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COMBINED MODE I - MODE III FRACTURE TOUGHNESS OF A HIGH-STRENGTH LOW-ALLOY STEEL

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of the Ohio State University

By
James Gregory Schroth, B.S.E., M.S.

* * * *
The Ohio State University
1985

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1985
Dedicated to

Bruce M. Brothers
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INTRODUCTION

Limited experimental data are available describing fracture under combined mode I (tension) and mode III (transverse shear) load components. This is unfortunate since it is well known that initially-flat cracks in tough materials, particularly in thin sections, often reorient to oblique planes during growth and then continue to grow under combined mode I - mode III loading conditions.

The purpose of this investigation was to characterize the mixed mode I - mode III fracture resistance of NiCop, a copper-containing steel that is similar in composition to ASTM A710 grade A. Material was provided by the Armco Steel Corporation in the form of 12.7mm (1/2 inch) plate. The steel was subjected to two heat treatments which produce similar grain sizes and carbide distributions in the final microstructures, but different final copper precipitate morphologies.

Despite the fact that combined mode fracture is associated with high toughness, most of the prior experimental research has been confined to materials with limited ductility. Several investigators have attempted to characterize fracture toughness with critical values of
the stress intensity factors, $K_{IC}$ and $K_{IIIC}$, at crack initiation. Fracture initiation loci consisting of functional relations between $K_I$ and $K_{III}$ at initiation have been suggested.

NiCop exhibits elastic-plastic fracture behavior in the section thicknesses used in this program. This invalidates the use of linear elastic fracture mechanics concepts. In this work, special formulations of the $J$-integral, denoted $J_1$ and $J_{iii}$, were used to characterize crack initiation. The tearing modulus, as introduced by Paris, was used to characterize $J$-resistance behavior during stable crack extension.
CHAPTER I

LITERATURE REVIEW

Parameters Characterizing Fracture Resistance

The discipline of fracture mechanics has developed largely because traditional design considerations which prevent failure of engineering structures by yielding, buckling or excessively large elastic deformations may not preclude failures caused by cracks propagating from pre-existing flaws. The major achievement of the fracture mechanics approach to materials characterization has been the identification of well defined properties which describe the conditions leading to crack extension. In order of historical development, the strain energy release rate \( G \), the stress intensity factor \( K \), and the \( J \)-integral have all been shown to characterize crack tip conditions in particular situations. Critical values of these parameters correspond to the attainment of a local stress/deformation field in the vicinity of the crack tip for which stable or unstable crack extension is imminent.

In the most general case, a cracked body can be loaded in three different ways. These are commonly designated the opening mode or mode I, the sliding mode or mode II, and
the tearing mode (antiplane strain mode) or mode III, and are illustrated in Figure 1. In theory, parameters could be defined which characterize crack extension for any of these three loading modes, individually as well as for cases of any simultaneous combination of the three. In this investigation, special formulations of the $J$-integral, designated $J_I$ and $J_{III}$, have been used to characterize crack extension for combined mode I - mode III loading conditions. The use of these resolved $J$ components is best understood if the accepted parameter from which they are derived, the mode I $J$-integral, and its limitations are discussed. To this end a brief review of mode I fracture mechanics is presented.

**Mode I Fracture Mechanics**

**Strain Energy Release Rate**

Cracks present within a loaded body act as stress concentrators and may produce local stresses sufficient for material separation at nominal stress levels well below those producing unacceptable deflection or yielding. For the infinite elastic plate of Figure 2, loaded in uniaxial tension and containing an elliptical hole, Inglis (1) showed that the maximum stress is produced at the ends of the major axis of the ellipse:

$$\sigma_{22} = \sigma(1 + 2 \sqrt{a/\rho})$$

[1]
Figure 1. The three modes of crack loading.

Figure 2. An elliptical hole through a plate loaded remotely in tension -- Inglis' stress concentration solution.
where $\sigma$ is the nominal applied stress at infinity, $a$ is the semi-major axis of the elliptical hole, and $\rho$ is the radius of curvature at the tip of the ellipse. For a mathematically sharp crack ($\rho = 0$) this relation predicts an infinite stress at the crack tip even for a vanishingly small applied stress, while an atomically sharp crack of major axis equal to 1 um produces a local stress of the order $100\sigma$. The observed fracture resistance of real materials containing macroscopic flaws proves that the local stress elevations predicted by stress concentration considerations alone are not a sufficient criterion for fracture.

The early work of Griffith on brittle fracture in glass (2) provides insight as to why elevated local stresses alone cannot produce crack extension. In a linear elastic body loaded under fixed-grip conditions, potential energy is stored in the form of elastic strain energy $U$. For a crack extension increment $da$, a portion of this energy is released since stresses along the new crack surfaces are relaxed. At the same time energy $W$ is required to break atomic bonds and form new free surfaces for crack growth to occur. For this situation the Griffith criterion

$$- \frac{dU}{da} \geq \frac{dW}{da}$$

[2]

must be satisfied for spontaneous crack extension. This
condition insures that the elastic strain energy released during crack extension, \( dU \), is at least sufficient to supply the energy \( dW \) required for formation of the new crack increment \( da \). The quantity \( dU/da \) is denoted by \( G \) and given the name "strain energy release rate" or "crack driving force." The energy input rate \( dW/da \) required for crack propagation is designated \( R \) and often assumed constant. In terms of this nomenclature, fracture is possible when

\[
G \geq R \quad [3]
\]

and the critical value of \( G \) producing crack extension is designated \( G_c \) or \( G_{IC} \) for mode I plane strain conditions.

For perfectly brittle materials \( R \) should be the surface energy of the newly formed crack increments. However, in practice \( R \) is always greater than the surface energy. For ductile materials such as metals, large stresses at the crack tip produce yielding and the formation of a crack tip plastic zone. The energy dissipated in extending this yielded zone during crack extension is the predominant component of \( R \) for metallic alloys. (3)

Since \( R \) is assumed to be a material-dependent constant, \( G_{IC} \) is a material property and was the first single-parameter measure of a material's fracture resistance. It is notable that the Griffith energy
criterion for fracture applies equally well to crack extension under fixed load conditions (4) if the quantity $-\frac{dU}{da}$ in Equation [2] is replaced by $(-dU + dX)/da$ where $X$ is the work done on the body by externally applied loads.

Stress Intensity Factor

Irwin (5) led the development of linear elastic fracture mechanics (LEFM) by showing that the global strain energy release rate parameter $G$ is directly related to conditions at the crack tip in a linear elastic body. In particular, the near-tip stress field can be represented by infinite series of the form (6):

$$\sigma_{ij} = \frac{K_I f_{ij}(\theta)}{\sqrt{2\pi r}} + \text{additional terms} \quad [4]$$

where $K_I$ is the mode I stress intensity factor. Similar series containing the characteristic $1/\sqrt{r}$ stress singularity result for mode II and mode III loadings.

Over some region around the crack tip, the singular term containing $K$ dominates and the stress field is adequately described using only the first term of the series. The stress intensity factor $K$ therefore characterizes the amplitude of the stress and strain fields at the crack tip. This near-tip deformation field is of the same form in all linear elastic bodies with the particular geometry and loading pattern entering the
expression only through the stress intensity factor $K$.

The relevance of $K$ in characterizing fracture behavior in real materials relies on a boundary layer approach. The inability of real materials to support infinite stresses means that the elastic solutions for stress and strain are not applicable very near the crack tip. There is always some finite plastic zone in which yielding has occurred. However, if this area is very small relative to geometric dimensions it is embedded within an elastic region which is dominated by the elastic singularity, that is, the first term of Equation [4] containing $K$.

Therefore, the states of stress and strain within the plastically deformed region are controlled by the stress intensity factor with the critical condition at the onset of crack extension being denoted $K_c$.

Under mode I loading conditions the crack tip stress field depends strongly on through-thickness constraint. Thin specimens develop plane stress ($\sigma_3 = 0$) conditions with a relatively large plastic zone size while thick components develop a predominately plane strain field $\sigma_3 = \nu(\sigma_1 + \sigma_2)$ with a smaller yielded zone. For both cases, plastic zone size is dependent on $\sigma_y$, being proportional to $K_I^2/\sigma_y^2$. The fundamentally different stress fields for these two extreme cases produce different planes of maximum shear-stress as shown in Figure 3. If voids initiate preferentially on these shear planes, plane
Figure 3. Planes of maximum shear stress in mode I elastic plane stress and plane strain loading.
stress conditions may lead to slant fracture. The critical mode I stress intensity factor $K_c$ for fracture initiation is consequently thickness dependent for the transition in constraint between plane stress and plane strain conditions.

Experimental experience shows that if specimen thickness $B > 2.5 \frac{K_c^2}{\sigma_y^2}$, a thickness-independent plane strain value of $K_{IC}$ is determined. This value is a lower bound for mode I fracture toughness and may be regarded as a material property. This condition is coupled with the restriction $a > 2.5 \frac{K_c^2}{\sigma_y^2}$ which ensures $K$-dominance in the ASTM standard procedure for plain strain fracture toughness testing, E-399. (8)

The J-Integral

Although the stress intensity factor has enjoyed great success in characterizing fracture under predominately LEFM conditions, the restriction to situations of very limited yielding make it of little use in characterizing the fracture behavior of tough alloys which fracture only after extensive yielding. This class of materials includes such important alloys as linepipe, construction, and nuclear-pressure-vessel steels.

The extension of fracture mechanics concepts to situations where large-scale yielding precedes crack extension was aided greatly by the introduction of the
J-integral by Rice. (9) He considered a homogeneous cracked body subjected to a two-dimensional deformation field. Rice showed that for an isotropic, linear or non-linear elastic material for which the strain energy density can be defined as

\[ W = \int_0^\varepsilon \sigma_{ij} \varepsilon_{ij} \]  

[5]

a special integral \( J \) is defined by

\[ J = \int \left( W d\gamma - \bar{T} \cdot \frac{\partial \bar{U}}{\partial x_1} ds \right). \]  

[6]

This integral is path independent for any contour around a crack tip as in Figure 4, so long as there are no singularities present between alternate paths. Rice has also pointed out that the J-integral is a special case of Eshelby's (10) energy-momentum tensor.

A second interpretation of \( J \) is that it may be defined as the energy release rate per increment of crack advance (in its original plane) of an elastic body held at fixed total displacement or at constant applied load (11)

\[ J = - \frac{d(PE)}{da}. \]  

[7]

This definition is illustrated in Figure 5 where the load-displacement curves of two non-linear elastic bodies containing infinitesimally different crack lengths are plotted together on the same axes. The shaded area between
Figure 4. Two-dimensional geometry for definition of $J$ in terms of a contour integral.
Figure 5. Definition of J for a non-linear elastic body in terms of load-displacement behavior.
the curves is equal to the energy released during crack
extension in the elastic body from length a to length a+da.

Rice has proven the equality between J and this shaded
area, normalized by crack front width (11), and therefore
additional definitions of J for a cracked body are

\[
J = - \int_{0}^{\delta} \frac{dP}{da} \, d\delta = \int_{0}^{P} \frac{d\delta}{dp} \, dp
\]  

[8]

as is apparent from Figure 5.

This energy release rate definition is identical to
that of G for linear elastic materials. J analysis is
therefore a generalized treatment extended to non-linear
elastic bodies which can be directly related to the LEFM
parameters \( K_I \) and \( G_I \) for linear elastic situations

\[
J_I = G_I = \frac{K_I^2}{E'}
\]  

[9]

where \( E' = E \) for plane stress and \( E' = E/(1-\nu^2) \) for plane
strain.

The above discussion has centered on the applicability
of the J-integral to elastic conditions. The more
important utility of J in treating elastic-plastic fracture
problems is realized by modeling the elastic-plastic
behavior of real materials by non-linear analogs. This is
equivalent to imposing a deformation theory of plasticity
and is allowable as long as proportional loading is
maintained in plastically deformed regions.
The line integral definition of $J$ allows any integration path starting on the lower surface of the crack and terminating on the upper surface. For a smooth ended notch this path may be taken directly along the tip (9) where the tractions disappear so that $J$ simplifies to

$$J = \int_{\Gamma_{\text{tip}}} W \, dy.$$  \[10\]

$J$ is thus an averaged measure of strain at the notch tip although its definition allows it to be evaluated at paths any desired distance from the large strain, near-tip region.

This procedure of shrinking the integration path for evaluation of the $J$-integral is not applicable to a mathematically sharp crack. Hutchinson (12) and Rice and Rosengren (13), HRR, have related $J$ to the local crack tip deformation field by assuming a deformation theory of plasticity applies. If a material's stress-strain behavior can be described by the relation

$$\frac{\varepsilon}{\varepsilon_0} \propto \left(\frac{\sigma}{\sigma_0}\right)^n$$  \[11\]

the near-tip asymptotic stress and strain fields are of the HRR forms

$$\sigma_{ij} \propto \left(\frac{J}{r}\right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta, n)$$  \[12A\]

$$\varepsilon_{ij} \propto \left(\frac{J}{r}\right)^{n/(n+1)} \tilde{\varepsilon}_{ij}(\theta, n).$$  \[12B\]
J is therefore the amplitude of the singular fields in the vicinity of the crack tip and may be considered as a parameter characterizing the crack tip region in a manner analogous to K.

Again, a boundary layer approach in viewing the crack tip region is beneficial, with three regions defineable as in Figure 6. In the outermost region 3, linear elastic conditions prevail. Region 1, just ahead of the crack tip, is an intensely strained volume containing holes, microcracks, and non-proportionally loaded material. It is in region 2 that the J-integral value is important. This area contains the elastic-plastic stress field which is dominated over some portion by the HRR singular field which has an amplitude dependent on J. If region 1, also called the fracture process zone, is completely embedded in a region with a deformation field which is closely approximated by the HRR singular expressions, the J-integral, which controls the amplitude of this field, may be presumed to be the parameter controlling fracture initiation.

The appropriateness of describing near-tip conditions with the single parameter J is assured when the process zone size is very small compared to the J-dominated region. This is completely analogous to limiting linear elastic descriptions of material behavior to situations where the plastic zone is much smaller than the K-dominated near-tip
Figure 6. Boundary layer representation of crack tip stress and strain fields for an elastic-plastic material.
elastic field.

Experimental Determination of J

Experimental methods for determining the J-integral as well as limits to its applicability have evolved greatly since its introduction and have been largely documented in the United States in conference proceedings and technical publications of the American Society for Testing and Materials. Landes and Begley (15) first evaluated J experimentally for real alloys and demonstrated its applicability as a failure criterion. They invoked the energy release rate interpretation of J:

$$J = \frac{1}{B} \frac{\partial U}{\partial a}$$  \[13\]

By measuring the energy input (area of load-load point displacement curve) while loading specimens of different initial crack lengths to common displacement values, slopes of the resulting $U$ versus $a$ relationship for a given displacement $\delta$ yield $J=f(\delta)$. The critical value of $J$, $J_C$ is the value at crack initiation. Critical $J_{IC}$ values for A533B and a Ni-Cr-Mo-V rotor steel correlated well with valid $K_{IC}$ tests on much larger specimens if the relation $J_{IC}=K_{IC}^2(1-\nu^2)/E$ was used for purposes of comparison.

McClintock's (16) observation that near crack tip slip line fields in perfectly plastic materials are not unique
for all geometries (Figure 7) suggests that $J$ may be of limited value in characterizing crack tip stress fields under fully plastic conditions. Landes and Begley (17) determined $J$ values for two geometries exhibiting different slip line fields at gross yield and found critical values to be identical for a given material. McMeeking (18) and others have used finite element calculations to show that for fully plastic conditions the near-tip deformation fields are always characterized by $J$ over some region if strain hardening exists as predicted by the HRR solution. Minimum size requirements of specimens which insure $J$-dominance around a fracture process zone comparable in size to the crack tip opening displacement may, however, differ greatly as a function of geometry. (19)

The compliance method of Landes and Begley has been replaced by more efficient techniques of determining the $J$-integral. Rice et al (20) showed that $J$ can be evaluated from the load displacement record of a deeply cracked bend-type specimen by

$$ J = 2 \int_0^{\delta_{\text{crack}}} \frac{P \, d\delta_{\text{crack}}}{b} $$

where $b$ is the remaining ligament, $P$ the load, and $\delta_{\text{crack}}$ the displacement due to the presence of the crack if the total displacement is $\delta = \delta_{\text{crack}} + \delta_{\text{no crack}}$. For compact
Figure 7. Perfectly plastic slip line fields for several loading geometries of cracked bodies. (17)
tension specimen geometries \( \delta_{\text{no crack}} \) is negligible and a useful approximation is

\[
J = 2 \int_{\delta=0}^{\delta} P \, d\delta . \tag{15}
\]

For compact tension geometries with \( a/w \) ratios less than about 0.9, a tensile load component in the uncracked ligament reduces the validity of Rice's assumption of pure bending. Merkle and Corten (23) used a limit load analysis to correct for this tensile load and suggest the relation

\[
J = \frac{AU}{Bb} \tag{16}
\]

where \( U \) is the area under the load-displacement curve and \( \lambda=g(a/w) \). Hickerson (24) determined \( \lambda \) using the exact compliance method of \( J \) calculation for seven alloys. Values of \( \lambda \) fell between the pure bending approximation of Rice (\( \lambda=2 \)) and the limit load analysis of Merkle and Corten. The pure bending approximation was excellent for \( a/w \geq 0.75 \).

Equation [15] was used by Landes and Begley (21) in their multiple specimen technique for \( J_{IC} \) determination. In this method, several identical deep-notched specimens are loaded to differing degrees, their cracks having undergone varying amounts of extension. \( J \) is calculated according to Equation [15] for each specimen and plotted versus the specimen's final measured crack length on a
Figure 8. Schematic J-resistance curve for a tough material loaded in mode I.

Figure 9. Mode I crack blunting prior to real material separation.
resistance curve as in Figure 8.

In ductile, tough materials crack extension as a result of blunting precedes crack growth by actual material separation (Figure 9). If the blunted shape can be approximated by a half circle, crack extension during the blunting process will be defined by (21)

\[ \Delta a = \frac{1}{2} \cos \frac{C}{J} = \frac{1}{2} \frac{J}{\sigma_f}. \]  

[17]

This blunting line of \( J=2\sigma_f \Delta a \) is constructed as a straight line intersecting the origin on the \( J \)-resistance curve and the initiation of real crack extension is expected to occur at the intersection of the blunting line and the experimental \( J \)-resistance curve.

This procedure has been adopted with minor alterations by the American Society for Testing and Materials in their standard recommended practice E-813. (22)

Methods of determining the \( J \)-resistance curve from a single specimen have also been developed. Most use the Rice approximation for calculation of \( J \) from load-displacement areas, but additionally provide means of monitoring crack length to produce continuous \( J \) versus \( \Delta a \) data. Experimental techniques for monitoring crack extension include the elastic compliance method, the electric potential drop method, the application of ultrasonic transducers, and measurement of specimen resonant frequency. (25) The elastic compliance method of
Clarke et al (26) has found the greatest acceptance and is suggested in the current ASTM recommended practice for single specimen $J_{IC}$ determination. (22) As an alternative to monitoring crack length, if key curves are known relating $P$, $\delta$, and $a$ for a particular material and specimen geometry, $J$ can be evaluated directly from experimental load-displacement results. (27)

In summary, extensive experimental experience has shown that the $J$-integral values $J_{IC}$ measured in subsized specimens correspond accurately with valid $K_{IC}$ values determined in much larger geometries (28), if the size requirements

$$B, b, a \geq 25 \frac{J_{IC}}{\sigma_f}$$

are met in bend specimens to insure $J$ dominance and plane strain constraint. The $J$-integral is equally valuable alone as a field parameter useful in characterizing fracture under large scale yielding conditions. (29,30)

**Tearing Modulus**

$J$-resistance curves evaluated for common engineering materials often rise steeply from the point of crack initiation, suggesting that design on the basis of $J_{IC}$ alone may be overly conservative for some applications. Paris et al (31) suggested that initial stable crack growth
can be characterized by the slope of the $J_R$ curve. The tearing modulus

$$T = \frac{\Delta J}{\Delta a} \frac{E}{\sigma_0^2}$$  \[19\]

was introduced as a dimensionless, temperature independent material property describing R-curve behavior.

Of course from its strict definition for non-linear elastic material, the $J$-integral is only applicable to elastic-plastic situations when proportional loading exists in plastic regions. The process of crack propagation necessarily coincides with unloading of material as the crack tip passes, forming a non-proportionally loaded "wake." Hutchinson and Paris (32) stated that the conditions for $J$-controlled crack extension are

Condition I  $\Delta a \ll R$  \[20A\]
Condition II  $D \ll R$  \[20B\]

where $D = J_{IC}/(\Delta J/\Delta a)_C$ and $R$ is a characteristic dimension over which the deformation field is dominated by $J$.
Condition I ensures that the elastically unloaded wake is embedded in a region which is describable in terms of $J$. Condition II results from consideration of the strain increments that occur with simultaneous increases in $J$ and $a$, and ensures that the increase of $J$ with $\Delta a$ is sufficiently great that predominantly proportional loading
takes place within the dominant singularity region. Equation [20B] can be restated in terms of the dimensionless parameter $W$

$$W = \frac{b \, dJ}{J \, da} \gg 1$$

where $b$ is the remaining ligament length or characteristic distance from the crack tip to the nearest boundary or loading point. The condition

$$r \gg D$$

also sets the limit on the minimum distance from the crack tip to possible integration paths for which the contour integral definition of $J$ is essentially path independent for the incremental plasticity theory. (32)

If the conditions of [20A] and [21] are met, resistance to crack extension under elastic-plastic conditions is describable by $T_{\text{mat}}$, considered a material property. Instability is predicted if $T_{\text{applied}}$ exceeds $T_{\text{mat}}$, where $T_{\text{applied}}$ is a function of the compliance of the structure containing the crack, or in the case of fracture toughness testing, the combined compliance of the specimen and loading machine. This treatment is completely analogous to LEFM $R$-curve analysis. Experiments using variable compliance elements within the loading system have verified the relation $T_{\text{mat}} = T_{\text{applied}}$ at instability (33),
but several questions persist concerning the use of $T_{\text{mat}}$ as a material property.

Begley and Landes found that $J_R$ curves were geometry dependent with the measured $T$ values larger for center-cracked panels than for compact tension specimens. (34) Further, under fixed load there is more inherent stability in bend configurations than in tensile geometries for a shrinking ligament at limit load. Also, the presence of side grooves may change $T$ for specimens tested in any geometry even for identical thickness for specimens smaller than those producing fully plain strain conditions. (35,36)

Some of these discrepancies are eliminated by imposing size limitations on specimens. For bend-type geometries, experience has shown that $J$-controlled crack extension may be limited to about 6% of the remaining ligament length and values of $W > 10$ (37) although some investigators find consistent behavior for values of $W$ down to ~ 1 - 2. (38) These conditions may be much more restrictive for purely tensile geometries just as for $J_{IC}$ determination.

The introduction of the tearing modulus in characterizing resistance to unstable crack extension has led to several refinements in $J$-calculation methods in the presence of crack growth. The load-displacement curve for a crack extending from $a_0$ to $a_1$ lies above the load-displacement curve for the same geometry containing an initial crack of length $a_1$ with no growth (Figure 10).
Figure 10. Actual and postulated load-displacement curves for calculating $J$-integral values in the presence of crack growth.
Garwood et al (39) showed that if the area between those curves is the energy consumed during crack extension, $J_{\text{ave}}B\Delta a$, for each such growth increment

$$ J_n = J_{n-1} \frac{W-a_n}{W-a_{n-1}} + \frac{2U}{B(W-a_{n-1})} \tag{23} $$

for a bend specimen and $U$ as defined in the figure. The same result is obtained if $J(a, \theta_C)$ is treated as a state function, in which case analysis of a deeply cracked specimen subjected to pure bending yields (32)

$$ J = 2\int_0^{\theta_C} \frac{M}{b} \, d\theta_C - \int_a^b J \, da. \tag{24} $$

This expression can be differentiated and reintegrated (40) to give the more compact form

$$ J = 2b_i \int_0^{\theta_C} \frac{M}{b^2} \, d\theta_C \tag{25} $$

or for compact specimen geometries of thickness $B$

$$ J = \frac{2b_i}{B} \int_0^\delta \frac{P}{b^2} \, d\delta. \tag{26} $$

Additional Crack Growth Parameters

Substantial recent work has been done to characterize crack extension beyond the range of $J$-controlled growth. Consideration of stable crack growth in elastic-perfectly
plastic materials has led to the conclusion that the crack tip strain field for a moving crack is dominated by a logarithmic singularity that is weaker than the $1/r$ singularity for the stationary case. (41) Thus, an increase in $J$ is necessary in the transition from a stationary to a moving crack if the maintainance of a critical strain over a characteristic distance is required for crack extension.

Numerical calculations coupled with experiments have shown that in some materials crack extension occurs under conditions of a constant crack tip opening angle (CTOA). (42,43) The CTOA is expected to reflect local crack tip conditions and a constant value during crack growth supports the contention that $dJ/da > 0$ does not reflect increases in local material toughness at the crack tip. Instead, screening of the tip by the developing plastic zone and stress relaxation in the unloaded wake diminish the energy input to the near tip region that is necessary for material separation. (44) The possibility of coupling the $J_c$ criterion for crack initiation with the constant CTOA criterion for extensive stable growth is an attractive possibility for applications beyond the limitations of the tearing modulus. (96)

For the conditions [20A] and [20B] however, the tearing modulus concept remains an analytically valid approach and though limited to small crack growth
increments, is a valuable means for characterizing the resistance to stable crack extension.

**Ductile Fracture in Metals**

The influence of microstructure on ductile fracture behavior can only be understood once the operative micromechanisms of crack extension are identified. For ductile fracture in metallic alloys, crack extension results from sequential processes that may be broadly described as: (1) void initiation, (2) void growth, and (3) void coalescence and linkage with the crack front. Microstructural constituents increase resistance to fracture directly in proportion to the extent that they inhibit these three processes.

Void Initiation

Voids typically initiate at second phase particles contained in the strain field ahead of the crack tip. (45) Void initiation sites have been observed directly in interrupted fracture test specimens as well as through post-mortem fractographic investigations. In the latter case, surface dimple spacings often correspond to bulk particle spacings determined metallographically, and the void-initiating particles are commonly visible at the dimple bottoms. The suppression of void nucleation in highly strained, high purity materials provides further
evidence of the role of second phase particles in the fracture process. Uniaxial tensile testing of such materials may produce reductions in area at fracture approaching 100%. (46)

Broek (47) observed void nucleation at precipitate particles in a variety of aluminum alloys and concluded that exceptionally strong matrix-particle bonding prevents surface decohesion until substantial strains are achieved in the fracture process zone. For systems exhibiting such strong bonding (which would include carbide-containing steels), the stress concentrations produced during plastic deformation by dislocations piled up against uncuttable particles are necessary for interface decohesion. Broek suggested that once decohesion occurs, several pileups intersecting the void nucleus empty into the newly formed cavity, producing spontaneous growth into a void.

A second model proposed by Ashby (48) treats the situation where a rigid spherical particle is contained in a slip band and is unable to deform with the sheared continuum around it. Plastic accommodation is accomplished by the formation of prismatic dislocation loops and large tensile stresses produced at the particle poles may cause decohesion and subsequent void formation. This model correctly predicts that holes form preferentially at large particles for a given strain (in that a large particle is more likely to intercept a slip band).
Well-bonded particles have also been observed to crack internally, initiating voids prior to interface decohesion. Conversely, poorly bonded particles such as manganese sulfide inclusions in steel may in effect act as pre-existent voids so that growth processes proceed without the need for any void nucleation strain.

Void Growth

Once voids exist within a material, subsequent growth processes depend upon the history of the surrounding deformation field and the matrix flow properties. Several investigators have modelled void growth in homogeneous deformation fields using continuum mechanics approaches. McClintock (49) treated cylindrical holes with elliptical cross sections growing in perfectly plastic and viscous materials. Although incorporating many simplifying assumptions, including the neglect of void-void interactions, the analysis correctly predicts that the macroscopic fracture strain: (1) decreases in the presence of a tensile hydrostatic stress and (2) increases with the extent of material work hardening.

Rice and Tracey (50) examined the behavior of an isolated spherical cavity growing in a tensile stress field with superimposed hydrostatic tension and confirmed the deleterious effect of hydrostatic stress on fracture strain, predicting an exponential dependence of void growth
rate on the mean normal stress for perfectly plastic materials. Tracey (51) extended consideration to strain hardening materials and approximately accounted for interactions between neighboring voids. In homogeneous fields, strain hardening decelerates void growth while void–void interactions accelerate it. Tracey has pointed out, however, that the HRR solution for the near-tip crack stress field predicts that the local hydrostatic stress component is increased by strain hardening so the net effect of material hardening on void growth in the crack tip region is not clear. Large gradients in crack tip stress and strain fields may also modify growth behavior relative to the above predictions based on models that include the assumption of homogeneous applied deformation fields.

Void Coalescence

Damage in the fracture process zone accumulates by void expansion until voids coalesce or link with the crack tip, either through an internal necking mechanism or by fast shear. (52) Internal necking, the mechanism whereby voids grow until they impinge upon one another, is a common fracture process in low strength, strongly hardening alloys. In higher strength, weakly hardening materials, deformation can be localized into intense shear bands linking voids together or with the crack tip, and fracture
Mutual Void Impingement

A model for ductile crack extension in the absence of shear localization was developed by Rice and Johnson. (53) Consideration of the (small geometry change) Prandtl field for mode I plain strain conditions reveals that there is no region of intense straining directly ahead of the crack tip where damage processes are expected to transpire. Rice and Johnson included large geometry effects in an analysis of the near tip stress field and showed that for conditions of crack tip blunting, the centered fan of the Prandtl slip line field is replaced by a non-centered fan which focuses intense strains into a small crack tip zone (Figure 11). If the crack blunts into a semicircular arc, this region is bounded by logarithmic spirals which extend $2 \delta_t$ ($\delta_t =$ crack tip opening displacement) ahead of the crack tip. In this stress field, the free surface at the crack tip limits the triaxiality (defined as $\sigma_m/\sigma$) to only 0.58 and the full constraint of the Prandtl field ($\sigma_m/\sigma = 2.4$) is only reached at the juncture of the logarithmic spirals approximately $2 \delta_t$ ahead of the crack tip.

In the Rice-Johnson model, void nucleation is neglected and the growth of an initial spherical hole (as from a poorly bonded inclusion) is followed through the
Figure 11. Details of the slip line field at a blunted crack tip in mode I.

Figure 12. Void growth behavior predicted by the Rice-Johnson model. (53)
position dependent deformation history as if the instantaneous deformation field was applied homogeneously and remotely using the Rice-Tracey (50) analysis. The effect of the hole on the local field is neglected as are the interactions among neighboring free surfaces. Fracture initiation is assumed to occur when the remaining ligament between the crack tip and growing void is reduced to the size of the void's major semi-axis. In applying the model to material behavior, \( X_0 \) is taken as the mean inclusion spacing and \( R_0 \) as the mean inclusion radius. The resulting relationships between \( \delta_t, X_0, \) and \( R_0 \) at fracture are displayed in Figure 12. This analysis shows approximate agreement with real behavior for microstructures containing a single, dominant population of poorly bonded, void-nucleating inclusions.

For complex microstructures containing varied precipitates and complex grain morphologies, however, the features relevant to the fracture process are less apparent and the model cannot be applied directly.

Void Linkage by Localized Shear

In high strength materials that exhibit low work hardening rates, mutual void impingement is not achieved. Instead, neighboring voids link up by shear decohesion along concentrated deformation bands (Figure 13). Shear decohesion is the principal mode of material separation in
Figure 13. Void linkage by concentrated shear in discrete deformation bands.
high strength alloys (54,55), but may also significantly affect the final fracture process in lower strength materials if the flow stress saturates in the near-tip region because of plastic straining. Clear evidence for this effect was presented by Clayton and Knott (56), who showed that increased prestrain of HY-80 steel produced increased contributions of localized shear in void coalescence.

From a continuum viewpoint, localized plastic flow is manifested as a tangential velocity discontinuity along characteristic directions (directions of pure shear). (57) A necessary condition for this behavior is that the material exhibit an ideal plastic state. This occurs when there is no work hardening during continuing deformation. For an ideal plastic solid deformed in plane strain, characteristic directions are indicated by slip-line fields for a given geometry. Thus, the Rice-Johnson (53) field for plane strain deformation at a blunted crack tip suggests shear localization along logarithmic spirals, while Drucker (58) has presented slip line fields indicating characteristic shear directions between neighboring cylindrical holes and those joining a hole with an adjacent free surface. Experimental evidence confirms that localized shear along characteristic directions ahead of notched geometries contributes to the fracture process in mildly-hardening materials. (59,60)
Specific microstructural elements have been associated with shear localization. In particular, the nature of hardening particles is important. Precipitation strengthened alloys exhibit flow localization and inferior fracture characteristics if the particles responsible for hardening are cut by successive dislocations emitted from a single source on a given slip plane. Uncutable dispersions, on the other hand, are bypassed by dislocations with the resulting dislocation debris contributing to rapid work hardening and development of "back stresses" which inhibit dislocation emission from an operating source. In this way new dislocation sources are continually activated, leading to more homogeneous strains throughout the region. In general, flow localization is encouraged when substantial dislocation emission is produced by a single source for small stress increases. This behavior is expected for polycrystalline solids as well as single crystals, although additional effects including grain boundary compatibility considerations would also play a role. (61)

Williams and Hirth (62) considered additional microstructural features and noted that diffuse strengthening mechanisms (including solid solution strengthening, precipitation strengthening, and grain size strengthening) which raise the flow stress without changing the work hardening rate enhance the possibility of plastic
instability. Dispersion strengthening, on the other hand, increases the work hardening rate as well as the flow stress and should inhibit flow localization and fracture if the dispersoids resist cracking and decohesion.

Flow Localization in Steels

Several investigations have reported shear localization contributions to ductile fracture in steels containing both large, poorly bonded MnS inclusions, and fine well-bonded carbides. (54,56,63) The observed fracture mechanisms were all similar. At low strains, voids nucleate and grow around the large MnS particles. Prior to mutual void impingement, flow localizes into narrow bands linking the crack tip and neighboring voids. The preferential formation of shear bands between adjacent free surfaces may be encouraged by attractive image forces which the surfaces exert on nearby dislocations. (56) Free surfaces also allow easy accommodation of displacements at the shear band ends. (63) Within each narrow deformation band, local shearing continues until interface stresses amplified by dislocation pileups at the carbides are sufficient to cause decohesion and "unzipping" of all the particles in the band. (56) Alternatively, a second population of voids much smaller than those associated with the inclusions may nucleate at the fine carbides and grow in the rapidly deforming shear band until coalescence.
McClintock (64) considered void growth in shear and found, just as for tensile fields, that fracture strains decrease with applied hydrostatic stress and increase if the surrounding matrix strain hardens during deformation.

**Combined Mode Crack Extension**

Experimental and theoretical developments in the field of fracture mechanics have been largely restricted to mode I conditions. As a result, standardized procedures are available for determination of the lower bound plane strain fracture toughness values $K_{IC}$ and $J_{IC}$. These quantities are regarded as material properties and are valuable for design purposes. Real structures, however, are subjected to complex stress states and may contain initial flaws having varied orientations. Thus, in most applications combined mode loading is the rule rather than the exception, and a closer look at both fracture initiation and stable crack propagation under mixed - mode conditions is warranted.

A common observation is that initially flat cracks in tough materials, particularly in thin sections, often reorient to oblique planes during subsequent growth. It is not always appreciated that propagation of such fractures occur under combined mode I - mode III conditions. The extent of such behavior in different materials may be considered to differ in degree rather than in kind, since
even in lower toughness, thick sections the formation of "shear lips" is commonly observed adjacent to specimen free surfaces.

The combined mode I - mode III fracture problem considered in this work is only one of several mixed-mode possibilities. To this time, very limited effort has been expended in treating any of the cases of mixed mode crack propagation. Theoretical and experimental results which are available suggest relations between instantaneous values of field parameters (K's, J's, and G's) and the crack initiation event. Two possible ways of predicting fracture initiation in elastic bodies subjected to combined mode loading involve consideration of the total strain energy release rate, or alternatively, the strain energy density.

Combined Mode Energy Release Rate

Returning to the work of Griffith, one possible combined mode fracture relation can be developed by applying an energy balance to an elastic body loaded simultaneously in all three modes. (65) The general strain energy release rate contains contributions from all possible loading modes and may be represented as

\[ G_{\text{tot}} = G_I + G_{II} + G_{III}. \]  \[27\]

For the special case of plane strain loading, this energy
release rate may be formulated in terms of the three independent stress intensity factors as

$$G_{tot} = \frac{1}{E} \left( (K_I^2 + K_{II}^2)(1-\nu^2) + K_{III}^2(1+\nu) \right). \quad [28]$$

Just as for the mode I case, fracture is postulated to occur when the total strain energy release rate reaches a critical value, $G_{totc}$, equal to the energy consumption rate during crack extension. If this "crack resistance" value, $R$, is constant irrespective of the loading mode, then

$$G_{totc} = R = \frac{1}{E} K_{IC}^2 (1-\nu^2) \quad [29]$$

where $K_{IC}$ is the critical stress intensity value at fracture measured under mode I conditions. For the specific case of combined mode I - mode III fracture,

$$K_{IC}^2 (1-\nu^2)/E = \frac{K_I^2(1-\nu^2)}{E} + \frac{K_{III}^2(1+\nu)}{E} \quad [30]$$

or,

$$K_{IC}^2 = K_I^2 + K_{III}^2/(1-\nu) \quad [31]$$

and the fracture locus appears as an ellipse in $K_I$-$K_{III}$ space. Alternatively, reformulation in terms of $J$ ($G$) values yields a linear fracture initiation locus in $J_I$-$J_{III}$ space,

$$J_{IC} = J_I + J_{III} \quad [32]$$

and it follows that $J_{IC}=J_{IIIc}$ from the assumption that $R$ is
independent of mode. Chiang (66) used a linear elastic
calculation to extend this general treatment to cases
where \( K_{\text{IIIc}} = K_{\text{IC}} \) and predicted an elliptical failure locus
in \( K_{\text{I}} - K_{\text{III}} \) space which also agrees in form with the earlier
dimensionless expression of Wu (67) developed for combined
mode I- mode II loading:

\[
\left( \frac{K_{\text{I}}}{K_{\text{IC}}} \right)^2 + \left( \frac{K_{\text{III}}}{K_{\text{IIIc}}} \right)^2 = 1
\]  \[33\]

or alternatively,

\[
\frac{J_{\text{I}}}{J_{\text{IC}}} + \frac{J_{\text{III}}}{J_{\text{IIIc}}} = 1.
\]  \[34\]

The above fracture criteria [27]-[34] all contain the
implicit assumption that cracks propagate in a self similar
fashion, that is, maintaining the original crack plane.

Several investigators have shown conclusively that
cracks under combined mode loading often grow at an
inclined angle to the initial crack. (68-71,80) Combined
mode I - mode II results of Erdogan and Sih (68) suggested
that crack extension occurs along the planes subjected to
the maximum tensile stress as originally postulated by
Griffith. The orientations of these planes depend on the
relative magnitudes of the two loading modes as

\[
K_{\text{I}} \sin \phi + K_{\text{II}} (3 \cos \phi -1) = 0
\]  \[35\]
where $\phi$ is the angle between the original crack plane and the plane of the initial crack extension. This criterion is identical to that invoking the generalized strain energy release rate but assuming crack propagation in the direction which maximizes the loss of elastic energy during crack extension. (65)

Strain Energy Density Criterion

Sih has subsequently proposed a more general theory of fracture which predicts crack extension when the strain energy density field in the crack tip region attains a critical magnitude. (69-71) In a linear elastic body, the strain energy density takes the form

$$
\delta W = \frac{(a_{11}K_{I}^2 + 2a_{12}K_{I}K_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2)}{\pi r} + \text{terms bounded at the crack tip}
$$

where

$$
a_{11} = \frac{((3-4 \nu - \cos \theta)(1+\cos \theta))/16G}
a_{12} = 2\sin \theta (\cos \theta -(1-2\nu))/16G
a_{22} = 2(1-\nu)(1-\cos \theta) + (1+\cos \theta)(3\cos \theta -1))/16G
a_{33} = 1/4G.
$$

This function contains a 1/r singularity at the crack tip and the quantity

$$
S = \frac{(a_{11}K_{I}^2 + 2a_{12}K_{I}K_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2)}{\pi}
$$

[37]
is the amplitude of this energy density within the crack tip region where singular terms dominate.

Sih has hypothesized that crack extension takes place in the direction of minimum strain energy density, \( \theta_0 \), defined by the conditions

\[
\frac{dS}{d\theta} = 0 \quad \text{and} \quad \frac{d^2S}{d\theta^2} = 0
\]

when \( S \) reaches a critical value \( S_c \). \( S \) is dependent on the applied stress field and the elastic constants of the material,

\[
S_c = S(\sigma_{ij}) = S(K_{I,II,III}, \mu, \nu). \tag{39}
\]

For the mode I - mode III situation

\[
S = \left[ \left( (3-4\nu - \cos \theta) (1+\cos \theta) \right) K_I^2 + 4K_{III}^2 \right] / 16 \pi G \tag{40}
\]

and \( S \) is always a minimum at \( \theta = 0 \) as in the pure mode I case and as usually observed experimentally, since the term containing \( K_{III} \) is independent of \( \theta \). The expression for \( S_c \)

\[
S_c = \frac{1}{G\pi} \left\{ \left( \frac{4-8\nu}{16} \right) K_I^2 + \frac{1}{4} K_{III}^2 \right\} \tag{41}
\]

leads to predicted relations between simultaneously applied mode I and mode III load components of the forms

\[
K_I^2 + A K_{III}^2 = \text{constant} \tag{42}
\]

\[
J_i + B J_{iii} = \text{constant} \tag{43}
\]
where the constants $A$ and $B$ depend on Poisson's ratio and are given in Table 1. These expressions can also be constructed as normalized forms identical to Equations (33) and (34)

\[
\left(\frac{K_I}{K_{IC}}\right)^2 + \left(\frac{K_{III}}{K_{IIIC}}\right)^2 = 1
\]

\[
\frac{J_i}{J_{IC}} + \frac{J_{iii}}{J_{IIIc}} = 1.
\]

TABLE 1. Combined Mode Fracture Initiation Criteria

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>.2</td>
<td>1.67</td>
<td>1.34</td>
</tr>
<tr>
<td>.25</td>
<td>2.00</td>
<td>1.50</td>
</tr>
<tr>
<td>.3</td>
<td>2.5</td>
<td>1.75</td>
</tr>
<tr>
<td>.35</td>
<td>3.33</td>
<td>2.16</td>
</tr>
<tr>
<td>.4</td>
<td>5.00</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Experimental Observations in Combined Mode I - Mode III Fracture

Limited experimental data are available describing fracture under combined mode I - mode III loading. Despite the fact that combined mode fracture is associated with large toughness, much of the experimental work reported in the literature has been confined to materials with limited ductility. It is generally concluded that an imposed mode III load contribution lowers the required mode I component
to initiate fracture (66,72,73,80), although Pook found that the mode I toughness was insensitive to transverse shear for several high strength alloys. (74)

To preface the following discussion, it is worthwhile to point out that conversions between $K$ and $J$ values may appear to affect relative "toughnesses" in mode I versus mode III. In a material for which $J_{IC}=J_{IIIc}$, conversion to the LEFM equivalent $K$ values (plane strain)

$$K_{IC} = \left[ \frac{EJ_{IC}}{(1-\nu^2)} \right]^{1/2}$$  \[46\]

$$K_{IIIc} = \left[ \frac{EJ_{IIIc}}{(1+\nu)} \right]^{1/2}$$  \[47\]

leads to the result that $K_{IC}(1-\nu)=K_{IIIc}$, or $K_{IIIc}<K_{IC}$. In the subsequent discussion, "toughnesses" will be compared on the basis of $J$-integral values.

Conflicting results are reported for the relative toughnesses for pure mode I and pure mode III cracks in a given material. Yoda (75) found that $J_{IC}=J_{IIIc}$ for two plain carbon steels and the same equality was reported by Wilson (80) for a high strength aluminum alloy. Shah (72) and Tsangarakis (76) report that $J_{IIIc}>J_{IC}$ for two high strength steels although Gupta and Banerjee (77) have since suggested that the experimental procedure of Tsangarakis may have led to overestimates of mode III toughness. Work by Liu and Shen (78) and by Miglin et al (79) on medium strength steels suggest that $J_{IIIc}<J_{IC}$ and cloud the
picture even further. It appears likely that there is no simple correlation between $J_{IC}$ and $J_{IIIc}$ which holds for the variety of alloys, microstructures, and mechanical properties displayed in the aforementioned results. The lack of a standard practice for either $K_{IIIc}$ or $J_{IIIc}$ determination also contributes to the apparent scatter since a variety of specimen sizes and experimental techniques were applied in obtaining the above results. All available data suggest that $J_{IC}$ and $J_{IIIc}$ have comparable magnitudes (within a factor of 2) although it is not clear if either is uniformly greater than the other.

If the pure mode I and mode III fracture toughnesses are not directly relatable, it is reasonable to include both of these values as normalization factors in any proposed fracture criterion. Three investigators have explored the entire range of mode I–mode III behavior and their results can be compared after such normalization is applied. Shah (72), Liu and Shen (78), and Wilson (80) all used combined tension and torsion of circumferentially cracked cylindrical specimens to produce fracture under a variety of mode I–mode III conditions. For this type of specimen, LEFM analysis shows that the value of $K_I$ depends only on the tensile load while $K_{III}$ depends only on the applied torque. Independent variation of those two parameters allows controlled application of mixed mode load conditions. In these experiments, $K_I$ and $K_{III}$ were
calculated at the same point in each experiment, although that critical point was detected using different methods by each investigator.

Both Shah and Wilson reported that records of torque versus angular displacement showed much greater non-linearity than did the tensile load versus extension behavior. Extensive mode III shear flow prior to fracture initiation is indicated by these observations and is further expected in light of the fact that the nominal shear stress levels exceeded the shear yield stress values by 48-84% at fracture in pure mode III while nominal tensile stresses 15-32% below tensile yield values produced fracture in pure mode I.

This behavior indicates that specimen size restrictions may differ appreciably with the loading mode so that comparisons between relative contributions of the two modes differ as a function of specimen size. Such a size effect on a mixed mode fracture criterion would likely be reduced if raw data were normalized by dividing by the appropriate pure mode toughnesses measured in the same specimen size. This correction was applied to the above data (72,78,80) and plotted in Figure 14. The resulting behavior supports the possible fracture criterion

\[ \left[ \frac{K_I}{K_{IC}} \right]^2 + \left[ \frac{K_{III}}{K_{IIIc}} \right]^2 = 1 \]
Figure 14. Dimensionless combined mode I - mode III fracture initiation data for three metal alloys.
as a lower bound to combined mode behavior with the exponents of 2 replaced by 3 or 4 to give slightly better overall data fitting. Significant scatter prevents a more refined description of the material behavior. The lower bound exponent of 2 agrees with the strain energy release rate and strain energy density criteria for self similar crack extension. However, this result is coincidental to some extent since the observed combined mode cracks tended to break up into discontinuous angled cracks which propagated at an inclined angle of approximately 45° to the original crack plane.
CHAPTER II

EXPERIMENTAL PROCEDURES

Heat Treatment and Microstructures

The material chosen for study in this investigation was Armco NiCop, an HSLA steel similar in composition to ASTM A710 Grade A. Alloy plate of 12.7 mm (1/2 inch) thickness was supplied by Armco Corporation with the composition listed in Table 2. The as-received material was heat treated according to two schedules previously developed by Lin (81) to produce microstructures with similar grain sizes and carbide distributions, but containing different copper precipitate morphologies. The heat treating procedures are presented in Table 3 along with the principal resultant microstructural elements as determined by Miglin. (82)

Both heat treatments begin with a reaustenitization step followed by air cooling. The temperature of 900°C is sufficiently low that only about 25 percent of the Nb initially present as Nb(C,N) goes into solution. The remaining carbides grow to 10-20 nm during the eight hour austenitization time, losing much of their strengthening effect, but helping to retard grain growth. During air
TABLE 2. Composition of NiCop Steel
Supplied by Armco Corporation.

<table>
<thead>
<tr>
<th>Element</th>
<th>Content, Weight Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>0.04</td>
</tr>
<tr>
<td>Niobium</td>
<td>0.04</td>
</tr>
<tr>
<td>Copper</td>
<td>1.17</td>
</tr>
<tr>
<td>Nickel</td>
<td>0.90</td>
</tr>
<tr>
<td>Manganese</td>
<td>0.49</td>
</tr>
<tr>
<td>Silicon</td>
<td>0.27</td>
</tr>
<tr>
<td>Chromium</td>
<td>0.70</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>0.20</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.007</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>0.010</td>
</tr>
<tr>
<td>Sulfur</td>
<td>0.009</td>
</tr>
</tbody>
</table>

cooling, the Nb in solution precipitates as fine carbides (~1 nm) in the austenite-ferrite interface during the austenite-to-ferrite transition as well as in the ferrite matrix below the transformation temperature. The resulting microstructure thus contains a bimodal distribution of Nb(C,N).

During the aging treatment, supersaturated Cu forms clusters or coherent bcc precipitates within the ferrite matrix which grow with time until they transform to semi-coherent or incoherent fcc precipitates of almost pure copper. The "peak aged" condition of Table 3 contains principally coherent bcc Cu precipitates within a fine-grained (4.5 μm) polygonal grain structure. The higher aging temperature and longer holding time of the second
heat treatment produces an "overaged" condition containing both coherent bcc and semi-coherent fcc copper precipitates. The longer treatment also results in a slight coarsening of the grain size to 5.6 μm. Throughout the remainder of this work the two heat treated conditions will be referred to simply as "peak aged" and "overaged."

Inclusion sizes and spacings are also given in the Table 3 and their significance in fracture behavior described in a subsequent section.

**Mechanical Properties**

Tensile properties and Charpy V-notch impact energies of peak aged and overaged material were determined by Lin. (81) To minimize the effect of elongated inclusions produced during rolling of the plate, specimen orientations were chosen as shown in Figure 15. The orientations of fracture toughness specimens used in combined mode fracture toughness testing are also displayed here. Charpy, Mode I, and Mode III crack planes lie in the short transverse–long transverse plane so that sulfides elongated in the longitudinal direction intersect the propagation plane at right angles. Although combined mode I–mode III crack planes are oriented obliquely to the ST-LT plane, elongated sulfides do not lie directly in the crack propagation plane so that their effect on fracture behavior is minimized.
<table>
<thead>
<tr>
<th>Heat Treatment (Number)</th>
<th>Peak Aged (C1)</th>
<th>Overaged (C3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austenitizing Temp., °C</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>Austenitizing Time, Hrs.</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Cooling</td>
<td>Air Cool</td>
<td>Air Cool</td>
</tr>
<tr>
<td>Aging Temp., °C</td>
<td>535</td>
<td>675</td>
</tr>
<tr>
<td>Aging Time, Hrs.</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Ferrite Morphology</td>
<td>Polygonal</td>
<td>Polygonal</td>
</tr>
<tr>
<td>Ferrite Mean Intercept Grain Size, µm</td>
<td>4.5</td>
<td>5.6</td>
</tr>
<tr>
<td>Fraction Nb in Interphase Precipitates</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Fraction Nb in Austenite-Nucleated Precipitates</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Coherency of Copper Precipitates</td>
<td>Coherent</td>
<td>Coherent &amp; Semi-Coherent</td>
</tr>
<tr>
<td>Structure of Copper Precipitates</td>
<td>bcc</td>
<td>bcc &amp; fcc</td>
</tr>
<tr>
<td>Sulfide-Oxide Complex Diameter</td>
<td>1.4µm</td>
<td>1.4µm</td>
</tr>
<tr>
<td>Sulfide-Oxide Complex Spacing</td>
<td>290µm</td>
<td>290µm</td>
</tr>
<tr>
<td>Elongated Sulfide Size</td>
<td>9µm (LT) x 1.3µm (ST)</td>
<td></td>
</tr>
<tr>
<td>Sulfide Spacing</td>
<td>200µm</td>
<td>200µm</td>
</tr>
<tr>
<td>Small Equiaxed Sulfides</td>
<td>≤0.1µm</td>
<td>≤0.1µm</td>
</tr>
</tbody>
</table>
Figure 15. Specimen orientations with respect to the plate fabrication process.
Tensile and Charpy V-notch impact energy results are given in Table 4. The overaged steel displays lesser yield and ultimate tensile strengths as well as greater total elongation to failure. Tensile data fitted to the power law relation,

\[ \frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \beta \left[ \frac{\sigma}{\sigma_u} \right]^{1/n} \]  

where \( \varepsilon \) is true strain, \( \sigma \) is true stress, \( \varepsilon_0 \) and \( \sigma_0 \) are yield values, \( \sigma_u \) is the ultimate strength, and \( \beta \) is a material constant, gave very similar strain hardening exponents for the two heat treated conditions. This was expected in view of the similar uniform elongations for the two heat treatments.

**Mode I and Combined Mode I - Mode III Fracture Toughness Testing**

**Specimen Design**

Both pure mode I and combined mode I - mode III fracture toughness tests were carried out using variations of a single modified compact tension specimen geometry. The dimensions of the specimen are given in Figure 16. The angle denoted as \( \theta \) in the figure is subsequently referred to as the crack inclination angle. This angle is defined as the complement of the angle formed between the specimen load line and the crack plane normal. Pure mode I conditions are obtained when \( \theta = 90^\circ \), as in standard
Figure 16. Modified compact specimen geometry.
compact tension specimens. On rotation of the initial crack plane so that $\theta$ is reduced from its pure mode I value of $90^\circ$, mode III components of load are introduced which produce displacements parallel to the crack front. By systematically decreasing the initial crack inclination angle in a set of specimens, increasing mode III components of load are introduced, and the complete range of combined mode I - mode III fracture behavior can be investigated.

In the plate-type specimen of Figure 16, as $\theta$ is varied from $90^\circ$ the crack front lengthens for a constant

TABLE 4. Tensile and Charpy V-Notch Results for Two Heat Treated Conditions of NiCop. (81)

<table>
<thead>
<tr>
<th>Heat Treatment</th>
<th>Peak Aged</th>
<th>Overaged</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 Percent Offset Yield Strength (MPa)</td>
<td>501</td>
<td>411</td>
</tr>
<tr>
<td>Ultimate Tensile Strength (MPa)</td>
<td>661</td>
<td>475</td>
</tr>
<tr>
<td>Uniform Elongation in 25 mm (percent)</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>Total Elongation to Failure (percent)</td>
<td>34</td>
<td>48</td>
</tr>
<tr>
<td>Strain Hardening Exponent n</td>
<td>0.216</td>
<td>0.213</td>
</tr>
<tr>
<td>Charpy V-Notch Impact Energy at 23°C (Joules)</td>
<td>225</td>
<td>&gt;325</td>
</tr>
<tr>
<td>47-Joule Transition Temperature (°C)</td>
<td>-105</td>
<td>-157</td>
</tr>
</tbody>
</table>
specimen thickness \( B \), according to the relation
\[
\text{width} = B / \sin \theta.
\]
The larger crack front widths at low angles require larger applied loads for crack propagation. In the specimens of NiCop used here, a practical lower limit on \( \theta \) is approximately 35°. For lower angles, the applied loads necessary for crack extension cause yielding in the ligament between the loading holes and the clip-gage cutout at the front of the specimen. Testing was carried out for both heat treated conditions using crack inclination angles of 90°, 75°, 55°, 45°, and 35° to obtain a variety of mode I – mode III conditions.

Considering its thickness, the specimen geometry of Figure 16 has a larger height and width than the standard compact tension configurations recommended for \( J_{lc} \) testing. (22) The initial crack length, measured from the load line, of nominally 38 mm corresponds to an \( a/w \) ratio of 0.375 which also lies outside the recommendation of \( a/w \geq 0.5 \). These deviations from standard \( J \) testing practice are necessitated by the elastic-plastic behavior of NiCop in 12.7 mm thicknesses which results in the formation of a large plastic zone. The oversized height and width of the specimen coupled with the initially short crack allow contained yielding behavior over several millimeters of crack extension before the plastic zone intersects the back surface of the specimen.
Although not shown in the figure, side grooves of 20 percent of total section were incorporated in all opening and combined mode specimens. Side grooves increase the triaxiality at the edges of the advancing crack, promoting ductile crack extension near the free surface and thereby reducing tunnelling. In the mixed-mode configurations side grooves are advantageous for additional reasons. Initially slanted cracks propagating in NiCop display tendencies for crack plane rotation or double shear crack formation in ungrooved specimens. The twenty percent side grooves act to guide combined mode cracks during crack extension, eliminating complications in the analysis of experimental records arising from continuously changing crack plane orientation and morphology.

In order to quantify combined mode I - mode III behavior it is necessary to measure load line displacements both parallel and perpendicular to the applied load. Razor blades were positioned at the load line within the cutout so that parallel displacement components could be monitored using a standard double cantilever beam clip gage instrumented with strain gages. Extension rods were tack-welded onto both sides of the specimen to facilitate measurement of normal displacements with a hand-held micrometer placed across the mouth of the specimen.
Electric Potential Drop Crack Measurement Technique

To generate a J-resistance curve from a single specimen, instantaneous crack length values must be known throughout the experimental record. In the fracture toughness testing of the mode I and combined mode I - mode III compact tension specimens, crack length was monitored by means of a direct current electric potential drop (potential difference) method. The electric potential drop technique involves applying a constant current through the cracked specimen in such a way that changes in crack length alter the potential difference between two potential probes suitably placed near the crack. (84) The instantaneous crack length is then related to potential difference measurements through theoretical or experimentally determined calibrations.

Johnson (85) originally solved Laplace's equation

\[ V^2 U = 0 \]  \[2\]

for the center cracked panel (CCP) geometry and found the relationship between crack length and potential difference to be

\[ U(a) = A \arccosh\left(\frac{\cosh \frac{\pi y}{2W}}{\cos \frac{\pi a}{2W}}\right) \]  \[3\]

where

- U = Potential difference for crack length a
- 2W = Center cracked panel width
- y = Probe location above center of crack
- A = Constant.

If the potential difference U is divided by the potential difference \( U_0 \) present for an initial crack length \( a_0 \), the
proportionality constant $A$ is eliminated in the resulting normalized relation

$$\frac{U}{U_0} = \frac{\text{arccosh} \left[ \frac{\cosh(\pi y W)}{\cos(\pi a_0 W)} \right]}{\text{arccosh} \left[ \frac{\cosh(\pi y W)}{\cos(\pi a W)} \right]} \quad [4]$$

By means of such dimensionless ratios, calibration curves of the form $U/U_0$ versus $a/W$ become independent of material properties, specimen thickness, and applied current. (86)

Schwalbe and Hellmann (87) noted that with regard to potential distribution, the CT specimen is nearly equivalent to half of a CCP, and showed that Equation [4] closely matches experimental results when applied directly to CT geometries.

For optimum application of the d.c. EP method, care must be exercised in choosing the locations of current leads and potential probes. Schwalbe et al (88) suggest that the current lead positions given in Figure 17 are optimum for direct application of Equation [4] to the CT specimen and that configuration was adopted for all modified CT specimen testing.

Potential distributions in graphitized electrical analog paper models suggest that placement of the potential probes near the crack tip (placement $B$, Figure 18) gives maximum sensitivity to crack extension. (89) This apparent advantage is offset by poor reproducibility since precise lead placement is necessary to obtain consistent
Figure 17. Potential probe and current lead positions used in modified compact tension fracture toughness testing.
Figure 18. Alternative suggested lead placements for the electric potential drop method.
results. In order to allow comparison between experimental calibrations and the theoretical equation, potential probe placement according to configuration A (Figure 18) was chosen. Aronson and Ritchie (86) have determined that probe placement within a few millimeters of the crack mouth on the front of the specimen maximizes both sensitivity and reproducibility for this geometry. The large cutout at the front of slant notch specimens (Figure 16), necessary for the attachment of the load-line displacement clip gage, precludes this placement. A value of \( y = 19 \text{mm} \) was used instead (attachment just next to the cutout) which has the only disadvantage that the signal to noise ratio is somewhat reduced. (86) For this reason extra care was taken in eliminating extraneous contributions to the measured potential drop.

Electric Potential Drop Calibration

The modified CT geometry used in this investigation differs from the single edge notched bend specimen to which Equation [4] directly applies. Calibration curves of \( \frac{U}{U_0} \) versus \( a/W \) were determined for the cases \( \theta = 90^\circ \) and \( \theta = 30^\circ \) to allow comparison with the theoretical prediction. Mild steel specimens were machined to the dimensions of Figure 16, with identical potential and current leads as those to be used in subsequent NiCop testing. A constant applied current of 20 A was maintained and cracks were advanced
using a sawcut. For a short while following a sawcut, the measured potential drifted downward before stabilizing. This was probably caused by resistivity fluctuations due to heating and subsequent cooling between crack extension increments as well as recovery processes in deformed regions. Only the stable (cool) potential drop values were recorded.

The results of this calibration procedure are plotted in Figure 19 along with the theoretical relationship. Although the differences between the curves are not large, the geometry dependent shift from the theoretical curve was accounted for in all data analysis. Behavior for specimens in the range \(30^\circ < \theta < 90^\circ\) was approximated by interpolating between the experimental bounds as a function of crack angle.

Experimental Precautions

During fracture toughness testing, a constant d.c. current of 20 amperes was applied to modified CT specimens through copper leads spot welded in the locations of Figure 17. A larger value of applied current would have the advantage of increasing the output voltage but would also generate greater resistive heating. (86) The potential drop between two probes positioned as in Figure 17 was fed to a digital voltmeter and recorded to a sensitivity of 0.1 \(\mu\)V.
Figure 19. Electric potential drop calibration curves for modified compact tension geometries compared with theoretically predicted behavior.
Iron wire was used in place of copper for the potential probe leads to prevent thermal emf's otherwise produced at the dissimilar metal probe-specimen junctions. (84) This assured that temperature fluctuations at the probe positions could not introduce extraneous potential contributions. Since the iron wire - copper lead connections at the voltmeter are subject to the same problem, copper leads run from the voltmeter were attached to the iron wire leads and isolated in a Dewar flask packed with tissues. The constant temperature of the dissimilar metal junctions guaranteed that induced thermal emf's would always cancel.

Care was taken to avoid bending or stretching the iron potential probe wires once an experiment was underway because resistivity changes produced by cold working the probe wires could introduce spurious potential contributions. The upper clevis holding the top loading pin was also electrically isolated from the loading machine frame to prevent secondary current circuits through the loading system.

Experimental Data Aquisition

All modified CT specimens were pulled in a 60,000 lb capacity Baldwin Universal Testing machine. Signals from the double cantilever beam clip gage were amplified and the resulting values of vertical displacement plotted as the X
component by an X-Y$_1$-Y$_2$ recorder. The potential drop measured between the iron wire leads was plotted on the recorder as Y$_1$ while the load cell signal was amplified and recorded as the Y$_2$ component on the recorder.

Load control was used during initial loading until the P versus δ$_v$ curve began to flatten significantly. From that point on, displacement control was invoked with applied increments of 0.010" (2.5 mm) vertical displacement. After each loading increment, horizontal displacements measured with a micrometer at the specimen mouth and potential drop values recorded from the digital voltmeter to 0.1 μV were recorded manually to supplement the chart recorder output. As maximum load was approached and exceeded, the applied load was dropped approximately 2000 lb (~9000 kN or ~10% of load) after each displacement increment so that stable values of δ$_h$ and U could be recorded.

Mode III Fracture Toughness Testing

Specimen Design

Pure mode III conditions are most readily produced using a circumferential crack in a cylindrical specimen loaded in torsion. Such torsion specimens, however, do not meet the requirements of this investigation. In particular, stable crack growth under continuing mode III
conditions is not usually obtained in torsion specimens since the crack front may break up into discontinuous segments during extension, each rotating varying degrees and continuing to grow under combined mode I - mode III conditions. Torsion specimens also contain radial gradients in nominal stress and strain.

In this work, mode III fracture toughness testing was carried out in the plate type "triple-pant-leg" specimen geometry of Figure 20. The 12.7 mm plate dimensions are sufficient that the crack tip plastic zones are well contained during initiation and initial growth. Side grooves occupying 80% of total section thickness reduce the growing crack front widths to 2.54 mm to allow crack propagation prior to general yielding in the cantilever beam arms of the specimen. These grooves also encourage a linear crack front.

With the incorporation of two cracks in the specimen a symmetrical loading arrangement is obtained which minimizes out-of-plane bending. However, small induced mode I stress components, ahead of the crack tips, which are tensile in the lower half of the specimen and compressive in the upper half are undoubtedly present.

Finally, during stable crack growth, crack lengths in the plate-type specimen are more readily monitored than in torsion-type geometries. Optical observation of the crack is possible in the plate-type specimen and is expected to
Figure 20. Mode III test specimen.
be more reliable than indirect monitoring techniques (e.g. electric potential drop or unloading compliance) which may be affected by contact of the specimen halves during mode III deformation.

Compliance Calibration

Because of its non-standard nature, a compliance calibration was carried out on the mode III plate specimen geometry to characterize its elastic behavior. 6061-T651 aluminum alloy plate of 12.7mm thickness was chosen for calibration purposes because of its relatively high strength and low modulus of elasticity. Compliance testing was done with the mode III fixture on a 20,000 lb capacity Instron testing machine. Initial crack lengths of 1.75" (44 mm) were extended by 2.54 mm increments by bandsaw cuts between loading cycles. Both crack lengths were measured to a precision of 0.0001 inch with a microscope incorporating a stage coupled with a LVDT and digital readout.

Loading produced an essentially linear slope at the upper load limit of 400 lb with no yielding in the plate samples. Slopes of the resulting load-displacement curves were fit visually to obtain compliances. Three compliance measurements were recorded at each crack length and an average value calculated. The effective crack length was taken to be an average of the two measured values.
Experimental Run

All mode III testing was done using the test fixture and loading geometry of Figure 21 in a 20,000 pound capacity Instron testing machine. Load line displacements were measured by affixing a double cantilever beam clip gage along the load line beneath the central leg of the specimen between two knife edges. The length of the knife edge attached to the specimen was kept as small as possible so that rotation of the specimen leg about its juncture with the specimen body was not magnified unnecessarily at the load line measurement point.

Clip gage and load cell signals were amplified and fed to an X-Y recorder. Apparent crack extension was monitored visually using a microscope eyepiece in combination with a mirror mounted over the specimen crack tip. To enhance the visibility of the crack tip the side groove root radii were polished using 3 um diamond paste.

Post Fracture Inspection

All incompletely broken specimens were cooled to -196°C in liquid N₂ and fractured. Crack lengths resulting from stable ductile fracture were measured and fracture surfaces were coated with clear lacquer to minimize oxidation. Fracture surfaces were removed using a bandsaw and viewed in the SEM using secondary electrons to generate images. Stereo pairs were taken to aid in topographic
Figure 21. Mode III test fixture and loading geometry.
characterization.

Some fractured specimens were sectioned in a plane normal to the crack plane to aid characterization. The surfaces were plated using an electroless nickel plating procedure which produces a thin nickel alloy layer of hardness >50Rc. This plated layer helped to minimize edge deformation during subsequent sectioning using a diamond saw. These cross sections were mounted in bakelite, diamond polished and viewed optically to determine surface roughness and the extent of subsurface microcracking.
CHAPTER III

RESULTS

Electric Potential Drop Data Analysis

The electric potential drop records obtained during CT specimen testing all displayed the general features given in Figure 22. The initial linear change in potential drop with vertical displacement $\delta_v$ is attributed to changes in geometry and resistivity changes in the newly formed plastic zone rather than to changes in physical crack growth. (84) This "EP blunting" region is followed by a smooth transition during which the slope of the $\Delta U$ vs $\delta_v$ plot first changes, and then stabilizes so that a second nearly linear region is produced.

Under these circumstances it is generally believed that crack initiation occurs at the first break from linear behavior. In generating crack length values from calibration curves the commonly used procedures are to take $U_0$ as the value where $U$ vs $\delta_v$ first deviates from linearity (90) or to define a $U_0$ base line which defines $U_0$ values increasing with load (88) as in Figure 23. Because the initial linear portion was not always well defined in the various slant notch geometries, and because the
Figure 22. Record of electric potential drop versus vertical displacement for the modified compact tension specimen.
Figure 23. Alternate methods of defining $U_o$ for electric potential crack length determination.
relative slopes of the linear portions depended on crack inclination angle and heat treatment, an alternate definition of \( U_0 \) was taken.

The chosen procedure consisted of fitting lines to both the initial and terminating linear portions of data. The intersection of these lines was taken to define \( U_0 \) and that single value was used for all subsequent crack length determinations (Figure 23). This procedure was more reproducible than attempting to find the first deviation from linearity and the relatively low initial slope of Figure 22 meant that a variably defined "base line" \( U_0 \) value would not have greatly affected the results.

To verify the validity of this procedure, final values of \( U \) were combined with measured final and initial crack lengths to extract \( U_0 \) values. \( U_0 \) (two-line interpolation) and \( U_0 \) (from final crack length) differed, on average, by 3.5% of the total potential range encountered (\( U_{\text{final}} - U_0 \)).

Fracture Resistance

Mode I J-Integral Calculation

For opening mode specimens, values of \( J \) were calculated from the relation

\[
J_p = \frac{f_0 b_p}{B_{\text{net}}} \int_0^\delta P \, d\delta
\]  

[1]
by numerical integration using the experimentally determined values of $P$, $\delta$, and $a$. In this equation $B_{\text{net}}$ is the actual crack front width. The subscripts $p$ refer to present values. The constant $f_0$ was taken to be 2.0 which is the exact value if pure bending exists in the remaining ligament. Although that condition is not rigorously true for the relatively low $a/w$ ratio of 0.375, use of the Merkle-Corten correction of $f_0 = 2 + (0.522)b/w$ has been proven to overestimate $J$ in some cases (24), and the application of such a corrected $f_0$ value, or even more rigorously the inclusion of a variable $f_0$ within the integral, is not justified in view of other sources of experimental error.

$J$-resistance curves were constructed in a manner analogous to that of ASTM E813 (22) as shown in Figure 24. The electric potential drop method used for monitoring crack length did not yield precise $\Delta a$ values in the initial blunting region. It was assumed that the blunting behavior followed the relation $J = 2\sigma_f \Delta a$ with $\sigma_f = (\sigma_y + \sigma_u)/2$. Exclusion lines which define $J$ values valid for $J_c$ calculation were offset 0.3 and 3.0 mm from the blunting line. The values of $J$ between these lines correspond to crack extension increments falling within the range $0 < a < 6\%b$ for the oversized specimen used. Linear regression analysis was used to fit a straight line to data in this region and the intersection of the regression line with the
Figure 24. J-resistance curve construction.
blunting line taken as $J_{ic}$.

This value of $J_{ic}$ may not precisely correspond to real crack initiation, but is believed to be a reproducibly determinable value of $J$ corresponding to some small amount of stable crack extension.

Representative plots of load versus load line displacement and crack extension versus load line displacement are given in Figure 25. The points of crack initiation and maximum load are designated with the symbols I and P respectively in the figure. In all cases crack initiation preceded maximum load although initiation values were well beyond the linear load-displacement region. It is worth repeating that the $\Delta a$ versus $\delta$ values prior to crack initiation were estimated from the theoretical relation between $J$ and $\Delta a$ and not determinable from the electric potential drop method used in this work. The point on the experimental record where the exclusion line was intersected by the $J_R$ curve is indicated in Figure 25 by the symbol E. This point always fell beyond the maximum load for mode I conditions.

The $J_{ic}$ values obtained were less than those of Miglin et al (82) using smooth-sided 1/2T specimens and may reflect the increased crack tip constraint imposed by the side grooves. The use of different in-plane specimen dimensions and crack length monitoring methods may also account for the discrepancies.
Figure 25. Load and crack extension versus load-line displacement for a mode I test.
Calculated values of $J_{ic}$ for both the overaged and peak-aged heat treated conditions violate the plane strain requirement of ASTM E813:

$$B > 25 \frac{J_{ic}}{\sigma_f}$$

[2]

$B = \text{specimen thickness}$

$b = \text{uncracked ligament length}$

$\sigma_f = \text{effective flow strength} = (\sigma_y + \sigma_u)/2.$

Combined Mode I J-Integral Calculation

Since there is no standard method for combined-mode $J$ analysis, the definition of $J$ as an energy measure was applied in analyzing the slant notch specimen geometries. Figure 26 is a schematic view of the front of the modified compact tension specimen with the general crack inclination angle $\theta$ defining the crack plane orientation. For a general load $P$ applied along the specimen axis, a mode I load component exists perpendicular to the crack plane and a mode III load component exists parallel to the crack front. From the geometry of Figure 26 these load components depend on the angle $\theta$, and can be represented as

$$P_I = P \sin \theta$$

[3]

$$P_{III} = P \cos \theta.$$  

[4]

Similarly, the horizontal and vertical load-line displacements measured experimentally define a total displacement $\delta$ which is resolvable into the mode I and mode
\[ \delta_I = \delta_v \sin \theta - \delta_h \cos \theta \]
\[ \delta_{III} = \delta_v \cos \theta + \delta_h \sin \theta \]

\[ P_I = P \sin \theta \]
\[ P_{III} = P \cos \theta \]

**Figure 26.** Resolution of applied load and measured displacements into mode I and mode III components.
III displacement components

\[ \delta_I = \delta_v \sin \theta - \delta_h \cos \theta \]  
\[ \delta_{III} = \delta_v \cos \theta + \delta_h \sin \theta \]

as shown in the figure. These resolved load and displacement values define energy expended parallel and perpendicular to the crack front and can be used to define the mode I and mode III components of \( J \)

\[ J_I = 2b_p \int_{0}^{\delta_I} \frac{p_I}{B_{net}} d\delta_I \]  
\[ J_{III} = 2b_p \int_{0}^{\delta_{III}} \frac{p_{III}}{B_{net}} d\delta_{III} \]

In these equations \( B_{net} \) is the actual crack front width and is related to the specimen thickness through

\[ B_{net}(\sin \theta) = 0.8B \]

with the factor 0.8 appearing because of the presence of side grooves totalling 20% of section thickness.

Both mode I and mode III resistance curves were constructed for each sample using the same procedure as that used in pure opening mode calculations (Figures 27 and 28). Again the intersection of the best fit regression line and the blunting line define critical \( J_{IC} \) and \( J_{IIIC} \) values, and the intersection with the 3 mm line was taken to define \( J_{IIEC} \) and \( J_{IIIEC} \). When 4 data points did not fall within the
Figure 27. Mode I J-resistance curve for combined mode conditions.
Figure 28. Mode III J-resistance curve for combined mode conditions.
exclusion lines, the first four data points beyond crack initiation were used to define the regression line.

Typical load versus load line displacement, and crack extension increment versus load line displacement data are given in Figure 29. In all mixed mode tests at crack inclination angles less than 75°, crack initiation occurred very close to maximum load. For θ=75°, crack initiation preceded maximum load slightly, but not as much as for the pure mode I case of θ=90°. Again, $J_{exc}$ values occurred beyond maximum load.

Because blunting behavior in mode III is controlled by shear stresses, mode III blunting behavior was assumed to follow the relation

$$J_{iii}^{bl} = 2\tau_y \Delta a = \sigma_f \Delta a$$

prior to real crack extension.

For combined mode specimens, the relation between horizontal and vertical displacements were nearly linear after an initial transient (Figure 30). The slopes of these linear relations varied with specimen and crack angle with most values falling between 0.15 and 0.35. In several specimens initial changes in horizontal displacements were "negative". This phenomena is shown in Figure 31 as corresponding to the situation where the first displacement increments were predominantly the mode I type before the subsequent mode III component took over.
Figure 29. Load and crack extension versus load-line displacement for a combined mode test.
Figure 30. $\delta_h$ versus $\delta_v$. 
Figure 31. Consecutive load line displacement increments during combined mode loading.
Mode III J-Integral Calculation

Although stable crack extension along the desired path was achieved in mode III testing, optical observation of the extending crack tip did not provide sufficient resolution for single specimen J determination. Visual examination was also limited to the upper surface of the specimen and post-mortem inspection of the mode III fracture surfaces revealed that some cracks led slightly at the upper surface. For these reasons another technique, the multiple specimen method, was used to determine $J_{III}$ values. For each of the heat treatments, three specimens were loaded to different total displacements using visual examination to estimate the extent of crack advance. The incompletely fractured specimens were heat tinted at $350^\circ\text{C}$ for 90 minutes and broken open at $-196^\circ\text{C}$ (liquid $\text{N}_2$). Actual crack lengths were measured on the heat tinted surfaces.

$J_{III}$ values were calculated using the assumed relation

$$J_{III} = \frac{f_0 U}{b B_{\text{net}}} \quad [11]$$

where $f_0$ was taken equal to 1.0 for the three-legged specimen since it contains 2 cracks, $U$ is the area under the load-displacement curve, $b$ is the remaining specimen ligament length, and $B_{\text{net}}$ is the net crack front width of 2.54 mm.
The load-displacement curves for all mode III tests appeared qualitatively like Figure 32. Crack initiation occurred well into the plastic, nearly flat region of the curve. The limited number of specimens tested prohibit a full characterization of mode III J-resistance behavior. One peak aged specimen, for example, exhibited a markedly larger maximum load than any of the other specimens, as well as undergoing extensive crack growth at small total displacements. Although this behavior does not greatly influence the apparent $J_{\text{IIIc}}$ value, it does change the apparent tearing modulus and was not used in that calculation.

$J_{\text{IIIc}}$ values were estimated by extrapolating the resistance curves to zero crack extension. Since most specimens underwent little crack growth, these values are probably reasonable estimates. However, $J_{\text{IIIexc}}$ could only be calculated by extrapolation to relatively large crack growth increments and consequently are not reported. Similarly, $T$ values were calculated but may differ appreciably from real behavior because of scatter in the few data available.

Interpretation of Calculated $J$ Values

Although the use of the $J$-integral in characterizing mode I fracture initiation resistance is well developed, the extension of $J$-integral methodology to combined mode
Figure 32. Typical mode III specimen load-displacement behavior.
fracture in NiCop, an elastic-plastic material, is not a standard procedure. In particular, it is not clear how the components of J for the combined mode loading case relate to the deformation field ahead of the crack tip since the "mode I" and "mode III" stress fields are not independent except for linear elastic conditions. However, the energy resolution process used in formulating $J_i$ and $J_{iii}$ components does separate the energy expended in each of the two loading modes so that the J values might be more generally thought of as effective R values which correspond to the material's resistance to fracture in each mode.

Table 5. J-Integral Results (kJ/m²).

<table>
<thead>
<tr>
<th>Mode</th>
<th>$J_{ic}$</th>
<th>$J_{iexc}$</th>
<th>$J_{iiic}$</th>
<th>$J_{iiiexc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Aged</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mode I ($\theta=90^\circ$)</td>
<td>467</td>
<td>1405</td>
<td>40</td>
<td>108</td>
</tr>
<tr>
<td>$\theta=75^\circ$</td>
<td>391</td>
<td>1019</td>
<td>183</td>
<td>339</td>
</tr>
<tr>
<td>$\theta=55^\circ$</td>
<td>230</td>
<td>373</td>
<td>237</td>
<td>364</td>
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<td>$\theta=45^\circ$</td>
<td>137</td>
<td>175</td>
<td>270</td>
<td>633</td>
</tr>
<tr>
<td>$\theta=35^\circ$</td>
<td>71</td>
<td>124</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mode III</td>
<td></td>
<td>328*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overaged</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mode I ($\theta=90^\circ$)</td>
<td>563</td>
<td>1788</td>
<td>60</td>
<td>148</td>
</tr>
<tr>
<td>$\theta=75^\circ$</td>
<td>526</td>
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<td>265</td>
<td>394</td>
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<td>290</td>
<td>409</td>
<td>329</td>
<td>529</td>
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<tr>
<td>$\theta=45^\circ$</td>
<td>164</td>
<td>248</td>
<td>384</td>
<td>886</td>
</tr>
<tr>
<td>$\theta=35^\circ$</td>
<td>105</td>
<td>217</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mode III</td>
<td></td>
<td>376*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Estimated values from limited data.
Tearing Moduli

Values of tearing modulus were calculated for all specimen configurations from the Paris relation

\[ T = \frac{dJ}{da} \frac{E}{\sigma_f^2} \] \hspace{1cm} [12]

where \( E \) = Young's Modulus, \( \sigma_f \) = Effective Flow Strength, \( \sigma_f = (\sigma_Y + \sigma_u)/2 \), \( dj \) = slope of the J-resistance curve, \( da \) = between exclusion lines.

Tearing modulus values for all crack orientations are compiled in Table 6.

<table>
<thead>
<tr>
<th></th>
<th>Overaged</th>
<th>Peak-Aged</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T_i )</td>
<td>( T_{iii} )</td>
</tr>
<tr>
<td>( \theta = 75^\circ )</td>
<td>246</td>
<td>125</td>
</tr>
<tr>
<td>( \theta = 55^\circ )</td>
<td>184</td>
<td>94</td>
</tr>
<tr>
<td>( \theta = 45^\circ )</td>
<td>33</td>
<td>23</td>
</tr>
<tr>
<td>( \theta = 35^\circ )</td>
<td>23</td>
<td>6</td>
</tr>
<tr>
<td>mode III (( \theta_{eff} = 0^\circ ))</td>
<td>31</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Overaged</th>
<th>Peak-Aged</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( T_i )</td>
<td>( T_{iii} )</td>
</tr>
<tr>
<td>( \theta = 75^\circ )</td>
<td>106*</td>
<td>9</td>
</tr>
<tr>
<td>( \theta = 55^\circ )</td>
<td>173*</td>
<td>50</td>
</tr>
</tbody>
</table>
| \( \theta = 45^\circ \) | 9 |*
| \( \theta = 35^\circ \) | |*

*Estimated values from limited data.
Fractography

Macroscopic Examination

After separation of the incompletely broken specimen halves, all fracture toughness specimens were examined optically at low magnification. Representative examples with varying crack inclination angles are pictured in Plate I for both the peak aged and overaged conditions. In all mode I and combined mode tests, the crack front is not linear, but rather tends to lead at the free surfaces where the crack front intersects the machined side grooves. The same tendency is exhibited by the final positions of the fatigue precracks. In all cases the apparent fatigue crack lengths measured optically at the surface do not extend to the specimen centers. Further, total fatigue crack length decreases monotonically with decreasing crack inclination angle. In mode III specimens, the crack fronts are relatively straight, leading only very slightly at the surfaces. The side grooves effectively guided the advancing cracks in all geometries.

The macroscopic appearance of all specimens suggests that ductile fracture mechanisms are responsible for crack extension in NiCop. Extensive plastic deformation is indicated by the macroscopically rough surface topographies of the $\theta=90^\circ$ (pure mode I) and $\theta=75^\circ$ specimens. Through thickness displacements are also apparent in these geometries (Plate I). The overaged specimens, in
PLATE I. Macroscopic Features of Mode I and Combined Mode Fracture Specimens.

Top left: Peak-aged specimens containing crack inclination angles (from top to bottom) of $90^\circ$, $55^\circ$, $45^\circ$, and $35^\circ$.

Top right: Overaged specimens with the same geometries as photo at left. Note the larger through-thickness displacements in the mode I specimen than in its peak-aged counterpart. Arrows indicate that crack fronts led at the side groove locations.

Bottom: This typical $\theta=45^\circ$ surface contains highly reflective "flat" areas visible to the naked eye.
particular, display pronounced necking or "sucking in" in the crack tip region. These displacements lead to differences in the amount of light reflected off the specimen sides when viewed obliquely, making the thinned regions visible. Plate II contains photos of this effect for two $\theta=75^\circ$ specimens. If these regions at least roughly correspond to the size and shape of the plastic zone, yielding appears to be contained in these specimens. The larger, better defined "plastic zone" of the overaged condition is expected in view of its lower yield strength.

For crack inclination values decreasing from $\theta=75^\circ$, there is a transition in the macroscopic appearance of the fracture surfaces. Variations in surface topography decrease with decreasing crack angle; the gross surface irregularities become smaller and the crack surfaces appear more nearly planar. Simultaneously, highly reflective "flat" appearing facets show up on the surfaces, particularly for $\theta=35^\circ$ and $\theta=45^\circ$ geometries (Plate I).

Fatigue Precracks

SEM examination of the fatigue precrack regions reveal variations in fatigue crack morphology as a function of crack angle. Although a sharp, flat precrack is obtained for mode I loading, addition of a concurrent mode III component changes the fatigue crack appearance. In combined mode configurations the fatigue precrack breaks up
PLATE II. Specimen Plastic Zones.

In these $\theta = 75^\circ$ specimens, through-thickness displacements indicate approximate plastic zone size and shape.

Top: Peak-aged specimen exhibits ill-defined plastic zone.

Bottom: Overaged specimen displays a nominally circular plastic zone indicating contained yielding to this extent of crack extension.
Plate II
into many alternate segments, each of which is inclined to the nominal desired crack path (Plate III). This "factory roof" crack morphology has been observed earlier in combined mode I - mode III fatigue work by Pook. (93) This behavior is similar for both heat treatments. As is apparent in Plate III, the extent of fatigue crack extension at the specimen center line decreased monotonically with crack angle. The initial ductile fracture regions in this plate also show qualitatively the decreased surface roughness with decreasing crack angle. Although the local stress concentration produced at the irregular combined mode fatigue cracks is undoubtedly different than those produced by sharp cracks, the extensive plasticity prior to crack initiation in NiCop should minimize any effect on initiation behavior.

Mode I Ductile Fracture

SEM investigation of the mode I fracture surfaces reveals that ductile fracture occurred by the processes of void initiation, void growth, and coalescence. Mode I surfaces are very similar in the peak aged and overaged specimens, although the latter display somewhat greater surface roughness, with vertical elevation changes over the surface of 1-2 mm. Plate IV illustrates typical opening mode fracture features. Several distinct populations of voids are present. Deep pits 50-100 μm across are spaced
Plate III. Typical Fatigue Precrack Morphologies.

Crack propagation was from left to right in all photos.

Top left: Mode I loading produces a very flat, sharp fatigue crack.

Top center: The presence of a small mode III component produces small ridges (arrow) at the onset of fatigue cracking.

Top right: Larger mode III components produce a highly "stepped" fatigue crack morphology.

Bottom left: Total fatigue crack length decreases with crack angle.

Bottom right: "Stepped" fatigue precrack.
PLATE IV. Mode I Fracture Surface Morphologies.

Crack propagation was from left to right.

Top left: Heterogeneous mode I fracture surface has many dimple sizes.

Top right: Void initiating particles are visible at the dimple bottoms.

Bottom left: Mode I dimples are predominantly equiaxed.

Bottom right: Fracture morphology near the side grooves exhibits elongated dimples, suggesting a change in fracture micromechanism or extensive straining after passage of the crack front.
300-500 μm apart. The initiation sites of these voids are not visible but are probably large inclusions carried over from the rolling operation since their population is similar for both heat treatments. These large pits are linked by densely dimpled areas containing shallower dimples 1-20 μm in diameter. In all opening mode specimens, dimples are nearly equiaxed or only locally elongated because of local topographical variations. Void initiating particles of 0.5-1 μm are visible within the small dimples. This size corresponds to the fine equiaxed sulfides present in the microstructure. Secondary cracks (usually aligned with the plane of the plate) were also visible.

In marked contrast, the fracture areas in the vicinity of the side grooves suggest a change in local fracture micromechanisms (Plate IV). In these regions dimples are highly elongated in the direction perpendicular to crack propagation. The dimple density is greatly reduced and dimples no longer impinge directly on one another. Instead, nearly smooth, featureless areas cover much of the surface between widely spaced dimples. Because this is the same region where fracture initiated, it is possible that crack growth occurred by normal microvoid coalescence in the amplified triaxial stress state near the side grooves. Subsequent plastic deformation in these outer regions prior to eventual crack extension at the specimen centerline may
lead to the unusual final appearance in the areas adjacent to the side grooves.

Combined Mode I - Mode III Fracture Surfaces

Combined mode fractures exhibit morphologies that vary strongly with crack angle. Again, however, specific features are similar for the two NiCop conditions. The surfaces of $\theta=75^\circ$ specimens most closely resemble those of the pure opening mode geometry (Plate V). Again, several dimple populations exist, with the majority of dimples nearly equiaxed. One new feature type not found in the mode I specimens is the "macrovoid" of Plate V. These macrovoids have dimpled walls and are several hundred micrometers long, in contrast to the deep pits which have nearly smooth interior walls. "Splits" are also found on the surfaces (Plate V). These result from decohesion along planes of weakness parallel to the plane of the plate. They are distinguishable from macrovoids by the striated appearance of their internal walls and have been commonly observed in HSLA steels. (94) These splits probably originate when there is a large sulfide density along planes in the plate midsection from the rolling process. In one $\theta=75^\circ$ specimen, a very large such decohesion zone opened, producing a secondary crack parallel to the sides of the plate reaching several millimeters ahead of the ductile fracture plane.
PLATE V. Fracture Surfaces for Low Mode III Component.

Top left: $\theta=75^\circ$ surface contains many dimple sizes.

Top center: Dimple shape is similar to pure mode I.

Top right: Side groove at photo bottom. As for pure mode I specimens, fracture appearance changes in the vicinity of the side grooves.

Bottom left: A "macrovoid" (arrow) with dimpled interior.

Bottom right: A "split" where delamination has occurred along an internally weak plane.
Plate V
For crack inclination angles $\leq 55^\circ$ the fracture surfaces exhibit noticeably elongated dimples (Plate VI), with the aspect ratio increasing with decreasing angle (increased mode III). All such dimples are elongated in a direction parallel to the crack front. This dimple morphology has been observed previously in ductile fractures by Beachem (95) when a mode III load component was present. Novel features found only in the low $\theta$ specimens are flat-appearing dimple-free regions which correspond to the highly reflective "facets" observed at lower magnification (Plate VI). These features are irregularly shaped and vary in size from tens to hundreds of micrometers in diameter. Stereo pairs revealed that these areas appear to lie at angles inclined to the nominal crack propagation plane. In many cases striations aligned with the mode III displacement direction suggest that these areas are produced by rubbing of the specimen halves during crack extension.

Mode III Fracture Surfaces

The mode III fractures produced in the "pant-leg" specimen geometry closely resemble the $\theta=35^\circ$ specimens subjected to mixed mode loading. Both elongated dimples and the "flat" areas are visible on fracture surfaces (Plate VII). Stereo pairs confirm that as in combined mode loading there are still local elevation variations from the
PLATE VI. Combined Mode Fracture Surfaces.

Crack propagation was from left to right.

Top left: \( \theta = 45^\circ \) surfaces contain "flat" appearing features (arrows).

Top right: Elongated dimples result from combined modes I and III.

Bottom left: \( \theta = 55^\circ \) surfaces contain a greater number of "flat" features.

Bottom center: \( \theta = 55^\circ \) surfaces exhibit very elongated dimples.

Bottom right: Close-up of dimple-free region.
PLATE VII. Mode III Fracture Surfaces.

Crack propagation was from left to right.

Top left: Typical mode III fracture surface.
Top center: Elongated dimples resulting from mode III fracture.
Top right: Deformed surface material has been forced over the side groove during deformation.
Bottom left and right: Stereo pair reveals that substantial surface roughness persists to mode III fracture.
nominal fracture plane. Large local mode III strains are indicated by the appearance of "smeared" material which overhangs the original notch root position in the direction of specimen-half travel. This phenomenon was also observed in combined mode loading. Because of concern that out-of-plane bending (resulting in local mode I tensile or compressive components) might affect the results, the upper and lower halves of the fracture surface were compared. No permanent difference is visible in the two regions; thus, out-of-plane bending is small. Supporting evidence is also provided by the nearly straight crack fronts obtained in mode III testing.

**Mode III Compliance Analysis**

In the plate-type specimen used for mode III fracture characterization, displacements imposed at a position remote from the crack tips produce shear stresses in the ligament regions. The relationship between load line displacements and the crack tip stress intensity factors are determinable for elastic behavior by compliance analysis. Such an analysis was carried out to better quantify the stress state in the mode III geometry.

The load point displacement $\delta$ in a linear elastic body is related to the magnitude of the applied load $P$ through the relation
\[ \phi = \frac{\delta}{P} \]  \hfill [13]

where \( \phi \) is the compliance. The compliance is a geometry-dependent quantity and for the fixed gross dimensions of the mode III specimen may be considered a function of crack length and elastic constants only.

For linear elastic conditions the energy rate, or energy per unit of new crack area generated, available for crack extension is (91)

\[ G = \frac{P^2}{2} \frac{d\phi}{dA} \]  \hfill [14]

where \( dA \) is the total increase in crack increment or \( 2B_{net} da \) for the mode III geometry (2 cracks). For the most general case the available energy rate may be divided into three individual components for the three possible loading modes

\[ G = G_I + G_{II} + G_{III} \]  \hfill [15]

since direct superposition of the elastic stress components is allowed. This relation may be represented equivalently in terms of the three stress intensity factors

\[ G = \frac{(1-\nu^2)}{E} K_I^2 + \frac{(1-\nu^2)}{E} K_{II}^2 + \frac{(1+\nu)}{E} K_{III}^2 \]  \hfill [16]

for plane strain conditions.
With the assumption that mode I and mode II stress components are negligible in the plate-type specimen, Equation [16] reduces to

$$G = G_{III} = \frac{(1+v)}{E} K_{III}^2$$  \[17\]

and the elastic mode III stress intensity factor is determinable from [14] and the compliance versus crack length relation.

Compliance testing was done using a 6061-T651 aluminum alloy plate specimen. The relatively low elastic modulus of aluminum allows greater displacements during elastic loading and as a result, compliances can be determined more accurately. When the results are transformed to dimensionless quantities, the compliance versus crack length results can be applied to any alloy system for which the elastic constants are known.

Because no data exist describing the elastic behavior of this type of specimen, approximations of the expected behavior were developed, for comparison, by modelling the cracked portion of the mode III specimen as three parallel cantilever beams with cross sections equivalent to the specimen legs and lengths equal to the crack length a. For the beam geometry of Figure 33 where u and v are displacements in the x and y directions respectively, two possible boundary conditions may be imposed at the fixed
Figure 33. Assumed cantilever beam geometry for approximate compliance analysis.
end of the beam. The beam may have zero slope at this position

\[
\text{Condition I } \left[ \frac{dy}{dx} \right]_{x=a, y=0} = 0 \tag{18}
\]

or displacements in the x-direction may be taken equal to zero

\[
\text{Condition II } \left[ \frac{du}{dy} \right]_{x=a, y=0} = 0 \tag{19}
\]

For both cases the total deflection analogous to that of the actual 3 legged specimen is composed of two parts, \( \delta_c \) resulting from a load \( P \) applied to the middle beam, and \( \delta_o \) resulting from the load \( P/2 \) applied to each of the outside beams.

For condition I at the position \( x=a \) (92)

\[
\delta_t = \delta_c + \delta_o = \frac{Pa^3}{3EI} + \frac{Pa^3}{6EI} = \frac{Pa^3}{2EI} \tag{20}
\]

\[
\phi = \frac{\delta}{P} = \frac{a^3}{2EI} \tag{21}
\]

For condition II

\[
\delta_c = \frac{Pa^3}{3EI} + \frac{Pc^2a}{2IG} \tag{22}
\]

\[
\delta_o = \frac{Pa^3}{6EI} + \frac{Pc^2a}{4IG} \tag{23}
\]
\[ \delta_t = \frac{Pa^3}{2EI} + \frac{3Pc^2a}{4IG} \] [24]

\[ \phi = \frac{a^3}{2EI} + \frac{3c^2a}{4IG} \] [25]

Dimensionless compliances, EB\(\phi\), are plotted versus dimensionless crack length, a/S, in Figure 34 for the experimental and cantilever beam analog results. For short crack lengths, the mode III specimen is more compliant than either model system. This is plausible if the bulk specimen ahead of the notched region contributes an effective beam length over and above the actual crack length. The convergence of the experimental and beam-model behavior for longer crack lengths supports this view since for the case of large a/W the contribution of the beam legs would dominate any contributions from the uncracked region. Of course, the mode III specimen outside legs are really subjected to distributed loads so the cantilever beam analogs provide only an approximation to the real specimen behavior. The behavior of the dimensionless compliance derivatives as functions of crack length is presented in Figure 35. Again, the difference between the mode III specimen values and those of the two analog models decreases with increasing crack length.

In order to calculate the stress intensity factor as a function of load for the actual specimen configuration, the presence of the side grooves must be taken into account:
Figure 34. Dimensionless compliance versus dimensionless crack length for the mode III specimen and cantilever beam analogs.
Figure 35. Dimensionless compliance derivative versus dimensionless crack length for the mode III specimen and cantilever beam analogs.
\[ G_{\text{III}} = \frac{P^2}{2} \frac{d\phi}{da} = \frac{P^2}{2} \frac{d\phi}{2B_{\text{net}} da} \]  \[26\]

and for the mode III specimen, the mode III stress intensity factor is
\[ K_{\text{III}} = \frac{P}{2} \sqrt{\frac{1}{B_{\text{net}}} \frac{d\phi}{da} \frac{E}{(1-\nu)}} \] \[27\]

In using the mode III specimen for testing purposes in NiCop, the load-displacement curve exhibits significant plasticity, indicating extensive yielding prior to crack extension. For this reason the elastic compliance relation determined above cannot be used to determine a specimen \( K_{\text{IIIc}} \). However, the relationship would be useful in testing materials that exhibit linear elastic behavior in the same geometry, as long as the elastic modulus is known.
CHAPTER IV

DISCUSSION

J Components in Mixed Mode Loading

The use of the modified compact tension specimen in obtaining mixed-mode fracture toughness data warrants discussion since it has not been used previously for materials exhibiting significant plasticity during fracture toughness testing. Unlike tension-torsion techniques applied to linear elastic materials (72,80) where the instantaneous values of tensile load and applied torque may be unambiguously related to the mode I and mode III stress intensity factors respectively, the relative contributions of opening mode and transverse shear to fracture in the plate geometry can only be estimated prior to testing. Two schemes allow calculation of approximate applied \( \frac{J_{III}}{J_i} \) ratios as a function of crack angle, and these were used in selecting the geometries used here in combined mode testing.

One method of estimating \( \frac{J_{III}}{J_i} \) values relies on previous observations (97) that to first order, horizontal displacements at the load line are negligible up to crack initiation in the slant notch specimen. Recalling the
earlier J formulations obtained using resolution of energy components, we now define an apparent J value

\[ J_a = 2b_p \int_0^\delta_v \frac{P}{b^2} d \delta_v \]  \[1\]

which would be the magnitude of J calculated neglecting the presence of the crack inclination. Now, neglecting horizontal displacements,

\[ \delta_I = \delta_v \sin \theta \]  \[2\]

\[ \delta_{III} = \delta_v \cos \theta \]  \[3\]

and, as discussed previously

\[ P_I = P \sin \theta \]  \[4\]

\[ P_{III} = P \cos \theta \]  \[5\]

Calculation of the mode I J component as before, including the crack orientation, yields

\[ J_i = 2b_p \sin^2 \theta \int_0^{\delta_v} \frac{P}{b^2} d \delta_v \sin \theta \]  \[6\]

or

\[ J_i = J_a \sin^3 \theta \]  \[7\]

Similar application to the mode III J component yields

\[ J_{III} = J_a \sin \theta \cos^2 \theta \]  \[8\]

Thus, during monotonic loading the \( J_{III}/J_i \) ratio is
expected to remain constant at
\[
\frac{J_{iii}}{J_1} = \frac{\cos^2 \theta}{\sin^2 \theta}
\]  

for negligible horizontal displacements. This predicts a linear trajectory emanating from the origin in $J_1$-$J_{iii}$ space for each $\theta$ value as displayed in Figure 36.

A second prediction of $J_{iii}/J_1$ ratios is obtained from linear elastic calculation of resolved stress intensities. In Figure 37 a remote tensile stress is applied to the slant notch geometry. The crack plane is inclined to the load direction at an angle $\theta$. The normal and shear stresses $\sigma_{22}'$ and $\sigma_{12}'$ acting on the crack plane are (98)
\[
\sigma_{22}' = \sigma \sin^2 \theta \\
\sigma_{12}' = \sigma \sin \theta \cos \theta
\]

and the stress intensities are
\[
K_I = \sigma \sin^2 \theta \sqrt{\pi a} f(\text{geometry}) \tag{12}
\]
\[
K_{III} = \sigma \sin \theta \cos \theta \sqrt{\pi a} g(\text{geometry}) \tag{13}
\]

These values of stress intensity are related to an apparent $K_a$ equal to $\sigma \sqrt{\pi a}$ (which would be the globally measured value neglecting crack orientation) if the functions $f$ and $g$ are near unity. This assumption leads to the relations used by Pook (74)
Figure 36. Predicted loading trajectories in $J_i$-$J_{iii}$ space.
Figure 37. Slant notch configuration for stress intensity resolution.
and predicts that the mode III/mode I ratio during remote loading is

\[
\frac{K_{III}}{K_I} = \frac{\cos \theta}{\sin \theta} \quad [16]
\]

or, through conversion to small scale yielding, plane strain J values

\[
\frac{J_{iii}}{J_I} = \frac{\cos^2 \theta \cdot 1}{\sin^2 \theta (1-\nu)^2} \quad [17]
\]

These predicted J ratios are somewhat larger than those calculated from the energy resolution method neglecting horizontal displacements (Figure 36). But the differences are rather small considering the assumptions invoked, and even better agreement might be obtained if the edge notched tension result that \(f > g\) applies for the deeply cracked bend specimen used here.

In actual testing, small horizontal displacements occurred prior to crack extension and the resulting loading paths typically followed curved trajectories as shown in Figure 38. In all specimens containing cracks oriented at \(\leq 55^\circ\), the \(J_{iii}/J_I\) ratio increased substantially throughout testing. Because the data resolution method assumes that
Figure 38. Experimental loading trajectories in $J_1$-$J_{1ii}$ space.
the load contributions parallel and perpendicular to the crack plane are fixed, the increase in relative transverse shear/opening mode results from deformation producing larger relative mode III displacements. This behavior is clearly illustrated in Figure 39. Conversely, the specimens containing cracks oriented at $\theta = 75^\circ$ exhibited little change in the relative mode I and mode III contributions (Figure 40).

Macrosopic Fracture Parameters

J-Integral Behavior

$J$ values at fracture initiation (normalized to the true crack front width) are plotted in Figure 41 for both of the heat treated conditions. In view of experimental uncertainties, both fracture initiation loci are adequately described by linear expressions and regression analysis yields

Peak Aged $J_{iiic} = -0.715 \ J_{ic} + 331 \ \text{kJ/m}^2$ [18]

Overaged $J_{iiic} = -0.808 \ J_{ic} + 473 \ \text{kJ/m}^2$ [19]

It is evident that for all load combinations the lower strength, overaged material exhibits greater toughness than the high strength, peak aged condition. The presence of any mode III component also appears to decrease the mode I contribution necessary for crack initiation over the range
Figure 39. $J_{iii}/J_i$ ratio versus $\delta_v$ for a small $\theta$ specimen.
Figure 40. $J_{iii}/J_1$ ratio versus $\delta_v$ for a large $\theta$ specimen.
Figure 41. Combined mode fracture initiation locus. Open symbols designate mode III specimen data.
of geometries tested. This agrees with most other mode I - mode III results for small scale yielding and elastic behavior.

Values of J calculated at the exclusion line (nominal crack growth of 3mm) do not appear to maintain linear behavior for low $\theta$ specimens (Figure 42). However, the large $J_{III}$ contributions for all $\theta = 35^\circ$ specimens may result from effects other than damage within the fracture process zone and will be discussed subsequently.

Opening and Tearing Mode Contributions in Combined Mode Loading

Individual mode I and mode III J values at crack initiation and after 3 mm of stable growth are plotted as a function of specimen angle for both heat treated conditions in Figures 43 through 46. The choice of abscissa was arbitrary, but allows convenient comparison between heat treatments, geometries, and cracking modes. Figures 43 and 44 show that $J_1$ values decrease with increasing applied mode III load components both for crack initiation, and more strongly, after 3 mm of growth. Over the same range of loading modes, $J_{III}$ values increase with increasing mode III component, again with larger changes in exclusion J values (Figures 45 and 46).

The apparent similarity in mixed-mode behavior for the peak aged and overaged materials is confirmed by
Figure 42. J-integral values following 3mm of crack growth.
Figure 43. Mode I J vs. \( \theta \), peak aged condition.
Figure 44. Mode III J vs. $\theta$, peak aged condition. Arrow indicates mode III specimen datum.
Figure 45. Mode I $J$ vs. $\theta$, overaged condition.
Figure 46. Mode III $J$ vs. $\theta$, overaged condition. Arrow indicates mode III specimen datum.
normalizing the data of 43-46 and plotting them together on the same graph. By dividing all $J_i$ and $J_{iii}$ values by the maximum $J_{iexc}$ and $J_{iiiexc}$ values respectively for the individual heat treatments, normalized fracture behavior in the slant notch specimens proves to be identical over the range of load conditions studied for both peak aged and overaged conditions (Figure 47). Thus, although the overaged material is universally tougher for differing mode I - mode III combinations, the relative opening mode and tearing mode contributions are individually the same for both microstructures.

Tearing Moduli

The widely different vertical separations between initiation and exclusion $J$ values in Figures 43-46 result from strong variations in tearing moduli as a function of mode and crack angle (Figure 48). Mode I tearing moduli drop off rapidly with decreasing crack angle, reaching a weakly defined minimum roughly near $\theta=45^\circ$. Mode III tearing moduli increase with decreasing crack angle, but less strongly. In comparing heat treated conditions, tearing moduli results mirror fracture toughness comparisons. For all load conditions the overaged microstructure displays greater resistance to stable fracture for both opening and tearing mode components. Further, normalizing the $T$ data also produces identical
Figure 47. Normalized $J/J_{exc}$ vs. $\theta$. Arrows indicate mode III specimen data.
Figure 48. Tearing moduli vs. $\theta$. Arrows indicate mode III specimen data.
relative $T_i$ and $T_{iii}$ values for the two microstructures. The general observation in pure mode I testing that large tearing modulus values accompany large fracture initiation resistance (99) is shown to extend to the individual components of combined-mode fracture as well.

Although only limited pure mode III testing was done, the estimated values of $J_{iiic}$ and $T_{iii}$ agree favorably with the trends established in modified CT specimens. This is somewhat surprising in view of the different geometries and crack detection methods used in the respective measurements. Apparent pure mode III tearing moduli were larger than any $T_{iii}$ components measured under combined mode, continuing the trend that $T_{iii}$ increases with mode III component. $J_{iiic}$ values also fell in the range expected from extrapolation of combined mode data.

**Energetics of Crack Advance**

Initially mode I cracks propagating in NiCop and other tough materials often reorient to oblique planes during stable crack growth. Since the crack front is expected to adopt the orientation which minimizes the propagation energy, mixed-mode measurements of stable crack growth resistance for fixed crack inclination angles could, in principle, yield predictive capabilities regarding the energetically optimum crack plane.
Globally determined J-integral values are directly proportional to the energy input to the cracked body and serve directly as relative energy measures in the following discussion. In preface to the subsequent considerations it is reiterated that all J's in this work are normalized with respect to the crack front width and hence reflect energy input per unit of physical crack area formed.

For the case of a crack propagating through a component with plate geometry, two factors determine the energetically optimum propagation plane. First, simple geometry reveals that a mode I crack which rotates to take on a mode I – mode III orientation increases its width and surface area according to

\[
\text{Thickness \ } B_{\text{actual}} = \frac{B_{\text{plate}}}{\sin \theta}
\]

where \( \theta \) is the crack inclination angle. This reciprocal sine function is of course minimum for a pure mode I crack (\( \theta = 90^\circ \)) and approaches infinity as \( \theta \to 0 \). Secondly, because the material damage processes leading to ductile fracture may vary significantly with loading mode, the energy input necessary for crack extension is also expected to be a function of \( \theta \).

To treat the total energetics of crack propagation, a new parameter \( J_{\text{total}} \) is introduced. This parameter is defined by
where the vectors \( \vec{P} \) and \( \vec{\delta} \) denote total values of load and displacement respectively. This quantity is equal to the global J value obtained from measuring load and vertical displacements in modified compact tension testing without resolution of mode I and mode III components, but taking the actual crack front width into account. The parameter \( J_{total} \) is numerically equivalent to the algebraic sum of \( J_I \) and \( J_{III} \) since these are proportional to the elements of the vector sum in the dot product \( \vec{P} \cdot \vec{\delta} \).

Figure 49 reveals that at crack initiation, \( J_{total,c} \) decreases only weakly with \( \theta \). Since the total propagation energy is proportional to \( J_{total,c} / \sin \theta \), this behavior suggests that the energy for crack initiation is minimum for the pure mode I geometry. Consideration of \( J_{total} \) values after 3 mm of crack extension, however, show markedly different behavior. Figure 50 reveals that \( J_{total,exc} \) first decreases strongly as \( \theta \) is varied from the pure mode I configuration, reaches a minimum value, and then increases. Multiplication by the reciprocal sine geometric factor does not change this behavior qualitatively, it merely increases the sharpness of the minimum and shifts it to slightly larger \( \theta \) values. Because of the limited number of geometries tested in this
Figure 49. $J_{\text{total, } C}$ vs. $\theta$. Arrows indicate mode III specimen data.
Figure 50. $J_{\text{total,exc}}$ vs. $\theta$. 
investigation, exact minima positions were not obtained, but for both material conditions they reside in the vicinity of $45^\circ < \theta_{\text{min}} < 55^\circ$. This result compares favorably with the findings of Miglin (100) where for stable crack propagation in non-grooved plate specimens, crack rotation also produced $45^\circ < \theta_{\text{min}} < 55^\circ$ in the peak aged condition; however, the overaged specimens converted to blunted double shear in that investigation.

The slope of the $J_{\text{total}}$-resistance curve may be defined as

$$T_{\text{total}} = T_i + T_{\text{iii}}$$

and behavior of $T_{\text{total}}$ versus $\theta$ also supports minimum propagation resistance for the two conditions of NiCop at crack inclination angles of $45^\circ - 55^\circ$ (Figure 51).

**Superdislocation Model**

The linear form of the fracture initiation criterion in $J_i$-$J_{\text{iii}}$ space (Figure 41) is predicted by a relatively simple model of combined mode crack loading under elastic-plastic conditions. Consider the dislocation array of the mode I - mode III plastic zone replaced by two superdislocations in an elastic continuum as in Figure 52. The presence of the edge superdislocation decreases the crack tip stress intensity factor $K_T$ in a manner analogous to the diffuse shielding contribution of the plastically
Figure 51. $T_{\text{total}} = T_1 + T_{\text{III}}$. Arrows indicate mode III specimen data.
Figure 52. Assumed crack tip configuration for application of the superdislocation model of crack initiation.
deformed region it represents. Similarly, the screw superdislocation acts to shield the crack tip stress field and to decrease the local $K_{III}$ value. Since the mode I and edge dislocation stress fields share no common non-zero stress components with the mode III and screw dislocation stress fields, the shielding contributions of the superdislocations are uncoupled and the dislocations do not interact with one another. The assumption of linear elastic conditions allows linear superposition of all deformation field contributions.

To mirror real behavior, the shielding contributions of the superdislocations should increase with the externally applied stress field, just as shielding is expected to increase with plastic zone size in real materials. This condition is met if the superdislocation Burgers vectors are taken to be equal to the corresponding values of the crack tip opening displacements. It is assumed that these values are expressible as

$$b_1 = \frac{\alpha K_{I,ext}^2}{\sigma_y}$$
$$b_3 = \frac{\beta K_{III,ext}^2}{\mu \sigma_y}$$

where $K_{I,ext}$ and $K_{III,ext}$ are the amplitudes, respectively, of the mode I and mode III crack tip stress fields resulting from externally applied stresses, $\sigma_y$ is the uniaxial yield stress, $E$ and $\mu$ are values of Young's
Modulus and the shear modulus respectively, and $\alpha$ and $\beta$ are constants.

Hirth and Wagoner (101) have determined the stress components produced in an elastic body by the interaction of a crack with neighboring dislocations lying parallel to the crack front. The assumed geometry is given in Figure 53 in the complex plane with the origin defined at the crack tip and the dislocation placed at position $\xi$. For the current problem, the stresses of interest are $\sigma_{22}$ and $\sigma_{23}$, the values of which are represented by

$$\sigma_{22} = \text{Re} \left\{ \phi'(z) + \phi'(\bar{z}) + (z-\bar{z}) \phi''(z) \right\} \quad [24A]$$

$$\sigma_{23} = \text{Re} \left\{ w'(z) \right\} \quad [24B]$$

for the field point position $z$, where

$$\phi(z) = -\frac{\mu b_1}{\pi(k+1)} \ln(z^{1/2} + \xi^{1/2})^2 \quad [25A]$$

$$w(z) = -\frac{\mu b_3}{\pi} \ln(z^{1/2} + \xi^{1/2})^2 \quad [25B]$$

and $\kappa=3-4\nu$ for plane strain and $\kappa=(3-\nu)/(1+\nu)$ for plane stress.

By differentiation of [25] and substitution into [24], the stress components $\sigma_{22}$ and $\sigma_{23}$ at the superdislocation positions are determined to be
Figure 53. Crack tip geometry used by Hirth and Wagoner (101) to determine dislocation-crack interactions.
at $z = r_1$ \[ \sigma_{22} = \frac{-\mu b_1}{\pi (\kappa + 1)r_1} \] \[ \text{[26A]} \]

at $z = r_3$ \[ \sigma_{23} = \frac{-\mu b_3}{4\pi r_3} \] \[ \text{[26B]} \]

Hirth and Wagoner also showed that the stress intensity factors, or amplitudes of the near-tip singular stress terms, produced by the interactions between the crack and superdislocations of Figure 52 are

\[ K_I = -2\sqrt{2} \frac{\mu b_1}{\pi (\kappa + 1)\sqrt{r_1}} \] \[ \text{[27A]} \]

\[ K_{III} = -\frac{\mu b_3}{\sqrt{2}\pi r_3} \] \[ \text{[27B]} \]

The stresses and stress intensity factors of [26] and [27] result solely from the interaction of the dislocation stress fields with the crack in the absence of externally applied stresses and will subsequently be subscripted with the letter "i" to denote that origin.

Continuing with the superdislocation model, it is assumed that the dislocations remain in the same plane as the crack and, further, that they will be stationary if the local stresses at the dislocation positions are

\[ \sigma_{22} = \sigma_y \text{ at } r_1 \text{ (edge dislocation)} \] \[ \text{[28A]} \]

\[ \sigma_{23} = \frac{\sigma_y}{2} \text{ at } r_3 \text{ (screw dislocation)} \] \[ \text{[28B]} \]

Since, for a loaded elastic body, the stresses in the near tip region consist of terms reflecting externally applied
stresses as well as the stresses produced by dislocation stress fields and dislocation-crack interaction terms, the total stress is

\[ \sigma_{\text{tot}} = \sigma_{\text{ext}} + \sigma_i \]  

were the subscript "ext" corresponds to stresses resulting from externally applied loads. Since \( \sigma_{\text{ext}} \) magnitudes may be related to globally applied \( K_{\text{ext}} \) values, combination of [28] and [29] yields

\[ \frac{K_{\text{Iext}}}{\sqrt{2 \pi r_1}} - \frac{\mu b_1}{\pi(k+1)r_1} = \sigma_y \]  

\[ \frac{K_{\text{IIext}}}{\sqrt{2 \pi r_3}} - \frac{\mu b_3}{4 \pi r_3} = \sigma_y/2. \]  

By multiplying through by the appropriate \( r \) values and completing the squares, one can calculate equilibrium values of \( r_1 \) and \( r_3 \) in terms of externally applied stress intensity factors, yield strength, and Burgers vector magnitudes.

Now, if attention is fixed at the crack tip, fracture initiation is assumed to occur when the local condition

\[ \frac{K_{I,\text{loc}}^2 + K_{\text{II,loc}}^2}{E} = \frac{2 \gamma_{\text{eff}}}{2\mu} \]  

is met. Superposition allows this to be rewritten

\[ (K_{I,\text{ext}}^2 - K_{I,i}^2)/E + (K_{\text{III,ext}}^2 - K_{\text{III,i}}^2)/2\mu = 2 \gamma_{\text{eff}} \]  

[32]
With the substitution of the equilibrium dislocation positions $r_1$ and $r_3$ into [27] along with the superdislocation Burgers vectors from equation [23] in terms of $K_{ext}$ values, [32] may be formulated in terms of applied $K$ values only.

Simplification leads to the final expression

$$J_1 \left[ 1 - \frac{q^2}{\sqrt{1-q+1}} \right]^2 + J_{iii} \left[ 1 - \frac{\beta^2}{\sqrt{1-\beta+1}} \right]^2 = 2 \gamma_{eff}$$  \[33\]

where $J_1$ and $J_{iii}$ are the applied $J$ values (=G values for this linear elastic treatment) and $q = 4a / (1+\nu)(\kappa+1)$. The linear form of [33] agrees qualitatively with the fracture initiation criteria obtained experimentally.

Evidently, for this expression to be valid, the constants $q$ and $\beta$ must both be less than one. An elastic-plastic solution by Rice (11) suggests that, for conditions of small scale yielding and perfect plasticity, $\beta = 1.1$ while $a$ may range from approximately 0.5 to 0.7 (108). Table 7 gives the slope of the linear fracture criterion predicted by equation [33] for several combinations of $a$ and $\beta$. The results show that several combinations of $a$ and $\beta$ give values for the slope $dJ_{iii}/dJ_1$ in the range of the values found experimentally (0.72 and 0.81 for the peak-aged and overaged conditions respectively), so that a unique correlation with experiment is not possible. Experimental results do suggest that $\beta < a$ and show that values of $a$, $\beta < 1$ would be consistent with
the data.

Table 7. Negative of the Slope of the Fracture Envelope in $J_{1}J_{11}$ Space Predicted by the Superdislocation Model.

<table>
<thead>
<tr>
<th>$q$</th>
<th>$\beta = 0.4$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
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<tbody>
<tr>
<td>0.5</td>
<td>0.96</td>
<td>1.00</td>
<td>1.06</td>
<td>1.15</td>
<td>1.32</td>
<td>1.72</td>
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<tr>
<td>0.6</td>
<td>0.91</td>
<td>0.95</td>
<td>1.00</td>
<td>1.09</td>
<td>1.25</td>
<td>1.62</td>
</tr>
<tr>
<td>0.7</td>
<td>0.84</td>
<td>0.87</td>
<td>0.92</td>
<td>1.00</td>
<td>1.15</td>
<td>1.49</td>
</tr>
<tr>
<td>0.8</td>
<td>0.73</td>
<td>0.76</td>
<td>0.80</td>
<td>0.87</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td>0.9</td>
<td>0.56</td>
<td>0.58</td>
<td>0.62</td>
<td>0.67</td>
<td>0.77</td>
<td>1.00</td>
</tr>
</tbody>
</table>

While the values of Table 7 are in the range of the predictions by Rice (11), they are not consistent with the small-scale yielding result that $\alpha < \beta$. Of course, in the present work the plastic zone is quite large, and the above inconsistency could arise if $\alpha$ and $\beta$ values deviate significantly from the small-scale yielding results, perhaps as a function of strain hardening exponent as well as plastic strain.

In review of the superdislocation model, the shielding of the crack tip by a large plastic zone is modelled by the presence of two superdislocations, one edge (mode I component) and one screw (mode III component), ahead of the crack tip. Because it is assumed that these dislocations and their associated stress fields do not interact, their Burgers vectors and shielding contributions depend only on the externally applied stresses with components which
correspond to their respective stress fields. If all assumptions hold, this implies that measurements of \( \text{CTOD} = f(K_{\text{app}}) \) for pure mode I and pure mode III conditions would give the slope of the fracture initiation criterion in \( J_1 - J_{\text{III}} \) space through the constants \( a \) and \( \beta \), and material elastic properties.

This approach uses a local strain energy release rate condition imposed at the crack tip in which the mode I and mode III \( J(G) \) contributions are equally effective in producing crack initiation. The shielding effect of the superdislocations, however, coupled with the assumption of equilibrium positions of the dislocations leads to different abilities of the applied \( J \) values to initiate crack growth. This elucidation of possible shielding effects is really the strength of the model.

This model is unrealistic in several ways. The assumption that the plastic zone can be modeled by superdislocations constrained to remain in the crack plane is made using only qualitative arguments. The use of elastic stress field approximations containing only \( 1/\sqrt{r} \) singular terms in the crack tip vicinity is not a realistic approximation nor is the application of dislocation stress fields to superdislocations with Burgers vectors of macroscopic proportions. Finally, the near tip fracture criterion [31] is unlikely to apply in this form for all mode I - mode III combinations. Differences in the
contributions of anti-plane shear, in-plane shear, and hydrostatic stress elevation would be likely to produce vastly different damage processes in the intensely strained crack tip region which lead to fracture. Thus the assumption of a constant energy \((\gamma_{\text{eff}})\) required for crack propagation is tenuous.

Still, the model explains why globally applied \(J_{\text{ic}}\) and \(J_{\text{illc}}\) values need not be equal even if similar strain energy release rates lead to fracture, if the extent of shielding varies with the mode of loading.

**Combined Mode Fracture Mechanisms**

The mechanisms producing fracture under elastic-plastic combined mode I - mode III conditions are not fully understood, but particular features may be inferred from fractographic observations as well as measurement of global fracture toughness values.

**Mode I**

As discussed in the Results, mode I fracture of NiCop occurred entirely by microvoid coalescence. Although both heat treated microstructures contain carbides, sulfides, and copper containing precipitates as second phase particles, the initiation of voids ahead of the crack tip undoubtedly originated at the largest elongated sulfides. The metallographically determined spacing of \(\sim 200 \, \mu\text{m}\)
between these inclusions corresponds closely with the spacing of the largest pits visible on the surfaces for both heat treated conditions. The large size of these pits indicates early initiation and the longest growth period prior to fracture of any of the surface voids. The small 0.5 to 1 µm particles found at the bottoms of the finer dimples correspond to the equiaxed sulfides present in both microstructures, while no void initiating particles were visible at all in the finest submicrometer sized dimples.

Since mode I fracture was initiated by similar sulfide inclusions in both NiCop conditions, the differences in mode I toughness result primarily from differences in continuum properties of the two materials. The greater toughness of the overaged material is expected in view of its lower yield strength and consequently larger plastic zone size. These may have contributed to lower effective constraint in the crack tip region so that the conditions were more nearly plane stress than for the higher strength peak aged material. The greater ductility and strain hardening exhibited in tensile testing of the overaged material also agree with the excellent toughness of the overaged condition.

Both mode I crack surface topographies displayed large elevation changes although they followed the side grooves at the surface. This is largely a result of the large fracture strains in NiCop, but may also be caused by some
voids initiating early in the load history at sulfides positioned 45° to the mode I crack plane in the high strain region of the plane strain field. McMeeking's numerical analyses (102) confirm this position as a preferred void nucleation site, but show that the greater hydrostatic stresses in the crack plane promote faster void growth directly ahead of the crack tip.

The larger mode I toughness for the overaged condition was accompanied by a larger tearing modulus. Relationships between fracture initiation toughness and crack growth resistance are common and may result from several sources. (99) Kahn et al (103) compiled J-resistance data from several sources and suggested that a universal Jr curve exists in which J and T are related as

\[ T = A J_{ic} \frac{E}{\sigma_0}^2 \]  

expressed in SI units, with A=857. \( J_i \) and \( T_i \) values measured in this program yield \( A \) values of 541 and 513 for the peak aged and overaged conditions respectively.

Combined Mode I - Mode III

Fracture surface features produced by combined mode loading are quite similar for the two heat treated conditions for any given specimen geometry, but change greatly with crack inclination angle. For \( \theta = 75^\circ \) specimens \( (J_{iii}/J_i \approx 10\%) \) the ratio of \( \delta_{iii}/\delta_i \) and consequently of
$J_{iii}/J_1$ is quite constant. Fracture occurs by microvoid coalescence and the fracture surfaces closely resemble the mode I case with nearly equiaxed dimples. For smaller crack inclination angles, both NiCop conditions displayed a tendency for shear flow. This is emphasized in the continual increase in $J_{iii}/J_1$ during loading produced by preferred shear displacements in the mode III direction. The fine dimples also became increasingly elongated in the shear direction in geometries with larger mode III components.

Although the details differ appreciably with loading mode, all combined mode fractures were caused by microvoid coalescence. Thus all fracture modes considered here incorporate the subprocesses of void initiation, growth and coalescence. Differences in fracture behavior with transverse shear/opening mode load ratios result from differences in these subprocesses as a function of crack angle.

A substantial barrier to the understanding of combined mode fracture under conditions of extensive plasticity is the lack of an available solution for the near tip stress and strain fields. A qualitative picture of the near-tip conditions, however, can be obtained by consideration of the elastic-plastic deformation fields for the individual mode I and mode III configurations.
Rice (104) has recently reviewed the construction of near-tip fields for both tensile and anti-plane shear loading. The slip line fields were presented for both stationary and moving cracks assuming isotropic, elastic-perfectly plastic mechanical behavior.

The stationary mode I crack tip (plane strain) is enclosed by the familiar Prandtl slip line field. This field is composed of constant stress sectors ahead of and behind the crack tip, with centered fan sectors above and below it. The corresponding field for the moving tensile crack is similar but an additional elastic sector separates the centered fan and the trailing constant stress region. The stationary mode III crack tip field, on the other hand, consists of a plastic centered fan sector directly ahead of the crack tip covering the angular range $-\pi/2 < \theta < \pi/2$, and elastic sectors joining the plastic regions and the trailing crack free surfaces. The corresponding field for the moving anti-plane strain crack consists of narrow centered fan sectors ahead of the tip and bordering the crack free surfaces with wide elastic sectors joining the two.

The noteworthy difference between the tensile and antiplane shear fields is that each predicts different locations of strain concentration. For both stationary and growing cracks, mode III loading produces concentrated
strains directly ahead of the crack tip while tensile loading produces strain concentrations above and below it. Of course, crack blunting accompanying tensile loading (as in the Rice-Johnson model) causes intensely focussed strains which extend 1-2 CTOD's ahead of the crack tip. However, the general observation that mode III fields predict concentrated slip in the original crack plane is interesting, particularly since cracks extending in the plate-type slant notch specimens are constrained by side grooves to propagate in the original crack plane.

Although the deformation fields of the two loading modes are not uncoupled as they were in the simplified superdislocation model presented earlier, it is reasonable to suggest that the relative components of each mode in the full elastic plastic mixed mode fields correspond roughly with the amplitudes of the respective components of J.

Thus for large $\theta$ specimens $J_i > J_{iii}$ and the crack tip stress field is dominated by the mode I components of stress (i.e. large hydrostatic and in-plane stress values), but small out-of-plane shear stresses. During loading the near tip region is enclosed by a "mode I" process zone and this effective "mode I" deformation field is felt by material at a much greater distance from the crack tip than an effective "mode III" deformation field consisting of out-of-plane shearing which has a much smaller magnitude, and consequently is only felt very near the crack tip.
This very qualitative picture supports the fractographic observation that small mode III components do not greatly change the micromechanisms of fracture from the pure mode I case. That is, void initiation and growth proceed in predominantly "mode I" type deformation fields.

However, at some point (relative to changing geometry or load pattern), as the mode III component is increased (low $\theta$) the physical region influenced by the "mode III" deformation field becomes larger than that of the mode I field. Now, during loading, transverse shear strains extend farther ahead of the crack tip than other deformation field components and hence void initiation may be produced predominantly by transverse shear. This would be the operative initiation mechanism for large $J_{III}/J_I$ geometries although growth processes are not so clear. In cases approaching pure mode III, material damage accumulates by intense shearing alone, while if a significant mode I component is present, void growth would be assisted by the "mode I" hydrostatic stress field. (49)

In the modified compact specimen, side grooves provide some hydrostatic stress elevation at the crack edges well ahead of the crack tip. This provides assistance for all ductile fracture mechanism in NiCop, as all extending cracks lead significantly at the groove positions.

The relative contributions of "mode I" and "mode III" deformation fields are reflected in measured $J$ values as a
function of geometry. It was noted earlier that $J_i$ values decreased with increasing mode III component. This is not surprising if fracture is strain controlled as the addition of transverse shear stresses should increase the effective strain in the near tip region, contributing to material damage. With this viewpoint Pook's (74) finding that $K_I$ was insensitive to simultaneously applied $K_{III}$ at initiation is surprising. Pook himself points out that the additional mode III component enlarges the plastic zone. The materials studied by Pook, however, exhibited very little plasticity and hence fracture may not have been entirely strain controlled.

Just as prominent as the decrease in mode I $J$ with decreasing $\theta$, was the decrease in $T_i$. This also agrees with the earlier discussion. If material damage is primarily by transverse shear, not only is little mode I component necessary for initiation, but little increases are necessary for propagation. In large $J_{III}/J_i$ geometries, damage ahead of the crack tip is envisioned to occur by mode III alone, while the opening mode simply acts to separate the extensively damaged specimen halves. The converse is true for large $\theta$ values, where "mode I" deformations dominate material damage and both $J_{III}$ and $T_{III}$ are quite low.

$J_{exc, total}$ and $T_{total}$ both display minima near $\theta=45^\circ$ and suggest that the micromechanisms leading to slant
fracture require less energy than other fracture geometries. When coupled with the observation that shear displacements increase at a larger rate than tensile displacements for all values of $\theta \leq 55^\circ$, a tendency toward shear instability in both of the microstructures is postulated.

Frictional Effects

The sharp upturn of total $T$ and $J$ values at very low crack angles is probably not entirely caused by increased energy expended within the plastic zone for conditions approaching mode III. Fractographic observation of combined mode specimens reveals that many of the "flat" appearing facets on the specimen surfaces in fact appear to be smeared regions where the specimen halves have contacted one another. There is consequently an increasing component of energy used in producing such frictional rubbing which increase measured values of $J_{iii}$ and $T_{iii}$.

To more quantitatively assess the extent of this rubbing, montages of fracture surfaces for all combined mode geometries were assembled from SEM fractographs taken at 20X. On these montages, nominally dimple-free regions were identified and their approximate sizes were determined from quantitative metallography on a Zeiss image analysis system. The average dimple-free feature size is seen to increase with decreasing specimen angle in Figure 54. For all geometries the average feature size was larger in the
Figure 54. Dimple-free feature size vs. $\theta$. 
Figure 55. Total area covered by dimple-free regions vs. $\theta$. 

- Peak Aged
- Overaged
overaged material. Figure 55 also shows that the total area of surface covered by these nominally dimple-free features increased with increasing mode III component. This procedure was very approximate as the features were sometimes hard to differentiate from very elongated dimpled regions in the high angle specimens, but the qualitative result that the effects of crack face rubbing increase with decreasing mode I component is clear.

Macrovoid Formation

Superimposed on the considerations of micromechanisms of fracture, the presence of macroscopic depressions in the specimen surfaces indicate another source of energy expended during the fracture process. Miglin et al (105) have previously postulated that such depressions originate from macrovoids initiated at inclusion sheets lying in the plane of the plate as a result of the rolling process. As the crack tip strain field engulfs these regions, the particles decohere and the voids grow prior to linking with the advancing crack tip. Plate VIII illustrates such macrovoids revealed by transverse sectioning of 45° specimens of both heat treatments. As is apparent from the plate, macrovoid depth was maximum in the overaged condition (Figure 56). This trend was followed for all crack angles, although at \( \theta = 35^\circ \), overall surface roughness in transverse sections was very limited and such
PLATE VIII. Macrovoid Observations.

Top left: Transverse section displays typical surface roughness for peak aged $\theta = 45^\circ$ specimen. Only shallow "macrovoids" (arrow) are visible. Dark layer is nickel plating.

Top right: Overaged $\theta = 45^\circ$ specimen exhibits greater surface roughness and deeper steps which correspond to "macrovoids" (arrow).

Bottom left and right: Stereo pair of $\theta = 45^\circ$ specimen surface. Leftmost arrow indicates a "macrovoid" or deep depression in the surface. The rightmost arrow indicates a steeply sloped region that appears to have been smeared by contact between the specimen halves.
Figure 56. Macroscopic surface depressions formed in combined mode fracture.
macroscopic depressions were rare.

Mode III

There was little discernable difference between the mode III fracture surfaces and those of $\theta=35^\circ$ combined mode specimens. Fracture occurred by void coalescence in the shear field ahead of the crack tip. Just as for $\theta=35^\circ$ cases, the mode III surfaces contained elongated dimples and "smeared" areas. Stereo pairs revealed only small elevation changes. This is the continuation of a general decrease in surface roughness as the crack path changes from mode I to mode III.

Effect of Cu Precipitation on Fracture Toughness

The two heat treatments applied to NiCop in this program produce very similar grain sizes and carbide distributions, but different copper precipitate morphologies. Thus, differences in fracture toughness between the peak aged and overaged conditions result chiefly from this difference.

There is still considerable controversy surrounding the precise strengthening mechanisms of copper in steel (106), as well as effects on dislocation motion during straining. In the peak aged condition, bcc Cu-containing clusters are coherent and can be cut by dislocations. Hornbogen (107) has postulated, however, that in addition
to particle cutting, strengthening is enhanced by a concentration of vacancies in the vicinity of Cu-rich clusters. As screw dislocations move through the lattice, these vacancies may produce superjogs which hinder further movement.

It is possible that the fcc epsilon particles in the overaged condition are cut by dislocations also since their slip directions are epitaxially related to those of the matrix, but there is also evidence that they are bypassed. (107) As copper is depleted in the matrix during overaging, cross slip becomes easier. (107) Miglin (100) has proposed that easy cross slip may promote blunting of pileups and delay void initiation.

The superior toughness exhibited by the overaged material for all loading modes is consistent with its resistance to plastic instability as evidenced by the large elongation to failure in uniaxial tension. It is somewhat surprising, however, that the relative $J_i$ and $J_{iii}$ components measured in combined mode loading are nearly identical for the overaged and peak aged conditions. The normalization procedure discussed earlier (Figure 47) shows that, aside from the general difference in toughness level, the two heat treatments display similar tendencies toward mode III flow and normalized J components were similar for all loading modes.
This behavior may result from the nature of the fracture mechanisms producing crack extension. All cracks grew by microvoid coalescence, though the character of the gross fields changed with the loading mode as evidenced by the transition from equiaxed to elongated dimples in changing from mode I to mode III. Because all fracture modes involved extensive plastic flow prior to failure, material resistance to plastic instability (as in ligaments separating adjoining growing voids) is the dominant material property limiting fracture. Both conditions exhibit moderate strain hardening and show good ductility at room temperature. Therefore, differences in fracture resistance appear to be a matter of degree rather than kind.

For alloy systems exhibiting shear localization tendencies, the application of a large transverse shear stress might lead to very localized deformation and premature fracture. At upper shelf temperatures, however, both NiCop conditions examined in this work display excellent fracture resistance in both tension and shear, although mode III shear flow appears to be favored under combined mode loading.
(1) Both peak aged and overaged conditions of ASTM A710 steel fracture in a ductile manner at room temperature. Both materials undergo extensive stable crack extension for pure mode I, combined mode I - mode III, and pure mode III loading conditions. In all cases, fracture proceeds by the mechanism of microvoid coalescence, although the deformation fields ahead of the crack tip change with loading mode.

(2) For all loading modes the overaged microstructure containing incoherent fcc epsilon copper precipitates was more resistant to both crack initiation and to stable extension than the peak aged microstructure containing bcc copper containing clusters. This difference in toughness is primarily a result of the overaged material's lower yield strength and greater resistance to plastic instability, as evidenced by uniaxial tensile properties.

(3) Fracture initiation values of the J-integral components $J_1$ and $J_{III}$ exhibit a linear fracture initiation locus in $J_1$-$J_{III}$ space for each of the alloy heat treatments. This
confirms that the presence of any mode III load component lowers the necessary mode I component at crack initiation (and vice versa).

(4) The linear form of the fracture initiation envelope is predicted by a simple model of combined mode elastic-plastic behavior in which the plastic zone is represented by two superdislocations, one of edge character and one of screw character, lying ahead of the crack tip.

(5) In traversing the combined mode loading spectrum, mode I J-integral components decrease with increasing applied mode III load component. This behavior is followed by values measured both at crack initiation and those measured after 3mm of growth. Concurrently, mode III J-integral components measured at initiation and after 3mm of growth increase.

(6) When total energy input to the specimen is measured as a function of loading mode, a minimum propagation energy is found which corresponds to the geometry of slant fracture where the crack plane is oriented at approximately 45° to both the load line and the plate surface.

(7) During combined mode loading, both alloy conditions displayed a marked tendency toward mode III shear flow. This tendency led to pronounced mode III displacements and continual increases in the $J_{III}/J_1$ ratio during loading.
(8) If individual combined mode J-integral components for each heat treated condition are normalized to account for the general toughness differential between the microstructures, both heat treatments exhibit identical combined mode behavior.
LIST OF REFERENCES


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