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A LABORATORY-BASED APPROACH FOR THE EVALUATION OF PAVEMENT RELIABILITY

The Ohio State University

Ph.D. 1982

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A LABORATORY-BASED APPROACH

FOR

THE EVALUATION OF PAVEMENT RELIABILITY

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Mohamed El Magzoub Abdallah, B.S.C.E., M.S.C.E.

* * * * *

The Ohio State University
1982

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DEDICATION

This dissertation is dedicated, in loving memory, to my mother.
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CHAPTER I
INTRODUCTION

On the occasion of the 26th anniversary of the U.S. interstate highway system this year (1982), the following update has recently been issued (61):

"A 1958 cost analysis indicated the system would be finished in 1972 at a cost of about $40 billion. However, 23 years after that estimate and expenditures of nearly $82 billion, the Highway Users Federation now estimates that an extra 1,726 miles remains to be finished, that 100 construction gaps exist, and that the final tab to finish up will total nearly $54 billion. As of 1982, some 40,438 miles of interstate were being traveled."

It is clear that the interstate system is almost complete, and that more emphasis should be put into developing maintenance approaches that manage to give serviceable highways with the optimum allocation of resources.

This, however, should not detract us from noticing that sizeable amounts of money are still planned to be spent on interstate construction alone. In 1981, the total expenditure of the entire nation on roads was about $36 billion, roughly equally divided between construction and maintenance (62).

This figure amounts to about half of what was spent on highways per passenger-mile ten years ago (62). This shrinkage in highway
funding, which is taking place in all levels of government, is due to the rapid inflation in the highway construction industry on the one hand, and the reduced income that resulted from the bad economic conditions and the more fuel-efficient motor vehicles that may keep multiplying in the future, on the other hand (62).

Recent studies by the U.S. Department of Transportation revealed the following dismaying situation (62):

(1) Ten percent of the interstate highway system requires immediate and substantial surface reconstruction, and the percentage increases annually.

(2) About one out of every five bridges requires rehabilitation or reconstruction.

(3) Twenty-five percent of the major roads in urban areas are chronically overloaded.

(4) Accident statistics have begun to grow worse after a long period of decline.

Combining this with the funding shortages mentioned earlier, these facts put pavement management programs to great challenges. Some of the decisions that a pavement management has to wrestle with are (63):
* economic efficiency for the agency,
* economy, safety and serviceability for the user,
* limited pollution during construction,
* use of local materials and labor,
* limiting physical deterioration,
* conservation of aggregates, and
* adequate load-carrying capacity.

To optimize on capital investments and maintenance costs of pavements with respect to the above-mentioned and other pertinent criteria, pavement management should adopt a design philosophy that is able to recognize explicitly not only the progressive nature of pavement deterioration and the age at which failure occurs, but also the significant degree of uncertainty that has to be faced when dealing with the real world.

The elements of uncertainty may best be handled with the technique of reliability analysis, and the consideration of reliability as a design parameter. This had been recognized as early as 1969 (see Reference 64), and the reliability function of pavements was incorporated into systems developed in the last decade. In these developments, failure distributions are usually approximated through mathematical techniques, e.g., estimating means and variables of functions by expansion series.

The purpose of this research is to investigate the direct application of statistical inference techniques to lives of representative
samples of flexible pavements, in an attempt to explain the behavior of these pavements, in fatigue, when subjected to different stress levels and configurations, as well as different levels of temperature. The specific objectives of this work can be summarized as follows:

1. Review and analysis of the various existing models of pavement reliability;

2. Development of a mathematical model for pavement reliability that relies on the distribution of a material property;

3. Mathematical derivations relating the pavement reliability function to thickness variations and foundation conditions;

4. Experimental verification of the developed model; and,

5. Analysis of the pavement reliability model and outlining some application procedures for it.

Fatigue failure in flexible pavements is recognized as one of the major causes of pavement distress in the field (65), and has been the subject of extensive research. Due to the fact that a relationship can be established between fatigue failures and functional failures in
flexible pavements, together with the availability of reasonable sample sizes of data, fatigue failure was chosen for this study, although the procedure can be applied to any other mode of failure.

This study starts with a short account on the theory of reliability of components and systems. Some statistical distributions of interest in fatigue failure are described and the concept of the pavement as a system, in the physical sense, is then outlined. Pavement reliability is discussed in Chapter III, where the nature of variability in pavements is looked into. Pavement reliability applications are then described and assessed. Chapter IV deals with the mathematical development of the pavement reliability, while Chapters V and VI deal with the verification of the model in both the level of laboratory samples and the pavement. The sensitivity of the model to variations of load, thickness and climate is conducted and some applications of the reliability model are outlined. Finally, a brief summary of this work, together with conclusions and recommendations for future work are given in Chapter VII. Appendix A contains the experimental procedures followed to generate the laboratory data, and Appendix B contains computer plots of the Weibull probability distribution function, probability density function and failure rate for each of the eleven samples used in this research.
CHAPTER II

MATHEMATICAL ASPECTS OF RELIABILITY

2.1 Introduction

The intent of this chapter is to provide some insights into the concept of reliability and its connections with the theory of probability. The characteristics of some statistical distributions, of interest in fatigue failure, are described. Reliability of systems is, then, dealt with in a general treatment of systems analysis. Finally, the concept of the pavement as a system, in the physical sense, is highlighted.

It must be mentioned that the reliability formalism addressed here is essentially only applied to electronic components and systems -- except for the unavoidable inclusion of some electromechanical devices (21).

There is no basic theoretical reason against applying this concept to other engineering devices (civil or mechanical), if components and systems can be well-defined, and if appropriate distributions describing the component behavior are available. In fact, it may even be preferable to adopt this concept since, as we shall see later, reliability is, by definition, a time-based function.
This is not directly seen in the concepts adopted by mechanical and structural disciplines.

2.2 The Concept of Reliability

The concept of reliability, in an abstract sense, has been known to mankind from the time human beings began living in societies. Trustworthy and dependable men are those who can be relied upon by the community. In this context, i.e., reliability as a human attribute, there is no way that a person's level of reliability could be measured (there is still no way to do so now).

In engineering, and in mathematical statistics, however, reliability is more defined; as it can be quantified. To quote Bazovsky (18), "...not only can it be exactly defined, but it can also be calculated, objectively evaluated, measured, tested and even designed into a piece of engineering (device)."

In an engineering sense, then, reliability can be conceived as a measure of the degree of successful performance of a functional device under the required conditions of operation (19).

Before proceeding to a formal definition of reliability, a historical summary of its development is in order.

There is a general agreement that reliability, as a science, was developed during World War II as a means to explain and remedy the severe problems of maintenance, repair, and field failures of the
military equipment (see, for example, Greene and Bourne (19), Shooman (20) and Goldberg (21).) Nevertheless, it was inherent in industries like the weapons industry and structures, from their early inception, to the extent that one cannot help thinking about it when contemplating the development of design as such (21).

2.2.1 Reliability Defined

Concisely defined, reliability is the probability of a device performing its purpose adequately for a specified period of time under encountered operating conditions (18, 19, 21, 22, 23, 24).

Compared to quality, which is the degree of conformance of the product to the engineering and manufacturing requirements that achieve the purpose of the client, the reliability characteristics of a product are concerned with certain techniques, of design and utilization, that determine the capability of this product to achieve a specified standard of performance and maintain this standard for a period of time under operating conditions (25).

In this context, quality is mainly concerned with the standards of workmanship, and the element of time does not influence its characteristics.

Reliability is concerned with four parameters, one of which is time. In fact, the whole concept of reliability is time-based — as is evident from the definition. The other parameters are: the standards of performance, environmental conditions, and the probability of maintaining performance (25).
Performance, generally, is related to the capability of a device to fulfill an allotted task. In some cases, degradation of performance is permitted to some tolerance level consistent with the overall requirement whereas, in other cases, failure to perform the task is looked upon as catastrophic.

Environmental conditions at which the reliability of the device is sought must also be stated, since factors like temperature, humidity, and type of loading are contributing to failure.

2.2.2 Failure

Fundamentally, a device fails when its strength becomes incompatible with the load imposed on it. If we take, for example, a working device and assume that both the strength and the load are subject to variation — as is always true to some extent (22). If the weakest part of this device can endure the most severe load, then the device is completely reliable. Otherwise, failures are to be expected.

The strength of the device is that of its material (22). A number of different material properties are usually involved in defining strength, except for geometrical variations of the device that usually affect the variability of the load, since they affect the stress imposed on the device and not the material strength.

The above-mentioned concepts are adopted in structural reliability, to define the probability of failure, $P_f$, of a structure as the probability that the demand or load, $D$, on the structure exceeds its capacity, or strength, or resistance, $C$, (26):
\[ P_f = P (C - D < 0) \]  \hspace{1cm} (2.1)

From there, a safety factor concept can be included:

\[ P_f = P (C/D < 1) \]  \hspace{1cm} (2.2)

Figure 2.1 shows the situation that arises when the strength (capacity) and load (demand) distributions overlap. The probability of failure may be measured by the shaded area of overlap of these curves.

This method of evaluating the probability of failures, hence reliability, was found to be generally ultra-conservative, and the corresponding reliability excessively pessimistic (26).

2.3 Reliability Formulation

Equation (2.1) shows the probability of failure of a structure. Reliability, by definition, is the probability of no failure. Hence, it can be given, for the above-mentioned case, as:

\[ R = 1 - P_f \]  \hspace{1cm} (2.3)

Generally, a failure distribution is a mathematical attempt to provide the engineer with information about the life of a device under particular conditions of stress and strain (25). This information is provided by plotting the frequency of failure against the operational time to failure.
C = capacity; and

D = demand.

Figure 2.1 Graphical Presentation of Failure (after Reference 60)
The mean of the distribution so represented gives the mean life of the device, which is sometimes called the mean time to failure (MTTF) or the mean time between failures (MTBF).

Referred to the total number of items tested, the failure distribution, \( f(t) \), gives the fraction which is expected to fail at time \( t \). This is often called the mortality distribution function, or simply "mortality" (25).

The cumulative mortality, or the cumulative probability distribution of failure, gives the proportion of items expected to fail at or before time \( t_1 \), and is given by:

\[
F(t) = \int_{-\infty}^{t} f(t) \, dt \quad (2.4)
\]

But, since the life of a device cannot take negative values, this expression becomes:

\[
F(t) = \int_{0}^{t} f(t) \, dt \quad (2.5)
\]

Reliability, or the reliability function, or the survival function, is given as the complementary of the above distribution, i.e.,

\[
R(t) = 1 - F(t) \quad (2.6)
\]

This function, by definition, gives the portion of items surviving the time \( t_1 \).

It follows that:

\[
R(t) = \int_{t_1}^{\infty} f(t) \, dt \quad (2.7)
\]
Failure rate is usually defined as:

\[
\frac{\text{Number of Failures}}{\text{Length of time during which failures can occur}}
\]

and this is usually expressed in terms of failures per unit time or per number of operations or cycles. Bompas-Smith (22) states that this definition is "very misleading," since the failure rate may change appreciably with the life of the device being considered. So, it is usually required to know, not only the overall failure rate, but the failure rate during any particular period of life.

Redefined, our failure rate would be:

\[
\frac{\text{No. of failures expected during a unit of failure time at a given lifetime}}{\text{No. of items exposed to failure at the same lifetime}}
\]

from which our first definition is a special case, i.e., when the failure rate remains constant.

Failure rate, let us call it \( Z(t) \), is given a number of different names in the literature, like the hazard rate, force of mortality, local failure rate, and instantaneous failure rate.

A strong relationship can be found between the reliability function and the failure rate. Consider, for example, Figure 2.2, which shows a distribution, \( f(t) \), of failures at time, \( t \), and the proportion surviving, \( R(t) \), at the same time. Between \( t \) and \( t+\delta t \), the proportion expected to fail is:
Figure 2.2 Derivation of Local Failure Rate

\[ f(t) \]

\[ R(t) \]

\[ t \]

\[ t + \delta t \]
and since the proportion $R(t)$ is surviving, the failure rate, by definition, is given by:

$$Z(t) = \frac{\int_{t}^{t+\delta t} f(t) \, dt}{R(t)}$$

$$= \frac{[F(t + \delta t) - F(t)]}{R(t)}$$

$$= \frac{f(t)}{R(t)} \quad (2.8)$$

(by dividing by $\delta t$ and letting $\delta t \to 0$)

We can now express our failure distribution in a general form.

Equation (2.8) can be written as:

$$f(t) = Z(t) \cdot R(t)$$

$$\frac{dF(t)}{dt} = Z(t) \cdot R(t)$$

$$- \frac{dF(t)}{dt} = -Z(t) \cdot R(t)$$

$$\frac{dR(t)}{dt} = -Z(t) \cdot R(t) \quad (2.9)$$

Integrating Equation (2.9) from 0 to $t$,

$$\ln R(t) = - \int_{0}^{t} Z(t) \, dt + C \quad (2.10)$$

Since $R(t) = 1$ at $t = 0$, $C = 0$, and
\[ R(t) = \exp \left\{ -\int_{0}^{t} Z(t) \, dt \right\} \] (2.11)

and

\[ F(t) = 1 - \exp \left\{ -\int_{0}^{t} Z(t) \, dt \right\} \] (2.12)

Differentiating Equation (2.12),

\[ f(t) = Z(t) \exp \left\{ -\int_{0}^{t} Z(t) \, dt \right\} \] (2.13)

Before leaving this section, it is interesting to examine the curve of the failure rate with time for a general case of failure. Figure 2.3, which is called "the bath tub curve," shows such a representation.

Regions A, B and C of this figure may be interpreted as:

A: infant mortality, running-in or burning-in period; this is the period where failures are attributed to causes such as poor design or production processes.

B: chance mortality, or constant failure rate period; this is the period of normal utilization when failure is due to chance alone.

C: wear-out, or deterioration period; where the incidence of failure becomes greater than the capacity for repair.
Figure 2.3  The "Bathtub" Curve
No clear physical distinction is generally available between these regions (25). This is reason enough to think of this curve as only illustrative.

2.4  Statistical Distributions of Interest

Most of the textbooks on elementary or advanced reliability theory, or even probability and statistics, include statistical distributions used in reliability. (See, for example, References 24, 27, 28 and 29). A brief account of some of these distributions is in order.

2.4.1 The Exponential Distribution

This is the most popular distribution in reliability, due to the fact that it is easy to work with, and the property it has of a constant failure rate. The constant failure rate represents the stage of random or chance failures in the "bath tub" curve (Figure 2.3). This stage is considered to be the useful life of the device under test (3).

Let the failure rate, then, be:

\[ \lambda(t) = \lambda = \text{constant} \quad (2.14) \]

It follows, from Equations (2.11), (2.12) and (2.13), that

\[ R(t) = \exp \left[ - \int_0^t \lambda \, dt \right] \]
\[ = e^{-\lambda t} \quad , \quad (2.15) \]
\[ F(t) = 1 - e^{-\lambda t} \quad , \quad (2.16) \]
The mean life is given by:

\[ \text{MTBF} = \bar{t} \]

\[ = \int_0^\infty t \cdot f(t) \, dt \]

\[ = \int_0^\infty \lambda t \, e^{-\lambda t} \, dt \]

\[ = \frac{1}{\lambda} \left[ \lambda t \, e^{-\lambda t} + e^{-\lambda t} \right]_0^\infty \]

\[ = \frac{1}{\lambda} \]

(2.18)

Figure 2.4 shows the cumulative distribution, reliability function and failure rate of this distribution.

2.4.2 The Weibull Distribution

In many other cases, failure processes have failure rates that either increase or decrease with time. This property should essentially be reflected in the process distribution.

Weibull (31) claimed that the simplest empirical distribution that could represent a wide variety of actual data could have the following failure rate:

\[ Z(t) = \frac{8}{\eta} \left( \frac{t - t_0}{\eta} \right)^{\beta-1} \]

(2.19)

where
Figure 2.4 The Probability Density Function, Reliability Function, and Hazard Rate for the Exponential Distribution (after Reference 66)
\( t_0 \) = a location constant that defines the starting point or origin of the distribution;

\( \eta \) = a scale constant that stretches the distribution along the time axis; and,

\( \beta \) = a shape constant that primarily controls the shape of the curve.

The value of \( \beta \) determines whether the failure rate is decreasing, constant, or increasing, depending on whether this value is less than unity, unity, or greater than unity, respectively. The original equation of Weibull was, in fact:

\[
\int Z(t) \, dt = \left( \frac{t - t_0}{\eta} \right)^\beta
\]

so that

\[
R(t) = \exp \left[ -\left( \frac{t - t_0}{\eta} \right)^\beta \right], \quad (2.20)
\]

\[
F(t) = 1 - \exp \left[ -\left( \frac{t - t_0}{\eta} \right)^\beta \right], \quad (2.21)
\]

and

\[
f(t) = \frac{\beta}{\eta} \left( \frac{t - t_0}{\eta} \right)^{\beta-1} \exp \left[ -\left( \frac{t - t_0}{\eta} \right)^\beta \right] \quad (2.22')
\]

Usually, \( t_0 \) is taken as zero in fatigue life distribution.

So, the distribution can be given, only with two parameters, as:

\[
R(t) = \exp \left[ -\left( \frac{t}{\eta} \right)^\beta \right], \quad (2.21')
\]

\[
F(t) = 1 - \exp \left[ -\left( \frac{t}{\eta} \right)^\beta \right], \quad (2.22')
\]

\[
f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \exp \left[ -\left( \frac{t}{\eta} \right)^\beta \right], \quad (2.23')
\]
\[ Z(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \]  

(2.19')

\( \eta \) is also called the characteristic life of the distribution; when \( t \) is equal to \( \nu \), the reliability is given by:

\[ R(t) = e^{-(\frac{t}{\eta})^\beta} \]

\[ = e^{-1} \]

\[ = 0.3679 \]  

(2.24)

which means that \( \eta \) is the time, measured from zero, by which 63.21 percent of the population can be expected to fail, no matter what value \( \beta \) has.

The Weibull distribution was derived by Fisher and Tippett, in 1928, as the third asymptotic distribution of extreme values (29). Consequently, in some applications, there may be theoretical reasons for choosing this distribution based on extreme value theory which, in this case, deals with minimum extremes.

As an example, suppose that \( t \) represents the life of a chain of \( n \) links. \( t_i \) is the life of the \( i^{th} \) link. The life of the whole chain is governed by the life of its weakest link, \( t = \min (t_i) \). Consequently, the distribution of \( t \) is the distribution of a minimum. For many different types of \( t_i \) variables, the limiting distribution of the minimum approaches a Weibull distribution as \( n \to \infty \).
Figure 2.5 shows the probability densities, cumulative distributions, and failure rates of this distribution for different values of $\beta$.

### 2.4.3 The Gamma Distribution

The Gamma distribution is a classical distribution which has appeared in the literature since the early 1800's. The Gamma density is given by:

$$f(t) = \frac{1}{\Gamma(\lambda)} \left(\frac{t}{\theta}\right)^{\lambda-1} \cdot \text{EXP}\left(-\frac{t}{\theta}\right)$$

(2.25)

where

- $\theta$ = the scale parameter;
- $\Gamma$ = the shape parameter; and,
- $\Gamma(\lambda) = \int_0^\infty t^{\lambda-1} \cdot \text{EXP}(-t) \, dt$, which is the gamma function.

Like the Weibull distribution, when $\lambda = 1$, the distribution yields the exponential distribution. Unlike the Weibull, however, when $\lambda$ is less than unity, the failure rate is increasing from zero to an asymptote given by the value of the constant failure rate of the exponential distribution. When $\lambda > 1$, the failure rate decreases from to the same asymptote (see Figure 2.6).

Bain (29) postulated that since the gamma distribution results when considering the time to the $R^{th}$ occurrence of a Poisson process, then a product or a system may fail after it has received $k$ shocks of some sort.
Figure 2.5 The Probability Density Function, Reliability Function and Hazard Rate for the Weibull Distribution (After Reference 66)
Figure 2.6 The Probability Density Function, Reliability Function and Hazard Rate for the Gamma distribution (After Reference 66)
One disadvantage of the gamma distribution is that its failure rate and cumulative distribution function (cdf) cannot be evaluated in closed form (29). But this should not be of big concern, since the same is true for the normal distribution (32).

The gamma distribution function is:

\[
F(t) = \left[\frac{1}{\theta^\lambda \Gamma(\lambda)}\right] \int_0^t t^{\lambda-1} \cdot \exp\left(-\frac{t}{\theta}\right) dt
\]  

(2.26)

2.4.4 The Normal Distribution

This distribution may be the most celebrated distribution in statistics. It often describes dimensions of parts made by automatic equipment, natural physical and biological phenomena, and certain types of life data (27). Part of the power of this distribution comes from the Central Limit Theorem that states that the normal distribution may be applicable when each data value is the sum of a large number of random contributions.

Although its failure rate is strictly increasing, which may mean that it is appropriate for the life of products with this type of failure rate, it covers the range from \(-\infty\) to \(+\infty\), which is not acceptable in a life distribution. Life must, of course, be positive. It may be suitable, however, if the mean of the distribution is greater than three times its standard deviation, or if the variate is transformed.

The probability density function (pdf) of the normal distribution is:
\[ f(t) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left[ -\frac{1}{2} \left( \frac{t - \mu}{\sigma} \right)^2 \right] \]  

(2.27)

where

\[ \mu = \text{the mean of the distribution, and} \]

\[ \sigma = \text{the standard deviation.} \]

The cdf of the distribution is:

\[ F(t) = \frac{1}{\sqrt{2\pi} \sigma} \int_{-\infty}^{t} \exp\left[ -\frac{1}{2} \left( \frac{u - \mu}{\sigma} \right)^2 \right] \, du \]  

(2.28)

A more workable form of this distribution is the standard normal distribution, \( Z \), where \( Z = \frac{C - \mu}{\sigma} \). The pdf of this distribution is:

\[ \phi(Z) = \frac{1}{\sqrt{2\pi}} \exp\left( -\frac{Z^2}{2} \right) \]  

(2.29)

This corresponds to a normal distribution with \( \mu = 0 \) and \( \sigma = 1 \) (see Figure 2.7).

The cdf is:

\[ \Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z} \exp\left( -\frac{u^2}{2} \right) \, du \]  

(2.30)

The reliability function and the failure rate are given in terms of the standard normal distribution functions as follows (10):

\[ R(t) = 1 - \Phi\left[ \frac{(t - \mu)}{\sigma} \right] \]  

(2.31)

and

\[ Z(t) = \frac{1}{\sigma} \Phi\left[ \frac{(t - \mu)}{\sigma} \right]/\{1 - \Phi\left[ \frac{(t - \mu)}{\sigma} \right]\} \]

(2.32)
Figure 2.3. Normal cumulative distribution.

Figure 2.4. Normal hazard function.

Figure 2.7

NORMAL DISTRIBUTION

(After Reference 27)
2.4.5 The Lognormal Distribution

Because of its logarithmic nature, this distribution is often used when the range of the data is several powers of 10 (27). Usual fits are on economic data, data on the response of a biological material to stimulus, and certain types of life data. It is, generally, the model of a random variable whose logarithm follows a normal distribution (33).

The lognormal pdf is:

\[ f(t) = \frac{1}{\sqrt{2\pi} \sigma} \cdot \frac{1}{t} \cdot \exp \left\{ -\frac{[\ln(t) - \mu]^2}{2\sigma^2} \right\} \]

\[ t > 0 \]  

(2.33)

If log_{10}(t) is taken instead of ln(t), then the whole expression should be multiplied by 1/ln (10) = 0.4343 (27).

Compared to the normal distribution, which arises as a result of the addition of effects, the lognormal distribution arises as a result of multiplication of many small effects (33).

The expected value and the variance of this distribution are given by (33):

\[ E(T) = \exp (\mu + \frac{1}{2}\sigma^2) \]  

(2.34)

and

\[ \text{VAR}(T) = \exp (2\mu + \sigma^2) \left[ \exp (\sigma^2) - 1 \right] \]  

(2.35)

where, in all the above equations,

\[ \mu = E[\ln(T)], \]

\[ \sigma^2 = \text{VAR}[\ln(T)], \] and
$T$ = the random variable, the realization of which is $t$.

From the above, one can conclude that the lognormal distribution is the same as the normal distribution with the data transformed to the logarithmic form.

Hence, the cumulative distribution is:

$$F(T) = \Phi\left\{\frac{\ln(t) - \mu}{\sigma}\right\}, \ t > 0$$  \hspace{1cm} (2.36)

and the reliability function and the failure rate are:

$$R(t) = 1 - \Phi\left\{\frac{\ln(t) - \mu}{\sigma}\right\}, \ t > 0$$  \hspace{1cm} (2.37)

and

$$Z(t) = \frac{1}{(\sigma t)} \Phi\left\{\frac{\ln(t) - \mu}{\sigma}\right\}\{1 - \Phi\left\{\frac{\ln(t) - \mu}{\sigma}\right\}\}, \ t > 0$$  \hspace{1cm} (2.38)

Figure 2.8 shows the functions of this distribution.

2.5 System Reliability

A system is defined as "an integrated assembly of interacting elements, designed to carry out cooperatively a predetermined function" (24). Another, less detailed definition is given by Sargious (34) as "...a device or scheme which behaves according to some description to accomplish an operational process."

Each element in the system converts an input (energy, control signal, information, etc.) into an output (performance). The "performance function" of the element relates output to input. The performance functions of all elements in the system combine in a predetermined way
Figure 2.8  LOGNORMAL DISTRIBUTION  
(After Reference 27)
to produce the desired system function (system performance). The system performance, then, depends on the element performances as well as the system structure (the way in which components are functionally "connected") (24).

2.5.1 The System Structure

The relationship between the elements, or components, of a system is governed by two basic forms. The elements may be arranged either in series or in parallel (see Figure 2.9). The series arrangement is a sequential arrangement, such that the output of the first element feeds into the second, whose output provides the input for the third, and so on.

The elements may also be arranged in another way, to function concomitantly, i.e., while one element performs its function, a second (third, etc.) element functions alongside with the first. This relationship is a parallel arrangement (for examples, see References 18, 20, 24).

A complex system can be considered as an arrangement of subsystem "blocks," each one of which may represent one of the basic component structures (24).

In addition to its technical performance, the life length of the system is another measure of its value. Thus, a high-performance, short-lived system may be worth less, in a given application, than a lower-performance, longer-lived system (24).

The life length of a system cannot be considered in the same way as other performance variables, as the functional element relation-
Figure 2.9 Basic System Structures

Figure 2.10 Non-redundant and redundant structures
ships determining the system output may be different from the element relationships that determine the system life length. That is, the functional system structure is not necessarily the same as the life structure of the system, although often the two are, indeed, identical (24). The failure to distinguish between the two types of structure may lead to confusion in the system analysis.

The life structure is determined by the way element failures affect the system life length. Hence, the system is a series life structure if the failure of an element invokes the failure of the system, no matter how the functional connections of the elements are. On the other hand, a system may feature redundant elements, the functioning of which is not critical to the system operation so long as a sufficient number of elements function. The life structure of this type is, therefore, a parallel life structure, regardless of their functional connections.

Bury (24) cites the following examples for both cases:

If we take a mechanical system that consists of a power subsystem (an engine) and a lubrication subsystem, these two subsystems function in parallel with each other, in that lubricating oils are continuously supplied to various points of the power subsystem. Whereas, when it comes to failure analysis, both subsystems must function for the system to operate. Therefore, the life structure of these subsystems is a series type.

One should notice, however, that Bury is assuming that the lubricating subsystem does not depend on the engine subsystem in its operation, as most of them do.
Consider, as an example of the parallel life structure, a redundant civil engineering structure, such as a concrete deck supported by a number of columns. If some of the columns fail, the structure will not fail so long as the remaining columns are able to support the deck. The life structure of the columns is, therefore, a parallel type.

The concept of redundancy is well known in structural engineering, and is self-explained by Figure 2.10 (19).

2.5.2 System Reliability

The probabilistic model of the system life length, \( X_\text{s} \), embodies all statistical information on \( X_\text{s} \). This model may take several equivalent forms, like the pdf, \( f(x_\text{s}) \), cdf, \( F(x_\text{s}) \) or \( R(x_\text{s}) = 1 - F(x_\text{s}) \), where \( R(x_\text{s}) \) is the reliability function of the system. In many cases, it may be convenient to only establish the system reliability value at some specific age \( x_\text{s}' \). Below are simple derivations of the system reliability of the life structures discussed.

**Series Life Structures**: As mentioned earlier, a series system, in a life structure configuration, functions only when all its constituent elements function. System success, at a system age \( x_\text{s} \), is, therefore, the intersection of the element success events. From this, the system reliability (the probability of success) is given by:

\[
R_\text{s}(x_\text{s}) = P(E_1 \cap E_2 \cap \ldots \cap E_k) \quad (2.39)
\]

where

- \( E_1 \) = the event of success of the \( i^{th} \) element at the time \( x_\text{s} \), and
- \( k \) = the total number of elements in the system.
When the life lengths of elements are independent, Equation (2.39) gives:

\[ R_s(x_s) = \prod_{i=1}^{k} P(E_i) = \prod_{i=1}^{k} R_E(x_s) \]  

(2.40)

where

\[ R_E(x_s) = \text{element reliability at time } x_s. \]

**Parallel Life Structures:** This structure fails only at the failure of the last surviving element. System failure, at age \( x_s \), is therefore the intersection of the element failure events, and is given by:

\[ R_s(x_s) = 1 - P(\overline{E_1} \cap \overline{E_2} \cap \ldots \cap \overline{E_k}) \]  

(2.41)

where

\[ \overline{E_i} = \text{the event of failure of the } i^{th} \text{ element at time } x_s. \]

When independence is assumed, Equation (2.41) gives:

\[ R_s(x_s) = 1 - \prod_{i=1}^{k} P(\overline{E_i}) \]

\[ = 1 - \prod_{i=1}^{k} (1 - R_E(x_s)) \]  

(2.42)

**k-out-of-n-Life Structures:** Some parallel systems function as long as any number, \( k \), out of \( n \) elements, function. The reliability of such a system of independent elements is given by:

\[ R_s(x_s) = \sum_{i=k}^{n} \binom{n}{i} (R_E(x_s))^i (1 - R_E(x_s))^{n-i} \]  

(2.43)
2.6 Decomposition of Complex Structures

In practical reliability analysis, the procedure often followed in dealing with systems is to decompose the system to sub-systems, compute the reliability of each of these sub-systems and then compute the overall system reliability from these subsystem reliabilities (20, 24, 35). To do this in a proper form, one has to introduce the concept of modules. In engineering terms, a module refers to either a unit or a component in a system, or a group of units or components that can be handled as one unit.

The idea of modules can better be clarified by the diagram given in Figure 2.11. In this diagram, the following sets are identified as modules of this system:

1. \((2,3)\)
2. \((4,5)\)
3. \((2,3,4,5)\)
4. \((1,2,3)\)
5. \((1,4,5)\)
6. each of the individual units, and
7. the whole system.

Given a modular decomposition of a system, one may decide to refine this decomposition by more decomposition of each of the modules into smaller modules, and so on.

Generally, refinements follow a hierarchy in this order: A system is decomposed into its major sub-systems. Then each major sub-system is decomposed into components and each component, into parts.
Figure 2.11  A System Representation (after ref. 35)
reliability of each part is determined and from there, the reliabilities of each of the components, sub-systems and finally, the system, are determined.

2.7 The Pavement as a System

Figure 2.12 shows a conceptual pavement system developed by Finn and others (see Ref. 2, 34). This figure is intended to systematize the many variables included in the structural design, operation, and management of a total pavement system.

Because of the difficulty in working with this system, a Texas research team simplified the problem by developing a "working" pavement design system, as shown in Figure 2.13. In this system, inputs are related to only one output: deflection. Instead of weighting functions, the model proposes the use of a "Road Test Deflection-Based Performance Equation." This equation, developed by Scrivner (and which shall be presented in the next chapter) relates deflection, in terms of a Dyna-flect index, and climate to the AASHO serviceability equation.

Both models defined the output of the structural system only in terms of the natural responses of that system to the input variables, i.e., deflection, stress, strain, permanent deformation. They both failed to identify the functional output of the pavement system, which may be listed as:

- Distribution of the concentrated wheel load to stress values that can be sustained by the subgrade.
Figure 2.12  An Ideal Pavement System (after ref. 34)
Figure 2.13  A Pavement System That Works (after ref. 34)
- Protection of the subgrade and lower pavement structures from detrimental climatic conditions.

- Provision of a surface with a suitable texture that allows the transmission of the wheel torque into vehicle displacement with convenient and safe rideability at the same time.

The situation, here, may be similar to that of, for example, a brake subsystem in a mechanical assembly. The function of this subsystem is to reduce the speed of a moving body by friction. In doing so, it dissipates a lot of energy in the form of heat, which is a natural response. The contact surfaces of both the brake subsystem and the moving body tend to wear with repeated use. This is also a natural response. The subsystem functional output is, still, the speed reduction of the moving body, not the heat or wear.

From what was mentioned above, it can be seen that the pavement system features multi-dimensional performance functions. Four performance specifications or conditions should be met simultaneously with some priorities and tolerances that differ from one system to another. These performance specifications are mentioned, again, very briefly:

1. load distribution,
2. protection of lower layers,
3. surface with suitable roughness for wheel traction, and
4. surface with suitable "smoothness" for comfortable and safe rideability.

The system responses (deflection, stress, etc.) may be indicative of how well the system is prepared to perform. The limits of these responses indicate the system's failure to perform certain functions. A cracked pavement, for example, may not be able to adequately distribute the applied loads to the subgrade. Moreover, it may not be able to protect the lower layers from rainwater. So, cracking is indicative of the inefficiency of the system to perform the first and second functions cited above.
CHAPTER III
THE CONCEPT OF RELIABILITY IN PAVEMENT ANALYSIS

3.1 Introduction

It has long been recognized by materials engineers that the material's properties vary to a great extent from point to point in a specimen, be it steel or a mud brick. In pavements, material variability, together with the variability of other influential factors like traffic and environment, has the sole effect on the random, sometimes unpredictable behavior of the pavement system. The situation is well described by Harr (1) as follows:

"...a pavement consists of distinct layers with unknown contacts at their interface; the layers may or may not be in contact in space or in time. Imposed loadings (wheels) are relatively large in area compared with the thickness of the surface area; consequently, Saint-Venant's principle cannot be invoked to change the system to an equivalent homogeneous and isotropic body. Ambient conditions greatly alter the properties of the layers, which range from thermal plastic, temperature-sensitive materials to granular soils whose actions depend greatly on their voids. Each layer is composed of complex conglomerations of discrete particles of varying shapes, sizes, and orientations. In addition, loads are variable in both magnitude and time and are dynamic in nature. It is not surprising how poor predictions of the transmission of induced energy through such systems have been. Randomness alone dictates the probabilistic (casual) rather than deterministic (causal) treatment."

44
The need for the probabilistic treatment of pavement analysis was felt, in an earnest way, in 1970, when a high-level workshop listed the lack of predicting variations in the behavior of the pavement system due to the statistical variations in factors such as load, thickness, material properties and environmental conditions as one of the most important problems facing pavement engineers (2).

3.2 Magnitude and Nature of Variations

Variability, or randomness, is introduced in the pavement system through material properties, geometry, loading and environment, as mentioned earlier. A brief account of each follows:

3.2.1 Material Properties

(a) Bituminous Materials: The variation of bituminous materials, evident from normal construction practices, has been under study by several researchers (2). The main purpose of these studies was to introduce the statistical quality control techniques in road construction. Consequently, the parameters measured were the simple physical characteristics and strength properties used in today's construction specifications. Properties investigated included binder content, air voids, Marshall stability, Marshall flow and aggregate gradation. It was found out that, in 28 states in the USA, more than 50% of the variation in these properties was due to the sampling and testing (4), with the rest being the material variability.

This variability was found to account for 3 to 13% of the material stiffness (2).
Fell and Taylor (5) accounted for the variability in fatigue life of asphalt mixes by fitting the logarithms of fatigue lives of specimens to a normal distribution.

Paterson (2) pointed to the seriousness of the implications that this distribution has on fracture analysis due to the large variability it creates. Reviewing fatigue data published then, he found out that, for example, a material with a mean life at a given strain level of one million applications has a five percent probability that its fatigue life may be as low as 0.3 million or as high as 3 million applications.

Together with the inherent variability in the fatigue relationship (S-N or ε-N), the construction control of air voids in the asphalt concrete accounted for most of the variability in the fatigue life. It was shown, by a sensitivity analysis, that an increase in air voids of three percent above the optimum reduced the fatigue life of the asphalt concrete by 85 percent. Minor changes in aggregate properties, like grading, shape and texture, had little influence (2).

(b) Granular and Cohesive Materials: Materials, used in the construction of lower layers, like crushed stone, natural gravel and soils, show a wide range of variability, depending on factors like material homogeneity, property being investigated, and environmental and loading conditions (2).

(c) Cemented Materials: The variability of these cement or lime-stabilized materials derives from their semi-brittle behavior. Another important item that contributes to their variability is possibly the variability in the content and distribution of the stabilizing agent.
(d) **Concrete**

Rigid pavement variability can be attributed to a host of varying parameters, including material properties like the flexural strength, and quality control variables like the water/cement ratio, slump, air voids and cement content, and other variables like pouring temperature, pouring conditions and workmanship (6).

3.2.2 **Geometry**

From studies on layer thicknesses and lane widths in pavements, it was concluded (2) that:

* the distribution of measurements of road dimensions is normal;

* the variability in layer thickness and surface shape are highly dependent on the quality control standards that are grouped as follows:

  - **very good control**: largely automatic or assisted manual control
  - **good control**: graded with manual leveling control
  - **fair control**: by eye to taped pegs, and
  - **low control**: non-surveying set-out of pegs.

3.2.3 **Traffic**

Traffic input to the pavement system can be represented by three factors: wheel load or tire pressure, duration of loading and frictional force due to traction. All three factors may carry significant variation to the pavement.
3.2.4 Environment

(a) Moisture: The moisture conditions in a pavement are a function of the moisture suction which is itself a function of the depth of water table and climate (2). The importance of variation in the moisture level in the pavement is due to the significant effects it has on the strength of subgrade and, generally, the lower layers.

(b) Temperature: Maqdisi (7) showed the dramatic influence of the variation in temperature levels on the stiffness of flexible pavements. Temperature variations also affect the strength of the lower layers by influencing the moisture content in these layers (2).

Factors that affect the variability of pavement temperature are (8):

* Air temperature
* Subsurface temperature
* Solar radiation
* Weather conditions:
  (1) degree of sunshine,
  (2) amount of rain,
  (3) wind velocity,
  (4) cloud cover, and
  (5) snow
* Time of day
* Site features.
(c) **Weathering**: Asphalt materials usually undergo weathering through oxidation that causes hardening in the bituminous binder. At air void contents higher than five percent, and a long period of exposure, this hardening will be significant on the fatigue life of the pavement (2), since it was observed that, contrary to common knowledge, fatigue cracking frequently starts at the surface, rather than the bottom, due to the combined effects of traffic and hardening. This idea led to the adoption of a finite surface life in South Africa, depending on material type and climate, which would override fatigue prediction methods (2).

From this brief description, it can be concluded that variability in pavements is a product of the existence of many sources. When the variability of the pavement material is under investigation, much care must be given to the sampling and testing procedures used; as they may be the cause of a significant portion of the overall material variability (9). Figure 3.1 shows the structure and the components of this overall variability, related to quality control procedures in a manufacturing (construction) process.

3.3 **Probabilistic Approaches**

Accounting for this variability in the pavement in a rational manner has aroused growing interest over the last decade to apply probabilistic, more specifically stochastic, techniques to pavement analysis (2,3). Applications of these techniques varied from, simply, assigning a standard deviation to deflection values -- as the California
Figure 3.1 Sources of Variance (After Reference 9)

\[
\begin{align*}
\sigma_0^2 &= \sigma_b^2 + \sigma_L^2 \\
\sigma_b^2 &= \sigma_a^2 + \sigma_t^2 + \sigma_s^2 \\
\sigma_o^2 &= \sigma_a^2 + \sigma_t^2 + \sigma_s^2 + \sigma_L^2
\end{align*}
\]
Division of Highways and the Asphalt Institute did in their overlay design procedures (3) -- to incorporating more sophisticated techniques, like the Markov processes and reliability functions, in pavement design systems, as the FPS of Texas, PDMAP of California and the VESYS programs of the FHWA (2,3).

3.3.1 The FPS System

The main theme on the statistical treatment of these systems is the calculation of the pavement reliability. The earliest of these systems, FPS (Flexible Pavement System), took the definition of pavement reliability as 'the probability that the pavement will have an adequate serviceability level for the design performance period' (2). Mathematically speaking, reliability was stated as:

\[ R = 1 - \text{(failure probability)} \]  \hspace{1cm} (3.1)

From this, it was deduced that reliability was the probability that the number of load applications to failure, \( N \), exceeded the actual traffic, \( n \), that would travel over the pavement, or:

\[ R = P[(\log N - \log n)] > 0 \]  \hspace{1cm} (3.2)

The logarithms of \( N \) and \( n \) were considered, as it was assumed that the distribution of both \( N \) and \( n \) was log normal (2).

Letting:

\[ D = \log N - \log n \]  \hspace{1cm} (3.3)

the distribution of \( D \) was, essentially, normal, and the reliability could be read directly from the standard normal distribution table, if:
\[
Z = \left[ \log N - \log n \right] / \sqrt{S^2_{\log N} + S^2_{\log n}}
\]  

where

- \( \log N \) = the mean of \( \log N \),
- \( S^2_{\log N} \) = the variance of \( \log N \),
- \( \log n \) = the mean of \( \log n \), and
- \( S^2_{\log n} \) = the variance of \( \log n \).

The means of \( \log N \) and \( \log n \) were assumed known and the variances were estimated by the partial derivative method which stated that the variance of a function \( g(x_1, x_2, \ldots, x_m) \), where \( x_1, x_2, \ldots, x_m \) are independent random variables, is (2):

\[
V[g(x_1, x_2, \ldots, x_m)] = \sum_{i=1}^{m} \left( \frac{\partial g}{\partial x_i} \right)^2 S^2_{x_i}
\]  

Since \( N \), for example, is given by:

\[
N = Q \cdot \alpha \cdot 10^6 / K \cdot S^2
\]  

where

- \( Q \) = function of serviceability loss = \( \sqrt{P_2} - \sqrt{P_1} \),
- \( \alpha \) = temperature parameter,
- \( S \) = surface curvature index of pavement,
- \( K \) = regression coefficient (constant), and
- \( P_1, P_2 \) = initial and terminal serviceability indices.

Then the variability of \( N \) is related to the variabilities of the functions in Equation (3.6) by Equation (3.5).
3.3.2 The PADS and the VESYS Programs

The second system was initially developed at the Massachusetts Institute of Technology. In this system, reliability was defined by Lemer and Moavenzadeh as the 'probability the serviceability will be maintained at adequate levels from a user's point of view throughout the design life of the facility,' (2), a definition that is similar to that of the FPS. However, they utilized the Markov process as follows:

To decide whether or not a facility was satisfactory, it was necessary to predict the behavior of that facility. This prediction involved uncertainty; so, it could be approximated by a Markov process, i.e., one in which the facility had a probability of being in a known state of serviceability now, and for which the transitional probabilities described the chance of the system being in another state of serviceability at the next observation. Thus, the probability of being in any one state depended only on the state preceding the trial. With some variation, the time spent at a given state could be included as a probabilistic variable in order to include the effect of aging. This process (semi-Markov, in fact) provided the reliability of the facility with respect to a given serviceability level as a function of time.

Some of the interstate transitions in the semi-Markov process may represent maintenance operations, which is the way that the facility's improvement in serviceability would take place.

The application of a Markov process to pavement aging is shown by a transition diagram and the equivalent probability matrix in
Figure 3.2. In this figure, maintenance operations are shown by broken lines. Each entry $P_{ij}$ in the matrix is the probability that it will be in the state $j$ after the next transition, given that it is now in the state $i$.

Because of the complexity of the model, simulation was suggested as a solution technique. The elements of the simulation process are described below (3);

1. Randomly determine the initial state of each subsection as normal aging or accelerated aging.

2. For each section, randomly determine the time in the initial state and the next state to be occupied.

3. Upon transition of a section, randomly determine the time to remain in the new state and the next state to be occupied. After each transition, test whether it is time for an overlay. After the overlay, repeat Steps 1 through 3. Repeat the entire process until the allowed number of overlays has been exceeded, or the design life is completed.

In this process, the interrelationships between events should be considered. For example, the transition of a subsection after overlay could be a function of:
Figure 3.2 A Markov Process for a Pavement
(After Reference 2)
1. The \textit{a priori} probabilities of state transitions and the time distribution in the state to be transferred from.

2. The states of nearby subsections.

3. Convenient measures of the performance of the subsection before the overlay.

For an illustration of the basic information available from the simulation, the following hypothetical list of events is given by McCullough (3):

<table>
<thead>
<tr>
<th>Time (months)</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Begin simulation; all subsections aging normally.</td>
</tr>
<tr>
<td>20</td>
<td>Section 6 enters accelerated aging state.</td>
</tr>
<tr>
<td>23</td>
<td>Section 23 enters accelerated aging state.</td>
</tr>
<tr>
<td>26</td>
<td>Section 6 enters maintenance.</td>
</tr>
<tr>
<td>26.05</td>
<td>Section 6 enters normal aging state (1.5 days required for maintenance)</td>
</tr>
<tr>
<td>128.7</td>
<td>Section 47 enters accelerated aging state, percentage of sections in accelerated aging state exceeds threshold values; begin overlay.</td>
</tr>
</tbody>
</table>
Thus, detailed statistics could be collected on things like maintenance costs, maintenance time, and percentage of sections in accelerated aging in any given time.

This reliability model was first applied in a system called PADS (Pavement Analysis and Design System). The way the pavement reliability is incorporated in the model is shown in Figure 3.3.

The input parameters to the serviceability-maintenance subsystem (in which reliability is involved) are the means and variances of the parameters of the AASHO serviceability index (PSI), i.e., rut depth, fatigue cracks and roughness (slope variance). These parameters are the output of the structural subsystem, determined through probabilistic closed-form solutions.

The serviceability-maintenance subsystem is subdivided into two sub-models: a serviceability-reliability (S-R) sub-model and a maintenance sub-model (10). After obtaining the statistical estimate of the serviceability index (i.e., the mean and variance of PSI), the probabilities of having a certain value of PSI at a particular time could be determined by assuming a normal distribution for PSI. These probabilities are the state probabilities. By definition, the reliability is given as the probability, at any time, that the pavement system is above some unacceptable value of PSI. This unacceptable limit, which is defined by the user, defines some threshold for failure, from which the life expectancy of the system can be determined (10).

In the maintenance sub-model, various maintenance strategies are generated and their consequences evaluated through dynamic feedbacks that loop into the S-R sub-model and a cost model.
Figure 3.3 The PADS Stochastic Pavement Model (After Reference 10)
Examples of some outputs of reliability as a function of quality control and material quality, and quality control and different temperature levels, are shown on Figures 3.4 and 3.5.

The PADS system was modified into a system called VESYS II M by the Federal Highway Administration. The modifications included the distress models, i.e., the rut depth, roughness and fatigue models (2).

3.3.3 The PDMAP System

In the third system, the PDMAP (Prediction of Distress Modes in Asphalt Pavements), the incorporation of parameter variabilities appears to be confined to certain material properties, like the elastic modulus, Poisson ratio and fatigue function (2). The output of the design system of this program includes predictions of behavior at various reliabilities.

The mathematical method used is a closed-form probabilistic solution of the N-layered system, developed by Huffered at the University of Utah (11). This solution assumes that the elastic moduli are the only random variables involved in the structural analysis of an N-layered pavement system, and uses the Taylor series to approximate the means and variances of the response random variables as:

\[ \bar{s}_1 = g(\bar{E}_1, \bar{E}_2, \ldots, \bar{E}_n) + \frac{1}{2} \sum_{j=1}^{n} \left( \frac{\partial^2 g}{\partial E_j^2} \right) \frac{\partial g}{\partial E_j} \sigma_{E_j}^2 \]  

(3.7)

and

\[ \sigma_{s_1}^2 = \sum_{j=1}^{n} \left( \frac{\partial g}{\partial E_j} \right) \sigma_{E_j}^2 \]  

(3.8)

where
Figure 3.4 Reliability as a Function of Quality Control and Time for (a) Weak, (b) Medium, and (c) Strong Material Quality.

Figure 3.5 Reliability as a Function of System Quality and Time for the Environment Differing from the Reference Temperature (70°F/21°C) by (a) -10, (b) 0, and (c) +10 Degrees.
\[ S_i = g_i(E_1, E_2, \ldots, E_n) \] is the structural response (i.e., stress, strain or deformation), at the \( i \)th layer, due to a unit load.

For a circular load distributed over radius, \( a \), the expected value and the variance of the total response, \( S_{T_i} \), for the \( i \)th layer, are, respectively, given by:

\[
E[S_{T_i}] = a \int_0^\infty J_1(ma) S_i \, dm \\
\text{and} \\
\text{VAR}[S_{T_i}] = a^2 \left[ \int_0^\infty J_1^2(ma) \, dm \right] \sigma_{S_i}^2
\]

The probabilistic prediction of fatigue cracking is arrived at by assuming that the pavement fatigue life depends on the maximum tensile strain in the asphalt concrete layer, and that Miner's Law applies for the estimation of the cumulative fatigue damage. Hence, the mean and variance of the logarithm of the load cycles to failure, \( N_f \), can be calculated as:

\[
E(\ln N_f) = K_1 + K_2E(\ln \varepsilon_H) + K_3E(\ln |E^*|) \\
\text{and} \\
\text{VAR}(\ln N_f) = K_2^2 \text{VAR}(\ln \varepsilon_H) + K_3^2 \text{VAR}(\ln |E^*|) + \sigma^2
\]

where \( \varepsilon_H \) = maximum horizontal strain in lower fibers of asphalt concrete;

\( |E^*| \) = dynamic modulus of asphalt concrete;

\( K_1, K_2, K_3 \) = regression coefficients; and,
\[ \sigma^2 \] = variance of a random error term.

The cumulative fatigue damage, \( F_m \), at the end of \( m \) time periods is, thus, given by:

\[
F_m = F_0 + \sum_{i=1}^{m} \frac{n_i}{N_r} \tag{3.13}
\]

where

\( F_0 \) = initial fatigue damage;

\( n_i \) = number of load applications during \( i \)th period; and,

\( N_{r_i} \) = number of load applications to failure during the \( i \)th period.

Reliability, as a measure of confidence is assessed in connection with fatigue life as follows:

1. The annual cumulative fatigue damage is calculated (using the means and variances). Assuming that the cumulative damage is normally distributed, a reliability factor, \( K \), is selected from the normal distribution tables (for a given reliability level) such that the damage that is equal to (mean + \( K \) x standard deviation) corresponds to the reliability level. The reliability factors for different reliability levels are given below (11):

<table>
<thead>
<tr>
<th>Reliability Level (%)</th>
<th>Reliability Factor (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>0.675</td>
</tr>
<tr>
<td>90</td>
<td>1.280</td>
</tr>
<tr>
<td>95</td>
<td>1.645</td>
</tr>
<tr>
<td>99</td>
<td>2.330</td>
</tr>
</tbody>
</table>
2. A plot is prepared of cumulative damage at a selected reliability level versus the number of years.

3. From the plot, the first time at which the cumulative damage exceeds 1 is found. This provides the estimate of fatigue life with the selected reliability level.

3.3.4 Pavement Reliability Applications Elsewhere

Korsunsky and Telyaev (12) reported incorporating pavement reliability, as a safety factor, in the flexible pavement design method adopted in the USSR. This function is given by:

\[ K_{rel} = 0.5 + \phi \left[ \frac{20(1.15K - 1.0)}{\sqrt{1.15K^2 + 1.0}} \right] \]  

where

\( \phi \) is a normalized Laplace function, and

\( K \) is a function of the pavement strength.

Figure 3.6 shows this reliability function with \( K \). It was shown that deformations and failures had formed almost through all the pavement structures that had at least one value of \( K \) below 1.0, and that more deformation took place in pavements that did not comply with the requirements of frost resistance and drainage capacity.
Figure 3.6  Reliability of Structures Designed by the Russian Method (After Reference 12)
3.4 **Assessment**

Nothing much is known about the Russian system. So, the comparison shall be limited to the three systems developed in the USA, in the way they handle pavement reliability.

The three systems agree in adopting a closed-form solution of a series to develop the means and variances of functions. Rauhut and co-workers (13) used the stochastic formulation developed by Moavenzadeh and others (see, for example, References 10 and 14) and concluded that it resulted in severely decreased fatigue life predictions as the coefficients of variation for \( K_1 \) and \( K_2 \) increased, \( K_1 \) and \( K_2 \) being the coefficient and the exponent of the fatigue relationship, given by (10):

\[
N_f = K_1 (1/\Delta \varepsilon)^{K_2}
\]

where

- \( N_f \) = number of cycles to failure
- \( \Delta \varepsilon \) = tensile strain amplitude.

On investigating the cause for the 'surprisingly large decreases in expected fatigue life', it was found (13) that the Taylor series gave very poor results for the function approximated, except when the arguments were very near to their mean values.

FPS adopted a definition that the reliability, \( R \), of a pavement design is 'the probability that the allowable load applications of the pavement-subgrade system, \( N \), will exceed the traffic loads to be applied, \( n \)', (15), and it was postulated that this definition is compatible with the statement that reliability is the probability that the serviceability
level of pavement will not fall below the minimum acceptable level before the performance period is over. Kher and Darter (16) and later, Paterson (2), claimed that the assumption that all the loss of serviceability could be associated only with traffic was not valid.

The other two systems assumed the validity of Miner's Law (11, 14), which is given by:

\[ D = \sum_{i=1}^{n} \frac{n_i}{N_i} < 1 \]  

(3.16)

where

- \( n_i \) = the number of load applications at the state \( i \),
- \( N_i \) = the number of cycles to failure for the state \( i \),
- \( m \) = the total number of states.

Miner's Law, which is the most classical damage theory, is basically a linear cumulative damage theory (17). Linearity, here, refers to the method of summing the fractions of consumed life.

Some later developments in damage processes resulted in the emergence of nonlinear cumulative damage theories. The main difference is that the late theories assumed that an interaction existed between load repetitions and the amount of damage (17). According to these theories, the amount of damage, experienced at any one load level, for example, is dependent on both the magnitude and the number of occurrences of prior loads.

One such theory, Shanley's theory (17), involves an exponential relationship for fatigue crack growth, and recognizes that fatigue crack growth tends to increase with crack depth.
The extensive work on fatigue fracture of flexible pavements, which was conducted at The Ohio State University, shows that this theory may be a closer representative to cumulative damage in flexible pavements than Miner's Law.

The reliability assessment in the three systems is based on the normal distribution assumption, either for the logarithms of the values of N and n, or for the serviceability. This has a crucial effect on the final results (2,13), as the distributions chosen should be based on a more adequate basis.
4.1 Introduction

In this chapter, an attempt to develop a model for the pavement is made, based on the distribution of the fracture toughness of the pavement material.

After listing the assumptions upon which the development is based, the failure distribution of fatigue specimens is derived. Then, the pavement reliability model is developed, based on the assumption that the pavement is a system that can be decomposed repeatedly to smaller modules, to a stage where a module can be represented by a laboratory specimen. At that stage, the information extracted from the laboratory specimens can be conveniently related to this basic module and, from there, system characteristics are developed according to the adopted hierarchy.

Modifications for operating conditions are carried, when possible, or explained. Then an improvement for the developed model is undertaken by re-examining, or relaxing some assumptions.

4.2 Assumptions

Let us make the following assumptions:

1. Laboratory specimens (cylinders, beams, slabs, etc.) represent a small part of the pavement. Therefore,
the information extracted from these specimens
with respect to failure apply to this part more
closely than to the whole pavement.

2. The size of this part, ΔL, is such that each part,
when loaded, can be considered as independent from
other parts, although there is no physical barrier
between them.

3. All parts are similar.

4. Parts are decomposition products of a larger module,
called section, of a finite size, L, containing a
finite number, n, of parts.

5. The failure of a section is reached only when a
finite number of parts, k, fails.

6. Sections are also decomposition products of a sub-
system of a finite size.

7. The failure of a section in the sub-system results
in a failure of the sub-system.

8. The sub-systems are decomposition products of the
pavement system, which is of a variable size.

9. The failure of a sub-system results in the failure
of the system.
10. The number of cycles to failure can be re-presented by a continuous variate, x.

4.3 Development

We can now proceed to develop a model for pavement reliability.

4.3.1 Laboratory Samples

The extensive work carried out at The Ohio State University over the last decade (7, 37-42) confirmed that the following relationship holds, for fatigue failure, for specimens of sand-asphalt and asphalt concrete:

\[
\frac{dc}{dN} = A K^n
\]  

(4.1)

where

A, n = material constants, \(n > 1\);

K = stress intensity factor; and,

dc = crack growth per increments of dN of load repetitions.

K is given by the following polynomial (43):

\[
K_1 = \sigma_{\text{max}} \sqrt{d} \times 6.898 \times \left(\frac{C}{d}\right) - 17.425 \times \left(\frac{C}{d}\right)^2 + 22.438 \times \left(\frac{C}{d}\right)^3
\]  

(4.2)

where

\(\sigma_{\text{max}}\) = maximum tensile stress at the bottom of the specimen due to a distributed applied load;

d = the depth of the specimen;
$c =$ the crack length; and,

$K_1 =$ the opening mode stress intensity factor.

If we plot the algebraic summation of the second and third terms in Equation (4.2) against the ratio of $(c/d)$ — Figure 4.1 — we can safely conclude that no significant loss in accuracy is suffered if we remove them from the equation. This will simplify our formulation in the coming steps.

Taking the first term in Equation (4.2),

$$K_1 = \sigma_{s_{\text{max}}} \times \sqrt{d} \times 6.898\left(\frac{c}{d}\right)$$

$$= M C \quad (4.3)$$

where

$M =$ constant.

Substituting into Equation (4.1):

$$\frac{dc}{dN} = A \cdot M \cdot C^n$$

$$= QC^n \quad (4.4)$$

Integrating,

$$\int_{c_0}^{c} \frac{dc}{C^n} = Q \int_{N_0}^{N} dN$$

$$\frac{c^{1-n}}{1-n} \bigg|_{c_0}^{c} =QN \bigg|_{N_0}^{N}$$
Figure 4.1 Relationship of $G$ versus $c/d$

$G$ = gain in $K_1$ if higher order terms are left in Equation (4.2).
(c) - (c_0) = \phi Q N \tag{4.5}

where

\[ \phi = 1 - n. \]

From Equation (4.5),

\[ c = (Q_0 + c_0) \tag{4.6} \]

into Equation (4.3),

\[ K_l = M(Q_0 + c_0) \tag{4.7} \]

At failure, \( c \) reaches the limit of \( c_f \), \( N \) reaches the limit of \( N_f \) and \( K_l \) reaches the limit of \( K_{lc} \) (where \( c_f \), \( N_f \) and \( K_{lc} \) are the critical crack length, the number of cycles to failure and the material fracture toughness, respectively).

\[ K_{lc} = N_f \tag{4.8} \]

(since \( N_f \) assumes a value that renders the term \( c_0 / N_f \) tend to zero.)

The distribution of the fracture toughness is given by:

\[ P(K_{lc} \leq x) = P(L \cdot N_f \leq x) \]
\[ = P(N_f \leq x/L) \]
\[ = P(N_f \leq (x/L)^\phi) \]
\[ = P(N_f \leq \zeta) \tag{4.9} \]
Landes and Shaffer (44) proved experimentally that the fracture toughness \( K_{lc} \) of rotor steel specimens is a random variable with a Weibull distribution, i.e.,

\[
F_{K_{lc}}(x) = 1 - \exp\left(-\left(\frac{x}{b}\right)^a\right), \quad x > 0
\] (4.10)

This result supported their rationale, which was stated as follows:

(a) The fracture toughness of a given heat of material is variable, differing throughout the material and particularly along the crack front,

(b) The point or region of lowest toughness controls the fracture toughness of the specimen.

We can conclude, then, from Equations (4.9) and (4.10), that the number of cycles to fatigue failure of specimens of asphaltic materials is a random variable with a Weibull distribution. Stated mathematically,

\[
F_{N_f}(x) = 1 - \exp\left(-\left(\frac{x}{\eta}\right)^\beta\right)
\] (4.11)

and the corresponding reliability of these specimens is:

\[
R(x) = \exp\left(-\left(\frac{x}{\eta}\right)^\beta\right)
\] (4.12)

4.3.2 The Pavement System

By the first assumption, the failure model developed above represents failure in the first level of the system hierarchy: the pavement part, or element. Rewriting Equations (4.11) and (4.12),
the pavement part, or element. Rewriting Equations (4.11) and (4.12),

\[ F_p(x) = 1 - \exp\left(-\left(\frac{x}{\eta}\right)\beta\right) \]  
(4.11')

and

\[ R_p(x) = \exp\left(-\left(\frac{x}{\eta}\right)\beta\right) \]  
(4.12')

where

\[ F_p(x) = \text{the failure distribution of a pavement element, and} \]
\[ R_p(x) = \text{the reliability of a pavement element.} \]

A section of \( n \) independent and identical elements fails only when \( k \) elements fail. Hence the life structure of a section can be considered as an \( r \)-out-of-\( n \) structure, where \( r = n - k \).

A straightforward approach to formulate a section model is, then, to use the binomial distribution (38, 39, 45). Success of exactly \( r \) out of \( n \) items is

\[ B(r:n) = \binom{n}{r} p^r (1-p)^{n-r} \]  
(4.13)

where

\[ B(r:n) = \text{the binomial PDF and} \]
\[ r:n \text{ stands for } r \text{ out of } n . \]

The success of at least \( r \) out of \( n \) items is:

\[ P_s = \sum_{i=r}^{n} \binom{n}{i} p^i (1 - p)^{n-i} \]  
(4.14)

Applying Equation (4.14) to our reliability formulation gives:
A subsystem of \( m \) sections, that fails on the failure of any section, has a life structure of a series configuration.

The reliability formulation of this module is:

\[
R_{ss}(x) = \prod_{i=1}^{m} R_s(x_i)
\]

\[
= [R_s(x)]^m
\]  

(4.16)

The buildup can, ideally, continue to formulate a system reliability, having the specified number of subsections in series. The procedure would be similar to that of the subsystem.

The failure rates of the three levels discussed above are given by:

\[
h_p(x) = \frac{\beta}{\eta} \left( \frac{x}{\eta} \right)^{(\beta-1)}
\]  

(4.17)

\[
h_s(x) = \frac{(\sum_{i=1}^{n}[R_p(x)]^i[1-R_p(x)]^{n-i})^{-\frac{1}{\gamma}}}{\sum_{i=1}^{n}[R_p(x)]^i[1-R_p(x)]^{n-i}}
\]  

(4.18)

and

\[
h_{ss}(x) = m[h_s(x)]
\]  

(4.19)

4.4 Modifications for Operating Conditions

The model developed in Equation (4.16) represents the reliability of an asphaltic pavement layer of the same thickness as the laboratory
specimens, subjected to the same type of loading and under the same environmental conditions of these specimens.

4.4.1 Modification for Thickness

To modify this model for thickness differences, we have to go back to the element or specimen level, and assume that we have two specimens of thickness $t_1$ and $t_2$. The life distribution of the first specimen is a known, $N_f_1$. We are required to find the life distribution, $N_f_2$, of the second specimen, bearing in mind that these specimens are from the same material and the only difference between them is the thickness.

The specimen stress intensity factor is given by Equation (4.3) as:

$$K_1 = \sigma_{\text{max}} \cdot \sqrt{d} \times 6.898 \left(\frac{c}{d}\right).$$

Let $K_1'$ be the stress intensity factor for the first specimen and $K_1''$ be the stress intensity factor for the second specimen. Substituting in the equation above and dividing, we obtain

$$\frac{K_1'}{K_1''} = \frac{\sigma_1}{\sigma_2} \times \frac{(d_1)^{1/2}}{(d_2)^{1/2}} \times \frac{(c/d)_1}{(c/d)_2} \quad (4.20)$$

$(c/d)_1$ = the crack/depth ratio of the first specimen, and
$(c/d)_2$ = the crack/depth ratio of the second specimen, and
$\sigma_1$, $\sigma_2$ = the $\sigma_{\text{max}}$ for the first and second specimens, respectively.
A suitable basis for comparing the two cases is to take the situation when the crack depth ratios are equal, i.e.,

\[(c/d)_1 = (c/d)_2\]  \hspace{1cm} (4.21)

At this situation, Equation (4.20) becomes:

\[K_1'/K_1'' = \frac{\sigma_1}{\sigma_2} \times \frac{d_1}{d_2}^{1/2}\]  \hspace{1cm} (4.22)

\(\sigma_{s\text{max}}\) has the following with the geometry of the loaded specimen (39, 45):

\[\sigma_{s\text{max}} = \frac{6M_{\text{max}}}{Bd^2}\]  \hspace{1cm} (4.23)

where

- \(B\) = specimen width, and
- \(d\) = specimen thickness.

It has been shown that \(\sigma_{s\text{max}}\) for a concentrated load can be calculated from (39, 45):

\[\sigma_{p\text{max}} \cdot B \cdot d^2/P = 1.7424 \left(\frac{d_f}{a}\right)^{0.2438}\]  \hspace{1cm} (4.24)

where

- \(a = \frac{E_b \cdot I_b}{E_f \cdot B/2}^{1/3}\)  \hspace{1cm} (4.25)
- \(d_f\) = foundation thickness
- \(E_b, I_b\) = the modulus and moment of inertia of the beam, \(I_b\) being given as \(Bd^3/12\)
- \(P\) = the load
- \(E_f\) = the foundation modulus.
a can be expressed, in terms of the specimen thickness,
as:

\[ a = \lambda_1 d \], where \( \lambda_1 \) = constant \hspace{1cm} (4.26)

Substituting into Equation (4.24),

\[ \sigma_{P_{\text{max}}} \cdot B \cdot d^2 / P \cdot \lambda_1 d = 1.7424(d_f / \lambda_1 d)^{0.2438} \] \hspace{1cm} (4.27)

Cleaning up,

\[ \sigma_{P_{\text{max}}} \cdot B \cdot d / P \cdot \lambda_1 = \lambda_2 \cdot d^{-0.2438} \]

and

\[ \sigma_{P_{\text{max}}} = \lambda_3 \cdot d^{-1.2438} \] \hspace{1cm} (4.28)

For the special case when the foundation is semi-infinite, it was shown (45) that:

\[ \sigma_{P_{\text{max}}} \cdot B \cdot d^2 / P \cdot a = 2.3094 \] \hspace{1cm} (4.29)

giving

\[ \sigma_{P_{\text{max}}} = \lambda_4 / d \] \hspace{1cm} (4.30)

where

\( \lambda_2, \lambda_3, \lambda_4 \) are constants.

To extend this derivation to a uniform loading condition, we refer to the procedure of transforming Equations (4.24) and (4.29), by numerical integration, to obtain the uniform loading condition, in (45).
Since this integration, that was carried on $M_{\text{max}}$, does not change the power of $d$, we can assume that only the constants $\lambda_3$ and $\lambda_4$ in Equations (4.28) and (4.30) change when the uniform case is sought, i.e.,

$$\sigma_{\text{max}} = \lambda_5 \cdot d^{-1.2438}$$

(4.31)

for a finite foundation, and

$$\sigma_{\text{max}} = \lambda_6 / d$$

(4.32)

for a semi-infinite foundation, where $\lambda_5$ and $\lambda_6$ are constants.

Substituting these relationships in Equation (4.22), we get

$$K_1' / K_1'' = \left(\frac{d_2}{d_1}\right)^{1.2438} \cdot \left(\frac{d_1}{d_2}\right)^{0.5}$$

(4.33)

for the finite foundation, and

$$K_1' / K_1'' = \left(\frac{d_2}{d_1}\right) \cdot \left(\frac{d_1}{d_2}\right)^{0.5}$$

(4.34)

for the semi-infinite foundation.

Let us take the case of the finite foundation, and let

$$d_2 / d_1 = r_t$$

From Equation (4.33),

$$K_1' / K_1'' = (r_t)^{1.2438} \cdot (1/r_t)^{0.5}$$

$$= (r_t)^{0.7438}$$

(4.35)
A material characteristic that would account for failure in both cases is the fracture toughness, $K_{lc}$, which is given by

$$K_{lc} = L N_f^{1/\phi} \quad (4.8)$$

Decomposing the constant $L$ gives:

$$L = M(\phi Q)^{1/\phi} \quad \text{(from Equation 4.8)}$$

and

$$Q = A M^\eta \quad \text{(from Equation 4.4)}$$

Also,

$$M = 6.898 \sigma_{max} / \sqrt{a} \quad \text{(from Equation 4.3)}$$

$$L = M(\phi A M^\eta)^{1/\phi}$$

$$= M^{(n/\phi + 1)} (A \phi)^{1/\phi}$$

and

$$K_{lc} = M^{(n/\phi + 1)} (A \phi)^{1/\phi} N_f^{1/\phi} \quad (4.36)$$

Since this value is the same for both cases, it follows that:

$$L_1 N_f_1^{1/\phi} = L_2 N_f_2^{1/\phi} \quad (4.37)$$

and

$$N_f_2^{1/\phi} / N_f_1^{1/\phi} = L_1 / L_2$$

$$= (M_1 / M_2)^{(n/\phi + 1)} \quad (4.38)$$

giving:
\[ N_{f2} = \left( \frac{M_1}{M_2} \right) \left( \frac{n}{\phi} + 1 \right) \cdot N_{f1} \]
\[ = \left( \frac{M_1}{M_2} \right) (n + \phi) \cdot N_{f1} \]
\[ = \left( \frac{M_1}{M_2} \right) \cdot N_{f1} \]
\[ = \frac{\sigma_1}{\sqrt{d_1}} \cdot \frac{\sqrt{d_2}}{\sigma_2} \cdot N_{f1} \]
\[ = \frac{k_1'}{k_{1m}} \cdot \frac{d_2}{d_1} \cdot N_{f1} \quad \text{(by Equation 4.22)} \]
\[ = (r_t)^{1.7438} \cdot N_{f1} \quad \text{(by Equation 4.35)} \]

(4.39)

The distribution of \( N_{f2} \) is given as:

\[ F_{N_{f2}}(x) = P(N_{f2} \leq x) \]
\[ = P(N_{f1} \leq x/(r_t)^{\alpha}) \quad (4.40) \]

where

\[ \alpha = 1.7438 \quad (4.41) \]

or

\[ F_{N_{f2}}(x) = 1 - \exp\left(-\left(\frac{x}{\eta(r_t)^{\alpha}}\right)^\beta\right) \quad (4.42) \]

where \( \eta \) and \( \beta \) are the distribution parameters of the first specimen.
The same relationship holds for semi-infinite foundations, with \( \alpha = 1.5 \).

Carrying this upwards to the pavement model of Equation (4.16), and provided that \( n \) is known, we can compare the reliability functions of pavements of different thicknesses. Moreover, we can conveniently incorporate the reliability function in thickness design.

4.4.2 Modification for Traffic

Unlike the controlled repetition of loads in the laboratory, load repetitions in the actual pavement, due to traffic, are random.

Classically, the distribution of traffic arrivals is considered as a Poisson process (46, 47, 48).

Adopting this distribution for load repetitions on the pavement, the element reliability can be modified as follows (35):

Let

\[
P(x) = e^{-\lambda x} \left(\frac{(\lambda x)^k}{k!}\right)
\]

be a Poisson probability that the pavement element experiences exactly \( k \) shocks in a period \( (0,x) \). Then the resulting survival probability for this period is:

\[
R_p(x) = \sum_{k=0}^{\infty} e^{-\lambda x} \frac{(\lambda x)^k}{k!} \cdot R_k(x)
\]

where

- \( R_k \) = the probability of surviving \( k \) shocks; and
- \( \lambda \) = the intensity of the Poisson process.

This modification, however, cannot be used unless the time \( x \) is measured in time units, not in load repetitions (which should represent the number of shocks here).
4.5 Model Improvement

The model developed in Equation (4.16) holds if the number of sections, \( m \), in the subsystem is finite and known. The task of estimating \( m \) is not easy since the element, section and subsystem sizes, which can be the bases of estimation, are arbitrarily chosen.

Let us have another look at our assumptions and start by relaxing the sixth assumption by letting the subsystem size vary, so that \( m \) increases without bound. This may be the case when the subsystem is replaced by the system in the hierarchy. Since the system is in series in the section level, then the system life length corresponds to the smallest life length of the sections (35). That is, if the section lives, \( s_1, s_2, ..., s_n \) have the distribution \( F_s \), then the system (pavement) life distribution, \( F_{ss} \), is that of:

\[
\xi_n = \min (s_1, s_2, ..., s_n)
\]  

As \( n \to \infty \), \( F_{ss} \) converges to an asymptotic distribution that represents minimum extremes, which is the Weibull distribution, as mentioned earlier.

Applying this to our case, \( R_{ss}(x) \) of Equation (4.16) should converge to a Weibull when \( m \) increases without bound.

A necessary and sufficient condition for an initial distribution to converge to a Weibull model is (24):

\[
\lim_{x \to 0^+} \frac{F(nx)}{F(x)} = \eta^\beta, \quad 0 < \eta, \beta
\]
where $n$ and $b$ are the Weibull parameters and $F(x)$ is the initial distribution. This condition is equivalent to (24):

$$\lim_{x \to 0} \frac{n f(n x)}{n x} = n^b$$

(4.47)

A distribution satisfies this condition if it can be approximated near the origin as a power function $(x/n)^{-b}$, or if,

$$f(x) = (b/n^b) \cdot x^{b-1}$$

(4.48)

near the origin.

In our case, the initial distribution is given by Equation (4.15) as:

$$R_s(x) = \sum_{i=1}^{n} \binom{n}{i} R_p(x)^i (1-R_p(x))^{n-i}$$

which is the section reliability, $R_p(x)$ being the element reliability or the laboratory-derived reliability.

The p.d.f. of this binomial is given by

$$f_{R_s}(x) = \binom{n}{r} R_p(x)^r (1-R_p(x))^{n-r}$$

(4.49)

For small values of $x$, $[R_p(x)]^r \approx 1$.

Thus we have near the origin,

$$f_{R_s}(x) \approx \binom{n}{r} [1 - R_p(x)]^{n-r}$$

(4.50)
Comparing this with the Weibull p.d.f.,

\[ f_w(x) = \frac{\beta}{\eta} \left(\frac{x}{\eta}\right)^{\beta-1} \cdot \exp \left[ -\left(\frac{x}{\eta}\right)^{\beta} \right] \quad (4.51) \]

which becomes

\[ f_w(x) = \frac{\beta}{\eta} \left(\frac{x}{\eta}\right)^{\beta-1} \quad (4.52) \]

near the origin — as \( \exp \left[ -\left(\frac{x}{\eta}\right)^{\beta} \right] \approx 1 \) for small values of \( x \) — we get

\[ \beta - 1 = n - r \]

or

\[ \beta = n - r + 1 \quad (4.53) \]

and

\[ \beta/\eta^\beta = \binom{n}{r} \]

or

\[ \eta = \left[ \frac{\beta}{\binom{n}{r}} \right]^{1/\beta} \quad (4.54) \]

which are the Weibull parameters of the asymptotic function approximating the system (or the pavement) state.

Since the variate of our initial equation is a probability function, namely,

\[ 1 - R_p(x) = 1 - \exp \left[ - \left(\frac{x}{\eta_0}\right)^{\beta_0} \right] \quad (4.55) \]

where \( \eta_0 \) and \( \beta_0 \) are the Weibull parameters of the element distribution, our asymptotic distribution is expected to reflect this fact.
The asymptotic distribution, or system distribution, is given by:

$$F_w(x) = 1 - \exp[-(\frac{x}{\eta})^\beta]$$

(4.56)

where $\beta$ and $\eta$ are given by Equations (4.53) and (4.54), respectively and $x$ is given by Equation (4.55).

$$R_{ss}(x) = \exp[-(\frac{x}{\eta})^\beta]$$

(4.57)

According to Hasofer (49), the conditions that hold for an initial distribution of independent sequences to converge to a limit distribution automatically hold for m-dependent sequences, i.e., sequences where variables which are distant more than m units are independent.

If we examine our second assumption, we find that there is a possibility of association between elements, since the failure of one element can result in load redistribution in the neighboring elements. Evidently, this case is an m-dependent case, since the effect of failure of one item is not instantaneously felt by all the remaining items in the system.

Hence, the second assumption holds, in effect, for our case when m is small.

As to the identicality assumption, the pavement thickness is expected to be a fixed value, by design. If the material is the same, then the pavement elements can be assumed to be identical.
5.1 Introduction

A derivation of a statistical distribution based on plausible physical considerations is, by itself, not a conclusive argument for the validity of this distribution. This is especially so in fatigue testing, where the variability is usually high and the amount of data, usually, is small. A distribution can only be accepted for use in fatigue life studies when it is confronted with actual fatigue data obtained under various conditions, and is shown to adequately represent the life lengths obtained.

The scarcity of abundant fatigue data, in general, poses a significant problem in the way of finding the appropriate fatigue life distribution, as mentioned earlier, since almost any parametric distribution of two or more parameters can be made to fit the fatigue data reasonably well.

To overcome this, the idea of a data bank to accumulate historical data on fatigue failures seems promising. A similar idea of testing data collected from different laboratories has already been implemented for metals (see, for example, Reference 36).

In the case of fatigue failures in flexible pavements, the situation is even worse. One of the very few known sources, for example, of
laboratory-generated fatigue data of a reasonable size is The Ohio State University and, maybe, it is the only source so far which has applied fracture mechanics concepts to the analysis of fatigue failure in flexible pavements (see, for example, references 4, 7, 37, 38, 39, 40, 41 and 42).

In what follows, a brief description of the data selected to verify the model given by Equation (4.11) is mentioned. Different statistical models are constructed for each sample and, then, statistical methods are applied to select models that fit the data best.

Selected models are, then, used in their own right, for data interpretation, when possible, to establish trends of the behavior of these models.

5.2 Experimental Data

The selected data is generated from different research projects at The Ohio State University. The criteria followed in selecting the data are the availability of samples of reasonable sizes and diversity to cover changes due to the following different conditions:

- load-induced changes,
- environment-induced changes,
- material changes, and
- geometrical changes.

Load-induced changes are accounted for in two ways: different load intensities and different sequences of loading patterns. The only
environmental effect considered is the temperature effect. The material and geometrical effects are catered to by selecting samples of different shapes (i.e., beams and slabs), and of different paving material constituents (i.e., sand asphalt and asphalt concrete) among samples from the loading and temperature groups cited above. These groups are detailed as follows:

5.2.1 The Sequential Loading Group

The main purpose of this test series was to verify the effect of load sequence in the failure of bituminous paving mixtures (41). In these tests, haversine loading functions with 0.1 second duration and 0.8 seconds rest period were used. The function was programmed by the MTS function generator, with the aid of a data track, such that the load amplitude was varied in a sequence of 40, 30 and 20 pounds, therefore forming a block of three cycles, each one with a different intensity of load (Figure 5.1). In the second series, the sequence of load application was selected as 20, 30 and 40 pounds (Figure 5.2). A third sequence was used in which the block consisted of eight cycles of 20 pounds and one cycle of 40 pounds (Figure 5.3). These blocks of loads were applied to the specimen until failure occurred.

Specimens tested in this group consisted of sand asphalt beams, 2" x 2" x 24" in size. (Details on experimental procedures in all groups are given in Appendix A.)
Figure 5.1 Load Function Used in the 40/30/20 lbs. Sequence Tests
Figure 5.2 Load Function Used in the 20/30/40 lbs. Sequence Tests
Figure 5.3 Load Function Used in the 20/40 lbs. Sequence Tests
5.2.2 The High Stress Levels Group

The investigation was concerned, here, with the analysis of slabs when subjected to repeated loads of high level stress that might induce some permanent deformation (41). In the two series performed, the MTS was utilized with a haversine loading function with a duration of 0.1 second and a rest period of 0.8 seconds also. These tests were carried out at room temperature (as were the sequential loading series).

In the first of the series, a load of 500 pounds (corresponding to 25.5 psi under the loaded area) was applied to the asphalt slab. This load level was selected because it was found experimentally (41) that stresses under the load in excess of 20 psi were sufficient to cause permanent deformation under the load. The second set was performed using a high load of 600 pounds for 3000 cycles so as to develop some permanent deformation under the load, and then the load was changed to 300 pounds, and the test was continued until sample failure.

The slabs used in this group were, again, made of sand asphalt with a diameter of 44 inches and a thickness of 1.5 inches.

5.2.3 The Temperature Group

This group contained 44 specimens of bituminous concrete beams of the standard fatigue size (2" x 2" x 24"). These specimens were divided into six samples (five consisting of seven specimens, and one sample consisting of nine specimens) to be tested at temperature levels of 35, 45, 60, 75, 85 and 95°F. Load levels for these groups
were chosen so that the resulting fatigue lives of beams would be comparable in magnitude (42).

In these tests, a dynamic haversine load function (Figure 5.4) was again used to apply the required cyclic fatigue loads on the specimen. Two cyclic repetitions per second of the applied load were used for the tests, which were carried out on the MTS as usual.

5.3 Statistical Analysis

5.3.1 Model Fitting

The eleven samples collected were prepared to be fitted to the following distributions:

- the exponential,
- the normal,
- the Gamma,
- the Weibull, and
- the log normal.

These are the main distributions that were cited as the most likely candidates for fatigue failure data, as mentioned earlier.

5.3.2 "Goodness of Fit" Tests

A "goodness of fit" test is any test of the agreement between a theoretical probability distribution and that of a set of the sample observations. The test is usually run on an established hypothesis that the random process under investigation can be adequately
Figure 5.4 Loading Function Used in Fatigue and Dynamic Modulus Tests
described by a given theoretical probability distribution. This is called the null hypothesis (23, 33, 50).

Some of the techniques developed for this type of testing are the Chi-square (which is the most widely used), Kolmogorov-Smirnov and Cramer-Von Mises goodness of fit tests (33).

One problem that poses a major concern in the statistical modeling of fatigue lives is the small sample size usually encountered with fatigue data. With a small sample size, several different distributions may appear acceptable by the technique utilized; yet tail probabilities from these distributions may vary considerably. Thus, relatively large samples are usually required, by some techniques, to verify the validity of a specified model at some probability level: however, even with smaller sample sizes, it may be possible to eliminate some models from consideration (29).

The Chi-square test is of the type that requires large sample sizes. It is very powerful for samples on the order of $n > 100$, although effective results have also been claimed for sample sizes down to 30 (with some statisticians, still considering it unreliable for samples under 40) (33).

On the other hand, the Kolmogorov-Smirnov and the Cramer-Von Mises tests have been shown, in many cases, to be more discriminating with respect to their power for rejection of an incorrect hypothesis concerning the distribution of sample data (33).

The power of these two tests mainly comes from their ability to individually examine each data point in the sample, while the Chi-
square test requires that the data be grouped into cells. This grouping usually results in a less sensitive test (33) and, it follows, that the Kolmogorov-Smirnov and the Cramer-Von Mises tests can be satisfactorily used for smaller samples. The Cramer-Von Mises test is especially recommended when working with small samples, as it was primarily developed for this purpose (33). (The reader is referred to, for example, References 50, 23, 24, 28, 29, and 33 and its valuable bibliography for more details on these tests.)

5.3.3 Results

The statistical package GOF (Goodness of Fit) was used to calculate the model parameters and run the goodness of fit tests. This package was developed in 1972 for the American Institute of Industrial Engineers. It has the capability of working with ten statistical models and running four types of goodness of fit tests (see Reference 33).

The tests used were the Kolmogorov-Smirnov and the Cramer-Von Mises. The other two tests, the Chi-square and the Moments, were not used, as the sample sizes fitted were smaller than the requirements of the first and, since the second is only used for the test of normality, it was felt that the two tests used would do that too.

Table 5.1 gives the values of the first two tests used for the five models tested. Table 5.2 gives the critical values of the Kolmogorov-Smirnov (KS) test for different levels of significance and different degrees of freedom. In our case, the degrees of freedom are equal to the sample size, since no cells are used.
TABLE 5.1
CALCULATED CRAMER-VON MISES AND KOLMOGOROV-SMIROV VALUES FOR ALL CASES

<table>
<thead>
<tr>
<th>Temperature Group</th>
<th>EXPONENTIAL</th>
<th>NORMAL</th>
<th>WEIBULL</th>
<th>GAMMA</th>
<th>LOGNORMAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CM</td>
<td>KS</td>
<td>CM</td>
<td>KS</td>
<td>CM</td>
</tr>
<tr>
<td>35</td>
<td>0.10912</td>
<td>0.24674</td>
<td>0.28044</td>
<td>0.27871</td>
<td>0.12692</td>
</tr>
<tr>
<td>45</td>
<td>0.25513</td>
<td>0.20970</td>
<td>0.21433</td>
<td>0.29721</td>
<td>0.15451</td>
</tr>
<tr>
<td>60</td>
<td>0.15004</td>
<td>0.19101</td>
<td>0.21766</td>
<td>0.33837</td>
<td>0.04397</td>
</tr>
<tr>
<td>75</td>
<td>0.31742</td>
<td>0.30931</td>
<td>0.34664</td>
<td>0.32260</td>
<td>0.07483</td>
</tr>
<tr>
<td>85</td>
<td>0.32490</td>
<td>0.32148</td>
<td>0.21864</td>
<td>0.29724</td>
<td>0.03567</td>
</tr>
<tr>
<td>95</td>
<td>0.56665</td>
<td>0.40713</td>
<td>0.29892</td>
<td>0.31795</td>
<td>0.09823</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Stress Level Group</th>
<th>EXPONENTIAL</th>
<th>NORMAL</th>
<th>WEIBULL</th>
<th>GAMMA</th>
<th>LOGNORMAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = 600/3000</td>
<td>0.29723</td>
<td>0.20591</td>
<td>0.43458</td>
<td>0.28370</td>
<td>0.03771</td>
</tr>
<tr>
<td>P = 5000</td>
<td>1.03539</td>
<td>0.35967</td>
<td>0.71436</td>
<td>0.27918</td>
<td>0.23518</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sequential Loading Group</th>
<th>EXPONENTIAL</th>
<th>NORMAL</th>
<th>WEIBULL</th>
<th>GAMMA</th>
<th>LOGNORMAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = 20/30/400</td>
<td>0.20140</td>
<td>0.19177</td>
<td>0.33622</td>
<td>0.29713</td>
<td>0.03742</td>
</tr>
<tr>
<td>P = 40/30/200</td>
<td>0.32767</td>
<td>0.28320</td>
<td>0.23920</td>
<td>0.31541</td>
<td>0.02429</td>
</tr>
<tr>
<td>P = 20/400</td>
<td>0.29742</td>
<td>0.29122</td>
<td>0.17351</td>
<td>0.38169</td>
<td>0.03579</td>
</tr>
</tbody>
</table>

* CM = Cramer-Von Mises Test
** KS = Kolmogorov-Smirnov Test
### TABLE 5.2

CRITICAL VALUES OF THE KOLMOGOROV-SMIRNOV STATISTIC*

<table>
<thead>
<tr>
<th>Degrees of Freedom** (N)</th>
<th>Level of Significance (α)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>1</td>
<td>0.900</td>
</tr>
<tr>
<td>2</td>
<td>0.684</td>
</tr>
<tr>
<td>3</td>
<td>0.565</td>
</tr>
<tr>
<td>4</td>
<td>0.494</td>
</tr>
<tr>
<td>5</td>
<td>0.446</td>
</tr>
<tr>
<td>6</td>
<td>0.410</td>
</tr>
<tr>
<td>7</td>
<td>0.381</td>
</tr>
<tr>
<td>8</td>
<td>0.358</td>
</tr>
<tr>
<td>9</td>
<td>0.339</td>
</tr>
<tr>
<td>10</td>
<td>0.322</td>
</tr>
<tr>
<td>11</td>
<td>0.307</td>
</tr>
<tr>
<td>12</td>
<td>0.295</td>
</tr>
<tr>
<td>13</td>
<td>0.284</td>
</tr>
<tr>
<td>14</td>
<td>0.274</td>
</tr>
<tr>
<td>15</td>
<td>0.266</td>
</tr>
<tr>
<td>16</td>
<td>0.258</td>
</tr>
<tr>
<td>17</td>
<td>0.250</td>
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<tr>
<td>18</td>
<td>0.244</td>
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<td>19</td>
<td>0.237</td>
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<tr>
<td>20</td>
<td>0.231</td>
</tr>
<tr>
<td>25</td>
<td>0.21</td>
</tr>
<tr>
<td>30</td>
<td>0.19</td>
</tr>
<tr>
<td>35</td>
<td>0.18</td>
</tr>
<tr>
<td>Over 35</td>
<td>$\sqrt{N}$</td>
</tr>
</tbody>
</table>

* From Reference (33)

** The degrees of freedom are defined to be equal to the sample size or the number of cells used, whichever is smallest.
Table 5.3 (which is compiled from References 33 and 29) gives the critical values of the Cramer-Von Mises (CM) statistic at different significance levels.

Although the critical values at the 0.2 significance level on both tables are already considered conservative by some statisticians (33), still most of the computed values of the KS statistic were within these values, as shown in Table 5.4. A look at the temperature group in this table reveals that, of all the distributions that would fit our data without some transformation (that would take it away from the comparison range), only the exponential was rejected by the KS test at the 0.2 level of significance ($\alpha$) when the $95^\circ F$ sample was fitted. While the sequential loading group all passed this test, the high stress level group was less fortunate. The exponential, normal and log normal distributions did poorly with the 500 lb. sample, while only the normal and the log normal were rejected for the 600/300 lb. sample.

The CM test seems to be more discriminating at $\alpha = 0.2$. Of all the three groups, only the $60^\circ F$ sample fitted the five distributions without rejection at this level. One can also notice that all the distributions rejected by the KS test at $\alpha = 0.2$ were also rejected by the CM test at the same level. This puts the CM test at an advantage, since nothing is lost by replacing the KS test and moreover, the CM test is more discriminating.

Again, even when the CM test was used at $\alpha = 0.2$, distributions that passed the test ranged from 2 to 5. This means that an even more conservative significance level is needed.
TABLE 5.3
CRITICAL VALUES FOR THE CRAMER-VON MISES STATISTIC

<table>
<thead>
<tr>
<th>α</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.119</td>
</tr>
<tr>
<td>0.20</td>
<td>0.241</td>
</tr>
<tr>
<td>0.15</td>
<td>0.284</td>
</tr>
<tr>
<td>0.10</td>
<td>0.347</td>
</tr>
<tr>
<td>0.05</td>
<td>0.461</td>
</tr>
<tr>
<td>0.025</td>
<td>0.581</td>
</tr>
<tr>
<td>0.02</td>
<td>0.620</td>
</tr>
<tr>
<td>0.01</td>
<td>0.743</td>
</tr>
</tbody>
</table>

Complied from References (33) and (29)
### Table 5.4

**Test of Hypothesis H₀: The Given Sample is from a Particular Distribution**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Distribution*</th>
<th>Kolmogorov-Smirnov</th>
<th>Cramer-Von Mises</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \alpha = 0.20 )</td>
<td>( \alpha = 0.20 )</td>
</tr>
<tr>
<td>( T = 35^\circ F )</td>
<td>1</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( R )</td>
<td>( R )</td>
</tr>
<tr>
<td>( T = 45^\circ F )</td>
<td>1</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td>( T = 60^\circ F )</td>
<td>1</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>( A )</td>
<td>( A )</td>
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<tr>
<td></td>
<td>4</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td>( T = 75^\circ F )</td>
<td>1</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td>( T = 85^\circ F )</td>
<td>1</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( A )</td>
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</tr>
<tr>
<td></td>
<td>3</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>5</td>
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<td>( A )</td>
</tr>
<tr>
<td>( T = 95^\circ F )</td>
<td>1</td>
<td>( R )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
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<td>3</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td>( P = 20/30/40^# )</td>
<td>1</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
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<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>( A )</td>
<td>( R )</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>( A )</td>
<td>( R )</td>
</tr>
</tbody>
</table>
Table 5.4 (continued)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Distribution*</th>
<th>Kolmogorov-Smirnov</th>
<th>Cramer-Von Mises</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\alpha = 0.20$</td>
<td>$\alpha = 0.20$</td>
</tr>
<tr>
<td>P = 40/30/20#</td>
<td>1</td>
<td>A</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>A</td>
<td>R</td>
</tr>
<tr>
<td>P = 20/40#</td>
<td>1</td>
<td>A</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
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<td>A</td>
<td>A</td>
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<tr>
<td></td>
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<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>P = 500#</td>
<td>1</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>P = 600/300#</td>
<td>1</td>
<td>A</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>R</td>
<td>R</td>
</tr>
</tbody>
</table>

* Numbers in this column refer to the following distributions:
  1 = exponential
  2 = normal
  3 = Weibull
  4 = gamma
  5 = lognormal

** A = Accepted, i.e., the calculated value is less than critical value
  R = Rejected.
The last column of Table 5.4 shows the test results of CM for $\alpha = 0.5$. This gave an improvement in the discriminating power of the test. No more than two distributions passed the test for any one sample, and for the $P = 500\#$ sample of the high stress level group, all distributions were rejected at this level. In the remaining samples, however, only the Weibull and the Gamma distributions were not rejected, except for the $35^\circ F$ sample of the temperature group, which had the exponential and the Weibull distributions (the Gamma distribution was not available for this sample), and the $95^\circ F$ sample of the same group.

At this point, the distributions that gave good fits to our samples were:

- the exponential, which occurred once;
- the Gamma, which occurred eight times; and
- the Weibull, which occurred ten times.

Since the exponential distribution can be a special case of either the Weibull or the Gamma distributions, the dispute is only between the two of them, and so will be the discussion.

Table 5.5 shows the difference between the computed values of the CM test for the Gamma and Weibull distributions for all the appropriate samples. Figure 5.5 is a graphical representation of this table. It is clear, from this figure, that all the cases -- except for one or maybe two clear-cut cases -- are marginal, with a difference ranging between $\pm 0.023$. 
<table>
<thead>
<tr>
<th>NO.</th>
<th>SAMPLE</th>
<th>CM(Gamma) - CM(Weibull)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Temperature Group</strong></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
<td>0.00757</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>0.02332</td>
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<tr>
<td>4</td>
<td>75</td>
<td>-0.02319</td>
</tr>
<tr>
<td>5</td>
<td>85</td>
<td>-0.00656</td>
</tr>
<tr>
<td>6</td>
<td>95</td>
<td>0.20069</td>
</tr>
<tr>
<td></td>
<td><strong>High Stress Level</strong></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>500</td>
<td>-0.07835</td>
</tr>
<tr>
<td>8</td>
<td>600/300</td>
<td>-0.00200</td>
</tr>
<tr>
<td></td>
<td><strong>Sequential Loading</strong></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>20/30/40</td>
<td>-0.00558</td>
</tr>
<tr>
<td>10</td>
<td>40/30/20</td>
<td>0.01547</td>
</tr>
<tr>
<td>11</td>
<td>20/40</td>
<td>0.01448</td>
</tr>
</tbody>
</table>
Figure 5.5 CM(Gamma) - CM(Weibull) for all Samples
A reason that might have influenced selection in the marginal cases would be the choice of the plotting position. Program GOF uses the following plotting position:

\[ p_i = \frac{i - 0.5}{n} \]  \hspace{1cm} (5.1)

where

\[ i = \text{the order of the observation, and} \]
\[ n = \text{the sample size}. \]

Figure 5.6 shows the Gamma and the Weibull cdf's for the 20/30/40 sample (from the sequential loading group) whose observations were plotted with all the plotting positions surveyed in Reference (51), and shown in Table 5.6. It is clear, from this figure, that the plotting positions cover a range surrounding the two theoretical distributions. Hence, the choice of plotting position may make some changes when the decision statistic values are in the marginal zone.

Another approach that might prove useful in discriminating between these two distributions is to adopt the assumption that both distributions come as special cases from a general family of distributions, namely, Stacy's generalized gamma distribution (GGD), which is given by (see References 29 and 52):

\[ f(x) = \frac{\beta}{\Gamma(k)} \theta^{-\beta k} x^{\beta k - 1} e^{-(x/\theta)^{\beta}} \]

\[ x > 0; \quad \theta, \beta, k > 0. \]  \hspace{1cm} (5.2)
Figure 5.6 The Range of Different Plotting Positions
TABLE 5.6
 THE EFFECT OF DIFFERENT PLOTTING POSITIONS ON THE C.D.F.
 (Sequential Group: 20/30/40#)

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.09</td>
<td>0.06</td>
<td>0.06</td>
<td>0.10</td>
<td>0.07</td>
<td>6.00</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>0.18</td>
<td>0.16</td>
<td>0.15</td>
<td>0.20</td>
<td>0.16</td>
<td>6.30</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>0.27</td>
<td>0.26</td>
<td>0.25</td>
<td>0.30</td>
<td>0.26</td>
<td>12.00</td>
</tr>
<tr>
<td>4</td>
<td>0.35</td>
<td>0.36</td>
<td>0.35</td>
<td>0.34</td>
<td>0.40</td>
<td>0.36</td>
<td>13.00</td>
</tr>
<tr>
<td>5</td>
<td>0.45</td>
<td>0.45</td>
<td>0.45</td>
<td>0.43</td>
<td>0.50</td>
<td>0.45</td>
<td>15.50</td>
</tr>
<tr>
<td>6</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.52</td>
<td>0.60</td>
<td>0.55</td>
<td>15.60</td>
</tr>
<tr>
<td>7</td>
<td>0.65</td>
<td>0.64</td>
<td>0.65</td>
<td>0.61</td>
<td>0.70</td>
<td>0.64</td>
<td>17.80</td>
</tr>
<tr>
<td>8</td>
<td>0.75</td>
<td>0.73</td>
<td>0.74</td>
<td>0.70</td>
<td>0.80</td>
<td>0.74</td>
<td>24.10</td>
</tr>
<tr>
<td>9</td>
<td>0.85</td>
<td>0.82</td>
<td>0.84</td>
<td>0.79</td>
<td>0.90</td>
<td>0.84</td>
<td>26.50</td>
</tr>
<tr>
<td>10</td>
<td>0.95</td>
<td>0.91</td>
<td>0.94</td>
<td>0.88</td>
<td>1.00</td>
<td>0.93</td>
<td>40.50</td>
</tr>
</tbody>
</table>

1. \( F_1(x_i) = \frac{i - 0.5}{n} \)

2. \( F_2(x_i) = \frac{i}{n + 1} \)

3. \( F_3(x_i) = \frac{i - 3/8}{n + 3/4} \)

4. \( F_3(x_i) = \frac{1 - \alpha}{n - \alpha - \beta + 1} \) \( (\alpha = \beta = 0.3) \)

5. \( F_5(x_i) = i/n \)

6. Median as a plotting position: \( R_1 = \frac{n - i + 0.7}{n + 0.4} \)

\[ F_6(x_i) = 1 - R_1 = \frac{1 - 0.3}{n + 0.4} \]
If \( k = 1 \), this distribution would give a Weibull distribution with parameters \( \beta \) (shape parameter) and \( \theta \) (scale parameter). On the other hand, \( \beta = 1 \) gives a gamma distribution with the shape parameter, \( k \), and the scale parameter, \( \theta \). Thus, a test of \( H_0 : k = 1 \) would be equivalent to a test of \( H_0 \) that the distribution under test is Weibull.

Bain (29) developed a test statistic for testing this null hypothesis, provided that \( \beta \), or its estimate, is known. This statistic is given by:

\[
S(\beta) = \frac{n^{1/n} \left( \prod_{i=1}^{n} x_i^{\beta} \right)}{\sum_{i=1}^{n} x_i^\beta}
\]

(5.3)

where \( n \) = sample size.

In this procedure, then, a size \( \alpha \) test of \( H_0 : k = 1 \) against \( H_1 : k > 1 \) is to reject \( H_0 \) if \( S(\beta) > S_{1-\alpha} \). A table was also developed for the critical values \( (S_{1-\alpha}) \) for different sample and test sizes.

\( S(\beta) \) values for our samples are given in Table 5.7. Figure 5.7 is generated from Bain’s table for selected test sizes, and extrapolated to accommodate sample sizes down to 5. The \( S(\beta) \) values, calculated for the 11 samples, are shown scattered at different \( \alpha \) levels. At a level of significance of 0.01, the null hypothesis cannot be rejected for eight samples, the rejected samples being 2, 3 and 11. Surprisingly, the three samples were marginal Weibulls in the previous analysis (see Figure 5.5).
### TABLE 5.7
*S(β) VALUES FOR ALL SAMPLES*

<table>
<thead>
<tr>
<th>Temperature Group (°F)</th>
<th>DESIGNATION</th>
<th>S(β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>1</td>
<td>0.46298</td>
</tr>
<tr>
<td>45</td>
<td>2</td>
<td>0.62754</td>
</tr>
<tr>
<td>60</td>
<td>3</td>
<td>0.63661</td>
</tr>
<tr>
<td>75</td>
<td>4</td>
<td>0.52103</td>
</tr>
<tr>
<td>85</td>
<td>5</td>
<td>0.59829</td>
</tr>
<tr>
<td>95</td>
<td>6</td>
<td>0.61866</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>High Stress Level (lb.)</th>
<th>DESIGNATION</th>
<th>S(β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>7</td>
<td>0.59454</td>
</tr>
<tr>
<td>600/300</td>
<td>8</td>
<td>0.49972</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sequential Loading</th>
<th>DESIGNATION</th>
<th>S(β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20/30/40</td>
<td>9</td>
<td>0.57052</td>
</tr>
<tr>
<td>40/30/20</td>
<td>10</td>
<td>0.61757</td>
</tr>
<tr>
<td>20/40</td>
<td>11</td>
<td>0.68580</td>
</tr>
</tbody>
</table>
Figure 5.7  Critical Values of GGD. Taken from Bain's Table (29)
One last point that may support this analysis is that the failure rate of the pavement material is expected to take the shape of a Weibull's failure rate, i.e., a rate that would start from the origin and increase to infinity as time goes on (when $\beta > 1$). The gamma failure rate (for a shape factor > 1) also starts from the origin and increases, but to the constant exponential failure rate as the limit (see Figures 2.5 and 2.6).

For what remains on this analysis, the Weibull distribution shall be assumed valid for all the samples.

5.4 Sensitivity of the Statistical Model

To see how the statistical models behave under different conditions, a quick comparison between them and the original deterministic models shall be made.

The general restrictions made for the development of the deterministic model are that it should "...be sensitive to rises and falls in the load, show sensitivity to the sequence of rises and falls in the load, ... and be capable of incorporating the secondary effects (e.g., environmental factors) within its constants" (41). Hence, our comparison shall be made with these restrictions in mind.

5.4.1 The Temperature Group

Since the shape parameter of a statistical model contains more information about the model behavior, it is singled out, in this study, as the parameter that reflects the sensitivity of the model toward any changes.
Figure 5.8 is a representation of the shape parameter (β) with temperature and load levels for the six temperature samples.

The crack growth process may be expressed in a simplified power law:

$$\frac{dc}{dN} = A K_1^n$$  (4.1)

where

$K_1 =$ the stress intensity factor, and

$A, n =$ material constants.

This model was found to explain satisfactorily the fatigue performance of bituminous materials (53). In his investigation of the temperature effects on the fatigue life of asphalt concrete pavements, Makdisi found that the model that provided the best fit to his data was a four-term model in the form:

$$\frac{dc}{dN} = A_1 K_1 + A_2 K_1^2 + A_3 K_1^4 + A_4 K_1^6$$  (5.4)

In this form, this model is not convenient for comparison of fatigue properties for different materials or different conditions. Since the coefficient $A$ of the one-term model, Equation (4.1), is more sensitive to variations, it is usually picked for comparison. For this purpose only, an equivalent value to the coefficient $A$ in the one-term model can be obtained from the four-term model by substituting for $K_1 = 1$, i.e., by algebraically summing the coefficients of the four terms to produce:
Figure 5.8 \textit{Variation of $B$ With Temperature and Load Levels}
\[ A_{EQ} = A_1 + A_2 + A_3 + A_4 \]  \hspace{1cm} (5.5)

where

\[ A_{EQ} = \text{the equivalent of } A \text{ (the one-term model coefficient).} \]

Table 5.8, second column, shows the values of \( A_{EQ} \) at each temperature. To get rid of the negative sign, for ease of drawing, a simple transformation was done on these values by adding a constant (5.5 x 10^{-7}) to each one. The transformed, or shifted values appear in column 4 of the sample table. Figure 5.9 shows the relationship between \( A_{EQ} \) and both temperature and \( \beta \). From this figure, one can say that there is some sort of relationship between the temperature and \( \beta \) through \( A_{EQ} \).

Majidzadeh et al. (41) reported that the stress intensity factor, \( K \), depends linearly on the intensity of the applied load, \( P \), and is also proportional to a function, \( f(c) \), reflecting the influence of the geometric configuration, including the crack size \( c \). This relationship is given, mathematically, as:

\[ K(t) = f(c) \cdot P(t) \]  \hspace{1cm} (5.6)

where

\[ K(t) = \text{the time-history of the stress intensity factor, and} \]

\[ P(t) = \text{the time variation of the loading function.} \]

From this equation,

\[ f(c) = \frac{K(t)}{P(t)} \]  \hspace{1cm} (5.7)
<table>
<thead>
<tr>
<th>Temperature °F</th>
<th>$A_{EQ}$ (10$^{-7}$)</th>
<th>$A(P)_{EQ}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real Value</td>
<td>Shifted Value</td>
</tr>
<tr>
<td>35</td>
<td>-3.93</td>
<td>1.57</td>
</tr>
<tr>
<td>45</td>
<td>-4.17</td>
<td>1.33</td>
</tr>
<tr>
<td>60</td>
<td>-4.65</td>
<td>0.85</td>
</tr>
<tr>
<td>75</td>
<td>-5.28</td>
<td>0.22</td>
</tr>
<tr>
<td>85</td>
<td>-2.34</td>
<td>3.16</td>
</tr>
<tr>
<td>95</td>
<td>0.836</td>
<td>6.34</td>
</tr>
</tbody>
</table>

*The values of $A_{eq}$ are taken from Reference (42), Table 5.8*
Figure 5.9 The Relationship between $A_{EQ}$ and both $\beta$ and Temperature
Hence, the \( \frac{dc}{dN} - (K/P) \) relation represents the variation of the damage rate in terms of the crack function, \( f(c) \).

This relationship, for the four-term model of Equation (5.4), is:

\[
\frac{dc}{dN} = A_1 P \left( \frac{K}{P} \right) + A_2 P^2 \left( \frac{K}{P} \right)^2 + A_3 P^4 \left( \frac{K}{P} \right)^4 + A_4 P^6 \left( \frac{K}{P} \right)^6
\]

(5.8)

Again, an equivalent coefficient for this equation would be:

\[
A(P)_{EQ} = A_1 P + A_2 P^2 + A_3 P^4 + A_4 P^6
\]

(5.9)

The real and shifted values of \( A(P)_{EQ} \) are given in the last two columns of Table 5.8. Figure 5.10 shows the relationship between \( A(P)_{EQ} \) and temperature, and \( A(P)_{EQ} \) and \( \beta \). The relationship between the logarithm of \( A(P)_{EQ} \) and temperature may be approximated by a straight line, as shown in that figure. That is, the variations in the value of \( A(P)_{EQ} \) may be attributed to temperature alone. Since the relationship between \( A(P)_{EQ} \) (in logarithmic form) and \( \beta \) is non-linear, the relationship between \( \beta \) and temperature is expected to be similar. This relationship is shown in Figure 5.11.

Another material constant, available in the studies, that may help us verify our statistical model, is the fracture toughness, \( K_{1c} \). This is a measure of the material's resistance to fracture. For asphaltic mixtures, the fracture toughness depends on factors such as the rate of loading, asphalt content and test temperature (42).
Figure 5.10 Relationship between $A(P)$ and both $\beta$ and Temperature
Figure 5.11 A Plot of $\beta$ Against Temperature
Values of fracture toughness obtained for each of the six samples are plotted against $\beta$ in Figure 5.12.

5.4.2 The Sequential Loading Group

Although the data available for the other two groups would not allow us to run the sample analysis of the temperature group, we may, still, be able to establish some trend of behavior for the statistical model, through its shape parameter, $\beta$. Conclusive evidence needs data samples, of bigger sizes, that cover more variations than what is covered here, the matter which requires extensive, long-range experimentation.

Unlike monotonic loading (constant amplitude), in which the damage accumulation was found to be independent of the applied stress levels, sequential loading gives a different behavior (41).

In Figure 5.13, the rate of crack growth, $\frac{dc}{dN}$, was plotted against $K/P$ values for the three series of the load sequences applied. The parameter that accounts for the difference, $A(P)$, is calculated, from this figure, and plotted against $\beta$ as before. Figure 5.14 shows the result.

5.4.3 The High Stress Level Group

Similar results of this group are shown in Figure 5.15. From only two observations in a curve, one can only "feel" the trend, or the direction of change, if there is any.

Although the trend, in this figure, differs from that in the sequential loading group (Figure 5.14), it is in the same direction
Figure 5.12 Relation between $\beta$ and Fracture Toughness

$K_{IC}$ (lbs/in. $^{3/2}$)

$\beta$
Figure 5.13 \( \frac{dc}{dN} \) versus \( \frac{K}{P} \) for Sequential Loads
Figure 5.14  Relation between A(P) and \( \beta \) for Sequential Loads
Figure 5.15 Relationship between \( A(P) \) and \( \beta \) for High Stresses
of the upper portion of the A(P) - B curve of Figure 5.10. A look at Figure 5.16 (which gives the mean time to failure, in load repetitions, against temperature changes) shows that the beginning of the upper portion in Figure 5.10 (about 80°F) is indeed the beginning of the downfall of the mean time to failure. This may suggest that there is some sort of similarity in the process of failure in the two cases, like another failure mode, i.e., creep in case of the temperature group and permanent deformation in case of the high stress level group.

Bompas-Smith shows an example of such a failure, Figure 5.17, for ball bearings, and outlines a graphical procedure by which these two failures can be separated (22).

Figure 5.18 shows plots of two samples from our data on Weibull probability paper; one is the 95°F sample from the temperature group and the other is the "P = 500#" sample from the high level stress group. Although the points on each set can be approximated by a straight line, one can still see the similarity between the two plots and the plot of Figure 5.17. Due to the steep slopes and the small sample sizes of these distributions and, maybe, the fact that the influence of the second failure mode is not high, it was felt that applying Bompas-Smith's technique would be somewhat difficult. However, one would mention the fact that the lower part of each of the curves is attributed to the primary mode, which is fatigue in our case, while the upper part is attributed to both the primary and secondary modes.
Figure 5.16 Relationship between Mean Time to Failure (in Load Repetitions) and Temperature
Figure 5.17 Bearing Failures with Bi-Modal Distribution (After Reference 22)
Figure 5.18 Weibull Probability Plots of Samples Expected to be Bi-modal
CHAPTER VI
INVESTIGATION OF THE SYSTEM MODEL

6.1 Introduction

In this chapter, some insights are given on the pavement reliability function. An explanation of the indications of parameters is given, together with developed equations for some model parameters that make it easier to use. Also, a short description of the concept of pavement failure, as pertaining to its cracking index, is given.

A sensitivity analysis is, then, run by changing values of various parameters in the model, or exposing the model to different conditions, and observing its behavior.

Finally, a brief account on some possible applications of the model is given to illustrate its potential.

6.2 Choice of Parameter Values

The system reliability model, developed in Chapter IV, is given by:

\[ R_{SS}(x) = \exp \left( - \frac{x^\beta}{\eta} \right) \] (4.57)

where

\[ \beta = n - r + 1 \] (4.53)

\[ \eta = (\beta/r)^{1/\beta} \] (4.54)

\[ x = 1 - \exp \left( - \frac{\theta}{\eta_0} \right) \] (4.55)
n = the number of elements in a section,

r = the number of elements that did not experience failure (r = n - k, where k = the number of elements that failed),

\( \beta_0, \eta_0 \) = the shape and scale parameters of the element distribution, and

t = the time to failure (given, here, as the number of cycles).

To be able to estimate the system reliability, values for x and for the parameters \( \beta \) and \( \eta \) have to be known. x is found from the element model. For \( \beta \), and consequently \( \eta \), one has to know the values of n and r.

Since this model is an asymptote, the larger the number of sections in the system, i.e., the closer the number to infinity, the closer we are to a true representation by the model. This can be achieved by choosing small values for n. The value of 10 was found more convenient to work with; as the failed portion of the pavement would be represented as a percentage (in area, for example) of the total pavement area, so one element, in this case, would represent one out of ten, or ten percent, and so on.

The knowledge of r is synonymous with the knowledge of k, the number of elements that failed in each n elements.

Adopting these concepts, a computer program was developed, by which Table 6.1 was generated. The first column gives the cracking index (CI), found by:

\[
CI(%) = \frac{k}{n} \times 100
\]  

(6.1)
<table>
<thead>
<tr>
<th>Cracking Index (%)</th>
<th>n</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.4472</td>
<td>2.00</td>
</tr>
<tr>
<td>20</td>
<td>0.4055</td>
<td>3.00</td>
</tr>
<tr>
<td>30</td>
<td>0.4273</td>
<td>4.00</td>
</tr>
<tr>
<td>40</td>
<td>0.4735</td>
<td>5.00</td>
</tr>
<tr>
<td>50</td>
<td>0.5364</td>
<td>6.00</td>
</tr>
<tr>
<td>60</td>
<td>0.6152</td>
<td>7.00</td>
</tr>
<tr>
<td>70</td>
<td>0.7128</td>
<td>8.00</td>
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<tr>
<td>80</td>
<td>0.8363</td>
<td>9.00</td>
</tr>
<tr>
<td>90</td>
<td>1.0000</td>
<td>10.00</td>
</tr>
</tbody>
</table>
while the second and third columns give corresponding values of $\eta$ and $\beta$ as calculated by Equations (4.54) and (4.53), respectively.

Figure 6.1 gives the relationship between CI and $\beta$, which is given mathematically by:

$$\beta = 0.1(CI) + 1. \quad (6.2)$$

The slightly more complicated relationship of $\eta$ and CI is shown in Figure 6.2. This relationship is approximated by:

$$\eta = 0.413205 \exp(4.55288 \times 10^{-5} (CI)^2) \quad (6.3)$$

This relationship was arrived at by regression analysis (with $R^2 = 0.9849$). Since no extrapolations would be needed with the range of CI selected, no problems would be encountered by using it.

It should be stressed that the cracking index referred to here is the cracking index at failure, i.e., the cracking index at which the pavement fails. This leads us to the vital issue of defining failure.

6.3 A Definition of Failure

Usually, in systems like pavements that do not exhibit catastrophic failure (i.e., either functioning or not functioning), failure starts some time in the system's life and 'drifts' gradually to a limit. In pavements, this limit is defined, by the analyst, according to a number of criteria, such as the classification of the road, the amount of traffic it carries, economy, etc.
Figure 6.2 Cracking Index at Failure versus $\eta$
Generally, there are two main approaches to the analysis of pavement failure: the structural (or distress) approach and the functional (or performance) approach. Figure 6.3 illustrates the difference between them.

Part (a) of the figure shows the structural representation of failure. As seen from the figure, a pavement may not show any distress (i.e., cracking) until the number of load repetitions reaches a level denoted by \( N_{Os} \).

Part (b) shows the functional approach. In this approach, the performance of the pavement is given as a rating index (i.e., PSI) that starts with an initial value, \( P_0 \), prior to opening the pavement to service, and decreases with load repetitions until it reaches a limiting value, \( P_f \), where the pavement is considered as functionally failed.

It is clear, from the figure, that the crack initiation (indicated by \( N_{Os} \)) does not imply that the pavement has functionally failed, i.e., that it no longer provides a rideable surface. At the time of initiation of structural failure, the pavement serviceability, \( P_t \), is still well above \( P_f \).

After the onset of cracking, however, the pavement distress will increase progressively, until a physical distress state, \( D_f \), associated with the functional failure level, \( P_f \), is reached.

The difference between the levels of failure (\( N_{ff} \) and \( N_{fs} \)), called the 'Distress to Performance' relationship, is a function of a host of parameters, like traffic volume, wheel loads, pavement structure, environment, etc. (54).
Figure 6.3 Structural and Performance Based Design Approaches.
While Pell (55) indicates that a considerable amount of crack propagation should take place over a wide area of the pavement before any serious deterioration of the pavement structure results, Terrel (56) reports that:

"Fatigue cracks were frequently developed during the summer and autumn months when test pavements received the first phase of loading. Provided there was no accompanying change in subgrade or base courses, the number of load repetitions supported were surprisingly large ... Intrusion of water through the cracked surface and the usual spring thaw tended to weaken the underlying material. When testing was resumed in the spring, total failure quickly resulted. That was a result, not of further fatigue distress, but of subgrade failure."

This difference in opinions emphasizes the need for more work to uncover the distress-performance relationships. However, there is a general agreement among engineers that 40% structural cracking of a pavement brings it to a probable functional failure condition (54, 57).

This means, in terms of our model, that the pavement reliability at functional failure can be obtained by assigning a cracking index, Cl, of 40%.

Different cracking indices can be assumed for other situations. For example, when a rehabilitation technique, like recycling, is to be selected based on the surface condition of the pavement, defining a
certain cracking index that relates to selecting a certain technique, as a failure level would allow us to estimate the pavement reliability at that level.

To clarify this point, take, for example, Epps' criteria for selecting a recycling technique, given in Table 6.2 (see, for example, Reference 58). This table has all the usual types of distress displayed across the top and major recycling alternatives along the left margin. However, we are only interested now in the "alligator cracking" type of distress. For this type, decisions are given, on the recycling alternative to select, for three ranges of cracking indices, namely, from 1% to 5%, from 6% to 25%, and above 25%.

Taking a cracking index of 10%, for example, and assuming a moderate extent of cracking, we can construct a reliability curve upon which we can base a decision.

6.4 Sensitivity of the Model

To see how our model behaves under different conditions, relative parameters were varied and reliability curves were generated for different values of these parameters. Following is a short account of the results.

6.4.1 Variation of CI

As mentioned earlier, the cracking index is used as a failure criterion. If it is decided that the pavement is considered failed at 20% cracking index, then this failure takes place earlier than
TABLE 6.2

SELECTION OF RECYCLING TECHNIQUES BASED ON SURFACE CONDITION OF ASPHALT CONCRETE (58)

<table>
<thead>
<tr>
<th>Surface Condition of Existing Pavement</th>
<th>Recycling Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean, Aligned, without Structural Deficits</td>
<td>A1</td>
</tr>
<tr>
<td>Clean, Aligned, with Structural Deficits</td>
<td>A2</td>
</tr>
<tr>
<td>Clean, Unaligned, without Structural Deficits</td>
<td>A3</td>
</tr>
<tr>
<td>Clean, Unaligned, with Structural Deficits</td>
<td>A4</td>
</tr>
<tr>
<td>Dirty, Aligned, without Structural Deficits</td>
<td>A5</td>
</tr>
<tr>
<td>Dirty, Aligned, with Structural Deficits</td>
<td>A6</td>
</tr>
<tr>
<td>Dirty, Unaligned, without Structural Deficits</td>
<td>A7</td>
</tr>
<tr>
<td>Dirty, Unaligned, with Structural Deficits</td>
<td>A8</td>
</tr>
</tbody>
</table>

Central Plant

<table>
<thead>
<tr>
<th>Recycling Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold Processed Under Structural Deficits with New Binder</td>
</tr>
<tr>
<td>Cold Processed Under Structural Deficits without New Binder</td>
</tr>
<tr>
<td>Hot Processed Under Structural Deficits with New Binder</td>
</tr>
<tr>
<td>Hot Processed Under Structural Deficits without New Binder</td>
</tr>
<tr>
<td>Hot Processed Under Structural Deficits with New Binder</td>
</tr>
<tr>
<td>Hot Processed Under Structural Deficits without New Binder</td>
</tr>
<tr>
<td>Hot Processed Under Structural Deficits with New Binder</td>
</tr>
<tr>
<td>Hot Processed Under Structural Deficits without New Binder</td>
</tr>
</tbody>
</table>
if, for example, a 40% cracking index is chosen as a failure criterion. Figure 6.4 shows reliability curves, for a pavement at 60°F, for cracking indices of 20%, 40% and 60%.

This figure indicates that the reliability loss with time is slower in the 20% level of failure criterion than, for example, in the 40% level, whereas the 40% and 60% levels have approximately the same rate of loss.

Although the second part of the above statement cannot be physically justified at the present state of the model development, the first part may hold, at least up to the workable level of 40%.

6.4.2 Thickness Variation

Recall that the modification of the model for changes in thickness of the asphalt layer was proposed to take place, in the element level, by:

\[ F_{Nf_2}(x) = 1 - \exp\left(-\frac{x}{\eta(r_t)^\alpha}\right)^\beta \]  \hspace{1cm} (4.42)

where

- \( F_{Nf_2} \) = the life distribution of the pavement with the new thickness \( (d_2) \)
- \( \eta, \beta \) = the Weibull parameters of the pavement with the original thickness \( (d_1) \). Typically, this thickness would be that of the laboratory investigated samples,
- \( r_t = d_2/d_1 \)
- \( \alpha \) = 1.7438 (for a finite (bounded) foundation)
- \( \alpha \) = 1.5 (for semi-infinite foundation)
Figure 6.4  Pavement Reliability for Different Cracking Indices
The sample representing 60°F, from the temperature group, was again chosen for verification. The thicknesses of the laboratory-manufactured specimens were found to vary between 2.00 and 2.27 inches with an overall average of 2.12 inches for the temperature group (42). Shown below is the range of variation, the average thickness for each and all samples of the temperature group. For a conservative design, the specimen thickness is taken as 2.5 inches. Three more thicknesses were chosen (d_2 = 4, 6, 8) to give values of r_t of 1.6, 2.4 and 3.2, respectively.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Av. Depth (in.)</th>
<th>Range (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>2.11</td>
<td>2.0 - 2.16</td>
</tr>
<tr>
<td>45</td>
<td>2.11</td>
<td>2.04 - 2.18</td>
</tr>
<tr>
<td>60</td>
<td>2.09</td>
<td>2.01 - 2.15</td>
</tr>
<tr>
<td>75</td>
<td>2.14</td>
<td>2.07 - 2.27</td>
</tr>
<tr>
<td>85</td>
<td>2.14</td>
<td>2.06 - 2.22</td>
</tr>
<tr>
<td>95</td>
<td>2.10</td>
<td>2.02 - 2.15</td>
</tr>
<tr>
<td>Overall</td>
<td>2.12</td>
<td>2.00 - 2.27</td>
</tr>
</tbody>
</table>

Figures 6.5 and 6.6 show the reliability curves for the three values of r_t for the finite and semi-infinite foundations, respectively. In both figures, lives are increasing in r_t, with the semi-infinite case being less reliable as expected.

6.4.3 Variation of Temperature

As mentioned earlier, environmental, loading and material variations are explained by variations in the parameters of the element model (i.e., that derived from laboratory samples).
Figure 6.5 Pavement Reliability for Different Thicknesses (Bounded Foundation)
Figure 6.6 Pavement Reliability for Different Thicknesses (Semi-Infinite Foundation)
Three levels of temperature were used to investigate the influence of temperature changes on the reliability function, namely, 60°F, 75°F and 85°F. $r_t$ was fixed at 1.6.

Figures 6.7 and 6.8 show the above-mentioned influence for both types of foundation. Although these figures do not depart from the trend of increase in life with temperature until 85°F (see Figure 5.16), it is of interest to see that the reliability curves of 60°F and 75°F are very close to each other.

A justification of this behavior may be found if we look, again, into Figure 5.16. This figure shows that the mean number of cycles to failure (or mean time to failure, MTTF) values are very close for the 60°F and 75°F samples (with that of the 60°F sample being a little higher), whereas the 85°F sample had a higher value. A lot of factors might have contributed to the mix-up between the 60°F and 75°F samples. The most prominent factor could be the difference in load intensities, which was intended to bring the lives of the samples into a comparable range.

6.4.4 Variation in Load Sequence

The three load sequence samples (20/30/40 lb., 40/30/20 lb., and 20/40 lb.) were run, with $r_t$ fixed at 1.6. The results are shown in Figures 6.9 and 6.10 for both types of foundation. The MTTF values were found to be:

- 20/30/40 lb. $1.781 \times 10^4$
- 40/30/20 lb. $2.22 \times 10^4$
- 20/40 lb. $3.377 \times 10^4$
Figure 6.7 Pavement Reliability at Different Temperature (and loading) Levels (Bounded Foundation)
Figure 6.8 Pavement Reliability at Different Temperature (and loading) Levels (Semi-Infinite Foundation)
Figure 6.9 The Influence of Load Sequence on Pavement Reliability (Bounded Foundation)
Figure 6.10 The Influence of Load Sequence on Pavement Reliability (Semi-Infinite Foundation)
which support the results shown in both figures.

6.5 Applications

According to Winfrey (59), physical properties — buildings, pavements, bridges, etc. — have three measures of life.

Service life is that period of time (or of service measured in some unit of production) extending from the date of installation into service to the date of retirement from service. It is a measure of the total actual usage.

Physical life is that period of time for which the property exists, not necessarily in a usable condition or in use.

Economic life is that period of time, or service, extending from the date of installation into service to that date when the property is no longer economically profitable to use.

Usually, service life and economic life are the focus of interest of the decision-maker when it comes to allocation of funds for the construction and up-keeping of highways. A tool that helps him predict both measures of life, with some assurance factor associated, would be of value to him.

The economic life may be predicted, with an associated reliability, using the relationship between pavement reliability and costs, shown in Figure 6.11 (60). This figure shows that the cost of construction increases while the user costs decrease, with the increase in reliability. The algebraic sum of these costs may show that there exists a minimum total cost related to a certain value of reliability. If this relationship is superimposed upon the relationship between reliability and life,
Figure 6.11 Relationship Between Pavement Costs and Reliability (After Ref. 60)
as shown in Figure 6.12, then the economic life can be predicted as the value, \(L_E\), corresponding to the economic reliability, \(R_E\), at which the total cost is minimum.

At this level of life, \(L_E\), the decision-maker may opt to improve the system, or delay improvement until certain decision criteria, like sufficient funds, are satisfied.

He may decide to perform minimal routine maintenance until the reliability drops to a certain level, \(R_S\), then take up major improvements, which end up a service life cycle for the pavement, at a life, say, \(L_S\).

A decision-maker has a host of rehabilitation alternatives to pick from. Knowing the laboratory-derived parameters, \(\eta_0\) and \(\beta_0\), of the material he intends to use, and the cost of each alternative, he can compare the reliabilities of these alternatives for the required analysis period and come out with an economically feasible option.

Suppose it is decided to go for recycling, before the above analysis is done, as the district has a sizeable quantity of stock-piled reclaimed material.

Due to the condition of the pavement, it is decided to exclude surface recycling. However, the condition of the sub-layers does not require any work on them. So, it is decided to go to the obvious: hot-mix recycling.

A choice should be made between two rejuvenators and soft asphalt. The following procedure is recommended:
Figure 6.12 Pavement Reliability, Cost and Life Relationships
1. Proper mix designs are made for each of the three alternatives. The amount of modifier, reclaimed material and virgin material (if any) are known, and also their prices.

2. A suitable number of specimens is manufactured from each mix (ten is recommended).

3. Fatigue testing is conducted under environmental and loading conditions that simulate design conditions.

4. For each mix, $\eta_0$, $\beta_0$ and $n$ are determined.

5. Also, for each mix, and for a given analysis period, or a number of load repetitions, a total cost-reliability curve is generated by varying the thickness ratio, $r_t$, and calculating both the reliability and cost for each value.

6. An optimum point, corresponding to a certain thickness ratio, is found for each mix.

7. Another plot for the coordinates of these options is made, from which the optimum alternative is picked.

Another use of this model could be the investigation of what can be called 'the structural adequacy loss' of a pavement. This follows naturally from the fact that a pavement's structural adequacy is a function of its thickness. Since the pavement thickness is accommodated for in our model, then for a particular life of the pavement,
if the reliability is known, a thickness that supports the pavement at the existing level of structural adequacy can be estimated. A "present structural adequacy index" can be defined as the ratio of thickness supporting the present situation to the original thickness.

The supporting thickness in terms of \( r_t \) can be estimated from Equations (4.42) and (4.57) as:

\[
(r_t) = \left[ \frac{1 - \exp\left(-\left(\frac{N_F}{\eta_0}\right)^{\beta_0}\right)}{\eta (-\log R)^{1/\alpha}} \right]^{1/\alpha}
\]  

(6.4)

where

\( N_F \) = the life of the pavement investigated,
\( \eta_0, \beta_0 \) = the laboratory-derived Weibull parameters,
\( \eta, \beta \) = system Weibull parameters,
\( R \) = reliability associated with \( N_F \) and \( r_t \), and,
\( \alpha \) = 1.7438 or 1.5, according to the type of foundation.

For any life, \( N_{f_1} \), the present structural adequacy index is given by:

\[
\text{PSAI} = \frac{(r_{t_1})}{(r_{t_0})} \times 100
\]

(6.5)

where

\( (r_{t_1}) \) = the original thickness ratio, or
\( (r_{t_0}) \) = the design thickness ratio.

To illustrate, take a simple example of a pavement initially represented by the 60°F of the temperature group, i.e., \( \beta_0 = 1.887 \),
\( \eta_0 = 2.436 \). Furthermore, if \((r_t)_0 = 1.6\), \( N_f = 0.8 \times 10^4 \), and \( \text{DI} = 40\% \), i.e., \( \beta = 5 \), \( \eta = \eta_0 \times 1.4735 \), then PSAI can be calculated for any value of the pavement reliability. Table 6.3 gives these values for reliability varying from 0.9998 to 0.90, and Figure 6.13 gives the graphical representation.

The sharp change in the relationship at the higher values of \( R \) may be due to the fact that failure occurs, by definition, at a cracking index of 40\%. This means that when the pavement reaches 40\% CI, it has already experienced loss in the structural capacity, but its reliability is still high at the beginning of the curve.
<table>
<thead>
<tr>
<th>R</th>
<th>( r_t )</th>
<th>PSAI (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9998</td>
<td>1.56</td>
<td>97.5</td>
</tr>
<tr>
<td>0.9990</td>
<td>1.02</td>
<td>63.75</td>
</tr>
<tr>
<td>0.99</td>
<td>0.55</td>
<td>34.38</td>
</tr>
<tr>
<td>0.98</td>
<td>0.45</td>
<td>28.13</td>
</tr>
<tr>
<td>0.97</td>
<td>0.41</td>
<td>25.63</td>
</tr>
<tr>
<td>0.96</td>
<td>0.37</td>
<td>23.13</td>
</tr>
<tr>
<td>0.95</td>
<td>0.35</td>
<td>21.88</td>
</tr>
<tr>
<td>0.94</td>
<td>0.33</td>
<td>20.63</td>
</tr>
<tr>
<td>0.93</td>
<td>0.32</td>
<td>20.00</td>
</tr>
<tr>
<td>0.92</td>
<td>0.31</td>
<td>19.38</td>
</tr>
<tr>
<td>0.91</td>
<td>0.30</td>
<td>18.75</td>
</tr>
<tr>
<td>0.90</td>
<td>0.29</td>
<td>18.13</td>
</tr>
</tbody>
</table>
Figure 6.13 Relationship between $R$ and PSAI at CI = 40%
CHAPTER VII
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary

A pavement reliability function was developed for the bituminous layer of a flexible pavement, based on the statistical distribution of the material fracture toughness and laboratory-derived fatigue data. Central to the development of the model was the idea of treating the pavement as a system, the components of which (called elements) could be represented by laboratory specimens. The life structure of the system was taken as a series configuration of a large number of similar modules (or sections). The life structure of these modules was considered to be an r-out-of-n configuration. The extreme value theory was used to determine the system reliability function.

A procedure was also developed to account for thickness changes in the pavement system. This was accomplished by a derivation of the failure distribution of a pavement element of a different thickness from that of the original element, by applying transformation techniques to the failure distribution of the latter.

The validity of the model was, then, tested at the element level — represented by laboratory samples. Distributions of fatigue lives were found for samples manufactured from different materials and tested at different levels of loading, loading sequences and temperatures. Five
families of distributions, namely the normal, lognormal, exponential, gamma and Weibull, were tried. Goodness of fit tests were done using the Kolmogorov-Smirnov and the Cramer-Von Mises methods.

An attempt was then made to relate variations of the shape parameter of the selected distribution (Weibull) to variations of parameters from models derived from the original tests -- that accounted for differences in materials and testing conditions.

The system reliability function was considered. The flexibility of the model to represent different levels of failure was shown. Sensitivity analysis was run to check the behavior of the model with respect to changes in thickness, temperature and loading conditions. Some applications of the model were then outlined.

7.2 Conclusions

Despite the fact that the samples tested were small, the developed model was found to be well-behaved in both the element (or laboratory specimens) level and the system (pavement) level. How closely this model would predict or represent the pavement condition in real life is yet to be tested.

Although all the indications lead to the postulation that fatigue lives of laboratory specimens are Weibull-distributed, chances are there that, through different conditions, it might be proved otherwise. These chances are, however, slim. But nevertheless, the structure of the system model would not be affected, as the condition at the element level is represented at the system level only through the probability of failure of the former.
The definition of functional pavement failure, as the condition of the pavement at 40% Cracking Index, is arrived at, subjectively, by a number of practitioners. In the absence of an objective way to bridge the distress-to-performance gap, the definition mentioned above is thought of as reasonable. Furthermore, the model can accommodate any development in this direction.

7.3 **Recommendations**

This development is an attempt to set trends in a new concept that aims at a meaningful utilization of statistical and system analysis applications to pavement data, in pavement management. More work is needed to ensure the consistency of the model. The following are some examples of the apparent points that need investigation:

1. Applications of the system model are only outlined here. To illustrate these applications numerically, some extra work has to be done. Experiments should be carried out to determine the failure distributions of specimens of the material intended for use. Moreover, some work is needed to formulate the total cost of each condition or strategy. A method that takes into account the idea of variability in costs (instead of the deterministic approach) would be preferable.
2. More verification of the failure distribution and the behavior of the verified models is needed. Different materials can be used, some of which are:
   a. asphalt concrete (no additives)
   b. sand-asphalt
   c. sulfur-extended asphalt
   d. recycled materials (different additives)
   e. other additives (cement coating, rubber, etc.)

3. Similar verification at different environmental conditions, like:
   a. stress variations
   b. climatic variations.

The crux of these types of analyses is the abundance of data in all required levels. Since fatigue data are usually expensive (one reason why there are few fatigue data available), the idea of a data bank that accumulates data, on both the laboratory level and the field level, for all types of pavement failures or material characterizations, seems appealing.

4. This model is developed only for the asphalt layer in the pavement. An extension of the model can be made available by finding the failure distributions of the other layers and treating the pavement elements as a series module of elements, from all the layers. The rationale behind this is clear, since the failure of the element at any layer constitutes a failure of the pavement element.
5. Again, the model was developed for the fatigue mode of failure. Models for other modes, like rutting, can similarly be developed. Or, all the possible competing modes of failure can be integrated in one model.

6. Applications of other associated fields in reliability technology should be looked into. The renewal theory, for example, that is used extensively for maintenance and replacement in the mechanical and electrical systems, can be a great asset to the growing field of pavement management and rehabilitation.

7. A study is required for the lack of fit of the system model, in terms of its representation to the real life data. This would only be possible if samples of reasonable sizes of pavement failures are available, something that adds to the appeal of the data bank idea.
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APPENDIX A

EXPERIMENTAL PROCEDURES
APPENDIX A
EXPERIMENTAL PROCEDURES

A.1 Introduction

In this section, a general description of the experimental procedures followed in specimen manufacture and testing is given. The sources of this information are from References (39) and (53).

A.2 Sand-Asphalt Beams

The main characteristics of the sand-asphalt mixture used are given in Table A.1. Two types of support, simulating foundation conditions, were used: one consisted of a very soft foam rubber and the other of somewhat harder art-gum rubber. The stress/strain diagrams for both supports were linear at all stress levels.

Figure A.1 is a schematic diagram of the experimental set-up, showing the dimensions of the beam and the rubber support, and the mode of loading.

A.3 Sand-Asphalt Slabs

The specifications for the sand-asphalt mixtures are similar to those shown in Table A.1. The mix was prepared in a mechanical mixer in three batches in succession, then weighed into smaller pre-determined amounts in separate containers which were then stored in
TABLE A.1

Test Characteristics of Sand-Asphalt Mixtures

A. Asphalt Cement

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Gravity</td>
<td>1.010</td>
</tr>
<tr>
<td>Softening Point, Ring and Ball</td>
<td>123°F</td>
</tr>
<tr>
<td>Ductility 77°F</td>
<td>150 + cm</td>
</tr>
<tr>
<td>Penetration, 100 gm., 5 sec., 77°F</td>
<td>63</td>
</tr>
<tr>
<td>Penetration, 200 gm., 60 sec., 39.4°F</td>
<td>23.5</td>
</tr>
<tr>
<td>Flash Point, Cleveland Open Cup</td>
<td>455°F</td>
</tr>
<tr>
<td>Viscosity, 140°F = 2970 Poises</td>
<td></td>
</tr>
<tr>
<td></td>
<td>275°F = 4.68 Poises</td>
</tr>
</tbody>
</table>

B. Aggregate

<table>
<thead>
<tr>
<th>Sieve Number</th>
<th>% Passing (Samples)</th>
<th>ASTM Specification D1663-59T</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>100.0</td>
<td>85-100</td>
</tr>
<tr>
<td>30</td>
<td>88.0</td>
<td>70-95</td>
</tr>
<tr>
<td>50</td>
<td>64.2</td>
<td>45-75</td>
</tr>
<tr>
<td>100</td>
<td>30.1</td>
<td>20-40</td>
</tr>
<tr>
<td>200</td>
<td>15.7</td>
<td>9-20</td>
</tr>
</tbody>
</table>

C. Mixture

| % Asphalt Cement (by weight of aggregate) | 6%                |
| Average Density                           | 2.052 ± 0.011 gm/cm³ |
| Average Air Voids                         | 17.0 ± 0.5%        |
Figure A.1. Test Specimen Arrangement for Beams on Elastic Foundation
an oven at a temperature of approximately 300°F. After a lapse of about 20 minutes, the containers were taken out of the oven and each was poured into one of six compartments in the compaction mold, which was coated with a mild soap solution to prevent sticking of the asphalt slab to the mold. The mix was then carefully levelled in each compartment after which the compartments were removed, followed by raking across the joints and finally, by levelling the entire surface with a trowel.

A rigid compaction head was then placed over the mix, and a 12 h.p. vibrator mounted over it. The vibrator was then started up and rotated continuously on the compaction head during the compaction operations to prevent the mix from shifting in any one direction. The compaction operation was completed when the present thickness adjustment screws made contact with the rim of the mold. This operation usually lasted about one minute.

The slab dimensions and the experimental set-up are shown in Figure A.2. The foundation consisted of two layers bonded together by Bondmaster rubber cement, the upper layer being a slab of aromatic rubber (one inch thick) and the lower layer being a slab of commercial art-gum rubber (twelve inches thick). Both layers were four feet in diameter.

A.4 Asphalt Concrete Beams

The mixture specifications are given in Table A.2. Four (4.0) Kg of pre-heated aggregate (350°F) were mixed with the required amount
Figure A.2  Schematic Diagram of the Testing System
### TABLE A.2

**COMPOSITION OF AGGREGATE MIX USED IN SERIES II AND III**

<table>
<thead>
<tr>
<th>U.S. Sieve Size</th>
<th>% Passing by Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Series II</td>
</tr>
<tr>
<td>3/4&quot;</td>
<td>100</td>
</tr>
<tr>
<td>1/2&quot;</td>
<td>100</td>
</tr>
<tr>
<td>3/8&quot;</td>
<td>95</td>
</tr>
<tr>
<td>No. 4</td>
<td>60</td>
</tr>
<tr>
<td>No. 6</td>
<td>50</td>
</tr>
<tr>
<td>No. 50</td>
<td>12.5</td>
</tr>
<tr>
<td>No. 200</td>
<td>4</td>
</tr>
<tr>
<td>% Asphalt</td>
<td>6.5</td>
</tr>
</tbody>
</table>

*Series III is A x 6.5*
of asphalt heated to pouring consistency (280 - 300°F). A mechanical mixer was used for mixing the aggregate and the asphalt. The mixing pan was also pre-heated. One minute mixing time insured that the final temperature was not less than 280°F.

The beam mold (2" x 2" x 24") was pre-heated to about 350°F. After compaction, the specimens were stored at 140°F in an oven for 24 hours. Finally, they were left at room temperature for at least 24 hours before testing. The test set-up was similar to that shown in Figure A.1.
APPENDIX B

CUMULATIVE DISTRIBUTIONS, DENSITY FUNCTIONS, AND
FAILURE RATES OF ALL GROUPS
Figure B.1  cdf for 35°F Temperature Group
Figure B.2 pdf for 35°F Temperature Group
B.3 Failure Rate for 35°F Temperature Group
Figure B.4 cdf for 45°F Temperature Group
Figure B.5 pdf for 45°F Temperature Group
Figure B.6 Failure Rate for 45°F Temperature Group
Figure B.7 cdf for 60°F Temperature Group
Figure B.8 pdf for 60°F Temperature Group
Figure B.9 Failure Rate for 60°F Temperature Group
Figure B.10  cdf for 75°F Temperature Group
Figure B.11 pdf for 75°F Temperature Group
Figure B.12 Failure Rate for 75°F Temperature Group
Figure B.13 cdf for 85°F Temperature Group
Figure B.14 pdf for 85°F Temperature Group
Figure B.15 Failure Rate for 85°F Temperature Group
Figure B.16 cdf for 95°F Temperature Group
Figure B.17 pdf for 95°F Temperature Group
Failure B. 18 Failure Rate for 95°F Temperature Group
Figure B.19  cdf for 20/30/40# Sequential Loading Group
Figure B.20 pdf for 20/30/40# Sequential Loading Group
Figure B.21 Failure Rate for 20/30/40# Sequential Loading Group
Figure B.22  cdf for 40/30/20# Sequential Loading Group
Figure B.23 pdf for 40/30/20# Sequential Loading Group
Figure B.24  Failure Rate for 40/30/20# Sequential Loading Group
Figure B.25 cdf for 20/40# Sequential Loading Group
Figure B.26 pdf for 20/40# Sequential Loading Group
Figure B.27 Failure Rate for 20/40# Sequential Loading Group
Figure B.28 cdf for Stress Group P = 500#
Figure B.29 pdf for Stress Group, P = 500#
Figure B.30 Failure Rate for Stress Group, P = 500#
Figure B.31 cdf for Stress Group, P = 600/300#
Figure B.32 pdf for Stress Group, P = 600/300#
Figure B.33 Failure Rate for Stress Group, $P = 600/300$