INFORMATION TO USERS

This reproduction was made from a copy of a document sent to us for microfilming. While the most advanced technology has been used to photograph and reproduce this document, the quality of the reproduction is heavily dependent upon the quality of the material submitted.

The following explanation of techniques is provided to help clarify markings or notations which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting through an image and duplicating adjacent pages to assure complete continuity.

2. When an image on the film is obliterated with a round black mark, it is an indication of either blurred copy because of movement during exposure, duplicate copy, or copyrighted materials that should not have been filmed. For blurred pages, a good image of the page can be found in the adjacent frame. If copyrighted materials were deleted, a target note will appear listing the pages in the adjacent frame.

3. When a map, drawing or chart, etc., is part of the material being photographed, a definite method of “sectioning” the material has been followed. It is customary to begin filming at the upper left hand corner of a large sheet and to continue from left to right in equal sections with small overlaps. If necessary, sectioning is continued again—beginning below the first row and continuing on until complete.

4. For illustrations that cannot be satisfactorily reproduced by xerographic means, photographic prints can be purchased at additional cost and inserted into your xerographic copy. These prints are available upon request from the Dissertations Customer Services Department.

5. Some pages in any document may have indistinct print. In all cases the best available copy has been filmed.
THE EFFECTS OF TABLE-BUILDING PROBLEM-SOLVING PROCEDURES ON STUDENTS' UNDERSTANDING OF VARIABLES IN PRE-ALGEBRA

The Ohio State University

Copyright 1984 by Keller, James Edward

All Rights Reserved
PLEASE NOTE:

In all cases this material has been filmed in the best possible way from the available copy. Problems encountered with this document have been identified here with a check mark □.

1. Glossy photographs or pages □
2. Colored illustrations, paper or print □
3. Photographs with dark background □
4. Illustrations are poor copy □
5. Pages with black marks, not original copy □
6. Print shows through as there is text on both sides of page □
7. Indistinct, broken or small print on several pages ✔
8. Print exceeds margin requirements □
9. Tightly bound copy with print lost in spine □
10. Computer printout pages with indistinct print □
11. Page(s) _______ lacking when material received, and not available from school or author.
12. Page(s) _______ seem to be missing in numbering only as text follows.
13. Two pages numbered ___________, Text follows.
14. Curling and wrinkled pages □
15. Other _____________________________________________________________
THE EFFECTS OF TABLE-BUILDING PROBLEM-SOLVING PROCEDURES
ON STUDENTS' UNDERSTANDING OF VARIABLES IN PRE-ALGEBRA

DISSER TATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

James Edward Keller, B.S., M.A.

* * * *

The Ohio State University
1984

Reading Committee:
Dr. Jon L. Higgins
Dr. Alan R. Osborne
Dr. Lorren L. Stull

Approved By

Advisor
Department of Science and Mathematics
Education
Dedicated to my parents, Mr. and Mrs. James Keller, sisters, Virginia, Pauline, Deloris, and brother, Thomas, all who have provided me encouragement, moral support, and love throughout my doctoral program.
ACKNOWLEDGMENTS

I sincerely express my appreciation to everyone who have helped to produce this document and made it possible for me to complete my doctoral program.

To my advisor, Dr. Alan Osborne, I am especially grateful for all his advice, guidance, and support throughout this investigation and my degree program. He has contributed to both my professional and personal development. I am appreciative to Dr. James Altschuld for his advice and support. Dr. Jon Higgins and Dr. Lorren Stull deserve my sincerest thanks for their advice and assistance in reviewing and critiquing this document.

Appreciation is extended to the cooperation and assistance of the administration and faculty of East High School, Columbus, Ohio. I am indebted to the principal, Mr. Willis, and Judy Silbaugh who were a source of help and motivation throughout this study. A special thanks is extended to Miriam Mertens and Raymond Schaefer, who allowed me to use their classes for this study, and who were very cooperative and patient throughout the study. A special note of thanks goes to the students who participated in the study. Without them my efforts would have been futile.
A heartfelt appreciation goes to the staff and laboratory assistants of Edgar Dale Media Center Microcomputer Laboratory at The Ohio State University. I am deeply grateful to all of them who provided assistance and allowed me to use their equipment and facilities during the countless hours spent on typing and printing this document.

To my family and friends, I am grateful for their interest, encouragement, motivation, and support throughout my years of study. I am thankful for their prayers and the incentives they have given me to trust God's wisdom and guidance.

It is most important for me to thank the Lord Who has provided me the strength and courage to produce this document, and Who has been a source of guidance throughout my degree program and my entire life, and through his faith I have learned to trust him each day of my life.
May 9, 1949 ....................................................... Born - Saint Matthews, South Carolina

1967 .......................................................... High School Diploma, John Ford High School, Saint Matthews, South Carolina

1971 .......................................................... B.S., Mathematics Education, South Carolina State College, Orangeburg, South Carolina

1971-1972 ...................................................... Graduate Teaching Assistant, Department of Mathematics, Clemson University, Clemson, South Carolina

1972 .......................................................... M.A., Mathematics Education, Clemson University, Clemson, South Carolina

1973-1980 ...................................................... Mathematics Instructor, Department of Mathematics, South Carolina State College, Orangeburg, South Carolina

1981-1982 ...................................................... Graduate Research Associate, Department of Science and Mathematics Education, The Ohio State University, Columbus, Ohio

1982-1984 ...................................................... Graduate Teaching Associate, Department of Mathematics, The Ohio State University, Columbus, Ohio

1983 .......................................................... Mathematics Instructor, Department of Developmental Education, Franklin University, Columbus, Ohio
FIELDS OF STUDY

Major Field: Mathematics Education


Studies in Research and Evaluation.
Professor James W. Altschuld.

# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEDICATION</td>
<td>II</td>
</tr>
<tr>
<td>ACKNOWLEDGMENTS</td>
<td>III</td>
</tr>
<tr>
<td>VITA</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xi</td>
</tr>
</tbody>
</table>

## Chapter

1. INTRODUCTION ........................................ 1
   - Need for the study ................................ 1
   - Problem Statement ................................ 11
   - Assumptions and Limitations .................... 12
   - Definition of Terms ................................ 13
   - Hypotheses ........................................ 15
   - Plan of the Report ................................ 16

2. REVIEW OF LITERATURE .............................. 17
   - Students' Thinking About Variables .......... 17
   - Students' Understanding of Variables ........ 28
   - Students' Understanding of Algebra .......... 32
   - Problem-Solving Procedures .................... 38

3. METHODS AND PROCEDURES .......................... 46
   - Design ........................................... 46
   - Population ...................................... 46
   - Assignment of groups to treatments .......... 46
   - Description ...................................... 48
   - Treatments ....................................... 49
   - Materials ........................................ 49
   - Groups ........................................... 49
   - TB Treatment .................................... 50
   - T Treatment ...................................... 53
Instrumentation ................................................................. 56

Pretest .............................................................................. 56
Problem-Solving Test ....................................................... 59
Posttest ............................................................................. 60
Procedures ........................................................................... 61
Group Sessions ................................................................. 61
Statistical Procedures ...................................................... 64
Scoring of Instruments ..................................................... 64
Reliability of Instruments .................................................. 64
Multivariate Analysis of Variance Procedures .............. 65
Correlational Analyses ...................................................... 66

IV. DATA ANALYSIS AND RESULTS ................................................................. 67

Subjective Analyses of Qualitative Data ......................... 68
Table-Building Instruction .................................................. 68
Translation Instruction ....................................................... 73
Summary of Qualitative Data ............................................. 76
Statistical Analyses of Quantitative Data ......................... 78
Results of Reliability Estimates of Instruments ............... 78
Analysis of Pretest Data ..................................................... 79
Analysis of Posttest Data .................................................... 81
Analysis of Problem-Solving Test Data ......................... 85
Analysis of Cognitive Development Levels .................... 90
Correlational Analyses on Selected Variables .................. 97
Summary of Quantitative Data ............................................ 99

V. SUMMARY AND CONCLUSIONS ................................................................. 101

Summary ............................................................................. 101
Results of Null Hypotheses .............................................. 104
Discussion ........................................................................... 106
Conclusions ........................................................................ 109
Implications ........................................................................ 112
Recommendations for Further Study ............................... 116

APPENDICES

A. TB Treatment Materials ................................................. 118
B. T Treatment Materials .................................................... 147
C. Test Instruments ............................................................. 166
D. Item Analyses of Test Instruments ................................. 179

BIBLIOGRAPHY ................................................................. 183
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Time and Days of Treatment Group Sessions</td>
<td>62</td>
</tr>
<tr>
<td>2. Pretest Means and Standard Deviations</td>
<td>79</td>
</tr>
<tr>
<td>3. Analysis of Variance for Pretest Data</td>
<td>81</td>
</tr>
<tr>
<td>4. Posttest Means and Standard Deviations</td>
<td>82</td>
</tr>
<tr>
<td>5. Analysis of Variance for Posttest Data</td>
<td>82</td>
</tr>
<tr>
<td>6. T-Values for Contrasts in Treatment Group Means on the Posttest</td>
<td>84</td>
</tr>
<tr>
<td>7. Means and Standard Deviations for the Problem-Solving Variables</td>
<td>86</td>
</tr>
<tr>
<td>8. Multivariate and Univariate Analysis of Variance of Problem-Solving Variables</td>
<td>87</td>
</tr>
<tr>
<td>9. T-Values for Contrasts in Treatment Group Means on the Problem-Solving Variables</td>
<td>89</td>
</tr>
<tr>
<td>10. Levels of Understanding by Treatment Groups</td>
<td>90</td>
</tr>
<tr>
<td>11. Cell Means on Problem-Solving Scores by Treatment and Cognitive Blocks</td>
<td>92</td>
</tr>
<tr>
<td>12. Two-Way Analysis of Variance of Problem-Solving Scores by Treatment and Cognitive Blocks</td>
<td>93</td>
</tr>
<tr>
<td>13. Dunn's Post-Hoc Comparison Procedure on Problem-Solving Cell Means of Treatment Condition by Cognitive Blocks</td>
<td>95</td>
</tr>
</tbody>
</table>
14. Cell Means on Problem-Solving Scores by Treatment and Reasoning Ability Blocks ........................................ 96
15. Two-Way Analysis of Variance of Problem-Solving Scores by Treatment and Reasoning Ability Blocks .......... 96
16. Pearson Product Moment Correlation Matrix for Selected Variables ......................................................... 98
17. Item Analysis of Pretest ......................................................................................................................... 180
18. Item Analysis of Posttest ...................................................................................................................... 181
19. Item Analysis of Problem-Solving Test ............................................................................................... 182
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Small group assignments</td>
<td>47</td>
</tr>
<tr>
<td>2. Completely randomized one-factor design</td>
<td>48</td>
</tr>
<tr>
<td>3. The research design</td>
<td>48</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

Need For The Study

A major concern of many colleges and universities is a large enrollment of students without adequate preparation in mathematics. Test data have shown a decline in the mathematics preparation of freshmen entering colleges and universities across the country. As indicated by Jones (1981), Scholastic Aptitude Test (SAT) scores and American College Test (ACT) mathematics scores had shown a monotonic decline within the last decade. As a result of the decline in SAT and ACT mathematics scores, a large number of students have been enrolled in basic skills mathematics at many institutions.

Many students who have been enrolled in basic college mathematics courses have misconceptions of the concept variable and cannot solve word problems successfully. Many students are afraid, become frustrated, and develop mathematics anxiety because of the difficulty they experience in understanding and solving word problems. Being able to solve problems successfully is a vital and central role in learning and applying mathematics. In order that students might overcome these difficulties, there is a need to strengthen or provide appropriate instructional strategies for the concept of variable.
Research evidence indicates that students do have misconceptions about variables and difficulty in solving word problems (Clement, 1982; Rosnack, 1981; Herscovics & Kleran, 1980). Algebra, which is in the central core for mathematically literate people, is a useful tool for solving problems. A necessary feature of algebra is the understanding and use of variables. Conceptual understanding of variables needs to be developed until students feel comfortable with variables and can use them to solve problems successfully.

Some factors that might contribute to the lack of success with variables and problem-solving in algebra are: (a) teaching strategies, (b) students' thinking processes, (c) levels of intellectual or cognitive development, (d) reading abilities, (e) mathematics aptitude, and (f) attitudes toward mathematics and problem-solving.

In order that students might perform well in the mathematics of problem-solving, factors such as the ones listed above need to be investigated. Because a student normally imitates or models a teacher and tries to learn or understand a concept based on a particular method of presentation, the approach teachers use to teach the concept of variable and its use in word problems is a factor of primary importance. The question of concern is which methods or strategies are most effective for students' levels of intellectual development and mathematics aptitudes or abilities? Since effective performance should be the result of effective teaching, teachers need to know the effect different teaching strategies will have on students' achievement. Is it possible for teachers to increase students' competence in solving problems and understanding the concept of variable by using different
Instructional strategies?

Several studies have compared different strategies for teaching the solution to verbal problems. Studies conducted by Richardson (1975) and Bassler, Beers, and Richardson (1975) compared a step method with a translation method. The step method directed students to complete the solution to word problems by the use of a number of steps. The translation method directed students to translate each phrase as it appeared in the problem. Findings indicate that both methods were about equally effective in producing student achievement. Significant retention effects were indicated by both methods. Richardson's research suggested that teachers who became familiar with these strategies used them to produce significantly greater pupil performance in solving problems than teachers who had not been presented these strategies.

Settle (1977) compared a "guess and test" approach requiring students to reason via analogy and with numbers to a "traditional" approach requiring students to reason deductively and with variables. He stated that "the difficulty students experience in learning to solve verbal problems in elementary algebra is largely due to a lack of experience useful in developing skill in writing relevant equations" (p. 2). The "guess and test" procedure required the student's initial response to be a proposal (guess) of the problem's solution. A test of this guess was made to determine its correctness, and then an equation was constructed for the problem. The use of a variable was avoided in the initial thinking. The equation was constructed by analogy as opposed to abstract translations and deductive reasoning.
Thus, the Guess and Test approach was a more intuitive or inductive approach to solving word problems. Students' reasoning through the problem was in terms of a specific number rather than a variable. The traditional approach was deductive in nature and required students to think abstractly and reason initially with the use of a variable.

Settle concluded from his study that the Guess and Test approach was superior to the Traditional approach in instructing subjects to write relevant equations to verbal problems of the type used in instruction. This superiority could not be established for transfer to similar types of verbal problems or for retention.

Although much research has been done in trying to determine the more effective strategies to use in teaching word problems, students still experience serious difficulties. If these difficulties are to be corrected or overcome, alternative approaches need to be investigated further to determine what might be a suggested or recommended teaching strategy for the existing problem. "Table-Building", a teaching strategy used by Leltzel & Demana (1981) in an experimental course, is pertinent to the issue at hand. This strategy enables students to generate a table of values in determining a solution to a problem by a "guess and check" approach. This approach allows students to adjust their guesses by checking them and as a result approximate solutions to problems. Students need to see that problems require reasoning with quantities that vary over some set of numbers. The intent is to help students understand the concept of variable. In this context they need to know that a variable, which is usually a letter, stands for the
numbers in some set and that variables can be used to describe a
general situation concisely.

The mathematics education literature revealed that the majority of
the studies comparing different strategies for solving word problems
yielded mixed results and did not give definitive conclusions about
which strategy should be used when teaching the solution of word
problems. Most of the studies conclude by suggesting that more
research is needed.

In order that students might conceptually understand variables,
the intent of this study was to investigate the effectiveness of a
Table Building (TB) teaching method. Since many students have trouble
solving word problems, the TB method was used to teach the solution to
word problems. The effects of this method were determined by comparing
performance of students instructed by this method with students
instructed by a Translation (T) method.

These two methods are exemplified for the following word problem:
A parking-meter coin-box contained $14.95 in dimes and
nickels. If there were six times as many dimes as nickels,
how many of each coin were in the box?

Table-Building Method

1. Read the problem carefully and determine the given and
wanted information.

2. Label appropriate column headings in the table expressing
relationships between the given and wanted information.

3. Use the following problem-solving strategies to determine
the solution.

(a) Guess and check a particular number. If the number is not the correct answer, adjust your guess by making a better guess.

(b) Examine numerical relationships in the table.

(c) Look for patterns.

(d) Use approximations to determine the answer.

(e) Use a letter to express relationships between the given and wanted information.

<table>
<thead>
<tr>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
<th>Value in Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6(5)=30</td>
<td>5(5)+10(30)=325</td>
</tr>
<tr>
<td>10</td>
<td>6(10)=60</td>
<td>5(10)+10(60)=650</td>
</tr>
<tr>
<td>15</td>
<td>6(15)=90</td>
<td>5(15)+10(90)=975</td>
</tr>
<tr>
<td>20</td>
<td>6(20)=120</td>
<td>5(20)+10(120)=1300</td>
</tr>
<tr>
<td>25</td>
<td>6(25)=150</td>
<td>5(25)+10(150)=1625</td>
</tr>
<tr>
<td>X</td>
<td>6(X)</td>
<td>5(X)+10(6X)</td>
</tr>
</tbody>
</table>

This problem situation indicates that the number of dimes is determined by the number of nickels. If the number of nickels is 5, then the number of dimes is 30, and the value of the coins in cents is 325. Since 325 cents is less than 1495 cents, 5 nickels is not a solution. Continuing this process, the student should see a pattern developing. If the number of nickels is denoted by the variable X,
then the algebraic phrase 5(X)+10(6X) represents the total value of the money. The portion of the problem statement that permits the student to write an equation is "the value of the money is 1495 cents." Thus, the equation is "5(X)+10(6X)=1495." This equation gives a general description of the situation and summarizes the information in the problem.

Using the TB method is more complex for the teacher than it appears on the surface. One critical factor in using the TB procedure is deciding the best or most appropriate time to introduce the variable in the table. No evidence in the literature has revealed this for the TB procedure indicated above or the Translation method that will be discussed. The literature revealed mixed results about which strategy is most appropriate to use when teaching solutions to word problems. These results may have been mixed because the timing factor was not investigated for introducing a variable in solving word problems. Thus, the amount of numerical exploration needed before students can conceptualize and use variables successfully is unanswered.

Translation Method

1. Read the problem carefully and then determine the given information and the wanted information.

2. Choose a variable to represent the information wanted, and then use this variable to express the relationship between the given information and the wanted information.

3. Write an equation expressing the relationship between the given and the wanted information.
Solution

1. Given Information: Value of the coins in cents is 1495.
   Wanted Information: The number of dimes and nickels.

2. If \( X \) = the number of nickels, then \( 6X \) = the number of dimes.
   The value of the nickels is \( 5(X) \) and the value of the dimes is \( 10(X) \).

3. Equation: \( 5(X) + 10(6X) = 1495 \).

This method introduces the use of a variable initially, allowing students to think abstractly by translating words and phrases into algebraic symbols, and then writing an equation describing the given situation.

If students can think abstractly, an advantage of the translation method is that they might be able to translate quickly, write the equation, and come up with a solution. However, if students cannot think abstractly, they may never come up with an appropriate equation, and if they have not been exposed to a systematic way of guessing and checking, or do not know equation solving procedures, the solutions may never be found.

Both methods discussed have advantages and disadvantages. A disadvantage of using the table-building method is that the process can be laborious if students do not have access to a calculator. This method however has a number of advantages and allows students to explore a variety of techniques in determining the solution to a
problem. Some advantages of this method are: (a) seeing numerical relationships, (b) guessing and checking possible solutions, (c) using successive approximation, (d) seeing that a variable represents numbers from some set, (e) using analogy rather than abstract reasoning in writing an equation, (f) recognizing patterns, (g) moving from specific values to generalizations. The table allows students to determine a numerical solution even if the equation is written incorrectly. However, if the equation is written correctly it can be solved to check the solution obtained by the table-building process.

It is a general belief that certain thinking patterns would be used in the thinking processes of students exposed to both methods. Dalton (1974) found that patterns do exist in the thinking processes used by students in solving word problems with at least two modes of thinking, deduction and trial-and-error being used. He found that subjects who used trial-and-error tended to be more effective problem solvers. There were implications that prompting students through the use of "guiding questions" led to the effective use of certain heuristic processes.

Understanding the underlying thinking processes leading to effective mathematics performance should be of theoretical and practical interest to mathematics educators. Researchers need to know and understand thinking processes used by students in performing problem-solving tasks so that improved and appropriate methods of mathematics instruction might be designed to meet students' needs.

Though not of primary concern in this investigation, the
Investigator wanted to identify some thinking processes students were utilizing in both methods. The intent was to recognize patterns that were used in thinking when students were solving word problems that might be useful for further investigation.

In addition to the thinking processes used by students, the manner and the time students are taught a concept might affect their understanding and use of variables in problem-solving. Since students have misconceptions of variables and difficulties in solving word problems, the manner in which they are taught the concept of variable and the time of introduction of a variable in solving problems might affect their understanding and use of variables.

Many students at the beginning algebra level are forced to use variables when they actually are not ready to conceptualize the idea. Because of certain methods or procedures used by teachers, students are introduced to variables, and they form many misconceptions about variables. Some students need more inductive experiences than others to understand the concept.

Does the amount of numerical exploration students experience in solving word problems before a variable is introduced affect their conceptual understanding of variables? This question was investigated in this study by forming two experimental groups for the TB method and one comparison group for the T method. Each group was taught the same type of word problems but the manner or approach was different, and the time the variable was introduced was different for each group. Descriptions of these groups are given in the treatment section.

Prior to treatments, students were given a pretest to assess their
Initial understanding and use of variables. This test assessed students' use and understanding of variables at four levels of understanding. These levels of understanding are based on Piagetian intellectual or cognitive levels of concrete and formal thought. After the treatments, the criteria for assessing students' achievement and understanding were a problem-solving test and a posttest (a parallel form of the pretest). The problem-solving test covered the treatment materials and assessed a student's ability to use variables and problem-solving procedures to solve word problems. The posttest did not cover directly what was taught in the treatment groups, but allowed students to apply or transfer their learning from the treatment materials. The test also assessed their abilities to use and understand variables. These tests are described in more detail in the Instrumentation section.

Thus, the essential difficulties students possess, need to be corrected in order that these students are prepared to solve problems.

**Problem Statement**

The objective of this study was to investigate the effects a Table-Building (TB) Method of teaching word problems will have on a student's ability to understand and use variables. A Translation (T) Method of teaching word problems was compared to the TB Method. Related to the objective were two primary and two secondary research questions.
Primary Research Questions

1. Will students who were exposed to Instruction in the Table-Building Method be more successful in understanding and using variables than students who were exposed to Instruction in the Translation Method?

2. Does the time at which a variable was introduced in the table-building process affect a student's ability to use variables and to solve word problems?

Secondary Research Questions

1. Will students who were exposed to Table-Building Instruction have better problem-solving performance than students exposed to Translation Instruction?

2. What relationship will exist between students' overall levels of cognitive development and their ability to solve word problems?

Assumptions and Limitations

The population was limited to ninth grade pre-algebra students enrolled at East High School, an Inner-city public senior high school in Columbus, Ohio. The students for the study were of average to below average abilities in mathematics. Thus, the generalizability of the study is limited.

It was assumed that the full range of Piagetian levels of reasoning abilities would be present in the classes. Because of the difficulties many students have experienced with algebra and the solving of word problems, many high school students have developed
negative attitudes and prejudices against algebra and word problems before beginning the study of a course in algebra. Because of the crossection of the student population that attend public high schools it was further assumed that different ability levels would exist among students in a pre-algebra course.

The word problems used for the study were limited to number and coin problems. It was also assumed that the motivation or interest levels of some students would not be as high as others because of factors such as previous mathematics experience, teachers, performance in mathematics, self-concept, and mathematical abilities. Treatment materials were developed by the Investigator. Treatments for each group were limited to one session per week for eight weeks and were administered by the Investigator.

Definition of Terms

1. Variable: A symbol, usually a letter, that represents or stands for the numbers in some set.

2. Word Problems: This term often refers to verbal, story, or thought problems. Such problems require students to think, use logical reasoning and understanding in writing equations, and to use problem-solving procedures to determine numerical solutions. For the purpose of this study, word problems were restricted to number and coin problems.

3. Mathematics Achievement: Raw scores attained by students on a problem-solving test designed to measure a student's ability to use variables and to find the solution to word problems.
4. **Table-Building (TB) Method**: An instructional method used to teach the concept of variable and solutions to word problems. This method required students to think inductively by generating a table of specific values and by using a guess and check technique to test the correctness of the values. Students were required to look for patterns and use successive approximations to determine the solutions. The use of analogy rather than abstract reasoning was used in approaching a generalization to write an algebraic equation to find the solution to the problems.

5. **Translation (T) Method**: An instructional method used to teach the concept of variable and the solution to word problems. This method required students to think deductively by using a set of rules which involved the use of variables initially, translation, and then writing equations to determine the solution. Students approached the solution from a generalization rather than from specific numbers.

6. **Concrete Level Reasoner/Thinker**: An individual who is able to think in a logically coherent manner about objects that do exist and have real properties and about actions that are possible; who can perform the mental operations involved both when asked purely verbal questions and when manipulating objects (Sinclair, 1971). Understanding Levels I and II on the pretest operationally define early and late concrete thinkers respectively.

7. **Formal Level Reasoner/Thinker**: An individual who has fully mastered the operations described for the concrete stage and, in addition, who has the capability of reasoning hypothetically, possesses a
system of combinatorial operations which give rise to the hypothetical thought and has the ability to use proportional reasoning schemes (Sinclair, 1971). Understanding Levels III and IV on the pretest operationally define early and late formal thinkers respectively.

8. **Treatment Materials**: Those instructional materials, handouts, worksheets, and activities which were designed to teach the concept of variable and the solution to word problems for both TE and T methods of instruction. (See Appendices A and B).

**Hypotheses**

**Hypothesis 1**: All three treatment groups will perform at the same levels on the pretest.

**Hypothesis 2**: The two experimental groups will perform at the same levels on the posttest.

**Hypothesis 3**: The TB treatment groups will perform at the same levels as the T treatment group on the posttest.

**Hypothesis 4**: The two experimental groups will perform at the same levels on the problem-solving test.

**Hypothesis 5**: The TB treatment groups will perform at the same levels as the T treatment group on the problem-solving test.

**Hypothesis 6**: The T treatment group will perform significantly better than the TB treatment groups on translation.

**Hypothesis 7**: The TB treatment groups will score significantly higher than the T treatment group when only numerical solutions to problems
are assessed.

**Hypothesis 8:** There will be no significant differences among the treatment groups on the problem-solving test at each of the four levels of understanding (cognitive development).

**Hypothesis 9:** Formal reasoners will perform significantly better than non-formal reasoners on the problem-solving test.

**Hypothesis 10:** Hypothesis ten was considered in three parts:

**H10A:** There will be a significant positive correlation between students' posttest and problem-solving test scores.

**H10B:** There will be a significant positive correlation between students' pretest and problem-solving test scores.

**H10C:** There will be a significant positive correlation between students' pretest and posttest scores.

---

**Plan of the Report**

This report is presented in five chapters. Chapter I was to acquaint the reader with the research problem being investigated. Chapter II presents a review of related literature. Chapter III describes the methods and procedures for this study. Descriptions of the design, instruments, treatments, and research and statistical procedures are given. Chapter IV gives the data analysis and results. Subjective analyses of qualitative data and statistical analyses of quantitative data are presented. Chapter V discusses research findings, gives a summary, conclusions, implications, and states recommendations for future research.
CHAPTER II
REVIEW OF LITERATURE

Conceptual understanding and use of variables are of prime importance for the successful study of algebra. Algebraic understanding depends on understanding variables, and is a useful tool for solving problems. The ability to solve problems successfully is vital and central in learning and applying mathematics. The literature reviewed for this study includes research in four areas:

(1) students' thinking about variables
(2) students' understanding of variables
(3) students' understanding of algebra
(4) problem-solving procedures

Students' Thinking About Variables

Understanding thinking processes used by students in performing such mathematical tasks as understanding and using variables should be of theoretical and practical interest to researchers and mathematics educators so that improved or appropriate methods of instruction might be designed to match students' abilities. Some factors affecting students' thinking about variables are: (1) reasoning abilities, (2) information-processing, and (3) understanding of mathematics.
Reasoning Abilities

Reasoning, which is a form of thinking, is the mental process of drawing conclusions from known or presumed facts.

A variable, which is usually a letter representing numbers in some set, can be used to describe a general situation concisely. In this context one might consider variables as generalized numbers. To generalize means to derive a broad conclusion or principle from particular instances or facts. Traditionally and in most algebra classes the concept of variable has been developed from a generalization. Students are given a variable to translate words and phrases into an equation describing a general situation. The equation is then solved for a specific number. This allows students to think from a general situation to a specific instance, which is considered by many as deductive thinking. It is the investigator's opinion that many students have not reached the appropriate level of intellectual development or readiness to conceptualize variables using such an approach by the time they enroll in algebra.

According to Emans (1973), learning theorists are greatly concerned with the nature and process of learning that leads to the development of logical thought. He indicated that learning theorists have always been interested in the process of mental development that occurs over time in individuals and will enable them to function successfully in their environment. It was the investigator's intent that the concept of variable be developed from specific instances (numbers) leading to a generalization from these numbers. Students'
readiness for the use and understanding of a variable will depend on their development of thinking. Emans stated that the achievement of truly logical thought depends on a qualitatively and quantitatively changing series of interactions of the child with the environment.

Kamill (1979) summarizes Piaget's study of the development of thought in children. Piaget indicated that the development of mental abilities occurs in four stages: sensorimotor, preoperational, concrete operations, and formal operations. He stated that individuals go through the stages in an invariant order, but there is a variability as to when a given stage is achieved. Based upon the works of Piaget as summarized by Kamill, these four stages of the development of mental abilities are described.

The sensorimotor period, the first stage, begins at birth. At this stage the child begins to attach some permanence to seen objects. The second stage, preoperational begins at about eighteen months, and the acquisition of language is the prime characteristic of this stage. The third stage is concrete operations. In this stage the child identifies concrete situations, but does not extrapolate beyond the immediate factual situations. These operations are mentally internalized, and reversible systems are based on manipulation of classes, relations, and quantities of objects. This level of thought begins at about age seven and ends at about age eleven, or will continue until the last and highest level is reached. The highest level of thought is formal operations. At this level reasoning is done with both concrete and formal concepts. Relationships, properties, and theories are all used. Individuals are capable of abstractions at this
level. The concrete patterns used are applications of simple operations to real objects and experience. The abstractions in formal thought, however, are seen as patterns of relationships, hypotheses, or postulated properties.

Based upon the levels of thinking identified by Piaget, understanding the concept of variable requires students to think abstractly. This is a formal concept requiring students to reason concretely as well as formally. The investigator's intent was to allow students to use a table of numerical values in developing the concept of variable. The amount of numerical exploration students receive in table-building should provide them with the necessary concrete experience needed to conceptualize a variable, but a critical factor in using table-building may be deciding the best or most appropriate time to introduce the variable. Students' readiness in terms of thinking will play a major factor in making this decision.

In addition to understanding the concept of variable, the use of the table requires students to think a certain way. They have to know the appropriate headings to use, the operations to perform, as well as the use of approximations, pattern searching and guessing and checking. It appears that students need to use a combination of concrete and formal concepts in developing the concept of variable and determining the solutions to problems.

Information-processing

Knowing what the learner does when performing a task or acquiring a new concept such as understanding and using variables is important in identifying students' thinking processes. According to Hiebert
(1981), the short-term memory is the center of all thinking or Information that can be processed at the same time. Hiebert indicated that many researchers suggest that children's restricted processing capacity has considerable consequences for the curriculum because it may place severe constraints on children's abilities to profit from instruction. Instructional tasks developing the concept of variable require children to receive, encode, and integrate information. In many cases children may possess all of the necessary skills for particular tasks in developing the concept of variable and still fail the tasks. The reason for this failure may be children's restricted capacity to deal with all the information needed to complete the task (Case, 1945). Brown (1978) suggested that individuals might be taught strategies to process information more efficiently and push back the limits that might otherwise be imposed by their restricted processing capacity. For more information on information-processing, see Duren (1980, p.18).

Understanding Of Mathematics

The understanding of mathematics is a factor which might influence students' thinking concerning variables in mathematics. It is generally accepted that students' initial understanding of specific concepts, rules, relationships, notations, operations, and symbols greatly influences their performance in mathematics. What does it mean to understand the concept of variable in mathematics? When students say they understand the concept of variable do they actually understand? Many students often seem not to understand the mathematics with which they are dealing, but instead they tend to follow memorized
rules or procedures. If teachers are to find out what students are thinking, they need to know what students are understanding.

There exists no universal definition for understanding in mathematics, but several definitions will be stated that might enable individuals to know when they do understand some concept, idea, or principle in mathematics. Consider the following definition of understanding which is a combination of many ideas:

We presume it is: comparing input data with many existing things you already know; looking for patterns, contrasts, comparisons; looking for apparent inconsistencies or contradictions; making careful note of the cues which can be used in the future to guide future selection of solution methods; trying to identify and retrieve an appropriate "assimilation paradigm" or schema, and to synthesize a new one if no appropriate old one can be found in memory; making careful critical appraisal of how well the present situation matches the retrieved schema that has been selected; and trying to develop appropriate "meta-language" in order to be able to analyze mathematical situations effectively (Davis, Jockusch, & McKnight, 1978b).

In an attempt to help teachers come to grips with understanding, Holt (1964) suggested the following:

It may help to have in our minds a picture of what we mean by understanding. I feel I understand something if and when I can do some, at least, of the following:

1) state it in my own words; (2) give examples of it; (3) recognize it in various guises and circumstances; (4) see connections between it and other facts or ideas; (5) make use of it in various ways; (6) foresee some of its consequences; (7) state its opposite or converse.

He suggested that this list is only a beginning, but it may help
teachers in the future to find out what their students really know as opposed to what they can give the appearance of knowing, their real learning as opposed to their apparent learning. It may help in planning instruction for developing the concept of variable and assessing students' understanding of this concept.

According to Van Engen (1953), understanding is described as a process of organizing and integrating knowledge according to a set of criteria. The definitions stated by Holt and Van Engen, suggested that students do not always understand completely. Teachers need to know the degree of students' understanding and supply them with additional or the sufficient knowledge that might enable them to understand completely.

Davis (1978a) proposed that the moves teachers make in teaching mathematics can serve as a basis for defining understanding in mathematics. He suggested that teachers view moves as ways to assess students' knowledge and to plan for instruction. He further stated that moves occur as a result of questions, exercises, problems, explanations, demonstrations, directions on task cards, and almost any other teacher-student Interactions. Davis said teachers can use moves to evaluate and diagnose the degree of understanding held by a particular student for a given item of mathematical knowledge. He stated that students can be said to have understanding to the extent that they can make a complete set of moves.

Cooney, Davis, and Henderson (1975) identified the following types of mathematical knowledge commonly taught in school mathematics: (1) concepts, (2) generalizations, (3) procedures, and (4) numerical facts.
As a result of the mathematical knowledge stated above, Davis (1978a) proposed the following model of understanding in mathematics. For each type of knowledge the model is based upon two levels of understanding:

1. **Concept**
   - Level 1: Examples and nonexamples of the concept.
   - Level 2: Characteristics of the concept.

2. **Generalization**
   - Level 1: Understanding what the generalization says.
   - Level 2: Understanding why the generalization is true.

3. **Procedure**
   - Level 1: Understanding how the procedure works.
   - Level 2: Understanding why the procedure works.

4. **Number Fact**
   - Level 1: Understanding what the fact says.
   - Level 2: Understanding why the fact is true and realizes its significance.

The model that has been described is applicable to instruction involving students' understanding and use of variables and problem-solving procedures. It is important that students understand basic number facts, procedures that can be used in solving problems, the concept of variable, and are able to approach generalizations. According to Cooney et al. (1975), statements of relationships between concepts are called generalizations. Developing the concept of variable by a table-building procedure requires that students start from specific numerical relationships and then approach
generalizations. The intent here is to teach for generalization. According to the indicated model, students need to understand what the generalization says and why the generalization is true. Cooney et al. (1975) indicated in the model that children understand a generalization to the extent that they can make the following moves: (1) show understanding of the concepts in the generalization, (2) state the generalization in their own words (paraphrase it), (3) create or recognize instances of the generalization, (4) tell when the generalization can be used or state conditions under which it is true, and (5) apply the generalization in exercises and simple problems. A specific example of teaching for generalization in the context of developing the concept of variable and problem-solving will be considered.

The sum of two numbers is 97. The larger of the two numbers is 5 more than 3 times the smaller number. What is the smaller and larger number? Write an equation using the variable X to express that the sum of the smaller and the larger number equals 97.

A table-building procedure can be used in teaching for this generalization. On the basis of the relationship between the wanted and the given information in the problem the following labels for table headings are appropriate: (1) smaller number, (2) 3 times smaller plus 5, (3) larger number, (4) smaller number plus larger number, and (5) sum. The students' goal is to determine the solution to the problem and write an equation satisfying the given conditions. Students will gain numerical experience with numbers in the table by guessing and
checking possible answers and examining relationships between various quantities as indicated by the headings in the table. Students may eventually come up with the answers, but may not understand the generalization. An assessment of students' understanding of the generalization can be made. The following might be used in making such an assessment: (1) Explain how to find the larger number if the smaller number is given, (2) Explain how to find the smaller number if the larger number is given, (3) Explain how to find the larger number if the smaller number and the sum are given, and (4) Explain how to find the sum if the smaller and the larger number are given. An attempt was made to focus on the generalization part of this model since it is pertinent to this study.

Skemp (1976) described "understanding" as relational ("knowing both what to do and why") and instrumental (the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works). Byers and Herscovics (1977) and Herscovics (1979) expanded on Skemp's definition. They suggested that there are four different kinds of understanding of mathematics:

1. **Instrumental understanding** is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.

2. **Relational understanding** is the ability to deduce specific rules or procedures from more general mathematical relationships.

3. **Intuitive understanding** is the ability to solve a problem without prior analysis of the problem.
(4) **Formal understanding** is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning.

On the basis of the different kinds of understanding indicated, it appears that students more often apply remembered rules to the solution of a problem without knowing why the rules work rather than knowing both what to do and why. The four kinds of understanding, Instrumental, relational, intuitive, and formal are all applicable at some point in understanding and using the concept of variable and solving problems. Intuitive understanding might best apply to situations where students are moving from specific instances and approaching generalizations. An example would be developing the concept of variable in table-building through problem-solving procedures. However, Instrumental and formal understandings may be applied in approaching such generalizations. As described by Byers and Herscovics it appears that Instrumental, relational, and formal understanding might be applied to situations where students move from a general situation to a specific instance. Traditional methods of teaching word problems and developing the concept of variable might apply here.

Thus, there exists no one specific way to understand mathematics. The definition of understanding is not universal in the context of understanding and learning mathematics. The definition should be based on a specific situation at hand, and teachers must assure that the students whose thinking processes they are trying to understand, know
and understand the definition that is most applicable in performing a specific mathematical task.

**Students' Understanding Of Variables**

**Meaning Of The Concept Variable**

A variable, which is usually a symbol or letter from the alphabet takes on a variety of meanings based on its use. It usually stands for the numbers in some set, or can take on values of any of the numbers of some set, such as the set of real numbers, the rational numbers, numbers between two given numbers, or all numbers. The set of values which a variable may assume is sometimes stated explicitly and sometimes only implied.

Collins (1975) found that students had different meanings for variables based upon what appeared "real" to them. They mapped a variable into a specific number, tried a set of numbers by guess and test; and viewed variables as generalized numbers.

Kuchemann (1978) expanded on the research of Collins. He found six different ways letters are used in beginning algebra: letter evaluated, letter ignored, letter used as an object, letter used as a specific unknown, letter used as a generalized number, and letter used as a variable.

**Misconceptions Of Variables**

Research evidence indicates that students have misconceptions and difficulties with variables (Firth, 1975; Wagner, 1978, 1981, 1983; Rosnick, 1981). Firth studied 15 year olds' concepts of variable. He
said errors that occurred in writing mathematical statements which involved variables were caused by a misunderstanding of the task or the meaning of variables. The confusion between the linear order of the alphabet and the linear ordering of whole numbers is a common mistake for students just being introduced to variables (Wagner, 1981). She stated that some pre-algebra and beginning algebra students seem to think that changing the literal unknown in an equation may also change the solution.

Rosnick (1981) stated that first-year engineering students had difficulty with translation. He indicated that the misconceptions students had concerning the use of letters in equations contributed to this difficulty. Students make certain kinds of errors by thinking of say x as representing pencils, when in fact, x represents the number of pencils. They need to know the different ways letters can be used in equations. It could be that many students have not reached the level of intellectual development to make the distinction between the uses of variables. Since equations are essential in mathematics and are used through algebra, students need to have a conceptual understanding of equations.

According to Wagner (1983), literal symbols are easy to use but hard to understand. Wagner pointed out that recent research has identified several factors that make literal symbols easy to use but hard to understand. These factors can be grouped under two main headings:

1. Literal symbols are like numerals, only they are different.
2. Literal symbols are like words, only they are different. Based on these factors, Wagner indicated that as students move to the level of algebra, they should begin to realize that a letter behaves like a numeral in that it may represent a single number and may be subject to numerical operations and relations; they must also realize that a letter behaves very different from a numeral with regard to the juxtaposition convention and the sign of the number. She said they should recognize that a letter is similar to a word in that it can mean different things in different contexts, but a letter is different from a word in that it must refer to the same thing throughout a single context.

Wagner suggested that if teachers are to help students develop better understanding of literal symbols teachers must first be aware of the myriad ways variables are used and recognize the particular characteristics they exhibit in various contexts. She said teachers then need to alert students to the properties of literal symbols and point out which characteristics are unique to the literal symbols themselves. Wagner also said if teachers want students to gain a real appreciation for the power of literal symbols and yet not be overwhelmed by a lengthy, and probably meaningless, discourse on their various characteristics, then teachers need to introduce these ideas gradually as different uses of literal symbols appear in the curriculum.

On the basis of the possible ways that letters appear to students, teachers need to make use of these ways based on students' levels of understanding or cognitive development. As indicated earlier,
Kuchemann (1978) found six different ways letters can be used in beginning algebra: a letter evaluated, ignored, used as an object, used as a specific unknown, used as a generalized number, used as a variable. The six ways of interpreting and using letters were identified from testing and interviewing secondary school algebra students. Hart (1981) described these ways of using letters that were identified. She said if a letter was assigned a numerical value from the outset it was considered a letter evaluated. She indicated that children ignored a letter or its existence but without giving it a meaning. The letter was considered not used. According to Hart a letter used as an object was regarded as a shorthand for an object or as an object in its own right. A letter used as a specific unknown was regarded as a specific but unknown number that could be operated upon directly. A letter used as a generalized number was seen as representing or at least as being able to take several values rather than just one. A letter used as a variable is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values. Students need to have a conceptual understanding of variables since variables make up equations and functions, the tools of applying mathematics and solving problems. Letters from the alphabet are often used in equations to represent specific things. They are sometimes evaluated, used to represent specific unknowns, and in most cases used to represent a variable.

Freudenthal (1973) suggested that students need to know that letters in mathematics occur as ambiguous names rather than as elements of a set of letters and that different letters can represent the same
numbers. He argued that the initial experience with variables should be in terms of indeterminates and that treatment of variables as unknowns should be delayed until considerable experience and sophistication in mathematics has been attained.

Herscovics and Kieran (1980) indicated that many researchers found that introducing the concept of equations through problem-solving managed to reach many students. Teachers are often confronted with the question of choosing between a general or more specific presentation. They stated that general approaches tend to be more formal and reach fewer students, whereas restricted approaches are relatively more concrete and accessible to more students.

Introducing the concept of equations through problem-solving has not been very successful for some students. Misunderstanding of the nature of variables is a major factor; thus the concept of variable needs to be developed through problem-solving procedures. The last section of this review is devoted to problem-solving procedures. The next section will focus on students' understanding of algebra.

Students' Understanding Of Algebra

Factors such as students' reasoning abilities, information-processing, and understanding of mathematics that have been discussed previously, have an effect on students' understanding and performance in algebra. According to Piaget, at the formal level of thinking, students' reasoning involves concrete and formal concepts. In order for students to be successful in algebra, they need to be at the formal level of thinking. It is evident that many students taking
their first course in algebra have not reached the formal level of thinking.

When students perform well on a particular topic in algebra, do these students understand the concept being taught? Could it be that performance here was based on memorized rules or procedures rather than understanding relationships or concepts? When students perform poorly, is this due to their not understanding the subject matter, or could there be other factors such as errors in computation, procedures, or simplification? What does it mean to say that a student understands algebra?

In order to understand algebra, students need to cultivate a high degree of competency in abstract thinking and make use of higher mental processes. Breslich (1939) stated that mental processes are not only compactly integrated but as thinking matures the resulting combinations of elements also mature; the organization of mental processes becomes more and more complex. Breslich stated that from interviewing high school algebra students, every case revealed the fact that all algebraic thinking is complex. Teachers might ask: Do students feel the same way today about algebra? Are students different from those that existed five decades ago? Have algebraic concepts and thinking processes changed any?

Rachlin (1981) stated that the literature on the understanding of algebra has largely developed from three theoretical sources: information-processing theory, extension of Piagetian theory of cognitive development, and extension of Vygotskian theory of cognitive development. Rachlin's review was a compilation of the literature
applicable to the study of reversibility, flexibility, and the ability to generalize and transfer for learning algebraic processes and operations.

Results of students' performance in algebra from the National Assessment of Educational Progress (NAEP) give a national representation of students' performance in algebra from the two assessments administered in 1972 and 1978 respectively (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981).

The NAEP data reported by Carpenter et al. (1981) has provided information about the importance of students' understanding of mathematical concepts and processes. The age groups of students assessed were 9, 13, and 17-year-olds, and also young adults. This assessment was a test of retention for some of the students, especially the 17-year-olds and young adults who had at least a year of algebra (Carpenter, et al., 1981).

Each content area of the test was assessed at four levels: knowledge, skill, understanding, and application. In general, results indicated that students failed to master a broad range of basic skills. Students' performance showed a lack of understanding of basic concepts and processes in many content areas, such as measurement and computation with fractions. The assessment focused on several algebraic topics: symbolism, algebraic representation or translation, algebraic manipulations, equation solving, use of formulas, graphing, and word problems.

As a result of the performance on algebraic exercises, several general observations were made. As described by Carpenter et al.
(1981), students who had a full year of introductory or pre-algebra scored very close to the mean of the 17-year-olds. Those who took a year of general, business or consumer math scored about 5-10 percentage points below the mean. Students with one year of algebra scored about 10 points above the mean. Those who had a second year of algebra and a year of geometry scored 5-20 points above the average for all 17-year-olds.

The results revealed that students' performance on exercises requiring translation to an algebraic formula by symbols was not high. Those with formal algebra course work scored higher. The high school students showed an intuitive knowledge of simplifying algebraic expressions, but showed little ability to manipulate rational expressions.

Carpenter et al. (1981) stated that one-third of those students with a year of Introductory algebra, less than one-half with one year of algebra, and less than two-thirds with two years of algebra solved standard linear equations correctly. Students could graph ordered pairs on the xy-plane, but could not show a knowledge of the relationship between equations and their graphs. Students' ability to substitute values into formulas was dependent on their algebra background.

According to the NAEP data, applying algebraic skills to solve problems appeared to be a major area of difficulty for most students. The majority of students at all age levels had difficulty with nonroutine problems that required some analysis.

Students' performance on algebra from the NAEP data as reported by
Carpenter et al. (1981) indicated that students had not reached the point of mastery, but did not suggest complete failure. Little evidence suggests any significant change in students' performance between the first and second assessments. Evidence did show that a cluster of 15 to 20 percent of the 17-year-olds showed mastery of algebraic skills and concepts, and about 30 to 40 percent of the population showed an intuitive knowledge of algebraic processes.

Results of the third mathematics assessment of the National Assessment of Educational Progress (NAEP) provided a basis for examining students' performance in mathematics between 1978 and 1982 (Carpenter, Lindquist, Matthews, & Silver, 1983). Exercises that required variables to be used to express relationships showed a relatively consistent pattern of decline, and problem-solving continued to be a major area of concern. Very little change occurred in problem-solving performance between 1978 and 1982. The data suggested that students did not carefully analyze the problems they were asked to solve. They generally tried to use all the numbers given in a problem statement in their calculation, without regard for the relationship of either the given numbers or the resulting answers to the problem situation. The data suggested that more attention needs to be given to increasing students' understanding of mathematical problems. Students should be given opportunities to reflect on the problem situation and the relationships among the physical situation, the data, the unknown, the computation, and the answer.

According to Shevarev (1975), pupils' errors in solving algebraic problems can be divided into three groups:
(1) Sometimes a pupil does not know the rule that must be applied or does not know it precisely.
(2) Sometimes they know the rule well but still do not know how to apply it.
(3) Finally, there are cases where a pupil knows the rule, is able to apply it, but nevertheless acts contrary to it.

He suggests that the third is the most important and most interesting. Shevarev indicated that in the first two cases, the basic causes of errors are known; hence, the practical questions are easily resolved. He further stated that the third instance is different; we have no satisfactory explanation of the causes of errors. In this connection, practical questions of how to prevent such errors, and what to do if they have already occurred remain unanswered, and is worthy of consideration.

The limited capacity of the short-term memory is an important factor that might affect students' thinking and performance in algebra. With the amount of abstract thinking that exists in algebra, students need to be able to remember certain concepts and facts so that they might perform effectively. Since the short-term memory is so limited, students need to retrieve information from their long-term memories in some proper form and integrate it with existing knowledge in the short-term memory so that some logical thought process might be formed, and can be used in performing mathematical tasks. It is not always easy to retrieve information from the long-term memory by simple recognition and recall.
Duren (1980) taught rules to seventh graders by using three different methods of instruction: deductive problem-solving, inductive problem-solving, and rule-example-practice. Results from Duren's study indicated that students who had learned rules taught either by the deductive or inductive approaches exhibited greater ability to recall the rules, and to reconstruct the rules if not recalled, than those taught by the rule-example-practice method.

The study of students' thinking processes and performance in algebra raises questions about their understanding which is the key to successful performance.

Karplus and Karplus (1974) suggest that many adolescents are still operating at a level of concrete reasoning patterns. Many students can not handle the manipulation of symbols and abstractions because they have not reached the formal operations level of thinking.

**Problem-Solving Procedures**

Problem-solving skills are essential in learning and applying mathematics. The research on problem-solving will focus on procedures for solving word problems. Misconceptions and difficulties with word problems, and techniques for solving word problems are factors that will be considered.

**Misconceptions and Difficulties With Word Problems**

Research evidence indicates that students have misconceptions and difficulties in solving word problems (Stright, 1938; Yeshurun, 1955; Alexander, 1960; Martin, 1963; Kulm, 1973; Maffei, 1974; Knifong & Holtan, 1975; Herscovics & Kieran, 1980; Clement, 1982). A summary of
research findings conducted by Settle (1977) indicates that there exist a variety of methods students can follow in solving verbal problems. For most of the approaches he said equation construction is far and away the major source of difficulty for students. Much of the research he cited indicated that difficulties learners have in solving word problems are vocabulary deficiencies, reading difficulties, translation difficulties, and equation construction. Settle indicated that the reading difficulties appear to be not as serious as some educators contend. He found that equation construction difficulties are much more serious than is commonly realized, especially when traditional approaches to writing the equation are used.

Techniques For Solving Word Problems

According to Yeshurun (1979), for years two main techniques have been used for solving word problems leading to equations: the Intuitive technique and the translation technique. He said that the purpose of the Intuitive technique is to show the students the solution to various problems leading to equations, explaining all the steps in hope that the student will be able to solve the next problem unaided even though no general explanation is given of what to do, when to do it, and why to do it. As indicated by Yeshurun the translation technique originated in the period about 1910 to 1930 when the logical nature of both grammar and mathematics was stressed. O'Brien (1956) stated that "to be able to translate from one language to another, a person must understand what is said in the first language and (must know) the vocabulary, syntax, and idiomatic structure of the second. Yeshurun said the transformation of the verbal problem into a mathematical
equation or equation system is based on the cognition (or awareness) of the connection existing among the quantities involved, referred to as the cognitive method.

A review of research by Settle (1977) suggested two major viewpoints for the attack used by the problem solver on a verbal problem: Gestalt psychology and Information Processing System (IPS). Settle stated that the Gestalt view of verbal problem solving is that of getting to know the problem as a whole, that is, understanding the problem before formulating the solution. The Gestalt depends on the semantics of the problem. It is mainly concerned with the generalization and transfer aspects of the problem. He further stated that the IPS concept places its emphasis on the syntax of the problem. The solver begins immediately pursuing a translation of the verbal problem to mathematical symbols. According to Kinney (1958), in reviewing the literature, there is no one "best" procedure for solving problems, but he stated that some systematic approach is better than none at all. Based on the ways students view problems according to the Gestalt and IPS viewpoints, students should be given instruction in some problem-solving procedure.

Traditional methods of solving word problems which are commonly used in most algebra classes require students to translate words and phrases into mathematical symbols by the initial use of some variable (letter). Such traditional methods are deductive in nature, requiring students to start with a generalization and approach a specific solution. This method of solving word problems may be viewed from the IPS standpoint. Research has indicated that students have difficulty
In constructing the equation and may not have reached the appropriate level of Intellectual development needed to make the correct translation.

According to Hendrix (1961) the Inductive method is a method of learning by discovery. Hendrix said that "the fallacy in the Inductive method lies in its confusion of verbalization of discovery with the advent of discovery itself." She said in some cases correct verbalization of a discovery does emerge immediately after the dawning of awareness, but these cases are rare. Hendrix (1961) indicated that the only teachers who have been really successful with the Inductive method are the rare ones who have acquired the knack of making the learner aware of generalizations before calling for verbalizations. She became conscious of the nonverbal awareness stage in discovery learning after experience with the Inductive method. The following are three mistakes Hendrix indicated that beginners are likely to make when they plan and attempt Inductive teaching:

1. They begin to call for generalizations, exerting anxious pressure on their classes, long before the students have noticed any basic similarity among the examples the teacher has presented. The discussion degenerates into a guessing contest during which the students try to find out "what it is the teacher wants me to say."

2. The teacher often calls for statements of generalizations when the students do not possess the vocabulary and rules of sentence formation necessary for a precise verbalization of the generalization, even if they have "seen it."
3. The teacher often confuses generalizations, which had to be discovered in the first place and hence, are appropriate subject matter for rediscovery, with situations in which the generalization is arbitrary - something that is merely a matter of definition.

Hendrix (1961) said a major difficulty in teaching inductively has been finding some way of telling when students are far enough along in an inductive approach to ask them to state the generalization. The problem which Hendrix worked on for nearly a decade was as follows: To what extent, if any, does the way in which one learns a generalization affect the probability of his recognizing a chance to use it? Hendrix (1947) stated that even in a field as abstract as mathematics, formation of a generalization can be, and often is, a sub-verbal process which changes the groping for a new clue into a directed inquiry. She adopted the following assumption as a criterion:

The best evidence that one possesses a generalization is behavior that would have been impossible without that generalization.

Hendrix's first attempt at quantitative experimentation on the problem occurred in November, 1946, with eleven volunteer subjects in a psychology-of-learning course at the University Of Chicago. According to Hendrix the subjects were placed in three groups of four, four, and three, and the same generalization was taught to each group by a different method. Two weeks later all subjects were tested and were allowed to answer the question by recognizing a chance to use the generalization taught. Results from this experiment suggested such
decided differences that the experiment was repeated twice in February, 1947, once with eleventh- and twelfth-grade boys in University High School at the University of Illinois and once with college girls enrolled in primary education at MacMurray College, Jackson, Illinois. According to Hendrix (1947), both experiments involved teaching three different groups the same generalization by a different method. A description of the methods used are as follows:

1. **Method I** - the usual pedagogical procedure in which the teacher or textbook states a generalization before, after, or accompanying a clarifying illustration and makes sure that the newly learned statement is applied to several examples.

2. **Method II** - involved teaching for unverballized awareness. In this method the stage must be set in such a way that as soon as the generalization dawns, the learner will begin to apply it.

3. **Method III** - involved the conscious generalization procedure. No one was asked to state the generalization until after his or her behavior indicated that the generalization had already taken form on the unverballized level.

From both experiments, Hendrix (1947), found that the highest transfer effects were achieved in the group taught by the unverballized awareness procedure, Method II, and the lowest transfer effects came from the group taught by Method I, the method in which the generalization was stated first. The groups who learned by conscious generalization
(Method III) performed between the other two groups.

Hypotheses emerging from Hendrix's data are:

(1) For generation of transfer power, the unverbalized awareness method of learning a generalization is better than a method in which an authoritative statement of the generalization comes first.

(2) Verbalizing a generalization immediately after discovery may actually decrease transfer power.

A study conducted by Schwartz (1948) supported the Hendrix hypothesis. Schwartz's results indicated that performance was better for subjects when no oral verbalization of the generalization was required. Conclusions of studies conducted by Hanson (1967), Retzer (1969), and Sowder (1974) are in conflict with those of Hendrix and Schwartz. The results suggest that how, or whether, one verbalizes a recent discovery does not differentially affect the subsequent ability to use the discovery. Hendrix's studies were questioned by Ausubel (1963) on methodological grounds and evaluative techniques. He suggested that the foundation for her hypothesis was shaky, and some of the measurement, evaluation, and controls reported by Hendrix made her conclusions tenuous. It was pointed out by Sowder (1975) that the most conspicuous difference between Hendrix's study and the studies of Hanson, Retzer and Sowder is that she apparently required very precise verbalizations of the discoveries, whereas their studies accepted whatever the subject offered unless it was incorrect.
Summary

The evidence of the literature review supports the idea that effective problem-solving procedures are of vital importance in mathematics instruction. Students' thinking abilities, understanding of algebra, concept of variable, and levels of cognitive development are factors related to solving problems successfully. According to the literature, mixed results have been revealed about which strategy is most appropriate to use when teaching solutions to word problems. Often the inadequacy of generalization is the major difficulty whatever the researcher's approach. That is to say, students have difficulty in dealing with variables.

Research evidence has indicated that students have misconceptions about variables and hence have difficulty in solving word problems. In order that students might overcome these difficulties, there is a need for an alternative approach in developing the concept of variable. Thus, developing the concept of variable and finding solutions to word problems depend on the process of generalization.
CHAPTER III
METHODS AND PROCEDURES

Design

Population

The population for this study consisted of 97 ninth grade pre-algebra (Math 9) students from East High School, an inner-city public senior high school in Columbus, Ohio. Math 9 is a two-semester (year) course designed to teach general arithmetic topics through basic algebraic concepts. The students were enrolled in four classes of the Math 9 course.

East High School was arbitrarily selected from the sixteen senior high schools in Columbus, Ohio. The classes for this study were selected from all of the Math 9 classes at East High School. The selection was based on the availability of Math 9 classes and the willingness of two teachers to allow their classes to participate in the study.

Assignment of groups to treatments

Students were assigned to treatments by creating eight small groups of approximately 13 students. Each of the four intact classes was divided into two groups. Each group was assigned to one of three treatments: the Table-Building (TB) treatment consisting of two
experimental groups (TB1, TB2) and the Translation (T) treatment (a comparison group). An illustration of the small group assignments is given in Figure 1.

<table>
<thead>
<tr>
<th>Classes</th>
<th>Treatment Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T n=12</td>
</tr>
<tr>
<td>2</td>
<td>TB1 n=13</td>
</tr>
<tr>
<td>3</td>
<td>T n=13</td>
</tr>
<tr>
<td>4</td>
<td>TB1 n=10</td>
</tr>
</tbody>
</table>

TB1=34, TB2=38, T=25, Total = 97

Figure 1. Small group assignments

To assure that backgrounds were comparable, previous math test scores and grades from students' records were used to identify groups with similar mathematics achievement. To control for differences in ability among the groups, students from a particular intact class were randomly assigned to treatments subject to their pretest scores. A combination of low and high pretest scores was used to assign students to groups so that the group means were equalized as much as possible. It was assumed that limited instruction on solving word problems had occurred prior to the study, and that students had not been exposed to teaching about variables.
Description

The design for this study was a completely randomized, one-factor design containing three levels of treatments, TB1, TB2, and T groups. An illustration of the factorial design is given in Figure 2, and the research design is illustrated in Figure 3.

<table>
<thead>
<tr>
<th>Treatment Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>.</td>
</tr>
<tr>
<td>Subjects</td>
</tr>
<tr>
<td>34</td>
</tr>
</tbody>
</table>

Figure 2. Completely randomized one-factor design

R1, R2, and R3, represent the random assignment of groups to treatments. The treatments for TB1, TB2, and T groups are represented by X1, X2, and X3 respectively. T1, T2, and T3 represent the pretest, problem-solving test, and posttest respectively.

The primary independent variables were treatment group membership and levels of cognitive development. The pretest, problem-solving test, and posttest scores were primary dependent variables.
Treatments

Materials

The treatment materials consisted of a set of translation exercises and a set of units developing table-building problem-solving procedures. Some of the word problems that appeared in the materials were used by Settle (1977). These problems were typical of number and coin problems found in beginning algebra textbooks. The translation exercises were used to develop vocabulary and background for the word problems. Some of Settle's materials were modified for use with the Translation group. The Investigator modified the problems to produce the materials for the Table-Building experimental groups.

Groups

The TB treatments were administered to two experimental groups and the T treatments were administered to one comparison group. All groups received treatments for eight sessions over an eleven week period. The first, tenth, and eleventh sessions were used for testing. The treatments consisted of presenting the students with discussions and supportive instructional materials and handouts designed to teach the concept of variable and the solution to word problems. Major topics consisted of numerical exploration with mathematical expressions, translation, and word problem solving procedures. Each group received the same amount of instructional time and the same word problems on the same days. The written description of the problems was identical for each group. The TB and T treatment differences occurred in the translation instruction and the problem-solving procedures.
IB Treatment

Experimental groups I and II received the TB treatment. The difference between the instruction of these groups was the amount of numerical exploration each group received in solving word problems before a variable was introduced in table-building. During the first treatment session, the groups who received this treatment completed tables using numerical exploration with mathematical phrases and sentences. No variables were used in table-building at this time. The TB problem-solving procedures were introduced during the second treatment session. Subsequent sessions consisted of review exercises of numerical exploration with mathematical phrases and sentences and solving word problems. After students gained experience in completing tables that were already constructed with headings, they were guided in constructing their own tables with the appropriate headings. The introduction of the variable in table-building for TB treatment groups I and II was made during the second and sixth sessions respectively.

Example of numerical exploration exercises

The second of two numbers is 4 more than twice the first number.

(a) Complete the following table.

<table>
<thead>
<tr>
<th>First number</th>
<th>2 times first number plus 4</th>
<th>Second number</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>2(9) + 4</td>
<td>22</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>32</td>
</tr>
</tbody>
</table>
(b) Explain in words how to find the second number if the first number is known.
(c) Explain in words how to find the first number if the second number is known.

Description of TB problem-solving procedures

The TB method was an intuitive or inductive approach to solving word problems. The objective was to determine the solution to problems by completing or constructing tables of numerical values. The goal was to approach a generalization. Numerical relationships between the given and wanted information in the problem were observed within the display of the table. In determining the solutions, intuitive techniques were used such as (a) guessing and checking, (b) successive approximations, and (c) recognition of patterns.

The numerical relationships in the table represented numbers in a specific set. The intent was to develop the concept of variable. After receiving numerical exploration in table-building, the intentions were that students would see that any letter, a variable, could be used to represent any of the numbers in the set.

This method is exemplified for the following word problem:

A collection of nickels, dimes, and quarters is worth $9.15. The number of nickels is 4 times the number of dimes. The number of
quarters is 5 less than the number of nickels. How many of each coin are there in the collection?

Determine the solution to this problem by completing the following table:

<table>
<thead>
<tr>
<th>No. of Dimes</th>
<th>No. of Nickels</th>
<th>No. of Quarters</th>
<th>Value of Coins In cents</th>
<th>Total Value In cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>12</td>
<td>7</td>
<td>10(3)+5(12)+25(7)</td>
<td>265 cents</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>39</td>
<td></td>
<td></td>
<td>915 cents</td>
</tr>
<tr>
<td>X</td>
<td>4(X)</td>
<td>4X-5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Guided questions were asked the students in building the table.

Some examples of these questions were:

1. Based on the information given in the first row of the table, how are the coins related to each other?
2. What is the value of the number of each coin and what is the total value?
3. If the number of dimes is given, how can the number of nickels be determined?
4. If the number of nickels is given, how can the number of dimes and the number of quarters be determined?
Students were then instructed to fill in the second through the fourth rows in the table. They were asked to look for patterns in the table. The goal was to find the number of coins that would give the total value, 915 cents. They were guided in using approximations to determine the solution.

Students were then asked the question: Suppose the number of dimes is expressed by a variable $X$, how is this related to the other coins and the total value? They were guided in discovering that the total value of the coins can be expressed using the variable $X$, and this expression is the same as the total value, 915 cents. Thus $10(X)+5(4X)+25(4X-5)=915$ can be solved to determine the solution.

Note that the variable $X$ in the last row of the table was not introduced in TB group II until the sixth session. After students gained experience in building the table, they constructed their own tables and used intuitive techniques discussed to determine solutions.

I Treatment

The T group students were instructed to translate English phrases and sentences to mathematical expressions and sentences by the initial use of variables. This is a traditional method that has been used in most algebra classes. During the first treatment session this group translated mathematical phrases and sentences to numerical phrases with the use of variables. The T problem-solving procedures were introduced during the second treatment session.

Examples of translation exercises
1. 5 Increased by twice $x$ __________________________
2. The value in cents of $X$ quarters and $3X$ dimes

3. The difference of $15$ and $X$ equals $5$

Description of the Problem-Solving Procedures

(Translation of English Expressions To Mathematical Expressions)

1. Read the problem carefully and then determine the given and the wanted information.

2. Choose a variable to represent the information wanted, then use this variable to express the relationship between the given information and the wanted information.

3. Write an equation expressing the relationship between the given and the wanted information.

Once the equation has been determined, solve it to determine the solution.

This method will be exemplified for the following word problem:

A collection of nickels, dimes, and quarters is worth $9.15. The number of nickels is 4 times the number of dimes. The number of quarters is 5 less than the number of nickels. How many of each coin are there in the collection?

Solution

1. Information given: Value of the coins in cents is $9.15. Information wanted: Number of dimes, nickels, and quarters.

2. If $X =$ the number of dimes, then $4X =$ the number of nickels and $4X - 5 =$ the number of quarters. The value of the dimes in cents is $10X$, the value of the nickels in cents is $5(4X)$, and the value of the
quartets In cents Is $25(4X-5)$.

3. Equation Is: $10(X)+5(4X)+25(4X-5)=915$.

At first, students were guided through the problem-solving procedures and were given partial solutions to be completed. The partial solutions included representation for the wanted information and a "key" sentence from the story. Students were expected to read the story and the partial solutions, and then construct the equation by translating the key sentence. The equation was then solved and the solution stated. Typical of this type of exercise were:

**Problem:** Twice a certain number Increased by 30 equals 120.

Find the number.

**Solution** If $X = \text{ the number}$, then $2X$ Increased by 30 equals 120

The equation Is ______________

**Solve:**

The number Is ______________

After practice, students were expected to write the equation without the presence of a key sentence.

**Example** James has 50 coins consisting of quarters and dimes.

The value of all the coins is $7.25. Find the number of each type of coins he has.

**Solution** If $X = \text{the number of quarters}$, then $50-X = \text{the number of dimes}$.

**Equation:** ________________
James has ___________quarters and ___________dimes

After students gained experience with writing the equation and determining the solution when partial solutions were given, they were expected to solve the problems without any key sentence given or partial solutions. They were to utilize the problem-solving procedures of this treatment.

Instrumentation

Three data collecting instruments were used for this study: a pretest, a problem-solving test, and a posttest.

Pretest

A modification of the algebra test developed by the Concepts in Secondary School Mathematics and Science (CSMS) Team was used to assess students' use and understanding of variables prior to treatments. This test was developed in London and has been used with thousands of secondary school students in England. The test was developed by first interviewing children individually and then trying pencil and paper versions on a few classes at a time. In all, the test went through ten drafts (Kuchemann, 1978).

The CSMS Algebra Test is a half-hour paper-and-pencil test designed to measure adolescents' understanding of six different ways that letters are used as variables in algebra. The test is based on the research of Collis (1975). As described by Hart (1981), the
following six different ways of interpreting and using letters were identified:

1. **Letter evaluated**: This category applies to responses where the letter is assigned a numerical value from the outset.

2. **Letter not used**: Here the children ignore the letter or at best acknowledge its existence but without giving it a meaning.

3. **Letter used as an object**: The letter is regarded as a shorthand for an object or as an object in its own right.

4. **Letter used as a specific unknown**: Children regard a letter as a specific but unknown number, and can operate upon it directly.

5. **Letter used as a generalized number**: The letter is seen as representing, or at least as being able to take several values rather than just one.

6. **Letter used as a variable**: The letter is seen as representing a range of unspecified values, and a systematic relationship is seen to exist between two such sets of values.

This test assesses understanding of variables at four different levels. The test was constructed within a Piagetian framework and the items classified in terms of Piaget's stages of concrete and formal operational thought. The four levels of understanding I, II, III, and IV correspond to Piagetian sub-stages, early-concrete, late-concrete, early-formal, and late-formal respectively.
According to Hart (1981), twenty-one of the original 51 test items were rejected, either because they came from a question with a large number of similar items or because their correlations were relatively low. The remaining 30 items were sorted into four groups covering a different facility range, with cut-off points adjusted according to the complexity of the item and the nature of its elements. The groups represent four different levels of understanding of generalized arithmetic as described by Hart (1981):

**Level 1:** The significance of letters need not be realized; letters can be ignored, evaluated, or seen as objects. *Items* from original test: 51, 61, 711, 8, 91, 131.

**Level 2:** Letters used as objects or evaluated but there is need to use mathematical syntax. *Items* from original test: 7111, 911, 9111, 111, 1111, 1311v, 151.

**Level 3:** Letters as specific unknowns. Lack of closure must be accepted. *Items* from original test: 411, 5111, 91v, 1311, 13v111, 14, 1511, 16.

**Level 4:** The manipulation of specific unknowns where this involves a coordination of operations or where the letters represent numbers or objects or their cost et cetera rather than the objects themselves. Letters are used as variables. *Items* from original test: 3, 4111, 71v, 13v, 17, 1811, 20, 21, 22.
The level of understanding displayed by children on the test was assessed by determining the most difficult group of items on which they showed competency. A criteria of correctly answering two-thirds of the items within a group were used. If students did not show competency on any group of items they were considered to be at the zero level of understanding. A child who, in this sense 'passes' a harder group of items without passing all easier groups was considered to be an 'error type' (Hart, 1981).

The algebra test had correlations ranging from .681 to .788 with other CSMS mathematics tests, and a correlation of about .70 with raw scores on the Calvert DH test of non-verbal reasoning. Phi coefficients, Item-item measures of association ranging from .28 to .40 were reported for the four levels of understanding on the algebra test. Measures of goodness of fit and other test statistics are explained in detail in Hart's book, Secondary School Children's Understanding Of Mathematics (1980 ed.). Reliability estimates of this test reported by the investigator for this study using KR-20 formula ranged from .845 to .875. A copy of the original test and the modified version used for this study can be found in Appendix C, pages 167 and 171 respectively.

**Problem-Solving Test**

This test was designed to measure a student's ability to use variables and determine the solution to word problems. More specifically a student's ability to translate and determine a numerical solution was assessed. A verbal problem test used by Settle (1977) was modified to meet the needs of this study. The original test consisted of 10 verbal problems, each followed by a request for a relevant
equation for the story. A reliability estimate of .69 was reported for Settle's test. For the purpose of this study, not all of the problems were used. Students were requested to write the equation or the numerical solution, or both.

The test for this study consisted of 7 word problems, and required 25 responses. Items which required only a numerical answer were considered numerical items, and items which required translation with letters and numbers or writing an expression or equation were considered translation items. Of the 25 items, 8 were translation items (2b, 2c, 3e, 4c, 4d, 5b, 5c, 7c), 14 were numerical items (1a, 1c, 2a, 2d, 3c, 3d, 4a, 4b, 4e, 4f, 5a, 6a, 7a, 7b), and 3 items were neither numerical or translation items (1b, 2a, 3b). The problems were based upon the materials covered in the treatment sessions. A reliability estimate of .885 was reported by the Investigator for this test. A copy of this test can be found in Appendix C, page 177.

Posttest

A parallel form of the pretest was used for posttesting. Based on an item analysis of the pretest, some of the items were improved because of low correlations or bad item discrimination. For some items only the structure or format was improved, for others the numbers were changed, and some items remained the same. The posttest was designed to assess a student's ability to use and understand variables. This test did not cover directly what was taught in the treatment groups, but the intent was that students would apply or transfer their learning from the treatment materials. A reliability estimate of .824 was reported by the Investigator for this test. A copy of this test can be
found in Appendix C, page 174.

**Procedures**

**Group Sessions**

The study consisted of eleven 40-minute sessions for each of the eight groups assigned to the three treatments. Each group met with the Investigator once a week over an eleven week period. The first, tenth, and eleventh sessions were used for testing. Treatments were administered during the second through the ninth sessions. The study began on Wednesday, November 30, 1983, with the administration of the pretest to the four intact classes during their respective class periods. The Investigator administered the test and students were allowed the entire class period to complete the test. The test was then scored by the Investigator and each of the four classes was divided into two groups and then randomly assigned to one of three treatments. Of the eight groups, the Investigator arbitrarily assigned three, three, and two groups to the TB1, TB2, and T treatments respectively.

During the following week, on Tuesday, December 6, the treatment sessions began. The Investigator met with each treatment group once a week for eight weeks. The first four groups met every Tuesday during their respective class time and the second four groups met every Wednesday during their class time. On the days the Investigator met with one half of a particular class, the two regular mathematics teachers each met with the other half of their respective classes in
different class rooms and continued with the regular class work. Table 1 gives the times and days each treatment group met with the Investigator.

Table 1

Times and Days of Treatment Group Sessions

<table>
<thead>
<tr>
<th>Class Periods</th>
<th>Time</th>
<th>Tuesday</th>
<th>Wednesday</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7:48-8:30 A.M.</td>
<td>T</td>
<td>TB1</td>
</tr>
<tr>
<td>2</td>
<td>8:33-9:15 A.M.</td>
<td>TB1</td>
<td>TB2</td>
</tr>
<tr>
<td>3</td>
<td>9:18-10:00 A.M.</td>
<td>T</td>
<td>TB2</td>
</tr>
<tr>
<td>4</td>
<td>10:03-10:45 A.M.</td>
<td>TB1</td>
<td>TB2</td>
</tr>
</tbody>
</table>

The group sessions format were similar for all groups. During each session the Investigator checked the roll for attendance and then handed out worksheets for that particular session. A brief discussion or explanation of the lesson for that day was given to help students start on their worksheets. The Investigator individually gave assistance to those students who needed and requested it, and periodically checked students' work to assure that they understood what they were doing. Prior to the end of each session students were given homework assignments to be completed and handed in to their regular mathematics teachers the following day. Many students started, completed, and handed in their homework at the end of the session. At the end of each week the Investigator collected homework sheets from
the teachers, graded them, and returned them to the students the following week. At the first treatment session all students were given a folder to keep their in-class worksheets and homework assignments. They were told that their homework grades would count a certain percentage of their mathematics grade, and that they would be tested at the end of the eight weeks.

Prior to the Christmas holidays students were administered three sessions of treatments. School was dismissed from Wednesday, December 21, through Tuesday, January 3. Prior to the vacation the Investigator met with the Wednesday groups on Monday, December 19, and the Tuesday groups on its respective day, December 20. After the vacation the Investigator met with the Wednesday groups on its respective day, January 4, and the Tuesday groups on Thursday, January 5. The week of December 26 was the only week when treatments were not administered.

For those students who were absent or missed a treatment session, the Investigator met with them during their free periods and gave them worksheets and the homework assignments, or they were given these two sheets at the next treatment session before receiving the sheets for that particular session.

The treatment sessions ended on Wednesday, February 1, 1984. At the end of the treatment sessions, students were given the problem-solving test (on February 7 and 8) and the posttest (on February 14 and 15). The problem-solving test assessed students' problem-solving skills based on the treatment materials, and the posttest was a parallel form of the pretest used to assess students' use and understanding of variables. The Investigator administered and
scored both tests. These tests were given during the small group sessions and students were allowed the entire 40 minutes to complete each test.

The TB and T treatment materials can be found in Appendices A and B respectively.

**Statistical Procedures**

The statistical design for this study was multivariate analysis of variance (MANOVA) with three levels (TB1, TB2, and T groups). The dependent variables were the pretest score, problem-solving (total, translation, numerical) score, and the posttest score. The independent variables were treatment group membership and levels of cognitive development.

The statistical procedures for this study were categorized into four parts: (1) scoring of instruments, (2) reliability of instruments, (3) multivariate analysis of variance procedures, and (4) correlational analyses.

**Scoring of Instruments:**

The pretest, problem-solving test, and posttest were hand-scored by the investigator. Frequencies, test statistics, and item analyses for these tests were provided by The Ohio State University (OSU) STATPACK computer program. (See Appendix D).

**Reliability of Instruments**

Reliability estimates were determined to assess the accuracy (consistency and stability) for each instrument. The Kuder-Richardson 20 (KR-20) formula was used to estimate the internal consistency.
reliability estimates for all tests. The OSU STATPACK computer program was used. Results are provided in the data analysis and results section, Chapter IV.

**Multivariate Analysis Of Variance Procedures**

Multivariate Analysis of Variance (MANOVA), using three dependent variables and univariate analysis of variance (ANOVA), using only one dependent variable were used to determine whether significant differences existed among the treatment groups. Hypotheses one through seven were tested, and the primary research questions and the first secondary research question were investigated by the use of MANOVA and ANOVA procedures with contrasts and post-hoc comparisons.

A one-way ANOVA was used on pretest scores to determine if initial differences existed among the treatment groups. The Biomedical Computer Programs P-Series (BMDP), the Statistical Package for the Social Sciences (SPSS), and the Statistical Analysis System (SAS), all one-way ANOVA procedures were used to analyze the pretest data to assure that the data were being analyzed correctly.

The computer program, MANOVA distributed by Clyde Computing Service (1969), the SAS General Linear Model Procedure (PROC GLM), and the SPSS MANOVA procedures were used to analyze posttest and problem-solving test data.

A randomized treatment by block design was performed on the problem-solving test scores. Students' levels of cognitive development were used as a blocking variable. Hypotheses eight and nine were tested by SPSS, SAS, and BMDP ANOVA procedures using a randomized
block design. Post-Hoc comparisons were performed using Dunn's procedure.

Correlational Analyses

Correlational analyses were used to determine relationships between independent and dependent variables. The second secondary research question was investigated and hypothesis 10 was tested. SAS Correlation Procedure (PROC CORR) and the SPSS subprogram CORR were used to compute the correlation coefficients, both using Pearson Product-Moment correlations.

Results and statistical analyses of the data indicated in the statistical procedures are reported in Chapter IV.
CHAPTER IV
DATA ANALYSIS AND RESULTS

Analysis of data for this study is divided in two parts: (1) subjective analyses of qualitative data and (2) statistical analyses of quantitative data.

This study is categorized as a teaching experiment. According to Kantowski (1978), the Soviets performed teaching experiments to observe qualitative effects of various forms of instruction. These methods allowed the study of changes in mental activity, identification of thinking and problem-solving processes, and the determination of how instruction influenced these processes.

The nature of the experiment for this study allowed the investigator to observe and investigate qualitative effects of instruction on students' thinking and problem-solving processes. The small sample size and the small groups used in the study enabled the observation of qualitative effects of treatments and comparison of these results with the quantitative data.
Subjective Analyses of Qualitative Data

The qualitative data include the investigator's observations of students' thinking processes, understanding of subject content, and problem-solving processes. These factors were observed and reported for both Table-Building and Translation Instruction.

Table-Building Instruction

Both experimental groups (TB1 and TB2) received TB Instruction. The only difference between treatments was the time the variable was introduced in table-building. The TB1 group was exposed to use of variables for seven (second through eighth) sessions and the TB2 group was exposed to use of variables for three (sixth through eighth) sessions.

Thinking Processes

During the first few sessions of treatments students appeared afraid to think for themselves when they were requested to do so. The majority of the students appeared to be thinking at a concrete level. Observing and asking students questions while they performed problem-solving tasks allowed the opportunity to observe their thinking processes. Many students could recall basic facts they had learned previously in mathematics, but some could not recall or retrieve the information presented during previous treatment sessions. It appeared that many students had the necessary basic skills to solve problems but did not know how to use these skills. Some of the students could think through a problem situation, but others needed assistance constantly. Toward the end of the eight week sessions many students
were operating more independently, but some still had problems thinking for themselves.

**Understanding of Subject Content**

The majority of the students indicated that they had never worked with the type of number and coin problems used in the study. Many students had trouble understanding the relationship between an English and mathematical phrase or sentence. Some students had reading and vocabulary problems. Many students were confused about which operation to use in performing mathematical computations. Students could perform single operations correctly, but had trouble with problems requiring use of multiple operations. Symbols used for various operations created a problem for many students. For example, expressions like $3n$, and $3(2)$ were ignored as multiplication operations. Many students would treat the expression $3n$ as $32$ rather than $6$ when $n=2$. The parentheses did not indicate to them a multiplication operation. They needed to see the symbol "x" for multiplication. The symbol "x" was used to represent a variable so it was not used as times, but the investigator made the students aware of this.

At first some students did not understand what was wanted in the problem. For example, consider the following number problem that was used in this study:

The sum of two numbers is 97. The larger of the two numbers is 5 more than 3 times the smaller number. Find the numbers.

When students were asked to find the numbers in this problem as the wanted information, some were confused until they saw a table
constructed with appropriate headings (See Appendix A, page 130).

Thus, table-building simplified the situation for them. Students saw numerical relationships and sought for patterns in table-building which enabled them to determine the wanted information.

When the TB1 group was first introduced to the variable, the investigator indicated the purpose and use of the variable, but many students asked questions and could not conceptualize with the letter that was being used. The investigator indicated to students that a variable could be used to represent any number in some set. It was indicated that a variable could be used to write an equation expressing relationships between the given and wanted information in a problem. Students did not feel comfortable with and could not at this point successfully write expressions or equations using letters. When asked to write an expression or an equation, many students ignored the letter and used numbers that made the equation a true statement. This indicated that some had a feeling that letters are used to represent numbers, but did not completely understand the concept of variable. Many students were turned off or confused when asked to respond to a written statement to see if they had reached a generalization of the problem situation. Instead of generalizing they could and did respond by using specific examples.

When the TB2 experimental group was introduced to the variable, the investigator explained the reason and purpose of the variable. Students in the TB2 group did not ask many questions at this point. When asked to write the equation some students used numbers instead of letters. At this point in both groups some students gave two
expressions, one with letters and the other with numbers. The students who used only letters in writing an equation appeared to have reached a generalization of the problem situation. Those who used two equations, one with numbers and the other one with letters appeared to have been aware of a generalization but they were not sure how to state the generalization. They did know that they could replace the letter with some number. Students who used only numbers appeared not to have reached a generalization of the problem situation. They had not developed a feeling for letters yet and appeared to still be exploring with numbers. The TB1 group having been exposed to variables longer than the TB2 group used variables more in expressing the relationship but did not always give the correct equation, whereas the TB2 group used numbers more in expressing an equation but in most cases had the correct equation. This indicated that the time variable which was introduced in table-building did have some effect on students' use and understanding of variables.

Problem-Solving Processes

Two factors affecting the success of students in using the table-building process were: (1) figuring out how to label columns in the table, and (2) the guessing process — which number to try first. The investigator gave brief explanations and discussions about the purpose of each session and directed students in their problem-solving procedures. Students were to use the skills and problem-solving procedures on their own. After students began working, the investigator observed them to see if they were completing the tables correctly. During the first few sessions the investigator had
appropriate table headings already constructed for the students. Some students made errors in computations in the table. The majority had no trouble with simple number problems requiring use with single operations. Many had difficulty with the coin problems. They confused the number of coins with the value of the coins. At first many students could not recognize patterns in the table, determine approximate answers, or see numerical relationships, but with practice and experience after a few sessions they had improved on these tasks. During each session some students completed their tasks early and started their homework or assisted others. After students gained experience in completing the table, they were given partial tables where not all of the headings were given and where they had to use their own guesses. Some students completed the missing headings but at times were confused about where to start their guesses. When students had to construct their own tables and headings, the majority were totally lost and did not know how many headings to use or the appropriate order in which to place them. The Investigator suggested to them that they needed: (1) headings for the wanted information in the problem, and (2) headings that expressed relationships between the wanted and given information in the problem. For example, consider the same word problem discussed in the previous section. The table with appropriate headings is provided (in Appendix A, page 130). When students were guided in how to construct their own tables, some constructed and completed their tables correctly, but others had difficulty and tried to force incorrect answers in many cases.

In many cases when students could not construct their own tables
they played around with numbers until they came up with the correct answer. If they did not get the correct answer they requested assistance in construction of the table. The majority of the students performed their own calculations using paper and pencil. However, some requested permission to use calculators. Some brought their own calculators and shared them with others or requested the use of my calculator. Since all students did not have calculators, they were not allowed to use them on tests.

Translation Instruction

The Translation (T) treatment group was a comparison group which received Translation Instruction. To get started on the solution of a problem, students were expected to: (a) use a variable to express relationships between the wanted and given information, and (b) write an equation describing the problem situation. This group was exposed to variables all eight treatment sessions.

Thinking Processes

During the first few treatment sessions, students appeared afraid to think for themselves. The majority of the students appeared to be at a concrete level of thinking. Since students were introduced to variables initially, and had to use translations with letters, some of them could not think abstractly because they were not at the formal level of thinking. Observations and questioning of students while they performed problem-solving tasks gave the investigator an opportunity to identify various thinking patterns exhibited by the students. Many students knew basic facts from previous mathematics
courses but had trouble recalling or retrieving information presented from previous sessions. Since students in this group had to reason initially with variables rather than numbers, some of them could not successfully reason with or see the purpose for the variable. Students would say "why use a letter," "what does it mean," "where did it come from." Many students could follow a set of rules when guided through them but could not get started by themselves. Many of these students requested assistance constantly. Toward the end of the eight weeks of treatment many students could think for themselves, but others had trouble conceptualizing the variable or feeling comfortable while working problems. Several of the students could not recall or use all the steps of the Translation problem-solving procedures that were presented.

**Understanding Of Subject Content**

The majority of the students in this group had never worked with the number and coin problems used in the study. Understanding the relationship between English and mathematical phrases or sentences created difficulty for many students. This group had trouble translating, and exhibited reading and vocabulary problems. Students knew basic mathematical operations but had trouble with key words indicating the operations to perform. The group did not have trouble with single operation problems but multiple operations created many difficulties. This group confused the variable symbols used with some operations. Many students would express 4 times x as 4 x (x) rather than 4x. Students ignored 4x as indicating a multiplication operation. A few students had seen or evidently worked with variables but did not
understand how to use them. When given problems, students had trouble at first understanding what to do or what to find in the problem. Trying to determine the wanted and given information created a problem for many. The students needed constant practice translating because this was where most of them had trouble.

**Problem-Solving Processes**

Brief explanations and discussions concerning the purpose of each session were given. Students were initially given key words to use in translating. During the first few sessions a few examples were given to follow. Students translated examples using worksheets. Students completed their worksheets and answers were discussed with the students. Mistakes were corrected when correct answers were written on the board. Some of the errors created were attributable to students not knowing the appropriate operations to use or to incorrect translation. Students were given procedures to follow in writing the equations but when they were asked to write the equation or solve a problem they did not follow the rules or procedures. At first, in helping students to translate, key sentences from the problems were given. Typical examples are provided in Appendix B, page 151.

Many students had trouble writing the equation. Once the equation was obtained students were taught how to solve the equation for the wanted information in the problem. Some students had trouble at first solving the equation. Errors were created such as addition or subtraction of terms to both sides of the equation. Once they got the equation down to the form "ax=b", they had no trouble determining the value for "x". With experience in solving equations most students
gained confidence. After experience students solved problems on their own and without the identification of a key sentence. Many students did not know how to get started. The Investigator suggested to them that they must use the rules suggested for solving the problems and for setting up the equations. Many students became frustrated when they could not get started on their own. However, there were some who could and did get started on their own.

Summary of Qualitative Data

Observations of all treatment groups indicated that similarities as well as differences existed among the groups in terms of thinking, understanding, and problem-solving. Since all groups were selected to be equal in abilities, initially it appeared that the overall understanding of problems for all groups was about the same.

All groups demonstrated similar attitudes and patterns of behavior. Initially students had positive attitudes, were motivated and enthusiastic about learning to solve problems. This was a new experience, an opportunity for the students to learn something different from their regular class sessions. Their attitudes and behavior patterns were no different from typical ninth grade mathematics students. Some days students were eager to learn, had positive attitudes about doing in-class worksheets and homework assignments, but other days some students were lazy, bored, had negative attitudes, and were not motivated to do anything. In addition to attitudes and behavior patterns, both TB and T treatment groups exhibited reading and vocabulary problems and could not recall or
retrieve successfully Information from previous treatment sessions.

In drawing any conclusions about the effects of the treatments, many factors or variables not controlled in the study need to be considered. Factors such as reading difficulty, attitudes toward problems and mathematics, and the time and the way treatments were administered may have had different effects on students' performance or achievement. Some students were administered treatments early in the morning and others later during the morning. Students who were absent were administered treatments by the investigator during students' free time. The conditions of treatments were varied a bit for these students. Since students' attitudes were not objectively assessed in this study, a determination could not be made as to the effects of attitudes toward treatments, problems, and mathematics upon students' performance. Nor could a determination be made of the effects of students' reading abilities on their performance.

Differences existed among groups in the treatments. Based on a particular treatment, groups had to think a certain way and use different problem-solving procedures. Students exposed to T treatment had to think initially with letters and in using problem-solving procedures started with generalizations and approached specific solutions. Students exposed to TB treatments reasoned initially with numbers instead of variables and in using problem-solving procedures started with specific numbers and approached generalizations.

The observations of students' reactions to the TB and T treatments revealed the following results:

1. The TB groups had initial difficulties in correctly labeling table
headings, and the T group had initial difficulties in the identification and use of variables in translation.

2. The TB groups had initial difficulties in guessing and checking, seeking patterns, and using approximations, and the T group had initial difficulties in recalling and being able to use the problem-solving rules or procedures needed to write an equation and solve problems.

3. The TB groups had initial difficulties in finding solutions to problems, and the T group had initial difficulties in understanding and solving equations correctly.

4. The TB groups could get started on the solution of a problem if the table was given. The T group had no comparable entry to a problem.

Statistical Analyses Of Quantitative Data

The quantitative data include the observed scores on students' pretest, posttest, and problem-solving test. Originally, 97 students were pretested, but because of excessive absences and reasons such as dropping out or changing to other schools, only 84 students, the final sample, were used in all analyses. All hypotheses in this section were stated in the null form and were tested using a .05 level of significance for all statistical tests unless reported otherwise.

Results Of Reliability Estimates Of Instruments

All instruments were hand-scored by the investigator and frequencies, test statistics, item analyses, and reliability estimates were provided by the OSU STATPACK computer program. Each item was scored dichotomously for each instrument. The KR-20 Internal
consistency reliability estimates for pretest, posttest, and problem-solving test were 0.873, 0.824, and 0.885 respectively. Other test statistics for all three instruments are given in Appendix D.

Analysis Of Pretest Data

The pretest was used to determine students' initial understanding of variables. This test consisted of 30 items, each was graded right or wrong. The possible range of scores was from 0 to 30. The observed range of pretest scores was from 2 to 26. Analysis of pretest data for 84 students revealed an overall mean of 11.98 with a standard deviation of 4.95. Means and standard deviations for the pretest are provided in Table 2.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>29</td>
<td>12.76</td>
<td>5.05</td>
</tr>
<tr>
<td>TB2</td>
<td>33</td>
<td>11.67</td>
<td>4.81</td>
</tr>
<tr>
<td>T</td>
<td>22</td>
<td>11.41</td>
<td>5.12</td>
</tr>
<tr>
<td>Total</td>
<td>84</td>
<td>11.98</td>
<td>4.95</td>
</tr>
</tbody>
</table>

In order that equal abilities existed among the groups, equivalent treatment groups were originally formed by randomly assigning subjects to treatment groups based on pretest scores. A balance between low and high scores was maintained for each group in order to equalize groups.
as much as possible. A one-factor analysis of variance (ANOVA) was performed on the pretest data for the 84 subjects included in the final sample to determine if initial differences existed among treatment group means.

The underlying assumptions for analysis of variance are that the sampled observations are independent and the basic population from which the sample is drawn is normally distributed with equal variances. These assumptions were met. The random assignment of subjects to treatment groups and the original formation of treatment groups by pretest results indicated that the independence and homogeneity of variance assumptions were met. The test statistics from the pretest data indicated that the group was not perfectly normal, but the slight deviation from the normal curve indicated that the normality assumption was not totally violated. Table 3 presents the results of a one-way ANOVA on pretest data for the final sample. The following hypothesis was tested:

H1: There will be no significant differences among all three treatment groups on the pretest.

Results indicated no significant differences existed among treatment group means (F(2, 81) = 0.565, p < 0.5708). H1 cannot be rejected at the .05 level of significance. Thus, since the cell means did not differ significantly, the treatment groups were similar in abilities.
Table 3

Analysis Of Variance For Pretest Data

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>2</td>
<td>27.99</td>
<td>13.995</td>
<td>0.565</td>
<td>0.5708</td>
</tr>
<tr>
<td>Error</td>
<td>81</td>
<td>2007.96</td>
<td>24.790</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>83</td>
<td>2035.95</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analysis Of Posttest Data

The posttest consisted of 30 items which paralleled the 30 pretest items. The purpose of the posttest was to assess students' use and understanding of variables. The possible range of scores was from 0 to 30. The observed range of posttest scores was from 4 to 27. The overall mean for the final sample on the posttest was 13.5 with standard deviation of 5.09. Means and standard deviations for the posttest are provided in Table 4. A one-way ANOVA was performed on the posttest data to determine if significant differences existed among treatment groups on their use and understanding of variables after treatments. The ANOVA summary for the posttest is provided in Table 5. Results indicate no significant differences existed among treatment groups on posttest means ($F(2,81)=2.419, p<.0955$).
Table 4

Posttest Means And Standard Deviations

<table>
<thead>
<tr>
<th>Treatment</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>29</td>
<td>15.14</td>
<td>4.86</td>
</tr>
<tr>
<td>TB2</td>
<td>33</td>
<td>12.45</td>
<td>5.03</td>
</tr>
<tr>
<td>T</td>
<td>22</td>
<td>12.91</td>
<td>5.19</td>
</tr>
<tr>
<td>Total</td>
<td>84</td>
<td>13.50</td>
<td>5.09</td>
</tr>
</tbody>
</table>

Table 5

Analysis Of Variance For Posttest Data

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>2</td>
<td>121.55</td>
<td>60.776</td>
<td>2.419</td>
<td>0.0955</td>
</tr>
<tr>
<td>Error</td>
<td>81</td>
<td>2035.44</td>
<td>25.129</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>83</td>
<td>2156.99</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Although no significant difference existed among treatment group means on the posttest, the overall mean for the posttest was higher than that of the pretest. The increase was 1.52 points. There was an increase for each treatment group mean from pretest to posttest. TB1, TB2, and T group means increased by 2.38, 0.78, and 1.5 respectively. This indicates that the groups did improve on their use and understanding of variables. Prior to the analysis of data, planned comparisons using contrasts were to be performed on the treatment group means. The contrasts made were: (1) TB1 versus TB2, (2) TB1 versus T, (3) TB2 versus T, and (4) TB1 and TB2 combined versus T. The investigator originally set the hypothesiswise error rate at .05. The specific error rate for each of the four separate comparisons was (.05/4=.0125). Thus, each of the separate comparisons was made at the .0125 level of significance. Table 6 reports the T-values, corresponding probabilities, and coefficients for the contrasts among group means on posttest data. The following hypotheses were tested:

H2: There will be no significant difference between the two experimental groups on the posttest.

H3: There will be no significant difference between the TB treatment groups and the T treatment group on the posttest.

The results indicate that no significant difference in group means exists for contrast number 1, (t(81)=2.1031, p < 0.039). Thus, H2 cannot be rejected at the .0125 level of significance. No significant difference existed for contrast number 4, (t(81)=0.7133, p< 0.478). Thus, H3 cannot be rejected at the .0125 level of significance. Therefore, the results indicate that there existed no statistically
significant difference between the two experimental groups in using and understanding variables, and no statistically significant difference existed in the use and understanding of variables between the combined experimental groups and the comparison group.

Table 6

T-Values For Contrasts in Treatment Group Means On The Posttest

<table>
<thead>
<tr>
<th>Contrasts No.</th>
<th>DF</th>
<th>T-Value</th>
<th>P(T)</th>
<th>TB1</th>
<th>TB2</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>81</td>
<td>2.103</td>
<td>0.039</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>81</td>
<td>1.573</td>
<td>0.120</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>81</td>
<td>-0.329</td>
<td>0.734</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
<td>0.713</td>
<td>0.478</td>
<td>1</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>
Analysis Of Problem-Solving Test Data

The purpose of the problem-solving test was to measure a student's ability to use variables and determine solutions to word problems. A student's ability to translate and determine numerical solutions was also assessed. The test consisted of 25 items. From this test three scores (total problem-solving (PS), translation (TS), and numerical (NS)) were determined and were called the problem-solving variables. The problem-solving score consisted of 25 items with a possible range of 0 to 25, the translation score comprised 8 items with a possible range of 0 to 8 and the numerical score comprised 14 items with a possible range of 0 to 14. Three items were neither numerical nor translation but were a part of the total problem-solving score. Items which required only a numerical answer or solution were considered to be numerical items, and items which required translation with variables and numbers or writing an expression or equation were considered as translation items. The observed ranges for the PS, TS, and NS dependent variables were from 1 to 22, 0 to 8, and 1 to 14 respectively. The overall means and standard deviations (SD) for the three variables were mean 10.19 with SD 5.12, mean 2.02 with SD 1.76, and mean 6.58 with SD 3.19 for the PS, TS, and NS variables respectively. Means and standard deviations for the problem-solving variables for all treatment groups are provided in Table 7.
Table 7

Means And Standard Deviations For The Problem-Solving Variables

<table>
<thead>
<tr>
<th>Treatment</th>
<th>N</th>
<th>PS</th>
<th>TS</th>
<th>NS</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>29</td>
<td>Mean</td>
<td>11.55</td>
<td>2.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>5.45</td>
<td>2.10</td>
</tr>
<tr>
<td>TB2</td>
<td>33</td>
<td>Mean</td>
<td>11.52</td>
<td>2.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>3.95</td>
<td>1.47</td>
</tr>
<tr>
<td>T</td>
<td>22</td>
<td>Mean</td>
<td>6.40</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>4.46</td>
<td>1.45</td>
</tr>
<tr>
<td>Total</td>
<td>84</td>
<td>Mean</td>
<td>10.19</td>
<td>2.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SD</td>
<td>5.12</td>
<td>1.76</td>
</tr>
</tbody>
</table>

A one-way multivariate analysis of variance (MANOVA) with univariate analysis of variance was performed on the problem-solving variables to determine if significant differences existed among the three problem-solving dependent variables for the three treatment groups. Results of the MANOVA and corresponding univariate analyses are reported in Table 8.
The multivariate analyses showed an overall significant ($p < .001$) difference among treatment groups on the three problem-solving dependent variables. The significant multivariate statistic suggested an investigation of the separate univariate F-tests. An inspection of the univariate ANOVA tests indicated that significant F-tests existed for all three dependent variables with significance at the .05 level. To determine which groups contributed to the significant univariate results, prior to the analysis of data, planned comparisons with contrasts were to be performed on all three problem-solving dependent variables for each treatment group. Separate one-way ANOVAS with contrasts were performed on all dependent variables. Each separate comparison was made at the .0125 level of significance for each of the problem-solving variables. Table 9 reports the T-values, corresponding probabilities, and coefficients for the contrasts among group means on
the problem-solving variables. The following hypotheses were tested:

H4: There will be no significant difference between the two experimental groups on the problem-solving test.

H5: There will be no significant difference between the TB treatment groups and the T treatment group on the problem-solving test.

H6: There will be no significant difference between the T treatment group and TB treatment groups on the translation scores.

H7: There will be no significant difference between the TB treatment groups and the T treatment group on numerical scores.

Results from Table 9 indicate that for the significant PS variable comparisons among groups suggest that TB1 and TB2 groups were significantly ($p < .001$) different from the T group in favor of TB1 and TB2 groups. The combined TB1 and TB2 groups were also significantly ($p < .001$) different from the T group. However, there was no significant ($p < .975$) difference between the TB1 and TB2 experimental groups. Thus, H4 cannot be rejected at the .0125 level, but H5 can be rejected at the .0125 level of significance. The significant TS variable indicated that the combined TB1 and TB2 groups differed significantly ($p < .012$) from the T group favoring the TB groups.
Table 9  
T-Values For Contrasts In Treatment Group Means  
On The Problem-Solving Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Contrast No.</th>
<th>DF</th>
<th>T-Value</th>
<th>P(T)</th>
<th>TB1</th>
<th>TB2</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS</td>
<td>1</td>
<td>81</td>
<td>0.031</td>
<td>0.975</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>81</td>
<td>3.917</td>
<td>0.001</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>81</td>
<td>3.995</td>
<td>0.001</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>81</td>
<td>4.444</td>
<td>0.001</td>
<td>1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>TS</td>
<td>1</td>
<td>81</td>
<td>1.061</td>
<td>0.292</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>81</td>
<td>2.746</td>
<td>0.007</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>81</td>
<td>1.839</td>
<td>0.070</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>81</td>
<td>2.583</td>
<td>0.012</td>
<td>1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>NS</td>
<td>1</td>
<td>81</td>
<td>-1.211</td>
<td>0.229</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>81</td>
<td>3.266</td>
<td>0.002</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>81</td>
<td>4.475</td>
<td>0.001</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>81</td>
<td>4.340</td>
<td>0.001</td>
<td>1</td>
<td>1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Although no significant \( p < .292 \) difference existed between the TB1 and TB2 experimental groups, from Table 7 the mean of 2.55 on the TS variable for TB1 had a slight favor over the 2.09 mean for the TB2 group. Thus, these results indicate that H6 can be rejected at the .0125 level. The significant NS variable indicated that the combined TB1 and TB2 groups were significantly \( p < .001 \) different from the T group favoring TB1 and TB2. No significant \( p < .229 \) difference existed between TB1 and TB2 groups on the NS variable, but from Table 7 the mean of 7.82 on the NS variable for TB2 had a slight favor over the
6.93 mean for TB1 group. Results indicate that H7 can be rejected at the .0125 level of significance.

Analysis Of Cognitive Development Levels

Students' levels of cognitive development were based on four levels of understanding as assessed by the pretest. The levels were determined by the criteria explained in the instrumentation section on the pretest data. The four levels correspond with Piagetian levels of cognitive development. Levels I, II, III, and IV correspond to early-concrete, late-concrete, early-formal, and late-formal respectively. The number of students at each level of understanding by treatment groups is reported in Table 10. Results reveal that 90.5% of the students in the study was at the non-formal level of thinking.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>17</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>TB2</td>
<td>22</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>33</td>
</tr>
<tr>
<td>T</td>
<td>14</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>Total</td>
<td>53</td>
<td>23</td>
<td>5</td>
<td>3</td>
<td>84</td>
</tr>
</tbody>
</table>
It was assumed that students' levels of understanding were directly related to their problem-solving performance. The correlation reported (on page 98) supported this claim. The problem-solving test data were originally analyzed by the single factor ANOVA using a completely randomized design. According to Myers (1979) a treatment x blocks design is more efficient than a completely randomized one factor design. He stated that if individual difference variables could somehow be removed, the error variance would be reduced and it would be easier to detect the effects of the independent variable. According to Myers some advantages of the treatment x blocks design are: (1) treatment groups are roughly matched for at least one measure that should affect performance, (2) since the design is essentially a two-factor design, the treatments x blocks interaction effects can be investigated. In this study, level of cognitive development is a concomitant variable which is correlated ($r=.40$, $p < .001$) with the dependent (problem-solving) variable. Myers suggests that if the correlation of a concomitant variable with a dependent variable is about .40, blocking is preferred rather than covariant analysis. The investigator was interested in an interaction between levels of cognitive development and treatment groups, that is, were differences in the effects of different treatments (teaching methods) greater at one level of cognitive level than at another? Thus, blocking on cognitive development levels by treatment groups was considered in analyzing the total problem-solving scores. Cell means on problem-solving scores by treatment and cognitive blocks are reported in Table 11.
A two-way ANOVA was performed on students problem-solving scores with treatment condition and cognitive blocks as independent variables. Results of the two-way ANOVA are presented in Table 12. The following hypothesis was tested:

H8: There will be no significant differences among all treatment groups on the problem-solving test at each of the four levels of cognitive development.

A significant main effect due to both cognitive blocks and treatment existed (p < .001). No significant interaction existed.
between treatment condition and cognitive blocks. This indicates that the effect of treatment conditions is constant over cognitive blocks. Since significant treatment condition and cognitive blocks existed, simple effects for each treatment group at each level of cognitive block were investigated. An examination of the total means for all treatment groups at each level of cognitive blocks ranged from 8.79 to 16.67 for levels one to four respectively (See Table 11). The means of 13.80 and 16.67 favor the formal reasoners over the non-formal reasoners with means of 8.79 and 11.78. Post-hoc comparisons were used to test simple effects of treatment groups at each level of cognitive blocks. Dunn's procedure was used with pairwise comparisons between the treatment groups.

Table 12
Two-Way Analysis Of Variance Of Problem-Solving Scores By Treatment and Cognitive Blocks

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive Blocks (A)</td>
<td>3</td>
<td>354.690</td>
<td>118.230</td>
<td>6.577</td>
<td>0.001</td>
</tr>
<tr>
<td>Treatment (B)</td>
<td>2</td>
<td>428.054</td>
<td>241.027</td>
<td>11.905</td>
<td>0.001</td>
</tr>
<tr>
<td>AB</td>
<td>6</td>
<td>97.654</td>
<td>16.276</td>
<td>0.905</td>
<td>0.496</td>
</tr>
<tr>
<td>Error</td>
<td>72</td>
<td>1294.380</td>
<td>17.977</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>83</td>
<td>2174.778</td>
<td>26.200</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
According to Kennedy (1978) when few pairwise comparisons are desired, Dunn’s procedure is more powerful than other hypothesis-wise techniques. Three pairwise comparisons were performed at each cognitive block level. The specific error rate for each of the three separate comparisons was (.05/3 = .017). Thus, each of the separate comparisons was made at the .017 level of significance. Results of post-hoc comparisons using Dunn’s procedure on the problem-solving cell means of treatment by cognitive blocks are provided in Table 13. Results revealed that TB1 and TB2 were both significantly different ($p < .017$) from the T group at Level I of understanding. There were no significant differences among the treatment groups at Levels II, III, and IV. Since there were no significant differences between treatment groups at each of the four levels of cognitive development, $H_8$ cannot be rejected at the .017 level of significance.

Further investigation was performed on the problem-solving scores by using two levels of blocks: (1) non-formal reasoning ability and (2) formal reasoning ability. The cell means for levels I and II were collapsed across all treatment groups and cell means for levels III and IV were collapsed across all treatment groups to form the two blocks indicated above respectively. Cell means on problem-solving scores by treatment and reasoning ability blocks are reported in Table 14.

A two-way ANOVA was performed on the problem-solving scores with treatment and reasoning ability blocks as independent variables. Results of the two-way ANOVA are presented in Table 15. The following hypothesis was tested:

$H_9$: There will be no significant difference between the
performance of formal and non-formal reasoners on the problem-solving test.

Results from the ANOVA revealed that a significant main effect was due to both reasoning ability blocks and treatment condition. Examination of cell means from Table 14 indicated that the means for all treatment groups at the formal level were higher than the means for all treatment groups at the non-formal level. The overall mean for each level favored the formal reasoners with mean of 14.88 over the non-formal reasoners with mean of 9.70. Thus, H9 was rejected at the .05 level of significance.

Table 13
Dunn's Post-Hoc Comparison Procedure on Problem-Solving Cell Means Of Treatment Condition by Cognitive Blocks

<table>
<thead>
<tr>
<th>Cognitive Blocks</th>
<th>Comparison</th>
<th>Results</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>TB2 vs. TB1</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td></td>
<td>TB1 vs. T</td>
<td>T &lt; TB1</td>
<td>.017</td>
</tr>
<tr>
<td></td>
<td>TB2 vs. T</td>
<td>T &lt; TB2</td>
<td>.017</td>
</tr>
<tr>
<td>II</td>
<td>TB2 vs. TB1</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td></td>
<td>TB1 vs. T</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td></td>
<td>TB2 vs. T</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td>III</td>
<td>TB2 vs. TB1</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td></td>
<td>TB1 vs. T</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td></td>
<td>TB2 vs. T</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td>IV</td>
<td>TB2 vs. TB1</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td></td>
<td>TB1 vs. T</td>
<td>No Difference</td>
<td>NS</td>
</tr>
<tr>
<td></td>
<td>TB2 vs. T</td>
<td>No Difference</td>
<td>NS</td>
</tr>
</tbody>
</table>

NS = Non Significant
### Table 14

Cell Means On Problem-Solving Scores By Treatment And Reasoning Ability Blocks

<table>
<thead>
<tr>
<th>Treatment</th>
<th>(Non-Formal)</th>
<th>(Formal)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean</td>
</tr>
<tr>
<td>TB1</td>
<td>27</td>
<td>11.11</td>
</tr>
<tr>
<td>TB2</td>
<td>29</td>
<td>11.10</td>
</tr>
<tr>
<td>T</td>
<td>20</td>
<td>5.75</td>
</tr>
<tr>
<td>Total</td>
<td>76</td>
<td>9.70</td>
</tr>
</tbody>
</table>

### Table 15

Two-Way Analysis Of Variance of Problem-Solving Scores By Treatment And Reasoning Ability Blocks

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p&lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning Ability (A)</td>
<td>1</td>
<td>190.443</td>
<td>190.443</td>
<td>9.680</td>
<td>0.003</td>
</tr>
<tr>
<td>Treatment (B)</td>
<td>2</td>
<td>422.625</td>
<td>211.312</td>
<td>10.741</td>
<td>0.001</td>
</tr>
<tr>
<td>AB</td>
<td>2</td>
<td>21.684</td>
<td>10.842</td>
<td>0.551</td>
<td>0.579</td>
</tr>
<tr>
<td>Error</td>
<td>78</td>
<td>1534.598</td>
<td>19.674</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>83</td>
<td>2169.350</td>
<td>26.137</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Correlational Analyses On Selected Variables

The Investigator was interested in the relationship that existed between the variables. Pearson Product-Moment correlations were performed on the following selected variables: pretest (PRE), posttest (PST), problem-solving scores (PRB), levels of cognitive development (COG), and reasoning ability levels (REA). The correlation matrix for all selected variables is shown in Table 16. The following hypothesis was tested:

H10: This hypothesis was considered in three parts which were tested separately:

H10A: There will be no significant positive correlation between students' posttest and problem-solving test scores.

H10B: There will be no significant positive correlation between students' pretest and problem-solving test scores.

H10C: There will be no significant positive correlation between students' pretest and posttest scores.

Significant positive correlations existed between the following variables: PST and PRB with $r = .60, p < .01$, PRE and PRB with $r = .56$, $p < .01$, and PRE and PST with $r = .69, p < .01$. The results indicate that students' understanding and use of variables before and after treatments were directly related to problem-solving performance. The results revealed that students' understanding and use of variables before treatment were directly related to their use and understanding
of variables after treatment. Thus, hypotheses H10A, H10B, and H10C were rejected.

Table 16
Pearson Product-Moment Correlation Matrix For Selected Variables

<table>
<thead>
<tr>
<th></th>
<th>PRE</th>
<th>PRB</th>
<th>PST</th>
<th>COG</th>
<th>REA</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRE</td>
<td>1.00</td>
<td>.56**</td>
<td>.69**</td>
<td>.80**</td>
<td>.55**</td>
</tr>
<tr>
<td>PRB</td>
<td>1.00</td>
<td>.60**</td>
<td>.40**</td>
<td>.30**</td>
<td></td>
</tr>
<tr>
<td>PST</td>
<td>1.00</td>
<td>.44**</td>
<td></td>
<td>.30**</td>
<td></td>
</tr>
<tr>
<td>COG</td>
<td>1.00</td>
<td></td>
<td>.80**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>REA</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

** Significance at the .01 level
* Significance at the .05 level

The significant positive correlations between COG and PRE (.80), PRB (.40), and PST (.44) suggest that students' levels of cognitive development were directly related to problem-solving performance and understanding and use of variables before and after treatments. REA correlated significantly with each of the variables (PRE, .55; PRB, .30; PST, .30; and COG, .80). These results indicate that students' reasoning abilities were directly related to their levels of cognitive development, problem-solving abilities, and understanding and use of variables before and after treatments.
Summary Of Quantitative Data

The results of the statistical analyses are summarized as follows:
1. No significant ($p < .057$) differences among Treatment groups TB1, TB2, and T on the pretest were detected. H1 could not be rejected at the .05 level of significance.
2. No significant ($p < .039$) difference between Treatment groups TB1 and TB2 on the posttest was detected. H2 could not be rejected at the .0125 level of significance.
3. No significant ($p < .478$) difference between the combined TB1 and TB2 treatment groups and the T treatment group on the posttest was detected. H3 could not be rejected at the .0125 level of significance.
4. No significant ($p < .975$) difference between Treatment groups TB1 and TB2 on problem-solving was detected. H4 could not be rejected at the .0125 level of significance.
5. A significant ($p < .001$) difference between the combined TB1 and TB2 treatment groups and the T treatment group on problem-solving was detected. This difference favored the TB treatment groups. H5 was rejected at the .0125 level of significance.
6. A significant ($p < .012$) difference between the combined TB1 and TB2 treatment groups and the T treatment group on translation was detected. This difference favored the TB treatment groups. H6 was rejected at the .0125 level of significance.
7. A significant ($p < .001$) difference between the combined TB1 and TB2 treatment groups and the T treatment group on numerical solutions was detected. This difference favored the TB treatment...
groups. H7 was rejected at the .0125 level of significance.

8. No significant differences among Treatment groups TB1, TB2, and T on problem-solving at each of the four levels of cognitive development were detected. H8 could not be rejected at the .017 level of significance.

9. A significant (p < .003) difference between non-formal and formal reasoners on problem-solving for TB1, TB2, and T treatment groups was detected. This difference favored the formal reasoners. H9 was rejected at the .01 level of significance.

10 a. A significant positive correlation (r = .60, p < .01) between students' posttest and problem-solving test scores was detected. H10A was rejected at the .01 level of significance.

b. A significant positive correlation (r = .56, p < .01) between students' pretest and problem-solving test scores was detected. H10B was rejected at the .01 level of significance.

c. A significant positive correlation (r = .69, p < .01) between students' pretest and posttest scores was detected. H10C was rejected at the .01 level of significance.

Significant positive correlations between levels of cognitive development and pretest, posttest, and problem-solving test scores were detected. Pretest, posttest, and problem-solving test scores were significantly positively correlated with reasoning ability.
Summary

Developing the concept of variable and solving problems successfully is a goal of the school mathematics programs. The review of literature indicated that students have misconceptions about variables and difficulty in solving word problems. In order that students might overcome these difficulties, there is a need to strengthen or provide appropriate instructional strategies for the concept of variable and problem-solving.

Algebra is a useful tool for solving problems. A necessary feature of algebra is the understanding and use of variables. Conceptual understanding of variables needs to be developed until students feel comfortable with variables and can use them to solve problems successfully. Approaches used in most algebra classes in developing this concept require students to use translation and deductive reasoning.

The purpose of this study was to investigate an alternative approach for developing the concept of variable. The objective was to determine the effects on a student's ability to understand and use variables of a Table-Building (TB) procedure of teaching word problems.
A Translation (T) method of teaching word problems was compared with the TB procedure.

The amount of numerical exploration students experience in solving word problems before a variable is introduced in the table-building process might affect their conceptual understanding of variables. Two primary and two secondary research questions were examined in this study:

(1) Will students who were exposed to instruction in the Table-Building Method be more successful in understanding and using variables than students who were exposed to instruction in the Translation Method?

(2) Does the time a variable was introduced in the table-building process affect a student's ability to use variables and to solve word problems?

(3) Will students who were exposed to Table-Building instruction have better problem-solving performance than students exposed to Translation Instruction?

(4) What relationship existed between students' overall levels of cognitive development and their ability to solve word problems?

The population consisted of 97 ninth grade pre-algebra (Math 9) students from East High School, an inner-city public senior high school in Columbus, Ohio. The students were assigned to treatments by creating eight small groups of approximately 13 students. Each of the four intact classes was divided into two groups and each group was
assigned to one of three treatments: the Table-Building (TB) treatment consisting of two experimental groups (TB1, TB2) and the Translation (T) treatment (a comparison group). The difference in the TB groups was the timing of the introduction of a variable in the table-building process. The TB approach required students to reason with numbers initially and determine solutions to problems by using intuitive techniques such as: (a) guessing and checking, (b) successive approximation, and (c) recognition of patterns. The T approach required students to introduce a variable initially and use translation to write an equation to determine the solution.

Eleven 40-minute group sessions were used for each treatment group. Each group met with the investigator once a week over an eleven week period. Treatments were administered during the second through the ninth sessions. The first, tenth, and eleventh sessions were used for testing. The investigator instructed all group sessions. Prior to treatments students' initial understanding and use of variables were assessed by a pretest. The test assessed students' use and understanding of variables at four levels of understanding based on Piagetian intellectual or cognitive levels of concrete and formal thought. After the treatments, achievement and understanding were assessed by a problem-solving test and a posttest. The problem-solving test assessed a student's ability to use variables and problem-solving procedures. The posttest paralleled the pretest and assessed students' abilities to use and understand variables.

The experimental design was a completely randomized one-factor design with three levels of treatments, TB1, TB2, and T groups. The
primary independent variables were treatment group membership and levels of cognitive development. The dependent variables were the pretest, posttest, and problem-solving (total, translation, numerical) test scores. Data analysis for this study was divided into two parts: (1) subjective analyses of qualitative data and (2) statistical analyses of quantitative data.

Results Of Null Hypotheses

This section provides a list of the null hypotheses that were tested and the outcomes.

H1: There will be no significant differences among all three treatment groups on the pretest.

Results: H1 could not be rejected at the .05 level of significance.

H2: There will be no significant difference between the two experimental groups on the posttest.

Results: H2 could not be rejected at the .0125 level of significance.

H3: There will be no significant difference between the TB treatment groups and the T treatment group on the posttest.

Results: H3 could not be rejected at the .0125 level of significance.

H4: There will be no significant difference between the two experimental groups on the problem-solving test.

Results: H4 could not be rejected at the .0125 level of significance.

H5: There will be no significant difference between the TB treatment groups and the T treatment group on the problem-solving test.

Results: H5 was rejected at the .0125 level of significance. This difference favored the TB treatment groups.
H6: There will be no significant difference between the T treatment group and the TB treatment groups on the translation scores.

**Results:** H6 was rejected at the .0125 level of significance. This difference favored the TB treatment groups.

H7: There will be no significant difference between the TB treatment groups and the T treatment group on numerical scores.

**Results:** H7 was rejected at the .0125 level of significance. This difference favored the TB treatment groups.

H8: There will be no significant differences among all treatment groups on the problem-solving test at each of the four levels of cognitive development.

**Results:** H8 could not be rejected at the .017 level of significance.

H9: There will be no significant difference between the performance of formal and non-formal reasoners on the problem-solving test.

**Results:** H9 was rejected at the .01 level of significance. This difference favored the formal reasoners.

H10A: There will be no significant positive correlation between students' posttest and problem-solving test scores.

**Results:** H10A was rejected at the .01 level of significance.

H10B: There will be no significant positive correlation between students' pretest and problem-solving test scores.

**Results:** H10B was rejected at the .01 level of significance.

H10C: There will be no significant positive correlation between students' pretest and posttest scores.

**Results:** H10C was rejected at the .01 level of significance.
Discussion

The purpose of the investigation was to determine the effects of a Table-Building Method of teaching problems on students' ability to understand and use variables. This method was compared with a Translation method. That is, would students exposed to TB Instruction be more successful in understanding and using variables than those exposed to T Instruction, and would the time a variable was introduced in the table-building process affect students' abilities to use variables and to solve word problems. On the basis of the investigator's observation of treatment groups, it appeared that students exposed to TB treatment would better understand variables than those exposed to T treatment, but the investigator did not think any significant differences would exist among the groups. The failure to reject hypothesis one indicated that originally no significant differences existed among treatment groups on understanding and using variables. The increase in mean performance from 11.98 on the pretest to 13.50 on the posttest indicated that students had improved on their understanding and use of variables. A failure to reject hypothesis three indicated that no significant difference existed between the combined TB treatment groups and the T treatment group on their abilities to understand and use variables after treatments. The investigator's opinion was that students exposed to T and TB1 treatments would use variables more than the TB2 group because of the more exposure to variables. The means of 15.14 and 12.91 for TB1 and T treatment groups on the posttest were higher than the mean of 12.45 for the TB2 group, favoring the TB1 group. Although no statistically
significant differences existed among all treatment groups on the posttest, the mean of 15.14 for the TB1 group and the Investigator's opinion and observations indicate that the TB1 group could better understand and use variables than T and TB2 treatment groups.

The Investigator's opinion was that the time variables were introduced in the table-building process appeared not to affect a student's ability to solve problems but would have some effect on a student's use of variables. The TB1 group was exposed to variable use longer than the TB2. This might have affected students' use of variables, but appeared not to make any difference on their understanding and use of variables in solving problems.

Would students exposed to TB Instruction have better problem-solving performance than students exposed to Translation Instruction? What relationship existed between students' overall levels of cognitive development and their ability to solve word problems? The rejection of hypothesis five indicated that students exposed to TB Instruction had better problem-solving performance than those exposed to T Instruction. The rejection of hypotheses six and seven indicated that students exposed to TB Instruction were more successful than students exposed to T Instruction in translating and obtaining numerical solutions. A mean of 2.55 for the TB1 group was favored for translation and a mean of 7.82 for the TB2 group was favored for numerical solutions.

The TB1 group translated better than the other groups because translation was implicit in the table. The TB1 students first translated using just numbers, then gradually built to a generalization
where variables were used. The TB2 group was favored on numerical solutions possibly because that group had more numerical exploration than the other two groups. It was hypothesized that the T treatment group would translate better than the TB groups because of their exposure to the continuous and direct translation. The TB1 group that was favored on translation did a different kind of translation than the T group. The translation was built in the table by the labeling of column headings. Students had more practice with the numbers in translating and if a variable was used, the translation was more or less by analogy.

It was predicted that students exposed to TB instruction would have better problem-solving performance than students exposed to T instruction. A variety of strategies for TB instruction were used by the students. Strategies such as approximating, pattern seeking, numerical exploring, and guessing and checking were used by TB students whereas the T treatment group had a set of steps or rules to follow in solving problems. The problem-solving procedures used by the different treatment groups may have made the difference in the problem-solving performance between the groups.

It was the Investigator's prediction that students' overall levels of cognitive development would be directly related to their problem-solving performance. The significant positive correlation between students' levels of cognitive development and problem-solving test scores indicated that as levels of cognitive development increased their problem-solving performance increased. This result supported the prediction made by the Investigator. The rejection of hypothesis nine
Indicated that formal reasoners had better problem-solving performance than non-formal reasoners. The results indicated that there were no significant differences among all three treatment groups on problem-solving at the formal level of reasoning, but the TB treatment groups were significantly different from the T treatment group on problem-solving performance at the non-formal level of reasoning favoring the TB groups. This suggests that students at the concrete level were less successful with problem-solving using translation instruction than those exposed to table-building instruction.

Conclusions

On the basis of analyses of qualitative and quantitative data results, four conclusions can be drawn:

1. There were no statistically significant differences between students who had received Table-Building (TB) Instruction and Translation (T) Instruction on their abilities to understand and use variables. However, subjective analyses and the group mean performance indicated that the TB1 group had a better understanding of variables than TB2 and T groups, but the TB1 and T groups used variables more than the TB2 group.

2. The time a variable was introduced in the table-building process did affect students' use of variables but did not affect their understanding of variables. However, no significant effect was made on a student's ability to use variables to solve problems.

3. Students exposed to Table-Building (TB) Instruction had significantly better problem-solving performance than those
exposed to Translation (T) Instruction. The TB groups were more successful than the T group in translating and obtaining numerical solutions. The TB1 group was favored for translation and the TB2 group was favored for numerical solutions.

4. There was a significant positive relationship between students' overall levels of cognitive development and their ability to solve word problems. Formal reasoners had better problem-solving performance than non-formal reasoners. There were no significant differences among all three treatment groups on problem-solving at the formal level of reasoning, but the TB treatment groups were significantly different from the T treatment group at the non-formal level, favoring the TB groups. Thus, students at the concrete level were less successful with problem-solving using the translation method than concrete thinkers who used the table-building method.

The following opinions were formed from the Investigator's observations of students' reactions to treatments:

1. TB and T groups had similar attitudes and behavior patterns during the administrations of treatments. The attitudes and behavior patterns for students exposed to both TB and T treatments were typical of ninth grade mathematics students.

2. TB and T groups had similar reading and vocabulary problems.

3. TB and T groups had difficulties in showing relationships between the wanted and given information in problems. Because of this, the TB groups had initial difficulties in correctly labeling
table headings, and the T group had initial difficulties in the identification and use of variables in translation.

4. TB and T groups had trouble in successfully recalling or retrieving information from previous sessions. Since all of the treatment problems were not of the same type, and students could use a variety of strategies with the table to solve each problem, some of the students were confused about which strategies were most appropriate for certain problems. Because of this, they had initial difficulties with the guessing process -- which number to try first, identification of patterns in the table, and the use of approximations. The T group had initial difficulties in recalling the problem-solving rules. Some students did not use the problem-solving procedures correctly in translating and writing equations to solve problems.

5. The TB groups had initial difficulties in finding specific solutions to problems. Some students made computational errors in the table and tried to force numbers to fit the solution. In many instances, if students could not successfully approximate or identify patterns in the table, they were not successful with finding the answers to problems by the use of the table. Some students used their own guessing process without the use of the table and in some cases were successful with the solutions. The T group had initial difficulties in understanding and solving the equation. Some students could solve equations requiring single operations, but they had trouble with equations requiring multiple operations. If students were not successful in solving the
equations, they could not determine the solutions to problems.

On the basis of qualitative and quantitative data, the investigator concluded that the Table-Building Method was more effective than the Translation Method in developing the concept of variable and teaching the solution to word problems.

**Implications**

Results indicated that students exposed to table-building instruction had significantly better problem-solving performance than those exposed to translation instruction. The TB groups were more successful in translating than the T group, and the TB1 group had a better understanding of variables than the T group. The evidence suggests table-building instruction is an appropriate method in teaching the concept of variable and solutions to word problems. Classroom teachers need to consider this alternative in their teaching methods. If a goal in mathematics is to help students become successful problem solvers, teachers need to provide appropriate instructional techniques for students that will develop meaning and understanding in mathematics and result in increased achievement. That is to say, students need experience in the table-building guess and check process.

The majority of classroom teachers presently teach the concept of variable and solutions to word problems by the traditional translation method. This study indicates that concrete reasoners exposed to translation instruction were not successful in solving problems.
Teachers need to be aware of the fact that not all of the students in their classes are at the same level of cognitive development and cannot learn or understand concepts that are taught at the formal level of thinking. This study reveals that students exposed to table-building instruction were more successful problem solvers than those exposed to translation instruction. For students who cannot think at the formal level, teachers need to provide intuitive approaches for problem-solving.

The table-building method investigated in this study is intuitive in nature and takes an inductive approach to solving word problems. Students are asked to reason through the problem with specific numbers rather than variables. The method allows students to explore a variety of techniques in determining the solution to a problem. Advantages of this method are: (a) seeing numerical relationships, (b) guessing and checking possible solutions, (c) using successive approximations, (d) seeing that a variable represents numbers from some set, (e) using analogy rather than abstract reasoning in writing equations, (f) recognizing patterns, (g) moving from specific values to generalizations.

This study dealt specifically with coin and number problems and involved average to below average ninth grade mathematics students in an inner-city setting. Teachers should incorporate this method with a variety of students at different ability levels at all grade levels in the mathematics curriculum but should use a variety of problem situations.
In order that classroom instruction might be more effective, appropriate instructional materials need to be developed. If teachers are willing to use alternative teaching methods, teaching aids and instructional materials to support those methods need to be developed.

The materials were developed for this study to meet the needs of a specific population of students. These materials can be modified to meet the needs of different ability level students and of different grade levels with variations in the types of problems used and the type questions asked.

The textbook has been the primary source of instruction throughout the school mathematics curriculum. In mathematics teaching, workbooks or in-class work sheets are used to supplement the textbook for many teachers, whereas others rely heavily on the text and no additional instructional materials other than the blackboard. For teaching concepts such as variables and solutions to word problems, it is important that teachers use instructional materials that will enable students to understand these concepts. The use of textbooks and the blackboard is not always the best source of instruction for all students. The TB treatment worksheets used in this study were appropriate for some students in teaching the concept of variable and solutions to word problems.

Considerable time is spent in reteaching the same concepts from grade level to grade level. If appropriate instructional aids are provided at certain levels in developing certain concepts such as understanding and using variables in solving problems, perhaps some of the time spent in reteaching these concepts can be minimized. Some
materials appear to be effective with certain content at all age levels and with all types of children. Although the use of the calculator was not maximized for use with the instructional materials in this study, this instructional aid can be used throughout the curriculum to develop the concept of variable and problem-solving. The use of the calculator will enable students to minimize errors in computation and to solve a variety of problems.

The use of a microcomputer is another tool or instructional aid that can be used with the TB treatment materials from this study. An appropriate microcomputer program such as VisiCalc may be used to generate tables of numerical values that might enable students to better understand the concept of variable and determine solutions to word problems. By applying problem-solving strategies and rules for table-building, students can use VisiCalc to generate a table of numerical values. VisiCalc is a powerful "electronic worksheet" that replaces paper, pencils, and the calculator, and allows one to solve involved number problems in just seconds that usually would take hours to do by hand. (Williams & Taylor, 1981; VisiCalc, 1982; Visicalc, 1983). This worksheet can be used to construct similar tables that were used in this study, allowing students to use inductive problem-solving procedures.

The instructional materials mentioned may not be appropriate or applicable in all situations and for all students at all levels. Teachers must observe and make decisions as to which instructional materials are most applicable to various situations.
Recommendations for Further Study

On the basis of the conclusions from this study there is evidence that more research is needed in the area of problem-solving and concept development of variables. Some factors affecting students' understanding of variables and problem-solving performance have been investigated in this study and many other factors have been identified which need further investigation.

The following areas are recommended for future research:

1. Design a study using both TB and T treatments controlling for reading ability and attitude.

2. Using only TB treatment, design a study with three experimental groups investigating the effects of the time at which a variable is introduced in the table-building process upon students' use and understanding of variables in solving problems.

3. Using only TB treatments investigate further the effects of the following:
   (a) Students' ability to label table headings appropriately expressing relationships between the wanted and given information in the problem.
   (b) Students' use of inductive problem-solving strategies such as the guessing process, identification of patterns, and use of approximations.
   (c) Students' abilities to write equations.

4. Investigate the effects a microcomputer electronic spreadsheet used in generating tables of numerical values will have on students' problem-solving performance. It is important to
Investigate which aspects or variants of the table will influence students' behavior, and what effect the computer will have on their understanding and use of variables.

5. Design a study using TB treatments for above average students using a wide variety of problem situations for elementary, middle, and high school students in an inner-city or suburban school setting.
APPENDIX A

TB TREATMENT MATERIALS
1. The second of two numbers is 13 more than the first.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>First Number</th>
<th>13 more than the first</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7 + 13</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>20 + 13</td>
<td></td>
</tr>
</tbody>
</table>

(b) Explain in words how to find the second number if the first number is known

(c) Explain in words how to find the first number if the second number is known

2. The second of two numbers is 4 less than the first number.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>First Number</th>
<th>4 less than the first</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9 - 4</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>16 - 4</td>
<td></td>
</tr>
</tbody>
</table>

(b) Explain in words how to find the second number if the first number is known

(c) Explain in words how to find the first number if the second number is known
3. You have 30 coins in nickels and dimes.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
<th>Value of Nickels</th>
<th>Value of Dimes</th>
<th>Value of Nickels plus value of dimes</th>
<th>Total Value in cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25</td>
<td>5(5)</td>
<td>10(25)</td>
<td>25 + 250</td>
<td>275.2</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td>190.5</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Explain in words how to find the number of dimes if the number of nickels is known ____________________________

(c) Explain in words how to find the value of the coins if the number of nickels and dimes is known ____________________________

4. The second of two numbers is three times the first.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>First Number</th>
<th>three times the first</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3(2)</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>3(15)</td>
<td></td>
</tr>
</tbody>
</table>

(b) Explain in words how to find the second number if the first number is known ____________________________

(c) Explain in words how to find the first number if the second number is known ____________________________
Directions: Complete the tables with the correct number, and fill in the indicated blanks with a correct statement.

1. The second of two numbers is 3 less than the first number.
   (a) Complete the following table:

<table>
<thead>
<tr>
<th>First Number</th>
<th>3 less than the first</th>
<th>Second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4-3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>28-3</td>
<td></td>
<td>47</td>
</tr>
</tbody>
</table>

   (b) Explain in words how to find the second number if the first number is known ________________________________________________

   (c) Explain in words how to find the first number if the second number is known ________________________________________________

2. You have 50 coins in dimes and quarters
   (a) Complete the following table

<table>
<thead>
<tr>
<th>Number of Dimes</th>
<th>Number of Quarters</th>
<th>Value of Dimes</th>
<th>Value of Quarters</th>
<th>Value of Dimes plus value of Quarters</th>
<th>Total Value in cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>50</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>$0.10</td>
<td>$0.25</td>
<td>$0.15</td>
<td>0.35</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>$0.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>$0.30</td>
<td></td>
<td></td>
<td>$0.30</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>$0.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>$0.50</td>
<td></td>
<td></td>
<td>$0.50</td>
</tr>
</tbody>
</table>

   (b) Explain in words how to find the number of quarters if the number of dimes is known ________________________________________________

   (c) Explain in words how to find the value of the coins if the number of dimes and quarters is known ________________________________________________

3. The first of two numbers is 5 more than the second. (a) Complete the table

<table>
<thead>
<tr>
<th>First Number</th>
<th>5 more than second Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>17</td>
</tr>
<tr>
<td>41</td>
<td>46</td>
</tr>
</tbody>
</table>

   (b) Explain in words how to find the second number if the first number is known ________________________________________________
Many different strategies can be used to solve story problems. In this unit you will determine the solutions to story problems by constructing a table of numerical values. Numerical relationships will be established between the given and wanted information in the problem. A strategy that will be used in building the table is to "guess" the answer to the problem, then "check" whether the guess is correct. The results of one guess can help you make a better guess. The purpose here is to help you approximate the answer and recognize any patterns in the table. The variable $x$ will be introduced in the last row of the table. This variable can be used to write an equation expressing relationships between the given and wanted information in the problem.

**Story**

1) The sum of a certain number and 87 is 253. Find the number.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number plus 87</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>120 + 87</td>
<td>207</td>
</tr>
<tr>
<td>160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td></td>
<td>253</td>
</tr>
<tr>
<td>$x$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. a) Is the number 120?  
   b) What is the sum of 87 and 120?  
   c) Is this sum greater or less than the sum 253?  

2. Fill in the second and third rows of the table
   a) Is 160 the number?  
   b) Is 180 the number?  

3. What pattern do you recognize in the table?
   a) 253 must lie between what two sums in the table?  
   b) Thus the number must lie between what two numbers?  

4. a) From the sum 247, how many units are needed to produce the sum 253?  
   b) What number in the table produces the sum 247?  
   c) How many units must be added to this number?  
   d) Thus the number is?  
   e) Now fill in the fourth row of this table  

5. a) Suppose this number is $x$, how would you express the sum of this number and 87?  
   b) This expression is the same as what number in the table?  
   c) Write an equation expressing this relationship  
   d) This equation can be solved to find the number.
Now let us try another example. Use some of the same strategies you used in the first problem.

2) The sum of four times a certain number and 20 equals 236. Find the number.

Determine the solution to this problem by completing the table:

<table>
<thead>
<tr>
<th>No.</th>
<th>4 times No. plus 20</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>4(50)+20</td>
<td>220</td>
</tr>
<tr>
<td></td>
<td>4(52)+20</td>
<td>228</td>
</tr>
<tr>
<td></td>
<td>4(56)+20</td>
<td>236</td>
</tr>
<tr>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Fill in the 2nd & 3rd rows.
2. a) What patterns do you recognize?
   b) As the sums increase by 8 what happens to the No.?
   c) As the sums increase by 16 what happens to the No.?
3. Fill in the 4th and 5th rows.
4. Write an equation.
5. What is the number?

3) The sum of twice a certain number and 28 equals 96. What is the number?

<table>
<thead>
<tr>
<th>No.</th>
<th>2 times No. plus 28</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>2(31)+28</td>
<td>90</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td></td>
<td>96</td>
</tr>
</tbody>
</table>

1. Fill in all the information in the table.
2. What patterns do you recognize?
3. What is the number?
4. Write an equation.
1) The sum of a number and 56 is 200
   a) Complete the table:

<table>
<thead>
<tr>
<th>Number plus 56</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>120+56</td>
<td></td>
</tr>
<tr>
<td>130+56</td>
<td>200</td>
</tr>
<tr>
<td>X</td>
<td>X+56</td>
</tr>
</tbody>
</table>

   b) If the sum is given, explain how to find the number

   c) The expression $X+56$ is the same as what number in the problem

   d) Write an equation expressing this relationship

Solve the following story problems:

2) The sum of a certain number and 70 is 350. Find the number.
   a) Complete the table:

<table>
<thead>
<tr>
<th>Number plus 70</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>150+70</td>
<td>320</td>
</tr>
<tr>
<td>250+70</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>350</td>
</tr>
</tbody>
</table>

   b) If the sum is 350, explain how to find the number

   c) What is the number?

   d) Write an equation expressing the relationship between $X+70$ and 350

3) Twice a certain number increased by 30 equals 120. Find the number.
   a) Complete the table:

<table>
<thead>
<tr>
<th>Twice the number plus 30</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(30)+30</td>
<td>90</td>
</tr>
<tr>
<td>2(40)+30</td>
<td>110</td>
</tr>
<tr>
<td>2(50)+30</td>
<td>130</td>
</tr>
<tr>
<td>X</td>
<td>120</td>
</tr>
<tr>
<td>2(X)+30</td>
<td></td>
</tr>
</tbody>
</table>

   b) If the sum is given, explain how you would find the number

   c) What is the number

   d) Write an equation expressing the relationship $2(x)+30$ and 120 in the given problem

   ___________________________________________
Many different strategies can be used to solve story problems. In this unit you will determine the solutions to story problems by constructing a table of numerical values. Numerical relationships will be established between the given and wanted information in the problem. A strategy that will be used in building the table is to "guess" the answer to the problem, then "check" whether the guess is correct. The results of one guess can help you make a better guess. The purpose here is to help you approximate the answer and recognize any patterns in the table.

**Story: 1)** The sum of a certain number and 67 is 253. Find the number.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number plus 67</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>120 + 67</td>
<td>207</td>
</tr>
<tr>
<td>160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td></td>
<td>253</td>
</tr>
</tbody>
</table>

1. a) Is the number 120?
   b) What is the sum of 67 and 120?
   c) Is this sum greater or less than the sum 253?

2. Fill in the second and third rows of the table
   a) Is 160 the number?
   b) Is 180 the number?

3. What pattern do you recognize in the table?
   a) 253 must lie between what two sums in the table?
   b) Thus the number must lie between what two numbers?

4. a) From the sum 257, how many units are needed to produce the sum 253?
   b) What number in the table produces the sum 257?
   c) How many units must be added to this number?
   d) Thus the number is
   e) Now fill in the fourth row of this table
Session 2

Now let us try another example. Use some of the same strategies you used in the first problem.

**Step 2**
The sum of four times a certain number and 20 equals 236. Find the number.

Determine the solution to this problem by completing the table:

<table>
<thead>
<tr>
<th>No.</th>
<th>4 times No. plus 20</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>h(50)+20</td>
<td>220</td>
</tr>
<tr>
<td>52</td>
<td>h(52)+20</td>
<td>228</td>
</tr>
<tr>
<td>56</td>
<td>h(56)+20</td>
<td>236</td>
</tr>
</tbody>
</table>

1. Fill in the 2nd & 3rd rows.
2. a) What patterns do you recognize?
   b) As the sums increase by 8, what happens to the No.?
   c) As the sums increase by 16, what happens to the No.?
3. Fill in the 4th row.

**Step 3**
The sum of twice a certain number and 28 equals 96. What is the number?

<table>
<thead>
<tr>
<th>No.</th>
<th>2 times No. plus 28</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>2(31)+28</td>
<td>90</td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
<td>96</td>
</tr>
</tbody>
</table>

1. Fill in all the information in the table.
2. What patterns do you recognize?
3. What is the number?
1) The sum of a number and 56 is 200
   a) Complete the table:

<table>
<thead>
<tr>
<th>Number</th>
<th>Number plus 56</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>176</td>
<td>200</td>
</tr>
</tbody>
</table>

   b) If the sum is given, explain how to find the number:

Solve the following story problems:

2) The sum of a certain number and 70 is 350. Find the number.
   a) Complete the table:

<table>
<thead>
<tr>
<th>Number</th>
<th>Number plus 70</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>250</td>
<td>350</td>
</tr>
</tbody>
</table>

   b) If the sum is 350, explain how to find the number:

   c) What is the number?

3) Twice a certain number increased by 30 equals 120. Find the number.
   a) Complete the table:

<table>
<thead>
<tr>
<th>Number</th>
<th>Twice the number plus 30</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>35</td>
<td>115</td>
<td>115</td>
</tr>
</tbody>
</table>

   b) If the sum is given, explain how you would find the number:

   c) What is the number?
In this unit we will use the same method we used in the last unit to solve the story problems:

**Story 1)** The sum of two numbers is 97. The larger of the two numbers is 5 more than 3 times the smaller number. Find the numbers.

a) Complete the table

<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>3 times smaller plus 5</th>
<th>Larger Number</th>
<th>Smaller plus Larger</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3(20)+5</td>
<td>65</td>
<td>20+65</td>
<td>85</td>
</tr>
<tr>
<td>22</td>
<td>3(22)+5</td>
<td>71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>3( )+5</td>
<td></td>
<td>X + (3X+5)</td>
<td>97</td>
</tr>
</tbody>
</table>

b) Explain how to find the larger number if the smaller number is given.

c) Explain how to find the smaller number if the larger number is given.

d) Explain how to find the larger number if the smaller number and the sum is given.

e) Explain how to find the sum if the smaller and the larger number is given.

f) What is the smaller number __________

g) What is the larger number __________

h) Write an equation using the variable X to express the sum of the smaller and the larger number equals 97

```
X + (3X+5) = 97
```
1. The sum of two numbers is 90. The larger number equals 14 more than 3 times the smaller number. **Find the numbers.**

   a) Complete the table

<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>3 times smaller plus 14</th>
<th>Larger Number</th>
<th>Smaller plus Larger</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3(5) + 14</td>
<td>29</td>
<td>5 + 22</td>
<td>34</td>
</tr>
<tr>
<td>10</td>
<td>3(10) + 14</td>
<td>59</td>
<td>10 +</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>3(20) + 14</td>
<td></td>
<td></td>
<td>90</td>
</tr>
<tr>
<td>X</td>
<td>3(X) + 14</td>
<td>X + (3X + 14)</td>
<td></td>
<td>90</td>
</tr>
</tbody>
</table>
In this unit we will use the same method we used in the last unit to solve the story problems:

**Story 1)** The sum of two numbers is 97. The larger of the two numbers is 5 more than 3 times the smaller number. Find the numbers.

a) Complete the table

<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>3 times smaller plus 5</th>
<th>Larger Number</th>
<th>Smaller plus Larger</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3(20)+5</td>
<td>65</td>
<td>20+65</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>3(22)+5</td>
<td></td>
<td></td>
<td>71</td>
</tr>
<tr>
<td>24</td>
<td>3( )+5</td>
<td></td>
<td></td>
<td>101</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>97</td>
</tr>
</tbody>
</table>

b) Explain how to find the larger number if the smaller number is given.

c) Explain how to find the smaller number if the larger number is given.

d) Explain how to find the larger number if the smaller number and the sum is given.

e) Explain how to find the sum if the smaller and the larger number is given.

f) What is the smaller number

g) What is the larger number
1. The sum of two numbers is 90. The larger number equals 11 more than 3 times the smaller number. Find the numbers.
   a) Complete the table
   
<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>3 times smaller + 11</th>
<th>Larger Number</th>
<th>Smaller plus Larger</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3(5) + 11</td>
<td>29</td>
<td>5 + 29</td>
<td>34</td>
</tr>
<tr>
<td>10</td>
<td>3(10)+11</td>
<td></td>
<td></td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>3(15)+11</td>
<td>59</td>
<td></td>
<td>94</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td>90</td>
</tr>
</tbody>
</table>

   b) Explain how to find the larger number if the smaller number is given

   c) Explain how to find the smaller number if the larger number and the sum is given

   d) Explain how to find the sum if the smaller and the larger number is given

   e) As the sum increases by ________ the smaller increases by ________ and the larger number increases by ________

   f) What is the smaller number ________

   g) What is the larger number ________
Solve the following story problems using procedures from previous units:

**Story: 1)** A jar contains a number of nickels. The value of the coins is $15.65. How many nickels are in the jar?

<table>
<thead>
<tr>
<th>Number of Nickels</th>
<th>Total Value in Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1000</td>
</tr>
<tr>
<td>250</td>
<td>1250</td>
</tr>
<tr>
<td>300</td>
<td>1500</td>
</tr>
<tr>
<td>X</td>
<td>5X</td>
</tr>
</tbody>
</table>

a) Fill in the third and fourth rows
b) As the number of nickels increases by 50, how much does the total value increases by ________
c) 1565$ falls between what two values in the table?
d) From 1500$ how many cents is needed to produce 1565$?
e) How many nickels is this ________
f) How did you obtain this ________
g) How many nickels will give 1500$ ________
h) From this number add ________
i) Thus the number of nickels in the jar is ________
j) Fill in the fifth and sixth rows

**Story: 2)** A jar contains a number of nickels and 27 dimes. The value of the coins is $9.75. How many nickels are in the jar?

<table>
<thead>
<tr>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
<th>Value of Nickels</th>
<th>Value of Dimes</th>
<th>Value of Nickels plus value of Dimes</th>
<th>Total Value in Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>27</td>
<td>5(100)</td>
<td>10(27)</td>
<td>500 + 270</td>
<td>770$</td>
</tr>
<tr>
<td>120</td>
<td>27</td>
<td>5(120)</td>
<td>10(27)</td>
<td>620 + 270</td>
<td>890$</td>
</tr>
<tr>
<td>140</td>
<td>27</td>
<td>5(140)</td>
<td>10(27)</td>
<td>740 + 270</td>
<td>970$</td>
</tr>
<tr>
<td>165</td>
<td>27</td>
<td>5(165)</td>
<td>10(27)</td>
<td>865 + 270</td>
<td>975$</td>
</tr>
<tr>
<td>X</td>
<td>27</td>
<td>5( )</td>
<td>10( )</td>
<td>5X + 270</td>
<td>975$</td>
</tr>
</tbody>
</table>

a) Fill in rows two through four
b) Look for any patterns in the table and then determine the answer by approximation.
c) Thus the number of nickels in the jar is ________
d) How fill in row five with the correct numbers
e) From the last row in the table suppose X represent the number of nickels and the number of dimes is 27, fill in the blank spaces
f) Write an equation expressing the relationship in the problem between the value of the nickels and the 27 dimes and the total value 975$.
1) A box contains a number of quarters. The value of the coins is $19.75. How many quarters are in the box?

   a) Complete the table

<table>
<thead>
<tr>
<th>Number of Quarters</th>
<th>Value of Quarters</th>
<th>Total Value In Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>25(50)</td>
<td>1250$</td>
</tr>
<tr>
<td>60</td>
<td>25(50)</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>25(70)</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>25(75)</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>25(100)</td>
<td>2000$</td>
</tr>
<tr>
<td>X</td>
<td>25X</td>
<td>1975$</td>
</tr>
</tbody>
</table>

   b) The number of quarters in the box is _____

   c) Write an equation using the variable X to express the relationship in the problem between the value of the quarters and the total value 1975$.

2) A box contains a certain number of dimes and 30 nickels. The value of the coins is $12.00. How many dimes are in the box?

   a) Complete the table

<table>
<thead>
<tr>
<th>Number of Dimes</th>
<th>Number of Nickels</th>
<th>Value of Dimes</th>
<th>Value of Nickels</th>
<th>Value of Dimes plus value of Nickels</th>
<th>Total Value In Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>30</td>
<td>10(80)</td>
<td>5(30)</td>
<td>800 + 150</td>
<td>950$</td>
</tr>
<tr>
<td>90</td>
<td>30</td>
<td>10(90)</td>
<td>5(30)</td>
<td>900 + 150</td>
<td>1050$</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>10(100)</td>
<td>5(30)</td>
<td>1000 + 150</td>
<td>1200$</td>
</tr>
<tr>
<td>110</td>
<td>30</td>
<td>10(110)</td>
<td>5(30)</td>
<td>1100 + 150</td>
<td>1260$</td>
</tr>
<tr>
<td>X</td>
<td>30</td>
<td>10X</td>
<td>5(30)</td>
<td>150 + (10X + 150)</td>
<td>1200$</td>
</tr>
</tbody>
</table>

   b) The number of dimes in the box is _____

   c) Write an equation using the variable X to express the relationship in the problem between the value of the dimes, the value of the nickels and the total value 1200$.
Solve the following story problems using procedures from previous units:

**Story: 1)** A jar contains a number of nickels. The value of the coins is $15.65. How many nickels are in the jar?

<table>
<thead>
<tr>
<th>Number of Nickels</th>
<th>Value of Nickels</th>
<th>Total Value in cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>$200</td>
<td>1000</td>
</tr>
<tr>
<td>250</td>
<td>$250</td>
<td>1250</td>
</tr>
<tr>
<td>300</td>
<td>$300</td>
<td>1500</td>
</tr>
<tr>
<td>350</td>
<td>$350</td>
<td>1750</td>
</tr>
</tbody>
</table>

- a) Fill in the third and fourth rows.
- b) As the number of nickels increases by 50, how much does the total value increase by?
- c) $1565 falls between what two values in the table?
- d) From 1500 to how many cents is needed to produce $1565?
- e) How many nickels is this?
- f) How did you obtain this?
- g) How many nickels will give 1500?
- h) From this number add
- i) Thus the number of nickels in the jar is
- j) Fill in the fifth row.

**Story: 2)** A jar contains a number of nickels and 27 dimes. The value of the coins is $9.75. How many nickels are in the jar?

<table>
<thead>
<tr>
<th>Number of Nickels</th>
<th>Number of Dimes</th>
<th>Value of Nickels</th>
<th>Value of Dimes</th>
<th>Value of Nickels plus value of Dimes</th>
<th>Total Value in Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>27</td>
<td>$100</td>
<td>270</td>
<td>570</td>
<td>770</td>
</tr>
<tr>
<td>120</td>
<td>27</td>
<td>120</td>
<td>270</td>
<td>570</td>
<td>770</td>
</tr>
<tr>
<td>140</td>
<td>27</td>
<td>140</td>
<td>270</td>
<td>570</td>
<td>770</td>
</tr>
<tr>
<td>160</td>
<td>27</td>
<td>160</td>
<td>270</td>
<td>570</td>
<td>770</td>
</tr>
<tr>
<td>180</td>
<td>27</td>
<td>180</td>
<td>270</td>
<td>570</td>
<td>770</td>
</tr>
</tbody>
</table>

- a) Fill in rows two through four.
- b) Look for any patterns in the table and then determine the answer by approximation.
- c) Thus the number of nickels in the jar is
- d) Now fill in row five with the correct numbers.
1) A box contains a number of quarters. The value of the coins is $19.75. How many quarters are in the box?

a) Complete the table

<table>
<thead>
<tr>
<th>Number of Quarters</th>
<th>Value of Quarters</th>
<th>Total Value In Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>25(50)</td>
<td>1250f</td>
</tr>
<tr>
<td>60</td>
<td>25( )</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>2000f</td>
</tr>
</tbody>
</table>

b) The number of quarters in the box is

2) A box contains a certain number of dimes and 30 nickels. The value of the coins is $12.00. How many dimes are in the box?

a) Complete the table

<table>
<thead>
<tr>
<th>Number of Dimes</th>
<th>Number of Nickels</th>
<th>Value of Dimes</th>
<th>Value of Nickels</th>
<th>Value of Dimes plus value of Nickels</th>
<th>Total Value In Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>30</td>
<td>10(50)</td>
<td>5(30)</td>
<td>800 + 150</td>
<td>950f</td>
</tr>
<tr>
<td>90</td>
<td>30</td>
<td>10(140)</td>
<td>5(30)</td>
<td>130 + 150</td>
<td>1250f</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>10(190)</td>
<td>5(30)</td>
<td>180 + 150</td>
<td>1350f</td>
</tr>
<tr>
<td>110</td>
<td>30</td>
<td>10(240)</td>
<td>5(30)</td>
<td>230 + 150</td>
<td>1280f</td>
</tr>
</tbody>
</table>

b) The number of dimes in the box is
During this session you will determine the solution to the story problems by guessing and checking your own numbers, looking for patterns, and using approximations. In the first story I have constructed the table for you. In the second story you will have to construct your own table and label the correct headings from the story and then determine the solution to the problem by procedures previously used:

**Story 1:** The sum of two numbers is 168. The numbers are such that one number is 3 times the other. Find the numbers.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>First No.</th>
<th>3 times First No.</th>
<th>Second No.</th>
<th>First No. plus Second No.</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>168</td>
</tr>
<tr>
<td>X</td>
<td>3(X)</td>
<td>3X</td>
<td></td>
<td>X + 3X</td>
</tr>
</tbody>
</table>

(b) Look in the fourth row of the table. We are looking for two numbers whose sum is 168, and the second number should be 3 times the first number. If this is correct in the table you can now write the correct answers to the given story.

The first number is ____ The second number is ____

(c) Using some of the information in the last row and the sum 168, write a true equation for the given story problem

**Story 2:** The difference of two numbers is 23. The larger number is 5 more than four times the smaller number. Find the numbers.

(a) You need to construct your own table and determine the solution to the given story. (b) Smaller Number is ____ Larger Number is ____
Solve the following story problems by guessing and checking your own numbers until you find the solution. Look for patterns in the table and use approximations to determine the answer to the problem:

1. The sum of two numbers is 99. The larger number is 3 more than twice the smaller number. Find the numbers.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>Smaller</th>
<th>Twice the smaller</th>
<th>Larger</th>
<th>Smaller number plus</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>plus 3</td>
<td>Number</td>
<td>Larger Number</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>2(X+3)</td>
<td>2X+3</td>
<td>Xt(2X+3)</td>
<td>99</td>
</tr>
</tbody>
</table>

(b) Look in the fourth row of the table. We are looking for two numbers, a smaller number and a larger number whose sum is 99, and the larger number should be 3 more than two times the smaller number. Check to see if these numbers are true in the table. If so, you can write the correct answers for the given story problem.

The smaller number is _____ The larger number is _____

(c) Write a true equation for the given problem ________________

2. The difference of two numbers is 30. The larger number is 4 more than three times the smaller number. Find the numbers.

(a) You need to construct your own table and determine the solution to the given story problem.

(b) Smaller number is _____ Larger number is _____
During this session you will determine the solution to the story problems by guessing and checking your own numbers, looking for patterns, and using approximations. In the first story I have constructed the table for you. In the second story you will have to construct your own table and label the correct headings from the story and then determine the solution to the problem by procedures previously used:

**Story 1:** The sum of two numbers is 168. The numbers are such that one number is 3 times the other. Find the numbers.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>First No.</th>
<th>3 times First No.</th>
<th>Second No.</th>
<th>First No. plus Second No.</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>168</td>
</tr>
</tbody>
</table>

(b) Look in the fourth row of the table. We are looking for two numbers whose sum is 168, and the second number should be 3 times the first number. If this is correct in the table you can now write the correct answers to the given story.

The first number is _____  The second number is _____

**Story 2:** The difference of two numbers is 23. The larger number is 5 more than four times the smaller number. Find the numbers.

(a) You need to construct your own table and determine the solution to the given story. (b) Smaller Number is _____  Larger Number is _____
Solve the following story problems by guessing and checking your own numbers until you find the solution. Look for patterns in the table and use approximations to determine the answer to the problem:

1. The sum of two numbers is 99. The larger number is 3 more than twice the smaller number. Find the numbers.

(a) Complete the following table:

<table>
<thead>
<tr>
<th>Smaller Number</th>
<th>Twice the smaller number plus 3</th>
<th>Larger number</th>
<th>Smaller number plus Larger number</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>99</td>
</tr>
</tbody>
</table>

(b) Look in the fourth row of the table. We are looking for two numbers, a smaller number and a larger number whose sum is 99, and the larger number should be 3 more than two times the smaller number. Check to see if these numbers are true in the table. If so, you can write the correct answers for the given story problem.

The smaller number is ____  The larger number is ____

2. The difference of two numbers is 30. The larger number is 4 more than three times the smaller number. Find the numbers.

(a) You need to construct your own table and determine the solution to the given story problem.

(b) Smaller number is ____  Larger number is ____
Session 6

Solve the following problems using procedures from the last session:

1. A carpenter and his helper got $260 for a job. The carpenter got $20 more than twice as much as the helper. How much did each person earn? (a) complete the table

<table>
<thead>
<tr>
<th>Helper's Amount</th>
<th>2 times Helper's Amount + 20</th>
<th>Carpenter's Amount</th>
<th>Helper's Amt. + Carpenter's Amt.</th>
<th>Total Amt. in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>2(X) + 20</td>
<td>2X + 20</td>
<td></td>
<td>260</td>
</tr>
</tbody>
</table>

(b) Amount earned by Helper ______ Amount earned by Carpenter ______

(c) Write an equation for the problem: _________________________________

2. If five times a certain number is increased by 20 the result is 160. Find the number

(a) Construct your own table to determine the solution to the problem:
(b) What is the number ______
(c) Write an equation for the problem ________________
TB2 Session 6

Solve the following problems using procedures from the last session. Note that the variable \( X \) has been introduced in the last row of the table. This variable \( X \) can represent any number, and can be used to write an equation expressing relationships between the given and wanted information in the problem.

1. A carpenter and his helper got $260 for a job. The carpenter got $20 more than twice as much as the helper. How much did each person earn? (a) complete the table

<table>
<thead>
<tr>
<th>Helper's Amount</th>
<th>2 times Helper's Amount + 20</th>
<th>Carpenter's Amount + Carpenter's Amt.</th>
<th>Total Amt. in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X )</td>
<td>( 2(X+20) )</td>
<td></td>
<td>( 260 )</td>
</tr>
</tbody>
</table>

(b) Amount earned by Helper \_________ \ Amount earned by Carpenter \_________

(c) Look in the last row of the table. Suppose \( X \) represent the amount earned by the Helper. Express in the table the Carpenter's amount which is $20 more than 2 times the Helper's amount.

(d) Fill in the table the Helper's amount plus the Carpenter's amount

(e) From the last row in the table, if the total amount is represented by \( X+(2X+20) \), this expression is the same as what number in the table?

(f) Write an equation expressing this relationship

(g) Note this equation can be solved to determine the solution to the problem.

2. If five times a certain number is increased by 20 the result is 160. Find the number.

(a) Construct your own table to determine the solution to the problem

(b) What is the number \_________

(c) Write an equation for the problem \_________
Solve the following problems:

1. A father and his son got $350 for a job. The father got $50 more than three times as much as the son. How much did each person earn?
   (a) First complete the headings for the table and use “guess” and “check” to determine the solution:

<table>
<thead>
<tr>
<th>Son's Amount</th>
<th>Father's Amount</th>
<th>Total Amount in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>$3(X+50)</td>
<td>$350</td>
</tr>
</tbody>
</table>

   (b) Amount earned by the son ______ Amount earned by the father ______

   (c) Write an equation for the problem _________________________________

2. If four times a certain number is increased by 25 the result is 225. Find the number.
   (a) Construct your own table to determine the solution to the problem
   (b) What is the number ______
   (c) Write an equation for the problem _____________________________
Solve the following problems using procedures from previous sessions:

1. Sam has $2.05 in quarters and dimes. He has 4 more quarters than dimes. Find the number he has of each type of coin.
   a) Complete the table to determine the solution to the problem
   
<table>
<thead>
<tr>
<th>No. of dimes</th>
<th>No. of quarters</th>
<th>Value of dimes plus value of quarters</th>
<th>Total value in cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 + 4</td>
<td>10(1) + 25(5)</td>
<td>135</td>
</tr>
<tr>
<td>X</td>
<td>X + 4</td>
<td>10(X) + 25(X + 4)</td>
<td>35X + 100</td>
</tr>
</tbody>
</table>

   b) Sam has ________ dimes and ________ quarters
   
   c) From the information in the table, write an equation or true statement for the given problem

   For problems 2 and 3, construct a table on another sheet of paper in order to solve the problems. Write your answers on the indicated blank spaces:

2. A jar contains 5 times as many dimes as quarters. The value of the coins is $9.00. How many of each coin is in the jar?
   a) There are ________ quarters and ________ dimes in the jar.
   
   b) Write an equation for the problem

3. Mary has $7.00 consisting of quarters and nickels. He has 6 more than twice as many quarters than nickels. Find the number of each type of coin.
   a) Mary has ________ nickels and ________ quarters
   
   b) Write an equation for the problem
Construct your own tables to solve the following problems:

1. A jar contains 4 times as many dimes as nickels. The value of the coins is $4.50. How many of each coin is in the jar?

There are _____ nickels and _____ dimes in the jar.
Equation: _______________________

2. A jar contains four more than three times as many dimes as quarters. The value of the coins is $18.55. Find the number of each type of coin.

There are _____ quarters and _____ dimes in the jar
Equation: _______________________
Session 8
(REVIEW)

This session is to review you for the test you will take during the next session. Use procedures you have learned from previous sessions to solve each problem. You should be able to solve each problem without any help from me. Use another sheet of paper to do your computation for each problem and write the answers on the blank lines.

1. If the sum of a certain number and 5 equals 35 can be expressed by the equation \( H + 5 = 35 \), what number \( N \) will make the equation a true statement? \( \)  
   \( \)  

2. If four times a certain number is increased by 30 the result equals 90.  
   a) What is the number? ___________  
   b) Write an equation or true statement for the problem ___________  

3. The sum of two numbers is 70. The larger number is 6 less than 3 times the smaller number.  
   a) If the smaller number is given, how would you find the larger number ___________  
   b) What is the smaller number? _________ Larger number is _________  
   c) Write an equation for the problem ___________  

4. James has 12 dimes and some quarters. The total value of the coins is $5.20.  
   a) How much money he has in dimes? ___________  
   b) How many money has he? _________ How much money has he in quarters _________  
   c) Write an equation for this problem ___________  

5. Mary has 3 more nickels than dimes. The value of these coins is $1.05.  
   a) If she had 4 dimes how many nickels she would have _________  
   b) If she had \( X \) dimes how many nickels she would have _________  
   c) Mary has _________ dimes and _________ nickels  
   d) Write an equation expressing the relationship between the value of the nickels and the dimes in cents and the total value 105¢ ___________
6. The difference of 4 times a certain number from 150 equals 50. Find the number.

The number is ______________________

7. Two numbers are such that the larger number is 30 more than the smaller number. Also, the larger number increased by 5 equals 8 times the smaller number.

a) If the smaller number is 3, what is the larger number ______

b) If the larger number is 34, what is the smaller number ______

c) The smaller number is _____ and the larger number is ______

d) Write the equation that will lead to the solution for the problem ________________________________
APPENDIX B

T TREATMENT MATERIALS
Session 1

Directions: Write a mathematical phrase or sentence for the word phrase or sentence.

Examples: a) 7 more than $X$  
           $X + 7$
           
           b) $X$ less than 12  
            $12 - X$
            
           c) The value in cents of $X$ nickels  
            $5X$
            
           d) The sum of 5 and $X$ equals 10  
            $5 + X = 10$
            
1. 5 less than $X$  
2. $X$ less than 5  
3. 8 divided by $X$  
4. $X$ divided by 8  
5. 4 subtracted from $X$  
6. $X$ subtracted from 4  
7. 5 more than $X$  
8. $X$ more than 5  
9. Twice $X$  
10. 7 more than twice $X$  
11. 5 less than twice $X$  
12. 5 increased by $X$  
13. Twice $X$ increased by 5  
14. The sum of $X$ and 20  
15. Twice the sum of $X$ and 20  
16. 357 times as big as $X$  
17. 3 less than half $X$  
18. The value in cents of $X$ dimes
T Session 1

19. The value in cents of 2X dimes

20. The value in cents of X quarters

21. The value in cents of X quarters and 6 dimes

22. The value in cents of X quarters and (X+3) dimes

23. The value in cents of 5 pounds of coffee costing 12 cents per pound

24. The value in cents of X pounds of coffee costing 12 cents per pound

25. 3 times the difference of 12 from X

26. 4 times the difference of X from 56

27. The difference of 15 and X equals 5

28. 5X more than 10 is 97

29. (X+3) more than twice X equals 61

30. The value in cents of (17-X) pounds of 50 cents per pound coffee is 560 cents
Directions: Write a mathematical phrase or sentence for the word phrase or sentence.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>7 less than X</td>
</tr>
<tr>
<td>2.</td>
<td>X subtracted from 8</td>
</tr>
<tr>
<td>3.</td>
<td>6 more than X</td>
</tr>
<tr>
<td>4.</td>
<td>Three times N</td>
</tr>
<tr>
<td>5.</td>
<td>Four times X</td>
</tr>
<tr>
<td>6.</td>
<td>6 more than twice X</td>
</tr>
<tr>
<td>7.</td>
<td>3 less than twice X</td>
</tr>
<tr>
<td>8.</td>
<td>Three times X increased by 9</td>
</tr>
<tr>
<td>9.</td>
<td>The sum of X and 15</td>
</tr>
<tr>
<td>10.</td>
<td>Twice the sum of X and 15</td>
</tr>
<tr>
<td>11.</td>
<td>The value in cents of X pennies</td>
</tr>
<tr>
<td>12.</td>
<td>The value in cents of 3X dimes</td>
</tr>
<tr>
<td>13.</td>
<td>The value in cents of X nickels and 5 dimes</td>
</tr>
<tr>
<td>14.</td>
<td>The value in cents of (X+4) quarters</td>
</tr>
<tr>
<td>15.</td>
<td>The value in cents of X pounds of nuts costing 15 cents per pound</td>
</tr>
<tr>
<td>16.</td>
<td>5 times the difference of 13 from X</td>
</tr>
<tr>
<td>17.</td>
<td>30 times the difference of X from 50</td>
</tr>
<tr>
<td>18.</td>
<td>The difference of 20 and X equals 6</td>
</tr>
<tr>
<td>19.</td>
<td>4X more than 12 is 42</td>
</tr>
<tr>
<td>20.</td>
<td>26 equals X increased by 7</td>
</tr>
<tr>
<td>21.</td>
<td>Y less than 5</td>
</tr>
<tr>
<td>22.</td>
<td>Twice W increased by 25</td>
</tr>
<tr>
<td>23.</td>
<td>(N+3) more than twice N equals 63</td>
</tr>
<tr>
<td>24.</td>
<td>5N decreased by 2 is 23</td>
</tr>
<tr>
<td>25.</td>
<td>The sum of N, 3N, and 5</td>
</tr>
</tbody>
</table>
In this unit we want to look at a method of solving story problems. This method involves translation of English expressions to mathematical expressions. The following steps will be used:

1. Read the story carefully and determine what is given and wanted.
2. Choose a variable to represent the wanted information, and then use this variable to express the relationship between the given and wanted information.
3. Write an equation.

In this section we will practice (a) reading the story and most of the solution; (b) solving the equation; and (c) stating the answer. Later you will be expected to solve the story problems without any help.

**Story 1:** The sum of a certain number and 87 is 253. Find the number.

**Solution:** We are looking for a number. "The sum of this number and 87 is 253."

If \( X \) = the number, then the sum of \( X \) and 87 is 253.

\[ \text{_____________} = 253 \]

So, the equation is ________________

Solve the equation:

Thus the number is ________________

Now, let's try another example.

**Story 2:** The sum of four times a certain number and 20 equals 236. Find the number.

**Solution:** We are looking for a number. We know, "four times the number plus 20 equals 236."

If \( X \) = the number, then the sum of 4 times \( X \) and 20 equals 236.

\[ \text{_____________} = 236 \]

So, our equation is, ________________

Solve:

Thus, the number is ________________.
Session 2

**Story:** 3) The sum of twice a certain number and 28 equals 96. What is the number?

**Solution:** We are looking for a number. We know, "twice it plus 28 equals 96."

If $x$ is the number, then \[ \text{the sum of twice } x \text{ and } 28 \text{ equals 96.} \]

\[ 2x + 28 = 96 \]

So, our equation is, \[ 2x + 28 = 96 \]

**Solve:**

Thus, the number is \[ \text{______}. \]
Complete the following:

1) If $X$ is the number, then the number plus 65 is 148 can be written
   $____ + 65 = 148$

2) If $X$ is the number, then the sum of the number and 56 is 200 can
   be written $______________ = 200$

Solve the following story problems:

3) The sum of a certain number and 70 is 350. Find the number.
   If $X$ is the number, then the sum of $X$ and 70 is 350
   Write the equation: ____________________
   Solve the equation:

   The number is ______

4) Twice a certain number increased by 30 equals 120. Find the number.
   If $X$ is the number, then $2X$ increased by 30 equals 120 can be
   written as $______________ = 120$
   Write the equation: ____________________
   Solve the equation:

   The number is ______
In this unit we want to use the same method we used in the last unit to solve the story problems.

I. Before solving the story problem we want to write a few mathematical phrases for the word phrases:

Example: The sum of two numbers is 8. If one number is \( x \) the other number is \( 8-x \)

1. The sum of two numbers is 9. If one number is 5 the other number is ______

2. The sum of two numbers is 9. If one number is \( x \) the other number is ______

3. The sum of two numbers is 18. If the smaller number is \( x \) the larger number is ______

II. Story: 1. The sum of two numbers is 97. The larger of the two numbers is 5 more than 3 times the smaller number. Find the numbers.

Solution: If \( x \)=the smaller number, then \( 97-x \)=the larger number

Also, larger is 5 more than 3 times smaller

or _______ = ______________________

Equation: ______________________

Solve:

The smaller number is ______ The larger number is ______
I. Write a mathematical phrase for each of the word phrases:

1. The sum of two numbers is 20. If one of the numbers is 7 the other number is ____________

2. The sum of two numbers is 20. If one of the numbers is X, the other number is ____________

3. The sum of two numbers is 78. If the smaller number is X, the larger number is ____________

II. Solve the following story problem:

1. The sum of two numbers is 90. The larger number equals 14 more than 3 times the smaller number. Find the numbers.

Solution: If X = the smaller number, then 90 - X = the larger number
Also larger = 3 times smaller plus 14

Equation: ______________

Solve the equation:

The smaller number is ____________
The larger number is ____________
Session 4

Name ____________________________

Date ____________________________

Solve the following story problems using procedures from previous units:

**Story: 1**
A jar contains a number of nickels. The value of the coins is $15.65. How many nickels are in the jar?

**Solution:**
The value in cents of X nickels is ________________

If X = the number of nickels in the jar, then
the value of the nickels is 1565

________________________ = 1565

So our equation is ____________________________

Solve:

Thus the number of nickels in the jar is ______

**Story: 2**
A jar contains a certain number of nickels and 27 dimes. The value of the coins is $9.75. How many nickels are in the jar?

**Solution:**
The value in cents of X nickels is ________________

The value in cents of 27 dimes is ________________

If X = the number of nickels in the jar, then the
value of the nickels + value of the dimes is 975

________________________ + ________________ = 975

So, our equation is ____________________________

Solve:

Thus the number of nickels in the jar is ______
1) A box contains a number of quarters. The value of the coins is $19.75. How many quarters are in the box?
   a) The value in cents of \( X \) quarters is ________
   b) If \( X \) = the number of quarters in the box, then the value of the quarters is 1975
      ________ = 1975
   c) Our equation is __________________________
   d) Solve the equation:
   e) The number of quarters in the box is ________

2) A box contains a certain number of dimes and 30 nickels. The value of the coins is $12.00. How many dimes are in the box?
   a) The value in cents of \( X \) dimes is ________
   b) The value in cents of 30 nickels is ________
   c) If \( X \) = the number of dimes in the box, then the value of the dimes + value of the nickels is 1200
      ________ + ________ = 1200
   d) Our equation is __________________________
   e) Solve the equation:
   f) The number of dimes in the box is __________
Session 5

Solve the following problems using procedures from previous sessions:

**Story: 1.** The sum of two numbers is 168. The numbers are such that one number is 3 times the other. Find the numbers.

**Solution:** We are looking for two numbers. We know (1) their sum is __________ and (2) one number is _______ times the other number.

If \( X \) = one of the numbers, then \( 168 - X \) = the other number.

Now, one number is 3 times the other number.

\[
\begin{align*}
\text{So, our equation is:} & \quad \phantom{=_{\text{90}}} \\
\text{Solve:} & \\
\text{One number is} & \quad \phantom{=_{\text{90}}} \\
\text{The other number is} & \quad \phantom{=_{\text{90}}}
\end{align*}
\]

Check your answers:

**Story: 2.** The difference of two numbers is 23. The larger number is 5 more than four times the smaller number. Find the numbers.

**Solution:** We are looking for two numbers. We know (1) their difference is _______ and (2) the larger number is 5 more than _______ times the smaller number.

If \( X \) = the smaller number, then \( X + 23 \) = the larger number.

Now, larger number is 5 more than four times the smaller number.

\[
\begin{align*}
\text{So, our equation is:} & \quad \phantom{=_{\text{90}}} \\
\text{Solve:} & \\
\text{Smaller number is} & \quad \phantom{=_{\text{90}}} \\
\text{Larger number is} & \quad \phantom{=_{\text{90}}}
\end{align*}
\]

Check your answers:
Solve the following story problems:

1. The sum of two numbers is 99. The larger number is 3 more than twice the smaller number. Find the numbers.

Solution: We are looking for two numbers. We know (1) their sum is _____ and (2) the larger number is ____ more than ____ times the smaller number.

If $x = $ the larger number, then $99 - x = $ the smaller number

Now the larger number is 3 more than twice the smaller number

Se our equation is __________________________

Solve:

The smaller number is _____ The larger number is _____

Check your answers:

2. The difference of two numbers is 30. The larger number is 4 more than three times the smaller number. Find the numbers.

Solution: We are looking for two numbers. We know (1) their difference is _____ and (2) the larger number is 4 more than ____ times the smaller number.

If $x = $ the smaller number, then $x + 30 = $ the larger number

Now, the larger number is 4 more than 3 times the smaller number.

Se our equation is __________________________

Solve:

Smaller number is _____ Larger number is _____

Check your answers:
Session 6

Name ___________________________

Date ________________

I. Write a mathematical sentence for the following:

1. The value of X quarters is 650 cents

2. The value of X dimes is 190 cents

3. The sum of 10X and 25(6X) is 360

4. If 4 times X is increased by 10 the result is 30

5. The sum of X and 27 equals 7 more than twice X

6. The difference of 118 and X equals 8 less than twice X

II. Solve the following problems using procedures from previous sessions:

1. A carpenter and his helper got $260 for a job. The carpenter got $20 more than twice as much as the helper. How much did each person earn?

   If X = the number of dollars for the helper, then

   _______ = the number of dollars for the carpenter

   Now, carpenter's amount is 20 more than twice the helper's amount

   Equation: _______________________________________________________

   Solve:

   Helper's amount of money _______ Carpenter's amount of money _______ 

2. If five times a certain number is increased by 20 the result is 160. Find the number.

   Try to solve this problem on your own:

   First you need to let X represent the number you are looking for, then

   write an expression for 5 times X increased by 20

   Write your equation: ____________________________________________

   Solve this equation:

   Answer: The number is _______ 

   Check your answer:
I. Write a mathematical sentence for the following:

1. The value of $N$ quarters is 750 cents
2. The value of $N$ dimes is 250 cents
3. The sum of $5X$ and $10(7X)$ is 285
4. 6 times $N$ increased by 5 is 35
5. The sum of $X$ and 40 equals 5 more than 8 times $X$

II. Solve the following problems:

1. A father and his son got $350 for a job. The father got $50 more than three times as much as the son. How much did each person earn?

   If $X =$ the number of dollars for the son, then _____ = the number of dollars for the father

   Equation: 

   Solve:

   Amount earned by the son _____ Amount earned by the father _____

2. If four times a certain number is increased by 25 the result is 225. Find the number.

   Solve this problem using procedures discussed in class:

   What is the number _____ Check your answer:
Today you are expected to solve the story problems with little help from me. You have been using the following steps to solve problems. See if you can apply them to solve these problems:

**STEPS:**
1. Read the story carefully and determine what is given and wanted.
2. Choose a variable (some letter) to represent the wanted information, and then use this letter to express the relationship between the given and wanted information.
3. Write an equation, solve it, then state your answer.

1. Sam has $2.05 in quarters and dimes. He has 4 more quarters than dimes. Find the number he has of each type coin.
   
   If ______ = the number of dimes ______ = the number of quarters
   
   Express the value of dimes ______ Express the value of quarters ______
   
   Equation: _______________________
   
   Solve:

   Sam has _______ DIMES and _______ QUARTERS

2. A jar contains 5 times as many dimes as quarters. The value of the coins is $9.00. How many of each coin is in the jar?
   
   Follow the steps above to solve this problem:

   The are _______ QUARTERS and _______ DIMES in the jar

3. Mary has $7.00 consisting of quarters and nickels. He has 6 more than twice as many quarters than nickels. Find the number of each type of coin. Solve using the STEPS above:

   Mary has _______ NICKELS and _______ QUARTERS
Solve the following problems using procedures discussed in class:

1. A jar contains \( \frac{1}{4} \) times as many dimes as nickels. The value of the coins is $4.50. How many of each coin is in the jar?

   Equation: ____________

   There are _____ nickels and _____ dimes in the jar.

2. A jar contains four more than three times as many dimes as quarters. The value of the coins is $18.55. Find the number of each type of coin

   Equation: ____________

   There are _____ quarters and _____ dimes in the jar.
This session is to review you for the test you will take during the next session. Use procedures you have learned from previous sessions to solve each problem. You should be able to solve each problem without any help from me. Use another sheet of paper to do your computation for each problem and write the answers on the blank lines.

1. If the sum of a certain number and 5 equals 35 can be expressed by the equation $N + 5 = 35$, what number $N$ will make the equation true statement? $N =$

2. If four times a certain number is increased by 30 the result equals 90.
   a) What is the number? __________
   b) Write an equation or true statement for the problem ________________

3. The sum of two numbers is 70. The larger number is 6 less than 3 times the smaller number.
   a) If the smaller number is given, how would you find the larger number ______________
   b) What is the smaller number? _______ Larger number is _______
   c) Write an equation for the problem __________________________

4. James has 12 dimes and some quarters. The total value of the coins is $5.20.
   a) How much money he has in dimes? ___________
   b) How many quarters has he? ___________ How much money he in quarters ___________
   c) Write an equation for this problem __________________________

5. Mary has 3 more nickels than dimes. The value of these coins is $1.05.
   a) If she had 4 dimes how many nickels she would have ________
   b) If she had $X$ dimes how many nickels she would have ______
   c) Mary has ________ dimes and ______ nickels
   d) Write an equation expressing the relationship between the value of the nickels and the dimes in cents and the total value 105¢ ______
6. The difference of 4 times a certain number from 150 equals 50. Find the number.
   The number is _____________________

7. Two numbers are such that the larger number is 30 more than the smaller number. Also, the larger number increased by 5 equals 8 times the smaller number.
   a) If the smaller number is 3, what is the larger number ______
   b) If the larger number is 34, what is the smaller number ______
   c) The smaller number is ______ and the larger number is ______
   d) Write the equation that will lead to the solution for the problem ____________________________
APPENDIX C

TEST INSTRUMENTS
Trial Item 1
What number does \(a + 4\) stand for if \(a = 2\) ....
What number does \(4a\) stand for if \(a = 2\) ....

Trial Item 2
\[
\begin{align*}
x & \rightarrow 3x \\
2 & \rightarrow 6 \\
x + 3 & \rightarrow 7x \\
3 & \rightarrow 9
\end{align*}
\]
Fill in the gaps:
(work down the page)
\[
\begin{align*}
5 & \rightarrow \text{ } \\
4 & \rightarrow \text{ } \\
n & \rightarrow \text{ }
\end{align*}
\]

Now check your answers against the answers on the back page.

1. Fill in the gaps:
\[
\begin{align*}
x & \rightarrow x + 2 \\
2 & \rightarrow 3 \\
x & \rightarrow 4x \\
3 & \rightarrow 6 \\
x & \rightarrow Var.1 \\
3 & \rightarrow Var.2 \\
\end{align*}
\]

2. Write down the smallest and the largest of these:
\[
\begin{align*}
n + 1, & \hspace{1cm} n + 4, & \hspace{1cm} n - 3, & \hspace{1cm} n, & \hspace{1cm} n - 7.
\end{align*}
\]

3. Which is the larger, \(2n\) or \(n + 2\)?
Explain: ................................................................................................................................................................

4. A added to \(n\) can be written as \(n + 4\). Add 4 onto each of these:
Multiply each of these by 4:
\[
\begin{align*}
6 & \rightarrow 8n \\
3 & \rightarrow 12n \\
2 & \rightarrow Var.6 \\
\end{align*}
\]

5. If \(a + b = 43\)  
If \(n - 246 = 762\)  
If \(a + f = 8\)
\[
\begin{align*}
a + b + 2 & \rightarrow Var.10 \\
n - 247 & \rightarrow Var.11 \\
a + f + g & \rightarrow Var.12
\end{align*}
\]
6. What can you say about $a$ if $a + 5 = 8$ 

What can you say about $b$ if $b + 2$ is equal to $2b$ 

7. What are the areas of these shapes?

![Shapes with areas labeled: A, 15, 16, 17, 18.]

8. The perimeter of this shape is equal to $6 + 3 + 4 + 2$, which equals 15. 

Work out the perimeter of this shape. $p =$ 

9. 

This square has sides of length $g$. So, for its perimeter, we can write $p = 4g$.

What can we write for the perimeter of each of these shapes?

![Various shapes with perimeters labeled: p = Var.20, p = Var.21, p = Var.22, p = Var.23.]

11. What can you say about $w$ if $w = v + 3$ and $v = 1$ 

What can you say about $m$ if $m = 3n + 1$ and $n = 4$
12. If John has J marbles and Peter has P marbles, what could you write for the number of marbles they have altogether?  

13. \( a + 3a \) can be written more simply as \( 4a \).

Write these more simply, where possible:

\[
\begin{align*}
2a + 3a &= 5a \quad \text{Var. 29}\ldots \\
2a + 3b &= 5b \quad \text{Var. 30}\ldots \\
(a + b) + a &= a + b \quad \text{Var. 31}\ldots \\
2a + 5c &= 5a + 5c \quad \text{Var. 32}\ldots \\
(a - b) + b &= a \quad \text{Var. 33}\ldots \\
3a - (b + a) &= 2a \quad \text{Var. 34}\ldots \\
a + 4 - a + 4 &= 8 \quad \text{Var. 35}\ldots \\
3a - b + a &= 4a - b \quad \text{Var. 36}\ldots \\
(a + b) - (a - b) &= 2b \quad \text{Var. 37}\ldots \\
\end{align*}
\]

14. What can you say about \( r \) if \( r = s + c \) and \( r + s + c = 30 \)  

\[\text{Var. 38}\ldots\]

15. In a shape like this, you can work out the number of diagonals by taking away 3 from the number of sides.

So, a shape with 5 sides has 2 diagonals;  
a shape with 7 sides has \( \ldots \text{Var. 39}\ldots \) diagonals;  
a shape with \( k \) sides has \( \ldots \text{Var. 40}\ldots \) diagonals.

16. What can you say about \( c \) if \( c + d = 10 \) and \( c \) is less than \( d \) \( \quad \text{Var. 41}\ldots\)

17. Mary's basic wage is £20 per week.  
She is also paid another £2 for each hour of overtime that she works.

If \( h \) stands for the number of hours of overtime that she works, and  
if \( W \) stands for her total wage (in £'s)  
write down an equation connecting \( W \) and \( h \) \( \quad \text{Var. 42}\ldots\)

What would Mary's total wage be if she worked 4 hours of overtime? \( \quad \text{Var. 43}\ldots\)
17. When are the following true—always, never, or sometimes?

Underline the correct answer:

- **A + B + C = C + A + B**
  - Always
  - Never
  - Sometimes, when ...
  - Var. 44

- **L + M + N = L + P + K**
  - Always
  - Never
  - Sometimes, when ...
  - Var. 45

19. **a = b + 3. What happens to a if b is increased by 27**

- Var. 46

- **f = 3g + 1. What happens to f if g is increased by 27**

- Var. 47

20. Cakes cost c pence each and buns cost b pence each.

If I buy 4 cakes and 3 buns, what does

- **4c + 3b** stand for?

- Var. 48

21. If this equation

- \((x + 1)^3 + x = 349\)

is true when **x = 6**, then what value of \(x\) will make this equation

- \((5x + 1)^3 + 5x = 349\)

true?

- \(x = \) .................

- Var. 49

22. Blue pencils cost 5 pence each and red pencils cost 6 pence each.

If I buy some blue and some red pencils and altogether it costs me 90 pence.

If **b** is the number of blue pencils bought, and if **r** is the number of red pencils bought, what can you write down about **b** and **r**?

- Var. 50

---

**Trial Item**

**ANSWERS**

- What number does \(a + 4\) stand for if **a = 2** ...

- What number does \(4a\) stand for if **a = 2** ...

- \(x \rightarrow 3x\)

- \(x \rightarrow x+3\)

- \(x \rightarrow 7x\)

- \(x \rightarrow x+8\)

- \(2 \rightarrow 6\)

- \(5 \rightarrow 8\)

- \(2 \rightarrow 14\)

- \(3 \rightarrow 11\)

- \(4 \rightarrow 7\)

- \(n \rightarrow n+3\)
PRETEST

Name ___________________________ School ___________________________

Date ___________________________

Directions: Enter all answers on the blank lines provided on the test.

1. Which is the larger, 2n or n + 2? __________

   Explain: __________________________________________________________________________

2. a) h added to n can be written as n + h.

   b) i added to 3n can be written as __________

3. a) If a + b = 43, a + b + 2 = ______

   b) If e + f = 8, e + f + g = __________

4. What can you say about a if a + 5 = 8 ______

5. What are the areas of these shapes?

   a) __________

   b) __________

   c) __________

6. The perimeter of this shape is equal to 6 + 3 + 4 + 2, which equals 15.

   Work out the perimeter of this shape

   P = __________
7. This square has sides of length $g$. So, for its perimeter, we can write $p=4g$.

What can we write for the perimeter of each of these shapes?

- **a)** $P = \phantom{0}$
- **b)** $P = \phantom{0}$
- **c)** $P = \phantom{0}$
- **d)** $P = \phantom{0}$

8. a) What can you say about $u$ if $u=v+3$ and $v=1$ 

b) What can you say about $m$ if $m=3n+1$ and $n=4$ 

9. $a+3a$ can be written more simply as $4a$.

Write these more simply, where possible:

- a) $2a + 5a = \phantom{0}$
- b) $2a + 5b = \phantom{0}$
- c) $2a + 5b + a = \phantom{0}$
- d) $(a-b)+b = \phantom{0}$
- e) $3a - b + a = \phantom{0}$

10. What can you say about $r$ if $r=s+t$ and $r + s + t = 30$

11. In a shape like this you can work out the number of diagonals by taking away 3 from the number of sides. So, a shape with 5 sides has 2 diagonals;

- a) a shape with 57 sides has __________ diagonals
- b) a shape with $k$ sides has __________ diagonals

12. What can you say about $c$ if $c+d=10$ and $c$ is less than $d$
PRETEST

13. Mary's basic wage is $130 per week. She is also paid another $5 for each hour of overtime that she works.

If \( h \) stands for the number of hours of overtime that she works, and if \( w \) stands for her total wage, write down an equation connecting \( w \) and \( h \).

14. When is the following statement true? Always, Never, or Sometimes

Underline the correct answer:

\( L + M + N = L + P + N \) Always, Never, Sometimes, When __________

15. Cakes cost \( c \) cents each and buns cost \( b \) cents each. If I buy 4 cakes and 3 buns, what does \( 4c + 3b \) stand for? __________

16. If the equation \((X+1)^2 + X = 349\) is true when \( X = 6 \), then what value of \( X \) will make the equation \((5X+1)^2 + 5X = 349\) true?

\( X = \) __________

17. Blue pencils cost 5 cents each and red pencils cost 6 cents each. I buy some blue and some red pencils and altogether it costs me 90 cents.

If \( b \) is the number of blue pencils bought, and if \( r \) is the number of red pencils bought, what can you write down about \( b \) and \( r \)?

___________
POSTTEST

NAME ___________________________ SCORE ______

DATE ___________________________

DIRECTIONS: This test is similar to the pretest you took earlier during the year. It does not cover exactly what you were taught during the group sessions, but you should apply what you have learned during the sessions to answer each question. Use another sheet of paper to do your computation for each problem and write all answers on the blank lines provided on the test.

1. If \(n\) can represent any number, write down the smallest and the largest of these: \(n+1\), \(n-4\), \(n-3\), \(n\), \(n-7\)
   a) smallest ________  b) largest ________

2. If 5 added to \(n\) can be written as \(n+5\), 5 added to \(4n\) can be written as ________

3. a) If \(a+b=57\), \(a+b+3=______\)  b) If \(e+f=10\), \(e+f+g=______\)

4. What can you say about \(a\) if \(a+7=9______\)

5. This rectangle has length \(l\) and width \(3\). So for its area, we can write \(A=3(l)=12\)
   What are the areas of these shapes?

   \[
   \begin{array}{ccc}
   \text{7} & \text{x} & \text{5} \\
   \text{10} & \text{y} & \text{+2} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{A} & \text{A} & \text{A} \\
   ______ & ______ & ______ \\
   \end{array}
   \]

6. The perimeter of this shape is equal to 5+2+3+1, which equals 11.
   Work out the perimeter of this shape.
   \[
   \begin{array}{ccc}
   \text{3} & \text{2} & \text{5} \\
   \text{1} & \text{6} & \text{8} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{10} & \text{3} \\
   \text{4} & \text{8} \\
   \end{array}
   \]
   \[
   \begin{array}{ccc}
   \text{P} = ________ \\
   \end{array}
   \]
POSTTEST

7. This square has sides of length \(s\). So, for its perimeter, we can write \(P = s + s + s + s = 4s\).

What can we write for the perimeter of each of these shapes?

8. a) What can you say about \(u\) if \(u = v + 4\) and \(v = 2\) ________

b) What can you say about \(m\) if \(m = 3(n) + 1\) and \(n = 4\) ________

9. \(a + 3a\) can be written more simply as \(4a\). Write the following more simply, where possible:
   a) \(4a + 5a = \) ________
   b) \(2a + 5b + b = \) ________
   c) \(2a + 5b + 3a = \) ________
   d) \((2a - 2b) + 2b = \) ________
   e) \(3a - b + a = \) ________

10. What can you say about \(r\) if \(r = s\) and \(r + s = 10\) ________

11. In a shape like this, you can work out the number of diagonals by taking away 3 from the number of sides. So, a shape with 5 sides has 2 diagonals.

   a) A shape with 60 sides has ________ diagonals
   b) A shape with \(X\) sides has ________ diagonals

12. What can you say about \(c\) if \(c + d = 10\) and \(c\) is less than \(d\) ________
13. Mary's basic wage is $180 per week. She is also paid another 
$5 for each hour of overtime that she works. If h stands for 
the number of hours of overtime that she works, and if W stands 
for her total wage, write down an equation for the total wage, 
expressing the basic wage plus the amount received for overtime. 
Thus, W=____________________

14. If the statement L + M = L + P is true sometimes, when is 
it true? __________________________

15. Cakes cost c cents each, and buns cost b cents each. If you 
buy 4 cakes and 3 buns, how would you express the total cost 
of the cakes and buns? ________________________

16. If \( x^3 = 64 \) when \( x = 4 \), when \( x = 3 \), \( x^5 + 1 = \) ____________

17. Blue pencils costs 5 cents each and red pencils cost 6 cents 
each. I buy some blue and some red pencils and altogether it 
costs me 90 cents. If b is the number of blue pencils bought, 
and if r is the number of red pencils bought, write an equation 
expressing the relationship between the cost of the blue pencils 
plus the cost of the red pencils and the total cost. 
______________________________________
PROBLEM-SOLVING TEST

NAME_________________________________  SCORE ________

DATE_________________________________

DIRECTIONS: Use problem-solving procedures you have learned from
the group sessions to solve each problem. Use another
sheet of paper to do your computation for each problem
and write all answers on the blank lines provided on
the test.

1. If the sum of a certain number and 10 equals 15 can be expressed
by the equation \( N + 10 = 15 \), answer the following:
   a) If the number \( N \) is 3, what would be the sum of this
      number and 10?
   b) From the sum 15 that is given, how would you find the number \( N \)?
   c) What number \( N \) will make \( N+10=15 \) a true statement?

2. If twice a certain number is increased by 20, the result
   equals 184. Answer the following:
   a) If twice the number 60 is increased by 20, what would
      the result be?
   b) If twice the number \( X \) is increased by 20, how would you
      express the result?
   c) Write an equation or true statement for the problem
   d) Twice what number increased by 20 will equal 184.

3. The sum of two numbers is 80. The larger number is 7 less than
twice the smaller number. Answer the following:
   a) If the smaller number is given, how would you find the larger
      number?
   b) If the sum 80 and the larger number is given, how would you
      find the smaller number?
   c) The smaller number is
   d) The larger number is
   e) Write an equation for the problem
PROBLEM-SOLVING TEST

4. Bill has 7 dimes and some quarters. The total value of the coins is $6.45. Answer the following:
   a) How much money has he in dimes? __________
   b) What will be the value of 20 quarters? __________
   c) What will be the value of X quarters? __________
   d) If X represents the number of quarters, express the total number of coins __________
   e) How many quarters has he? __________
   f) How much money has he in quarters? __________

5. Thomas has 5 more nickels than dimes. The value of these coins is $1.60. Answer the following:
   a) If he had 7 dimes and has 5 more nickels he would have __________ nickels
   b) If he had X dimes he would have __________ nickels
   c) Write an equation expressing the relationship between the value of the nickels and dimes in cents and the total value 160¢ __________

6. The difference of 5 times a certain number from 284 equals 69. Find the number.
   a) The number is __________

7. Two numbers are such that the larger number is 26 more than the smaller number. Also, the larger number increased by 6 equals 5 times the smaller number. Answer the following:
   a) If the smaller number is 4, what is the larger number? __________
   b) If the larger number is 36, what is the smaller number? __________
   c) Write the equation that will lead to the solution for the problem __________
Table 17

Item Analysis of Pretest

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>67</td>
</tr>
<tr>
<td>Number of Items</td>
<td>30</td>
</tr>
<tr>
<td>Mean Test Score</td>
<td>11.91</td>
</tr>
<tr>
<td>Median</td>
<td>12</td>
</tr>
<tr>
<td>Mode</td>
<td>13</td>
</tr>
<tr>
<td>Maximum</td>
<td>26</td>
</tr>
<tr>
<td>Minimum</td>
<td>2</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.87</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.13</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.63</td>
</tr>
<tr>
<td>Range</td>
<td>24</td>
</tr>
<tr>
<td>KR-20 (Reliability Estimate)</td>
<td>0.873</td>
</tr>
</tbody>
</table>

Item Difficulty Distribution

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of Items</th>
<th>Percentage of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>.81 - 1.00</td>
<td>7</td>
<td>23</td>
</tr>
<tr>
<td>.61 - .80</td>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td>.41 - .60</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>.21 - .40</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>.00 - .20</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Mean</td>
<td>.603</td>
<td></td>
</tr>
</tbody>
</table>

Item Discrimination Distribution

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of Items</th>
<th>Percentage of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>.81 - 1.00</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>.61 - .80</td>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td>.41 - .60</td>
<td>11</td>
<td>37</td>
</tr>
<tr>
<td>.21 - .40</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>.00 - .20</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Below .00</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Mean</td>
<td>.504</td>
<td></td>
</tr>
</tbody>
</table>
### Table 18

**Item Analysis of Posttest**

<table>
<thead>
<tr>
<th>Number of Students</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Items</td>
<td>30</td>
</tr>
<tr>
<td>Mean Test Score</td>
<td>13.55</td>
</tr>
<tr>
<td>Median</td>
<td>14</td>
</tr>
<tr>
<td>Mode</td>
<td>10</td>
</tr>
<tr>
<td>Maximum</td>
<td>27</td>
</tr>
<tr>
<td>Minimum</td>
<td>4</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.10</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.25</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.09</td>
</tr>
<tr>
<td>Range</td>
<td>23</td>
</tr>
<tr>
<td>KR-20 (Reliability Estimate)</td>
<td>0.824</td>
</tr>
</tbody>
</table>

### Item Difficulty Distribution

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of Items</th>
<th>Percentage of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>.81 - 1.00</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>.61 - .80</td>
<td>8</td>
<td>27</td>
</tr>
<tr>
<td>.41 - .60</td>
<td>7</td>
<td>23</td>
</tr>
<tr>
<td>.21 - .40</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>.00 - .20</td>
<td>5</td>
<td>17</td>
</tr>
</tbody>
</table>

**Mean** 0.548

### Item Discrimination Distribution

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of Items</th>
<th>Percentage of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>.81 - 1.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.61 - .80</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>.41 - .60</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>.21 - .40</td>
<td>14</td>
<td>47</td>
</tr>
<tr>
<td>.00 - .20</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Below .00</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Mean** 0.396
Table 19

Item Analysis of Problem-Solving Test

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>68</td>
</tr>
<tr>
<td>Number of Items</td>
<td>25</td>
</tr>
<tr>
<td>Mean Test Score</td>
<td>10.25</td>
</tr>
<tr>
<td>Median</td>
<td>9</td>
</tr>
<tr>
<td>Mode</td>
<td>13</td>
</tr>
<tr>
<td>Maximum</td>
<td>22</td>
</tr>
<tr>
<td>Minimum</td>
<td>1</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.51</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.33</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.66</td>
</tr>
<tr>
<td>Range</td>
<td>21</td>
</tr>
<tr>
<td>KR-20 (Reliability Estimate)</td>
<td>0.885</td>
</tr>
</tbody>
</table>

Item Difficulty Distribution

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of Items</th>
<th>Percentage of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>.81 - 1.00</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>.61 - .80</td>
<td>9</td>
<td>36</td>
</tr>
<tr>
<td>.41 - .60</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>.21 - .40</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>.00 - .20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>.590</td>
</tr>
</tbody>
</table>

Item Discrimination Distribution

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of Items</th>
<th>Percentage of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>.81 - 1.00</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>.61 - .80</td>
<td>9</td>
<td>36</td>
</tr>
<tr>
<td>.41 - .60</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>.21 - .40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>.00 - .20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Below .00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>.576</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


Emans, R. *Reading, reasoning, and reality*. In J. Frymler (Ed.), *Theory Into Practice: Teaching The Young To Think*, 1973 12(5), The College of Education, The Ohio State University, Columbus, Ohio.


Leitzel, J.R. & Demana, F. *A Numerical Problem Solving Course For Underprepared College Intending High School Seniors: An Experimental Course*. The Ohio State University, Columbus, Ohio, 1981.


Rachiin, S.L. *Processes used by college students in understanding basic algebra*. Doctoral dissertation, University of Georgia, 1981.


Wagner, S. Conservation of equation, conservation of function, and their relationship to formal operational thinking (Doctoral dissertation, New York University, 1977). *Dissertation Abstracts*


