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FORCE AND COMPLIANCE CONTROL FOR ROUGH-TERRAIN LOCOMOTION
BY MULTI-LEGGED ROBOT VEHICLES

The Ohio State University

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by

Tae-Sang Chung, B.S., M.S.E.E.

* * * * *

The Ohio State University
1985

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To my family
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Chapter 1
INTRODUCTION

1.1 Background

Over the past two decades a considerable amount of research has been done in the field of artificial legged locomotion because of its wide range of potential applications, generally in environments too hazardous for humans or in those inaccessible to conventional wheeled or tracked vehicles [1,2,3,4,5]. Some possible applications of legged locomotion are in the areas of land and under-water exploration, remote planet exploration, fire fighting, and hazardous nuclear power plant operation.

The major design objectives of legged vehicles are to achieve superior terrain adaptability and maneuverability on rough terrains. These capabilities result from the flexibility due to the large number of controllable degrees of freedom in multi-link leg systems. However, in order to realize the advantages of legged vehicles, a highly sophisticated joint coordination problem needs to be solved in real time [6,7], which is far from a human operator's capability [2].

The original walking machines were simple, finite-state machines operated by either purely mechanical or electronic circuit-controlled linkages [2,8]. As computer technology advanced, making real-time control possible, computer-controlled legged vehicles came into being and the concept of supervisory control was born [6,7,9]. However these
vehicles' capabilities were limited to implementing gaits and developing, control algorithms, mostly on fairly even terrains. Recent advances in the speed and capabilities of digital computers and research on control algorithms utilizing various sensory devices have made it possible to achieve terrain adaptability in legged vehicle locomotion [10,11,12, 13].

A major research project in legged locomotion, conducted at the Ohio State University, resulted in the development of an experimental prototype, the OSU Hexapod, plus various control schemes and gait selection algorithms for the vehicle. The vehicle was fully equipped with vector force sensors at each leg, and an algorithm was developed to implement actively compliant motion for adaptation of the vehicle to uneven terrains [14]. This algorithm, using force measurements at the foot-tips, actively balances positional and force errors in Cartesian space.

With the algorithm implementing active compliance, it was demonstrated that the OSU Hexapod is capable of walking adaptively over irregular terrain while maintaining body orientation. However, with force feedback, the vehicle experienced unexpected and theoretically inexplicable oscillation in certain leg postures. The following factors were considered as possible causes: (1) the force feedback control law itself (2) passive compliance in the system (3) force interaction of the legs (4) insufficient control frequency (5) excitation of system resonance and (6) uneven control intervals.

The objectives of this dissertation are to investigate the sources of system instability in force feedback control, to understand
the necessary considerations in force feedback control, and thus to improve the force control algorithm for actively compliant motion of legged vehicles. Therefore, this dissertation will cover most of the topics listed in the previous paragraph.

1.2 Organization

Previous work performed in the area of legged locomotion is reviewed in Chapter 2. The purpose of this chapter is to provide an overview of the control scheme of legged vehicles, thus creating a basis for subsequent discussions. The evolution of legged vehicles is outlined, and some recent developments are discussed. The overall control structure of walking machines is reviewed, in a hierarchical manner, with the related mathematics.

Chapter 3 covers active compliance. It investigates and solves the stability problem resulting from force feedback, and focuses on methods of resolving Cartesian force errors into joint space. Several different force control algorithms for general manipulator systems are surveyed, and their applications in the control of legged vehicles are studied.

Compliances can be implemented either in Cartesian space (Cartesian compliance) or joint space (joint compliance). It will be shown that, when decomposed into individual actuator systems, Cartesian compliance can cause difficulties in resolving force errors into joint space. Depending on the postures of a manipulator, Cartesian compliance may require positive feedback of the joint torque error to a certain joint actuator. Thus, the stability of the control of a certain joint is mainly dependent on the control of other joints. This requires heavy
interaction among individual actuator systems for overall system stability. As a solution to the stability problem, it is suggested that compliance be carried out in joint space instead of Cartesian space.

In Chapter 4, methods of establishing force setpoints are studied. In order to implement active compliance for the vehicle's locomotion, it is necessary to define the contact forces of the foot-tips against their environment so that the vehicle both maintains its balance on the supporting terrain and achieves the commanded motion. The force constraints usually result in an underdetermined system of equations allowing an infinite number of solutions. The minimum-norm solution based on the pseudoinverse technique has previously been considered the optimal choice [14]. In this dissertation, however, a different criterion is suggested for optimizing the force setpoint: choosing the force setpoint nearest on the solution plane to the current force measurement. With this force setpoint the system is minimally excited by force errors, while force constraints are still satisfied. The discontinuity of force setpoint during alternations of leg phases is smoothed with this technique.

The effect of discrete control on system stability is discussed in Chapter 5. In this discussion, three digital control issues will be studied. The first is the minimum frequency of discrete control necessary for an accurate simulation of the analog control model. In computer controlled systems, usually an analog controller is designed and then simulated by a digital computer. Thus, for an accurate simulation, a certain control frequency range, based on the system's time constant, is required.
The second issue involves the excitation of the system's mechanical resonance by impulsive digital control. Digital control impulses are applied to the system periodically, requiring that the control frequency chosen does not excite system resonance.

The third issue deals with the effect of passive compliance on system stability when encountering discrete-time force feedback. Usually it is understood that accurate position control requires a stiff system. However theoretical examination suggests that, if the control includes discrete-time force feedback, some passive compliance is necessary for system stability. A theoretical analysis and numerical simulation, confirming the above conclusion will be introduced. For all three issues, a quantitative guideline for the amount of passive compliance is suggested.

In Chapter 6, an experimental controller for the OSU Hexapod is proposed, based on the algorithm developed in Chapter 3 for implementing joint compliant motion. A compliance model in the form of a spring-damper system is examined, and from this model two parameters for compliant response are defined: the active spring constant and the damping factor. Servo gains are then expressed in terms of these two parameters. System resonance and control frequency are both considered in imposing constraints on the two free parameters. Experimental values will be assigned to these parameters which satisfy constraints, thus making it possible to obtain a set of servo gains.

Chapter 7 will develop an algorithm for distributing active compliance over some legs by using a hybrid control philosophy. This control scheme is achieved when some of the supporting legs are controlled by position only, and others are controlled by both
position and force. This strategy reduces force interactions among supporting legs.

When all the supporting legs are programmed for compliant motion, an interaction in force among legs may take place with active and/or passive compliance, causing the vehicle to roll and/or pitch. Numerical simulation will be presented which shows a possible interaction in force among legs.

A second simulation will be presented to demonstrate that interactions in force can be reduced using hybrid control allocated by legs. Three considerations will be discussed regarding the implementation of hybrid control. One is the dynamic selection of non-compliant supporting legs. The second is the timing of a mode switch between compliant and non-compliant supports, and the last is an assignment of relative leg phases suitable for implementing the hybrid control.

Finally, the contributions of this dissertation are summarized in Chapter 8, and suggestions for possible future research are discussed.
Chapter 2

REVIEW OF PREVIOUS WORK AND RELATED MATERIALS

2.1 Introduction

This chapter gives a brief overview of previous work done in the field of artificial legged locomotion and reviews some related mathematics. Earlier works provided a theoretical formalization of gaits of walking animals, and have been well established in the literature [15,16,17]. Control algorithms to implement the gaits are still under development, utilizing recent advances in computer architecture and sensory devices, such as artificial vision systems and ranging devices, that allow the vehicle to adapt to various terrain conditions.

In Section 2.2, some of the currently developed legged vehicles will be reviewed briefly, in historical order, showing the evolution of walking machines. Emphasis will be placed on the more recently developed vehicles equipped with sophisticated technological devices. More detailed information on most of the vehicles can be found in the literature [1,18,19,20].

In Section 2.3, several aspects of legged vehicle control are reviewed in a hierarchical manner, along with their related mathematics. The major topics are supervisory control for man-machine interface, leg motion planning and gait selection for implementing supervisory control, resolution of Cartesian variables into joint variables, motion execution
at the joint level, and compliant motion and attitude and altitude control for terrain adaptability.

In Section 2.4, the system configuration of the OSU Hexapod vehicle is provided. This vehicle has served as a test bed for evaluating various control schemes and gait selection algorithms, and was the model for experiments leading to the theory presented in this dissertation.

2.2 Existing Legged Vehicles

A number of legged-locomotion vehicles have been developed, ranging from those based on purely mechanical-linkage control to those controlled by electronic circuits, to those which are, at present, controlled by computers. The latter have on-board computers capable of performing complex high-speed computations and arriving at logical decisions using various sensory devices. Most have been constructed to implement gaits and develop control algorithms in a laboratory environment. But, some specialized machines, utilizing principles of legged locomotion, have been built for commercial purposes. Typical examples of the former are the G.E. Quadruped Transporter [2], the "Phoney Pony" [1], the OSU Hexapod [20], the Tokyo Institute of Technology Quadruped [11], the Moscow State University Hexapod [6], and the Carnegie-Mellon University six-legged vehicle [21]. The latter include the Menzi Muck Climbing Hoe [22], the Kaiser Spyder [23], and the ODEX 1 [4]. The development of walking machines is outlined in historical order in Table 2.1 [24].

Some recent developments in walking machines are of particular interest for several reasons. A quadruped vehicle developed at the Tokyo Institute of Technology utilized a pantograph leg design [11].
Table 2.1
Existing legged vehicles [24]

<table>
<thead>
<tr>
<th>Year</th>
<th>Researcher</th>
<th>Vehicle Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>J.E. Shigley</td>
<td>Quadruped vehicle with mechanical linkages. Impractical because of noncircular gears and complex linkages.</td>
</tr>
<tr>
<td>1960</td>
<td>Space General Corporation</td>
<td>Six-legged and eight-legged machines. Both used mechanical linkages and had limited adaptability to terrain conditions.</td>
</tr>
<tr>
<td>1966</td>
<td>McGhee and Frank at University of Southern California</td>
<td>The first legged vehicle to walk autonomously under computer control. A four-legged machine referred to as the &quot;Phoney Pony.&quot;</td>
</tr>
<tr>
<td>1968</td>
<td>General Electric Corporation</td>
<td>A four-legged machine weighing 3,000 pounds. The operator was required to use hands and feet to manually control the twelve joints for movements of the vehicle legs.</td>
</tr>
<tr>
<td>1969</td>
<td>Bucyrus-Erie Company</td>
<td>A four-legged machine weighing 13,500 tons, used for coal mining operations. The largest walking machine ever built.</td>
</tr>
<tr>
<td>1972</td>
<td>Vukobratovic at Institute Mihailo Pupin, in Yugoslavia</td>
<td>Powered biped exoskeletons for use in locomotion of paraplegics.</td>
</tr>
<tr>
<td>1972</td>
<td>Waseda University in Tokyo, Japan</td>
<td>A series of biped robots, pneumatically or hydraulically controlled. Used pressure sensors at front and rear of feet to detect ground reaction.</td>
</tr>
<tr>
<td>Year</td>
<td>Researcher</td>
<td>Vehicle Features</td>
</tr>
<tr>
<td>------</td>
<td>------------------------------------------------</td>
<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1972</td>
<td>University of Rome, in Italy</td>
<td>Six-legged vehicle. Two degrees of freedom per leg, using interactive computer control.</td>
</tr>
<tr>
<td>1976</td>
<td>University of Wisconsin</td>
<td>Exoskeleton for use by paraplegics, controlled by analog computer.</td>
</tr>
<tr>
<td>1977</td>
<td>Ohio State University</td>
<td>Six-legged vehicle referred to as the OSU Hexapod. Three degrees of freedom per leg, using interactive computer control.</td>
</tr>
<tr>
<td>1980</td>
<td>Tokyo Institute of Technology</td>
<td>Quadruped vehicle using pantograph leg geometry and interactive computer control.</td>
</tr>
<tr>
<td>1983</td>
<td>Odetics, Inc.</td>
<td>Six-legged vehicle, referred to as ODEX. Seven on-board computers controlled by the external operator.</td>
</tr>
<tr>
<td>1985</td>
<td>Ohio State University</td>
<td>Six-legged vehicle referred as the Adaptive Suspension Vehicle (ASV). Fully self-contained and automatically terrain adaptive vehicle. Sixteen computers controlled by the on-board operator. Three degrees of freedom per leg and pantograph leg geometry.</td>
</tr>
</tbody>
</table>
With this configuration the motion of the foot-tip in each direction on the leg geometry plane appeared in a fixed ratio, to be proportional to that of the corresponding prismatic actuator. This allowed for simpler calculations and easy executions of positional variables. When the foot-tip moved in the direction parallel to one of the prismatic actuators, only that actuator was used and energy was conserved. This pantograph configuration influenced the leg design of the Adaptive Suspension Vehicle [25,26] recently completed at The Ohio State University. The directions of two prismatic joints of the vehicle were chosen to be vertical and parallel to the vehicle body. Thus actuation of the vertical joint is not necessary in maintaining the height of the vehicle during forward locomotion. This conserves energy [11,27].

The first self-contained walking vehicle was the six-legged one, built at Carnegie-Mellon University [21]. Its leg motion used a hydraulic actuator system powered by a body mounted gasoline engine. A finite state, supervisory control of the vehicle was initiated by an on-board operator, with a microprocessor monitoring the operator's commands and the states of each leg system to help facilitate steering.

The Adaptive Suspension Vehicle (ASV), constructed at The Ohio State University, is a six-legged, fully self-contained, automatically terrain adaptive walking vehicle, capable of carrying its own operator, payload, power supply, and control computers [26]. A photograph of the assembled vehicle is shown in Figure 2.1. Except for a left-to-right reversal, all six legs are identical, and each has two degrees of freedom using a pantograph geometry in a plane [27]. The lift and drive hydraulic assembly is attached to the body by hinges, allowing each leg a third kinematic degree of freedom, abduction/adduction
Figure 2.1. The Adaptive Suspension Vehicle being assembled at The Ohio State University.
motion. In order to increase the static stability of the vehicle, the rear two legs are attached backward. Each of the six legs is controlled by three independent, hydraulically powered actuators. A distributed computer system is used for real-time control, decoupling various tasks hierarchically and functionally among computers [28]. The functional processing groups monitor operator commands, compute and execute individual leg motions for coordinated movements, monitor motion execution, scan terrains, etc.

2.3 Hierarchical Control Structure for Legged Vehicles

2.3.1 Introduction

This section gives, in a rather hierarchical manner, a summary of several stages in control of legged vehicles necessary to achieve their ultimate goal, "stable walking." It is adequate for efficiency and feasibility for the control structure of a multi-manipulator machine to be hierarchical [7,28]. Figure 2.2 shows a hierarchical control structure used with the OSU Hexapod Vehicle.

The highest level of control is achieved using man-machine cooperation, or supervisory control. The desired motion of the vehicle is determined by human intelligence, by having the person choose the control mode, vehicle speed and direction, and/or vehicle destination, which is most conveniently expressed in the body-fixed Cartesian coordinate system.

The next level is motion planning or coordination of limbs to carry out supervisory commands. This involves the trajectory generation of both the supporting and transferring feet, which involves solving problems of gait selection, foot placement, and foot force control or
Figure 2.2. Supervisory control structure used with OSU Hexapod Vehicle [7].
distribution of vehicle weight. Among these three problems, the gait
selection and foot placement are matters of kinematics, while the
problem of force control is related to dynamics.

The third level is resolved motion control [29]. In this level, the
joint motion to produce a prescribed foot movement in Cartesian
coordinates is obtained. While several other alternatives such as
master-slave control [19] and table look-up [30,31,32] exist, the most
effective means to achieve this function seems to be the use of Jacobian
relations [29,33,34,35].

2.3.2 Supervisory Control

It is desirable that the control commands given to a multi-legged
vehicle by a human operator be supervisory, high level control commands
such as operational mode, direction, and speed of the vehicle body
[6,7,9,28]. For a desired task, the operator initiates Cartesian motion
commands according to his visual feedback and/or other complementary
information from various sophisticated control panel devices such as a
video display, voice synthesizer, etc. The commands are then
conditioned by an on-board computer, considering the capability and the
current status of the system. This type of control relieves the
operator of the complicated coordination problem of leg sequencing.

Generally, the motion of the vehicle body has six degrees of
freedom: three degrees of translational motion and three degrees of
rotational motion. Therefore the velocity of the vehicle body can be
expressed by translational velocity, \( \mathbf{v} \), and rotational velocity, \( \mathbf{\omega} \), with
components expressed in the body-fixed coordinate system:

\[
\mathbf{v} = [v_x \ v_y \ v_z]^T
\]  

(2.1)
and

\[ \omega = [\omega_x \omega_y \omega_z]^T \]  \hspace{1cm} (2.2)

where \( v_x \) is the longitudinal velocity; \( v_y \) is the lateral velocity; \( v_z \) is the vertical velocity; \( \omega_x \) is the roll rate; \( \omega_y \) is the pitch rate; and \( \omega_z \) is the yaw rate. Similarly, translational and rotation accelerations are expressed respectively as follows:

\[ a = [a_x \ a_y \ a_z]^T \]  \hspace{1cm} (2.3)

and

\[ \alpha = [a_x \ a_y \ a_z]^T \]  \hspace{1cm} (2.4)

It is not realistic to expect the ordinary operator to fully utilize the combination of all six degrees of freedom. Practically, vehicle motion is flexible enough with only three degrees of freedom \((v_x, v_y, \text{ and } \omega_z)\), specified by the operator [7]. The other three undefined velocity commands can be actively set to zero or some other value by the control software, to control the altitude and attitude of the vehicle on an uneven terrain.

2.3.3 Motion Planning

2.3.3.1 Cartesian Velocity and Acceleration of the Vehicle Legs

When the vehicle body moves with translational velocity \( v \) and rotational velocity \( \omega \) in its body-fixed coordinate system, the velocity of a supporting foot \( i \) fixed on the ground is seen to be \(-v\) for translation and \(-\omega\) for rotation, both in the body-fixed coordinate system. Thus the resulting velocity vector of foot \( i \) is

\[ v_i = -v + (-\omega) \times p_i \]  \hspace{1cm} (2.5)
where \( \mathbf{p}_i \) is the position vector of foot 1 from the origin of the body-fixed coordinate system, and \( \times \) denotes the vector cross product. Conversely, if the supporting legs are actuated to give their feet the velocity expressed in (2.5), the vehicle body will be propelled by both translational velocity \( \mathbf{v} \) and rotational velocity \( \mathbf{\omega} \). It is assumed that the supporting feet do not slip on the ground.

Substituting (2.1) and (2.2) into (2.5) and performing the cross product gives a matrix form of (2.5):

\[
\begin{bmatrix}
  v_{ix} \\
  v_{iy} \\
  v_{iz}
\end{bmatrix}
= -
\begin{bmatrix}
  v_x \\
  v_y \\
  v_z
\end{bmatrix}
- 
\begin{bmatrix}
  \omega y_p - \omega z_p \\
  \omega z_p - \omega x_p \\
  \omega x_p - \omega y_p
\end{bmatrix}. \tag{2.6}
\]

The expression of acceleration of a supporting foot 1 in the body-fixed coordinate system is obtained by differentiating (2.5) with respect to time:

\[
\mathbf{a}_i = - \mathbf{a} + (-\mathbf{\omega}) \times (-\mathbf{v}) + (-\mathbf{a}) \times \mathbf{p}_i + (-\mathbf{\omega}) \times \mathbf{v}_i \\
= - \mathbf{a} + (-\mathbf{a}) \times \mathbf{p}_i + 2\mathbf{\omega} \times \mathbf{v} + \mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{v}) \tag{2.7}
\]

where the term \((-\mathbf{\omega}) \times (-\mathbf{v})\) is added in the first expression, since the body-fixed coordinate system rotates with \( \mathbf{\omega} \). Notice that the term \(2\mathbf{\omega} \times \mathbf{v}\) is Coriolis acceleration, and the term \(\mathbf{\omega} \times (\mathbf{\omega} \times \mathbf{v})\) is centripetal acceleration.

The foot trajectory of a supporting leg is determined uniquely by (2.5) as long as the foot can stay on the ground kinematically. However, leg phases should be alternated between support and transfer.
for continuous locomotion. Thus the control of legged locomotion requires an algorithm to synchronize alternation of leg phases and generate the trajectory of transfer feet. The synchronization and trajectory of the transfer foot are not necessarily unique, which produces both a complicated problem and great flexibility. The problem to be considered is the concept of gait selection, i.e., when to lift off, when and where to touch down, and how the leg will travel in the air.

2.3.3.2 Gait Selection

2.3.3.2.1 Stability

Several factors should be considered in selecting proper gaits: stability of the vehicle body during locomotion, speed, and terrain adaptability. It is essential that the vehicle not fall over during locomotion. This problem has led to all studies of stability in legged vehicle locomotion. The stability of a vehicle may be measured either statically or dynamically during locomotion, but the dynamical stability test is a very complicated one, thus making the static stability concept the one usually employed. In order to deal with the stability problem in a mathematical manner, McGhee and Frank [36] introduced the following concepts:

**DEFINITION 1:** The support pattern associated with a given support state is the convex hull of the point set of the vertical projections of the supporting feet on a horizontal plane.

A support pattern is considered statically stable if the vertical projection of the vehicle center of gravity is an interior point on the
support pattern. A quantitative measurement of stability is provided by Definition 2.

DEFINITION 2: The longitudinal stability margin is the shortest distance from the projection of the vehicle center of gravity to the front or rear boundary of the support pattern in the direction of travel as measured over an entire cycle of locomotion.

2.3.3.2.2 Periodic Gaits

The total number of distinct gaits in a legged system is extremely large, and this number increases dramatically as the number of legs increases [37]. However, gaits can be categorized, largely into two classes: periodic gaits and non-periodic gaits [1]. A gait is periodic if every limb of the vehicle operates with the same cycle time [8].

The following definitions were developed by McGhee [8] for a mathematical description of periodic gaits:

DEFINITION 3: The period, $\tau$, is the time required for one complete locomotion cycle.

DEFINITION 4: In straight line locomotion, the stride length, $\lambda$, of a gait is the distance by which the center of gravity of the system is translated during one complete locomotion cycle.

DEFINITION 5: The duty factor, $\beta_i$, is the fraction of a locomotion cycle that leg $i$ spends in contact with the supporting surface.

The criterion for an optimally statically stable gait would be that it maximize the longitudinal stability margin. It is necessary, for applying this criterion in a mathematically feasible way, that all the legs of the vehicle operate with the same duty factor [8].
DEFINITION 6: The relative leg phase, $\phi_i$, is the fraction of a locomotion cycle by which the contact of leg $i$ with the supporting surface lags the contact of leg 1.

For a constant leg duty factor, $\beta$, the gait longitudinal stability margin is dependent on the relative leg phases. Bessonov and Umnov [38] have shown that the optimal periodic gait providing the maximum longitudinal stability margin for a hexapod can be obtained by using the following relative leg phases:

$$
\begin{align*}
\phi_3 &= \beta \\
\phi_5 &= 2\beta - 1
\end{align*}
$$

where $\phi_3$ is the phase delay of the left middle leg, and $\phi_5$ is the phase delay of the left rear leg. Both $\phi_3$ and $\phi_5$ are measured as a fraction of a total locomotion cycle and are relative to the placing of leg 1, the left front leg. The relative phases of any right-left pair are exactly half a cycle out of phase to each other. This type of periodic gait shows a wave motion of stepping from the rear to the front, on either side of the vehicle, and thus is referred to as a wave gait [39].

2.3.3.2.3 Free Gaits

For periodic gaits over regular terrain, every leg is lifted and placed exactly one time within one locomotion cycle time, with a preset relative leg phase. Therefore, periodic gaits are relatively simple to implement. Also, they are optimally efficient for straight line locomotion at a constant speed over level terrain [36,38].

However, since the periodic gait does not offer terrain adaptability in relative leg phases, it may lose efficiency and optimality as the degree of terrain irregularities increases. Under certain terrain
conditions, locomotion is difficult or impossible using fixed relative leg phases and leg duty factors. Thus it may be advantageous to utilize a non-periodic, or free gait under those conditions.

Non-periodic gaits are characterized by the absence of fixed relative phase relationships among legs and by the necessity of terrain information for preview. Considerations in gait selections account for stability and continuity of motion. Therefore, an infinite number of gaits are possible, allowing the free gait control algorithms to choose an optimal gait for flexible locomotion over rough terrain.

The complete theoretical formalization of free gait problems has not yet been established. Rather, heuristic algorithms have been developed and are still under simulation study. Kugushev and Jaroshevskij [40] partially formalized the problem of free gaits where the trajectory of the center of gravity is specified in advance and the terrain covered by the trajectory predefined, designating certain regions unsuitable for support. McGhee and Iswandhi [41] completed a formalization of this problem, developed a heuristic algorithm for its solution, and evaluated the algorithm in a computer simulation study. Recently, Kwak [42] refined this algorithm and implemented it in a computer simulation.

2.3.3.3 Foothold Selection and Trajectory of a Foot in the Transfer Phase

Since the body motion of a vehicle does not depend on the motion of a foot in the transfer phase, its trajectory in the air is quite flexible as long as it synchronizes leg phase alternation and avoids collisions with surrounding obstacles. To generate transfer
trajectories, a desired foothold must be defined, then the curve must be determined along which the transfer foot travels. The components of foot position and rate along the ground plane and the vertical direction are determined so synchronization can be maintained through leg phase alternation.

Several criteria that must be considered in determining desired footholds exist. The two most important require that: (1) the foothold stay within its kinematic limit and not interfere with other legs, and (2) the static stability of the vehicle be maintained during the subsequent support phase [24,43,44]. One other criterion is the frequency of the alternation between leg phases [24,45]. It is most desirable to achieve the commanded body movement with the least amount of leg sequencing. If the motion command does not change rapidly, then the best foothold can be predicted dynamically at the front in the reachable area of the foot. However, for computational simplicity and for allowing rapid velocity changes, the same point always can be selected in the reachable area of the foot, usually at the center point [45].

The movement of a foot in the transfer phase should be performed so as to avoid collisions with surrounding obstacles. Thus the curve of the foot-tip, with respect to the earth-fixed coordinate system, should be lifted vertically to some predetermined height and lowered vertically [24,43]. The curve of the transfer remains at the height $h$ above the ground. The trajectory based on this curve should be transformed in the body-fixed coordinate system through the earth-to-body homogeneous transformation updated as the vehicle moves. When the terrain is fairly even, the curve can be defined directly in the body-fixed coordinate
system, usually as a half-sine curve [44] or a rectangular curve [45]. In this case the homogeneous transformation is not necessary.

2.3.3.4 Force Trajectory

When the motion of the vehicle center of gravity is specified, the resultant vector force and moment are accordingly determined. The net vector force and moment are due to external forces and torques resulting from ground reaction forces on the supporting legs. Therefore, six scalar equations can be formed by applying the equilibrium conditions for the resultant body force and moment, each for one component of force and moment along and about the body x, y, and z coordinates.

The equilibrium equations of force and moment are usually underdetermined. Thus, for a given resultant body force and moment, the ground reaction forces on the supporting legs cannot be determined uniquely. This implies that there is an infinite number of solutions to the force equilibrium equations. However, not all can be physically realizable because, for example, a negative vertical force cannot be implemented if the foot-tip cannot grasp the supporting ground. Therefore the inequality constraints imposed by vertical forces larger than a certain positive threshold should be added in the solution. Other constraints require that the ratio of lateral force and vertical force be within certain ranges to prevent the supporting leg from slipping on the supporting surface.

In order to maintain the dynamic stability of the vehicle during locomotion with acceleration and deceleration control, the force solution must be constrained such that the center of pressure (ground reaction force) [46] be within the support pattern maintaining a limit
of stability margin [1]. If the locomotion of a vehicle is controlled by acceleration and deceleration, such as for the ASV control, the center of pressure is not generally the same as the projection of the center of gravity. Lee [43] considered this method in his simulation studies of the ASV.

2.3.4 Resolving Cartesian Variables into Joint Variables

2.3.4.1 Jacobian Relation

For a commanded body motion, Cartesian positional variables of the vehicle feet are obtained in the body-fixed coordinate system. The velocities of supporting feet are determined as the sum of the body translational velocity and velocity due to body rotation. The velocities of the feet in the transfer phase are generated by the chosen gait selection algorithm, and the forces and moments of the supporting feet are determined to produce the desired resultant body force and moment. The next step is to resolve these Cartesian variables into those of individual joints for the execution of desired motion.

Essential to the resolution of Cartesian motion of a foot (end-effector motion for a general manipulator) into individual joint motions is the Jacobian relation. From linkage kinematics of any n-link manipulator it is possible to obtain the Jacobian matrix which relates the terminal link rectilinear velocity $\dot{\mathbf{x}}$ to the corresponding joint rates $\dot{\mathbf{q}}$ as follows [29]:

$$\dot{\mathbf{x}} = \mathbf{J} \dot{\mathbf{q}}. \quad (2.9)$$

In three-dimensional space, $\dot{\mathbf{x}}$ is maximally of dimension six, with three components for translational and three for rotational velocity. Notice
that not all the six Cartesian velocity components are independent if
the row rank of \( J \) is less than six. The joint rate vector \( \dot{\theta} \) is
n-dimensional, and its components are revolute and/or prismatic joint
velocities.

The element \( J_{ij} \) of the Jacobian matrix can be interpreted as the
differential change in the \( x_i \) coordinate due to differential change in
\( \theta_j \) coordinate, alone. Therefore, for the position variables, it is the
partial derivative of the position \( x_i \) relative to the joint position \( \theta_j \),
such that

\[
J_{ij} = \frac{\partial x_i}{\partial \theta_j}.
\]  (2.10)

Thus, the Jacobian actually relates the differential change in joint
spaces to those in Cartesian spaces. Therefore another form of the
Jacobian relation in (2.9) is as follows:

\[
dx = Jd\theta
\]  (2.11)

where \( dx \) and \( d\theta \) are differential changes in Cartesian space and joint
space, respectively.

Differentiating both sides of (2.9) with respect to time gives
acceleration relations:

\[
\ddot{x} = J\ddot{\theta} + J\dot{\theta}.
\]  (2.12)

Equation (2.12) can be interpreted such that the Cartesian acceleration
of the manipulator is the sum of the acceleration due to the joint
acceleration (the first product term in (2.12)), and that due to
centripetal and Coriolis effect of joint velocities (the second product
term in (2.12)). Notice that the centripetal accelerations are
accelerations proportional to the square of a joint velocity, and that
the Coriolis accelerations are proportional to the product of joint velocities from two different links [47].

When the Jacobian matrix is square and has an inverse, the joint rate which will result in a given Cartesian velocity can be obtained uniquely by (2.9), and the joint acceleration from (2.12), respectively, as follows:

\[ \dot{\theta} = J^{-1} \ddot{x}, \quad (2.13) \]

and

\[ \ddot{\theta} = J^{-1}(\ddot{x} - J \dot{\theta}), \quad (2.14) \]

When the Jacobian matrix is not square, an inverse does not exist. However, if the Jacobian matrix has full row rank, the joint variables can still be solved. In this case the joint rate and joint acceleration cannot be determined uniquely from the given Cartesian variables. Using pseudoinverses [48,49,50,51], (2.13) and (2.14) are modified respectively as follows:

\[ \dot{\theta} = J^+ \ddot{x} + (I - J^+J) \dot{\theta}_r, \quad (2.15) \]

and

\[ \ddot{\theta} = J^+(\ddot{x} - J \dot{\theta}) + (I - J^+J) \ddot{\theta}_r, \quad (2.16) \]

where \( J^+ \) denotes the pseudoinverse, and \( \dot{\theta}_r \) and \( \ddot{\theta}_r \) are any arbitrary vectors compatible to \( \dot{\theta} \) and \( \ddot{\theta} \), respectively. Notice that the matrix \( (I-J^+J) \) becomes null when it is premultiplied by \( J \) [49,51], thus the second terms in (2.15) and (2.16) are homogeneous solutions no matter what \( \dot{\theta}_r \) and \( \ddot{\theta}_r \) are.

By varying the homogeneous terms in (2.15) and (2.16), there actually exist an infinite number of solutions, allowing the application
of a criterion for optimality in their solutions. Studies on redundant manipulators are based on applications of various criteria for optimality in the resolution of Cartesian motion into joint space, while introducing homogeneous solutions in their pseudoinverse solutions [52,53,54,55].

2.3.4.2 Forces, Moments, and Equivalent Joint Torques

The effect of static forces and moments applied to the end effector of a manipulator at its individual joints can be expressed by using the same Jacobian matrix in (2.9). The relation can be derived by equating the virtual work performed by the force and moment applied at the end effector, to the virtual work performed by the equivalent torque (revolute joint) or force (prismatic joint) at the joints [56]. That is

$$dW = F^T dx = \tau^T d\theta$$

where $F^T = [F_x F_y F_z M_x M_y M_z]$ and $\tau$ is the vector of joint torques and/or forces.

Substituting (2.11) into (2.17) gives

$$dW = F^T J d\theta = \tau^T d\theta$$

Since (2.18) is true for any virtual displacement, the term $d\theta$ can be removed, giving

$$\tau = J^T F$$

This relation gives the torques for revolute joints and forces for prismatic joints, which must be applied at the manipulator joints in
order to maintain equilibrium against an applied force and moment at the end effector.

2.3.4.3 Jacobian Resolved Motion Control

There are various methods of providing position control of a manipulator, depending on the time derivative order of the Cartesian position variables under control [57]: resolved motion position control (RMPC) [33], resolved motion rate control (RMRC) [29,34], and resolved motion acceleration control (RMAC) [35]. All resolved motion control schemes resolve the control into joint coordinates specifying \( \theta, \dot{\theta}, \) or \( \ddot{\theta} \). In RMPC, \( x \) is specified, thus \( \dot{\theta} \) may be also specified by inverse kinematics (generally inverse kinematics is not unique), and \( \ddot{\theta}, \) and \( \dddot{\theta} \) are obtained numerically. In RMRC, \( \ddot{\theta} \) is specified by (2.13), \( \dddot{\theta} \) obtained numerically, and \( \dot{\theta} \) is measured. In RMAC, \( \dddot{\theta} \) is specified by (2.14), and \( \dddot{\theta} \) and \( \ddot{\theta} \) are measured.

Depending on the motor model of each joint, the appropriate motor input signal is generated from the given \( \theta, \dot{\theta}, \ddot{\theta}, \) and the joint torque applied externally. If the manipulator is modelled as having inertia, joint torques can be obtained by the Lagrangian formulation [58] or Newton-Euler formulation [59,60] of manipulator dynamics.

2.3.5 Motion Execution and State Feedback Control

The result of supervisory control and motion planning is a sequence of desired positions, velocities, and/or forces of the manipulator or leg end effector. This sequence is then converted to the corresponding joint variables. A sequence of force and torque signals for each actuator then is determined to execute motion. However, the simple application of the planned command sequence to the manipulator actuators
most often fails to produce the desired motion because of the cumulative
effects of unpredictable disturbances. The sources of disturbances may
be unpredictable errors arising from inaccuracies of kinematic and
dynamic models of the system under control, limitations of computational
precision and control frequency of digital control, mechanical effects
such as friction and vibration, and uncertainty of the environment on
which the system works. In order to execute a planned sequence of
motions correctly under these disturbances, a feedback control
measuring actual motions is necessary during motion execution.
Subsequent inputs to the system are modified according to the errors
between desired and actual motions.

Feedback loops can be closed in joint space or in Cartesian space.
In Cartesian space control the actual Cartesian states are determined
from measured joint states through the direct kinematics and/or the
Jacobian. Thus, errors between desired and actual states are computed
in Cartesian space, and resolved into joint space after being multiplied
by a proper feedforward compensation. In joint space control, the
desired Cartesian states are transformed into joint variables through
reverse kinematics transformation and/or the inverse Jacobian. Errors
are then computed from desired and measured joint states in joint space.
As before, these errors are applied to the joint actuators through
proper feedforward compensation.

The two different feedback control structures may result in
different computational complexities and accuracies. Also their control
performances and sensitivities may differ, since the stages where the
gain matrix is applied differ.
2.3.6 Rough-Terrain Adaptability

2.3.6.1 Introduction

When a legged vehicle moves on an even surface, there is little need to sense and analyze the terrain; all it must do is specify footholds within its kinematic limits and stride toward them. Actually, the use of legged locomotion is not justified by such a simple terrain. Its usefulness is based instead on its potential rough-terrain adaptability on off-road conditions. Rough terrain may be defined as a surface in three-dimensional space in which the heights of surface points are not constant. The surface may contain slopes, ditches, holes, obstacles, etc. Thus the ultimate goal of algorithms in legged vehicle control is to obtain a high degree of terrain adaptability.

Obviously, it is almost impossible for an artificial legged vehicle to achieve adaptability equal to that of human beings and legged animals, which have superior sensory organs, such as balancing and vision systems, and life-long experience in their environments. However, applications of various sensor devices in the control make it possible for legged vehicles to have some terrain adaptability. Thus, algorithms for sensor applications controlling rough-terrain locomotion are necessary and important.

Terrain adaptabilities can be classified into two different types. The first is a control system, programmed to react to the interaction between a vehicle and its environment without prior information about the environment. Sensor information may be classified as contact forces between supporting legs and the terrain, proximity measurements, or body attitude measurements. In this case adaptability is obtained in a feedback-and-correct manner. Later, actively compliant motion by
force feedback will be discussed, as well as attitude control based on measurements of the pitch and roll angles of the vehicle body.

The second type of terrain adaptability is achieved when the control system previews the terrain lying ahead using an artificial vision system [61] or a terrain scanner [12]. This allows the vehicle to prepare and make proper adjustments in anticipation of variation of terrain surface. When incorporating terrain preview, a free gait algorithm is suitable for achieving rough-terrain adaptability. It should be noted that simulation studies of free gait algorithms generally assume that terrain preview is possible [12,40,41,42].

2.3.6.2 Force Feedback and Compliant Motion

When the end effector of a manipulator comes into contact with its uncertain working environment, it is necessary for the manipulator to adapt itself to it. To accomplish this, positional commands are modified according to the contact force developed between the end effector and the working environment [62,63]. The basic strategy of accommodation involves the vector position commands in a coordinate system being deduced with appropriate gains directly from vector position and force measurements. The modification of positional variables can be programmed either in Cartesian space [64,65] or in joint space [66]. The former applies to Cartesian compliant motion, and the latter applies to joint compliant motion. These two methods of achieving compliance will be discussed in detail in Chapter 3.

The same strategy is necessary for the supporting legs of a walking vehicle adapting to terrain irregularity. If the foot-tip is controlled only by position variables and the terrain presents an unexpected
irregularity, the vehicle's load may prove ill-distributed among the supporting legs. Thus some legs are loaded excessively, while the feet of other legs may not even touch the supporting ground. This is not a desirable situation in locomotion control. Furthermore, the vehicle attitude may not be in the desired status. In order to avoid this situation the positional variables of the supporting feet are modified according to the measured ground reaction forces so as to shift the vehicle load from excessively loaded legs to insufficiently loaded legs.

Briggs [67] first designed, constructed, and installed a force sensing unit on one leg of the OSU Hexapod. The sensing unit provided a vector force information for the one leg, and this force information was utilized to demonstrate that the vehicle leg could adapt to terrain conditions through the use of active compliance [67,68].

Pugh [14] implemented and experimentally evaluated force feedback control for all six legs of the OSU Hexapod. In this work, the desired velocity was modified linearly by the position error and force error, with constant gains, as follows:

$$\dot{x}_c = \dot{x}_D + K_p(x_D - x_A) + K_F(F_D - F_A) \quad (2.20)$$

where variables with the subscript D denote desired motion commands derived externally, and those with the subscript A represent the actual motion measured. The gain matrices $K_p$ and $K_F$ are diagonal, and the ratio of corresponding elements on the diagonals gives a measure of active compliance. The desired position $x_D$ is often generated by numerically integrating the externally derived velocity command. The desired foot force $F_D$ is computed from the force constraints with which the vehicle body maintains its static equilibrium. The overall Jacobian
control structure implementing the actively compliant motion is shown in Figure 2.3.

2.3.6.3 Attitude and Altitude Control

The primary function of active compliance is to provide a suspension system for the legged vehicle. It enables all legs in the support phase to maintain contact with the terrain surface, even if it is uneven, and to support a proportionate amount of the vehicle weight. Therefore, it can be expected that actively compliant motion can maintain the vehicle body's current attitude during the whole period of support of a given set of legs. Actively compliant motion can also reduce rolling and pitching of the vehicle in the event of unexpected impact between the ground surface and the foot of the transfer phase [10].

However, when the variations of terrain height are large, the effect of vertical active compliance on vehicle attitude is not predictable. Usually the vertical velocity is not defined externally. Thus, the control software sets it dynamically in order to maintain the pitch and roll angles of the vehicle body at around zero and to maintain a desired amount of ground clearance by the vehicle body. This implies that body attitude information is fed back in motion control.

One of the earliest investigations on body attitude and altitude control was performed by Orin [20]. He implemented automatic body height, pitch, and roll regulations in his computer simulation study. The control algorithm relied on the vehicle support points to calculate a least-squares planar surface, which was then used as a representation of the terrain. The height, pitch, and roll of the body were adjusted
Figure 2.3. Jacobian control structure with force feedback [14].
to maintain the vehicle's position parallel to and at a constant height above this plane. A similar study was done by Lee [43] in his simulation study of the ASV vehicle which used a software simulation of foot proximity sensors.

Chang [44] implemented Orin's algorithm for the OSU Hexapod vehicle. He utilized force sensors to check whether the supporting feet contacted the ground and estimated the supporting surface from the contact points. Pugh [14] used a vertical gyroscope and a pair of gravitational pendulums to sense attitude of the OSU Hexapod. This resulted in the development of a closed-loop attitude regulation scheme to maintain the body in a position parallel to the level plane, rather than the estimated support plane. Broerman [69] developed an ultrasonic rangefinder for the OSU Hexapod to measure the proximity between the foot and the terrain. In that study, altitude of the foot was modified by proximity measurements.

In the work of Pugh [14], the feedback loop for body attitude regulation was closed by converting body attitude error into corrective vertical displacements of the supporting feet. From small angle approximations of trigonometric functions, the displacement of leg $i$ is represented in a linear equation as follows [14]:

$$\Delta z_i = -x_i \gamma_A + y_i \delta_A$$ (2.21)

where $x_i$ and $y_i$ are the $x$ and $y$ components of the foot of leg $i$ relative to the body's center of gravity, and $\gamma_A$ and $\delta_A$ are the body pitch and roll angles measured about the body $y$ and $x$ axes, respectively. The linear equation (2.21) has been derived on the principle that for small
angles the result of two successive rotations about two different coordinate axes is independent of the order of rotations [70].

If the vertical displacement in (2.21) is servoed with a control system with a time constant of \(1/k\), the expression is rewritten as follows:

\[\Delta z_i(t) = (1 - e^{-kt})(-x_i \gamma_A + y_i \delta_A)\] (2.22)

Differentiating (2.22) with respect to time \(t\) and setting \(t = 0\) gives

\[\dot{z}_D^A = k(x_i \gamma_A - y_i \delta_A)\] (2.23)

where \(\dot{z}_D^A\) represents a corrective velocity in the vertical direction derived by the attitude control algorithm. The desired vertical velocity is then obtained by adding the corrective term of (2.23) into the desired velocity, \(\dot{z}_D^T\), generated by the motion planning or trajectory generation step; that is,

\[\dot{z}_D = \dot{z}_D^T + \dot{z}_D^A\] (2.24)

The attitude control system associated with the Jacobian control structure is revealed in Figure 2.4.

2.4 System Configuration of the OSU Hexapod

A major research work in the area of legged locomotion, conducted at The Ohio State University, resulted in the development of an experimental prototype, the OSU Hexapod, and various control schemes and gait selection algorithms. The accumulated knowledge and experience gained from this vehicle made it possible to construct the Adaptive Suspension Vehicle (ASV), a full scale self-contained walking vehicle. Although this dissertation is not dedicated solely to the OSU Hexapod, experiments supporting the theory are based on it. Therefore a basic
Figure 2.4. Attitude control associated with the Jacobian control structure in Figure 2.2 [14].
review of the vehicle is given in this section, and a photograph of the prototype is shown in Figure 2.5. More detailed specifications can be found in references [14,18,71,72] for mechanical hardware, instrumentation, motor control circuitry, instrumentation electronics, computer interface, and control software.

The vehicle frame and leg segments are constructed of aluminum. Recently, two modifications were made to the mechanical structure: the body frame was strengthened by attaching two aluminum plates, and a spring block was attached through a ball and socket joint at the end of each foot to reduce impact on the feet and reduce the digital control frequency requirement. Further discussion of these modifications is included in Chapter 5.

Each of the six legs of the OSU Hexapod has three independently powered angular joints arranged in an arthropod configuration. All joints are powered by the same type of industrial-grade, series-wound, electric drill motor, containing a gear reduction unit [71]. The second stage of the gear reduction is accomplished by a non-backdriveable worm gear, which insures that the joints lock in position when power is removed.

The kinematic model of the OSU Hexapod is shown in Figure 2.6. The legs are spaced evenly on the left and right sides of the vehicle. They are numbered from front to rear on the left side as 1,3,5 successively, on the right side as 2,4,5. As shown in this figure, four dimensions characterize each leg: length of the upper limb segment ($l_1$), length of lower limb segment ($l_2$), elevation joint offset ($l_4$), and knee joint offset ($l_5$). The three joint angles are hip elevation (angle measured from the body x-y plane upward to the upper limb segment), $\theta_1$, knee
Figure 2.5. The USU Hexapod vehicle overcoming obstacles. Force sensors in legs measure vector ground reaction forces. Spring blocks at foot-tips eliminate an undesirable vehicle vibration experienced with force feedback control.
Figure 2.6. Kinematic model of the OSU Hexapod [24].
elevation (angle measured outward from the extension of the knee joint offset to the lower limb segment), $\theta_2$, and leg azimuth (angle measured forward from the body y-z plane to the elevation joint offset), $\psi$. The foot position of each leg is determined by direct kinematics, from the lengths of the limb segments and the three joint angles. Details of this kinematic model can be found in references [20,73].

Each of the eighteen joints is instrumented with a potentiometer measuring its angular position and a tachometer measuring its angular rate [71]. Each leg is equipped with three force sensors measuring the vector ground reaction force. Lateral forces are measured by two semiconductor strain gauges mounted to the sides of the lower segment of each leg. Axial force is measured by a piezoelectric load cell mounted in the lower limb segment [14,18]. Each leg is also equipped with an ultrasonic rangefinding sensor to measure the proximity between its foot and the terrain on which it steps [69].

Two additional sensors on the OSU Hexapod are the attitude sensors and the vision system. They are designed to obtain system level information (attitude and directional vision), while the previously mentioned are for local leg information. A vertical gyroscope has been installed as the primary attitude sensor, and a pair of gravitational pendulums installed as a backup sensor [14]. The attitude information secured from these sensors is used to regulate the vehicle body parallel to a level surface. A binocular vision system consisting of two solid-state TV cameras views the terrain from the front top of the vehicle. However the vision system is used basically to find a location marked by a laser beam, for implementing a "Follow-The-Leader" gait [61] in operation of the vehicle over rough terrain.
The control computer of the vehicle is a PDP-11/70, and is interfaced with the vehicle via an optically isolated digital data link [18]. The link multiplexes data words and controls the flow of data both from the computer to the vehicle (feedforward path) and from the vehicle to the computer (feedback path). Each word of feedforward data consists of eight data bits (256 step resolution) for specifying an input voltage to control the associated joint actuator motor, and five bits for addressing eighteen different motors. One digital-to-analog converter, located on each of the eighteen joints, is used to convert the eight bits of a feedforward data word to an analog voltage. Each word of feedback data consists of 10 bits (1024 step resolution) for sensor measurement and six bits for addressing. All sensor signals are multiplexed by an analog multiplexer to sample-and-hold circuitry, and an analog-to-digital converter transforms the held analog signal into digital form.

2.5 Summary

This chapter has given an overview of previous work done in the field of artificial legged locomotion.

Some of the existing legged vehicles were reviewed briefly, in historical order, showing the evolution of walking machines. Emphasis was placed on the more recently developed vehicles which are of particular interest. These were the Tokyo Institute of Technology Quadruped which utilized a pantograph leg design, the Carnegie-Mellon University six-legged vehicle which was the first self-contained walking machine, and the Adaptive Suspension Vehicle which was constructed at the Ohio State University and is fully self-contained, automatically
terrain adaptive, and will be capable of carrying its own operator, payload, power supply, and control computers.

Several aspects of legged vehicle control were reviewed in a hierarchical manner, along with their related mathematics. The major topics were supervisory control for man-machine interface, leg motion planning and gait selection for implementing supervisory control, resolution of Cartesian variables in joint variables, motion execution at the joint level, and compliant motion and attitude and altitude control for terrain adaptability.

The system configuration of the OSU Hexapod vehicle was provided. This vehicle has served as a test bed for evaluating various control schemes and gait selection algorithms, and was the model for experiments leading to the theory presented in this dissertation.

The remainder of this dissertation will discuss a stability analysis for force and compliance control algorithms and a modification on the existing active compliance strategy. A method of establishing force setpoints, the effect of passive compliance on the system stability, and an algorithm of distributing active compliance over some legs for avoiding leg interactions in force will also be discussed.
Chapter 3
FORCE CONTROL AND COMPLIANT MOTION

3.1 Introduction

In this chapter, several different force control algorithms for the compliant motion of general manipulator systems are surveyed, and their applications in the control of legged vehicles considered. Compliances can be implemented either in Cartesian space (Cartesian compliance) or in joint space (joint compliance), where their classifications are based on the method of resolving Cartesian force errors into joint space. To resolve Cartesian force errors, Cartesian compliance requires the Jacobian inverse, while joint compliance uses the Jacobian transpose. The Cartesian compliance was first introduced by Whitney [64] for achieving manipulator fine motion in the task-oriented Cartesian space. The joint compliance was implied in the feedback structure of the "hybrid force control" scheme which was introduced by Raibert and Craig [66]. Tsai [74] applied the hybrid control scheme to the joint pressure control of the ASV leg system, modifying its computational structure in order to save control computation time.

In the stability analysis of the two methods of compliant motion, a simple two-link manipulator will be considered. Based on this model, it will be proven that, depending on the leg posture of the OSU Hexapod, Cartesian compliance may require positive feedback of the joint torque error to a certain joint actuator. It will be shown that the control of
such a joint can locally be unstable if the environment which the leg system contacts is not compliant enough. This negatively affects the overall stability of the system, since the control and stability of any given joint is mainly dependent on the other joints, requiring heavy interactions among individual actuator systems.

To eliminate instability caused by force feedback, it will be proposed that compliance be in joint space instead of Cartesian space. Using as a model the existing control scheme of the OSU Hexapod [14], which suffers instability during force feedback in some situations, joint compliance will be implemented through proposed modifications of the Jacobian control structure.

## 3.2 Compliant Motion

### 3.2.1 Introduction

When the end effector of a manipulator comes into contact with its working environment, the trajectory is modified to accommodate contact forces or tactile stimuli during the motion. If compliance is defined as the ability of a manipulator to react to its known or unknown environment, it occurs either because the control system is programmed to react to contact force using active compliance or because of passive compliance inherent in the manipulator linkage.

Every element in a manipulator is compliant to some extent. The motors are not perfect position sources, independent of force; nor are the mechanical elements of the manipulator structure perfectly rigid. While an accurate positioning system may require an extremely stiff structure, compliances occurring within the manipulator structure such as structural compliance (e.g. distortion of the links and gears) or
intentional compliance (aided by devices such as Remote Center Compliance systems) may be used to implement compliant motion or to reduce the impact due to collision of the manipulator with its environment. A mathematical modeling of a robot collision with its environment was studied by Zheng and Hemami [75]. In assembly, RCC systems are sometimes mounted between the extremities of the robot and the gripper to solve accommodation problems in assembly operations [76,77].

The use of structural compliance or special compliance devices does not afford much flexibility. Since positional errors are not detected by the control system, even though they are allowed to adapt to the environment, such compliant motion is neither programmable nor controllable. In cases where programmability of compliance is required, compliant actuation or active compliance may be used, often with force sensors in the control loop [77].

Although compliant motion can be implemented either by structural passive compliance or by servo gains in the positional servo, these two sources cannot be separated completely. For example, if active compliance is implemented by digital control, some passive compliance is necessary to reduce the system's bandwidth, and thus stabilize the control system. The effect of passive compliance on the stability of the digital control system will be discussed in Chapter 5.

Actively compliant motion is the most realistic choice for the stable control of legged vehicle [10]. While the control system actuates leg action to follow the planned trajectory, the foot-tips negotiate uncertain terrain by controlling foot contacts during transfer phases and by reacting contact forces during support phases. Thus, active compliance acts as an active suspension system for a legged vehicle.
3.2.2 Actively Compliant Motion

Mason [77] has specified two types of compliant motion: explicit feedback, which specifies a linear relation between end effector force/torque and end effector position/orientation (or end effector velocity/angular velocity); and hybrid control, which controls position/orientation along specified degrees of freedom and separately controls force/torque along the remaining degrees of freedom. The basic concepts and terminology of compliant motion will be studied in this section. Topics to be considered will include center of compliance, the compliant motion coordinate frame, hybrid control, explicit force feedback control, and Cartesian and joint compliance.

3.2.2.1 Center of Compliance and Compliant Motion Coordinate Frame

The center of compliance is a point such that a force applied at the point will cause a translation in the direction of the force, and a pure moment applied about a line will cause only a rotation about the axis which is parallel to the line and passes through the point. The active compliance center may be positioned as desired, which characterizes the behavior of a compliant motion. It is possible to specify compliances for which there is no compliance center [77]. For instance, the response to a force in the x-axis may be commanded to be a motion with a negative component in the x-axis and a positive component in the z-axis, so that the manipulator will climb over obstacles. It is also possible to specify compliances about joint axes. A control scheme using this type of compliant motion will be developed in Section 3.5.

The compliant motion frame defines in space an orthogonal coordinate system for specifying both a manipulator's motion and
interactive forces with its environment. The coordinate system in which the end effector motion is specified is usually designated as the compliant motion coordinate frame, since the end effector of the manipulator contacts its environment and the degrees of freedom of a manipulator are characterized by those of the end effector. However, in the case where the coordination of multiple manipulators is necessary, a reference coordinate system should be chosen as the compliant motion frame. For example, in a multi-legged vehicle, the body-fixed coordinate system may be referred to in defining its motion, and therefore this coordinate system may be appropriate as the compliant motion frame. There are other cases in which compliance is fixed with respect to some object in the task, or not fixed in any predefined coordinate system with its own trajectory [77].

Under most circumstances, three axes of the compliant motion frame are designated parallel or orthogonal to the direction in which the natural and artificial constraint of the manipulator's task is specified. Thus the higher level control implementation is of a much simpler form [62]. In most cases, the origin of the compliance frame will be that compliance center which makes the matrix which relates force variables to positional variables diagonal.

3.2.2.2 Hybrid Control by Directions

A hybrid controller is defined as one that allows force to be commanded along certain degrees of freedom, and position commanded along the remaining degrees of freedom in the compliance frame [63,66]. Once freedoms in the compliance frame are designated, it can be decided which freedoms are position-controlled, and which force-controlled. However,
all external position constraints must be identified first. Each independent constraint eliminates one degree of freedom, and contributes one degree of constraint to the effector. Once the degrees of freedom and constraint have been identified, force commands are designated for the degrees of constraint, and a position trajectory designated for the degrees of freedom.

For instance, if the end effector of the manipulator is to slide on the environmental surface, compliant motion can be accomplished by controlling the position in the sliding direction parallel to the surface, and force in the direction normal to the surface. However, this kind of control scheme will not work where the motion of the base block is affected by, or relies upon, the force exerted by the manipulator. In legged vehicle control, force control should be includable for each degree of freedom, in order to insure desired locomotion and support of the body at a specified height.

3.2.2.3 Explicit Feedback Control

The explicit feedback scheme is based on the idea of generalized stiffness or generalized damping. In this scheme, a linear vector function is defined with a gain matrix to relate the position or velocity vector of the end effector to the force vector. The gain matrix represents stiffness of the control system. The feedback law is written using Hooke's law in vector form:

$$\mathbf{f} = \mathbf{K_S} \Delta \mathbf{x} \quad (3.1)$$

where \( \mathbf{f} \) is a vector of force and torque; \( \mathbf{K_S} \) is a matrix of stiffness; \( \Delta \mathbf{x} \) is a vector of six entries, giving manipulator deviation in position and
orientation. The gain or stiffness matrix $K_s$ is fixed by the programmer.

If the compliance center is positioned at the origin of the compliant motion frame, the matrix of stiffness becomes diagonal. In this case the motion along axes with larger entries on the diagonal of the stiffness matrix are primarily position controlled, while those along axes with smaller entries are primarily force controlled. In extreme cases, such as when entries are zero and the others are very large, explicit feedback becomes hybrid control.

The stiffness matrix is not always diagonal. In particular, this is the case if the origin of the compliance frame does not coincide with the compliance center. In this case, the stiffness matrix can be obtained through a congruence transformation. Let $C$ be a coordinate frame at the compliance center, oriented so that the desired stiffness matrix $K_{SC}$ is diagonal. Expressed in this coordinate system, the desired control law is

$$f_C = K_{SC} \Delta x_C \quad (3.2)$$

Let $M$ be the Jacobian which transforms the differential translation and rotation vectors in the compliance frame to $C$ frame $[70]$. Then the Jacobian relation is

$$\Delta x_C = M \Delta x \quad (3.3)$$

The corresponding transformation on force can be expressed by using the same Jacobian matrix $M$. The relation can be derived by equating the virtual work in both coordinate systems $[56,70]$. That is, let
\[ dW = f_c^T \Delta x_c = f^T \Delta x. \tag{3.4} \]

Likewise, substituting (3.3) into (3.4) gives
\[ dW = f_c^T M \Delta x = f^T \Delta x. \tag{3.5} \]

Since (3.5) is true for any virtual displacement, the term \( \Delta x \) can be removed, giving
\[ f = M^T f_c. \tag{3.6} \]

Substituting (3.2) into (3.6) for \( f_c \) and then (3.3) for \( \Delta x_c \) gives
\[ f = M^T K_{sc} M \Delta x. \tag{3.7} \]

By comparing (3.1) with (3.7), the generalized stiffness matrix in the compliance frame is expressed as the congruence transformation of \( K_{sc} \) by the Jacobian \( M \):
\[ K_s = M^T K_{sc} M \tag{3.8} \]

which is not, in general, a diagonal matrix.

### 3.3 Cartesian Compliance and Resolved Motion Control

To implement an explicit feedback scheme, a compliance center and a compliance frame are first defined. The compliance is defined second, by a linear vector equation with a diagonal stiffness matrix in a coordinate system whose origin is at the compliance center (as in (3.2)). The stiffness matrix involved in this relation characterizes the Cartesian compliant motion. This matrix is then transformed into the stiffness matrix expressed in the compliance frame, by (3.8), giving the final compliance law as (3.1). By this compliance relation,
a force error is transformed into a corrective position variable. The position variables including corrective terms are then resolved into joint variables of the manipulator, by the Jacobian relations described in Section 2.3.4. Cartesian compliance is realized by actuating each individual joint according to the resolved joint variables.

The successive transformations of the compliance relation in Cartesian space and the Jacobian relation from Cartesian space to joint space, seem intuitively and geometrically suitable for force control when considered separately and sequentially. This is possible because vector motion commands in the compliance coordinate system are deduced directly from vector force measurements reflected in that same coordinate system. However, the result of the transformations may not be suitable for the control stability in implementing the compliance control, since the dynamics are not considered in compensating force errors. In the following paragraphs, using a simple two-link manipulator, stability analysis will be performed in conjunction with these successive transformations.

Consider Figure 3.1 of a two-link manipulator. The robot has two rotary joints, whose positions are represented by the angles $\theta_1$ and $\theta_2$. The joint axes are both parallel to the x-axis, so that the robot moves in the y-z plane. The Jacobian matrix and its inverse are 2x2 matrices, and can be explicitly derived in order to relate the rectilinear velocities (differential positions) of the terminal link, and joint angular velocities (differential joint positions):

\[
\begin{bmatrix}
\frac{dy}{dz}
\end{bmatrix} =
\begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{bmatrix}
\begin{bmatrix}
\frac{d\theta_1}{d\theta_2}
\end{bmatrix}
\]

(3.9)
Figure 3.1. A two-link planar manipulator with rotary joints. The joint positions are denoted by $\theta_1$ and $\theta_2$, and measured clockwise. The lengths of links are $l_1$ and $l_2$. 
and

\[
\begin{bmatrix}
\frac{d\theta_1}{d\theta_2}
\end{bmatrix} =
\begin{bmatrix}
J^{-1}_{11} & J^{-1}_{12}
\end{bmatrix}
\begin{bmatrix}
dy
\end{bmatrix}
\begin{bmatrix}
dz
\end{bmatrix}
\]  \hspace{1cm} (3.10)

where

\[
J_{11} = -\ell_1\sin(\theta_1) - \ell_2\cos(\theta_1 + \theta_2);
\]  \hspace{1cm} (3.11)

\[
J_{12} = -\ell_2\cos(\theta_1 + \theta_2);
\]  \hspace{1cm} (3.12)

\[
J_{21} = \ell_1\cos(\theta_1) - \ell_2\sin(\theta_1 + \theta_2);
\]  \hspace{1cm} (3.13)

\[
J_{22} = -\ell_2\sin(\theta_1 + \theta_2);
\]  \hspace{1cm} (3.14)

\[
\Delta = \ell_1\ell_2\cos(\theta_2);
\]  \hspace{1cm} (3.15)

\[
J^{-1}_{11} = \frac{[- -\ell_2\sin(\theta_1 + \theta_2)]}{\Delta};
\]  \hspace{1cm} (3.16)

\[
J^{-1}_{12} = \frac{[\ell_2\cos(\theta_1 + \theta_2)]}{\Delta};
\]  \hspace{1cm} (3.17)

\[
J^{-1}_{21} = \frac{[- \ell_1\cos(\theta_1) + \ell_2\sin(\theta_1 + \theta_2)]}{\Delta};
\]  \hspace{1cm} (3.18)

\[
J^{-1}_{22} = \frac{[- \ell_1\sin(\theta_1) - \ell_2\cos(\theta_1 + \theta_2)]}{\Delta};
\]  \hspace{1cm} (3.19)

The transpose of the Jacobian matrix in (3.9) also relates the terminal link static forces (static force errors) to the corresponding joint torques (joint torque errors) [56]:

\[
\begin{bmatrix}
\tau_1
\tau_2
\end{bmatrix} =
\begin{bmatrix}
J^T_{11} & J^T_{12}
\end{bmatrix}
\begin{bmatrix}
f_y
f_z
\end{bmatrix}.
\]  \hspace{1cm} (3.20)

It is desired to control the tip of the terminal link to contact the ground, while maintaining a certain amount of contact force. Assume, however, that there is a static force error of $\Delta f_z$ in the $z$ direction. This force error is subject to control, thus may be related to the position error $\Delta z$ in $z$-direction. This position error then is resolved into joint variables, using the Jacobian relation of (3.10). The entries of the second column of the inverse Jacobian matrix in (3.10) resolve the differential position error $\Delta z$ into joint 1 and 2.
Notice that the second column of the Jacobian transpose in (3.20) resolves the static force error $\Delta f_z$ into equivalent static torque errors of joint 1 and 2.

Figure 3.2 and Figure 3.3 each show two different postures of the two-link manipulator, for which the inverse and transpose of the Jacobian matrix will be compared. The sign of entries of the Jacobian inverse and the Jacobian transpose can easily be found from the geometry of the manipulator configurations. It seems obvious that $J^{-1}_{12}$ is positive and $J^{-1}_{12}$ is negative for both postures of Figure 3.2 (a) and 3.2 (b). The signs of these entries are represented in Figure 3.2, with the clockwise sign being considered a positive direction. The term $J^T_{12}$ is positive for both postures of Figure 3.3 (a) and 3.3 (b). However, $J^T_{22}$ is negative in Figure 3.3 (a), while being positive in Figure 3.3 (b). By comparing Figure 3.2 and 3.3, it can be observed that $J^T_{22}$ and $J^{-1}_{12}$ are of different signs in posture (b). This means that if the manipulator is near the posture (b), the force and differential position of the tip of the terminal link are resolved in opposite directions of each other.

For the corrective position goal $\Delta z$, joint 1 and joint 2 are to be activated in opposite direction in both postures. With this activation, the corrective position error $\Delta z$ may be controlled in both postures, thus the force error might also be controlled. However, there is a significant difference between the two postures in achieving a force goal. In posture (a) both joints are activated consistently to achieve the force goal. In posture (b), however, there is a joint activated in the direction opposite to that of the torque necessary to reduce force error at the tip of the terminal link. This condition may cause both
Figure 3.2. Directional signs of two entries of the inverse Jacobian matrix. Two different postures of the two-link manipulator are considered.
Figure 3.3. Directional signs of two entries of the transpose of the Jacobian matrix. Two different postures of two-link manipulator are considered.
joint 1 and 2 to be excessively activated for a small force goal, and further, the system stability may be affected. Notice that this situation occurs, not because the magnitude of active stiffness (compliance) is not well programmed, but because the signs of entries of the Jacobian transpose, and those of corresponding entries in the Jacobian inverse, are different to each other. In the next section, the effect of Cartesian compliance on system stability will be studied in the context of this section.

3.4 Stability Analysis of Cartesian Actively Compliant Motion in the OSU Hexapod

3.4.1 Individual Joint Controllers in the Jacobian Control Structure

In order to analyze the stability of force feedback control, an existing Jacobian control system for the OSU Hexapod vehicle will be considered. All six legs of the OSU Hexapod vehicle are identical, and have three degrees of freedom each. The leg geometry is simple enough that the inverse kinematics is solvable. The Cartesian velocities of the foot-tip and the corresponding joint rates are related by the 3x3 Jacobian matrix, or by the inverse of the Jacobian:

\[ \dot{x} = J \dot{\theta} \]  \hspace{1cm} (3.21)

or

\[ \dot{\theta} = J^{-1} \dot{x} \]  \hspace{1cm} (3.22)

The control system is basically a rate servo, and its schematic diagram is shown in Figure 3.4. The closed-loop control is based on the idea of reducing the position and force errors of the foot-tip. If the velocity of the body is specified, then the Cartesian velocity and static force of the foot-tip are computed exactly with respect to the
Figure 3.4. A Jacobian control structure for actively compliant motion. Compliance is programmed to occur in the body-fixed Cartesian coordinate system.
base coordinates (body coordinates). Thus, the joint motors are actuated in a way that will make the actual linear velocity of the foot-tip satisfy

$$\dot{x}_c = \dot{x}_d + k_p(x_d - x_a) + k_f(f_d - f_a) \quad (3.23)$$

where $k_p$ and $k_f$ are scalar constants.

Substituting (3.23) into (3.22) gives

$$\ddot{\theta}_c = J^{-1}[\dot{x}_d + k_p(x_d - x_a) + k_f(f_d - f_a)]$$

$$= \dot{x}_d + k_pJ^{-1}(x_d - x_a) + k_fJ^{-1}(f_d - f_a) \quad (3.24)$$

Assuming that the position error $\Delta x = (x_d - x_a)$ is very small, an approximation can be made in the second equation of (3.24):

$$J^{-1}(x_d - x_a) = \theta_d - \theta_a \quad (3.25)$$

Substituting the approximation of (3.25) into the second equation of (3.24) gives

$$\ddot{\theta}_c = \dot{x}_d + k_p(\theta_d - \theta_a) + k_fJ^{-1}(f_d - f_a) \quad (3.26)$$

Equation (3.26) serves as the basis of the closed-loop algorithm. Since the legs are modelled as massless and feedforward computation is largely inaccurate accordingly, the burden of control system is shifted to the feedback controller, through increased stiffness caused by increase of compensation gains for the error signal:

$$\dot{e}_v = \ddot{\theta}_c - \ddot{\theta}_a$$

$$= (\dot{x}_d - \dot{x}_a) + k_p(\theta_d - \theta_a) + k_fJ^{-1}(f_d - f_a) \quad (3.27)$$

If the compliant motion is desired only in the vertical direction, the z-direction force is chosen to modify the velocity command. In this
case, only the last column of the Jacobian matrix inverse is used in (3.23); thus (3.27) can be re-written as follows:

\[
\mathbf{e}_v = (\dot{\mathbf{q}} - \dot{\mathbf{q}}_{\text{d}}) + k_p(\dot{\mathbf{q}}_{\text{d}} - \dot{\mathbf{q}}_{\text{a}}) + k_f [J^{-113} J^{-123} J^{-133}]^T (f_{\text{zd}} - f_{\text{za}}) .
\] (3.28)

Equation (3.28) is a vector equation, and thus it can be broken into three decoupled scalar equations:

\[
e_{vi} = (\dot{\theta}_i - \dot{\theta}_{a_i}) + k_p(\dot{\theta}_{d_i} - \dot{\theta}_{a_i}) + k_f J^{-113}(f_{dz} - f_{az}), \text{ for } i = 1 \ldots 3 .
\] (3.29)

As derived by (2.19), the joint torque which must be applied at the manipulator joints, in order to maintain equilibrium against the applied force at the foot-tip, is expressed as

\[
\tau = JTf
\] . (3.30)

Therefore the joint torque due to vertical force only is obtained from (3.30) as follows:

\[
\tau_{iz} = J_{T13} f_z
\] (3.31)

where \(\tau_{iz}\) is the torque at joint \(i\) due to the vertical force.

Applying (3.31) into (3.29), the error signal to joint \(i\) is expressed as

\[
e_{vi} = (\dot{\theta}_{d_i} - \dot{\theta}_{a_i}) + k_p(\dot{\theta}_{d_i} - \dot{\theta}_{a_i}) + k_f (J^{-113}/J_{T13})(\tau_{d_i} - \tau_{a_i}) .
\] (3.32)

This signal is applied to the \(i\)-th joint motor through a compensation gain. Notice that the control to joint \(i\) is almost decentralized, or independent of other joints, since the state variables of joint \(i\) are
made available to the control of joint \( i \) only, except for the torque terms. The torque variables result from the interaction between the foot-tip and the supporting ground. Thus, the control of an individual joint interacts with that of other joints through contact forces. From (3.32), a block diagram for signal flow to joint \( i \) can be drawn as Figure 3.5. From the Jacobian control structure, three identical models of joint control have been derived.

### 3.4.2 Stability Analysis for Cartesian Compliance

#### 3.4.2.1 Torque Gains of Joint Controllers

As shown in Figure 3.5, the rectilinear position gain \( k_p \) which is constant acts as the angular position gain. However, the gain for the joint torque is not the same as the rectilinear force gain \( k_f \). The torque gain is \( k_f(J^{-1}_{i3}/J^T_{i3}) \), and varies non-linearly according to the posture of the leg. The linkage structure of the OSU Hexapod shown in Figure 2.6 is the same as that of the two-link manipulator shown in Figure 3.1, except for the hip azimuth joint of the OSU Hexapod leg. It can be noticed that the hip azimuth joint does not affect the vertical position and force of the foot-tip, because the entries of \( J_{31} \), \( J^{-1}_{i3} \), and \( J^T_{i3} \) are all zero for any posture of the leg system. Thus, as explained in the previous section, the sign of the term \( (J^{-1}_{i3}/J^T_{i3}) \) can either be positive or negative, depending on joints. Furthermore, it may change for a given joint depending on the posture of the leg.

Figure 3.6 through 3.11 are plots of the \((2,3)\) and \((3,3)\) entries of both the Jacobian inverse and the Jacobian transpose, and their ratios forming torque feedback gains. The data were derived from the geometry of the OSU Hexapod in pure forward and crab motion. As shown in
Figure 3.5. Block diagram of an independent joint controller for constant Cartesian compliance. Leg coupling effects are ignored in this diagram. The state variables of joint $i$ are made available to only joint $i$. The torque gain $k_T$ varies non-linearly with the posture of the leg system.

\[ k_T = k_f \frac{J^{-1}_{13}}{J^T_{13}} \]
Figure 3.6. Plot of two entries of the inverse Jacobian matrix ($J^{-1}_{23}$ and $J^{-1}_{33}$) of the OSU Hexapod leg system. The vehicle was in forward locomotion.
Figure 3.7. Plot of two entries of the transpose of the Jacobian matrix ($J^T_{23}$ and $J^T_{33}$) of the OSU Hexapod leg system. The vehicle was in forward locomotion.
Figure 3.8. Plot of $J_{13}^T J_{13}^{-1}$ during forward locomotion. This term is the feedback gain of joint torque.
Figure 3.9. Plot of two entries of the inverse Jacobian matrix ($J^{-1}_{23}$ and $J^{-1}_{33}$) of the OSU Hexapod leg system. The vehicle was in crab motion.
Figure 3.10. Plot of two entries of the transpose of the Jacobian matrix ($J_{23}^T$ and $J_{33}^T$) of the OSU Hexapod leg system. The vehicle was in crab motion.
Figure 3.11. Plot of $J_{13}^T J^{-1}_{13}$ during crab motion. This term is the feedback gain of joint torque.
Figure 3.8, the torque gains for both the elevation joint and the knee joint are almost constant for the support phase of forward motion. However, the gain for the knee joint is negative. Figure 3.10 shows that the gains vary largely when the vehicle is in crab motion, causing the leg system to vary its posture between those of Figure 3.2 (a) and 3.2 (b).

When the sign of the torque gain, \( \left(\mathbf{J}^{-1} \mathbf{J}^T \right) \), is negative the joint torque is fed back through positive gain. This implies that a positive feedback loop exists in the individual joint controller shown in Figure 3.5. Since the errors of contact forces can still be reduced stably because of the interactions of the other joints in contact force, it cannot simply be concluded that the control of such a joint is unstable. However, the stability of control of a certain joint is dependent on the performance of the other joints, meaning the controls of the individual actuators are heavily interconnected.

3.4.2.2 Passive Compliance and Stability of Joint Controllers

When the postures of the leg system enter a region where any joint control requires positive force feedback for Cartesian compliance, the stability of that joint is primarily dependent on the performance of the other joints, or the responses of the contact force of the foot-tip. The other joints cannot always perform perfectly, since, for instance, they are controlled by a digital computer and the word size of the computer is limited. Thus, the control of such a joint easily becomes unstable unless the positional feedback gains (position gain and rate gain) are large enough to attenuate the positive torque feedback.
In a case where control interactions are neglected, it can be shown that the stability of the individual joint having positive torque feedback is dependent on both passive compliance between the foot-tip and the supporting ground, and servo gains. When the posture of any of the Hexapod legs is the same as that in Figure 3.3 (b), its knee joint experiences positive feedback. Figure 3.12 illustrates this, where the spring constant of passive spring between the foot-tip and the supporting ground is $K_S$. Based on the physical structure in Figure 3.12 and the control structure of individual joint in Figure 3.5, a signal flow graph as in Figure 3.13 can be drawn. The actual motor model is used in this graph.

Using the state variables assigned on Figure 3.13, the state equations can be formulated as

\[
\dot{x}_1 = x_2 \tag{3.33}
\]

and

\[
\dot{x}_2 = -G(k_p + J_{33}J^{-1}J_{33}k_fk_s)x_1 - (G + 3)x_2 + u \tag{3.34}
\]

The system matrix is then expressed as:

\[
A = \begin{bmatrix}
    0 & 1 \\
   -G(k_p + J_{33}J^{-1}J_{33}k_fk_s) & -(G + 3)
\end{bmatrix} \tag{3.35}
\]

Thus the characteristic equation of the system matrix is expressed as

\[
f(s) = | sI - A | = 0 \tag{3.36}
\]

or

\[
f(s) = s^2 + (G + 3)s + G(k_p + J_{33}J^{-1}J_{33}k_fk_s) = 0 \tag{3.37}
\]

and the system poles are the roots of this equation.
Figure 3.12. The geometry of a situation in which a positive force feedback can occur in the control of the knee joint when the stiffness $K_{sp}$ is too high.
Figure 3.13. Signal flow graph of the local control system for the knee joint.
The axis of symmetry of the parabola graph, \( y = f(s) \), is in the left half plane, because

\[
s = -\frac{3 + \sqrt{\Delta}}{2} < 0.
\]  
\[\text{(3.38)}\]

Therefore the real parts of the two roots of (3.37) are non-positive only when

\[
f(0) = G(k_p + J_{33}J^{-1}33k_fK_s) \geq 0.
\]  
\[\text{(3.39)}\]

This is illustrated graphically in Figure 3.14. When the product of \( J_{33}J^{-1}33 \) is non-negative, the condition in (3.39) is always true and the system is stable regardless of the servo gains and passive spring constant.

However, when the term \( J_{33}J^{-1}33 \) is negative, the condition of (3.39) is satisfied only if

\[
\frac{K_s}{k_p/k_f} \leq -\frac{1}{J_{33}J^{-1}33}.
\]  
\[\text{(3.40)}\]

Notice that the denominator of the lefthand side of (3.40), \( k_p/k_f \), is the active spring constant. Thus the lefthand side of (3.40) is the ratio of the passive spring constant to the active spring constant.

Figure 3.15 is the plot of the passive spring constant \( K_s \), in the unit of the active spring constant which satisfies the equality condition in (3.40) (with varying knee joint angle). The upper part of the graph is the stable region, while the lower part is the unstable region.

Inequality (3.40) implies that there is an upper limit in the active compliance, \( k_f/k_p \), for a given joint angle and the passive spring constant. This inequality can be satisfied by decreasing \( K_s \) (making the system more compliant passively) or increasing the active spring
Figure 3.14. Determination of the signs of two roots of a quadratic equation by utilizing parabola graphs.
Figure 3.15. Upper limit of passive spring constant in terms of active spring constant in stabilizing the local control of the knee joint with positive force feedback. The limit is dependent on the postures of leg system.
constant (making the system stiffer actively). However, either of these two measurements sacrifices the system performance in the continuous control. The first decreases the accuracy of position control, while the second decreases the degree of active compliance.

The interactions among legs occur through the vehicle body they support and the ground on which they step. These interactions are so complicated to analyze that it becomes difficult to formulate dynamics equations for the multi-legged vehicle. Thus, control is usually at the leg control level. The vehicle is modelled as a system of one leg and the effective mass (or worst case mass), and the controller is designed based on this model. For the overall system stability, an experimental tuning then may be necessary. Although stable control of individual legs does not guarantee overall system stability, it is a necessary condition for the control stability of a multi-legged vehicle.

The same argument can be applied to the leg controller and its decentralized joint controllers. The stability of individual joint control is an important and necessary condition for the control stability of a leg system, and thus overall system stability. As explained previously, a force control scheme which relates Cartesian force variables to positional variables may cause a certain joint to have a positive feedback joint torque. If the leg system is in a posture which causes positive feedback, and its contribution is significant, the control of the leg system may become unstable. Accordingly, the overall system may become unstable in force feedback.

It has been shown that control signals for the correction of position error (derived from force errors by a linear relation), may be resolved into joint space in a way contradictory to the force goal,
depending on the posture of the manipulator. This is not a problem of servo gains, but of the force feedback law itself. There is no way to control each joint separately or locally to achieve the desired force goal. Thus to maintain system stability, the rectilinear force control scheme of Figure 3.4 requires strong interactions among individual joint controllers, via external contact force. This stability problem may be solved by adjusting passive or active compliance, which may sacrifice the performance of the active compliance. In the next section, a force control algorithm will be presented which does not cause positive feedback. In that force control, compliance will be implemented in joint space.

3.5 Joint Compliant Motion

3.5.1 Explicit Force Feedback in Joint Space

The desired active compliance control can be implemented directly in joint space, without experiencing the stability problem pointed out for rectilinear force control. The feedback control for joint compliant motion is similar to rectilinear force control. Joint velocity modification is based on the idea of reducing the sum of position and torque errors of the joint. Thus the joint motor is activated to make the actual joint velocity satisfy the equation

$$\dot{\theta}_c = \dot{\theta}_d + k_p(\theta_d - \theta_a) + k_T(\tau_d - \tau_a) \quad (3.41)$$

where $\tau_d$ and $\tau_a$ are the desired and actual joint torques derived by the relation of (3.30), and $k_p$ and $k_T$ are joint position and torque gains, respectively.
This modified desired rate serves as the basis of the closed-loop control. The error signal for feedback control is then obtained as follows:

\[ e_{vi} = (\dot{\theta}_d - \dot{\theta}_a) + k_p(\theta_d - \theta_a) \\
+ k_T(\tau_d - \tau_a) \]  

Equation (3.42) is the same as (3.32) except that the torque gain is a constant instead of a non-linearly varying gain.

The block diagram of the control structure implementing (3.42) is shown in Figure 3.16. Figure 3.17 is the three-dimensional expansion of the joint control structure in Figure 3.16, showing rectilinear variables. By comparing Figure 3.17 with Figure 3.2, it can be noted that the control structure of the joint compliant motion is exactly the same as that of the Cartesian compliant motion, except for the force loop. The force loop goes through the Jacobian transpose instead of the Jacobian inverse in joint compliant motion.

In addition to better stability, the force control structure for joint compliant motion performs better than that for Cartesian compliance. The rectilinear force control does not compensate for system disturbances due to modelling error, and mechanical effects such as internal frictions of the manipulator, changing joint inertia, and gravity loading. Rather, this scheme allows the manipulator rectilinear accommodation to its contacting environment. In joint torque control, however, system disturbances may be compensated to some extent, since the torque terms converted from the measured force between the terminal link and the working environment contain the dynamics of the manipulator [57].
Figure 3.16. Block diagram of an independent joint controller for constant joint compliance. Leg coupling effects are ignored in this diagram. The state variables of joint $i$ are made available to only joint $i$. The torque gain $k_T$ is a constant.
Figure 3.17. A Jacobian control structure for constant joint compliance. Compliance is programmed to occur in all joints.
In control systems where feedforward computation is largely inaccurate or completely ignored, the burden of system control is shifted to the feedback controller, through increased stiffness caused by gain increase in the feedback loop. On the OSU Hexapod, for example, legs are assumed massless and the gravity loading of its body is not counted. Thus, basic position and velocity commands are servoed for locomotion. If the control system depends mainly on the feedback loops, actuators may be easily saturated when the required corrective torques are too large. Thus control instability may result. High servo gains may also limit the range of safe operating speed of the manipulator because the dynamic interactions with speed may increase errors. Furthermore, high positional servo gains are incompatible with compliant motion control.

3.5.2 Joint Compliance and Its Equivalent Cartesian Compliance

3.5.2.1 Compliance Center and Stiffness Matrix

The body-fixed coordinate system serves as the compliance frame for Cartesian compliant motion of the OSU Hexapod, and the origin of this frame is chosen as the compliance center. However, the control structure for joint compliance does not have such a fixed compliance frame. Instead there are local compliance frames, one for each joint, since the compliances are about the joint axes.

The spring constant of the joint compliant motion can be obtained from (3.42). If the servo is perfect and thus the left-hand side of (3.42) approaches zero, the equation becomes the general equation of a spring-damper system:

\[ (\tau_{di} - \tau_{ai}) = -\left(\frac{1}{k_T}\dot{\theta}_{di} - \dot{\theta}_{ai}\right) - \left(\frac{k_p}{k_T}\right)(\theta_{di} - \theta_{ai}) \]  

\( (3.43) \)
where \((1/k_T)\) is considered as a damping coefficient, and \((k_p/k_T)\) as a spring constant. By considering only the spring constant, and extending it to three dimensions, (3.43) becomes a linear joint compliance system:

\[
(I_d - I_a) = -(k_p/k_T)(\theta_d - \theta_a).
\]  

(3.44)

Applying relations of (3.22) and (3.30) into (3.44) gives the Cartesian compliance equation:

\[
J^T(f_d - f_a) = -(k_p/k_T)J^{-1}(x_d - x_a)
\]

or

\[
\Delta f = -(k_p/k_T)(JJ^T)^{-1}\Delta x.
\]

Thus, the equivalent Cartesian stiffness matrix of the joint space compliant motion is expressed as

\[
K_s = (k_p/k_T)(JJ^T)^{-1}
\]

in the Cartesian coordinate system.

3.5.2.2 Equivalent Vertical Stiffness of Joint Compliance

The equivalent Cartesian stiffness matrix of joint compliance, which was derived in (3.47), is a function of the joint positions. Thus degrees of stiffness depend on the posture of the leg system. Since vertical active compliance is of interest in this work, it is helpful to plot vertical stiffness in terms of joint positions. From (3.46) the vertical stiffness is expressed as

\[
K_{sz} = (k_p/k_T)(JJ^T)^{-1}_{33}.
\]

(3.48)

This expression is a function of the joint positions of only elevation and knee joints, since, on the OSU Hexapod, the hip azimuth joint of a given leg does not contribute any vertical motion of the foot-tip.
Figure 3.18 shows the plots of vertical stiffness for general joint positions, normalized by the vertical stiffness at the midstance position of the leg system. As shown in the figure, the vertical stiffness varies largely depending on the postures of leg system, a condition not present with Cartesian compliance. This variation is a shortcoming of joint compliance. At the same time, if the stiffness does not vary much in the working range of joint positions of the vehicle, varying stiffness may not be a serious problem.

Figure 3.19 is the plot of vertical stiffness computed by using joint positions of leg geometry when the vehicle was walked forward. As shown in the figure, stiffness does not change much during the support phase. Thus the effect of joint compliance and Cartesian compliance is almost the same in forward and backward locomotion. Notice that the system becomes a little more compliant at the both ends of the support phase. This is because, at the beginning and end of each support phase, the lower link is stretched out and the moment arm about the elevation joint increased. Accordingly, this increases the corresponding entry of the Jacobian transpose, which transforms the vertical foot-tip force to the elevation joint torque.

Figure 3.20 is the plot of the vertical stiffness while the vehicle was walked in crab motion. The lower limb segment swings inward and outward for crab motion, and thus the moment arm about the elevation joint varies accordingly. The moment arm becomes small when the lower limb is in an inward position. In this case the system becomes stiffer. Conversely, the moment arm becomes large when the lower limb is an outward position, making the system more compliant. These two situations are clearly shown in Figure 3.20. As shown in the figure, the system
Figure 3.18. Plots of equivalent vertical stiffness, at general joint positions, to implement active joint compliant motion. The angles of the elevation joint and knee joint are used as parameters. The stiffness varies depending on the leg posture.
Figure 3.19. Plots of equivalent vertical stiffness computed by using joint positions of the leg geometry while the vehicle is walked forward and normalized by that of the midstance. Variations of the stiffness are not significant during straight-line locomotion.
Figure 3.20. Plots of equivalent vertical stiffness computed by using joint positions of the leg geometry while the vehicle is walked in crab motion and normalized by that of the midstance. Variations of the stiffness are significant during side-step locomotion.

\[ K_{\text{vertical}} \text{ (normalized)} \]

\[ 1.2 \times 10^0 \]
\[ 1.0 \times 10^0 \]
\[ 8.0 \times 10^{-1} \]
\[ 6.0 \times 10^{-1} \]
\[ 4.0 \times 10^{-1} \]
\[ 2.0 \times 10^{-1} \]

TIME (SEC)

Support  Transfer  Support
becomes almost twice as compliant when the lower limb is stretched in the outermost position. This implies that the force gain becomes very high at this posture. Thus it may be necessary to limit the torque gain during side-step motion in order to provide adequate system stability.

3.5.2.3 Compliance Frame of Joint Compliance

The stiffness matrix of (3.47) is a real symmetric matrix, but generally is not diagonal except when the columns or rows of the Jacobian matrix are mutually orthogonal. Only when the manipulator consists of three translational joints whose axes are mutually orthogonal, will the columns or rows of its Jacobian matrix be mutually orthogonal. On the other hand, the stiffness matrix of Cartesian compliant motion is diagonal, with \( \frac{k_p}{k_f} \) as its diagonal elements. This can be obtained from (3.23) by assuming that the servo is perfect; i.e.,

\[
K_S = \frac{k_p}{k_f} I \quad (3.49)
\]

It is interesting to determine whether there is a Cartesian coordinate system in which the stiffness matrix of (3.47) is transformed into diagonal form. There is a theorem stating that by using a similarity transformation, any real symmetric matrix can be diagonalized by a real orthogonal matrix [78]. The diagonal elements, which are eigenvalues of the real symmetric matrix, are all real. Since the inverse of an orthogonal matrix is the transpose of that matrix, this theorem indicates that, for any real symmetric matrix \( S \), an orthogonal matrix \( U \) can be found such that

\[
U^T S U = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) \quad (3.50)
\]
where \( \lambda_i \)'s are real eigenvalues of the matrix \( S \).

The stiffness matrix \( K_S \) in (3.47) is real symmetric, and is positive definite since

\[
[(JJT)^{-1}]^T = (JJT)^{-1}
\]

and, for any non-zero vector \( x \),

\[
x^T(JJT)^{-1}x = x^T[(J^T)^{-1}J^{-1}]x = (J^{-1}x)^T(J^{-1}x) > 0.
\]  

(3.52)

Therefore an orthogonal matrix \( U \) can be found such that

\[
U^T(JJT)^{-1}U = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n),
\]

(3.53)

with \( \lambda_i \)'s being positive real.

From expression (3.8), the diagonal stiffness matrix \( K_{SC} \) in the \( C \) coordinate system can be expressed in terms of the stiffness matrix in the compliance coordinate system and the Jacobian matrix:

\[
K_{SC} = (M^T)^{-1}K_SM^{-1}.
\]

(3.54)

Comparing (3.53) with (3.54), it can be confirmed that if the transpose of the orthogonal matrix \( U \) in (3.53) is a Jacobian matrix form, there exists a coordinate system, i.e., a \( C \) frame, in which the stiffness matrix for joint compliant motion becomes diagonal. However, this match cannot always be made, since the form of Jacobian matrices is fixed as follows [70]:

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\[
M = \begin{bmatrix}
  n_x & n_y & n_z & (p \times n)_x & (p \times n)_y & (p \times n)_z \\
  o_x & o_y & o_z & (p \times o)_x & (p \times o)_y & (p \times o)_z \\
  a_x & a_y & a_z & (p \times a)_x & (p \times a)_y & (p \times a)_z \\
  0 & 0 & 0 & n_x & n_y & n_z \\
  0 & 0 & 0 & o_x & o_y & o_z \\
  0 & 0 & 0 & a_x & a_y & a_z 
\end{bmatrix}
\] (3.55)

where the entries are from the homogeneous transformation \( C \) which describes the \( C \) frame with respect to the reference frame.

\[
C = \begin{bmatrix}
  n_x & o_x & a_x & p_x \\
  n_y & o_y & a_y & p_y \\
  n_z & o_z & a_z & p_z \\
  0 & 0 & 0 & 1
\end{bmatrix}
\] (3.56)

Thus a Cartesian frame in which the stiffness matrix of the joint compliant motion is diagonal may not exist. Accordingly, a single compliance center may not exist in joint compliant motion. Every joint axis serves as a rotational compliance center.

3.6 Experimental Results

3.6.1 Introduction

This section presents experimental results confirming the force control algorithms developed in this chapter. Experiments were performed on the OSU Hexapod, and data were acquired from real-time operation of the vehicle. Based on the Hexapod Control Program version 3.4 [14], modifications were made for active joint compliant motion. The control program was written in the Pascal language, except for a few
routines which were coded in Macro assembly language for the lowest level machine interface. The program runs on the PDP 11/70, under the RSX-11 operating system, with a control frequency of around 52 Hz.

To implement the control structure of active joint compliance shown in Figure 3.17, the following servo gains were used:

\[
\begin{align*}
    k_p &= 2.23 \quad \text{[sec}^{-1}] , \\
    k_T &= 0.00042 \quad \text{[rad/(sec} \cdot \text{lbf} \cdot \text{in})] , \\
    G &= 64.35 \quad \text{[volts/(rad/sec)] .}
\end{align*}
\]

The design method for obtaining these servo gains will be discussed in Chapter 6.

The force setpoints were generated based on the static equilibrium condition of the vehicle. Generally, the system of equations for force constraints are underspecified if the vehicle is supported by more than three legs. Thus there is an infinite number of solutions; the pseudoinverse solution is the one with the minimum norm. The pseudoinverse solution is used for experiments in this section. Solution methods of force constraint equations and optimization of force setpoints will be discussed in Chapter 4.

3.6.2 Experiments for Active Joint Compliance

3.6.2.1 Position-only Control and Passive Foot Force Responses

Before evaluating the performance of the active compliance servo, it was necessary to observe the responses of foot position and force when the Hexapod was walked with the force feedback loop disabled. Figure 3.21 shows the z-axis position response obtained while the vehicle was walked with a leg duty factor of 2/3. Figure 3.22 is the corresponding passive foot force. The plot of position response shows
Figure 3.21. Foot position with the force feedback loop disabled. The Vehicle was walked forward with a leg duty factor of 2/3.
Figure 3.22. Passive force distribution with a leg duty factor of 2/3. The Vehicle was walked forward with the force feedback loop disabled.
close agreement between desired and actual positions, except for a slight amount of phase lag and a steady-state position error of approximately 0.4 inches. It can be noticed that exponential decays followed the half-sine wave trajectories, at which time the desired position reached the predefined vehicle height and the desired velocity command was set to zero. This confirms that the system is highly damped, thus explaining the phase delay in position response.

As shown in Figure 3.22, the uncontrolled force responses (passive foot force) did not agree with the desired foot force generated by the pseudoinverse technique, although the corresponding position response was stable and accurate. The system of equations of force constraints was underspecified with the leg duty factor of 2/3. Thus it has an infinite number of solutions. However, small uncertainties in leg positions and passive compliances add more constraints on the system, determining a valid one among the infinite number of solutions. Thus, the setpoint of the minimum-norm solution did not agree with the actual passive force distribution, one of an infinite number of possible distributions.

3.6.2.2 Disturbance in Force and Active Foot Force Responses with the Vehicle Stationary

The active compliance servo is implemented based on the control structure of Figure 3.17. The first test of the control algorithm was performed with the Hexapod stationary. After the vehicle system was settled with four legs on the ground (leg duty factor of 2/3), a force disturbance was applied to the control system by externally pushing down the upper limb segment of a leg system. This was done to demonstrate
active compliance and test the stability of the force control loop. Figure 3.23 represents actual and desired forces of a leg on which a force disturbance was applied. Figure 3.24 is the position response of the leg. Comparing the two figures, it can be observed that when a force was applied, the vehicle did exhibit active compliance and maintain stability.

When the force disturbance was removed, the force and position settled exponentially. As this experiment demonstrated, the system did not return to its original state. This was not due to the support pattern, which was underspecified for force constraints of the supporting legs. To verify this conclusion, exactly the same experiment was performed with the tripod gait, which is completely specified for foot force constraints. The results, plotted in Figure 3.25 and Figure 3.26, show that the system did not return to the original state where it had been before a disturbance was applied. All these results can be explained by motor and gear frictions. Steady-state position errors in Figure 3.21 can thus be explained by motor stalling. This possibility should be considered in evaluating the subsequent experimental results.

As a result of the first experiment, the following conclusion can be drawn: First, the control implementation showed active compliance, confirming the validity of the active compliance algorithm with a step disturbance. Second, the response settled to a stable state with exponential decay, verifying that the servo gains were properly chosen. Finally it can be concluded that the system has friction in both the motors and gears not responding to small control signals. The validity of the first two conclusions should be further verified with different postures of the vehicle and with different types of
Figure 3.23. Foot force disturbance. The control system for active compliant motion was disturbed by loading one of the vehicle legs externally.
Figure 3.24. Position responses against foot force disturbance. The Vehicle was supported by four legs. The leg control system was actively compliant to the external force disturbance. The leg system did not return to the original state when the disturbance was removed, confirming the existence of motor and gear friction.
Figure 3.25. Foot force disturbance. Three legs supported the vehicle.
Figure 3.26. Position response for foot force disturbance. Three legs supported the vehicle. The leg system did not return to the original state when the disturbance was removed, even with tripod support.
disturbances. This is the task of the next experiments done with the vehicle walking.

3.6.2.3 Actively Compliant Motion of the Vehicle in Walking

For more practical evaluation of the control algorithm for actively compliant motion, experiments were performed with the Hexapod vehicle walking. Under these circumstances, it was judged, the control system would be activated by a dynamically changing input, with almost all the possible postures of the legs occurring in walking. The experiment was performed with the vehicle walking forward with a leg duty factor of 2/3, meaning that four legs were in the support phase at all times. Figure 3.27 is the plot of the force tracking of leg 1. Figure 3.28 is the plot of position tracking. It can be observed that large discontinuities in force setpoints existed whenever the support pattern was changed. The desired positions were obtained by numerically integrating the desired velocity command given to the system externally.

As shown in Figure 3.27, there exists a small approximately constant gap and small-amplitude chattering in force responses. However, in general, the actual force tracked the desired force quite well, even with large discontinuities in force setpoints. The force gap resulted from using an inaccurate total vehicle weight and an inaccurate center of gravity in formulating force constraint equations. Even with these inaccuracies, the gap in force was compensated by allowing position errors, as shown in Figure 3.28. Thus the existence of errors can be interpreted as homogeneous components in stable states of the control system in terms of position and force. The presence of small-amplitude chattering can be explained two ways. First, that the
Figure 3.27. Foot force tracking using active joint compliance in an underspecified gait. The vehicle was walked forward with a leg duty factor of 2/3, meaning that four legs were in the support phase all the times. The foot force setpoint was generated as the pseudoinverse solution of the force constraint equations.
Figure 3.28. Foot position tracking using active joint compliance in an underspecified gait. The vehicle was walked forward with a leg duty factor of 2/3.
controls were applied on discrete points with limited resolutions, and second, that the whole vehicle was vibrating through passive compliance. It should be noticed that the chattering was also observed on the passive force response shown in Figure 3.26.

Another discrepancy in force tracking is apparent in Figure 3.27. Note that the actual force became non-zero before the commanded force when the legs alternated their phases between support and transfer. This was caused by the trajectory of desired force having been generated according to the leg kinematic cycle phase, not according to the ground contact of the leg. It can be observed from Figure 3.28 that the force discrepancy was compensated by position errors, as was programmed. With this capability, the vehicle can negotiate with or accommodate itself to irregular terrains.

Since the experiment was performed on level terrain, the force discrepancies at both sides of the support phase indicate something about the physical characteristics of the vehicle. Notice that the rising and falling of actual forces do not occur instantaneously, but have slopes. This is because the vehicle was compressed as a whole in the vertical direction, through its passive compliance. Each leg system was compressed or freed when lowered onto the ground or lifted off the ground. Actually, structural compression was observed visually during the experiment, although it was not reflected in position measurements. This situation is quite desirable in legged locomotion, in the sense of both comfort and control of the suspension system, since the force impulses from repeated collisions of feet on the supporting ground are smoothed. This is one reason that the legged system needs some amount of passive compliance in its physical structure. The effect of passive
compliance on the system performance will be discussed theoretically in Chapter 5.

Figure 3.29 and Figure 3.30 are plots of force and position responses of the same experiments as above, but with the leg duty factor of 5/6, in which five legs were in the support phase at all times. These figures are for reference purposes, and apply to the situations discussed above.

3.6.3 Comparison of Joint Compliance versus Cartesian Compliance

It is desirable to compare, experimentally, the performance of joint compliant motion with that of Cartesian compliance. Therefore, the servo gains of Cartesian compliant motion, comparable to those of joint compliance, first must be obtained. When the control structure of Cartesian compliance shown in Figure 3.4 is compared with that of joint compliance shown in Figure 3.17, it can be observed that positional loops are exactly the same. The only difference exists in the traveling of the force signal through the Jacobian inverse, in Cartesian compliance, and its passing through the Jacobian transpose, in joint compliance. Thus, the positional gains and feedforward compensation gain in Cartesian compliance are set to those of joint compliance given in (3.57) and (3.59). It will be shown in Chapter 6 that the elevation joint contributes most to vertical compliance. Thus, by comparing Figure 3.4 with Figure 3.17, the force gain of Cartesian compliance is set to the torque gain of joint compliance given in (3.58), times the value of $J_{23}/J_{123}^-$ at the midstance of the leg. Notice that term $J_{23}^T/J_{123}^-$ becomes the square of the upper limb segment at the midstance
Figure 3.29. Foot force tracking using active joint compliance in an underspecified gait. The vehicle was walked forward with a leg duty factor of 5/6, meaning that five legs were in the support phase at all times. The foot force setpoint was generated as the pseudoinverse solution of the force constraint equations.
Figure 3.30. Foot position tracking using active joint compliance in an underspecified gait. The vehicle was walked forward with a leg duty factor of 5/6.
of the leg. The two control structures are then matched closely at the normalized posture of the leg systems (midstance).

Figures 3.31 and 3.32 are the force and position responses, respectively, of the experiment implementing Cartesian compliance. The vehicle was walked forward with a leg duty factor of 2/3; thus the experimental results are comparable with those of Figures 3.27 and 3.28. In tracking both the force and position commands, Cartesian compliance showed generally good performance. Thus, no significant differences can be pinpointed between the two different algorithms of compliant motion. However, a close examination of force responses in Figure 3.31 shows that the actual force of Cartesian compliance resulted in undecayed oscillations with non-negligible amplitude. Similar oscillations are shown in position responses in Figure 3.32, although its amplitude is not significant.

Actually it was observed, during the experiment, that the entire vehicle vibrated or shook. However, as can be seen from Figure 3.32, this kind of vibration did not appear clearly in the response of any individual foot position. The vehicle oscillated through its passive compliance, with a frequency of vibration similar to that of the force response. It might be concluded that the vibration resulted from passive compliance. However, this is not the case, since the vibration was not observed during the experiment of joint compliant motion. The effect of passive compliance on the system performance will be discussed theoretically in Chapter 5.

From the comparison of the experimental results of two different compliance algorithms, it can be concluded that joint compliance showed better performance than the Cartesian compliance in maintaining system
Figure 3.31. Foot force tracking using active Cartesian compliance in an underspecified gait. The vehicle was walked forward with a leg duty factor of 2/3. This figure is comparable to Figure 3.27.
Figure 3.32. Foot position tracking using active Cartesian compliance in an underspecified gait. The vehicle was walked forward with a leg duty factor of 2/3. This figure is comparable to Figure 3.28.
stability. This seems reasonable, since, in resolving Cartesian force errors into joint space, Cartesian compliance uses the Jacobian inverse from geometric consideration, while joint compliance uses the Jacobian transpose from dynamic consideration. It should be noticed that in the joint compliance algorithm, the dynamics of each leg system is properly resolved into joint space.

3.7 Conclusions

The purpose of compliant motion is to accommodate the end effector of a manipulator to external forces in its unknown environment. In this chapter, it has been shown that Cartesian compliance may require a positive feedback in force for some joints, which in turn may cause stability problems in Jacobian control. To avoid the latter, a method of obtaining joint compliant motion was suggested. In this method, every joint is programmed to achieve compliance via the explicit feedback of joint torque into the control of position variables.

In explicit force feedback, in joint space, compliant motion occurs about each joint. It has been shown that the stiffness matrix of the joint compliance is positive definite in the body-fixed rectilinear coordinate system; thus its eigenvalues are positive. Therefore compliance in Cartesian space is always positive.

It has also been shown that the stiffness matrix of joint compliant motion is not diagonal in the body-fixed coordinate system, and may not be so in any Cartesian coordinate system. Thus, the equivalent vertical compliance is non-uniform and is dependent on the postures of the leg system. This is a shortcoming of joint compliance. It does not necessarily have to be a rectilinear compliance, however, as long as it
is performed in a way which accommodates the environment without any
instability problem with force feedback control.

It has been shown experimentally that the algorithm of active joint
compliance is effective. A comparison has been made between the
performance of joint compliance and that of Cartesian compliance, with
the former showing smoother and less oscillatory response than the
latter.

From theoretical analysis and physical experiment, the following
conclusion can be drawn: The legged vehicle will walk better with joint
compliance than with Cartesian compliance, since the former achieves
more natural compliance considering the linkage kinematic structure,
without causing a control instability in force feedback, and also the
former uses the Jacobian transpose which will compensate for error in
the dynamic model of the leg system.

The next chapter will discuss methods of establishing force
setpoints. In order to implement active compliance for the vehicle's
locomotion, it is necessary to define the contact forces of the
foot-tips against their environment so that the vehicle both maintains
its balance on the supporting terrain and achieves the commanded motion.
The force constraints usually result in an underdetermined system of
equations allowing an infinite number of solutions. Several criteria
for optimality will be considered in solving underdetermined force
constraint equations.
4.1 Introduction

The state variables involved in the implementation of actively compliant motion in joint space are joint position, joint rate, and joint torque. These variables are calculated either directly or in the form of errors from the corresponding Cartesian foot-tips terms, using the inverse Jacobian or the Jacobian transpose. Desired position and velocity of the foot-tips are specified by the foot trajectory planning algorithm. Thus, the force setpoint for the foot-tips remains to be specified. During the transfer phase foot-tips do not make contact with the ground. Thus the desired forces for foot-tips in that phase are simply set to zero. Leg control, then, is a pure position control as long as each foot is in the air, since zero force errors are fed back.

In the support phase, the force setpoint specification determines the action and reaction force between the feet and supporting terrain, allowing the vehicle to remain balanced and also achieve the commanded horizontal motion. When the force and moment of the vehicle body are specified, six constraint equations are formed for the required forces at the supporting foot-tips: three scalar equations for force components along three body axes, and three for moment components about
these axes. Usually the system of equations is underspecified, allowing the existence of an infinite number of solutions.

However, not all the solutions of the system of force equations are physically realizable. Solutions are subject to the constraints that the vertical forces should be non-negative and, in order for the supporting foot not to slip on the ground, the ratio of the horizontal force to the corresponding vertical force should be less than the friction coefficient of the supporting ground.

This chapter describes the set-up of constraint equations for only vertical forces, thus statically balancing the vehicle, and solves them by applying a criterion for optimality. The constraint for non-negative vertical forces and the friction constraint will not be considered in this discussion.

4.2 Formulation of Force Constraints

In formulating force constraints, the following two assumptions are made: First, the control system implements actively compliant motion in the direction of gravity, by servoing only the measurement of vertical forces. Second, since the vehicle moves relatively slowly, acceleration forces in all directions are ignored. Therefore the problem of force setpoint specification is reduced to distributing the vehicle's weight among supporting feet so that the ground reaction forces do not result in any moment about the vehicle's longitudinal and lateral axes (x and y axes, respectively) [14]. To allow computation in the body-fixed coordinate system, it is also assumed that the vehicle body is approximately parallel to the level surface of the ground.
In order for the vehicle body to maintain static equilibrium with respect to vertical ground reaction forces, both the resultant force and the resultant couples of all the z-directional forces acting on the vehicle should be zero. Using these two equilibrium conditions, three scalar equations can be obtained: one from the resultant force equation, and the other two from resultant moment equations. If the reference coordinate system is arranged so that the origin is at the center of gravity, these three scalar equations are expressed as follows:

\[ \sum_{i} f_{iz} = W, \quad (4.1) \]
\[ M_y = \sum_{i} x_i f_{iz} = 0, \quad (4.2) \]
\[ M_x = \sum_{i} y_i f_{iz} = 0, \quad (4.3) \]

where \((x_i, y_i)\) are the coordinates of foot \(i\), \(f_i\) is the vertical ground reaction force to foot \(i\), \(W\) is the total weight of the vehicle, and the index \(i\) runs over legs in the support phase. These three equations are combined into a single matrix notation:

\[
\begin{bmatrix}
1 & 1 & \ldots & 1 \\
x_1 & x_2 & \ldots & x_n \\
y_1 & y_2 & \ldots & y_n
\end{bmatrix}
\begin{bmatrix}
f_{1z} \\
f_{2z} \\
f_{3z} \\
\vdots \\
f_{nz}
\end{bmatrix}
= \begin{bmatrix}
W \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}, \quad (4.4)
\]

At any given time the vehicle is supported by three to six legs, with the foot-tips forming a stable support pattern on the ground. Therefore there are three to six unknowns with three equations. When only three feet are on the ground (three unknowns), the above system of
equations is uniquely solvable. However, if more than three legs support the vehicle, the system of force constraints is underspecified. In this case there exist an infinite number of solution sets, thus it is possible to apply an optimal concept in determining solutions of the force constraints.

Several approaches have been used previously in determining force setpoint from underspecified force constraints equations, adapting various criteria for optimization. Orin [20] and Chao [78] used a linear programming technique with appropriate extremal limits. However this approach required long computation times to calculate optimal solutions. Klein and Wahawisan [79] used the Moore-Penrose pseudoinverse to obtain the minimum norm solution. The pseudoinverse solution was calculated numerically, in real time, using Greville's method [80] in a separate processor. Since then, Pugh [14] has derived the analytic solution to the problem in closed form and included it efficiently in the body motion planning block. Klein, Olson, and Pugh [10] have proved that the pseudoinverse solution for commanded force changes linearly with vehicle motion, when a given set of feet are on the ground. Thus if coefficients of the linearity are calculated at the beginning of a stance, solutions can be updated efficiently by performing only superposition.

In general, any solution of an underdetermined linear system can be uniquely decomposed into the pseudoinverse solution (the minimum-norm solution) and a homogeneous solution [80]. The set of homogeneous solutions forms the null space of the system. Adding any combination of the homogeneous solutions to the pseudoinverse solution gives another solution. Thus homogeneous solutions can be used to optimize the
criterion of a secondary goal of the system [9,52]. Maciejewski [53] has used homogeneous solutions for obstacle avoidance in Jacobian control of a kinematically redundant manipulator. Blaho [55] has used homogeneous solutions for optimizing the dexterity measurement of a kinematically redundant manipulator. For the problem of force setpoint, Klein, Olson, and Pugh [10] have suggested using homogeneous solutions to minimize discontinuities in commanded forces when the leg phase is alternating between support and transfer.

While an introduction of components of homogeneous solutions into the pseudoinverse solution increases the norm of the solution, it may decrease the distance between the solution and a specific vector point of the same dimension. In certain applications, minimizing the distance of the solution to a specific point other than the origin (zero vector) is of more interest. For example, to resolve a given end effector trajectory into the joints of a kinematically redundant manipulator, the joint solution nearest to the present state of joints may be chosen instead of the one nearest to the origin. In this case, the current state of the joint is a reference point for the shortest distance criterion, and the joints will be minimally actuated in following the trajectory. The next discussion derives a simple solution method of an underspecified system of equations for the solution which is nearest to a given vector point.

4.3 Solutions of Underdetermined System of Equations

4.3.1 Generalized Inverses and Pseudoinverse

For matrices which are singular, or are not square and thus do not have inverses in ordinary sense, the concept of generalized inverses
can be introduced by the following four matrix equations [48]:

\[
\begin{align*}
AGA &= A \quad (4.5) \\
GAG &= G \quad (4.6) \\
(GA)^* &= GA \quad (4.7) \\
(AG)^* &= AG \quad (4.8)
\end{align*}
\]

where \(A\) is an \(m \times n\) matrix, and the operator \(^*\) denotes the complex conjugate transpose. If the matrix \(G\) satisfies (4.5), it is a generalized inverse of \(A\). If the matrix \(G\) satisfies (4.5) and (4.6), then it is called a reflexive generalized inverse of \(A\). In addition to (4.5) and (4.6), if \(G\) satisfies (4.7), the Hermitian condition of \(GA\), then it is called a left weak generalized inverse of \(A\). Likewise, if \(G\) satisfies (4.5), (4.6), and (4.8), then it is called a right weak generalized inverse of \(A\). If the matrix \(G\) satisfies all the above conditions in (4.5) through (4.8), then it is called the pseudoinverse or the Moore-Penrose inverse. In order to distinguish it from other generalized inverses, the pseudoinverse is denoted by \(A^+\) in this work.

For an example, the pseudoinverse of a real matrix \(A\) of full row-rank can be calculated as

\[
A^+ = A^T(AA^T)^{-1} \quad (4.9)
\]

while the pseudoinverse of a real matrix \(A\) of full column-rank can be calculated as

\[
A^+ = (ATA)^{-1}AT \quad (4.10)
\]

It can be easily shown that (4.9) and (4.10) satisfy the conditions in (4.5) through (4.8).
The concept of generalized inverses can be applied, for an example, in solving an under- or over-determined system of equations. When a system of equations is described by a non-invertible matrix, a generalized inverse of that matrix can be used to find the minimum-norm solution or the solution of least-square error. Properties and applications of generalized inverse can be found in the literature [48,49,50], and various methods for calculating them from given matrices are explained in the literature [51,81,82,83].

4.3.2 Solution Method for the Shortest-Distance Solution from a Reference Point

Consider the following underspecified system of linear equations:

\[ Ax = y \] (4.11)

where \( A \) is an \( m \times n \) matrix of rank \( m \) (\( m < n \)), and \( x \) and \( y \) are \( n \) and \( m \) dimensional vectors, respectively. Since the system is underspecified, there exist an infinite number of solutions, the most desirable being the one which is nearest to a given vector \( p \). The given vector \( p \) is of dimension \( n \), which is compatible to that of solutions of (4.11). A solution formula of this problem can be found in the literature [51]. In this section, a simple derivation of the solution formula will be presented for the convenience of readers.

Since the rows of \( A \) are assumed to be linearly independent, it is possible to find a matrix \( A^\perp \), of dimension \( (n-m) \times n \), such that all of the rows are linearly independent and each row vector is orthogonal to those of \( A \). Note that this matrix is shown only for the purpose of explanation and will never be computed. Rows of \( A^\perp \) are complementary to those of \( A \), thus rows of \( A \) and \( A^\perp \) together span the \( n \)-dimensional row
space. Therefore, for any solution \( x \) of (4.11) and an arbitrary vector \( p \) of dimension \( n \), the \( n \)-dimensional vector \( x - p \) can be uniquely expressed as a linear combination of columns of \( A^T \) and \( (A^\perp)^T \) as follows:

\[
x - p = A^T s + (A^\perp)^T q
\]

where \( s \) and \( q \) are column vectors of dimension \( m \) and \( (m-n) \), respectively. Transferring \( p \) to the right side of (4.12) gives

\[
x = p + A^T s + (A^\perp)^T q.
\]

Rows of \( A^\perp \) were chosen to be orthogonal to those of \( A \) matrix, thus

\[
A(A^\perp)^T = 0.
\]

Therefore \( (A^\perp)^T q \) is a homogeneous solution of (4.11). Substituting (4.13) into (4.11) and applying (4.14) gives

\[
A p + A A^T s = y,
\]

or

\[
A A^T s = y - A p.
\]

Since the columns of \( A^T \) are linearly independent, \( y = A^T u \) vanishes only if \( u \) is a zero vector. Therefore, \( u^T A A^T u = y^T v > 0 \) whenever \( u \neq 0 \).

Thus, \( A A^T \) is positive definite and therefore nonsingular. By premultiplying by \((A A^T)^{-1}\) on both sides of (4.16), the vector \( s \) then can be obtained:

\[
s = (A A^T)^{-1}(y - A p).
\]

Notice that \( s \) is always the same for any solution \( x \) of (4.11), with a given vector \( p \). Thus solutions of (4.11) are only different in their homogeneous terms, and can be expressed as
\[ x = p + A^T(AA^T)^{-1}(y-Ap) + (A^Tz) \]  \hfill (4.18)

where \( z \) is any arbitrary column vector of dimension \((n-m)\).

Now consider the distance of the solution from the given point \( p \).

From (4.18),

\[ x - p = A^T(AA^T)^{-1}(y-Ap) + (A^Tz) \]  \hfill (4.19)

Thus the square of the distance is

\[
||x - p||^2 = ||A^T(AA^T)^{-1}(y-Ap)||^2 + ||(A^Tz)||^2
\]

\[
\geq ||A^T(AA^T)^{-1}(y-Ap)||^2
\]  \hfill (4.20)

where the equality holds only with \( z = 0 \). Therefore the solution of the shortest distance from the given vector \( p \) is expressed uniquely as

\[ x = p + A^T(AA^T)^{-1}(y-Ap) \]  \hfill (4.21)

or, in another form,

\[ x = A^T(AA^T)^{-1}y + [I-A^T(AA^T)^{-1}A]p \]  \hfill (4.22)

Equations (4.21) and (4.22) contain the expression for the pseudoinverse of matrix \( A \), which was given in (4.9). Thus these factors can be replaced with \( A^+ \) in (4.21) and (4.22), resulting in the following expressions, respectively:

\[ x = p + A^+(y-Ap) \]  \hfill (4.23)

or

\[ x = A^+y + (I-A^+A)p \]  \hfill (4.24)

The first term on the right side of (4.24) does not depend on the given vector \( p \), and is actually the shortest-distance solution from the
origin (solution with \( p = 0 \)). This term is common to all solutions, and usually is called the pseudoinverse solution. The second term of (4.24) vanishes when it is pre-multiplied by the system matrix \( A \). Thus it is the homogeneous portion of the solution and is added in order to make the solution nearest to the given vector \( p \).

Figure 4.1 shows the vector diagram which represents the relationship of the pseudoinverse solution and the homogeneous solution in the formula of (4.24). The pseudoinverse solution is the projection of the origin on the solution plane, while the homogeneous solution is the projection of a given vector \( p \) on the plane spanned by rows of \( A^\dagger \) (homogeneous plane). The total solution nearest to a given point \( p \) is the vector sum of the pseudoinverse solution and the homogeneous solution, and is the projection of the point \( p \) on the solution plane.

From Figure 4.1, the orthogonal vector from the point \( p \) to the solution plane can be obtained as

\[
\mathbf{h} = A^+\mathbf{y} + (I - A^+A)p - p
\]

\[
= A^+(\mathbf{y} - Ap) \quad .
\]

(4.25)

Thus the square of the minimum distance from the point \( p \) to the solution plane is expressed as follows:

\[
D^2 = \mathbf{h}^T\mathbf{h}
\]

\[
= (\mathbf{y} - Ap)^T(AA^T)^{-1}(\mathbf{y} - Ap)
\]

\[
= \mathbf{y}^T(AA^T)^{-1}\mathbf{y} - 2\mathbf{y}^T(AA^T)^{-1}Ap + p^TAA^T(AA^T)^{-1}Ap
\]

\[
= \mathbf{y}^T(AA^T)^{-1}\mathbf{y} - 2\mathbf{y}^T(AA^T)^{-1}Ap + p^Tp
\]

\[
- [p^Tp - p^TAA^T(AA^T)^{-1}Ap]
\]

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Figure 4.1. Representation of the solution which is nearest to a given vector $p$ as the sum of the pseudoinverse solution, $A^T(AA^T)^{-1}y$, and the homogeneous solution, $[I-A^T(AA^T)^{-1}]p$. 
\[ \begin{array}{l}
\gamma^T(A^T)^{-1}\gamma - 2\gamma^T(A^T)^{-1}Ap + p^Tp - p^THp \\
= \gamma^T(A^T)^{-1}\gamma - 2\gamma^T(A^T)^{-1}Ap + p^Tp - (Hp)^T(Hp) \\
\end{array} \tag{4.26} \]

### 4.3.3 Projection Matrix and Selection of a Reference Point

The matrix in the second term of (4.24),

\[ P = I - A^TA, \tag{4.27} \]

is the orthogonal projection operator, since it maps any vector \( p \) into the null space of the system matrix \( A \) so the distance from point \( p \) to the null space is the shortest. This matrix is symmetric and, when multiplied by itself, the product is the same matrix.

In conjunction with the projection operator in (4.27), an optimality criterion for solving an underspecified system of equations can be achieved with the proper choice of the reference point \( p \). Liegeois [9] has shown that if one sets

\[ p = -\alpha\nabla H(\theta), \tag{4.28} \]

where \( \alpha \) is a real positive scalar gain and where \( \nabla H \) is the gradient of a smooth function \( H \) of a generalized coordinate vector \( \theta \), then the total solution in (4.24) minimizes the function \( H \). The constant \( \alpha \) is an arbitrary gain affecting the rate that the homogeneous solution optimizes \( H \); when this equation is implemented with a time-sampled controller, then the value will need to be limited to maintain stability. In the Jacobian control of a kinematically redundant manipulator, Liegeois defined the function \( H \) as the deviation of the joint angles relative to the center positions of the joints in order to achieve the maximal joint
availability around their center positions. Maciejewski [53] determined dynamically the reference vector \( p \) in order for the point on a redundant manipulator which is closest to an obstacle to move away from that obstacle without moving the end effector.

When the direct kinematics of the kinematically redundant manipulator are expressed in the form of \( f(\theta) = x \), where \( \dim \theta > \dim x \), the inverse kinematics problem is usually based on the differential form of the direct kinematics or \( \dot{x} = J \dot{\theta} \), with \( J \) being the Jacobian matrix. The solution to this Jacobian relation can be derived by the method revealed in this section: calculating the pseudoinverse of the Jacobian and manipulating homogeneous terms to optimize an object function. A different approach for solving the inverse kinematics problem was proposed by Baillieul [84]. He invented a method of augmenting the direct kinematic equations in order to optimize an objective function \( g(\theta) \). The equations for augmentation are defined as \( \forall g \cdot n_j = 0 \), where \( n_j \) is an independent vector in the null space of the rows of the original Jacobian matrix. With the augmentation of the kinematic equations, the Jacobian matrix is also extended to a square matrix. When the extended Jacobian is not singular, the joint solutions are uniquely determined and thus optimize the objective function \( g(\theta) \).

4.4 Force Setpoint Optimization

By applying (4.24) to the force constraint of (4.4), the following expression for a force setpoint is obtained:

\[
f_{z} = p + A^+(w-Ap)
\]  

(4.29)
where \( f_z = [f_{1z} f_{2z} \ldots f_{nz}]^T; \ w = [W 0 0]^T; \) \( A \) is of 3 x n matrix and defined as shown in (4.4). If a reference point \( p \) is given, then the equation uniquely specifies foot forces nearest to that point. The choice of vector \( p \) is still open. Thus the choice of the reference point \( p \) will be considered next, for optimizing force setpoint.

4.4.1 Minimum-Norm Solution

There may be several possible points which can be used as the reference point \( p \) in (4.29). Previously, the force setpoint was based on the reference point at the origin \( (p = 0) \). In this case, the force setpoint is the pseudoinverse solution, and its expression is simplified, without a homogeneous term:

\[
\mathbf{f}_{zd} = \mathbf{A}^+ \mathbf{w},
\]

where \( \mathbf{f}_{zd} \) is the desired force. The force setpoint is not affected by the current state of forces of the supporting feet. This solution was a generally acceptable choice for the OSU Hexapod Vehicle.

This choice of \( p \) vector gives the solution of minimum norm, and the square of the norm is expressed as

\[
||\mathbf{f}_{zd}||^2 = \mathbf{w}^T (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{w}. \tag{4.30}
\]

The error signal for force feedback becomes

\[
\mathbf{e}_f = \mathbf{f}_{zd} - \mathbf{f}_{za} = \mathbf{A}^+ \mathbf{w} - \mathbf{f}_{za}. \tag{4.32}
\]

Thus the square of the error signal is

\[
\mathbf{e}_f^T \mathbf{e}_f = \mathbf{w}^T (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{w} - 2 \mathbf{w}^T (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A} \mathbf{f}_{za} + \mathbf{f}_{za}^T \mathbf{f}_{za}. \tag{4.33}
\]
4.4.2 Minimum-Deviation Solution

Another good reference vector may be that of equal distribution of vehicle weight to the supporting feet:

\[ p^\text{E}_\text{E} = [W/n \ W/n \ldots \ W/n]^T \]  \hspace{1cm} (4.34)

It is expected that each foot on the ground will actually support some portion of the weight of the vehicle. Solutions based on this reference point will distribute the weight of the vehicle to the supporting feet as equally as possible.

It should be noticed that the pseudoinverse solution and the solution which is nearest to the equal distribution point in (4.34) are identical for the system of equations in (4.4). This can be proved as follows:

(1) Equation (4.1) defines a \((n-1)\)-dimensional hyper-plane in the \(n\)-dimensional space, and a normal vector to this plane as \([1 \ 1 \ldots \ 1]^T\). Thus the line connecting the origin and the equal distribution point \(p^\text{E}_\text{E}\) is perpendicular to the hyper-plane of (4.1).

(2) The solution plane of the system is the intersection of three \((n-1)\)-dimensional hyper-planes of (4.1), (4.2) and (4.3). Thus the solution plane is a sub-plane of the hyper-plane of (4.1).

(3) From conclusions of (1) and (2), it has been confirmed that the line connecting the origin and the point \(p^\text{E}_\text{E}\) is perpendicular to the solution plane of the system of equations in (4.4). Thus the projection of the origin on the solution plane (pseudoinverse solution) coincides with the projection of the point \(p^\text{E}_\text{E}\) on the solution plane.
4.4.3 Minimum-Perturbation Solution

The Hexapod controller was designed from a single leg model, meaning force coupling among legs may cause instability. It has been pointed out that the system, as a whole, began to oscillate with gains normally reasonable for a single leg [10]. Although the cause was not completely uncovered, the introduction of force feedback in experiments caused instability. Therefore if the force setpoint is specified so that the force error is minimal, but still maintains overall force constraints of (4.4), the vehicle system is activated by force feedback minimally. This may reduce possible interactions in force.

From this observation, the vector of current measurement of forces of supporting feet may be used as the reference vector $p$. With this choice of $p$, force errors will be minimal for sums of squares, resulting in minimal from perturbation in control by force feedback. The force setpoint expression will then be

$$ f_{zd} = f_{za} + A^T(w - Af_{za}) \quad \text{(4.35)} $$

or, in a different form,

$$ f_{zd} = A^Tw - (I - A^TA)f_{za} \quad \text{(4.36)} $$

where $f_{za}$ is the actual force. In this case, the force setpoint is affected by the current force states; thus a kind of adaptability results during force control. Notice that the other two methods mentioned previously use constant reference points with a given set of supporting feet during operation of the vehicle.

The signal actually used in implementing force feedback is the force error. The error signal for force feedback is determined as
\[ \text{ef} = \text{fzd} - \text{fza} \]
\[ = A^+(w - Af_{za}) \]  \hspace{1cm} (4.37)

Notice that the signal \( \text{ef} \) can be interpreted as the pseudoinverse solution of the following system of equations:
\[ A\text{ef} = (w - Af_{za}) \]  \hspace{1cm} (4.38)

Applying the expression in (4.26), the square of the error signal is expressed as
\[ \text{ef}^T \text{ef} = w^T(AAT)^{-1}w - 2w^T(AAT)^{-1}Af_{za} + f_{za}^Tf_{za} \]
\[ - (Pf_{za})^T(Pf_{za}) \]  \hspace{1cm} (4.39)

where \( P = [I - A^T(AAT)^{-1}A] \). When (4.39) is compared with (4.33), the force solution derived by projecting the actual force on the solution plane gives a lesser error signal than the pseudoinverse solution by the last term in (4.39).

Another improvement with this method is the smoothing of discontinuity force setpoint during alternations of the leg phase. This results from the subsequent setpoint being the nearest to the current force state close to the present setpoint in stable control. The force solutions obtained by the projection technique from a fixed reference point (the origin or the equal distribution point) show major discontinuities in force setpoints for all supporting legs whenever the leg support set is changed by the phase alternations. These discontinuities cause large control impulses to the system.

4.4.4 Physically Realizable Solution

Mathematically, it can be expected that some solutions may have negative components in certain support patterns. If force setpoints are
derived repeatedly by (4.29) while the vehicle is moving, it might be possible to move the force setpoint gradually away on the solution plane from a point whose components are of all positive, so that some components increase and the others become negative. However, since the support pattern of the vehicle legs are reset at each transition of the foot phase, this situation might not occur in the real operation range. In any case, care should be taken that none of the supporting feet is commanded to achieve a negative vertical force.

In real experiments of the OSU Hexapod, it has been observed that none of the force setpoint methods create negative components in the force solution. This is mainly because the vehicle's center of gravity is almost at its geometrical center. A possible solution to the problem is as follows: If the computation results in negative vertical force components for some feet, the force setpoints for such feet should be set to some positive values (or, zero) to make the feet contact the ground. The solution procedure is then performed again only on those feet whose force are not yet specified. The procedure is repeated until a complete solution is found.

4.5 Force Setpoint Computation

Pseudoinverse solutions for a system of equations can be calculated numerically using Greville's method [48], or a numerical inversion of $A A^T$ may be obtained first, then the solution obtained using equation (4.35). Since the dimension of matrix $(A A^T)$ in (4.35) is $3 \times 3$, an analytic inversion can be obtained and used efficiently in the force setpoint expression of each foot. The matrix to be inverted can be calculated as follows:
\[
\begin{pmatrix}
 n & \sum_i x_i & \sum_i y_i \\
 \sum_i x_i & \sum_i x_i^2 & \sum_i x_i y_i \\
 \sum_i y_i & \sum_i x_i y_i & \sum_i y_i^2
\end{pmatrix}
\]

(4.40)

where the index \(i\) runs over supporting feet.

4.6 Evaluation of Force Setpoint Optimization

Two strategies for force setpoint determination were developed in this chapter: the minimum-norm solution (pseudoinverse solution) and the minimum-perturbation solution (nearest point to current force measurement). In this section, the performance of these two strategies will be compared by the experiments performed on the OSU Hexapod. Since the first algorithm was used in the previous experiments in Chapter 3 (Figure 3.27 and Figure 3.28), an additional experiment for testing the second strategy of force setpoint generation was performed. To match the conditions of the previous experiment, the vehicle was walked forward with a leg duty factor of 2/3. The results of this experiment were plotted in Figure 4.2 and Figure 4.3. Figure 4.2 plots force setpoints and corresponding responses, while Figure 4.3 shows position responses. These figures are comparable to Figures 3.27 and 3.28.

Using the minimum-norm solution method, the curve of the force setpoint is piecewise linear, as proved theoretically by Klein, Olson, and Pugh [10] and shown in Figure 3.27. Notice that the curve shows discontinuities whenever the set of support legs is changed. This is because force setpoint is dependent on the horizontal geometry of the foot positions, and not on the performance of active compliance.
Figure 4.2. Foot force tracking using active joint compliance in an underspecified gait. The force solution of the minimum distance from the current force measurement (minimum-perturbation solution) was used for the foot force setpoint. The vehicle was walked forward with a leg duty factor of 2/3. This figure is comparable to Figure 3.27.
Figure 4.3. Foot position tracking using active joint compliance with a leg duty factor of 2/3. The minimum-perturbation solution was used for the force setpoint. This figure is comparable to Figure 3.28.
When the minimum-perturbation solution method is used, the force setpoint curve no longer shows piecewise linearity. The curve is dependent on the geometry of the supporting legs, but is also affected by force measurement or the control system's response as the vehicle moves. As shown in Figure 4.2, actual force responses follow the desired setpoint more closely and smoothly than those in Figure 3.27. This is the result of adaptability in force setpoint strategy.

Adaptation resulting from the minimum-perturbation solution method can be observed especially at the transitions of leg phases, where large discontinuities in force setpoint were smoothed. Since the legged vehicle cannot avoid repeated force disturbances from leg phase transitions and terrain irregularity, this method of force setpoint will excite the control system minimally in controlling leg force.

The degrees of freedom in force constraint equations increase with the number of legs supporting the vehicle. Thus, more adaptability is expected with the higher leg duty factor. Figure 4.4 shows the force response for the leg duty factor of 5/6, meaning that five legs were in the support phase at all times. A close agreement between the actual force and the force setpoint can be observed in this figure.

4.7 Summary

In implementing active compliance in the locomotion of a legged vehicle, it is necessary to define the contact forces of the foot-tips against their environment. This chapter is concerned with the formulation of equations for vertical force constraints and their solution method.

Force constraint equations are formulated such that the vehicle maintains its balance on the supporting terrain and achieves the
Figure 4.4. Foot force tracking using active joint compliance in an underspecified gait. The force solution of the minimum distance from current force measurement (minimum-perturbation solution) was used for the foot force setpoint. The vehicle was walked forward with a leg duty factor of 5/6.
commanded motion. Usually the solution of the force constraint
equations are not unique, allowing an infinite number of possible
solutions. The minimum-norm solution based on the pseudoinverse
technique has previously been considered the optimal choice [14].

In this dissertation, however, a different criterion was suggested
for optimizing the force setpoint: choosing the force setpoint nearest
on the solution plane to the current force measurement. With this force
setpoint the system is minimally excited by force errors, while force
constraints are still satisfied. It has been experimentally
demonstrated that, using this technique, the discontinuity of force
setpoint during alternations of leg phases is smoothed, and the actual
force measurements respond to the force setpoint quite closely.

The next chapter will discuss primarily the effect of passive
compliance on system stability when encountering discrete-time force
feedback. Usually it is understood that accurate position control
requires a stiff system. However theoretical examination suggests that,
if the control includes discrete-time force feedback, some passive
compliance is necessary for system stability. A theoretical analysis
and numerical simulation, confirming the above conclusion will be
introduced.
Chapter 5
DIGITAL CONTROL AND EFFECT OF PASSIVE COMPLIANCE ON SYSTEM STABILITY

5.1 Introduction

Control of a manipulator implies the use of digital computers, since this allows programmability in manipulator operation. By being programmable, manipulators can perform very flexible and complex tasks. Usually the controller design of a manipulator is based on an analog control model, because such control theory is well developed. The control computer then simulates the analog controller by updating the excitation at a fixed rate, and maintaining it as constant for the duration of the interval between two consecutive control points.

In this chapter, three issues affecting digital control will be studied. The first is the optimum control frequency for accurate digital simulation of an analog control system, the second issue is the excitation, by impulsive digital controls, of the resonance of a physical system, and the third is the effect of passive compliance on system stability while performing force feedback control. The first two issues are related to the control frequency, while the third regards the interaction of force gain and system stiffness in force control.
5.2 Digital Simulation and Control Frequency

In digital simulation of a continuous-control model, control is updated at a fixed rate and maintained as a constant input between consecutive control points. Thus, the performance of digital control will be degraded, compared to that of its analog counterpart. It is expected, however, that digital control simulates its analog counterpart closely when the control frequency is very high. The faster the discrete control is, the more closely it will simulate its analog counterpart. The frequency of digital control, however, cannot be increased over a certain range because of the hardware limitations of control computers and numerical resolution problems encountered with digital computers. The computation speed of a control computer is limited, since the word sizes of the computer and data channels between computer and plant are fixed, and a high control frequency beyond the resolution of the word sizes only increases round-off errors. Therefore a compromise must be made between needing more expensive, faster computers with increased word size, and the acceptable degradation in control performance.

The control frequency of a digital system is dependent upon a designed speed of system response. In order to simulate a continuous control model without significantly degrading system performance, the control interval should be between one sixth and one tenth of the smallest system time constant [85]. This means control is applied six to ten times in the smallest system time constant. Conversely, when the computation speed for required computation of one control is known, the fastest system pole is limited accordingly. This guideline will be used in the controller design for the OSU Hexapod, in Chapter 6.
5.3 Control Impulses and Resonance of Physical Systems

5.3.1 Digital Control Frequency and Resonant Frequency

Vibration of a mechanical system caused by external excitation is called forced vibration. When the excitation is oscillatory, the system is forced to vibrate at the excitation frequency. Therefore, if the frequency of excitation coincides with one of the resonant frequencies of the system, a condition of resonance is encountered, and a dangerously large oscillation may result [86].

When a physical system is under digital control, excitation is maintained as constant by the hold circuitry between two consecutive control points, with changes being made only at control points. This implies that steps are applied to the system periodically at the control points, and these impulses may excite the resonant frequency of the control plant, causing the system to go out of control. If the force measurement is fed back in control, particularly, system resonance is more easily excited because the two are directly coupled, while the deflections of linkages are not reflected in the measurements of joint position and velocity. Therefore care must be taken to prevent impulsive digital control from exciting the resonant frequency.

Since the control impulses are due to discrete control, their magnitude and impact on the system may be reduced by increasing the control frequency. To prevent this, it is recommended that the control frequency be more than ten times the resonant frequency [70]:

\[ f_c > 10 f_r \]  \hspace{1cm} (5.1)

where \( f_c \) is the control frequency and \( f_r \) is the resonant frequency.
This suggests that if the system becomes stiffer, the control should be accomplished with the higher frequency. Thus, the inequality of (5.1) imposes another limit on the frequency of digital control.

The structure of a serial link manipulator is extremely complicated, and varies depending on the manipulator posture. Thus it is difficult to provide a model for analyzing its resonance. Instead a simple model of this complex kinematic structure is provided in the form of a mass-spring-damper system to model the vertical motion. With a periodic forcing function, the system dynamics can be expressed as

\[ M \ddot{x} + B \dot{x} + K_{sp} x = F \cos(\omega t) \]  

(5.2)

where the parameters \( M, B, \) and \( K_{sp} \) represent mass, damping coefficient, and spring constant of the resonance model, respectively, and the right-hand side is the forcing function. The undamped natural frequency of the homogeneous part of this equation is

\[ \omega_n = \sqrt{\frac{K_{sp}}{M}} \]  

(5.3)

or,

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{K_{sp}}{M}} \]  

(5.4)

When the damping coefficient \( B \) is small, the resonant frequency is expressed as [84]

\[ \omega_r = \sqrt{\omega_n^2 - \frac{1}{2} \frac{B}{M}} \]  

(5.5)

or
It can be observed from (5.5) that, the less the damping in a given system, the closer is the resonant frequency to the natural undamped frequency $\omega_n$. Thus, when the damping coefficient is very small, the resonant frequency can be approximated to the undamped natural frequency. In this case, the condition in (5.1) can be rewritten as

$$f_r = \sqrt{f_n^2 - \frac{B^2}{2\pi M}}. \quad (5.6)$$

The inequality in (5.7) provides a lower bound of the control frequency in terms of system mass and spring constant. In order to reduce the danger of system resonance, use of the digital control frequency higher than this bound is recommended. Notice that, in the actual system where there is some amount of damping, the lower bound is lower than that in (5.7).

Solving the inequality in (5.7) for $K_{sp}$ gives

$$f_c > \frac{5}{\pi} \sqrt{\frac{K_{sp}}{M}}. \quad (5.7)$$

Conversely to (5.7), this inequality defines, in terms of control frequency, an upper bound on the structural stiffness of a manipulator system. Once the maximum possible computation frequency for manipulator control is determined, the manipulator should be designed with some capacity for passive compliance, at least satisfying the condition in (5.8). This controls the danger of resonance due to digital control.
impulses. In the actual system, where there is some amount of damping, the upper bound of the spring constant is higher than that in (5.8).

5.3.2 Resonance of the OSU Hexapod

The OSU Hexapod system is flexible due to its multi-legged, linkage design structure. Furthermore, a spring block was deliberately attached on each foot-tip (the purpose of which will be explained later). Therefore it is difficult to determine the vehicle's resonant frequencies.

Based on the fact that, for the purposes of this dissertation, force control is to be performed only in the vertical direction, the Hexapod structure can be modelled as a mass-spring-damper system, as shown in Figure 5.1. This is a passive compliance model, only for approximating the frequency of vibration in the vertical direction of the vehicle. In this model the compliance sources, soft spring blocks being the major ones, are lumped at the foot-tips of legs.

Assuming that the vehicle body does not rotate and only vibrates in the vertical direction, the dynamics of the passive compliance model can be expressed as

\[ M \ddot{z} + nB_p \dot{z} + nK_{sp} (z - z_0) = Mg, \quad (5.9) \]

where

- \( M \) = total mass of the vehicle,
- \( K_{sp} \) = passive spring constant of each leg,
- \( B_p \) = passive damping coefficient of each leg,
- \( z_0 \) = height of the vehicle without any compression or stretch,
Figure 5.1. Passive compliance model for the OSU Hexapod. Compliance parameters are lumped at each foot-tip. The number of supporting legs is between three and six, inclusively.
\[ g = \text{acceleration of gravity, and} \]
\[ n = \text{number of supporting legs}. \]
The homogeneous portion of this expression is identical to that of (5.2) except for the scaling of passive terms. Similar expressions, as inequalities of (5.7) and (5.8), are obtained by assuming that the damping coefficient is small, and thus by replacing \( K_{sp} \) with \( nK_{sp} \) in (5.7) and (5.8):

\[
f_c > \frac{5}{\pi} \sqrt{\frac{nK_{sp}}{M}}, \tag{5.10}
\]
and
\[
K_{sp} < \frac{\pi^2 M f_c^2}{25n}. \tag{5.11}
\]

The inequality in (5.10) defines a lower limit on control frequency in terms of the total vehicle mass, spring constant of each leg, and number of supporting legs. Notice that the limit is related to the square root of the number of supporting legs.

Conversely, (5.11) describes an upper limit on the stiffness of each leg system, based on control frequency and the total vehicle mass. The limit is inversely related to the number of supporting legs.

Substituting the actual values of the vehicle mass,
\[
M = 310 \text{ [lbm]} = 0.803 \text{ [lbf sec}^2/\text{in}] , \tag{5.12}
\]
and a possible control frequency,
\[
f_c = 60 \text{ [Hz]} , \tag{5.13}
\]
in the right side of (5.11) gives

\[ K_{sp} < 380.4 \text{ [lbf/in]} \quad (5.14) \]

for the case of tripod gait \((n = 3)\). If all the six legs are on the ground \((n = 6)\), supporting the vehicle, the limit is reduced to half of (5.14), or

\[ K_{sp} < 190.2 \text{ [lbf/in]} \quad (5.15) \]

Notice that if the control frequency is dropped to around 50 Hz, the limit falls to around 132.1 [lbf/in], requiring a very flexible system. The stiffness limit in (5.15) is quite low. The linkage structure of the vehicle legs cannot be made that flexible, since the effect of joint deflections on the foot-tip position varies accordingly to the posture of the leg system, thus the position control is quite inaccurate.

The spring constant of the aforementioned spring blocks is approximately 240 [lbf/in]. This is a major source of the vertical compliance, although the leg system itself is somewhat flexible. When the flexibility of the leg system itself and the damping coefficient of both the leg system and the spring block are all considered, the spring block may either approximately satisfy the condition set in (5.15), or be around the borderline. However, if the control frequency drops too much or if the spring block is removed, then the condition is not satisfied.

In this section an upper limit on the Hexapod's structural stiffness was expressed in terms of control frequency. This suggests that some amount of passive compliance is necessary in digital control of velocity servos, in order to prevent control-induced impulses from exciting the system resonant frequency.
5.4 Linear Force Feedback and Effect of Passive Compliance on System Stability

5.4.1 Passive Compliance Model

When the end effector of a manipulator comes into contact with its working environment, compliant motion is required to control the end effector position. This is necessary because the manipulator, itself, is not a perfect positioning device and because the working environment is not completely known. The interactive force between the end effector and the environment is measured and used in modifying positional command in Cartesian space or joint space, usually by a linear gain. This is an actively compliant motion allowing the manipulator to adapt to its working environment.

Since the action and reaction force results from interaction between two contacting objects, the force feedback gain obviously depends on the characteristics of the two objects, in this case their stiffness. This section will consider the effects of the manipulator's passive compliance and the environment on system stability, in conjunction with digital control. In regard to digital control, Whitney [64,87] pointed out that higher force feedback gains can be used if the environment is more compliant, since less force would build up over a fixed control time period.

In order to study the effects of passive compliance, a one-dimensional suspension model will be proposed in this chapter. Based on it, the effects of continuous control and digital control will be compared.

Figure 5.2 is a model of the suspension system, with two legs supporting the mass block of 2M. This model is very similar to a
Figure 5.2. A model of the vehicle suspension system. Passive compliance parameters of both the suspension system and the supporting ground are lumped at the contact points.
reduced model of the OSU Hexapod. Each leg consists of two links with an actuator attached at their joint. Passive compliance sources of the suspension system and the ground are lumped at the contact points, as spring constant $K_{sp}$ and damping coefficient $B_{p}$. In this model the velocity and position of the mass block are controlled according to the input command and the force measured at the foot-tip.

It is assumed that both legs of the model in Figure 5.2 are controlled identically and thus the mass block moves only vertically, without interactions between legs. Therefore the passive compliance model can be simplified to a single leg model with mass of $M$ as shown in Figure 5.3. This simplification shows only one leg, which is all that is necessary for the purpose of stability analysis.

5.4.2 Passive Compliance and Stability with Continuous Control

Two possible ways exist to analyze the stability of a control system: one is a numerical simulation using appropriate input from the state equations of the control system, and the other is by observation of the eigenvalues of the system state equation matrix. State equations are essential in either case, and thus will be defined first.

In the system model in Figure 5.3, the force measured at the foot-tip is gravity and acceleration force of the mass block. Thus it can be expressed as

$$f = M(g + \ddot{x}_3) \quad (5.16)$$

This force is exerted on the spring-damper block, with an equal amount of force being reacted upon by the block. Thus the force $f$ can also be expressed as
Figure 5.3. One-leg model of the vehicle suspension system. Passive compliance parameters of both the suspension system and the supporting ground are lumped at the contact point. The motor block controls the position $x_1$. 
\[ f = K_{sp}[(t_2 + x_1) - x_3] + B_p(x_1 - x_3). \quad (5.17) \]

Equating right sides of both (5.16) and (5.17) then gives an expression for \( \ddot{x}_3 \):

\[ \ddot{x}_3 = \frac{1}{M} \left( K_{sp}[(t_2 + x_1) - x_3] + B_p(x_1 - x_3) \right) - g. \quad (5.18) \]

For completion of the state equations, the actuator model from the OSU Hexapod is included. Its transfer function from input voltage to the motor angular position was identified [36] as

\[ \frac{\Theta(s)}{V(s)} = \frac{.14}{s(s+3)}. \quad (5.19) \]

a formula which has been found to give accurate predictions of actuator performance. In order to combine the actuator model into (5.18), the term \( x_1 \) can be approximated as

\[ x_1 = \ell_1 \theta, \quad (5.20) \]

by linearizing for \( \theta \) near zero.

By combining (5.18) and (5.20) on the actuator model in (5.19), a signal flow graph is formed and is drawn as Figure 5.4. This figure represents a control structure with linear force feedback and it is assumed that the feedback gains and the system forward gain are all positive. Scaling by a factor of \( \ell_1 \), Figure 5.4 is redrawn as in Figure 5.5. Based on this signal flow graph, state equations can then be obtained as follows:

\[ \dot{x}_1 = x_2, \quad (5.21) \]
Figure 5.4. A signal flow graph of the control of the suspension system. Linearization for \( x_1 \) was applied for \( \theta_1 \) near zero.
Figure 5.5. A modified signal flow graph of the control of the suspension system. Scale factor $\lambda_1$ was eliminated.
\begin{align*}
x_2 &= -G(k_p + k_f k_{sp})x_1 - [G(k_v + k_f b_p) + 3]x_2 \\
&
+ k_f k_{sp} x_3 + k_f b_p x_4 \\
&+ G[k_f f d + k_v x_2 d + k_p x_2 d] + G k_f k_{sp} x_2, \quad (5.22) \\
x_3 &= x_4, \quad (5.23) \\
x_4 &= \frac{k_{sp}}{M} x_1 + \frac{b_p}{M} x_2 - \frac{k_{sp}}{M} x_3 - \frac{b_p}{M} x_4 \\
&+ \frac{k_{sp}}{M} x_2 - g. \quad (5.24)
\end{align*}

The 4 x 4 system matrix is formulated by the coefficients of state variables in the right sides of (5.21) through (5.24). The eigenvalues of this matrix can then be obtained numerically, with various conditions of passive compliance, in order to observe the effect of passive compliance on system stability. A Fortran program was written for two purposes on a PDP-11/70 computer. The loci of eigenvalues of the system matrix are drawn in Figure 5.6, varying the passive spring constant parameter and setting the damping coefficient $b_p$ at a value greater than $1/k_f$ (damping coefficient of active compliance). The loci in Figure 5.7 represent a case where the damping coefficient $b_p$ is less than $1/k_f$. If the passive spring constant is increased, two loci branch away in both directions parallel to the imaginary axis, meaning that the system responses have high frequency oscillations. In all cases, however, the loci of eigenvalues are in the left half of the s-plane. Since the real parts of eigenvalues are negative, the system will be stable for all values of spring constant $k_f$, if analog control is performed.

Since the system matrix is of order four, it is difficult to compare and analyze, simply, the effects of passive compliance on system
Figure 5.6. Loci of eigenvalues of the system matrix for the case of $B_p > 1/k_f$. The loci were drawn by varying the passive spring constant $K_{sp}$. $B_p$ is the passive damping coefficient, while $1/k_p$ is the active damping coefficient.
Figure 5.7. Loci of eigenvalues of the system matrix for the case of $B_p < 1/k_f$. The loci were drawn by varying the passive spring constant $K_sp$. $B_p$ is the passive damping coefficient, while $1/k_p$ is the active damping coefficient.
stability in cases of continuous control and digital control. Therefore a gross simplification is made, reducing the system order. Assumptions for this are as follows:

(a) \( x_3 = \text{constant} \)
(b) \( B_p = 0 \)
(c) \( k_v = 0 \).

Condition (a) implies that the mass block is fixed relative to the supporting ground. Assumptions (b) and (c) are only for simplicity. The following case would also be valid without these assumptions; the only difference being that the related equations would be much more complex.

Figure 5.8 is a signal flow graph for the reduced model. The reduced-order state equations are

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-G(k_p + k_f k_{sp}) & -3 \\
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
\end{bmatrix} + \begin{bmatrix}
0 \\
G(u + k_f k_{sp} x_e) \\
\end{bmatrix}.
\]

(5.25)

The system matrix \( A \) is obtained as

\[
A = \begin{bmatrix}
0 & 1 \\
-G(k_p + k_f k_{sp}) & -3 \\
\end{bmatrix}.
\]

(5.26)

Thus the characteristic equation is defined from this matrix as

\[
\det(sI - A) = s^2 + 3s + G(k_p + k_f k_{sp}) = 0.
\]

(5.27)

Eigenvalues are the roots of (5.27), and they are expressed as

\[
s = \frac{-3 \pm \sqrt{9 - 4G(k_p + k_f k_{sp})}}{2}.
\]

(5.28)
Figure 5.8. A signal flow graph of a reduced model of the suspension system.
If the sign of the discriminant or the term inside the square root in (5.28) is positive, i.e.

$$9 > 9 - 4G(k_p + k_fK_{sp}) > 0$$

(5.29)

then the system has two real negative poles. In this case, the system is stable without any oscillation. If the discriminant becomes negative with a change in any one of $G$, $k_p$, $k_f$, and $K_{sp}$, then the system has poles forming a complex conjugate pair. In this case the response is a damped oscillation, but the system is still stable. The radian frequency of the oscillation is

$$\omega = \sqrt{4G(k_p + k_fK_{sp}) - 9}$$

(5.30)

and the damping envelope is $e^{-3t/2}$. If the spring constant $K_{sp}$ is increased or the spring is made stiffer with all the other parameters fixed, the frequency of oscillation is increased, accordingly, by (5.30). However, the damping term is not changed; thus the system is still stable. The reduced model coincides with its original in terms of its stability in passive compliances.

5.4.3 Passive Compliance and Stability with Digital Control

5.4.3.1 Sampled-Data Control System and Discretization

This dissertation originally considered the stability analysis of a one-legged suspension system model whose state equations are represented by (5.21) through (5.24). The goal was to investigate the effect of passive compliance on system stability in actively compliant motion by determining whether the flexibility of the system has a beneficial or detrimental effect on the performance of force feedback control. In order to analyze this problem, the state equations first
need to be modified for digital control. In digital control the state variables fed back through the digital computation loop are regarded as constant input for a whole interval between two control points. Thus the continuous state equations are modified into hybrid state equations for digital control:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t), \\
\dot{x}_2(t) &= -3x_2(t) - G(k_p + k_fK_s)x_1(kT) - G(k_v + k_fB_p)x_2(kT) \\
&+ k_fK_s x_3(kT) + k_fB_p x_4(kT) \\
&+ G(k_f d + k_v x_2 d + k_p x_2 d) + G k_f K_s x_2, \\
\dot{x}_3(t) &= x_4(t), \\
\dot{x}_4(t) &= \frac{K_s}{M} x_1(t) + \frac{B_p}{M} x_2(t) - \frac{K_s}{M} x_3(t) - \frac{B_p}{M} x_4(t) \\
&+ \frac{K_s}{M} d_2 - g,
\end{align*}
\]

where \( kT \leq t < (k+1)T \), and \( d, x_2 d, \) and \( x_2 d \) are for \( t = kT \).

Pure discrete state equations are obtained through discretization of the above hybrid state equations. If the above hybrid state equations are in the form

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t)
\end{align*}
\]

with

\[
\begin{align*}
u(t) &= u(kT), \quad \text{for } (k+1)T > t > kT,
\end{align*}
\]

then their solution is expressed \[89\] as

\[
\begin{align*}
x[(k+1)T] &= e^{AT}x(kT) + \int_0^T e^{As}Bds \ u(kT), \\
or \\
\begin{align*}
\dot{x}[(k+1)T] &= Gx(kT) + Hu(kT).
\end{align*}
\]

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with
\[ G = e^{AT} \]  \hspace{1cm} (5.39)
and
\[ H = \int_0^T e^{AS}Bds \]  \hspace{1cm} (5.40)

In this case, the control \( u(kT) \) can be computed as
\[ u(kT) = Cx(kT) + Dv(kT) \]  \hspace{1cm} (5.41)

with the matrices \( C \) and \( D \) for the control law. Thus the closed loop discrete state equation can be represented as
\[ x[(k+1)T] = [G + HC]x(kT) + HDv(kT) \]  \hspace{1cm} (5.42)

If there is a one-step computation delay in the feedback loop, the control \( u(kT) \) is computed based on the system input \( v[(k-1)T] \) and the state measurement \( x[(k-1)T] \):
\[ u(kT) = Cx[(k-1)T] + Dv[(k-1)T] \]  \hspace{1cm} (5.43)

Substituting (5.43) into (5.38) gives
\[ x[(k+1)T] = Gx(kT) + HCx[(k-1)T] + HDv[(k-1)T] ; \]  \hspace{1cm} (5.44)

thus the state equation can be augmented as
\[ x[(k+1)T] = \begin{bmatrix} G & H \end{bmatrix} \begin{bmatrix} x(kT) \\ z(kT) \end{bmatrix} + \begin{bmatrix} HD \\ 0 \end{bmatrix} z(kT) \]  \hspace{1cm} (5.45)

with substitutions of
\[ y_i(kT) = x_i(kT) \]  \hspace{1cm} (5.46)
\[ y_{i+n}(kT) = x_i[(k-1)T] \]  \hspace{1cm} (5.47)

and
\[ z(kT) = v[(k-1)T] , \]  \hspace{1cm} for \( i = 1 .. n \).  \hspace{1cm} (5.48)
The stability of the discrete system can be tested by determining whether the eigenvalues of the system matrix are within a unit circle on the z-plane. As shown in (5.45), the order of the system was doubled with a one-step computation delay in the feedback loop. This makes it a little hard to test the system stability analytically. Therefore, for simplicity of the analysis, it is assumed here that there is no computation delay, and the system matrix in (5.42), \([G + HC]\), is used in the following analysis. However, the effect of computation delay on the system stability will be included in the simulation to follow.

The loci of eigenvalues of the discrete system matrix in (5.42) are drawn in Figure 5.9, varying the sampling time \(T\). The system parameters were tabulated in Table 5.1. Keeping \(T\) very small, which almost amounts to continuous control, all the eigenvalues are within the unit circle. As \(T\) is increased from zero, some loci move toward the unit circle boundary. When the sampling time \(T\) is 0.1 [sec], an eigenvalue reaches the unit circle boundary. A further increase in \(T\) causes the eigenvalue to cross over the unit circle as expected, resulting in the system instability.

Figure 5.10 is the loci of eigenvalues with spring constant \(K_{sp}\), only, varying. As \(K_{sp}\) is increased, a pair of loci move away from the real axis. This implies that the frequency of damped oscillation is accordingly increasing as is the case with continuous control. A further increase in \(K_{sp}\) makes some loci cross over the unit circle, making the system unstable, which is not the case with continuous control. The critical passive spring constant was \(K_{sp} = 390 \text{ [lbf/in]}\).
Table 5.1
Parameter values for computer simulation of a one-leg suspension system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>body mass</td>
<td>$M$</td>
<td>3.680 lbf·sec$^2$/inch</td>
</tr>
<tr>
<td>gravitational acceleration</td>
<td>$g$</td>
<td>385.5 inch/sec$^2$</td>
</tr>
<tr>
<td>position error gain</td>
<td>$k_p$</td>
<td>1.17 sec$^{-1}$</td>
</tr>
<tr>
<td>velocity error gain</td>
<td>$k_v$</td>
<td>1.0</td>
</tr>
<tr>
<td>force error gain</td>
<td>$k_f$</td>
<td>0.146 inch/(sec·lbf)</td>
</tr>
<tr>
<td>compensator gain</td>
<td>$G$</td>
<td>56.6 volts/(rad/sec)</td>
</tr>
<tr>
<td>passive spring constant</td>
<td>$K_{sp}$</td>
<td>varying, lbf/inch</td>
</tr>
<tr>
<td>passive damping coefficient</td>
<td>$B_p$</td>
<td>5.0 lbf·sec/inch</td>
</tr>
<tr>
<td>sampling time</td>
<td>$T$</td>
<td>varying, sec</td>
</tr>
</tbody>
</table>
Loci of eigenvalues of the system matrix for the discrete control system. The loci were drawn by varying the sampling time $T$. As $T$ increases, some eigenvalues cross over the unit circle boundary, resulting in an unstable system.
Figure 5.10. Loci of eigenvalues of the system matrix for the discrete control system. The loci were drawn by varying the passive spring constant $K_{SP}$. As $K_{SP}$ increases, some eigenvalues cross over the unit circle boundary, resulting in an unstable system.
A similar stability test can be accomplished analytically on a reduced model of discrete control, as was done in the case of the reduced model implementing continuous control. Based on the simplified model in Figure 5.8, a discrete control model with control period \( T \) can be drawn as in Figure 5.11. When the control \( u(t) \) is constant between 0 and \( T \), \( x_2(t) \) is given by

\[
x_2(t) = u(0) \frac{1 - e^{-3t}}{3} + x_2(0)e^{-3t},
\]

and then \( x_1(t) \) is computed as

\[
x_1(t) = \int_0^t x_2(t) dt + x_1(0)
\]

\[
= u(0) \frac{3t + e^{-3t} - 1}{9} + x_2(0) \frac{1 - e^{-3t}}{3} + x_1(0).
\]

Now substituting \( t = T \) in (5.49) and (5.50) gives

\[
x_1(T) = u(0) \frac{3T - 1 + e^{-3T}}{9} + x_2(0) \frac{1 - e^{-3T}}{3} + x_1(0),
\]

\[
x_2(T) = u(0) \frac{1 - e^{-3T}}{3} + x_2(0)e^{-3T}.
\]

When the sampling time \( T \) is very small, an approximation of

\[
e^{-3T} = 1 - 3T
\]

can be valid. Applying this approximation into (5.51) and (5.52) gives

\[
x_1(T) = x_1(0) + Tx_2(0),
\]

\[
x_2(T) = (1 - 3T)x_2(0) + Tu(0).
\]
Figure 5.11. A signal flow graph for discrete control of the reduced model of the suspension system.
These two equations are generalized without any difficulty, with time index \( k \), as follows:

\[
\begin{align*}
  x_1[(k+1)T] &= x_1(kT) + T x_2(kT), \quad (5.56) \\
  x_2[(k+1)T] &= (1 - 3T)x_2(kT) + T u(kT). \quad (5.57)
\end{align*}
\]

The control \( u(kT) \) is computed from system input and state feedback, by the designed control law. Thus state feedback may be delayed, by computation, in real operation. However, if computation delay is included, it doubles the system order, making it too hard to handle analytically. Therefore, for simplicity of analysis, it is assumed here that there is no computation delay. The control \( u(kT) \) then is expressed as

\[
u(kT) = G[v(kT) - (k_p + k_f K_{sp})x_1(kT) + k_f K_{sp} x_e]. \quad (5.58)
\]

By substituting (5.58) into (5.57), final discrete state equations are obtained:

\[
\begin{align*}
  x_1[(k+1)T] &= x_1(kT) + T x_2(kT), \quad (5.59) \\
  x_2[(k+1)T] &= - GT(k_p + k_f K_{sp})x_1(kT) + (1 - 3T)x_2(kT) \\
  &\quad + GT[v(kT) + k_f K_{sp} x_e]. \quad (5.60)
\end{align*}
\]

The stability of discrete state equations can be tested by checking whether the eigenvalues of the system matrix are within a unit circle. The system matrix for state equations of (5.59) and (5.60) is defined as

\[
A = \begin{bmatrix}
  -1 & T \\
  - GT(k_p + k_f K_{sp}) & 1 - 3T
\end{bmatrix}. \quad (5.61)
\]
The characteristic equation is then expressed as

\[ \text{det}(zI - A) = z^2 - (2 - 3T)z + \left(1 - 3T\right) + T^2G(k_p + k_fK_{sp}) = 0. \] (5.62)

The eigenvalues are the roots of (5.62), and are expressed as

\[ z = \frac{2 - 3T \pm \sqrt{(2 - 3T)^2 - 4\left(\left(1 - 3T\right) + T^2G(k_p + k_fK_{sp})\right)}}{2}. \] (5.63)

In order for the system response to be stable and non-oscillatory, the discriminant (the inside of the square root in (5.63)) should be positive and the eigenvalues in (5.63) should be in \(0 < z < 1\). These conditions are expressed as follows:

\[ (2 - 3T)^2 - 4\left(\left(1 - 3T\right) + T^2G(k_p + k_fK_{sp})\right) > 0, \] (5.64)

and

\[ 0 < z < 1. \] (5.65)

If any one of \(T, G, k_f, k_p, \) and \(K_{sp}\) is increased from a certain value satisfying the condition expressed in both (5.64) and (5.65) with remaining parameters, the conditions of (5.64) and (5.65) are violated. However the system is still stable (with oscillation) as long as the eigenvalues are within the unit circle in z-plane. If the parameters listed are increased further over a certain range, the eigenvalues are driven out of the unit circle, resulting in an unstable system. This was not true with the continuous control case. Notice that in any case decreasing the control period \(T\) makes the system stable, as in the case of the continuous control system.

It is worthwhile to notice that the passive spring constant \(K_{sp}\) affects the system stability. Actually, the product of the force gain
kf and the spring constant Ksp acts as a single parameter. This implies that higher force feedback gain can be used if the system itself, or the environment, is more compliant; or if Ksp becomes smaller. If the system is very stiff (Ksp is very large), then the force feedback gain should be decreased so as to stabilize the system. However this is not compatible with actively compliant motion. Although the discussion here does not give any quantitative suggestions, it clearly explains why some passive compliance is necessary for stability in controlling contact force by digital control. Since a manipulator usually is highly non-linear and its order is high, it may be reasonable to adjust its passive compliance by empirical means.

5.4.3.2 Numerical Simulation of the Sampled-Data System

Based on the hybrid state equations (5.31) through (5.34), a numerical simulation was performed with a step input. Although the vehicle will not experience a step input in normal walking, its responses to it will still reveal how system stability is affected by passive compliance. The differential equations were numerically integrated by the Runge-Kutta fourth order technique [90]. The frequency of numerical integration (reciprocal of integration interval) was chosen to be an integer multiple of the control frequency so that no integration interval crossed a control point. Parameters tested with each simulation were sampling time T and passive spring constant Ksp.

Figure 5.12 shows simulation results for various sampling times, with other parameters fixed. It clearly shows that, with the sampling time decreased, oscillation of the response is reduced as expected. The
Figure 5.12. Simulation results for the sampled-data control system, with each graph for a different sampling time. The passive spring constant is fixed at 400 [lbf/inch]. As the sampling time increases, the response begins oscillation. The unstable oscillation begins with a control frequency of approximately 14 Hz.
step response becomes unstable with the control frequency less that 15 [Hz]. However, the analytic test showed that the minimum control frequency is 10 [Hz] when no computation delay is assumed. Thus, it can be concluded that the computation delay causes a detrimental effect on the stability.

Figure 5.13 is another simulation result, varying spring constant $K_{sp}$ only. The control frequency is fixed around 50 Hz. As the spring constant $K_{sp}$ is increased, the response enters into oscillation. This result corresponds to the stability analysis given in this section. The simulation shows that the maximum passive stiffness is around 400 [lbf/inch] for the system stability.

5.5 Experimental Results

It was determined theoretically, in this chapter, that some passive compliance is necessary for system stability in implementing force control by digital computer. All the experiments discussed in the previous two chapters were performed with the spring blocks attached at the foot-tip of each leg. These blocks are quite soft and thus are major sources of vertical compliance of the vehicle. The spring constant of these blocks is approximately 240 [lbf/in], allowing about 0.65 inch compression when the half of the vehicle mass is loaded on one leg. These spring blocks were deliberately attached to eliminate a undesirable vibration experienced with force feedback control.

It had been believed that the Hexapod's oscillation, resulting from force gains reasonable for a single leg, was due to internal passive compliance in the vehicle, and interaction in force between legs [10]. Thus a digital low-pass filter on the force error had been used to damp
Figure 5.13. Simulation results for the sampled-data control system, with each graph for a different passive spring constant. The control frequency is fixed at 50 Hz. As the spring constant increases, the response begins oscillation. The unstable oscillation begins with the spring constant of approximately 400 [lbf/inch].
out leg interactions in force. A pole of 2.0 [rad/sec] was chosen empirically for the low-pass filter. However, it had not still been understood whether passive compliance might be beneficial or detrimental to the stability of force control for actively compliant motion.

To clarify, experimentally, the confusion about the effect of passive compliance on system stability, the Hexapod body first was stiffened with the attachment of aluminum plates to its frame. However, the effect turned out to be worse, with the system showing more oscillation than before. A spring block was attached at the foot-tip of each leg to decrease vertical stiffness of the Hexapod. Experiments with these spring blocks at the foot-tips showed increased system stability.

In this section, the result of the experiment performed with the spring block detached from the vehicle legs, will be presented as confirmation of the validity of the stability analysis based on the passive spring constant and force feedback gain.

Figure 5.14 shows the force responses of the experiment without the spring blocks at the leg system. The vehicle was walked forward with a leg duty factor of 2/3. Thus, the result in Figure 5.14 can be directly compared with that in Figure 3.27. As plotted in Figure 5.14, the force response with the stiffer system shows undamped large-amplitude oscillations. This is quite different from the force response of the more compliant system shown in Figure 3.27. The oscillations were so strong that considerable impact was applied to the system, causing the vehicle to vibrate as a whole.

The comparison of two experiments with different passive compliances suggests that some passive compliance is necessary to make
Figure 5.14. Foot force tracking using active joint compliance with a leg duty factor of 2/3. The vehicle was walked without spring blocks on each leg. The control frequency was 48 Hz. This figure is comparable to Figure 3.27. The force response shows undamped large-amplitude oscillation.
the system stable when force feedback control is implemented by a
digital computer. It should be noted, however, that position control is
less accurate with a more compliant system. The system stability can
still be maintained by reducing force feedback gain instead of making
the system more compliant. However this makes the system actively
stiffer. Thus a criterion must be determined for optimal setting of the
passive compliance for trading between position accuracy and system
stability with force control.

5.6 Conclusion

The purpose of this chapter was primarily to investigate the effect
of passive compliance on system stability with regard to force feedback
implementation of actively compliant motion. Emphasis was placed on the
problem of stability with discrete control. In order to illustrate the
problem a model of an active suspension system and a continuous
controller for this model were represented. Based on this model of a
continuous control system, the effect of passive compliance was
investigated by deriving eigenvalues of the system matrix. It was found
that increasing the spring constant of passive compliance or the force
feedback gain also increases the frequency of oscillation. However the
damping envelope is not affected. Thus system stability is maintained
with a change of passive compliance or force feedback gain.

In order to determine whether the above was true with digital
control, a time-sampled model of a control system, based on the model of
a continuous control system, was derived. Both analysis and simulation
were performed on this model in order to investigate system stability.
In contrast to the case of continuous control, it was found that the
system stability is closely related to the product of force feedback
gain, the passive spring constant, and the square of the sampling time. When this product term increases, the system becomes unstable. In any case, the system can be made stable by decreasing the sampling time, which makes it act more nearly as a continuous system. This result also has been shown by experiments on the OSU Hexapod. Another noticeable point is that the computational delay causes a detrimental effect on the system stability. When there is a computational delay, the system must be controlled with a somewhat higher frequency than for the case of no computation delay.

In summary, it can be concluded that in implementing actively compliant motion by force feedback with discrete control, some passive compliance is necessary for stabilization of the system. Quantitative guidelines are not yet well established, but should be dependent on the force feedback gain and the desired accuracy of position control. The introduction of passive compliance also reduces the required control frequency for avoiding system resonance due to discrete-control-induced impulses.

In the next chapter an experimental controller for the OSU Hexapod, based on the algorithm developed in Chapter 3 through Chapter 5 for implementing joint compliant motion, will be designed. A compliance model in the form of a spring-damper system is examined, and from this model two parameters for compliant response are defined: the active spring constant and the damping factor. Servo gains are then expressed in terms of these two parameters. System resonance and control frequency are both considered in imposing constraints on the two independent parameters. Experimental values will be assigned to these parameters which satisfy constraints, thus making it possible to obtain a set of servo gains.
Chapter 6

CONTROLLER DESIGN FOR JOINT COMPLIANCE

6.1 Introduction

In this chapter, an experimental controller will be designed for the OSU Hexapod to implement actively compliant motion in joint space based on the discussions in Chapter 3 through Chapter 5. A controller for implementing Cartesian compliant motion was designed by Pugh [14]. Experiments for evaluating force control algorithms will be performed with servo gains designed in this chapter.

It is difficult or almost impossible to fully analyze the dynamics of the Hexapod with its multi-legged kinematic structure. Therefore several approximations will be adopted in order to simplify the problem. The method described here does not employ any formal optimality criteria, and the major concern is to provide stability and accurate behavior.

A control law for force accommodation has been discussed in Chapter 3. The joint torque error is combined linearly with errors of position and velocity to form a joint input signal. Assuming that a linear combination of errors is controlled to arrive at zero quickly, a compliance model will be expressed in terms of joint torque error, velocity error, and position error. The mass of the vehicle will first be related to torque error, then to the compliance model of a
joint which is most effective in reducing the vertical force error at the foot-tip.

From the compliance model, two parameters, which characterize the compliant response, will be defined: one is the active spring constant; the other the damping factor. These will be regarded as free variables, and the servo gains will be expressed in terms of them. Accounting for the vehicle's mechanical resonant frequency, a constraint will be imposed on the active spring constant in terms of the vehicle's passive compliance. Several other constraints will also be imposed on controller gains to avoid invalidating the compliance model as the digital computer implements control laws. The controller design will be completed by assigning experimental values to the two compliance parameters which will achieve desirable compliant motion and satisfy constraints on feedback gains.

6.2 Compliance Model

6.2.1 Control Law for Joint Compliance

To develop a model for compliant motion, the concept of a hybrid state vector is introduced for each joint. The hybrid state of each joint can be defined by a three dimensional vector consisting of joint position, velocity, and applied torque. The term "hybrid state" is justified by noting that the applied torque is specified by the output of a digital to analog converter and is therefore a discrete state variable while joint position and velocity are continuous time state variables. With subscript notation, desired and actual states of the joint hybrid state variables can be represented as follows:
The basic control law insures that the desired velocity of each joint is modified by position and torque errors of the joint in the following linear fashion:

\[
S_d = [\dot{\theta}_d \ \ddot{\theta}_d \ \tau_d]^T, \quad (6.1)
\]

\[
S_a = [\dot{\theta}_a \ \ddot{\theta}_a \ \tau_a]^T. \quad (6.2)
\]

where \( k_p \) and \( k_T \) are scalar constants. This linear relation is the source of actively compliant motion of each joint. Each joint is to be controlled to follow this modified velocity command.

In order to activate each joint motor so the actual joint velocity follows the modified velocity command, a feedback loop is closed around the commanded velocity and the actual one. The error signal is then expressed as

\[
e_{vi} = (\dot{\theta}_d - \dot{\theta}_a) + k_p(\theta_d - \theta_a) + k_T(\tau_d - \tau_a). \quad (6.4)
\]

Figure 6.1 is the block diagram of a joint controller implementing the feedback control law of (6.4), where \( G \) represents a compensator block. The task of controller design is to specify two scalar gains, \( k_p \) and \( k_T \), and the compensator \( G \), so that the error signal \( e_{vi} \) in (6.4) asymptotically converges to zero.

If the error signal actually converges to zero with a given set of gains, (6.4) is approximated as

\[
k_T(\tau_d - \tau_a) + (\dot{\theta}_d - \dot{\theta}_a) + k_p(\theta_d - \theta_a) = 0. \quad (6.5)
\]
Figure 6.1. Block diagram of an independent joint controller for constant joint compliance. Leg coupling effects are ignored in this diagram. The state variables of joint $i$ are made available to only joint $i$. The torque gain $k_T$ is a constant.
When a desired state of a joint is specified, (6.5) constrains the range of the actual state of the joint. Since the relation is linear, (6.5) describes a plane containing the desired setpoint in three-dimensional space. The range of the actual state is near the desired setpoint on this plane. This is illustrated in Figure 6.2, where plane I passes through point D of the desired setpoint.

For a given value of $e_{vi}$, (6.4) describes a plane which is parallel to plane I. As the value of $e_{vi}$ approaches zero, the range of actual joint states converges to plane I, passing through a family of planes, each of which is parallel to plane I. The primary task of the controller is to make the range plane converge to plane I as quickly as possible in the event of disturbances, while maintaining system stability.

The actual state of a joint actuator can exist anywhere on plane I. It may not be fixed to a certain point, such as point D, but rather traces its own trajectory on the plane. The trajectory of the actual state on the range plane is governed by (6.5). It will be shown in Section 6.2.4 that the dynamics of (6.5) can be approximated as a second-order inertia-spring-damper system. Therefore the dynamics of the compliant motion can be approximated as that of the second-order system.

Since the controller gains are parameters of the second-order system, the compliant motion is characterized by the controller gains. Thus the controller gains should be chosen in a way insuring a desired compliant motion. Another consideration in designing controller gains is that the mode of the compliant response should be comparatively slower than the mode which gives the approximation of (6.5), since the
Figure 6.2. Hybrid-state control space model of the force control problem. Actual joint state traces its own trajectory on a plane parallel to, and approaching plane I.
inertia-spring-damper system response relies on the approximation of (6.5). When this condition is not satisfied, the compliant motion predicted by the second-order system is not valid.

### 6.2.2 Identification of a Joint Most Effective for Control of the End Effector Force

Generally, the static joint torques are related to the foot-tip force by the Jacobian transpose;

\[ \tau = J^T \vec{F} \tag{6.6} \]

where \( \tau \) is the joint torque vector and \( \vec{F} \) is the Cartesian foot-tip force. The contact force \( \vec{F} \) is the result of the loading of the vehicle mass due to gravity and body acceleration. Thus, in order to couple the vehicle mass into (6.5), the Jacobian transpose should be obtained from the actual kinematics of the vehicle.

Figure 6.3 shows the geometry of a right-side Hexapod leg and the orientation of the body-fixed coordinate system. The Jacobian matrix of each leg, and its inverse, have been derived by Chao [73]. Since then the leg geometry was slightly modified by eliminating the offset link at the hip azimuth joint. Thus the entries of the new Jacobian matrix and its inverse can be obtained from the corresponding original ones by dropping the terms containing the hip-joint offset, \( \xi_3 \).

Since the z-direction force is used in compliant motion control, only the last column of the Jacobian transpose will be considered in modeling compliant motion. The three entries for the last column of the Jacobian transpose are
Figure 6.3. The geometry of a right-side leg of the OSU Hexapod.
\[ J_{31} = J_{13} = 0 \]  
\[ J_{32} = J_{23} = -x_1 \cos \theta_1 - x_2 \sin(\theta_1 + \theta_2) - x_5 \sin \theta_1 \]  
and
\[ J_{33} = J_{33} = -x_2 \sin(\theta_1 + \theta_2) \]

where \( x_1, x_2, \) and \( x_5 \) are lengths of the upper limb segment, the lower limb segment, and the offset segment between the upper and lower limb segments, respectively, and \( \theta_1 \) and \( \theta_2 \) are joint angles of elevation joint and knee joint, respectively. These entries relate the z-directional foot-tip force to the joint torques following the relation in (2.19).

A physical interpretation of Jacobian entries of (6.7), (6.8), and (6.9) may be helpful in understanding an approximation which will be made later. As shown in Figure 6.4, (6.7), (6.8), and (6.9) represent three moment arms about axes of hip azimuth joint, elevation joint, and knee joint, respectively, relative to the z-direction force. These moment arms are signed quantities, implying the directional sensitivities of joint torques against the z-direction force of the foot-tip with respect to the fixed directions of joint axes. The magnitudes and signs of the arms vary according to leg posture. Notice that the moment arm about the hip azimuth joint is zero and that all the moment arms are independent of the rotation angle of this joint, \( \psi \).

It can be observed from (6.8) and (6.9) that, when the rotation angles of elevation joint and knee joint about their initial positions (\( \theta_1 = 0 \)) are very small, the magnitude of the moment arm about the knee joint is comparatively smaller than that about the elevation joint.
Figure 6.4. Moment arm for each joint with respect to vertical force at the foot-tip.
Actually, the operating ranges of two joint angles are small during support phases of the corresponding leg. Thus (6.8) and (6.9) can be approximated as follows:

\[ J_{32} = J^T_{23} = -x_1 \]  
\[ J_{33} = J^T_{33} = 0 \]

From approximations in (6.7), (6.10), and (6.11), it can be concluded that the elevation joint is the most dominant in feedback control of vertical force at the foot-tip. The elevation joint plays the major role in force feedback, thus the compliance model and the control system model will be built for this joint. The controller based on models of this joint can be used for the other two joints, since the force feedback which is the main cause of instability has little effect on these two joints.

6.2.3 Mass Coupling

In Chapter 4 methods of defining force setpoints for the desired forces at the foot-tips of supporting legs were discussed. Desired forces are defined as those which allow the vehicle body to maintain its static equilibrium relative to the vertical ground reaction forces on the feet of supporting legs. Usually the system of force constraint equations is underspecified, and thus there exists an infinite number of solutions. Three different criteria were discussed for optimal force setpoints.
The torque error at the elevation joint is expressed as follows:

\[ e_{T2} = \tau_{2d} - \tau_{2a} = JT_{23}(F_d - F_a) \]  

(6.12)

Since the desired force reflects only a distribution of the vehicle's weight on its supporting feet, the error between desired force and actual force at the foot-tip is primarily related to the force created by the vehicle's vertical acceleration. Applying this observation to (6.12), an approximate torque error of the elevation joint is then formulated as follows:

\[ e_{T2} = -JT_{23}(m\ddot{z}) \]  

(6.13)

where \( m \) is effective mass loaded on the leg, approximated as a portion of the total mass of the vehicle.

From the Jacobian relation between the Cartesian and joint velocities, the z-direction velocity component is expressed in terms of joint velocities as follows:

\[ \dot{z} = J_{31}\dot{\psi} + J_{32}\dot{\theta}_1 + J_{33}\dot{\theta}_2 \]  

(6.14)

By differentiating both sides of (6.14) with respect to time, the acceleration of the foot-tip in z-direction is obtained:

\[ \ddot{z} = (J_{31}\ddot{\psi} + J_{32}\ddot{\theta}_1 + J_{33}\ddot{\theta}_2) + (J_{31}\dddot{\psi} + J_{32}\dddot{\theta}_1 + J_{33}\dddot{\theta}_2) \]  

(6.15)

The first three terms shown on the right side of (6.15) are centripetal and Coriolis accelerations, since each term of time-derivatives of the Jacobian entries contains a factor of time-derivatives of joint angles. The centripetal acceleration is proportional to the square of a joint's
velocity, and the Coriolis acceleration is proportional to the product of joint velocities from two different joints. The other three terms on the left side of (6.15) are accelerations caused by the joint accelerations.

Since the velocities of joints are small in real operation, their mutual or self products become even smaller. Therefore the increase due to centripetal and Coriolis effects of joint velocities can be neglected from the z-direction acceleration. An approximation of (6.15) is then obtained by dropping the first three terms:

$$\ddot{Z} = (J_{31}\ddot{\phi} + J_{32}\ddot{\theta}_1 + J_{33}\ddot{\theta}_2) .$$

(6.16)

Further simplification can be made on (6.13) and (6.16) by substituting (6.7), (6.10), and (6.11) as

$$e_{T2} = \ell_1 m \ddot{Z}$$

(6.17)

and

$$\ddot{z} = - \ell_1 \ddot{\theta}_1 ,$$

(6.18)

respectively. Substituting (6.18) into (6.17) gives a final approximate expression of torque error at the elevation joint, taking into account the acceleration of the joint and effective mass loaded to the leg in question:

$$e_{T2} = - m l_1^2 \ddot{\theta}_1 .$$

(6.19)

Substituting the torque error term with an approximation of (6.19) enables a mass term to be coupled into the compliance equation, as follows:

$$- k_T m l_1^2 \ddot{\theta}_{la} + (\dot{\theta}_{1d} - \dot{\theta}_{la}) + k_p (\theta_{1d} - \theta_{1a}) = 0 .$$

(6.20)
6.2.4 Model of Joint Compliance

Equation (6.20) is a second-order system, and the dynamics of compliant motion is governed by this equation. The homogeneous part of the equation is then separated in order to analyze the dynamics of compliant motion, as follows:

\[ k_T m_1^2 \ddot{\theta}_1 + \dot{\theta}_1 + k_p \theta_1 = 0 \quad (6.21) \]

or, in another form,

\[ m_1^2 \ddot{\theta}_1 + (1/k_T) \dot{\theta}_1 + (k_p/k_T) \theta_1 = 0 \quad (6.22) \]

When the spring constant, damping coefficient, and moment of inertia are defined as

\[ k_s = \frac{k_p}{k_T} \quad (6.23) \]

\[ B = \frac{1}{k_T} \quad (6.24) \]

and

\[ I = m_1^2 \quad (6.25) \]

respectively, then (6.22) is reduced to a general equation of inertia-spring-damper system, i.e.,

\[ I \ddot{\theta}_1 + B \dot{\theta}_1 + k_s \theta_1 = 0 \quad (6.26) \]

This equation is a model of the joint compliant motion. Its physical representation can be drawn as in Figure 6.5.

6.2.5 Parameters of Active Compliance

The compliance model of (6.26) is a second order system. Thus the undamped natural frequency and the damping factor of the system can be defined as follows:
Figure 6.5. A model of active joint compliance.
\[ \omega_n = \sqrt{\frac{K_s}{I}} = \sqrt{\frac{K_s}{mz_1^2}} \tag{6.27} \]

and

\[ \zeta = \frac{B}{2\omega_n I} = \frac{1}{2kpk_Tmz_1^2} \tag{6.28} \]

Given these parameters, (6.26) can be rewritten as

\[ \ddot{\theta}_1 + 2\zeta\omega_n \dot{\theta}_1 + \omega_n^2 \theta_1 = 0 \tag{6.29} \]

These parameters, the undamped natural frequency and the damping factor of the system, are important indices in characterizing the response of the system. Note from (6.29) that the parameter \( \omega_n \) turns out to be the radian frequency of oscillation when \( \zeta = 0 \) (undamped response). As \( \zeta \) increases in value from 0, the oscillation decays and becomes more damped. When \( \zeta > 1 \), oscillation does not occur, however, the speed of the system is decreased accordingly. With \( \zeta = 1 \), the response exhibits no overshoot which is described as critical damping.

So far, several parameters have been defined in conjunction with the active compliance model. Among them, the active spring constant \( K_s \) is an index of the different accommodations to the environment, and the damping factor \( \zeta \) represents dynamic characteristics of compliant motion. Therefore, these parameters will be chosen as independent variables in designing the joint controller. When this has been done, the two servo
gains $k_T$ and $k_p$ can be solved for the two parameters from (6.23) and (6.28) as follows:

$$k_p = \frac{K_s}{2\zeta \sqrt{mz_1^2}} \quad (6.30)$$

and

$$k_T = \frac{1}{2\zeta \sqrt{mz_1^2} k_s} \quad (6.31)$$

Later, constraints will be defined for these two parameters. The servo gains can be determined by assigning quantitative values to the parameters while observing their constraints.

6.3 Resonance Model of the OSU Hexapod

In designing the undamped natural frequency $\omega_n$, care should be taken that the mechanical system is not excited around its resonant frequencies. It has been suggested that the undamped natural frequency $\omega_n$ should be less than one half of the resonant frequency $\omega_r$ [70,90], or

$$\omega_n < \frac{\omega_r}{2} \quad (6.32)$$

A model for passive compliance of the OSU Hexapod as pictured in Figure 6.6 was constructed in Chapter 5. In this model, the compliance sources are lumped at the foot-tips of legs. The term $M$ is the total mass of the vehicle, and $K_{SP}$ and $B_p$ are the spring constant and damping coefficient of the spring block at each foot. Subscript $p$ denotes that the parameters are from passive compliance sources.

Assuming that the vehicle body does not rotate and moves freely in the z-direction only, the homogeneous portion of the dynamics equation
Figure 6.6. Passive compliance model for the OSU Hexapod.
of the passive compliance model can be expressed as

\[ M \ddot{z} + B \dot{z} + nK_{sp} z = 0 \]  \hspace{1cm} (6.33)

where \( n \) is the number of supporting legs. The resonant frequency is then obtained from (6.33) as follows:

\[ \omega_r = \sqrt{\frac{nK_{sp}}{M}}. \]  \hspace{1cm} (6.34)

Since the vehicle is supported by at least three legs, \( n \) varies from three to six. Thus the range of the resonant frequency also varies, depending on the number of supporting legs. Equation (6.34) can be rewritten as

\[ \sqrt{\frac{3K_{sp}}{M}} < \omega_r < \sqrt{\frac{6K_{sp}}{M}}. \]  \hspace{1cm} (6.35)

Combining conditions in (6.32) and (6.35), an upper limit of the undamped natural frequency can be obtained:

\[ \omega_n < \sqrt{\frac{3K_{sp}}{4M}}. \]  \hspace{1cm} (6.36)

Substituting \( \omega_n \) from (6.27) into (6.36) defines an upper limit on the spring constant of active compliance, in terms of the passive spring constant, total vehicle mass, effective mass loading at each leg, and the moment arm on the major compliant joint:

\[ K_s < \frac{3m_{el}^2K_{sp}}{4M}. \]  \hspace{1cm} (6.37)
6.4 Actuator Model

In order to narrow down feedback gains, the joint actuator model must be considered. Each joint actuator consists of an industrial grade, series-wound, electric drill motor and gear reduction unit. The second stage of the gear reduction is a non-backdriveable worm gear. This insures that the joints lock into position when power is removed, but makes the relationship between motor shaft torque and applied foot force highly nonlinear.

The motors are powered by half-wave AC phase control. The power control circuitry consists of a bridge rectifier and a triac for each motor. Each triac is controlled by an analog trigger generator circuit, which compares the input voltage signal with a reference wave-form, and sends a turn-on pulse to the triac gate when the line voltage is at the proper phase. The reference waveform is generated by a separate circuit, and partially removes the nonlinearity associated with AC phase control. Complete schematics for the motor power controller circuitry can be found in [71].

Accounting for the linearizing reference waveform and the high gear reduction ratio, a simple linear transfer function from input voltage for the power control circuitry to the motor angular position has been identified [36] as

\[
\frac{G(s)}{V(s)} = \frac{0.14}{s(s+3)}
\]  \hspace{1cm} (6.38)

which has been found to give accurate predictions of actuator performance.
6.5 Design of Servo Gains

As shown in Figure 6.1, there are three feedback loops in the joint motor controller: position, velocity, and torque feedback. Among them, position and velocity variables are directly measurable states of a joint motor, related to the system input voltage by a transfer function of (6.38). However, the motor and gearbox output torque is not directly related to the joint input voltage. This variable is derived from the measured foot-tip force through the Jacobian transpose. Notice that the actual force at each foot-tip is a result of interaction between the foot-tip and its supporting environment, and therefore, the torque feedback variable can be regarded as an input to the system. The control structure of a joint actuator is thus drawn as in Figure 6.7. Here the input $u$ represents the combination of all the inputs, desired position, desired velocity, and torque error.

In the control structure of Figure 6.7, block $G$ represents a compensator. In order to achieve certain specifications for stability, accuracy and speed, a complicated compensator which provides phase lag, phase lead, or a combination of both, may be designed. In this work, however, a simple gain is chosen for simplicity. When a compensator gain $G_1$ is substituted for the compensator block and the feedback loop of the motor back emf is explicitly shown, Figure 6.7 is redrawn as Figure 6.8.

If state variables $x_1$ and $x_2$ are assigned to the position and velocity of the joint motor, respectively, the state equations of the joint actuator are expressed as

$$ x_1 = x_2, \quad (6.39) $$
Figure 6.7. Control structure of a single joint of the OSU Hexapod.
Figure 6.8. Control structure for a single joint with a constant compensator gain.
\[ \begin{align*}
\dot{x}_2 &= -k_p G x_1 - (3 + G)x_2 + u \\
\end{align*} \]  
(6.40)

with the substitution of
\[ G = 0.14 G_1 . \]  
(6.41)

The matrix notation of the state equations is then
\[ \begin{align*}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} &= 
\begin{bmatrix}
0 & 1 \\
-k_p G & -(3 + G)
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
0 \\
u
\end{bmatrix} . \\
\end{align*} \]  
(6.42)

The 2 X 2 matrix on the right side of (6.42) is the system matrix.

To obtain the desired response from the control system, the closed-loop system poles should be placed properly. Since it is a basic requirement that the system response be stable, the real parts of the system poles should be negative. The characteristic equation of the system matrix in (6.42) is expressed as
\[ f(s) = s^2 + (3 + G)s + k_p G = 0 , \]  
(6.43)

and the system poles are the roots of this equation.

Since the position feedback gain \( k_p \) and the system forward gain \( G \) are always positive for negative feedback, the following condition is always true:
\[ f(0) = k_p G > 0 . \]  
(6.44)

Further, the axis of symmetry of the parabola graph, \( y = f(s) \), is in the left half plane, because
\[ s = -\frac{3 + G}{2} < 0 . \]  
(6.45)
Therefore, roots of (6.43) are either both negative or are two complex roots with a negative real part. This implies that the control system is always stable for positive $k_p$ and $G_1$. The condition necessary for the roots of (6.43) to be negative real is

$$\begin{align*}
(G + 3)^2 - 4k_pG &> 0, \\
\text{(6.46)}
\end{align*}$$

and in this case, the system is non-oscillatory.

It has been mentioned that the servo error signal of (6.4) should vanish quickly, in order for the compliance model of equation (6.5) to be valid. This can be achieved simply by increasing the feedforward compensator gain $G_1$. Notice that inequality (6.46) can also be satisfied with a large $G_1$ (or $G$), ideally with an infinite $G$. A rearrangement of (6.40) is

$$\begin{align*}
x_2 + k_p x_1 &= - \frac{(\dot{x}_2 + 3x_2 + u)}{G}, \\
\text{(6.47)}
\end{align*}$$

and, as $G$ approaches infinity in this equation, a limit is obtained:

$$\begin{align*}
x_2 &= - k_p x_1. \\
\text{(6.48)}
\end{align*}$$

When (6.48) is substituted into (6.39), the limit of the slow eigenvalue is determined as

$$\begin{align*}
s_1 &= - k_p. \\
\text{(6.49)}
\end{align*}$$

In this case, the other eigenvalue approaches $-G$ (i.e. negative infinity), since the product of two eigenvalues is $k_pG$ from (6.43).

Practically speaking, the compensator gain and position feedback gain cannot be too large, since there are limitations on the system hardware and the control computer. Also motor input signal saturation
would occur easily with high gains, introducing an extra nonlinearity. This control is performed by a digital computer with limited word size and computation speed. Data channels of A/D and D/A converters also have limited resolutions due to their word size. Thus the actual system eigenvalues can now be designed by considering these limitations.

If the servo gains are increased, the system time constant decreases accordingly. This requires a higher control frequency in order to simulate continuous control. Although the control frequency is limited primarily by computation speed, even computation rates exceeding the limit do not improve the control performance, since there are word size limitations in the computer's memory and in A/D and D/A converters. Actually the control frequency of the current control computer can reach approximately 85 Hz with code optimization of the control program and a table look-up method of trigonometric functions. However it should be limited to 60 Hz, since the power to the motors comes from the phase-controlled 60 Hz AC source.

In order to simulate the continuous control model, the interval of digital control is recommended to be between one sixth and one tenth of the smallest system time constant \( [85] \). This can be applied conversely to limit the fastest system pole. Since the control frequency is limited at 60 Hz, the fastest time constant is designated at

\[
\tau_{\text{min}} = \frac{1}{60} \cdot 6 = 0.1 \text{ [sec]} . \quad (6.50)
\]

This determines one system pole at

\[
p_1 = -10 . \quad (6.51)
\]
The other eigenvalue is then determined as

\[ p_2 = -\frac{kpG}{10}, \quad (6.52) \]

since the product of the two eigenvalues from (6.43) is \( kpG \). Later, the actual value of this pole will be compared with that of an ideal case in (6.49).

Since the pole in (6.51) should be a solution of (6.43), substituting (6.51) into (6.43) gives

\[ kpG - 10G + 70 = 0, \quad (6.53) \]

or

\[ G = \frac{70}{10 - kp}. \quad (6.54) \]

Up to this point, all the feedback gains have been defined in terms of the spring constant of active compliance, \( K_s \), and the damping coefficient, \( \zeta \) by (6.30) and (6.31). Also, an upper limit has been defined for the active spring constant by the inequality in (6.37), considering the system's resonant frequency. An eigenvalue of the control system was fixed at -10, based on the computation frequency of the control computer. From this the other eigenvalue is expressed as a function of the position feedback gain and the system forward gain by (6.52). The compensator of the controller is also a function of position gain as expressed by (6.54). Therefore, when the active spring constant \( K_s \), and the damping coefficient \( \zeta \) are fixed, the other system parameters are determined accordingly. These two independent variables thus will be selected for actual experiments.
In order to choose a reasonable active spring constant for joint coordinates, a Cartesian spring constant is first selected for z-direction compliant motion, and converted into joint space. A trial value for the Cartesian spring constant is assigned as

\[ K_s' = 8 \text{ [lbf/in]} \quad (6.55) \]

With this value, an eight-pound force error is compensated with a one-inch position error. Since the major compliant joint is the elevation joint, the corresponding spring constant in joint space is obtained for this joint. Figure 6.9 illustrates the above relation.

An eight-pound force error at the foot-tip exerts approximately \( 8\varepsilon_1 \) lbf-inch torque to the elevation joint by (6.12). A one-inch accommodation is equivalent to \( 1/\varepsilon_1 \) radian rotation of the joint. Thus the corresponding spring constant is

\[ K_s = 8\varepsilon_1^2 \text{ [lbf-in/rad]} \quad (6.56) \]

The passively compliant motions of wheeled vehicles are usually set to be slightly underdamped both for mechanical safety and for the on-board operator's comfort. The Hexapod has some passive compliance, since its structure is not perfectly stiff, and additional spring shoes are attached at its foot-tips. Accounting for these passive compliances, the damping factor of actively compliant motion is set to achieve the critical damping condition; i.e.,

\[ \zeta = 1 \quad (6.57) \]

resulting in the fastest non-oscillatory compliant response.
Figure 6.9. Relationship between the Cartesian active spring constant and its corresponding joint spring constant.
It is also necessary to determine the effective mass for each leg. The total vehicle mass $M$ is approximately

$$M = 310 \text{[lbm]} = 0.803 \text{[lb f*sec}^2/\text{in]} , \quad (6.58)$$

and the effective mass at one leg is a fraction of the total mass. Since there could be cases when only one leg supports its side of the vehicle, the effective mass $m$ is chosen as $1/2$ of the total mass of the Hexapod. Thus, for the most extreme case,

$$m = M/2 = 0.402 \text{[lb f*sec}^2/\text{in]} \quad (6.59)$$

Substituting values in (6.56), (6.57), (6.58), and (6.59) into (6.30) and (6.31) yields

$$k_p = 2.23 \text{[sec}^{-1}] \quad (6.60)$$

and

$$k_T = 0.00042 \text{[(rad/sec)/(lb f*inch)]} \quad (6.61)$$

The system's forward gain then is obtained from (6.54):

$$G = 9.009 \text{[sec}^{-1}] \quad (6.62)$$

The compensator gain is determined from (6.41):

$$G_1 = 64.35 \text{[volts/(rad/sec)]} \quad (6.63)$$

Substituting (6.60) and (6.62) into (6.52) determines the system dominant pole:

$$p_2 = -2.01 \quad (6.64)$$

All the system parameters now are specified. The dominant pole with the ideal servo would be $-2.23$, bound by substituting (6.60) into (6.49). Comparing this value with the value in (6.64) indicates the
actual pole is slower than that of the ideal servo model by only a ten percent difference in the magnitude of their eigenvalues. Thus the compliance model is expected to be valid in real control.

At this point a controller for the elevation joint has been designed. It is equally serviceable for the other joints, since all joints have the same control model (as in Figure 6.8), and torque terms act as input.

It is necessary in implementing joint control to specify trajectories of joint position and velocity. When the operator command is specified in Cartesian space, the corresponding joint velocity can be obtained by Jacobian relation. For the joint position, it is necessary to perform inverse kinematics. Since only the error of joint position is used, however, inverse kinematics is not necessary in implementing joint controllers. If the error of joint position is small, it is related to the Cartesian position error as follows:

$$\theta_d - \theta_a = J^{-1}(x_d - x_a).$$

(6.65)

With this observation, the existing Jacobian control structure basically is maintained. The overall control structure is represented in Figure 6.10.

6.6 Summary

In this chapter, an experimental controller was designed for the OSU Hexapod to implement actively compliant motion in joint space, based on the algorithm developed in Chapter 3 through Chapter 5. A compliance model in the form of a spring-damper system was built, and from this model the active spring constant and the damping factor were defined to characterize the active compliance. These two variables were regarded
Figure 6.10. Jacobian control structure for constant joint compliance.
as independent parameters, and all the servo gains were expressed in terms of these parameters. Mechanical resonance and the possible control frequency were both considered in imposing constraints on the two independent parameters. By assigning experimental values to these parameters so as to satisfy the constraints, a set of servo gains was designed.

Several experiments were performed using the controller designed in this chapter, and their results were evaluated in Chapter 3 through Chapter 5. As conclusions of these experiments, the following can be stated. First, the control implementation actually showed active compliance confirming the validity of the algorithm of active compliance in joint space. Second, the control system showed stable responses for various practical walking commands, verifying that the servo gains were properly chosen.

To this point, discussions on algorithms of active compliance were for individual legs. The next chapter will develop an algorithm for distributing active compliance over some legs by using a hybrid control philosophy. This control scheme is achieved when some of the supporting legs are controlled by position only, and others are controlled by both position and force. This strategy reduces force interactions among supporting legs.
Chapter 7
HYBRID CONTROL ALLOCATED BY LEGS

7.1 Introduction

A hybrid controller is defined as one that allows force to be commanded along certain degrees of freedom, while allowing position to be commanded along the remaining freedom in the compliance frame [63,66]. For instance, if the end effector of a manipulator is to slide on the environmental surface, the compliant motion can be accomplished by simultaneously controlling the position in the sliding direction parallel to the surface, and the force in the direction normal to the surface. This type of control will be specifically called "hybrid control by directions" to be distinguished from another type of hybrid control in a multi-leg system (or a multi-manipulator system), "hybrid control allocated by legs."

In the control of a legged vehicle, legs in the support phase should not slide, and their positions should be controlled in all directions in order to achieve a desired locomotion and support the vehicle body at a specified height. The positions of the legs in the transfer phase, also should be controlled in all directions in order to prepare for their support phase. However, since the terrain on which the vehicle walks is irregular and uncertain, a means is necessary for the vehicle to adapt itself to various terrain conditions and for the vehicle weight to be well distributed among supporting legs. To
accomplish this, the vertical direction of the hexapod is programmed to be controlled by both force and position of all legs. The ground reaction forces are measured at the supporting feet and their errors are balanced with position errors in order to accommodate to the environment. This type of force control is explicit feedback. All previous discussion of OSU Hexapod control in previous chapters was based on this explicit force feedback.

When all the supporting legs are programmed for compliant motion, interaction in forces among legs may arise in conjunction with active and/or passive compliances [10]. This interaction can excite the vehicle to roll and pitch, causing instability in vehicle attitude. In Section 7.2, a one-dimensional model of a suspension system with three supporting legs will be built, and state equations on this model will be formulated. A numerical simulation will be performed to show that force interactions occur among legs in conjunction with passive compliance and insufficient discrete control frequency. It will be shown that it is possible for force interaction to occur among legs without exciting the mass block they support.

In order to reduce or avoid possible interaction in force, a modification in the force control scheme is considered. Instead of controlling the forces of all the supporting legs, some are scheduled to be controlled by position only, and the others by both position and force. These modifications can be implemented by excluding force feedback loops from the controllers of certain leg systems. This breaks some possible paths of interaction in force among supporting legs and thus reduces some instabilities due to force interaction. This control scheme is called "hybrid control allocated by legs." A numerical
simulation of the suspension system will be performed in order to verify the validity of this scheme.

In Section 7.3 the concept of hybrid force control allocated by legs will be applied to the OSU Hexapod. Several provisions will be made in implementing this control scheme on the OSU Hexapod. For a given set of supporting legs, a schedule dynamically determines which legs will be controlled by position and which will be controlled by both position and force. Also, the timing of switching between the two modes of compliant and non-compliant supports will be determined. For smooth switching between the two support modes, a modification will be made on the assignment of relative leg phases.

7.2 Leg Interactions in Force
7.2.1 Simulation of Force Control for a Three-Leg Suspension System
7.2.1.1 A Two-Dimensional Model of the Three-Leg Suspension System

In order to investigate the existence of force interactions among legs, a simple two-dimensional model of a three-leg suspension system will be built. Based on this physical model, the hybrid state equations of an explicit force feedback control system will be formulated. Figure 7.1 is a model of the suspension system, where three legs support the mass block M. Each leg consists of one prismatic joint which is actuated linearly. It is assumed that all the sources of vertical compliance are lumped at the contact point of the foot-tip of each leg and the supporting ground. It is also assumed that the force measurement is performed at the contact point and is used in the position control of the suspension system.
Figure 7.1. A one-dimensional model of a three-leg suspension system. The passive compliance parameters are lumped at the contact points. The mass block is constrained to move only in a vertical direction.
In order to make it simple to formulate and analyze the state equations, it is assumed that the mass block is constrained to move only in the vertical direction, through a frictionless guide hole, as shown in Figure 7.1, and not to rotate about any axis parallel to the supporting ground. Even with this assumption, the force balance equation is redundant by two degrees of freedom, since the sum of three leg forces is constrained to be the acceleration and gravity force of the mass block.

By defining the height of the mass block from the ground as $x_1$ and joint extensions as $x_3$, $x_5$, and $x_7$ for the three legs, the force measurements for each leg are expressed as

\[ f_{1a} = K_{s1}(x_3 - x_1) + B_1(x_3 - x_1), \]  
\[ f_{2a} = K_{s2}(x_5 - x_1) + B_2(x_5 - x_1), \]  
\[ f_{3a} = K_{s3}(x_7 - x_1) + B_3(x_7 - x_1). \]  

From the geometric constraint on the mass block, the result of all the leg forces are equated as

\[ F = M(\ddot{x}_1 + g) = f_{1a} + f_{2a} + f_{3a}. \]  

Solving the acceleration of the mass block from (7.4) gives

\[ \ddot{x}_1 = \frac{1}{M} (f_{1a} + f_{2a} + f_{3a}) - g. \]  

For complete state equations, it is necessary to introduce an actuator model and define a control law for it. For the advantage of familiarity, the same actuator model as is used in the OSU Hexapod joint
is used in this exercise. The transfer function of the actuator is

\[
\frac{\theta(s)}{V(s)} = \frac{0.14}{s(s+3)}.
\]  

(7.6)

The control law is defined similarly as that discussed previously. For example, the error signal for the actuator of leg 1 is defined as

\[
e_v1 = (x_{1d} - x_{1a}) + k_{p1}(x_{1d} - x_{1a}) + k_{f1}(f_{1d} - f_{1a}),
\]

(7.7)

where the terms \(k_{p1}\) and \(k_{f1}\) are position and force gains, respectively, and the variables with subscript \(d\) represent the desired setpoints and those with \(a\) are for actual measurements.

Combining (7.5), (7.6) and (7.7), an overall signal flow graph is obtained, as shown in Figure 7.2. As shown in the graph, control of any individual leg is coupled to other legs through the common mass block. Defining the derivatives of \(x_1, x_3, x_5,\) and \(x_7\) as \(\dot{x}_2, \dot{x}_4, \dot{x}_6,\) and \(\dot{x}_8,\) respectively, the state equations for the continuous control system are formulated as follows:

\[
\dot{x}_1 = x_2, \\
\dot{x}_2 = -\frac{K_{s1} + K_{s2} + K_{s2}}{M} x_1 - \frac{B_1 + B_2 + B_3}{M} x_2
+ \frac{K_{s1}}{M} x_3 + \frac{B_1}{M} x_4 + \frac{K_{s2}}{M} x_5 + \frac{B_2}{M} x_6 + \frac{K_{s3}}{M} x_7 + \frac{B_3}{M} x_8, \\
\dot{x}_3 = x_4, \\
\dot{x}_4 = Gk_{f1}K_{s1}x_1 + Gk_{f1}B_1x_2
- G(k_{f1}K_{s1} + k_1)x_3 - [3 + G(k_{f1}B_1 + 1)]x_4
+ G(k_1x_{3d} + x_{4d} + k_{f1}f_{1d}).
\]

(7.8)  

(7.9)  

(7.10)  

(7.11)
Figure 7.2. Signal flow graph of the suspension system control. Leg systems are coupled to each other through the mass block.
\[
\dot{x}_5 = x_6, \quad (7.12)
\]
\[
\dot{x}_6 = \frac{gk_2k_s^2x_1 + gk_2B_2x_2}{1 + G(k_2x_5 + x_6d + k_2f_2d)}
- G(k_2k_s^2 + k_2)x_5 - [3 + G(k_2B_2 + 1)]x_6
+ G(k_2x_5d + x_6d + k_2f_2d), \quad (7.13)
\]
\[
\dot{x}_7 = x_8, \quad (7.14)
\]
\[
\dot{x}_8 = \frac{gk_3k_s^2x_1 + gk_3B_3x_2}{1 + G(k_2x_5 + x_6d + k_2f_2d)}
- G(k_3k_s^3 + k_3)x_7 - [3 + G(k_3B_3 + 1)]x_8
+ G(k_2x_5d + x_6d + k_2f_3d). \quad (7.15)
\]

The state equations (7.8) through (7.15) define the 8x8 system matrix A and the 8x3 input matrix B; thus the vector state equation is expressed as
\[
\dot{x} = Ax + Bu. \quad (7.16)
\]

The controllability matrix then is formulated from these two matrices as
\[
C = [B : AB : \ldots : A^7B]. \quad (7.17)
\]

It can be shown that an analytic computation of the controllability matrix is of full rank, and thus the control system is fully controllable. Furthermore, with the actual values of feedback gains and any positive values of compliance parameters, all the eigenvalues of the system matrix are on the left half of the complex plane. Therefore, in the case of continuous control, the system is stable with a bounded
input. The situation may be different, however, in discrete control. In order to investigate the differences, a simulation will be performed on hybrid state equations to be defined from the continuous control model. In discrete control the state variables fed back through digital computation loops are held as constants for a whole interval between two control points. In discrete control one step of computation delay is usually introduced. Thus the state equations for \( nT \leq t < (n+1)T \) contain the feedback states at \( t = (n-1)T \). From the continuous state equations (7.8) through (7.15), the hybrid state equations are obtained with a minor modification on state feedback terms passing digital computation loops, as follows:

\[
x_1 = x_2 , \quad (7.18)
\]
\[
x_2 = - \frac{K_{s1} + K_{s2} + K_{s3}}{M} x_1 - \frac{B_1 + B_2 + B_3}{M} x_2 \\
+ \frac{K_{s1}}{M} x_3 + \frac{B_1}{M} x_4 + \frac{K_{s2}}{M} x_5 + \frac{B_2}{M} x_6 + \frac{K_{s3}}{M} x_7 + \frac{B_3}{M} x_8 , \quad (7.19)
\]
\[
x_3 = x_4 , \quad (7.20)
\]
\[
x_4 = - 3x_4 \\
+ G_{k}f_{1}K_{s1}x_1[(n-1)T] + G_{k}f_{1}B_1x_2[(n-1)T] \\
- G(k_{f1}K_{s1} + k_{i})x_3[(n-1)T] - G(k_{f1}B_1 + 1)x_4[(n-1)T] \\
+ G(k_{1}x_3d[(n-1)T] + x_4d[(n-1)T] + k_{f1}f_{1d}[(n-1)T]) , \quad (7.21)
\]
\[ x_5 = x_6, \quad (7.22) \]
\[ x_6 = -3x_6 \]
\[ + Gk_2K_{s2}x_1[(n-1)T] + Gk_2B_2x_2[(n-1)T] \]
\[ - G(k_2K_{s2} + k_2)x_5[(n-1)T] - G(k_2B_2 + 1)x_6[(n-1)T] \]
\[ + G(k_2x_5d[(n-1)T] + x_6d[(n-1)T] + k_2f_2d[(n-1)T]), \quad (7.23) \]
\[ x_7 = x_8, \quad (7.24) \]
\[ x_8 = -3x_8 \]
\[ + Gk_3K_{s3}x_1[(n-1)T] + Gk_3B_3x_2[(n-1)T] \]
\[ - G(k_3K_{s3} + k_3)x_7[(n-1)T] - G(k_3B_3 + 1)x_8[(n-1)T] \]
\[ + G(k_2x_7d[(n-1)T] + x_8d[(n-1)T] + k_2f_3d[(n-1)T]) \], \quad (7.25)\]

where \( nT \leq t < (n+1)T \). If there is no computation delay, the time index of the variables of state feedback and input will be changed from \((n-1)\) to \( n \).

**7.2.1.2 Numerical Simulation**

Using the Runge-Kutta fourth-order integration technique [90], a numerical simulation is performed on the hybrid state equations (7.18) through (7.25). In implementing the simulation it is necessary to define initial conditions of state variables and an input function. In order to define initial conditions, a temporary constant input vector \( u_1 \) first is defined, then substituted in the continuous vector state equation (7.16).

\[ \dot{x} = Ax + Bu_1 \]. \quad (7.26)
Assuming that the system comes to rest with this input, the steady state is obtained from (7.26) with setting $\dot{x} = 0$. Thus an initial state is defined as

$$\mathbf{x}_i = -\mathbf{A}^{-1}\mathbf{B}\mathbf{u}_i \quad (7.27)$$

For system input, a step input $\mathbf{u}_f$ differs only by a constant from $\mathbf{u}_i$. The actual system input, of course, may be more complicated; however the step input is simple and sufficient for the purpose of this section.

If the discrete control system is stable, the steady state response will be the same as that of continuous control. Thus, in this case, the steady state response of the discrete system is expected to be

$$\mathbf{x}_f = -\mathbf{A}^{-1}\mathbf{B}\mathbf{u}_f \quad (7.28)$$

The actual responses need to be evaluated, comparing with that in (7.28).

The force responses of the simulation, with a step input in position command and a control frequency of 100 Hz are illustrated in Figure 7.3. The system parameters were tabulated in Table 7.1. Initially the force responses are the gravity loading of the mass block. At the beginning of the step command the forces increase sharply with acceleration of the mass block. Later they return to the values of gravity loading. As shown in the Figure, the force responses of all three legs are stable, which was also the case with the continuous control. Since the control frequency is quite high, the responses can be regarded as those of continuous control. These responses will be compared with responses of a different control frequency.
Table 7.1

Parameter values for computer simulation of a three-leg suspension system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>body mass</td>
<td>M</td>
<td>0.736 lbf·sec²/inch</td>
</tr>
<tr>
<td>gravitational acceleration</td>
<td>g</td>
<td>385.5 inch/sec²</td>
</tr>
<tr>
<td>position error gains</td>
<td>kₚ₁, kₚ₂, kₚ₃</td>
<td>1.17 sec⁻¹</td>
</tr>
<tr>
<td>velocity error gains</td>
<td>kᵥ₁, kᵥ₂, kᵥ₃</td>
<td>1.0</td>
</tr>
<tr>
<td>force error gains</td>
<td>k₟₁, k₟₂, k₟₃</td>
<td>0.146 inch/(sec·lbf)</td>
</tr>
<tr>
<td>compensator gain</td>
<td>G</td>
<td>56.6 volts/(rad/sec)</td>
</tr>
<tr>
<td>passive spring constants</td>
<td>Kₛ₁, Kₛ₂, Kₛ₃</td>
<td>varying, lbf/inch</td>
</tr>
<tr>
<td>passive damping coefficients</td>
<td>B₁, B₂, B₃</td>
<td>5.0 lbf·sec/inch</td>
</tr>
<tr>
<td>sampling time</td>
<td>T</td>
<td>varying, sec</td>
</tr>
</tbody>
</table>
Figure 7.3. Force responses of the suspension system. Simulation was performed with a control frequency of 100 Hz. The responses can be assumed to be near those of continuous control.
Figure 7.4 illustrates the force responses with a control frequency of 20 Hz. The responses are oscillatory, which is reasonable with a low frequency control. Considering only the first part of the responses, the oscillations are damped out within a short period. Thus the system responses can be regarded as stable. However, strangely, new oscillations arise again, with rapidly increasing amplitude, after the responses are settled down completely. This phenomenon does not seem to be physically possible. Thus, at first glance, computational error, caused by round off error with an improper integration interval, might be blamed. However, evidence can be gathered against this conclusion: The same simulations performed with larger or smaller integration intervals than that of the simulation in Figure 7.4, but with the same control frequency, still show the same recurrence of diverging oscillations as in Figure 7.4. However, the diverging oscillations do not occur when a higher control frequency is used with the same integration interval, as shown in Figure 7.3.

Contrary to the force response of individual leg, the resultant force of all three legs does not oscillate after it is settled down, and is equal to the gravity loading of the mass block, as is shown in Figure 7.4 (d). Physically, this implies that the mass block is not activated, being maintained at a constant height. Since the mass block does not move, the position and velocity feedbacks of the mass block to each leg controller are constant and zero, respectively. Thus they can be considered as input to the leg control system. Applying this to the signal flow graph of Figure 7.2, the control model is reduced to an individual leg control model as in Figure 7.5.
Figure 7.4. Force responses of the suspension system. Simulation was performed with a control frequency of 20 Hz. The step responses are oscillatory, and unexpected oscillations recur after the system is settled down.
Figure 7.5. Individual leg control system simplified from Figure 7.2 resulting after the mass block remains at rest.
From the reduced model of the leg control system, it is clear that
the passive spring constant and the damping factor act as position and
velocity feedback gains, respectively. As was discussed in Chapter 5,
it can be easily shown that the modes of the reduced control model are
all stable with continuous control, no matter what the passive
compliance parameters are. However, as shown in Figure 7.6, the force
response of the simulation is unstable with discrete control. This is
basically due to the insufficient control frequency and excessive
stiffness of the system. This verifies the cause of the unexpected
oscillations in Figure 7.4.

7.2.2 Simulation of Hybrid Force Control Allocated by Legs

As shown in Figure 7.4, the unexpected oscillation occurs without
activating the mass block the legs support, representing an interaction
in force among legs. Basically this interaction can be avoided by
increasing the control frequency and/or making the system more
compliant. However, this remedy has limitations, since the computation
speed is limited by hardware and the position control is inaccurate with
a quite flexible system.

The mechanism of interaction in force suggests a solution for
avoiding it. The interaction in force occurs because all the legs are
programmed for force control, which allows a path of force interaction.
Thus a possible solution is to break the path. In other words, the
force feedback loop is removed in the control of a certain leg,
resulting in a pure position control. With this strategy, it is
expected that a disturbance in control of a position-and-force control
leg is not amplified by the control of a position-control leg. This is
Figure 7.6. Force response of the reduced leg control system. The control frequency is 20 Hz. The unstable response verifies the divergence of oscillation in Figure 7.4.
the basic idea of the "hybrid force control allocated by legs." In this case, the force response of the leg controlled by position is not involved in its force control, reducing one degree of redundancy in the force constraints.

Figure 7.7 illustrates the force responses of hybrid force control allocated by legs. The simulation was performed with exactly the same condition except for setting the force feedback gain of leg 1, $k_f_1$, to zero. The responses no longer show unexpected oscillations. One can compare the force response of leg 1 of Figure 7.7 (a) with that of Figure 7.4 (a). Since, in the case of the hybrid control, the force of leg 1 is not controlled, its curve is increased more sharply with the uncontrolled acceleration of the mass block than with force control, but is smoother.

It is not enough, in avoiding interactions in force, to remove force feedback loop of one leg only. Although leg 1 is removed from the sources of force interactions, there still exists a path of interaction between leg 2 and leg 3. This situation is illustrated in Figure 7.8; another simulation result with a different spring constant. The force interactions occur between leg 2 and 3, both of which are controlled by force and position. Thus it can be concluded that interactions in force are always possible whenever there exists any path of interactions or any redundancy in the force constraints, although the coupling may also be reduced with the reduction of the order of redundancy.

An actual suspension system can roll or pitch without constraint. Thus, for the actual vehicle, interactions in force may occur more easily than in the case of one-dimensional constrained motion. If the body attitude of the vehicle is measured with appropriate sensors
Figure 7.7. Force responses of the suspension system with the hybrid force control scheme. The passive spring constant used is $K_s = 284$ [lbf/inch]. Simulation was performed with a control frequency of 20 Hz. The step responses are oscillatory. However the unexpected oscillations do not recur after the system is settled down.
Figure 7.8. Force responses of the suspension system with hybrid force control scheme. The passive spring constant used is $K_s = 71 \text{[lbf/inch]}$. Simulation was performed with a control frequency of 20 Hz. The step responses are oscillatory. The unexpected oscillations occur between leg 2 and leg 3.
and is used in modifying the positional control for attitude regulation [10], the interactions in force may also be damped out. In this chapter, however, a different approach, the hybrid force control allocated by legs, will be proposed in avoiding interactions and maintaining the vehicle at the desired attitude.

7.3 Hybrid Force Control Allocated by Legs for the OSU Hexapod

In this section, the method of hybrid force control allocated by legs will be applied to the control of the OSU Hexapod. This is based on the logical conclusion that reducing the number of force-control legs also reduces interactions in force among supporting legs, while forces are regulated for all supporting legs no matter how they are controlled.

7.3.1 Tripod Support and Force Responses

For the OSU Hexapod vehicle to maintain its static balance, the sum of forces of its supporting legs should be equal to its weight, and their moments about longitudinal and lateral directions should be zero. These constraints define three scalar equations, with force setpoints as their solutions. If the vehicle is supported by a tripod, then the equilibrium condition is deterministic. However, when the number of supporting legs is more than three, the equations of the system are underdetermined and an infinite number of solutions are possible. Thus an optimality criterion can be applied in determining force setpoint.

In the tripod gait, the force setpoint is determined uniquely from the geometry of the supporting tripod. If the height of the vehicle body is controlled so the body does not have any acceleration, the force responses are also deterministic. Therefore force control may not be
necessary in tripod gait locomotion if it is only for the purpose of load distribution. Experimentally, it has been observed that the actual force responses are close to the theoretical equilibrium condition, even without force feedback, in case the vehicle is operated in the tripod gait on a level terrain [10]. Figure 7.9 illustrates the force response of the vehicle in the tripod gait with force feedback control, while Figure 7.10 illustrates the tripod gait without force feedback control. In the case with no force feedback, force errors are large at touch-down and lift-off points, due to the uncontrolled collision of the ground and foot-tips. In the case when force feedback is present, the force response shows more chatter due to the active force control, than without force control. However, the two cases are generally very close.

The results of experiments with the tripod gait suggests that for non-tripod gaits the supporting legs are divided into two groups: a tripod set which will be controlled by position only and the others which will be controlled by both position and force. In this case force setpoints for the supporting legs are determined in the usual manner, among which those for active compliance are actually used in feedback control. If the legs controlled by both force and position follow their force setpoints closely, and if the vehicle system is stable, the force responses of the tripod legs controlled by position only will be deterministic. They will follow their unused force setpoints determined from the equations of force constraint, in addition to others for compliant legs. If this new force control technique works, the number of force feedback loops is reduced to less than half the amount used for
Figure 7.9. Force responses of the OSU Hexapod in the tripod gait. The responses were obtained with force feedback control.
Figure 7.10. Force responses of the OSU Hexapod in the tripod gait. The responses were obtained without force feedback control, yet they are quite close to those pictured in Figure 7.9 with force feedback control.
full force feedback control. This may reduce leg interactions in force significantly.

7.3.2 Selection of the Non-compliant Tripod and Its Timing

In order to implement hybrid force control allocated by legs, a selection policy for a non-compliant tripod set among supporting legs, as well as timing for switching from one non-compliant tripod to another, should be programmed. These problems are closely related to the pattern of leg sequencing or gaits for locomotion. Thus it is necessary to consider several definitions concerned with periodic gaits. The definitions characterizing periodic gaits have been set forth by McGhee and Frank [36], and others [7,37,41], and are summarized in Chapter 2.

Tripod sets for non-compliant support may be programmed in advance, according to various leg duty factors; alternatively, such sets can be determined dynamically according to the current supporting pattern. Similarly, the timing of leg switching can also be programmed in conjunction with the kinematic cycle variable. In any case, it is desirable for any two consecutive tripod sets to have two legs in common, for smooth shifting of non-compliant tripods. Since only one leg changes its status of compliance from active compliance to non-compliance at any instant, this switching method can minimize disturbances due to the adjustment between force and position errors.

The purposes of force control in legged-vehicle locomotion are to allow the vehicle to adapt itself to uncertain terrain conditions during leg phase alternations between support and transfer, and to evenly distribute the weight of the vehicle on its supporting legs during
locomotion. Therefore, the legs are actually required to do active compliant motion during their transfer phase, and some fraction of active compliant motion in the beginning of the support phase. The force measurement at the foot-tip of a leg is zero when in the air for transfer. Thus, during the transfer phase, active compliance is performed with the force setpoint of zero. Actually, it is purely position control until the transfer foot comes into contact with the ground or an unexpected obstacle. When a transfer leg enters the support phase, it still requires control by both force and position to regulate the vehicle's weight distribution. Later, when the load to the leg is sufficiently settled down, it can be controlled by position, only, to support the vehicle stably and to refrain from interaction with other legs by performing active compliant motion.

7.3.3 Leg Duty Factor and Compliance Duty Factor

Hybrid force control can be implemented only in cases where more than three legs support the vehicle at any time. The number of supporting legs, during locomotion, is an integer between three and six inclusively, and is dependent on the leg duty factor $\beta$. The number may or may not be constant. The average number of supporting legs during locomotion is obtained as

$$s = 6 \beta . \quad (7.29)$$

Since the number $s$ should be greater than three, the range of leg duty factor, $\beta$, for implementation of the hybrid force control, is determined as

$$1 > \beta > 1/2 . \quad (7.30)$$
The fraction of the kinematic cycle required for active compliant motion during the support phase is defined as the **compliance duty factor**, $\gamma$. The fraction of a kinematic cycle for non-compliant supporting is defined as the **non-compliance duty factor**. The non-compliance duty factor becomes $\beta - \gamma$. Since the legs using non-compliant control are required to form a tripod, the non-compliance duty factor should satisfy the following equality:

$$6(\beta - \gamma) = 3$$  \hspace{1cm} (7.31)

or,

$$\gamma = \beta - 0.5$$  \hspace{1cm} (7.32)

Figure 7.11 shows the phase sequencing of leg 1. The leg is controlled with a non-compliant mode when the kinematic phase is within $\gamma$ and $\beta$. For all other durations, it is controlled with an active compliant mode. Phase sequences of other legs are exactly the same as leg 1, but with phase shift by their relative leg phases.

### 7.3.4 Relative Leg Phases

In order to sequence the non-compliant tripod set smoothly, as discussed above, it is necessary to adjust the relative leg phases. The current assignment of relative leg phases is based on the optimization of the gait longitudinal static stability margin [36]. For a given leg duty factor, $\beta$, the gait longitudinal stability margin is dependent on relative leg phases [36]. Bessonov and Umnov [38] have derived an assignment of the optimal relative leg phases in maximizing the gait longitudinal static stability margin for a hexapod vehicle. With the assumptions that the legs are evenly spaced in right-left pairs along a longitudinal motion axis and that each foot contacts the ground at a
Figure 7.11. Phase sequence diagram of leg 1 for the hybrid force control.
single point, the assignment is described with the following relative leg phases:

\[ \phi_1 = 0, \]
\[ \phi_3 = \beta_1 \text{ for } 1 > \beta_1 > 0.5, \]
\[ \phi_5 = 2\beta_1 - 1, \]

where the leg numberings are as shown in Figure 7.12. Legs on the left side of a hexapod are numbered as 1, 3, and 5 from front to rear, and those on the right side as 2, 4, and 6. The relative phases of any right-left pair are exactly half a cycle out of phase with each other. The assignment of (7.33) has been used in locomotion control of the OSU Hexapod. This optimally stable gait is known as a wave gait [39].

In the optimal assignment of the relative leg phases in (7.33), the differences in kinematic phases among the three legs in each side are not equal to each other, except in the case where duty factor \( \beta = 2/3 \) is used. Therefore, if the active compliance status of supporting legs is switched by the kinematic cycle shown in Figure 7.11, the set of non-compliant tripod legs changes more than one leg at a time or it becomes one-sided. Figure 7.13 shows the sequences of the support pattern for the duty factor \( \beta = 3/4 \) and the compliance duty factor \( \gamma = 1/4 \). In this figure, for example, the non-compliant tripod changes from \((2, 3, 5)\) to \((1, 3, 6)\) at \( \phi = 1/4 \). Figure 7.14 illustrates the case of \( \beta = 5/6 \) and \( \gamma = 1/3 \). In this case, the non-compliant tripod changes one leg at a time. However, it contains one-sided sets, \((1, 3, 5)\) and \((2, 4, 6)\).

In order to achieve a smooth change of non-compliant leg set, it is desirable that the leg placing events be distributed equally over a
Figure 7.12. Schematic top view of the vehicle, showing leg numbering and strides.
Figure 7.13. Sequence of the leg support pattern for the optimal relative leg phase for \( \phi = 3/4 \). The dotted lines represent the transfer phase, and the thicker solid lines are for the compliant supporting.
* Circled numbers represent legs for compliant support.

Figure 7.14. Sequence of the leg support pattern for the optimal relative leg phase for $\phi = 5/6$. The dotted lines represent the transfer phase, and the thicker solid lines are for compliant supporting. Non-compliant tripods change smoothly, but the tripod sets (1,3,5) and (2,4,6) are all on one side of the vehicle.
locomotion cycle. Thus the differences in kinematic phases among the three legs on each side should be always 1/3 and equal to each other. An assignment satisfying this condition can be described as follows:

\[
\begin{align*}
\phi_1 &= 0.0 \\
\phi_3 &= 2/3 \\
\phi_5 &= 1/3
\end{align*}
\]

(7.34)

Relative phases of right side legs always lag to those of their correspondents in the right-left pair by one-half cycle. With this provision, the event of leg placing cycles from that of a left-side leg to that of a right-side leg and back to that of a different left-side leg. This cycling is illustrated in Figure 7.15. This assignment does not depend on a specific leg duty factor \( \beta \), which is always constant. This gait is called an equal phase (EPH) gait [27].

It is necessary to show that the EPH gait sequences the non-compliant tripod set smoothly, and further that it does not become one-sided. By the symmetric characteristics of leg kinematic phase cycle, if the requirements are satisfied for the interval of any consecutive two events of leg placing, they can be satisfied for the entire kinematic cycle. Thus the proof will be given for the interval of \( 0 < \phi < 1/6 \).

As shown in Figure 7.15, the six events of leg placing are spaced evenly and alternatively for the left and right sides, on the kinematic phase cycle. Thus the six events of leg lifting should also be spaced evenly and alternatively. Accordingly, within the interval of \( 0 < \phi < 1/6 \), at the most only one event of leg lifting occurs, and, depending on the leg duty factor, the leg for the lifting event may
Figure 7.15. Event sequence of leg placement for an equal phase (EPH) gait. Events are equally distributed over a full kinematic cycle. The value $\phi_d$ is the phase of the leg lifting event occurring in $0 \leq \phi < 1/6$. 

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differ. Actually the lifting event of leg 2, leg 5, or leg 4 will occur with the interval for $3/6 < \phi < 4/6$, $4/6 < \phi < 5/6$, or $5/6 < \phi < 1$, respectively. In any case, the kinematic phase of the lifting event, denoted as $\phi_d$, can be computed as in the third column of Table 7.2.

By the observations of the previous paragraph, the supporting legs were listed in the leg order of 4, 5, 2, 3, 6, 1 (clockwise listing of leg placing events on the phase circle in Figure 7.15) in Table 7.2, separately for the interval of $0 < \phi < \phi_d$ and $\phi_d < \phi < 1/6$. For the first phase segment, the first number in the list is that of the leg whose lifting event occurs within $0 < \phi < 1/6$, while, for the second phase segment, it is the leg number which follows that of the first segment. In the both cases the first three numbers represent the non-compliant tripod legs and the others represent compliant legs.

By extending Table 7.2 for other kinematic intervals of two consecutive events of leg placing, it can be concluded that the leg set of the non-compliant tripod sequences through (4, 5, 2), (5, 2, 3), ..., (1, 4, 5) as the kinematic phase cycles. This sequence consists of three consecutive events of leg placement, thus changes only leg at a time. Further, none of the sets in the sequence are one-sided, since each set contains one leg which is from the opposite side of the other two.

Figure 7.16 shows the sequences of the support pattern for the duty factor $\beta = 3/4$. As shown on the figure, the non-compliant tripod changes one leg at a time smoothly. Figure 7.17 illustrates the case of $\beta = 5/6$.

Although the equal phase assignment of relative leg phases results in a smooth change in the set of non-tripod legs, the gait longitudinal static stability margin is reduced in comparison to the original one.
Table 7.2
Sequence of supporting legs in EPH gait

<table>
<thead>
<tr>
<th>leg duty factor $\beta$</th>
<th>compliant duty factor $\gamma = \beta - 0.5$</th>
<th>leg lifting phase $\phi_d$</th>
<th>list of supporting legs $0 &lt; \phi &lt; \phi_d$</th>
<th>$\phi_d &lt; \phi &lt; 1/6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/6 &lt; \beta &lt; 4/6$</td>
<td>$0 &lt; \gamma &lt; 1/6$</td>
<td>$\beta - 3/6$</td>
<td>2 3 6*1</td>
<td>3 6 1</td>
</tr>
<tr>
<td>$4/6 &lt; \beta &lt; 5/6$</td>
<td>$1/6 &lt; \gamma &lt; 2/6$</td>
<td>$\beta - 4/6$</td>
<td>5 2 3 6 1</td>
<td>2 3 6 1</td>
</tr>
<tr>
<td>$5/6 &lt; \beta &lt; 1$</td>
<td>$2/6 &lt; \gamma &lt; 3/6$</td>
<td>$\beta - 5/6$</td>
<td>4 5 2 3 6 1</td>
<td>5 2 3 6 1</td>
</tr>
<tr>
<td>$\beta = 4/6$</td>
<td>$\gamma = 1/6$</td>
<td>0</td>
<td>2 3 6 1</td>
<td></td>
</tr>
<tr>
<td>$\beta = 5/6$</td>
<td>$\gamma = 2/6$</td>
<td>0</td>
<td>5 2 3 6 1</td>
<td></td>
</tr>
</tbody>
</table>

*The sets of three legs underlined represent the non-compliant tripod, and the others represent compliant legs.*
Figure 7.16. Sequence of the leg support pattern for the equal phase gait with $\beta = 3/4$. Non-compliant tripods change smoothly, and none of them are all on one side of the vehicle.
Figure 7.17. Sequence of the leg support pattern for the equal phase gait with $\beta = 5/6$. Non-compliant tripods change smoothly, and none of them are all on one side of the vehicle.

* Circled numbers represent legs for compliant support.
described in (7.33). However, since the leg duty factor will be greater than 2/3 for hybrid force control, and thus at least four will be on the ground, the longitudinal static stability margin will not be a serious concern. Figure 7.18 shows plots of static stability margins of the optimal and the new relative phase assignment for a whole period of phase with various leg duty factors. In this plot, it has been assumed that the gait stride is constant and equal for all feet, and that the rear foot always touches the point from which its front foot lifts off in the vehicle's body coordinate system, as shown in Figure 7.12. Thus, this gait is an example of a follow-the-leader gait [13].

7.4 Experiments with Hybrid Force Control Allocated by Legs

All the experiments presented previously were performed with force feedback control for all the supporting legs. The operation of the vehicle was generally stable for these experiments. Thus it seems that the control system did not introduce any interactions in force among legs. Actually, there is no proper way to measure interactions in force; thus it is difficult or nearly impossible to prove whether hybrid force control actually reduces interactions. Thus, alternatively, the force responses of position-only-controlled legs are compared with their static force constraints, not controlled actually. If the actual responses generally follow their possible setpoints, then hybrid control is assumed to be valid.

Experiments were performed under combinations of different leg duty factors and different strategies of force setpoint. Duty factors considered were 2/3, 3/4, and 5/6; all typical statically indeterminate
Figure 7.18 Longitudinal static stability margins with their optimal and equal-phase relative leg phase assignments.
hexapod gaits. For each leg duty factor, force solutions of the shortest distance from both the origin and the current force measurement were used. The relative leg phases of equal-phase gaits are assigned.

Figure 7.19 shows the force responses with the force setpoint of the minimum-norm solution, while Figure 7.20 shows the force setpoint of the minimum-perturbation solution. For the time interval at the end of each support phase of a leg, corresponding to one half of the total period of one kinematic cycle, the leg is controlled only by position. For the rest of the time interval at the beginning of the support phase, the leg is controlled by both position and force. However, in either of the two force setpoint strategies, the actual force responses track their setpoints quite well for the entire period of support. Further, they are not more oscillatory than those of full force feedback control. Thus it can be concluded that the algorithm of hybrid force control by allocated legs works, and that, if the operational conditions are severe, this control strategy may be beneficial.

7.5 Conclusions

In order to reduce force interactions among supporting legs in the active compliance control of legged vehicles, an algorithm for distributing active compliance over some legs by using hybrid control philosophy was developed.

It has been illustrated by simulations that, when all the supporting legs are programmed for compliance motion in legged vehicle control, interactions in force among supporting legs can occur with discrete-time force feedback. This interaction may cause the legged vehicle to excessively roll or pitch.
Figure 7.19. Foot force tracking using hybrid force control by legs for leg 1. Force setpoint of the minimum-norm solution was used. Some portion at the beginning of each support phase was for active compliance, and the left of the support phase was for position-only control.
Figure 7.20. Foot force tracking using hybrid force control by legs for leg 1. Force setpoint of the minimum-perturbation solution was used.
It has also been shown by simulation that interactions in force could be avoided when some of the supporting legs are controlled by position only, and others are controlled by both position and force. This can be explained that the disabling the force feedback loops for some legs break the possible path of excitation of force interactions.

In applying the hybrid force control scheme in Hexapod control, several preparations were made: a selection method of non-compliant supporting legs, switching between compliant and non-compliant modes, timing of the mode switch, and an assignment of relative leg phases of equal-phase gaits. Three legs among supporting legs were always programmed to be non-compliant and the others were programmed to be compliant. Each leg begins supporting the vehicle with compliance control for some portion of its support phase, and then it supports the vehicle without compliance control for the rest of its support phase.

Another advantage of hybrid force control allocated by legs can be pointed out in addition to their function of reducing interactions in force. The legged vehicle can be suspended in a more stable manner by a non-compliant tripod. If the supporting legs adapt themselves to the uncertain terrain in the beginning of the support phase, it would not be necessary for them to do so in the remaining time of their support phases. Rather it is desirable for them to support the body stably.

The philosophy of the hybrid force control allocated by legs can also be applicable to the different multi-manipulator systems, such as hand systems with multiple fingers. Multi-fingered hand systems are intended to hold or grasp an object without dropping and breaking it, which requires force control for fine motion. Thus it is expected that interactions in force may occur among fingers.
The next chapter will summarize the contributions of this dissertation and it will also discuss some suggestions for future research extensions.
Chapter 8
SUMMARY AND CONCLUSIONS

8.1 Research Contributions

The objective of this dissertation was to study algorithms for implementing actively compliant motion, allowing legged vehicles to adapt to uneven terrains. Special emphasis was given to investigating system stability problems and understanding necessary considerations in force feedback control, and thus improving the existing force feedback control algorithm for actively compliant motion of legged vehicles.

The main contribution of this research has been the development of the algorithm of joint compliant motion. Compliances can be implemented either in Cartesian space (Cartesian compliance) or joint space (joint compliance). It has been shown that, when decomposed into individual actuator systems, Cartesian compliance can cause difficulties in resolving force errors into joint space. Depending on the postures of a manipulator, Cartesian compliance may require positive feedback of the joint torque error to a certain joint actuator. Thus, the stability of the control of a certain joint is mainly dependent on the control of other joints. This requires heavy interaction among individual actuator systems for overall system stability.

As a solution to the stability problem, it has been suggested that compliance be carried out in joint space instead of Cartesian space. It has been proved that, unlike Cartesian compliance, joint compliant
motion does not cause negative compliance in joint space. The algorithm for joint compliant motion was implemented on the OSU Hexapod and experimentally verified as giving better performance in system stability than Cartesian compliant motion.

In order to implement active compliance in the vehicle's locomotion, it is necessary to define the contact forces of the foot-tips against their environment so that the vehicle both maintains its balance on the supporting terrain and achieves the commanded motion. The force constraints usually result in an underdetermined system of equations allowing an infinite number of solutions. The minimum-norm solution based on the pseudoinverse technique has previously been considered the optimal choice [14].

In this dissertation, however, a different criterion was suggested for optimizing the force setpoint: choosing the force setpoint nearest on the solution plane to the current force measurement. With this force setpoint the system is minimally excited by force errors, while force constraints are still satisfied. It has been experimentally demonstrated that, using this technique, discontinuities of force setpoints during alternations of leg phases are smoothed, and the actual force measurements respond to the force setpoint quite closely.

The effect of discrete control on system stability was investigated. Special emphasis was placed on the system's mechanical resonance resulting from impulsive digital control, and the effect of passive compliance on system stability when encountering force feedback.

Usually it is understood that an accurate position control requires a stiff system. However a theoretical analysis and numerical simulation
suggested that, if the control includes force feedback and is time discretized, some passive compliance is necessary for system stability. A quantitative guideline for the amount of passive compliance was suggested. An experiment was performed on the OSU Hexapod, confirming the above conclusion.

One of the important contributions of this research has been the development of the algorithm of "hybrid force control allocated by legs". Hybrid control allocated by legs is achieved when some of the supporting legs are controlled by position only, and others are controlled by both position and force. This strategy reduces force interactions among supporting legs, and further, the vehicle is suspended in a more stable manner by non-compliant supporting legs.

When all the supporting legs are programmed for compliant motion, a force interaction among legs may take place with active and passive compliance, causing the vehicle to roll and pitch. A numerical simulation was presented to show a possible interaction in force among legs. A second simulation was presented to demonstrate that interactions in force can be reduced using hybrid control allocated by legs.

Regarding the implementation of the hybrid force control algorithm, three considerations were discussed. One is the dynamic selection of non-compliant supporting legs. The second is the timing of a mode switch between compliant and non-compliant supports, and the last an assignment of relative leg phases suitable for implementing hybrid control.
8.2 Research Extensions

8.2.1 Limitation on Effective Torque Gains in Implementing Joint Compliance

In explicit force feedback, in joint space, compliant motion occurs about each joint, and considering the linkage kinematic structure, is achieved more smoothly than rectilinear compliant motion. It has been shown that the stiffness matrix in the body-fixed coordinate system is positive definite; thus its eigenvalues are positive. Therefore compliance in Cartesian space is always positive.

However, the stiffness matrix of joint compliant motion is not diagonal in the body-fixed coordinate system, and may not be so in any Cartesian coordinate system. Therefore the behavior of joint compliant motion cannot be visualized easily. Furthermore, equivalent vertical compliance is non-uniform and is dependent on the postures of the leg system. This situation is caused by variation of the moment arms which resolve end effector force measurement into joint torques.

If the moment arm of a given joint varies much in the working boundary of a foot-tip, the control of the joint may cause instability due to the large torque gain and the stiffness of environment [87]. A joint whose moment arm is large (i.e., the elevation joint of the OSU Hexapod in the stretched-leg posture), experiences feedback of an excessively large torque due to the increased moment arm. Thus if the torque gain is not so small, a stability problem may arise, especially with digital control. Actually it was observed in experiments performed on the OSU Hexapod that a leg system oscillated locally when it was in stretch-out postures.
To solve this problem it is suggested to put a limit on the effective feedback control gain for torque errors. With this strategy, instability due to high torque gain and feedback saturation may be avoided. Further, the degree of non-uniformity of the effective Cartesian compliance in implementing joint compliance may be reduced. Suggestions for future work are the investigation of the feasibility of limiting torque errors to avoid instability and achieve almost-uniform compliance, and determination of the appropriate limit.

8.2.2 Solution Method for Physically Realizable Force Setpoint

Recently, a fully self-contained, automatically terrain adaptive walking vehicle was constructed at The Ohio State University. It is a full scale, hydraulically powered hexapod called the Adaptive Suspension Vehicle (ASV) [25,28]. One of the proposed control algorithms for the ASV controls the vehicle's acceleration and deceleration according to both the operator's velocity command and a terrain preview provided by the terrain scanner [12]; thus foot coordination in force is necessary for acceleration and deceleration of the body.

When the force and moment of the vehicle body are specified, six constraint equations are formed for the required forces at the supporting foot-tips: three scalar equations for force components along three body axes, and three for moment components about these axes. Usually the system of equations is underspecified, allowing the existence of an infinite number of solutions. However, not all are physically realizable. Solutions are subject to the constraints that the vertical forces should be non-negative and, in order for the supporting foot not to slip on the ground, the ratio of the horizontal
force to the corresponding vertical force should be less than the friction coefficient of the supporting ground.

A mathematical formalization of the problem of physically realizable force setpoints is to find an optimal $x$ for the linear system of equations of

$$Ax = y,$$  \hspace{1cm} (8.1)

subject to scalar constraint inequalities of

$$g_i > 0, \quad i = 1 \ldots k.$$  \hspace{1cm} (8.2)

Two methods may be considered in finding an optimal $x$ for (8.1) and inequalities (8.2): one is a linear programming technique introducing slack and surplus variables for constraints [91]; the other is finding a homogeneous solution directing a common direction for all the gradients of constraint scalar functions needing improvement and adding it to the pseudoinverse solution [92].

The first technique is not realistic for the force set problem since the inequalities of friction-cone constraint are not linear. The friction cone may be approximated to a friction pyramid, introducing four linear inequalities for each friction cone [93,94]. However, the linear programming technique is still not realistic for the problem, since augmentation of the system of equations increases its dimension greatly. Thus, further research on the second strategy, which is called a gradient technique [92], is suggested as a research extension.
8.2.3 Optimal Choice of Passive Compliance

Research extensions are also required in determining the most desirable amount of passive compliance. It has been shown, analytically, that some passive compliance is necessary to stabilize the control system implementing force control. However, an accurate position control requires a stiffer system. Thus a criterion for optimizing the amount of passive compliance is necessary.

Another consideration is whether to lump passive compliance at a certain point in the manipulator system or distribute it all over the kinematic structure. It may also be necessary to investigate how system performance will be affected using different locations of passive compliance.

8.2.4 Refinement on the Strategy of Hybrid Force Control Allocated by Legs

More work is necessary to generate position commands when a leg system is switched from position and force control to position-only control. During compliant motion, position errors are compensated with force errors, resulting possibly in errorless feedback control. However, if the force control loop is disabled suddenly, the position errors will cause an impact on the control system. Thus active modification of position command is necessary to achieve a smooth change.

One of the particularly interesting future applications of hybrid force control allocated by legs would be in the field of multi-fingered hand systems. Multi-fingered hand systems are intended to hold or grasp an object without dropping and breaking it, which requires force control!
for fine motion. It is expected that interactions in force may occur among fingers. Thus it is suggested that, by controlling some fingers by position and force and others by position only, the concept of hybrid control developed in this dissertation could be applied.
REFERENCES


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