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DOMAIN WALL RESONANCE IN MAGNETIC GARNET THIN FILM AND THE ASSOCIATED ACOUSTIC PHENOMENA

The Ohio State University

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DOMAIN WALL RESONANCE IN MAGNETIC GARNET THIN FILM AND THE ASSOCIATED ACOUSTIC PHENOMENA

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Ren-jye Yeh, B.S., M.S.

* * * * *

The Ohio State University

1982

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ACKNOWLEDGMENTS

I want to thank my adviser, Prof. P. E. Wigen, for his guidance, encouragement and all the support during my stay at The Ohio State University. I would also like to thank Dr. Horst Dötsch of Philips Laboratory, Germany, and Prof. J. Morkowski from Institute of Molecular Physics, Poland for their assistance in this research project. I have to express my deep appreciation to the whole group working in the laboratory for the support in every aspect and constant discussion. I also enjoy working with them as a team. I would like to thank the staff members in the electronic shop and Mr. R. Tucker for the technical assistance. I have to thank my wife, Minzu, without her patience my life would be a disaster. Also, I want to thank Miss Carol Hugenberg for her assistance in typing this dissertation in a short time. I would like to gratefully acknowledge NSF Grant #80-07642 for support as a research associate from 1979-1982.
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"Domain Wall Resonances in CrBr₃ at Low Temperatures," E. Jedryka, R. J. Yeh, M. Ramesh, and P. E. Wigen, (to be published.)
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CHAPTER 1

INTRODUCTION

It was predicted\textsuperscript{(1)} and experimentally confirmed that in ferromagnetic materials the distribution of the magnetization is composed of different domains, the spins are parallel inside each domain and the orientation of the magnetization of different domains are oriented in different directions. At each local point in a ferromagnetic material, the spontaneous magnetization is strongly coupled to its neighboring magnetic moment by the exchange interaction so that the spins will line up parallel to each other. When the magnetic moment is uniform throughout the sample, the resulting magnetostatic energy is very large. The magnetostatic energy can be decreased by breaking the magnetization into domains. In the process of developing the magnetic domains, domain walls are formed which are regions in which the magnetization changes from the orientation in one domain to that of the neighbor domain. At equilibrium, the increase of the energy from the domain walls will offset the decrease of the magnetostatic energy. The domain structure is thus determined by minimizing the total energy of the system.
In the magnetic garnet thin films, the uniaxial anisotropy, $K_u$, will have a preferred or easy axis normal to the film plane. If $Q = K_u / 2\pi M$ is greater than unity, the magnetization, $\mathbf{M}$, in the domains will lie parallel to the easy axis. Usually, the magnetic domain shape is a meander type parallel strip as shown in Fig. 1.1, or a cylindrical type domain as shown in Fig. 1.2 which is called a magnetic bubble.\(^{(2)}\)

Faraday effect.\(^{(3)}\) As shown in Fig. 1.3, when polarized light passes through the magnetization, the plane of the polarization is rotated by an angle $\phi$ or $-\phi$ depending on the orientation of the magnetization and the Faraday rotation coefficient. Consequently, by using another polarizer, the light beams from domains with different orientation will have different intensity and can be observed.

Because of the exchange interaction between the neighboring spins, the neighboring spins within the domain wall region will have small changes in their orientation. The width of the domain wall which depends on the competition between the exchange interaction, $J$, and the uniaxial anisotropy energy, $K_u$, is proportional to $(J/K_u)^{1/2}$. In magnetic garnet thin film, the typical domain wall thickness is in the range of 100 nm. The study of the domain wall structure
FIGURE 1.1 An array of stripe domains. The arrows indicate the direction of the magnetization in each domain, (taken from reference 2.).
FIGURE 1.2 Bubble domains. The arrows indicate the direction of the magnetization in each domain, (taken from reference 2.)
In the Faraday effect, the polarization of the light beam is rotated by an angle $\phi$ or $-\phi$, depending on the direction of the magnetization in the domain.
is one of the major topics in this work. If the orientation of the spins is assumed to vary only in the direction normal to the wall, the structure of the domain wall can be determined. The most typical wall structure is the 180° Bloch wall as shown in Fig. 1.4.

When the spins in a domain wall are excited, the spins will change their orientation. The collective effect of this changing of the orientation of the spins is equivalent to a change in the position of the domain wall. The response of the domain wall movement to the force arising from applied magnetic fields depends on the internal structure of the domain wall. Domain wall resonance (DWR) is one technique for investigating the structure and the dynamic properties of the domain wall system. The concept of an inertia of the domain wall was introduced by Doring\textsuperscript{(4)} in 1948. A restoring force arises from the demagnetization field generated by the domain patterns when the domain wall is moved out of its equilibrium position\textsuperscript{(3)} Thus in a demagnetized media, the domain wall motion can be understood in terms of a natural resonance frequency.

By studying this resonance frequency, the internal structure of the domain wall and the motion of the spins about their equilibrium position can be evaluated. For magnetic garnet thin films used in this study, the frequency is in the range of 100 MHz and a microstrip transmission\textsuperscript{(5)} device is used to detect the resonance. A coplanar strip-
FIGURE 1.4 The spin orientation as a function of position in the 180° Bloch domain wall.
line of width about 1mm is used to excite the DWR and an asymmetrical slot line is used as the detector. Since the transmitted signal from DWR is attenuated by about 70dB, the asymmetrical rf-structure is essential to eliminate rf power coupling across the rf-structure itself.

When an external magnetic field, $H_p$, is applied in the plane of the film, the internal structure, as well as the dynamic process of the domain wall, are modified and the inertia, or effective mass, of the domain wall is changed. The concept of the effective mass of the domain wall arises from the increase of the internal wall energy of the moving wall due to the demagnetization field developed normal to the wall. This increase in energy can be treated as an effective kinetic energy of the domain wall. Details of the restoring force and the effective mass are given in Chapter 2.

In the magnetic garnets, the crystal structure is cubic. Hence the crystalline anisotropy energy has cubic symmetry. The influence of the cubic anisotropy energy on the effective mass of the domain wall was evaluated as part of this project. The detail of this calculation are in Section 2.1.3.

In the process of exciting the magnetic domain walls, an elastic wave was observed to be generated through
magnetoelastic interaction. The magnetoelastic interaction was first observed in ferromagnetic resonance (FMR) by Dotsch, et. al. In the FMR experiments in magnetic garnet films, the magnetoelastic interaction will excite both longitudinal and transverse elastic waves which propagate into the GGG, $\text{Gd}_3\text{Ga}_5\text{O}_{12}$, substrate where they are reflected back from the nonfilm side. A series of standing elastic modes are produced when the frequency of the standing modes satisfy a condition determined by the speed of the elastic wave in GGG and the thickness of the sample. In this work, the excitation of elastic waves by DWR was observed and analyzed. When the magnetic film is in the demagnetized state, the interaction between the magnetization and the elastic waves becomes more complex due to the nonuniformity of the magnetization. In the excitation from DWR, the standing modes of the transverse elastic wave are also observed with the same frequency splitting observed in the FMR, but the intensity of the longitudinal modes are negligible. In this work a fine structure spectra with a frequency splitting in the 50 kHz range was observed to be associated with each of the fundamental standing modes excited by DWR. The amplitude of the fine structure decreases with the increase of the frequency and the dispersion of the fine structure can be linear or quadratic depending on the mode of excitation. This fine structure has been identified
as due to multidimensional elastic waves. These fine structure spectra will be discussed in detail in Chapters 2 and 3.

In a pulsed rf experiment an echo due to the standing elastic normal modes was observed. In the FMR mode, the echo time is 0.3 μsec,\(^{(8)}\) the time required for the elastic wave to travel round trip across the thickness of the film/substrate system. In the DWR, the echo time is in the range of 10 μsec. This time corresponds to a beating of the fine structure modes discussed above.

The agreement between the experimental data and the calculations based on a theoretical model indicates that a good physical understanding of the static domain structure and the dynamic processes of the domain wall has been obtained.
CHAPTER II
THEORY OF DOMAIN WALL DYNAMICS

2.1 Theory for Effective Mass of Domain Walls

2.1.1 Introduction

The magnetic moment in the ferromagnetic materials is treated as a continuous function of space and time in this work. The dynamical system is based on the phenomenological theory of the Landau-Lifshitz model. The time variation of the magnetic moment is equal to the total torque acting on its magnetic moment,

\[
\frac{d\hat{M}}{dt} = -\gamma T + \alpha \hat{M} \times \frac{d\hat{M}}{dt},
\]

where \( \gamma \) is the gyromagnetic ratio, \( \hat{T} \) is the torque and \( \alpha \) is the damping parameter. For the samples investigated in this work the damping effect is small and it will be neglected in the further developments. In a magnetic system the torque on the magnetic moment is

\[
\hat{T} = \hat{M} \times \hat{H}_{\text{eff}} ,
\]

where \( \hat{H}_{\text{eff}} \) is the effective magnetic field acting on the magnetic moment \( \hat{M} \), or,
in which \( \epsilon \) is the total energy density of the system and \( \frac{\delta}{\delta \dot{M}} \) is the functional derivative. The magnitude of the magnetic moment in the ferromagnetic materials is considered as constant at each temperature independent of the orientation.

In the spherical coordinate system, Eq. 2.1 is written:

\[
\frac{d\dot{M}}{dt} = \dot{M}(\frac{\partial}{\partial \dot{\theta}} \dot{\theta} + \frac{\partial}{\partial \dot{\phi}} \dot{\phi}) = -\gamma \frac{\dot{M}}{M}
\]

In rectangular coordinate, as shown in Fig. 2.1,

\[
\frac{\delta \epsilon}{\delta \dot{M}} = \frac{\delta \epsilon}{\delta \dot{M}_x} \dot{x} + \frac{\delta \epsilon}{\delta \dot{M}_y} \dot{y} + \frac{\delta \epsilon}{\delta \dot{M}_z} \dot{z}
\]

By the coordinate transformation:

\[
\dot{x} = \dot{r} \sin \theta \cos \phi + \dot{\theta} \cos \theta \cos \phi - \dot{\phi} \sin \phi,
\]
\[
\dot{y} = \dot{r} \sin \theta \sin \phi + \dot{\theta} \cos \theta \sin \phi + \dot{\phi} \cos \phi,
\]
\[
\dot{z} = \dot{r} \cos \theta - \dot{\theta} \sin \theta.
\]

The functional derivatives can be transformed from the rectangular coordinate system to the spherical coordinate system. Eq. 2.5 then becomes:
FIGURE 2.1  (a) The domain wall lies in the xz-plane and the in-plane field is along the x-axis producing an equilibrium orientation \( \theta_0 \). (b) For the magnetization far from the domain wall. (c) The polar angle for the magnetization.
From Eq. 2.2, 2.4, and 2.6, Eq. 2.1 becomes:

\[
\frac{\delta \epsilon}{\delta M} = \frac{\delta \epsilon}{\delta \hat{r}} + \frac{1}{M} \frac{\delta \epsilon}{\delta \theta} + \frac{1}{M \sin \theta} \frac{\delta \epsilon}{\delta \phi}.
\]

From Eq. 2.2, 2.4, and 2.6, Eq. 2.1 becomes:

\[
\frac{\partial \mathbf{M}}{\partial t} = M(\dot{\theta} \hat{\phi} + \sin \theta \dot{\phi}) = -\gamma M \hat{r} \times (\frac{\delta \epsilon}{\delta M} \hat{r} + \frac{1}{M} \frac{\delta \epsilon}{\delta \theta} \hat{\theta} + \frac{1}{M \sin \theta} \frac{\delta \epsilon}{\delta \phi} \hat{\phi})
\]

\[
= +\frac{\gamma}{\delta \theta} \hat{\phi} - \frac{\gamma}{\sin \theta} \frac{\delta \epsilon}{\delta \phi} \hat{\phi}.
\]

So the Landau-Lifshitz eq. can be written in component form as:

\[(2.9a) \quad M \hat{e} = \frac{-\gamma}{\sin \theta} \frac{\delta \epsilon}{\delta \phi} \hat{\phi}, \]

\[(2.9b) \quad M \sin \theta \hat{\phi} = \gamma \frac{\delta \epsilon}{\delta \phi}. \]

2.1.2 The Energy Contribution

From Eq. 2.8, it is clear that the dynamics of the ferromagnetic system is determined by the total internal energy density. In the magnetic garnet thin film, the major energy terms are the anisotropy energy, the exchange energy, the demagnetization energy and the Zeeman energy. In order to simplify the physical picture, this section considers the system having only one isolated domain wall. The demagnetization energy generated by the domain structure will be treated in detail in Section 2.2.

A. The Anisotropy Energy

A.1. The Uniaxial Anisotropy Energy

The magnetic anisotropy energy represents that internal magnetic energy that depends on the orientation of
the magnetization. In general, the anisotropy energy energy depends on the crystal structure and represents the internal energy as the magnetization rotates away from a special orientation which is called the easy axis. The simplest form is that of the uniaxial anisotropy energy, which can be expressed as a series expansion of \( \sin^2 \theta \):

\[
(2.10) \quad E_u = K_{u1} \sin^2 \theta + K_{u2} \sin^4 \theta + \ldots.
\]

In the magnetic garnet used in this research, the first term dominates and \( K_{u1} \) will be written as \( K_u \). It is called the uniaxial anisotropy energy constant.

In general, there are two mechanisms to induce the uniaxial anisotropy in the magnetic garnet films grown by the LPE (Liquid-Phase Epitaxial) technique. One is the growth induced uniaxial anisotropy and the other is due to the stress which is generated by the lattice mismatch between the film and the GGG (Gadolinium Gallium Garnet) substrate.

The lattice mismatch between the film and the substrate will produce a stress, \( T \), which is parallel to the film plane and gives rise to the uniaxial anisotropy through the magnetostriction effect. For a \((111)\) oriented film,

\[
(2.11.a) \quad K_{u, \text{st}} = -\frac{3}{2} \gamma^{[111]} T.
\]

For a \((100)\) oriented film,
where the $\gamma$'s is the magnetostriction coefficient for the appropriate orientation.

The growth induced anisotropy may be due to single-ion anisotropy, dipolar energy or the anisotropic exchange interaction. This contribution is not completely understood at this time.

A.2 The Cubic Anisotropy Energy

Since the magnetic garnet has a cubic structure, the magnetocrystalline anisotropy energy has a cubic symmetry. The cubic anisotropy energy can be expanded as a polynomial series of the direction cosines:

\[(2.12.a) \quad \varepsilon_{\text{cubic}} = \sum_{i,j,k=0}^{\infty} A_{ijk} a_i^4 a_j^4 a_k^4,\]

where $A_{ijk}$ are the coefficients of cubic anisotropy and $a_i$ are the direction cosines ($a_1, a_2, a_3$) of the spontaneous magnetization with respect to the three cubic axes.\(^{(10)}\)

When $a_1$ is changed to $-a_1$, the cubic anisotropy energy should be invariant. Consequently the odd powers will vanish, i.e., $i, j, k$ can only be even integers. Also, the energy, $\varepsilon_{\text{cubic}}$, is invariant under the transformation of interchange of any two directions, $a_i$ and $a_j$. So the cubic anisotropy energy can be written as:
\[(2.12.b) \quad \varepsilon_{\text{cubic}} = A_{200} (a_1^4 + a_2^4 + a_3^4) + A_{220} (a_1^2 a_2^2 + a_1^2 a_3^2 + a_2^2 a_3^2) + A_{400} (a_1^4 + a_2^4 + a_3^4) + A_{222} a_1^2 a_2^2 a_3^2 + A_{440} (a_1^4 a_2^4 + a_1^4 a_3^4 + a_2^4 a_3^4) + \ldots \ldots \]

Using the identity relations for \(a_1, a_2\) and \(a_3\),

\[(2.13) \quad a_1^2 + a_2^2 + a_3^2 = 1
\]

\[a_1^4 + a_2^4 + a_3^4 = 1 - 2(a_1^2 a_2^2 + a_2^2 a_3^2 + a_1^2 a_3^2)\]

\[a_1^4 a_2^2 + a_2^4 a_3^2 + a_3^4 a_1^2 + a_1^2 a_2 a_3 + a_2^2 a_3 a_1 + a_3^2 a_1 a_2\]

\[= 3(a_1^2 a_2^2 + a_2^2 a_3^2 + a_3^2 a_1^2) - 5 a_1^2 a_2^2 a_3^2\]

etc.,

all the terms in Eq. 2.11 can be expressed as the combination of

\[a_1^2 a_2^2 + a_2^2 a_3^2 + a_1^2 a_3^2\]

and

\[a_1^2 a_2^2 a_3^2\]

Thus

\[(2.14) \quad \varepsilon_{\text{cubic}} = \text{const.} + K_1 (a_1 a_2^2 + a_2 a_3^2 + a_1 a_3^2) + K_2 a_1^2 a_2^2 a_3^2 + \text{h.o.t.} \]

In the magnetic garnets, \(K_1\) tends to be dominant and is usually referred to as the cubic anisotropy constant. The detailed calculation of the cubic anisotropy energy for (100) and (111) films will be reviewed in section 2.2.
B. The Exchange Energy.

The exchange interaction energy between two atoms having spins $\mathbf{S}_i$ and $\mathbf{S}_j$ at the lattice sites $r_i$ and $r_j$ can be expressed by the relationship

$$(2.15.a) \quad E_{\text{fx}} = -2J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j,$$

where $J_{ij}$ is the exchange integral between the $i^{\text{th}}$ and $j^{\text{th}}$ atomic orbital wave functions. For $J_{ij} > 0$, the atomic moments have a minimum energy when they line up parallel causing long range magnetic ordering. In most magnetic materials, only the nearest neighboring atoms have a significant contribution to the exchange interaction and $J_{ij} = J$ is assumed to be the same at every site. In the domain wall the angle between $S_i$ and $S_j$ deviate from parallel by a very small angle, $\theta_{ij}$. The exchange energy between two spins in the domain wall is then given by:

$$(2.15.b) \quad E_{\text{ij}} = -2JS^2\cos^2\theta_{ij}$$

$$= JS^2\theta_{ij}^2 + \text{higher order terms.}$$

In the continuum model, this exchange energy density is expressed as:

$$(2.15.c) \quad E_{\text{ex}} = \frac{A(\nabla M)^2}{M^2},$$
where $M$ is the magnetization and $A$ is the exchange constant related to $J$ and the crystal structure. In the cubic crystal structure, \( A \) is

\[
A = \frac{JS^2}{a} \quad \text{for simple cubic lattice,}
\]

(2.16.a)  
\[
A = \frac{2JS^2}{a} \quad \text{for body-center cubic lattice,}
\]

\[
A = \frac{4JS^2}{a} \quad \text{for face-center cubic lattice,}
\]

where $a$ is the lattice constant.

In the magnetic garnet system, the lattice structure is a complex cubic structure composed of locations that have point symmetries which are octahedral, "a" sites, dodechedral, "c" sites, and tetrahedral, "d" sites. For yttrium iron garnet the exchange constant can be expressed as an appropriate average of the exchange integral between spins located at the various sites. Thus

\[
(2.16.b) \quad A = \frac{5S^2}{4a} (8 J_{aa} - 5 J_{ad} + 3 J_{dd}) .
\]

For rare earth substituted iron garnets, the expression is still more complex but an average value of the exchange constant can be determined.
In the polar coordinate system and when the amplitude of the magnetization is uniform, the exchange energy density in Eq. 2.15 is expressed as

\begin{equation}
E_{\text{ex}} = A \left[ (v\theta)^2 + \sin^2 \theta(v\phi)^2 \right] ,
\end{equation}

where \(\theta\) and \(\phi\) are the polar coordinate of M as indicated in Fig. 2.1.

C. The Magnetostatic Energy.

The magnetostatic energy is given by the expression

\begin{equation}
E_M = \frac{1}{8\pi} \int H^2 dV
\end{equation}

where \(H\) is the magnetic field and the integral is over all space. The magnetic field, \(H\), for the whole system is determined by the Maxwell's equations. The magnetic field can be separated into two terms.

\begin{equation}
\hat{H} = \hat{H}_a + \hat{H}_d ,
\end{equation}

where \(\hat{H}_a\) is the applied field supplied by the external current source and \(\hat{H}_d\) is the demagnetization field which is due to the discontinuity of \(\hat{M}\) at the boundary of the sample.

The magnetostatic energy is then expressed by the

\begin{equation}
E_M = \frac{1}{8\pi} \int H_a^2 dV + \frac{1}{4\pi} \int \hat{H}_a \cdot \hat{H}_d dV + \frac{1}{8\pi} \int H_d^2 dV .
\end{equation}

\(H_d\) has to satisfy the Maxwell's equation:
and the boundary condition $H_d \to 0$ as $r \to \infty$. From Eq. 2.21, the divergence of the magnetic field is equivalent to the divergence of the magnetization. In general, $\vec{H}_d$ is related to $\vec{M}$ by:

$$\vec{H}_d = -4\pi \hat{N} \cdot \vec{M} \quad (2.22)$$

Where $\hat{N}$ is the demagnetization tensor depending on the geometry of the sample. For a thin film lying in the $x$-$y$ plane with $\vec{M}$ in the $z$ direction,

$$\vec{H}_d = -4\pi \vec{M} \quad (2.23)$$

Eq. 2.20 then reduces to:

$$E_m = \frac{1}{8\pi} \int H_a^2 dV - \int \vec{M} \cdot \vec{H}_a dV + 2\pi \int M^2 dV \quad (2.24)$$

in which $\vec{M} \cdot \vec{H}$ is the Zeeman energy and the last term is the demagnetization energy.

Inside the domain wall, if $\vec{M}$ is a function of $y$ only as in Fig. 2.1, the demagnetization field is

$$\vec{H}_d = -4\pi M_y \hat{y} \quad (2.25)$$

and the demagnetization energy is
(2.25.b) \[ E_d = 2\pi \int M_y^2 dV. \]

The total magnetic energy density is given by

\[ (2.26) \quad \varepsilon_{\text{tot}} = E_u + E_{\text{cubic}} + E_d + E_{\text{Zeeman}} + E_{\text{ex}} \]

2.1.3 The Static Wall Structure.

The static domain wall structure is determined by setting the time variation of the magnetic moment to zero, \( \dot{M} = 0 \). From the Landau-Lifschitz eq. (2.8 & 2.9) this is equivalent to

\[ (2.27) \quad \left( \frac{\delta \varepsilon}{\delta \theta} \right)_{\theta_s, \phi_s} = \left( \frac{\delta \varepsilon}{\delta \phi} \right)_{\theta_s, \phi_s} = 0, \]

where \( \theta_s, \phi_s \) are the static polar angles of \( M \).

In a Bloch domain wall structure, only the uniaxial anisotropy and the exchange energy are considered. The influence of the cubic anisotropy, Zeeman Energy, and demagnetizing energy is included later. The energy density for these terms is

\[ (2.28) \quad \varepsilon = K_u \sin^2 \theta + A \left[ (\nabla \theta)^2 + \sin^2 \theta (\nabla \phi)^2 \right]. \]

From Eq. 2.27, and if \( M \) does not vary along the plane of the domain wall,
(2.29) \[ \frac{\partial^2\phi}{\partial y^2} \sin^2\theta + \frac{\partial\phi}{\partial y} \frac{\partial\theta}{\partial y} \sin 2\theta = 0 \]

and

(2.30) \[ 2A \frac{\partial^2\theta}{\partial y^2} - \sin 2\theta K_u - A \left( \frac{\partial\phi}{\partial y} \right)^2 \sin^2\theta = 0 \]

The solution to Eq. 2.29 is \( \phi = \text{constant} \).

Solving Eq. 2.30 gives

\[ \theta = 2 \tan^{-1}(\exp(\frac{\sqrt{\Delta}}{\Delta})) \]

where \( \Delta = \sqrt{\frac{K_u}{A}} \), \( q \) is a function of time.

Eq. 2.31 gives the angle dependence of the magnetization through the domain wall. This wall structure, known as a Bloch Wall, is shown in Fig. 1.4.

If the Zeeman and demagnetization energies are taken into account, the energy density is:

\[ E = K_u \sin^2\theta + A(\nabla\theta)^2 + A \sin^2\theta (\nabla\phi)^2 + 2\pi M^2 \sin^2\theta \sin^2\phi \]

\[ - H_p M \sin\theta \cos\phi. \]

Here \( H_d \) is the demagnetization field in the domain wall and \( H_p \) is the applied field along the x axis. As calculated in Appendix I, \( H_d = -4\pi M_y \) in the domain wall.

From Eq. 2.27,

\[ \frac{\partial}{\partial y} \left[ 2A \sin^2\theta \frac{\partial\phi}{\partial y} \right] - 2\pi M^2 \sin^2\theta \sin 2\phi - H_p M \sin\theta \sin\phi = 0, \]

and
The solution to Eq. 2.33, is $\phi = 0$ and Eq. 34 reduces to

\begin{equation}
2A \frac{\partial^2 \theta}{\partial y^2} - K_u \sin 2\theta - A \sin 2\theta \left( \frac{\partial \phi}{\partial y} \right)^2 - 2\mu M^2 \sin 2\theta \sin^2 \phi + H_p M \cos \theta \cos \phi = 0.
\end{equation}

By using the relation,

\begin{equation}
\theta'_{\theta} = \frac{\partial \theta}{\partial \phi},
\end{equation}

where $\theta_{\theta} = \frac{\partial \theta}{\partial \theta},$

\begin{equation}
A \frac{\partial^2 \theta}{\partial \theta^2} = K_u \sin 2\theta - H_p M \cos \theta.
\end{equation}

On integration,

\begin{equation}
\Delta^2 \left( \frac{\partial \theta}{\partial y} \right)^2 = \sin^2 \theta - \sin^2 \theta_o - 2\mu (\sin \theta - \sin \theta_o),
\end{equation}

where $\theta_o$ is the angle of $M$ at a position far away from the center of the domain wall, and $\mu = H_p/H_u$, where $H_u = 2K_u/M$.

The solution to Eq. 2.38 gives the static structure of the wall in the presence of an in-plane magnetic field and their solutions can be calculated numerically. By using the program given in Appendix II, the domain wall
structure for various values of the in-plane field are shown in Fig. 2.2. If the domain wall effective thickness is defined by the interception of the tangent line of the \( \theta \) vs. \( y \) curve at the origin, or \( y=0 \), and the tangent curve at \( y=\infty \), then the domain wall effective thickness is given by

\[
\Delta_{\text{eff}} = \frac{\pi - 2\theta_0}{(3\frac{\delta \theta}{\delta y})_{y=0}}.
\]

From Eq. 2.38, the domain wall thickness is given by

\[
(2.40) \quad \Delta_{\text{eff}} = \Delta (\pi - 2\theta_0) / [4(1-\sin^2 \theta_0) - 2\nu (1-\sin \theta_0)]^{1/2}.
\]

In the case of high Q materials, \( \sin \theta_0 = \frac{H_p}{H_u - 4\pi M} \approx \frac{H_p}{H_u} = \mu \). Then

\[
(2.41) \quad \Delta_{\text{eff}} = \Delta \frac{\pi - 2\theta_0}{1 - \mu}.
\]

This domain wall thickness is shown in Fig. 2.3 as a function of the applied field.

The static angle \( \theta_0 \) is the equilibrium angle of the magnetization far away from the center of the domain wall. This angle is determined by the static equation of motion

\[
(2.42) \quad \left( \frac{\delta \xi_D}{\delta \theta} \right)_{\theta_0, \phi_0} = \left( \frac{\delta \xi_D}{\delta \phi} \right)_{\theta_0, \phi_0} = 0,
\]
FIGURE 2.2 The domain wall structure of different applied in-plane fields, $\mu = H_p / H_u$. 

1: Bloch wall
2: $\mu = .2$
3: $\mu = .4$
4: $\mu = .6$
Figure 2.3 The thickness of the domain wall as a function of the applied in-plane field
where $c_D$ is the energy density in the domain, and is given by:

\[ (2.43) \quad c_D = K_u \sin^2 \theta + 2\pi M^2 \cos^2 \theta - H_p M \sin \theta \cos \phi. \]

In the magnetic garnet films, the demagnetization field in the domain is $4\pi M^2$. In Eq. 2.43, $2\pi M^2 \cos^2 \theta$ is used as the demagnetization energy. From Eq. 2.42,

\[ (2.44) \quad K_u \sin 2\theta_1 - 2\pi M^2 \sin 2\theta_1 - H_p M \cos \theta_1 \cos \phi_1 = 0. \]

Taking $\phi_1 = 0$ as a trivial solution, Eq. 2.44 becomes

\[ (2.45) \quad 2K_u \sin \theta_1 - 4\pi M^2 \sin \theta_1 = H_p M, \]

or

\[ \sin \theta_1 = \frac{H_p M}{2K_u - 4\pi M^2}, \]

or

\[ (2.46) \quad \sin \theta_1 = \frac{H_p}{K_u - 4\pi M}. \]
If $H_u > 4\pi M$, i.e., for high $Q$ materials,

$$\sin \theta_0 = \frac{H_p}{H_u} = \nu.$$  \hspace{1cm} (2.47)

2.1.4 The Cubic Anisotropy of (111) & (100) Films

A. For (111) Films

From Eq. 2.12, the cubic anisotropy energy is

$$\varepsilon_{\text{cubic}} = K_1(a_1^2a_2^2 + a_2^2a_3^2 + a_3^2a_1^2) + K_2a_1^2a_2^2a_3^2 + \ldots.$$  \hspace{1cm} (2.48)

In the magnetic garnet films used in this project, $K_1$ is much larger than the higher order terms. Consequently only the first term is calculated. In Eq. 2.48, $a_1$, $a_2$, $a_3$ are the direction cosines of the magnetization along the crystal axes. As shown in Fig. 2.4, in the (111) film the magnetization in the $x,y,z$ coordinate has to be transformed into the crystal axes. The transformation matrix from the $(x,y,z)$ coordinate to the crystal axes of the (111) film is given by

$$\begin{pmatrix}
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}}
\end{pmatrix}.$$  \hspace{1cm} (2.49)
Figure 2.4 Coordinate system of the crystal axes and the rectangular axes with the z-axis lying along the easy axes for (100) and (111) films.
Using this transformation and the fact that $\hat{\mathbf{M}} = (M \sin \theta \cos \phi, M \sin \theta \sin \phi, M \cos \theta)$, in the coordinate system, the components of $\hat{\mathbf{M}}$ in the crystal axes are

\begin{equation}
\hat{\mathbf{M}} = M \begin{pmatrix}
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}}
\end{pmatrix}
\begin{pmatrix}
\sin \theta \cos \phi \\
\sin \theta \sin \phi \\
\cos \theta
\end{pmatrix}
\end{equation}

Consequently, the direction cosines are:

\begin{equation}
\begin{pmatrix}
\frac{1}{\sqrt{3}} \sin \theta \cos \phi + \frac{1}{\sqrt{2}} \sin \theta \sin \phi + \frac{1}{\sqrt{6}} \cos \theta \\
\frac{1}{\sqrt{3}} \sin \theta \cos \phi - \frac{1}{\sqrt{2}} \sin \theta \sin \phi + \frac{1}{\sqrt{6}} \cos \theta \\
\frac{1}{\sqrt{3}} \sin \theta \cos \phi - \frac{2}{\sqrt{6}} \cos \theta
\end{pmatrix}
\end{equation}
\begin{align}
(2.51) \quad a_1 &= \frac{1}{\sqrt{3}} \sin \theta \cos \phi + \frac{1}{\sqrt{2}} \sin \theta \sin \phi + \frac{1}{\sqrt{6}} \cos \theta \\
& \quad a_2 = \frac{1}{\sqrt{3}} \sin \theta \cos \phi - \frac{1}{\sqrt{2}} \sin \theta \sin \phi + \frac{1}{\sqrt{6}} \cos \theta \\
& \quad a_3 = \frac{1}{\sqrt{3}} \sin \theta \cos \phi - \frac{2}{\sqrt{6}} \cos \theta
\end{align}

and

\begin{align}
(2.52) \quad a_1^2 a_2^2 + a_2^2 a_3^2 + a_1^2 a_3^2 &= \frac{1}{4} \sin^4 \theta + \frac{1}{3} \cos^4 \theta \\
& \quad - \frac{\sqrt{2}}{3} \sin^3 \theta \cos \theta \sin 3 \phi.
\end{align}

From Eq. 2.48,

\begin{align}
(2.53) \quad \varepsilon_{\text{cubic}} &= K_1 \left( \frac{1}{4} \sin^4 \theta + \frac{1}{3} \cos^4 \theta - \frac{\sqrt{2}}{3} \sin^3 \theta \cos \theta \sin 3 \phi \right).
\end{align}

Including this cubic anisotropy energy, the total energy density in the (111) film is

\begin{align}
(2.54) \quad \varepsilon_{[111]} &= \frac{A}{M^2} (\nabla M)^2 + K_u \sin^2 \theta + 2M^2 \sin^2 \theta \sin ^2 \phi - MH_p \sin \theta \cos \phi \\
& \quad + K_1 \left( \frac{1}{4} \sin^4 \theta + \frac{1}{3} \cos^4 \theta - \frac{\sqrt{2}}{3} \sin^3 \theta \cos \theta \sin 3 \phi \right).
\end{align}

B. The (100) Films

Since the crystal axes coincide with the rectangular coordinate system in this orientation, the direction cosines are shown in Fig. 2.4:
(2.55) \[ \alpha_1 = \sin \phi \cos \theta \]
\[ \alpha_2 = \sin \theta \sin \phi \]
\[ \alpha_3 = \cos \theta . \]

The cubic anisotropy energy is, from Eq. 2.48:

(2.56) \[ \varepsilon_{\text{cubic}} = K_1 \left[ \frac{1}{4} \sin^4 \theta \sin^2 \phi + \sin^2 \theta \cos^2 \theta \right] . \]

The total energy density is:

(2.57) \[ \varepsilon(100) = \frac{A}{M^2} (\nabla M)^2 + K_u \sin^2 \theta + 2\pi M^2 \sin^2 \theta \sin^2 \phi \]
\[ - MH_p \sin \theta \cos \phi + K_1 (\frac{1}{4} \sin^4 \theta \sin^2 \phi + \sin^2 \theta \cos^2 \theta) . \]

2.1.5 Effective Domain Wall Mass

A. General Theory

When the spins in the domain wall are excited, the spins will precess along the internal magnetic field. This precessing produces a demagnetizing field within the domain wall which will cause the spins to change their polar angle. This precessing of spins will result in the oscillatory motion of the domain wall. The precession of the spin within the moving domain wall produces a \( \phi = 0 \) component of the magnetization and the wall is no longer a simple Bloch wall. This distortion of the wall structure produces a localized demagnetization field normal to the wall and the domain energy is increased. This increase of domain wall energy
is interpreted as the increase of the domain wall kinetic energy. (12) Under this interpretation, the domain wall effective mass per unit area of the wall, \( m \), is defined as the change of the domain wall energy, \( \delta \sigma \). \( \delta \sigma \) is equal to the change of the "kinetic energy" of the domain wall when the change of the velocity of the wall, \( \delta v \), is infinitesimal. That is, the domain wall effective mass is defined from the relation

\[
(2.58) \quad \delta \sigma = (\hbar mv^2) = mv \delta v .
\]

In order to calculate the domain wall mass, the equation of motion of the magnetization in the domain wall is derived from the Landau-Lifshitz equation:

\[
(2.58) \quad \frac{d\mathbf{M}}{dt} = -\gamma \mathbf{H} .
\]

From Eq. 2.8 & 2.9, the equation of motion in the polar coordinate is:

\[
(2.8) \quad \sin \theta \dot{\phi} = \frac{Y \delta \mathbf{e}}{M \delta \theta} ,
\]

\[
(2.9) \quad \dot{\theta} = -\gamma \frac{1}{M \sin \theta} \frac{\delta \mathbf{e}}{\delta \phi} .
\]

When the domain wall motion is changed by only an infinitesimal change in its velocity, the domain wall is assumed to remain a one dimensional problem. That is, the spins still
remain uniform in the x and z direction, where the coordinate of the domain system is shown in Fig. 2.1. In addition, the \( \theta(y) \) dependence of the domain wall structure is also assumed to have the same structure as the static wall. The additional energy of motion develops as the spin is pulled out of the plane of the domain wall, i.e., \( \phi = \phi(y) \). This \( \phi(y) \) is approximated to first order of the infinitesimal velocity as \( \phi = \phi_s + vf(y) \), where \( \phi_s \) is the angle in the static wall structure and \( f(y) \) is the function of \( y \).

The domain wall energy density per unit area is:

\[
\sigma = \int_{-\infty}^{+\infty} dy \delta \varepsilon = \int_{-\infty}^{+\infty} dy \left[ \frac{\delta \varepsilon}{\delta \theta} \theta_s + v \frac{d}{dy} \phi_s + v f(y) \right] \\
= \int_{-\infty}^{+\infty} dy \left\{ \left[ \frac{\delta \varepsilon}{\delta \phi} \phi_s + v \left[ \frac{\partial}{\partial y} \left( \frac{\delta \varepsilon}{\delta \phi} \right) \theta_s + \phi_s + v f(y) \right] \right\} f v \delta v.
\]

By the relations \( \frac{\delta \varepsilon}{\delta \phi} \theta_s, \phi_s = 0 \) and \( \frac{\delta \varepsilon}{\delta \phi} = -\frac{M}{\gamma} \sin \theta \) in Eq. 2.9, Eq. 2.60 becomes

\[
\sigma = \int_{-\infty}^{+\infty} dy \left[ \frac{\partial}{\partial y} \left( -\dot{\theta} \frac{M}{\gamma} \sin \theta \right) \right]_{v=0} f v \delta v.
\]

From Eq. 2.58 the effective mass of the domain wall is:

\[
m = \int_{-\infty}^{+\infty} dy \left[ \frac{\partial}{\partial y} \left( -\dot{\theta} \frac{M}{\gamma} \sin \theta \right) \right]_{v=0} f(y).
\]

If the center of the domain wall is defined by \( q(t) \), i.e., \( \theta(y) = \theta(y-q(t)) \), then

\[
\dot{\theta} = -q \frac{\partial \theta}{\partial y} = -v \frac{\partial \theta}{\partial y}.
\]
Eq. 2.62 becomes:

\[
(2.64) \quad m = \frac{M}{\gamma} \int_{-\infty}^{+\infty} dy \frac{\partial \theta}{\partial y} \sin \theta f(y),
\]

or

\[
(2.65) \quad m = \frac{M}{\gamma} \int_{\theta_0}^{\pi - \theta_0} \theta \sin \theta f(\theta),
\]

where \( \theta_0 \) is the equilibrium angle of the magnetization far away from the center of the wall. It is necessary to evaluate the term \( f(\theta) \) before the effective mass of the moving domain wall can be determined.

B. Uniaxial Anisotropy Energy

In most of the magnetic garnet thin films used in this work, uniaxial anisotropy energy dominates the other crystalline anisotropy energy terms. If only the uniaxial anisotropy, exchange, Zeeman and demagnetizing energies are taken into consideration in the magnetic system, the total energy density of the domain wall is

\[
(2.66) \quad \epsilon = A(\nabla \theta)^2 + A \sin^2 \theta (\nabla \phi)^2 + K_u \sin^2 \theta + 2\pi M^2 \sin^2 \theta \sin^2 \phi
\]

\[- H_p \frac{M}{\sin \theta \cos \phi},
\]

where \( H_p \) is the applied in-plane field and the polar angles of the magnetization are shown in Fig. 2.1. For the one
dimensional wall, \( \psi \) and \( \psi \) can be replaced by \( \sin \theta \) and \( \sin \phi \) respectively. From the equation of motion of the domain wall, Eq. 2.8 & 2.9.

\[
(2.67) \quad \dot{\psi} = \frac{\gamma}{M} \frac{1}{\sin \theta} \left\{ 2A \frac{\partial}{\partial y} (\sin^2 \theta \frac{\partial \phi}{\partial y}) - 2\pi M^2 \sin^2 \theta \sin 2\phi - MH \sin \theta \sin \phi \right\}
\]

\[
(2.68) \quad \dot{\phi} = \frac{\gamma}{M_s \sin \theta} \left\{ -2A \frac{\partial^2 \theta}{\partial y^2} + K_u \sin 2\theta + A \sin 2\theta \left( \frac{\partial \phi}{\partial y} \right)^2 \\
+ 2\pi M^2 \sin 2\theta \sin^2 \phi - MH \cos \theta \cos \right\}.
\]

As discussed before, when the wall is moving, the \( \psi(y) \) dependence of the wall structure is assumed to be the same as that of the static wall, and only the component \( m_y \), whose magnitude is proportional to \( \psi = \psi(y) \), is modified. Using the following relationships

\[
(2.69) \quad \dot{\phi} = -\frac{3\theta}{\partial y} = -\psi \frac{\partial \psi}{\partial y}, \quad \frac{3\psi}{\partial y} = \frac{1}{A} (\sin \theta - \mu)
\]

the equations of motion from Eq. 2.07 are linearized to become

\[
(2.70) \quad (H_p + 4\pi M \sin \theta) \psi - \frac{2A}{M_s} \frac{1}{\sin \theta} \frac{\partial}{\partial y} (\sin^2 \theta \frac{\partial \phi}{\partial y}) = \frac{1}{\gamma} \frac{\partial}{\partial y} \psi,
\]

or

\[
(2.71) \quad Q^{-1}(Q \psi + \sin \theta) \frac{\sin \theta - \mu}{\sin \theta} \frac{\partial}{\partial \theta} \sin^2 \theta (\sin \theta - \mu) \frac{d\theta}{d\theta} = \zeta \left( \sin \theta - \mu \right),
\]

where \( \zeta = 1/(\gamma H_u \Delta) \) and \( Q = K_u/(2\pi M^2) \).
Professor Morkowski proposed that the variation of the function \( f(y) \) is very small and the second term can be neglected. The effective mass from this assumption is found to be in reasonable agreement with the exact solution. The function \( f(\theta) \) can be calculated numerically from Eq. 2.71. The program and the procedure for this numerical calculation is in Appendix III. Using the boundary condition,

\[
\left. \frac{\partial f}{\partial \theta} \right|_{\theta = \frac{\pi}{2}} = 0 \quad \text{and} \quad f(\theta) \bigg|_{\theta = \theta_0} = 0,
\]

the solution to \( f(y) \) is shown in Fig. 2.5. From \( f(\theta) \) the effective mass is evaluated using Eq. 2.65 and it is shown in Fig. 2.6 as a function of the applied in-plane field.

C. Cubic Anisotropy Energy

For the (111) oriented films, the total energy is given in Eq. 2.54. Using the equations of motion, Eq. 2.8 & 2.9, the equations of motion become:

\[
\dot{\theta} = \frac{\gamma}{M \sin \theta} \left[ 2A \frac{\partial}{\partial y} (\sin^2 \theta \frac{\partial \phi}{\partial y}) - 2\pi M^2 \sin^2 \theta \sin 2\phi - MH_p \sin \theta \sin \phi + \sqrt{2} K_1 \sin^3 \theta \cos \theta \cos^3 (\phi + \delta) \right],
\]
FIGURE 2.5  The solutions obtained for $f(\theta)$ obtained for various values $\mu$. The curves (---) are obtained by neglecting the exchange term in Eq. 2.71 while the curves (____) are obtained from numerical solutions of Eq. 2.71.
FIGURE 2.6 Reduced mass vs. reduced in-plane field. Curve 1 is the mass calculated from Eq. 2.71, Curve 2 is from the same equation but neglecting the exchange term.
\[
\sin \theta = \frac{1}{M} \left\{ A \sin 2\theta \left( \frac{\partial^2 \phi}{\partial y^2} \right)^2 + K_u \sin 2\theta + 2\pi M^2 \sin 2\theta \sin^2 \phi \\
- MH_p \cos \theta \cos \phi - 2A \frac{\partial^2 \phi}{\partial y^2} + K_1 \left[ \sin^3 \theta \cos \theta - \frac{4}{3} \sin \theta \cos^3 \theta \\
- \frac{\sqrt{2}}{3} \sin \theta \sin 3\theta \sin 3(\phi + \beta) \right]\right\},
\]

where the angle \( \beta \) is the angle between the y-axis and [011] as shown in Fig. 2.4.

The static wall structure is evaluated from the conditions \( \dot{\phi} = \ddot{\phi} = 0 \). If \( \frac{\partial^2 \phi}{\partial y^2} \), and \( \frac{\partial \theta}{\partial y} \) are neglected, the static wall structure is determined by

\[
(2.75) \quad \dot{\phi} = 0 = 2\pi M^2 \sin \theta \sin 2\phi + H_p M \sin \phi \\
- \sqrt{2} K_1 \sin^2 \theta \cos \theta \cos 3(\phi + \beta)
\]

and

\[
(2.76) \quad \ddot{\phi} = 0 = K_u \sin 2\theta + 2\pi M \sin 2\theta \sin^2 \phi - MH_p \cos \theta \cos \phi - 2A \frac{\partial^2 \phi}{\partial y^2} \\
\quad + K_1 \left[ \sin \theta \cos \theta (\sin^2 \theta - \frac{4}{3} \cos^2 \theta) - \frac{\sqrt{2}}{3} \sin \theta \sin 3\theta \sin 3(\phi + \beta) \right].
\]

From Eq. 2.75 and assuming \( \phi \) is very small, \( \frac{\partial \theta}{\partial y} \) is determined from the expression

\[
(2.77) \quad \Delta^2 \left( \frac{\partial \theta}{\partial y} \right)^2 = (\sin^2 \theta - \sin^2 \theta_o) - 2 \mu (\sin \theta - \sin \theta_o) + \frac{K_1}{K_u} F(\theta),
\]
where \( F(\theta) = \frac{1}{4}(\sin^4\theta - \sin^4\theta_0) + \frac{1}{3}(\cos^4\theta - \cos^4\theta_0) \)
\[ + \frac{\sqrt{2}}{3}\sin3\theta(\sin^3\theta_0\cos\theta_0 - \sin^3\theta\cos\theta). \]

The value of \( \theta_0 \) is determined by minimizing the domain energy density, \( \varepsilon_D \), as in Eq. 2.43, and is solved numerically from the condition \( \delta_D/\delta\theta = 0 \), i.e.,

\[
(2.78) \quad k_u\sin2\theta_0 - 2\pi M^2\sin2\theta_0 - MH_p\cos\theta_0 + \frac{k_1}{k_u}\left[\sin^3\theta_0\cos\theta_0 - \frac{4}{3}\cos^3\theta_0\sin\theta + \sqrt{2}(3\cos^2\theta_0 + \sin^2\theta_0)\sin^2\theta_0\sin3\theta\right] = 0
\]

Using \( \dot{\theta} = -\frac{\partial \varepsilon}{\partial \theta} \) and \( \phi = \phi_s + \nu \), Eq. 2.73 becomes

\[
(2.79) \quad -\nu M \frac{\partial \theta}{\partial y} = \frac{2A}{\sin\theta} \left( \frac{3}{\partial y} \sin^2\theta \frac{\partial \phi}{\partial y} \right) - 2\pi M^2\sin\theta\cos2\phi_s x 2\nu
\]
\[-MH_p\cos\phi_s \nu - K_1 \sqrt{2} \sin^2\theta\cos\theta\sin3(\phi_s + \theta) + 3\nu
\]
\[ - 2M^2\sin\theta\sin2\phi_s - MH_p\sin\phi_s
\]
\[ + K_1 \sqrt{2} \sin^2\theta\cos\theta\cos3(\phi_s + \theta) + \frac{2A}{\sin\theta} \left( \frac{3}{\partial y} \sin^2\theta \frac{\partial \phi}{\partial y} \right) \phi_s.
\]

Comparing Eq. 2.79 and Eq. 2.75 and assuming \( \phi_s \to 0 \),

\[
(2.80) \quad -\frac{M}{\gamma} \frac{\partial \phi}{\partial y} = \frac{2A}{\sin\theta} \frac{3}{\partial y} (\sin^2\theta \frac{\partial \phi}{\partial y}) - \left[ 4\pi M^2\sin\theta + MH_p
\]
\[ + 3\sqrt{2} K_1 \sin^2\theta\cos\theta\sin3\theta \right].
\]
Since the exchange term has a small affect on the effective mass \(^{(13)}\), the expression for \(f\) can be simplified by neglecting the exchange term

\[
f(\theta) = \frac{M}{K_u y} \left[ Q u + \sin \theta + \frac{3\sqrt{2}}{2} \frac{K_1}{K_u} \sin^2 \theta \cos \theta \sin 3\theta \right]^{-1}.
\]

From Eq. 2.65 and Eq. 2.81, the domain wall effective mass is calculated. The program is given in Appendix IV.

For the (100) films, the calculation is similar to that of the (111) films. Using the total energy density from Eq. 2.57 and from the calculation of the static domain wall structure,

\[
\frac{\partial \phi}{\partial y} = \frac{1}{A} \left[ (\sin \theta - \sin \theta_0)^2 + \frac{K_1}{K_u} F(\theta) \right]^{\frac{1}{2}}
\]

where \(F(\theta)\) is calculated to the lowest order of \(K_1/K_u\),

\[
F(\theta) = \frac{1}{4} \sin^2 2\theta (\sin^4 \theta - \sin^4 \theta_0) + \sin^2 \theta \cos^2 \theta - \sin^2 \theta_0 \cos^2 \theta_0.
\]

Similarly, \(f(\theta)\) is determined from the relation

\[
\left[ Q u + \sin \theta + \frac{K_1}{K_u} \cos 4\theta \sin^3 \theta \right] f(\theta) = Q A^2 \frac{1}{\sin \theta} \frac{\partial}{\partial y} \left( \sin^2 \theta \frac{\partial f(\theta)}{\partial y} \right)
\]

\[
= \frac{1}{4\pi M y} \frac{\partial \phi}{\partial y}.
\]
When the exchange term is neglected on the same basis as before,

\[ f = \frac{1}{4\pi \gamma M} \frac{\partial^2}{\partial y^2} \left[ Q u + \sin \theta + \frac{K_1}{K_u} \cos 4\theta \sin^3 \theta \right]^{-1}. \]

From \( f(\theta) \) and \( \theta_0 \), the domain wall effective mass is calculated as a function of the applied in-plane field. The calculated effective frequency with cubic anisotropy energy is shown in Fig. 2.7.

2.2 Restoring Force Constant

2.2.1 Introduction

In the magnetic garnet films, the magnetic domain structure can be arranged in the parallel strip pattern as shown in Fig. 2.1 by the external applied field. When a very small excitation is applied by a rf electromagnetic field perpendicular to the film, the domain wall will be displaced from its static equilibrium position. Because the equilibrium state of the domain structure is determined by minimizing the total free energy, the small motion excited by the rf field is a small motion around the minimum of a potential energy. Consequently for small oscillations this motion can be treated as a harmonic oscillation. In
FIGURE 2.7 The reduced frequency with the cubic anisotropy taken into account. The parameters used in the calculation are given for film number 1 in Table 3.1. Curve 1 & 2 are for the stripes along [011] and [110] respectively.
this small oscillation, the restoring force constant is determined by the second order derivative of the energy density with respect to the spacial displacement.

In order to calculate the equilibrium position of the domain structure, as well as the restoring force constant, the energy contributed from the demagnetized state must be determined. The domain structure considered in this work will be limited to the stripe patterns. In this domain pattern, as shown in Fig. 2.1, the magnetization is assumed to be uniform across the thickness of the film. The domain wall is assumed to be one dimensional Bloch wall and the wall thickness in the garnet films is assumed to be much smaller than the domain periods, so that the domain wall contributes an energy density, \( \sigma_w \), per unit wall area.\(^{(2)}\)

2.2.2 Energy Contribution from the Parallel Domain Pattern

In a unit cell of the domain pattern as shown in Fig. 2.1, the following energy contributions are taken into account:

(1) The domain wall energy,

(2) The Zeeman energy, and
The demagnetization energy arising from the domain pattern.

The domain wall energy in the unit cell is given as:

\[
\frac{2\sigma_w L t}{L(L_1+L_2)} = \frac{2\sigma_w t}{L_1+L_2} = \frac{\xi\sigma_w}{\pi},
\]

where \(\sigma_w\) is the domain wall energy per unit area\(^{(2)}\), the \(L_1, L_2\) are defined in Fig. 2.1, and \(\xi\) is defined below.

The Zeeman energy per unit area is given by

\[
E_z = \frac{tL(\hat{L}_1 \cdot \hat{M} - \hat{L}_2 \cdot \hat{M})}{L(L_1+L_2)} = t \frac{L_1-L_2}{L_1+L_2} \hat{M} \cdot \hat{H}
\]

\[
= t \hat{M} \cdot \hat{H} \cdot n,
\]

where \(n = \frac{L_1-L_2}{L_1+L_2}\) and \(\xi = \frac{2\pi t}{L_1+L_2}\).

The most complex contribution is the demagnetization energy arising from the domain pattern. The first calculation scheme was described by Kittel.\(^{(14)}\) In the following calculation the method used is adopted from Kooy and Enz.\(^{(15)}\)
The domain pattern is assumed to be a parallel domain array with the domain wall lying in the z-x plane as shown in Fig. 2.1. In the calculation the reorientation of the magnetization due to the demagnetizing fields is neglected and only the uniform surface magnetic charge distribution is considered. Because of the parallel array domain pattern, the surface magnetic charge is a square wave function with amplitude of \( \pm 4\pi M_z \) and this function can be represented by:

\[
\rho_s = \frac{M}{L_1 + L_2} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi L_1}{L_1 + L_2} \cos \frac{2n\pi y}{L_1 + L_2}
\]

where \( y \) is the coordinate in the y-direction.

According to the Maxwell’s equations, the magnetic field and magnetization satisfy the following equations:

\[
\begin{align*}
\nabla \times \mathbf{H} & = 0 \\
\nabla \cdot \mathbf{H} & = -4\pi \nabla \cdot \mathbf{M}
\end{align*}
\]

So, the magnetic field can be represented by a magnetic potential \( \mathbf{x} \).

\[
\mathbf{H} = -\nabla \mathbf{x}
\]

The potential is chosen as:

\[
\begin{align*}
\mathbf{x}_{in} & = B_0 z + \sum B_n \sin \frac{n\pi L}{L_1 + L_2} \cos \frac{2\pi y}{L_1 + L_2} \sinh \frac{2\pi nz}{L_1 + L_2} \\
\mathbf{x}_{out} & = A_0 + \sum A_n \sin \frac{n\pi L}{L_1 + L_2} \cos \frac{2\pi y}{L_1 + L_2} \exp \left( -\frac{2\pi nz}{L_1 + L_2} \right)
\end{align*}
\]
The boundary conditions are:

(A) \( x_{\text{in}} \) and \( x_{\text{out}} \) are continuous at the boundary.

(B) On the surface the \( z \)-component of the magnetic field is discontinuous by:

\[
\left( \frac{\partial x_{\text{out}}}{\partial z} - \frac{\partial x_{\text{in}}}{\partial z} \right)_{\text{surface}} = -4\pi \rho_s ,
\]

where \( \rho_s \) is the magnetic surface charge as represented in Eq. 2.88. From the magnetic potential, Eq. 2.90, 2.92 and the boundary conditions the coefficient of \( x \) can be determined. The magnetic field \( H \) is then obtained from Eq. 2.90.

The demagnetization energy is given from the magnetic field:

\[
E_D = \frac{1}{8\pi(L_1+L_2)} \int_{\text{unit cell}} H^2 dV ,
\]

or

\[
E_D = 2\pi t M^2 n^2 + \frac{16t M^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^3} \sin^2 \frac{n\pi}{2}(1+n) (1-e^{-n\xi}) .
\]

In the case of an applied in-plane field, the magnetic moment is canted so that the magnetization in the Zeeman energy and demagnetization energy should be replaced by \( M_z \). If the system remains uniform, as assumed in Eq. 2.88, the total energy per unit cell, relative to the uniformly magnetized state of the film is
where $M \cos \theta$ is the $z$-component of the magnetization. Because the domain wall is assumed to be very thin, the angle $\theta$ can be considered as the canted angle in a uniform film with an applied in-plane field. This angle is determined by minimizing the domain energy and is given by:

$$\sin \theta = \frac{H_D}{H_u - 4\pi M} \quad .$$

2.2.3 Domain Structure and the Collapse Field

From the total energy density, Eq. 2.96, the domain period can be evaluated. The domain wall energy density in the applied in-plane field is

$$\sigma_w = \sigma_0 \left( 2 \cos \theta_0 - (\pi - 2\theta_0) \sin \theta_0 \right) \quad ,$$

where $\sigma_0 = \sqrt{\kappa T}$, $\theta_0 = \sin^{-1} \left( \frac{H_D}{H_u - 4\pi M} \right) \quad .$

The conditions that the static domain structure must satisfy are:
\[ \frac{\partial^2 E_{\text{tot}}}{\partial \eta^2} > 0 \]

From Eq. 2.98, and Eq. 2.96, these conditions yield the relations

\[ \frac{\partial E_{\text{tot}}}{\partial \eta} = 0 = -tM_H c\theta_H \cos \theta_o + 4\pi M^2 \cos^2 \theta_o \eta \]

\[ + \frac{16tM^2 \cos^2 \theta_o}{\pi \xi} \sum_{n=1}^{\infty} \frac{n \sin n \pi (1+\eta)}{2n^2} (1 - e^{-n \xi}) \]

\[ \frac{\partial E_{\text{tot}}}{\partial \xi} = 0 = \frac{\xi \omega}{\pi} + \frac{16tM^2 \cos^2 \theta_o}{\xi^2 \pi} \sum_{n=1}^{\infty} \frac{\sin^2 \frac{n \pi}{\xi}}{n^3} ((n \xi + 1)e^{-n \xi} - 1) \]

where \( \theta_H \) is the angle of applied field with respect to the z-axis. Eq. 2.99 and Eq. 2.100 are solved simultaneously by the computer, the program to find the \( \eta \) and \( \xi \) for each applied field \( H \) which is given in Appendix V. From the solution, the domain width, \( L_1 \) and \( L_2 \), are determined by both the amplitude and the angle \( \theta_H \) of the applied external field. For each angle \( \theta_H \), the period of the domains is a function of the amplitude of \( H \). For example, for the angle equal to 0 degrees, i.e., the applied field is normal to the plane of the film surface, the dependence of \( L_1 + L_2 \) on \( H \) is shown in Figure 2.8. The field where \( L_1 + L_2 = \cdots \)
FIGURE 2.8 The domain period as a function of the applied field perpendicular to the film surface. The dash line indicates the collapse field, (taken from reference 15) $4\pi M = 4335 \text{ G}$, $\sigma_w = 2.8 \text{ erg/cm}^2$. 
is called the collapse field. By this method, the collapse field can be calculated as a function of the angle $\theta_H$. This is shown in Fig. 2.9.

2.2.4 Restoring Force Constant of the Strip Domain Pattern

When the domain wall system is excited by an rf-field, the spins will precess around the internal field. This precession will produce a component of magnetization perpendicular to the domain wall. This component of the magnetization will generate a demagnetizing field which in turn changes the polar angle $\theta$ of the magnetization. In this process, the angles of the spin are changed and the center of the domain wall is moving as shown in Figure 2.10. In the case that the excitation is infinitesimal, the system is considered as perturbed from its equilibrium state. For any physical system which is infinitesimally perturbed from its equilibrium state can be approximated dynamically as harmonic motion. The restoring force constant of the system is the second derivative of the total potential energy with respect to the space coordinate.

In the magnetic stripe domain system, the space coordinates are $L_1$ and $L_2$. When the bias field is zero, i.e., $L_1 - L_2 = 0$, the restoring force constant density for the small domain wall motion is given by:

\[
(2.101) \quad k = \frac{d^2E_{\text{tot}}}{dL_1^2}.
\]

In order to calculate the restoring force constant in the
FIGURE 2.9 The collapse field as a function $\theta_H$, which is the angle between the applied field and the easy axis of the film.
FIGURE 2.10 The Curve 1 shows orientation of the polar angle of the spins in a static domain wall. When an external field is applied along the positive z-axis, the spins will reorient producing a domain wall defined by Curve 2.
variables \( m \) and \( \xi \), the change of coordinate from \( L_1 \) to \( \eta \) is necessary.

\[
(2.102) \quad \eta = \frac{L_1 - L_2}{L_1 + L_2} = 2\frac{L_1}{L_1 + L_2} - 1
\]

For normal rf-field excitation, \( L_1 + L_2 = 0 \), then

\[
(2.103) \quad \frac{d}{dL_1} = \frac{2}{L_1 + L_2} \frac{d}{d\eta}
\]

Consequently the force constant for \( \eta = 0 \), i.e., no bias field, is given by:

\[
(2.104) \quad k = \frac{d^2E_{\text{tot}}}{dL_1^2} = \frac{4}{(L_1 + L_2)^2} \frac{L_1 + L_2}{2t} E_{\eta\eta} = \frac{2E_{\eta\eta}}{(L_1 + L_2)t}
\]

From Eq. 2.96

\[
(2.105) \quad E_{\eta\eta} = 4\pi M_z^2 t \left( 1 + \frac{2}{\xi} \sum \frac{(-1)^n}{n} \left( 1 - e^{-\xi} \right) \right) = 4\pi M_z^2 t \left[ 1 - \frac{2}{\xi} \left( \log \frac{2}{1 + e^{-\xi}} \right) \right]
\]

and

\[
(2.106) \quad k = \frac{8\pi M_z^2}{L_1 + L_2} \left[ 1 - \frac{2}{\xi} \left( \log \frac{2}{1 + e^{-\xi}} \right) \right]
\]

From this force constant, shown in Fig. 2.11, and the domain wall effective mass, the normal mode frequency of the domain wall motion can be evaluated by:

\[
(2.108) \quad \omega = \sqrt{\frac{k}{m}}
\]
FIGURE 2.11 The restoring force constant as a function of the applying field, $H_p$. 

Reduced Force Constant

$H_p/H_u$
This frequency is in the MHz range for the magnetic garnets and can be detected experimentally.

The typical value of the effective mass in the material used is about $3.0 \times 10^{-11}$ g/cm$^2$ and the frequency is about 17 MHz, for the applied field equal to zero.

2.3 Magnetoelastic Interaction and Elastic Waves in the Film/Substrate System

2.3.1 Introduction

Magnetoelastic interactions in the ferromagnetic materials have been known since 1958. In the interaction between spin waves and elastic waves, the primary interest is in the range where both the spin waves and the elastic waves have the same wave vector and same frequency. This effect has been used to excite the elastic waves in the microwave frequency range. Later it was found that ferromagnetic resonance (FMR) was a more efficient method of exciting elastic waves. Through this magnetoelastic interaction in the elastically isotropic media, the effective elastic constant $c_{\text{eff}}$ is given by:

\[
(2.109) \quad c_{\text{eff}} = c_{44} + \frac{\gamma \beta_2^2}{M(f - f_k)}
\]

where $c_{44}$ is the elastic constant, $f_k$ is the spin wave frequency with wave vector $k$, $f$ is the frequency of the rf-field, $\gamma$ is the gyromagnetic ratio and $\beta_2$ is the second magnetoelastic interaction constant. These interactions have
been observed in the FMR of magnetic garnet films\(^{(7)}\) and in the weak ferromagnet FeBO\(_3\)\(^{(21)}\).

This magnetoelastic interaction also modifies the different modes of elastic waves in the ferromagnetic materials. The Rayleigh wave and the Love waves in magnetic films have been studied by R.E. Camley\(^{(22)}\).

All the phenomena described above are in the microwave frequency range and excited by the spin waves in the uniformly magnetized materials. In the experiments of Basu, et al.\(^{(23)}\) the magnetoelastic interaction was observed in the demagnetized state in the YIG films. In this work the elastic waves were excited by the magnetoelastic interaction in the demagnetized films through domain wall resonances. The frequency of these elastic modes are in the 100 MHz range.

In this work, the domain wall resonance will excite elastic waves which have also been observed\(^{(24)}\). The elastic waves are confirmed to be transverse waves. Longitudinal waves are weakly excited by DWR. This property is different from the FMR excited elastic waves in which both longitudinal and transverse waves can be excited\(^{(7)}\). The most interesting part of this DWR excited elastic waves is that there is a fine structure on the high frequency side of each elastic resonance spectra. These fine structure modes have a frequency splitting in the 50 kHz range and in the shorted
slot line rf-structure the spectra shows a linear dispersion. The complexity of the spectra is resolved by a pair of parallel strip line rf-structure as a multidimensional elastic waves propagating in the film/substrate system. Using a pulse experiment in the shorted slot line rf-structure, there is an interesting delay time equal to the reciprocal of the frequency splitting of the fine structure. The frequency splitting of the fine structure can be manipulated by moving the sample position with respect to the rf-structure. The details of this fine structure will be presented in sections 2.3.3 and 3.4.

2.3.2 Equation of Motion for Magnetoelastic Interaction.

In thin magnetic films having a thickness of about 5 μm, the coupling between magnetic and elastic waves can be given by (16):

\[ E_{me} = b_1 (a_x^2 e_{xx} + a_y^2 e_{yy} + a_z^2 e_{zz}) + 2b_2 (a_x a_y e_{xy} + a_y a_z e_{yz} + a_z a_x e_{xz}), \]

where \( a_i \) are the direction cosine of the magnetization \( \mathbf{M} \) and the \( e_{ij} \) are the elements of the strain tensor defined by

\[ e_{ij} = \frac{1}{2} \left( \frac{\partial U}{\partial x_i} + \frac{\partial U}{\partial y_j} \right), \]
where $\vec{U}$ is the displacement vector of the atom.

Because the GG3 is an elastically isotropic material, the energy contribution from the pure elastic interaction is given by (25):

(2.112)

$$E_e = \frac{1}{2} C_{11} (e_{xx}^2 + e_{yy}^2 + e_{zz}^2) + \frac{1}{2} C_{44} (e_{xy}^2 + e_{xz}^2 + e_{yz}^2) + C_{12} (e_{xx} e_{yy} + e_{yy} e_{zz} + e_{zz} e_{xx}),$$

where $C_{ij}$ are the elastic constants and $C_{12} = C_{11} - 2C_{44}$ for the isotropic media. So, the total energy density of the magnetic garnet film including the magnetoelastic effects is:

(2.113)

$$E_{tot} = K_u a_z^2 + A \left[ (\nabla a_x)^2 + (\nabla a_y)^2 + (\nabla a_z)^2 \right] - a \cdot \vec{H} + b_1 (a_x^2 e_{xx} + a_y^2 e_{yy} + a_z^2 e_{zz})$$

$$+ 2b_2 (a_x a_y e_{xx} + a_x a_z e_{xz} + a_y a_z e_{yz})$$

$$+ \frac{1}{2} C_{11} (e_{xx}^2 + e_{yy}^2 + e_{zz}^2) + \frac{1}{2} C_{44} (e_{xy}^2 + e_{xz}^2 + e_{yz}^2)$$

$$+ C_{12} (e_{xx} e_{yy} + e_{xx} e_{zz} + e_{yy} e_{zz}).$$
The equations of motion are:

\[
\frac{dM}{dt} = \gamma M \times \frac{\delta E_{\text{tot}}}{\delta M},
\]

for magnetic waves and

\[
\frac{\alpha^2 U_k}{\alpha t^2} = \frac{1}{\rho} \frac{\partial}{\partial x'} \frac{\partial E_{\text{tot}}}{\partial x_k},
\]

for elastic waves, where \(\rho\) is the mass density of the media.

From the total energy density, Eq. 2.115, and assuming the domain wall structure is uniform in the \(x-z\) plane, the equations of motion for the elastic waves in the film are:

\[
\rho \frac{\alpha^2 U_x}{\alpha t^2} - C_{11} \frac{\partial}{\partial x'} (\nabla \cdot \mathbf{U}) - C_{44} \left[ \frac{\alpha^2 U_x}{\alpha y^2} + \frac{\alpha^2 U_x}{\alpha z^2} - \frac{\alpha^2 U_y}{\alpha x \alpha y} - \frac{\alpha^2 U_z}{\alpha x \alpha z} \right] = 2b_1 \alpha_x \frac{\partial^2 U_x}{\partial x^2} + 2b_2 \left[ \alpha_x \left( \frac{\alpha_x}{\alpha y} + \frac{\alpha_z}{\alpha z} \right) + \alpha_y \frac{\partial^2 U_x}{\partial y^2} + \alpha_z \frac{\partial^2 U_x}{\partial z^2} \right],
\]

\[
\rho \frac{\alpha^2 U_y}{\alpha t^2} - C_{11} \frac{\partial}{\partial y'} (\nabla \cdot \mathbf{U}) - C_{44} \left[ \frac{\alpha^2 U_y}{\alpha z^2} + \frac{\alpha^2 U_y}{\alpha x^2} - \frac{\alpha^2 U_x}{\alpha x \alpha y} - \frac{\alpha^2 U_z}{\alpha y \alpha z} \right] = 2b_1 \alpha_y \frac{\partial^2 U_y}{\partial y^2} + 2b_2 \left[ \alpha_y \left( \frac{\alpha_x}{\alpha x} + \frac{\alpha_z}{\alpha z} \right) + \alpha_x \frac{\partial^2 U_y}{\partial x^2} + \alpha_z \frac{\partial^2 U_y}{\partial z^2} \right],
\]

\[
\rho \frac{\alpha^2 U_z}{\alpha t^2} - C_{11} \frac{\partial}{\partial z'} (\nabla \cdot \mathbf{U}) - C_{44} \left[ \frac{\alpha^2 U_z}{\alpha x^2} + \frac{\alpha^2 U_z}{\alpha y^2} - \frac{\alpha^2 U_x}{\alpha x \alpha z} - \frac{\alpha^2 U_y}{\alpha y \alpha z} \right] = 2b_1 \alpha_z \frac{\partial^2 U_z}{\partial z^2} + 2b_2 \left[ \alpha_z \left( \frac{\alpha_x}{\alpha x} + \frac{\alpha_y}{\alpha y} \right) + \alpha_x \frac{\partial^2 U_z}{\partial x^2} + \alpha_y \frac{\partial^2 U_z}{\partial y^2} \right].
\]
When there is no domain wall motion, or no magnetic excitations, Eq. 2.116 is a homogeneous differential equation for the elastic waves. When the domain wall is moving, the right hand side of the equation, Eq. 2.116, will not be zero. Consequently, the elastic waves are excited according to the driving force generated by the magnetic excitations. When the domain wall structure is assumed to be a simple Bloch wall and the wall structure does not change when the wall is moving, then the essential driving force is

\[ F = \frac{\partial a_y}{\partial y} \]  

(2.117)

2.3.3 The Pure 2-Dimensional Elastic Waves

From the previous section, the elastic waves are generated by DWR which creates the driving force $\frac{\partial a_y}{\partial y}$. In magnetic films on top of GGG substrates the ratio of film to substrate thickness is about 5\(\mu\)m to 500\(\mu\)m. When the magnetoelastic wave propagates into the substrate, the essential component is the pure elastic wave inside the substrate and the thin film region becomes a perturbation source in generating the elastic waves. Therefore the elastic wave in this film/substrate system is governed by the normal mode of the geometry of the GGG substrate. In general, the equation of motion of the pure elastic wave without any body force is:
Under the conventional notation, the elastic constants are expressed in $xx=1$, $yy=2$, $zz=3$, $yz,zy=4$, $xz,xz=5$ and $xy,yx=6$. For the isotropic media, $C_{xxxx} = C_{yyyy} = C_{zzzz}$, i.e.,

$$C_{11} = C_{22} = C_{33}, \text{ also, } C_{12} = C_{13} = C_{23} \text{ and } C_{44} = C_{55} = C_{66} = \frac{1}{2}(C_{11} - C_{12}). \text{ All the other } C_{ij} = 0 \text{ due to the cubic symmetry}^{(24)}.$$

Using these notations, the equations of motion for the pure elastic waves in GGG can be written as:

$$(2.118) \quad \rho \frac{\partial^2 \mathbf{U}}{\partial t^2} - C_{ijkl} \frac{\partial^2 \mathbf{U}}{\partial x_i \partial x_j} = 0$$

For the plane wave solutions,

$$\mathbf{\ddot{U}} = \begin{pmatrix} U_x \\ U_y \\ U_z \end{pmatrix} e^{i(k \cdot x - \omega t)}, \quad (2.120)$$
and Eq. 2.119 can be expressed as:

\[
(2.121) \begin{bmatrix}
C_{11}k_1 + C_{44}(k_2^2 + k_3^2) & (C_{11} - C_{44})k_1k_2 & (C_{11} - C_{44})k_1k_3 \\
(C_{11} - C_{44})k_1k_2 & C_{11}k_2^2 + C_{44}(k_1^2 + k_3^2) & (C_{11} - C_{44})k_2k_3 \\
(C_{11} - C_{44})k_1k_3 & (C_{11} - C_{44})k_2k_3 & C_{11}k_3^2 + C_{44}(k_1^2 + k_3^2)
\end{bmatrix}
\]

\[
\begin{pmatrix}
U_x \\
U_y \\
U_z
\end{pmatrix} = \rho \omega^2 \begin{pmatrix}
U_x \\
U_y \\
U_z
\end{pmatrix}
\]

For a one dimensional wave, i.e., \( k_2 = k_1 = 0 \), the dispersive relation is:

\[
(2.122) \quad \omega = v_t k_3, \quad \text{where} \quad v_t^2 = C_{44}/\rho.
\]
For a two dimensional wave, or \( k_2 = 0 \), Eq. 2.121 becomes

\[
(2.123) \begin{bmatrix}
C_{11} k_1^2 + C_{44} k_3^2 - \rho \omega^2 & 0 & k_j k_3 (C_{11} - C_{44}) \\
0 & C_{44} (k_1^2 + k_3^2) - \rho \omega^2 & 0 \\
(C_{11} - C_{44}) k_1 k_3 & 0 & C_{44} k_1^2 + C_{11} k_3^2 - \rho \omega^2
\end{bmatrix}
\begin{bmatrix}
U_x \\
U_y \\
U_z
\end{bmatrix} = 0
\]

The dispersion relations from Eq. 2.123 are

\[
(2.124) \quad C_{44} (k_1^2 + k_3^2) - \rho \omega^2 = 0
\]

for transverse waves, and

\[
(2.125) \quad C_{11} (k_1^2 + k_3^2) - \rho \omega^2 = 0
\]

for longitudinal waves.

For each frequency, \( \omega \), and propagation constant \( k_1 \),

\[
(2.126) \quad k_3^2 = \frac{\omega^2}{v_T^2} - k_1^2 \quad \text{or} \quad k_3^2 = \frac{\omega^2}{v_L^2} - k_1^2
\]

where \( v_L^2 = \frac{C_{11}}{\rho} \) and \( v_T^2 = \frac{C_{44}}{\rho} \) are the
longitudinal and transverse velocity of the elastic waves respectively.

The relationships between \( U_x, U_y, U_z \) can be solved by putting the dispersion relations into Eq. 2.123. From Eq. 2.123, \( U_y \) is not coupled to \( U_x \) and \( U_z \). So, \( U_x = U_z = 0 \) and the transversal wave polarized in the \( y \) direction is one solution.

The other solution is \( U_y = 0 \) and

\[
\begin{bmatrix}
-k_3 \\
\frac{k_3}{k_1}
\end{bmatrix}
\] or

\[
\begin{bmatrix}
0 \\
1
\end{bmatrix}
\]

(2.127)

\[
\begin{bmatrix}
\frac{k_3}{k_1}
\end{bmatrix}
\]

for transversal waves, where \( A \) is an arbitrary constant, or

\[
\begin{bmatrix}
1 \\
0 \frac{k_3'}{k_1}
\end{bmatrix}
\] or

\[
\begin{bmatrix}
1 \\
0 \\
\frac{-k_3'}{k_1}
\end{bmatrix}
\]

(2.128)

for longitudinal waves, where

\[
k_3 = \left[ \frac{\omega^2}{v_t^2} - k_1^2 \right]^{1/2}
\]

and

\[
k_3' = \left[ \frac{\omega^2}{v_L^2} - k_1^2 \right]^{1/2}
\]
3.1 Experimental System

The domain wall resonance (DWR) is detected by an rf spectrometer in the 10 MHz to 4 GHz range. The spectrometer is controlled by an HP 9825A desktop computer. A block diagram of the system is shown in Fig. 3.1. The rf-signal is generated by a sweeper which is programmable directly by HP-IB. The sweep range, the sweep mode, the power level and the sweep rate are all controlled by the HP 9825A. The signal generated by the sweeper is transmitted through the coaxial cable to the rf-structure where the rf-power will couple to the domain wall motion in the sample. The output power transmitted across the rf-structure is about 70 db below the input power. The output signal is amplified by a low noise high gain broad band rf-amplifier. When the rf-power level is amplified to about-20 dbm, it is converted to a D.C. voltage by a diode detector. The frequency response of the diode is shown in Fig. 3.2. The D.C. voltage signal is read by a DVM (digital volt meter), which will in turn transmit the D.C. signal into the memory of the computer. The program for the system was written for the HP 8350A sweeper as a signal source.
FIGURE 3.1 The block diagram of the transmission CW mode experimental system, (taken from reference 2).
FIGURE 3.2 The frequency response of the diode detector.
The amplitude of the applied D.C. magnetic field is monitored by a Bell 640 gaussmeter. This field is compared with the preset field amplitude by the computer which determines the current of the power supply in order to set the programmed D.C. applied in-plane magnetic field. In this spectrometer, the sweeper will generate the rf-signal in the programmed range and sweep rate. So that the spectra are taken into the memory as a function of frequency at each applied in-plane field. When the in-plane field is increased from zero to the saturation field, the spectra is either recorded into the tape or plotted onto a piece of paper. From the spectra, the resonance frequency for each applied field can be analyzed. A typical spectrum is shown in Fig. 3.3.

In the system, the rf-structure is the component where the coupling between rf-signal and DWR occurs. The rf-structure, as shown in Fig. 3.4, is composed of two parts. One is the coplanar strip-line, the other is the slot-line. This structure is made of a thin conductor (about 100 μm copper layer) on a high dielectric constant substrate. It is the ε-10 of 3M company. Due to the high dielectric constant substrate, most of the energy of the rf-wave is confined in the local area instead of radiating away. The flow of the current in the rf-structure is shown in Fig. 3.5. The asymmetric structure between the coplanar strip-line and the slot-line minimizes the coupling between the
FIGURE 3.3 A typical DWR spectra.
FIGURE 3.4  Layout of the rf structure. The light areas are copper. In the dark areas, the copper has been etched away, forming a short circuited slot line on the right side and a short circuited coaxial transmission line on the left side.
FIGURE 3.5  RF currents around the short circuited ends of the slot line and the coaxial line.
two lines. Fig. 3.6 shows the frequency response of this rf-structure from 20MHz to 500 MHz. This high isolation makes it possible to observe the signal from DWR which, as shown in Fig. 3.3, has an amplitude of 2-3 dbm after an amplification of 60 db.

The procedures for making the rf-structure can be summarized by the following:

1. A mask of the pattern of the rf-structure is obtained by usual photographic processes. The pattern drawing in Fig. 3.4 & 3.7 is designed onto a 50x50 cm² Ruby plastic paper. A photograph of this pattern sheet is taken on high contrast film producing a mask of 2.5x2.5 cm².

2. A piece of ε-10 circuit board is cut to 2.5x2.5 cm² and cleaned. A positive photoresist coating is applied on top of the conductor.

3. The mask is put on top of the ε-10 board with the photoresist layer and exposed.

4. The ε-10 board plate was then developed.

5. The ε-10 board was etched by chemical etchant or ion mill. The chemical etching usually introduces an undercut of about 45°, thus in the case where a fine structure is needed, an ion mill, which gives no undercut, was used.

The spectrometer can be modified to operate in a pulse experiment as well. The block diagram of the pulse experiment is shown in Fig. 3.8. A pulse generator, HP 8011A, is used
FIGURE 3.6 The frequency response of the shorted slot line rf-structure.
FIGURE 3.7 The layout of the parallel stripline rf-structure. The bright area is the dielectric material and the dark area is the conductor.
FIGURE 3.8 The block diagram of the pulse experiment system.
to drive a driver HP 33190B and a microwave switch, an HP 33144A. When the output from the pulse generator is at zero volt, the switch is open and rf-power is transmitted, otherwise the switch is closed. The rise time and fall time of this switch are 5 ns and 9 ns respectively. The frequency range of this switch is from 100 MHz to 18 GHz and can transmit a 0.1 µsec pulse without difficulty. In the DWR experiment, the pulse width was varied from 5 µsec to 10 µsec, and the echo has about 20 µsec delay.

In order to detect the magnetoelastic wave with a well defined orientation of propagation, a parallel pair stripline structure was constructed as shown in Fig. 3.7. The dark area is the conductor and the white area is ε-10. Except for the two strip-lines in the center of the rf-structure, the conductor is grounded. The rf-power at one strip-line will excite the DWR, which is coupled to the elastic wave which will propagate to the other strip-line. This elastic wave will then re-excite the DWR which will be detected at the 2nd strip-line. The procedures for sweeping the frequency and data storage are the same as for the shorted slot-line structure. Because the frequency range of the elastic modes associated with a given \( k_z \) (see Eq. 3.2) is only about 1 MHz, the frequency response from the background is relatively uniform. In this case, the data can be plotted by the x-y recorder directly. A typical spectrum is shown in Fig. 3.9.
FIGURE 3.9 A typical spectra for the DWR generated elastic wave detected by the parallel stripline rf-structure.
3.2 The Magnetic Garnet Thin Film

The samples used in this study are grown by LPE at the Philips Laboratories, Hamburg, Germany, and have the composition $Y_{2.96}\text{La}_{0.14}\text{Fe}_{3.75}\text{Ga}_{1.25}O_{12}$. The material parameters are listed in Table 3.1. The saturation magnetization was measured by a vibrating sample magnetometer and the uniaxial anisotropy and cubic anisotropy energies were measured by ferromagnetic resonance at OSU. The thickness and characteristic length are measured by infrared interference and bubble collapse measurement at Philips.

3.3 Experimental results of DWR

A typical domain wall resonance spectrum is shown in Fig. 3.3. Because of the frequency response of the rf-structure and the other components in the system, the signal is relatively weak and difficult to be observed directly. This difficulty is overcome by using the computer. A set of data points for the background spectrum is stored in the memory, when the film is saturated, i.e., an in-plane field larger than $H_u$ is applied. Since the film is saturated, this spectra produces the frequency response of the system only. After this spectrum is stored, the in-plane field is reduced to the desired value and a new spectra is obtained.
<table>
<thead>
<tr>
<th>Sample No.</th>
<th>$4\pi M$ (G)</th>
<th>$H_u$ (Oe)</th>
<th>$t$ (μm)</th>
<th>$t/t$</th>
<th>$H_c$ (Oe)</th>
<th>$\gamma$</th>
<th>$m_D (x10^{-11} g/cm^2)$</th>
<th>$\frac{m}{m_D}$</th>
<th>Film Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57</td>
<td>351</td>
<td>4.5</td>
<td>0.24</td>
<td>59</td>
<td>2.8</td>
<td>9.95</td>
<td>0.48</td>
<td>(111)</td>
</tr>
<tr>
<td>2</td>
<td>107</td>
<td>430</td>
<td>5.4</td>
<td>0.16</td>
<td>65</td>
<td>2.8</td>
<td>2.84</td>
<td>1.35</td>
<td>(001)</td>
</tr>
</tbody>
</table>
including the DWR effects. (See Fig. 3.3) This spectra is then subtracted from the background spectra and a typical result is shown in Fig. 3.10. Using this process, the resonance frequency as a function of applied field can be obtained. The data shown in Fig. 3.11 is the reduced frequency as a function of the reduced in-plane field. The reduced frequency is defined as \( \omega / \omega_0 \) where \( \omega_0 \) is the frequency when the applied field is zero and the reduced in-plane field defined by \( H_p / H_u \). The absolute value of the experimental and the calculated value of \( \omega_0 \) vary by a factor of 0.48 and 1.35 for (100) and (111) as shown in Table 3.1. However, the slope from the theoretical curve is very close to that of the experimental data as shown in Fig. 3.11.

From Eqs. 2.81 & 2.85 and from the value of cubic anisotropic constant in Table 3.1, the effects of the cubic anisotropy is expected to be very small. The results shown in Fig. 3.12 are the experimental result for the (111) film. For the data points shown by + and by x the inplane field is oriented along the (011) and (110) directions respectively. The calculated curves 2 and 3 are also calculated for these same orientations.

In the (100) film, a similar result is shown in Fig. 3.13. As expected the effect from cubic anisotropy is small but more significant in this film.
FIGURE 3.10 Domain wall resonance signal for $H = 80$ Oe. Here, the baseline has been subtracted, leaving only the wall resonance signal.
3.11 The reduced frequency of the DWR as a function of the reduced applied in-plane field. The data are taken for sample #2 with stripes parallel to [110] and the curve is calculated from Eq. 2.71.
FIGURE 3.12  The reduced frequency vs. reduced in-plane field applied parallel to the stripes of sample number 1.

The (+) are data obtained when the stripes are parallel to the [011] direction and Curve 1 is the theoretical prediction from Eq. 2.85 for $K_u / K_A = 0.25$. The (x) are data obtained when the stripes are parallel to the [110] direction and Curve 2 is the theoretical prediction. Curve 3 is the theoretical prediction for $K_A$ equal 0 from Eq. 2.85.
FIGURE 3.13 The reduced frequency vs. reduced in-plane field applied a parallel to the stripes of sample number 2. The experimental points are observed for $H_p$ and stripe lattice parallel to the [100] direction (+) or to the [110] direction (X).
3.4 Experimental Results of Magnetoelastic Interaction

When the frequency of the DWR is in the 100 MHz range, a series of spikes can be observed in the spectra as shown in Fig. 3.14. Using the shorted slot-line rf-structure, these spikes have also been observed in FMR\(^7\) and have been understood as the elastic standing modes across the thickness of the film/substrate system. When the resolution of the spectra is increased, a series of fine structure spikes is observed on the high frequency side of the acoustic peak. This fine structure, which is shown in Fig. 3.15 and in more detail in Fig. 3.16, shows a dependence on the angle between the rf-structure and the crystal axes. The linear dispersive spectra, as observed in Fig. 3.16, is obtained only in a certain orientation. When the sample is rotated, the fine structure usually shows a more complex spectra. At these frequencies, the Q of the acoustic modes, \(\nu/\Delta\nu\), is about 5000. The FMR and DWR have a Q of about 15, so several of these standing modes are observed within the magnetic resonance linewidth as shown in Fig. 3.14.

A series of experiments were conducted to show that these fine structure lines are independent of the domain pattern of the sample, i.e., the fine structure is about the same when the domain pattern is a bubble structure, parallel stripe lattice, or meander domain pattern. The sample was cut into
FIGURE 3.14  The spectra of the DWR and DR, the fmr within the domain. The standing elastic modes are shown as spikes on the spectra.
Figure 3.15 The DWR excited magnetoelastic standing modes with fine structures shown at higher resolution.
FIGURE 3.16 The details of the fine structure of the magnetoelastic modes excited by DWR.
different shapes: disk, rectangular, etc., and the fine structure shows the same characteristics. Usually, the surface magnetoelastic wave\textsuperscript{[26]} can be damped by a drop of water. However, the fine structure observed in this experiment can only be damped out by very viscous materials. When the sample is properly oriented to show a linear dispersion and then translated in the direction parallel to the slot-line, the frequency splitting shows a continuous change.

The pulse experiment, as described in the previous section, can be used to determine the traveling time of the excited waves. At FMR frequencies, when the rf power is pulsed at the frequencies of the magnetoelastic standing mode, an echo with a delay time related to the elastic wave traveling back and forth across the thickness of the sample is observed. In the sample used, the thickness is 0.5 mm and the velocity of elastic wave in GGG, given in Table 3.2, is $v = 3.5 \times 10^5$ cm/sec. Consequently, a delay time of about 0.3 µsec is in very good agreement with the observation. When the rf-power is pulsed at the fundamental frequency of the fine structure, the delay time is related to the inverse of the frequency splitting. A typical delay time is about 20 µsec as shown in Fig. 3.17. When the sample is translated along the slot-line, the frequency splitting is changed continuously and the delay time is changed correspondingly. The relation between the delay time and frequency splitting
TABLE 3.2. The Elastic Parameters for GGG

<table>
<thead>
<tr>
<th>density of mass (g/cm³)</th>
<th>elastic constant (x10¹¹ erg/cm³)</th>
<th>elastic velocity (x10⁵ cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C₁₁</td>
<td>C₄₄</td>
</tr>
<tr>
<td>7.102</td>
<td>28.7</td>
<td>9.04</td>
</tr>
</tbody>
</table>
FIGURE 3.17 The experimental result for the pulse experiment.
is shown in Fig. 3.18.

These fine structures have been identified as acoustic waves by using the simple acoustic wave model as described in Chapter 2. The dispersion of the 2-dimensional transversal acoustic wave is given by:

\[ \omega^2 = v_t^2 k_y^2 + k_z^2 , \]

where \( v_t^2 = \frac{C_{\text{uw}}}{\rho} \). For the case of \( k_y \ll k_z \),

\[ \omega = \frac{v_t k_z}{v_t} = k_z + \frac{k_y^2}{2k_z} . \]

Let \( v_t k_z = \omega_0 = 2\pi f_0 \), where \( f_0 \) is the resonance frequency when the elastic wave has only a \( k_z \)-component, or \( k_y = 0 \). Then the dependent on \( k_y \) is given by

\[ \omega - \omega_0 = \Delta \omega = 2\pi \Delta f = \frac{v_t^2 k_y^2}{2} . \]

In order to have a well-defined orientation of the propagation, the parallel strip-line is used to observe the fine structure. For these structures, one strip-line is used to excite the domain wall motion which in turn drives the elastic wave with \( k_y = 0 \) modes. These elastic waves will propagate in the \( y \) as well as \( z \) direction. Upon arrival at the output strip-line, these elastic waves excite DWR...
FIGURE 3.18 The correlation between the delay time of the pulse experiment and the frequency splitting of the corresponding fine structure of the elastic modes.
which couples electromagnetically to the output strip-line. These two strip-lines are parallel and have a separation, \( D \), that is large in comparison to the propagation distance of the domain wall motion. Thus all the magnetic dependent energy received in the second strip-line is transmitted through the elastic wave process. Because the waves propagate at an angle \( \theta \), \( \tan \theta = k_z/k_y \), only elastic waves which are reflected from the non-film surface and propagate to the second strip-line will be detected. This limits the condition on \( k_y \) to \( 2m\pi/D \), where \( m \) is an integer.

From Eq. 3.3,

\[
\Delta f_m = \frac{v_t^2 m^2}{2f_0 D^2} = A(f_0, D)m^2,
\]

where \( A(f_0,D) \) depends on the values of \( f_0 \) and \( D \) and can be compared directly with the experimental results.

As shown in Fig. 3.9 & 3.19, the \( \Delta f_m \) is a quadratic function of \( m \), the mode number. In Fig. 3.20, the calculated values of \( 1/A(f_0,D) \) are compared with the experimental values.
FIGURE 3.19 The quadratic dispersion of the elastic wave excited by DWR in the parallel stripline rf-structure.
FIGURE 3.20 The coefficient of the quadratic dispersion of the elastic wave.
CHAPTER 4
ANALYSIS AND DISCUSSION

4.1 The Singularity of DWR at $H_p \rightarrow H_u$

From Fig. 3.11, the experimental data and the calculated curve for DWR is in good agreement up to a reduced field value of 0.7. The model also predicts the dependence of the cubic anisotropy energy in Eq. 2.81 and 2.85. When the cubic anisotropy energy is relatively small, the calculation predicts the change of the effective mass to first order is by a factor of $H_c / H_u$, where $H_c$ is the cubic anisotropy field. This is confirmed by the experimental data as shown in Fig. 3.12 and Fig. 3.13. From these experimental results, the simple harmonic oscillation indeed properly describes the domain wall motion, which originates from the spin excitation in the moving domain wall.

When the in-plane field is increased to $H_p > 0.8H_u$, the theory based on small displacements predicts a singularity point. As shown in Fig. 2.6 and Fig. 2.11, the effective mass and the restoring force constant approach zero when the in-plane field is increased to $H_u$. An interesting question is the behavior of the resonance as the critical field is

100
approached.

The asymptote of the resonance frequency as $H_p$ approaches $H_u$ can be estimated to first order in

$$\left(1 - \frac{H_p}{H_u}\right) .$$

The following analysis is valid in the magnetic garnet system having a high $Q$ value. From Eq. 2.106, the restoring force constant is

$$k = \cos \theta_D ,$$

where $\theta_D$ is the polar angle of the magnetization. In the materials with $Q \gg 1$, $\theta_D$ can be approximated as

$$\sin \theta_D = \frac{H_p}{H_u} = \mu .$$

From Eq. 2.65, the effective mass is

$$m = \int_{0}^{\pi/2} \sin \theta f(\theta) \, d\theta ,$$

where

$$f(\theta) \sim \frac{\sin \theta - \mu}{\sin \theta + \mu} .$$

In a high $Q$ material, $\frac{\sin \theta}{Q} \ll \mu$, and

$$f(\theta) \sim (\sin \theta - \mu) / \mu .$$

From Eq. 4.3,

$$m \sim \left(\pi - \theta_D\right) - \cos \theta_D ,$$

and since $\omega^2 = k/m$,
\begin{align}
\omega^2 &\approx \frac{\cos^2 \theta_D}{(\frac{\pi}{2} - \theta_D) \cos \theta_D} \\
\text{Let } \theta_o &= \frac{\pi}{2} - \theta_D, \text{ then } \cos \theta_D = \sin \theta_o.
\end{align}

As \( H_p \) approaches \( H_u \), \( \theta_D \to \frac{\pi}{2} \) or \( \theta_o \to 0 \) and
\begin{align}
\cos \theta_D &= \sin \theta_o \\&= \theta_o - \frac{\theta_o^3}{3!} + \frac{\theta_o^5}{5!} + \ldots
\end{align}

From Eq. 4.7 and 4.8,
\begin{align}
\omega^2 &\approx \frac{\left( \theta_o - \frac{\theta_o^3}{6} + \frac{\theta_o^5}{5} + \ldots \right)^2}{\theta_o - \theta_o + \frac{\theta_o^3}{6} - \ldots} \\
&\approx \frac{1}{\theta_0} \approx \frac{1}{\frac{H_p}{H_u}}
\end{align}

This asymptotic form of Eq. 4.9 shows that the simple harmonic model is expected to produce a singularity at \( H_p = H_u \).

For a physical system with an infinite normal mode frequency the energy required to excite this system is infinite. This is not physical nor is it observed in the garnet system. As shown in Fig. 3.11, 3.12, & 3.13, the resonance frequency tends to increase as a linear function of \( H_p \) instead of showing a singularity as predicted by the calculated curve in the same figures. One possible reason is that the restoring force due to the surface charge is reduced by a factor of \( \theta_o \) when the in-plane
field reaches $H_u$. When $H_p + H_u$, the domain pattern is observed to no longer be that a parallel array pattern. This meander structure is presumably produced as a result of the demagnetization energy due to the magnetic surface charge being different from that calculated. And other possibility is that the contribution from the cubic anisotropy will increase the effective domain wall mass. As shown in Eq. 2.81 & 2.85, when $K_c$ is included, the domain wall mass does increase, but in the samples investigated, $K_c$ is too small to increase the mass sufficiently to overcome the slower rate of decrease of the restoring force constant. When $H_p + H_u$, the thickness of domain wall becomes so wide as shown in Fig. 2.3 that the restoring force constant is not valid anymore.

4.2 The method to measure Cubic Anisotropy

In order to determine the cubic anisotropy constant, an FMR technique is used. For the (100) sample, the total energy density, including the cubic anistotropy, is given by:

$$E = 2\pi M^2 \cos^2 \theta - k_u \cos^2 \theta - MH\sin \theta \cos (\phi - \phi_H)$$

$$+ k_c \left( \sin^4 \theta \cos^2 \phi \sin^2 \phi + \sin^2 \theta \cos^2 \theta \right),$$

where the coordinate system is shown in Fig. 4.1, and the energy contributions are from the demagnetizing energy, the uniaxial anisotropy energy, the Zeeman energy and the cubic anisotropy energy respectively. The dispersion
FIGURE 4.1 The coordinate system used to evaluate the cubic anisotropy constant.
relation for FMR is given by (27):

\[
\left(\frac{\omega}{\gamma}\right)^2 = \left(\frac{E_{\theta\theta} + Dk^2}{M}\right) \left(\frac{E_{\phi\phi}}{M\sin^2\theta} + Dk^2\right) - \left(\frac{E_{\theta\phi}}{M\sin\theta}\right)^2,
\]

where \(D\) is the exchange constant, \(k\) is the spin wave vector and \(E_{\theta\theta}, E_{\phi\phi}, E_{\theta\phi}\) are the second order derivative with respect to the angles \(\theta\) and \(\phi\). For the film thicknesses used in this work, the exchange energy, \(Dk^2\), is negligible and the uniform mode, i.e. \(k = 0\), is assumed. Then Eq. 4.11 becomes

\[
\left(\frac{\omega}{\gamma}\right)^2 = \left[ \frac{2K'}{M} + H\cos(\phi - \phi_H) + \frac{K_c}{M} (2 - 4\sin^2\phi\cos^2\phi) \right] \times \left[ H\cos(\phi - \phi_H) + \frac{2K_c}{M} (1 - 8\sin^2\phi\cos^2\phi) \right],
\]

where \(K' = 2\pi M^2 - K_u\) and \(\phi\) is the static azimuthal angle of the static magnetization. The angle \(\phi\) can be determined by the static condition

\[
\frac{\partial E_{\text{tot}}}{\partial \phi} = 0,
\]

which gives

\[
H\sin(\phi - \phi_H) + H_c\sin\phi\cos\phi \cos 2\phi = 0,
\]

where \(H_c = 2K_c/M\). The coordinate system for this expression is shown in Fig. 4.1, and the applied field is in the plane of the film, i.e., \(\theta = 90^\circ\).
Experimentally, the D.C. magnetic field is applied in the plane of the film and the azimuthal angle, $\phi_H$, is changed. For each $\phi_H$, a resonance frequency is obtained and by using the computer to fit these data to Eq. 4.12 a cubic-anisotropy constant for the (100) and the (111) films are determined.

4.3 The Magnetoelastic Modes

In 1976, Dötsch(7) used the shorted slot line rf-structure to detect the FMR and observed the standing elastic waves across the thickness of the film/substrate system. Similar elastic modes also due to the magnetoelastic interaction are observed in the DWR.(5) The Q value of the elastic modes is much larger than the Q value of the magnetic resonance, so that several elastic modes are imbedded in the magnetic resonance as shown in Fig. 3.14.

When the frequency of DWR is greater than about 100 MHz, a series of fine structure modes, as shown in Fig. 3.15 & 3.16, gives rise to a complex spectra stimulated by the magnetoelastic interaction in DWR. If this fine structure were due to the standing modes associated with a component of the wave vector in the plane of the film, the dispersion would be quadratic. If it were a surface mode, usually it could be damped out upon the applications of a drop of water on the surface of the sample. In addition, the
velocity of the surface Rayleigh wave in the GGG is expected to be about $3.0 \times 10^5$ cm/sec so the dimensions of the sample would have to be on the order of 2 cm to have the frequency splitting, $f = v/2\ell$, equal to 75 kHz. The sample used in this work has less than 1 cm dimensions and also the fine structure shows only slight differences for different sample geometries. From these experiments, it is confirmed that the fine structure is not due to a surface wave. When the domain structure is changed from a bubble domain structure to a stripe domain structure, the fine structure shows nearly the same characteristics. This indicates that the fine structure is not due to a mode localized inside the film itself.

When a parallel strip line rf-structure is used, a plane wave pattern of elastic modes is excited which has the quadratic spectra predicted for an elastic wave. Because of the symmetry of the parallel strip line rf-structure, the cross talk between the input and output lines is so large that the transmission signal due to domain wall excitation can't be detected. However, the energy converted into elastic waves has a lower damping factor and can be observed. This experiment demonstrates that the DWR can excite not only standing modes normal to the film plane but also elastic waves having a propagation component in the plane of the film.
As shown in Fig. 4.2, when the DWR is excited at stripline 1, the DWR will excite elastic waves with \( k = k_z \hat{z} + k_y \hat{y} \). The wave number \( k_z \) is that of the fundamental standing modes. When \( k_y \neq 0 \), this elastic wave is a plane wave propagating at the angle \( \theta = \tan^{-1}(k_z/k_y) \) with respect to the film normal. In order to detect this wave at the stripline 2, a distance \( L \) from stripline 1, \( k_y \) must satisfy the condition \( k_y = 2\pi n/L \). For this condition the spectra will satisfy a quadratic dispersion relation as observed in Figure 3.9.
FIGURE 4.2 The propagation of elastic wave generated by DWR.
CHAPTER 5
CONCLUSION

The domain wall oscillations are experimentally detected by a transmission type rf-structure. The resonance frequency is observed to be in the range of 10 MHz to 300 MHz for the magnetic garnet films. In the spectrometer, the rf-signal is swept in frequency with a fixed applied in-plane field. The resonance frequency is observed as a function of the applied in-plane field and compared with the theoretical calculation.

Theoretically, the DWR is modeled as a simple harmonic oscillator. The model predicts the dependence of the resonance frequency vs. applied field which agrees with the experimental result up to $H_p/H_u \approx 0.7$.

The cubic anisotropy energy has been incorporated into the theory. The calculation predicts a small angle dependence of the effective mass due to this cubic anisotropy. This effect is observed experimentally.

The elastic waves excited by DWR have a very interesting character. A fine structure having a frequency splitting of about 50 kHz is observed. This fine structure has been identified as bulk elastic waves having a multiple
dimensional nature. When the orientation of the propagation of the elastic waves is well defined along a certain crystal symmetry axis, the fine structure shows a nearly linear dispersion relation. Using rf pulses having a width of 5-10 μsec in the shorted slot line rf-structure, an echo is observed which has a delay time given by the reciprocal of the frequency splitting of the fine structure. This indicates that the fine structure is due to normal modes of bulk elastic modes generated by DWR and whose boundary conditions are determined by the rf-structure and the sample geometry.

In order to simplify the geometry of the rf power distribution, a pair of parallel strip lines was used to excite and detect the elastic waves generated by the DWR. From the model of plane elastic waves, a quadratic dispersion of the spectra is expected and was found to compare favorably with the experimental results. The agreement between the model and the experimental results confirms the proposal that the bulk elastic waves having well controlled propagation directions can be generated by DWR in the magnetic garnet film/GGG system.
LIST OF REFERENCES

8. H. Dötsch, private communication.
APPENDIX I

DEMAGNETIZATION ENERGY IN THE DOMAIN WALL
According to Maxwell's equations,

\[ \nabla \cdot \mathbf{E} = 0 \]

so that, if \( \mathbf{H}^D \) is the demagnetization field,

\[ \nabla \cdot \mathbf{H}^D = -4\pi \nabla \cdot \mathbf{M} \]

If \( \mathbf{M} \) is uniform in the \( x-z \) plane, then

\[ \nabla \cdot \mathbf{M} = \frac{\partial M_y}{\partial y} \]

so

\[ \frac{\partial \mathbf{H}^D}{\partial y} = -4\pi \frac{\partial M_y}{\partial y} \]

and thus

\[ H_y^D = -4\pi M_y + C \]
The integration constant $C$ is determined by the boundary condition that $H^D = 0$ and $y \to \mp \to$ so

$$C = 4\pi M_y^0$$

and

$$= 4\pi (M_y^0 - M_y)$$

The demagnetization energy is

$$w_D = \frac{1}{2} H^D \cdot \vec{M}$$

$$= 2\pi H^D M_y$$

$$= 2\pi (M_y M_y^0 - M_y)$$

Since

$$M_y = M \sin \theta \sin \psi$$

and

$$M_y^0 = M \sin \theta_0 \sin \psi_0$$

so

$$w_D = 4\pi M^2 (\sin \theta_0 \sin \theta \sin \psi \sin \psi_0 - \sin^2 \theta \sin^2 \psi)$$
APPENDIX II

PROGRAM FOR DOMAIN WALL STATIC STRUCTURE
Calculate the static wall structure:

1. \( \text{rad; dim } Q[S][0]; \text{dim } L[S][20] \)
2. \( \text{ent } "4\pi M=", 0; 0/4\pi \rightarrow 0 \)
3. \( \text{ent } "H_0", K; K/0 \rightarrow K \)
4. \( \text{ent } "\text{characteristic length}, r_1; 16\pi K/r_1 (4\pi)^2/2=0; 1/0 \rightarrow 0 \)
5. \( \text{cfg } 13; \text{ent } "\text{MAX NO OF ITER?}"; M; \text{if } flg_{13}; \text{cfg } 13; \text{gto } +0 \)
6. \( \text{dim } T[0][0] \)
7. \( \text{ent } "H_p", H; \text{aen } (OH/ (2K-4\pi 0))/C; C+.001 \rightarrow A \)
8. \( \text{sol } 0; 100; 0, 100 \)
9. \( \text{ent } "\text{EPSILON}"; E; \text{aep } ; \text{if } flg_{13}; \text{cfg } 13; 1e-6 \rightarrow E \)
10. \( \text{cfg } 3; \text{ent } "\text{PRINT TABLEAU?}"; R; \text{aep } 2; \text{if } flg_{13}; \text{cfg } 3 \)
11. \( \text{for } B=A+.01 \text{ to } \pi/2+.02 \text{ by } \pi/20 \)
12. \( \text{pens } \text{plt } 0, 100, 0; \text{aplt } 0, -B/(\pi/20) \)
13. \( \text{geb } "\text{ROMBERG}" \)
14. \( \text{if } flg_{2}; \text{step } \)
15. \( \text{aep } 2; \text{pnt } "\text{---------------}"; \text{aep } 2 \)
16. \( \text{pnt } "\text{INTEGRAL FROM"}, A, \quad \text{TO"}, B, \quad \text{"IS"}, T[0] \)
17. \( \text{fux } 2 \)
18. \( \text{lbl } "\text{Hp=}"; \text{str } (H) \rightarrow Q[S]; \text{lbl } Q[S]; \text{lbl } A="; \text{str } (A) \rightarrow Q[S]; \text{lbl } Q[S] \)
19. \( \text{str } (B) \rightarrow Q[S]; \text{lbl } Q[S]; \text{fux } 10; \text{str } (T[0]) \rightarrow L[S]; \text{lbl } L[S]; \text{fux } 2; \text{next } B; \text{end } \)
20. \( "\text{ROMBERG}"; \text{cfg } 2; \text{if } A>B; \text{cfg } 2; \text{dep } "\text{BOUNDS REVERSED}"; \text{ret } \)
21. \( \text{0} \rightarrow S[S]; \text{A} \rightarrow X[S]; \text{geb } "\text{EVAL}" \)
22. \( \text{Y} + S[S]; \text{B} - X[S]; \text{geb } "\text{EVAL}" \)
23. \( \text{Y} + S[S]; (B-A)/2 \rightarrow S \rightarrow T[0]; 0 \rightarrow N \)
24. \( \text{if } (N+1) \rightarrow M[S]; \text{aep } 2; \text{gto } +6 \)
25. \( (2(2(2(N-1) \rightarrow R) \rightarrow Q) \rightarrow P \rightarrow P \)
26. \( -1 \rightarrow I[S]; 0 \rightarrow S \)
27. \( \text{if } (I+2) \rightarrow P[S]; ((B-A)/2) \rightarrow S \rightarrow T[N-1]/2 \rightarrow T[N]; \text{gto } -3 \)
28. \( A+(B-A)/2 \rightarrow S[S]; \text{geb } "\text{EVAL}" \)
29. \( S[Y] \rightarrow S[S]; \text{gto } -2 \)
30. \( 1 \rightarrow J \)
31: if (J+1-J)>M+1; ret
32: -1-N; sfg 6; if flg3; gsb "PRINT"
33: if (N+1-N)>M+1-J; gto -2
34: T[N] = F; ((4↑(J-1)-G)T[N+1]-T[N])/(G-1) = T[N]; if flg6; sfg 6; gto -1
35: if abs(F-T[N])<E; T[N] = T[0]; ret
36: gto -3
37: "EVAL"
38: D/f (sin(X)↑2-sin(C)↑2-H0(sin(X)-sin(C))/K) = Y
39: ret
40: "PRINT"; 0→I
41: prt T[I]; jmp (I+1-I)>M+1-J
42: spc; ret
APPENDIX III

PROGRAM FOR FUNCTION $f$
"calculate f"
1: dim F[0:400];dim M[400,4];dim A$[5];dim H[0:400]
2: dim G[00]
3: rad
4: ent "start of Hp/Hu",r24
5: ent "end of Hp/Hu",r25
6: cfg 4;ent "To plot func?1=mass,2=no plt".A5 if A=1;efg 4
7: ent "Q",Q5 if flg4;efl 0,π/2,0,000006
8: ent "1/Hu*Gamma*Delta",r1
9: ent "# of points to calculate func",N
10: cfg 1;ent "Calculate for Hp/Hu=0 to 1?".A5 if flg13=0;efg 1
11: if flg4;fxl Ø;xax 0,π/18;efl 0,π/2,0,6;yax 0,2,0,6,1
12: if flg4;efl 0,π/2,0,6e-6.10-A
13: if flg4;A+10-A;pen;plt An/180,0,Ø;opn t-1.5,-1;str (A)-A$;1bl A$
14: if flg4;if A<90;gto -1
15: ent "4=xM",r20;ent "Hu",r21;r20/(r1*r21) r22
16: efl 'EVAL4';efl 'plot'
17: efl 'Fmo'
18: if flg1 and (rØ+.1+rØ)<1;gto -2
19: "END":end
20: "Fmo":pen# 2;for I=0 to N;r18+DI=X;if X=Ø;gto +2
21: plt X,Qr1(sin (X)-rØ)/(sin (X)+QrØ) r15
22: next I;pen;opn t 1,0;fxd 2;str (rØ)-A$;1bl "u=";1bl A$
23: pens;ret
24: "plot":
25: pens# 3
26: for I=0 to N
27: plt r18+DI,F[I]
28: next I;pen;opn t 1,0;fxd 2;str (rØ)-A$;1bl "u=";1bl A$
29: pens;ret
30: "Al":eit (X) * (eit (X)-rØ) r2-p1;ret p1
122

31: "B1": \( \cos(X) \ast (\sin(X) - r0) \) \((3\sin(X) - 2r0)\) -p1;ret p1
32: "C1": \( -(r0 + \sin(X)/Q) + p1; ret p1 \)
33: "D1": \( -1(\sin(X) - r0) - p1; ret p1 \)
34: "H1": \( \sin(X) - p1; ret p1 \)
35: "H": \( \sin(X) (r0 + \sin(X)/Q) / (\sin(X) - r0) - p1; ret p1 \)
36: "EVAL4":
37: \( \sin(r0) - r18; (\pi/2 - r18) / N = o; r0 = H[0] \)
38: for I=1 to N
39: r18+ID=X
40: 'A1' = A; 'B1' = B; 'C1' = C; 'D1' = E; 'H1' = H[I]
41: A/D\(T2\)-B/D\(M[I, 1]\)
42: -2A/D\(T2\)+B/D+C=M[I, 2]
43: A/D\(T2\)-M[I, 3]
44: E=M[I, 4]
45: next I
46: 0=M[I, 2]-M[I, 2]
47: for I=2 to N-1
48: M[I, 2]=M[I, 1]M[I-1, 3]/M[I-1, 2]-M[I, 2]
50: next I
51: M[N-1, 2]=M[N-1, 3]=M[N-1, 2]
52: M[N-1, 4]/M[N-1, 2]+F[N-1]+F[N]
54: for I=N-2 to 1 by -1
55: (M[I, 4]-F[I+1]M[I, 3])\(M[I, 2]-F[I]\)
56: G=F[I]=D*H[I]+G
57: next I
58: 0=F[0]
59: fxd 10; dep G, rl(\(\pi/2 - r18\))/2
60: ret
APPENDIX IV

PROGRAM FOR DOMAIN WALL EFFECTIVE MASS
0: "m/mo for [111] cubic anisotropy";
1: dim M[80]; rad
2: 0->r20
3: ent "4\pi M", r6; r6/4\pi+r7
4: ent "Hu", r8; r7r8/2->r8
5: ent "Ka/Ku", r1
6: ent "Beta angle", r10
7: ent "characteristic length", r9
8: "Gamma=2.8MHz/0e": 2.8e6+r5
9: (r9*r6r2)/2/(4\pi)r2+r16
10: f(r7r2/r15)->r16
11: r8/2\pi->r7r7->r3
12: ent "which file to store?", r21
13: ent "MAX NO OF ITER?", M; dim T[0:M]
14: ent "Epsilon", E
15: cfg 3; ent "Print tableau?", R; if flg13; cfg 3
16: "applied field": for H=0.0001 to 400 by 5; Hr7/2r8->r2
17: cfg 1; cll 'THITA'; if flg15; gto "END"
18: r4->A
19: \pi/2->B
20: gb "ROMBERG"
21: if flg2; stp
22: prn "H=", H; "M/M0=", T[0]; T[0]>M[r20+1->r20]
23: next H
24: "END": trk 1; fdf r21; rcf r21, M[*]; trk 0
25: end
26: "ROMBERG": cfg 2; if A>B; cfg 2; dep "BOUNDS REVERSED"; ret
27: 0->S; A->X; gb "EVAL"
28: Y+S->S; B->X; gb "EVAL"
29: Y+S->S; ((B-A)/2)S+T[0]; 0->N
30: if (N+1->N)->M; epc 2; gto +6
31: \((2(2^{(N-1)}-R)\cdot Q)-1-P\)
32: 
33: if \((I+2+I)>P; ((B-A)/R)S+T[N-1])/2+T[N]; gto -3
34: \(A+(B-A)/Q)\cdot X; gto "EVAL"
35: S+Y+S; gto -2
36: 1-J
37: if \((J+1-J)>M+1; ret
38: -1-N; efg 6; if flg3; gto "PRINT"
39: if \((N+1-N)>M+1-J; gto -2
40: T[N]+=F; ((4(J-1)-G)T[N+1]-T[N])/(G-1)\cdot T[N]; if flg6; cefg 6; gto -1
41: if abs(F-T[N])<E; T[N]=T[0]; ret
42: gto -3
43: "EVAL"
44: 'F2'(X)*sin(X)/(sin(X)+r3r2+.5*f2*3r1*sin(X)\cdot 12\cdot cos(r3)*sin(3r10))-Y
45: ret
46: "PRINT": 0+1
47: prt T[I]; jmp (I+1-I)>M+1-J
48: epc; ret
49: "THITA": 0+r11; 'FTH'(r11)--r12
50: r11+.01+r11; if r11>1.57; gto "RTH1"
51: if r12=0; r11-.01+r4; gto "RTH"
52: 'FTH'(r11)--r13; if r12-r13<0; r11-.01+r15; gto 'loop1'
53: r13+r12; gto -3
54: 'loop1': if abs(r11-r15)<.00001; r11+r4; gto "RTH"
55: (r11+r15)/2+r14
56: 'FTH'(r14)--r14; if abs(r14)<.00001; (r11+r15)/2+r4; gto "RTH"
57: if r12+r14<0; (r11+r15)/2+r11; r14+r13; gto 'loop1'
58: (r11+r15)/2+r15; r14+r12; gto 'loop1'
59: "RTH1": prt "Hp=", H
60: prt "NO root for THITA 0"
61: efg 1
62: "RTH": ret
63: "FTH": cos(p1)*sin(p1)*3-4*sin(p1)*cos(p1)*3/3-p2
64: p2-f2*sin(3t10)*(2cos(p1)*3+1)*sin(p1)*2/3-p2
65: sin(2p1)*(1-1/r3)-2r2cos(p1)+r1p2+p3
66: ret p3
67: "F1": f2*sin(3t10)*(cos(r4)*sin(r4)*3-cos(p1)*sin(p1)*3)/3-p2
68: (sin(p1)*4-sin(r4)*4)/4+(cos(p1)*4-cos(r4)*4)/3+p2+p3
69: ret p3
70: "F2": sin(p1)*2-sin(r4)*2-2r2(sin(p1)-sin(r4))+r1*F1(p1)+p2
71: if p2<0: pr "Error in line 71, p2=", p2; etp
72: f+p2-p2; ret p2
APPENDIX V

PROGRAM FOR COLLAPSE FIELD OF MAGNETIC GARNET THIN FILMS
EXTERNAL F, XF, XG
DIMENSION X(2), PAR(10)
DOUBLE PRECISION X, PAR, EPS, H, F, XF, XG, FF, FG
DOUBLE PRECISION X1, X2, X3, X4, X5, X6, X7, Y1, Y2, FF1, FF2, FF3

C ANGLE OF APPLIED FIELD = THETA
THETA = 6 * 5.1416 / 18
C H IS THE APPLIED FIELD
H = 530
1 EPS = .00001
KTE = 0
ITMAX = 100000
C PAR(1) = H = APPLIED FIELD
C PAR(3) = ITMAX
PAR(1) = H
PAR(2) = THETA
PAR(3) = ITMAX
WRITE(1, 10) H, THETA
X(2) = -.99
15 X(2) = X(2) + 1.0
IF (X(2) .GT. 5.5) GO TO 96
X1 = XF(X, PAR)
1F (DABS(X1) .EQ. 2.) GO TO 17
X2 = XG(X, PAR)
IF (DABS(X2) .EQ. 3.) GO TO 17
GO TO 19
17 WRITE(1, 90) X
GO TO 15
19 X3 = X1 - X2
Y1 = X(2)
IYY = X(2) * 100 + 10
DO 210 IY = IYY, 545, 10
IF (DABS(X3) .LE. EPS) GO TO 30
X(2) = (IY + 0.) * 0.01
X4 = XF(X, PAR)
IF (DABS(X4) .EQ. 2.) GO TO 80
X5 = XG(X, PAR)
IF (DABS(X5) .EQ. 3.) GO TO 80
X6 = X4 - X5
X8 = X6
IF (X3 * X6) 212, 212, 209
212 X3 = X6
GO TO 210
212 IF (DABS(X6) .LE. EPS) GO TO 30
Y1 = X(2) -.1
Y2 = X(2)
213 X(2) = (Y1 + Y2) / 2.
IF (DABS(Y1 - Y2) .LE. EPS) GO TO 30
X1 = XF(X, PAR)
X2 = XG(X, PAR)
X7=X1-X2
IF(DABS(X7).LE.EPS) GO TO 30
IF(X7*X6) 214,30,216
214 Y1=X(2)
   X3=X7
   GO TO 213
216 Y2=X(2)
   X6=X7
   GO TO 213
10 FORMAT(/,2X,'FIE L DS',0 10.5,2X,'THETA=',F10.2)
30 WRITE(1,20)X
   X3=X8
   KTE=KTE+1
20 FORMAT(/,2X,'X=',020.6,2X,'Y=',020.6)
   FF=F(X,1,PAR)
   FG=F(X,2,PAR)
   WRITE(1,60)FF,FG
60 FORMAT(/,2X,'F=',020.6,2X,'G=',020.6)
GO TO 210
80 WRITE (1,90)X
   IXX=X(1)
   GO TO(30,92,9^,96),IXX
92 WRITE(1,93)
   GO TO 210
93 FORMAT(/,2X,'F HAS NO ROOT')
94 WRITE(1,95)
   GO TO 210
95 FORMAT(/,2X,'G HAS NO ROOT')
210 CONTINUE
   IF(KTE .GT. 0) GO TO 99
   X(1)=4.
   WRITE(1,97)
   GO TO 100
96 WRITE(1,98)
98 FORMAT(/,2X,'NO XF OR GX')
97 FORMAT(/,2X,'XF,XG HAS NO INTERCEPTION')
99 FORMAT(/,2X,'X=',020.6,2X,'Y=',020.6,6X,'NO ROOT')
   H=H+20
   IF(X(2) .LE. 0.005) GO TO 100
   GO TO 1
100 STOP
END
FUNCTION F(X, K, PAR)
EXTERNAL DDEXP
DIMENSION X(2), PAR(10)
DOUBLE PRECISION X, PAR, B, D, F, F1, F2, F3, DDEXP
DOUBLE PRECISION FF1, FF2, FF3
ESP = .0001

C MATERIAL PARAMETER START FROM HERE, KU IS UNIAXIAL CONSTANT, IN UNIT ERGS/CM**3, M IS MAGNETIZATION, T IS THICKNESS IN MICRO METER, PAR(1) IS APPLIED FIELD, PAR(2) IS ANGLE OF APPLIED FIELD, PAR(3) IS ITMAX, D IS DOMAIN WALL ENERGY

T = .00052
KU = 70600
PI = 3.1416
M = 480/(4*PI)
HU = 2*KU/M
B = DARSIN(PAR(1)*DSIN(PAR(2))/(HU-4*PI*M))
D = .38526026
GO TO (110, 120), K

C FUNCTION F START HERE
110 N = 0
F1 = 0
DO 112 N = 1, 2000
F3 = 1 - DDEXP(-N*X(2))
F2 = DSIN(N*PI*(1 + X(1)))*F3/(N**2*X(2))
F1 = F1 + F2
IF(DABS(X(1)) .EQ. 0.) GO TO 112
IF(DABS(F2) .LE. ESP) GO TO 115
112 CONTINUE
111 FORMAT(/, 2X, 'F', 3(2X, D20.6))
115 F = PAR(1)*DCOS(PAR(2))/(4*PI*M)
F = F-X(1)*DCOS(B)-2*F1*((DCOS(B)))/PI)
RETURN
118 WRITE(1, 119)
119 FORMAT(/, 2X, 'FUNCTION F DOES NOT CONVERGE')
RETURN

C FUNCTION G START HERE
120 N = 0
F1 = 0
DO 122 N = 1, 2000
F3 = 1 - (1 + N*X(2))*DDEXP(-N*X(2))/N**3
F2 = ((DSIN(N*PI*(1 + X(1))/2)**2)**2) / (X(2)**2)
F1 = F1 + F2
IF(DABS(X(1)) .EQ. 0.7) GO TO 122
IF(DABS(F2) .LE. ESP) GO TO 125
122 CONTINUE
125 F = F1*((DCOS(B))**2)
F = D*(2*DCOS(B)-(PI-2*B)*DSIN(B))/(32*T*(M**2))-F
RETURN
130 WRITE(1, 131)
131 FORMAT(/, 2X, 'FUNCTION G DOES NOT CONVERGE')
C FUNCTION OF EXP START HERE

FUNCTION DDEXP(X)
DOUBLE PRECISION XDDEXP,X11,X12
EPS1=0.000001
N=0
X11=1.
X12=0.
1110 N=N+1
X11=X11*X/N
X12=X11+X12
1F(X11 .LE. ESP1) GO TO 1111
GO TO 1110
1111 RETURN
END

C FIND ROOT OF FUNCTION F

FUNCTION XF(X, PAR)
DIMENSION X(2),PAR(10)
K=1
CALL FINDR(X, K , PAR)
XF=X(1)
RETURN
END

C FIND ROOT OF FUNCTION G

FUNCTION XG(X, PAR)
DIMENSION X(2),PAR(10)
K=2
CALL FINDR(X, K , PAR)
XG=X(1)
RETURN
END

C SUBROUTINE OF FINDING ROOT START HERE

SUBROUTINE FINDR(X,K,PAR)
EXTERNAL F
DIMENSION X(2),PAR(10)
DOUBLE PRECISION X,PAR,FF1,FF2,FF3,EPS1,X1,X2
EPS1=.000001
X(1)=0.0
FF1=F(X,K,PAR)
IF(DABS(FF1).LE.EPS1) GO TO 180
DO 164 IX=1,10
  X(1)=IX*.1
  FF2=F(X,K,PAR)
FORMAT(/2(2X,D20.6),2X,12)
  IF(FF1*FF2) 170,170,162
  FF1=FF2
CONTINUE
  X(1)=K+1.
  GO TO 180
IF(DABS(FF2).LE.EPS1) GO TO 180
X1=X(1)-.1
X2=X(1)
X (1)=(X1+X2)/2.
  FF3=F(X,K,PAR)
  IF(DABS(X1-X2) .LE. EPS1) GO TO 180
  IF(DABS(FF3).LE.EPS1) GO TO 180
  IF(FF3*FF2)174,180,176
  X1=X(1)
  FF1=FF3
  GO TO 173
X2=X(1)
  FF2=FF3
  GO TO 173
RETURN
END