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VARIANCE BOUND TEST:
A NEW APPROACH

DISSERTATION

Presented in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy
in the Graduate School of The Ohio State University

By

Chang Soo Hur, B.B.A., M.A.

****

The Ohio State University
1985

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1985
To My Family
ACKNOWLEDGEMENTS

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I could not share enough time with my lovely wife, Gwysuk, because the dissertation started right after the marriage. My son, Derrick, had to give up a considerable amount of care from me, too. I hope that their patience will be compensated in the future.

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>VITA</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. VALUATION MODELS</td>
<td>8</td>
</tr>
<tr>
<td>III. EARLIER VARIANCE BOUND TESTS AND</td>
<td></td>
</tr>
<tr>
<td>SUBSEQUENT CRITICISMS</td>
<td>20</td>
</tr>
<tr>
<td>IV. ADDITIONAL PROBLEMS</td>
<td></td>
</tr>
<tr>
<td>IN EARLIER EMPIRICAL TESTS</td>
<td>35</td>
</tr>
<tr>
<td>V. PLANNING HORIZON</td>
<td>46</td>
</tr>
<tr>
<td>VI. EMPIRICAL STUDIES</td>
<td>60</td>
</tr>
<tr>
<td>VII. CONCLUSION AND INPLICATIONS</td>
<td>81</td>
</tr>
<tr>
<td>FOOTNOTES</td>
<td>84</td>
</tr>
<tr>
<td>TABLES</td>
<td>93</td>
</tr>
<tr>
<td>FIGURES</td>
<td>98</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>116</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. General Description of Data</td>
<td>93</td>
</tr>
<tr>
<td>based on the First-Order Autoregression Model</td>
<td></td>
</tr>
<tr>
<td>2. General Description of Data</td>
<td>94</td>
</tr>
<tr>
<td>based on the Ordinary Least Squares Model</td>
<td></td>
</tr>
<tr>
<td>3. Test for (Random-Walk) Nonstationarity</td>
<td>95</td>
</tr>
<tr>
<td>based on the First-Order Autoregression Model</td>
<td></td>
</tr>
<tr>
<td>4. Test for (Random-Walk) Nonstationarity</td>
<td>96</td>
</tr>
<tr>
<td>based on the Ordinary Least Squares Model</td>
<td></td>
</tr>
<tr>
<td>5. Partial Replication of the Shiller's Method</td>
<td>97</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.</td>
<td>Total Market Value of Stocks on the New York Stock Exchange (NYSE)</td>
</tr>
<tr>
<td>2.</td>
<td>Equal-Weighted Average Price of Stocks in SAMPLE ONE</td>
</tr>
<tr>
<td>3.</td>
<td>Detrended Equal-Weighted Average Price of Stocks in SAMPLE ONE</td>
</tr>
<tr>
<td>4.</td>
<td>Equal-Weighted Average Dividend Paid to Stocks in SAMPLE ONE</td>
</tr>
<tr>
<td>5.</td>
<td>Summary Information about Normality of the Average Price Time-Series in SAMPLE ONE</td>
</tr>
<tr>
<td>6.</td>
<td>Residuals from the Ordinary Least Squares Model with the Average Price Time-Series in SAMPLE ONE</td>
</tr>
<tr>
<td>7.</td>
<td>Summary Information about Normality of the Average Dividend Time-Series in SAMPLE ONE</td>
</tr>
<tr>
<td>8.</td>
<td>Residuals from the Ordinary Least Squares Model with the Average Dividend Time-Series in SAMPLE ONE</td>
</tr>
<tr>
<td>9.</td>
<td>The Average Dividend Time-Series in SAMPLE ONE Capitalized by Treasury Bill Rate</td>
</tr>
</tbody>
</table>
10. Comparison of Volatilities of the Ex-ante and Ex-post Average Price Time-Series in SAMPLE ONE .......................... 107
11. Total Wealth of Stocks in SAMPLE TWO ................................. 108
12. Gross Dividend paid to Stocks in SAMPLE TWO ..................... 109
13. Summary Information about Normality of the Total Wealth Time-Series in SAMPLE TWO ..................... 110
14. Residuals from the Ordinary Least Squares Model with the Total Wealth Time-Series in SAMPLE TWO ..................... 111
15. Summary Information about Normality of the Gross Dividend Time-Series in SAMPLE TWO ..................... 112
16. Residuals from the Ordinary Least Squares Model with the Gross Dividend Time-Series in SAMPLE TWO ..................... 113
17. The Gross Dividend Time-Series in SAMPLE TWO Capitalized by Treasury Bill Rate ................................. 114
18. Comparison of Volatilities of the Ex-ante and Ex-post Total Wealth Time-Series in SAMPLE TWO ..................... 115
CHAPTER I
INTRODUCTION

Many empirical tests of market efficiency have been performed since research interest in this area was reborn in the late 1950's and early 1960's, especially under the framework of rational expectations theory. Conventional methods of testing consist of regressions of excess returns on some variables representing the current information set. As summarized in Fama (1976) and Copeland and Weston (1983), the greater part of the empirical results from the conventional methods tend to report failure to reveal a statistically significant violation of market efficiency. On the other hand, only a few tests appear to reject market efficiency. Even in the case of rejection, the fundamental nature of joint hypothesis generally provides an excellent excuse to save market efficiency. In the meantime, the Efficient Market Hypothesis is widely accepted as a dominant paradigm in the area of financial economics.

However, some recent literature in the area [for example, Shiller (1981a), LeRoy and Porter (1981)] shows a seemingly strong argument against the Efficient Market Hypothesis by using a variance bound or volatility test, which is a very different approach from regression testing. These studies conclude that the movements in stock price
indexes are too volatile to be consistent with efficient markets. They assert that the volatility test provides more significant empirical evidence about the efficiency hypothesis than does the regression method. In fact, the variance bound test addressed by these studies appears to have more power in the statistical sense than do the conventional methods. Unlike the latter, the former takes advantage of information existing outside of the sample as well as within the sample. In particular, Shiller (1981b) criticizes the conventional tests as being inadequate in that these tests might make it too difficult to reject the null hypothesis of market efficiency, even if the market actually were inefficient. More importantly, the dividend or earnings valuation model has theoretical and empirical advantages, as a maintained hypothesis, over the two-parameter asset pricing model.

It is true that a few pieces of anomalous evidence regarding specific examples of possible inefficiency [for example, Ball (1978), Watts (1978)] were reported before the recent literature of the volatility test. However, these findings are not necessarily cause for as much concern as the anomalies found in the variance bound test. This is because these earlier results were usually obtained from regression methods based on the two-parameter capital asset pricing model, which may bring about the same problem as in cases of positive evidence of market efficiency. In other words, those anomalies would not escape Shiller's criticism in terms of data misalignment, either. Furthermore, authors reporting evidence casting doubt on market efficiency tend to
attribute those anomalies to misspecification of the asset pricing model used as a joint hypothesis, rather than to market inefficiency.

After the anomalous results from the variance bound test were reported, various forms of criticism were made on the test and its statistical procedures in order to save the Efficient Market Hypothesis. Some authors [for instance, Grossman and Shiller (1981), LeRoy and La Civita (1981), Michener (1982), Hansen and Singleton (1983)] argue that a great portion of the excess volatility of stock prices could be explained by significant fluctuations in future discount rates. Other authors [for example, Meese and Singleton (1982), Marsh and Merton (1983)] try to make the variance bound test itself invalid and meaningless, by showing that the generating processes of stock prices and dividends are nonstationary. In addition to criticisms on the two important assumptions usually made in empirical studies, inappropriateness of statistical procedures such as small sample bias [for instance, Flavin (1982), Kleidon (1983)] is likely to distort statistical results obtained from the volatility test.

However, each criticism above is just a theoretical argument and does not provide empirical evidence on how much of the reported "excess volatility" would be explained by each criticism. Thus, it seems to be an insufficient answer to the questions raised by the findings of the variance test. Besides addressing the three criticisms, this paper will bring out additional discussions which may combine to reinstate the hypothesis of market efficiency in financial economics. Here the basic idea to pose the testable hypothesis is almost the same as that of
Shiller (1981a). However, at the stage of empirical examination, careful adjustments should be prepared in the model specification and statistical procedures to minimize various significant problems in Shiller's methodology which will be explained in detail later. A good starting point may well be the nature of the joint hypothesis, to which almost all empirical investigations in this area are subject. In other words, the empirical violation of variance bounds required by the Efficient Market Hypothesis may be attributed to inadequacy of the model tested and/or inappropriateness of statistical procedure performed rather than to market inefficiency per se.

First of all, the valuation model employed by Shiller (1981a) as a maintained hypothesis may have to be modified in view of the concept of planning horizon. This consideration of the planning horizon gives a theoretical justification to the liquidation price specified in the modified valuation model. The price is a natural way to relieve a significant problem stemming from lack of infinite series of dividend data. In accordance with the rational expectations theory assumed, the liquidation price takes into account expectations of events subsequent to the specific planning horizon. More to the point, the modified model provides a well-defined measure of variance estimate, because of the identical structure of discount factors used in computing perfect foresight stock prices. In the case of Shiller, the heteroscedasticity problem over computed ex-post stock prices makes his measure of volatility unreliable for the variance bound test.
In addition, the modified valuation model may present a reasonable and effective solution to the problem of management control over dividend payments which is a main point of Marsh and Merton (1983). As the modified model theoretically implies, impacts of management control over dividend payouts may be shifted to and absorbed in the statistical behavior of the liquidation stock prices. In this sense, the estimation of ex-post rational stock prices would be left almost free of supposed disturbance by management. This would mean that the modified model is satisfactory, despite the assertion of Marsh and Merton, to represent ex-post rational stock price under the assumption of market efficiency, regardless of whether management control over dividend payments exists.

Next, as a procedural problem, the Standard and Poor's Composite Stock Price Index and its associated dividends or earnings data, which were usually employed in earlier literature, are not an acceptable sample for the empirical study. With these data, the essential matching relation underlying the valuation model is not clearly satisfied. Further, adjustment of the data to account for growth seems inappropriate because a significant portion of the growth in the market index may reflect the growth of the economy size itself rather than investment or production performed by the market as an entity at a given time. Also, the assumption that the real discount rate is constant through time is not reasonable for the empirical study because of the stochastic nature of real rates.
With the valuation model modified and the procedural problems corrected, empirical tests of market efficiency are carefully constructed. The statistical results of these tests indicate that the comparison of volatilities between the ex-ante and ex-post rational stock price series is sensitive to the length of planning horizon. Unlike the strong rejection of the Efficient Market Hypothesis reported in LeRoy and Porter (1981) and Shiller (1981a), the hypothesis is successfully supported for certain ranges of investment period. In other words, for those horizons, the volatility of the ex-post price series is greater than that of the ex-ante price series. This inequality is derived under the assumption of market efficiency.

However, the statistical results obtained on the basis of the modified model fail to give a consistent conclusion for the hypothesis, even though some of the results are clearly contrary to Shiller's assertion and suggest a "great hope" to save numerous theories developed on the basis of the Efficient Market Hypothesis. Before forming a conclusion that the degree of market efficiency may depend on the length of the planning horizon, it is necessary to examine a functional specification which explains dividend payments in terms of economy-wide random variables beyond managerial control, such as inflation rates or gross national product. At present, signaling behaviors of executive management seem to provide the strongest support for this specification.

In the following chapter, several important properties underlying the valuation models are discussed in relation to the variance bound
test. Chapter III presents testable hypotheses derived and statistical procedures used in the earlier literature. In Chapter IV, problems disregarded in the earlier empirical studies are demonstrated in view of the fundamental ideas of joint hypothesis testing. Chapter V introduces the concept of planning horizon to adjust the original form of the valuation models and then explains its theoretical and empirical effects on the volatility test. In Chapter VI, new null hypotheses on the market efficiency are formulated and tested on actual market data of stock price and dividend series. The final chapter presents conclusion and implications.
As Miller and Modigliani (1961) and Fama and Miller (1972) theoretically derive and prove, it is possible to formulate the valuation of a firm or an investment project in a number of equivalent ways. In the variance bound test, two kinds of valuation models are generally employed as an important maintained hypothesis. One is from the dividend (outflow) stream approach and the other from the earnings (inflow) stream approach. Both models treat price as the present value of an infinite series of such future cash flows. According to the accounting identity relation between cash inflow and outflow, the two models should give the same price to a particular asset. That is, an equivalent conclusion about the Efficient Market Hypothesis would be expected from empirical evidence obtained in either way.

In the previous tests of the market efficiency, various versions of the two-parameter capital asset pricing model were used as a maintained hypothesis. By using estimated values of parameters in the asset pricing model, excess returns (prediction errors) are calculated. Then the behavioral patterns of the excess returns are examined. However, as Roll (1977) argues, the validity of the capital asset
pricing model should be seriously questioned. Among other problems, the lack of knowledge about the true composition of the market portfolio is considered as a critical weakness of the two-parameter pricing model. Further, Brenner (1977) and Summers (1982) seem to be doubtful of empirical evidence obtained from employing the two-parameter valuation model, due to possible effects of model misspecification on tests of market efficiency and the unknown power of these tests arising from the inability to form a specific alternative hypothesis of market inefficiency. Also, Ball (1978) and Watts (1978) consider misspecification of the two-parameter valuation equations as a main cause of their empirical results, which lead to rejecting the efficiency hypothesis at a conventional significance level.

In this sense, the inflow or outflow valuation models used in the variance bound tests appear to be a good trial and give a fresh implication about market efficiency. In contrast to the two-parameter asset pricing model, the validity of the dividend or earnings valuation model is well established and the ideas underlying the models are widely accepted. Furthermore, testable hypotheses induced from the latter seem to be simple and easily understandable relative to those from the former. Also, unlike the conventional methods, no specific statistical distribution except the property of stationarity is necessarily required for empirical studies with this valuation model. The only critical trouble for empirical research with the inflow or outflow valuation equation is the unavailability of an infinite series of earnings or dividends data. Since several types of testable
hypotheses about the rationality of stock prices are nicely developed
and presented in Shiller (1981a, 1981b) and LeRoy and Porter (1981),
the key to obtaining appropriate empirical evidence about the market
efficiency by implementing the variance bound test is how to minimize
problems stemming from the lack of an infinite series of the data.

However, in Fama's (1970) context, the inflow and outflow
valuation models appear to posit weak-form (constant) return models, in
that there exists no significant variable other than lagged values in
the model specifications. Unlike the case of the two-parameter capital
asset pricing model, the semi-strong form and strong-form of market
efficiency may not be tested by using the dividend or earnings
valuation model. In other words, empirical evidence about only the weak
form of market efficiency can be obtained in the context of the
variance bound test. Nevertheless, the variance bound tests are
implemented with ex-ante and ex-post stock price data which provides
stronger empirical evidence about the weak form of market efficiency
than stock return data.

Several hints of misunderstanding or making little of the
fundamental ideas underlying the dividends or earnings valuation models
are apparent in the empirical procedures taken in both the Shiller and
the LeRoy and Porter papers. In addition to increasing doubt on the
validity of the two-parameter pricing model, these several hints become
another good motivation to present and discuss the valuation models in
one independent chapter here. First of all, in the case of perfect
certainty, the dividend stream approach expresses the firm value as a function of dividend and discount rate time-series:

\[
V(t) = \sum_{s=t}^{\infty} \frac{D(s)}{\Pi_{k=t}[1+r(k)]}
\]

where \(V(t)\) is a firm value at date \(t\) and \(r(k)\) is a one period discount rate in period \(k\). \(D(s)\) is a portion of total dividend payments in period \(s\) (known at date \(t\)) accruing to shareholders as of date \(t\). Here the dividend payments mean the sum of any kind of cash distribution inclusive of regular dividends during the period. Furthermore, it is very important to note that only the subset of the total payments is reasonable in assessing the asset value. The importance of this point becomes clearer in Equation (2.2).

The outflow valuation equation can be also written on a per share basis:

\[
p(t) = \sum_{s=t}^{\infty} \frac{d(s)}{\Pi_{k=t}[1+r(k)]}
\]

where \(p(t) = V(t)/n(t)\), \(d(s) = D(s)/n(t)\), and \(n(t)\) is the number of shares outstanding at date \(t\). This equation implies that an asset's price is a long moving average of its dividend series with discount factors as weights.

Similarly, the earnings stream approach results in a function of earnings and investments to express the firm value:
(2.3) \[ V(t) = \sum_{s=t}^{\infty} \frac{R(s)-I(s)}{\prod_{k=t}^{s}[1+r(k)]} \]

where \( R(s) \) is total earnings in period \( s \) and \( I(s) \) is investment undertaken in period \( s \). \( R(s) \) and \( I(s) \) are known as of date \( t \). The total earnings represent the sum of any kind of operating income. Unlike the dividend valuation equation, it is not necessarily a subset of the total earnings accruing to shareholders as of date \( t \). The numerator in (2.3) is the net amount after investments \([R(s)-I(s)]\) to avoid the double counting of the earnings. As in the case of the outflow stream approach, this earnings valuation model can also be written on a per share basis:

(2.4) \[ p(t) = \sum_{s=t}^{\infty} \frac{[R(s)-I(s)]/n(t)}{\prod_{k=t}^{s}[1+r(k)]} \]

One important point to note in (2.4) is that the correct earnings variable is not \([R(s)/n(s)]\) but \([R(s)/n(t)]\), as in (2.2). That is, on a per share basis, the earnings per share figure \([R(s)/n(s)]\) available in usual data bases like the Compustat Tapes is not relevant in (2.4) to compute \( p(t) \), while the dividend per share figure available in the usual data bases is relevant in (2.2) to compute \( p(t) \).

All variables are real in the above four equations. When an economic entity is assumed to grow at a constant rate, the growth should be accounted for by adjusting the discount factor in the denominator of the valuation model. Besides, the most critical relation
to keep the valuation models valid is matching between the value of an entity and its cash flows. In other words, an asset's price is the discounted value of dividends or earnings, which should have a definite relation to the asset as outward distribution or operating income. These simple but fundamental ideas underlying the valuation models provide significant implications to empirical designs of the variance bound test.

As Shiller (1981a) also implies, treatment of the discount rate \( r(k) \) is of interest and importance in both forms of the valuation model. Theoretically speaking, this should be a one-period holding rate of return for each period; namely, for period \( s \), the rate is defined as \( [p(s+1)-p(s)+d(s)]/p(s) \). The numerator in this definition signifies return from a transaction which buys a stock at the beginning of the period \( s \) and sells it at the beginning of the period \( s+1 \). This return consists of capital gain \( [p(s+1)-p(s)] \) and dividend income \( [d(s)] \). In the case of perfect certainty, the holding rate of the return should be identical, at market equilibrium, for all kinds of investment opportunities existing in the capital markets for a given period. Hence, the going interest rate equals the holding rate of return. In turn, the discount rate is the same as the market interest rate and accounts for the time value of money alone.

Until two decades ago, the Fisher effect had been the most dominant theory for the time-series behavior of the (expected) real rate of interest; namely, the real rate remains the same in a rational capital market over time because the nominal rate of interest would
fully adjust for the (expected) rate of inflation. In arguing against the Fisher effect, however, several papers [for instance, Mundell (1963), Tobin (1965), Fama (1981)] indicate that the real rate varies over time. In principle, there seems to be no good and necessary reason for the rate to be constant through time. Recently, it has become generally accepted that the real rate is stochastic over time.

More importantly, it must be kept in mind that the alternate valuation approaches are equivalent in pricing a firm as a capital asset in the market. This is based on the accounting identity relation; that is, the firm value measured in terms of cash flows into the firm should be the same as that measured in terms of cash outflows from the firm to its shareholders. Otherwise, arbitrage transactions take place to reach market equilibrium and restore the equivalence. Thus, neither approach is more correct in computing V(t) or p(t). On the other hand, to get the equivalence, there needs to be no specific relation between the patterns of cash flows \([(R(s)-I(s))/n(t)]\) and \([d(s)]\) through time. In other words, the equivalence between the dividend and earnings valuation models makes sense only in terms of the present value concept, not necessarily in terms of cash flow patterns.

In the case of uncertainty, R(s), I(s), D(s), and r(k) in the above equations can not be known as of date t; that is, future earnings, investments, dividend payments, and interest rates are stochastically varying through time. Besides, the discount rate accounts for the time value of money (pure interest rate) and uncertainty of future cash flows (risk premium) alike. One more point
to note here is that the discount rate usually employed in actual empirical studies does not generally take into consideration the rate of time preference whether under certainty or uncertainty. The rate of time preference is generally and implicitly assumed, in both cases, to be constant over time. This assumption indicates that the utility of one unit of the numeraire remains the same over time. Furthermore, the rationally expected holding rate of the return net of the risk premium should be identical, at market equilibrium, across investments for a given period by arbitrage transactions, and should not necessarily remain the same through time. In other words, because of the existence of the risk premium to compensate for future uncertainty, unlike the case of perfect certainty, the discount rate does not need to equal the market risk-free rate of return.

As long as rational expectations hold in the markets, unbiased expected values of the random variables above are available at data t. Therefore, V(t) or p(t) in case of uncertainty could be expressed in terms of the rational expectation of the random variables conditional on the information set available to market participants as of date t, \( \Phi(t) \). Even though the external forms of the present value equation in the case of uncertainty are similar to those in case of certainty, the underlying implications are quite different between the two cases. For instance, the former case allows forecasting errors while the latter case does not. The rational valuation equations corresponding to (2.1) through (2.4) are:
where $E$ denotes the mathematical expectation operator. $\phi(t)$ is random but its distribution is assumed to remain identical and independent through time. According to Michener (1982) and Kleidon (1983), such equations as (2.5) through (2.8) are consistent with the general equilibrium. A set of discount rates in accordance with the general equilibrium could be obtained under the additional assumption of complete markets. However, the set is not unique.

Even in the case of uncertainty, the equivalence relation between the two approaches still holds. If $\{\tilde{p}_d(t)\}$ and $\{\tilde{p}_e(t)\}$ are defined as the price sequences given by (2.6) and (2.8) respectively, then the two price sequences are identical stochastic processes for all $t$. Investors evaluate the firm or a share of the firm by discounting future cash flows expected only on the basis of the currently available information set $[\phi(t)]$, which is unique at the moment. As long as the two
approaches take advantage of the same information to form expectations, the present values such as \( V(t) \) and \( p(t) \) must be identical between them.

Again, as in the case of perfect certainty, to gain the equivalence above, \([\ddot{d}(s)]\) does not have to be the same as \([\ddot{R}(s) - \ddot{I}(s)]/n(t)\) for any period \( t \). Only the present value relation theoretically matters in the two versions of the valuation model. Nevertheless, in an empirical sense, the possibility of differences in patterns of dividends and earnings flows may provide a major implication for the tests of the rationality of stock prices on the data from a finite sample period. If the infinite series of the dividends and earnings data are available in empirical studies, the possibility posits no problem, because either variable contains the identical information for stock prices. However, all empirical tests have made use of a finite series of the data rather than an infinite one. Whenever the two variables show a different pattern of cash flows in the finite sample period, the equivalence of the two price series \( \{\ddot{p}_d(t)\} \) and \( \{\ddot{p}_e(t)\} \) does not hold any longer for the sample period. Thus, in the case where one relies solely on either a finite dividend or earnings series to make an empirical judgment about the market efficiency, the judgment would not be complete.

In the real world, there exists management control over dividend payments. Lintner (1956) suggests that management tends to create a smooth short run dividend series which does not necessarily reflect exactly the earning power or true value of the firm. In contrast,
management can not exert much control over earnings, which are
determined mainly by uncontrollable factors like economy-wide
variables. In this sense, the earnings series may be a better proxy for
the information set \( \phi(t) \) than the dividend series. Since an infinite
series of data is impossible to obtain for empirical studies, one
appears to get more appropriate results from using finite earnings data
rather than finite dividend data.

However, economic earnings and investment can not be observed in
the markets. Even though accounting earnings are currently available as
a substitute, these seem unlikely to be very reliable because they can
be easily manipulated by clever accounting practices, whether legally
or illegally. What is worse, generally accepted accounting principles
(GAAP) allow several legitimate accounting methods for one economic
event or transaction. So the earnings data are also imperfect in an
empirical sense, in that they appear to be only a crude way to convey
useful information to capital markets.

In consequence, the less discretionary net cash flow data \( \bar{R}(s) - \bar{I}(s) \)
are not directly observed and must be estimated, thus allowing
another source of uncertainty and errors, while the more discretionary
dividend data are observable and need not be estimated. This is an
explicit dilemma within the variance bound tests of the rationality of
stock prices. In this situation, the unobservability and the accounting
manipulation in relation to the inflow stream approach seems to be
worse than the problem of management control in relation to the outflow
stream approach. Perhaps the most desirable way at the moment to solve
the dilemma may be a signaling model which relates dividend payments to random and observable variables which are not controlled by the management. These would include such things as national income and the inflation rate. This model may solve the discretion problem in the dividend valuation equation, and still use finite series of data. However, the functional specification of the signaling relation is not simple to figure out in real capital markets. In addition to the signaling model, a proper estimation method for the stochastic discount rate series in the valuation models should be developed to make statistical results more appropriate, which would allow one to discard the usual simplifying assumption of a constant discount rate that is made in most empirical research.
CHAPTER III
EARLIER VARIANCE BOUND TESTS
AND
SUBSEQUENT CRITICISMS

The literature to date on the variance bound test is based on either the dividend stream model or the earnings stream model, as explained in the previous chapter. Earlier papers such as Shiller (1981a, 1981b) and LeRoy and Porter (1981) test the Efficient Market Hypothesis with the valuation models, under the assumption of constant discount rate where \( r(k) \) remains the same through time. It is reported that the sample variance of the time-series stock price index violates the bounds implied by the efficiency hypothesis. They argue that the variance bound test using these valuation models is likely to have stronger power of test in the empirical study of market rationality than has the conventional regression approach with the two-parameter asset pricing model. Also, the former seems to be less sensitive to data misalignment than the latter. However, the above papers are not confident enough to conclude that stock prices are irrationally determined in the capital market. A plausible clue is suggested by these authors themselves in their attempts to reinstate the market
efficiency which has been generally supported in empirical work before them; namely, the discount rates in the real world may vary over time greatly enough to explain virtually all excess volatility in stock prices.

LeRoy and Porter (1981) test the volatility restrictions with the earnings valuation model. Because the net cash inflow is not observable directly, they adjust the earnings valuation model (2.8) to minimize the problem stemming from this unobservability. By introducing a new variable $k(t)$, the future investment variable is removed from (2.8), while the problem of double counting generated by earnings retention continues to be accounted for. Unlike the case of Shiller (1981a), LeRoy and Porter pay attention to aggregation bias caused by using the market index data. For this reason, they extend the volatility test to three individual firms as well. Very interestingly, the statistical results turn out to be different in the cases of the market index and two individual stocks. This difference in empirical evidence among the sample selections is a good motivation, in view of aggregation bias and matching relation, for the portfolio approach employed in this paper.

Shiller (1981a, 1981b) develops the variance bound test on the basis of the dividend stream valuation model. In his tests of market efficiency, the most important assumptions are constancy of discount rates through time and stationarity of stock prices and dividend time series. Under these assumptions and following the notations in Chapter II, the basic model as a maintained hypothesis is:
where \( r \) is an assumed constant discount rate for a sample period. In Equation (3.1), \( p(t) \) is an ex-ante rational stock price, since it is the present value of (rationally) expected distributions. On the other hand, an ex-post rational stock price \( p^*(t) \) is defined as the value which the stock price would take if investors in the markets had perfect and complete information concerning the future path of the price movement. Therefore, the equation for this price does not need an expectation operator. The perfect foresight price is expressed in the following way:

\[
(3.2) \quad p^*(t) = \sum_{s=t}^{\infty} \frac{\bar{d}(s)}{(1+r)^{s-t}}
\]

As the two equations above imply, if the dividend and price series are formed rationally and the discount rate is correctly estimated, then the constructed ex-post price series expected at date \( t \) should be identical to the ex-ante price series. In a world of certainty, this is self-evident. Under the Efficient Market Hypothesis, the meaningful relation between the ex-ante and ex-post rational stock prices follows immediately from (3.1) and (3.2):

\[
(3.3) \quad p(t) = \mathbb{E}[p^*(t) \mid \phi(t)]
\]
As indicated in (3.3), the ex-ante rational stock price is the expectation of the perfect foresight stock price conditional on the current information set \( \Phi(t) \). As mentioned in the previous chapter, information variables contained in \( \Phi(t) \) are allowed to change stochastically over time. However, it is assumed here that those distributions remain identical and independent through time. This is why \( \Phi(t) \) is used in the valuation equation in stead of \( \Phi(s) \). The conditional expectation implies:

\[
(3.4) \quad \tilde{p}^*(t) = E[p^*(t) | \Phi(t)] + \tilde{\epsilon}(t)
\]

\[
= p(t) + \tilde{\epsilon}(t)
\]

where \( \tilde{\epsilon}(\cdot) \) is forecast error. This is an independently and identically distributed random variable and is orthogonal to \( p(t) \), as long as the ex-ante stock price is rationally determined to exploit fully the information contents in \( \Phi(t) \). From a principle of elementary statistics, the variance of the sum of uncorrelated variables is sum of their variances. Then, positive semidefiniteness of a variance measure gives the following testable inequality relation:

\[
(3.5) \quad \text{Var}[p^*(t)] \geq \text{Var}[p(t)]
\]

This is a simple but clear restriction on the relationship between the population moments of the two kinds of stock price series. In variance
bound tests with the market data in a sample period, comparing a point estimate of the upper bound with a point estimate of the variance being bounded provides empirical evidence regarding the market efficiency hypothesis. If their distributions are stationary, the population variance of the price series can be consistently estimated from a sample period of realization of the process over time. In cases where the price series follow a nonstationary process, critical statistics such as variance measure can not be defined for the volatility test.

Shiller (1981b) derives some similar inequalities that put limits on the standard deviation of the innovation in price or the standard deviation of the change in dividend payments. Depending on the assumption of stationarity, Shiller (1981b) derives the second testable inequality equation:

\[(3.6) \sigma[\Delta d(t)/\sqrt{2r}] \geq \sigma[\Delta p(t)]\]

where $\Delta$ is a first difference operator and $\sigma(\cdot)$ is standard deviation. This relation is obtained basically from the orthogonality property between the stock prices and a forecast error which is defined as $p(t+1)+d(t)-(1+r)p(t)$ or $\Delta p(t+1)+d(t)-r\cdot p(t)$. Again, the orthogonality comes from the rationality of stock prices.

Unlike the above two testable inequalities, the third one is derived without the stationarity assumption about the time series of stock prices and dividends. It requires only that $\Delta d(t)$ and $\Delta p(t)$ are jointly stationary with information variables contained in $\phi(t)$. This
is a weaker assumption than the stationarity of the price and dividend
series themselves. The market efficiency restricts the relation between
standard deviations of innovations in stock price and its corresponding
dividend payments in the following way:

\[(3.7) \frac{o[\Delta d(t)]}{\sqrt{2\pi(1+2\sigma^2)}} \leq o[\Delta p(t)]\]

This is obtained basically from the orthogonality property between the
forecast error \([\Delta p(t+1)-(r*p(t)-d(t))]\) and an information variable
which is defined as \([r*p(t)-d(t)]\). In turn, the orthogonality stems
from market efficiency. Testing (3.7) may give empirical evidence of
efficiency relatively free from criticism on the stationarity
assumption.

Shiller (1981a) tests the above hypotheses with two data sets; one
is the Standard and Poor's Composite Stock Price Index and its
associated dividend series for a period of 109 years from 1871 through
1979, and the other is a modified Dow Jones Industrial Average and its
associated dividend series for a period of 52 years from 1928 through
1979. He adjusts the data with Bureau of Labor Statistics Wholesale
Price Index to account for inflation during the sample periods. Then he
assumes an exponential growth in the stock price series; this rate is
also assumed to be constant over his sample periods. After estimating
the growth rate of the stock price series, he applies this estimated
rate to both stock price and dividend data as a detrending procedure.
With these inflation-adjusted detrended data, he computes the perfect-foresight stock price series, using Equation (3.2) altered with an arbitrarily set terminal condition:

\[ p^*(t) = \sum_{s=t}^{T} \frac{d(s)}{(1+r)^{s-t}} + \frac{p^*(T)}{(1+r)^{T-t}} \]

where \( t \) goes annually from 1871 to 1979. The terminal price \( p^*(T) \) in the above equation is devised, without any theoretical implication, to cope with empirical problems arising from lack of infinite series of the data. While Equation (3.2) is theoretically correct without the terminal condition, the statistical procedures with the altered equation (3.8) result in a crucial problem. Since the discount factor applied to the terminal price varies depending on \( t \), it is very difficult to define a variance measure of the perfect foresight price series, even when the stationarity assumption holds completely. Furthermore, the terminal price used in Shiller is \( p^*(1979) \) set at the detrended real stock price averaged over the sample period. So this averaging technique will not work satisfactorily to serve the original purpose of the introduction of the terminal condition. These problems will be discussed in detail in Chapter IV. Then \( p^*(t) \) is calculated recursively from the terminal date. This kind of backward determination of ex-post stock prices could be concisely expressed in the equation (3.8).
Following the data adjustments discussed above, Shiller (1981a) performs empirical tests of the variance bounds (3.5) and (3.6). For both hypotheses, he reports flagrant violation of the inequality relation imposed by the market efficiency. That is, in accordance with his empirical evidence, the stock market is not rational in the price formation. However, as Shiller (1981a) also notes, the two samples vary over time in weights and composition of firms. This variation seems to be a critical contradiction to one of the fundamental ideas underlying the valuation model; the most important requirement for the validity of the model is strict correspondence of entity and its cash distribution to investors.

The validity of the variance bound tests presented in the previous two papers depends on the assumed stationarity of stock prices and dividends (or earnings) time series. Also, both papers base the tests of market efficiency on the constant discount rate versions of the valuation model as a maintained hypothesis. The inflow and outflow stream approaches report the same empirical evidence that movements in stock price indexes are too volatile to accord with the Efficient Market Hypothesis. This anomalous evidence is explicitly in contradiction to the greater part of statistical results from other type of empirical work which bases the test of market efficiency on the two-parameter capital asset pricing model by way of regression methods. Importantly, formal significance tests of the excess volatility are not carried out in either of the two papers. This is true of the empirical regression-based tests as well.
Shiller (1981a, 1981b) and LeRoy and Porter (1981) claim that the variance bound test has greater robustness to data misalignment and stronger power of test over alternatives such as the conventional regression methods. Their empirical studies seem to provide a good reason to believe that the market prices fail to exploit all information contained in \( \phi(t) \). If their arguments are true, there could exist some lucrative trading rule to take advantage of market inefficiency. The reported 'flagrant violation' of the restrictions implied by the hypothesis hints that minor disincentives such as transaction commission and taxes may not make this trading rule impracticable. Nevertheless, how to formulate the trading rule is still open to question.

Most empirical work related to market efficiency is a joint test of a particular valuation model and market efficiency. The property of the joint tests indicates that the reported excess volatility may be caused by problems in empirical test designs and their implementation. Understandably, after the anomalous results from the variance bound test were reported, various forms of criticism have been made on the test and its statistical procedures in order to save the Efficient Market Hypothesis, which has been a dominant paradigm for a long time. One group [for instance, Grossman and Shiller (1981), LeRoy and La Civita (1981), Michener (1982), Hansen and Singleton (1983)] argues that the assumption of constant discount rate in the test is the primary cause of the rejection of the hypothesis. On the other hand, another group [for instance, Flavin (1982), Kleidon (1983)] insists
that a severe small-sample bias in the test brings about inappropriate rejection of market efficiency. A third group [for example, Meese and Singleton (1982), Marsh and Merton (1983)] suggests that the assumption of stationarity in stock price and dividend (or earnings) time series makes the variance bound test itself invalid and meaningless. However, each group seems unsuccessful in accounting for the reported "excess volatility" in a sufficient way.

First of all, Grossman and Shiller (1981) examine a possible impact of changing discount rates on empirical evidence from the variance bound test. They insist that the adequate discount rate applied to future dividends should be the marginal rate of substitution between consumption today and consumption in the future. Then it is empirically shown in the paper that as the coefficient of relative risk aversion becomes larger, the behavioral pattern of the ex-post stock prices gets closer to that of the ex-ante ones. Similarly, LeRoy and La Civita (1981) argue that the degree of risk aversion may affect the stock price variability by way of consumption variability. Further, they call into question the property of general equilibrium in the valuation model with a constant discount rate. However, the discussion concerning the constancy assumption addressed in the papers appears to be too theoretical. In other words, they just prove the existence of some bias in statistics related to the variance bound test. The empirical effect on the tests of the existing bias and the significance of the effect are not presented specifically.
Flavin (1982) shows a possible effect on the empirical result of small-sample bias. The volatility measured by taking deviations of the price series from sample means tend to be downward biased estimates of those computed from the population means. According to her, the excess volatility reported in the literature may be attributed to the statistical properties of the volatility measures in small samples, rather than to a failure of market efficiency. This bias comes from the fact that the asymptotic distributions of the test statistics may not be close approximations to the finite sample distributions. The question is whether the magnitude of the downward bias is large enough to provide a potential explanation of the apparent violation of the variance bounds. The direction of the bias seems to be correct, as Flavin argues. Nevertheless, since the true population means are not known, the magnitude of the bias can not be found and as a result the bias effect on the empirical results of the variance bound tests can not be measured. As in the case of the constancy assumption, the discussion concerning small sample bias seems to be merely theoretical.

Finally, Marsh and Merton (1983) cast strong doubt on the empirical result of the variance bound tests. In particular, they choose Shiller (1981a, 1981b) for the discussion. They suggest that the flagrant violation of the variance bounds required by market efficiency, as reported by Shiller, is attributed to a misspecification of the dividend process [the earnings process in case of LeRoy and Porter (1981)] rather than to market inefficiency per se. In Marsh and Merton, a new testable hypothesis is formulated based on the "stylized
facts" presented in Lintner (1956). They base the test of the hypothesis on the regression model. Because they provide interesting empirical evidence for management control over dividend payments with a nonstationarity assumption, their arguments deserve more detailed discussion than do the previous cases.

As a convincing rationale against steady-state distributions, Marsh and Merton single out the actual existence of management control over dividend payments, which is one of the stylized facts of Lintner (1956). Their empirical work begins with assuming a geometric Brownian motion process for stock prices and dividends. This process is a continuous-time version of the random walk process. Either version is nonstationary over time, whether under the framework of continuous time or discrete time. The nonstationarity assumption does not allow a critical statistic like variance to be defined and so makes the variance bound test invalid and meaningless.

It is concluded in Marsh and Merton (1983) that the excess volatility of the stock prices reported in Shiller can be reinterpreted in favor of market efficiency as long as executive management relates the decision on dividend payout to a "permanent component" of earnings or other cash inflow variables in a stable fashion. This management control brings about the smoothness and stability of the ex-post rational stock price series $p(t)$ whose variance becomes a bound implied by market efficiency, as shown in Shiller (1981a, 1981b). Thus, according to Marsh and Merton, the direction of the inequality relation (3.5) should be reversed. However, their reinterpretation of excess
variance in favor of market efficiency may be considerably limited. It seems unreasonable that they take direct advantage of the excess volatility calculated by Shiller because they find primary fault with assumptions of exogeneity and stationarity which play a critical role in Shiller's computing of the excess volatility itself. Any discussion of the idea of variance is contradictory to their basic assumption of nonstationarity. What is worse, the variance calculated in Shiller can not be statistically justified as an appropriate measure for the volatility test. This will be discussed in detail in the next chapter.

Marsh and Merton (1983) fail to explain the empirical result of LeRoy and Porter (1981) where earnings data is used instead of dividend data. The earnings variable is almost free of management control. As explained in the previous chapter, the equivalence between the inflow and outflow valuation models should be taken into careful consideration. Besides, in the equations used in Marsh and Merton (1983) to show empirical evidence for existence of management control, several kinds of errors in variables are found; for instance, accounting earnings data, estimated (long-run) target ratio, and estimated (long-run) dividend-yield are employed in their estimation process. 7 These accounting data and estimated variables are known to be very inaccurate, so the statistical significance of their empirical results is quite questionable. Also, their equations indicate that changes in dividend payments are caused by changes in stock prices. This kind of causality relation is sharply inconsistent with a well-established theory in financial economics. Their empirical evidence
looks like a matter of statistical illusion, because their regression model says nothing about causality relation between dependent variable and regressors.

In spite of their strong arguments, the problem of management control may not be very serious in the variance bound test with the modified valuation model; that is, it seems to be just a matter of empirical design. Even though the discretionary control is possibly significant in a finite sample period, the consequent trouble will disappear either if an infinite series of dividend data is available or if problems coming from lack of an infinite series data are practically minimized. Nevertheless, it appears very likely that Marsh and Merton provide a convincing rationale to save market efficiency, taking into account the plausible existence of management control over dividend payments in the empirical analysis of market efficiency.

Perhaps the nonstationarity arguments pointed out by Meese and Singleton (1982) and Marsh and Merton (1983) may be a critical factor to make the variance bound test senseless. This is because the variance is explosive under this circumstance and can not be defined. However, as Kleidon (1983) implies, the nonstationarity alone seems unlikely to eliminate dividends or earnings data from a set of significant information variables for explaining changes in stock prices. Long (1981) and Lintner (1981) also support the validity of the variance bound tests by considering the assumption of stationarity as another subsidiary hypothesis. Furthermore, either argument of varying discount rate and small sample bias is unlikely to be sufficient to invalidate
the variance bound test and its empirical evidence. As a result, the discussions above for opposing the variance bound test seem unsuccessful to make little of the test and its empirical evidence about the Efficient Market Hypothesis. Understandably, no empirical work is free from econometric and/or data limitations.
Most empirical tests are actually joint tests of the economic model (primary hypothesis) and the statistical model (subsidiary or maintained hypothesis). For this reason, the significance of the apparent anomaly reported in Shiller (1981a) and LeRoy and Porter (1981) is questionable. Prior to the variance bound test, the dividend or earnings valuation model had not been used as a maintained hypothesis for testing the market efficiency. In an attempt to introduce the new method, the papers commit several mistakes in relation to empirical studies. These mistakes result from misunderstanding of the valuation model and/or ignorance of fundamental ideas underlying the model. These adverse factors may lead to improper subsidiary hypotheses for the purpose of simplification. Still, most of the procedural problem comes from lack of infinite series of the dividends or earnings data.

In light of the fundamental properties of the valuation models discussed in chapter II, the market index data (which are frequently used in the variance bound tests) are not suitable to the tests and are misleading the interpretation of the resultant empirical evidence. One of the basic ideas underlying the valuation models is the exact
matching between an entity and its dividends or earnings flow. In the literature of the volatility test, the theorems and testable hypotheses are derived and proved for individual firms. Without this matching relation, the validity of the valuation models and the testable hypotheses based on the models becomes seriously questionable. When the Standard and Poor aggregate data are applied to the valuation model as in LeRoy and Porter (1981) and Shiller (1981a), the matching with the market and its associated aggregate dividends or earnings data becomes very doubtful. As well observed in the real world, the composition of the market and the weight of the member firms change very frequently through time. An index price at a point in time may have little relation to that in another point in time. The same is true for the associated dividends or earnings data. So empirical evidence obtained from these time-series data tends to be misleading.

More importantly, an infinite series of data is not available for empirical studies. When a finite sample period and the market index are selected for the variance bound tests, possibility for deficiency of the matching relation appears to be much higher. For example, new firms entering the market are reasonably expected to reinvest their entire net income for a period before starting to pay dividends. However, the entries of these new firms affect the price index of the market for the non-distribution period by way of a weighting system, while the dividends associated with the price index remain unchanged at least up to the cutoff point in time for the period. It is very unclear whether the undistributed dividends are completely reflected in the
prices during the sample period. Perhaps the price at the end of the sample period (whose length should be reasonable in terms of investors' ability for rational expectation) may capture future information about pricing forces of the remaining portion of the undistributed dividends. Under the imperfect matching relation, this kind of problem becomes more complicated and more serious. What is worse, with the terminal condition used in Shiller (1981a), a limited chance to lessen this problem through the reaction between price and undistributed dividends tends to disappear; in other words, the terminal price would not necessarily contain intrinsic information for the period outside the sample. In fact, rather than employing all firms in the capital market, the papers use the Standard and Poor's market index. Even though incoming firms substituting for ones leaving the index are not newly-established as in the case of new entry firms above, the effects on the empirical results from the variance bound tests could be quite comparable between the two cases. This is because member firms of the market index are generally considered to be mature relative to nonmember firms.

Another problem caused by using the market index data is aggregation bias in relevant statistics. In the aggregate, significant information for individual firms tend to wash out each other in the market. In particular, this happens among firms in different industries. That is, dividend data associated with the market index is unlikely to represent (unexpected) changes in the information set in a desirable way. The aggregate bias may be especially serious here, since
the test is about the Efficient Market Hypothesis. This problem motivates LeRoy and Porter (1981) to apply the variance bound test to individual firms as well as the Standard and Poor market index. As a good support for the above arguments about the various empirical weaknesses arising from employing the market index data, LeRoy and Porter (1981) demonstrate that the market and three individual firms give different statistical results for the market efficiency. They question the validity of testing the hypotheses with cross-section averages or market index data.

Shiller (1981a) introduces a terminal condition to relieve the unavoidable problem of lack of an infinite data series. With this introduction to the original form of the dividend valuation equation, he tries to compute the ex-post rational stock price time-series data for the purpose of the comparison with its ex-ante counterpart. That is,

$$p^*(t) = \sum_{s=t}^{T} \frac{d(s)}{(1+r)^{s-t}} + \frac{p(T)}{(1+r)^{T-t}}$$

According to him, \( r \) in (3.8) is the average dividend yield, in real detrended terms, over the sample period of 109 years, and the terminal price \( p^*(1979) \) is the real detrended stock price averaged over the sample period. However, this kind of terminal condition brings about several critical problems in relevant statistics for the volatility
test. For expositional purpose, computational examples for five perfect foresight stock prices are taken in the following way:

\[
\begin{align*}
\hat{p} (1875) &= \sum_{s=1875}^{1979} \frac{d(s)}{\alpha^{s-1875}} + \frac{\hat{p} (1979)}{\alpha^{104}} \\
\hat{p} (1900) &= \sum_{s=1900}^{1979} \frac{d(s)}{\alpha^{s-1900}} + \frac{\hat{p} (1979)}{\alpha^{79}} \\
\hat{p} (1925) &= \sum_{s=1925}^{1979} \frac{d(s)}{\alpha^{s-1925}} + \frac{\hat{p} (1979)}{\alpha^{54}} \\
\hat{p} (1950) &= \sum_{s=1950}^{1979} \frac{d(s)}{\alpha^{s-1950}} + \frac{\hat{p} (1979)}{\alpha^{29}} \\
\hat{p} (1975) &= \sum_{s=1975}^{1979} \frac{d(s)}{\alpha^{s-1975}} + \frac{\hat{p} (1979)}{\alpha^{4}}
\end{align*}
\]

where \( \alpha \) is \( 1+r \). As the examples in (4.1) illustrate, the ex-post price at each year is computed from the equation which has apparently different structure of discount factors. First of all, the number of the discount terms and dividend payments is not the same over the equations; for instance, the price at 1875 has 105 annual dividend incomes and discount terms. In contrast, the price at 1975 has 5 annual dividend incomes and discount terms. Each calculated ex-post price contains a different amount of information. In addition to the different number of dividend terms in (4.1), the discount factor for
the uniformly-set terminal price is also unequal over the equations; for example, the price at 1900 has the exponent 79 in the second term, while the price at 1950 has the exponent 29.

Because of the different structure of discount factors, each perfect foresight price computed following Equation (3.8) may not have an identical distribution; in other words, each and every price which belongs to the set of the calculated time-series prices for the sample period has its own statistical distribution. When the distributions of dividend and price data are stationary over time, it is obvious that each and every variance measure of the time-series perfect foresight price data computed in Shiller's way is unlikely to be identical, even though possibly independent. This may become a good explanation for the difference in statistical behavior. Therefore, the variance measure of \{p(t)\} in Shiller is beguiling and misleading.

In addition to the poorly-defined variance measure, the terminal condition in Equation (3.8) will not work to serve the original purpose of its introduction, which is to minimize the problem coming from lack of relevant infinite series data. Theoretically speaking, the stock price at the end of the sample period should completely reflect, under the assumption of market efficiency, all information existing outside the sample. In effect, market participants' ability to form an unbiased expectation is understandable for a reasonable length of time. However, the average of real detrended ex-ante rational stock prices has no good reason to represent the outside-sample information in a sufficient fashion. Even when the actual ex-ante index price at the end of 1979 is
used for the terminal condition rather than the average, the situation may not get any better, either. This is because his sample period of 109 years is too long for investors to form rational expectation. Even though the rational expectation is allowed over the long period, each computed ex-post price turns out to have different amount of forecasting error. This kind of trouble might induce Shiller to prefer the average to the actual year-end price.

The terminal condition employed in Shiller (1981a) may fail to account for the problem of management control over dividend payments presented in Marsh and Merton (1983). As mentioned in Chapter III, the plausible existence of this management control seems to be important for empirical studies with finite sample data, even though it is just a matter of experimental design. Usually, nondistributed dividends affect stock price. By way of reaction between asset price and retained dividends, the problem of the management discretion may be alleviated in a way. However, the Shiller's average price which is fixed and set uniformly for all ex-post prices allows no way to make the reaction possible. There is no mechanism in the equation (3.8) to absorb the impact of the plausible management control; that is, once some dividends are not distributed, these go out of the sample information forever. His terminal condition tends not to work in a desirable fashion to obtain appropriate empirical evidence.

Annual data used in Shiller (1981a) may fail to capture most of the unexpected change over the year in the information set in an efficient fashion, because of periods as long as one year. If several
events happen to an capital asset at different points in time within one year, there is a high possibility that their information effects wash out each other as in case of aggregation bias. Furthermore, the annual data do not take into account the time value of money between a dividend paid in January and one in December. These problems associated with the annual data seem likely to cause a kind of measurement error in the dividend variable in the valuation model.

Actual time-series data must be corrected with respect to inflation, growth, and time value of money before its application to statistical tests. First, Shiller (1981a) adjusts time-series data of dividends and stock prices with respect to growth and inflation. Then he discounts the detrended real time-series data by the dividend yield averaged over the sample period (d/p in real and detrended terms). As for inflation, it would cause little trouble. However, at the stage of detrending data, the method seems unlikely to be acceptable. Shiller supposes that the generating process of the stock price follows a constant exponential growth through time. He detrends the observed time-series data of stock price and its associated dividends, after estimating this growth rate from the stock price data. As Figure 1 shows, the Standard & Poor market index appears to demonstrate a constant exponential growth. However, this kind of growth is far from the original concept of growth implicit in the dividend valuation model. Precisely speaking, the growth should be increase in asset value caused by performance of investment and production. But the rapid growth shown in Figure 1 does not imply this sort of performance alone.
It also seems to reflect simple and natural growth in economy size itself possibly caused by growth in population. So there is a good chance that the true concept of market growth may not follow the exponential path. As a strong support for this argument, Figure 2 indicates no exponential growth in the price series of a portfolio of 91 firms. Figure 11 plots the wealth series of another portfolio, and implies the same thing as Figure 2. Neither of them shows any specific type of growth path. As a result, the growth path of dividends or asset value appears to vary depending on specific samples.\footnote{11}

Even though the growth path in Shiller (1981a) makes sense, his detrending procedure is not in line with the theory as long as the growth rate is assumed to be constant through time. The valuation model employed as a maintained hypothesis for the empirical tests is usually expressed in terms of real variables, with assumptions of constancy of discount and growth rates like the following:

\[
p = \sum_{t=1}^{\infty} \frac{d(t)}{(r-g)t}
\]

As shown in the equation (4.2), the constant growth rate should be accounted for in the denominator rather than in the numerator of the model. In this sense, his method for detrending data is likely to distort true information contained in the raw data. Moreover, his 109-year period seems to be unreasonably long to make a constancy assumption. Consequently, the problems arising from his growth
adjustment may even reduce the questionable usefulness of discount rate (the average real detrended dividend yield) and of terminal condition (the average real detrended index price) in computing the perfect foresight rational stock prices.

After reporting the anomalous evidence about the market efficiency, Shiller (1981a, 1981b) and LeRoy and Porter (1981) suggest that the assumption of constant discount rate could explain a good part of the reported excess volatility. At first, subsequent papers such as Grossman and Shiller (1981), LeRoy and La Civita (1981), and Michener (1982) focus on this assumption to reinstate the Efficient Market Hypothesis in the area of financial economics. The papers theoretically demonstrate that investors' risk aversion and concave utility cause the discount factor to change through time. On the other hand, Mundell (1963) and Tobin (1965) take the Keynesian macroeconomic position to show why the long-standing Fisher effect is not correct; in other words, the inflation rate and the real rate vary in opposite directions by way of changes in real money balances and real demand for bonds. Finally, Fama (1981) argues that the real rate varies directly with the anticipated level of real activity; that is, it is procyclical while the expected rate of inflation is countercyclical. In the meantime, the real rate is generally considered to change through time. However, how to figure out the stochastic path of the real rate is still open to question at the moment.

In addition, Shiller (1981a) discounts both dividend payments and terminal stock price by using the common discount rate (d/p) which is
expressed in terms of real detrended values averaged over his sample period. However, when dividend payments are controlled by management, as Marsh and Merton (1983) strongly argue, the (rationally expected) riskiness of the dividend process is no longer comparable to that of stock price series or net cash flow including the terminal condition in the equation (3.8). Thus, theoretically speaking, discount rates should be different between the two categories to get better estimates of ex-post rational stock prices $P^*(t)$. Unfortunately, a quantitative theory to figure out the difference between the two discount rates has not been formulated yet. Perhaps the difference may be worked out according to the true functional equation which actually governs management control over dividend payout. Nevertheless, unless the controlled dividend process maintains a simple linear relation to one or more observable economic variables, its true statistical nature may not be clearly identified. It seems most likely that the process follows a complicated nonlinear function of some random variables (say, earnings or investments). It would be a good idea to approach and try to solve this problem in a signaling context.
The criticisms of the earlier version of the variance bound tests such as nonstationarity, small sample bias, and nonconstant discount rate are summarized in Chapter III, as possible factors to mislead the empirical results of Shiller (1981a) and LeRoy and Porter (1981) to strong rejection of the Efficient Market Hypothesis. However, it seems unlikely that each of these criticisms independently give an clear and complete explanation to the excess volatility reported in the papers. Also, additional problems discussed in Chapter IV are not accounted for in the criticisms. Hence, each of the criticisms may just become a partial clue to reinstate the hypothesis as a dominant paradigm in financial economics. Perhaps the combined effects on the tests of the criticisms may successfully save the market efficiency. But a formal significance test of these combined effects is not practically available.

This chapter focuses on how to minimize empirical disturbance caused by the problems discussed in Chapter IV. The introduction of the concept of planning horizon seems likely to minimize the disturbance and improve the relevance and significance of the empirical
examinations of the volatility test. Shiller (1981a) and LeRoy and Porter (1981) do not take into consideration the possible effects on investment decisions of the idea of planning horizon. These papers seem to overlook the importance of the idea in the variance bound test as a possible factor influencing the empirical results. Unlike the case of conventional tests of market efficiency based on two-parameter asset pricing models, the concept of planning horizon is most likely to play an important role in the volatility tests with a finite sample of data. In particular, this concept provides a nice solution to the critical problems of the poorly-defined variance measure and the inappropriate terminal condition.

The two-parameter capital asset pricing model is the market-equilibrium version of the two-parameter portfolio theory. Markowitz (1959) and Tobin (1958) derive this theory on the basis of investors' optimizing behavior. Unlike the capital asset pricing model, the derivation of the dividend or earnings valuation model is not based on the optimizing behavior of market participants. Perhaps this optimizing behavior may not be a significant factor for the valuation model, if infinite series of data are available in perfect and efficient capital markets. However, real markets are not perfect and only a finite series of data is available for empirical studies. Thus, the optimizing behavior should be accounted for in the valuation model with a finite data series. Without this adjustment, appropriate conclusions for the hypothesis are unlikely to be expected from empirical studies. One of
possible methods to account for this is the introduction of the planning horizon to the dividend valuation model.\textsuperscript{14}

Theoretically speaking, individuals make investment decisions so that the discounted value of expected utility of net cash flows (or consumption) from investments over the horizon is maximized. However, since the expected utility and the true path of discount rates are not known in a quantitative way, mathematical derivation of the optimal horizon is not easy to obtain. At most, it could be said that there exists a finite optimal planning horizon over which expected utility is maximized. In this sense, the introduction of the planning horizon is an ad-hoc approach to take into account the optimizing behaviors observed in the markets. Nevertheless, in addition to market imperfectness and lack of infinite series of data, there exist several more plausible arguments to support the existence of a finite planning horizon in the real capital markets. The horizon may be limited by scheduled demand for cash and change in the investment opportunity set. One more convincing reason for a finite period of investment is tax benefit from capital gains. This finite planning horizon requires market participants, at the time of investment, to make rational expectations about liquidation prices of the assets purchased. Hence, the ad-hoc approach tends to improve the previous empirical studies of the variance bound test, by providing a type of asset pricing model consistent with optimizing behavior as well as with rational expectations.
First of all, change in the investment opportunity set through time could be a good motivation to liquidate the assets at some point in the future. When, very possibly, economic circumstances change over time, current investment may turn out to be suboptimal, even though it was optimal at the time of purchase. The current assets must be sold for reinvestment to recover an optimal position. This may happen even where rational expectation holds in a nice way, since forecasting error is always possible. Also, forecasting error is a function of time. More difficulty in long-range expectation about price and dividend processes in the far future may become another rationale for a finite investment period. So it is quite unlikely, especially under the actual imperfect capital markets, that planning horizons of market participants are infinite as indicated in the original forms of valuation models.

Next, scheduled demands for cash could be easily and frequently observed in the real world. When an amount of cash is bound to be spent at some time in the future, an asset which was purchased as investment for a period from fund availability to final consumption must be liquidated to meet the compulsory demands. In this case, the utility of one unit of numeraire is not identical through time. Thus, timing of cash flows becomes relevant in determining the level of expected utility in an empirical sense, while the original forms of the present value relations in the previous chapters do not make the most of the intertemporal pattern of cash flows in a theoretical sense. A nonconstant rate of time preference induced by the scheduled
consumption appears to make the planning horizon matter in the process of investment decision.

Finally, investors desire to realize capital gains for tax benefits. According to the current tax system in the United States, sixty percent of a "realized" capital gain is exempt from taxable income in the year when the capital gains are realized. In other words, as long as capital assets are not sold, the attractive tax benefits do not materialize. This is a reasonable incentive to liquidate capital assets at the end of a finite planning horizon. Consequently, at the time of investment, market participants try to form rational expectations about the future distribution of the liquidation price.

To get a better maintained hypothesis for the empirical examinations of the variance bound test, the original forms of the dividend stream valuation model such as (3.1) and (3.2) may have to be modified according to the concept of planning horizon as in the following:

\( P(t) = \sum_{s=t}^{t+h} \frac{E[\tilde{d}(s) \phi(t+h)]}{(1+r)^{s-t}} + \frac{P(t+h)}{(1+r)^h} \)

\( P(t+h) = \sum_{s=t+h}^{\infty} \frac{E[\tilde{d}(s) \phi(t+h)]}{(1+r)^{s-t-h}} \)

\( \hat{P}(t) = \sum_{s=t}^{t+h} \frac{\tilde{d}(s)}{(1+r)^{s-t}} + \frac{P(t+h)}{(1+r)^h} \)
As shown in the modification, the model is divided into two terms; one stands for dividend income and the other represents capital gains. \( P(t) \) and \( P^*(t) \) in Equations (5.1) and (5.2) are ex-ante and ex-post stock prices respectively, after the concept of planning horizon is accounted for. \( P(t) \) and \( P^*(t) \) in (5.1a) and (5.1b) represent the same as \( p(t) \) and \( p^*(t) \) in (3.1) and (3.2), except for the fact that the beginning date is changed to \( t+h \) from \( t \). In the above equations, the length of planning horizon for each of \( P(t) \) and \( P^*(t) \) is \( h \). This length moves successively over time for each date \( t \) so that the size of \( h \) (investment period) is uniform for all \( P(t)'s \) and \( P^*(t)'s \) in the sample. The modified models appear more appropriate for the variance bound test, as long as market prices used in empirical works are actually determined by investment decisions based on the idea of planning horizon.

Once capital assets are liquidated at the end of the investment period, the proceeds from the sale consist of dividend income and capital gains. In view of the previous arguments for the existence of the finite planning horizon, the validity of the outflow stream version of the valuation model seems most likely to be confined to a group of investors who expect only dividend income over their entire life and do not necessarily want capital gains income. Hence, the equations such as (2.5) and (2.6) may not hold, at least in the real world, for another

\[
(5.2a) \quad \tilde{P}(t+h) = \int_{s=t+h}^{s=\infty} \frac{d(s)}{(1+r)^{s-t-h}}
\]
group of investors who expect both types of income by liquidating the investments after the supposed planning horizons lapse. Once more, (2.5) and (2.6) becomes less reasonable for a third group of investors who expect only capital gains income possibly because of a tax burden from dividends income. There may exist all the three groups in real capital markets. However, the first group has the least possibility of existing in the real markets, while both the second and third groups have high chances of being seen.

In most cases, investors are reasonably expected to realize their wealth not only in the form of dividends, but also in the form of capital gains. That is, the second group appears to be most popular in the capital markets. The modified models stemming from the acknowledgement of importance of planning horizon for investment decisions is suitable especially for the pricing mechanism for this group of investors. Besides, in the limit, the modified models account well for investment behaviors of the first and the third groups. When h approaches infinity, P(\infty) in the second term of the modified equations is near zero in value. In this case, (5.1) and (5.2) are reduced to (3.1) and (3.2) respectively. On the other hand, for the third group, the first terms have only to be eliminated from (5.1) and (5.2). This means either that dividend income is reinvested as a whole, or that there is no dividend income during the investment period. Therefore, the modified models such as (5.1) and (5.2) seem very successful, at least in empirical sense, to complete the validity of the dividend stream valuation model. That is, unlike the equations such as (2.5) and
(2.6), the modified models most likely hold for any kind of group of investors.

Without this kind of modification in the dividend valuation models, several critical problems tend to distort empirical examinations of the volatility test. Shiller (1981a) introduces the "terminal condition" to compute the perfect foresight prices with a finite sample of dividend data. Interestingly, he applies the identical terminal condition to calculate all $P^*(t)$'s. This idea implies that individuals are expected to liquidate their investments at the identical time in the future as specified by the terminal condition, no matter when they purchased capital assets. So the resultant planning horizons turn out to be unreasonably long for some investors and very short for others; namely, the planning horizon varies over the investments in a single empirical study. For example, investor A buys common stock AAA in 1871 and sells it in 1979; another investor B buys common stock BBB in 1978 and also sells it in 1979. The investment period for the investor A happens to be as long as 109 years, while that for the investor B happens to be as relatively short as 1 year. This varying investment period results in non-uniform statistical distribution of ex-post rational stock price.

In addition, Shiller's method for computing perfect foresight stock prices leads to a case of partial sum, because of his failure to take the idea of the planning horizon into consideration. The situation of the partial sum aggravates dependency or overlap of observations in computing $P^*(t)$'s with the finite sample of the dividends data. It also
suggests that the degree of the dependency varies over individual $P^*(t)$'s as $t$ changes. In other words, for all cases where $t_1$ is greater than $t_2$, the set of observations used in computing $P^*(t_1)$ is a perfect subset of those used in computing $P^*(t_2)$. Further, the number of observations for computing an ex-post rational stock price decreases, as $t$ approaches the terminal year 1979. For example, the observations used in calculating $P^*(1975)$ become a part of those used in calculating $P^*(1925)$. The number of observations for the former is 4, while that for the latter is 54.

Consequently, as discussed in Chapter IV, the resultant varying investment periods imply that each perfect foresight price is computed as a total of dividends weighted with a 'different set' of discount factors as shown in the example equations (4.1). This difference indicates that each ex-post rational stock price has its own statistical distribution which is different from that of other prices in the sample. In other words, variance measure as a representative parameter of the distribution change over perfect foresight prices. Thus, the variance of the $\{p^*(t)\}$ series computed in accord with Shiller (1981a) is improperly defined for the variance bound test. Moreover, his computation procedure results in the case of partial sum. This partial sum constitutes dependency of informant observations (dividend data) in the sample. As $t$ approaches the terminal date (1979), the variance measure of the ex-post price tends to decrease because the quantity of information decreases. Also, overlap of observations means loss of independent information and this loss in
turn reduces variability. Hence, the variance of the perfect foresight price series may be downward biased, because each and every stock price shares a part of common information about subsequent changes in dividend payments.

The original purpose of the terminal condition is to relieve inevitable problems arising from lack of an infinite series of data, by designing the terminal condition to include necessary information existing beyond the sample period under the assumption of market efficiency. Shiller (1981a) defines the terminal condition to be the real detrended Standard and Poor price index averaged over his sample period from 1871 to 1979. However, this definition is unlikely to serve the supposed purpose. There is no apparent and good reason for the average to contain the information for the after-sample period. His inappropriate detrending procedure with the market index data as explained in Chapter IV makes the real detrended average much less convincing. Understandably, there seems to exist no way to set the actual 1979-end price index as a terminal condition under the situation of the resultant varying planning horizon and the partial sum relation among the computed ex-post stock prices. For example, it would be quite unreasonable to make rational expectation about the 1979-end liquidation price at the end of the year 1871. In addition, the terminal condition fixed for calculating all perfect foresight prices allows no mechanism to alleviate empirical trouble caused by the plausible management control advocated in Marsh and Merton (1983).
In contrast, the modified models such as (5.1) and (5.2) provide valuable advantages, especially in empirical sense, over the approach taken in Shiller (1981a). That is, the problems discussed in the above are solved with introducing the idea of planning horizon. First of all, perfect foresight prices computed on the basis of such modified models as (5.2) and (5.2a) have identical distributions. In turn, these identical distributions provide a well-defined variance measure of the price time-series. For example, in case that h is 7 years:

\[
P^*(t) = \sum_{s=t}^{t+h} \frac{d(s)}{(1+r)^{s-t}} + \frac{P(t+h)}{(1+r)^{h}}
\]

\[
P^*(1900) = \sum_{s=1900}^{1907} \frac{d(s)}{(1+r)^{s-1900}} + \frac{P(1907)}{(1+r)^{7}}
\]

\[
P^*(1910) = \sum_{s=1910}^{1917} \frac{d(s)}{(1+r)^{s-1910}} + \frac{P(1917)}{(1+r)^{7}}
\]

\[
P^*(1920) = \sum_{s=1920}^{1927} \frac{d(s)}{(1+r)^{s-1920}} + \frac{P(1927)}{(1+r)^{7}}
\]

When a sample period goes from 1900 through 1979, t in the first equation of (5.3) moves annually from 1900 to 1972 (=1979-h). So, seventy three ex-post stock prices are calculated with this length of planning horizon. The same number of observations is used to obtain each price of them; that is, each of the three examples in (5.3) has 8 observations which are seven dividend payments and one liquidation
price. Also, given a length of planning horizon [in the example (5.3), h=7], a set of discount factors is same for all members of the \( P'(t) \) series. As a result, variance measure of the ex-post prices from the modified models seems to be well defined and have little problem for the volatility test, unlike the case of Shiller (1981a).

Further, the modified models do not cause the partial sum relation over the computed perfect foresight prices. Even though some overlap of observations can not be avoided due to a limited amount of available data, the models do not allow a complete subset in observations for computing the prices. With a given length of planning horizon and resultant different liquidation prices, each and every ex-post price contains a certain amount of independent information about subsequent changes in both dividend payments and liquidation price. The amount of the independent information is uniform, given a specific planning horizon, for all perfect foresight stock prices. Hence, the estimate of variance measure of the \( P'(t) \) series is still unbiased and consistent, even though the information used for calculating each price is not perfectly independent of one another.

As emphasized in the previous chapters, most of the trouble with empirical studies of the variance bound test comes from the lack of an infinite series of data. The consideration of planning horizon provides a theoretical and technical support for liquidation prices to relieve a significant problem stemming from finite sample data. An infinite series of data is not necessarily required in the modified models, unless an infinite planning horizon is assumed. In such models as (5.1)
and (5.2), the terminal condition in Shiller is naturally replaced with a liquidation price which may represent currently unavailable information about the capital asset beyond sample period. This liquidation price is allowed to be different depending on investment assets given a planning horizon and depending on lengths of planning horizon given an investment asset. Thus, unlike the case of the fixed averaged terminal condition, the liquidation price turns out to be successful to contain, under the null hypothesis of market efficiency, necessary information existing beyond the sample period.

One more advantage obtained from the modified models is a partial solution to the problems caused by the plausible existence of management control over dividend payments. When management control results in too little or too much dividend distribution to make smoothness, the difference between the actual and normal dividend payments is going to bring about additional variation in the corresponding asset price. In other words, the liquidation prices in the modified models are most likely to absorb empirical distortion produced by the management control. However, this kind of self-adjustment mechanism can not be expected from the terminal condition which is the fixed real detrended stock price averaged over the sample period of Shiller (1981a).

The modification of the valuation models successfully accounts for effects on both ex-ante and ex-post price series of the concept of planning horizon, without changing the basic ideas and fundamental properties underlying the valuation models employed in the earlier
empirical examinations of the variance bound test. Therefore, this modification does not have to change the inequality relationship between variances of the ex-ante and ex-post rational stock price series. The relationship is determined by market efficiency and (3.5) remains intact as a testable hypothesis. In fact, the variance of the \( \{P(t)\} \) series in (5.1) is the same as the variance of the \( \{p(t)\} \) series which would result if the concept of planning horizon were not accounted for. The reason for this equality of the variances is that a null hypothesis like (3.5) is formulated under the assumption that the Efficient Market Hypothesis holds. On the other hand, the variance of the \( \{P'(t)\} \) series in (5.2) is quite different from that of the \( \{p'(t)\} \) series in (3.8) in both numeric and semantic terms. This difference may reverse the empirical results reported in Shiller (1981a).
Shiller (1981a) found a standard deviation of stock prices six to twelve times its theoretical upper limit. This result implies that stock prices are too volatile to be consistent with the Efficient Market Hypothesis. However, misspecification of the valuation model and/or inappropriate empirical design in the previous test may be principal factors that caused his empirical studies to reject the joint hypothesis of market efficiency and the validity of the valuation model employed. In this respect, Shiller also concludes either that expectations are not rational in the stock market, or that discount rates fluctuate greatly over time. In this chapter, efforts are made to correct for procedural and modeling problems discussed in Chapter IV and Chapter V. Then a new version of the variance bound test is designed for empirical studies.

The previous chapter explained the several plausible justifications for the introduction of the concept of planning horizon to the dividend valuation model. However, as long as capital markets are perfect and efficient, the concept of planning horizon may not matter in the valuation model with an infinite series of dividend data. At the moment, it is not clear, with real markets that are imperfect,
how much the concept matters, in both an empirical and a theoretical sense, to the valuation model with finite series of dividend data. Besides, it is not certain how much the problems brought about by management control are solved in the modified models with a finite series of dividend data. Thus, three alternative null hypotheses are considered for a single statistical test.

First of all, the modified models in Chapter V could be considered to hold as a correct valuation relation only for relevant planning horizons observed in capital markets. In other words, individuals with these planning horizons base their investment decisions on the modified models. For investors with other horizons, the models may not function as an appropriate pricing relation. A null hypothesis for this case can be summarized as follows:

Null Hypothesis (NH1): For some relevant planning horizons, the modified models are correctly specified for the valuation process. The variance measures computed from these models meet the inequality relation if the stock markets are efficient.

In principle, the length of planning horizon could be continuous from zero to infinity. The modified models are able to explain the pricing mechanism for capital assets for any period of planning horizon. If a given planning horizon is relevant for a number of investors and the number of investors is large enough to influence the
pricing mechanism in the real markets, the modified models are unlikely to be a source of rejection when the joint hypothesis of the principal hypothesis of market efficiency and other subsidiary hypotheses is rejected in an empirical study.

On the contrary, there may exist some ranges of planning horizon which are not relevant for any group of investors. Similarly, even though a group of investors exists for these ranges, the size of the group may not be significant enough to affect the pricing relation in the real markets. For the ranges of planning horizon, either of the original models or the modified models may be a main source of rejection of the joint hypothesis. In this case, a decision on the principal hypothesis (market efficiency) can be reserved for another type of empirical research which uses a different set of subsidiary hypotheses.

In consequence, when the variance bound required by the Efficient Market Hypothesis is violated for some ranges of planning horizon in empirical work, it could be concluded either that the stock markets are inefficient or that the modified models for valuation are not valid for these ranges of planning horizons (or that these ranges do not exist or are not significant in the real markets). This argument seems to indicate a kind of segmentation of market efficiency in terms of planning horizon.

Next, the concept may always be correct for any period of planning horizon in a theoretical sense. But it is very difficult to form rational expectations about liquidation prices, especially in the far
future. The resultant forecasting errors are a function of time; the errors for short planning horizons are most likely to be less than those for long planning horizons. Hence, it is possible that the markets are efficient for any length of planning horizon, but that for some range of planning horizon the empirical results may show irrationality because of difficulties in forming expectations about liquidation prices. That is, a high degree of forecasting error may significantly distort the empirical study. For this situation, a null hypothesis would be set up in the following way:

Null Hypothesis (NH2): For some horizons for which rational expectations about liquidation prices are allowed with a reasonable extent of forecasting errors, the modified models are valid not only in a theoretical sense but also in an empirical sense. If the capital market is efficient, the variance measure of the ex-post price series exceeds that of the ex-ante price series.

Investors in real markets have limited ability in the formation of rational expectations about liquidation prices. As planning horizon gets longer, investors' expectations become more and more uncertain. For example, if a period of planning horizon is thirty years, reasonable accuracy of expectations about the liquidation price is very difficult to obtain. Even though the modified valuation models are
correct and the stock markets are efficient, significant inaccuracy of forecasting for this period may mislead empirical examination of the variance bound test.

So far, the effects of the concept of planning horizon have been examined under the assumption that management control over dividend payments does not exist, or under the assumption that the modified models absorb the factor of the management control into the change in liquidation prices of the modified models. However, it is plausible for the control to exist in the real world. The existence of management control over dividend payout seems to induce some difference in the expected riskiness of dividend income and liquidation price. The expected riskiness of liquidation price is likely to be higher than that of dividend income. Strictly speaking, different discount rates should be applied to the two terms in the valuation equations to account for the effects of management control. Therefore, how much of the problems induced by the control can be solved with the change of liquidation prices in the modified models is not clear, unless a specific function to explain the management's behavioral decision process is assumed.

To account for this case of management control, the previous null hypotheses have to be changed:

Null Hypothesis (NH3): When the modified models properly reflect the effects of management control on pricing relations for some relevant planning horizons
(or for some periods of horizon with allowable extent of forecasting errors), the inequality relation holds if the stock markets are efficient.

As for management control, the extent of its solution in the modified models may be different according to length of the planning horizon. For very short periods its adjustment to liquidation prices tends to be ignored, while for very long periods the adjustment tends to be made in a partial way. As a result, it can reasonably be expected that for an intermediate range of planning horizons, the adjustment process is almost completed in the modified valuation equations such as (5.1) and (5.2).

The above three null hypotheses are identically summarized in terms of the testable hypothesis (3.5). If the testable hypothesis is not rejected, the interpretation of the empirical results may be positive for all assumptions contained in the null hypotheses. Hence, the three exchangeable hypotheses combine to show that expectations are rational in the capital markets. However, if the null hypothesis (3.5) is rejected, it is difficult to identify the exact source of the rejection (as always with joint hypothesis). Also, it is not even clear which of the null hypotheses is rejected. Caution should be exercised in the interpretation of negative results for the null hypotheses.

In the empirical study of the summarized hypothesis (3.5), this paper studies two kinds of samples, focusing on the matching relation
and the survival bias. Firms in SAMPLE ONE do not go bankrupt during the sample period. This requirement implies that the matching relation could hold in this sample for the whole period. However, at the time of investment decision, individuals can not be sure of whether a firm in which they invest will disappear from the market at some point in the future. So the survival requirement might not be consistent with rational expectations. To avoid this survival bias, firms in SAMPLE TWO are allowed to go bankrupt during the sample period. The number of firms which belong to SAMPLE TWO decreases over time; for instance, the sample includes 499 firms in December, 1926, but only 91 firms remain in December, 1983. Thus, SAMPLE TWO fails to keep the matching relation under control.

As explained in Chapter V, the modified valuation models give a well-defined measure of variance statistics with identical structure of discount factors. Given a length of planning horizon, each investment in the sample period has the identical investment length. As for discount rates for the sample period, monthly Treasury Bill rates are employed rather than assuming constancy of the rates as in the earlier tests. Since the risk premium could be reasonably assumed to be invariant through time, these changing Treasury Bill rates are most likely to be appropriate for the empirical study. If the risk premium is constant over time, the Treasury Bill rate could be considered to account for pure interest (real rate), inflation premium, and risk premium. More to the point, the Treasury Bill rate successfully reflects the nonconstancy property of the real rate discussed in
Chapter II and Chapter IV. Following the changing discount rates, Equations (5.1) through (5.2a) are revised as follows:

(6.1) \[ P(t) = \sum_{s=t+1}^{t+h} \frac{E[\tilde{d}(s)|\phi(t)]}{\eta_{k=t+1}^{s}[1+r(k)]} + \frac{P(t+h)}{\eta_{k=t+1}^{t+h}[1+r(k)]} \]

(6.1a) \[ P(t+h) = \sum_{s=t+h+1}^{t+h+1} \frac{E[\tilde{d}(s)|\phi(t+h)]}{\eta_{k=t+h+1}^{s}[1+r(k)]} \]

(6.2) \[ P'(t) = \sum_{s=t+1}^{t+h} \frac{\tilde{d}(s)}{\eta_{k=t+1}^{s}[1+r(k)]} + \frac{P'(t+h)}{\eta_{k=t+1}^{t+h}[1+r(k)]} \]

(6.2a) \[ \tilde{P}(t+h) = \sum_{s=t+h+1}^{t+h+1} \frac{\tilde{d}(s)}{\eta_{k=t+h+1}^{s}[1+r(k)]} \]

SAMPLE ONE consists of 91 firms each of which has no missing value in the data of stock price from January 1926 to December 1983. These firms have been listed on the New York Stock Exchange for this entire period. Experience of missing data generally implies occurrence of infrequent and irregular events. Such a long-period continuity of time-series suggests reasonable quality data with little noise. For a finite sample data, this kind of sampling criteria has special importance, in that new entry firms usually reinvest their entire operating income without paying dividends for a while. However, it should be noted that the sample is subject to survival bias.
Shiller (1981b) postulates that prices of some firms may be affected by "fads" or psychological factors; this is regarded as an alternative to market efficiency. The stability and maturity of the firms in SAMPLE ONE may reduce the chances of including such irrationally affected firms. Furthermore, the data from SAMPLE ONE is unlikely to include outliers which would disturb statistical analyses. The existence of outliers possibly indicates the impact of "fads" such as pessimism or optimism in the market. Those "fads" may significantly contaminate data with irrationality. Of course, as long as an infinite series of data is available for empirical studies, this sort of psychological alternative may not matter in the original form of the valuation relation.

The data from the sample is more appropriate than the market index data generally used in the literature of the volatility test. In addition to less serious aggregation bias, the fundamental property (matching) of the valuation models is successfully taken into account in this sample as a portfolio. The market index and its associated dividend data would not be properly related in the standard outflow valuation models. As explained in detail in Chapter II and Chapter IV, without an unambiguous matching relation between an entity and its outward distribution, the valuation models are wrong in definition and are misleading statistical research. In contrast, the portfolio consisting of 91 firms in the sample gives a clear relation between an entity and its outward distribution to investors. With this portfolio
approach, the valuation models work in a desirable way for empirical
tests of market efficiency.

The composition and weight distribution of the market is most
likely to change through time. It could be said that the market can be
thought of as an entity, and that firms entering and going out of the
market can be considered as new investments and liquidations
respectively. However, the real markets are too complicated to consider
those entries and exits as investments and liquidations of the entity
as in the case where a perfect market is assumed. Thus, the relation
between the entity and its dividend payments is very vague in the
aggregate approach. In this sense, the sample chosen here is much
better in securing a proper valuation relation for the volatility test
than the market index used in Shiller (1981a).

SAMPLE TWO starts with a portfolio of 499 firms existing in
December, 1925. This sample is devised to correct for the plausible
survival bias permitted in SAMPLE ONE. Hence, any member firm of the
sample can go bankrupt and disappear from the portfolio. When
bankruptcy happens to some firms at a point in the sample period, the
residual wealth of the bankrupt firms is supposed to be reinvested in
the surviving firms in a value-weighted fashion. By doing this, the
number (Factor) of shares held of the surviving firms may increase over
time. That is,

\[ \text{Initial Wealth (in December, 1925)} = \sum_{i=1}^{499} P_i \]
Wealth(t) = \sum_{m=1}^{S} P_m(t) \cdot \text{Factor}(t)

\text{Factor}(t) = \text{Factor}(t-1) + \frac{\sum_{j=1}^{B} P_j(t-1) \cdot \text{Factor}(t-1)}{\sum_{k=1}^{S} P_k(t-1) \cdot \text{Factor}(t-1)}, \quad t=2,232

\text{Factor}(1) = 1

where S = \# of surviving firms at each point in time.

B = \# of bankrupt firms at each point in time.

As suggested in (6.3), the composition and weights of the portfolio are allowed to change over time. That is, SAMPLE TWO does not keep the matching relation required by the valuation model in a nice way. For instance, dividend distributions have different corresponding entities over time. Nevertheless, it would be much less serious than the case in Shiller. By way of the reinvestment, the concept of 'pseudo-entity' can be applied to the sample. Thus, the problem of heteroscedasticity in price or wealth time-series could be minimized, unlike the case of the market-index sample. Besides, new firms are not allowed to enter the 'pseudo-entity' through time. In this sense, SAMPLE TWO remains a portfolio approach like SAMPLE ONE.

Price and dividend time-series data of the portfolios are computed on a quarterly basis from the master files of the Center for Research in Securities Prices (CRSP). Stock dividends and new issues are accounted for by CRSP. The data for SAMPLE ONE are equally-weighted
averages, while the data for SAMPLE TWO are total amounts of wealth and distributions. As explained in Chapter IV, annual data used in the earlier literature may not be very appropriate to account for information effects and time value of money. If nonregular dividends were paid earlier in a year, the magnitude of the bias explained in Chapter IV may become even larger in the outflow stream valuation model. Dividend data in this paper include all kinds of cash or cash-equivalent distributions in addition to regular dividends. Strictly speaking, irregular distributions depending on unexpected earnings or profits would be discounted separately from regular dividends, since the (rationally expected) riskiness of irregular dividends tends to be much closer to that of corresponding liquidation prices rather than to that of regular dividends. However, the impact of this all-inclusive approach on resultant present values may be negligible considering the infrequency and small amount of irregular distributions.

Nominal data are examined rather than real detrended data. Shiller (1981a) adjusts the data with respect to inflation and growth. However, his detrending procedures are suspicious, as discussed in Chapter IV, while his inflation adjustment appears reasonable. As LeRoy and Porter (1981) also acknowledge, whether the detrending step reinforces the assumed stationarity is questionable. Furthermore, as Figure 2 and Figure 4 show, no specific growth pattern is noticed in the plots of the nominal price and dividend time series of the portfolio. Figure 11 and Figure 12 also demonstrate no specific growth pattern in plots of the nominal wealth and gross distribution.
As long as the Treasury Bill rate is a good substitute for the true discount rate, nominal data may be better than real detrended data. Besides, when stationarity is already assumed for the time series data, a detrending procedure is self-contradictory.

Testing nonstationarity in time-series data is still an unfamiliar term in the area of statistics. Box and Jenkins (1970) introduce an ad-hoc method to test the random-walk model. Dickey and Fuller (1979) apply a simulation to study whether a first-order autoregressive model has a unit root or not. Recently, Meese and Singleton (1982) examine spot and forward exchange rates with the method of unit-root testing. All efforts are confined to the random-walk type of nonstationarity. Even though the unit root is rejected, it is still possible that the time-series data follow other unknown types of nonstationarity. Further, the validity of the testing method developed so far is not guaranteed, because the sampling distribution of the test statistics is not known.

The importance of stationarity for the volatility test leads us to even a limited study of behavioral patterns of the sample data over time. Table 1 and Table 2 demonstrate a general description of the data. Since time-series data usually have autocorrelation, the 1st-order autoregressive model is more legitimate than the ordinary least squares model. As Table 2 shows, the values of Durbin-Watson D in price and dividend data (SAMPLE ONE) and wealth and distribution data (SAMPLE TWO) are not very distant from 2.0. Thus, coefficient estimates are similar in Table 1 and Table 2. All estimates of coefficients (β, β',...
b, and b') for lagged stock price and dividend are less than 1, which implies that the time-series data are stationary over time, even though the statistical significance of sampling error underlying the estimates is not known.

Since estimates of coefficients for lagged price and wealth are around 0.94, it is necessary to examine the statistical significance of their distance from 1.0. Table 4 shows the standard test of parameter restriction based on the ordinary least squares model. Because Figure 2 and Figure 11 demonstrate no explosiveness, a one-side test is more adequate than a two-side one. The t statistics are calculated from the data in Table 2. The test result is that the random-walk form of nonstationarity is rejected in SAMPLE ONE and SAMPLE TWO at a conventional significance level. Besides, p-values for estimates of the constant terms in Table 2 show a conventional significance in both the samples. This significance is likely to support the rejection of the random-walk process in the price or wealth series, because the estimates of the constant terms should be statistically insignificant and small in magnitude if the random walk is a true process in the time-series.

As in the above t-test, the standard central limit theorem gives a statistical justification for the likelihood ratio test. Table 3 shows the likelihood ratio test based on the first-order autoregressive model. If both the null and alternative hypotheses displayed in Table 4 are stationary over time, the respective likelihood function need not be conditioned on the initial data P(1). Because the null hypothesis
indicates the random-walk process, this nonstationarity requires the likelihood function to be conditioned on something (here the initial price). The likelihood function of the alternative hypothesis is also conditioned to be comparable to that of the null hypothesis. Hence, the likelihood is expressed as the joint distribution of $P(2)$ through $P(232)$ conditioned on $P(1)$. This likelihood test rejects, at a conventional significance level, the null hypothesis of the random-walk form of nonstationarity in SAMPLE ONE and SAMPLE TWO.

In the test of the random-walk nonstationarity, the likelihood ratio test is more legitimate than the standard F-test or t-test. First of all, the likelihood ratio test is generally known to be robust to nonstandard atmosphere. In the likelihood ratio test, both the dependent variable $[P(t)]$ and the independent variable $[P(t-1)]$ are considered to be random. However, in the regular regression model for the F-test or the t-test, the independent variable is assumed to be fixed. Thus, the regular regression model does not, strictly speaking, apply. Here the two kinds of nonstationarity test lead to the same conclusion because of the large sample size and the low autocorrelation. However, in a small sample, they may result in different conclusions. For example, the existence of autocorrelation could play a significant role in the process of testing with the small sample.

Neither type of tests accepts the random-walk behavior in the price and wealth time-series in the samples. This statistical result supports stationarity for the variance bound test. However, this
support should be interpreted in a limited way. The likelihood ratio test is conditioned on the initial price. The conventional regression test for parameter restriction may not be reliable because of the autocorrelation of the time-series data. Most importantly, this result rejects the random-walk type of nonstationarity. Thus, it is still possible that the price or wealth time-series follow a more complicated nonstationary process other than the random walk. Nevertheless, the tests for stationarity of the time-series data provide a significant if limited support for the variance bound test with the data from SAMPLE ONE and SAMPLE TWO.

The ex-post rational stock prices are calculated on the basis of the equations (6.1) through (6.2a). Variances of the ex-ante and ex-post stock price series are computed for a consecutive period of planning horizons from 1 quarter through 36 years. As Figure 10 and Figure 18 display, the impact on the anomaly is sensitive to the length of planning horizon. SAMPLE ONE has a period of planning horizons when the inequality relation implied by the market efficiency holds. This period goes from 11 years through 31 years. SAMPLE TWO has such a period of planning horizons from 63 quarters (15 years and 3 quarters) through 93 quarters (23 years and 1 quarter). In both samples, only middle range of planning horizons demonstrates that the volatility comparison is consistent with the market efficiency. In short, this paper fails to obtain a consistent empirical evidence for the Efficient Market Hypothesis over the whole range of planning horizon.
SAMPLE ONE and SAMPLE TWO provide different ranges of the planning horizon which supports the hypothesis of market efficiency. The range of planning horizons showing positive evidence is wider in SAMPLE ONE than in SAMPLE TWO. This difference may be interpreted to come from the trade-off mechanism between survival bias and matching relation. SAMPLE ONE allows survival bias, while it keeps the matching relation between an entity and its distribution. SAMPLE TWO does not allow survival bias, while it fails to keep the matching relation over the sample period. Interestingly, behavioral patterns of the standard deviations are quite similar in both the samples.

As mentioned earlier, the inconsistent evidence requires much care and prudence to draw an appropriate conclusion on market efficiency. For periods when the joint hypothesis is not rejected, it could be interpreted that the modified valuation models are properly specified for investments having these horizons, and that the market for these investments is efficient. However, for periods when the joint hypothesis is rejected, its interpretation is not clear because the exact source of rejection is difficult to identify.

First of all, in relation to NH1, for the periods demonstrating rejection, the modified valuation equations such as (5.1) and (5.2) are considered to be misspecified as an important maintained hypothesis. Alternatively, if the models function as a proper valuation relation, the capital markets are interpreted to be inefficient for these periods of planning horizon. On the other hand, NH2 implies that the capital markets are efficient even for the periods, and that expectations about
liquidation prices suffer from significant forecasting errors. That is, the expected liquidation prices for the periods seem unsuccessful to contain necessary information existing beyond the sample period.

Finally, for periods when the summarized testable hypothesis (3.5) is rejected, it could be indicated from the third null hypothesis that management is irrational for the periods while expectations are still rational in the capital markets. Alternatively, the modified models do not satisfactorily shift the impact of management control over dividend payout into variation in liquidation prices. Once more, the failure to apply different discount rates to controlled dividend payments and uncontrolled liquidation prices may be a principal source of rejection. When NH3 is related to the previous NH1 and NH2, the interpretation of the negative results gets more complicated. What is worse, additional subsidiary hypotheses such as Treasury Bill rates and nominal data are involved in the empirical examination of the variance bound test.

If a significance test for each period of the planning horizon is practically possible, the explanation of the empirical results could be clearer and more convincing. For this kind of empirical study, an F-test or a simulation approach would be thought of as appropriate methods. As long as an infinite series of dividend data is available, the F-test is the best way to obtain the value of significance for each period. In addition, with the significance value for each period, a Bonferroni test is possible to get statistical information for simultaneous significance of market efficient over the whole (or a part of) consecutive periods of the planning horizon examined. However,
since only a finite series of data is available, variance measures of ex-ante and ex-post stock price series are not independent of each other. Thus, the F statistics can not be defined in a proper way with finite series of data.

A simulation approach remains as a second best test for significance, when sampling distributions of test statistics are not known. Again, this is not desirable in an practical sense. For a simulation study, statistical distributions of stock price and dividend time-series should be specifically figured out to generate random numbers. As mentioned in Chapter II, one of the advantages of the volatility test over the conventional regression test is no need to make use of limited knowledge of the distributions. The normal distribution is generally available in the current computer packages to generate random numbers. However, as Figure 5 and Figure 13 indicate, the normality of the true shape of the price or wealth distribution is questionable. Even though it follows normality, the normal distribution is contradictory to the limited liability inherent in properties of stock price series. The limited liability indicates that the price distribution is truncated at zero.

Unfortunately, the only inference that can be made about the significance tends to be very rough. Due to limited availability of dividend data, the number of informant observations used in computing perfect foresight stock prices decreases as the supposed period of planning horizon gets longer and longer; namely, the number of the observations is 232 quarterly dividends minus the period of the
planning horizon. The decrease in useful information is most likely to reduce statistical significance of the variance bound test. The period of planning horizon would still be unknown when the resultant evidence begins to become doubtful. This statistical argument could be a convincing explanation to rejection of the Efficient Market Hypothesis for the periods of planning horizon longer than 31 years (for SAMPLE ONE) or 93 quarters (for SAMPLE TWO). Furthermore, even though forecasting errors become larger and larger as planning horizon is extended, the present value of liquidation price gets less and less important. In this case, the modified models do not work very well to account for the artificial smoothness of dividend series caused by management control. That is, the undistributed portion of earned income seems unlikely to be shifted to the liquidation price in the modified valuation models. Thus, the extent of downward bias in volatility of the ex-post price time-series becomes larger and larger as planning horizon gets longer and longer. This argument may explain the downward sloping behavior in long planning horizons of the standard deviations of the ex-post price series shown in Figure 10 and Figure 18. As a result, the rejection of market efficiency in long horizons could be caused by management smoothing of the dividend stream rather than market irrationality.

Even for periods which support the hypothesis of market efficiency, detailed information about the significance of the positive evidence is open to question as well. Similarly, for the shorter part of the planning horizon, the volatility of the ex-post price series
tends to be more affected by that of the ex-ante price series than for the longer part of the planning horizon. This statistical argument could be a good explanation for the negative evidence in planning horizon shorter than 11 years (for SAMPLE ONE) or 93 quarters (for SAMPLE TWO). That is, the market may be efficient but a distorted evidence of rejection may be acquired. In sum, these two statistical arguments above could successfully reinstate the Efficient Market Hypothesis in financial economics over the entire range of planning horizon. However, these are little more than tentative excuses for the inconsistent empirical evidence obtained in this paper.

The contribution of this paper to the literature is in finding the fact that the empirical evidence obtained from the volatility test is subject to period of planning horizon and adjustment of management control. It is suggested that the strong rejection of market efficiency reported in the earlier papers comes from inaccurate subsidiary hypotheses. The paper singles out and tries to correct for significant drawbacks disregarded in the earlier papers, and presents several successful ways to minimize the problems caused by the lack of infinite series of data in the market. Hence, the paper provides an empirical, though partial, explanation to reinstate the Efficient Market Hypothesis in financial economics.
CHAPTER VII
CONCLUSION AND IMPLICATIONS

The earlier literature of the variance bound test reports empirical evidence that the stock market is not weak-form efficient. Even though the earlier version of the variance bound test has several advantages over the conventional regression method based on the two-parameter capital asset pricing model, the empirical evidence would be suspicious in view of the fundamental properties of the joint hypothesis.

This paper points out critical weaknesses overlooked in the earlier literature. The weaknesses are related to inappropriate valuation models and/or inadequate statistical procedures. Shiller's equation for computing ex-post stock prices results in a poorly-defined measure of volatility in the ex-post price series. His terminal condition fails to minimize the problems coming from the lack of an infinite data series. This terminal condition provides no mechanism to absorb the impact of the supposed management control over dividend payments. His market data are not consistent with the matching relation required by the basic ideas underlying the valuation model. In connection with the market sample, his assumption of constant exponential growth to adjust data is not correct. His assumption that
the real discount rate is constant through time is not reasonable because of the stochastic nature of real rates.

The introduction of the planning horizon to the volatility test results in the solution, to a considerable extent, to the problems caused by the weaknesses. Among others, the concept of planning horizon provides a well-defined measure of volatility which is due to the identical structure of discount factors. The valuation model modified by the concept allows some mechanism to alleviate the problems caused by management control. Unlike Shiller's terminal condition, the liquidation price related to the concept successfully relieves the problems caused by the unavailability of an infinite data. Two kinds of samples are designed considering the matching relation and the survival bias. Nominal and quarterly data are used, since the Treasury Bill rate is available as a good substitute for the true discount rate. The likelihood ratio test rejects the random walk behavior in the price and wealth data of the samples. These improvements in statistical design combine to enhance the quality and significance of the empirical research about the Efficient Market Hypothesis.

In principle, the lack of the infinite series of data is a main source of the empirical disturbances. Efforts of this paper are oriented to how to minimize the unavoidable problems in empirical studies of the variance bound test. Three alternative null hypotheses in Chapter VI are a good demonstration of the difficulty to interpret the statistical results produced by the minimizing efforts. For this reason, great care is required for the interpretation.
The empirical evidence shown in this paper is not consistent in terms of the period of planning horizon. For some periods, the testable inequality relation implied by the Efficient Market Hypothesis is met while for other periods the relation is not satisfied. The interpretation of this inconsistent evidence tends to differ according to the three null hypotheses. Besides, statistical significance of either of the positive or negative evidence is still open to question.

Nevertheless, this paper gives an important clue to reinstate the Efficient Market Hypothesis in the areas of finance and economics. The reported sensitivity of the volatility test to the period of the planning horizon implies that the strong rejection of the hypothesis shown in Shiller (1981a) is quite doubtful and that the strong rejection could be due to improper design of the empirical study. In this sense, the paper makes a significant contribution to the literature, even though the lack of infinite series of the data does not allow consistent evidence.

If a quantitative function is specifically formulated to relate the controlled dividend payout to uncontrolled economy-wide random variables, the inconsistency of the empirical evidence could be reduced. A recent concern for a signaling model may play a desirable role in deriving the function. Also, with increased knowledge of investors' utility functions and true structure of discount factor, the variance bounce test could provide a better empirical result. These are interesting and recommendable for future research.
FOOTNOTES

1. Strictly speaking, the market efficiency is about the pricing mechanism of stock itself. Hence, the empirical result obtained by using stock prices is more meaningful than that obtained by using stock returns. The absence of abnormal stock returns does not necessarily mean that the corresponding stock prices reflect all available information in the market. In this sense, the variance bound test provides stronger evidence about the market efficiency than the conventional regression test.

2. When too high a dividend is paid, its stock price is likely to go down. When too low a dividend is paid, its stock price is likely to go up. In fact, no dividend policy can affect the fundamental value of the asset. To keep the stock price unchanged over time, under the assumption of no inflation, the dividend payment would be (both realized and unrealized) net income per share earned from new investment decisions made during the current period. Therefore, it would be true, in a finite sample period of new investment or partial liquidation, that the less variable is the dividend stream, the more volatile will be the stock price. It is because management can not change the total return. See Chapter V and Chapter VI for more discussion.
3. See Fama and Miller (1972).


5. $k(t)$ is a kind of quantity index. However, its computation causes serious trouble in view of one of the basic ideas underlying the valuation models. See Kleidon (1983) for more discussion. Perhaps this trouble eloquently demonstrates a trade-off situation between unobservability and better quality (less discretion) of the earnings data in comparison to the dividend data.

6. For further discussion about the nonconstant rates and the property of general equilibrium, see Lucas (1978), Michener (1982), and Kleidon (1983).

7. For example, if there exists a long-term dividend yield, according to Marsh and Merton (1983), actual dividends capitalized by using Treasury Bill rates should be plotted around actual prices over the sample period of 58 years. However, Figure 9 and Figure 17 do not support this existence. These figures start at the year 1942. For years from 1926 through 1941, some capitalized values are explosive and cannot be plotted. The years do not support the existence of the long-term dividend yield, either.

8. In addition, as Miller and Modigliani (1961) originally posit, for a given investment policy the current market value of the firm or its security would not be changed according to management choice of dividend payments. A number of researchers [for instance, Black and
Scholes (1974), Litzenberger and Ramaswamy (1979), Hess (1983), and Miller and Scholes (1983)] have tried to get empirical results about this Miller and Modigliani proposition. Even though they failed to obtain consistent evidence, it could be safely said that the empirical result to date is at least inconclusive for rejecting the irrelevance proposition. This argument may play an important role in providing both Shiller's (1981a) model and the modified model presented in Chapter V with a theoretical support, even in the presence of Marsh and Merton's (1983) emphasis on the existence of management control over dividend payments. Also refer to footnote 2.


10. The usual formulation for computing a variance measure is derived under the implicit assumption that each and every element follows an independent and identical distribution; that is, it should belong to a random sample. In Shiller's method for obtaining the measure, this i.i.d. property does not hold.

11. For very mature firms, even a negative growth rate is reasonably expected; for instance, without new investment generating net income, the value of an asset tends to decrease with dividend distributions from regular operating revenues.

12. For this reason, the additional hypotheses of Shiller (1981a, 1981b) such as (3.6) and (3.7) will not be examined because they are derived on the basis of the constancy assumption. They fail to attract much concern from the literature of the variance bound test, either.
13. In the conventional tests of market efficiency on the basis of regression methods, one month or one quarter is generally and implicitly assumed to be a planning horizon for investment decision. Usually an appropriate length of the planning horizon for the tests tends to be selected solely in consideration of sample size of available data. Nonetheless, whatever length happens to be selected, it is generally considered as a reasonable investment period, even though little empirical work has been done to find a widely accepted investment period in real stock markets. Refer to note 4 for empirical examples.

14. For convenience, in this chapter, discussion on the planning horizon in relation to the volatility test tends to be confined to Shiller (1981a). One possible reason for this choice, as discussed in Chapter II, is that the dividend stream model is likely to be better in an empirical sense than the one based on inflow streams. Also, the concept of the planning horizon appears less appropriate for the earnings valuation model. Like the examination of impacts on the tests of small sample bias and nonconstancy of discount rates, the discussion proceeds with assuming the stationarity of stock prices and dividends time series.

15. As mentioned in Chapter IV, this leads to a non-uniform distribution over the ex-post stock prices. In this case, the amount of information about the capital entity varies depending on the stock prices. The variability causes heteroscedasticity over the calculated
ex-post prices. So the resultant variance measure of the calculated price series is not appropriate for the empirical test.

16. Perhaps the adjustment to (liquidation) price takes place not immediately but gradually. When executive management decides not to distribute dividends, investors' interpretation of this decision may not be uniform. Some consider it as positive and others as negative. Further, depending on the size of reliability of the management behavior, investors will make up their mind on how much of retained dividends is switched to adjust the stock price. So it may not be reasonable to expect full and immediate adjustment in asset prices, even under market efficiency.

17. Generally speaking, the discount rate \[ R(t) \] consists of real rate \[ r(t) \], inflation rate \[ i(t) \], and risk premium \[ p(t) \]. The Treasury Bill rate \[ TR(t) \] is considered to include the real and inflation rates in a satisfactory way. However, it does not account for the remaining portion of the risk premium. That is, \[ R(t) = r(t) + i(t) + p(t) = TR(t) + p(t) \]. If the risk premium is constant over the sample period, it would not change the slope of the capital market line, but just move the level of the line. Therefore, the Treasury Bill rate may be sufficient as a discount rate for the purpose of the variance bound test. In particular, it allows the real rate to vary through time.

18. In fact, 93 firms meet this criterion of the continuous survival. Two of them have data in terms of Canadian dollars.

19. There exist 503 firms in December 1925. Four of them have data in terms of Canadian dollars.
20. Partial replication of the original Shiller method produces a surprising and interesting result for the variance bound test, even though this replication still permits the inappropriate measure of variance explained in Chapter IV. As shown in Table 5, the hypothesis of market efficiency is supported in both SAMPLE ONE and SAMPLE TWO; the volatility of the ex-ante rational price series is less than that of the ex-post rational price series. The table also demonstrates the specific procedure employed here and its difference from the original one. One aspect of the difference is no adjustment to the data with respect to growth and inflation. However, the exact source of the reversed result is not clear at this stage. This would be a challenging topic for further research.

21. The plots display no apparent nonstationarity. Figure 3 tends to reinforce this argument. The figure shows plots of real detrended prices of SAMPLE ONE which were adjusted following the method of Shiller (1981a). In comparison to Figure 2, there is no outstanding improvement in terms of growth and nonstationarity.

22. The values of adjusted R-Square in the dividend data of both samples are very low. This implies that the first-order autoregressive model is not properly fitted to the dividend data. Thus, stationarity is called into question, even though estimates of \( b \) and \( b' \) in Table 1 and Table 2 are much less than 1.0, which suggests the random walk property. Apart from the inadequacy of the first-order autoregressive model, the estimates much lower than 1.0 reject only the random-walk type of nonstationarity. Figure 8 and Figure 16 demonstrate that the
dividend data follow other forms of nonstationarity in both SAMPLE ONE and SAMPLE TWO. Therefore, tests of random-walk nonstationarity will be confined to price and wealth time-series data.

23. Even though the actual price or wealth data do not follow a normal distribution, the maximum likelihood estimates \( \hat{\alpha} \) and \( \hat{\beta} \) in Table 3 and Table 4 approximately follow a normal distribution due to the central limit theorem. Also, the standard large sample arguments imply that estimates of \( \hat{o}_0^2 \) and \( \hat{o}_1^2 \) are asymptotically equal to the true variance of the population, respectively. Strictly speaking, this large sample theory is based on the constancy over time of the true variance. Paradoxically, the constancy is what will be examined in the two types of tests. Therefore, in the case where trend is clear in a time-series data, a test of nonstationarity would be almost impossible. In a sense, the clear trend may eliminate even the necessity of the test. However, Figure 6 and Figure 14 suggest that the trend of price or wealth series is marginal and unclear except for some outliers. In particular, the time series data of price and wealth appear quite stationary after about 50 quarters. Hence,

\[
\frac{\hat{\alpha} - \alpha}{s} \quad \text{and} \quad \frac{\hat{\beta} - \beta}{y} = N(0,1)
\]

where \( s \) is standard error of \( \hat{\alpha} \)

\( y \) is standard error of \( \hat{\beta} \)
24. The null hypothesis demonstrates the random walk. Thus, the variance of $P(t)$ is not constant over time. This is why the likelihood should be conditioned. As Figure 6 and Figure 14 show, the degree of nonconstancy seems to be insignificant for the likelihood ratio test. However, it is not known how well the samples chosen here for the period represent the population. In other words, there is possibility that the rejection of the random walk process may be reversed for a different sample period. Also refer to footnote 23.

25. The null and alternative hypotheses would be:

\[ H_0: \text{Var}(P) = \text{Var}(P^*) \]
\[ H_1: \text{Var}(P) > \text{Var}(P^*) \]

For this, the $F$-statistics is defined as:

\[
F = \frac{\hat{\text{Var}}(P)}{\text{Var}(P^*)}
\]

Because the ex-post rational price is computed on the basis of the dividends data and the liquidation price, the variances of the ex-post and ex-ante price series are not independent. However, the most important requirement for the validity of the $F$ statistics is independence of the two variance measures.

26. In the figures, bar charts show negative skewness and high kurtosis in both the samples. Further, normal probability plots demonstrate that price or wealth series do not follow normality by forming curves rather than straight lines. As for dividends data, the degree of deviation from normality appears greater, as seen in Figure 7 and Figure 15. Normal probability plots display more cyclic curves than those in the
case of price or wealth series. Also, skewness is more serious in dividends data than that in price or wealth data. Once simulation is undertaken, the same distribution should be assumed for both the dividends and price (or wealth) time-series.
TABLE 1

General Description of Data
based on
the First-Order Autoregression Model

(1) Equations

\[ p(t) = a + \beta p(t-1) + \epsilon(t) \]
\[ d(t) = a + b d(t-1) + \omega(t) \]

(2) Statistics

SAMPLE ONE: \[ a=2.78426 \ (t=2.544, \ p=0.0119) \]
\[ \beta=0.93816 \ (t=42.32, \ p=0.0001) \]
\[ a=0.16949 \ (t=5.003, \ p=0.0001) \]
\[ b=0.68905 \ (t=12.35, \ p=0.0001) \]

SAMPLE TWO: \[ a=669.921 \ (t=2.515, \ p=0.0128) \]
\[ \beta=0.94699 \ (t=52.95, \ p=0.0001) \]
\[ a=46.2409 \ (t=4.163, \ p=0.0001) \]
\[ b=0.73987 \ (t=14.28, \ p=0.0001) \]
<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
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General Description of Data based on the Ordinary Least Squares Model

(1) Equations

\[
P(t) = \alpha' + \beta' P(t-1) + e'(t)
\]

\[
d(t) = \alpha' + b' d(t-1) + \omega'(t)
\]

(2) Statistics

**SAMPLE ONE**:

\[\alpha' = 2.66991 (t=2.497, p=0.0132)\]

\[\beta' = 0.94171 (t=45.10, p=0.0001)\]

\[F = 2034.199 (p=0.0001)\]

\[R\text{-Square} = 0.8988, \text{ADJ } R\text{-Square} = 0.8984\]

\[\text{Durbin-Watson } D = 1.977\]

1st Order Autocorrelation = 0.015

\[\alpha' = 0.16360 (t=6.024, p=0.0001)\]

\[b' = 0.69473 (t=14.68, p=0.0001)\]

\[F = 215.562 (p=0.0001)\]

\[R\text{-Square} = 0.4826, \text{ADJ } R\text{-Square} = 0.4826\]

\[\text{Durbin-Watson } D = 2.509\]

1st Order Autocorrelation = -0.251

**SAMPLE TWO**:

\[\alpha' = 533.503 (t=1.984, p=0.0484)\]

\[\beta' = 0.95820 (t=56.49, p=0.0001)\]

\[F = 3191.251 (p=0.0001)\]

\[R\text{-Square} = 0.9330, \text{ADJ } R\text{-Square} = 0.9328\]

\[\text{Durbin-Watson } D = 1.928\]

1st Order Autocorrelation = 0.042

\[\alpha' = 45.0267 (t=5.241, p=0.0001)\]

\[b' = 0.73938 (t=16.60, p=0.0001)\]

\[F = 275.720 (p=0.0001)\]

\[R\text{-Square} = 0.5463, \text{ADJ } R\text{-Square} = 0.5443\]

\[\text{Durbin-Watson } D = 2.572\]

1st Order Autocorrelation = -0.264
Table 3

Test for (Random-Walk) Nonstationarity based on the First-Order Autoregression Model

Due to the central limit theorem, the normal distribution of the price time-series data is not necessarily required for this testing.

[Likelihood Ratio Test]

\[ H_0: P(t) = \alpha_0 + P(t-1) + \epsilon(t) \]
\[ H_1: P(t) = \alpha_1 + \beta_1 \cdot P(t-1) + \epsilon_1(t) \]

Likelihood Ratio (LR) = \[ \frac{L_0[\alpha_0, 1, \epsilon_0]}{L_1[\alpha_1, \beta_1, \epsilon_1]} \]

SAMPLE ONE: \(-2 \cdot \log(LR) = 7.718\)
SAMPLE TWO: \(-2 \cdot \log(LR) = 6.033\)
CRITICAL VALUE: \(\chi^2(1, 0.05) = 3.84\)

Likelihoods and the ratio are computed on the basis of the data shown in Table 1.
TABLE 4

Test for (Random-Walk) Nonstationarity
based on
the Ordinary Least Squares Model

Due to the central limit theorem, the normal distribution of the price or wealth time-series data is not necessarily required for this testing.

[Standard test of parameter restriction]

$H_0: \beta = 1$

$H_1: \beta < 1$

Because only one restriction is tested, the standard F-test becomes equivalent to the t-test.

SAMPLE ONE: $t = (0.941713-1)/0.020880 = -2.791523$

SAMPLE TWO: $t = (0.958209-1)/0.016952 = -2.463801$

CRITICAL VALUE: $t (229, 0.05) = -1.65$

Test statistics are computed on the basis of the data shown in Table 2. The t distribution approximates the normal distribution in this large sample size. The critical value is calculated by interpolation from the table for normal distribution.
TABLE 5

Partial Replication of the Shiller's Method

When applying part of Shiller's method to the nominal and quarterly data, comparison of 'volatility measures' of the price series supports the hypothesis of market efficiency.

\[ P^*(t) = \sum_{s=t+1}^{232} \frac{d(s)}{\prod_{k=t+1}^{232}[1+r(k)]} + \frac{M}{\prod_{k=t+1}^{232}[1+r(k)]}, \quad t=0,231 \]

where \( M \) is mean of nominal and quarterly price series. \( M \) for SAMPLE ONE is 8.63. \( M \) for SAMPLE TWO is 14568.19.

<table>
<thead>
<tr>
<th></th>
<th>S.D. of P</th>
<th>S.D. of P*</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAMPLE ONE</td>
<td>16.09</td>
<td>20.87</td>
</tr>
<tr>
<td>SAMPLE TWO</td>
<td>6206.51</td>
<td>7108.03</td>
</tr>
</tbody>
</table>

Differences in methods of the original Shiller and the above:

a. No adjustment to data with respect to growth and inflation.
b. Terminal condition is set as nominal quarterly average rather than real detrended annual average.
c. Sample (market vs. portfolio) and its period (109 vs. 58 years).
d. Discount Rate: Treasury Bill rates allowed to change over time.
Total Market Value of Stocks on the New York Stock Exchange

Figure 1
Figure 2

MARCH 1926 TO DECEMBER 1983
Equal-Weighted Average Price of Stocks in SAMPLE ONE

Figure 2
Detrended Equal-Weighted Average Price of Stocks in SAMPLE ONE

Figure 3
MARCH 1926 TO DECEMBER 1983
Equal-Weighted Average Dividend Paid to
Stocks in SAMPLE ONE
Figure 4
Summary Information about Normality of the Average Price Time-Series in SAMPLE ONE

Figure 5
Residuals from the Ordinary Least Squares Model with the Average Price Time-Series in SAMPLE ONE
Figure 6
Summary Information about Normality of the Average Dividend Time-Series in SAMPLE ONE
Figure 7
Residuals from the Ordinary Least Squares Model with the Average Dividend Time-Series in SAMPLE ONE

Figure 8
The Average Dividend Time-Series in SAMPLE ONE
Capitalized by Treasury Bill Rate
Figure 9
Comparison of Volatilities of the Ex-ante and Ex-post
Average Price Time-Series in SAMPLE ONE
Figure 10
MARCH 1926 TO DECEMBER 1983

Total Wealth of Stocks in SAMPLE TWO

Figure 11
Gross Dividend paid to Stocks in SAMPLE TWO

Figure 12
Summary Information about Normality of the Total Wealth Time-Series in SAMPLE TWO

Figure 13
Residuals from the Ordinary Least Squares Model with the Total Wealth Time-Series in SAMPLE TWO
Figure 14
Summary Information about Normality of the Gross Dividend Time-Series in SAMPLE TWO

Figure 15
Residuals from the Ordinary Least Squares Model with the Gross Dividend Time-Series in SAMPLE TWO

Figure 16
The Gross Dividend Time-Series in SAMPLE TWO
Capitalized by Treasury Bill Rate
Figure 17
Comparison of Volatilities of the Ex-ante and Ex-post Total Wealth Time-Series in SAMPLE TWO

Figure 18
BIBLIOGRAPHY


