=.48$ yields an adequate fit to the nickel data only when the spherical or monopole part of the lithium density is
Figure 21: Carbon coupled-channels fits with quadrupole coupling

The dashed and solid curves are the best microscopic double-folding model fits to the data with $N=0.54$ with and without the inclusion of quadrupole effects in a coupled-channels calculation respectively. The dotted curve is the coupled-channels prediction with $N=1$. The potential parameters are given in Table 6.
TABLE 6
Potential parameters and deformation lengths for $^7\text{Li}$ quadrupole transition.

<table>
<thead>
<tr>
<th>Target Nucleus</th>
<th>Transition N W</th>
<th>$N_W$</th>
<th>$a_W$</th>
<th>$\delta_2$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C}$</td>
<td>$3/2^- \rightarrow 3/2^-$</td>
<td>0.535</td>
<td>23.983</td>
<td>2.034</td>
<td>0.669</td>
</tr>
<tr>
<td>$^{58}\text{Ni}$</td>
<td>$3/2^- \rightarrow 3/2^-$</td>
<td>0.480</td>
<td>37.500</td>
<td>1.500</td>
<td>0.950</td>
</tr>
</tbody>
</table>

considered, unlike the case for the carbon data. The only objectional part to the fit provided by this model, shown by the solid line in Figure 22, is the overly deep minima predicted by the model for the angular range between 20 and 50 degrees. Since the quadrupole contribution oscillates out of phase with the cross section belonging to the spherical term, this contribution tends to fill in the minima in the angular distribution (PETR77). When the quadrupole contributions are included in a coupled-channels calculation, the minima are indeed filled in, but now the oscillations are both too small and out of phase with the data as illustrated by the dashed curve in Figure 22. The dotted curve in Figure 22 shows the results for the quadrupole coupling when the renormalization factor is set equal to one. As in the case for the carbon data, the inclusion of the quadrupole contributions to the elastic
Figure 22: Nickel coupled-channels fits with quadrupole coupling

The dashed and solid curves are the best microscopic double-folding fits to the nickel data with N=0.48 with and without the inclusion of quadrupole effects in a coupled-channels calculation respectively. The dotted curve is the coupled-channels prediction with N=1. The potential parameters are given in Table 6.
scattering fails to remove the renormalization required by the double-folding model in order to produce a reasonable fit. The parameters used to generate the coupled-channels calculation curves can be found in Table 6.

4.7 Coupled-Channels Analysis: Projectile-Excitation Effects

Another possible explanation for the failure of the double-folding model to correctly predict the angular distribution of the elastically scattered data without renormalization may be due to a strong coupling between the $3/2^-$ elastic channel and the $1/2^-$ inelastic channel belonging to the 0.478 MeV first excited state of the lithium projectile. If this inelastic contribution is the primary reason for the failure of this model, then the inclusion of its effects in a coupled-channels calculation should remove the necessity for the renormalization of the real part of the double-folded potential.

The procedure for obtaining the form factors for the inelastic channel again followed the one used by Hnizdo et al. The real form factor for the projectile excitation contribution was determined in a similar way to that for the reorientation term. The inelastic transition density had the same form given by equation 90, with the deformation length $\delta_{2,01}$ set by normalizing the transition density to the reduced electric transition probability $B(EL)$ value.
Using the experimental B(E2) value of $6.7 \, e^2 \cdot fm^4$ (NIFT71) for the transition to the first excited state of $^7$Li, the value $\delta_2^{01} = 2.57 \, fm$ was obtained for the projectile excitation deformation length. The imaginary part of the transition potential was again taken to be of a derivative Woods-Saxon form given by equation 92. For consistency, the same renormalization was used for the real transition density as was determined for the monopole term in the optical potential calculations of section 4.5, with the imaginary potential parameters having the same values that were determined in the same section. The real and imaginary deformation lengths were also set equal to each other, and Coulomb excitation with a radius value of $R_c = 1.3A_T^{1/3}$ was included in the calculations.

The deformation parameter $\beta$, which can also be used as a fitting parameter in the coupled-channels calculations, was determined by setting the deformation length $\delta = \beta R_T$, with $R_T = R_0 A_T^{1/3}$. If the value of $\beta$ is less than 0.1 (MARM70), the elastic cross section will not be affected by the inclusion of the inelastic channel. If this is not the case, then there will be a need to reoptimize the Woods-Saxon optical-model parameters. For the fits to the inelastic data in this investigation, the only parameter

\begin{equation}
B(EL,J_i,J_f) = \frac{e^2}{2J_i + 1} \left( \frac{Z}{A} \int p_2^{01}(r) r^{L+2} dr \right)^2. \tag{93}
\end{equation}
that will be adjusted is that belonging to the strength of the imaginary depth parameter \( W \) since it was the only parameter that required an adjustment in the \(^7\text{Li} \) analysis performed by Hnizdo et al. (HNIZ81). The values for the potential parameters and the inelastic deformation length that were used in the analysis for this section are given in Table 7.

<table>
<thead>
<tr>
<th>Target Nucleus</th>
<th>Transition</th>
<th>( N )</th>
<th>( W ) (MeV)</th>
<th>( r_w ) (fm)</th>
<th>( a_w ) (fm)</th>
<th>( \delta_2 ) (fm)</th>
<th>( \beta_2 ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{12}\text{C} )</td>
<td>( 3/2^- \rightarrow 1/2^- )</td>
<td>0.535</td>
<td>23.983</td>
<td>2.034</td>
<td>0.669</td>
<td>2.57</td>
<td>0.552</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.000</td>
<td>5.000</td>
<td>1.000</td>
<td>35.973</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{58}\text{Ni} )</td>
<td>( 3/2^- \rightarrow 1/2^- )</td>
<td>0.480</td>
<td>37.500</td>
<td>1.500</td>
<td>0.950</td>
<td>2.57</td>
<td>0.443</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.000</td>
<td>5.000</td>
<td>1.000</td>
<td>56.250</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The usual procedure for obtaining a fit to the inelastic data using coupled-channels calculations is to fit the elastic channel first, and then add or couple in the
inelastic channels one-by-one. If the coupling to the inelastic channels is not very strong, the introduction of the extra channels should produce very little change in the fitting parameters. The only parameter that needs to be adjusted for the above case is the deformation parameter $\beta$, since the magnitude of the inelastic cross section is proportional to $\beta^2$ (MARM70). When the deformation length $\delta=2.6$ fm was set equal to $\beta R_I$, a value of $\beta=.55$ was obtained when using the value of the imaginary radius $R_I=4.66$ fm found from fitting the carbon data with the double-folding model. This value of $\beta$ is larger than the approximate weak-coupling upper limit of $\beta<0.3$ (MARM70), and it should lead to a perturbation of the fit to the elastic channel.

The effect of coupling the projectile excitation channel to the elastic channel can be seen in Figure 23. The solid curve represents the original double-folding model fit to the elastic channel, while the dashed curve shows the altering of the fit as a result of coupling the inelastic channel to the elastic channel with the renormalization parameter $N=.54$. The fit has been changed slightly, with the major difference occurring for angles larger than $40^\circ$. If the inelastic channel has been properly coupled, then a suitable fit to the inelastic angular distribution also should result. As can be seen by the solid curve in Figure 24, the prediction has roughly the correct magnitude, but it yields a fit that becomes progressively out of phase with
Figure 23: Carbon coupled-channels fits with inelastic coupling - elastic channel

The dashed and solid curves are the double-folded fits to the carbon elastic channel with $N=0.54$ with and without the inclusion of projectile-excitation effects in a coupled-channels calculation respectively. The dotted curve is the prediction for $N=1$ with the modified value of $W$ found in Table 7.
Figure 24: Carbon coupled-channels fits with inelastic coupling - inelastic channel

The solid curve is the resulting fit to the carbon inelastic angular distribution produced by a coupled-channels calculation using the double-folded parameters found for the elastic channel. The dashed and dotted curves show the predictions with N=1 for the regular and modified values of W in Table 7.
increasing angle. The fact that the inelastic fit has the correct magnitude but wrong phase suggests that the original optical model parameters for the imaginary part of the double-folding model potential may have to be changed.

Calculations were next carried out with the renormalization parameter $N=1$. The resulting fit to the inelastic data is shown by the dashed curve in Figure 24. Although the predictions are now off in slope for angles beyond $40^\circ$, it appears that the calculations with $N=1$ improve the phase fit to the data. A set of calculations was also attempted with the value of $W$ increased by 50%, as required by Hnizdo et al. for their fits with the normalization constant $N=1$. These calculations did yield slight improvements in the fits to both the elastic and inelastic data, dotted curves in Figures 23 and 24 respectively, but the fits still are not very good for either distribution.

$^{58}\text{Ni}$

The fits to the inelastic scattering data belonging to the nickel target proceeded in the same manner as those for the carbon target. Again, a large value for the lithium deformation parameter ($\beta = 0.44$) was determined, although it is closer to the transitional value of 0.3. When the inelastic channel was coupled to the elastic channel with
Figure 25: Nickel coupled-channels fits with inelastic coupling - elastic channel

The solid curve is the prediction for the elastic channel using the double-folded parameters with $N=0.48$ when the inelastic channel was coupled to the elastic channel. The dashed curve is the resulting fit using the modified value of $W$ with $N=1$. The potential parameters are given in Table 7.
Figure 26: Nickel coupled-channels fits with inelastic coupling - inelastic channel

The solid curve is the coupled-channels predictions for the nickel inelastic angular distribution using the double-folded parameters found for the elastic channel without coupling. The dashed curve is the resulting fit using the modified value of W given in Table 7 with N=1.
N=0.48, a good description of the elastic data was obtained for angles out to 40° as shown by the solid curve in Figure 25, but the predictions for the inelastic data are generally out of phase with the experimental data as illustrated by the solid curve in Figure 26. When the renormalization parameter was set equal to one, this action resulted in poor fits to both sets of data. As in the case for the carbon data, increasing the value of the imaginary potential depth parameter W by 50% did improve the fits to the elastic and inelastic data over the fits with the unaltered value of W. The fits obtained with N=1 are shown by the dashed curves in Figures 25 and 26, and were produced with the modified value of W.

A search on the values for optical parameters might be one alternative for obtaining a better fit to the angular distributions since the value for the deformation parameter β indicated strong coupling between the elastic and inelastic channels. A combination of the inelastic effects and the quadrupole effects might also lead to an improved fit to the elastic angular distribution. This possible solution will be discussed in the next section.

4.8 Coupled-Channels Analysis: Quadrupole Moment + Projectile-Excitation Effects

When Hnizdo et al. (HNI81) included contributions to the elastic scattering from both the first excited state and
the ground state quadrupole moment of $^7$Li, they obtained their best fit to the elastic data. Since neither the quadrupole effects nor the inelastic effects individually resolved the renormalization problem in this investigation, contributions from a combination of the two effects were then investigated. The procedures for determining the form factors for these two effects were the same as those used in the previous two sections, and the combined contributions to the elastic scattering were determined in a coupled-channels calculation using the appropriate parameters found in Tables 6 and 7 for the quadrupole and projectile-excitation transitions respectively.

Calculations that include both the reorientation and projectile-excitation effects, with the value of the renormalization parameter $N=0.54$, are shown by the solid line in Figures 27 and 28 for the elastic and inelastic channel respectively. The predictions have approximately the correct slope and oscillation amplitude, but they grow progressively out of phase with the data. Setting the renormalization parameter $N=1$ improves the phase fit to the data as shown by the dashed lines in Figures 27 and 28, but now the slope is much too small to fit the data.

The main difference between the fitting procedure used by Hnizdo et al. and that employed in this investigation was
Figure 27: Carbon coupled-channels fits with inelastic coupling including quadrupole effects - elastic channel

The solid curve is a coupled-channels calculation for the elastic channel when quadrupole effects were included using the double-folded potential. The dashed and dotted curves are the predictions with $N=1$ for the standard and modified value of $W$ respectively. The parameter values are given in Tables 6 and 7.
Figure 28: Carbon coupled-channels fits with inelastic coupling including quadrupole effects - inelastic channel

The solid curve is a coupled-channels calculation for the inelastic channel when quadrupole effects are included. The dashed and dotted curves are the predictions with N=1 for the regular and modified value of \( W \), which are given in Tables 6 and 7, respectively.
that the imaginary optical potential parameters were adjusted in the coupled-channels calculations in order to obtain a best fit in the former investigation. The additional degrees of freedom introduced by searching on the imaginary parameters resulted in an increase in the strength of the imaginary potential parameter \( W \) by 35%. This increase occurred for the case where both the quadrupole and inelastic contributions were included in the coupling with the renormalization factor fixed at \( N=1 \). The parameter \( W \) was unchanged when only coupling between the ground state and the reorientation terms were considered, while the other two imaginary parameters retained their original values regardless of the coupling scheme. For the other case considered by Hnizdo et al., \(^9\)Be + \(^{40}\)Ca at 40 MeV, the value of \( W \) increased by 50% and 75% for the reorientation coupling and the reorientation plus projectile-excitation coupling respectively. These increases in the strength of \( W \) again occurred for the case when \( N=1 \).

To check whether a change in the strength of the imaginary potential might significantly improve the fit to the data for the \( N=1 \) cases, the value of \( W \) was increased by 50%. The prediction for the \( N=1 \) case with the modified value of \( W \) is given by the dotted curve in Figures 27 and 28. These results were obtained when contributions to the elastic scattering from both the quadrupole term and the inelastic term were included. Although this modification
yielded a slight improvement in the fit to the data, it does not remove the renormalization requirement that is needed to produce a reasonable fit. This result tends to counter the conclusion reached by Hnizdo et al. (HNIZ81) for 48 MeV $^7$Li on $^{54}$Fe, which stated that the double-folded potential without renormalization adequately describes the elastic and projectile-excitation data when reorientation effects are included in the analysis.

$^{58}$Ni

Figures 29 and 30 respectively contain the fits to the elastic and inelastic scattering data provided by including both the reorientation contributions and the projectile-excitation contributions in the coupled-channels calculations. The solid curve in both figures represents the predictions when the renormalization parameter $N=0.48$. As in the case for the carbon data, the slope and minima depth of the oscillations for the predictions nearly matches that exhibited by the data in the two distributions, but both predictions are off in phase. Setting $N=1$ resulted in poor fits to both sets of data, as illustrated by the dashed curve in the same two figures. Increasing the strength of the imaginary potential parameter $W$ by 50% did produce slight improvements in the fits (dotted line in Figures 29 and 30), but the fits to the data remain poor.
Figure 29: Nickel coupled-channels fits with inelastic coupling including quadrupole effects - elastic channel.

The solid curve is a coupled-channels calculation for the elastic channel when quadrupole effects were included. The dashed and dotted curves are the predictions with N=1 for the fitted and modified value of W respectively. The calculations were performed using the double-folded potentials whose parameter values are given in Tables 6 and 7.
Figure 30: Nickel coupled-channels fits with inelastic coupling including quadrupole effects - inelastic channel

The solid curve is a coupled-channels calculation for the inelastic channel when quadrupole effects are included. The dashed and dotted curves are the predictions with \( N=1 \) for the normal and modified value of \( W \) respectively. The calculations were performed using double-folding model potentials, with the parameter values listed in Tables 6 and 7.
Since the value of the deformation parameter $\beta$ for the lithium projectile indicated strong coupling between its low-lying first excited state and its ground state when scattered from either target, acceptable simultaneous fits to both the elastic and inelastic distributions probably require an iteration procedure in which the optical parameters, and possibly the nuclear deformation parameter $\beta$, are searched on until the best fit to both channels is acheived. It appears that this procedure should produce a good fit for the inelastic nickel data with $N=.48$ since it is primarily off in phase, but it is not obvious that searching on the parameters will result in a good simultaneous fit to both the elastic and inelastic data, especially for the double-folded potential without renormalization. If a good fit to both sets of data can be obtained using the double-folding model with $N=1$, then conclusions can be made as to which effects, quadrupole and/or projectile-excitation, are important in elastic scattering since the double-folding model can explicitly account for each contribution through coupled-channels calculations.
5.1 Summary of Results

Data pertaining to the scattering of $^7$Li ions from targets of $^{12}$C and $^{58}$Ni at 99 MeV have been taken with the intention of comparing this scattering to that of $^6$Li from the same targets at the same energy (SCHW81) in order to investigate possible quadrupole moments effects in $^7$Li scattering. The elastic scattering data has been analyzed by phenomenological optical model potentials with Woods-Saxon form factors and double-folding model potentials. Contributions to the elastic scattering from the large ground-state quadrupole moment of the lithium projectile and its low-lying first excited state have also been investigated through the use of coupled-channels formalism. The major results for this analysis are summarized below.

$^{58}$Ni

Good fits to the nickel data ($\chi^2/N < 6$) were found using either model, although the strength of the real part of the double-folded potential had to be reduced by
approximately one half (N=.48) in order to obtain a reasonable fit. A surprisingly good fit was achieved using the optical model parameters determined for the $^6$Li data. When coupling between the elastic channel and either the ground-state reorientation channel or the projectile-excitation channel was included, in which the double-folding model was used to describe the individual states, the predictions generally had the correct shape and amplitude, but tended to be out of phase with the data. The inclusions of these contributions failed to remove the renormalization requirement for the double-folded potential as the coupled-channels predictions with N=1 exhibited too small a slope to properly fit the data.

$^{12}\text{C}$

A good fit was never found for the carbon data, but reasonable fits were obtained using either a renormalized (N=.54) double-folding model potential or the optical model with a Woods-Saxon squared form factor for the real part of this potential. When a fit was attempted with the same parameters used to fit the $^6$Li data on $^{12}\text{C}$, the fit was off in both magnitude and slope. The coupled-channels predictions for the elastic scattering with either the quadrupole or inelastic contributions had an oscillatory structure that was generally out of phase with the data and usually oscillated too strongly for angles greater than
fifty degrees. Setting $N=1$ in the coupled-channels calculations resulted in poor fits to the data, and hence these contributions do not appear to remove the need for the large renormalization of the double-folded potential.

5.2 Discussion of Results and Suggestions for Future Analysis

Microscopic double-folded potentials are used in coupled-channels calculations to allow the contributions arising from strongly coupled channels to be isolated. The only adjustable parameter in the real part of the potential is a renormalization factor $N$. $^7\text{Li}$ has a large ground-state quadrupole moment, and since the density distributions of the ground states of the interacting nuclei are taken to be spherical in the double-folding model, it was not surprising that the double-folding model failed to predict the scattering distributions with $N=1$. On the other hand, if the quadrupole moment contributions of the lithium projectile were important to the elastic scattering, then the inclusion of its effects in a coupled-channels calculation should have allowed the use of the double-folded potential without requiring that its strength be renormalized. The fact that the Woods-Saxon potential parameters derived from the elastic scattering analysis provided, or can provide, a good fit to the $^7\text{Li}$ elastic scattering implied that they already contained the
contributions (if they were significant) to the scattering from other channels. An alternate solution to the renormalization problem, which was not investigated here, has been found to be due to projectile breakup channels (NAGA82). Since $^7\text{Li}$ is weakly bound with a low breakup energy of 2.47 MeV, improvements in the fits to their $^7\text{Li}$ data were obtained when coupling to breakup channels was included in a coupled-channels calculation.

Probably the most difficult result of the investigation to explain is why the $^6\text{Li}$ parameter sets of Schwandt et al. were able to provide a reasonable fit to the $^7\text{Li}$ data on the nickel target, but failed to even approximate to carbon distribution, especially since both parameter sets provided excellent fits ($\chi^2/N < 2$) to both $^6\text{Li}$ distributions. One possible explanation may be that some other effect, such as a strong coupling to the reorientation channel, plays an important role in the scattering of $^7\text{Li}$ projectiles from light target nuclei, with its effects becoming less important as the mass of the target nucleus increases.

Searches on the Woods-Saxon parameters did provide improved fits to the distributions, but a fit to the carbon data was never found using the standard Woods-Saxon form factors. This lack of fit with the standard optical model is not a common occurrence, since Zeller et al. (ZELL80) were able to produce reasonable fits to their $^7\text{Li}$ data on $^{12}\text{C}$ at 63 and 79 MeV. One observation about their fits is that they were
good only for data at angles less than 50°.

When attempting the optical model fits, it was noticed that the starting parameters played an important role in achieving a decent fit. The best fit to the carbon data with standard Woods-Saxon parameters was obtained when the global parameters of Cook (COOK82a) were used as the starting parameters in the search routine of the optical model code SNOOPY. Also, no acceptable fit was found with the Woods-Saxon squared form factor until a search attempt was performed using the Cook parameter set. This implies that the version of SNOOPY (SCHW77) used for this analysis is deficient when it comes to dealing with lithium projectiles. Alternate codes such as HERMES (COOK84c), which are written to handle more than just light ion (A < 4) scattering and which can handle spin 3/2 projectiles, may resolve some of the difficulties encountered in this analysis and needs further investigation.

Another possible reason for the problems that have occurred in fitting the carbon data is that the carbon data may be poor. It was noted that discrepancies existed in the value of the differential cross section for a particular angle, with the difference increasing with time; i.e. the later into the carbon run that the particular data point was repeated, the greater the difference between the measured values. This would indicate the possibility of some type of systematic error which was not removed from the data. One
possible source of such an error is that due to target deterioration. This problem could have been seen by using additional detectors at some fixed angle, and noting any change in the yield. Since such monitors were not utilized in the taking of the data, it was assumed that the target did not change significantly during the bombardment.

The lack of a good fit to either distribution when using the double-folding model without renormalization indicates that this model is deficient in describing the elastic scattering of $^7\text{Li}$ projectiles, and that other effects have to be explicitly accounted for. Neither the ground-state quadrupole contribution nor the projectile-excitation contribution, when coupled to the elastic channel, were able to remove the renormalization requirement of the real part of the double-folded potential. On the surface, this implies that the quadrupole effects are not very important in $^7\text{Li}$ scattering. This conclusion would contradict that reached by Hnizdo et al. (HNIZ81), in which it was found that the inclusion of contributions to the elastic scattering from the large ground-state quadrupole moment of $^7\text{Li}$ removed the need to renormalize the double-folded potential. If one carefully examines the coupled-channels procedure, one will find that the deformation parameter $\beta$ was large ($\beta \approx 0.5$). This large value for $\beta$ indicates that there exists a strong coupling between the reorientation channel and the elastic channel. This
strong coupling may considerably affect the elastic scattering results and may necessitate numerically different optical model parameters to obtain good fits to the experimental data. This need to alter the parameters used for the imaginary potential in the double-folding model was verified when a simultaneous fit using coupled-channels calculations was attempted for both the elastic scattering data and the data belonging to the first excited state of the $^7$Li projectile. The coupled-channels predictions had approximately the correct magnitude, indicating a reasonable value for $\beta$ had been used, but the predictions had the wrong phase and slope. Therefore, an iterative procedure needs to be implemented in the coupled-channels calculations such that the optical model parameters, and possibly the deformation parameter $\beta$, are adjusted in order to obtain an improved fit to each distribution under consideration. Only after the above procedure has been completed can one possibly make any meaningful conclusions as to whether quadrupole moment effects are important for the understanding of $^7$Li scattering.
Appendix A

DATA MANIPULATION

In determining the area under a peak in a spectrum, which is a measure of the intensity of the scattering into a particular angle, a fit to the data is usually performed using a Gaussian peak plus a quadratic-polynomial background of the form

\[ y(x) = a_1 \exp\left(-\frac{1}{2}\left(\frac{x-a_3}{a_2}\right)^2 \right) + a_4 + a_5 x + a_6 x^2. \]  \hspace{1cm} (94)

The goodness of the fit is measured by the quantity

\[ x^2 = \sum_{i=1}^{N} \frac{(y_i - y(x_i))^2}{\sigma_i^2}, \]  \hspace{1cm} (95)

where \( \sigma_i \) is the uncertainty in the point \( y_i \) (taken to be \( \sigma_i^2 = y_i \)). The optimum values of the parameters \( a_i \) are obtained by simultaneously minimizing the value of \( x^2 \) with respect to each of the other parameters. When a spectrum contains more than one peak, and these peaks are not well separated, this fitting procedure usually provides a good method for extracting the area information from the data.
When the data has poor statistics, the above method usually does not provide unique results since the starting values for the parameters $a_i$ seem to have a bearing on their values for the "best" fit. In this case, some of the gross structures in the spectrum are hidden by the statistical fluctuations, thus making it difficult to obtain the correct values for the fitting parameters. A sample spectrum with poor statistics is shown in Figure 31. In order to obtain a more accurate fit, especially when two or more peaks overlap, some type of manipulation on the data must be performed. This manipulation must conserve the information pertaining to the desired fitting parameters, which in this case yield the area under the peak. The most common form of manipulation is averaging the data over adjacent channels, and this procedure is known as smoothing.

The validity of the smoothing procedure is based on the fact that a Gaussian distribution can be approximated by a binomial distribution with probability 1/2 provided that the width of the distribution is larger than 1 (BEVI69). When the smoothing is taken over three adjacent channels, the smoothing function is given by

$$y'(n) = \frac{1}{4}y(n-1) + \frac{1}{2}y(n) + \frac{1}{4}y(n+1),$$

(96)

where $y(n)$ is the number of counts in the $n^{th}$ channel and $y'(n)$ is the averaged number of counts in channel $n$. The
Figure 31: Spectrum with poor statistics - original spectrum

This is the fit that was obtained for the unmodified spectrum.

\[ ^{58}\text{Ni}(^{7}\text{Li},^{7}\text{Li})^{58}\text{Ni} \]

99 MeV

\[ \theta_{\text{lab}} = 47^\circ \]
Figure 32: Spectrum with poor statistics - 1 smoothing

This is the resulting spectrum and the fit that was obtained after the original spectrum underwent 1 smoothing operation.
Figure 33: Spectrum with poor statistics - 3 smoothings

This is the resulting spectrum and the fit that was obtained after the original spectrum underwent 3 consecutive smoothing operations.
results for 1 and 3 smoothings on the data from Figure 31 are shown in Figures 32 and 33 respectively. Note that even after only one smoothing the fluctuations of the data about the "fit" curve have been significantly reduced.

Another type of data manipulation is called summing, and in this process you mathematically decrease the resolution of the spectrum by compressing its range. The summed distribution is given by

\[ y'(n) = \sum_{i=0}^{N-1} y(Nn-i+o), \quad (97) \]

where \( N \) is the "summing factor", \( o \) is the offset or starting channel, \( y(Nn-i+o) \) is the number of counts in channel \((Nn-i+o)\), and \( y'(n) \) is the summed number of counts in the \( n \)th channel. The "summing factor" is defined as the number of channels in the original spectrum that are to be combined or summed into a single channel in the summed spectrum; e.g. a summing factor of 4 means that the contents of the first 4 channels in the original spectrum are to be added together and placed in the first channel of the new or summed spectrum, with the process continuing until every channel in the original spectrum has been involved in a sum. To check the equivalency of the peak areas found for a summed spectrum with the areas obtained for the same unsummed spectrum, summing by integral factors ranging from 2 to 16 was performed on a spectrum with good statistics. The
original spectrum is shown in Figure 34. Also included in this comparison was a check to see if the starting channel for the summing process has an effect on the peak area results. The areas for both the elastic and inelastic-projectile peak for the summing attempts, along with the results obtained with up to 4 three-channel smoothings, are given in Table 8. As can be seen in this table, all of the manipulated peak areas fall well within the statistical uncertainty associated with the peak areas for the unmodified spectrum. The conclusions that can be made from examining these results are that summing conserves the peak area and that the offset has no influence on the summing results. A summed spectrum with a summing factor of 4 and a 2 channel offset is shown in Figure 35. Note that this spectrum has an identical shape when compared with the original spectrum in Figure 34, except that the number of channels has been decreased by a factor of 4.

The smoothing and summing procedures were then applied to the poor statistical spectrum of Figure 31, with their resultant least-squares fit peak areas given in Table 9. For this case, these two processes appear to yield substantially different results. Since the peak fitting actually depends on the ratio of peak areas, equation 72 of section 4.2, it can be seen in the last two columns of Table 9 that the ratio of peak areas provided by both methods are very similar and are well within the statistical uncertainty
Figure 34: Spectrum with good statistics - original spectrum

This is the fit that was obtained for the unmodified spectrum.

\[ ^{58}\text{Ni}(^7\text{Li},^7\text{Li})^{58}\text{Ni} \]

99 MeV

\[ \theta_{\text{lab}} = 25^\circ \]
The spectrum and the fit that was obtained after the number of counts in every 4 successive channels was resummed into a single channel.
$^{58}\text{Ni}(^{7}\text{Li},^{7}\text{Li})^{58}\text{Ni}$

99 MeV

$\theta_{\text{lab}} = 25^\circ$

Figure 36: Spectrum with good statistics - sum by 8

This is the resulting spectrum and the fit that was obtained after the number of counts in every 8 successive channels in the original spectrum was resummed into a single channel.
of each other. Based upon visual examination of the fit to the summed-by-8 spectrum shown in Figure 36 (similar to the 3-smoothed spectrum of Figure 33) and the knowledge that both methods yield identical results when applied to data with good statistics, the summing process was chosen as the method to use in order to obtain better fits to the spectra with poor statistics.

### TABLE 8

**Comparison of data manipulation methods**

The following are the fitted peak areas obtained after manipulating a spectrum with good statistics by one of the two methods mentioned in this section.

<table>
<thead>
<tr>
<th>Number of Smoothings</th>
<th>Summing Factor</th>
<th>Offset</th>
<th>Elastic Area</th>
<th>Inelastic Area</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>16005</td>
<td>3366</td>
</tr>
<tr>
<td>1</td>
<td>&quot;</td>
<td>0</td>
<td>16000</td>
<td>3367</td>
</tr>
<tr>
<td>2</td>
<td>&quot;</td>
<td>0</td>
<td>16009</td>
<td>3382</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
<td>0</td>
<td>16004</td>
<td>3385</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>16014</td>
<td>3377</td>
</tr>
<tr>
<td>&quot;</td>
<td>4</td>
<td>0</td>
<td>15994</td>
<td>3380</td>
</tr>
<tr>
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<td>1</td>
<td>16014</td>
<td>3382</td>
</tr>
<tr>
<td>&quot;</td>
<td>4</td>
<td>2</td>
<td>16032</td>
<td>3392</td>
</tr>
<tr>
<td>&quot;</td>
<td>4</td>
<td>3</td>
<td>16039</td>
<td>3395</td>
</tr>
<tr>
<td>&quot;</td>
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<td>0</td>
<td>16010</td>
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</tr>
<tr>
<td>&quot;</td>
<td>8</td>
<td>2</td>
<td>16011</td>
<td>3378</td>
</tr>
<tr>
<td>&quot;</td>
<td>8</td>
<td>3</td>
<td>16022</td>
<td>3379</td>
</tr>
<tr>
<td>&quot;</td>
<td>8</td>
<td>4</td>
<td>16027</td>
<td>3383</td>
</tr>
<tr>
<td>&quot;</td>
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</tr>
<tr>
<td>&quot;</td>
<td>8</td>
<td>6</td>
<td>16027</td>
<td>3378</td>
</tr>
<tr>
<td>&quot;</td>
<td>16</td>
<td>0</td>
<td>15997</td>
<td>3392</td>
</tr>
</tbody>
</table>
The choice for using the summing procedure can be justified in part by noting that the data was obtained using more channels than were necessary in order to extract sufficient information concerning the widths of the peaks. Had the data been taken with lower energy resolution, one of the consequences would have been an increase in the amplitudes of the peaks in this spectrum. The summing process essentially increases the amplitude information at the expense of information pertaining to the widths of the peaks.

### TABLE 9

Peak area comparison for spectrum with poor statistics

The following are the fitted peak areas obtained after manipulating a spectrum with poor statistics by one of the two methods mentioned in this section.

<table>
<thead>
<tr>
<th>Number of Summings</th>
<th>Summing Factor</th>
<th>Elastic Area (E)</th>
<th>Inelastic Area (I)</th>
<th>( \frac{I}{E} )</th>
<th>( \frac{E}{E+I} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>66.2</td>
<td>37.1</td>
<td>.56</td>
<td>.64</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>90.2</td>
<td>45.1</td>
<td>.50</td>
<td>.67</td>
</tr>
<tr>
<td>2</td>
<td>&quot;</td>
<td>90.7</td>
<td>44.2</td>
<td>.48</td>
<td>.67</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
<td>91.1</td>
<td>43.2</td>
<td>.44</td>
<td>.68</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>76.8</td>
<td>33.8</td>
<td>.44</td>
<td>.69</td>
</tr>
<tr>
<td>&quot;</td>
<td>4</td>
<td>119.3</td>
<td>51.1</td>
<td>.42</td>
<td>.70</td>
</tr>
<tr>
<td>&quot;</td>
<td>8</td>
<td>128.5</td>
<td>57.6</td>
<td>.44</td>
<td>.69</td>
</tr>
<tr>
<td>&quot;</td>
<td>16</td>
<td>130.9</td>
<td>61.5</td>
<td>.47</td>
<td>.68</td>
</tr>
</tbody>
</table>
Appendix B

STRONG-ABSORPTION RADIUS

The elastic scattering of low and medium energy projectiles by a nucleus can be described by many models, with the optical model being one of the most commonly used models. Many different sets of potential parameters can usually be found that will result in equivalently good fits to the data provided by the optical model calculations. The ambiguities between potentials of different depths for alpha-particle scattering have been studied by Igo (IGO 59), and he found that potentials with the same magnitude in the surface region predict the same scattering. Jackson and Morgan (JACK68) state that the relationship between the depth and shape parameters can be expressed as

\[ V_o \exp((R_o - r)/a) = \text{constant, } \quad r > R_o \]  \hspace{1cm} (98)

which can be reduced to

\[ V_o \exp(R_o/a) = \text{constant} \]  \hspace{1cm} (99)

provided that the diffuseness parameter \( a \) is unchanged.
Jackson and Morgan also proposed an alternative criterion for equivalent potentials which states that the real parts of the potentials be equal at the strong-absorption radius (SAR) and that the imaginary parts be small and approximately equal at this radius.

The SAR was first introduced by Blair (BLAI54) in conjunction with his model that he used to account for the general behavior of the elastic scattering cross section as a function of energy for alpha particles incident on various nuclei. For this semiclassical model, when the apsidal distance (distance of closest approach) of the alpha particle's classical trajectory is greater than its interaction radius, the particle will be elastically scattered with only a Coulomb phase shift. On the other hand, the particle will be absorbed when the apsidal distance is less than the interaction radius. These assumptions can alternatively be expressed in terms of the reflection coefficients $\eta_L$ by

$$
\eta_L = 0 \quad \text{for } l < l' \\
\eta_L = \exp(2i\delta_l) \quad \text{for } l > l'.
$$

(100)

The critical $l'$ value is that at which the classical turning point of the motion in a Coulomb field is equal to the radius of the projectile-target interaction. The value for $l'$ is related to the interaction radius $R$, which is assumed
equal to the apsidal distance, through the expression

\[ l' (l' + 1) \hbar^2 = 2mR^2(E-E_C), \tag{101} \]

where \( m \) is the mass of the projectile with a center-of-mass energy \( E \), and \( E_C = Z_T Z_p e^2 / R \) is the Coulomb potential energy (Coulomb barrier) at the classical turning point \( R \). This strong absorption model has come to be known as the "Sharp Cutoff Model," and the apsidal distance \( R \) is now called the "strong-absorption radius".

The validity of the assumptions for the sharp cutoff model has been explored by many groups. Both Kerlee et al. (KERL57) and Cheston et al. (CHES57) have graphically examined the partial wave contributions to the scattering as a function of \( l \) and have found that the transition from pure absorption to pure Coulomb scattering occurs over a small band of partial waves with angular momenta around the critical \( l' \) value. This property of the sharp cutoff model, as well as other associated properties, leads to the conclusion that the properties of the nuclear surface are important for the elastic scattering of alpha particles.

Similarities between results from the sharp cutoff model and the optical model were explored by Blair (BLAI57), as he compared the results of his model's analysis with the parameters of optical model calculations. For a nuclear potential with a tail, unlike the sharp potential radius of
the sharp cutoff model, the classical turning point is located at a distance several surface thicknesses beyond the mean radius $R_0$. The critical angular momentum, $\ell m$, such that the classical barrier for the total optical model potential was safely surmounted resulted in a value for $l_m$ that generally agreed with the $l'$ value obtained with the sharp cutoff model. Blair also noted that the nuclear part (i.e. Woods-Saxon part) of the potential, evaluated at the interaction distance provided by $l_m$, was always small; and therefore, this radius corresponded to a distance considerably far out in the nuclear tail. He concluded that $l_m$ is an "effective" quantity. This quantity is non-uniquely related to the parameters of a model or the model itself, which for the Woods-Saxon potential is a function of $r$, $a$, and $V$. Since this quantity is a property of the nuclear surface, the primary role of the full optical model potential is to furnish a means for describing the nuclear surface.

Jackson and Morgan also investigated the behavior of the optical model wave function as a function of distance along the incident beam direction. By plotting $|\psi|$ along the beam direction, they noted that the moduli of the wave functions on the "illuminated" side of the nucleus show very similar behavior, with the effects due to absorption starting in the vicinity of the SAR. They conclude that the SAR has a real physical significance, and it is as a measure
of the distance from the origin at which the absorption process starts to take effect.

The SAR for a given potential, along with the value for the potential at this distance, was determined with the aid of the optical model code SN00PY6 (SCHW77). The program used an Woods-Saxon optical potential or a microscopic potential to generate the reflection coefficients $n_L$, from which the critical $l^*$ value was found by determining at which $l$ value the $\text{Re}(n_L)=1/2$. This angular momentum value was then inserted into equation 101, which was then solved for $R$. The comparison between the real parts of the potentials was accomplished by plotting the real parts versus their radial distance, as seen in Figure 18 of section 4.5 from the double-folding analysis, and noting the value of the potentials at the SAR. This figure shows that all three potentials have nearly the same value and slope at the strong-absorption radius of 6.3 fm for 99 MeV $^7\text{Li}$ ions on a $^{12}\text{C}$ nucleus, and therefore should predict comparable scattering distributions.
THE 99 MEV $^7$LI SCATTERING DATA

The following is a tabulation of the experimental cross sections for the elastic and inelastic scattering of 99 MeV $^7$Li from targets of $^{12}$C and $^{58}$Ni. In the first column are the center of mass scattering angles, which are known to approximately 0.2 degrees. The second and third columns contain the elastic cross sections and their errors respectively. The fourth column lists the cross sections for the scattered $^7$Li particles in their first excited state. The last column contains the errors associated with column four. The errors or uncertainties are relative errors and reflect not only the statistical uncertainty but also the point-to-point uncertainty discussed in section 4.3.
<table>
<thead>
<tr>
<th>CM ANGLE (deg)</th>
<th>ELASTIC SIGMA CM (mb/sr)</th>
<th>ELASTIC ERROR (mb/sr)</th>
<th>INELASTIC SIGMA CM (mb/sr)</th>
<th>INELASTIC ERROR (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.83</td>
<td>1.23E+03</td>
<td>7.26E+01</td>
<td>3.05E+02</td>
<td>1.81E+01</td>
</tr>
<tr>
<td>11.62</td>
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<td>1.28E+02</td>
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<td>8.70E+01</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>16.36</td>
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<td>1.40E+02</td>
<td>7.53E+01</td>
</tr>
<tr>
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<td>19.52</td>
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<td>6.81E+01</td>
<td>3.77E+01</td>
</tr>
<tr>
<td>21.09</td>
<td>3.47E+02</td>
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<td>2.29E+01</td>
<td>1.36E+01</td>
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<td>22.67</td>
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<td>8.34E+00</td>
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