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Ramesh, Mahadevan

COUPLED OSCILLATIONS OF THE MAGNETIC DOMAIN-DOMAIN WALL SYSTEM IN SUBSTITUTED GARNET THIN FILMS

The Ohio State University

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COUPLED OSCILLATIONS OF THE MAGNETIC DOMAIN-DOMAIN
WALL SYSTEM IN SUBSTITUTED GARNET THIN FILMS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Mahadevan Ramesh, M.Sc.

The Ohio State University
1985

Reading Committee: Approved by

Prof. P.E. Wigen
Prof. J. T. Tough
Prof. C.A. Ebner

Dr. Philip E. Wigen, Advisor
Department of Physics
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VITA

October 21, 1955 Born, India

1977 M.Sc., Indian Institute of Technology, Kanpur, India

1978 - 1979 University Fellow, Department of Physics, The Ohio State University, Columbus, Ohio.

1979 - 1983 Teaching Assistant, Department of Physics, The Ohio State University, Columbus, Ohio.

1983 - 1985 Research Assistant, Department of Physics, The Ohio State University, Columbus, Ohio.

PUBLICATIONS


"The Effect of Cubic Anisotropy on the Coupled Domain-Domain Wall Oscillations" M. Ramesh, L. Pust and P.E. Wigen. (To be published).

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ACKNOWLEDGEMENTS</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>VITA</td>
<td>111</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xv</td>
</tr>
</tbody>
</table>

## CHAPTERS

I. INTRODUCTION. ............................................ 1

II. THEORY. .................................................. 15

1. Introduction. .......................................... 15

2. Domain Energy Terms. ................................. 24

   2.1 Uniaxial Anisotropy Energy and Zeeman Energy. . 24

   2.2 Demagnetization Energy. ............................. 26

3. Domain Wall Energy Terms. ............................. 36

   3.1 180° Bloch Wall. .................................. 37

   3.2 Bloch Wall Under $H_p$ . ........................... 42

   3.3 Domain Wall Energy: Dynamic Term. ................ 45

4. Static Equilibrium Conditions. ....................... 48

   4.1 Equilibrium Orientation of Domain Magnetization. 50

   4.2 Static Wall Energy and Domain Spacing. ............ 55
5. Dynamics of Domain-Domain Wall System
   5.1 Equations of Motion
   5.2 Domain Ferromagnetic Resonance
   5.3 Domain Wall Resonance
   5.4 Mode Coupling

6. Cubic Anisotropy Energy

## III. EXPERIMENTAL
1. Materials
2. Experimental Set Up
3. Experimental Observations

## IV. DISCUSSION
1. Domain Resonances
2. Domain Wall Resonance and Mode Coupling

## V. CONCLUSION

LIST OF REFERENCES

APPENDIX A. TO FIND AN EXPRESSION FOR THE ROTATIONAL PERMEABILITY OF PARALLEL STRIPE DOMAIN STRUCTURE

APPENDIX B. CALCULATION OF THE DEMAGNETIZATION ENERGY DUE TO $M_y$ COMPONENTS IN THE WALL AND THE DOMAINS

APPENDIX C. TO COMPUTE THE RATIO

\[ \frac{(f_{DWR})_{H_p=H_{sat}}}{(f_{DWR})_{H_p=0}} \]

APPENDIX D. PROGRAM TO CALCULATE THE FREQUENCIES OF COUPLED OSCILLATIONS
**LIST OF FIGURES**

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1</td>
<td>Parallel stripe domain pattern. (Ref. 17)</td>
<td>3</td>
</tr>
<tr>
<td>Fig. 2</td>
<td>Bubble domain pattern. (Ref. 17)</td>
<td>4</td>
</tr>
<tr>
<td>Fig. 3</td>
<td>Bloch and Néel Walls.</td>
<td>7</td>
</tr>
<tr>
<td>Fig. 4</td>
<td>A schematic diagram of a displaced wall, represented by 2. The undisplaced wall is labeled 1. (Ref. 18)</td>
<td>10</td>
</tr>
<tr>
<td>Fig. 5</td>
<td>Domain pattern under the influence of $H_p$. (Ref. 18)</td>
<td>12</td>
</tr>
<tr>
<td>Fig. 6</td>
<td>Coordinate system used to define the domain variables.</td>
<td>20</td>
</tr>
<tr>
<td>Fig. 7</td>
<td>Coordinate system used to study the domain wall. The wall variables, $\theta$ and $\psi$ are shown. (Ref. 17)</td>
<td>22</td>
</tr>
<tr>
<td>Fig. 8</td>
<td>Coordinate system used to derive the demagnetization energy term. The origin is the center of domain of type 1. Magnetic charge distributions are also shown.</td>
<td>27</td>
</tr>
<tr>
<td>Fig. 9</td>
<td>A plot of the demagnetization factor $N_{zz}$ vs. $\ln (L/T)$, where $L/T$ is the domain aspect ratio, defined following Eq. 22.</td>
<td>33</td>
</tr>
<tr>
<td>Fig. 10</td>
<td>Static wall profile for a 180° Bloch wall. Two solutions exist and the magnetization can change its orientation by either a clockwise or counter clockwise spiral. (Ref. 17)</td>
<td>40</td>
</tr>
</tbody>
</table>
Fig. 11. A plot of static equilibrium value of $\theta_{\perp}$ as a function of $H_p$ for samples. The solid curve is obtained from Eq. 66 and the dotted curve is from Eq. 65 which includes the domain wall correction term. ........................................... 53

Fig. 12. A plot of reduced static domain wall energy $\sigma_{WO}/\sigma_0$ as a function of $H_p$, for sample 5. The dotted curve is the high $Q$ approximation for the sample. ........................................... 57

Fig. 13. A plot of reduced static domain wall energy, $\sigma_{WO}/\sigma_0$ as a function of $H_p$, for sample 1. The dotted curve is the high $Q$ approximation. While the approximate curve reaches zero asymptotically, the actual curve goes to zero, more sharply. ........................................... 58

Fig. 14. A plot of the equilibrium domain spacing $L$ as a function of $H_p$ for sample 5. .............................. 60

Fig. 15. A plot of $N_{zz}$ as a function of $H_p$ for sample 5. .............................. 62

Fig. 16. A plot of the effective domain wall width $\Lambda_{eff}$ as a function of $H_p$. $\Lambda_{eff}$ has a singularity at $H_p = H_{sat}$. ........................................... 63
Fig. 17. Domain resonance frequencies $f^\pm$ vs. $H_p$ for sample 5 when $H_p$ is along the x axis ($\alpha=0$). The uniform FMR is also shown. The two modes $\omega^+$ and $\omega^-$ are seen to intersect. ........................ 74

Fig. 18. Domain resonance frequencies $f^\pm$ vs. $H_p$ for sample 5. $H_p$ is along y ($\alpha=\pi/2$). The uniform FMR is also shown. The two modes $\omega^+$ and $\omega^-$ do not intersect, in this case. ...................... 75

Fig. 19. "In-Phase" precession of the magnetization vectors in the two domains, corresponding to $\omega^+$ mode. At the instant shown in Fig. 19(a) both the vectors tend to come out of the page ($\Delta\phi_1 = \Delta\phi_2$). A half cycle later, as shown in Fig. 19(b) both the magnetization vectors precess into the page. ...................... 77

Fig. 20. "Out of phase" precession of the magnetization vectors in the two domains corresponding to the $\omega^-$ mode. For the instant shown in Fig. 20(a) one vector is coming out of the page while the other is going into the page. Fig. 20(b) shows the situation a half a cycle later. Again one vector is going into the page while the other is coming out. ...................... 79
Fig. 21. A plot of the domain resonance frequencies $f^\pm$ and FMR frequency as functions of $H_p$ for $\alpha=10^\circ$, for sample 5. A coupling and consequent mode repulsion can be seen between the $\omega^+$ and $\omega^-$ modes, due to the coupling term $\beta$. .............. 81

Fig. 22. A plot of the force constant $k$ vs. $H_p$ for sample 5. Although not shown, $k$ goes to zero when $H_p$ equals $H_{\text{sat}}$. ................. 83

Fig. 23. A plot of reduced domain wall mass $m/m_0$ where $m_0$ is the "zero in-plane" field mass, shown as a function of $H_p$ for sample 5. The dotted curve is obtained using the high Q approximation. ............... 87

Fig. 24. A plot of reduced domain wall mass $m/m_0$ plotted as a function of $H_p$ for sample 1. The dotted curve corresponds to the high Q approximation. While the actual curve goes to zero more sharply as $H_p \to H_{\text{sat}}$ the approximate curve goes to zero more asymptotically. ................. 88

Fig. 25. A plot of absolute domain wall resonance frequency $f_{\text{DWR}}$ vs $H_p$ for sample 5. ............... 90
Fig. 26. A plot of $f_{\perp}^t$, FMR frequency and $f_{\text{DWR}}$ as functions of $H_p$ for sample 5. The interaction terms between DR and DWR are not included and $\alpha = 0^\circ$. 

Fig. 27. A plot of the coupled mode frequencies as functions of $H_p$ for sample 5. The frequencies are obtained by solving Eq. 122. 

Fig. 28. The coordinate system used in the study of the effects of cubic anisotropy energy term. The crystal axes [11\bar{2}] and [\bar{1}10] are at an angle $\tau$ with respect to the x and y axes. (Ref. 18) 

Fig. 29. A plot of the normalized cubic anisotropy coupling constant $(Q_K)_{12}/K_1$ as a function of $\theta_1$ for $\phi = \tau = \pi/2$. 

Fig. 30. A plot of $(Q_K)_{15}/K_1$, where $(Q_K)_{15}$ is a coupling term due to cubic anisotropy energy term plotted as a function of $\theta_1$ for $\phi = \tau = 0^\circ$. 

Fig. 31. A schematic sketch of a unit cell of pure YIG showing some of the cation arrangement. (Ref. 34) 

Fig. 32. Block diagram of the RF spectrometer used in the experiments.
Fig. 33. A schematic diagram of the rf structure. The strip-line is on the top and the slotline at the bottom. The light areas are gold and the dark areas are etched. (Ref. 17) ................. 112

Fig. 34. Rf current distribution in the rf structure. The directions of flow of the net current are mutually perpendicular. (Ref. 17) ......................... 113

Fig. 35. A typical signal obtained in the derivative detection. This signal corresponds to the DWR of sample 2, at \( H_p = 70 \text{ Oe} \) for the sweep range 400 - 700 MHz ............. 117

Fig. 36. A typical frequency swept spectrum for sample 1 showing DWR, \( \omega^- \) and \( \omega^+ \) at \( H_p = 60 \text{ Oe} \). ............. 119

Fig. 37. The mode repulsion between DVR and \( \omega^+ \) in sample 1. As \( H_p \) is increased, the two signals come toward each other and repel away from each other without crossing. ......................... 121

Fig. 38. Experimentally observed resonance frequencies vs. \( H_p \) showing the various resonance modes for sample 1. 122

Fig. 39. Experimental resonance frequency vs. \( H_p \) for sample 2. ................................. 123

Fig. 40. Experimental resonance frequency vs. \( H_p \) for sample 3. 124
Fig. 41. Experimental resonance frequency vs. $H_p$ for sample 4. 125
Fig. 42. Experimental resonance frequency vs. $H_p$ for sample 5. 126
Fig. 43. The $\omega^+$ and $\omega^-$ modes of sample 5. The theoretical values shown as solid lines are obtained for the magnetic parameters of the film that are obtained by other methods (The DWR mode is not shown for clarity). 130
Fig. 44. Theory (solid lines) vs. experiment for the $\omega^+$ and $\omega^-$ frequencies as a function of $H_p$ for sample 5. A value of $H_1=800$ Oe is assumed for the parameter $H_1$ defined in Eq. 143. 132
Fig. 45. The $G_{12}$ coupling terms arising from the cubic anisotropy energy (solid line) and for $H_p$ applied at an angle $\alpha=10^\circ$ with the domain structure (dotted line), as a function of $H_p$. The values $K_1/K_u = 0.1$ and $\phi_1 - \tau = 5^\circ$ were used in the calculation. 136
Fig. 46. Theory (solid lines) vs. Experiment for the $\omega^+$ and $\omega^-$ frequencies as functions of $H_p$ for sample 5. $H_1 = 800$ Oe and $\alpha=10^\circ$. 137
Fig. 47. Theory (solid lines) vs. Experiment for the frequencies of all three modes as a function of $H_p$ for sample 5 ($H_1 = 800$ Oe and $\alpha = 10^\circ$). Q value of the sample is 1.74.

Fig. 48. Theory (solid lines) vs. Experiment showing the frequencies of all three modes as a function of $H_p$ for sample 4 ($H_1 = 790$ Oe and $\alpha = 14^\circ$). Q value of the sample is 1.41.

Fig. 49. Theory (solid lines) vs. Experiment showing the frequencies of all three modes as a function of $H_p$ for sample 3 ($H_1 = 440$ Oe and $\alpha = 7$ degrees). Q value of the sample is 0.66.

Fig. 50. Theory (solid lines) vs. Experiment showing the frequencies of all three modes as a function of $H_p$ for sample 2 ($H_1 = 370$ Oe and $\alpha = 7$ degrees). Q value of the sample is 0.43.

Fig. 51. Theory (solid lines) vs. Experiment, showing the frequencies of all three modes as a function of $H_p$ for sample 1 ($H_1 = 275$ Oe and $\alpha = 10$ degrees). Q value of the sample is 0.35.
Fig. 92. Theory (solid lines) vs. Experiment, showing the
frequencies of all three modes as a function of
$H_p$ for sample 1. Phenomenological factors $(f_o)_{DWR}$
and $H_2$ are introduced in addition to $H_1$ and
a. $(H_1 = 350 \text{ Oe}, H_2 = 300 \text{ Oe}, (f_o)_{DWR} = 455 \text{ MHz}$
and $\alpha = 10 \text{ degrees}$).
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Material properties of samples used in the experiments.</td>
<td>109</td>
</tr>
<tr>
<td>2</td>
<td>Experimental Observations</td>
<td>128</td>
</tr>
<tr>
<td>3</td>
<td>Fitting parameters $H_1$ and $\alpha$ for the samples</td>
<td>133</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

The concept of a ferromagnetic sample spontaneously breaking into domains of uniform magnetization in the absence of external fields, has been known since 1907. In crystalline samples, due to the effects of the anisotropy energy, magnetization would tend to orient along certain preferred magnetocrystalline axes called "easy axes" and thus domains of specific orientations of magnetization result. These domains are distributed in such a way that the total magnetization of the sample is zero. By forming the domains, the sample reduces its long range dipolar self-energy associated with the magnetization vectors and thus the total energy. But along with the creation of the domains, transition regions between adjacent domains, called the "domain walls", where the magnetization changes its orientation continuously, are also formed. The presence of domain walls contribute to an increase in the energy of the sample and hence the equilibrium domain structure is reached when the marginal decrease in the demagnetization self energy of the domains is exactly equal to the energy required to create a domain wall.
In this work, single crystal thin films are examined. Besides being easy to grow on a nonmagnetic substrate, a thin film is simpler to model theoretically than the bulk samples. These crystals usually possess a uniaxial anisotropy energy with the easy axis coincident with the normal to the film plane. Its magnitude can be controlled by adjusting the crystal growth parameters. If the anisotropy energy is larger than the magnetostatic energy, the magnetization will point "up" or "down", resulting in two kinds of domains. The resulting domain pattern can either be in the form of long, essentially parallel stripes as shown in Fig. 1 or in thin cylindrical domains called "bubble domains" as shown in Fig. 2. Depending on the parameters of the film, magnetic domains having a width on the order of 10 μm have been obtained in films of comparable thickness. Magnetic domains can be observed using the Faraday rotation effect under an optical microscope with a polarizing light source and suitable analyzers.

The magnetization of the material is expressed with a multiplicative factor of \( 4\pi \), as \( 4\pi M \), and is given in units of Gauss. The uniaxial anisotropy energy is expressed in terms of \( K_u \), in units of \( \text{ergs/cm}^2 \), where \( K_u \) is the energy difference between the state where the magnetization vector lies along the easy axis and when it is perpendicular to that direction. An alternate way of expressing the anisotropy energy is in terms of a field, \( H_u \), which is defined as
Fig. 1. Parallel stripe domain pattern. (Ref. 17)
\[ H_u = 2K_u / M. \] The physical interpretation of \( H_u \) is that when the magnetization \( M \) is oriented at an angle \( \theta \) relative to the easy axis, the torques acting on the spin due to the uniaxial anisotropy energy are equivalent to the presence of an internal field along the easy axis, \( H_u \), of magnitude \( H_u \cos \theta \). Anisotropic magnetic materials are characterized by a parameter \( Q \), which is defined as \( H_u / 4\pi M \). Those materials with \( Q \gg 1 \) are called "bubble materials", where this limit on \( Q \) is a sufficient condition for the presence of parallel stripe domains. In the other extreme, where \( Q \ll 1 \), the domain structure is no longer simple and other kinds of domains such as the "closure domains" are formed.\(^5\) Thus far, only the bubble materials have been studied extensively, due to their theoretical simplicity and largely due to their importance as potential candidates for applications such as in the magnetic bubble memory devices, etc.\(^6\) The present dissertation extends the work on the bubble type materials and a theory is developed which is general enough to be applied to all cases of stripe domains, even if the materials do not have a very large \( Q \).

The domain wall in a thin film, where the magnetization undergoes a reversal, extends over several atomic spacings and in this region, the change in the magnetization is continuous. The energy of the wall arises from the following two sources; (1) uniaxial anisotropy energy effects and (2) exchange energy effects.
The anisotropy energy is a minimum when the wall is narrow, with few spins pointing away from the easy directions. However, the exchange energy is minimized when the angle between adjacent spins is kept to a minimum, thus favoring a long domain wall. The competition between the exchange and the anisotropy effects determine the width of the wall and the wall energy. Of the possible wall structures, two simple cases are important. One is the "Bloch Wall" where the spins in the wall change their orientations in such a way that they always remain parallel to the plane of the wall. The case where the wall spins are perpendicular to the wall as they change their orientation is called a "Néel Wall". Both types of walls are shown Fig. 3. Since the spin components perpendicular to the wall increase the demagnetization energy of the wall and hence its total energy, the Néel wall is not energetically favorable, except in cases of very thin samples, where the sample thickness is comparable with the wall width. 

The dynamic response of the system of domains and the domain walls is determined by the motion of the spins in these regions. Inside the domains, the spins are essentially aligned along the easy axis and thus under the influence of an external oscillating field, the magnetization precesses about its equilibrium orientation. When the external frequency equals the natural frequency of precession, the phenomenon of ferromagnetic resonance takes place. Since there are two kinds of domains in thin films, two modes of ferromagnetic resonance, denoted
A 180° Bloch Wall

A 180° Néel Wall

Fig. 3 Bloch and Néel walls.
henceforth as domain resonances or simply DR, are possible, depending on the coupling between the domains. This was first studied by Smit and Beljiers\textsuperscript{9} in barium ferrites. The theory proposed by them was revised by Artman and Charap\textsuperscript{10} who studied this phenomenon in rare earth ion garnets and by Rachford, et. al.\textsuperscript{11} The magnetostatic modes associated with DR were also studied by Artman and Charap.\textsuperscript{12}

The other dynamic phenomenon that occurs is the domain wall resonance, denoted as DWR henceforth. If an oscillating field greater than the coercive field of wall motion is applied normal to the film, the domains that are oriented favorable to the field grows in size while the others shrink. Thus the domain width oscillates, suggesting an oscillatory motion of the domain wall. Microscopically, the spins in the wall experience dynamic torques due to the drive fields and this results in a dynamical reorientation of the spins along the azimuthal and polar directions. The variation in the azimuthal angle $\theta$ is interpreted as the wall displacement and a similar variation in the polar angle $\psi$ is interpreted as the inertia of the wall. As the Bloch wall moves, it loses its Bloch nature due to the development of a dynamical component of the magnetization normal to the wall due to the change in $\psi$. This also increases the demagnetization energy of the wall and for small wall velocities, this incremental energy can be shown to be proportional to $v^2/2$. The coefficient of proportionality
is defined to be the intertial wall mass density $m$. A schematic diagram of a displaced wall is shown in Fig. 4.

As the walls are displaced, there is also a dynamical variation in the domain structure and the demagnetization self energy of the pattern, the effect of which can be shown to act as a restoring force density on the wall, with a force constant $k$. While mass $m$ encompasses the complexity of the structure of a moving wall, $k$ is a measure of the dynamic change in the energy of the domain pattern. The wall motion then is studied as a simple harmonic oscillator problem, with an oscillation frequency of $\sqrt{\frac{k}{m}}$. The DVR has been investigated by several authors in various materials. Experimental observations of DVR are used to estimate the domain wall mass, since the force constant $k$ is relatively easier to evaluate. The values of $m$ in turn yield information on the validity of the model for the wall structure and the approximations introduced in the derivation of the formula for the mass. Various models have been proposed to compute the domain wall mass.

In this dissertation, most of the earlier work on domain wall mass is reexamined and generalized. The other dynamical properties associated with the wall motion, such as the wall mobility, coercive field, damping, etc. are not covered in the present work. A review of these properties is presented in Ref. 22.
Fig. 4. A schematic diagram of a displaced wall, represented by 2. The undisplaced wall is labeled 1. (Ref. 18)
A dc magnetic field, applied in the plane of the sample and denoted as $H_p$, is an important parameter in the study of domain dynamics. The initial random domain pattern is "straightened" into parallel stripe domains by the application of $H_p$, thus making the theoretical analysis easier. In the presence of $H_p$, the orientations of the domain magnetizations are no longer simply "up" and "down", but at an angle $\theta_o$ as shown in Fig. 5. The magnetization vectors across the domain walls are rotated through an angle of $\pi - 2\theta_o$, instead of $\pi$. Since the equilibrium orientations of the domain magnetization are determined by $H_p$, the DR frequencies are also functions of $H_p$. Also, the wall structure and hence the wall mass and the restoring force constants are functions of $H_p$, thus making the DVR frequencies dependent on $H_p$. The domain pattern is destroyed as $H_p$ is increased to the in-plane saturation field value $H_{sat}$, which causes the magnetization of the entire sample to lie in the plane of the film. A new result of this work is that $H_{sat}$ is related to $H_u$ and $4\pi M$ through a demagnetization factor $N_{22}$, which is a measure of the demagnetization energy of the domain structure. As $H_p$ approaches $H_{sat}$, both the domain wall mass and the restoring force constant approach zero while their ratio and hence the DVR frequency, remains finite. This prediction differs from the earlier models, where $m$ approaches zero faster than $k$, predicting a singularity in the frequency space of DVR at $H_p = H_{sat}$. The physical reason for this difference will be explained in Chapter 2 and in Appendix III.
Fig. 5  Domain pattern under the influence of $H_p$. (Ref. 18)
It is essentially due to the domains disappearing abruptly at $H_{\text{sat}}$, instead of their width slowly going to zero.

In the past, the domain ferromagnetic resonance and the domain wall resonance have been studied as separate phenomena. An additional feature of this work is that a more general viewpoint is taken and the domains and the walls are treated as a single system. The average specific free energy of the system is written down by examining the domains and the walls. The equations of motion for the variables of the system are then obtained in terms of the derivatives of this free energy functional. It is shown that even the static properties of the domains and the walls are interrelated and further that the normal modes of oscillations of the system are actually coupled domain-domain wall resonance modes. This coupling is illustrated by considering the resonance frequency vs. $H$ curves of the various modes. In the earlier models, where the DR and DWR are treated independently, the resonances would intersect, whereas in the experiment, the coupled modes are observed to repel each other, resulting in frequency gaps at the crossover points. The theoretical development leading to these frequency gaps is outlined in Sections 2.1 through 2.5.

The materials studied in this work are substituted Yttrium Iron Garnet (YIG) and their crystal structure has a cubic symmetry giving rise to cubic anisotropy effects in addition to those due to uniaxial
anisotropy energy. The influence of cubic anisotropy energy on the DR has been examined by Artman and Charap\textsuperscript{23} and the corrections to DWR have been developed by Yeh.\textsuperscript{13} The role of cubic anisotropy energy in mode coupling will be examined in Section 2.6. For example, cubic anisotropy energy has been proposed as a possible mechanism that couples the two domain resonance modes together. The DR-DWR coupling due to cubic anisotropy energy is also examined.

Chapter 3 deals with the experimental part of the work. The samples used in the present study is discussed, along with an outline of the experimental set-up that was employed. The experimental data and a brief summary of the observations, especially those arising from the mode coupling, are also included in this chapter.

A detailed discussion of the experimental results, in light of the proposed theory is presented in Chapter 4, along with the data analysis. It is seen that the agreement between the theory and the experiment is very good, demonstrating that the coupling between the domain wall modes and domain modes are understood.
2.1 INTRODUCTION

In the present study of domain phenomena, the domains and domain walls of a magnetic thin film are treated as components of a single system. A generalized theory to describe the dynamics of such a system is presented in this chapter. Earlier investigations, such as the theory of domain resonances, as proposed by Smit and Beljers, Slonczewski's theory of domain wall resonance and Morkowski's model for domain wall mass are modified and adapted to conform to the framework of the proposed formulation. Besides the predictions of the existing theories, new features such as the mode coupling are shown to emerge.

The theoretical approach used in this dissertation is based on the following general assumptions:

(1) The length scales involved in a system of domains and walls, like the domain wall width and the domain spacing, are several orders of magnitude larger than the atomic distances. This suggests that the system can be treated as a classical, magnetic continuum. Quantum
mechanical entities like the spin operators and exchange constants are replaced by their classical equivalents such as magnetization, $\mathbf{M}$, and exchange energy constant per volume, $\Lambda$, and the problem is treated classically. Quantum mechanical analysis of the domain wall resonance in the simple case of no externally applied magnetic fields, has been performed by Winter. A quantum theory of domain resonances has not been developed yet, since the presence of the long range dipole interaction term in the Hamiltonian complicates the problem.

(2) The magnetic thin film is assumed to divide itself into parallel stripe domains with straight, planar walls. Such a domain pattern is not formed at low fields in those samples with very low values of $Q$ and the present theory is not applicable in such cases. A DC magnetic field, applied in the plane of the sample, makes the domains long and straight and thus the walls planar. This results in the simplicity of the domain wall problem reducing essentially to a one dimensional problem, the wall profile changing only with the coordinate perpendicular to the plane of the wall. The coordinate axes in the plane of the wall are axes of translational symmetry and the domain wall parameters are invariant under such translations. Some authors have relaxed this assumption and have considered the possibilities of a variation of the wall profile along the thickness of the sample. Such corrections are important in cases of very thin samples.
(3) The domain wall is assumed to be a perfect Bloch wall. This condition is approximately met in typical garnet samples, except in cases of very thin samples where the sample thickness compares with the wall width. The moving wall is assumed to be rigid and to have essentially the same structure as a stationary wall. The small changes that do arise, due to the wall motion are treated as a perturbation. This condition is not satisfied when the wall velocities are large i.e. when excited by large drive fields and in such cases, distortions do occur. The flexural modes of a non-rigid wall are discussed in Ref. 28 and 29.

(4) The domains are assumed to be homogeneous and each domain is characterized by a uniform magnetization vector \( \vec{N} \). The transition region between the wall and the domains is assumed to be vanishingly small.

(5) The thickness of the domain wall is typically of the order of a few hundred angstrom units and is therefore assumed to be small compared with the width that of the domain, which is of the order of a few microns. Consequently, in the calculation of parameter densities, the volume occupied by the wall is neglected.

(6) Only the linear response of the system is studied and the non-linear, high power phenomena are not addressed.
(7) Cubic anisotropy effects are neglected at first as being small. The corrections due to cubic terms are however discussed in Section 2.6.

Other assumptions are introduced as necessary.

The basis for the dynamics of a magnetic medium is the Landau-Lifshitz equation of motion. This can be written in the following form, with the Gilbert damping parameter,

\[
\frac{d\vec{m}}{dt} = -\left(\frac{\gamma}{\mu_0}\right)\vec{T} + \alpha(\vec{m} \times \vec{m}) ,
\]

(1)

\(\vec{m}\) is the magnitude of magnetization at the region of interest, \(\vec{m}\) is a unit vector in the direction of the magnetization, \(\gamma\) is the gyromagnetic ratio, \(\vec{T}\) is the torque acting on the magnetization vector and \(\alpha\) is the damping parameter. For the samples studied, the Gilbert damping parameter \(\alpha\) is found to be negligibly small and is thus omitted in the present discussion.

The torque \(\vec{T}\) in turn, is assumed to be derived from the gradient of the potential energy of the system \(G\), i.e.,

\[
\vec{T} = -\vec{m} \times \nabla G .
\]

(2)

It is seen that the magnetic system is composed of three distinct
regions - the two kinds of domains and the domain wall. This is illustrated in Fig. 6. The two domains are in general characterized by their widths \( L_1 \) and \( L_2 \) and by their magnetization vectors, \( \hat{M}_1 \) and \( \hat{M}_2 \), which are oriented at angles \( \theta_1 \) and \( \phi_1 \) in the domains of the first kind, referred to as type 1 domains, and at angles \( \theta_2 \) and \( \phi_2 \) in the domains of reverse magnetization, referred to as type 2 domains. In the absence of external magnetic fields, the magnetization in type 1 domains would point "up" or along the positive z direction and the type 2 domains are the "down" domains. In the absence of any wall displacement, the two domain widths are equal and the equilibrium value of domain width is denoted by \( L \). When the walls are displaced by a distance \( q \), the new domain widths are given in terms of \( L \) and \( q \),

\[
L_1 = L + 2q
\]

and

\[
L_2 = L - 2q
\]

where the adjacent walls are assumed to be displaced in the opposite directions. The in-plane field, \( H_p \), is also shown in Fig. 6 and in general, \( H_p \) can be at an angle \( \alpha \) to the domain pattern.

Inside the domain wall, the magnetization is not uniform, but changes along the y direction. A new set of variables \( \Theta(y) \)
Fig. 6 Coordinate system used to define the domain variables.

\[ L_1 = L + 2q \]

\[ L_2 = L - 2q \]
and \( \psi(y) \) are introduced to describe the orientation of \( \hat{M} \). These are shown in Fig. 7. It will be shown that \( \Theta(y) \) is related to the wall displacement \( q \) and \( \psi(y) \) is related to the wall velocity, \( \dot{q} \), through the equations of motion. The center of the wall is taken to be the origin of the coordinate system chosen in Fig. 7. The magnetic system, then, is described by the six coordinates \( \Theta_1, \Theta_2, \phi_1, \phi_2, \Theta(y), \psi(y) \).

The next step in the procedure is to examine the domain structure and the domain walls and identify the various contributions to the energy, \( G \). Since \( G \) will be expressed as a function of the coordinates of the system, it will be in a "functional" form, the minimum of which corresponds to the actual energy of the system. In the calculation of \( G \), the elemental volume is taken to be made up of one type 1 domain, one type 2 domain and two domain walls. The volume of this element is \( DT(L_1 + L_2) \) where \( T \) is the sample thickness and \( D \) is the length of the domains (\( D \) is also the sample length). For the purposes of deriving an expression for \( G \), it is convenient to separate it into,

\[
G = G_{\text{domains}} + G_w ,
\]  

(4)

where \( G_w \) is the potential energy associated with the walls. Both terms on the right side are in general functions of all the six
Fig. 7 Coordinate system used to study the domain wall. The wall variables, \( \theta \) and \( \psi \), are shown. (Ref. 17)
variables of the system. The domain part of the free energy can be further decomposed into,

\[ G_{\text{domains}} = G_u + G_z + G_D, \quad (4a) \]

where \( G_u \) is the uniaxial anisotropy energy term, \( G_z \) is the Zeeman energy and \( G_D \) is the demagnetization energy. It is difficult to separate the \( G_u \) term, in a similar fashion. It will also be shown that it is not easy to write a general functional form of \( G_w \) and it is convenient to separate it into a static and a linearized dynamic term.

\[ G_w = G_{w0} + G_{w1}, \quad (4b) \]

where \( G_{w0} \) is the static term and \( G_{w1} \) is the dynamic wall energy density functional.

In the subsequent sections, the various contributions to \( G \) are evaluated and the static and dynamic behavior of the system are determined. In this chapter, the theoretical predictions are illustrated by computing the predicted quantities for sample 5, whose parameters are listed in Table I. (Chapter 3) The symbol manipulation program MACSYMA was used in deriving and verifying several steps of the present theory.
2.2 DOMAIN ENERGY TERMS:

2.2.1 UNIAXIAL ANISOTROPY ENERGY AND ZEEMAN ENERGY:

It was shown that the contributions to the domain part of $G$ arise due to uniaxial anisotropy energy, Zeeman energy and the demagnetization energy terms. In this section, the average uniaxial anisotropy energy, Zeeman energy and the demagnetization energy terms in the elemental volume defined in the previous section are evaluated.

The samples acquire a uniaxial anisotropic character from the stresses induced due to lattice mismatch and by the chemical substitution during the growth process. The uniaxial anisotropy results in an "easy axis" along which the magnetization vector would prefer to point. A satisfactory microscopic picture of this anisotropy effect is not yet developed. However, it is possible to describe the effects using a phenomenological contribution to the energy. In thin films for the coordinate system shown in Fig. 6, the $z$-axis is typically the easy axis and if the magnetization vector is assumed to orient at an angle $\theta$ from the easy axis, the energy due to this effect is given generally by a power series,

$$G_u = K_u \sin^2 \theta + K_{u1} \sin^4 \theta + \ldots$$  \hspace{1cm} (5)
In garnet samples, the first term is usually sufficient to describe the system and the higher order terms may be neglected. Then $K_u$ is identified as the uniaxial anisotropy constant, defined in the previous chapter. The average uniaxial anisotropy energy for the elemental volume, $G_u$, is,

$$G_u = \frac{L_1}{L_1 + L_2} K_u \sin^2 \Theta_1 + \frac{L_2}{L_1 + L_2} K_u \sin^2 \Theta_2$$

(6a)

$$= \frac{L + 2g}{2L} K_u \sin^2 \Theta_1 + \frac{L - 2g}{2L} K_u \sin^2 \Theta_2$$

This can be rewritten as,

$$G_u = \frac{K_u}{2} [\sin^2 \Theta_1 + \sin^2 \Theta_2] + \frac{qK_u}{L} [\sin^2 \Theta_1 - \sin^2 \Theta_2]$$

(6b)

Similarly the Zeeman energy term can be written down by inspection as,

$$G_2 = \frac{L_1}{2L} \vec{H} \cdot \vec{H}_1 - \frac{L_2}{2L} \vec{H} \cdot \vec{H}_2$$

$$= -\frac{H_p M}{2} \cos \alpha [\sin \Theta_1 \cos \phi_1 + \sin \Theta_2 \cos \phi_2] - H_p M \cos \alpha \frac{q}{L} (\sin \Theta_1 \cos \phi_1 - \sin \Theta_2 \cos \phi_2)$$
The demagnetization energy term arises due to the structure of the domain pattern. This term is usually very difficult, if even possible to evaluate, except for simple domain configurations such as the parallel stripes. Each domain is a magnetic dipole and the demagnetization energy is simply the net energy due to the dipolar interaction of all these dipoles with all the other dipoles. For the purposes of derivation of this energy term, a coordinate system located at the center of domain of type 1, as shown in Fig. 8, is chosen. The approach used here is similar to that of Kooy and Enz.\textsuperscript{14}

Inside the two kinds of domains, the $z$ component of magnetization vectors are,

\begin{align*}
M_z &= M \cos \theta_1 \quad \text{for type 1 domains}, \\
M_z &= M \cos \theta_2 \quad \text{for type 2 domains}.
\end{align*}

(8)

$M_z$ is interpreted in terms of a classical magnetic charge density. The potential due to this distribution of charges is then found and
Fig. 8 Coordinate system used to derive the demagnetization energy term. The origin is the center of domain of type 1.

$L_1 = L + 2q$
$L_2 = L - 2q$
finally, from the charge density and the potential, the magnetostatic potential energy is calculated. Since the domains are distributed periodically, it is possible to express quantities such as the magnetic charge density and the potential as Fourier series. The magnetic charge density, in particular, is only a function of $y$, due to earlier assumptions and can be written as:

$$\rho(y) = \text{div} \frac{\mathbf{H}}{M}$$

$$= M \cos \theta_1$$  
for $-\frac{L_1}{2} < y < \frac{L_1}{2}$

$$= M \cos \theta_2$$  
for $-\frac{1}{2} < y < \frac{L_1}{2}$  \hspace{1cm} (9)

and

$$\frac{L_1}{2} < y < \frac{L_1 + L_2}{2} .$$

Here $\rho(y)$ is only a surface charge density since $M$ is uniform inside the domain (Fig. 8). In this approach, the demagnetization contributions from the surface magnetic charges alone are taken into account. Stray fields and the canting of the magnetization at the surface of the film in the walls are neglected. The Fourier series for $\rho(y)$ is found to be,

$$\rho(y) = \frac{M(L_1 \cos \theta_1 + L_2 \cos \theta_2)}{L_1 + L_2} + \sum_{n=1}^{\infty} \frac{2M}{\pi n} \frac{\sin \pi n L_1}{L_1 + L_2} \cos \frac{2\pi n y}{L_1 + L_2} \times$$

$$\left( \cos \theta_1 - \cos \theta_2 \right) ,$$

(10)
Since the magnetic potential due to this charge density will be different for regions inside and outside the magnetic specimen, two potential functions, \( V_1 \) for volumes within the sample and \( V_e \) for volumes outside, are defined. The demagnetization field everywhere is then simply the gradient of the respective potentials, i.e.,

\[
\mathbf{H}_d = -\nabla V_1 .
\]  

(11)

For regions inside the sample, \( V_1 \) also satisfies the Poisson's equation,

\[
\frac{\partial^2 V_1}{\partial y^2} + \frac{\partial^2 V_1}{\partial z^2} = 4\pi \rho .
\]  

(12)

Due to the symmetry of the problem, \( V_1 \) and \( V_e \) are independent of \( x \). The term \( \mu_y \) is the rotational permeability of the sample and since this is the only component of the permeability tensor that is important here, the subscript \( y \) will be dropped. A discussion on \( \mu \) is presented in Appendix A. From Appendix A, it is seen that \( \mu = 1 + 4\pi M/H_{\text{int}} \), where \( H_{\text{int}} \) is the internal field that will be defined later. \( V_e \) obeys the Laplace's equation. The potentials \( V_1 \) and \( V_e \) are written in the following series form:

\[
V_1(y,z) = b_0 z + \sum_{n=1}^{\infty} b_n \sin \frac{\pi n L_1}{L_1 + L_2} \cos \frac{2\pi y}{L_1 + L_2} \sinh 2\pi n z \sqrt{\mu}.
\]  

(13)
The values of the coefficients $a_n$, $b_n$ and $c_n$ are obtained by matching the boundary conditions at the two surfaces $z = \pm T/2$.

\[ V_e(y,z) = a_0 + \sum_{n=1}^{\infty} a_n \sin \frac{\pi n L_1}{L_1 + L_2} \cos \frac{2\pi n y}{L_1 + L_2} \exp \left( \frac{-2\pi n z}{L_1 + L_2} \right) \]

for $z > T/2$

\[ = c_0 + \sum_{n=1}^{\infty} c_n \sin \frac{\pi n L_1}{L_1 + L_2} \cos \frac{2\pi n y}{L_1 + L_2} \exp \left( \frac{-2\pi n z}{L_1 + L_2} \right) \]

for $z < -T/2$.  \hspace{1cm} (14)

The second equation merely restates the condition that the $z$ component of the field is discontinuous by the amount of surface charge density. Application of the first boundary condition gives,

\[ V_1 \left|_{z = \pm T/2} = V_e \left|_{z = \pm T/2} \right. \right. \]

\hspace{1cm} (15)

and

\[ \frac{\partial V_1}{\partial z} \left|_{z = \pm T/2} = -4\pi \rho \right. \]

\[ - \frac{\partial V_e}{\partial z} \left|_{z = \pm T/2} \right. \]

\hspace{1cm} (16)

The second equation merely restates the condition that the $z$ component of the field is discontinuous by the amount of surface charge density. Application of the first boundary condition gives,

\[ a_0 = b_0 T/2 \]

\[ a_n = b_n \sinh \left( \frac{\pi n T}{L_1 + L_2} \right) \exp \left( \frac{\pi n T}{L_1 + L_2} \right). \]  \hspace{1cm} (17)
Similar conditions can be established for the coefficients \( c_n \) from the second boundary condition. Making use of the series form of the magnetic charge distribution, Eq. (10), and comparing term by term, the coefficients \( b_n \) are evaluated.

\[
b_0 = \frac{4\pi M (L_1 \cos \theta_1 + L_2 \cos \theta_2)}{L_1 + L_2},
\]

\[
b_n = \frac{4\pi M (L_1 + L_2) (\cos \theta_1 - \cos \theta_2)}{\pi n^2 (\sinh \Lambda + \sqrt{\mu} \cosh \Lambda)}
\]

where \( \Lambda = \frac{\pi T \mu H}{L_1 + L_2} \).

The demagnetization energy for the total volume is then given by,

\[
G_DZ = - \frac{1}{4\pi (L_1 + L_2)} \frac{1}{T} \left( \frac{(L_1 + L_2)^2}{T/2} \right) \int \int \int \hat{H}_d \cdot \delta \delta B dz dy ,
\]

where

\[
\delta \delta B = \mu \delta \delta H_d
\]

Carrying out this integral after substituting for \( \hat{H}_d \), it is seen that,

\[
G_{DZ} = 2\pi M^2 \left( \frac{L_1 \cos \theta_1 + L_2 \cos \theta_2}{L_1 + L_2} \right)^2 + \frac{4(L_1 + L_2)M^2}{\pi T} (\cos \theta_1 - \cos \theta_2)^2 \times
\]
The first term in the above expression is interpreted as the demagnetization energy of a uniformly magnetized slab due to the net magnetization and can be written as,

\[ G_{DO} = \frac{\pi M^2}{L^2} \left[ (L + 2q) \cos \theta_1 + (L - 2q) \cos \theta_2 \right]^2. \]  

The series part of \( G_{DZ} \) is interpreted in terms of a demagnetization factor \( N_{zz}(q) \) for the stripe domain structure, defined by:

\[ G_{DS} = \frac{\pi}{2} N_{zz}(q) M^2 (\cos \theta_1 - \cos \theta_2)^2 \]  

where

\[ N_{zz}(q) = \frac{16}{N^3} \left( \frac{L}{T} \right) \sum_{n=1}^{\infty} \frac{1}{n^3} \sin^2 \left( \frac{n\pi(L+2q)}{L_1 + L_2} \right) \frac{\sinh A}{\sinh A + \sqrt{\mu} \cosh A}. \]

The term \( N_{zz}(q=0) \), for the undisplaced wall, will be simply referred to as \( N_{zz} \) and is used in the calculations. It follows that \( N_{zz} \) is only a function of the domain aspect ratio, the ratio of the domain period to the sample thickness, i.e., \( L/T \), and the rotational permeability \( \mu \). A plot of \( N_{zz} \) as a function of \( L/T \) is shown in
Fig. 9 A plot of the demagnetization factor $N_{zz}$ vs. $\ln(L/T)$, where $L/T$ is the domain aspect ratio, defined following Eq. 22.
Fig. 9. It is seen that in very thin films, where \( L \gg T \), \( N_{zz} \) approaches 1, corresponding to a uniformly magnetized film. The demagnetization field in Eq. (11) can then be shown to be equal to,

\[
H_d = -4\pi M N_{zz} = -4\pi M.
\]

In the opposite limit when \( L \ll T \), \( N_{zz} \) and thus, the demagnetization fields are close to zero. In general, the internal field can be written as,

\[
H_{int} = H_u + H_d = H_u - 4\pi M N_{zz},
\]

where \( H_u = 2K_u/M \) was defined in the earlier chapter as the effective field due to uniaxial anisotropy energy.

It will be shown in Chapter 4 that experimentally, \( H_{int} \) is found to be linearly scaled as a function of \( H_p \). A phenomenological explanation is also provided. Henceforth, in this treatment, such a phenomenological scaling is included in the expression for \( H_{int} \).

From Fig. 9 it is also seen that the maximum changes in \( N_{zz} \) occur when the domain width \( L \) becomes comparable to the sample thickness \( T \). In the presence of \( H_p \), it will be shown in Section 2.4 that the domain spacings change and thus \( N_{zz} \) and \( H_{int} \) will also depend on \( H_p \). This dependence is strong if the material parameters of the sample produce domain spacing \( L \) on the same order as the sample.
thickness. This important dependence has hitherto been neglected by previous authors.\textsuperscript{10,11}

For $\theta_1=0$ and $\theta_2=\pi$, when no $H_p$ is present, all formulas derived in this section reduce to those obtained by Kooy and Enz. In the other limit, when $H_p$ is present, but walls are stationary, i.e. $L_1 = L_2$, the formulas reduce to those obtained by Artman and Charap.\textsuperscript{10} The formula for the demagnetization factor $N_{zz}$ also agrees, in this limit, with that given by Gemperle and Kaczer.\textsuperscript{31}

The energy term $G_{DZ}$ as given in Eq. (20) can be used to study the dynamics of the demagnetization process of the parallel stripe domain structure in general. For example, it will be used to obtain an expression for the equilibrium domain spacing $L$ in Section 2.4. In Section 2.5 it will be shown that this is intimately connected to the concept of the restoring force on the domain wall, in domain wall resonance.

The other contributions to the demagnetization energy come from the presence of the $x$ and $y$ components of magnetization and are given in terms of the demagnetization factors $N_{xx}$ and $N_{yy}$. The evaluation of the demagnetization energy for a thin, magnetized slab is given in Appendix B, which yields the rest of the terms:
\[ G_{DX} + G_{DY} = \frac{\pi}{2} N_{xx} M^2 (\sin \theta_1 \cos \phi_1 - \sin \theta_2 \cos \phi_2)^2 + \frac{\pi}{2} N_{yy} M^2 \times \]

\[ (\sin \theta_1 \sin \phi_1 - \sin \theta_2 \sin \phi_2)^2 . \quad (25) \]

Since the length of the domains \( D \) is considerably larger than the other two dimensions, \( T \) and \( L \), the factor \( N_{xx} \) is negligibly small. Since \( N_{xx} + N_{yy} + N_{zz} = 1 \), as shown for example in Ref. 32, (See Appendix B) \( N_{yy} = 1 - N_{zz} \) and thus,

\[ G_{D} = \frac{\pi M^2}{2L^2} [(L + 2q) \cos \theta_1 + (L + 2q) \cos \theta_2]^2 + \]

\[ \frac{\pi}{2} N_{zz} M^2 (\cos \theta_1 - \cos \theta_2)^2 + \]

\[ (1 - N_{zz}) \frac{\pi}{2} M^2 (\sin \theta_1 \sin \phi_1 - \sin \theta_2 \sin \phi_2)^2 . \quad (26) \]

Thus the total contribution to the domain free energy is given by the expression \( G_{\text{Domains}} = G_u + G_z + G_D \) where \( G_u \), \( G_z \) and \( G_D \) are given by expressions 6, 7, and 26 respectively.

2.3 DOMAIN WALL ENERGY TERMS

There has been a lot of work already done on the dynamics of the domain wall.\(^{17,18} \) So, in this section, only a brief discussion is given on the fundamentals. The results that are used from the other sources are sometimes merely quoted, rather than derived from first
principles. It will be shown that, unlike the energy terms of the domains, the domain wall terms are complicated, since the variables used to describe the wall motion, the azimuthal angle $\theta$ and the polar angle $\psi$ are themselves functions of the coordinates $x$, $y$ and $z$ in general. In this section, Slonczewski's approach to domain wall motion is followed. A simple 180° degree Bloch wall is discussed first and then the influence of $H_p$ on the wall is studied. Finally, the energy terms of a moving Bloch wall with $H_p$ are obtained.

2.3.1 180° BLOCH WALL

The simplest Bloch wall occurs in case of no applied fields, when the direction of magnetization changes 180° across the wall width. Using the coordinate system of Fig. 7, the Landau-Litvishitz equations (Eq. 1) can be rewritten in terms of the spherical coordinates $\theta$ and $\psi$ as,

\[
\Theta = \frac{\gamma}{M \sin \theta} \delta W
\]

\[
\psi \sin \theta = \frac{\gamma}{M} \delta W
\]

Here $W$ is the local energy density of the wall and $\delta W/\delta \psi$ etc. are the functional derivatives, due to $\psi$ etc. being functions of the coordinates. The functional derivatives are defined by,
Following Slonczewski, it can be shown that the contributions of the uniaxial anisotropy energy and the exchange energy in the wall result in the following expression for the energy density $W$,

$$W = K_u \sin^2 \theta + A\left(\frac{\partial \theta}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 \sin^2 \theta,$$

(30)

where $A$ is the exchange energy constant in units of ergs/cm. Substituting this form of $W$ in the equations of motion 27 and 28 and evaluating the static equilibrium of the wall, i.e. $\dot{\theta} = 0$, and $\dot{\psi} = 0$, result in the following equations,

$$\frac{\partial^2 \psi}{\partial y^2} \sin^2 \theta + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial y} \sin \theta = 0,$$

(31)

$$2A \frac{\partial^2 \theta}{\partial y^2} - [K_u + A \left(\frac{\partial \psi}{\partial y}\right)^2] \sin 2 \theta = 0,$$

(32)

where it has been assumed that the variables $\theta$ and $\psi$ are only functions of $y$. The solution of the above equations are the following,

$$\psi = \psi_0(x, z),$$

(33)
\[ \theta = \theta_0 \{ y - q(x, z) \} \]  
\[ \tan \frac{\theta_0}{2} = e^{(y - q)/\Delta} \]

where \( \Delta = \sqrt{\lambda/\kappa} \) and \( \Delta \) is a measure of the wall width.

In general, the static and dynamic solutions can be functions of \( x \) and \( z \). But due to the assumptions stated in Section 2.1, such dependence on \( x \) and \( z \) will no longer be considered. The static values, for example, will be considered constants within the wall, along \( x \) and \( z \). Since the origin of the coordinate system is located at the center of the wall, the term \( q \) can be interpreted as the displacement of the center of the wall, along the \( y \) direction and is identified as the same \( q \) defined previously. Eq. 34, which relates \( \theta(y) \) and \( q \), makes it possible to rewrite the equations of motion in terms of \( q \), rather than \( \theta \). This is useful since \( q \) already occurs in the domain energy terms. Also, integrating the equation of motion over the width of the wall will be shown to result in elegant equations in terms of \( q \) and \( \dot{q} \) and in the final form of \( G, q \), rather than \( \theta \), will be used. \( \theta_0(y) \) is plotted in Fig. 10 as a function of \( y \) and from Eq. 35, it is readily seen that,

\[ \theta_0 = \frac{\partial\theta}{\partial y} \sin \theta = \frac{\theta_0}{\Delta} \]  

(37)
Fig. 10 Static wall profile for a 180° Bloch wall. Two solutions exist and the magnetization can change its orientation by either a clockwise or counter clockwise spiral. (Ref. 17)
The contribution of the wall energy to that of the elemental volume of domains and walls is then given by,

$$G_w = \frac{1}{L} \int_{-\infty}^{\infty} \left[ A \theta_{cy}^2 + K_u \sin^2 \theta_0 \right] dy$$

(38)

The limits of the integral are actually the boundaries of the wall, but the integration is carried out to ±∞ for mathematical simplicity, without any significant errors. Squaring both sides of Eq. 37 it can also be seen that,

$$A \theta_{cy}^2 = K_u \sin^2 \theta_0$$

(39)

This equation merely restates the condition, that in a static domain wall, the wall energy is equipartitioned between the two sources of energy, namely the exchange and uniaxial anisotropy interactions. If Eq. 39 is substituted in Eq. 38 and the integral over y is converted to an integral over θ using Eq. 37, the following expression for $G_w$ results,

$$G_w = \frac{4\sqrt{(AK_u)}}{L}$$

(40)

The term $4\sqrt{(AK_u)}$ is simply written as $\sigma$, and is a measure of the energy of a 180° static Bloch wall.
2.3.2 BLOCH WALL UNDER $H_p$

The energy of a domain wall under the influence of an external field can be written easily, since it is just the difference in the energy of the system with and without the wall. For example, the local domain wall energy density for the wall to the right of domains of type 1 can be expressed as,

$$W(\theta_1, \phi_1, \theta(y), \psi(y)) = A\theta_y^2 + A\psi_y^2 \sin^2 \theta + 2\pi M^2 (\sin^2 \psi \sin^2 \theta - \sin \theta_1$$

$$\times \sin \theta \sin \psi \sin \phi_1) - \int \frac{H M}{\cos \theta} \left[ (\psi - \phi) - \sin \theta_1 \right] + K_u [\sin^2 \theta - \sin^2 \theta_1].$$

Here, for simplicity $H_p$ is assumed to be along the $x$ axis, although the results of this derivation can easily be generalized to arbitrary $\alpha$. The third term of Eq. 41 is the dynamical demagnetization energy of the wall, introduced for the first time. To obtain this, the domain wall is treated as a magnetic medium in the shape of a rectangular slab and the results of Appendix B are applied. The energy density due to the walls then is,

$$d_W(\theta_1, \theta_2, \phi_1, \phi_2, \theta, \psi) = \frac{1}{2L} \int_{-\infty}^{\infty} W(\theta_1, \phi_1, \theta, \psi)$$

$$+ W(\theta_2, \phi_2, \theta, \psi) \, dy,$$
where the presence of two walls (one going from domains of type 1 to type 2 and the other going from type 2 domains to type 1) is accounted for. In this general form, the functional form of Eq. 42 is very difficult to evaluate, unlike in the case of domain energy terms. Various approximations will have to be introduced to evaluate Eq. 42 without sacrificing much physics. Firstly, the two kinds of walls are assumed to be symmetric, i.e., when the $W$ for one is known, the $W$ for the other wall can be written from inspection. Also, it is postulated that the wall energy divides itself into a static part and dynamic part, i.e.,

$$q_w = q_{w0} + q_{w1}, \tag{43}$$

where both parts in general depend on all the variables $\theta_1$, $\phi_1$, $\theta_2$, $\phi_2$, $\theta$ and $\psi$. The static part is dealt with first. It can be shown that, even in the presence of $H_p$, the static value of $\psi$ is constant, by explicitly solving the equations of motion i.e., the $A\psi^2 \sin^2 \theta$ term in Eq. 41 drops out.

The other conditions that results from the solution of the equation of motion is the generalization of Eq. 39.

$$A\theta_y^2 = 2\pi a^2 \left( \sin^2 \theta \sin^2 \psi - \sin \theta_1 \sin \phi_1 \sin \theta \sin \psi \right) +$$

$$K_u \left( \sin^2 \theta - \sin^2 \theta_1 \right) - H_p M \left( \sin \theta \cos (\psi - \phi_1) - \sin \theta_1 \sin \phi_1 \right). \tag{44}$$
Eq. 41 is then simplified using this result to give,

\[
W(\theta_1, \phi_1, \theta, \psi) = 2(2\pi M^2 (\sin^2 \psi \sin^2 \theta - \sin \theta_1 \sin \phi_1 \sin \theta_1 \sin \psi) \\
+ K_u (\sin^2 \theta - \sin^2 \theta_1) - H_p M (\sin \theta \cos(\psi - \phi_1) \\
- \sin \theta_1 \sin \phi_1)) .
\]  (45)

This can be substituted in the integral in Eq. 42, with a symmetric term added for the presence of the other wall. Eq. 44 can be used to obtain a relationship between \( \theta \) and \( \psi \) and in principle, the integral can be evaluated. Even then, an analytical solution is almost impossible, in general, to obtain. However, analytical solutions are possible for samples with \( Q \gg 1 \). This is obtained in Ref. 17, Appendix B, for the case when \( \psi = 0 \). With a little generalization and once again letting \( H_p \) orient at an angle \( \alpha \) to the \( x \) axis, instead of being parallel to it, the following expression for \( Q_{w0} \) is realized, for \( Q \gg 1 \)

\[
Q_{w0} = \frac{a_o}{4L} \left[ [2 \cos \theta_1 - (\theta_2 - \theta_1) \sin \theta_2 \cos(\phi_2 - \alpha)] \\
- [2 \cos \theta_2 - (\theta_1 - \theta_2) \sin \theta_1 \cos(\phi_1 - \alpha)] \right].
\]  (46)

Although most of the samples investigated in this work do not belong to this regime of \( Q \), it is instructive to study this limit, as this is
the only case where analytical expressions for $Q_{\text{WO}}$ can be obtained. For example, the effect of $Q_{\text{WO}}$ on the static equilibrium orientations of the domain magnetization can be studied using Eq. 46 and qualitative arguments can be presented for the case of general values of $Q$. It is seen that $Q_{\text{WO}}$ depends on the domain variables $\theta_1$, etc. The domain variables in turn depend on $Q_{\text{WO}}$ through the equations of motion and hence, in principle, the domain and the domain wall equations will have to be solved in a self consistent manner to obtain the equilibrium values of the variables. A discussion on this is postponed until the next section. In the following, the dynamical part of the domain wall energy is derived assuming that the static orientations are already evaluated.

2.3.3 Domain Wall Energy: Dynamic Term

The equations of motion, Eqs. 27 and 28, are used to obtain the dynamic terms of the domain wall energy. It must be remembered that during the process of deriving this term, the equations of motion are linearized.

The variation in $\theta$ can be converted to a variation in $q$, the displacement of the wall,

$$\delta \theta = \frac{\partial \theta}{\partial y} \delta y = -\frac{\partial \theta}{\partial y} \delta q,$$

(47)
A new variable $\xi$ is introduced, which is a measure of the average variation of $\psi$ in the wall (A discussion on the interpretation of $\psi$ will be presented later) such that,

$$
\theta = \frac{\partial \psi}{\partial y}.
$$

(48)

Using Eqs. 27 and 48 the integration above can be rewritten as,

$$
\frac{\delta w_0}{\delta \xi} = \frac{1}{L} \int_{-\infty}^{\infty} \frac{\delta w}{\delta \psi} \, dy = \frac{1}{L} \int_{\theta_1}^{\theta_2} \frac{\partial w}{\partial \psi} \left( \frac{\partial \theta}{\partial \psi} \right)^{-1} \, d \theta.
$$

(49)

In the presence of $H_p$, an expression can be written for $\theta_y$ from Eq. 44 and from the assumption that the moving wall has the same profile as a stationary wall

$$
\theta_y = \frac{1}{\delta} \left[ \sin^2 \theta - \sin^2 \theta_1 - 2 \mu (\sin(\phi_1 - \psi)(\sin \theta - \sin \theta_1))^{1/2} \right],
$$

(51)

where $\mu = H_p / H_u$.

The evaluation of Eq. 50 can actually be carried out without any explicit knowledge of the form of $\theta_y$ to yield,

$$
\frac{\delta g_w}{\delta \xi} = \frac{\left( \cos \theta_1 - \cos \theta_2 \right) M_q}{\gamma L}.
$$

(52)
or,

\[ q = \frac{\gamma L}{(\cos \theta_1 - \cos \theta_2)} M \frac{\delta Q}{\delta \xi} \]

Similarly,

\[ -\frac{M(\cos \theta_1 - \cos \theta_2)}{\gamma L} \xi = \frac{\delta Q}{\delta q} \] \hspace{1cm} (53)

Eqs. 52 and 53 are recognized as the Hamilton's equations of motion written for the conjugate variables \( q \) and \( \xi \), where \( \xi \) is now recognized to be proportional to the momentum conjugate to \( q \). That is,

\[ p = \rho q = \frac{(\cos \theta_1 - \cos \theta_2) M \xi}{\gamma L} \] \hspace{1cm} (54)

where \( \rho \) is the mass density per unit volume of the sample associated with the domain walls. The kinetic energy of the system, which is also the dynamical part of the domain wall energy, is,

\[ Q_{WL} = \frac{p^2}{2\rho} = \frac{1}{2\rho} \left[ \frac{M(\cos \theta_1 - \cos \theta_2)}{\gamma L} \right]^2 \xi^2 \] \hspace{1cm} (55)

In terms of the more familiar concept of domain wall mass per area of the wall, \( m \), Eq. 55 can be rewritten as,

\[ Q_{WL} = \left[ \frac{M - \cos \theta_2}{\gamma} \right]^2 \frac{\xi^2}{2mL} \] \hspace{1cm} (56)
The evaluation of the domain wall mass itself was a subject of several investigations and one of the best models, proposed by Morkowski\textsuperscript{20} will be discussed in Section 2.5.

2.4 STATIC EQUILIBRIUM CONDITIONS

Assembling the various components of G together, from Eqs. 6, 7, 26, 43 and 56 the following expression can be written,

$$G(\theta_1, \phi_1, \theta_2, \phi_2, q, \xi) = \frac{1}{2} K_u (\sin^2 \theta_1 + \sin^2 \theta_2)$$

$$+ \frac{q K_u}{L} (\sin^2 \theta_1 - \sin^2 \theta_2)$$

$$- \frac{H M}{2} \cos \alpha [\sin \theta_1 \cos \phi_1 + \sin \theta_2 \cos \phi_2]$$

$$- \frac{H M \sin \alpha}{2} (\sin \theta_1 \sin \phi_1 + \sin \theta_2 \sin \phi_2)$$

$$- \frac{H M q \cos \alpha}{L} (\sin \theta_1 \cos \phi_1 - \sin \theta_2 \cos \phi_2)$$

$$- \frac{H M q \sin \alpha}{L} (\sin \theta_1 \sin \phi_1 - \sin \theta_2 \sin \phi_2)$$

$$+ \frac{\mu M^2}{2L^2} [(L + 2q) \cos \theta_1 + (L - 2q) \cos \theta_2]^2$$

$$+ \frac{\Phi}{2} N_{zz} M^2 (\cos \theta_1 - \cos \theta_2)^2$$

$$+ (1 - N_{zz}) \frac{\Phi}{2} M^2 (\sin \theta_1 \sin \phi_1 - \sin \theta_2 \sin \phi_2)^2$$
From the above equation, the static equilibrium configurations are obtained by setting the first derivative of \( G \) equal to zero.

The conditions for static equilibrium are,

\[
\frac{\partial G}{\partial X_1} = 0 ,
\]

where \( X_1 \) is any of the six arguments of the functional \( G \). Once the static values are determined, \( G \) is then written as a Taylor series for small variations about the equilibrium values, to give the dynamic response.

Some of the static values can be obtained quite easily, by exploiting the symmetries of the system. For example, in static equilibrium (when no fields are applied in the \( z \) direction) the walls are not displaced, i.e. \( q=0, \theta_1=0 \) and through Eq. 54, \( \xi = 0 \). Also, in the absence of the cubic anisotropy energy terms, there is a symmetry between the two kinds of domains, for reflection in the \( xy \) plane, resulting in the following conditions,

\[
\theta_1 + \theta_2 = \pi
\]
These relationships can be proved rigorously by solving the equations obtained for static equilibrium conditions.

2.4.1 EQUILIBRIUM ORIENTATION OF DOMAIN MAGNETIZATION

The equilibrium orientation of the domain magnetization, in the two domains are determined by the static values of the variables \( \theta_1 \), \( \theta_2 \), \( \phi_1 \) and \( \phi_2 \). These are obtained from Eq. 58, i.e.,

\[
\frac{\partial G}{\partial \theta_1} \bigg|_{q=0, \xi=0} = K_u \sin 2\theta_1 \left( -\frac{H \cdot M}{2} \cos \theta_1 \cos \phi_1 - \frac{H \cdot M}{2} \sin \theta_1 \sin \phi_1 \right) \\

\frac{H \cdot M}{2} \sin \alpha \cos \theta_1 \sin \phi_1 \\
+ (1 - N_{zz}) \omega M^2 \cos \theta_1 \sin \phi_1 \left[ \sin \theta_1 \sin \phi_1 - \sin \theta_2 \sin \phi_2 \right] \tag{61}
\]

\[
- N_{zz} \omega M^2 \sin \theta_1 \left( \cos \theta_1 - \cos \theta_2 \right) - \frac{\omega M^2}{L^2} \left( \cos \theta_1 + \cos \theta_2 \right)
\]

\[
\frac{\partial G}{\partial \theta_1} \bigg|_{q=0, \xi=0} = 0
\]

and

\[
\frac{\partial G}{\partial \phi_1} \bigg|_{q=0, \xi=0} = \frac{H \cdot M}{2} \cos \alpha \sin \theta_1 \sin \phi_1 - \frac{H \cdot M}{2} \sin \alpha \sin \theta_1 \cos \phi_1
\]
\[ (1 - N_{zz}) M^2 \cos \phi_1 (\sin \theta_1 \sin \phi_1 - \sin \theta_2 \sin \phi_2) + \frac{\partial \omega_0}{\partial \phi_1} = 0. \]  

(62)

Corresponding equations can also be obtained for \( \theta_2 \) and \( \phi_2 \).

After using the conditions stated in Eqs. 59 and 60, the above equations reduce to,

\[ K_u \sin \theta_1 \frac{H M}{2} \cos \alpha \cos \theta_1 \cos \phi_1 - \frac{H M}{2} \sin \alpha \sin \theta_1 \sin \phi_1 \]

\[- 2M^2 N_{zz} \sin \theta_1 \cos \theta_1 + \frac{\partial \omega_0}{\partial \theta_1} = 0 \]  

(63)

and

\[ \frac{M H}{2} \cos \alpha \sin \theta_1 \sin \phi_1 - \frac{M H}{2} \sin \alpha \sin \theta_1 \cos \phi_1 + \frac{\partial \omega_0}{\partial \phi_1} = 0. \]  

(64)

The solutions of these equations cannot proceed unless the analytical form \( \omega_0 \) is obtained. As discussed in the previous section, \( \omega_0 \) is known only in the restrictive case of high \( Q \) (\( Q \gg 1 \)). As a result, the equilibrium orientation of high \( Q \) samples are considered first.

Substituting Eq. 46 for \( \omega_0 \), it can easily be seen that \( \phi_1 = \alpha \) and

\[ \sin \theta_1 = \frac{H_p - \frac{\partial \sigma}{\partial H} (2 \theta_1 - \pi)}{H_u - 4 \pi M H_{zz}}. \]  

(65)

If the contribution of the wall term \( \frac{\partial \omega_0}{\partial \theta_1} \) is neglected as being small, the above equation reduces to,
Eq. 65 is more complex to solve than Eq. 66. Both these equations are solved numerically and a graph of $\theta_1$ as a function of $H_p$ is shown in Fig. 11 for high $Q$ materials. The solid line in Fig. 11 is from Eq. 65 and includes the wall energy term. The dotted line is the solution obtained by neglecting this term.

It is seen that neglecting the wall energy term does not result in large errors. The correction due to this term is a maximum at $H_p=0$ and decreases as $H_p$ is increased. This also yields the interesting result that even in the absence of $H_p$, the magnetization vectors in the domains do not lie along the $z$-axis, but are canted toward the $x$-axis, subtending a small angle $\theta_1$, dictated by the wall energy. This is interpreted as due to the exchange effects in the domain wall producing a small, in-plane magnetic field in the $x$ direction, thus causing $M$ to tilt. This field in turn is produced as a result of all the spins inside the Bloch wall having an $x$ component. As $H_p$ is increased, this field produced by the walls is masked by $H_p$ and gives rise to vanishingly small corrections to $\theta_1$. The above analysis assumes that the magnetization in all the Bloch walls are oriented in the same fashion so that the effective fields produced by them add up. If, however, the two adjacent domains walls are twisted in the
Fig. 11 A plot of static equilibrium value of $\theta_1$ as a function of $H_p$ for samples. The dotted curve is obtained from Eq. 66 and the solid curve is from Eq. 65 which includes the domain wall correction term.
opposite sense, their fields mutually cancel each other resulting in no correction to the static value of $\theta_1$.

For the samples used in this work, where $Q$ is not very high, a qualitative argument can be presented for neglecting the wall energy corrections to $\theta_1$. It was shown that the corrections are maximum at $H_p = 0$ and become smaller as $H_p$ is increased. This argument is valid for all values of $Q$ for which parallel stripe domains exist. Also, it can easily be shown that for all values of $Q$, the static wall energy and thus the correction terms are identical, for $H_p = 0$ i.e. the absolute upper bound of the wall energy corrections are the same for any value of $Q$ and occur at $H_p = 0$. Since the maximum correction itself is not appreciable, Eq. 66 can be used to evaluate $\theta_1$. In general, for smaller $Q$, the corrections to $\theta_1$ at any field $H_p$ is larger than the corresponding correction in the $Q \gg 1$ limit. Also, from Eq. 66, it can be seen that $H_{int} = 4\pi M_N \sigma_{zz}$ can be identified as the $H_{int}$ defined by Eq. 24 and is the field acting in the direction of the magnetization vector. Equations 65 and 66 are valid only when $H_p \leq H_{sat}$, where $H_{sat}$ is the in-plane saturation field, at which the domains disappear and the magnetization of the sample is pulled into the plane of the sample.
2.4.2 STATIC WALL ENERGY AND DOMAIN SPACING

Once the static values of $\theta_1$ and $\phi_1$ are obtained, it is possible to numerically evaluate the static wall energy, $G_{WO}$, for any value of $Q$. The starting point of such a derivation is Eq. 45. The polar angle $\psi$ of magnetization inside the wall also points along $\alpha$, i.e. $\phi_1 = \psi = \alpha$. The static demagnetization energy term of the wall in Eq. 45 is neglected as being small, for small values of $\alpha$. The static wall energy is then,

$$G_{WO} = \frac{1}{L} \int_0^\infty \left[ K_u (\sin^2 \theta - \sin^2 \theta_1) - H_p M (\sin \theta - \sin \theta_1)^{1/2} \right] dy,$$

where Eq. 66 is used to evaluate $\theta_1$. This integral is converted to an integral over $\theta$ using Eq. 51 and since $dy = \frac{3y}{2\theta} d\theta$,

$$G_{WO} = \frac{2}{L} \int_{\theta_1}^{\pi-\theta_1} \left[ K_u (\sin^2 \theta - \sin^2 \theta_1) - H_p M (\sin \theta - \sin \theta_1)^{1/2} \right] dy.$$

Substituting Eq. 66 for $\theta_1$ this relation gives,

$$G_{WO} = \frac{d_0}{2L} \left[ \sin \theta - \sin \theta_1 \right]^2 + 2 \left( \frac{4\pi M}{H_u} \right) \sin \theta_1 (\sin \theta - \sin \theta_1)^{1/2} \theta.$$

$$= \frac{G_{WO}}{2L}.$$
In the high Q limit, and with the appropriate substitution, this reduces to Eq. 46

$$q_{\omega_0} = \frac{\sigma_0}{2L} \left[ 2 \cos \theta_1 - (\pi - 2\theta_1) \sin \theta_1 \right].$$  \hspace{1cm} (70)

Eq. 69 can be solved numerically and a plot of $q_{\omega_0}/q_0$ as a function of $H_p$ is given in Fig. 12. For comparison, $q_{\omega_0}/q_0$ is also evaluated in the high Q limit, using Eq. 70 and is shown as a dotted line in the same diagram. While Fig. 12 is for sample 5, a similar diagram for sample 1, which has a smaller Q than Sample 5, is given in Fig. 13. It is seen from these two diagrams, that as Q gets smaller, the difference between the exact calculation and the high Q limit becomes more appreciable. For small Q values, $q_{\omega_0}/q_0$ is seen to be almost linear in $H_p$, while the high Q evaluation gives it a curvature, with $q_{\omega_0}$ going asymptotically to zero, as $H_p \rightarrow H_{\text{sat}}$.

Although the equilibrium domain spacing $L$ itself is not a dynamical variable in the problem, the values of $L$ can also be computed by minimizing $G$ with respect to $L$, i.e.

$$\frac{\partial G}{\partial L} = 0.$$  \hspace{1cm} (71)

This results in the following condition,
Fig. 12 A plot of reduced static domain wall energy $\sigma W_0/\sigma_0$ as a function of $H_p$, for sample 5. The dotted curve is the high $Q$ approximation for the sample.
Fig. 13 A plot of reduced static domain wall energy, $\sigma_{W0}/\sigma_0$ as a function of $H_p$, for sample 1. The dotted curve is the high Q approximation. While the approximate curve reaches zero asymptotically, the actual curve goes to zero more sharply.
where $\sigma_{WO}$ and $N_{zz}$, given by Eq. 23, are both functions of $L$. The above condition is recognized as the criterion for the stability of the domain pattern where the gain in the wall energy due to further breakdown into domains, is offset by the decrease in the demagnetization energy. The differentiation can easily be carried out by introducing a new variable $\eta = \pi T/L$. Since $N_{zz}$ is expressed in terms of a power series, its derivative is also a power series given by,

$$
\frac{\sigma_{WO}}{4T} = \frac{16}{3 \eta} \sum_{n \text{ odd}}^{\infty} \left(\frac{1}{3}\right)^n \left[ \frac{1}{\eta} \left(\frac{1}{1 + \sqrt{\mu \coth B}} + \frac{\nu \mu}{2 \sinh B + \sqrt{\mu \cosh B}}\right) \right],
$$

(73)

where $B = \frac{\nu \mu}{2}$.

This equation is solved numerically, after evaluating $\sigma_{WO}$ for various values of $H_p$. A plot of the domain width as a function of $H_p$ is shown in Fig. 14. It is seen that $L$ decreases with $H_p$ in a nearly linear fashion and at the saturation field when $H_p = H_{sat}$, the domain width still remains finite. This gives rise to an interesting behavior of the domain wall resonance as $H_p \to H_{sat}$, which will be discussed in 2.5.3.
Fig. 14 A plot of the equilibrium domain spacing L as a function of $H_p$ for sample 5.
The demagnetization factor \( N_{zz} \) is also a function of \( H_p \), through its dependence on \( L/T \). A plot of \( N_{zz} \) as a function of \( H_p \) is shown in Fig. 15.

The effective domain wall width \( \Delta_{\text{eff}} \) can be defined to be,

\[
\Delta_{\text{eff}} = (\pi - 2\phi_1) \left( \frac{\partial \phi}{\partial y} \right)_{y=0}^{-1}.
\]

This has also been computed as a function of \( H_p \) and a plot of \( \Delta_{\text{eff}} \) vs \( H_p \) is shown in Fig. 16. From this diagram, it can be seen that the domain wall width defined by Eq. 74 increases dramatically as \( H_p \) nears saturation values.

2.5 DYNAMICS OF DOMAIN-DOMAIN WALL SYSTEM

2.5.1 EQUATIONS OF MOTION

The variables that are used to describe the system are relabeled in this section as, \( \theta_1 = X_1 \), \( \phi_1 = X_2 \), \( \theta_2 = X_3 \), \( \phi_2 = X_4 \), \( q = X_5 \) and \( \xi = X_6 \), so that there is generality between the different variables.

The equations of motion of the system are then written down. For example, the equations describing the magnetization in the domains can be obtained from the Landau-Lifshitz equation, Eq. 1 and by
Fig. 15 A plot of $N_{zz}$ as a function of $H_p$ for sample 5.
Fig. 16 A plot of the effective domain wall width $\Delta_{\text{eff}}$ as a function of $H_p$. $\Delta_{\text{eff}}$ has a singularity at $H_p = H_{\text{sat}}$. 
transforming the gradient term, \( \frac{\partial \phi}{\partial x} \), in Eq. 2 to spherical coordinates. The free energy is presumed to be of the form expressed in Eq. 57. The resulting equations are,

\[
\phi_1 = \frac{-2Y}{M \sin \theta_1} \frac{\partial \phi}{\partial \theta_1},
\]

(75)

or in terms of the generalized notation,

\[
\phi_1 = \frac{-2Y}{M \sin \theta_1} \frac{\partial \phi}{\partial \theta_1},
\]

(76)

The factor of 2 that appears in the two equations above arises because the specific free energy \( \phi \) is averaged over the elemental volume that contains both kinds of domains, whereas the variables \( \theta_1 \) and \( \phi_1 \) appear in only one domain, i.e., half the elemental volume. Similarly an equation can be written for \( \phi_1 \),

\[
\phi_1 \sin \theta_1 = \frac{2Y}{M \sin \theta_1} \frac{\partial \phi}{\partial \theta_1},
\]

(77)

or

\[
\phi_1 \sin \theta_1 = \frac{2Y}{M \sin \theta_1} \frac{\partial \phi}{\partial \theta_1},
\]

(78)

The other domain equations are,

\[
\phi_3 = \frac{-2Y}{M \sin \theta_3} \frac{\partial \phi}{\partial \theta_4}.
\]

(79)
The equations of motion for the wall coordinates, Eqs. 52 and 53, can similarly be transformed. The wall coordinates $q$ and $\xi$ are average quantities for the wall, and since $G$ is not a function of the spatial derivatives of $\xi$ and $q$, the gradient term in the functional derivatives can be ignored, resulting in the following versions of Eqs. 52 and 53,

\begin{align*}
\dot{X}_4 \sin X_3 &= \frac{2\gamma \frac{\partial G}{\partial X_3}}{M}, \\
\dot{X}_5 &= \frac{\gamma L}{(\cos X_1 - \cos X_3)} \frac{\partial G}{\partial X_6}, \\
&\quad - \frac{(\cos X_1 - \cos X_3) M}{\gamma L} \dot{X}_6 = \frac{\partial G}{\partial X_5}. \tag{82}
\end{align*}

The dynamics of the system follow from the solutions of Equations 76, 78, 79, 80, 81 and 82. In general, once the equilibrium orientations are determined, the free energy, $G$, can be expanded as a Taylor series about the equilibrium configuration, i.e.

\[ G(X_1) = G_0 + \frac{1}{2} \sum_{i,j} \frac{\partial^2 G}{\partial X_i \partial X_j} \bigg|_{eq} \Delta X_i \Delta X_j + \ldots \tag{83} \]

where $G_0$ is the equilibrium value of $G$ and the derivatives are evaluated at the equilibrium values, denoted by the suffix "eq". The
linear terms are absent due to static conditions. The linear response of the system is given by the small oscillations about the equilibrium position. In this limit, only the quadratic terms are retained and the higher order terms are neglected. This truncated expression for $Q$ can be substituted into the equations of motion, Eq. 76 through 82 and if a harmonic time dependence is assumed, the following set of homogeneous, linear algebraic equations result, in terms of $G_{ij}$, where $G_{ij}$ are second derivatives of $G$ with respect to $X_i$ and $X_j$ evaluated at the static equilibrium values of the variables.

$$\begin{align*}
\omega \Delta X_1 &= -\frac{2\gamma}{M \sin X_1} \left[ G_{21} \Delta X_1 + G_{22} \Delta X_2 + G_{23} \Delta X_3 + G_{24} \Delta X_4 + G_{25} \Delta X_5 + G_{26} \Delta X_6 \right], \\
\omega \Delta X_2 \sin X_1 &= \frac{2\gamma}{M} \left[ G_{11} \Delta X_1 + G_{12} \Delta X_2 + G_{13} \Delta X_3 + G_{14} \Delta X_4 + G_{15} \Delta X_5 + G_{16} \Delta X_6 \right], \\
\omega \Delta X_3 &= -\frac{2\gamma}{M \sin X_3} \left[ G_{41} \Delta X_1 + G_{42} \Delta X_2 + G_{43} \Delta X_3 + G_{44} \Delta X_4 + G_{45} \Delta X_5 + G_{46} \Delta X_6 \right], \\
\omega \Delta X \sin X &= \frac{2\gamma}{M} \left[ G_{41} \Delta X_1 + G_{42} \Delta X_2 + G_{43} \Delta X_3 + G_{44} \Delta X_4 + G_{45} \Delta X_5 + G_{46} \Delta X_6 \right], \\
\cos X_1 - \cos X_3 &= \frac{1}{\gamma L} \Delta X_5 = \left[ G_{61} \Delta X_1 + G_{62} \Delta X_2 + G_{63} \Delta X_3 + G_{64} \Delta X_4 + G_{65} \Delta X_5 + G_{66} \Delta X_6 \right], \\
-\omega \left( \frac{1}{\gamma L} \right) \Delta X_6 &= \left[ G_{51} \Delta X_1 + G_{52} \Delta X_2 + G_{53} \Delta X_3 + G_{54} \Delta X_4 + G_{55} \Delta X_5 + G_{56} \Delta X_6 \right].
\end{align*}$$
The solutions of the set of the above equations determine the normal modes of the system and their eigenfrequencies. These frequencies are the various resonance frequencies of the system that are observed experimentally. The condition for the existence of non-trivial solutions to this set of equations is that the determinant of this system of equations be zero. The above equations can be transformed to the following matrix equation.

\[
\begin{bmatrix}
G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\
G_{21} & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} \\
G_{31} & G_{32} & G_{33} & G_{34} & G_{35} & G_{36} \\
G_{41} & G_{42} & G_{43} & G_{44} & G_{45} & G_{46} \\
G_{51} & G_{52} & G_{53} & G_{54} & G_{55} & G_{56} \\
G_{61} & G_{62} & G_{63} & G_{64} & G_{65} & G_{66}
\end{bmatrix}
\begin{bmatrix}
\Delta X_1 \\
\Delta X_2 \\
\Delta X_3 \\
\Delta X_4 \\
\Delta X_5 \\
\Delta X_6
\end{bmatrix} = 0
\]

(85)

where \( z_1 = \frac{\omega \sin \gamma_1}{2\gamma} \) and \( z_2 = \frac{\gamma_2 \cos \gamma_1 - \cos \gamma_2}{\gamma L} \).

The expansion of the determinant of this matrix results in a sixth order polynomial in \( \omega \) and the roots of this polynomial
correspond to the condition where the determinant vanishes, i.e. to the normal mode frequencies.

The elements of the above matrix can be explicitly evaluated by using Eq. 57, carrying out the differentiations and substituting the static values. Reverting back to the earlier notation, the $G_{ij}$ terms are,

$$G_{11} = G_{33} = \frac{\cos^2 \theta}{2} \left[ (H_u - 4\pi MN) + \frac{4\pi MN \sin^2 \alpha}{2} \right] + \frac{\sin^2 \theta}{2} \left[ \frac{4\pi MN \cos^2 \alpha}{2} \right], \quad (86)$$

$$G_{22} = G_{44} = \frac{\sin^2 \theta}{2} \left[ \frac{4\pi MN \cos^2 \alpha}{2} \right] + \frac{\pi MN \sin \theta}{2}, \quad (87)$$

$$G_{13} = G_{31} = \frac{\cos^2 \theta}{2} \left[ \frac{4\pi MN \sin^2 \alpha}{2} \right] + \frac{\sin^2 \theta}{2} \left[ \frac{4\pi MN \sin \alpha}{2} \right],$$

$$G_{24} = G_{42} = -\frac{\sin^2 \theta}{2} \left[ \frac{4\pi MN \cos^2 \alpha}{2} \right], \quad (88)$$

$$G_{12} = G_{21} = G_{23} = G_{32} = -G_{14} = -G_{41} = -G_{43} = -G_{34} =$$

$$\frac{(4\pi MN \sin 2\theta \sin 2\alpha)}{16} = \beta, \quad (89)$$

$$G_{15} = G_{35} = G_{51} = G_{53} = \frac{-4\pi^2 N \sin 2\theta}{2L} = \lambda, \quad (90)$$
The other elements are zero. In the above equations, the equilibrium orientation of $\theta_1$ is simply written as 0 and the equilibrium value of $\phi_1 = \phi_2 = \alpha$. The equilibrium domain spacing is $L$ and the values of $q$ and $\Psi$ at equilibrium are zero and $\alpha$. The terms $n$ and $B$ are introduced in connection with Eq. 73. Filling in the values of the elements, the matrix Equation 85 now has the form

$$
G_{55} = \frac{(16\pi M^2) \cos^2 \theta}{L^2} \left[ 1 + \frac{5}{n} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left( \frac{1}{1 + \sqrt{\mu} \coth B} \right) \right], \quad (91)
$$

and

$$
G_{66} = \frac{(2M \cos \theta)^2}{\gamma} \frac{1}{\gamma} \frac{n}{mL}. \quad (92)
$$

Here the old notations for the variables are used. Since the equilibrium value of $q$ is zero, the symbol $q$, instead of $\Delta q$, is
used to denote variations in $q$. By exploiting the symmetry of the above matrix, where many elements have the same values, the rows and columns can be rearranged in terms of new variables defined by,

$$\Delta \theta^\pm = \Delta \theta_1 \pm \Delta \theta_2$$ (94)

and

$$\Delta \phi^\pm = \Delta \phi_1 \pm \Delta \phi_2.$$ (95)

The resulting equation is,

$$\begin{bmatrix}
A & -iz_1 & 0 & 2\beta & 2\lambda & 0 \\
iz_1 & C & 0 & 0 & 0 & 0 \\
0 & 0 & B & -iz_1 & 0 & 0 \\
2\beta & 0 & iz_1 & D & 0 & 0 \\
\lambda & 0 & 0 & 0 & G_{55} & iz_2 \\
0 & 0 & 0 & 0 & -iz_2 & G_{66}
\end{bmatrix}
\begin{bmatrix}
\Delta \theta^+ \\
\Delta \phi^+ \\
\Delta \theta^- \\
\Delta \phi^- \\
\Delta \xi
\end{bmatrix} = 0, \text{ (96)}$$

where $A = g_{11} + g_{13}$, $B = g_{11} - g_{13}$, $C = g_{22} + g_{24}$ and $D = g_{22} - g_{24}$.

It is seen that the above matrix separates into three parts. The top left $4 \times 4$ matrix represents the domain ferromagnetic resonance behavior. The bottom right $2 \times 2$ matrix describe the domain wall resonance and the $\lambda$ terms represent the coupling between the domain
magnetization precession and the domain wall motion. In the absence of the coupling terms \( \lambda \) the matrix becomes quasidiagonal,

\[
\begin{bmatrix}
A & -iz_1 & 0 & 2\beta & 0 & 0 \\
iz_1 & C & 0 & 0 & 0 & 0 \\
0 & 0 & B & -iz_1 & 0 & 0 \\
2\beta & 0 & iz_1 & D & 0 & 0 \\
0 & 0 & 0 & 0 & G_{55} & iz_2 \\
0 & 0 & 0 & 0 & G_{66} & \Delta \xi
\end{bmatrix}
\begin{bmatrix}
\Delta \Theta^+ \\
\Delta \Phi^+ \\
\Delta \Theta^- \\
\Delta \Phi^-
\end{bmatrix} = 0. \quad (97)
\]

Before solving the 6 x 6 determinant in its complete form, it is instructive to examine the 4 x 4 matrix of the domain resonance and the 2 x 2 matrix of the domain wall resonance separately and acquire some insights into their physical nature.

2.5.2 DOMAIN FERROMAGNETIC RESONANCE

The matrix equation that accounts for the domain ferromagnetic resonance can be written as,

\[
\begin{bmatrix}
A & -iz_1 & 0 & 2\beta \\
iz_1 & C & 0 & 0 \\
0 & 0 & B & -iz_1 \\
2\beta & 0 & iz_1 & D
\end{bmatrix}
\begin{bmatrix}
\Delta \Theta^+ \\
\Delta \Phi^+ \\
\Delta \Theta^- \\
\Delta \Phi^-
\end{bmatrix} = 0. \quad (98)
\]
When the domain walls are perfect Bloch walls, the walls align with $H_p$ and thus the angle $\alpha$ and by virtue of Eq. 91, the term $\beta$, are both zero. If the wall is Néel type, it aligns perpendicular to $H_p$, making $\alpha = \pi/2$ and hence $\beta = 0$. For both these special cases of $\alpha$, the determinant of the matrix in Eq. 92 can be evaluated easily resulting in,

$$\frac{M^2 \sin^2 \theta}{4Y^2} = AC \text{ or } BD \tag{99}$$

Substituting for $A$, $B$, $C$ and $D$ gives,

For $\alpha = 0$,

$$(\frac{\omega}{Y})^2 = \left[ (H_u - 4\pi MN_{zz})^3 - H_p^2 (H_u - 4\pi MN_{zz} - 4\pi MN_{yy}) \right] / (H_u - 4\pi MN_{zz}) \tag{100}$$

and for $\alpha = \pi/2$,

$$(\frac{\omega^+}{Y})^2 = (H_u - 4\pi MN_{zz}) (H_u - 4\pi MN_{zz} + 4\pi MN_{yy}) - H_p^2 \tag{102}$$

$$(\frac{\omega^-}{Y})^2 = (H_u - 4\pi MN_{zz})^2 - H_p^2 \tag{103}$$

For $H_p > H_{sat}$ however, the domains disappear and the magnetization vector lies in the plane of the sample. In this case,
the ferromagnetic resonance frequencies are given by the familiar expression for a uniformly magnetized slab,

$$\left(\frac{\omega}{\gamma}\right)^2 = H_p \left[ H_p - (H_u - 4\pi M) \right] .$$

(104)

The behavior of these resonance frequencies as functions of $H_p$, is shown in Fig. 17 for $\alpha=0$ and in Fig. 18 for the case of $\alpha=\pi/2$.

The physical reason for the presence of two modes, $\omega^+$ and $\omega^-$, is due to the coupling of the two types of domains via the demagnetization field of the domain structure. From Fig. 17 and Fig. 18 it can be seen that $\omega^+$ mode joins the uniform FMR as the domains disappear and the $\omega^-$ mode disappears at the in-plane saturation. It is seen that $\omega^+$ mode has a higher frequency for $H_p=0$ in case of $\alpha=\pi/2$ while for $\alpha=0$, $\omega^-$ mode has a higher zero field resonance frequency.

The two modes can be analyzed in greater detail by considering the dynamic components of the magnetization vector $\Delta \mathbf{m}$ in the respective domains.

For the $\omega^+$ mode it can be seen that,

$$\Delta \theta_1 = \Delta \theta_2 \quad \text{and} \quad \Delta \phi_1 = \Delta \phi_2 .$$

(105)
Fig. 17. Domain resonance frequencies $f_\pm$ vs. $H_p$ for sample 5 when $H_p$ is along the $x$ axis ($\alpha=0$). The uniform FMR is also shown. The two modes $\omega^+$ and $\omega^-$ are seen to intersect.
Fig. 18 Domain resonance frequencies $f^\pm$ vs. $H_p$ for sample 5. $H_p$ is along $y$ ($\alpha = \pi/2$). The uniform FMR is also shown. The two modes $\omega^+$ and $\omega^-$ do not intersect, in this case.
Thus, this mode is recognized as the "in-phase" mode, where the magnetizations, apart from differences in their static orientation, precess in phase.

Also,

\[ \Delta m_{1z}, y_1 = \Delta m_{2z}, y_2 \quad (106) \]

and

\[ \Delta m_{1x} = - \Delta m_{2x}, \quad (107) \]

The above equations state that the dynamic power coupling from external sources that excite this mode occurs through the \( z \) and \( y \) components.

The "in-phase" mode is illustrated in Fig. 19. At the instant shown in Fig. 19a, the two magnetization vectors in the two domains are trying to come out of the plane of the paper (\( \Delta \phi_1 = \Delta \phi_2 \)). The instantaneous position of the vectors are shown by a broken arrow and the solid arrow is the static equilibrium orientation. Fig. 19b shows the situation a half cycle later where both vectors tend to go into the page. It is easy to see that as \( H_p \rightarrow H_{\text{sat}} \), when \( \theta_1 \rightarrow \theta_2 \rightarrow \pi/2 \), the two magnetization vectors would lie parallel to \( x \) axis and precess in phase.
Fig. 19 "In-Phase" precession of the magnetization vectors in the two domains, corresponding to \(\omega^+\) mode. At the instant shown in Fig. 19(a) both the vectors tend to come out of the page \((\Delta \phi_1 = \Delta \phi_2)\). A half cycle later, as shown in Fig. 19(b) both the magnetization vectors precess into the page. The solid curve at the tip of the vector corresponds to the trajectory above the plane of the paper and the dotted curve is below.
The $\omega^-$ mode corresponds to the "out of phase" precession and can be treated similarly. In this case, the following relations hold true,

$$\Delta \theta_1 = -\Delta \theta_2, \quad \Delta \theta_1 = -\Delta \theta_2,$$

(108)

$$\Delta m_{x_1}, y_1 = -\Delta m_{x_2}, y_2$$

(109)

and

$$\Delta m_{x_1} = \Delta m_{x_2}.$$

(110)

This mode is illustrated schematically in Fig. 20. It can be seen that for the two positions shown in Fig. 20a and Fig. 20b that if one magnetization vector tends to go into the page, the other tends to go in the opposite direction. In this case, the coupling to external sources occurs through the dynamic $x$ components of magnetization.

In general, when the walls are neither parallel nor perpendicular to the applied field, the evaluation of the determinant is complicated by the $2\beta$ terms and has to be performed numerically. It can be seen that, when $\alpha < \pi/4$ the curves for $\omega^+$ and $\omega^-$ as functions of $H_p$, cross (Fig. 17). However, they actually intersect only in the case when $\alpha=0$ and at the field value where the crossing occurs, both the "in-phase" and
Fig. 20 "Out of phase" precession of the magnetization vectors in the two domains corresponding to the \( \omega^- \) mode. For the instant shown in Fig. 20(a) one vector is coming out of the page while the other is going into the page. Fig. 20b shows the situation a half a cycle later. Again one vector is going into the page while the other is coming out.
"out of phase" resonances have the same frequency. When $\alpha$ is not zero, the presence of the $2\beta$ terms cause such degeneracy to be lifted and the two curves do not intersect at all. The domain resonance frequencies as functions of $H_p$, for $\alpha=10^\circ$ is shown in Fig. 21.

2.5.3 DOMAIN WALL RESONANCE

It has been shown in the previous subsection that the bottom four elements of the matrix of Eq. 97 yield the domain wall resonance oscillations. This can be written explicitly, as

$$
\begin{bmatrix}
G_{95} & 1z_2 \\
-1z_2 & \left(\frac{2M_0\cos\theta}{\gamma}\right)^2 \frac{1}{mL} \\
\end{bmatrix}
\begin{bmatrix}
q \\
\Delta \xi \\
\end{bmatrix} = 0.
$$

(111)

The solution of this equation, after substituting for $z_2$, is simply,

$$
\frac{\omega_{\text{DWR}}^2}{L^2} - \frac{G_{95}}{mL} = 0
$$

(112)
Fig. 21 A plot of the domain resonance frequencies $f^\pm$ and FMR frequency as functions of $H_p$ for $\alpha=10^\circ$, for sample 5. A coupling and consequent mode repulsion can be seen between the $\omega^+$ and $\omega^-$ modes, due to the coupling term $B$. 
From the above expression it is clear that $LG_{55}$ is the restoring force constant, $k$, acting on a unit area of the domain wall due to the presence of other domains and $k$ can be obtained by using the form of $G_{55}$ given in Eq. 91. The resulting expression for $k$ is valid for any general value of $Q$ and in the high $Q$ limit, i.e. $Q \gg 1$, it can be shown that the $k$ thus obtained reduces to those derived in Ref. 15 and 17.

Once the equilibrium value of $L$ and thus $\eta = \pi T/L$ are known, $G_{55}$ and thus $k$, can be evaluated by summing the series in Eq. 91 numerically. Since $L$ and $\eta$ are functions of the applied in-plane field $H_p$, the force constant is also a function of $H_p$. The behavior of $k$ as a function of $H_p$ is shown in Fig. 22. It is seen that initially $k$ increases with the $H_p$, reaching a maximum and then decreases to zero as $H_p \rightarrow H_{sat}$.

The concept of domain wall mass yields information about the structure of the domain wall and should be computed next. In order to obtain $m$, the assumption that the moving wall has the same structure as the stationary wall will be invoked, i.e., the variation in $\Theta(y)$ due to wall velocities is unimportant. The only extra energy term for a moving wall arises due to the dynamical $y$ component of the magnetization $\hat{M}$, induced by the torques on $\hat{M}$, resulting in dynamic
Fig. 22 A plot of the force constant $k$ vs $H_p$ for sample 5. Although not shown, $k$ goes to zero when $H_p$ equals $H_{sat}^p$. 
demagnetization self energy of the wall. I.e.

\[ \delta G_{W1} = \frac{1}{L} \int_{-\infty}^{\infty} dy \ \delta W \]  \hspace{1cm} (113)

or \[ \delta G_{W1} = \frac{1}{L} \int_{-\infty}^{\infty} \left[ \frac{\partial W}{\partial \theta} \delta \theta + \frac{\partial W}{\partial \psi} \delta \psi \right] dy . \]  \hspace{1cm} (114)

where the terms \( G_{W1}, W \) etc. have been defined earlier in Section 2.3. Since the variations in \( \theta \) are considered unimportant, the first term is neglected. Then the above equation, after substituting the equation of motion, Eq. 27, becomes,

\[ \delta G_{W1} = \frac{1}{L} \int_{-\infty}^{\infty} dy \ \left[ - \dot{\psi} \frac{\text{Main} \theta}{y} \right] \delta \psi . \]  \hspace{1cm} (115)

A new function \( f(y) \) is introduced such that

\[ \psi(y) = \dot{q} f(y) , \]  \hspace{1cm} (116)

where \( f \) can now be interpreted as a function describing the \( y \) dependence of the dynamic polar angle \( \psi \). (The static polar angle = \( \alpha \), for magnetization within the wall). Therefore,

\[ \delta \psi = f(y) \ \delta \dot{q} , \]  \hspace{1cm} (117)

and Eq. 107 can be rewritten as,

\[ \delta G_{W1} = \frac{M}{\gamma L} \int_{\theta_1}^{\theta_2} [d \ \theta \ f (\theta) \sin \theta] \ \dot{q} \ \delta \dot{q} . \]  \hspace{1cm} (118)
This is of the form

\[ \delta Q_{W_1} = \frac{m}{L} q \delta q = \delta \left( \frac{1}{2} \frac{m}{L} q^2 \right), \]  

(119)

where \( m \), which was previously introduced phenomenologically, is the domain wall mass per unit area of the wall defined by the equation,

\[ m = \frac{M}{\gamma} \int_{\gamma_1}^{\gamma_0} d\theta f(\theta) \sin \theta. \]  

(120)

Additionally, it can be shown that Eq. 54 can be obtained from the definition of \( f(y) \), Eq. 117, and integrating it over the width of the domain wall.

Following Ref. 18, it can be shown that the equation of motion for the wall is given by

\[ \ddot{\Theta} = -\gamma \left[ 2M \frac{\partial^2}{\partial y^2} \sin^2 \Theta \frac{\partial \psi}{\partial y} - 2M^2 \sin \Theta \sin 2\psi - MH \sin \Theta \sin \psi \right]. \]  

(121)

In the Morkowski model, the first term due to exchange energy is neglected as small. This is because \( \frac{\partial \psi}{\partial y} \) is proportional to \( \frac{\partial f}{\partial y} \) and hence to \( \frac{\partial f}{\partial \theta} \). This is assumed to be a slowly varying
function of $y$. Gathering the other terms and since the dynamic values of $\psi$ are small, Eq. 48 yields

$$\frac{1}{y} \frac{d\psi}{dy} = [H_p + 4\pi M \sin \theta] \psi.$$  \hspace{1cm} (122)

Substituting for $\frac{d\psi}{dy}$ from Eq. 51 and using the definition of $f(y)$, the following expression for $f(\theta)$ is deduced,

$$f(\theta) = \frac{\sin^2 \theta - \sin^2 \theta_0 - 2\mu (\sin \theta - \sin \theta_0)^{1/2}}{Q^{-1}(Q\mu + \sin \theta)}.$$  \hspace{1cm} (123)

where, as defined earlier, $\mu = H_p/H_u$. This is substituted in Eq. 120 and the resulting integral is evaluated numerically to obtain the domain wall mass, $m$. When $Q \gg 1$, $\sin \theta_0$ can be approximated to $\mu$ and $f(\theta)$ takes on a simpler form,

$$f(\theta) = \frac{1}{\gamma H_u A} \frac{\sin \theta - \mu}{Q^{-1}(Q\mu + \sin \theta)}.$$  \hspace{1cm} (124)

A plot of $m$ as a function of $H_p$ is shown in Fig. 23 where Eq. 115 is used. The high $Q$ approximation, as given in Eq. 124 is also shown in Fig. 23 by the dotted line for comparison. It is seen that in the exact computation of $m$, its value goes to zero as $H_p \rightarrow H_{sat}$, more sharply than the high $Q$ approximation of $m$, where it goes more slowly and asymptotically to zero at saturation. Fig. 24 shows a similar plot for sample 1, whose $Q$ value is less than that
Fig. 23 A plot of reduced domain wall mass $m/m_0$ where $m_0$ is the "zero in-plane field" mass, shown as a function of $H_p$ for sample 5. The dotted curve is obtained using the high Q approximation.
Fig. 24 A plot of reduced domain wall mass $m/m_0$ plotted as a function of $H_p$ for sample 1. The dotted curve corresponds to the high $Q$ approximation. While the actual curve goes to zero more sharply as $H_p^{\rightarrow H_{\text{sat}}}$, the approximate curve goes to zero more asymptotically.
of sample 5 for which Fig. 23 was obtained. It is seen that \( m \) behaves almost linearly with \( H_p \).

It is interesting to note that although both \( m \) and \( k \) go to zero as \( H_p \) approaches \( H_{\text{sat}} \), their ratio, the domain wall resonance frequency, remains finite. The ratio of

\[
\left( \frac{f_{\text{DWR}}}{H_p} \right)_{H_{\text{sat}}} = \frac{f_{\text{DWR}}}{H_p = 0}
\]

is computed in Appendix C. In the earlier works where the high \( Q \) limit was used in evaluating \( k \) and \( m \), a singularity was obtained in \( \omega_{\text{DWR}} \) as \( H_p \to H_{\text{sat}} \) since \( m \) went to zero faster than \( k \). However, even for high \( Q \) materials, for fields close to \( H_{\text{sat}} \), the approximation \( \sin \theta = \mu \) is incorrect. This is because, as shown in Section 2.4, the domain spacing \( L \) remains finite all the way up to \( H_{\text{sat}} \) and thus \( N_{zz} \) which is a function of \( L/T \), also remains finite. Thus the saturation field, \( H_u = 4\pi MN_{zz} \), is not just \( H_u \). This removes the singularity obtained in the frequency by earlier workers.\(^{17,18}\) A plot of \( f_{\text{DWR}} \) vs. \( H_p \) is shown in Fig. 25. It is seen, from this diagram that \( f_{\text{DWR}} \) is almost linear in \( H_p \), especially at small values of \( H_p \). In Fig. 26 all three modes of oscillation, namely \( \omega^\pm \) and DWR are plotted as functions of \( H_p \), where the coupling between the DR and DWR has not yet been included.
Fig. 25 A plot of absolute domain wall resonance frequency $f_{DWR}$ vs $H_p$ for sample 5.
Fig. 26 A plot of $f^+$, FMR frequency and $f_{DWR}$ as functions of $H_p$ for sample 5.

The interaction terms between DR and DWR are not included and $\alpha = 0^\circ$. 

\[ \text{FREQUENCY (GHz)} \]

\[ \text{H}_p \text{ (KOe)} \]
2.5.4 MODE COUPLING

The coupling of the domain resonance modes and the domain wall resonance mode are due to the $\lambda$ terms, where $\lambda$ can now be defined in terms of a coupling field $h_c$ as,

$$\lambda = \frac{-h_M \sin \theta}{2L}.$$  \hfill (125)

From Eq. 90 it can be seen that,

$$h_c = 4\pi MN_{yy}.$$  \hfill (126)

The $\sin \theta$ dependence of $\lambda$, on the equilibrium orientation $\theta$, implies that the coupling is unimportant at $\theta=0$ ($H_p=0$) and $\theta = \pi/2$ ($H_p = H_{sat}$) but important everywhere else. The coupling term also depends strongly on $N_{yy}$ and is thus very important in the case of low $N_{zz}$ ($N_{yy}$ being just $1 - N_{zz}$), i.e. when thin parallel stripes exist for $L << T$. In the opposite extreme, when $L >> T$, $N_{zz} \sim 1$ and thus $N_{yy}$ and the coupling are relatively unimportant.

The frequencies of the coupled oscillations of the DR and DWR modes are obtained by solving the zeros of the determinant of Eq. 96. A computer program is written to evaluate the determinant numerically.
and a listing of this program is included in Appendix D. The resulting coupled frequencies are plotted as functions of $H_p$ and are shown in Fig. 27.

One of the roots of this equation is $BD=Z_1^2$ and it can be easily shown using the results of Section 2.5.2 that this corresponds to the "out of phase" resonance, $\omega^-$. This means that the coupling between the DR and DWR does not affect $\omega^-$ but only changes the $\omega^+$ and DWR modes. The new coupled modes can be obtained by solving

$$(AC - Z_1^2)(k/m - \omega^2) - 2\lambda^2 C/m = 0 \quad (129)$$

This is a quadratic equation in $\omega^2$ and can easily be solved. In the absence of the coupling term $\lambda$, the $\omega^+$ mode and the DWR mode may cross each other when $\omega_{DWR} = \omega^+$ or when,

$$AC = \frac{M^2\sin^2\theta}{4\gamma^2} = \frac{k}{m} \quad (130)$$

In the presence of the coupling, the roots of Eq. 129 can be evaluated at this value of $H_p$ (or equivalently, $\theta$) and the frequencies are found to be no longer degenerate, thus giving rise to a frequency gap.
Fig. 27 A plot of the coupled mode frequencies as functions of $H_p$ for sample 5. The frequencies are obtained by solving Eq. 122.
The magnitude of this frequency gap, where the dispersion curves for $\omega^+$ and DVR as functions $H_p$ would have otherwise intersected, can be obtained by substituting the condition of Eq. 130 into the solutions of Eq. 129 and is given by the following equation,

$$\Delta(\omega^2) = (\omega^+)^2 - (\omega_{DVR})^2 = \frac{2\gamma_e \sin \theta}{L} \left[ \frac{M(H_e - 4\pi M N \omega^2)}{m} \right]^{1/2}.$$  (131)

This relationship shows that the frequency gap depends essentially on the coupling field $h_e$, the equilibrium orientation of the magnetization in the domains $\theta$, the equilibrium domain spacing $L$ and the domain wall mass, $m$ at this field value. The inclusion of the $2\beta$ terms in Eq. 128 does not alter the qualitative nature of the above discussion.

It is seen that accounting for the experimentally observed frequency gaps depends crucially on the theories used to evaluate $L$ and $m$. Also, by including other contributions to $G$, such as due to cubic anisotropy energy (which will be considered next), more coupling elements in the matrix in Eq. 96 can be non zero, resulting in a better expression for the frequency gap.
2.6 EFFECTS OF CUBIC ANISOTROPY ENERGY

The magnetic garnet structure has cubic symmetry. So, in addition to the uniaxial anisotropy energy terms, cubic anisotropy energy terms will also exist in the energy expression. These are expressed in terms of the direction cosines along the axes as,

\[ Q_K = K_1 (a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2) + K_2 (a_1^2 a_2^2 a_3^2) + \ldots \]  \hspace{1cm} (132)

Where \( K_1 \) and \( K_2 \) are defined as the cubic anisotropy energy constants analogous to \( K_u \) and the \( a_i \) are the direction cosines. The cubic anisotropy energy removes the isotropy in the plane of the sample. It is therefore necessary to identify the \( x \) and \( y \) axes with respect to the crystalline axes. For this formulation, the coordinate system used is shown in Fig. 28. The \( z \) axis is along [111] and \( x \) axis and the \( y \) axis are at an angle \( \tau \) with respect to the crystal \([112]\) and \([\bar{1}10]\) directions respectively. It can easily be seen that the leading term \( K_1 \) is negative if it is assumed that there exists an easy axis along [111]. Keeping only the leading term and substituting for all \( a_i \), the energy density due to cubic anisotropy energy \( Q_K \) is, \(^{12} \)
Fig. 28 The coordinate system used in the study of the effects of cubic anisotropy energy term. The crystal axes [111] and [110] are at an angle $\tau$ with respect to the $x$ and $y$ axes. (Ref. 18)
and the corrected free energy of the system $G'$ is then,

$$G' = G + Q_k.$$  \hfill (134)

The inclusion of $Q_k$ results in several changes. The static configurations, $\theta_1, \phi_1$ etc. now have to be re-evaluated with the following two equations.

$$\frac{\partial g'}{\partial \theta_1} = \frac{\partial g}{\partial \theta_1} + K_k [\sin^3 \theta_1 \cos \theta_1 - \frac{2}{3} \cos^3 \theta_1 \sin \theta_1 + \frac{\sqrt{2}}{3} \cos \frac{\theta_1}{2} \cos \frac{\phi_1}{2} \cos \frac{\phi_1 - \tau}{2}] \times (3 \sin^2 \theta_1 \cos^2 \theta_1 - \sin^4 \theta_1) = 0$$  \hfill (135)
These are modified versions of Eqs. 61 and 62. The above equations are coupled equations in \( \theta_1 \) and \( \phi_1 \) and can be solved numerically to obtain the static values of \( \theta_1 \) and \( \phi_1 \), using the forms of \( \frac{\partial g}{\partial \theta_1} \) and \( \frac{\partial g}{\partial \phi_1} \) derived in Section 2.4. In the simple case of the \( x \) axis coinciding with [11\( \bar{2} \)] direction (\( \tau=0 \)) and \( \alpha=0, \phi_1=0 \) is the equilibrium value of \( \phi_1 \) and only Eq. 129 needs to be solved to obtain \( \theta_1 \).

The second derivatives of the free energy will also have contributions from the cubic anisotropy energy term and these are,

\[
\frac{\partial g}{\partial \phi_1} = \frac{\partial g}{\partial \phi_1} + K_1 \sqrt{2} \sin \theta_1 \cos \theta_1 \sin 3(\phi_1 - \tau) = 0. \tag{136}
\]

\[
\frac{\partial^2 g}{\partial \theta_1^2} + \frac{\partial^2 g}{\partial \phi_1^2} + 2 \frac{\partial^2 g}{\partial \theta_1 \partial \phi_1} = K_1 [7 \cos^2 \theta_1 \sin^2 \theta_1 - \sin^4 \theta_1 - \frac{4}{3} \cos^4 \theta_1 +
\frac{2\sqrt{2}}{3} \cos 3(\phi_1 - \tau)(3 \sin \theta_1 \cos^3 \theta_1 - 5 \sin^3 \theta_1 \cos \theta_1)], \tag{137}
\]

\[
\frac{\partial^2 g}{\partial \phi_1^2} = -3K_1 \sqrt{2} \sin^3 \theta_1 \cos \theta_1 \cos 3(\phi_1 - \tau), \tag{138}
\]

\[
\theta_{11} = g_{11} + (g_{K})_{11} \tag{139}
\]
The cross derivative terms \((Q_{13})\) and \((Q_{24})\) are zero. Now \(G_{11}'\) and \(G_{22}'\) are terms to be included with the modified values of the static orientations \(\theta_1\) and \(\phi_1\) in Eq. 66 to obtain the corrected domain resonance frequencies. This modification has been studied by Charap and Artman in Ref. 23, where they show the shift in the resonance frequencies.

Cubic anisotropy terms also introduce changes in the values of the static domains wall energy \(Q_{WO}\), which in turn affects the domain spacing and thus the restoring force. This has been studied by Chung et al. The corrections to the domain wall mass \(m\) due to cubic anisotropy energy has been found to be not very large and are obtained by Yeh.

The \(G_{K}\) terms also give rise to the following coupling terms, which are more interesting from the perspective of the present study:

\[
(G_{K})_{12} = -K_1 \sqrt{2} \left[ 3 \sin^2 \theta_1 \cos^2 \theta_1 - \sin^4 \theta_1 \right] \sin 3(\phi_1 - \tau) \quad (141)
\]

\[
(G_{K})_{15} = \frac{K_1}{L} \left[ \sin^3 \theta_1 \cos \theta_1 - \frac{4}{3} \cos^3 \theta_1 \sin \theta_1 \right] + \]

and

\[
G_{22}' = G_{22} + (Q_{K})_{22} \quad (140)
\]
\[ \nu^2 \left( 3 \sin^2 \theta_1 \cos^2 \theta_1 - \sin^4 \theta_1 \right) \cos 3 (\phi_1 - \tau) \] \quad (142)

and

\[ (q_{K'}_{12}) = - \frac{K_1}{L} \sqrt{2} \sin^3 \theta_1 \cos \theta_1 - \sin \theta_1 \cos 3(\phi_1 - \tau). \] \quad (143)

These extra terms are to be added to the coupling terms already discussed in Section 2.5. The coupling term \((q_{K'}_{12})\) suggests that even if \(H_p \) lies parallel to the x-axis, i.e. \(\alpha = 0\), and thus \(q_{12} = 0\), \((q_{K'}_{12})\) can still remain finite (if \(\tau\) is not zero) and can introduce a coupling between the two modes of domain resonance. A plot of \((q_{K'}_{12})/K_1\) vs. \(\theta_1\) is shown for \(\phi_1 - \tau = \pi/2\) in Fig. 29.

\((q_{K'}_{15})\) is the correction to the coupling terms \(q_{15}\) between the domain resonances and the domain wall resonance. The angular dependence of \((q_{K'}_{15})\) for \(\phi_1 - \tau = 0\) is plotted in Fig. 30.

\((q_{K'}_{25})\) is the coupling term between \(\omega^-\) and DWR.

When the cubic anisotropy energy constant \(K_1\) is comparable to \(K_u\), complications begin to arise. As shown in Ref. 33, the "up-down" symmetry between the two types of domains is then broken and it can be shown that \(\theta_1\) and \(\theta_2\) at static equilibrium do not add up to \(\pi\). Since the orientations are assymmetric, the local energy densities and consequently the equilibrium domain spacings are different for the two domains. The consequences of large cubic anisotropy energy through the above effects on the domain resonance, the domain wall resonance and the coupling terms are proposed as a future extension of the present project.
Fig. 29 A plot of the normalized cubic anisotropy coupling constant \( (G_K)_{12} / K_1 \) as a function of \( \theta_1 \) for \( \phi - \tau = \pi/2 \).
Fig. 30 A plot of \((G^1)_{15} L/K\), where \((G^1)_{15}\) is a coupling term due to cubic anisotropy energy term plotted as a function of \(\theta_1\) for \(\phi = \tau = 0\).
3.1 MATERIAL

The material used to study the various resonances of the coupled domain-domain wall system was Ga substituted Yttrium Iron Garnet (YIG). The samples are single crystal thin films and were grown by Dr. M. Shone of Airtron division of the Litton Corporation using Liquid Phase Epitaxy method on non-magnetic GGG substrates. This technique is described in great detail in Ref. 34. The crystalline [111] direction is normal to the plane of the film. Due to the growth conditions, as well as the stresses induced by the lattice mismatch between the film and the substrate, the film develops an easy axis of magnetization along this direction.

The chemical formula for pure YIG is \( \text{Y}_3\text{Fe}_5\text{O}_{12} \). The unit cell has a cubic symmetry with a lattice constant of 12.376 Å and contains eight formula units. This structurally complex unit cell is shown schematically in Fig. 31. The oxygen ions form the corners of three kinds of polyhedra, namely tetrahedra, octahedra and dodecahedra and the cations occupy the polyhedral centers. Each unit cell has the following cation sites;
Fig. 31 A schematic sketch of a unit cell of pure YIG showing some of the cation arrangement. (Ref. 34)
24 Tetrahedral sites (d sites) occupied by Fe$^{3+}$ ions
16 Octahedral sites (a sites) occupied by Fe$^{3+}$ ions
24 Dodecahedral sites (c sites) occupied by T$^{3+}$ ions

Since Y$^{3+}$ ions are nonmagnetic, the ferrimagnetic nature of YIG arises from the Fe$^{3+}$ ions. The d and a sublattice magnetizations are in mutually opposite directions and thus effectively only one Fe$^{3+}$ per formula unit contributes to the net magnetization of the system. Each Fe$^{3+}$ ion has 5$\mu_B$ units of magnetic moment at absolute zero, and thus the unit cell has 40 $\mu_B$ units, where $\mu_B$ is the Bohr magneton. This microscopic analysis of the magnetization of YIG agrees well with the experimentally observed value of 4$\mu_M = 2160$ G at 0°K.

Pure YIG by itself is not well suited to study magnetic domain phenomena because the 4$\mu_M \gg H_u$, i.e., $Q \ll 1$ where $Q = H_u/4\mu_M$ and the in-plane saturation field is very small (~ 55 Oe for a sample of thickness around 10 μm). This condition on $Q$ would mean that the domain pattern is not simply parallel stripes, but more complex with perhaps closure domains, which are more difficult to model theoretically. When Ga is substituted in YIG it occupies the tetrahedral or d-sites preferentially and thus decreases the net magnetization of the system. This in turn increases $Q$ and provides a condition for the existence of parallel stripe domains. The substitution also increases the value of the uniaxial anisotropy field, $H_u$ for reasons not yet completely understood, thus increasing $Q$. 
As $x$ increases, La is added to the film to minimize the lattice mismatch between the YIG and the GGG substrate. It is possible to substitute all the Fe$^{3+}$ ions with Ga$^{3+}$ ions, i.e., $x$ can be made as large as 5. In the present study however, $x$ does not exceed 0.3.

The magnetization of the samples was measured using the vibrating sample magnetometer and $H_u$ was measured using the K-band ferromagnetic resonance spectrometer. These experimental techniques are described in Ref. 4. A total of five samples were studied and the samples with the larger sample number have a larger value of $x$, i.e., a larger Ga concentration. The thickness of the samples ranged from 10 to 15 $\mu$m and was measured at Airtron. The $g$ value was found to be 2.00 and the exchange energy constant $A$ is taken to be $3 \times 10^{-7}$ ergs/cm. The various exchange integrals and mechanisms that give rise to $A$ are also described in Ref. 4. The in-plane saturation field was measured both by domain wall resonance observations and by magneto-optical techniques developed in this laboratory.\textsuperscript{35}

It is interesting to note that unlike typical bubble materials, which are characterized by $Q \gg 1$, these samples have low $Q$, with some even having $Q < 1$. However all of these films demagnetize into parallel stripes that could be observed under an optical microscope using the Faraday rotation effect. The exact condition for stability
of parallel stripe domains in terms of the values of $Q$ is not clearly understood yet. The in-plane saturation field occurs at

$$H_{\text{sat}} = H_u - 4\pi M N_{zz}.$$  \hspace{1cm} (144)

In bubble materials, since $H_u \gg 4\pi M$, the demagnetization factor $N_{zz}$ was considered unimportant. For the samples used in this study, however, the factor $N_{zz}$ plays an important role. The parameters of the samples are given in Table 1.

In bubble materials, where extensive research has been done, the DVR and domain resonances occur at two different regimes. The DVR is at lower frequencies (from 10 to 50 MHz) and the domain resonances occur at much higher frequencies (over 4 GHz). As a result, the interaction between these modes has not been experimentally observed prior to this work. In Ga substituted YIG however, the material parameters are such that the DVR and the domain resonance modes occur in the same frequency range (0 to 4 GHz) making it possible to observe the various coupled modes using rf techniques.

3.2 EXPERIMENTAL SET-UP

A transmission rf spectrometer of the kind described in Ref. 5, is used to excite and detect the rf response of the sample. A block diagram of the spectrometer is given in Fig. 32.
<table>
<thead>
<tr>
<th>Sample #</th>
<th>Doping ions</th>
<th>$4\pi M$ (G)</th>
<th>$H_u$ (Oe)</th>
<th>Q</th>
<th>$H_{sat}$ (Oe)</th>
<th>$N_{zz}$ at Saturation</th>
<th>Thickness, T ($\mu$m)</th>
<th>$i/T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ga</td>
<td>1275</td>
<td>450</td>
<td>0.35</td>
<td>190</td>
<td>0.20</td>
<td>12.6</td>
<td>0.0020</td>
</tr>
<tr>
<td>2</td>
<td>Ga</td>
<td>1150</td>
<td>500</td>
<td>0.43</td>
<td>295</td>
<td>0.18</td>
<td>14.5</td>
<td>0.0022</td>
</tr>
<tr>
<td>3</td>
<td>Ga</td>
<td>725</td>
<td>475</td>
<td>0.66</td>
<td>385</td>
<td>0.12</td>
<td>12.3</td>
<td>0.0049</td>
</tr>
<tr>
<td>4</td>
<td>Ga,La</td>
<td>560</td>
<td>790</td>
<td>1.41</td>
<td>620</td>
<td>0.30</td>
<td>12.7</td>
<td>0.0092</td>
</tr>
<tr>
<td>5</td>
<td>Ga,La</td>
<td>475</td>
<td>825</td>
<td>1.74</td>
<td>690</td>
<td>0.28</td>
<td>12.5</td>
<td>0.0122</td>
</tr>
</tbody>
</table>

EXCHANGE CONSTANT $A = 3 \times 10^{-7}$ ergs/cm

$g = 2.00$
The source of rf excitation is a HP 8350 Sweeper - Oscillator, which has a frequency range of 10 MHz to 4 GHz and can operate in both cw and sweep modes. The various sweep parameters such as the sweep range, sweep rate, power level etc. are controlled by a HP 9825A computer via the HP-IB bus. The output of the Sweeper is connected to the input of a device called an "rf structure", through coaxial cables. The sample is placed at the center of the rf structure where the magnetic excitation of the sample takes place. Fig. 33 shows the construction of the rf structure. It consists of a coaxial strip-line on the one side and a coplanar slotline on the other, with a central line separating the two sides. It is made by evaporating or painting gold on a glass substrate and etching away the portions shown as dark areas in Fig. 33. Both the stripline and the slotline waveguides are shorted at the ends by grounding. The thin central line etched between the two sides enhances the low frequency isolation between the slotline and the strip-line sides. The rf current distribution in the rf structure is shown schematically in Fig. 34. It is seen that the rf currents on the two sides of the structure are in mutually perpendicular directions, thus producing rf fields and flux linkages which are assymmetric. It is this assymmetry that provides a large isolation of the order of 70 - 90 db between the two sides when the sample is not present or when the frequencies are far from the magnetic resonance frequencies. At resonance, the elliptically polarized rf fields on the one side are rotated by the
Fig. 32. Block diagram of the RF spectrometer used in the experiments.
Fig. 33 A schematic diagram of the rf structure. The strip-line is on the top and the slotline at the bottom. The light areas are gold and the dark areas are etched. (Ref. 17)
Fig. 34. Rf currents distribution in the rf structure. The directions of flow of the net current are mutually perpendicular. (Ref. 17)
large amplitude oscillations, inducing rf fields and hence rf currents on the other side and thus effectively coupling the two sides. At the resonance frequencies, an increase in transmission of the order of 10 db is observed. The asymmetry of the rf structure makes it possible for the strip-line (or the slot line) side to be used interchangeably as the input or the output. Ideally, the rf magnetic fields are perpendicular to the rf structure, but in practice, the rf fields are inhomogeneous and in-plane rf components are also present. Different resonances couple to different components of the rf fields producing varying amounts of transmission. For example, the DVR and $\omega^+$ couple to the perpendicular rf fields and thus produce larger signals, compared with the $\omega^-$ which couples to the weaker in-plane rf magnetic fields.

The output from the rf structure is amplified by passing it through a low noise, wide band rf amplifier (Avantek, 40 db gain, for frequencies greater than 1 GHz and Trontek, 60 db gain, for frequencies less than 1 GHz). The amplified signal is detected by a Wiltron crystal detector and a dc voltage proportional to the power level is produced. This then passes through a phase sensitive detection scheme.

The in-plane dc field is provided by a Varian 4 inch magnet, which can also be made to operate in the "field sweep" mode for cw experiments. Although the magnet could produce fields up to 6.5 KOe,
all experiments were performed in the range 0 - 1 KOe, which was enough
to cause the in-plane saturation in all samples. The field was
measured by a Bell 640 Gaussmeter using an axial Hall probe.

The basis of the phase sensitive detection is the Lock-in
amplifier (PAR model 126). The reference source of the Lock-in
provides a small ac voltage, which is amplified by a power amplifier
(McIntosh M75) and is fed into a pair of modulation coils. The coils
in turn produce a small ac in-plane magnetic field on the order of a
few 0e, superimposed on the larger dc field, \( H_p \), thus modulating
the response at this frequency. The noise spectrum of the signal
determines the choice of modulation frequency.\(^3\)\(^6\) The main source of
noise is from the crystal detector which varies as \( 1/f \) suggesting that
the modulation frequency be kept large. However, the frequency cannot
be increased limitlessly, because of the constraint imposed by the
audiofrequency instrumentation and performance of typical audio
components. The most suitable frequency is found to be in the range,
100 - 1000 Hz.

The impedance of the coils is chosen to match that of the output
impedance of the power amplifier. Capacitors are added in series such
that at the modulation frequency, the capacitive reactance cancels the
inductive reactance of the coils and the circuit is in resonance. The
modulation frequency used in the present study was 400 Hz.
The modulated signal from the detector goes through an ac amplifier in the Lock-in and then a mixer unit which mixes the amplified signal with the reference frequency. This then passes through a differential amplifier and a bandpass filter module of the Lock-in, which only retains the dc voltage, thus eliminating random, thermal and instrumental noise at all other frequencies except at the modulation frequency. The signal is further refined by detecting only the coherent signal that is "in phase" with the reference signal. This kind of detection produces the first derivative of the signal and the scheme is thus also called "derivative detection". A typical signal produced by this method is shown in Fig. 35. In the absence of phase sensitive detection, the resonance signal is superimposed on a nonmagnetic baseline which is characteristic of the rf structure and the cables used. This background subsequently has to be subtracted out to extract the signal and even then, the signals obtained are not sharp enough to yield the desired details. In the derivative detection scheme, since the background is not modulated, the resulting spectrum has only the magnetic response, thus making it easier to identify the signal, besides improving the signal to noise ratio by eliminating the random noise.

The detected signal from the Lock-in goes to a HP model 3437A digital voltmeter, where it is digitized and the numerical data is
Fig. 35. A typical signal obtained in the derivative detection. This signal corresponds to the DWR of sample 2, at $H_p=70$ Oe for the sweep range 400 - 700 MHz.
stored in the computer for analysis. A program was written to integrate the derivative signal numerically.

Experiments were done in both the in-plane field swept (at cw frequencies) and frequency swept (at constant $H_p$) configurations and were repeated over three to five different sweep ranges to ascertain the signal centers. In the frequency swept mode, the experimental errors were further reduced by averaging the signal over a hundred samplings at each frequency-field point. This averaging, as well as the frequency sweeps, plotting and manipulating the data, were controlled by the HP 9829 A computer.

3.3 EXPERIMENTAL OBSERVATIONS

Fig. 36 represents a typical frequency swept data, corresponding to an in-plane field of 60 Oe, for sample 1. Three groups of signals are easily identified. The lowest frequency signal at about 600 MHz is the DWR. The intermediate frequency signal corresponds to the "in phase" $\omega^+$ mode and the broad band of signals centered around 1400 MHz corresponds to the "out of phase" $\omega^-$ mode and the associated magnetostatic modes. Although the signals are labelled DWR, $\omega^-$ and $\omega^+$ for simplicity, it must be remembered that at this field value, they are actually complex, coupled modes of the domain-domain wall resonances.
Fig. 36. A typical frequency swept spectrum for sample 1 showing DWR, $\omega^-$ and $\omega^+$ at $H_p = 60$ Oe.
As the in-plane field, $H_p$, is increased, the DVR signal moves toward higher frequencies, while the $\omega^\pm$ signals move toward lower frequencies. At the point where they would cross each other in the absence of the coupling, the two signals are observed to "repel" each other, resulting in frequency gaps in the frequency-field dispersion relationship. This repulsion is shown in Fig. 37. The resonance frequencies and the associated error bars plotted as functions of $H_p$ for sample 1 is shown in Fig. 38, where the frequency gap can be seen more clearly. Fig. 39 through Fig. 42 show similar experimentally observed relationships for the other samples. In the field swept experiments, the signals were observed to disappear for frequencies lying in the frequency gap.

The main source of error in determining the resonance frequencies was in locating the centers of the signals. This was especially complicated when two signals happened to be close in frequency, i.e., near regions of repulsion, or when there was a rich structure associated with the signal. In some experiments, the input power levels were intentionally kept low, so as to excite and detect only the principal peaks.

A summary of the observed experimental results is given in Table 2. It is seen that the most significant effect is the frequency gap between the DVR and $\omega^\pm$. This gap is seen to vary from about
Fig. 37. The mode repulsion between DWR and $\omega^+$ in sample 1. As $H_p$ is increased, the two signals come toward each other and repel away from each other without crossing.
Fig. 38. Experimentally observed resonance frequencies vs. $H_p$ showing the various resonance modes for sample 1.
Fig. 39. Experimental resonance frequency vs. $H_p$ for sample 2.
Fig. 40. Experimental resonance frequency vs. $H_p$ for sample 3.
Fig. 41. Experimental resonance frequency vs. $H_p$ for sample 4.
Fig. 42. Experimental resonance frequency vs. $H_p$ for sample 5.
70 MHz in sample 1 to about 800 MHz for sample 5. A coupling and consequent repulsion is also observed between the two domain resonance modes themselves and between DVR and $\omega^-$. But these are seen to be a small effects compared to the DVR - $\omega^+$ coupling.

At $H_p$ larger than the in-plane saturation fields, the magnetization of the sample is pulled into the plane of the sample and the $\omega^+$ mode continues as the uniform FMR. The results of the FMR obtained from the rf spectrometer were consistent with those obtained by the K-band microwave spectrometer, although the uncertainties are larger. This was due to the Sweeper generating weak spurious frequencies and thus not being as monochromatic as the klystron.
## TABLE 2

### EXPERIMENTAL OBSERVATIONS

<table>
<thead>
<tr>
<th>Sample</th>
<th>( f^+ ) (MHz)</th>
<th>( f^- ) (MHz)</th>
<th>( f_{DWR} ) (MHz)</th>
<th>( \Delta f(\omega^+ - DWR) ) (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>941</td>
<td>1633</td>
<td>455</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>1128</td>
<td>1941</td>
<td>446</td>
<td>390</td>
</tr>
<tr>
<td>3</td>
<td>1300</td>
<td>1907</td>
<td>301</td>
<td>680</td>
</tr>
<tr>
<td>4</td>
<td>2123</td>
<td>---</td>
<td>220</td>
<td>1160</td>
</tr>
<tr>
<td>5</td>
<td>2220</td>
<td>---</td>
<td>192</td>
<td>1050</td>
</tr>
</tbody>
</table>
CHAPTER 4

DISCUSSION

4.1 DOMAIN RESONANCES

In order for the discussion to be effective, the DR data is analyzed first at regions which are far away from interaction with DWR, such as when \( H = 0 \). The coupling and the DWR are discussed in the next section. In Fig. 43 the absolute values of experimentally observed DR frequencies and those obtained theoretically from section 2.5 are plotted as functions of \( H_p \) for sample 5. The in-plane field is assumed to be parallel to the domain pattern and the DWR is not shown for clarity.

It is seen that there is a qualitative agreement between the theory and the experiment, although the theoretically predicted frequencies are seen to be lower than the experimental values. This difference is a maximum at \( H = 0 \). In order to achieve a better agreement between the theory and the experiment, \( H_{\text{int}} \), defined by Eq. 24, is phenomenologically scaled in terms of a fitting parameter, \( H_1 \), such that

\[
H_{\text{int}} = H_1 - (H_1 - H_{\text{sat}}) \frac{H_p}{H_{\text{sat}}},
\]

(145)

-129-
Fig. 43. The $\omega^+$ and $\omega^-$ modes of sample 5. The theoretical values shown as solid lines are obtained for the magnetic parameters of the film that are obtained by other methods (The DWR mode is not shown for clarity).
where $H_1$ can be interpreted as the corrected internal field that is obtained from the experiment for $H_p=0$. The above equation suggests that $H_{\text{int}}$ decreases linearly from the value $H_1$ at $H_p=0$ to $H_{\text{sat}}$ at $H_p=H_{\text{sat}}$. Fig 44 shows the DR frequencies as a function of $H_p$, where $H_1$ is now incorporated in calculating the frequencies. It can easily be seen that a relationship such as Eq. 149 cannot result from the theory proposed here. It was seen in section 2.4 that as $H_p$ is increased, the equilibrium domain spacing $L$ decreases, thus reducing $N_{zz}$ by virtue of Eq. 23 and from Eq. 24 it can be seen that such a decrease in $N_{zz}$ should result in an increase in $H_{\text{int}}$ as $H_p$ is increased, rather than a decrease as suggested by the scaling equation, Eq. 145. This is resolved by remembering that Eq. 23 was derived without incorporating the cubic anisotropy term in the expression for the potential energy $G$. It is shown\textsuperscript{33} that if the domains and walls are aligned along the [$11\bar{2}$] direction, and if $H_p$ has a component along [$11\bar{2}$], then the equilibrium domain width can actually increase with $H_p$.

The increase is especially significant for large values of $K_1/K_u$, where the anisotropy constant $K_1$ was defined in Section 2.6. Such a behavior could give rise to the observed scaling of $H_{\text{int}}$, although the exact form may not be linear as suggested by Eq. 145. Such changes in $H_{\text{int}}$ can be very sensitive functions of $H_p$ in regions where the domain spacing $L$ is comparable to sample thickness $T$ as $N_{zz}$ changes rapidly under such condition, as can be seen from Fig. 9. The values of $H_1$
Fig. 44. Theory (solid lines) vs. experiment for the $\omega^+$ and $\omega^-$ frequencies as a function of $H_p$ for sample 5. A value of $H_1 = 800$ Oe is assumed for the parameter $H_1$ defined in Eq. 145.
TABLE 3
FITTING PARAMETERS $H_1$ AND $\alpha$ FOR THE SAMPLES

<table>
<thead>
<tr>
<th>Sample</th>
<th>$Q$</th>
<th>$H_1$ (Oe)</th>
<th>$\alpha$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.35</td>
<td>275</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0.43</td>
<td>370</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0.66</td>
<td>440</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>1.41</td>
<td>790</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>1.74</td>
<td>800</td>
<td>10</td>
</tr>
</tbody>
</table>
for different samples are listed in Table 3. Another, perhaps a more plausible origin of \( H_1 \) could be that the domain structure is no longer an ideal parallel stripe pattern but contains some closure domains,\(^5\) where Eq. 24 is no longer applicable as an expression for the internal field. The field computation in such structures is very complex and has not yet been done. A comparison of all the \( H_1 \) values, shown in Table 3 seems to corroborate this hypothesis, where sample 1 with the lowest \( Q \) and thus with the most deviation from parallel stripe domain structure has the maximum corrections to \( H_{\text{int}} \) and thus the largest percentage difference between \( H_1 \) and \( H_{\text{sat}} \), whereas sample 5 has the minimum corrections.

It is also seen from Fig. 43 and Fig. 44 (and all the Figures showing experimental data for the other samples, Fig. 39 through Fig. 42) that experimentally there is a repulsion between the \( \omega^+ \) and \( \omega^- \) modes of the DR where they are theoretically predicted to intersect. This suggests that the coupling terms between the \( \omega^+ \) and \( \omega^- \) modes, namely the \( G_{12} \) elements of the matrix in Eq. 96, are nonzero.

It was shown in the previous chapter that \( G_{12} \) can arise from two possible mechanisms, (1) cubic anisotropy energy and (2) \( H_p \) being inclined to the domain structure at an angle \( \alpha \) instead of being parallel to it.

The contribution to \( G_{12} \) arising from cubic anisotropy energy is given in Eq. 141. When \( H_p \) is applied along \( \bar{[10]} \), i.e. \( \tau = 90^\circ \), the
magnetization in the domains do not necessarily line up at an angle \( \phi_1 = 90^\circ \), but deviate from \( \tau \) by a few degrees,\(^\text{33}\) depending on \( K_1/K_u \) and \( H_p \). Again, \( G_{12} \) terms are large for large \( K_1 \) values. The contribution to \( G_{12} \) due to \( H_p \) applied at an angle \( \alpha \) to the wall plane is given by Eq. 89. The \( G_{12} \) terms arising from these two different sources are plotted as functions of \( H_p \) for sample 5 in Fig. 45. The value \( \alpha = 10^\circ \) is used in Eq. 89 and is shown as a dotted line and the values \( K_1 = 0.1 K_u \) and \( \phi_1 - \tau = 5^\circ \) degrees are used for the \( G_{12} \) term obtained from Eq. 141 and is shown as a solid line. Since the origin of \( G_{12} \) is not clearly established, the term was assumed to arise from Eq. 89 for simplicity and \( \alpha \) is chosen as a fitting parameter to obtain agreement between the theory and the experiment. Fig. 46 shows the DR curves with the best fitting values of \( \alpha \) and \( H_p \). The values of \( \alpha \) used for the other samples are also listed in Table 3. From this table it is difficult to infer any pattern to \( \alpha \) as a function of \( Q \). It can be deduced that if \( \alpha \) is not zero, then the domain walls are not ideal Bloch walls, but have an admixture of a Néel component, since pure Bloch walls line up with \( H_p \). The theory assumes all walls to be of the Bloch type, and, hence, this small Néel component will have to be incorporated into the theory of resonance and the equations will have to be revised. Even in the case of the \( G_{12} \) term resulting from cubic anisotropy energy, it is shown\(^\text{33}\) that the walls have a Néel component. Conversely, if the origin of \( G_{12} \) is well
Fig. 45. The $G_{12}$ coupling terms arising from the cubic anisotropy energy (solid line) and for $H_p$ at an angle $\alpha=10^\circ$ with the domain structure (dotted line), as a function of $H_p$. The values $K_1/K_u = 0.1$ and $\phi_1 - \tau = 5$ degrees were used in the calculation.
Fig. 46. Theory (solid lines) vs. Experiment for the $\omega^+$ and $\omega^-$ frequencies as functions of $H_p$ for sample 5. $H_1 = 800$ Oe and $\alpha = 10^\circ$. 
understood, the fitting parameter that explains the $\omega^+ - \omega^-$ splitting, $\alpha$, can be used as a measure of the Néel component to wall structure. The accurate fitting of $\alpha$ is obscured by the presence of magnetostatic modes associated with $\omega^+$ and $\omega^-$ as they make the location of the exact centers of the signals difficult.

4.2 DOMAIN WALL RESONANCE AND MODE COUPLING

The two parameters discussed above do not seem to affect the characteristics of DVR and the mode coupling very much, probably because the above corrections were significant far from the DVR and hence for weak mode coupling. The complete frequency vs $H_p$ relationship for all the resonances, DR, FMR and DVR, is shown in Figs. 47 through 51 for all the samples. The theoretical curves with the fitting terms are the solid lines. The agreement between the theoretical and experimental DVR frequencies are excellent in samples with higher $Q$ especially at low $H_p$ and for the first time quantitative agreement has been achieved between the theory and the experiment. The reason for this is probably due to more accurate evaluation of the domain wall parameters where high $Q$ approximations were not used. The agreement however is poor in sample 1, the one with the smallest $Q$ value of all the samples studied. The data for sample 1 will be discussed later.
Fig. 47. Theory (solid lines) vs. Experiment for the frequencies of all three modes as a function of $H_p$ for sample 5 ($H_f=800$ Oe and $\alpha=10^\circ$) Q value of the sample is 1.74.
Fig. 48. Theory (solid lines) vs. Experiment showing the frequencies of all three modes as a function of $H_p$ for sample 4 ($H_1=790$ Oe and $\alpha=14^\circ$). Q value of the sample is 1.41.
Fig. 49. Theory (solid lines) vs. Experiment showing the frequencies of all three modes as a function of $H_p$ for sample 3 ($H_1 = 440$ Oe and $\alpha = 7$ degrees). $Q$ value of the sample is $0.66$. 
Fig. 50. Theory (solid lines) vs. Experiment showing the frequencies of all three modes as a function of \( H_p \) for sample 2 (\( H_1 = 370 \) Oe and \( \alpha = 7 \) degrees). Q value of the sample is 0.43.
Fig. 51. Theory (solid lines) vs. Experiment, showing the frequencies of all three modes as a function of $H_p$ for sample 1 ($H_1=275$ Oe and $\alpha=10$ degrees). $Q$ value of the sample is 0.35.
In general, there is excellent agreement between the theory and the experimentally observed mode coupling and frequency gaps in samples 3, 4 and 5. In sample 2 the agreement is only qualitative and in sample 1 it is poor. The samples with higher Q, (samples 3, 4 and 5) despite excellent agreements, do not demonstrate the mode coupling as dramatically as the lower Q samples such as sample 1, although the latter were difficult to deal with theoretically. It is seen that in the low Q samples the $\omega^+ -$ DVR mode repulsion occurs prior to the $\omega^+ - \omega^-$ crossover (Fig. 51) whereas this feature is reversed in the high Q samples. This is reproduced by the theory.

One area where big disagreement between the theory and the experiment occurs is in the description of the high field behavior of $\omega^+$ after the repulsion with DVR. Experimentally $\omega^+$ merges smoothly with the uniform FMR, whereas theory seems to predict a discontinuity between $\omega^+$ and FMR, where $\omega^+$ appears to go to infinity as $H_p \rightarrow H_{sat}$. The corrected theoretical $\omega^+$ frequencies have the term $\sin 2\theta / \sqrt{m}$ in them. As $H_p$ approaches $H_{sat}$, both $m$ and $\sin 2\theta$ go to zero. Although $\sqrt{m}$ may go to zero faster than $\sin 2\theta$ initially, for physical reasons, it will have to go slower than $\sin 2\theta$ as saturation nears, since the coupling should go to zero as domains saturate. This suggests that $\omega^+$ will have a local maximum after being repelled by the DVR and then it should come back and merge with the FMR. Such a behavior is experimentally observed in sample 1 and
to some extent in sample 2 supporting the essential correctness of the above analysis. In the other sample such local maxima are not readily observed experimentally. This mismatch between the theory and the experiment for $\omega^+$ in high $H_p$ is probably due to the inadequacy in the model for wall mass at high fields. It may be that the exchange term in Eq. 121, which was neglected in computing the mass may be important. There could also be other limiting effects near saturation such as the increase in wall width (Fig. 16) which have not been considered in the theory but could be dominant. It can easily be seen that frequency shifts due to mode repulsion should be more significant in the case of DWR, being at lower frequencies, than in the case of $\omega^+$ which is at higher frequency, whereas the present theory exaggerates the $\omega^+$ shifts due to incorrect evaluation of $m$ at high values of $H_p$.

Another feature that is observed experimentally but not accounted for theoretically, is the repulsion between the $\omega^-$ and DWR modes observed experimentally in Fig. 47 and Fig. 48 for samples 4 and 5. This was found to be absent in the other samples, although in samples where it occurred it introduced significant corrections in $\omega^-$. Such interactions will occur via the elements of the type $Q_{25}$ in the matrix of Eq. 96 and is possible with the cubic anisotropy energy term, which has not been explicitly taken into account in the data analysis.
Why such repulsion occurs in only high Q samples is not clear at present.

Of all the samples studied, sample 1 with the smallest Q value was the most difficult to analyze. This sample deviates from the theory significantly, probably because the preconditions for the theory are not completely met. Because of the low value of Q, the domains are probably no longer of parallel stripe pattern and the walls may deviate significantly from the ideal Bloch wall structure. Exactly how such deviations might become important as a function of Q and how to include them in the theory are not understood at present. Also, sample 1 is closer to pure YIG in stoichiometry than the other samples. Consequently, it has a low value of $H_u$ and a comparatively significant cubic anisotropy energy, $(K_1/K_u = 0.25)$ which has to be taken into account. The differences between the theory and the experiment could be due to both of these factors and the importance of each should be investigated further. The data for sample 1 can be fitted to the theory in a phenomenological way by introducing new fitting parameters $H_2$ and $(f_{DWR}^0)$ in addition to $H_1$ and $\alpha$. The new phenomenological parameters $H_2$ and $(f_{DWR}^0)$ can be of help in understanding the role of cubic anisotropy and the deformation of the domain lattice. It was seen that although $(f_{DWR}^0)$ the DWR frequency for $H_p=0$, is predicted to be higher than that observed experimentally, the relative variation of $f_{DWR}$ with $H_p$ is observed to be well
accounted for theoretically. This incorrect estimation of $m$ and $k$ at $H = 0$ could be due to deformed walls and domains. If the domain structure is still assumed to be a periodic stripe lattice, but the walls are not 180° Bloch walls, then the domain wall mass should increase, thus lowering the theoretical value of $(f_0^*)_{DWR}$. But in addition to over-estimating the $(f_0^*)_{DWR}$, the present theory also underestimates the zero field frequency of the $\omega^+$ mode for this sample and this cannot be accounted for by assuming non-Bloch walls alone. For this sample, the domain structure and walls are taken to be almost perpendicular instead of being parallel, to $H$, i.e. $\alpha = 80°$. This probably results from the onset of closure domains and is substantiated by direct observations under the microscope for small applied in-plane fields. $H_2$ is a phenomenological coupling parameter between $\omega^+$ and $DWR$ introduced in place of the theoretically predicted values and is found to be smaller. The results are shown in Fig. 52 with the values of the parameters $H_2$ and $(f_0^*)_{DWR}$.

In order to account for the data completely, a systematic study will have to be done on the onset of closure domains and into the effect of cubic anisotropy energy, on the various quantities such as the domain spacing, domain wall mass, force constant, the coupling constants etc.
Fig. 52. Theory (solid lines) vs. Experiment, showing the frequencies of all three modes as a function of $H_p$ for sample 1. Phenomenological factors $(f_0)^{DWR}$ and $H_2$ are introduced in addition to $H_1$ and $\alpha$. ($H_1=350 \text{ Oe}$, $H_2=300 \text{ Oe}$ ($f_0)^{DWR} = 455 \text{ MHz}$ and $\alpha=80 \text{ degrees}$).
CHAPTER 5

CONCLUSION

The two modes of domain ferromagnetic resonance $\omega^+$ and $\omega^-$ and the domain wall resonance mode were observed in thin Ga substituted YIG films using a transmission rf spectrometer where an rf structure was used to couple the signal to the sample. The experiments were conducted in the frequency range 10 - 2400 MHz for applied in-plane fields ranging from 0 to 1 KOe for five different samples with varying amounts of Ga substitution. For the first time, the DR and DWR signals were observed in the same field-frequency regime and as the field was increased, the $\omega^+$ and DWR modes were found to repel each other, resulting in frequency gaps, suggesting that these two modes may be coupled. A similar mode coupling, although not as pronounced, was observed between the $\omega^+$ and $\omega^-$ modes themselves and in some samples between $\omega^-$ and DWR.

A unified theory of domain phenomena, treating all the different resonances on the same footing, is developed. In the present theory, a detailed account of the demagnetization process is given and exact calculations to determine domain parameters such as the static domain
wall energy, domain wall mass etc. are carried out, to include the
effects of $Q$ of the samples explicitly. The coupling between the
modes is shown to arise as a natural consequence of such a generalized
theory and the magnitude of the coupling constants and the resulting
frequency gaps are computed. The proposed theory is seen to explain
the observed experimental behavior adequately with the introduction of
two fitting parameters $N_1$ and $\alpha$, whose origins could be due to
cubic anisotropy energy or closure domains. Excellent agreement is
obtained between the theory and the experimental results for samples
with relatively high $Q$ and qualitative agreement was achieved for
samples with low $Q$. The theory is then generalized to include some
cubic anisotropy energy effects.

Data for sample 1, which had the lowest $Q$ value of all the
samples studied, proved to be the most difficult to analyze
theoretically. This may be because this sample has the maximum cubic
anisotropy energy and the maximum deviations from ideal parallel stripe
domain structure. A more systematic study of these phenomena needs to
be undertaken and such additional data as the electron micrographs of
the domains and domain walls and domain width as a function of the
in-plane field, as can be observed under a microscope, are needed to
shed more information on the underlying physical processes. For the
present, the data of sample 1 is analyzed using two additional
phenomenological fitting parameters $N_2$ and $(f_0)_{DWR}$. 
Further studies can be carried out to verify the present theory, such as conducting experiments on other magnetic materials and at other temperatures. On the theoretical side, a more detailed investigation of the effect of cubic anisotropy energy and closure domains are in order. The fine structure associated with DR due to magnetostatic modes and the acoustic modes associated with the interacting resonances promise interesting behavior and should be looked into. The intensity and line shape studies, relaxation processes, non-linear phenomena etc. offer yet another interesting direction for future research.
LIST OF REFERENCES

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152
17. A. A. Parker, Dissertation, The Ohio State University (1980) [See other references quoted therein]

35. T.B. Mitchell, Private communication.


APPENDIX A

TO FIND AN EXPRESSION FOR THE ROTATIONAL PERMEABILITY OF PARALLEL STRIPE DOMAIN STRUCTURE
For a parallel stripe domain structure, there are two contributions to permeability, one due to wall motion and the other due to rotation of magnetization. The dispersion of $\mu$ due to wall motion occurs at domain wall resonance and is considered in Chapter 2.5. At low frequencies, however, the contribution to $\mu$, arises from the rotation of $M$.

If $K_{\text{int}}$ is assumed to be the internal energy due to the internal field $H_{\text{int}}$ and if a small field $h$ is applied along the $y$ direction, the torque on the magnetization $M$ can be written in terms of the derivative of $K_{\text{int}}$, i.e.,

$$hM\sin\theta = \frac{3(K_{\text{int}} \sin^2 \theta)}{d\theta}$$  \hspace{1cm} A-1

or

$$hM\sin\theta = K_{\text{int}} \sin 2\theta \ d\theta$$  \hspace{1cm} A-2

or,

$$hM = 2K_{\text{int}} \cos \theta \ d\theta.$$  \hspace{1cm} A-3

The change in the magnetization $M$ is,

$$M = \frac{3}{2} \ M\sin\theta \ d\theta$$  \hspace{1cm} A-4

$$= \ M\cos\theta \ d\theta.$$  \hspace{1cm} A-5
\[ \mu = 1 + 4\pi H_\text{int} \]

and the rotational permeability is

\[ \mu = 1 + 4\pi x = 1 + \frac{4\pi M}{H_\text{int}}. \]
APPENDIX B

CALCULATION OF THE DEMAGNETIZATION ENERGY
DUE TO $M_y$ COMPONENTS IN THE WALL AND THE DOMAINS
The demagnetization energy of a magnetic specimen is analogous to the depolarization energy of dielectric. A detailed account of this is given in Ref. 37.

According to Maxwell's equations,

\[ \vec{v} \cdot \vec{B} = 0 \]  
\[ \vec{v} \cdot \vec{D}_D = -4\pi \vec{V} \cdot \vec{M} \]

where \( \vec{D}_D \) is the demagnetization field. Writing the above equation in component form,

\[ (H_D)_1 = -4\pi M_1 N_{11} + C \]

where \( i \) can be \( x, y \) or \( z \) and \( (H_D)_1 \), \( N_{11} \) and \( M_1 \) denote the respective components of \( \vec{D}_D \), \( \vec{M} \) and the demagnetization tensor \( \vec{N} \), where \( x, y, z \) are chosen to lie along the symmetry axes of demagnetization tensor; \( \vec{N} \). From Ref. 37 it can be seen that

\[ N_{xx} + N_{yy} + N_{zz} = 1. \]

Considering the \( y \) component of Eq. B-3,

\[ (H_D)_y = -4\pi M_y N_{yy} + C \]

\[ B-4 \]
The constant $C$ is determined by the boundary conditions at $y = \pm \infty$, where $(H_{Dy})$ vanishes. Subscript 0 is used to denote variables in regions at $y = \pm \infty$,

$$(H_{Dy}) = 4\pi N_{yy} (M_{y0} - M_y) \quad B-5$$

and the demagnetization energy is,

$$(Q_D)_{y} = -\frac{M_y}{M_{oy}} (H_{Dy}) \delta M_y \quad B-6$$

$$2\pi N_{yy} (M_{oy} - M_y)^2 \quad B-7$$

The above equation can be applied to the domains 1 and 2 where 1 can denote the region of interest and 2 the region at $y = \pm \infty$ i.e.,

$$M_{oy} = M_{1y} = M_{y} \quad B-8$$

$$M_y = M_{2y} = M_{y} \quad B-8$$

and thus $(Q_D)_{y} = 2\pi N_{yy} M^2 (\sin \theta_1 \sin \phi_1 - \sin \theta_2 \sin \phi_2)^2 \quad B-9$

Eq. B-7 can also be applied to evaluate the demagnetization energy of the domain wall. Since the wall width is much smaller than the other dimensions of the wall $T$ and $D$, only $N_{yy}$ is important and is close
to unity, from the discussion in Section 2.3. In this case, the region at $Y = \pm \infty$ is the domain of type 1 i.e.

$$M_y = M \sin \theta \sin \psi,$$

$$M_{oy} = M \sin \theta \sin \phi_1,$$

and the domain wall demagnetization energy is,

$$W_D = 2\pi M^2 (\sin \theta \sin \phi_1 \sin \theta \sin \psi - \sin^2 \theta \sin^2 \psi).$$
APPENDIX C

COMPUTING THE RATIO

\[
\frac{(f_{DWR})^H_p = H_{sat}}{(f_{DWR})^H_p = 0}
\]
From the definition of the frequencies,

\[
\begin{align*}
\frac{(f_{DWR})_{H_p}}{H_{\text{sat}}} &= \frac{\sqrt{(k/m)}_{H_p}}{H_{\text{sat}}} \\
(f_{DWR})_{H_p} &= 0 = \frac{\sqrt{(k/m)}_{H_p}}{H_{\text{sat}}} = 0
\end{align*}
\]  

\[C-1\]

\[
\frac{k_{\text{sat}}}{k_0} = \left( \frac{m_{\text{sat}}}{m_0} \right)^{1/2} \left( \frac{k_{\text{sat}}}{k_0} \right)^{1/2},
\]

\[C-2\]

where the subscripts "sat" and 0 refer to \(H_p\) being equal to \(H_{\text{sat}}\) and zero respectively.

From Eq. 107 for the force constant, it can be seen that the force constant \(k\) is simply proportional to \(\frac{4\pi^2 M^2 \cos^2 \theta}{L}\).

At \(H_p = 0\), \(k\) is proportional to \(\frac{4\pi^2 M^2}{L_0}\) and when \(\theta_1 \approx \frac{\pi}{2} - \alpha_1\),

where \(\alpha_1\) is a small angle, the sample is almost at saturation. Then,

\[
k_{\text{sat}} \approx \frac{4\pi^2 M^2 \cos^2 \theta_1}{L_{\text{sat}}} = \frac{4\pi^2 M^2 \sin^2 \alpha_1}{L_{\text{sat}}} = \frac{4\pi^2 M^2}{L_{\text{sat}}},
\]

\[C-3\]

so,

\[
\frac{k_{\text{sat}}}{k_0} = \frac{\alpha_1^2}{L_{\text{sat}} L_0},
\]

\[C-4\]
Using equations 115 and 118, a formula can be derived for \( \frac{m}{m_0} \)

\[
\frac{m}{m_0} = \frac{\pi/2 \sin \theta [\sin^2 \theta - \sin^2 \theta_1 - 2\mu (\sin \theta - \sin \theta_1)]^{1/2}}{(Q\mu + \sin \theta)}
\]

where the terms \( Q, \mu \) etc. have been defined earlier.

Introducing \( a = \pi/2 - \theta \approx 0 \), the above integral can be rewritten as

\[
\frac{m_{sat}}{m_0} = \int_{0}^{\alpha_1} \frac{(1 - \alpha^2/2)(1 - \alpha^2/2) - (1 - \alpha_1^2/2) - 2\mu[\frac{\alpha^2}{2} - (1 - \alpha_1^2/2)]^{1/2}}{(Q\mu + 1 - \alpha^2/2)} \, d\alpha
\]

where \( \sin \theta = \cos \alpha = 1 - \frac{\alpha^2}{2} \) and \( \sin \theta_1 = 1 - \frac{\alpha_1^2}{2} \).

\[
\frac{m_{sat}}{m_0} = \int_{0}^{\alpha_1} \frac{(1 - \alpha^2/2)(\alpha_1^2 - \alpha^2)^{1/2} (1 - \mu)^{1/2}}{(Q\mu + 1 - \alpha^2/2)} \, d\alpha
\]

If \( \alpha^2/2 \) is small compared to 1, the above expression becomes,

\[
\frac{m_{sat}}{m_0} = \frac{(1 - \mu)^{1/2}}{(Q\mu + 1)} \int_{0}^{\alpha_1} (\alpha_1^2 - \alpha^2)^{1/2} \, d\alpha = \frac{(1 - \mu)^{1/2}}{4} \frac{\pi \alpha_1^2}{(Q\mu + 1)}.
\]
Therefore, using Eq. C-7 and C-4, the limit of \( \frac{(f_{DWR})_{sat}}{(f_{DWR})_o} \) is obtained

\[
\frac{(f_{DWR})_{sat}}{(f_{DWR})_o} = \frac{L_o}{L_{sat}} \left( \frac{4(Q_u + 1)}{\pi(1 - \mu)^{1/2}} \right)^{1/2}
\]

This expression is finite since \( L_{sat} \) is not zero at saturation as shown in Section 2.4 and \( 1 - \mu = \frac{H_u - H_p}{H_u} \) approaches \( \frac{4MN_{zz}}{H_u} \) as \( H_p \) approaches \( H_{sat} = H_u - 4MN_{zz} \). By virtue of \( L_{sat} \) being finite, \( N_{zz} \) too is finite and hence the denominator and thus the limit in Eq. C-9 are finite.
APPENDIX D

PROGRAM TO CALCULATE THE FREQUENCIES OF COUPLED OSCILLATIONS
0: "6 X 6 determinant for domain-domain wall system"
1: dim F[4,60], M[62], K[60], A[60], B[60], C[60], D[60]
2: dim U[60], P[6,20], Q[6]
3: ent "sample number", r20; r20+Q[2]
4: ent "Are parameters in file? yes=1 no=0", r72
5: if r72=1; gto 9
6: if r72=1; gto 9
7: ent "File for mass", r21
8: ent "File for force constant", r22
9: ent "File for domain width", r23
10: 1df r21, N[*]
11: 1df r22, K[*]
12: 1df r23, D[*]
13: if r72=1; gto 19
14: ent "4πN", N; N/4π-H
15: ent "Hs", U; U/4πH-Q[3]
16: ent "Nbx", X; 4πX-X
17: ent "Alpha (degree)", A; A-Q[4]
18: ent "Heat. (Ds)", S; "H1 (Ds)", r30
19: 1.76e7-Q; S-Q[1]; r30-Q[5]
20: ent "coupling const. in data? yes=1 no=0", r41
21: if r41=0; gto +2
22: ent "Upper coupling G15", C; "Lower coupling G25", D
23: fixd 4i degree 1
24: UM/2-r24; f(3e-7/r24)-r25
25: 1/2e00-r25-K
26: expr KM-H
27: S/60-L
28: f(K[1]/M[1]) /2e-F[1,1]; F[1,1] /1e6-F[1,1]
29: Qf (-30 (-30+NX sin(A) f2+ (4πN-NX-(U-r30)) cos(A)f2) /2e-F[3,1]
30: Qf (-30 (-30+NX cos(A) f2+ (4πN-NX-(U-r30)) sin(A)f2) /2e-F[2,1]
31: F(2, 1)/1e6+F(2, 1); F(3, 1)/1e6+F(3, 1)
32: if "Linear DWR? Yes=1 no=0", r97
33: if r97=1, "Zero frequency", r98, "Expt. slope", r99
34: if r97=1, r98+1
35: for I=2 to 60
36: (I-1)L-V
37: r30-(r30-S) V/S-N
38: aen(V/N)-T
39: (U-N)/M-Z
40: 4n-(O+Z)-Y
41: (Y-X) Msin(2T) sin(2A)-B; B/16-B
42: .5Mcos(T) t2 (U-ZM+XNcos(A) t2 + YNsin(A) t2) + .5Msin(T) t2 (NX+HY) -E
43: .5Mcos(T) t2 (U-ZM)+F
44: .5Msin(T)+G
45: .5Msin(T)+.5Msin(T) t2 (NXsin(A) t2 + MYcos(A) t2) -H
46: 4MV/D[I]QW[I]-W
47: if r97=1, M[I]= (r98+r99) t2*1e12-K[I]
48: K[I]/D[I]-R
49: if r41=1, goto +3
50: =M(.5Msin(2T))/D[I]-Q; Y=Q[5]
51: Q-P
52: if r41=1, Ncosin(2T)/20[I]=O; 2Ncosin(2T)/20[I]=P
53: O=r24; 1+K
54: for J=0 to 2400 by 10
55: 'Root' (J)-r3
56: if J=0, goto +2
57: if J3=r4<0, geb "Findroot"
58: r3=r4
59: dep V, J
60: next J
62: next I
63: ent "Upper level for uniform FMR", r57; r57~Q[0]
64: (r57-S)/20=r58
65: for I=1 to 20
66: S+(I-1)r58=V
67: Q=((V-(V-(U-4nM)))/(2pi*1e6))~F[4, I]
68: next I
69: for I=1 to 60
71: next I
72: ent "File to store frequencies", r27
73: ref r27, F[*]
74: ent "File for A", r27; ref r27, A[*]
75: ent "File for B", r27; ref r27, B[*]
76: ent "File for C", r27; ref r27, C[*]
77: ent "File for U", r27; ref r27, U[*]
78: ref 155, Q[*]
79: stp
80: "Findroot": r3=r9
81: r4=r8
82: 0=r11; J=r6; J=r10=r5
83: (r6+r5)/2=r7
84: 'Root' (-7)=r10
85: if r8<10<0, gto +3
86: r7=r5; r10=r8
87: gto +2
88: r10=r9; r7=r6
89: r11=r11; if r11<4, gto -6
90: geb "Entry"