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THE PRICING OF PRIVATE MORTGAGE DEFAULT INSURANCE: AN APPLICATION OF THE MODERN OPTION PRICING MODEL

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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THE OHIO STATE UNIVERSITY
1983

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This Dissertation is dedicated to Nancy, whose encouragement sustained me and whose sacrifices exceeded mine.
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PUBLICATIONS


FIELDS OF STUDY

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Studies in Real Estate Development and Financing, Feasibility Analysis, and Urban Economics

Minor Field: Finance

Studies in Corporate Financial Theory and Investment Theory
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Chapter I

INTRODUCTION

In a competitive mortgage lending market, rational borrowers and lenders have conflicting incentives that result in a complex set of contractual mortgage provisions. Lenders recognize that after a loan is made, the borrower will seek to maximize his wealth. If no protective provisions are implemented by the lender, the borrower's maximizing efforts will become the lender's loss. There are essentially three sources of such borrower-lender conflict which are directly analogous to the investment, dividend, and financing decisions of corporate finance. These are the house use policy, the consumption policy, and the subordination policy on future debt, respectively. This thesis asserts that mortgage default insurance is an efficient vehicle for partially resolving these borrower-lender conflicts, and, like any financial claim, is rationally priced.

The borrower's use of the house, the investment decision, is presumed by the lender to be for a stated purpose for which he charges an appropriate risk adjusted rate. If the borrower subsequently alters the projected usage to a state with a higher variance rate, the value of the loan falls and the borrower successfully extracts wealth from the lender.
The borrower's consumption policy is equivalent to the dividend decision. For example, the lender grants the loan with the expectation that the borrower will provide a minimum level of maintenance. However, the borrower could subsequently decrease his maintenance expenditures thereby increasing his net service flow (dividends). The market value of the property falls from deterioration and the borrower extracts wealth from the lender because the value of the loan also falls.

The borrower's financing decision, as in the corporate case, is a matter of potential claim dilution. By incurring additional unsubordinated debt, unanticipated by the lender, the borrower can increase the risk of the lender's loan. The value of the loan will fall, and the borrower will extract wealth from the lender.

The solutions to these borrower-lender conflicts are numerous and complex. For the idealized case in which contracting costs (i.e., negotiating, monitoring, and enforcing) are zero, these contentions are efficiently resolved through contractual provisions. For example, loan provisions that prohibit the rental of property without the lender's approval will control the borrower's investment policy.

---

¹ Net service flow is defined as the value of the gross rental services minus maintenance costs.

² For a more complete analysis, within a contingent claim framework, of the borrower-lender conflicts and efficient solutions see C.W. Smith's (1980) "On the Theory of Financial Contracting."
icy. Loan collateralization will protect the lender from claim dilution, and escrow accounts will help guarantee that the borrower provides minimum maintenance. In short, the contractual technology can always be made sufficient to remedy any potential conflicts by simply adding another provision.

With positive contracting costs, each contractual provision is a costly attempt to control the borrower-lender conflicts. In a competitive mortgage market, the lender becomes a specialist in the negotiating, monitoring, and administering of mortgage contracts. His costs will be passed on in the form of a higher interest rate charged to the borrower.

However, unlike the corporate bond market in which the borrowers bring their contracts to the lending market, the mortgage loan market is dominated by lender-supplied contracts. Competition establishes a maximum interest rate that can be charged to the borrower. As a result, the lender has incentive to offer the most efficient set of contractual provisions in order to minimize contracting costs. Standardized contracts, escrow accounts, maintenance specifications, and asset usage limitations are efficient contractual provisions.

* The lender's opportunity cost is the purchase of treasury bills, which involve virtually no contracting costs. Therefore any increase in the administrative costs of the loan must be paid by the borrower. See Jensen and Meckling (1976) for a discussion of this point within the corporate setting.
In this setting, the default potential within the borrower-lender conflicts is isolated and priced separately by an additional market mutation — mortgage default insurance. As a result, a unique opportunity exists for the examination of the pricing of this security claim under assumptions of rational market participants. This thesis proposes that mortgage default insurance is isomorphic to a common stock put option. If so, the recent advances in option pricing should provide the appropriate model(s) for examining the pricing of such coverage. In the following chapters this one to one mapping equivalence of the put option to default insurance will be developed and model simulations presented.

In chapter 2, institutional aspects and nuances of the mortgage default insurance market are discussed. Prices, participants, and a brief history of the market are provided as necessary background information. In addition, a rational theory for default risk is contrasted with the competing view.

In chapter 3, the contingent claim literature is briefly reviewed and its relevance to financial security pricing under assumptions of capital market equilibrium are addressed. In particular, the relevance of the option pricing model to the Modigliani/Miller propositions and the efficient market hypothesis is developed. Recent extensions and applications of the option pricing model are presented to demonstrate its usefulness in analyzing a diverse mix of financial instruments.
In chapter 4, the similarities and dissimilarities between default insurance and a stock put option are examined. In the process a general contingent claim model of mortgage default insurance is derived under a restrictive set of assumptions. The model also clarifies the distinction between delinquency behavior, default behavior, and default insurance pricing. Each assumption is relaxed in chapter 5 and the theoretical implications discussed. Simulation results from the final model are presented in chapter 6. Chapter 7 outlines the discrepancies in price behavior between the simulated prices and the observed prices. Possible explanations are presented with simulation tests and a discussion of the theoretical implications. Chapter 8 concludes the thesis with a brief discussion of the areas in which future research might possibly be directed.
Chapter II
THE MORTGAGE DEFAULT INSURANCE MARKET

2.1 FUNCTION AND HISTORY

Real estate mortgage default insurance indemnifies the first mortgage lender to the extent of his direct and consequential losses resulting from the borrower’s default. Suppliers include the federal government agencies (Federal Housing Administration and Veterans Administration) who usually insure the entire balance, and private mortgage insurance companies who generally insure the top 12 to 30 percent of the outstanding loan balance. Regulated lenders require the borrower to purchase default insurance in order to allow their lending of a percentage amount greater than the statute ceilings. In addition, the attachment of default insurance to a mortgage contract facilitates its sale in the secondary market.

Prior to the 1930’s most mortgage insurance companies were bank affiliated (Rapkin, 1981). With the depression of that decade, most insurers collapsed creating a void in the supply of mortgage guarantees. For the ensuing two decades, mortgage insurance became the exclusive domain of the federal government under the National Housing Act of 1934.
However, due to the rigidity of the application procedures, the dictation of non-market interest rates, and restrictions on the housing quality, government supplied insurance was not adequate in meeting market demands. Thus, in 1957, after a brief experimental period, the Mortgage Guaranty Insurance Corporation (MGIC) emerged as the first private mortgage insurance company since the 1930's. Numerous private insurers have since entered the market. In the aggregate, their annual supply of mortgage insurance now exceeds FHA and VA guaranteed loans.

2.2 PRICING AND ORIGINATION PROCEDURES

The pricing and policy procedures tend to be uniform across all private insurers. Prices are expressed as a function of the loan to value (L/V) ratio, percentage of principal subject to coverage, and time to maturity of coverage. Unlike the 100 percent coverage guaranteed by the government agencies, private insurers provide optional coverages ranging from the top 12 to top 30 percent of the mortgage balance. The only financing variable restricting private mortgage insurance is a maximum 30 year limit on the loan's years to maturity. The mortgage interest rate, discount points, closing costs, service charges, and application fees are determined by the lender with the insurer having no requirements.
The insurer charges only an insurance premium which is expressed as a percent of the loan balance. An initial premium is stated at closing plus an annual renewal rate. The typical 90 percent L/V mortgage with 20 percent coverage carries a .50% premium at closing plus a 0.25% annual renewal rate. Single premium plans are also available. Figure 1 shows a complete schedule of premium rates.

Under the annual premium plan, the lender is free to cancel coverage at his option, with the typical term to cancellation being five to six years (Erick, et al, 1979). Given the lender's right to cancel, an optimal cancellation strategy must exist as an implicit formulation from any default insurance pricing model.

An immediate question of interest for the researcher is the price dynamics of the insurance rates appearing in figure 1 across time, regions, and suppliers. MGIC quote schedules for all fifty states have been obtained and reviewed. For the stipulated variables (i.e. L/V ratio, coverage percentage, and time to maturity of coverage) the quoted prices were found to be identical for identical coverages. Certain coverages were unique to specific states.

A review of the files at the Ohio Insurance Commission was conducted to determine the extent of price changes across time. The findings indicate that prices are constant

---

* Michigan, for example, offers coverages for 20 years, 25 years, 30 years, 35 years, and 40 years. Most states only have coverages to a maximum of 15 years.*
### MGIC Default Insurance Rates for MGIC

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**NOTES**

(1) Annual renewal premium rates are 25% of the declining balance except for 17% coverage of 80% and under LTV which is 30% and for 17% coverage of 85% and under LTV, which is: 15% for 81%-85% LTV and 14% for 80% and under LTV. Five-year renewals are available as an option to the annual renewals in the 10 and 15 year programs. The five-year renewal rates are 50% of the principal balance.

(2) For seasoned loans on which, prior to application, (i) 12 or more monthly payments have been received, (ii) no contractual delinquencies exist, (iii) no payments have been received after the expiration of a 30-day grace period, and (iv) company underwriting standards are met, the premium rate shall be determined by subtracting 25% from the initial premium rate. However, no annual plan may be reduced below a minimum of 25%, and no single premium may be reduced by an amount greater than that applicable to the annual premium plan in the same loan-to-value and coverage percentile grouping.

### TRUTH IN LENDING LAW

With regard to the federal and/or state Truth in Lending Law, the MGIC insurance premium constitutes a "finance charge." Accordingly, MGIC has prepared informational materials and tables which indicate how to reflect the MGIC premium on the truth in lending disclosure form. Samples of these materials are included in the Appendix.

---

**Figure 1: Default Insurance Rates for MGIC**
across time.\textsuperscript{5} In conjunction with this review, a comparison of rates charged by different insurance carriers was also conducted. There appeared to be virtually no difference in quoted rates among suppliers for identical coverages. In summary, default insurance rates are constant across regions, time, and suppliers. Only the L/V ratio, coverage maturity, and depth of coverage affect quoted prices.

2.3 RELATED RESEARCH

The default literature is divided between work on the isolation of default determinants and work on the pricing of the default risk. Of these two, substantially more research has centered on the isolation of default determinants. Only recently, Swan (1982), has the pricing issue been directly addressed. Swan approached the problem by characterizing the insurance company's operation amidst a set of macro and micro factors. He first considered a monopolistic setting and was able to show that a profit maximizing firm will set prices such that the present value of marginal revenues will equal the present value of marginal costs. In competitive markets prices will reflect marginal cost and expected claim factors. The important factors, he claims, in pricing insurance are the cost of writing the insurance and the frequency of claims. He concludes his analysis by noting that

\textsuperscript{5} State law requires that records only be maintained for the past seven years. Records for years prior to 1974 had been destroyed.
option pricing models offer another approach to evaluating mortgage default insurance. Some additional work has examined the risk premia assessed in mortgage interest rates.6

Gau (1978) expanded the default determination research with a taxonomic model for classifying mortgages into homogeneous risk classes. The consensus determinants of default risk are grouped into three categories: property, financial, and borrower characteristics. Property characteristics involve expected price movements, age, design, and regional location. Financial variables include the loan's term to maturity and the L/V ratio. The borrower's contribution to default risk is measured in relation to his income. Proxies for income level and stability are found in the mortgagor's occupation, number of years employed at present job, and ratio of head of household's income to total family income.

These studies have found that the borrower's characteristics are significant determinants of the likelihood of default. This finding is in direct competition with a rational theory of consumer behavior. If borrowers act rationally, and the mortgage is a non-recourse loan, their decision to default should be strictly based on a comparison of the financial costs associated with the continuance or discontinuance of mortgage payments. The optimal choice

6 Examples of recent studies in mortgage default risk and interest risk premia include Von Furstenberg (1969, 1970), Sandor and Sasin (1975), and Jackson and Kaserman (1980).
will maximize the gains or minimize the losses from a default decision. Whether or not an individual has sufficient income to meet the payments is irrelevant to the rational decision. Consequently, a rational pricing model should specifically omit the borrower's characteristics as input variables.

Recent advances in financial decision theory provide a new framework for modeling the price of default risk. While the studies cited above have been useful in isolating the determinants of default behavior, there has been no attempt to explicitly price insurance protection from mortgage default under assumptions of rationality. In the following sections, default insurance will be presented as a contingent claim on the underlying house. By drawing from the option pricing literature, a model is developed that specifically excludes borrower characteristics. The model is capable of generating a schedule of insurance rates for comparison with the rate schedule of private insurers.
Chapter III
PRICING OF CONTINGENT CLAIMS

3.1 INTRODUCTION

Investors/consumers make a purchase decision based on the utility maximizing features of the selected commodity. In a multiperiod setting of uncertain economic conditions, the commodity is defined not only by its physical characteristics but also by the time period and states of the world upon which its production flow is contingent. A futures market effectively develops for the trading of these contingent commodities.

Fortunately, for the efficiency of the market participant, the market has devised an alternative to actually having to take delivery of contingent commodities. Security instruments with contractual agreements written against the contingent production flows are traded instead of the contingent commodities. The need for a vast array of contingent commodities is replaced by a single commodity of account against which contingent claims are written.

With this invention the investor is able to allocate his endowment over a mix of securities or contingent claims to yield the highest value of claims on future commodity production.
In the housing and mortgage markets the underlying commodity of account is the house which produces future service flow. The elemental argument of this thesis is that the market participants are free to trade contingent claims against the house which are value contingent upon the time interval, states of the world, and physical characteristics of the house.

3.2 CONTINGENT CLAIMS AND THE MODIGLIANI-MILLER PROPOSITIONS

In 1958 Modigliani and Miller (MM) proved that, in equilibrium, equivalent packages of financial claims on the same assets must command the same price. Using perfect market assumptions they proved that such claims are simply alternative modes of ownership on the same underlying income stream. As a consequence, the aggregate value of these claims is independent of their complexity and must equal the value of the assets themselves.

About the same time that Modigliani and Miller were formalizing their work on the valuation of claims to a firm's income stream, Samuelson (1965) and others were examining the valuation of stock options, a specialized financial claim. Expanding on the dissertation of the French economist Bachelier (1900), Samuelson posited that a stock's returns could be assumed to follow a particular stochastic process (Geometric Brownian Motion). By requiring an option on that stock to have a constant expected return at each
instant of time, he formally linked the value of the two securities. However, the assumption of constant expected returns does not have any economic justification for the pricing of securities under conditions of capital market equilibrium. For a brief period option pricing was relegated to the experimental state.

A revival occurred in 1973 with the path-breaking work of Black and Scholes (BS) which dramatically improved the economic focus of option pricing. By bringing to bear the MM proposition that equivalent financial claims must earn the same rate of return, BS were able to formulate a pricing model that depends only on observable variables. MM posit that a package of claims, no matter how complex, is equivalent to a simple equity position on a given income stream. Likewise, BS assert that a properly proportioned (hedged) portfolio of stock and options on that stock can at any point in time be made riskless. If the option is a function of the stock price and its time to maturity, then any gain or loss from a long stock position can be offset by adjusting a short position in the option. Continual adjustment of the hedge position will make it riskless, so it must earn the same rate as a riskless bond. This critical economic observation provides a rational theme for pricing the contingent claim.*

*A more detailed history of option and warrant pricing is presented in C. W. Smith's (1976) review of the literature.
3.3 **CONTINGENT CLAIMS AND EFFICIENT MARKETS**

A second critical element of the BS option pricing solution is the assumption that the stock returns follow a particular stochastic process. Specifically, they assume that changes in the price are governed by a diffusion process of the form

\[
\frac{dS}{S} = \mu dt + \sigma dz
\]  

(3.1)

where \( S \) is the value of the stock, \( \mu \) is the drift term, \( dz \) is a Wiener process, and \( \sigma^2 \) is the instantaneous variance of the stock returns.

This process may be directly linked to the weak form property of the efficient market hypothesis which says that only past price histories are impounded in current security prices. No attempt will be made to amplify the efficient market hypothesis which is elaborately developed and documented by Fama (1970). Let it be sufficient to note that equation (3.1) is the limiting case of a Markov process in continuous time (Figlewski, 1977). This Markov process is important, because it models the efficient market hypothesis by requiring that the probability of a variable attaining a particular value in the future, given its current value, be independent of past values. Within the framework of option pricing, the specific Markov process of random walk in con-

* For the formal derivation see Black and Scholes (1973).
tinuous time was employed by BS to solve their pricing model. The critical point is that their solution is grounded in an efficient market hypothesis which allows for empirical testing of the consistency between theoretical and observed prices.

3.4 THE BLACK-SCHOLE'S MODEL

In deriving the BS option pricing formula the following "ideal conditions" must be assumed about the market for the stock and the option: (1) the markets are "frictionless", i.e., there are no transaction costs, (2) there are no hindrances to borrowing, short-selling, and divisibility of ownership rights in either the stock or the option, (3) the stock price follows a Geometric Brownian Motion through time, causing the distribution of possible future stock prices to be log-normal, (4) the market operates continuously, (5) the risk-free interest rate is known and constant, (6) the stock pays no dividends, and (7) the option can be exercised only at maturity.

The prior assumptions are necessary for the creation and continual adjustment of the hedged position in the stock and option as discussed in section 3.2. Any unanticipated price movement in the underlying stock (or option) may be costlessly hedged against by simply adjusting the proportion of the claim (or stock) held in the portfolio.*

* The continual adjustment process is often criticized as being unrealistic. However, as Buser (1979) noted, it is
The following notation set was standardized by Smith (1976) and will be used throughout this thesis. Additional terms will be added when new issues are addressed.

- $t$ - current date
- $t^*$ - expiration date
- $T$ - time to expiration ($t^* - t$)
- $B$ - price of a default-free pure discount bond
- $C$ - price of an American call option at $t$
- $c$ - price of a European call option at $t$
- $P$ - price of an American put option at $t$
- $p$ - price of a European put option at $t$
- $r$ - risk free interest rate
- $S$ - stock price at $t$
- $\rho$ - expected average rate of growth in the stock price ($e^{\rho T} = E(S^*/S)$)
- $X$ - exercise price of the option

The starred variables such as $P^*$, $p^*$, $S^*$, etc. refer to prices at $t^*$, the expiration date of the option.

In introducing the BS Model, the following observations from option pricing theory deserve mentioning. The major focus in deriving a specific formula for valuing a derivative security (i.e. one that is a function only of prices of other securities) initially centered on the stock call and important to realize that, in equilibrium, there is no need, indeed no incentive, to form the hedged position because it can earn no more than a riskless government bond. It is only necessary that market participants respect the pricing discipline of the model by realizing that even in the midst of transaction costs arbitrage profits can result if prices depart too far from model values.
put option -- especially the call option. These options are of two basic varieties: the European option which is exercisable only at the contract's maturity date and the American option which may be exercised at any date prior to expiration of the contract. Of the two, the European option is more easily solved, eventually resulting in a closed-end solution.

The essential element of the call option is that the holder has the right to buy (call) the stock for a specified exercise price from the seller. The put option holder, in contrast, has the right to demand the seller's purchase of the stock at a specified exercise price (i.e. he "puts" the stock to the seller). Because this thesis builds on the put option, it will be used as the example contingent claim.

For the European case, the put owner will only exercise his option to sell if the price of the security is below the exercise price. If the price of the security is above the exercise price at maturity, he will not exercise and, indeed, the contract is worthless. Notationally, the value of a European put on one share of stock is

\[ p = p(S, X, T) \]  \hspace{1cm} (3.2)

which at the date of expiration is

\[ p^*(S^*, X, 0) = \max (0, X - S) \]  \hspace{1cm} (3.3)
Because the value of the put option is a function of the stock price at the date of expiration, its value must depend on the probability distribution over the range of possible stock prices as of the expiration date. A critical assumption, therefore, for the pricing solution, is the selection of the stochastic process that will govern this distribution of stock prices. BS assumed the Geometric Brownian Motion diffusion process because it results in a log-normal distribution of prices at the end of any finite interval. Equipped with this assumption, a frictionless market, and the realization noted earlier that a perfectly hedged portfolio can only earn the riskless rate of interest, BS had the necessary ingredients for an analytical solution. By use of the Fundamental Theorem of Stochastic Calculus (Ito's lemma), BS were able to convert the total derivative of the hedged portfolio function into a differential equation that is solved by transformation into the heat exchange equation from physics. This rather advanced mathematical procedure results in a final solution of the following form

\[ p(S,X,T,r,\sigma^2) = X e^{-rT} \phi(d_2^P) - \Phi(d_1^P) \]  

where:

\[ d_1^P = \frac{-\ln(S/X) - (r + \sigma^2/2)T}{\sigma \sqrt{T}} \]
\[ d_2^P = d_1^P + \sigma \sqrt{T} \]

10 A complete derivation will be presented in Chapter 4.
$N(.)$ is the cumulative normal density function and $\sigma$ is the variance rate per unit time for the logarithmic rate of return on the stock. Just as the derivation of equation (3.4) is quite formidable, the underlying economic principles are equally impressive in their implications regarding market behavior.

3.5 **RATIONAL DEFAULT BEHAVIOR**

Observation of equation (3.4) reveals that only five input variables are necessary to obtain a solution: the stock price, the riskless rate of interest, the exercise price, the time until expiration, and the variance on the underlying asset. These variables are directly observable except for the asset's variance rate which can be reasonably approximated. Thus, the formula can be empirically tested and, if proven satisfactory, used to price securities that are equivalent to or contain a put option.

From a theoretical point, particularly in this thesis, it is even more important to note the absence of either the expected return on the stock or investors' preferences from the solution. This is central to the theme of rationally priced mortgage default insurance. If equation (3.4), or some hybrid thereof, can be equated to default insurance, then a formal argument will exist for the omission of the mortgagor's income (or any other proxy for the borrower's preference structure) in modeling and explaining the default
premium. In the next chapter mortgage default insurance will be specifically equated to the put option.

3.6 EMPIRICAL TESTS

Before examining the insurance/put parity, the favorable results from empirical tests of the option pricing formula are given as credence for further pursuit of this analysis. Two studies have produced generally positive results. BS (1972) used the diaries of an option broker from 1966 to 1969 to test whether (1) model prices are on average too high or too low (2) market efficiency exists and (3) the model can be used to price options. Although difficulties may be encountered when estimating the variance of the stock ex ante, it appears that the model performs quite well when using the observed variance ex post. Galai (1975) repeated and extended the study of BS, also with favorable results.

3.7 EXTENSIONS OF OPTION PRICING

Solving the option valuation equation was not the only contribution of Black and Scholes' (1973) seminal paper. Equally important was the assertion that the call or put formula could be extended for the analysis of other important issues in finance.

For example, the equity of a levered firm may be viewed as a call option purchased from the bondholders. After
selling the firm to the bondholders, the stockholders purchase an option to buy the firm back on or before the bond's maturity date. As a result, changes in the capital structure of the firm will effect the exercise price and the distribution of the firm's value between the stockholders and the bondholders. BS also note that the default risk in corporate bonds may be isolated by subtracting the value of the bonds given by their formula from the value of a riskless bond with the same maturity date and face value. The list of applications continues to grow.\footnote{Examples of recent applications of the option pricing model include the pricing of: corporate debt (Merton, 1974), savings, callable, and retractable bonds (Brennan and Schwartz, 1977), dual purpose funds (Ingersoll, 1976), and mortgages (Asay, 1978).}

3.8 CONCLUSIONS

This chapter has briefly traced the development of the BS option pricing model from its origin in contingent claim analysis. It was shown to be fundamentally grounded in the tradition of the MM propositions and the efficient market hypothesis. The model is empirically robust with observable parameters. Only the underlying asset's variance is indeterminate, but is easily estimated from historical price movements. Tests of the model have generated favorable results.
Extensions and applications of the model have recently been initiated into the literature. The diversity and depth of model applications offers promising contributions to the understanding of complex financial issues. The next chapter will extend the model to the pricing of mortgage default insurance.
Chapter IV
MORTGAGE DEFAULT INSURANCE AS A PUT OPTION

4.1 INTRODUCTION

In applying the BS model to the pricing of mortgage default insurance, this chapter will utilize a set of assumptions about the housing, mortgage, and insurance markets that are compatible with the BS assumptions about security markets. Although these initial assumptions may not appear to realistically capture all aspects of the housing related markets, they are important for making the analogous link to the BS option model. The following chapter will relax these assumptions to accommodate the institutional restrictions and dynamics of the new markets.

4.2 ASSUMPTIONS

Extending equation (3.4) to the modeling of default insurance requires the following assumptions:

1. Default insurance covers 100% of the value of the mortgage

2. Underwriting and initial processing of the insurance application is costless.

3. Claims against the house are payout protected.\(^\text{12}\)

\(^{12}\) Merton (1973) defined an option to be payout protected if, for a fixed investment policy and fixed capital structure, the value of the option is invariant to the
4. Default and the subsequent foreclosure process is instantaneous with no transactions costs to the insurer, borrower, or lender.

5. Retirement or default of the mortgage before maturity is prohibited.

6. The mortgage debt is equivalent to a pure discount bond (i.e., there is no periodic amortization of principal and interest) with a fixed maturity date.

7. The riskless rate of interest for the coverage period is constant.

8. House price changes follow a diffusion process with a known and constant variance.

8.3 OPTION STRATEGIES IN THE HOUSE TRANSACTION

Given these assumptions, consider the potential homeowner who purchases a house without borrowing, but hedges his position by simultaneously selling an option against the house. The option may be either a call or a put. Under the assumptions made, the value of the option will be a function of only the house price \( H \) and time \( t \), or \( W(H, t) \). If the number of options sold is equal to

\[
\frac{1}{W_H}
\]

(4.1)

choice of payout policy. For claims against the mortgaged house to be payout protected, any extraction of rental service flow must be exactly offset by maintenance expenditures. The capital structure is fixed by assumptions (1) and (2). Therefore, changes in the value of claims against the house must be due to expected real gains or losses from its price distribution at the end of some finite time period.
where the subscript refers to the partial derivative of $W(H,t)$ with respect to its first argument, then the homeowner's equity position will be

$$H - \frac{W}{W_H}. \quad (4.2)$$

The homeowner's return on his equity over a short time interval is

$$\Delta H - \frac{\Delta W}{W_H}. \quad (4.3)$$

BS note that the ratio of the change in the option value to the change in the stock, $\Delta W/\Delta H$, is approximated by $W_H$ when $\Delta H$ is small. Therefore, if the house price changes by $\Delta H$, as in (4.3), the option's price will change by $W_H \Delta H$. Because the homeowner has sold $1/W_H$ options, the price change in his long house position is exactly offset by his short option position

$$\frac{1}{W_H} W_H \Delta H = \Delta H. \quad (4.4)$$

At this juncture BS made the revolutionary observation that if the hedged equity position (4.2) is continuously maintained, then its return (4.3) is no longer risky, but in fact certain.\footnote{Even if the position is not adjusted continuously, BS note that the risk is small and consists entirely of risk} Because the homeowner can hedge his position
(4.2) to make it riskless, the returns (4.3) are certain and can only earn the riskless rate:

\[ \Delta H - \frac{\Delta W}{W_H} = (H - \frac{W}{W_H}) r \Delta t. \]  \hspace{1cm} (4.5)

If the house price is assumed to follow the diffusion process given by (3.1), then \( \Delta W \) can be expanded by Ito's lemma, the stochastic analog of Taylor's expansion in calculus, to give:

\[ \Delta W = W_H \Delta H + \frac{1}{2} W_{HH} \sigma^2 H^2 \Delta t + W_L \Delta t. \]  \hspace{1cm} (4.6)

Substituting (4.6) into (4.5) the return on the homeowner's hedged equity position is:

\[-(\frac{1}{2} W_{HH} \sigma^2 H^2 \Delta t + W_L \Delta t)/W_H = (H - \frac{W}{W_H}) r \Delta t \]  \hspace{1cm} (4.7)

which is a function of only known parameters. Simplifying (4.7) results in the partial differential equation governing the value of a European option on a non-dividend paying house:

\[ W_t = \frac{1}{2} \sigma^2 H^2 W_{HH} + r H W_H - W \]  \hspace{1cm} (4.8)

that can be diversified away by forming a portfolio of a large number of such hedged positions.
Equation (9.8) is a parabolic linear partial differential equation of the second order which requires boundary specifications to obtain an analytical solution. If the option is a call (c) it must abide by the boundary conditions:

\[
\begin{align*}
    c(H^*, t^*) &= H^* - X^* & \text{for } H^* > X \\
    &= 0 & \text{for } H^* \leq X
\end{align*}
\]  \hspace{1cm} (4.9)

BS solved equation (9.8) subject to (9.9) by transforming it into the heat-transfer equation of physics which has a known closed form solution:

\[
c(H, T) = H N(d_1^C) - X e^{-rT} N(d_2^C)
\]  \hspace{1cm} (4.10)

where

\[
\begin{align*}
    d_1^C &= \frac{\ln(H/X) + (r + \sigma^2/2)T}{\sigma \sqrt{T}} \\
    d_2^C &= d_1^C - \sigma \sqrt{T}
\end{align*}
\]

If the option is a put option (p) it must abide by the boundary conditions:

\[
p(H^*, t^*) = 0 \quad \text{for } H^* \geq X
\]

\[
=p(H^*, t^*) = X - H^* \quad \text{for } H^* < X
\]  \hspace{1cm} (4.11)

To solve equation (4.8) subject to (4.11), BS note that if a put and call on the same stock have duplicate maturities and strike prices then the difference between their values must also obey (4.8) but with the boundary condition:
\[
\begin{align*}
c(H^*, t^*) - p(H^*, t^*) &= H^* - X \\
(4.12) 
\end{align*}
\]

The solution is

\[
\begin{align*}
c(H, T) - p(H, T) &= H - Xe^{-rT} \\
(4.13) 
\end{align*}
\]

and the value of the put upon rearranging is

\[
\begin{align*}
p(H, T) &= c(H, T) - H + Xe^{-rT} \\
(4.14) 
\end{align*}
\]

Substituting the value of \(c(H, t, X)\) from (4.10) into (4.14) results in the European put valuation formula:

\[
\begin{align*}
P(H, T) &= Xe^{-rT}N(d^2) - HN(d_1^P) \\
(4.15) 
\end{align*}
\]

where

\[
\begin{align*}
d_1^P &= \frac{-\ln(H/X) - (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\
d_2^P &= d_1^P + \sigma\sqrt{T} 
\end{align*}
\]

Now, consider the case of an individual who borrows funds to buy a house by issuing a single homogeneous mortgage bond. From section 3.7 on extensions of option pricing, the homeowner can be perceived as having arranged the sale of the house to the banker who simultaneously writes (sells) a call to the borrower on the house.\(^{14}\) If on the ma-

\(^{14}\) Although the lender is the true homeowner, the borrower,
turity date of the mortgage, the value of the house, $H^*$, is greater than the promised repayment on the mortgage, $M^*$, the homeowner will exercise his call by paying off the mortgage (even if contracting to sell the house is necessary to raise the funds). However, if on the maturity date the value of the house is less than the promised mortgage payoff, the rational homeowner will default on the mortgage. By not paying off the mortgage, the borrower would forego a loss equal to the difference between the loan payoff and the proceeds from sale of the house.

Because the mortgage's value is dependent on the price distribution of the house at the date of maturity, it, too, may be priced as a contingent claim. Notationally, the mortgage value at maturity is

$$M^* = \min (H^*, M^*)$$

(4.16)

As long as a positive probability exists that the value of the house may fall below the mortgage balance at maturity, there is a positive probability of default. Because the banker is a lending specialist and not a speculator in real estate markets, he prefers to have an underwriter, the mortgage insurer, incur the trading risk. The insurer provides

who owns the call option, will be referred to as the homeowner throughout this thesis.

15 See Asay (1978) and Buser (1979) for a contingent claims approach to pricing mortgages.

16 This is not to imply that the mortgage insurer incurs all the trading risk. He underwrites only that portion of
a guarantee with terms promising to provide the mortgage repayment in the event the homeowner decides to default his home to the mortgage insurer. Like any insurance policy, this guarantee has value to the insured (the lender) which must be at least actuarially equal to the insurer's cost.

Two points on the above discussion are worthy of note. First, the homeowner and not the banker (the insured) will pay the insurance premium. This is a contracting cost incurred by the banker which, like all other contracting costs, will be passed on to the borrower in competitive lending markets. Second, observe that the banker is no different from a security trader who "straddles" his long position in a stock by simultaneously writing a call and purchasing a put. He cannot benefit from possible upside price movements of the house because the homeowner will "call" the house by paying off the mortgage. Neither can he lose from downside price movements because he has the right to "put" the house to the insurance company if its value falls below the mortgage value and the borrower defaults. As a result, the banker earns a lending rate of return equal to the riskless rate of interest plus a premium to cover the costs of efficiently contracting the loan.

The probability distribution which lies to the left of \( N^* \). The homeowner is the primary speculator because he owns the call which is on that portion of the distribution to the right of \( N^* \).
To determine the cost of the guarantee, reexamine the payoffs to the lender’s claim at maturity, equation (4.16), after purchase of the default insurance. If the value of the house exceeds the mortgage balance, then the lender receives $\text{H}^*$ and the homeowner receives $\text{H}^* - \text{M}^*$. If the value of the house is less than the mortgage balance, then the lender still receives $\text{H}^*$, the homeowner receives nothing, and the mortgage insurance company receives $\text{H}^* - \text{M}^*$ which is negative, or a positive loss equivalent to $\text{M}^* - \text{H}^*$.

In essence the homeowner owns a call, $c(\text{H}, \text{M}^*, T)$ which at maturity pays

$$c^* (\text{H}^*, \text{M}^*, 0) = \max(0, \text{H}^* - \text{M}^*)$$

(4.17)

The mortgage insurance company has a non-positive claim on the house, $I(\text{M}, \text{M}^*, T)$, which is $\min(0, \text{H}^* - \text{M}^*)$ at maturity. In effect, if the house is put to the insurer, he will pay out $-\min(0, \text{H}^* - \text{M}^*)$ to the banker at maturity, which may be rewritten as

$$I(\text{H}^*, \text{M}^*, 0) = \max(0, \text{M}^* - \text{H}^*)$$

(4.18)

plus the proceeds from selling the house if $\text{H}^* < \text{M}^*$. The banker receives $\text{M}^*$ from either the homeowner or the insurance company; his debt is riskless.

By comparing equation (4.18) with equation (3.3) we observe that the payoff structures are identical. Under the simplifying assumptions, mortgage insurance must be equiva-
lent to a put option. The mortgage balance due, $M^*$, corresponds to the exercise price, $X$, for the stock option, and the value of the house, $H$, corresponds to the value of the stock, $S$. The mortgage insurance company has sold a put option to the banker, which gives the banker the right to "put" the house to the insurer for a guaranteed selling price, $M^*$, on the maturity date of the mortgage if its value is less than the promised mortgage payoff.

Using the identical arguments employed by Black and Scholes to derive the value of a European put option, equation (4.15), one can derive a valuation formula to price mortgage default insurance:

$$I = (H, M^*, T, \sigma^2) = \frac{M^* e^{-rT} N(d_2^P) - HN(d_1^P)}{M^*}$$

(4.19)

where $M^*$ is equivalent to $X$.

$H$ is the current value of the house, $M^*$ is the mortgage balance both now ($t$) and at the payoff date ($t^*$), and $\sigma^2$ is the variance rate per unit time for the logarithmic changes in the value of the house.\(^{17}\) Equation (4.19) is identical to

\(^{17}\) The assumption of short selling and divisibility of ownership rights in real estate could be accomplished through the trading of positions taken by partnerships or Real Estate Investment Trusts.

\(^{18}\) Merton (1977) proposed a similar formulation for the pricing of deposit insurance for commercial banks. In his equation, $H$ would represent the aggregate value of the
(3.4) except for the additional transformation to standardize the results as a percentage of the outstanding mortgage balance.

4.4 **COMPARATIVE STATICS**

The comparative statics of the default insurance contract are:

\[
\frac{\partial I}{\partial H} < 0; \frac{\partial I}{\partial r} > 0; \frac{\partial I}{\partial \sigma} > 0; \frac{\partial I}{\partial T} < 0. \tag{4.20}
\]

The economic intuition for each case is argued along the following lines. A rise in the value of the house diminishes the likelihood that the outstanding mortgage balance will exceed the value of the house, thus reducing the value of "putting" the house to the insurer. Likewise, an increase in the riskless rate of interest will lower the present value of the mortgage, widening the difference between the house value and mortgage value and lowering the value of the put. If the mortgage value is increased, then the gap between house and mortgage value is lessened. The put's value increases since the likelihood of the homeowner defaulting is increased. As the variance on the house returns increases, the area under the lognormal distribution of terminal house values is shifted. The region under the tail to the left of the mortgage balance (exercise price) expands.

The bank's assets, with a variance of \( \sigma^2 \), and \( M_\ast \) would represent the aggregate value of the bank's liabilities.
The probability of default increases and the value of the insurance premium is increased.

Increasing the time to maturity has two effects. Sufficient time is necessary to allow for the probability of the house value falling below the mortgage value, which increases the value of the insurance premium. However, increasing the time to maturity also lowers the present value of the mortgage or exercise price, which lowers the value of the insurance premium.¹⁹

4.5 DELINQUENCY, DEFAULT, AND DEFAULT PRICING

Adapting the option pricing model to default insurance clarifies the relationship between payment delinquency behavior, default behavior, and the actual pricing of default protection. Delinquent payments may result from liquidity constraints imposed on the homeowner. This occurrence is often associated with loss of income from job disposition. Numerous studies (see footnote 6) have attempted to estimate and predict delinquency behavior by including proxy variables for the homeowner's exposure to such liquidity constraints.

However, it must be realized that a delinquent payment simply triggers a contractual mortgage clause which allows the homeowner to impose short term lending performance on

¹⁹ This ambiguity only exists for the European case. As will be seen in the simulation results, the possibility of early exercise on an American put causes the sign to always be positive.
the banker. For such performance the banker earns a "penalty fee" rate of return. The homeowner is generally given 90 days to remedy (most likely by arranging alternative financing) his delinquent account before foreclosure proceedings are commenced.

Although the illiquidity of the homeowner may result in delinquent payment behavior, the rational homeowner will not default. Default is the allowance of foreclosure proceedings. Such action will only occur if the value of the insurance in equation (9.18) is positive which requires \( M^* \) to be greater than \( H^* \). In essence the default decision is a disinvestment decision which, in conformity with the MM separation theorem, is independent of the delinquency (financing) decision.

The critical argument of this thesis states that only the property and financing variables need be considered when pricing default insurance. Because of the argument presented above, the borrower characteristics are irrelevant and thus omitted from the default insurance pricing model. However, it can be shown that even if the borrower were to irrationally default due to liquidity considerations, the price of the default insurance would be unaffected.

Consider the homeowner who has become unemployed. The value of his home is unaffected such that \( H^* > M^* \). If he acts irrationally and allows foreclosure, then the banker owns an insurance put, equation (4.18), with zero value:
The banker will not put the house to the insurance company upon foreclosure. Instead, he will keep the house, sell it, and earn an arbitrage profit in the amount of \((H^* - M^*)\). The price of I, sold by the insurance company, remains unaffected.\(^{20}\)

If borrowers act rationally, the findings in prior studies of default behavior that borrower characteristics are significant explanatory variables of default behavior would appear overstated.\(^{21}\)

For the most part these studies have focused on micro factors. The major limitations to using the results of these studies to estimate the factors... is that actual foreclosure experience, while clearly a function of micro factors, will be conditioned by the macro environment. Specific coefficients attached to specific micro variables are likely to be different macro environments.\(^{22}\)

\(^{20}\) In most states the "arbitrage profit" must be returned to the homeowner after the banker's foreclosure expenses have been deducted. Even so, the price of I remains unaffected, because the insurer has incurred no default costs.

\(^{21}\) Borrower characteristics will be examined in Chapter VIII to determine if they might provide rational incentives for default. The initial model will assume they only impose costless liquidity constraints.

\(^{22}\) See Swan (1982).
By adapting the option pricing model, the macro variables are implicitly captured in the variance rate on house returns. For example, exogenous economic shocks will shift demand and cause greater house price variance from the expected mean growth rate. The default decision must remain a consideration of only the property and financing variables. More importantly, from the perspective of this thesis, even the indiscriminate inclusion of borrower characteristics in the pricing model would have no impact on prices. Because of the arbitrage profit opportunities conveyed to the banker in the event of irrational default, insurance premia can only be priced under rational assumptions.

4.6 CONCLUSIONS

This chapter has formally equated mortgage default insurance to that of a European put option written on the underlying asset, the house. As a result its price behavior should be governed by only the property and financing variables.

Assumptions regarding the nature of underwriting, mortgage amortization, coverage depth, housing service flow, and default timing were overly simplified in order to conform to the assumptions of the BS model. Even under this strict set of assumptions, the application of the option pricing model provides a new focus to default behavior. Borrower characteristics are shown to be irrelevant for the prediction of default as well as the pricing of default insurance.
Chapter V
MODEL MODIFICATIONS

5.1 INTRODUCTION

The default insurance model presented in Chapter 4 was developed under a rigid set of simplifying assumptions. This allowed a one to one mapping of the assumptions made by the BS model. It also allowed for a simple intuitive explanation of the put option nature of the default insurance contract. This chapter will relax the prior assumptions to accommodate the functions of the default insurance market, mortgage market, and housing market. The impact of each market restriction on pricing behavior may then be investigated.

5.2 UNDERWRITING AND APPLICATION PROCESSING COSTS

A closer examination of the MGIC quotation schedule (figure 1) reveals specific information about the underwriting and processing costs incurred by the insurer during the application process. Note (2) of the MGIC schedule states that a seasoned loan, without any history of delinquent payments may be insured at the rates quoted in the annual column less 0.25%. This reduction is allowable across all coverage levels except that no derived premium may fall below a
minimum of 0.25%. Since this allowance is a fixed amount, it seems logical to conclude that underwriting and processing costs incurred on new, unseasoned mortgages must be at least 0.25%. By comparing the annual rates quoted in figure 1 to a maximum underwriting fee of .25 percent, the following heuristic was used to price the underwriting costs (φ):

\[
\phi = \begin{cases} 
0.25 & \text{for } I_0 \geq 0.50 \\
I - 0.25 & \text{for } 0.25 \leq I_0 < 0.50 \\
0 & \text{for } I_0 \leq 0.25.
\end{cases}
\]  

(5.1)

The first year's insurance premium including underwriting costs is now:

\[
I = I(H, M^*, T, r, \sigma^2) + \phi.
\]  

(5.2)

5.3 **LIMITED COVERAGE**

As noted in Chapter 2, private mortgage default insurance provides only limited coverage to the banker in the event the borrower defaults on his mortgage. Referring to figure 1, observe that the insurance premia are quoted, in part, as a function of the coverage percent. Assuming that the lender incurs no other expenses in the process of filing a default claim, the amount of reimbursement paid by the insurer is limited to the quoted percentage times the outstanding mortgage balance. This partial coverage on the top
0 percent of the mortgage is equivalent to the difference between full coverage on the mortgage and full coverage on a second mortgage with an outstanding balance equal to \((1-\theta)\) percent times the balance of the first mortgage (Masulis, 1981). This is simply the difference between two put options with the same maturity written on the same asset. In the jargon of the option trader, this strategy is referred to as a "vertical spread." If the price moves against the short option position losses are limited by the gains from the long position on the second "far out" option. In general form this may be expressed as:

\[ I = I'(H, M^*, T, r, \sigma^2) - I''(H, (1-\theta)M^*, T, r, \sigma^2) + \phi \] (5.3)

which at expiration is

\[ I^* = \text{Max}[0, M^*-H^*] - \text{Max}[0, (1-\theta)M^*-H^*] + \phi \] (5.4)

or a present value today of

\[ I = \frac{M^*e^{-rTN(d_2^P)} - N(d_1^P)}{M^*} - \frac{(1-\theta)M^*e^{-rTN(d_1^P)} - N(d_1^P)}{M^*} + \phi \] (5.5)

which reduces to:

\[ I = \frac{M^*e^{-rT}[N(d_2^P) - (1-\theta)N(d_2^P)] - H[N(d_1^P) - N(d_1^P)]}{M^*} + \phi \] (5.6)

where

\[ d_1^* = [-\ln(H/(1-\theta)M^*) - (r + \sigma^2/2)T]/\sigma\sqrt{T} \]
\[ d_2^* = d_1^* + \sigma\sqrt{T} \]
For deep coverage percentages the value of the second put option will be minimal and the value of the limited insurance coverage (I) will not differ significantly from full insurance coverage (I'). Nevertheless, partial coverage prevents the lender from totally transferring the default risk to the insurer. This residual risk should partially explain why different debit rates are observed on different L/V mortgages even though they are "insured" against default.

5.4 SERVICE FLOW ON THE HOUSE

The original BS option pricing formulation assumed the underlying stock paid no dividends. For the home ownership case this is unreasonable, since the homeowner is clearly extracting some economic benefit, net rental services, which are analogous to dividends on stock. Merton (1973) has extended the BS model to include a special dividend policy - continuous payment at a fixed rate.

While such a policy is not totally acceptable for home-ownership, it does appear to have reasonable merit. For instance the homeowner, unlike the stockholder, possesses an asset that continually provides dividends. Whether one is sleeping, recreating, or working, the house is rendering a service to the owner. The fixed payout rate is not as realistic for two reasons. First, the value of incremental units of service flow will diminish to the homeowner as in-
creessional units of house are purchased. Unlike common stock investments, more housing units do not generate constant rates of return. Marginal service flow from more house is a diminishing return for nonpecuniary reasons. At some point another bedroom provides little benefit to the homeowner. Second, it is possible for the amount of net cash service flow extracted to be increased (decreased) by simply decreasing (increasing) the amount of house maintenance per unit time.

Examination of the homeowner's user cost of capital will help integrate these different dividend considerations into the BS model. Consider, for example, Hendershott and Hu's (1982) expression for the real user cost-of-capital:

$$ R = \frac{[ (1-\tau) r_m - q + \frac{H_k}{H} \hat{d} + (1-\tau) \tau_c ] H_k / P}{P} \tag{5.7} $$

where

- $R$ = implicit rent during first period
- $r_m$ = mortgage interest rate
- $q$ = periodic inflation rate
- $\hat{d}$ = periodic depreciation rate
- $H_k$ = purchase price of house including land
- $H$ = current house value
- $P$ = general price level
- $\tau$ = homeowner's marginal income tax rate

---

23 This is their simplified expression which assumes the after-tax rate of return equals the after-tax mortgage rate, the general inflation rate and housing inflation rate are equal, and there are no selling costs.
Setting taxes equal to zero, one can obtain:

\[ \frac{R}{H_k} - \delta + q = r_m \]  

where \( \delta \) equals \( \frac{H_k}{H} \hat{d} \), a gross depreciation rate (economic depreciation plus maintenance). This gross depreciation rate is a function of a maintenance rate \( (m) \). For this reason it is helpful to write:

\[ \delta = d(m) + m \]  

where \( d \), the economic depreciation rate, is a negative function of \( m \) the maintenance rate. Optimizing homeowners will select the level of maintenance that minimizes the gross depreciation rate.\(^2\) Initially \( \frac{\partial \delta}{\partial m} \) is < 0, but at some level of \( m \frac{\partial \delta}{\partial m} \) becomes > 0. By forming the same riskless hedge position as developed in equation (4.2) \( r_m \) becomes \( r \). Writing the net service flow, \( \frac{R}{H_k} - \delta \), as \( s \), and rearranging (5.6) results in:

\[ r = q + s. \]  

---

\(^2\) This optimizing condition is only true if the homeowner expects the probability of his defaulting to be zero. If the expected probability of default increases (i.e. if his equity position were to diminish), the homeowner will perform less maintenance at the expense of more economic depreciation. The possibility of this endogenous maintenance response will be explored in section 7.5.
Herton (1973) extended the option pricing model to include a dividend rate that was constant and continuous. He utilized an expression similar to (5.10) where \( q \) equaled the expected rate of growth in the stock and \( s \) equaled the dividend rate. The resulting solution is the same as equation (4.19) with two differences. First, a discount factor of rate \( s \) is multiplied times the house value. Second, \( r \) is adjusted by \( s \) in the cumulative normal density function. If the gross depreciation rate is assumed constant, the insurance premium, including limited coverage and underwriting costs, becomes:

\[
I = \frac{M^s e^{-rT}N(d_2) - He^{-sT}N(d_2')}{M^*} - \frac{(1-\theta)M^s e^{-rT}N(d_2') - He^{-sT}N(d_2')}{M^*} + \phi \quad (5.11)
\]

where:

\[
d_1^P = [-\ln(H/M^*) - (r - s + \frac{1}{2} \sigma^2)T]/\sigma \sqrt{T}
\]

\[
d_2^P = d_1^P + \sigma \sqrt{T}
\]

\[
d_1^' = [-\ln(H/(1-\theta)M^*) - (r - s + \frac{1}{2} \sigma^2)T]/\sigma \sqrt{T}
\]

\[
d_2^' = d_1^' + \sigma \sqrt{T}
\]

The comparative static for the net service flow variable is \( \partial I/\partial s > 0 \). Holding maintenance and nominal interest rates constant, increases in the net service flow rate occur only in response to a decrease in the expected inflation rate. Thus, the expected value of the house at the
maturity date of the mortgage is lower, default is more probable, and the value of the insurance is increased.

Because net service flow extracted from the house cannot be empirically observed, it must be estimated. Asay (1978) in his work on mortgage pricing, estimated $s$ by selecting that value which minimized the mean squared error term between predicted mortgage debit rates and observed mortgage debit rates. On average, that rate was found to be 5.8 percent. Future investigation and simulation work will assume initially that the annual dividend rate is fixed at six percent.

5.5 **TAXES**

Schles (1976), Ingersoll (1976), and Black (1975) have addressed the impact of taxes on the pricing of options. Scholes, in particular, has shown that taxable writers of options should price call options (put options) lower (higher) than the ES model value. "However, tax exempt investors and dealers in securities will still price options using the ES model."  

Assume initially that gains and losses on the sale of options are taxed at the ordinary rate, and that gains and losses on the sale of the underlying house are not taxed. For this case the returns to the writer of the option, equation (4.3), becomes:

\[ \text{---} \]

\[ \Delta H - \Delta W(1-\tau)/W_H \]  

which, after taxes, is no longer a hedged position. The government takes \( \tau \) of the gains and \( \tau \) of the losses, and thus \( \tau \) of the risk of such gains and losses. If instead the option writer sold \( 1/W(1-\tau) \) options, his return would be:

\[ \Delta H - \Delta W(1-\tau)/W_H(1-\tau) \]  

Substituting the expansion of \( \Delta W \) from equation (4.6) into (5.13) results in

\[-(\frac{1}{2}W_{HH}\sigma^2H^2\Delta t + W_t\Delta t)/W_H(1-\tau) \]

which is the same change in the hedged equity position as (4.7), except that the equity position is defined as:

\[ H = W/W_H(1-\tau) \]

In equilibrium, this change should equal the after tax riskless rate of return:

\[-(\frac{1}{2}W_{HH}\sigma^2H \Delta t + W_t\Delta t)/W_H = (H-W/W_H(1-\tau))r(1-\tau)\Delta t \]

which simplifies to:
Merton's (1973) examination of the proportional and continuous dividend case resulted in the same expression as (5.1) except that $\tau r$ was the dividend term. The solution is:

$$p(H, T) = M e^{-\tau r T N(d_2^p)} - H e^{-\tau r T N(d_1^p)}$$  \hspace{1cm} (5.18)$$

where

$$d_1^p = [-\ln(H/U*) - (r - \tau r + \frac{1}{2} \sigma^2)T]/\sigma\sqrt{T}$$

$$d_2^p = d_1^p + \sigma\sqrt{T}$$

for put options, and

$$c(H, T) = H e^{-\tau r T N(d_1^c)} - M e^{-\tau r T N(d_2^c)}$$  \hspace{1cm} (5.19)$$

where

$$d_1^c = [\ln(H/\text{M}*) + (r - \tau r + \frac{1}{2} \sigma^2)T]/\sigma\sqrt{T}$$

$$d_2^c = d_1^c - \sigma\sqrt{T}$$

for call options. In this world of ordinary taxes on option gains and losses, put options would be systematically priced higher and call options would be systematically priced lower.

If capital gains taxes are considered on the gains and losses from the sales of the house, then Scholes has shown that (5.17) would become:

$$W_t = rW - (r(1-\tau)/(1-\tau_C))HW_H - \frac{1}{2} \sigma^2 H^2 W_{HH}$$  \hspace{1cm} (5.20)$$
which can be rewritten:

\[ W_t = rW - \left[ r - (\tau - \tau_c) \frac{r}{(1-\tau_c)} \right] W H^{-\frac{1}{2}} \sigma^2 H^2 W_{HH} \quad (5.21) \]

where \( \tau_c \) equals the long term capital gains tax rate. The discount factor for \( H \) in the closed form solution to (5.21) would be \( e^{-[\tau - \tau_c] r / (1 - \tau_c)} \). The existence of capital gains taxes in combination with ordinary taxes continues to cause call options to be systematically underpriced and put options to be systematically overpriced. It is important to note that if the capital gains tax rate \( \tau_c \) equals the ordinary tax rate \( \tau \), equation (5.20) will reduce to the no tax case of equation (4.8). For a dealer in options and the underlying asset there would be no tax effect.

If the underlying asset pays proportional dividends that are taxable, the returns to owning the stock and writing \( 1/W (1-\tau) \) options are, after adjusting equation (5.13):

\[ \Delta H = \delta H (1-\tau) - \Delta W (1-\tau) / W_H (1-\tau). \quad (5.22) \]

Substituting for \( \Delta W \), the value of the option is

\[ W_t = rW - (r - \tau r - \delta + \tau \delta) \frac{\sigma^2}{2} H W^{-\frac{1}{2}} \sigma^2 H^2 W_{HH} \quad (5.23) \]
which reduces to the no tax dividend case if the dividend yield is equal to the rate of interest. In the analytical solution to (5.23) $H$ would be discounted by $e^{-(\tau r+\delta - r\delta)T}$; calls are underpriced and puts are overpriced.

If the dividends are not taxable, then (5.23) becomes

$$W_t = rW - (r-\tau r-\delta)HWH - \frac{1}{2} \sigma^2 H^2 W_{HH}$$

and the discount factor for $H$ in the analytical solution is even smaller, $e^{-(\tau r + \delta)T}$.

These are critical observations for the pricing of mortgage default insurance. Table 1 summarizes the alterations made to the discount factors for each tax case presented above. In order to evaluate the impact on the insurance price, the tax position of the banker, the insurer, and the homeowner must be considered.

The banker, who owns a covered call position in the house, sells a call to the homeowner and buys a put from the insurer. The call is the downpayment on the house, and is not considered taxable income to the banker. The purchase price of the put is treated as an expense. However, this insurance expense is passed on to the borrower in the form of a higher debit rate. The resulting increase in interest revenue is exactly offset by the expense of the insurance purchased; the two are a net wash. The banker is effectively a tax exempt agent on transactions involving the insur-
### Table 1: Impact of Taxes on Model Discount Factors

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No Dividends</td>
<td>Taxed</td>
<td>No Dividends</td>
<td>Taxed</td>
<td>Dividends not</td>
<td>Taxed</td>
<td>Cap. Gains</td>
</tr>
<tr>
<td>( \Pi^t )</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( \Pi^h )</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>e^{-rT}</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

### Numerical Example

\( T = 1, \ r = .05, \ \delta = .10, \ \gamma_H = .5, \ \gamma_L = .2, \ \gamma_C = .4(t) \)

<table>
<thead>
<tr>
<th>Homeowner</th>
<th>( \gamma_H )</th>
<th>( \gamma_L )</th>
<th>( \gamma_H )</th>
<th>( \gamma_L )</th>
<th>( \gamma_H )</th>
<th>( \gamma_L )</th>
<th>( \gamma_H )</th>
<th>( \gamma_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H-He = -.025</td>
<td>H-He = -.01</td>
<td>H-He = -.125</td>
<td>H-He = -.11</td>
<td>H-He = -.075</td>
<td>H-He = -.075</td>
<td>H-He = -.01</td>
<td>H-He = -.01</td>
</tr>
</tbody>
</table>

**Notes:**
- \( \gamma_L \) = Low tax bracket
- \( \gamma_H \) = High tax bracket
- \( H = He^{-rT} \)
The insurer is the option writer. He sells the insurance contract and recognizes revenues on the sale. Analyzing the price impact of taxes to the insurer requires consideration of how gains and losses would be taxed to the insurer who literally forms the riskless hedge. By going long perfect substitute houses and writing the insurance put options, the insurer would receive rent (dividends) from the long house position. Unlike the homeowner, the insurer would incur taxes on the rent revenues. In addition, discrete adjustments in the hedge position, to keep it riskless, would result in short term gains and losses from the sale of substitute houses, taxed at the ordinary rate. In summary, the insurer would incur ordinary taxes on rents, option (insurance) sales, and gains or losses from house sales.

The homeowner buys two options, a call from the banker and a put from the insurer. He also receives the dividends from the underlying house. The service flow or dividends are well known to be tax exempt benefits. If he exercises the call or sells it (by selling the house or paying off the mortgage) there are also no tax consequences. From sections 1034 and 121 of the internal revenue code, gains may be deferred by buying a new house or exercising the one time capital gain exclusion on a residential home.
If the insurance expires, the cost of the insurance cannot be recognized as a tax deductible expense. If he exercises his insurance contract, he puts the house to the banker. Unlike options traded on stock securities, there is no gain recognized for tax purposes even though the house value is less than the exercise price. In such a case he will actually recognize a loss equal to his equity position in the house. The tax benefits of such a loss should be priced in the call option, via the mortgage debit rate, not the put option.

If a homeowner thought insurance rates were mispriced, he too could form the riskless hedge and earn arbitrage profits. However, utilizing the hedge ratio is more difficult for the homeowner. In order to retain "homeowner status" for tax purposes, he could only go long his primary residence. Going long perfect substitute houses would not qualify for tax deferral on rollover purchases or the one time tax exclusion. Assuming he does go long one primary house, he could capture arbitrage profits by writing mispriced put options against the house. In addition, his service flow rents (dividends) would be tax free as well as any capital gains incurred from adjusting the hedge by selling the house and buying another appropriate size replacement. In summary, the homeowner pays no taxes on the dividends or service flow and no taxes on gains or losses from house sales. He would pay taxes on options written against the house.
5.5.1 Tax Specific Cases

Referring to table 1, columns three and seven depict the tax case for the insurer as discussed above. Columns four and six show the tax case for the homeowner. That is, the insurer incurs ordinary taxes on rents and capital gains, whereas the homeowner does not. The numerical example in table 1 assumes that the dividend rate is greater than the riskfree rate. Compare column four results to column three results (the dividend situation) and column six results to column seven results (the capital gains situation). For both cases the homeowner will price the insurance higher than the insurance company. Thus, the homeowner has incentive to be a buyer of insurance coverage and the insurance company has incentive to be a seller of insurance coverage.

For the high-low tax bracket specific cases of the homeowner the results agree with the analysis of Miller (1977), and Rosen and Rosen (1980). High tax bracket individuals have greater incentive to take on mortgage interest expense which includes the default insurance premium. The special tax treatment of capital gains and service flow is also more valuable to the high tax bracket individual.

One unique results of the special tax treatments to a homeowner is that low tax bracket homeowners have more incentive (although to a lesser extent than high tax bracket homeowners) to buy default insurance than to write default
insurance. This is in contrast to the Scholes (1976) analysis where high tax bracket investors have incentive to be option writers and low tax bracket investors have incentive to be buyers. It appears that an insurer will always price the insurance lower than the homeowner; he has incentive to the option writer.

5.5.2 Implications for Insurance Premia

As shown above taxes are indeed relevant to the pricing of default insurance. However, because all market participants are not in the same tax bracket, it is impossible to know exactly which tax bracket should be used in the pricing model. Nevertheless, it is possible to show that homeowners always have incentive to be buyers of insurance and insurers always have incentive to be writers of insurance. Establishing that insurers always have incentive to be writers of insurance is a critical point for insurance pricing. Given this fact, and assuming that insurance markets are competitive, insurers become the price setters while homeowners are price takers. The equilibrium condition used in (5.10) to price insurance is actually

\[ r = q^f + s^f \]  \hspace{1cm} (5.25)

where the superscripts denote supply side determination. Insurers will price the insurance premia based on their opportunity costs of forming the alternative hedge position.
The net service flow, \( s^5 \), is no longer the homeowner's service flow but rather a landlord's rental flow. It captures both explicit rents as well as the cash flow benefits of the depreciation shield. The impact of alternative tax brackets is beyond the scope of this thesis. However, two points are established by this section's analysis of taxes:

1. The insurer sets the insurance price because of the unique tax treatment accorded homeowners.

2. The higher the insurer's marginal tax rate, the lower he will price the insurance.

In the simulation work the tax rate is set equal to zero. The reader should realize that this assumption will give an overpriced bias to the results presented in Chapter VI.

5.6 NON-ININSTANTANEOUS DEFAULT AND FORECLOSURE

In reality, the time duration from the moment of default to actual foreclosure is quite lengthy. In the process of foreclosing, the lender, the insurer, and the borrower incur significant transaction costs. According to the terms of the insurance policy, certain of these costs are added to the unpaid principal balance in the claim loss report filed with the insurer. As a result, the exercise price of the insurance put option is effectively increased by the addition of accrued interest, attorney's fees, accrued real estate taxes, and minimize property preservation expenses. These foreclosure costs may be categorized according to those incurred by the lender, the lender and the insurer, and the borrower.
5.6.1 Lender and Insurer Incurred Default Costs

It is important to note that while the additional default costs increase the exercise price that the lender may "put" to the insurer, they also increase the exercise price that the insurer may "put" to the lender via the second put option in the model. This second put will, therefore, also increase in value which reduces the net price of the insurance coverage to the lender.26

With foreclosure costs, the value of the insurance put at expiration would be:

\[ I^* = \text{Max}[0,(1+\alpha)M^*-H^*) - \text{Max}[0,(1-\varphi)(1+\alpha)M^*-H^*) + e^{\gamma T_\varphi} } \hspace{1cm} (5.26) \]

where \( \alpha \) = foreclosure costs as a percent of the mortgage balance.

The closed form solution for current value would be:

\[ I = (1+\alpha)M^*e^{-\gamma T_\varphi}N(d_2^D) - He^{-\gamma T_\varphi}N(d_1^P) + \frac{(1-\varphi)(1+\alpha)M^*e^{-\gamma T_\varphi}N(d_1^P)-He^{-\gamma T_\varphi}N(d_1^P)}{M^*} + \varphi \hspace{1cm} (5.27) \]

The comparative static \( \frac{\partial I}{\partial \alpha} > 0 \) is confirmed by simulation results which show a significant jump in the value of I.

26 Discussions were held with Mr. Richard Lynch, Vice president for claims at MGIC, in which he indicated that the additional foreclosure costs were generally about twelve percent of the mortgage balance.
5.6.2 **Lender Incurred Default Costs**

A portion of the lender incurred default costs (i.e. accrued interest, attorney's fees, etc.) captured by the variable \( \alpha \) are explicitly passed on to the insurer in the form of a higher exercise price. Certain of the default costs, however, cannot be passed on by the terms of the insurance policy. Specifically, the time the lender's employees spend on foreclosure procedures cannot be included in the claim loss report. Ceilings also exist in the policy as to how much in legal expenses and property maintenance costs may be maximally claimed. Because these costs cannot be charged to the insurer, they cannot affect the valuation of the first put. Lender costs do, however, affect the value of the second put by effectively lowering the net proceeds recovered from the insurer. Alternatively, these unclaimed costs could be considered an increase in the exercise price of the second put; coverage is effectively less than the nominal level.

At this point a more intuitively appealing theory of the second put in the pricing model deserves exploration. As originally presented, the limited coverage (\( \theta \)) of the insurance policy is adjusted for in the model by simply subtracting a second put from the first, but with a \( \theta \% \) lower exercise price. The significance of this second put is the realization that the insurer is hedging his short put posi-

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27 Mr. Lynch of MGIC conveyed the opinion of bankers that they "lose money" on defaults even with the insurance.
tion by going long a put from the lender. The price of the put purchased from the lender has a lower value than the put written by the insurer, which always result in a positive premium being paid to the insurer. The price of this second put must be passed on to the borrower in the form of a higher mortgage debit rate. The price will vary with coverage levels and L/V ratios. For a given coverage depth, the second put price explains why mortgage rates will vary across L/V ratios even with insurance coverage. However, it should be noted that risky high L/V loans can be compensated for with deeper coverage levels. In such a case the debit rates will be the same even though the L/V ratios are different.

Assuming that the lender's default costs are more a function of the value of the house than a function of the mortgage balance, the value of the insurance at expiration would be:

\[
I^* = \max[0, (1+\lambda)H^* - H^*] - [\max 0, (1-\phi)(1+\lambda)H^* - (1-\lambda)H^*] + \phi \tag{5.28}
\]

where \(\lambda\) = lender's default costs, as percent of \(H\), not chargeable to the insurer.

5.6.3 **Borrower Default Costs**

The final transaction costs considered are those incurred by the borrower. If the borrower defaults and he faces foreclosure on his property, then a rational decision must be made as to whether or not to "walk away" from the proper-
ty. In a world of frictionless markets the homeowner will only allow foreclosure to occur if

$$H^* - M^* < 0.$$  \hspace{1cm} (5.29)

With the transaction costs ($\alpha$) incurred by the lender and insurer upon default, as noted in section 5.6.1, the borrower's default will cost the insurer

$$H^* - (1 + \alpha)M^*$$  \hspace{1cm} (5.30)

The exercise price is higher and the default is more costly to the insurer and lender. Countering this increased cost of foreclosure to the lender and insurer is the borrower's consideration of any costs he might incur by allowing foreclosure. The borrower must consider what, if any, costs he will incur by allowing foreclosure. Moving costs, opportunity costs on time spent searching and moving to a new residence, and the psychic costs of default are all positive costs of incurring foreclosure.\(^\text{29}\) As a result the borrower will only default and allow foreclosure if

$$H^*(1 + \Delta) - M^* < 0$$  \hspace{1cm} (5.31)

\(^{29}\) Higher interest costs on future borrowing are intentionally omitted. In a rational market, lenders know that the borrower's default decision is based on a rational set of decision rules. Therefore, default does not reflect on the borrower's character and will not be impounded in his future borrowing costs. Future borrowing costs, just as historical borrowing costs, will be based only on the property and financial variables.
where $\Delta$ is the borrower's default costs as a function of $H$. This effectively reduces the cost of foreclosure to the lender and insurer to

$$H^*(1 + \Delta) - H^*(1 + \alpha). \quad (5.30')$$

At expiration the value of the insurance becomes

$$I = \max[0, (1+\alpha)H^* - (1+\Delta)H^* - \max[0, (1-\theta)(1+\alpha)M^* - (1+\Delta)(1-\lambda)H^*] + \phi \quad (5.32)$$

The borrower incurred default costs $\Delta$ offset the lender and insurer incurred default costs $\alpha$ driving the price of the insurance coverage back down. Naturally the offset may not be exact. However, for purposes of this thesis it is initially assumed that the borrower, lender, and insurer transaction costs upon default net out to a zero sum.

The existence of borrower default costs provides an interesting intuitive explanation for the observation that underwriting procedures used by bankers and insurers include the review of borrowers' credit reports. A poor credit report may serve as a proxy for lower borrower costs. Lenders and insurers prefer borrowers with high default costs. If the value of $\Delta$ in equation (5.31) were low, then the price of an insurance put for that application should be higher. However, because insurance premia are fixed, the insurer will observe the possibility of selling an under-priced put to the applicant. The rational insurer will not write the put under such conditions; default insurance be-
comes a rationed good. 29

5.7 DEFAULT PRIOR TO THE LOAN'S MATURITY

Most mortgage defaults occur in the early years of a mortgage's life. At such time the lender exercises his put option against the insurer prior to maturity. Thus, rather than owning a European put, the lender must possess an insurance contract equivalent to the American put. Merton (1973) evaluated the perpetual American put on a non-dividend paying stock and noted that for a sufficiently low stock price it becomes advantageous to exercise the put. He also noted that there was no closed-form solution for a finite time to maturity American put.

Brennan and Schwartz (1977) and Parkison (1977) have presented numerical procedures for solving the American put with finite maturity on a dividend paying security. This thesis will utilize the Brennan and Schwarz procedure to continue the investigation of default insurance.

Following the theory set forth in section 4.3, the value of the American put must, by arbitrage arguments, obey the partial differential equation

\[ k^2S^2p_{SS} + rSP_S - rP + P_t = 0 \]  

(5.33)

29 Default insurance is an integral part of the mortgage package. If insurance is a rationed good, then mortgage debt, by requiring insurance attachment, could become rationed.
where the subscripts denote partial differentiation. Brennan and Schwartz devised a numerical solution to (5.33) by approximating solutions to the partial derivatives with finite differences. The numerical integration is taken over (n) number of discrete increments (h) in the stock price and over (m) number of discrete increments (k) in the time to maturity of the option.

The put at each time to maturity is a function of the stock price:

\[ P(S, T) = P(S_i, T_j) = P(ih, jk) = P_{ij} \]  

where \( i = 1, 2, \ldots, (n-1) \)

\( j = 1, 2, \ldots, (m) \)

By starting at the date of expiration where

\[ P(S, T) = P_{i0} = \max(X^*-S, 0) \]  

Brennan and Schwartz rewrite (5.33) in terms of time to maturity (T) rather than calendar time (t) and devise a set of \( 'n' \) linear equations in \( (n+1) \) unknowns \( P_{ij} \):

\[ a_iP_{i-1,j} + b_iP_{ij} + c_iP_{i+1,j} = P_{i,j-1} \]  

(5.36)

\[ P_{n-1,j} - P_{n,j} = 0 \]  

(5.37)

---

\(^{30}\) See McCracken and Dorn (1964) for a discussion of these techniques.
where

\[ a_i = \kappa r k_i - k_i \sigma^2 k_i^2 \]
\[ b_i = 1 + r k + \sigma^2 k_i^2 \]
\[ c_i = -k r k_i - k_i \sigma^2 k_i^2 \]

By repeated solution of (5.36) and (5.37) subject to the initial boundary condition (5.34), a solution for \( P_{ij} \) is found in terms of \( P_{i'j-1} \).

The set of equations (5.36) and (5.37) is subject to four additional boundary conditions. The put can never fall below its potential exercise value,

\[ P_{ij} > \text{Max}(X_t - S_i, 0) \]  \hspace{1cm} (5.38)

it cannot exceed its current striking price,

\[ P_{ij} \leq X_t \]  \hspace{1cm} (5.39)

and its value can never fall below zero,

\[ P_{ij} \geq 0 \].  \hspace{1cm} (5.40)

Since the put is known to be a convex function of the stock price, and given boundary conditions (5.38) and (5.39), the value of the put must approach zero as the value of the stock greatly exceeds the striking price,

\[ P_{ij} = 0, \quad \lim_{i \to 0} \]  \hspace{1cm} (5.41)
Discrete dividends introduce a final boundary condition. The value of the put is equal to the greater of its immediate exercise value and its value after the dividend, (D):

\[ P_{ij} = \max[X_t - S_i, P(S_i - D, j^+)] \quad (5.42) \]

where \( j^- \) = the last instant cum dividend
\( j^+ \) = the first instant ex dividend.

In order to apply this procedure to the pricing of default insurance rates, certain modifications and boundary constraints must be added to the Brennan and Schwartz model. The most notable change results from the equivalence of the amortizing mortgage balance to the striking price.

5.8 PERIODIC AMORTIZATION OF THE MORTGAGE

The mortgage debt is clearly not a pure discount bond as originally assumed. Over time the borrower serially repays the obligation. Normally, on a monthly basis, the borrower makes a mortgage payment of which a portion is accrued interest expense and the remainder is principal repayment. This discrete reduction in principal is also equivalent to a reduction in the striking price of the put. The new exercise price after each payment is the familiar mortgage constant:

\[ M_t = 1 - \frac{(1+r_m)^t - 1}{(1+r_m)^n - 1} \cdot M_0 \quad (5.43) \]
where \( r_m \) = mortgage debit rate
\( t \) = the \( t \)th payment
\( n \) = total number of mortgage payments.

Integration over each point in time is now subject to the modified arbitrage condition:

\[
P_{ij} \geq \text{Max} (M_t - S_i, 0).
\] \hspace{1cm} (5.44)

In the initial years the striking price will decline only minimally, while in the latter years the striking price will fall dramatically. As a result, the declining exercise price should have negligible impact on the default insurance rate for the initial years of coverage.\(^{31}\)

Other modifications to the numerical procedure are the changes to the BS model noted in sections 5.2 and 5.3. Specifically, a second put to adjust for limited coverage of 0% is subtracted from the first put for each L/V category. Each put is also transformed to a percentage of the original

\(^{31}\) Geske (1979) analyzed the impact of leverage on stock options. He noted that as the debt-equity ratio of a firm changed due to the change in maturity of the firm's debt, so does the riskiness of the return to the firm's stock. Because stock options are written on the firm's stock (which is equivalent to a call option itself) rather than on the firm's assets, changes in leverage will affect the stock variance and thus the option price. Default insurance, however, is written against the underlying house asset and its returns are independent of any financing changes. Therefore, mortgage amortization which reduces leverage, will not affect the model variance rate.
mortality balance and underwriting costs are incorporated in
cconformity with the heuristic (5.1). The numerical procedure appears in Appendix 1.

5.9 STOCHASTIC INTEREST RATES

The riskless rate of interest is assumed to be known and constant. Merton (1973) has shown that if interest rates are assumed to be stochastic, then the variance in the pricing formula arises from two sources - the bond and the stock. The new instantaneous variance was shown to be:

\[ \hat{\sigma}^2 = \sigma_H^2 + \sigma_B^2 - 2\hat{\rho}_{HB}\sigma_H\sigma_B \]  \hspace{1cm} (5.45)

where \( \sigma_H^2 \) = the instantaneous variance on house returns
\( \sigma_B^2 \) = the instantaneous variance on returns of a discount bond
\( \hat{\rho}_{HB} \) = the instantaneous correlation coefficient between the returns to the house and bond.

Whether the insurance price increases or decreases will depend on both the variance of the bond returns and the correlation of its returns to the house returns. A high bond variance would increase \( \hat{\sigma}^2 \) and raise insurance premia.32 However, a higher correlation between bond returns and house returns would lower \( \hat{\sigma}^2 \) and lower insurance premia. Table 2 summarizes a few scenarios and the impact on \( \hat{\sigma}^2 \). Because the parameters used in this thesis are purely arbi-

---

32 If the mortgage has a non-assumable clause, the homeowner can only capture gains from rising interest rates if he continues to hold the mortgage.
trary, a reference to \( \hat{\sigma}^2 \) could also be thought of as \( \hat{\sigma}^2 \). The correlation between bond returns and house returns remains an empirical question and is left for future work.

**TABLE 2**

Impact of Stochastic Interest Rates on Model Variance

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<th>( \sigma_R^2 )</th>
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<th>( \sigma_{HB} )</th>
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</table>
5.10 ALTERNATIVE STOCHASTIC PROCESSES

Cox and Ross (1976, 1975) and Cox, Ross, Rubinstein (1976) have examined option valuation formula's written on assets with alternative stochastic processes. The Poisson and binomial processes have received particular attention. They specify the process as:

\[ \frac{dS}{S} = \mu dt + (k-1) \sigma d\pi \]  

where \( \pi \) is a jump process and \( d\pi \) has zero value with probability of \((1-\gamma) dt\) and has the value of one with \(\gamma dt\) probability. \( \gamma \) is referred to as the "intensity" or "rate of probability flow" for the jump. If a jump occurs, \( S \) changes by the "jump amplitude" of \(k-1\) to \(kS\). If there is no jump, then \( S \) changes by the rate \( \mu \).

By shortening the time interval of the jump, they have shown that the BS continuous model is simply the limiting case of the more general jump process model. Therefore, for valuation purposes, it appears that the results from the BS diffusion model would differ insignificantly from the results of a Cox, Ross, Rubinstein approach. This thesis will assume the house price follows a continuous diffusion process.
5.11 CONCLUSIONS

This chapter has examined each assumption required for the simple model presented in Chapter 4. Those assumptions considered overly restrictive were appropriately modified. The revised model assumes the following:

1. Initial underwriting and processing costs are approximately 0.25% of the original mortgage balance.

2. Limited coverage (θ) is equivalent to a second put with a (1−θ) exercise price. It is subtracted from the first put which has an exercise price equivalent to the mortgage balance.

3. The house pays constant dividends (service benefits).

4. Transaction costs incurred by the lender and insurer upon default are offset and effectively negated by the transactions costs incurred by the borrower if he defaults.

5. The insurance contract is American in nature due to the possibilities of default prior to mortgage maturity.

6. The mortgage debt is amortized monthly. Thus the exercise price declines monthly.

7. The riskless rate of interest is known and constant.

8. House price changes follow a diffusion process with a known and constant variance.

9. Ordinary and capital gain tax rates are equal to zero.

10. The variance rate on bond returns is zero.

In general form the model for the default insurance rate is:

\[ I = \frac{I'(S_i, M_T, T) - I'(S_i, (1-\theta)M_T, T) + \phi}{M_0} \]  \hspace{1cm} (5.47)
where $M_c$ declines according to (5.43), $\phi$ follows (5.1) and $I'$ and $I''$ satisfy the differential equation (5.33) subject to boundary conditions (5.35), (5.39), (5.40), (5.41), (5.42), and (5.44).
Chapter VI
SIMULATION RESULTS

6.1 INTRODUCTION

This section will present a sampling of numerical results from the initial model (equation 5.6) through the extended model summarized in general form by equation (5.48). The results discussed will focus on the pricing impact of (1) modeling default insurance as an American put versus a European put, (2) a declining mortgage balance, and (3) limited insurance coverage. Discrepancies between the simulated price patterns and observed price patterns are isolated. The next chapter will focus on these discrepancies in an effort to understand the market's expectations about house price movements. In addition, the comparative statics will be investigated in this chapter for sensitivity of the model to changes in the mortgage interest rate, the risk free interest rate, the variance rate of house price returns, and the dividend rate.
6.2 PRELIMINARY RESULTS - THE EUROPEAN CASE

In Table 3, the simulation results of using Equation (5.6) are presented. A comparison is made between the theoretical value of the insurance premium and the observed values of the premium quoted by MGIC (Figure 1). It should be noted that in this first run the variable values of $a^2 = 0.04$ and $r = 0.13$ were strictly arbitrary.

Setting the value of $T$ equal to one was not arbitrary. The prices quoted by MGIC in the annual premium column of Figure 1 are the prices for the first year's insurance premium. Coverage beyond one year, under the annual plan, must be purchased in each subsequent year as outlined by note 1 of Figure 1. The lender, if he elects the annual premium plan, buys an option with the right to purchase a future option at a guaranteed price. The prices quoted for subsequent years' premia are, for most coverages, a constant 0.25% of the declining balance or 0.24% of the original loan balance. Since the right to purchase the second option, the third option, the fourth option, etc., rests with the lender, there must exist an optimal cancellation strategy to follow which is a function of the original L/V ratio.

A study of Table 3 leads to certain assumptions about the borrowers' behavior in the pricing of insurance rates. In the lending process the borrower is confronted with the choice of selecting (via a downpayment) the L/V ratio of his choosing. However, insurance rates are assessed by L/V rang-
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<th>MGIC Coverage L/V Ratio</th>
<th>MGIC 1 Yr Premium</th>
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es of 5% (i.e., 90-95%, 85-90%, etc.). Equation (5.6) requires a specific L/V ratio. It is assumed that borrowers act rationally when confronted with an insurance requirement and always pay down just enough equity to fall in the top portion of the L/V range. For example, a borrower endowed with approximately 10% in equity and confronted with a 22% coverage requirement would be expected to always find sufficient funds for a 90% L/V mortgage. Failure to do so could result in a 91% L/V ratio with an insurance premium almost 50 percent higher. For an additional .01 in downpayment he gains, instantaneously, a savings of .0025. As a result, the computed values are all based on the top L/V ratio in each range.

The second observation from table 3 is that $I''$, the second put in equation (5.6), is indeed significantly smaller than $I'$, the first put. Thus, the simulation confirms the earlier conjecture that limited coverage is not significantly different from full coverage. However, as the coverage becomes more limited, the second put increases in value, lowering the cost of the net protection. For the most limited coverage (12%), it appears that the contractual agreement to not insure 88% of the mortgage balance reduces the first year's insurance premium by about 25%. For the highest coverage (30%), the contracting away of coverage on the bottom 70% appears to reduce the price of insurance by virtually nothing. In essence, the bottom 70% of a mortgage
loan, under the initial assumptions, would appear to be virtually riskless. Naturally, this finding may change as assumptions 3 through 8 are relaxed.

The third observation from the simulation results is the positive correlation between predicted values and observed values across coverage percentages and L/V ratios. It appears that even under the rather strict assumptions imposed, the model may serve as a viable predictor of observed values, if the assigned input values are reasonable. If true, and assuming no misspecification, then this first set of simulations seems to suggest some possible distortions are occurring in the pricing procedure. Specifically, the 22%, 20%, and 16%, coverages at the 91-95% L/V ratio appear to have the greatest distortions. Further investigation revealed that the government agencies FNMA (Federal National Mortgage Association) and FHLMC (Federal Home Loan Mortgage Corporation) require certain minimum levels of insurance on mortgages before they will purchase them in the secondary market. Interestingly, their minimum requirements exactly correspond to the coverages noted earlier. In addition, the underwriter of the Columbus, Ohio office of MGIC speculates that at least three-fourths of all mortgages insured by them are covered at one of the three minimum levels:

- 91-95% L/V ratio with 22% coverage,
- 86-90% L/V ratio with 17% coverage, and
- 81-85% L/V ratio with 12% coverage.
Government requirement of minimum insurance coverage is a potentially important element of the default insurance market. Most insurance transactions may conceivably transpire at these coverage levels. Given the volume of transactions, the insurance market at these levels should be most efficient and elemental to future study.

Using the 22/91-95, 17/86-90, and 12/81-85 coverages an implicit variance rate on annual house returns may be derived. For example, using the 22/91-95 price of .80% and a riskless rate of 13%, the implicit variance rate is approximately .023.

Asay (1978) estimated the variance rate of return on houses in Los Angeles County over the period of 1966 to 1976. His findings appear in figure 2. The implicit variance of .023 derived above appears too high when compared to Asay's estimated variance of .014. Using Asay's lower variance rate significantly reduces the predicted premium rates and suggests further investigation is needed to improve the models specification.
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Los Angeles County

*The Los Angeles County $\sigma^2$ and $\gamma$ were used for this sample in the study.*

Figure 2: Asay's Estimated Variance Rate
6.3 **EUROPEAN VERSUS AMERICAN PUT MODELING**

For the American put case the Brennan and Schwartz numerical procedure must be used. While selection of parameter values was arbitrary, consideration was given to current economic conditions and the findings of prior research (Asay, 1979). The parameter values assigned appear in Table 4.

**TABLE 4**

Initial Parameter Values and Variables

Used in Numerical Approximation

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<th>Value</th>
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<td>$\sigma_m^2$</td>
<td>0.0142</td>
</tr>
<tr>
<td>$D$</td>
<td>$5.5\text{/mo.}$</td>
</tr>
<tr>
<td>$H$</td>
<td>$80$</td>
</tr>
<tr>
<td>$T$</td>
<td>30 years</td>
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<td>1431</td>
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<tr>
<td>$h$</td>
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<tr>
<td>$m$</td>
<td>$n = 360\text{ mos.}$</td>
</tr>
<tr>
<td>$k$</td>
<td>0.0833</td>
</tr>
</tbody>
</table>

Note that: $nm = \text{maximum house price} = 143.10$

The interest rates selected were those in existence when this research initially began in October 1981, and the...
MGIC quote schedule was obtained. The variance rate and dividend rate were those estimated by Asay (1979) in his work on rational mortgage pricing.

Achieving a good approximation by the numerical procedure requires that the house price change (h) be significantly less than the dividends (D). The maximum house value (nh) chosen must provide a good approximation of boundary condition (5.40). In order to check the reasonableness of the model developed in chapter 5 as well as simulate the premium for early exercise rights, the model results were compared to prices calculated by the BS model. The exercise price was held constant and only the first put prices were calculated and compared. Results for the 90 L/V level appear in Figure 3.

The prices for one year to maturity, as expected, are only slightly higher for the American put. However, as maturity is extended to several years, the American put's value continues to increase and becomes significantly greater than the European put. Most notably, the American put is an ever increasing function of time to maturity while the European put actually declines in value after the third year. Intuitively, the American put is a more flexible claim. Because it gives the right to exercise at any time, it must be

---

The author realizes that interest rates were extremely high during 1981. However, it is the intent of this thesis to examine the impact of different parameter values on price patterns rather than specific price levels. These values will simply serve as a reference point. Analyzing specific price levels is left for future work.
Figure 3: European Versus American Model of First Put Prices
equal, at a minimum, to the maximum value of a European put. This continual exercise privilege also requires its to have an ever increasing value. For the European put, the present value of the exercise price falls with time. At some point the declining present value of the exercise price dominates the variance rate and the price will fall. This point confirms the comparative static noted in section 4.4. Clearly the American put is a better model for default insurance.

6.4 IMPACT OF DECLINING MORTGAGE BALANCE

Equity build-up is an often cited disincentive for default on the home mortgage (Vandell, 1978 and Von Furstenberg, 1969). However, simulation results show that the equity accumulation from periodic pay-down of the principal balance does not contribute significantly to the reduction of default insurance rates. Figure 4 graphically compares the theoretical price of the first put for a 90 L/V mortgage that amortizes versus the rates for a non-amortizing mortgage. Predicted rates for each appear in table 5 and table 6, respectively. Figure 5 compares the impact of amortization versus non-amortization on the net rate at different coverage levels. Three points are noted:

1. The put’s value on the amortizing loan and non-amortizing loan are equal for the initial two years. The put’s value on the amortizing mortgage then increases at a slower rate.

2. The put’s value on an amortizing mortgage becomes constant whereas the put’s value on the non-amortizing loan continues to increase.
3. Lower coverages mitigate the impact of amortizing debt.

Point three is an intriguing observation. Figure 6 illustrates the impact of amortization on the first and second puts. The first put is more sensitive to the declining mortgage balance than the second put, which models the limited coverage. In fact, the first put's value levels out in year 5, whereas the second put's value, depending on the coverage level, could continue to increase in value until year 14. Intuitively, this suggests that the second put, which is equivalent to a far-out-of-the-money option, is rather insensitive, in the short run, to a gradual (diffusion) erosion in price. Sensitizing the second put to short maturity terms would require a large, unexpected dividend (i.e., moral hazard) or a very high variance rate. The modest declines in the mortgage balance during the first ten years offer little equity protection against waste. Moreover, the lower the coverage the greater the risk of moral hazard transferred to the banker. Low coverage rates are less sensitive to the declining mortgage balance.
Figure 1: Impact of Mortgage Amortization on First Put Prices

\[ R_f = 0.13 \]
\[ R_m = 0.16 \]
\[ \sigma^2 = 0.0142 \]
### TABLE 5

Predicted Insurance Rates for Amortizing Mortgage

<table>
<thead>
<tr>
<th>Cov.</th>
<th>L/V</th>
<th>Annual</th>
<th>3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
<th>10 yrs.</th>
<th>12 yrs.</th>
<th>15 yrs.</th>
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<tr>
<td>30%</td>
<td>91-95%</td>
<td>1.40</td>
<td>1.95</td>
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<td>2.10</td>
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<tr>
<td></td>
<td>86-90%</td>
<td>.66</td>
<td>1.08</td>
<td>1.15</td>
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<td>1.22</td>
<td>1.23</td>
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<td>.62</td>
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<td>.73</td>
<td>.74</td>
<td>.74</td>
<td></td>
</tr>
<tr>
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\( D = .5 \text{ per month} \)

\( \sigma^2 = .0142 \)

\( t = .13 \)

\( R_m = .16 \)
## TABLE 6

Predicted Insurance Rates for Non-Amortizing Mortgage

<table>
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<th>Cov.</th>
<th>L/V</th>
<th>Annual</th>
<th>3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
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<tr>
<td>17%</td>
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<td>1.93</td>
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</table>

D = .5 per month
\( \sigma^2 = .0142 \)
\( r = .13 \)
\( R = .16 \)
Figure 5: Impact of Mortgage Amortization on Net Insurance Rates
Figure 6: Comparison of Predicted Prices on First Put versus Second Put
6.5 LIMITED COVERAGE

Insurance rates were predicted by modeling the limited coverage as a subtracted second put. The results, based on the assigned parameter values, appear in table 5. A comparison of the predicted rates and the observed rates of figure 1 indicates the model is less sensitive to changes in coverage levels. For the first year's rates, the predicted rates are virtually the same for identical L/V ratios regardless of coverage limitations. As the time to maturity lengthens, coverage levels become distinguished by predicted rates that vary by only .01% to .02% for high coverages and by .05% to .10% for low coverages. Observed rates tend to vary across coverage levels by .10% to .50%.

Although the price differentials are greater for observed rates, there appears to be a consistent set of elasticities for both predicted and observed insurance rates. First, for a given L/V ratio, the "coverage elasticity of price" indicates that rates have greater sensitivity to 5% increments in coverage at low coverage levels than at high coverage levels.34

The apparent differences at the low coverage levels are due to the arbitrary heuristic, equation (5.1), used to assign the initial underwriting costs.

These are not true elasticities since a 5% increment at the 12% coverage level is actually a 42% relative coverage increase, whereas it is only a 20% relative coverage increase at the 25% coverage level.
Secondly, simulated rates and observed rates also have a consistent "L/V elasticity of price." In general, both model and observed rates, at a given coverage level, show greater sensitivity of price to changes in high L/V ratios. However, observed rates consistently, and as yet inexplicably, have a unique aberration occurring at the middle L/V category, (81-85) to (86-90). The change between these two levels has the lowest elasticity.

A third consistency is the "time elasticity of price." For a given coverage level and L/V ratio, rates are more sensitive to changes in maturity in the initial years of coverage. In addition, both simulated rates and observed rates are more sensitive to maturity changes at low coverage levels than at high coverage levels for the shorter maturities. The behavior of observed rates becomes somewhat inconsistent at longer maturities. In general, it appears that the model is more consistent in behavior across parameters than observed rates.36

A final point, with respect to the time parameter, is the fact that observed rates are an ever increasing function of time to maturity. In contrast, simulated rates on an amortizing mortgage become constant. Even the simulated

36 Undeveloped markets would explain the aberrative behavior of observed rates at certain parameter levels. Discussion with the MGIC underwriter of the Columbus, Ohio office confirmed this possibility. She indicated that the long maturity rates were seldom, if ever, used. It was her contention that these long-term rates were implemented for insuring balloon mortgages.
rates on a non-amortizing loan do not increase as dramatically as industry rates. Table 6 shows simulated rates for a non-amortizing mortgage. The contention of the Columbus, Ohio underwriter that the single premium long maturity rates were instituted for balloon mortgages agrees with the model simulations.

6.6 COMPARATIVE STATICS

6.6.1 Variance Rate on House Returns

Variance rate increases will increase the probability of the house value falling below the value of the mortgage. Protection from such a possibility becomes more valuable and the insurance rates increase. The opposite effect holds true for variance rate decreases. Figure 7 summarizes the impact of changing variance rates. Interestingly, the elasticity of price appears to be constant.
Figure 7: Impact of Changes in the Variance Rate

L/V = 90
Coverage = 17%
6.6.2 Appreciation Rate

Unlike the impact of changes in the mortgage interest rate, changes in the risk-free interest rate cause significant inverse movements in insurance rates. Rearranging equation (5.10), we see that the appreciation rate is the nominal interest rate less the net service flow on the house:

\[ g = r - s. \]  

(6.1)

In the option pricing model (6.1) is an identity relationship. Therefore, holding the service flow rate constant, an increase in the nominal interest rate implies an increase in the expected house appreciation rate. The homeowner anticipates equity build-up and the expected probability of default diminishes; the price of default insurance declines. Figure 8 summarizes the impact of increasing the risk-free rate given a constant net service flow rate. The elasticity of price appears to be an increasing function of decreases in the risk-free rate.

Referring again to (6.1), an alternative approach to examining the appreciation rate would be to hold the nominal interest rate constant and allow the service flow rate to vary. For this case, an increase in the service flow rate implies a decrease in the house appreciation rate. The
probability of default remains higher because the homeowner's equity is growing at a slower rate. Figure 9 reflects the impact of an increasing service flow rate given a constant nominal interest rate. The price pattern that emerges as dividends are increased is, as expected, almost identical to the pattern that occurred in figure 8 when nominal interest rates were decreased. The implicit change is not exact because the numerical procedure uses the service flow rate to set boundary conditions on the insurance price function which is calculated as a function of r.
Figure 8: Impact of Changes in the Risk-free Interest Rate
Figure 9: Impact of Changes in the Service Flow Rate

\[ D = \$1.00/\text{mo.} \]

\[ D = \$.75 \]

\[ D = \$.50 \]

\[ D = \$.25 \]

\( H = \$100 \)

\( L/V = 90 \)
6.7 RATE LEVELS

The simulation results presented in this chapter have assumed a 16 percent mortgage debit rate, a 13 percent risk-free rate, and a 6 percent service flow rate, which, with a zero marginal tax rate, implies an appreciation rate of 7 percent in house prices. The nominal interest rates selected are representative of the economic environment existing during the 1981 to mid 1982 period. For periods prior to 1981 and subsequent to August 1982, interest rate levels were substantially lower. Such a decline in interest rates, particularly the risk-free rate, will result in significantly higher simulated insurance rates, ceteris paribus. As a result, simulation tests were conducted to determine the offsetting adjustments necessary in the variance rate, service flow rate, and the implied appreciation rate such that simulated insurance prices would remain substantially unchanged.

The drop in the riskless rate to 10.5 percent by late 1982 is a decline of approximately 19 percent from the 13 percent level. It was found that a 25 percent reduction in the variance rate would roughly offset the drop in the riskless rate. Simulated insurance premiums with the lower variance and risk-free rate appear in Table 7. Prices start at a slightly lower level than the original predictions (Table 5), but increase to a somewhat higher level.
The service flow rate would have to drop by 35 to 40 percent to offset the fall in the risk-free rate. Results for each case appear in table 8 and table 9.

A reduction in both the dividend rate and variance rate of 19 percent approximately offset the decline in the risk-free rate. This implies an appreciation rate of 5.6 percent. These results appear in table 10.

During the 1977-1978 period nominal interest rates were even lower, and the disparity between real and nominal rates was much different. The risk-free rate was approximately 7 percent, but the house price appreciation rate was approximately 9 percent. For this period the service flow rate would have to be a negative 2 percent. In order for insurance rates to remain unchanged over this period, the simulation work suggests that the variance rate would have to increase by approximately 20 percent to .0171. The results appear in table 11.
### Table 7

**Simulated Insurance Rates with 25% Variance Adjustment**

<table>
<thead>
<tr>
<th>Cov. L/V</th>
<th>Annual</th>
<th>3 yrs</th>
<th>6 yrs</th>
<th>9 yrs</th>
<th>12 yrs</th>
<th>15 yrs</th>
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<td>1.35</td>
<td>2.01</td>
<td>2.09</td>
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<td>2.11</td>
<td>*</td>
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<tr>
<td>86-90%</td>
<td>.59</td>
<td>1.12</td>
<td>1.21</td>
<td>1.25</td>
<td>1.26</td>
<td>*</td>
</tr>
<tr>
<td>81-85%</td>
<td>.33</td>
<td>.63</td>
<td>.71</td>
<td>.75</td>
<td>.78</td>
<td>*</td>
</tr>
<tr>
<td>&lt;80%</td>
<td>.26</td>
<td>.39</td>
<td>.44</td>
<td>.47</td>
<td>.50</td>
<td>*</td>
</tr>
<tr>
<td>91-95%</td>
<td>1.35</td>
<td>2.00</td>
<td>2.07</td>
<td>2.08</td>
<td>2.08</td>
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<td>1.11</td>
<td>1.20</td>
<td>1.24</td>
<td>1.25</td>
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</tr>
<tr>
<td>81-85%</td>
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<td>.63</td>
<td>.70</td>
<td>.75</td>
<td>.77</td>
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<tr>
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<td>.11</td>
<td>.39</td>
<td>.44</td>
<td>.47</td>
<td>.50</td>
<td>*</td>
</tr>
<tr>
<td>91-95%</td>
<td>1.34</td>
<td>1.98</td>
<td>2.05</td>
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<td>1.19</td>
<td>1.23</td>
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<td>.63</td>
<td>.70</td>
<td>.74</td>
<td>.76</td>
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<td>1.20</td>
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<td>.63</td>
<td>.59</td>
<td>.73</td>
<td>.75</td>
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<td>.68</td>
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<td>.73</td>
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<tr>
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<td>.34</td>
<td>.43</td>
<td>.46</td>
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<td>.67</td>
<td>.78</td>
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<td>.28</td>
<td>.37</td>
<td>.44</td>
<td>.46</td>
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</table>

D = 9.50

S^2 = 0.01065

r = .105

R^2 = .13

* These prices were not calculated due to computational expense and irrelevance
### TABLE 8
Simulated Insurance Rates with 35% Service Flow Adjustment

<table>
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<tr>
<th>Cov.</th>
<th>L/V</th>
<th>Annual</th>
<th>3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
<th>10 yrs.</th>
<th>12 yrs.</th>
<th>15 yrs.</th>
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<tr>
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<td>86-90%</td>
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<td>1.27</td>
<td>1.35</td>
<td>1.38</td>
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<td>*</td>
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</tr>
<tr>
<td></td>
<td>81-85%</td>
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<td>.73</td>
<td>.82</td>
<td>.86</td>
<td>.87</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
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<td>*</td>
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<td>2.18</td>
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<td>*</td>
<td>*</td>
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<tr>
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<td>.73</td>
<td>.82</td>
<td>.93</td>
<td>.86</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
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<td>.29</td>
<td>.47</td>
<td>.52</td>
<td>.55</td>
<td>.37</td>
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<td>*</td>
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<tr>
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<tr>
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<td>1.25</td>
<td>1.31</td>
<td>1.33</td>
<td>1.33</td>
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<td>*</td>
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<td>.74</td>
<td>.81</td>
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<td>.51</td>
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<td>*</td>
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<td>1.23</td>
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<td>.73</td>
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<td>.83</td>
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<tr>
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<td>.13</td>
<td>.47</td>
<td>.52</td>
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<td>*</td>
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<tr>
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<td>.72</td>
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<td>.14</td>
<td>.34</td>
<td>.43</td>
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</tbody>
</table>

D = 5.323  
\( z^2 = 0.0142 \)  
\( \tau = 0.105 \)  
\( R^2 = 0.13 \)

* These prices were not calculated due to computational expense and irrelevance
TABLE 9
Simulated Insurance Rates with 40% Service Flow Adjustment

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<th>Cov.</th>
<th>L/V</th>
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<th>5 yrs.</th>
<th>7 yrs.</th>
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<th>12 yrs.</th>
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<td>1.76</td>
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</tr>
<tr>
<td></td>
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<td>.99</td>
<td>1.04</td>
<td>1.04</td>
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<tr>
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<td>.63</td>
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<td>.63</td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
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<td>.25</td>
<td>.39</td>
<td>.42</td>
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<td>.44</td>
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<td>.63</td>
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<td>.39</td>
<td>.42</td>
<td>.43</td>
<td>.44</td>
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<td>*</td>
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<td>.42</td>
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<td>.41</td>
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<td>81-85%</td>
<td>.11</td>
<td>.49</td>
<td>.57</td>
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<td>.58</td>
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D = 5.30
\( r^2 = .0142 \)
\( r = .105 \)
\( P = .13 \)

* These prices were not calculated due to computational expense and irrelevance.
### TABLE 10

Simulated Rates with Variance and Service Flow Adjustment

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<th>Cov.</th>
<th>L/Y</th>
<th>Annual</th>
<th>1 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
<th>10 yrs.</th>
<th>12 yrs.</th>
<th>15 yrs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>51-95%</td>
<td>1.45</td>
<td>2.15</td>
<td>2.27</td>
<td>2.29</td>
<td>2.29</td>
<td>2.29</td>
<td>2.29</td>
<td>2.29</td>
<td>2.29</td>
</tr>
<tr>
<td>86-90%</td>
<td>.65</td>
<td>1.24</td>
<td>1.35</td>
<td>1.39</td>
<td>1.40</td>
<td>2.27</td>
<td>2.29</td>
<td>2.29</td>
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</tr>
<tr>
<td>81-85%</td>
<td>.33</td>
<td>.70</td>
<td>.79</td>
<td>.84</td>
<td>.86</td>
<td>1.24</td>
<td>1.35</td>
<td>1.39</td>
<td>1.40</td>
</tr>
<tr>
<td>&lt;80%</td>
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<td>.53</td>
<td>.56</td>
<td></td>
<td>1.24</td>
<td>1.35</td>
<td>1.39</td>
<td>1.40</td>
</tr>
</tbody>
</table>

| 91-95%| 1.45 | 2.15   | 2.24   | 2.26   | 2.26   | 2.26   | 2.26    | 2.26    |        |
| 86-90%| .65  | 1.23   | 1.33   | 1.38   | 1.38   | 1.38   | 1.38    | 1.38    |        |
| 81-85%| .35  | .70    | .79    | .84    | .87    | 1.24   | 1.35    | 1.39    | 1.40   |
| <80%  | .12  | .43    | .53    | .56    |        | 1.24   | 1.35    | 1.39    | 1.40   |

| 22%  | 1.45 | 2.14   | 2.21   | 2.21   | 2.21   |        |         |         |        |
| 86-90%| .65  | 1.22   | 1.32   | 1.36   | 1.36   |        |         |         |        |
| 81-85%| .30  | .70    | .78    | .83    | .85    |        |         |         |        |

| 20%  | 1.45 | 2.11   | 2.17   | 2.17   | 2.17   |        |         |         |        |
| 86-90%| .65  | 1.21   | 1.30   | 1.34   | 1.34   |        |         |         |        |
| 81-85%| .20  | .70    | .78    | .82    | .86    |        |         |         |        |
| <80%  | .02  | .43    | .52    | .55    |        | 1.24   | 1.35    | 1.39    | 1.40   |

| 172% | 1.44 | 2.02   | 2.05   | 2.05   | 2.05   |        |         |         |        |
| 86-90%| .39  | 1.09   | 1.14   | 1.16   | 1.16   |        |         |         |        |
| 81-85%| .10  | .60    | .71    | .74    | .74    |        |         |         |        |
| <80%  | .02  | .31    | .41    | .48    | .50    |        |         |         |        |

\[ D = 5.405 \]
\[ \sigma^2 = .0115 \]
\[ r = .105 \]
\[ R^2 = .13 \]

*These prices were not calculated due to computational expense and irrelevance.*
TABLE 11
Simulated Rates for 1977-1978 Period

<table>
<thead>
<tr>
<th>Cov.</th>
<th>L/V</th>
<th>Annual</th>
<th>3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
<th>10 yrs.</th>
<th>12 yrs.</th>
<th>15 yrs.</th>
</tr>
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<td>1.42</td>
<td>1.41</td>
<td>1.40</td>
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<tr>
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<td>86-90%</td>
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<td>&lt;80%</td>
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<td>81-85%</td>
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<td>.37</td>
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</tr>
</tbody>
</table>

D = -5.17
σ² = .0142
r = .07
R² = .09

* These prices were not calculated due to computational expense and irrelevance
6.8 CONCLUSIONS

Numerical examples from the modeling of institutional restrictions have been presented in this chapter. Although many of the results were generally predictable, the simulation work was useful in demonstrating the sensitivity of prices over a range of parameter values. In particular, the model has been shown to be extremely sensitive to changes in the variance rate, risk-free rate, and dividend rate. This is important because insurance prices are known to be constant over time. Therefore, changes in these parameter values, if they are not offsetting, must be priced elsewhere. Most likely, these changes will be reflected in mortgage debit rates or in services rendered by either the insurer or the lender.

In addition, the simulation work exposed discrepancies between modeled price behavior and observed price behavior. The unique pricing structure existing within the default insurance industry deserves further exploration. These discrepancies are detailed in the next chapter with possible explanations. Simulation results based on these possible explanations are also presented.
Chapter VII
DEFAULT INSURANCE PRICE PATTERNS

7.1 INTRODUCTION

The simulation work presented in Chapter 6 provides an indication of what default insurance price patterns should exist in an efficient market. Observed prices demonstrate some notable deviations from these patterns. This chapter will focus on these deviations.

If model prices are correct, then one possible explanation for the price behavior discrepancies is that government regulation interferes with the market pricing mechanism. In this case it is possible that market participants only trade the quoted insurance contracts that are efficiently priced. Alternatively, if demand exists for all offerings, incorrect prices may be adjusted through mortgage debit rates or service performance by either the insurer or lender.

If model prices are incorrect, it may be due to its failure to capture certain market expectations. The following sections will outline the specific discrepancies between observed and simulated prices. Possible expectations not captured by the model are considered with plausible modifications.
7.2 PRICE DIFFERENTIALS

There are three variables of domain over which quoted insurance rates vary: the initial L/V ratio, the depth of coverage, and the time duration of coverage. Observed prices react differently from modeled prices over each of these parameters.

7.2.1 L/V Price Differentials

High L/V ratios expose the lender to greater default risk which results in more costly insurance protection. In the put option analogy, the L/V ratio is a ratio of the strike price to the house price. A higher strike price relative to the house value results in a higher probability that the put option will be exercised.

The MGIC quote sheet (figure 1) indicates that 95 L/V insurance premia are, on average, about two times more costly than the 80 L/V premia. This spread narrows somewhat over time. For the simulated insurance premia (table 5) the spread is four to five times greater. As in the observed case, the spread narrows over time. The model, therefore, predicts that high L/V loans are exposed to much greater default risk than low L/V loans and should carry much higher insurance premia.
7.2.2 Coverage Depth Price Differentials

The depth of coverage determines how much of a loss from default will be paid by the insurer. By contract, the insurer will pay 100% of all losses up to the ceiling of a coverage percent times the outstanding loan balance. The model captures this ceiling restriction by simulating the insurer as having created a vertical option spread. He has gone short a put to the banker with an exercise price equal to the outstanding mortgage balance. However, his loss exposure is limited by having gone long a put issued by the banker with an exercise price equal to one minus the coverage percent times the outstanding mortgage balance. A lower coverage percent increases the strike price of this second put, reducing his loss exposure, and therefore resulting in a lower insurance premium.

The MGIC quote sheet (figure 1) shows insurance premiums for 30% coverage to be about three times greater than premiums for 12% coverage. This percentage spread narrows over time. For simulated insurance premiums (table 5), the spread between 30% coverage and 12% coverage is virtually nonexistent. The model implies that coverage depth is immaterial in setting the price of default insurance.
7.2.3 Time Price Differential

The length of coverage is also an important determinant in setting insurance prices. Longer coverage periods extend the insurer's time of exposure to the variance rate, which operates on the probability of the house value falling below the mortgage value. If the variance rate dominates the offsetting house appreciation rate, then insurance premia should increase over time.

The MGIC quote sheet (figure 1) indicates that time is very expensive in setting insurance rates. Ten year coverage is, on average, approximately three times the price of one year coverage. For simulated premia the price of time to maturity is much less significant. Ten year simulated coverage is, on average, approximately one and a half times the price of one year coverage. The model predicts that default risk is significantly greater in the initial years, and that the marginal price of protection over time should fall substantially.

7.2.4 Interperiod Price Differentials

One might expect prices to follow some constant linear relationship over time. However, the MGIC quote sheet (figure 1) reveals a unique price change between the third and fourth years. Price differentials before the third year and after the fourth year are substantial changes (10 to 50 basis points). The price change between the third and fourth
year is uncharacteristically small (5 basis points). Simulated prices have no such anomaly. In fact, simulated prices have declining marginal changes over time which eventually become flat. A graph of these marginal prices appears in figure 10.
Figure 10: Marginal Insurance Rates Over Time

30% Coverage, 95 L/V

$\text{D} = 3.50$

$\sigma^2 = 0.142$

$r = 0.13$

$R_m = 16$
7.2.5 **Summary of Price Pattern Discrepancies**

The disparities between observed prices and simulated prices are summarized as follows:

1. Model prices have greater spreads across L/V ratios.
2. Model prices have lesser spreads across coverage depths.
3. Model prices have lesser spreads across time.
4. Model prices do not exhibit an inflection point between the third and fourth periods.

The remainder of this chapter will explore some explanatory arguments that might lead to the resolution of these discrepancies.

7.3 **Explanatory Arguments**

7.3.1 **House Investment Risk**

The risk of the house investment is measured by the variance rate of house price returns. This is a critical parameter to which the model is quite sensitive. Simulation tests indicate an increase in the variance rate will narrow the price spreads over L/V ratios, increase the spread over coverage levels, and have virtually no impact on price spreads over time.

Increasing the variance rate causes greater price change in far-out-of-the-money options than in close-to-the-money options. Default insurance on a low L/V mortgage is equivalent to a far-out-of-the-money option. Therefore, in-
creasing the variance rate causes low L/V insurance premia to increase more dramatically than the high L/V mortgage insurance; the spread is narrowed.

The spread in insurance rates over coverage levels is increased by a high variance rate. All insurance rates, regardless of the mortgage L/V ratio, include an implicit far-cut-of-the-money option - the subtracted second put. With a low variance rate this put is virtually worthless. By increasing the variance rate, the value of the second put becomes more discernable in the net insurance rate.

Table 12 summarizes the strike prices of the second puts for all coverages and L/V ratios. Compare, for example, the second put's strike prices on a 30% coverage 95 L/V loan versus the second put's strike price on a 16% coverage 95 L/V loan. The strike prices are substantially different, 0.67 versus 0.80, respectively. However, their far-cut-of-the-money nature makes their impact on insurance rates insignificant when the house variance rate is low.

As the variance rate is increased the prices of these second puts become significant and discernable across their strike prices. This phenomenon will cause the insurance rates to spread out over coverage depths. Table 13 shows insurance rates with a 50% increase in the variance rate. The spreading over coverage levels increases for longer coverage periods.
** Equivalent to strike price ratio of first put in model

* Calculated by taking \((1 - \text{Coverage \%})(L/V)\)
### Table 13

Simulated Insurance Rates for High Variance House Returns

<table>
<thead>
<tr>
<th>Cov.</th>
<th>L/V</th>
<th>Annual</th>
<th>3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
<th>10 yrs.</th>
<th>12 yrs.</th>
<th>15 yrs.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>91-95%</td>
<td>2.11</td>
<td>2.73</td>
<td>2.79</td>
<td>2.79</td>
<td>2.79</td>
<td>2.79</td>
<td>2.79</td>
<td>2.79</td>
</tr>
<tr>
<td>30%</td>
<td>86-90%</td>
<td>1.12</td>
<td>1.78</td>
<td>1.86</td>
<td>1.88</td>
<td>1.88</td>
<td>1.88</td>
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<td>1.88</td>
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<tr>
<td></td>
<td>81-85%</td>
<td>.45</td>
<td>1.13</td>
<td>1.22</td>
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<td>&lt;80%</td>
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<tr>
<td>25%</td>
<td>91-95%</td>
<td>2.11</td>
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<td>.72</td>
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</tbody>
</table>

D = .5

$C_r^2 = .0213$

$r = .13$

$l = .16$
These results suggest the house investment may be a riskier project than estimated by Asay (1978) in his study on mortgage pricing. Asay's surrogate variance was a group variance for houses located according to narrowly defined regions. Such a proxy variance is acceptable if:

1. perfect positive correlation exists between the returns on the individual members of the group, and
2. the member variances are equal.

When individual returns are not perfectly correlated, the proxy group variance is biased downward. Because individual homeowners set their own maintenance policies, this "diversification effect" seems probable. Individual variance rates may indeed be higher.

7.3.2 Maintenance Policies

Until now two assumptions have been made regarding maintenance policy on the house. First, the maintenance policy is given by the equilibrium condition (5.10), and second, the maintenance level is selected in order to minimize the so-called "gross depreciation rate" of equation (5.9).

Rather than taking the maintenance policy to be fixed, consideration should be given to the possibility that the homeowner may alter his maintenance rate. Unlike a corporation where the dividend rate is formally set by the board of directors with stockholder approval, the homeowner acts alone, with conceivable swiftness, in setting his mainte-
nance rate. Substituting (5.9) into (5.8) and rearranging, results in

\[ r = [q - d(\bar{m})] + [(R/H) - m] \] (7.1)

or

\[ r = q' + s' \] (7.1a)

where \( q' \) and \( s' \) are alternative definitions for the house appreciation rate and net service flow rate. In this expression it is easy to see that the homeowner may increase his service flow rate by lowering his maintenance rate.

If it is assumed that \( \delta \) in equation (5.9) is minimized for a broad range of \( m \) values, then the homeowner is free to substitute values of \( s' \) for values of \( q' \). Substituting a higher service flow rate for a lower appreciation rate will, however, have significant impact on the price of default insurance.

Simulation runs with a 50% and 100% increase in the service flow rate were conducted. The results of a 50% dividend increase appear in table 14. The increased dividend does not narrow price spreads over \( I/V \) ratios or widen spreads over coverage levels. However, large dividend rates do cause simulated insurance rates to increase over time. Intuitively, the dividend rate counters the appreciation rate. If less maintenance is performed (a higher dividend extracted) by the homeowner, the house price will remain
closer to the mortgage value over time and the insurance rate will continue to increase.

Although the high dividend rate causes prices to increase longer over time, it also results in unrealistically high premia in all years. An alternative dividend policy would be for dividend rates to increase over time. Such a policy would capture an increasing gross depreciation rate, even a possible "decay" rate.

Referring to (7.1), if \( m \) decreases over time due to less maintenance, and \( r \) and \( q \) are assumed fixed, then \( d(m) \) will increase over time. Depending on the initial values of \( r \) and \( q \), a reduction in \( m \) could result in \( d(m) > q \); the house would decay.

Using the same parameter values initially assumed, the service flow rate was allowed to increase by

\[
D_t = D_{t-1} + .002
\]

where each \( t \) interval is equal to one month. The results are significant as seen in table 15. The price of the insurance continues to increase over time. The simulation results appear to support a theory that insurers expect appreciation rates to decline over time due to declining maintenance rates. That is, "older" houses are expected to appreciate less rapidly than newer houses.
### TABLE 14

Simulated Insurance Rates with High Service Flow Rate

<table>
<thead>
<tr>
<th>Cov.</th>
<th>L/V</th>
<th>Annual 5 yrs.</th>
<th>4 yrs.</th>
<th>3 yrs.</th>
<th>2 yrs.</th>
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\[ D = 5.75, \quad \chi^2 = 0.42, \quad r = 0.13, \quad Rs = 0.16 \]

* These prices were not calculated due to computational expense and irrelevance.
## TABLE 15

Insurance Rates With Service Flow a Function of Time

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<th>Cov. L/V</th>
<th>Annual</th>
<th>3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
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<td>1.92</td>
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<td>.87</td>
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<td>.53</td>
<td>.57</td>
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<td>.72</td>
<td>.89</td>
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\[ D_n = D_{n-1} + 5.002 \]
\[ s^2 = .0142 \]
\[ r = .13 \]
\[ R_n = .16 \]
7.4 LENDER'S DEFAULT COSTS

In section 5.6.2 it was noted that certain default costs incurred by the lender could not be passed on to the insurer. The effect of these lender-absorbed costs is to lower the net proceeds received from "putting" the house to the insurer. Alternatively, these costs effectively raise the strike price of the second put at which the insurer may "put" the house back to the lender. The model was adjusted to raise the exercise price of the second put by \( \% \) of the mortgage balance to reflect these costs. The results of considering 20\% for the banker's default costs are shown in table 16. Insurance rates spread out over coverage levels, particularly beginning in the third year.

This adjustment has theoretical appeal. The value of the second put is the price of the residual default risk retained by the banker. The price of this put will be passed on to the borrower and should account for different mortgage debit rates being charged on different L/V mortgages even though they are "insured." If the second put has value, it would also explain why the banker performs underwriting services, such as house inspection.
### Table 16

Insurance Rates with Banker's Default Costs Considered

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<th>Cov.</th>
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</table>

- \( D = 1.5 \)
- \( e = 0.012 \)
- \( \varepsilon = 0.13 \)
- \( \beta = 0.16 \)
- \( \beta_c = 0.20 \times e \)
A final consideration in the pricing of insurance rates is the issue of moral hazard. This phenomenon has been recognized in the health insurance field for years (see Arrow (1963)). In general, moral hazard may be defined as an altered activity state due to changed incentives. For the specific case of default insurance, the homeowner may have incentive to extract large dividends from the house if he has the contractual right to put the house to the lender/insurer at a specified price. As noted in Mayers and Smith (1981), insurance firms recognize that policyholders' incentives change when insurance is purchased; premia must reflect these changes.

The lender cannot continuously enforce the mortgage covenant that requires adequate house maintenance. House inspections are seldom performed periodically, much less continuously. Therefore, it may be rational for a liquidity constrained homeowner to dismantle the house. Such a "gutting" of the house would transfer wealth from the lender to the homeowner if legal recourse is ineffective.

In modeling the possibility of house dismantling, the service flow rate becomes an endogenized variable. That is the homeowner has incentive to "gut" the house as the possibility of default increases. The increase in default probability will occur as the homeowner's equity diminishes. His incentive to "gut" increases because any value removed
from the house, with zero homeowner's equity invested, is
his gain at the lender's expense. With positive equity any
"gutting" dividends are simply a reduction of the home-
owner's equity: he gains nothing. In general the mainte-
nance policy might be modeled as \( M_t = f(M_t, R_t) \) where \( \partial M/\partial H < 0 \), and \( \partial M/\partial H > 0 \).

Simulations were conducted with a specific maintenance
policy of the form

\[ M_t = k (m_t/R_t) \]  \hspace{1cm} (7.3)

where \( k \) is an arbitrary constant. Now the economic depreci-
ation in equation (5.9) becomes \( \psi d(\bar{M}) \) where \( \psi \) is equivalent
to \( (1 - m_t/H_t) \). For \( \partial \psi/\partial m = 0 \), decreases in \( m \) are offset by
increases in \( \psi d(\bar{M}) \). If the homeowner's equity is zero, sub-
stitution of \( \psi d(\bar{M}) \) for less \( m \) is a transfer of wealth from
the insurer to the homeowner. The results from setting \( k \)
equal to \$.50 appear in table 17. Because this formulation
captures the incentive to decrease maintenance as equity de-
clines, the insurance prices for high L/V ratio mortgages
increases while the insurance price for low L/V mortgages
remain unchanged. High L/V mortgages are exposed to greater
default risk with an endogenized maintenance policy because
the borrower has less incentive to maintain the property
with less equity invested.
### TABLE 17

**Insurance Rates With Service Flow as a Function of Equity**

<table>
<thead>
<tr>
<th>Cov.</th>
<th>L/V</th>
<th>Annual 3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
<th>10 yrs.</th>
<th>12 yrs.</th>
<th>15 yrs.</th>
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</thead>
<tbody>
<tr>
<td>30%</td>
<td>91-95%</td>
<td>1.49</td>
<td>2.07</td>
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<td>.37</td>
<td>.66</td>
<td>.71</td>
<td>.73</td>
<td>.73</td>
<td>*</td>
<td>*</td>
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<tr>
<td></td>
<td>&lt;80%</td>
<td>.27</td>
<td>.40</td>
<td>.44</td>
<td>.46</td>
<td>.45</td>
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<td>*</td>
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<tr>
<td>25%</td>
<td>91-95%</td>
<td>1.49</td>
<td>2.06</td>
<td>2.10</td>
<td>2.10</td>
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<td>.69</td>
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<td>1.21</td>
<td>1.23</td>
<td>1.23</td>
<td>*</td>
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</tr>
<tr>
<td></td>
<td>81-85%</td>
<td>.37</td>
<td>.65</td>
<td>.70</td>
<td>.73</td>
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<tr>
<td></td>
<td>&lt;80%</td>
<td>.12</td>
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<td>.44</td>
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<td>2.04</td>
<td>2.08</td>
<td>2.08</td>
<td>2.08</td>
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<td>*</td>
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<tr>
<td></td>
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<td>.69</td>
<td>1.14</td>
<td>1.21</td>
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<tr>
<td></td>
<td>81-85%</td>
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</tr>
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<td>.43</td>
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<tr>
<td>17%</td>
<td>86-90%</td>
<td>.58</td>
<td>1.11</td>
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<td>1.17</td>
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<td>.68</td>
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<tr>
<td></td>
<td>&lt;80%</td>
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<td>1.47</td>
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</tr>
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<td></td>
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<td>1.02</td>
<td>1.06</td>
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<td>.02</td>
<td>.39</td>
<td>.36</td>
<td>.43</td>
<td>.43</td>
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</tr>
</tbody>
</table>

D = \((N_s/S_c) \times 5.30\)

\(\gamma^2\) = .0142

r = .13

\(R_s\) = .16

* These prices were not calculated due to computational expense and irrelevance.
House dismantling could also be conceived of as a large discrete dividend. The option pricing models of the BS tradition are not intended to capture large discontinuous jumps in asset value or dividend payout. However, using the Brennan and Schwartz numerical procedure introduced in section 5.7, a surrogate attempt was made to capture this event. An arbitrary house value (greater than the mortgage balance) was selected for each of the first three years and a large dividend extracted.\(^{37}\) The dividend was sufficiently large to cause the house value to fall 30% below the outstanding mortgage balance.

The results of this experiment appear in table 18. Although the approach is admittedly tenuous, the results are striking enough to deserve comment. The simulated insurance rates become much tighter across L/V ratios and spread cut more across coverage levels. It must also be noted that some inconsistent price patterns emerge, reflecting an instability in the numerical procedure created by this approach.

\(^{37}\) A "gutting dividend" might result if the homeowner faced a liquidity constraint. Because the house value is above the mortgage balance, the rational homeowner would surely not allow foreclosure. Extracting a large dividend might be a rational response to offset the mortgage payment burden.
### TABLE 18

Insurance Rates With Moral Hazard

<table>
<thead>
<tr>
<th>Cov.</th>
<th>L/V</th>
<th>Annual</th>
<th>3 yrs.</th>
<th>4 yrs.</th>
<th>5 yrs.</th>
<th>7 yrs.</th>
<th>10 yrs.</th>
<th>12 yrs.</th>
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<td>1.16</td>
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<td>.76</td>
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<td>.53</td>
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<tr>
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<td>86-90%</td>
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<td>1.08</td>
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<td>81-85%</td>
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<tr>
<td></td>
<td>&lt;80%</td>
<td>.23</td>
<td>.42</td>
<td>.50</td>
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<tr>
<td>16%</td>
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<td>1.19</td>
<td>1.72</td>
<td>1.77</td>
<td>1.78</td>
<td>1.78</td>
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<td>81-85%</td>
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<tr>
<td></td>
<td>&lt;80%</td>
<td>.12</td>
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<td>.40</td>
<td>.46</td>
<td>.46</td>
<td>*</td>
<td></td>
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</tr>
</tbody>
</table>

D = 5.50  with a "putting" dividend to house prices 102, 107.5, 112.5

\( r^2 = 0.0142 \)

\( r = 0.13 \)

\( R^2 = 0.16 \)

* These prices were not calculated due to computational expense and irrelevance.
This chapter has investigated the discrepancies between model price patterns and observed price patterns. Four major behavior discrepancies are summarized in section 7.2.5. Based on the adjustments presented and the accompanying simulation tests, the following explanations of market expectations are postulated:

1. Insurers perceive the house investment to be riskier than Asay's (1978) estimates suggest. This may partially reflect an awareness of exposure to moral hazard waste. Both higher variance rates and arbitrary house "guts" narrow price spreads over I/V ratios and expand price spreads over coverage levels.

2. Insurers anticipate real house deterioration over time. Making the dividend rate an increasing function of time, causes the house price to remain closer to the mortgage balance. The variance rate then operates on the probability of the house value falling below the mortgage value for a longer time period. This results in simulated prices continuing to increase over time in a pattern similar to observed prices.

3. Insurers and bankers realize that certain default costs incurred by the banker are not passed on to the insurer by contractual agreement. These costs result in greater residual default risk being incurred by the banker. Insurance prices reflect this residual risk by price adjustments across coverage levels. Raising the exercise price of the second put in the model to account for the unreimbursed banker's default costs, leads to a partial mimicking of the observed price adjustments over coverage levels (i.e., prices spread out over coverage levels).

This chapter has put forth some plausible explanations for the behavior of default insurance price patterns. The simulation work provides a useful reference point for the
analysis of this complex financial instrument. Some new theories have emerged regarding the homeowner's behavior and house price patterns. Many questions (e.g., what is the efficient level of insurance prices as opposed to what is the efficient price pattern) cannot be resolved until further investigation is conducted at both the theoretical and empirical level. The next chapter will address some possible avenues of future research.
Chapter VIII
FUTURE RESEARCH

Further study in the application of option pricing to mortgage default insurance could proceed along four veins of research: (1) Theoretical model adjustments, (2) Theoretical implications for market behavior, (3) Applications of the model to alternative financing instruments, and (4) Empirical tests. Each is briefly discussed below.

8.1 THEORETICAL MODEL ADJUSTMENTS

8.1.1 IMPACT OF BORROWER CHARACTERISTICS

An assumption of rationality has been the analytical theme of this thesis. Early in the paper it was asserted that rational actions mitigated the impact of borrower characteristics on the decision to default. It was even shown that an irrational default would have no impact on the price of default protection. This section will reexamine borrower characteristics to determine if they might provide rational incentives to default. The analysis will focus on five borrower characteristics: income stability, income level (tax bracket), moving costs, attachable assets, and occupation.

Income stability is often cited as a critical factor in the underwriting of mortgage loans. The "banker's logic"
argues that a homeowner without income cannot make mortgage payments. It was proposed in section 4.5 that such a liquidity constraint should not result in default. However, the lost income could give the homeowner incentive to be delinquent if he is willing to incur the late payment fees. The delinquency imposes a lending function on the banker holding the mortgage. By forcing the banker to "carry" him the typical 90 day grace period, the homeowner effectively raises the exercise price of the insurance put option by the amount of accrued interest.

Because the accrued interest over 90 days cannot be too substantial, it would appear to have only marginal impact on the default decision. Delinquency from income instability should have the greatest significance on low downpayment purchases. If the value of the house were to decline, given only minimal homeowner equity invested, the homeowner may indeed default. The mortgage balance would have increased from the delinquent payments to an amount in excess of the declined house value.

The impact of taxes on default insurance pricing was investigated in section 5.5. In general it was shown that high tax bracket homeowners will price default insurance higher than low tax bracket homeowners. In a similar fashion, taxes will also partition homeowners' incentives to default.
Adjusting equation (5.11) for taxes, the homeowner's user cost of capital would be:

\[ r = \left( \frac{(R/H_k - m)/(1 - \tau) + (q - d(m))/(1 - \tau)}{1 - \tau} \right) \]

Given that \( r \) is fixed in equilibrium, the values of the variables on the right hand side are determined by \( \tau \). As long as a household can satisfy the "burden" rate \( r \), they will remain invested in the house and make the mortgage payments. Because the grossed up variable values increase with \( \tau \), high tax bracket homeowners have greater incentive to stay in the house longer.

The analysis can be extended to each of the three cost of capital components. Like \( r \), \( q \) is given by market conditions. Maintenance, therefore, is the only remaining cost determinant, because economic depreciation, \( d \), is a negative function of maintenance. This is a critical point because maintenance is not necessarily market determined; the homeowner may set his own maintenance rate. For the typical case, the homeowner is expected to establish some optimal maintenance policy that will minimize gross depreciation. For the atypical case, when default has a high probability of occurring, the homeowner may alter (perhaps drastically) the maintenance rate.

If the value of the house declines and approaches the mortgage balance, the maintenance policy becomes a critical
factor in whether or not the house value falls below the mortgage value. If the value of the house equals the value of the mortgage, the homeowner has a perverse incentive to stop maintenance, or worse yet, make it negative (gut the house). He gains the maintenance expenses saved, and the banker incurs the cost of economic depreciation. This is true only for $H < M$.

Taxes are important in this scenario because maintenance expenses saved are more valuable to the high tax bracket investor. He has incentive to stay in the house longer due to lower after-tax mortgage payments and more valuable after-tax maintenance savings. Continuing the logic of this analysis to the extreme case, taxes will not only encourage maintenance stoppage, but actually entice the homeowner to dismantle the house. The greater the tax bracket, the greater the incentive to "gut" or extract large tax-free dividends. In summary, the banker should realize that although high tax bracket homeowners have greater incentive to stay in the house and meet their mortgage payments, they may also have the perverse incentive to commit moral hazard while staying in the house. In a declining real estate market, it is actually cheaper (and therefore rational) for the homeowner to stay in the house and extract dividends. Normally households choose $m$ to minimize $m + d(m)$. When $H < M$, the household minimizes $m$ and will stay in the house as long as $\{(R/H_k) - m\} > (1 - r)r(M/H_k)$. The
higher the tax bracket, the greater the value and, thus, the greater the risk of such hazard being committed.

The borrower's selling costs may also affect his decision to default. However, these costs are only relevant if a move must occur. For example, if \( H > M \) and the homeowner is able to make the mortgage payments, then he has incentive to continue to make the payments and retain his positive equity of \( H - M \). If the homeowner is instead unable to make the mortgage payments due to unemployment or other reasons, he must incur the selling costs \( (A) \) in order to realize his equity position. In this case, it is possible for \( H > M \) but for \( H(1-A) < M \). Default becomes cheaper than selling, and the homeowner will simply walk away from the house.

Throughout this thesis it has been maintained that the house is the only collateral available to the banker for recourse action in the event of default. If the mortgage instrument also sanctifies a personal obligation on the part of the homeowner, then his other assets become a critical factor. Under these conditions the homeowner would only choose to default if

\[
H + n \leq M
\]  

(8.2)

where \( n \) represents other attachable assets. The banker would certainly be interested in the magnitude of \( n \) as well as its stability level.
Note, however, that large other assets offer no protection to the banker if state law prevents the bank from attaching such assets to remedy its losses. Several states have such anti-deficiency legislation of which California provides the greatest protection to the mortgagee. In summary, the banker should first be concerned with the legal constraints imposed by state statute. If personal assets are attachable, then they are certainly important for the default decision.

A final borrower characteristic to consider is the homeowner’s occupation. Not only do occupational skills sensitize income stability to economic shocks, they also increase the value of “gutting” dividends. A skilled blue collar worker, for example, may have greater aptitude and physical resources available for dismantling the house. If he were unemployed, the opportunity cost of his time is lowered, and gutting becomes valuable. He has greater incentive to extract wealth from the banker. Given the possibility of such atypical (and yet rational) patterns of behavior, borrower characteristics could affect default and the price of protection.

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38 Other states with varying degrees of protective legislation are Idaho, New York, North Carolina, Oklahoma, Pennsylvania, South Carolina, North Dakota, South Dakota, Washington, and Utah. See Osborne, et. al., (1979) for a discussion on this point.
8.1.2 Stable Paretoian Distributions of House Returns

The option pricing model utilized in this thesis assumes that the random value of the house is log normally distributed. However, the work of Mandelbrot (1963), and Fama and Roll (1969, 1971) suggests that the distribution of most prices appears to be too leptokurtic to be consistent with the log normal assumption. An alternative approach would be to assume that the random house value is distributed according to the symmetric Paretoian stable class.

Instead of being governed by two parameters, these symmetric stable distributions are characterized by three parameters: the mean; the characteristic exponent, which determines how fast the tail tapers off; and the standard scale which is analogous to the variance. The variance itself is actually infinite for all values of the characteristic exponent except the limiting value of 2. For this limiting case, the normal distribution results.

The theoretical appeal for using a Paretoian stable distribution is its ability to sensitize prices to the possibility of a "deep" discontinuous change in the value of the asset. For mortgage default insurance, such a sudden and substantial change in house value could happen if the homeowner decided to extract a large dividend (i.e., dismantle the house). Comparison of the simulated prices presented in Chapter 6 with the observed prices in figure 1 appears to support the possibility of such discontinuous changes in
house value. Insurance rates on low L/V mortgages are quoted at higher values than the model predicts.

This thesis has utilized the seminal ES model because of its wide acceptance, and because of the applicable extensions made by Merton (1973) on stochastic interest rates, Scholes (1977) on taxes, and Brennan and Schwartz (1977) on the pricing of the American put. The Brennan and Schwartz procedure is particularly important in order to consider the amortizing feature of the mortgage debt and the service flow of the house.

A disadvantage to utilizing the stable distribution assumption is its limited applications and extensions appearing in the literature. However, any future investigation into the specific impact of moral hazard should consider the stable distribution assumption. McCulloch (1978) has developed an option pricing formula based on the log symmetric stable distribution. He has applied the formula (1981) to the pricing of deposit insurance for commercial banks as did Merton (1977) under the rival normal assumption. Their results are substantially different. As the bank's capital becomes larger (i.e., as the put's exercise price becomes farther out) the differences in their results become striking. For 15 percent capital and a twenty year portfolio maturity, Merton's estimate for insurance was below one basis point. McCulloch's estimate was 83.5 basis points. Such a discrepancy warrants further exploration.
8.2 FURTHER APPLICATIONS

The default insurance rates under review in this thesis are those quoted for fixed-rate mortgage instruments (FRMs). The recent offerings by lenders of various alternative mortgage instruments (AMIs), such as variable rate mortgages, graduated payment mortgages, etc., provide alternatives for study. For example, one could determine:

1. How standard insurance coverages for different L/V ratios should be altered to leave the residual default risk (thus the total default risk for the lender) the same for various AMIs as for the FRMs, and

2. What differential risk premia should be charged by the lenders of AMIs if the insurance coverage requirements established by GNMA, FNMA, and FHLMC remain unchanged.

The variance rate, initial L/V ratio, and risk-free interest rate are the same for the AMIs as the FRMs. Therefore, the critical variables that should alter the residual risk of the AMIs are the amortized loan schedule and the maintenance behavior of the borrower/occupant.

8.3 EMPIRICAL WORK

The typical approach to empirical testing in the option pricing literature (Black and Scholes, 1972, and Galai, 1975) has been to form a portfolio of hedged contracts. The quantities of options needed to satisfy the riskless hedge position are purchased under alternative price strategies. The pricing efficacy of the model is tested by buying "undervalued" calls and selling "overvalued" calls at model
prices. Using the true stock variances calculated ex post, Black and Scholes found the portfolio returns to be insignificantly different from zero.

To test market efficiency, the portfolio strategy is to buy "undervalued" calls and sell "overvalued" calls at market prices. Excluding transactions costs, the portfolios generated returns significantly greater than zero. However, after taking transactions costs into account, Black and Scholes conclude that no excess profits can be earned.

Conducting empirical tests in the tradition of Black and Scholes requires a free market setting in which bids on the underlying asset and contingent claims are called out continuously and prices are allowed to settle at their equilibrium level. Unfortunately, the underlying asset in the real estate market is not a continuously traded security. In the default insurance market, the contingent claim cannot equilibrate within narrow time intervals due to state regulatory involvement. In fact as noted in section 2.2, insurance rates appear to be unchanged over the past seven years. Due to these natural and imposed constraints, the empirical tests devised for the default insurance model cannot be as powerful in assessing market or pricing efficiency as those devised for testing options written on common stock securities.

Nevertheless, several interesting market phenomena could be tested. A few of these empirical questions are presented below with a brief discussion.
8.3.1 The Variance Rate

A fundamental area of empirical research should focus on the variance rate of the underlying house returns. This is a critical variable in setting the price of default insurance. However, default insurance prices are constant across time and regions (see section 2.2). Therefore, if the variance rate is not also constant, then the mortgage rate, house appreciation rate or service flow rate must be adjusting.

Asay (1978) estimated the variance rate on house returns, but only in the southern California region. Because default insurance is offered nationwide, future studies should concentrate on estimating variances across national regions. The results of a regional study in combination with time series analysis would provide direction to other empirical work.

8.3.2 Price and Supply Adjustments

If the variance rate is indeed dynamic, then one would expect some form of price or supply adjustments to be occurring. An implicit price is the underwriting service provided. By increasing or decreasing response time to loan applications, insurers can compete for customers. This response time should increase with a declining variance rate.
Instead of making price adjustments, insurers could simply choose to withdraw from the market if the variance rate becomes too high. A study of market participation among insurance companies should isolate a greater concentration of firms in competition when and where the variance rate is lower and more stable.

8.3.3 Exogeneous Adjustments

In addition to the price and/or supply responses to a dynamic variance rate, there might be certain exogeneous adjustments occurring in the economy across time and regions. The dividend rate and the implicit appreciation rate are known to be equally important in setting the price of default insurance. To offset the effects of an increasing variance rate, for example, the appreciation rate would have to increase and the dividend rate would have to decrease. The Merton (1973) adjustment for stochastic interest rates offers an appealing empirical test. From section 5.9 it has been shown that as the correlation between house returns and bond returns increases, the model variance rate declines. An estimate of this correlation between bond returns and house returns would be particularly useful. It might be, for example, that when house price volatility is high, house price appreciation is high, or there is greater correlation between bond and house returns than when house price volatility is low.
8.3.4 Mortgage Pricing

The pricing model can also be expanded to examine the residual default risk retained by the backer. This "second put" should be the major explanatory variable for rate differentials on different L/V mortgages. Inserting the model price of the second put in a linear equation should contribute significantly to the explanation of rate spreads across different L/V insured mortgages. As an empirical observation, the spread over different L/V commitment rates narrowed as interest rates simultaneously increased during the 1974-1975 period.\(^\text{39}\) This observation is consistent with the model's prediction that increasing interest rates will result in lower put prices.

8.3.5 Moral Hazard and Borrower Characteristics

A final area of empirical work might focus on the moral hazard/borrower characteristic issue. The results of such a study should be useful to the industry for setting lending policies. As discussed in section 8.1.1, state law and asset attachment have significant impact on the magnitude of losses incurred by the lender/insurer in the event of default. If the theory holds, a case study of defaulted mortgages involving substantial resell losses should reveal that the borrowers were without attachable personal assets or had protection under state statutes against such attachment. If

\[\text{39 see Zabranski (1978).}\]
true, then such borrower characteristics and state statutes would deserve consideration in the lending process. A survey of state statutes, court rulings, and enforcement records with a discussion of lending implications might be developed in an agency theory format.

8.4 CONCLUSION

The option pricing model offers a new approach to the examination of mortgage default. Its assumption of rational behavior mandates the reconsideration of why borrower characteristics might affect default behavior and default premia.

By focusing on the property and financial characteristics, the model proposes new variables for consideration in the analysis of default. There is a resulting void of empirical information on the estimation of these new parameters. A host of questions regarding the interaction of the exogeneous variables and the possible reactions of certain endogeneous variables remain unanswered and left for future research.
Appendix A

NUMERICAL PRICING PROCEDURE FOR MORTGAGE DEFAULT INSURANCE

MORTGAGE DEFAULT INSURANCE RATES WITH DISCRETE DIVIDENDS AND A DECLINING EXERCISE PRICE

PRICES FOR 1981 INTEREST RATES

DESCRIPTION OF PARAMETERS

\( \text{G} \) = FUT PRICE

\( \text{V}_0 \) = VARIANCE RATE FOR THE RETURN

\( \text{BE} \) = BISKLESS INTEREST RATE

\( \text{BE} \) = MORTGAGE RATE OF INTEREST

\( \text{EZ} \) = EARY EXERCISE PRICE (BEGINNING MORTGAGE BALANCE) EXRESSED AS A PERCENT (E.G. 60 PERCENT \( = 0.60 \))

\( \text{D} \) = DISCRETE DIVIDEND PER INTERVAL (USUALLY ZERO)

\( \text{IX} \) = HOUSE PRICE INCENTIVE

\( \text{IK} \) = TIME INCENTIVE (NO. OF YEARS \( / (\text{IK} \times 100) \))

\( \text{NS} \) = NUMBER OF HOUSE PRICES (MODE GREATEST OF EQUAL TO \( \text{IK} \))

\( \text{NM} \) = NO. OF INTERVALS (MORTGAGE PAYMENTS PERIODS) TO BE CONSIDERED

\( \text{H} \) = VECOR OF THE NO. OF TIME PERIODS CONSIDERED IN BETWEEN EACH MORTGAGE PAYMENT

DIMENSION G(1500,2), PG(1500), CC(1500), A(1500), B(1500), C(1500), 1AA(1500), BB(1500), CC(1500), EF(1500), HS(360), ST(1500), 2X(100), Y(100), Z(1500), FA(1500), FG(1500), CG(1500)

DIMENSION SC(500), IC(500)

READ (5,1) G, P, C, B, A, E, X, Y, Z, NS

FORMAT (1,10,5/I4)

READ (2,2) HK, (H(1) = 1, HK)

FORMAT (3/I4(10I4))

WRITE (6,100) G, P, C, B, A, E, X, Y, Z, NS

FORMAT (1,10,4/OX, 'FUT AND CRITICAL STOCK PRICES',/20X, 'VARIANCE RATE =',/20X, 'MORTGAGE RATE =',/20X, 'DISCRETE DIVIDEND =',/20X, 'HOUSE PRICE STEP =',/20X, 'TIME STEP =',/20X, 'NUMBER OF HOUSE PRICES =',/20X)

WRITE (6,200) HK

FORMAT (20X, 'NUMBER OF TIME INTERVALS = ',I4/)

WRITE (6,201) (H(1) = 1, HK)

FORMAT (20X, 'TIME STEPS PER INTERVAL = ',I4/10I4))

HK1 = HK - 1

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INITIALIZE ECONOMIC VALUES OF G

CALCULATION OF DECLINING EXERCISE PRICE

MK1=NS+1
NW=(1+(ER/12))**360-1
DO 3 IB=1, MK
Z=(1+(ER/12))**I-1
BFAC=1-(Z/2Z)
IN1=ER+1
IN2=BFAC*1E
3 CONTINUE
E(1)=1E
WRITE(6,150) (E(IB), IB=1, MK)
150 FG=1.0/(1.0, IC, DEG). BAL. = *12F10.5/
NT=NT(1)*NT(2)
DO 6 IN=1, NS
ST(I)=IS*FLOAT(I-1)
IF(ST(I) .GE. (NS1)) G(I, 1)=0.0
6 IF(ST(I) .LE. (NS1)) G(I, 1)=E(NS1) - ST(I)
I=0
DO 8 K=1, MK
NTK=NT(K)*1
IF(K .GE .1) GC TO 160
DO 170 I=1, NS
170 F(1)=CG(I)
160 DO 9 J=2, NTK
IF(K .GE .1) GC TO 161
DO 180 I=1, NS
180 F(I)=CG(I)
C SOLVE SIM ECONS FGS J
161 Y1=0.5*EF*IX
Y2=0.5*WB*IX
NS1=NS-1
NS2=NS-2
DO 10 I=1, NS2
A(I)=Y1*I-Y2*I
B(I)=1.0+2.0*I+2.0*I+1
C(I)=-1.0-I-2*I
10 F(I)=G(I+1, J-1)
BB(1)=G(1)
CC(1)=C(1)
FF(1)=F(1)
G(I, J)=G(I, J-1)*G(1, J-1)
F(1)=F(1)-A(1)*G(I, J)
A(NS1)=-1.
B(NS1)=1.
C(NS1)=1.
F(NS1)=C.

TRANSFORM EQUATIONS

DO 11 I=2,NS1
   IF (ABS(A(I)) .LT. 0.0001) GO TO 12
   BB(I)=EE(I-1)+B(I)/A(I)-CC(I-1)
   CC(I)=BB(I-1)*C(I)/A(I)
   FF(I)=EE(I-1)*F(I)/A(I)-FF(I-1)
   IF (ABS(EE(I)) .LT. 1.0000) GO TO 11
   BB(I)=0.0001*EB(I)
   CC(I)=0.0001*CC(I)
   FF(I)=0.0001*FF(I)
   GO TO 11
11 CONTINUE
   BB(I)=E(I)
   CC(I)=C(I)
   FF(I)=F(I)
   CONTINUE

SOLVE FOR G

G(NS,J)=FF(NS1)/EE(NS1)
DO 13 L=1,NS2
   I=NS-IX
13 G(I,J)=(FF(I-1)-CC(I-1)*G(I+1,J))/EB(I-1)
171 IF (K.EC.1) GO TO 14
   T=PLC(J-1+MYK)*IK
   GO TO 15
14 T=PLC(J-1)*IK
   MYK=0
15 CONTINUE
   I=L+1

ARBITRAGE CONDITION

K1=K+1
   NK2=NK+2
   HKD=HK2-K
   IZ=IZ/XB
   DO 32 I=2,IX
      ARB=E(HKD)-SZ(I)
      IF (ARB.LT.0.0) GO TO 31
      CONTINUE
31 SC(I)=SZ(I-1)+XB*(E(HKD)-SZ(I-1)-G(I-1,J))/ (G(I,J)-G(I-1,J)+XB)
   IC(I)=7
   DO 16 J=1,IX
      ARB=E(HKD)-SZ(I)
16 IF (ARB.EQ.0.0) G(I,J)=ARB
      IZ1=IZ+1
      DO 17 I=IZ1,NS
17 IF (G(I,J).LT.0.0) G(I,J)=0.0
      IF (J.EQ.MYK) GO TO 14
C
IF (T.GT.23.99.AND.T.LT.24.01) GO TO 128
IF (T.GT.24.99.AND.T.LT.25.01) GO TO 128
IF (T.GT.25.99.AND.T.LT.26.01) GO TO 128
IF (T.GT.26.99.AND.T.LT.27.01) GO TO 128
IF (T.GT.27.99.AND.T.LT.28.01) GO TO 128
IF (T.GT.28.99.AND.T.LT.30.01) GO TO 128
GO TO 8

128 WRITE (6,23) ST(843),ST(890),ST(942),ST(1001)
23 FORMAT (/IX,'$D CUR-DIV',$,F16.4)
WRITE (6,24) G(843,1),G(890,1),G(942,1),G(1001,1)
24 FORMAT (/IX,'$D CUR-DIV',$,F16.4)
WRITE (6,45) RATE(843),RATE(890),RATE(942),RATE(1001)
45 FORMAT (/IX,'$D CUR-DIV',$,F16.4)

WRITE PRICES OF SECOND E.T.

WRITE (6,210) ST(1204),ST(1124),ST(1081),ST(1054),ST(1004)
210 FORMAT (/IX,'$D CUR-DIV',$,F16.4)
WRITE (6,245) RATE(1204),RATE(1124),RATE(1081),RATE(1054),RATE(1004)
245 FORMAT (/IX,'$D CUR-DIV',$,F16.4)
WRITE (6,211) ST(1271),ST(1166),ST(1112),ST(1072),ST(1011)
211 FORMAT (/IX,'$D CUR-DIV',$.F16.4)
WRITE (6,246) RATE(1271),RATE(1166),RATE(1112),RATE(1072)
246 FORMAT (/IX,'$D CUR-DIV',$.F16.4)
WRITE (6,202) ST(1366),ST(1256),ST(1208),ST(1178),ST(1135),ST(1071)
202 FORMAT (/IX,'$D CUR-DIV',$.F16.4)
WRITE (6,247) RATE(1366),RATE(1256),RATE(1208),RATE(1178),RATE(1135),RATE(1071)
247 FORMAT (/IX,'$D CUR-DIV',$.F16.4)
WRITE (6,203) ST(1430),ST(1334),ST(1251),ST(1206),ST(1137)
203 FORMAT (/IX,'$D CUR-DIV',$.F16.4)
WRITE (6,248) RATE(1430),RATE(1334),RATE(1251),RATE(1206),RATE(1137)
248 FORMAT (/IX,'$D CUR-DIV',$.F16.4)

COMPUTE AND WRITE THE NET INSURANCE PREMIUM RATES

A1=RATE(843)-RATE(1204)*.25
A2=RATE(843)-RATE(1256)*.25
A3=RATE(843)-RATE(1208)*.25
A4=RATE(843)-RATE(1178)*.25
A5=RATE(843)-RATE(1135)*.25
B1=RATE(890)-RATE(1271)*.25
B2=RATE(890)-RATE(1256)*.25
B3=RATE(890)-RATE(1208)*.25
B4=RATE(890)-RATE(1178)*.25
B5=RATE(890)-RATE(1135)*.25
E5=RATE(890)-RATE(1072)*.15

IF (T.GT.1.90) GO TO 280
GO TO 281

280 B5=B5A
281 B6=RATE(890)-RATE(1011)
B6A=RATE(890)-RATE(1004)*.25
IF(T.GT.1.90) GO TO 282
GO TO 283
282  B6 = B6
283  C1 = BATE(942) - BATE(1396) + .25
     C2 = BATE(942) - BATE(1256) + .25
     C3 = BATE(942) - BATE(1208) + .20
     C3A = BATE(942) - BATE(1208) + .25
     IF(T.GT.1.9) GO TO 284
     GO TO 285
284  C3 = C3A
285  C4 = BATE(942) - BATE(1178) + .10
     C4A = BATE(942) - BATE(1178) + .25
     IF(T.GT.1.9) GO TO 286
     GO TO 287
286  C4 = C4A
287  C5 = BATE(942) - BATE(1135) + .05
     C5A = BATE(942) - BATE(1135) + .25
     IF(T.GT.1.9) GO TO 288
     GO TO 289
288  C5 = C5A
289  C6 = BATE(942) - BATE(1071)
     C6A = BATE(942) - BATE(1071) + .20
     C6AA = BATE(942) - BATE(1071) + .25
     IF(T.GT.1.9 .AND. T.LT.3.1) GO TO 290
     IF(T.GT.1.9) GO TO 291
     GO TO 292
290  C6 = C6A
     GO TO 292
291  C6 = C6A
292  D1 = BATE(1001) - BATE(1430) + .25
     D2 = BATE(1001) - BATE(1334) + .10
     D2A = BATE(1001) - BATE(1334) + .25
     IF(T.GT.1.9) GO TO 293
     GO TO 294
293  D2 = D2A
294  D3 = BATE(1001) - BATE(1251)
     D3A = BATE(1001) - BATE(1251) + .25
     IF(T.GT.1.9) GO TO 295
     GO TO 296
295  D3 = D3A
296  D4 = BATE(1001) - BATE(1206)
     D4A = BATE(1001) - BATE(1206) + .20
     D4AA = BATE(1001) - BATE(1206) + .25
     IF(T.GT.1.9 .AND. T.LT.3.9) GO TO 297
     IF(T.GT.3.9) GO TO 298
     GO TO 299
297  D4 = D4A
     GO TO 299
298  D4 = D4A
299  D5 = BATE(1001) - BATE(1137)
     D5A = BATE(1001) - BATE(1137) + .15
     D5AA = BATE(1001) - BATE(1137) + .20
     D5AAA = BATE(1001) - BATE(1137) + .25
     IF(T.GT.1.9 .AND. T.LT.3.9) GO TO 300
151

IF(T.C1.3,9.AND.T.L3.4.9) GC TO 301
IF(T.C1.4,9) GC TC 302
GO TO 310
300 DS=DSAA
GO TO 310
301 DS=DSAAA
GO TO 310
302 DS=DSAAAA
310 WRITE(6,311) A1, B1, C1, D1
311 FORMAT(/IX,.30 COVERAGE 95 L/V,1P10.4,
          1/IX, 90 L/V,1P10.4,
          2/IX, 85 L/V,1P10.4,
          3/IX, 80 L/V,1P10.4)
WRITE(6,312) A2, B2, C2, D2
312 FORMAT(/IX,.25 COVERAGE 95 L/V,1P10.4,
          1/IX, 90 L/V,1P10.4,
          2/IX, 85 L/V,1P10.4,
          3/IX, 80 L/V,1P10.4)
WRITE(6,313) A3, B3, C3
313 FORMAT(/IX,.22 COVERAGE 95 L/V,1P10.4,
          1/IX, 90 L/V,1P10.4,
          2/IX, 85 L/V,1P10.4,
          3/IX, 80 L/V,1P10.4)
WRITE(6,314) A4, B4, C4, D4
314 FORMAT(/IX,.20 COVERAGE 95 L/V,1P10.4,
          1/IX, 90 L/V,1P10.4,
          2/IX, 85 L/V,1P10.4,
          3/IX, 80 L/V,1P10.4)
WRITE(6,315) B5, C5, D4
315 FORMAT(/IX,.17 COVERAGE 90 L/V,1P10.4,
          1/IX, 65 L/V,1P10.4,
          2/IX, 60 L/V,1P10.4)
WRITE(6,316) A5
316 FORMAT(/IX,.16 COVERAGE 90 L/V,1P10.4)
WRITE(6,317) B6, C6, D5
317 FORMAT(/IX,.12 COVERAGE 90 L/V,1P10.4,
          1/IX, 85 L/V,1P10.4,
          2/IX, 80 L/V,1P10.4)
IF(K.EC.NK) GC TC 21
8 CONTINUE
21 CONTINUE
STOP
END

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