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MUSCULO-SKELETAL DYNAMICS AND MULTIPROCESSOR CONTROL OF A BIPED MODEL IN A TURNING MANEUVER

The Ohio State University

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MUSCULO-SKELETAL DYNAMICS AND MULTIPROCESSOR
CONTROL OF A BIPED MODEL IN A TURNING MANEUVER

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate School
of The Ohio State University

by

Ben-Ren Chen, B.S. Physics, M.S.E.E.

* * * * *

The Ohio State University
1985

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Department of Electrical Engineering
ACKNOWLEDGEMENTS

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Chapter 1

INTRODUCTION

1.1 Overview

In this dissertation, three essential components of an on-line strategy to control a multilink biped are developed. These components are dynamics of the musculo-skeletal system, actuators with variable stiffness, and a multiprocessor for distributed on-line control implementation. These studies may help in generating appropriate command signals for the design of prosthetic and orthotic devices for the handicapped by providing suitable inputs to muscular and neural stimulation systems.

The first topic of this dissertation is to deal with the development of methodology for the control of a multilink biped in a turning maneuver. Turning is one of the common activities of humans, but little work on human turning has been reported in the literature. The skeletal model used in this study consists of seven links in order to approximate locomotion characteristics similar to those of the lower extremities of the human body. The equations of motion of the model are established for describing the dynamic behavior of the system. The kinematic trajectories of the turning movement as functions of time are developed from photographs. For simplicity, the lower extremity muscles are replaced by force actuators. These actuators are idealized by lines joining the muscle point of origin to the point of insertion and assumed to produce tensile forces only. This system can be considered as a first order approximation to a musculoskeletal system. In order to guide the skeletal system along the desired turning trajectories and to control the ground reaction forces, a suitable control scheme is studied. The results of this study present a feasible set of musculoskeletal forces which could be utilized as a guide in electrical or neuromuscular stimulation used to help the handicapped to regain mobility.

For the implementation of efficient prosthetic and orthotic devices, it is desirable that sophisticated muscle-like force generators with controllable stiffness be interfaced with signals from the nervous system. The reason for the controllable stiffness is that humans adaptively and automatically adjust the stiffness of their muscles to improve their performance in almost all of their movements. For
example, in grasping an object and carrying it along a desired trajectory, the stiffness of the forearm muscles will be low initially such that impact forces of the hand are minimized. In this sense, the transmission of impact to the rest of the body is then eliminated. In order to carry the object along the desired trajectory, the stiffness of the forearm muscles can be increased such that the biosystem is relatively insensitive to external disturbance. Thus, the regulation of stiffness is a second topic of study. In humans, the muscle stiffness is an intrinsic property of muscle and is regulated by the central nervous system. The theoretical development of muscle stiffness is based on electron microscopic and X-ray diffraction findings of the ultra-structure of muscle fibers and the sliding filament theory. A mechanical muscle model is proposed for a planar forearm. Three main considerations for the design of such a model are as follows: first, the model should have the capability of controlling its stiffness, second, the model should be simple and easy to construct and third, the model should be applicable to a muscle-limb system with neuromuscular control. Applications of neuromuscular control of the mechanical muscle model are studied by considering a planar forearm model.

In real time control of biped robots or in providing real time computer control for the prosthetic devices, the computation of the control algorithm has to be performed on line. The next topic to be considered is the real time computer implementation of the control algorithm. The computation time of the control algorithm is a very crucial factor in real-time implementation. To achieve good stability and performance of any applied control algorithm in real time implementation, the computation of the control algorithm must be repeated very frequently. It was suggested by Luh et al. [1] that a sampling rate of no less than 60 Hz is required. Hence, a multiprocessor system is suggested for implementing the control algorithm. The fundamental advantages of a multiprocessor architecture is that separate tasks can be carried out simultaneously by several processors so that computation time can be significantly reduced. A multiprocessor-based system must integrate a set of computer models with an efficient environment. In real-time systems this environment is usually called the system executive and the application programs are referred to as tasks. The design goals for the multiprocessor executive are simplicity, flexibility, high operating speed, and small system overhead. The simplicity and flexibility can be achieved by modular design. Since the executive is only dedicated to dynamic control of multilink systems, high operating speed and small system overhead will result. The design of the multiprocessor system includes both hardware and software. The task decomposition and task scheduling of the control algorithm are also studied in order to utilized the multiprocessor system most effectively.
1.2 Organization of the Dissertation

In Chapter 2 of this dissertation, the related literature about this study is selectively reviewed. Four areas will be covered: dynamics and control strategies for multilink systems, biomechanisms of lower-limb movements, mechanical models of muscle, and multiprocessor systems. Chapter 3 deals with estimating actuator forces during turning. The kinematical motion trajectories and the dynamical equations of the skeletal model are developed. The turning motion involves a sequence of single and double stance phases which are treated as on-off constraints. These on-off constraints are considered in the dynamical system. Force actuators are implemented into the skeletal model and a nonlinear control scheme is employed. Digital computer simulations are presented to demonstrate the effectiveness of the control strategy and to estimate the actuator forces.

The stiffness property of muscle is studied in Chapter 4. The theoretical development of muscle stiffness is based on electron microscopic and X-ray diffraction findings of the ultra-structure of muscle fiber and the sliding filament theory. A corresponding mechanical model is proposed. The neuromuscular control applications are demonstrated by considering a planar forearm model.

A loosely coupled multiprocessor system dedicated to dynamic control of multilink systems is developed in Chapter 5. The multiprocessor system includes one central processor and a group of satellite processors. For simplicity and flexibility, both the system hardware and software are designed in a modular fashion. The scheduling of parallel computation among satellite processors and the central processor is also studied. A two link robotic system is used to demonstrate the efficiency of the multiprocessor system.

The results, limitations, and contributions of this dissertation are summarized in Chapter 6. Recommendations for further research and applications of this work are also presented. The dynamical equations of the seven-link biped system are derived in Appendix A. The Newton-Euler state space formulation developed by Hemami [2] is used to derive the equations of motion. Three different phases are involved in the turning motion, i.e., single stance phase with left or right foot on the ground and double stance phase. The dynamics for these three cases are studied separately. A graphics package for generating views of three-dimensional scenes is presented in Appendix B. The function of this package is to accept a viewing position and direction and then calculate the proper viewing transformations.
Chapter 2

SURVEY OF LITERATURE

2.1 Introduction

This chapter selectively reviews the previous studies which are related to this dissertation. The survey is composed of four sections. In section 2.2, dynamic formulations and control strategies for multilink systems are surveyed. Multilink systems are systems which are composed of many links such as human skeletal models, robotic manipulators, and prosthetic devices. In section 2.3 biomechanisms of human lower-limb movements are surveyed. Two aspects, i.e., human gait studies and estimation of muscular forces are considered in this section. In section 2.4 the literature on muscle models is reviewed. The final section is devoted to a survey of the different multiprocessor systems.

2.2 Dynamics and Control Strategies for Multilink Systems

The dynamic equations of motion of a multilink system are a set of equations describing the dynamic behavior of the system. The importance of dynamics stems from its use in simulation, analysis, and design of suitable control laws for the system. The two main approaches toward deriving the dynamics of multilink systems are the Lagrangian and Newton-Euler formulations. Uicker [3] and Kahn [4] used the Lagrangian formulation to derive the dynamics of a multilink system. Hollerbach [5] showed that the Uicker/Kahn formulation gives an order of $n^4$ dependence on the number of multiplications and additions and thus is too time consuming to compute in real time where $n$ is the number of joints. Hollerbach developed an efficient Lagrangian formulation of multilink system dynamics. The algorithm is based on recurrence relations for the velocities, accelerations, and generalized forces. In this algorithm the number of additions and multiplications varies linearly with the number of joints, as opposed to the former Lagrangian formulations with an $n^4$ dependence.

Stepanenko and Vukobratovic [6] derived the equations of motion of human locomotion using the Newton-Euler formulation. Orin et al. [7] proposed that the forces and moments in the Newton-Euler formulation are referred to the link's internal coordinate system. Luh et al. [1] further extended this idea by calculating the angular and linear velocities and accelerations in link coordinates as well [5]. Thus, an
efficient recursive Newton-Euler formulation was obtained by Luh et al. In a robot, the method involves the successive transformation of velocities and accelerations from the base of a manipulator out to a gripper, link by link. Forces are then transformed back from the gripper to the base to obtain the joint torques. A further advantage of using this method is that the computation is linearly proportional to the number of links [1].

A Newton-Euler state space formulation is suggested by Hemami [2]. In his formulation, \([\Theta, W]\) are used as the state variables for the rotation of a free body rather than the more conventional \((\Theta, \dot{\Theta})\). Where \(\Theta\) is the vector of three rotational Euler angles with respect to an inertial frame and \(W\) is the vector of angular velocity of the principal axes of a body... This choice eliminates elaborate and often difficult computation of \(\Theta\) that arises when there is more than one rigid body in the system. Furthermore, the recursive relationship for calculating angular velocities and accelerations is eliminated [8].

In the following, different strategies for controlling multilink systems will be surveyed. Since multilink systems are nonlinear systems in which joint motions are highly coupled, controlling these systems is very difficult. According to [9], multilink control is categorized into two different control aspects. The first is the gross motion control in which the arm moves from an initial position along a preplanned trajectory. The second is the fine motion control in which the end-effector of the multilink system dynamically interacts with the object, using sensory feedback information from the external sensors to complete the task. Only the gross motion control of a multilink system is considered here.

The linear feedback control has been studied extensively. In this method, the system dynamic equations are linearized about static operating points. State feedback matrices are then calculated to obtain the desired properties of the system. Centralized feedback control and independent joint control for calculating the feedback matrices were discussed by Golla et al. [10]. Linear feedback that simultaneously decouples the motion of each link and assigns the poles of the system were proposed by Hemami et al. [11].

The use of nonlinear feedback components to minimize the effects of the nonlinear coupling terms in a nonlinear system was suggested by Hemami and Camana [12]. Thus, theoretically, the performance of the system could be as good as that obtained by a linear system. A similar method was proposed by Paul [13] and Raibert and Horn [14].

The resolved motion rate control was suggested by Whitney [15]. In this method, the problems of coordinated rate control and position control of multidegree-of-freedom arms are treated together. A solution to the end-effector position control problem was presented, allowing
the arm to be driven to a final position specified in a Cartesian coordinate system. Luh et al. [16] proposed a resolved acceleration control method. This method adopted the idea of nonlinear control and extended the results of resolved motion rate controls. Essentially, the joint accelerations are first computed from the measured velocities and displacements, and these results are used to compute the input torques and forces to the multilink system. The convergence of the control is assured if the sampling frequency is sufficiently high.

Dubowsky and DesForges [17] proposed a simple model referenced adaptive control for mechanical manipulators. A linear second-order, time invariant, differential equation is selected as the referenced model for each degree of freedom of a robot arm. The manipulator is controlled by adjusting the position and velocity feedback gains to follow the model so that its closed-loop performance characteristics closely match the set of desired performance characteristics in the referenced model [9]. Koivo and Guo [18] presented an approach to the position and velocity control of a manipulator by using an adaptive controller of the self-tuning type for each joint. The parameters of the models were determined by an on-line recursive algorithm.

2.3 Biomechanisms of Lower-limb Movements

Numerous studies have been involved in the area of lower-limb movements. This survey concentrates on only two subjects in this area. These are human gait studies and estimation of muscular forces. Human gait studies deal with the motions of the joints and limbs during locomotion. They also deal with the time relationship between foot-fall and foot-rise, with rotation of limbs, with the division of gait into various phases, and with any other factors that add to the overall consideration of gait [19].

Muybridge [20] photographed a large number of phases of different kinds of human locomotion. The results of his work deeply affected gait studies of human locomotion. Murray and associates [21, 22] have measured and compiled data on many major movements of human locomotion such as the hip, knee, and ankle rotation patterns for one leg. Saunders et al. [23] have defined a set of six determinants of gaits. The determinants represent adjustments made by the body that minimize the movement of the body's center of gravity. The six determinants are: (1) pelvic rotation, (2) pelvic tilt, (3) knee flexion, (4) foot mechanics, (5) knee mechanics, and (6) lateral pelvic displacement. Hartrum [19] has combined all gait aspects into a complete mathematical model of locomotion. His primary contributions are the parametric representation of the kinematic motion of biped locomotion, incorporating all aspects of biped gait and a computer implementation of this representation. Several investigators such as Beckett and Chang [24] and Chow and Jacobsen [25] have studied human gait with the idea
that an optimal performance criterion exists which governs the overall gait function. Inman and associates [26, 27] have made extensive studies of body motion during level walking for both normal and prosthetic gait. Principal results have come from careful analysis of stick diagrams and force plates used to measure the reaction of the foot with the ground. Vukobratovic [28] had studied hip, knee, and ankle angle patterns and ground reaction forces for different kinds of legged biped locomotion.

Several works related to estimation of muscular forces are reviewed. Since there is no convenient experimental method for direct quantitative measurement of muscle forces in vivo, the pattern of EMG signals can serve as a qualitative measurement. Electromyographic (EMG) techniques have been used extensively [29, 30, 31] to estimate muscle forces. EMG is the recording of electrical changes in muscle. In resting muscle there is little or no electrical activity. When the muscle begins to contract, an increasing number of anterior horn cells and their corresponding muscle fibers are recruited into the contraction process. During recruitment the electromyogram first shows individual motor unit action potentials; as more tension is exerted by stimulating more motor units and increasing the firing frequency, the number of potentials in the electromyogram increases. Thus, EMG signals can be used as qualitative measurement of the activity of the muscle.

Seireg and Arvikar [32] developed a mathematical model for the musculoskeletal system capable of evaluating muscle forces and joint reactions for different static postures. Different criteria were studied for evaluation of the muscle forces necessary to maintain the human body equilibrium in standing, leaning, and stooping. They claimed that the feasibility of their approach for quantitative analysis of musculoskeletal forces was demonstrated by good correlation with EMG data. In a further study [33], they extended the previous mathematical modelling of the lower extremities. Load sharing of the muscles and the corresponding hip, knee, and ankle joint reactions during walking can be predicted when the motion pattern is known. McLeish and Charnley [34] determined the force in adductor muscles by estimating the load sharing of the muscles. The idea of using the concept of optimization of total muscular effort in determination of muscular forces was proposed by Ghista et al. [35]. Chow and Jacobson [25] considered the general problem of optimization of motion where the performance criterion was the minimization of total work done during all three phases of walking motion. Pedotti et al. [36] studied the muscular-force optimization problem by formulating four biologically meaningful optimization criteria for the muscular forces. Their experimental EMG data was compared with the muscular-force patterns obtained computationally via various performance criteria.
2.4 The Mechanical Models of Muscle

Various muscle models have been developed in order to interpret the functional characteristics of the skeletal muscle. The earliest muscle model is just a spring [37]. This is based on the idea that muscle behaves like a stretched spring when activated. This over-simplified model was improved by the classical viscoelastic model of muscle introduced by Gasser and Hill [38] and Levin and Wyman [39].

The next major development in muscle modelling was due to a combination of improved mechanical and thermal measurements by Hill [40]. Hill found that under a wide variety of conditions a muscle developed more heat while shortening than it did in isometric contractions. The amount of extra heat produced was solely dependent on the amount of shortening. Hill further showed experimentally that the rate of heat production plus the rate of mechanical work depends linearly on the load p acting on the muscle and shortening velocity V. From these two relationships he obtained

$$(p + a)V = b(P_0 - p) \quad (2.1)$$

where $P_0$ is the maximum isometric tension, $a$ is the heat constant and $b$ is the rate constant of energy liberation. Eq. (2.1) is normally termed the Hill equation.

Hill further suggested that muscle could be adequately represented by an active contractile component whose properties were described by the Hill equation in series with an elastic component. The contractile component of his model represents the active force generated by a muscle when it is stimulated. In relaxed muscle the contractile component generates only a negligible force. However, tension realistically exists in the relaxed muscle when stretched. Thus a third component in parallel with the contractile component was added.

Modification of the Hill's model has been proposed. Pinto and Fung [41] and Glantz [42] have shown that unstimulated muscle exhibited not simply elastic but viscoelastic behavior. McLaughlin et al. [43] specifically addressed the question of whether or not the passive element in series with the contractile element exhibits viscoelastic behavior and concluded that it did. The measurements performed by Jewell and Wilkie [44] indicated that Hill's equation is not obeyed instantaneously but only applies to the steady state. Thus, it is necessary to modify Hill's equation for the contractile element if the muscle is not tetanized. A more basic approach was made by A.F. Huxley [45, 46]. His model took account of the known structures of the muscle, as well as the mechanical and thermodynamic observations. The main features are that cross-bridges act cyclically, attaching and detaching according to rate constants, which are functions of their position.
A detailed mathematical muscle model which contains the physiological control stimulation rate and motor unit recruitment as explicit parameters was proposed by Hatze [47]. He claimed that his model correctly predicts the initial excitation-contraction delay, the errors in the force measurements, and several other contraction phenomena. Zheng et al. [48] proposed a mathematical model of skeletal muscle activation. A direct relationship between the stimulus rate and the active state was established. The tension-length relation was represented by a piecewise linear function and Hill's equation was used for the force-velocity relation. A threshold theory and a corresponding physical implementation were also proposed that consider the size principle and their physiological manifestations as well as their exceptions.

As the mechanical properties of muscle depend on its level of neural activation, any investigation of in-vivo muscle mechanics should seek quantitative relations between mechanical response and some appropriate measure of neural activities. Presently the only readily available measure are EMG patterns. The quantitative relations between EMG and mechanical parameters under isotonic and static conditions have been studied by Lippold [49] and Vredenbregt and Rau [50]. A second order linear relation between the EMG of human triceps and the corresponding isometric force was proposed by Coggshall and Bekey [51]. A quasi-linear muscle model has been developed by Zahalak and Heyman [52]. Two inputs of the model are muscle length perturbation and EMG perturbation.

2.5 Multiprocessor Systems

A multiprocessor system is a computer system employing two or more processing units under integrated control. Thus a multiprocessor system includes a number of processing units which may have different hardware architectures and software structures. Since the field of multiprocessor systems is very extensive, in this section only the papers related to Chapter 5 of this dissertation are reviewed. The applications of multiprocessor systems to multilink systems include data acquisition, computation of system dynamics and control algorithms as well as simulations.

A high speed, portable multiprocessor system for data acquisition was designed by Gaglianello [53]. He used a shared bus type of hardware architecture. The microprocessors utilized were Intel 8086's. A decentralized control scheme was used to develop the operating system. The multiprocessor system was developed such that each processor executes the same operating system. The task and the self-dispatching software module were developed for the purpose of software transparency. A multiprocessor-based instrument for data acquisition was presented by Silverman et al. [54]. The Multibus designed by Intel was used for the system bus. The operating system was roughly divided into two groups.
The first group includes routines which manage processes and resources and handle I/O and interrupt. The second group of routines is used for carrying out user commands.

Klein and Wahawisan [55] designed a multiprocessor system for controlling a walking machine. A tree hardware architecture was used to implement the control algorithm. Fault-tolerant features are added to the multiprocessor system so that failed units can be replaced by spare units. For modularity and simplicity, the multiprocessor was designed to be a loosely coupled system. Luh and Lin [56] suggested using a multiprocessor system for calculating the inverse dynamic algorithm for a multilink system. They proposed that one processor is used for the computation of the dynamics of one joint. Hence an n joint manipulator requires a multiprocessor system with an equal number of processors. A method has been developed for determining an optimum ordered schedule for each of the processors. Zheng and Hemami [8] suggested a multiprocessor system for implementing the inverse dynamics computations. The multiprocessor system is composed of a central processor and a group of satellite processors. The job of each satellite processor is to calculate the total external forces and moments acting on the corresponding link. The central processor is for computing the required applied torques of all the joints. Each satellite processor utilizes the same software module.

A multiprocessor system dedicated to simulation of multilink systems was designed by Grygier and Hemami [57]. General properties of the dynamics of multilink systems, communication protocol, and task decomposition were discussed. The efficiency of using the multiprocessor system has been demonstrated by simulating two different multilink systems.
3.1 Introduction

This chapter deals with the first objective of this dissertation which is to develop a conceptual methodology for the control of a seven link biped. This study allows us to estimate actuator forces of the lower extremities of a seven link biped during turning when the ground reaction forces and the trajectories of motion are specified. The kinematics of turning, dynamics of the seven link model, force actuator model, and a control strategy are involved in this study. This ability to predict the actuator forces and to simulate human locomotion could provide a basis for the design of prostheses and orthoses for the handicapped and help development of effective muscular and neural functional electrical stimulators. It may also lead to a better understanding of gait and muscle disorders and might eventually allow computerized analysis of pathological gaits and muscles to aid the physician in diagnosing the disorder.

Turning is one of the common activities of humans, but little work on human turning has been reported [58, 59, 60] in the literature. Many studies have been devoted to normal level walking [22, 33, 61, 62]. The development of a mathematical model for the musculo-skeletal system capable of evaluating muscle and joint reaction forces for different static postures was studied in [32, 33]. However, the force patterns in [32, 33] were obtained under the conditions that the system is in equilibrium at all times and inertia forces and moment are neglected. Most of the papers on dynamic analysis of human locomotion use ideal torque generators and ignore muscle and ligament actions. In this study, the ligament forces are treated as those that maintain holonomic connection constraints and can be eliminated from the dynamic equations of motion if the connections are always maintained and if they are considered to be inconsequential.

There are single and double stance phases involved in turning. Single stance phase is the period with one foot on the ground. Double stance phase is the period with both feet on the ground. Turning, like walking, involves lifting the foot from the ground and lowering the foot to the ground. So the motion of turning may be considered as a sequence of constrained and unconstrained movements [63].
In order to describe a turn kinematically, it is necessary to have a complete set of kinematic trajectories. The kinematics of the turning movement have been studied and are referred to the literature in Section 3.2. Section 3.3 discusses the force actuator model used in this study. Actuator forces are assumed to be directed along straight lines joining the corresponding points of origin and insertion. The Newton-Euler state space formulation developed in [2] is used to set up the equations of motion of this system in Section 3.4. Many control strategies such as linear feedback control, nonlinear feedback control [64] and model-referenced adaptive control [17, 18] can be employed to compensate for possible deviations of the system from the desired trajectories and stabilize the system. A nonlinear control strategy is considered in Section 3.5. Based on the control strategy, the actuator forces are obtained in Section 3.6. Digital computer simulations are carried out to estimate the actuator forces and to demonstrate the effectiveness of the control strategy.

3.2 Development of Kinematic Trajectories and Connection Constraints

The model consists of seven links (Figure 3.1) in order to approximate locomotion characteristics similar to those of the lower extremities of the human body. The foot, the leg, and the thigh each are considered as one rigid body. Consequently three connected rigid bodies represent the lower limb on one side. Another three rigid bodies define the other limb. A seventh rigid body represents the hip, the torso, the hands, the neck and the head. The mass, length, moment of inertia, and center of gravity parameters are listed in Table 3.1 [65, 66].

The procedure for making a right turn involves several steps [19, 23] that are performed sequentially. Although in a real human turn some steps may overlap. Initially both feet are on the ground. In turning to the right, the support gradually shifts to the left leg so that the right foot can be lifted. The next step is the rotation of the pelvis in the transverse plane (xy plane in Figure 3.2) about a longitudinal axis (z axis). A rotation of the pelvis in the frontal plane (xz plane) about a coronal axis (y axis) follows. The knee of the right foot bends to allow the foot to lift. The right foot turns and is lowered until it touches the ground. At this moment, both feet again are on the ground. The support gradually shifts from the left leg to the right leg. The next steps include knee flexion to lift the left leg. Moving, turning, and lowering of the left foot are the next steps until the left foot touches the ground and the right turn is completed. From the experimental data, it takes about three to five seconds for making a complete slow right turn. Figure 3.3 shows the swing and stance phases of both feet during turning and the duration of turning is 4.3 seconds.

For the model shown in Figure 3.1, the number of rotational degrees of freedom of the hip, knee and ankle joints are respectively three, one, and two. In single stance phase with the left foot on the
Fig. 3.1 The seven link biped model and the body coordinate of the 6th link
### Table 3.1 Parameters of the seven link biped model

<table>
<thead>
<tr>
<th>Link number</th>
<th>Mass</th>
<th>$M_1$</th>
<th>$I_{11}$</th>
<th>$I_{12}$</th>
<th>$I_{13}$</th>
<th>$l_1$</th>
<th>$K_1$</th>
<th>$C_3$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (pelvis)</td>
<td>6.1</td>
<td>.049</td>
<td>.049</td>
<td>.0067</td>
<td>.165</td>
<td>.165</td>
<td>.27</td>
<td>.095</td>
<td></td>
</tr>
<tr>
<td>3 (left thigh)</td>
<td>7.0</td>
<td>.089</td>
<td>.089</td>
<td>.0276</td>
<td>.247</td>
<td>.184</td>
<td>.25</td>
<td>.172</td>
<td></td>
</tr>
<tr>
<td>2 (left leg)</td>
<td>3.1</td>
<td>.105</td>
<td>.105</td>
<td>.0088</td>
<td>.267</td>
<td>.235</td>
<td>.264</td>
<td>.125</td>
<td></td>
</tr>
<tr>
<td>1 (left foot)</td>
<td>1.0</td>
<td>.004</td>
<td>.004</td>
<td>.0015</td>
<td>.123</td>
<td>.118</td>
<td>.26</td>
<td>.11</td>
<td></td>
</tr>
<tr>
<td>8 (trunk)</td>
<td>42</td>
<td>1.75</td>
<td>1.75</td>
<td>.4</td>
<td>.43</td>
<td>.37</td>
<td>.252</td>
<td>.276</td>
<td></td>
</tr>
<tr>
<td>5 (right thigh)</td>
<td>7.0</td>
<td>.089</td>
<td>.089</td>
<td>.0276</td>
<td>.184</td>
<td>.247</td>
<td>.25</td>
<td>.172</td>
<td></td>
</tr>
<tr>
<td>6 (right leg)</td>
<td>3.1</td>
<td>.105</td>
<td>.105</td>
<td>.0088</td>
<td>.235</td>
<td>.267</td>
<td>.264</td>
<td>.125</td>
<td></td>
</tr>
<tr>
<td>7 (right foot)</td>
<td>1.0</td>
<td>.004</td>
<td>.004</td>
<td>.0015</td>
<td>.118</td>
<td>.123</td>
<td>.26</td>
<td>.11</td>
<td></td>
</tr>
</tbody>
</table>
Frontal plane view  

Sagittal plane view

Fig. 3.2 The skeletal biped model
Fig. 3.3 The different phases of both feet of biped model during turning
ground, the system has a total of twelve degrees of freedom. In double stance phase, five holonomic constraints are added to the system. Three of these constraints are translational and the remaining two are rotational constraints. Therefore, the total system degrees of freedom are reduced to seven. For more detail see Appendix A.

The twelve degrees of freedom of the skeletal system are defined here by twelve angles denoted as $\phi_1, ..., \phi_{12}$. The anatomical correspondence of these angles is given in Table 3.2. The trajectories of $\phi_1$ through $\phi_{12}$ may be obtained experimentally using video cameras or cine-film cameras. In this study the trajectories of $\phi_1$ through $\phi_{12}$ as a function of time are obtained from a sequence of photographs of the turning motion [20, 67, 68]. The discrete points on the trajectory are interpolated to obtain continuous time functions by power series approximations [69]. These power series are differentiated in order to derive the corresponding angular velocities and accelerations. The results are shown in Figure 3.4 (a)-(l) where the angles are expressed in degrees.

In order to verify the kinematic trajectories $\phi_1, ..., \phi_{12}$, kinematic equations of the biped model are derived to generate a sequence of stick figures that are shown on a graphic display. Kinematic equations of the biped model are derived as follows. The inertial coordinate system (ICS) is defined to be fixed on the ground (Figure 3.2). A body coordinate system (BCS) coinciding with the principal axes is defined for each link of the model. Euler angles correspond to pitch, roll, and self rotation as defined in [70, 71]. The orthogonal transformation from the inertial system to the body coordinate system and vice versa are defined as follows.

For a given vector in the ith BCS, i.e., $\text{BCS}_i$, the orthogonal transformation to the ICS is given by

$$A(\theta_{i1}, \theta_{i2}, \theta_{i3}) = A_i =$$

$$\begin{bmatrix}
\cos\theta_{i1} & \sin\theta_{i1} & 0 \\
-sin\theta_{i1} & \cos\theta_{i1} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos\theta_{i2} & -\sin\theta_{i2} \\
0 & \sin\theta_{i2} & \cos\theta_{i2}
\end{bmatrix}
\begin{bmatrix}
\cos\theta_{i3} & 0 & \sin\theta_{i3} \\
0 & 1 & 0 \\
-sin\theta_{i3} & 0 & \cos\theta_{i3}
\end{bmatrix}$$

(3.1)

where $(\theta_{i1}, \theta_{i2}, \theta_{i3})^T$ is the vector of three rotational Euler angles of $\text{BCS}_i$ with respect to the inertial frame. The inverse transformation from the ICS to $\text{BCS}_i$ is given by $A_i^T$. 

17
<table>
<thead>
<tr>
<th>leg</th>
<th>The function of the angle</th>
<th>the anatomical correspondence</th>
</tr>
</thead>
<tbody>
<tr>
<td>left</td>
<td>$\phi_1 - \phi_6$</td>
<td>lateral and medial rotation of the hip joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_2 - \phi_7$</td>
<td>abduction and adduction of the hip joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_5$</td>
<td>flexion and extension of the hip joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_4 - \phi_5$</td>
<td>flexion and extension of the knee joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_2$</td>
<td>lateral and medial rotation of the ankle joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_3 - \phi_4$</td>
<td>flexion and extension of the ankle joint</td>
</tr>
<tr>
<td>right</td>
<td>$\phi_6 - \phi_8$</td>
<td>lateral and medial rotation of the hip joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_7 - \phi_9$</td>
<td>abduction and adduction of the hip joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_{10}$</td>
<td>flexion and extension of the hip joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_{10} - \phi_{11}$</td>
<td>flexion and extension of the knee joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_9$</td>
<td>lateral and medial rotation of the ankle joint</td>
</tr>
<tr>
<td></td>
<td>$\phi_{11} - \phi_{12}$</td>
<td>flexion and extension of the ankle joint</td>
</tr>
</tbody>
</table>

Table 3.2 The anatomical correspondence of the angles $\phi_1$ through $\phi_{12}$. 
Fig. 3.4(a) Kinematic trajectory of turning
Fig. 3.4(c)
Fig. 3.4(f)
Fig. 3.4(g)
Fig. 3.4(j)
Fig. 3.4(k)
Let $X_i$ be the position of the center of BCS$_i$ with respect to ICS, and $K_i$ and $L_i$ be the coordinates of contact points between links in the body coordinate system (Figure 3.2). For any two connected links $i$ and $i+1$, there exist holonomic connection constraints expressed by

$$X_i + A_i K_i = X_{i+1} + A_{i+1} L_{i+1}$$

The connection constraint equations can be specifically written

$$X_1 = X_2 + A_2 L_2$$
$$X_2 + A_2 K_2 = X_3 + A_3 L_3$$
$$X_3 + A_3 K_3 = X_4 + A_4 L_4$$
$$X_4 + A_4 K_4 = X_5 + A_5 L_5$$
$$X_5 + A_5 K_5 = X_6 + A_6 L_6$$
$$X_6 + A_6 K_6 = X_7$$

It is assumed here that each link only can rotate along its principal axes. This means that the relative rotations of the limbs are about axes that are parallel to principal axes of the bodies. It is also assumed that the ICS and all BCSs coincide initially. With these assumptions, the set of independence variables $\phi$ is related to the Euler angles as follows

$$\theta_{11} = \phi_1$$
$$\theta_{12} = 0$$
$$\theta_{13} = \phi_3$$
$$\theta_{21} = \phi_1$$
$$\theta_{22} = \phi_2$$
$$\theta_{23} = \phi_4$$
$$\theta_{31} = \phi_1$$
$$\theta_{32} = \phi_2$$
$$\theta_{33} = \phi_5$$
$$\theta_{41} = \phi_6$$
$$\theta_{42} = \phi_7$$
$$\theta_{43} = 0$$
$$\theta_{51} = \phi_8$$
$$\theta_{52} = \phi_9$$
where $\theta_1 = \theta_2 = \theta_3 = \theta_4$ due to the ankle and knee joints of the left leg not being allowed to rotate along z-axes of their body coordinates. We assume that the upper body (link 4) always remains in an erect posture during turning, then $\theta_4 = 0$. We also assume that $\theta_1 = \theta_2 = \theta_3 = 0$ during the entire motion. A perspective of the turning motion is shown in Fig. 3.5. The perspective transformations are derived in Appendix B.

3.3 Actuator Dynamics

This chapter deals with qualitative study of selective muscular actuators of the lower extremities. The choice of actuators in this planar model is determined by the particular analysis to be performed. Table 3.3, taken from anatomical literature [72, 73, 74], lists the muscles which are considered to be important in turning. The frontal and sagittal views of the attachment of these muscle-like actuators for the lower limbs are shown in Figure 3.6. The points of origin and insertion are obtained from the available anatomical data [72, 74] and are listed in Table 3.3 where the subscript indicates the particular body coordinate system (BCS). The lower limb actuators have been idealized by lines joining the muscle point of origin to the point of insertion and are assumed to produce tensile forces only. For some two-joint muscles such as the rectus femoris and gastrocnemius, a straight line representation is not possible due to an interposing structure. In this case, two line segments are used to circumvent the interposing structure, and approximate the shape. The intricate and highly nonlinear dynamics of the muscles are not included in the study. The length variation of a muscle is defined as the difference between the length of a muscle and its reference length, the length measured when the model stands on the ground in an erect posture. Note that the reference length defined here is different from physiological rest length because in human turning motion we usually start from an erect posture.
Fig. 3.5 The sequences of turning 90 degree to the right
<table>
<thead>
<tr>
<th>leg</th>
<th>No.</th>
<th>Muscle</th>
<th>origin point (m)</th>
<th>insertion point (m)</th>
<th>midpoint</th>
<th>ref. length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>iliacus (IL)</td>
<td>(.08,.165,.085)</td>
<td>(0.0,.33)</td>
<td></td>
<td>.2492</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>rectus femoris (RF)</td>
<td>(.08,.165,.085)</td>
<td>(.02,.183)</td>
<td>(.069,.226)</td>
<td>.583</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>gluteus maximus (GM)</td>
<td>(-.08,.165,0)</td>
<td>(0.0,.032)</td>
<td></td>
<td>.23</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>biceps femoris(BFLH)</td>
<td>(-.08,.165,0)</td>
<td>(-.056,.215)</td>
<td></td>
<td>.452</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>vastus lateralis (VAL)</td>
<td>(.034,0,-.136)</td>
<td>(.02,.183)</td>
<td>(.069,.226)</td>
<td>.1845</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>gluteus medius (GME)</td>
<td>(0,.165,12)</td>
<td>(.0,.052,.046)</td>
<td>(0,.069,.15)</td>
<td>.274</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>adductor longus (AD)</td>
<td>(0,.065,-.069)</td>
<td>(0,.0,.076)</td>
<td></td>
<td>.216</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>biceps femoris(BFSH)</td>
<td>(-.052,0,-.206)</td>
<td>(-.06,.155)</td>
<td></td>
<td>.121</td>
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<tr>
<td></td>
<td>9</td>
<td>gastrocnemius (GA)</td>
<td>(-.052,0,-.206)</td>
<td>(-.018,0)</td>
<td>(-.06,.155)</td>
<td>.545</td>
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<td></td>
<td>10</td>
<td>tibialis anterior(TA)</td>
<td>(0,0,-.115)</td>
<td>(-.049,0)</td>
<td></td>
<td>.167</td>
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<td>11</td>
<td>soleus (SO)</td>
<td>(-.02,0,.215)</td>
<td>(-.038,0)</td>
<td></td>
<td>.0794</td>
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</tr>
<tr>
<td>left</td>
<td>1</td>
<td>iliacus</td>
<td>(.08,.165,.085)</td>
<td>(0.0,.33)</td>
<td></td>
<td>.2492</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>rectus femoris</td>
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<td>(.02,.183)</td>
<td>(.069,.226)</td>
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<td>(0.0,.032)</td>
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<td>.452</td>
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<td>(.069,.226)</td>
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<td>(0,.069,.15)</td>
<td>.274</td>
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<td>(0,.0,.076)</td>
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<td>.216</td>
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<td>(-.06,.155)</td>
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<td>(-.049,0)</td>
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<td>(-.02,0,.215)</td>
<td>(-.038,0)</td>
<td></td>
<td>.0794</td>
</tr>
</tbody>
</table>

Table 3.3 Parameters of the muscle actuators of the lower extremities
right leg in the sagittal plane

right leg in the frontal plane

Fig. 3.6 A musculo-skeletal planar model for the lower extremities
Based on the turning sequences shown in Figure 3.5, the length variations of all the actuators can be derived as functions of time and are shown in Figure 3.7 and 3.8.

3.4 **The Dynamics of the System**

The dynamic equations of motion are a set of equations describing the dynamic behavior of the musculo-skeletal model. The importance of dynamics stems from its use in simulation and design of a suitable control law for the system. The Newton-Euler state space formulation developed in [2] is used to derive the equations of motion. Holonomic constraints are involved in the formulation. Holonomic constraints result when a prescribed point on the body is restricted to move on a surface, a curve, or be stationary. Equations (3.3) to (3.8) are eighteen permanent holonomic constraints acting on this model. In addition to holonomic constraints, other constraints are applied to the system when both feet are in contact with the ground.

From Figure 3.3, there are three different cases which have to be considered:

**Case 1.** Single stance phase with left foot on the ground.
In this phase, from 0 to 1.7 seconds as shown in Figure 3.3, the corresponding holonomic constraints are

\[ X_1 = c_1 \]  

(3.10)

where \( c_1 \) is a constant vector that describes the position of the left foot relative to the ICS.

**Case 2.** Single stance phase with right foot on the ground.
As shown in Figure 3.3, this phase is from 2.3 to 4 seconds. The holonomic constraints for this case can be expressed as

\[ X_7 = c_2 \]  

(3.11)

where \( c_2 \) is a constant vector that describes the position of the left foot relative to the ICS.

**Case 3.** Double stance phase.
In this phase, both constraint equations (3.10) and (3.11) must be applied.

The holonomic constraint forces act on the system while the constraints are in effect and become impulsive forces at the instant of contact. These forces result in the swing foot hitting the ground and the velocity of the foot being instantaneously changed.

36
Fig. 3.7 Muscle length variations of the left leg
Fig. 3.7 (left leg)
Fig. 3.8 Muscle length variations of the right leg
Fig. 3.8 Muscle length variations of the right leg

- Adductor longus
- Biceps femoris (short head)
- Gastrocnemius
- Tibialis anterior
- Soleus
In Figure 3.1, each of the knee joints is assumed to have only one rotational degree of freedom and each of the ankle joint is assumed to have two rotational degrees of freedom. In addition to holonomic constraint forces, actuator forces are acting on the system. There are twenty-two actuators, as listed in Table 3.2. Using all the forces discussed above, the equation of motion of the biped system can be obtained. The actual degrees of freedom of this system are twelve for the single stance phase and seven for the double stance phase [2, 64].

The projection transformations for reducing the dimensionality of the system for the three different cases are discussed separately.

Case 1. Single stance phase with left foot on the ground. For this case there are totally twenty-one holonomic constraints, namely Eqs. (3.3) through (3.8) and Eq. (3.10), acting on the system. The first step of projection is to use these constraints and solve for \((X, \dot{X})\) in terms of \((\theta, W)\) where \(W\) is the components of the angular velocity expressed in a body coordinate system. It is shown in [64] that the corresponding constraint forces can be eliminated due to projection.

In the second step of projection, the state space \((\theta, W)\) is projected to \((\phi, \dot{\phi})\). From Eqs. (3.9) and the above constraints, we can have

\[
\theta = f(\phi). \quad (3.12)
\]

Then the reduced equations of the seven link biped system for this case may be written as

\[
D(\phi)\ddot{\phi} + H(\phi, \dot{\phi}) + G(\phi) + E(\phi, \dot{\phi})\Gamma = T \quad (3.13)
\]

where \(D(\phi)\) is the inertial matrix, \(H(\phi, \dot{\phi})\) represents Coriolis and centrifugal terms, \(G(\phi)\) denotes the gravity loading term and \(E(\phi, \dot{\phi})\) describes the effect of the contact forces of the right foot. A detailed derivation of Eq. (3.13) is given in Appendix A.

Case 2. Single stance phase with right foot on the ground.

For this case, the holonomic constraints acting on the system are Eqs. (3.3) through (3.8) and Eq. (3.11). Following the same procedures of the steps of projection for the single stance phase with the left foot on the ground renders

\[
D'(\phi')\ddot{\phi}' + H'(\phi', \dot{\phi}') + G'(\phi') + E'(\phi', \dot{\phi}')\Gamma' = T' \quad (3.14)
\]
Case 3. Double stance phase

The dynamic equations for the double stance phase have already been included in Eqs. (3.13) and (3.14). The constraint force $\Gamma$ of Eq. (3.13) act on the system while the constraints are in effect, i.e., the right foot contacts the ground. Here forces of constraint are ground reaction forces. So $\Gamma$ is equal to zero when the right foot is not on the ground. Thus, the equations of motion for the seven link biped model with either one or two feet on the ground are formulated as an on-off constrained system. These equations are essential to both in the design of the control strategy and to the digital computer simulations of the turning motion.

3.5 Control Strategy

Due to the complexity of the system and the desired motion a nonlinear control strategy is proposed. Additional and different control functions are relegated to individual actuators. The muscles of the lower extremities collectively have three major functions:

1. To perform and guide the turning motion
2. To control the ground reaction forces
3. To stabilize the posture during the motion

Each function requires a system of forces. Let $T_1$ be the vector of inputs which are generated to control the ground reaction forces when contact constraints are in effect. Let $T_2$ be the vector of forces generated to guide the model motion and let $T_3$ be the vector of forces to stabilize the system during the motion. For the single stance phase with the left foot on the ground, then

$$T = T_1 + T_2 + T_3 \quad (3.15)$$

and

$$T_1 = E(\phi, \dot{\phi}) \Gamma \quad (3.16)$$

where $E(\phi, \dot{\phi})$ is defined in Eq. (3.13).

$T_2$ can be computed by the inverse plant method. Assume that the components of $D(\phi)$, $H(\phi, \dot{\phi})$, $G(\phi)$ and $E(\phi, \dot{\phi})$ can be accurately computed, then

$$T_2 = D(\phi)\ddot{\phi} + H(\phi, \dot{\phi}) + G(\phi) \quad (3.17)$$
where subscript d denotes the desired value.

In order to stabilize the system, \( T_3 \) is generated from position and velocity feedback as

\[
T_3 = D(\phi)[K_v(\dot{\phi}_d - \dot{\phi}) + K_p(\phi_d - \phi)]
\]

(3.18)

Therefore, the total inputs which are generated by actuators are

\[
T = D(\phi)[\ddot{\phi}_d + K_v(\dot{\phi}_d - \dot{\phi}) + K_p(\phi_d - \phi)] + H(\phi, \dot{\phi}) + G(\phi) + E(\phi, \dot{\phi})
\]

(3.19)

Substituting the above equation in Eq. (3.13) renders

\[
(\ddot{\phi}_d - \ddot{\phi}) + K_v(\dot{\phi}_d - \dot{\phi}) + K_p(\phi_d - \phi) = 0
\]

(3.20)

Equation (3.20) represents a linear system. The block diagram is shown in Figure 3.9. By selecting diagonal matrices \( K_v \) and \( K_p \), one is able to decouple the motion of each link in the biped system from that of the other links. If the desired eigenvalues associated with \((\phi_i, \dot{\phi}_i)\) are \( \lambda_1 \) and \( \lambda_2 \), the diagonal elements of matrices \( K_v \) and \( K_p \) are

\[
K_v(i, i) = \lambda_1 + \lambda_2
\]

\[
K_p(i, i) = \lambda_1 \cdot \lambda_2
\]

(3.21)

The decoupled motion of the ith link is then described by

\[
\ddot{\Delta \phi}_i + (\lambda_1 + \lambda_2) \dot{\Delta \phi}_i + \lambda_1 \cdot \lambda_2 \Delta \phi_i = 0
\]

(3.22)

where

\[
\Delta \phi_i = \phi_d - \phi_i.
\]

Assigning the eigenvalues to the system with negative real parts, the gain matrices \( K_v \) and \( K_p \) can be obtained to stabilize the system. The same methods can be used to generate inputs by muscles for the single stance phase with the right foot on the ground.

3.6. Results and Discussions

Referring to [28], the desired ground reaction forces of \( \Gamma \) and \( \Gamma' \) are assumed as given in Figure 3.10 because the ground reaction forces can not be inferred from the photographic references. With zero input and a symmetrical equilibrium state, the
Fig. 3.9 The control block diagram of the system
Fig. 3.10
The desired ground reaction force patterns
Fig. 3.10 (cont'd)
vertical ground forces at the initial and final points are equal to one-half the total biped weight. The ground reaction forces in x and y directions at these two points are zero. At the instant of take-off, the ground reaction forces go to zero and stay at zero until the next heel strike occurs. Based on the desired kinematic data and ground reaction forces, the control torques are calculated by using Eq. (3.19), and are plotted in Figure 3.11. There are more unknowns than equations in computing actuator forces. This implies that many feasible solutions are possible. One therefore has to set up a criterion to choose a suitable one from these solutions. The performance criterion used here is to minimize the total sum of squares of forces required to produce these torques. It is assumed here that coactivation of muscles is not admissible. The calculated force patterns of the actuators which are listed in Table 3.2 are shown in Figure 3.12 and 3.13. The actuator forces are very small at the initial, middle, and final phases of motion because of the two legs support. All actuator forces go to zero at the final state of the system because a symmetric double static stance with zero bias torque is assumed. All the actuator forces are positive and in reasonable range. The magnitudes of forces are relatively small which is due to the slow turning motion. The forces for fast turning motion can be obtained by rescaling the time axis.

Since there is no convenient experimental method for direct quantitative measurement of muscle forces, EMG data can serve as a qualitative measure. As in [36], the force patterns not only depend on a particular criterion but also depend on the kinematic data and the ground reaction forces. Therefore, different actuator torques and EMG patterns may result for different kinematic data. So far, applicable EMG data are not available for checking the correctness of the proposed control strategy and criterion. Further, the relegation of different control functions to different force actuators is not justified for natural systems. No evidence for such a control strategy exists. The main goal here is to present a feasible set of musculo-skeletal forces that can be utilized as a guide in electrical or neural muscle stimulation and could serve as a first order approximation in helping the physically impaired to regain mobility.

3.7 Summary

A mathematical model which is capable of quantitative computation of muscle forces of the human-skeletal system was presented in this chapter. Newton-Euler state space formulation was used to derive the equations of motion of the seven link biped system. This formulation is convenient for including arbitrary actuators, connection constraints, contact constraints, nonholonomic constraints, and Lyapunov stability function in this model as in [75]. If the forces of constraint are eliminated by projection, then the dimension of the system is reduced. As a result, a well structured set of equations of motion is obtained for the system.
Fig. 3.11
The input torques of the system.
Fig. 3.11 (cont'd)
Fig. 3.11 (cont'd)
Fig. 3.12 Actuator force patterns of the left leg
Fig. 3.12 (left leg)
Fig. 3.12 (left leg)
Fig. 3.12 (left leg)
Fig. 3.13 Actuator force patterns of the right leg
Fig. 3.13 (right leg)
Fig. 3.13 (right leg)
Fig. 3.13 (right leg)
Nonlinear feedback was employed for stabilizing the system and decoupling the degrees of freedom of the system. The advantage of using nonlinear feedback is that of simplicity. Theoretically, the performance of the biped system with nonlinear feedback is as good as a linear system. So it is possible that the nonlinear control strategy is used by the human body. In practice, there are some problems. First, the masses and moments of inertia are contained in the nonlinear term. Usually, for biped systems it is not easy to accurately measure these parameters. Second, a predictor has to be included to account for computation delay.

A set of kinematic trajectories was developed for a 90° turn to the right. Based on these trajectories and nonlinear control strategy, the required muscle force patterns are obtained. These results can be used to design assistive devices for helping the physically impaired. The present study can help in simulating the turning motion. In conjunction with existing programs in electrical stimulation of muscles that facilitate walking, this approach can also lead to computer analysis of different muscular injuries or atrophies.

To reduce the amount of computation, the muscle model used here is an ideal force actuator which ignores muscle characteristics. A more human-like muscle model which may include muscle dynamics, size principles and stiffness could enhance qualitative results of the estimated muscle forces reported in this chapter. One possible muscle model which includes the property of controllable stiffness is considered in the next chapter.
4.1 Introduction

To improve the performance of the biped and to make its behavior more human-like, it is desirable to develop actuator models with variable and controllable stiffness. In view of the significance of this subject, the issue is independently considered in this chapter. Many movements such as inserting a peg into a hole and installing a light bulb are easily performed by humans, but are generally difficult for robots. One reason is the ability to adaptively adjust the stiffness which considerably simplifies the performance of such tasks. For example suppose one is asked to grasp an object and then carry it along a desired trajectory. While the hand is touching the object, the total degree of stiffness of the forearm muscles is low such that impact effects are minimized. In this sense, the transmission of impact to the rest of the body is eliminated completely. In order to carry the object along the desired trajectory, the stiffness of the forearm muscles can be increased so that the biosystem is relatively insensitive to external disturbance. Hence the ability of a manipulator to react to contact forces or tactile stimuli, which is defined as compliance by Mason [76], is very crucial for the many applications of robot manipulators. Mason suggested that compliance can be implemented either because the control system is programmed to react to force or tactile stimuli, or because of passive compliance inherent in the manipulator linkage or in the servo or drive motor. Therefore, a sophisticated mechanical force generator with controllable stiffness is desirable. The study of how the central nervous system modulates the muscle stiffness can help the design of such a force generator.

In humans, the muscle stiffness is regulated by the central nervous system. Hogan [77, 78] proposed and experimentally proved that an important function of the activity of antagonist muscle groups is to modulate mechanical stiffness. Coactivation of the antagonist muscle pairs will increase the net stiffness of the joint, while the net torque about the joint is not changed. In this manner it is possible for biological systems to implement a programmable stiffness at each joint.
The studies by Huxley [45], Podolsky et al. [79], Morgan [80], and Mason [81] use a crossbridge model to make predictions about the variation of the stiffness under different muscle contraction conditions. Katbab [82] and Bavarian [83] proposed that the gain parameters of the muscle spindle are functions of the operating conditions of muscle. However, the stiffness property of muscle and coactivation of muscles have not been considered in their tuning procedure. Various kinds of muscle models have been suggested, for example Hill [40, 84], Bahler [85], Huxley and Simmons [86], Gottlieb and Agarwal [87], Hatze [88], Zahalak and Heyman [52], Akazawa [89], and Zheng et al [48], in order to interpret the functional characteristics of the muscle. These models range from a simple spring, a viscoelastic component and a force generator to a complex and rather complete myoactuator model which includes recruitment, excitation dynamics and contraction dynamics. However, these models are either too complicated to be implemented in musculo-skeletal models or have no clear correspondence between the muscle model and the microstructure of the muscle fibers.

The purpose of this chapter is to elucidate some mechanisms of the stiffness in the contractile element of muscles. Further, from the engineering point of view it is desired to design a mechanical model of a muscle which provides the capability of controllable stiffness. Based on electron microscope and X-ray diffraction findings of the ultra-structures in muscle fiber by H.E. Huxley [90] and the sliding filament theory by A.F. Huxley et al. [91], the stiffness of the muscle is studied. The results of this investigation are presented in Section 4.2. A corresponding mechanical model is proposed in Section 4.3. A planar forearm model is considered in Section 4.4 as a simple example of a neuromuscular control application.

### 4.2 Theoretical Development

In this study, the stiffness, \( S \), of the skeletal muscle is defined [92], as the instantaneous force change per unit length change, i.e.,

\[
S = \frac{dF}{dT}
\]  

(4.1)

This type of stiffness is categorized as series stiffness of muscle by Zahalak [92]. Soechting et al. [93] and Zahalak et al [94] used the isotonic quick release method, while Neilson [95] and Zahalak and Heyman [52] used frequency response technique for measuring the stiffness. The stiffness measured in isotonic quick-release tests is apparently a property of muscle because the measurements were made over a very short time interval, and the muscle reflexes could not significantly affect the stiffness measurements. In the frequency-response tests, some reflex loop modulations are unavoidably
present. However, the results of quick-release tests coincide approximately with those measured by frequency-response tests [92]. This implies that stiffness is an intrinsic property of the muscle.

The series stiffness of the isolated muscle resides partly in the tendon and partly in the contractile tissue. In this section, only the stiffness generated by the contractile element is modeled. Electron microscopic and X-ray diffraction findings of the ultra-structure of muscle fibers and the sliding filament theory [45] are used to develop the model. According to the sliding filament theory, muscle contractile force is generated by interacting myosin filament crossbridges themselves. So, crossbridge attachments directly affect the stiffness of the muscle. From the electron density maps shown in [96], the relative mass of the actin with respect to the myosin filaments (A/M), can be measured for different states of the muscle. In relaxed, active, and rigor states, the estimated A/M values are .2, .35 -.5, and .55 -.6 respectively [97]. Because of crossbridge attachments the variations in the relative mass A/M are due to a reduction of mass on each myosin filament and a corresponding increase of mass on each actin filament. Since these attachments impede the relative filament movements, the muscle stiffness is proportional to the number of active crossbridges at any time.

Based on the above discussions, a theoretical development of muscle stiffness is presented. If $F_N$ is the total normal force generated by the crossbridges, (Figure 4.1), the stiffness can be modelled as a linear function of $F_N$:

$$S = aF_N + b \quad (4.2)$$

where $a$ and $b$ are proportional constants.

This is because that the normal forces generated by the crossbridges increase the pressure between filaments so that the movements between filaments become more restrained. To derive the relationship between $F_N$ and muscle contractile force $F$, let $F_i$ be the force generated by the $i$th fiber of the muscle. Thus, $F$ is the summation of the longitudinal components of all $F_i$'s, while $F_N$ is the summation of the normal components of all $F_i$'s. The ratio of $F_N$ and $F$ is

$$\frac{F_N}{F} = \frac{\sum_i F_i \sin \phi_i}{2 \sum_i F_i \cos \phi_i} \quad (4.3)$$

The coefficient 2 in the denominator is presented because both the upper and lower halves of the muscle generate $F$ and the angle $\phi_i$ is indicated in Figure 4.1. If it is assumed that each muscle fiber
Fig. 4.1 The force diagram of muscle fibers
generates the same force $F_i$, then $F_i$ in Eq. (4.3) can be cancelled.
Since a muscle consists of many fibers, the summation can be
approximated as an integration. Eq. (4.3) now becomes

$$\frac{F_N}{F} = \frac{\int_{0}^{\phi} \sin \phi_i \, d\phi_i}{2 \int_{0}^{\phi} \cos \phi_i \, d\phi_i}$$

Integrating Eq. (4.4) yields

$$F_N = \left(\frac{1 - \cos \phi}{\sin \phi}\right) F$$

If, for most skeletal muscles, the angle $\phi$ is small then a Taylor
series expansion can be used to simplify Eq. (4.5) rendering

$$F_N = \frac{\phi}{4} F$$

Substituting Eq. (4.6) into (4.2), then

$$S = a' \phi F + b$$

From the above equation, it can be seen that the intrinsic muscle
stiffness is a function of both $\phi$ and $F$. If the muscle contractile
force $F$ equals to zero, it becomes evident that $b$ is the stiffness of
relaxed muscle and its surrounding tissue. Thus $b$ is the passive
stiffness of the muscle. Based on the sliding filament theory, it is
obvious that the stiffness of a muscle increases with increasing
muscular contraction. The proportionality of stiffness to $\phi$ in Eq.
(4.7) is analogous to the spring constant being a function of the cross
section of the spring wire. A muscle with a large $\phi$ corresponds to a
stiff spring, while a muscle with a small $\phi$ corresponds to a soft
spring. Usually, the variation of $\phi$ under contraction is not
significant. Therefore intrinsic stiffness is a function of $F$ if the
variation of $\phi$ is ignored.

From the experimental data obtained by Cannon and Zahalak [98],
the values of $a'$ and $b$ for both extensor and flexor muscles of the human
forearm are calculated. The stiffness estimated in the former study
is rotational stiffness, $K$, which is defined as

$$K = \frac{dM}{d\theta}$$

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where \( M \) is the muscle moment and \( \theta \) is the angular position of the forearm as shown in Figure 4.2. The relation between the muscle length change and the angular position change is

\[
dl = r d\theta
\]  

(4.9)

where \( r \) is the moment arm of both extensor and flexor muscles. Substituting Eq. (4.9) into Eq. (4.8) renders

\[
K = \frac{rdF}{dl/r}
\]  

(4.10)

By using Eq. (4.7), it implies

\[
K = (ra'\phi) M + (r^2b)
\]  

(4.11)

The values of \( r, \phi, a' \) and \( b \) for the extensor and flexor muscles are obtained from [98] and listed in Table 4.1. The values of proportionality factor, \( a'' \), were estimated by using the least square approximation method. Figure 4.3 shows the value of \( K \) for the extensor and flexor muscles versus moment \( M \) with the assumption that \( \phi \) is constant.

### 4.3 A Mechanical Model with Controllable Stiffness

Based on the previous section, a mechanical model of a skeletal muscle can be developed. The three main considerations for the development of such a model are as follows. First, this model should have the capability of controlling its stiffness. Second, the model should be simple and easy to construct. Third, the model should be applicable to a muscle-limb system with neuromuscular control. Taking account of the above considerations, the microstructure of the muscle fibers, and the sliding filament theory, a mechanical muscle model is considered as shown in Figure 4.4. The longitudinal force generator \( F \) in the model corresponds to a sliding-force generator at the sites of the myofilament overlap. The force generator suggested by Huxley and Simmons [86] or by Akazawa [89] can be employed.

The normal force generator \( F'N \) in the model corresponds to the normal forces generated by the crossbridge attachments. A spring with spring constant \( k \) is installed between two myosin filaments for maintaining a fixed gap when \( F \) equals zero. Because of this restoring spring, an additional force is needed for \( F'N \).

The relationship between \( F \) and \( F'N \) is

\[
F'N = \frac{\phi}{4} F + F_k
\]  

(4.12)
Fig. 4.2 The planar forearm model
<table>
<thead>
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<th>Parameter</th>
<th>Muscle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Extensor</td>
</tr>
<tr>
<td>r</td>
<td>.262 m</td>
</tr>
<tr>
<td>φ</td>
<td>.2094 rad.</td>
</tr>
<tr>
<td>a'</td>
<td>76.554</td>
</tr>
<tr>
<td>b</td>
<td>5.15</td>
</tr>
</tbody>
</table>

Table 4.1  Numerical parameters of normal human forearm
Fig. 4.3 Rotational stiffness for the extensor and flexor muscles versus moment
Fig. 4.4 A mechanical model of muscle
where $F_k$ is the force generated by the spring and is proportional to the length variation of the spring. From Eq. (4.2), the stiffness of this mechanical muscle, $S_m$, is

$$S_m = a(\phi F + F_k) + b. \quad (4.13)$$

The inputs of the muscle model are the stimulation rate from the central nervous system, muscle length, the rate of muscle length change, and the angle $\phi$. However $\phi$ is a function of muscle length. The outputs of the model are muscle contractile force and stiffness.

Theoretically, the filaments in the muscle could be realized in the mechanical model by many layers of wave-like plastic material. When this wave-like material is compressed by $F'_N$, the stiffness and viscosity of the system will increase. This is similar to the elastic and viscous properties of a muscle which vary with contractile activity of the muscle. In order to avoid the high heat generated by relative motion of plastic like material, lubricating fluid would be necessary between the layers of plastic material.

The mechanical properties of the muscle's passive elements such as tendon, membrane, sarcoplasm and connective tissue are represented by a viscoelastic component which is in series with the contractile element as suggested by Pinto and Fung [41] and Glantz [42].

### 4.4 Neuromuscular Control Applications

A planar forearm model in the sagittal plane with the upper arm fixed is considered here to demonstrate a neuromuscular control application of the mechanical muscle model developed in the previous section. In order to keep the system as simple as possible, all muscles acting on the forearm are combined and represented by only one extensor and one flexor muscle as shown in Figure 4.2. The forearm and hand is modeled as one rigid body having inertia $I$ and mass $m$ and rotating about a fixed axis. The angle-dependent variation of the moment arms of the extensor and flexor muscles are ignored. Hence moment arms have the constant value $r$. The angular displacement $\theta$ fully describes the motion of the model. Here $\theta$ is defined as zero when the forearm is in a vertical position. The Newtonian equation describing the forearm's motion is

$$I\ddot{\theta} = rF_e - rF_f + mg l \sin \theta \quad (4.14)$$

where $g$ is the acceleration due to gravity, $l$ is the distance from the center of gravity to the axis of rotation, and $F_e$ and $F_f$ are the forces generated by the extensor and flexor muscles, respectively.
A voluntary movement with a desired angular position \( \theta_d \), velocity \( \dot{\theta}_d \), and acceleration \( \ddot{\theta}_d \) is considered. For the details of how the central nervous system may generate \( \theta_d \), \( \dot{\theta}_d \), and \( \ddot{\theta}_d \), see [99]. The bias input forces \( F_e \) and \( F_f \) are required in order to guide the system to follow the desired trajectory. The inverse system is used to compute and produce the desired inputs to the musculoskeletal system. This means that the inverse system has replaced higher control centers, i.e., the spinal circuits, the midbrain, and the basal ganglia [100]. From Eq. (4.14), it is clear that only \( F_e - F_f \) can be obtained by the inverse system. Thus, one more condition has to be added in order to obtain unique bias forces for each muscle.

Because of gravitational effect, the forearm system is an unstable system. Thus, the antagonist muscle pair not only has to guide the motion but also has to stabilize the system. The two main components of this postural stability mechanism are the stretch reflex feedback loop and the bias inputs. Recent findings of Gottlieb and Agarwal [101] indicated that the main functions of the reflex loop of the muscles are load compensatory and sensory-motor integration, and not stability of the overall system. This means that the muscle reflex feedback loops are not adequate for postural stability. Hence, the bias inputs of the extensor and flexor muscles have to be utilized to stabilize the system which requires a high degree of stiffness of these muscles. This is in agreement with the observation by Hogan [77] that the neural input to a muscle simultaneously determines the contractile force and the stiffness of the muscle. Therefore, it is believed that an input command to determine the total stiffness of the system is required from the central nervous system. It is known that the total stiffness about a joint is determined by the sum of the stiffness of the extensor and flexor muscle. Thus, the total stiffness of the forearm model is

\[
K_t = K_f + K_e
\]  

(4.15)

Combining Eq. (4.1) and (4.15) renders

\[
K_t = a'_{e}\theta e r^2 F_e + r^2 b_e + a'_{f}\theta f r^2 F_f + r^2 b_f
\]  

(4.16)

With the desired trajectory and the desired total stiffness, \( F_e \) and \( F_f \) can then be determined uniquely.

From the results obtained by Houk [102], the muscle stiffness is not only modulated by the central nervous system but also by the spindle reflex loops. Thus, the action of the neural stretch reflex feedback is included in the system. The stretch reflex is a closed loop consisting of muscle spindle, group Ia afferent fibers, and alpha motoneurons. A muscle spindle, is an adjustable stretch receptor which detects length of the muscle and rate of its change. In addition to the muscle
spindle, the Golgi tendon organs sense the total tension of muscle. Thus these sensors may provide the information needed for regulating the muscle stiffness. In this study, simple position and velocity feedback have replaced the stretch reflex feedback so that the spindle response is

$$T = Ksp \Delta \theta + Bsp \Delta \dot{\theta}$$  \hspace{1cm} (4.17)

where

$$\Delta \theta = \theta - \theta_d.$$

Since the position and velocity sensitivities of muscle spindles are regulated by gamma efferent signals descending from higher nervous centers, the gains of $Ksp$ and $Bsp$ both depend on the gamma efferent. Considering the relationship between the stiffness and the contractile force, the compensation of random load disturbances by the stretch reflex feedback can be thought to maintain the desired stiffness of the system. The block diagram of the system is shown in Figure 4.5.

### 4.5 Summary and Conclusion

The property of muscle stiffness was studied based on the ultra-structure of muscle fibers and the sliding filament theory. It was found that the stiffness of the contractile element of the muscle is proportional to both the angle $\phi$ and the contractile force $F$. Since the variation of $\phi$ is quite small under contraction, the stiffness of the muscle can be considered as a function of contractile force only. This is consistent with the experimental results obtained by Morgan [80]. The numerical values of proportionality constants, i.e., $a'$ and $b$, were estimated by using the least square approximation method of the experimental results obtained by Connon and Zahalak [98].

A mechanical skeletal muscle model was developed. This model consists of two force generators which correspond to the longitudinal and normal forces generated by crossbridge attachments. A spring is installed between myosin filaments for maintaining a fixed gap when the muscle is relaxed. It is suggested that the filaments in the model could be realized theoretically by using many layers of wave-like plastic material. However, the actual material which could be used is not determined and remains an open problem for further investigation. The passive elements of muscle are represented by a viscoelastic component which is in series with the contractile part of the muscle.

Applications to the muscle-limb systems with the neuromuscular control were also discussed. The inverse dynamic system was shown to be insufficient to uniquely determine the required muscle input forces for a planar forearm model. Since the stiffness is directly related to the muscle force, it was suggested that the total stiffness about a joint be
Fig. 4.5 The theoretical neuromuscular control mechanisms of the planar forearm model
the result of an input command from the central nervous system. Many experiments have shown that the stiffness about a joint can be adjusted over a considerable range by the central nervous system. Combining the input command with the inverse system, the required input forces then can be determined uniquely. The stretch reflex feedback was also considered in the control system.
Chapter 5

A MULTIPROCESSOR SYSTEM FOR DYNAMIC CONTROL OF THE SEVEN LINK BIPED

5.1 Introduction

For the real time control of the biped motion studied in Chapter 3, a computer implementation of the control scheme is studied in this chapter. The computation time of the control algorithm is a very crucial factor for successful real time implementation. As mentioned in Chapter 1, the computation of control algorithm must be repeated frequently in order to achieve good stability and performance. In the control of multilink systems, multiprocessor systems have a number of potential advantages over uniprocessors. These advantages include the possibility of modular expansion and increased speed [55].

This chapter describes both the hardware and software design of a multiprocessor system which is dedicated to control the multilink biped system. A multiprocessor-based system must integrate a set of computer modules within an efficient "environment". In a real-time system this environment is usually called a "system executive" and the application programs are referred to as "tasks". Silverman et al. [54], Fathi and Krieger [103] and Heath [104] discussed the design of real-time executives for multiprocessor systems. However, none of them discussed how to use the designated multiprocessor to control a multilink system.

The design goals for the multiprocessor executive are: simplicity, flexibility, high operating speed, and small system overhead. Several approaches have been taken to meet these goals. The simplicity and flexibility can be achieved by modular design. The executive is composed of sub-executives which are identical and reside in each individual processor. Since the executive is designed to dedicate to the implementation of multilink system control, high operating speed and small system overhead will then result.

In Section 5.2, both the hardware and software design of the multiprocessor system are presented. The task decomposition and task
scheduling are studied in Section 5.3 for implementation of the control
algorithm in the multiprocessor system. The effectiveness of the
multiprocessor system is studied in Section 5.3 as well.

5.2 Design of the Multiprocessor System

Two aspects in the design of the multiprocessor system are
included in this section. These are the system hardware and software
executive. From the hardware point of view, a multiprocessor system can
be either tightly coupled, meaning that communication between processors
may occur through shared memory, or loosely coupled, meaning that
communication between different processors is accomplished through
input/output channels. For the purpose of simplicity, flexibility, and
modularity of the hardware design, a loosely coupled system was chosen.
Based on the above considerations and the available laboratory
resources, the system hardware architecture is designed as follows.

A PDP-11/23 computer is chosen for the central processor and
PDP-11/03's are used to serve as satellite processors. Each satellite
processor has data links with the central processor and its adjacent
satellite processors. The organization of the system is shown in Figure
5.1. The reason for such a connection is because this multiprocessor is
dedicated to control the multilink system. In multilink systems,
direct dynamic coupling only exists between adjacent links. Since each
satellite processor will be assigned to control one link, connections
are only needed for adjacent satellite processors. A DRV11 parallel
interface unit manufactured by Digital Equipment Corporation is used as
a communication channel between processors. From Figure 5.1, one may
notice that only the PDP-11/23 is associated with a terminal, disk
drive, and printer. These peripheral devices will be used for
communication between the system and the external world.

5.2.1 Structure of Executive

In the design of the executive, simplicity and flexibility are
emphasized. No matter in which processor the task resides, the
directive requiring executive services should be the same. In other
words, from the tasks point of view, the environment of the
multiprocessor system is the same as that of a uniprocessor system. The
flexibility arises from the consideration that the executive can be
modified and expanded. As a result, the executive is easily adapted to
control different multilink systems.

To provide simplicity and flexibility, the structure of the
executive has the following features:
Fig. 5.1 The hardware architecture of the multiprocessor system
1. The executive is composed of identical sub-executives each residing in the satellite processors.
2. Each sub-executive has a multilevel hierarchical structure.
3. The sub-executive is designed in a modular fashion.
4. Interprocessor communication provides linkage between sub-executives and merges the local environments created by individual sub-executives into a global environment.

The executive also includes the system manager and task managers which will be discussed in the next section. In summary, the structure of the executive can be expressed as shown in Figure 5.2.

5.2.2 System manager and task manager

The function of the system manager and the task manager is to provide channels for user tasks to enter an execution environment. The system manager is developed to reside in the central processor and the task manager is developed to reside in each satellite processor. The system manager is further divided into two portions: task allocator and task loader. The task allocator is used to communicate with the external world via the central processor and the task loader is used to communicate with satellite processors.

Each task is originally stored in the secondary memory; i.e., the disc storage of the PDP 11/23 computer, with a file name. The system user should indicate to the task allocator which task should be down loaded to satellite processor by identifying the file name. The task allocator then finds the task from the secondary memory and transfers it to the main memory of the central processor. The task allocator also needs other information from the user such as the task ID which determines the priority of execution and the satellite processor to which the task will be assigned. After all the tasks are transferred, a task/processor table is created which indicates the locations of all the tasks in satellite processors. The table is down loaded to satellite processors. This table is needed by each sub-executive for directing messages between tasks.

The task loader cooperates with the task manager in each satellite processor to down load tasks and the task/processor table to the memories of individual satellite processors. After all the tasks are loaded into the local memories, the task manager initializes the execution of the tasks.

5.2.3 Hierarchical Structure of Sub-Executives

The main feature of the sub-executive is the hierarchical structure. The hierarchical concept was first used by Dykstra [105] to
design an operating system. Today it has become the most appealing design concept [106]. The hierarchical approach consists of breaking the executive into a number of layers each built on top of lower layers. The bottom layer is the hardware; the highest layer is the task management. Between them are the memory, processor, and interprocessor communication managements. The block diagram of a sub-executive is shown in Fig. 5.3. User tasks ride on the top of the task management. Each management consists of several service routines. The routines on each layer can only request the service of those on lower layers. The processor management provides processor multiplexing as well as basic primitive operations. This module has the same structure as a basic processor management [107]. Therefore, detailed discussion of the module is not presented here. The memory management modules perform the operations necessary for dynamic allocation and freeing of memory. Since this module has a basic structure of a general memory management the details are not further discussed here. The task management and interprocessor communication management modules are more complex than the memory management module. The detailed structures are presented in the following subsections.

Task Management Module

A task-scheduler is included in the task management. The task-scheduler design may be grouped into two general categories [104]: preemptive and non-preemptive. A task running under a preemptive scheduler may be suspended if a task of higher priority is awakened by an interrupt handler. By contrast, a task running under a non-preemptive scheduler may not be suspended even though a task of higher priority is awakened. Except for interrupt servicing, the running task has control of the processor until it is finished. An objective of this design is to keep the executive as simple as possible so that executive overhead is small. In addition, the multibody control application does not require the immediate processing of data received from interrupt handlers. As a result, a non-preemptive type scheduler is chosen.

At any time, a task is in one of the following states within the executive:

1. Sleeping state - the task only exists in the PDP-11/23 hard disk. It is not scheduled for execution.
2. Ready state - the task is ready to be executed. A sleeping state may move into a ready state by a "wake up" signal.
3. Waiting state - a ready task that needs input parameters which have not come yet. The waiting state will be moved into the ready state after the required parameters arrive.
4. Running state - the task is in current control of one of the processors in the system.

A state transition diagram, with all the actions taken by the executive is shown in Figure 5.4.
Fig. 5.2 The block diagram of the executive structure
Fig. 5.3 The block diagram of hierarchical structure of the sub-executive
Fig. 5.4 The state transition diagram end executive service routines
The status of each task is maintained in a task control block. For the purpose of convenience, all the TCB's in the same state are linked together. The scheduler and each task maintain their own stack areas. When a task is interrupted, its content is stored on its stack. Similarly, when the scheduler transfers control to a task, the scheduler's content is stored on its stack. The scheduler is composed of four routines: WAKE-UP, RUN, WAIT, and SLEEP. In addition to the scheduler, two routines, SEND-A-MESSAGE and READ-A-MESSAGE, are also included in the task management model. The functions of the above six routines are described by their names and need no discussion.

The task to be activated may or may not reside in the same local memory as the task management. The same situation happens when a task wants to send a message to another task. A mechanism called service transfer is developed to solve this type of problem. As it is known, each processor has a sub-executive that resides in its local memory. Whenever a task requests the service of a task management routine, e.g., SEND-A-MESSAGE, the routine must first try to find the location of the destination task. If the destination task is found in the local memory, the routine is then continued. If the task is found from the task-processor table, residing in another processor's local memory, then the routine is interrupted by the interprocessor communication module. After the specified information has been transferred, the SEND-A-MESSAGE routine in the sub-executive of the destination processor continues. The restarting point of the routine is the point where this routine was interrupted. However, two routines which reside in two different local memories are involved in completing the service. The flowchart of a SEND-A-MESSAGE routine is shown in Fig. 5.5. As it can be seen, the interprocessor communication module works like a bridge linking all sub-executives together.

Interprocessor Communication Management Module

This module consists of two routines: 1. INTERPROCESSOR-COMMUNICATION-SENDING, 2. INTERPROCESSOR-COMMUNICATION-RECEIVING as shown in Fig. 5.5. When a routine in the task management module recognizes that the destination task resides in the other processor's local memory, the INTERPROCESSOR-COMMUNICATION-SENDING routine is called. This routine activates that interprocessor interrupt line which is one of the lines composing the parallel interface unit. The interrupted processor goes into the interrupt service routine which is the INTERPROCESSOR-COMMUNICATION-RECEIVING routine. After the interrupted processor acknowledges the interrupt, two routines begin an asynchronous handshake procedure to transfer the information. A parallel interface exists between every adjacent processor, only the processor where the destination task resides is interrupted. The rest of the system retains the normal activity. The message is sent word by word and checksum is used to detect transmission errors. If a transmission error is detected, then a block of words is retransmitted.
The flowchart of SEND-A-MESSAGE routines

Fig. 5.5 The flowchart of SEND-A-MESSAGE routines
5.3 Task Decomposition and Scheduling For Torque/Force Computation

How to use the designed multiprocessor system to dynamic control of the seven link biped is studied in this section. The control scheme studied in Chapter 3 is used for position control of the multilink biped system. Position control of the system involves computing the correct input torques applied to the joints for a set of desired position, velocities, accelerations and ground reaction forces.

Many different control schemes as mentioned in Section 2.2 have been suggested. Since the resolved-acceleration control [16] is not simple to implement and other methods using linear feedback control require adaptation or adjustment of feedback gains, a nonlinear feedback control scheme is used in Chapter 3 for position control of multilink systems. The structure of the nonlinear feedback control law is described in Eq. (3.19).

To efficiently use a multiprocessor system, it is necessary to decompose the application program into a number of tasks. These tasks must be distributed among the processors so as to minimize computation time. The computation time defined here is the time from the data acquisition of joint positions and velocities to the time when the desired torques are obtained.

Another problem that has to be considered in task scheduling is the amount of data communication between processors. In multiprocessor systems, communication times are generally much slower than instruction execution times, especially for a loosely coupled system. Therefore, an optimization based on maximum parallel operation may not be at all optimal when communication times are taken into account, especially if a large amount of data is to be shared.

Let define \( F_i \) and \( \bar{N}_i \) are the total external force and moment acting on the ith link respectively. From Eq. (A2), one obtains

\[
\begin{align*}
\bar{F}_i &= M_i \ddot{x}_i \\
\bar{N}_i &= J_i \dot{\omega}_i - f(\omega_i)
\end{align*}
\]  

(5.1)

In [56], a method of variable branch-and-bound has been developed which determines an optimum ordered schedule for each processor. This method is based on the inverse dynamics computation developed in [1] which involves a forward recursion from the base to the open-end of the chain of the multibody system to compute \( \bar{F}_i \)'s and \( \bar{N}_i \)'s and then a backward recursion to compute \( T_i \)'s where \( T_i \) is the input torque at ith joint. However, in this chapter the torque computation method described
in [8] is employed for the following reasons: first, the forward recursive relation for computation of $\bar{R}_i$'s does not exist; all the $\bar{R}_i$'s can be computed in parallel. Second, the recursive relation for computation of $\bar{F}_i$'s can be simplified to two steps of a serial precedence. According to [8], the algorithm for calculating the $T_i$'s needs a large amount of information from the other links. Then, if the calculation of $T_i$ is assigned in each satellite processor, heavy communication between satellite processors is unavoidable. An alternative approach is to assign to the central processor the calculation of $T_i$'s and to each satellite processor the calculation of $\bar{F}_i$ and $\bar{N}_i$ of its corresponding link. The procedures of calculating $\bar{F}_i$ and $\bar{N}_i$ are exactly the same for each link. As a result, the software for each satellite processor is uniform and heavy communication between satellite processors is reduced.

Based on the above assignments for satellite processors and the central processor, task decomposition and task ordering for each satellite processor are as follows.

**Task 1.** Data acquisition of $\phi_i$ and $\dot{\phi}_i$.

It is assumed here that a multilink system has a sufficient number of sensors to determine at any time the joint positions, $\phi_i$'s, and velocities, $\dot{\phi}_i$'s.

**Task 2.** Compute $\ddot{\phi}_i = \ddot{\phi}_{di} + K_v(\dot{\phi}_{di} - \dot{\phi}_i) + K_p(\phi_{di} - \phi_i)$, where $K_v$ and $K_p$ are the $(i, i)$ elements of $K_v$ and $K_p$ respectively. In this study, $K_p$ and $K_v$ are chosen to be diagonal matrices with positive elements for stabilizing and decoupling the system.

**Task 3.** Calculate $\theta_i$, $\dot{\theta}_i$, and $\ddot{\theta}_i$ and send $\theta_i$ to the central processor. The relationship between the vector $\phi$ and $\theta$ can be found in [8].

**Task 4.** Calculate $\check{W}_i$ by Eq. (A1).

**Task 5.** Calculate $\dot{\check{W}}_i$ by taking time derivative of both sides of Eq. (A1).

The following tasks deal with the calculation of $\bar{F}_i$ and $\bar{N}_i$. In order to calculate $\bar{F}_i$, one needs to know $X_i$. For any two connected link $i$ and $i+1$, there exists a holonomic connection constraint (see Figure 6) as mentioned in Eq. (3.2)

Differentiating Eq. (3.2) with respect to time twice renders

$$
\ddot{X}_{i+1} + A_{i+1}(WW_{i+1})^2K_{i+1} - A_{i+1}KK_{i+1}\dot{W}_{i+1} - [\ddot{X}_i + A_i(WW_i)^2L_i - A_iLL_i\dot{W}_i] = 0
$$

(5.2)
where $WW_i$ is a $3 \times 3$ skew symmetric matrix whose elements are related to the vector $\dot{W}_i$ as defined in (A7). The terms of $KK_i$ and $LL_i$ are similarly defined. From Eq. (5.2), the computation of $F_i$ can be divided into the following tasks.

**Task 6.** Calculate $AWK_i$ and $AWL_i$ and send $AWL_i$ to the next satellite processor.

where

$$AWK_i = A_i(WW_i)^2K_i - A_iKK_i\dot{W}_i \quad (5.3)$$

and

$$AWL_i = A_i(WW_i)^2L_i - A_iLL_i\dot{W}_i \quad (5.4)$$

**Task 7.** Wait for $\bar{x}_{i-1}$ from the previous satellite processor.

**Task 8.** Calculate $\bar{x}_i$ by Eq. (A2) and send it to the next satellite processor.

**Task 9.** Calculate $F_i$ by Eq. (5.1) and send $F_i$ to the central processor.

**Task 10.** Calculate $\bar{R}_i$ by Eq. (5.1) and send it to the central processor.

The formula for computing $T_i$ is [8]

$$T_i = A_i^TA_iA_i^T(\bar{R}_i + F_i - KK_i\Gamma_i + LL_iA_i^T\Gamma_{i+1}) \quad (5.5)$$

where

$$\Gamma_i = F_i + F_{i+1} + \ldots + F_n + m_ig + \ldots + m_ng \quad (5.6)$$

The task decomposition and ordering for the central processor are as follows.

**Task 1.** Calculate $A_i^TA_i$, $KK_iA_i^T$ and $LL_iA_i^T$.

**Task 2.** Get $F_i$'s from satellite processors.

**Task 3.** Compute $\Gamma_i$'s by Eq. (5.6).

**Task 4.** Get $\bar{R}_i$'s from satellite processors.

**Task 5.** Compute $T_i$'s by Eq. (5.5).
Fig. 5.6 The two link robot system
The main goal of this chapter is to utilize a multiprocessor system for dynamic control of the seven link biped. From the previous study, seven satellite processors and one central processor are required for dynamic control of the seven link biped. Due to the limitation of our laboratory facilities, the implementation of a multiprocessor which could control the seven link biped was not possible. Thus, a two link robot system (Fig. 5.6) is used to test the effectiveness of the task decomposition and scheduling developed previously. A Fortran program has been written for the computation of the desired input torques. The computation time is about 47.1 mseconds, of which 5.8 mseconds were spent on transferring information. For a uniprocessor system, the computation time is about 91.4 mseconds. So, the computation time is reduced about half by using one central processor and two satellite processors. It can be further reduced if floating point hardware processors are used for arithmetic computations.

Further expansion of the present multiprocessor system for dynamic control of the seven link biped can be developed as follows. Seven satellite processors and one central processor will be required. The interconnection of these processors will be the same as Figure 5.1. The previously discussed software executive can be employed to this multiprocessor system. The concept of the task decomposition and scheduling for position control of multilink systems can be directly extended to the seven link biped system. The computation times for the seven link biped can be estimated as follows. For the multiprocessor system with eight processors, the computation time is about 101 mseconds. About 25 mseconds were spent on data transformation between each of the seven satellite processors and the central processor. About 52 mseconds were spent for the computation of the applied torques. During this time interval each of the satellite processors spent 33 mseconds in computation of the external forces and moments. A final segment of 24 mseconds was needed to calculate the final applied torques in the central processor:

\[ 25 + 52 + 24 = 101 \text{ mseconds} \]

The corresponding total time for a uniprocessor is

\[ 7 \times 33 + 52 + 24 = 307 \text{ mseconds} \]

5.4 Summary and Conclusions

A multiprocessor which is dedicated to the dynamic control of multilink systems was described. Based on the dynamic method and control scheme discussed in Chapter 3, the issues of task decomposition and task scheduling were developed. One satellite processor was
assigned to each link of a multilink system to calculate the total external forces and moments of that link. The central processor was assigned to compute the applied torques of all links. For real time applications, computation times for the multiprocessor system and a uniprocessor system were estimated.
Chapter 6

SUMMARY AND FUTURE STUDIES

6.1 Introduction

An on-line strategy to control a multilink biped has been studied in this dissertation. Three issues were considered: dynamics of the musculo-skeletal system, actuators with variable stiffness, and a multiprocessor for distributed on-line control implementation. The results of this work are applicable to control of robots, development of prosthetic and orthotic devices, and the design of better mechanisms for functional electrical stimulation. In section 6.2 the results and contributions of this dissertation are summarized. The suggestions for further studies are presented in section 6.3.

6.2 Summary

In this study, a mathematical model for quantitatively computing the muscle forces of a musculoskeletal system has been presented. Kinematic models, skeletal dynamics, muscle models, and neural control were included. It was assumed that the bias torques are all zero at the initial and final posture because the biped was in a symmetric double static stance. A feasible set of muscle force patterns for the desired turning motion were obtained. It has been found that the forces are very small at the initial, middle, and final stages of the motion because of double stance phases. These results can be used to design assistive devices for helping the handicapped in bringing about turning. In conjunction with existing results of EMG studies of human muscles during walking, this approach can also lead to computer analysis of different muscular injuries and atrophies.

A mechanical model of skeletal muscle is proposed in Chapter 4. This model has the capability of controlling its stiffness. The property of the muscle stiffness is studied. It was found that the stiffness of the contractile element of the muscle is proportional to both the angle $\phi$ and the contractile force $F$ of the muscle. In Chapter 4 it was shown that the variation of $\phi$ is quite small, the stiffness of the muscle can be considered as a function of $F$ only.
A planar forearm model was used to study the application of the muscle model in the neuromuscular control system. The inverse system was shown to be insufficient to uniquely determine the desired muscle forces. Since the neural input to a muscle simultaneously determines contractile force and the stiffness of the muscle, it was suggested that the total stiffness about a joint is also an input command from the central nervous system. Combining the stiffness input and the inverse system, the required input forces can then be determined uniquely.

A multiprocessor system dedicated to the dynamic control of the multilink system was designed in Chapter 5. The system hardware and software executive were developed in a modular fashion. The system executive consisted of sub-executives linked by the system manager and task manager. This executive provided an entire environment to user tasks.

Based on the Newton-Euler state space formulation and nonlinear feedback control scheme, the issue of task decomposition and task scheduling of the control algorithm was developed. One satellite processor was assigned to each link of the multilink system for calculating the external forces and moments of that link. The central processor was dedicated to compute the required torques of all the links. The reason for these assignments was to maintain the software transparency of each satellite processor and reduce data communication between processors.

6.3 Further Studies

It is recognized that to make the results of this study more applicable further investigations are needed. The following are suggestions for further extension of this work and for additional future studies. The following are direct extensions of this:

1. Integration of the methodologies of Chapter 3, 4, and 5 in a comprehensive model. In Chapter 3 the human turning motion was studied by using ideal force actuators as the muscles in the human lower extremities. The results reported in Chapter 3 can be improved by implementing the muscle model developed in Chapter 4. This further study would allow the coactivation of muscles to improve disturbance rejection and stability during human turning.

2. Application to neural and muscular electrical stimulation to help the physically handicapped.

In addition other areas of research are suggested:

1. Since there is presently no convenient experimental method for direct quantitative measurement of muscle forces, EMG data can
serve as a qualitative measure. To date applicable EMG data for human turning is not available. It is necessary to obtain such data in order to further study suitable criteria for solving uniquely the muscle forces because more than one set of mathematical solutions for muscle forces are possible. The criteria used should have physical meanings. The criteria may include the minimization of performance indices $J_1$, $J_2$, $J_3$ and $J_4$ introduced by Pedotti et al. [36].

2. It was suggested that the filaments in the model presented in Chapter 4 could be realized by many layers of wave-like material with specific properties which would include low friction and viscoelasticity which should approximately satisfy Eq. (4.7). However, the actual material to be used is still undetermined. This is an area that should be considered for further investigation.

3. In Chapter 4 the relationship between the stiffness of muscle and its angle $\phi$ and contraction force $F$ was derived. However, although the control mechanism of the central nervous system that may regulate the muscle stiffness is still unknown, it has been shown that Golgi tendon organs, located in the muscle tendons sense the total tensions of muscle. In addition to the Golgi tendon organs, the muscle spindle senses the muscle length and the rate of muscle length change. Thus these sensors may also provide the information needed for regulating the muscle stiffness. How this sensory information is processed by the central nervous system should be studied in more detail.

4. In order to have a better performance for the real time implementation discussed in Chapter 5, a further reduction of the computation time of the control algorithm is required. The following two approaches may be utilized to achieve such an improvement. First, a tightly coupled multiprocessor system should be used for the real time implementation of the control algorithm. A commercially available multiprocessor system with shared memory could be employed. No data transfer is necessary between processors in a multiprocessor system with shared memory. Thus the data communication time for tightly coupled systems is much faster than that of the loosely coupled system. Second, the use of floating point hardware processors for arithmetic computations are suggested.
APPENDIX A

DYNAMICS OF THE SEVEN-LINK BIPED SYSTEM

The Newton-Euler formulation developed in [2] is used to derive the equations of motion of the seven-link biped system. An inertial coordinate system and a body coordinate system for each link are employed in this method. The translation of the center of gravity of the ith link is described in the inertial coordinate system. The rotation of the ith link is described by the vector of Euler angles, \( \theta_i \) and the vector of angular velocity of its principal axes, \( W_i \). The relationship between angular velocity \( W_i \) and the rate of change of Euler angle vector \( \dot{\theta}_i \) is

\[
\begin{bmatrix}
\dot{\theta}_{i1} \\
\dot{\theta}_{i2} \\
\dot{\theta}_{i3}
\end{bmatrix} =
\begin{bmatrix}
-sin\theta_{i3}/cos\theta_{i2} & 0 & -cos\theta_{i3}/cos\theta_{i2} \\
\cos\theta_{i3} & 0 & \sin\theta_{i3} \\
-tan\theta_{i2} \sin\theta_{i3} & 1 & -tan\theta_{i2} \cos\theta_{i3}
\end{bmatrix}
\begin{bmatrix}
W_{i2} \\
W_{i2} \\
W_{i3}
\end{bmatrix}
\]

or

\[
\dot{\theta}_i = B(\theta_i)W_i
\]

With the state variables defined above, the Newton-Euler state space equations of the ith link are

\[
\begin{align*}
\ddot{X}_i &= \ddot{X}_i \\
\ddot{X}_i &= M_i^{-1}(F_i + G_i) \\
\ddot{\theta}_i &= B(\dot{\theta}_i)W_i \\
\ddot{W}_i &= J_i^{-1}[f(W_i) + M_i + N_i]
\end{align*}
\]
where $J_i$ is the moment of inertia matrix, $M_i$ is the identical mass matrix, $F_i$ and $G_i$ are applied and constraint forces expressed in the ICS respectively, and $M_i$ and $N_i$ are applied torques and torques due to holonomic and nonholonomic constraints respectively. The vector $f(W_i)$ is defined as

$$f(W_i) = [W_i W_i^2 (I_{12} - I_{13}) W_i^3 W_i (I_{13} - I_{11}) W_i W_i^2 (I_{11} - I_{22})]^T$$

Two kinds of constraints, holonomic and nonholonomic, are involved in the system. Holonomic constraints result when a prescribed point on the body is restricted to move on a surface, a curve, or be stationary. Nonholonomic constraints result if rotation of the link is restricted along certain axes. Equations (3.3) to (3.8) are 18 permanent holonomic constraints acting on this model. In addition to these holonomic constraints more constraints are applied to the system due to the foot contacting the ground. From Figure 3.3, there are three different cases which have to be considered:

**Case 1.** Single stance phase with left foot on the ground.

In this phase, from 0 to 1.7 seconds as shown in Figure 3.3, the corresponding holonomic constraints are

$$X_1 = c_1$$

where $c_1$ is a constant vector.

**Case 2.** Single stance phase with right foot on the ground.

As shown in Figure 3.3, this phase is from 2.3 to 4 seconds. The holonomic constraints for this case can be expressed as

$$X_7 = c_2$$

**Case 3.** Double stance phase

In this case, both constraint equations (A4) and (A5) have to be applied.

Let $\Gamma$ be the vector of holonomic forces of constraint acting on the body at the joints connecting to other links or the points contacting the ground. If $\Gamma$ is described in ICS, the moment of these forces with respect to the center of gravity is

$$N = LLAT(\theta)\Gamma$$

or

$$N = KKAT(\theta)\Gamma$$

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where \( L \) and \( K \) are the skew symmetric matrices of \( L \) and \( K \) respectively. The matrix \( K \) is defined as

\[
K = \begin{bmatrix}
0 & -k_3 & k_2 \\
k_3 & 0 & -k_1 \\
-k_2 & k_1 & 0
\end{bmatrix}
\]

for \( K = [k_1 \ k_2 \ k_3]^T \) \hspace{1cm} (A7)

The holonomic constraint forces act on the system while the constraints are in effect and are impulsive at the instant of contact. The impulsive forces result in the swing foot hitting the ground and the velocity of the foot changing instantaneously.

Each of the knee joints is assumed to have only one rotational degree of freedom about the \( y \)-axis in ICS. Then nonholonomic constraints result due to this restriction. Let

\[
Q_1 = [0 \ 1 \ 0]^T 
\]

in ICS. The nonholonomic constraints are

\[
W_3 = A_2^T e_2 Q_1 + W_2 \\
W_6 = A_5^T e_3 Q_1 + W_5 
\]

where \( e_2 \) and \( e_3 \) are vectors of proportionality constants.

Let \( R_1^T \) be a 2x3 constant matrix (in ICS) whose rows are orthogonal to vector \( Q_1 \). In this case,

\[
R_1^T = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(Eq. (A9) may be rewritten as

\[
R_1^T (A_2^T A_3 W_3 - W_2) = 0 \\
R_1^T (A_5^T A_6 W_6 - W_5) = 0
\]

(A11)
Each of the ankle joints in the system is assumed to have two rotational degrees of freedom. Similarly, we can have

\[ R^T_2(A_1^T A_2 w_2 - w_1) = 0 \]  
\[ R^T_2(A_6^T A_7 w_7 - w_6) = 0 \]

where

\[ R^T_2 = [0 \ 0 \ 1] \]  \hspace{1cm} (A13)

The nonholonomic constraint forces, the forces which have to be exerted by the constraints in order to compel the system to fulfill such conditions, have to be added to the system.

In addition to holonomic and nonholonomic constraint forces, muscular forces are acting on the system. There are 22 muscles employed in this system as listed in Table 3.2. It is assumed here that muscles are represented by lines. Let \( F_i \) be the total muscular forces acting on the \( i \)th link described in ICS, and let \( T_i \) (in BCS\(_i\)) be the torque generated by \( F_i \). From the musculo-skeletal model shown in Figure 3.6, the total muscular forces and torques acting on each link are as follows.

\[ F_1 = u_9 + u_{10} + u_{11} \]
\[ F_2 = u_2 + u_4 + u_5 + u_8 - u_{10} - u_{11} \]
\[ F_3 = u_1 + u_3 + u_6 + u_7 - u_5 - u_8 - u_9 \]
\[ F_4 = -\tau_1 - \tau_2 - \tau_3 - \tau_4 - \tau_6 - \tau_7 - u_1 - u_2 - u_3 - u_4 - u_6 - u_7 \]
\[ F_5 = \tau_1 + \tau_3 + \tau_6 + \tau_7 - \tau_5 - \tau_8 - \tau_9 \]
\[ F_6 = \tau_2 + \tau_4 + \tau_5 + \tau_8 - \tau_{10} - \tau_{11} \]
\[ F_7 = \tau_9 + \tau_{10} + \tau_{11} \]  \hspace{1cm} (A14)

\[ T_1 = RR^T g A_1 u_9 + RR^T 10 A_1 u_{10} + RR^T 11 A_1 u_{11} \]
\[ T_2 = RR^T 2_2 A_2 u_2 + RR^T 4_2 A_2 u_4 + RR^T 5_2 A_2 u_5 + RR^T 8_2 A_2 u_8 - RR^T 10_2 A_2 u_{10} - RR^T 11_2 A_2 u_{11} \]
\( T_3 = R R^3 A_3^T u_1 + R R^3 A_3^T u_3 + R R^3 A_3^T u_5 + R R^3 A_3^T u_6 + R R^3 A_3^T u_7 \)
\[- R R^3 A_3^T u_8 - R R^3 A_3^T u_9 \]

\( T_4 = -(r r^4 A_4^T 1 + r r^4 A_4^T 2 + r r^4 A_4^T 3 + r r^4 A_4^T 4 + r r^4 A_4^T 6 + r r^4 A_4^T 7 + R R^4 A_4^T u_1 + R R^4 A_4^T u_2 + R R^4 A_4^T u_3 + R R^4 A_4^T u_4 + R R^4 A_4^T u_6 + R R^4 A_4^T u_7) \)

\( T_5 = r r^5 A_5^T 1 + r r^5 A_5^T 3 - r r^5 A_5^T 5 + r r^5 A_5^T 6 + r r^5 A_5^T 7 - r r^5 A_5^T 8 \)
\[- r r^5 A_5^T 9 \]

\( T_6 = r r^6 A_6^T 2 + r r^6 A_6^T 4 + r r^6 A_6^T 5 + r r^6 A_6^T 8 - r r^6 A_6^T 10 \)
\[- r r^6 A_6^T 11 \]

\( T_7 = r r^7 A_7^T 9 + r r^7 A_7^T 10 + r r^7 A_7^T 11 \)

where

\( u_i \) and \( \tau_i \) are the forces generated by the ith muscle of the left and right legs respectively and are expressed in ICS.

\( \mathbf{R R} \) and \( \mathbf{rr} \) are the skew symmetric matrix of a vector from the origin of BCS\(_j\) to the origin or insertion point of the ith muscle at the jth link of left and right legs respectively.

With all in forces discussed above, the equations of motion of the biped system can be written as follows.

\[
\begin{align*}
m \dddot{x}_1 &= [0, 0, -m_1 g]^T + \tau_1 - \tau_2 + \mathbf{f}_1 \\
I_1 \dddot{\mathbf{w}}_1 &= \mathbf{f}_1 + \mathbf{T}_1 + A_1^T A_2 R_2 \Lambda_1 \\
m_2 \dddot{x}_2 &= [0, 0, -m_2 g]^T + \tau_2 - \tau_3 + \mathbf{f}_2 \\
I_1 \dddot{\mathbf{w}}_2 &= \mathbf{f}_2 + LL_2 A_2^T \tau_2 - KK_2 A_2^T \tau_3 + T_2 + A_2^T A_3 R_1 \Lambda_2 - R_2 \Lambda_1
\end{align*}
\]
\[ m_3\dot{x}_3 = [0, 0, -m_3g]^T + \Gamma_3 - \Gamma_4 + F_3 \]

\[ I_3\ddot{w}_3 = f_3 + LL_3\dot{A}_3^T\Gamma_3 - KK_3\dot{A}_3^T\Gamma_4 + T_3 - R_{1A2} \]

\[ m_4\dot{x}_4 = [0, 0, -m_4g]^T + \Gamma_4 - \Gamma_5 + F_4 \]

\[ I_4\ddot{w}_4 = f_4 + LL_4\dot{A}_4^T\Gamma_4 + KK_4\dot{A}_4^T\Gamma_5 + T_4 \]

\[ m_5\dot{x}_5 = [0, 0, -m_5g]^T - \Gamma_5 + \Gamma_6 + F_5 \]

\[ I_5\ddot{w}_5 = f_5 - LL_5\dot{A}_5^T\Gamma_5 + KK_5\dot{A}_5^T\Gamma_6 + T_5 - R_{1A3} \]

\[ m_6\dot{x}_6 = [0, 0, -m_6g]^T - \Gamma_6 + \Gamma_7 + F_6 \]

\[ I_6\ddot{w}_6 = f_6 - LL_6\dot{A}_6^T\Gamma_6 + KK_6\dot{A}_6^T\Gamma_7 + A_6^T\dot{A}_5\Gamma_3 - R_{2A4} + T_6 \]

\[ m_7\dot{x}_7 = [0, 0, -m_7g]^T - \Gamma_7 + \Gamma_8 + F_7 \]

\[ I_7\ddot{w}_7 = f_7 + A_7^T\dot{A}_6 R_{2A4} + T_7 \]

where \( \Gamma_1 \) is in effect when the left leg contacts the ground while \( \Gamma_8 \) is in effect when the right leg contacts the ground.

The above equations can be expressed in more compact form

\[ U_1\ddot{z} = U_2 + U_3\Gamma_1 + U_4\Gamma_3 + U_5 + U_{12} + U_{13} \]  

(A16)

where

\[ U_1 = \text{Diagmat}_{42} \begin{bmatrix} m_1, m_2, m_3, m_4, m_5, m_6, m_7, I_1, I_2, I_3, I_4, I_5, I_6, I_7 \end{bmatrix} \]

\[ M_i = \text{Diagmat}_3 \begin{bmatrix} m_i, m_i, m_i \end{bmatrix} \]

\[ I_i = \text{Diagmat}_3 \begin{bmatrix} I_{i1}, I_{i2}, I_{i3} \end{bmatrix} \]

\[ Z = [\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4, \dot{x}_5, \dot{x}_6, \dot{x}_7, W_1, W_2, W_3, W_4, W_5, W_6, W_7]^T \]

\[ \dot{x}_i = [\dot{x}_{i1}, \dot{x}_{i2}, \dot{x}_{i3}]^T \]

\[ W_i = [W_{i1}, W_{i2}, W_{i3}]^T \]
\[ U_2 = [m_{1g}, m_{2g}, m_{3g}, m_{4g}, m_{5g}, m_{6g}, m_{7g}, f_1, f_2, f_3, f_4, f_5, f_6, f_7]^T \]

\( f_i \) is defined as in Eq. (A3)

\[ G = [0, 0, -g]^T \]

\[
\begin{bmatrix}
-I & 0 & 0 & 0 & 0 & 0 & 0 \\
I & -I & 0 & 0 & 0 & 0 & 0 \\
0 & I & -I & 0 & 0 & 0 & 0 \\
0 & 0 & I & -I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I & I & 0 \\
0 & 0 & 0 & 0 & 0 & -I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -I \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
U_3 = 
\begin{bmatrix}
\text{LL}_2A_2^T & -\text{KK}_2A_2^T & 0 & 0 & 0 & 0 & 0 \\
0 & \text{LL}_3A_3^T & -\text{KK}_3A_3^T & 0 & 0 & 0 & 0 \\
0 & 0 & \text{LL}_4A_4^T & \text{KK}_4A_4^T & 0 & 0 & 0 \\
0 & 0 & 0 & -\text{LL}_5A_5^T & \text{KK}_5A_5^T & 0 & 0 \\
0 & 0 & 0 & 0 & -\text{LL}_6A_6^T & \text{KK}_6A_6^T & 0 \\
0 & 0 & 0 & 0 & 0 & -\text{LL}_7A_7^T & 0 \\
\end{bmatrix}
\]

where

\[ I = \text{Diag}3 [1, 1, 1], \text{a unit matrix}. \]

\[ r = [r_2, r_3, r_4, r_5, r_6, r_7]^T \]

where

\[ r_i = [r_{i1}, r_{i2}, r_{i3}]^T. \]
\[
\begin{align*}
U'_{3} & = \\
& \begin{bmatrix}
I \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \\
U''_{3} & = \\
& \begin{bmatrix}
I \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]

\[
U_{4} = \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{align*}
& A_{2}^{T}A_{3}\bar{R}_{2} \\
& A_{2}^{T}A_{3}\bar{R}_{1} \\
& -\bar{R}_{2} \\
& -\bar{R}_{1} \\
& -\bar{R}_{1} \\
& A_{6}^{T}A_{5}\bar{R}_{1} \\
& A_{7}^{T}A_{6}\bar{R}_{2}
\end{align*}
\]
\[ A = [A_1 \ A_2 \ A_3 \ A_4]^T \]

where

\[ A_i = [A_{i1} \ A_{i2}]^T. \]

\[ \tau = [\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7, \tau_8, \tau_9, \tau_{10}, \tau_{11}, u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}]^T \]

where

\[ \tau_i = [\tau_{i1}, \tau_{i2}, \tau_{i3}]^T \]

\[ u_i = [u_{i1}, u_{i2}, u_{i3}]^T \]

The dimension of the system is 42 now. But the actual degrees of freedom of this system is much less than 42. For the single stance phase, the system has 12 degrees of freedom while the total system degrees of freedom are reduced to 7 for the double stance phase. So it is desirable to project the large dimensional state space onto a smaller subspace. These projection transformations will be discussed as follows.

As mentioned previously, three different cases involved during the turning motion. The project transformations for reducing the dimensionality of the system for these cases will be discussed separately.

Case 1. Single stance phase with left foot on the ground

For this case there are totally 21 holonomic constraints, namely Eqs. (3.3) through (3.8) and Eq. (A4), acting on the system. The first step of projection is to use these constraints and solve for \( X \) in terms of \( \theta \) and \( W \). [64] proved that the corresponding constraint forces can also be eliminated due to projection. Differentiating these holonomic constraint equations with respect to time twice one gets

\[ \ddot{X}_1 - [\ddot{X}_2 + A_2(WW)^2L_2 - A_2LL\dot{W}_2] = 0 \]
\[ \ddot{X}_2 + A_2(WW)^2K_2 - A_2KK\dot{W}_2 = [\ddot{X}_3 + A_3(WW)^2L_3 - A_3LL\dot{W}_3] = 0 \]
\[ \ddot{X}_3 + A_3(WW)^2K_3 - A_3KK\dot{W}_3 = [\ddot{X}_4 + A_4(WW)^2L_4 - A_4LL\dot{W}_4] = 0 \]

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\[ \dddot{x}_4 + A_4(ww_4)^2k_4 - A_4kk_4w_4 - [\dddot{x}_5 + A_5(ww_5)^2l_5 - A_5ll_5w_5] = 0 \]
\[ \dddot{x}_5 + A_5(ww_5)^2k_5 - A_5kk_5w_5 - [\dddot{x}_6 + A_6(ww_6)^2l_6 - A_6ll_6w_6] = 0 \]
\[ \dddot{x}_6 + A_6(ww_6)^2k_6 - A_6kk_6w_6 - \dddot{x}_7 = 0 \]
\[ \dddot{x}_7 = 0 \]

The above equations can be arranged in a more compact form as

\[ U_8 \dddot{w} + U_q \dddot{x} = U_7 \]

where

\[ U_8 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & A_2ll_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -A_2kk_2 & A_3ll_3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -A_3kk_3 & A_4ll_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -A_4kk_4 & A_5ll_5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -A_5kk_5 & A_6ll_6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -A_6kk_6 & 0 & 0 \\
\end{bmatrix} \]

(A17)

\[ U_7 = \begin{bmatrix}
0 \\
A_2(ww_2)^2l_2 \\
-A_2(ww_2)^2k_2 + A_3(ww_3)^2l_3 \\
-A_3(ww_3)^2k_3 + A_4(ww_4)^2l_4 \\
-A_4(ww_4)^2k_4 + A_5(ww_5)^2l_5 \\
-A_5(ww_5)^2k_5 + A_6(ww_6)^2l_6 \\
-A_6(ww_6)^2k_6 \\
\end{bmatrix} \]
Eqs. (A16) can be rewritten as

$$\begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & -I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & -I \end{bmatrix}$$

Substituting (A17) into (A18) renders

$$U_1 \begin{bmatrix} \dot{X} \\ \dot{W} \end{bmatrix} = U_2 \begin{bmatrix} U_3 & U_3 \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} + U_3 \Gamma_8 + U_4 \Lambda + U_5 \tau$$

(A18)

Substituting (A17) into (A18) renders

$$U_1 \begin{bmatrix} -U_q^{-1} U_8 \\ I \end{bmatrix} \dot{W} = U_2 + [U_3^{-1} U_3] \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} + U_3 \Gamma_8 + U_4 \Lambda + U_5 \tau$$

$$+ U_1 \begin{bmatrix} -U_q^{-1} U_7 \\ 0 \end{bmatrix}$$

(A19)

Here I is a 21x21 identity matrix.

Let \( \begin{bmatrix} -U_q^{-1} U_8 \\ I \end{bmatrix} = U_{10}. \)

It is shown in [64] that \( U_{10}^T \) is orthogonal to \( [U_3' U_3] \). Therefore \( \Gamma \) will be eliminated by multiplying both sides of Eq. (A19) by \( U_{10}^T \). So it ends up with
\[ U_{10}^T U_1 U_{10} \dot{W} = U_{10}^T \left\{ U_2 + U_3 \Gamma_8 + U_4 A + U_5 \tau + U_1 \begin{bmatrix} -U^{-1}_q & U_7 \\ 0 & 0 \end{bmatrix} \right\} \]

(A20)

In the second step of projection, we would like to eliminate \( A \), the nonholonomic forces, by replacing state \((\theta, W)\) with a vector of quasi coordinates. In this case, the left foot is on the ground. So the foot has no rotational degree of freedom. From Eqs. (3.9) and the constraints that the left foot is on the ground, one gets

\[
\begin{align*}
\theta_1 &= [0, 0, 0]^T \\
\theta_2 &= [0, \phi_2, \phi_4]^T \\
\theta_3 &= [0, \phi_2, \phi_5]^T \\
\theta_4 &= [\phi_6, \phi_7, 0]^T \\
\theta_5 &= [\phi_8, \phi_9, \phi_{10}]^T \\
\theta_6 &= [\phi_8, \phi_9, \phi_{11}]^T \\
\text{and} \\
\theta_7 &= [\phi_8, 0, \phi_{12}]^T
\end{align*}
\]

(A21)

From Eqs. (A21) and (A1), one can compute

\[ W = W(\phi, \dot{\phi}) \]

(A22)

where \( \phi = [\phi_2, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8, \phi_9, \phi_{10}, \phi_{11}, \phi_{12}]^T \)

Differentiating Eq. (A22) with respect to time one gets

\[ \ddot{W} = U_{11} \ddot{\phi} + U_{12} \]

(A23)

Substituting Eq. (A23) into (A20) renders

\[ U_{10}^T U_1 U_{10} U_{11} \ddot{\phi} = U_{10}^T \left\{ U_2 + U_3 \Gamma_8 + U_4 A + U_5 \tau + U_1 \begin{bmatrix} -U^{-1}_q & U_7 \\ 0 & 0 \end{bmatrix} \right\} \]

\[ -U_1 U_{10} U_{12} \]

(A24)
Multiplying both sides of Eq. (A24) by $U_{11}^{T}$ can eliminate $\Lambda$, then

$$U_{11}^{T}U_{10}^{T}U_{10}U_{11}\ddot{\phi} = U_{11}^{T}U_{10}^{T}(U_{2} + U_{3}^{\prime}\Gamma_{8} + U_{5}\tau + U_{1})\begin{bmatrix} -U_{9}^{-1}U_{7} \\ 0 \end{bmatrix} - U_{1}U_{10}U_{12}$$

(A25)

The above equation describes the dynamic of the system at the single stance phase with left foot on the ground.

Case 2. Single stance phase with right foot on the ground

For this case, the holonomic constraints act on the system on Eqs. (3.3) through (3.8) and Eq. (A5). Following the same procedures of the first step of projection for the single stance phase with left foot on the ground, one gets

$$U_{10}^{T}U_{10}^{T}\ddot{\phi} = U_{10}^{T}(U_{2} + U_{3}^{\prime}\Gamma_{1} + U_{4}\Lambda + U_{5}\tau + U_{1})\begin{bmatrix} -U_{9}^{-1}U_{7} \\ 0 \end{bmatrix}$$

(A26)

where

$$U_{8} = \begin{bmatrix} 0 & A_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -A_{22}^{2} & A_{32} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -A_{32}^{2} & A_{42} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -A_{42}^{2} & A_{52} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -A_{52}^{2} & A_{62} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -A_{52}^{2} & A_{62} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -A_{62}^{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -A_{62}^{2} \end{bmatrix}$$
Similarly, the quasi-coordinates of the second step of projection are

\[ \theta_1 = [\phi_1, 0, \phi_3]^T \]
\[ \theta_2 = [\phi_1, \phi_2, \phi_4]^T \]
\[ \theta_3 = [\phi_1, \phi_2, \phi_5]^T \]
\[ \theta_4 = [\phi_6, \phi_7, 0]^T \]
\[ \theta_5 = [\phi_9, \phi_9, \phi_{10}]^T \]
\[ \theta_6 = [\phi_9, \phi_9, \phi_{11}]^T \]
\[ \theta_7 = [\phi_9, 0, 0]^T \]  

(A27)
Similarly, the dynamic equation of the system at the single stance phase with right foot on the ground can be expressed as

\[ U_{11}^T U_{10}^T U_{10} U_{11} \phi' = U_{11}^T U_{10}^T \{ U_2 + U_{11}^T U_5 + U_1 \begin{bmatrix} -U_{10}^T \end{bmatrix} \} \]

\[-U_{10} U_{10} \}

where

\[ \phi' = [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_9, \phi_{10}, \phi_{11}]^T \]  

(A28)

Case 3. Double Stance Phase

The dynamic equation for the double stance phase has already been included in Eqs. (A25) and (A28), since the constrained motion of these equations corresponds to double stance phase, while the unconstrained motion corresponds to single stance phase. The constraint forces \( \Gamma_8 \) of Eq. (A25) act on the system while the constraints are in effect, i.e., the right foot contacts the ground. Here forces of constraint are ground reaction forces. So \( \Gamma_8 \) is equal to zero when the right foot is not on the ground any more in Eq. (A25). The forces of constraint \( \Gamma_8 \) can be expressed as functions of the state and inputs:

\[ \Gamma_8 = p^{-1} U_3^T U_{10} U_{11} [\phi - Q^{-1} U_{11}^T U_{10} \{ U_2 + U_5 \tau + U_1 \begin{bmatrix} -U_{10}^T \end{bmatrix} \} \]

\[-U_{10} U_{10} \}] \]  

(A29)

where

\[ p = U_3^T U_{10} U_{11} Q^{-1} U_{11}^T U_{10} \]

and

\[ Q = U_{11}^T U_{10} U_{10} U_{11} \]

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Similarly, $I_1$ are also functions of the state and inputs. Eq. (A25) is used as the dynamic equation of the system during $t = 0$ to $t = 1.9$ seconds, while Eq. (A28) is used for the rest of the turning motion.
Appendix B

A GRAPHICS PACKAGE FOR GENERATING VIEWS OF THREE-DIMENSIONAL SCENES

A graphical representation of a set of data helps in both the qualitative and quantitative analyses. This is especially true in the case of a three dimensional object that is easier to analyze if it can be displayed on a terminal. In generating a view of a scene, geometric transformations are used to achieve the effect of different viewing positions and directions. The perspective transformation is used in many three-dimensional visualization techniques to project the three-dimensional scene onto a two dimensional screen. A graphics package is developed which provides functions that accept a viewing position [108] and viewing direction and calculate the proper viewing transformations. This package involves:

1. A matrix multiplication that performs all the transformations used in modeling the objects in the scene.

2. A matrix multiplication to transform from world coordinates, a right-handed three dimensional cartesian coordinate system used to record the locations of points of an object, to the clipping coordinate system.

3. A clipping step.

4. A proportional division.

When a view of a three-dimensional scene is generated, a viewpoint, viewing direction, and aperture must be specified as shown in Fig. B1. These parameters are analogous to the adjustments made by a photographer when taking a picture of a real scene.

Transformations are extremely helpful in establishing the effect of different viewing parameters on the display of a scene. In order to calculate the position on the display screen of the image of a point on some object, we must first transform the point from the world coordinate system into the eye coordinate system, which has its origin fixed at the viewpoint and its ze axis pointed in the direction of view as shown in Fig. (B2). Assume we shall observe an object from a point (x₀, y₀,
Fig. B.1 Three viewing parameters which describe the view of a scene
Fig. B.2 The eye coordinate system
z_0), with the viewing axis \( z_e \) pointed directly at the origin of the world coordinate and assume that the \( x_e \) axis lies in the \( z = z_0 \) plane. A transformation \( V \), the viewing transformation, is used to convert points in the world coordinate system \((x_w, y_w, z_w)\) into points in the eye coordinate system \((x_e, y_e, z_e)\)

\[
[x_e \ y_e \ z_e \ 1]' = V[x_w \ y_w \ z_w \ 1]' \quad \text{(B1)}
\]

This transformation may be built up from several translations and rotations as follows:

1. The coordinate system is translated to \((x_0, y_0, z_0)\), as shown in (Fig. B3a).

\[
T_1 = \begin{bmatrix}
1 & 0 & 0 & x_0 \\
0 & 1 & 0 & y_0 \\
0 & 0 & 1 & z_0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

2. Rotate the coordinate system about the \( x' \) axis by 90°, as shown in Fig. B3b.

\[
T_2 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

3. Rotate about the \( y' \) axis by an angle \( \theta \) so that the point \((0, 0, z_0)\) which lies on the \( z' \) axis, as shown in Fig. B3c.

\[
\cos \theta = \frac{-y_0}{(x_0^2 + y_0^2)^{1/2}}
\]

\[
\sin \theta = \begin{cases} 
1 - \cos^2 \theta & \text{if } x_0 > 0 \\
-1 - \cos^2 \theta & \text{if } x_0 < 0
\end{cases}
\]
\[
T_3 = \begin{bmatrix}
\cos \theta & 0 & \sin \theta & 0 \\
0 & 1 & 0 & 0 \\
-sin \theta & 0 & \cos \theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

4. Rotate about the \( x' \) axis by an angle \( \phi \) so that the origin of the original coordinate system will lie on the \( z' \) axis, as shown in Fig. B3d.

\[
\sin \phi = \begin{cases}
-z_0 & \text{if } z_0 > 0 \\
\sqrt{x_0^2 + y_0^2 + z_0^2} & \text{if } z_0 < 0
\end{cases}
\]

\[
\cos \phi = (1 - \sin^2 \phi)^{1/2}
\]

\[
T_4 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi & \sin \phi & 0 \\
0 & -\sin \phi & \cos \phi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

5. Finally, reverse the sense of the \( z' \) axis in order to create a left-handed coordinate system that conforms to the conventions of the eye coordinate system, as shown in Fig. B3e.

\[
T_5 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Fig. B.3 Five steps in establishing the viewing transformation

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A perspective display can be generated by simply projecting each point of an object onto the plane of the display screen, as in Fig. B4. The coordinates \((x_s, y_s)\) of the projected image of the point \(P\) measured in eye coordinates \((x_e, y_e, z_e)\) are easily computed. Consider the \(y_ez_e\) plane drawn in in Fig. B5. The triangles \(OQ'P'\) and \(OOP\) are similar, giving the relation \(y_s/z_e = y_e/z_e\). A similar construction in the \(x_ez_e\) plane yields \(x_s/z_e = x_e/z_e\). The numbers \(x_s\) and \(y_s\) can be converted into dimensionless fractions by dividing by screen size:

\[
x_s = \left(\frac{Dx_e}{S_z_e}\right) x_e, \quad y_s = \left(\frac{Dy_e}{S_z_e}\right) y_e.
\]

Alternatively, we can convert to screen coordinates by including a specification of the location of the viewport in which the image is displayed

\[
x_s = \left(\frac{Dx_e}{S_z_e}\right) V_{sx} + V_{cx}, \quad y_s = \left(\frac{Dy_e}{S_z_e}\right) V_{sy} + V_{cy}
\]

The four viewport parameters are given in center-size notation: the viewport is centered at \((V_{cx}, V_{cy})\), is \(2V_{sx}\) unit wide and \(2V_{sy}\) unit high.

The simple application of Eqs. (B1) and (B2) to produce a perspective image has two undesirable effects: objects behind the viewpoint may appear on the screen, and objects may exceed the prescribed limits of the viewport given in Eq. (B2). These effects can be eliminated by testing each point in eye coordinates against a viewing pyramid, which defines the portion of eye coordinate space which the viewer can actually see. Two conditions must be met for an individual point to be visible within the pyramid:

\[
-z_e < (D/S) x_e < z_e \quad \text{and} \quad -z_e < (D/S) y_e < z_e
\]

For the convenience of the clipping task, we define a new coordinate system (the clipping coordinate system, subscript \(c\)) in terms of the eye coordinate system.
Fig. B.4 The perspective projection of point $P$ onto the display screen.

Fig. B.5 The $y_s z_s$ plane showing details of the perspective projection.
Start

Clear CRT

Input $x_o, y_o, z_o, D$ and $S$

Calculate $N$ and $V$

Open a data file

yes

Open a data file

no

use keyboard to enter point in world coord.

Calculate $x_s$ and $y_s$

graphic display

continue

yes

same viewpoint

no

Stop

yes

no

Fig. B.6 The flowchart of program - PROJ 1
The conditions of Eq. (B3) become simply

\[-z_c < x_c < z_c \text{ and } -z_c < y_c < z_c.\]

If the clipping process yields a visible line segment, we must still apply the inverse of the transformation (B4) followed by the perspective transformation (Eq. B2) to calculate the screen coordinates of the endpoints of the line. Eq. (B2) can be rewritten to perform both operations:

\[
x_s = \left(\frac{x_c}{z_c}\right) V_{sx} + V_{cx} \quad y_s = \left(\frac{y_c}{z_c}\right) V_{sy} + V_{cy}
\]  

The FORTRAN program PROJ 1 was created to provide calculations for generating views of three-dimensional scenes. The flowchart of PROJ 1 is shown in Fig. B6.
References


