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MODELING AND SIMULATION OF ZERO SEQUENCE CURRENT DISTRIBUTION ALONG UNDERGROUND CABLES

The Ohio State University

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MODELING AND SIMULATION OF ZERO SEQUENCE CURRENT DISTRIBUTION ALONG UNDERGROUND CABLES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

by

Ali Nezih Guven, B.S., M.Sc.E.

* * * * *

The Ohio State University
1984

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LIST OF PRINCIPAL SYMBOLS

\( f \) : frequency, Hz

\( \rho \) : soil resistivity, ohm-m

\( n \) : shielding factor

\( \text{GMR} \) : geometric mean radius, m

\( \text{GMD} \) : geometric mean distance, m

\( \text{GPR} \) : ground potential rise

\( R_C \) : conductor resistance

\( R_S \) : sheath resistance

\( R_{ti} \) : \( i \)th tower footing resistance, ohm

\( Z_{pi} \) : transmission line phase conductor impedance in span \( i \)

\( Z_{wi} \) : transmission line ground wire impedance in span \( i \)

\( Z_{gi} \) : \( i \)th span ground return path impedance

\( Z_S, Z_S' \) : equivalent impedances of external system, ohm

\( Z_{st}, Z_{st}' \) : terminal station grounding grid impedance, ohm

\( I_f, I_f' \) : fault current

\( I_{wi} \) : ground wire current in \( i \)th span

\( R_{gi} \) : resistance of the \( i \)th grounding rod, ohm

\( Z_{ci}, Z_{si} \) : equivalent phase conductor and sheath self impedance with ground return in segment \( i \), ohm/m

\( Z_{mi} \) : mutual impedance between phase conductor and sheath with common ground return in segment \( i \), ohm/m
Y_{Si} \quad \text{: shunt admittance of the sheath to the ground in segment } i, \text{ ohm/m}

Z_{Cap i} \quad \text{: shunt capacitive reactance between the phase conductor and the sheath in segment } i, \text{ ohm-m}

Z_{C Si} \quad \text{: equivalent shunt capacitive reactance at the } i \text{th node connected between the phase conductor and the sheath, ohm}

Z_{i} \quad \text{: driving point impedance matrix at the } i \text{th node}

Z_{11 i}, Z_{12 i}, Z_{21 i}, Z_{22 i} \quad \text{: entries of } Z_{i}, \text{ ohm}

V_{Si}(x), I_{Si}(x) \quad \text{: sheath voltage and current in segment } i \text{ at a distance } x \text{ from the grounding rod}

I_{Ci} \quad \text{: conductor current in segment } i

d_{i} \quad \text{: length of the } i \text{th segment, m}

I_{Si 0} \quad \text{: sheath current in segment } i \text{ at } x = 0^+

I_{Si} \quad \text{: sheath current in segment } i \text{ at } x - d_i-

V_{Si} \quad \text{: sheath voltage at the } i \text{th node}

V_{Ci} \quad \text{: conductor voltage at the } i \text{th node}

\beta_{i} \quad \text{: propagation constant in segment } i

M_{i}, N_{i}, D_{i}, F_{i} \quad \text{: current coefficients defined by } i \text{th segment cable parameters and } Z_{i-1}
LIST OF PRINCIPAL SYMBOLS CONTINUED

\( Q_{1i}, Q_{2i}, Q_{3i}, Q_{4i} \)
\( Q_{5i}, Q_{6i}, T_i, P_i, H_i \) : complex coefficients at segment \( i \)
\( \omega = 2\pi f \) : radian frequency
\( \epsilon_0 = 8.842 \cdot 10^{-12} \text{ F/M} \) : the permittivity of the free space
\( \eta_0 = 4\pi 10^{-7} \text{ H/M} \) : the permeability of the free space
CHAPTER 1

INTRODUCTION

A consequence of the continuous growth of power systems is the increase of fault currents in overhead and underground transmission systems. The type of fault that most likely occurs in a power transmission network is the ground (line-to-ground or double-line-to-ground) fault. In the case of a ground fault, the fault current (zero sequence current) will return to the source through the neutral conductors (overhead ground wires, cable sheaths, etc.) and through the ground path which is connected to the system neutral by grounding electrodes.

An accurate and convenient method is needed to determine fault currents that flow in neutral conductors and grounding electrodes. Determining the magnitude of the fault current and its distribution in the ground and neutral conductors is of prime importance for the following reasons:

1) To accurately calculate the ground potential rise and the electromagnetic induction on telecommunication lines, pipelines and railways in the vicinity of power lines.

2) To design safe and reliable grounding installations and to calculate step and touch voltages along the power line.
3) To calculate the optimum settings of the protective relays, specifically ground relays whose operation depend upon the zero sequence current flowing through the substation neutral.

4) To select the proper size of neutral conductors to withstand the thermal stresses imposed by large fault currents.

In problems where a proper knowledge of the fault current distribution among various return paths is necessary, the conventional short circuit computations are often inadequate. Conventional methods have tended to make assumptions that oversimplify the problem. As a result, their solutions are not accurate and alternative methods are needed.

An accurate analysis of the zero sequence current distribution between ground and neutral conductors is difficult due to the complexities of power systems. Modern power systems are solidly grounded through hundreds of grounding structures (towers and grounding rods). The metallic return paths which are electromagnetically coupled to the phase conductors are bonded to these structures. Moreover, the ground resistance of the structures and the soil resistivity are not constant and generally vary between wide limits along the route of the transmission line or cable. Span-by-span distribution of the fault current is influenced by a variety of factors. These factors can be summarized as:

1) Soil resistivity,
2) Resistance, configuration and number of grounding structures,
3) Transmission system (overhead lines or underground cables) geometry and electrical parameters,
4) Distance between source station and ground fault,
5) Number of source stations and source impedance,
vi) Substation grounding grid impedance.

The main objective of this study is to develop a systematic calculation method for zero sequence current distribution and neutral conductor voltages along underground cables. The proposed method will consider all of the principal factors listed above and it will not be restrictive in application. In addition, the methodology will be efficient and computer-oriented.

Before introducing the proposed method, background information on the structure and the electrical characteristics of underground cables will be presented in Chapter 2. Chapter 3 will review the basic components of interference on the telecommunication circuits due to power cables. Chapter 4 will describe pertinent studies published previously. Due to the similarities between overhead lines and cables, some methods in the literature related to transmission lines will also be discussed. The assumptions and the features of the proposed method will be presented in that chapter.

The new calculation method proposed to examine the fault current distribution along underground cables will be explained in Chapter 5. The method will use both lumped and distributed parameters. First an equivalent circuit representation of the cable will be developed. The model will consist of an equivalent phase conductor, the cable sheath and grounding structures (rods) if they are applied. Then the zero sequence current distribution between the sheath and the ground will be solved through the computation of driving point impedances. Sheath voltages will also be computed so that a qualitative analysis of touch voltages and the ground potential rise at the grounding structures can be made.
In Chapter 6, the results of calculations related to the model, and the various case and parametric studies conducted will be presented. Finally, Chapter 7 will discuss the potential applications of the proposed method and possibilities for future work.
CHAPTER II

ELECTRICAL CHARACTERISTICS OF UNDERGROUND AC CABLES

2.1 INTRODUCTION

Underground cables are used mainly in urban and suburban areas, and for special requirements such as water crossings, airports, and under highways. The reason is economics and there are two factors which bring this situation about. The first is the cost of installation which for cables is typically 6 to 20 times that of an equivalent overhead transmission line [1]. The second relates to the thermal and insulation requirements involved in underground transmission. Unlike overhead lines which are cooled by convection in the surrounding air, the ground acts as a thermal blanket around underground cables and limits the transmission capability.

Due to growing environmental and esthetic concerns, and increasing population densities, there has been a tendency in recent years to place more transmission underground. Therefore, we are seeing a greater effort to overcome the major drawbacks of underground cable systems. Development projects focus on reducing the manufacturing and installation costs, and improving the reliability and efficiency of underground cables.

Cables consist of three basic components, the conductor, the sheath and the insulation. Figure 1 illustrates the cross-sectional
structure of a single phase power cable. Cables are classified accord­
ing to their type of insulation as paper, rubber, plastic, or gas.

The first cables used for transmission were oil-impregnated paper-
insulated type cables, often with three conductors contained in a single
sheath. The three conductors were stranded and insulated separately and
then laid up spirally together. The space between and around the insu-
lated conductors was packed with paper or jute to form a cylindrical
surface which was then further wrapped with insulation. This is called
the belted type cable. As a solution of the problem of high electric
stresses set up tangentially to the paper insulation surfaces in these
cables, a new form of construction, known as the H type or shielded type
was originated by Hochstadter in the 1920's. In these cables, each core
is wrapped with a conducting layer of metallized paper which converts
the cable electrically into three single core cables with the electric
stress completely radial.

As system voltages increased above 33 kV, the solid type paper-oil
cable became prone to breakdown because of the voids formed in the
insulation.

In 1926, Emanueli introduced the self-contained low pressure oil-
filled (LPOF) cable which is still extensively used today at voltage
levels up to 500kV. Single conductor oil-filled cables, widely used in
Europe, consist of a concentric stranded conductor built around an open
helical spring core, which serves as a channel for the flow of oil. The
cable is insulated and sheathed in the same manner as solid cables.
Three conductor oil-filled cables are all of the shielded design, and
have three oil channels composed of helical springs that extend through
Figure 1 Cross section of a single phase power cable [1].
the cable in spaces normally occupied by filler material. The oil in
the cable and its connected reservoirs is maintained under moderate
pressure so that the development of voids or excessive pressure in the
cable is prevented. The usual voltage range for three-conductor oil-
filled cable is from 23 kV to 132 kV.

Another type of cable which found wide-spread acceptance espe-
cially in the U.S. after World War II, is the high-pressure oil-filled
pipe type cable (HPOF). In this system, the conductor is formed in seg-
ments without the central oil channel, insulated with paper, and
shielded with a combination of metal and synthetic tapes, which also act
as a barrier against casual moisture. Three of these cables are pulled
into a previously buried pipe which is eventually filled with oil at
high pressure (e.g., 13-15 atm.). Voids are suppressed mechanically by
the high pressure, thus assuring good electrical performance. Instal-
lation of this type of cable requires great care to prevent moisture
from entering the pipe, and oil from leaking from the system. The per-
formance of pipe type cables is very good compared to self-contained
LPOF cables. Pipe type cables are commercially available up to 550 kV
and rating exceeding 1000 MVA for self-cooled systems and 2000 MVA for
forced-cooled systems appear possible in the near future [2]. The first
installation of 345 kV pipe type cables of 24 km length was in New York
in 1964. Since then, pipe type cables have superseded self-contained
cables for underground transmission. However, strong limitations exist
in employing the conventional LPOF and HPOF cables at extra high voltage
transmission. Those are the high cost of reactive compensation and the
thermal considerations.
Since 1970, an increasing amount of extruded solid insulated
cables has been installed in the voltage range between 46 and 138 kV.
The materials mostly used for insulation are polyethylene, butyl rubber,
and cross-linked polyethylene. Solid insulation is usually extruded
onto the conductor.

Intense research and development activities have been observed in
the last decades in the areas of compressed gas insulated cables and low
temperature (cryogenic) cables. Considerable progress has been made in
developing both of these technologies, though problems still remain.

For a detailed description of the progress in the design and tech­
nology of underground power cables, refer to [1-5].

2.2 AC RESISTANCE OF CONDUCTORS

The metals predominantly used in cable conductors at the present
time are copper and aluminum. Most of the single conductor cables are
of the concentric-strand type. They may also be compact-round, annular
stranded, segmental or hollow-core. Compacted-sector conductors are
widely used in three conductor cables.

The DC resistance, \( R_{dc} \), of a conductor is given by:

\[
R_{dc} = R_0 [1 + \alpha(T-20)] \text{ ohms per unit length} \quad (1)
\]

where \( R_0 \) is the DC resistance at 20°C per unit length, \( \alpha \) is the temper­
ature coefficient at 20°C and \( T \) is the operating temperature in °C.

The effective AC resistance, \( R_{ac} \), of the conductor is strongly
influenced by skin effect and proximity effect. It is well known that
the resistance of a conductor to alternating current is larger than its
resistance to direct current. When alternating current flows in the conductor, there is a nonuniform distribution of current, with the outer filaments of the conductor carrying more current than the filaments closer to the center. This is commonly called skin effect. In small conductors, the skin effect is negligible. Skin effect decreases the internal inductance of the conductor but does not affect the external inductance. It is independent of the circuit configuration and the sequence of the current. It depends upon the frequency, and the dimensions and materials of the conductor. On the other hand, proximity effect is the phenomenon of non-uniform current distribution over the cross section of a conductor caused by the variation of current in a neighboring conductor. It depends upon the flux distribution both inside and outside the conductors. In addition to frequency and conductor dimensions and materials, it is influenced by the circuit configuration and the relative magnitudes and phase relations between the currents in the conductors. Proximity effect causes an apparent increase of the resistance of a conductor. While it is negligible for small conductors, it can become important under certain conditions, i.e., when cables are laid parallel and close to metal pipes, walls, etc.

Hence, the effective AC resistance can be written as:

\[ R_{ac} = R_{dc} (1 + Y_{cs} + Y_{cp}) \] ohms per unit length  \hspace{1cm} (2)

where \( Y_{cs} \) and \( Y_{cp} \) are the conductor skin and proximity effect factors, respectively.

The mathematical treatment of skin and proximity effects is complicated. By applying Maxwell's equations to straight conductors of
circular cross section, a differential equation, called the diffusion equation, can be obtained:

\[ \frac{d^2i}{dr^2} + \frac{1}{r} \frac{di}{dr} + k^2i = 0 \]

where \( i(r) \) is the value of conductor current at a radial distance \( r \) from the center of the conductor, and \( k \) is the wave number. The solution is expressed in Bessel functions which were approximated further to obtain some practical equations for the \( R_{ac}/R_{dc} \) ratio [4, p.38]. For non-circular stranded conductors, the analytical evaluation of \( Y_{CP} \) and \( Y_{CS} \) is more complicated and it is necessary to use the finite element method. There are many empirical equations suggested in the literature for the values of \( Y_{CP} \) and \( Y_{CS} \) [4-6].

Skin effect has a significant influence on \( R_{ac} \) of large conductors at power frequencies. For example, \( R_{ac}/R_{dc} \) ratio for 2000 mm\(^2\) conductors in LPOF cables is almost equal to 1.2. Skin effect increases the conductor losses and reduces current carrying capability. Milliken's construction scheme is widely applied to large conductors to reduce skin effect. The Milliken conductor consists of a number of individually stranded segments insulated from each other. Each segment is produced by concentric stranding with transposition of wires, i.e., each wire occupies every radial position over a complete lay. In the latest large conductors, the individual wires are also insulated, and \( R_{ac}/R_{dc} \) ratios of less than 1.05 are obtained.

Additional losses are observed in the cables when a metal structure is placed close to conductors. These losses are due to hysteresis...
and eddy-current effects within the metal. A material having high permeability and very high resistivity promotes hysteresis loss because the flux produced by cable currents concentrates within the low-reluctance metal and the action of circulating currents in the metal structure to counteract the incident flux is comparatively small. At one time, steel armor wires were used extensively for reinforcing lead covered LPOF cables. The high magnetic hysteresis losses discouraged this practice for single conductor cables. However, in three conductor cables, the resultant magnetic field is almost zero and there is no heat developed in the armor. In practice, hysteresis losses are mostly neglected for LPOF cables.

In HPOF cables, sheath is made of magnetic steel and due to its proximity to the phase conductors, it is subject to a significant degree of magnetization. In the strong field of a single conductor cable, the hysteresis losses in the pipe are unacceptably large and sometimes greater than conductor losses. In the case of three conductor pipe type cables, the scheme most commonly used in HPOF cable systems, the losses in the pipe are comparable to the losses caused by all the other AC effects combined.

An approximate measure of the magnetic hysteresis loss, \( W_h \), can be obtained from the Steinmetz formula [3,p.108];

\[
W_h = 0.002 f V B_{\text{max}}^{1.6} 10^{-7} \text{ watts}
\]

where \( f \) is the frequency, \( V \) is the volume of the magnetic material in cm\(^3\), and \( B_{\text{max}} \) is the peak value of the magnetic induction in gauss.
On the other hand, eddy-current losses are caused by the $I^2R$ losses of currents that circulate in the steel pipe tending to oppose the change of flux density. The eddy-current effects in a single conductor pipe type cable can be illustrated with the help of Figure 2.

The current flowing in the phase conductor induces a longitudinal emf, $e_s$, in the pipe wall. The magnitude of $e_s$ has a maximum on the inside surface and a minimum on the outside surface of the pipe. The

![Figure 2 Single conductor in a metal pipe [3].](image)
difference between the maximum and minimum emf's results in a current flowing down the inner portion of the pipe wall and up in the outer parts of it. The energy losses associated with this current are customarily described as eddy-current losses. If the pipe is grounded at both ends, another loss may arise from circulating currents through a ground loop (i.e., $i_S$ in Figure 2). This circulating current is caused by the average induced emf $e_S$ and, in general, is much larger than eddy currents [3,p.58]. Although the electromagnetic field in the pipe can, in principle, be calculated by solving Maxwell's equations, an exact analytical solution for eddy-currents is extremely difficult. Very recently, new analytical methods to calculate eddy-current losses in the steel pipes of HPOF cables have been suggested [7,8].

For the purpose of practical calculations, AC resistance of pipe type cables which takes the effect of the pipe into account can be determined in terms of incremental loss ratios. Several empirical approximate expressions have been proposed and the most commonly adopted formula is that suggested by Neher and McGrath [9]:

$$R_{ac} = (1 + 1.7Y_{cp} + 1.7Y_{cs} + Y_p + 1.7Y_{sh}) R_{dc}$$  \hspace{1cm} (3)

where $Y_{cs}$ and $Y_{cp}$ arise from conductor skin and proximity effects, respectively, $Y_p$ is due to the losses in the pipe, and $Y_{sh}$ is a resistance component for the losses in the shield and skid wire assembly.
2.3 GEOMETRY OF CABLES

The physical separation of sheaths and conductors in a cable circuit is a major factor in determining inductance, capacitance, charging current, insulation resistance, dielectric loss, and thermal resistance. Figure 3 shows the symbols for various cable dimensions that are going to be used throughout this chapter. The following paragraphs cover some important geometric definitions such as geometric mean radius (GMR) and geometric mean distance (GMD).

GMR is a property usually applied to the conductor alone, and depends on the geometry, material and stranding used in the construction of the conductor. The essence of introducing GMR is to replace the original conductor by an equivalent tubular cylindrical conductor with very thin wall, so that the sum of internal and external flux linkages of the original conductor, and the external flux linkage of the equivalent conductor will be the same. The radius of the equivalent conductor is called the geometric mean radius of the original conductor and will be denoted by \( GMR_c \). For a solid cylindrical, nonferromagnetic conductor of radius \( r \),

\[
GMR_c = 0.7788 \, r
\]

The factor by which the actual radius must be multiplied to obtain \( GMR_c \) varies with stranding or hollow-core construction. Sometimes in calculations involving zero sequence reactances, the three phase conductors comprising a three phase circuit are considered as a group and converted to a single equivalent conductor of radius denoted by \( GMR_{3C} \) [10].
Figure 3  Cable geometry [10].
GMD is a term that is used in the expression for external flux linkages (i.e., flux linkages extending outward from a cylinder of a radius of unit length to the current return path). The main idea here is to represent the separations between the conductor(s) and the return path(s) by an equivalent distance. The positive and negative sequence reactance of a three phase circuit depends on separation among phase conductors. If the conductors are equilaterally spaced as in Figure 3.a,

$$GMD_{3c} = S$$

If the conductors are asymmetrically spaced as in Figure 3.c but transposed along their length, the circuit can be treated as a balanced circuit by defining:

$$GMD_{3c} = \frac{3}{\sqrt{3}} S_{ab} S_{bc} S_{ca}$$  \hspace{1cm} (4)

The zero sequence reactance of a three phase circuit may depend on the spacing among conductors and sheath as well as among conductors. The equivalent spacing between a group of conductors and the enclosing sheath can be represented as GMD.

Finally, we will state an important result on the subject of GMD between a pipe and a circular conductor (or another pipe). In this case, GMD is equal to the distance between the centers of the pipe and the conductor [4,p.54].

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2.4 SHEATH VOLTAGES AND CURRENTS

In this section, mainly the sheath phenomena in power cables, including the function of bonding and cross-bonding, and the induced voltages and currents in sheaths will be presented.

2.4.1 Shielding

Shielding of an electric power cable, as defined by the Insulated Power Cable Engineers Association, "...is the practice of confining the dielectric field of the cable to the insulation of the conductor or conductors. It is accomplished by means of conductor and insulation shields..." [11]. Some important functions of shielding are:

a) To confine the electric field within the cable,
b) To obtain symmetrical radial distribution of voltage stress within the insulator,
c) To provide increased mechanical protection for the dielectric,
d) To protect the cable from induced voltages from adjacent transmission lines,
e) To reduce the hazards of shock at the outer surfaces of the cable,
f) To conduct charging, unbalanced neutral, and fault currents.

All metal shields are made of a nonferrous material to avoid magnetic losses. Some possible forms of metallic shielding are:

a) Cylindrical metal sheaths,
b) Copper tape applied helically with an overlap,
c) Concentric copper wires applied helically and equally spaced apart,
d) Corrugated copper strips applied longitudinally with an overlap.

For safe and effective operation, the shielding should be grounded at each end of the cable and at each splice. Metal shields bonded or grounded at more than one point have circulating currents flowing in them, the magnitude of which depends on the mutual inductance to the other cables, the current in these conductors, and the resistance of the shield. This circulating current does not depend on the length of the cables nor the number of bonds, providing there are bonds at each end. The only effect of this circulating current is to heat the shield and thereby reduce the effective current carrying capacity of the cable. Shields bonded or grounded at only one point will have a voltage built up in the sheath. The magnitude depends on the mutual inductance to other cables, the current in all the conductors, and the distance to the grounded point.

2.4.2 Cross-bonding

When three separate single conductor cables are used, relatively high longitudinal voltages are induced in each of the sheaths, even under balanced current conditions. This is due to the unequal spacing of the sheaths relative to any one conductor. If the cable is long, the induced voltages can reach a level to form a considerable hazard to life as well as possibly leading to arcing and consequent deterioration of the cable. Hence, it is necessary both to bond the sheaths of each cable together at the various joint shells and to ground them at intervals to insure that there are no large potentials present on them.
Such a bonding arrangement is illustrated in Figure 4.a where phase conductors and sheaths are denoted by solid and dotted lines, respectively. However, that arrangement will result in large circulating currents through the bonded sheaths and ground. The losses due to this circulating current are generally much large than the eddy-current losses.

One of the various arrangements to eliminate the sheath losses is the Kirke-Searing scheme which is used extensively in England [12]. In this arrangement, the sheaths are insulated from ground and interrupted at regularly spaced intervals and transposed. After three lengths have been traversed, the sheaths are bonded to each other and to ground as shown in Figure 4.b. The effect is to add the sheath voltages in the adjacent sections, and for balanced loads, the sum will be zero. High sheath currents are thus avoided and induced voltages kept under reasonable limits.

2.4.3 Sheath Phenomena in LPOF Cables

As mentioned in Section 2.2, the effective AC resistance of the conductor for LPOF cables (i.e., Eq. (2)) does not include the component representing the sheath losses. In this section, the equivalent resistance that represents the sheath losses in single conductor and three conductor cables will be derived. The effect of various sheath arrangements on the induced sheath voltages and currents will also be investigated.

a) Single conductor cables:

When single conductor cables are used for AC transmission, induced emfs in the sheaths due to the magnetic fields of the conductor currents
Figure 4  Sheath bonding schemes for three phase cables.

a) Normal sheath bonding

b) Kirke-Searing cross-bonding arrangement
may produce large sheath currents. The induced sheath currents can be classified into two kinds:

i) Local eddy-currents,

ii) Circulating or sheath circuit currents, whose outward and return paths are formed by the sheaths of separate cables.

The type of sheath currents flowing and hence the sheath losses will be determined by the sheath arrangement. The resulting sheath losses are manifested in the main circuit by an increase of resistance and a decrease in inductance. The following two cases will be presented separately:

1 - Sheaths open circuited at one or both ends,

2 - Sheaths grounded at both ends.

1 - Sheaths open circuited at one or both ends:

In this situation, the only currents which can flow in the sheath are the local currents resulting from the proximity of the return conductors. The magnitude of the local sheath currents and the consequent power losses are greatest when the sheaths are close to each other, as there is then the greatest variation in flux density over the sheath cross section.

The voltages induced in the sheaths of three phase single conductor cables can be determined readily if the local sheath currents are neglected. Let a, b, and c denote the cable conductors in a symmetrical formation and x, y, z their respective sheaths. With the sheaths open circuited, the voltage drops $V_x$, $V_y$, and $V_z$ induced in the sheaths can be written as:
\[ V_x = Z_{ax}I_a + Z_{bx}I_b + Z_{cx}I_c \]
\[ V_y = Z_{ay}I_a + Z_{by}I_b + Z_{cy}I_c \]
\[ V_z = Z_{az}I_a + Z_{bz}I_b + Z_{cz}I_c \]  

(5)

where Z's indicate mutual impedances between the conductor and the sheath specified by the subscripts. \( I_a, I_b \) and \( I_c \) are the phase conductor currents.

With positive sequence currents flowing in the phase conductors, \( I_a, I_b, \) and \( I_c \) in Eq. (5) are replaced by \( I_a, a^2I_a \) and \( aI_a \), respectively, where \( a = \cos 120^\circ \). This yields

\[ V_x = I_a(Z_{ax} + a^2Z_{bx} + aZ_{cx}) \]
\[ V_y = I_a(Z_{ay} + a^2Z_{by} + aZ_{cy}) \]
\[ V_z = I_a(Z_{az} + a^2Z_{bz} + aZ_{cz}) \]  

(6)

The mutual impedances in Eq. (6) have no resistive components, and their reactance components are \( 2\pi f \) times their inductances. The mutual impedances between two parallel coaxial and non-coaxial conductors are given as [13, p. 7]:

\[ Z_{ax} = Z_{by} = Z_{cz} = j2\pi f (-1 + \ln \frac{2\pi}{r_m}) 2 \cdot 10^{-7} \text{ ohm/m} \]  

(7)

\[ Z_{bx} = j2\pi f (-1 + \ln \frac{2\pi}{S_{ab}}) 2 \cdot 10^{-7} \text{ ohm/m} \]  

(8)

where \( \ell \) is the length of the cable in m, \( r_m \) is the average sheath radius in m, \( S_{ab} \) is the distance between conductors a and b in m. Other mutual
impedances can be determined similarly. Inserting the mutual impedances in Eq. (6), the following voltage relations are obtained:

\[ V_x = jI_a 0.1736 \frac{f}{60} \left( \log \frac{\sqrt{S_{ab}S_{ac}}}{r_m} - j \frac{3}{2} \log \frac{S_{ac}}{S_{ab}} \right) \text{ V/km} \]

\[ V_y = ja^2I_a 0.1736 \frac{f}{60} \left( \log \frac{\sqrt{S_{ab}S_{bc}}}{r_m} - j \frac{3}{2} \log \frac{S_{ab}}{S_{bc}} \right) \text{ V/km} \]  

\[ V_z = jaI_a 0.1736 \frac{f}{60} \left( \log \frac{\sqrt{S_{ac}S_{bc}}}{r_m} - j \frac{3}{2} \log \frac{S_{bc}}{S_{ac}} \right) \text{ V/km} \]

where \( S \) is the distance between axes of conductors indicated by the subscripts. The three sheaths \( x, y, \) and \( z \) may be regarded as a three phase circuit parallel to conductors \( a, b, \) and \( c. \) Selecting \( x \) as the reference phase, the induced voltages in Eq. (9) can be resolved into their symmetrical components:

\[ V_{x1} = jI_a 0.1736 \frac{f}{60} \left( \log \frac{2S}{r_m} \right) \text{ V/km} \]

\[ V_{x2} = jI_a 0.1736 \frac{f}{60} \left( \log \frac{\sqrt{S_{ab}S_{ac}}}{S_{bc}} - j \frac{3}{2} \log \frac{S_{ac}}{S_{ab}} \right) \text{ V/km} \]  

\[ V_{xo} = -jI_a 0.1736 \frac{f}{60} \left( \log \frac{\sqrt{S_{ab}S_{ac}}}{S_{bc}} + j \frac{3}{2} \log \frac{S_{ac}}{S_{ab}} \right) \text{ V/km} \]

where \( V_{x1}, V_{x2}, \) and \( V_{xo} \) are the positive, negative, and zero sequence induced sheath voltages, respectively. \( S \) is the geometric mean distance between the sheaths and is equal to \( (S_{ab}S_{bc}S_{ac})^{1/3}. \)
Sheath voltages in Eq. (10) reveal that in an unsymmetrical circuit, positive sequence currents flowing in the conductors induce positive, negative, and zero sequence voltages in the sheaths.

On the other hand, if the circuit is symmetrical (i.e., $S_{ab} = S_{ac} = S_{bc}$), it will be seen from Eq. (10) that:

$$V_{x2} = V_{x0} = 0$$

Hence, the sheath voltages induced by positive sequence currents will be positive sequence voltages.

2. Sheaths grounded at each end:

In the previous sections, it was stated that bonding the sheaths together and grounding them at each end or at intervals will suppress the induced voltages along the sheaths but will cause large circulating currents. The losses caused by the circulating currents, unlike those due to the local currents in the sheath, are minimized by laying the cables as close as possible.

The effect of the local currents in the sheath is superimposed on the circulating current. However, the losses due to local currents are mostly neglected and the losses due to circulating currents are represented by an increment in the AC resistance.

The equivalent circuit formed by the conductor and the bonded sheaths of a three phase cable is shown in Figure 5. Writing the voltage drop equations for both the conductor and the sheath results in:
\[ V = (R_C + jwL_C) I_C + jwM I_S \]  \hspace{1cm} (11)

\[ 0 = jwM I_C + (R_S + jwL_S) I_S \]  \hspace{1cm} (12)

where \( V \) is the voltage applied to the conductor in volts, \( R_C \) is the conductor AC resistance defined by Eq. (2), \( L_C \) and \( L_S \) are the conductor and sheath self inductances, respectively, and \( w = 2\pi f \). \( M \) is the mutual inductance between the conductor and the sheath and is given by:

\[ M = 2 \times 10^{-7} \ln \frac{S}{r_m} \text{ H/m} \]  \hspace{1cm} (13)

where \( S \) is the geometric mean distance between cables in m, and \( r_m \) is the average sheath radius in m. Sheath resistance, \( R_S \), is:

\[ R_S = \frac{\rho_S}{\pi(r_0-r_1)(r_0+r_1)} [1 + \alpha_S (T-20^\circ)] \text{ ohm/m} \]  \hspace{1cm} (14)

*Figure 5 Equivalent circuit formed by conductor and bonded sheaths.*
where $\rho_s$ is the resistivity of the sheath material in ohm-m, $\alpha_s$ is the temperature coefficient per degree C° at 20°, and $r_o$ and $r_i$ are the outer and inner radii of the sheath in m, respectively. For a lead sheath, the sheath resistance per unit length is:

$$R_s = \frac{0.8 \cdot 10^{-7}}{r_o^2 - r_i^2} \text{ ohm/m}$$

The self inductance of a cable sheath is in most practical cases almost equal to the mutual inductance between the sheath and the enclosed conductor. Hence, using Eq. (12), the sheath current can be solved as:

$$I_s = \frac{jwM}{R_s + jwM} I_c \quad (15)$$

Inserting Eq. (15) into Eq. (11) yields:

$$V = (R_c + jwL_c - \frac{w^2M^2}{R_s + jwM}) I_c$$

$$= R_c + \frac{w^2M^2}{R_s^2 + w^2M^2} R_s + jwL_c - \frac{w^3M^3}{R_s^2 + w^2M^2} I_c \quad (16)$$

It is thus seen that the apparent resistance of the conductor is increased by the amount:

$$R_s' = \frac{w^2M^2}{R_s^2 + w^2M^2} R_s \quad (17)$$

and the apparent reactance is decreased by:
\[ X_s' = \frac{w^3M^3}{R_s^2 + w^2M^2} \]  

Hence, neglecting the local current losses:

\[ \lambda = \frac{\text{sheath losses}}{\text{conductor losses}} = \frac{R_s}{R_c} \left( \frac{w^2M^2}{R_s^2 + w^2M^2} \right) \]

b) Three conductor cables:

The sheath losses in a three conductor cable are usually negligible except for very large cables and when it is important to make accurate calculations. The sheath losses in the largest cables are approximately 3 to 5 percent of the conductor losses. The sheath losses in the three conductor cables can be calculated from the equivalent sheath resistance, \( R_s' \), as [10,p.71]:

\[ R_s' = \frac{6.861 S_1^2}{R_s \cdot r_m^2} \cdot 10^{-6} \text{ ohm/phase/km} \]  

where \( R_s \) is the sheath resistance from Eq. (14),

\[ S_1 = \frac{1}{\sqrt{3}} (d+2T), \]  

and is the distance in m between conductor center and sheath center for three conductor cables made up of round conductors. \( d \) is the conductor diameter in m, and \( T \) is the insulation thickness in m.

For sector shaped conductors an approximate value for \( R_s' \) can be obtained by (19), except that \( d \) should be 0.82 to 0.86 times the diameter of an equivalent round conductor having the same cross sectional area.
2.5 **POSITIVE AND NEGATIVE SEQUENCE IMPEDANCES**

In this section, the resistance and reactance of three phase underground cables to positive and negative sequence currents will be presented. It will be assumed that the single conductor cables forming a three phase circuit are solidly bonded and grounded at each end, and cross-bonded. The presence of adjacent cable circuits other than the three phase cable system under consideration is neglected in the calculations. The AC resistances will be assumed to include skin and proximity effects, and the influence of conductor operating temperature. Also the negative sequence impedance of a three cable system will be equal to its positive sequence impedance. The positive sequence impedance of single conductor cables, three conductor cables, and pipe type HPUF cables will be covered separately below.

a) **Single conductor cables**

The resistance of single conductor cables to positive sequence currents, $R_1$, is the effective AC resistance which is the sum of the conductor AC resistance and the resistance representing the sheath losses:

$$R_1 = R_{ac} + \frac{x_m^2}{R_s^2 + x_m^2} R_s \text{ ohm/phase/km} \quad (20)$$

where $R_{ac}$ is the conductor AC resistance from Eq. (2), $R_s$ is the sheath resistance per km, and $x_m$ is the mutual reactance between conductors and sheath per km. $x_m$ is given as:
\[ X_m = wM = 0.1736 \frac{f}{60} \log \left( \frac{GMD_3c}{r_m} \right) \text{ohm/phase/km} \]  

where \( GMD_3c = \frac{3}{\sqrt[3]{S_{ab}S_{bc}S_{ac}}} \) is the geometric mean distance between conductors in m, \( r_m = \frac{r_o + r_i}{2} \) average sheath radius in m, \( f \) is the frequency.

The positive sequence reactance of single conductor cables is equal to the effective reactance of the cable which takes into account the effect of sheath currents:

\[ X_1 = 0.1736 \frac{f}{60} \log \left( \frac{GMD_3c}{GMR_c} \right) - \frac{X_m^3}{R_s + X_m^2} \text{ohm/phase/km} \]  

where \( f \), \( GMD_3c \), \( X_m \), and \( R_s \) are as defined in Eq. (20). \( GMR_c \) is the geometric mean radius of one conductor in m.

b) Three conductor cables

The positive sequence resistance, \( R_1 \), is given by:

\[ R_1 = R_{ac} + R_s' \]

\[ = R_{ac} + \frac{6.861 S_i^2}{R_s r_m^2} \cdot 10^{-6} \text{ohm/phase/km} \]  

where \( R_{ac} \) is the conductor AC resistance per km as defined in Eq. (2), and \( R_s' \) is the equivalent sheath resistance defined in Eq. (19). Since only negligible sheath current effects are present in three conductor cables, the positive and negative sequence resistances can also be assumed equal to \( R_{ac} \).
The positive sequence reactance can be calculated similarly:

\[
X_1 = 0.1736 \times \frac{f}{60} \cdot \log \frac{\text{GMD}_3}{\text{GMR}_C} \text{ ohm/phase/km} \quad (24)
\]

c) **Pipe type HPOF cables**

The positive and negative sequence resistances of HPOF cables are given as:

\[
R_1 = R_{ac} \quad (25)
\]

where \( R_{ac} \), as defined in Eq. (3), is the AC resistance including the skin and proximity effects, the influence of the steel pipe, shielding and skid wire assemblies. Hence, \( R_1 \) is expressed in terms of various incremental loss ratios which are to be determined experimentally. Analytical determination of \( R_1 \) is quite complicated.

The same difficulty applies for the reactance calculations. The usual approach is to determine the positive sequence reactance of pipe type cables as for self-contained LPOF cables in triangular formation and then increase it by 15% to account for the presence of the steel pipe. The following equation is suggested by Neher [14]:

\[
X_1 = 0.1736 \times \frac{f}{60} \cdot 1.15 \cdot \log \frac{\text{GMD}_3}{\text{GMR}_C} \text{ ohm/phase/km} \quad (26)
\]
where $G M D_{3C} = D_s$ (for close triangular form)

$$= 1.26 D_s \frac{6\sqrt{1 - \left( \frac{D_s}{D_p-D_s} \right)^2}}{D_p-D_s^2}$$

(for cradled form)

$D_s$ is the cable diameter in m, and $D_p$ is the inside diameter of the pipe in m.

2.6 ZERO SEQUENCE IMPEDANCE

When zero sequence currents flow through the phase conductors of a three phase cable circuit, they return to the source station in either the ground, or the sheaths, or in the parallel combination of ground, sheaths and sheaths of adjacent cables. The resistance that the zero sequence current flowing in a phase conductor encounters, includes the AC resistance of that conductor and the resistance of the existing return paths. The zero sequence reactance, similarly, involves the self inductances of the conductor and the return paths, and the mutual inductances between them. Reactance calculations will utilize the GMR and GMD approach by which a paralleled conductor group is represented by an equivalent conductor.

As possible return paths for the zero sequence currents, we will consider:

i. sheath alone,

ii. ground alone,

iii. sheath and ground in parallel,

iv. sheath, ground and sheaths of adjacent cables.

Cable sheaths are assumed to be bonded and grounded at several points. Hence, a three phase circuit of single conductor cables will
have a total of six metallic conductors plus the ground. In such cases, the most satisfactory procedure is to consider that one of the conductors is the return for loop circuits each consisting of one of the other conductors and the common return. Here, it is more convenient to consider the ground as the common return because: a) it is of a different character than the other six conductors, b) the equations for determining the impedance of any circuit consisting of a conductor with ground return, and for the mutual impedance between any two such circuits are available.

The most widely used formulas on the theory of ground return circuits are based on the work of J.R. Carson [15,16]. The self and mutual impedance expressions are obtained from Carson's solution of the ideal case of an infinite line carrying a constant current which returns through a homogeneous semi-infinite ground with a uniform resistivity. In Appendix A, Carson's formulation of ground return impedances, simplified equations and the expressions for parallel lines of finite length are presented in detail. As shown in Appendix A, Carson's equation for the mutual impedance between two close conductors reduces to a simple equation which is valid for conductors above the ground or conductors buried close to the surface of the ground. The mutual impedance $Z_m$ between two conductors, $a$ and $b$, with common ground return (i.e., the voltage between $b$ and ground for unit current in $a$ and ground return) may be written as:

$$Z_m = 0.988 \cdot 10^{-3} f + j2.894 \cdot 10^{-3} f \log \frac{658.4 \sqrt{b/f}}{d_{ab}} \text{ ohm/km} \quad (27)$$
where \( f \) is the frequency in Hz, \( \rho \) is the soil resistivity in ohm-m, \( d_{ab} \) is the distance between \( a \) and \( b \) in m. Similarly, the self impedance \( Z_c \) of conductor \( a \) with ground return (i.e., the voltage between \( a \) and ground for unit current in conductor \( a \)) is:

\[
Z_c = R_c + 0.988 \times 10^{-3} f + 32.894 \times 10^{-3} f \log \frac{658.4 \sqrt{f}}{GMR_a} \text{ ohm/km} \tag{28}
\]

where \( R_c \) is the resistance of conductor \( a \) per km, \( GMR_a \) is the geometric mean radius of \( a \) in m.

A very useful physical conception in the analysis of ground return paths is to concentrate the ground currents in a fictitious return conductor at some considerable depth below the conductor under consideration. The equivalent depth, resistance and reactance of the fictitious ground return conductor are represented by \( D_g \), \( R_g \), and \( X_g \), respectively and they can be defined as follows by utilizing Eq. (27):

\[
D_g = 658.4 \sqrt{\frac{\rho}{f}} \text{ m} \tag{29}
\]

\[
R_g = 0.988 \times 10^{-3} f \text{ ohm/km} \tag{30}
\]

\[
X_g = 2.894 \times 10^{-3} f \log \frac{D_g}{GMR_a} \text{ ohm/km} \tag{31}
\]

An inspection of the equations above shows that the reactance of the ground return is a function of soil resistivity and frequency, whereas the resistance of the ground return is a function of frequency.
Rudenberg and others have also arrived at the same equations by a different method of reasoning [17,p.401].

Rewriting Carson's simplified equations in terms of the variables defined in (29) - (31):

$$Z_c = R_c + R_g + jX_g \text{ ohm/km} \quad (32)$$

$$Z_m = R_g + j2.894 \times 10^{-3} f \log \frac{D_g}{d_{ab}} \text{ ohm/km} \quad (33)$$

To convert Eqs. (32) and (33) to zero sequence quantities, the phase conductors of a three phase system should be replaced by a single equivalent conductor. It is clear that for one unit of zero sequence current flowing in each phase conductor, three units of zero sequence current flow through the ground return and the equivalent conductor. Therefore, the corresponding zero sequence self and mutual impedances per phase are three times the values in Carson's simplified equations. Denoting the zero sequence self and mutual impedances by $Z_0$ and $Z_{om}$, respectively:

$$Z_0 = R_c + 3R_g + j3X_g \text{ ohm/phase/km} \quad (34)$$

$$Z_{om} = 3R_g + j8.682 \cdot 10^{-3} f \log \frac{D_g}{d_{ab}} \text{ ohm/phase/km} \quad (35)$$

where $R_c$ is the resistance of a conductor equivalent to the three conductors in parallel, and subscript $a$ denotes the equivalent conductor.
The following sections will cover the zero sequence self and mutual impedances of three conductor cables, single conductor cables and pipe type cables separately.

a) Three conductor cables:

Actual and equivalent circuits for a single circuit three conductor cable with a solidly grounded sheath are shown in Figure 6. The zero sequence impedance of three paralleled conductors will be determined by considering the presence of the ground return but ignoring the presence of the sheath. Hence, the zero sequence impedance of the fictitious conductor equivalent to the three conductors can be written as [10, p. 74]:

\[ Z_c = R_c + 3R_g + j0.521 \frac{f}{60} \log \frac{D_g}{GMR^3_c} \text{ ohm/phase/km} \quad (36) \]

where \( R_c \) is the AC resistance of one conductor in ohms per km, \( R_g \) is the AC resistance of ground return in ohms per km as defined in (30), \( D_g \) is the distance to equivalent ground return path in m as given in (29), and \( GMR^3_c \) is the geometric mean radius of the conducting path made up of the three conductors:

\[ GMR^3_c = \sqrt[3]{GMR_c GMD^2_{3c}} \quad m \quad (37) \]

Similarly, the impedance of the sheath will be determined by considering the presence of ground return path but ignoring the conductors:

\[ Z_s = 3R_s + 3R_g + j0.521 \frac{f}{60} \log \frac{D_g}{r_m} \text{ ohm/phase/km} \quad (38) \]
Figure 6 Representations of a three conductor cable.
where $R_s$ is the sheath resistance in ohms per km and $r_m$ is the average radius of the sheath in m.

The mutual impedance between conductors and sheath, considering the presence of the ground return path which is common to both conductors and sheath, in zero sequence terms is:

$$Z_m = 3R_g + j0.521 \frac{f}{60} \log \frac{D_g}{r_m} \text{ ohm/phase/km} \quad (39)$$

The equivalent circuit in Figure 6.c is obtained from Figure 6.b by representing the mutual impedance as a common series element. From this circuit, the following observations are possible:

1) If the current returns in the sheath only (i.e., none in the ground):

$$Z_0 = (Z_c - Z_m) + (Z_s - Z_m) = Z_c + Z_s - 2Z_m$$

2) If the current returns in ground only:

$$Z_0 = (Z_c - Z_m) + Z_m = Z_c$$

3) When both ground and sheath return paths exist, the total zero sequence impedance is:

$$Z_0 = Z_c - Z_m - \frac{(Z_s - Z_m)Z_m}{Z_s} = Z_c - \frac{Z_m^2}{Z_s}$$

4) If the return current flows through the sheath, ground and sheaths of adjacent cables, determing the total zero sequence impedance with the help of an equivalent circuit which incorporates mutual impedances as
series elements, becomes complicated. An approach that can be applicable to any number of additional metallic return paths is to use voltage drop equations in matrix form. As an example, assume that there is a single conductor auxiliary cable in the close vicinity of the three conductor cable. The equivalent circuit in terms of zero sequence quantities will be as seen in Figure 7. Subscripts c, s, and x denote the equivalent conductor, sheath and the sheath of the auxiliary cable, respectively. Hence, $Z_x$ denotes the zero sequence self impedance of the auxiliary cable with ground return and $Z$ with two subscripts denotes the mutual impedance with common ground return. It is assumed that both sheaths are solidly grounded at the ends.

![Figure 7](image_url)

**Figure 7** Equivalent circuit of a three conductor cable and an auxiliary cable.
The voltage drop equations can be written as:

\[
\begin{bmatrix}
E \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
Z_c & Z_{cs} & Z_{cx} \\
Z_{cs} & Z_s & Z_{sx} \\
Z_{cx} & Z_{sx} & Z_x
\end{bmatrix}
\begin{bmatrix}
I_o \\
I_s \\
I_x
\end{bmatrix}
\]  \hspace{1cm} (40)

Where all impedances are defined with ground return. Our purpose is to find the system zero sequence impedance, \( Z_0 \), which is:

\[
Z_0 = \frac{E}{I_0}
\]

where \( E \) is the conductor voltage and \( I_0 \) is the total zero sequence current flowing through the phase conductors. From Eq. (40), \( I_s \) and \( I_x \) can be solved in terms of \( I_0 \):

\[
\begin{bmatrix}
I_s \\
I_x
\end{bmatrix} = -
\begin{bmatrix}
Z_s & Z_{sx} \\
Z_{sx} & Z_x
\end{bmatrix}
^{-1}
\begin{bmatrix}
Z_{cs} \\
Z_{cx}
\end{bmatrix} I_0
\]  \hspace{1cm} (41)

and then:

\[
Z_0 = Z_c - [Z_{cs} \quad Z_{cx}]
\begin{bmatrix}
Z_s & Z_{sx} \\
Z_{sx} & Z_x
\end{bmatrix}
^{-1}
\begin{bmatrix}
Z_{cs} \\
Z_{cx}
\end{bmatrix}
\]  \hspace{1cm} (42)
Hence, the matrix approach is more practical and computer oriented. If there are more than one auxiliary cable sheaths, the dimensions of the matrices will increase correspondingly. It is also possible to replace the additional auxiliary sheaths by an equivalent sheath if the auxiliary cables are identical and equidistant from the main cable.

The method suggested to find the zero sequence impedance of a circuit can also be utilized to find the distribution of zero sequence current between return paths by assuming for convenience that \( I_0 = 1.0 + j0.0 \). However, results will yield constant currents flowing through the metallic return paths. A more rigorous method, which takes into account the sheath admittance to ground and grounding rod configurations, will be presented in Chapter 5.

b) Single conductor cables:

The impedance expressions applying to a three phase circuit of single conductor cables differ in some respects from those for three conductor cables. Here, the three sheaths are also replaced by an equivalent sheath. It is assumed that the three cables are perfectly transposed and their sheaths are solidly bonded and grounded. Zero sequence impedances are given as [10]:

\[
Z_c = R_c + 3R_g + j0.521 \frac{f}{60} \log \frac{D_g}{GMR_{3c}} \text{ ohm/phase/km} \quad (43)
\]

where \( R_c, R_g, D_g \) and \( GMR_{3c} \) are as defined in Eq. (36).
Similarly:

\[
Z_s = R_s + 3R_g + j0.521 \frac{f}{60} \log \frac{D_g}{GMR_{3s}} \text{ ohm/phase/km} \tag{44}
\]

and \(Z_m = 3R_g + j0.521 \frac{f}{60} \log \frac{D_g}{GMD_{cs}} \text{ ohm/phase/km} \tag{45}\)

where \(GMR_{3s} = \sqrt[3]{r_m GMD_{3c}^2}\), geometric mean radius of three sheaths in parallel,

\(GMD_{cs} = \sqrt[3]{r_m GMD_{3c}^2}\), geometric mean of all separations between sheaths and conductors.

The same discussions applied to three conductor cables in finding the circuit zero sequence impedance are valid also for the single conductor cables. The equivalent circuit in Figure 6.b represents the circuit of single conductor cables as well.

c) HPOF pipe type cables:

The positive or negative phase sequence impedances of a pipe type cable may be calculated by conventional formulae, suitably modified to include the effect of magnetic properties of the pipe. However, the calculation of zero sequence impedance is considerably more complicated, involving not only the permeability of the steel pipe, which depends upon the zero sequence current, but also the resistance of shield and skid wire assemblies.
Numerous tests have been made under both field and laboratory conditions to determine the zero sequence impedance and the effects of parameters involved. However, for a long time, there has been no experimental procedure nor empirical method which has been adopted uniformly.

In 1964, Neher indicated that the current density in the pipe is not radially uniform [14]. Although there is no radial current flow through the pipe wall, this results in differing voltages along the inner and outer surfaces of the pipe. Neher suggested that the impedance of the pipe return can be obtained by dividing the voltage along the inner surface of the pipe by the pipe current.

He assumed that the relative permeability $\mu_r$ is constant over the pipe section and the entire return current flows in the pipe. The current density $J_x$ at a radius $x$ from the center of the pipe is given by:

$$J_x = J_0 e^{-cx} e^{-jcx}$$

where $J_0$ is the current density on the inner surface of the pipe of thickness $t_p$ and of internal diameter $D_p$,

$$c = \frac{1}{\text{(skin depth)}} = \left(\frac{\pi f \mu_0 \mu_r}{\rho_p}\right)^{1/2}$$

and $\rho_p$ is the resistivity of the pipe in ohm-m. Since total current flowing in the pipe is $I_p$. 

43
\[ I_p = \pi D_p \int_0^{t_p} J_x \, dx \]

\[ = \pi D_p J_0 \left( \frac{1-j}{2c} \right) [1-e^{c t_p (1+j)}] \]

The voltage drop along the pipe per unit length,

\[ V_p = \rho_p J_0 [1-e^{-c t_p (1+j)}] \, \text{v/m} \]

Hence, the impedance of the pipe return path is:

\[ Z_p = \frac{V_p}{I_p} = \rho_p c \left( \frac{1+j}{\pi D_p} \right) \]

\[ = \left( \rho_p f \mu_0 \mu_r \right)^{1/2} \frac{1+j}{\pi D_p} \, \text{ohm/m} \]  \hspace{1cm} (46)

However, the permeability of the steel pipe, \( \mu_r \), will vary throughout the cycle because of the shape of the magnetization curve.

Once the system parameters including the pipe return impedances are known, the return currents in the pipe and the shields can be determined using circuit analysis.

### 2.7 Sequence Parameters

The use of single conductor cables for underground transmission and distribution requires a method of analysis which includes the effects of cable sheaths upon the positive and negative sequence network.
impedances. Circulating currents in the sheaths cannot be ignored in any of the sequence networks and the asymmetry in cable spacing substantially increases the effects.

For a three phase system of single conductor cables there will be a total of six circuits, each with ground return, three for the phase conductors and three for the sheaths. For other cases, there may be a different number of individual circuits. In the following analysis, a general method which can easily be applied to all cases is going to be developed. The method and discussions below are adapted from [5,19].

The basic assumptions are that the sheaths of the three cables are solidly bonded and grounded at both ends, and that the three cables under consideration are identical. The circuit can be in an asymmetrical configuration. The basic circuit arrangement is shown in Figure 8. Subscripts a,b,c denote the phase conductors, x,y,z denote their corresponding sheaths. Z with repeated subscripts denotes the self impedance of that conductor with ground return. The mutual impedances are shown by Z with two different subscripts. The impedance terms may be derived automatically from basic cable geometry and conductor spacings. Also, the following identities exist due to identical cables:

\[
\begin{align*}
Z_{aa} &= Z_{bb} = Z_{cc} = Z_a \\
Z_{xx} &= Z_{yy} = Z_{zz} = Z_x \\
Z_{ax} &= Z_{by} = Z_{cz} \\
Z_{ay} &= Z_{ab} = Z_{xb}
\end{align*}
\]
If the voltage drop equations are written in matrix form:

\[
\begin{bmatrix}
V_a - V_{a'} \\
V_b - V_{b'} \\
V_c - V_{c'} \\
0 \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
z_a & z_{ab} & z_{ac} & z_{ax} & z_{ab} & z_{ac} \\
z_{ab} & z_a & z_{bc} & z_{ab} & z_{ax} & z_{bc} \\
z_{ac} & z_{bc} & z_a & z_{ac} & z_{bc} & z_{ax} \\
z_{ax} & z_{ab} & z_{ac} & z_x & z_{ab} & z_{ac} \\
z_{ab} & z_{ax} & z_{bc} & z_{ab} & z_x & z_{bc} \\
z_{ac} & z_{bc} & z_{ax} & z_{ca} & z_{bc} & z_x
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c \\
I_x \\
I_y \\
I_z
\end{bmatrix}
\]

(47)
or in terms of block matrices:

\[
\begin{pmatrix}
\vec{V}_p \\
0
\end{pmatrix} =
\begin{pmatrix}
Z_C & Z_M^T \\
Z_M & Z_N
\end{pmatrix}
\begin{pmatrix}
\vec{I}_p \\
\vec{I}_s
\end{pmatrix}
\] (48)

where \(\vec{I}_p = \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix}\), \(\vec{I}_s = \begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix}\)

The definitions of the submatrices \(Z_C, Z_M, \text{ and } Z_N\) are obvious from Eq. (47). Since \(Z_N\) is always a square matrix and invertible, from Eq. (48)

\[
\vec{I}_s = - [Z_N]^{-1} [Z_M] \vec{I}_p
\]

And hence:

\[
\vec{V}_p = ([Z_C] - [Z_M^T] [Z_N]^{-1} [Z_N]) \vec{I}_p
\] (49)

The vector \(\vec{V}_p\) gives the set of voltage drops in the phase conductors when the set of currents \(\vec{I}_p\) is substituted. The matrices in the parantheses may be combined to produce a new matrix, \(Z_p\),

\[
[Z_p] = [Z_C] - [Z_M]^T [Z_N]^{-1} [Z_N]
\] (50)
and Eq. (49) can be written as:

\[ V_p = [Z_p] I_p \] (51)

Thus, it is possible to replace the system of Figure 8 with an equivalent one in which effects of neutral conductors have been incorporated in the equivalent phase impedance matrix \( Z_p \). If \( I_p \) consists of currents corresponding to one of the sequences, \( V_p \) will represent the voltage drops in each phase for that sequence of currents. The entries of \( Z_p \) are equivalent phase self and mutual impedances. To determine the equivalent sequence impedances, the positive, negative and zero sequence components of \( V_p \) must be determined when \( I_p \) consists of unit sequence currents. Defining \( T \) as the matrix of transformation from sequence voltages to phase voltages:

\[ \bar{V}_p = [T] \bar{V}_{012} \] (52)

where \([T] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\)

\[ \bar{V}_{012} = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \]
and $a = 120^\circ$, the matrix of sequence impedances, $Z_{SEQ}$, is obtained as:

$$[Z_{SEQ}] = [T]^{-1}[Z_p][T]$$  \hspace{1cm} (53)

where the entries of $Z_{SEQ}$ are the sequence impedances, that is, the voltage drop of the sequence indicated by the first subscript when unit currents of the sequence represented by the second subscript are introduced. $Z_{00}$, $Z_{11}$, and $Z_{22}$ are sequence self impedances.

The matrix computations to reach Eq. (53) are quite impractical to carry out algebraically unless important simplifications apply. When the network is asymmetrical, the mutual terms exist in $Z_{SEQ}$. These mutual terms are usually ignored if the unbalance is not substantial. For symmetrical circuits, the mutual terms in $Z_{SEQ}$ are all zero. Since,

$$Z_{ab} = Z_{bc} = Z_{ac}$$

the equations simplify and an algebraic solution may be obtained. The resulting equations are:
It also turns out that if $Z_{ab}$ in the equations above is set equal to

$$Z_{ab} = \frac{Z_{ab} + Z_{bc} + Z_{ca}}{3}$$

those equations are also valid for the asymmetrical case [19].

If the sheaths are open circuited, or are nonexistent, the neutral conductor currents $I_x$, $I_y$, and $I_z$ become zero. The sequence impedances become simply:

$$Z_{11} = Z_{aa} - Z_{ab}$$
$$Z_{22} = Z_{aa} - Z_{ab}$$
$$Z_{00} = Z_{aa} + 2Z_{ab}$$

When the network is asymmetrical but the sheaths are perfectly cross-bonded, the cables can be treated as symmetrically spaced with an axial separation of $\text{GMD}_{3c}$. Hence, the mutual sequence impedances will be zero. Then, if only positive sequence currents are flowing in the phase conductors, only positive sequence currents will be induced into the sheaths (i.e., no current will flow through ground).
The effects of parameters such as cable spacing and soil resistivity upon the sequence impedances can be investigated in detail by implementing the presented approach on a computer.

2.8 CABLE SHUNT ADMITTANCES

The capacitance of a pair of electrodes (i.e., conductor and sheath for cables), C, is defined as the electrical charge Q that will be set free at the electrode surfaces when a potential difference V is applied between them. It is determined by:

$$C = \frac{Q}{V} \text{ F}$$

where Q is in coulombs and V is in volts. The capacitance of a pair of electrodes depends on the geometry of the electrodes and the permittivity of the insulation between them. The capacitance of the conventional cable is significantly larger than that of the overhead lines. The capacitance of a coaxial conductor-sheath arrangement is:

$$C = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln \frac{2r_i}{d}} \text{ F/m}$$  \hspace{1cm} (54)

where $\varepsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$, $\varepsilon_r$ is the relative permittivity of the insulation, $r_i$ is the inner radius of the sheath in m, and d is the diameter of the conductor in m. This capacitance has two major implications for power cables:

a) The charging currents on AC are substantial and could, for reasonable lengths, be equal to the rated current.
b) Voltage control problems may result especially in urban areas at time of light load when the voltage rises are considerable. The charging current per meter length of a cable fed from one end is:

\[ I_C = \frac{V}{\sqrt{3}} \cdot \omega C \quad \text{A/m} \]

where \( V \) is the rms value of the line to line voltage applied. When the length of the cable is such that the capacitive current at the sending end for no-load equals the rated current, the length is termed critical. At critical length, there is no possibility of passing any load current through the cable without excessive temperature rise. For example, the critical length of a 345 kV HPOF cable with \( \varepsilon_r = 3.5 \) is almost 42 km.

The total three phase reactive power is given by:

\[ Q_{3\phi} = \sqrt{3} \cdot V \cdot I_C = \frac{4\pi^2 V^2 \varepsilon_0 \varepsilon_r 10^{-6}}{\ln \frac{2r_j}{d}} \quad \text{MVAR/m} \quad (55) \]

The reactive power can be decreased by a reduction in \( \varepsilon_r \), e.g., instead of paper-oil insulation (\( \varepsilon_r = 3.5 \)), polyethylene (\( \varepsilon_r = 2.2 \)) or gas (\( \varepsilon_r = 1.0 \)) insulation can be utilized. Decreasing the capacitance by increasing the thickness of insulation is costly and not preferable.

To overcome the problem of high cable capacitances, EHV cable circuits have shunt inductive reactors connected to reduce the overall capacitance. In self-contained systems, this reactive compensation increases the cost by roughly 20% [5,p.20].
The positive, negative and zero sequence shunt capacitances of conventional cables are all equal and given by Eq. (54). Three conductor belted cables without any conductor shielding, however, have somewhat different sequence capacitances [10, p. 77].

The calculation of cable insulation resistance is difficult because the properties of the insulation are generally predictable only within a wide range. The equations below are therefore dependent upon an accurate knowledge of the insulation power factor. For single conductor and three conductor shielded cables, the insulation resistance is given by:

\[ R_1 = R_2 = R_0 = \frac{\rho_d}{2\pi} \ln \left( \frac{2r_i}{d} \right) \cdot 10^{-5} \text{ ohm-km} \]

where \( \rho_d \) is the resistivity of the insulation in ohm-cm. The energy losses occurring in the insulation of cables are mainly due to leakage and dielectric hysteresis effects. The former loss, \( P_L \), is caused by the conduction current and is independent of frequency. The hysteresis loss, \( P_h \), on the other hand occurs only with alternating voltages, and under normal operating conditions is much larger than the leakage losses. In an ideal dielectric, both losses are proportional to the square of the impressed voltage. The per phase dielectric losses can be calculated as the product of the voltage and the component of the charging current in phase with the voltage. The following expression for dielectric losses can be obtained with the help of Figure 9:

\[ P_d = P_L + P_h = \frac{V}{3} I_c \cos \phi_d = \frac{W}{3} CV^2 \tan \delta \text{ W/m} \]
where $V$ is the line to line rms voltage, $\cos \phi_d$ is the dielectric power factor, and $\delta$ is the dielectric loss angle and is given by:

$$\delta = 90^\circ - \phi_d$$

An average value of $\cos \phi_d$ for modern cables is 0.003-0.004, and the dielectric losses of low and medium voltage cables are negligible. The dielectric power factor varies with temperature and voltage.

Finally, the electric stress between the conductor and the sheath will be presented. The stress at a point $x$ meters away from the conductor axis is:
\[ E = \frac{V}{x \ln \left( \frac{2r_1}{d} \right)} \text{ V/m.} \]

Hence, the stress is maximum at the surface of the conductor and is further increased by the effects of conductor stranding.
CHAPTER III

SHIELDING AND INTERFERENCE ASPECTS OF CABLES

3.1 INTRODUCTION

The electrical parameters and characteristics of conventional power cables have been examined in detail in Chapter 2. Effects of shielding and bonding, and sheath currents and voltages were also introduced. This chapter will investigate the coupling phenomenon, the shielding effectiveness, and the power frequency interference aspects of underground cables. These subjects are closely related, and form an area of mutual concern to the electric power and telephone industries. The main reason for the concern is that telephone lines frequently experience considerable interference from power lines which may lead to operating problems. Despite corrective measures to eliminate the problem, the following recent trends in these industries have increased the susceptibility of telephone lines to interference:

1) Higher capacity power systems with increased ground fault currents,
2) Increased use of common corridors (or trenches) by the utilities due to ecological restrictions,
3) The tendency for conversion of delta connected three phase power systems to multi-grounded neutral systems, which have a ground return path,
iv) The presence of harmonic currents in the power system due to the nonlinear properties of transformers, loads, and rectifier circuits,

v) Increased range of telephone loops and increased development of telephone plants in rural communities where single phase power systems are likely to be present.

The factors involved in the interaction between underground power systems and telecommunication circuits can be organized into three principle categories: influence (power system spectral components), coupling, and susceptibility (the reaction of the telephone system, i.e., noise, malfunction) [20]. A discussion of susceptibility is beyond the scope of this study, and only influence and coupling will be reviewed below.

Although we will concentrate on the interference on a buried communication cable due to underground cable circuits, the following discussions can be generalized to cover overhead transmission lines as well. Throughout this chapter, the term 'power line' will refer to both overhead and underground power systems.

3.2 INFLUENCE AND COUPLING

Influence is the group of factors in the power system which has the capacity to cause an effect on the interference phenomenon. Some of these factors are the geometry and the grounding configurations of the power line, harmonic content and magnitude of currents, and the number of phases.

Under certain conditions, voltages sufficient to cause high noise levels may be induced into the telephone lines when they run parallel
to power lines. This may be caused by electromagnetic and electrostatic unbalance in power lines, especially if harmonics are present in the phase currents. When the phase currents are balanced, the uni-grounded Y-system and the delta system do not contribute significantly to the interference under normal operating conditions. The three phase multi-grounded neutral systems, on the other hand, produce induction in both balanced and unbalanced conditions due to the fact that the ground provides an alternate path for the flow of current. The odd triple harmonics that exist in the phase currents, or large zero sequence currents produced by ground faults, combine directly in the neutral and may create interference problems. The division of the fault current between the neutral and ground return circuits depends on the power system characteristics, grounding and soil resistivity. The following chapters will present a comprehensive analysis of zero sequence current distribution along underground cables.

Single phase power systems with ground return are most undesirable from the point of view of interference because the harmonic cancellation which takes place in three phase systems is absent.

The coupling between two circuits may be in one of the following forms: electric induction, magnetic induction or conductive coupling. These three kinds of coupling may occur simultaneously, but they can be separated experimentally. It is thus justifiable to study them separately in order to simplify the analysis.

Electric induction is normally associated with the voltage on a power system which is coupled to the communication line (or cable) by the capacitances between the two systems. If the telephone plant utilizes shielded cables that are effectively grounded, the shielding
has the effect of discharging the capacitance. Hence the line pairs within the shield are almost unaffected by the capacitive coupling. This form of coupling does not contribute significantly to the interference problems in today's telephone circuits [20].

Magnetic induction is associated with the current flow in the power system and is a primary source of concern. The alternating magnetic field caused by the current flow will produce induced voltages along nearby conductors. These voltages will be distributed along the length of the exposed line. In the case of a shielded cable, both the conductor and its shield are exposed to the flux generated by a power line and will experience an induced voltage. If the shield is grounded at more than one point, a counteracting current will flow and induce an opposing voltage in the cable. This cancellation of some portion of the induced voltage due to the nearby wires and sheaths is commonly called "shielding".

In power systems with a multi-grounded neutral, the ground is used as a return path parallel to the neutral conductor and reduces the current returning through the neutral. This decreases the shielding effect of the neutral conductor and thus results in a higher net field acting on a telephone cable close to the power line.

The third form of coupling is conductive coupling which depends on the current flow in the ground. The portions of the fault current in the power system, flowing into the ground through grounding structures, raise the surrounding ground to a high potential with respect to remote ground. The voltage drop in the ground due to the diffused currents over the large volume may be measured on the surface of the ground. This potential difference is called a ground potential rise (GPR).
Since telephone lines are also referenced to ground, the potential differences from power system ground return currents are coupled to the telephone circuits. The magnitude of the GPR depends mainly upon the grounding resistance of the electrodes, the magnitude and location of the fault, and the presence or absence of other metallic return circuits or grounding structures in the area. The GPR in the vicinity of a single electrode can be calculated if the electrode has a simple form. As an example, the GPR of a hemispherical electrode of radius r meters, buried in soil of resistivity \( \rho \) ohm-m, with a current I flowing into ground is given in terms of I and the grounding resistance, R, as:

\[
GPR = IR = \frac{I\rho}{2\pi r}
\]  

Similarly, a point on the surface of the ground, d meters from the mid-point of the hemisphere has \( \frac{I\rho}{2\pi d} \) volts with reference to the remote ground. Hence, the potential of the ground around the grounding structures with respect to the remote ground falls off with distance. There are several methods in the literature to determine the grounding resistances of more complicated grounding electrodes [21,22,23]. However, GPR coupling does not contribute significantly to the induced voltage and can be neglected in practice except in some cases where the telephone cable shields are connected to the substation ground.

The study of interference problems requires the calculation of mutual inductance between two single conductor circuits, both with ground return. The following assumptions usually are made in
determining the self and mutual power frequency impedances in ground return circuits:

i. The cables are laid in parallel with separations much smaller than the length of the circuits.

ii. The soil in which the circuits are laid is homogeneous.

iii. The capacitances are considered negligible.

iv. The impedance expressions are based on Carson's solution of the ideal case of an infinite line carrying a constant current which returns through a homogeneous semi-infinite ground [15,16]. Carson's equations are simplified for power frequency applications. Different simplified versions of Carson's equations based on different assumptions have been suggested in the literature [20,23,24,25]. Which version to use depends on the accuracy desired and the system configuration. Appendix A reviews some of these versions.

The analysis of the basic concepts of the coupling phenomenon can be done using the simplified circuit representation shown in Figure 10. The circuit model consists of a power cable with ground and metallic return and a telephone wire with ground return. The transverse admittance parameters and capacitive coupling are assumed to be negligible. The telephone wire is assumed to be open circuited at one end, and the shielding effect of the telephone cable sheath is neglected.

Before investigating the coupling mechanisms represented in the model, a new notation which will be valid throughout this chapter is going to be introduced. Assume that the subscript "1" always identifies the disturbing (inducing) line whose current causes induction into the disturbed circuit with subscript "4". Subscripts "2" and "3" simply
denote the sheaths (or shielding conductors) in general. Generally, "2" will denote the sheath of the conductor "1".

The definitions of the parameters in Figure 10 are as follows:

- $Z_{ii}$, $i = 1, 2, 4$, is the self impedance of the $i$th conductor,
- $Z_{ij}$, $i, j = 1, 2, 4$, is the mutual impedance between the $i$th and $j$th conductors.

Figure 10 Simplified coupling model [20].
$Z_L$ is the load impedance,

$R_1, R_{22}$ and $R_{44}$ are the resistances to remote ground at terminals $c, d,$ and $f,$ respectively,

$R_{24} = \frac{\rho}{2\pi x_{24}}$ is the mutual resistance of the electrodes at terminals $d$ and $f$ with a distance $x_{24}$ between them. The sheath of the telephone cable has been neglected in the model.

The open circuit voltage to ground, $V_{OC},$ of the telephone wire can be written as the resultant of the two dominant coupling mechanisms;

$$V_{OC} = V_{GPR} + V_p$$ (57)

where $V_{GPR}$ is the ground potential rise at the grounding terminal of the telephone system due to the ground return current in the power system,

$$V_{GPR} = (I_1 + I_2) R_{24}$$ (58)

and $V_p$ is the voltage induced on the telephone wire itself due to the inductive coupling,

$$V_p = Z_{14}I_1 + Z_{24}I_2$$ (59)

$I_2$ can be expressed in terms of $I_1$ by writing the voltage drop equation for the neutral conductor,

$$(I_1 + I_2) (R_1 + R_{22}) + Z_{22}I_2 + Z_{12}I_1 = 0$$
Hence;

\[ I_2 = - \frac{R_1 + R_{22} + Z_{12}}{R_1 + R_{22} + Z_{22}} I_1 \]  

(60)

An effective mutual impedance expression which relates the total induced voltage in the disturbed circuit to the disturbing power system phase current can now be developed. Using

\[ V_{oc} = (I_1 + I_2)R_{24} + Z_{14}I_1 + Z_{24}I_2 \]  

(61)

and Eq. (60) yields:

\[ Z_m = \frac{V_{oc}}{I_1} = Z_{14} + R_{24} - (R_{24} + Z_{24}) \frac{R_1 + R_{22} + Z_{12}}{R_1 + R_{22} + Z_{22}} \]

\[ = Z_{14} + R_{24} \frac{Z_{22} - Z_{12}}{R_1 + R_{22} + Z_{22}} - Z_{24} \frac{R_1 + R_{22} + Z_{12}}{R_1 + R_{22} + Z_{22}} \]  

(62)

where \( Z_m \) is the effective mutual impedance.

Finally, the following two cases can be deduced from Eq. (62):

1. If the sheath ("2") is open, \( Z_{22} = 0 \) and \( I_2 = 0 \). Therefore,
\[ Z_m = Z_{14} + R_{24} \]

2. If the power system is not grounded (i.e., \( R_1 \) and \( R_{22} = 0 \)), there will be no ground return current component. Hence,
\[ Z_m = Z_{14} - Z_{24} \]
3.3 SHielding FACTOR

Inductive coupling, as explained in the previous section, is the major concern in interference studies. Shielding, a related concept, will be investigated in this section.

The main function of shielding is to reduce the longitudinally induced voltage, $V_p$, on a disturbed circuit by decreasing the induction. The primary voltage, $V_p$, is the voltage on the disturbed circuit by the current in the disturbing circuit when no shielding is present. The resultant voltage, $V_r$, is the voltage induced on the disturbed circuit with the shielding interaction. A measure of the shielding effectiveness is the shielding factor, $n$. It is defined as the ratio of the resultant voltage $V_r$ to the primary voltage $V_p$ for a constant value of disturbing current. Thus, a shielding factor of 1.0 represents no shielding, and shielding factors very close to zero represent very effective shielding. It is desirable to minimize the shielding factor.

The simplest form of inductive shielding is illustrated by the classical shielding model in Figure 11 [20]. The model considers a cable shield (conductor "3") and an enclosed telephone wire-pair (conductor "4") subjected to magnetic induction from a nearby parallel power line. The shield admittances to ground are taken to be zero, since they are shunted by the low-resistance grounding terminations $R_a$ and $R_b$. The transverse coupling is assumed to negligible. $V_p$ is the primary voltage induced along the shield and the shielded wire by the longitudinal coupling. It is also assumed that the shield and the shielded wire are physically close to each other in comparison to the distance from other power line conductors. Hence, the primary voltages induced on them will be accepted as identical.
The mutual impedance between the cable shield and the enclosed wire-pair is defined as $Z_{34}$. This mutual impedance gives rise to a shielding voltage, $V_s$, on the wire-pair due to the flow of the current on the shield. The resultant voltage, $V_r$, on the wire-pair is the phasor difference of $V_p$ and $V_s$. Since zero current flow is assumed on the wire-pair, the transformer action of $Z_{34}$ results in no comparable shielding voltage being induced onto the shield from the pair.

Figure 11 Classical inductive shielding circuit.
In regard to the shield circuit, the grounding impedances, $R_a$ and $R_b$, at the terminal points of the shield are assumed to be resistive. The shield current can be written as:

$$I_3 = \frac{-V_p}{R_T + R_{33} + j\omega L_{33}}$$

(63)

where $R_T = R_a + R_b$.

and $(R_{33} + j\omega L_{33})$ is the self impedance of the shield. The shielding voltage is:

$$V_s = -Z_{34}I_3$$

$$= Z_{34} \frac{V_p}{R_T + R_{33} + j\omega L_{33}}$$

(64)

The concern is to minimize $V_r$ on the enclosed wire-pair. Solving for $V_r$ yields:

$$V_r = V_p - V_s$$

$$= V_p \left(1 - \frac{Z_{34}}{R_T + R_{33} + j\omega L_{33}} \right)$$

(65)

From the definition of the shielding factor, $n$,

$$n = \left| \frac{V_r}{V_p} \right| = \left| 1 - \frac{j\omega L_{33}}{R_T + R_{33} + j\omega L_{33}} \right|$$

(66)

can be found by assuming $Z_{34} = j\omega L_{34}$. In a more rigorous treatment, a mutual resistance term can be included as $Z_{34} = R_{34} + j\omega L_{34}$ where the
R\textsubscript{34} term arises from dissipative coupling associated with the longitudinal mutual impedance.

Equation (66) shows that if the grounding resistances and R\textsubscript{33}, the DC resistance of the shield circuit, can be decreased, the shielding performance improves. At high frequencies, the shielding circuit impedance is dominated by the inductive portion and hence the shielding factor would approach

$$\eta = 1 - \frac{L_{34}}{L_{33}}$$

The effect of soil resistivity on the shielding factor is hard to generalize because both Z\textsubscript{34} and Z\textsubscript{44} are influenced.

The sheath of a cable can be the most efficient shielding conductor because of its small separation from the disturbed lines. The mutual inductance between the sheath and the core is practically equal to the self inductance of the sheath, both circuits having a ground return. With this additional assumption, L\textsubscript{33} = L\textsubscript{34}, the shielding factor determined in Eq. (66) becomes:

$$n = \left| \frac{R_T + R_{33}}{R_T + R_{33} + j\omega L_{33}} \right|$$

The induced voltage in the cable conductor is:

$$|V_R| = \left| n \cdot j\omega L_{14} I_1 \right| = \left| \frac{j\omega L_{13} I_1}{R_T + R_{33} + j\omega L_{33}} (R_T + R_{33}) \right|$$

$$= \left| I_3 \right| (R_T + R_{33})$$
This means that the resultant induced voltage in the cable conductor is equal to the resistive voltage drop on the sheath circuit.

A more accurate analysis than that leading to Eq. (68) shows that the induced e.m.f. per unit length is equal to the product of the current density on the inner surface of the sheath and the DC resistivity of the sheath circuit. This observation is of special importance at high frequencies. Because of the skin effect, the induced sheath current flows mainly on the outer surface, and the interference becomes very insignificant [25, p.57]. In addition to the skin effect, the classical shielding model as applied to shielded cables neglects the magnetic field ducting and fringing effects, and the intrinsic field penetration/coupling/mitigation mechanisms. However, those mechanisms are out of the scope of this study and will not be covered.

In a steel armored cable, the inductance $L_{33}$ will be increased depending on the type of the armoring. Furthermore, the losses in the armor must be taken into account. The permeability of the armoring depends on the magnetic induction and, thus, on the sheath current. Consequently, the shielding factor depends upon the induced e.m.f. It is high (i.e., undesirable) at low induced e.m.f., decreases with increased induction, passes a minimum and increases again when the steel is saturated [25].

The discussion above is also true for the shielding effect of the pipe type cable. It is found that pipe type cable is much more effective in reducing the electromagnetic induction than the aluminum sheathed cable [26].
In practice, auxiliary cables often are laid in parallel and adjacent to underground power cables. They may be subjected to longitudinal induced voltages when a ground fault occurs in the power circuit. The insulation of the auxiliary cable is governed by this induced voltage, which in turn depends on the efficiency of the shielding employed. To determine the induced voltages on auxiliary cables or the shielding factor when there are more than one shielding conductors, each grounded at more than one point, the best approach is to write the loop equations for each circuit involving the ground return path. The current distribution between the parallel shielding conductors and the ground return path can be found using the matrix and the geometric mean distance approach presented in Chapter 2. If there are several sections along the route such that the exposure of the auxiliary cable if different, each section can be treated separately and the total induced voltage for the whole route is calculated as the sum of the individual induced voltages. References [27] and [28] present two studies on induced voltages in auxiliary cable systems. It was found that the variations of inducing current, number of grounding points, grounding resistance and soil resistivity have insignificant effects on the resultant shielding factor within the values used. The shielding factors for the auxiliary cables installed along pipe type cables were better than those along oil-filled cables due to the very low zero sequence impedance of the steel pipes.

Before proposing a method which uses a generalized matrix approach to determine the shielding factor when there are multiple shielding conductors, a simpler situation, with only two shielding conductors, will
be presented first. Figure 12 shows the four conductors considered, with the notations "1" and "4" indicating inducing and disturbed conductors, respectively, and "2" and "3" denoting shielding conductors. The basic assumptions in the approach are:

1) The self and mutual impedances of each circuit with ground return path are known,

2) Soil is homogeneous,

3) The cables are laid in parallel with separations much smaller than the length of the system,

4) The current in the disturbing conductor, $I_1$, is known and it is assumed that $I_1$ is not altered by the currents in the shielding conductors. This assumption is not always valid because the reaction of the induced current reduces the effective reactance and increases the effective resistance of circuit 1,

5) No conductors, pipes or steel structures, other than those considered, exist in the vicinity of the auxiliary cable,

6) The effect of $I_4$ on the other circuit is neglected.
The following loop equations can be written for each conductor except the inducing conductor:

\[
\begin{bmatrix}
0 \\
0 \\
-v_r
\end{bmatrix} =
\begin{bmatrix}
Z_{12} & Z_{22} & Z_{23} \\
Z_{13} & Z_{23} & Z_{33} \\
Z_{14} & Z_{24} & Z_{34}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

(69)
where the entries of the impedance matrix (i.e., $Z_{ij}$, $i=1,2,3$, $j=2,3,4$) are self and mutual impedances per unit length with the ground return path, and $V_r$ is the resultant voltage on the disturbed conductor. Also note that $Z_{ij} = Z_{ji}$ for passive networks.

The currents in the shielding conductors, $I_2$ and $I_3$ can be written in terms of $I_1$ from Eq. (69):

$$\begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = -\begin{bmatrix} Z_{22} & Z_{23} \\ Z_{23} & Z_{33} \end{bmatrix}^{-1} \begin{bmatrix} Z_{12} \\ Z_{13} \end{bmatrix} I_1$$

(70)

Considering the third equation in Eq. (69):

$$V_r = -Z_{14}I_1 - [Z_{24} \quad Z_{34}] \begin{bmatrix} I_2 \\ I_3 \end{bmatrix}$$

$$= -I_1 \left( Z_{14} - [Z_{24} \quad Z_{34}] \begin{bmatrix} Z_{22} & Z_{23} \\ Z_{23} & Z_{33} \end{bmatrix}^{-1} \begin{bmatrix} Z_{12} \\ Z_{13} \end{bmatrix} \right)$$

$$= -Z_{14}I_1 \left( 1 - \frac{Z_{24}(Z_{12}Z_{33} - Z_{13}Z_{23}) + Z_{34}(Z_{13}Z_{22} - Z_{12}Z_{33})}{Z_{14}(Z_{22}Z_{33} - Z_{23})^2} \right)$$

(71)

Without circuits 2 and 3, $I_1$ would produce the e.m.f. of

$$V_p = -Z_{14}I_1$$

(72)

Hence, the shielding factor, the modulus of the ratio $V_r/V_p$ is simply equal to the term in brackets in Eq. (71).
If there is only one shielding conductor (i.e., conductor 2), then the shielding factor becomes:

$$\eta = \left| 1 - \frac{Z_{24}Z_{12}}{Z_{14}Z_{22}} \right|$$

Equation (71) can be further simplified if conductor 2 is near to conductor 1, and 3 is near to 4. Then, $Z_{13} = Z_{14} = Z_{23} = Z_{24}$. Hence, the shielding factor in this case is:

$$\eta = \left| \frac{(Z_{22}-Z_{12})(Z_{33}-Z_{34})}{Z_{22}Z_{33}-Z_{23}^2} \right|$$

Furthermore, if the separation between 1 and 4 is large enough to render $Z_{23}^2$ negligible compared to $Z_{22}Z_{33}$ (i.e., the reaction between conductors 2 and 3 is neglected),

$$\eta = \left| \frac{Z_{22}-Z_{12}}{Z_{22}} \cdot \frac{Z_{33}-Z_{34}}{Z_{33}} \right| = \eta_2 \cdot \eta_3$$

where $\eta_2$ and $\eta_3$ are the individual shielding factors of conductors 2 and 3, respectively [25]. Conventionally, the resultant shielding factor of multiple shielding conductors is represented by the product of the shielding factors of the individual conductors. The results found this way are often too optimistic because they neglect the mutual interactions between shielding conductors. On the other hand, the exact expressions for the induced voltage are too complicated and impractical, especially if there are more than two shielding conductors.
Sakai [29] presented a method to simplify the shielding factor expressions, and still obtain more accurate values for the shielding factor than by multiplying the individual shielding factors. Assuming that $Z_{12} = Z_{13} = Z_{14}$ and $Z_{24} = Z_{34} = Z_{23}$ for the two shielding conductor case, and using

$$n_2 = 1 - \frac{Z_{24}Z_{12}}{Z_{14}Z_{22}}$$

$$n_3 = 1 - \frac{Z_{34}Z_{13}}{Z_{14}Z_{33}}$$

he obtains:

$$n = \frac{n_2 n_3}{n_2 + n_3 - n_2 n_3}$$

He also extends this result to $n$ shielding conductors. However, the assumptions he makes are very restrictive and not always valid.

What follows is a generalized matrix method which can be easily implemented using a computer. With this method, it is not necessary to find the current distribution between shielding conductors. Assume that the subscripts 2, 3, 5, ..., $n$ denote the $(n-2)$ shielding conductors which are perfectly grounded. Then, the following matrix equation can be written for all circuits involved except the inducing conductor:
where the \((n-1)\) entries of the current vector are the conductor currents except the disturbed conductor current. The disturbed conductor is assumed to be open circuited (i.e., \(I_4 = 0\)). The entries of the impedance matrix \(Z_{ij}\), \(i=2,3,4,...,n\), \(j=1,2,3,5,...,n\), denote self and mutual impedances with ground return of the conductors specified by the subscripts. From the first \((n-2)\) equations in (75), the current vector \([I_2\ I_3\ I_5\ ...\ I_n]\) can be solved in terms of \(I_1\). Considering the last equation:

\[
V_r = -Z_{41}I_1 - [Z_{42} Z_{43} ... Z_{4n}] \begin{bmatrix} I_2 \\ I_3 \\ I_5 \\ \vdots \\ I_n \end{bmatrix}
\]

Hence,

\[
V_r = -Z_{41}I_1 + [Z_{42} Z_{43} ... Z_{4n}] \begin{bmatrix} -1 \\ Z_{22} Z_{23} ... Z_{2n} \\ Z_{32} Z_{33} ... Z_{3n} \\ \vdots \\ Z_{n2} \cdots Z_{nn} \end{bmatrix} \begin{bmatrix} Z_{21} \\ Z_{31} \\ \vdots \\ Z_{n1} \end{bmatrix}
\]

\[(76)\]
Since:
\[ V_p = -Z_{41} I_1 \]  

the shielding factor can be written as:
\[
\eta = 1 - \frac{1}{Z_{41}} [Z_{12} \ Z_{43} \ ... \ Z_{4n}] \begin{bmatrix}
Z_{22} & Z_{23} & \ldots & Z_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{n2} & Z_{n3} & \ldots & Z_{nn}
\end{bmatrix}^{-1} \begin{bmatrix}
Z_{21} \\
\vdots \\
Z_{n1}
\end{bmatrix}
\]  

The suggested method needs the inversion of the impedance matrix of the shielding conductors and two array multiplications. The impedances and hence the matrices can easily be obtained using the building block approach explained in Chapter 2 once the cable types, geometry and soil resistivity are known.

3.4 GROUNDING

The importance of effective grounding in modern power systems is continuously increasing with the increases in both voltage and short circuit levels. The metallic return circuits (i.e., neutral conductors) of the power systems are connected to the ground with grounding structures at specific locations. Hence, the ground is used as an alternate return path for power frequency fault and unbalanced currents, as well as for lightning and switching surge currents. Various grounding configurations are used in practice. Reference [21] presents a detailed study of transmission line grounding configurations and performances. The preferable grounding configuration depends on several factors, such
as resistance to ground, ease of installation and available space, possibility of high potential on the ground, and high potential gradients in the earth near ground. The distribution of the fault current between the sheath of a cable, and the grounding structures and ground will be the main topic of the following chapters. The effect of grounding schemes on the current distribution along an underground cable will also be covered.

The effect of the neutral conductor grounding impedances on shielding effectiveness has been demonstrated with the use of the classical shielding model. From a shielding effectiveness viewpoint, low grounding impedances are found to be beneficial.

The shielding conductors are often grounded at other locations as well as at the terminal points. It is preferable to ground such conductors at several not too distant points distributed fairly uniformly over its length. The main reasons are the lack of knowledge of the exposure site and the possibility of supply of fault current from two directions.

3.5 **INDUCTIVE COORDINATION**

The principal factors that influence the coupling mechanism between power lines and telecommunication circuits are [20]:

1. The influence of the power system, especially the magnitude and frequency composition of balanced and zero sequence currents,
2. Length, separation and orientation of the telephone line exposure relative to the disturbing power line,
3. Soil resistivity,
4. Shielding effectiveness of the communication cable shield, which depends on the sheath continuity and low resistance grounding on the sheath,
5. The performance of the grounding structures applied to the neutral conductors, and existence of bonds between cable sheath and power system neutral,
6. Existence of other low impedance metallic return circuits in the vicinity of either power or telephone wires.

To limit the influence and susceptibility, both power system and telephone companies are pursuing common inductive coordination techniques at the planning and operating stages. From the power system viewpoint, the possible coordination methods are:

1. Power circuits and associated apparatus should be designed, built and maintained to minimize the harmonics and zero sequence currents. Special attention is to be given to the location of power factor correction capacitors which present a low impedance path for higher harmonics.
2. Underground neutral conductors (sheaths) and overhead ground wires should be designed and built to maximize their shielding effectiveness. The continuity of these conductors should be maintained.
3. The transient disturbances are minimized by protection and switching devices. Some special devices such as booster transformers, filters, and phase-reversal transformers can be applied.
4. At the planning stage, systems that have less inductive interference, such as pipe type cables, concentric neutral cables, or a higher voltage system, should be considered.

5. Resonant shunts and filters should be applied to loads that contribute significantly to harmonic generation.

On the other hand, the telephone companies can take the following measures to reduce the interference from power lines to their circuits [20]:

1. Increased separation between power lines and telephone lines by using different rights-of-way,

2. Improved shielding, adequate sheath grounding and proper conductor configurations,

3. Utilizing special devices to minimize the generation of harmonics or other malfunctions which result from the influence of power systems, applying devices such as neutralizing transformers, ringer isolators, chokes, and drainage coils,

4. Utilizing additional shielding such as iron conduits or tape armor on cables,

5. Designing and using systems that have improved immunity to inductive exposures, e.g., a carrier system instead of a voice-frequency trunk system.

To summarize, the measures to limit interference require the cooperation and coordination of the power and telephone industries.
4.1 REVIEW OF PREVIOUS WORK

In conventional short circuit studies, the ground return impedance is assumed to be zero. In cases where this assumption is not realistic, it is possible to include the return path impedance in the equivalent power system circuit. Although this procedure considers the influence of the ground return impedance on the magnitude of the fault current, the division of the fault current at the faulted structure between ground and the metallic return conductors of the circuit remains unknown. Hence alternative methods must be used.

Methods proposed in the technical literature to determine the fault current distribution along transmission lines can be classified into two main categories, namely, the constant line parameter and varying line parameter methods. The former methods assume constant parameters along the transmission lines, whereas the latter methods allow the parameters to change from span to span. The following sections will investigate in detail these two categories and a recent study on the simulation of underground cables as well.

4.1.1 Constant Line Parameter Methods

A classical example of this approach is the study presented by
DeSieno et al. [30]. Their method uses uniformly distributed parameters and is valid for long transmission lines. It is based on forming differential equations for the currents and voltages along transmission line ground wires. This is done by treating the finite conductance to ground at each grounding point as a uniformly distributed conductance between the ground wire and ground. Figure 13 represents a three phase single-ground-wire transmission line of length \( l \) with the following parameters:

- \( Z_w \): Self impedance of the ground wire in ohms per unit length,
- \( Z_m \): Average mutual impedance between a phase conductor and the ground wire in ohms per unit length,
- \( R_t \): Average tower footing ground resistance in ohms,
- \( N \): Number of towers per unit length,
- \( \Delta x \): Average distance between towers,
- \( V(x) \) and \( I(x) \): Ground wire voltage and current, respectively, at a distance \( x \),
- \( I_e \): Current into ground at the faulted tower.

Assuming that \( \Delta x \) is very small compared to the length of the transmission line, the following differential equations relating the voltage and current of the ground wire can be written:

\[
\frac{dl(x)}{dx} = \lim_{\Delta x \to 0} \frac{I(x+\Delta x)-I(x)}{\Delta x} = \frac{V(x)}{R_t \Delta x} \tag{79}
\]

\[
\frac{dV(x)}{dx} = \lim_{\Delta x \to 0} \frac{V(x+\Delta x)-V(x)}{\Delta x} = \frac{I(x)Z_w-I_fZ_m}{l} \tag{80}
\]
where \( I_f \) is the fault current in the phase conductor.

From Eqs. (79) and (80), the following second order differential equations are obtained:

\[
\frac{d^2 I(x)}{dx^2} - \alpha^2 I(x) = -\alpha^2 \frac{Z_m}{Z_w} I_f' \tag{81}
\]

\[
\frac{d^2 V(x)}{dx^2} - \alpha^2 V(x) = 0 \tag{82}
\]

where \( \alpha^2 = \frac{N Z_w}{R_t} \tag{83} \)

The general solutions of (81) and (82), respectively, are:
The A' and B' are arbitrary constants to be determined from boundary conditions which are:

At the faulted tower: \( V'(d) = R_t I_e \)  \( (86) \)
At the left terminal: \( V'(0) = 0 \)  \( (87) \)

Solution of the boundary conditions \( (86) \) and \( (87) \) yields:

\[
A' = B' = \frac{N I_e}{2a \sinh(\alpha d)}
\]

Thus, inserting A' and B' in Eqs. \( (84) \) and \( (85) \) results in:

\[
V'(x) = R_t I_e \frac{\sinh(\alpha x)}{\sinh(\alpha d)}
\]  \( (88) \)

\[
I'(x) = \frac{N I_e}{\alpha} \frac{\cosh(\alpha x)}{\sinh(\alpha d)} + \frac{Z_m}{Z_w} I'_f
\]  \( (89) \)

The same approach can be applied to the right side of the fault location to solve for \( V''(x) \) and \( I''(x) \). Hence, at the faulted tower, the following node equation can be written:

\[
I_f = I'_f + I''_f = I'(d) + I''(l-d) + I_e
\]  \( (90) \)
where \( I_f \) if the total line to ground fault current.

From this equation, the current flowing to ground in the faulted tower, \( I_e \), can be determined as:

\[
I_e = \frac{(1-Z_m/Z_w) I_f}{1 + \frac{N}{\alpha} \left\{ \frac{1}{\tanh(\alpha d)} + \frac{1}{\tanh[\alpha(z-d)\alpha]} \right\}}
\]  

(91)

and it can be eliminated from Eqs. (88) and (89). Since the method assumes constant line parameters and very short span length, the results are generally inaccurate for short lines or when the faulted structure or an adjacent structure had a ground resistance significantly different from the average value.

4.1.2 Varying Line Parameter Methods

Some of the better known methods in this category are Sebo's matrix and equivalent star methods [31], and Dawalibi's single and double sided elimination methods [32]. Very recently another computer implemented method employing driving point impedance matrices was suggested by Gooi [33]. All of these methods allow varying conductor and ground wire impedances, soil resistivity and tower footing resistance along the transmission line. Since these parameters change from span to span in the real power systems, the varying line parameter methods are more accurate than the constant line parameter methods. The common points of the methods mentioned above are:

i) Impedances are considered as lumped parameters in each span of the transmission line,

ii) They are valid for one or two generating sources.
(iii) They are mainly designed to determine the fault current distribution along transmission lines and are only applicable to underground cables with insulated sheaths.

(iv) Shunt capacitive reactance is neglected.

In the following sections, Sebo's equivalent star method and Gooi's driving point impedance method will be discussed in detail.

a) Sebo's Equivalent Star Method [31]

Figure 14 represents the equivalent circuit of a transmission line energized by two source stations. The impedance of the ground return path, \( Z_g \), which is computed by means of the simplified Carson equations [10], is taken into account as an auxiliary quantity. \( Z_{C1} \) and \( Z_{W1} \) in the figure represent the zero sequence self impedance of the equivalent phase conductor and the ground wire in the 1st span, respectively. \( Z_{M1} \) is the zero sequence mutual impedance between the phase conductor and the ground wire in the ith span. \( R_t \), \( Z_s \) and \( Z_{st} \) are the tower footing resistance, source impedance and substation grounding grid impedance, respectively. As seen from Figure 14, each span of the line is represented by a six terminal network. The mutual impedance between the phase conductor and the ground wire in each span can be transformed into the equivalent circuit shown in Figure 15 which consists of only self impedances. It could be shown that the representations of a span in Figures 14 and 15 are identical in terms of driving point and transfer impedances.
Figure 14 Equivalent circuit of a faulted transmission line fed by two source stations.
Figure 15 Six terminal network which represents the kth span, closed with an impedance star.

Figure 16 Resultant network.
The algorithm of the star network reduction starts at the source station and successively (span-by-span) proceeds up to the faulted structure. At starting, the left source station can be considered as a star network by assigning:

\[ Z_{xn+1} = Z_s, \; Z_{yn+1} = 0, \; Z_{zn+1} = Z_{st} \]

where \( Z_{xi} \), \( Z_{yi} \) and \( Z_{zi} \) denote the equivalent impedances of the star network which represents the basic equivalent circuits of \( i \) spans. Then by applying three delta-star transformations at each span, that span and the equivalent star network connected to the left terminal of that span can be replaced by another star network (Figure 16). The process is

\[ \text{Figure 17 Resultant star network at the fault location.} \]
repeated until finally a resultant star is obtained at the fault location for each side of the fault. The equivalent network obtained at the fault location is shown in Figure 17. It is assumed that the magnitude of the fault current is one per-unit. Using node equations, the currents in the resultant network can be easily solved and then proceeding back from the fault toward the feeding points with the knowledge of the resultant impedances, the ground wire current in each span can be readily obtainable.

Sebo implemented this algorithm to a digital computer and compared the results of his computer program with the measurements obtained from field tests. It was found that the computed values were within ±15% of the measured values.

b) Gooi's Driving Point Impedance Method [33].

In 1983, Gooi suggested a new computational technique for the calculation of fault current distribution between ground and ground wires. The method utilizes a simplified transmission line model where each span is represented by a four terminal network. Figure 18 shows n spans of a transmission line with the equivalent impedance of an external system, $Z_s$, at the source station and a one per-unit fault current injected into the fault location. In the figure, $Z_{ci}$ and $Z_{wi}$ denote the zero sequence impedance of the equivalent phase conductor and ground wire with ground return in the ith span, respectively. $Z_{mi}$ is the mutual impedance between the equivalent phase conductor and the ground wire with common ground return in the ith span. Starting from the source station, a matrix equation relating the voltages and currents is written:
Figure 18 Transmission line zero sequence equivalent circuit with injected fault current.

\[
\begin{bmatrix}
V_{cn+1} \\
V_{wn+1}
\end{bmatrix} =
\begin{bmatrix}
-(Z_s+Z_{st}) & Z_{st} \\
-Z_{st} & Z_{st}
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_{wn+1}
\end{bmatrix}
\]  \hspace{1cm} (92)

Here the 2 x 2 matrix is referred to as the driving point impedance matrix at the source station. Assume that the driving point impedance
matrix at the ith span, \( Z_i \) is known as:

\[
\begin{bmatrix}
V_{ci} \\
V_{wi}
\end{bmatrix} =
\begin{bmatrix}
Z_{11i} & Z_{12i} \\
Z_{21i} & Z_{22i}
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_{wi}
\end{bmatrix}
\]  

(93)

Then it is possible to obtain the driving point impedance matrix at the (i-1)st span, \( Z_{i-1} \), using the circuit equations and Eq. (93) as follows:

\[
\begin{bmatrix}
V_{ci-1} \\
V_{wi-1}
\end{bmatrix} =
\begin{bmatrix}
Z_{11i-1} - Z_{ci-1} & Z_{21i+Z_{mi-1}} \\
Z_{21i-1} - Z_{mi-1} & Z_{22i+Z_{wi-1}}
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_{wi}
\end{bmatrix}
\]  

(94)

\[
I_{wi} = I_{wi-1} - \frac{V_{wi-1}}{R_{ti-1}}
\]

(95)

From (95), \( I_{wi} \) can be solved in terms of \( I_{wi-1} \) and \( I_c \) and inserted in Eq. (94). Hence \( V_{ci-1} \) and \( V_{wi-1} \) can be written in terms of the phase conductor current, \( I_c \), and the ground wire current in the (i-1)st span, \( I_{wi-1} \). The result is:
\[
\begin{bmatrix}
V_{ci-1} \\
V_{wi-1}
\end{bmatrix} =
\begin{bmatrix}
Z_{11i} - Z_{ci-1} & - \frac{(Z_{21i} - Z_{mi-1}) H_i}{R_{ti-1}} & H_i \\
- H_i & \frac{Z_{22i} + Z_{wi-1}}{F_i} & \frac{F_i}{Z_{22i} + Z_{wi-1}}
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_{wi-1}
\end{bmatrix}
\] (96)

where \( F_i = 1 + \frac{Z_{22i} + Z_{wi-1}}{R_{ti-1}} \)

\[ H_i = \frac{Z_{12i} + Z_{mi-1}}{F_i} \]

The 2 x 2 matrix in Eq. (96) is \( Z_{i-1} \) and it is skew-symmetric if \( Z_i \) is skew-symmetric. Hence starting with \( Z_{n+1} \) at the source station which is skew-symmetric, the driving point impedance matrices of all spans up to the fault location can be obtained successively. The current distribution at the first span of the line is solved using the boundary conditions at the fault location. Once the first span current values are known, the current values for the rest of the spans can then be determined.

Substantial simplifications and thus reductions in computer memory requirements and computational time are possible in this approach. Results obtained by the computer program TEST which utilizes this method were compared with those published in the literature and were found to be in agreement.
4.1.3 Simulation of Faulted Underground Cables

Unlike overhead transmission lines, there have not been many attempts to solve the fault current distribution along underground cables. In many cases, methods primarily suggested for transmission lines were applied to the underground cables. The only study on the simulation of faulted underground cables that the author is aware of is a recent research conducted at Georgia Institute of Technology as an EPRI project. The method suggested [34,35] computes the shield potentials of underground directly buried solid dielectric cables and the ground potentials in the vicinity of the cable. The method is based on modeling a short length (typically less than 50 meters) of the buried cable and associated grounding rods as a set of interconnected cylindrical segments located in the ground, with the ground itself represented by two homogeneous layers. The total number of segments obtained, as shown in Figure 19, is equal to \( N = n + 2m + p \), where \( n \) and \( m \) are the number of cable segments and grounding rods, respectively, and \( p \) is the number of grounding grid segments. First, an equivalent circuit representation of the ground is obtained using the following assumptions: a) Constant voltage over the outer surface of a segment; b) The current which flows from the surface of a segment into the ground is concentrated at the center of the segment. Because of the short lengths simulated, DC or quasi-static analysis is judged to be sufficient. A numerical solution of Laplace's equation,

\[
\nabla^2 v(x,y,z) = 0
\]
Figure 19 Cable and substation grounding grid segments.
to account for ground currents results in the matrix equation:

\[ V = Z_{\text{LAP}} I \]  \hspace{1cm} (97)

where \( V \) is an \( N \times 1 \) vector of average voltages of each segment, \( I \) is an \( N \times 1 \) vector of total currents flowing into ground from each segment and \( Z_{\text{LAP}} \) is a \( N \times N \) matrix obtained from the numerical solution of Laplace's equation. Hence the equivalent ground circuit is defined by a set of network elements connected between the cable segments and remote ground. These network elements are determined from:

\[ Y_{\text{LAP}} = Z_{\text{LAP}}^{-1} \]  \hspace{1cm} (98)

where \( Y_{\text{LAP}} \) is the admittance matrix of the equivalent ground circuit.

Then an equivalent circuit representation of the substation, overhead line and cable is developed using the buried segment models in Figure 20. Load currents and capacitive coupling effects are neglected. Inductive coupling between the phase conductor and the sheath is represented with current dependent voltage sources. Self impedances in the model are computed using the simplified version of Carson's formulae.

Finally, the individual equivalent circuit representations of the system components (i.e., ground, substation, cable, etc.) are connected to each other. The resultant network is solved by utilizing modified nodal analysis technique. This network analysis method is suited to the problem because of the presence of current dependent voltage sources and
Figure 20 Equivalent representation of a cable segment and a grounding rod segment.
mutual coupling. The method yields the following matrix equation:

\[
\begin{bmatrix}
Y & A & 0 \\
C & D & E \\
0 & G & H
\end{bmatrix}
\begin{bmatrix}
V \\
V' \\
I'
\end{bmatrix}
= 
\begin{bmatrix}
-I \\
e_S E_S \\
0
\end{bmatrix}
\quad (99)
\]

where \( V \) and \( I \) are vectors of \( N \times 1 \) as defined in Eq. (97),

- \( V' \) is the vector of the voltages at the nodes of segments,
- \( I' \) is the vector of currents on the branches of each segment,
- \( E_S \) is the supply voltage,
- \( e_S \) is a vector which has 1.0 at the entry corresponding to node \( s \), and zero elsewhere,
- \( Y, A, C, D, E, G, \) and \( H \) are submatrices of equivalent network admittances and impedances obtained from node equations.

The simultaneous solution of Eqs. (97) and (99) yields the currents flowing into ground from the segmented cable system:

\[
I = - \left[ I_d + (Y - AKC) Z_{\text{LP}} \right]^{-1} AK e_S E_S
\quad (100)
\]

where \( K = (D - E H^{-1} G)^{-1} \)

and \( I_d \) is the \( N \times N \) identity matrix.

Substituting Eq. (100) into Eq. (97) will give the average segment voltages. Then the voltages \( V' \) and the currents \( I' \) can also be obtained. A computer program has been developed and simulations of various cable segments in various soil environments have been performed.
to obtain step and touch potentials along the cable route. In the analysis, only single phase cables are considered and the coupling between phases is neglected. Sparcity techniques in matrix manipulations were employed because proportional to the length of the cable and the number of the grounding rods, the size of the matrices will greatly increase. Due to the assumptions in the segmentation process, the accuracy of the solutions will decrease when the number of cable segments, n, is kept low in order to limit the number of computations and memory requirements. As seen from Eq. (100), truncation errors will be inevitable due to many matrix inversions and multiplications. Also, the methodology requires implicitly that all segments be in the same type of soil.

4.2 PROPOSED METHOD OF ANALYSIS

Figure 21 shows the general network configuration to be simulated. A length of underground cable with or without an overhead transmission line is supplied from the terminal substations at each end. Each source at the terminal substation may be an existing generator or may represent the external power system by an equivalent Thevenin impedance in series with an ideal voltage source. The substations are assumed to be grounded with a grounding grid. Only a three phase or a single phase cable will be considered in the network. Grounding rods are connected to the cable neutral at specified locations. If a fault between the cable conductor and sheath is assumed to exist at some location on the cable, a portion of the fault current will return to the source through the sheath and another portion will flow through the ground.
Figure 21 Underground cable system.
The main purpose of this study is to find a direct and computer oriented method to simulate the fault current (i.e., zero sequence current) distribution.

The salient features of the proposed method for underground cables are the following:

1. The method accepts variable line parameters for each segment (i.e., soil resistivity, cable impedances, grounding rod resistances, segment lengths, cable geometry).
2. Fault can be assumed to occur anywhere along the cable.
3. One or two generating source stations are possible.
4. Cable to be simulated can have an insulated or a bare sheath, or a sheath with a semiconducting layer.
5. Source impedances and substation grounding grid impedances are represented in the model.
6. Inductive and capacitive coupling between phase and neutral conductors are taken into consideration.

It is clear from the features listed above that the cable network configuration in Figure 21 is not restrictive and the method can solve for a variety of cases such as cables without grounding rods, cables with an open neutral, substations without a grounding grid, overhead transmission lines or underground cables with overhead line extensions between terminal stations.

The basic differences between overhead lines and underground cable networks are the following:
1) The neutral conductor of a cable (i.e., cable sheath) may have a finite and distributed conductance to earth as in the case of a cable with either a bare neutral or a neutral covered with a semiconducting jacket.

2) The capacitive reactance between the conductor and the sheath in the case of a cable can be significant.

If the cables have insulated sheaths and are grounded only at several locations along the route, the problem of finding the zero sequence current distribution becomes very similar to the case of transmission lines. Hence the proposed method should be able to solve the fault current distribution along cables and overhead lines as well.

The proposed model will utilize and combine DeSieno's distributed parameter method and Gooi's driving point impedance approach which uses lumped parameters. Both of these methods were primarily proposed for fault current distribution in transmission line ground wires and were explained in the previous section.

The general methodology which will be introduced in Chapter 5 is based on the following assumptions:

1. The model will consist of zero sequence self and mutual impedances which are computed using Carson's formulae [10].

2. Load currents will be neglected.

3. The fault current is known from system studies.

4. The network is assumed to be linear and in the sinusoidal steady state. Only the fundamental frequency is considered.
5. For this study only the specific single phase or three phase cable in question is represented directly in the model. The effects of other buried conductors or cables in the vicinity of the specified cable, such as pipelines, railroads, auxiliary cables, are neglected.

6. Inductive coupling between grounding rods is neglected (i.e., it is assumed that the length of segments is much larger than the length of grounding rods).

The next chapter will introduce the equivalent network representation and the mathematical model of an underground cable network.
CHAPTER V

MODELING OF AN UNDERGROUND CABLE

The first step in deriving the mathematical model of the general cable system shown in Figure 21 is to identify all significant parameters that influence the fault current distribution. Because of the number of variables involved, the problem will be reduced to its indispensable elements. This reduces the amount of data to a manageable level which, however, will be sufficient to model and obtain the results with an acceptable accuracy. The variables which will be considered in developing the equivalent circuit representation of the underground cable are parameters related to cable, grounding system and substation parameters, soil resistivity, and fault location.

It is assumed that the ground fault occurs at a point along the underground cable between two terminal stations (i.e., substations). The terminal stations will be modeled by the equivalent circuit of the network which exists at either end of the cable. The grounding structures, connected to the cable sheath at specified intervals, will be shown in the model as resistances to ground. Since the zero sequence currents flowing through the phase conductors of a three phase system are in phase and equal in magnitude (each one of them is $I_0$), the three conductors can be replaced by a single equivalent conductor which will conduct $3I_0$. The corresponding zero sequence self and mutual impedances
related to the equivalent conductor can be derived using Carson's simplified equations.

In the following sections, starting with the definition and the model of a cable segment, a general methodology to determine the zero sequence current distribution between the neutral conductor and ground will be presented.

5.1 MODEL OF A CABLE SEGMENT

5.1.1 Equivalent Circuit of a Cable Segment

The distributed parameter approach, proposed for the transmission lines by DeSieno [30], cannot be applied to the entire length of the underground cable due to the irregularly spaced grounding rods of different lengths and in different types of soil. The grounding rods represent discontinuities in the sheath current vs. length along the cable function, and they should be modeled as lumped resistances. The sheath current and voltage between two grounding structures are continuous and that part of the sheath can be analyzed in the distributed parameter form.

The cable portion between two consecutive grounding rods with the inclusion of the grounding rod at the end looking toward the fault will be referred to as a segment. The zero sequence equivalent circuit of the ith cable segment is shown in Figure 22. The two terminals on each side will be referred to as nodes and they will be numbered according to the subscripts of the sheath voltages at the terminals. In terms of notations, $V_{Si}$ and $V_{Si}(x)$ should not be confused with each other: the former is the sheath voltage at the ith node (the terminal in the ith segment which is looking towards the substation) and the latter is the
sheath voltage in the ith segment at a distance of x unit length from the ith grounding rod. \( Z_{si} \) and \( Z_{ci} \), in Figure 22, are zero sequence self impedances, with ground return of the sheath and of the equivalent phase conductor, respectively. \( Z_{mi} \) is the mutual impedance between the sheath and the equivalent phase conductor with common ground return and \( Y_{si} \) is the shunt admittance of the sheath to the ground per unit length. \( Z_{cap,i} \) is the total shunt impedance between the sheath and the phase conductor in the ith segment and it mainly represents the capacitive reactance in a \( \pi \) network form. It is determined from the shunt capacitive reactance per unit length, \( X_{cap,i} \), as:

\[
Z_{cap,i} = -j \frac{X_{cap,i}}{d_i}
\]

where \( d_i \) is the length of the ith segment. \( R_{gi} \) is the grounding resistance of the ith grounding rod. \( Z_{si}, Z_{ci} \) and \( Z_{mi} \) are given in ohms per unit length. \( I_{si}(x) \) and \( I_{ei}(x) \) denote the values of sheath current and spatial current, respectively, both at x unit length away from the grounding rod. Throughout this section the subscript \( i \) will denote the ith segment of the cable as shown in Figure 22.
Figure 22 Zero sequence equivalent circuit of a cable segment.
The basic assumptions are:

i) the zero sequence equivalent circuit representing a cable segment is composed of an infinitely large number of infinitesimal elements,

ii) the parameters are of distributed type (except the grounding rod resistance and the shunt capacitive reactance which are at the ends of the segment),

iii) the ground is assumed to be uniform within this segment so that the shunt admittance of the sheath to the ground can be treated as uniformly distributed between the sheath and the ground.

Now, consider an infinitesimal element of the sheath at any point \( x \) measured from the grounding rod. The sum of currents entering and leaving this sheath element, and the current flowing to ground is zero,

\[
I_{sf}(x+\Delta x) - I_{sf}(x) = - I_{e1}(x) \Delta x
\]  

(101)

Also, the voltage drop over the infinitesimal loop is zero,

\[
V_{sf}(x+\Delta x) - V_{sf}(x) = - I_{sf}(x)Z_{sf}(x)\Delta x + Z_{mi}\Delta x I_{ci}
\]

\[
= \frac{1}{\gamma_{sf}\Delta x} (I_{e1}(x+\Delta x) - I_{e1}(x)) \Delta x
\]  

(102)

Dividing (101) and (102) by \( \Delta x \) and taking the limit of both sides as \( \Delta x \to 0 \) yields:
\[
\frac{dI_{S_i}(x)}{dx} = -I_{e_i}(x) \quad (103)
\]

\[
\frac{dI_{e_i}(x)}{dx} = -Y_{S_i}Z_{S_i}I_{S_i}(x) + Y_{S_i}Z_{m_i}I_{C_i} \quad (104)
\]

From (103) and (104), the differential equations for the sheath current, \(I_{S_i}(x)\), and sheath voltage, \(V_{S_i}(x)\), when \(0 < x < d_i\), are obtained as:

\[
\frac{d^2I_{S_i}(x)}{dx^2} - Y_{S_i}Z_{S_i}I_{S_i}(x) = -Y_{S_i}Z_{m_i}I_{C_i} \quad (105)
\]

\[
\frac{d^2I_{S_i}(x)}{dx^2} = -Y_{S_i} \frac{dV_{S_i}(x)}{dx} \quad (106)
\]

The general solution of these differential equations can be written as:

\[
I_{S_i}(x) = A \cosh \beta_i x + B \sinh \beta_i x + \frac{Z_{m_i}}{Z_{S_i}} I_{C_i} \quad (107)
\]

\[
V_{S_i}(x) = -\frac{AB_i}{Y_{S_i}} \sinh \beta_i x + \frac{BB_i}{Y_{S_i}} \cosh \beta_i x \quad (108)
\]

where \(\beta_i\) is the propagation constant in the \(i\)th segment and

\[
\beta_i = \sqrt{Y_{S_i}Z_{S_i}} \quad (109)
\]
A and B are arbitrary constants to be found from the boundary conditions in the ith segment.

5.1.2 Solving for Segment Voltages and Currents

Assume that the following boundary conditions are known for the ith segment:

\[
V_{s_i}(d_i^-) = V_{s_i} \quad \text{(110)}
\]
\[
I_{s_i}(d_i^+) = I_{s_i} \quad \text{(111)}
\]

where \(d_i\) is the length of the ith segment (i.e., the distance between (i-1)st and ith grounding rods counting from the left terminal station towards the fault), and \(d_i^-=d_i-\epsilon\), \(\epsilon\) being very small.

In addition to Eqs. (110) and (111), the following definitions will be given to clarify the notations that will be used throughout this chapter:

\[
V_{s_i}(0) = V_{s_{i+1}} \quad \text{(112)}
\]
\[
I_{s_i}(0^+) = I_{s_{i0}} \quad \text{(113)}
\]

Using the boundary conditions assumed, the arbitrary constants A and B can be determined. At \(x = d_i\):

\[
I_{s_i} = A \cosh \beta_i d_i + B \sinh \beta_i d_i + \frac{Z_{m_i}}{Z_{s_i}} I_{c_i}
\]
\[
V_{s_i} = -\frac{A \beta_i}{Y_{s_i}} \sinh \beta_i d_i - \frac{B \beta_i}{Y_{s_i}} \cosh \beta_i d_i
\]
Define:

\[ C_i = \cosh \beta_i d_i \]  \hspace{1cm} (114)

\[ S_i = \sinh \beta_i d_i \]  \hspace{1cm} (115)

Hence,

\[
\begin{bmatrix}
C_i & S_i \\
\frac{-\beta_i S_i}{Y_{si}} & \frac{-\beta_i C_i}{Y_{si}}
\end{bmatrix}
\begin{bmatrix}
A \\
B
\end{bmatrix} =
\begin{bmatrix}
I_{si} - \frac{Z_{mi}}{Z_{si}} & I_{ci} \\
V_{si}
\end{bmatrix}
\]

\[
\begin{bmatrix}
A \\
B
\end{bmatrix} =
\begin{bmatrix}
C_i & S_i \\
\frac{-\beta_i S_i}{Y_{si}} & \frac{-\beta_i C_i}{Y_{si}}
\end{bmatrix}
\begin{bmatrix}
I_{si} - \frac{Z_{mi}}{Z_{si}} & I_{ci} \\
V_{si}
\end{bmatrix}
\]
Therefore:

\[
\begin{bmatrix}
  A \\
  B
\end{bmatrix} = \frac{-Y_{si}}{\beta_i(C_i Z_{si} - S_{si}^2)} \begin{bmatrix}
  -\frac{\beta_i C_i}{Y_{si}} & -S_{si} \\
  \frac{\beta_i S_{si}}{Y_{si}} & C_i
\end{bmatrix} \begin{bmatrix}
  I_{si} - \frac{Z_{mi}}{Z_{si}} I_{ci} \\
  V_{si}
\end{bmatrix}
\]

\[A = C_i(I_{si} - \frac{Z_{mi}}{Z_{si}} I_{ci}) + \frac{Y_{si} S_{si}}{\beta_i} V_{si}\]  \hspace{1cm} (116)

\[B = -S_{si}(I_{si} - \frac{Z_{mi}}{Z_{si}} I_{ci}) - \frac{Y_{si} C_i}{\beta_i} V_{si}\]  \hspace{1cm} (117)

Inserting A and B into Eqs. (107) and (108) results in:

\[I_{si}(x) = [C_i(I_{si} - \frac{Z_{mi}}{Z_{si}} I_{ci}) + \frac{Y_{si} S_{si}}{\beta_i} V_{si}] \cosh \beta_i x\]

\[+ [S_{si}(I_{si} - \frac{Z_{mi}}{Z_{si}} I_{ci}) - \frac{Y_{si} C_i}{\beta_i} V_{si}] \sinh \beta_i x + \frac{Z_{mi}}{Z_{si}} I_{ci}\]  \hspace{1cm} (118)

\[V_{si}(x) = [-\frac{\beta_i C_i}{Y_{si}} (I_{si} - \frac{Z_{mi}}{Z_{si}} I_{ci}) - S_{si} V_{si}] \sinh \beta_i x\]

\[+ \frac{\beta_i S_{si}}{Y_{si}} (I_{si} - \frac{Z_{mi}}{Z_{si}} I_{ci}) + C_i V_{si}] \cosh \beta_i x\]  \hspace{1cm} (119)
At $x = 0+$:

$$I_{s_i}(0+) = I_{s_i0} = C_i I_{s_i} + (1-C_i) \frac{Z_{m_i}}{Z_{S_i}} I_{c_i} + \frac{Y_{s_i} Z_{s_i}}{\beta_i} V_{s_i}$$  \hspace{1cm} (120)$$

$$V_{s_i}(0) = V_{s_i+1} = \frac{\beta_i S_i}{Y_{s_i}} I_{s_i} - \frac{\beta_i S_i}{Y_{s_i}} Z_{m_i} I_{c_i} + C_i V_{s_i}$$  \hspace{1cm} (121)$$

Similarly, the conductor voltage at $x = 0$, $V_{c_i+1}$, can be written in terms of $V_{c_i}$, $V_{s_i}$, $I_{c_i}$ and $I_{s_i}$, where

$$V_{c_i} = V_{c_i}(d_i^-)$$  \hspace{1cm} (122)$$

$$V_{c_i}(0) = V_{c_i+1} = V_{c_i} - Z_{c_i} d_i I_{c_i} + Z_{m_i} \int_0^{d_i} I_{s_i}(x)dx$$  \hspace{1cm} (123)$$

Defining the effective sheath current in segment $i$ as:

$$I_{s_i,\text{eff}} = \frac{1}{d_i} \int_0^{d_i} I_{s_i}(x)dx$$  \hspace{1cm} (124)$$

will yield:

$$V_{c_i+1} = V_{c_i} - Z_{c_i} d_i I_{c_i} + Z_{m_i} d_i I_{s_i,\text{eff}}$$  \hspace{1cm} (125)$$

Inserting $I_{s_i}(x)$ from (118) into (124):

$$I_{s_i,\text{eff}} = \frac{A_S_i}{d_i \beta_i} + \frac{B(C_i - 1)}{d_i \beta_i} + \frac{Z_{m_i}}{Z_{S_i}} I_{c_i}$$  \hspace{1cm} (126)$$
where \( A \) and \( B \) are the constants determined using the boundary conditions in Eqs. (110) and (111). Hence:

\[
I_{\text{eff}} = \frac{S_i}{d_i \beta_i} [C_i I_{\text{eff}} - C_i \frac{Z_{\text{mi}}}{Z_{\text{si}}} I_{\text{ci}} + \frac{S_i Y_{\text{si}}}{\beta_i} V_{\text{si}}] + \frac{Z_{\text{mi}}}{Z_{\text{si}}} I_{\text{ci}}
\]

\[
+ \frac{(C_i-1)}{d_i \beta_i} [-S_i I_{\text{eff}} + S_i \frac{Z_{\text{mi}}}{Z_{\text{si}}} I_{\text{ci}} - \frac{C_i Y_{\text{si}}}{\beta_i} V_{\text{si}}]
\]

Since \( \beta_i^2 = Y_{\text{si}} Z_{\text{si}} \)

\[
cosh^2 \beta_i d_i - \sinh^2 \beta_i d_i = 1.0
\]

it follows that:

\[
I_{\text{eff}} = \frac{S_i}{d_i \beta_i} I_{\text{si}} + \left( 1 - \frac{S_i}{d_i \beta_i} \right) \frac{Z_{\text{mi}}}{Z_{\text{si}}} I_{\text{ci}} + \frac{(C_i-1)}{d_i Z_{\text{si}}} V_{\text{si}} \quad \text{(127)}
\]

Inserting Eq. (127) into (128) gives the equation desired:

\[
V_{\text{ci+1}} = V_{\text{ci}} + \left( \frac{Z_{\text{mi}}}{Z_{\text{si}}} \right) \frac{S_i}{\beta_i} \frac{Z_{\text{mi}}}{Z_{\text{si}}} - Z_{\text{ci}} d_i \right) I_{\text{ci}}
\]

\[
+ \frac{Z_{\text{mi}} S_i}{\beta_i} I_{\text{si}} + \frac{(C_i-1)}{Z_{\text{si}}} \frac{Z_{\text{mi}}}{Z_{\text{si}}} V_{\text{si}} \quad \text{(128)}
\]

Hence, as derived in Eqs. (121) and (128), the sheath and conductor voltages at \( x = 0 \), \( V_{\text{ci+1}} \) and \( V_{\text{si+1}} \), are expressed in terms of voltages and currents at \( x = d_i \). In matrix form:
\[
\begin{bmatrix}
V_{c1+1} \\
V_{s1+1}
\end{bmatrix} =
\begin{bmatrix}
1 & H_i \\
0 & C_i
\end{bmatrix}
\begin{bmatrix}
V_{c1} \\
V_{s1}
\end{bmatrix} +
\begin{bmatrix}
(1-P_i)d_iZ_{m1} & Z_{mi}S_i \\
-Z_{m1} & T_i
\end{bmatrix}
\begin{bmatrix}
I_{c1} \\
I_{s1}
\end{bmatrix}
\]

(129)

where 
\[T_i = \frac{B_iS_i}{Y_{si}}\]

\[H_i = (C_i-1)\frac{Z_{mi}}{Z_{si}}\]

\[P_i = \frac{S_i}{d_i\beta_i}\]

5.2 ZERO SEQUENCE EQUIVALENT CIRCUIT

The zero sequence equivalent circuit of a cable segment can be extended to obtain a representation for the cable network with both a source and a substation grounding grid at each end. This is shown in Figure 23. The total number of segments in the representation is \((n+k)\) such that the fault occurs at the \(n\)th grounding rod between the phase conductor and the sheath. Hence, the total number of grounding rods is \((n+k-1)\). \(Z_s\) and \(Z_{st}\) represent the equivalent impedance of the external system, and the substation grounding grid impedance at the left terminal station, respectively. \(Z_{c1}, Z_{s1}, \) and \(Z_{mi}\) are zero sequence self and mutual impedances in the \(i\)th segment and were defined in the previous section.
$Z_{csi}$ represents the equivalent shunt impedance (related to the capacitance reactance) at the $i$th node and it is determined from the segment data as follows:

$$Z_{csi} = \begin{cases} 
2Z_{cap,i}, & i = 1 \text{ or } n+k+1 \\
0, & i = n+1 \\
2\frac{Z_{cap,i-1}Z_{cap,i}}{Z_{cap,i-1} + Z_{cap,i}}, & \text{otherwise}
\end{cases}$$

The load current will be neglected in the analysis. A one per-unit fault current will be injected into the fault location. Hence, the currents through the sheath and grounding rods will all be obtained in per-unit.

### 5.3 DRIVING POINT IMPEDANCE METHOD

#### 5.3.1 General Methodology

Assume that a matrix equation relating to the voltages and currents at $x=0$ in $(i-1)$st segment may be written as:

$$\begin{bmatrix} V_{ci} \\ V_{si} \end{bmatrix} = \begin{bmatrix} Z_{11i} & Z_{12i} \\ Z_{21i} & Z_{22i} \end{bmatrix} \begin{bmatrix} I_{ci} \\ I_{si} \end{bmatrix}$$

where $1<i<n$. In terms of notation, the approach will be limited to the left side of the fault. Since the segments at the left and right sides of the fault are structurally mirror images of each other, the approach is easily applicable to the right side of the fault by changing the subscripts properly.
Figure 23 Zero sequence equivalent circuit of the cable network.
The 2x2 matrix in Eq. (130), $Z_i$, is referred to as the driving point impedance matrix at the (i-1)st segment (at the ith node) and the values of its entries depend upon the parameters of the (i-1)st segment and $Z_{i-1}$. The purpose of this section is to derive the driving point impedance matrix at the ith segment, $Z_{i+1}$, in terms of ith segment parameters and $Z_i$. Hence, it will be possible to determine the driving point impedance matrix of successive segments starting from the terminal station.

At the left terminal station:

\[
\begin{bmatrix}
V_{c1} \\
V_{s1}
\end{bmatrix} = \begin{bmatrix}
Z_{111} & Z_{121} \\
Z_{21} & Z_{221}
\end{bmatrix} \begin{bmatrix}
I_{c1} \\
I_{s1}
\end{bmatrix}
\]

or from Figure 23:

\[
\begin{bmatrix}
V_{c1} \\
V_{s1}
\end{bmatrix} = \begin{bmatrix}
-(Z_a+Z_{st}) & Z_{st} \\
-Z_{st} & Z_{st}
\end{bmatrix} \begin{bmatrix}
I_{c1} \\
I_{s1}
\end{bmatrix} \quad (131)
\]

where $Z_{st}$ is the substation grounding grid impedance and $Z_a$ is the equivalent impedance at the 1st node representing $Z_s$ and $Z_{cS1}$ in parallel.
Now, considering the ith segment where $1 < i < n$, assume that

\[ \begin{bmatrix} V_{ci} \\ V_{si} \end{bmatrix} = \begin{bmatrix} Z_{11i} & Z_{12i} \\ Z_{21i} & Z_{22i} \end{bmatrix} \begin{bmatrix} I_{ci} \\ I_{si} \end{bmatrix} \] (132)

is known. Then $V_{ci}$ and $V_{si}$ of the ith segment can be eliminated from Eqs. (120) and (129) using (132):

\[ V_{ci+1} = (Z_{11i} - Z_{ci}d_{i} + (1-P_{i})d_{i} \frac{Z_{m_{i}}^{2}}{Z_{si}} + H_{i}Z_{21i}) I_{ci} \]
\[ + (Z_{12i} + P_{i}d_{i}Z_{m_{i}} + H_{i}Z_{22i}) I_{si} \] (133)

\[ V_{si+1} = (C_{i}Z_{21i} - T_{i}) \frac{Z_{m_{i}}}{Z_{si}} I_{ci} + (C_{i}Z_{22i} + T_{i}) I_{si} \] (134)

\[ I_{si_{0}} = ((1-C_{i}) \frac{Z_{m_{i}}}{Z_{si}} + \frac{Y_{si_{i}}S_{i}}{\beta_{i}} Z_{21i}) I_{ci} + (C_{i} + \frac{Y_{si_{i}}S_{i}}{\beta_{i}} Z_{22i}) I_{si} \] (135)

where $P_{i}$, $H_{i}$ and $T_{i}$ are complex constants at the ith segment defined in Eq. (129).

Or in matrix form:

\[ \begin{bmatrix} V_{ci+1} \\ V_{si+1} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{21} \\ Q_{31} & Q_{41} \end{bmatrix} \begin{bmatrix} I_{ci} \\ I_{si} \end{bmatrix} \] (136)
\( I_{s10} = Q_5 I_{C1} + Q_6 I_{s1} \) \hspace{1cm} (137)

where the definitions of \( Q_k \), \( k=1...6 \), can be obtained from Eqs. (133) - (135).

Also, the following node equations at the \((i+1)\)st node can be written:

\[
\begin{align*}
I_{s1i+1} &= I_{s10} + \frac{V_{si+1}}{R_{gi}} - \frac{V_{ci+1} - V_{si+1}}{Z_{csi+1}} \hspace{1cm} (138) \\
I_{ci+1} &= I_{ci} - \frac{V_{ci+1} - V_{si+1}}{Z_{csi+1}} \hspace{1cm} (139)
\end{align*}
\]

Inserting \( V_{ci+1} \) and \( V_{si+1} \) from Eq. (136) into Eq. (138) will yield:

\[
\begin{align*}
I_{s1i+1} &= I_{s10} + \left( \frac{1}{R_{gi}} + \frac{1}{Z_{csi+1}} \right) (Q_{3i} I_{C1} + Q_{4i} I_{s1}) \\
&\quad - \frac{1}{Z_{csi+1}} \left( Q_{1i} I_{C1} + Q_{2i} I_{s1} \right) \\
&= I_{s10} + \left( G_i Q_{4i} - \frac{Q_{2i}}{Z_{csi+1}} \right) I_{s1} + \left( G_i Q_{3i} - \frac{Q_{1i}}{Z_{csi+1}} \right) I_{ci} \hspace{1cm} (140)
\end{align*}
\]

where \( G_i = \frac{1}{R_{gi}} + \frac{1}{Z_{csi+1}} \) \hspace{1cm} (141)

Eliminating \( I_{s10} \) from Eq. (140) using Eq. (137):

\[
I_{s1i+1} = \left( G_i Q_{4i} + Q_6 \right) I_{s1} + \left( G_i Q_{3i} + Q_5 \right) I_{ci} \hspace{1cm} (142)
\]
Or:

\[ I_{si} = M_i I_{si+1} + N_i I_{ci} \]  \hspace{1cm} (143)

where

\[ M_i = \frac{1}{G_i Q_{4i} + Q_{6i} - \frac{Q_{2i}}{Z_{csi+1}}} \] \hspace{1cm} (144)

\[ N_i = M_i \left( \frac{Q_{1i}}{Z_{csi+1}} - G_i Q_{3i} - Q_{5i} \right) \] \hspace{1cm} (145)

Similarly, \( V_{ci+1} \) and \( V_{si+1} \) in Eq. (39) can be eliminated using Eq. (136):

\[ I_{ci} = I_{ci+1} + \frac{Q_{1i} I_{ci} + Q_{2i} I_{si}}{Z_{csi+1}} - \frac{Q_{3i} I_{ci} + Q_{4i} I_{si}}{Z_{csi+1}} \]

Using Eq. (143)

\[ I_{ci} = I_{ci+1} + \frac{Q_{1i} - Q_{3i}}{Z_{csi+1}} I_{ci} + \frac{Q_{2i} - Q_{4i}}{Z_{csi+1}} (M_i I_{si+1} + N_i I_{ci}) \]

and the following expression can be obtained easily:

\[ I_{ci} = D_i I_{ci+1} + F_i I_{sti+1} \] \hspace{1cm} (146)

where

\[ D_i = \frac{1}{1 - \frac{Q_{1i} - Q_{3i}}{Z_{csi+1}} - \frac{Q_{2i} - Q_{4i}}{Z_{csi+1}} N_i} \] \hspace{1cm} (147)
Inserting Eq. (146) into Eq. (143) will yield the following matrix equation:

\[
\begin{bmatrix}
I_{ci} \\
I_{si}
\end{bmatrix} =
\begin{bmatrix}
D_i & F_i \\
D_i N_i & M_i + F_i N_i
\end{bmatrix}
\begin{bmatrix}
I_{ci+1} \\
I_{si+1}
\end{bmatrix}
\]  

(149)

Rewriting Eq. (136) using (149) will result in the desired expression:

\[
\begin{bmatrix}
V_{ci+1} \\
V_{si+1}
\end{bmatrix} =
\begin{bmatrix}
Q_{1i} & Q_{2i} \\
Q_{3i} & Q_{4i}
\end{bmatrix}
\begin{bmatrix}
D_i & F_i \\
D_i N_i & M_i + F_i N_i
\end{bmatrix}
\begin{bmatrix}
I_{ci+1} \\
I_{si+1}
\end{bmatrix}
\]

(150)

Hence, the driving point impedance matrix at the \((i+1)\)st segment, \(Z_{i+1}\), has been obtained as:

\[
\begin{bmatrix}
D_i(Q_{1i}+Q_{2i}N_i) & F_i(Q_{1i}+Q_{2i}N_i)+Q_{2i}M_i \\
D_i(Q_{3i}+Q_{4i}N_i) & F_i(Q_{3i}+Q_{4i}N_i)+Q_{4i}M_i
\end{bmatrix}
\begin{bmatrix}
I_{ci+1} \\
I_{si+1}
\end{bmatrix}
\]
\[
\begin{pmatrix}
V_{ci+1} \\
V_{si+1}
\end{pmatrix} =
\begin{pmatrix}
Z_{11i+1} & Z_{12i+1} \\
Z_{21i+1} & Z_{22i+1}
\end{pmatrix}
\begin{pmatrix}
I_{ci+1} \\
I_{si+1}
\end{pmatrix}
\]  
\tag{151}

where the entries of \( Z_{i+1} \) are defined by Eq. (150) as:

\[
Z_{11i+1} = D_i(Q_{1i} + Q_{2i}N_i)
\]  
\tag{152}
\[
Z_{12i+1} = F_i(Q_{1i} + Q_{2i}N_i) + Q_{2i}M_i
\]  
\tag{153}
\[
Z_{21i+1} = D_i(Q_{3i} + Q_{4i}N_i)
\]  
\tag{154}
\[
Z_{22i+1} = F_i(Q_{3i} + Q_{4i}N_i) + Q_{4i}M_i
\]  
\tag{155}

Hence, the entries of \( Z_{i+1} \) are defined in terms of the entries of \( Z_i \) and the cable parameters related to the \( i \)th segment. Therefore, starting from the left source station, the driving point impedance matrices of the successive segments can be computed up to the fault location. Figure 24 shows the general algorithm of the driving point impedance method. A similar approach can be applied to the right side of the fault. The fault current distribution at the fault location can be determined using the driving point impedance matrices of both sides at the fault location and it will be presented in Section 5.3.

In the most general form, the driving point impedance matrix is not a symmetric matrix. As seen from Eqs. (152) - (155), substantial savings in computation time are possible because of the common terms in the expressions.
Figure 24 Driving point impedance method.
5.3.2 Short Cable Case

The driving point impedance matrix method can be simplified considerably if the shunt capacitive reactance between the phase conductor and the neutral conductor is neglected. This is a valid assumption for transmission lines and short underground cables. Reconsidering the equations derived in the previous section if \( Z_{CS1} \rightarrow 0 \) will result in:

\[
D_i = 1 \\
F_i = 0 \\
M_i = \frac{1}{Q_4i + \frac{R_{gi}}{Q_6i}} \\
N_i = M_i (-Q_{5i} - \frac{Q_{3i}}{R_{gi}}) \\
Z_{i+1} = \begin{bmatrix} Q_{1i} + Q_{2i}N_i & Q_{2i}M_i \\ Q_{3i} + Q_{4i}N_i & Q_{4i}M_i \end{bmatrix} 
\]

(156)

or in terms of \( i \)th segment cable parameters and the entries of \( Z_i \):

\[
Z_{11i+1} = Z_{11i} + N_i Z_{12i} - Z_{C1i}d_1 + H_i (Z_{21i} + N_i Z_{22i}) \\
+ Z_{m1i}d_1 [(1-P_i) \frac{Z_{mi}}{Z_{si}} + N_i P_i] 
\]

(157)
\[ Z_{12i+1} = (Z_{12i} + H_i Z_{22i} + P_i d_i Z_{m_i}) M_i \]  
(158)

\[ Z_{21i+1} = (Z_{21i} + N_i Z_{22i}) C_i + T_i (N_i - \frac{Z_{m_i}}{Z_{s_i}}) \]  
(159)

\[ Z_{22i+1} = (C_i Z_{22i} + T_i) M_i \]  
(160)

where \( T_i, H_i \) and \( P_i \) are as defined in Eq. (129).

5.3.3 Transmission Line Case

The driving point impedance approach and the model presented in Section 5.3.1 are equally applicable to overhead transmission lines. In that case, a segment (i.e., span) is defined as the line section between two adjacent towers. The tower footing resistance is included in the model instead of the grounding rod resistance. Also, the shunt admittance between the ground wire and the ground and the shunt capacitive reactance between the phase conductor and the ground wire are neglected. Hence, the proposed model and the algorithm will be simplified further as zero shunt admittance to ground is inserted in the equations. In order to check the validity of the expressions derived, the resultant equations for the driving point impedance matrix will be compared to the equations obtained in Ref. [33] in which a similar approach was used for transmission lines.

The values of all variables in Section 5.3.2 when \( Y_{s_f} = 0 \) is inserted will be given below. In case of any uncertainty, the limit of the expressions as \( Y_{s_f} \) approaches 0 will be determined and L'Hospital limit rule will be applied when necessary.
\[ \beta_i = \sqrt{Y_{s_i} Z_{s_i}} = 0 \]
\[ C_i = \cosh\beta_i d_i = 1 \]
\[ S_i = \sinh\beta_i d_i = 0 \]

\[
\lim_{Y_{s_i} \to 0} P_i = \lim_{Y_{s_i} \to 0} \frac{S_i}{d_i \beta_i} = \lim_{Y_{s_i} \to 0} \frac{\sinh(\sqrt{Y_{s_i} Z_{s_i}} d_i)}{d_i \sqrt{Y_{s_i} Z_{s_i}}} = 1
\]

\[
\lim_{Y_{s_i} \to 0} T_i = \lim_{Y_{s_i} \to 0} \frac{\sqrt{Y_{s_i} Z_{s_i}} \cdot \sinh(\sqrt{Y_{s_i} Z_{s_i}} d_i)}{Y_{s_i}} = Z_{s_i} d_i
\]

\[
\lim_{Y_{s_i} \to 0} H_i = (C_i - 1) \frac{Z_{m_i}}{Z_{s_i}} = 0
\]

\[
\lim_{Y_{s_i} \to 0} \frac{Y_{s_i} S_i}{\beta_i} = 0
\]
Hence:

\[ Q_{3i} = Z_{21i} - Z_{mid} \]
\[ Q_{4i} = Z_{22i} + Z_{sid} \]
\[ Q_{5i} = 0 \]
\[ Q_{6i} = 1 \]

\[ M_i = \frac{R_{gi}}{R_{gi} + Z_{22i} + Z_{sid}} \]

\[ N_i = M_i \frac{Z_{mid_i} - Z_{21i}}{R_{gi}} = \frac{Z_{mid_i} - Z_{21i}}{R_{gi} + Z_{22i} + Z_{sid_i}} \]

The entries of the driving point impedance matrix given in Eqs. (157) - (160) turn out to be:

\[ Z_{11i+1} = Z_{11i} - Z_{sid_i} + (Z_{12i} + Z_{mid_i}) \frac{Z_{mid_i} - Z_{21i}}{R_{gi} + Z_{22i} + Z_{sid_i}} \]  
(161)

\[ Z_{12i+1} = (Z_{12i} + Z_{mid_i}) \frac{R_{gi}}{R_{gi} + Z_{22i} + Z_{sid_i}} \]  
(162)

\[ Z_{21i+1} = Z_{21i} - Z_{mid_i} + (Z_{22i} + Z_{sid_i}) \frac{Z_{mid_i} - Z_{21i}}{R_{gi} + Z_{22i} + Z_{sid_i}} \]  
(163)

\[ Z_{22i+1} = (Z_{22i} + Z_{sid_i}) \frac{R_{gi}}{R_{gi} + Z_{22i} + Z_{sid_i}} \]  
(164)

all of which agree completely with Eq. (94) when it is considered that \( Z_s, Z_m, \) and \( Z_C \) here are given in terms of ohms per unit length. The driving point impedance matrix for the transmission line case will be skew-symmetric since at the first span:
\( Z_{211} = -Z_{121} \)

and at the \((i+1)\)st segment:

\( Z_{21i+1} = -Z_{12i+1} \)

5.3.4 Fault Supplied by One Source Station Only

If there is no generating source at one of terminal stations, determining the driving point impedance matrices of that side of the fault may be simplified. It will be assumed that there is no generating source and no grounded star-delta transformer at the right terminal station shown in Figure 23. Hence, the total fault current will be supplied by the left terminal station. In this case:

\[ I_{cj} = 0 \quad \text{for } j > n \]

and \( I_{cn} = 1.0 \)

The equivalent circuit of the right side of the fault is a ladder network as shown in Figure 25. The driving point impedance matrix at the \( j \)th segment, \( Z_{j+1} \), is defined as:

\[ V_{sj+1} = Z_{j+1} I_{sj} \tag{165} \]

where \( Z_{j+1} = Z_{22j+1}, \ n < j < n+k \)

Hence, all entries of the driving point impedance matrix except \( Z_{22} \) are zero and need not be computed. Starting from the right terminal station where
The driving point impedance of all the other segments up to the fault location can be determined successively. At the jth segment, assume that \( Z_{j+1} \) of Eq. (165) is known. The following node equation can be written at the (j-1)st grounding rod:

\[
I_{sj-1} = I_{sj0} + \frac{V_{sj}}{R_{gj-1}}
\]  

(167)

From Eqs. (120) and (121):

\[
I_{sj0} = C_j i_{sj} + \frac{Y_{sj} S_j}{\beta_j} V_{sj+1}
\]  

(168)

\[
V_{sj} = \frac{\beta_j S_j}{Y_{sj}} I_{sj} + C_j V_{sj+1}
\]  

(169)

can be obtained by properly changing the subscripts and by inserting \( I_{Cj}=0 \). Then, using Eq. (165)

\[
I_{sj0} = \left(C_j + \frac{Y_{sj} S_j}{\beta_j} Z_{j+1}\right) I_{sj}
\]  

(170)

\[
V_{sj} = \left(\frac{\beta_j S_j}{Y_{sj}} + C_j Z_{j+1}\right) I_{sj}
\]  

(171)

Hence:
\[ I_{s_j-1} = (C_j + \frac{Y_{s_j}S_j}{B_j} Z_{j+1} + \frac{B_jS_j}{Y_{s_j}R_gj-1} + \frac{C_j}{R_gj-1} Z_{j+1}) I_{s_j} \] (172)

or

\[ I_{s_j} = M_j I_{s_j-1} \] (173)

where \( M_j \) is the reciprocal of the expression in parenthesis in Eq. (172). Eqs. (171) and (173) result in:

\[ V_{s_j} = M_j(T_j+C_jZ_{j+1}) I_{s_j-1} \] (174)

where

\[ T_j = \frac{B_jS_j}{Y_{s_j}} \]

Therefore, the driving point impedance at the jth segment, \( Z_j \), can be found from jth segment cable parameters and \( Z_{j+1} \) as follows:

\[ Z_j = M_j(T_j+C_jZ_{j+1}) \] (175)
Figure 25  Zero sequence equivalent circuit of the ladder network.
5.4 SOLUTION AT THE FAULT LOCATION

As described in the previous sections, starting from the terminal stations, the driving point impedance matrix of the segments can be obtained successively up to the fault location for both sides of the fault. The equivalent circuit representation at the fault is shown in Figure 26 where the fault is assumed at the nth grounding rod. Let \( I_c \) and \( I'_c \) denote the fault current contributions through the equivalent phase conductor in the nth and \((n+1)\)st segments, respectively. Assume that primed variables refer to the right side of the fault. Using the left and right side driving point impedance matrices, \( Z_{n+1} \) and \( Z'_{n+1} \), the fault current distribution at the fault location can be computed.

The following equations can be written based on Figure 26:

\[
I_c + I'_c = 1.0 \quad (176)
\]

\[
I_{s_{n+1}} + I'_{s_{n+1}} = 1.0 \quad (177)
\]

![Figure 26](image-url)  
Figure 26 Equivalent circuit at the fault location when the fault is supplied by two sources.
\[
\begin{bmatrix}
V_{cn+1} \\
V_{sn+1}
\end{bmatrix}
= 
\begin{bmatrix}
Z_{1n+1} & Z_{12n+1} \\
Z_{21n+1} & Z_{22n+1}
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_{sn+1}
\end{bmatrix}
= 
\begin{bmatrix}
Z_{1n} & Z_{12n} \\
Z_{21} & Z_{22n+1}
\end{bmatrix}
\begin{bmatrix}
I_c' \\
I_{sn+1}'
\end{bmatrix}
\]
(178)

Dropping the subscript \((n+1)\) for simplicity and eliminating \(I_{jn+l}\) in Eq. (178);

\[Z_{11}I_c' + Z_{12}I_s - Z_{11} (1-I_c) - Z_{12} (1-I_s) = 0\]
\[Z_{21}I_c' + Z_{22}I_s - Z_{21} (1-I_c) - Z_{22} (1-I_s) = 0\]

\[
\begin{bmatrix}
Z_{11} + Z'_{11} & Z_{12}+Z_{12}' \\
Z_{21}+Z_{21}' & Z_{22}+Z_{22}'
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_s
\end{bmatrix}
= 
\begin{bmatrix}
Z_{11} + Z_{12}' \\
Z_{21} + Z_{22}'
\end{bmatrix}
\]

Hence,

\[
\begin{bmatrix}
I_c \\
I_s
\end{bmatrix}
= \frac{1}{D}
\begin{bmatrix}
Z_{22}+Z_{22}' & -(Z_{12}+Z_{12}') \\
-(Z_{21}+Z_{21}') & Z_{11}+Z_{11}'
\end{bmatrix}
\begin{bmatrix}
I_c' \\
I_s'
\end{bmatrix}
\]
(179)

where \(D = (Z_{11}+Z_{11}')(Z_{22}+Z_{22}')-(Z_{12}+Z_{12}')(Z_{21}+Z_{21}')\).

\(I_c'\) and \(I_s'\) (i.e., \(I_{sn+1}'\)) can be found easily using Eqs. (176) and (177).

5.5 SOLVING FOR SHEATH CURRENTS AND VOLTAGES ALONG THE CABLE

Assume that the equivalent network at the fault location (Figure 26) is solved and the currents \(I_c, I_c', I_{sn+1}\) and \(I_{sn+1}'\) are determined.
$I_C$ and $I_C'$ are the equivalent phase conductor currents in the $n$th and $(n+1)$st segments, respectively, and $I_{Sn+1}$ and $I_{Sn+1}'$ are the sheath currents entering into the $n$th and $(n+1)$st segments, respectively. Using these currents, the sheath voltages and currents in the $n$th and $(n+1)$st segments can be computed. Here, a systematic algorithm, which solves for the sheath currents and voltages in the $i$th segment when $I_{Ci+1}$ and $I_{Si+1}$ are known, will be presented.

Consider the $i$th segment and the node where the $i$th grounding rod is connected. If $I_{Ci+1}$ and $I_{Si+1}(d_{Ti+1}) = I_{Si+1}$

are known, the following equations will yield the phase conductor and sheath currents and voltages at $x = 0$ and $x = d_i$ in the segments:

\begin{align}
V_{Ci+1} &= Z_{1i+1}I_{Ci+1} + Z_{2i+1}I_{Si+1} \\
V_{Si+1} &= Z_{1i+1}I_{Ci+1} + Z_{2i+1}I_{Si+1} \\
I_{Si}(0) &= I_{Si0} = I_{Si+1} - \frac{V_{Si+1}}{R_{gi}} + \frac{V_{Ci+1} - V_{Si+1}}{Z_{csi+1}} \\
I_{Ci} &= I_{Ci+1} + \frac{V_{Ci+1} - V_{Si+1}}{Z_{csi+1}} \\
I_{Si}(d_{Ti}) &= I_{Si} = M_i I_{Si+1} + N_i I_{Ci+1}
\end{align}

where $M_i$, $N_i$ and the entries of $Z_{i+1}$ are assumed to be known.
If required, the sheath voltage and current at \( x \) unit length away from the \( i \)th node in the \( i \)th segment can also be determined from Eqs. (107) and (108). Since the sheath voltage is continuous along the cable, the arbitrary constants \( A \) and \( B \) will be computed using \( V_{Si} \) and \( V_{Si+1} \) as boundary conditions. From Eq. (108):

\[
V_{Si}(u) = V_{Si+1} = - \frac{B_i}{Y_{Si}} B
\]

\[
V_{Si}(d_i) = V_{Si} = - \frac{B_i}{Y_{Si}} (S_i A + C_i B)
\]

one obtains:

\[
B = - \frac{Y_{Si}}{B_i} V_{Si+1}
\]

\[
A = - \frac{Y_{Si}}{B_i S_i} (V_{Si} - C_i V_{Si+1})
\]

Consequently:

\[
V_{Si}(x) = - (V_{Si} - C_i V_{Si+1}) \frac{S_i \sinh B_i x}{S_i} + V_{Si+1} \cosh B_i x
\]

(185)

Similarly, \( I_{Si}(x) \) can be obtained in terms of \( V_{Si} \) and \( V_{Si+1} \):

\[
I_{Si}(x) = \frac{Z_{mi}}{Z_{Si}} I_{Ci} - (V_{Si} - C_i V_{Si+1}) \frac{Y_{Si}}{B_i S_i} \cosh B_i x
\]

\[
- \frac{Y_{Si}}{B_i} V_{Si+1} \sinh B_i x
\]

(186)
Hence, starting from the nth segment, the node voltages and the sheath currents in the segments towards the left terminating station can be determined. Then the approach is applied to the right side of the fault.

In this chapter, an equivalent circuit representation and a mathematical model of the underground cable have been developed and a new methodology to determine the zero sequence current distribution between the cable sheath and the ground has been presented. The proposed method considers all significant variables involved. It is equally applicable to transmission lines and is implementable on an appropriate computer. The next chapter will describe the computer program that has been written and present the simulation results for different cases that the method and the computer are capable of solving.
6.1 DESCRIPTION OF THE COMPUTER PROGRAM

The underground cable model and the driving point impedance method presented in Chapter 5 have been simulated by a Fortran computer program. The program, named CABLE, accepts varying line parameters for each segment as input data. Basic input data variables are: the number and resistances of grounding structures, length of each segment, terminal station impedances, and cable segment data including the soil resistivity. The number of source stations (i.e., 1 or 2) and the distance of the fault to the source station are also specified. Detailed description of the input data variables and format is given in Appendix B. The input variables are mainly employed to simulate the equivalent circuit model in Figure 23.

Cable self and mutual impedances with ground return are either determined by the program according to the cable system or supplied by the user.

The proposed model and the computer program CABLE can be applied to the following problems:

1) Zero sequence current distribution along the sheath of a three conductor cable by replacing the three phase conductors by an equivalent conductor.
2) Fault current distribution along the sheath of a single phase (one conductor) cable,

3) Zero sequence current distribution along a group of three single phase cables assuming that the sheaths are transposed and bonded at regular intervals. In this case, the conductors and the sheaths will be represented by their equivalents in the model.

If the user of the program is supplying raw cable segment data, it is necessary to identify the problem so that the self and mutual impedances with ground return in the model (i.e., \(Z_C\), \(Z_S\) and \(Z_M\)) are correctly determined by the program. Carson's formulae, in which the ground return path is represented by an equivalent conductor, are utilized by the program to determine self and mutual impedances. These formulae constitute a useful and straightforward way to compute the zero sequence network for the system under consideration because the theory of geometric mean distance (GMD) can be employed easily.

The program outputs include the sheath voltages at each node and sheath currents at both ends of each segment. Fault current contributions of each terminal station and the sheath voltage at the fault location are also printed. When necessary, sheath current and voltage at the center of each segment may be calculated and printed.

If preferred, results can be stored on a disk and be plotted on the OSU Versatec plotter by executing another Fortran program which reads the stored results as input data.

Due to the double precision computations to prevent truncation errors and complex hyperbolic functions involved, H-extended Fortran
compiler is used. The source program is first compiled and stored on a
disk. Another program with the input data is linked to the load module
to obtain the results. The listing and JCL of the source program and
the program for the plotting routine are shown in Appendix C.

The simulated cable can have several sections. A section is
defined as a cable portion with one or more segments which have the same
soil resistivity, cable impedances and geometry. The program is capable
of solving the fault current distribution along 90 cable segments and 10
different cable sections. If the system under consideration makes it
necessary to go beyond these upper limits, the dimension statements in
the source program should be modified and their associated parameters
should be reinitialized.

It should be emphasized that the assumptions regarding the model
and the method are not restrictive. Although the general methodology
assumes that the fault occurs at a ground rod and it is fed by two
source stations, the program can automatically handle other situations
as well. In the case of one source station, the model and corresponding
expressions are simplified as derived in Section 5.3.4. If there is
only one source station, it is assumed to be the left terminal station
as shown in Fig. 5.2. For the ladder network extending beyond the fault
location towards the right terminal station, only $Z_{22}$ entry of the driv­
ing point impedance matrix is to be calculated. If the fault is between
two consecutive grounding rods, the program assigns a fictitious ground­
ing rod with a very high resistance at the fault location. The program
is flexible and by properly changing one or more variables in the input
data other cases such as a cable with no ground rods or a cable with
open neutral may also be simulated. Overhead transmission lines can be
handled easily by the program. In this case, all definitions and vari-
ables related to the cable sheath and grounding rods should be inter-
preted as referring to the transmission line ground wire and towers,
respectively. Section 6.3 will cover these special cases in detail.

The computer program has been found to be numerically stable for
all cases and parametric studies simulated, within the reasonable and
practical values of the parameters.

6.2 BASE CASE AND RESULTS

The segment-by-segment distribution of the fault current along an
underground cable is influenced by a number of variables. In order to
determine the sensitivity of the distribution to each significant vari-
able, first a specific cable system will be defined as the base case and
simulated using the computer program CABLE. Then the influence of each
parameter will be investigated separately by changing the value of that
parameter in the base case input data and leaving all other parameters
constant.

The cable system chosen for the base case is a three conductor
oil-filled paper insulated cable extending between two terminal
stations. The network configuration is as shown in Figure 21. The
cable is 6300 m long. The rated voltage is 69 kV. The cable sheath is
grounded with grounding rods of resistance 20 ohms at every 300 m. A
uniform soil resistivity of 100 ohm-m is assumed. The cable specifi-
cations are taken from Reference [10,p.81] and are as follows:
Conductor resistance = 0.113 ohm/mile = 0.070x10^{-3} ohm/m.
Sheath resistance = 0.442 ohm/mile = 0.274x10^{-3} ohm/m.
GMR (of a conductor) = 0.327 inches = 8.31x10^{-3} m.
GMD (of the conductors) = 1.244 inches = 3.16x10^{-2} m.
Average sheath radius = 1.655 inches = 4.20x10^{-2} m.
Capacitive reactance = 4740 ohm-mile = 7.62x10^6 ohm-m.

Each terminal station is represented with a source impedance of 0.1+j1.0 ohms and a grounding grid impedance of 1.0 ohm. It is assumed that the sheath has a semiconducting jacket on it and the sheath admittance to ground is 0.001 mho/m. The input data for the base case in metric units is given in Appendix 8.3. The zero sequence self and mutual impedances with ground return of the network model in Figure 23 are determined by CABLE.

A ground fault between a phase conductor and the sheath at the 8th grounding rod location has been simulated on the program CABLE. The computer output is illustrated in Appendix C.4. The sheath current distribution along the cable is plotted and shown in Figure 27 (curve 2). The sheath current profile of the same cable with an insulated sheath ($Y_s = 10^{-11}$ mho/m) is also shown for comparison purposes (curve 1). The magnitude of the sheath currents are given in per-unit in terms of the fault current. Actual sheath currents can be calculated by multiplying the per-unit values by the magnitude of the fault current. Distances in Figure 27 are measured from source #1 which is the left source station shown in Figure 23.

The results show that the greatest changes in the magnitude of the sheath current occur in the close vicinity of the fault location and at the terminal stations. These sections will be referred to as end effect
sections. For both cases, end effects last 3 to 6 segments at the fault. End effects at the terminal stations are less pronounced for the cable with an insulated sheath than the one with a semiconducting jacket on the sheath. The vertical jumps on the profiles denote the sheath current leakage to the ground through the grounding rods. The insulated sheath results in a steplike function for the sheath current distribution. If the cable sheath is not well insulated from the ground, as seen in Curve 1 in Figure 27, some portion of the sheath current leaves (or enters) the sheath between the grounding rods as well.

Figure 28 illustrates the angle of the sheath currents vs. distance from the left source station for two values of sheath admittances. The angle of the fault current should be added to those values in the figure in order to determine the actual angle of the sheath currents. The real and imaginary components of the sheath current if the fault current is assumed to be 1.0 + j0.0 p.u. are illustrated in Figures 29 (for $Y_S = 10^{-3}$ mho/m) and 30 (for $Y_S = 10^{-11}$ mho/m). Figures 31 and 32 show the sheath voltage profiles of the cables with noninsulated sheath ($Y_S = 10^{-3}$ mho/m) and insulated sheath ($Y_S = 10^{-11}$ mho/m), respectively. The sheath voltages are also given in per-unit. In order to calculate the actual voltages, the values in per-unit should be multiplied by the magnitude of the fault current. Unlike sheath current profiles, sheath voltages produce a continuous profile along the cable. The analysis of Figures 31 and 32 reveals that:

1) Sheath voltages are highest at the faulted grounded rod,

2) In the vicinity of the terminal stations, the sheath voltage increases slightly due to the end effect phenomena.
Figure 27 Base case sheath current distribution.
Figure 28  Angle of sheath currents, base case.
BASE CASE THREE CONDUCTOR CABLE
LENGTH OF CABLE = 6300 M.
FAULT DISTANCE = 2400 M
NO. OF SOURCES = 2
SHEATH CURRENT: (1) REAL COMPONENT
(2) IMAGINARY COMPONENT

Figure 29  Real and imaginary components of sheath currents,
Y_s = 10^{-3} mho/m.

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Figure 30 Real and imaginary components of sheath currents,
\( Y_s = 10^{-11} \) mho/m.
BASE CASE THREE CONDUCTOR CABLE
LENGTH OF CABLE=6300 M.
FAULT DISTANCE=2400 M
NO. OF SOURCES=2
SHEATH ADMITTANCE=0.1E-02 MHO/M

Figure 31 Sheath voltage profile, $Y_s = 10^{-3}$ mho/m.
**Figure 32** Sheath voltage profile, $Y_s = 10^{-11}$ mho/m.
ii) Grounding rod currents can be calculated by dividing the sheath voltages at grounding points by the rod resistances. Hence, the segments where the sheath voltages are very low have grounding rods with very low currents flowing through them. Furthermore, these segments correspond to the flat portion of the sheath current profiles where there are no significant changes (i.e., vertical discontinuities) in the magnitude of the sheath currents.

iv) Sheath voltages for the cables with insulated sheaths are higher than the cables with noninsulated sheaths.

The influence of the capacitive coupling between the phase conductors and the cable sheath on the fault current distribution has also been investigated by utilizing the base case cable system introduced in this section. The program CABLE was executed for the data in which the only difference from the base case is the value of the capacitive reactance. The results obtained for a very high capacitive reactance (i.e., $10^{12}$ ohm-m) are compared to those in Figure 27 which corresponds to the actual capacitive reactance. It was found that the effect of capacitive reactance on the sheath currents is negligible.

6.3 CASE STUDIES AND RESULTS

The following cases can be simulated using the computer program CABLE:

- Cable with one source station,
- Fault between two adjacent grounding rods,
- Transmission lines,
- Cable without grounding rods along the sheath,
- Cable with an open neutral.
- Single phase cable.

Each case represents a situation possibly encountered in the real world. The simulations of these cases will be done by using the base case test system which has been introduced in the previous section. The modifications to be done on the input data of the base case will be described separately.

6.3.1 Cable with One Source Station

If only one of the terminal stations is a source station, the entire zero sequence current will be supplied from that station (assuming there is no grounded star-delta transformer at the other terminal station).

The only change in the input data is inserting 1 for the value of the variable which defines the number of source stations (NSOUR). All the other data will be identical to the base case cable system. The source station is assumed to be the terminal station on the left side in Figure 23.

The sheath current distribution for two different values of sheath admittances introduced in the base case are shown in Figure 33. The results reveal that a large portion of the fault current returns to the source station through the sheath. The sheath current flowing towards the terminal station beyond the fault decays to a very small value due to the current leakage through the grounding rods and, if the sheath is not insulated, through the continuous contact between the ground and the sheath. For the insulated cable considered (curve 1), the terminal
station beyond the fault experiences a ground potential rise due to the current leaking into the ground through the grounding grid.

6.3.2 Fault Between Two Adjacent Grounding Rods

The ground faults in a cable occur almost randomly at a location anywhere along the cable. The main reasons for the faults are insulation failures and accidental diggings.

Once the fault location is specified by the user, the computer program determines if it occurs at a grounding rod. If not, the program inserts a fictitious grounding rod at the fault location. The resistance of that rod is assigned a very high value. The input data, in this case, are rearranged for the simulation before the method is applied. However, the results are given in terms of the actual segment numbering as defined by the user.

The base case cable system is simulated for a ground fault between the 9th and 10th grounding rods at a location 2800 m away from the source station #1. The sheath current profile is illustrated in Figure 34. The shape of the curves is similar to that of the base case shown in Figure 27. The fault distance in the base case was assumed as 2400 m. Thus, the length of the cable beyond the fault and the zero sequence impedance of that side seen from the fault location decreases. This results in a higher portion of the fault current to flow towards the terminal station #2 which is beyond the fault (the increase referred to the base case is approximately 5%).
TWO CONDUCTOR CABLE

LENGTH OF CABLE=6300 M.
FAULT DISTANCE=2400 M
NO. OF SOURCES=1
RESISTANCE OF RODS=20. OHMS
SOIL RESISTIVITY=100 OHM-M
SHEATH ADMITTANCE= (1):0.1E-10 MHO/M
(2):0.1E-02 MHO/M

Figure 33 Sheath current distribution, one source station.
Figure 34 Sheath current distribution when fault distance is 2800 m.

LENGTH OF CABLE = 6300 M.
RESISTANCE OF GR. RODS = 20 OHM
FAULT DISTANCE = 2800 M
NO. OF SOURCES = 2
SOIL RESISTIVITY = 100 OHM-M
SHEATH ADMITTANCE = 0.1E-02 MHO/M
6.3.3 Transmission Line

The proposed model can also represent a three phase transmission line with a ground wire. The sheath parameters in the model should be replaced with those of the ground wire. In addition to the input variables necessary for a cable case, the geometric mean distance between the phase conductors and the ground wire should also be defined in order to find self and mutual impedances with ground return.

To check the validity of the program, first a test transmission line will be defined. Then the same line will be simulated on both the program CABLE and the computer program, TEST, which was written for a study of fault current distribution along transmission lines [33]. Since TEST also assumes a fault current magnitude of 1.0, the results can be compared easily.

For this study, the tower spacing and tower resistance along the line are assumed to be uniform and they are equal to 268 m and 10 ohms, respectively. Rated source voltage is 345 kV and the length of the transmission line simulated is 15 miles. Soil resistivity is assumed to be uniform along the line and is equal to 10 ohm-m. The shunt admittance of the ground wire to ground is neglected. In the program, this is achieved by inserting a very small value for $Y_g$ (e.g., $1.0 \times 10^{-12}$). The tower, the phase conductor and ground wire are the same as the test system in Ref. [33]. They are tower type P-2, Bluebird and Drake conductors, respectively, and their data are taken from Ref. [36]. The input data cards for this case are shown in Appendix B.3.

The results of the simulations for a fault at the 30th tower and with two source stations are tabulated in Table 1. The ground wire current and tower voltage values are given only at some arbitrarily
chosen spans. The results from both programs show excellent agreement and the maximum discrepancy is less than 1%. This discrepancy is due mainly to truncation errors which arise from the difference in methodology: The proposed method solves a differential equation to determine the ground wire current distribution in each segment, whereas Gooi accepted a constant ground wire current in each segment.

Table 1 Transmission line simulation results, ACSR ground wire.

<table>
<thead>
<tr>
<th>Span</th>
<th>RESULTS OF CABLE</th>
<th>RESULTS OF TEST [33]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ground wire</td>
<td>Tower voltage</td>
</tr>
<tr>
<td></td>
<td>Current</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.2450</td>
<td>0.03624</td>
</tr>
<tr>
<td>2</td>
<td>0.2414</td>
<td>0.03149</td>
</tr>
<tr>
<td>20</td>
<td>0.2349</td>
<td>0.1205</td>
</tr>
<tr>
<td>29</td>
<td>0.4753</td>
<td>0.6687</td>
</tr>
<tr>
<td>30</td>
<td>0.5284</td>
<td>0.8094</td>
</tr>
<tr>
<td>31</td>
<td>0.4120</td>
<td>0.6687</td>
</tr>
<tr>
<td>90</td>
<td>0.1227</td>
<td>0.02117</td>
</tr>
</tbody>
</table>

The results of the ground wire current distribution along the transmission line are shown in Figure 35. Different case studies have also been conducted on the transmission line system presented. Some representative current profiles are shown in Figures 36-39. These figures reflect modifications made on the input data of Figure 35 as follows:
Figure 35 Transmission line ground wire currents.
Figure 36: Same ACSR ground wire and fault location as for Figure 35, but with only one source station.

Figure 37: ACSR ground wire and with only one source station, fault at the 70th tower.

Figure 38: Steel ground wire (EHS steel wire of diameter \( \frac{3}{8} \) inch = 0.95 cm, data taken from [36,p.82]),

Figure 39: Steel ground wire with only one source station.

From the previous studies, it is well known that the span-by-span distribution of the fault current between return paths shows a changing pattern along the sections of the transmission line in the vicinity of the fault and source station. These line sections are called end effect sections. End effect phenomena can be clearly observed in Figures 35-39. The following conclusions can also be stated by examining those plots:

i) Different ground wire materials result in different values for the constant ground wire current (represented by the practically flat portion of the curves). The ground wire current in the case of ACSR ground wire is higher than the steel ground wire case due to the higher steel wire impedance. Also, the end effects are more pronounced when ACSR ground wire is utilized; the length of the end effect section is 10-20 spans for ACSR ground wire, and 4-8 spans for steel ground wire.
Figure 36 Ground wire currents, one source station.
Figure 37  Ground wire currents when fault occurs at the 70th tower.
Figure 38  Ground wire currents, steel ground wire.
Figure 39  Ground wire currents, steel ground wire, one source station.
ii) The constant value of the ground wire current becomes more explicit as the distance between the source station and the fault increases over 20-30 spans depending on the ground wire material.

iii) The length and the degree of end effects at the terminal stations are much smaller than those at the fault location. Similar results were obtained by Sebo [31] and Gooi [33].

6.3.4 Cable Without Grounding Rods

A cable which does not have any grounding rods along the sheath can be simulated by introducing at least one fictitious grounding rod of very high resistance along the cable route. These fictitious rods can be assumed to exist at arbitrary locations, but it is suggested to distribute them uniformly. They will not influence the current distribution but will play a role in the process of segment numbering. Since the computer output gives two values for the sheath current in each segment (i.e., the initial and final segment currents), more points for the current profile will be obtained by utilizing the fictitious grounding rods.

The current profiles for a cable without any grounding rods are given in Figure 40 for two different sheath admittances. For a cable with an insulated sheath (Curve 1), the sheath current magnitude is constant in both directions of the fault. The entire fault current returns to the terminal stations through the sheath. On the other hand, for a cable with a semiconducting layer on the sheath (Curve 2), some portion of the fault current will leak to the ground, and consequently return
Figure 40 Sheath currents in a cable without grounding rods.
to the terminal station through the ground and the substation grounding grid. This will cause a higher potential at the terminal station neutral. The results reveal that the ground potential rise at the substation with respect to the remote ground is 0.011 p.u. for the first cable and 0.131 p.u. for the second.

For an insulated cable without any grounding rods there will be no ground leakage currents and this situation will result in higher touch voltages and lower step voltages along the cable than for a cable with noninsulated sheath.

6.3.5 Cable with an Open Neutral

An open neutral is a discontinuity in the neutral conductor. For cables, particularly bare buried cables, the open neutral is the result of sheath corrosion. The simulation of an open neutral in a cable segment can be achieved by increasing the sheath resistance of that segment to a very high value such that negligibly small sheath current will flow through that segment.

Figures 41-44 illustrate the sheath current profiles for the following cases:

1) Cable with a semiconducting layer on the sheath ($Y_s = 10^{-4}$ mho/m), open neutral in the 7th segment, fault fed by two source stations.

2) Insulated cable ($Y_s = 10^{-11}$ mho/m), open neutral in the 7th segment, fault fed by two source stations.

3) Cable with a semiconducting layer on the sheath ($Y_s = 10^{-4}$ mho/m), open neutral in the 5th segment, fault fed by one source station.
iv) Insulated cable \((Y_s = 10^{-11}\, \text{mho/m})\), open neutral in the 5th segment, fault fed by one source station.

The other relevant data are identical to the base case cable system.

The results show that in the presence of an open neutral on one side of the fault, significantly greater portion of the fault current will flow towards the terminal station on the other side of the fault. The segment with an open neutral has zero sheath current, as expected. The portion of the cable sheath between the open neutral and the terminal station picks up some current from the ground through grounding rods. For noninsulated cables, the magnitude of sheath current returning into the source station is higher than the insulated cables. This is due to the additional ground current picked up through the sheath surface. Thus, the current distribution and the ground potential rise at the terminal station are strongly influenced by the location of the open neutral, number of source stations, and the sheath admittance to ground.
THREE CONDUCTOR CABLE
LENGTH OF CABLE=6300 M
FAULT DISTANCE=2400 M
NO. OF SOURCES=2
OPEN NEUTRAL IN 7TH SEGMENT
SHEATH ADMITTANCE=0.1E-03 kH/II

Figure 41 Sheath currents in a cable with an open neutral.
Figure 42 Sheath currents, open neutral in an insulated cable.
THREE CONDUCTOR CABLE
LENGTH OF CABLE=6300 M
FAULT DISTANCE=2400 M
NO. OF SOURCES=1
OPEN NEUTRAL IN 5TH SEGMENT
SHEATH ADMITTANCE=0.1E-03 MHO/M

Figure 43 Open neutral, one source station.
Figure 44  Open neutral in an insulated cable, one source station.
6.3.6 Single Phase Cable

In this section, two example problems covered in Reference [35] will be simulated by using the computer program CABLE, and the results will be discussed and compared to those obtained in [35].

The data related to these two cases are as follows:

1) A 800 m long, 8660 V, single phase cable with concentric neutral is connected to a substation with an overhead line extension which is 400 m long. The cable sheath has a semi-conducting jacket on it and it is grounded with eight grounding rods of length 3 m each and equally spaced along the cable. Soil resistivity is 100 ohm-m.

2) The same configuration applies to a cable with an insulated sheath which is grounded with eight grounding rods of length 4.2 m long, equally spaced.

The general network configuration is as shown in Figure 19. The data related to the source station, overhead line and cable parameters are also specified. Fault is assumed to occur at the first grounding rod (i.e., at the source end of the cable). The sheath voltage at the fault is given for both cases and the magnitude of the fault current can also be computed by using a simplified equivalent circuit suggested in [35,p.C-1]. These are:

For the first cable: $I_f = 4538.7 \, A$, $V_f = 226 \, V$.

For the second cable: $I_f = 4494.2 \, A$, $V_f = 839.6 \, V$.

where $I_f$ and $V_f$ are the magnitudes of the fault current and sheath voltage at the fault location, respectively.
The two cases presented above are simulated on CABLE. The input data format is shown in Appendix B.3. The values of grounding rod resistances in the input data are obtained from the following equations [37]:

\[
R = \frac{\rho}{2\pi e} \left( \ln \frac{4l}{d} - 1 \right) \text{ ohms}
\]  

(187)

where \( \rho \) is the soil resistivity in ohm-m, \( l \) is the length of the rod in m, and \( d \) is the diameter of the rod in m. The diameter of the rods is assumed to be 1.25 cm. However, this assumption does not influence the rod resistances significantly. The grounding rod resistances are determined as 34.3 ohms for a 3 m long rod and 25.7 ohms for a 4.2 m long rod.

The sheath voltage profiles for the cable with a semiconducting jacket on the sheath (Case 1) and the insulated cable (Case 2) are shown in Figure 45. Sheath voltages are given in terms of volts because the fault current magnitudes of each case are known. The results reveal that:

1) The sheath voltage at the fault for the insulated cable is higher than that for the other cable. The fault voltages found by CABLE are:

\[ V_f = 182.1 \text{ volts in Case 1,} \]
\[ V_f = 767.6 \text{ volts in Case 2.} \]

The differences between the fault voltages computed by CABLE and those given in [35] are 19% and 9% for the Case 1 and Case 2, respectively.
SINGLE PHASE CABLE
LENGTH OF OHL=400. M
LENGTH OF CABLE=800. M.
FAULT DISTANCE=400. M
SHEATH VOLTAGES: (1) CASE 1
(2) CASE 2

Figure 45 Sheath voltages, single phase cables.
ii) The ground potential rise at the substation neutral for the cable with insulated sheath is lower than that for the other cable. However, the insulated cable has high sheath voltages.

6.4 PARAMETRIC STUDIES AND RESULTS

A relatively large number of parameters influences the fault current distribution between neutral conductors and the ground. It is important for system studies to identify these parameters and to understand their relative importance. Furthermore, this information is of practical value because some of these parameters are controllable. The controllable parameters which will be covered in this section are: resistance of grounding rods, sheath admittance to ground, and substation grounding grid impedance. In addition to these factors, the effect of soil resistivity and fault location on the fault location will also be investigated. Sheath voltage at the fault location gives a relative indication to the overall sheath voltage profile, and hence to step and touch voltages along the cable route. All the parametric studies will be performed on the base case cable system.

6.4.1 Soil Resistivity

The soil under the surface of the earth is not homogeneous and its resistivity varies within extremely wide limits, between 1 and 10,000 ohm-m. In problems involving the ground as a return conductor, such as in the studies of the grounding and inductive interference, it is usually necessary to determine the soil resistivity in the geographic area under consideration. The soil resistivity may sometimes be estimated
from geological maps based on the results of tests for similar
geological formations. In the technical literature, there are numerous
experimental methods to measure the soil resistivity [21].

The main factors which determine the soil resistivity are: type of
soil, moisture content, and chemical composition of salts in the soil.
Table 2 lists typical values of resistivity of some soils [37].

Table 2 Typical values of soil resistivity.

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Soil resistivity in ohm-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garden soil</td>
<td>5 - 50</td>
</tr>
<tr>
<td>Clay</td>
<td>8 - 50</td>
</tr>
<tr>
<td>Clay, sand and gravel</td>
<td>40 - 250</td>
</tr>
<tr>
<td>Sand and gravel</td>
<td>60 - 100</td>
</tr>
<tr>
<td>Sandstone</td>
<td>10 - 500</td>
</tr>
<tr>
<td>Rocks</td>
<td>200 - 1000</td>
</tr>
</tbody>
</table>

Figures 46-48 show the sheath current profiles for two values of
soil resistivity, i.e., 10 and 1000 ohm-m. Each figure represents the
results for a different sheath admittance. The following observations
are made from these figures:

1) Whatever the sheath admittance to ground is, larger soil
resistivities result in higher sheath current magnitudes.
In other words, a larger portion of the fault current
enters the source station through the sheath if the soil
resistivity is increased.
LENGTH OF CABLE=6300 M.
SHEATH ADMITTANCE=0.1E-02 MHO/M
FAULT DISTANCE=2400 M
NO. OF SOURCES=2
SOIL RESISTIVITY= (1):10 OHM-M
(2):1000 OHM-M

Figure 46 Sheath currents, effect of soil resistivity,
\[ Y_s = 10^{-3} \text{ mho/m}. \]
Figure 47  Sheath currents, effect of soil resistivity, $Y_s = 10^{-4}$ mho/m.
Figure 48 Sheath currents, effect of soil resistivity.

\[ Y_s = 10^{-11} \text{ mho/m.} \]
(ii) The soil resistivity does not influence the shape of the current profile and the length of the end effect section significantly.

(iii) The magnitude of the ground current depends not only on the soil resistivity but also on the sheath admittance to ground. Insulated sheath (i.e., $Y_s = 10^{-11}$ mho/m) presents a high resistance to the flow of currents from the sheath into ground. Hence, steplike functions corresponding to the sheath current profile are obtained for insulated cables (Figure 48).

6.4.2 Resistance of Grounding Rods

A common type of ground for underground transmission consists of one or more vertical ground rods, mostly 1.5 to 3 meters long and 1.25 and 2.5 cm in diameter. The self resistance of a vertical rod when the radius is small compared to the length is given by Eq. (187). Rewriting it:

$$R = \frac{\rho}{2\pi d} \ln \left( \frac{4\pi l}{d} - 1 \right) \text{ ohms}$$

where $\rho$, $l$, and $d$ are as defined in (187). The diameter of the rod has only a small effect on the resistance. On the other hand, an increase in length always produces a lower resistance. The resistance of rods of various lengths and diameters, as calculated from the above equation for $\rho = 100$ ohm-m, is given in Table 3.

The base case cable system has been simulated for four different values of sheath admittances to ground and the results are illustrated in Figures 49-52. In each figure, the sheath current profiles for two
LENGTH OF CABLE=6300 M.
SHEATH ADMITTANCE=0.1E-02 MHO/M
FAULT DISTANCE=2400 M
NO. OF SOURCES=2
RESISTANCE OF GR. RODS= (1): 10 OHM
(2): 1000 OHM

Figure 49 Sheath currents, effect of grounding rod resistance, $Y_s = 10^{-3}$ mho/m.
Figure 50 Sheath currents, effect of grounding rod resistance, $Y_s = 10^{-4}$ mho/m.
LENGTH OF CABLE = 6300 M.
SHEATH ADMITTANCE = 0.1E-04 MHO/M
FAULT DISTANCE = 2400 M
NO. OF SOURCES = 2
RESISTANCE OF GR. RODS = (1) : 10 OHM
                      (2) : 1000 OHM

Figure 51 Sheath currents, effect of grounding rod resistance, $Y_s = 10^{-5}$ mho/m.
LENGTH OF CABLE = 6300 M.
SHEATH ADMITTANCE = 0.1E-10 MHO/M
FAULT DISTANCE = 2400 M
NO. OF SOURCES = 2
RESISTANCE OF GR. RODS = (1): 10 OHM
(2): 100 OHM

Figure 52 Sheath currents, effect of grounding rod resistance, $Y_s = 10^{-11}$ mho/m.
Table 3 Typical values for grounding rod resistances in ohms ($p = 100$ ohm-m).

<table>
<thead>
<tr>
<th>Radius (cm)</th>
<th>Length of the rod (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td>0.625</td>
<td>132</td>
</tr>
<tr>
<td>1.25</td>
<td>113</td>
</tr>
<tr>
<td>2.5</td>
<td>95</td>
</tr>
</tbody>
</table>

different grounding rod resistances are shown. The results can be summarized as follows:

i) The grounding rod resistance has decreasingly less influence on the current distribution as the sheath admittance to ground increases. As seen in Figure 49, the sheath current profiles for rod resistances of 10 and 1000 ohms look almost identical for a bare cable sheath.

ii) Insulated cables with grounding rods of very high resistances will tend to behave as if there are no grounding rods along the sheath. The same observation is valid also for bare cables with grounding rods of reasonable resistances (refer to Figure 40).

iii) For a given sheath admittance to ground, grounding rod currents are inversely proportional to grounding rod resistances. Hence, increasing grounding rod resistance
results in a more smooth current profile with less currents in the grounding rods.

iv) The effect of the grounding rod resistance on the step and touch voltages along the cable route should be analyzed in connection with the sheath admittance. For insulated cables, decreasing the grounding rod resistance yields lower touch voltages and higher step voltages. Bare cables, on the other hand, present a low impedance to ground, and hence they result in lower touch voltages than the insulated cables for a given grounding rod resistances.

6.4.3 Fault Location

Figure 53 shows the sheath current profiles when a ground fault occurs at two different locations along the cable. The specified distances between the fault and the left source station are 1500 m and 3000 m. The division of the fault current at the fault location between the possible return paths (i.e., towards the left or right source stations) depends upon the relative magnitude of zero sequence impedance of each path. In general, higher portion of the fault current will flow through the return path which has a lower impedance.

It is also observed from Figure 53 that changing the fault location does not influence the shape of the profile at the end effect sections significantly.
Figure 53 Sheath currents, effect of fault location.
6.4.4 Substation Grounding Grid Resistance

Recently, an increasing emphasis has been given to the design of safe substation grounding. The design objective of substation grounding is to attain acceptable ground potential rise at the grounding grid under short circuit conditions on the power system. The grounding grid potential rise is equal to the grounding resistance multiplied by the current flowing from the grounding system into ground. However, a low resistance substation grounding grid is not, by itself, a guarantee for safety. An accurate calculation of the portion of the fault current which flows through the substation grounding grid is also necessary. The driving point impedance method proposed in Chapter 5 is capable of determining the currents through the substation grounding grid and grounding structures.

Substation grounding grid resistance mainly depends on the configuration of the grid and soil resistivity. Typically its value is between 0.5 and 10 ohms.

The base case cable system is simulated with the program CABLE for two different values of substation grounding grid resistances, 0.1 and 10 ohms. Figures 54 and 55 contain the sheath current profiles for two different values of sheath admittance. These figures reveal that:

i) Changing the substation grounding grid resistance does not change the current profile in the vicinity of the fault significantly, but influences the current distribution close to substations.

ii) Increasing the grid resistance will result in a higher portion of the fault current returning to the substation neutral through the cable sheath. This could also be clarified by
the following numerical results:

<table>
<thead>
<tr>
<th>$I_{st}$</th>
<th>$Z_{st}$</th>
<th>$Y_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.5%</td>
<td>0.1</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>4.4%</td>
<td>10</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>16.9%</td>
<td>0.1</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>4.4%</td>
<td>10</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

where $I_{st}$ is the grounding grid current in percent of the total fault current, $Z_{st}$ is the grounding grid resistance in ohms, and $Y_s$ is the sheath admittance in mho/m.
Figure 54 Sheath currents, effect of substation grounding grid resistance, $Y_s = 10^{-3}$ mho/m.
Figure 55 Sheath currents, effect of substation grounding grid resistance, $Y_s = 10^{-11}$ mho/m.
6.5 SHIELDING FACTOR COMPUTATIONS

In Chapter 3, the shielding factor has been defined as:

\[
\eta = \left| \frac{V_r}{V_p} \right| \tag{188}
\]

where \(V_r\) is the resultant voltage on a telecommunication wire and \(V_p\) is the voltage induced when no shielding is present. If it is assumed that there is a telecommunication wire parallel to an underground power cable and the mutual impedance between the wire and cable sheath is equal to the mutual impedance between the wire and cable conductor, then Eq. (188) can be rewritten as:

\[
\eta = \left| \frac{I_c - I_s}{I_c} \right| \tag{189}
\]

where \(I_c\) and \(I_s\) are conductor and sheath currents, respectively. If the wire exposure is limited to the \(i\)th segment,

\[
\eta_i = \left| \frac{I_{c_i} - I_{s_i, \text{eff}}}{I_{c_i}} \right| \tag{190}
\]

where \(\eta_i\) is the effective shielding factor in the \(i\)th segment. \(I_{s_i, \text{eff}}\) is the effective sheath current in the \(i\)th segment and it was defined in (124) as:

\[
I_{s_i, \text{eff}} = \frac{1}{d_i} \int_{0}^{d_i} I_{s_i}(x) \, dx
\]
Since the conductor and sheath currents change from segment to segment, a new shielding factor definition which takes the end effects into account will be given. The shielding effectiveness of the cable sheath for an exposure between the kth and jth segments will be measured with an effective shielding factor which is:

\[ n_{\text{eff}}(k,j) = \frac{\sum_{i=k}^{j} d_i (I_{ci} - I_{si,\text{eff}})}{\sum_{i=k}^{j} d_i I_{ci}} \] (191)

where \( d_i \) is the length of the ith segment in m.

The proposed method and the computer program CABLE are capable of solving conductor and sheath currents in each segment. The effective sheath current in the ith segment, \( I_{si,\text{eff}} \), can also be determined (See Eq. (127)). In order to calculate the effective shielding factor, there is no need to know the magnitude of the fault current. Sheath and conductor currents in Eq. (191) can be in per-unit.

The effective shielding factors of the two cables introduced in the base case cable system are determined using the base case simulation results. Assuming that the telecommunication wire exposure always starts at the source station, the effective shielding factors with respect to the exposure length have been calculated for two different cables. The network configuration is identical to the base case. One of the cables has a sheath admittance of \( 10^{-3} \) mho/m and the other one
Since the segment lengths are assumed to be uniform in the base case, Eq. (191) turns out to be:

\[ n_{\text{eff}}(1, j) = \frac{\sum_{i=1}^{j} (I_{ci} - I_{si,\text{eff}})}{\sum_{i=1}^{j} I_{ci}} \]  \hspace{1cm} (192)

Figures 56 and 57 show the effective shielding factors with respect to the length of exposure which is equal to \( j \) times the segment length. The end effect phenomena observed in sheath current profiles are apparent in these figures as well. The shielding factor decreases if more current flows through the sheath. Shielding factors for both cables reach maximums at the segments which yield constant sheath currents (i.e., minimums in Figure 27). It is also observed that the cable with insulated sheath has lower shielding factors, and hence is more effective in reducing the inductive interference on the telecommunication circuits.

In summary, this chapter has investigated various case and parametric studies conducted on the computer program developed. A possible application of the proposed method and the program CABLE, calculating the shielding factor of underground cables, has also been covered and illustrated on a cable system.
Figure 56 Shielding factor, $Y_s = 10^{-3}$ mho/m.
Base Case Three Conductor Cable
Shielding Factor
Fault Distance=2400 M
No. of Sources=2
Sheath Admittance=0.1E-10 mho/m

Figure 57 Shielding factor, Y_s = 10^{-11} mho/m.
CHAPTER VII

CONCLUSIONS AND SUGGESTED FUTURE WORK

7.1 CONCLUSIONS

In this dissertation, segment-by-segment distribution of the ground fault current along the sheath of an underground cable has been investigated. First, a new zero sequence equivalent circuit of a cable segment was developed. The equivalent circuit consists of an equivalent phase conductor, the cable sheath, and grounding structures, and it utilizes both lumped and distributed parameters. The distribution of sheath currents in a cable segment was determined by solving differential equations relating sheath currents and voltages. Then, a new systematic method which starts at both terminal stations and successively approaches toward the fault location was proposed. The suggested method solves the piecewise continuous sheath current distribution along the cable through the computation of driving point impedances.

Calculations related to the model have been performed on a computer; various case studies and parametric studies have been conducted. The effects of soil resistivity, sheath admittance to ground, grounding rod resistances, fault location, number of source stations and substation grounding grid impedance on the sheath current distribution have been examined. It was found that:
i) The number, location and length of grounding rods should be carefully designed by considering the sheath admittance to ground and soil resistivity. For some values of the sheath admittance, the grounding rods may turn out to be redundant and useless. For example, the current leakage into ground through grounding rods of resistances larger than 100 ohms is negligible when the sheath admittance is 0.001 mho/m.

ii) Larger soil resistivities result in higher sheath current magnitudes.

iii) The substation grounding grid impedance does not influence the end effects in the vicinity of the fault, but plays an important role on the ground potential rise at the substation neutral.

iv) The more insulated the cable sheath from ground is, the higher the sheath voltage is. The parameters which determine the degree of the sheath insulation from the ground are the sheath admittance, and the number and resistances of the grounding rods.

v) Cables with insulated sheaths have higher touch voltages than the noninsulated cables. A qualitative analysis of the results also shows that the more insulated the cable sheath from the ground is, the less the step voltages on the ground surface are. For an insulated cable sheath without any grounding rods, step voltages are practically equal to zero because ground leakage currents are negligibly small.

vi) Cables with insulated sheaths are more effective in reducing the inductive interference on the telecommunication circuits.
SUGGESTED FUTURE WORK

The proposed method can be applied to a variety of problems, where both lumped and distributed parameters exist, with proper modifications of the main program. One of the possible applications, computing the shielding factor of the cable sheath, has been covered in Section 6.5. Other possibilities are as follows:

1. The fault current can be computed directly by the model. This can be achieved by replacing the current source in the equivalent circuit by a voltage source. This capability will enhance the practical value of the computer program.

2. The grounding rod currents which can be determined by the proposed method can be accepted as known driving functions (i.e., injections injections into the ground). The ground potential at a point at the ground surface along the cable route can be computed by summing the potentials which result from each injection. Hence, step and touch voltages at the ground surface can be easily obtained.

3. The equivalent circuit representation of a cable and the method can be utilized for the computation of neutral conductor currents under transient or impulse conditions. The program CABLE is capable of calculating the model parameters at frequencies different than 60 Hz by utilizing Carson's simplified equations for ground return impedances (Refer to Appendix A). Those equations are valid for all frequencies used in power transmission, including higher harmonics in the range of audio frequencies (e.g., up to about 5 KHz). At high frequencies where the assumptions in simplifying Carson's original formulation are not valid, the user should compute and supply the accurate self and mutual
impedances with ground return. Carson’s original formulation neglects displacement currents in the ground and is valid up to 100 KHZ.

4. The general methodology can be extended to include the mutual coupling effects between phase conductors. Instead of replacing the phase conductors by an equivalent conductor, the phase conductors can be shown separately in the equivalent circuit. Hence for a three conductor cable, the resultant induced voltage along a sheath segment will depend on the phase conductor currents and the mutual impedances between the conductors and the sheath. Sheath current in the ith segment at a distance of x unit length from the ith grounding rod (in this case) will be:

\[ I_s(x) = A\cosh\beta_i x + B\sinh\beta_i x + \frac{\sum(Z_{mi}i_{mi})}{Z_{s_i}} \]  

(193)

where \( \sum(Z_{mi}i_{mi}) = Z_{asi}i_{ai} + Z_{bsi}i_{bi} + Z_{csi}i_{ci} \)  

(194)

and the subscripts a,b,c, and s denote the three phase conductors and the sheath, respectively. \( Z \) with any two of these subscripts is the mutual impedance per unit length between the conductors specified by the subscripts. \( Z \) with one subscript is the self impedance per unit length of that conductor. The following equations can be derived at \( x = 0^+ \), similar to Eqs. (120) and (121):

\[ I_s(0^+) = I_{s_0} = C_1i_{s_1} + \frac{Y_{s_1}S_i}{B_i}v_{s_1} + \frac{1-C_1}{Z_{s_1}}\sum(Z_{mi}i_{mi}) \]  

(195)
The phase conductor voltages at the (i+1)st node can be written in terms of the conductor voltages and currents at the ith node. For the sake of completeness, the approach presented in Chapter 5 will be applied to phase a. The effective sheath current in the ith segment will be:

\[
V_{ai+1} = V_{ai} - Z_{ai} I_{ai} - Z_{as} I_{si,\text{eff}} + Z_{ab} I_{bi} + Z_{ac} I_{ci}
\]  

Writing similar equations for phases b and c, and considering them with Eqs. (196) and (198), yield a matrix equation which relates the voltages at the (i+1)st node to the voltages and currents at the ith node. This equation is:

\[
\vec{v}_{i+1} = Z_A \vec{v}_i + Z_B \vec{i}_i
\]

where

\[
\vec{v}_{i+1} = [V_{ai+1} \ V_{bi+1} \ V_{ci+1} \ V_{si+1}]^T
\]

\[
\vec{v}_i = [V_{ai} \ V_{bi} \ V_{ci} \ V_{si}]^T
\]

\[
\vec{i}_i = [I_{ai} \ I_{bi} \ I_{ci} \ I_{si}]^T
\]
$Z_A$ and $Z_B$ are $4\times 4$ matrices whose entries are determined from the $i$th segment cable parameters.

Defining $Z_i$ as the driving point impedance matrix at the $i$th node, and assuming that this $4\times 4$ matrix is known at the $i$th node as:

$$\tilde{V}_i = Z_i \tilde{I}_i$$  \hspace{1cm} (200)

$\tilde{V}_{i+1}$ in Eq. (199) can be written only in terms of $I_i$. Writing node equations at the $i$th node and expressing $\tilde{I}_{i+1}$ in terms of $\tilde{I}_i$, will result in the driving point impedance matrix at the $(i+1)$st node, $Z_{i+1}$:

$$\tilde{V}_{i+1} = Z_{i+1} \tilde{I}_{i+1}$$

such that the entries of $Z_{i+1}$ can be determined from the entries of $Z_i$ and the $i$th segment cable parameters.

Hence, the driving point impedance method can be utilized to compute the sheath currents and phase conductor currents in each segment. Although this approach will result in an improvement for the method, it should also be noted that the memory requirements will increase. For symmetrical (or perfectly transposed) circuits, the results of the proposed method will be sufficiently accurate.
APPENDIX A

IMPEDANCE OF GROUND RETURN CONDUCTORS

The most widely known derivation for the impedance of ground return conductors was developed by Carson [15,16]. His formulation considers a primary conductor of infinite length carrying a constant current, I, which returns through a homogeneous semi-infinite earth with a uniform soil resistivity, \( \rho \). Carson's general equation for the mutual impedance between the primary conductor and another parallel conductor is [20]:

\[
Z_m = \frac{j\omega \rho}{2\pi} \ln \frac{D}{d} - 2j \int_0^\infty \sqrt{u^2 + j^2 - u} e^{-u(a_1 + a_2)} \cos(ua) du
\]

where
- \( Z_m \) = mutual impedance per m,
- \( a \) = horizontal separation between conductors, m
- \( h_1 \) = height of the primary conductor above ground (negative if below ground), m
- \( h_2 \) = height of the second conductor above ground (negative if below ground), m
- \( d \) = distance between conductors, m
- \( D \) = distance between the primary conductor and the image of the second conductor on the surface of the ground, m
- \( \rho \) = soil resistivity, ohm-m
\[ \omega = 2\pi f, \text{ where } f \text{ is the frequency} \]

\[ \alpha = \sqrt{\frac{\mu_0 \omega}{\rho}} \]

\[ \mu_0 = 4\pi \times 10^{-7} \text{ H/m.} \]

Carson also presented methods for the numerical evaluation of the complex integral above. These methods employ infinite series expansions. For power frequency applications and close separations between conductors, simplified approximate equations can be used [24]. The mutual impedance with common ground return between conductors, \( Z_m \), is given as,

\[
Z_m = \frac{j \omega \mu_0}{4\pi} \left[ 2 \ln \left( \frac{2}{g \alpha d} \right) + 1 - \frac{j}{2} \frac{2\sqrt{2}}{3} \right] \alpha (h_1 + h_2) \quad \text{(A1)}
\]

where \( Z_m \) is in ohm/m and \( g = 1.781 \). This equation is valid as long as \(|\alpha| < 0.5 \) and \( \alpha (h_1 + h_2) < 0.1 \). The self impedance of a nonferromagnetic conductor of radius \( r \) at a height \( h_1 \) is:

\[
Z_c = R_c + \frac{\omega \mu_0}{4\pi} \left( \frac{\pi}{2} + \frac{4\sqrt{2}}{3} \alpha h_1 \right) + \frac{j \omega \mu_0}{4\pi} \left( 2 \ln \frac{2}{g \alpha r} + \frac{3}{2} + \frac{4\sqrt{2}}{3} \alpha h_1 \right) \quad \text{(A2)}
\]

where \( Z_c \) and \( R_c \) are the self impedance and resistance of the conductor per m, respectively. This equation is valid provided that \( \alpha |h_1| < 0.1 \).

Equations (A1) and (A2) will be referred to as Hand-Calculable Carson (HCC) equations. HCC equations can be further simplified because the effect of the heights of the conductors on the self and mutual
impedances with ground return is negligible. For overhead transmission lines and cables buried close to the surface of the earth, the simplified Carson (SC) equations are given as [18,p.143]:

\[
Z_m = 0.988 \cdot 10^{-3} f + j2.894 \cdot 10^{-3} f \log \frac{D_g}{d} \text{ ohm/km} \quad (A3)
\]

\[
Z_c = R_c + 0.988 \cdot 10^{-3} f + j2.894 \cdot 10^{-3} f \log \frac{D_g}{GMR_c} \text{ ohm/km} \quad (A4)
\]

where \(GMR_c\) is the geometric mean radius of the conductor in m, and \(D_g\) is the equivalent depth of the fictitious conductor in which ground return currents are assumed to flow. \(D_g\) is determined as:

\[
D_g = 658.4 \sqrt{\frac{\rho}{f}} \text{ m.}
\]

For 60 Hz applications, Eqs. (A3) and (A4) become:

\[
Z_m = 0.0593 + j0.1736 \log \frac{D_g}{d} \text{ ohm/km}
\]

\[
Z_c = R_c + 0.0593 + j0.1736 \log \frac{D_g}{GMR_c} \text{ ohm/km}
\]

The analysis of the mutual and self impedances with ground return of parallel lines of finite length is discussed in detail in [24]. The mutual impedance of two parallel conductors at ground level, which are insulated from each other except at their ends and are of equal length, is given as:
\[ Z_m = j2\omega \left[ \frac{\mu_0}{4\pi} M_C(0,\alpha d) + \frac{\mu_0 F(\alpha l,\alpha d) - \mu_0 F(0,\alpha d)}{4\pi \alpha l} \right] \text{ ohms} \quad (A5) \]

where \( \alpha \) is the length of each conductor in m. \( M_C(\alpha l,\alpha d) \) and \( F(\alpha l,\alpha d) \) are functions of \( \alpha l \) and \( \alpha d \) and they represent the numerical values of complex integrals attained in the analysis. The self impedance \( Z_C \) of the circuit formed by a conductor of length \( \alpha \) and radius \( r \) placed on but insulated from ground except at its ends (where it is connected to ground through resistances \( R_1 \) and \( R_2 \)) is given by:

\[ Z_C = (R + j\omega \frac{\mu_0}{8\pi} \alpha l) + j\omega \frac{\mu_0 \alpha l}{4\pi} \left[ 2 \ln \frac{2}{g_{ar}} + 1 - j \frac{\pi}{2} \right. \]

\[ - \frac{2}{\alpha l} (F(0,\alpha r) - F(\alpha l,\alpha r)) \right] \alpha l + R_1 + R_2 - \frac{\rho}{\pi \alpha l} \text{ ohms} \quad (A6) \]

The functions \( M_C(0,z) \) and \( F(y,z) \) are available in a tabulated form for different values of \( y \) and \( z \) \([24, p.210]\). In order to implement the short-length Carson equations, Eqs. (A5) and (A6), on a computer, linear regression analysis has been applied to the given values of those functions by utilizing SAS General Linear Models Procedure. The best fitting curves for the real and imaginary components of \( M_C(0,z) \), \( F(0,z) \), and \( F(y,z) \) are as follows:

For \( 10^{-5} < z < 10^{-2} \) and \( 0 < y < 9 \):

\[ M_C(0,z) = M_R + jM_I \]

\[ M_R = 1031.3 - 3.96 \cdot 10^5 z + 8.97 \cdot 10^7 z^2 - 5.5 \cdot 10^9 z^3 \]

\[ M_I = \begin{cases} -79 & , \text{if } z < 0.01 \\ -79 + 7.88z + 11.88z^2 & , \text{if } z > 0.01 \end{cases} \]
\[ F(0, z) = F_{OR} + jF_{OI} \]

\[ F_{OR} = \begin{cases} 
-141, & \text{if } z < 0.001 \\
-141 - 102.6z + 36.1z^2 - 11.5z^3, & \text{if } z > 0.01 
\end{cases} \]

\[ F_{OI} = \begin{cases} 
-141, & \text{if } z < 0.1 \\
-142.2 + 10.9z + 4.8z^2, & \text{if } z > 0.1 
\end{cases} \]

\[ F(y, z) = F_R(y) + jF_I(y) \]

\[ F_R(y) = \begin{cases} 
0, & \text{if } y > 3.5 \\
126.6 - 168.2y + 75.1y^2 - 10.8y^3, & \text{if } y < 3.5 
\end{cases} \]

\[ F_I(y) = -137.8 + 62.2y - 10.8y^2 + 0.6y^3 \]

Then, self and mutual impedances of a single conductor cable with ground return have been determined by utilizing three different approaches; HCC, SC, and SLC (short-length Carson) equations. The data related to conductor and sheath resistances, and geometry of the cable are given in Section 6.3.6. Table 4 shows the self and mutual impedances for three different cable lengths. When utilizing HCC equations, it is assumed that the cable is buried 1m below the earth surface. The soil resistivity is taken as 10. ohm-m.

The numerical results in Table 4 reveal that as the length of the cable increases, the differences between these methods diminish. HCC and SC equations result in almost same values for all cable lengths considered. The maximum difference between the values obtained with these two methods is less than 2.5%. Self and mutual impedances computed by SLC equations are smaller than those computed by SC
equations. The maximum discrepancy between the results of SC and SLC equations occurs in the real component of the mutual impedance when the cable length is 300 m. However, the differences between other components are only about 10% and they decrease as the length of the cable increases.

**TABLE 4**

Self and mutual impedances with ground return of a single conductor cable in ohms.

<table>
<thead>
<tr>
<th>Length</th>
<th>Method</th>
<th>( Z_c )</th>
<th>( Z_m )</th>
<th>( Z_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 m</td>
<td>HCC</td>
<td>0.076+j0.246</td>
<td>0.018+j0.224</td>
<td>0.417+j0.230</td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td>0.076+j0.246</td>
<td>0.018+j0.224</td>
<td>0.417+j0.224</td>
</tr>
<tr>
<td></td>
<td>SLC</td>
<td>0.066+j0.231</td>
<td>0.008+j0.210</td>
<td>0.407+j0.215</td>
</tr>
<tr>
<td>700 m</td>
<td>HCC</td>
<td>0.177+j0.574</td>
<td>0.042+j0.523</td>
<td>0.973+j0.536</td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td>0.177+j0.574</td>
<td>0.041+j0.523</td>
<td>0.973+j0.523</td>
</tr>
<tr>
<td></td>
<td>SLC</td>
<td>0.163+j0.559</td>
<td>0.029+j0.516</td>
<td>0.959+j0.521</td>
</tr>
<tr>
<td>1300 m</td>
<td>HCC</td>
<td>0.328+j1.066</td>
<td>0.078+j0.971</td>
<td>1.807+j0.996</td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td>0.328+j1.066</td>
<td>0.077+j0.972</td>
<td>1.806+j0.972</td>
</tr>
<tr>
<td></td>
<td>SLC</td>
<td>0.314+j1.050</td>
<td>0.064+j0.961</td>
<td>1.790+j0.981</td>
</tr>
</tbody>
</table>
The effect of the self and mutual impedance calculating methods on the fault current distribution has been investigated by applying SC and SLC expressions to a cable network and implementing on CABLE. The cable simulated is assumed to be 6300 m. long and have a sheath admittance of 0.001 mho/m. The cable sheath is grounded by 20 rods of 20 ohms at every 300 m. The fault current distribution along the cable has been computed for the following two cases:

1) Raw cable data are given as input to CABLE which calculated the self and mutual impedances by utilizing SC equations.
2) Self and mutual impedances per unit length calculated by SLC equations for a cable length of 300 m are given as input to CABLE.

The sheath current profiles obtained for these two cases are shown in Figure 58. The input data cards for both cases are given in Appendix B.3.

Figure 58 reveals that the choice of SC or SLC equations in calculating the self and mutual impedances with ground return for the cable segment length assumed does not influence the fault current distribution significantly. Both approaches yield almost identical end effects. The magnitudes of sheath currents at the flat portions of the curves differ by about 5%. This discrepancy can be explained by:

1) the assumptions and simplifications utilized in SC expressions,
2) the errors in \( M_C \) and \( F \) function values which have been rounded [24].
iii) the errors due to extrapolating and modeling the $M_C$ and $F$
function values (i.e., the errors due to best fitting curves
utilized).

The choice between SC and SLC expressions in determining the self
and mutual impedances depends on the cable parameters and length of the
cable segment. It can be stated that SC equations will yield the fault
current distribution with sufficient accuracy for cables longer than 300 m.
Figure 58 Sheath current distribution, self and mutual impedances determined by two different methods.
APPENDIX B

INPUT DATA IDENTIFICATION

B.1 INPUT DATA VARIABLES

The computer program CABLE is written in FORTRAN. The input data should be given in metric units. If the user does not provide the zero sequence self and mutual impedances related to the power line (overhead transmission line or underground cable), the program has the capability of computing these by utilizing simplified Carson equations. The program, as it is, can accept data for up to 90 line segments (spans) and 10 sections. Input data variables are as follows:

- NSOUR : Number of source stations, 1 or 2,
- NGR : Number of grounding structures,
- FR : Frequency, Hz,
- ZSL, ZSR : Equivalent source impedance of the left and right terminal station, respectively, ohms,
- ZSTL, ZSTR : Substation grounding grid impedance of the left and right terminal station, respectively, ohms,
- RGR(I) : Resistance of Ith grounding structure, I=1, ..., NGR, ohms,
- DF : Distance between the fault location and the left terminal station, m,
- DSPAN(I) : Length of the Ith segment, I=1, ..., NGR+1, m,
IIMP : An integer identifier; it is positive if the user is supplying raw cable data, an it is negative if the user is supplying zero sequence self and mutual impedances,

IPLOT : An integer identifier; it is 1 if the user wants to store the results for plotting purposes, and it is 0 (or blank) otherwise.

If IIMP > 0, then the following data for the line section between (FROM)th and (TO)th segments are also needed:

IID : An integer identifier; it is positive if the following data are for a cable section, and it is negative if they are for a transmission line section,

NCON : An integer identifier, it is 1 if the data are for three single conductor cables, it is 2 for a single conductor cable (or single phase transmission line), and it is 3 for a three conductor cable (or a three phase transmission line with one ground wire),

SOILR : Soil resistivity in that section, ohm-m,

GMDCW : GMD between phase conductors and ground wire in m, defined only if IID < 0 and NCON = 3,

RC : Resistance of the phase conductor, ohm/m,

RS : Resistance of the neutral conductor, ohm/m

YS : Shunt admittance from the neutral conductor to ground, mho/m,

GMR : Geometric mean radius of the phase conductor, m,

GMD : Geometric mean distance between phase conductors, m,

AVER : Average sheath radius if it is a cable section, and geometric mean radius of the ground wire if it is a transmission line, m
ZCAP : Shunt impedance between the phase and neutral conductors, m.

If IIMP < 0, in addition to IID, NCON, SOILR, YS and ZCAP defined above, the following data for the line section between (FROM)th and (TO)th segments are needed:

ZC, ZS : Zero sequence self impedance with ground return of the equivalent phase conductor and the neutral conductor, ohm/m.

ZM : Zero sequence mutual impedance with common ground return between the phase and neutral conductors, ohm/m.
B.2 INPUT DATA FORMAT

Input data for the base case cable system (Figure 27, Curve 2):

<table>
<thead>
<tr>
<th>ZSL, ZSTL, ZSR, ZSTR</th>
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</thead>
<tbody>
<tr>
<td>NSOUR, NGR, FR, IIMP, IPLOT</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>RC, RS, YS, GMR, GMD, AVER, ZCAP</th>
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</thead>
<tbody>
<tr>
<td>FROM, TO, IID, NCON, SOILR</td>
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</tbody>
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B.3 Input Data for Test Cases

Figure 27, Curve 1:

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Figure 35:

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999

215
Figure 38:

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999
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Figure 40, Curve 2:

```
2 20 60.0 1  1
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1 21 1 3 100.
7.02E-05 .0002747 1.07E-03 .00306 .03160 .043 0.0 -7.63D+06
999
216
### Figure 45, Curve 2:

|     | 1.7600E+01 | 2.3 | 0.0 | 0.0 | 0.0 | 34.3 | 34.3 | 34.3 | 34.3 | 34.3 | 34.3 | 34.3 | 34.3 | 114.3 | 114.3 | 114.3 | 114.3 | 114.3 |
|-----|------------|-----|-----|-----|-----|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|
| 0.18| 1.42       |     |     |     |     | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 25.7 | 0.0  |     |     |       |       |       |       |       |
| 25.7| 25.7       | 25.7| 25.7| 25.7| 25.7| 25.7 | 25.7 | 25.7 | 25.7 | 25.7 | 25.7 | 25.7 |     |     |       |       |       |       |       |
| 399.9|399.9       |     |     |     |     | 114.3| 114.3| 114.3| 114.3| 114.3| 114.3| 114.3|     |     |       |       |       |       |       |
| 114.3|114.3       |     |     |     |     |     |     |     |     |     |     |     |     |     |       |       |       |       |       |

### Figure 45, Curve 1:

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218
Figure 58, Curve 1:

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Figure 58, Curve 2:

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APPENDIX C

SOURCE PROGRAM AND CONTROL WORDS

C.1 SOURCE PROGRAM AND ITS JCL

// JOB ,
// REGION=512K,MSGCLASS=A,TIME=(0,05)
//*JOPARM LINES=8000,DISKIO=1362
//61 EXEC FORTOQCL
//SYSPRINT DD SYSOUT=A
//FORT,SYSIN DD *

C******************************************************************************
C
C******************************************************************************
C
C STUDY OF ZERO SEQUENCE CURRENT DISTRIBUTION ALONG
C UNDERGROUND CABLES
C
C******************************************************************************

DIMENSION RCR(90),DSPAN(91),ISPI(10),NCON(10),IID(10)
DIMENSION RC(10),RS(10),YR(10),CMR(10),CMD(10),AVER(10),GMDCM(10)
DIMENSION ROCR(92),OOSP(92),IOSP(92)
INTEGER PROM(10).TO(IO)
INTEGER NCR,NSPAN,NX,NR,XN,NOCR,NOSP,NNL,NNR
REAL XS,XM,DE,ZC
COMPLEX ZSL,ZSR,ZSTL,ZSTR,ZCAP(10)
COMPLEX*16 ZC(I0),ZH(10),ZS(10)
COMPLEX*16 Z11(94),Z12(94),Z21(94),Z22(94),MN(92),MN(92),CNJ,CNR
REAL VS(92),VH(91),SC(91),SCM(91),SCM(91),VCM(92)
REAL VMAC(95),VMAC(95)
COMPLEX*16 A1,A2,A3,A4,B1,B2,B3,C1,C2,C3,C4,C5,C6,G1,G2,D2,AZ,ZCS(93),CC(91)

C******************************************************************************
C
C******************************************************************************

CREAD INPUT DATA, ALSO DETERMINE: NSPAN-MGR+1=NO. OF SEGMENTS
MSE=NO. OF SECTIONS
ISF(NSPAN)

C******************************************************************************

READ(5,11) NSOUR,NCR,FR,IMP,IPLOT
11 FORMAT(213,F5.1,213)
READ(5,12) ZSL,ZSTL,ZSR,ZSTR
12 FORMAT(8C9.2)
READ(5,13) (NCR(I),I=1,MGR)
13 FORMAT(7F10.4)
NSP=NSP+1
READ(5,14) DF,(DSPAN(I),I=1,NSPAN)
14 FORMAT(7F10.2)
NS=1

220
IF(IIK.LT.0) GO TO 852
16 READ(5,18) FROM(NS),TO(NS),IID(NS),NCON(NS),SOILR(NS),CHMDW(NS)
18 FORMAT(413,F10.4,F10.4)
IF(FROM(NS).EQ.999) GO TO 22
READ(5,851)RC(NS),RS(NS),TB(NS),CHM(NS),CHMD(NS),AVEX(NS),ZCAP(NS)
851 FORMAT(6F9.4,2C9.2)
GO TO 855
852 READ(5,853)FROM(NS),TO(NS),IID(NS),NCON(NS),SOILR(NS)
853 FORMAT(413,F10.4)
IF(FROM(NS).EQ.999) GO TO 22
READ(5,854)RF(NS),TS(NS),ZH(NS),ZCAP(NS)
854 FORMAT(4G9.2,F8.2,4G9.2)
C
855 IFR=FROM(NS)
IT=TO(NS)
DO 20 I=IFR,IT
ISP(I)=NS
20 CONTINUE
NS(NS+1)
GO TO 16
22 NSEC(NS-1)
C
23 CONTINUE
WRITE(6,21)
21 FORMAT(15X,'--------------------------------------------------------------------------------------
* DISTRIBUTION OF ZE SEQUENCE CURRENTS ALONG UNDERGROUND CABLES'//6X,'--------------------------------------------------------------------------------------')
C
C CHECK IF FAULT OCCURS AT A CR. ROD. THEN KF=0, OTHERWISE KF=1.
C USING DF AND DSPAN(NSPAN), DETERMINE KF, KFNN, FDIST.
C
D$=0.
K=0
DO 24 I=1,NSPAN
IF(DSPAN(I).EQ.0.) GO TO 46
IF(ISP(I).EQ.0.) GO TO 48
DSUM=DSUM+DSPAN(I)
KFN=I
IF(DF.EQ.DSUM) GO TO 30
IF(DF.LT.DSUM) GO TO 25
24 CONTINUE
GO TO 40
25 KFN=I-1
KF=1
FDIST=DF-(DSUM-DSPAN(I))
C
C CHECK IF NCON(I) IS 1, 2 OR 3 IN EACH SECTION
C SINGLE PHASE CABLE IF NCON=2
39 GO TO 55
40 WRITE(6,41)
41 FORMAT(//6X,'ERROR : MISTAKE IN INPUT DATA'//6X,'FAULT DISTANCE IS
# GREATER THAN THE CABLE LENGTH')
GO TO 900
221
42 WRITE(6,43)
43 FORMAT(6X,'ERROR : NUMBER OF SOURCES IS NOT EQUAL TO 1 OR 2'/)
        GO TO 900
44 WRITE(6,45)
45 FORMAT(6X,'ERROR : # GROD OR # SECTIONS ARE HIGHER THAN LIMIT
        8TS'/6X,'INCREASE THE DIMENSIONS ACCORDINGLY')
        GO TO 900
46 WRITE(6,47) I
47 FORMAT(6X,'ERROR : ',I2,'TH SEGMENT LENGTH IS ZERO'/)
        GO TO 900
48 WRITE(6,49) I
49 FORMAT(6X,'ERROR : ',I2,'TH SEGMENT DATA IS UNDEFINED'/)
        GO TO 900

C ****************************************************************************************************************
C PRINT THE INPUT DATA
C
C ****************************************************************************************************************
53 CONTINUE
54 WRITE(6,56) NSOURL,NGR
55 FORMAT(6X,'NUMBER OF SOURCES= ',I1//6X,'NUMBER OF GROUNDING RODS= '
        0,12)
56 IF(KF.EQ.1) GO TO 62
57 WRITE(6,61) KFN,D
58 FORMAT(6X,'FAULT OCCURS AT THE ',I3,'TH GROD '/6X,'DISTANCE FROM THE FAULT FROM SOURCE 1 = ',F10.1,' M'/)
        GO TO 64
59 IF(KFN.EQ.0) GO TO 864
60 WRITE(6,63) KFN,D
61 FORMAT(6X,'FAULT OCCURS AFTER THE ',I3,'TH GROD '/6X,'DISTANCE FROM THE FAULT FROM SOURCE 1 = ',F10.3,' M'/)
        GO TO 64
62 VRITE(6,65)
63 FORMAT(6X,'FAULT OCCURS BETWEEN THE TERMINATING STATION 1 AND T
        HE 1ST GROD '/6X,'DISTANCE OF THE FAULT FROM SOURCE 1 = '
        X,F10.1,' M'/)
64 WRITE(6,65)
65 FORMAT(6X,'RESISTANCE OF GROD (OHMS) : ')
        WRITE(6,66) (I,RCR(I),I=1,NGR)
66 FORMAT(6X,'LENGTH OF EACH SEGMENT (M) : ')
        WRITE(6,67) (J,DSPAN(J),J=1,NSPAN)
67 FORMAT(6X,'RESISTANCE OF GROD (OHMS) : ')
        WRITE(6,68) (J,DSF(J),J=1,NSSPAN)
68 FORMAT(6X,'LENGTH OF EACH SEGMENT (M) : ')
        WRITE(6,69) (J,ZT(J),J=1,NTZ)
69 FORMAT(6X,'INPUT SEGMENT DATA: '/6X,'FROM TO WCON RC (OHMS/
        PH) RS (OHMS/M) YS (OHMS/M) SOIL RES. CMR-COND.\,9X
        #,?,GND-3C',4X,'AVE. SH. RAD.' )
        DO 74 J=1,NSEC
        WRITE(6,70) FROM(J),TO(J),WCON(J),RC(J),RS(J),YS(J),SOILR(J),GMR(J)
        GND(J),AVS(J)
70 FORMAT(4X,12,2X,12,6X,11,7(4X,D11.5))
74 CONTINUE
C ****************************************************************************************************************
C IF KF=1, ADD A FICTITIOUS GROD AT THE FAULT LOCATION
C REARRANGE DATA TO OBTAIN NOSP,NGR,NOCR,DOSP,NOSP
C NF*KF+1 IS THE FAULTED RED GROD
C
C
115 IF(NCON(I).EQ.1) GO TO 116
  XS=DLOG10(DE/AVER(I))
  XM=XS
  ZS(I)=DCMPLX(DBLE(IM*RE),IM*CST*XH)
  ZM(I)=DCMPLX(DBLE(IM*RS(I)+3*RE),IM*CST*XM)
  GO TO 120
116 G3S=(AVER(I)*GHD(I)**2)**(1./3)
  XS=DLOG10(DE/G3S)
  XM=XS
  ZS(I)=DCMPLX(DBLE(RE(I)+3*RE),3*CST*XM)
  ZM(I)=DCMPLX(DBLE(3*RE),3*CST*XM)
C
120 CONTINUE
C
C PRINT SECTION ZERO-SEQ. IMPEDANCES
C
129 WRITE(6,130)
130 FORMAT(///6X,'SEGMENT ZERO-SEQUENCE IMPEDANCES :','/2X,'FR
90, FROM TO',10X,'ZC',21X,'ZM',23X,'ZS',21X,'ZCAP',16X,'YS '/17X,'("REA
2L IMG",12X),"REAL")
  DO 132 I=1,NSEC
    WRITE(6,131) FROM(I),TO(I),ZC(I),ZM(I),ZS(I),ZCAP(I),YS(I)
131 FORMAT(4X.I2,2X.I2,1X,4(C10.A.1X,G10.4,4X),D10.A)
132 CONTINUE
C
135 FORMAT(///6X,'IMP',NOSP,NSEC,NF,NP/IPLOT
90, IMP = ',I2,' NOSP = ',I2,' NSEC = ',I2,' NF = ',I2,' IPLOT =
2L 0',11//)
140 CONTINUE
C
C DRIVING-POINT IMPEDANCE METHOD
C
C NEED : GHD,ZS,ZM,ZS,ZSRC,ZSTR
C
C FIND : MN AND MH FOR EACH SEGMENT
ZCS,Z11,Z12,Z21,Z22 FOR EACH NODE
C
--------------------------------------------------------------------------------------------------------------------------
NNT=NOSP+2
NML=NF+1
NMR=NOSP-NP+1
C
C APPLY THE METHOD TO THE LEFT SIDE OF THE FAULT
C
C INITIALIZE AT THE LEFT SOURCE
C
K=IOSP(1)
ZCS(1)=ZCAP(K)**2/DOSP(1)
Z11(1)=(ZCS(1)*ZSL/(2ZCS(1)+ZSL))
Z12(1)=ZSTL
Z21(1)=ZSTL
Z22(1)=ZSTL
C
I=NODE AT THE JTH CR.ROD , J-SEGMENT COUNTER
DO 320 I=2,NML
  KK=IOSP(I)
  J=I-1
  K=IOSP(J)
  BM(K)=CDMSQRT(YS(K)*ZS(K))
  U=DOSP(J)
  BMU=BM(K)**U
  CN(J)=(CDEXP(BMU)+CDEXP(-BMU))/2.
  SN(J)=(CDEXP(BMU)-CDEXP(-BMU))/2.
  PH=SN(J)/BMU

224
\[ VN = BN(K) \times SN(J) / YS(K) \]
\[ AUX1 = ZCAP(K) \times 2 / DOSP(J) \]
\[ AUX2 = ZCAP(KK) \times 2 / DOSP(J) \]
\[ ZCS(I) = AUX1 \times AUX2 / (AUX1 + AUX2) \]
\[ IF(I.EQ.NNT) \quad ZCS(I) = 1.0D+24 \]
\[ ZMS = ZH(K) / ZS(K) \]
\[ MH = (CN(J) - 1) \times ZMS \]
\[ AUX1 = YS(K) \times SN(J) / BN(K) \]
\[ C1 = CN(J) + AUX1 \times Z22(J) \]
\[ C2 = (1 - CN(J)) \times ZMS + AUX1 \times Z21(J) \]
\[ C3 = VH + CN(J) \times Z22(J) \]
\[ C4 = CN(J) \times Z21(J) - VN \times ZMS \]
\[ C5 = Z21(J) - ZC(K) \times U \times (1 - PN) \times U \times ZM(K) + BH \times Z21(J) \]
\[ G1 = 1 / (ROGR(J) + 1) / ZCS(I) \]
\[ MH(J) = 1 / ((C1 + C3 + C5) / ZCS(I)) \]
\[ MN(J) = MN(J) \times (-C2 - C1 \times C4 + C6 / ZCS(I)) \]
\[ D1 = 1 / ((C4 + C3 + MN(J) - C6 \times MN(J)) / ZCS(I)) \]
\[ D2 = D1 \times (C5 \times MN(J) - C3 \times MN(J)) / ZCS(I) \]
\[ AUX1 = C5 \times MN(J) + C6 \]
\[ AUX2 = C3 \times MN(J) + C4 \]
\[ Z11(I) = D1 \times AUX1 \]
\[ Z12(I) = D2 \times AUX1 + C5 \times MN(J) \]
\[ Z21(I) = D1 \times AUX2 \]
\[ Z22(I) = D2 \times AUX2 + C3 \times MN(J) \]

320 CONTINUE
C
C PRINT DR.-PT. IMP. MATRIX OF THE LEFT SIDE IF NEEDED
C
330 CONTINUE
C
C APPLY THE METHOD TO THE RIGHT SIDE
C
K = IOSP(MOSP)
ZCS(NNT) = ZCAP(K) \times 2 / DOSP(MOSP)
ZAUX = ZCS(NNT) \times ZSR / (ZCS(NNT) + ZSR)
Z11(NNT) = (ZAUX + ZSTR)
Z12(NNT) = ZSTR
Z21(NNT) = ZSTR
Z22(NNT) = ZSTR

DO 350 IB = 2, NNR
I = NNT - IB + 1
J = I + 1
K = IOSP(J)
JJ = J - 1
KN(K) = CDQRT(YS(K) \times ZS(K))
U = DOSP(J)
BN = BN(K) \times U
CN(J) = (CDEXP(BNU) + CDEXP(-BNU)) / 2.
BN(J) = (CDEXP(BNU) - CDEXP(-BNU)) / 2.
PN = SN(J) / BNU
VN = BN(K) \times SN(J) / YS(K)
IF(NSTK.EQ.1) GO TO 345
C
AUX1 = ZCAP(K) \times 2 / DOSP(J)
AUX2 = ZCAP(KK) \times 2 / DOSP(J)
ZCS(I) = AUX1 \times AUX2 / (AUX1 + AUX2)
IF(IB.EQ.NNR) \quad ZCS(I) = 1.0D+24
ZMS = ZH(K) / ZS(K)
MH = (CN(J) - 1) \times ZMS

225
380 CONTINUE

SOLVE SHEATH SEGMENT CURRENTS AND NODE VOLTAGES
STARTING FROM THE FAULT TOWARDS THE TERMINAL STATIONS

FIND THE SHEATH VOLTAGE AT THE FAULT LOCATION
VS(NNL) = Z11(NNL) * CCL + Z22(NNL) * SCL
VC(NNL) = Z11(NNL) * CCL + Z12(NNL) * SCL
VSMAG(NNL) = DSQRT((DREAL(VS(NNL)))**2 + DIMAG(VS(NNL)))**2)
WRITE(6,485) VSMAG(NNL)

485 FORMAT(//6X,'SHEATH VOLTAGE AT THE FAULT = ',$D11.4/)

LEFT SIDE, TOWARDS TERMINAL STATION 1:

WRITE(6,488)

488 FORMAT(//6X,'SEGMENT DISTANCE FROM',6X,'SHEATH CURRENT',10X,'SHEATH CURRENT',12X,'SHEATH VOLTAGE'/9X,'NO.',6X,'STATION #1:',5X,'IMAG.:',14X,'REAL',14X,'IMAG',14X,'MAG.'//)

UU=DF
AY=CCL
AZ=SCL
DO 500 IB=1,WF
J=WF+1-IB
I=I+1
IF(IB NE.1) AZ=SCE(I)
IF(IB NE.1) AY=CC(I)
CC(J)=AY+(VC(I)-VS(I))/ZCS(I)
SCI(J)=AZ-VC(I)/ROGR(J)+((VC(I)-VS(I))/ZCS(I)
SCE(J)=BN(J)+AZ-NN(J)*CC(J)
VSC(J)=Z11(J)*CC(J)+Z22(J)*SCE(J)
VC(J)=Z11(J)+CC(J)+Z12(J)*SCE(J)
VSMAG(J)=DSQRT((DREAL(SCI(J)))**2+(DIMAG(SCI(J)))**2)
VSMAG(J)=DSQRT((DREAL(SCE(J)))**2+(DIMAG(SCE(J)))**2)

FIND MID-SEGMENT SHEATH VOLTAGE AND CURRENT
USING NODE VOLTAGES AT THE ENDS AS BOUNDARY CONDITIONS

489 K=10SP(J)
B=VS(K)-VS(I)/BN(K)
A=VS(J)-CM(J)+VS(I)*VS(K)/(BN(K)*SN(J))
U=DOSP(J)/2.
BNU=BN(K)*U
GU=(CDEXP(BNU)+CDEXP(-BNU))/2.
SU=(CDEXP(BNU)-CDEXP(-BNU))/2.
SCH(J)=ACU+B*SU+ZM(K)*CCL/ZS(K)
VM(J)=(ASU+B*CU)-(-BN(K)/TA(K))
VSMAG(J)=DSQRT((DREAL(SCH(J)))**2+(DIMAG(SCH(J)))**2)
VSMAG(J)=DSQRT((DREAL(VH(J)))**2+(DIMAG(VH(J)))**2)

WRITE(6,493) J, UU, VSMAG(J), SCI(J), VSMAG(1), UUM, SCH(J), SCH(V)
500 CONTINUE
380 CONTINUE

DO 150 J=1,9

IF (KJ.LE.15) GO TO 12

IF (KJ.LE.15) GO TO 15

IF (KJ.LE.15) GO TO 18

IF (KJ.LE.15) GO TO 21

IF (KJ.LE.15) GO TO 24

IF (KJ.LE.15) GO TO 27

IF (KJ.LE.15) GO TO 30

IF (KJ.LE.15) GO TO 33

IF (KJ.LE.15) GO TO 36

IF (KJ.LE.15) GO TO 39

IF (KJ.LE.15) GO TO 42

IF (KJ.LE.15) GO TO 45

IF (KJ.LE.15) GO TO 48

IF (KJ.LE.15) GO TO 51

IF (KJ.LE.15) GO TO 54

IF (KJ.LE.15) GO TO 57

IF (KJ.LE.15) GO TO 60

IF (KJ.LE.15) GO TO 63

IF (KJ.LE.15) GO TO 66

IF (KJ.LE.15) GO TO 69

IF (KJ.LE.15) GO TO 72

IF (KJ.LE.15) GO TO 75

IF (KJ.LE.15) GO TO 78

IF (KJ.LE.15) GO TO 81

IF (KJ.LE.15) GO TO 84

IF (KJ.LE.15) GO TO 87

IF (KJ.LE.15) GO TO 90

IF (KJ.LE.15) GO TO 93

IF (KJ.LE.15) GO TO 96

IF (KJ.LE.15) GO TO 99

IF (KJ.LE.15) GO TO 102

IF (KJ.LE.15) GO TO 105

IF (KJ.LE.15) GO TO 108

IF (KJ.LE.15) GO TO 111

IF (KJ.LE.15) GO TO 114

IF (KJ.LE.15) GO TO 117

IF (KJ.LE.15) GO TO 120

IF (KJ.LE.15) GO TO 123

IF (KJ.LE.15) GO TO 126

IF (KJ.LE.15) GO TO 129

IF (KJ.LE.15) GO TO 132

IF (KJ.LE.15) GO TO 135

IF (KJ.LE.15) GO TO 138

IF (KJ.LE.15) GO TO 141

IF (KJ.LE.15) GO TO 144

IF (KJ.LE.15) GO TO 147

IF (KJ.LE.15) GO TO 150

CALL END

END
C
C ***********************************************************************
C STORE DATA FOR PLOTTING PURPOSES
C XMAG: DISTANCE FROM SOURCE #1, YMAG: VARIABLE TO BE PLOTTED
C NM: NUMBER OF DATA POINTS
C
C
SUM=0.
NM=0
DO 595 I=1, NMSP
NM=NM+1
XMAG(NM)=SUM
YMAG(NM)=SCEMAG(I)
IF (I.GT.NF) YMAG(NM)=SCIMAG(I)
NM=NM+1
SUM=SUM+DOSP(I)/2.
XMAG(NM)=SUM
YMAG(NM)=SCEMAG(I)
SUM=SUM+DOSP(I)/2.
590 NM=NM+1
SUM=SUM+DOSP(I)
XMAG(NM)=SUM
YMAG(NM)=SCIMAG(I)
IF (I.GT.NF) YMAG(NM)=SCEMAG(I)
595 CONTINUE
C
C 650 WRITE(8,653) NM
653 FORMAT(13)
WRITE(6,654) NM
654 FORMAT(//6X,'FOR PLOTTING PURPOSES','13,' POINTS ARE STORED'/)
DO 660 I=1, NM
WRITE(8,655) XMAG(I), YMAG(I)
655 FORMAT(2E15.3)
660 CONTINUE
C
C 900 STOP
END

/*
//LED.SYSLMOD DD DSN=TS2B19.XX.LOAD(CABLE),DISP=(NEW,CATLC),
// UNIT=USEROA
*/
C.2 JCL FOR EXECUTING THE SOURCE PROGRAM

Figure 27, Curve 2:

// JOB
/*JOBPARM LINES=1000,DISKIO=1362
// EXEC PGM=BASE,REGION=192K
//STEPLIB PD DSN=TS2819.XX,LOAD,DISP=SHR
//FTP01 DD SYSOUT=A,DCB=(RECFM=VA,LRECL=137,BLKSIZE=141,BUFNO=1)
//FTP01 DD DSN=TS2819.PLOT1.DATA,DISP=(NEW,CATLG,DELETE),
// SPACE=(TRK,(1,1),RLSE),UNIT=USERDA,
// DCB=(RECFM=FB,LRECL=80,RLFSIZE=3120)
//FTP01 DD *
  2 20 60.0 1 1
  0.1 1.0 1.0 0.1 1.0 1.0
  20.  20.  20.  20.  20.  20.
  20.  20.  20.  20.  20.  20.
  20.  20.  20.  20.  20.  20.
  2400. 300. 300. 300. 300. 300.
  300.0 300. 300. 300. 300. 300.
  300.0 300. 300. 300. 300. 300.
  300.0 300. 300. 300. 300. 300.
  300.0 300. 300. 300. 300. 300.
    1 21 1 3 100.
7.023E-05 .0002747 1.0E-03 .008306 .03160 .043 0.0 -7.63D+06
999
/*
*/
//
C.3 PLOTTING ROUTINE AND ITS JCL

// JOB
// *JOB PARM LINES=1000
// EXEC PLOT
//GO.FT08F001 DD DSN=TS2819.PLOT1.DATA,DISP=SHR
//GO,SOURCE DD *

REAL*D(200),SC(200)
CALL PLOTS(0,0,0)
READ(8,100) M
100 FORMAT(13)
DO 200 I=1,M
READ(8,150) D(I),SC(I)
200 CONTINUE

N1=N+1
N2=N+2
CALL PLOTS(1,1.5,-3)
CALL SCALE(SC,7.,M,1)
CALL SCALE(D,4.,M,1)
CALL AXIS(0,0,'PER-UNIT SHEATH CURRENT',23,7.0,90.,SC(N1),SC(N2))
CALL LINE(D,SC,M,1,1,75)
CALL NEVPEN(2)
CALL SYMBOL(2.5,6.2,0.10,'THREE CONDUCTOR CABLE',0.,21)
CALL SYMBOL(2.5,6.0,0.10,'LENGTH OF CABLE=6300 M',0.,23)
CALL SYMBOL(2.5,5.8,0.10,'RESISTANCE OF GR. RODS=20 OHM',0.,29)
CALL SYMBOL(2.5,5.6,0.10,'FAULT DISTANCE=2400 M',0.,21)
CALL SYMBOL(2.5,5.4,0.10,'NO. OF SOURCES=2',0.,16)
CALL SYMBOL(2.5,5.2,0.10,'SOIL RESISTIVITY=100 OHM-M',0.,26)
CALL SYMBOL(2.5,5.0,0.10,'SHEATH ADMITTANCE=0.1E-02 MHO/M',0.,31)
CALL PLOTS(0.,0.,4999)
STOP
END

*
**Distribution of Zero Sequence Currents Along Underground Cables**

<table>
<thead>
<tr>
<th>Number of Sources</th>
<th>Number of Grounding Rods</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>20</td>
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</table>

Fault occurs at the 8th GR. Rod

Distance of the fault from source 01 = 2400.0 M

### Resistance of Grounding Rods (OHMS):

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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<th>10</th>
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<td>20.0</td>
<td>20.0</td>
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### Length of Each Segment (M):

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### Source Impedance 1:

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<th>Terminating Station 01</th>
<th>Terminating Station 02</th>
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<td>1.00</td>
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### Input Segment Data:

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<th>From</th>
<th>To</th>
<th>IACM (OHMS)</th>
<th>RC (OHMS)</th>
<th>RS (OHMS)</th>
<th>VS (MV)</th>
<th>Soil Res.</th>
<th>CHS-COMD</th>
<th>GP-HC</th>
<th>AVE-SN-RA</th>
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<tr>
<td>1</td>
<td>2</td>
<td>0.70230E-01</td>
<td>0.27470E-01</td>
<td>0.1000E-02</td>
<td>0.1500E+03</td>
<td>0.8306E-02</td>
<td>0.3160E-01</td>
<td>0.4300E-01</td>
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### Segment Zero-Sequence Impedances:

<table>
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<th>From</th>
<th>To</th>
<th>ZC (OHMS)</th>
<th>IMAG (OHMS)</th>
<th>ZM (OHMS)</th>
<th>IMAG (OHMS)</th>
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<th>IMAG (OHMS)</th>
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<td>-2400E-03</td>
<td>-17780E-03</td>
<td>-22380E-02</td>
<td>-10028E-02</td>
<td>-22380E-02</td>
<td>-7630E+07</td>
<td>1000E-02</td>
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</table>

Time = 1 NSP = 21 NSEC = 1 MW = 8 IPODT = 0
ASSUMED FAULT CURRENT IS 1.0 P.U.

ZERO SEQUENCE CURRENT

FROM SOURCE #1: -0.5966 -j.2360 -01

FROM SOURCE #2: -0.4034 -j.2360 -01

SHEATH VOLTAGE AT THE FAULT = 0.2471E+00

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<tr>
<th>SEGMENT</th>
<th>DISTANCE FROM</th>
<th>SHEATH CURRENT</th>
<th>SHEATH CURRENT</th>
<th>SHEATH VOLTAGE</th>
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<td>1.2340 -01</td>
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REFERENCES


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28. G.B. Maund and J.R. Osterfield, "Calculation and Treatment of Induced Voltage in Pilot and Auxiliary Cables Associated with
REFERENCES CONTINUED

275 kV and 400 kV Power Circuits", IEEE Conf. on Progress in Overhead Lines and Cables for 220 kV and Above, No:44, 1968, pp. 413-422.


REFERENCES CONTINUED
