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Q-NUCLEOSYNTHESIS: IMPLICATIONS FOR STELLAR EVOLUTION

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Q-Nucleosynthesis: Implications for Stellar Evolution

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By
Craig L. Joseph, B.S., M.S.

*

The Ohio State University
1985

Reading Committee:  
George W. Collins, II
Gerald H. Newsom
Richard N. Boyd

Approved By
George W. Collins, II
Department of Astronomy
DEDICATION

To Julie
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Last but not least, I would like to express my heartfelt thanks to the faculty, staff and students of the OSU
Department of Astronomy for making my stay at Ohio State both pleasant and productive. I shall remember them with fondness as the Big Telescope nears completion.
VITA

October 29, 1955 .......... Born-North Canton, Ohio

1978-1979 ................. Undergraduate Research Assistant, Department of Physics, The Ohio State University, Columbus, Ohio

1979 ....................... B.S., The Ohio State University, Columbus, Ohio

1979-1984 .................. Graduate Teaching Associate, Department of Astronomy, The Ohio State University, Columbus, Ohio

1984 ....................... M.S., The Ohio State University, Columbus, Ohio

1984-1985 .................. Graduate Teaching Associate, Department of Astronomy, The Ohio State University, Columbus, Ohio

FIELDS OF STUDY

Major Field: Theoretical Astrophysics

Studies in Stellar Structure and Evolution: Dr. George W. Collins, II

Studies in Nuclear Astrophysics: Dr. Richard N. Boyd

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The basic features of stellar evolution from the main sequence to the giant branch were first elucidated by the pioneering work of Sandage and Schwarzschild (1952) and Hoyle and Schwarzschild (1955) nearly three decades ago. Although numerous refinements have been made to stellar interior models since then, the basic picture has remained essentially unchanged. The most recent comprehensive study of the core hydrogen-burning evolution of population II stars is that of VandenBerg (1983). Using improved radiative opacities and model stellar atmospheres to provide more realistic surface boundary conditions, VandenBerg has been remarkably successful in reproducing the observed color-magnitude diagrams of both open and globular clusters. This good agreement reinforces the widely held notion that the early (hydrogen-burning) stages of stellar evolution are sufficiently well understood that only "fine tuning" remains to be done.

Despite these successes, persistent problems remain. Perhaps the most notorious of these is the apparent inabil-
ity of the theory to account for the observed flux of neutrinos from the sun. The Brookhaven $^{37}$Cl experiment (Davis et al. 1968), now in its 15th year of operation, measures the flux of high-energy neutrinos emitted by nuclear reactions in the solar interior. This experiment places a firm upper limit of approximately 2 SNU (1 SNU = $10^{-36}$ captures per $^{37}$Cl atom sec$^{-1}$) on the flux of neutrinos to which the detector is sensitive. Models of the solar interior, however, predict a capture rate of approximately 7 SNU (Bahcall et al. 1982). As shown by several investigators (Bahcall et al., 1982; Filippone and Schramm, 1982), this discrepancy between theory and observation cannot be accommodated by known errors in the observations or in the physics upon which the models are based. Although at least forty possible explanations have been proposed in the literature, most are either ad hoc or introduce other difficulties at the expense of solving the solar neutrino problem. More recently, the identification of the five minute solar oscillations with acoustical normal modes beneath the photosphere (Ulrich, 1970; Wolff, 1972; Ando and Osaki, 1975) has provided additional observational constraints on the Sun's internal structure. The agreement between the predicted eigenfrequencies of the standard solar model and the observed acoustical spectrum of the Sun is qualitatively quite good. Like the solar neutrino problem, however,
the discrepancies which remain are still larger than can be accounted for by observational errors or errors in the input physics (Ulrich and Rhodes, 1983). It is somewhat disturbing that the only direct probes of the solar interior yield results that are discordant with theoretical predictions. These failures of the standard solar model may well be symptoms of a common flaw in our understanding of the physics of the solar interior.

Since the Sun is presumably a non-exceptional main sequence star, these symptoms may be apparent in other stars as well. The most fundamental test of stellar evolution theory is the comparison of predicted evolutionary sequences with the observed color-magnitude diagrams of star clusters. Attempts to fit theoretical isochrones (time-constant loci) to the color-magnitude diagrams of old galactic clusters have proved moderately successful. However, clear and systematic discrepancies exist between the predicted and observed characteristics of several well-studied old open clusters. Theoretical models indicate that the rapid contraction phase following core hydrogen exhaustion will produce a characteristic blueward hook and a gap near the main sequence turnoff. The hook is not apparent in any of the data, and the gap is displaced vertically relative to the predicted location (Maeder, 1974). Furthermore, the subgiants are not found at the positions
indicated by the isochrone fits. Maeder (1974, 1976) has attempted to explain these discrepancies in terms of the "overshooting" of convective elements beyond the formal convective core boundary as determined by the Schwarzschild criterion (Schwarzschild, 1958). Although models which incorporate convective overshoot are better able to reproduce the observations, there is no consensus as to whether overshooting to the extent required is energetically possible. Anomalous gaps also appear on the subgiant and giant branches of some globular clusters (Da Costa and Villumsen, 1981; Buonanno et al., 1984). In several clusters for which good photometric data exist, these gaps are wide and distinctly defined. However, conventional stellar interior models fail to predict an episode of rapid evolution in this part of the H-R diagram.

A reappraisal of the predictions of stellar evolution theory may also be suggested by cosmological considerations. As has been noted by several investigators (Janes and Demarque, 1983; Cannon, 1983; Pilachowski et al., 1983; VandenBerg, 1983) the ages of the oldest globular clusters, as inferred from isochrone fits, are in disagreement with the maximum age of the Universe as inferred from some recent estimates of the Hubble constant $H_0$. From a comparison of VandenBerg's (1983) isochrones with the color-magnitude diagrams of M15 and M92, Sandage (1983) deter-
mired an age of $18 \times 10^9$ years for both clusters; Carney (1980) obtained the same result using the isochrones of Ciardullo and Demarque (1977). This lower limit to the age of the Universe requires $H_0 < 60 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ for a Friedman model with $q_0 = 0$. Although Sandage and Tamman (1975a,b; 1976) have presented arguments in favor of $H_0 = 50-55 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, more recent determinations appear to be converging on a larger value. Indeed, the average of all observational determinations of $H_0$ published since the work of Sandage and Tamman is approximately $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ (van den Berg, 1975; Bottinelli and Gouguenheim, 1976, 1977; de Vaucouleurs, 1977; Corwin, 1977; Tulley and Fisher, 1977; Lynden-Bell, 1977; Lynden-Bell and Liller, 1978; de Vaucouleurs and Bollinger, 1979; Aaronson et al., 1980; Aaronson and Mould, 1983). Such a large value of $H_0$ has not been widely accepted largely because of the conflict with globular cluster ages. But in view of the known difficulties with the stellar models, as exemplified by the standard solar model, it may be premature to place the blame on cosmology. If the Hubble constant is as large as recent estimates suggest, a reassessment of stellar evolution theory is clearly in order.

A new mode of hydrogen-burning nucleosynthesis, which may have an important bearing on the problems discussed above, has recently been proposed by Boyd et al. (1983).
This mechanism is based upon the possible existence of anomalous nuclei in which the nuclear density, and the binding energy per nucleon, are increased relative to normal nuclei. A mass of five nuclei meeting these requirements could then be stable to baryon emission, thereby permitting nuclear reactions which would otherwise not be possible. Plausible candidates which have been discussed in the literature include superdense nuclei (Ohnishi, 1981) and nuclei containing an extra embedded hadronic particle, for example a primordial quark (DeRujula et al. 1978, Chapline, 1982, Boyd et al. 1983) or a superheavy X-particle (Cahn and Glashow, 1981). The most well-studied case is that of an unconfined quark bound to a nucleus; accordingly, all such hypothetical nuclei have been dubbed "Q-nuclei". As the results presented in this investigation are based upon the quark model of a Q-nucleus, a pertinent question to consider is whether such entities might exist in nature.

In most interpretations of Quantum Chromodynamics (QCD), the quark-quark interaction is so strong (and so long-ranged) that quark confinement is absolute, thereby precluding the existence of quarks outside of hadrons. This concept of confinement is based largely upon the well-known observation that electric charge appears to be quantized in units of the charge of the electron. However, Lackner and Zweig (1983) have shown that the unusual chemi-
cal properties of free fractional charges would render their detection exceedingly difficult, and the question of whether free quarks exist in nature remains of fundamental importance.

Several theoretical studies have addressed the possibility of "broken" QCD. Wagoner and Steigman (1979) explored the consequences of incomplete confinement by assuming that the effective potential between quarks reaches a limiting value at a finite quark separation. They found that the number of quarks which survive the quark-hadron phase transition following the Big Bang is a function of the quark mass, but is less than about $10^{-14}$ quarks per baryon. DeRujula et al. (1978) developed a picture of broken QCD in which the gluon has a finite (small) mass; in the limit of zero gluon mass, their model recovers the conventional picture of confinement. An important conclusion of their study is that the interaction between quarks and nuclei would be strongly attractive; consequently, primordial quarks left over from the Big Bang would presumably accrete nucleons. In another study, Chapline (1982) showed that the embedded quark would significantly increase the binding energy of the resulting Q-nucleus. In a model of broken QCD proposed by Slansky et al. (1981), observable fractionally-charged states are not single quarks, but "diquarks", i.e., two quarks bound together. Diquarks would also bind with nuclei, though perhaps only weakly.
The important theoretical implications of unconfined quarks has motivated numerous searches for free fractional charges. The majority of these searches have proved unsuccessful (Lyons, 1982). The only investigation that so far has yielded a positive result is the controversial LaRue experiment (LaRue et al., 1977, 1979, 1981). The experimental apparatus consists of small superconducting niobium spheres which are suspended magnetically between two horizontal capacitor plates. The net charge on the surface of each sphere is measured by observing the oscillations produced when an alternating electric field is applied between the capacitor plates. LaRue and his co-workers claim that residual surface charges of \( \pm e/3 \) were found on several of the spheres which were measured; the abundance of free fractional charges implied by these results is roughly one in \( 10^{16} \) nuclei. Although these results have not been confirmed by other investigators, it is clear that if fractionally-charged nuclei exist in nature their abundance is exceedingly small.

In the hydrogen-burning cycle suggested by Boyd et al. (1983), \(^4\text{He} \) is produced by successive proton captures on \( \text{He}, \text{Li} \) and \( \text{Be} \) nuclei containing an extra embedded quark (or other suitable hadronic particle). Such a cycle is possible because the additional quark increases the nucleon binding energy, thereby permitting a mass five nucleus
which is stable to baryon emission. Although this HeLiBe Q-nuclear cycle is analogous to the conventional CNO cycle, the Q-nuclear reaction rates would be expected to be much larger because of the low Q-nuclear charge and reduced Coulomb barrier. Using a simple model of a Q-nucleus and the statistical formulation of Woosley et al. (1975) for calculating nuclear reaction rates, Boyd et al. (1983) show that the slowest reaction in the HeLiBe cycle has a $<\sigma v>$ value 14 to 15 orders of magnitude larger than that of the proton-proton (p-p) reaction at temperatures characteristic of the solar interior. Consequently, even the minute abundance of Q-nuclei implied by the LaBue experiment may be sufficient for the HeLiBe cycle to compete effectively with the proton-proton cycle in the Sun. Furthermore, consideration of beta-decay lifetimes suggests that few high-energy neutrinos would be produced by the HeLiBe Q-nuclear cycle, thus providing a possible resolution to the solar neutrino problem. While solar models incorporating the Q-nuclear cycle have been shown to be consistent with the observed solar parameters (Joseph, 1984a; Boyd et al., 1985), the effect of Q-nucleosynthesis on the observable properties of other stars has not yet been explored. Clearly, the existence of an energy source such as that described could have profound implications on our understanding of stellar structure and evolution.
The purpose of this dissertation is to determine the extent to which $Q$-nucleosynthesis alters the predictions of conventional stellar interior models, and whether these predictions are supported by the available observational data. Furthermore, since we will be assuming that the required increase in binding energy is provided by an embedded quark, an additional objective will be the estimation of an upper limit on the abundance of free fractional charges in nature. Such an estimate would be an important independent contribution to the upper limits imposed by other quark search experiments.

This investigation will focus primarily on the evolution during and immediately following the core hydrogen-burning phase, for several reasons. First, the hydrogen-burning evolution is characterized by relatively simple and well-understood physics, so that uncertainties in the equation of state and other physical inputs will not introduce unnecessary complications. In addition, the only star for which accurate values of the age, mass, radius and luminosity are available is the Sun. Consequently, the observed solar parameters provide important constraints for main sequence evolutionary models. Furthermore, an abundance of good observational data exists for the main sequence and giant branches of many clusters. Finally, as discussed earlier, the agreement between theory and observation for
evolution on the main sequence and giant branch is already qualitatively quite good. Consequently, these early evolutionary stages should prove to be a sensitive testing ground for the observable effects of Q-nucleosynthesis.

The HeLiBe Q-nuclear cycle, other possible Q-nuclear cycles, and Q-nuclear reaction rates will be discussed in Chapter II, and relevant details of the stellar evolution code will be described in Chapter III. Chapter IV will deal with the effect of Q-nucleosynthesis on the structure and evolution of the Sun, with particular emphasis on the interior of the present Sun as inferred from the high-energy neutrino flux and global solar oscillations. In Chapter V we will investigate the effects of Q-nucleosynthesis on the properties of the zero age main sequence and on the main sequence and sub-giant branch evolution of low mass stars. A discussion of the results obtained in this study will be presented in the final chapter.
2.1 THE HE-LI-BE CYCLE

The properties of Q-nuclei and the manner in which they might participate in stellar nucleosynthesis have been described in detail by Boyd et al. (1983, 1985). In this and the following sections we will summarize the results of those investigations which have a bearing on the present study, and develop the formalism required for the numerical calculations to be presented in subsequent chapters.

The simplest Q-nuclear cycle is the HeLiBe cycle, illustrated in Figure 1. (In that figure, as in the remainder of this dissertation, a Q-nucleus is identified by a superscript Q). In this cycle, \(^{4}\text{He}\) is produced by successive proton captures on the Q-nuclei \(^{4}\text{He}^Q\), \(^{7}\text{He}^Q\), \(^{6}\text{Li}^Q\), and \(^{7}\text{Li}^Q\), a process which clearly cannot occur with normal nuclei because of the mass five barrier. It is worth noting at the outset that this cycle is formally equivalent to the CN portion of the CNO bi-cycle, with the He, Li and Be Q-nuclei playing the roles of C, N and O, respectively. Indeed, assuming that the embedded quark merely increases
the depth of the nuclear potential well without otherwise altering the nuclear structure, many features of the HeLiBe cycle can be understood, at least qualitatively, by analogy with the CN cycle. It is clear, for example, that the Q-nuclei, like the CN nuclei, act as catalysts with the seed nucleus $^4\text{He}$ being returned at the end of the cycle. Consequently, the total number of Q-nuclei participating in the cycle will remain constant in time and, once equilibrium is established, the relative abundances will be determined by the relative proton capture and beta-decay lifetimes. The slowest reaction in the CN cycle is the radiative proton capture on the nucleus with the largest charge, $^{14}\text{N}$, and as a result $^{14}\text{N}$ is by far the most abundant nucleus in the equilibrated CN cycle. Similarly, the slowest reaction in the HeLiBe cycle is expected to be the $^6\text{Li}^Q (p,\gamma)^7\text{Be}^Q$ reaction, in which case the total Q-nuclear abundance is essentially that of $^6\text{Li}^Q$. The rate of energy production, $\epsilon_Q$, is also limited by the $^6\text{Li}^Q (p,\gamma)^7\text{Be}^Q$ reaction rate and is given by

$$\epsilon_Q = q_a \left[ ^6\text{Li}^Q \right] \left[ ^1\text{H} \right] \langle \sigma v \rangle^Q_6 \tag{1}$$

where the notation $[X]$ denotes the number density (cm$^{-3}$) of nuclear species $X$, $\langle \sigma v \rangle$ is the product of cross section and velocity averaged over the Maxwellian velocity distribution, and $q_a$ is the net energy released in the formation of
a \(^4\)He nucleus, excluding the energy lost in the form of neutrinos.

Since the Coulomb barrier is the dominant feature of most charged particle reactions, the HeLiBe Q-nuclear cycle has an obvious advantage over the CN cycle because of the lower Q-nuclear charges. However, this advantage is essentially negated by the very low upper limit to the Q-nuclear abundance imposed by experiment, so the HeLiBe cycle would not be expected to generate appreciable energy at temperatures characteristic of equilibrium CN burning. On the other hand, Boyd et al. (1983, 1985) have shown that a tiny abundance of Q-nuclei can make a significant contribution to the energy production at temperatures for which the proton-proton (PP) cycle is the dominant mode of hydrogen-burning nucleosynthesis. The rate of helium production via the PP cycle is limited by the very slow p-p reaction. Because it is mediated by the weak interaction, the p-p reaction has a very small cross section; at a temperature of 15x10^6 K, the lifetime of protons against the p-p reaction is approximately 10^10 years. By comparison, the lifetime of \(^6\)Li against proton capture at the same temperature is only about 10^-1 years, despite the larger Coulomb barrier. Since the \(<\sigma v>\) value for the \(^6\)LiQ+p reaction is not expected to differ from that of the \(^6\)Li+p reaction by more than two or three orders of magnitude, it is clear that the
Figure 1: The HeLiBe Q-nuclear cycle.
HeLiBe cycle can compete effectively with the PP cycle despite the expectedly small Q-nuclear abundance. The fractional abundance of Q-nuclei required to produce energy at a rate comparable to the PP cycle is approximately equal to the ratio of the $\langle \sigma v \rangle$ values for the p-p and $^6\text{Li}Q + p$ reactions.

In order for the HeLiBe Q-nuclear cycle to operate as depicted in Figure 1, the increase in binding energy required to render $^5\text{Li}Q$ stable to proton decay must not be so large as to inhibit the $^7\text{Li}Q(p,\gamma)$ reaction. Normally in nuclear reactions, when a particle channel is energetically allowed it will dominate over a gamma channel. For example, the $^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction, which completes the CH cycle, occurs about $10^4$ times more frequently than the $^{15}\text{N}(p,\gamma)^{16}\text{O}$ reaction, which leads to the NO portion of the CNO bi-cycle. The nuclear structure of $^7\text{Li}Q$, in which one proton and two neutrons are in the least-tightly bound ($^1p_{3/2}$) orbit, would also be expected to favor a $(p,\gamma)$ reaction, unless the increase in binding energy is so large as to render this reaction endothermic. The $^7\text{Li}(p,\alpha)^4\text{He}$ reaction is exothermic by 17.35 MeV; consequently, the increase in binding energy of the $^1p_{3/2}$ nucleons in $^7\text{Li}Q$ must not exceed 17.35 MeV, or approximately 5 MeV per nucleon (some kinetic energy must be available for the alpha-particle to penetrate the Coulomb barrier). As di-
cussed by Boyd et al. (1985), the expected range in Q-nuclear parameters is sufficiently large to easily accommodate these binding energy requirements.

Another consideration of some importance is the energy available for the \(^{\text{Li}}\text{Q}\) and \(^{\text{Be}}\text{Q}\) beta-decays. Specifically, the lifetime of \(^{\text{Be}}\text{Q}\) against beta-decay should be sufficiently short to ensure that a \((p,\gamma)\) reaction on \(^{\text{Be}}\text{Q}\) will be highly improbable. This upper limit on the \(^{\text{Be}}\text{life-time}\) is necessary for two reasons. First, a \((p,\gamma)\) reaction on \(^{\text{Be}}\text{Q}\) could initiate another Q-nuclear cycle in which heavier Q-nuclei are built up, via successive \((p,\gamma)\) reactions, until an allowed particle channel is reached, for example a \((p,\alpha)\) reaction on \(^{\text{C}}\text{Q}\) or \(^{\text{O}}\text{Q}\). This is undesirable because the resulting depletion of nuclei from the HeLiBe cycle could reduce the energy-producing efficiency by a considerable amount. It would then be necessary to assume a larger initial abundance of light Q-nuclei, in order to attain a given rate of Q-nuclear energy production, than would be the case if there were no attrition of the light Q-nuclei. The possible consequences of such attrition will be discussed in more detail in the following section. Secondly, a \((p,\gamma)\) reaction on \(^{\text{Be}}\text{Q}\) would give an \(^{\text{Be}}\text{Q}\), which would presumably beta-decay to \(^{\text{Be}}\text{Q}\). The resulting neutrino would be expected to have an energy comparable to that of the neutrino produced in the decay of \(^{\text{Be}}\text{Q}\).
in the PP cycle. Thus the $^7\text{Be}^Q$ beta-decays must occur rapidly enough to ensure that very few of these high-energy neutrinos are produced. As shown by Boyd et al. (1983), the increase in electrostatic energy and decrease in the size of the Q-nuclear charge distribution resulting from the embedded quark would be expected to increase the energy available for beta-decay above the positron emission threshold. Above this threshold the beta-decay lifetime decreases rapidly with increasing decay energy. Consequently, the beta-decay lifetime of $^7\text{Be}^Q$ may well be several orders of magnitude less than the lifetime of $^7\text{Be}$ against electron capture (which leads to the high-energy neutrinos in the PP cycle). Boyd et al. (1983) concluded that an upper limit of approximately 1000 seconds for the lifetime of $^7\text{Be}^Q$ will ensure that few high-energy neutrinos will be produced via a $^7\text{Be}^Q(p,\gamma)^8\text{B}^Q(e^+\nu)^8\text{Be}^Q$ reaction. A similar line of reasoning suggests a lower limit to the beta-decay lifetime of $^7\text{Be}^Q$ (and $^8\text{Li}^Q$); if these beta-decays occur too rapidly, the accompanying neutrinos may be more energetic than those produced in the $^8\text{B}$ decay, thereby aggravating the solar neutrino problem.
2.2 ADDITIONAL Q-NUCLEAR CYCLES

At high temperatures, the lifetime of \(^7\text{BeQ}\) against proton capture may be comparable to or less than the beta-decay lifetime, resulting in leakage of Q-nuclei out of the HeLiBe cycle. The possibility of additional Q-nucleosynthesis involving heavier Q-nuclei depends upon the stability of the Q-nucleus \(^8\text{Be}\). If the \(^4\text{HeQ}\) nucleus is very tightly bound, and the effective interaction range of the embedded particle is comparable to the size of \(^4\text{HeQ}\), the binding energy of \(^8\text{BeQ}\) will not be much affected in which case it would decay to \(^4\text{He}\)+\(^4\text{HeQ}\). However, in view of the considerable uncertainties involved, it is prudent to examine the Q-nuclear reactions which may occur if \(^8\text{BeQ}\) is in fact stable to particle emission. The most likely branching possibilities are illustrated in Figure 2, in which the HeLiBe cycle is shown with heavy connecting lines. Assuming that \(^8\text{BeQ}\) is bound, the \((p,\gamma)\) reactions and beta-decays would be expected to proceed on to heavier and heavier Q-nuclei until an energetically favorable particle channel is reached. At temperatures which favor the beta-decay of \(^{12}\text{CQ}\) over another \((p,\gamma)\) reaction, one such channel might be a \((p,\alpha)\) or \((p,3\alpha)\) reaction on \(^{11}\text{BQ}\). At still higher temperatures, the rate of proton capture on \(^{12}\text{CQ}\) will exceed the beta-decay rate, and the cycle might then proceed on to \(^{15}\text{NQ}\). A \((p,^4\text{HeQ})\) or \((p,\alpha)\) reaction on \(^{15}\text{NQ}\) would then return a \(^4\text{HeQ}\) or \(^{12}\text{CQ}\) nucleus to the cycle.
Figure 2: Additional Q-nuclear cycles. The HeLiBe cycle is indicated with heavy connecting lines.
When the Q-nuclear cycle depicted in Figure 2 is operating in equilibrium, the most abundant Q-nuclear species would be \(^{6}\text{Li}^Q\), \(^{10}\text{Be}^Q\), and \(^{14}\text{N}^Q\), since these nuclei have the largest lifetimes against the \((p,\gamma)\) reactions which lead up to the allowed particle channels. The steady-state abundances of these nuclei will depend upon the competition between beta-decays and proton radiative captures, and can be found by solving the appropriate equilibrium rate equations. In the discussion which follows, let \(f_\beta^7\) and \(f_\beta^{11}\) denote, respectively, the fraction of \(^7\text{Be}^Q\) and \(^{11}\text{Be}^Q\) nuclei which undergo beta-decay to \(^7\text{Li}^Q\) and \(^{11}\text{B}^Q\), and let \(\tau_7^\beta\) and \(\tau_{11}^\beta\) denote the corresponding beta-decay lifetimes. Then

\[
f_\beta^7 = \frac{\tau_7^\beta}{(\tau_7^\beta + \tau_p)} = \frac{1}{(1 + \tau_7^\beta)} \frac{[\text{^1H}\langle\text{^0}\gamma\rangle]}{[\text{^7Li}\langle\text{^0}\gamma\rangle]} - 1
\]

and

\[
f_\beta^{11} = \frac{\tau_{11}^\beta}{(\tau_{11}^\beta + \tau_p)} = \frac{1}{(1 + \tau_{11}^\beta)} \frac{[\text{^1H}\langle\text{^0}\gamma\rangle]}{[\text{^11C}\langle\text{^0}\gamma\rangle]} - 1
\]

where \(\tau_7^\beta\) and \(\tau_{11}^\beta\) are the lifetimes of \(^7\text{Be}^Q\) and \(^{11}\text{Be}^Q\) against proton capture. With these definitions, the equilibrium rate equations for the Q-nuclei \(^7\text{Be}^Q\), \(^6\text{Be}^Q\), and \(^{12}\text{C}^Q\) take the form

\[
\frac{2}{\text{d}t}[\text{^7Be}^Q] = [\text{^6Li}^Q][\text{^1H}\langle\text{^0}\gamma\rangle] - \frac{[\text{^7Be}^Q]}{\tau_7^\beta} - \frac{[\text{^7Be}^Q][\text{^1H}\langle\text{^0}\gamma\rangle]}{[\text{^7Li}\langle\text{^0}\gamma\rangle]} = 0
\]

\[
\frac{2}{\text{d}t}[\text{^8Be}^Q] = [\text{^7Be}^Q][\text{^1H}\langle\text{^0}\gamma\rangle] + (f_\alpha^{11}f_\alpha^{01} - 1)[\text{^10}^\alpha^Q][\text{^1H}\langle\text{^0}\gamma\rangle] = 0
\]
In the above equations, $f^{11}_a$ and $f^{15}_a$ denote, respectively, the fraction of $^{11}\text{B}Q$ and $^{15}\text{N}Q$ nuclei which undergo a (p,$\alpha$) reaction. Unfortunately, it is not possible to estimate these ratios with any degree of confidence without a more detailed understanding of the quark-nucleon interaction. Consequently, for all model calculations presented here we simply assume that $f^{11}_a = f^{15}_a = 0$, so that the $^4\text{He}Q$ seed nucleus is always returned to the HeLiBe cycle. We will, however, retain these factors in all equations which follow.

Equations (4), (5) and (6) can be solved for the steady-state $^7\text{Be}Q$, $^{10}\text{B}Q$ and $^{14}\text{N}Q$ abundances relative to the $^4\text{Li}Q$ abundance. The result is

$$
\frac{[^7\text{Be}Q]}{[^6\text{Li}Q]} = f^7_\beta \tau^7_\beta \frac{\langle Q \rangle}{6},
$$

$$
\frac{[^{10}\text{B}Q]}{[^6\text{Li}Q]} = \frac{1-f^7_\beta}{(1-f^{11}_\beta f^{11}_a)} \times \frac{\langle Q \rangle}{6}/\frac{\langle Q \rangle}{10},
$$

$$
\frac{[^{14}\text{N}Q]}{[^6\text{Li}Q]} = \frac{(1-f^7_\beta)(1-f^{11}_\beta)(1-f^{11}_a)(1-f^{15}_a)}{(1-f^{11}_\beta f^{11}_a)(1-f^{15}_a)} \times \frac{\langle Q \rangle}{6}/\frac{\langle Q \rangle}{14}.
$$

It is convenient to express the steady-state $Q$-nuclear abundances in terms of the total $Q$-nuclear abundance $[Q]$. 

$$
\frac{3}{2} \frac{[^{12}\text{C}Q]}{[^6\text{Li}Q]} = (1-f^{11}_\beta) \frac{[^{10}\text{B}Q]}{[^6\text{Li}Q]} \frac{\langle Q \rangle}{10} + (f^{15}_a - 1) \frac{[^{14}\text{N}Q]}{[^6\text{Li}Q]} \frac{\langle Q \rangle}{14} = 0.
$$

(6)
Since $^7\text{Li}Q$, $^{10}\text{B}Q$ and $^{14}\text{N}Q$ are expected to be the most abundant Q-nuclei which result from equilibrium Q-nuclear burning, we have, to a very good approximation,

$$[Q] = [^6\text{Li}Q] + [^{10}\text{B}Q] + [^{14}\text{N}Q] . \tag{10}$$

The equilibrium Q-nuclear abundances can then be written as

$$[^6\text{Li}Q] = [Q] / (1 + \Delta_6) , \tag{11}$$

$$[^{10}\text{B}Q] = [Q] / (1 + \Delta_{10}) , \tag{12}$$

$$[^{14}\text{N}Q] = [Q] / (1 + \Delta_{14}) , \tag{13}$$

where

$$\Delta_6 = \frac{(1-f_{15})^Q \langle \sigma v \rangle^Q_{14} + (1-f_{15}^{11})^Q \langle \sigma v \rangle^Q_{10}}{(1-f_{15})^{14} \langle \sigma v \rangle^{14}_{10} (1-f_{15}^{11})^Q \langle \sigma v \rangle^{14}_{10}} \tag{14}$$

$$\Delta_{10} = \frac{(1-f_{15})^{10} \langle \sigma v \rangle^{10}_{14} + (1-f_{15})^{11} \langle \sigma v \rangle^{11}_{10}}{(1-f_{15})^Q \langle \sigma v \rangle^Q_{14} (1-f_{15})^{11} \langle \sigma v \rangle^{11}_{14}} \tag{15}$$

$$\Delta_{14} = \frac{(1-f_{15})^{14} \langle \sigma v \rangle^{14}_{10} + (1-f_{15})^{11} \langle \sigma v \rangle^{11}_{14}}{(1-f_{15})^Q \langle \sigma v \rangle^Q_{10} (1-f_{15})^{11} \langle \sigma v \rangle^{11}_{14}} \tag{16}$$

By inspection of equations (7)-(16) and Figure 2 it is clear that there are three limiting cases which approximately correspond to Q-nuclear burning at low, intermediate and high temperatures. At low temperatures, the $(p,\gamma)$ reactions on $^7\text{Be}Q$ should be inconsequential, so that $f_{15}^7 = 1$. 
In this case, $\Delta_6 \ll 1$, $\Delta_{10} \gg 1$, $\Delta_{14} \gg 1$ and, as expected, $[\Omega] \approx [^6\text{Li}^Q]$. At higher temperatures, the proton radiative captures will proceed on to nuclei heavier than $^7\text{Be}^Q$, but not beyond $^1\text{C}^Q$ if $\gamma^1_B \ll \gamma^1_{\alpha}$. In this case $f_{\alpha}^{11} = 1$, $[^{1\alpha}\text{N}^Q] \approx 0$, and

$$\frac{[^{10}\text{Be}^Q]/[^{6}\text{Li}^Q]}{[^{10}\text{Be}^Q]/[^{6}\text{Li}^Q]} = \frac{1 - f_{\alpha}^{11}}{1 - f_{\alpha}^{11}} \times \frac{\langle \sigma v \rangle_6^{Q}}{\langle \sigma v \rangle_{10}^{Q}} . \quad (17)$$

Since $\langle \sigma v \rangle_6^{Q}$ is many orders of magnitude larger than $\langle \sigma v \rangle_{10}^{Q}$ (see Section 2.3), it is clear that a substantial fraction of the $^4\text{He}^Q$, $^7\text{Li}^Q$ and $^7\text{Be}^Q$ nuclei will be converted to $^8\text{Be}^Q$ unless the lifetime of $^7\text{Be}^Q$ against a $(p, \gamma)$ reaction exceeds the beta-decay lifetime by a similar amount, especially if the $^{11}\text{B}^Q + p$ reaction returns to $^8\text{Be}^Q$ much more frequently than to $^4\text{He}^Q$. At temperatures high enough for the $(p, \gamma)$ reactions to proceed on to $^{1\alpha}\text{N}^Q$, $f_{\alpha}^{11}$ will be close to zero, $[^{6}\text{Li}^Q] \ll [\Omega]$, and

$$\frac{[^{1\alpha}\text{N}^Q]/[^{10}\text{Be}^Q]}{[^{10}\text{Be}^Q]/[^{6}\text{Li}^Q]} = \frac{\langle \sigma v \rangle_6^{Q}}{\langle \sigma v \rangle_{10}^{Q}} \times (1 - f_{\alpha}^{15}) . \quad (18)$$

The rate of alpha production, $R_\alpha$, from the $Q$-nuclear cycles described above is

$$R_\alpha = f_{\beta}^{7}[^6\text{Li}^Q][^7\text{H}]\langle \sigma v \rangle_6^{Q} + f_{\beta}^{11}(2 - f_{\alpha}^{11}) [^9\text{Be}^Q][^7\text{H}]\langle \sigma v \rangle_{10}^{Q}$$

$$+ f_{\alpha}^{15}[^{1\alpha}\text{N}^Q][^7\text{H}]\langle \sigma v \rangle_{14}^{Q} . \quad (19)$$
At high temperatures, Q-nucleosynthesis may also produce carbon at the rate

$$R_c = (1-f_a^Q) [^{14}NQ] [^1H] \langle \sigma v \rangle _{_{14}}^Q.$$  \hspace{1cm} (20)

In normal composition stars, the abundance of $^{12}$C produced in this way would be negligibly small compared to the $^{12}$C already present. However, as suggested by Boyd et al. (1985), sufficient $^{12}$C may be produced to ignite CNO burning in first generation stars.

The rate of Q-nuclear energy production is difficult to estimate because the energy released in the formation of a $^4$He (or $^{12}$C) nucleus depends upon the relative probabilities of the various particle channels indicated in Figure 2, as well as the beta-decay lifetimes, none of which are known. However, the situation is simplified somewhat if the proton capture on $^{11}$B or $^{15}$N favors the return of the $^4$He catalyst. In this case the net result is just the conversion of 8 protons into two helium nuclei (an energy difference of 53.5 Mev), or the conversion of 12 protons into a carbon nucleus (releasing 87.5 Mev of energy). The net energy produced, excluding the neutrino energy lost in the Q-nuclear beta-decays, can be estimated by using the beta-decay energies for the corresponding (normal) nuclei. The rate of energy production (erg cm$^{-3}$ sec$^{-1}$) is then

$$c_Q = q_6 f_7^{[^6LiQ]} [^1H] \langle \sigma v \rangle _6^Q + 2q_{10} f_1^{[^{10}BQ]} [^1H] \langle \sigma v \rangle _{10}^Q + q_{14} [^{14}NQ] [^1H] \langle \sigma v \rangle _{14}^Q,$$  \hspace{1cm} (21)
where \( q_0 = 26.2 \text{ MeV}, \ q_{10} = 20 \text{ MeV}, \) and \( q_{14} = 60 \text{ MeV}. \) Although the assumptions leading to equation (21) are questionable at best, this is not an important consideration since the energy production by way of the heavy \( Q \)-nuclei is expected to be negligible compared to the energy produced by the HeLiBe cycle or by conventional nuclear cycles. What is important is the fraction of light \( Q \)-nuclei which are lost via the \( ^7\text{Be}^Q(p,\gamma)^8\text{B}^Q \) reaction, which depends primarily on the relative rates of proton capture on \( ^6\text{Li}^Q \) and \( ^{10}\text{B}^Q \), and the lifetime of \( ^7\text{Be}^Q \) against proton capture and beta-decay.

2.3 \( Q \)-NUCLEAR REACTION RATES

The rates of most charged-particle reactions involving light nuclei (the p-p reaction being a notable exception) are based upon a low-energy extrapolation of experimental measurements. Obviously, no such experimental data exist for reactions involving \( Q \)-nuclei; consequently, it is necessary to resort to some sort of general description which incorporates the most important features of such reactions. Boyd et al. (1983, 1985), in their investigation of \( Q \)-nuclear burning, adopted the statistical formulation of Woosley et al. (1975). Although this treatment is known to be rather inaccurate for light nuclei, since it relies on a high density of states in the vicinity of the interaction
energy, the dominant feature of charged particle reactions, the Coulomb barrier, is properly taken into account. Such a treatment is the best means presently available for estimating proton capture rates on Q-nuclei, and it is the approach adopted for this study.

Following Woosley et al. (1975) we write the reaction rate \( \langle \sigma v \rangle_{ij} \) between the target nucleus \( i \) and the projectile nucleus \( j \) as

\[
N_A \langle \sigma v \rangle_{ij} = 7.883 \times 10^9 \frac{(Z_i Z_j)^{4/3} T_{9a}}{A^{5/6} T_j^{3/2}} \exp(2\chi - \tau_j T_{9a}^{-1/3})
\]

where

\[
T_{9a} = T_9 (1 + 0.08167 a T_9)^{-1}
\]

\[
\tau_j = 4.2487 (Z_i^2 Z_j^2 A)^{1/3}
\]

\[
a = 0.1215 (AR^3/Z_i Z_j)^{1/2}
\]

\[
\chi = 0.52495 (AZ_i Z_j R)^{1/2}
\]

and

\[
R = 1.25 A_j^{1/3} + 0.1
\]

\( N_A \) is Avogadro's number, \( T_9 \) is the temperature in units of \( 10^9 \) K and \( A \) is the reduced mass of the interacting nuclei. Since the mass of a Q-nucleus is not known, we assume that it is the same as the corresponding nucleus. The Q-nuclear reaction rates predicted by equation (22) will then differ
from those predicted for the corresponding nuclei only in the value of the Q-nuclear charge. The proton capture rates on the Q-nuclei *Li^Q, 7Be^Q, 10B^Q and 14N^Q are given by

\[
N_A \langle\sigma v\rangle^Q_{\delta} = 1.83 \times 10^{11} T^{-2/3}_9 \exp(-9.60/T^{1/3}_9),
\]

(28)

\[
N_A \langle\sigma v\rangle^Q_{\gamma} = 3.91 \times 10^{11} T^{-2/3}_9 \exp(-11.35/T^{1/3}_9),
\]

(29)

\[
N_A \langle\sigma v\rangle^Q_{10} = 9.25 \times 10^{11} T^{-2/3}_9 \exp(-13.08/T^{1/3}_9),
\]

(30)

\[
N_A \langle\sigma v\rangle^Q_{14} = 3.56 \times 10^{12} T^{-2/3}_9 \exp(-16.14/T^{1/3}_9).
\]

(31)

The temperature dependence of these reactions is illustrated in Figure 3; also shown are the \(^1\text{H}(^1\text{H},\text{e}^+\nu)^2\text{H}\) and \(^{14}\text{N}(p,\gamma)^{15}\text{O}\) reactions.

The errors introduced by applying the statistical treatment of Woosley et al. (1978) to light nuclei can be roughly gauged by comparing the predictions of equation (22) and the experimentally determined reaction rates for the same nuclei. In Figure 4 are plotted the *Li(p,\gamma)\(^7\text{Be}, \(^{10}\text{B}(p,\gamma)^{11}\text{C}\) and \(^{14}\text{N}(p,\gamma)^{15}\text{O}\) reaction rates as given by Fowler et al. (1975); also shown are the same rates as given by equation (22). At a given temperature, the calculated rates are roughly three orders of magnitude larger than the experimentally measured values. However, as noted by
Figure 3: Q-nuclear reaction rates. The $^3\text{He}(p,\gamma)^4\text{He}$ reactions are indicated by the dotted and dashed curves, respectively. Individual rates have been multiplied by the scale factors indicated in parentheses.
Figure 4: $^6$Li(p,γ)$^7$Be, $^{10}$B(p,γ)$^{11}$C and $^{14}$N(p,γ)$^{15}$O reaction rates. Solid curves are the experimentally measured rates; broken curves are the rates predicted by the statistical treatment of Woosley et al. (1975). Note scale factors in parentheses.
Boyd et al. (1985), the statistical approach may be more accurate for Q-nuclei since the presence of the embedded quark may well increase the density of Q-nuclear states. It should also be noted that the reactions shown in Figure 4 are non-resonant; since the rates of many radiative capture reactions on light nuclei are dominated by resonances, the statistical treatment adopted here would be expected to over-estimate the rates of non-resonant reactions. In any event, the uncertainties associated with the Q-nuclear reaction rates are probably not less than about two orders of magnitude, and Q-nuclear abundances inferred from model calculations are expected to be uncertain by a comparable amount.
3.1 **INPUT PHYSICS**

The stellar evolution code used in this study is a substantially re-written version of the Paczynski code. Since many features of this program have been described in detail elsewhere (Paczynski, 1970; Joseph, 1984b), we will describe here only those changes in the constitutive physics and numerical approximations which have a bearing on the present study.

The radiative Rosseland mean opacity $\kappa_r$ is obtained by interpolation in the opacity tables of Cox and Stewart (1970a,b). The interpolation of $\log(\kappa_r)$ in $\log(\rho)$ and $\log(T)$ is a four point Lagrangian while the interpolation in composition is linear in the mass fractions $X$ and $\log(Z)$. At temperatures below $10^4$ K, the radiative opacity due to water vapor is calculated following Paczynski (1970). For the conductive opacity $\kappa_c$ the approximation formulae of Sweigert (1973) are used. These formulae are fitted to the data of Hubbard and Lampe (1969) and Canuto (1970) in the non-relativistic and relativistic regimes, respectively. The combined opacity is given by
\( k = \frac{\kappa T \kappa c}{(\kappa r + \kappa c)} \) \hspace{1cm} (32)

All thermonuclear reaction rates (Q-nuclear rates excluded) were computed according to the expressions given by Fowler et al. (1975), except for the reactions \( ^1H(p,e^+\nu)^2H \), \( ^4\text{He}(^3\text{He},\gamma)^7\text{Be} \), and \( ^7\text{Be}(p,\gamma)^8\text{B} \), for which we have used the expressions of Harris et al. (1983). The local rate of energy production (erg gm\(^{-1}\) s\(^{-1}\)) from the equilibrium PP cycle is given by the expression

\[
c_{PP} = Q_{11}R_{11}X_1^2 + Q_{33}R_{33}X_3^2 \\
+ Q_{34} fQ_{76} + (1-f)Q_{71} R_{34}X_3X_4
g(33)
\]

where \( R_{11}X_1X_1 \) is the reaction rate (gm\(^{-1}\) s\(^{-1}\)) between the nuclear species \( i \) and \( j \), and \( Q_{11}, Q_{33}, Q_{34}, Q_{76}, \) and \( Q_{71} \) are, respectively, the energy released (excluding neutrino losses) by the reactions \( ^1H(p,e^+\nu)^2H(p,\gamma)^3\text{He} \), \( ^3\text{He}(^3\text{He},2p)^4\text{He} \), \( ^3\text{He}(^4\text{He},\gamma)^7\text{Be} \), \( ^7\text{Be}(e^-\nu)^7\text{Li}(p,\alpha)^4\text{He} \), and \( ^7\text{Be}(p,\gamma)^6\text{B}(e^+\nu)^7\text{Be}(\alpha)^4\text{He} \). In equation (33) \( f \) is the branching ratio between the PPII and PPIII branches:

\[
f = \frac{R_{71}X_3}{(R_{71}X_3 + R_{71}X_1)} \hspace{1cm} (34)
\]

The \(^3\text{He} \) abundance in equation (33) is found by solving the equilibrium rate equation

\[
X_3 = R_{11}X_1^2 - 2R_{33}X_3^2 - R_{34}X_3X_4 = 0 \hspace{1cm} (35)
\]
Energy production from the equilibrated CNO cycle is considerably easier to treat than is the case for the PP cycle. The only significant side branch (the NO cycle) occurs approximately $10^4$ times less frequently than the dominant CN cycle, and can be safely neglected as far as energy production is concerned. The slowest reaction in the CN cycle is the proton radiative capture on $^{14}$N, and the rate of energy production is calculated by

$$e_{\text{CN}} = Q_{\text{CN}} R_{\text{14}} x_1 x_{14},$$

where $Q_{\text{CN}} = 25.2$ Mev, and the $^{14}$N abundance is approximately equal to the original C+N abundance if the temperature is too low for the NO cycle to have come to equilibrium. At temperatures high enough to ensure equilibrium of the NO reactions, the $^{14}$N abundance is set equal to the original C+N+O abundance. The reaction rates $R_{ij}$ are corrected for electron screening following the treatment of Gabroske et al. (1973). For the hydrogen-burning reactions, only the weak screening approximation is considered.

Two different formulations of the equation of state are employed depending upon the temperature region under consideration. In the outer envelope, at temperatures below approximately $10^5$ K, the physics is complicated by partial
ionization and superadiabatic convection. Under these conditions we treat in detail the ionization equilibria of hydrogen and helium, using ground state partition functions in the Saha equations. The ionization of elements heavier than helium is treated very crudely by considering two stages of ionization of a single "generic" metal with an abundance equal to the total heavy element abundance $Z$. At temperatures below $10^4$ K dissociation of the $\text{H}_2$ molecule is taken into account as well. The Saha equations are solved iteratively with a Newton-Raphson algorithm, and convergence is assumed when the correction to the electron density is less than one part in $10^4$. The total pressure is given by the sum of the (non-degenerate) ion and electron pressures, the blackbody radiation pressure, and a negative pressure contribution which takes into account Coulomb interactions in the Debye-Hückel approximation (Cox and Giuliani, 1968). The partial derivatives of the pressure, internal energy and other state variables are approximated numerically by calculating the changes in these quantities in response to small perturbations of the independent variable. Superadiabatic convection is treated within the framework of standard mixing length theory (Cox and Giuliani, 1968), with a constant ratio of mixing length to pressure scale height. The excess of the actual temperature gradi-
ent over the adiabatic gradient is found by solving the usual cubic equation with a Newton-Raphson algorithm. The boundary between radiative and convective zones is based upon the Schwarzschild criterion.

If helium is more than 99.9% doubly ionized, complete ionization is assumed for all constituents. The ions (except for the Debeye-Huckel correction) are assumed to obey an ideal gas equation of state. Partial degeneracy of the electron gas is taken into account in both the relativistic and non-relativistic regimes. The electron pressure and internal energy, and their derivatives with respect to temperature and density, were numerically integrated using the expressions given by Chandrasekhar (1939) and are accurate to one part in $10^4$. The resulting values are stored in tabular form as a function of temperature and density, and the electron contribution to the equation of state is obtained by four point Lagrangian interpolation in these tables. Convective energy transport is considered to be perfectly efficient, and the temperature gradient in a convective zone is set equal to the adiabatic gradient.
3.2 MODEL CONSTRUCTION

The program solves the usual scalar forms of the stellar structure equations using the Henyey relaxation method (Henyey et al. 1964). The dependent variables are \( \ln(T) \), \( \ln(P) \), \( \ln(r/R_0) \), and \( L_r/L_0 \), while \( M_r/M_0 \) is taken to be the independent variable. The changes in the dependent variables between adjacent Henyey mesh points are constrained so as not to exceed certain preset limits, and the distribution of grid points is examined frequently to ensure that these constraints are satisfied. When new grid points are added, the values of the dependent variables at the new points are obtained by four point Lagrangian interpolation. Convergence to the correct solution is assumed when the corrections to the dependent variables are less than some preset tolerance (typically \( 10^{-3} \)) at all mesh points.

In order to obtain the outer boundary conditions for the interior Henyey mesh points, the stellar structure equations are integrated inward from the photosphere (i.e., from a Rosseland mean optical depth \( \tau = 2/3 \)) to a prescribed mass point in the envelope with a fourth-order Runge-Kutta algorithm. The starting values at \( \tau = 2/3 \) are obtained from a separate atmosphere integration, using the scaled solar \( T(\tau) \) relation of Krishna Swamy (1966). The mass contained within the envelope integration is chosen to be large
enough to ensure that, at the bottom of the envelope, ionization is complete and convection, if present, is nearly adiabatic. This is necessary because the equation of state employed in the interior (i.e., below the point at which the outer boundary conditions are applied) is based upon these assumptions. The envelope defined in this way typically contains 5% of the stellar mass. The temperature, density and radius at the bottom of the envelope serve as outer boundary conditions for the stellar interior. In practice, envelope integrations are made for each corner of a rectangle in the H-R diagram within which the current stellar model is located. Typical dimensions of this rectangle are 0.05 in log(L/L_\odot) and 0.01 in log(T_eff). The outer boundary conditions for any given model are then obtained by linear interpolation.

The first model of an evolutionary sequence is assumed to be chemically homogeneous, with specified abundances (by mass) of hydrogen (X), helium (Y), and heavy elements (Z). We also assume that the entropy term in the energy balance equation is negligible compared to the rate of nuclear energy production in the zero age model. A first approximation for this model is obtained by the conventional method of integrating outward from the center and inward from the outer boundary, and joining these integrations at some
intermediate point. The structure of the model so obtained is taken to be a trial solution, and the Henyey method is used to obtain the final converged zero age main sequence model.

The size of the time step separating consecutive models of an evolutionary sequence is limited by several constraints, and if any of these constraints are violated the time step is reduced accordingly. Having selected a time step, a trial model for the new epoch t+Δt is obtained by a linear extrapolation of the two preceding models. The depletion of hydrogen as a result of nuclear burning is calculated with a fully implicit, backward differencing scheme. Although time-centered differencing may be more accurate, backward differencing ensures numerical stability as the hydrogen abundance approaches zero. At each grid point the rate equation describing hydrogen-burning nucleosynthesis is solved with a Newton-Raphson algorithm, with convergence assumed when the correction to the hydrogen abundance is less than one part in 10⁴. In a convective zone complete mixing of hydrogen and helium is assumed, and the change in composition is calculated by using the mean values of the reaction rate coefficients, averaged over the mass of the mixed region. After each iteration of the Henyey method, the change in chemical composition is recal-
culated at every grid point using the corrected values of the dependent variables.

In the following chapters we will use the program described above to investigate the consequences of Q-nucleosynthesis on stellar interior models. Unless specifically stated otherwise, the numerical methods and convergence criteria described above will be employed for all model calculations.
Chapter IV

Q-NUCLEOSYNTHESIS AND SOLAR EVOLUTION

4.1 THE STANDARD SOLAR MODEL

The discrepancy between the observed and predicted flux of high-energy neutrinos from the Sun is one of the most persistent and perplexing problems in contemporary astrophysics. The suggestion that Q-nucleosynthesis may contribute to stellar burning was, in fact, largely activated by the long-standing solar neutrino problem, and so the Sun is a logical place to begin an investigation of Q-nuclear energy production. In this section we will discuss the main features of the standard solar model adopted for this study. In the following sections we will examine departures from the standard model which result from Q-nuclear burning, both with and without attrition of Q-nuclei from the HeLiBe cycle. We conclude this chapter with a comparison of the observed solar parameters and those predicted by the standard model and models which incorporate Q-nuclear burning.

Models of the solar interior must precisely reproduce the known solar parameters. The most accurately determined of these parameters are the mass \( M_\odot = 1.989 \times 10^{33} \) gm, lumi-
nosity \( L_0 = 3.86 \times 10^{33} \text{ erg sec}^{-1} \), and radius \( R_0 = 6.96 \times 10^{10} \text{ cm} \). Furthermore, the age of the oldest meteorites is about \( 4.6 \times 10^9 \) years (Wasserburg et al. 1977, 1980). Although it is difficult to estimate the time elapsed between the formation of the Sun and the subsequent condensation of meteoroids, it is generally believed that this time difference is small compared to the main sequence lifetime of the Sun (Bahcall et al. 1982). The commonly accepted value of the age of the Sun is \( t_0 = 4.7 \times 10^9 \) years; this is the value we shall adopt for the present investigation.

The chemical composition of the Sun is less well known. Abundance determinations from photospheric spectra yield a heavy element abundance \( Z/X = 0.0228 \), with an estimated uncertainty of \( \sim 10\% \) (Boss and Aller, 1976). The solar helium abundance, on the other hand, is exceedingly difficult to measure because the photospheric temperature is too low to excite the helium spectrum. The usual procedure is to adopt the spectroscopically measured value of \( Z \) or \( Z/X \), and to determine an appropriate value of \( Y \) by imposing the constraint that the solar model must reproduce the observed solar luminosity; \( Y \) is therefore a theoretical prediction of the standard solar model.

All solar models presented here are based upon the standard assumptions of uniform chemical composition at \( t=0 \), no
rotation and no internal magnetic fields. Furthermore, we assume that all nuclei (except $^1$H and $^4$He) participating in the PP and CNO cycles have reached steady state (equilibrium) abundances. This approximation is justified by a consideration of the lifetimes of the relevant nuclear species at the temperatures and densities expected in the solar interior. The longest-lived PP reactant (excluding $^1$H) is $^3$He. According to Clayton (1968), at $T=10^7$ K, $^3$He reaches a steady state abundance after approximately $10^6$ years of hydrogen-burning; at $T=1.5 \times 10^7$ K, the equilibrium timescale is approximately $10^6$ years. Consequently, $^3$He comes to equilibrium after very little hydrogen-burning. The CN portion of the CNO bi-cycle cycle reaches equilibrium on a timescale comparable to the lifetime of $^{12}$C, which is about $10^6$ years at $T=1.5 \times 10^7$ K and $10^9$ years at $T=10^7$ K. Although this lifetime is comparable to the main sequence lifetime for temperatures less than about $10^7$ K, the energy production from the CN cycle at such low temperatures is negligible. The NO branch of the CNO bi-cycle is neglected entirely, as is justified by the long lifetime ($\sim 10^{11}$ year) of $^{16}$O against proton capture (Clayton, 1968). The validity of these assumptions will be demonstrated later in this section.

Under steady state conditions, the equations of nucleosynthesis reduce to the single equation
where \( X_{ij} \) is the abundance by mass of nuclear species \( i \), and \( R_{ij} X_i X_j \) is the reaction rate \( (gm^{-1}sec^{-1}) \) between species \( i \) and \( j \). In equation (37) we have assumed that all of the carbon and nitrogen originally present has been converted to \(^{14}\)N as a consequence of equilibrium CN burning. The \(^3\)He abundance is given implicitly by the equilibrium rate equation (35); at those points in the model where equation (35) yields an equilibrium abundance larger than 0.001, we use this value in place of the steady-state abundance. Since the peak in the \(^3\)He distribution occurs well outside the energy producing region, this approximation should not introduce any appreciable errors. The rate of neutrino production \( R_{\nu} \) (neutrinos per sec) from each of the neutrino-producing reactions of the PP and CNO cycles is given by an integral over the solar mass of the pertinent reaction rate. Under steady-state conditions we have

\[
R_{\nu}(pp) = \int R_{11} X_1^2 \, dm , \tag{38}
\]

\[
R_{\nu}(\text{pep}) = 5.51 \times 10^{-5} \int \rho (1+X_1) T_6^{-1.2} (1+0.02 T_6) R_{11} X_1^2 \, dm , \tag{39}
\]

\[
R_{\nu}(^{7}\text{Be}) = \int r R_{34} X_3 X_4 \, dm , \tag{40}
\]

\[
R_{\nu}(^{8}\text{B}) = \int (1-r) R_{34} X_3 X_4 \, dm , \tag{41}
\]
\[ R_{\nu}^{(13N)} = \int R_{14} \, X_1 X_{14} \, d\mu \]  \hspace{1cm} (42) 

and

\[ R_{\nu}^{(15O)} = R_{\nu}^{(13N)} \]  \hspace{1cm} (43) 

where the branching ratio \( f \) expresses the relative probabilities of electron capture and proton capture on \( ^7\)Be (c.f. equation (34)). The expression for the pep reaction rate is taken from Bahcall et al. (1982) and the \( ^7\)Be(e\(^-\),\(\nu\))^7Li reaction rate is corrected for the capture of continuum electrons following Bahcall and Moeller (1969). Except for the simplifications described above, the input physics are as described in Section 3.1.

A model of the present Sun is obtained by simultaneously varying \( Y \), the primordial solar helium abundance, and \( \alpha = 1/H_P \), the ratio of mixing length to pressure scale height (assumed constant), until the observed solar radius \( R_o \) and luminosity \( L_o \) are obtained after an elapsed time of \( 4.7 \times 10^9 \) years. A common practice when constructing a solar model is to assume a value of \( Z = 0.02 \) for the heavy element abundance, since this is close to the solar value and also because opacity tables are available for this abundance, thereby circumventing the need to interpolate the opacity in \( Z \). However, if \( Z \) is kept fixed, the resulting value for
the ratio $Z/X$ may differ considerably from the observed value of $Z/X=0.0228$. For this reason, $Z$ was varied together with $Y$ so as to keep the ratio $Z/X$ fixed at the observed value. Given trial values of $Y$ and $\alpha$, say $Y_0$ and $\alpha_0$, an evolutionary sequence is constructed which yields a luminosity $L_0$ and radius $R_0$ at $t=t_0$:

\begin{align*}
L(Y_0, \alpha_0) &= L_0 \\
R(Y_0, \alpha_0) &= R_0
\end{align*}

(44)  
(45)

In general, $L_0 \neq L_0$ and $R_0 \neq R_0$. Let $\Delta \alpha$ and $\Delta Y$ represent corrections which when added to the trial values $\alpha_0$ and $L_0$ yield the desired eigenvalues $Y$ and $\alpha$. Expanding $L$ and $R$ in Taylor series about $\alpha_0$ and $Y_0$, and retaining only first order terms, gives two equations for the unknown corrections $\Delta Y$ and $\Delta \alpha$:

\begin{align*}
\frac{\partial L}{\partial Y} |_{Y_0} \delta Y + \frac{\partial L}{\partial \alpha} |_{\alpha_0} \delta \alpha &= L_0 - L_0 \quad (46) \\
\frac{\partial R}{\partial Y} |_{Y_0} \delta Y + \frac{\partial R}{\partial \alpha} |_{\alpha_0} \delta \alpha &= R_0 - R_0 \quad (47)
\end{align*}

The derivatives in equations (46) and (47) were evaluated numerically by evolving a series of models with slightly perturbed values of $\alpha$ and $Y$. The improved trial values of $Y$ and $\alpha$ obtained from equations (46) and (47) were iterated until the model radius and luminosity at $t_0$ were equal to the observed solar values to within a fractional accuracy of 0.001.
The time steps between consecutive models in an evolutionary sequence were constrained by several criteria. The expected changes in $\log(T_{\text{eff}})$ and $\log(L/L_\odot)$ were not permitted to exceed 0.005 and 0.01, respectively. In addition, the maximum change in the central hydrogen abundance was limited to 0.01. The changes between Henyey mesh points in a particular model were limited to 0.05 in $\log(T)$, $\log(L)$ and $\log(r)$, and 0.15 in $\log(\rho)$. Furthermore, since most observable solar neutrinos originate within the inner 0.05$R_\odot$, the density of mesh points in this region was substantially increased relative to the norm in order to ensure that the neutrino flux was accurately calculated. Convergence was assumed when the corrections to the dependent variables had fallen to less than one part in $10^3$ at every mesh point. These criteria typically yielded evolutionary sequences of 25-30 models, and a final solar model with approximately 120 mass shells.

The important features of the standard solar model obtained in this way are shown in Figure 5 and Tables 1-3. The primordial solar helium abundance by mass is $Y = 0.23$, and the corresponding heavy element abundance is $Z = 0.017$. The mixing length parameter required to reproduce the solar radius is $l/H_p = 1.5$. The outer convective zone has a depth of 0.26$R_\odot$ (0.012$R_\odot$) and terminates at a temperature of $1.85 \times 10^6$K. With regard to energy production, the PP chains
contribute 98.5% of the solar luminosity, while the CNO cycle contributes the remaining 1.5%. Of the energy produced by the PP cycle, 89.8% originates in the PPI branch; the PPIII branch, through which most of the detectable solar neutrinos are produced, accounts for only 0.01% of the total energy output from the PP chains. One-half of the solar luminosity originates within the inner 0.09\(M_\odot\), while 95% originates within 0.35\(M_\odot\). As indicated in the last column of Table 2, one-half of the high-energy \(^8\)B neutrinos are produced within the inner 0.01\(M_\odot\), and 95% originate within 0.05\(M_\odot\).

The neutrino flux at the Earth, and the predicted capture rates for the on-line \(^{37}\)Cl detector and the proposed \(^{71}\)Ga detector, are shown in Table 3 for the reactions \(^1\)H(p e\(^+\nu\))\(^2\)H, \(^1\)H(e\(^-\)\ P\(\\nu\))\(^2\)H, \(^7\)Be(e\(^-\)\(\nu\))\(^7\)Li, \(^8\)B(e\(^+\nu\))\(^8\)Be, \(^13\)N(e\(^+\nu\))\(^{13}\)C, and \(^15\)O(e\(^+\nu\))\(^{15}\)N. The predicted capture rates for these two detectors are based upon the neutrino absorption cross sections of Bahcall et al. (1978). As expected, the neutrinos from the decay of \(^8\)B account for about 75% of the total count rate of 5.2 SNU for the \(^{37}\)Cl detector; the \(^7\)Be(e\(^-\nu\))\(^7\)Li reaction contributes most of the remaining detectable neutrinos.

It is important to verify that the standard solar model adopted for this study is consistent with other solar models reported in the literature, and that the assumptions
### Table 1

**Some fundamental parameters of the standard solar model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$1.989 \times 10^{23}$ gm</td>
</tr>
<tr>
<td>Luminosity</td>
<td>$3.86 \times 10^{33}$ erg sec$^{-1}$</td>
</tr>
<tr>
<td>Radius</td>
<td>$6.96 \times 10^{10}$ cm</td>
</tr>
<tr>
<td>Age</td>
<td>$4.7 \times 10^{10}$ yr</td>
</tr>
<tr>
<td>Heavy element abundance by mass</td>
<td>0.0172</td>
</tr>
<tr>
<td>Primordial $^4$He abundance by mass</td>
<td>0.229</td>
</tr>
<tr>
<td>Depth of convective envelope</td>
<td>$0.26 R_\odot (0.012 M_\odot)$</td>
</tr>
<tr>
<td>Central temperature</td>
<td>$15.0 \times 10^6$ K</td>
</tr>
<tr>
<td>Central density</td>
<td>$155$ gm cm$^{-3}$</td>
</tr>
<tr>
<td>Central $^1$H abundance by mass</td>
<td>0.389</td>
</tr>
<tr>
<td>Fraction of energy from PP cycle</td>
<td>0.985</td>
</tr>
<tr>
<td>Fraction of energy from CN cycle</td>
<td>0.015</td>
</tr>
</tbody>
</table>
Table 2

The standard solar model

The temperature and density are in cgs units. The last column gives the fraction of $^8$B neutrinos which originate within the corresponding mass fraction.

<table>
<thead>
<tr>
<th>$M(r)/M_0$</th>
<th>$r/R_0$</th>
<th>log T</th>
<th>log ρ</th>
<th>$L(r)/L_0$</th>
<th>$X(^7\text{H})$</th>
<th>$L_0(^8\text{B})$</th>
</tr>
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<td>0.0000</td>
<td>0.0000</td>
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</tr>
<tr>
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<td>0.1252</td>
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<td>1.0000</td>
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</tbody>
</table>
**Figure 5:** Variation with respect to mass of the temperature, density, radius, luminosity and hydrogen abundance in the standard solar model.
Table 3

*Predicted neutrino fluxes and capture rates*

All capture rates are given in solar neutrino units (SNU).

<table>
<thead>
<tr>
<th>Source</th>
<th>$\phi_{\text{cm}^{-2} \text{s}^{-1}}$</th>
<th>$\phi \text{ (}\text{^7Cl}\text{)}$</th>
<th>$\phi \text{ (}\text{^7Ga}\text{)}$</th>
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</thead>
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<td>69.50</td>
</tr>
<tr>
<td>pep</td>
<td>$1.59 \times 10^{8}$</td>
<td>0.246</td>
<td>2.62</td>
</tr>
<tr>
<td>$^7\text{Be}$</td>
<td>$3.37 \times 10^{9}$</td>
<td>0.792</td>
<td>22.64</td>
</tr>
<tr>
<td>$^8\text{B}$</td>
<td>$3.57 \times 10^{9}$</td>
<td>3.851</td>
<td>1.13</td>
</tr>
<tr>
<td>$^{13}\text{N}$</td>
<td>$4.00 \times 10^{8}$</td>
<td>0.264</td>
<td>2.38</td>
</tr>
<tr>
<td>$^{15}\text{O}$</td>
<td>$4.00 \times 10^{8}$</td>
<td>0.065</td>
<td>1.38</td>
</tr>
<tr>
<td>$\Sigma \phi \sigma$</td>
<td></td>
<td>5.217</td>
<td>99.65</td>
</tr>
</tbody>
</table>
and numerical approximations employed do not adversely affect the results. Table 4 compares selected parameters of several solar models which employ similar input physics, but were constructed with different stellar evolution codes. Consequently, differences in the parameters listed in Table 4 mainly reflect differences in the numerical techniques employed rather than the treatment of the constitutive physics. Models 1-15 are based upon the Cox and Stewart (1970a,b) opacity tables and, with the exception of Model 3, the nuclear reaction rates of Fowler et al. (1975). Model 1 is the standard solar model adopted for this investigation; Model 2 is similar to 1 except that the heavy element abundance was fixed at Z=0.02 in order to facilitate a comparison with the other models listed. Both of these models are in excellent agreement with other solar models which employ the same opacities and nuclear reaction rates. It is difficult to account for the rather large range (0.22-0.25) in the $Y$ eigenvalue. However, different workers have adopted different values for the solar constant, to which $Y$ is particularly sensitive. The method of opacity interpolation also influences the inferred value of $Y$: linear interpolation tends to overestimate the opacity, leading to an overestimate of $Y$. At any rate, the value of $Y$ obtained here lies well within the range of values predicted by other solar models. The disparity in the $1/H_p$
eigenvalues is probably the result of the many variations in mixing length theory adopted by different workers, as well as differences in the treatment of the photospheric layers. For example, when a gray $T(t)$ relation is used instead of the scaled solar $T(t)$ adopted here, the mixing length eigenvalue decreases from 1.55 to 1.4.

The level of numerical noise was checked by constructing a solar model for which the mesh point spacing was decreased by a factor of three, and the convergence tolerance increased by a factor of ten, relative to the values employed for the model described above. The parameters so obtained agreed with those listed in Tables 1 and 2 to within 2%; the difference in the neutrino count rate was 0.15 SNU. Consequently, errors arising from the finite zoning and numerical tolerances are small compared to other known sources of error. In order to justify the assumption of steady-state abundances, another solar model was constructed in which the abundances of $^1\text{H}$, $^3\text{He}$, $^4\text{He}$, $^{12}\text{C}$, $^{13}\text{C}$, $^{14}\text{N}$ and $^{16}\text{O}$ were followed in detail by solving the full set of nucleosynthesis equations in the manner described by Bodenheimer et al. (1965). As expected, at $t=t_0$ the PP and CN cycle nuclei were in equilibrium within the inner 0.5$M_\odot$. The parameters of this model differed from those in Table 1 by no more than 0.1%. Thus no appreciable errors are introduced by adopting local steady-state abundances for the minor nuclear constituents.
### Table 4

**A comparison of standard solar models**

<table>
<thead>
<tr>
<th>Model</th>
<th>Z</th>
<th>Y</th>
<th>1/Hp</th>
<th>$T_c/10^6$ K</th>
<th>$\phi_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.017</td>
<td>0.229</td>
<td>1.50</td>
<td>15.01</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>0.020</td>
<td>0.235</td>
<td>1.55</td>
<td>15.10</td>
<td>5.7</td>
</tr>
<tr>
<td>3</td>
<td>0.020</td>
<td>0.234</td>
<td>-</td>
<td>14.89</td>
<td>6.1</td>
</tr>
<tr>
<td>4</td>
<td>0.020</td>
<td>0.243</td>
<td>-</td>
<td>14.88</td>
<td>5.6</td>
</tr>
<tr>
<td>5</td>
<td>0.020</td>
<td>0.253</td>
<td>1.35</td>
<td>-</td>
<td>5.3</td>
</tr>
<tr>
<td>6</td>
<td>0.020</td>
<td>0.222</td>
<td>1.38</td>
<td>-</td>
<td>9.1</td>
</tr>
<tr>
<td>7</td>
<td>0.020</td>
<td>0.225</td>
<td>-</td>
<td>-</td>
<td>7.0</td>
</tr>
<tr>
<td>8</td>
<td>0.015</td>
<td>0.248</td>
<td>1.80</td>
<td>14.77</td>
<td>4.7</td>
</tr>
<tr>
<td>9</td>
<td>0.020</td>
<td>0.244</td>
<td>1.10</td>
<td>14.90</td>
<td>5.8</td>
</tr>
<tr>
<td>10</td>
<td>0.020</td>
<td>0.250</td>
<td>1.35</td>
<td>14.90</td>
<td>5.2</td>
</tr>
<tr>
<td>11</td>
<td>0.020</td>
<td>0.220</td>
<td>2.04</td>
<td>-</td>
<td>5.6</td>
</tr>
<tr>
<td>12</td>
<td>0.020</td>
<td>0.246</td>
<td>1.64</td>
<td>14.84</td>
<td>5.4</td>
</tr>
<tr>
<td>13</td>
<td>0.020</td>
<td>0.253</td>
<td>1.35</td>
<td>-</td>
<td>5.7</td>
</tr>
<tr>
<td>14</td>
<td>0.020</td>
<td>0.240</td>
<td>1.40</td>
<td>-</td>
<td>4.6</td>
</tr>
<tr>
<td>15</td>
<td>0.020</td>
<td>0.232</td>
<td>1.70</td>
<td>-</td>
<td>---</td>
</tr>
<tr>
<td>16</td>
<td>0.017</td>
<td>0.254</td>
<td>1.60</td>
<td>15.35</td>
<td>7.8</td>
</tr>
<tr>
<td>17</td>
<td>0.017</td>
<td>0.244</td>
<td>1.80</td>
<td>15.50</td>
<td>7.6</td>
</tr>
<tr>
<td>18</td>
<td>0.017</td>
<td>0.270</td>
<td>1.50</td>
<td>-</td>
<td>8.3</td>
</tr>
</tbody>
</table>

**REFERENCES:**

As is well known, the largest source of error in the constitutive physics, and hence in the predicted neutrino flux, stems from uncertainties in the radiative opacities. Models 17 and 18 in Table 4 are based upon the opacity data of the Los Alamos Astrophysical Opacity Library (Huebner et al. 1977). At temperatures and densities characteristic of the solar interior, these opacities are typically 15% larger than the Cox-Stewart opacities. As is apparent in Table 4, the main effect of these larger opacities on solar models is a larger inferred helium abundance, a larger central temperature, and a correspondingly larger neutrino flux. The influence of the opacity data is further demonstrated by Model 16, which is similar to Model 1 except that the radiative opacities were everywhere increased by a factor of 1.15. The resulting model is in qualitatively good agreement with Models 17 and 18. Although it would have been preferable to use the more recent Los Alamos opacities in place of the Cox-Stewart opacities, this should have little effect on the results of this study as long as it is understood that the solar model adopted here is only intended to serve as a calibration against which variations can be measured. However, in order to insure that relative variations are not much affected by the treatment of the opacities, we will on occasion compare results with models in which the opacity has been artificially increased by 15%.
4.2 NON-STANDARD MODELS

In this and the following sections we will use the formalism developed in Chapter II to examine the consequences of Q-nucleosynthesis on the structure and observable properties of the Sun. We begin by considering energy production from the HeLiBe cycle, ignoring any attrition of Q-nuclei from this cycle via a \((p,γ)\) reaction on \( ^7\text{Be}^0 \). From a computational standpoint, this is tantamount to setting \(τ_\text{He}^0 = 0 \) in Equation (2); from a physical standpoint, we are simply assuming that the beta-decay lifetime of \( ^7\text{Be}^0 \) is very much less than the lifetime against proton capture. We will relax this assumption in the following section and investigate the possible consequences of a significant attrition of Q-nuclei from the HeLiBe cycle.

With a finite Q-nuclear abundance, the equations describing hydrogen-burning nucleosynthesis and energy production take the form

\[
\frac{3}{2t}[^4\text{He}] = [^3\text{He}]2<σν>_{33} + [^3\text{He}][^4\text{He}]<σν>_{34} \\
+ [^1\text{H}][^1\text{H}]<σν>_{14,1} + [^1\text{H}][Q]<σν>_{6}^Q
\]

and

\[
ɛ_{\text{nuc}} = ɛ_{\text{pp}} + ɛ_{\text{CN}} + q_6[^1\text{H}][Q]<σν>_{6}^Q,
\]

where \([Q] = [^6\text{Li}^Q]\) if there is no branching at \( ^7\text{Be}^0 \), as we have assumed. Since the total Q-nuclear abundance is not known a priori, we leave it as a free parameter. If we
define $Q/N$ to be the ratio of Q-nuclei to normal nuclei in the chemically homogeneous, zero age Sun, we have

$$[Q] = \frac{N_{Q}}{\mu_{0} N}$$

(50)

where $\mu_{0}$ is the mean number of nucleons per ion corresponding to the solar surface composition. Note that the actual number of Q-nuclei, relative to normal nuclei, will change slightly with time (and with position in the Sun) as hydrogen is converted to helium, because the number of normal nuclei per unit mass decreases as a result of hydrogen-burning, while the number of Q-nuclei per unit mass remains constant. Consequently, the ratio $Q/N$ near the solar center may be as much as a factor of four larger than the parameter we have chosen to represent the Q-nuclear abundance.

A series of solar models, each characterized by different assumed values of the parameter $Q/N$, were constructed as described in the previous section using the same step-widths and numerical tolerances as for the standard model. The results of these calculations are summarized in Table 5 and Figures 6-12. In Figure 6 are shown the eigenvalues $y$ and $1/H_p$ required to reproduce the solar radius and luminosity, as a function of $Q/N$. Both of these eigenvalues increase with increasing Q-nuclear abundance, for the entire range of $Q/N$. This behavior is easily understood in
terms of the rather strong temperature dependence of the \( ^{\text{a}}\text{Li}^Q(p,\gamma)^{\text{Be}}^Q \) reaction rate. As shown in Figure 3, the rate of energy production from this reaction varies approximately as \( T^{13} \), compared to \( T^9 \) for the p-p reaction. Consequently, the effective energy-producing region is more centrally concentrated than would otherwise be the case (see Figure 9), and as \( \Omega/N \) increases the structure is more nearly described by a Cowling point source model (Cox and Giuli, 1968). The increased efficiency of the energy sources in such a model results in a lower luminosity, a lower central mass concentration and hence a larger radius than a model in which the energy sources are more uniformly distributed. Thus a zero-age model of one solar mass, containing a small abundance of Q-nuclei but with \( Y \) and \( 1/H_p \) the same as for the standard solar model, will be larger and less luminous than the zero-age Sun. It then follows that the radius and luminosity after \( 4.7\times10^9 \) years would be larger and smaller, respectively, than the observed solar values, assuming that the time dependence is not altered appreciably. Since the luminosity is strongly increasing function of the mean molecular weight, we can recover the solar luminosity by increasing the initial helium abundance. (An increase in \( Y \) also reduces the opacity, thereby increasing the luminosity.) The radius, on the other hand, is sensitive primarily to the mixing length parameter \( 1/H_p \), which determines the adiabat deep in the convection zone.
A larger value of $1/H_p$ corresponds to more efficient convection in the outer layers, a flatter temperature gradient and therefore a higher surface temperature. Because a change in $1/H_p$ has little effect on the luminosity, a higher surface temperature results in a smaller radius. Thus an increase in both $Y$ and $1/H_p$ are required to compensate for the structural changes induced by the additional energy source.

An increase in $1/H_p$ corresponds physically to a deeper convective envelope; the variation with $Q/N$ of the depth of the outer convective zone, and the temperature at the base of the convective zone, is illustrated in Figure 7. The necessity of a deeper convective envelope is also a natural consequence of the steeper central temperature gradient. According to the Schwarzschild condition, and ignoring any overshooting of convective elements, the outer convective zone terminates at the point where the (constant) adiabatic gradient in the envelope intercepts the radiative gradient in the interior. Because of the steeper central temperature gradient, this point will lie at a greater depth beneath the photosphere compared to the standard model. We will discuss the observational implications of an extended solar convection zone in Section 4.4.

It is interesting to note that all attempts to construct a solar model with $Q/N$ $\geq 10^{-10}$ failed, as no value of $1/H_p$
Table 5

Variation in solar parameters with increasing Q/H

<table>
<thead>
<tr>
<th>Q/H x 10^{15}</th>
<th>Z</th>
<th>X</th>
<th>L/H_o</th>
<th>L_o/L_o (10^6 K)</th>
<th>T_c (g cm^{-3})</th>
<th>X_c</th>
<th>R_c/R_o</th>
<th>B_b/R_o (10^6 K)</th>
<th>T_b (g cm^{-3})</th>
<th>L_o/L_o (SW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0172</td>
<td>0.229</td>
<td>1.50</td>
<td>0.724</td>
<td>15.01</td>
<td>0.392</td>
<td>0.743</td>
<td>1.86</td>
<td>0.000</td>
<td>5.2</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0171</td>
<td>0.232</td>
<td>1.53</td>
<td>0.734</td>
<td>15.14</td>
<td>0.348</td>
<td>0.739</td>
<td>1.89</td>
<td>0.066</td>
<td>5.2</td>
</tr>
<tr>
<td>0.32</td>
<td>0.0170</td>
<td>0.237</td>
<td>1.56</td>
<td>0.747</td>
<td>15.32</td>
<td>0.277</td>
<td>0.735</td>
<td>1.94</td>
<td>0.178</td>
<td>4.8</td>
</tr>
<tr>
<td>0.73</td>
<td>0.0169</td>
<td>0.244</td>
<td>1.64</td>
<td>0.767</td>
<td>15.45</td>
<td>0.290</td>
<td>0.0157</td>
<td>0.726</td>
<td>2.04</td>
<td>0.341</td>
</tr>
<tr>
<td>1.50</td>
<td>0.0167</td>
<td>0.253</td>
<td>1.74</td>
<td>0.786</td>
<td>15.28</td>
<td>0.297</td>
<td>0.0340</td>
<td>0.716</td>
<td>2.16</td>
<td>0.502</td>
</tr>
<tr>
<td>2.10</td>
<td>0.0166</td>
<td>0.257</td>
<td>1.83</td>
<td>0.795</td>
<td>15.11</td>
<td>0.301</td>
<td>0.0430</td>
<td>0.710</td>
<td>2.23</td>
<td>0.580</td>
</tr>
<tr>
<td>2.94</td>
<td>0.0165</td>
<td>0.262</td>
<td>1.90</td>
<td>0.802</td>
<td>14.91</td>
<td>109.3</td>
<td>0.303</td>
<td>0.0492</td>
<td>0.704</td>
<td>2.30</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0163</td>
<td>0.266</td>
<td>2.04</td>
<td>0.809</td>
<td>14.58</td>
<td>98.0</td>
<td>0.305</td>
<td>0.0500</td>
<td>0.695</td>
<td>2.42</td>
</tr>
<tr>
<td>10.00</td>
<td>0.0161</td>
<td>0.278</td>
<td>2.33</td>
<td>0.812</td>
<td>14.13</td>
<td>85.6</td>
<td>0.293</td>
<td>0.0650</td>
<td>0.679</td>
<td>2.66</td>
</tr>
<tr>
<td>50.00</td>
<td>0.0156</td>
<td>0.299</td>
<td>3.50</td>
<td>0.804</td>
<td>13.02</td>
<td>62.3</td>
<td>0.273</td>
<td>0.0692</td>
<td>0.652</td>
<td>2.90</td>
</tr>
</tbody>
</table>
Figure 6: The eigenvalues $Y$ and $1/H_p$ required to reproduce the observed solar radius and luminosity, as functions of $Q/N$. 
Figure 7: Radius and temperature at the bottom of the convective envelope, as functions of Q/N.
could be found which would reduce the radius to the solar value. A value of $1/H_p$ larger than five or ten corresponds to almost perfectly efficient convection, i.e., the temperature gradient in the convection zone differs little from the adiabatic gradient; increasing $1/H_p$ any further has a negligible effect on the structure. Consequently, the dependence of the stellar radius on $1/H_p$ disappears, and the eigenvalue problem has no solution. Newman and Fowler (1976) encountered a similar situation in their investigation of the possible consequences of a large error in the p-p reaction rate. They concluded that an enhancement of this rate by more than a factor of about 50 could not be accommodated by conventional solar models, for essentially the same reason. Although the inability to obtain a self-consistent solar model certainly suggests an upper limit of about $10^{-10}$ for the fractional Q-nuclear abundance (assuming no attrition of Q-nuclei from the HeLiBe cycle), we will see shortly that serious discrepancies with observation are apparent for much smaller abundances.

The central temperature, density and hydrogen abundance at $t=t_0$ are plotted in Figure 8 as a function of $Q/N$, and Figure 9 compares the run of variables with respect to mass for solar models with $Q/N=0$, $2\times10^{-15}$ and $10^{-14}$. A striking feature which is apparent in Figure 8 is the precipitous decline in the central hydrogen abundance for small values
of Q/N. This decrease is due in part to the larger initial helium abundance needed to obtain the solar luminosity at $t=t_0$, but the main reason is an actual increase in the rate of helium production brought about by the Q-nuclear cycle. This is shown clearly in column (5) of Table 5, which lists the zero-age solar luminosity, $L_0/L_0$, as a function of Q/N (see also Figures 12 and 13). As Q/N increases, so does $L_0/L_0$; consequently, the time-averaged luminosity increases as well. Since the main sequence luminosity derives almost entirely from nuclear energy production, a larger time-averaged luminosity implies an enhanced rate of hydrogen-burning (as well as a reduction in the main sequence lifetime). The larger central helium abundance, compared to the standard solar model, also contributes to the initial increase in central temperature (and density), since the temperature is proportional to the mean molecular weight. Furthermore, because the PP chains still provide most of the solar energy for small values of Q/N, and because the rate of the p-p reaction is proportional to the square of the hydrogen abundance, a lower hydrogen abundance requires a higher core temperature in order to maintain the solar luminosity.

This trend of increasing temperature and density, and decreasing hydrogen abundance, is reversed at a Q-nuclear abundance of about $5 \times 10^{-16}$. As a result of the strong
Figure 8: Central temperature, density and hydrogen abundance in the present Sun as functions of Q/N.
Figure 9: Variation with respect to mass of the luminosity, temperature, and density for solar models with $Q/N = 0$, $2.1 \times 10^{-15}$, and $10^{-14}$. Arrows indicate direction of increasing $Q/N$. 
temperature dependence of the $^6\text{Li}^Q(p,\gamma)^7\text{Be}^Q$ reaction rate, and the increasingly large contribution of this reaction to the rate of energy production as $Q/N$ is increased, solar models with $Q/N$ larger than about $5 \times 10^{-15}$ have a central convective zone, which increases in mass as $Q/N$ increases (Figure 10). This is not unexpected, as the core of the standard solar model is only marginally stable against convection, the adiabatic gradient exceeding the radiative gradient by only 20% at the center. Convective mixing resupplies the energy-producing regions with hydrogen, and the accompanying reduction in mean molecular weight results in a corresponding decrease in central temperature and density. As $Q/N$ is increased further, the hydrogen abundance in the core increases to about 30% by mass, and remains approximately constant thereafter. The central temperature, however, continues to decrease as the HeLiBe cycle rapidly replaces the PP cycle as the dominant mode of solar energy production. Note that a likely consequence of employing the larger Los Alamos opacities would be the onset of convection at a smaller $Q$-nuclear abundance.

Figure 11 shows the relative contribution of the HeLiBe cycle to the total energy production, and the predicted neutrino count rate for the $^{37}\text{Cl}$ detector. Although all of the reactions listed in Table 3 have been included in the total count rate, we have assumed that the $Q$-nuclear cycle
Figure 10: Convective core mass in the zero age Sun and in the present Sun, as a function of Q/N.
contributes no detectable neutrinos; consequently, the rates given in Table 5 and in Figure 11 should be considered lower limits. The neutrino count rate is lowered relative to that of the standard solar model for all values of Q/N, despite the initial increase in core temperature which would seem to favor a higher 8B neutrino flux. At Q/N = 5x10^-16, for example, the central temperature is nearly 5x10^5 K above that of the standard model. (Note, however, that the gradient of temperature with respect to mass is also larger, so that the average temperature of the region within which most of the neutrinos are produced is somewhat less than 5x10^5 K above that of the standard solar model.) The rate of neutrino production from the decay of 8B is equal to the 7Be(p,γ)8B reaction rate, which is proportional to the concentration of 1H and 7Be, and to a rather large power of the temperature (~T^10). Although the 3% increase in temperature at Q/N = 5x10^-16 implies a 54% increase in the rate of this reaction, the temperature effect is largely counteracted by the 40% decrease in the hydrogen abundance. The larger 4He concentration, on the other hand, favors the 3He(4He,γ)7Be reaction over the 3He(3He,2p)4He reaction, thereby increasing the 7Be abundance. The net result is that the relative contribution of the branch of the PP cycle involving 7Be, which yields most of the detectable solar neutrinos, is increased by about
15% relative to the standard solar model. However, the contribution of the PP cycle to the total rate of energy production is reduced by about 20% from that of the standard model, resulting in a net decrease in the flux of high-energy neutrinos. Hence even at a Q-nuclear abundance $Q/N=5\times10^{-18}$, the HeLiBe cycle already contributes a sufficiently large fraction of the solar energy that the neutrino count rate is reduced despite the higher core temperature. For larger values of $Q/N$, the core temperature decreases, as does the efficiency of the PP cycle; the high-energy neutrino flux decreases accordingly. The predicted count rate drops below the observed upper limit of 2 SNU at a Q-nuclear abundance of about $2\times10^{-13}$ Q-nuclei per normal nucleus. This is in agreement with the value found by Joseph (1984a) and Boyd et al. (1985) although the numerical approximations and constitutive physics employed in those studies were somewhat different than those adopted here.

The presence of the convective core also contributes to the reduction in the predicted count rate by altering the local equilibrium abundances of $^7\text{Be}$ and $^3\text{He}$. Although a mixed solar interior has been invoked previously as a possible solution to the solar neutrino problem (Ezer and Cameron, 1968; Iben, 1969; Shaviv and Salpeter, 1971), a reduction in the high-energy neutrino flux by more than a
Figure 11: Relative contribution of the HeLiBe cycle to the solar luminosity ($L_Q/L_\odot$), and the $^{37}$Cl capture rate ($\Sigma(\sigma\phi)$), as functions of $Q/N$. 
factor of two would require a mixed region containing over half the solar mass, an unlikely possibility. However, as pointed out by Iben (1969) and Falk and Mitalas (1977), it is nevertheless incorrect to use the locally defined equilibrium abundances of $^7$Be and $^3$He when calculating the neutrino production rate in a convectively mixed region. This is because at the temperatures and densities characteristic of the solar interior, the lifetimes of $^3$He and $^7$Be against proton capture and electron capture exceed the typical mixing timescale. Consequently, the abundance by mass of these elements will be constant throughout the convective region. In this case the mixed abundances must be calculated by equating the total production rate over the entire mixed region to the total destruction rate. The mixed $^3$He abundance, $<X_3>$, is found by integrating equation (35) over the mass of the convective core:

$$[X_1]^2/R_{11}dm - 2[X_3]^2/R_{33}dm - [X_3][X_4]/R_{34}dm = 0$$  \hspace{1cm} (51)$$

Similarly, the homogenized $^7$Be abundance is given by

$$[X_e][X_7]/R_{7e}dm + [X_1][X_7]/R_{71}dm - [X_3][X_4]/R_{34}dm = 0$$  \hspace{1cm} (52)$$

and the flux of $^8$B neutrinos which originate within the convective core is

$$\phi_\nu(^8B) = [X_1][X_7]/R_{71}dm$$  \hspace{1cm} (53)$$
Under conditions of local equilibrium, the concentration of 7Be decreases rapidly with increasing temperature and hence with distance from the solar center. The result of mixing, then, is an increase in the 7Be abundance (above the equilibrium value) near the edge of the core, and a decrease in the abundance near the center. Since most of the 8B neutrinos originate within the inner 0.01M☉, the predicted count rate will be less than if local steady-state values are used. The magnitude of this effect is illustrated in Figure 12, where the ratio 7Be(mixed)/7Be(equilibrium) at the solar center is plotted as a function of Q/N. As Q/N increases (and the convective core grows larger), the 7Be abundance at the center drops below the steady-state concentration by nearly a factor of 2.5, with a corresponding decline in the high-energy neutrino production.

Figure 13 shows the evolution in the H-R diagram of several model sequences characterized by different Q-nuclear abundances. As Q/N is increased, the zero age Sun becomes hotter and more luminous; for large abundances the main sequence evolution is toward the right in the H-R diagram, a behavior which is characteristic of stars possessing convectively-mixed cores. This behavior is brought out even more clearly in Figure 14, which shows the evolutionary tracks beyond the point of core hydrogen exhaustion.
Figure 12: Ratio of the $^7$Be abundance in the convective core to the central equilibrium abundance, as a function of $Q/N$. 
Track A represents the standard solar evolution \((Q/N=0)\), and track B shows the solar evolution for \(Q/N=2.5\times10^{-15}\).

The curve labeled C is the evolutionary track of a one solar mass star with the same Q-nuclear abundance as B, but with values of \(Y\) and \(1/H_p\) identical to the standard model. A comparison of tracks B and C shows the shift in temperature and luminosity required to offset the effects of Q-nuclear energy production and yield an acceptable solar model at the solar age. The requisite increase in temperature and luminosity is accomplished by increasing both \(Y\) and \(1/H_p\). Both evolutionary sequences with \(Q/N=2.5\times10^{-15}\) exhibit the characteristic blueward hook which occurs as hydrogen is exhausted in the convectively-mixed core.

The main sequence lifetime of the Sun (as measured by the age at which the central hydrogen abundance drops to zero) decreases with increasing Q-nuclear abundance. At \(Q/N=2.5\times10^{-15}\), the main sequence phase terminates at \(8.5\times10^9\) years, a reduction in lifetime of approximately 20% compared to the standard solar evolution. Note that this is a direct consequence of the larger zero age helium abundance (and correspondingly larger zero age luminosity) required to satisfy the constraints which define a solar model. As is shown in Figure 14, if the helium abundance (and mixing length parameter) are not so constrained, the effect of Q-nuclear energy production is to drive down the
Figure 13: Solar evolution in the H-R diagram. Curves are labeled with the value of $Q/N \times 10^{15}$. 
Figure 14: Solar evolution beyond core hydrogen exhaustion. See text for details.
luminosity and increase the evolutionary lifetime by a few percent.

5.3 NON-STANDARD MODELS WITH ATTRITION

In the previous section we intentionally ignored the possibility of leakage of Q-nuclei from the HeLiBe cycle. However, as discussed in Section 2.2, such attrition may in fact occur if 1) \(^7\)Be\(^Q\) is stable to baryon emission and 2) the lifetime of \(^7\)Be\(^Q\) against a \((p,\gamma)\) reaction is sufficiently short. In this section we will examine the consequences of an attrition of Q-nuclei from the HeLiBe cycle for two values of the \(^7\)Be\(^Q\) beta-decay lifetime \(\tau_B\). As representative values we adopt \(\tau_B = 100\) and 1000 seconds, the latter value representing the upper limit suggested by Boyd et al. (1983).

The steady state Q-nuclear abundances are given by equations (11)-(13), and the rates of energy generation and helium production are given by equations (19) and (21). The assumption that the Q-nuclear cycles depicted in Figure 2 operate in equilibrium is reasonable in view of the short lifetimes of the participating Q-nuclei. At solar temperatures and densities, the lifetime of \(^6\)Li\(^Q\) against proton capture is on the order of a few minutes; for \(^10\)B\(^Q\) and \(^14\)N\(^Q\) the respective lifetimes are of the order of a few years and a few hundred thousand years, much shorter than the
hydrogen-burning timescale. As was discussed in Section 2.2, we will assume for the sake of simplicity that the branching ratios and are both zero, so that the *He catalyst is always returned to the main cycle.

A series of solar models was constructed for the chosen values of the parameter $\tau_B$, and a range in values of the total Q-nuclear abundance. The results of these calculations are shown in Tables 6 and 7 and Figures 15-24. For small values of $Q/N$, these "leaky" solar models differ little from the standard model because most of the Q-nuclei in the energy-producing region have been converted to $^{10}B^Q$, and the HeLiBe cycle does not contribute significantly to the solar energy production. Figure 15 shows the $^6Li^Q$ abundance, relative to the total Q-nuclear abundance, as a function of mass fraction in several representative solar models. For $\tau_B = 100$ seconds, the ratio $[^6Li^Q]/[Q]$ at the center is only about 0.001 for $Q/N = 10^{-13}$; this ratio increases to about 0.1 as $Q/N$ is increased to $10^{-13}$. As expected, the attrition of light Q-nuclei is greater for $\tau_B = 1000$ seconds. For comparable values of $Q/N$, the $^6Li^Q$ concentration is one to two orders of magnitude smaller for the larger beta-decay lifetime, and the attrition occurs over a much larger mass fraction.

The fractional abundance of $^6Li^Q$ increases rapidly with increasing mass fraction because of the strong temperature
Table 6

Variation in solar parameters with increasing $Q/\bar{N}$, for $\tau_\alpha = 100$ seconds

<table>
<thead>
<tr>
<th>$Q/\bar{N} \times 10^{14}$</th>
<th>$Y$</th>
<th>$L/H_p$ ($10^6$ K)</th>
<th>$\rho$ (g cm$^{-3}$)</th>
<th>$X_c$</th>
<th>$R_b/R_e$ ($10^6$ K)</th>
<th>$T_b$</th>
<th>$\Sigma(\alpha\beta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.229</td>
<td>1.51</td>
<td>15.02</td>
<td>155.0</td>
<td>0.388</td>
<td>0.742</td>
<td>1.863 0.004 5.30</td>
</tr>
<tr>
<td>0.10</td>
<td>0.230</td>
<td>1.52</td>
<td>14.93</td>
<td>152.3</td>
<td>0.393</td>
<td>0.739</td>
<td>1.888 0.039 4.67</td>
</tr>
<tr>
<td>0.50</td>
<td>0.234</td>
<td>1.59</td>
<td>14.56</td>
<td>141.2</td>
<td>0.409</td>
<td>0.732</td>
<td>1.971 0.180 2.74</td>
</tr>
<tr>
<td>1.00</td>
<td>0.238</td>
<td>1.68</td>
<td>14.15</td>
<td>129.0</td>
<td>0.422</td>
<td>0.723</td>
<td>2.068 0.326 1.49</td>
</tr>
<tr>
<td>2.50</td>
<td>0.253</td>
<td>1.95</td>
<td>13.24</td>
<td>103.1</td>
<td>0.426</td>
<td>0.701</td>
<td>2.326 0.616 0.38</td>
</tr>
<tr>
<td>5.00</td>
<td>0.273</td>
<td>2.42</td>
<td>12.41</td>
<td>79.8</td>
<td>0.375</td>
<td>0.675</td>
<td>2.630 0.822 0.10</td>
</tr>
<tr>
<td>10.00</td>
<td>0.293</td>
<td>3.24</td>
<td>11.83</td>
<td>63.9</td>
<td>0.232</td>
<td>0.649</td>
<td>2.963 0.935 0.03</td>
</tr>
</tbody>
</table>
Table 7

Variation in solar parameters with increasing \( Q/H \) for \( \tau_B = 1000 \) seconds

<table>
<thead>
<tr>
<th>( Q/H \times 10^{13} )</th>
<th>( Y )</th>
<th>( L/H_p ) (10^6 K)</th>
<th>( \rho ) (g cm(^{-3}))</th>
<th>( \chi_c )</th>
<th>( R_b/H_0 ) (10^6 K)</th>
<th>( \Sigma(\phi) )</th>
<th>( L_Q/L_0 ) (S HU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.230</td>
<td>1.51</td>
<td>15.00</td>
<td>154.4</td>
<td>0.389</td>
<td>0.741</td>
<td>1.870</td>
</tr>
<tr>
<td>0.10</td>
<td>0.230</td>
<td>1.52</td>
<td>14.98</td>
<td>153.5</td>
<td>0.389</td>
<td>0.740</td>
<td>1.880</td>
</tr>
<tr>
<td>0.50</td>
<td>0.233</td>
<td>1.56</td>
<td>14.76</td>
<td>146.5</td>
<td>0.391</td>
<td>0.735</td>
<td>1.938</td>
</tr>
<tr>
<td>1.00</td>
<td>0.238</td>
<td>1.62</td>
<td>14.53</td>
<td>139.0</td>
<td>0.390</td>
<td>0.728</td>
<td>2.014</td>
</tr>
<tr>
<td>2.50</td>
<td>0.250</td>
<td>1.79</td>
<td>14.02</td>
<td>122.2</td>
<td>0.364</td>
<td>0.713</td>
<td>2.191</td>
</tr>
<tr>
<td>5.00</td>
<td>0.265</td>
<td>2.06</td>
<td>13.62</td>
<td>107.3</td>
<td>0.289</td>
<td>0.693</td>
<td>2.423</td>
</tr>
<tr>
<td>10.00</td>
<td>0.283</td>
<td>2.57</td>
<td>13.39</td>
<td>98.7</td>
<td>0.137</td>
<td>0.668</td>
<td>2.719</td>
</tr>
<tr>
<td>25.00</td>
<td>0.292</td>
<td>4.06</td>
<td>12.78</td>
<td>61.3</td>
<td>0.182</td>
<td>0.633</td>
<td>3.120</td>
</tr>
</tbody>
</table>
Figure 15: The $^6\text{Li}^0$ abundance distribution for several representative solar models. Solid and broken curves denote, respectively, models for which $t_\beta = 100$ and 1000 seconds. Curves are labeled with the value of $Q/N \times 10^{14}$. 
dependence of the $^7\text{Be}^Q(p,\gamma)^8\text{Be}^Q$ reaction rate. For a beta-decay lifetime of 1000 seconds and $Q/N = 10^{-12}$, the $^6\text{Li}^Q$ abundance increases by a factor of about $10^3$ within a region containing only 30% of the mass. This rapid change in the $^6\text{Li}^Q$ abundance in the leaky solar models substantially delays the onset of convection (as $Q/N$ is increased) because the $Q$-nuclear energy source is much less centrally concentrated than in the case for the "non-leaky" models. In Figure 16 are plotted $[^6\text{Li}^Q V]$ and the product of these two quantities, for $\tau_B = 1000$ seconds and $Q/N = 5\times10^{-14}$. In models with no attrition of $Q$-nuclei, the $^6\text{Li}^Q$ abundance is constant and the rate of energy production from the $\text{HeLiBe}$ cycle is proportional only to $<\sigma v>_0^Q$; the large temperature dependence of this reaction rate induces core convection even for very small $Q$-nuclear abundances. In the leaky models, however, the temperature dependence of the rate of energy production is much reduced; whereas $<\sigma v>_0^Q$ decreases rapidly with decreasing temperature, $[^6\text{Li}^Q]$ increases with decreasing temperature. Consequently, the rate of energy production from the $\text{HeLiBe}$ cycle, which is proportional to the product $<\sigma v>_0^Q[^6\text{Li}^Q]$, has a much smaller temperature dependence in the leaky models. As is shown in Figure 17, the net result is a slightly lower core temperature but no significant change in the temperature gradient or the luminosity profile.
Figure 16: Variation with mass of \([^6\text{Li}^Q]\), \(<\sigma v>^Q\), and the product \([^6\text{Li}^Q]<\sigma v>^Q\) for \(Q/N = 5 \times 10^{-14}\) and \(\tau_B = 1000\) seconds.
Figure 17: Variation with mass of the temperature, density and luminosity for the standard solar model (solid curves) and for a solar model with $Q/N = 5 \times 10^{-14}$ and $\tau_\beta = 1000$ seconds (broken curves).
Figures 18 and 19 show the effect of increasing the total Q-nuclear abundance. As Q/N is increased, the $^7\text{Li}^Q$ abundance in the core increases as well, although it remains severely depleted relative to the surface abundance. However, the increasingly large contribution of the HeLiBe cycle to the solar energy production results in a lower core temperature (and density), and this has the effect of reducing the $^7\text{Be}^Q(p,\gamma)$ reaction rate which is responsible for the leakage. Consequently, as Q/N is increased and the core temperature decreases in response, a larger and larger fraction of Q-nuclei remain in the form of $^7\text{Li}^Q$. Furthermore, the region over which the attrition occurs decreases (in mass) as Q/N is increased, as was shown in Figure 15. Hence as Q/N is increased the temperature dependence of the Q-nuclear energy source increases, and this energy source becomes more centrally concentrated (see Figures 18 and 19). Increasing the total abundance of Q-nuclei therefore reduces the effects of attrition, and the resulting structure is qualitatively similar to that of a non-leaky model characterized by a much smaller Q-nuclear abundance. In particular, the constraints which define a solar model require an increase in both Y and $1/\nu_p$ as Q/N is increased.

Figures 20-23 show the variation with Q/N of several solar parameters for three values of the $^7\text{Be}^0$ beta-decay lifetime ($\tau_B = 0$ corresponds to the case of no attrition).
Figure 18: Variation with mass of \([^6\text{Li}^Q]\), \(<\sigma v>^Q\), and the product \([^6\text{Li}^Q]<\sigma v>^Q\) for \(Q/N = 10^{-14}\) and \(\tau_B = 1000\) seconds.
Figure 19: Variation with mass of the temperature, density and luminosity for the standard solar model (solid curves) and for a solar model with \( Q/N = 10^{-12} \) and \( \tau_B = 1000 \) seconds (broken curves).
For values of $Q/N$ less than about $5 \times 10^{-15}$, the central temperature, density, and hydrogen abundance in the leaky solar models are approximately the same as in the standard model. Since the central $^7\text{Li}^Q$ abundance in the leaky models is only a small fraction of the total $Q$-nuclear abundance, a much larger $Q/N$ is necessary (compared to the non-leaky models) for the HeLiBe cycle to appreciably affect the internal structure. For a sufficiently large $Q/N$, the central $^7\text{Li}^Q$ abundance in the leaky models becomes comparable to that in the non-leaky models, and as $Q/N$ is increased further the temperature, density and hydrogen abundance in the core decrease as the HeLiBe cycle begins to contribute to the energy production. As shown in Figure 20, the central hydrogen abundance drops to very low values in the leaky models because the onset of convective mixing is delayed for the reasons discussed previously. Indeed, for $\tau_\beta = 100$ seconds a convective core does not develop until hydrogen is nearly exhausted at the center. It is likely, in fact, that convection is triggered when the hydrogen abundance drops to a very low value; the lack of free protons inhibits the $^7\text{Be}^Q(p,\gamma)$ reaction, less $^7\text{Li}^Q$ depletion occurs, and the rate of $Q$-nuclear energy production again becomes very temperature sensitive. Although the models for $\tau_\beta = 1000$ seconds were not extended beyond $Q/N = 10^{-12}$, it is probable that the decrease in core
hydrogen abundance in these models would also be reversed by convective mixing for a sufficiently large Q-nuclear abundance.

The consequences of an attrition of light Q-nuclei on the predicted neutrino count rate is shown in Figure 23, where the solid curves represent the neutrinos contributed by the PP and CNO cycles only. Clearly, the high-energy neutrino flux can be forced to arbitrarily low values for sufficiently large total Q-nuclear abundances, regardless of the extent to which the light Q-nuclei are drained from the HeLiBe cycle. The fact that the neutrino flux from the decay of $^8B$ can be reduced to near zero can be attributed to the very low core temperatures achieved for large values of $Q/N$, as well as to the small contribution of the PP cycle to the total energy output. For example, a solar model with $Q/N = 10^{-12}$ and $\tau_B = 1000$ seconds has a core temperature below $12 \times 10^6$ K, and the PP cycle accounts for less than 10% of the solar luminosity. In fact, for such large Q-nuclear abundances the most important contributor to the predicted neutrino flux is the electron capture on $^7\text{Be}$, rather than the $^8\text{B}$ decay. Also, as is apparent in Tables 6 and 7, a much lower core temperature is required in the leaky models to reduce the neutrino flux to a given value. This is because convective mixing, which contributes to the reduction of the $^8\text{B}$ neutrino flux in the non-
Figure 20: Variation in central hydrogen abundance with increasing $Q/N$, for $\tau_B = 0$, 100, and 1000 seconds.
Figure 21: Variation in central temperature with increasing $Q/N$, for $\tau_B = 0$, 100, and 1000 seconds.
Figure 22: Variation in central density with increasing Q/N, for $\tau_B = 0, 100, \text{and} 1000$ seconds.
Figure 23: Neutrino count rate, as a function of $Q/N$, for $\tau_B = 0$, 100, and 1000 seconds. Dotted curves are the count rates if the neutrinos produced by the $Q$-nuclear cycle are included.
leaky models, does not occur in the leaky models until the high-energy neutrino flux is already close to zero.

If a significant attrition of light Q-nuclei occurs through the $^7\text{Be}^Q(p,\gamma)^8\text{B}^Q$ reaction, it would be expected that the subsequent beta-decay of $^8\text{B}^Q$ would produce a neutrino of comparable energy to that produced in the decay of $^8\text{B}$ in the PP cycle, and that this source of neutrinos may contribute to the count rate. The rate of neutrino production from the decay of $^8\text{B}^Q$ is

$$R_v(^8\text{B}^Q) = (1-f_B^7)[^1\text{H}][^6\text{Li}^Q]\langle\sigma v\rangle^Q_6,$$

(54)

where $f_B^7$ is defined by equation (2), and we have assumed that the $^7\text{Be}^Q$ beta-decay lifetime is much smaller than the proton capture lifetime. The dotted curves in Figure 23 show the expected neutrino detection rate for the leaky models when this contribution to the neutrino flux is taken into account. As expected, for small to moderate Q-nuclear abundances the neutrino count rate is substantially increased by the $^8\text{B}^Q$ neutrinos. However, for sufficiently large values of $Q/N$ the core temperature is so low that even this source of neutrinos does not prevent the count rate from falling below the observed upper limit of 2 SNU.

Finally, we show in Figure 24 the evolution in the H--H diagram of a sequence of models with $\tau_B = 0$, 100 and 1000
seconds; the value of $Q/N$ for each sequence was chosen to yield a count rate of about 0.5 SNU (PP and CNO neutrinos only) at the solar age. Note that the sequence with $\tau_\beta = 0$ shows a significant evolution in temperature because of convective mixing, while the other models, which have radiative cores, more closely follow the standard evolution.
Figure 24: Solar evolution in the H-R diagram for $\tau_\beta = 0$, 100 and 1000 seconds. Curves are labeled with the value of $Q/N$ and $\tau_\beta$. 
4.4 DISCUSSION

We have seen that the expected flux of high-energy neutrinos from the Sun can be reduced to arbitrarily low levels for sufficiently large Q-nuclear abundances, even if we consider the contribution to the neutrino flux from the decay of $^8B^Q$. Although the neutrinos produced in the beta-decays of $^6Li^Q$ and $^7Be^Q$ are expected to have slightly higher energies than those produced by the p-p reaction, binding energy considerations suggest that their contribution to the $^{37}Cl$ detection rate would still be negligible (Sur and Boyd 1985). If there is no attrition of Q-nuclei from the HeLiBe cycle, the predicted detection rate for the $^{37}Cl$ experiment falls below the observed upper limit of 2 SNU at an abundance of about $2 \times 10^{-13}$ Q-nuclei per normal nucleus. If $^6Li^Q$ is depleted at high temperatures, the observational result can still be accommodated with a Q-nuclear abundance one to two orders of magnitude larger, assuming that the beta-decay lifetime of $^7Be^Q$ is no longer than about 1000 seconds. It appears, therefore, that the observed high-energy neutrino flux from the Sun is consistent with that predicted by solar models in which a substantial fraction of the solar luminosity is provided by a small abundance of Q-nuclei.

The method by which a solar model is constructed provides additional observational constraints. The require-
ment that the model radius and luminosity at $t=t_0$ agree with the observed solar values uniquely determines the zero age solar helium abundance, $Y$, and the ratio of mixing length to pressure scale height, $1/H_p$, in the outer convective zone. Since both of these parameters increase markedly with increasing Q-nuclear abundance, they potentially provide an important test of the solar models presented here.

The standard solar model adopted for this study predicts a zero age $^4$He abundance (by mass) of $Y=0.23$, a value typical of other standard models (c.f. Table 4). The range in derived abundances is $0.22 \leq Y \leq 0.25$ for models based upon the frequently used Cox and Stewart (1970a,b) opacity tables; solar models based upon the more recent data of the Los Alamos Astrophysical Opacity Library (Heubner et al. 1977) generally predict a slightly larger helium abundance. From a study of the sensitivity of the derived helium abundance on various physical inputs, Bahcall et al. (1982) conclude that the standard solar model predicts a helium abundance of $Y = 0.25 \pm 0.01$, with most of the uncertainty attributable to uncertainties in the opacity. A few standard models, however, seem to require a helium abundance as large as 0.27 (Endal and Sofia, 1981; VandenBerg, 1983). For the discussion which follows, we shall adopt the range $0.22 \leq Y \leq 0.27$ as representative of the values inferred from
model calculations, and we will assume that an increase in \( Y \) of 0.02 is indicative of the effect produced by using the Los Alamos opacity data in place of the Cox-Stewart data.

As is apparent in Figure 6 and Tables 5-7, the predicted solar helium abundance increases with increasing \( Q \)-nuclear abundance. For both leaky and non-leaky models, the largest derived helium abundance is about 0.30, for values of \( Q/N \) considerably larger than are needed to solve the solar neutrino problem. For the non-leaky models, a neutrino count rate of 1 SNU is obtained with a helium abundance of about 0.26; this value increases to 0.28 if we scale upward the value of Bahcall et al. (1982). A slightly lower (0.25 \( \leq Y \leq 0.27 \)) helium abundance gives a count rate of 1 SNU for models in which \( \text{Li}^0 \) has been depleted, but if we consider the neutrinos produced by the \( Q \)-nuclear cycles the required abundance probably does not differ much from the non-leaky models. It seems reasonable to conclude that the solar helium abundance can be no less than about 0.26 to 0.28 if the proposed mode of energy production is operating in the Sun.

Most unfortunately, direct measurement of the \( ^{4}\text{He} \) abundance in the solar photosphere is not possible because the temperatures are too low to excite the helium spectrum. Abundance estimates from solar cosmic rays and solar wind particles suggest a rather low value of \( Y \approx 0.20 \) (Lambert,
1967; Durgaprasad et al., 1968; Bertsch et al., 1972). It is doubtful, however, that this abundance is truly indicative of the photospheric abundance. The elemental composition of energetic particles in flare events varies widely from flare to flare and, for those elements for which photospheric abundances can be measured, the average flare composition differs considerably from that of the photosphere (Cook et al., 1967). Furthermore, the He/H ratio in the solar wind varies with the level of solar activity (Ogilvie and Wickerson, 1969). Abundance determinations from chromospheric spectra suggest a much larger value of Y = 0.28 (Heasley and Milkey, 1978). Although this tends to support the non-standard models presented in this study, the chromospheric helium lines are formed under conditions of extreme non-LTE, and the derived abundance is highly model dependent; the error bars are correspondingly large (ΔY = ±0.05). Solar helium abundance determinations are therefore of little help in discriminating between the standard and non-standard models.

Since the solar heavy element abundance is known rather accurately, another approach is to compare the predicted solar helium abundance with that expected from galactic chemical evolution in the solar neighborhood. Theoretical models of chemical evolution of the interstellar medium predict that both the helium and heavy element abundance
will increase with time. Estimates of the helium to heavy element enrichment ratio ($\Delta Y/\Delta Z$) depend, among other things, upon the initial mass function and stellar mass loss rates, both of which are uncertain. Although Maeder (1980) has found values as low as 0.73, most estimates appear to be in the range $2.0 \leq \Delta Y/\Delta Z \leq 3.3$ (Dearborn and Trimble, 1978; Chiosi, 1979; Chiosi and Caimmi, 1979; Chiosi and Matteucci, 1982). Since it is generally agreed that no metals were produced in the Big Bang, a solar metal abundance of $Z = 0.02$ implies a solar helium abundance 0.04 to 0.07 above that of the primordial (pre-galactic) value.

Attempts to determine the primordial helium abundance $Y_p$ from observation have produced conflicting results; comprehensive reviews can be found in Pagel (1982) and Yang et al. (1984). The usual procedure is to extrapolate to zero metallicity an observational $Y$ versus $Z$ relation derived from abundance determinations of galactic and extragalactic HII regions. In this way Peimbert and Torres-Peimbert and their collaborators find evidence for a $\Delta Y/\Delta Z$ ratio of about 3.0, in good agreement with theoretical models, and a primordial helium abundance $Y \approx 0.23$ (Peimbert and Torres-Peimbert, 1974, 1976; Peimbert et al., 1978; Lequeux et al., 1979). However, as emphasized by Kunth and Sargent (1983), such an extrapolation is hazardous at best in view of the large errors associated with the abundance determi-
nations. From spectrophotometry of 12 metal-poor emission line galaxies, Kunth and Sargent conclude that \( Y = 0.245 \pm 0.003 \).

As noted by Olive et al. (1981) and Yang et al. (1984), the solar helium abundance as inferred from standard model calculations is anomalously low for its metallicity. Indeed, the "best value" solar helium abundance suggested by Bahcall et al. (1982) differs little from the primordial "best value" found by Kunth and Sargent, a result which is difficult to reconcile with current theories of galactic chemical enrichment. Based upon estimates of the primordial helium abundance, and the observational and theoretical \( Y \) versus \( Z \) relation, the solar helium abundance would be expected to lie in the range \( 0.27 \leq Y \leq 0.32 \), well above that predicted by the standard model. The reality of this discrepancy is evident in Figure 25, where the helium abundance (by mass) is plotted against the oxygen to hydrogen ratio (by number) for the Sun and selected HII regions and emission line galaxies. (It is generally accepted that oxygen, being a primary product of nucleosynthesis, is an accurate metallicity indicator). Error bars have been drawn on some of the data to provide an indication of the uncertainties associated with individual abundance determinations. The \( O/H \) ratio for the Sun \( (O/H_\odot = 6.9 \times 10^{-4}) \) is taken from Ross and Aller (1976), and the estimated uncer-
tainty is indicated by the horizontal bar; the vertical error bar encompasses the range in helium abundance predicted by standard model calculations. Despite the large scatter, there is clearly a trend of increasing helium abundance with increasing O/H. With the exception of one abundance measurement for the Orion nebula, there are no helium abundances smaller than 0.285 for O/H > 2x10^{-4}. It is well known that HII regions in the Small Magellanic Cloud are metal deficient relative to the Sun, yet the average helium abundance for the three measurements in Figure 25 is virtually the same as the predicted solar value. The most extensively studied HII region in Figure 25 is the Orion nebula. Although it is slightly metal poor relative to the Sun, the helium abundance is 0.29 ± 0.01, well above the range predicted by the standard solar model.

There seems to be little doubt that the larger helium abundance predicted by solar models incorporating Q-nuclei is in better agreement with observation. However, it would be premature to claim that this represents a serious failure of the standard solar model, for two reasons. An extensive (though by no means exhaustive) search of the literature revealed no HII regions for which the O/H ratio was comparable to or larger than the solar value. Since the composition of the solar photosphere presumably reflects the composition of the interstellar gas 5x10^8
Figure 25: Helium and oxygen abundances for galactic and extragalactic HII regions. Dots represent emission line galaxies observed by Kunth and Sargent (1983). Crosses denote galactic HII regions observed by Hawley (1978). Abundance measurements for the Orion nebula (Dufour 1975; Peimbert and Torres-Peimbert 1977; Hawley 1978) are shown by open circles. Triangles and squares denote, respectively, HII regions in the SMC and LMC (Dufour 1975; Dufour and Harlow 1977; Lequeux et al. 1979). The Sun is represented by the encircled dot.
years ago, it is curious that there is little (if any) evidence for HII regions that are metal rich compared to the Sun. This suggests that the metal abundances in HII regions may be systematically underestimated, or that the composition of the solar photosphere does not accurately reflect the composition of the original solar nebula. The latter possibility is unlikely, as abundance determinations for carbonaceous meteorites are in generally good agreement with solar abundances (Ross and Aller, 1976). It seems more likely that the metal abundances in HII regions have been systematically underestimated, perhaps because the fraction of heavy elements bound up in dust particles has not been properly taken into account. Furthermore, spectroscopic abundance determinations for early type stars consistently yield $Y = 0.30 \pm 0.02$, with metallicities comparable to, or slightly greater than the Sun (Leckrone, 1971; Shipman and Strom, 1970). Although this also supports the case for a larger solar helium abundance, this value of $Y$ still lies below what would be expected from a crude extrapolation of the data in Figure 25. In view of the large uncertainties involved, perhaps the safest conclusion that can be drawn is that a solar helium abundance greater than about 0.26-0.28, as required by solar models with Q-nuclei, is not inconsistent with the available observational data.
A more definitive test of solar models with Q-nuclear burning is a comparison of the interior structure with that inferred from the observed solar oscillation frequencies in the 5 minute band. Ulrich and Rhodes (1983) have shown that there is a discrepancy between the observed and theoretically predicted frequencies of the spherical harmonic modes of low degree ($0 \leq \ell \leq 4$) that cannot be accounted for by any known uncertainties in the observations themselves or in their theoretical treatment; they conclude that this represents a serious failure of the standard solar model, of significance comparable to that of the solar neutrino problem. Specifically, the calculated frequencies for the standard model differ from the nearest observed frequency by 1 to 15 mHz. Uncertainties in the opacity, nuclear cross sections and other input physics produce a theoretical uncertainty of only about 1 mHz; observational errors are of this magnitude as well. More recently, Noels et al. (1984) have confirmed the results of Ulrich and Rhodes, and have extended their study to include the intermediate degree modes ($0 \leq \ell \leq 100$) which have recently been observed for the first time by Duvall and Harvey (1983). They find that the predicted frequencies are too small by 5-10 mHz for $\ell \leq 3$, and by 10-20 mHz for $\ell = 10$ and $\ell = 20$.

It has been known for some time that the discrepancy between the predicted and observed frequencies of both the
low and high degree modes can be considerably reduced if the Sun has a much deeper convective zone than predicted by the standard solar model. The convective envelope of the standard model adopted in this study extends to a depth of $0.74R_\odot$ and contains 1.2% of the solar mass; the temperature at the base of the convective zone is $1.85 \times 10^6$ K. These values are in excellent agreement with those predicted by other standard models (e.g., see Bahcall et al., 1982 and Ulrich, 1982). The effect on the calculated frequencies of increasing the depth of the outer convective zone is most apparent for the high degree modes ($l \geq 200$), because the power in these modes is confined primarily to the outer few percent of the Sun’s mass. Numerous investigators have computed oscillation frequencies of high degree for solar envelope models characterized by different values of the mixing length parameter, which determines the depth of the convection zone. All agree that a convection zone as shallow as that predicted by the standard models is ruled out by the observations (Rhodes et al., 1977; Berthomieu et al., 1980; Scuflaire et al., 1981; Noels et al., 1984). Berthomieu et al. (1980) find good agreement with the observed frequencies if the convection zone extends to $0.67R_\odot$. Scuflaire et al. (1981) find that the data can be well reproduced with a convection zone extending to $0.66R_\odot$ and to a temperature of $2.8 \times 10^6$ K. The low degree modes
are not quite as useful in probing the structure of the outer convective zone, since these modes penetrate all the way to the center and are therefore less sensitive to conditions near the surface. However, Gabriel et al. (1982) and Scuflaire et al. (1982) find that the discrepancy between the observed and predicted frequencies for $l \leq 3$ is substantially reduced (though not alleviated entirely) if the convective zone extends to $0.69 R_\odot$ and $2.5-2.6 \times 10^6$ K.

There are other independent lines of evidence which suggest that the solar convective zone extends to deeper, hotter layers than predicted by the standard model. From numerical simulations of convection in rotating spherical shells, Gilman (1979) finds that the observed rotation rate at high solar latitudes can only be explained by a deeper convective zone. However, a depth in excess of $0.4 R_\odot$ is necessary to reproduce the observed rotation velocities; a value this large is probably incompatible with the oscillation data, and an unreasonably large value of the mixing length parameter ($l/H_p \gtrsim 10$) would be needed to obtain such an extensive convection zone in a solar envelope model.

More compelling is the anomalously low lithium abundance in the solar photosphere. The Sun is deficient in lithium by about two orders of magnitude relative to the cosmic abundance and the abundance in Type I carbonaceous meteorites (Boesgaard, 1976; Ross and Aller, 1976). Lithium is rapid-
ly destroyed by the reactions \(^6\text{Li}(p, \alpha)^3\text{He}\) and \(^7\text{Li}(p, \alpha)^4\text{He}\) at temperatures in excess of about \(2.4\times10^6\) K, suggesting that lithium in the solar photosphere has been exposed to temperatures of this magnitude. This could occur if the base of the solar convective zone were at a temperature of at least \(2.4\times10^6\) K; the surface lithium abundance would be depleted as lithium is transported by convective motions to temperatures where it is destroyed. However, the outer convective zone of standard envelope models does not extend to temperatures higher than \(2\times10^6\) K. The e-folding time for lithium depletion at this temperature is approximately \(10^{12}\) years. The observed depletion of lithium in the solar photosphere is, however, consistent with the oscillation data which suggest that the solar convective envelope does in fact reach sufficiently high temperatures to burn lithium.

Figures 26 and 27 show the radius and temperature at the base of the convective zone, as a function of \(Q/N\), for solar models which derive part of their luminosity from Q-nuclear burning. The fact that these models have outer convective zones of the size required by the observed oscillation frequencies is strong supporting evidence for this mode of energy production. For models with no attribution of Q-nuclei, a Q-nuclear abundance of \(5\times10^{-15}\) \((1.0\times10^{-14})\) yields a convection zone extending to about
0.695 (0.680) R\(_{\odot}\) and to a temperature of about 2.4x10\(^6\) (2.7x10\(^6\)) K; the corresponding high-energy neutrino flux is 0.52 (0.35) SNU. Similar values are obtained for the "leaky" models for sufficiently large Q-nuclear abundances.

The requirement that lithium not be completely destroyed provides us with a means of establishing an upper limit to the abundance of Q-nuclei in the Sun. The rate of the \(^7\)Li(p, \(\alpha\)) reaction, and hence the \(^7\)Li lifetime, is highly temperature sensitive (~T\(^{22}\)). At a temperature of 2x10\(^6\) K the lifetime of \(^7\)Li is on the order of 10\(^{12}\) years; at a temperature of 3x10\(^6\) K the lifetime is reduced to about 10\(^7\) years. Since the lifetime (or e-folding time) of lithium in the solar convection zone must be no less than about 10\(^6\) years, the temperature at the base of this zone can be no greater than about 2.7x10\(^6\) K. For the non-leaky models, this restricts the Q-nuclear abundance to the range 5x10\(^{-15}\) \(\leq Q/N \leq 10^{-14}\), which is also compatible with the observed high-energy neutrino flux.

It must be emphasized that no attempt has been made to calculate the oscillation frequencies for the models presented here. Q-nuclear energy production substantially alters the run of variables throughout the solar interior, and the predicted acoustical spectrum may be different than what is expected from simply increasing the mixing length to simulate a deeper convection zone. A complete eigenfre-
Figure 26: Depth of the convective envelope, as a function of $Q/N$, for $\tau_B = 0$, 100, and 1000 seconds.
Figure 27: Temperature at the base of the convective envelope, as a function of $Q/N$, for $\tau_B = 0, 100, \text{ and } 1000 \text{ seconds.}$
quency analysis, for both the low and high degree modes, is clearly needed. It is interesting to note in this context that Ulrich and Rhodes (1983) have calculated the oscillation frequencies of several non-standard solar models which have received much attention in the literature, and all failed to reproduce the observed frequencies. However, they found that changes affecting the inner few percent of the solar mass produce detectable changes in the oscillation frequencies of low degree ($\ell \leq 4$). In particular, a mixed core with a mass $0.05M_\odot$ noticeably improves the agreement between the observed and calculated frequencies of the low degree $\ell$ modes of high radial order $n$. Although the frequency differences $\nu_{n,\ell} - \nu_{n-1,\ell+2}$ are much too large, this spacing is sensitive to both the temperature gradient and the mean molecular weight. It is encouraging that this spacing decreases with increasing helium abundance, as noted by Deubner and Gough (1984). At any rate, the envelope structure predicted by solar models with Q-nuclei is certainly supported by the observational data. Whether or not the overall structure is consistent with the entire spectrum of observed frequencies will have to await a more detailed analysis.

There is yet another independent line of evidence which may support, albeit indirectly, the case for Q-nuclear burning in the Sun. Conventional models of solar evolution
indicate that the Sun's luminosity has increased by nearly 40% since its arrival on the main sequence, and by approximately 24% over the past 3 billion years. An increase in the main sequence luminosity is an ubiquitous feature of stellar evolution, and is the direct result of the steadily increasing molecular weight of the core as hydrogen is converted to helium (e.g., see the discussion in Newman and Rood, 1977 and Endal, 1981). However, there is no evidence in the geologic or paleontological record that the solar constant has changed appreciably during the past 3 billion years. In fact, isotopic studies of Precambrian rocks indicate that the global mean temperature has actually decreased during this time (Knauth and Epstein, 1976; Endal, 1981). Furthermore, there is substantial evidence, including microfossils of blue-green algae, that liquid water existed in abundance as long ago as 3.2x10^9 years (Schopf and Barghoorn, 1967; Sagan and Mullen, 1972). Simple climatic models predict that a decrease of even 2% in the solar constant is sufficient to produce a glaciated Earth which, because of the high albedo of ice, would require a solar luminosity substantially larger than present in order to thaw out (Bodyko, 1969; Sellers, 1969; North, 1975). Models of the Earth's atmospheric evolution (Sagan and Mullen, 1972; Hart, 1978) suggest that an enhanced greenhouse effect prior to about 2x10^9 years ago
may have offset the effects of a reduced solar constant, thereby averting this "ice catastrophe". However, these same models, as well as paleontological evidence, show that the Earth's atmospheric chemistry changed from reducing to oxidizing about 2 billion years ago, and this would have removed the compounds responsible for the greenhouse effect (Cloud, 1976). Conventional solar models indicate an increase in the solar constant of about 16% since that time.

As was discussed in Section 4.2, a finite abundance of $Q$-nuclei results in a brighter zero age Sun. Figure 28 shows the time variation of the luminosity for the standard solar model ($Q/N=0$) and for two "non-leaky" models with $Q/N=2\times10^{-15}$ and $10^{-14}$. For the larger abundance, the predicted increase in luminosity over the Sun's lifetime is only 23%, compared to 39% for the standard model; during the past 2 billion years the corresponding changes are, respectively, 9% and 16%. For abundances greater than about $5\times10^{-15}$, there is relatively little change in zero age luminosity as the abundance is increased further, and it is likely that a luminosity increase of no less than about 20% is unavoidable if the Sun derives its energy from the conversion of hydrogen to helium. It may be significant that the "leaky" models show a luminosity increase comparable to that of the standard model (c.f. Figure 24);
it would be premature, however, to favor the non-leaky models on this basis alone. In any event, solar models which derive energy from Q-nuclear burning are certainly more consistent with the Earth's past climatic history than are conventional solar evolutionary models. Although Q-nuclear burning alone does not solve the faint early Sun problem, it at least provides some "breathing room" for theoretical models of global climate and atmospheric evolution.
Figure 28: Time variation in the solar luminosity. Curves are labeled with the value of \( Q/N \).
Chapter V

Q-NUCLEOSYNTHESIS AND STELLAR EVOLUTION

5.1 THE ZERO AGE MAIN SEQUENCE

In the previous chapter we examined the effects of Q-nuclear energy production on both the observed and inferred properties of the Sun, and showed that solar models which derive part of their energy from Q-nuclear burning are consistent with those properties. Although these results are encouraging, it remains to be demonstrated that Q-nuclear burning does not conflict with the observed properties of other stars. In this section we will investigate the effects of Q-nucleosynthesis on the interior structure and observable properties of zero age main sequence (ZAMS) models. In the following section we will study the evolutionary effects of Q-nuclear burning, with particular emphasis on age determinations based upon a comparison of cluster turnoff properties with the predictions of model calculations.

The interiors code described in chapter III was used to generate a series of homogeneous ZAMS models with composition parameters $(Y,Z)=(0.25,0.0167)$ and with masses in the
range $0.5 \leq M/M_\odot \leq 7.0$. A mixing length of 1.5 pressure scale heights was used for all model calculations. The input physics, integration stepwidths and other numerical constraints are the same as those employed in the construction of the solar models described in Section 4.1. In Tables 8-10 we present some basic properties of 1) the standard ZAMS, 2) the "non-leaky" ZAMS with $Q/N=2.5\times10^{-15}$, and 3) the ZAMS with $Q/N=10^{-14}$ and a $^7\text{Be}^0$ beta-decay lifetime of 1000 seconds. Tabulated in columns (1)-(12) are, respectively, the model mass in solar units, the logarithm of the central temperature and density, the mass of the convective core (solar units), the stellar radius (solar units), the logarithm of the effective temperature, the absolute bolometric magnitude (assuming $M_{\text{bol}_0}=4.72$), the $B-V$ color and absolute visual magnitude, and the fraction of the total luminosity contributed by the PP, CNO and Q-nuclear cycles. Except where otherwise noted, c.g.s. units are assumed. The $B-V$ colors and bolometric corrections used to transform from the $(M_{\text{bol}}, T_{\text{eff}})$ plane to the observational plane are taken from the values tabulated by VandenBerg (1983), which are based upon the model atmosphere results of Buser and Kurucz (1978) and Bell and Gustafsson (1978). These values are tabulated as functions of effective temperature, surface gravity and metallicity, and the values appropriate for a given stellar model are
obtained by interpolation. In order to gauge the accuracy of the predicted colors, the interpolated B-V colors of the theoretical ZAMS in Table 8 are compared, in Figure 29, with the empirical color-temperature relation of Bessel (1979) for normal dwarfs. The agreement is surprisingly good, considering the differences in age and chemical composition of the field dwarfs upon which the empirical relation is based. The theoretical relation, however, predicts a solar color approximately 0.03 magnitudes to the blue of the empirical relation, and this will have to be kept in mind when comparing predicted and observed main sequence properties.

The location of our fiducial ZAMS in the H-R diagram is shown in Figure 30. At high temperatures (log $T_{\text{eff}} \geq 3.75$), this main sequence is in excellent agreement with that of VandenBerg and Bridges (1984), which employs the opacity data from the Los Alamos Astrophysical Opacity Library, as well as model atmosphere grids to provide the surface boundary conditions. Primarily because of these improvements, their models are slightly redder than ours at low temperatures. Consequently, meaningful comparisons with observations will be restricted to temperatures above about 4800 K, or B-V $\leq 0.90$. The main sequence in Figure 30 shows the characteristic change in slope near log $T_{\text{eff}} = 3.8$, which roughly corresponds to the transition from PP to CNO burn-
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Table 10

**Leaky zero age main sequence models**

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Figure 29: Comparison of theoretical (solid curve) and empirical (open circles) color-temperature relations.
ing and the change from radiative to convective energy transport in the stellar core. The relative contributions of the PP and CNO cycles to the total luminosity are shown in Figure 31. Core convection begins near 1.2 M\(_{\odot}\) as the CNO cycle begins to make a substantial contribution to the energy output; the PP and CNO cycles contribute equally to the luminosity near 2 solar masses.

The effect of Q-nuclear burning on the position of the ZAMS is illustrated in Figures 32 and 33, with and without attrition from the HeLiBe cycle, respectively. For the non-leaky case, the Q-nuclear burning ZAMS (hereafter Q-ZAMS) begins to depart appreciably from our fiducial ZAMS near log T\(_{\text{eff}}\)=3.75 and M/M\(_{\odot}\)=1.0. For 3.9 log T\(_{\text{eff}}\)>4.1 the Q-ZAMS lies nearly 0.5 magnitudes above the ZAMS and is virtually parallel to it. As shown in Figure 33, the Q-ZAMS with leakage from the HeLiBe cycle also lies above the standard main sequence over a considerable range in temperature. In contrast to the non-leaky models, however, the upper main sequence is relatively unaffected. Note that the difference between the standard ZAMS and the Q-ZAMS is primarily a temperature effect. At a given mass, Q-nuclear burning results in a lower luminosity and a larger radius (c.f. Section 4.2); consequently, the surface temperature is reduced by a considerable amount.
Figure 30: The standard zero age main sequence.
Figure 31: Fraction of the stellar luminosity contributed by the PP and CNO cycles, as a function of mass.
Figure 32: The zero age main sequence for $Q/M=2.5 \times 10^{-15}$ and $\tau_B = 0$. Lower curve is the standard ZAMS.
Figure 23: The zero age main sequence for $Q/H=10^{-12}$ and $\tau_8$ = 1000 seconds. Lower curve is the standard ZAMS.
Figures 34 and 35 show the relative contributions of the PP, CNO and Q-nuclear cycles to the stellar luminosity, as a function of mass. For both the leaky and non-leaky case, the Q-nuclear cycle makes a significant contribution to the energy production over the entire range of masses considered here. The PP cycle dominates the energy production below about $1M_\odot$ if there is no attrition of Q-nuclei, but the HeLiBe cycle rapidly increases in importance with increasing mass; at $3M_\odot$, nearly 90% of the stellar luminosity derives from Q-nuclear burning (for the chosen abundance). At higher masses the Q-nuclear cycle decreases in importance as the highly temperature sensitive CNO reactions begin to play a role, but even at $7M_\odot$ the HeLiBe cycle still contributes most of the energy. The dominance of Q-nuclear burning over such a large range in mass is not unexpected, and is again a consequence of the temperature dependence of the Q-nuclear energy source, which is approximately $T^{13}$ compared to $T^4$ for the PP cycle and $T^{20}$ for the CNO cycle. Because its temperature dependence is intermediate between the PP and CNO cycles, the HeLiBe cycle begins to contribute to the energy production at a lower mass (or core temperature) than does the CNO cycle in conventional models. Similarly, Q-nuclear burning persists to higher masses than does the PP cycle in the absence of Q-nuclei. The onset of CNO burning is further inhibited by
the lower core temperatures. At $3M_\odot$ for example the central temperature is about $4 \times 10^6$ K less than in conventional models, and the efficiency of the CNO cycle is correspondingly reduced. If there is leakage from the HeLiBe cycle, the effective temperature dependence of the Q-nuclear energy source is considerably reduced (Figure 35) for the reasons discussed in Section 4.3. Consequently, the Q-nuclear energy contribution is confined to lower masses in the leaky stellar models than in the non-leaky models; this result is apparent in the ZAMS models shown in Figures 32 and 33.

The mass at which core convection appears depends critically on the extent of leakage (if any) from the HeLiBe cycle. If there is no attrition of Q-nuclei, core convection develops near $0.7M_\odot$ and, for $1.2 \leq M/M_\odot \leq 2.0$, the convectively-mixed region is considerably larger in extent than in conventional models. However, at larger masses Q-nuclear burning actually reduces the extent of convective mixing. This is because, at a given mass, the highly temperature sensitive CNO cycle contributes much less energy than in conventional models and the central temperature gradient is reduced accordingly. The situation is quite different if $^7\text{Li}Q$ is depleted in the core. As is evident in Table 10, models in the range $0.7 \leq M/M_\odot \leq 1.8$ do not have convective cores and, in contrast to standard stellar mod-
Figure 34: Fraction of the stellar luminosity contributed by the PP, CNO and Q-nuclear cycles, as a function of stellar mass, for Q/H=2.5x10^{-15} and $\tau_B = 0$. 
Figure 35: Fraction of the stellar luminosity contributed by the PP, CNO and Q-nuclear cycles, as a function of stellar mass, for $Q/N=10^{-14}$ and $\tau_B = 1000$ seconds.
els (or Q-nuclear models without leakage) core convection appears in stars of very low mass. As has been discussed previously, *LiQ may be lost from the HeLiBe cycle via a *BeQ (p,γ) reaction if *Be is a stable Q-nucleus, and since the resulting steady-state *LiQ abundance increases outward from the center, the effective temperature dependence of the rate of energy production is considerably reduced. However, at low temperatures the proton capture rate on *BeQ is much reduced, less attrition occurs and the temperature sensitivity of Q-nuclear energy production again approaches that of the non-leaky cycle. At low masses (and low core temperatures) the effects of attrition are therefore minimized, and the temperature gradient is sufficiently large to induce convection. (Note that in the non-leaky models the HeLiBe cycle does not make a large enough contribution to the energy production for M/M⊙ < 0.7 to induce convection, despite the large temperature sensitivity.) For masses greater than about 0.7M⊙, the higher core temperatures result in a larger degree of attrition and core convection disappears. Because the Q-nuclear cycle inhibits the onset of CNO burning, core convection does not reappear until near 1.8M⊙. This is an important result, because the color-magnitude diagrams of old galactic clusters show a gap near the main sequence turnoff which is almost certainly attributable to convective mixing in stars.
with masses less than $1.5\,M_\odot$. This strongly suggests that an attrition of $Q$-nuclei of this magnitude is unlikely. However, convection will set in at a mass less than $1.8\,M_\odot$ if $Q/N$ is decreased, or if $\tau_B$ is decreased, or both. Thus this observation alone does not necessarily rule out an attrition of light $Q$-nuclei, but does constrain the magnitude of the effect. Also note that if $Q/N$ and $\tau_B$ are decreased together the high-energy solar neutrino flux can still be maintained at or below the observed upper limit.

In Figure 36 we compare the predicted location of the theoretical ZAMS in the $(M_V, B-V)$ plane with the photoelectric observations of the Pleiades obtained by Johnson and Mitchell (1958). This cluster was chosen for comparison because it is young and therefore has a well defined main sequence extending over a large range in magnitude. Also, several studies have shown that the metallicity of the Pleiades is close to the solar value (Hesser and Henry, 1971; Chaffee et al. 1971). The observations have been corrected for a reddening of $E(B-V)=0.4$ (Crawford and Perry, 1976), and an apparent distance modulus of 5.4 magnitudes obtained by fitting the observations to the theoretical main sequence. This distance modulus is slightly less than the value of 5.6 found recently by VandenBerg and Bridges (1984). However, these authors adopted the empirical color-temperature relation of Bessel (1979) which pre-
dicts a solar color of \((B-V)_0 = 0.63\), whereas the model atmosphere colors adopted here predict \((B-V)_0 = 0.60\). Since the slope \(\Delta M_V / \Delta (B-V)\) of the main sequence near \(B-V = 0.6\) is about 7, a shift in \(B-V\) of 0.03 magnitudes would require an increase in distance modulus of about 0.2 magnitudes, in agreement with that of VandenBerg and Bridges.

As is evident in Figure 36, the observed Pleiades main sequence is very well reproduced by the models over a wide range in color. The failure to reproduce the data for \(B-V \leq 0\) is almost certainly due to evolutionary effects. For \(B-V \geq 0.8\) the models are a little too blue, but as mentioned earlier this is most likely the result of our inadequate treatment of low temperature opacities. The good agreement between the theoretical and observed ZAMS for the Pleiades severely constrains any mechanism which appreciably changes the predicted \(M_V - (B-V)\) relation. In Figure 37 we compare the Pleiades data with the Q-ZAMS with no attribution of Q-nuclei. There is little doubt that models with \(Q/N \geq 2.5 \times 10^{-15}\) provide a poor fit to the observed ZAMS. In fact, there is considerable ambiguity in the derived distance modulus since it is not clear what the "best fit" is; in the figure we have rather arbitrarily matched the theoretical and observed sequences near \(B-V = 0.6\). However, this places the models well above the observed points in the blue. Although the shape of the Q-ZAMS would fit the data
Figure 36: A fit of the theoretical ZAMS to the published observations of the Pleiades.
well for $B-V \approx 0.4$, the models would be nearly 0.5 mag too faint near $B-V=0.6$. Although the fit can be improved with a smaller Q-nuclear abundance, the abundance used here is already a factor of two smaller than what is required to reproduce the solar p-mode oscillations of high degree. Furthermore, a change in helium abundance or metallicity merely shifts the ZAMS in luminosity without affecting the overall shape. On the other hand, a change in the assumed mixing length parameter does alter the shape. If $1/H_p$ is decreased, the red portion of the main sequence is shifted to lower temperatures, and the magnitude of this shift increases with decreasing temperature. This would indeed improve the fit. However, a value of $1/H_p$ larger than the "canonical" value of 1.5 is required to obtain a self-consistent solar model with Q-nuclear burning; if we were to use a value of the mixing length parameter based upon a solar calibration ($2 \leq 1/H_p \leq 3$), this would merely aggravate the problem. This expectation is confirmed in Figure 38, which shows the Q-ZAMS for $Y=0.3$ and $1/H_p=2.0$.

The Q-ZAMS with attrition of Q-nuclei is shown in Figure 39. These models match the observed main sequence much better than the non-leaky models, although the fit is still not satisfactory. As before, a decrease in $1/H_p$ would improve the fit, but this seems inconsistent with the value suggested by solar model calculations. It appears, there-
fore, that the only way to improve the agreement between the observed and theoretical ZAMS is to reduce the Q-nuclear abundance below the value required by solar models.

In view of the young age of the Pleiades (~5x10⁷ years), a possible solution to this apparent inconsistency is that the cosmic abundance of He, Li and Be Q-nuclei has decreased over time. If this were the case, the abundance of these Q-nuclear species at the time of the Sun's formation could be significantly larger than the abundance at the present epoch. Indeed, it would be unreasonable to suppose that the relative abundances of Q-nuclei have remained constant since the Big Bang. Q-nuclei would certainly be returned to the interstellar medium by way of stellar mass loss, and a certain fraction would have undergone nuclear processing. The cosmic abundance of a given Q-nucleus would depend upon the nuclear burning processes which create or destroy it, the mechanisms which return this material to the interstellar medium, and the initial stellar mass function, among other things. Unfortunately, at the present level of approximation it is difficult to say whether the abundance of a particular Q-nuclear species would be expected to increase or decrease with time, let alone estimate the magnitude of this change over the past 5x10⁷ years. We have already seen that if ⁷BeQ is not a
Figure 37: A comparison of the non-leaky Q-ZAMS and the published data for the Pleiades. The Q-nuclear abundance is $2.5 \times 10^{-14}$. 
Figure 38: A comparison of the non-leaky Q-ZAMS and the published data for the Pleiades. The helium abundance and mixing length ratio are, respectively, 0.30 and 2.0. The Q-nuclear abundance is the same as in Figure 37.
Figure 39: A comparison of the leaky Q-ZAMS and the published data for the Pleiades. The Q-nuclear abundance is $5 \times 10^{-13}$, and $\tau_B = 1000$ seconds.
stable Q-nucleus, *LiQ is greatly increased in abundance as a result of equilibrium hydrogen-burning. If *BeQ is stable, *LiQ (and other light Q-nuclei) will be converted to heavier Q-nuclei, principally 10BQ and perhaps 14NQ. However, the relative Q-nuclear abundances which result from hydrogen burning may be considerably modified during the subsequent stage of core helium-burning. For example, it is possible that *LiQ is destroyed by a *LiQ(α,γ)10BQ reaction at sufficiently high core temperatures. Another interesting possibility is a "triple-alpha" reaction in which a *HeQ replaces a *He nucleus. This may be an important mode of attrition of this Q-nucleus, especially if the lifetime of *BeQ is of the order of or greater than that of *Be. Similarly, *HeQ may be destroyed by radiative capture reactions on 12C and 16O during core helium burning. At any rate it is plausible, though by no means certain, that the cosmic abundance of HeLiBe Q-nuclei (*HeQ and *LiQ in particular) might decrease with time as a result of normal galactic chemical evolution. Thus the failure to reproduce the observed ZAMS does not necessarily rule out the existence of Q-nuclei.
5.2 Evolutionary Models

In the previous section we showed that a *LiQ abundance on the order of $2 \times 10^{-13}$, which is the minimum required to alleviate several outstanding inconsistencies with solar models, is probably ruled out on the basis of the poor agreement between the predicted and observed ZAMS as defined by the Pleiades. It was suggested, however, that the cosmic abundance of *LiQ may decrease with time as the result of processes which destroy *LiQ during the course of normal stellar evolution. If this is the case, a more appropriate place to look for possible manifestations of Q-nucleosynthesis would be an old stellar population, for example old galactic or globular clusters.

In Figure 40 are shown the evolutionary tracks, with and without Q-nuclei, for stars with masses 0.8$\odot$ and 1.5$\odot$. The models with Q-nuclei have $Q/N=10^{-14}$, and we assume no depletion of *LiQ. All have the same composition $(Y,Z)=(0.25,0.017)$ and the mixing length is 1.5 pressure scale heights. The models with Q-nuclear burning evolve at lower temperatures and luminosities than conventional models with the same mass and composition. Consequently, the evolutionary track for a star with Q-nuclear burning is qualitatively similar to that of a "normal" star with a smaller helium abundance and/or mixing length. Because the Q-nuclear models evolve at a lower luminosity, their main
sequence lifetimes are somewhat larger than conventional models of the same mass. At 0.8\(M_\odot\), the age difference is only a small fraction of the main sequence lifetime, on the order of 2\(\times\)10\(^6\) years. At 1.5\(M_\odot\), however, the Q-ZAMS is nearly 0.3 magnitudes fainter than the standard ZAMS, and the main sequence lifetime is increased over 40%, from 1.6\(\times\)10\(^6\) years to 2.3\(\times\)10\(^6\) years. The larger convective core in the model with Q-nuclear burning also contributes to this increase in main sequence lifetime, because convective mixing supplies the energy-producing regions with more hydrogen than would otherwise be available.

It is important to note that an increase in age, at a given mass, does not necessarily imply that galactic or globular clusters are older than indicated by standard stellar models. A cluster age is normally estimated by comparing theoretical isochrones of different ages with the main sequence turnoff and subgiant branch of the cluster, based upon an assumed reddening and distance modulus. (An isochrone is the locus of points, in an H-R diagram or color-magnitude diagram, representing stars with different masses but a common age.) Both the temperature and luminosity of the turnoff from the main sequence decrease with increasing cluster age. Although the evolutionary tracks in Figure 40 are not isochronous curves, the main sequence luminosity is still indicative of the main sequence life-
Figure 40: Evolution in the H-R diagram for $M/M_\odot = 0.8$ and 1.5. The tracks are labeled by the value of $Q/M$. 
time. It is clear from Figure 40 that the main sequence luminosity of a conventional stellar model of given mass will be the same as that of a model with Q-nuclear burning and a slightly larger mass, and hence a shorter main sequence lifetime.

In order to examine the effect of Q-nucleosynthesis upon the characteristic features of the main sequence turnoff and subgiant branch of a typical globular cluster, a series of isochrones was constructed with and without Q-nuclear burning. Our "fiducial" isochrone, against which comparisons will be made, has an age of $16 \times 10^9$ years, a composition $(Y,Z) = (0.20, 0.001)$, and is based upon a mixing length parameter of 1.5 pressure scale heights. This particular choice of $Y$ and $1/H_p$ was motivated by the very good agreement between VandenBerg's (1983) isochrones, which are based upon these values, and the color-magnitude diagrams of several globular clusters for which good photometric data exist. The chosen heavy element abundance is typical of a globular cluster of intermediate metallicity, such as M13. Although no attempt was made to fit this isochrone to any real cluster, the shape of the turnoff and subgiant branch agrees well with similar isochrones constructed by VandenBerg (1983) and Ciardullo and Demarque (1977), which do in fact reproduce the observations well.
Our standard isochrone is illustrated in Figure 41 by the heavy curve. Also shown are three isochrones of ages 4, 10, and 16x10⁹ years, for which Q/H=10⁻¹⁰ and τ₂=0. All isochrones presented here are represented by no fewer than 20 points, and the turnoff region is covered by a sufficient number of points to clearly define the blueward hook resulting from core convection, if present.

It is clear from Figure 41 that, at a given age, Q-nuclear burning greatly reduces the luminosity of the main sequence turnoff and shifts it to the red. At 16x10⁹ years and Q/H=10⁻¹⁰, the difference in luminosity is nearly a magnitude, and the isochrone for 10¹⁰ years is still about 0.5 magnitudes below the fiducial isochrone. Clearly, if globular cluster stars contain Q-nuclei at an abundance level of 10⁻¹⁴, globular cluster ages based upon fits to standard isochrones are greatly overestimated.

As shown in Figure 42, increasing the helium abundance from Y=0.2 to Y=0.3 shifts the isochrones to the blue and increases slightly the turnoff luminosity. The overall shape near the main sequence turnoff more closely approximates that of the reference isochrone, and the location and slope of the main sequence is accurately reproduced. Although it is unlikely that the helium abundance in globular cluster stars is this large, a similar effect can be achieved with a smaller increase in Y if the mixing length
Figure 41: Isochrones for 4, 10, and $16 \times 10^9$ years. The light curves are isochrones with $Q/N=10^{-14}$, $Y=0.20$ and $1/H_p=1.5$. The heavy curve represents a fiducial isochrone with an age of $16 \times 10^9$ years, $Y=0.20$, and $1/H_p=1.5$. Number beside each curve is the age in billions of years.
parameter is also increased. In Figure 43 we show an isochrone with $Y=0.25$, $1/H_p=2.0$, $Q/N=10^{-14}$, and an age of $7 \times 10^9$ years. This isochrone provides a very good fit to the reference isochrone, suggesting that color-magnitude diagrams of globular clusters can be reasonably well approximated by models with $Q$-nuclear burning if $Y$ and $1/H_p$ are slightly larger than the values usually adopted.

Note that the non-standard isochrone in Figure 43 exhibits the blueward hook which is characteristic of stars with central convection zones. For younger clusters, this produces a gap in the stellar distribution near the main sequence turnoff. However, the evolutionary rate during core hydrogen exhaustion is less rapid for stars in the mass range represented by the isochrones in Figure 43, and the gap would not be as pronounced as that found in old galactic clusters. The presence of numerous unresolved binaries, as well as differences in composition and age among cluster stars would also tend to obscure this gap. Consequently, although the observation of a statistically significant gap in the stellar distribution near the main sequence turnoff in a globular cluster would provide a strong case for $Q$-nucleosynthesis, such a feature would be very difficult to observe.

Based upon a comparison of model isochrones with the color-magnitude diagrams of 15 globular clusters for which
Figure 42: Isochrones for 4, 10, and $16 \times 10^9$ years. The light curves have $Q/N=10^{-14}$, $Y=0.3$, and $1/H_p=1.5$. The heavy curve is the fiducial isochrone with $Y=0.2$, $1/H_p=1.5$ and an age of $16 \times 10^9$ years. Number next to each curve is the age in billions of years.
Figure 43: A 7x10^6 year isochrone with Q/H = 10^{-14}, Y = 0.25 and l/H_p = 2.0 (light curve). The heavy curve is the fiducial isochrone with Y = 0.2, l/H_p = 1.5, and an age of 16x10^6 years.
main sequence photometry is available. VandenBerg (1983) concludes that these clusters have ages in the range 15-18x10⁶ years. If we simply scale these values downward by the amount indicated by the isochrones in Figure 43, the maximum age of the globular clusters would be about 9x10⁶ years (for Q/N=10⁻¹⁴). This lower limit to the age of the universe would then require  \(H_0 \leq 120 \text{ km sec}^{-1} \text{ Mpc}^{-1}\) for a Friedman model with \(\Omega_0 = 0\), a value which is not inconsistent with recent estimates of the Hubble constant (e.g., see the review by Hodge, 1981). It must be emphasized that the turnoff luminosity, and therefore the age, is very sensitive to the assumed Q-nuclear abundance. If Q/N is decreased from 10⁻¹⁴ to 5x10⁻¹⁵, the 16x10⁶ year isochrone in Figures 41 and 42 is shifted upward in luminosity by about 0.3 mag, which would increase the age estimates by about 30%. At any rate, the reduction in globular cluster ages required by Q-nuclear burning does not conflict with the maximum age of the universe inferred from cosmological considerations.
Chapter VI
DISCUSSION AND CONCLUSIONS

Motivated by the failure of conventional stellar evolution theory to correctly predict the observed solar neutrino flux and p-mode oscillation frequencies, we have investigated the possibility that the Sun and other stars derive part of their energy from Q-nuclear burning. In this chapter we will summarize the principle conclusions of this investigation, and suggest possible directions of future work.

We have studied the effects of Q-nuclear energy production on the properties of the present Sun by constructing a number of solar models for a range in values of the assumed Q-nuclear abundance. It was found that the predicted neutrino capture rate for the \(^{37}\text{Cl}\) detector drops below the observed upper limit of 2 SNU at a Q-nuclear abundance of about \(2 \times 10^{-15}\) Q-nuclei per normal nucleus. At a slightly higher abundance (\(\sim 5 \times 10^{-15}\)) the outer convection zone extends to a depth of about 0.7\(R_\odot\) and a temperature of \(\sim 2.4 \times 10^6\) K, in agreement with the values inferred from the observed frequencies of the solar p-mode oscillations and
the low lithium abundance in the solar photosphere. If 
BeQ is stable, so that there is no attrition of light 
Q-nuclei during hydrogen burning, a solar abundance of 
Q-nuclei greater than about $10^{-14}$ is ruled out because the 
base of the convective envelope then extends to sufficiently 
high temperatures that lithium is destroyed completely. 
The predicted abundance of helium in the solar photosphere 
is increased by about 20%, a value which is not inconsis-
tent with other helium abundance determinations. The 
change in solar luminosity during the past $3 \times 10^9$ years is 
considerably less than is predicted by conventional models 
of solar evolution if no attrition of He, Li and Be 
Q-nuclei occurs, a result which is consistent with the 
paleontological record. Thus solar models in which 
Q-nuclear burning contributes to the energy production do 
not conflict with any observed properties of the Sun. Any 
one of these results in and of itself does not provide a 
convincing argument for a mode of energy production as 
unconventional as that suggested; collectively, however, 
they constitute strong supporting evidence for an energy 
source with the characteristics described.

In an attempt to ascertain whether or not the abundance 
of Q-nuclei suggested by solar model calculations is com-
patible with the observed properties of main sequence 
stars, zero age main sequence models with solar composition
and masses in the range $0.7 \leq M/M_\odot \leq 7.0$ were constructed with and without Q-nuclear burning. It was determined that a significant attrition of Q-nuclei from the HeLiBe cycle was unlikely, as these models predict radiative cores in stars with masses as large as $1.8 M_\odot$, in contradiction to observation; this is consistent with the expectation that the Q-nucleus $^9\text{BeQ}$ is probably unstable. Q-nuclear burning in ZAMS stars (without attrition) drives down the luminosity and increases the stellar radius, resulting in a substantial decrease in surface temperature for masses larger than about $1 M_\odot$. Consequently, the upper main sequence is considerably redder than predicted by conventional interior models. A comparison of the published observations of the Pleiades and the predicted location of the Q-nuclear burning main sequence rules out a Q-nuclear abundance of $\sim 2 \times 10^{-15}$ at the present epoch, the minimum value suggested by solar model calculations. The fact that the observed ZAMS agrees well with the predictions of standard interior models suggests an upper limit to the present abundance of light Q-nuclei about a factor of ten smaller than the indicated solar abundance. An obvious way to reconcile these apparently conflicting results is to assume that the cosmic abundance of these Q-nuclei has decreased since the Sun's formation. It is certainly plausible that nuclear burning processes during advanced stages of stellar evolution might
selectively destroy some or all of the Q-nuclei that participate in the HeLiBe cycle. It was suggested, for example, that during core helium burning $^7\text{He}^Q$ would probably be destroyed by reactions similar to those that destroy $^7\text{He}$, i.e., triple-alpha-like reactions and radiative capture reactions on carbon and oxygen nuclei (or Q-nuclei). An alpha-capture reaction on $^6\text{Li}^Q$ may similarly deplete this Q-nucleus during core helium-burning. On the other hand, at carbon-burning and oxygen-burning temperatures, protons and alpha-particles are liberated by the reactions $^{12}\text{C}(^{12}\text{C},\alpha)^{20}\text{Ne}$, $^{12}\text{C}(^{12}\text{C},p)^{23}\text{Na}$, $^{16}\text{O}(^{16}\text{O},\alpha)^{28}\text{Si}$ and $^{16}\text{O}(^{16}\text{O},p)^{31}\text{P}$. Analogous reactions involving Q-nuclei would be expected to liberate $^8\text{He}^Q$, and proton capture reactions on this Q-nucleus and other light Q-nuclei which survived helium-burning might "re-ignite" the HeLiBe cycle. At such high temperatures, however, the steady-state He, Li, and Be Q-nuclear abundances would likely be several orders of magnitude smaller than the abundances present at the end of core hydrogen-burning. At any rate, without a more detailed knowledge of the quark-nucleon interaction it is impossible to say if stellar burning could result in a depletion of the light Q-nuclei of the required magnitude.

Evolutionary calculations have shown that models with a Q-nuclear abundance of $10^{-13}$ have longer main sequence lifetimes than conventional models of the same mass and
chemical composition, and evolve at lower luminosities and surface temperatures. In contrast to standard interior models, core convection is present for masses as small as $0.7M_\odot$, and for masses in the range $0.7 < M/M_\odot < 2.0$ the convectively-mixed region is substantially larger than is indicated by standard model calculations. This may provide a natural explanation for the anomalous characteristics of the gap near the main sequence turnoff of several old galactic clusters; these characteristics have, in fact, been attributed to a mixed region larger in extent than predicted by stellar models (Maeder, 1974, 1976).

Model isochrones for an old stellar population indicate that, for a Q-nuclear abundance of $10^{-14}$, the luminosity near the main sequence turnoff is nearly a magnitude fainter than that for a standard isochrone of the same age. This implies a substantial decrease in age estimates of globular clusters based upon isochrone fits. The shape and color of the main sequence turnoff can be well reproduced by a small increase in both the helium abundance and mixing length parameter. Although the model isochrones predict a small gap near the main sequence turnoff, it is very unlikely that this gap would be apparent in a cluster color-magnitude diagram.

Within experimental and theoretical uncertainties, a fractional Q-nuclear abundance of $10^{-15} - 10^{-14}$ Q-nuclei per
normal nucleus is consistent with the upper limits imposed by quark search experiments and theoretical considerations. Assuming that the quark-quark interaction goes to zero at some finite quark separation, Wagoner and Steigman (1979) estimated that the free quark abundance following the Big Bang is less than about $10^{-14}$ free quarks per baryon. On the other hand, the fractional charges LaRue and his coworkers claim to have observed on their niobium spheres suggest an abundance of less than about $10^{-16}$ fractionally charged nuclei per normal nucleus. However, it is not clear whether these fractional charges (if they exist) originate within the spheres or from the thin layer of tungsten which coats them (Lyons, 1982). If it is the latter, the inferred abundance would be five orders of magnitude larger. Other experimental searches have produced similar upper limits ($Q/N \sim 10^{-21} - 10^{-15}$). However, many of these experiments are based upon assumptions regarding the chemical properties of free fractional charges. As discussed by Lackner and Zweig (1983), the actual abundance of such entities may differ greatly from experimental estimates due to their peculiar chemistry. Similar considerations may apply to other possible Q-nuclear candidates.

If we assume that nuclear burning processes during advanced stages of stellar evolution favor the destruction of the Q-nuclei which constitute the HeLiBe cycle, so that
their abundance decreases with time, Q-nucleosynthesis appears to produce no obvious conflicts with observations relating to evolution on the main sequence and subgiant branch. In particular, the success with which Q-nucleosynthesis accounts for several outstanding discrepancies between theory and observation suggests that further investigation is warranted. A complete eigenfrequency analysis of solar models with Q-nuclear burning, including a comparison with the recently observed intermediate degree p-mode oscillations, may determine whether the structure of the proposed model is compatible with the entire spectrum of observed acoustical frequencies. A particularly important test of this mode of energy production would be provided by a $^{71}$Ga or $^{116}$In solar neutrino detector. The neutrinos produced in the beta-decays of $^7$LiQ and $^7$BeQ are expected to be significantly higher in energy than the neutrinos produced by the p-p reaction. As discussed by Sur and Boyd (1985), the count rate for a $^{71}$Ga of $^{116}$In detector would therefore be larger than that predicted by the standard solar model, perhaps by as much as a factor of two. The models presented here may be unique in predicting a larger count rate for neutrino detectors with a low energy threshold, and confirmation of this prediction would be compelling evidence in favor of Q-nuclear burning. Experimental searches for possible Q-nuclear candidates, some of
which are now in progress, may also prove fruitful. Clearly, should the existence of Q-nuclei be substantiated by independent experiment, their role in all phases of stellar evolution should be thoroughly investigated.
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