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FUNDAMENTAL CHARACTERISTICS OF MULTISOLID PNEUMATIC TRANSPORT BED

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Sunil Satija, B.Tech., M.S.

*****

The Ohio State University
1984

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5.18 Fluctuations of the Pressure Signal acquired at 30.3 cm above the Distributor Plate

5.19 Variation of the Probability Density Function with the Pressure for Y2

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NOMENCLATURE

A  dimensionless group defined as \( r_0^2 u_p/L D_{eA} \)

Ar  Archimedes number defined as \( \rho_a (\rho_g - \rho_a) g d_s^3/\mu^2 \)

C  \( C_2/C_1 \)

C_0  inlet gas reactant concentration (mole/L³)

C_1  gas reactant concentration in the gas phase (mole/L³)

C_2  gas reactant concentration in the solid phase (mole/L³)

\( \bar{C}_2 \)  average gas reactant concentration in the solid phase (mole/L³)

C_{s0}  initial solid reactant concentration (mole/L³)

C_s  solid reactant concentration in the solid phase (mole/L³)

C_{Dm}  drag coefficient of a particle in a multiparticle system

C_{Dp}  drag coefficient of a single particle

C_{Ds}  drag coefficient of a dense particle

C_{Dp}  fine particle-dense particle interaction coefficient

d  diameter of particle (L)

d_s  average diameter of dense particles (L)

d_p  average diameter of fine particles (L)

D_s  diameter of a dense particle defined as \( 6/S \sqrt{\nu} \) (L)

D  inside diameter of the bed or reactor (L)

D_{eA}  effective diffusivity of the gas in the solid particle (L²/θ)


$D_S$ solid diffusion coefficient $(L^2/\theta)$

$D_{eA0}$ initial effective diffusivity of the gas in the solid particle $(L^2/\theta)$

$D_A$ $D_{eA0}/D_{eA0}$

$D_m$ molecular diffusion coefficient of the gas divided by tortuosity $(L^2/\theta)$

$D_S$ $D_S/D_{eA0}$

$D$ dimensionless group defined as $3(1-\varepsilon_b)L_D eA/U_g \varepsilon_b \tau^2_0$

$E$ dimensionless group defined as $1/N_{Sha}$

$f_1$ $C_1/C_0$

$f_2$ $C_2/C_0$

$f_p$ solid friction factor

$f, f'$ parameters defined by Eqn. (4.54)

$f_k$ interaction coefficient defined by Eqn. (4.14)

$f_{kk}$ particle-particle interaction coefficient

$f'$ friction factor in the bed without dense particles

$f$ slug frequency

$F_p$ friction factor in the bed with dense particles

$F$ dimensionless group defined as $3(1-\varepsilon_b)k_d L/U_g \varepsilon_b \tau^2_0$

$F_1$ volumetric feed rate of the particles $(L^3/\theta)$

$F_g$ gravitational force on a dense particle $(ML/\theta^2)$

$F_s$ buoyancy force on a dense particle $(ML/\theta^2)$

$F_{DG}$ drag force due to air on a single dense particle in a single particle system $(ML/\theta^2)$

$F_{DP}$ drag force due to fine particles on a single dense particle in a single particle system $(ML/\theta^2)$

$F_{DGM}$ drag force due to air on a dense particle in a multiparticle system $(ML/\theta^2)$
system (ML/θ²)

\[ F_{Dpm} \]

drag force due to fine particles on a dense particle in a multiparticle system (ML/θ²)

\[ df_c \]

interaction force due to collision between fine particle and dense particle (ML/θ²)

\[ df_d \]

drag force on fine particles due to air (ML/θ²)

\[ df_f \]

frictional force due to dense particles and bed wall (ML/θ²)

\[ df_g \]

net gravitational force (ML/θ²)

\[ g \]

gravitational acceleration (L/θ²)

\[ \gamma_c \]

conversion factor

\[ \gamma_1 \]

\[ \frac{C_s}{C_{s0}} \]

dimensionless group defined as \( k_v C_0^m L C_{s0}^{n-1} U_p \)

\[ H \]

height of the expanded bed (L)

\[ H_m \]

maximum height of the slugging bed (L)

\[ H_{mf} \]

height of the bed at minimum fluidization (L)

\[ k_v \]

volumetric reaction rate constant (L³/θ³ (mole))

\[ k_d \]

gas film mass transfer coefficient (L/θ)

\[ k_1, k_2 \]

parameters defined by Eqn. (5.1)

\[ K \]

parameter defined by Eqn. (4.29)

\[ l \]

height of the dense bed (L)

\[ L \]

length of the reactor (L)

\[ L_s \]

length of a slug (L)

\[ m \]

order of the reaction with respect to the solid reactant

\[ m' \]

loading ratio (\( N_p/N_a \))

\[ n \]

order of the reaction with respect to the fluid reactant

\[ n_0 \]

exponent in the Richardson and Zaki's Eqn.

\[ n' \]

exponent defined in Eqn. (4.52)

\[ xx \]
Sherwood No. defined as \( r_0 \frac{k_d}{D_eA_0} \)

number of data points used in calculating average pressure

probability of pressure signal \( Y_2 \) to be equal to zero

pressure in the bed (\( M/L\theta^2 \))

mean pressure fluctuation

pressure at any instant \( i \) (\( M/L\theta^2 \))

pressure drop in the dense bed (\( M/L\theta^2 \))

pressure drop due to fine particle acceleration (\( M/L\theta^2 \))

pressure drop due to air acceleration (\( M/L\theta^2 \))

frictional pressure drop due to fine particles on the dense particle and bed wall (\( M/L\theta^2 \))

frictional pressure drop due to air on the dense particle and bed wall (\( M/L\theta^2 \))

\( \Delta p_{fs} = \Delta p_{fds} + \Delta p_{fg} \) total frictional pressure drop in the dense bed (\( M/L\theta^2 \))

pressure due to static head of fine particles (\( M/L\theta^2 \))

pressure drop due to static head of air (\( M/L\theta^2 \))

radial position within the solid particle (L)

radius of the solid particle (L)

solid particle recirculation rate (\( M/\theta \))

Reynolds Number defined as \( d(U_g - U_p)\rho_g/\mu \)

Reynolds Number defined as \( d U_l\rho_g/\mu \)

Reynolds Number at minimum fluidization condition

particle Reynolds Number defined as \( \rho_a d_s U_p \rho_0/\mu_a \)

Reynolds Number defined as \( \rho_a d_s V_a/\mu_a \)

S, Sc defined as \( C_s/C_{s0} \)

Schmidt Number defined as \( \mu/\rho_g D_N \)
\( S_v \)  
surface area per unit volume of dense particles (L\(^{-1}\))

\( t \)  
time (\( \Theta \))

\( t_r \)  
residence time of solid particles in the dense section (\( \Theta \))

\( U_a \)  
superficial air velocity (L/\( \Theta \))

\( U_1 \)  
extrapolated air velocity at the bed voidage equal to one (L/\( \Theta \))

\( U_g \)  
linear gas velocity (L/\( \Theta \))

\( U_{g1} \)  
linear gas velocity in the dense section (L/\( \Theta \))

\( U_{g2} \)  
linear gas velocity in the dilute section (L/\( \Theta \))

\( U_{gs} \)  
superficial gas velocity (L/\( \Theta \))

\( U_m \)  
superficial velocity of air-fine particle mixture (L/\( \Theta \))

\( U_{mf} \)  
minimum fluidization velocity of the dense bed (L/\( \Theta \))

\( U_p \)  
linear particle velocity (L/\( \Theta \))

\( U_{p1} \)  
linear particle velocity in the dense section (L/\( \Theta \))

\( U_{p2} \)  
linear particle velocity in the dilute section (L/\( \Theta \))

\( U_{ps} \)  
superficial particle velocity (L/\( \Theta \))

\( U_{p0} \)  
linear fine particle velocity at the packed bed inlet (L/\( \Theta \))

\( U_t \)  
terminal velocity of a single particle (L/\( \Theta \))

\( U_{tp} \)  
terminal velocity of fine particle (L/\( \Theta \))

\( U_{ts} \)  
terminal velocity of dense particle (L/\( \Theta \))

\( U_s \)  
slug velocity (L/\( \Theta \))

\( U_{ss} \)  
single slug velocity (L/\( \Theta \))

\( U_{tr} \)  
transition to turbulent regime velocity (L/\( \Theta \))

\( V \)  
volume of the solid particle (L\(^3\))

\( V_1 \)  
volume of the dense section (L\(^3\))

\( V_a \)  
linear air velocity (L/\( \Theta \))

\( V_s \)  
volume of a single dense particle (L\(^3\))
$W_a$ mass flow rate of air (M/θ)

$W_s$ solid flow rate (M/θ)

$W_p$ mass flow rate of fine particles (M/θ)

$W_s'$ weight of dense particles in the packed bed (M)

$x$ $r/r_0$

$X$ gas reactant conversion at the reactor outlet

$X_s$ solid reactant conversion

$X_f$ overall final conversion of the solid reactant

$y$ $z/L$

$z$ axial distance along the reactor (L)

Greek Symbols:

$\rho_a$ density of air (M/L$^3$)

$\rho_p$ density of particles (fine) (M/L$^3$)

$\rho_s$ density of dense particles (M/L$^3$)

$\rho_m$ density of air-fine particle mixture defined by Eqn. (4.30) (M/L$^3$)

$\rho_m'$ density of air-fine particle mixture defined by Eqn. (4.40) (M/L$^3$)

$\phi$ sphericity factor

$\phi_p$ sphericity factor of fine particles

$\phi_s$ sphericity factor of dense particles

$\epsilon_a$ void fraction in the particle

$\epsilon_0$ initial void fraction in the particle

$\epsilon$ void fraction in the dense bed

$\epsilon_b$ void fraction in the reactor

$\epsilon'$ void fraction in the pneumatic transport bed without dense

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particles

$\varepsilon_{mf}$
void fraction in the dense bed at the minimum fluidization condition

$\varepsilon_p$
volume fraction of fine particles in the dense bed

$\varepsilon_s$
volume fraction of dense particles in the dense bed

$\varepsilon_{po}$
volume fraction of fine particles in the pneumatic transport bed

$\varepsilon_{b1}$
void fraction in the dense section

$\varepsilon_{b2}$
void fraction in the dilute section

$\varepsilon_{s1}$
volume fraction of the particles in the dense section

$\varepsilon_{s2}$
volume fraction of the particles in the dilute section

$\mu_a$
viscosity of air

$\mu$
gas viscosity

$\lambda$
an index associated with the geometry of a particle — $\lambda = 0$ for a flat plate, $\lambda = 1$ for a cylinder, $\lambda = 2$ for a sphere.

$\theta$
dimensionless time, $k_v C_{A0} t$

$\gamma_s$
local solid reaction rate (mole/L$^2$)$\theta$

$\gamma$
constant defined by Eqn. (2.3)
ABSTRACT

A multisolid pneumatic transport bed, which is obtained by superimposing a circulating bed of fine particles on a fluidized dense bed of large particles, is a new design of a combustor developed for efficient and pollution free combustion of coal, petroleum coke, paper mill wastes, wood, and municipal solid wastes combined with sewage sludge. While the system has been demonstrated successfully in pilot scale operation for combustion of different fuels and a commercial process for combustion of petroleum coke has also been in operation, little is known regarding the fundamental characteristics of the combustor. In this study, the sulfur retention capability and the hydrodynamic characteristics of this combustor are extensively analyzed.

The volume reaction model has been modified to account for the heterogeneous sulfation reaction of limestone/dolomite. The model has been verified with experimental sulfation data obtained by Fee et al. (1980) and other data available in literature. The reaction model is combined with the hydrodynamic characteristics of the vertical pneumatic transport reactor to account for limestone/dolomite sulfation reaction. The effect of different operating parameters on the sorbent utilization and sulfur dioxide conversion in the vertical pneumatic transport reactor is simulated. Experimental data obtained
from two multisolid pneumatic transport combustion units is utilized to verify the combined reaction and reactor model of limestone/dolomite sulfation in the vertical pneumatic transport reactor.

Fundamental characteristics of the multisolid pneumatic transport bed including the hold-up of fine particles in the packed and fluidized dense beds, pressure drop in the packed dense bed, minimum fluidization velocity of the dense bed, and terminal velocity of dense particles are also investigated in this study. The hold-up of fine particles in the fluidized dense bed is found to be higher by a factor as large as ten compared to that observed in the system without dense particles. The pressure drop in the packed bed established in a particular manner described in the study, is found to initially decrease significantly and then to increase with the increase of the fine particle flow rate at a given air flow rate. Similarly, the minimum fluidization velocity of the dense bed is found to decrease initially and then to increase with the increase of the fine particle flow rate. The terminal velocity of dense particles is found to decrease with the increase of the fine particles flow rate. Semi-theoretical correlations are developed to account for all the fundamental characteristics investigated in this study.

Experiments are also conducted to analyze the slugging characteristics of dense particles based on the real time pressure fluctuation data. Slug velocity, height of the slugging beds, slug frequency, and transition to turbulent regime velocity for dense particle fluidized beds are analyzed.

The information presented in this report can be utilized to
optimize the design and operation of the multisolid pneumatic transport combustor. Also, the analysis presented is rather general and can be applied to variety of other process applications.
CHAPTER 1

INTRODUCTION

A multisolid fluidized bed or a multisolid pneumatic transport bed (two names used interchangably) is a new design of a combustor developed for efficient and pollution free combustion of coal, petroleum coke, paper mill wastes, wood, and municipal solid wastes combined with sewage sludge. Multisolid fluidized bed combustion superimposes an entrained bed of fine particles upon a fluidized dense bed of large particles in the combustor. The dense bed contains 6x12 mesh size heavy particles which serve to retain and distribute coal particles creating adequate mixing for mass transfer and reaction. The entrained bed material (typically sand and limestone) circulates between the combustor and the external heat exchanger (EHE) to control the combustor temperature and to carry a portion of the heat to the EHE. The presence of dense bed in the combustor permits operation at high superficial velocities of 6 to 9 m/s while the EHE operates at low superficial velocities of 0.3 to 0.6 m/s. Limestone particles in the entrained bed material react with sulfur dioxide generated during the combustion reaction to produce sulfur free flue gases. This combustor has been shown to offer several advantages over the conventional combustion process including (1) high combustion
efficiency due to better control of temperature and the residence time, (2) reduction of combustor size due to the characteristic of being operated at high superficial gas velocity, (3) ability to burn high sulfur, high nitrogen solid or liquid fuels while meeting stringent air emission regulations for \( \text{SO}_2 \) and \( \text{NO}_x \) due to finely divided limestone being used, and (4) capability of processing wide variety of fuels and limestone types. The system has been demonstrated successfully at Battelle Columbus Laboratory (Nack et al., 1977) in a pilot scale operation for combustion and gasification of coal, petroleum coke, paper mill wastes, wood, and municipal solid waste combined with sewage sludge. While a commercial process utilizing multisolid pneumatic transport bed (MPTB) for combustion of petroleum coke has also been in operation (Johnson, 1982), little is known regarding the fundamental characteristics of the combustor. The fundamental information which dictates the optimum operation and proper design of this unit includes the prediction methods of the combustion efficiency, sulfur retention, limestone utilization, and the hydrodynamic characteristics of the dense bed and the pneumatic transport bed.

The sulfur retention and the limestone utilization in a combustor can be predicted with the knowledge of sulfation kinetics, the properties of the calcined limestone sorbents and the understanding of the hydrodynamic characteristics of the combustor. Although several models are available that describe the initial stages of the sulfation reaction (e.g., Borwardt, 1970; Borgwardt and Harvey, 1970; Hartman and Coughlin, 1974,1976; Ramchandran and Smith, 1977; Georgakis et al., 1979; Bhatia and Perlmutter, 1980), these models are unable to
accurately predict the final sulfation level of the sorbents. Also most of these studies utilized only one type of limestone and only the kinetics of that limestone were investigated. Recently, Fee et al. (1980) investigated the sulfation characteristics and the properties of various calcined limestones collected from different parts of the country. The kinetics and diffusion parameters of the calcined sorbents can be determined from these characteristics with an adequate model.

The fine particles in the MPTB before entering the fluidized dense bed and after exiting the dense bed, are conveyed pneumatically. Thus, to predict the sulfur retention and the sorbent utilization in the multisolid pneumatic transport bed, the hydrodynamic information of the dense bed and that of the pneumatic transport bed are of equal importance. The hydrodynamics of the pneumatic transport bed have been investigated by several researchers (e.g., Belden and Kessel, 1949; Konno and Saito, 1969; Hinkle, 1953; Yang, 1978). Yang (1976) has presented a unified theory on the hydrodynamics of the pneumatic transport bed which can be used to predict the acceleration length, fine particle velocity and hold-up of fine particles in the pneumatic transport bed. There are several models available to account for the reactant conversion in the vertical pneumatic transport reactor for catalytic reactions (Pratt, 1974; Paraskos et al., 1976; Varghese and Verma, 1979; Fan, 1980, 1981; Fan and Hwang, 1981). However, little has been done on modeling of non-catalytic gas solid reaction in vertical pneumatic transport reactors.

The hydrodynamic characteristics of the dense bed of the MPTB
which are of importance for the proper design and efficient operation of the unit include the hold-up of fine particles in the packed dense bed, pressure drop in the packed dense bed, minimum fluidization velocity of the dense bed, terminal velocity of the dense particles, and the hold-up of fine particles in the fluidized dense bed. The analysis of hold-up of fine particles in the packed bed is essential to the understanding of the pressure drop in the dense bed. Hold-up of fine particles in the packed dense bed is affected by the fine particle-dense particle interaction coefficient. Equations have been developed to elucidate the interaction effect of two different sizes of fine particles conveyed pneumatically (Welschof, 1962; Nakamura and Capes, 1976). However, no work has been reported on the interaction force on fine particles due to collision with dense particles in the packed bed. The pressure drop in the packed dense bed in combination with the pressure drop in the fluidized dense bed for air-fine particle flow are utilized to understand the minimum fluidization velocity behavior of the dense bed. The pressure drop in the packed dense bed with air flow only has been extensively analyzed in the literature, but no work has been reported for the pressure drop in the packed dense bed with air-fine particle flow. The operating range of gas velocity in the multisolid pneumatic transport bed is bounded by the terminal velocity and the minimum fluidization velocity of the dense particles in an air-fine particle medium. The minimum fluidization velocity of a dense bed of particles less than 5 mm in diameter with air as the fluidizing medium have been thoroughly documented in the literature (e.g., Wilhelm and Kwauk, 1948; Richardson, 1971; Babu et al., 1978). However, very few studies have
been reported for the minimum fluidization velocity of particles greater than 5 mm in diameter (Wen and Yu, 1966). Also, no study has been reported for the minimum fluidization velocity of a dense bed with air-fine particles as the fluidizing medium. The terminal velocity of dense particles in the air-fine particle medium is affected by the particle-particle interaction force on the dense particle. Recently, Arastoopour et al. (1982) determined the interaction coefficient between the fine particle and a dense particle, dropped countercurrently to the fine particle flow. The interaction coefficient determined by Arastoopour et al. (1982) can be utilized to estimate the terminal velocity of dense particles in an air-fine particle medium. However, only one type of fine particle was considered by them and the range of dense particle diameter considered was also small. The analysis of the hold-up of fine particles in fluidized dense bed is essential to the understanding of the residence time and the overall rate of reaction of fine sorbent in the dense bed. No analysis has been reported in the literature for interaction force on fine particles due to collision with dense particles in the fluidized dense bed. This estimation of this interaction force is important for prediction of hold-up of fine particles in the fluidized dense bed.

Three different types of fine particle-dense particle interaction coefficients are involved in the analysis of the hydrodynamic characteristics of the multisolid pneumatic transport bed. For the analysis of fine particle hold-up in the packed dense bed the effect of interaction on the fine particles due to dense particles, which are in packed state, is to be considered. Whereas the analysis of the
hold-up of fine particles in the fluidized dense bed involves the interaction of dense particles in fluidized state on the fine particles being conveyed through the dense bed. And for the analysis of terminal velocity of dense particles, the effect on a single dense particle of interaction of fine particles being conveyed pneumatically is to be considered.

The large diameter particle fluidized beds are normally operated in the slugging flow regime because the slugging flow regime is observed for a wide gas velocity range, which expands from approximately minimum fluidization velocity to the terminal velocity of dense particles. The information regarding the height of the slugging bed is essential for the design of the combustor so that sufficient length of the combustor is provided to allow the disengagement of dense particles. Also important is the slug velocity in the dense bed, to predict the gas conversion and combustion efficiency. The characteristics of fine particle slugging beds have been investigated (e.g., Davidson and Harrison, 1963; Ormiston et al., 1965; Matsen et al., 1969; Kohoe and Davidson, 1970; Theil and Potter, 1977; Baker and Geldart, 1978). However, very few studies have been reported for dense particle slugging beds (Cranfield and Geldart, 1974; Canada et al., 1978; Canada and McLaughlin, 1978; Miller et al., 1981; Glicksman et al., 1981). The dense particle slugging beds exhibit different gas flow characteristics as compared to fine particle slugging beds because of the slug velocity to the emulsion phase gas velocity ratio. This ratio is less than one for dense particle slugging beds and the gas moves freely from the dense phase through the slug and back to the dense phase. The ratio is
greater than one for fine particle slugging beds and the gas circulates around the slug. The gas flow pattern around the slug affects the slug velocity and the height of the slugging beds. The transition from the slugging dense bed to the turbulent dense bed is also of importance for the proper design of the unit. The transition to the turbulent regime has been investigated (Canada et al., 1978; Yerushalmi and Cankurt, 1979; Yang, 1982), but is not well defined and contradictory statements have been reported.

In this study, sulfation characteristics of limestone/dolomite sorbents are investigated, non-catalytic gas-solid reaction with structural changes in the solid particle in the pneumatic transport reactor is modeled, and the hydrodynamic characteristics of the dense bed of the multisolid pneumatic transport reactor are extensively investigated and analyzed. This information is presented in five chapters. All the chapters are written in such a manner that each of them is essentially self-contained.

In Chapter 2, the sulfation kinetics of calcined limestone/dolomite sorbents are investigated. The volume reaction model is modified to account for the heterogeneous sulfation reaction. The reaction parameters of several different sorbents characterized by Fee et al. (1980) are determined. Effects of particle size and sulfur dioxide concentration on the extent of sulfation is also analyzed.

The heterogeneous model developed to account for the non-catalytic gas-solid reaction in the vertical pneumatic transport reactor is presented in Chapter 3. The model takes into consideration both the positive and negative variations of the solid porosity and the variation of the gas diffusivity with the reaction. The model is
applied to the specific case of limestone/dolomite sulfation where negative variations of solid porosity and the gas diffusivity with the reaction are encountered. The verification with the experimental data on limestone sulfation obtained from a bench scale unit and a pilot plant unit is conducted.

Chapter 4 presents the hydrodynamic characteristics of the dense bed of the MPTB. The hydrodynamic characteristics investigated and analyzed include the hold-up of fine particles in the packed dense bed, pressure drop in the packed dense bed, minimum fluidization velocity of the dense bed, terminal velocity of dense particles, and the hold-up of fine particles in the fluidized dense bed. Semi-empirical models have been formulated to analyze these characteristics.

The characteristics of the large diameter particle slugging beds are presented in Chapter 5. Slug rise velocity, height of the slugging bed, pressure fluctuations in the slugging bed, transition to turbulent bed are extensively investigated and analyzed. Chapter 6 lists the conclusions.
CHAPTER 2

SULFATION KINETICS OF CALCINED LIMESTONE/DOLOMITE

ABSTRACT

The volume reaction model has been modified to account for the heterogeneous sulfation reaction of limestone/dolomite. Effects of particle size and sulfur dioxide concentration on the extent of sulfation have been analyzed, based on the model. Also the effect of wearing off the particle outer surface during sulfation has been simulated numerically. The model has been verified with experimental sulfation data obtained by Fee et al. (1980) and other data available in literature.

I. INTRODUCTION

Limestone absorbs $\text{SO}_2$ by a mechanism involving two consecutive steps—the calcination of calcium carbonate followed by the sulfation of CaO with sulfur dioxide to form calcium sulfate. Extensive porosity develops during the calcination of limestone, but the subsequent sulfation greatly diminishes the porosity of the reacting particles thereby limiting the total conversion.

Several models are available for heterogeneous gas-solid reactions (e.g., Yagi and Kunii, 1955; Cannon and Denbigh, 1957;

Borgwardt (1970, 1972) presented a model for the sulfation of limestone/dolomite particles which concluded that (1) the sulfation step is rate-controlling in the initial stages of the conversion and that (2) the rate equation is first order with respect to gas concentration. The decrease in reaction rate with the extent of conversion is explained by a decrease in frequency factor, a term which relates the reaction rate to the number of molecular collisions occurring per unit volume per unit time. When all the pores of the solid reactant are filled, the reaction is assumed to cease. An effectiveness factor was employed to represent the degree to which reaction occurs within the internal structure of the solid. A maximum effectiveness value of one indicates that the reaction occurs equally throughout the internal pore structure. Effectiveness factors less than one are associated with reaction primarily on the outer surface of individual particles. For an effectiveness factor of one, the model predicts the extent of conversion at the cessation of the
sulfation reaction. However, for an effectiveness factor of less than one, it fails to predict the extent of conversion at which sulfation will terminate.

The model developed by Hartman and Coughlin (1974,1976) is a version of a grain model that not only incorporates the chemical reaction resistance along with the diffusion resistances but also considers the decrease in porosity of the reacting particle with the extent of the reaction. Georgakis et al. (1979) proposed in their model that the changing grain size model, which accounts for density changes during the reaction, be combined with the pore plugging model (Chrostowski and Georgakis, 1978). The combined model was used to fit the experimental data of Borgwardt (1970) and Hartman and Coughlin (1974,1976). As shown in Georgakis et al. (1979), the prediction of the model does not agree very satisfactorily with the experimental data.

In Christman and Edgar's model (1980), once the initial pore size distribution is known, the pore size distribution of the solid particle as a function of time and location is obtained by population balance. The macroscopic properties of the solid are obtained by integrating over the entire pore size distribution. The predicted sulfation as a function of time fits the experimental data well for the initial stages of sulfation but not for the later stages and, thus, the model is unable to predict the ultimate sulfation of the sorbents accurately.

Bhatia and Perlmutter (1981) incorporated diffusional effects into their model for kinetically controlled reactions in solids with a randomly distributed pore size. In the sulfation of limestone the
incomplete conversion of calcium oxide was explained by the rapid, product-layer-induced closure of pores on the outer surface of sorbent particle. The optimal temperature of the lime-SO\(_2\) reaction, as noted earlier by Hartman and Coughlin (1976) was also simulated by the model.

The effect of initial structure of the porous solid on the solid conversion was studied by Lindner and Simonsson (1981). Three models with different geometric description of the initial solid structure were considered. All of these models take into account the effect of decreasing porosity and diffusion rate in the growing ash layer and the effect of decreasing gas-solid interfacial area during the reaction. These models which contain more parameters than the grain size variation model (Ramchandran and Smith, 1977 and Georgakis et al., 1979) were shown to predict the experimental data better than did the grain size variation models.

Utilizing the S-shaped breakthrough curves of SO\(_2\) concentration obtained by introducing a step input of SO\(_2\) to a packed column of calcined limestone, the kinetics of the reaction between calcined limestone and SO\(_2\) was analyzed by Orbey et al. (1982). In their analysis, the empirical expression for variation of surface area, particle density and reaction rate constant was used. Furthermore, no variation of the porosity and the diffusivity of gas in solid particles was considered.

The present study seeks to predict the sulfation reaction using a volume reaction model proposed by Wen (1968) and Fan et al. (1977, 1978). The model is generally applicable to heterogeneous reactions between gas and initially porous solids. The model takes into
consideration both diffusion and chemical resistance to reaction. The variation of both void fraction and diffusion resistance with increasing solid product concentration is taken into account at numerous points along the particle radius. Thus, analyses are made of the gas and solid concentration profiles within the particle. The model has been verified with the TGA (Thermogravimetric Analyzer) data obtained at Argonne National Labs. (Fee et al., 1980) and with the data reported in the literature. The effects on sorbent sulfation of particle size, sulfur dioxide concentration, and particle surface wear are examined.

II. VOLUME REACTION MODEL

The overall reaction of a limestone particle with sulfur dioxide in the presence of oxygen can be represented in two steps as

\[
\begin{align*}
\text{CaCO}_3(s) & \rightarrow \text{CaO}(s) + \text{CO}_2(g) & \text{I} \\
\text{CaO}(s) + \text{SO}_2(g) + 1/2 \text{O}_2(g) & \rightarrow \text{CaSO}_4(s) & \text{II}
\end{align*}
\]

At high temperatures, reaction I is relatively rapid in comparison to reaction II. Thus, the development which follows considers only the reaction II. Furthermore, it is assumed that the pore structure developed in reaction I does not change with time except by reaction II, i.e., sintering does not occur. The material balance over a differential element in the solid reactant phase \(S_s\) for the gas reactant \(\text{SO}_2\) (A) and the solid reactant is given by (Fan et al., 1977, 1978):
The reaction of CaO with $\text{SO}_2$ is accompanied by a substantial expansion of the volume of the solid phase because the molar volume of CaSO$_4$ is about three times that of CaO. Since the reacting particles usually retain original gross external volume, the reaction with $\text{SO}_2$ causes a decrease in porosity. The relation of void fraction to solid concentration for a given range of the solid conversion can be assumed to follow the equation (Wen, 1968):

$$
\varepsilon_a = \varepsilon_0 - \gamma (1 - \frac{C_s}{C_{s0}})
$$

Based on the data of Borgwardt and Harvey (1972) and Hartman and Coughlin (1974), the values of parameters $\varepsilon_0$ and $\gamma$ are estimated to be 0.52 and 0.84 respectively for limestone sulfation. These values of $\varepsilon_0$ and $\gamma$ are used in the present model simulation. When the value of $\gamma$ is not known, the model considers it to be an adjustable parameter whose value can be obtained by fitting the TGA data (as shown in Section IV-A).

Effective diffusivity, $D_{eA}$, which appears in Eqn. (2.1), accounts for the overall rate of gas penetration in a porous particle of solid reactant. It is a complex entity, constantly changing with (1) variations in the characteristics of the pore network such as pore shape, and size and (2) variations in the physical conditions such as temperature and pressure. An experimental study of gaseous diffusion in a porous particle (Currie, 1960) has revealed that the effective
diffusivity is changed proportionally to \( \varepsilon_{a}^{1.3} \) when molecular diffusion is predominant (Weisz and Schwartz, 1962). Furthermore, the micropore model (Wakao and Smith, 1962) asserts that the effective diffusivity is proportional to \( \varepsilon_{a}^{2} \) if micropores predominate in the system.

The diffusion of \( \text{SO}_2 \) in a calcium oxide particle occurs in the transition region between molecular diffusion and knudsen diffusion (Satterfield, 1970 and Szekely et al., 1976). Thus, none of the above relations can be used with confidence. Hartman and Coughlin (1976) used the simple relation:

\[
D_{eA} = D_{m} \varepsilon_{a}
\]

(2.4a)

where \( D_{eA} \) is the effective diffusion coefficient and \( D_{m} \) is the diffusion coefficient divided by the tortuosity. The value of \( D_{m} \) at 850 \(^\circ\)C was calculated by Hartman and Coughlin (1976) to be 0.075 cm\(^2\)/sec, based on the data of Campbell et al. (1970) for the diffusion (chiefly Knudsen) of carbon dioxide in the calcined limestone with an estimated pore diameter of 0.04 \( \mu \)m. For comparison, the estimated Knudsen diffusion coefficient of \( \text{SO}_2 \) at 850 \(^\circ\)C is 0.04 for a pore diameter of 0.02 \( \mu \)m and increases linearly with increasing pore diameter (Satterfield, 1970 and Szekely et al., 1976). For additional comparison, in the absence of a porous solid, the calculated molecular diffusion coefficient of sulfur dioxide in nitrogen at 850 \(^\circ\)C is 1.7 cm\(^2\)/sec (Hirschfelder et al., 1964), which agrees well with measured data (Ajersch and Toguri, 1972). Because of the unavailability of tortuosity of the solid particles, \( D_{m} \) is viewed as an adjustable parameter in the model.
In this model, $D_{eA}$ is related to the initial effective diffusion coefficient, $D_{eA0}$, and the solid diffusion coefficient, $D_S$, by the following equation:

$$D_{eA} = D_{Me} + D_S = D_{eA0} \frac{ca}{\epsilon_0} + D_S$$

(2.4b)

The value of $D_S$ is taken from Hartman and Coughlin (1976) as $6 \times 10^{-9}$ cm$^2$/sec which is a factor of approximately $10^{-7}$ less than the gas diffusion coefficient. Thus, in Eqn. (2.4b), gaseous diffusion is the dominant factor until the void fraction is reduced to $10^{-5}$ or less.

The rate of the solid reactant consumption, $\gamma_S$, for the sulfation reaction, can be written in the form

$$\gamma_S = k_V c_2^n c_S^m$$

(2.5)

Borgwardt (1970), Borgwardt and Harvey (1972), Yang et al. (1975), and James and Hughes (1976) concluded that the reaction is first order with respect to sulfur dioxide, thus, $n = 1$. The reaction is assumed to be first order with respect to the solid concentration, thus, $m = 1$. The remaining term in Eqn. (2.5), $k_V$, is the volumetric rate constant.

The initial conditions for Eqns. (2.1) and (2.2) are, respectively

$$t = 0 \quad C_2 = 0$$

$$t = 0 \quad C_S = C_{S0}$$

(2.6)
The boundary conditions for Eqn. (2.1) are

\[
\begin{align*}
    r &= 0 & \frac{\partial C_2}{\partial r} &= 0 \\
    r &= r_0 & D_{eA} \frac{\partial C_2}{\partial r} &= k_d \left( C_1 - C_2 \right)
\end{align*}
\]

(2.7)

When considering the effects of both particle size and \( \text{SO}_2 \) pressure on the extent of sulfation of the particle, the radius of the solid sphere, \( r_0 \), is fixed for each calculation. However, when studying the effect of wearing off of the particle outer layer on the extent of sulfation, \( r_0 \) is varied during the calculation.

Conversion of the solid reactant is defined as

\[
\chi_s = 1 - \frac{\int_{C_s} \frac{dV}{dV} \left( C_s \right)}{\int_{C_{s0}} \frac{dV}{dV} \left( C_{s0} \right)}
\]

(2.8)

The dimensionless forms of Eqns. (2.1) and (2.2) subject to the initial and boundary conditions represented by Eqns. (2.6) and (2.7) are

\[
\left( \frac{C_{s1}}{C_{s0}} \right) \phi_v \frac{\partial \left( C_{s0} \right)}{\partial \theta} = \frac{1}{x^\lambda} \frac{\partial}{\partial x} \left( D_{eA} \frac{x^\lambda}{\partial x} \left( C_s \right) \right) - \phi_v^2 C \frac{\partial C}{\partial \theta}
\]

(2.9)

\[
\frac{\partial S}{\partial \theta} = -C
\]

(2.10)

The initial condition in dimensionless form will be

\[
\begin{align*}
    \theta &= 0 & C &= 0 \\
    \theta &= 0 & S &= 1
\end{align*}
\]

(2.11)
The boundary conditions in dimensionless form will be

\[ x = 0 \quad \frac{\partial C}{\partial x} = 0 \]
\[ x = 1 \quad DA \frac{\partial C}{\partial x} = NSha (1 - C) \quad (2.12) \]

The dimensionless forms of Eqns. (2.4) and (2.8) are

\[ DA = \varepsilon_a / \varepsilon_0 + \tilde{D}_S \quad (2.13) \]
\[ \chi_s = 1 - (\lambda + 1) \int_0^1 S x^\lambda dx \quad (2.14) \]

The dimensionless groups are expressed as follows

\[ C = C_2 / C_1 \quad S = C_S / C_{S0} \quad DA = \frac{D_e A}{D_e A_0} \quad (2.15) \]
\[ NSha = k_d x_0 / D_e A_0, \quad \theta = k_v C_1 t, \quad \tilde{D}_S = \frac{D_S}{D_e A_0} \quad (2.16) \]
\[ \phi_v = R \sqrt{ \frac{k_v C_{S0}}{D_e A_0}} \quad \text{and} \quad x = r / r_0 \quad (2.17) \]

Here, \( \phi_v \) is termed the Thiele modulus.

III. METHOD OF SOLUTION

The numerical solution of nonlinear partial differential equations is often highly complicated and frequently unstable. However, this difficulty has been eased by the recent development of a useful and reliable software interface which can eliminate much of the
expensive and time consuming effort involved (e.g., Sincovec and Madson, 1975). The software interface, which is basically implemented by the so-called "method of lines" (Vichnevetsky, 1971), can provide centered differencing in the spatial variable for time dependent nonlinear partial differential equations. This gives a semidiscrete system of nonlinear ordinary differential equations which may then be solved utilizing the recently developed powerful ordinary differential equation integrators.

The present study employs this software interface for numerical calculations. Gear's (1971) backward differentiation is used as the formula for time integration. The modified Newton's method with internally generated jacobian matrix is utilized to solve the nonlinear equations. Both of them have been combined with the software interface as the load module.

The relative error bound for the time integration process in the present calculation is set at $10^{-2}$ for each time step, the integrator adjusts the time step size and/or order of time integration formula to achieve the specified error level. The error of $10^{-2}$ is checked to be adequate for the desired accuracy of the results. The number of spatial meshes specified for this study is 40.

IV. RESULTS AND DISCUSSION

This section presents the calculated results using the volume-reaction model. Validation of the model is based on the experimental data of Fee et al. (1980), Borgwardt and Harvey (1972), and Hartman and Coughlin (1976). Fee et al. (1980) have determined the reaction rate and capacity of different sorbents obtained from
<table>
<thead>
<tr>
<th>Sorbent ANL Classification</th>
<th>Source</th>
<th>Wt% CaCO$_3$ in the Sorbent</th>
<th>Wt% MgCO$_3$ in the Sorbent</th>
<th>Average Pore Dia. of Calcine, µm</th>
<th>Surface Area of Calcine, $m^2/g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5101</td>
<td>E.E. Duff &amp; Sons, Ohio</td>
<td>51.8</td>
<td>43.3</td>
<td>0.083</td>
<td>20.0</td>
</tr>
<tr>
<td>5301</td>
<td>Chas. Pfizer, Co., Ohio</td>
<td>53.4</td>
<td>45.4</td>
<td>0.047</td>
<td>36.7</td>
</tr>
<tr>
<td>6401</td>
<td>Gordon Inc., Pa.</td>
<td>64.2</td>
<td>29.5</td>
<td>0.067</td>
<td>19.1</td>
</tr>
<tr>
<td>7301</td>
<td>Denny &amp; Simpson, Illinois</td>
<td>73.06</td>
<td>12.5</td>
<td>0.172</td>
<td>7.59</td>
</tr>
<tr>
<td>8001</td>
<td>Greer Limestone, W. Va.</td>
<td>80.4</td>
<td>3.5</td>
<td>0.211</td>
<td>6.64</td>
</tr>
<tr>
<td>9202</td>
<td>Carbon Limestone, Ohio</td>
<td>93.2</td>
<td>1.6</td>
<td>0.106</td>
<td>17.7</td>
</tr>
<tr>
<td>9701</td>
<td>Greer Limestone, W. Va.</td>
<td>97.8</td>
<td>0.6</td>
<td>0.109</td>
<td>12.0</td>
</tr>
<tr>
<td>---</td>
<td>Hartman and Coughlin (1976)</td>
<td>98</td>
<td>0.4</td>
<td>1.0</td>
<td>--</td>
</tr>
<tr>
<td>---</td>
<td>Borgwardt and Harvey (1972)</td>
<td>91</td>
<td>8</td>
<td>0.60</td>
<td>1.8</td>
</tr>
</tbody>
</table>
Table 2.2. Numerical Values of the Parameters Employed

<table>
<thead>
<tr>
<th>Type of Sorbent</th>
<th>Experimental Conditions</th>
<th>Derived Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$S_2O_3$ Concentration</td>
<td>$CaO$ Concentration</td>
</tr>
<tr>
<td></td>
<td>$10^{-8}$ mole/cm$^3$</td>
<td>$mole/cm^3$</td>
</tr>
<tr>
<td>5301</td>
<td>3.2</td>
<td>0.014</td>
</tr>
<tr>
<td>9401</td>
<td>3.2</td>
<td>0.013</td>
</tr>
<tr>
<td>7301</td>
<td>3.2</td>
<td>0.019</td>
</tr>
<tr>
<td>0001</td>
<td>3.2</td>
<td>0.022</td>
</tr>
<tr>
<td>9731</td>
<td>3.2</td>
<td>0.027</td>
</tr>
<tr>
<td>5101</td>
<td>3.2</td>
<td>0.014</td>
</tr>
<tr>
<td>9231</td>
<td>3.2</td>
<td>0.024</td>
</tr>
<tr>
<td>Hartman &amp; Coughlin</td>
<td>1.4</td>
<td>0.017</td>
</tr>
<tr>
<td>Borgward &amp; Harvey</td>
<td>2.6</td>
<td>0.027</td>
</tr>
</tbody>
</table>
various parts of the country utilizing the thermogravimetric analyzer (TGA). The pore size distribution and the surface areas of the sorbents were also determined using porosimeter. The experimental results obtained with the TGA and the porosimeter are listed in Tables 2.1 and 2.2.

IV-A. Concentration Profiles

The dependence of the gas and solids concentration profiles and solids conversion on the parameters used in the volume reaction model is demonstrated in this subsection. The values of the parameters employed in the calculation are presented in Table 2.2.

In Fig. 2.1, the predicted concentration of gas inside the particle is plotted as a function of the distance from the center of the particle at various dimensionless times, $\theta$, and $\phi_v$ values of 1.0 and 10. As shown in Eqn. (2.17), $\phi_v$ is functionally dependent on $r_0$, $k_v$, $C_s0$ and $D_{eA0}$. Table 2.2 shows $C_s0$ to be relatively constant for limestones. Further, $k_v$ can be viewed as a fundamental rate constant which is independent of the particular limestone considered. As a result, increasing $\phi_v$ from 1.0 to 10.0 can be considered as either an increase in the sorbent particle radius, $r_0$, by a factor of 10 or a decrease in the initial effective diffusion coefficient, $D_{eA0}$, by a factor of 100. It is seen that the gas concentration inside the solid particle increases with $\theta$, but after a finite time, the concentration starts to decrease and lowers to zero value before all the solid (shown in Fig. 2.2) has been reacted. The increase in gas concentration in the particle is due to the amount of gas diffusing into the particle being greater than the amount reacted. Comparison of gas concentration profiles for $\phi_v = 1$ and $\phi_v = 10$ shows that this
Fig. 2.1 Computed Gas Concentration Profiles Inside Particle
$C_1 / C_{s0} = 1.0$
Fig. 2.2 Computed Solids Reactant Concentration Profiles Inside Particles. \( \frac{C_1}{C_{S0}} = 1.0 \)
phenomenon occurs less pronounced when the rate of reaction is high compared to the rate of diffusion (high $\phi_V$ values). The porosity of the particle decreases with conversion, resulting in a decrease in the rate of diffusion. Therefore, after a finite extent of conversion, the rate of diffusion becomes almost negligible and only the gas already entrapped in the solid reacts.

In Fig. 2.2, the predicted solid concentration profile is plotted for different values of dimensionless time at $\phi_V$ values of 1.0 and 10. The solids concentration decreases with increasing time until a certain time, beyond which, extrapolation would indicate that conversion remains unchanged. Because the rate of reaction is high compared to the rate of diffusion (implied by the large value of $\phi_V$), pores on the outer layer of the particle are blocked during the reaction. For the larger values of $\phi_V$, the calcium oxide near the center of the particle remains unreacted.

In Fig. 2.3, the effect of $\phi_V$, the Thiele Modulus, on total conversion (sulfation) of the solid reactant is shown. The overall conversion of the particle increases with time until a certain time, beyond which, conversion remains constant. The final conversion and the rate of increase of conversion decreases with increasing $\phi_V$.

In Fig. 2.4, the gas concentration profile in the solid particle for $C_1/C_{s0}$ of $1.5 \times 10^{-6}$ is plotted for various dimensionless times and for $\phi_V$ values of 1 and 10. In contrast to Fig. 2.2, for $C_1/C_{s0}$ as low as $1.5 \times 10^{-6}$, the gas concentration inside the solid particle increases to a certain value in infinitesimal dimensionless time and decreases thereafter until it reduces to zero value. It is noted that $C_1/C_{s0}$ for all the sorbents listed in Table 2.2 is close to the low
Fig. 2.3 Conversion of the Solid Reactant versus Time as a Function of Thiele Modulus. $C_1 / C_{s0} = 1.0$
Fig. 2.4 Computed Gas Concentration Profile Within the Particle for Low $C_1 / C_{s0}$, $C_1 / C_{s0} = 1.5 \times 10^{-6}$. 

\[ \phi_v \]
value of \( C_1 / C_{s0} \) used in Fig. 2.4. The solid concentration profile in the solid particle at various dimensionless times follows the same trend as exhibited in Fig. 2.2 even for low values of \( C_1 / C_{s0} \).

The smallest sorbent size considered in this study is 0.0098 cm in diameter. For the sorbents of this small size, Borgwardt and Harvey (1972) reported that the diffusion resistance is negligible and the reaction occurs evenly throughout the pore structure. The capacity of these sorbents can be determined from the total pore volume available for accumulation of the reaction product. Thus, the porosity variation constant, \( \gamma \), defined in Eqn. (2.3) for a given sorbent can be evaluated from the experimental data of the final conversion and initial pore volume for the sorbents of size 0.0098 cm in diameter. Specifically, \( \gamma \) can be calculated by

\[
\gamma = \frac{\varepsilon_0}{X_f} \quad (2.18)
\]

where \( X_f \) is the conversion of the sorbent at the end of reaction.

The initial reaction rate is determined by the slope of the initial conversion versus time relationship which is sensitive to the variation of the initial reaction conditions in the system. The volumetric reaction rate constant, \( k_v \), used in the rate expression is generally estimated from initial reaction rates of small sorbents. However, \( k_v \) determined in this manner would involve errors due to the fact that initial reaction rate is known to be fast and thus a constant concentration of \( SO_2 \) surrounding the particles would be difficult to be maintained. Nevertheless, it should be noted that the initial reaction rate obtained based on the slope of the initial
conversion versus time relationship does not correlate well with BET surface areas for the sorbents considered in this study. For example, the sorbents 8001 and 7301 with initial BET surface areas of 6.64 and 7.59 m²/g provide the initial reaction rates of 2.4 and 1.25 min⁻¹.

In this study, the volumetric reaction rate constant, $k_v$, and the diffusivity term, $D_m$, are treated as adjustable parameters. The values for $k_v$ and $D_m$ are obtained by fitting the model to the experimental data for a selected size of the sorbent. These values are then used to calculate the conversion of the sorbent at different sizes and in different $SO_2$ concentration environments. When the values

IV-B. Effect of Sorbent Particle Size

Hartman and Coughlin (1976) presented experimental data for the overall sulfation of CaO versus exposure time for three different particle sizes. The values of the parameters of the volume reaction model (which have been derived by fitting the experimental data at a particle radius of 0.0028 cm) are given in Table 2.2 along with the experimental conditions. Figure 2.5 shows the experimental values and the volume reaction model predictions which may be compared. The values for $k_v$, $C_{SO}$, and $D_m$ are held constant for the three values of $\phi_v$ considered. The curves for different particle radii, i.e., 0.045 and 0.056 cm, are obtained by changing the $\phi_v$ in the same proportion as the particle radius (see Eqn. 2.17). It is evident that the agreement is satisfactory.

Figure 2.6 compares the predicted sulfation versus time data for
Fig. 2.5  Sulfation of CaO Predicted by the Volume Reaction Model in Comparison with Data of Hartman and Coughlin (1976)
Fig. 2.6  Sulfation of CaO Predicted by the Volume Reaction Model in Comparison with Data of Borgwardt and Harvey (1972)
different $\phi_v$ values, based on the volume reaction model, with the experimental data of Borgwardt and Harvey (1972). The parameter values, derived by fitting the experimental data for particles of radius 0.0125 cm, are listed in Table 2.2. The same parameter values are used to calculate the sulfation for particles of radius 0.0048 cm and 0.065 cm. As required, the $\phi_v$ values for the three sets of experimental data are in the same proportion as the corresponding particle radii. The volume reaction model appears to adequately predict the experimental data.

The effects of sorbent particle size on sulfation have been analyzed, based on the TGA data obtained by Fee et al. (1980). Included under this consideration are sorbents 5301, 6401, 7301, 8001, and 9701. The experimental conditions and values of the parameters used in the volume reaction model are listed in Table 2.2. Figures 2.7 to 2.11 compare the experimental and predicted results for these five sorbents. Again, agreement is adequate for all the sorbents.

The present volume reaction model is used to predict sulfation for any intermediate size or by extrapolation to any particle size. In Fig. 2.12, final sulfation versus radius is plotted for sorbent 7301. The properties are listed in Table 2.2. It is seen that a nonlinear relationship exists between the final sulfation (i.e., conversion) and radius.

For the sorbents in Table 2.2, the values of $k_v$, the volumetric rate constant, and $D_m$ the diffusion coefficient divided by the tortuosity, show a considerable variation; from $3 \times 10^3$ to $68 \times 10^3$ cm$^3$/mole sec for $k_v$, from 0.008 to 0.4 cm$^2$/sec for $D_m$. Some of the variation in the $D_m$ values may be due to the differences in the pore
Fig. 2.7 Sulfation of CaO Predicted by the Volume Reaction Model in Comparison with data of the Present Study for Sorbent 5301
Fig. 2.8  Sulfation of CaO Predicted by the Volume Reaction Model in Comparison with data of the Present Study for Sorbent 6401
Fig. 2.9 Sulfation of CaO Predicted by the Volume Reaction Model in Comparison with data of the Present Study for Sorbent 7301
Fig. 2.10 Sulfation of CaO Predicted by the Volume Reaction Model in Comparison with data of the Present Study for Sorbent 8001
Fig. 2.11 Sulfation of CaO Predicted by the Volume Reaction Model in Comparison with data of the Present Study for Sorbent 9701
**Fig. 2.12** Effect of Particle Radius on Final Sulfation for Sorbent 7301

- Experimental data
- Predicted by the model
network between stones. Variations in tortuosity factors of more than an order of magnitude have been reported (Satterfield, 1970 and Szekely et al., 1976). In addition, some of the variation in $k_v$ and $D_m$ may arise from the transition from predominantly molecular diffusion to predominantly knudsen diffusion with decreasing pore size. Although there is considerable scatter in the values, a decreasing trend in the $k_v$ values and the $D_m$ values with decreasing pore size is observed; excluding, in the latter case, stones 5301 and 5101 which have a high MgCO$_3$ content. (The calcination of the MgCO$_3$, without the subsequent sulfation of the MgO, may give these sorbents significantly different pore networks and $D_m$ values.) Obviously, more data is needed to determine if this trend is significant.

IV-C. Effect of SO$_2$ Concentration

The sulfation versus time data obtained for sorbents 5101 and 9202 by Fee et al. (1980) were used to study the effect of SO$_2$ concentration on sulfation. The values of the parameters obtained by fitting the experimental data at an SO$_2$ concentration of 3000 ppm to the volume reaction model are listed in Table 2.2. These values are used to calculate the sulfation for other SO$_2$ concentrations. As is evident from Fig. 2.13, the model adequately predicts the experimental data.

IV-D. Effect of Particle Surface Wear

The removal of sulfated outer layers of a particle due to wear occurs during sulfation in the fluidized bed combustor. An attempt is made here to predict the effect of this wear on the extent of
Fig. 2.13 Comparison of the Data of the Present Study for Sorbents 5101 and 9202 with Predictions of the Volume Reaction Models for Different \( \text{SO}_2 \) Concentrations
Predicted sulfation for particle of radius 0.0049 cm
Predicted sulfation for particle of radius 0.0128 cm
Predicted sulfation for particle of radius 0.0049 cm which reduces to 0.00342 cm at 60 min
Predicted sulfation for particle of radius 0.0128 cm which reduces to 0.00896 cm at 60 min
Predicted sulfation for particle of radius 0.00342 cm
Predicted sulfation for particle of radius 0.0128 cm which reduces to 0.00512 cm at 60 min

Fig. 2.14 Effect of Reduction in Particle Size at Different Times for Sorbent 9701
sulfation. The present volume reaction model, which allows particle wear to be predicted for any time during the reaction and with any thickness of outer layer removal is utilized for simulation. Figure 2.14 shows the effect on sulfation of particle wear which, in a single step, removes 30% to 60% of the original radius at 60 min. The parameter values of the model are listed in Tables 2.1 and 2.2. The extent of sulfation of the peeled particle is calculated by integrating throughout the reduced particle. As shown in Fig. 2.14, at the time of peeling, the average sulfation of the peeled particle is much less than the average sulfation of the original particle. As a result, the sulfation versus time relation plot for the reduced particle shows a decrease at the time of particle wear off. Thereafter, the sulfation of the reduced particle increases with time.

Particle wear increases the extent of sulfation of the reduced particle. The final sulfation of the two particle sizes considered (initial radii 0.0049 cm and 0.0128 cm) increases as particle wear occurs during reaction for both 30 and 60% reductions in particle radius. Further, Fig. 2.14 shows that increasing the wear-off increases the final sulfation of the reduced particle (i.e., smaller reduced particles are sulfated to higher levels). However, the final sulfation of the worn particle is not as high as an "unworn" particle of the same final radius. In Fig. 2.14, the final sulfation for a particle whose radius has been reduced from 0.0049 cm to 0.0034 cm during reaction is not as high as that for a particle with an initial radius of 0.0034 cm. A similar trend is calculated for the other particle size (but not shown in Fig. 2.14).

Figures 2.15 and 2.16 show the variations of the solids and gas
Fig. 2.15 Sulfur Dioxide Gas Concentration Profile for Sorbent 9701 for Particles of Radius 0.049 cm which Reduce to 0.034 cm at 60 min
Fig. 2.16 Calcium Oxide Reactant Solid Concentration Profile for Sorbent 9701 for Particles of Radius 0.049 cm which Reduce to 0.034 cm at 60 min.
concentrations with time for particles of 0.0049 cm original radius and of reduced radius of 0.0034 cm. The peeling takes place after 60 min of reaction. The solid and gas concentration profiles change at this time. After the radius of the particle is decreased, the gas concentration in the particle increases, then decreases. (Similar effects were calculated for the other particle sizes). Figures 2.15 and 2.16 show why the extent of sulfation of the worn particle is less than for an "unworn" particle of the same radius, 0.0034 cm. The calcium oxide concentration profile in Fig. 2.14 shows that significant sulfation occurs at a radius of 0.0034 cm at 60 min. The same sulfation level and the consequent reduction in the effective diffusion coefficient of that location would, obviously, also occur during the sulfation of a particle with initial radius of 0.0034 cm. However, during its entire sulfation the unworn particle experiences the comparatively higher $\text{SO}_2$ levels and steeper $\text{SO}_2$ gradients (from radius 0.0035 cm to the particle core) that the worn off particle experiences only when the outer layer is removed. This is shown in the comparison of the $\text{SO}_2$ gas concentration profiles at 60 and 61 min in Fig. 2.15. As a result, more of the central core of the "unworn" particle is sulfated leading to the higher overall conversion shown in Fig. 2.14.

The effect of particle wear on the overall conversion of sorbent in a fluidized-bed coal combustor depends on the particular limestone sorbent, the sorbent particle size, and the extent of sulfation of the sorbent. For large particles, wear off of a nearly completely sulfated outer layer exposes a largely unsulfated inner core for further sulfation. This increases the extent of sulfation of the
original limestone particle (i.e., reduced particle plus removed outer layer) and decreases the limestone feed requirement for a fluidized bed coal combustor. In the other extreme, wearing off a largely unsulfated outer layer shown in Fig. 2.16) results in decreased sulfation of that original limestone particle and increased limestone feed requirements. This occurs because the fine particles arising from the breakup of the outer layer are carried out of the combustor and only the reduced particle is sulfated further.

V. CONCLUDING REMARKS

The volume reaction model is modified to account for the decrease in the solid porosity and the gas diffusivity in the solid particle with the reaction. The modified volume reaction model is shown to predict the extent of sulfation of limestone/dolomite sorbents. In particular, the model is shown to describe the frequently observed situation when the sulfation reaction does not go to completion because the high molar volume product blocks the essential gas transport pathways. The final sulfation level of the limestone/dolomite sorbent is shown to be fixed by the porosity of a particular sorbent and the effective diffusivity of SO\textsubscript{2} in that pore network. Sorbent specific data, which is obtained by fitting experimental TGA results at a particular particle size and SO\textsubscript{2} level, adequately predicts the sulfation behavior for different particle sizes and SO\textsubscript{2} levels. Thus for a particular sorbent, only a single TGA experiment must be performed in order to predict the performance of the wide range of particle sizes and SO\textsubscript{2} levels encountered in a fluidized bed combustor.
CHAPTER 3

NON-CATALYTIC GAS-SOLID REACTION IN A VERTICAL PNEUMATIC TRANSPORT REACTOR

ABSTRACT

A heterogeneous model is developed to account for non-catalytic gas-solid reactions in a vertical pneumatic transport reactor. The model takes into consideration both the positive and negative variations of the solid porosity and the variation of gas diffusivity with the reaction. The method of lines utilizing the second order centered finite difference scheme for the spatial discretization is employed to obtain the model solution. The model is employed to account for the sulfur dioxide conversion and the sorbent utilization in the vertical pneumatic transport reactor as a scrubber for flue gas desulfurization using limestone/dolomite sorbents. The sulfation characteristics of the sorbents in the reactor with or without the spent sorbent recycle are examined. Experimental data obtained from a 16.2 cm bench scale reactor and a 25.4 cm pilot scale reactor where limestone/dolomite reacts with sulfur dioxide generated from coal combustion, is utilized to verify the model.

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I. INTRODUCTION

The vertical pneumatic transport reactor, known as the riser, has been widely utilized for catalytic fluid reactions such as cracking of gas oil and residue to gasoline (Stemerdink, 1962; Bryson et al., 1972). It has been utilized for non-catalytic fluid-solid reactions such as gasification and combustion of coal and biomass (Nack et al., 1977) and the calcination of aluminum hydroxide (Reh, 1972).

There are several models available to account for the reactant conversion in the vertical pneumatic transport reactor for catalytic reactions (Pratt, 1974; Paraskos et al., 1976; Varghese and Verma, 1979; Fan, 1980, 1981; Fan and Hwang, 1981). Basically, they can be classified into two types, namely, the homogeneous model and the heterogeneous model. The homogeneous model (Paraskos et al., 1976; Fan, 1980, 1981) is used when the Thiele modulus, defined as $(d/2)(k_{v}/D_{eA})^{1/2}$, is less than 1.0. The models of Fan (1980, 1981) and Fan and Hwang (1981) incorporate the true hydrodynamic properties which are obtained empirically, including the linear velocity of the gas and solid particle, slip velocity and void fraction. Other models (Pratt, 1974; Paraskos et al., 1976; Varghese and Verma, 1979), however, impose simplified assumptions on the hydrodynamic properties, for example, the slip velocity is equal to zero or is equal to the particle terminal velocity. In contrast to the efforts on modeling catalytic reactions in the vertical pneumatic transport reactor, little has been done on modeling the non-catalytic reactions in the reactor. Recently, Hartke et al. (1981) performed experiments on the thermal decomposition of NaHCO$_3$ in a laboratory scale circulating fluidized bed reactor. The results for the rate constants obtained
were compared with those obtained in a rotary kiln, a conventional fluidized bed and a pneumatic transport reactor. High decomposition rates were found in the circulating fluidized bed reactor and in the pneumatic transport reactor. A simple, modified shrinking core model was formulated to account for the rate determining step of NaHCO₃ decomposition.

One of the important examples of non-catalytic gas-solid reactions in the vertical pneumatic transport reactor is limestone/dolomite sulfation in the reactor. Numerous combustion processes utilizing the pneumatic transport or circulating bed reactors are available in industry, e.g., Lurgi's Process (Peterson et al., 1980), and Battelle's process, which is known as the Multisolid Fluidized Bed Combustion Process (Nack et al., 1977). High sorbent utilization has been observed in these reactors compared to that in the conventional bubbling or turbulent bed reactors (Nack et al., 1977). The major factor contributing to high sorbent utilization in such a reactor is the sorbent size. The sorbent size for the pneumatic transport or circulating bed reactor is as small as 20 μm, while that for the bubbling or turbulent bed reactor is as large as 1000 μm or over. In addition, substantial amounts of spent sorbent are recycled in the operation of the pneumatic transport or circulating bed reactor. A key to widespread commercialization of these reactors is the ability to accurately predict the retention of sulfur dioxide by a given sorbent. Limestone/dolomite are the common sorbents used for fluidized bed combustion. Extensive analysis of the sorbent utilization in the bubbling or turbulent fluidized bed combustors has been conducted (e.g., Chen and Wen, 1981; Fan et al.,
1981a; Marroquin et al.,1982). However, little analysis has been done on sorbent utilization in the pneumatic transport or circulating bed combustors.

In this chapter, a heterogeneous model for a non-catalytic gas-solid reaction in the vertical pneumatic transport reactor is developed. The model takes into account the empirical correlations for the hydrodynamic properties and gas-film diffusion around the solid particle. Both the positive and negative variations of the solid porosity and the variations of the gas diffusivity with the reaction are considered in the model. The variations of solid porosity and the gas diffusivity in the solid particle are modeled in similar manner to limestone sulfation in Chapter 2. The effects of various operating parameters including the superficial gas velocity, reaction rate constant, length of the reactor, particle size, and effective diffusivity on the reactant conversion are analyzed. The model is also applied for specific example of limestone/dolomite sulfation and the conversions of sorbents and sulfur dioxide is extensively simulated. The experimental data obtained from a 16.2 cm bench scale reactor and a 25.4 cm pilot scale reactor where limestone reacts with sulfur dioxide generated from coal combustion, are utilized to verify the model.

II. THE MODEL

A general model for non-catalytic gas-solid reaction in pneumatic transport reactor with variations in solid properties is first developed. The model is then applied to the specific case of limestone/dolomite sulfation.
II-A. The General Model

Consider a gas-solid reaction of the following form:

\[
aA + B_s \rightarrow \sum_i b_i B_i + \sum_j c_j S_j
\]

The reaction takes place in an isothermal pneumatic transport reactor. Assume that the entrance effects can be neglected, the gas and solid flows follow the plug flow pattern, and the solid particles are spherical and uniform in size (Fan, 1981). A material balance of the gas reactant in the gas phase give rise to

\[
U_g \epsilon_b \frac{dC_1}{dz} + \frac{3(1-\epsilon_b)k_d}{r_0} (C_1 - C_2 \mid r = r_0) = 0
\]

where \( \epsilon_b \) is the void fraction in the bed.

The boundary condition for Eqn. (3.1) is

\[
z = 0; C_1 = C_0
\]

A material balance of the gas reactant and the solid reactant in the solid phase yields, respectively:

\[
U_p \frac{\partial (C_2 \epsilon_s)}{\partial z} = \frac{1}{r^2} \frac{\partial}{\partial r} (D_e A \frac{r^2 \partial C_2}{\partial r}) - a \gamma_s
\]

\[
U_p \frac{\partial C_s}{\partial z} = - \gamma_s
\]
The boundary conditions for Eqns. (3.3) and (3.4) are:

\[ z = 0; \quad C_2 = 0 \]
\[ C_s = C_{s0} \]  \hspace{1cm} (3.5)
\[ r = 0; \quad \frac{\partial C_2}{\partial r} = 0 \]  \hspace{1cm} (3.6)
\[ r = r_0; \quad D_{e \alpha} \frac{\partial C_2}{\partial r} = k_d (C_1 - C_2) \]  \hspace{1cm} (3.7)

\( \varepsilon_a \) in Eqn. (3.3) is the void fraction of the particle. It is assumed that over a given range of the solid conversion, relationship between \( \varepsilon_a \) and the solid concentration can be approximated by (Wen, 1968; Fan, et al., 1977)

\[ \varepsilon_a = \varepsilon_0 + \gamma (1 - \frac{C_s}{C_{s0}}) \]  \hspace{1cm} (3.8)

where \( \varepsilon_0 \) is the void fraction of the particle at the initial condition. \( \gamma \) is the parameter for void fraction variation, which is a positive or negative constant. Positive constant for \( \gamma \) accounts for the increase of the void fraction during the reaction. Examples of the reactions with \( \gamma \) as a positive value are ore roasting, coal gasification and combustion, coal desulfurization, calcination, catalyst deactivation, propellant combustion, coal liquefaction, and sodium thiosulfate manufacture. Negative constant for \( \gamma \) accounts for the decrease of the void fraction during the reaction. Examples of the reactions with \( \gamma \) as a negative value are sulfation of limestone,
and oxidation, halogenation and sulfidation of metal or alloy.

Effective diffusivity $D_{eA}$ in Eqn. (3.3) accounts for the overall rate of gas diffusion in the porous particle of solid reactant. The relationship between the effective diffusivity and the solid porosity was developed in Chapter 2. The same relationship is used here, which has the form

$$D_{eA} = D_m c_a$$

(3.9)

where $D_m$ is the molecular diffusivity divided by the tortuosity. $D_{eA}$ can be related to $D_{eA0}$ by

$$D_{eA} = D_{eA0} \frac{c_a}{c_0}$$

(3.10)

Equation (3.10) is used in this study to account for the diffusivity variation with the void fraction in the particle.

$\gamma_s$ is the rate of the solid reactant consumption and can be expressed by

$$\dot{\gamma}_s = k_v C_2^n C_s^m$$

(3.11)

where $k_v$ is the volumetric rate constant.

The conversion for the solid reactant is defined as

$$X_s = 1 - \frac{\int \frac{C}{C_s} \, dv}{\int \frac{C}{C_s0} \, dv}$$

(3.12)

The gas reactant conversion in the reactor outlet can be
evaluated by

\[ x = 1 - \frac{U_e e_b C_1 + U_p (1 - e_b)C_2}{U_g s C_0} \] (3.13)

where \( \bar{C}_2 \) is the average gas reactant concentration in the solid particles which can be expressed by

\[ \bar{C}_2 = \frac{3}{r_0^3} \int_0^{r_0} C_2 r^2 dr \] (3.14)

In dimensionless forms, Eqns. (3.1) through (3.7) respectively become:

\[ \frac{df_1}{dy} + F(f_1 - f_2) \bigg|_{x = 1} = 0 \] (3.15)

\[ y = 0; f_1 = 1 \] (3.16)

\[ A \frac{\partial (af_2)}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} \left( D_A x^2 \frac{\partial f_2}{\partial x} \right) - \phi_v^2 f_2^n g_1 \] (3.17)

\[ \frac{\partial g_1}{\partial y} = -G f_2^n g_1 \] (3.18)

\[ y = 0; f_2 = 0 \] (3.19)

\[ g_1 = 1 \] (3.20)

\[ x = 0; \frac{\partial f_2}{\partial x} = 0 \] (3.21)

\[ x = 1; D_A \frac{\partial f_2}{\partial x} = N_{sha} (f_1 - f_2) \] (3.21)

where

\[ A = \frac{r_0^2 U_p e_A}{L D_{eA0}}, f_2 = \frac{C_2}{C_0}, \phi_v = r_0 \sqrt{\frac{a_{kn} C_{m0} C_0^{-1}}{D_{eA0}}} \]
The dimensionless form of Eqn. (3.12) is:

\[ x_s = 1 - 3 \int_0^1 g_1 x^2 \, dx \]  

(3.22)

II-B. Limestone/Dolomite Sulfation Model

As described in Chapter 2, the limestone particles react with \( \text{SO}_2 \) in the presence of oxygen to form \( \text{CaSO}_4 \) with the reaction represented in two steps by

\[
\text{CaCO}_3 (s) \rightarrow \text{CaO}(s) + \text{CO}_2 (g) \quad (I)
\]

\[
\text{CaO}(s) + \text{SO}_2 (g) + \frac{1}{2} \text{O}_2 (g) \rightarrow \text{CaSO}_4 (s) \quad (II)
\]

At high temperatures, reaction I is relatively fast compared to reaction II. Thus, only reaction II is considered in the determination of overall rate of reaction of sulfation. The rate of sulfation, \( \dot{\gamma}_s \), can be expressed by Eqn. (2.11). As described in Chapter 2, \( \dot{\gamma}_s \) has been shown to be of the first order with respect to the sulfur dioxide concentration and can be assumed to be of the first order with respect to the solid reactant concentration.
Also noted in Chapter 2, the reaction of CaO with $SO_2$ is accompanied by a substantial expansion of the volume of grains which constitute the CaO particles. The volume expansion is due to the fact that the molar volume of CaSO$_4$ is about three times that of CaO. The reacting particles usually retain their original gross external volume and thus, the reaction with $SO_2$ causes decrease in the porosity. The relation between the void fraction and the solid concentration can be assumed to follow the Eqn. (3.8) with negative values of $\gamma$.

III. EVALUATION OF HYDRODYNAMIC AND MASS TRANSFER PROPERTIES FOR THE MODEL

The hydrodynamic properties for the pneumatic transport reactor can be obtained from the mass balance or empirical correlations available in literature as given below:

The mass balance for the solid particle in the reactor yields

$$\varepsilon_b = 1 - \frac{4W_s}{\rho_p \pi D^2 U_p}$$  \hspace{1cm} (3.23)

The linear particle velocity is obtained by (Yang 1977, 1978):

$$U_p = U_g - U_t \sqrt{\left(1 + \frac{f_p \rho_p U_p^2}{2g_c D}\right)} 4.7$$  \hspace{1cm} (3.24)

The solid friction factor, $f_p$, can be obtained by the following empirical correlation equations (Yang, 1978)

$$f_p = 0.0126 \left(1 - \varepsilon_b\right)^3 \left[(1 - \varepsilon_b) \frac{(Re)_t}{(Re)_p}\right]^{-0.979}$$  \hspace{1cm} for $U_g/U_t > 1.5$ (3.25)
The terminal velocity for the solid particle, $U_t$, can be evaluated by (Yang, 1973)

$$U_t = \frac{0.153 d 1.14 g 0.71 (\rho_p - \rho_g) 0.71}{0.43 \mu 0.29 \rho_g}, \text{ for } 2.0 < (Re)_p < 1000$$

and

$$U_t = \frac{d^2 (\rho_p - \rho_g) g}{18 \mu}, \text{ for } (Re)_p \leq 0.1$$

where

$$(Re)_p = \frac{d (U_t - U_p) \rho_g}{\mu}$$

Equations (3.23) through (3.26) are solved simultaneously to obtain the hydrodynamic properties including $\varepsilon_b$, $f_p$, and $U_p$. The values of $\varepsilon_b$ and $U_p$ are required for the model equations.

The gas film mass transfer coefficient, $k_d$, for the suspended solid particles can be obtained by the following empirical correlation equation (Satterfield, 1969):

$$\frac{k_d d}{D_M} = 2.0 + 0.63 (Sc)^{1/3} (Re)_p^{1/2}$$

where $Sc = \frac{\mu}{\rho_g D_M}$
IV. METHOD OF SOLUTION

IV—A. Method

The numerical scheme utilized to solve the partial differential equations is based on the same principle as that of Chapter 2.

IV—B. Comparison with Analytical Solution

The model equation for the special case of \( n = 1, m = 0 \), and \( \gamma = 0 \) can be solved analytically by Laplace Transforms. The analytical solutions for the gas reactant concentration in both the gas phase and the solid phase and the solid concentration can be expressed by (Fan and Hwang, 1981):

\[
\frac{C_1}{C_0} = 1 - \frac{D(\beta_0 - \text{tanh} \beta_0)(\exp(s_0 y) - 1)}{s_0 h_0} - \sum_{n=1}^{\infty} A' D(\gamma_n - \text{tanh} \gamma_n)(1 - \exp(s_n y)) \frac{1}{(\phi_n^2 + \gamma_n^2) h_n}
\]

(3.31)

\[
\frac{C_2}{C_0} = \frac{1}{x} \left( \frac{\sinh(\beta_0 x) \exp(s_0 y)}{G_0} + \sum_{n=1}^{\infty} \frac{\sin(\gamma_n x) \exp(s_n y)}{G_n} + \sum_{n=1}^{\infty} \frac{\sin(\delta_n x) \exp(\tau_n y)}{F_n} \right), \text{ for } x \neq 0
\]

(3.32a)
\[
\frac{C_2}{C_0} = \frac{\beta_0}{G_0} \exp(s_0y) + \sum_{n=1}^{\infty} \frac{\gamma_n}{G_n} \exp(s_ny) + \sum_{n=1}^{\infty} \frac{\delta_n}{F_n} \exp(\tau_ny),
\]
for \(x = 0\)

\[
\frac{C_s}{C_{s0}} = -\frac{k_v L C_0}{U_p C_{s0}} \left[ \frac{1}{x} \sinh(s_0x) \exp(s_0y) \right] + \sum_{n=1}^{\infty} \frac{\sin(\gamma_nx)\exp(s_ny)}{s_n G_n}
\]

\[
+ \sum_{n=1}^{\infty} \frac{\sin(\delta_nx)\exp(\tau_ny)}{s_n G_n} + \sum_{n=1}^{\infty} \frac{\sin(\delta_nx)}{\tau_n F_n} \right), \text{for } x \neq 0
\]

\[
\frac{C_s}{C_{s0}} = -\frac{k_v L C_0}{U_p C_{s0}} \left[ \frac{\beta_0}{G_0 s_0} \exp(s_0y) \right] + \sum_{n=1}^{\infty} \frac{\gamma_n}{s_n G_n} \exp(s_ny)
\]

\[
+ \sum_{n=1}^{\infty} \frac{\delta_n}{F_n \tau_n} \exp(\tau_ny) + 1 + \frac{k_v L C_0}{U_p C_{s0}} \frac{\beta_0}{G_0 s_0}
\]

\[
+ \sum_{n=1}^{\infty} \frac{\gamma_n}{s_n G_n} + \sum_{n=1}^{\infty} \frac{\delta_n}{F_n \tau_n} \right), \text{for } x = 0
\]
where:

\[ D = \frac{3(1-e_b)LD}{U e_b r_0^2}, \quad A' = \frac{e_0 \tau_0 U}{L D e A}, \quad E = \frac{1}{N_{Sha}} \]

\[ \beta_0 \] is the real root of the following equation:

\[ \{E(\beta_0^2 - \phi^2) + A'D\}\beta_0 = \{A'D + (E-1)(\beta_0^2 - \phi^2)\} \tanh \beta_0 \]

\[ \gamma_n \] is the root of the following equation:

\[ \tan \gamma_n \{A'D - (E-1)(\gamma_n^2 + \phi^2)\} = \{A'D - E(\gamma_n^2 + \phi^2)\} \gamma_n \]

\[ s_0 = \frac{\beta_0^2 - \phi^2}{A'}, \quad s_n = \frac{-\phi^2 + \gamma_n^2}{A'} \]

\[ h_0 = (1-E) \tanh \beta_0 + E\beta_0 + (EA' + D'A')/2\beta_0 + \]

\[ \{s_0(1-E) - D\} A'/(2\beta_0 \cosh^2 \beta_0) \]

\[ h_n = (1-E) \tan \gamma_n + E\gamma_n - (E+D)A'/2\gamma_n + \]

\[ \{(\phi^2+\gamma_n^2)(1-E) + D'A'\} / (2\gamma_n \cos^2 \gamma_n) \]

\[ \delta_n = (n-1/2) \pi \]

\[ \tau_n = -\phi^2 + \delta_n^2/A' \]
$$G_0 = h_0 \cosh \beta_0 + \{s_0[(1-E)\tanh \beta_0 + \beta_0] + D(\beta_0 - \tanh \beta_0)\} \frac{A'}{2\beta_0} \sinh \beta_0$$

$$G_n = h_n \cos \gamma_n + \{s_n[(1-E)\tan \gamma_n + \gamma_n] + D(\gamma_n - \tan \gamma_n)\} \frac{A'}{2\gamma_n} \sin \gamma_n$$

$$F_n = \{(1-E) + \frac{n}{2} A'\} (-1)^{n+1}$$

As shown in Fig. 3.1, the numerical solutions agree well with the analytical solutions for the special case considered for the model. The values of the parameters used for Fig. 3.1 are given in Table 3.1.

V. RESULTS AND DISCUSSION

The numerical simulation is first conducted to account for the general behavior of the non-catalytic gas-solid reactions in the vertical pneumatic transport reactor. Specifically, the general effects of the process variables represented by two major types of non-catalytic gas-solid reactions are considered. These two types of reactions are the solid porosity increasing with the reaction and the solid porosity decreasing with the reaction. The former is characterized by a positive value for \(\gamma\) in the model while the latter is characterized by a negative value for \(\gamma\) in the model. The model is then applied to the specific example of limestone/dolomite sulfation and the sulfation behavior of limestone/dolomite sorbents in a vertical pneumatic transport reactor with or without the spent sorbent recycle is simulated. The experimental results obtained for non-catalytic reaction of limestone with sulfur dioxide are utilized to partially verify the general mathematical model developed in this study.
Table 3.1. Nominal Values of the Parameters Employed for Figs. 3.1 through 3.11

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
</tr>
<tr>
<td>m</td>
<td>0 for Fig. 3.1, 1 for Figs. 3.2-3.11</td>
</tr>
<tr>
<td>n</td>
<td>1</td>
</tr>
<tr>
<td>\rho_p</td>
<td>2.5 g/cm³</td>
</tr>
<tr>
<td>\rho_g</td>
<td>0.001 g/cm³</td>
</tr>
<tr>
<td>D</td>
<td>7.62 cm</td>
</tr>
<tr>
<td>k_v</td>
<td>200 l/sec for Fig. 3.1, 8000 cm³/g-mole sec for Figs. 3.4-3.5, 3.7, 3.9-3.11, 500-8000 cm³/g-mole sec for Figs. 3.3, 3.8, 500-60000 cm³/g-mole sec for Figs. 3.2, 3.6</td>
</tr>
<tr>
<td>D_eA0</td>
<td>0.1 cm²/sec for Figs. 3.1-3.3, 3.8-3.10, 0.001-0.1 cm²/sec for Figs. 3.5, 3.7, 3.11, 0.0001-0.1 cm²/sec for Fig. 3.4, 0.01 cm²/sec for Fig. 3.6</td>
</tr>
<tr>
<td>W_g</td>
<td>50-500 g/sec for Figs. 3.1-3.2, 3.4, 300 g/sec for Figs. 3.3, 3.5, 3.7, 3.8, 3.11, 100 and 300 g/sec for Fig. 3.9, 50 g/sec for Figs. 3.6, 3.10</td>
</tr>
<tr>
<td>U_g</td>
<td>300-2000 cm/sec for Figs. 3.1, 3.3, 500 cm/sec for Figs. 3.2, 3.4-3.11</td>
</tr>
<tr>
<td>d</td>
<td>50 μm for Figs. 3.1-3.4, 3.7, 3.9-3.11, 20-200 μm for Figs. 3.5, 3.6, 100 μm for Fig. 3.8</td>
</tr>
<tr>
<td>C_s0</td>
<td>0.025 g-mole/cm³</td>
</tr>
<tr>
<td>C_0</td>
<td>3.25 x 10⁻⁵ g-mole/cm³</td>
</tr>
<tr>
<td>μ</td>
<td>1.8 x 10⁻⁴ g/cm sec</td>
</tr>
</tbody>
</table>
Table 3.1. contd.

| L     | 800 cm for Figs. 3.1-3.7, 3.9-3.11  
|       | 2000 cm for Fig. 3.8 |
| c_0   | 0.2 for Figs. 3.1-3.3, 3.7-3.11  
|       | 0.6 for Figs. 3.4-3.6 |
| γ     | 0.0 for Fig. 3.1  
|       | 0.8 for Figs. 3.2, 3.3, 3.6, 3.8-3.10  
|       | (-0.8) and 0.8 for Figs. 3.4, 3.5  
|       | 0.0-0.8 for Fig. 3.7  
|       | (-0.8)-0.0 for Fig. 3.11 |
Fig. 3.1 Comparison of the Analytical Solution and the Numerical Solution for the Model
V-A. Numerical Simulation of the General Non-Catalytic Gas-Solid Reaction

The results of the numerical simulation based on the general model are shown in Figs. 3.2 through 3.11. The nominal values of the parameters employed in these figures are given in Table 3.1. Note that the values of $\varepsilon_0$ and $\gamma$ given in Table 3.1 are applicable only for a limited range of solid conversion considered in this study. Extrapolation beyond which would result in values for $\varepsilon_0$ which are either negative or over one. Figures 3.2, 3.3 and 3.6 through 3.10 show the parametric effects of the superficial gas velocity, reaction rate constant, flow rate of the solid particles, length of the reactor, and variation constant of the solid porosity on the reactant conversion for cases involving positive values of $\gamma$. The parametric effects of the variation constant of the solid porosity on the reactant conversion for cases involving negative values of $\gamma$ is described in Fig. 3.11. While the parametric effects of the particle radius and diffusivity of the solid particles on the reactant conversion for cases involving both negative and positive values of $\gamma$ are described in Figs. 3.4 and 3.5.

Figure 3.2 shows the effect of the solid flow rate on the gas conversion as a function of the reaction rate constant. The gas conversion increases with the increase of the flow rate. The effect of the solid flow rate on the gas conversion is more pronounced at the high reaction rate constant. At the reaction rate constant of 8000 cm$^3$/g-mole sec, the increase in the solid flow rate from 100 g/sec to 500 g/sec, gives rise to an increase of the gas conversion from 40% to 93% while at the reaction rate constant of 500 cm$^3$/g-mole sec, the gas
Fig. 3.2 Effect of the Solid Flow Rate on the Gas Conversion as a Function of the Reaction Rate Constant
conversion increases only from 4% to 17%.

Figure 3.3 shows the effect of the superficial gas velocity on the gas conversion as a function of three reaction rate constants, i.e., 500, 2000, 8000 cm$^3$/g-mole sec. It is seen that the gas conversion decreases with the increase of the gas superficial velocity due to the decrease of the gas residence time in the reactor. It is noted that the increase in the superficial gas velocity from 300 cm/sec to 900 cm/sec reduces the gas conversion from 99% to 40% for the reaction rate constant of 8000 cm$^3$/g-mole sec.

Figure 3.4 shows the effect of the initial gas diffusivity in the solid, $D_{eA0}$, on the gas conversion as a function of the solid flow rate for both positive and negative values of $\gamma$. For positive values of $\gamma$, it is seen that the gas conversion increases as the solid flow rate increases at a given $D_{eA0}$. For a given solid flow rate, the gas conversion increases as $D_{eA0}$ increases to about 0.02 cm$^2$/sec, beyond which, the gas conversion remains practically constant. This signifies that the gas diffusion in the solid is not the rate determining step in the overall rate of reaction. Note that the Thiele modulus with $D_{eA0}$ of 0.02 cm$^2$/sec is 0.25 indicating that the homogeneous model can be used to account for the reactant conversion in the particle at $D_{eA0}$ beyond 0.02 cm$^2$/sec (Fan, et al., 1978). For negative values of $\gamma$, it is seen that the variation of the gas conversion with $D_{eA0}$ for a given solid flow rate is similar to that for positive values of $\gamma$. For $D_{eA0}$ as large as 0.1 cm$^2$/sec, the gas conversion for negative values of $\gamma$ is identical to that for positive values of $\gamma$ for a given solid flow rate. However, the differences are observed for the gas conversion between the two cases at $D_{eA0}$ as small
Fig. 3.3 Effect of the Superficial Gas Velocity on the Gas Conversion as a Function of the Reaction Rate Constant for Positive Values of $\gamma$
Fig. 3.4 Effect of the Initial Gas Diffusivity in the Solid Particle on the Gas Conversion as a Function of Solid Flow Rate
Figure 3.5 shows the effect of the particle diameter on the gas conversion as a function of three different values of the initial gas diffusivity in the solid, i.e., 0.001, 0.01 and 0.1 cm$^2$/sec for both positive and negative values of $\gamma$. For positive values of $\gamma$ it is seen that, at $D_{eA0}$ as small as 0.001 cm$^2$/sec, the gas conversion decreases as the particle diameter increases. This is due to the combined effect in that the increase in the particle diameter results in substantial decrease of overall rate of reaction and moderate increase of the hold-up and residence time of the particle. On the other hand, at $D_{eA0}$ as large as 0.1 cm$^2$/sec, the gas conversion increases as the particle diameter increases. This is due to the effect that the increase in the particle diameter results in the essentially unvaried overall rate of reaction and moderate increase of the residence time of the particle. For negative values of $\gamma$, the trend of the gas conversion variation with $d$ for a given $D_{eA0}$ is similar to that for positive values of $\gamma$. No difference of the gas conversion is observed for both negative and positive values of $\gamma$ at a small $d$.

The effect of the particle diameter on the gas conversion as a function of the reaction rate constant, $k_v$, is shown in Fig. 3.6. Three values for $k_v$ are considered, i.e., 500, 8000, and 60000 cm$^3$/g-moles sec. For the low values of $k_v$, the overall rate of the reaction is predominantly determined by the rate of the reaction. Thus, the increase in the particle diameter results in the essentially unvaried overall rate of reaction. However, as described previously, the increase in the particle diameter results in the increase of the
Fig. 3.5 Effect of the Particle Diameter on the Gas Conversion as a Function of the Initial Gas Diffusivity in the Solid Particle
Fig. 3.6 Effect of the Particle Diameter on the Gas Conversion as a Function of the Reaction Rate Constant for Positive Values of $\gamma$
residence time and the hold-up of the particles in the bed. Thus, it results in a slight increase of the gas conversion as the particle diameter increases for small values of $k_v$, i.e., 500 and 8000 cm$^3$/g-mole sec, as shown in Fig. 3.6. For the large values of $k_v$, the overall rate of reaction is predominantly determined by the rate of diffusion. Thus, the increase in the particle diameter results in the decrease of the overall rate of reaction. This effect combined with the effect of the increase of residence time and hold-up of particle in the bed yields the decrease of the gas conversion as the particle diameter increases for a given large value of $k_v$, i.e., 60000 cm$^3$/g-mole sec, as shown in Fig. 3.6.

Figure 3.7 shows the effect of the variation constant of the solid porosity, $\gamma$, on the gas conversion as the function of three values of initial gas diffusivity, i.e., 0.001, 0.01, and 0.1 cm$^2$/sec. It can be seen that for $D_{e AO}$ as small as 0.001 cm$^2$/sec, the increase in $\gamma$ substantially increases the gas conversion. This is because the overall rate of reaction is predominantly determined by the rate of diffusion. The increase in $\gamma$ increases the rate of diffusion. For $D_{e AO}$ as large as 0.1 cm$^2$/sec, however, the increase in $\gamma$ has negligible effect on the gas conversion. This is due to the fact that the overall rate of reaction is predominantly determined by the rate of reaction.

The axial profiles of the gas conversion as a function of three reaction rate constants, i.e., 500, 2000, and 8000 cm$^3$/g-mole sec are shown in Fig. 3.8. As expected, at a given location of the reactor, the increase in the reaction rate constant increases the gas conversion. The axial variation of the gas conversion
Fig. 3.7 Effect of the Positive Values of $\gamma$ on the Gas Conversion as a Function of the Initial Gas Diffusivity in the Solid Particle
Fig. 3.8 Axial Gas Conversion Profiles as a Function of Reaction Rate Constant for Positive Values of \( \gamma \)
decreases with an increase in distance. This variation is particularly profound for the reaction rate constant of 8000 cm$^3$/g-mole sec.

Figure 3.9 shows the solid and gas concentration profiles along the reactor. It is seen that both the solid and gas concentrations drop sharply along the reactor. Figure 3.10 shows the radial concentration distributions of the solid reactant and the gas reactant as a function of the axial distance. It is seen that, at a given particle radial distance, the radial variation of the solid reactant concentration remains unchanged or increases with the increase of the axial distance. It is seen in Fig. 3.10, however, that at a given particle radial distance, the radial variation of the gas reactant concentration in the solid particle remains unchanged or decreases with the increase of the axial distance.

The effect of negative values of $\gamma$ on the gas conversion as a function of three values of $D_{eA0}$, 0.001, 0.01, and 0.1 cm$^2$/sec, is shown in Fig. 3.11. In contrast to Fig. 3.7 for positive values of $\gamma$, at $D_{eA0}$ as small as 0.001 cm$^2$/sec, the gas conversion decreases as $(-\gamma)$ increases. The effect of $(-\gamma)$ on the gas conversion is substantially reduced as $D_{eA0}$ increases to 0.1 cm$^2$/sec.

V-B. Numerical Simulation of Limestone/Dolomite Sulfation

Four limestone/dolomite sorbents indexed by 5101, 8001, 9202, and 9701 which were previously characterized by Fee, et al., (1980) are used for the simulation. Note that these sorbents represent a wide range of sorbent reactivity properties reported by them. The properties which characterize these sorbents include the volumetric
Fig. 3.9 Axial Gas and Solid Concentration Profile in the Reactor for Positive Values of $\gamma$. 
Fig. 3.10 Concentration Profiles of the Gas and the Solid Reactant in the Particle at Various Locations in the Reactor for Positive Values of $\gamma$
Fig. 3.11 Effect of the Negative Values of $\gamma$ on the Gas Conversion as a Function of the Initial Gas Diffusivity in the Solid Particle
rate constant, $k_v$, the variation constant of solid porosity, $\gamma$, the solid initial effective diffusivity, $D_{eA0}$ and the initial solid porosity, $\varepsilon_0$. The values of these properties for calcined sorbents were determined in Chapter 2, and are listed again in Table 3.2. It is noted in the table that sorbent 5101 has the highest initial effective diffusivity which is followed in order by sorbents 8001, 9202 and 9701. Sorbent 8001 has the highest reaction rate constant which is followed in order by sorbents 9202, 9701 and 5101. The performance of these sorbents in the reactor, however, is described by a non-linear relationship among various parameters of the sorbent along with the transport properties of the reactor.

The results of the numerical simulation for various sorbents are shown in Figs. 3.12 through 3.18. The nominal values of the operating parameters employed in these figures are given in Table 3.3. It should be noted in the table that the sorbent density, $\rho_p$ of 2.5 g/cm$^3$ is used, which represents the density of uncalcined limestone particles. The density of the particle would reduce to 1.5 g/cm$^3$ upon complete calcination in the reactor. It would then increase with the sulfation reaction, with the maximum value of 3.4 g/cm$^3$ upon complete sulfation. The analysis conducted in this study indicates that under the dilute flow conditions, the change in the particle density has practically negligible effect on the hydrodynamic properties of the reactor. Thus, the volumetric flow rate of the solid particles practically remains constant throughout the reactor. Since $W_s$ in Eqn. (3.23) is taken as the uncalcined limestone feed rate, the corresponding density of 2.5 g/cm$^3$ would only yield an appropriate solid particle volumetric flow rate in the reactor. Parametric
Table 3.2. Properties of the Various Calcined Sorbents Used in the Model Simulation Obtained at 1123°K (Fee et al., 1980)

<table>
<thead>
<tr>
<th>Sorbent Identification Number</th>
<th>Type of Sorbent</th>
<th>( C_{80} ) g-moles/cm(^3)</th>
<th>( D_{8A0} ) cm(^2)/sec</th>
<th>( k_v ) cm(^3)/g-mole sec</th>
<th>( \varepsilon_0 )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5101</td>
<td>Dolomite</td>
<td>0.0138</td>
<td>0.402</td>
<td>3411.9</td>
<td>0.60</td>
<td>0.64</td>
</tr>
<tr>
<td>8001</td>
<td>Limestone</td>
<td>0.0221</td>
<td>0.095</td>
<td>70002.8</td>
<td>0.53</td>
<td>0.75</td>
</tr>
<tr>
<td>9202</td>
<td>Limestone</td>
<td>0.0245</td>
<td>0.0024</td>
<td>28684.1</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>9701</td>
<td>Limestone</td>
<td>0.0226</td>
<td>0.0008</td>
<td>7799.1</td>
<td>0.64</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Table 3.3. Nominal Values of the Parameters Employed for Figs. 3.12 through 3.18

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
</tr>
<tr>
<td>m</td>
<td>1</td>
</tr>
<tr>
<td>n</td>
<td>1</td>
</tr>
<tr>
<td>( \rho_p )</td>
<td>2.5 g/cm(^3)</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>0.0004 g/cm(^3)</td>
</tr>
<tr>
<td>D</td>
<td>7.62 cm</td>
</tr>
<tr>
<td>( W_s )</td>
<td>0-300 g/sec for Fig. 3.12&lt;br&gt;167 g/sec for Figs. 3.13-3.15&lt;br&gt;4.5-100 g/sec for Fig. 3.16-3.18</td>
</tr>
<tr>
<td>( U_{gs} )</td>
<td>500 cm/sec for Figs. 3.12, 3.14-3.18&lt;br&gt;100-1000 cm/sec for Fig. 3.13</td>
</tr>
<tr>
<td>d</td>
<td>50 ( \mu m ) for Figs. 3.12-3.13, 3.15-3.18&lt;br&gt;20-200 ( \mu m ) for Figs. 3.14</td>
</tr>
<tr>
<td>( C_0 )</td>
<td>3.25 \times 10^{-5} \text{ g-mole/cm}^3 \text{ for Figs. } 3.12-3.13&lt;br&gt;3.25 \times 10^{-8} \text{ g-mole/cm}^3 \text{ for Fig. } 3.14&lt;br&gt;0.1 \times 10^{-5} \text{ g-mole/cm}^3 \text{ for Figs. } 3.16-3.18&lt;br&gt;3 \times 10^{-8} - 10^{-4} \text{ g-mole/cm}^3 \text{ for Fig. } 3.15</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1.8 \times 10^{-4} \text{ g/cm sec}</td>
</tr>
<tr>
<td>L</td>
<td>800 cm</td>
</tr>
</tbody>
</table>
effects of the sorbent flow rate, superficial gas velocity, sorbent diameter, and the initial sulfur dioxide concentration on the sulfur dioxide conversion and the sorbent utilization without the spent sorbent recycle are shown in Figs. 3.12 through 3.15. In Figs. 3.16 through 3.18, effects of the recycle rate of spent sorbents and calcium-sulfur molar ratio (Ca/S) on the reactor performance are described.

The effect of the solid flow rate on the sorbent utilization and the SO$_2$ conversion for these sorbents is shown in Fig. 3.12. For sorbents 8001 and 9202, the complete conversion of the gas phase is achieved at the sorbent flow rate 170 g/sec and 220 g/sec, respectively, while for sorbents 9701 and 5101 the gas conversion monotonically increases with the sorbent flow rate. The sorbent utilization monotonically decreases with the sorbent flow rate for all the four sorbents. For a given solid flow rate, sorbent 8001 yields the highest SO$_2$ conversion and the sorbent utilization and is followed in order by sorbents 9202, 9701 and 5101. It is noted that sorbent 8001 has the highest volumetric reaction rate constant, $k_v$, and is followed in order by sorbents 9202, 9701 and 5101.

Figure 3.13 shows the effect of the superficial gas velocity on the sorbent utilization and the SO$_2$ conversion for these sorbents. For sorbents 8001 and 9202 the complete SO$_2$ conversion is reached at the gas velocity of 400 cm/sec or below and it decreases slightly as the gas velocity increases up to 1000 cm/sec. For sorbents 9701 and 5101, the SO$_2$ conversion dramatically decreases with the increase of the gas velocity from 100 cm/sec to 1000 cm/sec. At a given gas velocity, the SO$_2$ conversion decreases in the same sorbent order as
Fig. 3.12 Effects of the Sorbent Flow Rate on the SO\textsubscript{2} Conversion and the Sorbent Utilization for the Four Sorbents
Fig. 3.13  Effects of the Superficial Gas Velocity on SO₂ Conversion and Sorbent Utilization for the Four Sorbents.
does the reaction rate constant, $k_r$. The variation of sorbent utilization for sorbents 9701, 9202 and 5101 with the superficial gas velocity shows a maximum in the velocity range of 200-800 cm/sec. High gas superficial velocity gives rise to a relatively high bulk concentration of sulfur dioxide in the reactor which promotes the overall rate of reaction. However, a high gas superficial velocity also gives rise to a relatively low sorbent hold-up which retards the overall rate of reaction. The consequence of these two counteracting effects on the sorbent utilization yields a maximum in the relationship between sorbent utilization and superficial gas velocity.

Figure 3.14 shows the effect of the sorbent diameter on the SO$_2$ conversion for the four sorbents. The SO$_2$ conversion for sorbents 8001, 9202 and 9701 decreases with the increase in the sorbent diameter while it increases with the increase in the sorbent diameter for sorbent 5101. This is due to the combined effect in that for a small value of $D_{eA0}$, the increase in the sorbent diameter results in substantial decrease of the overall rate of reaction and moderate increase of the residence time of the sorbent and hence, the decrease of the SO$_2$ conversion. On the other hand, for a large value of $D_{eA0}$, the increase in the sorbent diameter results in the essentially unvaried overall rate of reaction and the moderate increase of the residence time of the sorbent and hence, the increase of the SO$_2$ conversion. It is noted that sorbents 9701 and 9202 have relatively low initial gas diffusivities while sorbent 5101 has a high initial gas diffusivity.

The effect of the inlet SO$_2$ concentration on the SO$_2$ conversion is described in Fig. 3.15. The increase of the inlet
Fig. 3.14 Effects of the Sorbent Diameter on SO$_2$ Conversion for the Four Sorbents
Fig. 3.15 Effects of the Inlet SO₂ Concentration on SO₂ Conversion

PARAMETER: type of sorbent
SO₂ concentration from $3 \times 10^{-8}$ g-moles/cm$^3$ to $10^{-4}$ g-moles/cm$^3$, decreases the SO₂ conversion for sorbents 9202 and 8001 by a considerable amount, while for sorbents 5101 and 9701, the decrease in the SO₂ conversion is not substantial.

In industrial operation, only part of the spent sorbent is discharged from the reactor with the remaining spent sorbent is recycled back to the reactor (Nack, et al., 1977). The amount of the sorbent discharged is then made up by the fresh sorbent. Calcium-sulfur molar ratio (Ca/S) is defined as the ratio of moles of Ca in fresh feed to the moles of S in flue gas entering the reactor.

Effect of different operational variables on the gas conversion in an isothermal pneumatic transport reactor with spent sorbent recycle is similar to that without spent sorbent recycle exhibited in Figs. 3.12 through 3.15. To account for the inlet sorbent concentration in the reactor with the spent sorbent recycle, it is assumed that the concentration of the solid reactant in the particle at the reactor inlet is uniform and can be evaluated from a simple material balance which considers the reactant concentration in the fresh sorbent and the recycle spent sorbent. Note that considerable non-uniformity of the solid reactant concentration and porosity in the particle would develop under the high extent of the solid conversion in the reactor and thus, the assumption employed here is valid only for the low extent of the solid conversion in the reactor. Figures 3.16 through 3.18 show the effect of Ca/S on the gas conversion and the sorbent utilization for the four sorbents in the pneumatic transport reactor operated with spent sorbent recycle. The recirculation ratio, defined as the ratio of the amount of the recycle spent limestone sorbent to
the amount of the make-up fresh feed, of 0, 5, and 10 are used for Figs. 3.16 through 3.18, respectively.

It is noted that the increase of the recirculation ratio at the same Ca/S increases the total Ca flow rate to the reactor. The SO$_2$ conversion and the sorbent utilization increase with the increase in Ca/S for the same recirculation ratio as exhibited in Figs. 3.16 through 3.18. The increase in recirculation ratio at the same Ca/S molar ratio, increases the solid hold-up in the reactor and thus, decreases the gas residence time in the reactor. The combined effect of these two phenomena results in the increase of the gas conversion and the sorbent utilization up to the recirculation ratio of ten as shown in Figs. 3.16 through 3.18.

VI. MODEL VERIFICATION

The present model is used to analyze the experimental data obtained from a bench scale unit and a pilot scale unit, on limestone sulfation in the vertical pneumatic transport reactor, or specifically, Battelle’s Multisoloid Fluidized Bed Combustor. The bench scale reactor is 16.2 cm ID and 610 cm in length and the pilot scale reactor is 25.4 cm ID and 838.2 cm in length.

In Battelle’s Multisoloid Fluidized Bed Combustion process, the reactor consists of two sections, namely, the dense section and the dilute section. In the dense section, coal and limestone are introduced pneumatically and combustion of coal takes place. The dense section contains crushed iron ore particles of 6 x 12 mesh size. These particles serve to retain and distribute coal particles creating adequate mixing for mass transfer and reactions. Sulfur dioxide is
Fig. 3.16 Effects of Ca/S on SO$_2$ Conversion and Sorbent Utilization with No Recycle of Calcined Sorbents
Fig. 3.17 Effects of Ca/S on SO₂ Conversion and Sorbent Utilization for Recirculation Ratio of Five
Fig. 3.18 Effects of Ca/S on SO₂ Conversion and Sorbent Utilization for Recirculation Ratio of Ten
generated during the combustion process, which subsequently reacts
with calcined limestone entrained through the dense section to the
dilute section of the reactor.

Heat generated from the combustion reaction is absorbed by sand
particles which are circulated out of the reactor along with the spent
sorbent and ash. The sand particles and part of the spent sorbents
are then separated from the fine particles. These particles are
collected in the hot recycle chamber where part of the particles are
recycled back to the reactor while others are transported to the
external heat exchanger unit prior to recycling back to the reactor.
Figures 3.19 and 3.20 show the schematic diagrams of the bench scale
and pilot scale units respectively. Tables 3.4 and 3.5 detail the
dimensions of the two systems and the operating conditions
employed in the experiments. The values of the parameters used
in the simulation are listed in Table 3.6.

The dense section of the reactor is substantially smaller than
the dilute section. However, the dense section provides higher
residence time for the fine sorbent than does the dilute section. The
residence time of the fine sorbent in the dense section, $t_r$, for the
operating condition considered in this study was experimentally
determined as 8 secs, as given in Table 3.4.

The volume fraction of the sorbent in the dense bed can be
calculated by

$$
\varepsilon_{sl} = \frac{F_1 \cdot t_r}{V_1}
$$

(3.23)

where $F_1$ and $V_1$ are the volumetric feed rate of the sorbent and the
Fig. 3.19 Schematic Diagram of the Bench Scale Reactor with Inside Diameter of 16.2 cm (Fan et al., 1983a).
Fig. 3.20 Schematic Diagram of the Pilot Scale Reactor with Inside Diameter of 25.4 cm (Fan et al., 1983b)
Table 3.4. Specification and Operating Conditions of Multisolid Fluidized Bed Combustion Bench Scale Unit

<table>
<thead>
<tr>
<th>Specification and Operating Conditions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustor Internal Diameter</td>
<td>16.2 cm</td>
</tr>
<tr>
<td>Combustor Height</td>
<td>610 cm</td>
</tr>
<tr>
<td>Height of Dense Bed (settled)</td>
<td>50.8 cm</td>
</tr>
<tr>
<td>Height of Dense Bed (expanded)</td>
<td>101.6 cm</td>
</tr>
<tr>
<td>Coal Feed Rate</td>
<td>6.3 g/sec</td>
</tr>
<tr>
<td>Sulfur Percentage in Coal</td>
<td>4%</td>
</tr>
<tr>
<td>Sand Circulation Rate</td>
<td>63.1 g/sec</td>
</tr>
<tr>
<td>Total Solids Circulation Rate</td>
<td>75.8-85.4 g/sec</td>
</tr>
<tr>
<td>CaCO Percentage in Limestone</td>
<td>83.6%</td>
</tr>
<tr>
<td>Limestone Sorbent Circulation Rate</td>
<td>9.47 g/sec for Ca/S = 2</td>
</tr>
<tr>
<td></td>
<td>13.89 g/sec for Ca/S = 3</td>
</tr>
<tr>
<td></td>
<td>15.78 g/sec for Ca/S = 4</td>
</tr>
<tr>
<td></td>
<td>19.73 g/sec for Ca/S = 5</td>
</tr>
<tr>
<td>Calcium-Sulfur Molar Ratio</td>
<td>2, 3, 4, and 5</td>
</tr>
<tr>
<td>Material of Dense Bed</td>
<td>Iron ore (crushed)</td>
</tr>
<tr>
<td>Average Bed Temperature</td>
<td>868°C</td>
</tr>
<tr>
<td>Average Diameter of Limestone Particles</td>
<td>16 μm</td>
</tr>
<tr>
<td>Residence Time of Limestone Particles in Dense Bed</td>
<td>~6 sec</td>
</tr>
<tr>
<td>Type of Limestone</td>
<td>Greer Limestone</td>
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Table 3.5. Specification and Operating Conditions of Multisolid Fluidized Bed Combustion Pilot Scale Unit

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combustor Internal Diameter</td>
<td>25.4 cm</td>
</tr>
<tr>
<td>Combustor Height</td>
<td>838.2 cm</td>
</tr>
<tr>
<td>Height of Dense Bed (settled)</td>
<td>30.5 cm</td>
</tr>
<tr>
<td>Height of Dense Bed (expanded)</td>
<td>152.4 cm</td>
</tr>
<tr>
<td>Coal Feed Rate</td>
<td>11.4 g/sec</td>
</tr>
<tr>
<td>Sulfur Percentage in Coal</td>
<td>4%</td>
</tr>
<tr>
<td>Sand Circulation Rate</td>
<td>433.3 g/sec</td>
</tr>
<tr>
<td>Total Solids Circulation Rate</td>
<td>502-555 g/sec</td>
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<tr>
<td>CaCO Percentage in Limestone</td>
<td>94.0%</td>
</tr>
<tr>
<td>Limestone Sorbent Circulation Rate</td>
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<tr>
<td></td>
<td>63 g/sec for Ca/S = 1.95</td>
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<tr>
<td></td>
<td>69 g/sec for Ca/S = 2.14</td>
</tr>
<tr>
<td></td>
<td>83 g/sec for Ca/S = 2.55</td>
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<tr>
<td></td>
<td>93 g/sec for Ca/S = 2.86</td>
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<tr>
<td></td>
<td>104 g/sec for Ca/S = 3.2</td>
</tr>
<tr>
<td></td>
<td>114 g/sec for Ca/S = 3.5</td>
</tr>
<tr>
<td>Calcium-Sulfur Molar Ratio</td>
<td>1.95-3.5</td>
</tr>
<tr>
<td>Material of Dense Bed</td>
<td>Iron ore (crushed)</td>
</tr>
<tr>
<td>Average Bed Temperature</td>
<td>843°C</td>
</tr>
<tr>
<td>Average Diameter of Limestone Particles</td>
<td>250 μm-350 μm</td>
</tr>
<tr>
<td>Residence Time of Limestone Particles</td>
<td>~8 sec</td>
</tr>
</tbody>
</table>
### Table 3.6. Numerical Values of the Parameters Employed for Figs. 3.21 through 3.24

| Parameter | Value
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
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</tr>
<tr>
<td>$m$</td>
<td>1</td>
</tr>
<tr>
<td>$n$</td>
<td>1</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>2.5 g/cm$^3$</td>
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<tr>
<td>$\rho_g$</td>
<td>0.0004 g/cm$^3$</td>
</tr>
<tr>
<td>$D$</td>
<td>16.2 cm for Figs. 3.21, 3.22, 25.4 cm for Figs. 3.23, 3.24</td>
</tr>
<tr>
<td>$k_v$</td>
<td>$0.65 \times 10^{-5}$ cm$^3$/g-mole sec for Figs. 3.21, 3.22, $0.295 \times 10^{-5}$ cm$^3$/g-mole sec for Figs. 3.23, 3.24</td>
</tr>
<tr>
<td>$D_{eA0}$</td>
<td>$0.095 \text{ cm}^2$/sec for Figs. 3.21, 3.22, 0.1 cm$^2$/sec for Figs. 3.23, 3.24</td>
</tr>
<tr>
<td>$U_{gs}$</td>
<td>850-1060 cm/sec for Fig. 3.21, 385-1060 cm/sec for Fig. 3.22, 609.6 cm/sec for Fig. 3.23, 797.2 cm/sec for Fig. 3.24</td>
</tr>
<tr>
<td>$d$</td>
<td>16 μm for Figs. 3.21, 3.22, 350 μm for Fig. 3.23, 250 μm for Fig. 3.24</td>
</tr>
<tr>
<td>$C_{s0}$</td>
<td>0.021 g/mole/cm$^3$</td>
</tr>
<tr>
<td>$C_0$</td>
<td>$3.63 \times 10^{-8}$-4.5 $\times 10^{-8}$ g/mole/cm$^3$ for Fig. 3.21, $3.3 \times 10^{-8}$-12.8 $\times 10^{-8}$ g/mole/cm$^3$ for Fig. 3.22, $4.6 \times 10^{-6}$ g/mole/cm$^3$ for Fig. 3.23, $3.5 \times 10^{-6}$ g/mole/cm$^3$ for Fig. 3.24</td>
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<tr>
<td>$\mu$</td>
<td>$4.5 \times 10^{-4}$ g/cm sec</td>
</tr>
<tr>
<td>$L$</td>
<td>610 cm for Figs. 3.21, 3.22, 838.2 cm for Figs. 3.23, 3.24</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.532 for Figs. 3.21, 3.22, 0.6 for Figs. 3.23, 3.24</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.75 for Figs. 3.21, 3.22, -0.70 for Figs. 3.23, 3.24</td>
</tr>
</tbody>
</table>
volume of the dense section, respectively. The particle linear velocity in the dense section, $U_p1$ can be calculated by

$$U_p1 = \frac{U_p}{\varepsilon_s1} \tag{3.24}$$

where $U_p$ is the superficial velocity of the sorbent.

In the dilute section, it is assumed that the entrained particles including limestone, coal ash, coal carbon, and inert material (sands) act independently. The hydrodynamic and mass transfer properties of each type of particle, in the dilute section, can be evaluated by Eqns. (3.23) through (3.30). The volume fraction of the fine sorbent in the dilute section, $\varepsilon_{s2}$, can be calculated by the hydrodynamic properties and the solid flow rate of all types of particles. Equations (3.1) through (3.7) are solved for both the dense and dilute sections. The term $(1 - \varepsilon_b)$ in Eqn. (3.1) and $U_p$ in Eqns. (3.3) and (3.4) are replaced by $\varepsilon_{s1}$ and $U_{p1}$ for the dense section and by $\varepsilon_{s2}$ and $U_{p2}$ for the dilute section. Note that the following relationship holds

$$U_{g1} \varepsilon_{b1} = U_{g2} \varepsilon_{b2} = U_{gs} \tag{3.25}$$

Most of the parameters listed in Table 3.6 are operating variables or properties of the sorbents. Operating variables include $D$, $U_{gs}$, $\rho_g$, $d$, $C_0$, $\mu$, and $L$ and the properties of the sorbent include $\rho_p$, $D_{s0}$, $C_{s0}$, and $\varepsilon_0$. The type of reactant used for the bench scale experiments is Greer Limestone which was identified as Sorbent 8001 by Fee et al. (1980). The values of the parameters
and $\gamma$ listed in Table 3.6 for bench scale data simulation are taken
directly from Table 2.2. The type of sorbent used for the pilot scale
experiments is not known and $\gamma$ and $k_v$ values are obtained by fitting
the model with one set of experimental data. The experimental
conditions for this selected set of data are superficial gas velocity
= 609.6 cm/sec, calcium-sulfur molar ratio (Ca/S) = 1.95, sulfur
retention = 84.9%. The same values of $\gamma$ and $k_v$ are then used in the
model to simulate the sulfur retention for remaining data. Note that
the values $\gamma$ (0.7) and $k_v$ (29500 cm³/g-mole sec) obtained in this
manner for the sorbent used in this study are in the same range as
that obtained for similar limestone sorbents under similar reaction
conditions.

The experimental data and model verification for the bench scale
unit are presented in Figs. 3.21 and 3.22. Figure 3.21 shows the
sulfur retention versus the superficial gas velocity as a function of
Ca/S molar ratio for sulfation in the vertical pneumatic transport
reactor. Note that variation of Ca/S molar ratio reflects that of Ca
circulation rate. It is seen that the comparison between the model
prediction and experimental data is satisfactory. Model prediction
with some additional experimental data is compared in Fig. 3.22. The
experimental conditions for Fig. 3.22 are the same as for Fig. 3.21
except that wider ranges of gas velocity (385-1060 cm/sec) and the
inlet gas reactant concentration ($3.3 \times 10^{-8} - 12.8 \times 10^{8}$ g-mole/cm³)
are considered. It is seen that all the predicted values lie within
\(\pm 15\%\) of the experimental data.

The comparisons between the experimental data and model
prediction for sulfur retention as a function of calcium-sulfur molar
Fig. 3.21 Comparison of the Model Prediction with Experimental Data in a Plot of X vs. \( U_{gs} \) for the Bench Unit
Fig. 3.22 Comparison of the Model Prediction with Additional Experimental Data in a Plot of $X_{\text{predicted}}$ vs. $X_{\text{experimental}}$ for the Bench Unit
ratio for pilot scale unit are presented in Figs. 3.23 and 3.24. As expected, it is seen that the sulfur retention increases with the calcium-sulfur molar ratio. The comparison is shown to be satisfactory.

VII. CONCLUDING REMARKS

A heterogeneous model for non-catalytic gas-solid reaction in the vertical pneumatic transport reactor is developed. Numerical simulation is made of the effects of various operating parameters on the conversion of the gas and solid reactants. The results indicate that the gas reactant conversion increases with the increase of the solid flow rate, the decrease of the gas superficial velocity and the increase of the reaction rate constant. Increasing the particle diameter would either increase or decrease the gas reactant conversion depending upon the rate of diffusion in the particle.

The limestone/dolomite sorbent utilization in a vertical pneumatic transport reactor as a scrubber for flue gas desulfurization is extensively analyzed by the heterogeneous model. Effects of various operating parameters on the sulfur retention and sorbent utilization is simulated for four representatative sorbents previously characterized by Fee et al. (1980) and evaluated in Chapter 2. The results indicate that effects of the type of sorbents on the sorbent utilization and the \( \text{SO}_2 \) conversion are significant. Furthermore, the sensitivity of different parameters including the sorbent flow rate, superficial gas velocity and particle diameter on the \( \text{SO}_2 \) conversion is affected by the properties of the sorbents. The sorbent utilization can be maximized by operating the reactor at the minimum
Fig. 3.23 Comparison of the Model Prediction with the Experimental Data Obtained from the Pilot Unit
Fig. 3.24 Comparison of the Model Prediction with the Experimental Data Obtained from the Pilot Unit
sorbent flow rate at which the desired $SO_2$ conversion is achieved. Increase in the spent sorbent recirculation ratio at a given Ca/S also results in an increase of the $SO_2$ conversion and sorbent utilization for the range of recirculation ratios considered in this study.

Experimental data obtained from a bench scale unit and a pilot scale unit in which limestone is reacted with $SO_2$ generated from coal combustion, is compared with model predictions. Model predictions are shown to match the experimental data at various operating conditions satisfactorily.
CHAPTER 4

HYDRODYNAMIC CHARACTERISTICS OF A MULTISOLID PNEUMATIC TRANSPORT BED

ABSTRACT

An apparatus made of plexiglas with 0.102 m I.D. and 6.46 m in height is utilized to study the fundamental characteristics of the multisolid pneumatic transport bed (MPTB). The dense bed section of the apparatus is situated at 1.92 m above the fines recycle joint to allow fines to accelerate prior to contact with the dense particles. Fundamental characteristics including the hold-up of fine particles in the dense bed, the pressure drop in the packed bed of dense particles, the minimum fluidization velocity of the dense bed, and the terminal velocity of dense particles are extensively investigated.

The hold-up of fine particles in the packed bed of dense particles in the multisolid pneumatic transport bed is experimentally examined. Three types of dense particles and three types of fine particles are used in the experiments. The hold-up of fine particles in the packed bed of dense particles is found to be higher by a factor as large as six compared to that observed in the system without dense particles. An interaction coefficient between fine particles and dense particles is defined based on the momentum balance equation of
fine particles in the packed bed. This interaction coefficient which is empirically correlated is utilized to account for the hold-up of fine particles in the packed bed.

Experiments are also performed to study the pressure drop behavior in the packed bed of dense particles in the MPTB. The packed bed under extensive analysis is established by fluidizing and then defluidizing the dense particle bed by increasing and then decreasing the air flow rate at a constant fine particle flow rate. The pressure drop in a packed bed established in this manner initially decreases significantly and then increases as the fine particle flow rate increases at a given air flow rate. This behavior signifies the apparent drag reduction phenomenon in the packed bed. The minimum fluidization velocity of the dense bed with air-fine particle as the fluidizing medium defines the lower operating limit of the gas velocity in the MPTB. The experimentally measured pressure drop in the packed dense bed and in the fluidized dense bed are utilized to determine the minimum fluidization velocity at different fine particle flow rates. The minimum fluidization velocity of the dense bed is found to decrease initially and then to increase with the increase of the fine particle flow rate. A semi-empirical correlation is developed to account for the minimum fluidization velocity.

The terminal velocity of dense particles in an air-fine particle heterogeneous medium defines the upper operating limit of the gas velocity in the multisolid pneumatic transport bed reactor. The terminal velocity of dense particles in an air-fine particle medium is experimentally determined from the turbulent fluidized bed voidage versus air velocity relationship in this study. It is found that the
terminal velocity of dense particles decreases with the increase of the fine particle flow rate. The fine particle and dense particle interaction coefficient is defined based on the momentum balance equation for a dense particle. The interaction coefficient is empirically correlated and utilized to account for the terminal velocity of dense particles in the air-fine particle heterogeneous medium.

The hold-up of fine particles in the fluidized dense bed of the multisolid pneumatic transport bed is also experimentally determined and analyzed based on the momentum balance equation for fine particles. The hold-up of fine particles is found to be as high as ten times the hold-up of fine particles without dense bed. An interaction coefficient between the fine particle and dense particle is defined and is empirically correlated. This interaction coefficient is utilized to account for the hold-up of fine particle in the fluidized dense bed.

I. INTRODUCTION

A multisolid pneumatic transport bed (MPTB) is a fluidized bed in which a circulating bed is superimposed on a dense-phase turbulent or slugging bed or simply the dense bed. The dense bed is made up of large dense particles, with a size range from 1 mm to 15 mm, whereas the fine particles which constitute the circulating bed, range in size from 50 μm to 500 μm. The MPTB is operated under the conditions that the gas velocity through the bed is greater than both the terminal velocity of the circulating particles and the minimum fluidization velocity of the dense particles, but is less than the terminal
velocity of the dense particles.

The MPTB by virtue of its unique ability to control the residence time of circulating particles, heat transfer rates and mass transfer rates, has a variety of process applications. The system has been demonstrated successfully in pilot-scale operations for the combustion and gasification of coal, petroleum coke, paper mill wastes, wood, and municipal solid waste combined with sewage sludge by Battelle Columbus Laboratory (Nack et al., 1977). While a commercial process using the MPTB for combustion of petroleum coke has also been in operation (Johnson, 1982), little is known regarding the fundamental characteristics of the MPTB (Liu et al., 1979).

Utilizing the Battelle cold model unit, Liu et al. (1979) investigated the heat transfer properties, residence time of fine particles and flow regimes in the dense bed of the MPTB. A systematic investigation on the fundamental characteristics of the dense particles and the circulating particles in the bed is conducted in this study utilizing a bench scale unit of the MPTB. Hold-up of fine particles in the packed dense bed, the pressure drop in the packed bed of dense particles, the minimum fluidization velocity of the dense bed with air only and air fine particle flow, the terminal velocity of dense particles in the presence of fine particles, and the hold-up of fine particles in the fluidized dense bed are investigated and analyzed in this study.

The analysis of the hold-up of fine particles in the packed dense bed is essential to the understanding of the pressure drop and minimum fluidization of the dense bed in the multisolid pneumatic transport bed (MPTB). This study is intended to examine the effects of the
physical properties of the fine particles and the dense particles, and the flow rates of both air and fine particles on the hold-up of fine particles in the packed bed. The hold-up of fine particles in the packed bed is experimentally measured and semi-empirically analyzed. A quantitative analysis is conducted to account for the interaction force between the fine particles and dense particles in the packed bed.

Also in this investigation, an extensive analysis of pressure drop in the packed bed of dense particles is conducted. The analysis is essential to the understanding of the minimum fluidization behavior of the dense bed. The effects of the properties of the fine particles, the air flow rate and the fine particle flow rate on the pressure drop in the packed bed are studied. A mathematical model is developed to account for the pressure drop in the packed bed for air and air-fine particle flows.

The operating range of the gas velocity in the multisolid pneumatic transport bed (MPTB) is limited by the terminal velocity and the minimum fluidization velocity of dense particles in an air-fine particle medium. The MPTB is normally operated under the condition in which the gas velocity through the bed is greater than the minimum fluidization velocity of dense particles but is less than the terminal velocity of dense particles for a given fine particle flow rate. Thus to define the operational range of the MPTB, it is required to know the minimum fluidization velocity and the terminal velocity of the dense particles in an air-fine particle medium. The minimum fluidization velocity of the dense bed with air only and air-fine particles as fluidizing media is explored in this study. Effects of
the dense and circulating particle properties and air and circulating particle flow rates on the minimum fluidization velocity of the dense bed are presented. A mathematical model is developed to account for these effects.

Also, an extensive analysis of dense particles terminal velocity in air-fine particle medium is presented. The terminal velocity of a dense particle in a heterogeneous air-fine particle medium is affected by fine particle and dense particle interaction force, among other factors. Theoretical expressions have been derived for particle and particle interaction force for vertical pneumatic conveying of binary particle mixtures by Nakamura & Capes (1976) and for fine particle flow across a coarse particle by Soo (1967). In all these studies, however, analysis of the particle and particle interaction coefficient was not made. The dense particle terminal velocity experimentally obtained in this study is utilized to evaluate the fine particle and dense particle interaction coefficient. A generalized expression for the interaction coefficient is also developed.

The analysis of hold-up of fine particles in the fluidized dense bed is essential to the understanding of residence time and the overall rate of reaction of fine particles. In this study the effects of the physical properties of fine particles and dense particles, and the flow rates of both air and fine particle on the hold-up of fine particles in the fluidized dense bed are examined. The hold-up of fine particles is experimentally measured at different flow conditions and is semi-empirically analyzed. A mathematical model is developed to account for the interaction coefficient of fine particles and dense particles.
II. EXPERIMENTAL

The schematic diagram of the experimental unit is shown in Fig. 4.1. Air at high pressure is sent through an oil filter, a humidifier and an orifice meter before it enters the bottom of the MPTB. The oil filter traps any oil mist contained in the air. The humidifier is a cyclone type construction where air enters tangentially and leaves at the center of the top surface. It is heated electrically by an external jacket heater. The humidity of air exiting from the humidifier depends upon the inlet humidity, air flow rate, water level, and water temperature. The relationship between these variables and humidity of air is established for the humidifier. The humidifier controls the humidity of inlet air to the MPTB to around 75% in order to minimize the electrostatic effects in the MPTB (Klinzing, 1981). There are several pressure taps located along the entire length of the MPTB. The inlet air flow rate is measured by the orifice meter with an orifice diameter of 3.81 cm I.D. The calibration of the orifice meter is presented in Appendix A. There are two thermometers placed at the inlet and outlet of the MPTB to measure the air temperature.

The MPTB is 0.102 m I.D. and 6.46 m in height and is made of plexi-glas. The circulating (fine) particles enter near the bottom of MPTB through a 45° inclined pipe. The feed rate of fine particles to the MPTB is controlled by a gate valve in the feed pipe. The dense section of the bed is located at about 1.92 m above the fines recycle joint. Fine particles are accelerated in this length before entering the dense section.

The height of the dense section when settled varies from 0.152 m
Fig. 4.1 Schematic Diagram of the Experimental Unit of the Multisolid Pneumatic Transport Bed
to 0.610 m. An adequate freeboard area is provided to allow disengagement of the dense particles. Fine particles exiting from the bed are separated by two cyclones. These fine particles then enter the stand pipe through a sampling box used to measure the circulating rate of fine particles in the MPTB. A flexible feed tube in the sampling box allows a quick switch of fines flow to the stand pipe to that to the sampling bottle. By measuring the weight of the fines collected in the sampling bottle and sampling time, the fine particle circulation rate in the MPTB can be obtained within an accuracy of ±0.05 g/sec. The stand pipe is of 3.81 cm I.D. and 4.25 m in height.

There are four shutters located along the column at the heights of 2.17 m, 2.34 m, 4.72 m, and 5.5 m from the fines recycle joint. Details of the shutter design are shown in Fig. 4.2. The shutter contains a galvanized iron sheet with an opening drilled on one side of the sheet. The opening is equivalent to the column diameter. On one end, the sheet is fastened to a pull spring and is held on the other end by a solenoid such that the opening on the sheet and the column align. All the four shutters and the solenoid valve in the air inlet line act simultaneously. When sections of the column were isolated by the shutters, the hold-up of fine particles in the top section (Section III) was first measured by opening the top shutter and removing the isolated fine particles by vacuum. These particles were then weighed. The hold-up of fine particles in Section II and Section I was proceeded in order in a similar manner. Section I is of height 0.17 m covering from 2.17 m above the fine particle recycle joint. Section II is of height 1.38 m located immediately above Section I. Section III is of height 0.78 m located immediately above
Fig. 4.2 Details of the Shutter Design
Section II. The bottom shutter is located immediately below the retaining grid which supports the dense bed. The operating manual for the multisolid pneumatic transport bed with shutters is presented in Appendix B.

Nine different types of dense particles made of glass, aluminum, and iron oxide were used in the experiments. The properties of the dense particles are listed in Table 4.1. Figure 4.3 shows the size distribution of aluminum and iron oxide particles. Glass particles are of narrow size distribution. Four types of circulating particles were used including fine sand of average diameter 155 μm, coarse sand with an average diameter of 245 μm, glass beads with an average diameter 105 μm, and FCC particles with an average diameter of 65 μm. The size distributions of fine sand, coarse sand and FCC particles are given in Fig. 4.4. The glass beads are of narrow size distribution with a size range from 100 μm to 110 μm. The properties of the fine particles are listed in Table 4.2. Note that the sphericity factor, φ for the dense particles was calculated from the following empirical relationship (Wen and Yu, 1966):

\[
\phi = \frac{1}{14} \varepsilon_{mf}^3
\]

(4.1)

where \(\varepsilon_{mf}\) is the minimum fluidization voidage.

The sphericity factor for fine particles was obtained from the relationship of the pressure drop versus the air velocity described by Ergun equation (Ergun, 1952) for air flow in the packed bed. The separate experiments were performed to obtain this relationship. The results obtained were found to be in reasonable agreement with those
<table>
<thead>
<tr>
<th>Type of Particle</th>
<th>Average diameter of particle (mm)</th>
<th>Density of Particles (g/cm³)</th>
<th>Sphericity factor (⁻)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass Beads</td>
<td>1.00</td>
<td>2.767</td>
<td>1.000</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.32</td>
<td>3.537</td>
<td>0.838</td>
</tr>
<tr>
<td>Glass Beads</td>
<td>3.00</td>
<td>2.520</td>
<td>1.000</td>
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<tr>
<td>Glass Beads</td>
<td>4.00</td>
<td>2.470</td>
<td>1.000</td>
</tr>
<tr>
<td>Aluminum</td>
<td>5.50</td>
<td>3.537</td>
<td>0.990</td>
</tr>
<tr>
<td>Glass Beads</td>
<td>6.00</td>
<td>2.225</td>
<td>1.000</td>
</tr>
<tr>
<td>Aluminum</td>
<td>6.96</td>
<td>3.537</td>
<td>0.920</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>11.08</td>
<td>4.140</td>
<td>0.632</td>
</tr>
<tr>
<td>Iron Oxide</td>
<td>12.67</td>
<td>4.140</td>
<td>0.500</td>
</tr>
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</table>
Table 4.2. Characteristics of Fine Particles

<table>
<thead>
<tr>
<th>Type of Particle</th>
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<th>Density of Particles (g/cm³)</th>
<th>Sphericity factor (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCC Particles</td>
<td>65</td>
<td>2.605</td>
<td>0.32</td>
</tr>
<tr>
<td>Glass Beads</td>
<td>105</td>
<td>2.500</td>
<td>1.00</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>155</td>
<td>2.446</td>
<td>0.66</td>
</tr>
<tr>
<td>Coarse Sand</td>
<td>245</td>
<td>2.446</td>
<td>0.72</td>
</tr>
</tbody>
</table>
Fig. 4.3 Cumulative Size Distribution of Various Dense Particles
Cumulative Size Distribution of Various Fine Particles

Fig. 4.4 Cumulative Size Distribution of Various Fine Particles
estimated by Eqn. (4.1). Examination of the sphericity of fine particles was also performed under the microscope to further verify the sphericity values obtained in this study.

The hold-up of fine particles was measured at different flow rates of air and fine particles in all the three sections of the bed isolated with the help of the shutters. The hold-up was first measured without any dense particles and without any retaining grid in the bed. The retaining grid was then placed at the normal location of the dense bed and the hold-up of fine particle was measured. Extensive measurements of hold-up of fine particles were made with dense bed located above the grid. Three types of dense particles were used in the experiments including Al particles of 5.50 mm and 6.96 mm in diameter and iron oxide particles of 11.08 mm in diameter. Three types of circulating particles were used including fine sand of an average diameter 155 μm, FCC particles of an average diameter of 65 μm, and glass beads of an average diameter of 105 μm.

The pressure drop in the packed bed was measured for air flow only and air-fine particle flow through the packed bed at different flow rates of air and fine particles. It should be noted that the pressure drop data reported in this study, unless otherwise noted, was obtained from a packed bed which was established by fluidizing and then defluidizing the dense particles by increasing and then decreasing the air flow rate while maintaining a constant fine particle flow rate. The pressure drop across the retaining grid was experimentally measured (shown in Appendix C) and is subtracted from experimentally measured pressure drop in the packed bed. Five types of dense particles were used including Al particles of 2.32 mm, 5.50
and 6.96 mm in diameter and iron oxide particles of 11.08 mm and 12.67 mm in diameter. All the four type of fine particles described in Table 4.2 were used for pressure drop measurements.

The minimum fluidization velocity of the dense bed was determined from the pressure drop versus superficial gas velocity relationship for air flow and air-fine particle flow through the dense bed. Minimum fluidization velocity of types of dense particles beds were measured including beds of Al particles of 2.32 mm, 5.50 mm, and 6.96 mm in diameter and iron oxide particles of 11.08 mm in diameter. All the four types of fine particles described in Table 4.2 were used for the experiments.

For dense particle terminal velocity measurements, a shallow bed of dense particles of 5 cm in height was utilized. The bed was made shallow so that high bed voidages can be obtained within the given length of the column. The expanded height of the fluidized bed at different air velocities can be determined by either visual observation method or pressure drop measurement method. In the pressure drop measurement method, the pressure drop is measured every 10 cm. in the test section. As shown in Figs. 4.5 & 4.6 the intersection of two lines of different slopes of the axial pressure drop profile is defined as the bed height in the pressure drop measurement method. The bed height determined by the visual observation method is also shown in the figures. It is seen in the figures that for all the cases, the difference of the bed height obtained by these two methods is less than 10%. In what is to follow the experimental results presented were obtained by the visual observation method. Seven different types of dense particles were
Dense Particle: GB 4.00 mm

Fig. 4.5 Pressure Drop Profile in the Fluidized Bed with Air as the Fluidizing Medium
**Fig. 4.6** Pressure Drop Profile in the Fluidized Bed with Air as the Fluidizing Medium
used for terminal velocity measurements including glass beads of 1.00 mm, 3.00 mm, 4.00 mm, and 6.0 mm in diameter and aluminum particles of 2.32 mm, 5.50 mm, and 6.96 mm in diameter. Three types of circulating particles were used including fine sand of an average diameter of 155 μm, coarse sand with an average diameter of 245 μm, and FCC particles with an average diameter of 65 μm.

The hold-up of fine particles was experimentally measured in the Section I, which is the section of height 0.17 m located immediately above the retaining grid, with the help of bottom two shutters. Four types of dense particles were used in the experiments including Al particles of 2.32 mm, 5.50 mm, and 6.96 mm and iron oxide particles of 11.08 mm in diameter. Three types of circulating particles were used including FCC particles of 65 μm in diameter, glass beads of 105 μm in diameter, and fine sand of 155 μm in diameter. The experiments were conducted at different fine particle and air flow rates.

III. HOLD-UP OF FINE PARTICLES IN THE PACKED DENSE BED

III-A. Results and Discussion

III-A-1. Hold-up of fine particles in the pneumatic transport bed

The hold-up of fine particles in the pneumatic transport bed in the absence of dense particles is examined for both systems with and without a retaining grid. For all cases, the hold-up of fine particles in Section I of the column for the system with a retaining grid is about 5% higher than that without a retaining grid, while the hold-up of fine particles in Sections II and III for the system with a
retaining grid is practically identical to that without a retaining grid. Thus, it can be assumed that the existence of the grid has a minimum effect on the hold-up of fine particles and hence on the particle velocity in the system.

The hold-up of fine particles in Sections II and III were measured and compared with several existing correlation equations in the literature. These correlations are given in the following publications: (i) Konno and Saito (1969)

\[ U_p = U_a - U_t \]  (4.2)

(ii) Yang (1978)

\[
U_p = U_a - U_t \sqrt{(1 + \frac{f_p U_p^2}{2gD})} \epsilon' 4.7
\]

\[ f_p' = 0.0126 \frac{1-\epsilon'}{\epsilon_{1.3}'} \left(\frac{(Re)_t}{(Re)_p}\right)^{-0.979} \]  (4.4)

for \( U_a/U_t > 1.5 \)

\[
f_p' = 0.041 \frac{1-\epsilon'}{\epsilon_{1.3}'} \left(\frac{(Re)_t}{(Re)_p}\right)^{-1.021} \]  (4.5)

for \( U_a/U_t < 1.5 \)

where \( (Re)_p = \rho_a(U_a - U_p)d_p/\mu_a \), \( (Re)_t = \rho_a U_t d_p/\mu_a \), \( \epsilon' = 1 - \frac{4W_p}{\rho_p \pi D^2 U_p} \)

and \( U_t = \frac{0.153 d_p^{1.14} \rho_a^{0.71} - 0.71 \rho_a^{0.43} \rho_a^{0.29}}{\mu_a} \)

for \( 2.0 < (Re)_p < 1000 \)

\[ U_t = \frac{d_p^2 (\rho_p - \rho_a)g}{18 \mu_a} \]  for \( (Re)_p \leq 0.1 \)
Figure 4.7 compares the hold-up of fine particles predicted by Eqn. (4.2) and experimentally obtained. As can be seen in the figure, the prediction is within 30% of the experimental values. The predicted values, however, are consistently lower than the experimentally obtained values. The prediction of the hold-up of fine particles is improved by the use of Eqns. (4.3)-(4.5) as shown in Fig. 4.8. However, it is again seen that the equations underpredict the experimental values of the hold-up of fine particles obtained in this study. The hold-up of fine particles predicted by other correlation equations, including Belden and Kassell's (1949), and modified Hinkle's (1953) by IGT (1978) were also examined. The results of the comparison of the experimental values with these correlations are unsatisfactory.

III-A-2. Hold-up of fine particles in the packed bed

The hold-up of fine particles in the packed bed of dense particles is affected by various factors including gravitational force, frictional force due to air, the dense particles and the bed wall, particle-particle interaction force and particle inertia. In this study, a one-dimensional model based on the force balance of fine particles is developed to account for the hold-up of fine particles in the packed bed. The model assumes that the voidage of the packed bed is uniformly distributed and the flow pattern of air and fine particles is plug flow in the bed. Furthermore, there is no cross-sectional concentration variation of fine particles and the frictional force due to the dense particles and the bed wall can be expressed by the Fanning equation.
Fig. 4.7 Comparison of the Experimental Hold-up Data with the Predicted Values from the Equation by Konno & Saito (1969) in the Pneumatic Transport Bed
Fig. 4.8 Comparison of the Experimental Hold-up Data with the Predicted Values from the Equation by Yang (1978) in the Pneumatic Transport Bed
The force balance of fine particles in a differential element \( dz \) of the packed bed results in the following equation:

\[
\frac{dU}{dm_p} \frac{dp}{dt} = df_d - df_g - df_f - df_c
\]

(4.6)

where \( dm_p \) is the mass of \( dN \) particles in \( dz \), \( df_d \) is the total drag force due to air, \( df_g \) is the net gravitational force, \( df_f \) is the frictional force due to the dense particles and the bed wall, and \( df_c \) is the interaction force due to collision with the dense particles.

The total drag force on \( dN \) particles in \( dz \) is the sum of the drag forces of each individual particle as shown in the following equation

\[
df_d = \frac{3}{4} \frac{\rho_a |v_a - U_p| \cdot (v_a - U_p)}{\rho_p d_p} \ dm_p
\]

(4.7)

where \( C_{Dm} \) is the drag coefficient of fine particles due to air in the packed bed. It is assumed that \( C_{Dm} \) can be expressed in terms of the drag coefficient for a single particle according to

\[
C_{Dm} = C_{Dp} \cdot \varepsilon^{-4.7}
\]

(4.8)

where \( \varepsilon \) is the void fraction of the packed bed which is equal to \( 1 - \varepsilon_p - \varepsilon_s \). Here \( \varepsilon_p \) and \( \varepsilon_s \) are the volume fractions of the fine particles and dense particles in the dense bed, respectively. \( \varepsilon \) varies from 0.4 to 0.5 for the the dense and fine particles employed in this study. As will be shown later, \( \varepsilon_p \) is considerably smaller than \( \varepsilon_s \). Thus, for practical purposes, \( \varepsilon \) can be approximated by \( (1 - \varepsilon_s) \).
The drag coefficient of a single particle can be given by (Schiller and Naumann, 1933):

\[ C_{Dp} = \frac{24}{(Re)_p} (1 + 0.15 (Re)_p^{0.687}) (Re)_p \leq 1000 \]

and \[ C_{Dp} = 0.44 \quad (Re)_p > 1000 \] (4.9)

The net gravitational force can be expressed as

\[ df_g = dm_p (1 - \frac{\rho_a}{\rho_p})g \] (4.10)

As indicated earlier, the frictional force of fine particles due to the dense particles as well as the wall can be defined by the Fanning equation as

\[ df_f = \frac{2F_p U^2}{D_p} dm_p \] (4.11)

The momentum balance equation of a single moving dense particle in the air-fine particle flow system was studied by Arastoopour et al. (1982). They defined the particle interaction coefficient to account for the particle-particle drag force. The particle interaction coefficient of their study, however, cannot be directly applied to account for the interaction of fine particles and dense particles in the packed bed considered in this study.

Welschof (1962) developed equations to elucidate the effect of collision on two different sizes of fine particles conveyed pneumatically. Modifying the Welschof (1962) equation to account for
the force on fine particles due to collision with the stationary dense particles in the present system yields

$$df_c = 2 f_{kk} \left[ -\frac{s}{1} \frac{U_p}{W_p} + 1 \right] \frac{U_p}{D} \frac{d \rho_p}{dm_p}$$

(4.12)

Substituting Eqns. (4.7) to (4.12) into Eqn. (4.6), we have

$$\frac{dU_p}{dt} = \frac{3}{4} C_{Dm} \frac{a_p}{d_p} \left| V_a - U_p \right| \left( V_a - U_p \right) - (1 - \frac{a}{p}) g$$

$$-2f_{kk} \left[ -\frac{s}{1} \frac{U_p}{W_p} + 1 \right] \frac{U_p}{D} - 2 \frac{F_p}{D}$$

(4.13)

The correlation equation for $f_{kk}$ developed by Welsch of (1962) cannot be extended to account for that in the present system. Consequently, $f_{kk}$ remains as an unknown in the equation. Furthermore, $F_p$ is an undetermined parameter. In practice, the last two terms of Eqn. (4.13) can be combined and, thus, Eqn. (4.13) can be simplified to

$$\frac{dU_p}{dt} = \frac{3}{4} C_{Dm} \frac{a_p}{d_p} \left| V_a - U_p \right| \left( V_a - U_p \right) - (1 - \frac{a}{p}) g - 2f_k \frac{U_p}{D}$$

(4.14)

where $f_k$ is the interaction coefficient.

In Eqn. (4.14), $t$ can be related to the linear particle velocity and the axial distance from the bottom of the packed bed by

$$dz = \frac{U_p}{d} \frac{dt}{U_p}$$

(4.15)
The boundary condition for Eqn. (4.15) is

\[ z = 0, \quad u_p = u_{p0} \quad (4.16) \]

Note that \( z = 0 \) is defined at the location immediately underneath the packed bed. Since adequate length (1.92 m) is provided for fine particle acceleration before entering the packed bed, \( u_{p0} \) in Eqn. (4.16) is taken as the steady state velocity of fine particles in the pneumatic transport bed without dense particles obtained experimentally in this study.

The quantity \( f_k \) in Eqn. (4.14) can be obtained by a trial and error procedure. For a specified value of \( f_k \) the axial particle velocity profile can be obtained by integrating Eqns. (4.14) and (4.15) subject to the boundary condition of Eqn. (4.16). The axial hold-up of fine particles can then be calculated from the axial particle velocity profile of fine particles. The axial average hold-up of fine particles is checked with the experimentally obtained hold-up of fine particles to justify the specified value of \( f_k \). Thus, the value of \( f_k \) is found to be a function of the loading ratio and the type of dense particles for a given type of fine particles.

Figure 4.9 shows the relationship of \( f_k \) with loading ratio \( m' \) for various types of dense particles for fine sand. For a given type of dense particles, it is seen that \( f_k \) decreases as the loading ratio \( m' \) increases. The rate of decrease, however, varies with the type of dense particles. Mathematically, variation of \( f_k \) with \( m' \) for various types of dense particles for fine sand can be expressed by
Fig. 4.9 Variation of the Interaction Coefficient with the Loading Ratio for Air-Fine Sand Flow through Packed Bed of various Dense Particles
In a similar manner, values of $f_k$ for glass beads and FCC particles flowing through the packed bed can be obtained. Figure 4.10 shows the variation of $f_k$ with the loading ratio $m'$ for the air-glass bead flow and air-FCC particle flow. Variation of $f_k$ in the figure can be mathematically represented by

$$f_k = 1050 - 400 \text{ m}', \text{ for aluminum particles of 6.96 mm} \quad (4.20)$$

$$f_k = 600 - 240 \text{ m}', \text{ for iron oxide particles of 11.08 mm} \quad (4.21)$$

$$f_k = 400 - 210 \text{ m}', \text{ for aluminum particles of 6.96 mm} \quad (4.22)$$

$$f_k = 200 - 130 \text{ m}', \text{ for iron oxide particles of 11.08 mm} \quad (4.23)$$

It is apparent that $f_k$ is a function of physical properties of the fine particle and the dense particle. A general relation for $f_k$ is finally reached which has the form

$$f_k = \frac{7.03 \times 10^{-5}}{\phi_S} \left(\frac{d_s}{D}\right)^{-2} \frac{d_p}{\phi_p D}^{-1.61}$$

$$- 1.94 \times 10^{-4} \phi_S^{1.36} \left(\frac{d_p}{\phi_p D^{2/3}}\right)^{-2.13} \text{ m}' \quad (4.24)$$

where $D$ is the diameter of the bed. It is noted that the diameter of the bed is not varied in this study. It is employed in Eqn. (4.24)
Fig. 4.10  Variation of the Interaction Coefficient with the Loading Ratio for Air-Glass Bead and Air-FCC Particle Flows through Packed Bed of various Dense Particles
solely for the purpose of creating a dimensionless group. Thus, caution should be exercised when using Eqn. (4.24) for other diameters of the bed.

Based on Eqns. (4.14) to (4.16) along with Eqn. (4.24), the hold-up of fine particles in the packed bed is calculated. The calculated values are compared with the experimental values of hold-up of fine particles as shown in Fig. 4.11. It is seen in the figure that all the calculated values lie within ±25% of the experimental values.

III-B. Concluding Remarks

The hold-up of fine particles in a packed bed of dense particles is higher by a factor as large as six compared with that observed in systems without dense particles.

The interaction coefficient defined in this study varies as a function of loading ratio, and physical properties of fine particles and dense particles. A general expression for the interaction coefficient is developed. This expression, in combination with the momentum balance equation, allows satisfactory prediction of the hold-up of fine particles in the packed bed.

IV. PRESSURE DROP IN THE PACKED DENSE BED

IV-A. Experimental Results

IV-A-1. Air flow only

The pressure drop in the packed bed was measured for all of the
**Fig. 4.11** Comparison of the Hold-up Data Obtained Experimentally with the Calculated Values for various Fine and Dense Particles.
five types of dense particles with air flow only at different superficial air velocities. The variation of the pressure drop as a function of the superficial air velocity is linear when plotted on a log-log scale for all of the dense particles as shown in Fig. 4.12.

**IV-A-2. Air-fine particle flow**

Figure 4.13 shows the effect of fine sand flow rates on the pressure drop in the packed bed for four types of dense particles. It is seen that increasing the fines flow rate at a constant superficial air velocity, decreases the pressure drop in the packed bed. Beyond a certain fines flow rate, however, increasing the fines flow rate increases the pressure drop in the packed bed. A similar reduction effect using iron oxide particles of an average diameter of 11.08 mm in the dense bed is also observed as shown in Fig. 4.14. It is noted from Fig. 4.14 that the pressure drop reduction at low superficial air velocities is small whereas at high superficial air velocities especially close to the minimum fluidization velocity of the dense particles, the pressure drop reduction is large and the maximum reduction can be as large as 15% of the pressure drop with air flow only.

The variation of the pressure drop in the packed bed with coarse sand flow rates follows the same trend as that with fine sand flow rates. Figure 4.15 shows the variation of the pressure drop for four types of dense particles with air-coarse sand flow rates. It is observed that the pressure drop reduction is generally more profound
Fig. 4.12 Variation of the Pressure Drop with the Superficial Air Velocity for Air Flow Only
Fig. 4.13 Variation of the Pressure Drop with the Fine Sand Flow Rate for various Dense Particles.
Dense particles
\(d_s=11.08 \text{[mm]}, \rho_s=4.140 \text{[g/cm}^3]\)

<table>
<thead>
<tr>
<th>Key</th>
<th>(U_a \text{[m/s]})</th>
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</thead>
<tbody>
<tr>
<td>○</td>
<td>3.7</td>
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<tr>
<td>△</td>
<td>3.3</td>
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<td>2.7</td>
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</tbody>
</table>

Fine particle: Fine sand

\(\Delta P \text{[kPa]}\)

\(W_p \text{[g/s]}\)

Fig. 4.14 Variation of the Pressure Drop with the Fine Sand Flow Rate as a Function of the Superficial Air Velocity
Fig. 4.15 Variation of the Pressure Drop with the Coarse Sand Flow Rate for various Dense Particles.
for the air-coarse sand flow than that for the air-fine sand flow. Figure 4.16 shows the effect of coarse sand flow rates on the pressure drop in the packed bed of iron oxide particles of an average diameter of 11.08 mm as a function of the superficial air velocity. The maximum pressure drop reduction observed for air-coarse sand flows is about 25% of the pressure drop with air flow only.

The pressure drop in the packed bed was also measured for air-fine glass bead flows for two types of dense particles, aluminum (Al) with an average diameter of 5.50 mm and aluminum (Al) with an average diameter of 6.96 mm. As shown in Fig. 4.17 for Al particles of an average diameter of 5.50 mm, the variation of the pressure drop as a function of fine glass bead flow rate follows the same trend as that observed for air-fine sand and air-coarse sand flows. It is noted, however, that the magnitude of the pressure drop reduction for air-fine glass bead flows is relatively small compared to air-fine sand and air-coarse sand flows.

The variation of the pressure drop in the packed bed with FCC particle flow rates as shown in Fig. 4.18, also follows the same trend as that observed for other fine particle flows. The pressure drop reduction observed for air-FCC particle flow is smaller than that for air-fine sand and air-coarse sand flows, but is slightly greater than that for air-glass bead flows. Figure 4.18 shows the effect of FCC particle flow rates on the pressure drop in the packed bed of Al particles of an average diameter of 5.50 mm as a function of the superficial air velocity. Again, the pressure drop reduction effect is observed to be more pronounced at large superficial air velocities.
Dense particles
d_s=11.08[mm], \rho_s=4.140[g/cm^3]

<table>
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<tr>
<th>Key</th>
<th>U_a [m/s]</th>
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<tr>
<td>O</td>
<td>3.7</td>
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<tr>
<td>Δ</td>
<td>3.3</td>
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Fig.4.16 Variation of the Pressure Drop with the Coarse Sand Flow Rate as a Function of the Superficial Air Velocity
Fine particle: Glass beads

Dense particles

d_{s} = 5.50 [mm], \rho_s = 3.537 [g/cm^3]

<table>
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<tr>
<th>Key</th>
<th>U_a [m/s]</th>
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<tr>
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<td>△</td>
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Fig. 4.17 Variation of the Pressure Drop with the Glass Bead Flow Rate as a Function of the Superficial Air Velocity
Fig. 4.18 Variation of the Pressure Drop with the FCC Particles Flow Rate as a Function of the Superficial Air Velocity
IV-B-1. Apparent drag reduction phenomenon

The pressure drop in the packed bed of dense particles for air flows only always increases with increasing air flow rate. When the air flow rate increases beyond the minimum fluidization velocity, the bed becomes fluidized. This fluidized bed can be defluidized by then decreasing the air flow rate to attain its packed bed state. This defluidized bed in the packed state denoted as bed I has a different particle arrangement and a different void fraction from the original packed bed. For packed bed of particles as large as those used in this study, the defluidized (packed) bed voidage is practically identical to the minimum fluidization voidage.

The variation of the pressure drop in bed I, with decreasing air flow rate is shown by line (a) in Fig. 4.19. When fine particles, e.g., fine sand, are introduced at a flow rate of 8 g/sec into the air stream in bed I, the pressure drop variation with increasing air velocity is shown by line (b) in Fig. 4.19. It is noted in this figure that the pressure drop for line (b) is consistently higher than that for line (a) at a given air velocity, indicating an increased pressure drop in a given packed bed due to the presence of fine particles in the air flow. It should be noted that to examine the possible effects of micro-movement of the dense particles on the pressure drop during the operation, separate experiments were performed in this study for a packed bed where dense particles were glued to each other. The results of these experiments also indicate that the pressure drop in the packed bed for air-fine particle flows
Fig. 4.19 Variation of the Pressure Drop in the Packed Bed with Superficial Air Velocity for the Air-Fine Sand Flow Rate of 8 g/sec through the Bed

- Fine particle: Fine sand
- Dense particle: 11.08 mm

**Key**

- **●** 0  Decreasing air flow rate in defluidized bed (Bed I)
- **○** 8  Increasing air flow rate in Bed I
- **△** 8  Decreasing air flow rate in defluidized bed (Bed II)
- **○** 8  Increasing air flow rate in defluidized bed (Bed II)

**Flow condition in packed bed**

- **0** Decreasing air flow rate in defluidized bed (Bed I)
- **8** Increasing air flow rate in Bed I
- **8** Decreasing air flow rate in defluidized bed (Bed II)
- **8** Increasing air flow rate in defluidized bed (Bed II)
is higher than that for air-flows only at a given air velocity.

Denote bed II as the packed bed established by initially fluidizing and then defluidizing the dense particles of either the original bed or bed I by increasing and then decreasing the air flow rate at a constant fine particle flow rate, e.g., 8 g/sec. The pressure drop variation with the air velocity in bed II is shown by line (c) in Fig. 4.19. Line (c) also represents the pressure drop variation in the packed bed with the increase of the air velocity to fluidize and then decrease of the air velocity to defluidize the dense particles of bed II at the fine particle flow rate of 8 g/sec. It is noted that the pressure drop for line (c) is consistently lower than that for line (a) at any given air flow rate. This phenomenon underlining the pressure drop reduction is characterized as "apparent drag reduction". The "apparent" here is used to signify that the packed beds denoted by beds I and II, for which the pressure drop is measured and compared, are in different bed configurations, while no "true" drag reduction phenomena is evident in the packed bed of identical configuration as noted by lines (a) and (b) for bed I.

A similar plot for the pressure drop variation with respect to air velocities in the packed bed is shown in Fig. 4.20 for a fine sand flow rate as large as 25 g/sec. It is noted that no apparent drag reduction is observed in this figure. The pressure drop in the fluidized bed for air-fine sand flows is slightly higher than that for air flow only. The increase in the pressure drop can be accounted for by the increase in the hold-up of fine particles in the bed.

It is noted that the height of the packed bed after defluidization for air flows only was found to be practically
Fig. 4.20  Variation of the Pressure Drop in the Packed Bed with Superficial Air Velocity for the Air-Fine Sand Flow Rate of 25 g/sec through the Bed
identical to that for air-fine particle flows despite entrapment of fine particles in the packed bed for air-fine particle flows. It appears that consideration of the void fraction variation alone in the packed bed cannot, by itself, adequately explain the apparent drag reduction phenomenon. Thus, rearrangement of the dense particles in the packed bed and the nature of the flow patterns of the air and fine particles due to the entrapment of fine particles possibly contribute to the phenomenon. The fine particles entrapped between dense particles might be reducing the stagnant wake behind the dense particles and increasing the flow area. Also, the entrapment of fine particles might be changing the shape of dense particles in such a manner that the air flow pattern becomes similar to that in packed bed of spherical particles. Apparently, the fluidization and subsequent defluidization of the bed with air flow at a given fine particle flow rate gives rise to a characteristic packed bed configuration which must be accounted for in the analysis of the pressure drop. More studies are required to provide a fundamental description of the apparent drag reduction phenomenon.

IV-B-2. Analysis with air flow only

The experimentally measured pressure drop in the packed bed with air flow only was compared with the predicted pressure drop based on the Ergun equation (Ergun, 1952) for turbulent flow as given in the following:

\[
\frac{\Delta P}{\frac{1}{\varepsilon}} = 1.75 \frac{1-\varepsilon}{\varepsilon^3} \frac{\rho_a}{\phi_s} \frac{U_a^2}{d_s^3} \tag{4.25}
\]
As shown in Fig. 4.21, the experimental values lie within \( \pm 25\% \) of the predicted values.

**IV-B-3. Analysis with air-fine particle flow**

The total pressure drop in the packed bed of the MPTB for air-fine particle flows can be expressed as

\[
\Delta p = \Delta p_{ag} + \Delta p_{ads} + \Delta p_{hg} + \Delta p_{hds} + \Delta p_{fg} + \Delta p_{fds}
\] (4.26)

Based on the momentum balance equation for fine particles in the packed bed developed in Section IV-A., the pressure drop due to the acceleration of fine particles and air was estimated to be insignificant compared to the total pressure drop. Ignoring the pressure head to air, and combining the frictional pressure drop due to air \((\Delta p_{fg})\) and fine particles \((\Delta p_{fds})\) on the dense particle and bed wall, Eqn. (4.26) can be reduced to:

\[
\Delta p = \Delta p_{fs} + \Delta p_{hds}
\] (4.27)

The hold-up of fine particles in the packed bed was measured and analyzed in the Section IV. Equations (4.14) to (4.16) when integrated over the height of the dense bed \( L \) with \( f_k \) values calculated from Eqn. (4.24), results in a velocity profile of fine particles in the packed bed. The hold-up of fine particles can then be calculated from the velocity profile. The pressure drop due to the hold-up of fine particles can be expressed by

\[
\Delta p_{hds} = \varepsilon_p (\rho_p - \rho_a) g L / (1 - \varepsilon_s)
\] (4.28)
Fig. 4.21 Comparison of the Pressure Drop in the Packed Bed Experimentally Obtained with the Calculated Pressure Drop for Air Flow Only
\( \Delta p \) can be expressed by a modified form of Ergun's equation for turbulent flow as

\[
\frac{\Delta p_{fs}}{1} = k \frac{1 - \varepsilon}{\varepsilon^3} \frac{\rho_m U_m^2}{\phi_m d} \tag{4.29}
\]

where \( \rho_m \) and \( U_m \) are defined as

\[
\rho_m = \frac{\dot{W}_p + \dot{W}_a}{\rho_p + \rho_a}
\]

\[
U_m = \frac{\dot{W}_p + \dot{W}_a}{\rho_p + \rho_a} \frac{\pi D^2}{4} \tag{4.31}
\]

Equation (4.29) defines the parameter \( K \) for air-fine particle flows. Note that for air flow only, Eqn. (4.29) reduces to Eqn. (4.25). Using Eqns. (4.14) to (4.16), (4.24) and (4.27) to (4.29) and the experimentally obtained pressure drop for the packed bed, the values of \( K \) can be calculated. The values of \( K \) were found to be a function of the fine sand flow rate only and are independent of the dense particle density, dense particle size and loading ratio, \( m' \), defined as \( \dot{W}_p/\dot{W}_a \). Figure 4.22 shows the calculated values of \( K \) as a function of the fine sand flow rate, \( \dot{W}_p \). It is seen that \( K \) can be related to \( \dot{W}_p \) for air-fine sand flows by

\[
K = 1.75 - 0.223 \ln (\dot{W}_p + 1) \tag{4.32}
\]

where \( \dot{W}_p \) has the units of g/sec.
Fine particle: Fine sand

Fig. 4.22 Variation of K with the Fine Sand Flow Rate for various Dense Particles
The values of $K$ are also calculated for the air-coarse sand flow. Again the values of $K$, thus calculated were found to be a function of fine particle flow rates only and are independent of the dense particle density, dense particle size, and loading ratio as shown in Fig. 4.23. It is seen that $K$ can be related to $W_p$ for air-coarse sand flows by

$$K = 1.75 - 0.247 \ln (W_p + 1) \quad (4.33)$$

Similar to that described above, experimentally measured pressure drop in the packed bed for the air-fine glass bead flow is also used to obtain the values for $K$. $K$ can be correlated as a function of fine glass bead flow rates as shown in Fig. 4.24 by

$$K = 1.75 - 0.141 \ln (W_p + 1) \quad (4.34)$$

The variation of the values of $K$ calculated from the experimental pressure drop data with the flow rate of FCC particles is shown in Fig. 4.25. The relationship between $K$ and fine particle flow rates can be expressed by

$$K = 1.75 - 0.187 \ln (W_p + 1) \quad (4.35)$$

Clearly, Eqns. (4.32)-(4.35) follow the following general form

$$K = 1.75 - A \ln (W_p + 1) \quad (4.36)$$
Fig. 4.23 Variation of $K$ with the Coarse Sand Flow Rate for various Dense Particles
Fig. 4.24 Variation of $K$ with the Glass Bead Flow Rate for various Dense Particles
Fig. 4.25 Variation of K with the FCC Particle Flow Rate for various Dense Particles
where A is a constant with a value of

- 0 for air flow
- 0.223 for air-fine sand flow
- 0.247 for air-coarse sand flow
- 0.141 for air-fine glass bead flow
- 0.187 for air-FCC particle flow

Furthermore, A can be related to the physical properties of the fine particles including shape factor and particle size according to

\[
A = 4.37 \left( \frac{d_p}{\phi_p D} \right)^{0.5} \]  

(4.37)

It is noted that the diameter of the bed, D, in Eqn. (4.37) is not varied in this study. It is introduced solely for the purpose of forming a dimensionless group.

Substituting Eqn. (4.37) into Eqn. (4.36) yields

\[
K = 1.75 - 4.37 \left( \frac{d_p}{\phi_p D} \right)^{0.5} \ln (W_p + 1) \]  

(4.38)

Using Eqns. (4.14) to (4.16), (4.24), (4.27) to (4.29), and (4.38), the pressure drop in the packed bed as functions of superficial air velocities and fine sand flow rates is calculated for dense particles including Al with an average diameter of 6.96 mm and iron oxide with an average diameter of 11.08 mm. The calculated values are compared with the experimental data in Fig. 4.26. It is seen that the variation of the calculated pressure drop with fine sand flow rates closely follows the trend of that experimentally observed and the agreement between them is good.
Fig. 4.26 Comparison of the Experimental Data with the Calculated Values for the Pressure Drop Variation with the Fine Sand Flow Rate.
Similarly, the pressure drop in the packed bed can be calculated for the air-coarse sand flow. The comparison of the predicted pressure drop and experimental pressure drop for Al particles of an average diameter of 6.96 mm and iron oxide particles of an average diameter of 11.08 mm is shown in Fig. 4.27. The agreement between the two is again satisfactory.

Figure 4.28 compares the experimental pressure drop with the predicted values from for air-fine particle flow. The observed pressure drop lies within \pm 15\% of the predicted pressure drop for four types of fine particle flows through the packed bed of four types of dense particles.

IV-C. Concluding Remarks

Experiments were conducted to account for the pressure drop in the packed beds of five different types of dense particles with air flow only, air-fine sand, air-coarse sand, air-glass beads, and air-FCC particles as flow media. When air-fine particles were used as flow media, a reduction in pressure drop was observed in the packed bed, which is established by fluidizing and then defluidizing the dense particle bed by increasing and then decreasing the air flow rate at a given fine particle flow rate. The reduction in pressure drop which occurs at low flow rates of fine particles characterizes the apparent drag reduction phenomenon.

The observed pressure drop in packed beds with air as a flow medium agrees well with the Ergun's equation. The Ergun equation was modified to predict the pressure drop in the packed bed with air-fine particles as flow media by replacing the coefficient 1.75 in the
Fig. 4.27 Comparison of the Experimental Data with the Calculated Values for the Pressure Drop Variation with the Coarse Sand Flow Rate.
Fig. 4.28 Comparison of the Pressure Drop Experimentally Obtained and Calculated for Various Fine and Dense Particles
equation with a parameter \( K \). The parameter \( K \) was found to be a function of the fine particles flow rate only for a given type of fine particles. A general empirical correlation for \( K \) as a function of flow rates and fine particle properties was obtained. The modified Ergun equation developed in this study satisfactorily predicts the observed apparent drag reduction effect.

V. MINIMUM FLUIDIZATION VELOCITY OF THE DENSE BED

V-A. Experimental Results

V-A-1. Air flow only

The variation of pressure drop with increasing or decreasing air velocity for various bed heights for aluminum particles of an average diameter 5.50 mm is shown in Fig. 4.29. It is seen that for a bed height as small as 7.62 cm, the pressure drop variation with increasing air velocity for a bed from the original packed state to the fluidized state is identical to that with decreasing air velocity for the same bed from its fluidized state to the packed state. When the bed height increases to 15.24 cm, however, a marked difference in the pressure drop variation with increasing and decreasing air velocity is observed. The pressure drop difference is even more pronounced when the bed is increased to as high as 22.86 cm. It is noted that the minimum fluidization velocity obtained from all of these bed heights, i.e., 7.62 cm, 15.24 cm and 22.86 cm is identical.

V-A-2. Air-fine particle flow

Figure 4.30 shows effects of the fine sand flow rate on the
Fig. 4.29 Effect of the Bed Height of the Dense Particles on the Minimum Fluidization Velocity
minimum fluidization velocity for four types of dense particles. It is seen from the figure that for all four types of dense particles, the minimum fluidization velocity of the dense bed for low loading rates of fine sand in air flow is greater than that for air flow only. This can be attributed to the significant apparent drag reduction effect occurring in the packed bed at the minimum fluidization condition due to the presence of fine particles in the air flow. It is seen that for all the four types of dense particles, the minimum fluidization velocity of the dense bed increases, reaches a maximum and decreases with the increase of the flow rates of fine particles.

The pressure drop in the dense bed with air-coarse sand as a fluidizing medium is obtained for all the four types of dense particles. The effect of the coarse sand flow rate on the minimum fluidization velocity is shown in Fig. 4.31. Similar to the behavior of air-fine sand flow in the packed bed, it is seen that by introducing coarse sand into air flow, the minimum fluidization velocity increases when the coarse sand flow (loading) rate is small. Beyond a certain coarse sand flow rate, however, the minimum fluidization velocity decreases. The maximum increase of the minimum fluidization velocity in the packed bed is found to be larger for the air-coarse sand flow than that for the air-fine sand flow.

The minimum fluidization velocity variation with the fine glass bead flow rate is shown in Fig. 4.32. As can be seen in the figure, it follows a similar behavior as that exhibited by air-fine sand and air-coarse sand flows. However the maximum increase of the minimum fluidization velocity for air-fine glass bead flow is seen to be
Fig. 4.30 Variation of the Minimum Fluidization Velocity with the Fine Sand Flow Rate for various Dense Particles
Fig. 4.31 Variation of the Minimum Fluidization Velocity with the Coarse Sand Flow Rate for various Dense Particles
Fig. 4.32 Variation of the Minimum Fluidization Velocity with the Glass Bead Flow Rate for various Dense Particles

Key: $d_s$ [mm] | $\rho_s$ [g/cm$^3$]
---|---
● | 5.50 | 3.537
⊕ | 6.96 | 3.537
○ | 11.08 | 4.170

Fine particle: Glass beads
smaller than that for air-fine sand or air-coarse sand flow.

Variations of the minimum fluidization velocity with the flow rate of FCC particles for various dense particles are shown in Fig. 4.33. It is seen that the minimum fluidization velocity variation for FCC particles follows the similar trend as that exhibited for other fine particles. The maximum increase of the minimum fluidization velocity for air-FCC particle flow is larger than that observed for air-glass bead flow, but is smaller than that for air-fine sand and air-coarse sand flows.

V-B. Analysis and Discussion

Studies of the minimum fluidization velocity for a dense bed with air flow only have been thoroughly documented in the literature (e.g., Wilhelm and Kwauk, 1948; Richardson 1971; Babu et al., 1978). Very few of these studies, however, involve a bed of dense particles of size greater than 5 mm (Wen & Yu, 1966). Correlation equations of the minimum fluidization velocity for a dense bed with gas or liquid as a fluidizing medium have been extensively reviewed by Babu et al. (1978) and Grewal and Saxena (1980). No experimental data or correlation equations are available to account for the minimum fluidization velocity for the dense bed with air-fine particles as a fluidizing medium.

V-B-1. Air flow only

The minimum fluidization velocity of the dense bed of particle sizes ranging from 2.32 mm to as large as 12.67 mm with air as a fluidizing medium is analyzed. Empirical correlation equations
Fig. 4.33 Variation of the Minimum Fluidization Velocity with the FCC Particles Flow Rate for various Dense Particles
proposed by Wen and Yu (1966), Babu et al. (1978), and Grace (1982) are selected for comparison with the experimental data obtained in this study. These equations express the minimum fluidization velocity in terms of minimum fluidization Reynolds number which varies as a function of the Archimedes number, Ar.

The comparison of the minimum fluidization velocity experimentally obtained and calculated by these correlation equations are shown in a plot of \( R_{mf} \) versus \( Ar \) given in Fig. 4.34. The values of \( Ar \) vary from about \( 2 \times 10^6 \) to \( 4 \times 10^8 \). It is seen that the correlation of Wen and Yu (1966) generally agrees well with the experimental data. The correlation by Grace (1982) agrees well with the experimental data at large \( Ar \), while it slightly over predicts the minimum fluidization velocity at small \( Ar \). The correlation by Babu et al. (1978) over predicts the minimum fluidization velocity at all \( Ar \) considered in this study.

V-B-2. Air-fine particle flow

The minimum fluidization velocity is related to the operating parameters through a force balance at the minimum fluidization condition in which the drag force by upward air or air-fine particle mixture equals the effective weight of the dense particles. Thus, we have:

\[
\frac{\rho_f}{\rho_s} \cdot l = (1 - \varepsilon_{mf}) (\varepsilon_s - \varepsilon_m) \frac{g}{l}
\]

(4.39)

where \( 1 - \varepsilon_{mf} \) is the volume fraction of the dense particles at the minimum fluidization condition and \( \rho'_m \) is defined as

\[
\rho'_m = \frac{\varepsilon_p \rho_p + \varepsilon_a \rho_a}{\varepsilon_p + \varepsilon_a}
\]

(4.40)
Fig. 4.34 Comparison of the Minimum Fluidization Velocity Obtained Experimentally and Calculated by Different Correlations for Air Flow Only.
\[ \frac{\Delta p_{fs}}{1} = K \left( \frac{1-\varepsilon_{mf}}{a_{mf}^3} \right) \frac{\varepsilon}{m_s} \frac{U_m^2}{d_s} \]  

where \( \rho_m \) and \( U_m \) are defined as

\[ \rho_m = \frac{(\rho_p + \rho_a) / (\rho_p \rho + \rho_a / \rho_a)}{U_m = \frac{(\rho_p / \rho + \rho_a / \rho_a) / (\pi D^2 / 4)}{U_{mf} = \frac{(\rho_p / \rho + \rho_a / \rho_a) / (\pi D^2 / 4)}} \]

At the minimum fluidization condition, Eqn. (4.41) becomes

\[ \frac{\Delta p_{fs}}{1} = K \left( \frac{1-\varepsilon_{mf}}{a_{mf}^3} \right) \frac{\varepsilon}{m_s} \frac{U_{mf}^2}{d_s} \]

As described in the Section IV of Chapter 4, for a given type of fine particles the value of \( K \) varies as a function of fine particle flow rates only. A general empirical correlation for \( K \) was also developed for fine sand, coarse sand, FCC particle and fine glass bead flow in the packed bed as given in the following

\[ K = 1.75 - 4.37 \diameter \rho_p / (\varepsilon D)^{0.5} \ln (\rho_{sp} + 1) \]
where \( W_p \) has the units of \( g/sec \).

From Eqns. (4.39) and (4.45), we have

\[
(1 - \varepsilon_{mf}) (\rho_s - \rho_m) g = K \left( \frac{\rho_m U_{mf}^2}{\varepsilon_{mf}^3} \right) \frac{\rho_m U_{mf}}{\varepsilon_{mf} \rho_s d_s} \quad (4.47)
\]

Under the low loading condition for fine particles, the following relationship holds

\[
\rho_m' \ll \rho_s \quad \text{and} \quad \rho_m \ll \rho_s \quad (4.48)
\]

Given that \((\rho_a/\rho_p) \ll 1\), we have \((U_a)_{mf} \approx U_{mf}\). Thus Eqn. (4.47) becomes

\[
\rho_p U_p U_{mf} + \rho_a U_{mf}^2 = \left( \frac{\rho_s^3 \varepsilon_{mf}^3}{K} \right) \frac{\rho_s g d_s}{\varepsilon_{mf} \rho_s} \quad (4.49)
\]

Finally from Eqn. (4.49), \( U_{mf} \) expressed in terms of \( Re_{mf} \) is developed which has the following form

\[
Re_{mf} = \left\{ 0.25 (\rho_p/\rho_a)^2 Re_p^2 + \left[ (\rho_s \varepsilon_{mf}^3) / K \right] (d_s^3 \rho_a \rho_s g/\nu_a)^2 \right\}^{1/2}
\]

\[
- 0.5 (\rho_p/\rho_a) Re_p' \quad (4.50)
\]

In this study, Eqn. (4.50) is used to calculate the minimum fluidization velocity of the dense bed in the MPTB. Figure 4.35 shows the comparison of the minimum fluidization velocity experimentally obtained and calculated based on Eqn. (4.50). It is seen that comparison is satisfactory for all the dense particles and air-fine
Fig. 4.35 Comparison of the Experimentally Measured Minimum Fluidization Velocity with Calculated Values for Air-Fine Particles Flow
particles employed in this study.

V-C. Concluding Remarks

The minimum fluidization velocity of the dense bed with air-fine particles as a fluidizing medium increases, reaches a maximum, and then decreases with the increase of the fine particle flow rate. Utilizing the defined properties for air-fine particle mixtures and the modified Ergun equation, a semi-empirical correlation is developed to account for the minimum fluidization velocity of the dense bed under low fine particle loading conditions.

VI. TERMINAL VELOCITY OF DENSE PARTICLES

VI-a. Experimental Results

VI-A-1. Air flow only

For a shallow bed of dense particles as large as used in this study, the slugging regime of the fluidized bed exists over a very narrow, low air velocity range. At air velocities sufficiently higher than the minimum fluidization velocity, the bed appears to be in turbulent flow regime with well defined upper layer though fluctuating about its mean position. The measurements of bed height in this study for particle terminal velocity determination were made under such high air velocity conditions.

The variation of the bed voidage with the superficial air velocity for Al particles of 2.32mm in diameter is shown in Fig. 4.36. When plotted on a log-log scale, the variation is linear up to the
Fig. 4.36 A Log-Log Plot of the Variation of the Fluidized Bed Voidage with the Superficial Air Velocity for Al Particles of 2.32 mm Diameter
voidage of 0.864 for Al particles of 2.32mm in diameter. It is then followed by a linear variation of different slope. Similar behavior is also obtained for all other dense particles used in this study. The voidage at which the two lines intersect, however, is different for different dense particles. The air velocity determined by extrapolating the line to the bed voidage of one, as indicated in the figure, characterizes the terminal velocity of dense particles.

VI-A-2. Air-fine particle flow

The variation of the fluidized bed voidage with the superficial air velocity for various fine sand flow rates is shown in Fig. 4.37. It is seen that the log-log variation of the bed voidage with the superficial air velocity for fine sand flow is also linear. Furthermore, the voidage at which two lines intersect ranges from 0.85 to 0.9. The linear variation extrapolated to the voidage of one is again utilized to characterize the dense particle terminal velocity at different fine sand flow rates. Similar plots are obtained for different dense particles and fine particles.

The variation of dense particle terminal velocity with the fine sand flow rate is shown in Fig. 4.38. It is seen that the terminal velocity monotonically decreases with the increase of the fine sand flow rate and the reduction in velocity is higher for large particles than for small particles.

The dense particle terminal velocity varies with the coarse sand flow rate in the same manner as that with the fine sand flow rate. Figure 4.39 shows the variation of the dense particle terminal velocity for six different types of dense particles with coarse sand
Fig. 4.37 A Log-Log Plot of the Variation of the Fluidized Bed Voidage with the Superficial Air Velocity for Air-Fine Sand Flow through the Bed of Al Particles of 2.32 mm Diameter

<table>
<thead>
<tr>
<th>Key</th>
<th>Fine sand flow rate (g/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>19.3</td>
</tr>
<tr>
<td>o</td>
<td>42.1</td>
</tr>
<tr>
<td>x</td>
<td>65.4</td>
</tr>
<tr>
<td>i</td>
<td>105.8</td>
</tr>
</tbody>
</table>
Fluidizing Medium: Air-Fine Sand

Fig. 4.38 Variation of the Dense Particle Terminal Velocity with the Fine Sand Flow Rate
Fig. 4.39 Variation of the Dense Particle Terminal Velocity with the Coarse Sand Flow Rate
flow rates. It is observed that the decrease in the terminal velocity with fine particle flow is generally more profound for coarse sand flow than for fine sand flow.

The terminal velocities of five different types of dense particles were also measured in an air-FCC particle medium. It is observed that variation of the dense particle terminal velocity with the FCC particle flow rate is similar to that with fine sand and coarse sand flow rates. However, the magnitude of the decrease in the terminal velocity for air-FCC particle flow is smaller than that for air-coarse sand flow but higher than that for air-fine sand flow, at same fine particle flow rate conditions.

VI-B. Analysis and Discussion

The terminal velocities of spherical or non-spherical particles in a single phase or in a pseudo-homogeneous multiphase medium have been extensively analyzed and well documented in the literature (e.g., Schiller & Naumann, 1933; Squires & Squires, 1937; Pettyjohn & Christiansen, 1948; Becker, 1959; Perry, 1975). The terminal velocity of spherical particles was correlated, as early as in 1933, by Schiller & Naumann. Squires & Squires (1937), Pettyjohn & Christiansen (1948) and Becker (1959), to name a few, studied the terminal velocity of geometrically well defined, non-spherical particles. However, little work has been reported on the terminal velocity of particles in heterogeneous two phase systems.

The classical technique of measuring the terminal velocity of a particle is to measure the steady state velocity of the particle in a stagnant medium. However, the experimental determination of the
terminal velocity of dense particles in an air-fine particle medium poses some difficulties. Fine particles have to be conveyed in a column of finite size and the dense particle velocity should be measured in this column. With air-fine particle flow, the visual observation of the dense particle is impaired unless the particle is close to the column wall in which case the column interaction may cause some errors.

The present study employs the method in which, the terminal velocity of dense particles in an air-fine particle medium is determined by measuring the variation of the shallow fluidized bed voidage with the superficial air velocity and extrapolating the linear variation to the voidage of one. The variation of the bed voidage with the air velocity for liquid-solid fluidization and sedimentation has been established in the literature (e.g., Richardson and Zaki, 1954). For particulate fluidization, the voidage-fluid velocity relation can be expressed as (Richardson & Zaki, 1954):

\[
\frac{U_a}{U_i} = e^{n_0}
\]

where

\[
n_0 = 4.4 \left( \text{Re}_T \right)^{0.03} \quad 0.2 < \left( \text{Re}_T \right) < 1
\]

\[
n_0 = 4.4 \left( \text{Re}_T \right)^{-0.1} \quad 1 < \left( \text{Re}_T \right) < 500
\]

\[
n_0 = 2.4 \quad \left( \text{Re}_T \right) > 500
\]

and

\[
\text{Re}_T = \frac{\rho_a U_{ts} d_s}{U_a}
\]

and \( U_i \) is the extrapolated velocity as the bed voidage approaches one. It has been shown by Avidan & Yerushalmi (1982) that similar relationship can be used for turbulent fluidized beds. Canada et al. (1978) also used this relationship to correlate the voidage in
turbulent fluidized beds of coarse particles. Capes (1974) and Yang (1982) modified Eqn. (4.51) by employing the cluster property in place of the fine particle property to account for bubbling and turbulent fluidized beds of fine particles. As indicated earlier, for air-dense particle turbulent fluidized beds employed in this study, the variation of the bed voidage with the superficial air velocity on a log-log scale yields two straight lines of different slopes which intersect at the voidage ranging from 0.85 to 0.9. The first straight line obtained for small voidages can be explained by Eqn. (4.51). The slope of this line is found to vary from 2 to 4 for all the dense particles utilized in this study. The second straight line obtained for high bed voidages can be approximated by:

\[
\frac{U_a}{U_{ts}} = \varepsilon^{n'}
\]  

(4.52)

To verify the experimental method, in which the terminal velocity of the dense particles is estimated by extrapolating the second straight line to the bed voidage of one, the terminal velocities of seven different types of dense particles in an air medium were measured and compared with the values predicted by existing correlations. As will be reported later, the experimental values are within \( \pm 6\% \) of the predicted values. The bed height was found to fluctuate by 5-10 cms when operated in the turbulent flow regime. It should be noted that the 10 cm fluctuation in the bed height amounts to only 5-10\% of the entire bed height which yields only 0.2-0.5\% deviation in the bed voidage. Thus, the error introduced in bed voidage measurements due to bed height fluctuations is minimal. In this study, the averaged
value of the fluctuating bed height is used to obtain the bed voidage.

The terminal velocity of dense particles in air and air-fine particle flow can be mathematically expressed in following manner:

The forces of fluid acting on each of the suspended dense particle must be in dynamic equilibrium:

\[ F_g = F_s + F_{Dgm} + F_{Dpm} \]  \hspace{1cm} (4.53)

where \( F_g \) is the gravity force acting on a dense particle
\( F_s \) is the buoyancy force acting on a dense particle
\( F_{Dgm} \) is the drag force exerted by air on a dense particle in a multiparticle system
\( F_{Dpm} \) is the drag force exerted by fine particles on a dense particle in multiparticle system

The drag forces in a multiparticle system \( F_{Dgm}, F_{Dpm} \) can be expressed in terms of the drag forces in a single particle system by:

\[ F_{Dgm} = f \times F_{Dg} \]

\[ F_{Dpm} = f' \times F_{Dp} \]  \hspace{1cm} (4.54)

where \( f \) is the parameter which relates the drag force on a dense particle due to air flow in a single particle system, \( F_{Dg} \), to that in a multi-dense particle system, \( F_{Dgm} \).
\( f' \) is the parameter which relates the drag force on a dense particle due to fine particle flow in a single dense particle system, \( F_{Dp} \), to that in the multi-dense particle system, \( F_{Dpm} \).
Substituting Eqn. (4.54) into Eqn. (4.53), we have

$$F_g - F_s = f * F_{Dg} + f' * F_{Dp} \quad (4.55)$$

As the air velocity approaches the dense particle terminal velocity, it yields that $f$, $f'$ and $f''$ approach one. Under this condition, Eqn. (4.55) can be simplified to

$$F_g - F_s = F_{Dg} + F_{Dp} \quad (4.56)$$

$F_g - F_s$ can be expressed as:

$$F_g - F_s = V_s (\rho_s - \rho_a)g \quad (4.57)$$

where $V_s$ is the volume of a single coarse particle.

For spherical particles $F_{Dg}$ can be written as:

$$F_{Dg} = \left(\frac{3}{4}\right) C_{Ds} \left(\frac{\rho_a}{D_s}\right) \frac{V_a}{V_s} \quad (4.58)$$

where

$$C_{Ds} = \begin{cases} \frac{24}{\text{Re}_s} (1 + 0.15 \text{Re}_s^{0.687}) & \text{Re}_s < 1000 \\ 0.44 & \text{Re}_s > 1000 \end{cases} \quad (4.59a)$$

and

$$\text{Re}_s = \frac{\rho_a D_s V_a}{\mu_a} \quad (4.59b)$$

The air velocity, $V_a$ is defined as

$$V_a = \frac{U_a}{(1-c_p)} \quad (4.60)$$
and under dilute fine particle flow rate condition \( V_A = U_A \). As stated earlier, several studies have been reported for drag force due to air on non-spherical particles of specific shapes, e.g., prisms, thin disks etc. (Squires & Squires, 1937; Pettyjohn & Christiansen, 1948; Becker, 1959).

For irregular particles, define the dense particle diameter as

\[
D_s = \frac{6}{S_v} \quad \text{(4.61)}
\]

where \( S_v \) is the surface area of solid per unit volume.

Define \( d_s \) as the diameter of a sphere having the same volume as that of the particle, or

\[
d_s = \left( \frac{6V_s}{\pi} \right)^{1/3} \quad \text{(4.62)}
\]

Define \( \phi_s \) as the ratio of the surface area of a sphere to that of a irregular particle of same volume. Then from Eqns. (4.61) & (4.62) we have

\[
D_s = \phi_s d_s \quad \text{(4.63)}
\]

Thus, for non-spherical particles, \( F_D \) can be expressed by:

\[
F_{Dg} = \left( \frac{3}{4} \right) C_{Ds} \rho_a \frac{d_s}{\phi_s} V_a^2 V_s \quad \text{(4.64)}
\]

with \( C_{Ds} \) given by Eqn. (4.59).
F_{dp} can be expressed by considering the loss of momentum of fine particles due to collision with the dense particles (Soo, 1967) as:

\[ F_{dp} = \left(\frac{3}{2}\right) C_{dp} \frac{\rho_p \varepsilon_p}{d_s} v_s U_p^2 \]  

(4.65)

Note that Eqn. (4.65) is defined in such a manner that \( \phi_s \) is implicitly included in \( C_{dp} \). For low fine particle loadings, \( U_p \) can be approximated by:

\[ U_p = U_a - U_{tp} \]  

(4.66)

Substituting Eqns. (4.57) through (4.64) into Eqn. (4.56) and setting the air velocity as the dense particle terminal velocity yields the following equation for the dense particle terminal velocity for air flow only:

\[ (\rho_s - \rho_a) g = \left(\frac{3}{4}\right) C_{Ds} \frac{\rho_a \phi_s}{d_s} U_{ts}^2 \]  

(4.67)

The terminal velocities of all the seven different types of dense particles used in this study were calculated by Eqn. (4.67). The calculated values are compared with the experimental values as shown in Fig. 4.40. It is seen in the figure that the calculated values lie within ±6% of the experimental values.

By substituting Eqns. (4.57) through (4.66) into Eqn. (4.56), an expression for the dense particle terminal velocity in a heterogeneous medium of air-fine particle is obtained as given in the following:

\[ (\rho_s - \rho_a) g = \left(\frac{3}{4}\right) C_{Ds} \frac{\rho_a \phi_s}{d_s} U_{ts}^2 + \frac{3}{2} C_{dp} \frac{\rho_p \varepsilon_p}{d_s} U_p^2 \]  

(4.68)
Fig. 4.40 Comparison of the Dense Particle Terminal Velocity Experimentally Obtained with the Predicted Values
Using Eqn. (4.68) and the experimentally obtained dense particle terminal velocities, the values of the fine particle-coarse particle interaction coefficient \( C_{dp} \) can be calculated. The variation of the interaction coefficient thus obtained with the dense particle diameter is shown in Fig. 4.41. It is seen in the figure that there is no definite trend in the interaction coefficient variation with the dense particle diameter. For a fine particle, all the values fluctuate about a certain mean value. The mean values of the interaction coefficient obtained from Fig. 4.41 are correlated with the fine particle properties as given by

\[
C_{dp} = 0.00845 \left( \frac{d_p}{\phi_p^{1.5}} \right)^{0.75}
\]

(4.69)

where \( d_p \) is in \( \mu \text{m} \).

Arastoopour et al. (1982) conducted experiments to measure the velocity of a single dense particle which was dropped countercurrently to the upward flow of air-fine particles. As indicated earlier, these experiments represent another method for determining the dense particle terminal velocity. The interaction coefficient determined from these dense particle velocity data was found to decrease with the increase of the dense particle diameter. The range of dense particle sizes in Arastoopour et al.'s (1982) study is smaller than that in the present study. Sand particles were used as fine particles in their study. Comparison of the interaction coefficient estimated for Arastoopour et al.'s (1982) system based on the correlation given by Eqn. (4.69) with that obtained by Arastoopour et al. (1982) is shown in Table 4.3. Two values of the sphericity factor for sand particles,
Fig. 4.41 Variation of the Dense Particle-Fine Particle Interaction Coefficient with the Dense Particle Diameter
Table 4.3. Estimation of Fine Particle-Dense Particle Interaction Coefficient of Arastoopour et al.'s (1982) Study based on the Present Correlation Equation

<table>
<thead>
<tr>
<th>Dense Particle Diameter used in Arastoopour et al.'s (1982) Study (cm)</th>
<th>Obtained by Arastoopour et al. (1982)</th>
<th>Estimated from Present Correlation with $\phi_p=0.72$</th>
<th>Estimated from Present Correlation with $\phi_p=0.66$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.476</td>
<td>1.013</td>
<td>0.882</td>
<td>0.972</td>
</tr>
<tr>
<td>0.635</td>
<td>0.973</td>
<td>0.882</td>
<td>0.972</td>
</tr>
<tr>
<td>0.794</td>
<td>0.947</td>
<td>0.882</td>
<td>0.972</td>
</tr>
</tbody>
</table>
i.e. 0.66 and 0.72, which are taken from the present study are used in the calculations. As can be seen in the table, the values calculated by Eqn. (4.69) and those obtained by Arastoopour et al. (1982) are comparable.

Based on Eqns. (4.68) & (4.69), the terminal velocity of dense particles in an air-fine particle medium is calculated and compared with the experimental values obtained in this study. As shown in Fig. 4.42, all the calculated values lie within ±10% of the experimental values.

VI-C. Concluding Remarks.

The terminal velocity of dense particles in air and air-fine particle media was determined from the turbulent fluidized bed voidage-air velocity relationship. It is demonstrated that the experimental method used in this study for measurement of the dense particle terminal velocity yields satisfactory results.

The fine particle-dense particle interaction coefficient measured in a single dense particle system, is found to be a function of fine particle physical properties only and to be independent of dense particle diameter and density. A general expression for the interaction coefficient was developed. This expression in combination with the force balance on dense particles allows satisfactory prediction of the dense particle terminal velocity in an air-fine particle medium.
Fig. 4.42 Comparison of the Dense Particle Terminal Velocity Experimentally Obtained with the Predicted Values for Air-Fine Particle Flow.
VII. HOLD-UP OF FINE PARTICLES IN THE FLUIDIZED DENSE BED

VII-A. Results and Discussion

Figure 4.43 shows the variation of the hold-up of fine sand in the fluidized dense bed experimentally determined with the loading ratio with four different dense particles. The hold-up is found to increase with the loading ratio. Also the hold-up of fine sand in the dense bed of Al 2.32 mm particles is more than the hold-up in the dense bed of larger particles. The hold-up of fine sand measured without dense bed is also shown in the figure. The hold-up of fine sand in the dense bed is higher by a factor as large as ten as compared to the hold-up without dense bed. The variation of the hold-up of FCC particles and glass beads with the loading ratio is shown in Fig. 4.44. Similar effects of loading ratio and dense particle diameter on the hold-up of FCC particles and glass beads are observed as that for fine sand.

The hold-up of fine particles in the fluidized dense bed is also affected by various factors, similar to the hold-up of fine particles in the packed dense bed, including gravitational force, frictional force due to air, the dense particles and the bed wall, fine particle-dense particle interaction force and particle inertia. The one dimensional model based on the force balance of fine particles developed in the Section III-A-2 to account for the hold-up of fine particles in the packed dense bed is utilized to account for the hold-up of fine particles in the fluidized dense bed also. The interaction coefficient, $f_k$, defined in Eqn. (4.14) is determined from the hold-up data for all the fine particles. Figure 4.45 shows the variation of the interaction coefficient with the loading ratio for
Fig. 4.43 Variation of the Hold-up of Fine Sand in the Fluidized Dense Bed with the Loading Ratio

<table>
<thead>
<tr>
<th>Key</th>
<th>Dense particle</th>
<th>Fine particle: Fine sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>△</td>
<td>2.32 [mm]</td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>5.50 [mm]</td>
<td></td>
</tr>
<tr>
<td>○</td>
<td>6.96 [mm]</td>
<td></td>
</tr>
<tr>
<td>○</td>
<td>11.08 [mm]</td>
<td></td>
</tr>
</tbody>
</table>

$E_p \times 10^3$ vs. $m'$
Fig. 4.44 Variation of the Hold-up of FCC Particles and Glass Beads in the Fluidized Dense Bed with the Loading Ratio
Fig. 4.45 Variation of the Interaction Coefficient with the Loading Ratio for Various Dense Particles.
various dense particles when fine sand is used as the fine particle.
The values of $f_k$ are found to oscillate around a mean value and no
definite trend of variation of $f_k$ with the loading ratio was observed.
Similar results were noticed for other fine particles utilized in this
study. The relationship of mean value of $f_k$ thus obtained with the
dense particle diameter is shown in Fig. 4.46 for all the three fine
particles. A general relation for $f_k$ can be derived from this figure as

$$f_k = 15214 \exp(-35.7 \frac{d_s}{D})$$

Based on Eqns. (4.14)-(4.16) along with the Eqn. (4.70), the
hold-up of fine particles in the fluidized dense bed is calculated.
The calculated values are compared with the experimental values of the
hold-up of fine particles as shown in Fig. 4.47. It is seen in the
figure that all the calculated values lie within ±15% of the
experimental values.

**VII-B. Concluding Remarks**

The hold-up of fine particles in the fluidized dense bed is
analyzed based on the experimental data. The hold-up is found to be
higher by a factor as large as ten compared with that observed in
system without dense particles.

The interaction coefficient between the fine particles and the
dense particles defined in this study is found to be independent of
the loading ratio and is found to vary with the fine particle and
dense particle properties. A general expression for the interaction
coefficient is developed. This interaction coefficient in combination
Fig. 4.46 Variation of the Interaction Coefficient with the Dense Particle Diameter
Fig. 4.47 Comparison of the Experimental Hold-up of Fine Particles in the Fluidized Dense Bed with the Predicted Values
with the momentum balance equation, allows satisfactory prediction of the hold-up of fine particles in the fluidized dense bed.
CHAPTER 5

CHARACTERISTICS OF THE SLUGGING BEDS OF DENSE PARTICLES

ABSTRACT

Slugging characteristics of four different types of dense particles in the size range of 1 mm to 7 mm are determined through the pressure signals acquired from the beds. Pressure fluctuations in the slugging beds, slug velocity, height of the slugging beds, and slug frequency of dense particles slugging beds determined in this study are compared to those of fine particle slugging beds. Dense particle slugging beds are found to behave much differently as compared to the fine particle slugging beds. Pressure fluctuations in the dense particle turbulent beds and the transition velocity to the turbulent beds are also determined.

I. INTRODUCTION

With the increased use of fluidized beds as coal combustors and gasifiers, the hydrodynamic information of dense particle fluidized beds is gaining importance. The hydrodynamic information is essential for successful and optimum design of these units. For example, the hydrodynamic information for the slugging bed of dense particles, defined as particles of diameter above 1 mm, in the multisolid
Pneumatic transport bed is of extreme importance in estimating the heat transfer coefficient, combustion efficiency as well as in determining the height of the combustor required to allow disengagement of dense particles. For fluidized beds of dense particles, the difference in the minimum slugging and the minimum fluidization velocity is a small factor of the minimum fluidization velocity (even for large diameter units). Therefore, for dense particles fluidized beds slugging regime is observed for a wide gas velocity range, from approximately minimum fluidization velocity to the onset of turbulence velocity.

Most of the data and correlations documented in the literature for slugging fluidized beds are for fine particle beds. Recently, some studies have been reported for the hydrodynamics of slugging beds of dense particles (e.g., Cranfield and Geldart, 1974; Baker and Geldart, 1978; Canada et al., 1978; Canada and McLaughlin, 1978; Miller et al., 1981; Glicksman et al., 1981). One of the major differences between a dense particle slugging bed and a fine particle slugging bed is the slug velocity to particulate phase gas velocity ratio. For dense particle slugging beds this ratio is less than one (slow slug condition) and for fine particle slugging beds it is greater than one (fast slug condition). This delineation is important because the two situations exhibit different gas flow characteristics. In the slow slug condition, the gas moves freely from the particulate phase through the slug and back to the particulate phase. Whereas, for fast slug condition, the gas circulates around the slug and there is cloud formed around the slug.

The two phase theory originally proposed by Toomey and Johnstone
(1952) for fluidized beds is applicable in the beds where the gas velocity in particulate phase is equal to the minimum fluidization velocity and there is no through flow of gas in bubble or slug phase. The two phase theory has been questioned by several investigators for fine particle bubbling fluidized beds (e.g., Lockett et al., 1967; Grace and Clift, 1974; Rowe et al., 1978), though it appears to be a good approximation for fine particle slugging beds (Grace et al., 1982). For dense particle slugging beds operating in slow slug condition, some through flow of gas in the slug phase will be present and the two phase theory can not be applied. Davidson and Harrison (1966), Lockett et al. (1967), and Grace and Harrison (1969) have derived expressions for the gas flow in slow bubble regime with through flow of gas in the bubble phase. Recently, Zhang et al. (1983) considered through flow of gas in the bubble or slug phase as well as the non uniform velocity in the particulate phase for dense particle bubbling and slugging beds.

Theil and Potter (1977), Canada et al. (1978), Yerushalmi and Cankurt (1979), and Yang (1982) measured the transition to turbulent regime velocity for particles of various different sizes. However, the transition from slugging or bubbling regime to the turbulent regime is not well defined in the literature and contradictory statements have been reported regarding this transition.

In this study, the pressure fluctuations and the bed expansion were measured for slugging fluidized beds of particles in the size range of 1 mm to 7 mm. The measured pressure signals were utilized to calculate the probability density function, slug velocity and the slug frequency. The onset of the turbulent regime velocity was also
II. EXPERIMENTAL

The experimental apparatus utilized to measure the slugging characteristics of the dense particle fluidized beds consists of a bed of 0.102 m I. D. and 6.46 m in height. The schematic diagram of the experimental apparatus is shown in Fig. 5.1. Air is sent through an oil filter and an orifice meter before it enters the bottom of the bed. The oil filter traps any oil mist contained in the air. The dense particles are located at about 2.0 m from the bottom of the bed. Any dense particles being entrained from the bed are separated by two cyclones. These dense particles then enter the stand pipe, which is 3.81 cm I.D. and 4.25 m in height.

Two pressure taps were installed vertically along the column. The outside opening of the taps were connected to differential pressure transducers which were connected to VAX-11/780 minicomputer through A/D converters for real time data acquisition. The transducers are of the type LX 1802DZ of National Semiconductor Co.. These transducers are linear in response from 0 psi to 15 psi differential pressure. The pressure data acquired through these transducers was utilized to calculate the probability density functions, probability distribution function, power spectral density function, auto-correlation function and the cross-correlation function. The computer programs used for real time data acquisition and calculations of the statistical functions from this data are listed in Appendix D.
Fig. 5.1 Schematic Diagram of the Experimental Unit
The pressure taps were located in the upper portion of the bed at 15.2 cm and 30.3 cm from the distributor plate. Static bed heights of 15 cm, 20.3 cm, 25.4 cm, and 30 cm at the minimum fluidization were used in the experiments. A wide range of air velocities, from minimum slugging velocity to beyond the onset of turbulence velocity were tested. The sampling interval of 0.01 seconds was used and 8192 samples were collected for each run. Four different types of dense particles were used including glass beads of 1 mm, and Al particles of 2.32 mm, 5.50 mm, and 6.96 mm in diameter. The physical characteristics of the particles are described in Table 5.1.

III. RESULTS

The slugging characteristics of dense particles were measured with four different bed heights of dense particles at minimum fluidization. The slug velocity was found to decrease with increase of the bed height from 15 cm (H/D = 1.5) to 20.3 cm (H/D = 2). However, the slug velocity remained constant with the increase of the bed height beyond 20.3 cm. Similarly, the transition to turbulent regime velocity was observed to increase with the increase of the bed height up to 20.3 cm and remained constant thereafter. Dense particle slugging beds have been reported to show less bed height dependence than the fine particle slugging beds (Canada et al., 1978) and the transition from shallow bed to the deep bed for dense particle slugging beds has been reported to occur at H/D of approximately two. All the data reported in this study is for bed heights greater than 20.3 cm.

For all the dense particles used in this study, square nosed
**Table 5.1. Characteristics of Dense Particles**

<table>
<thead>
<tr>
<th>Type of Particle</th>
<th>Average diameter of particle (mm)</th>
<th>Density of particles (g/cm³)</th>
<th>Sphericity factor (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass Beads</td>
<td>1.00</td>
<td>2.767</td>
<td>1.000</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.32</td>
<td>3.537</td>
<td>0.838</td>
</tr>
<tr>
<td>Aluminum</td>
<td>5.50</td>
<td>3.537</td>
<td>0.990</td>
</tr>
<tr>
<td>Aluminum</td>
<td>6.96</td>
<td>3.537</td>
<td>0.920</td>
</tr>
</tbody>
</table>
slugs were observed in the entire gas velocity range considered. Three types of slugs have been reported in the literature, represented by axisymmetric slugs, asymmetric slugs, and square nosed slugs. In fine particle slugging beds, axisymmetric slugs are the most commonly observed slugs. Large particle diameter to the bed diameter ratio, angular particle shapes and roughened walls promote formation of asymmetric slug flows. Square nosed slugs were initially observed in tubes up to 0.05 m in diameter only (Kehoe and Davidson, 1970, and Hovmand and Davidson, 1971). However, Theil and Potter (1977) observed square nosed slugs in columns up to 0.22 m in diameter and the tendency to form square nosed slugs was related to the internal angle of friction of the solid material. In dense particle slugging beds, square nosed slugs are normally observed.

The slugging characteristics of dense particles determined from the pressure fluctuations signals and the height of the slugging beds are discussed in the following sections. The pressure fluctuations in turbulent beds of dense particle and the transition from slugging to turbulent regime is also discussed.

III-A. Pressure Fluctuations in Slugging Beds

Pressure fluctuations in the slugging fluidized bed have been measured and documented in the literature by Kohoe and Davidson (1973), Brodhurst and Becker (1976), Canada et al. (1978), Yerushalmi and Cankurt (1979), Fan et al. (1983), and Yamazaki et al. (1983). The pressure fluctuations at a point in the slugging bed are caused by the bed height fluctuation and passage of slug across the point. A
typical pressure signal measured (Y1) in a slugging bed of Al particles of 2.32 mm in diameter, at the height of 15 cm is shown in Fig. 5.2. When a slug is formed at the bottom of the bed, the bed height increases and consequently, the pressure at any point above the slug increases. When the roof of the slug reaches the 15 cm height, the level at which the probe is located, the pressure (Y1) begins to decrease and is at its lowest when the base of the slug is at 15 cm height. As the slug moves across the probe, the pressure begins to rise again to the minimum fluidization pressure. It is noted in the figure that the signal has a definite constant frequency. Similar signals are recorded for all the dense particles considered in this study operated in slugging fluidized beds.

III-B. Slug Rise Velocity

Several techniques have been used for measuring the slug or bubble velocity in gas-solid fluidized beds. The techniques included: photographing with ordinary cameras as well as with X-ray cameras, sensing with capacitance probes and electroresistivity probes and by cross-correlation function of signals obtained from pressure transducers. In this study, the slug rise velocities were measured for dense particles in the size range of 1 mm to 7 mm by calculating the cross-correlation function of two signals obtained through transducers. Figure 5.3 shows a typical cross-correlation function obtained for Al particles of 2.32 mm in diameter at superficial air velocity of 2.246 m/sec. The cross-correlation function shows an absolute maximum at time, tB, and local maxima at times t_{s1}, t_{s2}, t_{s3} etc. The distance between two transducers divided by the time
Dense Particle: Al 2.32 mm

$U = 2.248 \text{ m/sec}$

Fig. 5.2 Variation of Pressure at 15.2 cm above the Distributor Plate with Time
Fig. 5.3 Variation of the Cross-Correlation function between Two Signals acquired at 15.2 cm and 30.3 cm with Time
t_s yields the slug velocity. Time t_{s1}, t_{s2} etc. are obtained by adding time t_s to the multiples of time period of the signal.

The variation of the slug rise velocities calculated from these cross-correlation functions with the superficial air velocity are shown in Figs. 5.4 and 5.5. It is noted in these figures that the slug rise velocity initially increases with the increase of the superficial air velocity. However, beyond a certain air velocity, the slug velocity decreases with the increase in the superficial air velocity.

Slug rise velocity has been measured and correlated by several investigators including Davidson and Harrison (1963), Ormiston et al. (1965), Matsen et al. (1969), Kehoe and Davidson (1970), Theil and Potter (1977), Baker and Geldart (1978), Geldart et al. (1978), and Fan et al. (1983). The slug rise velocity has been correlated by the equation of the form

$$ U_s = k_2 (U_a - U_{mf}) + k_1 (gD)^{0.5} \quad (5.1) $$

where k_1 value is found to be equal to 0.35 for axisymmetric slugs and 0.5 for asymmetric slugs. Velocities of square nosed slugs were observed to be in the same range as axisymmetric slugs and asymmetric slugs (Baker and Geldart, 1978 and Geldart et al., 1978).

The slug velocities experimentally determined in this study for \( U_a - U_{mf} < 1.5 \) can be correlated by Eqn. (5.1). k_1 and k_2 values thus
Fig. 5.4 Variation of the Slug Velocity with the Superficial Air Velocity for Glass Beads of 1.0 mm.
Fig. 5.5 Variation of the Slug Velocity with the Superficial Air Velocity for Al Particles of 2.32 mm and 5.50 mm
obtained are shown in Table 5.2. $k_1$ and $k_2$ values reported by several investigators including Ormiston et al. (1965), Theil and Potter (1977), Fan et al. (1983), for fine particle slugging beds are also shown in Table 5.2. $k_1$ values for dense particle slugging beds are found to be in the same range as for fine particle slugging beds, whereas $k_2$ values are significantly less than those obtained for fine particle fluidized beds.

The decrease in the slug velocity observed beyond superficial air velocities of $U_{mf} + 1.5$ m/sec can be explained by considering the particle movement. At low superficial air velocities, the maximum height of the expanded bed is such that as the slug passes through a plane, the solid particles in that plane move up in the bed, reach their maximum height and come back to their original position before the next slug is formed at the bottom of the bed. At high superficial air velocity, the bed expansion is such that the next slug is formed when particles are falling down. The downward motion of these solid particles encountered by the slug retards the slug and the decrease in velocity of slugs is observed. This decrease in slug velocity observed with the increase in the air velocity at high air velocities has not been reported in the literature. This may be due to the low range of gas velocities considered by them. Fan et al. (1983) observed decrease in the slug velocity for particles of 0.711 mm in diameter beyond air velocities of 0.76 m/sec. The decrease in slug velocity was explained as due to the change in regime of fluidization to the turbulent regime. The transition velocity to the turbulent regime thus determined is too low compared to the values determined in this study and those reported by Canada et al. (1978).
Table 5.2. The parameters of slug velocity correlation described by Eqn. (5.1)

<table>
<thead>
<tr>
<th>Type of Particle</th>
<th>Values Determined By</th>
<th>( k_1 ) and ( k_2 ) Values Obtained By Least Square Analysis</th>
<th>( k_2 ) Values Obtained by Least Square Analysis With ( k_1 = 0.35 )</th>
<th>( k_2 ) Values Obtained by Least Square Analysis With ( k_1 = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GB 1.0 mm</td>
<td>This Study</td>
<td>0.512 0.774</td>
<td>0.948</td>
<td>0.787</td>
</tr>
<tr>
<td>Al 2.32 mm</td>
<td>This Study</td>
<td>0.464 0.555</td>
<td>0.662</td>
<td>0.521</td>
</tr>
<tr>
<td>Al 5.50 mm</td>
<td>This Study</td>
<td>0.320 0.271</td>
<td>0.243</td>
<td>0.104</td>
</tr>
<tr>
<td>Particles Upto 254(\mu m)</td>
<td>Ormiston et al. (1965)</td>
<td>0.35 .87-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Particles Upto 359(\mu m)</td>
<td>Theil and Potter (1977)</td>
<td>.35-.5 1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Particles Upto 1.12 mm</td>
<td>Fan et al. (1983)</td>
<td>0.35 1-4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
III-C. Height of the Slugging Bed

The maximum bed height of slugging fluidized beds were measured for particles from 1 mm to 7 mm in diameter. The variations of the ratio of maximum bed height to the minimum fluidization height with the superficial air velocity for all the four types of dense particles are shown in Fig. 5.6. At the same \((U_a - U_{mf})\), the bed expansion ratio decreases with the increase in the particle diameter.

Matsen et al. (1969), Baker and Geldart (1978), Geldart et al. (1978), Babu et al. (1978), Canada et al. (1978) and Miller et al. (1981) have documented the bed expansion data for slugging fluidized beds. Most of the data reported is for fine particle slugging beds. Matsen et al. (1969) analyzed over 60 sets of data and correlated the data with the equation

\[
\frac{H_m}{H_{mf}} - 1 = \frac{(U_a - U_{mf})}{0.35 (gD)^0.5}
\]  

Equation (5.2) is the most widely accepted correlation for bed expansion of slugging fluidized beds. As shown in Fig. 5.6, the bed expansion data for glass beads of 1.0 mm in diameter correlates well with Eqn. (5.2). For Al particles of 2.32 mm in diameter, Eqn. (5.2) overpredicts the bed expansion slightly whereas, for Al particles of 5.50 mm and 6.96 mm in diameter Eqn. (5.2) overpredicts the bed expansion data considerably. Similarly the correlation of Babu et al. (1978) overpredicts the bed height data for particles larger than 1.0 mm in diameter.

III-D. Slug Frequency

The frequency of pressure signals acquired from fluidized beds
Fig. 5.6 Variation of the Bed Expansion with the Superficial Air Velocity for Four Dense Particles
can be determined by calculating the power spectral density function. Figure 5.7 shows the power spectral density function of the pressure signal acquired at 15 cm. above the distributor plate for Al particles of 2.32 mm in diameter at superficial air velocity of 2.246 m/sec. At this air velocity the bed is in slugging regime. It is noted that the power spectral density function has a sharp peak at the frequency of 1.23 cycles/sec, which is the dominant frequency of the signal. The dominant frequency of the signal thus calculated is defined as the slug frequency in this study.

The variation of the slug frequency determined in this study for glass beads of 1.0 mm diameter with the superficial air velocity as a function of the height of the bed is shown in Fig. 5.8. The slug frequency is found to decrease with the increase of the gas velocity and the bed height. The variation of the slug frequency with the superficial air velocity is low and frequency appears to be approaching a limiting value. Figure 5.9 shows the variation of the frequency with the gas velocity for different particles. The frequency is found to be affected by the particle diameter, i.e. at the same \( U_a - U_{mf} \), particles with larger diameter show higher frequencies compared with the particles of smaller diameters. Also, the frequency is found to decrease sharply with the increase of the air velocity and the decrease in slug frequency continues upto \( U_a - U_{mf} \) of 4.5 m/sec.

Slug frequency has been investigated by several investigators including Verloop and Hertjees (1974), Bayenes and Geldart (1974), Brodhurst and Becker (1976), Theil and Potter (1977), Baker and
Fig. 5.7 Variation of the Power Spectral Density Function with the Frequency for Al Particles of 2.32 mm
Dense Particle: Glass Beads 1.0 mm

A Bed Height 20.3 cm
ο Bed Height 25.4 cm
+ Bed Height 30.0 cm

A Values Calculated by Eqn. (5.3)
B Values Calculated by Eqn. (5.4)
for Bed Height of 30 cm

Fig. 5.8 Variation of the Slug Frequency with the Superficial Air Velocity as a function of the Bed Height
Geldart (1978), Canada et al. (1978), Sadasivan et al. (1980) at room temperature and by Yamazaki et al. (1983) at high temperatures. Baker and Geldart (1978) found the dominant frequency to decrease with the increase of the bed height and the superficial gas velocity. Also the tendency for the frequency to approach the limiting value at high \( U_a - U_{mf} \) was observed. Bayenes and Geldart (1974) also found the slugging frequency to decrease with the increase of the bed height. But, beyond a certain bed height, the frequency was found to be by the bed height. The slug frequency was also found to be independent of particle physical properties which is in contrast to the observations made in this study. The slug frequencies calculated from the equations of Bayenes and Geldart (1974) described by Eqn. (5.3) and Brodhurst and Becker (1976) described by Eqn. (5.4) are also shown in Figs. 5.8 and 5.9.

\[
f = 1.17 \text{d}^{0.143} \tag{5.3}
\]

\[
f = 0.34\left(\frac{g}{D}\right)^{0.5} \left(\frac{H_{mf}}{D}\right)^{-0.85} \tag{5.4}
\]

The estimated slug frequencies from Eqn. (5.3) are in the same range as the experimental values. Equation (5.4) overpredicts the slug frequencies considerably.

**III-E. Pressure Fluctuations in Turbulent Beds**

Pressure signal observed in a turbulent fluidized bed is more random in nature compared to that in the slugging fluidized bed. Also the amplitude of the pressure signal in turbulent bed is much
Fig. 5.9 Variation of the Slug Frequency with the Superficial Air Velocity for Al Particles of 2.32 mm, 5.50 mm, 6.96 mm in diameter.
smaller than that of the slugging bed. Figure 5.10 shows a typical pressure signal again acquired at 15 cm above the distributor plate for Al particles of 2.32 mm in diameter, when the bed is operated in turbulent flow regime. Pressure fluctuations in a turbulent bed have been documented in literature by Canada et al. (1978) and Yerushalmi and Cankurt (1979).

The power spectral density function of the pressure signal acquired at 15 cm above the distributor plate at the air velocity of 6.095 m/sec, when the bed is operated in the turbulent regime, is shown in Fig. 5.11. It is noted that there is no well defined dominant frequency for turbulent beds.

III-F. Transition to Turbulent Regime

As the air velocity through a slugging bed is increased, the fluctuations grow in amplitude and the bed expansion increases. However, beyond a certain air velocity, denoted as $U_c$ by Yerushalmi and Cankurt (1979), the pressure fluctuations begin to decrease and the randomness in the fluctuations increases. As the air velocity is further increased, Yerushalmi and Cankurt (1979) found the fluctuations to level off beyond a certain air velocity denoted as $U_k$. The air velocity, $U_k$, was characterized as the onset of turbulence regime velocity by Yerushalmi and Cankurt (1979). They measured the transition velocity for particles upto 268 μm in diameter and found both $U_c$ and $U_k$ to decrease with the particle diameter. Also the ratios $U_c/U_t$ and $U_k/U_t$ were found to be in excess of ten for the beds of fine particles.

Theil and Potter (1977) conducted experiments with cracking
Dense Particle: Al 2.32 mm

U = 6.085 m/sec

Fig. 5.10 Fluctuations of the Pressure Signal acquired at 15.2 cm above the Distributor Plate.
Fig. 5.11 Variation of the Power Spectral Density Function with the Frequency.
catalyst in three beds of diameters 5.1, 10.2, and 21.6 cm. They indicated that the transition velocity to the turbulent regime decreases sharply with the bed diameter. It should be noted that, the results by Theil and Potter (1977) are indicated as a single transition velocity, without any specification of how it was determined. Staub and Canada (1978) also marked the transition to turbulent regime by a single velocity which was determined by several keys including visual observation, bed height data, pressure trace chart recordings and bed pressure drop. They estimated the onset to turbulent regime velocity ($U_{tr}$) to be little higher than $U_k$. Transition velocities $U_{tr}/U_t$ of 0.65 for 650 $\mu$m glass beads and 0.35 for 2.6 mm beads were determined by Staub and Canada (1978) in the bed of 0.305 m x 0.305 m. Yang (1982), based on the pressure fluctuation data of Li and Kwauk (1980), reported that the gas velocity $U_c$, at which the fluctuations begin to level off may represent the end of turbulent regime instead of the onset to turbulent regime velocity defined by Yerushalmi and Cankurt (1979).

The pressure signals recorded in this study at various superficial air velocities were utilized to calculate the mean fluctuation, $P_m$, at a point in the bed. The mean fluctuation is defined as

$$P_m = \frac{\sum_i (P_i - \bar{P})^2/N}{\bar{P}}$$

(5.5)

where $N$ is the total number of data points
and $\bar{P}$ is the average pressure.

The variation of the mean pressure fluctuation with the
superficial air velocity for all the four particles is shown in Figs. 5.12 and 5.13. It is noted in the figures that for glass beads of 1.0 mm in diameter fluctuations seem to level off beyond a certain air velocity, whereas for other particles the fluctuations continue to decrease even at high air velocities where the bed is observed to be in turbulent regime. Thus, it is difficult to define $U_k$ for some particles from the fluctuations amplitude data.

IV. DISCUSSION

IV-A. Characteristics of Dense Particle Slugging Beds

As noted in Section III of this chapter, dense particle slugging beds behave much differently as compared to the fine particle slugging beds. The difference between these two beds may be because of the slug velocity to particulate phase gas velocity ratio. Dense particle slugging beds operate in slow slug condition whereas, fine particle slugging beds operate in fast slug condition. In fine particle slugging beds there is no through flow of gas in the slug phase and the air velocity in particulate phase has been determined to be approximately equal to the minimum fluidization velocity by various investigators. In slow slug regime, considerable through flow of gas in slug phase is expected and the gas velocity in the particulate phase may not be equal to the minimum fluidization velocity. The through flow of gas through the slug will reduce the slug velocity and the maximum bed expansion. Also different particles at the same gas velocity will have different amount of through flow gas and therefore different slug frequency.
Fig. 5.12 Variation of the mean pressure fluctuation with the Superficial Air Velocity for Glass Beads of 1.0 mm and Al Particles of 2.32 mm in diameter.
Fig. 5.13 Variation of the mean pressure fluctuation with the Superficial Air Velocity for Al Particles of 5.50 mm and 6.96 mm in Diameter.
The values of $k_2$ with $k_1$ equal to 0.35 in the slug velocity Eqn. (5.1) determined in this study and shown in Table 5.2, can be correlated by the equation of the form

$$ k_2 = 1.476 \left( \frac{d_s}{D} \right)^{-0.9} \left( \frac{\rho_s}{1000 \rho_a} \right)^{-4.2} $$

(5.6)

for $d_s > 2.32$ mm

Similar correlation for $k_2$ was used by Fan et al. (1983) for slug velocity of fine particle slugging beds. The correlation can be described as

$$ k_2 = 2.43 \left( \frac{d_s}{D} \right)^{-0.5} \left( \frac{\rho_s}{1000 \rho_a} \right)^{4.2} $$

(5.7)

for $d_s < 1.12$ mm

The slug velocities calculated from Eqns. (5.1) and (5.6) for Al particles of 2.32 mm and 5.50 mm in diameter are compared with the experimental values, as shown in Fig. 5.14. The agreement between the two values is satisfactory. Similarly, the maximum bed height of the slugging bed can be correlated by the equation of the form

$$ \frac{H_m}{H_{mf}} - 1 = k_2 \frac{(U_a - U_{mf})}{\{0.35 (gD)^{0.5}\}} $$

(5.8)

where $k_2$ is given by Eqn. (5.6). The maximum bed height calculated from Eqns. (5.6) and (5.8) for Al particles of 2.32 mm, 5.50 mm, and 6.96 mm in diameter are compared with the experimental values as shown in Fig. 5.15. The comparison is found to be satisfactory.

The slug frequency can be correlated by the equation of the following form for particles greater than 2.32 mm in diameter

$$ f = 100 \left( \frac{d_s}{(U_a - U_{mf}) H_{mf}} \right)^{0.5} $$

(5.9)
Fig. 5.14 Comparison of the Experimental Slug Velocities with the Predicted Values
Fig. 5.15 Comparison of the Experimentally determined Bed Height with the Predicted Values.
where \( d_s, H_{mf} \) are in m and \( U_a, U_{mf} \) are in m/sec.

The comparison of the slug frequency values calculated by the Eqn. (5.9) with the experimental values for three types of dense particles, as shown in Fig. 5.9, is found to be satisfactory.

IV-B. Transition to Turbulent Regime

The transition to turbulent regime can be more easily defined by considering the probability density function of the pressure signal acquired at the height which is greater than the minimum fluidization height, as discussed in the following. The pressure was recorded at two positions in the fluidized bed which were located at 15.2 cm and 30.3 cm from the distributor plate. For the bed of height of 20.3 cm at minimum fluidization, the minimum pressure at 30.3 cm from the distributor plate (denoted as Y2) is around zero. As the slug rises through the bed, Y2 increases and as the bed collapses to about minimum fluidization height, Y2 approaches zero. Figure 5.16 shows a typical Y2 profile obtained with the bed of Al particles of 2.32 mm in diameter and at superficial air velocity of 2.246 m/sec. The probability density plot obtained from this data is shown in Fig. 5.17. Clearly the Y2 pressure is around zero for about 10% of the time. As the flow regime of the bed changes to the turbulent regime, the fluctuations in the bed height decrease and if the bed height is higher than 30 cm, pressure Y2 remains above zero. Figure 5.18 shows the pressure profile obtained for Al particles of 2.32 mm in diameter at superficial air velocity of 5.476 m/sec. The variation of the probability density function with the pressure is shown in Fig. 5.19. It is noted that the probability of Y2 being zero
Fig 5.16 Fluctuations of the Pressure Signal acquired at 30.3 cm above the Distributor Plate.
Fig. 5.17 Variation of the Probability Density Function with the Pressure for Y2

Dense Particle: Al 2.32 mm

U = 2.248 m/sec
Fig. 5.18 Fluctuations of the Pressure Signal acquired at 30.3 cm above the Distributor Plate.
Dense Particle: Al 2.32 mm

$U = 5.476 \text{ m/sec}$

Fig. 5.19 Variation of the Probability Density Function with the Pressure for Y2
is almost negligible. Thus, onset of turbulence can be defined as the air velocity at which the probability of $Y_2 = 0$, denoted as $p_0$, is equal to zero. Figure 5.20 shows the variation of $p_0$ with the superficial air velocity for all the four dense particles utilized in this study. The onset of turbulence velocity calculated from this figure are shown in Table 5.3. It is noted that these velocities match closely to the visually observed onset of turbulence velocities.

The onset of turbulence velocities determined in this study (in column of diameter 10 cm) correspond closely to the values of Canada et al. (1978) (obtained in column of 0.305 m x 0.305 m). Thus, column diameter above 10 cm may not have any effect on the onset of turbulence velocity. This conclusion will have to be verified with additional experimental data obtained in different diameter columns.

It is noted that the probability density function of the signal acquired from the level above the minimum fluidization height of the bed at pressure equal to zero ($p_0$) will be zero in turbulent regime only if the height of the turbulent bed is higher than the location of the pressure transducer and the bed is operated at atmospheric pressure. The void fraction of the bed at transition to turbulent regime has been observed to be approximately 0.75 (Canada et al., 1978). Thus, if the pressure transducer is located between the heights, $H_{mf}$ and $3H_{mf}/2$, the bed height of the turbulent bed will be higher than the level of the pressure transducer and $p_0$ will be zero. In case, the transducer is located above the surface of the turbulent bed, $p_0$ will be approximately one in the turbulent bed and a sharp jump in $p_0$ at the transition will be noticed in contrast to the drop of $p_0$ observed in the alternative situation. If the bed is operated
Fig. 5.20 Variation of $p_0$ with the Superficial Air Velocity
Table 5.3. Transition to Turbulent Regime Velocity

<table>
<thead>
<tr>
<th>Type of Particle</th>
<th>Determined By</th>
<th>$U_{tr}/U_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GB 0.65 mm</td>
<td>Canada et al. (1978)</td>
<td>0.50</td>
</tr>
<tr>
<td>GB 1.00 mm</td>
<td>This Study</td>
<td>0.459</td>
</tr>
<tr>
<td>Al 2.32 mm</td>
<td>This Study</td>
<td>0.368</td>
</tr>
<tr>
<td>GB 2.60 mm</td>
<td>Canada et al. (1978)</td>
<td>0.35</td>
</tr>
<tr>
<td>Al 5.50 mm</td>
<td>This Study</td>
<td>0.343</td>
</tr>
<tr>
<td>Al 6.96 mm</td>
<td>This Study</td>
<td>0.338</td>
</tr>
</tbody>
</table>
at high pressure, this method of determining the transition to turbulent regime can be utilized by considering the probability density function at pressures approximately equal to the outlet pressure instead of zero pressure.

V. CONCLUDING REMARKS

Experiments are conducted to analyze the characteristics of slugging fluidized beds of dense particles. The slugging characteristics analyzed include pressure fluctuations in the slugging beds, slug velocities, maximum height of the slugging beds, and the frequency of slugs. Dense particles slugging beds are found to behave much differently as compared to the fine particle slugging beds. The slug velocities, height of the slugging beds, and the slug frequency for dense particle beds are semi-empirically correlated.

A new method of determining the transition velocity from slugging regime to the turbulent regime by determining the probability density function of the pressure signal acquired from the height above the minimum fluidization height is described. The transition velocities of the dense particles determined by this method are found to match with the visually observed transition velocities.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The key to widespread commercialization of fluidized bed combustors is to be able to accurately predict the fundamental properties such as hydrodynamic, mass transfer and heat transfer properties of these combustors. One of the more promising of these combustors, the multisolid pneumatic transport combustor is extensively analyzed in this study. The hydrodynamic characteristics of the dense bed and the pneumatic transport bed, sulfation kinetics of limestone/dolomite sorbents and the characteristics of slugging fluidized beds of large diameter particles are investigated and analyzed in this study. The important conclusions of this study are described in the following:

1. The modified volume reaction model is capable of accounting for the extent of sulfation during the entire course of the reaction as well as the final sulfation level of the calcined sorbents. The model can be utilized to determine the sorbent specific data including the effective gas diffusivity, reaction rate constant, and the porosity variation constant from the TGA data.

2. The reaction model of limestone/dolomite sulfation when combined with the reactor model of the vertical pneumatic transport reactor,
accounts for the limestone/dolomite sulfation in the vertical pneumatic transport reactor. The model predictions are shown to match with the experimental sulfur retention and sorbent utilization data obtained from units at Battelle Columbus Laboratories for limestone sulfation in the multisolid fluidized bed combustors.

3. The residence time of fine particles in the fluidized dense bed of the multisolid pneumatic transport bed is higher by a factor as large as ten, compared to the residence time obtained without dense bed in the system.

4. The general expressions of the interaction coefficient developed for both packed and fluidized dense beds used along with the momentum balance equation of the fine particles can be utilized to satisfactorily predict the hold-up of fine particles in dense beds of the multisolid pneumatic transport bed.

5. The pressure drop in the packed dense bed of the multisolid pneumatic transport bed, which is established by fluidizing and then defluidizing the dense particle bed by increasing and then decreasing the air flow rate at a given fine particle flow rate, decreases with the introduction of fine particle in the air flow. As the flow rate of fine particle is increased at a fixed air flow, the pressure drop in the packed dense bed decreases up to a certain fine particle flow rate, beyond which the pressure drop increases with the fine particle flow rate.

6. Modified form of Ergun equation developed in this study can be used to satisfactorily predict the pressure drop in the packed dense bed of the multisolid pneumatic transport bed with air-fine particle flow.

7. Pressure drop in the packed and fluidized dense beds of the
The multisolid pneumatic transport bed with air-fine particle flow can be utilized to determine the minimum fluidization velocity of the dense bed with air-fine particle as fluidizing medium.

8. The experimental method of determining the terminal velocity of dense particles in air and air-fine particle heterogeneous medium from the turbulent fluidized bed voidage-air velocity relationship developed in this study yields satisfactory results.

9. The terminal velocity of dense particles monotonically decreases with the increase of the fine particle flow rate in the multisolid pneumatic transport bed.

10. The slug velocity and height of the slugging beds for dense particle beds in the multisolid pneumatic transport bed are much smaller than that for fine particle beds.

11. The new method developed to determine the transition from slugging regime to turbulent regime by determining the probability density function of the pressure signal acquired from the height above the static bed height, defines the transition more sharply as compared to other methods available in literature.

Although the main focus of the study is to analyze the multisolid pneumatic transport combustor, the problems addressed are rather general in nature and can be applied to a variety of other processes. For example, the kinetic model for limestone/dolomite sulfation can be used for prediction of sulfur retention or the sorbent utilization in any unit. Only the hydrodynamic characteristics of the unit will have to be combined with the kinetic model. The model developed for non-catalytic gas-solid reaction in the vertical pneumatic transport reactor can be utilized to predict the sulfur retention, extent of
gasification or combustion of coal as well as biomass, and the calcination of aluminum hydroxide where the solid properties vary with the reaction. The correlation of interaction coefficient determined in the terminal velocity analysis can be utilized to study the elutriation of particles in fluidized bed reactors where both coarse particles and fine particles are present. In fact in a recent study by Geldart, it was mentioned that coarse particles with the terminal velocity greater than the operating gas velocity are found to be elutriated in the presence of fine particles. The elutriaition of these coarse particles can be estimated with the use of the interaction coefficient developed in this study.

Some recommendations for future work are described in the following:
1. The investigation of the apparent drag reduction phenomenon observed in this study should include the study of air and air-fine particle flow pattern across a dense particle and the arrangement of dense particles in the packed bed with air flow and air-fine particle flow.
2. The emulsion phase gas velocity and the through flow in the slug phase can be measured to understand the behavior of dense particle slugging beds. This investigation may explain the reason for dense particle slugging beds behavior being so different than fine particle slugging beds
3. Theoretical prediction of the transition velocity from slugging and bubbling regimes to the turbulent regime.
4. The effect of scale-up, including larger diameter unit, high temperatures, high pressures, on the hydrodynamic characteristics of
the multisolid pneumatic transport reactor.

LIST OF REFERENCES


40. Fan, L.S., S. Satija, B.C. Kim, and H. Nack,


54. Hartke, W., H. Helmrich, H. Kroger, and K. Schugeler, "Non-Catalytic Decomposition of Sodium Bicarbonate in a


69. Lee, D.C., and C. Georgakis, "A Single Particle Size Model for Sulfur Retention in Fluidized Bed Coal Combustors," AIChEJ,


APPENDIX A

CALIBRATION OF THE ORIFICE METER FOR PNEUMATIC TRANSPORT BED

The calibration of orifice meter was accomplished by measuring the flow rate of water flowing through the orifice and the pressure drop across the orifice. The pressure drop across the orifice thus measured was correlated to the volumetric flow rate of water as

\[ \Delta H = 69906.325 \bar{V}^2 - 5.7649 \bar{V} \quad (A.1) \]

where \( \Delta H \) is the pressure drop across the orifice in m \( \text{H}_2\text{O} \)
\( \bar{V} \) is the volumetric flow rate in m\(^3\)/sec

For flow through an orifice, the pressure drop across the orifice can be related to the fluid velocity as

\[ U = C_0 \left( 2 \, g_c \frac{(-\Delta P/\rho)}{(S_1^2/S_0^2 - 1)^{0.5}} \right) \quad (A.2) \]

where \( C_0 \) is the orifice coefficient which is a function of Reynolds number and the orifice dimensions.

The pressure drop equation can be simplified to

\[ (\Delta P/\rho) = U_1^2 \frac{a}{C_0^2} \quad (A.3) \]
where \( a \) is function of dimension of orifice and pipe only.

From the equation of pressure drop across the orifice for water flow described by Eqn. (A.1), \( a/c_0^2 \) can be calculated as

\[
a/c_0^2 = 69.9063 - 288.25 \, N_{Re} \tag{A.4}
\]

which should be valid for air flow also. Thus simplifying Eqn. (A.4) for air flow at NTP, the following relation is derived

\[
\Delta H \left( P/P_0 \right) \left( T_0/T \right) = a \frac{V_0^2}{\nu_0} - b \frac{V_0}{\nu_0} \tag{A.5}
\]

where \( P_0 \) is 1 atm.

\( T_0 \) is 273 K

\( a \) is 90.4557

\( b \) is 0.0798213

The volumetric flow rate of air at NTP can be written as

\[
\bar{V}_0 = b + \left\{ b^2 + 4a\Delta H(P/P_0)(T_0/T) \right\}^{0.5}/2a \tag{A.6}
\]

and the volumetric flow rate of air at \( P \) atm and \( T \) K can be described as

\[
V = \bar{V}_0 \left( P_0/P \right) \left( T/T_0 \right) \tag{A.7}
\]

The program used for calculating the air velocity is listed below. The program calculates the air velocity. Input to the program include orifice pressure drop, absolute pressure in the air inlet pipe,
temperature, average pressure in the column.

C
C THIS PROGRAM CALCULATE THE SUPERFICIAL AIR VELOCITY THROUGH
C THE COLUMN FROM THE ORIFICE PRESSURE DROP
C
C THE INPUT PARAMETERS INCLUDE ORIFICE PRESSURE DROP, ABS. PRESSURE
C IN THE INLET PIPE, ABS. PRESSURE AT THE BOTTOM AND TOP OF
C THE COLUMN AND THE TEMPERATURE INSIDE THE COLUMN.
C
C THE PROGRAM ALSO TABULATES THE LOG VALUE OF THE PRESSURE DROP
C ALONG WITH LOG OF SUPERFICIAL VELOCITY IF DESIRED.
C

REAL LGP2, LGP3, LP2, LP3
WRITE(6, 10)
10 FORMAT('1', 2X, 'INPUT DATA ON TERMINAL OR THROUGH A DATA FILE' //
1 5X, 'ENTER 1 FOR DATA TO BE ENTERED ON TERMINAL' //5X,
2 'ENTER 2 FOR DATA TO BE ENTERED THROUGH A FILE ASSIGNED FOR 003' //)
12 READ(5, *) IM
IF (IM.EQ.2) GO TO 30
IF (IM.NE.1 .AND. IM.NE.2) GO TO 40
20 IO=5
GO TO 50
30 IO=3
GO TO 50
40 WRITE(6, 60)
60 FORMAT('1', 2X, 'ENTER 1 OR 2' //)
GO TO 12
50 PI=22./7.
A=90.45557
B=7.98213E-2
PS=760.
TS=273.
D=0.10167
DH=.0001
IF(DH.LT.0.0001) GO TO 720
IF (IM.EQ.2) GO TO 85
WRITE(6, 70)
70 FORMAT('2X, 'INPUT DATA' //5X, 'DIAMETER OF DENSE PARTICLE (mm)' //,
1 'FINE PARTICLE FLOW RATE (g/sec)' //)
81 READ(IO, *) PD, GS
GO TO 86
85 READ(IO, *) IP, PD, GS
86 IF(PD.GT.10.) GO TO 315
WRITE(7, 310)
310 FORMAT('1', 30X, 'AL PARTICLE')
GO TO 320
315 WRITE(7, 330)
330 FORMAT('1', 30X, 'STEEL PARTICLE')
320 WRITE(7, 340) D, PD, GS
268

340 FORMAT(///10X, 'DIAMETER OF THE COLUMN = ', F9.5, ' M'
1  /10X, 'DIAMETER OF THE DENSE BED PARTICLE = ', F9.4, 'MM'
1  /10X, 'SOLID FLOW RATE = ', F9.4,
1  'GM/SEC')
IF (IM.EQ.2) GO TO 82
WRITE(6,71)
71 FORMAT(//2X, 'INPUT ATMOSPHERIC PRESSURE (mm Hg)')
82 READ(I0,*) AP
WRITE(7,100) AP
100 FORMAT(///10X, 'ATMOSPHERIC PRESSURE - ', F7.2/) 
WRITE(7,200)
200 FORMAT(///14X, 'ORIFICE METER', 30X, 'TEST SECTION'//
$ 5X, 'GAUGE PRESS.', 1X, 'PRESS. DIF.', 2X, 'TEMP.', 
$ 1X, 'GAUGE PRESS.', 1X, 'GAUGE PRESS.', 2X, 'AIR VEL.', 4X, 'DP5',
$ 4X, 'CDP', 4X, LN(U)', 4X, LN(DP5)', 4X, LN(CDP)',
$ 7X, '(in Hg)', 4X, DH (in H2O)', 2X, '( C)', 2X,
$ '2) (mm Hg)', 3X, '3) (mm H2O)', 1X, '(m/sec)', 2X, '(mm H2O)'
$ '1X, '(mm H2O)'///)
IF (IM.EQ.2) GO TO 83
WRITE(6,72)
72 FORMAT(/// 'INPUT ABS. PRES IN INLET PIPE (in Hg), ORIFICE PRES DROP (',
$ 'in H2O), TEMP (C)', '/ ABS.PRES IN COL. BOTTOM (mm Hg) AND TOP (mm H2O)
$ 'PRES DROP IN COL. (mm H2O)'/
$ 'PRES DROP ACROSS DENSE BED (mm H2O)'/
$ 'INPUT ABS. PRES. IN INLET PIPE AS ZERO TO READ NEW FLOW RATE'/
$ 'INPUT ORIFICE PRES DROP ALSO ZERO FOR END OF THE RUN')
83 DO 500 J=1,30
IF (GS.LT.1.E-5) GO TO 88
READ(I0,*) OGP,DH,TE,LGP2,LGP3,DP5,CDP
GO TO 89
88 READ(I0,*) OGP,DH,TE,LGP2,LGP3,DP5
89 IF (OGP.LT.0.0001) GO TO 700
OP=OGP*25.4+AP
ODH=DH*2.54/100.
T=TE+273.
LP2=LGP2+AP
LP3=LGP3/13.6+AP
VONT=(B+SQRT(B**2+4.*A*ODH*OP*TS/PS/T))/2./A
V=2.*VONT*PS*T/((TS*(LP2+LP3))
U=4.*V/(PI*U**2)
AU=ALOG(U)
ADP5=ALOG(DP5)
IF (GS.LT.1.E-5) GO TO 163
ACDP=ALOG(CDP)
163 WRITE(7,300) OGP,DH,TE,LGP2,LGP3,U,DP5,CDP,AU,ADP5,ACDP
300 FORMAT(5X,F8.3,5X,F8.3,2X,F7.1,4X,F7.2,7X,F7.2,3X,F7.3,2X,F7.1,
$ 1X,F7.1,1X,F9.4,1X,F9.4,1X,F9.4)
500 CONTINUE
700 GO TO 710
720 STOP
END
APPENDIX B

OPERATING MANUAL FOR THE MULTISOLID PNEUMATIC TRANSPORT BED WITH SHUTTERS

1. Load the stand pipe with the desired type of fine particles. Loading can be accomplished through the hopper on the second floor.

2. Check the support plate in the center of the bed. The opening size of the sieve of support plate should be smaller than the size of dense particles to be used.

3. Desired type of dense particles can be loaded by removing the top section of the bed on the second floor.

4. Fill the humidifier to about 1 in above the lower tap with hot water, 20 mins. before the scheduled beginning of the experiment.

5. Connect the jacket below the humidifier and set the rheostat to about 80.

6. Air at the superficial velocity of 2 m/sec should be allowed to flow through the bypass valve.

7. Check if all the solenoids and the solenoid valve in the air inlet line are connected.

8. Humidity of the air can be checked by positioning a wet and dry bulb thermometers above the bypass air line.

9. After the humidity level is above 70%, the desired amount of air flow and fine particle flow should be allowed to flow through the column.

10. Air flow can be determined by measuring the orifice pressure drop, pressure in the air inlet line, temperature, and average pressure in the column. Fine particle flow can be determined by collecting fine particles in the sampling bottle on the second floor for a measured length of time. The weight of the fine particles collected divided by the time interval yields the fine particle flow rate.
11. After activating the solenoids and isolating the column through shutters by pressing the common switch, the air inlet valve should be closed, the bypass valve should be opened as soon as possible and on/off switch on the same switch box should be turned off. This will open the solenoid valve.

12. The hold-up of fine particles and dense particles can be determined by taking off the top section of the bed, sucking the particles from the each shutter, top to bottom, through vacuum and collecting the particles in the vacuum bag.

13. After finishing the desired set of experiments, the jacket heater should be disconnected and the air flow should be maintained for about 20 minutes afterwards.
APPENDIX C

PRESSURE DROP IN THE MULTISOLID PNEUMATIC TRANSPORT BED WITH AIR FLOW

Pressure drop in the multisolid pneumatic transport bed was experimentally measured with retaining grid in the center of the bed. The experimentally measured pressure drop is correlated and described below.

The pressure drop between two points located at a distance of 2.1 m on the two sides of retaining grid is determined as

At low $U_a$, i.e., $U_a < 4.534$ m/sec

$$
\Delta P = 0.1594 \ (U_a)^{1.2809}
$$

(C.1)

for high $U_a$, i.e., $U_a > 4.534$ m/sec

$$
\Delta P = 0.0551 \ (U_a)^{1.984}
$$

(C.2)

The pressure drop between two points located at a distance of 3.9 m on the two sides of retaining grid is determined as

At low $U_a$, i.e., $U_a < 4.015$ m/sec

$$
\Delta P = 0.8456 \ (U_a)^{0.3106}
$$

(C.3)
for high $U_a$, i.e., $U_a > 4.015$ m/sec

$$\Delta P = 0.1181 \left(U_a\right)^{1.727} \quad \text{(C.4)}$$

where $U_a$ is in m/sec

and $\Delta P$ is in kg/m$^2$
APPENDIX D

COMPUTER PROGRAMS FOR REAL TIME DATA ACQUISITION AND
STATISTICAL ANALYSIS OF THE DATA

PROGRAM FOR REAL TIME DATA ACQUISITION

DATA IS ACQUIRED THROUGH TWO CHANNELS AD3 AND AD4 AND IS
STORED IN A BINARY FILE FOR007.DAT

INTEGER*2 N1(10000), N2(10000)
INTEGER*2 AD0, AD1, AD2, AD3, AD4, AD5, AD6, AD7
INTEGER*2 DA0, DA1, DA2, DA3, DA4, DA5, DA6, DA7
INTEGER*2 DOUT0, DOUT1
COMMON/ADDATA/AD0, AD1, AD2, AD3, AD4, AD5, AD6, AD7
COMMON/DOUT/D0UT0, D0UT1
WRITE(6, 100)
100 FORMAT('1 ', 'INPUT THE NUMBER OF DATA POINTS DESIRED ')
READ(5, *) N
WRITE(6, 110)
110 FORMAT('///', 'INPUT THE IDENTIFICATION RUN NO. ')
READ(5, *) NI
CALL RTPSETUP
CALL DELAY(5)
DO I=1, N
CALL AD
N1(I)=AD3
N2(I)=AD4
ENDDO
WRITE(7) NI
WRITE(7) N
WRITE(7) (N1(I), N2(I), I=1, N)
STOP
END

C
C

C THIS PROGRAM CALCULATES THE STATISTICAL FUNCTIONS FROM
C THE REAL TIME DATA OF PRESSURE FLUCTUATIONS
THE STATISTICAL FUNCTIONS CALCULATED INCLUDE PROBABILITY DENSITY FUNCTIONS, PROBABILITY DISTRIBUTION FUNCTIONS, MEAN, VARIANCE, AUTO CORRELATION FUNCTIONS, CROSS CORRELATION FUNCTION, AND POWER SPECTRAL DENSITY FUNCTIONS

DIMENSION X(10000), Y1(10000), Y2(10000)
DIMENSION XP1(10000), ACC1(10000), ACC2(10000), CC(10000)
DIMENSION PDF1(10000), PDF2(10000), XP2(10000)
DIMENSION PDSF1(10000), PDSF2(10000), G1(10000), G2(10000)
DIMENSION YF1(10000), YF2(10000), YF3(10000), YF4(10000)
DIMENSION YFI1(10000), YFI2(10000), YFI3(10000), YFI4(10000)
INTEGER*2 NL(10000), N2(10000)
CHARACTER A*10, B*10, C*10, NC*1, D*38
CHARACTER ANAME*15
COMMON/PLTIT/A,B,C,D

THE DATA FILE SHOULD BE ASSIGNED TO FOR003 AND THE BATCH CALCULATIONS FILE TO FOR004

PRINT *, ' HAVE YOU ASSIGNED RAW DATA FILE TO FOR003 '
PRINT *, ' AND THE BATCH CALCULATIONS FILE TO FOR004 '
READ (3) NI
READ (3) N
READ (3) (NL(I), N2(I), I=1, N)
CALL TAB(NI, D)
PI=3.14157
AN=FLOAT(N)
N=ALOG(AN)/ALOG(2.)+0.2

CALCULATION OF MINIMUM AND MAXIMUM VALUES

N1MIN=NL(1)
N1MAX=NL(1)
N2MIN=N2(1)
N2MAX=N2(1)
DO I=1, N
IF (NI(I).LT.N1MIN) N1MIN=NI(I)
IF (NI(I).GT.N1MAX) N1MAX=NI(I)
IF (N2(I).LT.N2MIN) N2MIN=N2(I)
IF (N2(I).GT.N2MAX) N2MAX=N2(I)
ENDDO
ND1=N1MAX-N1MIN
ND2=N2MAX-N2MIN

CALCULATION OF PROBABILITY DENSITY FUNCTIONS AND PROBABILITY DISTRIBUTION FUNCTIONS

I=1
DO II=1, ND1+1, 10
IPD=0
NN=NMIN+N+1
DO J=1,N
IF ((N1(J)==NN).LT.10 ) IPD=IPD+1
ENDDO
IF (IPD.LT.1) GO TO 20
IF (II.GT.1) GO TO 50
IPDO=0
50 PDF1(I)=FLOAT(IPD-IPDO)/FLOAT(N)
PDSF1(I)=FLOAT(IPD)/FLOAT(N)
XP1(I)=FLOAT(NN+5)*15./2047.
I=I+1
IPDO=IPD
20 CONTINUE
ENDDO
NP1=I-1
IF (NMIN.LT.-10 ) THEN
DO I=1,NP1
J=NP1-I+1
IF (J.EQ.NP1) THEN
PDSF1(J)=PDF1(J)
ELSE
PDSF1(J)=PDSF1(J+1)+PDF1(J)
ENDIF
ENDIF
ENDF
ENDDO
I=1
DO II=1,ND2+1,10
IPD=0
NN=N2MIN+N+1
DO J=1,N
IF ((N2(J)==NN).LT.10 ) IPD=IPD+1
ENDDO
IF (IPD.LT.1) GO TO 30
IF (II.GT.1) GO TO 60
IPDO=0
60 PDF2(I)=FLOAT(IPD-IPDO)/FLOAT(N)
PDSF2(I)=FLOAT(IPD)/FLOAT(N)
XP2(I)=FLOAT(NN+5)*15./2047.
IPDO=IPD
I=I+1
30 CONTINUE
ENDDO
NP2=I-1
IF (N2MIN.LT.-100 ) THEN
DO I=1,NP2
J=NP2-I+1
IF (J.EQ.NP2) THEN
PDSF2(J)=PDF2(J)
ELSE
PDSF2(J)=PDSF2(J+1)+PDF2(J)
ENDIF
ENDIF
ENDIF
ENDDO
CONVERSION OF INTEGER COUNTS TO THE PRESSURE AND CALCULATION OF MEAN

SUMY1=0.
SUMY2=0.
DO I=1,N
   Y1(I)=FLOAT(N1(I)+5)*15./2047.
   Y2(I)=FLOAT(N2(I)+5)*15./2047.
   X(I)=.01*FLOAT(I-1)
   SUMY1=SUMY1+Y1(I)
   SUMY2=SUMY2+Y2(I)
ENDO
SUMY1=SUMY1/FLOAT(N)
SUMY2=SUMY2/FLOAT(N)

CALCULATING THE RMS VALUE OF FLUCTUATIONS

RMS1=0.0
RMS2=0.0
DO I=1,N
   RMS1=RMS1+(Y1(I)-SUMY1)**2
   RMS2=RMS2+(Y2(I)-SUMY2)**2
ENDO
RMS1=SQRT(RMS1/FLOAT(N))
RMS2=SQRT(RMS2/FLOAT(N))
ARMS1=RMS1/SUMY1
ARMS2=RMS2/SUMY2
WRITE(6,63) RMS1, ARMS1, RMS2, ARMS2

PLOTTING THE RAW DATA

PRINT *, 'DO YOU WANT TO PLOT THE DATA? (Y/N)'
READ(5,1000) NC

1000 FORMAT(A1)
IF(NC.EQ.'N') GO TO 100
A= 'Y1 VS TIME '
B= 'TIME (sec)' 
C= 'PRES. (psi)'
PRINT *, 'INPUT THE MAXIMUM TIME TO BE PLOTTED'
READ(5,*) TMAX
NTT=TMAX*100+1
CALL PLOTT(X,Y1,NTT,2,0)
A= 'Y2 VS TIME ' 
CALL PLOTT(X,Y2,NTT,2,0)

PLOTTING THE PROBABILITY DENSITY FUNCTIONS


100 PRINT *, 'DO YOU WANT TO CALCULATE THE PROBABILITY DENSITY',
    1 'FUNCTION? (Y/N)' READ(5,1000) NC IF (NC.EQ. 'N') GO TO 110
A= 'PDF OF Y1'
B= 'PRES. (psi)'
C= 'PROB.FUNC.'
CALL PLOTT(XP1,PDF1, NP1,2,0)
A= 'PDF OF Y2'
CALL PLOTT(XP2,PDF2, NP2,2,0)
C
C PLOTTING THE PROBABILITY DISTRIBUTION FUNCTIONS C
110 PRINT *, 'DO YOU WANT TO CALCULATE THE PROBABILITY',
    1 'FUNCTION (Y/N)' READ (5, 1000 ) NC IF (NC.EQ. 'N') GO TO 111
A= 'CDF OF Y1'
B= 'PRES. (psi)'
C= 'CUMM.PROB.'
CALL PLOTT(XP1,PDSF1, NP1,2,0)
A= 'CDF OF Y2'
CALL PLOTT(XP2,PDSF2, NP2,2,0)
C
C CALCULATING AND PLOTTING THE POWER SPECTRAL DENSITY FUNCTIONS
C FROM THE RAW DATA
111 PRINT *, 'DO YOU WANT TO CALCULATE POWER SPECTRAL DENSITY',
    1 'FUNCTIONS FROM RAW DATA? (Y/N)' READ(5,1000) NC IF (NC.EQ. 'N') GO TO 112
F=0.D0.
DO I=1,N YF1(I)=Y1(I) YF2(I)=0.D0.
ENDDO
CALL FFT(YF1,YF2,N,NW,0.)
DO I=1,N G1(I)=YF1(I)**2+YF2(I)**2 X(I)=FLOAT(I-1)*100./FLOAT(N)
ENDDO
A= 'PSDF of Y1'
B= 'FREQ. (c/s)'
C= 'DENS.FUNC.'
PRINT *, 'INPUT THE MAXIMUM FREQUENCY TO BE PLOTTED'
READ (5,*) BB NN=BB*FLOAT(N)/100.
CALL PLOTT(X,G1,NN,2,1)
DO I=1,N YF3(I)=Y2(I) YF4(I)=0.
ENDDO
CALL FFT(YF3,YF4,N,NW,0.)
DO I=1,N
\[ G_2(I) = YF_3(I)^2 + YF_4(I)^2 \]
\[ X(I) = \text{FLOAT}(I-1) \times 100. / \text{FLOAT}(N) \]

ENDDO

A = 'PSDF of Y2'
CALL PLOTT(X,G2,NN,2,1)

C FILTERING THE NOISE

112 PRINT *, 'DO YOU WANT TO FILTER THE NOISE (Y/N)'
READ (5,1000) NC
IF (NC.EQ. 'N') GO TO 115
113 PRINT *, 'INPUT THE FREQ. FOR LOW PASS FILTER'
READ (5,*) ICUT
PRINT *, 'INPUT THE AMPLITUDE AFTER TRUNCATION'
READ (5,*) AMP
H = 0.01
FRQ = 1. / (H*FLOAT(N))
ICUT = ICUT / FRQ
DO I = 1, N
YFI1(I) = YF1(I)
YFI2(I) = YF2(I)
YFI3(I) = YF3(I)
YFI4(I) = YF4(I)
ENDDO
CALL FILTER(YFI1,YFI2,ICUT,AMP,N)
CALL FILTER(YFI3,YFI4,ICUT,AMP,N)

C CALCULATING THE INVERSE FOURIER TRANSFORM

CALL FFT(YFI1,YFI2,N,NW,2)
CALL FFT(YFI3,YFI4,N,NW,2)
PRINT *, 'DO YOU WANT TO PLOT THE FILTERED DATA (Y/N)'
READ (5,1000) NC
IF (NC.EQ. 'N') GO TO 115
A = 'FILT. DATA'
B = 'TIME (sec)'
C = 'Y1'
DO I = 1, N
X(I) = FLOAT(I-1) * 0.01
ENDDO
CALL PLOTT(X,YFI1,1000,2,0)
C = 'Y2'
CALL PLOTT(X,YFI3,1000,2,0)

C TRYING ANOTHER FILTER FREQUENCY

PRINT *, 'DO YOU WANT TO TRY ANOTHER FILTER FREQ. (Y/N)'
READ (5,1000) IC
IF (IC.EQ. 'Y') GO TO 113
PRINT *, 'DO YOU WANT TO KEEP THE ORIGINAL DATA (Y/N)'
READ (5,1000) IC
IF (IC.EQ. 'Y') GO TO 115
DO I = 1, N
Y1(I) = YF11(I)
Y2(I) = YF13(I)
ENDO

CALCULATING AND PLOTTING THE AUTO CORRELATION FUNCTIONS

DO I = 1, N
  X(I) = FLOAT(I-1) * 0.01
ENDO

PRINT *, ' DO YOU WANT TO CALCULATE AUTO CORRELATION',
1 ' FUNCTIONS (Y/N)'
READ (5, 1000) NC
IF (NC.EQ. 'N') GO TO 120
PRINT *, ' ARE THE CALCULATIONS DONE BY BATCH PRGM. (Y/N)'
READ (5, 1000) NB
IF (NB.EQ. 'Y') THEN
  PRINT *, ' ENTER THE FILE NAME OF CALCULATED DATA'
  READ (5, 119) NAME
  FORMAT (A15)
  OPEN(UNIT=4, FILE=NAME, STATUS='OLD')
  READ (4) NI
  READ (4) (ACC1(I), ACC2(I), I = 1, 200)
  CALL TAB(NI, D)
ELSE
  DO J = 1, N
    ACC1(J) = 0.
    ACC2(J) = 0.
  DO I = 1, N+1-J
    ACC1(J) = ACC1(J) + (Y1(I) - SUMY1) * (Y1(I+J-1) - SUMY1) * 0.01
    ACC2(J) = ACC2(J) + (Y2(I) - SUMY2) * (Y2(I+J-1) - SUMY2) * 0.01
  ENDDO
  ACC1(J) = ACC1(J) / (FLOAT(N+1-J) * 0.01)
  ACC2(J) = ACC2(J) / (FLOAT(N+1-J) * 0.01)
ENDDO
ENDIF
A = 'AUTOCOR.Y1'
B = 'TIME (sec)'
C = 'AUTOCORR.'
CALL PLOTT(X, ACC1, 200, 2, 0)
A = 'AUTOCOR.Y2'
CALL PLOTT(X, ACC2, 200, 2, 0)

CALCULATING AND PLOTTING THE CROSS CORRELATION FUNCTION

DO I = 1, N
  X(I) = FLOAT(I-1) * 0.01
ENDO

PRINT *, ' DO YOU WANT TO CALCULATE CROSS CORRELATION FUNCTION (Y/N)'
READ (5, 1000) NCC
IF (NCC.EQ. 'N') GO TO 130
IF (NC.EQ. 'N') THEN
  PRINT *, ' ARE THE CALCULATIONS DONE BY BATCH PRGM. (Y/N)'
  READ (5, 1000) NB
  IF (NB.EQ. 'Y') THEN
    READ (4) NI
```
READ (4) (ACC1(I), ACC2(I), I=1, 200)
CALL TAB(N1, D)
ENDIF
IF (NB.EQ. 'Y') THEN
READ (4) (CC(I), I=1, 100)
ELSE
DO J=1, 1000
CC(J)=0.
DO I=1, N+1-J
CC(J)=CC(J)+(Y1(I)-SUMY1)*(Y2(I+J-1)-SUMY2)*0.01
ENDDO
CC(J)=CC(J)/(FLOAT(N+1-J)*0.01)
ENDDO
ENDIF
AMAX=CC(1)
DO I=1, 100
IF (CC(I).GT.AMAX) THEN
AMAX=CC(I)
IMAX=I
ENDIF
ENDDO
TMAX=(IMAX-1)*0.01
A='CROSS CORR'
B='TIME (sec)'
C='CROSS CORR'
CALL PLOTT(X, CC, 100, 2, 0)
WRITE(6, 129) TMAX
129 FORMAT (10X, 'CROSS CORRELATION IS MAXIMUM AT ', F7.2, ' SECS')
READ (5, *) TAA
END PROGRAM
C
C CALCULATING AND PLOTTING THE POWER SPECTRAL DENSITY FUNCTIONS
C FROM AUTO-CORRELATION FUNCTIONS
C
130 PRINT *, 'DO YOU WANT TO CALCULATE POWER SPECTRAL DENSITY',
1 'FUNCTIONS FROM AUTO-CORR.FUNC. (Y/N)'
READ (5, 1000) NC
IF (NC.EQ. 'N') GO TO 140
PRINT *, 'DO YOU WANT THE CALCULATIONS DONE BY FFT (Y/N)'
READ (5, 1000) NF
IF (NF.EQ. 'N') GO TO 145
F=0.
CALL FFT(ACC1, ACC2, N, 13, 0)
DO I=1, N
G1(I)=ACC1(I)/2.+ACC2(N-I)/2.
G2(I)=ACC2(I)/2.+ACC1(N-I)/2.
X(I)=FLOAT(I-1)*100./FLOAT(N)
ENDDO
GO TO 146
145 IF (NB.EQ. 'Y') THEN
READ (4) (G1(I), G2(I), I=1, 1000)
ELSE
DO I=1, 1000
```

G1(I) = 0.0
G2(I) = 0.0
DO J=1,N
ANG = 2.0 * PI * F * X(J)
G1(I) = G1(I) + ACC1(J) * COS(ANG) * 0.04
G2(I) = G2(I) + ACC2(J) * COS(ANG) * 0.04
ENDO
F = F + 0.1
ENDO
DO I=1,N
X(I) = FLOAT(I - 1) * 0.01
ENDDO
ENDIF
146 A = 'PSDF of Y1'
B = 'FREQ. (c/s)'
C = 'DENS. FUNC.'
PRINT *, 'INPUT THE MAXIMUM FREQUENCY TO BE PLOTTED'
READ (5, *) BB
NN = BB * FLOAT(N) / 100
CALL PLOTT(X, G1, NN, 2, 0)
A = 'PSDF of Y2'
CALL PLOTT(X, G2, NN, 2, 0)
140 CONTINUE
STOP
END

SUBROUTINE PLOTT

THIS SUBROUTINE IS CALLED TO PLOT ALL THE STATISTICAL
FUNCTIONS CALCULATED IN THE PROGRAM

REAL

SUBROUTINE PLOTT(X, Y, N, NP, IC)

DIMENSION X(10000), Y(10000)
CHARACTER A*10, B*10, C*10, D*38
CHARACTER A1*30, A2*30, A3*30
COMMON/PLTIT/A, B, C, D
NA=0
11 FORMAT (A30)
PRINT *, 'DO YOU WANT TO SPECIFY THE AXIS'
READ (5, 1000) NC
IF (NC.EQ. 'Y') THEN
    PRINT *, 'ENTER XMIN, XMAX'
    READ (5, *) XMIN, XMAX
    PRINT *, 'ENTER YMIN, YMAX'
    READ (5, *) YMIN, YMAX
    NA=1
ENDIF
PRINT *, 'DO YOU WANT TO SPECIFY LOGS ON GRAPH'
READ (5, 1000) NCC1
IF (NCC1.EQ. 'Y') THEN
    PRINT *, 'ENTER XLG, YLG'
READ (5,*) XLG1, YLG1
PRINT *, 'ENTER THE DESCRIPTION'
READ (5,11) A1
PRINT *, 'ANOTHER LOG (Y/N)'
READ (5,1000) NCC2
IF (NCC2.EQ. 'Y') THEN
PRINT *, 'ENTER XLG, YLG'
READ (5,*) XLG2, YLG2
PRINT *, 'ENTER THE DESCRIPTION'
READ (5,11) A2
PRINT *, 'ANOTHER LOG (Y/N)'
READ (5,1000) NCC3
IF (NCC3.EQ. 'Y') THEN
PRINT *, 'ENTER XLG, YLG'
READ (5,*) XLG3, YLG3
PRINT *, 'ENTER THE DESCRIPTION'
ENDIF
ENDIF
ENDIF
19 CALL PLT$GEN(8,2,0.0)
IF (NA.EQ.1) GO TO 20
IF (IC.LT.1) THEN
  XMIN=X(1)
  YMIN=Y(1)
  XMAX=X(1)
  YMAX=Y(1)
  DO I=1,N
    IF (X(I).LT.XMIN) XMIN=X(I)
    IF (X(I).GT.XMAX) XMAX=X(I)
    IF (Y(I).LT.YMIN) YMIN=Y(I)
    IF (Y(I).GT.YMAX) YMAX=Y(I)
  ENDDO
ELSE
  XMIN=X(1)
  YMIN=Y(1)
  XMAX=X(1)
  YMAX=Y(20)
  DO I=1,N
    IF (X(I).LT.XMIN) XMIN=X(I)
    IF (X(I).GT.XMAX) XMAX=X(I)
  ENDDO
  DO I=10,N
    IF (Y(I).LT.YMIN) YMIN=Y(I)
    IF (Y(I).GT.YMAX) YMAX=Y(I)
  ENDDO
  DO I=1,3
    Y(I)=Y(4)
  ENDDO
ENDIF
WRITE (6,*) XMIN, XMAX, YMIN, YMAX
C READ (5,*) NN
20 CALL PLT$AX(XMIN, XMAX, 5, NP, YMIN, YMAX, 5, NP, 0, 0, 1)
CALL PLT$TL(A,B,C,1)
CALL PLT$PT(X,Y,N,0,0,1,1)
CALL PLT$HD(D,1)

IF (NCC1.EQ. 'Y') CALL PLT$LG(XLG1,YLG1,0,A1,0.,1)
IF (NCC2.EQ. 'Y') CALL PLT$LG(XLG2,YLG2,0,A2,0.,1)
IF (NCC3.EQ. 'Y') CALL PLT$LG(XLG3,YLG3,0,A3,0.,1)

CALL PLT$DN
CALL PLT$DMP
PRINT *, 'DO YOU WANT TO CHANGE THE AXIS (Y/N)'
READ(5,1000) NC

1000 FORMAT(A1)
IF (NC.EQ. 'Y') THEN
  PRINT *, 'DO YOU WANT TO CHANGE X-AXIS'
  READ(5,1000) NCX
  IF (NCX.EQ. 'Y') THEN
    PRINT *, 'ENTER XMIN AND XMAX'
    READ(5,*) XMIN,XMAX
  ENDIF
  PRINT *, 'DO YOU WANT TO CHANGE Y-AXIS'
  READ(5,1000) NCY
  IF (NCY.EQ. 'Y') THEN
    PRINT *, 'ENTER YMIN AND YMAX'
    READ(5,*) YMIN,YMAX
  ENDIF
NA=1
GO TO 10
ENDIF
RETURN
END

SUBROUTINE FFT
THIS SUBROUTINE PERFORMS THE FAST FOURIER TRANSFORM
OF THE ORIGINAL DATA OR THE CALCULATED AUTO-
CORRELATION FUNCTIONS

SUBROUTINE FFT(XREAL,XIMAG,N,NU,IC)
DIMENSION XREAL(10000),XIMAG(10000)
N2=N/2
NU1=NU-1
K=0
DO 100 L=1,NU
   DO 101 I=1,N2
      P=IBITR(K/2**NU1,NU)
      ARG=6.283185*P/FLOAT(N)
      C=COS(ARG)
      S=SIN(ARG)
      K1=K+1
      K1N2=K1+N2
      IF (IC.LT.1) THEN
         TREAL=XREAL(K1N2)*C+XIMAG(K1N2)*S
      ENDIF
TIMAG= XIMAG(K1N2)*C - XREAL(K1N2)*S
ELSE
TREAL= XREAL(K1N2)*C - XIMAG(K1N2)*S
TIMAG= XIMAG(K1N2)*C + XREAL(K1N2)*S
ENDIF
XREAL(K1N2)= XREAL(K1)- TREAL
XIMAG(K1N2)= XIMAG(K1)- TIMAG
XREAL(K1)= XREAL(K1)+ TREAL
XIMAG(K1)= XIMAG(K1)+ TIMAG
101  K=K+1
    K=K+N2
    IF (K.LT.N) GO TO 102
    K=0
    NU1=NU1-1
100  N2=N2/2
    DO 103 K=1, N
       I= IBITR(K-1,NU)+1
       IF (I.LE.K) GO TO 103
       TREAL= XREAL(K)
       TIMAG= XIMAG(K)
       XREAL(K)= XREAL(I)
       XIMAG(K)= XIMAG(I)
       XREAL(I)= TREAL
       XIMAG(I)= TIMAG
103  CONTINUE
    RETURN
END
FUNCTION IBITR(J,NU)
    J1=J
    IBITR=0
    DO 200 I=1,NU
       J2=J1/2
       IBITR= IBITR*2+(J1-2*J2)
200  J1=J2
    RETURN
END

SUBROUTINE FILTER

THIS SUBROUTINE IS CALLED TO FILTER THE NOISE
ONLY LOW PASS FILTER IS PROGRAMMED

SUBROUTINE FILTER(X1,X2,ICUT,AMP,N)
DIMENSION X1(10000), X2(10000)
CUT1=1.
CUT2=AMP
IC1=ICUT
IC2=ICUT+1
DO I=1,IC1
   X1(I)=X1(I)*CUT1
   X2(I)=X2(I)*CUT1
SUBROUTINE TAB

DESCRIPTION

SUBROUTINE TAB (NI,D)
CHARACTER D*38
IF (NI.EQ.000) D = 'SIMULATED SIN FUNCTIONS'
IF (NI.EQ.101) D = 'AL 2.32, AIR FLOW ONLY, UA=1.33, H=12.75'
IF (NI.EQ.102) D = 'AL 2.32, AIR FLOW ONLY, UA=1.42, H=12.75'
IF (NI.EQ.103) D = 'AL 2.32, AIR FLOW ONLY, UA=1.50, H=12.75'
IF (NI.EQ.104) D = 'AL 2.32, AIR FLOW ONLY, UA=1.58, H=12.75'
IF (NI.EQ.105) D = 'AL 2.32, AIR FLOW ONLY, UA=1.72, H=12.75'
IF (NI.EQ.106) D = 'AL 2.32, AIR FLOW ONLY, UA=1.98, H=12.75'
IF (NI.EQ.107) D = 'AL 2.32, AIR FLOW ONLY, UA=2.21, H=12.75'
IF (NI.EQ.108) D = 'AL 2.32, AIR FLOW ONLY, UA=2.51, H=12.75'
IF (NI.EQ.109) D = 'AL 2.32, AIR FLOW ONLY, UA=2.86, H=12.75'
IF (NI.EQ.110) D = 'AL 2.32, AIR FLOW ONLY, UA=3.10, H=12.75'
IF (NI.EQ.999) D = 'NOISE DATA'
IF (NI.EQ.200) D = 'GB 1.00, NOISE DATA'
IF (NI.EQ.201) D = 'GB 1.00, AIR FLOW ONLY, UA=0.911, H=12.75'
IF (NI.EQ.202) D = 'GB 1.00, AIR FLOW ONLY, UA=1.269, H=12.75'
IF (NI.EQ.203) D = 'GB 1.00, AIR FLOW ONLY, UA=1.773, H=12.75'
IF (NI.EQ.204) D = 'GB 1.00, AIR FLOW ONLY, UA=2.038, H=12.75'
IF (NI.EQ.205) D = 'GB 1.00, AIR FLOW ONLY, UA=2.280, H=12.75'
IF (NI.EQ.206) D = 'GB 1.00, AIR FLOW ONLY, UA=2.596, H=12.75'
IF (NI.EQ.207) D = 'GB 1.00, AIR FLOW ONLY, UA=1.062, H=12.75'
IF (NI.EQ.208) D = 'GB 1.00, AIR FLOW ONLY, UA=1.355, H=12.75'
IF (NI.EQ.209) D = 'GB 1.00, AIR FLOW ONLY, UA=1.909, H=12.75'
IF (NI.EQ.210) D = 'GB 1.00, AIR FLOW ONLY, UA=2.164, H=12.75'
IF (NI.EQ.211) D = 'GB 1.00, AIR FLOW ONLY, UA=2.392, H=12.75'
IF (NI.EQ.220) D = 'GB 1.00, AIR FLOW ONLY, UA=1.201, H=6.25'
IF (NI.EQ.221) D = 'GB 1.00, AIR FLOW ONLY, UA=1.618, H=6.25'
IF (NI.EQ.222) D = 'GB 1.00, AIR FLOW ONLY, UA=1.969, H=6.25'
IF (NI.EQ.223) D = 'GB 1.00, AIR FLOW ONLY, UA=2.320, H=6.25'
IF (NI.EQ.224) D = 'GB 1.00, AIR FLOW ONLY, UA=2.623, H=6.25'
IF (NI.EQ.225) D = 'GB 1.00, AIR FLOW ONLY, UA=2.936, H=6.25'
IF (NI.EQ.226) D = 'GB 1.00, AIR FLOW ONLY, UA=3.219, H=6.25'
IF (NI.EQ.227) D = 'GB 1.00, AIR FLOW ONLY, UA=3.513, H=6.25'
IF (NI.EQ.228) D = 'GB 1.00, AIR FLOW ONLY, UA=3.783, H=6.25'
IF (NI.EQ.229) D = 'GB 1.00, AIR FLOW ONLY, UA=4.033, H=6.25'
IF (NI.EQ.230) D = 'GB 1.00, AIR FLOW ONLY, UA=4.213, H=6.25'
IF (NI.EQ.231) D = 'GB 1.00, AIR FLOW ONLY, UA=3.888, H=6.25'
IF (NI.EQ.232) D = 'GB 1.00, AIR FLOW ONLY, UA=4.437, H=6.25'
IF (N I.E Q .233) D = 'GB 1.00, AIR FLOW ONLY, UA=4.782, H=6.25
IF (N I.E Q .240) D = 'GB 1.00, AIR FLOW ONLY, UA=1.222, H=9.50
IF (N I.E Q .241) D = 'GB 1.00, AIR FLOW ONLY, UA=1.609, H=9.50
IF (N I.E Q .242) D = 'GB 1.00, AIR FLOW ONLY, UA=1.958, H=9.50
IF (N I.E Q .243) D = 'GB 1.00, AIR FLOW ONLY, UA=2.457, H=9.50
IF (N I.E Q .244) D = 'GB 1.00, AIR FLOW ONLY, UA=2.743, H=9.50
IF (N I.E Q .245) D = 'GB 1.00, AIR FLOW ONLY, UA=3.037, H=9.50
IF (N I.E Q .246) D = 'GB 1.00, AIR FLOW ONLY, UA=3.235, H=9.50
IF (N I.E Q .247) D = 'GB 1.00, AIR FLOW ONLY, UA=3.495, H=9.50
IF (N I.E Q .248) D = 'GB 1.00, AIR FLOW ONLY, UA=3.768, H=9.50
IF (N I.E Q .249) D = 'GB 1.00, AIR FLOW ONLY, UA=4.080, H=9.50
IF (N I.E Q .250) D = 'GB 1.00, AIR FLOW ONLY, UA=4.310, H=9.50
IF (N I.E Q .251) D = 'GB 1.00, AIR FLOW ONLY, UA=4.585, H=9.50
IF (N I.E Q .252) D = 'GB 1.00, AIR FLOW ONLY, UA=1.829, H=9.50
IF (N I.E Q .253) D = 'GB 1.00, AIR FLOW ONLY, UA=2.253, H=9.50
IF (N I.E Q .254) D = 'GB 1.00, AIR FLOW ONLY, UA=2.606, H=9.50
IF (N I.E Q .255) D = 'GB 1.00, AIR FLOW ONLY, UA=1.452, H=9.50
IF (N I.E Q .256) D = 'GB 1.00, AIR FLOW ONLY, UA=1.718, H=9.50
IF (N I.E Q .257) D = 'GB 1.00, AIR FLOW ONLY, UA=2.141, H=9.50
IF (N I.E Q .258) D = 'GB 1.00, AIR FLOW ONLY, UA=2.362, H=9.50
IF (N I.E Q .301) D = 'A1 2.32, AIR FLOW ONLY, UA=2.246, H=9.50
IF (N I.E Q .302) D = 'A1 2.32, AIR FLOW ONLY, UA=2.690, H=9.50
IF (N I.E Q .304) D = 'A1 2.32, AIR FLOW ONLY, UA=3.293, H=9.50
IF (N I.E Q .305) D = 'A1 2.32, AIR FLOW ONLY, UA=3.539, H=9.50
IF (N I.E Q .311) D = 'A1 2.32, AIR FLOW ONLY, UA=2.011, H=9.50
IF (N I.E Q .312) D = 'A1 2.32, AIR FLOW ONLY, UA=2.449, H=9.50
IF (N I.E Q .313) D = 'A1 2.32, AIR FLOW ONLY, UA=2.897, H=9.50
IF (N I.E Q .315) D = 'A1 2.32, AIR FLOW ONLY, UA=5.476, H=9.50
IF (N I.E Q .318) D = 'A1 2.32, AIR FLOW ONLY, UA=8.189, H=9.50
IF (N I.E Q .319) D = 'A1 2.32, AIR FLOW ONLY, UA=1.962, H=7.50
IF (N I.E Q .320) D = 'A1 2.32, AIR FLOW ONLY, UA=2.464, H=7.50
IF (N I.E Q .321) D = 'A1 2.32, AIR FLOW ONLY, UA=2.839, H=7.50
IF (N I.E Q .322) D = 'A1 2.32, AIR FLOW ONLY, UA=3.428, H=7.50
IF (N I.E Q .323) D = 'A1 2.32, AIR FLOW ONLY, UA=3.989, H=7.50
IF (N I.E Q .324) D = 'A1 2.32, AIR FLOW ONLY, UA=4.455, H=7.50
IF (N I.E Q .325) D = 'A1 2.32, AIR FLOW ONLY, UA=4.878, H=7.50
IF (N I.E Q .326) D = 'A1 2.32, AIR FLOW ONLY, UA=5.268, H=7.50
IF (N I.E Q .327) D = 'A1 2.32, AIR FLOW ONLY, UA=5.523, H=7.50
IF (N I.E Q .328) D = 'A1 2.32, AIR FLOW ONLY, UA=5.896, H=7.50
IF (N I.E Q .329) D = 'A1 2.32, AIR FLOW ONLY, UA=6.264, H=7.50
IF (N I.E Q .331) D = 'A1 2.32, AIR FLOW ONLY, UA=7.029, H=7.50
IF (N I.E Q .332) D = 'A1 2.32, AIR FLOW ONLY, UA=7.466, H=7.50
IF (N I.E Q .333) D = 'A1 2.32, AIR FLOW ONLY, UA=7.466, H=7.50
IF (N I.E Q .400) D = ' AL 5.50, AIR FLOW ONLY, UA=3.166, H=9.50
IF (N I.E Q .401) D = ' AL 5.50, AIR FLOW ONLY, UA=3.387, H=9.50
IF (N I.E Q .402) D = ' AL 5.50, AIR FLOW ONLY, UA=3.802, H=9.50
IF (N I.E Q .403) D = ' AL 5.50, AIR FLOW ONLY, UA=4.113, H=9.50
IF (N I.E Q .404) D = ' AL 5.50, AIR FLOW ONLY, UA=4.569, H=9.50
IF (N I.E Q .405) D = ' AL 5.50, AIR FLOW ONLY, UA=4.856, H=9.50
IF (N I.E Q .406) D = ' AL 5.50, AIR FLOW ONLY, UA=5.312, H=9.50
IF (N I.E Q .407) D = ' AL 5.50, AIR FLOW ONLY, UA=3.284, H=9.50
IF (N I.E Q .408) D = ' AL 5.50, AIR FLOW ONLY, UA=3.518, H=9.50
IF (N I.E Q .409) D = ' AL 5.50, AIR FLOW ONLY, UA=3.671, H=9.50
IF (N I.E Q .410) D = ' AL 5.50, AIR FLOW ONLY, UA=3.995, H=9.50
IF (N I.E Q .411) D = ' AL 5.50, AIR FLOW ONLY, UA=4.292, H=9.50
IF (N I.E Q .412) D = ' AL 5.50, AIR FLOW ONLY, UA=3.680, H=7.50
IF (N I.E Q .413) D = ' AL 5.50, AIR FLOW ONLY, UA=4.005, H=7.50
IF (N I.E Q .414) D = ' AL 5.50, AIR FLOW ONLY, UA=4.303, H=7.50
IF (N I.E Q .415) D = ' AL 5.50, AIR FLOW ONLY, UA=4.848, H=7.50
IF (N I.E Q .416) D = ' AL 5.50, AIR FLOW ONLY, UA=5.845, H=7.50
IF (N I.E Q .417) D = ' AL 5.50, AIR FLOW ONLY, UA=7.026, H=7.50
IF (N I.E Q .418) D = ' AL 8.404, AIR FLOW ONLY, UA=8.327, H=7.50
IF (N I.E Q .419) D = ' AL 9.766, AIR FLOW ONLY, UA=4.183, H=7.50
IF (N I.E Q .420) D = ' AL 6.96, AIR FLOW ONLY, UA=4.300, H=7.50
IF (N I.E Q .421) D = ' AL 6.96, AIR FLOW ONLY, UA=4.410, H=7.50
IF (N I.E Q .422) D = ' AL 6.96, AIR FLOW ONLY, UA=4.525, H=7.50
IF (N I.E Q .423) D = ' AL 6.96, AIR FLOW ONLY, UA=4.745, H=7.50
IF (N I.E Q .424) D = ' AL 6.96, AIR FLOW ONLY, UA=5.000, H=7.50
IF (N I.E Q .425) D = ' AL 6.96, AIR FLOW ONLY, UA=5.416, H=7.50
IF (N I.E Q .426) D = ' AL 6.96, AIR FLOW ONLY, UA=5.909, H=7.50
IF (N I.E Q .427) D = ' AL 6.96, AIR FLOW ONLY, UA=6.935, H=7.50
IF (N I.E Q .428) D = ' AL 6.96, AIR FLOW ONLY, UA=7.834, H=7.50
IF (N I.E Q .429) D = ' AL 8.895, AIR FLOW ONLY, UA=8.895, H=7.50
RETURN
END

The equations used for statistical functions calculations are described in the following:

Auto-correlation \( R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} x(t) x(t+\tau) \, dt \) (D.1)

Cross-correlation \( R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t) y(t+\tau) \, dt \) (D.2)

Power Spectral D.F. \( G_{xx}(\omega) = 4 \int_{0}^{\infty} R_{xx}(\tau) e^{-j\omega \tau} \, d\tau \) (D.3)