INFORMATION TO USERS

This reproduction was made from a copy of a document sent to us for microfilming. While the most advanced technology has been used to photograph and reproduce this document, the quality of the reproduction is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help clarify markings or notations which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure complete continuity.

2. When an image on the film is obliterated with a round black mark, it is an indication of either blurred copy because of movement during exposure, duplicate copy, or copyrighted materials that should not have been filmed. For blurred pages, a good image of the page can be found in the adjacent frame. If copyrighted materials were deleted, a target note will appear listing the pages in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed, a definite method of “sectioning” the material has been followed. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For illustrations that cannot be satisfactorily reproduced by xerographic means, photographic prints can be purchased at additional cost and inserted into your xerographic copy. These prints are available upon request from the Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases the best available copy has been filmed.
Croushore, Dean Darrell

OPTIMAL GOVERNMENT FINANCIAL POLICY IN A TRANSACTIONS-COST MODEL OF MONEY

The Ohio State University

Ph.D. 1984

University Microfilms International 300 N. Zeeb Road, Ann Arbor, MI 48106

Copyright 1984 by Croushore, Dean Darrell

All Rights Reserved
OPTIMAL GOVERNMENT FINANCIAL POLICY IN A
TRANSACTIONS-COST MODEL OF MONEY

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Dean Darrell Croushore, A.B., M.A.

* * * * *

The Ohio State University
1984

Reading Committee:  Approved By
Edward J. Kane
J. Huston McCulloch
Richard G. Anderson

Edward J. Kane
Adviser
Department of Economics
To Claudette
ACKNOWLEDGEMENTS

I would like to acknowledge the valuable guidance given to me by Professor Edward J. Kane. His careful reading and many worthwhile suggestions over the last two years have helped to give this dissertation style and direction. It has been a pleasure to write this dissertation with his help. I would also like to thank Professors J. Huston McCulloch and Richard G. Anderson for contributing many interesting ideas which led me to undertake further investigations in certain areas.

I have benefited from many discussions with friends, colleagues and instructors at The University of Minnesota and The Ohio State University. In particular, I have been inspired by Professor Thomas J. Sargent and Professor Edward J. Kane.

Finally, thanks to my wife, Claudette, for helping me to survive the rigors of graduate school; and to my parents for encouraging my intellectual development.
VITA

December 3, 1956  Born, Chateauroux, France

March 1978  A.B. in Economics, Ohio University, Athens, Ohio

1978-79  Graduate Study, The University of Minnesota, Minneapolis, Minnesota

1980  Research Assistant, The Center for Human Resource Research, The Ohio State University, Columbus, Ohio

1980-81  University Fellow, The Ohio State University, Columbus, Ohio

June 1981  M.A. in Economics, The Ohio State University, Columbus, Ohio

1981-82  William Green Memorial Fund Fellowship, The Ohio State University, Columbus, Ohio

1982-1984  Teaching Assistant, The Ohio State University, Columbus, Ohio

FIELDS OF STUDY

Money and Banking
Public Finance
Labor Economics
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEDICATION</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>VITA</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>viii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>1. The Theory of Government Financial Policy</td>
<td>8</td>
</tr>
<tr>
<td>A. The Theory of Government Debt</td>
<td>11</td>
</tr>
<tr>
<td>B. Debt Versus Taxes</td>
<td>15</td>
</tr>
<tr>
<td>C. Optimal Monetary Policy</td>
<td>29</td>
</tr>
<tr>
<td>D. The Tradeoff Between Money and Debt</td>
<td>31</td>
</tr>
<tr>
<td>E. Macroeconomic Models and Criticisms</td>
<td>34</td>
</tr>
<tr>
<td>F. The Intergenerational Free Lunch</td>
<td>36</td>
</tr>
<tr>
<td>G. Modeling Money</td>
<td>41</td>
</tr>
<tr>
<td>H. Financing by Taxes, Bonds, and Money</td>
<td>46</td>
</tr>
<tr>
<td>I. Summary and Conclusions</td>
<td>48</td>
</tr>
<tr>
<td>2. The Empirical Evidence on the Effects of Government Financial Policy</td>
<td>50</td>
</tr>
<tr>
<td>A. The Effects of Government Debt on Savings and Consumption</td>
<td>51</td>
</tr>
<tr>
<td>B. Empirical Studies of Social Security</td>
<td>64</td>
</tr>
<tr>
<td>C. The Effects of Deficits on Inflation</td>
<td>66</td>
</tr>
<tr>
<td>D. Summary and Conclusions</td>
<td>69</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>3. The Neutrality of Optimal Government Financial Policy</td>
<td>71</td>
</tr>
<tr>
<td>A. The Individual Maximization Problem</td>
<td>73</td>
</tr>
<tr>
<td>B. Aggregate Equilibrium</td>
<td>77</td>
</tr>
<tr>
<td>C. The Intergenerational Free Lunch</td>
<td>81</td>
</tr>
<tr>
<td>D. Full Government Choice: Taxes, Bonds, Money</td>
<td>86</td>
</tr>
<tr>
<td>E. The Maximization Problem of the Government</td>
<td>91</td>
</tr>
<tr>
<td>F. The Neutrality Proposition</td>
<td>93</td>
</tr>
<tr>
<td>G. Conclusions and Extensions</td>
<td>102</td>
</tr>
<tr>
<td>4. Extensions of the Basic Model</td>
<td>106</td>
</tr>
<tr>
<td>A. An Alternative Utility Function</td>
<td>108</td>
</tr>
<tr>
<td>B. A Clower-Constraint Model</td>
<td>111</td>
</tr>
<tr>
<td>C. A Range-of-Incomes Model</td>
<td>115</td>
</tr>
<tr>
<td>D. Fiscal Illusion</td>
<td>122</td>
</tr>
<tr>
<td>A. The Individual Maximization Problem</td>
<td>128</td>
</tr>
<tr>
<td>B. Aggregate Equilibrium</td>
<td>135</td>
</tr>
<tr>
<td>C. Summary and Conclusions</td>
<td>156</td>
</tr>
<tr>
<td>A. The Individual Maximization Problem</td>
<td>160</td>
</tr>
<tr>
<td>B. Aggregate Equilibrium</td>
<td>172</td>
</tr>
<tr>
<td>C. Summary and Conclusions</td>
<td>185</td>
</tr>
<tr>
<td>7. Optimal Taxation, Monetization, and Debt in Financing Optimal Government Spending</td>
<td>187</td>
</tr>
<tr>
<td>A. Intergenerational Tradeoffs with Pure Public Goods and a Growing Population</td>
<td>189</td>
</tr>
<tr>
<td>B. The Optimal Growth Rate of Pure Public Goods</td>
<td>192</td>
</tr>
<tr>
<td>C. Taxes in the Transactions-Cost Model of Money</td>
<td>195</td>
</tr>
<tr>
<td>D. Optimal Government Spending and Financing</td>
<td>202</td>
</tr>
<tr>
<td>E. Government Policy Experiments</td>
<td>209</td>
</tr>
<tr>
<td>F. Summary and Conclusions</td>
<td>216</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>218</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Neutral Government Financial Policies for a One-Time Change in Bond Issuance and Monetization</td>
<td>98</td>
</tr>
<tr>
<td>2.</td>
<td>A Numerical Example Illustrating the Choice Between Using Money and Bonds</td>
<td>134</td>
</tr>
<tr>
<td>3.</td>
<td>Demonstrating the Intergenerational Free Lunch</td>
<td>153</td>
</tr>
<tr>
<td>4.</td>
<td>Illustration of the Choices of Capital and Store of Value</td>
<td>170</td>
</tr>
<tr>
<td>5.</td>
<td>Illustration of Eight Aggregate Equilibria</td>
<td>174</td>
</tr>
<tr>
<td>6.</td>
<td>Equilibrium Values of Variables</td>
<td>180</td>
</tr>
<tr>
<td>7.</td>
<td>Equilibrium Values of Variables for Alternative Methods of Government Intervention</td>
<td>205</td>
</tr>
<tr>
<td>8.</td>
<td>Government Policy Experiments</td>
<td>210</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Operational Bequests</td>
<td>19</td>
</tr>
<tr>
<td>2.</td>
<td>Nonoperational Bequests</td>
<td>21</td>
</tr>
<tr>
<td>3.</td>
<td>The Simple Equilibrium Concept of Bonds and Money</td>
<td>79</td>
</tr>
<tr>
<td>4.</td>
<td>The Full Equilibrium Concept of Bonds and Money</td>
<td>82</td>
</tr>
<tr>
<td>5.</td>
<td>Budget Constraints for Using Bonds and Money</td>
<td>117</td>
</tr>
<tr>
<td>6.</td>
<td>Examples of Possible Equilibrium Choices for Individuals</td>
<td>119</td>
</tr>
<tr>
<td>7.</td>
<td>Illustration of the Optimal Choice of Capital</td>
<td>163</td>
</tr>
<tr>
<td>8.</td>
<td>Illustration of the Returns to Capital</td>
<td>165</td>
</tr>
<tr>
<td>9.</td>
<td>Description of the Nine Aggregate Equilibria</td>
<td>173</td>
</tr>
</tbody>
</table>
INTRODUCTION

The current public debate over the effects of large government budget deficits highlights a long-running dispute among economists. Deficits are blamed for many of our macroeconomic ills, including high interest rates, inflation and unemployment. While there is general agreement that government deficits provide a stimulative effect on the economy in the short term, there is widespread disagreement about how large this effect is and about the long-term consequences.

When a government's expenditures exceed its annual receipts, the resulting deficit must be financed either by borrowing from the public (issuing government bonds) or by printing money. Increased borrowing by the government is commonly thought to cause interest rates to rise. This in turn causes a reduction in capital investment by business firms, a phenomenon known as crowding out. When the government finances its operations by printing money, an increased rate of inflation is the expected result.

Empirical research on the effects of these alternative methods of government finance has failed to reach positive conclusions. Increased monetization by the government has been historically associated with high inflation rates, but quantitative results vary over time and place. It is difficult to accurately predict how high inflation will be for any given growth rate of the money supply. For example, in the U.S. in both
the periods 1967-8 and 1978-9, the money supply grew at a 15% rate, but inflation was 4% in the earlier period and 11% in the later period.

There has been a lively dispute, both theoretical and empirical, about the effects on interest rates of government borrowing to finance the deficit. Classical economist David Ricardo (1820) showed how it was theoretically possible for government borrowing to have no effect at all on interest rates. However, he suggested that this neutrality of government debt was unlikely to occur in practice. The debate intensified in the 1950's and 1960's when Keynesian economists promoted the use of government deficits for reducing unemployment. Analysis was focused upon the "burden of the debt," in which Keynesians argued that future generations were not harmed by the existence of government debt, but benefited from it.

Empirical work provides little conclusive evidence on how interest rates are affected by government borrowing. Some studies support the neutrality of government debt, while others suggest that government borrowing has such a tremendous impact on interest rates that it crowds out borrowing by business firms dollar for dollar.

Previous theoretical research on the effects of monetization and government borrowing focuses on each method of government finance separately. That is, government borrowing is usually analyzed without considering the portion of the deficit which is financed by monetization. Similarly, studies of inflation focus solely on the growth of the money supply, ignoring changes in the amount of government bonds issued to the public. By looking at both methods of finance together, the interactions between them can be investigated. For
example, a high growth rate of the money supply may not be very inflationary when the deficit is fairly small.

One of the key considerations of the theoretical model to be employed in this dissertation is the manner in which money is modeled. Classical macroeconomic models employ an ad hoc money-demand function. These models assert that government's monetary policy has powerful effects. Recent research has shown, however, that these models imply irrational behavior on the part of individuals in the economy, because people allow themselves to be systematically fooled into expecting a lower inflation rate than actually occurs. The theory of rational expectations suggests that to build an effective model of money, individual behavior must be modeled so that people are assumed not to make systematic errors of this type. Consequently it is necessary to begin by building a microeconomic base for the theory of money.

A review of the theoretical literature on optimal government financial policy begins in chapter 1. The classical theory of debt, with its focus on the burden of the debt for future generations is reviewed. Theories about the tradeoffs in financing government spending by debt versus taxes are analyzed, including a discussion of the Ricardian Equivalence Theorem. Monetary policy and the optimal rate of inflation are examined, focusing on the differences between debt finance and money finance. The criticisms of macroeconomically based models of money are reviewed, and microeconomically based models of money are shown to be more useful in the analysis of policy. The possibility of financing debt perpetually is discussed. Finally, models which examine
government's full choice between taxation, monetization, and borrowing as sources of finance are reviewed.

Empirical studies which test theoretical hypotheses about government financial policy are reviewed in chapter 2. If debt is neutral, then people respond to a temporary increase in debt by increasing savings dollar-for-dollar. Consumption should not respond to debt, unless there is fiscal illusion. Empirical studies of these hypotheses have yielded a wide range of results, some supporting the debt-neutrality theory, others suggesting complete non-neutrality. There appear to be serious econometric problems in most of these studies, so the results are not very convincing. Also examined in this chapter are data-measurement problems and recent work examining the monetization of debt as government deficits grow.

To analyze the conditions under which debt issuance and monetization are neutral, a method similar to that used in developing the Modigliani-Miller theorem in corporate finance is employed in chapter 3. The Modigliani-Miller theorem is basically a mathematical proof that the choice of financing corporate activity by issuing bonds or issuing equity (stock) is irrelevant. The corresponding theorem for government policy is that the choice of financing government expenditures by taxation, debt issuance, and monetization is irrelevant.

The proof of this irrelevance theorem has a mathematical structure. Beginning with a series of equations which describe the economy, consumption and savings streams for individuals are derived. These are added together to find the aggregate demand for consumption goods and total savings in the economy. Equilibrium requires that supply and
demand be equated in all markets. The government faces a budget constraint equation which must be satisfied at all times. Combining these equations yields certain conditions which must be satisfied for government financial policy to be irrelevant. These conditions involve the amount of monetization, the amount of government borrowing, and the structure of taxation on different segments of the population in different generations.

The results of this analysis show that government financial policy can be neutral providing that the tax structure in the economy is adjusted appropriately. It suggests that government's finance choices may be nonneutral under two conditions: (1) if the government makes inappropriate changes in taxes, and (2) if people fail to see changes in the structure of taxation.

The major results of chapter 3 can be shown to hold under more general circumstances. Some alternative models are examined in chapter 4. The use of a less-restrictive utility function makes the analysis more complicated, but the results are unchanged. A Clower-constraint model which makes money work as a medium of exchange is shown to be nearly identical to the model in chapter 3. A model in which individuals have different incomes and must make a choice between using bonds or money as a store of value is examined. Finally, it is shown that the neutrality proposition of chapter 3 fails when people have fiscal illusion.

A major issue which a model of government financial policy must address is that of financing government spending by issuing bonds (borrowing) or by printing money (monetization). In order to analyze
the effects of such policies, it is vital for a theory to explain why bonds and money coexist, and how each is used. A transactions-cost approach to modeling money is employed for this purpose in chapters 5, 6, and 7. People choose between money and bonds based on the transactions costs involved with each. Bonds pay a higher interest rate than money, but require greater transactions expenses.

The basic transactions-cost model is set up in chapter 5. The maximization problem faced by individuals is examined closely. A weighted social-welfare function for government optimization is then derived. Aggregate equilibria are analyzed under conditions of: (1) no government intervention, (2) partial government intervention, (3) full government intervention, and (4) costly government intervention.

Because the major negative impact of government debt is thought to come about by reductions in the capital stock, the transactions-cost model is modified to include capital in chapter 6. Increased government debt leads to higher real interest rates, causing capital to be crowded out, thus reducing output. The effect on the capital stock is thus another consideration in determining the government's optimal financial policy. It is shown that the government should not try to maximize GNP if it wishes to maximize social welfare.

Pure public goods are added to the model in chapter 7. When the economy is growing, the government must decide how to trade off utility between generations in order to choose an optimal time-path of spending on pure public goods. Taxes are introduced in this chapter in the form of a proportional tax on nominal income. Optimal government financing by taxes, borrowing, and monetization is then determined. Finally,
departures from optimal spending and financing are examined in terms of their macroeconomic and social-welfare effects.

This research differs from previous work most significantly by: (1) analyzing government's full choice between taxation, monetization, and borrowing simultaneously, rather than separately, and (2) providing a model of money with a microeconomic base, so that the results are consistent with rational expectations. The results of this research suggest theoretical considerations for the conduct of public policy.
Chapter 1

The Theory of Government Financial Policy

Every government faces the question of how to finance its expenditures. Taxation is the predominant method of public finance for most governments, but there are other possibilities. A national government may borrow funds from its own citizens or from citizens of other nations. The resulting debt requires that the government make additional future expenditures in paying the interest and repaying the principal of the debt. A third method of finance is that of monetization, in which the government simply prints money to pay for its expenditures.

This chapter examines the theoretical literature on government's choice of finance. Section A covers the classical theory of debt. A key question is whether government debt imposes a burden on future generations. The debate over the differences between internal and external debt is discussed.

A key consideration in this literature is the fact that the government must obey a budget constraint. Because of this, increased government debt is often accompanied by decreased taxation. The tradeoff between financing government expenditures by taxation and financing by debt is examined in section B. If the issuance of government debt implies future tax liabilities which are equal in
present value to the amount of increased debt, then the Ricardian Equivalence Theorem may hold. This theorem suggests that the choice between debt and taxes is irrelevant for determining the state of the economy. Debt is said to be "neutral" in its effects. The conditions under which debt is neutral, and the possible reasons for nonneutrality of debt, are considered.

Section C looks at the theory of optimal monetary policy, considered by itself. The costs of monetization in terms of inflation are analyzed. This section includes a discussion of the optimal quantity of money. Monetization and debt are similar methods of finance in that the government buys goods and services by issuing pieces of paper in both cases. With debt, however, there are future government expenditure obligations. These do not exist under monetization. Section D looks at the money-versus-debt tradeoff.

In the 1960's and early 1970's, many economists in the United States felt that the major problems of macroeconomics had been solved. This conclusion came about due to the successful "fine-tuning" of the economy in the 1960's, and the development of large macroeconometric models for simulation and forecasting purposes. Section E examines these "macro-macro" models. Their poor performance in the 1970's and 1980's may be due to the lack of microfoundations. Criticisms of these models, especially of their predictions about the results of monetary and fiscal policies, are discussed in this section.

A recurring theme throughout the literature on the burden of the debt is the opportunity for perpetual debt finance. If debt is in some sense "cheaper" than taxes, why not roll over government debt
indefinitely? If the government borrows continuously, it is never required to increase taxes to pay off the debt. Then there are no future tax liabilities, so the Ricardian Equivalence Theorem is invalid. The possibility of using perpetual debt finance is examined in section F. It is clear that perpetual debt finance is possible only in a growing economy. The same factors that make perpetual debt finance possible have been used by others as a motivation for the existence of money. The "intergenerational free lunch" is the true reason for this possibility. This concept is examined in detail in this section.

When economists build a model for analyzing fiscal and monetary policies, the purpose of money in the model is of key importance. The policy implications of the model depend on what functions money serves in the model. Section G is a brief introduction into models of monetary economies. A key concern in this literature is to show how money and bonds can coexist, since bonds tend to dominate money in terms of explicit rate of return. The usefulness of the overlapping-generations model is also examined in this section.

The full choice between financing government spending by taxation, borrowing, and monetization is examined in section H. Difficulties in the optimization problem of the government are discussed here. The key idea is that money, debt, and taxes all have costs associated with them. At the optimum, the marginal cost for all three methods must be the same.

Section I summarizes issues in this literature which remain unsettled, and indicates how the following chapters answer some of these questions.
A. The Theory of Government Debt

The effects which the issuance of government debt has on an economy have been long debated. From Ricardo to Barro the discussion has been both lively and important, with far-reaching implications for government action.

Three major questions can be asked about government debt: (1) Does the debt have any economic effect or burden? (2) If the debt has a burden, who suffers and who gains, and how do they do so? (3) Should debt be used to finance government expenditures? Clearly the answer to the third question depends on the answers to the first two. The first question has received most of the attention in the literature.

In this section, the historical debate on the burden of the debt spanning two and one-half centuries is examined. Early discussion focused on whether debt was neutral because "we owe it to ourselves." The differences between the effects of internal debt and external debt were debated in the 1950's. Finally, the question of whether debt leaves a burden for future generations to pay was a major concern in the early 1960's.

The French economist Melon (1735) argued that internal debt had no burden: "Les Dette d'un Etat sont des Dette de la main droite a la main gauche." (The debts of a state are debts of the right hand to the left hand) [p. 274]. He was the first in a long line of economists to argue that debt is harmless because "we owe it to ourselves."

Say (1832), however, strongly disagreed. Comparing the borrowing of private individuals to the borrowing of a government, he felt that
the former was "for the purpose of beneficial employment," whereas the latter was simply "for the purpose of barren consumption and expenditure." [p. 442] He agreed that Melon was correct in that the interest on the public debt is burdenless, since it is basically a transfer between individuals in the nation. But because the capital originally borrowed would have been put to use in the absence of the public debt, it is the destruction of this capital which represents a burden of the debt.

Adam Smith (1776) agreed completely with Say. He felt that large debts would "in the long-run probably ruin all the great nations of Europe" [p. 863]. He argued that the "we owe it to ourselves" argument was fallacious because of the destruction of capital and because some borrowing comes from foreign sources. Frequently governments get rid of the debt only by a "pretended payment," by devaluing or adulterating coinage.

In addition to the destruction of capital caused by the debt, Smith argued that there are some psychological difficulties. If government is allowed to borrow it tends to do so. If a sinking fund is established to pay off old debts, it encourages the government to contract new debts. If the government borrows in wartime (rather than collecting taxes), people do not see how expensive war really is. All these effects, however, are forms of "fiscal illusion," which is discussed in detail in section C.

Ricardo (1817) basically agreed with Say and Smith, suggesting that "that which is wise in an individual is wise also in a nation" [p. 163]. Ricardo was the first to present the idea that debt could be neutral in
its effects if people responded in a certain way. However, Ricardo believed in fiscal illusion as well.

In general, then, the classical fiscal tradition opposes government debt largely on the grounds that it destroys capital. There were some dissenters to this view, although their reasoning was not always clear. Alexander Hamilton argued that the government debt somehow helps private individuals establish credit, so it can lead to increased capital formation. Malthus suggested that debt can increase the productive powers of a nation. For more on these views, see Harris (1947).

Until the mid-1900's, however, the debate focused on internal versus external debt, everyone agreeing that external debt was burdensome, and disagreeing on the effects of internal debt. According to Wicksell:

If the loan is secured from outside the economy, that is if the State borrows abroad, then clearly the man in the street is right when he thinks that future generations are thereby saddled with a burden which the current generation could shoulder just as well or better and hence should carry. If the money is borrowed domestically, this objection would for obvious reasons not be valid (as is often noted).

[Wicksell, 1896, p.105]

Keynesian theory led macroeconomists to question the notion that internal debt might be burdensome.² The view of the "new orthodoxy" (a term coined by Buchanan (1958)), as presented by Lerner (1944,1948) and Meade (1958,1959) is that the real cost of the debt is an opportunity cost [see also West (1975)]. Since current projects use current resources, the cost of such projects is already realized.³

Buchanan (1958,1960) and others [see Bowen, Davis, & Kopf (1961), and the discussions in Ferguson (1964)] objected to this idea, arguing
that while current generations take voluntary action in buying and selling government bonds, the higher tax rates in the future are a burden on (some) future taxpayers, since they did not voluntarily agree to the higher taxes.

A lively debate followed in the economic literature. Modigliani (1961) and Shoup (1962) finally made explicit the notion that a burden from public debt may arise because of a smaller capital stock. In this case, future consumption is reduced, while present consumption is increased.

This notion is now accepted by most economists as being the mechanism by which the debt is a burden, if it is a burden. An increase in government debt issue which causes an increase in current consumption must lower current savings, leading to a reduction in capital formation. With less capital at work, future consumption is lower.

This mechanism works if the issuance of government debt raises the (perceived) net wealth of individuals, so there are wealth effects on saving and consumption. The increase in savings is less than the increase in government demand for loanable funds, so the rate of interest is bid up. This causes private investment to decrease. Thus there is crowding out of private investment.
B. Debt Versus Taxes

The early literature on government debt often ignores the fact that the government faces a budget constraint. When debt is issued, the government's budget is affected today (because taxes must be reduced or government spending increased to satisfy the budget constraint) and in the future (because interest payments and principal repayments must be made). This section examines the tradeoff between debt and taxes, focusing on the Ricardian Equivalence Theorem.

The Ricardian Equivalence Theorem states that the choice of financing some part of government expenditures by debt rather than taxes has no effect on real economic variables such as consumption and investment. The theorem's main argument is that individuals see their future tax liabilities as being just equal in present value to the debt. Thus the reduction in current taxes can exactly offset the future tax liabilities if all of the "tax cut" is saved, and none is consumed. While Ricardo (1820) was the first to point this out, debt neutrality has received its strongest and most rigorous treatment from Barro (1974, 1976, 1978a).

Suppose current government expenditures of $X$ may be financed by raising the full amount of $X$ with current taxes, or by borrowing $X$. If $X$ is borrowed, debt is issued, and the interest payments on the debt are financed by taxes in the amount of $Y$ per year, forever. The Ricardian Equivalence Theorem (RET) states that the two options are equivalent if:
1. No escape from the perpetual taxes is possible (by moving or by dying or by tax avoidance activities).

2. Capital markets are perfect.

3. No uncertainty exists about future taxes.

4. Administrative costs are negligible.

5. Economic agents act rationally.

If all these assumptions hold, then economic agents, seeing the future tax liabilities due to government debt, save the complete amount that their taxes are cut. Their consumption plans over time are unchanged. There is no effect on the interest rate because, in terms of a loanable funds market, the demand and supply of funds are both increased by the same amount.

The Ricardian Equivalence Theorem was envisioned first by Ricardo, although as O'Driscoll (1977) points out, Ricardo did not believe in it. Barro (1978b) argues that the debt neutrality theorem should still be named after Ricardo, since Ricardo's belief in fiscal illusion is non-Ricardian!

Similar ideas have been suggested by others. Tobin (1952), in discussing whether government bonds should be included in net wealth, asks, "How is it possible that society merely by the device of incurring a debt to itself can deceive itself into believing that it is wealthier?" [p.117]. In replying to Buchanan (1958) and Bowen, Davis, & Kopf (1960) in the "new orthodoxy" debate, Lerner (1961), in discussing an increase in public expenditures financed by government debt, suggests that "any genuine impoverishment of future generations must be the
result of not reducing private consumption by the full amount of the resources used up..." [p.141].

Even though debt today implies future tax liabilities, all the assumptions of the Ricardian Equivalence Theorem must hold for neutrality to occur. Many economists do not believe that all of these assumptions apply. Much recent research concerns how bequests and gifts affect the neutrality of debt.

1. Bequests and Gifts

The assumption that people can not escape the payment of taxes (which are levied perpetually to cover the interest on the debt) has led to a lively debate. The only possibilities which exist for escaping taxes are moving, death and tax avoidance. Tax avoidance depends upon the structure of taxation, and is analyzed in the public-finance literature.

Ricardo (1817) suggested that a large debt might force people to move away, even from a native land they love. If their nation's debt becomes very large, "then it becomes in the interest of every contributor to withdraw his shoulder from the burthen...to remove himself and his capital to another country, where he will be exempted from such burthens" [p.163].

Barro (1978a) suggests that if continuing benefits are paid for by the debt (hence by the tax payments), moving will not occur.

Moving to another country generally entails great costs, so that a nation's debt would have to be quite large before an individual would go to such lengths to avoid tax payments.
It is much more likely, however, for someone to escape paying perpetual taxes by dying. (This is not to imply that any choice procedure would be involved here!). By increasing their bequests to make up for the future tax payments that their heirs must make, the current generation can leave everyone just as well off as they are in the case where no debt is contracted.

Again, Ricardo (1820) saw this clearly. "But if he leaves his fortune to his son, and leaves it charged with this perpetual tax, where is the difference whether he leaves him 20,000 £., with the tax, or 19,000 £. without it?" [p.187].

Barro (1974) presents this idea clearly and forcefully using a pure consumption-loan model (also called an overlapping-generations model) of the type used by Samuelson (1958) and Diamond (1965). As long as bequests (from old to young) or gifts (from young to old) are operational (positive) both before and after the size of government debt is altered, individuals completely offset any effects of the debt by changing the size of their bequests or gifts. The bequest effect makes finite-lived individuals act as if they lived forever.

A graphical explanation of the Barro (1974) result may help clarify the matter. Barro assumes that individuals maximize their own utility, which depends on the utility of their heirs. The utility of their heirs depends on the net bequest they receive. This net bequest is defined as the gross bequest given minus the government debt for which they are liable. Subject to certain (budget) constraints, utility is maximized as shown in figure 1.
This figure shows the situation when bequests are operational. Net bequests \((A-B)\) are shown to be unchanged by a change \((\Delta B)\) in government debt. The range of possible gross bequests is always \((0, \infty)\). The range of possible net bequests is \((-B, \infty)\) before the change in debt, and \((-B-\Delta B, \infty)\) afterwards.

Operational Bequests
Figure 1
Here $U^*$ represents the maximum achievable utility level for an individual, and $A - B$ is the net bequest given to an heir. By assumption, the gross bequest $A$ of actual assets to be transferred must be positive. $B$ is the per-capita share of total government debt, which is equal to the present value of all future tax payments on the debt. A member of the current generation maximizes utility ($U^*$) by choosing a net bequest over the range $-B$ to infinity. In the diagram, point 'a' is chosen.

When there is an increase in government debt and an equal reduction in current taxes, the diagram is affected in a very simple way. Only the minimum net bequest changes, from $-B$ to $-B-\Delta B$, since this is the net bequest when the gross bequest is at its minimum level of zero. The utility frontier is unaffected. In figure 1, point 'a' is still chosen. The increase in $B$ has no effect and the additional debt is neutral. If gross bequests had been unchanged (i.e., the tax cut was spent on consumption), point 'b' would have been achieved. This is clearly not a point of utility maximization.

Of course one can easily draw a graph showing the case where debt neutrality does not hold. In figure 2, which arises if an individual does not care for an heir (or has no heir), we see that the full amount of the increase in disposable income, caused by the reduction in taxes, is spent on consumption.

The neutrality of the debt rests on two issues:

1. The utility frontier does not change when bonds are issued.
2. The solutions before and after a change in debt are not corner solutions.
This figure shows the situation when bequests are not operational. The gross bequest is always at its minimum of 0. Net bequests change from -B to -B-ΔB.

Nonoperational Bequests
Figure 2
Vickrey (1961) saw the Ricardian Equivalence Theorem result: "If individuals of this generation have objections to imposing a burden on the future, it will always be open to them to avoid this result by increasing the amount they individually bequeath to the future generation by an amount sufficient to pay off the debt" [p.136]. Vickrey argues that since the addition of debt expands the opportunity set of the individual, debt financing is good—it allows greater freedom of choice. What this argument ignores is that an increase in debt reduces the opportunity set of future generations for gifts (from young to old).

Buiter (1979) presents a more complicated model in which both bequests and gifts may occur. He obtains the result that unless both gifts and bequests are possible, some government policy can affect real variables, such as consumption. This, however, is not very surprising and could be easily deduced from Barro's original model (for example, in figure 1, if gifts are not possible from young to old, and the government runs a surplus which exceeds $z_a$, then the current generation is made worse off). Buiter must achieve a corner solution somewhere to generate nonneutrality.

Government debt and a positive burden on future generations can be justified if the argument is made that the older generation pays for investment in human capital by the younger generation. In the absence of the possibility of leaving negative bequests, the debt may be used to transfer wealth from young to old to pay for the human capital investment [see Drazen (1978) and Feldstein (1976)].
Tobin (1978) argues that since the taxes on one's heirs are based on the heirs' incomes, and these are unknown, then the debt burden on them is unclear.

In addition, Carmichael (1982) emphasizes that debts and bequests may not be perfect substitutes because one does not know one's exact length of life, nor the number of children one might have. Also there are certain indivisibilities which must be overcome—in an economy with a population growth rate of \( n (0<n<1) \) one cannot have \((1+n)\) children. Of course, it is possible that people leave bequests not just to their own children, but to society as a whole. In that case the Ricardian Equivalence Theorem holds even for people who have no heirs.

2. Other sources of nonneutrality
   a. Imperfect capital markets

Barro (1974, 1978a) points out that if capital markets are not perfect, then there may be some gain in wealth due to debt issue. For example, if the private interest rate exceeds the interest rate at which the government can borrow, the issuance of debt increases net wealth (this is suggested also by Ferguson (1964b)). This is a simple arbitrage opportunity taken advantage of by the government. For this argument to hold the government must be more efficient at the margin in carrying out the loan process (some obvious counter evidence might be the student-loan program). Webb (1981) examines cases of asymmetric and imperfect information as causes of imperfections in capital markets.

By borrowing externally, in an international financial market into which private borrowers cannot enter, a government may obtain a net
wealth gain (via arbitrage profits) for its citizens if the foreign interest rate is lower than the domestic rate. This argument is presented by Diamond (1965). It is exactly the opposite from the beliefs in the earlier internal-external debate, which suggest that external debt causes an economic burden while internal debt does not.

b. Uncertainty about future taxes

If one is uncertain about one's future tax liabilities, it is difficult to offset government debt changes. Many argue that uncertainty about future tax liabilities will lead people to fail to capitalize them fully [Buchanan (1976), Feldstein (1976), Buchanan and Wagner (1977)]. Barro (1978b) sees no reason for this to occur. In fact, risk-averse people might overestimate their future tax liabilities under uncertainty, seeing bonds as a reduction in net wealth.

Ferguson (1964b) points out that uncertainty about future tax liabilities may arise not only because future tax rates are unknown, but also because the number of future taxpayers and general economic conditions in the future can not be known ahead of time.

c. Administrative costs

If the administrative costs of additional debt issue exceed those of additional taxation, a reduction in wealth occurs due to an increase in debt. In fact, if the Ricardian Equivalence Theorem holds otherwise, then the costs of debt versus taxes may enable the government to find an optimal debt-tax mix.
This is the approach taken by Barro (1979, 1980, 1984) in a series of recent papers. When taxes entail direct collection costs (for administration and enforcement), then any temporary change in government spending (e.g., due to war) or income (e.g., due to labor productivity shocks) ought to be accompanied by (nearly) unchanged taxes. Debt should change to accommodate most of any transitory shock. Barro's empirical work shows that government financial policy in the U.S. has responded in the manner predicted by this theory.

This same result, that taxes ought not to move when transitory shocks occur in the economy, is also suggested by Kydland and Prescott (1980). In a complex model of economic fluctuations, in which there are productivity shocks and monetary shocks, they find that countercyclical movements of taxes are improper. The benefits of stabilization are exceeded by costs in the form of additional excess burdens of taxation.

d. Fiscal illusion

The Ricardian Equivalence Theorem fails if people do not realize that debt issue today means higher future taxes. This is known as "fiscal illusion." It may occur because people are irrational, or because people lack information (perhaps due to costs of collecting and interpreting information) on the effects of debt. Since economists disagree widely on the effects of debt, the common man is likely to respond inconsistently to changes in government debt. Some would argue, as does Buchanan (1958, 1964, 1976), that there may be a psychological effect by which people favor debt over taxes. Debt seems "cheaper"
because the payment amounts are smaller and spread over a longer period of time.

Cavaco-Silva (1977) suggests that not only do people not act to neutralize government debt issue, they don't even know that it has been issued. In light of the size of the government debt, this must be irrational behavior.

Ricardo showed himself to be non-Ricardian by believing in fiscal illusion.

It would be difficult to convince a man possessed of 20,000 £., or any other sum, that a perpetual payment of 50 £. per annum was equally burdensome with a single tax of 1000 £. . . . If an individual were called upon to pay 1000 £. to the income-tax, he would probably endeavor to save the whole of it from his income. . . .

[Ricardo, 1820, p.186]

However, if a loan was raised, Ricardo believed that a person would only save the required interest on the loan.

The macroeconomic models of Patinkin (1965) include the term $k \frac{V_0}{r_p}$ as a part of households' wealth, where $\frac{V_0}{r_p}$ represents the real value of outstanding government debt, and $k$ is a factor of fiscal illusion. Patinkin assumes that $0 < k < 1$, because he assumes that people do not fully discount all future tax liabilities associated with government debt. Christ (1957) defends the assumption that $k > 0$ because "it seems unlikely that taxpayers really do alter their spending behavior in response to changes in the discounted future real interest burden of the public debt" [p.350].

Buchanan and Wagner (1977,1978) base their theory of public choice partly on fiscal illusion. They argue that because people have fiscal
illusion, they don't see the true cost of government spending. As a result, government spending is much higher than its optimal level from a public-choice standpoint.

The theory of rational expectations suggests that people do not suffer from fiscal illusion. There do not appear to be great costs involved in learning about the effects of government debt. The Lucas (1976) policy-evaluation critique suggests that if fiscal illusion occurred prior to the 1960's, then the exploitation of that fiscal illusion in the 1960's and 1970's by continuous deficit spending would lead people to undertake more rational behavior.

Because theory is unable to verify the existence of fiscal illusion, the concept has been put to the test econometrically. Empirical results are examined in chapter 2, sections A and B.

e. The David-Scadding hypothesis

David and Scadding (1974) argue that in the long run people see public debt as representing the value of public investment which has been made. They hypothesize that people see government investment and private investment as perfect substitutes. Consequently any increase in government debt does not lead to an increase in private saving. It is argued that Denison's Law (that the gross private saving rate (GPSR) is very stable over time) is evidence that their hypothesis is correct and that the Ricardian Equivalence Theorem is false.

Boskin (1978) suggests that the gross private saving rate is a useless concept, and Denison's Law merely a statistical artifact. More appropriate measures of saving are not stable over time. Buiter and
Tobin (1979) argue that the stability in the gross private saving rate is due to countercyclical government fiscal policy, not private saving behavior.

3. Formal models of neutrality

Attempts have been made recently to develop formal models in which the Ricardian Equivalence Theorem holds. These models have microeconomic bases, and begin with utility maximization by individuals. Wallace (1981) proves an irrelevance proposition, showing that a change in money growth may not lead to a change in the price level or in consumption. The conditions necessary for irrelevance to hold are examined. Under certain legal restrictions, irrelevance may fail to hold. Finn (1983) looks at a similar model with a slightly different market configuration, and confirms Wallace's results.

Gilles and Lawrence (1981) try to expand Wallace's hypothesis into a model which looks more realistic. They show that taxes, bonds, and money can be equivalent forms of government finance. However, the money and bonds in their model are perfect substitutes, so their results are not unexpected.

Stiglitz (1983a, 1983b) focuses on debt neutrality in a simple life-cycle framework. He establishes the conditions necessary for neutrality, noting that distributional effects destroy neutrality. Debt may cause inflation without having any real effects. If individuals aren't identical, and lump-sum taxes aren't possible, then neutrality fails to hold. Similar results are summarized by Bryant (1983).
C. Optimal Monetary Policy

Monetization is a possible method of government finance. When a government pays for goods by printing money, inflation seems to be the inevitable result. Rather than paying for government spending directly through taxes, people pay indirectly because their (nominal) money buys fewer real goods and services. This is known as the "inflation tax." In addition, the government benefits from unanticipated inflation if it has issued debt in nominal form. The market value of the debt falls as inflation rises.

M. Friedman (1969) finds that it is optimal for an economy to run a deflation. He suggests that people's desire to hold real money balances varies inversely with the inflation rate. As inflation rises, the costs of holding money balances rise (assuming that the rate of return on money doesn't rise with inflation, e.g., as in the case of currency), so people make more trips to the bank, and hold lower real balances. An individual's welfare is maximized when real balances are held to satiety. The government can achieve this by making the quantity of money such that a deflation occurs which is just equal to an individual's time discount rate.

Friedman's results are confirmed by others. Buiter (1982) comments that it is always Pareto-optimal to run a deflation as suggested by Friedman, as it gives everyone the same intertemporal exchange possibilities. Folkerts-Landau (1982) shows that this type of deflation is optimal, even in a transactions-cost model, if lump-sum taxation is
possible. The same result holds in the model of chapter 3 in this dissertation.

Phelps (1973) clarifies the tradeoffs faced by government in using the inflation tax by monetization. He does this utilizing the differential-incidence analysis of public finance theory, comparing the inflation tax to a tax on wage income. Conditions for the optimal tradeoffs between the two types of taxes are developed in welfare-loss terms.

Of course, many other costs are associated with inflation beyond those of the inflation tax and the change in the real value of government debt. Distribution effects between borrowers and lenders occur when inflation varies from its expected rate. Similar distribution effects are important for many other nominally-denominated contracts as well, especially labor contracts. Inflation also imposes excess burdens due to the nominal structure of taxes. See Feldstein (1982b) for an overview of these issues.
D. The Tradeoff Between Money and Debt

If the Ricardian Equivalence Theorem holds for government debt, does a similar neutrality theorem hold for monetization? Johnson (1962) suggests the possibility:

The existence of government debt implies the levying of taxes to pay interest on it, and in a world of reasonable certainty these taxes would be capitalized into liabilities equal in magnitude to the government debt; hence if distribution effects are to be ignored, a change in the real amount of government debt will have no wealth effect. Finally, if this logic applies to interest-bearing government debt, why should it not apply to the limiting case of noninterest-bearing government debt, which is equally a debt of the public to itself, and to commodity moneys, which are the same thing though based on custom rather than law?[p.343]

Bryant and Wallace (1979a) investigate the tradeoff between monetization and debt in a model in which the use of bonds entails transactions costs. They find that the optimal government financial policy is to pay for deficits by issuing money, since money doesn't use up resources. In more recent work, however, Bryant and Wallace (1979b, 1980) suggest that financing government spending in part by debt and in part by monetization may be social-welfare maximizing, as it allows the government to discriminate in price between groups who hold these alternative assets.

Sargent and Wallace (1981) analyze a model which suggests that fiscal and monetary authorities can not act independently. In this model, an upper limit on the public's demand for real bonds means that government debt can't grow indefinitely. If deficits grow faster than the economy, then the monetary authority is eventually forced to monetize them. The key issue examined in this paper is the monetarist
assumption that the money supply growth rate can be set independently of fiscal action, and that monetary policy alone determines the inflation rate. The debt limit is reached at an early date if monetary growth is currently low. Therefore, a tight monetary policy today means a higher inflation rate in the future. If people have rational expectations, and foresee this result, it is possible that current inflation rises as well.

P. Miller (1983a) suggests some additional reasons for which deficits may lead to inflation. He suggests that deficits may crowd out private investment, and that high government debt may lead to financial innovation which causes private monetization of bonds to occur.

Buitr (1982) suggests that a government which acts according to Sargent and Wallace's model is irrational. Optimal government policy is to lend, not borrow. A similar result is found in chapter 5 of this dissertation. Nonetheless, the idea that fiscal and monetary policies require coordination is important.

Blinder (1983) models the fiscal-authority versus monetary-authority conflict as a nonzero-sum game. The outcome is likely to be a Nash equilibrium, in which each authority expects the other to pursue its own (individual) optimal strategy. It is suggested that a policy of coordination would make everyone better off, but the Nash equilibrium does not get to this point. Blinder's solution is to require consultation between fiscal and monetary authorities, or, preferably, switching to a unified policy-making system.

McMillan (1981) suggests that the fiscal authority must model the monetary authority's systematic reaction to fiscal policy, to find the
optimal fiscal policy. This need arises because the two authorities have conflicting goals.

Empirical estimation of the monetary-fiscal tradeoff is examined in chapter 2, section C.
Macroeconomic models generally show that fiscal policy has powerful economic effects. These effects come about because of the assumption of fiscal illusion, either explicitly, as in Patinkin (1965), or implicitly as in Blinder and Solow (1973, 1974), Silber (1970), Stein (1974), and Butkiewicz (1979). The Keynesian consumption function assumes that consumption spending depends on current disposable income and wealth, and does not take into account the future tax liabilities associated with government debt. The theoretical models themselves preclude the possibility of debt neutrality. Meltzer (1963) presents empirical evidence supporting this view.

Fiscal policy works in macroeconomic models because people are assumed to suffer fiscal illusion. They fail to foresee future tax liabilities. This is the central point which has been attacked by rational-expectations theorists. Lucas (1976) argues that the attempt to use activist fiscal policy should lead to changes in the model:

Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models.

...it implies that comparisons of the effects of alternative policy rules using current macroeconomic models are invalid regardless of the performance of these models over the sample period or in ex ante short-term forecasting.

[p. 41]

Carmichael (1982) criticizes the macroeconomic models because there are no microfoundations behind the holding of assets. The assumptions
of "wealth" effects and "real balance" effects are inadequate in specifying the roles which assets play.

Keynesian macroeconomic models appear to ignore the fact that the government must face a budget constraint. Christ (1968) examines the issue and concludes that Keynesian multipliers are far off target because of it. However, Sargent (1979, pp. 107-111) defends the Keynesian results. The budget constraint of the government is implicit in these macro models. More recently, however, B. Friedman (1982) adds equations relating to the government budget constraint and market-clearing conditions for government securities to the MPS model, a large macroeconometric model. He finds that doing so greatly reduces the predicted stimulative effect of fiscal policy and enhances that of monetary policy. Unfortunately in a large model such as this, it is difficult to see what drives this result.

These arguments suggest that a better method of analyzing macroeconomic models is to build models with microeconomic foundations. Methods of modeling money with microeconomic considerations are examined in section G.
F. The Intergenerational Free Lunch

One of the most confusing facets in analyzing government financial policy is growth of an economy. In an economy in which the rate of growth exceeds the rate of interest, every generation can be made better off by the existence of government debt. This is a well-known implication of overlapping-generations models [see Samuelson (1958), Feldstein (1976), Barro (1976), and Kareken and Wallace (1980)].

Samuelson's (1958) seminal contribution demonstrates in a simple manner that in an infinitely-growing economy with finite-lived individuals, government can act as an infinitely-lived agent and make everyone better off. This can be accomplished through the use of fiat money, via a social contract, by an unfunded social-security system, by continuous government debt, or by a system of lump-sum taxation and transfers. Everyone in the economy is made better off because the chosen scheme rearranges people's intertemporal-consumption opportunities by making exchanges between generations possible. These exchanges are not possible under laissez-faire due to the lack of trading possibilities between generations (and because no asset has a real return which exceeds the growth rate). Because this is a Pareto-improvement for everyone due to the trades between generations, it is termed an "intergenerational free lunch." This is examined further in the theoretical model of chapter 3 in this dissertation.

Diamond (1965) adds capital goods to Samuelson's model, to investigate the effects which debt has on capital. Diamond's major finding is that the effects of debt depend upon the difference between
the growth rate of the economy and the real interest rate. There is nothing which guarantees these to be equal (i.e., at the Golden Rule\textsuperscript{6}). The results are that internal debt raises the real interest rate and lowers utility levels if the real interest rate exceeds the growth rate. Debt affects both the demand side and supply side of the capital market. The utility of individuals is affected because taxes are needed to pay interest on the debt and because a change in the real interest rate changes the value of existing debt as well as the value of factor payments.

Diamond's results seem powerful and plausible, but they rely on a restriction on the tax structure. Bierwag, Grove, and Khang (1969) note that a lump-sum tax scheme or social security transfer system may lead to a Pareto-optimum in Diamond's model. Taxes may be used to neutralize the effects of debt.

1. Social Security

Economic growth may justify the use of an unfunded social-security scheme. As Samuelson (1958,1959) shows, this type of scheme can give people the intergenerational free lunch. Lerner (1959) worries about the effects of such a "chain letter" scheme. If growth isn't infinite, then some generation must suffer. This may be the problem of the Social-Security System in the United States in the early 1980's. Another problem is that of cohort size. When an unusually large group of the population, such as a "baby-boom" generation, reaches retirement age, the current young may be forced to pay higher social-security taxes.
An unfunded social-security system has been compared to government debt, but may differ due to microeconomic effects. Social-security schemes generally encourage earlier retirement than that which would be chosen if people save privately, as shown by Munnell (1974) and Gultekin and Logue (1979). Laffer (1978) suggests that the present value of net social security benefits falls as an individual's wages rise. Enders and Lapan (1982) suggest that a social-security setup may encourage risk-sharing between generations, making everyone better off. Thus, the effects of social security differ from those of debt due to the way in which the system is implemented. For that reason, we look at the empirical results on the effects of debt in section A of chapter 2, and look separately at the empirical results for social security in section B.

2. Perpetual Debt Finance

In arguments that government debt has no burden, economic growth is often cited. When the growth rate exceeds the interest rate, interest on debt may be financed by additional debt, and yet the debt-to-GNP ratio will fall over time. This result has been remarked upon by many authors, beginning with the internal-external debate in the 1950's. Tobin (1952) suggests that this is a reason for fiscal policy's effectiveness: "...the government [can] pay the interest on its debt, not by taxes, but by incurring further debt. In this case the debt does have a net expansionary effect..." [p.117]. Bowen, Davis and Kopf (1960) suggest that neutrality of debt could occur if debt was passed on forever, and never repaid. Vickrey (1961) suggests that taxes need
never be raised to pay off the debt—it can grow forever. However, if this occurs, there may still be a burden to the debt due to the reduction of the capital stock. As Feldstein (1976) suggests, "the first generation—that is, the generation that receives the debt as a transfer from the government—knows that no future generation will be called upon to repay the debt. The first generation will therefore increase its own consumption and thus reduce capital accumulation" [pp.332-3].

A serious shortcoming exists in this analysis. In an economy with capital, it is inefficient for the growth rate to exceed the real interest rate, as shown by Phelps (1966). The real interest rate is not exogenous, but depends upon the growth rate. Having a real interest rate below the growth rate is inefficient. If the issuance of government debt then drives the interest rate upward, crowding out capital, this is an efficient result, as otherwise the capital stock is too high.

When the growth rate of the economy is less than the interest rate, then government debt can not be refinanced continuously. Eventually the debt limit of the desired real wealth held by individuals will be reached. This is the point made by Sargent and Wallace (1981) (see section D above). Darby (1984) suggests that the Sargent-Wallace results depend upon the relationship between the growth rate and the real interest rate. However, P. Miller and Sargent (1984) respond with a more reasonable model showing that the real interest rate is driven at least as high as the growth rate.
In chapter 3 of this dissertation, it is shown that government can drive the real interest rate to its golden-rule level (equal to the growth rate) by the use of financial instruments and taxes. Doing so provides the intergenerational free lunch to everyone, making everyone better off.
G. Modeling Money

Because monetization may be a source of revenue for the government, a model which is to deal with government financial-policy questions must model money adequately. Macroeconomic approaches, as described in section E, often assume some form for the demand function of money. It is more appropriate in the analysis of policy, however, to begin with a microeconomically based model, rather than "starting with curves" [see Bryant and Wallace (1980)].

1. Transactions-Cost Models

One of the requirements placed upon a model of money is that it be able to explain why money, which pays (in some forms) zero nominal interest, is not dominated by bonds, or investment in real goods, or capital goods, which usually pay positive nominal interest. The most common and natural explanation is because of transactions costs.

In transactions-cost models, a person incurs transactions costs when buying or selling bonds. Bonds yield a greater gross return than money, but the net return may be higher or lower than that of money. Naturally, the longer is the holding period and the larger the amount, the greater are the benefits to holding bonds over holding money, when there is a fixed cost of transacting. Transactions costs of this nature are the key factors in the important contributions of Baumol (1952), Tobin (1956), and Miller and Orr (1966).

A serious obstacle in modeling money with transactions costs arises because the transactions costs generally have a fixed component. For
example, a "trip to the bank" costs a given fixed amount, regardless of the value of transactions undertaken. This creates difficulties because it makes individual's budget constraints nonconvex. Heller (1974) and Heller and Starr (1976) prove the existence of a monetary equilibrium when there are transactions costs both when budget constraints are convex and when they are nonconvex. Chapter 5 of this dissertation presents a model which illustrates the problems with nonconvex budget sets.

Recent work with transactions-cost models has extended their scope, although tractability is often a problem in these models. Barro (1970, 1972) analyzes the costs of inflation when there are transactions costs. Especially interesting in his work is the model of the "payments period." In addition to the usual question of when to make "trips to the bank," the analysis also looks at the frequency of payments from employers to employees.

Fischer (1982) develops a complicated model of money and banking. Technological advances in the banking industry are shown to reduce the transactions-costs in banking services. The implications for the use of money can then be analyzed.

Bryant and Wallace (1980) argue that the transactions-cost approach to money is not sufficiently well developed to be useful. The lack of an underlying theory to support the transactions technology is a major concern. The results of the model hinge almost entirely on how the transactions technology is specified. Furthermore, Bryant and Wallace see the low spreads of mutual funds (whose spreads are much lower than
the spread between money and Treasury-bill interest rates) as direct evidence that there is more to the story than transactions costs.

2. Legal-restrictions

Bryant and Wallace (1980) argue that facts contradict transactions-cost models. They suggest that a reasonable model of money is one in which some people face legal restrictions which prevent them from buying bonds. These restrictions may be of the nature of a minimum-denomination requirement on bonds, ceilings on interest rates, or the requirement of zero explicit nominal interest on demand deposits. Other theoretical analyses from Minnesota economists develop similar models [see P. Miller (1983b), and Wallace (1983)].

In models with legal restrictions, the government can effectively discriminate by price because different groups of people are restricted to using certain financial instruments. Bryant and Wallace show that this may be the only way to make Pareto-improvements. Surprisingly, even those who are restricted may benefit from the restrictions.

3. Models with money in the utility function

It has long been popular to model money by placing it in an individual's utility function. An article of particular importance which models money in this way is Sidrauski (1967). In Sidrauski's model, a person's real money balances enter the utility function. When monetary growth is excessive, there is an increase in inflation, so the stock of real cash falls, as does utility.
The obvious criticism of this type of model comes about because money is worthless in and of itself. Money may provide services, such as liquidity, but yields no utility benefits itself. The services of money ought to be modeled in this case, and these are likely to affect the budget constraint, not the utility function. McCallum (1983) defends the money-in-the-utility-function approach, basing his claims for it on indirect utility. But this may require that more than real balances be in the utility function, as indicated in chapter 3 of this dissertation.

4. Overlapping-generations models

The work of Samuelson (1958) and Diamond (1965) shows that overlapping-generations models are quite useful in examining changes in an economy over time. These models have the advantage of being relatively simple, yet capable of yielding sophisticated results. Their advantage over other models arises because they model life-cycle effects, since every individual may be followed from birth to death. At the same time, they answer the terminal-value problem of modeling money. Bryant (1980) is convinced that overlapping-generations models are the best models of money extant, as they are capable of answering all the relevant questions of monetary theory.

5. Clower-constraint models

One of the most difficult problems faced by monetary theory is in modeling money which is used for transactions. Clower-constraint models assume that money is used in every transaction [see Clower (1967)].
However, without some restriction on the holding period of money, it is impossible to measure the transactions demand for money. This is because money disappears from most budget constraints. The budget constraints of most models make it impossible to tell whether money is used to buy goods or if goods are used to buy goods. To avoid this problem, Clower-constraint models assume that money must be received in advance in order to be spent today. Income received at time \( t \) can't be spent until time \( t+1 \). Kohn (1981) argues that if this assumption is not made, money cannot be modeled as a medium of exchange.

The major criticism which can be made about Clower-constraint models is that they overly restrict the holding period of money. This restriction is probably necessary for tractability, but may be unrealistic in certain situations, for example, when hyperinflation occurs. While the holding period should be determined as a choice variable, the assumption of a minimum holding period may be no problem as long as it is a short period of time (e.g., one day or one hour).

A simple Clower-constraint model is developed in chapter 4 of this dissertation. It is shown to be isomorphic to the restricted-transactions model of chapter 3.
H. Financing by Taxes, Bonds, and Money

To make a decision about how to finance its spending, the government must estimate the costs and benefits associated with each method. To do this, some technical difficulties must be overcome.

The first problem is that a plan which is made optimally at one time may not be optimal later. This is a case of "the inconsistency of optimal plans" [see Kydland and Prescott (1977)]. When optimal plans are inconsistent, optimal control theory is useless for planning. However, there are conditions on the objective function under which optimal plans are consistent. These are summarized by Stutzer (1984).

The next (and related) issue is that of implementing optimal policy. In an environment of uncertainty, the effectiveness of policy may depend upon whether the government follows rules (given ahead of time and known widely) or uses discretion (based on the judgment of government agents). The choice of rules versus discretion may have a strong effect on people's expectations. Barro (1983) and Barro and Gordon (1983ab) analyze the choice of rules versus discretion for government debt policy. They find that fully-contingent rules are best, and that the ranking of simple rules versus discretion depends upon circumstances.

Another consideration for government finance is that of the tax system. While public-finance theory suggests the use of progressive taxes for redistributitional purposes, P. Miller (1983c) finds that regressive taxes may be desirable for stabilization purposes. The structure of the tax system may also be important in determining the
effects of fiscal and monetary policies. The model in chapter 3 illustrates the necessity of considering the tax structure in analyzing government financial policy.

A complete analysis of government financial policy ought to examine the full choice which is available to the government between using taxation, borrowing, and monetization. A recent attempt to do so is given by Aiyagari and Gertler (1983). Their interesting work verifies, in a quite complicated model, that the Ricardian Equivalence Theorem can hold. However, it holds only if government monetizes none of the deficit. If some monetization occurs, then government bonds become part of net wealth, and government debt is not neutral. Depending upon the structure of taxation, real and nominal interest rates may be affected by open-market operations. These results are interesting and significant, but the model can be criticized for lacking solid microfoundations, as the authors utilize a money-in-the-utility-function approach.

The same criticism can be made of Helpman and Sadka (1979). They take into account public-finance considerations in a second-best optimization problem, and find that the optimal interest rate is positive. However, in their model money and bonds play the same roles as stores of value, but money is included in the utility function while bonds are not. This dissertation's chapter 5 reaches results similar to those of Helpman and Sadka, but without artificially distinguishing money from bonds by including money in the utility function.
I. Summary and Conclusions

The alternative methods of government finance must be analyzed in terms of their welfare costs. In this chapter, we examine the economic theory concerning the costs and benefits of taxation, monetization, and debt as sources of government revenue. Despite the long period over which economic theorists have analyzed these problems, there are few solid conclusions.

Three areas of theory need further development. Firstly, and most importantly, an analysis of government financial policy must model money in a reasonable manner. A model of money with microeconomic foundations is much richer than one with an ad-hoc money-demand function. Secondly, an understanding of how the intertemporal allocation of consumption is affected by government policy is vital, especially in a growth model. Confusion over the relation between growth rates and interest rates has complicated this analysis. Finally, more work is needed to analyze the full choice which government has between financing by money, debt, and taxes. As is shown dramatically in chapter 7 of this dissertation, analyzing the effects of a change in one government policy tool, without examining the others, is likely to yield deceptive results.

Many of the results of the theories discussed in this chapter have been tested empirically. Chapter 2 reviews the empirical literature.
FOOTNOTES TO CHAPTER 1

1 The debate over the idea of a sinking fund had an interesting history. Originally proposed in the late 1700's in Britain, the idea was to establish a fund to be used to pay off the debt. The earliest writers were very impressed with how compound interest could make such a fund grow. The absurdity of the notion was pointed out later by many authors. See McCulloch (1857), especially the article by Hamilton (1818), which Ricardo (1820) discussed in his paper "Funding System."

2 Hansen (1953) suggests that Keynes "never faced up to the debt problem" [p. 219]. Keynes's macroeconomic theory failed to examine any of the long-run implications of a growing national debt.

3 Klein (1966) suggests that distributional issues are involved as well. Since the rich are the major taxpayers and bondholders, they really just borrow from and repay each other. The poor are made better off as the public projects financed by the debt help to reduce unemployment.

4 This theory abstracts completely from risk. Consider the interest rate here to be either risk-adjusted or consider the model to be one of a risk-free world.

5 Inflation is not a necessary result of increased monetization, however, as shown by Wallace (1981) and demonstrated in chapter 3 of this dissertation.

6 The Golden Rule is followed when the interest rate equals the growth rate of the economy. This maximizes steady-state consumption, as shown by Phelps (1966).

7 This discussion refers to explicit interest only. An additional consideration is the implicit interest earned on money held at a financial institution. Implicit interest may be in the form of checking services, branch bank locations, automatic teller machines, or other services which are priced below marginal cost. See Rush (1980).

8 The legitimacy of doing so has long been disputed as well. For example, see Hicks (1935).

9 In a model with a finite-time horizon, money has no value in the last period. No one wants to hold money as a store of value, because there is no next period. But if money has no value in the last period, it will have no value in the next-to-last period, and so on. So in a finite-horizon model, money has no value. Since overlapping-generations models have infinite time horizons, this is not a problem. See the articles in Kareken and Wallace (1980) for a general discussion of these issues.
Chapter 2

The Empirical Evidence on the Effects
of Government Financial Policy

The economic theory discussed in chapter 1 presents testable hypotheses. This chapter looks briefly at the major empirical results of testing some of these hypotheses.

If the Ricardian Equivalence Theorem holds, then people should respond to an increase in government debt by increasing savings dollar for dollar. This is an easily testable proposition, but differing econometric approaches have produced a wide range of empirical results. These are presented in section A. A key issue in these empirical tests is the method used to measure the government debt. Potential adjustments in published figures are examined in this section.

An unfunded social-security system is much like government debt, as the analysis in chapter 1, section F suggests. Empirical work similar to that in section A, but looking at "social-security debt" rather than explicit government "treasury" debt is examined in section B. These results do not appear to be very robust.

When government debt is not neutral,¹ a change in the method of financing may affect inflation and interest rates. The mixed evidence and econometric difficulties involved in testing these effects are examined in section C.
A. The Effects of Government Debt on Savings and Consumption

The first study to try to take into account the future tax liabilities implied by government debt is that of Tanner (1970). He estimates a single-equation model for savings as a function of disposable income, the interest rate, and a real-balance variable designed to account for future tax liabilities. His results indicate that there is almost complete capitalization of future tax liabilities.

Tanner estimates the following model:

\[
S = 16.43 + 0.281 Y_D - 1.758 r - 3.058 M/p - 0.0335 \text{DEBT},
\]

\[\begin{align*}
(1.91) & & (8.52) & & (2.57) & & (3.52) & & (1.93)
\end{align*}\]

\[R^2 = .795, \text{ DW} = 1.34,\]

where \(S\) is real savings, \(Y_D\) is real disposable income, \(r\) is the nominal interest rate, \(M/p\) is real money balances, and \(\text{DEBT}\) is the real government debt. The proxy used for the interest rate is the T-bill rate. The data set consists of quarterly Canadian data for 1951-67. Lacking a good measure of real government debt, Tanner creates a proxy variable using real interest payments made by the government divided by the market interest rate.

Yawitz and Meyer (1976) criticize Tanner, arguing that his real-balance variable treats all government bonds as perpetuities. Thus he overestimates capital gains and losses on government debt due to changes in the interest rate. Yawitz and Meyer also suggest that Tanner should include net private wealth in the estimated equation. The use of currency by itself biases its coefficient upward, as it picks up some wealth effects.
Taylor (1971) finds that 90% of a tax reduction is added to savings. Using quarterly U.S. data for 1953-1969, he estimates an equation of the form:

\[(2) \quad S_t = 0.955 S_{t-1} + 0.449 \Delta LY - 0.277 \Delta PY + 0.893 \Delta TY \]
\[- 2.16 \Delta SSC - 0.901 \Delta TAX + 3.65 \Delta r, \]
\[(113.8) \quad (4.21) \quad (0.86) \quad (2.86) \]

\[- 2.16 \Delta SSC - 0.901 \Delta TAX + 3.65 \Delta r, \]
\[(3.30) \quad (4.87) \quad (2.08) \]

\[R^2 = .899, \quad SE = 2.01,\]

where \(S\) is savings, \(LY\) is labor income, \(PY\) is property income, \(TY\) is transfer income, \(SSC\) is social-security contributions, \(TAX\) is personal taxes, and \(r\) is the nominal interest rate. The variables are all in real aggregate terms.

The marginal propensity to save (MPS) out of transfers is near 90% as is the MPS out of tax reductions. Furthermore, the coefficients are not significantly different from one. The coefficient on \(SSC\) suggests that a dollar of increased social-security contributions reduces savings by two dollars, which makes sense due to the matching contributions of employers.

Kochin (1974) estimates a very simple regression equation with consumption as a function of disposable income and the federal deficit. He fits the equation:

\[(3) \quad \Delta C = 2.88 + 0.392 \Delta Y_D - 0.109 \Delta DEF + 0.218 \Delta C_{t-1}, \]
\[(3.44) \quad (7.86) \quad (2.95) \quad (2.42) \]

\[R^2 = .89, \quad SE = 1.26,\]

on annual U.S. data, 1952-1971. The variables are \(C\) = real consumption expenditures on nondurables, \(Y_D\) = real disposable income, and \(DEF\) = the real value of the deficit. The data are differenced to correct for
autocorrelation, a dubious procedure here unless special assumptions are made about the error term.

The results indicate that an increased deficit reduces consumption only by about 10% of the deficit amount. A change in disposable income raises current consumption by about 40% of the change in income, with a long-run marginal propensity to consume of about 50%, which seems rather low compared to usual results.

Kochin's study has been widely criticized. Yawitz and Meyer (1976) suggest using a government-debt variable and a measure of monetization, rather than a deficit measure, since money and bonds are likely to have different economic effects. The question which an empirical study should examine, argue Yawitz and Meyer, is whether any part of government bonds should be included in net wealth. But Kochin looks at the deficit, some of which might be monetized, and his regression includes no net-wealth variable. When Yawitz and Meyer include net wealth in Kochin’s equation and run the regression, the coefficient on the deficit variable becomes insignificant.

Yawitz and Meyer attempt to improve on Kochin’s equation by including the market value of privately-held U.S. government debt. They set up an econometric equation based on the Ando-Modigliani life-cycle model. They construct an estimate of the market value of government debt, correcting for the non-perpetuity of debt. Running a regression on U.S. data for 1953-1969, they obtain the following results:

\[
(4) \quad C_t = 0.75 \ Y_t + 0.03 \ A_t^{h_{t-1}} + 0.05 \ B_t^{g_{t-1}},
\]

\[
\begin{align*}
(17.3) & \quad (3.60) \quad (3.88) \\
R^2 &= 0.9999, \quad DW = 1.82,
\end{align*}
\]
where the variables are $C = \text{consumption}$, $Y = \text{income}$, $A^h_t = \text{household net wealth}$, net of holdings of government bonds, $B_g = \text{market value of government debt}$. An F-test, testing the hypothesis that the coefficients on $A^h_t$ and $B_g$ are equal, cannot reject equality. Equality of the coefficients means that there is no discounting of government debt—it is treated fully as net wealth.

Because of this result, Yawitz and Meyer combine the net wealth and government-debt variables, and estimate the following equation:

$$(5) \quad C_t = 0.78 \cdot Y_t + 0.03 \cdot A^h_{t-1} + 0.03 \cdot B^g_{t-1},$$

$$(16.4) \quad (2.91) \quad t-1$$

$R^2 = 0.9999$, $DW = 1.34$, where $A^h_{t-1} = A^h_{t} + B^g_{t}$.

Yawitz and Meyer thus come to the exact opposite conclusion from that suggested by the Ricardian Equivalence Theorem. Their empirical evidence suggests that people completely ignore future tax liabilities. However, they admit that over their sample period, the value of the government debt showed little variation. For this reason, their result is not very conclusive. Comparing equations (4) and (5), the big change in the Durbin-Watson statistic suggests possible misspecification. Equation (4) is probably a more reasonably specified equation, because it yields an error term which is not serially correlated. The test for equality of the coefficients on $A^h_t$ and $B^g_t$ has relatively low power.

Butter and Tobin (1979) suggest that if the Ricardian Equivalence Theorem holds, the coefficients on $Y_d$ and $DEF$ in equation (3) should be equal in magnitude and opposite in sign. So Kochin really provides evidence of non-neutrality. Kochin's work is flawed, according to
Buiter and Tobin, because of simultaneity in cyclical fluctuations of $C$, $Y_D$, and DEF. Fiscal policy moves the deficit countercyclically, so there is simultaneous-equations bias in equation (3). Buiter and Tobin are critical of Kochin's proxy choices as well, suggesting that state and local debts are important, that private, not personal, disposable income should be used, that the dependent variable should include the imputed value of durables, and that everything should be in per-capita terms. Furthermore, equation (3) has a constant term, implying a time trend in the undifferenced equation, which Kochin did not include when he ran his regression before differencing it.

Buiter and Tobin replicate Kochin's experiment fairly well over a different data set. They then perform sensitivity tests which show that Kochin's results are very sensitive to the sample period chosen. Adding the years 1949-51 and 1972-76 to the sample period deprives the deficit of any explanatory power. Buiter and Tobin then implement the suggestions made above on proxy choices. They find no support for debt neutrality although the robustness of the results is not great due to the presence of multicollinearity in all the independent variables. They then estimate a Keynesian consumption function with the following results:

(6) \[ C = 122.91 + 0.876 Y_D, \]

\[ (4.04) \quad (90.0) \]

\[ R^2 = .997, \ SE = 26.9, \ DW = 1.59, \ \hat{\rho} = .121, \]

where all variables are in per-capita terms and $\hat{\rho}$ is the estimated first-order autocorrelation coefficient. $C$ includes the imputed value of durables.
Buiter and Tobin argue that their results support Keynesian macroeconomic theory. The Ricardian Equivalence Theorem fails, and people don't respond to future tax liabilities. The marginal propensity to consume out of current disposable income is nearly 90%.

Finding that a Keynesian-type consumption function gives a fairly good fit, of course, says nothing about the Ricardian Equivalence Theorem. While Buiter and Tobin correct for autocorrelation, it seems likely that their low Durbin-Watson statistic is due to specification error.

Holcombe, Jackson, and Zardkoohi (1981) attempt to estimate a more complicated model of personal savings. Personal savings depends upon permanent income, transitory income, liquid assets, interest rates, the inflation rate and the expected inflation rate, government debt and expenditures, and the business cycle. They estimate a regression model with all these variables. However, multicollinearity in the variables makes estimation difficult. They run a smaller version of their model, leaving out some variables. Unfortunately, the variables left out are theoretically the most-important ones for savings behavior: the interest rate and permanent income. They find reason to reject the Ricardian Equivalence Theorem, because a one-dollar increase in government debt leads to only a 20-cent increase in savings.

Seater (1982) estimates a complex and interesting econometric model, based on the empirical work of Barro (1978c). One of his estimated equations is:
\[
(7) \quad C = 394.9 + 0.36 \left[ Y_D^t + RE_t \right] + 0.08 \left[ Y_D^{t-1} + RE^{t-1} \right] \\
(0.83) \quad (5.14) \quad (0.73) \quad (0.73) \quad (1.83) \quad (5.14) \\
- 0.016 U \cdot Y_D^t + 0.17 DUR + 0.01 \ W + 0.17 \ M \\
(0.06) \quad (0.77) \quad (1.11) \quad (0.81) \\
- 345.3 \left[ R_b (1-\tau)-\pi \right] - 0.007 \%AR_b \cdot W - 0.25 \left[ DEF-MON \right] \\
(2.08) \quad (1.17) \quad (1.39) \\
- 0.14 \ MON - 0.14 \left[ \Delta M-MON \right] - 0.002 \ DEBT \\
(0.23) \quad (0.25) \quad (0.03) \\
+ 0.108 \left[ \Delta DEBT - (DEF-DM) \right] + 0.05 \ SSW, \\
(1.08) \quad (1.25) \\
\]

\[ R^2 = .9995, \quad \hat{\rho} = -0.38, \quad Q = -0.14, \]

where all stocks and flows are in real per-capita terms, and the variables are $C =$ consumption, $Y_D =$ disposable income, $RE =$ corporate retained earnings, $U =$ unemployment rate, $DUR =$ stock of durable goods, $W =$ private wealth, $M =$ monetary base, $R_b =$ corporate bond rate, $\tau =$ average marginal tax rate, $\pi =$ expected inflation rate, $DEF =$ current government deficit, $MON =$ current government deficit which is monetized, $DEBT =$ market value of outstanding government debt, and $SSW =$ Feldstein's measure of social-security wealth. $Q =$ Quinn's statistic for testing for first-order autocorrelation (rejected after the initial correction). Annual U.S. data are used over the sample period 1947-1974.

Seater interprets the estimated coefficients on the last six variables as providing evidence in support of the theory that people take into account future tax liabilities. However, the insignificance of almost every variable except disposable income plus retained earnings suggests that there is a large standard error of estimate (which isn't reported), so that the hypothesis tests have relatively low power. This possibility is supported by the fact that coefficients which are
expected to be nonzero are also found to be insignificant. Furthermore, an additional regression on financial-asset demand yields inconsistent results. Seater blames this on uncorrectable autocorrelation, but the reason seems more likely to be specification error.

Kormendi (1983) attempts to test the neutrality of debt in a permanent-income model. His estimated equation is:

\[
\Delta C = 0.29 \Delta Y + 0.04 \Delta Y_{t-1} - 0.22 \Delta G + 0.04 \Delta W + 0.72 \Delta TR \\
+ 0.06 \Delta TX + 0.10 \Delta RE - 0.30 \Delta GINT, \\
\]

\[ R^2 = .890, \ SE = .0192. \]

This equation is estimated in first-difference form on annual U.S. data for 1930-76. The coefficients (all in real per-capita terms) are $C = \text{personal consumption expenditures}, Y = \text{net national product (not the usual disposable personal income)}, G = \text{government spending on goods and services}, W = \text{wealth}, TR = \text{transfer payments}, TX = \text{tax payments}, RE = \text{retained earnings}, GINT = \text{government interest payments}.$

If debt neutrality holds, then the coefficients on TX, RE, and GINT should be zero, and the coefficient on G should be negative. The regression results in (8) thus support the debt-neutrality hypothesis. The results are claimed to be robust to omitting war years, using alternative estimation techniques, using alternative proxies, and using disposable income instead of net national product as an independent variable.

Although these results seem strong, they don't necessarily support full debt neutrality. If debt is neutral (ignoring possible monetization), then the coefficient of G should be fairly close to -1,
because government spending should crowd out consumption and investment one for one. The results of equation (8) indicate that only 22% of government spending comes out of consumption, leaving the remaining 78% to come from investment. Also, first-differencing as a correction for autocorrelation is an improper procedure. Again, specification error seems to be a distinct possibility.

Carmichael (1984) suggests that the reason these studies result in such a wide range of parameter estimates is because they focus on the wrong variable. The effects on consumption of a change in government financing are likely to be fairly small. The real test is to see if private savings increases one-for-one with an increase in government debt.

Carmichael estimates the following regression equation on annual U.S. data for 1947-74:

\[
(9) \quad K = 7700 + 0.528 [\hat{Y} + \text{RE}] + 0.938 [\hat{Y} + \text{RE}]U + 0.577 r - 0.226 \Delta \text{DEBT} \\
\quad \quad \quad \quad + 0.137 \text{NW(DEBT)} - 0.662 \text{DEBT} - 11.04 \Delta \text{SSW} \\
\quad \quad \quad \quad + 4.901 \text{NW(SSW)} - 0.145 \text{SSW},
\]

\[R^2 = 0.993, DW = 1.78,\]

where \(K\) is the private capital stock, \(\hat{Y}\) is disposable income adjusted for changes in current and future taxes, \(\text{RE}\) is retained earnings, \(r\) is the real return on capital, \(\text{DEBT}\) is the market value of government debt, \(\text{NW(DEBT)}\) is the net wealth of government debt, and \(\text{SSW}\) is social-security wealth. All stocks and flows are in real per-capita terms. The net wealth of government debt and of social security is positive only if the growth rate of the economy exceeds the real interest rate.
These net-wealth terms represent measures of the intergenerational free lunch.

Carmichael's results lend partial support to the non-neutrality hypothesis. The coefficients on ΔDEBT, NW(DEBT), DEBT, ΔSSW, and SSW ought to be zero if debt is neutral, according to Carmichael. Under non-neutrality, the signs of the coefficients on ΔDEBT and NW(DEBT) ought to be positive, due to reduced current taxes when an increase in debt occurs. The signs on DEBT, ΔSSW, NW(SSW), and SSW ought to be negative. The estimated coefficients on debt support the hypothesis of complete non-neutrality, but the coefficient on SSW supports complete neutrality.

Three difficulties with Carmichael's specification must be raised. First, the significant coefficient on DEBT may be attributed to the strong negative time trend of real government debt (per-capita) and the strong positive time trend of the capital stock. The major reason for the decline in real debt is inflation's undermining the purchasing power of nominal debt. Second, there is likely to be simultaneity bias in the results. The interest rate is a market-clearing price for savings and investment. Government financial policy may well affect both sides of this market. This suggests that a simultaneous-equations system is the proper specification. Finally, the model ignores any possible monetization of government deficits, assuming implicitly that inflation is innocuous for real variables.

One key difficulty which may strongly influence the empirical results is the problem of developing an appropriate measure of government debt. Government debt is usually reported in nominal form
(par value). For most theoretical analysis what is needed is a measure of real debt. Accounting for inflation is not enough, however. Because government debt (most often) pays a fixed nominal rate of return, changes in market interest rates change the value of the debt.

Most empirical studies have failed to correct for changes in the market value of debt. Seater (1981) constructs a series on the market value of government debt by examining annual data on the par value of outstanding debt and adjusting for changes in interest rates. He finds that his constructed series is more accurate (in terms of its correspondence to data on actual market values of debt) than other series derived by alternative methods, such as that used by Tanner (1979).

There is a danger, however, in adjusting debt statistics to account for inflation and changes in interest rates. If changes in nominal debt are thought to change the inflation rate or cause changes in interest rates, then using real rather than nominal data may lead to a simultaneity problem. This is especially vital for regressions on savings where the interest rate is an independent variable, such as Tanner (1970), Seater (1982), and Carmichael (1984), as shown in equations (1) and (9) above.

Correcting the data to get the market value of debt is not the only consideration. As the theory presented later in chapters 5, 6, and 7 shows, the measure of the debt ought to be reduced by the amount which government lends. Grossman (1982) suggests this same idea. Another consideration is that of valuing government's tangible assets. Should government debt be measured net of government's capital stock? How
should the implicit liabilities of government programs, such as Social Security, FDIC and FSLIC insurance programs, corporation bailouts, and loan, pension, and deposit guarantees be measured, and should they be included in debt? These questions suggest that government-finance accounting needs to maintain several types of accounts including cash-flow accounts and net-wealth accounts, just as is done in the private sector.

When debt is adjusted for inflation, the resulting series often moves opposite to what is popularly believed. In the U.S., despite ever-growing deficits, the real value of federal government debt as a proportion of real GNP has fallen monotonically from the end of World War II to 1980. The large deficits of the Thatcher government in Great Britain are actually surpluses, according to M. Miller (1982). He adjusts for inflation and finds that the real British government debt has fallen in recent years when there were extremely high deficits (as conventionally measured).

In attempting to create a financial net-worth series for the U.S. government, Eisner and Pieper (1984) find some interesting results. They subtract the financial assets of the government from measured debt to get a "net debt" series. Government holdings of gold are included as assets. While adjustments for inflation and interest-rate changes are found to be important in determining changes in net debt, changes in asset prices also play a significant role. The large increase in the market price of gold in 1979 reduced the net debt of the government by $64 billion.
New measures of government net debt which follow the suggestions given above are worthwhile for accounting and budgeting purposes, but they can't measure fiscal impact on the economy, according to Buiter (1984). He suggests that any measure designed to tell whether fiscal policy is expansionary or contractionary depends upon an economic model. No model-free measure of fiscal impact can be found.
B. Empirical Studies of Social Security

Because the Social-Security System in the United States is an unfunded pay-as-you-go arrangement, it has effects very similar to government debt. Feldstein (1974, 1976) argues that his results which show that social security causes a decrease in savings, disprove the Ricardian Equivalence Theorem. A plethora of studies followed [Munnell (1974, 1976), Barro (1978c), Darby (1979), von Furstenberg (1979); see the summary in Danziger, Haveman and Plotnick (1981)].

Many of these studies have major problems. If both government debt and social security are important for their future tax-liability effects then both should be included in the same study. To the extent that social security is an "intergenerational free lunch" as suggested above, this must be accounted for, as suggested in chapter 1, section F. Account needs to be made of human capital as well as physical capital. Human capital bequests must be separated from other financial assets, since pensions may not be fully vested. Many of the studies use Feldstein's (1974) figures on perceived social-security wealth, which were shown to have been calculated incorrectly, by Leimer and Lesnoy (1982). Leimer and Lesnoy find that correcting a simple error in Feldstein's data significantly affects the estimated impact of social security on savings. They show that the empirical results on the effect of social security on savings are not robust. Results are highly sensitive to reasonable changes in assumptions about the structure of the economy, and to the sample period used.
The study by Gultekin and Logue (1979) attempts to correct for many of the problems mentioned above. However, severe econometric problems may exist in their results, since many coefficients end up with theoretically wrong signs.

There is an additional problem in comparing the effects of social security and government debt. In the United States, the Social-Security System may affect the labor-supply decisions of individuals [Munnell (1974)], changing their life-cycle consumption and savings patterns, due to the heavy effective tax imposed on working between ages 65 and 71. Furthermore, the Social-Security System can be seen as a type of retirement income insurance, reducing the uncertainty of consumption in retirement years. Clearly a proper empirical study would allow separate effects for social security and government debt, and attempt to control for the many other problems mentioned above.
C. The Effect of Deficits on Inflation

If government debt is not neutral, then the government deficit, which increases the debt, may affect the inflation rate. This effect may occur when the monetary authority buys some of the newly-issued government debt, or buys more existing debt on the open market. In this section, we look at the effects deficits have on the degree of monetization, which is closely related to the inflation rate.

Hamburger and Zwick (1981) report evidence that changes in U.S. government deficits affected the money supply only after Keynesian policy was implemented in the 1960's. Prior to the 1960's, monetization of deficits was insignificant (of course, deficits were small in those years as well).

McMillan and Beard (1982) attempt to replicate the results of Hamburger and Zwick, using revised national-accounts data. The estimated coefficients are considerably different. They find no relationship between deficits and monetization. Sensitivity tests show, however, large changes in estimated coefficients when the sample period is changed slightly. Hamburger and Zwick (1982) suggest that McMillan and Beard altered the time alignment of the variables in attempting to replicate their study. When the variables are properly aligned, the estimated coefficients differ insignificantly from their values using the earlier data. Hamburger and Zwick (1982) also find evidence that the change in operating procedures by the Federal Reserve Board in 1979 did not significantly change the degree to which the Fed monetizes deficits.
Dwyer (1982) tests hypotheses on the relationship between changes in government debt and inflation using vector-autoregressive techniques. He finds no support for the hypothesis that the Fed increases monetization as deficits grow. In fact he suggests that inflation causes deficits to grow, rather than vice-versa.

In estimating a reaction function for monetization by the Fed, Blinder (1983) finds that deficits affect monetization, and that the relationship between deficits and monetization has been stable since 1961. Significantly, he finds that monetization is inversely related to previous inflation and to the growth in government spending. The estimated equation is:

\[
\frac{AM}{GNP} = 0.0013 + 0.064 \frac{DEF}{GNP} - 0.733 \frac{DEF}{GNP} \pi_{t-1} \\
- 0.398 G \frac{DEF}{GNP} - 0.099 \frac{CURR}{GNP},
\]

\( R^2 = .56, DW = 2.16, \)

where \( M \) is adjusted bank reserves, \( GNP \) is nominal gross national product, \( DEF \) is the federal budget deficit, \( \pi \) is the inflation rate, \( G \) is nominal government purchases, and \( CURR \) is the stock of currency. The data set is annual U.S. data for 1961-1981.

The deficits in all of the studies mentioned above are annual deficits. P. Miller (1983a) argues that it is higher deficit policies, not higher deficits, which leads to inflation. The focus on deficits is incorrect, because deficits show strong business-cycle effects and are subject to random shocks. A higher deficit policy would be a change in some parameter in the government's policy-formation function. P. Miller
(1982c) uses vector-autoregressive techniques to determine that such a policy shift occurred in the United States in about 1967. The shift was towards the increased use of deficits to finance government spending.
D. Summary and Conclusions

This chapter examines the empirical literature on the effects of debt and deficits upon saving, consumption, and inflation. A considerable amount of empirical work has been performed which attempts to test for the neutrality of debt. The results vary widely from study to study, and seem not to be robust. Specification error appears to be a major problem, with simultaneity the possible culprit. The full choice of financing government spending by taxation, monetization, and debt is often ignored in the econometric frameworks of these studies.

These results suggest the need for additional theoretical work aimed at providing an improved framework for econometric estimation. This need motivates the next chapter, which presents an irrelevance theorem for government debt and monetization.
1 This refers to macro-neutrality. Under macro-neutrality, endogenous macroeconomic variables such as interest rates and inflation aren't affected by a change in government financing. However, individuals may be affected (in terms of their lifetime consumption streams). Chapter 3 of this dissertation discusses the difference between micro-neutrality and macro-neutrality.

2 In every equation in this chapter, the numbers in parentheses below the coefficients are the t-statistics, DW is the Durbin-Watson statistic, and SE is the standard error of estimate. DW and SE were not always reported by the authors. Time subscripts are omitted whenever the meaning is clear.

3 This may also be why Kochin found a low Durbin-Watson value in his non-differenced equation. It seems likely that the problem is specification error, not autocorrelation. RESET and Hausman tests for specification error should be usefully employed here.

4 Private disposable income includes the corporate sector, but personal disposable income does not.

5 The (implicit) time trend in equation (3) which is missing from Kochin's non-differenced specification probably explains the significantly different coefficient estimates between the differenced and non-differenced versions. This isn't surprising with a time-series data set in which all variables are growing.

6 Real federal debt has risen significantly during President Reagan's term in office, beginning in 1981.
Chapter 3
The Neutrality of Optimal Government Financial Policy

This chapter develops a neutrality theorem for government financial policy. In a framework in which the government acts as an optimizing agent, it is shown that the choice of financing government expenditures by taxation, debt issuance, and monetization is irrelevant. Neither monetary policy nor fiscal policy affects the interest rate or inflation.

In this chapter we set up an overlapping-generations model of two-period lived individuals, some of whom are poor and some of whom are rich. The poor hold their savings in the form of money, which pays no nominal interest, while the rich hold bonds which may pay a positive interest rate. Government chooses an optimal policy which maximizes the welfare of each individual when each generation receives the same real consumption stream. Following this optimal policy gives the government complete freedom to finance itself by any amount of debt and any amount of money issuance, as long as it properly adjusts taxes between rich and poor and between old and young.

The results of this chapter suggest that any analysis of fiscal and monetary policies must focus on the structure of taxation. Economic research often suggests that a change in the government's choice of financing between taxes, bonds, and monetization leads to changes in
interest rates and inflation. However, these studies generally ignore the effects of changes in the temporal structure of taxation.

This chapter presents a simple restricted-transactions overlapping-generations model. Individual maximization is examined in section A. Aggregate equilibrium is shown in section B. Optimal monetary and fiscal policy is analyzed in terms of providing an "intergenerational free lunch", which is a pure Pareto-improvement. This is done in a model without taxation in section C. In section D, the government's full choice between using taxes, bonds, and money is considered. The maximization problem faced by the government is examined in section E. A neutrality theorem, which shows that taxes can be used as perfect substitutes for bonds and money, is established in section F. The results are summarized and extensions are suggested in section G.

In this chapter, we model money as an intrinsically useless asset, held only because it serves as a store of value. Bonds coexist with money and promise a nominal return in the future. We choose the restricted-transactions approach, in which substitution between money and bonds is restricted in some way, but suggest that the same results hold in a Clower-constraint model, in which money serves explicitly as a medium of exchange.
A. The Individual Maximization Problem

This is a model in which people can hold money or bonds. There is a minimum denomination for bonds, so that they can only be purchased by people who have relatively high income or wealth. This minimum denomination can be justified by transaction costs (a theme developed in chapter 5) or by legal restrictions (see Wallace (1983)). In contrast to previous uses of this type of model (for example, by Sargent and Wallace (1981), Buiter (1982), and Bryant and Wallace (1980)), in which bonds pay a real return, we assume here that the return on bonds is in nominal terms.¹

People live two periods, working only during the first period of their lives. There is one good, which is used solely for consumption. The good can be costlessly stored until the following period.

At time \( t \), \( N_p(t) \) identical poor people are born, who produce \( \alpha \) units of output in their first year of life, and \( N_r(t) \) identical rich people are born, who produce \( \beta \) units of output while young. Total population is \( N(t) = N_p(t) + N_r(t) \), and the poor and rich populations grow at the same rate \( n \), so that:

\[
(1) \quad N_p(t) = (1+n)N_p(t-1) \quad \quad N_r(t) = (1+n)N_r(t-1).
\]

Bonds have a minimum real size \( k \), where \( \beta/2 > k > \alpha \), but are divisible above this minimum size. People are allowed to either buy or sell bonds, but not both.² Legal restrictions or transactions costs are assumed to prevent people from sharing ownership in bonds. Consequently, the poor never hold bonds.
Let $c_i^h(s)$ denote the consumption of a person at time $s$, in period of life $i$ ($i=y$ young, $i=o$ old), of population $h$ ($h=p$ poor, $h=r$ rich).

Each individual maximizes utility,

\[
U^h = \ln c_i^h(t) + \delta \ln c_i^h(t+1), \quad (h=p,r),
\]

where $0 < \delta \leq 1$. The term $\delta$ is the time discount rate.

Saving is defined as $s_i^h(t) = [w^h - c_i^h(t)]p(t)$, where $w^h$ is the real production by a person ($w^p = a$, $w^r = \beta$), and $p(t)$ is the price level at time $t$.

Denote the nominal rate of interest as $R(t)$. Bonds purchased at time $t$ in the amount $s_i^r(t)$ return a payoff of $[1+R(t)]s_i^r(t)$ at time $t+1$, if $s_i^r(t) > k \cdot p(t)$.

Holding money in the amount $s_i^p(t)$ at time $t$ yields a nominal return of $s_i^p(t)$ at time $t+1$, as money pays no nominal interest. Storing the consumption good in the real amount $s_i^p(t)/p(t)$ returns the same real amount, which has a nominal value at time $t+1$ of $s_i^p(t) \frac{p(t+1)}{p(t)}$.

Consequently money is used for savings by the poor only if $\frac{p(t+1)}{p(t)} \leq 1$ for all $t$. If $\frac{p(t+1)}{p(t)} > 1$, then storage is used instead. We assume for now that $\frac{p(t+1)}{p(t)} \leq 1$, so that storage is not used. Perfect foresight is assumed, so that expected changes in the price level equal actual changes in the price level.

1. The Individual Maximization Problem of the Poor

A poor person born at time $t$ maximizes utility by choosing consumption while young, consumption while old, and savings (in the form of money).
\[
\max_{c_Y(t), c_P(t+1), s^p(t)} U = \ln c_Y^p(t) + \delta \ln c_O^p(t+1)
\]
\[s.t. \quad s^p(t) = [\alpha - c_Y^p(t)]p(t)\]
\[c_P^p(t+1)p(t+1) = s^p(t).\]

This can be set up in Kuhn-Tucker form and solved to get:

(3) \[c_Y^p(t) = \frac{1}{1+\delta} \alpha.\]

(4) \[c_P^p(t+1) = \frac{\delta}{1+\delta} \frac{p(t)}{p(t+1)} \alpha.\]

(5) \[s^p(t) = \frac{\delta}{1+\delta} \alpha p(t).\]

For example, suppose the time discount rate is \(\delta = 0.9\), output by a poor person is \(\alpha = 6\), and the price level changes according to \(\frac{p(t)}{p(t+1)} = 2\). Then a poor person would choose \(c_Y^p = 3.16\), \(c_P^p = 5.68\), \(s^p(t) = 2.84\). If the minimum denomination on bonds is \(k = 7\), the poor will always choose to hold money and never hold bonds.

2. The Individual Maximization Problem of the Rich

The problem of the rich differs only in that they may earn interest on their savings.

\[
\max_{c_Y^r(t), c_O^r(t+1), s^r(t)} U = \ln c_Y^r(t) + \delta \ln c_O^r(t+1)
\]
\[s.t. \quad s^r(t) = [\beta - c_Y^r(t)]p(t)\]
\[c_O^r(t+1)p(t+1) = [1+R(t)]s^r(t).\]

This can be set up in Kuhn-Tucker form and solved to get:

(6) \[c_Y^r(t) = \frac{1}{1+\delta} \beta.\]

(7) \[c_O^r(t+1) = \frac{\delta}{1+\delta} [1+R(t)] \frac{p(t)}{p(t+1)} \beta.\]

(8) \[s^r(t) = \frac{\delta}{1+\delta} \beta p(t).\]
We assume that $\delta$ is such that $\frac{\delta}{1+\delta} \beta > k$, so that the rich always hold bonds and never hold money.

For example, with $\delta=0.9$, and $\frac{p(t)}{p(t+1)} = 2$ as in the last numerical example, suppose that output by a rich person is $\beta=20$, and the interest rate is $R(t)=0.5$. Then a rich person would choose $c^r_y=10.53$, $c^r_o=28.42$, $s^r(t)/p(t) = 9.47$. With a minimum denomination on bonds of $k=7$, the rich always choose bonds and never hold money.

In comparing equations (3)-(5) and (6)-(8), we note that consumption while young and the real savings of both rich and poor are unaffected by either the interest rate or the inflation rate. This is a feature of the simple Cobb-Douglas model used here, and is not a significant factor in the later results. That is, the irrelevance theorem proven later in this paper also holds up under more-complicated models in which the interest rate and inflation rate affect savings and first-period consumption.\(^3\)

Equations (5) and (8) give individual demands for money and bonds, respectively. The difficulty which arises is that these planned demands may not clear the markets for money and bonds at any levels of the inflation rate and interest rate. The achievement of market equilibrium is examined in the next section.
B. Aggregate Equilibrium

Now that we have solved the maximization problems of individuals, we examine the conditions necessary for market clearing to occur.

1. Consumption goods

At time \( t \), the total demand for consumption goods, \( D_c(t) \), is the sum of consumption of the young poor, young rich, old poor, and old rich.

\[
D_c(t) = N_p(t) c_y^p(t) + N_r(t) c_y^r(t) + N_p(t-1) c_o^p(t) + N_r(t-1) c_o^r(t).
\]

At time \( t \) the total supply of goods, \( S_c(t) \), consists of goods produced by the young poor plus goods produced by the young rich.

\[
S_c(t) = N_p(t) \alpha + N_r(t) \beta.
\]

At equilibrium, \( D_c(t) = S_c(t) \).

Notice that one possible solution (but not the only one) is a solution in which the poor trade only with the poor, and the rich trade only with the rich. We call this the segmented-society solution, and it is typical of restricted-transactions overlapping-generations models.

The segmented-society solution is given by:

\[
\begin{align*}
N_p(t) \alpha &= N_p(t) c_y^p(t) + N_p(t-1) c_o^p(t), \\
N_r(t) \beta &= N_r(t) c_y^r(t) + N_r(t-1) c_o^r(t). 
\end{align*}
\]

Equations (11) and (12) can be solved, using equations (1), (3)-(5), (6)-(8) to yield:

\[
\begin{align*}
\frac{p(t)}{p(t-1)} &= \frac{1}{1+n}, \quad \text{for all } t. \\
R(t) &= 0, \quad \text{for all } t.
\end{align*}
\]
The segmented-society solution occurs when the nominal interest rate is zero and price deflation occurs at a specific rate. Defining the inflation rate as:

\[ \pi(t) = \frac{p(t)}{p(t-1)} - 1, \]

we require the inflation rate to be \(-\frac{n}{1+n}\) when the segmented-society solution occurs. Defining the real interest rate as:

\[ i(t) = \frac{1+R(t)}{1+\pi(t)} - 1, \]

equations (13) and (14) yield a real interest rate of \(n\), which is available to both rich and poor when the segmented-society solution holds.

2. Bonds (saving by the rich)

Consider the bond market in a simple sense as a pool of loanable funds. Young people who purchase bonds \(D_b\) add loanable funds to the pool. Old people who receive interest and principal payments \(S_{rb}\) take funds out of the pool. This is illustrated in figure 3.

Total demand for bonds at time \(t\) is:

\[ D_b(t) = N_r(t) s^r(t) . \]

Total payoffs on existing bonds at time \(t\) are:

\[ S_{rb}(t) = [1+R(t-1)]D_b(t) = N_r(t-1) s^r(t-1)[1+R(t-1)]. \]

Simple equilibrium requires that inflows to the bond market equal outflows, or:

\[ D_b(t) = S_{rb}(t). \]

This can be solved using equations (1), (8), (17), (18) to get:

\[ (1+n) \frac{p(t)}{p(t-1)} = 1+R(t-1). \]

Note that the segmented-society solution holds here.
Simple equilibrium considers market clearing to occur when the inflow and outflow of funds are equal. In the bond market, funds flow in when bonds are purchased and flow out when the interest and principal on bonds is paid out. In the money market, old people supply money and young people demand money.

Figure 3
The Simple Equilibrium Concept of Bonds and Money
3. Money (saving by the poor)

Consider the money market as a pool of funds, as shown in figure.

3. Total demand for money by the young poor at time \( t \) is:

\[
D_m(t) = N_p(t) s^P(t).
\]

Total supply of money by the old poor at time \( t \) is:

\[
S_m(t) = N_p(t-1) s^P(t-1).
\]

Simple equilibrium requires that:

\[
D_m(t) = S_m(t).
\]

This can be solved using equations (1), (5), (21), (22) to get:

\[
(1+n) \frac{p(t)}{p(t-1)} = 1.
\]

Equation (24) is equivalent to equation (13) of the segmented-society solution.

Note that Walras's Law holds, that is:

\[
[D_c(t) - S_c(t)]p(t) + [D_b(t) - S_{rb}(t)] + [D_m(t) - S_m(t)] = 0.
\]

Consequently, market clearing in any two of the three markets gives us a solution, for when two markets clear, the third must clear as well.

Using the bond-market equilibrium condition, equation (20), and the money-market equilibrium condition, equation (24), it is clear that the only solution is the segmented-society solution, where \( \frac{p(t)}{p(t-1)} = \frac{1}{1+n} \) and \( R(t) = 0 \) for all \( t \).

This result should not be surprising. Poor people only trade goods for money while young, and money for goods when old. Rich people only desire to trade goods for bonds while young, and bonds for goods when old. Consequently, no trading occurs across economic classes.
C. The Intergenerational Free Lunch

In the previous section, we considered only simple equilibria in the bond market and the money market. The difficulty in attaining such equilibria, as was first pointed out by Samuelson (1958), is that private citizens alone are unable to make contracts with each other, due to the overlapping-generations structure.

A young rich person wishes to purchase a bond which yields income in old age. But the only people who are available to sell a bond to a young rich person are old rich people. Old rich people will not be alive in the next period, so they are unable to repay the interest and principal on such a bond.

A similar situation holds with money. A young poor person wants to give up goods and get money from an old poor person. But if money is a liability of an old poor person, such a trade would be unacceptable, as the old poor person would not be alive in the following period and could not repay the liability.

These problems suggest that the conditions for reaching full equilibrium are more stringent than for reaching simple equilibrium. Simple equilibrium is defined as an equilibrium in which inflows to markets equal outflows, as shown in figure 3. Full equilibrium actually looks at the contracts between people. For every purchaser of a bond, there must be a seller. Figure 4 illustrates the full equilibrium concept. Simple equilibrium must hold when full equilibrium holds, but the converse is not true. We assume that government can and does take actions which allow the achievement of full equilibrium. Otherwise, the
Explanation: Full equilibrium divides the simple markets into two parts. In the bond market, a person who wishes to buy a bond must find someone who wants to sell a bond in order for trade to occur. In order for a person to obtain money, money must be supplied by some outside agent, such as the government.

**Figure 4**
The Full Equilibrium Concept of Bonds and Money
absence of people with whom to trade would force people to use storage instead of bonds and money.

In the case of bonds, full equilibrium requires that the demand for bonds equal the supply of bonds and that interest and principal payments on bonds equal interest and principal receipts by bondholders. We write this as:

\[(26) \quad D_b(t) = S_b(t) \quad \text{[demand and supply of bonds]},\]

\[(27) \quad D_{rb}(t) = S_{rb}(t) \quad \text{[demand and supply of funds used to make interest and principal payments on bonds]},\]

where

\[D_b(t) = D_b^p(t) + D_b^g(t) \quad S_b(t) = S_b^p(t) + S_b^g(t)\]

\[D_{rb}(t) = D_{rb}^p(t) + D_{rb}^g(t) \quad S_{rb}(t) = S_{rb}^p(t) + S_{rb}^g(t).\]

The superscripts \(p\) and \(g\) denote the private and government sectors, respectively. From equations (8) and (17), \(D_b^p(t) = N_r(t) \frac{\delta}{1+\delta} \beta p(t)\).

From equations (8) and (18), \(S_{rb}^p(t) = N_r(t-1) [1+R(t-1)] \frac{\delta}{1+\delta} \beta p(t-1)\).

The government bond variables are written as flow variables here because private citizens only want to hold one-period bonds. Thus the entire stock of government debt (or lending) turns over every period, so the stock and flow are equivalent.

Government can ensure that full equilibrium is satisfied by setting:

\[(28) \quad S_b^g(t) = N_r(t) \frac{\delta}{1+\delta} \beta p(t).\]

Then \(D_b(t) = S_b(t)\) since \(D_b^g(t) = 0 = S_b^p(t)\), so the bond market clears. Lagging this once to get \(S_b(t-1) = N_r(t-1) \frac{\delta}{1+\delta} \beta p(t-1)\), the repayment obligation at time \(t\) becomes:

\[(29) \quad D_{rb}^g(t) = (1+R(t-1)) S_b^g(t-1) = N_r(t-1) [1+R(t-1)] \frac{\delta}{1+\delta} \beta p(t-1).\]
Notice that this satisfies equation (27), $D_{rb}(t) = S_{rb}(t)$ where $D_{rb}(t) = 0 = S_{rb}(t)$, so the bond repayment flow condition is satisfied.

A government following this policy has a deficit, as conventionally defined, of $N_r(t-1) \frac{\delta}{1+\delta} \beta p(t-1) R(t-1)$, at time $t$. The government has a debt at time $t$ of $N_r(t) \frac{\delta}{1+\delta} \beta p(t)$. Even though government debt exists, it is refinanced each period by new borrowing, so the government never has an unbalanced cash flow. This is a true Ponzi scheme, as the debt always exists but need never be repaid, since time never ends.

In the money market, full equilibrium can be reached by having the government issue (any amount of) money, which is a general obligation paying no interest. If we denote government money outstanding at time $t$ by $M(t)$, then full equilibrium requires that:

$$
(30) \quad D_m(t) = M(t) \quad S_m(t) = M(t-1),
$$

where $D_m(t)$ and $S_m(t)$ are defined in equations (21) and (22), respectively.

These equations say simply that money demand equals the money stock at time $t$, for all $t$. It is clear that $D_m(t) = S_m(t+1)$, because people who demand money at time $t$ supply it at time $t+1$. Note that if $M(t) = M(t-1)$, then simple equilibrium holds (equation (23)). Since we've assumed so far that government's actions are designed solely for the purpose of achieving full equilibrium while not interfering with simple equilibrium, we have $M(t) = M(t-1)$ and both simple and full equilibria hold.

When the government follows the debt and monetary policies suggested above, both simple and full equilibria hold in all markets. The nominal interest rate is zero and deflation occurs so that.
\[ \frac{p(t)}{p(t-1)} = \frac{1}{1+n} \]. Examination of equations (4) and (7) shows that second-period consumption is \( c_o^p(t+1) = \frac{\delta}{1+\delta} \alpha(1+n) \), \( c_o^r(t+1) = \frac{\delta}{1+\delta} \beta(1+n) \), for all \( t \). If government fails to provide money and bonds as suggested above, then people must rely on storage, which yields them second-period consumption of \( c_o^p(t+1) = \frac{\delta}{1+\delta} \alpha, c_o^r(t+1) = \frac{\delta}{1+\delta} \beta \), for all \( t \).

Everyone in all generations is clearly made better off when government issues debt and money, so we call this situation "an intergenerational free lunch." The same situation is found in many overlapping-generations models. One line of research shows that government debt is beneficial to society (see Tobin (1952), Feldstein (1976), McCallum (1982)) because interest payments can be financed by additional debt and not taxes. This illustrates the idea of the intergenerational free lunch.

We next investigate what happens when debt and money are changed from the levels which supply the intergenerational free lunch.
D. Full Government Choice: Taxes, Bonds, Money

Now we analyze the equilibrium which obtains when government revenue can be raised by any of three sources—taxation, borrowing, and money creation.

The government collects lump-sum per-capita taxes on the young and old, poor and rich, at time t. Total tax collections are equal to:

\[ X(t) = N_p(t)\tau_y^p(t) + N_p(t-1)\tau_o^p(t) + N_r(t)\tau_y^r(t) + N_r(t-1)\tau_o^r(t), \]

where \( \tau_i^h(t) \) represents the lump-sum tax levied on a person in the \( i^{th} \) year of life (\( i=y \) young, \( i=o \) old) of economic class \( h \) (\( h=p \) poor, \( h=r \) rich). We assume that people can not escape the taxes and that the tax system is costless to administrate. At time t, the government sells bonds in the amount \( S_b^g(t) \) and pays interest and principal on bonds issued the previous period in the amount:

\[ [1+R(t-1)]S_b^g(t-1) = D_{rb}^g(t). \]

The government issues new money (or retires old money, if negative) in the amount \( m(t) = M(t) - M(t-1) \), where \( M(t) \) is the total amount of government-issued money outstanding at time t.

The budget constraint faced by the government is:

\[ [1+R(t-1)]S_b^g(t-1) = S_b^g(t) + X(t) + M(t) - M(t-1). \]

Expenditures on bonds (interest and principal payments) are financed by issuing new bonds, collecting taxes, and issuing money. Note that total taxes, \( X(t) \), may be negative in the case of transfers. Government lending (investment) is permitted, but can be shown to be nonoptimal.
Taxes are given in nominal terms. The budget constraint faced at
time $t$ by a young poor person can be written in real present-value terms
as:

\[(33) \quad \alpha - \text{PV}_T(t) = c^p_y(t) + c^p_o(t+1) \frac{d(t+1)}{p(t)},\]

where

\[(34) \quad \text{PV}_T(t) = \frac{\tau^p_y(t)}{p(t)} + \frac{\tau^p_o(t+1)}{p(t)}.\]

$\text{PV}_T$ denotes the present value of taxes.

The budget constraint faced at time $t$ by a young rich person is:

\[(35) \quad \beta - \text{PV}_R(t) = c^r_y(t) + c^r_o(t+1) \frac{1}{1+R(t)} \frac{d(t+1)}{p(t)} .\]

\[(36) \quad \text{PV}_R(t) = \frac{\tau^r_y(t)}{p(t)} + \frac{\tau^r_o(t+1)}{[1+R(t)]p(t)}.\]

The maximization problems faced by individuals change because of
the introduction of taxes. Solving the maximization problems, we find
that the new choices of the rich and poor are given by:

\[(3') \quad c^p_y(t) = \frac{1}{1+\delta} [\alpha - \text{PV}_T(t)].\]

\[(4') \quad c^p_o(t+1) = \frac{\delta}{1+\delta} \frac{p(t)}{p(t+1)} [\alpha - \text{PV}_T(t)].\]

\[(5') \quad \frac{s^p(t)}{p(t)} = \frac{\delta}{1+\delta} [\alpha - \text{PV}_T(t)] - \frac{\tau^p_o(t+1)}{p(t)}.\]

\[(6') \quad c^r_y(t) = \frac{1}{1+\delta} [\beta - \text{PV}_R(t)].\]

\[(7') \quad c^r_o(t+1) = \frac{\delta}{1+\delta} [1+R(t)] \frac{p(t)}{p(t+1)} [\beta - \text{PV}_R(t)].\]

\[(8') \quad \frac{s^r(t)}{p(t)} = \frac{\delta}{1+\delta} [\beta - \text{PV}_R(t)] - \frac{\tau^r_o(t+1)}{[1+R(t)]p(t)}.\]

Notice that an increase in taxes on either rich or poor in either
period causes consumption spending in both periods to decrease. An
increase in taxes on the young decreases saving. An increase in taxes
on the old causes savings to increase, so that additional funds are
accumulated to pay the additional tax.
1. The Goods Market

The government is assumed for now to acquire no goods, so only private citizens demand and supply goods. Consequently the goods-market clearing condition is the same as it was previously, \( D_c(t) = S_c(t) \), where \( D_c(t) \) is given by (9) and \( S_c(t) \) is given by (10).

2. The Bond Market

Simple equilibrium in the bond market requires that:
\[
\text{(37)} \quad D_b^P(t) + D_{rb}^G(t) = S_{rb}^P(t) + S_b^G(t),
\]
where \( D_b^P(t) \) is given by (17), \( S_{rb}^P(t) \) by (18), \( S_b^G(t) \) is a choice variable of the government, and \( D_{rb}^G(t) \) is given by (29).

Full equilibrium in the bond market requires that:
\[
\text{(38)} \quad D_b^P(t) = S_b^G(t).
\]
\[
\text{(39)} \quad D_{rb}^G(t) = S_{rb}^P(t).
\]
Since \( D_{rb}^G(t) = [1+R(t-1)]S_b^G(t) \) and \( S_{rb}^P(t) = [1+R(t-1)]D_b^P(t) \), equation (39) holds for all \( t \) if equation (38) holds for all \( t \).

3. The Money Market

Simple equilibrium requires that:
\[
\text{(40)} \quad D_m(t) = S_m(t) + m(t) = S_m(t) + M(t) - M(t-1),
\]
where \( D_m(t) \) is given by (21), \( S_m(t) \) by (22), and \( M(t) \) is a choice variable of the government.

Full equilibrium requires that:
\[
\text{(41)} \quad D_m(t) = M(t).
\]
\[
\text{(42)} \quad S_m(t) = M(t-1).
\]
Since $S_m(t) = D_m(t-1)$, equation (42) holds for all $t$ if equation (41) holds for all $t$.

Again we confirm that Walras's Law holds, that is:

\[ (43) \quad [D_C(t) - S_C(t)]p(t) + [(D_P^b(t) + D^g_{rb}(t)) - (S^p_{rb}(t) + S^g_{rb}(t))] 
+ [D_m(t) - S_m(t) - M(t) + M(t-1)] = 0. \]

Consequently market-clearing in two markets assures (simple) market-clearing in the third market.

We now consider the solution to the model at full equilibrium, using money-market clearing and bond-market clearing. To get full equilibrium in the bond market, we must have $S^g_{rb}(t) = D^P_{rb}(t)$. Using substitutions from equations (38), (1), (17), (8'), (36), (39), (29), (18), (37), the bond-market clearing condition becomes:

\[ (44) \quad N_r(t-1) \frac{\delta}{1+\delta} p(t-1) [[1+R(t-1)](1+n)\frac{p(t)}{p(t-1)} - 1] - Z_b(t) = 0, \]

where

\[ (45) \quad Z_b(t) = S^g_{rb}(t) - [1+R(t-1)] S^g_{rb}(t-1) 
+ N_r(t-1) \frac{1}{1+\delta} [(1+n)\delta \tau_y(t) - \delta [1+R(t-1)] \tau_y(t-1) 
- (1+n) \frac{1}{1+R(t)} \tau_y(t+1) + \tau_y(t)]. \]

Equation (44) simply says that excess demand for bonds is zero at equilibrium. These equations split the excess demand for bonds into two parts, where the second part, $Z_b(t)$, can be affected by government's choices.

Full equilibrium in the money market requires that $M(t) = D_m(t)$.

Using substitutions from equations (21), (22), (5'), (1), (30), (34), (40), (41), (42), the money-market clearing condition becomes:

\[ (46) \quad N_p(t-1) \frac{\delta}{1+\delta} \alpha p(t-1) [(1+n)\frac{p(t)}{p(t-1)} - 1] - Z_m(t) = 0, \]

where
\( Z_m(t) = M(t) - M(t-1) \)

\[ + N^p(t-1) \frac{1}{1+\delta} \left[ (1+n)\delta\tau^p_y(t) - \delta\tau^p_y(t-1) \right] \]

\[-(1+n)\tau^p_o(t+1) + \tau^p_o(t) \].

Equations (46) and (47) show the excess demand for money (which is zero at equilibrium). Government's choices of financial and fiscal variables affect \( Z_m(t) \).
E. The Maximization Problem of the Government

We assume that the government pursues purposeful, rather than random, policies. The government has the variables $S_g(t), M(t), \tau_p^o(t), \tau_y^p(t), \tau_y^r(t), \tau^r_o(t)$ under its control at time $t$, subject to the budget constraint (32). We assume that government policy is designed to maximize the utility of average poor and rich people, subject to the availability of resources. Since technological growth is ruled out, government seeks a stationary solution in which every person in every generation has the same real consumption stream \{c^h_y(t), c^h_o(t+1)\} as everyone else in the same economic class. The government is assumed not to redistribute income from rich to poor (or vice-versa), but rather to maintain the segmented society.

Total real output at time $t$ by each class is $N_p(t)\alpha$ and $N_r(t)\beta$. Total real consumption is $N_h(t)c^h_y(t) + N_h(t-1)c^h_o(t)$, for $h = p, r$. This can be written $N_h(t)[c^h_y(t) + \frac{1}{1+n} c^h_o(t)]$. We wish to maximize $\ln c^h_y + \delta \ln c^h_o$, $h = p, r$, subject to $\alpha = c^p_y + \frac{1}{1+n} c^p_o$, $\beta = c^r_y + \frac{1}{1+n} c^r_o$. The solutions to these maximization problems suggest that the government should pursue policies which set:

- $c^p_y = \frac{1}{1+\delta} \alpha$,
- $c^r_y = \frac{1}{1+\delta} \beta$,
- $c^p_o = \frac{\delta}{1+\delta} (1+n) \alpha$,
- $c^r_o = \frac{\delta}{1+\delta} (1+n) \beta$.

These consumption paths are exactly the same as those for the segmented-society solution described above. Consequently, the government seeks to set $\frac{p(t)}{p(t-1)} = \frac{1}{1+n}$ and $R(t) = 0$ for all $t$. 
The government can achieve its goals by the appropriate use of its instruments $S^g_b(t)$, $M(t)$, $\tau^p_y(t)$, $\tau^r_o(t)$, $\tau^r_y(t)$. Equations (44) and (46) show that market-clearing conditions require $Z_b(t) = 0$ and $Z_m(t) = 0$ in order to get $\frac{p(t)}{p(t-1)} = \frac{1}{1+n}$ and $R(t) = 0$. Using (45) and (47), government must use its instruments such that:

\begin{align*}
(48) \quad S^g_b(t) - S^g_b(t-1) + N_r(t-1) \frac{1}{1+\delta} [((1+n) \delta \tau^r_y(t) - \delta \tau^r_y(t-1) - (1+n) \tau^r_o(t+1) + \tau^r_o(t) = 0.
\end{align*}

\begin{align*}
(49) \quad M(t) - M(t-1) + N_p(t-1) \frac{1}{1+\delta} [((1+n) \delta \tau^p_y(t) - \delta \tau^p_y(t-1) - (1+n) \tau^p_o(t+1) + \tau^p_o(t) = 0.
\end{align*}

The government is constrained by its budget, given by equation (32). Using the definition of total tax collections $X(t)$, the budget constraint requires that the instruments in equations (48) and (49) satisfy:

\begin{align*}
(50) \quad N_p(t-1) \frac{1}{1+\delta} [(1+n) \tau^p_y(t) + \delta \tau^p_y(t-1) + (1+n) \tau^p_o(t+1) + \delta \tau^p_o(t)] + N_r(t-1) \frac{1}{1+\delta} [(1+n) \tau^r_y(t) + \delta \tau^r_y(t-1) + (1+n) \tau^r_o(t+1) + \delta \tau^r_o(t)] = 0.
\end{align*}

Equations (48), (49), and (50) represent the conditions under which the bond market clears (48), the money-market clears (49), the government budget constraint is satisfied (50), and government's policy targets are achieved. These conditions are necessary for government to reach a first-best optimum equilibrium. Tradeoffs between taxes, bonds, and money can be made and are neutral as long as these three equations are satisfied. These tradeoffs are examined dynamically in the next section.
F. The Neutrality Proposition

We now prove the proposition that the government can choose any time path of bond sales and money issuance desired, provided that it adjusts taxes or transfers properly, and that neither inflation nor interest rates are affected.

This proposition seems intuitively obvious, given equations (48), (49), and (50). According to equation (48), any change in government debt issuance at time $t$ can be offset by changes in taxes on the rich at time $t$, $\tau^r_y(t)$ and $\tau^o_r(t)$. By equation (49), a change in government monetization can be offset by changes in taxes on the poor at time $t$, $\tau^p_y(t)$ and $\tau^o_p(t)$. By equation (50), these tax changes are restricted so that the government budget constraint remains satisfied.

Consider two time paths of government policy variables, the '-' path, given by $\{S_b^g(t), M(t), \tau^p_y(t), \tau^o_y(t), \tau^p_o(t), \tau^o_o(t)\}$, and the '-' path, given by $\{S_b^g(t), M(t), \tau^p_y(t), \tau^o_y(t), \tau^p_o(t), \tau^o_o(t)\}$. Suppose that $S_b^g(t) = S_b^g(t)$ and $M(t) = M(t)$ for $t < \hat{t}$. At time $\hat{t}$, $S_b^g(\hat{t}) - S_b^g(\hat{t}-1) = k_b + (\Delta S_b^g(\hat{t}) - \Delta S_b^g(\hat{t}-1))$, where $k_b$ may be positive or negative. That is, the '-' path differs from the '-' path by changing the amount of bond financing at time $t$. For all $t \geq \hat{t}+1$, $S_b^g(t) - S_b^g(t-1) = S_b^g(t) - S_b^g(t-1)$. Also, at time $\hat{t}$, $M(\hat{t}) - M(\hat{t}-1) = k_m + (M(\hat{t}) - M(\hat{t}-1))$, where $k_m$ may be positive or negative. For all $t \geq \hat{t}+1$, $M(t) - M(t-1) = M(t) - M(t-1)$. So the two paths differ only at $\hat{t}$ in the amount of monetization which occurs.

Assuming that the '-' path satisfies equations (48), (49), and (50), we show that the '-' path also satisfies these equations, when the
appropriate tax choices are made. This is shown by finding the necessary choices of taxes to satisfy equations (48), (49), and (50) for all \( t < \hat{t}-1 \), for \( \hat{t}-1 \), for \( \hat{t} \), and for all \( t > \hat{t} \).

With \( \hat{S}_D(t) = \hat{S}_D(t) \) and \( \hat{M}(t) = \hat{M}(t) \) for \( t < \hat{t} \), we choose \( \hat{\tau}_i(t) = \hat{\tau}_i(t) \) for \( t < \hat{t} \), \( i = y, o, h = p, r \). Consequently equations (48), (49) and (50) are satisfied for \( t < \hat{t}-1 \), as the '-' equations are identical to the '+' equations.

At time \( \hat{t}-1 \), equation (48) is satisfied only if \( \tau_o(t) = \tau_o(t) \), since all other terms are the same in both paths. Similarly, equation (49) is satisfied at \( \hat{t}-1 \) only for \( \tau^o(t) = \tau^o(t) \). Under these conditions, equation (50) is also satisfied at time \( \hat{t}-1 \).

At time \( \hat{t} \), equation (48) can be shown to hold only if:

\[
(51) \quad k_o = N_o(\hat{t}) \frac{1}{1+\delta} \{ [\tau^o(t+1) - \tau^o(t+1)] - \delta [\tau^o(t) - \tau^o(t)] \}.
\]

This suggests that an increase in government borrowing can be offset by a decrease in taxes now on the young rich and/or an increase in taxes on them in the following period, in order to be neutral.

At time \( \hat{t} \), equation (49) can be shown to hold only if:

\[
(52) \quad k_m = N_p(\hat{t}) \frac{1}{1+\delta} \{ [\tau^p(t+1) - \tau^p(t+1)] - \delta [\tau^p(t) - \tau^p(t)] \}.
\]

A neutral increase in the money supply requires decreased taxes on the young poor now, or increased taxes on them next period.

At any time \( t > \hat{t} \), equation (48) can be shown to hold only if:

\[
(53) \quad (1+n)\delta[\tau^p(t) - \tau^p(t)] + [\tau^o(t) - \tau^o(t)]
= \delta[\tau^r(t-1) - \tau^r(t-1)] + (1+n)[\tau^o(t+1) - \tau^o(t+1)].
\]

Similarly, at any time \( t > \hat{t} \), equation (49) can be shown to hold only if:

\[
(54) \quad (1+n)\delta[\tau^p(t) - \tau^p(t)] + [\tau^o(t) - \tau^o(t)]
\]
We now consider the market-clearing equations (53) and (54) at time \( t+1 \). Using these equations plus the market-clearing equations (51) and (52) at time \( t \), we get:

\[
\begin{align*}
(55) & \quad k_b = N_p(t+1) \frac{1}{1+\delta} \left[ \hat{\tau}_y(t+2) - \hat{\tau}_o(t+2) \right] - \delta \left[ \hat{\tau}_y(t+1) - \hat{\tau}_y(t+1) \right], \\
(56) & \quad k_m = N_p(t+1) \frac{1}{1+\delta} \left[ \hat{\tau}_o(t+2) - \hat{\tau}_o(t+2) \right] - \delta \left[ \hat{\tau}_y(t+1) - \hat{\tau}_y(t+1) \right].
\end{align*}
\]

Equations (55) and (56) imply that a change in government borrowing or monetization at time \( t \) will require changes in the tax streams of those born at time \( t+1 \). A comparison of equations (54) and (56) suggests that an increase in monetization at time \( t \) requires a change in the taxation stream of every generation born at or after time \( t \). Similarly a comparison of equations (53) and (55) suggests the same when government borrowing is increased. We can write this as:

\[
\begin{align*}
(57) & \quad k_b = N_p(t) \frac{1}{1+\delta} \left[ \hat{\tau}_o(t+1) - \hat{\tau}_o(t+1) \right] - \delta \left[ \hat{\tau}_y(t) - \hat{\tau}_y(t) \right], \\
(58) & \quad k_m = N_p(t) \frac{1}{1+\delta} \left[ \hat{\tau}_o(t+1) - \hat{\tau}_o(t+1) \right] - \delta \left[ \hat{\tau}_y(t) - \hat{\tau}_y(t) \right],
\end{align*}
\]

for all \( t \geq t \).

We have shown that equations (48) and (49) hold for all \( t \) when equations (57) and (58) hold for \( t \geq \hat{t} \), when \( \hat{\tau}_1^h(t) = \hat{\tau}_1^h(t) \) for \( t < \hat{t} \), and when \( \hat{\tau}_o^o(t) = \hat{\tau}_o^o(t) \) and \( \hat{\tau}_o^o(t) = \hat{\tau}_o^o(t) \). Now we must verify that taxes can be chosen to satisfy the government budget constraint, equation (50).

We have already established that equation (50) holds for \( t < \hat{t} \). At time \( \hat{t} \), equation (50) can be combined with (57) and (58) to get:

\[
(59) \quad k_b + k_m + N_p(t) \hat{\tau}_y(t) - \hat{\tau}_y(t) + N_p(t) \hat{\tau}_y(t) - \hat{\tau}_y(t) = 0.
\]

This equation requires that increases in non-tax government finance (i.e., bond issuance and monetization) must be accompanied by reductions
of current taxation on the young, with no change in the tax liability of the old generation, in order for the government budget constraint to be satisfied at time $t$. Following the same procedure as at time $t$, we find that for all $t \geq \hat{t}$:

$$k_b + k_m + N_r(t)[\tau^r_y(t) - \tau^p_y(t)] + N_p(t)[\tau^p_y(t) - \tau^p_y(t)] = 0.$$  

This analysis suggests that a neutral increase in bond issuance or monetization leads to a permanent reduction in taxes on people while young. When equation (60) is followed, the government budget constraint is satisfied at all times.

We have shown that the bond-market clearing equation (48), the money-market clearing equation (49), and the government budget constraint (50) hold for all time, when taxes follow the paths outlined above. Following these requirements is enough to ensure macro-neutrality. This means that any $k_b$ and any $k_m$ can be chosen without changing the inflation rate or the interest rate, which are the macroeconomic variables of this model. However, this is not enough to ensure micro-neutrality. Micro-neutrality holds if, in addition to no changes in macroeconomic variables, no redistribution occurs. This requires that no individual is affected in any way.

We must consider what happens to the real consumption streams of the rich and poor. By equations (3'), (4'), and (34), we see that the consumption stream of a poor person born at time $t$ is unaffected only if:

$$[\tau^p_y(t) - \tau^p_y(t)] + [\tau^p(t+1) - \tau^p(t+1)] = 0.$$  

Similarly, equations (6'), (7'), and (36) require that:

$$[\tau^p_y(t) - \tau^p_y(t)] + [\tau^p(t+1) - \tau^p(t+1)] = 0.$$
Using equations (57) and (58) with (61) and (62), it is clear that micro-neutrality requires:

\[ \begin{align*}
    (63) & \quad k_m = -N_p(t) \left[ \tilde{\tau}_y(t) - \tilde{\tau}_o(t) \right] = N_p(t) \left[ \tilde{\tau}_o(t+1) - \tilde{\tau}_o(t+1) \right], \\
    (64) & \quad k_b = -N_r(t) \left[ \tilde{\tau}_y(t) - \tilde{\tau}_o(t) \right] = N_r(t) \left[ \tilde{\tau}_o(t+1) - \tilde{\tau}_o(t+1) \right],
\end{align*} \]

for all \( t \geq t \).

By choosing the tax streams to follow (63) and (64), no individual's real consumption is affected, all markets clear, and the government budget constraint is satisfied. Table 1 summarizes the results.

We have shown that given a '−' path which clears markets at \( R(t) = 0 \) and \( \frac{p(t)}{p(t-1)} = \frac{1}{1+n} \) and satisfies the government budget constraint, a change at time \( t \) to a '−' path, involving a one-time change in bond issuance and monetization, is neutral when taxes are adjusted appropriately.

Now consider a third '−' path, given by \{\( S^g(t) \), \( M(t) \), \( \tilde{\tau}_y(t) \), \( \tilde{\tau}_o(t) \), \( \tilde{\tau}_p(t) \), \( \tilde{\tau}_r(t) \)\} where \( [S^g_b(t) - S^g_b(t-1)] = K_b(t) + [S^g_b(t) - S^g_b(t-1)] \) and \( [M(t) - M(t-1)] = K_m(t) + [M(t) - M(t-1)] \) for all \( t \geq t \), where \( K_m(t) \) and \( K_b(t) \) may be positive, negative or zero at any time \( t \). The '−' path differs from the '−' path at many points in terms of government bond issuance and monetization. By looking at each period separately, beginning with \( \tilde{t} \), the changes in \( \tilde{\tau}_i(t+s) \), \( i = y, o; h = p, r; s = 0, 1, 2, \ldots \), can be found which ensure neutrality. Using table 1, we infer that:

\[ \begin{align*}
    (65) & \quad \tilde{\tau}_y(t+s) = \tilde{\tau}_y(t+s) - \frac{1}{N_h(t+s)} \sum_{i=0}^{s} K_j(t+i) \\
    (66) & \quad \tilde{\tau}_o(t+1+s) = \tilde{\tau}_o(t+1+s) + \frac{1}{N_h(t+s)} \sum_{i=0}^{s} K_j(t+i),
\end{align*} \]
Table 1
Neutral Government Financial Policies
for a One-Time Change in Bond Issuance and Monetization

<table>
<thead>
<tr>
<th>Time(t)</th>
<th>t &lt; \hat{t}</th>
<th>t ≥ \hat{t}</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_b^g(t)</td>
<td>\hat{S}_b^g(t) = \tilde{S}_b^g(t)</td>
<td>\hat{S}_b^g(t) = k_b + \tilde{S}_b^g(t)</td>
</tr>
<tr>
<td>M(t)</td>
<td>\hat{M}(t) = \tilde{M}(t)</td>
<td>\hat{M}(t) = k_m + \tilde{M}(t)</td>
</tr>
<tr>
<td>\tau_y^p(t)</td>
<td>\hat{\tau}_y^p(t) = \tilde{\tau}_y^p(t)</td>
<td>\hat{\tau}_y^p(t) = \tilde{\tau}_y^p(t) - \frac{k_m}{N_p(t)}</td>
</tr>
<tr>
<td>\tau_o^p(t+1)</td>
<td>\hat{\tau}_o^p(t+1) = \tilde{\tau}_o^p(t+1)</td>
<td>\hat{\tau}_o^p(t+1) = \tilde{\tau}_o^p(t+1) + \frac{k_m}{N_p(t)}</td>
</tr>
<tr>
<td>\tau_y^r(t)</td>
<td>\hat{\tau}_y^r(t) = \tilde{\tau}_y^r(t)</td>
<td>\hat{\tau}_y^r(t) = \tilde{\tau}_y^r(t) - \frac{k_b}{N_r(t)}</td>
</tr>
<tr>
<td>\tau_o^r(t+1)</td>
<td>\hat{\tau}_o^r(t+1) = \tilde{\tau}_o^r(t+1)</td>
<td>\hat{\tau}_o^r(t+1) = \tilde{\tau}_o^r(t+1) + \frac{k_b}{N_r(t)}</td>
</tr>
</tbody>
</table>
for \( h = p \) and \( j = m \), and \( h = r \) and \( j = b \).

All that remains to be shown is that the proposition of neutrality is nonvacuous. That is, is there a '-' path which satisfies market-clearing and the government budget constraint? One such path is the intergenerational free-lunch path of the preceding section, where taxes are zero, some (any) quantity of money has been issued, and bonds are issued to achieve full equilibrium.

Numerical Illustration of Neutrality

The following numerical example is intended to help in understanding the neutrality proposition and its implications.

Let the parameters take the following values:

\[
\begin{align*}
\alpha &= 6 \quad \text{Output of the poor} \\
\beta &= 20 \quad \text{Output of the rich} \\
\delta &= .9 \quad \text{Time discount rate} \\
n &= 1 \quad \text{Population growth rate} \\
&\quad \text{(population doubles each period).}
\end{align*}
\]

Given these parameters, in the optimal simple equilibrium, the individual choice variables are stationary and are given by:

\[
\begin{align*}
\frac{c_{y}^{r}}{y} &= 10.53 \quad \frac{c_{o}^{r}}{p} = 18.95 \quad \frac{s_{p}^{r}}{p} = 9.47 \\
\frac{c_{y}^{p}}{y} &= 3.16 \quad \frac{c_{o}^{p}}{p} = 5.68 \quad \frac{s_{p}^{p}}{p} = 2.84.
\end{align*}
\]

Suppose that the following initial conditions hold at time \( t = 1 \):

\[
\begin{align*}
N_{p}(1) &= 16 \quad \text{Population of the poor} \\
N_{r}(1) &= 32 \quad \text{Population of the rich} \\
M(1) &= 45.47 \quad \text{Money supply}
\end{align*}
\]
\[ S^g_b(1) = 303.16 \quad \text{Government debt} \]
\[ p(1) = 1.0 \quad \text{Price level}. \]

We now consider the following situation. For \( t < 4 \), we let \( M(t) = 45.47 \) and \( S^g_b(t) = 303.16 \). At \( t = 4 \) there is a one-time increase in the money supply and in the debt, so that for \( t \geq 4 \), \( M(t) = 55.47 \) and \( S^g_b(t) = 403.16 \). The money supply is raised permanently by 10 and the debt is raised permanently by 100.

The lump-sum taxes and transfers which are needed to ensure neutrality are:

For \( t < 4 \)
\[ \tau^p_y(t) = \tau^r_y(t) = \tau^r_o(t) = 0. \]

For \( t = 4 \)
\[ \tau^p_y(4) = \tau^r_o(4) = 0 \]
\[ \frac{\tau^p_y(t)}{p(t)} = -0.625 \quad \frac{\tau^r_y(t)}{p(t)} = -3.125 \]
\[ X(t) = -110.00 \quad \text{(net tax income of the government)}. \]

For \( t \geq 4 \)
\[ \frac{\tau^p_y(t)}{p(t)} = -0.625 \quad \frac{\tau^r_y(t)}{p(t)} = -3.125 \]
\[ \frac{\tau^p_o(t)}{p(t)} = 1.25 \quad \frac{\tau^r_o(t)}{p(t)} = 6.25 \]
\[ X(t) = 0. \]

Neutrality occurs here because the government adjusts lump-sum taxes and transfers very precisely. The demand for bonds rises by exactly the same amount as the increase in government debt. The demand for money rises by exactly the amount of the increased money supply. Since there is no change in the present value of taxes, and no change in the interest rate or inflation rate, individual's budget constraints are unchanged. Taxes substitute perfectly for bonds and money.
Further numerical analysis shows that the simple equilibrium reached here is Walrasian stable when price adjusts according to excess demand for goods.
G. Conclusions and Extensions

A widely held notion about the effects of government financing is that monetization leads to inflation and that government borrowing leads to higher interest rates. The neutrality proposition presented in this chapter suggests that the optimal policy of the government is one which adjusts taxes on individuals so that choices about monetization and borrowing are irrelevant.

It is clear that when government follows optimal tax policies, neither fiscal policy nor monetary policy have any effect on inflation or interest rates. This suggests that Barro's (1979) attempt to examine the second-order effects of government taxation (when neutrality of debt is assumed) is likely to be fruitful for determining optimal approaches to government financing. These second-order effects are things such as the administrative costs of collecting taxes, which are fairly small but positive.

Any non-neutrality of government finance must be due to:

(1) frictions in the economy such as transactions costs;
(2) "fiscal illusion", in which people are fooled because they fail to perceive the changes in taxation which must occur when the government changes monetization or borrowing; or
(3) because the government does not maximize the welfare of average individuals but pursues some other goals.

A model with transactions costs is examined in chapter 5. Fiscal illusion is modeled in chapter 4. Alternative governmental objective functions are considered in chapter 7.
The model of this chapter includes money as a store of value. Money also solves what has been called the "overlapping-generations friction," which in this paper becomes an intergenerational free lunch. Some have argued that this friction leads to quite different results than those which occur when money is used as a medium of exchange. For that reason, a Clower-constraint model in which money explicitly serves as both a medium of exchange and a short-term store of value is presented in chapter 4.

The optimal nominal interest rate in this chapter is found to be zero, with a real interest rate of $n$, the population growth rate. This result is obtained because neither money nor bonds are productive beyond providing the "intergenerational free lunch", which takes advantage of the fact that population is growing, to provide additional consumption to everyone. If we consider a model in which capital exists and is used in production, and in which productivity growth occurs, then the real returns to money and bonds may differ. This is demonstrated in chapter 6.

A key assumption in the model is that the financial demands of the poor and rich are segmented—the poor only use money, and the rich only use bonds. We assume that government does not undertake policies in such a way as to allow the poor to use bonds. Suppose, however, that instead of having two classes of people, there is a range of real incomes. For example, let $a$ be the smallest real income received by anyone, and $\beta$ the largest, let population be $N(t)$, and let $w^h(t)$, $h=1,2,\ldots,N(t)$ be given by $w^h(t) = a + \frac{\beta - a}{N(t)}(h-1)$. Then with the minimum bond size being $k$, where $a \leq k \leq \beta$, government policy affects
the number of people using bonds and money. Optimal government policy may then not be as easy to formulate. This is shown in chapter 4.

An additional implication of this chapter is that models of money in which money is valued in the utility function cannot achieve neutrality results. For example, in Aiyagari-Gertler (1983), the utility function of the individual is of the form $U = c_1^\alpha c_2^\beta (M/p)^\gamma$. If the nominal money supply increases, then each person must hold more nominal money. A neutrality result will lead to $c_1, c_2,$ and the time path of $p$ remaining constant when the money supply increases, but this increases utility. Consequently no neutrality theorem holds. The MUF approach must either be rejected as a model of money, or it must include more things in the utility function, such as taxes.
FOOTNOTES FOR CHAPTER 3

1. The important question of who issues bonds and money is addressed in section B.

2. Otherwise, the poor could buy bonds, e.g., by buying \( k + \alpha/2 \) of them and selling \( k \) of them.

3. This is shown for a more general CES utility function in chapter 4.

4. For now we assume that the money supply is held constant. This assumption is relaxed in section D.

5. Government expenditures include interest payments on debt. Since the government here never collects tax revenues, it runs a perpetual deficit.

   In this model, it turns out that \( R(t-1) = 0 \), so that the deficit is zero. Chapter 5 shows the case of a positive deficit.

6. As we show later, \( M(t) = M(t-1) \) is required here in order to satisfy the government budget constraint, since government takes no net cash flow from bond sales and redemptions.

7. We assume throughout that taxes are never changed so much as to cause the poor to be able to hold bonds or to cause the rich to hold money instead of bonds.

8. The implicit social-welfare weights assigned to the rich and poor here are investigated in chapter 5.

Chapter 4

Extensions of the Basic Model

In this chapter specifications alternative to the model of chapter 3 are considered. In section A, the use of a more general utility function is examined. The logarithmic utility function of chapter 3 is a special case of a CES-type utility function. It is shown that the results of chapter 3 are not dependent upon having a logarithmic utility function.

The restricted-transactions model of chapter 3 appears to model money strictly as a store of value. Theorists such as Wallace (1980) argue that the distinction between money's roles as a medium of exchange and a store of value have been overdrawn. Nonetheless, it may be interesting to examine the neutrality theorem of chapter 3 in a model in which money appears to work as a medium of exchange rather than as a store of value. This is accomplished using a Clower-constraint model in section B.

One of the somewhat unrealistic features of the model of chapter 3 is its division of people into two classes. This means that effectively there is no substitution between money and bonds, since only the rich use bonds, and only the poor use money. This defect is remedied in section C, which presents a model in which everyone's income is
different. The choice between using money and bonds is an endogenous decision for everyone.

Section D considers the possibility of fiscal illusion. The existence of fiscal illusion clearly means that government financial policy can not be neutral in its effects. The implications for optimal government policy and for empirical studies are discussed in this section.
A. An Alternative Utility Function

The results of chapter 3 might be thought to trace to the choice of a logarithmic utility function. The logarithmic form imposes substitution effects whose magnitudes are exactly equal to income effects, but opposite in sign. Changes in the interest rate and the inflation rate don't affect the consumption of people in youth, their savings, or money-holding. In this section it is shown that a generalization to a constant-elasticity-of-substitution (CES) utility function does not change the results that the optimal nominal interest rate is zero and the optimal inflation rate is \(-\frac{n}{1+n}\).

We consider the CES utility function used by Enders and Lapan (1982). Modifying their function for use in our model, the utility function is:

\[
U = \frac{1}{\rho} \left[ c_y^\rho + \delta c_o^\rho \right], \quad \text{where } 0<\rho<1.
\]

As \(\rho\) goes to zero, the utility function takes the logarithmic form of chapter 3.

Using this utility function, individuals have the following consumption and savings functions:

\[
\begin{align*}
(2) & \quad c_y^P(t) = \frac{1}{1+A(t)} \alpha. \\
(3) & \quad c_o^P(t+1) = \frac{A(t)}{1+A(t)} \frac{p(t)}{p(t+1)} \alpha. \\
(4) & \quad s^P(t) = \frac{A(t)}{1+A(t)} \alpha p(t). \\
(5) & \quad A(t) = \delta^{1/(1-\rho)} \left( \frac{p(t)}{p(t+1)} \right)^{\rho/(1-\rho)}.
\end{align*}
\]
In comparing these equations to equations (3)–(8) of chapter 3, we notice that they are equivalent if \( \rho = 0 \), when \( A(t) = B(t) = \delta \).

The segmented-society solution is found by solving the equations:

\[
\begin{align*}
(10) \quad & a N p(t) = (1+n) N p(t-1) \frac{a}{1+A(t)} + N p(t-1) \frac{a \cdot A(t-1) \cdot p(t-1)}{1+A(t-1) \cdot p(t)} . \\
(11) \quad & B p(t) = (1+n) B p(t-1) \frac{\beta}{1+B(t)} + N p(t-1) \frac{\beta \cdot B(t-1) \cdot p(t-1)}{1+B(t-1) \cdot p(t)} [1+R(t-1)]. 
\end{align*}
\]

Assuming that the government seeks a stationary solution in utility and consumption, so that \( A(t) \) and \( B(t) \) are stationary, equations (10) and (11) can be solved to get:

\[
\begin{align*}
(12) \quad & \frac{p(t)}{p(t+1)} = 1+n . \\
(13) \quad & R(t-1) = 0 .
\end{align*}
\]

The segmented-society solution is the same as in chapter 3.

The condition for simple equilibrium in the bond market is:

\[
(14) \quad (1+n) \frac{B(t)}{B(t-1)} \frac{p(t)}{p(t-1)} = \frac{1+B(t)}{1+B(t-1)} [1+R(t-1)].
\]

The condition for money-market clearing is:

\[
(15) \quad (1+n) \frac{A(t)}{A(t-1)} \frac{p(t)}{p(t-1)} = \frac{1+A(t)}{1+A(t-1)} .
\]

When stationarity is imposed, the solutions to equations (14) and (15) are precisely the segmented-society solutions, given by equations (12) and (13).
The major results of chapter 3 thus hold under this alternative specification of the utility function.
B. A Clower-Constraint Model

The following section examines a Clower-constraint model which achieves results similar to the restricted-transactions model of chapter 3. The Clower constraint (see the discussion in chapter 1, section G) simply requires that money be used in all transactions. While this might seem to be an innocuous assumption, implementation of the Clower constraint puts an important restriction on people's budget constraints.

If one examines the budget constraints in chapter 3, it is impossible to tell whether goods are paid for by money, or if goods are bartered directly. To ensure that the Clower constraint is enforced, an assumption about the holding period on money is imposed. This assumption states that to spend money on anything (goods or bonds) at time $t$, a person must obtain that money at time $t-1$. In other words, money income received at time $t-1$ can not be spent until time $t$. Income must be held as money for at least one period.

The holding-period restriction on money can be justified by appealing to transactions-cost considerations. People find that it is too costly to go to the bank every day for every transaction. Consequently they keep money available constantly. It could be argued that what is desirable is an explicit model of transactions costs. After all, the Clower constraint literally means that transactions costs are infinite in the current period and zero next period. The advantage of the Clower constraint lies in its tractability. Models with
nonconvex transactions technologies are very difficult to solve analytically.

1. The Model

We now consider the following overlapping-generations model, in which each person lives two periods, working when young and being retired when old. Because of the Clower constraint, people are assumed to be able to borrow money before birth, so that they have money to spend when young. Because of the holding-period restriction, no one would ever borrow money for just one period, assuming a positive nominal interest rate.

Each person is assumed to produce $\alpha$ units of output in youth, with wages paid in money in the amount $\alpha p$, where $p$ is the price level. A person borrows money before birth to consume goods in youth. Denote the borrowed amount as $f$, which is in nominal terms. Borrowing the amount $f$ requires a payment of $(1+R)f$ in old age.

The budget constraints faced by an individual who is born at time $s$ are:

(16) $I(s-1) = f(s-1) \geq 0$
(17) $M(s-1) = I(s-1) \geq 0$
(18) $I(s) = \alpha p(s) \geq 0$
(19) $E(s) = c(s) p(s) \geq 0$
(20) $M(s) = I(s) + S(s) \geq 0$
(21) $S(s) = M(s-1) - E(s) \geq 0$
(22) $I(s+1) = 0$
(23) $E(s+1) = c(s+1) p(s+1) + [1+R(s-1)] f(s-1) \geq 0$
\( (24) \quad M(s+1) = 0, \)
\( (25) \quad S(s+1) = M(s) - E(s+1) = 0, \)

where \( I(t) \) is income from all sources at time \( t \), \( M(t) \) is money holding from time \( t \) to time \( t+1 \), \( E(t) \) is expenditures at time \( t \), \( S(t) \) is money which was held from \( t-1 \) to \( t \) which is not spent at time \( t \). Each of these variables is in nominal terms and must be nonnegative.

A person is assumed to maximize a utility function of the form:
\( (26) \quad U = \ln c(s) + \delta \ln c(s+1). \)

Assuming a nonnegative nominal interest rate, the solution to the maximization of (26) subject to (16) – (25) is given by:
\( (27) \quad f(s-1) = \frac{1}{1+\delta} \frac{1}{1+R(s-1)} \alpha p(s), \)
\( (28) \quad E(s) = c(s) p(s) = f(s-1), \)
\( (29) \quad c(s+1) p(s+1) = \frac{\delta}{1+\delta} \alpha p(s), \)
\( (30) \quad I(s-1) = M(s-1) = f(s-1), \)
\( (31) \quad I(s) = E(s+1) = \alpha p(s), \)
\( (32) \quad S(s) = 0. \)

2. Aggregate Equilibrium

We assume the population \( (N) \) grows constantly at rate \( n \):
\( (33) \quad N(t) = (1+n) N(t-1). \)

We now consider the conditions for market clearing at time \( t \):

a. Demand for consumption goods:
\( (34) \quad D_c(t) = N(t) c_y(t) + N(t-1) c_o(t), \)
where \( c_y(t) \) and \( c_o(t) \) represent real consumption by people who are young and old, respectively.

b. Supply of consumption goods:
(35) \[ S_c(t) = N(t) \alpha \].

c. Borrowing:

(36) \[ S_b(t) = N(t+1) f(t) \].

d. Repayments of borrowing:

(37) \[ D_b(t) = N(t-1) [1+R(t-2)] f(t-2) \].

e. Money holding:

(38) \[ D_m(t) = N(t+1) M_y(t) + N(t) M_o(t) \],

where \( M_y(t) \) and \( M_o(t) \) are the quantities of money held for spending when young and old, respectively.

f. Money supplied (exchanged for goods):

(39) \[ S_m(t) = N(t) M_y(t-1) + N(t-1) M_o(t-1) \].

Walras's Law holds here, as shown by the equation:

(40) \[ [D_c(t)-S_c(t)]p(t) + [D_b(t)-S_b(t)] + [D_m(t)-S_m(t)] = 0 \].

Consequently, market clearing in any two markets necessitates market clearing in the third market.

When stationarity is imposed on the model, the market-clearing solutions are:

(41) \[ R(t) = 0 \], for all \( t \).

(42) \[ 1 + \pi(t) = \frac{p(t+1)}{p(t)} = \frac{1}{1+n} \], for all \( t \),

where \( \pi(t) \) is the inflation rate. These results are identical to those in chapter 3.

A proof of the neutrality of government financial policy when lump-sum taxation is possible can be developed that follows exactly the steps in chapter 3. Because the steps involved are identical in form to those already given, the proof is not presented here.
C. A Range-of-Incomes Model

One of the facets of the model in chapter 3 which seems somewhat unrealistic is the lack of substitution between money and bonds. In chapter 3, the rich always hold bonds and the poor always hold money. A change in the relative rates of return to bonds and money does not affect the demands for bonds and money, because of the assumed lack of substitution.

We consider the following model in which a range of incomes exists. Let \( \alpha \) be the smallest real income received by anyone, and \( \beta \) be the largest. Incomes of other people are divided proportionately between \( \alpha \) and \( \beta \). With population \( N(t) \) growing at rate \( n \), \( w_h(t) \) is the real income of person \( h (h=1,2,...,N(t)) \) at time \( t \).

\[
(43) \quad w_h(t) = \alpha + \frac{\beta - \alpha}{N(t)-1} (h-1).
\]

We assume that there is a minimum size on bonds of \( k \) (in real terms). Individuals now have an additional decision to make. Very rich people choose bonds as a store of value, and very poor people choose money as a store of value, when \( \alpha k \leq \beta \). The very rich meet the minimum-denomination requirement easily, and the very poor have incomes below \( k \). Some people may have to consider the following maximization process. It may be possible for people to reduce consumption while young slightly in order to reach the minimum denomination on bonds, thus increasing consumption when old. The maximization problem can be written formally in two steps.
1. Maximization using bonds

When a person uses bonds as a store of value, an additional budget constraint exists, since savings must equal or exceed the minimum denomination.

The budget constraints are now:

\[(44) \quad b = (w-c_y)p_y - \tau_y , \]
\[(45) \quad c_0p_o = (1+R)b - \tau_o , \]

where \(b\) is nominal savings (bond purchases), \(w\) is income, \(c_y\) and \(c_o\) are consumption in youth and old age, respectively, \(p_y\) and \(p_o\) are the corresponding prices, \(\tau_y\) and \(\tau_o\) are lump-sum nominal taxes, and \(R\) is the nominal interest rate.

Figure 5 shows the consolidated budget constraint for the use of bonds. The solid part of the budget line is attainable. It represents the case where bond purchases exceed or equal the minimum denomination on bonds.

Two solutions arise that use bonds:

(a) An indifference curve may be tangent to the budget line. In this case, \(b \geq k p_y\). With a logarithmic utility function, the marginal conditions are met exactly at the point where 

\[ (1+i)\delta c_y = c_o , \]

where

\[ 1+i = \frac{1+R}{1+\pi} , \quad 1+\pi = \frac{p_o}{p_y} . \]

The choice variables are:

\[(46) \quad c_y = \frac{1}{1+\delta} (w-PVT_b) , \]
\[(47) \quad c_o = \frac{\delta}{1+\delta} (1+i) (w-PVT_b) . \]
\[(48) \quad \frac{b}{p_y} = \frac{\delta}{1+\delta} (w-\frac{\tau_y}{p_y}) + \frac{1}{1+i} \frac{\tau_o}{p_o} . \]
\[(49) \quad PVT_b = \frac{\tau_y}{p_y} + \frac{1}{1+i} \frac{\tau_o}{p_o} . \]
This diagram illustrates the budget lines a person faces. The outermost line is the budget line for bonds, but a minimum denomination \((k)\) must be purchased. The interior line is the budget line when money is used as a store of value.

**Figure 5**
Budget Constraints for Using Bonds and Money
This is illustrated in figure 6 (a).

(b) An indifference curve may pass through the end point of the bond budget line, but be above the money budget line. In this case \( b = kp_y \). The marginal conditions are not met as \( (1+i)\delta c_y \leq c_o \). Effectively a person gives up consumption in youth to save up to the minimum denomination on bonds, yielding greater consumption in old age. The choice variables are:

\[
\begin{align*}
(50) \quad & c_y = w - k - \frac{\tau_y}{p_y}, \\
(51) \quad & c_o = (1+i)k - \frac{\tau_o}{p_o}, \\
(52) \quad & b = kp_y.
\end{align*}
\]

This is illustrated in figure 6 (b).

2. Maximization using money

When a person uses money as a store of value, the budget constraints are:

\[
\begin{align*}
(53) \quad & m = (w-c_y) p_y - \tau_y, \\
(54) \quad & c_o p_o = m - \tau_o,
\end{align*}
\]

where \( m \) is nominal money holdings from youth to old age. The consolidated budget constraint is diagrammed in figure 5. When the indifference curve which runs through the end point of the bond budget constraint intersects the money budget constraint, it is optimal for money to be used. The individual then chooses:

\[
\begin{align*}
(55) \quad & c_y = \frac{1}{1+\delta} (w-PVT_m), \\
(56) \quad & c_o = \frac{\delta}{1+\delta} \frac{1}{1+\pi} (w-PVT_m).
\end{align*}
\]
This diagram illustrates the three possible equilibrium choices for an individual: (a) a non-corner solution using bonds, (b) a corner solution using bonds, and (c) a solution using money.

Figure 6
Examples of Possible Equilibrium Choices for Individuals
(57) \[ \frac{m}{p_y} = \frac{\delta}{1+\delta} (w - \frac{\tau_y}{p_y}) + \frac{1}{1+\delta} 1+\pi \frac{\tau_o}{p_o}. \]

(58) \[ PVT_m = \frac{\tau_y}{p_y} + 1+\pi \frac{\tau_o}{p_o}. \]

This is illustrated in Figure 6 (c).

3. Neutrality and Optimal Policy

We now consider a version of the neutrality theorem of chapter 3. A change in the amount of government debt or in the money supply is accompanied by a rearrangement of the lump-sum taxes, keeping the present value of an individual's taxes constant. If \( \frac{\tau_y}{p_y} \) is reduced while \( \frac{\tau_o}{p_o} \) is raised, this affects the bond budget constraint. The end point of the constraint moves downward and to the right as \( w - k - \frac{\tau_y}{p_y} \) rises and \( (1+i)k - \frac{\tau_o}{p_o} \) falls.

However, this creates a problem for government policy. Since people have a range of incomes, as \( \frac{\tau_y}{p_y} \) falls and \( \frac{\tau_o}{p_o} \) rises, more people begin to use bonds and fewer use money. To achieve neutrality, the government must now consider the increased demand for bonds and decreased demand for money. This requires additional adjustments in the timestream of taxes. The solution is much more difficult than the solution developed in chapter 3.

Of course, optimal government policy may be to reduce \( \frac{\tau_y}{p_y} \) and increase \( \frac{\tau_o}{p_o} \) enough to move the endpoint of the bond budget constraint
down to the $c_y$ axis. Then everyone is better off as all can use bonds as a store of value. The minimum-denomination requirement becomes inoperative. It may be, however, that governmental operations of this sort are costly to administer. In that case, the optimal government policy may not be to drive everyone to the use of bonds. A similar result, in which administrative costs of entering the bond market affect the degree of government intervention, is shown in chapter 5.
Economists such as Ricardo (1820), Buchanan (1976), and Feldstein (1976) argue that when the government incurs debt, people fail to foresee the future tax liabilities associated with that debt. This fiscal illusion may be examined using the model of chapter 3.

We first consider the neutrality theorem of chapter 3. At time $t$, the government issues some additional bonds and money, balancing its budget by adjusting taxes. Suppose, however, that young people see the current reduction in their taxes, but believe (erroneously) that taxes on them will be unchanged when they are old. If macro-neutrality is expected to hold, so that no change is expected in either the interest rate or the inflation rate, then people must perceive their budget constraints to be shifted outwards. This occurs because they believe that the present value of the taxes they face has been reduced.

In this case, it is impossible for the government to achieve micro-neutrality. Fiscal illusion leads people to believe that they are wealthier, as their budget appears to be bigger. They are then shocked in old age when they get hit with the additional taxes. Their utility levels, of course, must be lower than those that would be achieved in the absence of fiscal illusion. The government would then be wise, if it wishes to maximize individuals' welfare, to avoid any changes in government financial policy.³

The existence of fiscal illusion reduces the usefulness of financing government spending by issuing debt. For example in the model of Barro (1979, 1983, 1984), tax collecting involves administrative
costs. It is therefore optimal to smooth tax rates over time. This means that any random shock to government spending is optimally financed by a change in government debt, not taxation. When people are aware of the future tax liabilities associated with government debt, they respond in the manner suggested by the Ricardian Equivalence Theorem, so that no changes in interest rates or the inflation rate occur. However, if people suffer from fiscal illusion, then the policy of smoothing tax rates may not be optimal. There is a tradeoff between the administrative costs of taxation and the social-welfare costs caused by the changes in interest rates and the inflation rate when changes in government debt occur.

This analysis indicates that optimal government financial policy depends upon the structure of people's expectations. Because government financial policy is endogenous to expectations, to determine whether fiscal illusion occurs, it is not valid to focus on coefficients from single-equation regression models, such as a regression of savings or consumption on government debt. This inappropriate procedure is the method of every study examined in chapter 2, sections A and B. To settle this question of fiscal illusion, the structure of the economy and of optimal government financial policy need to be examined, and a simultaneous-equation econometric framework developed to discriminate among the alternative hypotheses. The typical econometric study implicitly assumes that the government sets policy independently of how people are expected to react to that policy. This is a more naive notion than the idea that people don't respond to changes in government
policy which affect them, an idea which has been successfully attacked by rational-expectations theorists.
FOOTNOTES TO CHAPTER 4

1 See Clower (1967). The existence of equilibria in Clower-constraint models with transactions costs is analyzed by Heller (1974) and Heller and Starr (1976).

2 This may seem unrealistic, but could be interpreted as meaning a person implicitly borrows from his or her parents.

3 Election politics may make a government myopic, in which case it does not seek to maximize social welfare. The government may desire time-inconsistency to maximize electoral chances.

4 This tradeoff may be visible politically. Opposing interest groups are likely to clash over these issues. Homebuilders' groups always want interest rates to be lower, while pensioners' groups prefer higher interest rates, for example.
Chapter 5

A Transactions-Cost Model of Money and
Optimal Government Financial Policy

The model of this chapter differs from the previous model by allowing both borrowing and lending by private individuals. The choice which a consumer faces between using bonds or money as a store of value is modeled explicitly using a transactions-cost technology.

The model is modified by changing the lifetime of an individual to three periods. A young person has no income and must borrow in order to buy consumption goods. In middle age, a person produces output, repays borrowings, buys consumption goods, and stores value for old age. Old people use all their savings to buy consumption goods.

The major decision faced by individuals is whether to use bonds or money as a store of value. Both lending and borrowing are assumed to be costly in this model. If a person saves wealth in the nominal amount $s$ by buying a bond (lending), then a real cost of $\gamma_s + \theta_s s/p$ is incurred (in units of the consumption good). When a person borrows the nominal amount $b$ by selling a bond, a real cost of $\gamma_b + \theta_b b/p$ is incurred. There are thus both fixed and variable components in the cost of borrowing and lending.
In this chapter we look first at the maximization problem of the individual. The nonconvexity of the budget constraint complicates individual choice considerably. Next, we examine aggregate equilibrium under alternative assumptions on the scope of government intervention. Stability considerations are used to rule out alternative equilibria. Numerical examples illustrate the results.
A. The Individual Maximization Problem

Each person's utility function is:

\[ U(t) = \ln c_y + \delta \ln c_m + \delta^2 \ln c_o, \]

where \( \delta \) is the time discount rate and \( c_y, c_m, \) and \( c_o \) represent consumption while young, middle-aged, and old.

When young, a person faces the budget constraint:

\[ (1-\delta_b)b = c_y p_y + \gamma_y p_y, \]

where \( \delta_b \) is the marginal cost of borrowing amount \( b \), \( p_y \) is the price level, and \( \gamma_y \) is the fixed cost incurred when \( b \) is positive. Because a young person has no income, borrowing is necessary in order to have positive consumption. This equation is in nominal terms, and shows that borrowing \( (b) \) pays for consumption \( (c_y p_y) \) and for transactions costs \( (\gamma_y p_y + \delta_b b) \).

In middle age, the budget constraint is:

\[ (3a) \quad p_m X = (1+R_y)b + c_m p_m + m + (1+\delta_s)s + \gamma_s p_m \quad \text{if } s>0, \]

or

\[ (3b) \quad p_m X = (1+R_y)b + c_m p_m + m \quad \text{if } s=0, \]

where \( X \) is output, \( p_m \) is the price level, \( R_y \) is the nominal interest rate last period, \( m \) is money holdings, \( s \) is bond purchases, \( \gamma_s \) is the fixed cost incurred when \( s \) is positive, and \( \delta_s \) is the marginal cost of borrowing amount \( s \). This equation shows that the nominal value of output \( (p_m X) \) is used to repay principal and interest on borrowings \([ (1+R_y)b ] \), to buy consumption goods \( (c_m p_m) \), to buy bonds \( (s) \), and to pay transactions costs on bonds \( (\gamma_s p_m + \delta_s s) \). Any money left over is held until next period \( (m) \).
In old age, the budget constraint is:

(4) \[ m + (1 + R_m)s = c_o p_o, \]

where \( R_m \) is the nominal interest rate last period and \( p_o \) is the current price level. This equation shows that all income and savings in the form of money and bonds \([m + (1 + R_m)s]\) are used to buy consumption goods. No bequest is left to heirs.

The major decision faced by individuals is whether to hold bonds or money as a store of value from middle age to old age. Because of the fixed-cost component of transactions costs, it is impossible to derive a marginal condition for the money-bonds choice. Instead, a comparison of utility levels achieved under each method must be made. This requires two maximization procedures, one assuming bonds are used, the other assuming bonds are not used.

1. Maximization Using Bonds

When bonds are used as a store of value \((s > 0)\), the transactions cost \( y_{sp_m} \) is incurred. Equation (3a) is relevant for middle age. The maximization can be solved by setting up a Lagrangian. The choice variables \( b, c_y, c_m, c_o \) must be positive, and \( s \) is assumed to be positive, while \( m \) must be nonnegative.

(5) \[
L = \ln c_y + \delta \ln c_m + \delta^2 \ln c_o \\
+ \lambda_1 [(1 - \theta_b) b - c_y p_y - \gamma_{bp_y}] \\
+ \lambda_2 [p_m x - (1 + R_y) b - c_m p_m - m - \gamma_{sp_m} - (1 + \theta_s)s] \\
+ \lambda_3 [m + (1 + R_m)s - c_o p_o].
\]

The Kuhn-Tucker condition on \( m \) is:

(6) \[
\frac{\partial L}{\partial m} = -\lambda_2 + \lambda_3 \leq 0 \quad m \geq 0 \quad (\frac{\partial L}{\partial m})_m = 0.
\]
The other first-order conditions of this maximization require
\[
\lambda_1 = \lambda_2 \frac{1 + R_y}{1 - \theta_b} \quad \text{and} \quad \lambda_2 = \lambda_3 \frac{1 + R_m}{1 - \theta_s}.
\]
If \(1 + R_m > 1 + \theta_s\), then (6) requires \(m = 0\). If \(1 + R_m = 1 + \theta_s\), or if \(1 + R_m < 1 + \theta_s\), then the yield on money is at least as large as the yield on bonds, and money has the advantage of lower transactions cost. So \(1 + R_m \leq 1 + \theta_s\) implies \(s = 0\). Assuming that \(1 + R_m > 1 + \theta_s\), the first-order conditions yield:
\[
\begin{align*}
\delta \frac{1 + R_y}{1 - \theta_b} c_y p_y &= c_m^p m, \\
1 + R_m \frac{1 + R_m}{1 + \theta_s} c_m p_m &= c_o p_0.
\end{align*}
\]

We may now derive a budget constraint in present-value terms by combining (2), (3a), and (4), eliminating \(b\) and \(s\).

\[
p_m X - \frac{1 + R_y}{1 - \theta_b} y_b p_y - y_s p_m = \frac{1 + R_y}{1 - \theta_b} c_y p_y + c_m^p m + \frac{1 + \theta_s}{1 + R_m} c_o p_0.
\]

Using (7), (8), and (9) we solve for the choice variables:

\[
\begin{align*}
c_y &= \frac{1}{1 + \delta + \delta^2} \left[ (X - \gamma_s) \frac{1 - \theta_b}{1 + R_y} p_m - \gamma_b \right], \\
c_m &= \delta \frac{1 + R_y}{1 - \theta_b} p_y c_y, \\
c_o &= \delta \frac{1 + R_m}{1 + \theta_s} \frac{p_m}{p_o} c_m.
\end{align*}
\]

\[
\begin{align*}
b &= b Y = \frac{1}{1 + \delta + \delta^2} \left( X - \gamma_s \right) \frac{1}{1 + R_y} p_m + \frac{\delta + \delta^2}{1 + \delta + \delta^2} \frac{1}{1 - \theta_b} \gamma_b, \\
1 + \delta + \delta^2 \left( X - \gamma_s \right) \frac{1}{1 + \theta_s} \frac{1}{1 + R_y} p_y & \quad \gamma_b \frac{p_y}{p_m}.
\end{align*}
\]

2. Maximization Using Money

When money is used as a store of value, there is no transactions cost, and equation (3b) is relevant for middle age. The interest rate
is now irrelevant, as bonds are not purchased. The budget constraint in present-value terms is now:

\[
p_m X - \frac{1+R}{1-\theta_b} b Y = \frac{1+R}{1-\theta_b} c_o y + c_m p_m + c_o p_o.
\]

Using the first-order conditions from the maximization procedure, the optimal values of the choice variables are found to be:

\[
c_y = \frac{1}{1+\delta+\delta^2} \left[ X \frac{1-\theta_b}{1+R} p_m - y_b \right].
\]

\[
c_m = \delta \frac{1+R}{1-\theta_b} \frac{p_y}{p_m} c_y.
\]

\[
c_o = \delta \frac{p_m}{p_o} c_m.
\]

\[
\frac{b}{p_y} = \frac{1}{1+\delta+\delta^2} X \frac{1}{1+R} \frac{p_m}{p_y} + \frac{\delta+\delta^2}{1+\delta+\delta^2} \frac{1}{1-\theta_b} y_b.
\]

\[
\frac{m}{p_m} = \frac{\delta^2}{1+\delta+\delta^2} \left[ X \frac{1}{1-\theta_b} \frac{y_b}{p_m} \right].
\]

3. The Choice between Money and Bonds

In deciding between the use of money and bonds, the individual simply compares the utility levels achieved under each of the two choices. Using bonds means less consumption while young and middle-aged than using money, but yields more consumption in old age. We denote utility obtained when money is used as \( U_m \), and utility when using bonds as \( U_s \), and let \( PV_s = (X - y_s) \frac{1-\theta_b}{1+R} \frac{p_m}{p_y} - y_b \). Then we have:

\[
c_y^s = \frac{1}{1+\delta+\delta^2} PV_s.
\]

\[
c_m^s = \delta \frac{1+R}{1-\theta_b} \frac{p_y}{p_m} c_y^s.
\]

\[
c_o^s = \delta^2 \frac{1+R}{1-\theta_b} \frac{1+R_m}{1+\theta_s} \frac{p_y}{p_m} \frac{p_m}{p_o} c_y^s.
\]
We define $V = U_s - U_m$. Bonds will be used when $V > 0$, and money will be used when $V < 0$.

\[
V = (1 + \delta + \delta^2) \left[ \ln PV_s - \ln PV_m \right] + \delta^2 \ln \frac{1 + R_m}{1 + \theta_s}.
\]

The first term of $V$ is negative since $PV_m > PV_s$. Thus we clearly need $\frac{1 + R_m}{1 + \theta_s} > 1$ to get $V > 0$ (i.e., $R_m > \theta_s$ is a necessary condition for the use of bonds).

$V$ is affected by the parameters $\delta, X, \gamma_s, \theta_b, R_y, \frac{p_m}{p_y}, \gamma_b$, and $\theta_s$. For any of these parameters a, if $\frac{\partial V}{\partial a}$ is positive, then bonds are more likely to be used as a rises, while if $\frac{\partial V}{\partial a}$ is negative, then money is more likely to be used instead of bonds. Using equation (21), we can find the signs of these partial derivatives. $\frac{\partial V}{\partial X}$ is positive because more income makes the bond transaction-cost less important. $\frac{\partial V}{\partial \gamma_s}$ is negative, as expected, since a rise in transactions costs means fewer bonds are used. $\frac{\partial V}{\partial \gamma_b}$ is negative because a higher borrowing transactions cost reduces $c^s_y$ proportionately more than it reduces $c^m_y$, encouraging money use. $\frac{\partial V}{\partial R_m}$ is positive and $\frac{\partial V}{\partial \theta_s}$ is negative because increasing the
returns to bonds encourages bond use. \( \frac{\partial V}{\partial \theta_b} \) and \( \frac{\partial V}{\partial R_y} \) are negative, and \( \frac{\partial V}{\partial p_m/p_y} \) is positive, but all three of these are very small effects.

4. A Numerical Example

Table 2 illustrates the effects of changes in the parameters \( X, \delta, R_y, R_m, p_y/p_m, Y_s, Y_b, \theta_s, \) and \( \theta_b \). Columns a and g serve as bases. For the parameters given in columns a and g, utility maximization implies the use of money rather than bonds. We may compare columns b through f with column a. Column b shows that an increase in output (income), \( X \), may lead to a switch from money to bonds. Column c shows the switch to bonds due to a reduction in the fixed cost of buying bonds \( (Y_s) \). Column d shows a similar result when the fixed cost of borrowing \( (Y_b) \) falls. Column e shows the switch to bonds when the interest rate on bonds \( (R_m) \) rises when a person is middle-aged. Column f demonstrates the switch when the variable cost of buying bonds \( (\theta_s) \) falls.

In column g, the parameters are such that money is held rather than bonds, but the choice is a close one, as the table indicates. We now compare columns h through j with column g. Column h shows that an increase in inflation \( \left( \frac{p_m}{p_y} \right) \) encourages the switch from money to bonds. Similarly column i shows that a decrease in the nominal interest rate \( (R_y) \) when a person is young leads to the use of bonds. Finally, a decrease in the variable cost of borrowing \( (\theta_b) \) leads to this same result.
Table 2
A Numerical Example Illustrating the Choice Between Using Money and Bonds

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>7.0</td>
<td>8.0</td>
<td>7.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ry</td>
<td>.333</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.29333</td>
<td>.333</td>
<td></td>
</tr>
<tr>
<td>Rx</td>
<td>.333</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.357</td>
<td>.333</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>py/pm</td>
<td>1.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.5</td>
<td>1.43</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ys</td>
<td>.4</td>
<td>.36</td>
<td>.4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Yb</td>
<td>.4</td>
<td>-</td>
<td>-</td>
<td>.2</td>
<td>.4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>es</td>
<td>.08</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.06</td>
<td>.08</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>eb</td>
<td>.08</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.04</td>
</tr>
<tr>
<td>c'sy</td>
<td>.973</td>
<td>1.142</td>
<td>.979</td>
<td>1.046</td>
<td>.973</td>
<td>.973</td>
<td>1.041</td>
<td>1.099</td>
<td>.077</td>
<td>1.092</td>
</tr>
<tr>
<td>c'sm</td>
<td>1.093</td>
<td>2.235</td>
<td>1.916</td>
<td>2.047</td>
<td>1.903</td>
<td>1.903</td>
<td>2.036</td>
<td>2.049</td>
<td>2.045</td>
<td>2.048</td>
</tr>
<tr>
<td>Us</td>
<td>1.486</td>
<td>1.922</td>
<td>1.505</td>
<td>12.69</td>
<td>1.500</td>
<td>1.502</td>
<td>1.669</td>
<td>1.735</td>
<td>1.7113</td>
<td>1.728</td>
</tr>
<tr>
<td>cm</td>
<td>1.041</td>
<td>1.210</td>
<td>1.041</td>
<td>1.114</td>
<td>1.041</td>
<td>1.041</td>
<td>1.108</td>
<td>1.170</td>
<td>1.147</td>
<td>1.163</td>
</tr>
<tr>
<td>cm*</td>
<td>2.036</td>
<td>2.368</td>
<td>2.036</td>
<td>2.180</td>
<td>2.036</td>
<td>2.036</td>
<td>2.169</td>
<td>2.182</td>
<td>2.177</td>
<td>2.181</td>
</tr>
<tr>
<td>cm</td>
<td>2.749</td>
<td>3.197</td>
<td>2.749</td>
<td>2.943</td>
<td>2.749</td>
<td>2.749</td>
<td>2.928</td>
<td>2.946</td>
<td>2.940</td>
<td>2.944</td>
</tr>
<tr>
<td>U^m</td>
<td>1.499</td>
<td>1.908</td>
<td>1.499</td>
<td>1.684</td>
<td>1.499</td>
<td>1.499</td>
<td>1.670</td>
<td>1.734</td>
<td>1.7112</td>
<td>1.727</td>
</tr>
<tr>
<td>b's/p</td>
<td>1.492</td>
<td>1.677</td>
<td>1.4989</td>
<td>1.355</td>
<td>1.492</td>
<td>1.492</td>
<td>1.566</td>
<td>1.629</td>
<td>1.606</td>
<td>1.554</td>
</tr>
<tr>
<td>s/p</td>
<td>1.586</td>
<td>1.863</td>
<td>1.597</td>
<td>1.706</td>
<td>1.586</td>
<td>1.616</td>
<td>1.697</td>
<td>1.708</td>
<td>1.704</td>
<td>1.707</td>
</tr>
<tr>
<td>m^p</td>
<td>1.566</td>
<td>1.750</td>
<td>1.566</td>
<td>1.429</td>
<td>1.566</td>
<td>1.566</td>
<td>1.640</td>
<td>1.706</td>
<td>1.682</td>
<td>1.628</td>
</tr>
<tr>
<td>m/p</td>
<td>1.832</td>
<td>2.131</td>
<td>1.832</td>
<td>1.962</td>
<td>1.832</td>
<td>1.832</td>
<td>1.952</td>
<td>1.964</td>
<td>1.960</td>
<td>1.963</td>
</tr>
</tbody>
</table>

Choice money bonds bonds bonds bonds bonds bonds bonds bonds bonds bonds

Explanation: A dash (-) means that the parameter takes the same value as it has in the preceding column. The results are interpreted in the text.
B. Aggregate Equilibrium

We now assume that we have an economy populated by two types of individuals, rich and poor, distinguished by their output $X$. The rich produce $X = \beta$, while the poor produce $X = \alpha$, and $\beta > \alpha$. Since we wish to study economies which use both money and bonds, we'll assume that the parameters are such that the poor hold money as a store of value, and the rich hold bonds. We write the individual choice variables in real terms and attach time subscripts to get:

\begin{align*}
(22) \quad c_y^r(t) &= \frac{1}{1+\delta+\delta^2} \left[ (\beta - \gamma_s) \frac{1-\theta_b}{1+R(t)} \frac{p(t+1)}{p(t)} - \gamma_b \right]. \\
(23) \quad c_m^r(t) &= \delta \frac{1+R(t-1)}{1-\theta_b} \frac{p(t-1)}{p(t)} c_y^r(t-1). \\
(24) \quad c_y^o(t) &= \delta^2 \frac{1+R(t-1)}{1+\theta_s} \frac{p(t-1)}{p(t)} c_m^r(t-1). \\
(25) \quad \frac{b^r(t)}{p(t)} &= \frac{1}{1-\theta_b} c_y^r(t) + \frac{1}{1-\theta_b} \gamma_b. \\
(26) \quad \frac{s^r(t)}{p(t)} &= \delta^2 \frac{1+R(t-1)}{1-\theta_b} \frac{1}{1+\theta_s} \frac{p(t-1)}{p(t)} c_y^{r(t-2)}. \\
(27) \quad c_y^m(t) &= \frac{1}{1+\delta+\delta^2} \left[ \alpha \frac{1-\theta_b}{1+R(t)} \frac{p(t+1)}{p(t)} - \gamma_b \right]. \\
(28) \quad c_m^m(t) &= \delta \frac{1+R(t-1)}{1-\theta_b} \frac{p(t-1)}{p(t)} c_y^m(t-1). \\
(29) \quad c_y^o(t) &= \delta^2 \frac{p(t-1)}{p(t)} c_m^o(t-1). \\
(30) \quad \frac{b^o(t)}{p(t)} &= \frac{1}{1-\theta_b} c_y^o(t) + \frac{1}{1-\theta_b} \gamma_b. \\
(31) \quad \frac{m^o(t)}{p(t)} &= \delta^2 \frac{1+R(t-1)}{1-\theta_b} \frac{p(t-1)}{p(t)} c_y^{o(t-2)}. 
\end{align*}
Assume that both rich and poor populations grow at the rate \( n \), so
\[
N_r(t) = (1 + n)N_r(t - 1) \quad \text{and} \quad N_p(t) = (1 + n)N_p(t - 1).
\]
The aggregate variables are as follows:

a. Aggregate demand for consumption goods:
\[
D_c(t) = N_r(t)c^r(t) + N_p(t-1)c^p(t) + N_r(t-2)c^r(t) + N_p(t)c^p(t) + N_p(t-1)c^p(t).
\]

b. Aggregate supply of consumption goods:
\[
S_c(t) = N_r(t-1)\beta + N_p(t-1)\alpha.
\]
c. Use of goods in transactions:
\[
L(t) = N_r(t)[Y_p(t) + \theta_s s^r(t)] + N_r(t-1)[Y_p(t) + \theta_s s^r(t)]
\]
\[
+ N_p(t)[Y_p(t) + \theta_p s^p(t)].
\]
d. Market-clearing condition for goods:
\[
D_c(t) + L(t) = S_c(t).
\]
e. Aggregate demand for bonds:
\[
D_b(t) = N_r(t-1)s^r(t).
\]
f. Aggregate supply of bonds:
\[
S_b(t) = N_r(t)b^r(t) + N_p(t)b^p(t).
\]
g. Aggregate expenditures on (payouts of) interest on bonds:
\[
D_{rb}(t) = (1+R(t-1))S_b(t-1).
\]
\[
S_{rb}(t) = (1+R(t-1))D_b(t-1).
\]
h. Aggregate demand for money:
\[
D_m(t) = N_p(t-1)m^p(t).
\]
i. Aggregate supply of money:
\[
S_m(t) = D_m(t-1) = N_p(t-2)m^p(t-1).
\]

It may now be confirmed, using algebra, that Walras's Law holds.

In this model, Walras's Law is expressed as:
\[
[D_c(t)p(t) + L(t) - S_c(t)p(t)] + [D_b(t) - S_b(t)]
\]
\[ + [D_{rb}(t) - S_{rb}(t)] + [D_{m}(t) - S_{m}(t)] = 0. \]

1. The government's optimization problem

In this model with transactions costs and borrowing it may no longer be possible to maintain the segmented-society solution of chapter 3. Consequently we must derive an objective function which the government maximizes. We assume first that every generation is to be treated equally. Thus a stationary solution is desirable, and time subscripts can be dropped from stationary variables.

We assume that the government seeks to maximize a social-welfare function of the form:

\[ W(t) = \phi N_r(t) U^r(t) + (1-\phi) N_p(t) U^p(t), \]

where \( \phi \) is the social-welfare weight assigned to a rich person and \( (1-\phi) \) is the social-welfare weight assigned to a poor person. Transforming this by dividing through by \( N_r(t) \), we get a stationary social-welfare measure:

\[ W^* = \phi U^r + (1-\phi) \frac{N_p}{N_r} U^p, \]

where \( U^r(t) = U^r, U^p(t) = U^p \), and \( \frac{N_p(t)}{N_r(t)} = \frac{N_p}{N_r} \) for all \( t \).

In chapter 3, the government is assumed to maintain the segmented-society solution, in order to assure that simple equilibrium holds. This decision can be reached by maximizing a social welfare function like (43). The government's maximization problem in chapter 3 thus implicitly involves social-welfare weights. These can be found by maximizing (43), subject to the goods-clearing constraint, and imposing the simple-equilibrium solution. That is:
The first-order conditions of this maximization require that:

\[
\max_{c_r, c_o, c_y, c_p} W = \phi U^r + (1-\phi) \frac{N_p}{N_r} U^p
\]

s.t. \[U^r = \ln c_y + \delta \ln c_o, \]
\[U^p = \ln c_y + \delta \ln c_o, \]
\[\beta + \frac{N_p}{N_r} \alpha = \frac{c_y}{1+n} + \frac{1}{1+n} c_r + \frac{N_p}{N_r} c_p + \frac{1}{1+n} \frac{N_p}{N_r} c_o. \]

The simple-equilibrium solution of chapter 3 yields:

\[\frac{1}{1+n} c_o = \delta c_r, \quad c_y = \frac{1-\phi}{\phi} c_r, \quad \frac{1}{1+n} c_p = \delta c_y. \]

(44) \[\beta + \frac{N_p}{N_r} \alpha = (1+\delta) (1 + \frac{1-\phi}{\phi} \frac{N_p}{N_r}) c_r. \]

The simple-equilibrium solution of chapter 3 yields:

(45) \[c_y = \frac{1}{1+\delta} \beta, \quad c_p = \frac{1}{1+\delta} \alpha. \]

Solving (44) and (45), we find that the imposition of simple equilibrium in chapter 3 would have arisen in a social-welfare maximization context with social-welfare weights:

(46) \[\phi = \frac{\beta}{\alpha+\beta}, \quad 1-\phi = \frac{\alpha}{\alpha+\beta}. \]

In the remainder of this chapter, the social-welfare weights given by equation (46) will be used.

2. Equilibrium when government provides a constant money supply and does not intervene in the bond market

We first assume that the government intervenes only in the money market. Intervention is necessary in this market because no private individual is capable of issuing money, as suggested in chapter 3. If a constant money supply is maintained, then \(D_m(t) = S_m(t)\). The stationary solution (interest rates and inflation constant) is given by:

\[D_m(t) = S_m(t).\]
the stationary solution requires that \( m(t) = \frac{m(t-1)}{p(t-1)} \).

Government intervention is not necessary in the bond market in this model. In the model of chapter 3, bond-market intervention is necessary because no private individual wants to issue bonds. In the present model, however, both borrowers and lenders exist.

The equilibrium solution occurs when demand and supply of bonds is equated, \( D_b(t) = S_b(t) \), for all \( t \). If this is satisfied, then \( D_{rb}(t) = S_{rb}(t) \), as inspection of equations (38) and (39) shows. Given this and money-market clearing, Walras's Law requires that the goods market clear as well. Thus clearing of the bond market assures full equilibrium.

The stationary solution is given by:

\[
D_b(t) = S_b(t) \\
N_r(t-1)s^r(t) = N_r(t)b^r(t) + N_p(t)b^p(t) \text{ from (36) and (37)}.
\]

Using equations (25), (26), (22), (31), (27), and simplifying, we get:

\[
[(1+n)^2 \delta^2 \frac{1}{1-\theta_b} \frac{1}{1+\theta_s} \gamma_b] (1+R)^2 \\
+ [-(1+n) \delta^2 \frac{1}{1+\theta_s} (\beta-\gamma_s) + (1+n)^2 (\delta+\delta^2) \frac{1}{1-\theta_b} (1+\frac{N_p}{N_r}) \gamma_b] (1+R) \\
+ [(1+n) (\beta-\gamma_s) + (1+n) \frac{N_p}{N_r} \alpha] = 0,
\]

where \( \frac{N_p}{N_r} = \frac{N_p(t)}{N_r(t)} \).
The analytical solution for R is too cumbersome to be useful. Since equation (44) is a quadratic, two solutions for R exist. However, for many values of the parameters, one root is so large that consumption according to equations (22)-(24) and (27)-(29) is negative. Therefore, that root is not sustainable. The smaller root is sustainable.

A Numerical Example

Let $\delta = .9$, $\delta_s = \delta_b = .005$, $\gamma_s = \gamma_b = .275$, $n = .5$, $\frac{N_p}{N_r} = .5$, $\beta = 10$, $\alpha = 2$.

Solutions to (48) are $R = 17.545$ and $R = .731$.

When $R = 17.545$, $c_y^r$ and $c_y^p$ are negative. Thus $R = 17.545$ is not sustainable.

When $R = .731$, the other variables are:

- $c_y^r = 1.274$
- $c_m^r = 2.991$
- $c_o^r = 6.955$
- $U^r = 2.799$
- $c_y^p = .181$
- $c_m^p = .426$
- $c_o^p = .575$
- $U^p = -2.924$
- $b_y^r = .1557$
- $b_m^r = .459$
- $s_y^r = 2.679$
- $m = .383$

- $D_c(t) = 10.079$
- $S_c(t) = 11.000$
- $L(t) = .921$

Social welfare $W = 2.089$.

Stability

It is necessary to ensure that the solution is stable. Two types of stability may be defined:

1. Sustainability, as discussed previously, means that individual choice is consistent with the aggregate equilibrium. In an
aggregate equilibrium in which the rich hold bonds and the poor hold money, it must be shown, given the interest rates and inflation rate which come out of the aggregate equilibrium, that indeed the rich prefer bonds to money and the poor prefer money to bonds.

In the numerical example given above, it can be shown that \( R = 0.731 \) is sustainable. If a rich person holds money as a store of value instead of bonds, the utility level obtained is 2.440. This is less than the utility level of 2.799 which is received when bonds are held, so every rich person prefers bonds to money. If a poor person holds bonds rather than money, a utility level of -3.137 is obtained. This is less than the level of -2.924 obtained under money holding, so money is preferred to bonds. Thus the solution is sustainable.

2. Aggregate stability of a solution requires that the solution be reached from any starting point. For example, in the aggregate equilibrium above, this means that given any \( a \) and \( b \) such that \( 0 \leq a \leq 1 \) and \( 0 \leq b \leq 1 \), where \( a \) represents the proportion of poor people holding bonds, and \( b \) represents the proportion of rich people holding bonds, the only sustainable solution occurs where \( a = 0 \) and \( b = 1 \).

Because in this model all agents are identical, the only possibilities at equilibrium are \( a = 1 \) and \( b = 1 \), \( a = 1 \) and \( b = 0 \), or \( a = 0 \) and \( b = 0 \). These equilibria are denoted the all-bond equilibrium, the mixed equilibrium, and the all-money equilibrium. For one of these equilibria to be considered
stable, it must be shown that the other two are not sustainable.\(^3\)

In the numerical example given above, the all-bond equilibrium is not sustainable. At the equilibrium interest rate of \( R = 0.584 \), the poor prefer to hold money. Their utility level for holding money is \(-2.713\), while that for holding bonds is \(-2.964\). An all-money equilibrium cannot be defined in this model, because unless someone holds bonds, consumption while young will be zero, and utility is driven infinitely negative. Thus aggregate stability holds for the mixed equilibrium in the numerical example.

3. Equilibrium when the government intervenes in the bond market to achieve simple equilibrium

If the government ignores the effects of transactions costs, for example by suggesting that they are of minor importance, it may try to achieve simple equilibrium as in chapter 3. In this model, simple equilibrium occurs when inflows to the bond market equal outflows. As before, the simple-equilibrium solution is \( R = 0, \frac{p(t)}{p(t+1)} = 1+n \). The condition \( \frac{p(t)}{p(t+1)} = 1+n \) can be met by setting \( M(t) = M(t-1) \), as in the previous section.

When \( R = 0 \), no one wants to hold bonds, as a greater return on investment is possible using money, since the use of money does not entail transactions costs. Consequently the only equilibrium possible is an all-money equilibrium.
Equations (22) through (41) describe the mixed equilibrium in which the rich use bonds and the poor use money. The all-money solution uses equations (27)-(31), but equations (22)-(26) must be modified as follows:

\[
\begin{align*}
(22') \quad c^r_y(t) &= \frac{1}{1+\delta+\delta^2} \left[ \beta \frac{1-\theta_b}{1+R(t)} \frac{p(t+1)}{p(t)} - \gamma_b \right], \\
(23') \quad c^r_m(t) &= \delta \frac{1+R(t-1)}{1-\theta_b} \frac{p(t-1)}{p(t)} c^r_y(t-1), \\
(24') \quad c^r_o(t) &= \delta^2 \frac{p(t-1)}{p(t)} c^r_m(t-1), \\
(25') \quad \frac{b^r(t)}{p(t)} &= \frac{1}{1-\theta_b} c^r_y(t) + \frac{1}{1-\theta_b} \gamma_b, \\
(26') \quad \frac{m(t)}{p(t)} &= \delta^2 \frac{1+R(t-1)}{1-\theta_b} \frac{p(t-1)}{p(t)} c^r_y(t-2).
\end{align*}
\]

Aggregate equations must be modified as follows:

\[
\begin{align*}
(34') \quad L(t) &= N_r(t)[\gamma_b p(t) + \theta_b b^r(t)] + N_r(t)[\gamma_b p(t) + \theta_b b^p(t)], \\
(36') \quad D^g_b(t) &= D^g_b(t), \\
(40') \quad D_m(t) &= N_r(t-1)m^r(t) + N_p(t-1)m^p(t), \\
(41') \quad S_m(t) &= D_m(t-1) = N_r(t-2)m^r(t-1) + N_p(t-1)m^p(t-1).
\end{align*}
\]

Aggregate equilibrium requires bond-market clearing, so \(D^g_b(t) = S_b(t)\).

\[
S_b(t) = N_r(t)b^r(t) + N_p(t)b^p(t), \\
S_b(t) \quad \frac{N_r(t)p(t)}{p(t)} = \frac{1}{1+\delta+\delta^2} \left[ \frac{1}{1+R(t)} \frac{p(t+1)}{p(t)} \left( \beta \frac{N_p}{N_r} \right) + (\delta+\delta^2)(1+\frac{N_p}{N_r}) \frac{1}{1-\theta_b} \gamma_b \right].
\]

Since \(D^g_b(t) = S^g_b(t)\), and we wish to find \(S^g_b(t)\) such that \(R(t) = 0\) and \(\frac{p(t)}{p(t+1)} = 1+n\), set \(D^g_b(t)\) equal to the value taken by \(S^g_b(t)\) when \(R(t) = 0\) and \(\frac{p(t)}{p(t+1)} = 1+n\). That is:

\[
\begin{align*}
(49) \quad \frac{D^g_b(t)}{N_r(t)p(t)} &= \frac{1}{1+\delta+\delta^2} \left[ \frac{1}{1+n} \left( \beta \frac{N_p}{N_r} \right) + (\delta+\delta^2)(1+\frac{N_p}{N_r}) \frac{1}{1-\theta_b} \gamma_b \right].
\end{align*}
\]
Notice that when (49) is followed, and \( M(t) = M(t-1) \) so that \( \frac{p(t)}{p(t+1)} = 1+n \), we get \( R(t) = 0 \) as the only solution which clears the bond market.

The government budget constraint is \( b^g(t) = [1+R(t-1)] b^g(t-1) + M(t)-M(t-1) \). With \( M(t) = M(t-1) \), and \( R(t) = 0 \), this requires \( b^g(t) = b^g(t-1) \). But:

\[
\frac{b^g(t)}{N_r(t)p(t)} = \frac{b^g(t-1)}{N_r(t-1)p(t-1)} = \frac{b^g(t-1) N_r(t)}{N_r(t)p(t)} \frac{N_r(t-1)}{p(t-1)} = \frac{b^g(t-1) N_r(t)}{N_r(t)p(t)}.
\]

Hence \( b^g(t) = b^g(t-1) \), and the government budget constraint is satisfied.

A Numerical Example

We let the parameters be the same as before, where \( \delta = .9 \), \( \theta_s = \theta_b = .005 \), \( Y_s = Y_b = .275 \), \( n = \frac{N_r}{N_p} = .5 \), \( \beta = 10 \), \( \alpha = 2 \).

Equation (49) requires \( \frac{b^g}{N_r p} = 2.967 \). This gives \( R(t) = 0 \) and \( \frac{p(t)}{p(t+1)} = 1.5 \) for all \( t \).

The other variables are:

\[
\begin{align*}
\frac{b^r}{p} &= 2.634 \quad \frac{b^p}{p} = .666 \quad \frac{m^r}{p} = 2.865 \quad \frac{m^p}{p} = .474 \\
N_r(t-1) &= 10.359 \quad S_c(t) = 11.000 \quad L(t) = .641 \\
Social welfare W &= 2.41328.
\end{align*}
\]

Comparison of these results with those of section 2 shows that everyone is made much better off by government intervention in the bond market. Social welfare increases by over 15%. This improvement comes
about for two reasons: (1) the elimination of the transactions costs by bondholders, and (2) the existence, as in chapter 3, of an intergenerational free lunch. The first reason is obvious, but the second one does not become clear until section 5 below.

4. Equilibrium when the government intervenes in bond and money markets to maximize social welfare

The previous section showed that the government can improve social welfare by achieving simple equilibrium. However, because of the presence of transactions costs, it is possible to increase social welfare even further. Simple equilibrium maximizes social welfare only in the absence of transactions costs.

The government is now assumed to maximize social welfare subject to the constraints of:

1. Achieving a stationary equilibrium,
2. Satisfying the government budget constraint,
3. Satisfying all market-clearing requirements, and

A stationary solution yields \( R(t) = R \) and \( \frac{p(t)}{p(t+1)} = C \) for all \( t \). Whenever possible we deal with stationary variables and eliminate time notations.

To check for a stable solution, we must look at sustainability and aggregate stability. Sustainability is checked by verifying that neither rich nor poor desire to switch between bonds and money at the aggregate equilibrium. Aggregate stability is checked by showing that
for any $\frac{D^g_b}{N_r p}$ level chosen by the government, either the mixed equilibrium or the all-money equilibrium is sustained, but not both.

The government budget constraint can be written as:

$$D^g_b(t) = [1+R(t-1)]D^g(t-1) + M(t) - M(t-1),$$

where

$$\frac{D^g_b(t)}{N_r(t)p(t)} = (1+R) \frac{D^g_b(t-1)}{N_r(t-1)p(t-1)} + \frac{M(t)}{N_r(t)p(t)} - \frac{M(t-1)}{N_r(t-1)p(t-1)} \frac{p(t-1)}{p(t)} + \frac{p(t-1)}{p(t)}.$$

We set $\frac{p(t-1)}{p(t)} = C$ for all $t$. Then the budget constraint becomes:

$$D^g_b(t) = [1 + \frac{(1+R)C}{1+n}] \frac{D^g_b}{N_r p} = [1 - \frac{C}{1+n}] \frac{M}{N_r p}.$$

In an all-money equilibrium, the following conditions must hold.

Bond-market clearing requires $\frac{D^g_b}{N_r p} = \frac{S_b}{N_r p}$. Using equations (25), (30), and (37), and solving for $(1+R)C$, we get:

$$C = \frac{\delta + N_p/N_r \alpha}{(1+\delta+\delta^2) N_r p/(1+n)} - \frac{(1+\delta^2)(1+N_p/N_r)}{(1+R)C} \frac{1}{1/(1-\theta_b)} \gamma_b.$$

Money-market clearing requires $\frac{M}{N_r p} = \frac{D_m}{N_r p}$. Using equations (26'), (31), and (40'), substituting to get $\frac{D^g_b}{N_r p}$ in the equation instead of $\frac{M}{N_r p}$ by using equation (50), and solving for $C$, we get:

$$C = (1+n) \left[ 1 - \frac{(1+\delta^2)[1-(1+R)C/(1+n)]D^g_b/N_r p}{1+\delta^2 (\delta + N_p/N_r \alpha) - \frac{1}{1+n} \delta^2 (1+R)C \frac{1}{1/1-\theta_b} \frac{N_p}{N_r} \gamma_b} \right].$$

In a mixed equilibrium in which the rich hold bonds and the poor hold money, the market-clearing conditions are more complicated. Bond-market clearing requires $\frac{D_b}{N_r p} = \frac{S_b}{N_r p}$ where:
using equations (25), (26), (30), (36'), and (37) we get:

\[(53)\quad [\delta^2 - \frac{1}{1+n} \gamma_b - \frac{1}{1+n} \gamma_s] [((1+R)C)^2 \]

\[+ [(\delta+\delta^2) \frac{1}{1-\theta_b} (1+\frac{N_p}{N_r}) \gamma_b \delta^2 \frac{1}{1+n} (1+\gamma_s) - (1+\delta^2) \frac{D_b}{N_r p} (1+R)C \]

\[+ [\beta - \gamma_s + \frac{D_p}{N_r} \alpha] = 0. \]

Money-market clearing requires \(\frac{M}{N_r p} = \frac{D_m}{N_r p}\). Using equations (31),

\[(40), \text{ and } (50), \text{ and solving for } C \text{ gives us:} \]

\[(54)\quad C = (1+n) \left\{ 1 - \frac{(1+\delta^2)[1-(1+R)C/(1+n)]D_b/N_r p}{\frac{1}{1+n} \delta^2 \frac{N_p a}{N_r} - \frac{1}{1+n} \delta^2 \frac{(1+R)C}{1-\theta_b} \frac{N_p}{N_r} \gamma_b} \right\}. \]

Equation (53) is a quadratic. Numerical simulation suggests that
the larger root of (53) is not sustainable as it leads to negative
consumption. It is possible to write out the sustainable (smaller) root
of (53) algebraically, but the result is too complex for useful
analysis.

The method of optimization by the government is as follows. First
examine the all-money equilibrium. Choose the value of \(D_b/N_r p\) which
maximizes social welfare, using (51) and (52) to determine \((1+R)\) and \(C\).
Check the solution for sustainability (i.e., ensure that both rich and
poor prefer money to bonds). Then, given this value of \(D_b/N_r p\), check
the sustainability of the mixed equilibrium. Second, choose the value
of \(D_b/N_r p\) which maximizes social welfare in the mixed equilibrium, using
(53) and (54) to determine \(1+R\) and \(C\). Check this solution for
sustainability. Given the optimal value of \(D_b/N_r p\), check the
sustainability of the all-money equilibrium. Finally, if both equilibria are sustainable, choose the one with the greatest social-welfare value.

We now return to the numerical example where \( \delta = .9, \theta_s = \theta_b = .005, \gamma_s = \gamma_b = .275, n = .5, N_p/N_r = .5, \beta = 10, \alpha = 2 \). Looking at the all-money equilibrium and using an iterative algorithm to maximize social welfare, the optimal demand for bonds by the government is \( D_b/N_r.p = 2.976 \). Equilibrium values of the macroeconomic variables are:

\[
R = .00140, \quad C = \frac{p(t)}{p(t+1)} = 1.493, \quad (1+R)C = 1.495, \]

Social welfare \( W = 2.41329 \).

The values of \( R \) and \( C \) have not changed much from their simple-equilibrium values of 0 and 1.5, respectively, but there is a slight improvement in social welfare. The values of the other variables are:

\[
\begin{align*}
&c^r_y = 2.354 \quad c^r_m = 3.184 \quad c^r_o = 4.278 \quad U^r = 3.0758 \\
&c^p_y = .390 \quad c^p_m = .527 \quad c^p_o = .708 \quad U^p = -1.7987 \\
&b^r_p = 2.642 \quad b^p_p = .668 \quad m^r_p = 2.865 \quad m^p_p = .474 \\
&D_c(t) \quad S_c(t) \quad L(t) \quad \frac{N_r(t-1)}{N_r(t-1)p(t)} = 11.000 \quad \frac{L(t)}{N_r(t-1)p(t)} = .641
\end{align*}
\]

The real interest rate, \((1+R)C\), has fallen from 1.5 to 1.495. This leads to additional borrowing and increased consumption by youth. Rich people lose a little and poor people gain a little, compared to the simple equilibrium.

This solution is stable. It is sustainable because the utility level of a rich person using money is only 2.994, and the utility level of a poor person using bonds is only -2.317. Aggregate stability holds because the mixed equilibrium is not sustainable.
5. Equilibrium when the government incurs transactions costs

The analysis of the preceding section may seem unrealistic because individuals incur transactions costs in the bond market but the government does not. In this section the government is assumed to incur transactions costs of $\theta\beta g(t)$ when it enters the bond market at time $t$.

When transactions costs are incurred by the government, the method of analysis of the previous section remains valid. The government budget constraint and the determination of $\frac{p(t)}{p(t+1)}$ are changed as follows.

The government budget constraint now becomes

$$(1+\theta g)\beta g(t) = (1+R(t-1))\beta g(t-1) + M(t) - M(t-1).$$

When stationarity is imposed we get:

$$(50') \quad [(1+\theta g) - \frac{(1+R)C}{1+n}] \frac{\beta g}{N r p} = [1 - \frac{C}{1+n}] \frac{M}{N r p}.$$

In the all-money equilibrium, the solution for $\frac{p(t)}{p(t+1)}$ becomes:

$$(52') \quad C = (1+n) \left[ 1 - \frac{(1+\delta+\delta^2)[(1+\theta g) - (1+R)C/(1+n)]\beta g/N r p}{\frac{1}{1+n} \delta^2 \beta g \frac{N p a}{N r} - \frac{1}{1+n} \delta^2 \frac{(1+R)C}{1-\theta b} \frac{N p}{N r} \gamma b} \right].$$

In the mixed equilibrium, the solution for $\frac{p(t)}{p(t+1)}$ becomes:

$$(54') \quad C = (1+n) \left[ 1 - \frac{(1+\delta+\delta^2)[(1+\theta g) - (1+R)C/(1+n)]\beta g/N r p}{\frac{1}{1+n} \delta^2 \frac{N p a}{N r} - \frac{1}{1+n} \delta^2 \frac{(1+R)C}{1-\theta b} \frac{N p}{N r} \gamma b} \right].$$

Equations (51) and (53) hold and determine the real interest rate

$$i = (1+R)C - 1 = (1+R) \frac{p(t)}{p(t+1)} - 1.$$
A Numerical Example—All-Money Equilibrium

As in the previous examples, we let \( \delta = .9, \theta_s = \theta_b = .005, \gamma_b = \gamma_s = .275, n = .5, N_p/N_r = .5, \beta = 10, \alpha = 2. \) We set the transactions cost for the government at \( \theta_g = 0.1. \)

Examining the all-money equilibrium, the optimal bond-market intervention is \( D_b^g/N_r^p = 2.731. \) The other macroeconomic variables are: social welfare \( W = 2.310, R = .102, C = 1.492, (1+R)C = 1.642. \) This equilibrium is sustainable.

The mixed equilibrium is also sustainable when \( \theta_g = .1. \) The optimal bond-market intervention is \( D_b^g/N_r^p = 0.835. \) In this case social welfare = 2.305, \( R = .130, C = 1.453, (1+R)C = 1.641. \)

When \( D_b^g/N_r^p \) is set at 2.731, the mixed equilibrium is not sustainable. When \( D_b^g/N_r^p \) is set at 0.835, the all-money equilibrium is not sustainable. Thus the all-money equilibrium is stable when \( D_b^g/N_r^p = 2.731, \) and the mixed equilibrium is stable when \( D_b^g/N_r^p = 0.835. \)

Because social welfare is higher in the all-money equilibrium than in the mixed equilibrium, the government's optimal policy is to set \( D_b^g/N_r^p = 2.731. \) Comparing this result to the result in the preceding section (when \( \theta_g = 0 \)), we see that an increase in government transactions cost leads to a decrease in government bond-market intervention. The nominal interest rate rises, inflation increases, and the real interest rate rises.

The values of the other variables are:

\[
\begin{align*}
&c^r_y = 2.132\quad c^r_m = 3.170\quad c^r_o = 4.255\quad U^r = 2.9698 \\
&c^p_y = .345\quad c^p_m = .513\quad c^p_o = .689\quad U^p = -1.965
\end{align*}
\]
Compared to the case where \( \theta_g = 0 \), everyone is worse off, consumption is lower in every period, borrowing is lower, money-holding is lower, and total transactions costs are higher.

A Numerical Example—Mixed Equilibrium

As transactions costs rise, optimal government intervention in the bond market diminishes in magnitude, causing interest rates to rise. As interest rates rise there is a switch from an all-money equilibrium to a mixed equilibrium in which the rich hold bonds.

Suppose that \( \theta_g \) increases to .2. The all-money equilibrium is no longer sustainable. A mixed equilibrium is obtained when the government sets \( D_b^g/N_r^p \) at 9.679, to maximize social welfare \( W \) at 2.269. This equilibrium is sustainable.

The other variables are:

\[
R = 0.336 \quad C = 1.317 \quad (1+R)C = 1.759
\]

\[
\begin{align*}
\frac{b^r}{p} &= 2.420 \quad \frac{b^p}{p} = 0.623 \\
\frac{m^r}{p} &= 2.853 \quad \frac{m^p}{p} = 0.462 \\
\frac{D_c(t)}{N_r(t-1)} &= 9.951 \quad \frac{S_c(t)}{N_r(t-1)} = 11.000 \quad \frac{L(t)}{N_r(t-1)p(t)} = 1.049.
\end{align*}
\]
The Intergenerational Free Lunch

By comparing the results of this section to those of section 2, in which the government does not intervene in the bond market, we see that government intervention provides an intergenerational free lunch. This was not obvious from sections 3 and 4, because the gains in social welfare could be attributed to gains from reducing transactions costs in the economy. However, the numerical examples in this section show that social welfare is increased by government intervention even when that intervention causes aggregate transactions costs to rise, as shown in table 3.

This table shows that government intervention yields increased social welfare due to an intergenerational free lunch rather than due to a change in transactions costs. Government intervention allows people to obtain a more desirable intertemporal allocation of consumption. Government intervention is necessary because no private citizen with a finite life can run the type of Ponzi scheme required here.

6. Optimal taxation

In chapter 3 it is shown that lump-sum taxes are perfect substitutes for bonds and money. When there are transaction costs present, lump-sum taxation may be used to substitute for bonds, eliminating those costs. When the government uses costless lump-sum taxation, it may reach a first-best optimum in terms of social welfare.

In this model both bonds and money may be eliminated by setting lump-sum taxes as:
Table 3

Demonstrating the Intergenerational Free Lunch

<table>
<thead>
<tr>
<th>Government Intervention</th>
<th>Social Welfare</th>
<th>Transactions Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Individual</td>
</tr>
<tr>
<td>$D_b^g/N_r p$</td>
<td>$W$</td>
<td>$\theta_g$</td>
</tr>
<tr>
<td>0</td>
<td>2.089</td>
<td>0</td>
</tr>
<tr>
<td>2.731</td>
<td>2.310</td>
<td>.1</td>
</tr>
<tr>
<td>0.679</td>
<td>2.269</td>
<td>.2</td>
</tr>
</tbody>
</table>
where $\tau$ is in nominal terms, so $\tau/p$ is in real terms. A subsidy is given to young and old, and taxes are imposed on the middle-aged.

Tax revenue received by the government is equal to:

$$N_r(t)\tau_y(t) + N_r(t-1)\tau_m(t) + N_r(t-2)\tau_o(t) + N_p(t)\tau_y(t) + N_p(t-1)\tau_m(t) + N_p(t-2)\tau_o(t) = 0.$$ 

Thus the government budget remains balanced at all times.

Faced with these taxes, individuals maximize utility. The results indicate that the demands for bonds and money are zero, and consumption levels are:

$$c^r_y = \frac{1}{1+\delta^2} \frac{\beta}{1+n}, \quad c^m = \delta(1+n)c^r_y, \quad c^o = \delta^2(1+n)^2 c^r_y,$$

$$c^p_y = \frac{1}{1+\delta^2} \frac{\alpha}{1+n}, \quad c^m = \delta(1+n)c^p_y, \quad c^o = \delta^2(1+n)^2 c^p_y.$$ 

Of course, since holding money does not entail transactions costs, the government could provide a constant nominal money supply (so that its budget remains balanced and $p(t)/p(t+1) = 1+n$) and charge taxes on the old and middle-aged equal to:

$$\frac{\tau_y}{p_y} = -\frac{1}{1+\delta^2} \frac{\beta}{1+n}, \quad \frac{\tau_m}{p_m} = \frac{1+\delta}{1+\delta^2} \frac{\beta}{1+n}, \quad \frac{\tau_o}{p_o} = -\frac{\delta^2}{1+\delta^2} \frac{(1+n)}{1+n},$$

$$\frac{\tau_y}{p_y} = -\frac{1}{1+\delta^2} \frac{\alpha}{1+n}, \quad \frac{\tau_m}{p_m} = \frac{1+\delta}{1+\delta^2} \frac{\alpha}{1+n}, \quad \frac{\tau_o}{p_o} = -\frac{\delta^2}{1+\delta^2} \frac{(1+n)}{1+n},$$

where $0 \leq \beta < 1, 0 \leq \alpha \leq 1$. This would result in the same stationary consumption levels, but money would be used.
In the numerical example in which $A = .9, \theta_s = \theta_b = .005, Y_s = Y_b = .275, n = .5, N_p/N_r = .5, \beta = 10, \gamma = 2$, the consumption levels are:

$c^r_y = 2.460 \quad c^m_r = 3.321 \quad c^r_o = 4.483 \quad U^r = 3.1968$

$c^p_y = .492 \quad c^m_p = .664 \quad c^p_o = .897 \quad U^p = -1.666.$

First-best social welfare is 2.566. The tax levels needed to eliminate bonds and money are:

\[
\begin{align*}
\frac{\tau_y}{p_y} &= -2.460 \\
\frac{\tau_m}{p_m} &= 6.679 \\
\frac{\tau_o}{p_o} &= -4.483 \\
\frac{\tau_y}{p_y} &= -.492 \\
\frac{\tau_m}{p_m} &= 1.336 \\
\frac{\tau_o}{p_o} &= -.897.
\end{align*}
\]

Money may be used, for example, by setting $x_r = x_p = .5$. Then the tax levels and money demands are:

\[
\begin{align*}
\frac{\tau_y}{p_y} &= -2.460 \\
\frac{\tau_m}{p_m} &= 5.185 \\
\frac{\tau_o}{p_o} &= -2.242 \\
\frac{m}{p} &= 1.494 \\
\frac{\tau_y}{p_y} &= -.492 \\
\frac{\tau_m}{p_m} &= 1.037 \\
\frac{\tau_o}{p_o} &= -.448 \\
\frac{m}{p} &= .299.
\end{align*}
\]
C. Summary and Conclusions

This chapter extends the model of chapter 3 to include borrowing by private individuals. It also models the choice between money and bonds using a transactions-cost technology.

The results indicate that government intervention may be optimal in the bond market. This intervention works by providing an intergenerational free lunch. By running a deficit which rises over time, monetizing a constant proportion of that deficit, and making increased loans over time, the government is able to transform the lifetime consumption opportunities of everyone, making them better off. This transformation comes about by changing the interest rate and the inflation rate.

Looking at equations (52') and (54') we see that government bond-market intervention causes \( \frac{p(t)}{p(t+1)} \) to fall below \( 1+n \). Since population grows at rate \( n \), \( N_r(t)p(t) \) is growing. Looking at equation (50'), the government budget constraint, we see that \( D_g(t) \) is growing, \( M(t) \) is growing, and there is a growing nominal budget deficit. The budget deficit, as conventionally defined, is government spending, including net interest payments, minus tax receipts. The nominal deficit is thus equal to \( [\theta_g - RC/(1+n)]D_g(t) \), which grows over time.\(^5\)

The method followed in choosing the optimal size of \( D_g/N_r \) in order to maximize social welfare involves a tradeoff between monetization and lending. This might not be apparent because the maximization process is not straightforward. Due to the complex nature of the equilibrium,
multipliers cannot be derived explicitly. However, because the government is constrained by its budget, the optimization method finds the point where $D^g_{\infty}/N_p$ and $M/N_p$ are optimal. Changes in $D^g_{\infty}/N_p$ and $M/N_p$ involve benefits and costs. Benefits come from the intergenerational free lunch. Costs come about due to changes in the interest rate and inflation rate, plus transactions costs. On the margin, at the optimum, the net benefits from a unit of additional lending are just equal in social welfare terms to the net cost of additional monetization.

We now have a model which shows realistic tradeoffs between bond financing and money financing of government spending. We have shown that the existence of perfect lump-sum taxation may lead to a first-best optimum. However, there are usually costs involved in taxation, and lump-sum taxes are not often used. In chapter 7 we model taxation more realistically, to show tradeoffs between all three methods of financing government spending.

In this model, an increase in interest rates causes people to reduce their borrowing and their consumption while young. In many economies, however, the main concern when interest rates rise is that capital investment may be reduced. An extension of the model of this chapter incorporating capital goods is presented in chapter 6.
FOOTNOTES TO CHAPTER 5

1This feature of the model is very realistic. For example, people in the United States holding wealth in savings accounts were restricted to earning 5 1/4% nominal interest throughout the 1970's, while interest rates on certificates of deposit rose as high as 14%. The higher interest rate is irrelevant to those whose accounts aren't large enough to buy CD's due to transactions costs of a fixed nature, or legal restrictions on account size. Of course, competition in the banking market has yielded implicit interest even to people holding accounts with interest-rate ceilings. This comes about by increased banking services priced below marginal cost, such as expansion of branch banking and automated teller services. These clearly reduce customers' transactions costs.

2By sustainable we mean that the solution for R is consistent with individual maximization. This is clearly not the case when consumption is negative. It is possible, of course, that neither root is sustainable. This would occur, for example, if Y_s and Y_b are so large that either borrowing or lending is impossible.

3Additional equilibria exist in which the rich or poor are just indifferent between holding money and bonds. These equilibria are unstable, however. They require people to be indifferent between holding money and bonds, yet if one indifferent person switches, the equilibrium collapses and becomes one of the three stable equilibria.

4The all-bond equilibrium is unachievable here. If both rich and poor hold bonds, then the government budget can only be balanced by having D^g(t) = 0 for all t. Thus the all-bond equilibrium is never sustainable.

5Tax receipts are zero and net interest payments are negative, since the government is a lender.
Chapter 6
A Capital-Goods Model of Money and
Optimal Government Financial Policy

One of the major concerns of economists about government financial policy is its effect on the capital stock. In this chapter, capital goods are introduced into the model. In addition to borrowing to support consumption, people borrow to purchase capital.

Individuals face two major decisions—how much capital to purchase and whether to hold money or bonds as a store of value. These decisions depend upon the values of interest rates for borrowing while young and lending while middle-aged, changes in prices, as well on the production and utility functions. As in the last chapter, transactions costs are involved in borrowing and lending.

This chapter proceeds in the same manner as chapter 5. In section A, the individual optimization problem is examined. Several possibilities for aggregate equilibria are developed in section B. A numerical example illustrates the optimization problems faced by the government. The results are summarized in section C.
A. The Individual Maximization Problem

Capital goods and consumption goods are assumed to be physically identical, merely put to different uses. Capital goods are completely used up in production in one period. The production function is \( F(k) \), where \( k \) is the amount of capital which a person uses. It is assumed that \( F(0) = a > 0 \), \( F'(0) \geq 0 \), \( F''(k) < 0 \) for all \( k \), and \( \lim_{k \to \infty} F'(k) \leq 0 \).

As in chapter 5, an individual borrows when young and stores value in middle age to finance consumption when old. Borrowing and using bonds as a store of value cause a person to incur transactions costs.

The maximization problem facing the individual is:

\[
\max_{c_y, c_m, c_o, k, s, b, m} U = \ln c_y + \delta \ln c_m + \delta^2 \ln c_o
\]

subject to:

\( (1) \quad (1-\theta_b)b = c_y p_y + \gamma b p_y + k p_y \),

\( (2a) \quad p_m F(k) = (1+R_y)b + c_m p_m + s + m + \gamma s p_m + \theta s \) if \( s > 0 \), or

\( (2b) \quad p_m F(k) = (1+R_y)b + c_m p_m + m \) if \( s = 0 \),

\( (3) \quad m + (1+R_m)s = c_0 p_o \).

These budget equations are identical to those in chapter 5 (equations (2)-(4)) except that now capital must be purchased when young, and output in middle age depends upon capital purchases.

The shape of the utility function requires that we get \( c_y > 0 \), \( c_m > 0 \), and \( c_o > 0 \), if utility is not to be driven infinitely negative. Using budget equations (1) and (3), nonnegativity of consumption requires \( b > 0 \) and either \( s > 0 \) or \( m > 0 \). Because of the nonconvexity of transactions
costs, the choice between holding bonds and money must be made by comparing utility levels under each. Because of the nature of the budget constraint, money and bonds are never used by the same person. If the marginal conditions on money and bonds are the same, money is used because bonds have a positive fixed-cost component.

1. Maximization using bonds

Assuming the individual uses bonds, equation (2a) is used in setting up the Lagrangian. The first-order conditions for the maximization are identical to those in chapter 5 for $c_y, c_m, c_o, b, s, m$. A necessary condition for the use of bonds is $\frac{1+R_m}{1+\theta_s} > 1$. The term $\frac{1+R_m}{1+\theta_s} - 1$ is the net nominal interest rate received by a bondholder. If it is less than zero, then money dominates bonds.

The Kuhn-Tucker condition on $k$ is:

$$\frac{\partial L}{\partial k} = -\lambda_1 p_y + \lambda_2 p_m F'(k) \leq 0 \quad k \geq 0 \quad \frac{\partial L}{\partial k} = 0.$$  

The other maximization conditions yield $\lambda_1 = \frac{1+R_m}{1-\theta_b} \lambda_2$, so (4) requires:

$$F'(k) \leq \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m},$$  

with equality if $k>0$. Equation (5) shows that the marginal product of capital must be less than or equal to the real marginal cost of borrowing.

If $F'(0) \leq \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m}$, then $F'(k) < \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m}$ for all $k>0$, since $F''(k)<0$. Then we must get $k=0$ to satisfy (4).
If $F'(0) > \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m}$ then $k > 0$. The value of $k$ is found by solving:

$$F'(k) = \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m}.$$  

(6)

This shows that when capital is used, the marginal product of capital equals the real marginal cost of borrowing. The optimal value of $k$ is finite since $\lim_{k \to \infty} F'(k) = 0$ and $\frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} > 0$. Notice that $\frac{\partial k}{\partial R_y} < 0$, $\frac{\partial k}{\partial \theta_b} < 0$, and $\frac{\partial k}{\partial (p_y/p_m)} < 0$. Figure 7 illustrates the optimal choice of capital. This diagram shows two production functions, $F_a$ and $F_b$. The marginal product of capital at $k=0$ is greater than the real marginal cost of borrowing, so the optimal capital stock is positive, and occurs at level $k^*$, for production function $F_a$. For production function $F_b$, the marginal product of capital is always less than the real marginal cost of borrowing, so it is not optimal to purchase capital.

A budget constraint is derived in present-value terms by combining equations (1), (2a), (3) and eliminating $b$ and $s$. Define this in terms of the present value at middle age.

$$F(k) = \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} k - \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} y_b - y_s$$  

(7)

$$= \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} c_y + c_m + \frac{1+R_m}{1+R_m} \frac{p_o}{p_m} c_o.$$  

The left-hand side (LHS) of (7) is the present value at middle age of income minus nonconsumption expenditures. The right-hand side (RHS) of (7) is the present value at middle age of consumption expenditures.
These graphs show the optimal choice of capital \( k \) for two different production functions \( F_a \) and \( F_b \). For production function \( F_a \),

\[
F'_a(0) > \frac{1+R_y p_y}{1-\theta_b p_m},
\]

and the optimal \( k \) is \( k^*_a \). For production function \( F_b \),

\[
F'_b(0) < \frac{1+R_y p_y}{1-\theta_b p_m},
\]

and the optimal \( k \) is \( k^*_b = 0 \).

**Figure 7**

Illustration of the Optimal Choice of Capital
The optimal choice of \( k \) maximizes the LHS of (7). As long as 
\[
F'(k) > \frac{1+R_y p_y}{1-\theta_b p_m} y, 
\]
an increase in \( k \) causes \( F(k) \) to rise relative to the term 
\[
\frac{1+R_y p_y}{1-\theta_b p_m} k. 
\]
Figure 8 illustrates the returns to capital.

The optimal choices for consumption, borrowing, and saving are given by:

(8) \[
c_y = \frac{1}{1+\delta+\delta^2} \left( \frac{1-\theta_b p_m}{1+R_y p_y} (F(k)-\gamma_s) - (k+\gamma_s) \right), 
\]

(9) \[
c_m = \delta \frac{1+R_y p_y}{1-\theta_b p_m} c_y, 
\]

(10) \[
c_o = \delta^2 \frac{1+R_y p_y}{1-\theta_b p_m} \frac{1+R_m p_m}{1-\theta_s p_o} c_y, 
\]

(11) \[
\frac{b}{p_y} = \frac{1}{1+\delta+\delta^2} \frac{1}{1+R_y p_y} p_m (F(k)-\gamma_s) + \frac{\delta+\delta^2}{1+\delta+\delta^2} \frac{1}{1-\theta_b} (k+\gamma_b), 
\]

(12) \[
\frac{s}{p_m} = \frac{\delta^2}{1+\delta+\delta^2} \frac{1+R_y p_y}{1-\theta_b p_m} \frac{1}{1+\theta_s} \left[ \frac{1-\theta_b p_m}{1+R_y p_y} (F(k)-\gamma_s) - (k+\gamma_s) \right]. 
\]

We now consider the production function given by:

(13) \[
F(k) = \alpha + \psi k - \frac{\beta}{2} k^2, 
\]

where \( \alpha > 0, \psi \geq 0, \beta > 0. \)

This function has the desired properties: (1) \( F(0)=\alpha > 0; \) (2) \( F'(0)=\psi \geq 0; \)

(3) \( F''(k)=-\beta < 0; \) (4) \( \lim_{k \to \infty} \psi - \beta k \leq 0. \) The marginal product of capital is:

(14) \[
F'(k) = \psi - \beta k. 
\]
Gross returns to capital are $F(k^*)-\alpha$, in units of the consumption good at middle age. The interest cost of capital is $A-\alpha = \frac{1+R_y p_y}{1-\theta_b} p_m k^*$. Net returns to capital are $F(k^*)-A = F(k^*)-\alpha - \frac{1+R_y p_y}{1-\theta_b} p_m k^*$.

Figure 8
Illustration of the Returns to Capital
Since $F'(0) = \psi$, then $k > 0$ if and only if $\psi > \frac{1 + R_y p_y}{1 - \theta_b p_m}$.

Using (14) in (6), we find that the optimal capital purchase is:

$$(15) \quad k = \frac{1}{\beta} \left[ \psi - \frac{1 + R_y p_y}{1 - \theta_b p_m} \right], \text{ for } \psi > \frac{1 + R_y p_y}{1 - \theta_b p_m}.$$  

Notice that $\frac{\partial k}{\partial \psi} < 0$, $\frac{\partial k}{\partial R_y} > 0$, $\frac{\partial k}{\partial \theta} < 0$, $\frac{\partial k}{\partial p} < 0$, $\frac{\partial (p_y / p_m)}{\partial \psi} < 0$. When (15) determines the capital purchase, output is:

$$(16) \quad \hat{F}(k) = \alpha + \frac{1}{2 \beta} \left[ \psi^2 - \left( \frac{1 + R_y p_y}{1 - \theta_b p_m} \right)^2 \right].$$

The net return to capital, as shown in figure 8, is given by

$$F(k) = \alpha - \frac{1 + R_y p_y}{1 - \theta_b p_m} k = \frac{1}{2 \beta} \left( \psi - \frac{1 + R_y p_y}{1 - \theta_b p_m} \right)^2.$$  

When the production function is given by (13), equations (8), (11), and (12) can be rewritten as either (18)-(20) or (21)-(23), depending on whether or not capital is used.

a. When $\psi \leq \frac{1 + R_y p_y}{1 - \theta_b p_m}$, then $k = 0$ and $F(k) = \alpha$.

$$(18) \quad c_y = \frac{1}{1 + \delta + \delta^2} \left[ \frac{1 - \theta_b p_m}{1 + R_y p_y} (\alpha - \gamma_s) - \gamma_b \right].$$

$$(19) \quad \frac{b}{p_y} = \frac{1}{1 + \delta + \delta^2} \left( \frac{1}{1 + R_y p_y} \frac{p_m}{p_y} (\alpha - \gamma_s) + \frac{\delta + \delta^2}{1 + \delta + \delta^2} \frac{1}{1 - \theta_b} \gamma_b \right).$$

$$(20) \quad \frac{s}{p_m} = \frac{\delta^2}{1 + \delta + \delta^2} \left( \frac{1}{1 + R_y p_y} \frac{1}{1 + \theta_s} \left[ \frac{1 - \theta_b p_m}{1 + R_y p_y} (\alpha - \gamma_s) - \gamma_b \right] \right).$$

b. When $\psi > \frac{1 + R_y p_y}{1 - \theta_b p_m}$, then $k > 0$, and (15)-(17) hold.

$$(21) \quad c_y = \frac{1}{1 + \delta + \delta^2} \left[ \frac{1 - \theta_b p_m}{1 + R_y p_y} \left( \alpha + \frac{1}{2 \beta} (\psi - \frac{1 + R_y p_y}{1 - \theta_b p_m})^2 - \gamma_s - \frac{1 + R_y p_y}{1 - \theta_b p_m} \gamma_b \right) \right].$$

$$(22) \quad \frac{b}{p_y} = \frac{1}{1 + \delta + \delta^2} \left[ \frac{1}{1 + R_y p_y} \frac{p_m}{p_y} (\alpha + \frac{1}{2 \beta} \psi^2 - \gamma_s) - \frac{1}{1 - \theta_b} \frac{1}{2 \beta} \frac{1 + R_y p_y}{1 - \theta_b p_m} \right].$$
2. Maximization Using Money

When money is used as a store of value, equation (2b) is used in setting up the Lagrangian. The first-order conditions for the maximization are identical to those in chapter 5 for $c_y$, $c_m$, $c_o$, $b$, $m$. Conditions for optimal $k$ are unaffected by the choice between money and bonds. The optimal capital purchase depends upon $R_y$, $p_y$, and $p_m$ and does not depend upon choice variables.

An individual's optimal choices for consumption, borrowing, and saving are given by:

\begin{align*}
(24) \quad c_y &= \frac{1}{1+\delta+\delta^2} \left[ \frac{1-\theta_b}{1+R_y} \frac{p_m}{p_y} F(k) - (k+\gamma_b) \right]. \\
(25) \quad c_m &= \delta \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} c_y. \\
(26) \quad c_o &= \delta^2 \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \frac{p_m}{p_o} c_y. \\
(27) \quad \frac{b}{p_y} &= \frac{1}{1+\delta+\delta^2} \frac{1}{1+R_y} \frac{p_m}{p_y} F(k) + \frac{\delta+\delta^2}{1-\theta_b} \frac{1}{1+\delta+\delta^2} (k+\gamma_b). \\
(28) \quad \frac{m}{p_m} &= \frac{\delta^2}{1+\delta+\delta^2} \left[ F(k) - \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} (k+\gamma_b) \right].
\end{align*}
When the production function takes the form of (13), the equations for \( c_y \), \( b/p_y \), and \( m/p_m \) depend on the choice of capital:

a. When \( \psi \leq \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \), then \( k=0 \), \( F(k)=\alpha \), and:

\[
(29) \quad c_y = \frac{1}{1+\delta+\delta^2} \left[ \frac{1-\theta_b}{1+R_y} \frac{p_m}{p_y} \alpha - \gamma_b \right].
\]

\[
(30) \quad \frac{b}{p_y} = \frac{1}{1+\delta+\delta^2} \left[ \frac{1}{1+R_y} \frac{p_m}{p_y} \alpha + \frac{\delta+\delta^2}{1+R_y} \frac{1}{1-\theta_b} \gamma_b \right].
\]

\[
(31) \quad \frac{m}{p_m} = \frac{\delta^2}{1+\delta+\delta^2} \left[ \alpha - \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \gamma_b \right].
\]

b. When \( \psi > \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \), then \( k>0 \), and (15)-(17) hold, and:

\[
(32) \quad c_y = \frac{1}{1+\delta+\delta^2} \left[ \frac{1-\theta_b}{1+R_y} \frac{p_m}{p_y} \alpha + \frac{1}{2\beta} \left( \psi - \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \right)^2 \right] - \gamma_b.
\]

\[
(33) \quad \frac{b}{p_y} = \frac{1}{1+\delta+\delta^2} \left[ \frac{1}{1+R_y} \frac{p_m}{p_y} \alpha + \frac{1}{2\beta} \psi^2 \right] - \frac{1}{1-\theta_b} \frac{1}{2\beta} \frac{1+R_y}{p_m} \frac{p_y}{p_m}

+ \frac{\delta+\delta^2}{1+\delta+\delta^2} \left[ \gamma_b + \frac{1}{2\beta} \psi - \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \right].
\]

\[
(34) \quad \frac{m}{p_m} = \frac{\delta^2}{1+\delta+\delta^2} \left[ \alpha + \frac{1}{2\beta} \left( \psi - \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \right)^2 - \frac{1+R_y}{1-\theta_b} \frac{p_y}{p_m} \gamma_b \right].
\]
3. Choices of Capital and Store of Value

We now partition individuals into four classes, based on their choices of capital purchases and whether they use bonds or money as a store of value. These classes are denoted:

I. Capital is purchased and bonds are used \((k>0, s>0, m=0)\),

II. Capital is purchased and money is used \((k>0, s=0, m>0)\),

III. Capital is not purchased and bonds are used \((k=0, s>0, m=0)\),

IV. Capital is not purchased and money is used \((k=0, s=0, m>0)\).

All of the parameters facing an individual \((\delta, \theta_s, \theta_b, \gamma_s, \gamma_b, \frac{p_y}{p_m}, \frac{p_m}{p_o}, R_y, R_m, \alpha, \beta, \psi)\) affect his or her decisions about capital and the choice of a store of value. Table 4 illustrates how changes in \(\psi\) (the marginal productivity of capital at \(k=0\)) and \(\gamma_s\) (the fixed-cost component in the transactions cost of buying bonds) affect these decisions.

In table 4, the second column can be used as a starting point. When \(\gamma_s = .275\) and \(\psi = 15\), capital is chosen in the amount \(.977\), and output is \(9.71\). The person is in class I, purchasing capital and using bonds as a store of value. When \(\psi\) falls to \(8\), capital is not as productive as before. Consequently capital purchases decline to \(.497\), with output falling to \(4.43\). The decline in income causes the person to switch from using bonds to using money as a store of value (class II).

A further reduction in the marginal productivity of capital causes the person to stop using capital entirely, as shown in the last two columns. When the fixed-cost component of the transactions cost on bonds is low \((\gamma_s = .10)\), the person uses bonds as a store of value, moving into class III. A higher level of \(\gamma_s\) causes the person to hold money,
Table 4
Illustration of the Choices of Capital and Store of Value

<table>
<thead>
<tr>
<th>δ = .9</th>
<th>θ_b = θ_s = .005</th>
<th>γ_b = .275</th>
<th>α = 2</th>
<th>β = 12.5</th>
<th>R_y = R_m = .333</th>
<th>( \frac{p_y}{p_m} ) = ( \frac{p_m}{p_o} ) = 1.333</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ_y</td>
<td>.275</td>
<td>.275</td>
<td>.100</td>
<td>.275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ</td>
<td>14</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>.977</td>
<td>.497</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F(k)</td>
<td>9.71</td>
<td>4.43</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_y</td>
<td>1.49</td>
<td>.574</td>
<td>.291</td>
<td>.255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_s</td>
<td>2.39</td>
<td>.923</td>
<td>.468</td>
<td>.410</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_m</td>
<td>3.81</td>
<td>1.469</td>
<td>.745</td>
<td>.652</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_o</td>
<td>2.26</td>
<td>-.317</td>
<td>-2.16</td>
<td>-2.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U^s</td>
<td>1.54</td>
<td>.630</td>
<td>.312</td>
<td>.312</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_y</td>
<td>2.48</td>
<td>1.014</td>
<td>.501</td>
<td>.501</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_m</td>
<td>2.98</td>
<td>1.217</td>
<td>.601</td>
<td>.601</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c_o</td>
<td>2.13</td>
<td>-.290</td>
<td>-2.20</td>
<td>-2.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U^m</td>
<td>2.75</td>
<td>1.35</td>
<td>.569</td>
<td>.532</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_s )</td>
<td>2.14</td>
<td>.826</td>
<td>.419</td>
<td>.367</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_m )</td>
<td>2.81</td>
<td>1.41</td>
<td>.590</td>
<td>.590</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_o )</td>
<td>2.23</td>
<td>.912</td>
<td>.451</td>
<td>.451</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Store of Value</td>
<td>bonds</td>
<td>money</td>
<td>bonds</td>
<td>money</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
while using no capital, thus becoming a member of class IV.

We now consider the conditions necessary for aggregate equilibrium in a model with rich and poor people who differ only in the productivity of their capital.
B. Aggregate Equilibrium

The aggregate equilibrium depends upon the proportions of the population in each class (I. $k>0, s>0$; II. $k>0, s=0$; III. $k=0, s>0$; IV. $k=0, s=0$). The equilibrium interest-rate and inflation-rate sequences both affect and are affected by the number of people in each class. Consequently, the equilibrium values of the endogenous variables are difficult to find, and checking for stability is more complicated than it is in chapter 5.

To keep things fairly simple, we assume again that there are two types of people, rich and poor, who are distinguished by the term $\psi$ in the production function (13). This term is the marginal productivity of capital at $k=0$. The well-endowed rich have $\psi^r$ and the poor have $\psi^p$, where $\psi^r>\psi^p$.

In chapter 5 three types of equilibria are possible--an all-bond equilibrium, a mixed equilibrium, and an all-money equilibrium. For each of these equilibria, three possibilities for capital exist in this model: (1) the all-capital equilibrium with $k^r>0$ and $k^p>0$; (2) the mixed-capital equilibrium with $k^r>0$ and $k^p=0$; and (3) the no-capital equilibrium with $k^r=0$ and $k^p=0$. This makes nine possible equilibria, each of which must be checked in order to ensure stability. Figure 9 and table 5 detail these equilibria.

In figure 9, the nine alternative equilibria are displayed. The diagram shows the class (I, II, III, or IV) which rich and poor are in when each aggregate equilibrium occurs. Equilibrium 8 is unattainable.
### Capital: All-capital Mixed No-capital

<table>
<thead>
<tr>
<th>Store of value</th>
<th>All-bond</th>
<th>Mixed</th>
<th>No-capital</th>
</tr>
</thead>
</table>
|                | 1. $k^r > 0 \ s^r > 0$
|                | II. $k^p > 0 \ s^p > 0$
|                | III. $k^r = 0 \ s^r > 0$
|                | 4. $k^r > 0 \ s^r > 0$
|                | 5. $k^r > 0 \ s^r > 0$
|                | 6. $k^r > 0 \ s^r > 0$
|                | 7. $k^r = 0 \ s^r > 0$
|                | 8. $k^r = 0 \ s^p = 0$
|                | 9. $k^r = 0 \ s^p = 0$

#### Note:
For equilibria 1, 4, and 7, only a private equilibrium is possible unless transactions costs can be reduced to zero (since no one wants to hold money, and the government budget constraint must be satisfied). For equilibria 3, 6, and 9, a private equilibrium is impossible, since when everyone uses money, no borrowing is possible, unless government intervenes. Equilibrium 8 is impossible in this model, because when capital isn't used, the rich and poor face identical budget constraints. Table 5 gives examples showing parameters for which each of these equilibria (except 8) holds.

**Figure 9**

Description of the Nine Aggregate Equilibria
Table 5
Illustration of Eight Aggregate Equilibria

For all examples, $\theta_s=\theta_b=.005$, $\beta=12.5$, $n=.5$, $\frac{N_p}{N_r}=.5$, $\theta_g=0$.

<table>
<thead>
<tr>
<th>Equilibrium:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>10</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\psi_r$</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\psi_p$</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma_b$</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.275</td>
<td>.275</td>
<td>.275</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td>.275</td>
<td>.275</td>
<td>.275</td>
<td>.1</td>
<td>.275</td>
<td>.275</td>
</tr>
<tr>
<td>$\frac{D^g_{bb}}{N_{r,p}}$</td>
<td>0</td>
<td>2.456</td>
<td>6.515</td>
<td>0</td>
<td>2.009</td>
<td>3.539</td>
<td>0</td>
<td>.8</td>
<td></td>
</tr>
<tr>
<td>$U^r$</td>
<td>4.1</td>
<td>3.5</td>
<td>4.5</td>
<td>3.9</td>
<td>2.4</td>
<td>2.5</td>
<td>-4.9</td>
<td>-2.0</td>
<td></td>
</tr>
<tr>
<td>$k^r$</td>
<td>.91</td>
<td>1.00</td>
<td>1.00</td>
<td>.89</td>
<td>1.00</td>
<td>1.00</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$F^r$</td>
<td>17.6</td>
<td>12.8</td>
<td>17.7</td>
<td>17.5</td>
<td>9.8</td>
<td>9.7</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$\frac{s^r}{p}$</td>
<td>4.4</td>
<td>3.3</td>
<td>0</td>
<td>4.1</td>
<td>2.3</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\frac{m^r}{p}$</td>
<td>0</td>
<td>0</td>
<td>4.6</td>
<td>0</td>
<td>0</td>
<td>2.3</td>
<td>0</td>
<td>.43</td>
<td></td>
</tr>
<tr>
<td>$U^p$</td>
<td>3.0</td>
<td>1.3</td>
<td>3.3</td>
<td>2.8</td>
<td>-1.9</td>
<td>-1.8</td>
<td>-4.9</td>
<td>-2.0</td>
<td></td>
</tr>
<tr>
<td>$k^p$</td>
<td>.19</td>
<td>.20</td>
<td>.28</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$F^p$</td>
<td>10.7</td>
<td>5.6</td>
<td>10.9</td>
<td>10</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$\frac{s^p}{p}$</td>
<td>2.9</td>
<td>0</td>
<td>0</td>
<td>2.7</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\frac{m^p}{p}$</td>
<td>0</td>
<td>1.5</td>
<td>3.1</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>.43</td>
<td></td>
</tr>
</tbody>
</table>
in this model, because when neither rich nor poor hold capital they have the same income, so they make the same choice of holding money or bonds.

Table 5 shows conditions under which each of the eight attainable equilibria holds. The table gives values to the parameters \( \alpha, \psi_r, \psi_p, \gamma_b, \gamma_s, \) and \( D^g_b/N_r p \). Changes in these parameters lead to a change in the type of equilibrium. Equilibrium 1, in which everyone uses capital and holds bonds, arises when \( \alpha=10, \psi_r=14, \psi_p=5, \gamma_b=\gamma_s=1, \) and \( D^g_b/N_r p=0 \). The equilibrium changes to an all-capital, mixed-bond equilibrium (2) when \( \alpha \) falls to 5, \( \psi_p \) decreases to 4, and \( D^g_b/N_r p \) increases to 2.456, as the poor prefer holding money to holding bonds. Restoring the values of \( \alpha \) and \( \psi_p \) to their original levels of 10 and 5, respectively, and increasing government lending to \( D^g_b/N_r p=6.515 \), the equilibrium changes to the all-capital, no-bond equilibrium (3). Government lending drives the interest rate so low that not even the rich wish to hold bonds.

When the marginal productivity of capital for the poor is decreased to \( \psi_p=1 \), the fixed components of the transactions costs on bonds are raised to \( \gamma_b=\gamma_s=.275 \), and government lending is reduced to zero, the mixed-capital, all-bond equilibrium (4) is attained. Reducing the zero-capital level of output to \( \alpha=2 \), and increasing government lending to \( D^g_b/N_r p=2.009 \), the poor switch to money (equilibrium 5). Increasing government lending still further to \( D^g_b/N_r p=3.539 \) yields the mixed-capital, all-money equilibrium (6).

When the marginal productivity of capital for the rich is decreased to \( \psi_r=2 \), a no-capital equilibrium is achieved. Rich and poor now face exactly the same parameters and make the same decisions about money versus bonds. With no government lending, both classes choose to use
bonds, and equilibrium 7 is attained. When government lending
sufficiently lowers the interest rate, and bond transactions costs rise,
people switch to using money, and equilibrium 9 occurs.

These examples demonstrate the wide range of possible equilibria in
this model. Stability of an aggregate solution requires that only one
equilibrium holds for a given set of parameters.

The aggregate variables are as follows:

a. Population of each generation:

\[
\begin{align*}
N_r(t) &= (1+n)N_{r(t-1)} \quad N_p(t) = (1+n)N_{p(t-1)}. \\
\end{align*}
\]

b. Aggregate demand for consumption and capital goods:

\[
\begin{align*}
D_c(t) &= \left[ N_r(t)c^c(t) + N_{r(t-1)}c^c_{m}(t) + N_r(t-2)c^c_o(t) \right. \\
&\quad + N_p(t)c^c(t) + N_{p(t-1)}c^c_{m}(t) + N_p(t-2)c^c_o(t) \\
&\quad + N_r(t)k^c(t) + N_p(t)k^c(t) \left. \right] p(t). \\
\end{align*}
\]

c. Aggregate supply of consumption and capital goods:

\[
\begin{align*}
S_c(t) &= \left[ N_r(t-1)F^c(k^c(t)) + N_{p(t-1)}F^c_k(k^c(t)) \right] p(t). \\
\end{align*}
\]

d. Aggregate use of goods in transactions:

\[
\begin{align*}
L(t) &= N_r(t) \left[ \gamma^c(t) + \theta^c(t) \right] + N_p(t) \left[ \gamma^c_p(t) + \theta^c_p(t) \right] \\
&\quad + z^c(t)N_r(t-1) \left[ \gamma^c_s(t) + \theta^c_s(t) \right] \\
&\quad + z^c_p(t)N_p(t-1) \left[ \gamma^c_{s_p(t)} + \theta^c_{s_p(t)} \right] + \theta^c_g \left[ D_g(t) + S_g(t) \right], \\
\end{align*}
\]

where \( z^c(t) = \begin{cases} 1 & \text{if } s^c(t) > 0 \\ 0 & \text{if } s^c(t) = 0 \end{cases} \)

\( z^c_p(t) = \begin{cases} 1 & \text{if } s^c_p(t) > 0 \\ 0 & \text{if } s^c_p(t) = 0 \end{cases} \).

e. Aggregate demand for bonds:

\[
\begin{align*}
D_b(t) &= N_r(t-1)s^b(t) + N_p(t-1)s^b_p(t) + D^b_g(t). \\
\end{align*}
\]

f. Aggregate supply of bonds:

\[
\begin{align*}
S_b(t) &= N_r(t)b^b(t) + N_p(t)b^b_p(t) + S^b_g(t). \\
\end{align*}
\]
g. Aggregate expenditures on (payments of) interest and principal on bonds:

\[ D_{rb}(t) = [1 + R(t-1)] S_b(t-1) . \]

h. Aggregate receipts of interest and principal on bonds:

\[ S_{rb}(t) = [1 + R(t-1)] D_b(t-1) . \]

i. Aggregate demand for money:

\[ D_m(t) = N_{r}(t-1)m^P(t) + N_{p}(t-1)m^P(t) . \]

j. Aggregate supply of money:

\[ S_m(t) = D_m(t-1) + [M(t) - M(t-1)] . \]

Walras's Law requires:

\[ [D_c(t) + L(t) - S_c(t)] + [D_b(t) - S_b(t)] + [D_{rb}(t) - S_{rb}(t)] \\
+ [D_m(t) - S_m(t)] = 0 . \]

Equations (39)-(42) imply that when \( D_b(t) = S_b(t) \) for all \( t \), then \( D_{rb}(t) = S_{rb}(t) \) for all \( t \). Consequently when the bond market clears \( (D_b(t) = S_b(t)) \) and the money market clears \( (D_m(t) = S_m(t)) \), the goods market also clears due to Walras's Law (equation (45)).

The real interest rate, \( i = (1+R)^C-1 \), which clears the bond market can be found by setting \( D_b(t) = S_b(t) \), substituting equations to eliminate all choice variables, and solving algebraically. The result is the following cubic expression:
\[ (46) \left\{ \frac{1}{1+n} \delta^2 + \frac{1}{1+\theta_s} \right\} \frac{1}{2\beta} \left[ z_{b}^{r} z_{k}^{r} + z_{b}^{p} z_{k}^{p} \frac{N_{r}}{N_{r}} \right] (1+i) \]

\[ + \left\{ \frac{1}{\beta} \right\} (\left[ z_{b}^{r} + z_{b}^{p} \frac{N_{r}}{N_{r}} \right] ) \]

\[ - \left[ \frac{1}{1+n} \delta^2 - \frac{1}{1+\theta_s} \right\} \frac{1}{2\beta} \left[ z_{b}^{r} z_{b}^{r} \psi_{r} + z_{k}^{p} z_{b}^{p} \frac{N_{r}}{N_{r}} \psi_{p} \right] \]

\[ + \left( z_{b}^{r} + z_{b}^{p} \frac{N_{r}}{N_{r}} \right) \gamma_{b} \right\} (1+i) \]

\[ + \left\{ \frac{1}{1+n} \delta^2 - \frac{1}{1+\theta_s} \right\} \left[ (z_{b}^{r} + z_{b}^{p} \frac{N_{r}}{N_{r}}) (\alpha - \gamma_{s}) \right] + \frac{1}{2\beta} \left( z_{k}^{r} z_{b}^{r} \psi_{r} + z_{k}^{p} z_{b}^{p} \frac{N_{r}}{N_{r}} \psi_{p} \right) \]

\[ + \left\{ (1+\delta^2) \beta \right\} \frac{D^{g}_{b}}{N_{r}^{p}} \left[ 1 - z_{b}^{r} z_{b}^{p} \right] \]

\[ - \left[ (\delta^2) \frac{1}{1+\theta_{s}} \right\} \left[ (1 + \frac{N_{r}}{N_{r}}) \gamma_{b} + \frac{1}{\beta} \left( z_{b}^{r} \psi_{r} + z_{b}^{p} \frac{N_{r}}{N_{r}} \psi_{p} \right) \right] (1+i) \]

\[ + \left\{ \gamma_{s} \left[ z_{b}^{r} + z_{b}^{p} \frac{N_{r}}{N_{r}} \right] - (1 + \frac{N_{r}}{N_{r}}) \alpha - \frac{1}{2\beta} \left[ z_{k}^{r} \psi_{r} + z_{k}^{p} \frac{N_{r}}{N_{r}} \psi_{p} \right] \right\} = 0, \]

where \( z_{k}^{r}(t) = \begin{cases} 1 & \text{if } k_{r}(t) > 0 \\ 0 & \text{if } k_{r}(t) = 0 \end{cases} \)

The ratio of consecutive price levels, \( C = \frac{p(t)}{p(t+1)} \), which clears the money market for a given real interest rate \( i \) is given by:

\[ (47) \quad C = (1+n) \left\{ 1 - \frac{\left( (1+\theta_{s}) - \frac{1}{1+n} \right) \frac{D^{g}_{b}}{N_{r}^{p}} \right\}_ {\text{DENOM}} \]

where the denominator is given by:

\[ (48) \quad \text{DENOM} = \frac{1}{1+n} \frac{\delta^2}{1+\delta^2} \left( [\alpha - \frac{1}{1-\theta_{b}} \gamma_{b}(1+i)](1-z_{k}^{r})+(1-z_{b}^{p}) \frac{N_{r}}{N_{r}} \right) \]

\[ + \frac{1}{2\beta} \left[ z_{k}^{r}(1-z_{b}^{r})(\psi_{r} - \frac{1}{1-\theta_{b}}(1+i)^2) + z_{k}^{p}(1-z_{b}^{p}) \frac{N_{r}}{N_{r}} (\psi_{p} - \frac{1}{1-\theta_{b}}(1+i)^2) \right] \]

The price level is undefined unless at least one group uses money as a store of value.
1. Equilibrium When Government Provides a Constant Money Supply and Does Not Intervene in the Bond Market

If the government fails to intervene in the bond market, and maintains a constant money supply, then the price level changes according to:

\[
\frac{p(t)}{p(t+1)} = 1+n.
\]

Equation (46) with \( \frac{D_g}{N_p} \) set equal to 0 determines the interest rate. Again, due to the complexity of the expression for \( i \), we examine a numerical example.

We let the parameters take the following values: \( \delta = .9, \theta_s = \theta_b = .005, \gamma_s = \gamma_b = .5, n = .5, \frac{N_p}{N_r} = .5, \alpha = 3, \beta = 50, \psi_r = 50, \psi_p = 10. \) Under these parameters, the only stable equilibrium is equilibrium 2, the all-capital, mixed-bond equilibrium. The endogenous variables take the values shown in table 6, column a.

In this equilibrium, the real interest rate is over 200%. The demand for borrowing is very high, since people wish to borrow both for consumption and for capital formation. A high real interest rate is necessary to equate the demand and supply for bonds.

The rich find it profitable to utilize much more capital than the poor. A considerable difference in output is apparent, with the rich producing 27.9 units and the poor producing only 3.9 units.

We now show that government intervention may be optimal by changing the interest rate and inflation rate to maximize social welfare.
Table 6
Equilibrium Values of Variables

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{b^0}{N_r p} )</td>
<td>0</td>
<td>8.58</td>
<td>7.55</td>
<td>.216</td>
</tr>
<tr>
<td>e</td>
<td>0</td>
<td>0</td>
<td>.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Eqbm.</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>l+i</td>
<td>3.17</td>
<td>1.49</td>
<td>1.73</td>
<td>2.94</td>
</tr>
<tr>
<td>p(t)/p(t+1)</td>
<td>1.50</td>
<td>1.48</td>
<td>1.61</td>
<td>2.72</td>
</tr>
<tr>
<td>kr</td>
<td>.936</td>
<td>.970</td>
<td>.965</td>
<td>.941</td>
</tr>
<tr>
<td>Fr</td>
<td>27.9</td>
<td>28.0</td>
<td>28.0</td>
<td>27.9</td>
</tr>
<tr>
<td>cr</td>
<td>2.64</td>
<td>6.36</td>
<td>5.41</td>
<td>2.90</td>
</tr>
<tr>
<td>cr</td>
<td>7.58</td>
<td>8.56</td>
<td>8.44</td>
<td>7.69</td>
</tr>
<tr>
<td>cm</td>
<td>21.5</td>
<td>11.4</td>
<td>12.2</td>
<td>20.2</td>
</tr>
<tr>
<td>cr</td>
<td>4.10</td>
<td>7.87</td>
<td>6.91</td>
<td>4.36</td>
</tr>
<tr>
<td>sr</td>
<td>6.79</td>
<td>0</td>
<td>0</td>
<td>6.89</td>
</tr>
<tr>
<td>mr/p</td>
<td>0</td>
<td>7.71</td>
<td>7.60</td>
<td>0</td>
</tr>
<tr>
<td>ur</td>
<td>5.28</td>
<td>5.75</td>
<td>5.64</td>
<td>5.34</td>
</tr>
<tr>
<td>kp</td>
<td>.136</td>
<td>.170</td>
<td>.165</td>
<td>.141</td>
</tr>
<tr>
<td>rp</td>
<td>3.90</td>
<td>3.98</td>
<td>3.97</td>
<td>3.91</td>
</tr>
<tr>
<td>cp</td>
<td>.216</td>
<td>.734</td>
<td>.599</td>
<td>.253</td>
</tr>
<tr>
<td>cm</td>
<td>.621</td>
<td>.988</td>
<td>.935</td>
<td>.671</td>
</tr>
<tr>
<td>cp</td>
<td>.838</td>
<td>1.32</td>
<td>1.35</td>
<td>1.64</td>
</tr>
<tr>
<td>b^p/p</td>
<td>.857</td>
<td>1.41</td>
<td>1.27</td>
<td>.898</td>
</tr>
<tr>
<td>sp/p</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>mr/p</td>
<td>.558</td>
<td>.889</td>
<td>.842</td>
<td>.604</td>
</tr>
<tr>
<td>up</td>
<td>-2.10</td>
<td>-.096</td>
<td>-.328</td>
<td>-1.33</td>
</tr>
<tr>
<td>( D_c(t)/N_r (t-1) )</td>
<td>28.15</td>
<td>28.78</td>
<td>27.64</td>
<td>28.11</td>
</tr>
<tr>
<td>( S_o(t)/N_r (t-1) )</td>
<td>29.85</td>
<td>29.97</td>
<td>29.95</td>
<td>29.87</td>
</tr>
<tr>
<td>L(t)/N_r (t-1)</td>
<td>1.69</td>
<td>1.19</td>
<td>2.31</td>
<td>1.76</td>
</tr>
<tr>
<td>W</td>
<td>4.22</td>
<td>4.79</td>
<td>4.67</td>
<td>4.34</td>
</tr>
</tbody>
</table>
2. Equilibrium When Government Intervenes Optimally in the Bond and Money Markets to Maximize Social Welfare

When the government tries to maximize social welfare,\(^1\) it must satisfy its budget constraint and achieve a stationary equilibrium which is stable. This can be done by finding the optimal money supply and government bond level for each of the eight possible equilibria. The equilibrium with the highest social-welfare value is then chosen. Finally, the stability of the equilibrium must be verified.

For the numerical example, the optimal government intervention is for the government to lend enough that it eliminates the use of bonds as a store of value. Equilibrium 3, the all-capital, all-money equilibrium is sustainable and maximizes social welfare when \(\frac{D_b^g}{N_rP} = 8.58\). The results are shown in Table 6, column b.

A Pareto-improvement occurs relative to the situation in which the government does not intervene. Both the rich and the poor are made better off. The real interest rate falls by over 50% of its previous value, with little change in the inflation rate. Output is increased because people purchase more capital than before. Consumption increases because of higher output and because of a reduction in resources used up in transactions.

The results are due primarily to an intergenerational free lunch. Government intervention provides funds to the bond market, lowering the interest rate significantly. Everyone is made better off due to the perpetual nature of the government loan, which need never be repaid.
3. Equilibrium Under Optimal Government Intervention with Transactions Costs

When the government incurs transactions costs, the scope of its optimal intervention in the bond and money markets is reduced. Because the $\theta_g$ term in equation (47) becomes positive, resources must be obtained by the government to pay these transactions costs.

In the numerical example, with $\theta_g = .1$, equilibrium 3 is still sustainable, but now $\frac{D^G}{N^P}$ is reduced to 7.55. The results are shown in table 6, column c. Compared with the case of no transactions costs (column b), this equilibrium has a higher interest rate, hence less capital and less output. Everyone is worse off.

Notice that in comparison with the private equilibrium (column a), everyone is better off. However, total consumption is lower. Output is higher because the reduced real interest rate has led to increased capital use. But transactions costs eat up a lot of output. As in chapter 5, it is the intergenerational free lunch which makes people better off. The reduction in consumption is overcome by a better intertemporal tradeoff of goods, and a Pareto improvement occurs.

As the transactions costs faced by the government rise, intervention becomes less valuable. As these costs rise, optimal government policy moves closer to laissez-faire, because as the cost of intervention rises, the benefits from the intergenerational free lunch decline.

This is illustrated in column d of table 6, where $\theta_g = .2$ (i.e., 20% of the funds government lends must go to transactions costs). The
equilibrium becomes a mixed-bond equilibrium. Optimal government intervention in reduced drastically to \( \frac{D^g}{N_{r, P}} = 0.216 \). The real interest rate is much higher than in the previous government-intervention example. The deflation rate is also very high.

4. Maximization of output

Governments sometimes follow a policy of maximizing their nation's gross national product (GNP) or some other measure of economic output. An increase in output is obtainable in this model, but only at the cost of decreased social welfare.

For example, when there are no transactions costs on government intervention, as in part 1 above, output can be increased from 29.966, as in column b of table 6, to 29.972. This is accomplished by an increase in government lending from 8.58 to 9.25, causing the real interest rate to fall from .49 to .37. Capital usage is increased by both rich and poor.

The amount of goods available for consumption increases more than the transactions costs of generating additional output. Despite the increase in consumption, social welfare falls. The changes in the real interest rate and the inflation rate change people's intertemporal consumption possibilities, making them worse off.

The implication of this result is clear—the government should not drive the interest rate as low as possible in order to increase investment and national product. It must trade off the gains from
additional output with the loss in terms of intertemporal tradeoffs available in consumption.
C. Summary and Conclusions

This chapter extends the model of chapter 5 to include the use of capital goods. This increases the number of potential equilibria, making the determination of stability more complex.

The results indicate that government intervention in markets affects social welfare in three ways: (1) By changing the resources spent on transactions costs; (2) By providing the intergenerational free lunch, which transforms people's intertemporal consumption possibilities, making them better off; and (3) By affecting the utilization of capital in the economy, thus affecting output.

All optima found in this chapter are second-best. As discussed in previous chapters, the existence of perfect lump-sum taxation may lead to a first-best optimum. To model more realistically the complete choice of government between debt, money, and taxes, chapter 7 introduces taxation into the model.
Social welfare is defined using the same concept of social-welfare weights used in the previous chapters. In the numerical example these weights are $5/6$ for the rich and $1/6$ for the poor, as they are in chapter 5.
Chapter 7

Optimal Taxation, Monetization, and Debt
in Financing Optimal Government Spending

In previous chapters we have examined the question of how the government may choose between monetization and debt to maximize social welfare. The solutions are second-best, due to the existence of transactions costs. If costless lump-sum taxation is available to the government, then first-best solutions become possible, as taxes may be used to eliminate all transactions costs in all markets.

Lump-sum taxes of the type needed to implement a first-best solution in this model have no counterpart in the real world. This is probably due to the tremendous administrative and information costs which lump-sum taxation would entail. To model taxation more realistically, we assume that a proportional tax on nominal income is established.1

Under a proportional income tax system, distortions exist. In the models of chapters 5 and 6, the intertemporal-consumption choice is distorted for any person who uses bonds as a store of value. This occurs because the nominal interest on the bonds is taxed. Since labor supply is assumed to be fixed in these models, no labor-leisure distortion develops.
In addition to taxes, this chapter introduces government expenditures on goods into the model. For simplicity, these goods are assumed to be pure public goods. The optimal choice of government spending is modeled. The optimal size of a government's spending on pure public goods clearly depends upon the costs of financing its purchases.
A. Intergenerational Tradeoffs with Pure Public Goods
and a Growing Population

In previous chapters, a stationary solution for the government's optimization problem is found. The equilibria in these models are stationary because a person of a given type (rich, poor) has the same consumption stream and utility level as someone of the same type living at any other time.

With a growing population, more people are available to pay for and receive the benefits of a pure public good as time passes. Consequently, the optimal amount of the pure public good is likely to rise over time. Utility levels are no longer stationary over time. The government's social welfare objective function must deal with this fact. This function must be capable of trading off the utility of people in different generations.

Consider the following social-welfare function:

\[
W = \sum_{t=1}^{\infty} \sum_{h=1}^{N(t)} \phi^h(t) \delta^t_u U^h(t),
\]

where \( t \) is a time index, \( h \) is an index for individuals which at time \( t \) runs from 1 to the population \( N(t) \), \( \phi^h(t) \) is the social-welfare weight assigned to person \( h \) at time \( t \), \( \delta^g \) is the social rate of time discount (it is assumed that \( \delta^g > 0 \) but it is not assumed that \( \delta^g \leq 1 \)), and \( U^h(t) \) is the utility of person \( h \) who is born at time \( t \). Intergenerational tradeoffs depend upon the parameter \( \delta^g \). This term is treated as a discount rate over time by raising it to the power \( t \) in equation (1). The social rate of time discount (\( \delta^g \)) shows what the tradeoff is between
individuals in different generations, while the social-welfare weight \( \phi^h(t) \) trades off welfare between individuals of the same generation.²

In chapter 5 the derivation of the social-welfare weights (\( \phi \)) is demonstrated by investigating the simple-equilibrium solution of chapter 3 (i.e., the solution which yields the optimal time paths of consumption to the rich and poor without redistributing income). For lack of any other guidance as to what the social rate of time discount should be, we use the rate which is implicit in the solution of the model in chapter 3, the properties of which are now examined.

For this model, the social-welfare function is:

\[
W = \phi \sum_{t=1}^{\infty} \delta^t g N_r(t) U^r(t) + (1-\phi) \sum_{t=1}^{\infty} \delta^t g N_p(t) U^p(t),
\]

where \( \phi \) is the social-welfare weight assigned to the rich, and \( 1-\phi \) is that of the poor. The social rate of time discount is the same for both rich and poor.

We consider first a dictatorship solution which maximizes social welfare (2) by choosing consumption levels for the rich and poor at all times \((c^r_y(t), c^r_o(t), c^p_y(t), c^p_o(t))\). Imposing stationarity and the segmented-society solution, we find that:

\[
\beta = \left[ 1 + \frac{1}{1+n} \frac{\delta}{g} \right] c^r_y,
\]
\[
\alpha = \left[ 1 + \frac{1}{1+n} \frac{\delta}{g} \right] c^p_y,
\]

where \( \beta \) and \( \alpha \) are the output levels of the rich and poor.

Under individual choice, people would choose to have:

\[
c^r_y = \frac{1}{1+\delta} \beta,
\]
\[
c^p_y = \frac{1}{1+\delta} \alpha,
\]

according to equations (3) and (6) in chapter 3.
To achieve the dictatorship solution in a model of individual choice, equations (3) and (5) must be made to be equivalent, as must (4) and (6). This may be accomplished by setting:

\[
\delta_g = \frac{1}{1+n}.
\]

This value of \( \delta_g \) is implicit in the simple-equilibrium solution in chapter 3. When the population is growing \((n>0)\), then \( \delta_g < 1 \). In this case the utility received by a person in the future has a social value less than that of a person who is currently alive. When the population is declining \((n<0)\), then \( \delta_g > 1 \), and the utility of an individual in a future generation is valued more highly.

In the social-welfare function given by (2), the discount rate given by (7) leads to the following simplified social-welfare function:

\[
W_r^* = \phi \sum_{t=1}^{\infty} U_r(t) + (1-\phi) \frac{N_p}{N_r} \sum_{t=1}^{\infty} U_p(t),
\]

where \( \frac{N_p}{N_r} = \frac{P(t)}{N_r(t)} \) for any \( t \). This is derived by dividing through equation (2) by \( \delta_g N_r(1) \), and noting that \( \delta_g^t N_r(t) \) is a constant, as is \( \delta_g^t N_r(t) \). In effect, the social rate of time discount \( \delta_g = \frac{1}{1+n} \) gives each generation equal weighting as a group, since population grows at rate \( n \).
B. The Optimal Growth Rate of Pure Public Goods

The optimization problem of the government becomes more complex with pure public goods and a growing population. Growth in population means more people exist to pay for public goods as time passes. Consequently, the demand for public goods may rise over time as public goods become relatively cheaper. Alternatively, taxes could be reduced over time so that people would have increased demand for consumption goods.

The choice of the growth rate for public goods can be investigated using the social-welfare weights and the social rate of time discount from section A. The following two-period overlapping-generations model includes public goods which are paid for entirely by taxes. These public goods are assumed not to be congested. Congestion of public goods does not change the method of analysis followed below, but adds additional parameters to the model. For simplicity, we assume congestion does not occur. For simplicity, we assume that the only store of value is storage of goods, and that everyone is identical and produces output only while young.

An individual's utility function takes the form:

\[ U(t) = \ln c_y(t) + \gamma \ln g(t) + \delta \ln c_o(t+1) + \gamma \delta \ln g(t+1), \]

where \( g(t) \) is the real government supply of pure public goods at time \( t \), and \( \gamma \) is the individual's public-good discount rate. \( \delta \) is the time discount rate for both public and private goods. \( \gamma \) represents the difference in utility between a unit of the public good and a unit of the private good.
With storage of goods as the only store of value, an individual chooses to have:

(10) \( c_y(t) = \frac{1}{1+\delta} [1-\tau(t)] \alpha, \)

(11) \( c_0(t) = \frac{\delta}{1+\delta} [1-\tau(t-1)] \alpha, \)

where \( \tau(t) \) is the tax rate on output at time \( t \), and \( \alpha \) is the individual's output which is produced while young.

The budget constraint of the government requires that taxes equal expenditures, or:

(12) \( g(t) = N(t) \tau(t) \alpha, \)

where \( N(t) \) is the size of the generation born at \( t \).

Following equation (8), the social-welfare function for this model can be written as:

(13) \( W = \sum_{t=1}^{\infty} U(t), \)

since we are considering identical individuals in this model.

The government seeks to maximize \( W \) subject to (9), (10), (11), and (12), by choosing sequences \( \{g(t)\} \) and \( \{\tau(t)\} \). The results of this constrained maximization are:

(14) \( \tau(t) = \frac{\gamma}{1+\gamma}, \)

(15) \( g(t) = N(t) \frac{\gamma}{1+\gamma} \alpha. \)

Thus the tax rate remains constant over time and government expenditures (in real terms) grow at the population growth rate \( n \). This result helps to simplify the model by making it stationary in tax rates and with simple growth in government spending.\(^3\)

Under these conditions, consumption is:

(16) \( c_y(t) = \frac{1}{1+\delta} \frac{1}{1+\gamma} \alpha, \)
We now consider the dictatorship optimum problem of maximizing (13) subject to (9) and (12), by choosing \( g(t) \), \( c_y(t) \), and \( c_o(t) \), for all \( t \). The results of this maximization are the same for \( g(t) \), \( \tau(t) \), and \( c_y(t) \), which follow (14), (15), and (16). But now \( c_o(t) \) is increased to:

\[
(18) \quad c_o(t) = \frac{\delta}{1+\delta} \frac{1}{1+\gamma} (1+n) \alpha.
\]

This result demonstrates the existence of the intergenerational free lunch. As suggested in chapter 3, this result can be obtained by using a financial instrument (such as bonds or money), a social-security system, or lump-sum taxation. Notice, however, that the income tax by itself is incapable of generating the intergenerational free lunch. The government needs an additional tool to reach a first-best optimum.
C. Taxes in the Transactions-Cost Model of Money

Now we return to the transactions-cost model of money from chapter 5. The model is modified to include government spending on pure public goods and proportional taxation on nominal income.

1. The Individual Maximization Problem

The individual's utility function is now modified to include utility received from pure public goods. It is assumed that the same intertemporal discount rate ($\delta$) holds for both private and public goods. Public goods and private goods are distinguishable by a discount rate ($\gamma$) on public goods. The utility function is:

$$U = \ln c_y + \gamma \ln g_y + \delta \ln c_m + \delta \gamma \ln g_m + \delta^2 \ln c_o + \delta^2 \gamma \ln g_o.$$  \hspace{1cm} (19)

The budget equations change slightly from those in chapter 5. In middle age, the budget equation is:

$$p_m X (1 - \tau_m) = (1 + R) b + c_p m + m + (1 + \theta_s) s + \gamma_s p_m$$ if $s > 0$, and

$$p_m X (1 - \tau_m) = (1 + R) b + c_p m + m$$ if $s = 0$.  \hspace{1cm} (20)\hspace{1cm} (21)

In these equations, as in chapter 5, $X$ is output in middle age, $R$ is the nominal interest rate, $\theta_b$ is the marginal transactions cost of borrowing, $\gamma_b$ is the fixed transactions cost incurred when borrowing, $b$ is nominal borrowing when young, $m$ is money holding, $\theta_s$ is the marginal transactions cost of buying bonds, $s$ is the nominal amount of bonds purchased, and $\gamma_s$ is the fixed transactions cost incurred when bonds are purchased.

In old age, the nominal interest earned on bonds is taxed, so the budget equation is:
\[(22) \quad m + (1+R_m(1-\tau_o))s = c_o p_o.\]

In these equations, \(\tau_m\) and \(\tau_o\) are the tax rates faced by the individual in middle age and old age.

When bonds are chosen as a store of value in middle age, the overall budget constraint may be written as:

\[(23) \quad p_m x(1-\tau_m) - \frac{1+R_y y_p}{1-\theta_b b_p} y_p - \gamma_s p_m = \frac{1+R_y y_p}{1-\theta_b b_p} y_p + c_m p_m + \frac{1+R_m(1-\tau_o)}{1+R_m(1-\tau_o)} c_o p_o.\]

The individual's optimal choices for consumption, borrowing, and saving are:

\[(24) \quad c_y = \frac{1}{1+\delta+\delta^2} \left[ (X(1-\tau_m)-\gamma_s) \frac{1-\theta_b p_m}{1+R_y y_p} - y_b \right].\]

\[(25) \quad c_m = \delta \frac{1+R_y y_p}{1-\theta_b b_p} c_y.\]

\[(26) \quad c_o = \delta \frac{1+R_m(1-\tau_o) p_m}{1+\theta_s} p_o c_m.\]

\[(27) \quad \frac{b}{p_y} = \frac{1}{1+\delta+\delta^2} \left[ X(1-\tau_m)-\gamma_s \right] \frac{1}{1+R_y y_p} p_m + \frac{\delta+\delta^2}{1-\theta_b} y_b.\]

\[(28) \quad \frac{s}{p_m} = \frac{\delta^2}{1+\delta+\delta^2} \frac{1+R_y y_p}{1-\theta_b} \frac{1}{p_o} \left[ (X(1-\tau_m)-\gamma_s) \frac{1-\theta_b}{1+R_y y_p} - y_b \frac{p_y}{p_m} \right].\]

When money is used as a store of value, the overall budget constraint is:

\[(29) \quad p_m x(1-\tau_m) - \frac{1+R_y y_p}{1-\theta_b b_p} y_p = \frac{1+R_y y_p}{1-\theta_b b_p} y_p + c_m p_m + c_o p_o.\]

The optimal choices for consumption, borrowing, and money holding are:

\[(30) \quad c_y = \frac{1}{1+\delta+\delta^2} \left[ X(1-\tau_m)-\gamma_s \right] \frac{1-\theta_b p_m}{1+R_y y_p} - y_b.\]

\[(31) \quad c_m = \delta \frac{1+R_y y_p}{1-\theta_b b_p} c_y.\]

\[(32) \quad c_o = \delta \frac{p_m}{p_o} c_m.\]
2. Aggregate Equilibrium

As in chapter 5, we consider a model in which rich people produce output $\beta$, and poor people produce output $\alpha$. The aggregate variables are:

a. Population:

\[
N_r(t) = (1+n)N_r(t-1) \quad N_p(t) = (1+n)N_p(t-1).
\]

b. Aggregate demand for private and public goods:

\[
D_c(t) = \left[ N_r(t)c^r(t) + N_r(t-1)c^r(t) + N_r(t-2)c^r(t) + N_p(t)c^p(t) + N_p(t-1)c^p(t) + N_p(t-2)c^p(t) \right] p(t) + G(t),
\]

where $G(t) = g(t)p(t)$ is nominal government spending on public goods.

c. Aggregate supply of private and public goods:

\[
S_c(t) = \left[ N_r(t-1)\beta + N_p(t-1)\alpha \right] p(t).
\]

d. Aggregate use of goods in transactions:

\[
L(t) = N_r(t) \left[ \gamma_b p(t) + \theta_b b^r(t) \right] + N_p(t) \left[ \gamma_b p(t) + \theta_b b^p(t) \right] + z^r_s(t)N_r(t-1) \left[ \gamma_s p(t) + \theta_s s^r(t) \right] + z^p_s(t)N_p(t-1) \left[ \gamma_s p(t) + \theta_s s^p(t) \right] + g \left[ D^g_b(t) + s^g_b(t) \right],
\]

where $z^r_s(t) = 1$ if $s^r(t) > 0$, $z^p_s(t) = 1$ if $s^p(t) > 0$.\]
All the other aggregate variables take the same definitions as in chapter 6 (see equations (39) to (44)). Walras's Law holds (see chapter 6, equation (45)).

We define the ratio of successive price levels in a stationary equilibrium as:

\[(39) \quad C = \frac{p(t)}{p(t+1)} \]

The real interest rate is:

\[(40) \quad i = (1+R) \frac{p(t)}{p(t+1)} - 1.\]

The budget constraint facing the government becomes:

\[(41) \quad X(t) + S^g_b(t) + [1+R(t-1)]D^g_b(t-1) + M(t) - M(t-1)\]

\[\quad = G(t) + \theta_g [D^g_b(t) + S^g_b(t)] + D^g_b(t) + [1+R(t-1)]S^g_b(t-1),\]

where \(X(t)\) is total nominal tax collections at time \(t\), \(S^g_b(t)\) is government borrowing at time \(t\), \(D^g_b(t)\) is government lending at time \(t\), \(M(t)\) is the money stock at time \(t\), and \(\theta_g\) is the marginal transactions-cost of entering the bond market as either a lender or a borrower. The left-hand side of \((41)\) is government income, and the right-hand side represents expenditure. The term \(M(t) - M(t-1)\) is net monetization, which may be positive or negative. From equation \((41)\), it should be obvious that when \(\theta_g > 0\), it would be inefficient for the government to enter more than one side of the bond market in any period. That is, either \(D^g_b(t) \geq 0\) and \(S^g_b(t) = 0\), or \(D^g_b(t) = 0\) and \(S^g_b(t) \geq 0\).

Because we are looking for equilibria which are stationary in individual consumption, as suggested by section B above, the government budget constraint is more usefully written in stationary form. Deflating the variables for population growth and putting them in real terms is done because the aggregate real variables all change at the population growth rate.
Total nominal tax collections at time $t$ are:

\[
X(t) = \frac{G(t)}{N_r(t)p(t)} + \theta \left( \frac{D^g_b(t) + S^g_b(t)}{N_r(t)p(t)} \right) + \frac{D^g_b(t) - S^g_b(t)}{N_r(t)p(t)}.
\]

In a stationary equilibrium, where $\tau(t)$ is constant, $\frac{X(t)}{p(t)}$ grows at the population growth rate $n$. Consequently $\frac{X(t)}{N_r(t)p(t)}$ is constant.

Since $G(t) = g(t)p(t)$, and $g(t)$ is assumed to grow at the population growth rate $n$ (following the analysis of section B above), $\frac{G(t)}{N_r(t)p(t)}$ is also a constant. Bond- and money-market equilibrium conditions require that all other terms in equation (42) be constant as well, for individual consumption to be stationary. Notice, however, that individual utility levels are not stationary. Utility depends on public goods ($g(t)$), the quantity of which is growing over time.

The real interest rate which clears the bond market is the root of the following quadratic equation:

\[
(44) \quad \left\{ \frac{1}{1+n} \frac{\delta^2}{1+\delta+\delta^2} \left( \frac{1}{1+\theta_b} \frac{1}{1+\theta_s} \gamma_b \left[ z^r_s + \frac{N_p}{N_r} z^p_s \right] \right) (1+i)^2 + \left\{ \frac{\delta+\delta^2}{1+\delta+\delta^2} \frac{1}{1+\theta_b} \gamma_b \left[ 1+ \frac{N_p}{N_r} \right] - \frac{D^g_b(t) - S^g_b(t)}{N_r(t)p(t)} \right\} \right. \\
\left. - \frac{1}{1+n} \frac{\delta^2}{1+\delta+\delta^2} \left[ (\beta(1-\tau)-\gamma_s) z^r_s + (\alpha(1-\tau)-\gamma_s) z^p_s \right] (1+i) \\
+ \frac{1}{1+\delta+\delta^2} \left[ \beta(1-\tau) + \frac{N_p}{N_r} \alpha(1-\tau) - z^r_s \gamma_s - \frac{N_p}{N_r} z^p_s \gamma_s \right] \right\} = 0,
\]
\[
\frac{N_r \cdot N^d_r}{N_r} = \frac{N^p_r}{N^r_r}, \text{ which is constant over time.}
\]

The government must choose \( \frac{G(t)}{N_r(t)p(t)}, \tau, \frac{D^G_b(t)}{N_r(t)p(t)}, \frac{S^G_b(t)}{N_r(t)p(t)}, \text{ and } M(t) - M(t-1), \text{ for all } t, \text{ subject to the budget constraint (42)}. \) Since there are five instruments and one constraint, only four of the instruments may be chosen independently. Using (42) we can solve for \( \frac{M(t) - M(t-1)}{N_r(t)p(t)} \), given the values of the other four instruments.

\[
(45) \quad \frac{M(t) - M(t-1)}{N_r(t)p(t)} = \frac{G(t)}{N_r(t)p(t)} + \theta \frac{D^G_b(t) + S^G_b(t)}{N_r(t)p(t)} + \left[ 1 - \frac{1+i}{1+n} \right] \frac{D^G_b(t) - S^G_b(t)}{N_r(t)p(t)} - \frac{X(t)}{N_r(t)p(t)}.
\]

Now the value of \( C = \frac{p(t)}{p(t+1)} \) which clears the money market can be found. We define \( \text{MNRP} = \frac{M(t) - M(t-1)}{N_r(t)p(t)} \), \( \text{GNRP} = \frac{G(t)}{N_r(t)p(t)} \),

\( \text{DBGNP} = \frac{D^G_b(t)}{N_r(t)p(t)} \), \( \text{SBGNP} = \frac{S^G_b(t)}{N_r(t)p(t)} \), \( \text{DBGNP}^* = \text{DBGNP} - \text{SBGNP} \),

\( \text{XNRP} = \frac{X(t)}{N_r(t)p(t)} \). \( \text{XNRP} \) depends upon the nominal interest rate \( R \), which may be found from the real interest rate \( i \) only when \( C \) is known. We write (45) as:

\[
(46) \quad \text{MNRP} = \text{GNRP} + \theta \left( \text{DBGNP} + \text{SBGNP} \right) + \left[ 1 - \frac{1+i}{1+n} \right] \text{DBGNP}^* - \frac{1}{1+n} \left[ \beta + \frac{N_p}{N_r} \alpha + \frac{1}{1+n} \left( 1 + C \right) \left( \frac{S^r_p}{p} + \frac{N_p}{N_r} \frac{S^p}{p} \right) \right] \tau.
\]

We now separate (46) as follows:

\[
(47) \quad A1 = \text{GNRP} + \theta \left( \text{DBGNP} + \text{SBGNP} \right) + \left[ 1 - \frac{1+i}{1+n} \right] \text{DBGNP}^* - \frac{1}{1+n} \left[ \beta + \frac{N_p}{N_r} \alpha + \frac{1+i}{1+n} \left( \frac{S^r_p}{p} + \frac{N_p}{N_r} \frac{S^p}{p} \right) \right] \tau.
\]

\[
(48) \quad \text{MNRP} = A1 + \frac{1}{(1+n)^2} \left[ \frac{S^r_p}{p} + \frac{N_p}{N_r} \frac{S^p}{p} \right] \tau C.
\]
The market-clearing value of \( C = \frac{p(t)}{p(t+1)} \) is found to be:

\[
(49) \quad C = \frac{(1+n)A2 - (1+n)^2A1}{A2 + \left[ \frac{Np}{p} + \frac{Ns}{p} \right] \tau},
\]

where:

\[
(50) \quad A2 = \frac{\delta^2}{1+\delta+\delta^2} \left[ \beta(1-\tau) - \frac{1}{1-\theta_b} (1+i)\gamma_b \right] (1-z_s^R) + \frac{Np}{N_r} \frac{\delta^2}{1+\delta+\delta^2} \left[ \alpha(1-\tau) - \frac{1}{1-\theta_b} (1+i)\gamma_b \right] (1-z_s^P),
\]

\[
(51) \quad \frac{s_r}{p} = \left[ \frac{\delta^2}{1+\delta+\delta^2} \frac{1}{1+\theta_s} \left[ \beta(1-\tau) - \gamma_s \right] - \frac{\delta^2}{1+\delta+\delta^2} \frac{1}{1-\theta_b} \frac{1}{1+\theta_s} \gamma_b (1+i) \right] z_s^R,
\]

\[
(52) \quad \frac{s_p}{p} = \left[ \frac{\delta^2}{1+\delta+\delta^2} \frac{1}{1+\theta_s} \left[ \alpha(1-\tau) - \gamma_s \right] - \frac{\delta^2}{1+\delta+\delta^2} \frac{1}{1-\theta_b} \frac{1}{1+\theta_s} \gamma_b (1+i) \right] z_s^P.
\]

Equation (49) now solves for \( C \) given the exogenous parameters and the values of government's policy instruments. The government uses its instruments to change the real interest rate and the inflation rate, according to equations (44) and (49), to levels which maximize social welfare.
D. Optimal Government Spending and Financing

The government must choose its spending level and make its financing decisions to maximize social welfare. Following the analysis of section A above, the social-welfare function is:

\[
W = \sum_{t=1}^{\infty} \frac{1}{1+n} t N_r(t) U^r(t) + \sum_{t=1}^{\infty} \frac{1}{1+n} t N_p(t) U^p(t).
\]

In developing a solution, a problem arises because \( U^r(t) \) and \( U^p(t) \) rise over time, since they depend on \( g(t) \). However, it is possible to modify the \( W \) given by (53) to get a measure of social welfare which is stationary. Call this new social-welfare function \( W^* \). If \( W^* \) is a monotonic transformation of \( W \), then maximizing \( W^* \) is equivalent to maximizing \( W \).

The modified social-welfare function \( W^* \) can be found by examination of the utility function. An individual's utility function is given by:

\[
U^h(t) = \ln c^h_y(t) + Y \ln g(t) + \delta \ln c^h_m(t+1) + \delta Y \ln g(t+1) + \delta^2 \ln c^h_o(t+2) + \delta^2 Y \ln g(t+2), \quad h = (r,p).
\]

Since real government spending rises at the population growth rate, this can be rewritten as:

\[
U^h(t) = U^h_c(t) + U^g(t), \quad h = (r,p).
\]

(56) \( U^h_c(t) = \ln c^h_y(t) + \delta \ln c^h_m(t+1) + \delta^2 \ln c^h_o(t+2), \quad h = (r,p). \)

(57) \( U^g(t) = Y (1+\delta+\delta^2) \ln g(t) + Y \delta(1+2\delta) \ln (1+n). \)

Since the government is assumed to seek a solution in which the consumption of private goods is stationary (i.e., \( c^h_i(t)=c^h_i \) for all \( t; \ h=r,p; \ i=y,m,o \)), \( U^h_c(t) = U^h_c \) for all \( t \).

Using (57) and (55), we see that:
\[ U^h(t+1) = U^h(t) + \gamma(1+\delta+\delta^2) \ln (1+n). \]

Using this in (53) and simplifying, we get:

\[ W = \frac{\beta}{\alpha+\beta} \sum_{t=0}^{\infty} N_r(1) [U^r(1) + t\gamma(1+\delta+\delta^2) \ln (1+n)] \]
\[ + \frac{\alpha}{\alpha+\beta} \sum_{t=0}^{\infty} N_p(1) [U^p(1) + t\gamma(1+\delta+\delta^2) \ln (1+n)]. \]

Since government's actions affect only \( U^r(1) \) and \( U^p(1) \) in the expression for \( U \) given by (59), the constant terms may be eliminated to get the modified social-welfare function:

\[ W^* = \frac{\beta}{\alpha+\beta} U^r(t) + \frac{\alpha}{\alpha+\beta} N_p \frac{N_r}{N_r} U^p(t), \text{ for any } t. \]

This means that maximizing the weighted social welfare function (60) for one generation leads to the same optimum as maximizing total social welfare for all generations. Consequently we need not worry about the growth of public goods, as it does not affect the maximization problem.

1. Equilibrium When Government Provides a Constant Money Supply and Does Not Intervene in the Bond Market

Suppose the government chooses to finance its spending solely by the use of taxation. Monetization (MNRP) and debt (DBGNP*) are both zero. The price level then changes according to:

\[ \frac{p(t)}{p(t+1)} = 1+n. \]

The government budget constraint is satisfied by setting:

\[ \text{GNRP} = \frac{1}{1+n} \left\{ \beta + \frac{N_p}{N_r} \alpha + \frac{1}{1+n} (1+1-C) \left[ \frac{s_r}{p} + \frac{N_p}{N_r} \frac{s}{p} \right] t \right\}. \]

The government's optimization problem is to maximize (60) subject to the government budget constraint (62) and the bond-market clearing condition (44). Because of the complex nature of the equilibria in this model, we
examine a series of numerical examples to illustrate the optimization process.

We let the parameters of the model take the following values: \( \delta = .9, \theta_s = \theta_b = \gamma_b = 0, \gamma_s = .55, n = .3, \theta_g = 0.2, \gamma = .377, \frac{N_D}{N} = .5, \beta = 14, \) and \( \alpha = 2. \) With these parameters, the equilibrium is a mixed equilibrium in which the rich hold bonds and the poor use money as a store of value. The endogenous variables take the values shown in table 7, column a.

In this case, optimal government spending is about 20% of the total output of the economy. The tax rate is nearly 25%. The interest rate is about 72%, with a constant deflation of 30%, yielding a nominal interest rate of 33%. The government budget is balanced, as tax revenues exactly equal expenditures.

2. Equilibrium When Government Intervenes Optimally in the Bond Market and Maintains a Constant Money Supply

Suppose the government chooses to finance its spending by using taxation and intervening in the bond market. The price level again changes according to (61), and \( MNRP = 0. \) Now the government budget constraint is satisfied by setting:

\[
GNRP = XNRP - \theta_g (DBGNP + SBGNP) - \left[1 - \frac{1+i}{1+n}\right] DBGNP^g,
\]

where \( XNRP \) is the term on the right-hand side of (62).

When the numerical exercise (using the same parameter values as in part 1) is run, the results are significantly different from those in the no-bond-market-intervention case, as shown in table 7, column b. Government spending is reduced slightly (by 2%). The optimal tax rate
Table 7
Equilibrium Values of Variables
for Alternative Methods of Government Intervention

<table>
<thead>
<tr>
<th>Variable</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G/N_p$</td>
<td>3.072</td>
<td>3.012</td>
<td>3.096</td>
<td>3.000</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.2500</td>
<td>0.2476</td>
<td>0.2292</td>
<td>0.2366</td>
</tr>
<tr>
<td>$D_g/N_p$</td>
<td>0</td>
<td>0.1209</td>
<td>0</td>
<td>0.3232</td>
</tr>
<tr>
<td>$m/N_p$</td>
<td>0</td>
<td>0</td>
<td>0.06794</td>
<td>0.04580</td>
</tr>
<tr>
<td>$X/N_p$</td>
<td>3.072</td>
<td>3.005</td>
<td>3.028</td>
<td>2.965</td>
</tr>
<tr>
<td>$1+i$</td>
<td>1.726</td>
<td>1.640</td>
<td>1.726</td>
<td>1.516</td>
</tr>
<tr>
<td>$p(t)/p(t+1)$</td>
<td>1.300</td>
<td>1.300</td>
<td>0.8016</td>
<td>0.9608</td>
</tr>
<tr>
<td>$R$</td>
<td>0.3276</td>
<td>0.2612</td>
<td>1.153</td>
<td>0.5774</td>
</tr>
<tr>
<td>$c_y^r$</td>
<td>2.127</td>
<td>2.247</td>
<td>2.190</td>
<td>2.468</td>
</tr>
<tr>
<td>$c_m^r$</td>
<td>3.305</td>
<td>3.316</td>
<td>3.401</td>
<td>3.367</td>
</tr>
<tr>
<td>$c_o^r$</td>
<td>4.816</td>
<td>4.642</td>
<td>4.634</td>
<td>4.195</td>
</tr>
<tr>
<td>$s^r/p$</td>
<td>2.974</td>
<td>2.984</td>
<td>3.061</td>
<td>3.030</td>
</tr>
<tr>
<td>$U^r$</td>
<td>4.251</td>
<td>4.258</td>
<td>4.282</td>
<td>4.280</td>
</tr>
<tr>
<td>$c_y^p$</td>
<td>0.3207</td>
<td>0.3387</td>
<td>0.3296</td>
<td>0.3717</td>
</tr>
<tr>
<td>$c_m^p$</td>
<td>0.4982</td>
<td>0.4998</td>
<td>0.5120</td>
<td>0.5071</td>
</tr>
<tr>
<td>$c_o^p$</td>
<td>0.5829</td>
<td>0.5847</td>
<td>0.3694</td>
<td>0.4385</td>
</tr>
<tr>
<td>$m^p/p$</td>
<td>0.4483</td>
<td>0.4498</td>
<td>0.4608</td>
<td>0.4564</td>
</tr>
<tr>
<td>$U^p$</td>
<td>-1.055</td>
<td>-1.015</td>
<td>-1.364</td>
<td>-1.146</td>
</tr>
<tr>
<td>$D_c(t)/N_c(t-1)$</td>
<td>14.45</td>
<td>14.4186</td>
<td>14.45</td>
<td>14.3660</td>
</tr>
<tr>
<td>$S_c(t)/N_c(t-1)$</td>
<td>15.00</td>
<td>15.00</td>
<td>15.00</td>
<td>15.00</td>
</tr>
<tr>
<td>$L(t)/N_r(t-1)$</td>
<td>0.55</td>
<td>0.5814</td>
<td>0.55</td>
<td>0.6340</td>
</tr>
<tr>
<td>$W$</td>
<td>3.653</td>
<td>3.663</td>
<td>3.662</td>
<td>3.673</td>
</tr>
</tbody>
</table>
falls by 1%, leading to a drop in tax collections of over 2%, and causing the budget to become unbalanced. The government enters as a lender in the bond market, causing the real interest rate to fall sharply (by 12%). The decline in the nominal interest rate is even steeper, falling by 20%.

Since the price level and population grow at the same rate, the national "debt" remains constant in nominal terms. Government interest receipts balance the budget. Total government spending is 3.043 (which equals spending on public goods plus transactions costs), which exactly equals tax receipts plus interest receipts.

Compared to the situation in part 1, both the rich and poor are made better off, due primarily to the reduction in interest rates plus the reduction in taxes. This is true even though more of the economy's output is spent on transactions costs.


Suppose that the government chooses to intervene in the money market but not in the bond market. The real interest rate is determined by equation (44), where $DBGNP = 0$. Of course, the real interest rate is the same as in part 1 above, when government intervenes in neither the bond nor money markets. Equation (49) then determines the inflation rate.

In the numerical example, government's intervention in the money market makes the rich better off and the poor worse off, as indicated by column c of table 7. Government spending is higher, with a lower tax
rate. The deficit is monetized entirely, causing inflation of nearly 25%. This causes the nominal interest rate to rise significantly, to 115%. Since the government taxes nominal interest receipts, tax collections don't fall much (comparing column c to column a in table 7), even though the tax rate is considerably lower. The poor are made much worse off, since they use money as a store of value, and inflation occurs.

4. Full Government Intervention

When the government intervenes using all of its tools, social welfare increases. The real interest rate is determined by equation (44), and the price level changes according to equation (49).

In the numerical example, we see an increase in social welfare, shown in column d of table 7. This is not a Pareto-improvement (although that would certainly be achievable here) because the poor are made worse off. The rich are considerably better off than in parts 1 and 2 above, but not part 3. This result comes about because monetization by the government allows it to lend a greater amount (even with lower taxes). Increased government lending reduces the real interest rate, making the rich better off (because it lets them smooth out their consumption more over time).

In this case, monetization by the government, although it appears to be small in magnitude, causes inflation of about 4%. Although the poor are happy to see the lower real interest rate, inflation causes a large reduction in old-age consumption, leading to decreased utility.
The optimal provision of public goods is reduced somewhat and the tax rate falls. The nominal value of the public goods now grows faster than the population growth rate, due to inflation. There is a growing nominal government deficit, which is monetized. The nominal interest rate has jumped significantly. This helps to increase tax revenues, since nominal interest receipts are taxed. It is again optimal to increase transactions costs because the benefits due to the intergenerational free lunch exceed the costs.
E. Government Policy Experiments

We may now consider the results which occur when the government does not behave optimally. Nonoptimal behavior might occur due to information costs, measurement error, or an imperfect decision-making process. Costs of gathering information may prevent the fiscal and monetary authorities from knowing precisely how their actions affect social welfare. They may not know the time-discount rate of the public, which is obviously vital if intertemporal-consumption opportunities are to be provided optimally. There may be errors in measuring the "amount" of government debt and monetization. In addition, there is the problem of revealed preference in the choice of quantity of the public good to be provided. The government may not know how individuals value public goods relative to the consumption of private goods. The political process is likely to be only an imperfect method of public choice. The monetary and fiscal policy authorities may maximize something other than social welfare, as suggested recently by Blinder (1983).

1. Nonoptimal Government Spending on Public Goods

Consider the case of full government intervention above, and the numerical example given in part 4 of section D. Here the optimal level of government spending, $G/N_p$, is found to be 3.000. Suppose this is raised to 3.2 or lowered to 2.8, and that the optimal levels of taxation, debt, and monetization are then found. The results are shown in table 8, columns b and c. The arrows show whether there is an
Table 8

Government Policy Experiments

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>G/N_\text{r}p</td>
<td>3.000</td>
<td>3.200+</td>
<td>2.800+</td>
<td>3.000</td>
<td>3.000</td>
<td>3.000</td>
<td>3.000</td>
</tr>
<tr>
<td>\tau \text{e}</td>
<td>.2366</td>
<td>.2519+</td>
<td>.2212+</td>
<td>.25+</td>
<td>.21+</td>
<td>.2366</td>
<td>.2366</td>
</tr>
<tr>
<td>D_{D}/N_\text{r}p</td>
<td>.3232</td>
<td>.3037+</td>
<td>.3428+</td>
<td>.4382+</td>
<td>.1458+</td>
<td>.35+</td>
<td>.30+</td>
</tr>
<tr>
<td>m/N_\text{r}p</td>
<td>.04580</td>
<td>.04679+</td>
<td>.04472+</td>
<td>.02031+</td>
<td>.1164+</td>
<td>.04855+</td>
<td>.04348+</td>
</tr>
<tr>
<td>X/N_\text{r}p</td>
<td>2.965</td>
<td>3.162+</td>
<td>2.768+</td>
<td>3.017+</td>
<td>2.876+</td>
<td>2.967+</td>
<td>2.964+</td>
</tr>
<tr>
<td>1+i 1</td>
<td>1.516</td>
<td>1.523+</td>
<td>1.509+</td>
<td>1.448+</td>
<td>1.627+</td>
<td>1.500+</td>
<td>1.529+</td>
</tr>
<tr>
<td>p(t)/p(t+1)</td>
<td>.9608</td>
<td>.9463+</td>
<td>.9753+</td>
<td>1.147+</td>
<td>.4670+</td>
<td>.9404+</td>
<td>.9779+</td>
</tr>
<tr>
<td>R</td>
<td>.5774</td>
<td>.6096+</td>
<td>.5467+</td>
<td>.2630+</td>
<td>2.485+</td>
<td>.5955+</td>
<td>.5635+</td>
</tr>
<tr>
<td>c_{y}^r</td>
<td>2.468</td>
<td>2.404+</td>
<td>2.532+</td>
<td>2.535+</td>
<td>2.383+</td>
<td>2.493+</td>
<td>2.447+</td>
</tr>
<tr>
<td>c_{m}^r</td>
<td>3.367</td>
<td>3.296+</td>
<td>3.438+</td>
<td>3.304+</td>
<td>3.490+</td>
<td>3.367</td>
<td>3.367</td>
</tr>
<tr>
<td>c_{o}^r</td>
<td>4.195</td>
<td>4.087+</td>
<td>4.303+</td>
<td>4.083+</td>
<td>4.347+</td>
<td>4.145+</td>
<td>4.238+</td>
</tr>
<tr>
<td>s_{r}/p</td>
<td>3.030</td>
<td>2.966+</td>
<td>3.094+</td>
<td>2.974+</td>
<td>3.141+</td>
<td>3.030</td>
<td>3.030</td>
</tr>
<tr>
<td>U_{r}</td>
<td>4.280</td>
<td>4.279+</td>
<td>4.275+</td>
<td>4.268+</td>
<td>4.306+</td>
<td>4.280+</td>
<td>4.279+</td>
</tr>
<tr>
<td>c_{y}^p</td>
<td>.3717</td>
<td>.3625+</td>
<td>.3810+</td>
<td>.3821+</td>
<td>.3583+</td>
<td>.3755+</td>
<td>.3685+</td>
</tr>
<tr>
<td>c_{m}^p</td>
<td>.5071</td>
<td>.4969+</td>
<td>.5173+</td>
<td>.4982+</td>
<td>.5247+</td>
<td>.5071</td>
<td>.5071</td>
</tr>
<tr>
<td>c_{o}^p</td>
<td>.4385</td>
<td>.4232+</td>
<td>.4540+</td>
<td>.5142+</td>
<td>.2205+</td>
<td>.4292+</td>
<td>.4463+</td>
</tr>
<tr>
<td>m_{r}/p</td>
<td>.4564</td>
<td>.4472+</td>
<td>.4655+</td>
<td>.4483+</td>
<td>.4723+</td>
<td>.4564</td>
<td>.4564</td>
</tr>
<tr>
<td>U_{r}^p</td>
<td>-1.146</td>
<td>-1.153-</td>
<td>-1.146-</td>
<td>-1.102-</td>
<td>-1.709-</td>
<td>-1.154-</td>
<td>-1.141-</td>
</tr>
<tr>
<td>S_{c}(t)/N_{r}(t-1)</td>
<td>15.00</td>
<td>15.00</td>
<td>15.00</td>
<td>15.00</td>
<td>15.00</td>
<td>15.00</td>
<td>15.00</td>
</tr>
<tr>
<td>L(t)/N_{r}(t-1)</td>
<td>.6340</td>
<td>.6290+</td>
<td>.6391+</td>
<td>.6639+</td>
<td>.5879+</td>
<td>.6410+</td>
<td>.6280+</td>
</tr>
</tbody>
</table>
increase (+) or decrease (-) in the value of the variable from its optimum level.

When government spending on public goods is higher than it should be, it is optimal for the government to raise taxes, reduce lending (i.e., increase government debt), increase deficits, and increase monetization of deficits. This causes the real interest rate to rise, and increases inflation. The nominal interest rate thus rises as well. Although the increased supply of public goods increases utility, the increased taxes and suboptimal interest rate and inflation rate cause reduced private consumption, thus reducing the utility of both rich and poor. Public goods crowd out private goods. Total consumption of goods increases, as transactions costs fall, but the goods are not efficiently allocated, so social welfare declines.

The opposite effects occur when public goods are underprovided. Taxes are decreased, as is monetization. The real interest rate and inflation rate both decline. In this case private consumption rises, but because goods are inefficiently allocated between public and private goods, social welfare falls.

Buchanan and Wagner (1977) argue that public goods are overprovided because debt financing is seen by the public as being cheaper than financing by taxes. When debt is used to finance government expenditures, people don't see the true cost of those expenditures, so they want more public goods than are optimal. Thus debt financing causes expenditures to be too high.

However, this hypothesis is empirically indistinguishable from a theory, such as that of Niskanen (1971), which argues that public goods
are oversupplied because bureaucrats face an optimization problem which is not equivalent to maximizing social welfare. Optimal financing of an inefficiently high level of government spending leads to increased deficits and debt, as shown above. In this case, however, high government expenditures cause increased government debt, which is the opposite direction of causation from that implied by Buchanan and Wagner.

2. Nonoptimal Taxation

We next consider what happens when government spending on public goods is at its optimal level, but the tax rate is set too high or too low. When the tax rate is set higher than its optimal level, as indicated in column d of table 8, increased lending and decreased monetization are optimal. This causes the real interest rate to fall, the inflation rate to decline (in fact a deflation occurs), and the nominal interest rate to fall significantly. The decline in the interest rate allows increased consumption for everyone when young, but decreases the old-age consumption of the rich. The increased taxes cause middle-age consumption to fall for everyone. The decline in inflation causes old-age consumption of the poor to rise. Although the poor are made better off, the rich are worse off, and social welfare declines.

The opposite effects occur when tax rates are set too low, as demonstrated in column e of table 8. It is then optimal for the government to increase deficits and the monetization of deficits, due to the shortfall of tax revenue. This leads to a rise in the real interest
rate and an increase in inflation (to over 100%). The nominal interest rate is driven upwards tremendously. The higher interest rates make the rich better off, as they get a higher return on their savings. However the poor, who use money as a store of value, have their savings eaten away by inflation, and their old-age consumption falls by nearly 50%. The poor are much worse off, and social welfare declines.

The focus of supply-side economics and the Reagan administration's policies is on the proper size of tax rates. The reductions in tax rates in the early 1980's may be seen as either: (1) reducing tax rates from a nonoptimally high level, as supply-siders and the proponents of Reaganomics contend, or (2) reducing tax rates to a suboptimally low level, as many critics argue. If tax rates were previously too high, and the tax cuts reduced them to their proper level, then we would expect, according to the results of this chapter, to see increases in interest rates and inflation, with an increase in social welfare. If tax rates are set too low, then increases in interest rates and inflation again occur, but social welfare declines.

Thus the Reaganomics tax cuts can be judged in terms of efficiency only when we know how social welfare is affected. This is information which is very difficult to discover. Election results correspond to changes in social welfare only if the political mechanism exactly reflects social welfare. Thus further investigation of political hypotheses (see footnote 2) is necessary before we can relate the concept of social welfare to real-world political events.

In the numerical example, a relatively small amount of monetization causes a tremendous amount of inflation. Monetization finances just 4%
of government spending on public goods, and does not even triple compared to the optimal case. Nonetheless the increase in monetization causes inflation to explode.

These results seem contrary to monetarist theory. Monetarists argue that the growth rate of the money supply should be chosen at a fixed level, regardless of the choices of tax rates, government spending, and government debt. From the point of view of maximizing social welfare, this is obviously incorrect. Furthermore, as equation (49) implies, the inflation rate depends not only on the growth rate of the money supply but also on government spending, deficits, and tax rates.

In this numerical example, where relatively little deficit monetization leads to a large increase in inflation, the reduction in tax rates and the increased rate of interest actually lead people to increase real money demand, despite the tremendous increase in inflation. This is the opposite of the expected result under monetarist theory, which suggests that real money demand should fall when inflation occurs.

It is interesting to compare column b to column d and column c to column e in table 8. In both columns b and d, taxes are higher than is optimal. However the macroeconomic implications are exactly the opposite in terms of interest rates and inflation. The same is true in comparing columns c and e. The implication is that it is not correct to focus on just one aspect (tax rates, in this case) of government financial policy. The relationship between all of the instruments of
government policy must be examined before macroeconomic results can be analyzed.

3. Nonoptimal Debt and Monetization

Suppose that government spending and taxation are at their optimal levels, but that the choice between debt (lending) and money is not made optimally. The substitution of money for debt is called an open-market operation.

Consider the effects when the government increases money issue and decreases debt (increases lending). The monetary authorities might accomplish this by buying securities on the open market. This is illustrated in column f of table 8. The decreased debt causes a decline in the real interest rate. However, the increase in monetization causes an increase in inflation. Inflation rises more than the real interest rate falls, so the nominal interest rate increases.8

The decline in the real interest rate makes the young better off, but the rich are worse off in old age. The poor are worse off in old age due to the increased inflation. Overall, the rich are slightly better off and the poor worse off, and social welfare falls. The exact opposite results occur for decreased monetization (an open-market sale), as shown in column g of table 8.
F. Summary and Conclusions

This chapter attempts to illustrate the full tradeoff the government faces between using taxation, borrowing, and monetization. Also, the choice of optimal provision of public goods is modeled. The government must be able to make interpersonal and intertemporal tradeoffs. This is accomplished by the use of a social-welfare function with weights which vary over time and across different types of people.

The choice of the optimal quantity of public goods depends upon their costs. In the second-best framework presented here, all three financing methods have opportunity costs in social welfare. The goal of policy is to find the optimal tradeoff between the methods of finance based on their social-welfare costs.

In the numerical examples presented here, the government finds it optimal to lend, rather than to borrow. This result is due to the structure of the model, which requires private individuals to borrow a considerable amount. Also, the working life is only one-third of a lifetime, and must support both young age and old age. In a model with two working periods, it may be optimal for the government to borrow, as it was in chapter 3.
FOOTNOTES TO CHAPTER 7

1. The choice of a tax "schedule", that is, whether to have taxes which are regressive, progressive or proportional, and the degree of progressivity, is considered by P. Miller (1983c).

Surprisingly, in a model with labor-leisure choice, labor-productivity shocks, and imperfect risk-sharing, he finds that the optimal tax schedule is regressive.

2. Hypotheses about the political structure may be investigated using alternative social-welfare weights \( \phi(t) \). That is, one group of voters may act in such a way as to increase its weight and reduce that of others. This may also lead to differences in \( \delta \) over time as well, although that may lead to inconsistency in optimal planning (Kydland-Prescott (1977)).

3. The use of an additively separable utility function seems to contribute to this result. Under a more-complicated utility function, we might have obtained a result where tax rates fall over time (but at a declining rate), and government spending grows over time, but the growth rate falls over time. In this case it would be necessary to analyze a long-run steady state, if one existed.

Bryant (1983) shows this in a simple two-period overlapping-generations model. When utility functions are separable in the public good, the allocation of the public good has no effect on private-sector decisions.

4. To simplify the model, the production function for public goods is assumed to be identical to the production function for private goods.

5. These interest rates seem high, but these are not annual interest rates. If the period of time is 20 years, then the annual real interest rate here is only 2.75%.

6. In this case the debt is negative--it is really a debt owed by people to the government.

7. In some cases, one of the tools (taxation, borrowing, monetization) may be either redundant or inferior. Redundancy occurs when only two tools need to be used to reach the social optimum, as in chapter 3. A tool is inferior when its use reduces social welfare. In the numerical example given here all three tools are useful.

8. In this case, the nominal interest rate rises, as inflation increases more than the fall in the real interest rate. However, with different parameter values, it may be that the nominal interest rate falls when there is an open-market purchase.
BIBLIOGRAPHY


Kohn, Meir. "'Forced Saving' and the Effect of Anticipated Inflation on a Production Economy." Dartmouth College working paper no. 82-5, September 1982a.


