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ELECTROMAGNETIC SCATTERING FROM INLETS AND PLATES MOUNTED ON ARBITRARY SMOOTH SURFACES

The Ohio State University

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ELECTROMAGNETIC SCATTERING FROM INLETS AND PLATES
MOUNTED ON ARBITRARY SMOOTH SURFACES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

John Leonidas Volakis

* * * * *

The Ohio State University
1982

Reading Committee:
Professor L. Peters, Jr.
Professor W.D. Burnside
Professor P.H. Pathak

Approved By

[Signature]
Department of Electrical Engineering
Ξτὸν Προστάτη καὶ Ζωοποιῶ Θεό

To God, the protecting and life giving.
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VITA

May 13, 1956 ..................... Born - Chios, Greece

1978 ............................. B.E., Youngstown State University, Youngstown, Ohio

1979 ................................ M.Sc., The Ohio State University, Columbus, Ohio

1978-1982 ........................ Graduate Research Associate, ElectroScience Laboratory, Department of Electrical Engineering, The Ohio State University.

PUBLICATIONS


FIELDS OF STUDY

Major Field: Electrical Engineering


Studies in Communication Theory. Professor C.E. Warren

Studies in Digital Systems. Professor F. Ozguner

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CHAPTER I

INTRODUCTION

While scattering of electromagnetic waves by material objects has been an active subject throughout the twentieth century, the advent of digital computers has made it practical to consider scattering from complex shapes. The goal of this research has been to investigate the high frequency backscattering or monostatic scattering from appendages mounted on a smooth surface or body. Only perfectly conductive structures are considered. The method of solution employs the so-called Uniform Geometrical Theory of Diffraction (UTD) [1] which is an outgrowth of the Geometrical Theory of Diffraction (GTD) [2]. The UTD is discussed in some detail in Chapter II.

The reader is referred to reference [3] for a general review of the Radar Cross Section (RCS) analysis prior to 1965. In addition, Ruck, etc. [4], and Crispin and Seigel [5] present a summary of the early RCS techniques as well as a summary of the RCS characteristics of canonical shapes. There are two noteworthy early efforts directed toward treating complex shapes. At the University of Michigan, a serious attempt was
made to treat the component parts [6] of a target such as an aircraft and then obtain the RCS of the complete vehicle via superposition [7]. There was also a substantial effort at The Ohio State University to measure the RCS of such vehicles and check the accuracy of these measurements by computations.

In the 1960's, Northrop Corporation became interested in developing computer programs for RCS analysis. At first their interest centered on the integral equation techniques [8] but it soon became evident that this method was only applicable to targets of small electrical size due to the limited memory capacity of the computers. In the 1970's, the development of the so-called Physical Theory of Diffraction (PTD) [9] followed. Using the fringe currents of Ufimtsev [10] as a correction to the Physical Optics (PO) [11], they developed a set of rather widely used computer codes for RCS analysis of complex shapes when the wavelength(\(\lambda\)) is much smaller than the body size. However, these codes are extremely expensive to use in terms of computer time because in general an integration is required over the surface. In addition, the cost becomes substantially greater when multiple scattering interactions are involved.

The uniform ray theory of diffraction or the UTD offers the advantage that it analyzes the high frequency scattering from the target in terms of the distinct scattering mechanisms associated with the specular or flash points on the target and avoids integration over surface currents required by the PTD. Thus it is expected that UTD will produce accurate solutions for the scattered fields and yet substantially minimize the required computer time.
In the late 1960's, Ryan [12] considered the RCS of several sectionally continuous bodies of revolution by including in his solution first order contributions such as the Geometrical Optics Field (GO) [11], Keller's [2] edge diffracted fields and the creeping wave mechanisms. Ryan and Peters [13] also formulated the Equivalent Current (EC) concept in order to correct the ray solution at axial caustics and obtain the scattered field from finite edges. This concept was originally introduced by Millar [14] and is more extensively discussed in Chapter II.

The present dissertation employs the UTD and EC method to obtain the monostatic RCS from much more complex shapes than those studied by Ryan. In particular, it considers the high frequency backscattering of a plane wave from perfectly conductive structures formed of inlets and plates over a smooth surface. The analysis is such that it is applicable to almost any surface. However, for testing purposes, it was only applied here to cylindrical and ogival surfaces. The inlets and plates or fins connected to this surface can also be of general shape and orientation (the internal portion of the inlet is not considered here but was studied elsewhere [15]).

The major contribution of this work is the application of the UTD to configurations with arbitrary smooth surfaces. Thus, accurate RCS results from complex structures can be obtained since such structures can be modeled accurately. Our solution will not only include the direct scattered field contribution from each part of the structure, i.e., the surface, the inlet or the plate, but also the backscattered
field caused by the interactions of the inlet with the surface or the plate with the surface. These higher order contributions are significant in certain regions, especially where the RCS is relatively low. Measured data will also be provided and it will be seen that they agree very well with the calculated results.

During the course of this study, in addition to the introduction of generalized numerical "raytracing routines", some new concepts will also be presented. Particularly, when in Chapter V, the RCS of an inlet with a curved rim over a surface is considered, a new set of equivalent line currents obtained from the GO scattering is derived to be used in caustic regions. These are integrated over a line on the rim, thus eliminating a physical optics approach which requires a surface integral. Also, in the analysis of a finite plate over a surface, the junction-corner and junction-edge fields are introduced. These are then used for the computation of the fields caused by the discontinuities formed at the connection of the plate with the surface. A brief description of the material in each Chapter is outlined below:

Chapter II gives a summary of the basic theory of the UTD as it applies to various canonical shapes such as the curved wedge, corner and convex surface. It also presents some additional basic theory to be used in our study. Therefore, it will serve as a reference to the following Chapters.

Chapter III derives a generalized solution of the backscattered fields from a general shape thin-edge inlet over an arbitrary smooth surface. In the lit region, the solution includes the GO field from the
arbitrary surface, the diffracted field from the inlet edge, the reflected-diffracted field between the surface and the inlet, and the reflected-diffracted-reflected field. The last three components are computed via the integration of a set of equivalent line currents over the edge (see Chapter II). These currents are expressed in terms of the UTD diffraction coefficients and in case of the reflected-diffracted and reflected-diffracted-reflected field, their evaluation required the development of a numerical routine to trace all the reflected rays from the arbitrary surface to the inlet edge. In the shadow region, only the surface diffracted-edge diffracted-surface diffracted field is included in the solution along with the GO field. One should also note that our solution in this region applies to convex surfaces only since the currently available surface diffraction coefficients are only valid for such geometries.

Chapter IV applies the analysis in Chapter III to a cylindrical inlet over an ogive and a solid cylinder. The equivalent current solution is also compared to that which includes the GO field from the surface and the field associated with the backscattered rays of the above mechanisms. The last approach is usually referred to here as the ordinary UTD solution which fails at caustics and does not incorporate the field contribution from the junction of the inlet with the surface. In addition, the calculated results from the model of the cylindrical inlet over the ogive is compared with measurements for the $E_\phi$ and $E_\theta$ polarizations of the incident wave.

Chapter V deviates from the analysis in Chapters III and IV, and is devoted to the backscattering of an inlet with a curved rim (instead of
a thin-edge) over an arbitrary surface. The same type of mechanisms discussed in Chapter III are included in this solution except that the diffraction in those mechanisms is now replaced by a reflection at the rim. Also, in this chapter, a new set of equivalent line current is derived in terms of the reflection coefficients and the radii of curvature of the reflected wave from the inlet rim.

Chapter VI develops the theory for computing the backscattered field from a plate formed of linear segments over an arbitrary surface. Only the lit region is considered here. Furthermore, the equivalent currents are not used in this problem. Instead, the ordinary UTD fields, which also include those diffracted through the corners (see Chapter II), are used. The presented solution includes the GO, the corner diffracted, the reflected-edge diffracted, the reflected-corner diffracted, the reflected-corner diffracted-reflected, the junction-corner and the junction-edge fields. The last two types of fields are caused by the discontinuities formed at the junction of the plate with the surface and are derived here for a three dimensional structure. It is noted that the calculation of the reflected-edge diffracted field required the development of a numerical routine to simultaneously trace the reflection point on the arbitrary surface and the corresponding diffraction point on the edges. In addition, the computation of the reflected-reflected and reflected-edge diffracted-reflected fields is discussed and developed in this chapter but these fields were not used here. Multiple edge and multiple corner diffracted fields are also not incorporated in this solution.
Chapter VII presents measured and calculated data from a plate over an ogive. Two types of models were considered. In one case, the plate was some distance, $d$, away from the surface and for the other, the plate was connected to the surface.

Finally, Chapter VIII summarizes the results and accomplishments of this study, and the scope for future work.
CHAPTER II
THEORETICAL BACKGROUND

A. INTRODUCTION

This chapter presents the basic formulation of the Uniform Geometrical Theory of Diffraction (UTD). The UTD is an extension of Keller's Geometrical Theory of Diffraction (GTD) and is valid at the shadow and reflection boundaries. The concepts of GTD and UTD are discussed and the diffraction coefficients associated with the various mechanisms are given. In addition, some standard notational form is introduced to be used throughout the dissertation. A brief description of the backscattered field is also presented from the GTD point of view. The computation of this type of field is required for the evaluation of the Radar Cross Section (RCS) of a particular structure. A more elaborate representation of the backscattered field, which is specialized to the case of scattering from inlets, fins, wings, or plates on an arbitrary surface will be given later in Chapters III and VI when such structures are studied in detail.
B. A SHORT HISTORICAL SURVEY AS NEEDED FOR THIS APPLICATION

The GTD was formally developed by Keller [2] and his colleagues in the 1950's as an extension of Geometrical Optics (GO) field. Earlier in time, Van Kampen [16] recognized the critical points in Kirchoff's aperture integral and he noted that the integral consisted of contributions from first, second, and third order effects. These correspond to the geometrical optics, edge diffracted and corner diffracted fields (see Figure 2.1). Keller [17,18] introduced explicit diffraction coefficients in his computation of the diffracted field scattered by a two-dimensional screen.

Keller's solutions, though, were invalid in the transition regions adjacent to the shadow and reflection boundaries (see Figure 2.2) of a curved wedge. In the early 1970's, Kouyoumjian and Pathak [19], motivated by the work of Pauli [20] and Oberhettinger [21], derived compact, uniform diffraction coefficients which were valid in these troublesome boundary regions. Lee and Deschamps [22] also derived a solution for these fields. Kouyoumjian and Pathak's solution is now generally known as the Uniform Geometrical Theory of Diffraction (UTD) due to its continuity throughout space including the transition regions.

When the incident field on the edge is rapidly varying, additional asymptotic terms are required in the diffraction coefficients. Lee and Deschamps [22] included such terms in their solution and simultaneously Hwang and Kouyoumjian [23] developed a practical uniform slope diffraction coefficient which accounted for these variations. Recently, Veruttipong [24] obtained a more rigorous dyadic slope diffraction
a). Reflection.

b). Edge diffraction.

Figure 2.1. Ray mechanisms.
c) Corner diffraction.

Figure 2.1. (Continued).

d) Surface diffraction.
Figure 2.2. Incident, reflected, edge diffracted and surface diffracted rays along with their associated boundaries projected onto the plane normal to the edge at $Q_e$. 
coefficient, valid in the lit and shadow regions, by employing the
method of Hwang and Kouyoumjian along with plane wave spectrum analysis.
However, it should be noted that Karp and Keller [18] originally used
slope diffracted fields when they treated the hard screen. Also,
Rudduck and Wu [25] used the concept of slope diffraction in their
study of radiation from parallel plate waveguides, as well as Mentzer,
et al., [26] who applied it to horn antennas.

The GTD is also used to analyze the diffraction by smooth convex
surfaces. In the early 1950's, Franz and Depperman [27] introduced the
concept of creeping or surface waves in their attempt to explain the
ripple in the RCS pattern of a circular cylinder. These are modeled by
rays which travel on the surface of a curved structure and continually
shed energy. Levy and Keller [28] later pursued a quantitative study to
develop surface diffraction coefficients in order to incorporate the
fields due to this mechanism in their solution similarly to the edge
diffracted fields. Voltmer [29] later extended the work of Hong [30] to
arrive at solutions which included up to second order terms.

The construction of a uniform solution for the diffraction by
smooth convex surfaces has been the task of long investigation. Buchal
and Keller [31], Zauderer [32,33], Brown [34], and Ludwig [35] obtained
uniform solutions which were valid in the transition regions but
cumbersome for engineering applications. Logan and Yee [36] and
Borovikov and Kinber [37] in their related papers gave article surveys
on the problem of diffraction by a convex surface. Advances in this
subject were drawn from the pioneering work of Fock [38]. Pathak [39]
obtained a uniform asymptotic solution to the canonical problem of a
smooth convex cylinder which supercedes the above limitations. Subsequently, Pathak, etc., [40] using the ansatz based on the cylinder solution in [39] arrived at a practical three dimensional UTD solution which is valid at all regions excluding the immediate vicinity of the surface in the shadow region. They also generated a more accurate reflection coefficient to maintain the continuity of the reflected and surface diffracted fields at the shadow boundary.

Solutions in the format of the UTD have also been developed for the problem of radiation and coupling associated with antennas on perfectly conducting convex curved surfaces (see Figure 2.3). Lee [41] also derived a solution for the coupling problem, but his result does not explicitly identify the effects of surface ray torsion since it is a heuristic modification of Fock's expression for the surface field of a source on a sphere. In addition, Mittra, and Safari-Naini [42] arrived at a uniform solution for the radiation problem. They obtained their results from just the canonical problem of the radiation from a source on a circular cylinder; therefore, they could not generalize their results to include torsion surface rays on more general convex bodies. Lately, Pathak, Wang, etc., [43,44] obtained uniform solutions for the radiation and coupling problems associated with antennas on a general convex surface. These explicitly show the effect of surface ray torsion and indicate that it is localized to the source on the surface and also at the field point if it lies on the surface.

The uniform diffraction coefficients as derived by Kouyoumjian, Pathak and their colleagues at The Ohio State University will be used in our study to evaluate the RCS of generalized structures. A summary of
Figure 2.3. Coupling and radiating rays from sources on a smooth, convex surface.
these techniques can be found in a book chapter by Kouyoumjian, Pathak and Burnside [1] and a detailed treatment in [45]. Their solutions have been proven quite successful on a variety of high frequency radiation and scattering applications. An accurate list of these applications is not our purpose but it includes the study of horn radiation [26,46,47], reflector radiation [48], and airborne antenna radiation [49,50,51,52]. It has also been used to compute RCS patterns of various surfaces [12], plates and fins [53,54]. One is referred to the IEEE Press, book [55] for an extensive treatment on the theory and applications of GTD as well as UTD.

Furthermore, one should realize that all ray techniques inherently fail at caustic regions. These occur when a congruence of rays are scattered at a common field point. Millar [14] first introduced the concept of Equivalent edge Currents (EC) which may be placed on the diffracting edge. These currents may be employed in the standard radiation integral to give a good approximation of the far zone scattered field at and away from caustics. Later, Ryan and Peters [13] formulated the equivalent edge current concept and clarified the nature of these currents. Knott and Senior [56,57] were at first critical of the equivalent current concept but later supported their use. Recently, Sikta and Peters [54] modified the placement for these currents in order to effectively treat scattering with oblique incidences. This matter will be discussed in Chapter IV.

The EC concept motivated Burnside and Pathak [58] to propose a numerically efficient corner diffraction coefficient in order to incorporate the corner effect in the framework of UTD. Chu [59]
improved this coefficient to account for edges with discontinuities in their second derivative.

Equivalent edge currents and the corner diffraction coefficient will be extensively used in this study. The following sections give a summarized description of the reflection and diffraction coefficients which are employed in the later chapters for RCS computations. A derivation of the equivalent edge currents is also presented here.

C. GEOMETRICAL OPTICS FIELD

Suppose that a high frequency field, \( E^i \), is incident on an arbitrary smooth surface as shown in Figure 2.4. This field may be generated from an electric or a magnetic source which produces a plane, cylindrical or spherical wave. In this study, a plane wave is assumed to be incident from the radar to the target. It can be generally expressed as

\[
\begin{align*}
E^i(x, y, z) &= [x E_x(x, y, z) + y E_y(x, y, z) + z E_z(x, y, z)] \frac{e^{-jks'}}{s}, \\
&= E_x(x, y, z) + y E_y(x, y, z) + z E_z(x, y, z) \frac{e^{-jks'}}{s}, \tag{2.1}
\end{align*}
\]

where \( s' \) is the distance from the source to the target and the \( e^{j\omega t} \) factor has been assumed and suppressed. The x, y, and z components of the incident field \( E^i \) are expressed by \( E_x^i, E_y^i, \) and \( E_z^i \), respectively and \( k = 2\pi/\lambda \) is the free space propagation factor. Also note that for RCS calculations \( s' \to \infty \) at the target.

The reflected field from the specular point (\( S_R \)) on a perfectly conducting smooth, convex surface is given by

\[
E^R(s') = E^i(S_R) \cdot \frac{\sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s')(\rho_2 + s')}}}{R} e^{-jks'} \tag{2.2}
\]
Figure 2.4. Geometry for reflection from a smooth arbitrary surface.
where $F^i(S_p)$ indicates the value of the incident field on the specular point, and $s^r$ is the distance along the reflected ray where $F^R$ is measured. $R$ is the dyadic reflection coefficient and is found by imposing the boundary conditions on the total field at the surface. Generally [40],

$$ \bar{R} = R_s \hat{e}_\perp \hat{e}_\perp + R_h \hat{e}_\parallel \hat{e}_\parallel $$

which in the deep lit region reduces to

$$ R_{s,h} = \mp 1. $$

The unit vector $\hat{e}_\perp$ (all symbols with a "hat" on the top will subsequently denote unit vectors) is perpendicular to the plane of incidence, i.e.,

$$ \hat{e}_\perp = \hat{e}_2 = -\hat{n} \times \hat{I}, $$

and $\hat{e}_\parallel$ is in the plane of incidence and perpendicular to the direction of incidence, $\hat{I}$. Therefore,

$$ \hat{e}_\parallel = \hat{I} \times \hat{e}_\perp. $$

The plane of incidence is defined as that containing the incident ray and the normal to the surface, $\hat{n}$. The unit vector $\hat{e}_\parallel$ is similar in nature to $\hat{e}_\perp$ and is given by
The unit vector $\hat{s}^r$ denotes the direction of the reflected ray and is determined by the following laws of reflection:

$$\hat{I} \cdot \hat{n} = \hat{s}^r \cdot \hat{n} = \cos \theta_i$$

and

$$\hat{I} \times \hat{n} = \hat{s}^r \times \hat{n} .$$

The first law simply requires that the angle, $\theta_i$, between the surface normal and the direction of incidence is equal to that formed by the same normal and the reflected ray. The second law satisfies the requirement that $\hat{s}^r$ must be in the plane of incidence. Consequently, from geometrical considerations,

$$\hat{s}^r = \hat{I} - 2(\hat{I} \cdot \hat{n})\hat{n} .$$

The square root portion of equation (2.2) is the spreading factor associated with the wavefront reflected by the surface. It was reduced from

$$A(s^r) = \sqrt{\frac{d\sigma^r}{d\sigma^r}} = \sqrt{\frac{\rho_1 \rho_2}{(\rho_1^2 + s^r)(\rho_2^2 + s^r)}} ,$$

$$(2.7)$$
where $\sigma_i$ and $\sigma_r$ are the projected areas to the incident and reflected wavefronts as illustrated in Figure 2.5. The principal radii of curvature of the reflected wavefront at the reflection point, $S_r$, are denoted by $\rho_1^r$ and $\rho_2^r$. These are associated with the principal surface directions $\hat{t}_1$ and $\hat{t}_2$, respectively. In addition, $\hat{t}_1$ and $\hat{t}_2$ are the surface tangents at $S_r$, in, and perpendicular to the plane of incidence, respectively.

A generalized expression for $\rho_{1,2}^r$ was found by Kouyoumjian [19] after diagonalizing the curvature matrix obtained by Deschamps [60] and are given as

$$\frac{1}{\rho_{1,2}^r} = \frac{1}{\rho_m^i} + \frac{1}{\cos \theta_i} \left[ \frac{\sin^2 \omega + \cos^2 \theta_i \cos^2 \omega + \cos^2 \omega \cos^2 \theta_i \sin^2 \omega}{R_1} + \frac{\cos^2 \omega \cos^2 \theta_i \sin^2 \omega}{R_2} \right]$$

$$\pm \frac{1}{2} \sqrt{\left( \frac{1}{\rho_1^i} - \frac{1}{\rho_2^i} \right)^2 + 4 \left( \frac{1}{\rho_1^i} - \frac{1}{\rho_2^i} \right) \left[ \frac{\sin^2 \omega - \cos^2 \theta_i \cos^2 \omega}{R_1} + \frac{\cos^2 \omega - \cos^2 \theta_i \sin^2 \omega}{R_2} \right] - \frac{4 \cos^2 \theta_i}{\cos^2 \theta_i} \frac{\left[ \frac{\sin^2 \omega + \cos^2 \theta_i \cos^2 \omega + \cos^2 \omega \cos^2 \theta_i \sin^2 \omega}{R_1} + \frac{\cos^2 \omega \cos^2 \theta_i \sin^2 \omega}{R_2} \right]^2}{R_1 R_2}}$$

or

$$\rho_{1,2}^r = \rho_{1,2}^r \left( \rho_1^i, \rho_2^i, R_1, R_2, \omega, \theta_i \right)$$

(2.12a)

(2.12b)

in which the "+" sign refers to $\rho_1^r$ and the "-" to $\rho_2^r$. The principal radii of the incident wavefront at the point of reflection are denoted by $\rho_1^i$ and $\rho_2^i$. They are associated with the principal directions $\hat{t}_1$ and $\hat{t}_2$, respectively (see Figure 2.6). In addition,

$$\frac{1}{\rho_m^i} = \frac{1}{2} \left( \frac{1}{\rho_1^i} + \frac{1}{\rho_2^i} \right).$$

(2.13)
Figure 2.5. Geometry associated with $d\sigma^i$ and $d\sigma^r$. 

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Figure 2.6. Relationship of the principal and soft, hard directions for the incident or reflected wavefront.

Note that for a plane wave incidence \( r_{1,2} \) and \( R_{1,2} \) are the radii of curvature of the surface at \( S_P \), and are associated with the principal surface directions \( \hat{\tau}_{1,2} \), respectively. Finally, \( \omega \) is the angle shown in Figure 2.4 and is defined by

\[
\omega = \begin{cases} 
\cos^{-1}(-\hat{\tau}_1 \cdot \hat{\tau}_2) = \cos^{-1}(\hat{\tau}_2 \cdot \hat{\tau}_1) & \text{for } -\hat{\tau}_1 \cdot \hat{\tau}_2 > 0 \\
\pi - \cos^{-1}(\hat{\tau}_2 \cdot \hat{\tau}_1) & \text{otherwise.}
\end{cases}
\]

(2.13)

Therefore, it is restricted to the first quadrant. When \( \omega = \pi/2 \) or \( \omega = 0 \) the incident plane coincides with one of the principal planes of the surface at \( S_P \). More precisely, in this case \( \hat{\tau}_1 = \pm \hat{\tau}_1 \) and \( \hat{\tau}_2 = \pm \hat{\tau}_2 \), respectively. When \( \hat{\tau}_1 = \pm \hat{\tau}_1 \), Equation (2.12) reduces to
\[
\frac{1}{\rho_1} = \frac{1}{\rho_1} + \frac{2}{R_1 \cos \theta_1} \quad (2.14a)
\]

and

\[
\frac{1}{\rho_2} = \frac{1}{\rho_2} + \frac{2 \cos \theta_1}{R_2} \quad . \quad (2.14b)
\]

Alternatively, when \( \hat{t}_1 = \pm \hat{t}_2 \),

\[
\frac{1}{\rho_1} = \frac{1}{\rho_1} + \frac{2 \cos \theta_1}{R_1} \quad . \quad (2.15a)
\]

and

\[
\frac{1}{\rho_2} = \frac{1}{\rho_2} + \frac{2 \cos \theta_1}{R_2} \quad . \quad (2.15b)
\]

The principal directions, \( \hat{x}_1^r, \hat{x}_2^r \), corresponding to the principal radii of curvature, \( \rho_1^r, \rho_2^r \), respectively, are expressed by

\[
\hat{x}_1^r = \frac{\hat{x}_1^\perp - (Q_{22}^r - 1/\rho_1^r) \hat{x}_2^\perp}{(Q_{22}^r - 1/\rho_1^r)^2 + (Q_{12}^r)^2} \quad (2.16a)
\]

and

\[
\hat{x}_2^r = - \hat{x}_1^r \times \hat{x}_1^r \quad . \quad (2.16b)
\]

where

\[
\hat{x}_{1,2}^r = \hat{x}_{1,2} - 2(\hat{n} \cdot \hat{x}_{1,2})\hat{n} \quad . \quad (2.17)
\]
The components of the curvature matrix are given by

\[ Q_{11}^r = \frac{1}{\rho_1^1} + \frac{2}{R_\tau_1 \cos \theta_1}, \]  

(2.18a)

\[ Q_{12}^r = Q_{21}^r = -\sin 2\omega \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]  

(2.18b)

and

\[ Q_{22}^r = \frac{1}{\rho_2^2} + \frac{2 \cos \theta_2}{R_\tau_2}. \]  

(2.18c)

By Euler's theorem [61], one notes that

\[ \frac{1}{R_\tau_1} = \sin^2 \omega \frac{R_1}{R_2} + \cos^2 \omega \]  

(2.19a)

and

\[ \frac{1}{R_\tau_2} = \cos^2 \omega \frac{R_1}{R_2} + \sin^2 \omega. \]  

(2.19b)

\( R_\tau_1 \) and \( R_\tau_2 \) are the surface radii of curvature associated with the directions \( \hat{\tau}_1 \) and \( \hat{\tau}_2 \), respectively. Furthermore, note that the inverse of \( Q_{ij}^r \) give the principal radii of the reflected wavefront (see Equations (2.14) and (2.15)) when \( \omega = 0 \) or \( \pi/2 \) in which case \( Q_{12}^r = Q_{21}^r = 0 \). \( Q_{12}^r \) then accounts for the torsion effect of the surface on the ray field.
D. DIFFRACTION BY A WEDGE

The diffraction problem from a three-dimensional wedge is illustrated in Figure 2.7. The analysis presented here is that given in [19].

The scattered field from a wedge due to an incident field, $\mathbf{E}^i$, can be asymptotically expressed as

$$
\mathbf{E}D_e(s) \sim \mathbf{E}^i(Q_e) \cdot \mathbf{n}_e A(s) e^{-jks}
$$

(2.20)

where $s$ is the distance from the diffraction point ($Q_e$) to the observation point. For a given source origin and direction of diffraction, this point is unique since the diffracted rays are contained in a cone of rays such that $\beta_0 = \beta_0^i$ (see Figure 2.7).

The dyadic edge diffraction coefficient, $D_e$, is diagonal if it is defined in a special ray-fixed coordinate system. The edge-fixed plane of incidence, contains the incident ray and the edge vector, $\hat{e}$. The plane of diffraction contains the diffracted ray and $\hat{e}$. The unit vectors $\hat{\psi}$ and $\hat{\psi}'$ are perpendicular to the plane of diffraction and incidence respectively. Alternatively, the unit vectors $\hat{\beta}_0$ and $\hat{\beta}_0^i$ are parallel to the corresponding plane of diffraction and incidence such that

$$
\hat{\beta}_0 = \hat{s} \times \hat{\psi}
$$

(2.21a)

and

$$
\hat{\beta}_0^i = \hat{s}' \times \hat{\psi}',
$$

(2.21b)

where $\hat{s}' = \hat{1}$. 

26
Figure 2.7. Geometry for three-dimensional wedge diffraction problem.
The diffraction coefficient can now be expressed as

$$D_e = -\beta_0^* \beta_0 D_S - \psi^* \psi D_h.$$  \hfill (2.22)

Note that $D_S$ is referred to as the soft diffraction coefficient obtained when the boundary condition,

$$(E|_{\text{wedge}}) = 0$$  \hfill (2.23a)

is imposed on the total field and $D_h$ is the hard diffraction coefficient associated with the boundary condition,

$$\left( \frac{\partial E}{\partial n} \right|_{\text{wedge}} = 0.$$  \hfill (2.23b)

They are given by

$$D_{s,h}(L^1, \psi, \psi', \beta_0) = -e^{-j\pi/4} \frac{\cot(\pi + \beta^-)}{2n \sqrt{2\pi k \sin \beta_0}} [\cot(\pi + \beta^-) F(kL^1 a^+(\beta^-))$$

$$+ \cot(\pi - \beta^-) F(kL^1 a^- (\beta^-))] \pm \{ \cot(\pi + \beta^+) F(kL^1 a^+ (\beta^+))$$

$$+ \cot(\pi - \beta^+) F(kL^1 a^- (\beta^+)) \}.$$  \hfill (2.24)

with

$$a^\pm (\beta) = 2 \cos^2 \left( \frac{2n \pi N^\pm - \beta}{2} \right),$$  \hfill (2.25)

$$\beta = \beta^\pm = \psi \psi'.$$  \hfill (2.26)
and \(N^\pm\) are integers which most nearly satisfy the equations:

\[
2\pi n^+ - \beta = \pi \quad (2.27a)
\]

and

\[
2\pi n^- - \beta = -\pi \quad (2.27b)
\]

The complex transition function is denoted by \(F(x)\) and accounts for the continuity of the coefficient at the shadow and reflection boundaries. It is defined by

\[
F(x) = 2j \frac{2}{|x|} e^{j\alpha} \int_{|x|}^{\infty} e^{-j\tau^2} d\tau \quad (2.28)
\]

and is plotted in Figure 2.8. This can be evaluated in terms of Fresnel Integrals as given by Boersma [62]. However, for practical applications [45], the following approximation may be used:

\[
F(x) \sim \begin{cases} 
\sqrt{\pi x} - 2xe^{\pi/4}e^{j(\pi/4 + x)} & \text{for } x < .3 \\
F(x_i) + [F(x_{i+1}) - F(x_i)] \frac{x - x_i}{x_{i+1} - x} & \text{for } .3 < x < 5.5 \\
F(x_i) + \frac{j}{2x} - \frac{3}{4} \frac{1}{x^2} - j \frac{15}{8} \frac{1}{x^3} + \frac{75}{16} \frac{1}{x^4} & \text{for } x > 5.5 
\end{cases}
\]

(2.29)

where the values of \(x_i\) and \(F(x_i)\) are tabulated in Table 2.1. In addition, when \(x < 0\) then \(F(x) = F^*(|x|)\) where the * indicates the complex conjugate. The value of \(n\) in \(D_{s,h}\) is related to the wedge angle by
Figure 2.8. Transition function.

\[ F(KLa) = 2i \sqrt{KLa} e^{jKLa} \int_{-\infty}^{\infty} e^{-jT^2} dT \]
TABLE 2.1
LINEAR INTERPOLATION DATA FOR F(x)

<table>
<thead>
<tr>
<th>x_i</th>
<th>F(x_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.5729 + j0.2677</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6768 + j0.2682</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7439 + j0.2549</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8095 + j0.2322</td>
</tr>
<tr>
<td>1.5</td>
<td>0.8730 + j0.1982</td>
</tr>
<tr>
<td>2.3</td>
<td>0.9240 + j0.1577</td>
</tr>
<tr>
<td>4.0</td>
<td>0.9658 + j0.1073</td>
</tr>
<tr>
<td>5.5</td>
<td>0.9797 + j0.0828</td>
</tr>
</tbody>
</table>

WA = (2-n)\pi, \quad (2.30)

and the terms associated with \(\psi-\psi'\) and \(\psi+\psi'\) account for the continuity across the shadow and reflection boundaries, respectively.

The quantity A(s) is the ray divergence factor given by

\[
A(s) = \begin{cases} 
\frac{1}{\sqrt{s'}} & \text{for plane, cylindrical and conical wave incidence,} \\
\sqrt{\frac{s'}{s(s'+s)}} & \text{for spherical wave incidence.}
\end{cases} \quad (2.31)
\]

The L parameter is defined by

\[
L^i = \frac{s(\rho_1 s + 1)p_1^i p_2^i \sin^2 \beta_0}{\rho_e^i (\rho_1^i s + 1)(\rho_2^i s + 1)} \quad ,
\quad (2.32)
\]
where \( \rho_{1,2} \) are the principal radii of the incident wavefront at \( Q_e \) and \( \rho_e \) is the radius of curvature of the incident field in the plane containing \( \hat{e} \) and \( \hat{I} \). According to Euler's theorem [61], it is found by

\[
\frac{1}{\rho_e} = \frac{\cos^2 \alpha_0 + \sin^2 \alpha_0}{\rho_1 \cdot \rho_2},
\]

(2.33)

where

\[
\alpha_0 = \cos^{-1} (\hat{\chi}_1 \cdot \hat{e})
\]

(2.34)

and \( \hat{\chi}_1 \) is the principal direction associated with \( \rho_1 \).

At the shadow boundary, the following limit should be used for numerical calculation of \( D_{s,h} \):

\[
\cot \left( \frac{\pi \pm \beta}{2n} \right) F[kL^1 a^\pm(\beta)] = n \left[ \sqrt{2\pi kL^1} \right] \text{sgn} \varepsilon \\
- 2kL^1 \varepsilon \exp(j \pi/4) \exp(j \pi/4),
\]

(2.35)

where

\[
\beta = 2\pi n N \pm (\pi - \varepsilon)
\]

and \( \varepsilon \) is small.

When \( n = 2 \), the wedge becomes a half plane and the form of \( D_{S,h} \) reduces to
\[ D_{s,h}(\psi, \psi', \beta_0) = \frac{-e^{-j\pi/4}}{2\sqrt{2\pi k \sin \beta_0}} \left[ \frac{F(k\pi a(\rho^-))}{\cos \beta^-/2} - \frac{F(k\pi a(\rho^+))}{\cos \beta^+/2} \right]. \]  

2.36

E. DIFFRACTION BY A CURVED WEDGE

The geometry of the curved wedge is shown in Figure 2.9. The general expression for the diffracted field is still given by Equations (2.20) - (2.22) but a more general form of the quantities \( A(s) \) and \( D_{s,h} \) is required which may be reduced to those in Equations (2.24) and (2.31) for a planar wedge.

According to [19], one obtains

\[ A(s) = \sqrt{\frac{\rho}{s(\rho+s)}}, \]  

2.37

where \( \rho \) is the caustic distance between the caustic at the edge and the apparent caustic away from the edge as defined by the astigmatic ray tube. It is found by using

\[ \frac{1}{\rho} = \frac{1}{\rho_e} - \frac{\hat{n}_e \cdot (\hat{r} - \hat{s})}{a_e \sin^2 \beta_0}. \]  

2.38

The unit vector \( \hat{n}_e \) is directed away from the center of curvature of the edge at \( Q_e \) and \( a_e > 0 \) is the radius of curvature of the edge at that point.
Figure 2.9. Geometry for three-dimensional curved wedge diffraction problem.
The diffraction coefficients for the curved edge are now given by

\[
D_{s,h}(L^i, L^n, L^o, \psi, \psi', \beta_0) = \frac{e^{-j\pi/4}}{2n \sqrt{2\pi k \sin \beta_0}} \left[ \frac{2 \sin(\pi/n) F[kL^i(\psi-\psi')] \cos(\pi/n) - \cos((\psi-\psi')/n)}{2 \sin(2\pi/n)} \right],
\]

\[
\pm \left\{ \frac{\cot(\pi + (\psi + \psi'))}{2n} F[kL^n a^+(\psi + \psi')] \right\}
+ \cot(\pi - (\psi + \psi')) F[kL^o a(\psi + \psi')]\right\}
\]

in which

\[
a(\gamma) = 2 \cos^2(\gamma/2)
\]

and

\[
a^+(\gamma) = 2 \cos^2(2\pi n - \gamma)
\]

One should note that the difference between the diffraction coefficients of the planar and curved wedge are simply the L parameters. For the curved case they have been modified to account for the two reflection boundaries associated with the "o" and "n" surfaces of the wedge.

In particular, the reflection distance parameters are given by

\[
L^o = \frac{s(\rho e^o + s) \rho_1 \rho^o \rho^o \sin^2 \beta_0}{\rho e^o (\rho_1 \rho^o + s)(\rho^o + s)}
\]

which accounts for the reflection boundary at $(\pi - \psi')$, and
for that at \([(2n-1)\pi-\psi']\). The principal radii of the reflected wavefront at \(Q_e\) are denoted by \(\rho_{r1,2}^r\) and are computed by Equation (2.12). The superscripts "o" or "n" may be applied when they are related with the "o" or "n" side of the curved wedge. In addition, the quantity \(\rho_{e}^r\), is the radius of curvature of the reflected wavefront at \(Q_e\) in the plane containing the reflected ray and \(\hat{ê}\). It can be found by

\[
\frac{1}{\rho_{e}^r} = \frac{1}{\rho_e^r} - \frac{2(\hat{n} \cdot \hat{n}_e)(\hat{I} \cdot \hat{n})}{a_e \sin^2 \beta_0} \tag{2.44}
\]

where \(\hat{n}\) is the surface normal at the "o" or "n" surfaces of the wedge.

The scalar diffraction coefficients for the half plane curved screen become

\[
D_{s,h}(L^i, L^r, L^r, \psi, \psi', \beta_0) = \frac{-\exp[-j(\pi/4)]}{2\sqrt{2\pi k \sin \beta_0}} \left\{ \frac{F[kL^i a(\psi-\psi')] \times F[kL^r a(\psi+\psi')]}{\cos[(\psi-\psi')/2]} \right\} \cos[(\psi+\psi')/2] \tag{2.45}
\]

since for this special case \(L^r = L^{ro} = L^r_0\).
Finally, it should be mentioned that although the high-frequency diffracted electric fields from a perfectly conducting curved wedge were written in a dyadic form, they can also be expressed in a matrix equation by

\[
\begin{bmatrix}
  E^D_{\psi} \\
  E^D_{\beta_0}
\end{bmatrix} =
\begin{bmatrix}
  -D_s & 0 \\
  0 & -D_h
\end{bmatrix}
\begin{bmatrix}
  E^I_{\beta_0} \\
  E^I_{\psi}
\end{bmatrix}
\frac{\sqrt{\rho}}{s(p + s)} \exp(-jks),
\]

with the high-frequency diffracted magnetic field

\[
H^D_{\psi} = \gamma_0 \hat{n} \times E^D_{\psi}
\]

and \( \gamma_0 \) the admittance of the medium.

F. CURVED SURFACE DIFFRACTION

The problem of diffraction by a smooth convex surface is illustrated in Figure 2.10. The following presentation is drawn from [40].

The shadow boundary separates the lit from the shadow region. In this region, the scattered rays are shed tangentially off the surface. On the surface, they propagate along geodesic paths which are determined by Fermat's principle [63]. The diffracted fields in the shadow region are more conveniently expressed in the coordinate system composed of the surface normal, \( \hat{n} \), the surface tangent, \( \hat{\tau} \), in the direction of incidence or diffraction and the binormal to the surface defined by \( \hat{b} = \hat{\tau} \times \hat{n} \). The diffracted fields in the shadow region can now be expressed as
Figure 2.10. Geometry for diffraction by a smooth convex surface.
where $s^d$ is the distance from $S_d$ to $P$ as shown in Figure 2.10.

The dyad, $\mathcal{T}$, is given by

$$\mathcal{T} = b_{d_1} b_{d_2} T_s + n_{d_1} n_{d_2} T_h ,$$

with

$$T_{s,h}(S_i, S_d, x^d) = \sqrt{m(S_i)m(S_d)} \sqrt{2/K} e^{-j(\pi/4+kt)} \left[ \frac{F(x^d)}{2\sqrt{\pi} \xi^d} \right] - \left\{ \frac{p^*(\xi^d)}{q^*(\xi^d)} \right\} \sqrt{\frac{dn(S_i)}{dn(S_d)}} .$$

The above coefficient is valid even in the transition region around the shadow boundary. In fact, the first term in the square brackets is similar to Kirchoff's edge diffraction coefficient and is dominant when the diffracted ray is close to the shadow boundary. In contrast, the second term becomes important in the deep shadow region. The functions $p^*(\xi)$ and $q^*(\xi)$ are the complex conjugates of the universal functions given by Logan [36]. The functions $e^{-j\pi/4}p^*(\xi)$ and $e^{-j\pi/4}q^*(\xi)$ are plotted in Figures 2.11 and 2.12, respectively. Other parameters appearing in $T_{s,h}$ are defined below:

$$m(S) = \left[ \frac{kR_x(S)}{2} \right]^{1/3} ,$$

(2.51a)
Figure 2.11. Plot of $e^{-j\pi/4}p^*(\xi)$ vs $\xi$ based on Logan's tabulated data [36] for $p(\xi)$. 
Figure 2.12. Plot of $e^{-i\pi q_*(\xi)}$ vs $\xi$ based on Logan's tabulated data [36] for $q(\xi)$. 
\[ \xi^d = \int_{S_i}^{S_d} \frac{m(t')}{R(t')} \, dt' \quad \text{(2.51b)} \]

\[ t = \int_{S_i}^{S_d} dt' \quad \text{(2.51c)} \]

\[ \chi^d = \frac{kL_d(\xi^d)^2}{2m(S_i)m(S_d)} \quad \text{(2.51d)} \]

and

\[ L^d = \frac{\rho^d_1 \rho^d_2}{(\rho^d_1 + s)(\rho^d_2 + s)} \cdot \frac{s^d(\rho^d_2 + s^d)}{\rho^d_2} \quad \text{(2.51e)} \]

The principal radii of the incident wavefront at \( S_i \) are denoted by \( \rho^i_1,2 \) and \( \rho^d_2 \) is the distance from the caustic at \( S_d \) to the second caustic as illustrated in Figure 2.10. \( R(\tau(S)) \) is the surface radius of curvature along the ray. The quantity \( \sqrt{dn(S_i)/dn(S_d)} \) is the spreading factor of the surface diffracted rays. For a general surface, its computation is difficult but for cylindrical structures it is given by [39]

\[ \sqrt{\frac{dn(S_i)}{dn(S_d)}} = \sqrt{\frac{\rho^i_{\tau_1}}{\rho^i_{\tau_1} + t}} \quad \text{(2.52)} \]

where \( \rho^i_{\tau_1} \) is the radius of curvature of the incident wavefront in the direction of \( \tau_1 \). Note that \( \rho^i_{\tau_1} \) can be computed using Equation (2.33). Finally, for plane wave incidence \( \rho^i_{\tau_1} \to \infty \) and therefore the spread
factor becomes unity for cylindrical surfaces. In addition, one finds

$$\rho_d^2 = \rho_{\tau_1}^0 + t$$

(2.53)

for such surfaces.

In the deep lit region the scattered fields correspond to the GO field as discussed in Section C of this chapter. Within the transition region, though, a more general form than Equation (2.4) is needed for $R_{s,h}$ and is given by [39]

$$R_{s,h}(S_r, \xi^L, \chi^L) = \sqrt{\frac{-4}{\xi^L}} e^{-j(\xi^L)^3/12} e^{-j\pi/4} \begin{bmatrix} F(XL) - \frac{p^*({\xi^L})}{2\pi \xi^L} \\ \frac{q^*({\xi^L})}{2\pi \xi^L} \end{bmatrix}$$

(2.54)

The transition function $F(XL)$ plays the same role in Equation (2.54) as that of $F(Xd)$ in Equation (2.50). They both go to unity away from the shadow region and ensure the uniformity of the fields at the shadow boundary. The various new parameters in the generalized expression of $R_{s,h}$ are defined as follows:

$$\xi^L = -2m(S_r)\cos\theta_1$$

(2.55a)

$$\chi^L = 2kL^L\cos^2\theta_1$$

(2.55b)

and

$$L^L = \frac{\rho_1 \rho_2}{(\rho_1^s + s^p)(\rho_2^s + s^p)} \frac{s^p(\rho_2^s + s^p)}{\rho_2^p}$$

(2.55c)
Note that for continuity, $p_2^d = \rho_2^d$ and $L^d = L^d$ at SB. When the incident wavefront is converging, then $L^L, d$ may become negative in which case the transition function is computed as $F(X^L, d) = F^*|X^L, d|$. Outside the transition region $R_{s,h}$ of Equation (2.54) reduces to that of Equation (2.4). Therefore, the generalized reflection coefficient may be used throughout all lit regions without employing Equation (2.4) even if this type of approach becomes slightly inefficient for computational purposes.

G. EQUIVALENT CURRENT FORMULATION

When a confluence of rays from an edge contribute to the far field scatter direction, then the caustic distance, $\rho$, in Equation (2.37) becomes infinite. This type of behavior causes a caustic and is illustrated in Figure 2.13 for a plane wave incidence. Obviously, in this direction, the UTD result for the edge diffracted fields becomes invalid in the far field ($s \rightarrow \infty$) and an integral approach is required for their evaluation.

One can generate a set of electric and magnetic equivalent currents (EC) [13,14] along the edge which can be treated as line sources in the radiation integral to compute the field in the vicinity of the caustic. This method is analogous to Physical Optics. The derivation of the EC is such that the fields resulting from the radiation integral for an infinite edge and the UTD are equal where $s$ is finite. An alternate derivation of these currents from those presented in [13,14,54] is given below [56].
Looking at Figure 2.14 for an electric current, $I^e$, along the edge, the radiated fields, with $r$ sufficiently large, are given by

\[
\overline{E} = \frac{j k Z_0}{4 \pi} \int_{\theta_0}^{\hat{\theta}_0} e^{i \theta - i k (r + z \cos \beta_0)} dz
\]

in which the phase term $k z \cos \beta_0$ is associated with the incident ray field. Evaluating the integral of Equation (2.56a) by the stationary phase approximation one obtains

\[
\overline{E} = \frac{j k Z_0}{4 \pi} \int_{\theta_0}^{\hat{\theta}_0} e^{i \theta} \frac{\sin \beta}{r} e^{-j kr} dz
\]
Figure 2.14. Illustration of the diffracted field at P by UTD and that radiated at P due to an equivalent line current $I^e$ or $I^m$. 

a). By UTD.

b). By equivalent currents.
\[ E = \hat{\theta}(z_s) \frac{j k z_o \sin \beta(z_s)}{4\pi r(z_s)} I_o \sqrt{\frac{z}{k \phi''(z_s)}} e^{-j[k \phi(z_s) + \pi/4 \text{sgn}\phi''(z_s)]} \] (2.57)

where

\[ \phi(z) = r + z \cos \beta_0 = \sqrt{x^2 + (x - z_p)^2} + z \cos \beta_0 \] (2.58)

and

\[ \phi'(z_s) = 0 \] (2.59)

Therefore, the stationary point, \( z_s \), satisfies the equation,

\[ \cos \beta_0 = \frac{z_p - z_s}{\sqrt{x^2 + (x - z_p)^2}} = \cos \beta \] (2.60)

which is the law of edge diffraction. In addition, at \( z_s \), \( \hat{\theta} = \hat{\beta}_0 \) and

\[ \phi''(z_s) = \frac{\sin^2 \beta_0}{s} \] (2.61)

Substituting the above expressions into Equation (2.57), one obtains

\[ E_{\beta_0} = \frac{j k z_o}{4\pi} I_o \sqrt{\frac{2\pi}{k s}} e^{-j k s} e^{-j k z_s \cos \beta_0} e^{-j\pi/4} \] (2.62)
Alternatively, by the use of the UTD, for a plane wave incidence, the diffracted field is given by (see Equations (2.20), (2.22) and (2.24))

\[ E_{\beta_0}^{\text{UTD}} = -E_{\beta_0}^i(z_s)D_s(L^i, \psi, \psi', \beta_0) \frac{e^{-jks}}{\sqrt{s}}. \]  \hspace{1cm} (2.63)

Comparing Equation (2.62) with (2.63), it is seen that

\[ I^e = -E_{\beta_0}^i(z_s) D_s(L^i, \psi, \psi', \beta_0) \frac{8\pi}{k} e^{-j\pi/4}, \] \hspace{1cm} (2.64)

and since \( E_{\beta_0}^i(z_s) = E_z^i \sin \beta_0 \), then

\[ I^e(z) = -\hat{z} \cdot E^i(z) D_s(L^i, \psi, \psi', \beta_0) \frac{8\pi}{k} e^{-j\pi/4} \] \hspace{1cm} (2.65)

where \( Z_0 \) is the impedance of the medium.

When an equivalent magnetic current is assumed on the edge, the same procedure can be followed to give

\[ I^m(z) = -\hat{\epsilon} \cdot (\hat{z} \times \hat{s}^i) \cdot E^i(z) D_s(L^i, \psi, \psi', \beta_0) \sqrt{\frac{8\pi}{k}} e^{-j\pi/4}. \] \hspace{1cm} (2.66)

In conclusion, the edge diffracted fields can be equivalently computed by integrating the above EC, \( I^e \) and \( I^m \), over an infinite line along the edge. The stationary point of the radiation integral
corresponds to the edge diffraction point which satisfies the law of diffraction. The EC method remains valid in the caustic regions and can also be used for obtaining the scattered field from finite edges. For the last case, the EC integral is only extended over the length of the edge. In the next section, the asymptotic contribution of the end points of this integral is discussed in order to derive a corner diffraction coefficient.

For a variable \( \hat{\alpha} \) (curved edges or wedges) the currents \( I_0^{E,m} \) will not be a constant and the expressions for \( I_0^{E,m} \) is still valid if the edge diffraction coefficient is replaced by its more general form in Equation (2.39), i.e.,

\[
\bar{I}^e = -\hat{\alpha} \frac{\hat{\alpha} \cdot \hat{E}^l}{Z_0 \sin \beta_0} D_s(L^i, L^o, L^n, \psi, \psi', \beta_0) \sqrt{\frac{8\pi}{k}} e^{-j\pi/4}
\]

and

\[
\bar{I}^m = -\hat{\alpha} \frac{(\hat{\alpha} \times \hat{s}) \cdot \hat{E}^l}{\sin \beta_0} D_h(L^i, L^o, L^n, \psi, \psi', \beta_0) \sqrt{\frac{8\pi}{k}} e^{-j\pi/4}.
\]

The parameters associated with the diffraction coefficients are now a variable at each curved edge (wedge) point and are as defined in Sections D and E of this chapter. Equations (2.67) and (2.68) can be derived by again considering the evaluation of the EC integral with the stationary phase approximation. When no stationary point exists and the edge is finite, the above current expressions can still be used, and the scattered field by the EC method will only be that due to the edge terminations. In addition, they are also valid for arbitrary sources.
In this case, \( s' \) and \( E^i \) can take any form. Chapter III deals with such cases along with general integral expressions for computing the scattered fields from any type of edge or incidence. Modification to the free space Green's function will also be discussed for treating caustic fields caused by multiple ray interactions.

H. DIFFRACTION BY A CORNER

The basic concept of diffraction by a corner was illustrated in Figure 2.1. An eigenfunction solution to this problem has been derived by Satterwhite [64] but is extremely inefficient for numerical calculations. An approximate corner diffraction coefficient was then proposed by Burnside and Pathak [50,58] in the context of UTD to efficiently compute the corner diffracted fields. This coefficient was derived by asymptotically evaluating the EC current integral of Equation (2.56) for a finite edge (wedge) of length \( l \) (see Figure 2.13) and considering its endpoint contributions.

According to this result the corner diffracted fields associated with one edge and a corner can be expressed by

\[
E^D_c(s) = E^i \cdot \overline{D}_c e^{-jks} \tag{2.69}
\]

Looking at Figure 2.15 the dyad \( \overline{D}_c \) is given by

\[
\overline{D}_c = -\hat{\phi}_0 \hat{\phi}_0 D_{cs} - \hat{\psi}' \hat{\psi} D_{ch} \tag{2.70}
\]
Figure 2.15. Geometry for corner diffraction problem.

in which the units vectors are computed with respect to the incident corner ray, \( \hat{s}_c \), and that diffracted from the corner, \( \hat{s} \), as given in Equation (2.21). \( D_{cs, ch} \) are the usual soft and hard corner diffraction coefficients. For a plane angular sector (n=2) these are given by

\[
D_{cs, ch}(L^1, L_c, \psi, \psi', \beta_c, \beta_{oc}, \beta_o) = \frac{j \sqrt{\sin \beta_c \sin \beta_{oc}}}{k \sin \beta_o (\cos \beta_{oc} - \cos \beta_c)} F[k L_c a(\pi + \beta_{oc} - \beta_c)] \cdot \\
\left\{ \begin{array}{c} F[k L^1 a(\beta^+)]/\cos(\beta^+/2) \quad F\left[\frac{L^1 a(\beta^-)/\lambda}{k L_c a(\pi + \beta_{oc} - \beta_c)}\right] \quad \pi \\
F[k L^1 a(\beta^-)]/\cos(\beta^-/2) \quad F\left[\frac{L^1 a(\beta^+)/\lambda}{k L_c a(\pi + \beta_{oc} - \beta_c)}\right] \end{array} \right\}
\]

(2.71)
where $F(x)$, $\lambda^1$, $a(\beta)$, $\beta^2$ and $\beta_0$ are associated with the edge diffraction point, $Q_e$, and were defined in Section D of this chapter. The new parameters, $\beta_c$ and $\beta_{oc}$ are shown in Figure 2.15 and $L_c$ [59] is given by

$$L_c = \frac{\rho_s}{\rho + s},$$  \hspace{1cm} (2.72)

where $\rho$ is the caustic distance computed by Equation (2.38) at the corner. For a planar edge $\rho + s_c$ and $L_c$ becomes

$$L_c = \frac{s_c s}{s_c + s},$$  \hspace{1cm} (2.73)

Note that $L_c$ enforces continuity across the shadow boundary of the edge diffracted field, i.e., edge diffracted fields exist only if the point of diffraction $(Q_e)$ occurs within the edge limits. In addition, the factor $\left| F\left[ \frac{L^1 a(\beta)}{\lambda} \right] \right| \left| F[kL^1 a(\beta)] \right|$ in $D_{cs, ch}$ is a heuristic function to ensure that the corner diffraction coefficient will not change sign when it passes through the SBs of the edge.

The complete solution for the corner diffracted fields must also include the contribution associated with the other edge which forms the corner in a similar manner as above.

Even though this solution is not rigorous, experimental results show that it is sufficiently accurate for engineering applications. In our study, it will be applied for calculating the fields from the corners of fins or general shape plates.
I. BACKSCATTERED FIELD

The backscattered field includes only those ray field contributions which generate a scattering ray, \( s \), such that \( \hat{s} = -\hat{I} \), where \( \hat{I} \) will always denote the direction of the incident plane wave to the target and \(-\hat{I}\) the backscatter direction. In general, this field may be expressed as (see Figures 2.16, 2.17 and 2.18)

\[
E_{\text{BSC}} = E_{\text{GO}} + E_{\text{UTD}} + E_{\text{GO-UTD}}.
\]

(2.74)

The first term includes all singly and multiply reflected fields as illustrated in Figure 2.16. The second term incorporates all first and higher order edge, corner and surface diffracted fields. Examples of these mechanisms are shown in Figure 2.17. The last term in Equation (2.74) contains all ray mechanisms which involve reflection and diffraction. Some examples are the Reflected-Edge Diffracted (R-D\(_e\)), Edge Diffracted-Reflected (D\(_e\)-R), R-D\(_e\)-R, Reflected-Corner Diffracted (R-D\(_c\)), D\(_c\)-R, or R-D\(_c\)-R, etc., fields as illustrated in Figure 2.18.

A representation of \( E_{\text{UTD}} \) and \( E_{\text{GO-UTD}} \) can be given as follows:

\[
\overline{E}_{\text{UTD}} = \sum \overline{E}^D_e + \sum \overline{E}^D_c + \sum \overline{E}^D_s + \sum \overline{E}^D_e \overline{E}^D_e + \\
\sum (\overline{E}^D_e \overline{E}^D_c + \overline{E}^D_s \overline{E}^D_s) + \sum (\overline{E}^D_e \overline{E}^D_s + \overline{E}^D_s \overline{E}^D_e) + \ldots.
\]

(2.75)
Figure 2.16. Examples of reflected mechanisms in the backscatter direction.

Figure 2.17. Examples of first and second order backscatter diffraction mechanisms.
Figure 2.18. Examples of backscatter mechanisms involving reflection and diffraction.
\[ E^{\text{GO-UTD}} = \sum (E^{\text{RD}_{e}} + E^{\text{DE}_{e}}) + \sum (E^{\text{RD}_{c}} + E^{\text{DE}_{c}}) + \sum E^{\text{RD}_{e}} + \sum E^{\text{RD}_{c}} + \ldots \]  

The symbolism \( E^{\text{De}} \) denotes fields due to edge diffraction, \( E^{\text{Dc}} \); due to corner diffraction, \( E^{\text{Ds}} \); due to surface diffraction, \( E^{\text{RD}_{e}} \); due to Reflection-Edge Diffraction, etc. The number of ray field components which can be included in the above expressions can become enormous if all backscatter interactions are included. However, one needs to only consider a few mechanisms in order to obtain a reasonably accurate result. In the high level RCS region, usually the lower order terms are sufficient as compared to those required in the low level RCS regions. The significance of a particular higher order mechanism to the total field can only be judged when compared to the next lower order mechanism. In general, when a difference of greater than 10 dB exists between the two mechanisms in all regions, then this higher order ray field may be ignored. Alternatively, when an important ray mechanism is neglected, the UTD result will inherently show a pattern discontinuity at its SB, RB or existance boundaries whenever a complementary component has been included. The size of the discontinuity is proportional to the importance of the neglected ray field. Usually, for RCS calculations, 1-2 dB pattern jumps are acceptable. One should develop a much better understanding of these matters in the next chapters.
In our study, the solution of $E^{BSC}$ in the lit region generally will include all mechanisms involving up to a single diffraction and any number of reflections. When within the shadow region, where RCS is usually low, second and third order diffraction terms may be required for an accurate result.

Finally, once $E^{BSC}$ has been computed, the Radar Cross Section, Echo Area, or Echo Width, $\sigma$, is given by [65]

$$\sigma = 4\pi s^2 \lim_{s \to \infty} \left| \frac{E^{BSC} \cdot \hat{E}}{E^I} \right|^2$$

(2.77)

where $s$ is the distance from the reference point on the target to the radar and $\hat{E}$; the polarization of $E^I$. 

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CHAPTER III
BACKSCATTERING FROM A THIN-EDGE INLET OVER
AN ARBITRARY SMOOTH SURFACE

A. INTRODUCTION

This chapter presents a generalized formulation for the computation of the RCS from a thin-edge inlet of any shape when placed over an arbitrary smooth surface. Such a structure is shown in Figure 3.1. This study is basically devoted to backscattering from the face of the inlet in the region of $0 < \phi < \pi$ (see Figure 3.1). The internal portion of the intake is neglected in this study, although it has been studied elsewhere \[15\]. In addition, the back of the inlet is assumed to be tapered, thus eliminating any of its significant contributions in the backscatter direction. The solution employs the integration of the EC over the knife edge of the inlet. These currents are generated by the various interacting ray field mechanisms.

The EC method has the capability of treating general inlet structures without major modifications, and unlike the ordinary UTD, it is valid at caustic regions which are often encountered in this
Figure 3.1. Sketch of an arbitrary thin-edge inlet structure.
structure. In addition, this method can inherently include the
contribution of the fields due to the corners formed by the termination
of the junctions of the inlet with the surface. The currently available
corner diffraction coefficient, discussed in section H of Chapter II,
can only be applied to simple edge terminations. However, this
coefficient will be used later to treat the corner scattering from a
plate or fin over the surface.

The EC technique requires that the following information about the
inlet structure be available:

At each point on the surface on which the inlet
is supported (see Figure 2.4)

a. the outward normal, $\hat{n}$,
b. the principal radii, $R_1$ and $R_2$,
c. The corresponding principal surface directions,
   $\hat{t}_1$, and $\hat{t}_2 = \hat{n} \times \hat{t}_1$,

and for the inlet (see Figure 3.2)

e. the locus of the leading knife edge,
f. the edge vector, $\hat{e}$, tangent to the edge,
g. the normal to the edge curvature (curvature vector)
   $\hat{n}_e = \frac{d\hat{e}}{dz} / |\frac{d\hat{e}}{dz}|$,
h. the normal to the surface containing the inlet edge,
   this is labeled as $\hat{n}_I$,
i. the principal radii of the surface of the inlet, $R_{I1}$
   and $R_{I2}$ at the position of the edge or rim, and
j. the corresponding principal directions, $\hat{t}_{I1}$ and
   $\hat{t}_{I2} = \hat{n}_I \times \hat{t}_{I1}$.
Figure 3.2. Definition of parameters associated with the inlet edge.
The edge locus can either be defined in functional or numerical form.

Different ray mechanisms exist in the lit and shadow region of the inlet structure as shown in Figure 3.3. In the lit region each diffraction point on the inlet rim can be seen directly by the incident rays. In contrast, when in the shadow region, the same points can only be reached by creeping waves (see Figure 2.10). These begin to travel on the surface from an earlier point and then shed energy toward the inlet. Each of the regions will be studied separately.

The reader is cautioned that our solution in the shadow region is only applicable to convex surfaces. In addition at the SB, the surface diffracted field is non-ray optical and one could then argue that the UTD cannot be used to compute the diffracted field from the edge. However, the UTD is valid if one first computes the edge diffracted field and then that diffracted by the surface. Therefore, by imposing reciprocity, the UTD can still be used in the first case even if the fields become non-ray optical.

B. RCS IN THE LIT REGION

In the lit region, only mechanisms involving a single diffraction are included for the calculation of the EC. It will be shown that ray mechanisms which involve multiple diffractions are not needed for a sufficiently accurate result in the case of inlet sizes with a mean radius of greater than a wavelength. The procedure, though, outlined in this chapter, is quite general so that additional field mechanisms may be incorporated to the total solution if desirable. This can be accomplished by altering the modified Green's function introduced later.
a). Projection in the plane containing $\hat{I}$ and $p_e^0$.

b). Projection in the plane containing $\hat{I}$ and $p_e^i$.

Figure 3.3. Definition of the lit and shadow regions for the inlet structures.
c). A three dimensional sketch.

Figure 3.3. (Continued).
Figure 3.4. Illustration of the dominant ray field mechanisms from the thin-edge inlet structures.
c). Reflected-Edge diffracted-Reflected rays.

Figure 3.4. (Continued).
Accordingly, there are four significant ray field mechanisms from the face of the inlet which contribute to the backscatter direction. These are illustrated in Figure 3.4 and are identified as follows:

a. Geometrical Optics (GO) field, $E^\text{GO}$

b. Singly edge diffracted field ($D_e$), $E^1$ e

c. Reflected-Edge diffracted field ($R-D_e$), $E^\text{RDe}$

d. Edge diffracted-Reflected field ($D_e-R$), $E^\text{DeR}$

e. Reflected-Edge diffracted-Reflected field ($R-D_e-R$), $E^\text{RDDeR}$

One, of course, could specifically include in the above list the field contributions from the two junction terminations where the inlet is attached to the surface. However, these are included implicitly by means of the finite limits of the EC integrals and therefore they will not be studied as a separate entity in this chapter.

By reciprocity, for a plane wave incidence and far field calculations,

$$E^\text{DeR} = E^\text{RDe} \quad (3.1)$$

Therefore, only the $R-D_e$ field component needs to be calculated. The total backscattered field in the lit region can then be given to this order of approximation as

$$E^{\text{BSC}} = E^\text{GO} + E^1 + 2E^\text{RDe} + E^\text{RDDeR} \quad (3.2)$$

and the RCS, $\sigma$, is given by Equation (2.77).
Some additional higher order diffracted terms which were neglected are indicated in Figure 3.5. These include the $D_e-D_e$ ray field from the rim of the inlet and possible $D_e-R-D_e$ mechanisms. Of the second types, one is associated with surfaces which contain a normal in the direction of the diffracted rays from the inlet edge and the other could exist for some inlet geometries where the surface has normals in the plane of the inlet rim. As discussed earlier, these must be incorporated in our solutions for a sufficiently accurate result for inlets with mean radius of less than or equal to a wavelength.

The GO field exists whenever a surface normal coincides with the direction of incidence ($\hat{I}$). In such a case, it is simply obtained from Equation (2.2) as $s \to \infty$ and is given by

$$E^{GO} = -E^i \sqrt{R_{1}^{GO} R_{2}^{GO}} e^{-j2k(S_{GO} - \overline{O_e}) \cdot \hat{I}} e^{-jks}.$$  \hspace{1cm} (3.3)

In the above, $\overline{O_e}$ denotes the vector from the origin 0 (see Figures 3.1 and 3.2) to the reference point $O_e$ (all points with a bar will always have this meaning herein), and $R_{1,2}^{GO}$ are the principal radii of the surface at the specular point, $S_{GO}$ (see Figure 3.4).

All other ray fields include an edge diffraction. As discussed in Chapter II, each of these mechanisms can be modeled by a set of EC, $I^e$ and $I^m$ on the inlet rim (see Equations (2.67) and (2.68)). These currents are then used in the radiation integral to obtain the appropriate scattered fields as follows:
Figure 3.5. Some higher order ray field mechanisms associated with the thin-edge inlet structures.
c). Other type of $D_e$-$R$-$D_e$ ray.
\[
\tilde{E}^e = -Z_0 \frac{e^{-jks}}{s} \int_{\text{rim}} \tilde{r}^e(\xi) \hat{e}(\xi) \cdot \tilde{g}^e(\xi) \, d\xi ,
\]
(3.4a)

and

\[
\tilde{E}^m = \frac{e^{-jks}}{s} \int_{\text{rim}} \tilde{r}^m(\xi) \hat{e}(\xi) \cdot \tilde{g}^m(\xi) \, d\xi ,
\]
(3.4b)

Note that \( \lambda \) is along the inlet edge (see Figure 3.2), and the total scattered field for the modeled mechanism is given by

\[
\tilde{E} = \tilde{E}^e + \tilde{E}^m .
\]
(3.5)

In addition, in the far field

\[
\tilde{E} = \frac{Z_0 H}{s} .
\]
(3.6)

The functions \( \tilde{g}^{e,m}(\lambda) \) denote modified electric and magnetic dyadic Green's functions and their form can be quite general in order to accommodate the multiple interactions of a single ray. One can then obtain the Green's function associated with the total scattered field as the sum of these individual ray Green's functions. The rest of this chapter is devoted to the computation of \( \tilde{I}^{e,m}(\lambda) \) and \( \tilde{g}^{e,m}(\lambda) \) for the various ray mechanisms to be used in computing the backscattered field from thin-edge inlets.
1. Singly Edge Diffracted Field

For an arbitrary edge, \( I^{e,m}(\xi) \) are still defined by (see Equation (2.67) and (2.68))

\[
I^{e,m}(\xi) = -\left( \frac{\hat{e} \cdot \hat{e}^I}{z_0} \right) e^{i(0_e)D_s} h(L^i, L^r, L^F, \psi, \psi', \beta_0^i) \frac{2\sqrt{\lambda}}{\sin\beta_0^i} e^{-j\pi/4} .
\]

However, the parameters associated with the diffraction coefficient must be found by more general expressions. Their pictorial definition is shown in Figure 3.2 and are computed as follows:

\[
\beta_0^i = \cos^{-1}(\hat{I} \cdot \hat{e}) ,
\]

\[
\psi' = \begin{cases} 
\cos^{-1}(-\hat{I}_p \cdot \hat{t}_e) & \text{for } -\hat{n}_I \cdot \hat{i}_p > 0 \\
2\pi - \cos^{-1}(-\hat{I}_p \cdot \hat{t}_e) & \text{for } -\hat{n}_I \cdot \hat{i}_p < 0 
\end{cases}
\]

and

\[
\psi = \psi' .
\]

Note that \( \hat{t}_e \) is the unit vector defined by

\[
\hat{t}_e = \hat{n}_I \times \hat{e} ,
\]

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and \( \hat{I}_p \) is the unit vector projection of \( \hat{I} \) on the plane normal to \( \hat{e} \). Its derivation is given in Appendix A, and the results are stated below:

\[
I_{px} = \csc \theta \left[ (e_y^2 + e_z^2)I_x - e_x (e_y I_y + e_z I_z) \right], \tag{3.11a}
\]

\[
I_{py} = \csc \theta \left[ (e_x^2 + e_z^2)I_y - e_y (e_x I_x + e_z I_z) \right], \tag{3.11b}
\]

and

\[
I_{pz} = \csc \theta \left[ (e_x^2 + e_y^2)I_z - e_z (e_x I_x + e_y I_y) \right], \tag{3.11c}
\]

where \((e_x, e_y, e_z)\) denote the \(x, y, z\) components of \( \hat{e} \) and likewise for the unit vectors \( \hat{I} \) and \( \hat{I}_p \). Furthermore, for RCS computations \((s, s' \to \infty)\) the \( L \) parameters given in Equations (2.32) and (2.43) reduce to

\[
L^i = \infty \tag{3.12a}
\]

and

\[
L^r = \frac{\rho_{11} r_{12}^{11}}{\rho_{61}^i} \sin^2 \theta \tag{3.12b}
\]

where the reflected radii \( \rho_{11}^{12} \) are associated with the surface of the inlet at the rim. (The notation \( \rho_{2m}^{rk} \) refers to the \( g \)th reflected radius at the \( m \)th interaction of the \( k \)th mechanism and will be used throughout this dissertation.) The parameters to be used for evaluating

\[
\rho_{11,21}^{r} = \rho^r (\infty, \infty, R_{11}, R_{12}, \omega_{11}, \theta_{11}^1) \tag{3.13}
\]
of Equation (2.12) are found by

$$\omega_1^I = \cos^{-1}(\frac{\hat{\mathbf{I}}_2 \cdot \hat{\mathbf{I}}_1}{|\hat{\mathbf{I}}_2 \times \hat{\mathbf{I}}_1|}) ,$$  \hspace{1cm} (3.14)$$

$$\theta_1^I = \cos^{-1}(|\hat{\mathbf{I}}_1|) ,$$  \hspace{1cm} (3.15)

and

$$\hat{\mathbf{I}}_2 = -\hat{\mathbf{n}}_I \times \hat{\mathbf{I}}_1 / |\hat{\mathbf{n}}_I \times \hat{\mathbf{I}}| .$$  \hspace{1cm} (3.16)

In addition, according to Equation (2.44),

$$\rho_{e1} = -\frac{a_e \sin^2 \beta_0}{2(\hat{\mathbf{n}}_I \cdot \hat{\mathbf{n}}_e)(\hat{\mathbf{I}} \cdot \hat{\mathbf{n}}_I)} ,$$  \hspace{1cm} (3.17)

where $a_e$ and $\hat{\mathbf{n}}_e$ are the curvature radius and normal, respectively, of the edge at $P_e$ which is the integration point along the rim. After substitution of the above parameters into Equation (2.39), the diffraction coefficient reduces to

$$D_1(\omega, \mathbf{L}^e, \mathbf{L}^r, \psi, \psi, \beta_0) = -\sqrt{\lambda} e^{-j\pi/4} \left[ 1 \pm \frac{F(2kL^r \cos^2 \psi)}{\cos \psi} \right] ,$$  \hspace{1cm} (3.18)

and for large $L^r$ one finds

$$D_1(\omega, \omega, \omega, \psi, \psi, \beta_0) = -\sqrt{\lambda} e^{-j\pi/4} \left( 1 \pm \frac{1}{\cos \psi} \right) .$$  \hspace{1cm} (3.19)
Finally, $g^{e,m}(z)$ take their far field representation given by

$$g^{e,m}(z) = \left\{ \begin{array}{l} \hat{e}(z) \hat{e}(z) \\ \hat{e}(z)(\hat{\mathbf{S}} \times \hat{e}(z)) \end{array} \right\} \frac{jk}{4\pi} e^{-jk(\hat{P}_e - \hat{O}_e)} \cdot \hat{I}_e \left\{ \begin{array}{l} \hat{e}(z) \hat{e}(z) \\ \hat{e}(z)(\hat{\mathbf{S}} \times \hat{e}(z)) \end{array} \right\} \frac{jk}{4\pi} e^{jkd_2} . \tag{3.20}$$

From the exact Wiener-Holf solution [66] Bowman, etc.[67], obtained an asymptotic expression for the high frequency backscattered field from a semi-infinite hollow cylinder. When the first order EC solution of this subsection is specialized to such a geometry the results from Equation (3.5) and those in [67] are compared in Figure 3.6 for the $\phi$ (H-plane) and $\theta$ (E-plane) polarizations of incidence. Pathak and Huang [15] also calculated the first order, backscattered field from a semi-infinite cylinder by the same method described here. Their results are, of course, identical to ours.

2. Reflected-Edge Diffracted Field

For the R-$D_e$ mechanism, the incident field to the edge is that reflected from the surface (see Figure 3.4). Therefore, Equation (3.7) now takes the form

$$I^{e,m}(z) = -\left\{ \begin{array}{l} \hat{e} \cdot \overline{E}_2^R(P_e) \\ \hat{e} \times \hat{S} \end{array} \right\} \frac{Z_0}{\overline{E}_2^R(P_e)} D_{s,h}(L^1, L^\gamma, L^\alpha, \psi, \psi') \frac{2\sqrt{\lambda}}{\sin \theta_0^1} e^{-j\pi/4} \tag{3.21}$$

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Figure 3.6. \( E_\phi \) and \( E_\theta \) RCS patterns of a semi-infinite hollow cylinder.
where $E^R_2$ is the reflected field from $S_r$ when measured at the inlet edge in the direction of the reflected ray ($s^r$). The parameters associated with the diffraction coefficient in Equation (3.21) are given by

$$\beta^i_0 = \cos^{-1}(\hat{s}^r \cdot \hat{e}) \quad (3.22)$$

$$\psi' = \begin{cases} 
\cos^{-1}(\hat{s}^r \cdot \hat{e}) & \text{for } -\hat{n}_I \cdot \hat{s}^r > 0 \\
2\pi - \cos^{-1}(\hat{s}^r \cdot \hat{e}) & \text{for } -\hat{n}_I \cdot \hat{s}^r < 0 
\end{cases} \quad (3.23a)$$

$$\psi = \begin{cases} 
\cos^{-1}(\hat{I}_p \cdot \hat{t}_e) & \text{for } -\hat{n}_I \cdot \hat{I}_p > 0 \\
2\pi - \cos^{-1}(\hat{I}_p \cdot \hat{t}_e) & \text{for } -\hat{n}_I \cdot \hat{I}_p < 0 
\end{cases} \quad (3.23b)$$

$$L^i = \frac{r^{12}_{12} r^{12}_{22}}{\rho^{12}_{e2}} \sin^2 \beta^i_0 = \frac{(\rho^{12}_{11} + s^r)(\rho^{12}_{21} + s^r)}{\rho^{12}_{e2}} \sin^2 \beta^i_0 \quad (3.24a)$$

$$L^r = \frac{r^{12}_{12} r^{12}_{22}}{\rho^{12}_{e2}} \sin^2 \beta^i_0 \quad (3.24b)$$

and

$$s^r = \rho_e - \overline{s}^r \quad (3.25)$$
Note that \( s_\text{p}^r \) is similar in nature to \( \hat{I}_\text{p} \) and is given by Equation (3.11) if \( \hat{I} \) is replaced by \( s_\text{p}^r \), whereas \( \rho_{11,21}^r \) are the principal radii of the reflected wavefront at \( S_r \) and are associated with the principal directions \( \chi_{11,21}^r \) as defined in Equation (2.16). Further, \( \rho_{12,22}^i \) are the principal radii of the incident wavefront at the inlet edge, and their notational description is similar to that used for the reflected radii. \( \rho_{\text{e}2}^1 \) (see Equation (2.33)) is the radius of the same wavefront when evaluated in the plane of \( (\hat{e}, s_\text{p}^r) \); and \( \rho_{12,22}^r \) are the principal radii associated with the wavefront reflected off the inlet at \( P_\text{e} \). The above radii are found by the following expressions:

\[
\rho_{11,21}^r = \rho^r(\alpha, \omega, R_1, R_2, \omega_1, \theta_1) \tag{3.26}
\]

with

\[
\omega = \cos^{-1}(|\hat{\tau}_2 \cdot \hat{\tau}_1|) \tag{3.27}
\]

\[
\theta_1 = \cos^{-1}(|\hat{\theta} \cdot \hat{\tau}|) \tag{3.28}
\]

and

\[
\hat{\tau}_2 = -\hat{n} \times \hat{I} / |\hat{n} \times \hat{I}| \tag{3.29}
\]

Note also, that

\[
\rho_{12,22}^r = \rho^r(\rho_{11}^r + s_\text{r}, \rho_{21}^r + s_\text{r}, R_{11}, R_{12}, \omega_2, \theta_1^2) \tag{3.30}
\]
with

\[ \omega_{I}^2 = \cos^{-1}(|\hat{\mathbf{I}}_2 \cdot \hat{\mathbf{I}}_1|), \quad (3.31) \]

\[ \theta_{I1}^2 = \cos^{-1}(|\hat{n}_I \cdot \hat{\mathbf{s}}|), \quad (3.32) \]

and

\[ \hat{\mathbf{I}}_2 = -\hat{n}_I \times \hat{\mathbf{r}}/|\hat{n}_I \times \hat{\mathbf{r}}|. \quad (3.33) \]

In addition,

\[ \frac{1}{\rho_{22}^2} = \frac{1}{\rho_{12}^2} - \frac{2(\hat{n}_I \cdot \hat{\mathbf{e}})(\hat{\mathbf{s}} \cdot \hat{n}_I)}{a_e \sin \beta_0}. \quad (3.34) \]

and according to Equation (2.19) or (2.33),

\[ \frac{1}{\rho_{12}^2} = \frac{\sin^2 \omega_e}{\rho_{12}^2} + \frac{\cos^2 \omega_e}{\rho_{22}^2}. \quad (3.35) \]

where

\[ \omega_e = \cos^{-1}(|\hat{\mathbf{e}} \cdot \hat{\mathbf{r}}_2^2|). \quad (3.36) \]

Note that in the computation of \( \hat{\mathbf{r}}_{11,21}^2 \) one can choose the principal directions of the incident plane wave as

\[ \hat{\mathbf{r}}_{12}^{11,21} = \hat{\phi}, \hat{\theta}. \quad (3.37) \]
Also, \( g_{e, m}(z) \) is again given by Equation (3.20).

The reflected field in the EC expressions, according to Equation (2.2), is given by

\[
E_2^R(p_e) = \hat{X} \cdot E^l(0_e) \cdot R_1 \sqrt{\frac{\rho_{11}^2 \rho_{12}^2}{(\rho_{11}^2 + s^2)(\rho_{12}^2 + s^2)}} e^{-jk(S_r - O_e) \cdot \hat{n} - jksr}
\]

(3.38)

in terms of the principal plane directions. The dyad \( R_1 \) was defined in Equations (2.3) and (2.54), and in this case

\[
L_1^L = \frac{s^2(p_{21}^2 + s^2)}{p_{21}^2}
\]

(3.39)

and

\[
\frac{1}{R_1} = \frac{\sin^2 \omega}{R_1} + \frac{\cos^2 \omega}{R_2}
\]

(3.40)

In addition,

\[
\hat{X} = \hat{\lambda}_r^2 \hat{\lambda}_{11} + \hat{\lambda}_r^2 \hat{\lambda}_{21}
\]

(3.41)

The computation of \( E_2^R \) requires that the position of the surface point, \( S_r \), associated with each integration point, \( p_e \), be known. This can generally be found by imposing the reflection laws (see Equations (2.8) and (2.9)) governed by

\[
(\hat{n} \times \hat{l})(\hat{s}^r \cdot \hat{n}) = - (\hat{n} \times \hat{s}^r)(\hat{l} \cdot \hat{n})
\]

(3.42)
The above equation must be reduced to three equations in order to theoretically solve for \( S_{px}, S_{py}, \) and \( S_{pz} \) which are the \( x, y, z \) components of \( S_r \). However, one notes that \( \mathbf{n} \) is also a function of the point on the surface where \( S_r \) is to be evaluated. Therefore, an analytic expression for \( \mathbf{n} \) is either not available or too complex to consider an analytical solution for \( S_r \). A major task in this study is the development of an efficient numerical solution of Equation (3.42). Once this has been accomplished, the ray theory can easily be applied in the lit region for computing the scattered fields in the bistatic or backscattering cases.

The following presentation outlines a numerical solution to Equation (3.42). This is an iterative procedure and has been found to work well for finding \( S_r \). The only limitation to this routine is that for each ray incidence an initial specular point, \( S^0_r \), which satisfies the reflection laws should be available. However, this limitation will be overcome later. In addition, the surface parameters, \( R^0_1, R^0_2, \hat{\alpha}_1, \hat{\alpha}_2 \) and \( \hat{\mu}^0 \) must also be known at this point as discussed earlier. Our purpose would now be that of determining the next reflection point, \( S^1_r \), when the previous diffraction point, \( P^0_e \), is moved a small distance along the rim to \( P^1_e \) as shown in Figure 3.7.

In the neighborhood of \( S^0_r \) the convex surface may be approximately represented as

\[
z' = f(x', y') = -\frac{1}{2} \left( \frac{x'^2}{R^0_1} + \frac{y'^2}{R^0_2} \right)
\]

in the primed coordinate system which has its origin at \( S^0_r \) and the following coordinate directions:
The above expression for $f(x',y')$ was found by using a Taylor series expansion about $S_0$ for the position vector on the surface and retaining up to quadratic terms only.

The normal at $S_1$ which is in the neighborhood of $S_0$ is now given by

$$\mathbf{n}_1 = -\frac{\partial f}{\partial x} \hat{x}' - \frac{\partial f}{\partial y} \hat{y}' + \hat{z}'$$  \hspace{1cm} (3.45)

which reduces to

$$\mathbf{n}_1 = \frac{x'}{R_1^0} \hat{x}' + \frac{y'}{R_2^0} \hat{y}' + \hat{z}'$$  \hspace{1cm} (3.46)

Thus, the reflected ray associated with $S_1$ can be represented as

$$\mathbf{s}_1 = (p_{ex}' - x') \hat{x}' + (p_{ey}' - y') \hat{y}' + (p_{ez}' - z') \hat{z}'$$  \hspace{1cm} (3.47)

where $p_{ex}', y', z'$ are the components of $p_e^1$ in the primed coordinate system. These are found from the components of $p_e^1$ in the main coordinate frame as follows.
\[
\begin{bmatrix}
\hat{p}_x' \\
\hat{p}_y' \\
\hat{p}_z'
\end{bmatrix} = [T(\phi_r, 0)][T(\phi_0, \theta_0)]
\begin{bmatrix}
\hat{p}_x - s_{rx}^0 \\
\hat{p}_y - s_{ry}^0 \\
\hat{p}_z - s_{rz}^0
\end{bmatrix}
\quad (3.48)
\]

Note that the above transformation process involves one translation and three coordinate rotations (see Figure 3.8). First, the translation from the origin, 0, of the main coordinate system to the surface point \(S_r\) is performed. Then the \(x, y, z\) coordinate directions are rotated by angles \(\phi_0\) and \(\theta_0\). These are the spherical angles of \(n^0\) with respect to the \(x, y, z\) coordinate system, i.e.,

\[
\phi_0 = \tan^{-1} \left( \frac{n_{y}^0}{n_{x}^0} \right), \quad (3.49a)
\]

\[
\theta_0 = \cos^{-1}(n_{z}^0) \quad (3.49b)
\]

and \(n_{x, y, z}^0\) are the \(x, y, z\) components of the unit normal, \(\hat{n}^0\).

Therefore,

\[
[T(\phi, \theta)] =
\begin{bmatrix}
\cos \phi \cos \theta & \sin \phi \cos \theta & -\sin \theta \\
-\sin \phi & \cos \phi & 0 \\
\cos \phi \sin \theta & \sin \phi \sin \theta & \cos \theta
\end{bmatrix}
\quad (3.50)
\]

At this point of the transformation process, the origin is at \(S_r^0\) and the rectilinear coordinate direction are
Figure 3.8 Relationships among the coordinate systems \((x,y,z)\), \((x',y',z')\) and \((\theta_0,\phi_0,z')\).

\[
\begin{align*}
\theta_0 &= x\cos\theta_0 \cos\phi_0 + y\cos\theta_0 \sin\phi_0 - z\sin\theta_0 , & (3.51a) \\
\phi_0 &= -x\sin\phi_0 + y\cos\phi_0 & (3.51b)
\end{align*}
\]

and

\[
\hat{z'} = \hat{n}_0 .
\]

Since \(\hat{\theta}_0\) and \(\hat{\phi}_0\) are normal to \(\hat{z'}\), then they are in the same plane with \(\hat{t}_1\) and \(\hat{t}_2\). Consequently, for transforming to the primed coordinate system (see Equation \((3.44)\)), an additional rotation by
\[ \phi_r = \cos^{-1}(\hat{e}_0 \cdot \hat{t}_1) \]  

is done. Note that this transformation is often referred to as Eulerian and it should be clear that Equation (3.48) is quite general to accomplish any forward coordinate transformation. The inverse transformation is simply given by

\[
\begin{bmatrix}
\begin{array}{c}
\dot{p}_{ex} \\
\dot{p}_{ey} \\
\dot{p}_{ez}
\end{array}
\end{bmatrix}
= \begin{bmatrix}
s_0 \\
s_0 \\
s_0
\end{bmatrix}
\begin{bmatrix}
\cos \theta \\
\sin \theta \\
-1
\end{bmatrix}
+ [T^{-1}(\phi, \theta)][T^{-1}(\phi, \theta)]
\begin{bmatrix}
\dot{p}_{ex}' \\
\dot{p}_{ey}' \\
\dot{p}_{ez}'
\end{bmatrix},
\]  

(3.53)

where

\[
[T^{-1}(\phi, \theta)] = \begin{bmatrix}
\cos \theta \cos \phi & -\sin \phi & \sin \theta \cos \phi \\
\cos \theta \sin \phi & \cos \phi & \sin \theta \sin \phi \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}.
\]  

(3.54)

One can now proceed to find \( x' \) and \( y' \) by substituting Equations (3.46) and (3.47) into (3.42) to obtain,

\[
(n^{-1} \cdot s_1^T)(\gamma_{Z}^z, -I_{y}^z) = -(n^{-1} \cdot \hat{I})(\gamma_{Z}^z, -z') - (p_{ez}', -y'),
\]  

(3.55a)

\[
-(n^{-1} \cdot s_1^T)(\gamma_{Z}^z, -I_{x}^z) = (n^{-1} \cdot \hat{I})(\gamma_{Z}^z, -z') - (p_{ex}', -x'),
\]  

(3.55b)

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and
\[
(n^{-1} \cdot s_1)(x' I_y - y' I_x) = -(n^{-1} \cdot I)[x' (p^{1}_{ey} - y') - y' (p^{1}_{ex} - x')]
\]

where
\[
\hat{I} = x'I_x' + y'I_y' + z'I_z'.
\]

The dot products are given by
\[
\begin{align*}
(n^{-1} \cdot \hat{I})_{\text{R}_1} &= x' \frac{\hat{I}_x}{\text{R}_1} + y' \frac{\hat{I}_y}{\text{R}_2} + z' \frac{\hat{I}_z}{\text{R}_2} \\
\end{align*}
\]

and
\[
(n^{-1} \cdot \hat{s}_1) = x' \frac{(p^{1}_{ex} - x')}{\text{R}_1} + y' \frac{(p^{1}_{ey} - y')}{\text{R}_2} + (p^{1}_{ez} - z').
\]

The last equation reduces to
\[
(n^{-1} \cdot \hat{s}_1) = x' \frac{p^{1}_{ex}}{\text{R}_1} + y' \frac{p^{1}_{ey}}{\text{R}_2} + p^{1}_{ez},
\]

after neglecting higher order terms with respect to \(x'\) and \(y'\) (see Equation (3.43)).
When Equations (3.57) and (3.58b) are substituted into Equation (3.55) and only first order terms are retained, both sides of Equation (3.55c) become approximately equal to zero, and we have

\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\frac{x'}{R_1} \\
\frac{y'}{R_2}
\end{bmatrix}
= \begin{bmatrix}
C_{x'} \\
C_{y'}
\end{bmatrix}
\] (3.59)

where

\[
A_{11} = -(I_x' p_{ey}' + I_y' p_{ex}') ,
\] (3.60a)

\[
A_{12} = -2I_y' p_{ey}' + 2I_z' p_{ez}' + I_z' R_2^0 ,
\] (3.60b)

\[
A_{21} = -(2I_x' p_{ex}' + 2I_z' p_{ez}' + I_z' R_1^0) ,
\] (3.60c)

\[
A_{22} = -A_{11} ,
\] (3.60d)

\[
C_{x'} = I_y' p_{ez}' + I_z' p_{ey}' ,
\] (3.61a)

and

\[
C_{y'} = -(I_x' p_{ez}' + I_z' p_{ex}').
\] (3.61b)
A first order approximation of the new reflection point is then given by

\[ \vec{S}_r^1 = x\hat{x} + y\hat{y} + z\hat{z}, \quad (3.62) \]

where \((x,y,z)\) are the \((x',y',z')\) primed components when transformed to the main coordinate system by use of Equation (3.53). Because of the approximate nature of the above routine, there is no assurance that \(S_r^1\) as given above will be precisely on the actual surface, albeit, it will be quite close to it. It is, of course, necessary that the newly calculated point must be on the surface if this approach is to be meaningful. In order to compute such a point, \(S_r^1\) is projected onto the surface point \(S_{rp}^1\). However, it should be understood that a projection of a point on an arbitrary surface is not a simple exercise. For the purpose of continuity, this problem is discussed in the next chapter when specific bodies are studied. In addition, an efficient numerical procedure is now available for the projection of a point on any surface for which only the information outlined earlier are required. This will be published at a later date.

Once \(S_{rp}^1\) is found, the associated surface parameter \(n^1, t_1^1, R_1^1\) and \(R_2^1\) are known precisely and

\[ \vec{S}_s^1 = \vec{p}_e - \vec{S}_{rp}^1 \quad . \quad (3.63) \]
These can now be used in the exact reflection law of Equation (3.42) to check the preciseness of $S_{rp}^1$. In lieu of all previous approximations, one enforces that

\[
\hat{\xi_0} c_{ex} + \hat{\eta} c_{ey} + \hat{\zeta} c_{ez} \neq 0,
\]

(3.64)

where

\[
c_{ex} = \left( \frac{x_p^2}{R_1^2} + \frac{y_p^2}{R_2^2} + 1 \right) \left[ (\hat{n}^1 \cdot \hat{s}_1^r) (n_{y y}^1 z \cdot n_{z y}^1) + (\hat{n}^1 \cdot \hat{n}_x^1) (n_{z s_1^r}^1 - n_{s_1^r}^1) \right],
\]

(3.65a)

\[
c_{ey} = \left( \frac{x_p^2}{R_1^2} + \frac{y_p^2}{R_2^2} + 1 \right) \left[ (\hat{n}^1 \cdot \hat{s}_1^r) (n_{x z}^1 y \cdot n_{s_1^r}^1 x) + (\hat{n}^1 \cdot \hat{n}_y^1) (n_{z s_1^r}^1 - n_{s_1^r}^1) \right],
\]

(3.65b)

\[
c_{ez} = \left( \frac{x_p^2}{R_1^2} + \frac{y_p^2}{R_2^2} + 1 \right) \left[ (\hat{n}^1 \cdot \hat{s}_1^r) (n_{x y}^1 z \cdot n_{s_1^r}^1) + (\hat{n}^1 \cdot \hat{n}_y^1) (n_{x s_1^r}^1 z - n_{s_1^r}^1) \right],
\]

(3.65c)

\[
x_p = S_{rx}^0 - (S_{rp}^1)_x,
\]

(3.66a)

\[
y_p = S_{ry}^0 - (S_{rp}^1)_y
\]

(3.66b)
and

\[ \hat{s}_{1} = \hat{x} \hat{s}_{1x} + \hat{y} \hat{s}_{1y} + \hat{z} \hat{s}_{1z} \quad (3.67) \]

In order to compute the reflection point, \( \hat{s}_{r} \), associated with the next integration point, \( \hat{p}_{e} \), the above process must be iterated. The reference point will now be \( \hat{s}_{r} \) (this is from the final projection) instead of \( s_{r} \). \( \hat{p}_{e} \) is replaced by \( p_{e} \), \( n^{0} \) by \( n^{1} \), \( n^{1} \) by \( n^{2} \), and so on. However, one should be quite careful in continuing this iterative process. The errors given in Equations (3.65) will be continuously increasing and the method will eventually produce results of unacceptable accuracy. The increase in error is due to the violation of our original assumption that the reflection law equation be satisfied at the reference point. But since this error is now available, Equation (3.42) can be modified to give

\[ (\hat{n} \times \hat{l})(\hat{s} \cdot \hat{n}) = -(\hat{n} \times \hat{s})(\hat{l} \cdot \hat{n}) + \hat{x}'C_{ex}' + \hat{y}'C_{ey}' + \hat{z}'C_{ez}' \quad (3.68) \]

where \( C_{ex}', y', z' \) are the components of the \( \vec{C}_{e} \) vector from the previous iteration in the primed coordinate system. Noting that \( C_{ez}' = 0 \) since Equation (3.55c) does not contain any first order terms, Equation (3.59) becomes
The above procedure can be used to trace the specular point throughout the EC integration and will therefore be generally called as the "raytracing routine". Although it was discussed only for the case when the reflected ray (equivalently \( P_e \)) was perturbed, Equation (3.69) also applies to the case when the incident ray is varied (by less than 3 degrees at a time) and \( P_e \) is kept constant. Therefore, only the knowledge of the specular point, \( S^K_r \), associated with a single incidence, \( \hat{\mathbf{I}}_K \), is required. \( S^0_r \) for the desired initial incidence is then obtained by tracing \( S^K_r \) to \( S^0_r \) when \( \hat{\mathbf{I}}_K \) is slowly perturbed to \( \hat{\mathbf{I}} \) (see Figure 3.9), while the diffraction point remains constant. Consequently, for each pattern point (constant \( \hat{\mathbf{I}}_r \)) \( S^0_r \) is traced by perturbing the diffraction point \( P_e \), as discussed. One such \( S^K_r \) can be chosen as the projection of \( P^0_e \) on the surface. Then \( \hat{\mathbf{I}}_K = -n^K \), where \( n^K \) is the surface normal at \( S^K_r \). Therefore, an initial specular point is not required to be analytically calculated for any pattern computation. A computer program has been written using the previous approach which always begins the raytracing routine at \( S^K_r \) and arrives to the desired reflection point at the expense of additional computer time.

When \( C_{ex}' \) or \( C_{ey}' \) are large, say greater than \( 5 \times 10^{-3} \) using a 32 bit computer representation of a real number, this is an indication of a poor approximation to the new specular point. For a better
Figure 3.9. Raytracing of the reflection point when the incident ray is varied from $\hat{I}_K$ to $\hat{I}$. 
estimation of $S_r$ the raytracing routine may be reiterated with the last computed specular point as the reference and with $P_e$ and $I$ kept constant. In this iteration, the resulting $x'$ and $y'$ will be much smaller than before, and therefore Equation (3.43) becomes a more accurate representation of the surface at the next reflection point. The re-evaluation of the resulted $S_r$ may be continued until $C_{ex}'$ and $C_{ey}'$ are both less than $5 \times 10^{-3}$. When close to the shadow boundary or when $s'$ is small, a large number of spot iterations may be required to satisfy the above upper bound on the errors. Also, in certain cases, because of the extensive number of coordinate transformations involved in a single iteration, the accuracy of the computer may be reached when the errors are still greater than the above bound. Under these circumstances, a larger upper bound on the errors is probably acceptable in order to limit computer time without jeopardizing the accuracy of the results.

The value of the errors $C_{ex}'$ and $C_{ey}'$ can indicate when the SB of a diffraction point is reached. Beyond the SB, Equations (3.42) and (3.68) are not valid and the errors are increasing at the spot iterations instead of decreasing. For arbitrary smooth surfaces, this seems to be the only way to determine when a diffraction point has been shadowed. From this pattern point and on, the ray fields are treated as discussed in Section C of this chapter.
3. Reflected-Edge Diffracted-Reflected Field

The equivalent edge currents associated with the \( R-D_e-R \) ray field are the same as those given for the \( R-D_e \) field in equation (3.21) except for the following modifications:

\[
\psi = \psi'
\]  
(3.70)

and

\[
L^r = \frac{s^r \rho_3^r + s^r \rho_2^r}{\rho_2^r \rho_3^r} \frac{r_3^2 r_1^2 \sin^2 \theta_0}{\rho_2^r \rho_2^r} 
\]  
(3.71)

where (see Equation (3.30))

\[
\rho_1^3 = \rho_2^3
\]  
(3.72)

and (see Equation (3.34))

\[
\rho_2^3 = \rho_2^3
\]  
(3.73)

However, \( g_{e,m} \) for this mechanism must be formulated to account for the retracing of the radiated rays back to the receiver.

In order to derive the modified ray Green's function, one visualizes an infinitesimal current element (electric or magnetic), \( dl \), on the inlet's edge as shown in Figure 3.10. In the direction of \( -s^r \), this element produces a field which is given by (see also Equation (3.4))
Figure 3.10. Radiation of equivalent currents in the backscatter direction associated with the R-D_e-R mechanism.

\[ (dE^\text{E}_\text{m}^\text{e})_3 = \left\{ \frac{Z_0 \hat{\theta}_\text{e}(\lambda) I^e(\lambda)}{\hat{s}_r \times \hat{\phi}_\text{e}(\lambda) I^\text{m}(\lambda)} \right\} \frac{(1+jks^r)}{4\pi(s^r)^2} e^{-jks^r \sin \theta^e} dx, \]

where

\[ \hat{\theta}_\text{e} = \hat{s}_r \times \hat{\phi}_\text{e}, \]  
\[ \hat{\phi}_\text{e} = \hat{\varepsilon} \times \hat{s}_p, \]
\[ \theta^e = \cos^{-1}(-\hat{\varepsilon} \cdot \hat{s}_r), \]
and \( s^r \) can become small. Note also that \( s^p \) is the same as that in Equation (3.23a). Consequently, after reflection, the backscattered field due to this current element is given by

\[
(dE_3^{RD_e}) e^{s^m}(s) = (dE_3^{RD_e}) e^{s^m}(s^r) - R_3 \sqrt{p_{13}^r r_{23}^r} \frac{e^{-jks^r}}{s} e^{-jk(S^R - O_e) \hat{r}}.
\]  

(3.76)

The total R-D\(_e\)-R ray field from the inlet rim can now be calculated by integrating the above expression over the rim, viz.,

\[
E_3^{RD_e} = \int_{\text{rim}} (dE_3^{RD_e}) e^{s^m} + \int_{\text{rim}} (dE_3^{RD_e}) e^{s^m}.
\]  

(3.77)

After comparing Equation (3.76) with Equations (3.4a) and (3.4b), it is concluded that one obtains

\[
\hat{g} e^{s^m}(\lambda) = \begin{cases} \hat{e}(\lambda) \hat{S} e^{s^m}(\lambda) & R_3 \frac{1 + jks^r}{4\pi(s^r)^2} \sin \theta e^{\sqrt{p_{13}^r p_{23}^r}} e^{-jks^r} e^{-jk(S^R - O_e) \hat{r}}. \\ -\hat{e}(\lambda) \hat{S} x \hat{S} e^{s^m}(\lambda) \end{cases}
\]  

(3.78)

The principal radii \( \rho_{13, 23}^r \) are associated with the wavefront reflected from \( S^R \) toward the backscatter direction and are found by (see Equation (2.12))

\[
\rho_{13, 23}^r = \rho_{13}^r \rho_{23}^r R_1 R_2(\omega_i, \theta_i).
\]  

(3.79)

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where \( \omega \) and \( \theta_i \) were defined in Equations (3.27) and (3.28). The incident radii \( p_{13,23}^{i3} \) are in this case associated with the wave radiated by the infinitesimal current source and are simply

\[
p_{13}^{i3} = p_{23}^{i3} = s^r. \tag{3.80}
\]

Note that \( \bar{R}_3 \) is the generalized dyadic reflection coefficient defined in Equations (2.3) and (2.54). It is associated with the ray scattered from the rim and then reflecting off \( S_R \) toward the backscatter direction. Therefore, its parameters are given by

\[
L^L_3 = \frac{(s^r)^2}{p_{23}^{p3}}, \tag{3.81}
\]

and \( R_t \) by Equation (3.40).

C. RCS IN THE SHADOW REGION

A limiting case occurs when the incident ray to the rim point \( P_e \) lies on the shadow boundary as indicated in Figure 3.3. The reflection point \( S_R \) then coincides with \( S_d \) \( D_s-D_e-D_s \) mechanism as shown in Figure 3.11. Following the same procedure as discussed for the \( R-D_e-R \) mechanism the backscattered field due to a infinitesimal current source on the inlet can be expressed as
Figure 3.11. Geometry of surface diffracted rays to and from the inlet.

\[
(dE^mD_s)e, m(s) = (dE^mD_s)e, m(s^d) \cdot \frac{1}{\sqrt{s^d}} e^{-jk(S_i - \delta o)} \cdot \hat{I} \cdot e^{-jks} ,
\]

where

\[
(dE^mD_s)e, m(s^d) = \left\{ \begin{array}{l}
Z_0 \hat{\theta}_e(\lambda) I_e(\lambda) \\
\frac{1}{s^d} \hat{\theta}_e(\lambda) I_m(\lambda) \end{array} \right\} \frac{(1+jksd)}{4\pi(s^d)^2} \cdot e^{-jksd} \cdot \sin \theta_e dx,
\]

(3.83)

\[
\hat{\theta}_e = \hat{s}_d \times \hat{\phi}_e ,
\]

(3.84a)

\[
\hat{\phi}_e = \hat{e} \times \hat{s}_p ,
\]

(3.84b)

\[
\theta_e = \cos^{-1}(-\hat{e} \cdot \hat{s}_d) ,
\]

(3.84c)
\[
I_{e,m}(\lambda) = \begin{cases} 
\hat{e} \cdot \overline{D_s(s_d)} \\
(e \times \hat{s}^d) \cdot \overline{D_s(s_d)} 
\end{cases} D_s, h(L_i^{Li}, L_i^{Lr}, L_r^{Li}, L_r^{Lr}, \psi, \psi, \beta_i) \frac{2\sqrt{\lambda}}{\sin \beta_0} e^{-j\pi/4},
\]

\[
\overline{D_s(s_d)} = E_i(0_e) \cdot \overline{T_1} \sqrt{\frac{\rho_{21}}{sd(\rho_{21}^2 + sd^2)}} e^{-jk(\overline{S_i} - \overline{O_e}) \cdot \hat{e}} e^{-jks^d}
\]

\[
\beta_i = \cos^{-1}(-\hat{s}^d \cdot \hat{e}),
\]

and

\[
\psi = \begin{cases} 
\cos^{-1}(-\hat{s}_p^d \cdot \hat{e}) & \text{if } -\hat{n}_i \cdot \hat{s}_p^d > 0 \\
2\pi - \cos^{-1}(-\hat{s}_p^d \cdot \hat{e}) & \text{otherwise.}
\end{cases}
\]

Following our usual notation, \(s_p^d\) is given by Equation (3.11) if \(s_d^d\) is used in place of \(I\). Note that \(T_1\) is defined by Equation (2.49), viz.,

\[
\overline{T_1} = \hat{b}_i \hat{b}_d T_s(S_i, S_d, x_i^d) + \hat{n}_i \hat{n}_d T_h(S_i, S_d, x_i^d)
\]

and

\[
\overline{T_3} = \hat{b}_d \hat{b}_i T_s(S_d, S_i, x_2^d) + \hat{n}_d \hat{n}_i T_h(S_d, S_i, x_2^d)
\]

In addition, \(\overline{T_1}\) applies to the surface diffraction of the incident ray toward \(P_e\) and \(\overline{T_3}\) corresponds to the surface diffraction of the backscattered ray from the infinitesimal current source. Whereas
\( \rho_{21} \) is the caustic distance associated with \( T_i \) and the L parameters are given by

\[
L^i = \frac{s_d (\rho_{e2} + s^d) (\rho_{21} + s^d) \sin^2 \beta_0}{2 \rho_{e2} (\rho_{21} + 2s^d)} \tag{3.91a}
\]

and

\[
L^r = \frac{s_d (\rho_{e2} + s^d) \rho_{12} \rho_{22} \sin^2 \beta_0}{\rho_{e2} (\rho_{12} + s^d) (\rho_{22} + s^d)} \tag{3.92b}
\]

with

\[
\frac{1}{\rho_{e2}} = \frac{\sin^2 \omega_e}{s^d} + \frac{\cos^2 \omega_e}{\rho_{21} + s^d}, \tag{3.93}
\]

\[\omega_e = \cos^{-1}(|e \cdot b_d|), \tag{3.94}\]

and

\[
\frac{1}{\rho_{e2}} = \frac{1}{\rho_{e2}^i} - \frac{2 (\hat{n}_1 \cdot e)(s^d \cdot \hat{n}_1)}{a_e \sin^2 \beta_0}. \tag{3.95}
\]

The radius of curvature of the inlet edge is referred to as \( a_e \) and \( \rho_{12,22} \) are the radii of the wavefront reflected from the inlet. It should also be noted that all of the above parameters are calculated at the diffraction point \( P_e \).
The computation of the surface diffraction coefficients requires the knowledge of the geodesic paths on the convex surface (see Figure 3.10) associated with each edge point. If one assumes that they are known, the backscattered field can be evaluated as

\[
\overline{E}_{\text{BSC}} = \overline{E}_0 + \overline{E}_s \overline{D}_e \overline{D}_s = \overline{E}_0 + \int_{\text{rim}} (\overline{d} \overline{E}_s \overline{D}_e \overline{D}_s)^e + \int_{\text{rim}} (\overline{d} \overline{E}_s \overline{D}_e \overline{D}_s)^m
\]

(3.96)

when the whole rim is shadowed. Based on our previous development, the modified Green's function for the \( \overline{E}_s \overline{D}_e \overline{D}_s \) field is easily found to be

\[
\overline{g}^{e,m}(x) = \left\{ \begin{array}{c}
\hat{e}(x) \hat{e}(x) \\
\hat{e}(x)(\hat{s} d \times \hat{e}(x))
\end{array} \right\}
\]

\[
\cdot \frac{-T_3}{4\pi(s d)^2} \sin \theta \sqrt{\frac{d}{23}} e^{-jk(s d)} e^{-jk(s d - \theta e)}. \]

(3.97)

One should note the similarity of this expression to that given in Equation (3.78).

If only a part of the inlet is shadowed, the backscattered field must be computed with the illuminated portion treated as discussed in section B of this chapter and the rest as developed above.

Mathematically,

\[
\overline{E}_{\text{BSC}} = \overline{E}_0 + \int_{\text{C}_1} \overline{d} \overline{E}_s \overline{D}_e \overline{D}_s + \int_{\text{C}_2} \overline{d} \overline{E}_e \overline{D}_e \overline{D}_e + \int_{\text{C}_3} \overline{d} \overline{E}_e \overline{D}_e \overline{D}_e
\]

(3.98)
where \( P^1, C_2 \) denote the left and right end points of the inlet edge, respectively, and \( P^2, e, m \) are the last lit points from either side of the inlet edge.

Presently, the EC method has not been specifically applied in the shadow region. This is due to the unavailability of the geodesic paths for an arbitrary smooth surface. These are, of course, unique once the direction of incidence and the diffraction point on the inlet are defined. A study is currently being pursued on this subject [68].

When a major part of the inlet is illuminated, the integrand of the EC integrals for the R-D\(_e\) and R-D\(_e\)-R ray components can be numerically extrapolated in the shadowed part. Thus, the contribution of the EC due to the surface or creeping waves can be approximately incorporated. Such an approach, of course, requires that the inlet rim as well as the surface be smooth and continuous. The technique, although approximate, rendered good results with respect to the measured when at least half of the inlet rim was illuminated and an appropriate predictor-interpolator algorithm was chosen. One such algorithm which remains bounded at all times uses a summation of shifted \( \sin x \) functions and is therefore referred as Shannon's predictor-interpolator [69]. Accordingly, the integrand of Equations (3.4) can be expressed as

\[
\sum_{i=0,N,2N,...} E^{e,m}_{i} \cdot \sin\left[\frac{\pi}{d^i} \left| P^j - P^i \right| \right]
\]

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where $\hat{e}_I$ is the polarization of the incident field and the parameter $p_{e}^i$ is the ith sampling (or integration) point on the inlet rim as illustrated in Figure 3.10. The calculated integrand is given by $E_{i}^{e,m}$ at $z=p_{e}^i$ and $E_{j}^{e,m}$ corresponds to the predicted component at $z=p_{e}^j$. The infinitesimal lengths $dz^i = |p_{e}^{i+1} - p_{e}^{i}|$ can be a variable but for best results, they should not vary substantially from their adjacent counterparts. The arc length,

$$|p_{e}^{i+N} - p_{e}^{i}| = \sum_{k=i}^{i+N} dx^k$$  \tag{3.100}$$

is the sampling interval of the predictor and $N$ is an integer greater or equal to one. When $p_{e}^{j}$ is far away from the last lit point of the inlet, it is advantageous to use $N>1$, provided Nyquist’s theorem is not violated. This choice of $N$ limits the number of already predicted field points to be used for the prediction of a following point, thus increasing the accuracy of the result.

When most of the inlet is shadowed, the above method is not applicable. In the next chapter where the ogival surface with a cylindrical inlet is studied, one can resort to the use of ordinary UTD by considering only the stationary ray from the top of the cylindrical inlet. Thus, the contribution of the junctions is neglected which may be significant as will be discussed. In the region where the inlet is totally shadowed, only one stationary surface ray exists. For principal plane incidences ($I_y=0$), this ray follows a geodesic path
which is a great circle. Therefore, the parameters associated with the surface diffraction coefficients are easily calculated.

D. SUMMARY

In this chapter, the backscattered fields from a structure with an inlet over an arbitrary surface (see Figure 3.1) were computed. The solution in the lit region was comprised of the $G_0$, $D_e$, $R-D_e$, $D_e-R$ and $R-D_e-R$ fields. The last four mechanisms include a diffraction through the inlet edge and were computed via the EC method (see Chapter II). This technique is valid at caustic regions and can also incorporate the contribution of the fields from the junctions of the inlet with the surface. However, since the EC method involves an integration over the inlet rim, a raytracing routine had to be developed in order to trace the surface reflected rays to the rim.

When in the shadow region (see Figure 3.3), the $D_e$, $R-D_e$, $D_e-R$ and $R-D_e-R$ are replaced by the $R-D_s-R$ mechanism. The computation of this field component by the EC method requires the knowledge of the geodesic paths on the arbitrary smooth convex surface. Although the general mathematics for the application of this technique were developed in section C of this chapter, it was not applied at present since the geodesic paths are not yet available. Alternatively, a numerical extrapolation of the edge currents will be used when a major part of the inlet is lit. Otherwise, only the ray through the stationary inlet point will be considered.
The development in this chapter will be applied here to the scattering associated with cylindrical inlets over an ogive and an infinite cylinder. The results from the EC method are compared with those obtained by the use of the ordinary UTD which considers only the stationary points of the inlet. The presence of caustics will be quite apparent for these structures and it will be seen that the ordinary UTD becomes inadequate in the caustic regions. Measured RCS patterns are in addition presented which agree quite well with the results by the EC method.
CHAPTER IV

RCS FROM STRUCTURES WITH
CYLINDRICAL THIN-EDGE INLETS

A. RCS OF A SEMI-INFINITE CYLINDRICAL INLET OVER AN
INFINITE CYLINDER

1. Introduction

In an attempt to introduce and discuss some of the problems
associated with the solution of the backscattered field from inlet
structures, the RCS from a purely theoretical model is discussed.
The structure, consisting of a semi-infinite hollow cylinder (inlet)
over a solid infinite cylinder (surface), is considered as shown in
Figure 4.1. This will be referred to as structure A, and its RCS is
computed for an arbitrary tilt angle, $\alpha$, of the inlet face. Such a
structure, although simple, involves the major difficulties associated
with the solution of the RCS from inlets over a surface. In fact,
because of its two-dimensional surface, there are broad ranges of angles
for which caustics exist, and the EC method is essential. The solution
will be developed in the $\phi=0$ plane for the region $0<\theta<\pi/2$ which is in
the lit region of this inlet structure, and one notes that $E_0=0$ here.
Figure 4.1. Structure A: Semi-infinite hollow cylinder over an infinite solid cylinder.

The backscattered field is first computed by the use of the ordinary UTD and then, the EC method is used to eliminate the erroneous computation in the vicinity of the caustics.

2. Solution by the Ordinary UTD

The circular (\(\alpha_t=0\)) or elliptical (\(\alpha_t\neq 0\)) inlet edge has only one stationary point in the backscatter direction. This is located at the top of the rim as demonstrated in Figure 4.2. Therefore, only the \(D_e, R-D_e, D_e-R\) and \(R-D_e-R\) (as well as \(D_e-R-D_e\) which is neglected) ray
which diffract through $P_0^e$ contribute to the backscatter direction. These are shown in Figure 4.3. According to the development in Chapter II, the following results are obtained in the far field for a plane wave incidence.

For the directly diffracted field (see Equation (2.20) with $s=\infty$),

$$\overline{E_t}(\theta) = \overline{E_t}(0_e) \cdot (\sum_{s=0}^{1} \alpha_{s} D_s^1) \rho_{1} e^{j2kd_{2}} e^{-jks} \frac{e^{-j\pi/4\sqrt{\lambda}}}{\sin \theta} \frac{1}{s}, \quad (4.1)$$

where (see Equation (2.45))

$$D_{s,h}^1 = D_{s,h}(\omega,\omega,\pi/2+\theta,\pi/2+\theta,\pi/2) = e^{-j\pi/4\sqrt{\lambda}} \frac{\sin \theta}{\sin \theta} (1 \pm \frac{1}{\sin \theta}) \quad (4.2)$$
a). Singly diffracted ray ($D_e$).

b). $R-D_e$ ray.

Figure 4.3. Geometry of the $D_e$, $R-D_e$ and $R-D_e-R$ stationary ray mechanisms associated with structure A.
c). R-D_e-R ray.

Figure 4.3. (Continued).

\[ d_2 = \frac{\cos(\theta + \omega_{\text{e}})}{\cos \alpha_{\text{t},\text{e}}} \]  \hspace{1cm} (4.3)

and (see Equation (2.38))

\[ \rho_1 = -\frac{a_0^e}{2a_0^e} = \frac{\cos \alpha_{\text{t},\text{e}}}{2\cos(\theta + \alpha_{\text{t},\text{e}})} \]  \hspace{1cm} (4.4)

with

\[ a_0^e = \cos \alpha_{\text{t},\text{e}} \]  \hspace{1cm} (4.5)

\[ \hat{n}_e^0 = -x\sin \omega_{\text{e}} + z\cos \alpha_{\text{t},\text{e}} \]  \hspace{1cm} (4.6)
\[ \hat{s} = -\hat{t} = \hat{x}\sin\theta + \hat{z}\cos\theta . \quad (4.7) \]

The R-D field is given by (see Equation (2.20))

\[ \bar{E}_{2}^{RD}(\theta) = \bar{E}_{2}^{R}(P_{e}^{0}) \cdot (\hat{\gamma}\hat{y} \omega_{s}^{2} \hat{\gamma}\hat{r} \omega_{h}^{2}) \sqrt{\frac{2}{\rho_{2}}} e^{jk\delta_{2}} e^{-jk\hat{s}} \]  

where (see Equation (2.22) and (2.45))

\[ \hat{\gamma}\hat{r} = -\hat{x}\cos\theta - \hat{z}\sin\theta \]

\[ D_{s,h}^{2} = D_{s,h}(\omega, \omega, \pi/2 + \theta, 3\pi/2 - \theta, \pi/2) = -\frac{e^{-j\pi/4\sqrt{\lambda}}}{4\pi} \left[ \frac{1}{\cos(\pi/4 - \theta)} \right] \pm 1 \],

\[ (4.9) \]

and \( E_{2}^{R}(P_{e}^{0}) \) is the reflected field from the cylindrical surface when measured at \( P_{e}^{0} \). It is given by (see Equation (2.2) with \( \rho_{1}^{r} = 0 \))

\[ \bar{E}_{2}^{R}(P_{e}^{0}) = \bar{E}_{1}^{(0_{e})} \cdot \bar{R}_{1} \sqrt{\frac{\rho_{21}^{2}}{\rho_{21}^{2} + \rho_{0}^{2}}} e^{jk\delta_{1}} e^{-jk\delta_{0}} . \quad (4.10) \]

Substituting for (see Equations (2.3), (2.4) and (2.14))

\[ \bar{R}_{1} = \hat{\gamma}\hat{r} - \hat{\gamma}\hat{y} \],

\[ (4.11) \]
\[ s_0^r = \frac{a}{\cos \theta} \]  
\[ d_1 = a(\tan \theta - \tan \alpha_{t2}) \sin \theta \]  
\[ \text{and} \]
\[ r_{21}^2 = \frac{R_2}{2 \cos \theta} \]  
Equation (4.10) reduces to
\[
E^R_{2}(P^0_e) = -(\hat{\alpha}E_{\theta}^\uparrow \cos \theta + \hat{\beta}E_{\theta}^\downarrow \sin \theta + \hat{\gamma}E_{y}^\downarrow) \sqrt{\frac{R_2}{R_2 + 2a}} \\
e^{-jka}[(\tan \alpha_{t2} - \tan \theta) \sin \theta + \frac{1}{\cos \theta}] ,
\]
where \( E_{\theta}^\uparrow \) and \( E_{y}^\downarrow \) are the \( \hat{\theta} \) and \( \hat{\gamma} \) components of \( E^\uparrow \) at the reference point \( 0_e \). In addition, the caustic distance, \( R_{2}^* \), is (see Equation (2.38))
\[
\frac{1}{R^2_2} = \frac{1}{\rho^2_{e2}} - \hat{\alpha}_e \cdot (\hat{s}_0^r - \hat{s}) = \frac{a(R_2 + 2a) \cos \alpha_{t2}}{2a \cos \alpha_{t2} \cos \theta - 2(R_2 + 2a) \sin \alpha_{t2} \sin \theta}
\]
with
\[
\rho_{e2}^i = \rho_{22}^i = \rho_{21}^i + s_0^r
\]
and
\[
\hat{s}_0^r = -\hat{\alpha} \sin \theta + \hat{\gamma} \cos \theta .
\]
Finally, the $R$-$D_{e}-R$ field from $p_{e}^{0}$ is given by (see Equations (2.2) and (2.20))

\[
\bar{E}_{3}^{R_{e}R}(\theta) = \bar{E}_{2}^{R}(p_{e}^{0}) \cdot (\hat{\gamma}_{D}^{3} \hat{\gamma}_{S}^{3} \hat{\gamma}_{R}^{3}) \sqrt{\frac{3}{\rho_{2}^{3} + s_{0}^{3}}} \cdot \rho_{1}^{3} \rho_{23}^{3} \cdot e^{jk(d_{1} - s_{1})/s_{1}}
\]

(4.19)

and the various new parameters in the above expression are found by (see Equations (2.45), (2.38), (2.3), (2.4) and (2.12))

\[
D_{s_{0}, h_{0}}^{3}(s_{0}^{r}, s_{0}^{r}, s_{0}^{r}, 3\pi/2 - \theta, 3\pi/2 - \theta, \pi/2) ,
\]

(4.20)

\[
\rho_{2}^{3} = \frac{a(R_{2} + 2a) \cos \alpha_{\text{L}}}{2a \cos \alpha_{\text{L}} \cos \theta - 2(R_{2} + 2a) \cos(\theta - \alpha_{\text{L}})} ,
\]

(4.21)

\[
\rho_{3}^{3} = \hat{\gamma}_{R} \hat{\gamma}_{S} \hat{\gamma}_{S}
\]

(4.22)

and

\[
\rho_{13, 23}^{3} = \rho_{13}^{3} \rho_{23}^{3} \rho_{1}^{3} \rho_{2}^{3} \rho_{3}^{3} R_{2}, \theta, \pi/2
\]

(4.23)

with

\[
\rho_{13}^{3} = s_{0}^{r} ,
\]

(4.24a)
\[ \rho_{23} = \rho_{e3} = \rho_2^0 + sr^0. \] (4.24b)

The RCS of structure A in the \( \phi=0 \) plane can now be expressed in the region of \( 0<\theta<\pi/2 \) as

\[ \sigma(\theta, \phi=0; \alpha_{t\ell}=0) = 4\pi s^2 \left| \frac{\Re E_1^\ell(\theta) + 2\Re E_0^\ell(\theta) + \Re E_0^R(\theta)}{|E_1^\ell|^2} \right|^2. \] (4.25)

Plots of \( \sigma(\theta, \phi=0; \alpha_{t\ell}=0) \) for the \( \hat{e}_I=\hat{\iota} \) (H-plane) and \( \hat{e}_I=\hat{\theta} \) (E-plane) polarizations of the incident wave are given in Figure 4.4 when \( R_2=7.68 \text{ in.}, a=2 \text{ in.} \) and \( f=c/\lambda=9.01 \text{Ghz} \). One observes that the UTD results for all the ray field mechanisms become indeterminate in the H-plane at the nose-on region (\( \theta=\pi/2 \)) for \( \alpha_{t\ell}=0 \). This is because all the inlet rim points are stationary when \( \theta=\pi/2 \). More precisely, one scattered ray associated with every edge point is in the backscatter direction, thus creating a caustic. To verify this, the UTD expressions for each ray field are examined as \( \theta=\pi/2 \) and \( \alpha_{t\ell}=0 \). In particular, from Equation (4.4),

\[ \rho_1^1 | \alpha_{t\ell}=0 = \frac{a}{2\cos\theta} \left| \theta=\pi/2 \right| \]

and from Equation (4.16)

\[ \rho_2^2 | \alpha_{t\ell}=0 = \frac{R_2+2a}{2\cos\theta} \left| \theta=\pi/2 \right| \]

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Figure 4.4. Calculated $E_{\phi}$ and $E_{\theta}$ RCS patterns from structure A by the ordinary UTD with $\alpha_{tx}=0^\circ$, $R_2=7.68''$, $a=2''$ and $\lambda=1.311''$. 

a). $E_{\phi}$, $\phi=0$ RCS pattern.
b). $E_\theta, \phi=0$ RCS pattern.

Figure 4.4. (Continued).
Therefore, $E_{\text{De}}^{\infty}$ and $E_{\text{RD}e}^{\infty} = E_{\text{De}R}^{\infty}$ at $\theta=\pi/2$. Also, from Equation (4.24a)

$$p_{13}^{\infty} = \frac{a}{\cos\theta} \bigg|_{\theta=\pi/2} + \infty$$

which causes $p_{13}^{\infty}$ and thus $E_{\text{RD}e}^{\infty}$ also becomes indeterminate as $\theta$ approaches $\pi/2$.

In the $E_\theta$ pattern, the caustic due to $E_{\text{De}}$ is not apparent because the soft edge diffraction coefficient goes to zero at $\theta=\pi/2$. But the UTD results for this pattern are incorrect even in the non-caustic regions due to strong junction-corner fields (see Figures 4.1 and 4.5). This occurs when the polarization of the incident field has a large component in the direction of the inlet edge close to the surface of the supporting body. In contrast, the $\hat{\phi}(\phi=0)$ polarization is almost perpendicular to the edge vector at the junction-corner, and thus these fields are negligible in the H-plane.

3. Solution by the EC Method

When the fields associated with the inlet present such a behavior, the EC method discussed in Chapter III must be applied. In this case, the effect of the spread factor associated with the diffracted rays is accounted for by the EC integration around the curved inlet rim. The numerous parameters required for the application of the EC technique to structure A become:
For the surface,

\[ \hat{n} = \hat{z}\cos\theta_y + \hat{y}\sin\theta_y , \quad (4.26) \]

\[ R_1 = \infty \text{ (was set to } 10^{20} \text{ for numerical calculations)}, \]

\[ R_2 = \text{radius of the infinite solid cylinder}, \]

\[ \hat{t}_1 = -\hat{x}, \quad (4.27a) \]

\[ \hat{t}_2 = \hat{y}\cos\theta_y - \hat{z}\sin\theta_y \quad (4.27b) \]

and

\[ \theta_y = \tan^{-1} \left( \frac{S_{ry}}{S_{rz}} \right) \quad (4.28) \]

where \( S_{rx}, S_{ry} \) and \( S_{rz} \) are the \( x, y, z \) components of the reflection point, \( S_r \). For the inlet (see Figure 4.5), one obtains

\[ \bar{p}_e = a[-\hat{x}\cos\theta \tan\alpha_e + \hat{y}\sin\theta + \hat{z}(\frac{R_2}{a} + \cos\theta)], \quad (4.29) \]

\[ \hat{e} = \frac{\hat{x}\sin\theta \tan\alpha_e + \hat{y}\cos\theta - \hat{z}\sin\theta}{\sqrt{1 + \sin^2\theta \tan^2\alpha_e}}, \quad (4.30) \]

\[ \hat{n}_e = \frac{d\hat{e}/d\theta}{|d\hat{e}/d\theta|} = \frac{\hat{n}_e}{|\hat{n}_e|} \quad , \quad (4.31a) \]
Figure 4.5. Definition of parameters associated with the inlet geometry.

\[ \hat{n}_e = \hat{\chi} \cos \theta_I \tan \alpha_{\perp} + \hat{\gamma} \frac{\sin \theta_I}{\cos^2 \alpha_{\perp}} + \hat{\zeta} \cos \theta_I , \]  

(4.31b)

\[ \hat{\chi}_I = \hat{\gamma} \sin \theta_I + \hat{\zeta} \cos \theta_I , \]  

(4.32)

\[ R_{II} = (=10^{20}) , \]  

(4.33a)

\[ R_{I2} = a , \]  

(4.33b)

\[ \hat{\chi}_{II} = -\hat{\chi} , \]  

(4.34a)

\[ \hat{\chi}_{I2} = \hat{\gamma} \cos \theta_I - \hat{\zeta} \sin \theta_I \]  

(4.34b)
and

\[ \theta_l = \cos^{-1}(\mathbf{\hat{r}} \cdot \mathbf{r}_e), \quad (4.35a) \]

where

\[ \mathbf{r}_e = (P_e \mathbf{e}_e - \mathbf{O}_e) / |P_e \mathbf{e}_e - \mathbf{O}_e| \quad (4.35b) \]

and

\[ \mathbf{O}_e = \mathbf{\hat{z}}R_2. \quad (4.36) \]

In addition, the above \( P_e \) corresponds to an edge radius of curvature given by

\[ a_e = \frac{a}{\cos^2 \alpha_{\|}} \left( \sin^2 \theta_{\|} + \cos^2 \theta_{\|} \cos^2 \alpha_{\|} \right)^{3/2}. \quad (4.37) \]

The point projection required in the raytracing routine is quite simple for a cylindrical surface. If \( S_p \) is a point slightly off this surface, its projection is

\[ \bar{S}_{rp} = x_{Sp} + yR_2 \sin \theta_{\|} + zR_2 \cos \theta_{\|}, \quad (4.38) \]

where \( \theta_{\|} \) was defined in Equation (4.28).

One should finally note that due to the structure's symmetry about the xz plane, the EC integrals are also symmetric about \( \theta_l = 0 \).
Therefore, the integration over the rim is only needed to be carried out from $\theta_1 = 0$ to

$$\theta_1 = \theta_{IC} = \cos^{-1}\left(-\frac{a}{2R^2}\right) \quad (4.39)$$

with

$$d\gamma = |\vec{p}_e^{i+1} - \vec{p}_e^{i}| \quad (4.40)$$

and then double the result.

The $E_\phi$ and $E_\theta$ RCS patterns by the EC method for the same dimensions, are shown in Figure 4.6. These are the more accurate results. It is observed that the $E_\phi$ pattern is almost identical to that obtained by the ordinary UTD when away from the caustic region. In fact, the UTD was used to compute the early portion of the pattern and then the calculation switched to the EC technique where the .5-1dB jumps are observed. The difference between the EC method and ordinary UTD when away from the caustic region is due to the junction-corner fields. The $E_\theta$ pattern by the EC method is drastically different from the same pattern by the UTD. When $\theta$ is of the order of 90 degrees, the unit vector $\hat{e}$ around the junction-corners has a strong $\theta$ component, and therefore, the backscattered fields as computed by the EC method peak at this region. However, for small $\theta$ angles, the $\hat{e}$ vector has a small component in the $\theta$ direction, and the EC and UTD results are identical for $\theta < 20^\circ$, which is a consequence of weak junction-corner fields in this region.
Figure 4.6. Calculated $E_{\phi}$ and $E_{\theta}$ RCS patterns from structure A by the EC method with $\alpha_{\phi}=0^\circ$, $R_2=7.68''$, $a=2''$ and $\lambda=1.311''$. 

a). $E_{\phi}$, $\phi=0$ RCS pattern.
Figure 4.6. (Continued).

b). $E_\theta$, $\phi=0$ RCS pattern.
It is also interesting to look at the phase plots (phase reference is at \( \theta_e=2\theta_2 \)) associated with each field mechanism as the shadow boundary of the inlet at \( \theta=\pi/2 \) is approached. These are shown in Figure 4.7 for the \( \phi \) and \( \theta \) polarization of incidence. One should note that in the H-plane, the \( D_e \) and \( R-D_e-R \) fields are completely out of phase with the \( R-D_e \) field. Thus, the sum \( -D_e+2RDe+RDeR \) is small at the SB in order that continuity be maintained with the surface diffracted fields. These are indeed small for this polarization of incidence (parallel to the surface). However, this is not the case with the \( \theta \) polarized wave where the surface fields can be quite strong since this polarization is perpendicular to the surface at the SB. In fact, all the fields are in phase at the SB for the \( \theta \) polarization of incidence.

Figures 4.8 and 4.9 present RCS patterns for nonzero \( \alpha_tz \). Figure 4.8 shows the results with \( \alpha_tz=-10^\circ \) for the \( \phi \) and \( \theta \) polarization of incidence. The results when \( \alpha_tz=10^\circ \) are given in Figure 4.9. The same trends hold for other values of \( \alpha_tz \).

B. RCS OF A SEMI-INFINITE CYLINDRICAL INLET OVER AN OGIVE

1. Introduction

In this section, the RCS from a structure consisted of a semi-infinite hollow cylinder over an ogive of radius \( R_1 \) and tip angle \( 2\alpha \) (see Figure 4.10) is studied. Such a model will be referred as
Figure 4.7. Phase patterns of the fields shown in Figure 4.6.

a). $E_{\phi}, \phi=0$ phase pattern.
b). $E_9$, $\phi=0$ phase pattern.

Figure 4.7. (Continued).
Figure 4.8. Calculated $E_{\phi}$ an $E_{\theta}$ RCS patterns from structure A by the EC method with $\alpha_{\xi \zeta}=-10^\circ$, $R_2=7.68''$, $a=2''$ and $\lambda=1.311''$. 

a). $E_{\phi}$, $\phi=0$ RCS pattern.
b). $E_\theta$, $\phi=0$ RCS pattern.

Figure 4.8. (Continued).
Figure 4.9. Calculated $E_\phi$ and $E_\theta$ RCS patterns from structure A by the EC method with $\alpha_z=+10^\circ$, $R_2=7.68''$, $a=2''$ and $\lambda=1.311''$. 
Figure 4.9. (Continued).

b). $\theta$, $\phi=0$ RCS pattern.
Figure 4.10. Geometry of structure B: A semi-infinite hollow cylinder over an ogive.
structure B. Unlike the cylinder, the ogive is a doubly curved surface thus representing a type of a more general surface. Solutions will be again obtained in the $\phi=0^\circ$ plane for $0<\phi<\pi/2+\alpha$ by the ordinary UTD (without the junction-corner fields) and the EC method as a function of various parameters. These are (see Figure 4.10) the ogive radius, $R_1$; the ogive tip angle, $2\alpha$; the inlet cylinder radius, $a$; the inlet tilt angle, $\alpha_t$; and the inlet lateral position with respect to the center of the ogive, $\Delta$.

2. Solution By the Ordinary UTD

The inlet of structure B is the same as that of structure A. Therefore, it is associated with one stationary point (see Figure 4.2), $P^0_0$, and the ray fields $D_e$, $R-D_e$ and $R-D_e-R$ are computed in a manner similar to that in the previous section. The only difference will be the parameters associated with the reflection and diffraction coefficients. In addition, for $0<\theta<\alpha$ the ogive also has a specular point as shown in Figure 4.11, and thus $E^O_0$ is not equal to zero in this case. For $\theta>\alpha$ the tip diffracted field is the dominant contribution for this ogive. There are also other mechanisms from the ogive such as diffraction from the back tip and creeping waves. However, these fields will be neglected since they are significantly weaker than the above two components. The GO backscattered far field from the ogive at any point in space is then given by [70]
Figure 4.11. Dominant backscattered rays from the ogive and their associated geometry.
where $\beta_0$ (see Figure 4.11) is related to $\phi$ by

$$
\beta_0 = \sin^{-1}(-I_x) = \sin^{-1}(\sin\theta\cos\phi), \tag{4.42}
$$

i.e., $\beta_0=\theta$ when $\phi=0$ which is the case in this section. The result in the region of $\alpha<\beta_0<\pi/2$ is still singular when $\beta_0$ is close to $\alpha$. Therefore, for practical purposes the value of $E^{\alpha}(\beta_0=\alpha)$ was used until $E^{\alpha}(\beta_0)<E^{\alpha}(\beta_0=\alpha)$. The definition of the various other parameters in the above equation are

$$
S_{\beta_0} = -\hat{x}R_1I_x\hat{y}\rho_x\sin\theta_y + \hat{z}\rho_x\cos\theta_y \tag{4.43}
$$

with

$$
\rho_x = h-R_1(1-I_x^2) = R_1(\cos\beta_0-\cos\alpha), \tag{4.44}
$$

$$
h = R_1(1-\cos\alpha), \tag{4.45}
$$

$$
\theta_y = \tan^{-1}(-I_y/I_x) \tag{4.46}
$$

and

$$
\bar{E}^{\alpha}(\beta_0) = -E'(0)\left\{ \begin{array}{ll}
\sqrt{\frac{R_1R_2}{2}}e^{-j2k(S_{\beta_0}-\bar{O}_e)\hat{I}e^{-jks}} & \text{for } 0<\beta_0<\alpha \\
\frac{R_1\sin\alpha}{4\pi}e^{-j2k(S_{\bar{O}_e})\hat{I}e^{-jks}} & \text{for } \beta_0=\alpha \\
\frac{\lambda\tan^2\alpha}{8\pi\sin^3\beta_0(1-\tan\frac{\alpha}{2})^2\cot^2\beta_0\frac{3}{2}}e^{-jks} & \text{for } \alpha<\beta_0<\pi/2
\end{array} \right. \tag{4.41}
$$
\[ \mathcal{S}_{\text{tip}} = \begin{cases} \hat{x}R_1 \sin \alpha \\ \hat{-x}R_1 \sin \alpha \end{cases} \]  

whichever is visible by the incident ray. Note that \( \Omega_e \) is the phase reference (see Figure 4.10) located at

\[ \Omega_e = x\Delta + zh \]  

The transverse principal radius of the ogive, \( R_2^{GO} \), at \( S_{GO} \) is well known to be

\[ R_2^{GO} = R_1 (1 - \cos^2 \theta_{GO}) \]  

but one can derive this by equating Equations (2.44) and (2.14b) since \( \rho_2^e = \rho_e^e \) for principal plane incidence. The convenience of this approach is that \( \hat{n}_e \) can be chosen so that \( \phi_e \) reduces to a simple function.

Proceeding as discussed, one obtains

\[ \frac{a_e}{2(\hat{n} \cdot \hat{n}_e)(\hat{I} \cdot \hat{n})} \bigg|_{\hat{I} = -\hat{n}_e} = \frac{R_2^{GO}}{2} \]  

Choosing

\[ \hat{n}_e = \hat{y} \sin \theta_y + \hat{z} \cos \theta_y \]  

then,

\[ \rho_x = R_1 (\cos \theta_{GO} - \cos \alpha) \]
and after substitution in Equation (4.50) the given result for \( R_G \) is obtained. The above method for computing the transverse surface radius seems to be quite general for application on other bodies.

The measured \( E_{\theta}, \theta=90^\circ \) RCS pattern from a bare ogive with \( R_1=12.073 \) in., \( \alpha=46.34^\circ \) and \( \lambda=1.311 \) in. before supplying it with a cylindrical inlet is shown in Figure 4.12 and compared to that calculated by use of Equation (4.41). It is observed that in the nose region, the theoretical result does not compare well with the measured. The discrepancy is introduced by the creeping wave mechanisms which have been neglected.

Currently, a simple numerical raytracing routine has been developed which is capable of locating \( S_G \) on an arbitrary smooth surface. Also, a computer program [68] is available for generating a wide variety of analytical surfaces by adjusting a set of curvature and size parameters and for calculating the surface parameters, \( \hat{R}_1, \hat{R}_2, \hat{\alpha}_1 \) and \( \hat{\alpha}_2 \) as discussed in Chapter III.

One now proceeds to calculate the other UTD fields from structure B. Each of these mechanisms is associated with a shadow boundary at \( \beta=\beta_s \) as shown in Figure 4.13. The angle \( \beta_s \) for the stationary ray is found by using

\[
\beta_s = \cos^{-1}\left\{ \frac{R_1}{\sqrt{(a+R_1)^2+(\tan \alpha \hat{\alpha} + \Delta)^2}} \right\} - \alpha_c^0
\]  

(4.53)

with

\[
\alpha_c^0 = \tan^{-1}\left\{ \frac{\Delta \tan \alpha \hat{\alpha}}{R_1 + a} \right\}.
\]  

(4.54)

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Figure 4.12. $E_\phi$, $\phi=90^\circ$ RCS pattern from an ogive of radius, $R_1=12.073''$ and tip angle, $\alpha_t=46.34^\circ$; $\lambda=1.311''$. 
a). Ray geometry.

Figure 4.13. Geometry of the shadow boundary and the surface rays associated with structure B.
b). Detail of the right triangle $O_{c}S_{d}^{0}P_{e}^{0}$.

Figure 4.13. (Continued).
The angle $\theta$ corresponding to the reflection point as shown in Figure 4.14, is unique for each incidence but cannot be explicitly expressed as a function of $\theta$. The inverse relationship, i.e., of $\theta$ in terms of $\beta$, is quite simple and through geometrical considerations of Figure 4.14 it is found for $\beta<\beta_s$ to be

$$\theta = 2\beta + \sin^{-1}(\lambda_3/s_0^r) \quad (4.55)$$

with

$$\lambda_3 = (R_1 + \lambda_1)\sin\beta, \quad (4.56)$$
$$\lambda_1 = (\Delta + \tan\alpha t_2)/\sin\beta \quad (4.57)$$

and

$$s_0^r = \left| P_e^o - S_r^o \right| \quad (4.58)$$

where

$$P_e^o = -\hat{x}(\tan\alpha t_2 + \Delta) + \hat{z}(h+a) \quad (4.59)$$

and

$$S_r^o = \hat{x}R_1\sin\theta - 2R_1(\cos\beta - \cos\alpha) \quad (4.60)$$

In addition,

$$\theta_i^o = \theta - \beta \quad (4.61)$$
Figure 4.14. Geometry for computing $\theta$ in terms of $\beta$. 
When \( \beta = \beta_s \), then
\[
s_0^d = s_0^d = \sqrt{(R_1 + a)^2 + (\text{atan} \alpha_{ld} + \Delta)^2 - R_1^2}
\]  
(4.62)
and the \( D_e, R-D_e \) and \( R-D_e-R \) backscatter rays all merge. For \( \beta > \beta_s \) (see Figure 4.13) only the \( D_s-D_e-D_s \) mechanism exists as discussed in Chapter II and
\[
\theta = \pi/2 + \beta.
\]  
(4.63)

In the lit region, the \( D_e \) field is again given by Equation (4.1) with all the parameters remaining unchanged since we are dealing with an identical inlet. However, the parameters for the \( R-D_e \) and \( R-D_e-R \) field take a more general form than those associated with Equations (4.8) and (4.19). Specifically,
\[
\tilde{E}_2^{R-D_e}(\theta) = E_2^R(p_e) \cdot (\hat{Y}_Y D_0^2 \hat{Y}_R^2 \hat{R}_D^2) \sqrt{\rho_2^2} e^{ikd_2} e^{-jks} \]  
(4.64)
and
\[
\tilde{E}_3^{R-D_e}(\theta) = E_3^R(p_e^0) \cdot (\hat{Y}_Y D_0^3 \hat{Y}_R^3 \hat{R}_D^3) \sqrt{\rho_3^3} \sqrt{\rho_2^2 + s_0^2} R_3 \sqrt{r_1^3 r_3^3} e^{ik(d_1 - s_0^1)} e^{-jks} \]  
(4.65)
where
\[
\hat{\Lambda} = \hat{\Lambda}_0 \cdot \hat{\Lambda}_2 \cdot \hat{\Lambda}_3 \cdot \hat{\Lambda}_4 \]  
(4.66)
The reflected field, $E_2^R$, now takes the form (see Equation (2.2))

$$E_2^R(p^0) = E_1^0(0_0) R_1 \sqrt{\frac{\rho_{11}^2 \rho_{21}^2}{(\rho_{12}^2 + s_0^2)(\rho_{21}^2 + s_0^2)}} e^{-jk(s_0^2 - d_1)} \quad (4.67)$$

and since $S_0$ in the principal plane of the ogive (see Equations (2.3) and (2.54)),

$$R_1 = \theta_R^0 \hat{R}_h(S_0^o, \xi_1^L, \chi_1^L) + \gamma_R S(S_0^o, \xi_1^L, \chi_1^L) \quad (4.68)$$

with

$$R_1 = R_1 \quad (4.69)$$

and

$$L_1^L = \frac{\rho_{11}^2 \rho_{21}^2}{\rho_{21}^2} \quad (4.70)$$

The remaining parameters are (see Equations (2.14) and (4.61))

$$\rho_{11}^2 = \frac{R_1 \cos \theta_1^0}{2} \quad (4.71a)$$

$$\rho_{21}^2 = \frac{R_2}{2 \cos \theta_2^0} \quad (4.71b)$$

$$R_2 = R_1 \left(1 - \frac{\cos \alpha}{\cos \beta}\right) \quad (4.72)$$
and

\[ d_1 = -\left( \mathbf{S}_r^0 \cdot \mathbf{O}_e \right) \cdot \mathbf{l} \]  

(4.73)

The diffraction coefficient associated with the R-De field is found by (see Equation (2.45))

\[ D_{s,h}^2 = D_{s,h} \left( \rho_{12}^2, \rho_{12}^2, \rho_{12}^2, \pi + \theta, \frac{3\pi - \theta + 2\beta}{2}, \pi/2 \right) \]  

(4.74)

with

\[ L^i = r^i = \rho_{12}^2 + s^0 \]  

(4.75)

and the caustic distance, \( \rho_{2}^2 \), is (see Equation (2.38))

\[ \rho_{2}^2 = \frac{\rho_{12}^2 \cos \alpha t_x}{\cos \alpha t_x - \rho_{12}^2 [\hat{n}_{e} \cdot (\mathbf{S}_r^0 \cdot \mathbf{S}_e^0)]} \]  

(4.76)

where

\[ \mathbf{S}_r^0 \cdot \mathbf{S}_e^0 = -\hat{x} \sin(\theta - 2\beta) + \hat{z} \cos(\theta - 2\beta) \]  

(4.77)

and \( \hat{n}_{e} \), \( \hat{z} \) and \( \rho_{12}^2 \) are defined by Equations (4.6), (4.7) and (4.17), respectively.
For the R-Dg-R field (see Equation (2.45)),

\[ D_{s,h}^3 = D_{s,h}(L^1_s, L^1_h, L^1_s, 3\pi/2-\theta+2\beta, 3\pi/2-\theta+2\beta, \pi/2) \]  \hspace{1cm} (4.78)

with

\[ L^1 = \frac{s_{0}\rho_{12}^{13}}{\rho_{12}^{13} + s_r} \]  \hspace{1cm} (4.79)

and

\[ \rho_{12}^{13} = \rho_{11}^{12} s_0 r \]  \hspace{1cm} (4.80a)

The caustic distance is (see Equation (2.38))

\[ \rho_2^3 = \frac{\alpha \rho_{12}^{13} \cos \alpha_{\ell 2}}{\cos \alpha_{\ell 2} - 2 \rho_{12}^{13} (\hat{m}_0^0 \cdot \hat{m}_0^0)} \]  \hspace{1cm} (4.81)

and the dyadic reflection coefficient is given by (see Equations (2.3) and (2.54))

\[ \hat{R}_3 = \hat{\theta}^r \hat{e} R_h(S_{\rho}, \xi_{3}^1, x_{3}^1) + \hat{\alpha} \hat{e} R_s(S_{\rho}, \xi_{3}^1, x_{3}^1) \]  \hspace{1cm} (4.82)
Finally, from Equation (2.12)

\[ \rho_{13,23}^3 = \rho(s_0^p + s_0^p R_1, R_2 + \theta_0^p, \pi/2) \]  (4.84)

In the shadow region, the \( D_e, R-D_e \) and \( R-D_e-R \) fields cease to exist. The total backscattered field is then only that due to the \( D_s-D_e-D_s \) mechanism through \( P_e^0 \) and tip diffraction. The field of the surface diffracted ray shown in Figure 4.13 (see also Equation (3.82)), can be expressed by (see Equation (2.48)),

\[ \bar{E}_{D_sD_eD_s}(\theta) = \bar{E}_{D_sD_e}(S^0_d) \cdot \frac{1}{\rho_{23}^3} e^{-jk(S^0_d - \bar{D}_e)} \cdot \frac{1}{s} e^{-jks} \]  (4.85a)

\( \bar{E}_{D_sD_e}(S^0_d) \) is the electric field measured at \( S^0_d = xR_1 \sin \beta_s - zR_1 (\cos \beta_s - \cos \alpha) \) after the incident ray has gone through surface and edge diffraction, viz. (see Equation (2.20)),

\[ \bar{E}_{D_sD_e}(S^0_d) = \bar{E}_{D_s}(P^0_e) \cdot (\rho_0 d \hat{A} \hat{A}_d d \hat{A}_d \hat{A} d h) \sqrt{\rho_{0}(\rho + s_0^d)} \cdot e^{-jks_0^d} \]  (4.85b)

where

\[ \hat{A}_d = \hat{A} (\beta_s) = x \sin \beta_s + z \cos \beta_s, \]  (4.86)

\[ L_3 = \frac{s_0^p(s_0^p + p_2^3)}{p_2^3} \]  (4.83)
provided the ray torsion, caused during surface diffraction, is neglected.

The field after the first surface diffraction, when ray torsion is neglected, is given by (see Equation (2.48) with $\rho_d \to \infty$)

$$E^d_s(p_e) = E^i(0_e) \cdot \frac{e^{-jk\frac{d}{s_0}}}{s_0} \cdot e^{-jk(S_1-O_e)\cdot \hat{n}}$$

with (see Equations (2.49) and (2.50))

$$\overline{T}_1 = \overline{M}_T(S_1^d,S_1^d,\chi_1^d) + \overline{n}_1^n d \cdot h(S_1^d,S_1^d,\chi_1^d) .$$

The unit vector $\overline{n}_1$ is simply

$$\overline{n}_1 = x \sin \beta + 2 \cos \beta ,$$

and the parameters associated with $T_{s,h}$ are
\[ t_1 = t = R_1 (\beta - \beta_s) \quad , \]
\[ m(S_i^0) = m(S_d^0) = (\frac{kR_1}{2})^{1/3} \quad , \]
\[ \varepsilon_1^{d} = \varepsilon_2^{d} = (\frac{kR_1}{2})^{1/3} t_1 \quad , \]
\[ L_1 = L_2^d \quad , \]
\[ \chi_1^{d} = \frac{kS_0^d}{2} (\beta - \beta_s)^2 \quad . \]

and
\[ \sqrt{\frac{dn(S_i^0)}{dn(S_d^0)}} = 1 \quad . \]

Finally, according to Equation (2.49)
\[ T_3 = \gamma \gamma T_S (S_d^0, S_i^0, x_3^d) + \gamma \gamma T_h (S_d^0, S_i^0, x_3^d) \quad (4.98) \]

with parameters
\[ t_3 = t \quad , \]
\[ \varepsilon_3^{d} = \varepsilon_2^{d} \quad , \]
\[ L_3 = \frac{s_0^d (\beta + s_0^d)}{r_{23}^d} \quad (4.101) \]
\[ \rho_{23} = \rho + s_0 + t, \quad (4.102) \]

\[ x_3^d = kL_3^d(\beta - \beta_s)^2 \quad (4.103) \]

\[
\sqrt{\frac{dn(s_0^d)}{dn(s_0^s)}} = \sqrt{\frac{\rho + s_0^d}{\rho + s_0^s + t}} \quad (4.104)
\]

From the above development, the RCS of structure B in the \( \phi=0 \) plane can be expressed by

\[
\sigma(\theta, \phi=0; \alpha_t, \Delta) = 4\pi s^2 \lim_{s \to \infty} \frac{1}{|E|^2} \left\{ \begin{array}{ll}
| \overrightarrow{E}^0(\theta) + \overrightarrow{E}_1^0(\theta) + 2\overrightarrow{E}_2^R(\theta) + \overrightarrow{E}_3^R(\theta) \cdot \hat{\theta}^I |^2 & \text{for } 0 < \theta < \theta_s \\
| \overrightarrow{E}^0(\theta) + \overrightarrow{E}_s^D \overrightarrow{E}_s^D(\theta) \cdot \hat{\theta}^I |^2 & \text{for } \theta_s < \theta < \frac{\pi + \alpha}{2}
\end{array} \right.
\quad (4.105)
\]

where \( \theta_s \) is the \( \theta \) angle corresponding to \( \beta = \beta_s \), according to Equation (4.55).

Figure 4.15 presents dB plots of \( \sigma(\theta, \phi=0; \alpha_t, \Delta=0) \) and the individual ray field components when \( \hat{\theta}^I \) is equal to \( \hat{\phi}, \phi=0 \) and \( \hat{\theta}, \theta=0 \).

The given data were calculated with \( R_1=12.073^\circ, \alpha=46.34^\circ, a=2.1^\circ, \) and \( \lambda=1.311^\circ \). It is seen that the UTD fields \( \overrightarrow{E}_D^R \) and \( \overrightarrow{E}_D^R \) again contain caustic regions in a manner comparable with the cylindrical surface.
Figure 4.15. Calculated $E_\phi$ and $E_\theta$ RCS patterns from structure B by the UTD with $R_1=12.073''$, $\alpha=46.34^\circ$, $a=2.1''$, $\alpha_{\phi}=0$, $\Delta=0$ and $\lambda=1.311''$. 

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b). $E_\theta, \phi=0$ RCS pattern.

Figure 4.15. (Continued).
These caustics, as discussed, occur when $\rho_1$ and $\rho_2$ given in Equations (4.4) and (4.76), respectively, become indeterminate. One should again observe that in the $E_\theta$ pattern, the caustic at $\theta=\pi/2$ due to $E_{1}^{D_e}$ is not evident because $D_2^t \to 0$. In addition, $\rho^{r3}_{13,23}$ in Equation (4.84) were bounded throughout the calculated region for this geometry. Therefore the field $E^{R D_e R}$ from the stationary point is reasonably accurate as shown. However, this field component is generally much weaker than those caused by the $D_e$ and $R-D_e$ rays. Similarly to the case of the cylindrical surface, the UTD $E_\theta$ pattern in Figure 4.15 is mostly erroneous (in the nose region $60^\circ<\theta<100^\circ$) due to the existence of strong junction-corner fields. The shadow boundary for this geometry is (from Equation (4.53) and Figure 4.13) at $\beta_s=31.6^\circ(\theta_s=121.6^\circ)$ and it should be noted that the patterns are continuous at this point.

3. Solution by the EC Method

The EC method must again be applied in order to treat the caustic regions associated with structure B and the junction-corner fields. Since the inlet is identical to that studied in section A of this chapter, all of the inlet parameters required for the application of the EC method remain the same except the edge locus. This is redefined as

$$F_e=-\hat{x}(\Delta \cos \theta \tan \alpha_z)+\hat{y} \sin \theta + \hat{z}(h+\cos \theta I) \quad (4.106)$$
to account for the lateral movement of the inlet.

The parameters describing the surface of structure B can easily be obtained if they are referred to a coordinate system with center at

$$\mathbf{c} = -y(R_1-h)\sin\theta_y + z(R_1-h)\cos\theta_y$$

(4.107)

and axes

$$\mathbf{X}_c = \hat{x}$$

(4.108a)

and

$$\mathbf{Z}_c = -y\sin\theta_y + z\cos\theta_y$$

(4.108b)

as shown in Figure 4.16. The angle $\theta_y$ was defined in Equation (4.28) and the $X_cZ_c$ plane coincides with the principal plane of the ogive. Therefore, the ogive surface normal is simply given by

$$\mathbf{n} = \mathbf{X}_c \sin \theta_c + \mathbf{Z}_c \cos \theta_c$$

(4.109)

where

$$\theta_c = \cos^{-1}\left\{ (S_r)z_c / |S_r| \right\}$$

(4.110)

If the $S_r$ is slightly off the surface, then its projection on the surface of the ogive is found by

$$S_{rp} = xR_1 \sin \theta_c + zR_1 \cos \theta_c$$

(4.111)
Figure 4.16. Geometry for computing the surface parameters of an ogive.
Finally, the principal radii of the ogival surface are \( R_1 \) and

\[
R_2 = R_1 \left( 1 - \frac{\cos \alpha}{\cos \theta_c} \right) \tag{4.112}
\]
corresponding to the principal directions given by

\[
\hat{t}_1 = x_c \cos \theta_c - z_c \sin \theta_c \tag{4.113}
\]

and

\[
\hat{t}_2 = y_c \tag{4.114}
\]

respectively.

For incidence in the \( \phi=0 \) plane, the ray geometry is symmetric with respect to \( \theta_I = 0 \) since the ogive is a body of revolution. Therefore, the EC integration can again be carried out from \( \theta_I = 0 \) to

\[
\theta_I = \theta_{IC} = \cos^{-1} \left\{ \frac{h_x^2 - h_1^2}{2ha} \right\} \tag{4.115}
\]

where

\[
h_x = \sqrt{R_1^2 - (\Delta x)^2 + h - R_1} \tag{4.116}
\]

and to a second order approximation for small \( \Delta x \),

\[
\Delta x = \begin{cases} 
\frac{hR_1 - \sqrt{(R_1h)^2 - R_1htan\alpha_{t2} a^2 tan\alpha_{t2} - 2h\Delta}}{htan\alpha_{t2}} & \text{for } \alpha_{t2} \neq 0 \\
-\Delta & \text{for } \alpha_{t2} = 0
\end{cases} \tag{4.117}
\]
provided $|\omega z|$ is not close to $\pi/2$. The value obtained from the integration must consequently be doubled.

The calculated $E_\phi$ and $E_\theta$ RCS patterns by the EC method are shown in Figure 4.17. The geometry associated with these patterns is the same as that used for the UTD patterns in Figure 4.15. The individual ray fields are also given, and it is noted that the result by the EC method is bounded at the caustic regions. The early portion of the $E_\phi, \phi=0$ pattern was again calculated by the UTD, and then the EC method was used from the point where the .5-1 dB jumps are indicated. For $\theta>90^\circ$, portion of the inlet begins to become shadowed, and therefore the extrapolation process (see Equation (3.99)) was used to compute the edge equivalent currents caused by the surface rays. This approach was maintained until $\theta$ was within about $5^\circ$ of the shadow boundary associated with the stationary ray. For this geometry $\theta_s=121.6^\circ$, and one can observe the failure of the extrapolation process in the $E_\theta, \phi=0^\circ$ pattern, causing a non-realistic ripple effect for the $E_\theta$ field. As discussed in Chapter III, the extrapolation routine is inadequate when most of the inlet has been shadowed. Therefore, the later portion of both patterns is the UTD result as given in Figure 4.15. Continuity is still maintained in the total $E_\phi$ pattern after switching from the EC method to the UTD, due to the weak junction-corner fields associated with this polarizaiton. A 3 dB discontinuity, though, is observed in the plot of $E_\theta R$, but this ray field is not dominant and therefore, it does not effect the total result. In the $E_\theta$ pattern, the change in the methods
Figure 4.17. Calculated $E_\phi$ and $E_\theta$ RCS patterns from structure B by the EC method and UTD; $R_1=12.073''$, $\alpha=46.34''$, $a=2.1''$, $\alpha_t=0$, $\Delta=0$ and $\lambda=1.311''$. 

a). $E_\phi$, $\phi=0$ RCS pattern.
b). $E_{\theta}, \phi=0$ RCS pattern.

Figure 4.17. (Continued).
of computation caused a great discontinuity. Again, this is due to strong junction-corner fields as can be seen by the difference between the $E_0$ patterns in Figures 4.15 and 4.17. The results by the EC method are obviously considered to be more accurate.

It is clear then, that except for the singularities caused by the caustics, the UTD (without junction-corner fields) can be used for the case where the electric field of the incident wave is parallel to the supporting surface. However, the equivalent currents are needed for the orthogonal polarization to account for the surface-inlet junction in addition to the singularities. It is noted that this is the dominant mechanism for near nose on incidence ($60^\circ < \theta < 100^\circ$).

In order to verify and check the preciseness of the EC method results, a model was hand-built and the RCS (backscatter) was measured for both polarizations with incidence angles contained in the $\phi=0$ plane. The dimensions and geometry of this model are given in Figure 4.18. All parameters describing this model of structure B are the same as those used for calculating the patterns in Figure 4.17. Of course, a cylindrical inlet of finite length had to be constructed. In order to reduce the backscattered fields from the back of the finite inlet, in the nose region, its face was tilted at an angle of $\alpha^b_{tl}=45^\circ$ as shown, and absorbing material was placed at two positions.

The back rim of the inlet is associated with two stationary points (see Figure 4.2). One is located at the top of the rim and the other at the bottom. The absorber at the bottom of the back inlet face serves
Figure 4.18. Geometry of the model of structure B used for measurements.
the purpose of eliminating the contribution of any backscatter mechanisms that diffract through the lower stationary point. The absorber inside the inlet reduces the effect of the ray mechanisms associated with paths through the inner portion of the inlet. Therefore, the only significant mechanisms that backscatter in the front region from the back of the inlet are those associated with the top stationary point and traveling outside the inlet. These are demonstrated in Figure 4.19 and include the directly diffracted ray from $P^b_{o}$ and the $D_e-D_e$ rays from $P^b_{o}$ to $P^o_e$ and conversely. The fields due to these rays must now be added to the patterns in Figure 4.17 before they are compared to the measurements. For the sake of continuity, the reader is referred to Appendix B for their calculation. Of course, these mechanisms exist only in the region of $\theta<\pi/2$, but at $\theta=\pi/2$, their fields sum up to zero for the $E_\theta$ polarization and the front edge is dominant for the $E_\phi$ polarization. Consequently, pattern continuity is maintained. Additional higher order ray mechanisms which are caused by the existence of $P^b_{e}$ are illustrated in Figure 4.20. However, these produce comparatively weak fields and were not considered.

The measured and calculated results for the model in Figure 4.18 are shown in Figures 4.21 for both polarization of incidence. An excellent agreement is observed considering the complexity of the model, the difficulty in the alignment and the approximations made in the analysis. Only a small discrepancy is observed in the $E_\phi$, $\phi=0$ pattern.
Figure 4.19. Dominant ray mechanisms associated with the top stationary point of the back inlet rim.
Figure 4.20. Some higher order ray mechanisms associated with the stationary point $p_{e}^{bo}$.
Figure 4.20. (Continued).
Figure 4.21. Comparison of measured and calculated $E_\phi$ and $E_\theta$ RCS patterns from structure B with $R_1=12.073''$, $\alpha=46.34^\circ$, $a=2.1''$, $\alpha_{t,z}=0$, $\Delta=0$, $\phi_{t,z}=45^\circ$, $\varepsilon=7''$ and $\lambda=1.31''$. 

a). $E_\phi$, $\phi=0$ RCS pattern.
b). $E_\theta$, $\phi=0$ RCS pattern.

Figure 4.21. (Continued).
where the caustic due to the R-D_e ray field occurs. This is in the non-specular region of the pattern and is probably due to measurement error or the D_e-D_e ray shown in Figure 3.5. The rippling effect in the later portion of the measured patterns is due to creeping wave diffraction along the major cross section of the ogive (see Figure (4.11)). As discussed, this component was not included in our solution.

The RCS of structure B can be easily studied as a function of its various geometrical parameters. Some results are presented here in order to demonstrate the capability of the numerical solution. Figures 4.22 and 4.23 show RCS patterns of structure B in the \( \phi = 0 \) plane, with \( \alpha_{t_{\perp}} \) equal to -10 and 10 degrees, respectively. The major pattern characteristics are similar to those presented before, especially for the E_\theta patterns. A lobe, though, appears in the non-specular region of the E_\phi pattern due to the larger inlet radius (a=3" for Figures 4.22 and 4.23) used for these patterns. Also it should be noted that the caustics contained to the R-D_e and D_e fields are still the cause of the large RCS but their location is primarily a function of the tilt angle, \( \alpha_{t_{\perp}} \). Figures 4.24 and 4.25 show RCS patterns for \( \Delta \) equal to -1.5" and 1.5", respectively. It is seen that positive \( \Delta \) and \( \alpha_{t_{\perp}} \) cause similar effects to the E_\phi pattern. However, the effect of \( \Delta \) is not as strong for this geometry due to large R_1.
Figure 4.22. Calculated $E_\phi$ and $E_\theta$ RCS patterns of structure B by the EC method and UTD; $R_1=26''$, $\alpha=45^\circ$, $a=3''$, $\alpha_{\phi}=-10^\circ$, $\Delta=0$ and $\lambda=1.311''$. 

a). $E_\phi$, $\phi=0$ RCS pattern.
b). $E_\theta$, $\phi=0$ RCS pattern.

Figure 4.22. (Continued).
Figure 4.23. Calculated $E_\phi$ and $E_\theta$ RCS patterns of structure B by the EC method and UTD; $R_1=26^\circ$, $\phi=45^\circ$, $a=3^\circ$, $\alpha=10^\circ$, $\Delta=0$ and $\lambda=1.311^\circ$. 

a). $E_\phi$, $\phi=0$ RCS pattern.
b). $E_{\theta}, \phi=0$ RCS pattern.

Figure 4.23. (Continued).
Figure 4.24. Calculated $E_{\phi}$ and $E_{\theta}$ RCS patterns of structure B by the EC method and UTD; $R_1=26''$, $\alpha=45^\circ$, $a=3''$, $\alpha_{\phi}=0^\circ$, $\Delta=1.5''$ and $\lambda=1.311''$. 

a). $E_{\phi}$, $\phi=0$ RCS pattern.
Figure 4.24. (Continued).

b). $E_\theta$, $\phi=0$ RCS pattern.
Figure 4.25. Calculated $E_\phi$ and $E_\theta$ RCS patterns of structure B by the EC method and UTD; $R_1=26''$, $\alpha=45^\circ$, $a=3''$, $\alpha_{\phi,2}=0^\circ$, $\Delta=1.5''$ and $\lambda=1.311''$. 
b). \( E_\theta, \phi=0 \) RCS pattern.

Figure 4.25. (Continued).
C. RCS IN NON-PRINCIPAL PLANES: A DISCUSSION

This chapter concentrated on computing the RCS from inlet structures in the \( \phi = 0 \) plane which was also the principal plane associated with the surface and the inlet of structures A and B. In this plane, the RCS obtains its maximum value due to the existence of the caustics associated with the individual ray fields. However, for incidence angles off the principal plane of the inlet, the RCS drops rapidly. Because of this behavior, we were not primarily concerned with the computation of the RCS in the non-principal planes of the inlet structures. Nevertheless, the EC method described in Chapter III can be applied in any plane of incidence. In this case, the ray geometry will not be symmetric with respect to \( \theta_i = 0 \), and therefore, the EC integration must be carried out from \( P_e^C_1 \) to \( P_e^C_2 \) (see Equation (3.98)).

It should be added that recently, as mentioned in Chapter II, Sikta and Peters [54] suggested that the edge vector, \( \hat{e} \), in the equivalent current expressions should be replaced by

\[
\hat{e}_* = \frac{\hat{I}_x \hat{n}_I}{|\hat{I}_x \hat{n}_I|} \quad (4.118)
\]

for the \( D_e \) and R-\( D_e \) fields in Equations (3.7) and (3.21), by

\[
\hat{e}_* = \frac{s \hat{r} \hat{r}_x \hat{n}_I}{|s \hat{r} \hat{r}_x \hat{n}_I|} \quad (4.119)
\]
for the R-De-R field and by

$$\hat{e}_n = \frac{s^d x \hat{n}_I}{|s^d x \hat{n}_I|}$$

(4.120)

for the Ds-De-Ds field in Equation (3.85). Thus, only the current

element component perpendicular to the plane of scattering is

considered. Such a modification does not have any appreciable effect

except within 5° of the broadside region of the inlet and possibly for

incidences outside the principal plane. Of course, our previous results

are uneffected by this change.

Although this chapter concentrated on the study of cylindrical

inlets placed over bodies of revolution, any other type of geometry can

be studied if the geometry subroutine in the existing solution is

modified appropriately. The purpose of this chapter was only to test

the solution by the EC method, developed in Chapter III, for generalized

inlet structures. Based on our development and results, the RCS for

arbitrary incidences on various inlets over smooth surfaces can now be

studied theoretically within the context of the assumptions stated in

the introduction of Chapter III.
CHAPTER V

RCS FROM STRUCTURES WITH CURVED RIM INLETs

A. INTRODUCTION

In many instances, the inlet rim may be composed of a curved surface in lieu of an edge. The RCS of such inlet structures is then of concern. The purpose of this chapter is to compute the RCS from a structure with a curved inlet rim. A toroidal shape with radius $p_I$ is used over the center of an ogive, as shown in Figure 5.1. This will be referred to as structure C. The backscatter mechanisms will be analyzed, discussed and compared to those associated with the thin-edge type of inlets. It should also be noted that more complex shaped inlet rims could be treated with the presented solution. This would simply require that the radius $p_I$ be replaced by the locus radius of the inlet reflection point being considered.

The region of interest will again be in the $\phi=0$ plane for $0<\theta<\pi/2+\alpha$ (see Figure 5.1). In this range, structure C has a shadow boundary (SB) as demonstrated in Figure 5.2. This SB is the same as that of structure B given in Equation (4.53) with an inlet radius identical to the variable $a$ as defined in Figure 5.1 and $\alpha_L=\Delta=0$. 

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Figure 5.1. Structure C: Geometry of a lipped cylindrical inlet over an ogive.
Figure 5.2. Geometry of the shadow boundary and surface rays associated with structure C; shown view is a cross-sectional cut in the $\phi=0$ plane.
In the lit region of structure C there exist four dominant backscatter ray mechanisms. One is the GO ray from the ogive and the other three are caused by the presence of the inlet as shown in Figure 5.3. These are specular rays and follow similar paths with the stationary rays discussed in relation with the thin-edge inlets. The only difference is that edge diffraction has been replaced with a reflection in the case of structure C. These mechanisms (RₚR-R and RₚR-R) generate purely GO fields. Note also that the reflection point is not the same for all three mechanisms as shown in Figure 5.3.

In the shadow region, there exists only the tip diffracted fields and the fields due to the Dₛ-R-Dₛ ray mechanism. This ray was illustrated in Figure 5.2 and when on the surface of the ogive it follows the same geodesic path as that of the Dₛ-Dₑ-Dₛ ray discussed earlier, which is a great circle for incidence in the φ=0 plane.

One should also note that the inlet of structure C has been designed so that there are not any significant diffraction mechanisms from the various inlet surface discontinuities. In particular, the top of the inlet has been tapered toward the back and the lip smoothly joins its inner portion in order to minimize diffraction due to inflection points. The junction of the inlet with the surface can cause additional diffraction in the backscatter direction but these rays produce weak fields in comparison with the GO fields from the top of the inlet. This statement now holds for both polarizations of incidence, although such junction diffraction field would be weaker for the φ than the θ polarization.
a). Singly reflected ray.

Figure 5.3. Geometry associated with the singly, doubly and triply reflected rays; shown views are cross-sectional cuts in the $\phi=0$ plane.
b). Doubly reflected ray.

Figure 5.3. (Continued).
c). Triply reflected ray.

Figure 5.3. (Continued).
The GO fields from the ogive are given by Equation (4.41) and the R, R-R and R-R-R ray fields associated with the inlet can easily be computed as discussed in Section C of Chapter II. However, it will be seen that caustic regions appear again. In order to obtain a bounded solution at these regions, the EC method is applied with a new set of Equivalent line Currents in place of the usual Equivalent edge Currents. This is used herein instead of a physical optics approach to avoid the difficulties of a surface integral. These currents are placed on a circular line along the toroidal lip and through the inlet specular point $P_r^0$, $P_r^2$ or $P_r^3$ (see Figure 5.3) depending on the mechanism under evaluation. Of course, they are terminated at the junction of the line with the surface and therefore the EC integral will also incorporate some type of junction-corner fields, the nature of which has not been analyzed thus far.

We now proceed to calculate the backscattered fields from structure C due to the dominant GO rays only.

B. SOLUTION BY THE GO APPROXIMATION

1. Solution in the Lit Region

The GO field from the ogive is given by Equation (4.41). The geometry for the remaining specular mechanisms in the lit region is shown in Figure 5.3.
The singly reflected field from the toroidal inlet rim can be expressed in the far zone as (see Equation (2.2))

\[ E_i^R(\theta) = E^i \mathbf{R} \sqrt{\rho_{11}^1 \rho_{21}^2} e^{j2kq_2^2} e^{-jks} \]

where

\[ \mathbf{R} = -\hat{\phi} \cdot \hat{y} \]

\[ q_2^1 = \rho_1 \cos \phi \]

and \( \rho_{11}^1, \rho_{21}^1 \) are the principal radii of the reflected wavefront. For normal incidence on a toroid in the \( \phi=0 \) plane, they are given by (see Equation (2.14) with \( \phi=0 \))

\[ \rho_{11}^1 = R_{11}/2 = \rho_I/2 \]

and

\[ \rho_{21}^1 = -\frac{a_e}{2(\hat{n}_r^1 \cdot \hat{n}_e)} = \frac{R_{12}(\theta)}{2} \]

as discussed earlier (see Equation (4.50)). The parameters \( R_{11} \) and \( R_{12} \) are the principal radii of the toroid as a function of \( \theta \). Choosing \( \hat{n}_e \), then

\[ a_e = a + \rho_1 \cos \phi \].
Note that $a_e$ is the major radius at the reflection point. Equation (5.4b) now reduces to

$$r_1 = \frac{a + \rho_1 \cos \theta}{2 \cos \theta}, \quad (5.6)$$

with

$$R_{12}(\theta) = \rho_1 + \frac{a}{\cos \theta}. \quad (5.7)$$

The above transverse principal radius is also derived in [71] by the use of differential geometry.

The R-R field can easily be computed in terms of the angle $\beta_{r2}$ as defined in Figure 5.3. The path of this ray is computed by satisfying the laws of reflection at both reflection points, $p_{r2}^0$ and $s_{r2}^0$. Since $p_{r2}^0$, $s_{r2}^0$ and the incident ray are already in the same plane, the only other requirement is that the normals satisfy the relation $\mathbf{n}_0 \cdot \mathbf{n}_2 = 0$, which implies that

$$\theta_1^2 + \theta_1^2 = \pi/2. \quad (5.8)$$

Therefore, for a given surface reflection point,

$$s_{r2}^0 = \rho R_1 \sin \beta_{r2} + 2 R_1 (\cos \beta_{r2} - \cos \alpha) \quad (5.9)$$
then

\[ F_2^0 = \hat{\chi} \rho_1 \cos \beta_2 r_2 + \hat{z}(h + a - \rho_1 \sin \beta_2), \]  

(5.10)

and from the geometry of the R-R ray in Figure 5.3,

\[ \theta(\beta_2) = 2 \beta_2 + \sin^{-1}(\lambda_3/s_0^2), \]  

(5.11)

where (see also Equation (4.55) and Figure 4.14)

\[ \lambda_3 = R_1 \sin \beta_r - \rho_1 \cos \beta_2, \]  

(5.12)

and

\[ s_0^2 = \sqrt{\lambda_3^2 + (\lambda_4 + a - \rho_1 \sin \beta_2)^2}, \]  

(5.13)

with

\[ \lambda_4 = R_1(1 - \cos \beta_2). \]  

(5.14)

The R-R field can now be found by

\[ \overline{E}_{2}^{RR}(\theta) = \overline{E}_{2}^{R}(s) \cdot (-\hat{\lambda} \wedge \hat{r}^2 \wedge \hat{r}) \sqrt{\rho_{12}^2 r^2} \cdot e^{jkd_2^2} \frac{e^{-jks}}{s}. \]  

(5.15)
where
\[ d_{r2}^2 = -(\overline{\rho_{r2} - \overline{0_e}}) \cdot \hat{r} \]  \hspace{1cm} (5.16)

and the phase reference point is
\[ \overline{0_e} = zh \]  \hspace{1cm} (5.17)

as usual. The field, \( E_2^R \), reflected from the surface point \( S_0^{r2} \) is given by Equation (4.67) with the use of the parameters \( \beta_{r2}, s_0^{r2}, \theta_{i1}^{r2} \) and
\[ d_{1}^{r2} = -(S_0^{r2} - \overline{0_e}) \cdot \hat{r} \]  \hspace{1cm} (5.18)
in place of \( \beta, s_0^r, \theta_i^0 \) and \( d_1 \), respectively. Also note that \( \theta_{r2}^{r2} \) and \( \theta_{i1}^{r2} \) are given by Equations (4.66) and (4.61) if \( \theta \) and \( \beta \) are replaced by \( \theta(\beta_{r2}) \) and \( \beta_{r2} \), respectively. Finally, \( p_{12,22}^{r2} \) are the principal radii of the wavefront reflected off \( P_0^{r2} \). Since the ray in the direction of \( S_0^{r2} \) is in the principal plane of the toroid, they are calculated by the use of Equation (2.14), viz.,
\[ \frac{1}{p_{12}^{r2}} = \frac{1}{p_{12}^{r2}} + \frac{2}{\rho_1 \cos \theta_{i1}^{r2}} \]  \hspace{1cm} (5.19a)
\[ \frac{1}{p_{12}^{r2}} = \frac{1}{p_{12}^{r2}} + \frac{2 \cos \theta_{i1}^{r2}}{R_{I2}(\pi/2 + \beta_{r2})} \]  \hspace{1cm} (5.19b)
with \( i_{12} = r_2^2 s_0^2, i_{22} = r_2^2 s_0^2, \theta_{i1}^{r2} = \pi/2 - \theta_{i1}^{r2} \) and \( R_{I2}(\cdot) \) is the
transverse principal radius of the toroid at \( p_{r2}^0 \), given by Equation (5.7).

The geometry associated with the triply reflected field is depicted in Figure 5.3. The ray path of this field satisfies the law of reflection at the reflection point \( S_{r3}^0 \), and \( \hat{n}_{r3}^3 = -\hat{s}_{r3}^3 \), where \( \hat{n}_{r3}^3 \) is the normal to the inlet rim at \( p_{r3}^0 \). For a given \( S_{r3}^0 \), where

\[
S_{r3}^0 = \hat{\chi} R_{1} \sin \beta_{r3}^3 + \hat{\chi} R_{1} \cos \beta_{r3}^3 - \cos \alpha_1, \tag{5.20}
\]

the corresponding incidence angle \( \theta \) satisfying the above conditions, is found by

\[
\theta_{(r_3)} = 2 \beta_{r3}^3 + \sin^{-1}\left(\frac{R_{1} \sin \beta_{r3}^3}{s_{r3}^0 + \rho_1}\right), \tag{5.21}
\]

in which

\[
s_{r3}^0 = \sqrt{2R_{1}(1-\cos \beta_{r3}^3)(R_{1}+a)+a^2} \tag{5.22}
\]

In addition,

\[
p_{r3}^0 = \hat{\chi} \rho_1 \cos \gamma_{r3}^3 + \hat{\chi} (h+a-\rho_1 \sin \gamma_{r3}^3), \tag{5.23}
\]

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where

$$\gamma_{r3} = \pi/2 + \beta_{r3} - \theta_{1}^3$$  \hspace{1cm} (5.24)$$

and

$$\theta_{1}^3 = \theta(\beta_{r3}) - \beta_{r3}^3$$  \hspace{1cm} (5.25)$$

The R-R-R field can now be expressed as

$$E_{r3}^{RRL}(\theta) = E_{3}^{r}(r_{3}) \cdot (-j\gamma + \hat{r}_{3} \hat{r}_{3} \gamma_{r3}) \sqrt{\frac{\rho_{12}^{r3} \rho_{22}^{r3}}{(\rho_{12}^{r3} + \rho_{22}^{r3}) (\rho_{12}^{r3} + \rho_{22}^{r3})}} \cdot \hat{r}_{3}$$

$$\sqrt{\frac{r_{3} r_{3}}{\rho_{13}^{r3}}} e^{jk(d_{1} r_{3} - s_{0} r_{3})} e^{-jks_{s}}$$  \hspace{1cm} (5.26)$$

In the above Equation (see Figure 5.3),

$$d_{1}^{r3} = R_{1} \sqrt{2(1 - \cos \beta_{r3})} \sin(\theta - \beta_{r3})$$  \hspace{1cm} (5.27)$$

the unit vector \( \hat{r}_{3} \) can be found by Equation (4.66) if \( \theta \) and \( \beta \) are replaced by \( \theta(\beta_{r3}) \) and \( \beta_{r3} \), respectively; the parameters

$$\rho_{13,23}^{r3}$$

are the radii of the wavefront reflected off \( S_{r3}^{0} \) toward the backscatter direction and are found from Equation (2.12) as

$$\rho_{13,23}^{r3} = \rho \left( \rho_{12}^{r3} + \rho_{22}^{r3}, \rho_{12}^{r3} + \rho_{22}^{r3} \right), R_{1}, R_{2}(\beta_{r3}), \theta_{1}^3, \pi/2;$$  \hspace{1cm} (5.28)$$

the dyadic reflection coefficient, according to Equation (2.3) and (2.54) is expressed by
$$R_3 = \theta^{r_3} \theta_R(S_{r_3}, \xi_3, \chi_3) + \theta_y R_S(S_{r_3}, \xi_3, \chi_3)$$  \hfill (5.29)$$

with

$$L_3 = \frac{s_0(r_2^3 + s_0^3)}{r_2^3} \hfill (5.30)$$

and

$$R_\tau = R_1 \hfill (5.31)$$

and the principal radii $\rho_{12,22}^r$ are associated with the wavefront reflected off $P_r^0$, viz., from Equation (2.14),

$$\frac{1}{\rho_{12}^r} = \frac{1}{\rho_{11}^r} + \frac{2}{s_0^r} \rho_1 \hfill (5.32a)$$

and

$$\frac{1}{\rho_{22}^r} = \frac{1}{\rho_{21}^r} + \frac{2}{\Gamma_1^r(Y_{r_3})} \hfill (5.32b)$$

In the above, $\rho_{11,21}^r$ are given by Equations (4.71) if $\beta_r^0$ and $\theta_1^r$ are used in place of $\beta$ and $\theta_1^0$, respectively. Finally, $E_3^r(p_{r_3}^0)$ is the reflected field from $S_{r_3}^0$ when measured at $p_{r_3}^0$. It is found by the same expression as $E_2^r(p_{r_2}^0)$ if all parameters with subscript or superscript $r_3$ are used in place of those with $r_2$, respectively.
It should be clear at this point that if one wishes to obtain an RCS pattern in the lit region of structure C as a function of θ, the ogive angles $\beta_R^2$ and $\beta_R^3$ must first be computed. As discussed in the previous chapter, for a given θ, the corresponding β angle must be found numerically. Therefore, for a chosen $\beta_R^3$, θ is determined by Equation (5.21) and one is now faced with the task of solving Equation (5.11) numerically to compute $\beta_R^2$ for this θ. Since $\beta_R^2$ is always close to and greater than $\beta_R^3$, an initial value for $\beta_R^2$ can be chosen equal to $\beta_R^3$ and appropriately incrementing until Equation (5.11) gives the desired value of θ. When the incident ray is close to the shadow boundary, extensive computer time is required for the evaluation of $\beta_R^2$. In this case, $\beta_R^2$ is set equal to $\beta_R^3$ since at the SB both angles merge and become equal to (see Figure 5.2)

$$\beta_S = \cos^{-1} \left( \frac{R_1}{R_1 + a} \right)$$

(5.33)

which corresponds to

$$\theta = \theta_S = \pi/2 + \beta_S.$$  

(5.34)

One further caution should be exercised in relation to the existence boundaries of the R-R and R-R-R rays. As shown in Figures 5.4 and 5.5, there are also minimum ogive angles, $\beta_R^2^{\text{min}}$ and $\beta_R^3^{\text{min}}$, below which the doubly and triply reflected rays are respectively non-existent. From geometrical considerations, it can be shown that
\[ \beta_{r2}^\text{min} = \sin^{-1}\left(\frac{\rho I}{R_1+a}\right) \]  

\( (5.35) \)

and

\[ \beta_{r3}^\text{min} = \theta(\beta_{r3}^\text{min}) - \frac{1}{2} \sin\left(\frac{\rho I}{S_0^3+\rho I}\right) \]  

\( (5.36) \)

Note that the last Equation must be solved numerically. For \( \beta_{r2} < \beta_{r2}^\text{min} \) the doubly reflected ray is compensated by a \( D_s-R-D_s \) mechanism as shown in Figure 5.4. Likewise, when \( \beta_{r3} < \beta_{r3}^\text{min} \), the triply reflected ray is replaced by a \( D_s-R-R-D_s \) mechanism (see Figure 5.5). Because \( \beta_{r2}^\text{min} \) and \( \beta_{r3}^\text{min} \) correspond to nearly broadside incidence, the calculation of the backscattered fields below these angles was not of concern here. Such a computation would be extremely tedious without contributing any information about the RCS of the structure in the forward region.

2. Solution in the Shadow Region

When \( \theta=\theta_s \) (see Equation (5.34)), the singly, doubly and triply reflected mechanisms, discussed in the previous subsection, merge to a single ray. For \( \theta>\theta_s \), the above rays are compensated by the \( D_s-R-D_s \) ray. As discussed in Chapter IV, at \( SB \) the phase of the fields due to these three mechanisms is such that continuity is maintained with the field caused by the \( D_s-R-D_s \) ray. The last is given by (see Equation (2.48) and Figure 5.2)
Figure 5.4. Geometry for determining $\beta_{r2}^{\text{min}}$; shown view is a cross-sectional cut in the $\phi=0$ plane.
Figure 5.5. Geometry for determining $\beta_{r3}^{\text{min}}$; shown view is a cross sectional cut in the $\phi=0$ plane.
The field \( E^{D}sR(s^0_d) \) is that measured at \( S^0_d = xR_1\sin \beta - 2R_1(\cos \beta - \cos \alpha) \) where \( \theta = \pi/2 + \beta \), after it has gone through surface diffraction and reflection, viz., from Equations (2.2) and (2.4),

\[
E^{D}sR(s^0_d) = E^{D}s(p^0_r) \cdot (-\hat{y}\cdot \hat{n}^0_d) \sqrt{\frac{\rho_{12}\rho_{22}}{(\rho_{12}^2 + s^d_0)(\rho_{22}^2 + s^d_0)}} e^{-jks^d_0} \tag{5.38}
\]

in which

\[
s^d_0 = s^r_0(\theta_s) = s^r_0(\theta_s) = R_1\tan \beta_s - \rho_I \tag{5.39}
\]

In addition, the principal radii, \( \rho_{12,22}^r \) of the wavefront reflected from \( p^0_r \) are given by (see Equation (2.14))

\[
\frac{1}{\rho_{12}^r} = \frac{1}{s^d_0} + \frac{2}{\rho_I} \tag{5.40a}
\]

and

\[
\rho_{22}^r = \frac{R_1(\pi/2 + \beta_s)}{2} \tag{5.40b}
\]

provided surface ray torsion is neglected, and \( \hat{n}^0_d \) was defined in Equation (4.86). The electric field, \( E^{D}s(p^0_r) \), is the same as that given in Equation (4.89) and with all parameters found by Equations (4.90) to (4.97), except that \( s^d_0 \) of Equation (5.39) must be used. Also the dyadic
surface diffraction coefficient, $T_3$, is given by Equation (4.98) with $t_3$, $s_3^d$ remaining unchanged, and the expressions for $p_{23}^d$, $x_3^d$ and

$$\sqrt{\frac{dn(s_0^d)}{dn(s_0^d)}}$$

in Equations (4.102) to (4.104) are still valid if the value of

$$L_3^d = \frac{(p_{22}^r+s_0^d)(p_{12}^r+s_0^d)}{p_{23}^r}$$

is used, and $p_{22}^r$ in place of the caustic distance $p$.

3. Results

The RCS from structure can now be expressed as

$$\sigma(\theta, \phi=0) = 4\pi^2 \lim_{s\to\infty} \frac{1}{|E|^2} \left\{ \left|\frac{E_0 - E_1 + 2E_2 + E_3}{E_0 + E_1 + E_2 + E_3}\right|^2 \right\} \begin{cases} \theta_{min} < \theta < \theta_s & \theta_s < \theta < \pi + \alpha \\ \end{cases}$$

Based on the above formulation, a plot of $\sigma$ with $R=12.073^\circ$, $\phi=46.34^\circ$, $\alpha=2.1^\circ$ and $\rho_l=\lambda/4=.3275^\circ$ is shown in Figure 5.6 for the $\phi$ and $\theta$ polarizations of incidence ($\phi=0^\circ$). The individual ray field components are also given and one observes that the singly and doubly reflected fields have caustics at the same positions where the caustics associated with the thin edge inlets were observed for identical size structures (see Figures 4.15 and 4.21 and note that the caustics are independent of $\rho_l$). The caustics occur because the locus of all inlet points on a
Figure 5.6. Calculated $E_\phi$ and $E_\theta$ RCS patterns from structure C by the GO approximation and UTD; $R_1=12.073''$, $\alpha=46.34^\circ$, $a=2.1''$ and $\rho_I=\lambda/4=.3275''$.

a). $E_\phi$, $\phi=0^\circ$ RCS pattern.
TOTAL FIELD

SINGLY REFLECTED FIELD

2 x (DOUBLY REFLECTED) FIELD

TRIPLY REFLECTED FIELD

Radar cross section in dB $> \lambda^2$

$\theta$ angle in degrees

b). $E_\theta$, $\phi=0^\circ$ RCS pattern.

Figure 5.6. (Continued).
circular ring through \( p_0^0 \) or \( p_0^1 \) become specular. This is also confirmed by the fact that the transverse principal radii, \( \rho_2^1 \) and \( \rho_2^2 \) (see Equations (5.6) and (5.19b)), become indeterminate at the caustics associated with the singly and doubly reflected rays, respectively.

In order to obtain a bounded solution at the caustic regions, the concept of equivalent currents can again be applied to generate a new set of EC. This is done by employing an approach similar to that outlined in Chapter II. The subject of the next section deals with the derivation of these currents and the application of the EC method by their use.

C. SOLUTION BY THE EC METHOD

The reflected field from a cylindrical surface due to a plane wave incidence, as shown in Figure 5.7, is given in the far field by (see Equation (2.2) with \( \rho_{2}^{r=\infty} \))

\[
E_{\parallel, \perp}^R = E_{\parallel, \perp}^i (\pm 1) \sqrt{\rho_1^r} \frac{e^{-jks}}{\sqrt{s}}
\]

(5.43)

where the subscript "\( \parallel \)" refers to the parallel polarization of incidence and likewise for the "\( \perp \)" subscript. The "+" sign is associated with the parallel polarization and the "-" with the perpendicular.

Equating the above \( E_{\parallel}^R \) to Equation (2.62) one obtains

\[
I^e = E_{\parallel}^i \sqrt{\frac{\rho_1^r}{Z_0}} \sqrt{\frac{8\pi}{k}} e^{-j\pi/4}
\]

(5.44)
a). Due to plane wave.

b). Due to an equivalent line current.

Figure 5.7. Illustration of the reflected field from a cylinder due to a plane wave incidence or an equivalent line current on the cylinder.
In the case of a magnetic current after employing the same procedure for $E^R$, it is found that
\[
I^m = -E_\perp \sqrt{\frac{\rho_1}{\sqrt{\frac{8\pi}{\kappa}}}} e^{-j\pi/4} \quad (5.45)
\]
Consequently, generalizing the above equivalent line currents to the case of an arbitrary specular line and direction of incidence as done with the equivalent edge currents in Equations (2.67) and (2.68), the following expressions are obtained:
\[
\overline{I}^e = \hat{p} \frac{\mathbf{E}^1}{\zeta_0 \sin \theta_0} \sqrt{\frac{\rho_2}{\sqrt{\frac{8\pi}{\kappa}}}} e^{-j\pi/4} \quad (5.46)
\]
and
\[
\overline{I}^m = -\hat{p} \frac{(\mathbf{p} \times \mathbf{s}^i) \cdot \mathbf{E}^1}{\sin \theta_0} \sqrt{\frac{\rho_2}{\sqrt{\frac{8\pi}{\kappa}}}} e^{-j\pi/4} \quad (5.47)
\]
where $\hat{p}$ is the unit vector tangent to the specular line (for the toroid this line will be a circle through the specular point); $\rho_2$ is the radius of curvature of the reflected wavefront at each path point perpendicular to $\hat{p}$ and $\beta^i_0 = \cos^{-1}(-\mathbf{s}^i \cdot \hat{p}) \quad (5.48)$

The above equivalent currents can now be used in Equations (3.4a) and (3.4b) in order to compute the singly and doubly reflected fields from structure C in the $\phi=0$ plane.
For the singly reflected field (see Figure 5.8)

\[ \hat{p} = \hat{r}_{12} = \hat{r}_{I2} = \hat{y} \cos \theta_I - \hat{z} \sin \theta_I, \]

(5.49)

\[ \hat{r}_{11,21} = \rho^r (w_{12}, \rho_{11}, \theta, \omega_{r1}, \theta_{11}), \]

(5.50)

\[ \theta_{r1} = \cos^{-1} (\hat{r}_{I1} \cdot \hat{r}_{11}), \]

(5.51)

\[ \hat{h}_{11} = \hat{x} \sin \theta + \hat{y} \sin \theta \cos \theta + \hat{z} \cos \theta, \]

(5.52)

\[ \omega_{r1} = \cos^{-1} (|\hat{r}_{I1} \cdot \hat{r}_{I2}|), \]

(5.53)

\[ \hat{r}_{12} = \frac{\hat{I} \times \hat{h}_{11}}{|\hat{I} \times \hat{h}_{11}|}, \]

(5.54)

\[ \hat{r}_{11} = \hat{r}_{I2} \times \hat{h}_{11} \]

(5.55)

and the locus of the specular line is given by

\[ \bar{p}_{r1} = \hat{x} \rho_{r1} \sin \theta (h + a + \rho_{r1} \cos \theta) \sin \theta + \hat{z} (h + a + \rho_{r1} \cos \theta) \cos \theta. \]

(5.56)

Note that \( \hat{r}_{I1} \) and \( \hat{r}_{I2} \) are the principal directions of the toroid at \( p_{r1} \), \( \hat{h}_{11} \) is the unit normal at \( p_{r1} \), and \( \hat{r}_{I2} \) is the unit vector normal to the plane of incidence. In addition, \( \rho_{11} \) was used in place of \( \rho_{r1} \) in Equations (5.46) and (5.47) since the computation of \( \rho_{r1} \) is tedious and this approximation does not cause any noticeable effect on the total result.
Figure 5.8. Illustration of some parameters associated with the calculation of the Equivalent line Currents, \( \Im^e \) and \( \Im^m \).

The modified Green's function for this field component has the same expression as that given in Equation (3.20) but \( P_e \) and \( \hat{e} \) must be replaced with \( P_{11} \) and \( \hat{\rho} \), respectively. Finally, due to symmetry, the integration is carried out from \( \theta_1 = 0 \) to

\[
\theta_1 = \theta_1 = \cos^{-1}\left\{ -\frac{h^2 - h_{11}^2 + h_{21}^2}{2hh_{21}} \right\} \tag{5.57}
\]

in which

\[
h_{11} = \frac{\sqrt{R_1^2 - (\rho_1 \sin \theta)^2 + h - R_1}}{h} \tag{5.58}
\]
and
\[ h_{z1} = a + p_1 \cos \theta. \] (5.59)

The result must then be doubled.

For the doubly reflected field, the EC expressions become
\[ \overline{I}^e = \frac{\hat{p} \cdot E^R_2(Pr_2)}{Z_0 \sin \beta_0} \sqrt{\frac{r^2}{\rho_{12} \sqrt{\frac{2\pi}{k}}}} e^{-j\pi/4} \] (5.60)

and
\[ \overline{I}^m = -\frac{\hat{p} \cdot (px \hat{r}^2) \cdot E^R_2(Pr_2)}{\sin \beta_0} \sqrt{\frac{r^2}{\rho_{12} \sqrt{\frac{2\pi}{k}}}} e^{-j\pi/4}. \] (5.61)

In the above, \( Pr_2 \) defines the locus of the specular line and is given by
\[ \bar{p}_{r2} = x(\rho_1 \cos \theta_{r2} + (h + a - \rho_1 \sin \beta_2) \sin \theta_1 + 2(h + a - \rho_1 \sin \beta_2) \cos \theta_1, \] (5.62)

where \( \beta_{r2} \) satisfies Equation (5.11) and is found as described in the previous section. The reflected field \( E_2^R \) is the same as that of Equation (3.38) and must be computed at each integration point, \( Pr_2 \), by the application of the raytracing routine discussed there. The unit vector, \( \hat{r}^2 \), denotes, as usual, the direction of the reflected ray off the ogive and toward the appropriate \( Pr_2 \). \( \rho_{12,22} \) are the principal radii of the wavefront reflected off \( Pr_2 \) and are found from Equation (2.12) as
\[ \rho_{12,22} = r (\rho_{12} r_{22}, \rho_{12}, R_{12} (\pi/2 + \beta_{r2}), \omega_{12}, \theta_{i2}) \]  

(5.63)

with

\[ \theta_{i12} = \cos^{-1} (-\hat{t} \cdot \hat{n}_{I2}) \]  

(5.64)

\[ \hat{n}_{I2} = \hat{r} \cos \beta_{r2} - \hat{\theta} \sin \beta_{r2} - \hat{z} \cos \theta_{r2} \]  

(5.65)

\[ \omega_{i12} = \cos^{-1} (|\hat{t}_{I1} \cdot \hat{t}_{I2}|) \]  

(5.66)

\[ \hat{t}_{I2} = \frac{\hat{I} \times \hat{n}_{I2}}{|\hat{I} \times \hat{n}_{I2}|} \]  

(5.67)

\[ \hat{t}_{I1} = \hat{t}_{I2} \times \hat{n}_{I2} \]  

(5.68)

and \( \rho_{12,22} \) are the principal radii of the wavefront incident on \( P_{r2} \) (see Equation (3.24a)). In addition, the unit vectors \( \hat{t} \) and \( \hat{t}_{I2} \) are given in Equation (5.49) and

\[ \beta_{i0} = \cos^{-1} (-\hat{t}_2 \cdot \hat{p}) \]  

(5.69)

Finally, the integration should be carried out from \( \theta_{I} = 0 \) to

\[ \theta_{I} = \theta_{Ic} = \cos^{-1} \left[ \frac{-h_{22}^2 - h_{22}^2}{2h_2} \right] \]  

(5.70)
where

\[ h_{x2} = \sqrt{R_1^2 - (\rho_1 \cos \beta_{r2})^2 + h - R_1} \]  \hspace{1cm} (5.71)

and

\[ h_{z2} = a - \rho_1 \sin \beta_{r2} \]  \hspace{1cm} (5.72)

The result must then be doubled.

The EC method and UTD (in the shadow region) can now be applied to recompute the RCS patterns shown in Figure 5.6. The results are shown in Figure 5.9 for the \( \phi \) and \( \theta \) polarizations (\( \phi = 0^\circ \)) of incidence. A number of observations in these patterns can now be discussed.

First, the \( E_\phi \) pattern is quite similar to that of structure B shown in Figure 4.17. This is essential because it validates the results of the EC method. In fact, as \( \rho_1 \) is decreased, the result from structure C should approach that of structure B. It must, of course, be realized that the GO optics approximation is questionable for \( \rho_1 < \lambda / 4 \) and therefore such a comparison could not be made.

Second, the \( E_\theta \) pattern of Figure 5.9 is drastically different from that of the thin-edge inlet in Figure 4.17. However, this was to be expected because the origin of the backscattered fields from either structure is also different. In fact, the \( E_\theta \) field near the nose of structure B is caused by the junction-corners at the base of the inlet and that of structure C by the specular points at the top of the inlet. The last are GO fields and therefore remain large throughout the pattern range.
a). $E_\phi$, $\phi=0$ RCS pattern.

Figure 5.9. Calculated $E_\phi$ and $E_\theta$ RCS patterns from structure C by the EC method and UTD; $R_1=12.073''$, $a=46.34^\circ$, $a=2.1''$ and $\rho_1=\lambda/4=.3275''$. 

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b). $E_\theta, \psi=0$ RCS pattern.

Figure 5.9. (Continued.)
Third, the fields associated with the singly and doubly reflected ray mechanisms as computed by the EC method are comparable to those obtained by the GO approximations. More precisely, the two results are basically identical within 1-2 dB when outside the caustic region and when at the caustics the EC method obtains a bounded result, whereas that by the GO becomes indeterminate. One should further note that the pattern of the singly reflected field is identical for both polarizations of incidence as expected, and ripples about the GO result when away from its caustic region. The last observation is because θIC in Equation (5.57), which defines the limits of integration, is varying as a function of the incidence angle θ.

Fourth, and most important, a quick comparison of the patterns in Figure 4.17 and 5.9 reveals that the scattered fields from structure C with ρI=λ/4 are 5-10 dB higher in the region near the nose (θ=π/2). In fact, for ρI=λ/2, the RCS in the same region is doubled (3 dB increase) for both polarizations of incidence as shown in Figure 5.10. Therefore, the RCS of structures associated with lipped inlets appears to be proportional to the radius of the lip.

It should be further mentioned that the extrapolation routine (see Equation (3.99)) was used in the patterns of Figures 5.9 and 5.10 whenever appropriate. Close to the shadow boundary, the EC method for the doubly reflected field was abandoned for the reasons discussed in the previous chapter and the GO approximation was then used up to the shadow.
Figure 5.10. Calculated $E_\phi$ and $E_\theta$ RCS patterns from structure C by the EC method and UTD; $R_1=12.073''$, $\alpha=46.34^\circ$, $a=2.1''$ and $\rho_1=\lambda/2=.655''$. 

<table>
<thead>
<tr>
<th>Angle in Degrees</th>
<th>Radar Cross Section in $\lambda^2$</th>
</tr>
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<tbody>
<tr>
<td>$	heta$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>10.00</td>
</tr>
<tr>
<td>5</td>
<td>15.00</td>
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<tr>
<td>10</td>
<td>20.00</td>
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<tr>
<td>15</td>
<td>25.00</td>
</tr>
<tr>
<td>20</td>
<td>30.00</td>
</tr>
</tbody>
</table>

- **TOTAL FIELD**
- **SINGLY REFLECTED FIELD**
- **2 X (DOUBLY REFLECTED) FIELD**
- **TRIPLY REFLECTED FIELD**
b). $E_{\theta}, \phi=0$ RCS pattern.

Figure 5.10. (Continued)
boundary. The pattern point where the switch was made, is indicated by a small jump in the region of 100-130 degrees.

Furthermore, there was no need to apply the EC method for the computation of the triply reflected field since it was not associated with a caustic for this geometry and is also very weak. Alternatively, the EC method developed here can be applied to the triply reflected field if the need appears.
CHAPTER VI
BACKSCATTERING FROM A FLAT PLATE OVER AN ARBITRARY SMOOTH SURFACE

A. INTRODUCTION AND PROBLEM FORMULATION

The problem of backscattering from a plate or fin over an arbitrary smooth surface (see Figure 6.1) is not, in actuality, a different one than that from the thin-edge inlet rim over a surface. If the plate is of general shape, the computation of the dominant field components \(D_e, R-D_e, D_e-R, R-D_e-R\) must again be accomplished by the use of the EC method. The presentation in Chapter III is then still valid when applied to the plate periphery instead of the inlet rim. In the case of flat plates, though, the parameters associated with the edge can be simplified. Particularly, the principal radii of the surface forming the edge, \(R_{11}\) and \(R_{12}\) are now infinite and therefore \(L^i=L^r\) in the EC expressions given in Equations (3.7), (3.21) and (3.85). Also, the normal to the plate, \(n_p=n_I\) (see Figure 3.2) is a constant throughout the EC integration. However, for a complete solution, when in the broadside
Figure 6.1. General shape plate over an arbitrary surface.
region of the plate, one must add the Reflected-Reflected (R-R) field to
the above components (see Section D of this Chapter).

For obvious reasons, a solution which does not involve numerical
integration is always desirable. If the periphery of the flat plate can
be described by piecewise linear segments (edges) then one can
incorporate the concept of corner diffraction (see Section H of Chapter
II) to obtain the backscattered field from a plate structure in the
context of the UTD. This is done by considering only stationary and end
point contributions. Recently, with the development of a diffraction
coefficient from an inflection point [59], such a solution can also be
obtained when the plate's periphery is defined by a finite number of
curvalinear segments (for example ellipsoidal or paraboloidal). One
should further note that at caustic regions when the ordinary UTD is
employed, the proper limit from the end point contributions must be taken
in order to obtain a bounded solution. Such matters will be discussed
in the following section.

This chapter develops the theory required for the computation of the
backscattered field at any point in the lit region of the plate
structure by the use of the UTD. The plate will be defined by a set of
corners which are assumed to be connected with straight edges (see Figure
6.2). In the next chapter measured and calculated results are presented
from a plate normally over an ogive for various pattern cuts with respect
to the $\theta$ and $\phi$ angles (see Figure 6.1). As discussed in Chapter III our
calculations will be limited to the lit region only due to the
unavailability of the required surface geodesic paths at the time of this
a). Plate is a minimum distance, $d$, away from the surface.

b). Plate is connected to the surface.

Figure 6.2. Plate formed of linear edge segments over an arbitrary smooth surface.
study. The computation of the backscattered fields in the shadow region is a matter of current investigation. It should be mentioned that the geodesic paths can at present be computed for a quite general surface [68].

Two types of plate structures will be studied. One involves the case when the plate is away from the surface, as shown in Figure 6.2, and the other when the plate is connected to the surface (again see Figure 6.2). The first geometry could be representative of cracks such as exist in control surfaces and can potentially have significant RCS values. However, the second geometry is also discontinuous at the junction of plate with the surface and therefore the study in this dissertation cannot be used to make any conclusions regarding the effect of the cracks. The analysis of the scattering of cracks is being currently pursued by other researchers at the ElectroScience Laboratory. Once the UTD solution of the backscattered field from the geometry with the crack is completed, it will then be extended to that where the plate is connected with the surface. This is accomplished by the incorporation of the Junction-Edge and Junction-Corner diffracted fields (see Figure 6.2). These fields are caused by the discontinuities at the junction of the plate with the surface. The junction-corner field is derived here by taking a limiting value of the fields diffracted through the bottom corners of the plate. However, the junction-edge field is dominant only when the plate is normal to the surface and in this case they are derived here by the application of image theory. A detailed treatment of these fields will be given in Section C of this chapter.
As discussed in the last section of Chapter II, our solution includes the mechanisms involving first order edge or corner diffraction. A list of these is given in Table 6.1 and one should note that the fields $E_{JE}$ and $E_{JC}$ exist only in conjunction with the structure in Figure 6.2 where the plate is connected to the surface. Also, note that in this table, the phenomena of corner and edge diffraction are associated with each edge and corner of the plate, and that of reflection with the surface upon which the plate is installed. Let us now discuss the backscatter ray mechanisms which are not shown in Table 6.1 and explain some of the reasons for neglecting them. These mechanisms include the edge diffracted and GO rays from the plate; the reflected-reflected ray between the surface and the plate; the R-D$_e$-R ray; as well as various diffraction mechanisms among the edges and corners of the plate.

The first order edge diffracted field from each plate edge contributes to the backscatter direction only when the incident wave is perpendicular to this edge as shown in Figure 6.3. In any other direction, the equivalent edge currents or the corner diffraction can be used to obtain the backscattered field from the respective edge. In the following, the corner diffraction is to be used. When the wave is directed perpendicular to the edge, all the diffracted rays from this edge merge at infinity. This constitutes a caustic. However, at this point, the corner diffracted fields from each corner also become indeterminate and the proper limit of their sum is bounded, rendering the correct result. This is, of course, a consequence of the required field
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DESCRIPTION</th>
<th>FIGURE</th>
<th>EQUATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{E}^{\text{GO}}_1$</td>
<td>Geometrical Optics field from the surface.</td>
<td>3.4a, 4.11</td>
<td>3.3, 4.41</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{DC}}_1$</td>
<td>Diffracted field from the corners of the plate.</td>
<td>6.8</td>
<td>6.12</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{RD}C}_2$</td>
<td>Reflected-Corner diffracted field (R-D$_C$).</td>
<td>6.13</td>
<td>6.12</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{DC}R}_2$</td>
<td>Corner diffracted-Reflected field (D$_C$-R).</td>
<td>6.13</td>
<td>6.22</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{RD}E}_2$</td>
<td>Reflected-Edge diffracted field (R-D$_E$).</td>
<td>6.14</td>
<td>6.61</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{DE}R}_2$</td>
<td>Edge diffracted-Reflected field (D$_E$-R).</td>
<td>6.14</td>
<td>6.61</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{RD}C}R_3$</td>
<td>Reflected-Corner diffracted Reflected field (R-D$_C$-R).</td>
<td>6.21</td>
<td>6.74</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{JC}}$</td>
<td>Junction-Corner diffracted field.</td>
<td>6.23</td>
<td>6.77</td>
</tr>
<tr>
<td>$\mathbf{E}^{\text{JE}}$</td>
<td>Junction-Edge diffracted field.</td>
<td>6.23</td>
<td>6.80</td>
</tr>
</tbody>
</table>
continuity and of the derivation of the corner diffracted fields as a residue of the EC integral over the finite edge. Thus, in doing so, one does not need to include the edge diffracted field as a separate quantity.

There could also exist a GO field from the plate in the backscatter direction as shown in Figure 6.4. This occurs at the reflection boundary associated with the singly corner and edge diffracted fields. It is obvious that the existence of such a field is restricted only to the pattern point when the incident wave is broadside to the plate. At this point the edge diffracted fields from all edges are at a caustic and it would appear that physical optics or equivalent edge current integration is the only way to calculate the total scattered field from the plate.
Figure 6.4. Illustration of plate reflection in the backscatter direction.

alone. However, the proper limit of the sum of the diffracted fields from all plate corners gives the correct result. In machine calculations, the limiting process is not accurately performed and therefore a "kink" will appear in the pattern. Alternatively, one may force the machine computation to occur at about 1/2 degree away from broadside and the correct result will again be obtained for all practical purposes since the pattern slope is zero at broadside.

The R-R backscatter ray between the plate and the surface occurs when the normal (\(\hat{n}\)) at the surface reflection point and that to the plate (\(\hat{n}_p\)) are orthogonal and in the plane of incidence (see Figures 6.5 and 5.3). Mathematically, this is stated by
Figure 6.5. Illustration of the Reflected-Reflected backscatter ray between the surface and the plate.

\[ \hat{n} \cdot \hat{n}_p = 0 \]  \hspace{1cm} (6.1a)

and

\[ \frac{-\hat{n} \times \hat{I}}{|\hat{n} \times \hat{I}|} = \frac{\hat{n}_p \times \hat{I}}{|\hat{n}_p \times \hat{I}|} \]  \hspace{1cm} (6.1b)

In the current study, plate structures were considered where the plate was placed normal to the surface. Such geometries (with smooth, monotonic surfaces) do not give rise to backscattered R-R fields but are associated with strong Junction-Edge diffracted fields when the plate is
connected to the surface. If the plate is not normal to the surface, the R-R fields become quite strong in certain regions while those from the Junction-Edge are weak. The R-R and Junction-Edge fields are discussed in detail in Section C of this chapter.

The R-D_e-R backscatter ray, shown in Figure 6.6, exists only when the reflected ray is in the plane perpendicular to the edge ($\beta_o=\pi/2$). Such a situation usually occurs when close to broadside of the plate. As seen in our study of the inlet structures, the field associated with this ray is negligible in the deep lit region and becomes essential only when within 5° of the shadow boundary. However, the computation of the R-D_e-R ray path is quite cumbersome and can be accomplished by the raytracing routine discussed in Section B(subsection 4) of this chapter with $\beta^j_o=\pi/2$. Since our calculations will be for the lit region only, the improvement (about 1 dB) due to the inclusion of this component does not justify by far the effort required for determining the ray paths. Therefore, the R-D_e-R field was at present neglected but must be considered (whenever it exists, see Equation (6.25) with $\beta^i_o=\pi/2$) if continuity is to be maintained at the shadow boundary.

Within the plate, there also exist an infinite number of higher order backscatter ray interactions. Some of these which involve a second order edge or corner diffraction may become important when the observation point is close to the plane of the plate. Examples are shown in Figure 6.7 for the D_e-D_e, D_e-D_c and R-D_e-D_e ray mechanisms. In
Figure 6.6. Illustration of the R-D_e-R backscatter ray mechanism.
a). Double edge diffraction.

b). Edge-Corner diffraction.

Figure 6.7. Examples of possible higher order ray mechanisms.
c). Reflection-Double edge diffraction.

Figure 6.7. (Continued).

addition, edge wave mechanisms, indicated in Figure 6.8, are of significant concern. A detailed treatment on the higher order ray mechanisms within the plate can be found in [54]. Our purpose here is to at first study the dominant ray fields given in Table 6.1.

B. PLATE NORMAL TO AND OFF THE SURFACE

1. Introduction

The structure with a plate normal to the surface and at a minimum distance, d, away from it (see Figure 6.2), is associated with the
a). Edge wave double corner diffraction.

b). Reflection and then edge wave double corner diffraction.

Figure 6.8. Examples of possible edge wave mechanisms.
The geometry of diffraction by a corner in the backscatter direction is shown in Figure 6.9 (see also Figure 2.15). Noting that for RCS computations (see Equation (2.71)) $L^i_{\to\infty}$, $L^j_{\to\infty}$, $\beta^j_{\text{ock}} = \pi - \beta^i_{\text{ck}}$, the corner diffraction coefficient of Equation (2.71) reduces to
Figure 6.9. Geometry of the backscatter rays from corners $C_j$ and $C_k$ ($k=j+1$) associated with the $j$th edge.
\[ D_{ck}^{ij} = \cos^{-1}(\hat{e}_e \cdot \hat{e}_j) \] (6.3)

and
\[ \psi_j^{ij} = \psi_i^j = \begin{cases} \cos^{-1}(-\hat{e}_e \cdot \hat{e}_j) & \text{if } -\hat{n}_p \cdot \hat{e}_j \geq 0 \\ 2\pi - \cos^{-1}(-\hat{e}_e \cdot \hat{e}_j) & \text{if } -\hat{n}_p \cdot \hat{e}_j < 0 \end{cases} \] (6.4)

where
\[ \hat{e}_j = \frac{\overline{C_j - C_k}}{|\overline{C_j - C_k}|} \] (6.5)

is the edge vector for the jth edge and will always be in the counterclockwise direction; \( \hat{e}_j \) is defined by Equation (3.11) if \( \hat{e}_j \) is used in place of \( \hat{e}_e \); and \( \hat{t}_e \) is the unit normal to the jth edge in the plane of the plate (same as \( \hat{t}_e \) in Figure 3.2). It is given by
\[ \hat{t}_e = \hat{n}_p \times \hat{e}_j \] (6.6)

with
\[ \hat{\mathbf{n}}_p = \frac{\mathbf{e}_j \times \mathbf{k}}{|\mathbf{e}_j \times \mathbf{k}|} \quad (6.7) \]

In addition, note that \( \beta^j_0 = \beta_0 = \pi/2 \) since edge diffraction is only possible for this angle.

The backscattered field from corner \( C_k \) when associated with the \( j \)th edge is given by (see Equation (2.69))

\[
\begin{align*}
\mathbf{E}^D_{1j} &= \hat{e}^T_0 (0_e) ( \alpha^j, \beta^j, \gamma^j, \Delta^j, \Phi^j, \varphi^j) e^{-j2k[\hat{\mathbf{r}} \cdot (\mathbf{c}_k - \mathbf{0}_e)]} e^{-jks} \\
&= \hat{e}^T_0 (0_e) ( \alpha^j, \beta^j, \gamma^j, \Delta^j, \Phi^j, \varphi^j) e^{-j2k[\hat{\mathbf{r}} \cdot (\mathbf{c}_k - \mathbf{0}_e)]} e^{-jks} \\
\end{align*}
\]

(6.8)

where \( 0_e \) is the usual reference point. The unit vectors \( \hat{\mathbf{e}}^j_0 \) and \( \hat{\mathbf{e}}^j \) are associated with the edge fixed plane of diffraction (see Equation (2.21) along with Figure 2.9 and note that for this mechanism \( \hat{\mathbf{e}}^j_0 = \hat{\mathbf{e}}^j \) and \( \hat{\mathbf{e}}^j = \hat{\mathbf{e}}^j \)) and are found for the \( j \)th edge by

\[
\hat{\mathbf{e}}^j = -\frac{\mathbf{e}_j \times \mathbf{k}_p}{|\mathbf{e}_j \times \mathbf{k}_p|} \quad (6.9)
\]

and

\[
\hat{\mathbf{e}}^j_0 = -\mathbf{i} \times \hat{\mathbf{e}}^j \quad (6.10)
\]

The reader is cautioned that our notation in this chapter will become slightly more complex in order to be specific and consistent. The major symbolism will remain the same as that introduced in Chapter II and III but in certain cases subscripts and superscripts will be added in order that reference can be made to a specific mechanism, edge or corner. A
careful look at the figures should give the reader the proper perspective
and definition of these symbols. For example, the angle $\beta^j_{ck}$ is the same
as $\beta_c$ in Figure 2.15, but refers to the jth edge at corner $C_k$; the angle
$\beta^j_{0ck}$ is the same as $\beta_{0c}$ in Figure 2.15, but refers to the jth edge at
corner $C_k$; $\psi^j$ is the usual $\psi$ angle but refers to the jth edge; etc.

Analogously, the corner diffraction coefficient associated with
corner $C_j$ and the jth edge is simply

$$ D^1_{cj,cjh} = -D^1_{cks,ckh} $$

since

$$ \beta^j_{cj} = -\beta^j_{ck} \quad (\text{see Figure 6.9}). $$

The sum of the fields from both corners associated with the jth edge
can now be expressed as

$$ E^1_{lj} = -E^i(0_e) \cdot (-\beta^j_{o}^i D^1_{lj} + \psi^j D^1_{nj}) \cdot \left[ 1 - e^{2j k d e \cos \beta^j_{ck}} \right] 
\frac{e^{-j k s}}{s} $$

where $d^i_{e} = |C_j - C_k|$ is the length of the jth edge and $O_e$ is the usual
reference point. Substituting for $D^1_{cks,ckh}$ from Equation (6.2), we have

$$ E^1_{lj} = \left[ \beta^j_{o}^i E^i(0_e) + \psi^j E^i(0_e) \right] \sin \beta^i_{ck} \frac{\sin(k d e \cos \beta^j_{ck})}{k \cos \beta^j_{ck}} 
\frac{1}{2 \pi \cos^2 \beta^j_{ck}} \left[ \frac{\cos^2 \psi^j}{\cos^2 \beta^j_{ck}} \right] $$

$$ = e^{-j k s} \left[ 2i (C_k - O_e) - d^i_{e} \cos \beta^j_{ck} \right] e^{-j k s} \frac{4 \pi s}{4 \pi s} $$

(6.11)
with \( \psi^j \) as given in Equation (3.23b) if \( \hat{I}_p^j, \hat{e}_e^j \) and \( \hat{I}_m^j \) are used in place of \( \hat{s}_p, \hat{e}_e \) and \( \hat{n}_I \), respectively.

When \( \beta_{ck}^j + \pi/2 \) the incident ray is normal to the \( j \)th edge and the factor

\[
\sin^j \frac{\sin(kd_e \cos \beta_{ck}^j)}{k \cos \beta_{ck}^j} \rightarrow d_e^j .
\]

If this limit is used in Equation (6.11), the correct maximum backscattered fields from the \( j \)th edge are obtained and the EC method is not required.

The RCS of a plate due to first order diffracted rays is simply the sum of the fields from each edge, viz.,

\[
\overline{E}_1^{Dc} = \sum_{j=1}^{Ne} \overline{E}^{Dc}_{1j} .
\]

The above equation was used to compute conical RCS patterns from a \( 2\lambda \times 2\lambda \) plate. The results for the \( \theta \) polarization of incidence are shown in Figure 6.10 and compared to those found by the EC method. Figure 6.11 also presents results for the \( \phi \) polarization. One observes that the results by the use of the corner diffraction coefficient or the EC method are identical near the broadside region of the plate. However, near the nose, the two methods deviate due to the heuristic factor included in the corner diffraction coefficient as discussed in Section H of Chapter II. Sikta [54] has noted that the corner diffracted fields are more accurate than the equivalent currents in this region.
Figure 6.10. $E_\theta$ conical RCS patterns of a $2\lambda x2\lambda$ plate due to singly diffracted fields.

Figure 6.11. $E_\phi$ conical RCS patterns of a $2\lambda x2\lambda$ plate due to singly diffracted fields.
When $\theta^j_0 = \pi/2$ and $\psi^j = \pi/2$ or $3\pi/2$ (broadside to plate) then an indeterminate factor appears in the diffraction coefficient associated with each edge and its evaluation requires a careful limiting process. As mentioned earlier, during machine calculations such a process is not properly done, and therefore, a spike will appear in the pattern. This is demonstrated in Figure 6.12 which shows the RCS pattern of a $2\lambda \times 2\lambda$ plate in the plane perpendicular to it. The EC method result is also presented and it is identical to that obtained by Equation (6.12), except at $\phi = 90^\circ$.

3. **Reflected-Corner Diffracted Fields**

The geometry for the R-D$_C$ ray mechanisms is shown in Figure 6.13. The fields due to corner $C_k$ when associated with the jth edge can be expressed as

$$E^{RD}_{ck} = E^R_k(C_k) \cdot (\beta^{2j}_o \hat{\beta}^{2j}_o \hat{\psi}^j_k \hat{D}^{2j}_C \hat{c} \hat{k}_h) e^{-jk(C_k - O_c) \cdot \hat{r} \over s}, \quad (6.13)$$

where $\beta^j$ and $\beta^j_o$ were defined in Equations (6.9) and (6.10), and $\hat{\beta}^{2j}_o$ with $\hat{\psi}^j_k$ are the unit vectors associated with the edge fixed plane of incidence for the jth edge at $C_k$. Therefore,

$$\hat{\psi}^j_k = -\frac{\hat{e}^{\alpha} x \hat{r}^{ij}_{ckp}}{|\hat{e}^{\alpha} x \hat{r}^{ij}_{ckp}|} \quad (6.14)$$

and

$$\hat{\beta}^{2j}_o = \hat{r}^{ij}_{ck} \hat{\psi}^j_k \quad (6.15)$$
Figure 6.12. $E_\theta$, $\theta=90^\circ$ RCS pattern of a 2x2\lambda plate.
Figure 6.13. Geometry of the Reflected-Corner Diffracted backscatter ray through corner $C_k$ when associated with the $j$th edge.
where \( \hat{s}^{rj}_{ckp} \) is the vectorial projection of \( \hat{s}^{r}_{ck} \) onto the plane perpendicular to the \( j^{th} \) edge (see Appendix A) and \( \hat{s}^{r}_{ck} \) is the direction of the ray reflected off \( S^{ck} \). The scalar corner diffraction coefficients \( D^{2j}_{cks,ckh} \) take their general form given in Equation (2.71), viz.,

\[
D^{2j}_{cks,ckh} = D_{cs, ch}(L_{j}^{12}, L_{ck}^{12}, Y_{j}^{12}, Y_{k}^{12}, B_{ck}^{12}, B_{ckh}^{12})
\]

and the various parameters are:

\[
L_{ck}^{12} = \rho_{e2}^{12}
\]

with \( \rho_{e2}^{12} \) as found by Equation (3.35) if \( S^{ck}, C_{k} \) and \( \hat{e}^{j} \) are used in place of \( S_{r}, P_{e} \) and \( \hat{e} \), respectively; \( Y_{j} \) was given in Equation (6.4);

\[
Y_{k}^{12} = \begin{cases} 
\cos^{-1}(-\hat{t}_{e} \cdot \hat{s}_{ckp}^{j}) & \text{if } -\hat{n}_{p} \cdot \hat{s}_{ckp}^{r} > 0 \\
2\pi - \cos^{-1}(-\hat{t}_{e} \cdot \hat{s}_{ckp}^{j}) & \text{if } -\hat{n}_{p} \cdot \hat{s}_{ckp}^{r} < 0 
\end{cases}
\]

with \( \hat{t}_{e}^{j} \) as defined in Equation (6.6);

\[
B_{ck}^{j} = \cos^{-1}(\hat{\hat{e}}^{j} \cdot \hat{s}_{ck}^{r})
\]

(note \( B_{ck}^{j} = \cos^{-1}(-\hat{\hat{e}}^{j} \cdot \hat{s}_{ck}^{r}) \));

\[
B_{ock}^{j} = \cos^{-1}(\hat{\hat{e}}^{j} \cdot \hat{I})
\]

(note \( B_{ock}^{j} = \cos^{-1}(-\hat{\hat{e}}^{j} \cdot \hat{I}) \));
and the distance parameter, $L_{j}^{i2}$, is associated with the diffraction point of the R-Dg mechanism along the $j$th edge. This ray mechanism is discussed in the next subsection and therefore, $L_{j}^{i2}$ will be given there.

The field, $E_{2}^{c}(C_{k})$, reflected off $S_{r}^{ck}$, is found by the general expression in Equation (3.38) with $S_{r}^{ck}$ and $s_{r}^{r}$ in place of $S_{r}$ and $s'$, respectively. Also note that the parameters used in Equation (3.26) for the computation of the reflected principal radii must be calculated with $\hat{n}_{ck}$ in place of $\hat{n}$. Of course, the location of the reflection point, $S_{r}^{ck}$, is required for a given incidence. Therefore, one is again faced with the finding of a solution to Equation (3.42) when the diffraction point is the plate corner, $C_{k}$. As discussed in Section B (subsection 2) of Chapter III, an initial reflection point corresponding to an incidence $\hat{I}_{K}$ must first be known. This may be chosen as the projection of $C_{k}$ on the surface (see Figure 3.9). Consequently, $\hat{I}_{k}$ is stepped to the desired incidence, $\hat{I}$, and thus tracing the associated reflection point. However, it should be realized that a substantial amount of computer time is required for the application of this routine since the steps are taken at 1-2 degrees and the angle between $\hat{I}_{k}$ and $\hat{I}$ can be quite large. In addition, the process must be applied individually to all plate corners. Note, though, that once the reflection point for the initial incidence is found, only one iteration is sufficient for tracing $S_{r}^{ck}$ from one pattern point to the next.
The total R-D field can now be found as the sum of all such fields from each corner, viz.,

\[
E_{2}^{\text{RD}} = \sum_{j=1}^{N_e} \sum_{k=j}^{j+1} E_{2j}^{\text{RD}c} \tag{6.22}
\]

where \( N_e \) is the number of edges describing the plate.

4. Reflected-Edge Diffracted Field

The geometry associated with the R-D\(_e\) ray from the jth edge is shown in Figure 6.14. The path of this ray is determined by satisfying the reflection laws at \( S^e_j \) and the diffraction law at \( p^j_e \).

Mathematically,

\[
(nx\hat{I})(\hat{s} \cdot \hat{n}) = -(nx\hat{r})(\hat{t} \cdot \hat{n}) \tag{6.23a}
\]

(see also Equation (3.42)) and

\[
\hat{e}_j \cdot \hat{s} = \hat{e}_j \cdot \hat{t} \tag{6.23b}
\]

where

\[
\hat{s} = \hat{s}_e^j - p^j_e - s^e_j \tag{6.24a}
\]

and

\[
\hat{n} = \hat{n}^e_j \tag{6.24b}
\]

as shown in Figure 6.14.
Figure 6.14. Geometry of the Reflected-Edge diffracted backscatter ray associated with the jth edge.
It is noted that this solution may not be unique if the surface is not smooth and monotonic. In addition, the calculated $p^j_e$ may not be within the limits of the $j$th edge. In this case, the R-$D_e$ field associated with the $j$th edge is zero but the $L_{j1}^{12}$ distance parameter must still be computed since it will be required in the calculation of the corner diffraction coefficient (see previous subsection).

Assuming a smooth monotonic surface, one can determine a priori the existence of an edge diffraction point within the limits of the edge. Looking at Figure 6.15, it is clear that $p^j_e$ is within the limits of the $j$th edge if

$$\beta^j_{CJ} < \beta^j_o < \pi - \beta^j_{ck} \quad (k=j+1) \quad (6.25)$$

(remember that $\hat{e}^j$ is always in the counterclockwise direction and $\beta^j_o = \cos^{-1}(\hat{e}^j \cdot \hat{l})$ throughout this chapter), otherwise $p^j_e$ does not lie on the actual edge being considered, but it may still be on an extension of this edge. Whenever Equation (6.25) is satisfied, one is faced with the major task of locating $p^j_e$ and consequently $S^e_j$ for a given incidence. An iterative solution to Equation (6.23a) was given in Chapter III provided that $p^j_e$ is known. This must now be found by the additional relationship in Equation (6.23b).

It is convenient to express $p^j_e$ as

$$p^j_{e0} = p^j_{eo} + e^j \Delta z \quad , \quad (6.26)$$

where $p^j_{e0}$ is an initial known edge diffraction point corresponding to
\[ k = j + 1 \]

\[ \theta_j < \theta_j < \pi - \theta_{ck} \]

a). Edge diffraction point within the limits of the jth edge.

Figure 6.15. Geometry for determining the existence of the edge diffraction point, \( p^j_e \).
$k = j + 1$

b). Edge diffraction point outside the limits of the jth edge.

Figure 6.15. (Continued).
a known reflection point, $S_{r}^{\text{ej}0}$, on the surface. The new diffraction point, $P_{e}^{\text{ej}1}$, corresponds to the surface reflection point, $S_{r}^{\text{ej}1}$, and is a small distance, $\Delta \xi$, along the edge from $P_{e}^{\text{jo}}$ (see Figure 6.16). Our problem now reduces to that of determining $S_{r}^{\text{ej}1}$ and $\Delta \xi$ for a given direction of incidence, $\hat{\mathbf{i}}$. For simplicity, in the following presentation, the superscript $j$ will be suppressed when referring to $P_{e}^{j}$, $\hat{n}^{j}$ or $\beta_{o}^{j}$. Also the reflection point and the associated direction of the reflected ray will be denoted as $S_{r}$ and $\hat{s}_{r}$, respectively. Thus, our nomenclature will be exactly the same as that introduced in Chapter III except that the superscript or subscript, $o$, will correspond to the variable value at the previous incidence instead of integration point. A similar definition holds for the superscript or subscript of $l$. One is referred to Figure 6.16 for the precise definition of each variable.

The introduction of the new unknown $\Delta \xi$ requires that a solution to a set of equations of the form (see also Equation (3.59))

$$
\begin{bmatrix}
    A_{11} & A_{12} & A_{13} \\
    A_{21} & A_{22} & A_{23} \\
    A_{31} & A_{32} & A_{33}
\end{bmatrix}
\begin{bmatrix}
    x' \\
    y' \\
    \Delta \xi
\end{bmatrix}
= 
\begin{bmatrix}
    C_{x'} \\
    C_{y'} \\
    C_{\Delta \xi}
\end{bmatrix}.
$$

(6.27)

be found. Substituting in Equations (3.60) and (3.61) for $P_{e}^{l}$ as given in Equation (6.26) and retaining first order terms, it is found that
Figure 6.16. Geometry associated with the simultaneous tracing of $P_e$ and $S_r$. 

(REFERENCE)
\[ A_{11} = -(I_y e_y^0 + I_y e_{ex}^0), \quad (6.28a) \]
\[ A_{12} = -2I_y e_y^0 + 2I_z e_z^0 + I_z R_2^0, \quad (6.28b) \]
\[ A_{13} = -(I_y e_z^0 + I_z e_y^0), \quad (6.28c) \]
\[ A_{21} = -(2I_x e_x^0 + 2I_z e_y^0 + I_z R_1^0), \quad (6.28d) \]
\[ A_{22} = -A_{11}, \quad (6.28e) \]
\[ A_{23} = I_x e_z^0 + I_z e_x^0, \quad (6.28f) \]
\[ C_{x'} = I_y e_z^0 + I_z e_y^0, \quad (6.29a) \]

and
\[ C_{y'} = -(I_x e_z^0 + I_z e_x^0). \quad (6.29b) \]

Note again that \((x', y', z')\) refer to the usual primed cartesian coordinate system as defined in Equation (3.44) and with origin at \(S_r^0\) (see also Equations (3.48) and (3.53) for generalized coordinate transformations). Therefore, \((e_{x'}, e_{y'}, e_{z'})\) are the components of the unit vector \(\hat{e}\) in the primed coordinate system. The other variables may be expressed in a similar manner.

The remaining matrix components \((A_{31}, A_{32}, A_{33}, \text{as well as } C_{x'})\) can be found by manipulating Equation (6.23b). In accomplishing this, the diffraction law is re-written as
\[ \tan \theta_0 = \tan \left[ \cos^{-1} (\hat{e} \cdot \hat{r}) \right] = \frac{L_1}{\lambda_1}, \quad (6.30) \]

where \( L_1 \) and \( \lambda_1 \) are defined in the Figure 6.16. We proceed now to express \( \lambda_1 \) and \( L_1 \) in terms of \( S_{r}^{1} \), \( P_{e}^{1} \) and \( \hat{e} \).

From geometrical considerations,

\[ \lambda_1 = |S_{p}^{1} - P_{e}^{1}| \quad (6.31) \]

with

\[ \overrightarrow{S_{p}^{1}} = [\hat{e} \cdot (\overrightarrow{S_{r}^{1}} - \overrightarrow{P_{e}^{1}})] \hat{e} + P_{e}^{1} \quad (6.32) \]

as the projection of \( S_{r}^{1} \) onto the edge. Substituting for (see also Equation (3.62))

\[ \overrightarrow{S_{p}^{1}} = x'x' + y'y' + z'z' \quad (6.34a) \]

and neglecting second order terms, one obtains

\[ \lambda_1 = x'e_x + y'e_y -(P_{ex}e_x + P_{ey}e_y + P_{ez}e_z) - \Delta \lambda \quad (6.34b) \]

The derivation of \( L_1 \) in terms of \( x' \), \( y' \) and \( \Delta \lambda \) is much more tedious [72]. From Figure 6.16,

\[ L_1 = L_0 + \Delta S_{r} - (\hat{e} \cdot \Delta S_{r}) \hat{e} = L_0 + \Delta L \quad (6.35) \]
with $\Delta s^r$ defined as

$$\Delta s^r = s^r_{1} - s^r_{0} \quad (6.36)$$

and $L_0$ is the smallest distance from $S^0_r$ to the edge, viz.,

$$L_0 = S^0_p - S^0_r = s^r - (\hat{s} \cdot \hat{s}^0)\hat{e} \quad (6.37)$$

(see Equation (3.24a) for definition of $\hat{s}^r$). Since $\Delta L = |\Delta L|$ is assumed to be much smaller than $L_0$, then $L_1$ can be approximated as

$$L_1 = L_0 + \frac{L_0 \cdot \Delta L}{L_0} \quad (6.38)$$

by retaining the first two terms of its binomial expansion. To obtain an expression for $\Delta L$ in the primed coordinate frame, one substitutes Equations (6.26) and (6.34a) into Equation (6.24a) (note that the $S^0_r$ is the origin of the primed coordinate frame) and then using Equation (6.36) into (6.35) to find

$$\Delta \hat{l} = \hat{x}'(-x'^t + x'e^2_{x^t} + y'e_{x^t}e_{y^t} + z'e_{x^t}e_{z^t})$$

$$\hat{y}'(-y'^t + y'e^2_{y^t} + x'e_{x^t}e_{y^t} + z'e_{y^t}e_{z^t})$$

$$\hat{z}'(-z'^t + z'e^2_{z^t} + x'e_{x^t}e_{z^t} + y'e_{y^t}e_{z^t}) \quad (6.39)$$

Therefore, the dot product $L_0 \cdot \Delta L$ in Equation (6.38) becomes
\[ \mathcal{L}_0 \cdot \Delta \mathcal{L} = - \left( x' \alpha_x' + y' \alpha_y' + z' \alpha_z' \right), \quad (6.40a) \]

where

\[ \alpha_x' = L_{ox} - e_x^2 L_{ox} - e_x e_y L_{oy} - e_x e_z L_{oz}, \quad (6.40b) \]
\[ \alpha_y' = L_{oy} - e_y^2 L_{oy} - e_x e_y L_{ox} - e_y e_z L_{oz}, \quad (6.40c) \]
\[ \alpha_z' = L_{oz} - e_z^2 L_{oz} - e_x e_z L_{ox} - e_y e_z L_{oy}, \quad (6.40d) \]

and

\[ \mathcal{L}_0 = x' L_{ox} + y' L_{oy} + z' L_{oz}. \quad (6.41) \]

Substituting the derived expressions for \( \alpha \) and \( L \) into Equation (6.30), after neglecting higher order terms, it is found that

\[ x'(e_x \tan \beta_0^1 + \alpha_x') + y'(e_y \tan \beta_0^1 + \alpha_y') - \Delta \tan \beta_0^1 = \mathcal{L}_0 + (\vec{P} \cdot \mathcal{E}) \tan \beta_0^1. \quad (6.42) \]

Thus,

\[ A_{31} = R_1^0 (e_x \tan \beta_0^1 + \alpha_x'), \quad (6.43a) \]
\[ A_{32} = R_2^0 (e_y \tan \beta_0^1 + \alpha_y'), \quad (6.43b) \]
\[ A_{33} = -\tan \beta_0^1 \quad (6.43c) \]

and

\[ C_{\Delta \kappa} = \mathcal{L}_0 + (\vec{P} \cdot \mathcal{E}) \tan \beta_0^1. \quad (6.44) \]
Because of the above approximations, neither the calculated value of $\Delta \lambda$ by Equation (6.27), nor will the position of the reflection point $S_r^1$ (as discussed detailly in Chapter III) be exact. The errors associated with the calculation of $S_r^1$ were given in Equation (3.65) and that associated with $\Delta \lambda$ is found by applying Equation (6.23b) to the newly calculated points, $S_r^1$ and $P_e^1$. Therefore,

$$C_{e\Delta \lambda} = \lambda_1 \tan \beta_0 - L_1$$  \hspace{1cm} (6.45)

with $\lambda_1$ and $L_1$ as given in Equations (6.31) and (6.35).

In order to avoid propagation of the above errors, Equation (6.27) must be rewritten in a manner analogous to Equation (3.69) as

$$\begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix} \begin{bmatrix}
x' \\
y' \\
\Delta \lambda
\end{bmatrix} = \begin{bmatrix}
\frac{x'}{R_0^1} + C_{x' + C_{ex'}} \\
\frac{y'}{R_0^2} + C_{y' + C_{ey'}} \\
C_{\Delta \lambda + C_{e\Delta \lambda}}
\end{bmatrix}$$  \hspace{1cm} (6.46)

where $C_{ex'}$, $C_{ey'}$ and $C_{e\Delta \lambda}$ are associated with the previous ray path. This equation can now be iteratively applied to every new $\hat{I}$ (successive incidence angles should not be more than 1-2 degrees apart) and thus simultaneously tracing the reflection and diffraction points. Note also that as discussed in Chapter III, if the errors become large, then more
than one spot iteration is required in order to obtain an acceptable solution.

The only limitation to the application of the above iterative process is the required knowledge of an initial R-D_e ray path for each edge. The calculation of this path is not as simple as that discussed for the R-D_C rays since the diffraction point is not known. Therefore, a numerical search routine is presented below in order to locate the R-D_e ray path for the first pattern point.

Step 1: An initial edge diffraction point, P^1_e, is assumed on the jth edge such that

\[ P^1_e = P^0_e + \hat{e}^j \pm \Delta \lambda \]  

(6.47)

where

\[ \hat{e}^j \pm = \begin{cases} \hat{e}^j & \text{if } |\beta^j_{cj} - \beta^j_0| < |\pi - \beta^j_{ck} - \beta^j_0| \\ -\hat{e}^j & \text{otherwise} \end{cases} \]  

(6.48)

and for the first iteration,

\[ P^0_e = \begin{cases} c_j & \text{if } \hat{e}^j \pm = \hat{e}^j \\ c_k & \text{if } \hat{e}^j \pm = -\hat{e}^j \end{cases} \]  

(6.49)

The length, \( \Delta \lambda \) (see Figure 6.17), can be approximately chosen to be proportional to the difference of the angle \( \beta_0^0 \) at \( P^0_e \) with the desired \( \beta_0^j \) at \( P^j_e \) which is being sought. Thus,
a). positive $\Delta \lambda$.

Figure 6.17. Geometry associated with the computation of $\Delta \lambda$ in Equation (6.47).
b). negative $\Delta l$.

Figure 6.17. (Continued).
\[
\Delta^2 = \frac{\Delta^j_0 \pm (\beta_0^j - \beta_c^j)}{|\Delta^j_0 \pm (\beta_0^j - \beta_c^j)|} \frac{d_j}{\beta_0^j} \tag{6.50}
\]

where

\[
\Delta^j_0 = \beta_c^j - (\pi - \beta_c^j) \tag{6.51}
\]

and

\[
\Delta^j_0 \pm = \begin{cases} 
\beta_c^j - \beta_0^j & \text{if } \hat{e}_j^\pm = \hat{e}_j \\
\pi - \beta_c^j - \beta_0^j & \text{if } \hat{e}_j^\pm = -\hat{e}_j
\end{cases} \tag{6.52}
\]

Note that the term inside the brackets simply determines the proper sign of \(\Delta^2\) as illustrated in Figure 6.17. The rest of the formula gives the magnitude of \(\Delta^2\) as a fraction of the total \(j\)th edge length, \(d_j\).

Step 2: The new reflection point, \(S_{r}^{e1}\), is now found by applying the raytracing routine described in Chapter III.

Step 3: Calculate the \(\beta_0\) angle corresponding to \(S_{r}^{e1}\) as

\[
\beta_0^1 = \cos^{-1}(-e_j^r \cdot s_{e1}^r) \tag{6.53}
\]

and check if

\[
|\beta_0^j - \beta_0^1| < .3^\circ \tag{6.54}
\]
If Equation (6.54) is satisfied, the search routine is terminated and
\[ \vec{p}_j = \vec{p'}_e. \]

Otherwise, return to Step 1 with
\[ \vec{p}_e = \vec{p'}_e. \]

The reader should note that if \( \hat{\Delta} \) of Equation (6.50) equals \( \Delta \) in Equation (6.26), then the above search routine is basically the raytracing routine of Equation (6.46). Of course, the search routine is called as such since \( \Delta \) in Equation (6.50) is only an approximation of the exact value of \( \Delta \). A typical itinerary of the successively calculated \( p_e^1 \) by the search routine is shown in Figure 6.18. At each next iteration, \( p_e^1 \) approaches closer to \( p_e^j \) and thus \( |\Delta \| \) becomes smaller. In this case, Equation (6.50) is a better representation of \( \Delta \) and therefore the final iteration will lead to \( p_e^1 = p_e^j \).

The reader is also cautioned on some problems associated with the above search routine. These are discussed below:

1. The distance parameter \( \Delta \) in Equation (6.50) cannot be too large; otherwise, the raytracing routine of Equation (3.69) will fail to trace the next reflection point. This limitation does not, of course, cause a problem since one simply sets a limit on the maximum value of
Figure 6.18. Typical itineraries of the edge diffraction point, $p_e^1$, during the search routine.

$|\Delta \lambda|$. Thus the number of iterations required for locating $p_e^j$ are increased. A value of $|\Delta \lambda|_{\text{max}} = \lambda/4$ was used.

2. When close to the end of the edge, the inaccuracy of Equation (6.50) may generate a point $p_e^1$ which is outside the limits of the finite edge. In this case, $p_e^1$ must be set equal to the nearest corner before continuing the process.

3. If the edges are long (large $d_{ej}$), it may occur that the iteration is not converging rapidly. For instance, $p_e^1$ may oscillate around $p_e^j$, as shown in Figure 6.19, without converging to it. In order to overcome this, the value of $\Delta \lambda$ from
Equation (6.50) can be checked if it remains the same in the first decimal place for more than one iteration. If so, a proportionality factor of 1/2 is introduced in Equation (6.50) during the execution of Step 1. This action may, of course, be repeated as necessary.

At this stage it should be realized that the initial reflection and diffraction points associated with the R-D_e mechanism of each edge will not be exact. Therefore, before continuing with the application of Equation (6.46), the corresponding errors \( C_{ex}' \), \( C_{ey}' \) and \( C_{e\Delta l} \) must be computed as usual. Alternatively, the search routine is equally
effective for tracing the successive reflection and diffraction points. This is easily accomplished by revising Step 1 so that \( \mathbf{P}^0_e \) is set equal to the previous \( \mathbf{P}^j_e \) during the first iteration. However, the search routine is time consuming. In fact, it was applied throughout the computation of patterns and was observed that 2-3 iterations are usually needed when the edges are not too long and the diffraction point is away from the corners.

One now proceeds to calculate the R-D \( \mathbf{E} \) field associated with the \( j^{th} \) edge (see Figure 6.14). This is generally expressed by

\[
E_{2j}^{\text{RD}} = E^{\text{R}}_{2j} (\mathbf{P}^j_e) \cdot (\mathbf{\hat{u}}_0 \mathbf{D}_s^j \mathbf{\hat{v}}_j \mathbf{\hat{w}}_j \mathbf{\hat{w}}_j) \sqrt{\rho^2_2} e^{-jk(\mathbf{P}^j_e - \mathbf{O}) \cdot \mathbf{I}} e^{-jks} . \quad (6.55)
\]

where \( \mathbf{\hat{v}}_j \) and \( \mathbf{\hat{w}}_j \) were defined in Equation (6.9) and (6.10), and \( \mathbf{\hat{u}}_0 \) with \( \mathbf{\hat{v}}_j \) are the unit vectors associated with the edge fixed plane of incidence for the \( j^{th} \) edge at \( \mathbf{P}^j_e \). Therefore,

\[
\mathbf{\hat{v}}_j = \frac{\mathbf{\hat{u}}_0 \times s^{r}_{ejp}}{|\mathbf{\hat{u}}_0 \times s^{r}_{ejp}|} \quad (6.56)
\]

and

\[
\mathbf{\hat{u}}_0 = s^{r}_{ej} \times \mathbf{\hat{v}}_j . \quad (6.57)
\]

with \( s^{r}_{ejp} \) denoting, as usual, the vectorial projection of \( s^{r}_{ej} \) onto the plane perpendicular to the \( j^{th} \) edge (see Appendix A). Also the scalar
edge diffraction coefficients, $D_{s}^{2j}$ and $D_{h}^{2j}$ take the form (see Equation (2.36))

$$D_{s,h}^{2j} = D_{s,h}(L_{1}^{12}, \psi, \psi', \beta)$$

with $\psi^j$ and $\psi'^j$ as defined in Equation (3.23) if $\hat{n}_{p}, \hat{t}_{e}, \hat{s}_{ej}$ and $\hat{s}_{ejp}$ are used in place of $\hat{n}_{I}, \hat{t}_{e}, \hat{s}_{r}$ and $\hat{s}_{p}$. The distance parameter $L_{12}^j$ is given by

$$L_{12}^j = \frac{(r_{11}^{2} + s_{e1}^{2})(r_{21}^{2} + s_{e2}^{2})}{\rho_{e2}^{2}} \sin^{2} \beta$$

where $\rho_{11,21}$ and $\rho_{e2}$ are found by Equations (3.26) and (3.35) if applied to the surface point $S_{e}^{ej}$; and the caustic distance $\rho_{2}^{2}$ (see Equation (2.38)) becomes

$$\rho_{2}^{2} = \rho_{e2}^{2}$$

Finally, the reflected field, $E_{2}^{R}(p_{e}^{j})$, is given by Equation (3.38) if $S_{r}^{ej}$ and $s_{e}^{ej}$ are used in place of $S_{r}$ and $s_{r}$, respectively.

The total R-D field from all edges is

$$E_{2}^{RD} = \sum_{j=1}^{N_{e}} E_{2j}^{RD}$$

and note that $E_{2j}^{RD} = 0$ if $p_{e}^{j}$ is not within the limits of the $j^{th}$ edge (see Figure 6.17). As discussed at the beginning of the subsection,
whenever this occurs, \( P_e^j \) must still be computed in order to be used in Equation (6.16). However, in such a case, the computation of \( P_e^j \) becomes extremely tedious and time consuming. Moreover, there may not even exist an edge diffraction point (it is at infinity).

Alternatively, since \( L_{ij}^2 \) is a slowly varying quantity and is only significant at the shadow and reflection boundaries then approximate expression can be derived to be used when \( P_e^j \) is outside the \( j^{th} \) edge. Such an expression must, of course, satisfy the continuity requirement at the corners.

One can assume that the R-D_e ray can be replaced by an equivalent ray which has an apparent (image) point source, \( S_s^j \), along the path of the reflected ray toward the nearest corner, \( C_{near} \), as shown in Figure 6.20. Note that for the \( j^{th} \) edge

\[
\overline{C}_{near} = \begin{cases} 
\overline{C}_j & \text{if } |\overline{B}_{cj} - \overline{B}_o| < |\overline{B}_{ck} - \overline{B}_o| \\
\overline{C}_k & \text{otherwise}
\end{cases}
\]  

(6.62)

The location of \( S_s^j \) is then found by

\[
\overline{S}_s^j = \overline{C}_{near} - d_s \overline{S}_{s_{near}}
\]  

(6.63)

where \( d_s \) denotes the distance of the apparent source from the nearest corner and must be computed for each new incidence. The above \( S_s^j \) can now be used to calculate a fictitious edge diffraction point outside the limits of the \( j^{th} \) edge. From the geometry in Figure 6.20, the projection of \( S_s^j \) onto the \( j^{th} \) edge is
Figure 6.20. Geometry for calculating the location of an apparent image source and the associated edge diffraction point, $p_e^j$. This point source is such that it generates a corner diffracting ray equivalent to the $R-D_c$ ray through $C_{near}$.
\[ \bar{C}_p = [\hat{e}^j \cdot (\bar{s}_s \cdot \bar{C}_{\text{near}})]e^j + \bar{C}_{\text{near}} \]  

(6.64)

and since \( S_s^j C_p e_j \) is a right triangle, it follows that

\[ \overline{p}_e^j = \frac{e^j}{C_p - e^j} |C_p - S_s^j| \cot \beta_0^j \]  

(6.65)

and

\[ s_{e_j}^s = |\overline{p}_e - S_s^j| . \]  

(6.66)

Therefore (see Equation (2.32) and note that for a point source

\[ \rho^i_1 = \rho^i_2 = \rho_e = s_{e_j}^s \),

\[ L_{ij}^{12} = \frac{s}{s + s_{e_j}^s} \sin^2 \beta_0^j \]  

At this point, the only remaining complication is the determination of

the distance \( d_s^j \) needed for calculating \( S_s^j \) in Equation (6.63). This can

be found by imposing the continuity of the approximate \( L_{ij}^{12} \) with its

exact value when \( \overline{p}_e^j - \bar{C}_{\text{near}} \). Thus,

\[ \frac{s}{d_s^j + s} \sin^2 \beta_0^j \]  

where \( \rho_{12,22,e_2} \) are the radii associated with the ray reflected off the

surface and toward \( C_{\text{near}} \).
These are given by Equations (3.24a) and (3.35) with $s_{\text{near}}^r$ and $s_{\text{near}}^r$ in place of $S_r$ and $s^r$. From Equation (6.68) since $s^r = 0$, it follows that

$$d_{s}^{j} = \frac{\rho_{12}^{12} \rho_{22}^{12}}{\rho_{e2}^{12}} \quad (6.69)$$

The above expression for $d_{s}^{j}$ can now be used to find $S_{s}^{1}$ and consequently $L_{j}^{i2}$ by Equation (6.67).

5. Reflected-Corner Diffracted-Reflected Field

The geometry associated with the R-Dc-R ray mechanism is shown in Figure 6.21. The field caused by this ray when associated with corner $C_k$ of the $j$th edge can be expressed as

$$\mathbf{E}_{E3j} = -\mathbf{E}_{2}(C_{k}) \cdot \left( -\beta_{ok}^{j} \hat{\beta}_{ok}^{j} \mathbf{R}^{3j} \hat{\psi}_{k}^{j} \mathbf{B}_{ck}^{j} \mathbf{D}_{ck}^{j} \right)$$

$$\mathbf{E}_{E3j} = -\mathbf{E}_{2}(C_{k}) \cdot \left( -\beta_{ok}^{j} \hat{\beta}_{ok}^{j} \mathbf{R}^{3j} \hat{\psi}_{k}^{j} \mathbf{B}_{ck}^{j} \mathbf{D}_{ck}^{j} \right)$$

$$\mathbf{E}_{E3j} = -\mathbf{E}_{2}(C_{k}) \cdot \left( -\beta_{ok}^{j} \hat{\beta}_{ok}^{j} \mathbf{R}^{3j} \hat{\psi}_{k}^{j} \mathbf{B}_{ck}^{j} \mathbf{D}_{ck}^{j} \right)$$

where the field $\mathbf{E}_{E2}(C_{k})$ is the same as that used in Equation (6.13). The unit vectors $\hat{\beta}_{ok}^{j}$ and $\hat{\psi}_{k}^{j}$ were defined in Equations (6.14) and (6.15), and

$$\mathbf{D}_{ck}^{3j} = \mathbf{D}_{ck}^{3j} \left( \hat{L}_{j}^{i2}, \hat{L}_{ck}^{i2} \hat{\psi}_{k}^{j}, \hat{\psi}_{k}^{j}, \hat{\psi}_{k}^{j}, \hat{\beta}_{ck}^{j}, \hat{\beta}_{ck}^{j}, \pi/2 \right) \quad (6.71)$$

In the above, the various parameters are as follows:
Figure 6.21. Geometry of the Reflected-Corner Diffracted-Reflected backscatter ray through corner $C_k$ when associated with the $j$th edge.
(see Equation (2.73) and (3.35)); \( \psi_k^j \) is defined by Equation (6.18); \( \beta_{ck} \) by Equation (6.19), and \( L_{ij} \) is associated with the R-D-R ray. As mentioned earlier, this ray exists in rare cases and was not incorporated in our solution. Therefore, \( L_{ij} \) was set to infinity.

Finally, the dyadic reflection coefficient \( R_3 \), and the associated principal radii, \( r_{13,23} \), are the same as those used in Equation (3.76). Obviously, the various surface parameters must be computed at \( S_{ij} \), and \( s_{ij} \) should be used in place of \( s \) in Equations (3.80) and (3.81).

The R-D-R field on the \( j \)th edge is

\[
E_{3j}^{RD} = \frac{1}{E_{3j}^{RD} + E_{3j}^{RDck}} \bigg|_{k=j+1}
\]

and therefore, the total R-D-R field associated with the plate structure is found by

\[
E_{3j}^{RD} = \sum_{j=1}^{N_e} \sum_{k=j}^{j+1} E_{3j}^{RDck}
\]

It is further noted that this field is comparatively quite weak and becomes significant only at the shadow boundary of the plate. This will be seen in the next chapter which presents RCS results for specific structures.
C. PLATE CONNECTED NORMALLY TO THE SURFACE

1. Introduction

This section deals with the backscattered fields from a plate formed of piecewise linear edges when connected to a surface (see Figure 6.2). Our study is concentrated on structures where the plate is normal to the surface as shown in Figure 6.22. The following is a detailed treatment of the backscattered fields from such a plate structure. In addition, possible modifications to our solution are discussed for application to more general geometries.

When the plate is connected to the surface, the fields associated with the edges and corners which do not lie on the surface are exactly the same as those given in Section B of this chapter. In order to obtain the total backscattered field from the plate structure of Figure 6.22, the field contribution from the Junction-Corners \( C_1 \) and \( C_{Ne} \), and the Junction-Edge \( C_{Ne}C_1 \) along the surface must also be considered.

2. Junction-Corner Diffracted Field

The backscattered fields from the junction-corners (see Figure 6.23) can only be associated with the plate edge directed away from the surface. This is based on the argument that such a discontinuity was only caused by the existence of this edge. More precisely, if the plate was of infinite extent, then the junction-corner fields would be non-existent. Therefore, the backscattered field from junction-corner \( C_1 \) is the sum of the fields \( E_{11}^{Dc1} \) (see Equation 6.8), \( 2E_{21}^{RDc1} \) (see...
Figure 6.22. A flat plate positioned normally onto a smooth, convex surface.
Figure 6.23. Illustration of the Junction-Corner and Junction-Edge backscatter rays and their associated geometry.

Equation (6.13)) and $E_{31}^{RDc1R}$ (see Equation (6.70)) with $s_{c1}^r \rightarrow 0$. However, when $s_{c1}^r \rightarrow 0$, Equation (6.70) is invalid. Alternatively, the R-Dc-R field is insignificant (except at the existance boundary of the corresponding R-Dc-R field) and therefore it may be completely neglected for the present study. Our results will show that such an approximation is acceptable.

Thus, when $s_{c1}^r \rightarrow 0$, the backscattered field from junction-corner $C_1$ is

$$E_{JC1} = E_{11}^{Dc1} + 2E_{21}^{RDc1} \bigg|_{s_{c1}^r \rightarrow 0} \tag{6.75}$$
Similarly, from the junction-corner $C_{Ne}$,

$$E_{JC_{Ne}} = E_{Dc(Ne-1)} + 2E_{DC(Ne-1)}$$

(6.76)

The total junction-corner field associated with the plate structure is

$$E = E_{J} + E_{JC_{Ne}}$$

(6.77)

Furthermore, the scalar reflection coefficients in the expression

of $E_{R}^{*}(C_{k})|_{s_{ck} \rightarrow 0}$ (see Equations (6.13), (3.39), (3.38), (2.54)

and (2.3)) become

$$R_{s,h} = -\frac{4}{\xi} e^{-j(\xi_{L}^{1})/12} \left\{ p^{*}(\xi_{L}^{1}) \right\} e^{-j\pi/4}$$

where

$$\xi_{L} = 2(k_{r} \tau) 1/3 - 2(k_{r} \tau) 1/3 \cos \theta_{1}$$

and $R_{t}$ with $\theta_{1}$ are defined in Equations (3.40) and (2.8). This form is still considered valid since it gives the GO result ($R_{s,h}=\mp 1$) in the deep lit region. However, for a detailed analysis of the reflected fields in the close neighborhood of the surface, the reader is referred to [39], [36] and [73].

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3. Junction-Edge Diffracted Field

The ray mechanism associated with the backscattered junction-edge fields is also shown in Figure 6.23. A solution is now developed for these fields when the junction (wedge) angle is \( \pi/2 \). In this case,

\[ \hat{n}_p \cdot \hat{n}_J = 0, \]

where \( \hat{n}_J \) is the normal to the surface along the junction-edge and \( \hat{n}_p \) denotes, as before, the normal to the plate. Therefore (see also Equation (6.1) and Figure 6.5) the two reflection points corresponding to the Reflected-Reflected ray coincide with the junction-edge and thus, give rise to quite strong junction-edge fields. According to the laws of diffraction (see also Equation (6.1b)) the backscattering ray associated with these fields can only originate from a point, \( S_J \), where the incident ray is normal to the junction-edge vector, \( \hat{n}_J \). The surface normal at this point must then be

\[ \hat{n}_J = \hat{n}_p \times \left( \frac{\hat{n}_J \times \hat{n}_p}{|\hat{n}_J \times \hat{n}_p|} \right) \quad (6.78) \]

and \( S_J \) can be found in the same manner as \( S_{60} \) (see discussion following Equation (4.43)). In order for \( S_J \) to lie within the limits of the junction-edge, \( \hat{n}_J \) must be within the angular span of the normals at the junction-corners. Mathematically, this occurs whenever

\[ |\hat{n}_c \cdot \hat{n}_J| > |\hat{n}_c \cdot \hat{n}_N_e| \quad (6.79a) \]
and
\[ |^{\wedge}c_{\text{Ne}} \cdot n | > |^{\wedge}c_{1} \cdot n^{\wedge}c_{\text{Ne}}|, \]  

(6.79b)

where \(^{\wedge}c_{1}\) and \(^{\wedge}c_{\text{Ne}}\) represent the surface normals at \(C_{1}\) and \(C_{\text{Ne}}\) (see Figure 6.23). Therefore, the existence of the junction-edge fields is restricted to an angular sector similar to the form of a mezzanine slice [74] as shown in Figure 6.24. The junction-edge fields are zero outside this region.

Whenever the incident ray is within this region, the junction-edge fields can be computed by introducing the well-known concept of image theory. Accordingly, the incident ray can be replaced by an equivalent ray in the absence of the plate as shown in Figure 6.25. The scattered field by the equivalent ray is simply that reflected by the surface. Therefore, the junction-edge fields are given in the far zone by

\[ E^{\text{JE} \cdot R} = \sqrt{p_{1} p_{2}} e^{-j2k(S_{J} - O_{e})} \cdot \frac{e_{j} e^{-jks}}{s}, \]  

(6.80)

where

\[ R = -e'_{1}^{\wedge}c_{1} R_{s} - e'_{1}^{\wedge}c_{1} R_{h}, \]  

(6.81)

with

\[ e'_{1} = e'_{1}^{\wedge}, \]  

(6.82a)

\[ e'_{1} = I e'_{1}, \]  

(6.28b)
Figure 6.24. Three dimensional sketch indicating the existence region of the junction-edge fields and their boundaries at \( \phi = \phi_{sl} \) and \( \phi = \phi_{SN_e} \) for a conical pattern cut (constant \( \theta \)).
Figure 6.25. Illustration of the application of image theory for computing the junction-edge fields.
and \( R_s, h \) are defined in Equation (2.54). The parameters in Equation (2.55) for this case become

\[
\xi^L = -2 \left( \frac{k R_T}{2} \right)^{1/6} \cos \theta_i,
\]

\[
\Theta_i = \cos^{-1}(-\hat{n}_i \cdot \hat{n}_J),
\]

\[
L^L = \infty
\]

and

\[
\frac{1}{R_T} = \frac{\cos^2 \omega + \sin^2 \omega}{R_1 R_2}
\]

with

\[
\omega = \cos^{-1}(|\hat{\mathbf{J}} \cdot \hat{\mathbf{t}}_1|).
\]

The reader is reminded that \( R_1 \) and \( R_2 \) as always denote the principal surface radii of curvature corresponding to the principal directions \( \hat{\mathbf{t}}_1 \) and \( \hat{\mathbf{t}}_2 \) at the point of interest (\( S_j \) in this case). Note also that the dyadic coefficient in Equation (6.81) is simply the negative of Equation (2.3) due to imaging. In addition, \( \rho_{1,2}^\perp \) denotes as usual, the principal radii of the wavefront reflected off \( S_j \) by the image wave. They are computed by Equation (2.12) with \( \rho_{1,2}^\perp \).

In order for our solution to be valid, it is imperative that the total field be continuous at the existence boundary of the junction-edge fields (see Figure 6.24). A conical pattern cut about a plate
normal to an ogive (see Figure 7.1 for dimensions) is shown in Figure 6.26 and demonstrates the continuity. At the angle $\phi = \phi_{S1} = 111^\circ$ which is also the existence boundary of $E_{21}^{RD_e}$, the continuity is maintained by the relation

$$E_{JC1}(\phi_{S1}^+) = E_{JC1}(\phi_{S1}^-) + 2E_{21}^{RD_e}(\phi_{S1}^-) + E_{JE}(\phi_{S1}^-)$$

(6.83)

where $\phi_{S1} = \phi_{S1} \pm \Delta \phi$, with $\Delta \phi \rightarrow 0$.

The above equality is accomplished by two simultaneous sign changes in the corner diffraction coefficient associated with $E_{21}^{RDc1}$ (see Equations (6.75) and (6.13)). The functions $\cos^1_{ocl} - \cos^1_{c1}$ and $a(\beta^T)$ (see Equations (2.71) and (6.16)) change sign in order to compensate for the non-existance of the R-D$e$ rays, respectively. The first sign change indicates the passage of the edge diffraction existence boundary and the second the passage of the reflection boundary. The later boundary is governed by Equation (6.1). When $\phi = \phi_{SN_e} = 69^\circ$, which is in the other side of the existence boundary of $E_{JE}$ and $E_{21}(\eta_{e}-1)$, then the field from the junction-corner $C_{Ne}$ plays the role of continuity.

If the angle $\phi$ is exactly or extremely close to $\phi_{S1}$ or $\phi_{SN_e}$, a "kink" may appear in the pattern as shown in Figure 6.26 at $\phi = \phi_{S1}$. This is either due to numerical errors in the raytracing routine during the determination of the R-D$e$ ray path (at this point $s_{e}^r \rightarrow 0$ and consequently Equation (6.23a) becomes meaningless), or due to the inability of the corner diffraction coefficient to simultaneously...
Figure 6.26. Conical $E_\phi$, $\theta=30^\circ$ RCS pattern cut for the plate structure in Figure 7.1.
treat two discontinuities. In any case, the next pattern point will
obviously be correct and the actual field value at \( \phi = \phi_{s1} \) or \( \phi_{sN_e} \) may
be obtained by smoothly joining the field values at the angles adjacent
to either side of \( \phi_{s1} \) or \( \phi_{sN_e} \). The "kink" can then be ignored.
Furthermore, because of the approximate expression of \( \varepsilon_m \) in Equations
(6.75) and (6.76), a slight jump may appear at \( \phi = \phi_{s1} \) or \( \phi_{sN_e} \) for
certain plate geometries when close to the shadow region of the
plate. The discussion in this paragraph may of course be applied to
other pattern cuts as well.

Up to this point, expressions were given for calculating all the
backscattered field components listed in Table 6.1. These can be used
for determining the RCS from plate structures such as those shown in
Figure 6.2 where the plate is normal to the surface. The next chapter
presents measured and calculated results based on the previous
development. A novel treatment of the junction-edge and junction-corner
backscattered fields was also given and expressions were derived to be
used for three dimensional plate structures where the plate is connected
normally to the surface.

In order to compute the RCS from plate structures in which the
plate is not normal to the surface, the doubly reflected (R-R) field
must be added to Table 6.1. When it exists (in the region broadside to
the flat plate, see Figure 6.24), this field component is quite strong
for either case where the plate is off or onto the surface. The
following section discusses an approach for calculating the R-R ray
path.
D. TRACING OF THE DOUBLY REFLECTED RAY

When the plate is tilted with respect to the surface, the expressions given in the previous section for the junction-corner fields are still valid; however, this is not the case for the junction-edge field. For this type of geometry the doubly reflected field (R-R) between the surface and the plate is existant and that from the junction-edge negligible. In fact, the later is equal to the field diffracted by the internal wedge formed by the plate and the surface (see Section E of Chapter II). This is known to be weak, and therefore, the junction-edge field can be neglected in the presence of the R-R field.

The backscatter ray mechanism associated with the R-R field was shown in Figure 6.5. As one may realize, the difficulty in computing this field is the determination of the ray path, i.e., the finding of the reflection points on the surface and the plate. Once these are known, it is only a simple exercise to calculate the GO field scattered from each point (see Section C of Chapter II).

The surface reflection point can be found by expressing the surface normal, \( \hat{n} \), as given in Equation (3.46) and solving Equation (6.1) for \( x' \) and \( y' \). These parameters were defined in Section B of chapter III (see Equation (3.43)) with reference to the reflection point associated with the previous incidence angle. Immediately, according to Equation (6.1a), it is recognized that the R-R ray is only possible when the incident ray (\( \hat{I} \)) is on the side of the plate which forms the smallest angle with the surface. In addition, it can only exist when \( \hat{I} \) is close to the broadside region of the plate (see Figure 6.24). This, of
course, is necessary so that continuity is maintained between the R-R and junction-edge field associated with the plate connected normally to the surface.

In order to solve Equation (6.1) for $x'$ and $y'$, it is convenient to combine Equation (6.1a) and (6.1b). To do this, Equation (6.1a) is re-written as

$$\mathbf{n} \cdot \mathbf{i} = -\sin \theta_p \quad (6.84a)$$

where $\mathbf{n}_p \cdot \mathbf{i} = -\cos \theta_p$ (see Figure 6.5).

Also, noting that $|\hat{n}_x\mathbf{I}| = \sin \theta_i = \cos \theta_p$ and $|\hat{n}_p x\mathbf{I}| = \sin \theta_p$, Equation (6.1b) becomes

$$-(\hat{n}_x\mathbf{I}) = (\hat{n}_p x\mathbf{I}) \cot \theta_p \quad (6.84b)$$

Dividing (6.84b) by (6.84a), it is obtained

$$\bar{n}_x\mathbf{I} = (\hat{n}_p x\mathbf{I}) \frac{(\mathbf{n} \cdot \mathbf{i}) \cot \theta_p}{\sin \theta_p} \quad (6.85)$$

where $\bar{n}$ is defined by Equation (3.46) in the primed coordinate system (see Equation (3.44)) with origin at the previous surface reflection point. Equating the $x'$ and $y'$ components from each side of Equation (6.85) it is found that
\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x' \\
y'
\end{bmatrix}
= \begin{bmatrix}
C_{x'} \\
C_{y'}
\end{bmatrix}
\]  \hspace{1cm} (6.86)

with

\begin{align*}
A_{11} &= I_x' \beta_{x'} \\
A_{12} &= I_y' \beta_{x'} - I_z' \\
A_{21} &= -(I_x' \beta_{y'} - I_z') \\
A_{22} &= -I_y' \beta_{y'} \\
C_{x'} &= -I_z' \beta_{x'} - I_y' \\
C_{y'} &= I_z' \beta_{y'} + I_x'
\end{align*}

\hspace{1cm} (6.87a) \hspace{1cm} (6.87b) \hspace{1cm} (6.87c) \hspace{1cm} (6.87d)

\begin{align*}
\beta_{x'} &= (I_z' \eta_{py'} - I_y') \frac{\cot \theta_p}{\sin \theta_p} \\
\beta_{y'} &= (I_z' \eta_{px'} - I_x') \frac{\cot \theta_p}{\sin \theta_p}
\end{align*}

\hspace{1cm} (6.89a) \hspace{1cm} (6.89b)

\[\hat{n}_p = x' \hat{n}_{px'} + y' \hat{n}_{py'} + z' \hat{n}_{pz'}\]

(6.90)

and \(I_x', y', z'\) are defined in Equation (3.55). Note also that the \(z'\) components of \(\hat{n}_p \hat{x'}\) and \(\hat{n}_p \hat{x'}\) are very small and therefore, these are not considered since they consist of second order terms which are neglected in this solution.
According to the development in Chapter III, the new surface reflection point can now be computed iteratively. As discussed there, this may have to be projected on the surface and the necessary errors be computed and incorporated in Equation (6.86) (see also Equation (3.69)). Consequently, the direction of the reflected ray, $\hat{\mathbf{r}}^2$, can be found by

$$\hat{\mathbf{r}}^2 = \mathbf{i} - 2\sin\theta_p \mathbf{n}\quad (6.91)$$

and the reflection point on the plate (if it exists) can be determined by intersecting this ray with the plate (see [51]).

The above raytracing routine has not at present been implemented. However, it is intended to be used when this research is extended to plate structures in which the plate has an arbitrary orientation with respect to the surface.
CHAPTER VII

MEASURED AND CALCULATED RCS RESULTS
FROM PLATE STRUCTURES

A. PLATE NORMAL TO AND OFF THE SURFACE

Based on the development in the previous chapter, the RCS from a plate structure, where the plate is not connected to the surface (see Figure 6.2), will be calculated in the lit region by

\[ \sigma(\theta, \phi) = 4\pi^2 \lim_{s \to \infty} \frac{|(E^{GO} + E_1 + 2E_2 + 2E_2 + E_3 + E^{RD}C + E^{RD}R)|^2}{|E_i|^2} \cdot \hat{e}_I. \] (7.1)

The various field components in the above expression are given by Equations (3.3), (6.12), (6.22), (6.61) and (6.74) (note that for the ogive, \( E^{GO} \) is given by Equation (4.41)). Also, as usual, the unit vector \( \hat{e}_I \) denotes the polarization of the incident wave, \( E_i \). As discussed, Equation (7.1) does not include the R-D_e-R field which is only significant close to the shadow boundary of the plate. In addition, the doubly reflected field (see Section D of Chapter VI) must
be included in case of structures with plates not normal to the surface. One is, of course, reminded that such a backscattered field can only be present in the broadside region of the plate toward its tilted side (see Figure 6.24).

In order to verify the accuracy of Equation (7.1), a model with a four-sided plate over an ogive was constructed as depicted in Figure 7.1. The plate was placed in a slit of a styrofoam pad ($\varepsilon_r=1.01$) and was then positioned on the surface of the ogive so that it would effectively look as shown in Figure 7.1 if the styrofoam was transparent. Note that the positioning for this model was such that the plate was at the center of and normal to the surface, and a minimum distance $d=1/2"$ away from it. The actual size and corner coordinates of the plate are shown in Figure 7.2. The ogive of this model has a tip angle of $\alpha=60^\circ$ and a radius of $R_1=10.34"$. Furthermore, all of our measurements and calculation in this chapter will be performed at 9.01 GHz ($\lambda=1.311"$).

First, let us examine the continuity of $\sigma$ at the existence boundaries of the R-D$_E$ field associated with each of the plate edges. This can easily be done by calculating a conical RCS pattern (constant $\theta$) of the model in Figure 7.1. Such a pattern is shown in Figure 7.3 with $\theta=45^\circ$ for the $\phi$ and $\theta$ polarizations of incidence. As seen, the results are quite continuous throughout all angles of $\phi$, even though the boundaries of the R-D$_E$ fields from all edges are intercepted by the pattern. In particular, at $\phi=37^\circ$ the R-D$_E$ backscatter ray from edge #1 comes into existence (see Figure 7.2). It is then replaced at $\phi=90^\circ$.
Figure 7.1. Geometry of the measurement model with the plate off the surface.
Figure 7.2. Actual size and corner coordinates (with respect to 0) describing the plate shown in Figure 7.1.
Existence region of the R-D_e field from edge # 1, # 3, # 4, # 2.

Conical and E \_\theta RCS patterns at f=9.01 GHz from the model shown in Figure 7.1 with \( \theta = 45^\circ \).

Figure 7.3. Conical E_\phi and E_\theta RCS patterns at f=9.01 GHz from the model shown in Figure 7.1 with \( \theta = 45^\circ \).
EXISTANCE REGION OF THE R-D_e FIELD FROM

EDGE #1
EDGE #3
EDGE #4
EDGE #2

Radar cross section in db = \lambda^2

\( E_\theta, \theta = 45^\circ \) pattern

\[ \phi \text{ angle in degrees (} \theta = 45^\circ) \]

b). \( E_\theta \) pattern.

Figure 7.3. (Continued)
by the same type of ray from edge #3 which ceases to exist at $\phi=143^\circ$. Also near the broadside region to the plate, the backscatter $R-D_e$ rays from edge #2 and #4 become existant. This occurs at $\phi=78.5^\circ$ and $\phi=85^\circ$ for edge #4 and #2, respectively. Of course, due to symmetry, they cease to exist at $\phi=101.5^\circ$ and $\phi=95^\circ$. The pattern continuity at these existance boundaries is maintained by a sign change occurring in the corner diffraction coefficient associated with the respective $R-D_C$ fields (see also Section C of Chapter VI). One should also note that if a significant component has been neglected then the pattern will contain a discontinuity at the existance boundary of this mechanism. The size of the discontinuity is proportional to the importance of the component. In particular, our results do not contain the field due to the $R-D_e-R$ ray. For the patterns shown in Figure 7.3, this ray exists only within $5^\circ$-$10^\circ$ of the broadside region and is associated with edges #2 and #4. Obviously, if the $R-D_C-R$ field was large, it would cause discontinuities at its existance boundaries since the respective $R-D_C-R$ field would change sign at this pattern point in an effort to counteract the existence of this field. Furthermore, some slight asymmetries in the calculated patterns are due to inaccuracies caused by the raytracing routines.

Measurements on the model of Figure 7.1 were made at three pattern cuts for the $\phi$ and $\theta$ polarizations of incidence. Two of them were normal to the plate (xy and yz planes) and the other was in the plane of the plate (xz plane).
The calculated and measured RCS patterns in the yz plane (\(\phi=90^\circ\))
are given in Figure 7.4 for the \(\phi \) and \(\theta\) polarizations of incidence. In
addition, Figure 7.5 presents calculated plots of the various field
components which comprise these patterns. Such breakdown is useful for
controlling the RCS of the structure. Good agreement between theory and
measurements is observed except at the first ten degrees and for angles
very close to the broadside of the plate. Failure of these calculations
at broadside is due to the same reason discussed in relation to the
pattern shown in Figure 6.12, i.e., an improper limiting process in the
singly corner diffracted fields (see Figure 7.5). For small angles of
\(\theta\) (close to the plane of the plate) the R-D_\(e\)-R fields from edges #2 and
#4 and other higher order terms (see Figure 6.7) become significant
particularly for the \(E_\phi\) polarization where the scattering from the
bottom edge of the plate tends to obscure the GO contribution. However,
these have not yet been included in our solution and thus the
discrepancy of the calculations with respect to the measurements. As
mentioned, the reason for neglecting the R-D_\(e\)-R field is because of
their extensive computation time without significantly contributing to
the total pattern, except at the SB and at angles near the plane of the
plate.

The lobing in the RCS patterns of Figure 7.4 is characteristic to
the interaction of two scattering sources. In our case these comprise
the GO field from the ogive and that caused by the existence of the
plate. The last type of field is mostly dominated by the R-D_\(e\) and
R-D_\(c\) field components as shown in Figure 7.5. The singly corner
Figure 7.4. Comparison of measured and calculated $E_\phi$ and $E_\theta$ RCS patterns at $f=9.01$ GHz from the structure of Figure 7.1. Shown patterns were taken in the $\phi=90^\circ$ plane.
b). $E_\theta$, $\phi=90^\circ$ RCS pattern.

Figure 7.4. (Continued).
a). $E_\phi$, $\phi=90^\circ$ RCS patterns.

Figure 7.5. Plots of the field components (except $E_{\phi0}$) comprising the calculated RCS patterns in Figure 7.4.
b). $E_\theta, \phi=90^\circ$ RCS pattern.

Figure 7.5. (Continued).
diffracted field from the plate is seen to become significant only near the broadside region to the plate and therefore the patterns in Figure 7.4 have a higher value in this region. However, the R-D<sub>C</sub>-R field is 30-40 dB below the value of the GO field and is consequently insignificant to the total result (except at the shadow boundary). Note also that the lobing occurs at about the RCS value of the bare ogive which is 19.9 dB above $\lambda^2(\lambda=1.311\")$ as shown in Figure 7.4. The SB of the plate is at about $\theta=118\degree$ for this model but after $\theta=100\degree$ the field contribution from the plate becomes small (see Figure 7.5). The total RCS then settles within about 1-2 dB to the RCS value of the ogival surface (this is more true for the $E_\theta$ than the $E_\phi$ pattern). It is also noted that for $\theta<90\degree$ a better agreement between measurements and calculations is observed in the $E_\theta$ than the $E_\phi$ pattern. This is due to strong R-D<sub>E</sub>-R fields from edges #2 and #4 of the plate for the $\phi$ polarization of incidence (compare R-D<sub>C</sub>-R fields in Figure 7.5) since it is parallel to these edges.

It is also interesting to examine the contribution of the R-D<sub>E</sub> fields, shown in Figure 7.5 for the patterns of Figure 7.4. When they exist, these are usually quite dominant. Since the patterns are taken broadside to the plate, the R-D<sub>E</sub> fields from edges #2 and #4 are present throughout the patterns and are very strong, especially for the $E_\phi$ polarization of incidence. The existence of the R-D<sub>E</sub> rays from the side edges (#1 and #3) is very much dependent on the geometry of the surface. For the patterns shown, they enter the calculations at about $\theta=17\degree$ and cease to exist at about $\theta=40\degree$. Moreover, note that the total
pattern is continuous at these transition points. In fact, as discussed, Figure 7.5 shows that the R-Dc fields are also discontinuous at these points in order to counteract the corresponding discontinuities in the R-De fields.

Next, let us examine the RCS patterns in the xy plane (θ=90°). Our measurements showed that these pattern cuts were very sensitive to the orientation of the structure in Figure 7.1, i.e., symmetry in the 360° measured patterns was always distorted. A simple study with our computer program proved that indeed the RCS in that region presented notable differences for small variations in the angle θ of the conical pattern cuts. This is demonstrated in the calculated RCS patterns shown in Figure 7.6 for θ=89, 90 and 91 degrees. It is also worthwhile to mention that small variation with respect to the orientation and positioning of the plate was not found to cause significant pattern changes. Figure 7.7 shows two measured RCS patterns attempted in the θ=90° plane and calculated for each incidence polarization. The measured results (A and B) in each graph were obtained by measuring the complete 360° pattern and plotting the portions of the pattern from each symmetric side of the plate. Figure 7.8 also presents the individual field components (except \( \vec{E}_0 \) which is shown in Figure 7.7) comprising the calculated patterns in Figure 7.7. Considering the discrepancies between the two measured patterns for the results in Figure 7.6, the comparison of the calculated with the measured patterns is thought to be sufficiently good. However, note that the calculated pattern for the \( E_\theta \) polarization shown in Figure 7.6 is at θ=88°. This was done since the
Figure 7.6. Calculated $E_\phi$ and $E_\theta$ RCS patterns at $f=9.01$ Ghz from the structure of Figure 7.1. Shown patterns are conical cuts at $\theta=89$, $90$ and $91$ degrees.

a). $E_\phi$ RCS patterns.
CALCULATED FOR:

\[ Q = 89 \]
\[ e = 90 \]

\[ \theta = 89^\circ \]
\[ \theta = 90^\circ \]
\[ \theta = 91^\circ \]

b). \( E_\theta \) RCS patterns.

Figure 7.6. (Continued).
Figure 7.7. Measured and calculated $E_{\phi}$ and $E_{\theta}$ RCS patterns at $f=9.01$ GHz from the structure of Figure 7.1. Measured patterns are attempted conical cuts at $\theta=90^\circ$ and the calculated is as indicated.

$a)$. $E_{\phi}$ RCS patterns.
Figure 7.7. (Continued).

b). $E_\theta$ RCS patterns.
a). $E_\phi$, $\theta=90^\circ$ RCS patterns.

Figure 7.8. Plots of the field components (except $E_0^0$) comprising the calculated RCS patterns in Figure 7.7.
b). \( E_0, \theta=88^\circ \) RCS patterns.

Figure 7.8. (Continued).
best agreement between measurements and calculations was obtained at this cut. For $E_\theta$ pattern cuts at other angles of $\theta$, the reader is referred to Figure 7.6.

It is also worthwhile to examine the contribution of the various components which comprise the calculated patterns in Figure 7.7. One observes that the GO field shown in the same figure is the dominant component throughout the pattern. The only other significant contributor is the singly corner diffracted field shown in Figure 7.8 which also causes most of the rippling effect on the total pattern. In addition, the $E_\phi$ pattern is higher in the non-specular region due to strong R-D$_g$ field from edge #1 of the plate. One should further note that the R-D$_c$-R field in Figure 7.8 contains a discontinuity at about $\phi=85^\circ$. Obviously then, this point is the existence boundary of the R-D$_g$-R field (from edge #4 in this case) which is not included. However, since the R-D$_c$-R field is very weak (40 dB down) such a discontinuity has no effect on the total pattern.

The measured and calculated $E_\theta$ RCS pattern in the $\phi=0^\circ$ plane is shown in Figure 7.9. Of course, one does not need to consider the $E_\phi$ pattern in this plane since it will simply be that of the ogive alone. In effect, the plate cannot be "seen" since the $\phi$ polarization of incidence is perfectly perpendicular to its plane [54]. Also note that the $E_\theta$ pattern had to be calculated at a nonzero $\phi$ angle (we used $\phi=1/4$ degrees), otherwise the results in Chapter VI are not accurate. More precisely when in the plane of the plate, most of the first order diffraction terms are nonexistent and therefore the edge waves and
Figure 7.9. Measured and calculated $E_0$, $\phi=0$ RCS pattern at $f=9.01$ Ghz from the structure of Figure 7.1.
other higher order (see Figures 6.7 and 6.8) terms must be considered for a more accurate result. Alternatively, one can compute the pattern value at an angle slightly away from the plane of the plate as was done for the pattern in Figure 7.9. Of course, in this case, the results of Equation (7.1) will be sufficient only if the first order diffracted terms are significantly stronger (about 10 dB) than the higher order ones. This is true if the incident polarization has a large parallel component to one or more plate edges. For example, the nonspecular region of the pattern in Figure 7.9 is dominated by the singly diffracted field from edge #3 (see Figure 7.10) which is strong and thus good agreement between measurements and calculations is obtained. For a better agreement, the higher order mechanism (see Figures 6.6, 6.7 and 6.8) must be included in our solution. This task is currently being undertaken. In general, when the incident ray is within five to ten degrees near the plane of the plate, then higher order mechanisms are required for an accurate result. Such a statement is more critical when dealing with small plate sizes \( (d_e < \lambda) \). It is also noted that the calculated pattern in Figure 7.9 was not completed for small \( \theta \) angles. In this region, one must also incorporate the R-D_e-R field from edge #4 in our solution for a more accurate result. This field will cancel some of the contribution due to the R-D_e field from the same edge and thus the pattern dip shown in the measured results will be obtained. Note also, that the extent of this cancellation occurs for about 5 degrees which corresponds to the existance region of the R-D_e-R rays (see Figure 6.24).
Figure 7.10. Plots of the field components (except $E_0$) comprising the calculated $E_0$, $\phi = 25^\circ$ RCS pattern in Figure 7.9.
B. PLATE CONNECTED NORMAL TO THE SURFACE

When the plate is connected to the surface, the junction-corner (\(\mathbb{E}^{JC}\)) and junction-edge (\(\mathbb{E}^{JE}\)) fields must be incorporated in the expression for \(a\). In addition, the \(D_c\), \(R-D_e\), \(R-D_c\) and \(R-D_c-R\) terms will only be associated with the edges and corners not connected to the surface. Specifically, when the plate is connected to the surface, it follows that

\[
\bar{E}_{Dc} = \bar{E}_{Dc2} + \bar{E}_{1c(N_e-1)} + \sum_{j=2}^{N_e-2} \sum_{k=j}^{j+1} \bar{E}_{Dck} \\
\bar{E}_{Dc} = \bar{E}_{Dc2} + \bar{E}_{1c(N_e-1)} + \sum_{j=2}^{N_e-2} \sum_{k=j}^{j+1} \bar{E}_{Dck} (7.2)
\]

(see Equation (6.8) and compare to Equation (6.12)),

\[
\bar{E}_{RDc} = \bar{E}_{RDc2} + \bar{E}_{2c(N_e-1)} + \sum_{j=2}^{N_e-2} \sum_{k=j}^{j+1} \bar{E}_{RDck} \\
\bar{E}_{RDc} = \bar{E}_{RDc2} + \bar{E}_{2c(N_e-1)} + \sum_{j=2}^{N_e-2} \sum_{k=j}^{j+1} \bar{E}_{RDck} (7.3)
\]

(see Equation (6.13) and compare to Equation (6.22)),

\[
\bar{E}_{RDc} = \sum_{j=1}^{N_e-1} \bar{E}_{2j} (7.4)
\]

(see Equation (6.55) and compare to Equation (6.61)) and

\[
\bar{E}_{RDcR} = \bar{E}_{RDc2R} + \bar{E}_{3c(N_e-1)} + \sum_{j=2}^{N_e-2} \sum_{k=j}^{j+1} \bar{E}_{RDckR} \\
\bar{E}_{RDcR} = \bar{E}_{RDc2R} + \bar{E}_{3c(N_e-1)} + \sum_{j=2}^{N_e-2} \sum_{k=j}^{j+1} \bar{E}_{RDckR} (7.5)
\]

(see Equation (6.70) and compare to Equation (6.74)). As usual, \(N_e\) denotes the number of plate edges including that connected to the surface (see Figure 6.22).
The RCS from a plate connected to a surface (see Figure 6.2) can now be expressed in the lit region by

\[
\sigma(\theta, \phi) = 4\pi s^2 \lim_{s \to \infty} \frac{|(E + E - JE - J - Dc - RDc - RDc - RDc) \cdot E|^2}{|E|^2}.
\]

Equation (7.6)

The various field components in the above expression are as given in Equations (3.3), (6.80), (6.77), (7.2), (7.3), (7.4) and (7.5), respectively. However, for the ogival surface, \( E^{GO} \) is specifically given by Equation (4.41). Furthermore, the discussion in relation to the R-Dg-R and R-R fields given in association with Equation (7.1) also applies to Equation (7.6). When the plate is tilted with respect to the surface, the expression for \( E^{JE} \) in Equation (6.80) is invalid. As mentioned there, this field can be neglected in such cases. In effect, the doubly reflected field takes over the role of continuity at the existance boundaries (see Equation (6.83), Figure 6.24 and Section D of Chapter VI).

The accuracy of Equation (7.6) was tested with the model shown in Figure 7.11. This has the same ogive and plate size as that in Figure 7.1 except that the plate's bottom edge has been modified slightly to fit the curvature of the surface and during measurements the plate was placed in a slit of a styrofoam pad in order to make it rigid.

RCS measurements to the model of Figure 7.11 were again made at the same three pattern cuts discussed in relation to the model of Figure 7.1, i.e., in the xy, yz and xz plane. Both the \( \phi \) and \( \theta \) polarizations

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Figure 7.11. Geometry of the measurement model with the plate connected to the surface.
of incidence were considered at \( f = 9.01 \) GHz. The reader is hereon cautioned not to make any conclusions regarding the scattering by the gap between the plate and the surface when comparing the results from the structures in Figures 7.1 and 7.11. Here, the structures are studied as whole, whereas the scattering of the gap requires an individual attention.

The calculated and measured RCS patterns in the yz plane (\( \phi = 90° \)) are given in Figure 7.12 for the \( \phi \) and \( \theta \) polarizations of incidence. Again, a good agreement between theory and measurements is generally observed throughout these \( E_\phi \) and \( E_\theta \) RCS patterns. The only discrepancy occurs as usual in the first 5-10 degrees of the patterns. The reason is the same as that discussed in relation to the patterns in Figure 7.4 since in this region, the direction of the incident wave is close to the plane of the plate. Note also that our calculations were only carried out to about \( \theta = 90° \) which is the shadow boundary of the junction-edge field and therefore, the limit of the lit region. From there on, the plate begins to become shadowed and after ten degrees the contribution from the plate is negligible, i.e., the total field is basically the GO field from the surface alone.

An interesting point to observe is that the measured patterns in Figures 7.4 and 7.12 are basically identical when away from broadside to the plate. In fact, a study proved that these patterns are unaffected in this region due to variations on the minimum distance \( d \) (see Figure 7.1) of the plate from the top of the ogive. However, when the plate is connected to the surface, the mechanisms involved in the calculation
Figure 7.12. Comparison of measured and calculated $E_\phi$ and $E_\theta$ RCS patterns at $f=9.01$ GHz from the structure of Figure 7.11. Shown patterns were taken in the $\phi=90^\circ$ plane.
b). $E_\theta$, $\phi=90^\circ$ RCS pattern.

Figure 7.12. (Continued).
of the patterns are different even though the total result remains the same. In particular, the R-D_e field from edge #4 associated with the model in Figure 7.1 is replaced by the junction-edge field in case of the structure in Figure 7.11. These are quite dominant as can be seen in Figure 7.13 in which the calculated φ=90° plane patterns are compared to those where the junction-edge field has been removed. The difference is drastic, not only in magnitude, but also in pattern shape for both polarizations of incidence.

Although measurements in the xy plane (θ=90°) for the model in Figure 7.11 were indeed performed, this pattern was not calculated since it falls in the shadow region of the plate. For the sake of record, the measured E_ϕ and E_θ RCS patterns from each side of the plate are given in Figure 7.14. One should also note that as mentioned in the last section, in relation to Figure 7.6, these patterns are quite sensitive to the angle θ and therefore, they cannot be taken with absolute accuracy. The largest θ angle for which the plate is in the lit region throughout the conical pattern is at about 78 degrees. Figure 7.15 shows the calculated E_ϕ and E_θ conical patterns at θ=78°, and one observes that these resemble in all respects those measured at θ=90°.

Lastly, the measured and calculated E_θ RCS patterns in the φ=90° plane (the calculated is at φ=1/4 degrees) are shown in Figure 7.16. This pattern is in the plane of the plate and therefore the same discussion applies as that given in relation to the pattern in Figure 7.9. In effect, higher order terms are required for better agreement.
a). $E_{\phi}$, $\phi=90^\circ$ RCS patterns.

Figure 7.13. Comparison of calculated $E_{\phi}$ and $E_{\theta}$ RCS patterns at $f=9.01$ GHz from the structure in Figure 7.11 with and without the junction-edge field. Shown patterns were taken in the $\phi=90^\circ$ plane.
b). $E_\theta$, $\phi=90^\circ$ RCS pattern.

Figure 7.13. (Continued).
Figure 7.14. Measured $E_\phi$ and $E_\theta$ RCS patterns at $f=9.01$ GHz from the structure of Figure 7.11. Shown patterns were taken in the $\theta=90^\circ$ plane.
b). $E_\theta$, $\theta=90^\circ$ RCS pattern.

Figure 7.14. (Continued).
Figure 7.15. Calculated $E_{\phi}$ and $E_{\theta}$ RCS patterns at $f=9.01$ GHz from the structure of Figure 7.11. Shown patterns are conical cuts with $\theta=78$ degrees.
b). \( E_\theta, \theta=78^\circ \) RCS pattern.

Figure 7.15. (Continued).
Figure 7.16. Measured and calculated $E_g$, $\phi=0$ RCS pattern at $f=9.01$ GHz, from the structure of Figure 7.11.
between theory and experiment. One is also cautioned that the jump in the calculated pattern at $\theta=10^\circ$ is not a discontinuity but simply the included fields in the expression of Equation (7.6) undergo severe changes at this point which is the existence boundary of the junction-edge and the R-$D_\Theta$ fields associated with edge #3. To verify this, the reader is referred to Figure 7.17 which shows the calculated $E_\Theta$ pattern at $\phi=10^\circ$. It is obvious then that the total field is actually continuous at the existence boundary of the junction-edge field as discussed in the previous chapter.

C. SUMMARY OF RESULTS

This chapter presented measured and calculated results in the $\phi=90^\circ$, $\theta=90^\circ$ and $\phi=0^\circ$ planes of the plate structures depicted in Figures 7.1 and 7.11. In one case, the plate was off the surface and in the other it was connected to it. Furthermore, the plate was normal to the surface for both structures.

It was demonstrated that the fields in Table 6.1 are indeed capable of predicting the RCS of such plate structures in the lit region as given by Equations (7.1) and (7.6). However, throughout this chapter, it was stressed that when the direction of the incident wave is within 5-10 degrees to the plane of the plate, then higher order terms (see Figures 6.7 and 6.8) must be added to our solution for better results. This task is currently being undertaken.
Figure 7.17. Calculated $E_\theta, \phi=10^\circ$ RCS pattern at $f=9.01$ GHz from the structure in Figure 7.11.
If the plate is tilted with respect to the surface, the doubly reflected field (see section D of Chapter VI) must also be included to our solution. However, in this case, the junction-edge fields are negligible and Equation (6.80) is not valid.
A. SUMMARY AND FURTHER WORK

The object of this research was to compute and analyze the high frequency RCS of fairly general structures involving more than one surface by the use of the UTD and EC methods. In particular, our investigation concentrated on curved surfaces with a plate or an inlet attached. Expressions were obtained for the computation of the RCS of a thin-edge inlet over an arbitrary smooth surface, a curved rim inlet over an arbitrary smooth surface, and a plate with piecewise linear edges over an arbitrary smooth surface. In addition, it was pointed out that the solution of the backscattered field for the thin-edge inlet structure can be directly applied to other configurations where the inlet is replaced by some general shape plate. Calculated results were also compared with measured results for specific models in order to verify the validity of our solutions.
Our solution included not only the backscattered field from each part of the structure, such as the surface, the inlet or the plate, but also the backscattered field caused by the interaction of the inlet with the surface or the plate with the surface. In case of the inlet structures, the computation of the above components required the application of the equivalent current (EC) concept. The basic principle of this method was reviewed in Chapter II and its application required the development of a routine to trace the reflected rays from the surface to the inlet rim. However, thus far, the EC concept was limited to scattering by edges. Therefore, a new set of equivalent currents was derived for application to the curved rim inlet structure. Alternatively, the backscattering of the plate with linear edges over an arbitrary surface did not require any EC integration which is obviously a time consuming process. Instead, the currently available corner diffraction coefficient was used to treat the scattering by the corners. This approach, though, required the introduction and development of the junction-corner and junction-edge fields. These are scattered by the junction of the plate with the surface and are usually quite dominant when the plate is normal to the surface. In addition, a number of generalized raytracing routines were developed during the study of this structure.

The EC solution of the thin-edge inlet over an arbitrary surface was developed in Chapter III. It was stated that the only information required at each surface point was the normal, the principal radii and the corresponding principal directions. In addition, the inlet locus
solution consisted of the GO (from the surface), the singly diffracted from the inlet, the reflected-diffracted between the inlet and the surface, and the reflected-diffracted-reflected fields. The evaluation of the last three of these involved the integration of an equivalent current over the knife edge of the inlet. The generation of these currents, in turn, required the development of a numerical routine to trace all the reflected rays from the surface to the edge. In the shadow region only the surface diffracted-edge diffracted-surface diffracted field was dominant. Although the EC solution for this component was indeed developed, it was not applied due to the unavailability of the surface geodesic paths. Instead, our solution in this region was limited to the field contribution of the inlet stationary point. However, the geodesic paths on a sufficiently general surface are now available and are currently being incorporated into the EC solution.

The above solution was consequently applied (see Chapter IV) to a cylindrical inlet over a solid cylinder and an ogive. In the case of the ogive, measured results were presented which agreed very well with our calculations. In addition, the results by the EC method were compared to those by the ordinary UTD. As discussed, the ordinary UTD failed at caustics and did not include the fields scattered from the junction of the inlet with the surface.

In Chapter V, the backscattered field from a structure consisting of an inlet with a curved rim over an arbitrary surface was studied. The caustic regions in this case were treated by the use of a new set of equivalent line currents. These were integrated over the specular
rim line similarly to the approach used for the thin-edge inlet structure. Our solution in the lit region included the GO field, that reflected from the inlet rim, the doubly reflected field between the inlet rim and the surface, and the triply reflected field. However, in the shadow region, only the surface diffracted-reflected-surface diffracted field was dominant. This was treated similarly to the case of the thin-edge inlet structure.

The backscattering of a general shape plate over an arbitrary surface could be treated by the EC solution outlined in Chapter III. However, if the plate is defined in terms of corners joined by linear edges then the currently available corner diffraction coefficient could be used in the context of the ordinary UTD, and the EC integration can be avoided. The UTD solution of such a structure was developed in Chapter VI and again, only the lit region was considered due to the unavailability of the geodesic paths. In this region, our solution included (see Table 6.1) the GO field, the corner diffracted field, the reflected-corner diffracted field, the reflected-edge diffracted field, the reflected-corner diffracted-reflected field, the junction-corner field, and the junction-edge field. In addition, the solutions for the doubly reflected and reflected-edge diffracted-reflected fields were also discussed for possible implementation if required. The junction-corner and junction-edge are new types of fields and were developed in section C of Chapter VI. Furthermore, a large part of section B of this chapter was devoted to the derivation of a numerical routine to simultaneously trace the reflection and diffraction point associated with the reflected-edge diffracted field.

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In Chapter VII, measured and calculated results were presented for two types of plate structures. In one case, the plate was connected to the surface and in the other the plate was a small distance off the surface. The junction-corner and junction-edge fields exist only in the first type of structure. Therefore, one can study the effect of these fields by comparing the results from the two structures. Our calculations agreed very well with the measurements for both structures except when the plane of incidence was near the plane of the plate. In this case it was noted that higher order mechanisms were needed for obtaining a better agreement.

It should be clear that this research demonstrates that one can practically consider calculating the scattered field from completely general structures including interaction terms by the use of UTD and EC methods. One of the greatest advantages of UTD is, of course, its efficiency. However, additional work is required before this goal is completely achieved.

The computation of the backscattered field must first be completely extended into the shadow region. This is now possible since the geodesic paths for a quite general surface are currently available [68]. The next step would be to incorporate the above solutions to a computer code capable of modeling a general structure consisting of a surface, inlets and plates. Such an achievement will extend our capability to accurately predict backscattering from even more complex structures, than have been considered here, and is not far from completion. Consequently, higher order terms can be added in this solution to improve the calculated results in certain regions, as pointed out in the text.
APPENDIX A

PROJECTION OF A VECTOR ONTO A PLANE

Let us assume a plane which is normal to the unit vector \( \hat{n} \) and some unit vector \( \hat{s} \) as indicated in Figure A.1. The unit vector projection of \( \hat{s} \) onto this plane is now desired. This unit vector, \( \hat{s}_p \), will then be normal to \( \hat{n} \) and in the plane of \((\hat{n}, \hat{s})\). Therefore, it will satisfy the relationships,

\[ \hat{n} \cdot \hat{s}_p = 0 \]  
(A.1a)

and

\[ \text{csc} \alpha (\hat{n} \times \hat{s}) = \hat{n} \times \hat{s}_p \]  
(A.1b)

The \( \text{csc} \alpha \) factor ensures that the magnitude of \( \hat{s}_p \) is unity.

From Equation (A.1a) it is obtained that

\[ s_{px} = \frac{ny_{spy} + nz_{spz}}{nx} \]  
(A.2)
where

$$\hat{n} = \hat{x}n_x + \hat{y}n_y + \hat{z}n_z$$  \hspace{1cm} (A.3)

and

$$\hat{s}_p = \hat{x}s_{px} + \hat{y}s_{py} + \hat{z}s_{pz}$$  \hspace{1cm} (A.4)

Also, Equation (A.1b) can be broken up into three equations by equating the $x,y,z$ components from each side of it. This gives

$$\csc\alpha(yz_x - nz_y) = yz_{sp} - nz_{sy},$$  \hspace{1cm} (A.5a)
\begin{align}
\text{csc}\alpha(n_x s_z - n_z s_x) &= n_x s_p z - n_z s_p x, \quad (A.5b) \\
\text{csc}\alpha(n_x s_y - n_y s_x) &= n_x s_p y - n_y s_p x, \quad (A.5c)
\end{align}

where
\[ \hat{s} = x s_x + y s_y + z s_z. \quad (A.6) \]

Equations (A.2) and (A.5) must now be solved simultaneously to find the components of \( \hat{s}_p \).

After substituting Equation (A.2) into Equations (A.5b) and (A.5c), they become
\begin{align}
n_x \text{csc}\alpha(n_x s_z - n_z s_x) &= n_y n_z s_p y + (n_x^2 + n_z^2) s_p z, \quad (A.7a) \\
n_x \text{csc}\alpha(n_x s_y - n_y s_x) &= (n_x^2 + n_y^2) s_p y + n_y n_z s_p z. \quad (A.7b)
\end{align}

The above, when coupled with Equation (A.5a) are redundant and therefore only one of them needs to be considered. After solving simultaneously Equations (A.5a) and (A.7a) or (A.5a) and (A.7b) it is found that
\begin{align}
\hat{s}_p z &= \text{csc}\alpha[(n_x^2 + n_y^2) s_z - n_z (n_x s_x + n_y s_y)] \quad (A.8a) \\
\hat{s}_p y &= \text{csc}\alpha[(n_x^2 + n_z^2) s_y - n_y (n_x s_x + n_z s_z)]. \quad (A.8b)
\end{align}

Consequently, substituting Equations (A.8a) and (A.8b) into equation (A.2) it is in addition obtained that
\[ \hat{s}_p x = \text{csc}\alpha[(n_y^2 + n_z^2) s_x - n_x (n_y s_y + n_z s_z)]. \quad (A.8a) \]
Appendix B

Scattering by the Back Edge of the Measurement Model of Structure B

It was mentioned in section B of Chapter IV that the backscattered field due to certain mechanisms through the top back edge stationary point of the measurement model in Figure 4.18 could not be neglected. These mechanisms were shown in Figure 4.19 and consisted of the $D_e$ ray from the edge point $P_e^{bo}$ and the $D_e-D_e$ rays through the edge points $P_e$ and $P_e^{bo}$. Other less significant mechanisms are shown in Figure 4.20. Note also that the bottom back edge stationary point of the inlet was covered with absorber and thus it is completely neglected in this study.

As discussed, the inner portion of the cylindrical inlet was partially filled with absorber and therefore the dominant ray mechanisms exist only up to $\theta=\pi/2$ (see Figure 4.10). The fields due to the $D_e$ and $D_e-D_e$ rays through $P_e^{bo}$ are easily found via the ordinary UTD since they are not associated with any caustic region for $0<\theta<\pi/2$. The sum of these fields is

$$E_b^{bsc} = E_{1b}^{De} + 2E_{2b}^{DeDe}$$

(8.1)
where $E_{1b}^D$ denotes the singly diffracted field from $p_{e}^{bo}$ in the backscatter direction and $E_{2b}^D$ that diffracted from $p_{e}^0$ to $p_{e}^{bo}$ and back to the receiver. In addition, the factor of two accounts for the reverse direction of the $D_e-D_e$ ray. The field $E_{1b}^{BSC}$ due to the back edge must be added to Equation (4.105) in order to obtain the calculated results shown in Figure 4.21. The derivation of the individual fields $E_{1b}^D$ and $E_{2b}^D$ follows.

A. SINGLY DIFFRACTED FIELD FROM $p_{e}^{bo}$

The singly diffracted field from $p_{e}^{bo}$ in the backscatter direction is expressed by (see Equation (2.20))

$$E_{1b}^D(\theta) = E_{1}(\theta) \cdot (\hat{\eta} \hat{\eta}^{b})_{s,0} \cdot \frac{1}{\sqrt{l_{1}}} e^{-j2k(E_{e}^{bo} - \eta_{e})} \cdot e^{-jks} \ , \ (B.2)$$

where from Equation (2.45),

$$p_{s,h}^{b} = D_{s,0} \left( \frac{\pi}{2} - \theta, \frac{\pi}{2} - \theta, \frac{\pi}{2} \right) = - \frac{e^{-j\pi/4\sqrt{2}}}{4\pi} \left( \frac{\sin \theta - 1}{\sin \theta} \right) \ (B.3)$$

and

$$(p_{e}^{bo} - \eta_{e}) \cdot \hat{\eta} = \lambda \sin \theta \cdot \cos \theta \ \ (B.4)$$

with $\lambda$ and $\alpha$ as shown in Figure B.1.
Figure B.1. Definition of parameters associated with the $D_e$ ray from $p_{e}^{bo}$.

The caustic distance, $\rho_1^{lb}$, is given by (see Equation (2.38))

$$
\rho_1^{lb} = -\frac{a_{e}^{bo}}{2\rho_{e}^{bo}} .
$$

From Figure B.1 it is seen that

$$
\hat{n}_e^{bo} = \hat{x}\cos\alpha_{t}^{b} + \hat{z}\sin\alpha_{t}^{b}
$$

and $a_{e}^{bo}$ is the radius of curvature of the back edge at $p_{e}^{bo}$ and is given by...
\[ a_{e}^{bo} = \sin \alpha_{e} \]  \hspace{1cm} (B.7)

Substituting Equations (B.6) and (B.7) into Equation (B.5), \( p_{1}^{1b} \) simplifies to

\[ p_{1}^{1b} = \frac{\sin \alpha_{e}}{2 \sin (\theta + \alpha_{e}^{D})} \]  \hspace{1cm} (B.8)

**B. DOUBLY DIFFRACTED FIELD THROUGH \( p_{e}^{o} \) AND \( p_{e}^{bo} \)**

The field diffracted from \( p_{e}^{o} \) toward \( p_{e}^{bo} \) is given by (see Equation (2.20))

\[ E_{2b}^{D} = E_{2b}^{0} \cdot (\frac{\Delta_{s}^{1b} \Delta_{h}^{1b}}{\rho_{1}^{2b}}) \sqrt{\frac{\rho_{1}^{2b}}{(\rho_{1}^{2b} + \lambda) \lambda}} e^{j \lambda \sin \theta} e^{-j \lambda} \]  \hspace{1cm} (B.9)

where, from Equation (2.45)

\[ D_{s,h}^{1b} = D_{s,h}(\xi, \xi, 0, \pi/2, \pi/2) = \]

\[ \frac{-e^{-j \pi/4}}{2^{\pi k}} \left[ \frac{F[2k \cos^2(\pi/4+\theta/2)]}{\cos(\pi/4+\theta/2)} \right] \right] (B.10) \]

Therefore,

\[ D_{s}^{1b} = 0 \]
and

$$D_{1b} = -e^{-j\pi/4} \frac{F[2k\cos^2(\pi/4+\theta/2)]}{\sqrt{2\pi k}} \cos(\pi/4+\theta/2).$$

From the above result it is seen that for H-plane incidence ($E_\phi, \phi = 0^\circ$ polarization) this ray mechanism does not cause any scattering. Such a conclusion is, of course, consistent with the required boundary conditions.

The caustic distance $\rho_{1b}$ is now found by (see Equation (2.38))

$$\rho_{1b} = -\frac{a_0}{n_e (1+x)} \quad (B.11)$$

where all parameters were defined in Equations (4.5) to (4.7) and are also shown in Figure B.2. Substituting these into Equation (B.11) the expression for $\rho_{1b}$ reduces to

$$\rho_{1b} = \frac{a_0 \cos \alpha_{t_2}}{\cos(\theta+\alpha_{t_2})+\sin \alpha_{t_2}} \quad (B.12)$$

The backscattered field from $p_{e0}^{bo}$ due to the field $E_{2b}^{e0}(p_{e0}^{bo})$ scattered at $p_{e0}^{bo}$ after diffraction from $p_{e0}^{o}$, can now be expressed as

$$E_{2b}^{e0}(\theta) = E_{2b}^{e0}(p_{e0}^{bo}) \cdot (\chi_{2b}^{2b} + \chi_{2b}^{h}) \sqrt{\frac{2b}{2b}} e^{-j2k\sin \theta} e^{-jks} \quad (B.13)$$

The diffraction coefficients are given by
Figure B.2. Definition of parameters associated with the D_e-D_e ray through \( p^0_e \) and \( p^{bo}_e \).

\[
D_{s,h}^{2b} = \begin{cases} 
0 & \\
\frac{e^{-j\pi/4}}{\sqrt{2\pi k}} \frac{F[2k_0^2 \cos^2(\pi/4-\theta)]}{\cos(\pi/4-\theta)} 
\end{cases} \tag{B.14}
\]

and the caustic distance \( \rho_2^{2b} \) is found by

\[
\frac{1}{\rho_2^{2b}} = \frac{1}{\rho_1^{2b}} - \frac{\hat{n}_e^{bo} \cdot (\hat{x} + \hat{I})}{a_e^{bo}}. \tag{B.15}
\]

The parameters \( \hat{n}_e^{bo} \) and \( a_e^{bo} \) were defined in Equations (B.6) and (B.7), and \( \rho_1^{2b} \) is given in Equation (B.12). Substituting these into (B.15) it is obtained that

\[
\rho_2^{2b} = \frac{a_1^{2b} \cos \theta + I}{a \cos \theta + x^{2b} \cos \theta + \sin(\theta + \phi)} \tag{B.16}
\]
REFERENCES


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