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Brown, Roger Keith

TRIANGULAR PROXIMITY-COUPLED ARRAYS: PHASE TRANSITION IN A MAGNETIC FIELD AND DYNAMICAL PROPERTIES

The Ohio State University

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Ph.D. 1985
TRIANGULAR PROXIMITY-COUPLED ARRAYS:
PHASE TRANSITION IN A MAGNETIC FIELD
AND DYNAMICAL PROPERTIES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of the The Ohio State University

By
Roger Keith Brown, B.A., B.S.

* * * * *

The Ohio State University
1985
To My Parents
ACKNOWLEDGEMENTS

First I would like to thank my advisor, Professor James C. Garland, whose guidance, encouragement, and patience (lots of it was needed.) have made this work possible. Secondly I want to thank Dr. Douglas J. Resnick, who, in the early dark days of my ignorance, provided daily guidance in the lab.

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And last (but not least) I want to thank my daily co-workers, Hu Jong Lee, Don Harris, Joe Calabrese, Lou Nanna, Brent Warner, and J.P. Pritchard, whose friendship and good humor have made my years at Ohio State among the most enjoyable of my life.
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PHASE TRANSITION IN A MAGNETIC FIELD
I. INTRODUCTION

Large two-dimensional arrays of Josephson junctions form a new class of superconductors whose critical properties differ markedly from those of either bulk or thin film superconductors. The arrays consist of superconducting grains or islands arranged on the sites of a regular lattice and separated from one another by a non-superconducting material, either an insulator or a normal metal. A number of experiments$^{1-4}$ have shown that the resistance of such arrays at low temperatures goes to zero in a broad transition ($\Delta T \sim 1K-1K$) quite unlike that of bulk superconductors but similar to that of high sheet resistance superconducting thin films. In the case of zero external magnetic field this similarity with superconducting thin films has been exploited to suggest explicit expressions for the temperature dependence of the electrical transport properties near the transition temperature $T_c^5,6$. In both systems the resistive properties are thought to be governed by the unbinding of thermally excited supercurrent vortex anti-vortex pairs according to the ideas of Kosterlitz and Thouless.$^7$

The variation of array properties with magnetic field is however quite different than that of high sheet
resistance films. A number of workers have reported that the resistance of square arrays of Josephson junctions above $T_c$ varies periodically in a perpendicular magnetic field with resistance minima occurring at field values corresponding to integer multiples of the magnetic flux quantum, $\Phi_0 = hc/2e$, threading each cell of the array. They also report the observation of smaller secondary minima at half integer $\Phi_0$. It is believed that there are $T_c$ oscillations corresponding to these resistance oscillations although a direct measurement of $T_c(H)$ has not been reported for Josephson junction arrays. However in a related system of square arrays of superconducting wires Pannetier et al have measured transition temperature oscillations directly. They observe $T_c$ maxima at integer $\Phi_0$ and secondary maxima at a number of rational fractions of a flux quantum $f = \Phi/\Phi_0$ including $f = 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4$.

These experimental findings have stimulated considerable theoretical effort to understand the properties of two-dimensional superconducting networks. Several workers have dealt specifically with the case of Josephson junction arrays. In their models they have assumed an energetic coupling between adjacent superconducting grains of the form $E_j = -J\cos(\Phi_j - \Phi_i - (2e/hc) \int A \cdot d\mathbf{l})$ where $\Phi_i$ and $\Phi_j$ are the phases of the Ginzburg-Landau superconducting order parameter of the $i$'th and $j$'th grains respectively, $\mathbf{A}$ is the vector potential due to the applied magnetic field, $H$, and the
integral is taken from the center of the i'th grain to the center of the j'th grain. The argument of the cosine is called the gauge invariant phase difference across the junction.\textsuperscript{16} The ground state properties of this model can be calculated exactly. Teitel and Jayaprakash\textsuperscript{11} have presented general exact arguments that predict the qualitative behavior of the transition temperature $T_c$ in a square lattice from a knowledge of the ground state properties. Furthermore $T_c$ has been calculated explicitly using both Monte Carlo simulations\textsuperscript{11,12,14,15} (MC) and mean field theory\textsuperscript{13,15} (MFT). These calculations predict a variation of $T_c$ with magnetic field that is in qualitative agreement with experiment. It should be noted that the predictions of the theory are lattice dependent while the experiments have been performed only on square lattices of Josephson junctions.

While it is well accepted that the transition in zero magnetic field is a Kosterlitz-Thouless (KT) transition the nature of the phase transition at arbitrary $f$ has not been established. There is some evidence from Monte Carlo simulations of the specific heat\textsuperscript{11,14} that the transition at a few values of $f$ is not a KT transition although the universality class has not been clearly established. Unfortunately the specific heat due only to the Josephson coupling degrees of freedom is immeasurable small. The only experimentally accessible signatures of the phase transition are the electrical transport properties for which there
exist no explicit predictions at arbitrary $f$.

In the first part of this thesis I report the results of an experimental study of the superconducting phase transition in triangular arrays of $10^6$ proximity-coupled Josephson junctions. Samples consisting of thin film superconducting islands arranged on the sites of a triangular lattice and covered with an overlay of a normal metal were produced using photolithographic techniques. The detailed temperature dependence of the electrical transport properties in the vicinity of the superconducting transition temperature $T_c$ have been measured in zero magnetic field and at various values of the parameter $f = \Phi / \Phi_0$. The dependence of these properties on $f$ is compared with theoretical predictions about the phase transition of Josephson junction arrays.
II. THEORY REVIEW

1. PHASE TRANSITION IN A MAGNETIC FIELD

Josephson junction arrays have been modeled in the following way. The individual superconducting grains or islands are characterized by a traditional complex Ginzburg-Landau order parameter \( \Psi = |\Psi| e^{i \phi} \) where \( |\Psi|^2 = n_s \) is the volume density of superconducting charge carriers. The coupling between the grains is then described by the following Hamiltonian:

\[
H = -J \sum_{\langle ij \rangle} \cos(\phi_j - \phi_i - A_{ij}).
\]

Where \( \phi_i \) and \( \phi_j \) are the phases of the \( i' \)th and \( j' \)th superconducting grains respectively; \( A_{ij} = (2e/\hbar c) \int_1^j \vec{A} \cdot d\vec{l} \) is the integral of the vector potential \( \vec{A} \) (times a constant) from the center of the \( i' \)th grain to the center of the \( j' \)th grain; and \( \langle ij \rangle \) denotes a sum over nearest neighbor pairs. The coupling constant \( J = (\hbar/2e)I_c^0 \) where \( I_c^0 \) is the Josephson critical current appearing in the dc Josephson relation

\[
i_s = i_c^0 \sin(\phi_j - \phi_i - A_{ij}).
\]
The quantity $\phi_j - \phi_i - A_{ij}$ is called the gauge invariant phase difference across the junction.\textsuperscript{16}

These models assume that the superconducting grains are small compared to the magnetic penetration depth $\lambda$ so that the phase $\phi_i$ and the vector potential $A_i$ do not vary over the diameter of the grain. In fact real proximity-coupled arrays typically have superconducting islands that are 10$\mu$m in diameter compared to a zero temperature penetration depth of tens of nanometers, and are thus in the opposite limit. However, as shown in appendix A the Hamiltonian (1.1) is still applicable if we take $\phi_i$ and $\phi_j$ to be the phases at the centers of the $i$'th and $j$'th islands respectively.

These models make the additional assumption that the array is in the weak-coupling limit in which the fields produced by screening currents are negligible compared to the applied field so that one can take the vector potential $A$ to be that due to a uniform external field $H$. These theoretical treatments do not specify under what conditions this limit is achieved. It seems reasonable, however, that the appropriate limit is $LIc^0 \ll \phi_0$ where $L$ is the geometrical inductance of a loop comprising a single cell of the array and $Ic^0$ is the Josephson critical current of a single junction as defined above. Since $LIc^0$ is the maximum flux that can be screened out of a single cell without exceeding the critical currents of the junctions in the loop, the limit given above insures that the flux threading a single cell is
The models treat the set of phases \( \{\phi_i\} \) as classical microscopic variables and apply classical statistical mechanics to study the Hamiltonian written down above. This model can be characterized in terms of the parameter \( f = \phi / \phi_0 \) where \( \phi \) is the flux threading each cell of the array (note that in the weak coupling limit the field penetrates uniformly.) and \( \phi_0 = \hbar c / 2e \) is the superconducting magnetic flux quantum. It can easily be shown that the Hamiltonian (1) is periodic in \( f \) with period 1 and is symmetric about \( f = 1/2 \). Thus it is sufficient to consider the behavior of the model on the interval \( f \in [0, 1/2] \).

Teitel and Jayaprakash\(^1\) (TJ) call this parameter the 'uniform frustration'. This terminology stems from the fact that in the \( f = 0 \) case the model is isomorphic to an X-Y model in which the variables \( \{\phi_i\} \) represent the angles that a set of classical spins constrained to lie in a plane make with respect to some fixed axis. The ground state of this hamiltonian is clearly the state with all the phase differences (or spin directions) \( \phi_j - \phi_i = 0 \pmod{2\pi} \). For \( f = 0 \) the obvious way to minimize the energy is to have all the gauge invariant phase differences \( \phi_j - \phi_i - A_{ij} = 0 \pmod{2\pi} \). Now, however, the ordinary phase difference \( \phi_j - \phi_i = 0 \). Since there is an additional constraint that \( \phi \) must change by \( 2\pi \) in going around any closed loop (this fact follows from single valuedness of the order parameter.) it is in general not possible to minimize simultaneously the energy of every...
possible to minimize simultaneously the energy of every bond. In terms of the X-Y model one would say that an individual spin cannot simultaneously minimize its interaction with each of its neighbors. In magnetism this phenomenon is called 'frustration' and hence the terminology of Teitel and Jayaprakash. Note that if some of the gauge invariant phase differences are not zero then the dc Josephson relation \( (1.2) \) implies that supercurrents will be flowing through some of the bonds.

For the case \( f=0 \) it is easy to see that the ground state energy per bond is \( E_g = -J \). For \( f \neq 0 \) the effect of frustration is to raise the ground state energy. That is \( E_g \) satisfies the relation

\[
0 \leq E_g (f \neq 0) \geq -J. \tag{1.3}
\]

The ground state energy, though negative, is smaller in magnitude than at \( f=0 \). Roughly speaking one would expect the coherence of the superconducting ground state of the array to be destroyed when the fluctuation in the coupling energy between adjacent islands is equal to the ground state energy per bond. Thus from the equipartition theorem one expects

\[
\ln T_c \sim |E_g(f)|. \tag{1.4}
\]

Combining (1.3) and (1.4) we see that the effect of a magnetic field is to depress the transition temperature from its \( f=0 \) value. Since the model is periodic in \( f \) with period...
1 the transition temperature returns to its unfrustrated value at integer $f$.

It was noted above that the model is symmetric about $f=1/2$. In the introduction it was noted that data on the magnetic field dependence of real arrays indicates that $T_c$ does not vary monotonically on the interval $f \in [0, 1/2]$. In particular, while $T_c$ is generally decreasing on this interval there is a sharp upward feature at $f=1/2$. If these models are reasonable representations of real arrays we expect them to reproduce this structure. They do so successfully and in fact they predict a far richer structure than is seen experimentally. These models which assume an ideal, perfectly homogeneous array, predict an infinite self-similar structure with features at all rational $f^{11-15}$. Presumably the amount of structure observed in a real array is limited by lattice inhomogeneities and inhomogeneities in the junction coupling strengths.

In order to understand the special role played by rational $f$ in this problem one must examine the structure of the ground state for $f=0$ in more detail. To do so it is convenient to introduce the concept of the vorticity of a unit cell. Single valuedness of the order parameter requires that the phase change around any closed path in the array must be an integer multiple of $2\pi$. In particular the phase change around a unit cell must be $\Delta \phi = 2\pi n$ where $n$ is an integer. The vorticity of a unit cell is then defined by $n$. When
a magnetic field is applied perpendicular to the plane of the array a phase change is induced around each cell in accordance with the second Ginzburg Landau equation

$$\tilde{J}_s = (e/m)|\Psi|^2[\tilde{\Phi}-(2e/c)\tilde{A}]$$ (1.5)

where in the weak coupling limit $\tilde{A}$ is just the vector potential due to the applied uniform field. If one considers a path deep within the superconducting islands then $\tilde{J}_s=0$ and the phase gradient is directly proportional to $\tilde{A}$. The phase drops across the junctions around a unit cell must then adjust themselves so that the total phase change around a the cell is an integer multiple of $2\pi$. For $f=0$ all the phases are aligned in the ground state and $n=0$ for every cell of the array. For $f\neq 0$ it is no longer true that $n=0$ for every cell of the array.

TJ11,12 have argued that the Hamiltonian (1.1) maps onto a Coulomb gas problem on a dual lattice with charges $q_n = n-f$. The neutral ground state thus consists of a lattice of charges $-f$ and $1-f$. That is a fraction of the cells $f$ have vorticity 1 and a fraction $1-f$ have vorticity 0. The ground state configuration of charges or vortices can be determined using Monte Carlo simulations. Once the vortex configuration is known the ground state energy per junction can be computed exactly from the Hamiltonian (1.1). It is found that for rational $f=p/q$ where $p$ and $q$ are coprime integers the array repeats itself with a unit of $nqxmp$ where
n and m are integers. For irrational f no such simple repeatable unit can be found, and it is not clear how to calculate the ground state energy.

Figure 1 shows the ground state configuration of vortices in a square lattice for several rational f as computed by TJ. The pluses denote cells with unit vorticity and the blank cells have zero vorticity. Note that for a square lattice the integers n and m are always equal to one. In a triangular lattice n and m are sometimes different from 1.

Figure 2 shows the ground state configuration of vortices in a triangular lattice for f=1/2 and 1/3 as computed by Shih and Stroud. In this figure the cross-hatched cells have unit vorticity and the blank cell zero vorticity. The arrow represents the direction of current flow. These illustrations make it clear that the ground state energy is changing in some complex fashion as a function of magnetic field. What is not clear from looking at these diagrams is whether or not the change in ground state energy is smooth and monotonic.

Figure 3 shows the absolute value of ground state energy per island, |E_g(f)| as a function of f for both square and triangular lattices as calculated from the vortex configurations. The lines connecting the points are merely a guide to the eye. The variation of E_g(f) is clearly not monotonic on [0,1/2]. Whether or not the curve is smooth cannot be determined from a finite number of points but it
FIGURE 1. The ground state configuration of vortices in a square lattice at several rational $f$. The pluses represent cells with unit vorticity while the empty cells have zero vorticity.
FIGURE 2. The ground state configuration of vortices in a triangular lattice of Josephson junctions at $f=1/2$ and $1/3$. The crosshatched cells have unit vorticity and the blank cells have zero vorticity. The arrows represent the direction of current flow.
FIGURE 3. The absolute value of the ground state energy per grain, $|E_g(f)|$, as a function of $f$ for triangular and square lattices.
is believed that the ground state energy has structure at all rational \( f \) so that if one were to blow up the scale between any two of the points shown one would see additional features of the same kind seen at this larger scale.

The curves for the two different lattices look very much the same. Both have a large feature at \( f=1/2 \). All other features are relatively small. In the square lattice the most notable secondary feature is at \( f=1/3 \) while in the triangular lattice the most notable feature is at \( f=1/4 \).

The same authors have also calculated the \( T=0 \) critical current as a function \( f \).\(^{12,14} \) In order to put an array of \( N \times N \) grains into a metastable state carrying a net supercurrent one imposes a twisted boundary condition on the phase \( \phi(x=N)-\phi(x=0)=N_\phi \) where \( x \) is the coordinate of a lattice site along the direction of current flow in units of the lattice constant. This boundary condition insures that as one goes from one end of the lattice to the other the phase changes by an average amount \( \delta \) between adjacent rows. The boundary condition can be explicitly incorporated into the Hamiltonian as follows:

\[
H_s = -J \sum_{\langle ij \rangle} \cos(\phi_j - \phi_i - \delta e_{ij} \cdot t) \quad (1.6)
\]

where \( e_{ij} \) is the unit vector pointing along the bond between the \( i \)'th and \( j \)'th island and \( t \) is the unit vector in the direction of the twist. One can calculate the set of phases
corresponding to the minimum of the twisted Hamiltonian, \( \{ \Phi_i^0 \} \), in the same way that \( \{ \Phi_i^0 \} = 0 \) was calculated.

From the dc Josephson relation one knows that the supercurrent passing between the i'th and j'th islands is given by

\[
is = (2e/h) J \sin(\Phi_j^0 - \Phi_i^0 - A_{ij} - \delta e_i \cdot \mathbf{t})
\]

By averaging over all bonds one can calculate the net current per lattice constant flowing through the array. One can slowly vary \( \delta \) and when the current per bond reaches a maximum value that current is defined as the critical current.

Figure 5 shows a plot of the calculated critical current per lattice constant, \( \text{ic}(f) \), in units of the single junction critical current \( \text{ic}^0 \) for both square and triangular lattices. We can see that the structure in \( \text{ic} \) is much more dramatic than that in \( \text{Eg} \). Note that for a triangular lattice the critical current at \( f = 1/2 \) is anisotropic; it is larger for current flowing parallel to a lattice direction than for current flowing perpendicular to a lattice direction.

TJ\textsuperscript{1,2} have generalized this behavior to arbitrary \( f = p/q \) for a square array by deriving the following bound on \( \text{ic}(f) \):

\[
\text{ic}(p/q) \leq |e| \text{Eg}(f) |/h | \pi /q.
\]

The presence of a factor of \( |\text{Eg}(f)| \) by itself shows that \( \text{ic} \) will vary nonmonotonically on \([0, 1/2]\), but it is the factor of \( 1/q \) which gives rise to really dramatic variations in
FIGURE 4. Calculated values of the zero temperature critical current per unit length as a function of $f$ for both square and triangular lattices. Also plotted are values of the transition temperature as calculated by Monte Carlo simulations at a few values of $f$. Note that the proposed relation 1.12 between $i_c(f)$ and $T_c$ is approximately obeyed.
ic. In fact the factor of $1/q$ suggests that $ic = 0$ for irrational $f$ since an irrational number can be thought of as the limit of a sequence of rational numbers in which the denominator and numerator become larger and larger integers as the sequence is continued.

One can relate the transition temperature to the $T=0$ critical currents by an equipartition argument. One expects that a transition to the normal state will occur if the fluctuations in the current through a single junction are equal to $ic(T=0)$. Combining this observation with the single junction relation $J = (\hbar/2e)ic$ and the equipartition theorem suggests the following relation for $T_c$:

$$k_B T_c(f) \approx (\hbar/2e)ic(f) \leq [e|E_g(f)|/\hbar]^{1/2}/q.$$  \hspace{1cm} (1.12)

Thus $T_c(f)$ exhibits the same kind of dramatic discontinuous behavior as $ic(f)$. Comparing the estimate of $T_c$ (1.12) with rough estimate (1.4) made earlier we see that the main difference in the two results is the factor of $1/q$. This factor of $1/q$ appearing in the critical current and consequently in the transition temperature is surprising and its physical origin is not intuitively clear.

$T_c$ has been calculated explicitly at a few values of $f$ using Monte Carlo simulations$^{12,14}$. In these simulations a quantity called the helicity modulus, which is a measure of the phase correlations in the system, is calculated as a function of temperature. When the helicity modulus goes to
zero so do the phase correlations and a resistive transition is expected to occur. Figure 4 shows in addition to the critical current $i_c(f)$ the calculated values of $T_c(f)$ in units of $(\hbar/2e\kappa) i_c^0$. We see that the relation (1.12) is approximately obeyed. The Monte Carlo values of $T_c$ are also plotted separately in Figure 5.

$T_c$ has been calculated at such a small number of values of $f$ because the Monte Carlo simulations are very time consuming and expensive. Shih and Stroud\textsuperscript{13-15} (SS) have developed a much simpler mean field theory (MFT) which allows the calculation of $T_c$ at a large number of $f$ values. MFT can not give correct quantitative results in these two-dimensional systems but it has the advantage of allowing one to examine the qualitative dependence of $T_c$ on $f$ in much greater detail than more exact methods.

This mean field theory is constructed in the following way. In the canonical ensemble the expectation value of an operator is given by:

$$
\langle 0 \rangle = \frac{1}{Z} \int_{0}^{2\pi} ... \int_{0}^{2\pi} \prod_{i=1}^{N} \phi_i \, 0 \exp(-\beta H) 
$$

where $\beta = 1/k_B T$ and $Z$ is the partition function given by:

$$
Z = \int_{0}^{2\pi} ... \int_{0}^{2\pi} \prod_{i=1}^{N} \phi_i \, \exp(-\beta H).
$$

A set of "phase order parameters", $\{\eta_i\}$, are defined by:
FIGURE 5. The transition temperature of square and triangular arrays as calculated by Monte Carlo simulations.
The Hamiltonian (1) is now rewritten as follows:

\[ H = -J \sum <ij> \{ \cos(\phi_i) [\cos(\phi_j) + A_{ij}] + \sin(\phi_i) [\sin(\phi_j) + A_{ij}] \} \]  

(1.17)

The mean field approximation consists of replacing \( H \) by the following effective Hamiltonian:

\[ H_{\text{eff}} = -J \sum <ij> \{ \cos(\phi_i) <\cos(\phi_j) + A_{ij}> + \sin(\phi_i) <\sin(\phi_j) + A_{ij}> \} \]  

(1.18)

The expectation values are determined self consistently from the set of coupled equations (1.16). The mean field transition temperature is defined as the highest temperature for which the coupled equations (1.16) have a non-trivial (i.e. non-zero) solution. Thus in the MFT approximation the transition is a long range phase ordering transition.

Figure 6 shows the MFT transition temperature, \( T_{c}^{MFT} \), as a function of \( f \) for square and triangular lattices as calculated by SS\textsuperscript{13,15}. Note that \( T_{c}^{MFT} \) is plotted in units of J/\( k_B \). As with the ground state energy the lines connecting the points are merely a guide to the eye. Note the lattice dependence of \( T_{c}^{MFT} \). Both square and triangular lattices have their largest feature at \( f=1/2 \). The triangular lattice has a large feature at \( f=1/4 \) almost equal in magnitude to the 1/2
FIGURE 6. The mean field transition temperature as a function of $f$ for triangular and square lattices.
feature. In a square lattice this large feature at $f=1/4$ is absent though there are various other secondary features the most notable occurring at $f=1/3$.

The MFT values of the transition temperature are considerably higher than the corresponding Monte Carlo values—a factor of 2 to 5 times larger. This difference in $T_c$ values is not surprising since MFT does not take account of fluctuations which are very important in determining $T_c$ in low dimensional systems. In this connection recall the comment made earlier that the mean field transition is a long range phase ordering transition. For the case $H=0$ in which the Hamiltonian of the array corresponds to the Hamiltonian of a two-dimensional X-Y model it is well known that long range ordering does not occur at finite temperatures. Thus it is clear that MFT cannot give correct quantitative values of $T_c$.

These calculations of $T_c$ do not make clear the nature of the transition from the $T=0$ superconducting ground state to a resistive state at finite temperature. Monte Carlo simulations of the specific heat at a few values of $f$ have shown evidence that in a few cases the transition is no longer a Kosterlitz Thouless transition. It is found that for $f=0$ as one increases the size of the system being modeled the specific heat has a small peak the magnitude of which eventually saturates and does not change as the size of the array is increased. On the other hand for $f=1/2$ in a
square lattice and for \( f = 1/2 \) and \( 3/8 \) in a triangular lattice the specific heat has a peak at \( T_c \) which continues to increase in magnitude as the size of the system increases suggesting a divergence in the specific heat as one goes to the thermodynamic limit. Such a divergence in the specific heat is characteristic of a more traditional second order phase transition rather than a KT transition. Unfortunately the specific heat due to the Josephson coupling degrees of freedom only is immeasurably small since there are only \( 10^6 \) junctions compared to \( 10^{17} \) atoms in the sample. The only experimentally accessible signatures of the phase transition are the electrical transport properties of the array in the vicinity of \( T_c \). No quantitative predictions exist for these properties except for the \( f = 0 \) case which will be discussed in the following section.

2. TRANSPORT PROPERTIES IN ZERO MAGNETIC FIELD

As has already been mentioned the Hamiltonian (1.1) in zero magnetic field is isomorphous to a two-dimensional XY model. It is well established that this model undergoes a special kind of phase transition called a Kosterlitz-Thouless vortex unbinding transition. There are a number of systems believed to undergo such a transition. The key ingredient in all such systems is the existence of pairs
of excitations of opposite polarities whose total energy goes as the logarithm of the separation $r$. Simple thermodynamic arguments can be used to show that there is a temperature $T_c$ at which the expectation value of the pair separation goes to infinity. Above this temperature there are "free" excitations of both polarities which change the response of the system to an external perturbation. In the XY model the excitations are vortices and antivortices. In the case of magnetic systems the change in the response of the system is in the susceptibility. For superconducting systems the change is in the resistance via the mechanism of flux flow resistance.

Explicit calculations of the resistive properties of Josephson junction arrays in the vicinity of the KT transition have not been performed. However in the related system of high sheet resistance superconducting films, another system believed to undergo a KT transition, a number of efforts have been made to calculate the temperature dependence of the transport properties. The most widely accepted of these calculations is that of Halperin and Nelson. They have derived expressions for both the resistance above $T_c$ and the nonlinear IV characteristics below $T_c$. Their expressions are calculated in the limit $\tau << T_c << 1$ where

$$\tau = (T - T_c) / T_c$$ (1.19)
and

\[ T_c = \frac{(T_c^0 - T_c)}{T_c}. \] (1.20)

\( T_c^0 \) is the BCS transition temperature of the film. For \( T > T_c \) they find an exponentially activated resistance of the form:

\[ R(T) = AR_n \exp[-B(\tau_0 / \tau)^{1/2}]. \] (1.21)

where \( R_n \) is the normal state resistance of the film and \( A \) and \( B \) are constants of order unity. HN find a nonlinear contribution to the resistance below \( T_c \) due to current induced unbinding of vortex anti-vortex pairs. This non-linear resistance is of the form

\[ R = \frac{V}{I \sim I^a(T)} \] (1.22)

with \( a(T) \) given by

\[ a(T) = 2 + \pi B'(\tau / \tau_0)^{1/2} \] (1.23)

where \( B' \) is another constant of order unity. Alternatively one can write

\[ V \sim I^a(T) \] (1.24)

where \( a(T) = 1 + a(T) \). Thus one expects the IV characteristics to have a power law form.

The validity of these expressions, derived in the context of Ginzburg Landau theory over a very narrow temperature range, when applied to Josephson junction arrays is
doubtful. In fact Lobb et al showed that a simple minded application of them is clearly incorrect for the following reason. We recall that $T_c \sim J$ where $J$ is the temperature dependent Josephson coupling constant. As the temperature is increased above $T_c$ the coupling constant is reduced from its value at $T_c$. That is $J(T>T_c) < J(T_c)$. Therefore the reduced temperature $\tau = (T-T_c)/T_c$ should be replaced by $(T-T_{c_{\text{eff}}})/T_{c_{\text{eff}}}$ where $T_{c_{\text{eff}}}$ is the temperature at which the transition would occur if the $J$ were temperature independent and had the value $J(T)$. As is shown in appendix B this expression for the reduced temperature can be rewritten as

$$\tau' = (T'-T_c)/T_c$$

(1.25)

where

$$T' = [J(T_c)/J(T)]T = [\text{ie}^0(T_c)/\text{ie}^0(T)]T.$$  

(1.26)

Actually the most interesting prediction of the HN theory concerns the qualitative behavior of the of the resistivity at $T=T_c$. From equations (1.23) and (1.24) we see that at $T=T_c$ the voltage should vary as the third power of the current. Just above $T_c$ the IV characteristics should exhibit linear ohmic behavior. Thus one expects a jump in the IV power law exponent $a(T)$ from 1 to 3 at $T=T_c$. This jump is believed to be universal and therefore may apply to Josephson junction arrays even though the detailed
temperature dependence of the HN theory may not. It should be noted here that Abraham et. al. have reported evidence of such a universal jump in square arrays of proximity-coupled junctions.
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III. EXPERIMENTAL DETAILS

1. SAMPLE DESCRIPTION

The samples consisted of triangular arrays of $10^6$ Pb islands on an oxidized silicon substrate covered with a thin overlay of Sn. Two different island shapes are used: a circle and a six-armed figure or asterisk. The typical thickness of the Pb islands is 1500Å and the typical thickness of the Sn overlay is 2000Å. For the samples with circular islands the lattice constant was 16µm and the spacing between islands at the point of closest approach was 2µm. For the samples with asterisk shaped islands the lattice constant was 10µm and the spacing between the edges of adjacent islands is .75µm. Figure 7 shows schematic top and side views for both types of samples.

The fabrication of the arrays of circular Pb islands using photolithographic techniques has been described in detail by Resnick\textsuperscript{20}. The fabrication of the arrays with asterisk shaped islands is almost identical except for making the optical mask. Instead of using photographic reduction, electron beam lithography was used to make the mask. One begins with a 4" by 4" glass slide covered on one
FIGURE 7. Schematic top and side views of both types of proximity-coupled arrays.
side with an 800Å layer of chromium. An organic electron beam resist (PMMA) is spun onto the chromium covered side of the slide. Controlling the speed of the spinner controls the thickness of the resist layer, and in this case a layer slightly less than 1μm thick was produced. It is typical of lithographic procedures that the thickness of the resist layer is the same size as the smallest feature of the pattern to be produced. $10^6$ asterisk shapes arranged on a triangular lattice are then exposed using a computer controlled scanning electron microscope. The exposed slide is placed in a developing solution consisting of equal parts of methylisobutyl ketone and isopropyl alcohol. The developer dissolves the exposed resist leaving the chromium underneath uncovered. This chromium is then dissolved with ceric acid. Afterward the unexposed resist is dissolved in methylene chloride. The result is a metalized glass slide with holes in the metal layer in the form of the desired lattice of islands. The pattern is 1cm² and contains $10^6$ islands.

The mask is then placed in contact with a three inch oxidized silicon wafer covered with a 1μm layer of AZ 4110 photoresist. The mechanical contact is brought about using a Karl Suss contact printer. The photoresist is exposed using an ultraviolet light source and the pattern is afterward developed in a 4:1 solution of AZ400K photoresist developer and deionized H₂O. The result is a three inch silicon wafer covered with a 1μm layer of photoresist except for a 1cm²
area in which there are holes in the form of the desired array. The metallization steps which follow are identical to those used in the production of the disk array as described by Resnick²⁰. For the disk arrays it was found possible to vary the disk diameter (and consequently the spacing between the edges of adjacent disks) by varying the exposure time. Spacings were produced in a range from 3.5μm to 1.3μm. In the electron beam arrays all of the samples produced had the same spacing as the mask (~.75μm).

In the rest of this thesis I will refer to the samples An or Dn where A and D refer to asterisk and disc shaped islands respectively and n is the number of the sample. Data was taken on two separate runs for sample A3. It was found that the sample characteristics had changed slightly between the two runs. Therefore data from the two runs are referred to by the designations A3a and A3b.

Figure 8 shows a photomicrograph of a small section of the photo mask and a micrograph of a small section of an array of Pb islands produced from the mask. Since reflection microscopy was used the transparent asterisk shaped portions of the mask appear dark in this picture and the chromium covered sections of the mask appear light. Note that the lattice is not a true triangular lattice but is slightly distorted. This is a peculiarity of the digital resolution of the electron beam writing system. At this level of resolution the machine cannot write figures with arbitrary
FIGURE 8. Photomicrographs of small sections of the optical mask and a finished array of Pb islands produced from the mask.
angles.

Figure 9 is a schematic representation of the lattice with the three lattice directions marked. The lattice direction 2 is chosen so that it has a slope of two. With this slope the angle $\theta=63.4^\circ$ rather than $60^\circ$. The lattice constant in direction 1 is $10\mu m$ and consequently the lattice constant in the other two directions is $11.2\mu m$. It is not clear how this distortion will alter the results of the theoretical predictions. It seems reasonable however that the qualitative results will not be affected. The topology of the lattice is the same leading one to expect that the differences between triangular lattices and other types of lattices will be preserved.

The final step in the sample preparation procedure is to evaporate Pb pads onto the ends of the sample. These pads are used for current injection and their superconducting nature insures uniform current injection. The pads are made from 99.999% pure Pb evaporated through an aluminum mask. The disk samples were 2.5cm long by 1cm wide and the Pb pads covered .6cm at each end of the sample thus shorting out half its length. The remaining half inch of sample still contained more than $10^6$ Pb islands. The asterisk arrays were only 1cm long and contained exactly $10^6$ Pb islands. In order to measure as large an array as possible I tried to overlap the current pads onto the sample as little as possible. Using a mechanical aluminum mask to make the pads about 10%
FIGURE 9. A schematic representation of the sample geometry.
of the length of the sample was shorted out.

Figure 10 shows a schematic top and side view of both types of sample with the Pb pads deposited. The edge of the pads forms an equipotential line across the width of the sample so that if the resistivity of the array is uniform across its width the current will be injected uniformly into the sample. The method of making current and voltage contacts to the samples will be discussed in the next section.

2. DATA ACQUISITION

All of the experiments were done in a standard helium 4 cryostat. Temperatures below 4.2K were achieved by pumping on the whole bath and regulating the pressure with a home built pressure regulator. The sample was placed in good thermal contact with a copper block that was suspended inside a vacuum can. The copper block was relatively weakly thermally linked to the bath and was electronically regulated to temperatures above the bath temperature. The regulation was good to within 1mK. The temperature was measured with a Lakeshore Cryotronics germanium resistance thermometer. The thermometer, which was encapsulated in a helium filled metal case, was soldered into the sample block and the leads were varnished to the sample block with GE 7031 varnish to further ensure good thermal contact. The
FIGURE 10. Schematic top and side view of both types of array with the Pb current injection pads deposited.
The resistance of the thermometer was monitored using a standard four terminal ac resistance bridge. When checked against a manometer the thermometer was about 15mK hotter. The bridge is sensitive enough to resolve temperature differences of about .5mK.

Figure 11 shows a schematic picture of the inside of the vacuum can. In addition to the two thermometers, the heater and the sample block the diagram shows the pin block used to make electrical contact to the sample. Shown here is the configuration used with the disk samples in which the four spring loaded pins come down in a line along the length of the sample. The inner pins are the voltage pins and the distance between them is 1cm so that this is the length of the sample whose electrical properties are being measured. The pin configuration for the asterisk samples is shown in the inset. Both current and voltage pins are in contact with the Pb pads on the ends of the sample. In this case the measurements are not true four terminal measurements. However, if the resistance between the pad and the sample is sufficiently small it will have no effect on the measurement.

The resistance, current voltage characteristics (IV), and critical current of the sample are measured using a bridge circuit in which a SHE system 330 SQUID acts as an extremely sensitive null detecting galvanometer. Figure 12 shows a schematic drawing of the bridge circuit. Electrical
FIGURE 11. Schematic drawing of the inside of the vacuum can showing the sample holder and the regulated copper block. The inset shows the pin configuration used with the asterisk samples.
Current Sampling Resistors

FIGURE 12. Schematic diagram of the SQUID galvanometric bridge.
Measurements are made in the following way. Both current supplies and the SQUID are zeroed. The sample current supply is then increased to some chosen measuring level, $I_*$. This current divides between the two current pathways according to their relative resistance thereby causing the SQUID to deflect. The standard resistor current supply is then increased until the SQUID output returns to zero. In this balanced condition the following relation holds

$$I_* R_* = I_* R_s + R_* = I_* R_s / I_*$$  \hspace{1cm} (3.1)$$

The voltage across the sample can be determined from

$$V = I_* R_s .$$  \hspace{1cm} (3.2)$$

The voltage resolution of the bridge will be limited by the minimum detectable deflection of the SQUID. The sensitivity of the SQUID in the range on which it is normally used is 5nA/mV. The noise level of the SQUID is typically a few millivolts. For practical purposes of balancing the bridge a minimum deflection of 10mV is used. This deflection corresponds to a current $I_* = 10\text{mV} \times 5\text{nA/mV} = 50\text{nA}$. This current in turn corresponds to a voltage $V = 50\text{nA} \times 1.813 \times 10^{-4} \Omega \times 10^{-11} \text{V}$. In fact $10^{-11} \text{V}$ is the experimental resolution of the bridge circuit. This resolution only obtains if the sample resistance is small compared to the standard. If the sample resistance is comparable to or larger than $R_s$ then only a fraction of the current $I_*$ passes through the SQUID pickup.
coil and the voltage resolution goes down. The SQUID sensor is enclosed in a Pb-Bi can which provides a superconducting shield against time varying magnetic fields.

For some of the magnetoresistance measurements the bridge was balanced using a feedback circuit to control the standard resistor current supply. This feedback circuit allowed one to vary the magnetic field continuously and monitor the resistance of the sample on an X-Y chart recorder. A detailed description of the feedback circuit is given in appendix C.

The sample can fits inside of a superconducting magnet. The magnet was designed to produce fields of a few tens of gauss. Calibration with an FW Bell model 640 Gauss-meter showed that the field varied by less than 1% within one inch (in the vertical direction on either side of the center of the magnet. Figure 13 shows a schematic drawing of the dewar with the probe and magnet in place.
FIGURE 13. Schematic diagram of the helium 4 cryostat with the probe and magnet in place.
IV. RESULTS

1. ZERO MAGNETIC FIELD

Using the bridge circuit described above the resistance of the arrays as a function of temperature is measured. Figure 14 shows a typical R(T) curve in zero magnetic field. Because of the sensitivity of the array properties to the component of the field perpendicular to the plane of the array it was necessary to zero out this component of the earth's field. Most of the zeroing was done with a double walled μ-metal shield which surrounded the entire dewar. Even with this shielding it was found that there was a residual perpendicular component of the field of a few milligauss. This remnant field was zeroed out with the superconducting magnet. The details of how the zeroing is done will be discussed in the section on magnetic field effects.

The resistive transition shown here is similar to that reported in a number of previous experiments on Josephson junction arrays.\textsuperscript{1-4} We see the characteristic double transition. Above the superconducting transition of the Pb islands the resistance of the sample as a function of temperature is flat as in normal dirty homogeneous thin metallic films. At
FIGURE 14. Resistance vs. Temperature in zero magnetic field for sample A3a.
the superconducting transition of the Pb islands (about 7.0K) there is a sharp drop in the resistance due to the fact that a large area of the sample is being shorted out. The resistance of the sample remains finite for a range of several degrees below the island transition temperature. Just below the transition there is a plateau region of slowly decreasing resistance. A couple of degrees below the Pb transition the resistance begins to decrease more rapidly. The resistance drop becomes steeper and steeper until an inflection point is reached, after which the resistance decreases more slowly in a broad resistive tail. Note that the resistance is plotted only down to .003Ω even though the bridge circuit can resolve resistances orders of magnitude smaller. Lower resistances are not shown because in the temperature range below the last data point plotted the current-voltage (IV) characteristics are nonlinear so that the resistance is not well defined. This crossover from linear ohmic IV characteristics to non-linear IV characteristics has been observed in previous experiments on proximity-coupled Josephson junction arrays.¹,³

Figure 15 plots the IV characteristics on a log-log scale over a range of temperatures in the tail region of the resistive transition. In a log-log plot a linear region of the IV characteristic corresponds to power law behavior. That is, the voltage is increasing as a power of the current where the power is given by the slope of the linear
FIGURE 15. A log-log plot of the IV characteristics for sample A3b.
region. We see that at sufficiently high temperatures, the IV characteristics plotted on a log-log scale are linear with slope 1, corresponding to ohmic behavior. As the temperature is lowered the IV characteristics show a bending behavior; at low currents, they have slope 1 but as the current is increased they bend upward and eventually enter a region of constant slope greater than 1. As the temperature is lowered further the IV characteristics have no linear region at all even down to the lowest voltages that can be resolved with the SQUID circuitry. In this low temperature region the IV characteristics have a power law form, \( V = I^a(T) \), although there is a change in the value of \( a(T) \) as one goes from low currents to high currents as manifested by a break in slope in the log-log plot. The low current value of \( a(T) \) is larger than the high current value.

Figure 16 shows a plot of \( a(T) \) for sample S3a. In the low temperature region where the IV characteristics show a break in slope on a log-log plot it is not clear which slope to use in determining \( a(T) \). The closed circles represent the slopes of the low current region and the open circles represent the slope of the high current region. There is more scatter in the slopes of the low current region than in those of the high current region, but in neither case is there a sharp drop in \( a(T) \). Any drop, if it occurs is closer to \( a=2 \) than to \( a=3 \). This drop at \( a=2 \) is in contrast to the data of Abraham et al who see a drop at \( a=3 \) in agreement
FIGURE 16. A plot of the IV exponent $a(T)$ for sample A3a. The closed circles represent the slopes of low current portion of the IV characteristics and the open circles represent the slopes of the high current portion.
with the modified HN theory. I have no explanation for this discrepancy.

At sufficiently low temperatures the sample appears to have a finite critical current. I say 'appears' because as with many superconducting systems the voltage increases continuously as a function of current so that the best one can say is that at some current, \( I_c \), the voltage across the sample becomes too small to be resolved by one's measuring apparatus.

In fact this how the critical current is measured. A voltage is chosen just within the resolution of the SQUID bridge circuit (10^{-11}V) and the current through the sample is slowly increased until that voltage is reached. The current which causes this voltage drop is defined to be the critical current. The difficulty with this definition of \( I_c \) is that it works at all temperatures, even in the regime where the sample is clearly resistive; If one uses a small enough current source one will not be able to resolve a voltage until some finite current is reached.

To get meaningful results from such a measurement, a self-consistent procedure is used. The critical current is measured in the way described above and its temperature dependence is fitted to some reasonable theoretical form. Above some temperature this fitting procedure breaks down. Below this temperature one can believe that one has measured a real critical current with reasonable accuracy.
Figure 17 shows a plot of critical current vs. temperature of sample S3a. Note the very strong temperature dependence of $I_c$. The data can be fitted to the single junction form of the critical current derived by DeGennes:

$$I_c = I_c^0 \left[1 - (T/T_{cs})\right]^2 \exp\left(-d/\xi_n\right) \quad (4.1)$$

where $d$ is the thickness of the junction and $\xi_n$ is the normal metal coherence length. $\xi_n$ is given by:

$$\xi_n = \left(\hbar v_F l / 6\pi k_B T\right)^{1/2} \quad (4.2)$$

This expression was derived in the limit $\xi_n \ll d$.

Measurements of the residual resistivity ratio of sputtered Sn films deposited in the same sputtering rig used to produce the Sn overlay in the arrays indicate a mean free path in the Sn of $l = 500\,\text{Å}$. The Fermi velocity of Sn is $v_F = 1.90 \times 10^8\,\text{cm/s}$. The critical current was not measured at temperatures lower than 4.2K, so I calculate $\xi_n(4.2K)$ to see if the junctions in these arrays are in the appropriate limit. The result is $\xi_n(4.2K) = 920\,\text{Å} = 0.092\,\text{µm}$. Since the width of the junctions is $d = 0.75\,\text{µm}$ they are in the limit $\xi_n \ll d$.

This result for the critical current was derived in part using Ginzburg Landau theory and is therefore strictly valid only in the limit $[1 - (T/T_{cs})] \ll 1$ where $T_{cs}$ is the transition temperature of the superconducting banks of the SNS junction. In particular the prefactor $[1 - (T/T_{cs})]$ comes from Ginzburg Landau theory and is approximately zero in the
FIGURE 17. Critical current as a function of temperature.

Sample A3b
range of its validity. Thus the magnitude of $I_c$ depends very sensitively on the magnitude of this prefactor near $T_{cs}$. Farther below $T_{cs}$ where the magnitude of the order parameter in the superconducting banks is changing slowly as a function of temperature this prefactor is not valid, and in the simplest approximation one can assume it is constant. The temperature dependence in the exponential factor depends only on the normal metal properties and therefore is still applicable when the temperature is far from $T_{cs}$. Thus the data will be fitted to the modified expression

$$I_c = I_{c0} \exp\left[-d/\xi(T)\right].$$

(4.3)

This expression can be rewritten as

$$I_c = I_{c0} \exp\left(-\alpha T^{1/2}\right)$$

(4.4)

where $\alpha = (6\pi k_B / \hbar v_f)^{1/2} d$.

Figure 18 shows a plot of $T^{1/2}$ vs. $\ln(I_c)$. Over a range of temperatures from 4.2K to 4.8K the data plotted in this way are reasonably linear. Above 4.8K the critical current bends down from this straight line. In this high temperature regime the array is approaching $T_c$ where the temperature dependence of the collective vortex excitation of the array becomes important and masks the single junction effects. From a least squares fit to the linear region of the critical current data we get the following values for the parameters $\alpha$ and $I_{c0}$:
FIGURE 18. A plot of $T^{1/2}$ vs. $\ln(I_c)$. The straight line through the data points is the result of a least squares fit.
\[ a = 29.0 \pm 1 \text{ K}^{-1/2} \]
\[ \ln(I_{c0}) = 55.6 \pm 2(I_{c0} \text{ in A}) \] (4.5)

From the parameter \( a \) the mean free path in the normal metal can be determined by \( l = (6\pi k_B/\hbar v_F)(d^2/q^2)^{1/2} \approx 250 \text{Å} \). This value of \( l \) with is within a factor of 2 of the value determined from the residual resistivity ratio of sputtered Sn films, thus giving some indication that this fitting procedure is reasonable. It should be noted that the form of the normal metal coherence length used here does not take account of the divergence in \( \xi \), which occurs as the Sn transition temperature is approached. However, since these measurements were taken nearly half a degree above \( T_c(\text{Sn}) \) it is reasonable to expect that the Sn is acting like a true normal metal.

As was discussed in chapter II analogy with high sheet resistance superconducting films has been used to suggest explicit expressions for the temperature dependence of the resistive tail and the non linear IV characteristics. We have already seen that the measured IV characteristic power law exponent \( a(T) \) does not have the predicted temperature dependence. As was seen in the theory chapter, Lobb et al have adapted the HN analysis to suggest an expression of the exponentially activated resistance above \( T_c \). However, considering that the resistance data in the tail region covers less than a decade and that \( T_c \) is an adjustable parameter an attempt to fit to this expression
seems unfruitful.

One other quantity which can be checked against theory is the transition temperature $T_c$. Kosterlitz and Thouless showed that for a 2-D X-Y model in which interactions between vortices are ignored the vortex unbinding transition temperature is given by

$$\kappa B T_c = \pi J.$$  \hfill (4.6)

In the X-Y model $J$ is the ferromagnetic coupling constant between adjacent spins. For Josephson junction arrays $J$ is the Josephson coupling constant between adjacent superconducting grains. When one takes account of interactions between the vortices one finds that the transition temperature is lowered. The gas of bound vortex anti-vortex pairs forms a polarizable medium which reduces the effective interaction between vortices by a factor of $1/\varepsilon$ where $\varepsilon$ is the dielectric constant of the medium. This reduction of the interaction energy leads to the result

$$\kappa B T_c = \pi J/\varepsilon(T_c).$$  \hfill (4.7)

$\varepsilon > 1$ so that $T_c$ is reduced from its value in the simplified theory. No one has succeeded in calculating $\varepsilon$ exactly so that $T_c$ is not known. Monte Carlo simulations by Shih and Stroud on a triangular lattice give the result

$$\kappa B T_c = 1.45J.$$  \hfill (4.8)
The Josephson coupling constant $J$, as has been noted above, is temperature dependent and is related to the single junction critical current by $J(T) = (\hbar/2e)I_c(T)$. Thus the above relationship for $T_c$ is an implicit relationship and one must know $I_c(T)$ in order to extract the value of $T_c$.

In principle one could calculate $I_c(T)$. In fact as we have seen in the discussion of the critical current data the temperature dependence of $I_c$ has been calculated with reasonable accuracy. Calculating the magnitude, however, requires an extremely accurate characterization of the junction, more accurate than can be obtained for these arrays.

Thus if one wants to use formula (4.8) one must use a self-consistent procedure. We have already seen that the critical current of the whole array well below $T_c$ fits reasonably well to the form

$$I_c = I_{c0} \exp(-qT^1/2).$$

(4.4)

Recalling the calculation of Shih and Stroud, we remember that the critical current per lattice constant across the width of the array is $2I_c$. The factor of 2 reflects the fact that there are more junctions than superconducting islands in the array. If one assumes that the current is distributed uniformly across the array then it is necessary to divide the array parameter, $I_{c0}$, by twice the number of lattice constants across the width of the array (2000) to get the
single junction value, i.e. The self-consistent expression for the transition temperature is

\[ k_B T_c = 1.45 \left( \frac{\hbar}{2e} \right) \left( \frac{I_c}{2000} \right) \exp \left( -e T_c^{1/2} \right). \]  

(4.9)

This equation can be solved numerically for \( T_c \). Actually it is convenient to deal with the reduced transition temperature

\[ t_c = \frac{k_B T_c}{J(T_c)}. \]  

(4.10)

The Monte Carlo simulation predict \( t_c = 1.45 \). The value of \( T_c \) which gives a reduced transition temperature of 1.45 is \( T_c = 4.65 \text{K} \). This value of \( T_c \) is considerable lower than the values extracted from the IV characteristic power law data \( (T_c = 5.08 \text{K}) \). The poorness of the estimate of \( T_c \) derived from equation (4.9) is accentuated if one calculates the reduced temperature at the higher value of \( T_c \) \( (T_c = 5.08 \text{K}) \) derived from the IV characteristic power law data. The result is \( t_c = 2.0 \pm 5 \). The reason for this discrepancy between theory and experiment is not known.

2. MAGNETIC FIELD DATA

In the previous section it has been shown that in zero magnetic field these arrays undergo a superconducting transition qualitatively similar to that reported in previous experiments on proximity-coupled arrays. This
transition has tentatively been identified as a Kosterlitz-Thouless vortex unbinding transition. In this section the effect of an applied magnetic field on the superconducting transition of the array is examined.

As was seen in the theory chapter one expects the superconducting transition to vary periodically in a magnetic field with the field period corresponding to one flux quantum per unit cell of the array. In fact the array properties do vary periodically in a magnetic field. From an optical micrograph of the array such as that shown in Figure 8 one can determine the area of a unit cell to within a few percent, and one can therefore determine the flux periodicity to within a few percent. The measured flux periodicity is \( \phi = (2.06 \pm 0.04) \times 10^{-7} \text{Gcm}^2 \). Comparing the known value of the flux quantum \( \phi_0 = \hbar c/2e = 2.07 \times 10^{-7} \text{Gcm}^2 \), we see that the experimental value matches the known value to within the uncertainty of the measurement. In the data presentation which follows magnetic field dependent quantities are plotted as a function of the parameter \( f = B/B_0 \) where \( B_0 \) is the measured field periodicity. The measured field period is \( B_0 = 0.415 \text{G} \).

Figure 19 shows the resistance as a function of temperature for several values of the parameter \( f \). Only the lower part of the resistive transition is plotted since the resistance at higher temperatures is not affected by these weak magnetic fields. It is clear from these data that as \( f \)
FIGURE 19. Resistance as a function of temperature for several values of the parameter $f$. The measuring current was 10μA.
is increased toward 1/2 the transition temperature is being depressed. The $f=1$ data show, however, that as $f$ is increased from 1/2 to 1 $T_c$ increases again. We see, however, that $T_c$ does not quite return to its $f=0$ value; it is slightly lower. This lack of strict periodicity in $f$ is a characteristic of all real arrays, and is believed to be a single junction effect; the magnetic field penetrating the finite width of the junction suppresses the single junction critical current, $i_c$, which in turn suppresses the transition temperature. (Recall that $T_c \sim J \sim i_c$)

The periodicity of the array properties will become more obvious if one plots the resistance as function of magnetic field at fixed temperature. In this case the sample will move continuously between the various $R(T)$ curves and the resistance will oscillate. First I show zero field cooled (ZFC) magnetoresistance. This data is obtained by setting the magnetic field to zero at temperatures well above the field dependent region of the $R(T)$ curve and then cooling the sample into the transition region. The magnetic field is then varied and the resistance is monitored using the SQUID bridge circuit. For this data taken at fixed temperature we use a feedback circuit to keep the bridge balanced automatically as the magnetic field is changed.

The upper trace in Figure 20 shows ZFC resistance at $T=5.2K$ as a function of $f$. Note the minima at integer values of $f$. Note also the sharp suppression in the depth of the
FIGURE 20. Magnetoresistance at 5.2K for sample A3a. The various traces were obtained by cooling the sample in different values of the magnetic field.
minima as \( f \) is increased from zero. The other traces in Figure 20 show magnetoresistance curves taken at the same temperature as the ZFC trace, but in each case the sample was cooled at a different value of applied field. Note the large difference between these latter curves and the ZFC curve; The symmetry about \( f=0 \) is destroyed and the relative magnitudes of the various minima are altered depending on the value of \( f \). This lack of reversibility above the transition temperature \( T_c \) is not well understood.

At this it is convenient to discuss how the perpendicular component of the magnetic is zeroed out. The \( \mu \)-metal shield reduces the ambient field to a few milligauss. However, the sensitivity of the sample to perpendicular fields is so great that even this small a field has an effect on the sample properties. This residual field is zeroed in the following way. The sample is cooled into the transition region and a magnetoresistance trace is taken. The magnetic field is set at the value where the minimum in the resistance occurs. The sample is then warmed up above the field dependent region of the resistance curve and cooled back down into the transition region. Another magnetoresistance trace is taken. It is found that the position of the resistance minimum has shifted slightly. The magnetic field is set to this new value and the procedure is repeated. After several iterations it is found that the position of the resistance minimum is no longer changing.
The belief is that this minimum corresponds to a zero perpendicular component of magnetic field.

Next I show field cooled (FC) magnetoresistance data. In order to take this data it was necessary at every field point to warm the sample up above the field dependent region of the R(T) curve, set the magnetic field, and cool the sample back down into the transition region. Figure 21 shows a magnetoresistance curve at 5.2K taken in this manner. Only a single period of the variation is shown. Note the relative magnitudes of the minima at \( f=0 \) and \( f=1 \). The minimum at \( f=1 \) is suppressed by about 20% from the \( f=0 \) minimum. This is in sharp contrast to the ZFC data in which the second minima is depressed by over 100% from the first. Outside of the steep depression of the resistance near integer values of \( f \) the magnetoresistance is relatively flat. There is a hint of substructure within this flat region but nothing is clearly defined in this data set.

Figure 22 shows another field cooled magnetoresistance trace at a lower temperature. We now see sharp substructure within a single flux period. We see resistance minima at \( f=1/4, 1/2, 3/4 \) as well as distinct features at \( f=1/3 \) and \( 2/3 \) (The resistance resolution of the bridge is smaller than the size of the data points plotted here so that the features are distinct and reproducible.). The data are nearly symmetric about \( f=1/2 \) in agreement with theory. Also in agreement with theory is the large feature at \( f=1/4 \) which
FIGURE 21. Field cooled magnetoresistance at 5.2K for sample A3a. Each data point was taken by cooling the sample into the transition region at the given value of magnetic field.
FIGURE 22. Field cooled magnetoresistance at 4.9K for sample A3a.
is nearly the same size as the feature at \( f=1/2 \). This feature is predicted only for a triangular lattice. Note that at 4.9K the sample is below the \( f=0 \) transition temperature so that the resistance goes to zero at \( f=0 \) and 1 rather than having a minimum.

For purposes of comparison Figure 23 shows Pannetier et al's data for the transition temperature of a square array of superconducting wires\(^1\). Such an array of wires is expected to show the same magnetic field structure as Josephson arrays of the same geometry\(^1, 2\). We see from the data that this is in fact the case. \( T_c \) is periodic in \( f \) with maxima at integer \( f \) and subsidiary maxima at various rational \( f \) within a single period. Note that the second largest feature is at \( f=1/3 \) rather than at \( f=1/4 \).

Thus the most obvious qualitative prediction of the magnetic field dependence of triangular Josephson junction arrays has been verified. To emphasize again the importance of field cooling in obtaining these results Figure 24 shows both the FC and ZFC magnetoresistance at 4.9K. In the ZFC data the symmetry about \( f=1/2 \) has been destroyed and the features at \( f=1/3, 2/3 \) and \( 3/4 \) are no longer detectable. Note also that the period of the ZFC data is slightly larger than one.

Figure 25 shows a ZFC trace for sample A1. At the time I made measurements on this sample I did not yet understand the necessity of field cooling so that there is no
FIGURE 23. Transition temperature of a square array of superconducting wires as measured by Pannetier et al.
FIGURE 24. Field Cooled and Zero Field Cooled magnetoresistance at 4.9K.
FIGURE 25. Field Cooled magnetoresistance for sample Al.
corresponding FC data. Again we see the lack of symmetry about \( f = 1/2 \) and the strong suppression of the minima at \( f = 1 \). Of particular interest in this data is the appearance of a feature between \( f = 1/3 \) and \( f = 1/2 \). I show two successive traces taken at the same temperature in order to demonstrate that the feature is reproducible. The feature is rather broad making it difficult to pinpoint its location. If one makes a guess as to the location of the feature as indicated by the arrow and computes \( f \) using measured ZFC period as a normalization one finds \( f = .397 \). Comparing this value to the rational fractions \( 3/7 \approx .428 \) and \( 3/8 \approx .375 \) we see that among nearby ration fractions with low \( q \) values \( 3/8 \) is the closest. Recalling Shih and Stroud's calculation of \( T_c \) (Figure 6) we see that there is a feature at \( f = 3/8 \) comparable in magnitude to the feature at \( f = 1/3 \). It is unfortunate that there is no field cooled data for this sample because it seems likely that even higher order structure would have appeared in such data.

These two samples have nominally the same characteristics. The size and spacing of the Pb islands is the same. The thickness of the Pb islands and the thickness of the Sn overlay is the same. The depositions of the Pb and Sn were done under the same nominal conditions. Thus it is not clear why one sample shows more structure than the other. The most likely factor limiting the amount of structure observed is inhomogeneity. The theories deal with arrays in
which each unit cell is exactly identical in a real array such perfect homogeneity is not possible. The photomask used to produce a photoresist pattern is itself not perfectly homogeneous. The subsequent processing steps required to produce an array of Pb asterisks—printing, developing, and liftoff—are not exactly reproducible. Therefore the amount of inhomogeneity may vary from sample to sample. At the present time I have no method of quantifying the inhomogeneity of a particular sample or of correlating such inhomogeneity to the amount of structure observed.

So far the data presentation has focused only on the magnetic field dependence of the resistance above the transition temperature \( T_c \). The theories can predict the behavior of this quantity only qualitatively. One would like to be able to measure the transition temperature as a function of \( f \) directly. The resistance vs. temperature curves in a magnetic field show that \( T_c \) is oscillating. However, the broad nature of the transition makes it unclear how to define \( T_c \).

Recall that in the zero field case we characterized the superconducting transition by the crossover from linear to power law IV characteristics. In order to better characterize the transition in a magnetic field the IV characteristics are measured as a function of temperature at various values of \( f \).

Figure 26 shows a log-log plot of the IV characteris-
FIGURE 26. A log-log plot of the IV characteristics at $f = 0.043$. 

Sample A3a

$f = 0.043$
tics for \( f = 0.043 \). As in the zero field case we see a crossover from linear to nonlinear IV characteristics. At sufficiently high temperatures the IV characteristic is linear over the whole current and voltage range measured. As the temperature is lowered the IV characteristics exhibit bending behavior; At low current they are linear and as the current is increased they bend upward into a power law dependence. At sufficiently low temperatures there is no linear region even at the lowest currents and voltages measured.

Notice, however, that in the region of pure power law behavior there is not the sharp break in slope seen in the zero field case. Since \( f = 0.043 \) for this set of IV characteristics we see that only a very small magnetic field is needed to eliminate this sharp break in slope at low temperatures. This feature of the data is not understood at this time. However, aside from this feature the IV characteristics show the same qualitative behavior as the IV characteristics in zero magnetic field.

Figure 27 shows another set of IV characteristics for \( f = 1/2 \). Again we see the same qualitative behavior as a function of temperature. This behavior is in sharp contrast to that of high sheet resistance superconducting films. As was mentioned in the theory chapter these films are also believed to undergo a K-T vortex unbinding transition. However, the application of a perpendicular magnetic field to
FIGURE 27. A log-log plot of the IV characteristics at f=1/2.
these films has a very different effect on the IV characteristics; All of the IV characteristics acquire a linear region at low current and no transition to a zero resistance state takes place\textsuperscript{23}. Within the theory of type II superconductivity this linear portion of the IV characteristic is easily understood; The magnetic field penetrates the sample in the form of vortices which give rise to a linear flux flow resistance in the presence of a transport current. This difference in the superconducting properties of the two systems is evidence that the response of Josephson junction arrays to a magnetic field can not be described by the Ginzburg Landau equations as has sometimes been assumed.

Figure 28 shows the power law exponent $q$ as a function of temperature at various values of $f$. The values for $f=0$ are taken from the smaller of the two slopes in the power law regime. The temperature where $q=1$ is determined by looking for bending behavior at low currents. The lowest temperature at which this bending is observed is taken to be the start of the regime of ohmic behavior. If we identify the transition temperature with the crossover from linear to non-linear IV characteristics it is clear from this data, as in the resistive transition data, that $T_c$ is depressed by the application of a magnetic field. In fact $T_c$ is quite steeply depressed near $f=1$; At $f=.043$ $T_c$ has been depressed to 75% of its maximum depression. Note that the form of the function $q(T)$ does not change much as a function of
FIGURE 28. The IV characteristic power law exponent $a(T)$ at several values of $f$. The lines are only a guide to the eye.
temperature. If the IV power law is supposed to contain information about the nature of the phase transition we see no indication here of qualitatively different behavior at different values of $f$.

Figure 29 shows $a(T)$ for $f=1/2$ and for two nearby values of $f$ on either side of $1/2$. From this plot it is clear that $T_c$ does not vary monotonically on $f\in[0,1/2]$; there is a dip in $T_c$ at $f=1/2$. It is difficult, however, to map out the function $T_c(f)$ in detail by measuring IV characteristics. Instead the following technique was employed to measure $T_c(f)$. A temperature within the region of power law behavior for $f=0$ is chosen. A test current is then chosen and the voltage drop along the sample at that current is noted. The sample is then warmed up above the field dependent region of the resistive transition, the magnetic field is changed, and the sample is cooled down again. Using the same test current the temperature of the sample is adjusted until the voltage drop along the sample is the same as at $f=0$. That temperature is then defined to be $T_c(H)$.

Figure 30 shows a plot of $T_c(f)$ measured by the technique just described. We can now clearly see fine structure in $T_c$ corresponding to the structure in the magnetoresistance above $T_c$. There are distinct maxima in $T_c$ at $f=1/4$, $1/2$ and $3/4$. I have also indicated possible structure in $T_c$ at $f=1/3$ and $2/3$. 
FIGURE 29. The IV characteristic power law exponent $a(T)$ at $f=1/2$ and at two nearby values of $f$. 

Sample A3a

- $f=0.11$
- $f=0.38$
- $f=0.50$

IV Exponent

Temperature (K)
FIGURE 30. $T_c$ as a function of $f$. Sample A3a
To complete the picture of the array transition in a magnetic field we will now examine the behavior of the critical current as a function of magnetic field. Figure 31 shows a plot of critical current as a function of $f$ at $T=4.2\,\text{K}$. The zero field critical current is $5.4\,\text{mA}$ and is not shown on this plot. As with the resistive tail, $R(T)$, and the IV characteristic power law, $a(T)$, we see evidence of suppression of the superconducting transition temperature, $T_c$, by the application of a magnetic field. Note the large depression of $I_c$ for relatively small values of $f$. For instance at $f=0.126$ the depression of $I_c$ has reached $90\%$ of its maximum value. Measuring critical currents is difficult for reasons discussed previously, and therefore this data is not as clean as the transition temperature data. Nevertheless we see the same basic structure as a function of $f$; $I_c$ has maxima at $f=1/4$, $1/2$ and $3/4$. The data is not good enough to determine if there are features at $f=1/3$ and $2/3$. 
FIGURE 31. The array critical current as a function of $f$. 
V. DISCUSSION

It has just been shown that all features of the superconducting transition of triangular proximity-coupled arrays—the resistive tail, the nonlinear IV characteristics, and the critical currents below $T_c$—show oscillatory behavior in an applied perpendicular magnetic field. The period of this oscillation corresponds to one flux quantum per unit cell of the array. Furthermore there is substructure at various rational fractions of flux quantum $f=\phi/\phi_0$ per unit cell within a single period. In this section I examine how good a qualitative and quantitative understanding of this behavior can be obtained in terms of current theories of Josephson junction arrays.

Before examining the data in detail it is necessary to understand the importance of screening. The theories described in chapter II all assumed that the arrays are in the weak coupling limit in which the magnetic fields due to screening currents are small compared to the applied field. It was suggested in that chapter that the appropriate limit given in terms of array parameters was $\Phi_c \sim L_{ic} \ll \Phi_0$ where $L$ is the geometrical inductance of a loop comprising a
unit cell of the array and $i_c$ is the critical current of a single junction.

The critical screening flux $\Phi_c$ may be estimated in the following way. The single junction critical current $i_c$ was estimated previously from the measurement of the critical current of the whole array well below $T_c$. The inductance of a unit cell of the array is estimated as follows. The self inductance of a circular loop of wire is given by

$$L = 4\pi \times 10^{-7} r \ln(r/a)$$  \hspace{1cm} (5.1)$$

where $r$ is the radius of the loop, $a$ is the radius of the wire and the inductance is in MKS units. Since the lattice spacing of the array is 10\,\mu m $r$ is taken to be 5\,\mu m. The radius of the wire is taken to be the zero temperature penetration depth of Pb, $\lambda_0=39\,\text{nm}$. Since the Pb is 150\,nm thick the current will be spread over the top and bottom of the film as well. However, the spreading out of the current will decrease the inductance so that the estimate derived from this assumption will represent an upper bound on $L$. The result is $L=3 \times 10^{-11} \text{H}$. $T_c$ is determined from the nonlinear IV characteristics as described previously. Using equation (5.1) one can determine that the critical current at $T_c$ is $i_c(T_c=5.08, H=0)=9.3\,\text{nA}$. Thus $\Phi_c=L i_c=2.79 \times 10^{-19} \text{Webers}$. The flux quantum in MKS units is $\Phi_0=2.07 \times 10^{-15} \text{Webers}$. Therefore $\Phi_c<<\Phi_0$ and the screening fields are negligible.
1. CRITICAL CURRENT

As we have seen the zero temperature critical current can be computed exactly for rational $f$. These calculations assume that the Josephson coupling constant $J$ and therefore the single junction critical current $i_c$ are constants independent of temperature. In this case at $T=0$ and $f=0$ the critical current per lattice constant across the width of the array is $i_c=2i_c^0$. The factor of 2 is a peculiarity of the geometry of triangular arrays reflecting the fact that there is more than one junction per lattice constant. Recall that for $f \notin [0,1/2]$ $i_c(f \neq 0) < i_c(f=0)$. If one computes the ratio $i_c(f)/i_c(f=0)$ one can compare this number to the experimentally measured values $I_c(f,T<T_c)/I_c(f=0,T<T_c)$ where $I_c$ is the critical current of the whole array. Of course $I_c$ is a finite temperature critical current rather than a zero temperature critical current. However if $k_BT \ll J(T)$ then the temperature will be effectively zero and one might expect that the measured ratio $I_c(f,T)/I_c(f=0,T)$ will equal the calculated ratio $i_c(f,T=0)/i_c(f=0,T=0)$.

Figure 32 shows a plot of the ratio $I_c(f)/I_c(f=0)$ at $T=4.2K$. In addition the ratio $i_c(f)/i_c(f=0)$ as calculated by Shih and Stroud is plotted. Note that there are two theoretical points at $f=1/2$. The lower point is for current injected perpendicular to a lattice direction corresponding to the experimental configuration. The upper point is for
FIGURE 32. The depression of the array critical current from its \( f=0 \) value (closed circles) compared to the predictions of Shih and Stroud (open circles).
current injected parallel to a lattice direction. In this case the critical current at $f=1/2$ is much larger than that at $f=1/4$. In the case of perpendicular injection the critical currents at $f=1/4$ and $1/2$ are of the same size in agreement with the experimental data. It may be seen however that the quantitative agreement between theory and experiment is not particularly good.

It is worthwhile to calculate the ratio $kBT/J(T)$ to see if it is indeed small compared to 1. The critical current of the whole sample at $f=0$ is $I_c=5.4$ mA corresponding to a single junction critical current of $i_c=2.5$ HA. Thus $kBT/J = kBT/(\hbar/2e)i_c = 0.070$. Thus the array is roughly in the low temperature limit.

2. TRANSITION TEMPERATURE

It has already been shown that the transition temperature has a magnetic field dependence which is in rough qualitative agreement with theory. In order to make a quantitative comparison with theory one must calculate the reduced transition temperature $t_c = kBTc/J(Tc)$. As we have already seen in the discussion of the zero field data the best experimental estimate of $t_c$ is an order of magnitude larger than that predicted by theory. The same problem persists when one calculates $t_c(f=0)$.

Recall that $J=(\hbar/2e)i_c$. Experimentally the array
critical current had the following temperature dependence

\[ I_c = I_c^0 \exp(-AT^1/2) \]

(4.4)

It was assumed that the single junction parameter \( I_c^0 \) was given by \( I_c^0/2000 \). This value of \( I_c^0 \) gave an experimental value of \( T_c \) that was an order of magnitude too large. One can attempt to fit the magnetic field dependence of \( T_c \) by regarding \( I_c^0 \) as an adjustable parameter and varying it until the experimental \( T_c(f=0) \) matches the theoretical value.

Figure 33 shows a plot of \( T_c \) as a function of \( f \) where \( I_c^0 \) has been adjusted so that \( T_c(f=0) \) equals the Monte Carlo value of 1.45. The theoretical values of \( T_c(f) \) as calculated from Monte Carlo simulations have also been plotted. Only a few values of \( T_c^{MC} \) have been calculated but it is clear even from these few values that the Monte Carlo simulations predict a much more dramatic variation of \( T_c \) than is observed experimentally.

Figure 34 shows another plot of \( T_c(f) \) where \( I_c^0 \) has been adjusted so that \( T_c(f=0) \) equals the mean field value of 3.0. In addition the mean field results of Shih and Stroud have been plotted. The quantitative agreement is not good; the theoretical values are 300% larger than the experimental values. It is in fact reasonable to expect that MFT will overestimate \( T_c \) since it does not take account of fluctuations which are vital for understanding the details of phase
FIGURE 33. A comparison between the measured transition temperature $T_c$ and the Monte Carlo calculations of Shih and Stroud. The parameter $i_0$ has been adjusted so that experiment and theory agree at $f=0$. 
FIGURE 34. A comparison between the measured transition temperature and the Mean Field Theory calculations of Shih and Stroud. The parameter $ic_0$ has been adjusted so that theory and experiment agree at $f=0$. 
transitions in two dimensions. However the relative sizes of the features at various rational \( f \) is in closer agreement than in the case of the MC simulations. In this sense the measured values of \( T_c \) are mean field like.

3. MAGNETIC FIELD HYSTERESIS

We have already seen how the state of the sample at a given magnetic field, \( H \), and temperature, \( T \), depends upon the path followed in the \( H-T \) plane; If one sets the field at an elevated temperature and then cools the sample into the transition region one reaches a different final value of resistance than if one cools the sample in zero field and then increases the field.

Such hysteretic effects have not been treated in detail by current theories of Josephson junction arrays although some indications of them have been found. In particular Shih and Stroud have found evidence for such irreversible effects in their calculations of the ground state configuration of phases. Their method of calculating these configurations involves solving the coupled mean field equations (1.16) which in principal are exact at \( T=0 \). The equations are solved self-consistently. One starts by generating a guess for the configuration of phases and then calculates the expectation values \( \langle \exp(i\hat{\Phi}_i) \rangle \) from equations (1.16). These expectation values are used in the effective Hamiltonian
(1.1) and the process is repeated. One iterates the equations until the input set of phases \( \{\phi_i\} \) is the same as the output set.

Shih and Stroud find that when the mean field equations are solved in this way at \( T=0 \) the solution depends sensitively on the initial guess for \( \{\phi_i^0\} \). In other words there are a multiplicity of metastable states. In order to find the true ground state they use a process called "mean field annealing". In this process the mean field equations are first solved at a finite temperature below the mean field transition temperature. Working at finite temperatures tends to avoid trapping in metastable states. The temperature is then lowered and the process is repeated with the set of phases computed at the higher temperature used as an initial guess for the lower temperature. The mean field equations are again iterated to convergence. In this way the sample is cooled to zero temperature. The solution obtained in this way is independent of the initial guess for the configuration of phases. It is clear that this process is the equivalent of field cooling.

The difficulty with applying this analysis to the the irreversible magnetoresistance is that the data in question is taken at temperatures above the superconducting transition temperature \( T_c \). At these temperatures there is no long range phase coherence in the array as is shown by the fact that the sample is resistive. It seems surprising then
that the sample can become trapped in a metastable minimum of the free energy.

However it is clear that phase coherence of some sort exists above $T_c$; if the phases of adjacent superconducting islands were completely uncorrelated then no magnetoresistance oscillations would be observed at all. Therefore phase coherence exists in the array on some length scale. In addition to these finite phase coherent regions there exists defects of some sort (vortices in the case of a KT transition) which destroy the long range order and which, in the presence of a transport current, propagate through the lattice and give rise to electrical resistance. The irreversible magnetoresistance data show that the nature of the finite regions of phase coherence depends upon path followed in the $H$-$T$ plane and that they are stable against being transformed into one another. That is if the sample is cooled in zero field and the field is then changed the configuration of the phase coherent regions is different than if the sample is cooled in a finite field. Furthermore the final state reached by ZFC followed by changing the field is stable against transformation into the state of presumably lower free energy reached by field cooling. An adequate theoretical treatment of this temperature regime does not exist at the present time.
LIST OF REFERENCES - PART1


PART 2

DYNAMICAL PROPERTIES
I. INTRODUCTION

As we have already seen in the first part of this thesis at temperatures well below the transition temperature of the array a large current can be passed through the array without any voltage drop along the array. That is to say there are no dissipative processes occurring so that a steady current can be maintained without putting any energy into the system. Above a maximum temperature dependent current called the 'critical current' dissipation does occur and there is a finite voltage difference between the ends of the sample.

Such behavior is observed in almost all superconducting systems. In general the appearance of a finite voltage between the ends of the superconductor does not correspond to the system being in the 'normal' metallic state. At sufficiently high current densities a superconductor will become normal. That is the, Cooper pair condensate is completely destroyed and the transport properties of the sample are described by traditional theories of metallic conduction. However, the current at which this occurs is far above the critical current. In the intermediate region the sample is in a complex nonequilibrium state in which the
superconducting order parameter can vary rapidly in space and time. The features of this intermediate state are not universal but are strongly dependent on the sample geometry as well as on material parameters. Furthermore once dissipation starts to occur, heating also occurs and must be taken into account if one is to understand the response of the sample to increasing currents. Thus the thermal properties of the sample and of its environment must be taken into consideration. The typical kind of data which are used to try and characterize the intermediate state are IV characteristics. In general the IV characteristics are highly non-linear in the intermediate regime.

In this part of my thesis I will report on some regular step structures observed in the IV characteristics of a few proximity-coupled Josephson arrays. At this time no fundamental understanding of the origin of theses steps exists so that the following presentation will be highly descriptive and qualitative. I begin with a discussion of the experimental techniques which are different in this high current regime than those used in studying the phase transition which required relatively small measuring currents. In the second section I present the data. The final section contains a discussion in which I compare the results of these experiments to similar results seen in other superconducting systems. Using this comparison as a basis I offer some qualitative suggestions as to what might
be happening in these arrays.
II. EXPERIMENTAL METHODS

As mentioned in the introduction a proper understanding of how the superconducting state is destroyed by the injection of large currents through the superconductor requires that the effects of heating be taken into account. Ideally one would like the sample to be in such good contact with a cold thermal reservoir that it will be heated by an insignificant amount above the temperature of the reservoir when it is in the dissipative state. In practice such ideal thermal anchoring cannot always be achieved.

I have tried to overcome some of the problems associated with heating by using a pulsed current technique. In this technique a short duration triangular current pulse is applied to the sample and the voltage response of the sample during the pulse is measured using a transient signal recorder.

Figure 35 shows a block diagram of the experimental setup. A wavetek 147 signal generator is manually triggered to produce a triangular voltage pulse. A precision resistor is placed in the line to limit the current. The resistance of the sample is current dependent but is at all times much smaller than that of the current limiting resistor so that
FIGURE 35. Schematic diagram of the circuit used for pulsed measurements of the IV characteristics.
the wavetek is acting as a current source.

At the same time that the wavetek is triggered the sync output of the wavetek is used to trigger a Nicolet digital oscilloscope. A Keithly 399 isolating amplifier is used to isolate the wavetek signal ground from the ground of the Nicolet. The Wavetek signal ground is floating and can be referenced to earth ground at any convenient point. In this experiment it is grounded to the copper sample block. One end of the sample is also grounded to the thermally regulated block.

The voltage drop along the sample is amplified with a PAR 113 preamplifier and then input into one channel of the Nicolet oscilloscope. The voltage across the current limiting resistor is input into the other channel. Thus both current and voltage are measured during the same pulse. From the two traces thus recorded it is easy to reconstruct the IV characteristics. An extra set of leads was attached to the current pins. (See Figure 11 for the pin configuration.) By monitoring the voltage between various combinations of these four pins we can determine if the region of current dissipation is localized along the length of the sample.

The IV characteristics taken in this pulsed manner do indeed differ from those taken at dc. As the pulse time is decreased the voltage at a given value of current also decreases indicating that the sample is being heated less
than at longer pulse times. For pulse times of about .05s the voltage response of the sample saturates; An IV characteristic taken with a pulse of .005s is identical to one taken with a pulse of .05s. Thus a source of heating has been eliminated. However, since there are a number of thermal impedances in the system including that between the phonons in the sample and the phonons in the substrate and that between the substrate and the regulated block, the elimination of heating with respect to one of these impedances does not mean that heating has been eliminated with respect to the others. In the discussion I will point out which of the possible sources of heating is most significant in understanding the form of the observed IV characteristics.
III. RESULTS

Figure 36 shows the resistive transition for sample D1. This sample undergoes the same characteristic double transition as the samples discussed in part 1. Note that $T_c=3.9K$ which is more than a degree lower than the $T_c$'s for the samples discussed in part 1. In this sample the Pb islands were in the form of disks rather than asterisks. The distance between the islands at the point of closest approach is 2.1μm compared to a spacing of 0.75μm for the asterisk samples. This comparatively wider spacing partially explains the low value of $T_c$. Another reason for the lower $T_c$ is that this sample was a recycled sample which had sat in the atmosphere for some months before this particular set of measurements was made. Empirically I have discovered that the effect of such aging is to increase the resistance of the sample in the plateau region above 7.2K and to decrease $T_c$.

Figure 35 also shows a plot of critical current as a function of temperature below $T_c$. The critical current is defined as the current at which the voltage across the sample exceeds the noise level of our preamplifier (about 10μV). The critical current rises rapidly and smoothly. Note
FIGURE 36. Resistance vs. temperature and critical current vs. temperature for sample D1.
that the temperature range of interest is very close to the transition temperature of bulk Sn, $T_c(Sn)=3.72K$. In fact the highest critical current plotted is for $T=3.71K$. Thus in analyzing the data one must ask what effect the incipient superconductivity of the Sn (usually expressed in terms of a divergent normal metal coherence length, $\xi_n(T)$) has on the observed IV characteristics. (Although one must ask this question, one will not find an answer to it in this thesis.) In fact I will later show the results of measurements on two other samples in which the IV characteristics show similar behavior to the sample under discussion at temperatures well below 3.72K. Unfortunately we have insufficient data to make strong correlations between sample characteristics and the nature of the low temperature IV characteristics.

Figure 37 shows a series of IV characteristics corresponding to the critical currents shown in Figure 35. Just below the transition $T_c=3.91K$ the IV characteristics are continuous and nonlinear. At about .1K below $T_c$, however, the form of the IV characteristics changes, developing a pronounced voltage step structure. In this temperature regime the voltage increases in discontinuous steps followed by regions of roughly constant differential resistance, $dV/dI$. Furthermore the voltage drop in the regime of the step IV characteristics is localized toward the center of the sample as determined by the fact that no voltage drop was measured between the current pins and the voltage pins.
FIGURE 37. Pulsed IV characteristics for sample D1 showing the crossover from continuous nonlinear behavior to step like behavior.
Figure 38 shows the IV characteristics in the step structure regime replotted on an expanded current scale. The lines drawn through the linear region above the voltage steps are the results of least squares fits to the data. Several regularities in the IV characteristics may be immediately noticed. First, the critical current, which in this temperature regime corresponds to the current at which the first voltage step occurs, increases linearly with temperature. This linearity is shown clearly in figure 39 which plots the step critical current as a function of temperature. Returning to figure 38 we see that the slope of the linear region above the first voltage step is a constant independent of temperature. The magnitude of $dV/dI$ is about $8 \times 10^{-3} \Omega$. Table 1 lists the value of the slope for all five temperatures as determined from a least squares fit. There is considerable scatter in the slopes probably due to the low signal to noise ratio of the data. Nevertheless the slopes do not vary by more than about 10% and there is no clear 'trend' in the data. The linear portion of the IV characteristic intersects the current axis at a large fraction of the critical current. This current intercept is called the "excess current" because it is the amount of extra current required to flow through the sample to produce a given voltage drop compared to an ohmic resistor with $R=dV/dI$. Because of the approximately constant slope of the IV characteristics above the first voltage step and the
FIGURE 38. Step structure in the IV characteristics of disk array D1.
linear dependence of the step critical current the "excess current" also has a linear temperature dependence.

At all temperatures there is also a second voltage step followed by a second region of roughly constant dynamic resistance, \( \frac{dV}{dI} \). In all cases the slope of this second linear region is steeper than that of the first linear region. In table 1 the values of the slope \((R_2)\) of this second linear region are listed. The data for \( T=3.74K \) are not good enough to extract a slope. In this case it is clear that the slopes are increasing with temperature. At the lowest temperature listed, \( T=3.71K \), there was clear hysteresis in the second linear region between the upward and downward parts of the pulse; The downward trace is displaced slightly upwards along the voltage axis. This hysteresis is evidence of heating and offers a possible explanation for the increasing slopes of the second linear region; Heating of the sample causes the voltage across the sample to increase more rapidly with current than it would in the absence of heating. Thus at lower temperatures where the second linear region of the IV characteristic corresponds to higher power dissipation, \( P=IV \), one might expect the voltage to increase faster with increasing current than at higher temperatures. At the two temperatures just below 3.71K there is no detectable hysteresis in the second linear region and it is interesting to note that the two values of \( R_2 \) for these two temperatures are approximately equal.
TABLE 1. Dynamic Resistance of the Linear Portions of the IV Characteristics for Sample D1.

<table>
<thead>
<tr>
<th>T(K)</th>
<th>R1(10⁻³Ω)</th>
<th>R2(10⁻³Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.75</td>
<td>7.74</td>
<td>15.7</td>
</tr>
<tr>
<td>3.74</td>
<td>8.52</td>
<td>---</td>
</tr>
<tr>
<td>3.73</td>
<td>8.23</td>
<td>34.2</td>
</tr>
<tr>
<td>3.72</td>
<td>7.70</td>
<td>36.5</td>
</tr>
<tr>
<td>3.71</td>
<td>8.00</td>
<td>42.9</td>
</tr>
</tbody>
</table>
Although there is no hysteresis in the linear portions of the IV characteristics there is critical current hysteresis. That is the critical current on the downward part of the current pulse is less than on the upward part. Figure 40 shows this hysteresis at $T=3.72K$. It should be noted that this hysteresis is reproducible. That is if a series of traces are taken the voltage steps up and down at the same values of current for each trace.

Similar behavior in the IV characteristics has been observed in two other samples. Figure 41 shows a series of IV characteristics for sample D2. This sample also had disk shaped islands and the spacing between the islands at the point of closest approach was $1.5\mu m$. Again we see a discontinuous voltage jump at the critical current followed by a region of constant differential resistance $R=dV/dI$. As before this linear portion of the IV characteristic intersects the current axis at a large fraction of the critical current. The magnitude of $R$ is roughly independent of temperature and for this sample has the value $R=30\times10^{-3}\Omega$. As with sample D1 the voltage drop is localized toward the center of the sample.

The voltage step is an order of magnitude larger in this sample than in sample than in sample D1 and the value of $R$ is a factor of four larger. Note that the temperature at which we observe this step like behavior are below the bulk Sn transition at $T=3.72K$. 
FIGURE 40. Critical current hysteresis in the IV characteristic of sample D1 at T=3.71K.
FIGURE 41. Pulsed IV characteristics for sample D2.

Sample D2

- $T = 3.60 \text{K}$
- $T = 3.55 \text{K}$
- $T = 3.50 \text{K}$
- $T = 3.45 \text{K}$
Figure 42 shows a plot of the step critical current as a function of temperature. Notice that for this sample the critical current does not vary linearly with temperature.

This sample also shows critical current hysteresis and in fact shows a much larger amount of hysteresis than sample D1. Figure 43 shows a typical example of this hysteresis. The difference between the upward and downward critical currents is 15mA compared to 3mA for the example of hysteresis given for sample D1. Note that there is a break in slope between the region above the critical current and the hysteretic region, the latter having a smaller slope. We also see a small amount of hysteresis in the linear region above the critical current. This hysteresis is reproducible from trace to trace and does not depend on the speed with which the current is ramped up and down.

Similar step structure was observed in one additional sample. This sample also had circular islands. The spacing between the islands at the point of closest approach was 3.0nm. In addition to a voltage step, critical current hysteresis and the phenomenon of excess current this sample showed an interesting new behavior. Figure 44 shows a plot of voltage vs. time for three different temperatures for sample D3. Each trace represents the response of the sample during a single triangular current pulse. The upper trace corresponds to a pulse length of .25ms while the two lower traces correspond to a pulse length of .5ms. The different
FIGURE 42. Step critical current as a function of temperature for sample D2.
FIGURE 43. Critical current hysteresis in the IV characteristic of sample D2 at T=3.55K.
FIGURE 44. Voltage response of sample D3 during a single triangular current pulse for three different temperatures.
positions of the peaks of the voltage response is due to different pulse lengths used for different traces. The new behavior we observe is the appearance of precursor voltage spikes before the region of constant dV/dI is reached. Note that the spikes are asymmetric, the rise being immeasurably fast on this time scale and the fall taking about 10μs. The number of precursor spikes increases as the temperature decreases. Note that at the lowest temperature shown where there are three spikes the last two are spaced closer together in time than the first two.

In fact if one passes a dc current through the sample and observes the amplified voltage output on an oscilloscope one finds that the sample outputs voltage spikes periodically with the period depending on the current. One can determine the period with a frequency counter. Figure 45 shows a plot of pulse period vs. current for T=3.45K. We see that the spike period increases smoothly as the current decreases with an apparent divergence at about 60mA.

This sample exhibits obvious hysteresis in that the spikes appear during the upward ramp but not during the downward one. It also exhibits the more familiar critical current hysteresis if the critical current during the upward ramp is defined as the point of minimum voltage between the last precursor spike and the region of constant dV/dI. Figure 46 shows the critical current hysteresis at T=3.50K. Note that the critical current is about 10mA, again consid-
FIGURES 45. Spike period as a function of current for sample D3.
Figure 46. Hysteresis in the voltage response of sample D3 at $T=3.50K$. 
erably larger than in sample D1. We also see that there is quite a large amount of hysteresis in the region of constant \( \frac{dV}{dl} \) above the voltage step; The slope \( \frac{dV}{dl} \) during the upward part of the current ramp is considerably larger than the slope during the downward part of the ramp. Note that this sample also exhibits the excess current phenomenon.

After sample D3 I measured a series of disk samples with a spacing between the edges of adjacent disks of 3\( \mu \)m or more. While all of these sample developed large critical currents below \( T_c \), no step structure was observed in the IV characteristics. Even at critical currents in excess of 100mA the voltage increased in a continuous nonlinear manner. The samples of closer spacing were made by pushing the photolithographic system available to us far beyond its 5\( \mu \)m limit. The batch of photoresist patterns with the smaller spacing proved to be a fluke and could not be reproduced even after months of effort to do so. Eventually the asterisk arrays with .75\( \mu \)m spacing were made at the submicron facility at Cornell University. However, these samples also showed no step structure even at very large critical currents. It appears that some intermediate range of coupling is required to produce these effects.

Finally I would like to comment on the fact that for samples D2 and D3 the phenomenon of interest occurred at temperatures well below the transition temperature of Sn. In this temperature regime the sample is an array of SS'S
junctions rather than SNS junctions. It is not clear how the physics of the two systems is related. As long as we are not too far from the transition of the Sn the critical current density in the junction region will be much lower than in the Pb islands so that the array is still an array of weak links. However, if the array properties depend strongly on the nature of the coupling between the superconducting islands then there is no reason to expect the behavior of the two systems to be similar. Without further systematic data on the dependence of these IV characteristics on sample parameters nothing further can be said about this matter.
IV. DISCUSSION

The simplest case in which the current induced breakdown of superconductivity is relatively well understood is that of a single Josephson junction. The response of a single Josephson junction to currents greater than the critical current has been successfully described by a phenomenological model known as the Resistively Shunted Junction (RSJ) model. In this model it is assumed that an ideal Josephson element obeying the Josephson current phase relation is shunted by a resistance $R_m$.

The total current passing through the junction is given by

$$I = I_c \sin \gamma + \frac{V}{R_m} \quad (4.1)$$

where $\gamma$ is the phase difference across the junction and $V$ is the voltage drop across the shunting resistor $R_m$. If $I < I_c$ then all of the current flows through the Josephson element and the second term in above equation is zero. When $I > I_c$ then $V > 0$ and the phase difference $\gamma$ is changing in time according to the ac Josephson relation

$$\frac{d\gamma}{dt} = \frac{2eV}{\hbar} \quad (4.2)$$
Substituting for $V$ in equation (4.1) one obtains
\[ i = i_c \sin \gamma + \left( \frac{n}{2eR_n} \right) \frac{dy}{dt}. \] (4.3)

This equation can be solved analytically for $\gamma(t)$ after which $V(t)$ is obtained from (4.2). The result is
\[ V(t) = R_n ic \left( \kappa^2 - 1 \right) \left\{ \kappa + \sin \left[ \left( \kappa^2 - 1 \right)^{1/2} + \Delta \right] \right\} \] (4.4)
where $\kappa = i/i_c$, $\theta = \left( 2eR_n i_c/n \right) t$, and $\tan \Delta = (\kappa^2 - 1)^{-1/2}$. It may be seen that $V(t)$ is a periodic function of time with period
\[ P = 2\pi \left( \frac{n}{2eicR_n} \right)[(\kappa^2 - 1)]^{-1/2}. \] (4.5)

Figure 47 shows a plot of $V(t)/icR_n$ for $\kappa = 1.1$. Note that the voltage occurs in steep spikes followed by regions of low slowly changing voltage. We may understand this behavior qualitatively in the following way. The region of lowest voltage will occur when the Josephson current element is carrying as much current as possible, i.e. the critical current $i_c$. Therefore the phase difference across the junction is $\pi/2$. Since the total current is greater than the critical current there will still be a voltage drop across the junction and the phase difference will be changing in time according to (4.2). As the phase difference changes the amount of supercurrent passing through the junction decreases and consequently the fraction of the current shunted through the resistor increases. At the same time the voltage drop across the resistor increases which increases the rate
FIGURE 47. The normalized voltage \( \eta \) as a function of the reduced time according to the RSJ model of a Josephson junction.
of change of the phase thus leading to snowballing effect in which one expects the voltage to change most rapidly when it is at its largest value. Thus one expects voltage spikes.

The dc voltage is the time average of $V(t)$ and is given by

$$V_0 = \langle V(t) \rangle = R \text{nic} [\kappa^2 - 1]^{1/2}.$$  \hfill (4.6)

This expression may be used to rewrite equation (4.3) for the instantaneous voltage as

$$V(t) = R \text{nic} (\kappa^2 - 1)/\{\kappa + \sin(\omega_0 t + \Delta)\}$$  \hfill (4.7)

where $\omega_0 = 2eV_0/h$. Thus we see that the frequency of the voltage variation is just the Josephson frequency corresponding to the dc component of the voltage. The constant of proportionality between the frequency $f$ and voltage $V_0$ is approximately $500$ MHz/µV. In most experimental situations the real time variation of the voltage is not measured but only the time averaged voltage as evinced by the dc IV characteristic. The IV characteristic is given by (4.6) and is plotted in Figure 48. We see that the voltage rises with infinite slope at the critical current and asymptotically approaches the line $V_0 = iR_n$. IV characteristics with this shape have been observed in real weak links in a variety of experiments.

In some experimental situations, however, critical current hysteresis is observed. For structures with large
FIGURE 48. The IV characteristic of a Josephson Junction according to the RSJ model.
capacitance such as SIS tunnel junctions hysteresis can easily be understood in the context of the RSJ model. However, in weak link structures which exhibit normal type conductivity, such as SNS junctions, it is believed that the capacitance is negligible so that the RSJ model predicts no hysteresis. Nevertheless critical current hysteresis has been observed in SNS junctions. It has been demonstrated fairly conclusively that this hysteresis is due to heating. As shown by Gubankov et al heating effects can be modeled in a simple way. One can assume that the junction is heated above the bath temperature according to the relationship

\[ \Delta T = \kappa i V \]  

(4.8)

where \( \kappa \) is the thermal conductance between the junction and the helium bath. The IV characteristic is still given by (4.6) with

\[ i_c = i_c(T + \Delta T) = i_c(T + \kappa i V_0) . \]  

(4.9)

Since \( V_0 \) appears in the argument of \( i_c \), equation (4.6) now only gives the VI characteristic implicitly. If one assumes a linear dependence of \( i_c \) on temperature as

\[ i_c(T + \Delta T) = i_c(T) + \alpha \Delta T \]  

(4.10)

one can solve for \( V_0 \) explicitly. In terms of the reduced variable \( v' = V / i_c R_w \) and \( i' = i / i_c \) the result is
where $\Theta = i_c(T)Rw/a_x$ is a dimensionless parameter which characterizes the severity of heating effects. Figure 49 shows a plot of $i'$ vs. $v'$ for several values of $\Theta$. We see that for large enough values of $\Theta$ heating gives rise to a region of negative resistance in the VI characteristic. If a current source is used rather than a voltage source the region of negative resistance give rise to critical current hysteresis; The voltage jumps to the asymptotic line $V_0 = iR_w$ at the critical current and jumps down to zero again at the beginning of the region of negative $dV/dI$.

The second simplest case in which some progress has been made toward understanding the breakdown of superconductivity by currents in excess of the critical current is that of thin superconducting wire or filaments with transverse dimensions less than $\xi$ and $\lambda$, the coherence length and penetration depth respectively. In this case the magnetic field may be neglected and one can take $A=0$. Then the second Ginzburg-Landau equation for the supercurrent density is given by

$$J_s = \frac{e\hbar}{m|\nabla|^2} q$$

(4.12)

where $\Psi(x) = |\Psi(x)|e^{i\Phi(x)}$ is the Ginzburg-Landau order parameter and $q(x) = d\Theta/dx$. $|\nabla|$ and $\Theta$ are functions only of the position, $x$, along the length of the filament. The usual
FIGURE 49. The IV characteristics of a Josephson junction with heating. The parameter $\beta$ is a measure of the severity of the heating.
current carrying solution of the Ginzburg-Landau equation in this one-dimensional case is

\[ \psi = \psi_0 e^{i q x} \]  \hspace{1cm} (4.13)

where

\[ \psi_0^2 = \psi^2 (1 - q^2 \xi^2) \]  \hspace{1cm} (4.14)

and \( \psi \) is the zero current value of \( \psi_0 \). The maximum stable supercurrent density occurs when \( q \xi = 1/\sqrt{3} \) and \( \psi_0^2 = (2/3) \psi^2 \). Any higher phase gradient \( q \) is unstable causing the super-electron density \( n_s = |\psi|^2 \) to collapse to zero. The collapse is expected to be localized in a length of order \( \xi \) with \( |\psi| \) driven to zero only at a point. Clearly the current at this point must be carried by the 'normal' electrons. The normal current gives rise to a voltage drop which in turn gives rise to a time variation of the phase gradient according to the ac Josephson relation. Thus after the collapse of the order parameter \( q \), \( |\psi| \), and \( J_s \) begin to increase again. After \( J_s \) reaches its maximum value again the process is repeated. The frequency associated with this cyclic process is just the Josephson frequency associated with the voltage drop along the filament.

The above description of the nonequilibrium time dependent state of a superconducting filament is quite sketchy and phenomenological. However, it has been accepted as reasonably correct because it gives a reasonably accurate
description of real experiments. As in the case of a single
Josephson junction a typical experiment consists of the
measurement of dc IV characteristics. Webb and Warburton
were the first to report the existence of regular step
structure in the IV characteristics of Sn whisker
crystals. The same system was subsequently studied in
greater detail by Meyer and Minnegerode, and Skocpol,
Beasley, and Tinkham observed similar behavior in thin film
Sn microbridges.

In all these experiments the voltage was seen to
increase in discontinuous jumps followed by regions of
constant dynamic resistance \( \frac{dV}{dI} \). The dynamic resistance is
found to increase by a constant amount, \( R \), after each
voltage step. These linear portions of the IV characteristic
are found to intersect the current axis at about 50% of the
critical current \( i \). Skocpol et. al. were the first to
establish that each step is associated with a spatially
localized voltage unit along the length of the bridge. Fur­
thermore, they find it is possible to induce horizontal
steps on the IV characteristics by the application of
microwaves. The steps occur whenever the voltage across any
voltage unit satisfies the ac Josephson relation
\[ 2\pi f = \frac{2e}{\hbar} V \]
where \( f \) is the microwave frequency.

Skocpol et. al. have also presented a phenomenological
model of this behavior which includes all of the ingredients
mentioned above. In addition they introduce a second length
scale to describe the disequilibrium between the superconducting and normal electrons. Recall that the order parameter $|\Psi|$ collapses to zero over a length scale of order $\xi$. The consequent flow of normal current causes the time averaged supercurrent $\langle J_s \rangle$ to be less than the critical current density $J_c$. It is not necessarily true that the length scale over which $\langle J_s \rangle J_c$ is of order $\xi$. In fact it is found experimentally found that in Sn microbridges the length over which the voltage drop occurs is of order $10\mu m$ which is much larger than the coherence length of Sn.

Scocpol et. al. identity this length with the quasiparticle diffusion length, $\Lambda$ first introduced by Pippard et. al. to explain the excess resistance at SN interfaces. It is a simple consequence of the model of Scocpol et. al. that each voltage step is associated with a differential resistance

$$R = \frac{dV}{dI} = 2\rho_n \Lambda / A$$

where $\rho_n$ is the normal state resistivity of the Sn and $A$ is the crosssectional area of the bridge. $R$ is just the normal state resistance of a $2\Lambda$ length of the bridge. Note that a consequence of this model is that in perfectly homogeneous wire much longer than one cannot nucleate a single phase slip center; If one moves more than a length $\Lambda$ from the phase slip center the supercurrent density $\langle J_s \rangle$ will again reach the critical value $J_c$ and one must nucleate a second phase slip center. In practice real superconducting filaments are inhomogeneous and some points along the wire have lower critical current than others. At
the weakest of these points the first phase slip center will nucleate.

The term "phase slip center" comes from the supposed time variation of the phase gradient \( q = \frac{d\phi}{dx} \). It is believed that when \( J_s \) and \( q \) reach their maximum values they are abruptly driven to zero and then build back up to their maximum values during the rest of the Josephson cycle. This abrupt loss of "phase coherence" is referred to as a phase slip event.

Phase slip centers have sometimes been modeled as Josephson elements with a non-sinusoidal current phase relation. That is \( i_s = i_c f(\gamma) \) where \( f \) is some 2\( \pi \) periodic function other than the \( \sin \). In this case the differential equation for the phase difference \( \gamma \) becomes

\[
i = i_s f(\gamma) + \frac{2e}{h R_m} \frac{d\gamma}{dt}. \quad (4.15)
\]

Again one solves for \( \gamma(t) \) and differentiates to get \( V(t) \). The IV characteristic is determined by computing the time average voltage \( V_0 = \langle V(t) \rangle \). The detailed behavior of the IV characteristic near \( i_c \) depends on the function \( f(\gamma) \). However, for \( i \gg i_c \) the IV characteristic approaches the asymptotic line

\[
V_0 = R_m (i - \langle f \rangle i_c) \quad (4.16)
\]

where \( \langle f \rangle \) is the average of the function \( f \) over a single period. Thus this generalized RSJ model exhibits the excess
current phenomenon with \( i_{\text{ex}} = \langle f \rangle i_c \). If \( f \) is the \( \sin \) then \( i_{\text{ex}} = 0 \). Figure 50 shows the IV characteristic for the case

\[
f(\gamma) = \gamma \left( \text{mod} 2\pi \right) / 2\pi
\]

(4.17)

which is just a sawtooth pattern. Note that the curve looks very similar to that of the original RSJ model. The main difference is the excess current given by \( i_{\text{ex}} = \langle f \rangle i_c = 0.5 i_c \). This model is of course phenomenological. We have no first principles method of calculating the function \( f(\gamma) \).

The relevance of these results to the data on proximity-coupled arrays is clear. The voltage drop associated with the steps we observe in the IV characteristics of proximity-coupled arrays is localized along the length of the array. By comparing the dynamic resistance associated with a voltage step to the resistance of the array in the 'normal' state we can estimate what fraction of the length of the array is involved in the dissipation. We take the resistance in the center of the plateau region below the transition temperature of the Pb islands to be the 'normal state' resistance. For sample D3 the ratio \( R/R_n \approx 0.044 \) corresponding to about 25 rows of junctions. For samples D1 and D2 we cannot determine \( R/R_n \) precisely because the pulsed IV data was taken on a separate run from the resistance data, and experience has shown that thermal cycling increases the resistance of the samples. Nevertheless we can estimate \( R/R_n \) for these two samples and we find \( R/R_n > 0.5 \).
FIGURE 50. The IV characteristic of a Josephson junction according to the GRSJ for the case $f(\gamma) = \gamma (\text{mod} 2\pi)/2\pi$. 

$$V = R_N(i' - i_{ex}')$$

$$i_{ex}' = 0.5$$

$$1/\ln(\frac{i'}{i - 1})$$
corresponding to about 300 rows of junctions. There is an order of magnitude difference in the scale of localization of the current dissipation between these two samples and that of the third sample. We also note that the size of the voltage step in these two samples is also an order of magnitude larger than in the third sample.

The fact that all three samples exhibit the excess current phenomenon shows that the dissipative state is not simply due to a section of the sample being driven normal by local heating. The sample is in some kind of nonequilibrium state in which current is being carried by both the superconducting electrons and the normal quasiparticles. We do not have enough systematic data on how the IV characteristics behave as a function of samples parameters to determine any of the details of this dynamic state.

The one clue we have is the precursor voltage spikes observed in the pulsed IV characteristics of sample D2. These spikes are reminiscent of the voltage peaks in the RSJ model of the dissipative state of a single Josephson junction. We can attempt to fit the current dependence of the spike period using the RSJ expression

$$P = (h/2eicR)(((i/ic)^2-1)^{-1/2}).$$

(4.18)

Figure 51 shows a plot of spike period vs $$((i/ic)^2-1)^{-1/2}$$ where ic has been used as an adjustable parameter. The fit
FIGURE 51. A plot of spike period vs. $[(i/i_c)^2 - 1]^{-1/2}$. 

Sample D3  
$I_c' = 60 mА$
is quite good. Note that $i_c$ in this expression is the current at which the divergence in the spike period occurs and not the current at which the IV characteristic begins to exhibit a constant dynamic resistance $dV/dI$. If we assume that the entire array is acting like a single Josephson junction then taking the slope of this line and multiplying by $2eic/h$ should give the characteristic normal state resistance $R_n$. In fact the slope $m=5.163E-05$ and $(2eic/h)m=1.06E-10Ω$. This resistance is many orders of magnitude smaller than the dynamic resistance $R$ of the IV characteristic above the upper critical current $i_c$. Another way of looking at this fact is that the characteristic voltage of the RSJ model is $V_c=i_cR_n=6.4X10^{-12}V$ where $i_c$ and $R_n$ have again been taken from the fit to the spike period data. This voltage is much smaller than the characteristic voltages of the IV characteristics in these arrays. Recalling the proportionality between voltage and frequency in the a.c. Josephson effect is $500MHz/μV$ we see that the frequency of these spikes is much too low to be associated with the Josephson effect in the individual SNS junctions. It is possible that the spikes are due to some collective effect of the array such as the passage of vortices across the width and that the time scale is set by the physics of this collective process (by the vortex mobility for instance). Again we do not have sufficient systematic data about the dependence of this property on array parameters to
speculate intelligently about the physics.

Another interesting issue is whether the critical current hysteresis observed in the IV characteristics is due to heating effects or to some other cause. We have already noted that for sufficiently fast pulse times the voltage response of the sample saturates. This fact implies that at worst the phonons in the sample are being heated by an amount $\Delta T = Q R_r$ where $Q$ is the heat flux per unit area and $R_r$ is the thermal boundary resistance between the film and the substrate (We are ignoring gradients across the thickness of the film.). We can estimate $R_r$ using the standard acoustic mismatch model\(^7\) in which the thermal boundary resistance between two solids is given by

$$R_r = \frac{\Pi^2 k_B^4}{30k^3} \left( \frac{\eta_1}{s_1^2} + \frac{2\eta_t}{s_t^2} \right) \quad (4.19)$$

where $s_1$, $s_t$ are the longitudinal and transverse sound velocities in the hotter of the two solids and $\eta_1$, $\eta_t$ are the longitudinal and transverse phonon transmission coefficients of the boundary between the two solids. (Note that $R_r$ also depends on $s_1$, $s_t$ of the cooler solid but this dependence is hidden in $\eta_1$, $\eta_t$.) If we assume that the dissipation is confined to the Sn between the superconducting discs then the needed values of $s_1$ and $s_t$ are those of Sn which are given in the literature\(^8\).

Furthermore the phonon transmission coefficients between Sn
been measured. The known values of these numbers are $s_1 = 3.5 \times 10^2 \text{m/sec}$, $s_2 = 1.82 \times 10^2 \text{m/sec}$, $\eta_1 = 0.44$, and $\eta_2 = 0.15$. The temperature dependence of $R_T$ does not vary much over the range of interest. We take $T = 3.8$ to get a typical value of $R_T$. We find $R_T = 1.212 \times 10^{-5} \text{Ksm}^2 / \text{J}$.

We now assume the worst case in which the dissipation is confined to a single row of junctions across the width of the array. We can then estimate the amount by which the Sn in that row is heated above the temperature of the substrate. The appropriate area for computing the heat flux $Q$ is the area of a single junction times the number of junctions across the width of the array. We take the area of a single to be $1.2 \mu\text{m} \times 8 \mu\text{m} = 9.6 \times 10^{-12} \text{m}^2$. In the 1 cm width of the sample there are approximately 1000 junctions. Thus the total interface area is approximately $10^{-6} \text{m}^2$. The characteristic power dissipated in the arrays is $P_c = I_c V_{\text{step}}$. In sample D3 the characteristic power dissipated when the array is in the resistive state is $P_c = I_c V_{\text{step}} \approx 30 \text{mA} \times 0.03 \text{mV} = 9 \times 10^{-7} \text{W}$. Again taking $T = 3.8 \text{K}$ we calculate the temperature difference between the Sn and the substrate. We find $\Delta T = \dot{Q} R_T = (P/A)R_T \approx 1 \text{mK}$. Figure 40 shows that the critical current hysteresis at 3.71K is about 3mA. Comparing this number with the slope of the critical current vs temperature curve, $m = 0.309 \text{mA/mK}$, shows that the Sn must be 10mK hotter than the substrate to account for the hysteresis thermally. Thus the amount of heating calculated from an estimate
of the thermal boundary resistance between the Sn and the substrate is an order of magnitude too small to account for the observed hysteresis. Recalling that the calculation assumes that the dissipation is confined to a single row of junctions when it is more reasonable from the magnitude of the dynamic resistance to assume that the dissipation is spread out over 25 rows makes a stronger case against heating as the cause of hysteresis.

There is another qualitative feature of the data for sample D3 which indicates that heating is not the cause of hysteresis. Up to the second voltage step the IV characteristics take the form:

\[
V = \begin{cases} 
0, & I < I_c \\
R_N (I - I_{ex}), & I > I_c 
\end{cases} \tag{4.20}
\]

where \( I_{ex} = I_{ex}(T) = I_c(T) - (V_{step}/R_N) \) and \( I_c(T) = A - BT \). \( A \) and \( B \) are determined from the fit to the data of Figure 52. We have already seen that \( V_{step} \approx 0.025 \text{mV} \) and \( R_N \approx 8 \text{m}\Omega \). Figure 52 shows a series of IV characteristics of this form. Suppose that this form is the true constant temperature form of the IV characteristics. Then if there is sufficient heating to move the critical current a significant amount along the current axis the IV characteristic above the voltage step will have to curve upwards with increasing current as shown by the curved line in the figure. It is interesting to compare this result with the RSJ model of a Josephson junction in which
FIGURE 52. A schematic plot of IV characteristics showing the effect of heating.
there is also a temperature dependent critical current. In this case, however, the IV characteristic tends to the same asymptotic line, $V=IR$ at all temperatures making it quite easy to understand the thermal origin of hysteresis as was seen earlier in the discussion.

For sample D2 the characteristic power is $P=icV \Delta e = 50\text{mA} \times 0.5\text{mV} = 2.5 \times 10^{-5}\text{ W}$. The temperature difference between the Sn and the substrate is given by $\Delta T = \Delta R = (P\Delta T/A) = 3.03 \times 10^{-2}\text{ K}$.

Is this amount of heating sufficient to account for the observed hysteresis? If we look again at Figure 43 we see that at $T=3.6\text{K}$ the voltage jumps up at $I=50\text{mA}$ and jumps down at $I=38\text{mA}$. Looking at the plot of $I_c$ vs $T$ in Figure 42 we see that this amount of hysteresis corresponds to a temperature difference of about $0.06\text{K}$. Considering that the formula (4.19) used to calculate $R_r$ assumes perfect defect free crystals and a perfect strain free interface between the two solids it will probably provide a somewhat low estimate of $R_r$. Thus when we make the extreme assumption that the dissipation is confined to a single row of junctions the heating of the Sn is sufficient to account for the hysteresis. However, in this case it is not clear why the dynamic resistance of the array should be so high. If the dissipation is spread out over the scale indicated by $R_d/R_m$ then the heating of the Sn is not sufficient to account for the observed hysteresis. It should also be noted that this
simple minded model of the critical current hysteresis does not account for the break in slope observed at the beginning of the hysteretic region as seen in Figure 43.

If the hysteresis is not due to heating its cause must be looked for in the dynamics of time dependent state of the superconductor. Again too little is known about the physics to speculate intelligently.

In conclusion the IV characteristics of triangular arrays of proximity-coupled junctions have been measured at temperatures well below the array transition temperature $T_c$. These IV characteristics show certain features in common with the IV characteristics of other low dimensional superconductors indicating that the arrays are in a dynamic time dependent state in which current is being shifted back and forth from the superconducting condensate to the normal electrons or quasiparticles. There exists insufficient systematic data to determine any of the details of this dynamic state.
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APPENDIX A

Applicability of Model Hamiltonian to Real Arrays

The correct Hamiltonian for the case in which we cannot ignore variations in $\phi$ and $A$ over the width of the grain is

$$ H = -J \sum_{<ij>} \cos[\phi_{i} - \phi_{j} - (2\pi/\Phi_{0}) \int_{iE}^{j} A \cdot d\mathbf{l}] $$

(A.1)

where the subscript $E$ denotes the edge of the SNS junction. In fact this Hamiltonian can be replaced by

$$ H = -J \sum_{<ij>} \cos[\phi_{iC} - \phi_{jC} - (2\pi/\Phi_{0}) \int_{iC}^{jC} A \cdot d\mathbf{l}] $$

(A.2)

where the subscript $C$ denotes the center of a superconducting island. We can easily see this since

$$ \int_{iC}^{jC} A \cdot d\mathbf{l} = \int_{iC}^{jE} A \cdot d\mathbf{l} + \int_{iE}^{jE} A \cdot d\mathbf{l} + \int_{jE}^{jC} A \cdot d\mathbf{l}. $$

(A.3)

Inside the superconducting islands the variation of the phase is governed by the Ginzburg-Landau equation

$$ \mathbf{j}_{S} = (e/m) n s^2 [\mathbf{\nabla} \phi - (2e/c) \mathbf{A}]. $$

(A.4)

Since the superconducting islands are large compared to $\lambda$ we
can take \( \vec{J}_0 = 0 \) in the interior so that

\[
\vec{\nabla} \phi = (2e/hc) \vec{A} = (2\pi/\hbar) \vec{A}
\]

(A.5)

where \( \hbar = hc/2e \) is the superconducting flux quantum. Thus the pieces of the integral over the vector potential inside the superconducting island give

\[
(2\pi/\hbar) \int_{i\text{c}}^{i\text{c}} \vec{A} \cdot d\vec{l} = \phi_{i\text{c}} - \phi_{i\text{c}}
\]

(A.6)

and

\[
(2\pi/\hbar) \int_{j\text{c}}^{j\text{c}} \vec{A} \cdot d\vec{l} = \phi_{j\text{c}} - \phi_{j\text{c}}.
\]

(A.7)

Thus the expression (A.2) can be written

\[
H = -J \sum \langle ij \rangle \cos[\phi_{i\text{c}} - \phi_{j\text{c}} + \phi_{i\text{c}} - \phi_{j\text{c}} - (2\pi/\hbar) \int_{i\text{c}}^{j\text{c}} \vec{A} \cdot d\vec{l}].
\]

(A.8)

This is just the Hamiltonian (A.1). Therefore the Hamiltonian used in theoretical treatments of Josephson junction arrays applies to real proximity-coupled arrays.
APPENDIX B

Derivation of the Reduced Temperature $T'$

In calculating the critical properties of the X-Y model it is typical to calculate physical quantities as a function of the reduced temperature

$$t = \frac{(T - T_c)}{T_c}.$$  \hspace{1cm} (B.1)

We recall that $k_B T_c = AJ$ where $J$ is the Josephson coupling constant between adjacent grains and $A$ is a constant of order unity. If $J$ is temperature dependent the for $T > T_c$ the sample will behave as if it had an effective transition temperature

$$T_{ceff} = \frac{AJ(T)}{k_B}.$$  \hspace{1cm} (B.2)

Thus the appropriate reduced temperature for calculating the critical properties of the model is

$$t = \frac{(T - T_{ceff})}{T_{ceff}}.$$  \hspace{1cm} (B.3)

Dividing numerator and denominator by $T_c/T_{ceff}$ we find

$$t = \frac{[(T_c/T_{ceff}) T - T_c]}{T_c}.$$  \hspace{1cm} (B.5)

Using the proportionality between $T_c$ and $J$ we find
Thus the temperature $T$ has been replaced by an effective temperature $T' = [J(T_c)/J(T)]T$ which is just the desired result.
APPENDIX C

SQUID Feedback Circuit

For some of the magnetoresistance measurements the galvanometric bridge was balanced automatically using a feedback circuit to control the standard resistor current supply. Figure 53 shows a schematic diagram of this circuit. The output of the SQUID is input into this circuit which is basically an integrator. The output of the integrator is used to control a voltage programmable Hewlet Packard 6177B constant current supply. A voltage divider is used on the output of the integrator to insure that the programming voltage never exceed the specified 1.2V rating of the 6177B.

Since the 6177B current supply is unipolar it was necessary to use an active switch to prevent the programming voltage from going negative. The switch was made using a 741 operational amplifier and a 3N128 MOSFET. When the output of the integrator is positive the output of the 741 hold the gate positive with respect to the source and the MOSFET conducts. When the output is negative the 741 holds the gate negative with respect to the source and the MOSFET does not conduct.
A Keithly isolating amplifier is used to float the programming voltage. This isolation is necessary because the load is grounded in the dewar.

The effect of this circuit is to automatically balance the bridge circuit. When the sample current supply is turned up a current passes through the SQUID pickup coil and causes the output to increase. This output is integrated and fed back to the programming terminals of the standard resistor current supply. The output of this current supply then increases driving the bridge back toward balance. The output of the standard current supply will continue to increase until the integrated voltage is no longer changing, i.e. when the SQUID output is zero.
FIGURE 53. A schematic diagram of the SQUID feedback circuit