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THE ANALYSIS AND ASSESSMENT OF TIME VARIANT LINEAR TRENDS IN ANNUAL ECONOMIC DATA SERIES WITH AN APPLICATION TO ENERGY FORECASTING FOR THE STATE OF OHIO

The Ohio State University

Ph.D. 1983

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THE ANALYSIS AND ASSESSMENT OF TIME VARIANT LINEAR
TRENDS IN ANNUAL ECONOMIC DATA SERIES WITH AN
APPLICATION TO ENERGY FORECASTING FOR THE STATE OF OHIO

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
Galip Feyzioglu, B.A.

* * * * *

The Ohio State University

1983

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To My Mother
ACKNOWLEDGMENTS

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LIST OF NOTATIONS AND ABBREVIATIONS

In the following thesis lower case Latin letters are used to represent real scalars. Unless otherwise specified in the text, upper case Latin letters or Greek letters are used to represent vectors or matrices defined over the field of real numbers.

1. "A" is a matrix consisting of m rows and n columns: $A_{m \times n}$
2. $i^{th}$ row of A: $A_i$
3. $j^{th}$ column of A: $A_j$
4. Element located at the $i^{th}$ row and $j^{th}$ column of A: $a_{ij}$
5. Identity matrix of dimension n: $I_{n \times n}$
6. Transpose of A: $A^t$
7. Product of A and B: $A \cdot B$
8. Kronecker product of A and B: $A \otimes B$
9. Generalized inverse of A: $\text{GINV}(A)$
10. Expected value of ".": $E(\cdot)$
11. e has a p-variate normal distribution with mean vector 0 and covariance matrix $W$: $e \sim N_p (0 \mid W)$
12. e is distributed as a sample of size n from a p-variate normal distribution with mean vector 0 and covariance matrix $W$: $e \sim N_{p,n} (0 \mid W \otimes I)$
13. Limit of $f(t)$ as $t \to t_a$ from the right hand side: $f(t_a)_+$
14. Limit of $f(t)$ as $t \to t_a$ from the left hand side: $f(t_a)_-$
15. The difference between the right hand side limit of $f(t)$ and the left hand side limit of $f(t)$: $f(t)_+ - f(t)_-$
16. $a$ is an element of set $A$: $a \in A$
17. $a$ is not an element of set $A$: $a \notin A$
18. Set $A$ is a subset of set $B$: $A \subseteq B$
19. Union of set $A$ and set $B$: $A \cup B$
20. Intersection of set $A$ and set $B$: $A \cap B$
21. Absolute value of scalar $a$: $|a|$
22. Statement $A$ is conditional upon state $B$: $A \mid B$
23. Ohio Department of Energy: ODOE
25. Billion kilowatt-hours: BKWH
26. Trillion British Thermal Units: TBtu
INTRODUCTION

Since 1973, Ohio law requires that major Ohio electric and natural gas utilities submit for review Ten-Year Forecast Reports of annual requirements of the residential, commercial, industrial, and other customer categories within their respective service areas. Such forecasts are generally produced on the basis of "demand" models whose statistical properties and theoretical underpinnings are deemed generally acceptable by conventional standards. The accuracy of the forecasts submitted by the utilities has, however, come increasingly under question as the forecast time path trajectories of annual requirements have consistently failed to approximate the realized time path trajectories.

That inaccurate forecasts and analyses will sooner or later result in nonoptimal investment policies for the utility companies which, in turn, will result in nonoptimal rate structures for the utility customers is evidenced on a national scale by the growing number of nuclear power plant construction projects abandoned in recent years at various stages of development and by the planned abandonment of more projects in the near future. The belated realization that the initially forecast levels of energy requirements, which occasioned the construction projects in the first place, will just not materialize in view of the increasing discrepancies between the time
paths of actual requirement levels and those of forecast requirement levels is obviously a major factor in these decisions. It is also quite clear that the rate payers will eventually bear the major burden of paying for such dead weight losses when utility forecasts and capacity planning are too high relative to established trends, and suffer the inconvenience of shortages and curtailments when utility forecasts and capacity planning are too low relative to established trends. Similar considerations are also relevant for many economic goods or services when there are significant lead times or indivisibilities involved in their production processes.

It has been the conventional wisdom that the historical behavior of observations on annual economic data series such as the annual requirements of electricity and natural gas should be explained in terms of, and their future behavior be forecast by, econometric demand models based on neoclassical economic theory. While this approach has a definite appeal because of its normative implications (1), a number of prominent economists have voiced their concern that the current success of such enterprise as a reliable source of useful information or as a relevant instrument of positive policy analysis leaves something to be desired (2).

In any society which prides itself on efficiency, rationality and fair play, administrators and executives in public and private sectors cannot realize their goals if their current plans and policies to meet future requirements are based upon patently inaccurate assessments of these requirements. Therefore, it seems appropriate to propose and develop methodologies which are more reliable and accurate as
predictors, even though they may be less appealing in terms of their normative implications. Indeed, it would be more responsible, and more becoming as a scientific discipline, to strive to understand, describe and assess more accurately the influences of relevant factors on the eventual consequences of policy measures than to preach the moral superiority of particular policy measures on the basis of conclusions drawn from speculative theories and then to demonstrate ex post facto why these measures had to fail under the prevailing circumstances after life, limb, and property of large numbers of people may have been severely constrained or manipulated. Until and unless the alleged moral and positive aspects of economic theories are successfully consolidated, the requirement for positive information inputs into public and private decision making processes and procedures require correspondingly positive, accurate and reliable methods of evaluating the dynamics of economic systems. This in turn requires a methodology for the analysis and assessment of discernible trends in the sequences of intertemporal magnitudes generated by the underlying socio-physical systems. In fact, the analysis and assessment of time path trajectories of observed data series have historically been a major source of useful information and insight concerning the regularities in the intertemporal behavior of the underlying systems which have generated the data series in question as witness Kepler's discovery of the laws of planetary motion in astronomy.

During the course of our association with the Ohio Department of Energy, we have come to observe that time path trajectories of all data series pertaining to the annual disposition or provision of energy
resources could be characterized in terms of time variant linear trends. Furthermore, it became quite clear to us that the magnitudes of historical trends in many cases produced far more accurate, and easy to interpret, results than those produced by various econometric models. We have, therefore, developed a conceptual framework and an analytical methodology for the analysis and assessment of time path trajectories of annual economic data series as a problem in intertemporal measurement of systemic flows. The conceptual framework and the analytical methodology are based upon similar methodologies of measurement and analysis employed in such positive domains of inquiry as astronomy, geodetics and physics and their validity is, therefore, independent of any ontological presumptions which are implicit in current economic analyses and econometric practices.

The purpose of this thesis is to formally present the conceptual framework and the analytical methodology for the analysis and assessment of time variant linear trends in the time paths of observed economic data series. The accurate and consistent analyses and assessments of time paths require, first of all, an understanding of the general conditions that characterize observed economic data series. Next, it requires a conceptual framework within which observed data can be related to mathematical concepts. Finally, it requires a methodology for the modeling and assessment of the time paths in question, which would represent the functional or definitional relations among the empirical data in terms of mathematical concepts and mathematical functions (3).
The plan of the thesis is as follows. In chapter I we review, in succession, (a) the historical problem of forecasting energy data series in Ohio, (b) the nature and implications of observed economic data series such as those pertaining to the annual consumption of energy in Ohio, and (c) certain critical concepts which are utilized in this thesis. In sections A and B of chapter II we develop and discuss a conceptual framework and an analytical methodology for a systemic treatment of time variant linear trends in a single aggregate observed economic data series. Section C contains an application of the methodology developed in this chapter to the analysis and assessment of the time path of commercial natural gas sales in Ohio between 1960 and 1981. We follow the same pattern of exposition in chapter III except that we consider the simultaneous analyses and assessments of time variant linear trends in the time paths of definitionally or functionally related data series. The methodology developed in this chapter is applied to the simultaneous characterization of the composition of sectoral electricity sales in Ohio between 1960 and 1981. In chapter IV we summarize the results established in this thesis and suggest a few areas where our methodology is expected to yield equally fruitful results.

Our preliminary attempts at the analysis and assessment of sectoral energy consumption patterns in Ohio have already led to a clearer understanding of the factors influencing the dynamic behavior of sectoral disposition of various fuels in Ohio (4). It has led to the establishment of our methodology as the official methodology of the Ohio Department of Energy in forecasting, analyzing and investigating Ohio's
energy requirements. Most recently, it was applied to make an independent assessment of the 1980 long term residential electric demand forecast of the American Electric Power System, in a case involving claims of deliberate model and parameter manipulations to arrive at predetermined forecast magnitudes. The clarity of the empirical implications of our results were instrumental in the resolution of the controversy (5). In 1982, the Ohio Legislature, in order to protect the public interest of Ohio rate payers from the undesirable consequences of unreliable utility forecasts, enacted into law Amended Substitute Senate Bill No. 378 which stipulates that public hearings be held on utility forecasts by the Ohio Department of Energy (6) and that independent forecasts be made of utility service area requirements for the electric and natural gas utilities in Ohio. The conceptual framework and analytical methodology developed in this thesis are also expected to be utilized as major tools in meeting these stipulations.
A. Forecasting Economic Data Series: The Case of Energy Data Series for the State of Ohio

The time paths of many annual data series pertaining to the provision of energy by fuel types, or to the disposition of energy by ultimate end user categories within the States of the Union as well as in the U.S. as a whole, have followed distinctly different linear trends in the sixties and the early seventies as opposed to the rest of the seventies and the early eighties. Many private and public agencies have tried to explain these trends using traditional methods of projecting energy data series. Most of these forecasts, however, have turned out to be matters of serious misinformation as is evidenced by the current plight of the electric utilities industries in many states.

In the early seventies many Ohio electric utilities started to employ econometric or end use demand models for forecasting purposes. Figure 1 shows the consolidated ten year forecasts of net electricity generation in Ohio for the decades 1974-1984 through 1982-1992. These forecasts were consolidated from the corresponding Ten Year Forecast Reports of Ohio electric utilities which were submitted annually to the Ohio Power Siting Commission, or to the Ohio Department of Energy, from
Source: Ten Year Forecast of Reporting Utilities and Ohio Department of Energy
1974 to 1981 (1). According to the 1974 ten year forecasts of utilities, the statewide net generation in 1984 was projected to be around 220 billion kWh. The actual 1982 level turned out to be closer to 110 billion kWh.

This kind of discrepancy is one reason why serious doubts have been raised about the reliability of traditional forecasting techniques. Figure 1 shows that the output forecasts of demand models have not been much different from straight line projections of the average trend in the preceding periods of analysis to the next ten years. However, the historical time path shows a discrete decline in the post-1973 marginal trend relative to the pre-1973 marginal trend. Despite such a clear-cut signal given from the data, the demand models have failed to predict that (a) the magnitude of average trends will be declining in the post-1973 period and (b) the forecast trajectory which is projected to move along the average trend will always be above the realized trajectory which follows the marginal trend. The failure of traditional forecasting techniques in these respects suggests a need for developing a new methodology in analyzing the time paths of economic data series.

From an empirical point of view, the time paths of economic data series may, in general, be regarded as reflecting the dynamic states of economic systems. Where this is the case, discontinuities or nondifferentiabilities in the time path of economic data series would be indications of disruptions or alterations in the historical course of events which have defined the operating conditions of the system in question. Hence, precise, accurate and consistent analyses and assessment of time variant linear trends in the time paths of economic data series would
provide useful information in understanding the dynamic behavior of economic systems.

Whether the measures currently taken to achieve future goals are appropriate or not is contingent upon the validity and accuracy of current analyses and assessments of the projected behaviors of relevant systemic trajectories. Hence, the information derived from analyses and assessments of historical time path trajectories of empirical economic systems may contribute significantly both to the understanding of positive economic problems and to the design, choice and implementation of suitable measures for their solution.

At present, no established methodology exists for the analysis and assessment of time path trajectories consisting of time variant linear trends. We, therefore, propose to develop, in this thesis, a conceptual framework as well as a mathematical method which permit us to treat the simultaneous modeling and assessment of time variant linear trends in the time paths of definitionally or functionally related economic data series. The conceptual framework to be developed is a systemic one within which relations among sets of definitionally or functionally related observed economic data series can be characterized in an empirically valid and analytically consistent manner. The mathematical method to be developed is a comprehensive one for the modeling and estimation of definitionally or empirically related linear trend functions that are continuous and differentiable over a specified domain of time except on a finite set of points.
Empirically valid applications of consistent analytical methods require a clear understanding of the nature and properties of the observations that are being analyzed and assessed (2). In the next section (section B) of this chapter, we establish two general characteristics of observed economic data series which must be explicitly satisfied by all positive analyses of economic observations. It is also necessary to have a clear understanding of the concepts of observation, model and adjustment as they are employed in positive sciences in general before these concepts are consistently applied to positive analyses and assessments of economic data series. In section C, therefore, we provide a review of these concepts as they are employed in positive sciences. In section D, we demonstrate that the "true" econometric forecasting model of an economic data series, if it exists, would have to have an explicit solution in the domain of time. We also discuss differences between econometric and mathematical models of economic observations to show why the search for the "true" econometric model may have been such an elusive one so far. In section E, we indicate the power and flexibility which empirically valid applications of the analytical tools to be developed in chapters II and III can bring to the field of positive economic inquiry and discuss some of the practical problems which they are specifically designed to address—forecasting energy data series for the State of Ohio.
B. The Nature of Economic Data Series

1. General Comments

The affairs of Man take place in a social setting defined both in space and in time. The existence in every human society of things or activities which, according to the traditions of the society under consideration, can be called goods or services will be taken as given along with the corresponding processes of provision or disposition of goods or services (3). The nature and existence of processes of provision and disposition of a particular commodity in a particular human society are, therefore, historical phenomena. The elements of an observed economic data series then reflect the magnitudes of physical or financial transactions that have taken place between the providers and the disposers of that commodity in that society during successive periods of time. Observations pertaining to the outcomes of such recurrent transactions of a commodity are usually compiled, consolidated and published, on a periodic basis, by accountants according to the principle of double entry bookkeeping. In this an economist is an observer of other people's data. The initial data about the levels of production and distribution are measured according to physical models of reality and are compiled and consolidated according to accounting theory. Hence the quantitative data that an economist tries to interpret almost always reflect an accountant's view of socio-physical reality.

Every model of data analysis prescribed by an economist must explicitly preserve and consolidate the conditions imposed by the corresponding accounting model which defines the data as well as the
physical nature of the observations which the data represent. The neglect of the former may lead to implicit or explicit denial of the basic axiom about the whole being the sum of its parts. The neglect of the latter may lead to postulation of operations which imply adding apples and oranges.

Analyzing the time paths of a related set of economic data series is an exercise in dynamic analysis. For such an analysis to be logically and empirically consistent, it must preserve the static relations that hold among the data at all times. This is a basic principle in positive dynamic analysis. In the case of economic data series, the static relations can, as discussed above, be classified into two general categories: accounting and physical relations.

2. Implications of the Accounting Nature of Economic Data Series

From an accounting point of view, the recurrent transactions during a specified accounting period give rise to corresponding flows of goods or services from recognized providers to recognized disposers. To each such physical flow during each accounting period there corresponds a compensatory flow, usually of a pecuniary nature, from the disposers to the providers.

Furthermore, during any given accounting period, the total disposition of any one type of good or service within a geographically designated society is equal to the actual receipts of all recognized end users plus losses, shrinkage and "unaccounted for" at all recognized sources of provision. Similarly, the total provision of a particular good or service is equal to the current domestic output from all
recognized providers plus net imports minus changes in inventories. By the very logic of double entry bookkeeping which underlies the above definitions, the total provision must necessarily be equal to the total disposition.

The importance of double entry bookkeeping to economic observations is like that of the Euclidean geometry to the Newtonian mechanics. Just as the Euclidean geometry forms a basis of perception and measurement in physical space, double entry bookkeeping forms a basis of perception and measurement in social transactions. Hence, just as measurements based upon an Euclidean model of physical space have to satisfy certain formal conditions (such as the interior angles of a triangle having to equal 180 degrees), observations based on an accounting model of social transaction have to satisfy the formal conditions imposed by the principle of double entry bookkeeping. Every dynamic analysis of observed economic data series must, therefore, start from an explicit accounting scheme, maintained over the domain of time under consideration, and must preserve the formal conditions imposed on observations by that accounting scheme. Otherwise, such analyses would be logically inconsistent.

3. **Implications of the Physical Nature of Economic Data Series**

It has been said that "To measure is to compare with a standard." (4) Hence, to measure an aspect of a thing is to compare the size of the aspect in question to a known standard. If such a comparison can be expressed by a numerical magnitude, then we can talk about a quantitative measurement. To talk of quantitative measurement in the absence
of an established standard and an established procedure for comparison is, therefore, a contradiction in terms.

To the extent that economic data series are ultimately based upon observations, they represent the dynamic behavior of the physical flows of goods or services provided, or disposed of, by recognized individual or institutional members of a geographically designated society. We have already seen that such flows are measured through established methods of physical measurement and are consolidated through established methods of double entry bookkeeping.

The people reading the gas or electric meter observe, on the basis of scientifically established principles of measurement, the cumulative level of flows disposed of by the metered customer unit at the time of the current reading. The customer is charged the amount between the current cumulative reading and the previous cumulative reading. The magnitude of this flow is, then, consolidated according to established accounting practices, and the consolidated amount reported under an appropriate general heading such as residential, industrial and commercial during the billing cycle.

It is possible that during, say, a particular month of December, commercial electricity disposition figures include energy used up during open heart surgeries, midnight masses, office parties, electrocution of convicted criminals and so on. The meter cannot distinguish among different usages of a specific commodity such as electricity, nor can it differentiate among different motives with which the commodity is used.
From a physical point of view, an economic time series is a historical record of the rates of flows of goods or services generated within a geographically specified society. The impact of events or circumstances, such as the 1973 oil price hikes, on the behavior of the time path of a particular physical flow rate, such as energy consumption in Ohio, must be determinable from a study of the historical time path of that good or service. Similarly, the variance or invariance of definitional or functional transformations among related rates of flows, such as the transformation of physical flows into financial flows with price defined as the coefficient of transformation, must also be determined from analyses and assessments of the component time paths involved in the transformations. Furthermore, these analyses and assessments must be consistent with the analyses of physical observations in general. That is, the terms "observation", "model" and "adjustment" cannot be treated as delicately intertwined esoteric mysteries signifying the unfolding of a particular ontological principle into behavioral laws which only a formal education in "economic theory" can unravel. Nor can relations implying the transformation of the time path of apples into that of oranges over a specific domain of time be construed as anything but wishful thinking, no matter what the "statistical properties" of such an exercise may indicate.

In summary, the physical nature of economic time series imposes the following restrictions on their analysis:

(1) Uncontrolled time series of observations on a physical flow can help determine the appropriate mathematical description of its
time path, and thereby provide hints as to the underlying factors which have influenced that time path.

(2) Analysis and assessment of the time path is an exercise in modeling and adjustment of observations on the physical volume of flows generated by a dynamic system.

C. A Review of the Concepts of Observation, Model and Adjustment

1. The Concept of Observation

Since our objective is to apply the concepts of observation, model and adjustment to positive analyses of economic data series and the systems that generate them, it would be useful to present a review of these concepts within the framework of positive analysis. We do this by quoting from Observations and Least Squares by Edward M. Mikhail, a standard text in geodetics and engineering. These quotations are intended to illustrate how the review of authoritative discussions and descriptions of the key concepts underlying positive data analyses are indispensable to their correct understanding. Such an understanding is, in turn, essential to the correct application of these concepts in the domain of positive economic inquiry.

To the extent that positive economic inquiry deals with observable magnitudes of flows of goods or services, we must begin our review with the concept of observation. According to Mikhail:

"The term 'observation' (or measurement) is often used in practice to refer to both the operation or process itself, as well as the actual outcome of such operation. With regard to adjustment the outcomes, and in particular numerical outcomes, will be designated as the 'observations.' Such numerical observational data are fundamental to science and engineering because they supply the instrument for analysis and manipulation." (Mikhail 1976, 1.2)
2. The Concept of Mathematical Model

   a. Mathematical Model. Observations are outcomes of physical procedures and processes (5). Such outcomes are then described by abstract fictional entities known as models. In the case of quantitative physical observations (which are the only kind with which we shall be concerned here), the models are known as mathematical models. The mathematical model consists of two separate models--functional and stochastic--which jointly express the observed magnitudes of the physical phenomena under consideration. The functional model describes the deterministic properties of the empirical reality which are of interest to the investigator, whereas the stochastic model represents and describes the nondeterministic properties of the variables involved, relative to a specific functional model. To quote Mikhail:

   "[T]he mathematical model ... is defined here as a theoretical system or an abstract concept by which one describes a physical situation or a set of events. Such a description is not necessarily meant to be complete or exhaustive, but to relate only to those aspects or properties that are under consideration. Since a model serves a particular purpose, its setup can vary widely from one point of view to another. Thus the same physical system may be described by more than one model. The model then replaces the physical situation for the purpose of assessing it.

   "The mathematical model is often thought of as being composed of two parts: the functional model and the stochastic model. The functional model will in general describe the deterministic properties of the physical situation or event under consideration. On the other hand, the stochastic model designates and describes the nondeterministic or stochastic (probabilistic) properties of the variables involved, particularly those representing the observations." (Mikhail, 1976, 1.2)

   b. Functional Model. The functional model is a concise mathematical description of the deterministic aspects of the empirical reality under investigation. Depending upon the aspects of the
situation to be investigated, or the perspective of the investigator, the deterministic aspects of the same empirical reality can be described by different functional models.

The empirical realities which we deal with in this thesis are geographically designated socio-physical systems, the dynamic behaviors of which result in the generation of material or financial flows that are "observed" by accountants. The functional model that we want to develop would then be a mathematical description of the trajectories of the magnitudes of these flows over time. Assessments of such trajectories would then constitute assessments of the functioning of the underlying socio-physical systems.

c. Relating Observations to the Model. Relating physical observations to a functional model may involve several intermediate stages or steps. For example, the classification and aggregation of individual customer meter readings into sectoral disposition levels are determined according to classificatory convention, such as two digit SIC codes. Conventions pertaining to the reporting or gathering of individual observations among the elements of each conventionally designated class as well as any adjustments that may be applied by accountants to account for the billing cycle, for example, conceptually define a subsystem of a socio-physical system to which physical observations are related. Since a functional time path model is a mathematical description of a particular conceptualization of a subsystem of a socio-physical system which transforms time into a flow magnitude, it is necessary that all observations which are related to a particular functional time path model be obtained under the same,
or compatible, conventions. To the extent that the underlying conventions have been changed over the course of history, data may have to be recomputed or reprocessed to make them internally consistent, before relating them to a functional time path model. Otherwise, it would not be possible to distinguish between the influence of environmental shocks and conditions on the time path of a uniformly conceptualized socio-physical system and the time paths of two or more differently conceptualized socio-physical systems.

The augmentation of the functional model to account for the properties of observations and observational methods is an important aspect of model assessment in the physical sciences:

"The augmentation of the functional model due to measurements and their properties is of basic importance. The evaluation of observations depends on how and by what instruments and methods they have been acquired. The 'measured' length of a line depends to a great extent on the measuring process, on whether the calibration of the instrument is considered to be known, on which reductions are to be applied, and so on. The same applies to the case of measuring angular directions. For instance, we must consider the zero direction and determine its place in the model, whether it is to be taken as known or unknown." (Mikhail, 1976, 1.3.2)

Consideration of this aspect of positive analysis will have to be developed for effective applications of time path analyses to the assessments of the dynamic behaviors of interrelated socio-physical systems.

d. The Stochastic Model. That physical observations are subject to variations which are due to influences not accounted for by the functional model is a common experience in physical sciences. In the past these variations were attributed to various errors. In more recent years the variability of observations has come to be recognized
as an inherent property of observations. Statistical concepts and models are thus introduced to explain the variations of actual observations from the levels inferred from the functional model.

The totality of the assumptions pertaining to the properties of probabilistic fluctuations in the observed magnitudes of variables in the functional model are called the stochastic model. In the case of the mathematical description of a time path generated by a socio-physical system, these fluctuations are due to physical as well as "social" and other unaccountable factors. The characterization of stochasticity as a property of observations as opposed to one of relations is a critical distinction between mathematical modeling and regression modeling of data. Note also that the functional model as the characterization of the deterministic aspects of observations is conceptually independent from the stochastic model as the characterization of the statistical aspects of observations relative to a specified functional model.

3. The Concept of Adjustment

a. Adjustment. The choice of a specific functional model depends on the purpose and the extent of information required of the system to be studied. In general, there is a minimum number of observations required to uniquely determine a chosen functional model. This minimum number is, in turn, determined by the minimum number of independent variables in the chosen model. Let \( n_0 \) be the minimum number of independent variables in a given model and \( n \) the number of independent observations on the \( n_0 \) independent variables. If \( n < n_0 \), there is
said to be a deficiency and the model cannot be uniquely determined on
the basis of given observations. If \( n = n_0 \), the number of observations
is said to be sufficient to uniquely determine the model provided that
the set of \( n \) observations in question consist of one observation on
each one of the "\( n_0 \)" independent variables. If \( n > n_0 \), then
\( r = n - n_0 \) is known as the redundancy. The redundancy \( r \) is meaningful
only if the observations and the functional model are mutually consis-
tent. That is, each one of the \( n (> n_0) \) observations must be consistent
with the underlying functional model.

In view of the stochastic nature of observations, whenever there
is redundancy in observed data, such data will be inconsistent in
the sense that different sufficient subsets of the data will yield
different assessments of the functional model. The term "adjustment"
refers to the choice and application of an external criterion, such as
the principle of least squares, in order to obtain a unique solution
from redundant data which satisfies the chosen criterion.

b. The Least Squares Principle. Since its application by
Gauss to an astronomical problem, the least squares adjustment has been
applied in a vast number of fields in science and engineering. Except
for the fact that we are dealing with the simultaneous adjustment of
definitionally related observed data vectors pertaining to a set
of definitionally related functional models, the concept of least
squares principle to be employed in our study is the one discussed
by Mikhail in the following quotation:
"The least squares principle states that
\[ v^T W v = \text{minimum} \]
where \( W \) is the weight matrix of the observation. ... The weight matrix \( W \) is square and of order equal to \( n \), the number of observations. Its elements reflect the stochastic properties, such as variation and correlation, of all the observations ..." (Mikhail, 1976, 5.2)

In positive analysis, the problem of adjustment such as the least squares principle arises because of the inherent inconsistency of redundant data. From among the infinite solutions to the mathematical model, one is sought which, in addition to satisfying the functional model, would also have certain optimal properties such as satisfying the least squares principle. In many cases such a solution will also be unique. It should be noted, however, that different approaches to the problem of adjustment do not constitute different approaches to the problem of modeling. Although a problem of unsatisfactory modeling of data may be revealed through adjustment results, it cannot be rectified by changing the technique of adjustment or the criterion of adjustment (see Mikhail 1976, chapter 5).

c. Relations Between the Model and Least Squares Techniques. Depending upon the nature and properties of the adjustment task at hand, the objectives of the researcher, and the nature of available information, alternative techniques of applying the least squares principle may be employed to adjust the observations or to estimate parameters (see Mikhail, chapters 6-9). It should be noted, however, that whatever technique is used for a given model and given data, the final results are always the same:
"Although for a specified model and a given set of data the least squares yields unique results, there are several techniques that can be employed. It should be emphasized that whatever technique is used, the final answers are always the same." (Mikhail 1976, 5.3)

If this comes as a surprise to some, it should be remembered that the mathematical model consists of the functional model and the stochastic model. The stochastic model consists of a completely specified weight matrix reflecting the current state of knowledge of the investigator concerning the stochastic properties of the observations. In general, if

$$ e_t \sim \mathcal{N}_N \begin{bmatrix} 0 | \sigma^2 V \end{bmatrix}, $$

then $\sigma^2 V$ is the covariance matrix of the distribution. $\sigma^2$ is an arbitrary positive scalar known as the reference variance, and $V$ is known as the cofactor matrix. $W = V^{-1}$ is called the weight matrix. Hence, knowledge of the weight matrix is tantamount to a knowledge of the covariance matrix of $e_t$ up to a scalar multiple. Hence a different specification of $W1 \neq c \cdot W$, where $c$ is an arbitrary positive scalar, is tantamount to a different stochastic model and, therefore, a different mathematical model, even if the respective functional models are specified to be the same. Furthermore, as is discussed by Mikhail, the application of the least squares principle does not actually require a knowledge of the distribution associated with the observations. All that is required is that the weight matrix $W$ be known, which in the case of mathematical modeling is always presumed to be known:

"It is important to note that the application of least squares principle given above does not require a prior knowledge of the distribution associated with the observations. All that is necessary is to have $W$ defined and known. In the past it has been erroneously stated that least squares adjustment requires
normal distribution. Perhaps the reason for this incorrect assertion is that when the observations are normally distributed, the least squares estimates will have some special properties, such as being identical to those from the method of maximum likelihood..." (Mikhail, 1976, 5.2)

In general, when there is no a priori estimate of a covariance matrix of observations, it is customarily characterized to be of the form $\sigma^2 I$, where $I$ is the $N \times N$ identity matrix and $0 < \sigma^2 < \infty$. Hence, the weight matrix $W = (I)^{-1} = I$. Upon appropriate analysis of the residual vector after adjustment, we may decide that the stochastic model is inadequate and specify a new stochastic model with covariance matrix $\sigma^2 V$, where $V = I$. Hence, the new weight matrix would be $W = V^{-1}$. Obviously, the least square residuals for the two cases will not be the same. It should be noted, however, that since a mathematical model is defined in terms of the functional and the stochastic model, and since we have specified different stochastic models to the same functional model in the above, we are actually dealing with two different models. It is, therefore, normal that the residuals from them not be equal.

It is thus clear that if econometric models were mathematical models, the so called two stage least squares or three stage least squares methods would not be considered as techniques of least squares adjustment, but as possible exercises in successive model modification, to the extent that the analysis of the residuals from initial models are deemed to warrant such modifications. Although the statistical evaluation of the preliminary outcomes of least squares adjustment techniques are instrumental in more accurate model specification, the important point is that a technique of least squares adjustment is not a method of validating a wishful thinking for a fact. The functional
model to which the data that are being adjusted relate must constitute a mathematical description of a bona fide empirical system known a priori. To illustrate this point, let \( X_1 \) be the numerical magnitude of all the corn in Kansas, \( X_2 \) the numerical magnitude of all the tea in China and \( X_3 \) the numerical magnitude of the rain in Spain that falls mainly on the plain. Then, the statement that the linear transformation

\[
X_1 = b_0 + b_1 X_2 + b_2 X_3
\]

is a "tentative" functional model describing implicitly the behavioral laws of social, natural, or ontological phenomena or mechanisms, as long as the least squares estimates of the parameters and the residual vector \( v = \hat{X}_1 - X_1 \) possess "desirable" statistical properties vis-a-vis an arbitrary weight matrix \( W \) during a period of observation exceeding three years, should be regarded as wishful thinking and not scientific hypothesis testing.

D. Econometric Models and Mathematical Models

An econometric model, as a formal proposition, is an assertion concerning the existence of time-invariant stochastic transformations among economic data series that, for all practical purposes, are considered to be measured nonstochastically. Assuming for the moment the possibility of existence of data series of the type mentioned above, let

\[
( y_t, x_t )
\]

be a completely specified vector of variables on which nonstochastic observations are available for \( t = 1, 2, \ldots, N \), where \( y_t = (y_{1t}, \ldots, y_{Mt}) \) is an \( M \)-element vector of dependent variables and \( x_t = (x_{1t}, \ldots, x_{Kt}) \) a \( K \)-element vector of independent variables. Assume further that
$0 < M < K < \infty$ and $M + K < N < \infty$. Let

$$e_t \equiv F ([ y_t \ x_t ]) = y_t - G (x_t)$$

such that

$$e_t \sim N_M \left[ 0 \mid Q \right]$$

where $Q$ is a completely specified, $M \times M$ positive definite symmetric matrix, and let

$$E (y_t) = G (x_t)$$

be an explicit function of $x_t$ involving $n_Q$ unknown parameters, where $0 < n_Q < M + K$. Under the conditions specified above, one can always find a unique solution for the $n_Q$ parameters of $G (x_t)$, such that the quadratic form

$$\sum_{t=1}^{N} (y_t - \hat{G} (x_t))^{-2} V (y_t - \hat{G} (x_t))$$

is minimized, where $V = \frac{1}{\sigma^2} Q^{-1}$, and $\sigma^2$ is an arbitrary positive scalar and $\hat{G} (x_t)$ is the explicit solution to $G (x_t)$ in terms of the parameters minimizing the quadratic form mentioned above.

Estimating the parameters of $G (x_t)$ is not, however, sufficient to predict the future magnitudes of $E (y_t)$; we are still faced with the problem of describing the deterministic trends in the dependent variables so as to be able to project their future magnitudes. Describing deterministic trends in the dependent variables, in turn, implies specifying the historical sequence of observations on the dependent variables as a transformation of the domain of time such that
\[ E(x_{1t}) = x_{1}(t) \]
\[ E(x_{2t}) = x_{2}(t) \]
\[ \cdots \cdots \cdots \cdots \]
\[ E(x_{Kt}) = x_{K}(t); \]

\[ u_{t} = x_{t} - E(x_{t}) = x_{t} - x(t) \sim N_{K} \left[ \begin{array}{c} 0 \end{array} \right] \]

where \( x(t) = \begin{bmatrix} x_{1}(t), x_{2}(t), \ldots, x_{K}(t) \end{bmatrix} \), and where \( \Omega \) is a completely specified \( K \times K \) positive definite symmetric matrix. Hence, for forecasting purposes, we are dealing with a stochastic transformation defined over \( t \) of the form \( E(y_{t}) = E(G[x(t) + u_{t}]) \). If

\[ E(G[x_{t} + u_{t}]) = G[E(x_{t} + u_{t})], \]

then we can deterministically solve for \( E(y_{t}) \) as

\[ E(y_{t}) = G[x(t)] = H(t). \]

If, on the other hand, we can relatively accurately describe the behavior of a series of observations \( y_{t} \) through a set of explicit functional models of the form

\[ y_{t} = H(t) + Z_{t}, \] where \( Z_{t} \sim N_{M} \left[ \begin{array}{c} 0 \end{array} \right] \),

then for the purposes of monitoring, predicting and explaining the intertemporal behavior of \( y_{t} \), considerations pertaining to the nature and existence of time invariant stochastic relations between \( y_{t} \) and some \( x_{t} \) become somewhat spurious. On the contrary, the only way to demonstrate the intertemporal variance or invariance of a relationship known to exist between any two series of observations would be to compare their respective time path models over the domain of time under consideration. In the case of energy data series, for example, this implies that we must characterize and measure their time paths in terms of time variant linear trends.
There are at least two levels at which there exist significant differences in the way a mathematical model is conceptualized as presented in section 1.C.2, and the way an econometric model is conceptualized. The first is with regard to the nature of observations certain aspects of which the corresponding models are presumed to describe.

In a mathematical model each observed magnitude is presumed to consist of a deterministic part, as characterized by a specific functional model, and a stochastic part, as characterized by the corresponding stochastic model. This aspect of positive analysis has been sufficiently elaborated in the preceding sections of this chapter. In fact, the objective of quantitative scientific theories is to explain the magnitude of observations within their respective domains of discourse. The accuracy of such theories can only be enhanced as the accuracy of the methods and models of quantitative measurement are themselves enhanced (6). For example, the differences in the Newtonian mechanics and quantum mechanics are possible in part due to the advances that were made in methods of physical measurement. The concept of a quantitative magnitude as consisting of a deterministic part and a stochastic part is a basic perception that makes possible such cumulative advances in quantitative empirical investigation. For, as the methods of measurement become more accurate, one gains the ability to distinguish between systemic error and stochastic error. And if a particular functional model of an antecedently and independently known physical system consistently yields residuals after adjustment which are significantly higher than those that would be expected from the
established stochastic properties of the measurements, then it is a signal for the need and the opportunity to improve the mathematical model of the physical system under consideration and hence a signal for the need and the possibility to learn more about the system in the process (7).

As opposed to the dualistic perception of observations outlined above, the economist is interested in observations as nonstochastic magnitudes, though occasional lip service is paid to the possibility that economic observations may include a stochastic component. A classic example of this position is as quoted below.

"In each behavior equation, the disturbance is interpreted as representing the joint effect, on the behavior described by that equation, of all variables of minor individual importance that have not been explicitly introduced into the system of equations. For instance, random variation in consumers' tastes will lead to a certain amount of shifting in the curve of consumers' demand. Similarly, in the technical relations between input and output, a certain amount of random shifting in the relationships is due to a large number of minor causes of variation not explicitly studied.

"It is important to note that in the interpretation of disturbances just given, each disturbance is associated with an equation of the system, and not with a variable. This excludes the interpretation of the 'disturbances in the equations' as errors of measurement. If errors of measurement occur to a marked degree, separate provision must be made for them in the probability distribution of observed variables by introducing additional 'disturbances in the variables.' In order to concentrate on the effect of disturbances in the equations, we shall assume in this study that all variables are measured without error. Systems in which 'disturbances in the equations' (also called 'shocks') and 'disturbances in the variables' (also called 'errors') occur side by side have been studied in [T. W. Anderson and Hurwicz]." (Koopmans, Rubin, and Leipnik 1950, pp. 57-58. See also Note (11).)

The important point is that given the economist's interest in "disturbances in relations", no interest is shown in the question of what functional model is to be used in determining the observational
errors so that the observations may be accordingly adjusted prior to econometric investigation of the "disturbances in relations".

Theoretically, the interest shown in observational errors is directed towards the determination of their influences on the asymptotic properties of the estimated parameters of econometric models. Such interest usually takes the form of concern about the asymptotic implications of considering stochasticity of observations along with the stochasticity of econometric relations. A typical example is provided in the following quotation from Johnston.

"So far we have implicitly assumed that the X variables have been measured without error and the only form of error admitted to the relation has been in the disturbance term u. The latter has generally been thought of as representing the influence of various explanatory variables that have not actually been included in the relation. It could of course also have a component representing measurement error in the dependent variable Y and the previous results would still be valid. We now have to ask the question of what happens if the X variables are subject to measurement error. We assume that the \( \beta \) vector represents the coefficients of the correctly measured X variables. What will happen if we apply our least-squares techniques to the actual measurements available of the X and Y variables? The answer is that OLS estimates will not only be biased but will also be inconsistent." (Johnston, J. 1978, pp. 281)

Econometric models currently utilized for the analysis or forecasting of economic data series do not explicitly consider the effects of the stochasticity of observations in the data series. The consideration of the stochasticity of observed data series has rather unpleasant implications for econometric analysis and estimation, as indicated in the above quotation. This may be one reason why the general practice seems to be to ignore the fact altogether.

We have already established in section I-C that economic observations are, in most cases, ultimately based upon physical
measurements and, therefore, are subject to stochastic fluctuations. The economist's implicit or explicit contention of nonstochastic observations presumes the existence of nonstochastic methods of physical measurement and, therefore, is contrary to the current state of the art and the accepted practice in other empirical sciences.

A practical consequence of this state of affairs is that in physical sciences observations are usually subjected to a series of previously established corrections for known observational or systemic anomalies, so that in effect the adjusted observations of one model may become the unadjusted observations of the next model. These correction models and processes greatly enhance the explanatory power and accuracy of the functional model to which the resulting observations are related. This becomes possible because, through successive model modifications implied in the above process, as many aspects of observations as deemed necessary and possible to the particular aspect which is being described by the functional model in question may be eliminated or accounted for.

The economist by denying the stochasticity of observations more or less condemns himself to dealing with raw data in almost all instances of econometric analysis. In search of metaphysically predicated invariant statistical regularities among raw data series, regularities due to social or physical events such as recessions, wars, severe climatic changes, or other "Acts of God" (such as the Arab oil embargo, the revolution in Iran, alterations in institutions, rules or frameworks of exchange, the breakdown of the international monetary system) go unheeded for all practical purposes. It is no wonder, then, that
models which do not account for such grave, recurrent facts of life and which have historically been known to significantly influence many aspects of economic activity should produce unreliable or inaccurate forecasts. This, in turn, implies trivial, irrelevant or unreliable conclusions pertaining to appropriate course of action required in achieving specified policy objectives (8).

At a deeper level the perceptions concerning observations reflect a more basic difference concerning the processes which are responsible for the observations in question. In empirical sciences there seems to be a meticulously maintained distinction between the physical concepts and operations and the mathematical or theoretical concepts and operations which represent them in a model (9). In fact, the determination of rules and procedures, including procedures for the measurement of the relevant physical magnitudes involved, through which one-to-one correspondences are established between the mathematical concepts and operations indicated in the functional model and the corresponding physical concepts and operations which they represent, seems to be a prerequisite of quantitative science in general (10).

As mathematical statements, econometric models may, in general, be characterized as representations of stochastic transformations among nonstochastic observations (11). However, no distinction is made between stochastic versus functional model in econometric jargon. It is also not clear what are the correspondences between the mathematical descriptions of stochastic transformations predicated to exist among mathematical variables representing magnitudes of physical observations and the physical transformations, if any, that presumably involve the
physical magnitudes and take place independently and antecedently of such mathematical descriptions.

In the final analysis, the stochastic transformations are alleged to be derived from neoclassical economic theory (12). This introduces at least one problem into the picture which we shall simply mention without discussion. The initial question of empirical import of mathematical symbolism, however, does not become clarified by referring its justification to what is essentially an axiom system in pure mathematics (13). The following rather classical distinction between the import of axiom systems in pure mathematics and the import of axiom systems in physics by Carnap is definitely one not sufficiently considered by many economists.

"... Sometimes an axiom system in mathematics is called a theory. Mathematicians speak of set theory, group theory, matrix theory, probability theory. Here the word 'theory' is used in a purely analytic way. It denotes a deductive system that makes no reference to the actual world. We must always bear in mind that such a use of the word 'theory' is entirely different from its use in reference to empirical theories such as relativity theory, quantum theory, psychoanalytical theory, and Keynesian economic theory.

"A postulate system in physics cannot have, as mathematical theories have, a splendid isolation from the world. Its axiomatic terms--'electron', 'field', and so on--must be interpreted by correspondence rules that connect the terms with observable phenomena. This interpretation is necessarily incomplete. Because it is always incomplete, the system is left open to make it possible to add new rules of correspondence. Indeed, this is what continually happens in the history of physics." (Carnap, 1966, pp. 10, 11, 236-237)

In addition to the initial problem mentioned above, we shall note that the concepts of goods and services as introduced, interpreted and represented in neoclassical utility theory are surely not the same as the everyday use of such concepts. A typical exposition of what an
The economist has in mind when he talks about commodities in a theoretical model is provided in the following quote from Hildebrand and Kirman.

"A commodity is anything which may be used or consumed. It may be a physical good such as bread or a service such as the use of some object. ..."

"A commodity must then be completely homogeneous, i.e. one unit of it must be completely indistinguishable from another in all respects. ... 'in all respects' means not only in terms of its physical characteristics, but also in terms of where it is located in time and space. ... Thus, a commodity is fully described by its physical characteristic and the time and place at which it is available." (Hildebrand & Kirman, 1976, pp. 36; see also Malinvaud, 1973, pp. 2-8)

It is clear from the above quotation that as a theoretical construct an economic good or service is identified with regard not only to the dimension of physical properties but also to the dimensions of space and time of availability. An empirical interpretation of such a theoretical construct would require not only quantification of conventional physical dimensions of every day goods such as weight, length, volume, etc. but also a coordinatization of space and time relative to which it may be determined whether two units of the same everyday commodity constitute two units of the same economic commodity or units of two different economic commodities (14). Until such coordinatization of space and time is developed for all everyday goods and services, the quantities of everyday goods and services cannot be transformed into quantities of economic goods and services. The utilization of the former magnitudes in alleged local estimations of theoretical laws derived from axiom systems designed to reflect the behavior of the latter magnitudes is, therefore, an arbitrary thought experiment at best.
E. Mathematical Models and the Time Paths of Economic Data Series

1. The Analysis and Assessment of Time Variant Linear Trends in the Time Paths of Economic Data Series

From the point of view of "positive" analysis, the objective of a forecasting model is to describe the time path to be followed by the future values of a specified data series. Hence, as a first approximation, extrapolation of the most recent trends in the time path, assessed from consistent, accurate and precise analyses of the historical behavior of the time path in question, is always a prudent and parsimonious alternative in short term or intermediate term forecasting. Furthermore, since the possibility of discontinuities in the time path are explicitly acknowledged, significant and sustained deviations of the most recent observations from the previously prevailing trend would be interpreted to indicate the establishment of a new trend in the time path. Hence, this possibility of modifying or updating the currently prevailing trend in the time path as new observations become available promises to make time path models rather accurate and sensitive forecasters of the short term or intermediate term values of the data series in question.

An important point to be recognized at the outset is the fact that many economic data series are definitionally related. Almost any economic data series could be decomposed into alternative sets of component data series, as is the case with the decomposition of national totals into state totals, into national sectoral totals and so on. Consistent analyses of the intertemporal decomposition of a total time path into the parts of its alternatively conceptualized sums or
equivalently the composition of a total time path in terms of its alternatively designated component parts, could provide useful information in understanding the historical behaviors of the magnitudes in question. In fact, in every case in which component as well as total behavior are to be described, it is a logical necessity that the sum of the component descriptions be identical to the total description. Since every whole imposes a definitional constraint on each and every component of its alternatively designated sums, these constraints should be explicitly introduced into the process of analysis. Furthermore, since every well-founded constraint on a data series is in fact an additional piece of a priori information pertaining to the historical behavior of the data series in question, the larger the number of definitional constraints (i.e., constraints that are well founded as a matter of logical necessity and are imposed on the description of a time path), the greater would be the empirical validity, logical consistency, and factual accuracy of the description in question.

Consider, for example, the problem of describing the time path of residential electricity sales in Ohio. This can be taken as an isolated problem, a problem within the context of describing the time paths of electricity sales in all sectors of Ohio simultaneously, or a problem in describing the time paths of different energy resources that make up the time path of residential energy consumption in Ohio. But all of these can be considered as embedded in one problem involving several levels of whole-part relations. It is obvious that a description of the time paths of residential electricity consumption in Ohio, which is consistent with the historical time paths of all possible relevant energy
data series in Ohio, would be a much more reliable, useful and sensitive description of the historical behavior than that of the single data series by itself. By the same token, the process of forecasting in this instance would require that the forecast of a single data series not be made in isolation, but in conjunction and conformity with the forecasts of all the parts and all the wholes. This way the historical time path of the data series can be made explicitly consistent with all the other series.

Positive analysis and assessment of functional relationships that are known to exist between two data series require the knowledge of their respective time paths. For example, if $y_t$ is the magnitude of electricity generated by coal fired plants in Ohio during calendar year $t$, and $x_t$ is the magnitude of coal burned in the process, then all the complex social, legal, economic, political, physical, chemical, administrative and other factors that are involved in the transformation of $x_t$ into $y_t$ can be abstracted to their bare bones and expressed as

1.1.a) $y_t = a_t x_t$

for all $t$ in $T$, where $T$ is a specified continuous interval of historical time. Since $y_t$ and $x_t$ are physical magnitudes, their time paths can be expressed in terms of a functional and a stochastic model. Hence, if

1.1.b) $y_t = y(t) + v_y(t)$

1.1.c) $x_t = x(t) + v_x(t)$
where \( y(t) \), and \( x(t) \) are functional time path models, assessed on the basis of an appropriate adjustment criterion, such as least squares, and the presumed statistical properties of the observations \( x_t \) and \( y_t \). The residuals \( v_y(t) \), and \( v_x(t) \), in turn, represent the magnitudes by which the corresponding observations must be corrected in order to become consistent with the estimated functional time path models.

From 1.1.a-c, the coefficient of transformation, \( \hat{a}_t \), can be calculated as

\[
\hat{a}_t = a(t) = \frac{y(t)}{x(t)}
\]

The historical behavior of other \textit{a priori} functional relations such as those between the transformation of, say, annual volume of kWh sales into corresponding magnitude of total revenues can also be assessed through a similar positive approach. Furthermore, the analysis of such transformations as elements in a hierarchical series of definitionally related transformations (i.e., the analysis of the transformation of total kWh sales into total revenues as a sum of the transformations of component sectoral sales into corresponding sectoral revenues) may greatly enhance our understanding of the intertemporal relations between sectoral sales and sectoral revenues as well as the intertemporal relations between the total transformation and its component transformations over specific historical periods of time.

There is no doubt that such an understanding would contribute to more efficient investment planning for generating capacity and would be a powerful tool, both for utility executives and for regulatory agencies, in presenting empirically valid arguments for or against rate
adjustment requests. There is no reason, of course, that the reasoning and arguments made in this, or any other section, should be confined, or interpreted to apply, only to electric utilities or to the provision or disposition of energy in particular. They are general in their application to any observational economic data series.

2. Analysis and Assessment of the Effects of Specific Historical Events, Circumstances or Policies on the Behavior of Trends in Single, or Definitionally or Functionally Related Time Paths

Important historical events, incidents, or circumstances of a non-recurrent, or recurrent but nonseasonal nature, such as wars, famines, recessions, droughts, embargos, breakdowns or establishments of international agreements, technological breakthroughs, establishment or abolishment of relevant federal or state policies, may have distinct and decided effects on the time paths of many economic data series at, during, or subsequent to, their occurrence. The identification of such historical influences and a quantification of their impact on the time paths of a particular set of economic data series through functional or definitional analyses of the relevant magnitudes involved could contribute immensely to the understanding of the nature of the intertemporal relations among the magnitudes being examined.

On the other hand even a simple demonstration that the recession of 1974-75, say, has had the effect of changing the sign of the linear trends in a host of economic data series during the same period of time, or has led to permanent reductions in the magnitude of the
post-1975 trends relative to the pre-1973 trends, may in itself constitute important information in forecasting the data series under consideration. Such an observation establishes the necessity as well as the possibility of specifying a finite set of events, incidents, or circumstances the presence or absence of which must be explicitly stated for the purposes of forecasting the data series within the historically determined confidence intervals of corresponding sizes with a specified probability. Furthermore, it opens up the possibility of generating alternative scenarios pertaining to the nature of the forecasting horizon by predicing different frequency, timing or duration for the occurrence of the relevant factors within the forecasting horizon. Obviously, the future behavior of a time path, extrapolated on the basis of its historical behavior under the predicated conditions, would be different under each different predication of the relevant factors. To the extent that such forecasts make accurate assessments of alternative states of the future possible, they would be very valuable information for those whose present actions must take into account realistic and reliable assessments of best, worst, or most likely, scenarios of availability or saleability of goods or services that are critical for the future success of their enterprises or for the attainment of pursued policy objectives.

3. **Long Term Implications of Time Path Analyses**

Positive investigation is a cumulative enterprise. It is through an accumulation of repeated and interrelated evidence that certain events, circumstances, or descriptions are accepted as valid
explanations of certain observations. Archimedes' law of buoyancy, Kepler's laws of planetary motion, and Boyle's law of gases are all relatively precise descriptions of repeatedly observed regularities in the respective phenomena. In the history of scientific enterprise, at least at the earlier stages of such established sciences as physics and astronomy, universal laws, such as those of Newtonian mechanics or the molecular theory of gases, have followed, and not preceded, more modest descriptions of locally observed regularities in certain quantifiable aspects of the phenomena which have happened to be of some interest and importance to their respective investigators (15).

The switch of emphasis from the music of the spheres to the trajectories of the spheres is a crucial event in the development of astronomy and, hence, of modern day "positive" science. Since in economics, as in astronomy, the observer has virtually no control on what is being observed, a similar change of venue from "normative" to "positive" investigation may require a shift of emphasis from presumptions of lofty pursuits such as the greatest good of the greatest number to clear, precise, accurate and consistent analyses and assessments of historical performances of quantitative magnitudes and of the systems and circumstances that are responsible for them. The conceptual framework and the mathematical model to be developed in this thesis are hoped to provide an empirical perspective and an analytical method required to initiate a conscious and conscientious switch from "normative" to "positive" analyses of dynamic economic systems and the dynamics of the circumstances which determine their historical behavior.
CHAPTER II

THE ANALYSIS AND ASSESSMENT OF TIME VARIANT LINEAR TRENDS IN THE TIME PATH OF AN AGGREGATE OBSERVED ECONOMIC DATA SERIES

A. System Theoretic Framework

1. Preliminary Remarks

The importance of analyzing the time paths of economic data series as a source of information concerning the dynamic behavior of the underlying socio-economic system has been discussed from a number of different perspectives in the previous chapter. In section A of this chapter we develop a formal model within which the analysis and assessment of an aggregate dynamic flow system will be established in terms of a nondifferentiable flow velocity time path generated by sums of sequences of discontinuous increments in accelerations. From an empirical point of view, the existence of such discernibly discontinuous accelerations would imply corresponding discontinuities in the dynamic forces determining the intertemporal movement of the system. Hence each anomaly in the time path of systemic flows is to be interpreted as an opportunity to learn more about the dynamic system under consideration in particular and economic systems in general.

In section B, we shall derive the general form of the functional model of time variant trends in time paths of systemic flow
velocities for a single data series. We shall consider the mathematical modeling and adjustment of a single systemic flow velocity time path in terms of time variant linear trends. In section C, we shall discuss the application of this methodology to the case of commercial natural gas disposition in Ohio between 1960 and 1981 as a demonstrative example.

2. The Analysis and Assessment of a Dynamic Aggregate Flow System
   a. Conceptual Framework. Let \( n \) be a positive integer such that

   \[ T = < 0, n > \]

   is an interval of time of size \( n \) consisting of \( n \) successive periods of equal duration. For the purpose of this thesis, we consider periods of calendar year duration only. For example, the interval January 1, 1960 through December 31, 1981 would be represented as \( T = < 0, 22 > \). We shall let

   \[ i = 1, 2, \ldots, n, \]

   be the \( i \)th calendar year in \( T \). Let

   \[ S \]

   be a geographically designated socio-physical system. An example of \( S \) would be the State of Ohio. Let

   \[ R_s \subseteq S \]
be a designated class of providers or end users in $S$, e.g. the class of commercial end users in Ohio. Finally, let

2.5) $X_{ai}$

be the aggregate magnitude relative to a uniform system of accounting over $T$, of a good or service $X$, disposed of among recognized or eligible members of $R_a$ or provided by recognized or eligible members of $R_a$ during the $i$th successive annual period in $T$. An example of $X_{ai}$ would be total natural gas sales to all end users in Ohio in calendar year 1981. It should be noted that as economic observations the units of magnitude of $X_{ai}$ are in "units of $X$ per year". Given $T = < 0, n >$, $S$, $R_a$ and $X$, $X_a$ is an "n" element vector of observations whose magnitudes reflect the outcomes of the intratemporal activities of the members of the socio-physical system $S$, in their capacity as members of $R_a \subset S$, during the $i$th annual period in $T$.

Our objective is to develop a methodology to describe the time path traced by the elements of $X_a$ over $T$ in terms of a functional model of time variant linear trends, $X_{at}$, and a stochastic model of residuals relative to the prescribed functional model, $e_t$. Furthermore, since the elements of $X_a$ reflect physical observations consolidated according to an underlying accounting system, we want the class of mathematical models that is to be proposed to be able to accomodate, preserve, and explicitly reflect, whenever necessary, the physical or accounting definitions, operations, transformations or aggregations which underlie the magnitudes of the elements of $X_a$. For the purpose of simplifying
the exposition, however, we shall first proceed with the consideration of a single time path trajectory and then indicate how the mathematical model proposed can be extended to include definitionally related time paths.

b. Systemic Implications of the Time Paths of Observations on Economic Flows. If, in a given human society $S$, $X_a \neq 0$, then those aspects of $R_a$ which, relative to a uniform system of accounting applied over $T$, have generated $X_a$ will be referred to as the system of aggregate disposition or provision of $X$ in $S$ over $T$, or $S_{xa}$. Since the units of the elements of $X_a$ are in terms of "units of $X$ per year", $S_{xa}$ can be conceptualized as a dynamic system which generates a continuous flow of $X$ through $R_a$ within each one of the $n$ calendar years in $T$. The elements of $X_a$ then measure the annual flows of $X$ through $R_a$, for $i = 1, 2, \ldots, n$.

The nature and existence of the flows reported in $X_a$ require the existence of mechanisms, institutions, and organizations in $S$ which have physically, socially, and historically supported the observed flows. Hence, $S_{xa}$ includes the sum total of all such existing mechanisms, institutions and organizations at every $t \in T$. Any changes in these underlying mechanisms, to the extent that they influence the observed magnitudes of the flows generated by $S_{xa}$, constitute temporal changes in $S_{xa}$. Such changes in the underlying structure may go on in a continuous manner over $T$, reflecting the reactions of centers of decision in the system to anticipated future magnitudes of the flows in question under a "normal" range of changes in operating conditions.
To the extent that the anticipated "normal" range of changes in operating conditions has actually been realized historically, the historical magnitudes of the deterministic aspect of the observed sequence of realized systemic flows in $X_a$ would be expected to move along some well-behaved and smooth time path over $T$. To the extent that "unanticipated", "extraordinary", or "unprecedented" operating conditions have prevailed at certain points, or over certain subdomains, of $T$, one would expect to observe correspondingly "unexpected", "extraordinary" or "unprecedented" behavior in the functional time path of the realized sequence of the systemic flows at such points, or over such subdomains, of $T$.

In general, the mechanisms, institutions, or organizations required to support the observed magnitude of aggregate flows of a major commodity or commodity group within any given society at a given interval in time do not develop at random, nor do they cease to exist at random. For example, the distribution of major fuels within any country, especially one with the size and complexity of present day U.S.A., is an immense enterprise requiring extensive infrastructure outlays, such as private pipeline systems, several levels of storage, distribution terminals, powerlines, railroads, and public highways. Such outlays have a tendency to become at least semi-permanent, for their amortization and overhead costs are spread over a long period of time. The development of particular patterns or levels of sectoral energy consumption distribution over the country over time can, in turn, be interpreted as reflecting the geographical, geological, and climatological characteristics of the states as well
as the attitudes of their administrations, institutions, and inhabitants
towards various types of economic activities, their system of values,
beliefs, and preferred life styles. These factors are also semi-
permanent for the most part. Accordingly, by "unanticipated", "extra-
ordinary" or "unprecedented" operating conditions, we have in mind such
definite and objectively identifiable events as civil or overseas wars,
large scale flood, famine, drought, blizzard, plague or other "Acts
of God", significant changes in levels or rates of growth of unemploy-
ment, incomes, populations, or prices from one period of observation to
the next, breakdown or reformulation of national or international laws,
treaties or agreements, embargoes on critical raw materials, etc.

\( S_{X_a} \) can, therefore, be conceptualized as a deterministic dynamic
system which has transformed time and circumstances into an observed
stream of aggregate flows of \( X \), in \( R_a \), over \( T \), such that the annual
rate of transformation of \( t, 1 < t < n \), into the stream of flows
in question may be described by a continuous and differentiable
function of \( t \), i.e.

\[
2.6.a) \quad \frac{d X_{at}}{dt} = f(t),
\]

and

\[
2.6.b) \quad | f(t) | < \infty,
\]

provided that the "operating conditions" or the "states" of the system
are considered to be "normal" or "customary" over the domain of \( t \),
relative to the initial period, $0 \leq t \leq 1$, in $T$.

It should be obvious that since the units of $X_{at}$ are in terms of 
"units of X per year", the units of $f(t)$ must be in terms of "units of 
X per year per year." Thus $f(t)$ is the magnitude of the acceleration 
at time (t) of the systemic flows generated by $S_{xa}$. Hence, $S_{xa}$ may be 
characterized as a physical flow system which moves with an acceleration 
whose magnitude is defined by $f(t)$ over the domain of $t$. The 
magnitude of $X_{at}$ at any point $t$ can then be determined by integration, 
i.e.

\[ \int_0^t f(t) \, dt + X_{a1}. \]

To the extent that substantial external "forces" are either known 
or claimed to have influenced or modified the time path of the flows of 
$S_{xa}$, we require that such deterministic influences or modifications be 
observable and representable in terms of discontinuities in the 
functional representation of the time path of $\frac{dX_{at}}{dt}$ at the beginnings 
of these annual periods within which the external forces in question 
are introduced into the environment of $S_{xa}$. Furthermore, some of these 
forces may be temporary in nature. For example, in the case of energy 
disposition, a severe winter may lead to a substantial deviation from 
an established time path trajectory for an observational period of one 
year only. Such temporary deviations can, in principle, be represented 
by a series of consecutive changes in the accelerations defining the 
time path of $X_{at}$, which sum up to zero. On the other hand, other forces 
acting on the system due to certain other more fundamental and longer
lasting circumstances, such as the breakdown of the international monetary system, a major devaluation of the dollar, OPEC price hikes and so on, may lead to more permanent alterations in the subsequent time path of $X_{at}$. That is, the net effects of irreversible or persistent alterations in the "operating conditions" in terms of the resulting sequences of systemic accelerations may not add up to zero at all, or may add up to zero within a frame of reference longer than one year.

In any case, whenever the "operating conditions" or the "states" of the system are altered for whatever reason, there should be a corresponding alteration in the acceleration of the systemic flows. Hence, if

2.8) $J = \{J_1, \ldots, J_2, J(\lambda + 1)\}$

is a subset of $i$ with

2.9) $J(\lambda + 1) = n, \lambda \geq 1,$

such that at the end of every annual period ($i$)

$i = J_k, \quad k = 1, 2, \ldots, \lambda,$

the state of the system is deemed to have changed relative to the state of the system prevailing during $i = J_k$; and if

2.10.a) $j = \{j_1, \ldots, j_2, j(\lambda + 1)\}$

is the set of end points of the elements of 2.8; and
2.10.b) \( j0 = 0, \)

then we shall require the functional representation of the accelerations in the systemic flows, \( \frac{d X}{dt} \), and the systemic flows, \( X_{at} \), to satisfy the following conditions:

2.11.a) \( \left| \frac{d X_{at}}{dt} \right| = \infty \) if \( t = jk \)

2.11.b) \( \left| \frac{d X_{at}}{dt} \right| < \infty \) otherwise

2.11.c) \( \left| X_{at} \right|_t = jk_+ - X_{at} \left| t = jk_- = 0, \right. \)

2.11.d) \( 0 < \left| \frac{d X_{at}}{dt} \right|_t = jk_+ - \frac{d X_{at}}{dt} \right|_t = jk_- \right| \right| < \infty \)

The requirements 11.a-c define the functional time path of the systemic flows \( X_{at} \) as being continuous over the interval \( 0 \leq t \leq N \), and nondifferentiable at a finite set of interior points, \( t = jk \), of the interval in question. The requirement 11.d defines the time path of systemic accelerations \( \frac{d X_{at}}{dt} \) as being discontinuous at \( t = jk \) (2). 

B. Mathematical Method

1. Mathematical Implications of Nondifferentiable Time Paths of Systemic Flow Velocities

Let

2.12) \( \frac{d X_{at}}{dt} = f_k (t) \) if \( jk < t \leq j(k+1) \).
\[ \frac{d X_{at}}{dt} = f_{a}(t) \quad \text{if} \quad 0 \leq t \leq j 1. \]

Then, the magnitude of \( X_{at} \) would be given by

\[ X_{at} = \int_{jk}^{t} f_{k}(t) \, dt + C_{k} \quad \text{if} \quad jk < t \leq j(k+1), \]

\[ X_{at} = \int_{o}^{t} f_{o}(t) \, dt + C_{o} \quad \text{if} \quad 0 \leq t \leq j 1. \]

Furthermore, by definition,

\[ X_{at} \bigg|_{t = j(k+1)}^{jk} = \int_{jk}^{j(k+1)} f_{k}(t) \, dt - C_{k} = C_{k}, \]

and, similarly,

\[ X_{at} \bigg|_{t = j(k-1)}^{jk} = \int_{j(k-1)}^{jk} f_{k-1}(t) \, dt + C_{k-1}. \]

From 2.11.c, we require that 2.14 equal 2.15. Substituting the right hand side of 2.14 into the left hand side of 2.15 we express the necessary condition of the nondifferentiability of \( X_{at} \) as

\[ C_{k} = \int_{j(k-1)}^{jk} f_{k-1}(t) \, dt + C_{k-1} \quad \text{if} \quad jk < t \leq j(k+1), \]

for all \( k = 1, 2, \ldots, 2 \).
We shall adopt as a methodological convention that (3)

2.17.a) \( \frac{dX_{at}}{dt} = f_0 = 0 \) if \( 0 \leq t \leq j_1 \).

Therefore

\[
X_{at} = \int_{0}^{t} f_0(t) \, dt + C_0 = \int_{0}^{0} 0 \, dt + C_0 = C_0 \text{ if } 0 \leq t \leq j_1.
\]

In particular, when \( t = 1 \leq j_1 \)

\[
X_{at} = X_{a1}, \text{ hence:}
\]

2.17.b) \( X_{at} = C_0 = X_{a1} \) if \( 0 \leq t \leq j_1 \).

Similarly, from 2.16,

2.17.c) \( C_1 = \int_{j_1}^{j_1} f_0(t) \, dt + C_0 = \int_{0}^{0} 0 \, dt + C_0 = C_0 \text{ if } j_1 < t \leq j_2. \)

Hence, from 2.17.a-c,

2.18.a) \( C_k = X_{a1} \) if \( 0 \leq t \leq j_2 \).
and from iterative replacement of $C_{k-1}$ in 2.16 (4)

\[
C_k = \left[ \sum_{j=1}^{k} \int_{q=2}^{j(q-1)} f_{q-1}(t) \, dt \right] + \chi_{al}
\]

if $jk < t < j(k+1)$, and if $2 \leq k \leq l$.

Through integration we could, in general, explicitly determine the functional representation of the time path of systemic flows, if the $f_k(t)$ mentioned in 2.12 and 2.13 were given. The process of integration could be easier to manipulate, and the implications of the process easier to interpret if the time paths of the $f_k(t)$ and the $C_k$ could be represented as a functional form defined over the domain $0 \leq t \leq n$. We shall try to accomplish this result in the next section. On the other hand, we can always solve for the time path of systemic accelerations if the explicit functional form of $\chi_{al}$ is given.

2. System State Representation of the Functional Model of a Single Nondifferentiable Time Path of System Flow Velocities

The equations 2.12 and 2.13 do not explicitly distinguish among the successive alterations in systemic flows, or in the accelerations that are introduced due to each successive change in the state of the system. Such a representation can easily be achieved, and is essential for measuring the magnitude of deterministic changes induced in the systemic time paths due to specific historical changes in the states of the system.
Let the functional representation of the net change in the systemic acceleration time path, subsequent to the end point \( j_k \) of the \( k \)-th subinterval in the domain of \( T \), be defined as \( \Delta_k \) where

\[
\Delta_k = f_k(t) - f_{k-1}(t) \quad \text{if} \quad j_k \leq t \leq j(k+1); \\
\]

or

\[
f_k(t) = f_{k-1}(t) + \Delta_k \quad \text{if} \quad j_k \leq t \leq j(k+1). \\
\]

From the methodological convention adopted in 2.17.a-c, during the initial subinterval in \( T \), i.e. \( 0 \leq t \leq j_1 \), \( f_0(t) = 0 \). Hence, from 2.20

\[
f_1(t) = f_0(t) + \Delta_1 = \Delta_1 \quad \text{if} \quad j_1 \leq t \leq j_2. \\
\]

In view of 2.21 and by iterative replacement of \( f_{k-1}(t) \) (5), we can rewrite 2.20 as

\[
f_k(t) = \sum_{m=1}^{k} \Delta_m \quad \text{if} \quad j_k \leq t \leq j(k+1), \\
\]

and
2.22.b) \[ f_0(t) = 0 \quad \text{if} \quad 0 \leq t \leq j1. \]

Similarly, we could define,

2.23.a) \[ f_{(k-1)}(t) = \sum_{m=1}^{k-1} \triangle_m \]

if \( j(k-1) < t \leq jk \) and \( 2 \leq k \leq \lambda; \)

and

2.23.b) \[ f_{(k-1)}(t) = f_0 = 0 \quad \text{if} \quad 0 \leq t \leq j1. \]

Substituting 2.23.a into 2.18.b, and repeating 2.18.a, we obtain an incremental description of \( C_k \), such that

2.24.a) \[ C_k = x_{a1} \quad \text{if} \quad 0 \leq t \leq j2 \]

2.24.b) \[ C_k = \sum_{q=2}^{k} \int [ \sum_{m=1}^{q-1} \triangle_m ] \ dt + x_{a1} \]

if \( jk < t \leq j(k+1) \) and \( 2 \leq k \leq \lambda. \)
2.24.b is in turn equal to (6)

\[ 2.24.c \] \[ \sum_{m=1}^{k-1} \int_{jm}^{\Delta_m} \cdot dt + \chi_{a1} \]

if \( jk < t \leq j(k+1) \), and \( 2 \leq k \leq \lambda \).

The objective of our exercise in this section is to express \( X_{at} \), as defined in 2.13, in terms of incremental changes in the time path of systemic accelerations. To achieve this objective, we substitute, successively, 2.24.a and 2.22.b, 2.24.a and 2.22.a, and 2.24.c and 2.22.a, into 2.13 to obtain

2.25.a) \[ X_{at} = \chi_{a1} \quad \text{if} \quad 0 \leq t \leq j1 \]

2.25.b) \[ X_{at} = \chi_{a1} + \int_{j1}^{j2} \Delta dt \quad \text{if} \quad j1 < t \leq j2 \]

2.25.c) \[ X_{at} = \sum_{m=1}^{k} \int_{jm}^{\Delta_m} \cdot dt + \sum_{m=1}^{k-1} \int_{jm}^{\Delta_m} \cdot dt + \chi_{a1} \]

\[ = \sum_{m=1}^{k} \int_{jm}^{\Delta_m} \cdot dt + \chi_{a1} \]

if \( jk < t \leq j(k+1) \) and \( 2 \leq k \leq \lambda \).
In general, 2.25.a-c can be rewritten as

2.26.a) \[ X_{at} = X_{a1} \] if \( 0 \leq t \leq j_1 \)

2.26.b) \[
X_{at} = \sum_{m=1}^{k} \int_{j_m}^{t} \Delta_m \, dt + \sum_{m=k+1}^{\lambda} \int_{j_m}^{0} \Delta_m \, dt
\]
if \( j_k < t \leq j(k+1) \) and if \( 1 \leq k \leq \lambda \).

If we introduce a set of dummy variables such that

\[
DUM_m = 0 \quad \text{if} \quad t \leq j_m ,
\]

\[
DUM_m = 1 \quad \text{if} \quad t > j_m ,
\]

\( m = 1, 2, \ldots \lambda, \quad 0 < t < n, \)

then we can express 2.26.a-b as

2.27.a) \[ X_{at} = X_{a1} \] if \( 0 \leq t \leq j_1 \)

2.27.b) \[
X_{at} = \sum_{m=1}^{\lambda} DUM_m \int_{j_m}^{t} \Delta_m \, dt + X_{a1} \quad \text{if} \quad j_1 < t \leq n
\]

2.27.a-b can, in turn, be expressed in the form of a vector product as
Thus we have demonstrated that, by construction, the general functional form of time path trajectories of instantaneous systemic flow velocities satisfying conditions 11.a-d can be expressed as a time dependent transformation of a dummy variable vector indicating, at every \( t \in T \), the occurrence or nonoccurrence of a predetermined set of consecutive alterations in the operating conditions, or the "states", of the system under consideration. The coefficients of transformation in 2.28 are time dependent functionals representing the changes in the dynamic flow velocity of the system between \( j_m \) and \( t \), due to the \( m \)th discontinuity in the time path trajectory of systemic accelerations.
The explicit characterization of 2.28 depends upon the explicit functional forms of the $\Delta_m$. When the $\Delta_m$ are constant over $j_l < t < n$, then the explicit characterization and interpretation of 2.28 becomes rather simple, as we shall discuss in the next section.


If, in addition to conditions 2.11.a-d and 2.17.a-c, the dynamic system under consideration satisfies the requirement that

2.29) $\frac{d\Delta_m}{dt} = 0$ if $jm < t < n$ and $1 \leq m \leq \ell$,

then the functional model of the trajectory of $X_{at}$ will be given from 2.28 as

2.30) $X_{at} = \left[ 1 \ DUM1 \ldots DUM\ell \right] t \cdot$ 

\[
\begin{bmatrix}
X_{a1} \\
\Delta_1 \cdot \frac{dt}{j1} \\
\Delta_1 \cdot \frac{dt}{j1} \\
\Delta_2 \cdot \frac{dt}{j2} \\
\vdots \\
\Delta_\ell \cdot \frac{dt}{j\ell}
\end{bmatrix}
\]
or

\[ X_{at} = \begin{bmatrix} 1 & DUM_1 \cdot (t - J_1) & \ldots & DUM_m \cdot (t - J_m) \end{bmatrix} \cdot \begin{bmatrix} X_{a1} \\ \Delta_1 \\ \Delta_2 \\ \vdots \\ \Delta_m \end{bmatrix} \]

or

\[ X_{at} = \begin{bmatrix} 1 & T_1 & T_2 & \ldots & T_n \end{bmatrix} \cdot \begin{bmatrix} X_{a1} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \]

where

\[ T_m = DUMm \cdot (t - Jm), \]

and

\[ a_m = \Delta_m. \]

Characterization of the time path of \( X_{at} \) is, then, tantamount to the solution of 2.32.a for \( X_{a1}, a_1, a_2, \ldots a_m \), given \( n > 1 \) observations, which is a straightforward problem in adjustment.
The vector of variables in 2.32.a indicate, for every $t \in T$, both the temporal sequence of the discontinuities in systemic accelerations, which are responsible for the deterministic aspects of the observed magnitudes of systemic flow velocities at $t$, and the duration of each individual discontinuity from its initial occurrence to $t$. We shall refer to this vector as the system state vector, and the variables in it as system state variables.

It should be noted that there are no units of measure for the first variable in the system state vector, which simply indicates the existence of non-zero systemic flows during the initial period of observation, as per 17.a-b. The units of measure of subsequent variables, which indicate the durations until $t$ of successive discontinuous changes in systemic accelerations, are in years. Similarly, the units of measure of the first coefficient, which indicate the magnitude of non-zero systemic flow velocity in the first period of observation, are in terms of units of $X$ per year; and the units of measure of subsequent coefficients, which indicate the magnitudes of the successive alterations in the structure of systemic accelerations, are in terms of units of $X$ per year per year:

4. Mathematical Modeling and Adjustment of a Single Time Path Trajectory

The mathematical model of the time path trajectory of a single economic data series will consist of a functional time variant trend model of the type described in the previous section and the specification of a weight matrix of observations defining the stochastic
properties of the observations in question relative to the specified functional time path model. It should be kept in mind that different specifications of the weight matrix of observations for the same characterization of the functional model implies a different mathematical model and hence a separate adjustment problem.

The mathematical model of the time path of a single economic data series will be of the form \( x_{at} - e_t = \hat{x}_{at} \), where \( x_{at} \) are the stochastic observations, and \( \hat{x}_{at} \) the deterministic aspects of \( x_{at} \), and where

\[
2.33.a) \quad x_{at} - e_t = [1 \ T_1 \ T_2 \ ... \ T_L] \cdot \begin{bmatrix} \hat{x}_{at} \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_L \end{bmatrix}
\]

The stochastic behavior of the corresponding sequence of \( n \) residuals will be characterized by an independently and identically distributed sample of size \( n \) from a normal distribution:

\[
2.33.b) \quad e = [e_1 \ e_2 \ ... \ e_n] \sim N_n \begin{bmatrix} 0 \\ \sqrt{\sigma^2} I \end{bmatrix}
\]

The application of least squares adjustment to a sequence of \( n \) observations of the form \( x_{ai} \), \( i = 1, 2, \ldots, n \), generated by the
mathematical model presented in 2.33.a-b, will require the minimization of the quantity, 

\[
2.34) \quad \left[ X_{a*} - Z \cdot B \right] I^{-1} \left[ X_{a*} - Z \cdot B \right]
\]

with respect to B, where \( I^{-1} = I \) is the weight matrix of observations and

\[
2.35) \quad B = \begin{bmatrix}
\hat{X}_{a1} \\
a_1 \\
a_2 \\
\vdots \\
a_{\lambda}
\end{bmatrix}
\]

Since the interior points \( j_m, m = 1, \ldots, \lambda \), of \( T \) are, by construction, elements of the set of end points, \( j_i \), of the annual periods of observation in \( T \), they will have integer values. Since \( X_{ai} \) refers to the magnitude of annual flows at the end of the \( ith \) calendar year in \( T \), the system state variables \( T_m \) will, also, have integer values:

\[
2.36) \quad T_m = 0 \quad \text{if} \quad j_i \leq j_m
\]

\[
T_m = j_i - j_m \quad \text{if} \quad j_i > j_m,
\]
where \( j_i \) is the end point of the \( i \)th annual period of observation in \( T \).
Hence, the observation matrix of system state variables \( Z, \) in 2.36
above, is an \( n \times (\lambda + 1) \) matrix of integers of the form

\[
2.37.a) \quad Z_1 = \begin{bmatrix}
1 \\
1 \\
. \\
. \\
. \\
1
\end{bmatrix}
\]

\( n \times 1 \)

\[
2.37.b) \quad Z_{(m+1)} = \begin{bmatrix}
Z_{1, (m+1)} \\
Z_{2, (m+1)}
\end{bmatrix}
\]

\( n \times 1 \)

where

\[
2.37.c) \quad Z_{1, (m+1)} = \begin{bmatrix}
0 \\
. \\
. \\
0
\end{bmatrix}
\]

\( m \times 1 \)

and
5. A Change in Notation

We shall first introduce a change in notation in designating the trend variables as defined in 2.32.b, whose observational properties are indicated in 2.37.b-d. In actual modeling, it is operationally easier to work with variables designated to indicate changes in established trends with reference to the sequence number of the observation, \( J_m \). This number signifies the \( m \text{th} \) change in long term acceleration and, hence, the corresponding discontinuity in the flow time path. We found this convention easier to remember than the sequence number of the discontinuity within the trajectory model under consideration, i.e. "m". Thus, from now on we shall replace

\[
2.38.a) \quad T_m \text{ in 2.32.b by } T \quad \text{ if } m = 1
\]

and we shall replace

\[
2.38.b) \quad T_m \text{ in 2.32.b by } T (J_m - 1) \quad \text{ if } 1 < m \leq k.
\]
6. **Representation of Shifts in Long Term Trends and Temporary Deviations from Long Term Trends**

It is shown in appendix B that certain sums and differences of trend variables can be consolidated into single special variables. These variables may be utilized to represent a specific type of anomaly in the time path trajectory, such as a temporary deviation from an established trend during a particular year, or a shift in the position of an established trend line during a particular year without a change in the rate of acceleration before or after the shift.

We shall simply note here that if there is a temporary deviation of one year duration from an established trend, the state of the system during that year, say, 19XX can be characterized by a special variable DXX such that the observations on DXX take the form

$$2.39) \quad \begin{align*}
DXX &= 1 \quad \text{if year} = 19XX \\
DXX &= 0 \quad \text{otherwise}.
\end{align*}$$

Similarly a shift of one year duration following a given year 19XX may be characterized by a special variable, say DUMQ such that

$$2.40) \quad \begin{align*}
DUMQ &= 0 \quad \text{if year} \leq 19XX \\
DUMQ &= 1 \quad \text{otherwise}.
\end{align*}$$
When variables of the type 2.39 and 2.40 are utilized in the system state vector of a time path trajectory, their corresponding coefficients would then measure the magnitude of the temporary deviation from, or the shift in, the long term trend prevailing in the neighborhood of the relevant subinterval of T.

The usefulness of introducing special variables to take care of special type of behavior within a relatively small subdomain of T is that it allows us to use T and T(Jm-1) to represent relatively stable long term trends, describing the actual course of the time path with relatively high accuracy, and, hence, to capture and characterize as much of the information as the observations on the trajectory may have to offer with relative syntactic economy. Other special system state variables may be introduced, which the investigator may deem necessary, useful, or desirable, to further distinguish between long term trends and special circumstances which may temporarily influence the position of subsequent observations relative to the underlying long term trends.

C. An Application: Commercial Natural Gas Sales in Ohio 1960-1981

1. Preliminary Remarks

The development of the conceptual framework and the analytical methodology expounded so far has been motivated by the realization that many sectoral energy flows in the U.S. and in Ohio have moved along two successive long term trends between 1960 and 1981. Hence, for the purpose of monitoring the current direction of systemic activity, such trends and recognizable anomalies around them could serve as
quantitative historical benchmarks. Similarly, for the purpose of evaluating the historical performance of the system with a view to forecasting its future behavior, such trends and recognizable anomalies around them may be interpreted to be symptomatic of the successively prevailing states of the system under consideration. Incorporating these trends and anomalies may thus lead to accurate, reliable, realistic, and useful characterizations of the nature, causes, and specific impacts of such systemic states, and may even suggest appropriate counterveiling remedies, if necessary.

The first step in such a comprehensive research program would be the characterization and measurement of the time variant trends in question. To provide a framework and a method to perform the tasks necessary for such a first step is the primary goal of this thesis.

The first task involved in the modeling of a time path trajectory is to plot the observations on the trajectory under consideration against the points in time at which they are obtained, or against some indicator of time such as their sequence numbers. The discontinuities to be included in the characterization of the time path model may be decided, in many cases, by inspection.

For further diagnostics, the investigator could difference the data by one lag and try to trace the time path of the acceleration function \( \frac{dX_{at}}{dt} \). One may also run preliminary stepwise regression models to decide, among contending points, to introduce the beginning of a new trend.

An understanding of the actual history of the system may lead to a more efficient modeling effort, to the extent that it enables the
investigator to look for specific anomalies or variations at or beyond specific points in time.

2. **Long Term Trends**

In figure 2, we present the time path of observations for the state of Ohio on the quantities of natural gas consumed by the commercial class of customers, provided by the natural gas utilities. The magnitude of these flows are designated as NGCXB0H, where the units of measure of NGCXB0H are in $10^{12}$ Btus/per year. Constant conversion factor of $1.032 \times 10^{12}$ Btus/10$^9$ cubic feet is employed for all years.

It is clear by inspection that whatever events, factors, or incidents may have produced the sequence of observations in figure 2, such determinants have maintained activity levels that have resulted in two relatively stable linear trends, one persisting from 1960 through 1972 and the other from 1973 through 1981. For the purpose of characterizing the trends and measuring their magnitude, it is not necessary that we know explicitly the reasons or the determinants which led to this behavior.

In terms of the methodology introduced in the preceding sections of this chapter, we may characterize the acceleration function of the time path in question by two discontinuities, $m = 1, 2$ at points $J_1 = 10, J_2 = 13$. According to the convention introduced in 2.38.a-b, the functional model of the time path trajectory of commercial natural gas flows in Ohio will, then, be given by:

$$NGCXB0H = a_0 + a_1 T + a_2 T^{12}.$$
Figure 2. Observations on the Time Path of Commercial Sector Natural Gas Sales in Ohio, 1960-1981
Source: Ohio Department of Energy
where $a_0$ is the initial magnitude of NGCXBOH at $t = 1960$, and $a_1$ and $a_2$ are the constant accelerations induced and maintained by the two successive states of the system under consideration, prevailing from the end of 1960 through 1972, and from 1973 through 1981, respectively.

The results of adjustment are presented in tables 1, 2 and figures 2, 3. The tabulated results indicate that over the 22 year period under consideration 92% of the variation in commercial natural gas sales has been accounted for by the two systemic trends, which, starting at an initial flow magnitude of 104.96 TBtu/year at the end of 1960, have increased at a constant acceleration of $8.06\ \text{TBtu/}(\text{year})^2$ between 1960 through 1972, and have decreased at a constant deceleration of $-4.1\ \text{TBtu/}(\text{year})^2$ between 1972 through 1981. Furthermore, the average magnitude of deviations from these two trends has been around 8.7 TBtu, so that 95% of the time the variation along the trends in question has remained within a band of $\pm 17.4$ TBtus. The model's forecast magnitude for 1982 is 160.68 TBtus, whereas the realized commercial sales in 1982 were 164.34 TBtus.

3. **Implications for Monitoring and Forecasting**

Given the fact that the only critical changes in the long term states of the system seem to have occurred only once in a 22 year period, it could reasonably be assumed that the recent trend is still in effect as long as current observations lie within the confidence limits of the model in question. Hence as long as realized magnitudes of sales beyond 1981 do not deviate from the magnitudes forecast by the model by more than $\pm 17$ TBtus for two or more years, one could
Table 1. Ohio Commercial Natural Gas Sales, 1960-1981, Time Path Model 1:
NGCXB0M = 104.96 + 8.06 * T - 12.16 * T^2

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<tr>
<th>SOURCE</th>
<th>DF</th>
<th>SS</th>
<th>SS OF SQUARES</th>
<th>MEAN SQUARE</th>
<th>F VALUE</th>
<th>PR &gt; F</th>
<th>R SQUARE</th>
<th>CV</th>
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<td>6226.2875162</td>
<td>111.66</td>
<td>0.0001</td>
<td>0.821590</td>
<td>5.2900</td>
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<td>69.1274698</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>CORRECTED TOTAL</td>
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<td>139.1391391</td>
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<td></td>
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<td></td>
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<td>1</td>
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<th>PR &gt;</th>
<th>TFD ERROR OF ESTIMATE</th>
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Figure 3. Observations on the Time Path of Commercial Sector Natural Gas Sales in Ohio, 1960-1981, Actual Versus Predicted by Time Path Model I

Source: Ohio Department of Energy
Figure 4. Acceleration Function of Time Path Model 1: INGCXOH1 = 8.06* DUMF - 12.16 * DUM7
Source: Ohio Department of Energy
Table 2. List of Actual Adjusted and Residual Values, and the System State Variables Associated with Time Path Model 1

<table>
<thead>
<tr>
<th>OBS</th>
<th>YEAR</th>
<th>I1111</th>
<th>T</th>
<th>HH2</th>
<th>HGCXBHH</th>
<th>P1HGC11</th>
<th>P1HGC11</th>
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<td>0</td>
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<td>104.960</td>
<td>6.409</td>
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<td>1</td>
<td>0</td>
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<td>113.018</td>
<td>6.598</td>
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<td>121.076</td>
<td>7.285</td>
</tr>
<tr>
<td>4</td>
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<td>3</td>
<td>0</td>
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<td>177.482</td>
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<td>185.540</td>
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<td>1972</td>
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<td>15</td>
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<td>2</td>
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<td>193.460</td>
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<tr>
<td>16</td>
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<td>199.362</td>
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<tr>
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<td>9</td>
<td>168.340</td>
<td>164.775</td>
<td>3.565</td>
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</table>
assume the system to be functioning stably under the same operating conditions prevailing since 1972.

On the other hand, the fact that the trends have changed in the past implies that they are expected to change in the future as well. For, just as it is nearly inconceivable for most economic goods that their consumption continues to increase by a constant annual rate of acceleration ad infinitum, it is also difficult to imagine that the consumption of currently popular commodities such as natural gas declines to zero at a constant annual rate of deceleration.

The implications of these considerations for the purpose of forecasting with the model at hand is that, since the only historical contingency the model explicitly considers is a major variation in the long term trend, it can be used to make forecasts on the basis of future scenarios involving specific future alterations on long term trends only. It can also be used to make forecasts on the basis of the currently identified trends until, through constant monitoring of the future realizations of flow magnitudes relative to their forecast magnitudes, it is determined that a new set of operating conditions has been activated. At that time the model must be updated to include the most recent temporal changes influencing the systemic environment in question and the forecasts be accordingly updated. The whole process of forecasting, monitoring, and learning must, then, start anew.

4. An Alternative Formulation of the Time Path Model

It may be observed that even from the point of view of interpreting the time path in question by inspection, further improvements could be
made in the predictive and monitoring accuracy of the time path model on the basis of statistical considerations alone. A review of figure 2, or the residual vector RNGCM in table 2, would indicate that most of the variation in the residuals and, hence, the standard error of the estimate occur around three specific subdomains of time, namely, those corresponding to years 1963-64, 1975 and 1977. Between 1963 and 1964, there seems to have been a downward shift in the prevailing trend, whereas during 1975 and 1977 there seem to have been temporary deviations of one year duration from the prevailing trend.

In accordance with the discussion in section 8.6 above, we introduce a special variable, which we shall call DUMC, to take care of the shift in 1963-64 where

\[\text{DUMC} = 0 \quad \text{if year < 1963}\]

and

\[\text{DUMC} = 1 \quad \text{if year > 1963} .\]

Similarly, we introduce two special variables, D75 and D77, to take care of the temporary deviations of one year duration in 1975 and 1977, where

\[\text{D75} = 1 \quad \text{if year = 1975}\]

\[\text{D75} = 0 \quad \text{otherwise}\]

and

\[\text{D77} = 1 \quad \text{if year = 1977}\]

\[\text{D77} = 0 \quad \text{otherwise} .\]

The time variant trend model is now given by
\[ \text{NGCXBOH} = a_0 + a_1 T + a_2 T^2 + a_3 \text{ DUMC} + a_4 D75 + a_5 D77 \]

where \( a_3 \) is the magnitude of the shift from 1963 to 1964, and \( a_4 \) and \( a_5 \) are the magnitudes of the respective temporary deviations in 1975 and 1977.

The results of adjustment are presented in table 3, 4 and figures 5, 6. The tabulated results indicate that the inclusion of the three anomalies in the time path along with the two long term trends increases the explanatory power of the model from 92% to 99%, and the mean square error is reduced from 75.1 to 11.7. Correspondingly, the standard error of the estimate is reduced from 8.7 TBtu to 3.4 TBtu, meaning that the future levels of systemic flows are expected to remain within \( \pm 6.8 \) TBtus of the values forecast by the model, except possibly under circumstances that have prevailed after 1963, during 1975, or during 1977. The results of the modified trend model as presented in table 3 clearly allow for a more precise monitoring and forecasting of future behavior of the system.

At the levels of accuracy indicated above, the long term behavior of NGCXBOH is now described in terms of two long term trends which, starting from an initial flow rate of 107.56 TBtu/year at the end of 1960, have increased at a constant acceleration of 10.68 TBtu/(year)^2 between 1960 through 1972 and decreased at a constant deceleration of -4.75 TBtu/(year)^2 between 1972 through 1982. The model's forecast for 1982 is 161.68 TBtus/year, whereas the actual for 1982, as previously

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F VALUE</th>
<th>PR &gt; F</th>
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<th>G.V.</th>
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<table>
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<td>1</td>
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<td>1</td>
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<td>13.67</td>
<td>0.0054</td>
<td>1</td>
<td>566.87367571</td>
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<td>23.07</td>
<td>0.0002</td>
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<td>1111.45687253</td>
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<th>STANDARD ERROR</th>
<th>T FOR H0: PARAMETER = 0</th>
<th>PR &gt;</th>
<th>T</th>
<th>SE ERROR OF ESTIMATE</th>
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Figure 5. Observations on the Time Path of Commercial Sector Natural Gas Sales in Ohio, 1960-1981, Actual Versus Predicted by Time Path Model 2
Source: Ohio Department of Energy
Figure 6. Acceleration Function of Time Path Model 2: \( \text{FNGCXML3} = 10.68 \times \text{DUMF} - 15.44 \times \text{DUM7} \\ - 20.21 \times (\text{DUM9} - 2 \times \text{DUM10} + \text{DUM11}) -18.71 \times (\text{DUM11} - 2 \times \text{DUM12} + \text{DUM13}) - 26.46 (\text{DUMC-DUMB}) \)

Source: Ohio Department of Energy
Table 4. List of Actual Adjusted and Residual Values, and the System State Variables Associated with Time Path Model 2

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<tr>
<th>OBS</th>
<th>YEAR</th>
<th>INT</th>
<th>I</th>
<th>T12</th>
<th>D75</th>
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<th>DUMC</th>
<th>HGCXB3</th>
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noted, was 164.34 TBTUs—well within the confidence limits of the model.

The major difference between equations 2.41 and 2.42 is that 2.42 is a significantly more powerful instrument for monitoring and, hence, detecting possible future alterations in the long term trends in a more timely and more accurate manner. On the other hand, 2.42 requires more detailed specifications of future scenarios to produce forecasts. In addition to possible alterations in long term trends, explicit consideration should be given to the occurrence or nonoccurrence of shifts in long term trends or of temporary deviations of one year duration from established trends.

It would of course be much more desirable if we could have some specific information about causally or definitionally interrelated processes which lead to changes in long term trends as well as those which lead to significant anomalies around established trends. Such information, if it could be made to bear on the modeling process, could yield more precise mathematical formulations of the relevant time paths and could, also, lead to scenarios of future states of the system that may be specified to be different from its current characterization.

For example, it is fairly certain that the temporary deviation observed in 1977 reflects the impact of the 1977 natural gas shortage in Ohio on the commercial sector. One could then tentatively presume, in the absence of further analysis or consideration, that similar differences in interstate versus intrastate prices of natural gas could lead to similar shortages in the future with a similar impact on the commercial sector. On the other hand, the behavior of the system in
1975 is a bit more problematic in terms of what it may imply. In view of the fact that there have been four recessions between 1960 and 1981 (in 1961, 1971, 1975 and from 1980 on), it would seem to require further investigation why the only systemic response took place in 1975. As far as the shift in 1963-64 is concerned, not even a tentative explanation is offered at this time. Thus appropriate empirical research is required both to strengthen or disconfirm reasonable tentative explanations of anomalies, or to suggest such reasonable tentative explanations in the first place.

Similarly, there is no doubt that among the major determinants of the change in trends after 1972 is the increasing discrepancy between unregulated interstate natural gas prices and the regulated prices that Ohio's natural gas distribution companies were permitted to charge to their customers within the state. In fact, between 1972 and 1978 a ban was imposed, upon approval by Public Utilities Commission of Ohio, on new natural gas hookups in Ohio. Furthermore, in view of the declining interstate gas supplies to Ohio, the regulated price of natural gas increased, on the average, fivefold between 1970 and 1981 (7). The impact of such an increase in energy costs, when combined with the increase in petroleum prices and a 100% devaluation of the dollar in 1971 would have a definite impact on the structure of costs, prices, incomes, and production relations in the U.S., including Ohio.

Reasonableness of presumption, however, does not constitute explanation. Positive assessments of the impacts of external or internal shocks on the kinematics of systemic flows as well as the reasons which precipitate the shocks require positive investigation
and analysis. In the next section, we shall enumerate some of the appropriate avenues of research which may be required to assess the nature, determinants, or specific impacts of shocks, or changes in the operating conditions, which may help explain the observed variations in the dynamic behavior of systemic trajectories.

5. Heuristic Implications

One way of increasing our understanding of the established behavior of a system is to consider the behavior of its definitional components. The definitional components of a time path of economic observations can be considered under two categories: accounting and functional identities. For example, in terms of the accounting conventions underlying the data, NGCXBOH is equal to the sum of commercial sales within the service areas of eight major Ohio natural gas utilities (8). Hence, a joint analysis of the accounting components of NGCXBOH, along with NGCXBOH, may lead to a better understanding of the behavior of both the whole and its constituent parts. Such a joint analysis may indeed suggest avenues of investigation which may not be obvious from an analysis of NGCXBOH alone.

In terms of physical conventions, NGCXBOH can be separated into various multiplicative components such as per commercial customer Btu usage from natural gas and the number of commercial natural gas customers. Joint analyses of such components may lead to a clearer understanding of the physical structures of long term trends as well as the variations or the regularities in their structural components. Such an understanding may contribute to a more rational management,
planning and utilization of time and resources on the part of legislators, regulators, utilities, and utility customers.

Considerations must also be given to the accounting and functional identities involving the financial flows associated with NGCXBOH. Among the more important of these is the time path of revenues generated from the sales of NGCXBOH. If there is a dynamically invariant relationship between revenues per unit of sales, this should be demonstrated by comparing the time path of the revenues in question with the time path of NGCXBOH. If there are alterations in the time path of average revenues within the domain under consideration, further research may be required as to the magnitude or the reasons for such alterations. It may also be important to consider the relation between income generated by the commercial sector and the proportion of that income that is paid to natural gas utilities.

In fact, the kind of analysis and investigation that can be carried on to increase the current level of our knowledge and understanding of the determinants of established economic trends is limited, for all practical purposes, by the imagination of the researcher and the resources and data available to him. Since such positive knowledge and understanding require meticulous assessments of the interconnections of definitional threads binding together the trajectories of the relevant functional or accounting identities implicit in the macro or micro structures of a given systemic trajectory, a methodology for the joint analysis and assessment of definitionally related time path trajectories is an indispensable
tool for the conduct of explanatory inquiry in economics. In the next chapter we shall extend the analytical results of this chapter to the analyses and assessments of the time paths of definitionally related data series.
CHAPTER III

THE ANALYSIS AND ASSESSMENT OF TIME VARIANT LINEAR TRENDS
IN DEFINITIONALLY RELATED ECONOMIC DATA SERIES

A. System Theoretical Framework
   1. Preliminary Remarks

In the preceding chapter we have developed a framework and methodology for the analysis and assessment of time variant trends in the time paths of aggregate economic flows. An aggregate economic flow magnitude implies, by virtue of its basis in accounting theory, a directional movement of ownership claims on goods or services from a specified set of individuals or institutions which are recognized providers of the goods or services in question to a specified set of individuals or institutions which are recognized disposers of the goods or services in question, within the period of observation under consideration. Hence, each economic flow magnitude, whether it is physical or financial in nature, has a dual aspect. It is an item of provision relative to its origin in one set of individuals or institutions, and an item of disposition relative to its destination to another set of individuals or institutions.

The importance of recognizing and explicitly investigating historical behavior of aggregate economic flow magnitudes in terms of their composition relative to their sources of provision, or in
terms of their composition relative to their destinations for disposition, has already been elaborated in chapter I. In this chapter we shall discuss some of the basic issues involved in the simultaneous analysis and assessment of the time paths of definitionally related provisional or dispositional economic flows in terms of an extension of the conceptual framework and the analytical methodology expounded in chapter II. We then provide a presentation of the current status of one of the ongoing lines of investigation we have been pursuing at the Ohio Department of Energy, as an example of the application of this methodology.


In many cases the investigator of systemic behavior may be interested not only in the analysis and assessment of the time path of a given system "S" but also in the dynamic composition of "S" in terms of its alternatively designable constituent parts, internal mechanisms, structural components, or subsystems, whatever the appropriate term may be in a given application.

As discussed in the final section of the last chapter, the investigator may be interested not only in dynamic behavior of statewide commercial natural gas sales in Ohio but also in the determinants of the dynamic composition of statewide sales in terms of individual utility service area sales. Similarly, one may want to investigate the behavior of commercial natural gas sales in relation to the behavior
of natural gas sales within residential, industrial, transportation and electric utilities sectors in Ohio, or in the U.S. at large. Another important case where joint analyses of time paths may be called for is when there exist both causal and accounting relations among two or more systemic trajectories. For example, in assessing the efficiency of coal burning generating plants in a given electric utility service area, one may want to investigate the joint trajectories of the Btu contents of coal deliveries to the service area, Btu equivalent of total generation from the coal burning units in the service area, and total generation losses defined as the difference between the former and the latter. Although these examples are stated in terms of physical flow systems, the analysis of corresponding financial flows along with the physical flows may, of course, be of vital importance to the planning and implementation of many public or private policies or programs, and to the outcomes of many regulatory decisions.

In general, then, depending upon the requirements of the problem at hand, a given economic system "S" may be designated as an element of many alternatively designable partitionings of higher order, i.e. more inclusive, or "macro", systems. Similarly, it may be partitioned into alternatively designable sets of constituent elements, or "micro" systems. In this chapter we shall extend our methodology to study the implications of the latter case and provide an application. Further extension of the methodology to include higher order systems is straightforward.

Let
3.1.a) \[ R = R_1, R_2, \ldots, R_h \]

be a partitioning of \( R_a \), i.e.

3.1.b) \[ R_p \cap R_r \text{ is empty for all } p \neq r, \quad p, r = 1, 2, \ldots, h; \quad 2 \leq h < \infty; \]

and

3.1.c) \[ \bigcup_p R_p = R_a. \]

If

3.2) \[ X_{at} = \sum_{p=1}^{h} X_{pt}, \]

where \( X_{pt} \) is the annual rate of flow of \( X \) generated by \( R_p \in R \) at \( t \in T \), and \( X_{at} \) is the corresponding rate of flow for \( R_a \), then we shall refer to a flow velocity vector of the form

3.3) \[ xD_t = [X_{at}, X_{1t}, X_{2t}, \ldots, X_{ht}] \]

as either a vector of decomposition of \( X_a \) over \( R_a \) or a vector of composition of \( X_a \) in terms of the elements of \( R_a \).

If

\[ X_{a*} = [X_{a1}, X_{a2}, \ldots, X_{an}] \]

and
\[ X_p^* = [ X_{p1} \ X_{p2} \ ... \ X_{pn} ], \]

are vectors of observations on the magnitude of \( X_{at} \) and \( X_{pt} \) for \( t = t_0 + i, \ i = 1, 2, \ ... \ n, \) and \( p = 1, 2, \ ... \ h, \) and if

\[ 3.4) \quad D = [ x_a^* \ X_1^* \ X_n^* ] \neq 0, \]

then we shall refer to those aspects of \( R_a \), which relative to a uniform system of accounting applied over \( T \) have generated 3.4, as a system of disposition of \( X \) in \( R_a \), or as a system of provision of \( X \) in \( R_a \), or simply \( DXR \). The systemic implications of observations on a single time path trajectory, as discussed in section II.A may be naturally extended to vectors of observations on definitionally related time path trajectories as well.

In summary, then, the elements of each \( x \) \( D_t \) in 3.3 are accounting magnitudes expressed in units of "units of \( X \) per year." Hence, each element in each \( x \) \( D_t \) can be interpreted as representing the annual rate of flow of \( X \) through a subset of \( R_a \) during the \( i \)th successive period in \( T \). Thus the vector \( x \) \( D_t \) can be interpreted as representing either the decomposition during the \( i \)th successive period in \( T \) of a total flow velocity into a set of component flow velocities, or the composition of a total flow velocity in terms of its component flow velocities. The vector \( D, \) then, is a set of observations on the above mentioned phenomena for the \( n \) successive annual periods in \( T \).

The nature and existence of the flows reported in \( D \) obviously require the existence of mechanisms, institutions, and organizations
in S which have physically, socially, and historically supported the observed flows. Hence, D X R includes the sum total of all such existing mechanisms, institutions and organizations at every t ∈ T. Any changes in these underlying mechanisms, to the extent that they influence the observed magnitudes of the flows generated by D X R, constitute temporal changes in D X R. Such changes in the underlying structure may go on in a continuous manner over T, reflecting the reactions of centers of decision in the system to anticipated future magnitudes of the flows in question under a "normal" range of changes in operating conditions. To the extent that the anticipated "normal" range of changes in operating conditions has actually been realized historically, the historical magnitudes of the systemic flows in D would be expected to change in some well-behaved and smooth manner over T. Moreover, to the extent that "unanticipated", "extraordinary" or "unprecedented" operating conditions or shocks have prevailed at certain points, or over certain subdomains, of T, one would expect to observe correspondingly "unexpected" or "extraordinary" or "unprecedented" variations in the time paths of realized patterns of disposition of the systemic flows over such points, or subdomains, of T. By "unanticipated" or "extraordinary" or "unprecedented" operating conditions or shocks, we have in mind, as mentioned in chapter II, such events as civil or overseas wars, large scale flood, famine, drought, blizzard, plague or other "Acts of God". We may also include significant changes in levels or growth rates of unemployment, incomes, populations, or prices from one period of observation to the next, embargoes on critical raw materials, technological breakthroughs and so on.
The measurement of the behavior of systemic flows over the domain of time $T$ is, hence, a prerequisite for an understanding, and possible explanations, of the relations between historical changes in prevailing operating conditions and the changes in the magnitudes of realized systemic flows. We may therefore conceptualize $D \times R$ as a dynamic system in $S$, which has transformed historical time and circumstances into a stream of aggregate flows of $X$ through $R_a$, moving at successively observed average annual flow rates, $X_{ai}$. The stream of aggregate flows of $X$ through $R_a$ is composed of substreams of component flows through the elements of a specified partitioning $R$ of $R_a$. The $p$th substream moving at successively observed average annual flow rates $X_{pi}$, with $X_{ai} = \sum_{p=1}^{h} X_{pi}$.

Our objective is to describe the deterministic aspects of the behavior of $D \times R$, in transforming historical time and circumstances into streams of successive annual flows $X_{ai}$ and $X_{pi}$, with a view to assess the behavior of these transformations over the domain of historical time $T$. If we could describe the deterministic magnitudes of the observations in successive annual patterns of disposition $x_{Di}$ in terms of a vector of functional time path models of the form:

$$3.5.a) \quad \dot{x}_t = [ \ddot{x}_{at}, \ddot{x}_{1t}, ..., \ddot{x}_{ht} ]$$

where

$$3.5.b) \quad x_t = \dot{x}_t + e_t,$$
then the explicit form of the vector of functional time path models 3.5.a along with a specified stochastic model for the corresponding residuals 3.4.c could replace D X R in order to assess its inter-temporal behavior.

To the extent that the information yielded from a process of analysis and assessment along the lines described above enables us to determine the accuracy of a particular explicit formulation of 3.5.a in explaining the observed historical behavior of D X R, it should also enable us to help predict the observable future behavior of D X R within the limits of accuracy historically determined. Hence, whenever the predictions and actual observations are seen to diverge by more than what is warranted by the historical performance of the particular formulation of 3.5.a and the corresponding stochastic model of 3.5.b, more information concerning the system can be obtained from an analysis of how the current conditions of performance of the system are different from its historically observed conditions of performance. Hence, updated reformulation of the functional model 3.5.a, or the corresponding stochastic model of 3.5.c, incorporating up-to-date information on the behavior of D X R can be utilized to make more reliable forecasts of the system's behavior in the near future, or under scenarios based upon the most recent information on the historical performance of D X R.
8. Mathematical Implications of the Simultaneous Analysis of
Definitionally Related Time Paths

1. General Implications On the Mathematical Model

By definition of the concept of a mathematical model as presented
in chapter I, the magnitude at time $t$ of a component flow velocity $X_{pt}$,
is identical to the magnitude yielded by the value of its functional
time path model at time $t$, plus a residual term:

$$3.6.a) \quad X_{pt} = \hat{X}_{pt} + e_{pt}.$$  

Similarly

$$3.6.b) \quad X_{at} = \hat{X}_{at} + e_{at}.$$  

Substituting 3.4.a and 3.4.b into 3.2, we get

$$3.7.a) \quad \sum_{p=1}^{h} [ \hat{X}_{pt} + e_{pt} ] = \hat{X}_{at} + e_{at}.$$  

Hence

$$3.7.b) \quad \sum_{p=1}^{h} \hat{X}_{pt} = \hat{X}_{at}, \quad \text{and}$$  

$$3.7.c) \quad \sum_{p=1}^{h} e_{pt} = e_{at}.$$
Since 3.7.b and 3.7.c are true by the physical and accounting conventions underlying the data, the functional time path model of the aggregate flow magnitude should equal to the sum of the time paths of its constituent parts. This, in turn, implies that, in the case of time variant linear time path models, the sum of the parameters of the system state variables in the time path models of the constituent parts must add up to the parameters of the system state variables in the time path model of the aggregate flow magnitude. When this condition is met by a given vector of functional time path models of the form 3.5.a:

\[ \hat{x}_t = [\hat{x}_a(t), \hat{x}_1(t), \hat{x}_2(t) \ldots \hat{x}_h(t)] \]

then condition 3.7.c will be met automatically.

2. Implications on the Functional Time Path Model

In the case of time variant linear trend models, which are the subject matter of this thesis, the vector of functional time path models which are responsible for the deterministic components of the magnitudes in a given observation vector of the form \( \hat{X} \) may be specified as

3.8.a) \[ \hat{x}_t = [Z_a(t) \cdot B_a, Z_1(t) \cdot B_1, \ldots, Z_h(t) \cdot B_h] = Z(t) \cdot B \]

subject to
3.8.b) \( R_2B = 0. \)

where \( Z_a(t), Z_p(t), p = 1, 2 \ldots h, \) are vectors, respectively, of \( m_a+1 \) and \( m_p+1 \) system state variables, of the type introduced in 2.32.b, 2.39.c, 2.41.c, or others similarly constructed, determining the respective time paths of \( X_{at} \) and \( X_{pt}, p = 1, 2, \ldots h. \) \( B_a \) and \( B_p \) are the corresponding parameter vectors, and

3.8.c) \( B = [B_a^T B_1^T \ldots B_h^T]^T. \)

\( R_2 \) is a completely determined matrix of integers of full row rank.

Let

3.9) \[
\begin{align*}
Z_a & \\
Z_p
\end{align*}
\]

be observation matrices on \( Z_a(t) \) and \( Z_p(t), \) respectively, for \( i = 1, 2, \ldots n, \) where

3.10.a) \( \text{rank}(Z_a) = m_a+1 < n \)

3.10.b) \( \text{rank}(Z_p) = m_p+1 < n \) for all \( p = 1, 2, \ldots h, \)

3.10.c) \( m_a+1 < \text{rank} \left[ \begin{array}{c} Z_a \\ U (U Z_p) \end{array} \right] = \text{rank}(R_2) < n \)
and where \( m_a \) and \( m_p \) are the number of nonintercept system state variables included in the time path models of \( X_{at} \) and \( X_{pt} \), \( p = 1, 2 \ldots h \). It should be clear that the dimensions of \( R^2 \) are \( \text{rank}(R^2) \times [m_a + h + 1 + \sum_{p=1}^{h} m_p] \).

The deterministic component of the observation vector \( D \) of the definitionally related time paths under consideration will then be given as

\[
3.11.a) \quad \hat{D} = \text{Diag} [ Z_a Z_1 \ldots Z_p ] \cdot B = D - e
\]

subject to

\[
3.11.b) \quad R^2 \cdot B = 0.
\]

3. Implications on the Stochastic Model

If a vector of residuals of the form

\[
e_t = [ e_{at} \quad e_{1t} \quad e_{2t} \ldots e_{nt} ]
\]

is associated with a vector of functional time path models of the form discussed in the preceding section, and if the marginal distribution of \( e_{pt} \) is specified as

\[
3.12.a) \quad e_{pt} \sim N \left( 0 : \sigma_p^2 \right)
\]

for all \( p = 1, 2, \ldots h \) and for all \( t \), then, from 3.7.c, the joint distribution of the vector \( e_t \) would be a singular multivariate normal
where the cofactor matrix of the distribution of \( e_t \),

\[
V = \frac{1}{\sigma^2} E (e_t^2) e_t \tag{3.13}
\]

would be positive semidefinite, satisfying the identity

\[
\begin{bmatrix}
-1 & 1 & 1 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}_{(h+1) \times (h+1)} V
\begin{bmatrix}
-1 \\
1 \\
\vdots \\
1
\end{bmatrix}_{1 \times (h+1)} = 0.
\]

Furthermore, a sample of size \( n \) from 3.12.b of the form

\[
e = \begin{bmatrix}
e_1^2 & e_2^2 & \cdots & e_n^2
\end{bmatrix}
\]

will have a singular multivariate normal distribution of the form

\[
N_{(h+1) \times n} \left[ 0 \mid \sigma^2 V \otimes I \right].
\]
For the purpose of making least squares adjustment, the stochastic model has to specify the weight matrix of observations which, as discussed in chapter I, is the inverse of the cofactor matrix of the distribution associated with the residuals.

Since V, as specified in 3.13, is singular by definition, $V^{-1}$ does not exist. Hence a generalized treatment of the problem requires the specification of the weight matrix of observations as (2)

$$\text{3.17) } W = \text{GINV}(V) \otimes I.$$ 

4. Implications for Adjustment

In general, neither the constraints on the functional model as specified in 3.8.b nor the constraints on the stochastic model as specified in 3.14 will be automatically satisfied by unconstrained least squares estimates of the parameters of the individual time path models, or by the unconstrained least squares residuals. Although alternative least squares adjustment techniques may be employed in the adjustment of an observation vector of the type $D$ (3), a generalized treatment of the problem requires the construction of the cofactor matrix $V$ to satisfy 3.14 before adjustment, and that the constraints on the parameters, 3.8.b, be imposed as explicit constraints on a joint adjustment problem. Such a problem would then entail the minimization with respect to $B$ of the quantity (4)

$$\text{3.18.a) } (D - Z \cdot B)^{-1} \cdot \text{GINV}(V) \otimes I \cdot (D - Z \cdot B)$$
subject to

3.18.b) \[ R1 \cdot B = \begin{bmatrix} R2 \\ R3 \end{bmatrix} \cdot B = \begin{bmatrix} 0 \\ g \end{bmatrix} \]

where

3.18.c) \[ Z = \text{Diag} [ Z_0 \ Z_1 \ldots \ Z_h ] \]

\( R3 \) above is a \( \text{rank}(R3) \times \left[ m_a + h + \frac{h}{m_p} \right] \) matrix of known elements, and \( g \) is a \( \text{rank}(R3) \times 1 \) vector of known elements, indicating \( \text{rank}(R3) \) independent nonidentity constraints on the parameters of the time paths under consideration. Hence,

3.18.d) \[ \text{rank}(R1) = \text{rank}(R2) + \text{rank}(R3), \]

and, therefore, \( R1 \) is a full row rank matrix.

In general, it will be assumed that

3.19) \[ 0 \leq \text{rank}(R3) \leq \left[ \frac{m_a + h + \frac{h}{m_p} - \text{rank}(R2)}{m_p} \right] \]

It is clear that, if \( \text{rank}(R3) = 0 \), then there are no nonidentity constraints imposed on the elements of \( B \). It should also be clear that, if \( \text{rank}(R3) = \) the right hand side limit in 3.19, then \( g \equiv 0 \), and the elements of \( B \) could be inferred from the constraints in 3.18.b alone.

Although this procedure may be utilized as an optional technique of adjustment with systems in physics, etc., there are very few, if any,
economic systems whose \textit{a priori} restrictions on the parameters would be sufficient for the estimation of the relevant parameters.

C. An Application: Sectoral Electricity Sales in Ohio 1960-1981

1. Objective

In this section we shall model and measure the composition of total electricity sales of the electric utilities industry in Ohio in terms of sales to end users in residential, commercial, industrial and transportation sectors. The objective of the exercise is to provide a demonstrated application of the framework and methodology expounded so far to the joint description of the time path trajectory of a "macro" system, i.e., the system of aggregate electricity disposition in Ohio, in terms of the "micro" systems constituting the elements of a sectoral partitioning of the above mentioned "macro" system. By studying the constituent parts of an additive identity, we hope to achieve a greater degree of understanding concerning the circumstances, factors or events which may have influenced the time paths of the parts and, hence, the time path of the whole. We also expect to be faced with questions concerning the dynamic behavior of the elements of the system under consideration which will require further analysis and investigation. Our main emphasis will be, however, on providing a demonstrated application of the technique rather than a final and exhaustive empirical analysis of the problem of sectoral electricity disposition in Ohio \textit{per se}. 
2. **System Designation**

Data definitions are presented in appendix C. All data are from *Edison Electric Institute Statistical Year Book, 1960-1981*. The following are the variable designations to be utilized.

3.20.a) \( E_{LSBOH} = \) Residential Sector Electricity sales

3.20.b) \( E_{CMBOH} = \) Commercial Sector Electricity sales

3.20.c) \( E_{INBOH} = \) Industrial Sector Electricity sales

3.20.d) \( E_{TRBOH} = \) Transportation Sector Electricity sales

3.20.e) \( E_{TTBOH} = \) Total Sales to Sectoral End Users, where

3.20.f) \( E_{TTBOH} = E_{LSBOH} - E_{CMEOH} + E_{INBOH} + E_{TRBOH}. \)

In terms of the theoretical terminology developed in sections III.B, and II.A, \( R_a \) = sectoral end users of the electric utilities industry in Ohio. \( R \), the partitioning of \( R_a \) into component customer categories, or micro systems, under consideration is

3.21) \( R \equiv \) Residential, Commercial, Industrial, Transportation.

The corresponding vector of composition, the joint time paths of the elements to be analyzed, is specified as
3.22.a) \[ \{ \text{ELRSBOH}_t \quad \text{ELCMBOH}_t \quad \text{ELINBOH}_t \quad \text{ELTRBOH}_t \quad \text{ELTTBOH}_t \} \],

with

3.22.b) \[ X_{at} = \text{ELTTBOH}_t \quad \text{and} \quad X_{pt} = \text{ELPBOH}_t, \quad p = \text{RS, CM, IN, TR}. \]

A list of realized values of the variables mentioned above are presented in appendix C, table 18. All units are in \(10^{12}\) Btu/year, where a constant conversion factor of 3.412 TBtu/10^9 kWh has been utilized in deriving the Btu equivalents from kWh sales.

3. **Analyses and Modeling of Sectoral Time Paths**

a. **Residential and Commercial Sectors.** Observations on the time paths of residential and commercial electricity sales are plotted in figures 7 and 8, respectively. As it can be discerned by inspection these time paths are quite similar in temporal structure except in 1981, when residential sales is less in a succeeding year relative to the preceding year for the first time in 22 years. Except for this peculiarity, both sectors may be characterized by successively increasing linear trends between 1960-66 and 1967-72, and declining trends set in after 1972 in both sectors, which reduce the previously established net rates of flows of electricity disposition, but do not change their respective signs. We have further indicated a tentative declining trend after 1980 in the residential sector. The magnitude of this final trend, being based on a single observation, should be subjected to more accurate reassessment as final observations on the next couple of years become available.
Figure 7. Observations on the Time Path of Residential Electricity Sales in Ohio, 1960-1981
Figure 8. Observation on the Time Path of Commercial Electricity Sales in Ohio, 1960-1981
The respective time path models are, hence, identified as

3.23) \( ELRSBOH = f(T, T6, T12, T20); \)

3.24) \( ELCMBOH = f(T, T6, T12). \)

Our choice of the particular time path models depends both on their statistical properties as well as the fact that the domains of time under consideration, i.e. 1960-1966, 1967-1972, and 1972-1981, seem to be periods within which many energy data series have followed stable trends. Hence, our choice of model represents our assessment of acceptable error, syntactic economy, and heuristic possibilities for the joint representation of the systems under consideration. This statement naturally applies to all models that we discuss in this thesis.

b. **Industrial Sector.** Observations on the time path trajectory of industrial sales are plotted in figure 9. It is clear that the behavior of industrial sales is much more prone to systemic shocks than the residential or the commercial sectors. In particular, we observe that immediately following the recession of 1971 there has been an upward shift in the time path of industrial electricity utilization levels. We indicate this shift by the variable \( DUM6. \)

Furthermore, the recession of 1975, during which the unemployment rate in Ohio has gone up to 9.1% from 4.8% in 1974, may be interpreted to have led to a temporary deviation of one year, which we indicate by the variable \( D75 (5). \) The recession of 1980, when the unemployment in Ohio has risen from 5.9% in 1979 to 8.4% (6), has led to what may
Figure 9. Observations on the Time Path of Industrial Electricity Sales in Ohio, 1960-1981
be characterized as a downward shift in industrial sales, which has held on through 1981, during which the unemployment rate has crept up to 9.6%. We shall represent the shift from 1979 to 1980 with the system state variable DUM14, and withhold judgment on the possible further impacts of the systemic shocks underlying this recent shift, such as a change in the net long term acceleration of the system and, hence, the onset of a different long term trend after 1980.

It is clear, then, that the description of the time path of industrial electricity sales since 1971 has been very closely related to the rate of unemployment which, in turn, is a reliable indicator of the prevailing economic climate and of industrial activity levels in general. The behavior of industrial energy utilization during the recession of 1971, as opposed to the recessions of 1975 and 1980, raises important questions concerning the impacts of the precipitating causes of the respective recessions on Ohio's economy, or the cumulative effects of temporary shocks on the structure and determinants of the dynamics of specific systems, which require further investigations beyond the objective of this thesis.

Two observations, however, now may be made. First, among the precipitating causes of the 1971 recession is the 100% devaluation in U.S. currency, whereas the 1975 and 1980-on recessions follow drastic increases in crude oil prices. Second, the unemployment rate in Ohio went up to 12.5% in 1982, from 9.6% in 1981. Given the sensitivity of industrial disposition to the unemployment rate, and the fact that a 2.5% annual rise in 1980 has led to a major drop in industrial
electricity disposition in the same year, we would expect a 2.9% rise in 1982 to produce similar effects.

Among these rather drastic shock-induced fluctuations, we have characterized the remainder of the observations in terms of two long term trends: one from 1960 through 1972 and the other from 1972 on. The time path model is then identified as

\[ 3.25 \quad \text{ELINBOH} = f(T, T12, DUM6, DUM14, D75). \]

c. Transportation Sector. Observations on the time path of transportation sector sales are plotted in figure 10. The time path model is specified as

\[ 3.26.a \quad \text{ELTRBOH} = f(T, T6) \]

\[ 3.26.b \quad \text{subject to } \frac{df}{dT} + \frac{df}{dT6} = 0. \]

d. Total Electricity Sales. Observations on the time path of total electricity sales in Ohio are plotted in figure 11. As discussed in section B of this chapter, the aggregate sales time path trajectory has to equal the sum of the component sales time path trajectories. We have determined during preliminary modeling and adjustment that the sum of the changes in trends after 1972 in the residential and commercial sectors may be predicated to be equal to and opposite in sign to the change in the industrial trend after 1972.
Figure 10. Observations on the Time Path of Transportation Electricity Sales in Ohio, 1960-1981
Figure 11. Observations on the Time Path of Total Electricity Sales in Ohio, 1960-1981
Taking this into consideration, the time path of total electricity sales is specified as

3.27.a) \( ELTTBOH = f(T, T6, T20, DUM6, DUM14, D75) \)

subject to

3.27.b) \[ \begin{bmatrix} R2 \\ R3 \end{bmatrix}_{9x1} \cdot B = 0, \]

where \( B \) is the parameter vector of the system of disposition under consideration. \( R2 \) is the matrix specifying identity constraints on the elements of \( B \), and \( R3 \) a vector specifying the constraint on the parameters of the transportation sector time path. The observations on system state variables are included in table 19 in appendix C.

4. **Adjustment**

We have run individual adjustments of the component models 3.23-3.26.b whose results are included in table 20 in appendix C. From the standard errors of the unconstrained component models, we have constructed a matrix "COV", where

\[ COV = \text{covariance matrix} = \text{cofactor matrix} \]

for the system of disposition under consideration, to satisfy condition 3.14. The sample covariance matrix of residuals after adjustment is designated as \( ELWI \) in appendix C, and the system parameter vector \( B \) as BETA.
The adjustment problem as presented in appendix C involves the minimization of the quantity

\[ (Y - Z \cdot BETA)^\top \cdot (GINV \ W \otimes I) \cdot (Y - Z \cdot BETA) \]

subject to

\[ R1 \cdot BETA = 0, \]

where

\[ Y = \begin{bmatrix} ELRSBOH \; ELCMOH \; ELINBOH \; ELTRBOH \; ELTTBOH \end{bmatrix} \]

is the counterpart of the vector 'D' as defined in 3.4.

\[ Z = \text{Diag} [ \begin{bmatrix} SELRS \; SELCM \; SELIN \; SELTR \; SELTT \end{bmatrix} ] \]

\[ \text{SELRS} = \begin{bmatrix} \text{INT} \; T \; T6 \; T12 \; T20 \end{bmatrix} \]

\[ \text{SELCM} = \begin{bmatrix} \text{INT} \; T \; T6 \; T12 \end{bmatrix} \]

\[ \text{SELIN} = \begin{bmatrix} \text{INT} \; T \; T12 \; \text{DUM}6 \; \text{DUM}14 \; D75 \end{bmatrix} \]
3.31.d) \( \text{SELTR} = [ \text{INT} \quad T \quad T6 ] \)
\[ 22 \times 3 \]

3.31.e) \( \text{SELTT} = [ \text{INT} \quad T \quad T6 \quad T20 \quad \text{DUM6} \quad \text{DUM14} \quad \text{D75} ] \)
\[ 22 \times 7 \]

3.32) \( \text{GINVW} = \text{GIN} \quad (\text{COV}) \)
\[ 5 \times 5 \]

3.33) \( \text{I} \quad = \text{identity matrix} \)
\[ 22 \times 22 \]

3.34) \( \text{R1} \)
\[ 9 \times 25 \]

is a completely specified matrix the elements of which are 1, -1, or 0.

3.35) \( \text{BETA} \)
\[ 25 \times 1 \]

is the unknown parameter vector of the systemic trajectories under consideration.

In terms of the theoretical framework previously developed,

3.36) \( \text{Z} \cdot \text{BETA} = \text{Y-e} \)

is the functional time path model of the system, and

3.37) \( \text{e} \sim [ \text{O} \quad \text{COV} \otimes \text{I} ] \)

is the stochastic model.
5. **Statistical Implications of Adjustment Results**

The results of adjustment are presented in tables 5-16. Tables 5 and 6 include the system parameter vector $\mathbf{B}$ along with the associated standard errors and $t$-statistics, the overall $R^2$ and mean square error for the system as a whole, and the $R^2$'s and mean square errors of individual time paths within the system. Due to labeling restrictions, parameters for DUM6 and DUM14 are abbreviated as M6 and M14, respectively.

The statistical properties of the adjusted system are rather exemplary. The lowest absolute value of the $t$-statistic for the system parameter vector is 3.61 with 92 degrees of freedom. The degree of freedom is estimated as the number of observations ($= 110$) less the number of estimated parameters ($= 25$) plus the number of independent linear constraints ($= 9$). Overall $R^2$ for the system is .9999, and the overall mean square error for the system is 9.1 TBtu/year.

The breakdown of the overall systemic $R^2$ and mean square magnitudes in terms of the constituent elements of the system under consideration are given in vectors $\mathbf{RSQI}$ and $\mathbf{MSESEC}$, respectively. Between 1960 and 1981, the time path models specified account for 99.99% of the observed variation in residential sector electricity sales, 99.96% of the observed variation in commercial sector electricity sales, 99.94% of the observed variation in industrial sector electricity sales, 98.78% of the observed variations in transportation sector electricity sales, and 99.98% of the observed variation in total electricity sales of the electric utilities industry in Ohio.
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<td>7.90886</td>
</tr>
<tr>
<td>EI.CM.112</td>
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<td>0.271872</td>
<td>-9.30181</td>
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<tr>
<td>EI.ITH.111</td>
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</tr>
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<td>EI.ITH.11</td>
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<td>5.64568</td>
</tr>
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<td>EI.ITH.112</td>
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<td>4.61367</td>
<td>5.42259</td>
</tr>
<tr>
<td>EI.ITH.M6</td>
<td>3.85514</td>
<td>0.337241</td>
<td>11.4314</td>
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<td>EI.ITH.M14</td>
<td>-19.307</td>
<td>5.34438</td>
<td>-3.61259</td>
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<td>EI.ITH.D75</td>
<td>-51.1025</td>
<td>4.5767</td>
<td>-11.1658</td>
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<td>EI.IK.111</td>
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<td>0.0142167</td>
<td>24.6588</td>
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<tr>
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<td>EI.IK.16</td>
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<td>EI.IK.120</td>
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<td>5.34438</td>
<td>-3.61259</td>
</tr>
<tr>
<td>EI.IK.M14</td>
<td>-51.1025</td>
<td>4.5767</td>
<td>-11.1658</td>
</tr>
<tr>
<td>EI.IK.D75</td>
<td>-6.36223</td>
<td>1.23315</td>
<td>-5.15933</td>
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Table 6. R Square and Mean Square Errors for the System and for Individual Time Paths Within the System

<table>
<thead>
<tr>
<th></th>
<th>R21 SYSTEM</th>
<th>R0M1</th>
<th>R01 J1</th>
<th>M5S1 SYSTEM</th>
<th>R0M1</th>
<th>M5S1</th>
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<tbody>
<tr>
<td></td>
<td>R Squared</td>
<td></td>
<td>Mean Square Error</td>
<td>R Squared</td>
<td></td>
<td>Mean Square Error</td>
</tr>
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<td>0.999602</td>
<td>0.99937</td>
<td>0.999756</td>
<td>0.99984</td>
<td>0.999756</td>
</tr>
<tr>
<td>ROW1</td>
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<td>0.999602</td>
<td>0.99937</td>
<td>0.999756</td>
<td>0.99984</td>
<td>0.999756</td>
</tr>
<tr>
<td>MSESEC</td>
<td>1.03583</td>
<td>1.03583</td>
<td>26.3517</td>
<td>0.000512331</td>
<td>35.221</td>
<td></td>
</tr>
</tbody>
</table>
Hence, on purely statistical grounds, the proposed time path models do constitute an accurate description of the dynamic structure of total electricity sales in Ohio, defined as the sum of sectoral electricity sales.

6. Implications of Adjustment Results for Monitoring and Forecasting

a. Residential Sector Electricity Sales. Parameters of residential sector electricity sales time path are presented in table 7, along with the associated standard deviations and t-statistics. The actual observations, adjusted observations and the residuals are presented in table 8.

Residential electricity sales, starting from an initial level of 34.95 TBtus/year in 1960, have increased at an annual acceleration of 2.48 TBtu/(year)² between 1961 and 1966, and at an annual acceleration of 5.50 TBt/(year)² between 1967 and 1972. In 1973 the rate of acceleration has dropped to 4.17 TBtu/(year)², and in 1981 it has become a net deceleration of -2.19 TBtu/(year)². Furthermore, the standard deviation of actual observations around the theoretical time path predicated by the estimated model has been 1.02 TBtus/year, so that we would expect the future realizations of the observed magnitudes of residential electricity sales to be within a band of ± 2.04 TBtu/year around the projected systemic trajectory with 95% probability. Significant deviations from the confidence limits established by the historical stochastic behavior of the system would be interpreted as symptomatic of functional changes in the dynamic
Table 7. Parameter Estimates for the Time Path Model of Residential Sector Electricity Sales in Ohio, 1960-1981

<table>
<thead>
<tr>
<th>ELRS</th>
<th>ESTIMATE</th>
<th>ERROR</th>
<th>T RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>111</td>
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<td>0.67201</td>
<td>52.088</td>
</tr>
<tr>
<td>7</td>
<td>2.48217</td>
<td>0.164097</td>
<td>15.1263</td>
</tr>
<tr>
<td>16</td>
<td>3.01686</td>
<td>0.269582</td>
<td>11.1909</td>
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<tr>
<td>112</td>
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<td>0.221995</td>
<td>-5.97421</td>
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<tr>
<td>120</td>
<td>-6.36223</td>
<td>1.23315</td>
<td>-5.15933</td>
</tr>
</tbody>
</table>
Table 8. List of Adjusted Actual and Residual Values Associated with the Time Path Model of Residential Sector Electricity Sales (TTU/Year)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>ADJUSTED</th>
<th>ACTUAL</th>
<th>RESIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
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<tr>
<td>1961</td>
<td>37.4321</td>
<td>37.7367</td>
<td>-0.304615</td>
</tr>
<tr>
<td>1962</td>
<td>39.9143</td>
<td>39.818</td>
<td>0.0962416</td>
</tr>
<tr>
<td>1963</td>
<td>42.3964</td>
<td>41.2852</td>
<td>1.11125</td>
</tr>
<tr>
<td>1964</td>
<td>44.8786</td>
<td>44.2195</td>
<td>0.659094</td>
</tr>
<tr>
<td>1965</td>
<td>47.3608</td>
<td>47.6656</td>
<td>-0.306841</td>
</tr>
<tr>
<td>1966</td>
<td>49.8429</td>
<td>51.2482</td>
<td>-1.40528</td>
</tr>
<tr>
<td>1967</td>
<td>55.342</td>
<td>54.899</td>
<td>0.442932</td>
</tr>
<tr>
<td>1968</td>
<td>60.841</td>
<td>60.0853</td>
<td>0.755761</td>
</tr>
<tr>
<td>1969</td>
<td>66.34</td>
<td>66.2951</td>
<td>0.044236</td>
</tr>
<tr>
<td>1970</td>
<td>71.8391</td>
<td>72.283</td>
<td>-0.392949</td>
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<tr>
<td>1971</td>
<td>77.3381</td>
<td>76.7017</td>
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<td>1972</td>
<td>82.8371</td>
<td>81.6491</td>
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<td>1973</td>
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<tr>
<td>1974</td>
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<td>91.1686</td>
<td>0.014363</td>
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<td>1975</td>
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<td>-0.248848</td>
</tr>
<tr>
<td>1976</td>
<td>99.5283</td>
<td>98.7774</td>
<td>0.750917</td>
</tr>
<tr>
<td>1977</td>
<td>103.701</td>
<td>105.874</td>
<td>-2.17332</td>
</tr>
<tr>
<td>1978</td>
<td>107.874</td>
<td>108.331</td>
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</tr>
<tr>
<td>1979</td>
<td>112.047</td>
<td>110.89</td>
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</tr>
<tr>
<td>1980</td>
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<td>115.45</td>
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</tr>
<tr>
<td>1981</td>
<td>114.03</td>
<td>114.03</td>
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</table>
structure of the systemic trajectory. This ability to distinguish between systemic and stochastic error makes it possible to evaluate and respecify, whenever necessary, the functional model of the systemic trajectory in view of the most recent observations and the most recent information concerning the determinants and circumstances responsible for these observations.

It may be noted that there seems to be a change in long term trends, followed by the trajectory of residential electricity sales, once every six to eight years. Hence, any ceteris paribus forecasts should take this behavior into consideration and either specify, on the basis of independent investigation, the nature and expected magnitude of such future alterations specifically or limit themselves to a period of six to eight years from the onset of the most recent long term trend.

The current model's forecast for 1982 is 111.84 TBtu/year whereas our initial estimate of the actual 1982 sales, based upon information available to date, is 113.80 TBtu/year. The difference of -1.96 TBtu/year is barely within the 95% confidence interval. It should therefore be emphasized again that the magnitude of the post-1980 trend definitely requires a reestimation or maybe even a remodeling of the systemic trajectory from 1979 on, as the final observations for 1982 and 1983 become available.

It is clear that the severity of the current recession in Ohio is having a negative effect on residential electricity sales. Whether this effect is due to a decline in the number of residential customers or a further acceleration in residential conservation or other
consumption reducing behavior will have to be investigated in detail before coming to a conclusion concerning the magnitude and the possible duration of the negative effect. Similarly, the relation between the share of electricity costs in residential sector fuel costs, the share of residential fuel costs in residential expenditures, or other such well-defined relations, may shed a light on the historical behavior of the physical flows under consideration.

b. Commercial Sector Electricity Sales. Parameters of commercial sector electricity sales time path are presented in table 9 along with the associated standard deviation and t-statistics. Actual observations, adjusted observations, and residuals are presented in table 10.

Starting from an initial level of 23.63 TBtu/year at the end of 1960, commercial electricity sales have increased subject to an acceleration of 2.29 TBtu/(year)$^2$ between 1961 and 1966, and 5.10 TBtu/(year)$^2$ between 1967 and 1972. In 1973 the systemic acceleration has dropped to 2.57 TBtu/(year)$^2$. The standard error of actual observations around the theoretical time path predicated by the estimated model has been 1.37 TBtu/year, so that we would expect future realizations of observations on the annual magnitudes of commercial electricity sales to be within an interval of ± 2.74 TBtu/year around the projected systemic trajectory with 95% probability.

The model's ceteris paribus forecast for 1982 is 93.74 TBtus/year, and our initial estimate of actual 1982 commercial sales is 92.57 TBtu/year; the residual 1.17 TBtu/year is well within the 95%

<table>
<thead>
<tr>
<th>ELCM</th>
<th>ESTIMATE</th>
<th>ERROR</th>
<th>T RATIO</th>
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<td>0.219236</td>
<td>10.4508</td>
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<td>16</td>
<td>2.81241</td>
<td>0.355603</td>
<td>7.90886</td>
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<tr>
<td>112</td>
<td>-2.5289</td>
<td>0.271872</td>
<td>-9.30181</td>
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### Table 10. List of Adjusted Actual and Residual Values Associated with the Time Path Model of Commercial Sector Electricity Sales (TBTU/Year)

<table>
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<th>RESIDUAL</th>
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<td>0.604489</td>
</tr>
<tr>
<td>1963</td>
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<td>29.582</td>
<td>0.916716</td>
</tr>
<tr>
<td>1964</td>
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<td>1.60426</td>
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<tr>
<td>1965</td>
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<td>36.5084</td>
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<tr>
<td>1966</td>
<td>37.3723</td>
<td>39.6474</td>
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<tr>
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<tr>
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<tr>
<td>1981</td>
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</tr>
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</table>
confidence interval. There are therefore no signs that the prevailing economic recession has had an effect on the dynamic behavior of commercial sector electricity consumption as of the end of 1982, or that the long term trend established after 1972 has been altered in any material manner.

c. **Industrial Sector Electricity Sales.** Electricity sales time path is presented in table 11, along with the associated standard deviations and t-statistics. Actual observations, adjusted observations, and residuals are presented in table 12.

Starting from an initial level of 136.55 T\(\text{Btu/year} \) at the end of 1960, industrial electricity sales have increased, subject to an acceleration of 2.12 T\(\text{Btu/(year)}^2 \) between 1961 and 1972. In 1973 the systemic acceleration has risen to 5.98 T\(\text{Btu/(year)}^2 \).

The change in the systemic acceleration of the industrial sector from 1972 to 1973 is equal, in absolute magnitude but opposite in direction, to the sum of the coincident changes in the systemic accelerations of the residential and the commercial sectors. The net impact of these changes on total sales, therefore, is characterized to be nil.

The behavior of long term systemic trends is, however, only a part of the story in characterizing the dynamic behavior of industrial sector sales. As the level of annual industrial electricity utilization has increased over time, it has also become increasingly more vulnerable to alterations in the general economic environment. From 1971 on every major fluctuation in the economy has had a major impact on the industrial electricity consumption levels in Ohio. Although a definitive
Table II. Parameter Estimates for the Time Path Model of Industrial Sector Electricity Sales in Ohio, 1960-1981

<table>
<thead>
<tr>
<th>ELIN</th>
<th>ESTIMATE</th>
<th>ERROR</th>
<th>T RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>INT</td>
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<td>2.51574</td>
<td>54.2776</td>
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<tr>
<td>T</td>
<td>2.11643</td>
<td>0.374876</td>
<td>5.64568</td>
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<tr>
<td>DUM6</td>
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<td>4.41367</td>
<td>5.42259</td>
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<tr>
<td>T12</td>
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<td>11.4314</td>
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<td>D75</td>
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<td>5.34438</td>
<td>-3.61259</td>
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<td>-11.1658</td>
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</table>
Table 12. List of Adjusted Actual and Residual Values Associated with the Time Path Model of Industrial Sector Electricity Sales (TBTU/Year)

<table>
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<th>RESIDUAL</th>
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<tr>
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</tr>
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<td>184.487</td>
<td>184.487</td>
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</tr>
<tr>
<td>1981</td>
<td>188.521</td>
<td>185.93</td>
<td>2.59078</td>
</tr>
</tbody>
</table>
characterization of these systemic responses and their underlying determinants would require detailed and extensive analysis and research, we have tentatively identified an upward shift in historical trend from 1971 to 1972, following the recession of 1971. Coincident with this upward shift in industrial electricity consumption, there is a downturn in the industrial utilization of coal and petroleum products (7).

Both of these alterations could also be presumed tentatively to reflect the systemic impact of the 100% devaluation in the U.S. currency between 1971 and 1973, the circumstances leading to which are probably the major factors responsible for the recession in the first place. The magnitude of this shift, indicated by the system state variable DUM6, is measured as 23.93 TBtu/year.

As opposed to the recession of 1971, the recession of 1975 and that of 1980 to date were probably induced by, and certainly followed within approximately a year of, major increases in crude oil prices. While the 1975 recession is characterized as having lasted for one year, the current recession which started in 1980 is still going on and, as already mentioned, deepened further in 1982. In the first four months of 1983 a slight upturn is indicated by reduced levels in the unemployment rate of the civilian labor force in Ohio (8). The impact of the 1975 recession on industrial sales, characterized as a temporary deviation of one year from the prevailing long term trend and indicated by the system state variable D75, is measured to be -19.31 Btu/year. The initial impact of the current recession, from 1979 to 1980, is indicated temporarily by a shift variable DUM14 (9), the magnitude of which is measured as -51.10 TBtu/year. The severity of the current
recession on Ohio's industrial sector relative to the 1975 recession should be obvious from the magnitudes of the respective impacts on the time path.

The standard error of actual observations, around the time path trajectory described by the estimated model, has been 5.13 TBtu/year. We would, therefore, expect that under the operating conditions similar to those that prevailed in 1981, or under predicated future scenarios of operating conditions that are actually realized, the future magnitudes of annual industrial sales would fall within an interval of $\pm 10.26$ TBtu/year around the projected systemic trajectory 95% of the time.

The present model's *ceteris paribus* forecast for 1982 is 194.5 TBtu/year, whereas our initial estimate of actual industrial utilization in 1982 is 151.75 TBtu/year. The residual of 42.75 is way over the 95% confidence limit.

As mentioned in our discussion of the industrial sector in section 6 of this chapter, the downward shift in industrial electricity consumption in 1980 of -51.10 TBtu/year was accompanied by a coincident increase of 2.5% in the civilian unemployment rate. In 1982 the increase in civilian unemployment rate is up by 2.9% relative to 1981. Our expectation of a coincident drop in industrial electricity utilization levels in 1982, which would be similar in magnitude to the drop in 1980 level of utilization, is thus confirmed.

Hence, from 1982 on, another shift variable should be introduced to the industrial model time path to take care of the impact of the changes in the relevant economic conditions in 1982, relative to 1981,
on the systemic trajectory. It should be clear that any forecast scenario for industrial sector electricity sales in Ohio must include a fairly specific and reasonably accurate description of the time path of unemployment in Ohio throughout the forecast horizon if further pathetic predictive performances such as those documented in appendix A are to be avoided. Furthermore, such scenarios must be constantly monitored and updated as actual events evolve. For example, any forecast scenario which predicted an increase of, say, 5.98 TBtu/year between 1979 and 1989 would, thereby, have called for an observed difference to 59.8 + 10.26 TBtu/year between the 1979 level of industrial electricity sales and 1989 level of electricity sales. Whereas if the economy stabilizes in 1983, with the unemployment level returning to the 1979 level in the same manner that it came down from the 1979 level to the 1982 level and the recession/depression not actually affecting the long term trend, we would expect 1989 level of industrial sales to be within 11.96 ± 10.26 TBtu/year of the 1979 level. Furthermore, given the fact that we have witnessed three increasingly serious recessions between 1970 and 1982 with the civilian unemployment rate rising over 8% in four of the 13 years considered, whether the economy can actually return to the 1979 level of activity by 1987 is an open question. Similarly, how industrial electricity sales will react to possible national policy changes such as natural gas deregulation, or international changes such as a possible breakdown of OPEC or a prolonged crisis in the Middle East, are questions not answerable at the present time.
All this confirms that modern day industrial systems such as Ohio's are not islands unto themselves. Therefore, any attempt to project or explain their behavior without due consideration of the shocks, alterations, or prevailing trends that originate in or influence the macro systems which include them, is likely to be misleading. There is sufficient evidence in the observed behavior of industrial electricity sales in Ohio to date that any reasonable forecast of, or attempt to understand, industrial electricity utilization requires a clear understanding of the impacts of specific shocks or alterations in the national or the international economic conditions on the Ohio economy.

d. Transportation Sector Electricity Sales. Electricity utilization in the transportation sector is reported by only one utility and is extremely small in magnitude. We have included it, however, for the sake of completeness.

The parameters of transportation sector electricity sales time path are presented in table 13, along with the associated standard errors and t-statistics. Actual observations, adjusted observations and residuals are presented in table 14.

Starting from an initial level of .35 TBtu/year at the end of 1960, transportation sector electricity sales have declined, subject to a deceleration of -.034 TBtu/(year)^2 between 1961 and 1966. The systemic acceleration is characterized as zero from 1967 on. The standard error of observations around the model is .022 TBtu/year. The model's forecast for 1982 is .147 TBtu/year, and our initial estimate
Table 13. Parameter Estimates for the Time Path Model of Transportation Sector Electricity Sales in Ohio, 1960-1981

<table>
<thead>
<tr>
<th>ELIR</th>
<th>ESTIMATE</th>
<th>ERROR</th>
<th>1 RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>III1</td>
<td>0.350567</td>
<td>0.0142167</td>
<td>24.6588</td>
</tr>
<tr>
<td>T</td>
<td>-0.0338966</td>
<td>0.00265457</td>
<td>-12.7691</td>
</tr>
<tr>
<td>16</td>
<td>0.0338966</td>
<td>0.00265448</td>
<td>12.7696</td>
</tr>
<tr>
<td>YEAR</td>
<td>TRBOHI</td>
<td>ADJUSTED</td>
<td>ACTUAL</td>
</tr>
<tr>
<td>------</td>
<td>--------</td>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>1960</td>
<td>0.360567</td>
<td>0.37532</td>
<td>-0.024753</td>
</tr>
<tr>
<td>1961</td>
<td>0.31667</td>
<td>0.3612</td>
<td>-0.0245296</td>
</tr>
<tr>
<td>1962</td>
<td>0.282774</td>
<td>0.27296</td>
<td>0.00981351</td>
</tr>
<tr>
<td>1963</td>
<td>0.248877</td>
<td>0.20472</td>
<td>0.0441572</td>
</tr>
<tr>
<td>1964</td>
<td>0.214981</td>
<td>0.1706</td>
<td>0.043806</td>
</tr>
<tr>
<td>1965</td>
<td>0.181084</td>
<td>0.1706</td>
<td>0.0106841</td>
</tr>
<tr>
<td>1966</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1967</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1968</td>
<td>0.147187</td>
<td>0.1706</td>
<td>-0.0236125</td>
</tr>
<tr>
<td>1969</td>
<td>0.147187</td>
<td>0.1706</td>
<td>-0.0236125</td>
</tr>
<tr>
<td>1970</td>
<td>0.147187</td>
<td>0.1706</td>
<td>-0.0236125</td>
</tr>
<tr>
<td>1971</td>
<td>0.147187</td>
<td>0.1706</td>
<td>-0.0236125</td>
</tr>
<tr>
<td>1972</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1973</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1974</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1975</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1976</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1977</td>
<td>0.147187</td>
<td>0.13648</td>
<td>0.0107075</td>
</tr>
<tr>
<td>1978</td>
<td>0.147187</td>
<td>0.1706</td>
<td>-0.0236125</td>
</tr>
<tr>
<td>1979</td>
<td>0.147187</td>
<td>0.17</td>
<td>-0.0228125</td>
</tr>
<tr>
<td>1980</td>
<td>0.147187</td>
<td>0.14</td>
<td>0.00718746</td>
</tr>
<tr>
<td>1981</td>
<td>0.147187</td>
<td>0.16</td>
<td>-0.0128125</td>
</tr>
</tbody>
</table>
for 1982 .16 TBtu/year. The residual .013 is well within the 95% confidence interval of ± .044 TBtu/year.

e. Total Electricity Sales. Total electricity sales is simply the sum of the constituent sectoral sales, and the parameters of total electricity sales are the sums of the respective parameters included in the constituent sectoral sales.

In general, the number of system state variables included in describing the aggregate time path will equal the number of distinct system state variables included in $U \bigcup Z_p(t)$, where $Z_p(t)$ is the system state variable vector for the $p$th component time path. Whenever deemed appropriate, desirable, or warranted by a priori information, the number of included system state variables in the aggregate time path model may be reduced through a priori restrictions which may specify that certain sums of component parameters be equal to zero. However, the number of variables characterizing the aggregate time path will in general be larger than the number of variables in any one of the components. Two observations may be made in this regard. First, since the parameters of the aggregate model are determined completely on the basis of definitional or other a priori constraints, the degrees of freedom for the aggregate time path should be specified as $n$ in computing the standard error of the estimate. Second, if there are reasons for keeping the number of variables with nonzero coefficients in the aggregate time path model to a minimum, one may first specify a time path model for the aggregate observations, and use the outcome of the unconstrained parameter estimates for the model as constraints.
to be obeyed by the component time path models.

Parameters of total electricity sales in Ohio are presented in table 15 along with the associates standard errors and t-statistics. Actual observations, adjusted observations and residuals are presented in table 16.

Starting from an initial level of 195.47 TBtu/year at the end of 1960, total electricity sales in Ohio have increased, subject to an acceleration of 6.86 TBtu/(Year)$^2$ between 1961 and 1966 and 12.72 TBtu/(year)$^2$ from 1967 to 1980. In 1980 the acceleration of the total system characterized in terms of residential, commercial, industrial, and transportation components is reduced to 6.36 TBtu/year, reflecting the recent downturn in residential long term trends. Alterations in residential, commercial and industrial long term trends occurring at the end of 1972 are designated to add up to zero.

In addition to the long term trends in the component time paths, the time path of total disposition, by definition, includes the shocks that the industrial sector has been subject to. Hence, the post-1971 shift in the industrial time path, indicated by the variable DUM6, is similarly indicated in the total time path model with the same coefficient. The same is true with the one year deviation during 1975, indicated as D75, and the decline in industrial utilization after 1979, indicated as DUM14.

The comments that have been made concerning component time path trajectories apply, by definition, to the total time path trajectory as well. In particular, the model's forecast for 1982, under a *ceteris paribus* scenario of general economic conditions in Ohio
Table 15. Parameter Estimates for the Time Path Model of Total Electricity Sales in Ohio, 1960-1981

<table>
<thead>
<tr>
<th>ELT</th>
<th>ESTIMATE</th>
<th>ERROR</th>
<th>T RATIO</th>
</tr>
</thead>
<tbody>
<tr>
<td>JMT</td>
<td>195.474</td>
<td>2.78495</td>
<td>70.1894</td>
</tr>
<tr>
<td>J1</td>
<td>6.85589</td>
<td>0.482133</td>
<td>14.2199</td>
</tr>
<tr>
<td>J16</td>
<td>5.86317</td>
<td>0.437128</td>
<td>13.4129</td>
</tr>
<tr>
<td>DUM6</td>
<td>23.9355</td>
<td>4.11367</td>
<td>5.42259</td>
</tr>
<tr>
<td>D75</td>
<td>-19.307</td>
<td>5.34438</td>
<td>-3.61259</td>
</tr>
<tr>
<td>DUM14</td>
<td>-51.1025</td>
<td>4.5767</td>
<td>-11.1658</td>
</tr>
<tr>
<td>J20</td>
<td>-6.36223</td>
<td>1.23315</td>
<td>-5.15933</td>
</tr>
</tbody>
</table>
### Table 16. List of Adjusted Actual and Residual Values Associated with the Time Path Model of Total Electricity Sales (TBTU/Year)

<table>
<thead>
<tr>
<th>Year</th>
<th>Adjusted</th>
<th>Actual</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>195.474</td>
<td>195.405</td>
<td>0.068986</td>
</tr>
<tr>
<td>1961</td>
<td>202.33</td>
<td>199.875</td>
<td>2.4651</td>
</tr>
<tr>
<td>1962</td>
<td>209.186</td>
<td>209.121</td>
<td>0.065007</td>
</tr>
<tr>
<td>1963</td>
<td>216.042</td>
<td>218.675</td>
<td>-2.63321</td>
</tr>
<tr>
<td>1964</td>
<td>222.898</td>
<td>225.397</td>
<td>-2.49891</td>
</tr>
<tr>
<td>1965</td>
<td>229.754</td>
<td>228.331</td>
<td>1.42261</td>
</tr>
<tr>
<td>1966</td>
<td>236.609</td>
<td>232.698</td>
<td>3.91114</td>
</tr>
<tr>
<td>1967</td>
<td>243.329</td>
<td>244.128</td>
<td>0.8002</td>
</tr>
<tr>
<td>1968</td>
<td>262.056</td>
<td>262.007</td>
<td>0.050281</td>
</tr>
<tr>
<td>1969</td>
<td>274.767</td>
<td>278.555</td>
<td>-3.7888</td>
</tr>
<tr>
<td>1970</td>
<td>287.486</td>
<td>288.211</td>
<td>-0.72574</td>
</tr>
<tr>
<td>1971</td>
<td>300.205</td>
<td>301.177</td>
<td>-0.97256</td>
</tr>
<tr>
<td>1972</td>
<td>336.857</td>
<td>330.554</td>
<td>6.30286</td>
</tr>
<tr>
<td>1973</td>
<td>349.576</td>
<td>363.31</td>
<td>-13.7332</td>
</tr>
<tr>
<td>1974</td>
<td>362.295</td>
<td>360.682</td>
<td>1.61305</td>
</tr>
<tr>
<td>1975</td>
<td>355.707</td>
<td>356.588</td>
<td>-0.880454</td>
</tr>
<tr>
<td>1976</td>
<td>387.734</td>
<td>384.073</td>
<td>2.86025</td>
</tr>
<tr>
<td>1977</td>
<td>400.453</td>
<td>411.794</td>
<td>-11.3413</td>
</tr>
<tr>
<td>1978</td>
<td>413.172</td>
<td>409.099</td>
<td>4.07303</td>
</tr>
<tr>
<td>1979</td>
<td>425.891</td>
<td>418.62</td>
<td>7.27085</td>
</tr>
<tr>
<td>1980</td>
<td>387.507</td>
<td>389.11</td>
<td>-1.60258</td>
</tr>
<tr>
<td>1981</td>
<td>393.864</td>
<td>390.97</td>
<td>2.89414</td>
</tr>
</tbody>
</table>
vis a vis 1981, is 400.22 TBtu/year, whereas our estimate for 1982 is
358.28 TBtu/year. The residual 41.94 is well beyond a conservatively
calculated 95% confidence interval of \(+11.87\) TBtu/year. Since the
origin of this discrepancy is the industrial sector, the suggestions
we made for updating the industrial sector sales time path for the
deepening of the recession in 1982 carry over to the updating of the
total sales time path as well.

D. Heuristic Implications

1. Implications for Tracing Intersystemic or Intrasystemic
Diffusion of Macro or Micro Level Shocks and Policy
Planning Considerations

The comments that we made under this heading in chapter II apply
to every time path in every system of disposition, including the system
of electricity disposition in Ohio, jointly and severally. It is
further clear from the behavior of the system analyzed and assessed in
this chapter, and the commercial natural gas disposition system analyzed
and assessed in chapter II, that the behavior of the energy disposition
time paths under consideration indicates major alterations in systemic
trends at a number of common points in time. In particular, residential,
commercial and industrial electricity sales as well as commercial
natural gas sales all indicate a nondifferentiability in trends at 1972.
All sectoral electricity sales display an upturn after 1966. Both
commercial natural gas and industrial electricity sales register
similar one year temporary deviations during 1975.
The prevalence of such common points of variation suggests that coincident "micro" level systemic responses which are similar in nature may arise from coincident "macro" level shocks or structural alterations in the successively more inclusive "macro" economic systems to which the "micro" systems are definitionally related. For example, it is reasonable to suspect that the recessions of 1975 and the current one, from 1980 on, represent to a considerable extent the outcomes of systemic adjustment on the part of Ohio and the U.S. economic sectors to severe and sudden increases in world crude oil prices. An understanding of the channels through which the magnitude of such shocks or structural alterations in the worldwide environment are transmitted to the U.S. and to Ohio, and the channels through which the magnitude of such shocks or structural alterations impact specific economic dimensions such as costs, prices, employment, incomes or the general economic environment in the U.S. and in Ohio, are extremely interesting exercises in empirical economic analysis and investigation. More importantly, they may be extremely informative exercises if the alterations in sectoral activity levels (as in the case of provision or disposition of natural gas and electricity) in response to specified measures of magnitude of external shocks of the types mentioned above are quantified. The understanding and the information which a thorough examination of the definitionally related dynamics of such successively inclusive or functionally dependent systems may yield would constitute a necessary foundation for any empirically valid analysis, assessment, and explanation of the historical behavior of systems. It would, thereby, provide a
reliable and valid foundation for the monitoring, forecasting or explaining of the magnitude or duration of alterations in projected systemic trajectories under corresponding scenarios of predicated alterations in future states of the world at large.

The importance of such understanding or information in circumstances of crisis management is undeniable. One of the problems tackled by the Carter Administration in the immediate aftermath of the 1978 OPEC price hikes was the preparation of a plan for state level gasoline rationing in the case of a possible oil embargo, or disruptions in deliveries from the Middle East due to other factors. The problem of finding an equitable solution to the question by how much a given state's gasoline consumption should be curtailed for a curtailment of X barrels in the total amount of gasoline available for distribution in the U.S. became a rather critical issue for state governments. At the Ohio Department of Energy, we demonstrated at the time that, between 1960 and 1973, annual gasoline consumption in Ohio could be expressed as a time variant linear transformation of annual gasoline consumption in the U.S. With the coefficient of transformation from U.S. gasoline consumption to Ohio gasoline consumption constant between 1960 and 1970, 1971 and 1975, and from 1976 on, this is given by

\[ 3.38.a) \quad GSTTPOH = a_0 \cdot GSTTPUS + a_1 \cdot WGSTTPUS + a_2 \cdot SGSTTPUS \]
3.38.b) GSTTPOH and GSTTPUS

are, respectively, Ohio and U.S. gasoline consumption levels in million gallons/year.

3.38.c) WGSTTPUS = DUM5 \cdot GSTTPUS,

3.38.d) SGSTTPUS = DUM10 \cdot GSTTPUS,

3.38.e) DUM5 = 0 \text{ if year} \leq 1970,

DUM5 = 1 \text{ if year} > 1970,

3.38.f) DUM10 = 0 \text{ if year} \leq 1975,

DUM10 = 1 \text{ if year} > 1975.

The model as estimated at the time is presented in table 17.

The utilization of similar transformation models under situations involving critical national supply shortages such as possible crude oil embargos, or involving statewide supply shortages similar to the 1977 natural gas crisis in Ohio, may provide a simple and rather objective first approximation in the search for an "equitable" allocation of the burden of specific supply disruptions among designated end user categories.

| VARIABLE  | DF | PARAMETER ESTIMATE | STANDARD ERROR | T RATIO  | PROB>|1| | VARIABLE LABEL |
|-----------|----|--------------------|----------------|----------|------|--------|--------------------|
| GSTTPUS   | 1  | 0.051215           | 0.0002371328   | 215.9758 | 0.0001|        | GSTTPUS            |
| WGSTTPUS  | 1  | -0.00230596        | 0.0003491172   | -6.6051  | 0.0001|        | WGSTTPUS           |
| SGSTTPUS  | 1  | -0.00154608        | 0.0003344739   | -4.6224  | 0.0002|        | SGSTTPUS           |

   a. **Elasticity of Consumption.** Another pressing problem for state regulatory agencies is that of determining the effect of rate hikes on the sales and, hence, on the revenues and profits of the regulated utilities. This is posed, in traditional practice, as a question of price elasticity which is very dear to the hearts of economists. The concept of price elasticity, however, suffers from the same shortcomings discussed in section D of chapter I which traditional demand analysis suffers as a tool of empirical analysis or as an instrument of policy planning or implementation. A dynamic concept of price elasticity of consumption will be briefly introduced, for it may prove to be more useful and reliable as a tool of positive analysis and as an instrument of policy planning or implementation.

   By definition, at any point in time $t$,

   $$3.39) \quad R_x(t) = P_x(t) \cdot X(t)$$

   where $R_x(t)$ is the revenues from the sales of $X(t)$, and $P_x(t)$ is the price of $X$ at time $t$. Hence, if any two of the three elements in the above definition are known over a historical or forecast domain in time, then the third is also known. Differentiating both sides of 3.39 we get
3.40) \[
\frac{d R_x(t)}{dt} = \frac{d P_x(t)}{dt} \cdot X(t) + \frac{d X(t)}{dt} \cdot P_x(t)
\]
or
3.41) \[
\frac{dR_x(t)/dt}{dP_x(t)/dt} = X(t) + \frac{dX(t)/dt}{dP_x(t)/dt} \cdot P_x(t)
\]
or
3.42) \[
\eta = \frac{1}{X(t)} \cdot \frac{dR_x(t)/dt}{dP_x(t)/dt} - 1 = \frac{d X(t)/dt}{dP_x(t)/dt} \cdot \frac{P_x(t)}{X(t)}
\]

Hence, as long as the historical time paths of \( P(t) \) and \( X(t) \) are known, so would \( \frac{d X(t)}{dt} \) and \( \frac{dP_x(t)}{dt} \), so that the right hand side of 3.42 could be utilized to investigate the historical behavior of the elasticity of disposition of \( X \) with regard to price. If this behavior seems to display any regularities over specific subdomains of the historical interval of time under consideration, further research may be conducted to investigate possible determinants of such behavior. Similarly, forecast magnitudes of the elasticity may be computed on the basis of provided scenarios for the projected time paths of \( P_x(t) \) and \( X(t) \).

b. **Price and Consumption.** It should be noted that 3.41 could also be expressed as

3.43) \[
X(t) = \frac{dR_x(t)/dt}{dP_x(t)/dt} - \frac{d X(t)/dt}{dP_x(t)/dt} \cdot P_x(t).
\]

We have already mentioned that if the time paths of \( R_x(t), P_x(t), \) and \( X(t) \) are known over the domain of time under consideration, then
one could compute the ratios on the right hand side of 3.43 and investigate what the implications of their behavior may be, provided that
d$P_x(t)/dt \neq 0$. If $dP_x(t)/dt=0$ for any subdomain in $T$, the implication, from 3.40, is that

$$3.44.a) \quad \frac{dR_x(t)}{dt} = \frac{dX(t)}{dt} \cdot P_x(t)$$

or

$$3.44.b) \quad \frac{dR_x(t)}{dt} = \frac{dX(t)}{dt} = P_x(t)$$

where $P_x(t)$ is constant within the subdomain of time in which $dP_x(t)/dt = 0$.

As long as the time paths of $X(t)$, $P_x(t)$, and $R_x(t)$ are known, the behavior and properties of the coefficients in the transformation of
$P_x(t)$ into $X(t)$, as expressed in 3.43, can easily be determined. Hence, if there are any a priori expectations of a "theoretical" nature concerning the behavior and properties of these coefficients, whether such expectations are actually satisfied or not can also be easily verified. It should be noted, however, that, in general, the coefficients in question may or may not be constant or even linearly dependent on time. We discuss the case where $X(t)$ and $R_x(t)$ are both characterized by time variant linear trends in appendix D. In fact, it is our contention that the objective of a positive economic theory is to try to explain the regularities in the relations among definitionally related economic time paths, after the existence of such regularities have been empirically demonstrated.
c. **Income and Consumption.** The analytical procedures introduced in the preceding section may be directly extended to the analysis and assessment of intertemporal regularities between the consumption of a given commodity \( X \), in a given system of disposition of \( X \), and a given measure of disposable income \( Y \), for the designated system of disposition. In general, such analyses may be conducted either in terms of the additive identity so that, at any given point in time, we have

\[
Y(t) = R_x(t) + D_{Nx}(t),
\]

where \( D_{Nx}(t) \) is income disposed of in a manner other than in the form of expenditures on \( X(t) \), or in terms of the multiplicative identity of the form

\[
R_x(t) = S(t) \cdot Y(t)
\]

where \( S(t) \) is the share, at time \( t \), of the expenditures on \( X(t) \) in \( Y(t) \).

It is presumed in the above designations that expenditures on \( X(t) \) and revenues from sales of \( X(t) \) are to be defined so as to equal each other. Hence

\[
P_x(t) \cdot X(t) = S(t) \cdot Y(t).
\]

Differentiating both sides of 3.47 we get
\[ P_x(t) \frac{X(t) + \frac{d X(t)}{d t}}{dt} \cdot P_x(t) = \frac{d S(t)}{dt} \cdot Y(t) + \frac{d Y(t)}{dt} \cdot S(t) \]

or

\[ X(t) = \frac{d S(t)/dt}{d P_x(t)/dt} \cdot Y(t) + \frac{d Y(t)/dt}{d P_x(t)/dt} \cdot S(t) \]

\[ - \frac{d X(t)/dt}{d P_x(t)/dt} \cdot P_x(t). \]

Considerations concerning the nature of coefficients of 3.49 will be similar to those discussed in the preceding section so that if \( \frac{d P_x(t)}{dt} \neq 0 \), 3.49 will be expressible as a transformation, from \( Y(t), \ S(t), \ P_x(t) \) to \( X(t) \). These implications, when \( X(t) \) is characterized in terms of time variant linear trends, may be developed along the lines of the discussion presented in appendix D. The researcher may, if deemed desirable or necessary, increase the number of factors to be included in a transformation such as 3.49 by further breakdown of \( Y(t) \) or \( R_x(t) \) into their respective definitional components.

The important point to note in the results obtained in this section is that when one is dealing with observations obtained under experimentally uncontrolled conditions, the definitions of coefficients in such relations as income-consumption, price-consumption etc., involve total derivatives only. This is to be contrasted with econometric relations where, in general, the coefficients are gratuitously presumed to represent magnitudes of partial derivatives.
CHAPTER IV

CONCLUSION

Compared to the decade of the 1960s, the state of the world economy from 1970 on has been characterized by a series of rather traumatic shocks. These shocks include: (a) the breakdown of the Bretton Woods Monetary Agreement in 1971, and the international monetary order that had prevailed since 1945; (b) the subsequent 100% devaluation of the U.S. currency by 1973; (c) the quadrupling of crude oil prices by OPEC in 1973, and the doubling of the same in 1978; (d) the winding down and termination of more than a decade of U.S. overseas military engagement in southeast Asia; (e) three successive recessions of increasing severity and duration in 1971, 1974-75, and 1980 to this writing.

Conventional econometric models and statistical methods have fared rather poorly when it comes to capturing and characterizing the different trends that the time paths of many economic data series have followed before, during and after these shocks.

The poor performance of conventional econometric models and statistical methods as forecasting tools raises a doctrinal as well as a practical issue. Although our major concern in this thesis has been a practical one, we may also examine the doctrinal issue as it is relevant to the alternative analysis we developed.
The doctrinal issue at hand can be summarized as follows. It has been the conventional academic wisdom to consider neoclassical theories as the only legitimate explanations of economic observations. Econometric models and statistical methods have then been employed as means of providing empirical interpretations and tests of these theories. If this is so, what are the logical implications of the poor performance of such models and methods with regard to the presumed ontological paradigm which they are thought to interpret and test?

Our answer to this question has been threefold. First, neoclassical economic theories are ultimately based upon axiomatic systems in pure mathematics. One of the theoretical implications of such axiomatic systems is known as the law of supply and demand. An econometric model is, then, asserted as an empirical interpretation, or a local specification and measurement, of such a general "theoretical" law. This view that scientific enterprise starts with axiomatic systems from which general theoretical laws are deduced and then empirically interpreted as observationally testable propositions belongs to the positivist school in the philosophy of science. The positivist view of science has, however, come to be questioned, especially in the past twenty years or so, and is now considered defunct among many philosophers of science. (2) Thus, the fact that an econometric model is traceable to a consistent system of axioms which carry a certain normative appeal is neither necessary nor sufficient for such a model to operate as a legitimate, let alone successful, explanatory or predictive tool of empirical import. (3)
Second, we have demonstrated in chapter I that if some true empirical interpretations of such a theoretical law of supply and demand did in fact exist, a true linear econometric model would possess an explicit solution in the domain of time. Hence, it would be much more economical, at least as far as forecasting the future transaction magnitudes of goods or services is concerned, to analyze and assess the behavior of their time path trajectories directly.

Third, the theoretical law of supply and demand is basically derived from the axiomatic system incorporating the neoclassical conception of behavior of "agents" with regard to choices they make subject to constraints. The rational behavior in question typically requires that such choices satisfy the maximization of a continuous, differentiable, and homothetic function of all "consumption bundles" subject to a linear budget constraint. The demand for some commodity X is then derived from a simultaneous solution to the first order conditions of the relevant Lagrangian equation. By the logic of mathematical maximization involved, these first order conditions are formulated in terms of partial derivatives only. Hence, in the consequent derivation of the theoretical demand function, the coefficients of the right hand side variables are also defined exclusively in terms of partial derivatives. Thus, any empirical interpretation of the coefficients of a theoretical demand function necessarily requires, among other things, a completely controlled experimental design whereby m-1 of the \( 1 < m < n-1 \) explicitly introduced explanatory variables
are kept constant and the one remaining variable varies within a wide enough range of values to yield observable responses in the "agent's" demand for X. The correlation between the observed magnitudes of the demand for X and the corresponding magnitudes of the uncontrolled experimental variable is then interpreted as an empirical counterpart of the corresponding coefficient in the underlying theoretical demand equation. It is only then that such a coefficient may be expected to conform to its theoretically predicated characteristics regarding sign, magnitude and so on. It is also under such circumstances that the results obtained are construed to confirm or disconfirm specific formulations of theoretical demand for X.

We have demonstrated at the end of chapter III that the definitional relations among the observed temporal sequences of transaction prices, quantities and income involve total derivatives in the coefficients involved. There is no way that a partial derivative can be estimated from uncontrolled sequences of historical observations. Since parameters of econometric models are estimated from such observations, neither the sign and magnitude of such estimates nor the performance of the forecast trajectories derived from them can be said to constitute a confirmation or a disconfirmation of the theoretical law of supply and demand, or for that matter, of the underlying neoclassical presumptions.

As an empirical statement, a linear econometric model is an assertion that the time path of the dependent variable is a weighted sum of the time paths of the independent variables, where the weights
are presumed to be time invariant. It may be that such a relation reflects an accounting identity. It may also reflect a specific law in a given situation such as the transformation of chemical energy from coal into electricity, or the thermodynamic necessity of increasing heat inputs to keep a given residential housing stock at a given internal temperature as the ambient temperature falls. In these cases, an econometric model can make an assertion not about the existence of these relations but only about their invariance over time. Thus, if an econometric model displays poor statistical properties, it is because the dynamics of these relations have not been time invariant over the period under consideration. The proper thing to do in such a situation is to examine the actual time paths of the function and its arguments and define the coefficients of the originally proposed relation from the estimated time path parameters as we have done in Chapter III. If, on the other hand, a proposed econometric model cannot be construed to represent any known definitional identity, or an established a priori law of the type discussed above, then it is at best an exercise in wishful thinking no matter what the statistical properties of the adjusted model may be.

If two unrelated transaction magnitudes such as, say, the sales of residential electricity and those of tomatoes in Ohio have displayed similarly structured trends over the period 1960-1981, one could get rather exemplary statistical results by regressing the sales of tomatoes upon the sales of residential electricity. The wisdom gained
from such an exercise is, however, doubtful at best. Whenever an arbitrary collection of "independent" variables is introduced to explain a set of "dependent" variables through regression analysis, it must almost always be possible to extract useful information concerning the dynamics of all the variables under consideration, as well as their joint or independent empirical determinants in the domain of time, through analyses and assessments of the time paths of each one of the variables, the components of their appropriately specified definitions, and the definitional relations that may exist among them.

The practical issue raised by the poor performance of econometric models and statistical methods as forecasting tools may be expressed as follows. To the extent that nonseasonal economic forecasting and analysis are conducted through these models and methods, many executives, administrators and planners in public or private sectors face a dilemma of choosing between the flipping of a coin and these sophisticated yet misleading mathematical analyses and projections. The undesirable consequences of such a state of affairs in a world of highly interdependent economic systems, including suboptimal allocations of scarce resources, inefficiency, waste, or prolonged deterioration in economic welfare, are too obvious to merit further comment.

There is, therefore, a practical need to provide a reasonably reliable and empirically valid alternative to econometric methods to those who require historical analyses and projections pertaining to the future magnitudes of observations on economic data series subject to alternative specifications of the relevant future states of the
world. Furthermore, since economic observations are not the outcomes of controlled experiments but rather reflect the aggregate outcomes of the decisions and actions of an ever-changing population in a socio-economic system, the appropriate methodologies introduced for the analyses and assessments of observed regularities in temporal sequences of such observations may be similar to the methodologies employed in nonexperimental physical sciences such as astronomy and geodetics. This is indeed the reason why we began our discussion in Chapter I with a review of the physical and metaphysical presumptions pertaining to the nature and properties of quantitative observations and mathematical modeling of these quantitative observations. Such an approach would provide a strictly positive basis for the analyses and assessments of economic observations.

To return to the issue at hand, annual observations on the magnitudes of economic flows generated in a geographically delineated system, such as Ohio or the U.S., reflect the level or intensity of systemic activities involving transactions of the good or service under consideration. Whatever the motives, constraints, and objectives of individual agents may be, they have maintained an aggregate level of transactions $X_a(t)$ between $t$ and $t-1$. In other words, they have sustained a temporal stream of recorded flows from recognized providers of $X$ to recognized disposers of $X$ equal to $X_a(t)$ units/year at time $t$.

In experimentally noncontrollable systems such as the systems of heavenly bodies, or complex socio-economic systems such as the U.S.
or Ohio, information concerning the determinants of the system's
dynamic behavior should be extracted from characterizations of the
time path of the observations in question, consistent with definiti-
tional or other a priori constraints. Since an observation on an
annual transaction magnitude of X can be interpreted as a flow
velocity, a sequence of such observations would characterize the time
path of flow velocities generated by the systemic activities involving
transactions of X over the domain of time.

In general, the first observation in such a sequence, \( X_a(t_0) \),
would indicate the cumulative impact of all preceding historical
sequences of actions, decisions, conditions, or determinants pertaining
to the aggregate transactions of X in the system \( S_{xa} \). The dynamic
behavior of \( X_a(t) \) within the domain of time over which observations are
available would then be determined by the behavior of \( dX_a(t)/dt \), which
along with \( X_a(t_0) \), would be sufficient to describe a deterministic
functional time path model for the system \( S_{xa} \).

The question of why a particular system of disposition, \( S_{xa} \), has
moved along a time path characterized by a particular \( X_a(t_0) \), and
\( dX_a(t)/dt = f_a(t) \), is a matter of independent investigation which can
generally be addressed only after the \( X_a(t_0) \) and \( f_a(t) \) have been
measured through consistent modeling and adjustment. Furthermore, to
the extent that economic flow magnitudes are accounting aggregations
of physical or financial flows, the systemic parameters \( X_a(t_0) \) and
\( f_a(t) \) are, by definition, equal to
\[
\sum_{\text{p=1}}^{h} X_p(t_0) \quad \text{and} \quad \sum_{\text{p=1}}^{h} f_p(t),
\]
respectively.
respectively, where for any given partitioning of $S_{xa}$ into constituent micro systems $S_{xp}$, $p=1, 2, ... h$, $X_p(t_0)$ and $f_p(t)$ are the corresponding parameters of the time path of the $p$th micro system of $S_{xa}$. Similarly, for every macro system in which $S_{xa}$ is included as a micro system, the time path parameters of the aggregate flows of $X$ in the higher order systems which include $S_{xa}$ would explicitly include the time path parameters of the flows of $X$ in $S_{xa}$.

One consequence of the accounting properties of economic flow magnitudes is that any anomaly in the time path of any one of the constituent micro systems of $S_{xa}$ must either be offset by compensating anomalies in other constituent micro system time paths or be explicitly included in the time path trajectory of $S_{xa}$. Similarly, every anomaly in any one of the higher order systems in which $S_{xa}$ is a constituent element must either be explicitly transmitted to the time path of $S_{xa}$, or be offset, completely or partially, or be further accentuated, by anomalies in other constituent micro system time paths.

The ability to trace intersystemic temporal anomalies due to micro or macro shocks, or structural changes through simultaneous modeling and adjustment of a macro time path and its constituent micro time paths, has powerful heuristic implications for positive analyses and assessments of economic systems at large and the measurement of regularities that may be observed among them. These implications have been discussed under the appropriate headings in chapters II and III.

The basic power and importance of a conceptual framework of the type outlined above is, therefore, that it permits the analyses and
assessments of economic observations as deterministic transformations of time and circumstances, with a stochastic observational residual. This approach allows empirical investigations of economic observations and the systemic activities that are responsible for them, with a view to discover the possible intertemporal regularities that may be observable in their time path parameters. Such demonstrated systemic regularities may then be utilized to investigate the nature of the intertemporal regularities in the parameters of definitionally or functionally related variables.

We noticed in the course of our own work with the ODOE that if observations on sectoral fuel disposition levels in Ohio and the U.S. were plotted against the domain of time over which they were obtained, the time path trajectories displayed certain regularities in that they could all be described in terms of a limited number of time variant linear trends, possibly accompanied by a limited number of temporary deviations from, or a limited number of shifts in, the trends. Furthermore, it also became clear that a preponderance of such shifts in, or temporary deviations from, the trends seemed to occur at a limited number of common points in the interval, 1960-1981. This raised the possibility of specific anomalies in trends being associated with specific events, occurrences, or shocks of nationwide, or worldwide, scope and implications such as the conduct and conclusion of an extensive overseas military engagement, a 100% devaluation in domestic currency, two OPEC price hikes, and the three recessions following the last three incidents. It became quite evident, therefore, that even
a simple quantitative description of the time paths under consideration could become independent and indispensible sources of useful information concerning the historical behavior of annual series of economic observations. Such information, in turn, could be utilized in monitoring the forecast behavior of such observations and in formulating alternative forecast scenarios. The nature and existence of dynamic regularities among a particular data series and the elements of its alternatively designated definitions in terms of accounting or functional identities may also be explored.

The basic task involved in achieving this objective has been the development of a method for modeling and measuring continuous time paths that are differentiable except at a finite number of points within the domain of time. The development of this methodology and its general interpretation was achieved for a single time path trajectory in Chapter II and for the elements of an additive identity in Chapter III. In these chapters, we also provided demonstrated applications of the methodology to the analysis and assessment of commercial natural gas and sectoral electricity sales in Ohio. In both studies, changes in the direction of systemic flows were observed, in varying degrees, after 1972. In the case of industrial electricity sales, the increasing importance of the business cycle on the volume of sales was dramatically underlined and quantified. In chapter III, we also indicated how empirical regularities among definitional relations between income, price, and consumption ought to be formulated and investigated. Although the discussion of dynamic regularities
among income, prices, and consumption was developed within the context of energy disposition and provision, the underlying methodology may be extended to the analysis and assessment of data on any systemic transactions. In particular, it could easily be extended to the analysis and assessment of national income data and its component parts.

The major contribution of this dissertation, in our assessment, is the introduction of a conceptual framework which starts from considerations pertaining to the nature and properties of quantitative observations in general and of quantitative economic observations in particular. This way we can provide a strictly positive basis for the measurement of economic flows and the analyses and assessments of their systemic implications. Considerations of a normative nature need not be a sine qua non of empirical economic investigations, for economic observations are indeed amenable to explanation, and economic magnitudes to measurement by the same concepts and methods that underlie the conduct of inquiry in such nonexperimental positive sciences as astronomy and geodetics.

Many investigators in positive sciences have general agreement as to what the concepts, observations and mathematical models mean. On the other hand, since the investigator himself is one of the "agents" in traditional economic analysis, it is not surprising that the ontological properties that he attributes to the population at large tend to be in conformity with his own image of himself. In fact, it could be observed, especially in the case of ontological
presumptions of a normative kind, that just as the eye cannot perceive itself except in a mirror, the investigator's perception of ontological norms are reflected to him or her in the mirrors of received knowledge, values and faith which are preserved, practiced, and transmitted by the culture and society to which he or she belongs. This observation may also explain why the normative conclusions of both Marxist and Neoclassical camps are both so complete, comprehensive and conclusive, yet diametrically opposite to each other, whereas in the domain of positive investigation what has actually been concluded by either side remains inconclusive and open to question. The utilization of the conceptual framework and the analytical methodology introduced in this thesis, hopefully, would become a first step towards an increased understanding of the behavior of empirical economic systems and a consequent transformation of the domain of empirical economic investigation into a field of positive inquiry.
APPENDIX A

Actual Versus Successive Ten Year Projections
of Service Area Requirements by Major Natural
Gas and Electric Utilities in Ohio

Figures 12 - 30 are based upon Ten Year Forecast Reports of Ohio Electric and Natural Utilities, indicating utility service area requirements as projected by the respective utilities, and filed with the Ohio Department of Energy. Such projections are based either on straightforward econometric demand models, or some definitional decomposition of the projected magnitudes along some end use criteria. In the latter case at least one definitional component is projected on the basis of econometric, statistical methods.

In the case of electric utilities, the forecasts presented include the successive decades 1974-1984 to 1982-92, for each one of the reporting utilities. In the case of natural gas utilities, all reporting utilities have forecasts for the successive decades 1979-89 to 1982-92. Some utilities have forecasts at hand for the successive decades 1974-84 to 1978-88 as well. We have presented all available forecasts of natural gas utilities in figures 12 - 20; the consolidated natural gas forecasts in figure 20 for the state as a whole, however, cover only the successive decades 1979-89 to 1982-92.
Source: Ten Year Forecast of Reporting Utility and Ohio Department of Energy
Source: Ten Year Forecast of Reporting Utility and Ohio Department of Energy
Source: Ten Year Forecast of Reporting Utility and Ohio Department of Energy
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Source: Ten Year Forecast Reporting Utility and Ohio Department of Energy
APPENDIX B

Special Constraints on the Parameters of a Single Time Variant Linear Trend Time Path

A. An Hypothetical Example:

We have already established a methodology for the mathematical modeling of the time paths of systemic annual flow velocities, moving along time variant linear trend trajectories, subject to temporal sequences of forces the existence of which are, either, known to have given rise to corresponding sequences of observed discontinuities in the accelerations in systemic flows; or are inferred from the discontinuities in question. In the latter case, the nature of the forces in question are to be determined as a separate research problem in system analysis.

In chapter III we have extended time variant trend techniques to the analysis and assessment of definitionally related data series. The basic proposition in definitional analysis is that the parameters of the whole must equal to the sum of the respective parameters of its parts. This proposition extends, naturally, to sums of any linear constraints that may have been imposed on specific parameters of individual time paths of the parts in questions, as well.

In general linear constraints imposed on the parameters of a functional model imply an increase in the redundancy of the model.
This is because the introduction of independent constraints on system parameters will reduce $n_0$, the minimum number of independent observations required for unique estimation, which, in general, is equal to the number of unknown parameters in the unconstrained model. For example if $a_1$ and $a_2$ are two unknown parameters in a mathematical model then at least two independent pieces of information are required to solve for them. Hence the redundancy of the model in question is $n - 2$. However, if it is specified that $a_1 + a_2 = c$ where $c$ is a specified constant then it is clear that knowledge of either $a_1$ or $a_2$ will be sufficient to solve for both $a_1$ and $a_2$. Hence in this case $n_0 = 2 - 1 = 1$ and the redundancy of the model is $n - 1$. In fact, if the number of independent restrictions on parameters is equal to $n_0$, then the system can be solved without reference to any observations. This is an established technique of least squares principle in adjustment (1).

In a number of special cases a number of constraints imposed on the parameters of successive system state variables $T_m, T_{(m+1)}$, etc., can be represented by replacing the subsequence of successive system state variables in question by a smaller number of special types of unconstrained variables; without altering the redundancy of the system in question (2).

Although the introduction of special variables does result in some syntactic economy, a clear understanding of the implicit constraints, and the original system state variables for which they stand is necessary, when time path models of individual parts of the same whole are brought together in the analysis of the composition of an
aggregate time path in terms of its individual parts or in the analysis of the decomposition of an aggregate time path into its individual parts.

The type and number of unconstrained special variables representing implicit constraints on subsequences of system state variables as indicated in 2.37.a-c, depends upon the objectives or preferences of the researcher as long as the definition, derivation and implications of the unconstrained new variables in terms of the initial system state variables, and the constraints on them, are, clearly and consistently, established and maintained throughout the analysis.

To provide an introduction to this technique of replacing certain constrained system state variables, with a smaller number of unconstrained variables, we shall consider two cases which occur frequently in sequences of energy disposition or provision data series; namely, temporary deviations of one year duration from long term trends, and displacements of established trends.

B. Deviations from Established Trends of One Year Duration:

Let figure 32 be the plot of a functional model of a time variant linear trend trajectory of a sequence of aggregate annual flow velocities on a specified commodity X. Let figure 31 be the adjusted magnitude of observations at \( t = 1, 2 \ldots 20 \), of the functional model graphed in 32. Let 33 be the graph of \( \frac{d X}{d t} \) at, i.e. of the systemic reactions to dynamic forces generated by alterations in operating conditions, which have taken the form of induced discontinuities in the accelerations of systemic flow velocities over \( Z = < 0, 22 > \).
Figure 31. Observations on the Annual Flows of X
Source: Ten Year Forecast Reporting Utility and Ohio Department of Energy
Figure 32. Time Path of Observations on the Annual Flows of X
Source: Ten Year Forecast of Reporting Utility and Ohio Department of Energy
Figure 33. Time Path of Systemic Accelerations of the Annual Flows of X
Source: Ten Year Forecast of Reporting Utility and Ohio Department of Energy
The convention 17.a-c is duly reflected in figure 32. There are altogether six points at which the time path trajectory of systemic flows becomes nondifferentiable indicating systemic reactions to alterations in dynamic configurations of forces operating on the system in question. The points in question are \( jm = 1, 4, 5, 6, 9, 10; \) and \( m = 1, 2, \ldots, 6. \)

The inclusion of \( j_1 = 1 \) among the points of discontinuity in systemic accelerations is due to the aforementioned convention adopted in 17.a-c. The trend established between \( j_1 \) and \( j_2 \) could, in effect, be interpreted to reflect the initial dynamic configuration of forces which determine the motion of the systemic flow velocity in question; whereas \( X_{a_1} \) reflects the initial level of the systemic flow velocity at \( t = 1. \)

The equation of motion of the trajectory in figure 32 is then given by a functional model of the type

\[
B.1) \quad X_{at} = X_a^1 + \sum_{m=1}^{6} a_m \cdot T_m
\]

where \( a_m \) are equal to the magnitude of the discontinuities indicated in figure 33.

We have observed in the course of our analyses of energy data for the U.S. and the State of Ohio that certain temporary events, which may take place within a specific calendar year, alter the behavior of systemic flows during the specific calendar year within which they take place, and within the two years immediately after they take place, in such a manner that observations on the magnitude of systemic flows for
the annual periods immediately preceding the calendar year in question and immediately succeeding the first calendar year after the calendar year in question are described by the same system state variables. Stated from a different perspective certain sequences of observations may indicate that their trajectory can be expressed, over a given subdomain of T, in terms of one underlying long term trend, and one or more temporary deviations from such an established long term trend, or one or more shifts in the position of the long term trend in question.

For example, in figure 32 the underlying long term trend between $1 < t \leq 22$ may be considered to be $a_1$. A temporary deviation of duration one year is observed at $t = 5$. The existence of such a behavior implies that if $a_2$ is the magnitude of the deviation in question then the equation of motion of the system is defined as

\begin{align*}
B.2.a) \quad & x_{at} = x_{a1} + a_1 T_1 \quad \text{if} \quad j_1 = 1 < t \leq 4 \\
B.2.b) \quad & x_{at} = x_{a1} + a_1 T_1 + a_2 T_2 \quad \text{if} \quad j_2 = 4 < t \leq 5 \\
B.2.c) \quad & x_{at} = x_{a1} + a_1 T_1 + a_2 T_2 + a_3 T_3 \quad \text{if} \quad j_3 = 5 < t \leq 6 \\
B.2.d) \quad & x_{at} = x_{a1} + a_1 T_1 + a_2 T_2 + a_3 T_3 + a_4 T_4 \quad \text{if} \quad j_4 = 6 < t \leq 9.
\end{align*}

Hence
In general, from 2.32.b,

\[ T_m = DUM_m \cdot (t - j_m) \]

Therefore

\[ T_2 = DUM_2 \cdot (t - j_2) \quad \text{if} \quad 0 \leq t \leq 22 \]
\[ T_3 = DUM_3 \cdot (t - j_3) \quad \text{if} \quad 0 \leq t \leq 22 \]
\[ T_4 = DUM_4 \cdot (t - j_4) \quad \text{if} \quad 0 \leq t \leq 22 \]

and

\[ T_2 = t - j_2 \quad \text{if} \quad 6 < t \leq 22 \]
\[ T_3 = t - j_3 \quad \text{if} \quad 6 < t \leq 22 \]
\[ T_4 = t - j_4 \quad \text{if} \quad 6 < t \leq 22 \]

Substituting 2.4.b into 2.3 we get

\[ a_2 (t - j_2) + a_3 (t - j_3) + a_4 (t - j_4) = 0 \]

\[ \text{if } j_4 = 6 < t \leq 22 \]
or

B.6) \((a_2 + a_3 + a_4) t - [a_2 j_2 + a_3 j_3 + a_4 j_4 ] = 0\)

if \(j_4 = 6 < t < 22\).

By construction

B.7) \(j_2 = j_3 + 1 = j_3 + 2\)

if \(j_4 = 6 < t < 22\)

Substituting into B.6 we get

B.8.a) \([ a_2 + a_3 + a_4 ] t - [ a_2 j_2 + a_3 [j_2-1] + a_4 [j_2-2] ] = 0\)

if \(j_4 = 6 < t < 22\)

or

B.8.b) \([ a_2 + a_3 + a_4 ] t - [ [a_2 + a_3 + a_4] j_2 - [a_3 + 2a_4] ]\)

\(= [ a_2 + a_3 + a_4 ] [ t - j_2 ] + [ a_3 + 2a_4 ] = 0\)

if \(j_4 = 6 < t < 22\).

Obviously the above equality will be true if and only if

\([ a_2 + a_3 + a_4 ] = 0\) and \(a_3 + 2a_4 = 0\)

hence
B.9.a) \[ a_3 = -2a_4, \]
and
B.9.b) \[ a_2 - 2a_4 + a_4 = 0 \]
or
B.10.a) \[ a_4 = a_2 \]
and
B.10.b) \[ a_3 = -2a_2. \]

Thus a temporal deviation of duration one year, from an established annual trend will be characterized by three consecutive discontinuities in the systemic accelerations of magnitude \( a_2, -2a_2 \) and respectively.

For purposes of estimation, however, we are dealing with discrete observations, where the observations on \( T_m \) take the form

B.11) \[ T_m = DUMm \cdot (i - jm) \]

so that

B.12.a) \[ Z_{(m+1)} = 0 \quad \text{if} \quad t < jm \]

B.12.b) \[ Z_{(m+1)} = (i - jm) \quad \text{if} \quad t > jm. \]

Hence, in the example discussed above where
B.13) \[ a_2 \cdot T_2 - 2a_2 \cdot T_3 + a_2 \cdot T_4 = a_2 \cdot 0 - 2a_2 \cdot 0 + a_2 \cdot 0 = 0 \]
if \( 1 < i \leq j_1 \)
\[ a_2 \cdot T_2 - 2a_2 \cdot T_3 + a_2 \cdot T_4 = a_2 \cdot (1) - 2a_2 \cdot 0 + a_2 \cdot 0 = a_2 \]
if \( j_1 < i \leq j_2 \).

The corresponding columns of the observation matrix \( Z \) of the system
state variables satisfy

B.14.a) \[ a_2 \cdot Z.3 - 2a_2 \cdot Z.4 + a_2 \cdot Z.5 = \]
\[ a_2 \cdot (0) - 2a_2 \cdot 0 + a_2 \cdot 0 = 0 \]
if \( 1 < i \leq j_1 \)

B.14.b) \[ a_2 \cdot Z.3 - 2a_2 \cdot Z.4 + a_2 \cdot Z.5 = \]
\[ a_2 \cdot (1) - 2a_2 \cdot 0 + a_2 \cdot 0 = a_2 \]
if \( j_1 < i \leq j_2 \)

B.14.c) \[ a_2 \cdot Z.3 - 2a_2 \cdot Z.4 + a_2 \cdot Z.5 = \]
\[ a_2 \cdot (2) - 2a_2 \cdot (1) + a_2 \cdot 0 = 0 \]
if \( j_2 < i \leq j_3 \)

and from B.12.b.

B.14.d) \[ a_2 \cdot Z.3 - 2a_2 \cdot Z.4 + a_2 \cdot Z.5 = 0 \]
if \( j_3 < i \leq j_4 \).
Hence we can replace the three system state variables $T_2 T_3 T_4$, and the specified restrictions on their parameters by a single unconstrained variable $D_{j2}$ such that

$$B.15.a) \quad D_{j2} = 1 \quad \text{if} \quad j_2 < t \leq j_3$$

and

$$B.15.b) \quad D_{j2} = 0 \quad \text{otherwise}$$

and similarly in the observation matrix we can replace $Z_{3,3}$, $Z_{4,5}$, and $Z_{5,5}$ by $\tilde{Z}_{3,3}$ where $\tilde{Z}$ is the observation matrix obtained after the above mentioned substitution, and where

$$B.16) \quad \tilde{Z}_{3,3} = 1 \quad \text{if} \quad i = j_3$$

$$\tilde{Z}_{3,3} = 0 \quad \text{otherwise}.$$ 

The introduction of $D_{j2}$ and $\tilde{Z}_{3,3}$ is tantamount to the representation of an annual deviation from established long term trends in terms of successive discontinuation in the time path of the flow velocity of the system, instead of successive nondifferentiabilities. Figures 34 and 35 are provided for a comparison of the two techniques of representation. Since both time path models will estimate the same parameters, the use of one or the other in parameter estimation is a matter of preference.
Figure 34. Representation of the Time Path of the Annual Flows of Discontinuity and Points of Non-differentiability.
Source: Ten Year Forecast of Reporting Utility and Ohio Department of Energy
Figure 35. Representation of the Annual Flows of X with Points of Discontinuity and Points of Non-differentiability.

Source: Ten Year Forecast of Reporting Utility and Ohio Department of Energy
The reason why the two models are observationally equivalent, is because, the only difference in the respective time paths described by the two different functional models in question occur in the interval \( j_2 < t < j_3 \) and \( j_3 < t < j_4 \). Both of these subintervals are of one period of observation in duration. Hence, the implication of observational equivalence simply boils down to the fact that differences in mathematical descriptions of a time path, pertaining to its behavior within an annual period, cannot be resolved on the basis of annual observations. That is, the description of the behavior of flows within an annual period requires observations of period less than one year; otherwise, we cannot choose between two alternative descriptions of the same time path trajectory which differ from each other only with respect to the characterization of the trajectory under consideration within a given annual period of observation.

C. Displacements of Established Trends for One Year Duration

In view of the preceding discussion the behavior of the trajectory in figure 32 and figure 34 is described by

\[
\begin{align*}
X_t & = X_{a1} + A_1 T1 + a_2 T2 + a_5 T5 + a_6 T6. \\
\end{align*}
\]

From the acceleration function plotted in figure 33, it is clear that

\[
\begin{align*}
a_5 + a_6 &= 0. \\
\end{align*}
\]
Hence:

\[ X_{at} = x_{a1} + a_1 T1 + a_2 DJ2 + a_5 (T5 - T6). \]

Observationally

\[ T5 - T6 = 0 \quad \text{if} \quad t \leq 9 \]
\[ T5 - T6 = 1 \quad \text{if} \quad t > 9. \]

We may therefore replace \( T5 - T6 \) for purposes of modeling and estimation with a binary variable, say, \( DUMj5 \) such that

\[ DUMj5 = 0 \quad \text{if} \quad t \leq 9 \]
\[ DUMj5 = 1 \quad \text{if} \quad t > 9. \]

The final form of the functional model becomes

\[ X_{at} = x_{a1} + a_1 T1 + a_2 DJ2 + a_5 DUMj5. \]

Thus we may express the trajectory under consideration in terms of a seven variable model as in B.1, subject to three independent parameter constraints, i.e. B.9.a-b and B.18. We may alternatively express the same trajectory in terms of a four variable model with no constraints as in B.22. The latter case is a lot easier to deal with in adjustment.
APPENDIX C

Appendix to Chapter III: Modeling and Adjustment of Ohio Sectoral Electricity Disposition System

A. Data Definitions

Sectoral sales data are from Edison Electric Institute's Statistical Year Book of the Electric Utilities Industry 1960-1981.

The following sales categories are reported in the Year Book:

- Residential
- Large Light and Power
- Small Light and Power
- Street and Highway Lighting
- Railroad and Railways
- Other Public Authorities
- Interdepartmental
- Total

The units are in gigawatt hours per year.

The variables analyzed in chapter III are related to the magnitudes reported by EEI through the following scheme which has been adapted from Federal Energy Data System Technical Documentation, June 1978 of U.S. Department of Energy.

\[ ELRSBOH = 3.412 \ (TBtu/BKWH) \times \text{Residential} \times 10^{-3} \]
$EL_{CMBOH} = 3.412 \text{(TBtu/BKWH)} \times [\text{small light and power and interdepartmental street and highway lighting and other public authorities}] \times 10^{-3}$

$EL_{INBOH} = 3.412 \text{(TBtu/BKWH)} \times \text{large light and power} \times 10^{-3}$

$EL_{TRBOH} = 3.412 \text{(TBtu/BKWH)} \times \text{railroad and railways} \times 10^{-3}$

$EL_{TTBOH} = EL_{RSBOH} + EL_{CMBOH} + EL_{INBOH} + EL_{TRBOH}$

The multiplication by $10^{-3}$ at the end is to transform gigawatt-hours into billion kilowatt-hours.

B. **SAS Matrix Program Utilized in the Simultaneous Adjustment of Sectoral Electricity Sales**

The SAS matrix program which we developed for the simultaneous adjustment problem discussed in chapter III is presented in the following computer printout pages.
PROC MATRIX;
TITLE;
FETCH SSVEL DATA=NEW1
(KEEP=INT T6 DUM6 T12 DUM14 T20)
COLNAME=CN
RN='1960' '1961' '1962' '1963' '1964' '1965' '1966'
LRNSSSRS=CN(,1 2 3 5 8);
LRNSSVCM=CN(,1 2 3 5);
LRNSSVIN=CN(,1 2 3 4 6 7);
LRNSSVTR=CN(,1 2 3);
PRINT SSVEL COLNAME=CN ROWNAME=RN;
FETCH ELSEC DATA=NEW1
(KEEP=ELR530H ELMBOH ELINBOH ELTRBOH ELTTBOH)
COLNAME=C;
PRINT ELSEC COLNAME=C ROWNAME=RN;
SELRS=SSVEL(,1 2 3 5 8);
DFELRS=NROW(SELRS)-NCOL(SELRS);
SELCM=SSVEL(,1 2 3 5);
DFELCM=NROW(SELCM)-NCOL(SELCM);
SELIN=SSVEL(,1 2 4 5 6 7);
DFELIN=NROW(SELIN)-NCOL(SELIN);
SELTR=SSVEL(,1 2 3 4 6 7 8);
DFELTR=NROW(SELTR)-NCOL(SELTR);
SELTT=SSVEL(,1 2 3 4 6 7 8);
Y1=ELSEC';
Y=SHAPEC(Y1,1);
Z=BLOCK(SELRS,SELCM,SELIN,SELTR,SELT);;
N=I(22);
R2I=I(25);
R1=J(9,25,0);
SYSINT: R1(1,)=R2I(1)+R2I(6)+R2I(10)+R2I(16)-R2I(19);
SYST: R1(2,)=R2I(2)+R2I(7)+R2I(11)+R2I(17)-R2I(20);
SYST6: R1(3,)=R2I(3)+R2I(8)+R2I(18)-R2I(21);
SDUM6 : R1(4,)=R2I(12)-R2I(22);
SYST12: R1(5,)=R2I(4)+R2I(9)+R2I(13)-R2I(16);
SYST25: R1(6,)=R2I(14)-R2I(23);
SDUM14: R1(7,)=R2I(15)-R2I(24);
SYST20: R1(8,)=R2I(5)+R2I(10)-R2I(25);
TRT6 : R1(9,)=R2I(17)+R2I(18);
PRINT R1;
VAR=1.0321 1.8611 24.9343 .0805;
VAR1=DIAG(VAR);
VAR2=VAR1/ VAR;
VAR3=(VAR1[27,3230]);
COV=VAR2[VAR3;
Q=(9,1,0);
GINV=GINV(COV);
PRINT COV GINV COLNAME=C ROWNAME=C;
B11=GINV(Z'*CGINVW*N)*Z);
B21=GINV(R1*B11*R1');
B31=B11W'*GINVW*N-Y;
BETA=B31+(B11*R1*B21*(Q-R1*B31));
PY1=Z*BETA;
V1=PY1-Y;
S1=SSQ(V1);
DF=NRW(Y)+NRW(R2I)-NCOL(R2I);
MS1=S1/DF;
R21=1-(S1/SQ(Y));
E1=SHAPE(V1,NCOL(N));
E1W=E1'*E1/NCOL(N);;
CVBETA=B11-E1W*B11*E1R1*B11;
PRINT E1W COLNAME=C ROWNAME=C;
OUTPUT E1W COLNAME=C ROWNAME=C OUT=E1W1;
S1=VECDIAG(CVBETA)**.5;
ST1=BETA*/S1;
SYSPARAM=BETA|S1|ST1;
ELRSBH=Y(1:NCOL(N));
RST1=BETA(1:NCOL(SELRS));
RSS1=S1(1:NCOL(SELRS));;
ST1=ST1(1:NCOL(SELRS));;
PSELRS=PY1(1:NCOL(N));;
VELRS=V1(1:NCOL(N));;
ELCMBOH=Y(1:NCOL(N):2*NCOL(N));
CMBETA=BETA(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN));
CMS1=S1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN));
CMS1=ST1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN));;
PSELCM=PY1(1:NCOL(N):2*NCOL(N));;
VELCM=V1(1:2*NCOL(N):3*NCOL(N));;
ELINBH=Y(1:2*NCOL(N):3*NCOL(N));;
INBETA=BETA(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTR));
IN1=S1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTR));;
INST1=ST1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTR));;
PELIN=PY1(1:2*NCOL(N):3*NCOL(N));;
VELIN=V1(1:2*NCOL(N):3*NCOL(N));;
ELTRBH=Y(1:2*NCOL(N):4*NCOL(N));
TRBETA=BETA(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTR)+NCOL(SELTR));
TR1=S1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTR)+NCOL(SELTR));;
TRST1=ST1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTR)+NCOL(SELTR));;
PELTR=PY1(1:2*NCOL(N):4*NCOL(N));;
VELTR=V1(1:2*NCOL(N):4*NCOL(N));;
ELTBOH=Y(1:2*NCOL(N):5*NCOL(N));
TTBETA=BETA(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTR)+NCOL(SELTR)+NCOL(SELTT));
TT1=S1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTT));
TTST1=ST1(1:NCOL(SELRS)+NCOL(SELCM)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELIN)+NCOL(SELTT));;
PELTTO=PY1(1:4*NCOL(N):5*NCOL(N));
VELTTOH1=V1(1+4*NCOL(N):5*NCOL(N));
RSR21=1-<SSQ(VELRSOH1)#/SSQ(ELRSB01));
CMR21=1-<SSQ(ELCMOH1)#/SSQ(ELCMOH1));
INR21=1-<SSQ(VELINOH1)#/SSQ(ELINOH1));
TRR21=1-<SSQ(VELTROH1)#/SSQ(ELTROH1));
TT21=1-<SSQ(VELTTOH1)#/SSQ(ELTTB01));
RSQ1=RSR21|CMR21|INR21|TRR21|TT21;
RSB0H1=VELRSOH1|ELRSB0H1|VELRSOH1;
CMB0H1=ELCMOH1|ELCMOH1|ELCMOH1;
INBOH1=ELINOH1|ELINOH1|ELINOH1;
TRBOH1=ELTROH1|ELTROH1|ELTROH1;
TTB0H1=VELTTOH1|ELTTB0H1|VELTTOH1;
DATAMAT=RN*|RSB0H1|CMB0H1|INBOH1|TRBOH1|TTB0H1;
ELRS=R2ETA|RSS1|R SST1;
ELCM=CMBETA|CMS1|CMST1;
ELIN=INBETA|INS1|INST1;
ELT=TRBETA|TRS1|TRST1;
ELTT=TTBETA|TTS1|TTST1;
MSERS=SSQ(VELRSOH1)/DFELRS;
MSEC=SSQ(ELCMOH1)/DFELCM;
MSETN=SSQ(ELINOH1)/DFELIN;
MSETR=SSQ(ELTROH1)/DFELTR;
MSETT=SSQ(VELTTOH1)/DFELT;
MSBSEC=MSESEC|MSEC|MSETN|MSETR|MSETT;
L2='ADJUSTED'='ACTUAL'='RESIDUAL';
L1='ESTIMATE'='ERROR'='T RATIO';
L3='SYSTEM';
L4='ELRS.INT'='ELRS.T'='ELRS.T6'='ELRS.TJ2'='ELRS.T20'='ELCM.INT'
'ELCM.T'='ELCM.T6'='ELCM.TJ2'='ELCM.INT'='ELIN.T'='ELIN.TJ2'='ELIN.M6'
'ELIN.M14'='ELIN.D75'='ELTR.INT'='ELTR.T'='ELTR.T6'='ELTT.INT'='ELTT.T'
'ELTT.T6'='ELTT.T20'='ELTT.M6'='ELTT.M14'='ELTT.D75';
NOTE PAGE TABLE C.1;
NOTE SKI P=2 OBSERVATIONS ON SECTORAL ELECTRICITY SALES IN OHIO 1960-1981;
NOTE SKI P=2 (T3TU/YEAR);
PRINT ELSEC COLNAME=C ROWNAME=RN;
NOTE PAGE TABLE C.2;
NOTE SKI P=2 OBSERVATIONS ON SYSTEM STATE VARIABLES 1960-1981;
PRINT SSVEL COLNAME=CN ROWNAME=RN;
NOTE PAGE TABLE C.4;
NOTE SKI P=2 A PRIORI COVARIANCE MATRIX OF RESIDUALS;
PRINT COV COLNAME=C ROWNAME=C;
NOTE SKI P=4 GENERALIZED INVERSE OF COV;
PRINT GINV COLNAME=C ROWNAME=C;
NOTE SKI P=4 SAMPLE COVARIANCE MATRIX OF RESIDUALS;
PRINT ELWI COLNAME=C ROWNAME=C;
NOTE PAGE TABLE C.5;
NOTE SKI P=2 CONSTRAINT MATRIX RI;
PRINT RI;
NOTE PAGE TABLE 3.1;
NOTE SKI P=2 PARAMETER ESTIMATES FOR ALL TIME PATHS;
PRINT SYSPARM COLNAME=L1 ROWNAME=L4;
NOTE PAGE TABLE 3.2;
NOTE SKI P=4 R-SQUARE AND MEAN SQUARE ERROR FOR THE SYSTEM AS A WHOLE;
PRINT R21 COLNAME=L5;
PRINT MSS1 COLNAME=L5;
NOTE SKI P=4 R-SQUARES AND MEAN SQUARE ERRORS FOR INDIVIDUAL TIME PATHS;
PRINT RSS1 COLNAME=C;
PRINT MSBSEC COLNAME=C;
NOTE PAGE TABLE 4.A;
NOTE SKIP=2 PARAMETER ESTIMATES: OHIO RESIDENTIAL ELECTRICITY SALES;
PRINT ELRS COLNAME=L1 ROWNAME=LRNSSVRS;
NOTE PAGE TABLE 4.B;
NOTE SKIP=2 OHIO RESIDENTIAL ELECTRICITY SALES 1960-1981;
NOTE SKIP=2 ADJUSTED AND ACTUAL OBSERVATIONS AND RESIDUALS (TBTU/YEAR);
PRINT RS0H1 COLNAME=L2 ROWNAME=RN;
NOTE PAGE TABLE 5.A;
NOTE SKIP=2 PARAMETER ESTIMATES: OHIO COMMERCIAL ELECTRICITY SALES;
PRINT ELCM COLNAME=L1 ROWNAME=LRNSSVCVM;
NOTE PAGE TABLE 5.B;
NOTE SKIP=2 OHIO COMMERCIAL ELECTRICITY SALES 1960-1981;
NOTE SKIP=2 ADJUSTED AND ACTUAL OBSERVATIONS AND RESIDUALS (TBTU/YEAR);
PRINT CM0H1 COLNAME=L2 ROWNAME=RN;
NOTE PAGE TABLE 6.A;
NOTE SKIP=2 PARAMETER ESTIMATES: OHIO INDUSTRIAL ELECTRICITY SALES;
PRINT ELIN COLNAME=L1 ROWNAME=LRNSSVIN;
NOTE PAGE TABLE 6.B;
NOTE SKIP=2 OHIO INDUSTRIAL ELECTRICITY SALES 1960-1981;
NOTE SKIP=2 ADJUSTED AND ACTUAL OBSERVATIONS AND RESIDUALS (TBTU/YEAR);
PRINT IN0H1 COLNAME=L2 ROWNAME=RN;
NOTE PAGE TABLE 7.A;
NOTE SKIP=2 PARAMETER ESTIMATES: OHIO TRANSPORTATION ELECTRICITY SALES;
PRINT ELTR COLNAME=L1 ROWNAME=LRNSSVTR;
NOTE PAGE TABLE 7.B;
NOTE SKIP=2 OHIO TRANSPORTATION ELECTRICITY SALES 1960-1981;
NOTE SKIP=2 ADJUSTED AND ACTUAL OBSERVATIONS AND RESIDUALS (TBTU/YEAR);
PRINT TR0H1 COLNAME=L2 ROWNAME=RN;
NOTE PAGE TABLE 8.A;
NOTE SKIP=2 PARAMETER ESTIMATES: OHIO TOTAL ELECTRICITY SALES;
PRINT ELTT COLNAME=L1 ROWNAME=LRNSSVTT;
NOTE PAGE TABLE 8.B;
NOTE SKIP=2 OHIO TOTAL ELECTRICITY SALES 1960-1981;
NOTE SKIP=2 ADJUSTED AND ACTUAL OBSERVATIONS AND RESIDUALS (TBTU/YEAR);
PRINT TT0H1 COLNAME=L2 ROWNAME=RN;
C. Tables 18 Through 22

Data matrices utilized as inputs to the SAS matrix program described in the preceding section are presented in tables 18 through 22. All data are internally generated by the Ohio Department of Energy. The source and generation of the data in table 18 is discussed in section A of appendix C.
Table 18. Observations on Sectoral Electricity Sales in Ohio (TWh/Year)

<table>
<thead>
<tr>
<th>ELSEC</th>
<th>ELSBDOH</th>
<th>ELCHEBHOH</th>
<th>ELMDBOH</th>
<th>ELTRBDOH</th>
<th>ELTRBDOH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>35.484</td>
<td>24.259</td>
<td>135.286</td>
<td>0.37532</td>
<td>195.405</td>
</tr>
<tr>
<td>1961</td>
<td>37.737</td>
<td>26.545</td>
<td>135.252</td>
<td>0.3412</td>
<td>199.875</td>
</tr>
<tr>
<td>1962</td>
<td>39.018</td>
<td>27.603</td>
<td>141.427</td>
<td>0.27296</td>
<td>209.121</td>
</tr>
<tr>
<td>1963</td>
<td>41.285</td>
<td>29.582</td>
<td>147.603</td>
<td>0.20472</td>
<td>218.675</td>
</tr>
<tr>
<td>1964</td>
<td>44.219</td>
<td>31.186</td>
<td>149.821</td>
<td>0.1706</td>
<td>225.397</td>
</tr>
<tr>
<td>1965</td>
<td>47.656</td>
<td>36.508</td>
<td>143.986</td>
<td>0.1706</td>
<td>228.331</td>
</tr>
<tr>
<td>1966</td>
<td>51.282</td>
<td>39.647</td>
<td>141.666</td>
<td>0.13648</td>
<td>232.698</td>
</tr>
<tr>
<td>1967</td>
<td>54.899</td>
<td>42.240</td>
<td>146.852</td>
<td>0.13648</td>
<td>244.128</td>
</tr>
<tr>
<td>1968</td>
<td>60.083</td>
<td>46.062</td>
<td>155.689</td>
<td>0.1706</td>
<td>262.007</td>
</tr>
<tr>
<td>1969</td>
<td>66.295</td>
<td>50.224</td>
<td>161.865</td>
<td>0.1706</td>
<td>278.555</td>
</tr>
<tr>
<td>1970</td>
<td>72.232</td>
<td>58.720</td>
<td>157.088</td>
<td>0.1706</td>
<td>288.211</td>
</tr>
<tr>
<td>1971</td>
<td>76.717</td>
<td>62.576</td>
<td>161.729</td>
<td>0.1706</td>
<td>301.177</td>
</tr>
<tr>
<td>1972</td>
<td>81.691</td>
<td>67.114</td>
<td>181.655</td>
<td>0.13648</td>
<td>330.554</td>
</tr>
<tr>
<td>1973</td>
<td>88.814</td>
<td>72.880</td>
<td>201.479</td>
<td>0.13648</td>
<td>363.31</td>
</tr>
<tr>
<td>1974</td>
<td>91.188</td>
<td>72.982</td>
<td>196.395</td>
<td>0.13648</td>
<td>360.682</td>
</tr>
<tr>
<td>1975</td>
<td>95.694</td>
<td>76.360</td>
<td>184.487</td>
<td>0.13648</td>
<td>356.588</td>
</tr>
<tr>
<td>1976</td>
<td>98.777</td>
<td>78.017</td>
<td>207.142</td>
<td>0.13648</td>
<td>384.873</td>
</tr>
<tr>
<td>1977</td>
<td>105.874</td>
<td>82.704</td>
<td>223.076</td>
<td>0.13648</td>
<td>411.794</td>
</tr>
<tr>
<td>1978</td>
<td>108.331</td>
<td>82.160</td>
<td>218.436</td>
<td>0.1706</td>
<td>409.099</td>
</tr>
<tr>
<td>1979</td>
<td>110.89</td>
<td>85.3</td>
<td>222.26</td>
<td>0.17</td>
<td>418.62</td>
</tr>
<tr>
<td>1980</td>
<td>115.45</td>
<td>88.38</td>
<td>185.14</td>
<td>0.14</td>
<td>389.11</td>
</tr>
<tr>
<td>1981</td>
<td>114.03</td>
<td>90.85</td>
<td>185.93</td>
<td>0.16</td>
<td>390.97</td>
</tr>
</tbody>
</table>
Table 19. Observations on the System State Variables in the Time Path Models of Sectoral Electricity Sales in Ohio

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<th>Row</th>
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<th>COL2</th>
<th>COL3</th>
<th>COL4</th>
<th>COL5</th>
<th>COL6</th>
<th>COL7</th>
<th>COL8</th>
<th>COL9</th>
<th>COL10</th>
<th>COL11</th>
<th>COL12</th>
<th>COL13</th>
<th>COL14</th>
<th>COL15</th>
<th>COL16</th>
<th>COL17</th>
<th>COL18</th>
<th>COL19</th>
</tr>
</thead>
<tbody>
<tr>
<td>R001</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>R003</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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Table 20. OLS Adjustment Results for Component Sectoral Electricity Sales Time Path Models

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| VARIABLE | DF | PARAMETER ESTIMATE | STANDARD ERROR | T RATIO | PROB>|T| |
|-----------|----|---------------------|----------------|---------|-------|
| INTERCEPT | 1  | 34.985322 | 0.672453 | 52.8939 | 0.0001 |
| T | 1  | 3.643817 | 0.143672 | 25.9248 | 0.0001 |
| T6 | 1  | 3.867314 | 0.273999 | 11.1543 | 0.0001 |
| T12 | 1  | -1.343302 | 0.228730 | -5.8672 | 0.0001 |
| T20 | 1  | -8.247196 | 1.238133 | -6.6454 | 0.0001 |

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| VARIABLE | DF | PARAMETER ESTIMATE | STANDARD ERROR | T RATIO | PROB>|T| |
|-----------|----|---------------------|----------------|---------|-------|
| INTERCEPT | 1  | 33.675772 | 0.663119 | 51.2259 | 0.0001 |
| T | 1  | 2.359043 | 0.221627 | 10.6382 | 0.0001 |
| T6 | 1  | 2.397694 | 0.343943 | 7.0330 | 0.0001 |
| T12 | 1  | -2.417529 | 0.225062 | -4.1862 | 0.0001 |

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| VARIABLE | DF | PARAMETER ESTIMATE | STANDARD ERROR | T RATIO | PROB>|T| |
|-----------|----|---------------------|----------------|---------|-------|
| INTERCEPT | 1  | 32.682273 | 2.711333 | 49.327 | 0.0001 |
| T | 1  | 2.306385 | 0.671587 | 9.0237 | 0.0001 |
| T12 | 1  | 3.223974 | 0.874636 | 3.6339 | 0.0033 |
| DUM6 | 1  | 24.977825 | 4.332194 | 5.7342 | 0.0021 |
| DUM14 | 1  | -47.972837 | 4.813314 | -9.8944 | 0.0021 |
| DUM5 | 1  | -19.473874 | 5.156233 | -3.7602 | 0.0001 |

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| VARIABLE | DF | PARAMETER ESTIMATE | STANDARD ERROR | T RATIO | PROB>|T| |
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| INTERCEPT | 1  | 0.339574 | 0.314627 | 24.9937 | 0.0001 |
| T | 1  | -0.331895 | 0.20261906 | -12.9418 | 0.0001 |
| T6 | 1  | 1.532893 | 0.02261206 | 12.9418 | 0.0001 |
| RESTRICTION | -1  | 0.435856 | 0.443229 | -0.9928 | 0.3226 |
Table 21. A Priori Covariance Matrix of Systemic Residuals = "COV", Generalized Inverse of "COV" = GINV, and Sample Covariance Matrix of Residuals After Adjustment = "CI.WL"

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Table 22. Constraint Matrix R1

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<td>1</td>
<td>19</td>
<td>13</td>
<td>1</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1980</td>
<td>1</td>
<td>20</td>
<td>14</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1981</td>
<td>1</td>
<td>21</td>
<td>15</td>
<td>1</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
D. **Figure 36: Energy Consumption from Coal and Petroleum Products in Ohio Industrial Sector, 1960-1981**

Figure 36 is presented on the following page.
Figure 36. Energy Consumption from Coal and Petroleum Products in Ohio Industrial Sector, 1960-1981

Source: Ohio Department of Energy
APPENDIX D

Appendix to Chapter III: Measurement of Income Consumption, Price Consumption and Elasticity of Consumption Concepts: A Special Case

If the time paths of $X(t)$ and $R_x(t)$ are, both, characterized by time variant linear trends then, from 3.32.a-b,

\[ D.1.a) \quad \frac{d}{dt} X(t) = \sum_{m_x=1}^{l_x} b_{m_x} \cdot \text{DUM}_m \]

\[ D.1.b) \quad \frac{d}{dt} R_x(t) = \sum_{m_r=1}^{l_r} c_{m_r} \cdot \text{DUM}_r \]

so that, by necessity

\[ D.2.a) \quad P_x(t) = a_0 + \sum_{m_x=1}^{l_x} a_{m_x} \cdot \text{DUM}_{m_x} \cdot X(t) \]

and

\[ D.2.b) \quad \frac{d}{dt} P_x(t) = 0 \]

except at a finite number of points $t = t_{xr}$, determined by the set

\[ D.3) \quad \{ \text{DUM}_{m_{xr}} \} = \{ \text{DUM}_{m_x} \} \cup \{ \text{DUM}_{m_r} \} \]
such that

D.4.a) \[ \text{DUM}_{x r} = 0 \text{ if } t \leq t_{x r} \]
\[ \text{DUM}_{x r} = 1 \text{ if } t > t_{x r}. \]

Furthermore, it should also be clear that,

D.4.b) \[ a_0 = \frac{c_0}{b_0}, \quad a_{x r} = \frac{c_{m_{r x}}}{b_{m_{x}}}. \]

Substituting D.2.a into 4.39 we can express the relation between revenues and volume of sales as

D.5) \[ R_{x}(t) = a_0 X(t) + \sum_{m_{r x}=1}^{\sigma_{r x}} a_{m_{r x}} \cdot X_{m_{r x}}, \]
where

D.6) \[ X_{m_{r x}} = \text{DUM}_{m_{r x}} \cdot X(t). \]

In the case of time variant linear trend trajectories, then, the relationship between \( R_{x}(t) \) and \( X(t) \) may be directly estimated from an adjustment of observations on \( R_{x}(t) \) as a linear transformation of \( X(t), X_{m_{r x}} \).
NOTES

A. Notes to Introduction

(1) The claims of economic analysis both to positive and to normative authority are widely known. The neoclassical version of this claim is explicitly stated in the following textbook quotations:

"So, in so far as it is a positive, that is, explanatory science, economics must analyse the behaviour of agents who enjoy some freedom but are subject to the constraints imposed on them by nature and institutions. It must investigate the consequences of such individual behaviour for the state of affairs which is realised in the community.

"In so far as it is a normative science, economics must also investigate the best way of organising production, distribution and consumption. It must give the conceptual tools which enable us to assess the comparative advantages of different forms of organisation.

"In its pursuit of this double activity, positive and normative, our science has come to attribute a central role to the prices which regulate the exchange of goods among agents. For the individual, these prices reflect more or less exactly the social scarcity of the products which he buys and sells. This is why the study of the price system is just as important as the study of production and consumption.

"In this general conceptual context, there are two types of objective for microeconomic theory. In the first place, it must describe the activity of agents, that is, it must provide models which explain in abstract terms how each consumer i determines \( x_i \) and how each producer j determines \( y_j \), and it must also describe how all the \( x_i \) and all the \( y_j \), and possibly also prices \( p_h \), are simultaneously determined. (It must therefore place
itself at the level of the individual agent in a partial perspective as well as at the level of the whole economy). This is the objective of equilibrium theory, first partial, then general equilibrium.

"In the second place, it must look for an optimal organisation of production, consumption and exchange, and then study the properties of a state of the economy in which this optimal organisation is realised. This is the objective of optimum theory, also called welfare theory. Malinvaud, E., Lectures on Microeconomic Theory, North-Holland Publishing Company, Amsterdam-London, 1973, pg. 2, 5.


(2) The empirical irrelevance of econometric models, and their unreliability as explanatory or predictive tools are well documented in the literature. The following are some of the better known comments in this regard. Although the august company mentioned below all have their own differences as to why econometric analysis fails, they all agree that it has generally been a failure so far. There is no indication that things have improved appreciably since these comments were made.

"The achievements of economic theory in the last two decades are both impressive and in many ways beautiful. But it cannot be denied that there is something scandalous in the spectacle of so many people refining the analyses of economic states which they give no reason to suppose will ever, or have ever, come about." Hahn, F.H., "Some Adjustment Problems" Econometrica, January 1970, (presidential address delivered at the December 1968 meeting of the Econometric Society).

"Far be it from me to argue that the fire should not be shifted when the target moves. The trouble is caused, however, not by any inadequate selection of targets, but rather by our inability to hit squarely any one of them. The uneasiness of which I spoke
before is caused not by the irrelevance of the practical problems to which present day economists address their efforts, but rather by the palpable inadequacy of the scientific means with which they try to solve them. . . . .

"Uncritical enthusiasm for mathematical formulation tends often to conceal the ephemeral substantive content of the argument behind the formidable front of algebraic signs. . . . .

"In no other field of empirical inquiry has so massive and sophisticated a statistical machinery been used with such indifferent results. Nevertheless, theorists continue to turn out model after model and mathematical statisticians to devise complicated procedures one after another. Most of these are relegated to the stockpile without any practical application or after only a perfunctory demonstration exercise. Even those used for a while soon fall out of favor, not because the methods that supersedes them perform better, but because they are new and different." Leontief, Wassily, "Theoretical Assumptions and Nonobserved Facts," American Economic Review, Vol. LXI (1971). (Presidential address delivered at the December 1970 meeting of the American Economic Association.)

"It will be remembered that my argument began with the divergence between the increasing power of economists to elaborate trains of subtle and rigorous reasoning and build complex models, on the one hand, and on the other the slow advance of their power to diagnose and prescribe for the problems of our day. I have ascribed this divergence basically to the developments of theory and econometrics alike having been built upon arbitrary assumptions about the behaviour of economic agents. In an experimental science assumption chosen a priori can be tested against facts of observation; but in economics there is little opportunity for experiment, and I have argued that the possibility of making this deficiency good through econometrics is also limited. For our knowledge of the behaviour of economic agents we must rely mainly on the patient accumulation of direct observations." Phelps Brown, E.H., "The Underdevelopment of Economics" The Economic Journal, March 1972. (Presidential Address to the Royal Economic Society, July 8, 1971.

"It will be guessed that the neoclassical system is not a description of reality. And this the ensuing pages will affirm. On what does its hold on the economic mind depend?
"That it performs an instrumental service in guiding attention away from inconvenient fact and action has already been stressed. Accordingly it is a formula for a quiet noncontroversial life. But this is not all—for economists, as for others, truth and self-respect have their claims. The neoclassical system owes much to tradition—it is not implausible as a description of a society that once existed. Nor is it entirely unsatisfactory as a picture of that part of the economy hereinafter called the market system.

"Additionally it is the available doctrine. Students arrive; something must be taught; the neoclassical model exists. It has yet another strength. It lends itself to endless theoretical refinement. With increasing complexity goes an impression of increasing precision and accuracy. And with resolved perplexity goes an impression of understanding. If the economist is sufficiently "caught up in his data and his techniques," he can overlook social consequences—his attention being elsewhere, he can even, without damage to conscience, "support a system that maltreats large numbers of people."

"It should not be supposed, however, that the present hold of the established or neoclassical system is secure. The link between doctrine and reality cannot be stretched too far. That the comparative development in housing and space travel is a manifestation of consumer will cannot be believed. Nor does anyone suppose that there is a tendency to equality in wage income as between different sectors of the economy. When belief is stretched too far, it snaps; the doctrine is rejected. The same is true of refinement without relevance. It comes, sooner or later, to seem but a game." Galbraith, John Kenneth, Economics and the Public Purpose, The New American Library, New York, N.Y., 1975, pg. 26-27.

"A great many of the extant presentations "pure" economic theories, or systems of universal economic premises, axioms, and definitions, have been left too crudely formulated in their undefined economic predicates to justify economic observers making a serious (and expensive) attempt to provide those "pure" economic theories with an observational interpretation that is adequate for technological application. The slow growth of an empirical foundation for such "speculative" theories (mentioned by Leontief) is hardly due to lack of competence or performance by economic observers; at least there is no reliable
evidence in the history of economic science for such a contention. In the United States, at least, the number of professional economists and econometricians in academic employ who are trained economic observers is too small relative to the number engaged in forming "pure" theories to undertake the additional deductive and definitional refinement that "speculative" economic theories (as currently presented by theorists) require in order that technological application may become feasible." Basmann, R.L. "Modern Logic and the Suppositious Weakness of the Empirical Foundations of Economic Science" Schweizerische Zeitschrift fur Volkswirtschaft und Statistik, April 1977.

"Year after year economic theorists continue to produce scores of mathematical models and to explore in great detail their formal properties; and the econometricians fit algebraic functions of all possible shapes to essentially the same sets of data without being able to advance, in any perceptible way, a systematic understanding of the structure and the operations of a real economic system." Leontief, Wassily, "Letters 'Academic Economics'" Science Vol. 217, 9 July 1982. See also Note (5) to Chapter II.

"'A dismal performance . . . . What economist revealed most clearly was the extent to which their profession lags intellectually.' (1) This editorial comment by the leading economic weekly (on the 1981 annual proceedings of the American Economic Association) says, essentially, that the 'king is naked.' But no one taking part in the elaborate and solemn procession of contemporary U.S. academic economics seem to know it, and those who do don't dare speak up . . . ." Leontief, Wassily, "Academic Economics", Science, Vol. 217, 9 July 1982. The reference (1) in the text is to Bus. Week 18, January 1982, pg. 124.

"The different varieties of economic policies we hear about and try to practice are usually justified by theoretical constructions such as Phillips Curves, Laffer Curves, full-employment budgets, "rational expectation" theorems, and similar abstract notions. The builders of more and more intricate econometric models try in vain to compensate for their lack of hard, systematically organized factual information by relying on increasingly ingenious but utterly unreliable methods of indirect statistical inference.
"While debate over these theories continues, the economy steadily deteriorates. A year ago when President Reagan issued his first budget I observed that the proposed combination of drastic tax cuts with unprecedented tightening of credit could very likely bring a slump threatening to lead into a deep depression. The explosive rise in productive investment so confidently predicted a year ago failed to materialize notwithstanding all the tax concessions passed last summer."


(3) "In general, the application of any mathematical theory to any realistic problem requires constructing a model of the problem in mathematical terminology. How each concept in the model corresponds to a concept in the problem requires understanding of both areas on the part of the person making the application. If the problem is physical, he must understand the physical facts that are to be related. He must also understand how the mathematical concepts are related so that he can establish a correspondence between the physical concepts and the mathematical concepts.

"If this correspondence has been established in a meaningful way, presumably the conclusions in the mathematical model will also have physical meaning. If it were not for this aspect of the use of mathematical models, mathematics could make little contribution to the problem for it could otherwise not reveal any fact or conclusion not already known. The usefulness of the model depends on how removed from obvious the conclusions are, and how experience verifies the validity of the conclusions.

"It must be emphasized that there is no hope of making any meaningful numerical computations until the model has been constructed and understood. Anyone who attempts to apply a mathematical theory to a real problem without understanding of the model faces the danger of making inappropriate applications, or the restriction of doing only what some one who does understand has instructed him to do. Too many students limit their aims to remembering a sequence of steps that "give the answer" instead of understanding the basic principles." Nering, Evar D., Linear Algebra and Matrix Theory, 1970, John Wiley and Sons, New York, pg. 219
What we would like to emphasize is that without understanding the basic sociophysical nature, origin, and attributes of the problem at hand no meaningful models can even be constructed.

(4) See, for example, An Assessment of the Supplies of Natural Gas and Fuels for Generating Electricity in Ohio July 1982-June 1983, Ohio Department of Energy, Columbus, August 15, 1982, pp. 41-52.

(5) "Econometric models that may be appropriate from the point of view of economic theory have performed relatively poorly in forecasting electrical consumption during the 1970's, a period characterized by structural changes in the U.S. economy. In view of this, A.E.P.'s introduction of a dummy variable to capture the alleged post coal strike conservation trend beginning in 1978, as well as a six percent across-the-board reduction applied to the output of the long term forecasting model with the dummy variable, are attempts based upon the judgments of American Electric Power Corporation's staff to modify model output which seems incompatible with recent empirical evidence.

"In view of the observed inadequacy of traditional econometric models as reliable forecasting tools, the Ohio Department of Energy has developed an alternative conceptual framework and a corresponding analytical methodology for the analysis and forecasting of economic data series which involve the measurement of discernible time variant trends in the time paths of the data series under consideration. Our independent assessment of the available evidence indicates that A.E.P.'s final forecast for total residential kWh consumption in 1991 is 2.8 percent lower than Ohio Department of Energy forecasts." Investigation of A.E.P. Long Range Residential Consumption Forecasting Methodology, Ohio Department of Energy, Columbus, 1982, pp. 1-2.

(6) Ohio Department of Energy was abolished in March 1983 by an Act of Ohio Congress. The responsibility for enforcing the provisions of Amended Substitute Senate Bill No. 378 with respect to the
holding of public hearings on utility forecasts were vested in the Division of Forecasting and Information of Ohio Department of Energy. The Division, its personnel and responsibilities were transferred to the Ohio Department of Development. The first hearings on electricity forecasts are scheduled to start on October 1, 1983.

B. Notes to Chapter I

(1) Individual electric utility forecasts on which figure 1 is based, along with similarly documented forecasts for the individual natural gas utilities are presented in Appendix A.

(2) See Note 3 to the Introduction.

(3) As opposed to the Robinson Crusoe and Man Friday type introductory examples in many economic texts, which interpret or characterize the nature, existence or emergence of socio-economic systems, or activities, in terms of the unconscious pursuit of "rational" appeasement of greed and avarice by individual "agents", an alternative explanation of the same phenomena is offered by the 14th century Muslim scholar Ibn Haldun, in his Al Muqaddimah, or the Introduction to History, which has been characterized by Toynbee, in the Observer, as: "Undoubtedly the greatest work of its kind that has ever been created by any mind in any time or place ... the most comprehensive and illuminating analysis of how human affairs work that has been made anywhere."

"Human social organization is something necessary. The philosophers expressed this fact by saying: 'Man is "political" by nature.' That is, he cannot do without
the social organization for which the philosophers use the technical term 'town' (polis).

"This is what civilization means. (The necessary character of human social organization or civilization) is explained by the fact that God created and fashioned man in a form that can live and subsist only with the help of food. He guided man to a natural desire for food and instilled in him the power that enables him to obtain it.

"However, the power of the individual human being is not sufficient for him to obtain (the food) he needs, and does not provide him with as much food as he requires to live. Even if we assume an absolute minimum of food - that is, food enough for one day, (a little) wheat, for instance - that amount of food could be obtained only after much preparation such as grinding, kneading, and baking. Each of these three operations requires utensils and tools that can be provided only with the help of several crafts, such as the crafts of the blacksmith, the carpenter, and the potter. Assuming that a man could eat unprepared grain, an even greater number of operations would be necessary in order to obtain the grain: sowing and reaping, and threshing to separate it from the husks of the ear. Each of these operations requires a number of tools and many more crafts than those mentioned. It is beyond the power of one man alone to do all that, or part of it, by himself. Thus, he cannot do without a combination of many powers from among his fellow beings, if he is to obtain food for himself and for them. Through co-operation the needs of a number of persons, many times greater than their own number, can be satisfied.

"Likewise, each individual needs the help of his fellow beings for his defense. When God fashioned the natures of all living beings and divided the various powers among them, many dumb animals were given more perfect powers than God gave to man. The power of a horse, for instance, is much greater than the power of a man, and so is the power of a donkey or an ox. The power of a lion or an elephant is many times greater than the power of man.

"Aggressiveness is natural in living beings. Therefore, God gave each of them a special limb for defense against aggression. To man, instead, He gave the ability to think, and the hand. With the help of the ability to think, the hand is able to prepare the ground for the crafts. The
crafts, in turn, procure for man the instruments that serve him instead of limbs, which other animals possess for their defense. Lances, for instance, take the place of horns for goring, swords the place of claws to inflict wounds, shields the place of thick skins, and so on. There are other such things. They were all mentioned by Galen in De usu partium.

"The power of one individual human being cannot withstand the power of any one dumb animal, especially the power of the predatory animals. Man is generally unable to defend himself against them by himself. Nor is his unaided power sufficient to make use of the existing instruments of defense, because there are so many of them and they require so many crafts and things. It is absolutely necessary for man to have the co-operation of his fellow men. As long as there is no such co-operation, he cannot obtain any food or nourishment, and life cannot materialize for him, because God fashioned him so that he must have food if he is to live. Nor, lacking weapons, can he defend himself. Thus, he falls prey to animals and dies much before his time. Under such circumstances, the human species would vanish. When, however, mutual co-operation exists, man obtains food for his nourishment and weapons for his defense. God's wise plan that mankind should subsist and the human species be preserved will be fulfilled.

"Consequently, social organization is necessary to the human species. Without it, the existence of human beings would be incomplete. God's desire to settle the world with human beings and to leave them as His representatives on earth would not materialize. This is the meaning of civilization, the object of the science under discussion.

"The aforementioned remarks have been in the nature of establishing the existence of the object in this particular field. A scholar in a particular discipline is not obliged to do this, since it is accepted in logic that a scholar in a particular science does not have to establish the existence of the object in that science. On the other hand, logicians do not consider it forbidden to do so. Thus, it is a voluntary contribution.

"God, in His grace, gives success.

"When mankind has achieved social organization, as we have stated, and when civilization in the world has thus become a fact, people need someone to exercise a restraining influence and keep them apart, for aggressiveness and injustice are in the animal nature of man. The weapons made for
the defense of human beings against the aggressiveness of dumb animals do not suffice against the aggressiveness of human beings toward each other. It could not come from outside, because all the other animals fall short of human perceptions and inspiration. The person who exercises a restraining influence, therefore, must be one of themselves. He must dominate them and have power and authority over them, so that no one of them will be able to attack another. This is the meaning of royal authority.

"It has thus become clear that royal authority is a natural quality of man which is absolutely necessary to mankind. The philosophers mention that it also exists among certain dumb animals, such as the bees and the locusts. One discerns among them the existence of authority and obedience to a leader. They follow one who is distinguished as their leader by his natural characteristics and body. However, outside of human beings, these things exist as the result of natural disposition and divine guidance, and not as the result of an ability to think or to administrate." Ibn Khaldun, The Muqaddimah, An Introduction to History


(4) Mikhail, Edward M., Observations and Least Squares 1.2.

(5) "All measurements require a definition of the units used. On the international level the "Bureau International des Poids et Mesures" in Paris is responsible for the establishment of the fundamental standards and scales of the principal physical quantities and makes the necessary comparisons with national standards for length, mass, temperature etc. An international organization of legal metrology was founded in Paris in 1955. (Organisation Internationale de Metrologic Legale, O.I.M.L.) and it is their responsibility to determine the general principles of legal metrology." Bjerhammar, Arne, Theory of Errors and Generalized Matrix Inverses, Elsevier Scientific Company, Amsterdam-London, 1973, pg. 7.

(6) "Modern society is highly dependent on advanced measuring techniques. We are all intrigued by the outstanding achievements of the astronauts and others who, together, have conquered the moon, and who perhaps will soon conquer other planets. These challenging results, all based on the most refined measuring techniques, have tremendously increased our interest in measurements and their mathematical treatment, during the last decade. Bjerhammar, Arne, Theory of Errors and Generalized Matrix Inverses, Elsevier Scientific Publishing Company, Amsterdam-London, 1973, pg. 1."
(7) See Mikhail, 1976, Section 1.3.2.

(8) "Not having been subjected from the outset to the harsh discipline of systematic fact finding, traditionally imposed on and accepted by their colleagues in the natural and historical sciences, economists developed a nearly irresistible predilection for deductive reasoning. As a matter of fact, many entered the field after specializing in pure or applied mathematics. Page after page of professional economic journals are filled with mathematical formulas leading the reader from sets of more or less plausible but entirely arbitrary assumptions to precisely stated but irrelevant theoretical conclusions.

"Nothing reveals the aversion of the great majority of the present-day academic economists for systematic empirical inquiry more than the methodological devices that they employ to avoid or cut short the use of concrete factual information. Instead of constructing theoretical models capable of preserving the identity of hundreds, even thousands, of variables needed for the concrete description and analysis of a modern economy, they first of all resort to "aggregation." The primary information, however detailed, is packaged in a relatively small number of bundles labeled "Capital," "Labor," "Raw Materials," "Intermediate Goods," "General Price Level," and so on. These bundles are then usually fitted into a "model," that is, a small system of equations describing the entire economy in terms of a small number of corresponding "aggregative" variables. The fitting, as a rule, is accomplished by means of "least squares" or another similar curve-fitting procedure.

"The procedure described above was standardized to such an extent that, to carry out a respectable econometric study, one simply had to construct a plausible and easily computable theoretical model and then secure--mostly from secondary or tertiary sources--a set of time series or cross section data related in some direct or indirect way to its particular subject, insert these figures with a program of an appropriate statistical routine taken from the shelf into the computer, and finally publish the computer printouts with a more or less plausible interpretation of the numbers. Leontief, Wassily, "Letters 'Academic Economics'" Science, Vol. 217, 9 July 1982.

(9) "You cannot arithmetically add two lines. What you add are not the lines, but numbers that represent the lengths
of the lines. The lines are not numbers; they are configurations in physical space. I have always stressed that a distinction must be made between arithmetical addition and the kind of addition that constitutes the physical operation of combining. It helps us to keep this distinction in mind if we follow Hempel (who has written at length about extensive magnitudes) in introducing a special symbol, a small circle, "ο", for the physical operation of joining. This provides a much more satisfactory way of symbolizing the additive rule for length:

\[ L(α \circ β) = L(α) + L(β). \]

The combining of lengths can be diagrammed:

\[ \begin{array}{c}
  a & b \\
  \hline
  L(α) & L(β) \\
  \end{array} \]

\[ L(α \circ β) \quad [\text{not } "L(α + β)"] \]

Although in the case of weight it does not matter exactly how the two bodies are placed together on the scale, it does matter in the case of length. Suppose that two line segments are placed like this:

\[ \begin{array}{c}
  A & a & b & C \\
  \end{array} \]

They are end to end, but not in a straight line. The distance between points A and C is not the sum of the lengths of a and b. We must always be careful, therefore, to specify exactly what we mean by the operation of joining." Carnap, R., An Introduction to the Philosophy of Science, Basic Books Inc., New York, 1966, pp. 72-73.

"The underlying spirit of this treatment of the theory of matrices is that of a concept and its representation. For example, the abstract concept of an integer is the same for all cultures and presumably should be the same to a being from another planet. But the various symbols we write down and carelessly refer to as "numbers" are really only representations of the abstract numbers. These representations should be called "numerals" and we should not confuse a numeral with the number it represents. Numerals of different types are the inventions of various cultures and individuals, and the superiority of one system of numerals over another lies in the ease with which they can be manipulated and the insight they give us into the nature of the numbers they represent.
"We happen to use numerals to represent things other than numbers. For example, we put numerals (not numbers) on the backs of football players to represent and identify them. This does not attribute to the football players any of the properties of the corresponding numbers, and the usual operations of arithmetic have no meaning in this context. No one would think of adding the halfback, 20, to the fullback, 40, to obtain the guard, 60.

"Matrices are used to represent various concepts with a wide variety of different properties. To cover these possibilities a number of different manipulations with matrices are introduced. In each situation the appropriate manipulations that should be performed on a matrix or a set of matrices depend critically on the concepts represented. The student who learns the formalisms of matrix "arithmetic" without learning the underlying concepts is in serious danger or performing operations which have no meaning for the problem at hand.

"In even the simplest problems matrices can appear as representing several different types of concepts. For example, it is typical to have a problem in which some matrices represent vectors, some represent linear transformations, and others represent changes of bases. This alone should make it clear that an understanding of the things represented is essential to a meaningful manipulation of the representing symbols." Nering, Evar D. Linear Algebra and Matrix Theory, John Wiley and Sons, New York, 1970, pp. v, vi.

(10) "It is important to understand that we cannot really say we know what we mean by any quantitative magnitude until we have formulated rules for measuring it. It might be thought that first science develops a quantitative concept, then seeks ways of measuring it. But the quantitative concept actually develops out of the process of measuring. It was not until thermometers were invented that the concept of temperature could be given a precise meaning. Einstein stressed this point in discussions leading to the theory of relativity. He was concerned primarily with the measurement of space and time. He emphasized that we cannot know exactly what is meant by such concepts as "equality of duration", "equality of distance (in space)", "simultaneity of two events at different places", and so on, without specifying the devices and rules by which such concepts are measured. Carnap, R., An Introduction to the Philosophy of Science, Basic Books Inc., New York, 1966, pg. 68."
An explanatory economic model is a system of logical deductions beginning in a conjunction of theoretical economic premises and warrantable factual statements of initial conditions and ending in a conjunction of prediction-statements that attribute definite probabilities to specified observable economic events. The events whose probabilities are deduced from the conjunction of premises and statements of initial conditions are described with the help of sample statistics, which are mathematical functions of observable economic quantities such as prices, incomes, and interest rates; once the forms of those mathematical functions have been determined, the joint and marginal distribution functions of sample statistics are derived from initial and background conditions with the help of theoretical economic premises.


"Generally, the equations of an econometric model are, except for identities, stochastic ones and hence the problem arises of how to specify the (joint) stochastic character of a number of random variables simultaneously. This leads us to consider the problem of the distribution of vector random variables, that is, the characteristics of the joint distribution of a number of random variables simultaneously and not "one at a time." Dhrymes, Phoebus J., Econometrics, Springer-Verlag, New York, 1974, pg. 1-2.

Although some lip service is paid in the literature to the possibility of accounting for observational errors in economic data series, utilized as inputs to or outputs of econometric models, the distinguishing feature of econometric analyses and models are the stochastic transformations of a given set of
nonstochastic measurements on a set of "independent variables" into a set of nonstochastic measurements on a set of "dependent variables". That is qua observations $[y_t \ x_t]$ is presumed to be nonstochastic for purposes of econometric analysis. However, an econometric model is tantamount to an assertion that there exists a time invariant transformation of the quantities of $x_t$ into quantities of $y_t$, over the domain of time $t = t_0 + 1, \ldots t_0 + N$; such that the difference $e_t = [y_t - F(x_t)] \sim N(0 \mid V)$ where $V$ is an $M \times M$ positive definite symmetric matrix, which is presumably known. The stochasticity of $e_t$ is presumed to reflect the fact that in "reality" $y_t = F([x_t \ Z_t])$ where the vector of variables of $Z_t$ is either not known or not explicitly introduced into the system, and that over a sufficiently long period of time $E[F([x_t \ Z_t]) - F([x_t])] = 0$. See also the quotation from Koopmans, Rubin and Leipnik 1950, in chapter I page 30 of the text.

(12) "There is, however, a methodological dualism in the mathematical study of economic phenomena. At one extreme, people called mathematical economists perform mathematical, but not particularly numerical, analysis of deterministic and nonstochastic systems, whereas statistically minded economists called econometricians are chiefly concerned with the application of modern statistical methods to the estimation of parameters relevant to economic relationships at the other extreme. In the judgment of a majority of mathematical economists, the intrinsic internal mechanisms of economic systems are deterministic in nature, though stochastic factors are not completely lacking. They argue that clear insight into the internal mechanisms can be better obtained by analyzing the effects of several interplaying factors of economic significance by means of mathematical reasoning, without worrying about nonessential stochastic features. Thus, they are primarily concerned with models
that are not necessarily posed in a form convenient for statistical testing. This is principally for the purpose of obtaining cognitive insight into the working of an economic system. They leave the statistical testing of their theories to econometricians who, on the basis of empirical data, try to statistically fit numerical equations to economic reality. While econometricians compromise with the limited availability of statistical data as well as with the applicability of statistical estimating methods by employing equations of qualitatively simpler type, equations of fuller economic implication are major subjects in mathematical economic theory. This book is concerned exclusively with the nonstochastic mathematical studies of economic phenomena. Therefore appeal is made to qualitatively oriented mathematical methods in the following chapters, rather than to statistical and numerical methods." Nikaido, Hukukane, Convex Structures and Economic Theory, Academic Press, New York, 1968, pp. 2-3.

"... an econometric model is a chain of logical deductions beginning in a set of economic propositions (employed as postulates) and ending in a set or propositions (theorems) whose terms are probability distributions of sample statistics. The sample statistics are of two kinds: firstly, the unconstrained estimators and test statistics associated with the reduced-form, e.g., coefficient estimates and identifiability test statistics; secondly, the estimators of the structural parameters themselves.

"A minimal subset of economic postulates (maintained hypothesis) is employed to specify which set of economic variables is to be understood as independent and which set as dependent in the reduced-form system; and, in connection with some principle of estimation, to provide a definition of reduced-form estimators and test statistics that does not presuppose the empirical truth of any other economic postulates or derived propositions. The simultaneous equations model qua null hypothesis makes the prediction that the distribution functions of such unconstrained estimators and test statistics will be found in repeated sampling under appropriate conditions to possess definite properties, a prediction that the unconstrained estimators and test statistics of the reduced-form are capable of falsifying." Basmann, R. L., "Remarks Concerning the Application of Exact Finite Sample Distribution Functions of GCL Estimators in Econometric Statistical Inference", American Statistical Association Journal, December 1963, Vol. 58, pg. 943.
"A word about terminology is in order. The statements we refer to here as definitions and postulates do not necessarily play the same kind of role everywhere in economic theory. For example, Propositions 1.2.1 and 1.2.2 can be derived as theorems from the utility function
\[
\psi(C, S; a, b) = \frac{(C - b)(S + b)}{a^2(C - b) + (S + b)} \quad \text{for } C > -b, S > b
\]
where \(a > 0\) and \(b > 0\). Maximizations of \(\psi(C, S; a, b)\) subject to the constraint
\[
C + S = Y
\]
yields the consumption function (1.1a), where
\[
a_1 = \frac{1}{1 + a} \\
\gamma_1 = b.
\]
Since \(a\) and \(b\) are positive, Proposition 1.2.2 follows.


"The analysis and explanation of economic fluctuations has been greatly advanced by the study of systems of equations connecting economic variables. The construction of such a system is a task in which economic theory and statistical method combine. Broadly speaking, considerations both of economic theory and of statistical availability determine the choice of the variables. Economic theory predominates in the definition of the "behavior equations" describing a certain type of economic decisions taken by a certain category of economic agents, and in the specification of the variables that may possibly enter each behavior equation (i.e., of the conditions that may affect that decision by that group of agents)... Theoretical preconceptions, statistical evidence, and sometimes mere assumption or approximation, are intermingled in the determination of the form of each equation, as regards linearity and as regards the occurrence and length of time lags. All these things being determined, it is almost entirely left to statistical methods to estimate the numerical
values of the coefficients in the equations, and to assess the possible degree of error in those estimates, subject to the assumptions made." Koopmans, Rubin, and Leipnik, 1950, pg. 55-56.

"The theory of utility is essentially logical in nature. It can be applied whatever are the motivations of consumer choices since the economist takes the function $S$ as given and does not attempt to explain how it is arrived at.

"Having reached this point, we have a better understanding of the purely logical nature of the 'theory of utility' on which our reasoning will be based. The consumer's system of preferences is given; we do not have to concern ourselves with the motivation of these preferences and we do not exclude a priori any individual ethical system. All that matters is that the axioms A.1 to A.4 should hold. They are philosophically and psychologically neutral, and express a certain internal consistency of choices."

(Malinvaud, E., 1973)

"There is no possible world in which the laws of group theory and the abstract geometry of Euclidean 3-space would not hold, because these laws are dependent only on the meanings of the terms involved, and not on the structure of the actual world in which we happen to be. The laws of logic and pure mathematics, by their very nature, cannot be used as a basis for scientific explanation because they tell us nothing that distinguishes the actual world from some other possible world.......

"Campbell and other authors often speak of the entities in theoretical physics as mathematical entities. They mean by this that the entities are related to each other in ways that can be expressed by mathematical functions. But they are not mathematical entities of the sort that can be defined in pure mathematics. In pure mathematics, it is possible to define various kinds of numbers, the function of logarithm, the exponential function, and so forth. It is not possible, however, to define such terms as "electron" and temperature" by pure mathematics. Physical terms can be introduced only with the help of nonlogical constants, based on observations of the actual world. Here we have an essential difference between an axiomatic system in mathematics and an axiomatic system in physics." (Carnap, An Introduction to the Philosophy of Science, op. cit.) pp. 10, 11, 236, 237.

"During the last five or six decades there have been important shifts in philosophical thinking about scientific discovery and the growth of scientific knowledge. The positivists distinguished the context of discovery and the context of justification, dismissing the former as the subject matter of history or psychology. The only aspects of the growth of scientific knowledge relevant to philosophy were the inductive justification or confirmation of knowledge claims and the incorporation of older theories into more comprehensive theories via intertheoretic reduction. The resulting view of scientific knowledge was a static one which, ignoring the dynamics of scientific progress and being tied to an untenable observational/theoretical distinction and associated epistemology, led to a highly distorted portrait of science and the knowledge it provided, which had little to do with the epistemic activities science actually was engaged in. Rejecting such a view a group of "young Turks" - including Hanson, Feyerabend, and Kuhn - started examining scientific practice and the history of science and developed Weltanschaungen views that, unfortunately, made scientific knowledge a social phenomenon in which science became a subjective and, to varying degrees, an irrational enterprise.

"More recently philosophers such as Lakatos, Toulmin, and Shapere have attempted to steer a middle course between these two extremes wherein science is a rational enterprise concerned with obtaining objective knowledge of the real world.

"Only when we get to Shapere's work do we find an account where reason is accorded a sufficiently detailed and central place in a philosophical account of the growth of scientific knowledge. Although very much in the state of developing work in progress, and much more work is needed, out of Shapere's work is emerging the outlines of a promising and coherent philosophical portrait of the growth of scientific knowledge; and it is an approach which goes beyond the mere attempt to steer a "middle course" between the extremes of positivism and the Weltanschaungen analyses, constituting a larger attack on the tradition in philosophy of science from whence those two extremes arise. Whether the particular details of his account prove correct, approximately correct, or end up by being replaced, his work does constitute a promising approach and perspective for work in philosophy of science; and it is my preception that, increasingly, philosophical work on the growth of scientific knowledge is coalescing in the general
direction and approach exemplified by Shapere's work."
Suppe, Frederick, The Structure of Scientific Theories,
University of Illinois Press, Urbana, 1979, pg. 704, 705.

C. Notes to Chapter II

(1) If the time path trajectory of $X_{at}$ is given by

$$X_{at} = C_o$$

if

$$0 \leq t \leq N$$

then $x = 0$, $k = 0$, $f_o(t) = 0$, $jO = 0$, and $j1 = N$.

We have simply excluded this case, which is, by definition, con­
tinuous and differentiable over $T$ from the scope of our analysis by
specifying $x > 1$.

(2) See, for example, Chiang 1974, section 6.7

or

Aramanovich, et al, 1965, Section I.1 for a definition and further
discussion of nondifferentiability, and discontinuity of mathemati­
cal functions.

(3) Let us note that since we have no direct observations prior to
$t = 1$, we are free to choose either $t = 0$ or $t = 1$ as our initial
level of systemic flows. The choice of "0" presumes that the
linear trend in the flow velocity observed after $t = 1$, was also
effective between, at least, $0 \leq t \leq 1$ as well. However, since we
do not have any direct observations prior to $t = 1$, this presump­
tion cannot be empirically verified.

Since $t = 1$ is the first direct observation available for
analysis, we have adopted by 17.a-c the convention that we shall
treat the level of systemic flows at $t = 1$ as our initial level of
observations and presume that $\frac{dX_{at}}{dt} = 0$ if $t < 1$. The convention in
The question is basically more conservative than treating \( t = 0 \) as the initial point of observation concerning the initial level of flow velocities for we lose one degree of freedom in the process. But we feel it is a more appropriate expression of our ignorance concerning the state of affairs prior to \( t = 1 \).

\[
C_k = \int_{j(k-1)}^{j(k)} f_{k-1}(t) \, dt + C_{k-1}
\]

\[
= \int_{j(k-1)}^{j(k-1)} f_{k-1}(t) \, dt + \left[ \int_{j(k-2)}^{j(k-1)} f_{k-2}(t) \, dt + C_{k-2} \right]
\]

\[
= \int_{j(k-1)}^{j(k-1)} f_{k-1}(t) \, dt + \ldots + \left[ \int_{j(1)}^{j(2)} f_1(t) \, dt + C_1 \right]
\]

\[
= \sum_{q=2}^{k} \int_{j(q-1)}^{j(q)} f_{q-1}(t) \, dt + x_{a1}
\]

\[
\text{if } 2 \leq k \leq 2 \quad \text{and if} \quad j_k < t \leq j(k+1).
\]
\[ f_k(t) = f_{k-1}(t) + \Delta_k \]

\[ = \left[ f_{k-2}(t) + \Delta_{k-1} \right] + \Delta_k \]

\[ = \left[ f_0(t) + \Delta_1 \right] + \sum_{m=2}^{k} \Delta_m \]

\[ = \sum_{m=1}^{k} \Delta_m \]

if \( jk < t < j(k+1) \) and if \( 1 \leq k \leq 2 \).

(6) 1. \[
C_k = \sum_{q=2}^{k} \int_{j(q-1)}^{j_q} \left[ \sum_{m=1}^{q-1} \frac{\Delta_m}{j(q-1)} \right] dt + x_{a1}
\]

if \( jk < t < j(k+1) \) and if \( 2 \leq k \leq 2 \).

2. \[
\int_{j(q-1)}^{j_q} \left[ \sum_{m=1}^{q-1} \frac{\Delta_m}{j(q-1)} \right] dt = \int_{j1}^{j2} \Delta_1 dt
\]

if \( q = 2 \)
\[ j_3 = \int \left[ \Delta_1 + \Delta_2 \right] \, dt \]
\[ j_2 \]

if \( q = 3 \)

-----

\[ j_k = \int \left[ \Delta_1 + \Delta_2 + \cdots + \Delta_{k-1} \right] \, dt \]
\[ j(k-1) \]

if \( q = k \)

\[
3. \sum_{q=2}^{k} \int \left[ \sum_{m=1}^{q-1} \Delta_m \right] \, dt = \sum_{m=1}^{k-1} \int \Delta_m \, dt, \]
\[ j(q-1) \, j_m \]

if \( 2 \leq k \leq \lambda \)

where the right hand side of 3 is obtained by summing over the corresponding values of \( q \) for successive values for \( q = 2, 3, \ldots, k \); which is tantamount to joining the successive domains of integration of common integrands at each step of the summation.

Substituting the right hand side of 3 into 1

\[
4. \quad C_k = \sum_{m=1}^{k-1} \int \Delta_m \, dt + X_{a1} \]
\[ j_m \]
\[ \text{if } jk < t \leq j(k+1) \]
\[ \text{and if } 2 < k < 2. \]

(7) ODOE Staff Report, An Assessment of the Supplies of Natural Gas and Fuels for Generating Electricity in Ohio, November 1982-October 1983, Columbus, The Ohio Department of Energy, December 15, 1983.


D. Notes to Chapter III

(1) See Kshirsagar, 1974, chapter 11, section 3.


(3) See Mikhail 1976, chapters 5-9. See also Theil chapter 6, section 9.

(4) See Note (2) above.


(6) See Note (5) above.

(7) See figure 36, Appendix C.

(8) See Note (5) above.
Due to the fact that the system state variables actually utilized in the models and the variable designation scheme of chapter II were developed independently the designation of DUM14 as a shift variable for the post-1979 recession is not consistent with the scheme developed in chapter II. It should be clear, however, that no matter how it is labeled

\[
\begin{align*}
\text{DUM14} &= 0 & \text{if } \text{year} \leq 1979 \\
\text{DUM14} &= 1 & \text{if } \text{year} > 1979.
\end{align*}
\]

The same comment applies to the designation of DUM6 as well.

E. Notes to Chapter IV

(1) See Note (2) to the Introduction. See also appendix A.

(2) See Note (14) to chapter I.

(3) See Note (3) to the Introduction. See also Note (9) to chapter I.

F. Notes to Appendix B

(1) See, for example, Edwards, Mikhail (1976) chapters 6-8.

(2) The major usefulness of such substitution, aside from a certain clarity and economy in exposition, is when utilizing canned statistical programs which do not adjust the degrees of freedom due to constraints, such as SAS PROC SYSREG. Although one can write a computer program to do constrained adjustment, or utilize SAS PROC MATRIX, to simulate the output format of PROC SYSREG is either too toilsome or may even be impossible in the latter case. There may, very well, be, however, other canned programs available, of which we are not aware, which may
automatically adjust the degrees of freedom for the constraints imposed.

(3) See B.14.a-d.
BIBLIOGRAPHY

Al-Ghazali, Abu Hamid Muhammad, Tahafut Al-Falasifah (Incoherence of the Philosophers), Sabih Ahmad Kamali (Trans.), Lahore, Pakistan Philosophical Congress, 1963.

Al-Ghazali, Abu Hamid Muhammad, "Kitab-ul Ilm" (The Book of Knowledge) Ihya Ulum-ud Din (The Revitalization of Religious Sciences), Faris, N.A. (Trans.), Lahore, Sh. Muhammad Ashraf, 1966


Goursat, Edouard, *A Course In Mathematical Analysis*, Boston, Ginn and Co., 1904


Kneale, W., "Propositions and Truth in Natural Languages", *Mind*, April 1972, Vol. LXXXI.


MacDuffee, C. C., Vectors and Matrices, Menasha, Wisconsin, Mathematical Association of America, 1943.


Tarski, A., Introduction to Logic and to the Methodology of Deductive Sciences, New York, Oxford University Press, 1941.


