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The Ohio State University

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PROTON THRESHOLD STATES IN $^{26}$Al AND
THEIR ROLE IN ASTROPHYSICS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
school of the Ohio State University

By

Vythilingam Wijekumar, B.Sc, M.S.

* * * * *

The Ohio State University
1985

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PROTON THRESHOLD STATES IN $^{26}$Al
AND THEIR ROLE IN ASTROPHYSICS

By

Vythilingam Wijekumar, Ph.D.

The Ohio State University, 1985

Professor Hershel J. Hausman, Advisor

The energy levels of $^{26}$Al between $E_x=6.3$ and 6.5 MeV, corresponding to proton threshold energies in the $^{25}$Mg+p reaction from $E_p=0$ to 200 keV, have been investigated using the reactions $^{27}$Al($^3$He,$\alpha$)$^{26}$Al and $^{24}$Mg($^3$He,p)$^{26}$Al. Despite early work reporting a doublet at $E_x=6346$ keV and $E_x=6362$ keV, most subsequent work reported a single state with conflicting spin and parity assignments. Our work clearly establishes the presence of the doublet and resolves the conflicts. We find that there are six states in this excitation energy region, including a new state at $E_x=6410\pm5$ keV. The $l$-values leading to possible spin and parity assignments for all these states have been made using DWBA.

By measuring the spectroscopic factors of these states from the $^{25}$Mg($^3$He,d)$^{25}$Al reaction, resonance strengths, $\omega_Y$, of the proton threshold states in $^{26}$Al
corresponding to the $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$ reaction have been deduced.

The limits on the branching ratios from these states to the ground state of $^{26}\text{Al}$ have been obtained by using the reaction $^{27}\text{Al}(^3\text{He},\alpha\gamma)^{26}\text{Al}$.

Finally by combining the results of the above experiments, stellar reaction rates of the $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$ reaction have been calculated. Our results conclude that the production rate of $^{26}\text{Al}$ in stellar environments from the reaction $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$ is substantially higher than what was calculated from other work.
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Publications and Communications


MAJOR FIELD OF STUDY

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CHAPTER I
INTRODUCTION

Every living body in this world depends upon the Sun either directly or indirectly, but hardly anyone thinks about how the Sun was formed. Even the few who spend time in researching the mystery of its formation, which happened about some billions of years ago, rarely find useful information to guide them. One such rare event is the Allende meteorite\(^1-3\) which fell in Mexico in 1969.

Meteorites are the best sources of primitive solar-system materials since meteorites apparently never formed aggregates sufficiently large for geochemical cycles, which could alter their isotopic composition as in the case of terrestrial materials. Primitive class meteorites consist of carbonaceous chondrites which contains carbon and some small round inclusions called chondrules showing evidence of having once been melted. It is believed\(^4\) that the carbonaceous chondrites solidified early in the history of the Solar system and then remained almost unaltered.

-1-
The Allende meteorite is a very large (about two tons) carbonaceous chondrite sufficient enough to show most of the isotopic anomalies\textsuperscript{1-3}) found between the primitive Solar system material and the terrestrial ones. An excess over normal abundance (terrestrial) of an isotope is called an overabundance or in more general, an anomaly of that isotope. Although all the anomalies\textsuperscript{1-3}) are important for describing the formation of the Solar system, the most important one at present is the anomaly found in $^{26}\text{Mg}$. The normal isotopic abundance of $^{26}\text{Mg}$ is 11.01\% (78.99\% of $^{24}\text{Mg}$ and 10\% of $^{25}\text{Mg}$). But some minerals of Allende have up to 11.5\% isotopic composition of $^{26}\text{Mg}$.

The excess of $^{26}\text{Mg}$ isotope is believed to be due to $^{26}\text{Al}$ since $^{26}\text{Al}$ is the radioactive parent nuclei of $^{26}\text{Mg}$ and it is further correlated by the ratio of the excess of $^{26}\text{Mg}/^{24}\text{Mg}$ being linearly proportional to the ratio of $^{27}\text{Al}/^{24}\text{Mg}$. The results of Lee, Papanastassiou and Wasserburg\textsuperscript{1,3}) showed that the inclusions had a ratio of $^{26}\text{Al}/^{27}\text{Al}=5\times10^{-5}$ at the time of the formation of the solar system. This opens up a series of questions about how $^{26}\text{Al}$ was produced in nucleosynthesis and how soon it was incorporated into the Allende inclusions since radioactive $^{26}\text{Al}$ has a cosmologically very short mean lifetime for $\beta$-decay of about $10^6$ years.
There are various models used by numerous authors, including the Nobel laureate William Fowler, to solve these problems and other nuclear anomalies found in the Allende meteorite. Most of the models used are based on one of the following category.

1). Supernova
2). Nova
3). A combination of Supernova and Nova
4). Other possible astrophysical scenarios containing $^{26}\text{Al}$ nucleosynthesis (Red giants, etc.).

The stellar explosions called supernovas are rare events. Probably about three supernovas every century occur among the 100 billion stars of our galaxy. According to the supernova theory, some freshly produced $^{26}\text{Al}$ was added to the solar system material just before the beginning of its condensation phase from a nearby supernova explosion which might have triggered the necessary gravitational collapse to form the solar system. The isotope $^{26}\text{Al}$ can be produced in the following zones of supernova:

a) Hydrostatic and/or explosive carbon burning (Arnett and Wefel, Truran and Cameron)

b) Oxygen burning

c) Neon burning (Weaver and Woosley)

d) Silicon burning

e) Helium burning (Chance and Harris)
f) Explosive hydrogen burning (Arnould et al\textsuperscript{10} and Wallace et al\textsuperscript{14})

The ratio of $^{26}\text{Al}/^{27}\text{Al}$ produced in all the above zones except that of the last one is smaller than $10^{-3}$ which lacks room for any dilution of material void of $^{26}\text{Al}$ from other zones or for any reasonable time delay for the condensation of the inclusions. The last zone, explosive hydrogen burning, gives a maximum ratio of one which is somewhat better than the results from other zones.

The isotope $^{26}\text{Al}$ is primarily produced by the capture reaction $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$ in the MgAl cycle (see Fig. 1.1) during hydrogen burning nucleosynthesis. Whether it is in a supernova\textsuperscript{7-11}, nova\textsuperscript{5,10,15-16} or in any other possible astrophysical scenarios, this is probably the only dominant reaction for the production of $^{26}\text{Al}$ that can explain the associated $^{26}\text{Mg}$ anomaly. The temperature and the proton density are the critical parameters that are different for the above mentioned various scenarios.

Figure 1.2 illustrates the importance of the proton threshold states of $^{26}\text{Al}$, formed by the proton capture on $^{25}\text{Mg}$, on astrophysical temperature related to the various scenarios. The excitation energies of the $^{26}\text{Al}$ states are marked in the middle column. The proton capture resonance energies are listed on the left column. The arrows in the
Fig. 1.1. Mg-Al cycle reactions
Fig. 1.2. Energy-level diagram of $^{26}\text{Al}$ above the proton threshold.
right column span the relevant states of $^{26}\text{Al}$ which can dominate the production rate of $^{26}\text{Al}$ at the various astrophysical temperatures written in the middle of the arrows.

At temperatures of $T_9<0.3$, corresponding to explosive hydrogen burning, all the resonances of energy $E_p<320$ keV can possibly contribute to the reaction rates. The information about the resonances for $E_p>200$ keV are well known from the work of De Neijs et al$^{17}$, Elix et al$^{18}$ and Anderson et al$^{19}$. Because of the Coulomb barrier involved, it is not practical to populate the states for $E_p<200$ keV with the direct reaction $^{25}\text{Mg}+p$. Therefore, other reactions must be utilized to identify the relevant nuclear properties of such states. The spin and parity assignments of these states are also required to identify the states with $J^\pi=2^+$ and $3^+$ since they are the only states of $^{26}\text{Al}$ that can be populated by the $l=0$ proton capture of $^{25}\text{Mg}$ which dominates the production rate of $^{26}\text{Al}$.

There were quite a number of experiments$^{18,20-25}$ done in the past to determine the nuclear level structure of $^{26}\text{Al}$. A summary of their results is listed in Table 1.1. As seen from the table, there are conflicting reports. Such as the possible existence of a doublet of states at $E_x=6346$
TABLE 1.1

The previously reported energy levels of $^{26}$Al in the region of interest

<table>
<thead>
<tr>
<th>$^{26}$Al $^{59a)}$ $(\tau,p)(\tau,p)$ $(\tau,p)$</th>
<th>$^{26}$Be $^{72b)}$ $(\tau,p)$ $(\tau,p)$</th>
<th>$^{26}$Be $^{73c)}$ $(\tau,a)$ $(\tau,a)$</th>
<th>$^{26}$Be $^{78d)}$ $(\tau,d)$ $(\tau,d)$</th>
<th>$^{26}$El $^{79e)}$ $(p,\gamma)$ $(p,\gamma)$</th>
<th>$^{26}$Ch $^{83f)}$ $(\tau,a)$ $(\tau,a)$</th>
<th>$^{26}$En $^{78g)}$ adopted value $(\text{keV})$</th>
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<td>$(\text{keV})$</td>
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<td>$(\text{keV})$</td>
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<td>$(\text{keV})$</td>
<td>$(\text{keV})$</td>
<td>$(\text{keV})$</td>
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<tr>
<td>$6335\pm10$</td>
<td>$6361\pm8$</td>
<td>$6358\pm10$</td>
<td>$6360\pm8$</td>
<td>$6343\pm1.6$</td>
<td>$6400\pm3$</td>
<td>$6494.8\pm1.3$</td>
</tr>
<tr>
<td>$6351\pm10$</td>
<td>$6388\pm10$</td>
<td>$6417\pm8$</td>
<td>$6409\pm10$</td>
<td>$6413\pm8$</td>
<td>$6400\pm3$</td>
<td>$6498\pm5$</td>
</tr>
<tr>
<td>$6424\pm10$</td>
<td>$6487\pm10$</td>
<td>$6518\pm8$</td>
<td>$6503\pm10$</td>
<td>$6508\pm8$</td>
<td>$6400\pm3$</td>
<td>$6498\pm5$</td>
</tr>
</tbody>
</table>

a) ref$^{22)}$ b) ref$^{21)}$ c) ref$^{22)}$ d) ref$^{23)}$ e) ref$^{18)}$ f) ref$^{24)}$ g) ref$^{25)}$
and 6362 keV reported by Hinds and Middleton\textsuperscript{20} which was not reported in later work. Also contradictory spin and parity assignments can be seen even for the few states which have assignments. The above results suggest that additional experimental work should be done to resolve these conflicts. Further, these additional studies\textsuperscript{2} should be also able to provide spectroscopic factors for proton capture and branching ratios of the states of interest so that the reaction rates for the production of $^{26}$Al can be calculated to a complete extent.

From the above information, one can easily anticipate that a single experiment cannot provide all the information required. In order to make step by step progress, one should first more accurately determine the number of states of $^{26}$Al near the proton threshold, their excitation energies and their spin and parity assignments.

The reaction $^{27}$Al(\textit{τ},\textit{α})$^{26}$Al is the best one for the above purpose since it can provide a spectrum which is almost free of any background lines and its high Q-value (7.5 MeV) provides a wide separation between the elastic peaks and the states of interest. The energy level density of $^{26}$Al near proton threshold is very high so a mass spectrometer is needed to separate these states. Further a high bombarding energy, 10 to 20 MeV, is
essential in order to utilize the direct interaction theory, Distorted Wave Born Approximation (DWBA), to extract spin and parity assignments. The nuclear structure laboratory at University of Notre Dame has both these facilities and the experiment was done there. The experimental details and the results are described in chapter 2.

Having obtained a clear picture of the nuclear states and possible spin assignments, the next thing was to obtain information about the proton width of the proton threshold states of $^{26}\text{Al}$ from the proton capture of $^{25}\text{Mg}$. Since the reaction $^{25}\text{Mg}(\tau,d)^{26}\text{Al}$ is similar to $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$ (the net transfer is only a proton in both reactions), the former reaction could be used to extract information about the proton width of the latter one. Chapter 3 describes the experimental details as well as the method used to deduce the proton widths. This experiment was again done at the nuclear structure laboratory at University of Notre Dame for the same reasons described in the last paragraph.

The branching ratios of these states are also needed for a complete calculation of the production rate of $^{26}\text{Al}$. However, the high level density of states near the proton threshold makes the job almost impossible. But
limits on branching ratios were obtained by using the reaction $^{27}\text{Al}(\tau,\alpha\gamma)^{26}\text{Al}$ and that was good enough to show the dramatic increase in the production rate of $^{26}\text{Al}$. Our laboratory, Van de Graaff laboratory at Ohio State University, with its new data acquisition program called UNICORN, was used to complete this final experiment. Chapter 4 describes all this in detail.

The theory used for calculating astrophysical reaction rates is described in chapter 5. Our results were also compared with the results of other authors$^{18,19,24}$ to show the remarkable increase in the production rate of $^{26}\text{Al}$ for various astrophysical temperatures over others.

The final chapter 6 concludes with a brief summary of everything achieved from our experiments. It also discusses room for any further improvement.
Chapter II

\( ^{27}\text{Al}(^{3}\text{He},\alpha)^{26}\text{Al} \) Experiment

This chapter describes the experimental set-up, procedure and the results of the analysis of the \( ^{27}\text{Al}(^{3}\text{He},\alpha)^{26}\text{Al} \) and \( ^{24}\text{Mg}(^{3}\text{He},p)^{26}\text{Al} \) reactions. Outgoing particles from each reaction were detected by using Position Sensitive Proportional Counters as well as Nuclear Emulsion Plates. However, the latter detection system was restricted to use only for calibration purposes and for obtaining maximum resolution of the excitation energies as counting the tracks on Nuclear Emulsion Plates is a time consuming process. The angular distribution data obtained from the proportional counter runs was analysed using the DWBA program code CHUCK\(^{26} \). The results are compared with the previous work at the end of the chapter.

Even though both these major reactions were performed at the Nuclear Structure Laboratory at the University of Notre Dame, a preliminary experiment was done at the Van de Graaff Laboratory at Ohio State University. A 15 \( \mu \text{g/cm}^2 \) self supporting natural Al target was bombarded with 6 MeV \( ^{3}\text{He}^+ \) ions from the Van de Graaff accelerator at O.S.U. Outgoing \( \alpha \)-particles were detected at 60 degrees in
a silicon surface barrier detector of 100 \( \mu \text{m} \) thickness which was thick enough to completely stop the \( \alpha \)-particles of interest but not the protons and deuterons of the same energy. The spectrum obtained with this detector indicated the doublet structure of the states at \( E_x=6346 \) and 6362 keV which was not seen in recent work of other authors\(^{21-24}\) and showed us that there is more nuclear structure involved about these states to be examined in later experiments.

2.1 Experimental Set-up for \( ^{27}\text{Al}(^{3}\text{He},\alpha)^{26}\text{Al} \)

A self supporting natural Al target of thickness 30 \( \mu \text{g/cm}^2 \) was used for this experiment. Details of target preparation are described in Appendix A. A beam of 12 MeV \( ^3\text{He} \) ions, produced from the Notre Dame Tandem Van de Graaff accelerator, was used to bombard the Al target at the center of an ORTEC chamber. Outgoing particles were momentum analysed in a magnetic spectrometer, equipped with a position sensitive dual proportional counter of dE-E type of 40 cm effective length. The function of the position sensitive proportional counter(PSPC) is described below.

The PSPC at the focal surface of the spectrograph allows for real time counting of particles scattered into the spectrograph, recording the position of the particles and identifying the particle type by acting
as a dE/dX detector. The description of a PSPC is given below.

The PSPC is a rectangular box of length about 50 cm and width of about 5 cm. with a carbon coated quartz fiber wire passing through its central longitudinal axis. There is a thin mylar window to allow particles to enter it. A P10 gas mixture (90% argon and 10% methane) is used to fill the cavity inside at a rate of about .5cc/sec at 1 atmospheric pressure. The continuous flow of gas avoids the build up of electronegative ions (O, H₂O) outgassing from the walls. A Voltage difference about 800 to 1400 Volts is applied between the wire and the case of the counter. Each charge particle entering the counter creates electron-positive ion pairs and loses about 30 keV (energy lost for 15 MeV protons) to several MeV of energy (for low energy alphas). Since the center wire has a relative positive potential, the electrons will be accelerated towards the center and create more ionization in the gas. This results in an amplified negative pulse of electrons collected in the center wire. Similarly the positive ions give a positive pulse in the case of the counter. Both the positive and the negative pulses can be used to determine the energy lost by the charged particle.
The dual proportional counter has two wires. The front wire with two preamplifiers at both ends is used to determine the position of the momentum analysed particles. The ionization is localized along the wire by the geometry when a charge is deposited at some position of the wire. This position can be detected in two ways. Since the resistive wire acts as a charge divider circuit (see Fig. 2.1), when a charge $Q$ is deposited at some point $X$ on the wire, a charge of $Q_a = XQ/L$ will be detected at A and a charge of $Q_b = (L-X)Q/L$ will be detected at B. The charge on the case (or back wire) is $-Q$ regardless of the position. $X$ can be deduced in several ways from this data. This method was not used in our experiment. However the charge division of this method is important. In the method we used, the front wire acts as a transmission line and the difference in arrival time of the two pulses is used to determine the position of the particle. The resistance of the wire (about 4 MΩ) and capacitance of the cavity (about $10^{-13}$ F) combines to form a filter, RC transmission line, that attenuates the high frequency components of the signal ($e^{-\omega}$). The position information is then contained in the rise time of the pulse which is about 2-5 μs in this case. After passing the pulse through a shaping amplifier, this information can be retrieved by detecting the zero crossing of a bipolar pulse, or a fractional height of the trailing edge of the unipolar pulse. For the parameters above and a 40 cm long
Fig. 2.1. Basic configuration of the position-sensitive detector.
wire the dispersion is about 10ns/mm and for the total length it's about 4 μs. The back wire, behind the first acts as a coincidence to reduce γ and neutron background and provides a good dE/dX signal to determine particle type.

2.2 Electronics

The electronic-circuit used should be able to extract the position information of a charged particle from the time difference between the two pulses coming from both ends of the front wire and the energy lost due to ionization of the same particle at the back wire.

The circuit was connected as shown in Fig. 2.2 and the electronic modules were set as indicated. Positive bias of about 500 V was applied to the wires and negative bias of about -500 V was applied to the case. The walk adjustments to the two front Timing Single Channel Analysers(TSCA) were done so that the output time pulse could be independent of the pulse height.
Fig. 2.2. Electronic circuit diagram.
Fig. 2.3. A typical Backwire Spectrum (schematic)
2.3 Back Wire Pulses

The relative energy loss for various types of particles could be estimated from a dE/dx chart. A typical spectrum of the amplified backwire pulses, displayed on a NS600 multichannel analyser is shown in Fig. 2.3. The gain of the amplifier and the bias to the back wire were adjusted to get the particles of interest (α-particles) in the center of the spectrum. The bias was adjusted in such a way so that the coarse gain of the amplifier could be kept at a minimum to get the best resolution. The TSCA windows of the back wire were adjusted by looking at the pulses coming from a linear gate, so that only the particles of interest could be seen on the display of the Multi Channel Analyser.

2.4 Front Wire Pulses

The front wire acted as a voltage divider and thus while a given particle type left approximately the same amount of energy in the counter regardless of the position, the signals from the amplifier were distributed in voltage. The TAC was started with the signal from one end of the front wire and stopped by the signal from the other end. The position information of a charge particle which was proportional to the time difference of the two
pulses was proportional to the TAC pulse height. All the linear signals from both ends of the front wire should fall in the range of all three (two front and one case) timing signals of the TSCAS. This was checked in the following way.

A linear signal from one end of the front wire amplifier was connected to the input of a linear gate and the gate pulse to the linear gate was taken from the back wire TSCA pulse. The delay was adjusted to get coincidence between both pulses and the amplifier gain was also altered so that the largest pulses of interest were just below 10 Volts. This procedure was repeated for the other end amplifier while maintaining the same amplifier gain as the earlier one.

The time at which the TAC pulses finish with respect to the dE/dX pulses depends upon the position of the event. Both pulses should reach the ADC'S at the same time. This was accomplished by using the Strobe coming from the PSPC module and by adjusting the delay in the back wire delay line amplifiers.

The programs THUNDR and LGHTNG were loaded and the two dimensional spectrum was inspected on a Tektronics display screen. Back wire gain and the delay in the stop
signal TSCA were adjusted to get the spectrum of all the particles of interest. By defining horizontal histograms on the screen of the 2-D spectrum, α-particles of different energy could be redisplayed as a single 1-D spectrum. The spectrum included all the states of $^{26}\text{Al}$ from $E_x=4598$ keV to $E_x=7348$ keV.

Alpha-particles from the $^{27}\text{Al}(^3\text{He},\alpha)^{26}\text{Al}$ reaction were collected at $2.5^\circ$ intervals from $10^\circ$ to $40^\circ$. An average value of the charge collected for each measurement was about 3mC. A typical α-spectrum taken at $25^\circ$ is shown in Fig. 2.4. The strongest state, $E_x=4705$ keV of $^{26}\text{Al}$, was fixed at a particular position for all the different angles by adjusting the magnetic field of the spectrograph. This helped us identify the peaks of $^{26}\text{Al}$, since peaks due to impurities, such as $^{12}\text{C}$, moved through the spectrum at different displacement rate than peaks due to $^{26}\text{Al}$. The strongest contaminant group in the entire spectrum was the α-group to the ground state of $^{11}\text{C}$. The peaks in the pulse height spectrum were fitted with Gaussian shapes to extract intensity and correct for background. The overall energy resolution of our detection system was found to be about 15 keV.
Fig. 2.4. Energy spectrum of the alpha particles from the reaction $^7\text{Li}(^3\text{He},)^{\alpha}\text{Al}$ at 12 MeV.
Fig. 2.5. The spectra corresponding to states of $^{26}$Al between $E_x=6346$ and 6498 keV (Spectra smoothed once).
The spectra corresponding to states of $^{26}\text{Al}$ between $E_x=6346$ and 6498 keV, which are of highest interest for astrophysics at temperatures of $T_9<0.3$, are expanded for several angles and shown in Fig. 2.5. The spectra are fit, with seven states, as shown. While the state labelled 6463 keV is weakly populated here, other work (later in our own work) supports the inclusion of this state in the analysis. Energy calibration of the proportional counter was done by using the well known and isolated bound states of $^{26}\text{Al}$ as well as $^{11}\text{C}$ ground state group. The uncertainties in the excitation energies are estimated to be $\pm 6$ keV. The existence of a doublet of states at $E_x=6346$ and 6362 keV supports the findings reported by Hinds and Middleton. Further the presence of a new doublet at $E_x=6398$ and 6410 keV removes the difficulty associated with the characteristic angular distribution of the previously reported state at $E_x=6413$ keV which will be considered later in this chapter.

2.5 Plate Runs with $^{27}\text{Al}(^{3}\text{He,}α)^{26}\text{Al}$ and $^{24}\text{Mg}(^{3}\text{He,p})^{26}\text{Al}$

To further confirm the existence of the levels and to measure their excitation energies more accurately, seven spectra were recorded with nuclear emulsion plates by replacing the Position Sensitive Proportional Counter. Three of them were taken with $^{27}\text{Al}(^{3}\text{He,}α)^{26}\text{Al}$ reaction and
four with $^{24}\text{Mg}(^3\text{He},p)^{26}\text{Al}$ reaction. For the first reaction, measurements were made at $E(^3\text{He})=12$ MeV for $\theta=20^\circ$ and $15^\circ$ and 9 MeV for $\theta=20^\circ$. Measurements for the second reaction were recorded at $E(^3\text{He})=12$ MeV for $\theta=40^\circ$ and $45^\circ$, at 15 MeV for $\theta=25^\circ$ and at 18 MeV for $\theta=25^\circ$. In order to relate our results to lower lying well known$^{25}$ levels of $^{26}\text{Al}$ (some of the excitation energies known to ±2 keV or better) in the region 4705 to 6083 keV excitation, we recorded the spectra from the present region of interest down to these lower levels. Our average projectile energy values were adjusted by using the accurately known excitation energies in this region. Some of these levels were included in Table 2.1. Standard deviations of our results are between 0.6 and 1.7 keV; agreement with the previously published$^{25}$) values lies between -0.3 and 0.8 keV (where their tabulated uncertainties range from 0.9 to 1.8 keV). Excitation energies of the states in the region of interest averaged over all our measurements are given in Table 2.1. The number of measurements for which each state is observed is also listed, as is the standard deviation of the mean excitation energy. Estimated internal errors include uncertainties in reaction angle, input energy, spectrograph field, spectrograph calibration, and beam spot position. The error in the differences of our new excitation energies and the tabulated energies used for reference is ±2 keV. The values of the excitation energies obtained from the
Proportional Counter runs agree with the Plate Runs to better than 6 keV in all cases and usually to better than 2 keV.

Spectra taken with plates which best illustrate the level structure in the region of interest are displayed in Fig. 2.6. The top panel, taken with the \(^3\)He,\(\alpha\) reaction using a thin (15 \(\mu g/cm^2\)) Al target, clearly establishes the doublet character of the 6346 and 6363 keV states, confirming the very early findings by Hinds and Middleton\(^{20}\)) and resolving conflicting assignments in later works. However, the energies of these states are about 10 keV lower in their findings since their values for all high excitation states are always lower as reported by Endt and Van der Leun\(^{25}\)). The new state, at \(E_{x}=6410\) keV reported above, from the \(^3\)He,\(\alpha\) reaction taken with the counter, is prominent also in the \(^3\)He,\(p\) measurements(lower panel). A new state, \(E_{x}=6463\) keV, which was weakly populated in the \(^3\)He,\(\alpha\) reaction(see Fig. 2.5), is required to fit the experimental data for the \(^3\)He,\(p\) run here.
TABLE 2.1

Excitation energies measured with the $^{27}$Al($^3$He,$\alpha$)$^{26}$Al and $^{24}$Mg($^3$He,$p$)$^{26}$Al reactions

<table>
<thead>
<tr>
<th>$E_x$ (keV)</th>
<th>$\delta$ (keV)</th>
<th>Tabulated values$^a$)</th>
<th>Present Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{27}$Al($^3$He,$\alpha$) runs</td>
<td>$^{24}$Mg($^3$He,$p$) runs</td>
<td>$E_x$ (keV)</td>
<td>$\sigma$ (keV)</td>
</tr>
<tr>
<td>4771.6</td>
<td>1.6</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4938.1</td>
<td>0.9</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>5393.7</td>
<td>0.9</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5454.9</td>
<td>1.8</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>6083</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>6346</td>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6362</td>
<td>10</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>6399</td>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6410.3</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6435</td>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6498</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

a) ref$^{25}$)
Fig. 2.6. Energy spectrum of the alpha particles from the reaction $^{27}$Al($^3$He,$\alpha$)$^{24}$Al and protons from the reaction $^{24}$Mg($^3$He,p)$^{26}$Al taken with nuclear emulsion plates.
2.6 DWBA Analysis

The angular distribution of the $\alpha$-particles for each of these states from the reaction $^{27}\text{Al}({}^3\text{He},\alpha)^{26}\text{Al}$, was analysed using the DWBA code CHUCK$^{26}$ to obtain spectroscopic information about these states. The program CHUCK calculates nuclear scattering amplitudes and differential collision cross-sections by numerically solving an appropriate set of coupled differential equations. However the well known program code DWUCK solves the problem by a T-matrix approach to the DWBA, which is again equivalent to the solution of a set of(two) coupled equations with coupling one way from the initial(elastic) channel to the final, outgoing channel.

The basic difference between the two program codes is that, in the coupled-equations approach two-way coupling can affect the distorted waves in both channels and therefore the final amplitudes in a non-linear fashion. It is necessary to include the coupling strengths and angular momentum dependance in the coupling explicitly rather than scaling the cross-sections at the end of the problem. When the spins of all channels and angular momentum transfers are properly described in the input file, the code completely accounts for the angular momentum
coupling and these variables need not be accounted for in any other way, i.e. by $2j+1$ terms in the spectroscopic factors, etc. On the other hand, coupling strengths (e.g. spectroscopic factors, deformation parameters, etc.) should be included explicitly in the two-way coupling input file.

The relationship between the experimental differential cross-section, $\sigma_{\text{exp}}$ and the differential cross-section calculated by DWUCK is given by

$$\sigma_{\text{exp}}(\theta) = \sum C^2 S(2J_\ell + 1) \cdot \sigma_{\text{DWUCK}}(\theta)/(2J_\ell + 1)(2j+1)$$

where $J_\ell$, $J_i$ and $j$ are the final (residual state), initial (target), and transferred particle angular momenta; $C$ is an isospin Clebsch-Gordan coefficient of the form $<T_iT_z|T_fT_z>(coupled \ the \ isospin \ of \ the \ target \ and \ transferred \ particle \ to \ that \ of \ the \ final \ state)$, $S$ is the spectroscopic factor and $N$ is an overall normalization factor which embodies the overlap between the projectile and the exiting and transferred particles together with the strength of the interaction producing the transfer.

In the case of CHUCK, this simply reduces to

$$\sigma_{\text{exp}}(\theta) = \sum C^2 S_{\text{chuck}}(\theta)$$

For the reaction $^{27}\text{Al}(^3\text{He},\alpha)^{26}\text{Al}$, we have $C^2 = 1$ for $T_f = 0$ and $C^2 = 1/3$ for $T_f = 1$. 
Fig. 2.7. Angular distribution of the 4705 keV state.
### TABLE 2.2
Optical-model parameters

<table>
<thead>
<tr>
<th></th>
<th>$^{27}\text{Al} + ^{3}\text{He}$</th>
<th>$^{26}\text{Al} + ^{4}\text{He}$</th>
<th>bound state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$(MeV)</td>
<td>130.0</td>
<td>200.0</td>
<td></td>
</tr>
<tr>
<td>$R_r$(fm)</td>
<td>1.31</td>
<td>1.35</td>
<td>1.25</td>
</tr>
<tr>
<td>$a_r$(fm)</td>
<td>0.67</td>
<td>0.60</td>
<td>0.65</td>
</tr>
<tr>
<td>$W$(MeV)</td>
<td>15.0</td>
<td>17.0</td>
<td></td>
</tr>
<tr>
<td>$R_i$(fm)</td>
<td>1.38</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>$a_i$(fm)</td>
<td>1.01</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>$U_{s.o}$(MeV)</td>
<td>10.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{s.o}$(fm)</td>
<td>1.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{s.o}$(fm)</td>
<td>0.67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\lambda=25$
Fig. 2.8. Angular distribution of the proton threshold states of interest.
TABLE 2.3
Spectroscopic information from the $^{27}$Al($^3$He,$\alpha$)$^{26}$Al reaction

<table>
<thead>
<tr>
<th>$E_x$ (keV)</th>
<th>$l_n$</th>
<th>$\sigma_{exp}/\sigma_{th}$</th>
<th>$J^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4705</td>
<td>2</td>
<td>.88</td>
<td>(0+5)$^+$</td>
</tr>
<tr>
<td>6346</td>
<td>1</td>
<td>.034</td>
<td>(1+4)$^-$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>.082</td>
<td></td>
</tr>
<tr>
<td>6363</td>
<td>0</td>
<td>.013</td>
<td>(2,3)$^+$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>.054</td>
<td></td>
</tr>
<tr>
<td>6398</td>
<td>1</td>
<td>.021</td>
<td>(1+4)$^-$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>.017</td>
<td></td>
</tr>
<tr>
<td>6410</td>
<td>0</td>
<td>.003</td>
<td>(2,3)$^+$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>.016</td>
<td></td>
</tr>
<tr>
<td>6436</td>
<td>1</td>
<td>.010</td>
<td>(1+4)$^-$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>.030</td>
<td></td>
</tr>
<tr>
<td>6498</td>
<td>0</td>
<td>.002</td>
<td>(2,3)$^+$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>.06</td>
<td></td>
</tr>
</tbody>
</table>
Optical model parameters, originally taken from Betts and Fortune\textsuperscript{23}) and Nurzynski et al\textsuperscript{27}), had to be modified slightly to fit the well known $l=2$ bound state transition to the $E_x=4705$ keV state of $^{26}$Al. The fit produced is shown in Fig. 2.7. The same optical model parameter values were used in our calculation for all the proton threshold states of interest and given in Table 2.2. Our states of interest cannot be fit properly by using a single $l$ transfer alone. However with mixed $l$ transfers, a technique used by Betts and Fortune\textsuperscript{22-23}), and others, we were able to fit the angular distribution reasonably well (see Fig. 2.8). Fits beyond the angle $\theta=35^\circ$, are not satisfactory since the compound nuclear reaction mechanism started to dominate at large angles for low bombarding energy. One might ask why we should not do this reaction at higher bombarding energies so that a good fit could be obtained for large angles also. In fact the energy lost in the target is also small at high energies. Unfortunately, at high beam energies, the energy spread associated with the production of the beam and guidance systems(Quadrupole magnets, etc) which direct the beam to the target, as well as the kinematic spread in the outgoing reaction particles all increases at a higher rate to offset the advantage in going to high energies.
The 1-transfers for all our states of interest are always an admixture of \( l=0 \) and \( l=2 \) or \( l=1 \) and \( l=3 \). The ratios of the admixtures for the states are given in Table 2.3. Each distribution has a unique admixture of 1-transfers.

The \(^{27}\text{Al}(^{3}\text{He},a)^{26}\text{Al} \) reaction is a neutron pick up one. Since the ground state of \(^{27}\text{Al} \) has spin-parity \( 5^+/2 \), \( 1d_{5/2} \) pickup can populate states having \( J^\pi=0^+-5^+ \) whereas with \( 2s_{1/2} \) pickup only \( J^\pi=2^+ \) and \( 3^+ \) are allowed. Therefore an admixture of \( l_n=0 \) and \( l_n=2 \) in angular distribution establishes the spin and parity of that state to be \( 2^+ \) or \( 3^+ \). This is the case for the states \( E_x=6363, 6410 \) and \( 6498 \) keV. Similarly \( l_n=1 \) and \( l_n=3 \) in angular distribution, lead to spin and parity assignment of that state to \( J^\pi=1^-+4^- \). The states \( E_x=6346, 6398 \) and \( 6436 \) keV all belong to this category.

2.7 Discussion and comparison with previous results

From our results, we find that a doublet of states at \( E_x=6346 \) and \( 6363 \) keV is essential to fit the data. This confirms the existence of the doublet seen by Hinds and Middleton\(^{20}\) but not in later work by Betts and Fortune\(^{21-23}\) or by Champagne, Howard and Parker\(^{24}\). It is quite interesting to note that the spin and parity
assignments given by Betts and Fortune\textsuperscript{22}) (using the same reaction $^{27}$Al($^3$He,$^\alpha$)$^{26}$Al, but at 18 MeV) for the state $E_x=6346$ keV is $(2,3)^+$. But when the same authors\textsuperscript{23}) used a different reaction, $^{25}$Mg($^3$He,d)$^{26}$Al, they assigned $J^\pi=(1$ to 4)$^-$ for the same state. In both experiments, they reported a single state instead of a doublet. It is possible in the first case that they would have seen a strong population of $E_x=6363$ keV state of the doublet and in the second case that of the other state $E_x=6346$ keV of the doublet. The same arguments can be given for the assignments made by Champagne et al\textsuperscript{24}) to these states.

Betts and Fortune reported that the angular distribution of the $E_x=6413$ keV state was not characteristic of a direct stripping reaction which is now understood in terms of there being a new doublet of states at $E_x=6398$ and 6410 keV with an admixture of odd 1-transfers and even 1-transfers respectively.

As mentioned earlier, only 1=0 tranfer to states with spins and parities $(2,3)^+$, are expected to contribute to the stellar reaction rates for the production of $^{26}$Al in MgAl cycle during hydrogen burning nucleosynthesis. We conclude that there are only three states that satisfy this requirement for $E_p<200$ keV. They are the states $E_x=6363$ keV, $E_x=6410$ keV and $E_x=6498$ keV. Since the first two
states are very close to the proton threshold of $^{25}\text{Mg}$ (with $E_p(\text{c.m.}) = 57$ and 104 keV) they can influence the production rate of $^{26}\text{Al}$ in significant ways in the stellar reaction [note; production rate due to each state is proportional to $\exp(-E_p/kT)$.]. We need further information about these states such as spectroscopic factors for proton capture and $\gamma$-ray branching ratios in order to estimate their contributions to the stellar reaction rate at $T_9 < 0.3$.

To obtain the resonance strengths for proton capture, we did the $^{25}\text{Mg}(^{3}\text{He},d)^{26}\text{Al}$ reaction which is described in the next chapter.
Chapter III

$^{25}\text{Mg}(^{3}\text{He,d})^{26}\text{Al}$ Experiment

The purpose of this chapter is to show how to extract the resonance strengths, $\omega \gamma$, for the $(p,\gamma)$ resonances on $^{25}\text{Mg}$. The quantity $\omega \gamma$ is defined by

$$\omega = (2J_r+1)/(2J_i+1)(2j+1)$$

$$\gamma = \Gamma_p \Gamma_\gamma / \Gamma$$

where $J_r$, $J_i$ and $j$ are the angular momenta of the resonance, initial target and transfered proton; $\Gamma_p$ and $\Gamma_\gamma$ are the corresponding proton and $\gamma$-ray partial widths for a state with total width $\Gamma$. However, the resonance strengths will be deduced from the $^{25}\text{Mg}(^{3}\text{He,d})^{26}\text{Al}$ proton transfer reaction for reasons described earlier. The usual experimental set-up and procedure which is similar to the earlier case are briefly stated in this chapter. The use of R-matrix theory to deduce the resonance strengths of the $(p,\gamma)$ reaction from the spectroscopic factors of the indirect reaction is also described in this chapter.

3.1 Experimental set-up and procedure for $^{25}\text{Mg}(^{3}\text{He,d})^{26}\text{Al}$

This experiment was done at the nuclear structure laboratory at University of Notre Dame. A thin($\approx 15$ $\mu$g/cm$^2$)
layer of enriched (99%) \(^{25}\text{Mg}\) was evaporated on a 10 \(\mu\text{g/cm}^2\) carbon foil at the evaporation facility at the Van de Graaff Laboratory at Ohio State University and was used for the experiment at Notre Dame (refer Appendix for details of target making). A beam of 15 MeV \(^3\text{He}\) ions, produced from the Tandem accelerator was used to bombard the \(^{25}\text{Mg}\) target which was at the center of an ORTEC chamber. Reaction particles were momentum analysed in the magnetic spectrometer, equipped with the position sensitive dual proportional counter. The function of the proportional counter was described earlier in Chapter 2. The electronics used were also similar, except for the windows of the TSCA's and the gains of the Amplifiers since the particles of interest were now deuterons.

Outgoing deuterons were detected in the proportional counter for eight different angles between 7.5\(^\circ\) and 42.5\(^\circ\). These angles were chosen so that the impurity peaks would have minimum obscurity on our peaks of interest. A typical deuteron spectrum taken at \(\theta=32.5^\circ\) is shown in Fig. 3.1. The strongest state of our states of interest, \(E_x=6363\) keV of \(^{26}\text{Al}\), was fixed at a particular position for all the different angles by adjusting the magnetic field of the spectrograph. This helped us identify all the peaks of \(^{26}\text{Al}\), since peaks due to impurities, such as \(^{16}\text{O}\), moved through the spectrum at different
Fig. 3.1. Energy spectrum of the deuterons from the reaction $^{25}\text{Mg}(^{3}\text{He},\text{d})^{26}\text{Al}$ at 15 MeV.
Fig. 3.2. The spectra corresponding to states of $^{26}$Al between $E_x=6346$ and 6498 keV (Spectra smoothed once).
displacement rates. The strongest contaminant groups in the entire spectrum were the deuteron groups to the ground and first excited states of $^{17}\text{F}$. The peaks in the pulse height spectrum were fitted with Gaussian shapes to extract the number of deuterons in each peak and correct for background. The overall energy resolution of our detection system was found to be about 15 keV.

The spectra corresponding to our states of interest are expanded for two different angles and shown in Fig. 3.2. The spectra are fit with seven states as shown. The state labelled 6463 keV which was weakly populated in the $^{27}\text{Al}(^{3}\text{He},\alpha)^{26}\text{Al}$ reaction, is prominent in this reaction and confirms it as a state of $^{26}\text{Al}$. Energy calibration of the proportional counter was done by using the well known and isolated bound states of $^{26}\text{Al}$ as well as the ground and first excited states of $^{17}\text{F}$. The uncertainties in the excitation energies are estimated to be ±6 keV.

The existence of a doublet of states at $E_x=6346$ and 6363 keV was already confirmed from our previous experiment. This experiment also supports the existence of the doublet. In addition, the spectrum at $\theta=7.5^\circ$ strongly suggests that there has to be a state of $^{26}\text{Al}$ at $E_x=6363\pm5$ keV since if there was only one state at $E_x=6346$ keV as noted by Champagne et al and others, then that peak could
not have been seen separately from the peak of $^{17}F_1(495 \text{ keV excitation})$ as their kinetic energies were within 15 keV (smaller than the resolution). The new doublet of states at $E_x=6398$ and 6410 keV found from our previous experiment is also needed here to fit the data.

To determine the target thickness and to verify the optical model parameters used for DWBA analysis, elastically scattered $^3\text{He}$ particles from the $^{25}\text{Mg}(^3\text{He},^3\text{He})^{25}\text{Mg}$ reaction were also collected at $5^\circ$ intervals from $15^\circ$ to $45^\circ$. The value of the charge collected varied from 10 µC for $\theta=15^\circ$ to 200 µC for $\theta=45^\circ$. The peaks in the pulse height spectrum were fitted with Gaussian shapes to extract the number of elastically scattered $^3\text{He}$ particles in the spectrum.

3.2 DWBA Analysis

The angular distribution of the $^3\text{He}$ particles from the $^{25}\text{Mg}(^3\text{He},^3\text{He})^{25}\text{Mg}$ reaction, was analysed using the DWBA code CHUCK to obtain the target thickness. The optical model parameters taken from Betts and Fortune$^{23}$ are used in our calculation and listed in Table 3.1. The calculated ratios of differential cross-section $\sigma(\theta)/\sigma_R(\theta)$ plotted against $\theta_{\text{cm}}$ is shown in Fig 3.3. The differential cross-section $\sigma(\theta)$ is given by
\[ \sigma(\theta) = \frac{Y}{(n_0 N_t d\Omega)} \]

where \( Y \) = yield of the elastic \(^3\text{He} \) particles collected in solid angle \( d\Omega \) for \( n_0 \) number of beam particles.

\( N_t = \text{Number of target atoms/cm}^2. \)

\( n_0 = Q/e \) where \( Q \) is the charge collected.

Dividing both sides of the equation by the Rutherford cross-section, \( \sigma_R \), we get

\[ \left( \frac{\sigma}{\sigma_R} \right)_{\text{cal}} = \left( \frac{\sigma}{\sigma_R} \right) \left( \frac{e}{N_t d\Omega} \right) \]

Define a quantity \( f = \frac{e}{N_t d\Omega} \)

Inserting \( f \) in the above equation,

\[ \left( \frac{\sigma}{\sigma_R} \right)_{\text{cal}} = \left( \frac{\sigma}{\sigma_R} \right) f \]

The experimental quantity, \( \frac{Y}{Q \sigma_R} \), has to be multiplied by the factor \( f \) to fit the theoretical quantity, \( \left( \frac{\sigma}{\sigma_R} \right) \), of the ratio of cross-section. From our fit,

\( f = 2.22 \text{ mbC/sr} = 2.22 \times 10^{-27} \mu\text{C.cm}^2/\text{sr} \)

Substituting \( d\Omega = 0.25\text{msr} \) and \( e = 1.6 \times 10^{-13} \mu\text{C} \)

\( N_t = \frac{e}{f \Omega} = (1.6/2.22 \times 0.25) \times 10^{17} \text{ atoms/cm}^2 \)

The number of \(^{25}\text{Mg} \) atoms in 1 gram is \((6.023 \times 10^{23}/25)\).

Using this, we get the target thickness

\( N_t = 12 \mu\text{g/cm}^2 \)

This value is in very good agreement with the expected value from target making measurements.

The angular distribution of the deuterons from each of the proton threshold states of \(^{26}\text{Al} \) was also analysed using the code CHUCK to obtain spectroscopic
<table>
<thead>
<tr>
<th></th>
<th>$^{25}\text{Mg}+^3\text{He}$</th>
<th>$^{26}\text{Al}+d$</th>
<th>bound state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$(MeV)</td>
<td>177.0</td>
<td>120.0</td>
<td></td>
</tr>
<tr>
<td>$r=r_{s.o}$(fm)</td>
<td>1.14</td>
<td>1.00</td>
<td>1.26</td>
</tr>
<tr>
<td>$a=a_{s.o}$(fm)</td>
<td>0.72</td>
<td>0.90</td>
<td>0.60</td>
</tr>
<tr>
<td>$r'_c$(fm)</td>
<td>1.40</td>
<td>1.30</td>
<td>1.26</td>
</tr>
<tr>
<td>$W$(MeV)</td>
<td>13.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r'$(fm)</td>
<td>1.60</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>$a'$(fm)</td>
<td>0.77</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>$V_{s.o}$(MeV)</td>
<td>8.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W'$(MeV)</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\lambda=25$

$W'=4W_D=(100-2.0E_X)\text{MeV}$
Fig. 3.3. Angular distribution of elastic scattering.

\[ S = 1.055 \]
\[ f = 2.222 \]
factors to these states. Optical model parameters, again taken from Betts and Fortune were used in the analysis and given in Table 3.1. A few bound state peaks, at $E_x = 5542$ and $5394$ keV, were fit respectively with $l=2$ and an admixture of $l=1$ and 3 to verify the optical model parameters used. The fits are shown in Fig 3.4. Since the program code CHUCK cannot handle unbound states for calculations, the excitation energies of our states of interest were replaced by the excitation energy of a bound state $E_x = 6270$ keV. The excitation energy $E_x = 6270$ keV was chosen only after verifying that the calculation of differential cross-sections would not change noticeably for a small fluctuation about 200 keV in excitation energy. As before, we have to use mixed $l$-transfers to fit the angular distributions of our states of interest except for one state at $E_x = 6498$ keV. The angular distributions of $l=2$ and $l=3$ are similar as noted by Betts and Fortune. Since we already had the possible spin assignments to these states from the $^{27}$Al($^3$He,α)$^{26}$Al reaction, we did not have any problems about identifying the $l$-transfers (see Fig. 3.5). A small mixture of odd $l$-transfer to an even $l$-transfer could mislead one in making an assignment of odd $l$-transfer to that state as was done by Betts and Fortune for the state at $E_x = 6360$ keV. Since they were not aware of the fact that there was a doublet of states at $E_x = 6346$ and $6362$ keV, with a little mixture of odd $l$-transfer from the
Fig. 3.4. Angular distribution of the states at $E_x = 5394$ and 5542 keV.
Fig. 3.5. Angular distribution of the proton threshold states of interest from the $^{25}\text{Mg}(^{3}\text{He,d})^{26}\text{Al}$ reaction.
state at $E_x=6346$ keV and a strong $l=2$ mixture from the state at $E_x=6362$ keV, they assigned an odd $l$-transfer value to the unresolved doublet.

Since the ground state of $^{25}$Mg has a spin and parity assignment $J^\pi=5/2^+$, the observation of $l=0$ restricts $J^\pi$ to $2^+$ or $3^+$. This implies the states at $E_x=6363$ and 6410 keV have $J^\pi=(2,3)^+$ assignments. The state, at $E_x=6498$ keV, populated only by a pure $l=2$ transfer, has an assignment of $J^\pi=(0+5)^+$. The observation of $l=1$ restricts $J^\pi$ to $(1+4)^-$. This is the case for the states at $E_x=6346$, 6398, 6436 and 6465 keV. All these results are consistent with the previous results obtained from the $^{27}$Al($^3$He,$\alpha$)$^{26}$Al reaction except for a small difference in the population of the state at $E_x=6498$ keV, which is now a pure $l=2$ state whereas it was a mixture of $l=0$ and $l=2$ in the previous case.

The most important part of this experiment is to obtain the spectroscopic factors for our states of interest. Earlier we defined

$$\sigma_{\text{exp}}(\theta) = \int_C^2 S \sigma_{\text{CHUCK}}(\theta)$$

Where $C^2=1/2$ for both possible values of $T_f$ (0 and 1)

For a single $l$-transfer, this reduces to

$$\sigma_{\text{exp}}(\theta) = C^2 S \sigma_{\text{CHUCK}}(\theta)$$

From fits of angular distribution, the ratio of $\sigma_{\text{exp}}/\sigma_{\text{CHUCK}}$, $R$, can be determined. Then the spectroscopic
factor S is simply given by
\[ S = \frac{R}{C^2} \]
In order to facilitate the calculations in our program, the ground state spin of \(^{25}\text{Mg}\) was assumed to be zero. Also the proper coupling strength, \(\beta = 4.42 \times 10^4\) was not used in our calculation. To correct these two factors, the equation has to be modified as
\[ \sigma_{\text{exp}} = N_C^2 \frac{S\sigma_{\text{CHUCK}}}{(2J_t + 1)} \]
Where \(N_C = \frac{4.42 \times 10^4}{(20)^2} = 110.5\)
\((2J_t + 1) = (2 \times 5/2 + 1) = 6\)
This implies \(R = N_C^2 S / (2J_t + 1)\)
In our plot program R is given by
\[ R = S'\times A\times f \]
Where \(f\) is equal to 2.222 as defined earlier in the elastic scattering fit. \(A\) is a multiplication factor for each partial wave and \(S'\) is the factor calculated by the plot program to fit the theoretical values to experimental points. Hence the spectroscopic factor S is given by
\[ S = S'\times A\times f(2J_t + 1) / (N_C^2) \]
\[ = .24AS' \quad \text{(since } f(2J_t + 1) / (N_C^2) = .24 \text{)} \]
Using the above equation spectroscopic factors for the \(^{(3}\text{He},d)\) reaction were calculated for all our states of interest and listed in Table 3.2. The next step is to deduce the proton widths for our states of interest of the \(^{25}\text{Mg} + p\) reaction from the above results.
TABLE 3.2

Spectroscopic information from the $^{25}\text{Mg}(^{3}\text{He},d)^{26}\text{Al}$ reaction

<table>
<thead>
<tr>
<th>$E_x$ (keV)</th>
<th>$l_n$</th>
<th>$\omega S$</th>
<th>$J^\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5394</td>
<td>1</td>
<td>0.0174</td>
<td>(0+4)$^-$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0696</td>
<td></td>
</tr>
<tr>
<td>5542</td>
<td>2</td>
<td>0.064</td>
<td>(0+5)$^+$</td>
</tr>
<tr>
<td>6346</td>
<td>1</td>
<td>0.0274</td>
<td>(1+4)$^-$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0091</td>
<td></td>
</tr>
<tr>
<td>6363</td>
<td>0</td>
<td>0.0896</td>
<td>(2,3)$^+$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0448</td>
<td></td>
</tr>
<tr>
<td>6398</td>
<td>1</td>
<td>0.0172</td>
<td>(1+4)$^-$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0172</td>
<td></td>
</tr>
<tr>
<td>6410</td>
<td>0</td>
<td>0.0139</td>
<td>(2,3)$^+$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0035</td>
<td></td>
</tr>
<tr>
<td>6436</td>
<td>1</td>
<td>0.0103</td>
<td>(1+4)$^-$</td>
</tr>
<tr>
<td>6465</td>
<td>1</td>
<td>0.0240</td>
<td>(1+4)$^-$</td>
</tr>
<tr>
<td>6498</td>
<td>2</td>
<td>0.0172</td>
<td>(0+5)$^+$</td>
</tr>
<tr>
<td>6678</td>
<td>0</td>
<td>0.0356</td>
<td>(2,3)$^+$</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0089</td>
<td></td>
</tr>
</tbody>
</table>
Using the R-matrix theory\textsuperscript{28}, the proton width $\Gamma_p$ of a state can be defined as

$$\Gamma_p = 2P_1(E)\gamma_p^2$$

where $P_1(E)$ is the penetrability factor, $E$ is in center of mass energy, $l$ is the orbital angular momentum, and $\gamma_p$ is the reduced width of that state. The spectroscopic factor $S$ can then be written as

$$S = \gamma_p^2 / \gamma_{sp}^2$$

where $\gamma_{sp}$ is the reduced width of a pure single-particle state. Since the nuclear potential is very large compared to the excitation energies, the change in nuclear potential caused by small changes (500 keV) in the excitation energy is much smaller than the average magnitude of nuclear potential. This implies that the states which are close in excitation energy (<500 keV), formed by the same partial wave with similar spectroscopic strengths will experience almost the same potential since the residual interactions which split the single particle strength for a given shell model orbital are on the order of the relative change in excitation energy. Hence their single-particle reduced width will be the same. This gives us the ratio of the spectroscopic factor of two such states as

$$S_1 / S_2 = \gamma_{p1}^2 / \gamma_{p2}^2$$

or

$$S_1 / S_2 = \left[ \Gamma_p(E_1) / \Gamma_p(E_2) \right] \left[ P_1(E_2) / P_1(E_1) \right]$$

$P_1(E_1)$ and $P_1(E_2)$ can be calculated from the penetrability program. Since $\Gamma_p(E)$ is known for the state at $E_x=6678$ keV
from previous work of other authors\(^{24}\) and the states at 
\(E_x=6362\) and \(6410\) keV are formed by the same partial wave 
with similar spectroscopic strengths, \(\Gamma_p(E)\) can be 
calculated for these two states of interest from the 
spectroscopic factors obtained from our experiment. Using 
the equation,
\[
\Gamma_p(E_1) = [S_1/S_2] \left[ \frac{P_1(E_1)}{P_1(E_2)} \right] \Gamma_p(E_2)
\]
\(\Gamma_p(E)\) was calculated for all our states of interest and 
listed in Table 3.3.

The \(\gamma\) of a resonance is given by
\[
\gamma = \frac{\Gamma_p \gamma \Gamma}{\Gamma}
\]
\(\Gamma\) is usually of the order of an electronvolt and it is 
independent of the resonance energy. But \(\Gamma_p\) is a rapidly 
decreasing quantity with decreasing resonance energy (due to 
Coulomb barrier) and is negligible compared to \(\Gamma\) for our 
proton threshold states (refer Champagne et al). Hence \(\gamma\) 
reduces to
\[
\gamma = \Gamma_p'
\]
or the resonance strengths can be calculated using the 
equation
\[
\omega \gamma = \omega \Gamma_p
\]
Table 3.3 also lists all the resonance strengths calculated 
from our experimental results.
Table 3.3
Resonance strength from $^{25}\text{Mg}(^3\text{He},d)^{26}\text{Al}$ Experiment

<table>
<thead>
<tr>
<th>$E_x$ (keV)</th>
<th>$S(l=0)$</th>
<th>$\Gamma_p$ (eV)</th>
<th>$\Gamma_\gamma$ (eV)</th>
<th>-- $\omega_\gamma$ cm (eV) --</th>
<th>Present (exp)</th>
<th>Elix $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6363 3$^+$</td>
<td>0.154</td>
<td>$2.03\times10^{-11}$</td>
<td>$1.2\times10^{-11}$</td>
<td>$&lt;2\times10^{-8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6410 2$^+$</td>
<td>0.033</td>
<td>$1.93\times10^{-6}$</td>
<td>$7.9\times10^{-7}$</td>
<td>$&lt;1\times10^{-7}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6678 2$^+$</td>
<td>0.084</td>
<td>460</td>
<td>$\Gamma_\gamma(0.14)$</td>
<td>$5.8\times10^{-2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Ref.18
From our results, the resonance strength, $\omega_\gamma$, of the state at $E_x=6363$ keV is about four orders of magnitude higher than the resonance strength of the state at $E_x=6346$ keV taken from results of Champagne et al\textsuperscript{24} (since the $P_1(E)$ decreases rapidly with low resonance energy) and hence the production rate of $^{26}$Al is much higher than that was calculated from previous work of other authors\textsuperscript{19} (assuming that this state also has similar branching ratio to the ground state).

The resonance strength, $\omega_\gamma$, of the state at $E_x=6410$ keV is also about four orders of magnitude higher than the resonance strength of the state $E_x=6400$ keV reported by Champagne et al since they used $l=2$ transfer for the angular distribution of the state (instead of $l=0$). It was not a direct first order reaction in their results as they were unaware of the presence of a doublet. This again increases the production rate of $^{26}$Al by a large factor (assuming it decays to ground state). Even a 10% branching ratio of these states to ground state can increase the production rate of $^{26}$Al by at least three orders of magnitude at $T_g=1$ (see Table 3.3).
3.3 Alternative approach to obtain the resonance strengths

Our results for the resonance strengths are based on the reference\textsuperscript{24} that $\Gamma_p$ is 460 eV for the state at $E_x=6678$ keV. When you compare our results with the deduced experimental upper limits for the resonance strengths from Elix et al\textsuperscript{18})(see Table 3.3), our results are higher by about an order of magnitude. As an alternative way to check our results, we used the theoretical equation\textsuperscript{29}

$$\Gamma_p = \frac{3}{2} TC^2 S$$

(T=transmission factor)

which was also used by Michael Wiescher\textsuperscript{30} and others to obtain proton widths. The results are listed in Table 3.4. They are within the experimental upper limits of Elix et al\textsuperscript{18} results. Also it shows that the proton width, $\Gamma_p$, is only about 2 eV for the state at $E_x=6678$ keV. Only this set of values for the resonance strengths will be used later in our reaction rate calculations.

So certainly the next step is to get the branching ratios of these two states(at $E_x=6363$ and 6410 keV) to the ground state.
Table 3.4
Spectroscopic factor and proton width of $^{26}$Al states.

<table>
<thead>
<tr>
<th>$E_x$ (keV)</th>
<th>$E_P$ (lab)</th>
<th>$S$</th>
<th>$T(1=0)$</th>
<th>$\Gamma_P$ (eV)</th>
<th>$\omega_P$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6363</td>
<td>60</td>
<td>0.154</td>
<td>$1.23 \times 10^{-11}$</td>
<td>$7.12 \times 10^{-13}$</td>
<td>$4.15 \times 10^{-13}$</td>
</tr>
<tr>
<td>6410</td>
<td>109</td>
<td>0.0334</td>
<td>$3.00 \times 10^{-6}$</td>
<td>$3.76 \times 10^{-8}$</td>
<td>$1.57 \times 10^{-8}$</td>
</tr>
<tr>
<td>6678</td>
<td>388</td>
<td>0.0854</td>
<td>60.70</td>
<td>1.94</td>
<td></td>
</tr>
</tbody>
</table>

$\Gamma_P = T C^2 S^{3/4}$ (3/4 is inserted in order to correct the factor 2 to 3/2 in Pene. program).
Chapter IV

$^{27}\text{Al}(^{3}\text{He},\alpha\gamma)^{26}\text{Al}$ Experiment

The branching ratio of each of our states of interest to the ground state is the only missing ingredient for determining the production rate of $^{26}\text{Al}$ in stellar environments. This chapter describes our attempt to obtain this by using the $^{27}\text{Al}(^{3}\text{He},\alpha\gamma)^{26}\text{Al}$ reaction. A silicon surface barrier annular detector at 180° to the beam was used in conjunction with a GMX detector at 90° to the beam for our $\alpha$-$\gamma$ coincidence experiment. A new Real Time Data Acquisition program named UNICORN was used to collect and analyse our data. The branching ratio of any number of strong and isolated peaks in the particle spectrum could be determined at the same time with the use of this program. This helped us to check our calibration and efficiency of the GMX detector while the experiment was in progress. Also any gain shift in the amplifiers could be determined and corrected with the use of this program by simply replaying the event tapes.
4.1 EXPERIMENTAL SET-UP FOR THE $^{27}$Al($^{3}$He,$^{6}$He)$^{26}$Al REACTION

A self supporting natural Al target of thickness 35 $\mu$g/cm$^2$ was used for this experiment. A beam of 6 MeV $^{3}$He ions, produced from the Van de Graaff Accelerator, was used to bombard the Al target through a very narrow set of collimators in our north 45 degree beam line. This beam line was chosen because one could place the GMX detector within 5 cms from the target and an annular surface barrier detector at 180° to the beam. The distances of both detectors were adjusted to get both good resolution and count rate. The bias of the particle detector was kept at 30 volts so that only the $\alpha$-particles from our states of interest could be stopped but not the protons or deuterons of the same energy. A negative 4500 Volt was applied to the $\gamma$-ray detector.

4.2 ELECTRONICS

The electronics circuit was set-up as shown in the diagram (Fig 4.1). The signal from the GMX preamplifier was split into two signals. One was sent to ORTEC 571 Amplifier for extracting the energy information and the other was sent to an ORTEC Timing Filter Amplifier(454) for timing purpose. Similarly the signal from the particle detector (after going through the 109A preamplifier) was
split into two; one to a 571 Amplifier and the other to a Fast Filter Amplifier 579. Signals from Timing Filter Amplifiers were sent through Constant Fraction Discriminators (473A) to reject the unwanted pulses and to provide Logic signals to the Time to Amplitude Convertors (TAC). The TAC was started with GMX pulses and stopped with particle detector pulses. The timing of the signals was adjusted with the delay in the Delay Line amplifiers so that the signals from the 571 Amplifiers (GMX and particle detector pulses) and TAC arrived at the same time to the inputs to the ADCS. A gate pulse to each ADC was taken from an Event Trigger whose input was controlled by a Timing Single Channel Analyser (TSCA) through the TAC. 

FAN IN/FAN OUT module was used to split signals from the EVENT TRIGGER. Using Level Adapters NIM signals were converted into TTL signals for the 3511 ADCS. The 2259 ADC was controlled by a NIM signal directly from the FAN IN/FAN OUT module. Data was collected in CAMAC mode using LeCroy 3500 system and the DATA GENERAL ECLIPSE Computer.

The VJACQ program in the LeCroy 3500 system was used to read the ADCS in the CAMAC mode and send the readings to the DATA GENERAL Computer. The program UNICORN enabled the DATA GENERAL to receive Data from the LeCroy and store them in tapes in real time memory for future sortings.
Fig. 4.1. Electronic circuit diagram.
The program VJACQ was a mini version of the "PEGASYS" program for general use. This defined the various modules (ADCS etc.) in appropriate slots in the crate for the CAMAC mode.

The program UNICORN required a parameter set file (Table 4.1) for its sorting operation. The input signals (originated from GMX, particle detector and TAC) to the ADCS were known as the Signal Sources in this program for our use. Signal Source Numbers 1, 2 and 3 were taken respectively from TAC, GMX and particle detector. Since the Crate had one EVENT Trigger, there was only one Coincidence Group with three Signal Sources. Histograms were defined as 4096 channels for 3511 ADCS and 2048 channels for 2259 ADCS. When all the 4096 channels are displayed on a spectrum, then the gain is one. But when the whole display is confined to 2048 channels, then the gain should be written as two and so on for other cases. If a certain initial portion of a spectrum is not desired then one could use the "OFFSET" to delete the undesired channels from the initial display. Conditions on Histograms could be defined by using windows on Signal Sources. Windows are required to specify the region of interest of a spectrum from a Signal Source. A zero condition implies that there is no condition on the display of signals from that source. Table 4.1 was the parameter set used for most part of our experiment.
**TABLE 4.1**

**UNICORN PARAMETER SET**

Run ID: 195107  Seq#: 1  Sort program: UNISORT.PR

User mode: OFF  Event buffer size: 512 words  Tape block size: 512 words

--- Histograms ---
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<th>Offsets</th>
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<th>Cond’n</th>
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--- Conditions ---
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<td>inside Window 3</td>
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<tr>
<td>4</td>
<td>inside Window 4</td>
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</tr>
<tr>
<td>6</td>
<td>C(1) . and. C(3)</td>
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<tr>
<td>7</td>
<td>C(1) . and. C(4)</td>
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<td>8</td>
<td>C(6) . or. C(7)</td>
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--- 1-D Windows ---
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</tr>
</tbody>
</table>

--- Coincident Groups ---
<table>
<thead>
<tr>
<th>#</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4</td>
</tr>
</tbody>
</table>
A gamma-ray spectrum from the decay of the doublet states at $E_x=6363$ and 6346 keV is shown in Fig 4.2. High energy gamma-rays are not marked in the spectrum since their statistics were poor. The two most important peaks in the spectrum are the 417 keV gamma-ray and 830 keV gamma-ray. The 417 keV gamma-ray came from the emission of the second excited state of $^{26}$Al ($E_x=417$ keV) decaying to the ground state. The 830 keV gamma-ray came from the emission of the third excited state ($E_x=1.06$ MeV) of $^{26}$Al decaying to the first excited state ($E_x=228$ keV). Since most of the excited states decay to the ground state through the second excited ($E_x=417$ keV), a measurement of the 417 keV gamma-ray from the decay of an excited state gives a reasonable lower limit to the branching ratio of that state to the ground state. Similarly, a measurement of the 830 keV gamma-ray gives a lower limit to the branching ratio of that state to the first excited state. Since the fate of any excited state (other than the first one) of $^{26}$Al is either to end up in the ground state or in the first excited state, a lower limit to the branching ratio to the first excited state gives an upper limit to the branching ratio of that state to the ground state. If a peak of an excited state in the particle spectrum is isolated or well resolved from the rest then the limits (lower and upper) can be easily obtained from the gamma-ray spectrum in coincidence with the particle spectrum. Our states of
Fig. 4.2. Gamma spectrum in coincidence with alpha peaks 6346 and 6363 keV.
interest were neither isolated nor well resolved from the rest of the peaks in the particle spectrum. Fortunately the program UNICORN could do many things to solve this problem. Replaying the event tapes, an $\alpha$-particle spectrum in coincidence with the 417 keV gamma-ray was obtained. If there was only one state($E_x=6346$ keV) present in the peak position of the doublet($E_x=6346$ and 6363 keV) then we would have certainly seen the state at $E_x=6398$ keV(and/or $E_x=6410$ keV) separately from the state at $E_x=6346$ keV. Since we could not see this, there was evidence that the state at $E_x=6363$ keV was present in the spectrum gated with the 417 keV gamma-ray. This was compared to the ungated gamma-ray spectrum as well as to the total gamma-rays gated spectrum. We did not see any significant difference in the population of our states of interest. Further, the windows on the position of the doublet(fixed window width) in the particle spectrum were readjusted to check the population of the 417 keV gamma-ray. It was present for all the different window settings. In brief, all our states of interest should have significantly decayed through the second excited state($E_x=417$ keV) to the ground state.

The lines from $^{133}\text{Ba}$, $^{137}\text{Cs}$, $^{60}\text{Co}$ and the well known and isolated states of $^{26}\text{Al}$ were used to obtain a calibration for gamma-rays. The $^{27}\text{Al}(p,\gamma)^{28}\text{Si}$ reaction was done at the resonant energy 1.910 MeV proton energy to
obtain an efficiency calibration for high energy gamma-rays. Since the statistics from high energy gamma-rays from our states of interest were poor, it was decided not to give the results of that experiment as it was not required for any analysis.

4.3 Calculation of the Branching Ratio Limits

During the whole experiment, ungated α-particle groups without any coincidence requirements were collected in a ND100 Multichannel Analyser for short periods of time. Let $N_1$ be the number of ungated α-particles corresponding to the second excited state ($E_x=417$ keV) of $^{26}$Al collected in the analyser (see Fig. 4.3) from the particle detector for $Q_N$ beam charge in the integrator. Also let $A_1$ be the number of α-particles corresponding to the same state collected in the LeCroy (Multichannel Analyser) in coincidence with 417 keV gamma-ray in the GMX detector for $Q_L$ beam charge in integrator. This shows that for every $A_1/Q_L$ gamma-rays of energy 417 keV detected in the GMX detector there should be $N_1/Q_N$ α-particles (from the excited state at $E_x=417$ keV) entering the charged particle detector. If it is assumed that the gamma-rays in coincidence with the α-particles detected at $180^\circ$ are isotropically emitted, then for a gamma-ray detector efficiency of 100%, $N_\gamma=$number of $\gamma$'s in coincidence for charge $Q_L$.
Fig. 4.3. Ungated and gated alpha particles spectrum (Smoothed).
\[ = (N_1/Q_N)(d\Omega_\gamma/4\pi)Q_L \]

\( A_1 = \text{actual number of } \gamma \text{'s detected} \)

\[ = (N_1/Q_N)(d\Omega_\gamma/4\pi)Q_L \varepsilon_\gamma \]

\( \varepsilon_\gamma = \text{efficiency of } \gamma \text{-detector for } 417 \text{ keV } \gamma \text{-ray} \)

This implies that

\[ \varepsilon_\gamma = (A_1/Q_L)/[(N_1/Q_N)(d\Omega_\gamma/4\pi)] \]

or

\[ \varepsilon_\gamma x(d\Omega_\gamma/4\pi) = (A_1/Q_L)/(N_1/Q_N) \]

The ratio \( (A_1/Q_L)/(N_1/Q_N) \) gives the probability of the system for detecting 417 keV gamma-rays in coincidence with \( \alpha \)-particles of any energy since the charge particle detector has always 100\% efficiency for all the \( \alpha \)-particles of interest with different energy. Similarly using the \( \alpha \)-particles corresponding to the state at \( E_x=1058 \text{ keV of } ^{26}\text{Al} \) (which decays 100\% to the first excited state at \( E_x=228 \text{ keV of } ^{26}\text{Al} \)), a probability of \( (A_2'/Q_L)/(N_2/Q_N) \) for the 830 keV \( \gamma \)-ray could be obtained where \( A_2' \) is the number of \( \alpha \)-particles corresponding to the state at \( E_x=1058 \text{ keV} \) in coincidence with the 830 keV \( \gamma \)-ray and \( N_2 \) is the number of ungated \( \alpha \)-particles corresponding to the same state. If \( A_i \) is the number of \( \alpha \)-particles from one of our states of interest collected in the LeCroy MCA in coincidence with the 417 keV gamma-ray for \( Q_L \) beam charge in the integrator, then the number of \( \alpha \)-particles which entered the charged particle detector from that state should be given as \( (A_i/Q_L)/[(A_1/Q_L)/(N_1/Q_N)] \). If \( N_i \) is the number of ungated \( \alpha \)-particles corresponding to the same state of interest
collected in ND100 for $Q_N$ beam charge then the fraction $F(417 \text{ keV})$ of $\alpha$-particles corresponding to the state of interest that decayed through the second excited state at $E_x=417 \text{ keV}$ to ground state is given by

$$F(417 \text{ keV}) = \frac{(A_1/Q_L)/[(A_1/Q_L)/(N_1/Q_N)]/(N_1/Q_N)}{A_1N_1/A_1N_1}$$

$F(417 \text{ keV})$ gives the lower limit $F_1$ of the branching of a state decaying to ground state. Similarly, the fraction $F(830 \text{ keV})$ of $\alpha$-particles corresponding to the state of interest that decayed through state at $E_x=1058 \text{ keV}$ to the first excited state at $E_x=228 \text{ keV}$ is given by

$$F(830 \text{ keV}) = A_1'/N_2/A_2N_1$$

where $A_1'$ is the number of $\alpha$-particles corresponding to the state of interest in coincidence with 830 keV gamma-rays collected in the LeCroy MCA for $Q_L$ beam charge.

As discussed earlier in section 4.2, the quantity $[1-F(830 \text{ keV})]$ gives the upper limit $F_u$ of the branching ratio of that state decaying to ground state. The values of $A_1$ and $A_1'$ are listed in Table 4.2 for the states at $E_x=6363, 6410$ and 6498 keV. $F=[(F_1+F_u)/2]$ gives the average value of the branching ratio to ground state for the states of interest. The value $F$ for the state at $E_x=6498 \text{ keV}$ is $0.44\pm0.22$ which is in agreement with the results from Elix et al$^{18}$ (0.66±0.9). This suggests that the average values of branching ratios to ground state from our states of interest are reasonable.
Table 4.2

Calculation of Branching Ratio Limits

<table>
<thead>
<tr>
<th>Excitation Energy $E_x$ (keV)</th>
<th>417</th>
<th>1058</th>
<th>6363</th>
<th>6410</th>
<th>6498</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_i$ ungated alpha-particles</td>
<td>1030</td>
<td>581</td>
<td>1341</td>
<td>399</td>
<td>1512</td>
</tr>
<tr>
<td>$A_i$ alphas gated with 417 keV gamma</td>
<td>90</td>
<td>0</td>
<td>91</td>
<td>20</td>
<td>41</td>
</tr>
<tr>
<td>$A_i'$ alphas gated with 830 keV gamma</td>
<td>0</td>
<td>35</td>
<td>44</td>
<td>13</td>
<td>40</td>
</tr>
<tr>
<td>$F(417 \text{ keV}) = \frac{A_i N_i}{A_i N_i}$</td>
<td>1</td>
<td>0</td>
<td>0.78</td>
<td>0.57</td>
<td>0.31</td>
</tr>
<tr>
<td>$F_1$ (lower limit)</td>
<td>1</td>
<td>0</td>
<td>0.78</td>
<td>0.57</td>
<td>0.31</td>
</tr>
<tr>
<td>$F(830 \text{ keV}) = \frac{A_i' N_i}{A_i N_i}$</td>
<td>0</td>
<td>1</td>
<td>0.54</td>
<td>0.53</td>
<td>0.44</td>
</tr>
<tr>
<td>$1 - F_1$</td>
<td>0</td>
<td>1</td>
<td>0.54</td>
<td>0.53</td>
<td>0.44</td>
</tr>
<tr>
<td>$F_1$ (upper limit)</td>
<td>1</td>
<td>0</td>
<td>0.46</td>
<td>0.47</td>
<td>0.56</td>
</tr>
<tr>
<td>$F = (F_1 + F_1)/2$, Average Branching Ratio (g.s)</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>0.52</td>
<td>0.44</td>
</tr>
<tr>
<td>Literature Branching ratio (g.s)</td>
<td>1</td>
<td>0</td>
<td>0.62</td>
<td>0.52</td>
<td>0.44</td>
</tr>
</tbody>
</table>
Fig. 4.4. Angular distribution of 417 and 830 keV gamma-rays.
In the above calculation for the branching ratios, we have implicitly assumed that the angular distribution of the $\gamma$-rays 417 and 830 keV are isotropic. This may not be true. The experiment was repeated to get the angular distribution of the above two $\gamma$-rays. Using the angular correlation coefficients of dipole and quadrupole strengths (Legendre polynomial coefficients), the data was fit and is shown in Fig 4.4. The angular distribution is not isotropic in both cases. But the difference is within 15% from isotropic for both gammas. The corrections are included in Table 4.3.

If one wants to get the actual (not just limits) branching ratios to the ground state from our states of interest then one should do a number of experiments such as the $^{25}\text{Mg}(^{3}\text{He},d)^{26}\text{Al}$ to determine a suitable angle and energy so that only one state from either one of our doublets could be populated. This certainly involves the use of a mass spectrometer with nuclear emulsion plates since very good resolution of about 14 keV is required to separate the doublets. A great deal of time is required to complete this task. Once the proper energy and angle are determined then the branching ratio of that state could be determined within a small uncertainty (e.g. about 5%) provided that state has a strong population over the background radiation. However in our case, it is redundant
Table 4.3
Calculation of Branching Ratio Limits

<table>
<thead>
<tr>
<th>Excitation Energy $E_x$(keV)</th>
<th>417</th>
<th>1058</th>
<th>6363</th>
<th>6410</th>
<th>6498</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_i$ ungated alpha-particles</td>
<td>1030</td>
<td>581</td>
<td>1341</td>
<td>399</td>
<td>1512</td>
</tr>
<tr>
<td>$N_i$ alphas gated with 417 keV gamma</td>
<td>0</td>
<td>91</td>
<td>20</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>$N_i$' alphas gated with 830 keV gamma</td>
<td>0</td>
<td>32</td>
<td>44</td>
<td>13</td>
<td>40</td>
</tr>
<tr>
<td>$F(417 \text{ keV}) = A_{1}N_{1}/A_{1}N_i$</td>
<td>1</td>
<td>0</td>
<td>0.68</td>
<td>0.50</td>
<td>0.27</td>
</tr>
<tr>
<td>$F_{1}$(lower limit)</td>
<td>0</td>
<td>0.58</td>
<td>0.57</td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>$F(830 \text{ keV}) = A_{i}'N_{2}/A_{2}'N_i$</td>
<td>0</td>
<td>1</td>
<td>0.58</td>
<td>0.57</td>
<td>0.48</td>
</tr>
<tr>
<td>$1-F_u$</td>
<td>0</td>
<td>0.42</td>
<td>0.43</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>$F_u$Upper limit</td>
<td>1</td>
<td>0</td>
<td>0.42</td>
<td>0.43</td>
<td>0.52</td>
</tr>
<tr>
<td>$F=(F_{1}+F_u)/2$, Average Branching Ratio(g.s)</td>
<td>1</td>
<td>0</td>
<td>0.55</td>
<td>0.47</td>
<td>0.40</td>
</tr>
<tr>
<td>Literature Branching ratio(g.s)</td>
<td>1</td>
<td>0</td>
<td></td>
<td>0.66</td>
<td></td>
</tr>
</tbody>
</table>

a) 14.7% increase due to isotropic correction
b) 9.3% decrease due to isotropic correction
to obtain such an accuracy for branching ratio since the resonance strengths of our states have about 100% uncertainty so that it would not make any major difference in the astrophysical calculation rate for the production of $^{26}\text{Al}$.

In summary, we have all the necessary experimental tools (energy levels, their proton resonance strengths and branching ratios) needed to calculate the reaction rate of the $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$ reaction in stellar environments. The next chapter describes the theory required for this calculation.
Chapter V

Thermonuclear Reaction Rates

The $^{26}$Al isotope is produced by thermonuclear reactions in stellar environments. Thermonuclear reaction can be classified into two major categories. They are non-resonant reaction and resonant reaction. Depending on the temperature of the stellar environment and the nuclear level structure of nuclei participating in the reaction, one process can dominate the other. The word cross-section is used as a measure of reaction rate. Consider a nuclear reaction $a+A\rightarrow B+b$. Assume that some nuclei of type $A$ are being bombarded by a uniform flux of particles of type $a$, then the cross-section for the reaction can be defined as
\[
\sigma (\text{cm}^2) = \frac{\text{number of reactions/nucleus } A/\text{unit time}}{\text{number of incident particles/cm}^2/\text{unit time}}.
\]
In a mixture of gases in the state of thermodynamic equilibrium in stellar interiors, the quantity $\langle \sigma v \rangle$ is used instead of $\sigma$. The term $\langle \sigma v \rangle$ represents the average value of the product of relative velocity between $A$ and $a$ and the cross-section. Using Maxwell-Boltzman distribution of velocities, one can show\(^{31}\) that the reaction rate per pair particles is given
by
\[ \langle \sigma v \rangle = \frac{4\pi}{\mu^2} (\frac{\mu}{2\pi kT})^{3/2} \int_0^\infty v^3 \sigma(v) \exp(-\frac{\mu v^2}{2kT}) dv \]
where \( \mu \) = reduced mass of A and a
\( k \) = Boltzman's constant

The details of \( \sigma(v) \) in the integrand should be known in order to evaluate the integral.

5.1 Nonresonant Reaction Rates

Nuclear reactions can proceed in stars only if the reacting particles penetrate the Coulomb barrier that separates them. The Coulomb barrier between two particles is given by

\[ V = \frac{Z_1 Z_2 e^2}{R} = \left[ \frac{1.44 Z_1 Z_2}{R \text{(fm)}} \right] \text{MeV} \]

But the kinetic energy of the interacting particles in stellar environment is given by Maxwell-Boltzmann distribution of velocities corresponding to a thermal energy

\[ kT = 8.62 \times 10^{-5} T \text{ keV} \]

By comparing both equations it is obvious that in stellar environments (10 to 1000 T\(_K^\circ\)), the average kinetic energy of interacting particles is many orders of magnitude smaller than the Coulomb barrier that separates them. This implies that the particles with the highest energies in Maxwell-Boltzmann distribution have the best chance of penetrating the Coulomb barrier. From the Maxwell-Boltzmann
distribution of velocities one can show that the probability for a pair of particles to have velocity \( v \) in the range of \( v+dv \) is
\[
\phi(v)dv = (\mu/2\pi kT)^{3/2} \exp(-\mu v^2/2kT) \cdot 4\pi v^2 dv
\]
which decreases rapidly with energy for \( \mu v^2/2\pi kT \). If the probability for barrier penetration is known then the product of this quantity with \( \phi(v)dv \) will give the optimum region for maximum cross-section. From the theory of \(\alpha\)-decay (Gamov), we know that the barrier penetration is proportional to \( \exp(-2\pi Z_1 Z_2 e^2/\hbar v) \). A similar nature for nuclear reactions can also be assumed since the nuclear particles have to overcome the Coulomb effect first. The quantum-mechanical interaction between two particles is always proportional to a geometric factor, \( \pi \lambda^2 \), where \( \lambda \) is the de Broglie wavelength.
\[
\pi \lambda^2 \propto (1/p)^2 \propto 1/E.
\]
Combining these two factors with nuclear interaction, one can write \( \sigma(E) \) as
\[
\sigma(E) = S(E) \cdot (1/E) \cdot \exp(-2\pi Z_1 Z_2 e^2/\hbar v)
\]
where \( S(E) \) represents the intrinsically nuclear parts of the probability for the occurrence of a nuclear reaction. It is also found to be a constant or at least a slowly varying function of energy over a limited energy range for nonresonant reactions. Replacing the velocities, \( v \), in terms of the energy \( E \), the reaction rate per pair particles becomes
\[
\lambda = \langle \sigma v \rangle = \int_0^\infty \sigma(E) v(E) \psi(E) dE \\
= \left( \frac{8}{\mu \pi} \right)^{1/2} \left( \frac{1}{kT} \right)^{3/2} \int_0^\infty S(E) \exp\left[ (-E/kT) - bE^{-1/2} \right] dE \ldots \ldots (A)
\]

where \( b = 31.28 z_1 z_2 A^{1/2} \) (keV)\(^{1/2} \), and the reduced atomic weight \( A = A_1 A_2 / (A_1 + A_2) = \mu / M_u \).

Since \( \exp(-E/kT) \) goes rapidly to zero for large \( E \) and \( \exp(-bE^{-1/2}) \) goes rapidly to zero for small \( E \), the major contribution to the integral will come from values of the energy that are such that the exponential factor is near its maximum. Most stellar reactions occur in a fairly narrow band of stellar energies and the factor \( S(E) \) will have a nearly constant value over the band of energies. If \( S(E) \) is replaced by \( S_0 \) (constant or average) in the last equation, we get reaction rate per pair

\[
\lambda = \langle \sigma v \rangle = \left( \frac{8}{\mu \pi} \right)^{1/2} S_0 \left( \frac{kT}{\mu} \right)^{-3/2} \int_0^\infty \exp\left[ (-E/kT) - bE^{1/2} \right] dE \ldots \ldots (B)
\]

This integral can be evaluated by approximating the integrand by an appropriate Gaussian (Known as the method of steepest descent).

The product of \( \exp(-E/kT) \) and \( \exp(-bE^{-1/2}) \) is plotted in Fig 5.1. The particles that are most effective in causing nuclear reactions are those pairs having energies near \( E_0 \). The value of \( E_0 \) is determined from the maximum of the term, \( \left[ (-E/T) + b / \sqrt{E} \right] \), in the exponent and we get \( E_0 = (bkT/2)^{2/3} = 1.22 (z_1^2 z_2^2 A_T^2)^{1/3} \) keV. \( E_0 \) is also known as the "most effective energy for thermonuclear reactions".

For the method of steepest descent, one can write
Fig. 5.1 The dominant energy-dependent factors in thermonuclear reactions.
\[ \exp\left[\frac{-E}{kT} - bE^{-1/2}\right] = \exp(-g) \exp\left[-\frac{(E-E_0)/(\Delta/2)}{2}\right] \]

where \( g = \frac{E_0}{kT} + bE_0^{-1/2} = 3E_0/kT \)

and \( \Delta = (4/\sqrt{3})(E_0kT)^{1/2} \) \((\Delta/2 = 1/e \text{ width of the exponential})\)

Substituting these in eq. A, one gets

\[ \lambda = \langle \sigma v \rangle = \left(\frac{8}{\pi n}\right)^{1/2} (1/kT)^{3/2} e^{-g} \int S(E) \exp\left[-\frac{(E-E_0)/(\Delta/2)}{2}\right] dE \]

When \( S(E) \) is constant, it can be replaced by \( S_0 \) and we get reaction rate per pair particles as

\[ \lambda = \langle \sigma v \rangle = 4.5 \times 10^{14} S_0 g^2 e^{-g} A_{1/2} \text{ cm}^3/\text{sec} \]

\[ = 7.2 \times 10^{-19} S_0 (\text{keV barns}) g^2 e^{-g} A_{1/2} \text{ cm}^3/\text{sec} \]

The total reaction rate per unit volume can be written as

\[ r_{12} = (1 + \delta_{12})^{-1} N_1 N_2 \lambda_{12} \]

To introduce local density into the reaction rate, one should replace \( N_i \) by

\[ N_i = \rho N_0 X_i / A_i \]

where \( X_i \) is the fraction by mass of species \( i \).

Hence we get,

\[ r_{12} = 2.6 \times 10^{29} \rho^2 X_1 X_2 S_0 (\text{keV barns}) g^2 e^{-g} / [(1 + \delta_{12}) A_{1/2} A_{A_{1/2}}] \text{ cm}^{-3}/\text{sec} \]

This equation represents the basic nonresonant stellar reaction-rate formulae in first approximation. For a detailed discussion with correction factors to \( S(E) \) and to the evaluation of integral, one should refer to Clayton^31).
5.2 Resonant Reaction Rates

This is classified into two categories.

a). Reactions when the resonances are far from the effective stellar energies, \( E_0 \) (known as reactions in the wings of resonances, \( E_r > E_0 \)).

b). Reactions when the resonances are in the range of stellar energies, \( E_r = E_0 \).

Using the Breit-Wigner single level formula for resonance cross-section\(^{33}\), we get

\[
\sigma_{\Gamma_1}(1,2) = (2J+1)\pi\lambda^2 \Gamma_1 \Gamma_2 / [(E-E_r)^2 + (\Gamma/2)^2]
\]

where \( \pi\lambda^2 = 656.6/\text{AE barns} \) and \( \Gamma_1 \) and \( \Gamma_2 \) are the partial widths of the compound state to the entrance and exit channels.

Introducing nuclear spin

\[
\sigma_1(1,2) = \pi\lambda^2 (2J+1)/[(2J_1+1)(2J_2+1)] \Gamma_1 \Gamma_2 / [(E-E_r)^2 + (\Gamma/2)^2]
\]

Using the above equation, one can easily show\( ^r \) that

\[
S(E) = (657/A)(\omega\Gamma_1(E)\Gamma_2)/[(E-E_r)^2 + (\Gamma/2)^2] \exp(31.28Z_1Z_2/\text{A}\sqrt{E}) \text{ keV barns}
\]

where \( \omega = (2J+1)/[(2J_1+1)(2J_2+1)] \)

A typical plot is shown in Fig 5.2. It is clear from the fit that if the resonance is very far from \( E_0 \), the reaction must proceed through the Wings of the resonance and the nonresonant reaction-rate formalism must be used.
Fig. 5.2. A schematic representation of $S(E)$. 
When the resonance is in the vicinity of $E_0$, the full height of the resonant cross-section must be employed and when the width $\Gamma$ is very small, the integration for $r_{12}$ can be performed. The reaction rate is then given by

$$r_{10} = N_1 N_0 \int \psi(E) \nu(E) \sigma(E) dE$$

where $\psi(E)$ is given by Maxwell-Boltzmann energy distribution,

$$\psi(E) dE = \frac{2 \sqrt{\pi}}{\sqrt{T}} \left( \frac{E}{kT} \right) \left( \frac{2}{\pi} \right) \frac{e^{-E/kT}}{\sqrt{T}} dE$$

when $\Gamma < \Delta E$, $\psi(E)$ and $\nu(E)$ change only very little and the integral can be written as

$$r_{12} = N_1 N_0 \psi(E) \nu(E) \int \sigma(E) dE$$

Again replacing $\lambda^2$ and $\Gamma_1$ by their values at $E_r$, the integration can be performed and the result is

$$\int \sigma(E) dE = 2 \pi \lambda^2 \omega \Gamma_1 \Gamma_2 / \Gamma$$ [since area under cross-section = $(\pi \Gamma / 2)$(peak value of $\sigma(E)$)]

Hence

$$r_{10} = N_1 N_0 (2/\sqrt{\pi}) (\sqrt{E_r} / kT)^{-3/2} \exp(-E_r / kT) \left( \sqrt{2 E_r / \mu} \right) 2 \pi \lambda^2 \omega \Gamma_1 \Gamma_2 / \Gamma$$

$$= N_1 N_0 \frac{h^2}{2\pi \mu kT} \frac{2 \pi \lambda^2 \omega \Gamma_1 \Gamma_2 / \Gamma}{E_r} \exp(-E_r / kT)$$

$$= 8 \times 10^{-12} \times N_1 N_0 \omega (A T_6)^{-3/2} \times (\Gamma_1 \Gamma_2 / \Gamma) \exp(-11.6 E_r / T_6) \text{ cm}^{-3} \text{ sec}^{-1}$$

$$= 2.94 \times 10^{36} \rho^2 (X_1 X_0 / A_1 A_2) \omega (A T_6)^{-3/2} (\Gamma_1 \Gamma_2 / \Gamma) \exp(-11.6 E_r / T_6) \text{ cm}^{-3} \text{ sec}^{-1}$$

$r$'s are in keV.

Finally for many isolated narrow resonances, we have

$$\langle \sigma \nu \rangle = (2 \pi h^2 / \mu kT)^{3/2} \frac{\Gamma_r}{\hbar} (\omega \Gamma_1 \Gamma_2 / \hbar \Gamma) \exp(-E_r / kT)$$
With this equation, we can calculate the thermonuclear reaction rates for our case. This will be considered in the next chapter.
CHAPTER VI

Reaction rate calculation and conclusion

In this chapter, we calculate the $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$ reaction rate in stellar environments as a function of temperature for $T_9<0.3$. Only low energy resonances are considered in the calculations since the contributions from high energy resonances or non-resonant reaction processes are negligible in this temperature range.

The reaction rate for narrow resonances is given by

$$N_A\langle\sigma v\rangle=N_A\left[\frac{2\pi}{\mu kT}\right]^{3/2}\left[\frac{h}{2\pi}\right]^2\sum_r(f\omega\Gamma_p\Gamma_\gamma/\Gamma)\exp(-E_r/kT)$$

where $N_A=$ Avogadroy's number

$E_r=$ The resonance energy (c.m.) for a system with reduced mass $\mu$ at temperature $T$

$k=$ Boltzmann constant

$f=$ branching ratio of each state to ground state

$\Gamma_\gamma=$ $\gamma$-ray width

$\Gamma_p=$ proton width

$\Gamma=$ Total width

$\omega=(2J_r+1)/[(2J_r+1)(2J_p+1)]$

$J_r=$ Angular momentum of the resonance
\[ J_t = \text{Angular momentum of the target} \]
\[ J_p = \text{Angular momentum of the transferred proton} \]

Substituting the values for our system, this reduces to

\[ N_A <\sigma v> = 1.63 \times 10^{11} \gamma T^{-3/2} \exp(-E_r/kT) \]

(where \( \gamma = \gamma \gamma_p / \gamma \))

Writing the contribution from each term separately,

\[ N_A <\sigma v> = 3.42 \times 10^{-6} f_1 T_9^{-3/2} \exp(-0.661/T_9) \]
\[ + 0.326 f_2 T_9^{-3/2} \exp(-1.218/T_9) \]
\[ + 0.0689 T_9^{-3/2} \exp(-2.230/T_9) \]
\[ + 2.84 \times 10^3 T_9^{-3/2} \exp(-3.534/T_9) \]
\[ + 2.60 \times 10^4 T_9^{-3/2} \exp(-4.405/T_9) \]

The first two terms correspond to the contribution from the resonances 58 keV and 105 keV c.m proton (bombarding) energies. The third and fourth term correspond to the contribution from the resonances at 192.2 keV and 304.5 keV taken from Champagne et al.\(^{24}\). The last term corresponds to the contribution from resonances up to \( E_{\text{cm}} = 1660 \) keV taken from Champagne\(^{24}\). The terms \( f_1 \) and \( f_2 \) correspond to the branching of the states \( E_x = 6363 \) keV and 6410 keV to the ground state. The final spin of these two states has been assumed to be \( J_f = 3^+ \)(see ref.\(^{33}\)) for the first one and \( J_f = 2^+ \)(see ref.\(^{33}\)) for the last one (either \( 2^+ \) or \( 3^+ \) only).

Calculations for \( N_A <\sigma v> \) were done for three different cases. In the first case, average values of the branching ratios were used for the first two states. For the second case, upper limits of branching ratios were used. Finally, for the third case lower limits of branching ratios were
used. These values are listed in Table 6.1. The fourth and fifth column give the values from Champagne et al\(^{24}\) and the contribution from high energy proton threshold states corresponding to the states at \(E_x > 6498\) keV. (taken from Anderson\(^{19}\) and Elix et al\(^{18}\)). Champagne et al\(^{24}\) considered the states at \(E_x = 6343\) keV and 6400 keV with \(J^\pi = 3^+\) and \(2^+\) respectively and did the calculation for reaction rates. According to our results, these states have \(J^\pi = (1+4)^-\) and their contribution to the reaction rate is negligible compared to the contribution from the nearby states at \(E_x = 6363\) keV and 6410 keV with \(J^\pi = (2,3)^+\). This implies that the reaction rate calculation done by Champagne et al\(^{24}\) are probably in error. Anderson et al\(^{19}\) and Elix et al\(^{18}\) did not consider any states below \(E_x = 6498\) keV for their calculation. The sixth column gives the statistical-model code (Hauser\(^{43}\)) calculated values taken from Anderson et al\(^{19}\). One could see the tremendous increase in reaction rate around \(T_9 = 0.1\) from our results by looking at the log reaction rates comparison (see Fig. 6.1 and 6.2). In view of the highly resonant nature of the experimental data (including both low and high energy resonances), the statistical-model estimates are surprisingly good for all the temperature range. Without our results (corresponding to the high energy resonances only), one would have had difficulties in explaining why the statistical-model calculations do not agree for
TABLE 6.1
Reaction rate calculation for $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}$

<table>
<thead>
<tr>
<th>$T_g$ (°K)</th>
<th>Average B.Ratio</th>
<th>High B.Ratio</th>
<th>Low B.Ratio</th>
<th>High energy Sta.</th>
<th>Improper Sta.</th>
<th>Statistical Model</th>
</tr>
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<td>0.04</td>
<td>3.15x10^{-13}</td>
<td>3.88x10^{-13}</td>
<td>2.47x10^{-13}</td>
<td>5.29x10^{-24}</td>
<td>1.34x10^{-13}</td>
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<tr>
<td>0.05</td>
<td>8.78x10^{-12}</td>
<td>1.04x10^{-11}</td>
<td>7.13x10^{-12}</td>
<td>2.63x10^{-19}</td>
<td>8.69x10^{-13}</td>
<td>2.90x10^{-12}</td>
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<td>0.06</td>
<td>1.65x10^{-10}</td>
<td>1.83x10^{-10}</td>
<td>1.45x10^{-10}</td>
<td>3.39x10^{-16}</td>
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<tr>
<td>0.07</td>
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<td>2.10x10^{-09}</td>
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<td>5.44x10^{-14}</td>
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</tr>
<tr>
<td>0.08</td>
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<td>1.43x10^{-08}</td>
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<td></td>
</tr>
<tr>
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<td>6.36x10^{-08}</td>
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<td>2.41x10^{-04}</td>
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<tr>
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<td>1.19x10^{-03}</td>
<td>1.19x10^{-03}</td>
<td>1.16x10^{-03}</td>
<td>1.16x10^{-03}</td>
<td>8.17x10^{-03}</td>
</tr>
</tbody>
</table>

a) Ref. 19  b) Ref. 18  c) Ref. 24  d) Ref. 33
Fig. 6.1. Thermonuclear reaction rates.
Fig. 6.2. A comparison of thermonuclear reaction rates.
temperatures \(T_g < 0.2\), but do for \(T_g > 0.2\).

The present work gives an almost complete stellar reaction rate for the production of \(^{26}\text{Al}\) via the \(^{25}\text{Mg}(p,\gamma)^{26}\text{Al}\) reaction. However this reaction rate does not help a lot in obtaining the \(^{26}\text{Al}/^{27}\text{Al}\) abundance ratio in the Mg-Al cycle, since the ratio is given by

\[
(26\text{Al}/27\text{Al})_{\text{equil}} = aN_A <\sigma\nu>_p (27\text{Al})\lambda_\beta (25\text{Al}) /
\]

\[
[N_A (\sigma\nu)_p (26\text{Al}) [p \times N_A <\sigma\nu>_p (25\text{Al}) + \lambda_\beta (25\text{Al})]]
\]

where \(a\) is the fraction of \(^{25}\text{Mg}\) proton captures to the \(^{26}\text{Al}\) ground state, all the other symbols having their usual meaning as defined in the previous chapter. Arnould et al.\(^{10}\) used a value for \(a=0.5\)(using only the high energy resonances) and they were able to get a reasonable \(^{26}\text{Al}/^{27}\text{Al}\) ratio of 0.1 to 1.0 which was sufficient enough to explain the \(^{26}\text{Mg}\) anomaly found in the Allende meteorite. If the low energy resonances from the present results are also included, the quantity \(a\) will take almost a similar value (≈0.47 to 0.55). In other words by including the low energy resonances into the above equation, we would still get the same value for the \((^{26}\text{Al}/^{27}\text{Al})\) ratio. However in deriving the above equation, a number of assumptions were implicitly made about the stellar reaction rate of the \(^{25}\text{Mg}(p,\gamma)^{26}\text{Al}\) reaction. According to Arnould et al.\(^{10}\) if the stellar reaction rate would be higher by an order of magnitude or more from what they had calculated earlier
(note: They had the information only about the high energy resonances available at that time) then these assumptions could be violated and they have to look for a different region\textsuperscript{10}) for the production of $^{26}$Al. Furthermore from the recent work of Schmalbrock et al\textsuperscript{35}), the destruction rate of the $^{26}$Al(p,$\gamma$)$^{27}$Si reaction could be considerably higher than what was predicted earlier. This again shows that the above equation cannot be used alone to describe the $^{26}$Mg anomaly or the amount of $^{26}$Al in the interstellar medium even at present\textsuperscript{16}) without the complete stellar reaction rates for the reactions $^{25}$Mg(p,$\gamma$)$^{26}$Al and $^{26}$Al(p,$\gamma$)$^{27}$Si. Since the present results give an almost complete stellar reaction rate for the $^{25}$Mg(p,$\gamma$)$^{26}$Al reaction and when the latter reaction rate is better determined, one will have all the necessary information about the complete Mg-Al cycle reactions in order to explain the $^{26}$Mg anomaly and the amount of $^{26}$Al in the interstellar medium.
Appendix

Target Preparation

The $^{27}$Al targets were prepared from natural aluminum sheets. They are easier to evaporate than using powder or pellets of aluminum. A tantalum boat of thickness .005" was wrapped by an aluminum sheet and was heated beyond the melting point of aluminum. The sheet quickly started melting and then tried to condense on Teepol coated glass slides kept inside the evaporator. The advantage of this method is the rapid exchange of heat from the tantalum boat to the aluminum sheet (more surface area in contact) which allows shorter time to heat the glass slide and thereby increasing the chance of condensing aluminum on it. Later, the target slides were floated on water and targets were taken on target frames. Very thin self supporting targets of thickness $10 \mu g/cm^2$ were prepared from this method as well as thick targets ($\approx 50 \mu g/cm^2$).

The enriched $^{24}$Mg and $^{25}$Mg oxide powder were bought from Oak Ridge National Laboratory. In each case the oxide powder was put inside a tantalum boat which has a
hole in it. The size of the hole is critical. If it is too large then most of the oxide will pop out before it has started melting in it (note: Rapid chemical reaction occurs between the magnesium oxide and tantalum metal). If it is too small then most of the reduced magnesium would stay inside the tantalum boat. I used a 0.043" hole tantalum boat (0.005" thick) and used joule heating method to evaporate the metal. The evaporated magnesium metal started condensing on carbon slides inside the evaporater. Later on the carbon slides were floated on water and the targets were taken on target frames. Again any thickness target can be made by this process.
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