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Dominek, Allen Keith

A UNIFORM ELECTROMAGNETIC REFLECTION ANSATZ FOR SURFACES
WITH SMALL RADII OF CURVATURE

The Ohio State University

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A UNIFORM ELECTROMAGNETIC REFLECTION ANSATZ
FOR SURFACES WITH SMALL RADII OF CURVATURE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Allen Keith Dominek

******

The Ohio State University

1984

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CHAPTER I

INTRODUCTION

The high frequency analysis and classification of scattered fields have been characterized in terms of the surface parameters of conducting scatterers. This relationship is due to the wave nature of the incident field and how it interacts with the geometry of the scatterer, especially when the incident wavelength is small compared to the characteristic length of the scatterer. One such parameter is surface curvature, i.e., how sharp or smooth is an edge-like structure. Conceptually, as the surface is deformed from a slowly to rapidly varying surface in terms of the wavelength, the scattered field must uniformly change from a reflected to a diffracted field. The topic of this dissertation is the uniform prediction of the scattered fields from smooth, convex, edge-like, perfectly conducting surfaces.

Edge-like surfaces are common in complex shapes such as wings and stabilizers of aircraft. There have been several computer codes developed to analyze the radiation and/or scattering from these complex shapes that are based upon the geometric theory of diffraction (GTD) [1] or physical theory of diffraction (PTD) [2]. The former uses
geometrical optics [3] as a base to account for the specular scattered fields and adds various edge and surface diffracted fields to complete the solution. The latter uses physical optics [3] as a base which includes both a specular field plus a crude edge diffracted term. However, the edge diffracted term inherent in physical optics is incorrect but is corrected in the physical theory of diffraction. Also, physical optics, in general, fails to satisfy the reciprocity theorem.

Using either the GTD or PTD to predict the specular scattered field from the leading edge of a wing has accuracy limitations. Geometrical optics, as expressed by an asymptotic Luneburg-Kline expansion [3] in inverse powers of k, correctly gives the reflected field when the surface curvature is sufficiently small. When the radius of curvature of the surface at the specular point becomes small, the geometrical optics solution fails. Though the first term in the geometrical optics is adequate for many applications, higher terms are required for larger surface curvatures. These higher order terms are unique for each surface and they are known only for a few canonical surfaces. While the physical optics approach is usually a more accurate representation for the specular term as the radius of curvature decreases than is geometrical optics, it also contains an erroneous term from the current discontinuity at the shadow boundary. Thus, to obtain a more accurate model, one is required to understand the physics involved in scattering from such an extremely common surface.

The scattered field can be calculated, in principle, by means of a radiation integral if the induced surface currents that exist over the
surface of the scatterer are known. The currents are dependent upon the total field that is present at the surface of the scatterer. These induced currents create the scattered field. At low frequencies where the wavelength of the incident field is very large compared to a characteristic dimension of the scatterer, the current over the entire surface contributes. However, at higher frequencies where the wavelength of incident field is very small compared to a characteristic dimension of the scatterer, only a localized current contributes due to the rapid phase variation of the current over most of the surface of the scatterer. It is the approximation of this current by $2\pi n xH^j$ in the lit region and its discontinuity at the shadow boundaries that limit the accuracy of the physical optics solution.

An asymptotic evaluation of the radiation integral for a given surface can result in several contributions (scattered field mechanisms) that each have a unique physical interpretation in various spatial regions of the scatterer. The common scattering mechanisms are reflection, edge diffraction and curved surface diffraction. The reflected field, as generated by specular reflection, is the contribution to the total scattered field that arises from current in a localized region on the illuminated side of the scatterer. The direction of the reflected field maximum is given by Snell's Law of Reflection. An edge diffracted field arises from a surface discontinuity where the localized current changes too rapidly. A surface diffracted field generates a creeping wave that is excited at an incident shadow boundary where the surface is smooth. The creeping wave
propagates around the surface along geodesic paths and is continually shed in tangential directions from the smooth surface.

The physical interpretation of the scattered field from an edge-like, convex surface is primarily dependent upon the radius of curvature at the surface scattering center. When the radius of curvature is roughly less than a tenth of a wavelength, the surface appears to be an edge since the scattered field can be predicted from edge diffraction theory. When the radius of curvature is roughly greater than two wavelengths, the surface can be represented as a smooth convex surface since the scattered field can be predicted as a reflected field from the leading term in a Luneburg-Kline expansion. Thus, a uniform scattering solution for the field from an edge-like surface should reduce to the diffracted field solution at one extreme and the reflected field for the other. Since the range of these two techniques do not overlap, there remains a region where a solution is still required. The goal of this dissertation has been to critically examine the more precise options available and demonstrate how this gap may best be filled.

The scattered, ray optical, field solution for the extreme cases of diffraction and reflection have matured over the past twenty years. The high frequency edge diffracted field was originally formulated by Keller [1]. Various asymptotic formulations have been developed but only the uniform theory of diffraction for edge diffraction by Kouyoumjian and Pathak [4] and for surface diffraction by Pathak, Burnside and Marhefka [5] are consistent with Keller's original approach of using the
diffracted field to modify the geometrical optics field to obtain an accurate result. The high frequency reflected field has been studied for electrically large bodies by a variety of investigators. Luneburg [6] and Kline [7,8] have formulated an asymptotic expansion, in inverse powers of $k$, for the reflected field. The vector Luneburg-Kline expansion for the reflected field has been described by Lee [9]. Scalar Luneburg-Kline expansions are presented by Keller, Lewis and Seckler [10] for many two and three dimensional surfaces. Pathak [11] has formulated another representation of the reflected field that is uniform across the incident shadow boundary where the Luneburg-Kline expansion fails.

Chapter II presents several total scattered far field curves in the frequency and time domains for plane wave incidence upon canonical surfaces. In most cases, the total scattered field is a composite of reflected and creeping wave fields.

Chapter III illustrates techniques to predict the reflected field. The traditional approach is to calculate the Luneburg-Kline asymptotic expansion. Reflected field expansions for several surfaces are presented. Two other techniques, which are numerically based, extract the reflected far field from the total scattered far field. The extraction techniques naturally present the reflected fields in two formats. In one format, the bistatic source-observation angle dependence of the reflected field is demonstrated for a fixed electrical size of the scatterer by subtracting the creeping wave contribution from the total scattered far field curves. In the other format, the
electrical size dependence of the reflected field is demonstrated for a fixed source-observation angle by subtracting the creeping wave contribution from the total scattered far field curves. The reflected field has also been isolated from the creeping wave contribution in the time domain by a time gating process.

Chapter IV presents heuristic expressions for the reflected field from smooth, convex two and three dimensional surfaces. The expressions are based upon the exact, closed form scattering expressions for a parabolic cylinder and paraboloid of revolution under axial, plane wave illumination. In these special cases, the total scattered field has a physical interpretation of being solely a reflected field. The functional properties and parameters of these exact expressions are physically interpreted and extended to surfaces of more general shapes to develop heuristic reflection coefficients. The geometric properties that are incorporated in the reflection coefficients are the radius of curvature of the surface at the specular point and the incident shadow boundary location with respect to the specular point. The functional role of the incident shadow boundary contribution has been recently interpreted from the Luneburg-Kline expansions for circular and elliptic cylinders. An important capability of the two dimensional reflection coefficient enables a smooth and continuous transition in the scattered field prediction from a scattering center that electrically behaves as a knife edge to one that electrically behaves as a smooth reflecting surface. This uniformity can then be incorporated into the UTD edge diffraction coefficient which results in a more general scattering coefficient.
Chapter V presents comparisons of the reflected fields generated by the techniques of Chapter III and IV. Aspect angle and swept frequency comparisons are done for two and three dimensional surfaces. Monostatic measurements are also shown and compared to these techniques for elliptic cylinders of two different eccentricities. Monostatic measurements of a smooth edge are also compared to the heuristic modification of the UTD edge diffraction coefficient solution.

Chapter VI summarizes this new reflection concept in terms of its accuracy and innate limitations.
CHAPTER II

SCATTERED FIELDS FROM CANONICAL SURFACES

Calculating the exact reflected field from a surface is not always directly possible. The scattered field from a smooth, convex, perfectly conducting surface usually is a composite of scattering mechanisms. The classification of scattering into unique mechanisms arises from the physical interpretation of how an incident field is scattered to the observation point. The scattered field in the illuminated region of a smooth convex scatterer can, in general, be considered to be a superposition of reflected and diffracted fields. Diffracted fields arise from edges formed by surfaces (discontinuities in slope and/or curvature) and from shadow boundaries on surfaces separating lit and shadowed regions.

The frequency region where this mechanism classification is valid begins at some lower frequency and extends upward. This lower frequency is somewhat vague in the sense of the desired accuracy required if the
mechanism is represented by an asymptotic expansion. There is a large parameter associated with an asymptotic expansion that limits the range of the expansion. The large parameter for a reflected field is often $kR$ where $k$ is the wavenumber of the field and $R$ is the radius of curvature for the surface at the reflection point. When a large parameter becomes too small, the expansion diverges and the scattering changes its physical interpretation. For small $kR$, scattering is generally denoted as Rayleigh [12,13] where the whole surface (the volume) controls the scattering unlike in the high frequency region where only local areas of the surface controls the scattering.

There are several canonical surfaces that may be used to obtain the reflected field from a convex surface. The importance of these surfaces is that the total scattered field can be calculated exactly. An "exact" reflected field can be obtained by removing the creeping wave contribution for the surfaces that have a finite physical cross section. The purpose in obtaining exact representations and/or curves of reflected fields from these surfaces is for the generalization in predicting reflected fields from arbitrary smooth, convex surfaces.

The canonical surfaces, their coordinate system that will be used throughout this dissertation, and their eigenfunction representation of the total scattered field are presented in this chapter. There will be three types of curves presented for the total scattered fields. The scattered field dependencies will be illustrated as a function of the
bistatic angle for a fixed surface size (fixed frequency), as a function of surface size (frequency) for a fixed aspect angle and finally as an impulse response [14] for a bandlimited, impulsive plane wave excitation.

The impulse responses are generated by numerically transforming windowed frequency spectra using a Fast Fourier Transform (FFT). The term windowed means that the spectra magnitudes were multiplied by a weight function to minimize the Gibb's phenomenon associated with the truncation of the spectra at some upper cutoff frequency. A Kaiser-Bessel window [15] is used for this purpose and it has a weight for the nth sample of N+1 total samples which is given by

\[
\omega(n) = \frac{I_0[\pi\alpha\sqrt{1 - \left(\frac{2n}{N}\right)^2}]}{I_0[\pi\alpha]} \quad 0 < |n| < N/2
\] (2.1)

where

\(I_0(x)\) is the modified Bessel function of the first kind, order zero and

\(\alpha\) is a parameter chosen to be 2 for the best performance.

The use of a window to minimize the effect of finite bandwidth distorts the transient waveform due temporal leakage from the window's side lobes and finite main beam width (which also limits the temporal resolution). A more desirable technique would be to extend the bandwidth from the
known spectrum in such a way that the extension does not perturb the
temporal information in the original spectrum and that it smoothly
terminates the abrupt cutoff in the original spectrum. An excellent
tutorial discussion for spectrum extension techniques is found in [16] but
these techniques were not applied here since they are not absolutely
necessary for this application.

All calculations will be for plane wave incidence with the
observation point in the far field. The frequency domain scattered
field magnitudes are normalized to the standard echo width and echo
area definitions for two and three dimensional fields, respectively
which are

\[
\text{Echo width} = \lim_{r \to \infty} 2\pi \left| \frac{E \cdot \hat{s}}{E \cdot \hat{p}} \right|^2
d\text{and}
\]

\[
\text{Echo area} = \lim_{r \to \infty} 4\pi r^2 \left| \frac{E \cdot \hat{s}}{E \cdot \hat{p}} \right|^2
\]  

(2.2)

(2.3)

where

\[E^i, E^s\] are the incident and scattered electric fields, and

\[\hat{p}^i, \hat{p}^s\] are unit vectors representing the incident and desired
scattered field polarization directions.
The time domain scattered field magnitudes follow Kennaugh's inverse Laplace normalization [14] of

\[ G_2(\omega) = \frac{\sqrt{2}\rho}{c} \frac{E^S \cdot p}{E^i \cdot p} e^{-j\omega(t-p/c)} \] (2.4)

**Impulse response** \( \mathcal{L}^{-1}\{G_2(\omega)\} \) (2.5)

for two dimensional surfaces, and

\[ G_3(\omega) = \frac{2r}{c} \frac{E^S \cdot p}{E^i \cdot p} e^{-j\omega(t-r/c)} \] (2.6)

**Impulse response** \( \mathcal{L}^{-1}\{G_3(\omega)\} \) (2.7)

for three dimensional surfaces. The constant \( c \) represents the speed of light with a time convention of \( \exp(+j\omega t) \) for \( \vec{E}^S \) and \( \vec{E}^i \). This particular normalization makes the response dimensionless since \( G(\omega) \) has dimensions of seconds. The actual numerical normalization is performed with the impulse response of a sphere. The monostatic reflection return for a sphere has a step character at \( t=0 \) with a value of one-half. This early time signature response is readily observed by manually transforming the Luneburg-Kline expansion (Chapter III) for the reflected field. Under this normalization, the step height is independent of the sphere radius. Another step normalization advantage arises from the insensitivity of using a bandlimited frequency spectrum.
because the frequency content for a step is inversely proportional to frequency. The two dimensional normalization involves just scaling the sphere normalization by a square root of two. A dimensional return is possible for two and three dimensional scattering with units of volt/(second/meter) volt/second, respectively. This is accomplished by scaling Equations (2.4) and (2.6) with factors of $cE/vp^2$ and $cE/vp^2/\sqrt{2}$, respectively for a unit incident plane wave.

The surfaces that have been chosen to obtain the reflected field from are the parabolic, circular and elliptic cylinders for the two dimensional surfaces and the paraboloid of revolution (acoustic solution only), sphere and prolate spheroid for the three dimensional surfaces. Though the circular cylinder and the sphere are special cases of the elliptic cylinder and the prolate spheroid, respectively, they are considered separately because of their a simpler scattered field representation. The monostatic scattered field for the parabolic cylinder and paraboloid of revolution has the advantage, for axial plane wave incidence, of being solely a reflected field since the infinite physical cross section doesn't allow a creeping wave mechanism and they have a simple closed form analytic representation.
A. TWO DIMENSIONAL SURFACES

Electromagnetic scattering from two dimensional surfaces can be uniquely described as a response to two principle incident field polarizations. These principle polarizations result from the scalarization of the vector Helmholtz equation into two scalar Helmholtz equations. When the cross section of the scatterer is oriented in the x-y plane, the incident field principle polarizations occur when there are only z directed electric or magnetic field components. Nomenclature commonly used is $TM_z$ for an incident z-polarized electric field and $TE_z$ for an incident z-polarized magnetic field. In acoustic scattering, the $TM_z$ and $TE_z$ cases correspond to the soft and hard boundary conditions at the surface of the scatter, respectively. A soft boundary case is where the total z component of the field is zero at the surface and a hard boundary case is where the normal derivative of the total z component of the field is zero at the surface.

The parabolic, circular and elliptic cylindrical surfaces have been chosen as canonical surfaces to obtain the frequency and bistatic angle dependence of the two dimensional reflected field. The surfaces and their parameters are illustrated in Figure 2.1. Described below are the eigenfunction expansions for the total scattered field from these surfaces due to an incident wave of either polarization. A time convention of $\exp(+j\omega t)$ for the field representations has been assumed and suppressed through out.
(a). Parabolic surface.

\[
\sqrt{2f} = \sqrt{2\rho} \sin \frac{\phi}{2}
\]

(b) Circular surface.

(c). Elliptic surface.

Figure 2.1. The two dimensional surface geometries.
1. Parabolic Cylinder

The parabolic cylindrical surface shown in Figure 2.1(a) is described in cartesian coordinates by

\[
x = \left( \xi^2 - \eta_1^2 \right) / 2 \\
y = \xi \eta_1
\]

(2.8)

The scattering surface is the parabolic cylinder with \( \eta_1 = \sqrt{2} \rho \sin \phi / 2 \) = \( \sqrt{2} f \) being defining the surface. The characteristic dimension for this surface is the focal length denoted by \( f \). A half plane surface is formed when the surface collapses on the positive z-axis. The half plane edge is along the z-axis and the surface exists for \( x > 0, y = 0 \).

An exact expression for the two types of scattered fields for a plane wave incident from \( \pi / 2 < \phi' < \pi \) [17] is

\[
\begin{bmatrix}
E_z \\
H_z
\end{bmatrix} = \frac{1}{\sin^2 \phi/2} \sum_{n=0}^{\infty} \frac{(-j \cot \phi/2)^n}{n!} \begin{bmatrix}
a_n \\
b_n
\end{bmatrix} D_n(-\gamma \xi) D_{n-1}(\gamma \eta)
\]

(2.9)

where

\[
\gamma = \sqrt{2} \sqrt{k} = \sqrt{2} k e^{j \pi / 4}
\]

\[
\xi = \sqrt{2} \rho \cos \phi / 2
\]

\[
\eta = \sqrt{2} \rho \sin \phi / 2
\]
\[ a_n = - \frac{D_n(\eta_1 \gamma^*)}{D_{-n-1}(\eta_1 \gamma)} \]

\[ b_n = - \frac{D_n'(\eta_1 \gamma^*)}{D_{-n-1}'(\eta_1 \gamma)} \quad \text{and} \quad \eta_1 = \sqrt{2f} \]

Note that \( D_n(z) \) are Weber-Hermite functions defined by

for \( n \), a non-negative integer,

\[ D_n(z) = 2^{-n/2} e^{-z^2/4} H_n(z/\sqrt{2}) , \]

and for \( n \), a negative integer,

\[ D_n(z) = e^{i\pi/4} \frac{(-1)^n}{n!} e^{-z^2/4} \frac{d^n}{dz^n} \left[ e^{-z^2/2} \tilde{F} \left( \frac{z}{\sqrt{2j}} \right) \right] . \]

\( H_n(z) \) is a Hermite polynomial of degree \( n \) and \( \tilde{F}(z) \) is defined as

\[ \tilde{F} \left( \frac{z}{\sqrt{2j}} \right) = \int_{\sqrt{2j} z}^{\infty} e^{-\xi^2} d\xi . \]

For axial incidence, \( \phi' = \pi \), Equations (2.9) and (2.10) reduce to
\[
E_z = -\frac{\sqrt{f}}{\sin \phi/2} \frac{F(2k \rho \sin^2 \phi/2)}{F(2k \rho)} \frac{e^{-jk(\rho-2\rho)}}{\sqrt{\rho}} \tag{2.11}
\]

and

\[
H_z = \frac{\sqrt{f}}{\sin \phi/2} \frac{F(2k \rho \sin^2 \phi/2)}{2 - F(2k \rho)} \frac{e^{-jk(\rho-2\rho)}}{\sqrt{\rho}} \tag{2.12}
\]

where

\(\rho, \phi\) define the observation point in circular cylindrical coordinates.

\(F(x)\) is the wedge transition function defined as

\[
F(x) = 2j \sqrt{x} e^{jx} \int_{\sqrt{x}}^{\infty} e^{-j\tau^2} \, d\tau \tag{2.13}
\]

with small and large argument forms as follows:

\[
F(x) = [\sqrt{\pi x} - 2xe^{j\pi/4} - \frac{2}{3} x^2 e^{-j\pi/4} + \ldots] e^{j(\pi/4 + x)}
\]

for \(x\) small \tag{2.14}

and

\[
F(x) \sim 1 + j \frac{1}{2} \frac{1}{x} - \frac{31}{4} \frac{1}{x^2} - j \frac{15}{8} \frac{1}{x^3} + \frac{105}{16} \frac{1}{x^4} + \ldots
\]

for \(x\) large. \tag{2.15}
The axial monostatic scattering spectra, soft and hard incident polarizations, for a parabolic cylinder with a focal length of .0381 m are shown in Figure 2.2. The bandlimited (6.5 GHz bandwidth) impulse response of these spectra are shown in Figure 2.3. These curves are characteristic of what is called a reflected field since the exact solution is composed of one scattering mechanism. Note that the scattered field values for the soft case are farther displaced from the high frequency limit (leading term in the Luneburg-Kline expansion) than the hard case. It is also interesting to note that as the focal length goes to zero, a half plane is formed and the scattered field is called an edge diffracted field. It is this surface and its exact representation that is exploited in Chapter IV to predict the reflected field from a surface with a very small radius of curvature.

2. Circular Cylinder

The circular cylindrical surface shown in Figure 2.1(b) is described in rectangular coordinates by

\[ x = \rho \cos \phi \quad \text{and} \quad y = \rho \sin \phi \]

(2.16)

The scattering surface is the circular cylinder with \( \rho = a \) defining the surface. The characteristic dimension for this surface is the radius of the cylinder denoted by \( a \).
Figure 2.2. Axial, far field, monostatic scattering spectra for a parabolic cylinder with a 0.0381 m focal length. Plane wave illumination.
Figure 2.3. Bandlimited impulse responses for a parabolic cylinder generated from the spectra in Figure 2.2. Impulsive plane wave approximation using 6.5 GHz bandwidth.
An exact expression for the two types of scattered fields for a plane wave incident from $\phi' = \pi$ [17] is

\[
\begin{bmatrix}
    E_z \\
    H_z
\end{bmatrix} = \sum_{n=0}^{\infty} \varepsilon_n (-j)^n \begin{bmatrix}
    a_n \\
    b_n
\end{bmatrix} H_n^{(2)}(k\rho) \cos \phi
\]  

(2.17)

where

\[
\varepsilon_0 = 1, \quad \varepsilon_n = 2 \quad \text{for} \ n \neq 0,
\]

\[
a_n = -\frac{J_n(ka)}{H_n^{(2)}(ka)} \quad \text{and} \quad b_n = -\frac{J_n'(ka)}{H_n^{(2)'}(ka)}.
\]

(2.18)

Note that $J_n(x)$, $J_n'(x)$, $H_n^{(2)}(x)$ and $H_n^{(2)'}(x)$ are the ordinary cylindrical Bessel functions.

In the far field, $H_n^{(2)}(k\rho)$ can be replaced with

\[
H_n^{(2)}(k\rho) \approx \sqrt{\frac{2j}{\pi k\rho}} \ j^n \ e^{-jk\rho} \quad \text{for} \ k\rho \gg 1; \ k\rho \gg n.
\]  

(2.19)
Frequency spectra curves are shown in Figures 2.4(a) and (b), for soft and hard polarizations, respectively, for bistatic angles of 0, 90 and 180 degrees and a cylinder radius of .0762 m. The impulse response (6.5 GHz bandwidth) based upon these spectra are shown in Figures 2.5(a) and (b). It is readily seen from the impulse response that the scattering is a superposition of mechanisms. The initial return is associated with the reflected field and the second return, as readily seen for the hard case, is accountable by a creeping wave mechanism. The soft case has a negligible creeping wave mechanism since tangential E is zero. Also, it is observed that the character of the mechanisms changes as the observation angle increases from 0 degrees (backscatter) to 180 degrees (forwardscatter). This information naturally implies a complex analytical representation is required to accurately predict the reflected field as a function of bistatic angle. Another interesting observation is that the forwardscattering from a cylinder is comparable to the backscatter or forwardscatter for a strip. An explanation of this is that the forward scattering is dominated by a shadowing phenomena.

3. Elliptic Cylinder

The elliptic cylindrical surface shown in Figure 2.1(c) is described in rectangular coordinates by

\[
\begin{align*}
x &= d \cosh u, \cos v = a \cos v \\
y &= d \sinh u, \sin v = b \sin v
\end{align*}
\]  \hspace{1cm} (2.20)
Figure 2.4. Scattered far field spectra of circular cylinders for bistatic angles of 0°, 90°, and 180° and a 0.0762 m radius.
(b) Plane wave illumination: hard

Figure 2.4. (Continued).
(a) Impulsive plane wave approximation using 6.5 GHz bandwidth: soft.

Figure 2.5. Bandlimited impulse responses for a circular cylinder generated from the spectra in Figures 2.4(a) and (b).
(b) Impulsive plane wave approximation using 6.5 GHz bandwidth: hard.

Figure 2.5. (Continued).
where

\[ d = a \sqrt{1 - (b/a)^2} \]

\[ \cosh u_1 = 1/ \sqrt{1 - (b/a)^2} \quad \text{and} \quad \sinh u_1 = b/a / \sqrt{1 - (b/a)^2} \]

The interfocal distance, the semi-major axis length and the semi-minor axis length are given by \[2d, a = dcosh(u_1) \quad \text{and} \quad b = dsinh(u_1)\], respectively. The scattering surface is the elliptic cylinder with \( u_1 = \) a constant. The characteristic dimension for this surface is the semi-major axis length. It is convenient to let \( \xi = \cosh(u) \) and \( \eta = \cos(v) \). A strip of width \( 2d \), centered at the origin, can be defined in this coordinate system when \( u_1 = 0 \).

An exact expression for the two types of scattered fields for a plane wave incidence at an angle \( \phi' \) [17] is given by

\[
\begin{bmatrix}
E_z \\
H_z
\end{bmatrix} = \sqrt{8\pi} \sum_{n=0}^{\infty} j^n \left[ \frac{1}{Ne_n} \{(ae_n^1) R^{(4)}(c,\xi_1) Se_n(c,\cos\phi')Se_n(c,\eta) \right.
\]

\[
+ \frac{1}{Ne_n} \{bo_n^1 \} R^{(4)}(c,\xi_1) So_n(c,\cos\phi')So_n(c,\eta) \right]
\]

\( (2.21) \)

\( (2.22) \)

where
\[ c = kd \]

\[ \xi_1 = \cosh(u_1), \text{ } u_1 \text{ is the observation surface} \]

\[ n = \cos(v), \text{ } v \text{ is the observation parameter angle} \]

\[ a e_n = - \frac{\text{Re}_{n}^{(1)}(c_1, \xi)}{\text{Re}_{n}^{(4)}(c_1, \xi)} \]

\[ a o_n = - \frac{\text{Ro}_{n}^{(1)}(c_1, \xi)}{\text{Ro}_{n}^{(4)}(c_1, \xi)} \]

\[ b e_n = - \frac{\text{Re}_{n}^{(1)'}(c_1, \xi)}{\text{Re}_{n}^{(4)'}(c_1, \xi)} \] and

\[ b o_n = - \frac{\text{Ro}_{n}^{(1)'}(c_1, \xi)}{\text{Ro}_{n}^{(4)'}(c_1, \xi)} . \]

Note that \( \text{Ne}_{e,n} \) are normalization constants defined in [18], with \( \text{Se}_{e,n}(c, n) \) representing the angular Mathieu functions, and \( \text{Re}_{e,n}(c, \xi) \) and \( \text{Ro}_{e,n}(c, \xi) \) representing the radial Mathieu functions. Reference [18] contains a description of these functions.
In the far field, \( \text{Re}_e o_n(c, \xi) \) can be replaced with

\[
\text{Re}_e o_n(c, \xi) \sim -\sqrt{\frac{3}{c \xi}} \cdot j^n e^{-jc \xi} \quad \text{for } c \xi >> 1, c \xi + k \rho \tag{2.23}
\]

Frequency spectra curves are shown in Figure 2.6(a) and (b), for the soft and hard polarization, respectively, for bistatic angles of 0, 90 and 180 degrees with the incident angle being zero degrees. The cylinder semi-major radius is 0.0762 m with a \( b/a \) ratio of 0.5. The \( b/a \) ratio is the ratio of semi-minor to semi-major axis lengths. The impulse response (6.5 GHz bandwidth) based upon these spectra are shown in Figures 2.7(a) and (b). Monostatic frequency spectra curves are shown in Figures 2.8(a) and (b), soft and hard polarization, respectively, for incidence angles of 0, 45 and 90 degrees and the same cylinder size as before. The impulse response (6.5 GHz bandwidth) based upon these spectra are shown in Figures 2.9(a) and (b). The same general comments can be made for the elliptic cylinder as was made for the circular cylinder.
(a) Plane wave illumination along the semi-major axis: soft.

Figure 2.6. Scattered far field spectra of elliptic cylinders for bistatic angles of 0, 90 and 180 degrees with a semi-major axis length of .0762 m and a b/a ratio of .5.
(b) Plane wave illumination along the semi-major axis: hard.

Figure 2.6. (Continued).
(a) Impulsive plane wave approximation using a 6.5 Ghz bandwidth: soft.

Figure 2.7. Bandlimited impulse responses for an elliptic cylinder generated from the spectra in Figures 2.6(a) and (b).
(b) Impulsive plane wave approximation using a 6.5 GHz bandwidth: hard.

Figure 2.7. (Continued).
(a) Plane wave illumination: soft.

Figure 2.8. Monostatic scattered far field spectra of elliptic cylinders for incidence at 0, 45 and 90 degrees from the semi-major axis with a semi-major axis length of 0.0762 m and a b/a ratio of 0.5.
(b) Plane wave illumination: hard.

Figure 2.8. (Continued).
Figure 2.9. Bandlimited impulse responses for an elliptic cylinder generated from the spectra in Figures 2.8(a) and (b).

Impulsive plane wave approximation using a 6.5 GHz bandwidth: soft.

(a) Impulsive plane wave approximation using a 6.5 GHz bandwidth: soft.
(b) Impulsive plane wave approximation using a 6.5 GHz bandwidth: hard.

Figure 2.9. (Continued).
B. THREE DIMENSIONAL SURFACES

Electromagnetic scattering from three dimensional surfaces often do not have scalarized solutions as do the two dimensional surfaces. Hence, their eigenfunction expansions are more difficult to calculate even if they exist. Because of this, a more limited examination of the scattered far fields is presented.

The paraboloidal, spherical and prolate spheroidal surfaces have been chosen as canonical surfaces to obtain the frequency dependence of the three dimensional reflected field. The surfaces and their parameters are illustrated in Figure 2.10. Described below are field expansions for the the total scattered field from these surfaces due to an incident plane wave illustrated with frequency spectra and bandlimited impulse responses.

1. Paraboloid of Revolution

The paraboloidal surface shown in Figure 2.10(a) is described in rectangular coordinates by

\[ x = \sqrt{\xi \eta_1} \cos \phi \]
\[ y = \sqrt{\xi \eta_1} \sin \phi \quad \text{and} \]
\[ z = (\xi - \eta_1)/2 \]

where
Figure 2.10. The three dimensional surface geometries.

(a) Paraboloidal surface.

(b) Spherical surface.

(c) Prolate spheroidal surface.
where

\[ \xi = 2r \cos^2 \theta/2 \quad \text{and} \]

\[ n = 2r \sin^2 \theta/2 \]

The scattering surface is the parabolic cylinder with \( n_1 = 2r \sin^2 \theta/2 = 2f \), a constant defining the surface. The characteristic dimension for this surface is the focal length denoted by \( f \) with the focus located at the origin.

The acoustic eigenvalue expansion for the scattered far fields, axial plane wave incidence, reduces [17] to

\[
U_s = -\frac{f}{\sin^2 \theta/2} \frac{F_3(2kr \sin \theta/2)}{F_3(2kf)} \frac{e^{-jk(r-2f)}}{r} \quad (2.25)
\]

\[
U_h = \frac{f}{\sin^2 \theta/2} \frac{F_3(2kr \sin \theta/2)}{2 - F_3(2kf)} \frac{e^{-jk(r-2f)}}{r} \quad (2.26)
\]

where

\[ r, \theta \text{ define the observation point (rotational symmetric)} \]

\[ F_3(x) \text{ is a transition function defined as} \]

\[
F_3(x) = 2jx e^{jx} \int \frac{e^{-j\tau^2}}{\sqrt{\pi} \tau} d\tau \quad (2.27)
\]
with small and large argument forms of

\[ F_3(x) = \left[ x(\gamma + j\pi/2 + zn) - jx^2 - \frac{x^3}{4} + j \frac{x^4}{18} + \ldots \right] e^{j(-\pi/2+x)} \]

for \( x \) small \hspace{1cm} (2.28)

and

\[ F_3(x) \sim 1 + j \frac{1}{x} - 2 \frac{1}{x^2} - j6 \frac{1}{x^3} + 24 \frac{1}{x^4} + \ldots \]

for \( x \) large. \hspace{1cm} (2.29)

The axial monostatic scattering spectra, soft and hard incident polarizations, for a paraboloid of revolution with a focal length of .0381 m are shown in Figure 2.11, respectively. The bandlimited (5.5 GHz bandwidth) impulse response of these spectra are shown in Figure 2.12. These curves are characteristic of what is called a reflected field since the exact solution is composed of one scattering mechanism. As in the two dimensional case, note that the scattered field values for the soft case are farther displaced from the high frequency limit (leading term in the Luneburg-Kline expansion) than the hard case. It is this surface and its exact representation that is exploited in Chapter IV to predict an electromagnetic reflected field for a three dimensional surface.

2. Sphere

The spherical surface shown in Figure 2.10(b) is described in rectangular coordinates by

\[ x = r \sin\theta \cos\phi \]
\[ y = r \sin\theta \sin\phi \]
\[ z = r \cos\theta \]

and

\[ (2.30) \]

\[ 42 \]
Figure 2.11. Axial, far field, monostatic, acoustic scattering spectra for a paraboloid of revolution cylinder with a 0.0381 m focal length. Plane wave illumination.
Figure 2.12. Bandlimited impulse responses for a paraboloid of revolution generated from the spectra in Figure 2.11. Impulsive plane wave approximation using 5.5 GHz bandwidth.
The scattering surface is the sphere with \( r=a \) defining the surface. The characteristic dimension for the surface is the radius of the sphere denoted by \( a \).

An exact expression for the two components of scattered far fields for a plane wave incident from \( \theta'=\pi \) and \(-\hat{x}\) polarized [17] is

\[
E_\theta = -j \frac{\cos \phi}{kr} e^{-jkr} \sum_{n=1}^{\infty} \left[ a_n \frac{P_n^1(\cos \theta)}{\sin \theta} - b_n \sin \theta P_n(\cos \theta) \right]
\]

\[
E_\phi = -j \frac{\cos \phi}{kr} e^{-jkr} \sum_{n=1}^{\infty} \left[ a_n \sin \theta P_n^1(\cos \theta) - b_n \frac{P_n^1(\cos \theta)}{\sin \theta} \right]
\]

where

\[
a_n = -\frac{2n+1}{n(n+1)} \frac{j_n(ka)}{h_n^{(2)}(ka)} \quad \text{and} \quad b_n = -\frac{2n+1}{n(n+1)} \frac{j_n'(ka)}{h_n^{(2)'}(ka)}
\]

Note that \( j_n(x), j'_n(x), h_n^{(2)}(x) \) and \( h_n^{(2)'}(x) \) are the ordinary spherical Bessel functions and \( P_n^1(\cos \theta) \) and \( P_n^{1'}(\cos \theta) \) are associated Legendre functions for the first kind of order \( n \) and degree 1. Prime denotes differentiation with respect to \( \cos \theta \).
The bistatic, 9 polarized, scattering spectra for .0762 m radius sphere are shown in Figure 2.13 for bistatic angles of 0, 90 and 180 degrees. The bandlimited (5.5 GHz bandwidth) impulse response of these spectra are shown in Figure 2.14. As with the two dimensional cylinders, there are two major returns shown in the impulse response which are due to the reflection and creeping wave mechanisms. The step portion of the reflected field is readily seen in the monostatic impulse return which has been used as stated earlier to normalize the transient responses.

Examination of the monostatic creeping wave time signature for the sphere indicates that a return exists before the expected time of arrival if the energy transverses a great circle path. This observation might imply that the mechanism is noncausal but examination of the surface creeping waves (unlike for a two dimensional cylindrical surface) reveals that a caustic is formed on the shadowed region of the surface. A caustic is generated because there is a common intersection point for an infinite number of great circle paths that creeping waves can follow to return to the source. This caustic introduces a 90 degree phase shift for the high frequency fields compared to the fields of a two dimensional cylindrical surface. It is this 90 degree phase difference that is the major difference between the creeping wave fields of a circular cylinder and sphere and is what forms a symmetrical cusp at the optical time of arrival for the creeping wave.
Figure 2.13. Monostatic, copolarized scattered far field spectra of a sphere for bistatic angles of 0, 90, and 180 degrees with a 0.0762 m radius. θ polarized plane wave incidence.
Figure 2.14. Bandlimited impulse responses for a sphere generated from the spectra in Figure 2.13. \( \theta \) polarized plane wave incidence.
3. Prolate Spheroid

The prolate spheroidal surface shown in Figure 2.10(c) is described in rectangular coordinates by

\[
\begin{align*}
  x &= d \sinh u_1 \sin v \cos \phi = b \sin v \cos \phi \\
  y &= d \sinh u_1 \sin v \cos \phi = b \sin v \sin \phi \\
  z &= d \cosh u_1 \cos v = a \cos v
\end{align*}
\]

The interfocal distance, semi-major axis length and semi-minor axis length are given by \(2d, a = d\cosh(u_1)\) and \(b = d\sinh(u_1)\), respectively. The scattering surface is the prolate spheroid \(\xi_1 = \cosh u_1 = \) constant. The parameters \(u\) and \(v\) have the same relationships with the semi-major and semi-minor axis lengths as in the elliptic cylinder section. The characteristic dimension for this surface is the semi-major axis length.

Scattered field calculations based upon an eigenfunction solution \([19,20]\) are possible but they were made using a method of moments approach \([21]\) since a program was readily available. Monostatic, far field frequency spectra calculations were done for two different spheroids. Figures 2.15(a) and (b) are spectra for \(\theta\) and \(\phi\) polarized, plane wave incidence upon a spheroid at polar \((\theta)\) angles of 0, 45 and 90 degrees with a semi-major axis length of .0762 m and a \(b/a\) ratio of .5. The \(b/a\) ratio is the ratio of semi-minor to semi-major axis lengths. The bandlimited (5.5 GHz bandwidth) impulse response of these spectra
Figure 2.15. Monostatic, copolarized scattered far field spectra of a prolate spheroid for incidence $\theta$ angles of 0, 45, and 90 degrees with a semi-major axis length of .0762 m and a $b/a$ ratio of .5.

(a) $\theta$ polarized plane wave incidence.
Figure 2.15. (Continued).
are shown in Figures 2.16(a) and (b). Figures 2.17(a) and (b) are spectra for $\theta$ and $\phi$ polarized, plane wave incidence upon a spheroid at polar ($\theta$) angles of 0 and 90 degrees with a semi-major axis length of .1524 m and a b/a ratio of .25. The bandlimited (5.5 GHz bandwidth) impulse response of these spectra are shown in Figures 2.18(a) and (b). The time signature for axial monostatic scattering from the 4:1 prolate spheroid has an interesting feature near the expected time of arrival of the creeping wave mechanism. Figure 2.19 shows a more complex response at this time than would be expected from the creeping wave alone. There is a small, sharp, positive return which is believed to be due to the creeping wave being partially reflected back and not traveling around the "tip" portion of the spheroid. As the spheroid becomes elongated, a "tip" becomes more pronounced. Ogival tip scattering has been modeled in the past [22] as creeping waves being reflected from and around an ogival tip.
(a) $\theta$ polarized impulsive plane wave illumination using a 5.5 GHz bandwidth.

Figure 2.16. Bandlimited impulse responses for a prolate spheroid generated from the spectra in Figures 2.15(a) and (b).
(b) \( \phi \) polarized impulsive plane wave illumination using a 5.5 GHz bandwidth.

Figure 2.16. (Continued).
Figure 2.17. Monostatic, copolarized scattered far field spectra of a prolate spheroid for incidence \( \theta \) angles of 0 and 90 degrees with a semi-major axis length of .1524 m and a \( b/a \) ratio of .25.
(b) $\phi$ polarized plane wave incidence.

Figure 2.17. (Continued).
(a) 0 polarized impulsive plane wave illumination using a 5.5 G Hz bandwidth.

Figure 2.18. Bandlimited impulse response for a prolate spheroid generated from the spectrum in Figure 2.17.
(b) φ polarized impulsive plane wave illumination using a 5.5 Ghz bandwidth.

Figure 2.18. (Continued).
Figure 2.19. Enlarged view of the axial, monostatic, bandlimited impulse response for the 4:1 prolate spheroid of Figure 2.18.
CHAPTER III

REFLECTED FIELDS FROM THE CANONICAL SURFACES

Three techniques, one analytical and two numerical, are presented to acquire the reflected field for the canonical surfaces of Chapter II. The techniques are 1) an asymptotic expansion in inverse powers of $k$, 2) subtraction of the creeping wave field (calculated from another asymptotic expansion) from the total scattered field and 3) numerically isolating and transforming the desired mechanism from the time domain representation of the total scattered field into the frequency domain. From these techniques, the bistatic angle and frequency dependencies of the reflected field can be examined to aid in the prediction of the reflected field from an arbitrary surface.

A. ASYMPTOTIC SOLUTIONS

High frequency scattered fields can be accurately modeled with an asymptotic expansion in inverse powers of $k$. Asymptotic solutions are strictly valid when the large parameter in the problem is large. The large parameter often encountered in electromagnetic problems is $kL$. 

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where \( k \) is the wavenumber and \( L \) is a characteristic dimension associated with the surface. The required largeness of this parameter is dependent upon the required accuracy. As the large parameter becomes small, the solution will "gracefully" fail. Asymptotic, reflected field solutions are presented since they give the correct high frequency limit of the reflected field in a ray optical format and information on the functional bistatic angle dependence even though they are of limited accuracy for surfaces with small radii of curvature.

There are two general methods for obtaining high frequency reflected field solutions. One method, called the Luneburg-Kline expansion, obtains the solution from the Helmholtz equation by expanding the field in a power series involving inverse powers of \( k \). The large parameter becomes small for small characteristic dimensions of the scatterer and near the shadow boundary. The other method involves asymptotically evaluating an integral representation of the scattered field. This method has an apparent benefit of being able to obtain a uniform solution between the illuminated and shadow regions of the scatterer.

1. Luneburg-Kline Expansions

The asymptotic expansion for the reflected field was originally developed by Luneburg and Kline [6,7,8]. Schensted [23] later solved for the bistatic reflected field (the first two terms) from a body of revolution for an electromagnetic plane wave axially incident. Lee [9]
generalized the reflected field expansion for an arbitrary polarized incident electromagnetic wave. Schensted's solution was questioned by Lee since Schensted's solution did not result in the correct expansion for monostatic, plane wave incidence, scattering from a sphere.

The methodology involved in obtaining a Luneburg-Kline expansion of a scalar reflected field for a scatterer is presented. The total field \( U \) must satisfy the scalar Helmholtz equation which is given by

\[
(\nabla^2 + k^2)U = 0 \tag{3.1}
\]

with the proper boundary conditions

\[
U = 0 \text{ on the surface for the soft case or} \tag{3.2}
\]

\[
dU/dn = 0 \text{ on the surface for the hard case}. \tag{3.3}
\]

The solution can be given as

\[
U = U^i + U^r \tag{3.4}
\]

with \( U^i \) representing the incident field and \( U^r \) the reflected field, \( U^i, r \) may be expanded as

\[
U^i, r \sim e^{-jk} \sum_{n=0}^{\infty} u_n^i, r (-jk)^{-n} \text{ as } k \to \infty \tag{3.5}
\]
where \( u^i_n \) are known coefficients and \( u^r_n \) are unknown coefficients.

Inserting Equation (3.5) into Equation (3.1) results in two equations that are known as the eikonal and transport equations which are, respectively, given by

\[
|\psi| = 1 \quad \text{and} \quad (3.6)
\]

\[
2\psi u^r_n - \psi - u^r_n \nabla^2 \psi = - \nabla^2 u^r_{n-1} \quad (3.7)
\]

The solution of Equation (3.1) is

\[
u^r_n = u^r_n(s_0) \left[ \frac{G(s)}{G(s_0)} \right]^{1/2} - \frac{1}{2} \left[ G(s) \right]^{1/2} \int_{s_0}^{s} \left[ G(\tau) \right]^{1/2} \nabla^2 u^r_{n-1}(\tau) d\tau
\]

(3.8)

where

\( s_0 \) is the initial point of the reflected ray on the surface

\( s \) is the distance along the reflected ray

\( G(\tau) = (\rho^r_1 + \tau - s_0)^{-1}(\rho^r_2 + \tau - s_0)^{-1} \) and

\( \rho^r_{1,2} \) are the principle radii of curvature for the reflected wavefront.
Equation (3.8) has a physical interpretation of describing optical rays that travel a distance $s$ from the surface and orthogonally transverse surfaces of constant $\phi$ which are determined by Equation (3.6). The leading term in Equation (3.8) describes the reflected field adequately for large parameters. This term physically requires the field to spread in a tube of rays. As the large parameter becomes small, higher order terms are required. They are determined through the recursive relationships of Equations (3.7) and (3.8). The calculation of the higher order terms becomes rapidly tedious. The solution indicated in Equation (3.8) becomes divergent if the large parameter becomes too small.

The Luneburg-Kline expansions [10,23,24] for the reflected far field from the canonical surfaces are expressed below. The incident field is a unit plane wave incident upon the convex side of the surfaces for both polarizations as shown in Figures 2.1 and 2.10. The reflected field expressions are phase referenced to the point of reflection and represented by the product of two terms. The first term is defined as the geometrical optics (GO) term that is multiplied by another term that is a correction factor. The correction factor accounts for the frequency and bistatic angle dependencies of the reflected field. Magnitude expressions of the fields are also provided that are based upon the first three terms of the expansion to be correct to $k^{-2}$.
The following is a summary of solutions for various surfaces:

a) Parabolic Surface (Axial, monostatic reflection); \( \phi' = \pi \)

\[
G_0 = \sqrt{\frac{\pi}{\rho}} \csc \phi/2 \ e^{-jk(\rho-2f)}; \ \phi = \pi
\]  

(3.9)

\[
E_z^r = -G_0 \left[ 1 + \frac{1}{4jkf} + \frac{1}{8(kf)^2} + \ldots \right]
\]

(3.10)

\[
H_z^r = G_0 \left[ 1 - \frac{1}{4jkf} - \frac{1}{4(kf)^2} + \ldots \right]
\]

(3.11)

The reflected field magnitudes are

\[
|E_z^r| = |G_0| \left[ 1 + \frac{5}{32} \frac{1}{(kf)^2} + \ldots \right]
\]

(3.12)

\[
|H_z^r| = |G_0| \left[ 1 - \frac{7}{32} \frac{1}{(kf)^2} + \ldots \right]
\]

(3.13)

b) Circular Surface

\[
G_0 = \sqrt{\frac{a}{2\rho}} \sin \phi/2 \ e^{-jk(\rho-2asin\phi/2)}
\]

(3.14)

\[
E_z^r = G_0 \left[ 1 + \frac{8}{\sin^2 \phi/2 - 3} + \frac{320}{\sin^4 \phi/2 - \sin^2 \phi/2 + 2} \right]
\]

(3.15)

\[
H_z^r = G_0 \left[ 1 - \frac{8}{\sin^2 \phi/2 + 3} - \frac{448}{\sin^4 \phi/2 - \sin^2 \phi/2 - 2} \right]
\]

(3.16)
The reflected field magnitudes are

\[ |E_z^r| = |G_0| \left[ 1 + \frac{352}{(\sin^2 \phi/2 - \sin^2 \phi/2 + 12)} \left( \frac{1}{16\kappa \sin \phi/2} \right)^2 + \ldots \right] \]  
\hspace{10cm} (3.17)

\[ |H_z^r| = |G_0| \left[ 1 - \frac{416}{(\sin^2 \phi/2 - \sin^2 \phi/2 - 12)} \left( \frac{1}{16\kappa \sin \phi/2} \right)^2 + \ldots \right] \]  
\hspace{10cm} (3.18)

c) Paraboloidal Surface (Axial, monostatic scattering); \( \theta' = \pi \)

\[ G_0 = \frac{f}{r} \csc^2 \frac{\theta}{2} e^{-jk(r-2f)} ; \theta = \pi \]  
\hspace{10cm} (3.19)

\[ U_s = -G_0 \left[ 1 + \frac{1}{2jkr} + \frac{1}{4(kr)^2} + \ldots \right] \]  
\hspace{10cm} (3.20)

\[ U_h = G_0 \left[ 1 - \frac{1}{2jkr} - \frac{3}{4} \frac{1}{(kr)^2} + \ldots \right] \]  
\hspace{10cm} (3.21)

The reflected field magnitudes are

\[ |U_s| = |G_0| \left[ 1 + \frac{3}{8} \frac{1}{(kr)^2} + \ldots \right] \]  
\hspace{10cm} (3.22)

\[ |U_h| = |G_0| \left[ 1 - \frac{5}{8} \frac{1}{(kr)^2} + \ldots \right] \]  
\hspace{10cm} (3.23)

d) Spherical surface \([E^1 = xe^{jke}]\)

\[ G_0 = \frac{a}{2\pi} e^{-jk(r-2\sin \theta/2)} \]  
\hspace{10cm} (3.24)
\[ E_\theta = - GO \left[ 1 - \frac{1}{2j k \sin^3 \theta/2} - \frac{7 \cos^2 \theta/2}{(2k \sin^3 \theta/2)^2} + \ldots \right] \]  

(3.25)

\[ E_\phi = GO \left[ 1 - \frac{1 - 2 \cos^2 \theta/2}{2j k \sin^3 \theta/2} + \frac{(7 - 2 \cos^2 \theta/2) \cos^2 \theta/2}{(2k \sin^3 \theta/2)^2} + \ldots \right] \]  

(3.26)

The monostatic reflected field magnitudes are

\[ |E_\theta| = GO \left[ 1 + \frac{1}{8(ka)^2} + \ldots \right] \]  

(3.27)

\[ |E_\phi| = 0 \]  

(3.28)

e) Prolate Spheroidal Surface (Axial, monostatic scattering)

\[ E^i = xe^{-jkz} \]

\[ GO = \frac{b^2}{2ar} e^{-jk(r-2a)} \]  

(3.29)

\[ E = -GO \left[ 1 - \frac{1}{2jka} + \ldots \right] \]  

(3.30)

The high frequency-early time, time domain representation of the reflected field can be readily obtained from the Luneburg-Kline expansion. Replacing the powers of \( k \) by \( s/(jc) \) in the expansion and Laplace transforming term by term yields a series with the first two terms being a delta function and a step function, each multiplied by their own constant. This early time functional behavior is demonstrated in the bandlimited impulse responses shown in Chapter II. However, the "impulse" heights are different for the prolate spheroid when the
incident angle is ninety degrees off axis for theta and phi polarizations. This differs from the Luneburg-Kline result since the delta function constant is equal to the negative of the square root of the product of the principle radii of curvature at the reflection point divided by two and is polarization independent. The difference in "impulse" heights is attributed to the lower frequencies in the frequency spectrum indicating the "impulsive" part of the reflected field is truly not solely due to high frequency information.

A further restriction has to be placed upon the Luneburg-Kline reflected field other than the largeness of the large parameter. As indicated by Equations (3.14) and (3.24), the other restriction limits the observation point to be in the deep lit region of the scatterer because as the observation point approaches an incident shadow boundary, these expansions become divergent. Defining the the large parameter not just to be \( kL \) where \( L \) is a characteristic dimension of the scatterer but to be \( 2kR\cos^2(\theta^i) \) where \( R \) is the radius of curvature of the reflected wavefront and \( \theta^i \) is the angle of incidence of the incident wavefront illuminating the surface would naturally indicate the validity of the expansion. Chapter IV utilizes such a large parameter definition.

2. Uniform Expansion

A uniform asymptotic solution for the scattered fields has been done [11] for a circular cylindrical surface. The uniformity of the solution pertains to the prediction of a continuous scattered field from
the deep light region through the transition region and to the deep shadow region. The reflected field in the far field, deep light region is described by a Pekeris function. The generalized, three dimensional, reflected field expression from a smooth, convex surface [5] based upon a circular cylinder is

\[ E^r(0) = E^i(Q_R) \star [R_s e_\perp e_\perp + R_h e_\parallel e_\parallel] \sqrt{\frac{\rho_1^r \rho_2^r}{(\rho_1^r + s^r)(\rho_2^r + s^r)}} e^{-jks^r} \]  

(3.31)

with

\[ R_{s,h} = \begin{bmatrix} \sqrt{-\frac{4}{\xi L}} e^{-j(\xi L)^{3/12}} \frac{e^{-j\pi/4}}{2 \sqrt{\pi \xi L}} [1 - F(x^L)] + \hat{\rho}_{s,h}(\xi^L) \end{bmatrix} \]  

(3.32)

where

- \( E^i(Q_R) \) is the incident field at the reflection point \( Q_R \)
- \( s^r \) is the reflected ray distance from \( Q_R \) to the observation point \( 0 \)
- \( \rho_1^i, \rho_1^r, \rho_2^i, \rho_2^r \) are the radii of curvature of the incident and reflected wavefront given by Equation (3.37)
- \( e_\perp, e_\parallel, e_\parallel \) are unit vectors defined in Figure 3.1
Figure 3.1. Illustration of uniform reflection parameters. (a) Astigmatic ray tube, and (b) Surface geometry with incident and reflected ray coordinates.
\[ p_{s,h} = \begin{bmatrix} p^*(x) \\ q^*(x) \end{bmatrix} e^{-j \pi/4} \frac{e^{-j \pi/4}}{2 \sqrt{\pi} x} \]

\( p^*(x) \) and \( q^*(x) \) are the soft and hard pekeris functions

\[ \xi^L = 2m \cos \theta^i \]

\[ m = \left( \frac{kR}{2} \right) \]

\( \theta^i \) is the angle of incidence

\( R \) is the radius of curvature of the surface at \( Q_r \) in the incidence plane

\( F(x) \) is the transition function defined in Equations (2.13)-(2.15)

\[ x^L = 2kL \cos^2 \theta^i \]

and

\[ L^L = \frac{\rho_1^2 + \rho_2^2 \rho_0^2 (\rho_2^2 + s^r)}{(\rho_1^2 + s^r) (\rho_2^2 + s^r)} \]

Some of the above parameters are illustrated in Figure 3.1. In the far field, for plane wave incidence, \( x^L \) goes to infinity, \( F(x^L) \) goes to unity and Equations (3.31) and (3.32) becomes
\[ E^r(0) = E^i(Q_R) \cdot \left[ R_s \hat{e}_1 \hat{e}_1 + R_h \hat{e}_1 \hat{e}_1 \right] \sqrt{\frac{r}{\rho_{1,2}}} \frac{e^{-jksc}}{sr}, \] (3.33)

where

\[ R_{s,h} = \sqrt{-\frac{-4}{\xi} \frac{e^{-j\xi^L}}{12} \hat{p}_{s,h}(\xi^L)} \] (3.34)

For \( \xi^L \gg 1 \) Equation (3.34) has an asymptotic expansion which is given by

\[ R_s \sim 1 + \frac{1}{2jkacos^3\theta^i} + \frac{5}{(2kacos^3\theta^i)^2} + \ldots \] (3.35)

\[ R_h \sim 1 - \frac{1}{2jkacos^3\theta^i} - \frac{7}{(2kacos^3\theta^i)^2} + \ldots \] (3.36)

The principle radii of curvature of the reflected wave for an incident spherical wavefront (see Figure 3.2) is given by

\[ \frac{1}{\rho_{1,2}^r} = \frac{1}{s^i} + \frac{1}{\cos^i} \left[ \frac{\sin^2\theta^i}{R_1} + \frac{\sin^2\theta^i}{R_2} \right] \pm \sqrt{\frac{1}{\cos^2\theta^i} \left[ \frac{\sin^2\theta^i}{R_1} + \frac{\sin^2\theta^i}{R_2} \right] - \frac{4}{R_1 R_2}} \] (3.37)
Figure 3.2. Surface geometry illustrating creeping wave parameters.
where

\( s^1 \) is the radius of curvature of the incident wave front at \( O_r \)

\( R_{1,2} \) are the principle radii of curvature for the surface at \( O_r \)

\( \theta_1 \) is the angle between the direction of the incident ray and \( \hat{U}_1 \)

and

\( \theta_2 \) is the angle between the direction of the incident ray and \( \hat{U}_2 \).

When \( \hat{U}_1 \) is in the plane of incidence, Equation (3.37) reduces to

\[
\frac{1}{\rho_1} = \frac{1}{s^1} + \frac{2}{R_1 \cos \theta^1} \quad \text{and} \quad (3.38)
\]

\[
\frac{1}{\rho_2} = \frac{1}{s^1} + \frac{2 \cos \theta^1}{R_2} \quad . \quad (3.39)
\]

A more general expression for the principle radii of curvature of a reflected wavefront due to an incident, astigmatic wavefront can be found in [4].

The generalized, two dimensional reflected field is obtained from the above expressions by suppressing the terms with subscripts of two.

It was commented in the previous chapter that the reflected field loses its physical interpretation near the forward scattering direction.
as indicated by the bandlimited impulse responses shown in Figure 2.6. 
This is confirmed in the comparison of Equations (3.15) and (3.16) to 
(3.35) and (3.36) which indicates that the higher order terms are not in 
agreement for the reflected field from a circular cylinder. However, 
the first two terms agree between Equations (3.10) and (3.11) to (3.25) 
and (3.36). This could be a consequence of obtaining a uniform solution 
between the lit and shadowed regions of the circular cylinder since the 
Fock fields, which are based upon a parabolic surface, are valid where 
the lit and shadowed solutions were made uniform as they must be. It 
must be commented that the second order term in the uniform solution may 
not be complete since the solution was generated only for uniformity to 
the first order. This solution still provides a correct uniform 
solution between the lit and shadowed regions of the scatterer when it 
has the required large radius of curvature.

B. FREQUENCY DOMAIN EXTRACTION TECHNIQUE

The total scattered field usually has a creeping wave mechanism 
added with the reflected field for most of the chosen canonical 
surfaces. The reflected field can be recovered by numerically 
subtracting the the creeping wave field from the total scattered field. 
The creeping wave field is highly dependent on the surface that it 
travels over as well as the reflected field with its own surface 
dependencies. The creeping wave field also has an asymptotic 
representation that is derivable from a Watson transformation of the
angular type eigenfunction solution. The asymptotic expansion was first cast into a GTD format for a circular cylinder by Keller [25]. The first three terms in the asymptotic expansion of the creeping wave field for a circular cylinder of radius $a$ was given by Franz and Galle [26] which is (see Figure 3.2), for one ray, given by

$$E_z^C = E_z^i (Q_1) \sum_{n=1}^{\infty} D_n^S (Q_1) D_n^S (Q_2) \exp \left[ -\int_{Q_1}^{Q_2} (jk + \alpha_n^S) \, dz \right] \left[ -\gamma(jh + \alpha_n^S) \, dz \right]^{-1} \frac{e^{-jks}}{\sqrt{s}}$$

(3.40)

and

$$H_z^C = H_z^i (Q_1) \sum_{n=1}^{\infty} D_n^h (Q_1) D_n^h (Q_2) \exp \left[ -\int_{Q_1}^{Q_2} (jk + \alpha_n^h) \, dz \right] \left[ -\gamma(jh + \alpha_n^h) \, dz \right]^{-1} \frac{e^{-jks}}{\sqrt{s}}$$

(3.41)

where

- $Q_1, Q_2$ are the initial and final location points of diffraction
- $dz$ is the differential path length of the ray on the surface
- $s$ is the distance between $Q_2$ and the far field observation point
- $\gamma$ is the closed path integration around the closed surface
- $D_n^S, h$ are the soft and hard surface diffraction coefficients given by
\[ D_{on}^{2s}(Q_1) = D_{on}^{2s} \left[ 1 + m^{-2} \frac{q_n}{30} e^{-j\pi/3} + m^{-4} \frac{3q_n}{1400} e^{j\pi/3} + \ldots \right] \] (3.42)

\[ D_{on}^{2h}(Q_1) = D_{on}^{2h} \left[ 1 + m^{-2} \left( \frac{-q_n}{30} - \frac{-2}{10} \right) e^{-j\pi/3} + m^{-4} \frac{3}{200} \left( \frac{-2}{q_n} - \frac{-4}{q_n} \right) e^{-j\pi/3} + \ldots \right] \] (3.43)

\[ D_{on}^{2s} = \frac{m}{\sqrt{2\pi k}} \frac{e^{-j\pi/12}}{[A_1(-q_n)]^2} \] (3.44)

\[ D_{on}^{2h} = \frac{m}{\sqrt{2\pi k}} \frac{e^{-j\pi/12}}{q_n[A_1(-q_n)]^2} \] (3.45)

\[ \alpha_{n}^{s,h} \text{ are the soft and hard attenuation constants given by} \]

\[ \alpha_{n}^{s} = \alpha_{on}^{s} \left[ 1 + m^{-2} \frac{q_n}{60} e^{-j\pi/3} + m^{-4} \frac{1}{140} \left( \frac{q_n}{10} - q_n^{-1} \right) e^{j\pi/3} + \ldots \right] \] (3.46)
\[ h_n = h_{on} \begin{bmatrix} 1 + m \frac{-2}{10} \left[ \frac{-q_n}{6} + q_n^2 \right] e^{-j\pi/3} + m^4 \frac{1}{200} \\
\left[ \frac{-q_n^2}{7} + 4q_n^1 + q_n^4 \right] e^{j\pi/3} + \ldots \end{bmatrix} \]

\[ \alpha_n^s = \frac{q_n}{R_g} m e^{j\pi/6} \quad (3.48) \]

\[ \alpha_{on} = \frac{q_n}{R_g} m e^{j\pi/6} \quad (3.49) \]

\[ m = \left[ \frac{kR_g}{2} \right]^{1/3} \quad (3.50) \]

\( R_g \) is the radius of curvature of the surface along the ray direction and

\( \text{Ai}(x) \) and \( \text{Ai}'(x) \) are Airy functions.

The Airy functions \( \text{Ai}(x) \) and \( \text{Ai}'(x) \) are defined as

\[ \text{Ai}(x) = \frac{1}{\pi} \int_0^\infty \cos \left( \frac{1}{3} t^3 + xt \right) dt \quad \text{and} \quad (3.51) \]

\[ \text{Ai}'(x) = \frac{-1}{\pi} \int_0^\infty tsin \left( \frac{1}{3} t^3 + xt \right) dt \quad . \quad (3.51) \]
The values \( q_n \) and \( \tilde{q}_n \) are, respectively, the roots to \( \text{Ai}(-q_n) = 0 \) and \( \text{Ai}'(-q_n) = 0 \). The derivatives of the radius of curvature are with respect to the path length.

The physical interpretation of this representation is that the incident energy is directed around the shadowed portion of the cylinder in an infinite number of modes denoted by the subscript \( n \). Each mode has its own diffraction coefficient and attenuation constant. Due to the surface boundary conditions, the soft modes shed off energy much more readily than due the hard ones. The factor involving the closed path integration accounts for multi-encirclements of the creeping wave which becomes important only for small cylinders. This expansion remains valid as long as the large parameter, \( m \), is large enough and the launch point is far enough away from the attachment point in terms of the path length. It has been shown [27] that this representation is accurate for relatively small large parameters as long as higher order terms and modes are included in the expansion. Usually the first two modes are sufficient for a convergent solution. An uniform, first order representation is also available [11] when the launch point approaches the attachment point.

The diffraction coefficients and attenuation constants for surfaces of nonconstant curvature has been studied by Keller and Levy [28] and by Voltmer [29]. Nonconstant curvature modifies the higher order terms in the diffraction coefficients and attenuation constants. Keller/Levy and Voltmer have essentially the same attenuation constants but Keller/Levy
performed an integration by parts on the attenuation constants and combined the endpoint contributions with the diffraction coefficients. Due to this fact, the Keller/Levy diffraction coefficient is essentially a first order coefficient while Voltmer does include more terms. Voltmer's diffraction and attenuation constants have the following form:

\[
D_{2n}^{2s}(Q_i) = D_{on}^{2s} \left[ 1 + m^{-2} \frac{q_n}{30} \left( 1 + \frac{R_g R_g}{6} \right) e^{-\frac{j\pi}{3}} + ... \right] (3.53)
\]

\[
D_{2n}^{2h}(Q_i) = D_{on}^{2h} \left[ 1 + m^{-2} \frac{q_n}{30} \left( 1 + \frac{R_g R_g}{6} \right) \right. \\
\left. - \frac{q_n}{10} \left( 1 - \frac{R_g R_g}{6} \right) e^{-\frac{j\pi}{3}} + ... \right] (3.54)
\]

and

\[
S_{2n}^{s} = S_{on}^{s} \left[ 1 + m^{-2} \frac{q_n}{15} \left( \frac{1}{4} - \frac{2}{3} \frac{R_g R_g}{g g} + \frac{4}{9} \frac{R_g^2}{R_g} \right) e^{-\frac{j\pi}{3}} + ... \right] (3.55)
\]
$$h_n = h_{on} \left[ 1 + m^{-2} \left[ \frac{q_n}{15} \left[ \frac{1}{4} - \frac{2}{3} R_g R_g + \frac{4}{9} R_g^2 \right] \right] \right]$$

$$+ \frac{q_n}{10} \left[ 1 - \frac{R_g R_g}{6} + \frac{R_g^2}{9} \right] e^{-j\pi/3} + ...$$

(3.56)

The variables $R_g$, $R_g^*$ and $R_g^{**}$ are the radii of curvature and its first and second derivatives with respect to path length, respectively.

A more convenient form for the attenuation constants for numerical evaluation is obtained by an integration by parts which yields

$$\int_{01}^{02} \alpha_n^s \, d\xi = \int_{01}^{02} \alpha_{on}^s \, d\xi + q_n^2 I_1$$

(3.57)

and

$$\int_{01}^{02} h_n \, d\xi = \int_{01}^{02} \alpha_{on}^s \, d\xi + \tilde{q}_n^2 I_1 + \tilde{q}_n^{-1} I_2$$

(3.58)

where

$$I_1 = \frac{e^{-j\pi/6}}{15} \left( \frac{2}{k} \right)^{1/3} \left[ \frac{1}{4} \int_{01}^{02} R_g^{-4/3} (1 + \frac{8}{9} R_g^2) \, d\xi - \frac{d}{d\xi} (R_g^{2/3}) \right]_{01}^{02}.$$
and
\[
I_2 = \frac{e^{-j \pi/6}}{10} \left( \frac{2}{k} \right)^{1/3} \left[ \frac{1}{4} \int_{q_1}^{q_2} R_g^{-4/3} (1 + \frac{1}{18} R_g^2) dz - \frac{1}{4} \frac{d}{dx} (R_g^{2/3}) \right]^{q_2}_{q_1}.
\]

(3.60)

Specializing the attenuation constants for an elliptic surface [30] yields
\[
\int_{q_1}^{q_2} \alpha_n^s \, dz = q_n \left( \frac{k b^2}{2a} \right)^{1/3} e^{j \pi/6} K(q_1, q_2) + q_n^2 I_1
\]

(3.61)

and
\[
\int_{q_1}^{q_2} \alpha_n^t \, dz = \bar{q}_n \left( \frac{k b^2}{2a} \right)^{1/3} e^{j \pi/6} K(q_1, q_2) - \bar{q}_n^2 I_1 + \bar{q}_n^{-1} J_2
\]

(3.62)

where
\[
J_1 = e^{j \pi/6} \left( \frac{2a}{k b^2} \right)^{1/3} \frac{1}{60} [(16 - 8 \beta) K(q_1, q_2) - 15 E(q_1, q_2)]
\]
\[
- \frac{1}{15} \left( \frac{2}{k} \right)^{1/3} \frac{d}{dz} (R_g^{2/3}) \bigg|^{q_2}_{q_1}
\]

(3.63)

\[
J_2 = e^{j \pi/6} \left( \frac{2a}{k b^2} \right)^{1/3} \frac{1}{20} (2 - \beta) K(q_1, q_2)
\]
\[
- \frac{1}{40} \left( \frac{2}{k} \right)^{1/3} \frac{d}{dz} (R_g^{2/3}) \bigg|^{q_2}_{q_1}
\]

(3.64)
\[ K(\Omega_1, \Omega_0) = \int_{\Omega_0}^{\Omega_1} \left[ 1 - \beta \cos^2 t' \right]^{-1/2} \, dt' \]  
(3.65)

\[ E(\Omega_1, \Omega_0) = \int_{\Omega_0}^{\Omega_1} \left[ 1 - \beta \cos^2 t' \right]^{1/2} \, dt' = \frac{1}{a} \int_{\Omega_0}^{\Omega_1} \, d\ell \]  
(3.66)

\[ \beta = 1 - \left( \frac{b}{a} \right)^2 \]  
(3.67)

\[ a \] is the semi-major axis length and \[ b \] is the semi-minor axis length.

The radius of curvature and its first and second derivatives are given by

\[ R_g = \frac{a^2}{b} (1 - \beta \cos^2 t')^{3/2} \]  
(3.68)

\[ \dot{R}_g = -3 \frac{a}{b} \beta \sin t' \cos t' \] and  
(3.69)

\[ R_g = 3 \frac{a}{b} (\cos^2 t' - \sin^2 t') \left[ 1 - \beta \cos^2 t' \right]^{-1/2} \]  
(3.70)

on the surface defined on the x-y plane as

\[ x = \alpha \cos t' \] and  
(3.71)

\[ y = \beta \sin t' \]  
(3.72)
Only the third order correction terms are known for the circular cylinder. It was found that they are important because the resulting reflected field from a subtraction of the creeping wave field from the total scattered field still exhibited an imperfect subtraction if they are not included. The imperfection is evident by a small amount of ripple in the magnitude of the frequency spectrum for the "reflected" field.

Voltmer also has an attenuation constant correction term to account for the transverse radius of curvature to the creeping wave path for a three dimensional surface. This term, however, is only valid when the transverse curvature to the creeping wave is greater than or equal to the radius of curvature along the creeping wave path. When this term is not valid, it is said that the creeping wave is in the paraxial region of the scatterer. An example of this is axial backscatter from a prolate spheroid. Levy and Keller [31] have shown that the acoustic, leading term, GTD solution for axial backscatter has the same diffraction coefficients and attenuation constants as the two dimensional cylinder solution.

C. TIME DOMAIN EXTRACTION TECHNIQUE

Chapter II contained several numerically generated, bandlimited impulse responses of the total scattered field for several convex surfaces when illuminated with a bandlimited impulsive plane wave. All of these transient responses demonstrated the isolation, in time, of the
various scattering mechanisms. A frequency spectrum of a desired mechanism can be obtained by extracting the transient response of the mechanism and numerically transforming back into the frequency domain using an FFT. This technique has the advantages of being applicable to bandlimited spectra that are either lowpass (DC to an upper cutoff frequency) or bandpass (a nonzero lower to an upper cutoff frequency) with the original spectrum being numerically generated or measured.

The actual steps involved in obtaining the reflected field is to first transform an unwindowed spectrum into the time domain. The response will contain Gibb's phenomenon since there was no windowing. A symmetric Hanning window [15] is applied to isolate or gate out the reflected field contribution. The Hanning weight for the nth sample for N+1 total samples is

\[
    w(n) = \begin{cases} 
        \frac{1}{2} \left[ 1 + \cos \left( \frac{2n\pi}{N} \right) \right] & n = -\frac{N}{2}, \ldots, -1,0,1, \ldots, \frac{N}{2} \\
        0 & \text{otherwise} 
    \end{cases} \quad (3.73)
\]

The window is centered about the mechanism and its maximum extent is determined by the maximum of the creeping wave mechanism. This extracted portion is then transformed back into the frequency domain. A Hanning window is used since it optimally partitions the transient contributions of the reflected and creeping wave fields based upon test cases involving a circular cylinder.
There are other limitations for this technique besides the artificial separation of the mechanisms. One is that the mechanisms must be separated enough in time so the roll off of the window magnitude does not significantly perturb the desired mechanism magnitude. Another limitation is perturbation of the endpoint regions for the resulting spectrum arises from two sources. The true low frequency portion of the spectrum can never be achieved because of the finite duration of the gating time window and the "smearing" effect of the window. This "smearing" effect is best understood by analyzing the dual situation in the frequency domain when the two functions (the impulse response and the window) are multiplied in the time domain. In the frequency domain, the transform of the two functions are convolved together. The transform of the window has one major lobe and several very low side lobes. As the transform of the time window is shifted along the spectrum, the finite width of the main lobe "smears" energy from spectrum areas of large relative magnitudes to spectrum areas of small relative magnitudes. This "smearing" can also be thought of as smoothing or averaging the spectrum as the window transform moves through the spectrum. Distortion of the spectrum endpoint regions also can occur from side lobe levels that are too high. Likewise, there is a "smearing" effect of the window at the high frequency portion of the resulting spectrum.
The roll off in the resulting spectrum from time gating can be eliminated by artificially extending the spectrum using the techniques in [16]. This occurs since the amplitude spectrum is smoothly terminated so when the transform of the gating window is effectively convoluted with the original spectrum, there exist no sharp discontinuities in the amplitude spectrum to be smoothed out.
The asymptotic expansions in Chapter III revealed a uniqueness of the reflected field for a given surface. An example of this observation is indicated by the monostatic reflection from a parabolic and circular cylinder. Here, the correction factors to the geometrical optics (GO) term are different when the length variables are expressed in terms of the as radii of curvature of the surface at the reflection point. Also the character of the reflected field is dependent upon whether the observation point is in the deep lit region or near the incident shadow boundary. This is indicated by the different correction terms for the circular cylinder that are derived by the Luneburg-Kline expansion (deep lit region) or the uniform expansion (shadow boundary transition region).

However, there are some generalizations that can be made for both the two and three dimensional reflected field based upon the Luneburg-Kline expansions shown in Equations (3.9)-(3.30). The expansions for the copolarized, reflected field have the following general form:
Field Value = [L.T.][C.F.][R.F.][P.F.] \quad (4.1)

where

[L.T.] is the leading term

[C.F.] is the correction factor

[R.F.] is the range factor and

[P.F.] is the phase factor

It is common to combine the leading term and range factor into a term called the spread factor (SF) which is given by

\[ SF = \sqrt{\frac{\rho}{\rho^r + s^r}} \]

for two dimensional surfaces and

\[ SF = \sqrt{\frac{\rho_1 \rho_2}{(\rho_1^r + s^r)(\rho_2^r + s^r)}} \]

for three dimensional surfaces. The spread factor has the physical interpretation of expressing the power flow from a reference surface to the observation surface for an astigmatic tube of rays as shown in Figure 3.1. The reflected field magnitude is accurately predicted for surfaces with large radii of curvature by the spread factor. As the
surface curvature becomes electrically small, the spread factor must be multiplied by an appropriate correction factor. The directional change from unity of this correction factor is dependent upon the polarization of the incident field in relation to the surface. The correction factor also contains information about the observation distance from the surface for near field calculations.

The correction factor is only asymptotically known for a few surfaces. However, for a parabolic cylinder and paraboloid of revolution, the correction factors are known exactly for axial plane wave incidence and an arbitrary observation location. These solutions were first found by Lamb [32]. Generalized two and three dimensional surface reflection coefficients are presented below based upon these two solutions by extending the physical interpretation of the parameters in these exact results. Although the predicted reflected field is not exact, it does mimic the reflected field from a parabolic surface. In addition, it accurately predicts reflected field values from surfaces with small radii of curvature especially when the illuminated surface is similar to the surface from which the solution evolved.

A. TWO DIMENSIONAL SURFACE

The generalized two dimensional reflection coefficient is based upon physical interpretation of the scattered field from a parabolic cylinder. The exact solution for the scattered field from a parabolic cylinder with axial, unit plane wave illumination (\(\phi' = \pi\)) as shown in Figure 2.1(a) is
\[ E_z = -\frac{\sqrt{f}}{\sin \phi/2} \frac{F(2k\cos \phi/2)}{F(2kf)} \frac{e^{-jk(\rho-2f)}}{\sqrt{\rho}} \text{ and } (4.2) \]

\[ H_z = \frac{\sqrt{f}}{\sin \phi/2} \frac{F(2k\cos \phi/2)}{2 - F(2kf)} \frac{e^{-jk(\rho-2f)}}{\sqrt{\rho}} \text{ (4.3)} \]

where

\( \rho, \phi \) are the polar coordinates of the observation point

\( f \) is the focal length of the parabolic cylinder and

\( F(x) \) is the transition function defined in Equation (2.13).

The expressions in Equations (4.2) and (4.3) follow the format indicated in Equation (4.1) where

\[
\begin{array}{|c|c|}
\hline
\text{Soft} & \text{Hard} \\
\hline
\text{[L.T.]}: & \frac{\sqrt{f}}{\sin \phi/2} & \frac{\sqrt{f}}{\sin \phi/2} \\
\text{[C.F.]}: & \frac{F(2k\cos \phi/2)}{F(2k\rho)} & \frac{F(2k\cos \phi/2)}{2-F(2k\rho)} \\
\text{[R.F.]}: & 1/\sqrt{\rho} & 1/\sqrt{\rho} \\
\text{[P.F.]}: & e^{-jk(\rho-2f)} & e^{-jk(\rho-2f)} \\
\hline
\end{array}
\]
The scattered fields of Equations (4.2) and (4.3) have a physical interpretation is that they represent a wave originating at the focus of the parabolic cylinder and spreading as a cylindrical wave. The focus location of the parabolic surface for the incident plane wave is actually the caustic location for the scattered wave which doesn't move with respect to the observation location. In general, the scattered (reflected) field caustic location is dependent upon the observation location for a fixed source location. The strength of the scattered field is dictated by several factors. The most obvious is the spread factor \( \sqrt{f/p} \) which is identified in Equations (4.2) and (4.3) along with several other functions that form the correction factor. The exact correction factor has both observation and surface dependencies. It is the transition function term in the denominator of this factor that allows the solution to blend smoothly from a curved surface reflection to an edge diffraction. Physically, as the caustic approaches the surface, the argument of the transition function becomes smaller and the transition function modifies the GO contribution.

The basic concept in predicting the reflected field from an arbitrary surface is to take into account the reflected ray caustic distance \( \rho^r \) and the incident shadow boundary distances from the surface specular point \( z^i_1 \) and \( z^i_2 \) as shown in Figure 4.1. These distances modify the GO field as parameters in a transition function just as the focal length did for a parabolic surface solution. Thus based on physical reasoning, the generalization of these distance parameters in the transition function for the parabolic surface will allow one to more accurately predict reflected fields. Any prediction
Figure 4.1. Two dimensional curved surface geometry and parameter definition.
of a scattered field based upon this generalization for another surface is in error since the true scattered field from a surface can only be given by its unique transition function. However, if based on many practical examples, it can be applied for a very wide variety of shapes as shown later.

The generalization of Equations (4.2) and (4.3) begins by replacing the spread factor with its general form which is given by

$$SF = \sqrt{\frac{\rho r}{\rho r + s^r}}$$  \hspace{1cm} (4.4)

The correction factor generalization is restricted to modifying the arguments in the transition functions. The transition function argument in the numerator and denominator controls the range and surface dependencies, respectively. The range dependence transition function has an argument that is very similar in form to the UTD wedge diffraction coefficient. This is reasonable since when the parabolic surface collapses into a half plane, they both give the same exact scattering solutions. Thus, the notation for the arguments of the UTD wedge diffraction coefficient has been adopted which is $kLa$. The parameters in this argument are "$k$" being the wavenumber of the incident field, "$L$" is length parameter and "$a$" is an angular parameter. The functional form of the parameters for half plane diffraction are

$$k = \frac{2\pi}{\lambda}$$ \hspace{1cm} (4.5)

$$L = \frac{s's}{s^r + s}$$ \hspace{1cm} and \hspace{1cm} (4.6)
\[ a = 2 \cos^2(\psi \pm \psi')/2 \]  \hspace{1cm} (4.7)

Figure 4.2 illustrates the variables \( s, s', \psi \) and \( \psi' \).

The numerator transition function parameter identification with Equation (4.5) involves equating the geometries in Figures 4.1 and 4.2. This is done by artificially positioning a half plane along the plane containing the inward normal unit vector with the edge positioned at the surface specular point. The distance dependence \( \rho \) in Equations (4.2) and (4.3) is replaced by \( s'/s'(s' + s) \) in order to include the effect of the source distance to the specular point. Note that \( s' \) and \( s \) are the source and observation distances to the point of reflection on the surface. The angle dependence \( 2\sin^2(\phi/2) \) in Equations (4.2) and (4.3)
is first modified to include the angle of the incident plane wave which
yields $2\cos^2(\phi + \pi)/2$. This form is readily related to $a(\psi + \psi')$
parameter for a half plane in the UTD solution. The parameters $\psi'$ and $\psi$
are angles measured from the plane containing the inward normal unit
vector to the incident and reflected rays, respectively.

The argument of the transition function for the surface is
generalized through the examination of the Luneburg-Kline expansions for
monostatic reflection from a parabolic and elliptic cylinders. The soft
and hard correction factors for the far field, axial monostatic
reflected fields from these surfaces are presented in Table 4.1 (see
Equations (3.10), (3.11) and Appendix A). The surface dimensions
indicated in Table 1 are illustrated in Figure 2.1. These dimensions
can be physically interpreted to have geometric meaning. An obvious
geometric parameter to associate with the surface dimensions is the
reflected field caustic distance, $\rho^r$. The reflected wavefront radius of
curvature (caustic distance) for a smooth surface due to an incident
plane wave is $\rho^r=R\cos(\theta^i)/2$ where $R$ is the surface radius of curvature
at the specular point and $\theta^i$ is the angle of incidence. The surface
radii of curvature for the specular points pertaining to Table 4.1 are
$2f$ (parabolic surface) and $b^2/a$ (elliptic surface). Table 4.2
incorporates the caustic distance extension into the correction factors
shown Table 4.1.
## Table 4.1

### Asymptotic Correction Factors

<table>
<thead>
<tr>
<th>Cylinder Type</th>
<th>Soft</th>
<th>Hard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parabolic</td>
<td>[1 + \frac{1}{4jk^T} + \ldots]</td>
<td>[1 - \frac{1}{4jk^T} + \ldots]</td>
</tr>
<tr>
<td>Elliptic</td>
<td>[1 + \frac{a}{2jkb^2} - \frac{3}{16jka} + \ldots]</td>
<td>[1 - \frac{a}{2jkb^2} - \frac{3}{16jka} + \ldots]</td>
</tr>
</tbody>
</table>
However, there is an additional term in the elliptic cylinder correction expansions that is not present in the parabolic cylinder correction expansions. The conjecture for the physical significance of this second term is that it represents the influence of an incident shadow boundary upon the reflected field. This influence is inversely related to the distance between the specular point and an incident shadow boundary. This distance is illustrated in Figure 4.1 by 11 and 12 for the general case of a bistatic reflected field. The distance is the difference between phase fronts for the rays that reflect from the surface and are tangent to the surface. To satisfy reciprocity, only the incident shadow boundaries that are visible from both source and observation and source points are used. The absence of such a term for the parabolic cylinder is apparent since the dependence is inversely related to such a distance which is infinite. For axial, monostatic reflection from an elliptic cylinder, both distances are equal to the distance \( a \). Further justification for the shadow boundary effects originate from endpoint contributions in an equivalent stationary phase evaluation of surface currents. Table 4.2 shows the functional dependence when there are two endpoint corrections as in the case of a closed, convex two dimensional surface.
### TABLE 4.2
ASYMPTOTIC CORRECTION FACTORS

**Cylinder Type**

<table>
<thead>
<tr>
<th>Type</th>
<th>Soft</th>
<th>Hard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parabolic</td>
<td>$1 + \frac{1}{4jk\rho r} + \ldots$</td>
<td>$1 - \frac{1}{4jk\rho r} + \ldots$</td>
</tr>
<tr>
<td>Elliptic</td>
<td>$1 + \frac{1}{4jk\rho r} - \frac{3}{32jk} \left[ \frac{1}{z_1^e} + \frac{1}{z_2^e} \right] + \ldots$</td>
<td>$1 - \frac{1}{4jk\rho r} - \frac{3}{32jk} \left[ \frac{1}{z_1^e} + \frac{1}{z_2^e} \right] + \ldots$</td>
</tr>
</tbody>
</table>
The examination of the asymptotic form of the correction factors finishes with the addition of the bistatic angle dependence to the generalized correction factors shown in Table 2. The functional form for the bistatic angle dependence is based upon the correction factors for a circular cylinder (Equations (3.15) and (3.16)) which is given by

\[(C.F.)_{s,h} = \left[ 1 \pm \frac{8/\cos^2 \theta}{16 j k a \cos \theta} + \ldots \right] \]  \quad (4.8)

Comparing this expression with the generalized correction factor expressions in Tables 1 and 2 yields

\[(C.F.)_{s,h} = \left[ 1 \pm \frac{8/\cos^2 \theta \frac{a}{b} \pm 3 \frac{b}{a}}{16 j k b \cos \theta} + \ldots \right] \]  \quad (4.9)

or

\[(C.F.)_{s,h} = \left[ 1 \pm \frac{1}{4 j k \rho \cos^2 \theta} \left[ 1 \pm \frac{3}{8} \rho \cos^2 \theta \cdot \left[ \frac{1}{x_1} \pm \frac{1}{x_2} \right] \right] + \ldots \right] \]  \quad (4.10)

Note that the true bistatic correction factor for an elliptic cylinder will contain contributions dependent upon first and second derivatives of the radius of curvature at the specular point (see Appendix A).
The objective now is to relate the parameters in Equation (4.10) to that of a $kL^c a^r$ form. The parameter assignment philosophy originates from the argument of the transition function in the denominator of the exact result (see Equations (4.2) and (4.3)) which is $2k\beta$. Here, $\beta$ can be thought as the $L^c$ parameter; whereas, and the factor $2$ is associated with the $a^r$ parameter. These parameter associations are extended further using the information in Equation (4.10). The $L^c$ parameter becomes $\rho^r$ times some multiplicative term which accounts for the finite distance the incident shadow boundaries are from the specular point and the \(^"a"\) parameter becomes $2\cos^2(\theta^i)$. Thus, the $L^c$ and $a^r$ parameters are

$$L^c = \rho^r \left[ 1 - \frac{3}{8} \rho \cos^2 \theta^i \left( \frac{1}{\varepsilon^c_1} + \frac{1}{\varepsilon^c_2} \right) \right]^{-1}$$

or

$$L^c = \rho^r \left[ \frac{\varepsilon^c_1 \varepsilon^c_2}{8 \varepsilon^c_1 \varepsilon^c_2 - 3 \rho \cos^2 \theta^i (\varepsilon^c_1 + \varepsilon^c_2)} \right]$$

(4.11)

and

$$a^r = 2\cos^2 \theta^i.$$

(4.12)
Note that only the minus sign in the parenthesis of Equation (4.10) has been included in Equation (4.11). This occurs because this type of correction is actually negative as shown in Table 4.1. When such a correction is performed for the soft and hard correction factors as in Equations (4.2) and (4.3) the proper correction results. When the distances are small (less than 1.5 \( \lambda \)), the ray optical reflection interpretation fails and there is not a sufficient stationary point to justify the GO. An estimate of the reflected field can still be obtained by suppressing this multiplicative term.

Hence, the reflected field for a convex two dimensional, perfectly conducting surface is given by

\[
\begin{align*}
E_z^r(Q) &= -E_z^i(Q_R) R_s \sqrt{\frac{\rho}{\rho + s}} e^{-jksr} \\
H_z^r(Q) &= E_z^i(Q_R) R_h \sqrt{\frac{\rho}{\rho + s}} e^{-jksr}
\end{align*}
\]

where

\[
R_s = \frac{F[kLa(\psi + \psi')]}{F[kLcaf(\psi + \psi')]}.
\]

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\[
R_h = \frac{F[kLa(\psi + \psi')]}{2 - F[kL^c a^r]}
\]

\[
L = \frac{s's}{s^2 + s}
\]

\[
a(\psi + \psi) = \frac{2\cos^2(\psi + \psi')}{2}
\]

\[
L^c = p^r \frac{8z^e_1 z^e_2}{8z^e_1 z^e_2 - 3p^r \cos^2 \theta^i (z^e_1 + z^e_2)}
\]

\[
a^r = 2\cos^2 \theta^i
\]

\[
\theta^i = \cos^{-1}(-\hat{e} \cdot \hat{n})
\]

One main advantage of the reflection coefficients in Equations (4.11) and (4.12) is the uniformity of being able to describe (predict) a reflected or diffracted field from a two dimensional surface depending upon the radius of curvature at its scattering center.

A further extension of the reflection concept can be applied to the UTD diffraction coefficient for a wedge [4]. The wedge geometry assumes a perfectly sharp tip which is not realistic when modelling physical structures. Hence, the modelling of relatively "sharp" edges can be improved by considering them as rounded edges with small but a finite radius of curvature. The conjecture for this extension originates from
the scattering from a parabolic cylinder as it collapses into a half plane (focal length goes to zero). Both the half plane scattering expressions from the parabolic cylinder and wedge structure yield the same, exact far field result for edge on, plane wave incidence, i.e.,

\[
E_d^z = -\frac{\sqrt{r}}{F(2k_0f)} e^{-jk_0} = D_s e^{-j\pi/4} e^{-jks} \quad (4.15)
\]

and

\[
H_d^z = \frac{\sqrt{r}}{2-F(2k_0f)} e^{-jk_0} = D_h \frac{e^{-jks}}{\sqrt{s}} = 0 \quad (4.16)
\]

The coefficients \(D_s, h\) are the soft and hard UTD diffraction coefficients for a half plane. Thus, the conjecture is to scale the diffraction coefficients by the surface factors in the parabolic cylinder scattering expressions. The scaled diffraction expressions based upon Equations (4.11) and (4.12) are

\[
E_s^z = \frac{\sqrt{r}}{F(kLc_a r)} \frac{\sqrt{2\pi k}}{e^{-j\pi/4}} \frac{e^{-jks}}{\sqrt{s}} \quad \text{and} \quad (4.17)
\]

\[
E_s^z = \frac{\sqrt{r}}{2-F(kLc_a r)} \frac{\sqrt{2\pi k}}{e^{-j\pi/4}} \frac{e^{-jks}}{\sqrt{s}} \quad . \quad (4.18)
\]
The eigenfunction expansion for the scattered fields from a parabolic cylinder also reduces to the UTD half plane result for other source/observation angles as the focal length goes to zero as demonstrated by Cherry [33].

B. THREE DIMENSIONAL SURFACES

The generalized three dimensional reflection coefficient is based upon physical interpretation of the scattered fields from both a parabolic cylinder and an acoustic paraboloid of revolution to provide uniformity between two and three dimensional solutions. The exact solution for the acoustic scattered field from a paraboloid of revolution with axial, unit plane wave illumination as shown in Figure 2.10(a) is given by

\[
U_s = -\frac{F_3(2kr \sin \theta/2)}{F_3(2kf)} \frac{e^{-jk(r-2f)}}{r} \quad (4.19)
\]

and

\[
U_h = \frac{F_3(2kr \sin \theta/2)}{2-F_3(2kf)} \frac{e^{-jk(r-2f)}}{r} \quad (4.20)
\]

where

- \( r, \theta \) are the polar coordinates of the observation point
- \( f \) is the focal length of the paraboloid of revolution and
- \( F_3(x) \) is the transition function defined in Equation (2.27).
The expressions in Equations (4.19) and (4.20) follow the format indicated in Equation (4.1) where

<table>
<thead>
<tr>
<th></th>
<th>Soft</th>
<th>Hard</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L.T.):</td>
<td>(-f \frac{\theta}{\sin \varphi})</td>
<td>(f \frac{\theta}{\sin \varphi})</td>
</tr>
<tr>
<td>(C.F.):</td>
<td>(\frac{\theta}{F_3(2kr \sin^2 \varphi)})</td>
<td>(\frac{\theta}{F_3(2kr \sin^2 \varphi)})</td>
</tr>
<tr>
<td>(R.F.):</td>
<td>(1/r)</td>
<td>(1/r)</td>
</tr>
<tr>
<td>(P.F.):</td>
<td>(e^{-jk(r-2f)})</td>
<td>(e^{-jk(r-2f)})</td>
</tr>
</tbody>
</table>

The scattered fields of Equations (4.19) and (4.20) have a physical interpretation of wave originating at the focus of the parabolic cylinder and spreading as a spherical wave. The focus location of the paraboloidal surface for the incident plane wave is actually the caustic location for the scattered wave which doesn't move with respect to the observation location. In general, the scattered (reflected) field caustic location is dependent upon the observation location. The strength of the scattered
field is dictated by several factors. The spread factor is readily identified in Equations (4.19) and (4.20) leaving a quotient of transition functions to form the correction factor. The exact correction factor has both observation and surface dependencies.

The generalization of Equations (4.19) and (4.20) for electromagnetic scattering follows the same procedure as for the two dimensional case, but the vector nature of the solution has to be added heuristically. Due to the limited amount information available from this acoustic solution, only a monostatic reflected field coefficient has been formulated. As before, the spread factor is given by

\[ SF = \sqrt{\frac{\rho_1^r \rho_2^r}{(\rho_1^r + s^r)(\rho_2^r + s^r)}} \]  

(4.21)

The three dimensional correction factor appears to be quite different from the two dimensional correction factor for the two exact solutions. However, they are almost the same when extending the two dimensional solution to a three dimensional solution. The extension is based upon the astigmatic ray tube concept in that the power flow in this tube is determined by two independent, principle wavefront curvatures. The form of the three dimensional correction factor is simply the product of two, two dimensional correction factors. This was arrived at because the three dimensional spread factor is just the product of two, two dimensional spread factors and that the spreading from each principle curvature should have its own correction factor.
The above two dimensional based correction factor can be compared to the exact three dimensional correction factor for a spreading spherical wave. Figure 4.3 shows such a three dimensional correction factor comparison between the exact three dimensional correction factor and a product of two, two dimensional correction factors for both the soft and hard cases. The exact correction factor is from the paraboloid of revolution scattering solution; whereas, the two dimensional correction factor is from the parabolic cylinder scattering solution. The agreement is excellent with a slight difference for smaller arguments for the hard case. It is interesting to note that the three dimensional transition function is nearly the square of the two dimensional transitional function. Figure 4.4 illustrates the magnitude and phase for the two and three dimensional transition functions.

An uniform, generalized correction factor that properly reduces to the exact correction factor, for either cylindrical and spherical spreading is

\[
[CF]^s = \left[ \frac{\mathcal{G}(2krsin^2\theta)}{\mathcal{G}(2kf)} \right]^{(2-\Lambda_c)/2} \quad \text{and} \quad (4.22)
\]

\[
[CF]^h = \left[ \frac{\mathcal{G}(2krsin^2\theta)}{2-\mathcal{G}(2kf)} \right]^{(2-\Lambda_c)/2} \quad . \quad (4.23)
\]
Figure 4.3. Three dimensional correction factor comparison between exact and extended two dimensional correction factors for soft and hard acoustic cases.
Figure 4.4. Comparison of two and three dimensional transition functions denoted by $F(x)$ and $F_3(x)$, respectively.
Note that \( G(x) \) is a generalized transition function defined by

\[
G(x) = 2jx e^{(\Lambda_c+1)/2 jx} e^{-j\tau^2} \int_{\sqrt{x}}^{\Lambda_c/\tau} e^{-\Lambda_c^2/\tau^2} d\tau
\]

(4.24)

where

\[
\Lambda_c = \frac{\min(R_1, R_2)}{\max(R_1, R_2)}
\]

In addition, \( R_1, R_2 \) are the principle surface radii of curvature with small and large argument forms given by

\[
G(x) = je^{jx} \left[ j^{q-1} x^q r(1-q) - \frac{x}{1-q} + j \frac{x^2}{2-q} + \frac{x^3}{3-q} - j \frac{x^4}{6(4-q)} + \ldots \right]
\]

for small \( x \) and

(4.25)

\[
G(x) \sim 1 + \frac{q}{jx} - \frac{q(q+1)}{x^2} - \frac{q(q+1)(q+2)}{x^3} + \frac{q(q+1)(q+2)(q+3)}{x^4} + \ldots
\]

for large \( x \) with \( q = (\Lambda_c+1)/2 \).

(4.26)

The blending factor \( \Lambda_c \) has been suggested by Pathak [34] as a possible means to achieve the limiting cases.
It was shown in Figure 4.3 that the product of two soft or two hard, two dimensional correction factors simulated the proper three dimensional soft and hard acoustic correction factors. However, for the vector case, a product of soft and hard correction factors seems appropriate because it can simulate the proper asymptotic correction factor trend for axial reflection from a prolate spheroid. The simulation is demonstrated by Equations (4.27)-(4.29) by multiplying the soft and hard asymptotic forms for axial reflection from an elliptic cylinder. The principle radii of curvature for the surface are \(b'/\sqrt{a}\) and \(b^2/a\); thus, the correction factor is given by

\[
[CF] = \left[ \frac{a}{8b' - 3a} + \ldots \right] \left[ 1 - \frac{a}{16jk b'} + \ldots \right] \quad (4.27)
\]

\[
[CF] = \left[ 1 + \frac{1}{16jk b' b} \left[ \frac{b}{8a(b' - b)} - \frac{6b' b}{a} \right] \right] + \ldots \quad (4.28)
\]

and for \(b = b'\)

\[
[CF] = \left[ 1 + j \frac{3}{8ka} + \ldots \right] \quad (4.29)
\]
As can be seen, the contribution in the second term due to the radius of curvature at the specular point vanishes leaving only the endpoint contribution. This explains why the asymptotic correction term is the same for axial monostatic reflection from a prolate spheroid (semi-major radius $a$) as is for monostatic reflection from a sphere (radius $a$).

Although there is a coefficient difference between the exact expressions for a sphere or prolate spheroid (Equations (3.25) and (3.30) to that of Equation (4.29) (1/2 compared to 3/8); however, the significant character is described.

Finally, the heuristic expression for the monostatic reflected field from a perfectly conducting, convex three dimensional surface is given by Equation (4.30). It is based upon the heuristic extensions for the two and three dimensional solutions presented. The functional form reduces to the limiting cases of a cylinder (Equations (4.13) and (4.14)) and sphere (Equation (4.29)). The expression is

$$E_r(0) = -E^i(0_R) \sqrt{\frac{\rho_1}{\rho_1 + s^r}} \sqrt{\frac{\rho_2}{\rho_2 + s^r}} \left[ R_h(\rho_1) \hat{s}_1 \hat{u}_1 + R_h(\rho_2) \hat{s}_2 \hat{u}_2 \right] e^{-jks^r}$$

(4.30)
where

\( O_1, O \) are the specular and observation points, respectively

\( \rho_1^r, \rho_2^r \) are the principle radii of curvature of the reflected wavefront

\( \hat{u}_1, \hat{u}_2 \) are the principle unit surface vectors as illustrated in Figure 3.1

\[
R_s(\rho_1^r) = \left[ \frac{(kLa)}{(kL_i a^r)} \right]^{(2 - \Lambda_c)/2}
\]

\[
R_h(\rho_1^r) = \left[ \frac{(kLa)}{2 - (kL_i a^r)} \right]^{(2 - \Lambda_c)/2}
\]

\[
L = \frac{s^s}{s^s + s}
\]

\( a = 2 \)

\[
L_i^c = \rho_1^r \frac{8\xi^e_{1,i} \xi^e_{2,i}}{8\xi^e_{1,i} \xi^e_{2,i} - 3\rho_1(\xi^e_{1,i} + \xi^e_{2,i})}
\]

\( a^r = 2 \).

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The accuracy of Equation (4.30) for several cases is very reasonable as will be demonstrated in Chapter V even though the vector solution was developed heuristically from an acoustic solution. Note that there are two $L^C_i$, one for each principle plane and that for a simple convex surface, there are two endpoint contributions for each principle plane.

Although, Equation (4.15) gives the proper asymptotic trend for the first two terms, it does not give the proper magnitude correction for when the principle radii of curvature are equal as will be illustrated in Chapter V. This observation can be seen in terms of an asymptotic expansion of Equation (4.30). In terms of a magnitude correction, the first three terms have to be considered and in Equations (4.27)-(4.29) just two terms were used to demonstrate the physical significance of the second term in the vector Luneburg-Kline expansion for the monostatic reflected field for a sphere and prolate spheroid. The correction of this inadequacy, if the technique of multiplying a soft and hard acoustic reflection coefficients together for a vector solution, is to have a different transition function with the proper asymptotic expansion so the third term is essentially zero.

Other expressions for the reflected field have been developed elsewhere that indicate the functional dependence upon the scatterer geometry. One attempt by Meckelburg, [35], using the Luneburg-Kline approach, yielded only partial information for the second term. Meckelburg, didn’t obtain the Laplacian required for the transport of the field from the surface (see Equation (3.8)). However, having the
second term is of only limited usefulness since the third term is really required for a magnitude correction. Foo [36] generated the polarization matrix for the reflected field from a smooth convex surface based upon the first order correction term to physical optics. The element of the matrix are, modulo scaling factors, such that

\[ S_{11} = \frac{(jk)^2}{2\pi} A_F(k) \left[ 1 - \frac{(K_1 - K_2)}{2jk \cos^2 \alpha} \right] \] (4.31)

\[ S_{22} = \frac{(jk)^2}{2\pi} A_F(k) \left[ 1 + \frac{(K_1 - K_2)}{2jk \cos^2 \alpha} \right] \] and (4.32)

\[ S_{12} = S_{21} = \frac{(jk)^2}{2\pi} A_F(k) \frac{K_1 - K_2}{2jk \sin^2 \alpha} \] (4.35)

where \(K_1\) and \(K_2\) are the principle surface curvatures.

The subscripts indicate the alignment of the transmit and receive polarizations with respect to the principle surface directions. \(A_F(k)\) is the Fourier transform of the silhouette area function for a scatterer as delineated by an incident impulsive plane wave moving at half the free space velocity. The angle \(\alpha\) is the angle between the plane of incidence and first principle surface direction. \(S_{11}\) and \(S_{22}\) reveal an increase or decrease of the monostatic return for the two principle
polarizations from the high frequency limit as it should be. However, the variation is the same which is undesirable as will be illustrated in Chapter V. It is believed that the correction is a result of how the surface current was obtained. The surface was expanded in a Taylor series about the specular point which automatically simulates a paraboloidal surface to first order. As presented earlier in the two dimensional case, this surface character is not taken into account in the endpoint effect of the shadow boundary. Thus, the $S_{11}$ and $S_{22}$ expressions are missing information. These expressions, for the case of a sphere, don't have a nonzero $k^{-2}$ correction term as they should.

The dependence of the reflected field upon polarization and the principle radii of curvature allows the generation of a cross polarized field. Foo has suggested a form for this component in Equation (4.33) which is reasonable for surfaces with finite principle radii of curvature, but it is missing shadow boundary information. Figure 4.5 illustrates the monostatic, co and cross polarized reflected field generated by a cylindrical, circular cross section surface with a radius of .0762 m. Figure 4.6 illustrates the monostatic, co and cross polarized reflected field from an arbitrary surface with principle radii of curvature of .1524 and .0381 m. The fields in both figures have been normalized with respect to the GO level. The normalized reflection expression used to obtain the co-polarized return is

$$R^{CO} = \cos^2 \alpha R^1 + \sin^2 \alpha R^2$$

(4.34)
Figure 4.5. Calculated monostatic, (a) co and (b) cross polarized reflected field return for a circular cylinder with a = 0.0762 m as a function of angle from a principle direction. Normalized to the G.O. return. Plane wave illumination.
Figure 4.6. Calculated monostatic, (a) co and (b) cross polarized reflected field return for a circular cylinder with \( a = 0.0762 \text{ m} \) as a function of angle from a principle direction. Normalized to the G.O. return. Plane wave illumination.
and for the cross polarized return, the expression is

$$R^{cr} = \sin \alpha \cos \alpha (R_1 - R_2)$$  \hspace{1cm} (4.35)

Note that $R_1$ and $R_2$ are the reflection coefficients for when the incident electric field are in the plane of the principle radii of curvature of the surface. The coefficients $R_1$ and $R_2$ for two dimensional surfaces are

$$R_1 = \frac{1}{F(kR)} \quad \text{and} \quad (4.36)$$

$$R_2 = \frac{1}{2 - F(kR)} \quad . \hspace{1cm} (4.37)$$

Whereas for three dimensional surfaces, they are

$$R_1 = \left[ \frac{1}{G(kR_2)(2 - G(kR_1))} \right]^{(2 - \Lambda_C)} \quad \text{and} \quad (4.38)$$

$$R_2 = \left[ \frac{1}{G(kR_1)(2 - G(kR_2))} \right]^{(2 - \Lambda_C)} \quad . \hspace{1cm} (4.39)$$

The angle $\alpha$ is measured from the plane containing the larger radii of curvature.
CHAPTER V

COMPARISON OF VARIOUS REFLECTED FIELD SOLUTIONS

A critical examination of the various for reflected field solutions for perfectly conducting, smooth, convex surfaces is presented here. This examination entails far zone reflected field comparisons for circular and elliptic cylinders, spheres and prolate spheroids which are illuminated by an unit plane wave polarized in the principle directions. Three approaches are used in making the reflected field comparisons which are (1) calculated monostatic and bistatic aspect angle comparisons, (2) calculated swept frequency comparisons and (3) measured monostatic aspect angle comparisons.

The comparisons demonstrate the agreement of the various techniques used to predict the reflected field from a surface to the "exact" reflected field from that same surface. The exact field values are numerically generated by two methods. Method one is the substraction of the surface diffracted fields (creeping waves) from the eigenfunction
function or moment method solution for the total scattered field. This method works well for the circular cylinder where the first three terms for the diffraction and attenuation coefficients of the creeping wave are known. Without the higher order terms, the subtraction result is not a monotonic varying function due to residual creeping wave fields. Method two is the time isolation of the reflected field from the creeping wave. Here, the frequency domain solution of the total scattered field is Fast Fourier transformed into the time domain and the reflected field mechanism is gated out with a Hanning window. The gated mechanism is then transformed back into the frequency domain. The limiting factor for the use of this method is an insufficient time separation between mechanisms. Method one is called eigenfunction minus creeping wave (EMC) and method two is called time domain extraction (TDE).

The notation used to identify the reflected field prediction techniques is LK for Luneburg-Kline expansion, PRC for Pathak reflection coefficient, PARC for Pathak asymptotic reflection coefficient and TRC for the parabolic based reflection coefficient. Table V.1 lists the notation for the techniques used to generate the curves in this chapter.

All calculated field values in the frequency domain are echo widths and areas in dB/m and dB/m², respectively. The phase centers for the reflected field phase plots have been adjusted to the surface reflection point.
### Table 5.1

<table>
<thead>
<tr>
<th>SOLUTIO TECHNIQUE NOTATION</th>
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<tbody>
<tr>
<td>EGF</td>
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<td>TDE</td>
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<tr>
<td>LK</td>
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<tr>
<td>PRC</td>
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<td>PARC</td>
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<td>TRC</td>
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#### A. Calculated Aspect Angle Comparisons

Aspect angle calculations have been performed for circular and elliptic cylinders. The bistatic angle dependence of the normalized reflected field for a circular cylinder is shown in Figures 5.1(a,b) and 5.2(a,b) for soft and hard incident polarizations, respectively. The sets (a) and (b) correspond to cylinder radii of .2 and 1 λ, respectively. The normalization is in regard to the GO multiplicative factor (Er=(a/2)cosθi for the cylinder), therefore it is the correction.
Figure 5.1. Bistatic, soft, reflected field for a circular cylinder normalized with respect to the G.O. field. Cylinder radius (a) 0.2\(\lambda\) and (b) 1.\(\lambda\). The normalization yields the correction factor.
term that is plotted. For each plot, the correction term for EMC, LK, PRC, PARC and TRC techniques are plotted. Two major reflection characteristics illustrated by these curves are that (1) the magnitude correction for the soft polarization is larger than unity and monotonically increasing for increasing bistatic angles while for the hard polarization it is smaller than unity and monotonically decreasing for increasing bistatic angles and (2) the net correction is smaller for the soft case than the hard. Other characteristics that are evident in these plots are that the LK and EMC solutions agree for larger cylinders and smaller bistatic angles. Whereas, the PRC and EMC results agree well for large bistatic angles. Finally, the TRC and EMC solutions agree for smaller bistatic angles. The TRC result would generally follow the PRC one if the argument of the transition function didn't have the endpoint correction due to the incident shadow boundaries. This is also seen in the swept frequency comparisons.

The monostatic, total scattered field, as a function of aspect, for an elliptic cylinder \((a=1m, b/a=.2)\) is shown in Figures 5.3(a) and (b) for soft and hard incident polarizations, respectively. Overlaid on these curves are curves for the reflected field predicted by GO, PRC and TRC. There is excellent agreement between the total scattered field and the TRC based reflected field for the soft case since the creeping wave field is very small. Similar agreement exists for the hard case when the effect of the creeping wave is visually averaged out. As with the
Figure 5.2. Bistatic, hard, reflected field for a circular cylinder normalized with respect to the G.O. field. Cylinder radius (a) 0.2 λ and (b) 1. λ. The normalization yields the correction factor.
Figure 5.3. Monostatic reflected field aspect angle scan for an elliptic cylinder at 6 GHz with $a = 1\,\text{m}$ and $b/a = 0.2$. Incident plane wave polarization: (a) soft and (b) hard.
circular cylinder case, there is a polarization dependence for the magnitude of the reflected field. The PRC based field attempts to properly predict the reflected field but does not quite make it. The TRC based field fails slightly when the source/receiver is broadside to the flatter portions of the cylinder. This is reasonable since that portion of the surface is not well modeled by a parabolic surface since there are incident shadow boundaries close to the specular point. As mentioned earlier, such a condition does not form a good stationary point for that portion of the surface, thus, the reflected field is not just determined by the local character of the surface.

B. CALCULATED SWEPTE FREQUENCY COMPARISONS

The frequency dependence of the reflected field calculated by the various techniques is examined for several surfaces. The studied surfaces are the circular cylinder, the elliptic cylinder, the sphere and the prolate spheroid. The techniques used to calculate the "exact" frequency dependence of the reflected field are EMC and TDE. The techniques used to predict the reflected field for comparison with the "exact" field are LK and TRC.

The monostatic reflected field frequency spectra for a .0762 m radius circular cylinder are shown in Figures 5.4(a) and (b) (soft and hard polarization, respectively) based upon the EMC, TDE, LK, PRC and TRC techniques. The TRC result has excellent agreement with the EMC and TDE results considering the TRC result is based upon the reflection from
Figure 5.4. Monostatic reflected far field spectra for a 0.0762 m radius circular cylinder. Incident plane wave polarization: (a) soft and (b) hard.
Figure 5.4. (continued)
different from a circular cylinder. The TRC curves are comparable to the PRC curves if its radius of curvature dependency is not modified to account for the incident shadow boundaries. The LK closely agrees with the exact result only for larger $ka$ values. As shown, all solutions approach the high frequency, GO limit.

As commented in Chapter III, TDE curves in Figure 5.4, exhibit a roll off at the upper end of the spectrum due to the finite bandwidth of the original spectrum and the time gating window smoothing. Another limitation of the technique is the reproduction of a true, monotonically rapid, field variation such as in the Rayleigh region of a scatterer. This is also limited by the smoothing effects of the effective convolution process. The first perturbation can be minimized by extending the spectrum as earlier mentioned but the recovering rapid variations is limited to the width and side lobe levels of the time gate window and how the window artificially separates the energy associated with the different mechanisms in time.

The monostatic reflected field frequency spectra for an elliptic cylinder are shown in Figures 5.5, 5.6, and 5.7, for aspect angles of 0, 45 and 90 degrees, respectively. Each plot has curves for the TDE, PRC and TRC techniques and for both polarizations. The ripple in the TDE results are due to residual creeping wave fields.
Figure 5.5. Monostatic reflected far field spectra for an elliptic cylinder with \( a = 0.0762 \) m, \( b/a = 0.5 \) and \( \phi = 0^\circ \) degrees. Incident plane wave polarization: (a) soft and (b) hard.
Figure 5.5. (continued)
As commented in Chapter III, the first three terms in the asymptotic representation of the diffraction and attenuation coefficients are required for a good prediction of these fields. A reasonable estimate of the reflected field from the TDE results can be obtained by visually averaging the ripple out. When done, the EMC and TDE are in excellent agreement. The TRC result matches well with the EMC and TDE results for 0 degrees; however as the aspect angle approaches 90 degrees, both the TRC and PRC fails to capture the dominant character of the reflected field. This is because the character of the reflected field changes between aspect angles of 0 and 90 degrees. At the lower frequencies, the amplitudes tends to increase for the soft and decrease for hard. The monostatic reflection at 90 degrees from an elliptic cylinder has just the opposite trend. This discrepancy is not unlikely since the reflected field is not well defined when the distance from the specular point to the incident shadow boundaries is small. The minimum distance before stationary phase is valid to interpret the scattered field as a reflected field is approximately \( \lambda/4 \). Without a sufficient stationary phase point, the scattering can't to be modeled ray optically.

A family of curves are presented in Figures 5.8 and 5.9 for the monostatic reflection from an elliptic cylinder with plane wave incident at 0 and 30 degrees off the semi-major axis, respectively. Plotted are the eigenfunction, TRC and PRC solutions for five b/a ratios ranging from 1 to .1 as a function of ka. The sets a,b are for the soft and
Figure 5.6. Monostatic reflected far field spectra for an elliptic cylinder with \( a = 0.0762 \text{ m} \), \( b/a = 0.5 \) and \( \phi = 45^\circ \) degrees. Incident plane wave polarization: (a) soft and (b) hard.
Figure 5.6. (continued)
Figure 5.7. Monostatic reflected far field spectra for an elliptic cylinder with \(a=0.0762\) m, \(b/a=0.5\) and \(\phi=90\) degrees. Incident polarization: (a) soft and (b) hard.
Figure 5.7. (continued)
Figure 5.8. Monostatic reflected far field spectra for elliptic cylinders as a function of ka for five different b/a ratios when \( \gamma = 0 \) degrees. Incident plane wave polarization: (a) soft and (b) hard.
hard cases, respectively. Though the eigenfunction solutions still have
the creeping wave component included, the ripple can be visually
averaged out to obtain the general level of the reflected field. There
is good agreement between the TRC and eigenfunction curves for the 0°
incidence case, which is especially noticeable for the soft case. The
agreement is not as good for the 30° incidence case. The disagreement
might be due to the electrically close distances between the specular
point and the incident shadow boundaries for the smaller b/a ratio
cases, but that may not be totally correct. The 0° incidence case
showed good agreement even for the .1 b/a ratio case and small ka
values. It could be concluded that information pertaining to the
symmetry of the scatterer with respect to the incident field is missing.
Such information can be represented in the form of the slope for the
surface at the specular point with respect to the phase front of the
incident field [38]. The PRC result fails for smaller radii of
curvature, as shown, since the smallness of the function parameter
assumes that the observation point is near a shadow boundary.

The monostatic, copolarized reflected field frequency spectrum for
a .0762 m radius sphere is shown in Figure 5.10 based upon the EMC, TDE,
LK and TRC techniques. The agreement between the EMC, TDE and LK
approaches are excellent. The Luneberg-Kline expansion of the
monostatic reflected field is an extremely good representation for only
two terms. The third and fourth order terms are identically equal to
Figure 5.9. Monostatic reflected far field spectra for elliptic cylinders as a function of $k\alpha$ for five different $b/a$ ratios when $\phi=30$ degrees. Incident plane wave polarization: (a) soft and (b) hard.
Figure 5.10. Monostatic, copolarized reflected far field spectra for a 0.0762 radius sphere.
zero for the monostatic case. As commented in Chapter IV, the TRC result fails due to the heuristic formulation when both the principle radii of curvature are equal.

The next set of plots are for the monostatic, copolarized reflected field spectra for two prolate spheroids. For this geometry, there are two principle incident polarizations, i.e., \( \hat{\theta} \) and \( \hat{\phi} \) directed with respect to the surface coordinates as shown in Figure 2.10(c). The spectra for a prolate spheroid with a semi-major axis of .0762 m and a b/a ratio of .5 are shown in Figures 5.11, 5.12 and 5.13 for incident \( \theta \) angles of 0, 45 and 90 degrees, respectively. Figures 5.14 and 5.15 are spectra for a prolate spheroid with a semi-major axis of .1524 m and a b/a ratio of .25 at incident \( \theta \) of 0 and 90 degrees, respectively. Note that the TDE results are predicted by the TRC solution though the agreement could be better.

Figure 5.16 illustrates a trend for the reflected field in the resonance region of a scatterer when the principle radii of curvature are not equal at the specular point. In this case the TDE reflected fields are, normalized with respect to the GO return to illustrate the effective correction factor, from Figures 5.13 and 5.15. Here, the incident \( \theta \) angle is 90 degrees and the principle surface radii of curvature are .1524 m and .0381 m for the smaller spheroid and .6096 m and .0381 m for the larger spheroid. Observation of the correction
Figure 5.11. Monostatic, copolarized reflected far field spectra for a prolate spheroid with $a=0.0762$ m, $b/a=0.5$ and $\theta=0$ degrees.
Figure 5.12. Monostatic, copolarized reflected far field spectra for a prolate spheroid with $a=0.0762$ m, $b/a=0.5$ and $\theta=45^\circ$ degrees.
Figure 5.13. Monostatic, copolarized reflected far field spectra for a prolate spheroid with $a=0.0762$ m, $b/a=0.5$ and $\theta=90$ degrees. Incident plane wave illumination.
Figure 5.14. Monostatic, copolarized reflected far field spectra for a prolate spheroid with \( a = 0.1524 \) m, \( b/a = 0.25 \) and \( \theta = 0^\circ \) degrees. Incident plane wave illumination.
Figure 5.15. Monostatic, copolarized reflected far field spectra for a prolate spheroid with $a=0.1524$ m, $b/a=0.25$ and $\theta=90$ degrees. Incident plane wave illumination.
Figure 5.16. Comparison of the correction factors for the reflected fields (both principle polarizations) obtained from TDE for two prolate spheroids in Figures 5.13 and 5.15.
factors in Figure 5.16 indicates that the reflected field has a soft character when the incident electric field is in the plane containing the larger principle radii of curvature and a hard character when the incident electric field is in the plane containing the smaller principle radii of curvature. The effect, if any, of the principle radii of curvature in the plane transverse to the incident electric field can not be determined from curves in Figure 5.16.

The final swept frequency comparison involves the scattering from the leading edge of a perfectly conducting disk with an edge-on incident plane wave, polarized in the plane of the disk. Although the specular point isn't located on a smooth, convex, three dimensional surface, the scattering can be modeled by letting one of the principle radii of curvature be very small Figure 5.17 compares the TDE result with the TRC result for a .0762 m radius disk. The magnitude agreement is excellent; whereas, the phase agreement is reasonable. The radius of curvature transverse to the plane of the disk was assumed to be .00001 m for the TRC calculation. The TDE result was generated from an eigenfunction solution [39].

C. MEASURED ASPECT ANGLE COMPARISONS

Aspect angle measurements were taken on three elliptic-like, finite length cylinders. The cross sections were not true ellipses due to fabrication inaccuracies. The cylinders were .305 m long and their semi-major axis lengths and b/a ratios are given in Table V.2.
Figure 5.17. Monostatic, copolarized scattered far field spectrum for the leading edge return for a 0.0762 m radius disk. Incident plane wave illumination, polarized in the plane of the disk.
TABLE 5.2

ELLIPSTIC-LIKE CYLINDER DIMENSIONS USED IN MEASUREMENTS

<table>
<thead>
<tr>
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<th>Semi-major axis (m)</th>
<th>b/a</th>
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<tbody>
<tr>
<td>CYL 1</td>
<td>.1</td>
<td>.2</td>
</tr>
<tr>
<td>CYL 2</td>
<td>.4</td>
<td>.05</td>
</tr>
</tbody>
</table>

The scattered field measurements were compared to the predicted GO, PRC and TRC solutions which were all scaled to account for the finite cylinder length. The cylinders were rotated about the axis that is parallel to the cylinder's generator. The incident field was polarized parallel and perpendicular to this axis for the soft and hard incident polarizations, respectively. Figure 5.18 is for the soft polarization case and Figure 5.19 is the the hard polarization case. The sets a,b corresponds to CYL1 and CYL2, respectively. The disagreement between theory and measurement is readily accounted by the non-elliptic cross section of the cylinders. The scaling of the two dimensional fields to account for the finite cylinder length, L, is shown below:

\[
\begin{bmatrix}
E_z^r \\
H_z^r
\end{bmatrix} = \begin{bmatrix}
E_z^i \\
H_z^i
\end{bmatrix} \frac{L}{\lambda} e^{in/4} \frac{e^{-jks}}{s} \text{ and (5.1)}
\]
Figure 5.18. Measurement comparison for the monostatic aspect angle scan of the soft reflected field for .305 m long elliptic cylinders. Cylinder cross section dimensions: (a) a=.1 m, b/a=.2 and (b) a=.4 m, b/a=.05.
Figure 5.19. Measurement comparison for the monostatic aspect angle scan of the hard reflected field for .305 m long elliptic cylinders. Cylinder cross section dimensions: (a) a=.1 m, b/a=.2 and (b) a=.4 m, b/a=.05.
\[ \text{Echo Area} = \frac{\text{Echo Width}}{2L^2} \quad (5.2) \]

Another measurement was performed to validate the smooth edge approximation for the UTD diffraction coefficient in Equations (4.9) and (4.10). The monostatic scattering from a 1/16 inch plate with dimensions .202 by .305 m was measured for the incident field parallel to an edge whose length was .305 m. This leading edge was rounded to have a radius of curvature of roughly .005 inch. Figure 5.20 shows the measurement with the effect of the rear edge processed out through time gating. The calculated return based upon a smooth edge and an infinitely thin edge are also shown. The TRC smooth edge return is in better agreement with the measured TDE return than the exact knife edge case even though the radius of curvature varied from .00254\(\lambda\) to .00765\(\lambda\). The ripple in the processed measurements is due to noise.
Figure 5.20. Measurement comparison for edge on, parallel electric field, scattered field from a smooth edge of a 1/16" plate. The rear edge was time gated out. Incident edge length of .305 m. Effective edge curvature of .005 inches.
A more complete, physical understanding of the scattering from curved, edge-like surfaces has been accomplished. There has been an uncertainty of how "sharp" a surface has to be before the scattered field changes its physical interpretation from a reflected to a diffracted field. The physical insight into this transition zone of field classification has come from examining the scattering from a two dimensional parabolic surface.

The parabolic surface yielded an exact, closed form expression for the axial scattering due to plane wave incidence for a surface whose radius of curvature can be smoothly varied from large values to zero, i.e., a half plane. From this exact expression, its parameters were physically interpreted to form a reflection (scattering) coefficient. The coefficient's dominant parameter was the caustic distance of the reflected ray system which is extremely dependent upon the surface radius of curvature at the specular point as well as the curvature of the incident wavefront.
The information in determining the reflected field was augmented by understanding the physical significance of higher order terms in the Luneburg-Kline expansion of the reflected field. There were terms in the reflected field expansions for circular and elliptic cylinders that were not present for a parabolic cylinder. This was geometrically attributed to phase difference distance between the specular and incident shadow boundary locations. Only the incident shadow boundaries that are visible from the observation point were used when the source and observation points are interchanged to satisfy reciprocity. This information was incorporated into a factor that multiplicatively modified the reflected ray caustic distance.

The heuristic reflection coefficient was further generalized for bistatic, reflected field predictions based upon the Luneburg-Kline expansion for the bistatic reflected field of a circular cylinder. Due to the optical bases of the angle range dependence, the observation point must be in the deep lit region of the scatterer. It was shown that as the observation point approached the incident shadow boundary, the heuristic reflection coefficient gracefully failed. This is understandable based upon studying the bandlimited, transient waveforms for the bistatic scattering from circular and elliptic cylinders shown in Chapter II. The transient waveforms for the reflected field showed a characteristic transition as the observation point went from the deep lit region to the incident shadow boundary regions. The prediction of reflected fields in the vicinity of the incident shadow boundary has to be done with an uniform field expansion such as Pathak’s uniform reflection coefficient [5].
All these heuristic extensions of the exact scattering from a parabolic cylinder, for both principle polarizations, has conveniently adopted the form of a $K\lambda a$ parameter which is a natural form for UTD edge scattering. In both cases, the $K\lambda a$ parameters are arguments of transition functions that uniformly blend between limiting cases. With respect to the reflection concept, the blending occurs between surfaces whose specular point has very large to very small radii of curvature. A modification of the UTD diffraction coefficient was proposed to account for non-ideal, perfectly sharp edges. Measurements of such a modification proved very promising.

The agreement between the reflected fields from this heuristic, two dimensional reflection coefficient and "exact" calculation for canonical surfaces was excellent. The regions where the predictions failed can be attributed to the scattering not being modeled very well from an optical viewpoint. In such cases, the incident shadow boundary is close to the specular point creating a poor stationary phase point. Another possible improvement maybe from a slope correction term. This term would take into account a nonsymmetric surface at the specular point with respect to the incident field phase front.

The concepts that were developed for two dimensional reflected field calculation were extended to three dimensional cases. A heuristic reflection coefficient was conjectured that uniformly went to the two dimensional reflection coefficient and correctly predicted the form for the first two terms in the asymptotic
expansion for the monostatic reflection at a specular point where both principle radii of curvature are equal. However, because of the higher order terms involved in the heuristic, three dimensional reflection coefficient, the proper trend did not prevail for that limiting case. This three dimensional reflection coefficient does result in reasonable monostatic reflected field predictions when the principle radii of curvature are unequal.

Improvements of this three dimension reflection coefficient may be made in obtaining a more desirable transition function for the equal radii of curvature case and extending the solution to include bistatic angles predictions. As with the two dimensional solution, the three dimensional solution will fail when a good stationary phase scattering point can not be established, i.e, when the incident shadow boundaries are too close to the specular point.
APPENDIX A

LUNEBURG-KLINE EXPANSION FOR AN ELLIPTIC CYLINDER

The derivation of the Luneburg-Kline expansions for an elliptic cylinder that were presented in Table 1 is performed here. The first three terms are generated for an axial plane wave illuminating the cylinder as shown in Figure A.1 with the soft boundary condition enforced on the cylinder surface. The general, bistatic expansion for the reflected field is reduced for far field, monostatic reflection. From this expression and the soft and hard reflected field expansions for a circular cylinder, the first two terms for the far field, monostatic reflected field from an elliptic cylinder with axial plane wave illumination is conjectured for the hard boundary condition.

The first three terms for the reflected field, represented as

\[ u^r \sim e^{-jk\psi} \sum_{n=0}^{2} \frac{u_n^r}{n} \quad \text{(A.1)} \]

will be found when the incident field is

\[ u^i = e^{-jkx} \quad \text{(A.2)} \]
Figure A.1. Geometry and parameter illustration for the Luneburg-Kline reflected expansion for an elliptic cylinder.
by matching the amplitude and phase constants on the elliptic surface for the boundary condition of \( U^t + U_s^r - U_0^t = 0 \) to obtain \( u^r_0 \) and \( \psi \). The higher order coefficients \( u^n_r \), are obtained by the transport equation

\[
2\eta u^n_r - \eta \psi + u^n_r \eta^2 \psi = -\eta u^{n-1}_r
\]

(A.3)

which has a solution of

\[
u^n_r = u^n_r(s_0) \left[ -\frac{G(s)}{G(s_0)} \right]^{-1/2} - \frac{1}{2} \left[ G(s) \right]^{-1/2} \int_{s_0}^{s} \frac{G(t)}{G(s)} \eta^2 u^{n-1}_r \, dt
\]

(A.4)

where

\[ G(\tau), s_0 \text{ and } s \text{ are defined in Equation (3.7).} \]

It is advantageous to express \( \eta^2 \) in ray coordinates of \( s \) and \( \beta \) (see Figure A.1) to simplify the integration in Equation (A.4). The ray coordinates have a physical interpretation of expressing the reflected field as a system of rays with an apparent source at \( s=0 \) which, in general, moves as a function of source-observation position for a given surface. Thus the Laplacian is given by

\[
\eta^2 = \left[ 1 + \frac{a}{s^2} \right] \frac{\partial^2}{\partial s^2} + \left[ \frac{1}{s} - \frac{c}{s^2} - \frac{a}{s^3} \right] \frac{\partial}{\partial s} - \frac{2b}{s^2} + \frac{a}{s^2 \beta} + \frac{a^2}{s^2 \beta^2} + \frac{b}{s^3} \frac{a}{\beta}
\]

(A.5)
where
\[ a = \xi + \eta = b^2 \]
\[ b = \xi \cos \beta + \eta \sin \beta = \frac{s}{s} = \psi (s_0) - s_0 \]
\[ c = \xi \cos \beta + \eta \sin \beta = b \]
\[ \xi \text{ and } \eta \text{ are the x and y coordinates of the caustic} \]
\[ \overline{s} \text{ in the distance along the ray } \beta \text{ from the caustic to the wavefront} \]

The dots refer to diffraction with respect to \( \beta \).

The Laplacian can be derived from tensor algebra.

From the surface boundary condition
\[ U_1 + U_s \sim 0 \quad (A.6) \]
and Equation (A.1) and A.2), yields
\[ n=0 \]
\[ e^{-jkx} + e^{-jk \psi(s_0, \beta)} \frac{u_n(s_0, \beta)}{(-jk)^n} \sim 0 \quad (A.7) \]

The magnitude and phase terms at the surface are
\[ u_0^r(s_0, \beta) = -1 \quad (A.8) \]
\[ u_0^r(s_0, \beta) = 0 \text{ for } n > 0 \quad (A.9) \]
\[ \psi(s_0, \beta) = -\frac{a_1^2 \sin \beta/2}{(a_1^2 \sin^2 \beta/2 + b_1^2 \cos^2 \beta/2)^{1/2}} \quad (A.10) \]
Extending $u^r_1$ and $\psi$ along the reflected ray path yields

$$u^r_0(s, \beta) = -\left[ \frac{so}{s} \right]^{1/2} \tag{A.11}$$

$$\psi(s, \beta) = \psi(s_0, \beta) + s = s_0 \tag{A.12}$$

where

$$s_0 = \frac{R}{2} \cos \theta \quad \text{and} \quad s_0 = \frac{R}{2} \sin \beta/2$$

$$R = a_1 \left[ \frac{\cot^2 \beta/2 + 1}{b_1 + a_1 \cot \beta/2 + 1} \right]^{3/2}$$

The distance, $s_0$, represents the reflected ray caustic distance which is the distance to the surface from the apparent source location the reflected field system of rays. $R$ is the radius of curvature at the specular point.

The higher order coefficients $u^r_1$ and $u^r_2$ can from Equation (A.4) which has the form for this application of

$$u^r_n = -\frac{1}{2\sqrt{s}} \int_{s_0}^{s} \sqrt{\frac{1}{2} \gamma^2 u^r_{n-1}(\tau, \beta)} d\tau \tag{A.13}$$
To obtain $u_1^r$, the Laplacian of $u_0^r$ is

$$\nabla^2 u_0^r(s, \beta) = -\frac{1}{2} \sqrt{\frac{s_0}{s}} \left[ (\frac{1}{2} + A_2) \frac{1}{s^2} + (c + 2bA_1) \frac{1}{s^3} + \frac{5}{2} a \frac{1}{s^4} \right]$$

$$A_1 = \frac{\dot{R}}{R} + \frac{\cot \beta/2}{2}$$

$$A_2 = -\frac{\dot{R}^2}{2R^2} + \frac{\dot{R}}{2R} \cos \beta/2 + \frac{\dot{R}}{R} + \frac{\cos^2 \beta/2 - 2}{8 \sin^2 \beta/2}$$

$$a = \frac{3}{16} b_1 \cos^2 \beta/2$$

$$b = -\frac{3}{4} b_1 \cos^2 \beta/2$$

$$c = \frac{3}{8} b_1 \sin \beta/2$$

The single and double dots over the symbols indicate first and second partial derivatives with respect to $\beta$.

Performing the integration as indicated in Equation (A.13) yields

$$u_1^r(s, \beta) = \tilde{u}_1^r(s, \beta) - \tilde{u}_1^r(s_0, \beta) \quad \text{(A.15)}$$

where

$$\tilde{u}_1^r(t, \beta) = -\frac{1}{4} \sqrt{\frac{s_0}{s}} \left[ (\frac{1}{2} + A_2) \frac{1}{t} + \frac{1}{2} (c + 2bA_1) \frac{1}{t^2} + \frac{5}{6} a \frac{1}{t^3} \right]$$
the third coefficient is given by

\[ u_2^r(s, \beta) = \frac{1}{2 \sqrt{s}} \int_{s_0}^S \tau^2 \left[ \left( \frac{\nabla^2 u_1^r(s, \beta) - \nabla^2 u_1^r(s_0, \beta)}{s} \right) \right] d\tau \]  

(A.16)

\[ u_2^r(s, \beta) = -\frac{1}{32} \sqrt{\frac{s_0}{s}} \]

\[ \left[ H_1 \frac{1}{\tau} + \frac{1}{2} H_2 \frac{1}{\tau^2} + \frac{1}{3} H_3 \frac{1}{\tau^3} + \frac{1}{4} H_4 \frac{1}{\tau^4} + \frac{1}{5} H_5 \frac{1}{\tau^5} + \frac{1}{6} H_6 \frac{1}{\tau^6} \right]^{s_0}_{s} \]

(A.17)

where

\[ H_1 = -\left[ F_1(1+A_1^2 + 2A_1) + 4A_1 \frac{d}{ds} + 4F_1 \right] \frac{1}{s_0} - \left[ F_1 4(A_1^2 + s_0) - \frac{\dot{f}_1}{\dot{s}_0} - F_2(1+A_1^2 + 2A_1) + 4A_1 \frac{d}{ds} + 4F_2 \right] \frac{1}{s_0^2} \]

\[ - \left[ F_1 R(A_1 \dot{s}_0 + s_0) - \frac{\dot{f}_1}{\dot{s}_0} - F_2 16s_0 + F_3(1+A_1^2 + A_1) + 4A_1 \frac{d}{ds} + 4F_3 \right] \frac{1}{s_0^3} \]

\[ - \left[ F_2 24(A_1 \dot{s}_0 + s_0) - \frac{\dot{f}_1}{\dot{s}_0} - F_3 24s_0 \right] \frac{1}{s_0^4} - F_3 48(s_0)^2 \frac{1}{s_0^5} \]
\[ H_2 = F_1[9+A_1^2+2A_1] + F_1^4A_1 + F_1^4 + \left[ -F_1 (c+2bA_1) - F_1^8b \right] \frac{1}{s_0} \]

\[ + [F_1^8bs_0 - F_2^2(c+2bA_1) - F_2^8b] \frac{1}{s_0^2} \]

\[ + [F_2^16b^4s_0 - F_3^8b^4] \frac{1}{s_0^3} + F_3^24bs_0 \frac{1}{s_0^4} \]

\[ H_3 = F_1^2[3c+4bA_1] + F_1^16b + F_2[25+A_1^2+A_1] \]

\[ + F_2^4A_1 + F_2^4-5a \left[ F_1 \frac{1}{s_0} + F_2 \frac{1}{s_0^2} + F_2 \frac{1}{s_0^3} \right] \]

\[ H_4 = F_1^21a + F_2^2 \left[ 5c+6bA_1 \right] + F_2^24b + F_3 \left[ 49+A_1^2+A_1^2 \right] + F_3^4A_1 + F_3^4 \]

\[ H_5 = F_2^45a + F_3^2[7c+8bA_1] + F_3^32b \]

\[ H_6 = F_3^77a \]

\[ F_1 = \frac{1}{2} + A_2 \quad F_2 = \frac{1}{2} (c+2bA_1) \quad F_3 = \frac{5}{6} a \]

The single and double dots over the symbols indicate first and second partial derivatives with respect to \( \theta_0 \).
Thus, the first three terms in the expansion for the reflected field is

\[
U^r_s \sim -\sqrt{\frac{s_0}{s}} e^{-jk\psi(s,\beta)} \left[ 1 + j \frac{u^r_1(s,\beta)}{k u^r_0(s,\beta)} - \frac{u^r_2(s,\beta)}{k^2 u^r_0(s,\beta)} \right] \tag{A.18}
\]

where

\[
u^r_0 = -\frac{s_0}{s}
\]

\[
s_0 = \frac{R}{2} \sin \beta/2
\]

\[
R = \frac{a_1^2}{b_1} \left[ \frac{\cot^2 \beta/2 + 1}{b_1^2 \cot \beta/2 + 1} \right]^{3/2}
\]

\[
\psi = \psi(s_0) + s - s_0
\]

\[
\psi(s_0) = \frac{-a_1^2 \sin \beta/2}{\left[ a_1^2 \sin^2 \beta/2 + b_1^2 \cos^2 \beta/2 \right]^{1/2}}
\]
\[ u_1^r = \frac{u_0^r}{4} \left[ \frac{1}{(2 + A_2)} \frac{1}{\tau} + \frac{1}{2} (c + 2b_1a_1) \frac{1}{\tau^2} + \frac{5}{6} a \frac{1}{\tau^3} \right]_{s_0}^s \]

\[ u_2^r = \frac{u_0^r}{32} \left[ H_1 \frac{1}{\tau} + \frac{1}{2} H_2 \frac{1}{\tau^2} + \frac{1}{3} H_3 \frac{1}{\tau^3} + \frac{1}{4} H_4 \frac{1}{\tau^4} + \frac{1}{5} H_5 \frac{1}{\tau^5} + \frac{1}{6} H_6 \frac{1}{\tau^6} \right]_{s_0}^s \]

For monostatic reflection \((\theta = \pi)\),

\[ u_1^r = \frac{u_0^r}{4b_1} \left[ \frac{a_1}{8} \frac{b_1}{b_1 - 3} \frac{1}{a_1} \right] \]  \(\text{(A.19)}\)

\[ u_2^r = \frac{u_0^r}{(16b_1)^2} \left[ 136 \left( \frac{a_1}{b_1} \right)^2 - 80 + 25 \left( \frac{b_1}{a_1} \right)^2 \right] \]  \(\text{(A.20)}\)

\[ u_s^r \sim -\sqrt{\frac{b_1^2}{2a_1}} \frac{b_1}{s-a_1 - \frac{2a_1}{2a_1}} \left[ \frac{a_1}{8b_1 - 3a_1} \frac{b_1}{16k_b} + \frac{136(b_1^2)}{(16kb_1)^2} - 80 + 7.5(b_1^2) \right] \]  \(\text{(A.21)}\)
Similar steps can be taken to obtain the reflected field for the
other principle polarization. The differences are that the boundary
condition is \( \frac{\partial u}{\partial n} = 0 \) and \( u_n(s, o, \beta) \), in general, is not equal to zero in
Equation (A.4). The second term presented in Table 1 was generated by
examining the differences in Equations (3.15), (3.16) and (A.21). The
conjectured, monostatic, reflected field for axial illumination in the
far field is

\[
U_s^r \sim \sqrt{\frac{b_1^2}{2a_1s}} e^{jk(s-a_1 - \frac{b_1^2}{2a_1})} \left[ 1 - \frac{\frac{a_1}{8b_1 + 3a_1}}{16kb_1} \right]
\]  

(A.22)
APPENDIX B

CREEPING WAVES ON THE CANONICAL SURFACES

Four sets of figures are presented to illustrate the far field, monostatic creeping wave field and the quality of the time domain extraction (TDE) technique to recover this mechanism from the total scattered field for four perfectly conducting surfaces. The surfaces are the circular cylinder, elliptic cylinder, sphere and prolate spheroid. All the plots also have the asymptotic expansions for the creeping waves plotted for comparison to the TDE results. The creeping waves are plotted as a function of frequency and normalized to echo widths/m for the two dimensional surfaces and echo areas/m$^2$ for the three dimensional surfaces. The phase center for all the phase plots have been adjusted to the half-way location that the creeping wave travels on the surface.

Figures B.1(a),(b) illustrates the monostatic creeping wave spectra for a circular cylinder of radius .0762 m due to a soft and hard incident plane wave incident. The error encountered in the TDE result at the high frequency end of the spectra is due to spectral leakage of the window side lobes. This leakage will be apparent in all the following spectra.
Figure B.1. Far field, monostatic creeping wave contribution from a circular cylinder of radius \(0.0762\) m for (a) soft and (b) hard plane wave illumination.
Figures B.2(a,b) illustrate the monostatic creeping wave spectra for an elliptic cylinder with a semi-major axis of 0.0762 m and a b/a ratio of 0.5 due to a soft and hard incident plane wave incident along the semi-axis.

Figure B.3 illustrates the monostatic creeping spectrum for a sphere of radius 0.0762. Two asymptotic approaches to calculate the monostatic creeping wave for this surface are possible. Voltmer [29] used the method of steepest descents on a Kirchhoff integral with Huygen sources based upon GTD. The creeping wave field is given by

\[
E^c = E^r a \sqrt{\frac{\pi k}{2}} \frac{e^{-jk(s+a)}}{s} e^{j3\pi/4} \sum_{n=0}^{\infty} \left[ D_n^{2s} e^{-a_n \pi a} - D_n^{2h} e^{-a_n h} \right] 
\]

where

\[
D_n^{2s} = D_{on}^{2s} \left[ 1 + m q_n \frac{1}{30} + \frac{1}{2} e^{-j\pi/3} + \ldots \right] 
\]

\[
D_n^{2h} = D_{on}^{2h} \left[ 1 + m -2 q_n \frac{1}{30} + \frac{1}{2} - q_n -2 \frac{1}{10 - 4} e^{-j\pi/3} + \ldots \right] 
\]
Figure B.2. Far field, monostatic creeping wave contribution from an elliptic cylinder with a semi-major axis of .0762 m and a b/a ratio of .5 (a) soft and (b) hard plane wave illumination along the semi-major axis.
Figure B.3. Far field, monostatic, copolarized creeping wave contribution from a sphere of radius .0762 m for plane wave illumination.
\[ \alpha_n^s = \alpha_{on} \left[ 1 + m^{-2} \frac{q_n}{60} e^{-j\pi/3} + \ldots \right] \]

\[ \alpha_n^h = \alpha_{on} \left[ 1 + m^{-2} \left( \frac{-q_n}{60} + \frac{-2}{30} \left( \frac{1}{10} - \frac{1}{4} \right) \right) e^{-j\pi/3} + \ldots \right] \]

Note that the coefficients for the positive terms in the soft and hard diffraction coefficients in [29] have been replaced by .5 instead of .25. Senior and Scott [17] independently generated the same asymptotic expansion by first setting the bistatic angle to zero before taking the Watson Transformation. A very accurate creeping wave representation can be obtained by using only one hard mode in Equation (B.1).

A quick comparison between the creeping wave return for a circular cylinder and a sphere reveals that only the phase is significantly different with the magnitude having the same trend. The reason for the phase difference is the point caustic that the creeping wave passes through and it is this phase difference that results in a symmetric creeping wave return about the optical time of arrival in the time domain as shown in Figure 2.14. The creeping wave return for the circular cylinder, as shown Figure 2.5(b), initiates at its optical time of arrival and hence, its return is unsymmetric.
Figure B.4 illustrates the axial, monostatic creeping wave spectra for a prolate spheroid. The spectra for four prolate spheroids are shown with three of them having the same semi-major axis length of .0762 m and b/a ratios of 1, .75 and .5. The other prolate spheroid has a semi-major axis length of .1524 m and a b/a ratio of .25. These curves are compared to calculations using Voltmer's approach. The agreement between the TDE and asymptotic results diminishes as the b/a ratio becomes smaller. Part of this occurrence is believed to be due to the paraxial effect of the surface. The asymptotic expressions were based upon the surface radius of curvature transverse to the creeping wave path to be much larger than the radius of curvature along the creeping wave path. This condition is violated for axial, monostatic scattering from a prolate spheroid with b/a ratios smaller than unity. Another reason for poor agreement arises from the small radii of curvature in the regions around the tips of the prolate spheroids. A fundamental criterion for good field representation with an asymptotic expansions requires that the radius of curvature along the creeping wave path be large. As the frequency is increased, the asymptotic results will converge to the TDE results.
Figure B.4. Far field, monostatic creeping wave contribution from a prolate spheroid for axial plane wave illumination.
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