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THE EFFECTS OF NEGATIVE INSTANCES AND FOCUSING STRATEGIES ON CONJUNCTIVE CONCEPT LEARNING

The Ohio State University

Ph.D. 1982

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THE EFFECTS OF NEGATIVE INSTANCES
AND FOCUSING STRATEGIES ON CONJUNCTIVE
CONCEPT LEARNING

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By
David Lee Stout, B.A., M.A.

* * * * *

The Ohio State University
1982

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CHAPTER I
INTRODUCTION

Counterexamples or negative instances are considered essential by mathematicians to the understanding of advanced mathematical concepts (Gelbaum and Olmsted, 1964; Steen and Seebach, 1970). Mathematics educators also support the use of negative instances for teaching mathematical concepts (Cohen and Carpenter, 1980; Dienes, 1964, 1971; Shumway, 1971, 1974, 1977; Skemp, 1971). Sowder (1980) has identified the use of negative instances in teaching mathematical concepts and principles as a critical area of needed research. Educational psychologists have stressed the need to include negative instances in instructional sequences designed for teaching concepts (Markle and Tiemann, 1970). Clark (1971) reviewed over 250 experimental studies by psychologists concerned with concept attainment and found, in marked contrast, negative instances adversely affected learning conjunctive concepts. However, Clark (1971) suggests concepts taught in schools are different from those researched in laboratories. Research concerning the use of negative instances in classroom-type instruction has shown negative instances to be helpful in teaching and learning concepts (Tennyson, et al., 1972; Sheppard, 1972; Shumway, 1971, 1974, 1977).
Recent research by mathematics and science educators (Shumway, White, and Wilson, 1981) has explored the seemingly contradictory evidence regarding the use of nonexamples in conjunctive feature identification tasks. Reviews of conjunctive concept research consistently support the use of learning sequences of only positive instances for conjunctive concepts (Bourne and Dominowski, 1972; Erickson and Jones, 1978). However, research done using conjunctive concepts from mathematics consistently supports the use of learning sequences of a mixture of positive and negative instances (Shumway, 1971, 1974, 1977; Cohen and Carpenter, 1980). In an attempt to explain the discrepancy, Shumway and White (1977) analyzed concepts from mathematics and science and hypothesized prevalent levels of irrelevant dimensions to be the confounding factor. Employing ideas similar to those of Bourne, et al., (1976), on frequency analysis, Shumway and White (1977) manipulated the frequencies of levels of irrelevant dimensions across positive and negative instances of a conjunctive concept. That is, the number of times a particular feature of an irrelevant dimension appeared in an instance was manipulated. The results supported the classical psychological research findings (i.e., favored all positive instances) when the frequency of levels of irrelevant dimensions was .5; however, when the frequency was .9, the results supported the use of both positive and negative instances. In a refinement of the experiment, Shumway, et al., (1981) have replicated their results.
When teaching concepts whose instances might have a large or an infinite number of irrelevant dimensions, educational psychologists have suggested the most efficient organization of positive and negative instances is in a rational set (Markle and Tiemann, 1969; Tennyson and Park, 1980). A rational set consists of two divergent positive instances each matched to a negative instance with prompting of the irrelevant dimensions. Positive instances are divergent when irrelevant features are as different as possible. Matching occurs when a positive instance and a negative instance differ only in one relevant feature. Divergency and matching help the learner focus attention on features which define the concept class and aid in concept learning (Tennyson and Boutwell, 1973; Tennyson, et al., 1972).

Stout (1981) attempted to use the results of Shumway, et al., (1981), and Tennyson, et al., (1972), to explore the effectiveness of three focusing strategies. After analyzing concepts from mathematics, it becomes apparent that many irrelevant dimensions are either never or, at best, rarely manipulated in the instances displayed. The learner can, therefore, erroneously ascribe relevancy to an irrelevant dimension because of the prevalency of one of its features. However, if teachers were able to focus the attention of students on the relevant dimensions, or limit the number of irrelevant dimensions which must be considered, then giving only positive instances of the concept would seem to be
adequate. It seems teachers must also be aware of irrelevant dimensions which exist and which are attractive to learners but which are impossible or inconvenient to manipulate. Some examples are the position of exercises in a text and the use of $f(x)$ as a functional value. It is in this area of impossible or inconvenient manipulation where negative instances would seem to be most powerful since the frequency of levels of such irrelevant dimensions approaches 1.0. That is, a level of an irrelevant dimension is present in both positive and negative instances most of the time. Thus, a negative instance of the concept would help the learner to correctly judge the irrelevant dimension as irrelevant. Because of this apparent interaction of the use of positive and negative instances with the use of a focusing strategy, it was hypothesized that as the intensity of focusing decreased, the value of negative instances would increase.

Before conducting an experiment with mathematics tasks, it seemed important to test the hypothesis in a psychological setting in order to determine the effects of focusing and negative instances on conjunctive concept learning (Stout, 1931). In such a "laboratory" type experiment, there is greater control of potential confounding factors. The two-letter conjunctive concept "D and K" of Shumway, et al, (1931), was used.

Sixteen seven-letter strings constituted the stimulus population. The seven letters in each string represented the
dimensions. Each dimension had two features, for a total of fourteen features. The features for dimension one were B or C; for two, D or F; for three, G or H; for four, J or K; for five, L or M; for six, P or Q; and for seven, R or S. Eight of the strings were positive instances and eight matching negative instances of the concept were chosen. Two learning sequences were constructed. One sequence consisted entirely of positive instances and the other mixed (positive and negative) instances. In each sequence, the positive instances were divergent. On each page there were two identical pairs. Thus, there were four instances per page.

Divergency has been defined to mean irrelevant features of two positive instances are as different as possible (Tennyson, et al., 1972). However, for the Stout (1931) study, divergent meant all irrelevant features, except two, changed from one positive instance to the next positive instance. The two irrelevant features left unchanged were P and R, representing those irrelevant features of which we may be unaware of unable to manipulate. Thus, the prevalence for one feature of each of the two unchanged irrelevant dimensions was 1, while all others had .5 prevalence.

The instances in the mixed sequence were arranged in a modified rational set. That is, the standard feature in a rational set of prompting irrelevant dimensions was not used. Each page contained two positive instances and two negative instances. The two positive instances were divergent and each was matched to a negative instance.
Three focus strategies were also developed: a three-focus strategy, a five-focus strategy, and a seven-focus strategy. The three-focus strategy required subjects to limit attention to three dimensions of two features each, for a total of six features. D and K represented two of the six features. In the five-focus strategy, the subject's attention was focused on five dimensions of two features each. Again, two of the features were D and K. The seven-focus strategy in reality was a no-focus strategy since the subjects were required to consider all seven dimensions with two features each. D and K represented two of the fourteen features.

The results (Stout, 1931) supported the usual psychological research findings for conjunctive concepts (i.e., the favoring of all positive instances over mixed sequences) when a three-dimension or five-dimension focus strategy is employed. However, when the seven-dimension focus strategy was used, results strongly supported the use of negative instances. These results corroborated the findings of Shumway, et al., (1981). Clearly, the value of negative instances was greatly enhanced by the presence of highly frequent, or prevalent, levels of irrelevant dimensions in the seven-dimension focus strategy.

The significance of the seven-dimension focus strategy was that it represented the situation where levels of irrelevant dimensions were very prevalent. Such a situation is common among mathematical concepts. Some examples are using
a's, b's, and c's in \( c^2 = a^2 + b^2 \) or frequently using \( x \) as the unknown.

The implications of the Stout (1931) study seem to be:

1) focusing the learner's attention on a narrow range of possible irrelevant dimensions reduced the value of negative instances; 2) not knowing about, or perhaps not being able to manipulate, some prevalent irrelevant features of a concept dictates the need to use negative instances of the concept; and 3) using negative instances can reduce the need to use focusing strategies.

One next step is to extend these results to a conjunctive mathematical concept in order to validate the results in a classroom-type setting.

**Problem Statement.** Negative instances should play an important role in the learning of conjunctive mathematical concepts. In particular, the purpose is to extend the results of Stout (1931) using a conjunctive mathematical concept.

Two mathematical concepts will be used: difference of squares and equilateral quadralateral. Each of these concepts will be studied in separate experiments. As in the Stout (1931) study, each concept will be delineated along seven dimensions of two features each, two relevant and five irrelevant. The five irrelevant dimensions will be chosen for their attractiveness to learners based on small group interviews with individuals from beginning algebra classes and the author's experience as a mathematics teacher. These individuals will
come from the same population of beginning algebra students as the subjects for the study.

Focusing strategies used by Stout (1981) will be translated into corresponding focusing strategies for the two mathematical concepts using the fourteen features of each. Similarly, as done by Stout (1981), two irrelevant features of each concept will go unchanged. Thus, each of these features will have a prevalency of 1. The purpose of these two unchanging features is to represent what the author believes to be a common occurrence in the mathematics classroom. That is, frequently teachers present concepts which have attractive irrelevant features which are difficult, inconvenient, or impossible to manipulate and which are incorrectly thought to be relevant by some students. Consequently, these students will misconceive the concept.

The objectives are two-fold: (1) to acquire information leading to an explanation of the seemingly contradictory results in conjunctive concept research; and (2) to obtain information useful for teaching conjunctive mathematical concepts.

Significance of the Problem. Laboratory and instructional conjunctive concept learning research results disagree concerning the use and value of negative instances for learning conjunctive concepts. Shumway, et al. (1981) discovered a factor concerning irrelevant features of conjunctive concepts. They found irrelevant features which were present
ninety percent (90%) of the time enhanced the need to use and value of negative instances. When irrelevant features were present only fifty percent (50%) of the time, as in laboratory-type research, they found negative instances delayed attainment of the concept. The ninety percent (90%) prevalency of irrelevant features seemed to explain the discrepancy between laboratory and instructional conjunctive concept learning research results. That is, in laboratory-type conjunctive concept research, results favor the use of only positive instances. However, instructional-type conjunctive concept research results favor the use of both positive and negative instances. The potentially important factor which seems to explain the discrepancy is the prevalency of irrelevant features.

Stout (1981) increased the prevalency factor of two irrelevant features to one-hundred percent (100%) in order to simulate situations, which the author felt to be common in mathematics teaching, where irrelevant features are impossible, inconvenient, or difficult to manipulate. Stout (1981) also employed focusing strategies whereby the learner's attention to irrelevant features was manipulated. The focusing strategies simulated teaching practices of varying degrees of limiting students' attention to various features of a concept. The learning sequences (Stout, 1981) were developed using divergent and matched instances as recommended by many educational psychologists.
Results (Stout, 1981) showed negative instances were of great value in concept attainment when subjects had to consider the two prevalent features. However, when subjects had to consider irrelevant features present only fifty percent (50\%) of the time, results supported laboratory research results.

The Stout (1981) study was a psychological-type study using letter strings employed by Shumway, et al. (1981). The present study will try to extend these results to classroom-type conjunctive concepts and further validate the importance of the prevalency factor.

Even though many results from classroom-type research consistently support the use of negative instances, there has been no research, to the author's knowledge, which has tried to isolate when negative instances are of greatest value or when their use is imperative. This study will try to provide some answers.

Subjects. Subjects will be students attending the same Northwest Florida junior college and enrolled in a beginning algebra course Spring or Summer term, 1982. These students have very little, if any, expertise in algebra as ascertained by their mathematics placement-test scores.

Assumptions. 1. The concepts "difference of squares" and "equilateral quadrilateral" are conjunctive. Relevant features are "difference" and "squared terms" and "four sides" and "sides equal in length," respectively.
2. Focusing strategies of letter strings employed by Stout (1981) can be extended to learning mathematical concepts.

3. Subjects' placement into the beginning algebra course by testing will be sufficient to insure they are not knowledgable concerning the concepts "difference of squares" and "equilateral quadralateral."

4. The procedures will insure randomness of assigning subjects to treatments.

5. The dependent variable trials to solution will be accurately reflected on the focus-strategy answer sheets.

6. The independent variables sequence condition and focus strategy are qualitative, active, fixed, and between-subjects.

7. The results will be generalizable to similar populations of college students enrolled in beginning algebra courses.

Definitions of Terms. 1. Tennyson, et al., (1972) defined divergency as follows: two positive instances of a concept are divergent when irrelevant features are all different. For the concepts "difference of squares" and "equilateral quadralateral," two positive instances will be divergent when all but two irrelevant features are different.

2. Tennyson, et al., (1972) defined matching instances as a pair of instances, one positive and one negative, where the negative instance differs from the positive instance in as few dimensions as possible. For the two concepts being
used, instances will be matched when the negative instance differs from its paired positive instance in only one (relevant) dimension.

3. Prevalency is the frequency of a feature, or level, of a dimension in the learning sequence.

4. A learning sequence consists of eight positive or eight positive and eight negative instances of the concept.

**Hypotheses.** Using the results of Stout (1981) as a basis or theory, the hypotheses are: (1) If a three-dimension or five dimension focus strategy is employed, then results will support the usual psychological findings in conjunctive concept research (i.e., the learning sequence of only positive instances will be favored over the sequence with both positive and negative instances). (2) If a seven-dimension (i.e., no-focus) strategy is used, the sequence of positive and negative instances will be favored over the all positive sequence.
CHAPTER II
REVIEW OF THE LITERATURE

This review will be restricted to research on conjunctive concepts since it is in the area of conjunctive concepts where disagreement concerning the use and value of negative instances exists. The disagreement exists in the results obtained in laboratory and instructional conjunctive concept research. Thus, the literature related to the use and effectiveness of negative instances in conjunctive concept learning is reviewed in two sections: laboratory research and instructional research. A third section will deal with background research for the present study.

Laboratory Research. Laboratory research usually involves concepts defined over finite universes and dimensions such as color, size, shape, and number. One of the earliest studies was done by Smoke (1933). He found subjects preferred to learn a concept via a mixture of positive and negative instances presented simultaneously. He also concluded:

although negative instances may not make for rapidity in learning they tend to make for accuracy, especially in the case of difficult concepts. It appears that in so far as negative instances assist concept learning they do so largely because of the way in which they prevent the learner from coming to one or more erroneous conclusions while he is still in the midst of the learning process. (p.588)
Thus, Smoak (1933) suggested concept learning may become more efficient with the use of negative instances. However, the amount of information which each instance transmitted was not considered. Hovland (1952) suggested the potential usefulness of an instance be measured by the number of possible concepts it eliminates upon presentation to the subject. Hovland and Weiss (1953) used the results of Hovland's (1952) analysis and compared the effectiveness of various combinations of positive and negative instances in conjunctive concept learning. Instances were chosen so the amount of information conveyed was equal for both positive and negative instances. Results showed more subjects attained the concept when presented with all positive instances than with all negative instances. However, a number of subjects did learn the concept using only negative instances (p. 182). Using one-dimensional concepts (so as to equalize the amount of information given by a positive or negative instance), Huttenlocher (1962) found sequences of only negative instances were more difficult for subjects to use than sequences of all positive instances. However, Huttenlocher (1962) concludes "with equivalent amounts of information per instance, negative instances in mixed series can result in highly efficient learning." (p. 39).

Haygood and Devine (1967) tried to clarify the apparent superiority of sequences containing all positive instances over sequences of negative instances in conjunctive concept
learning. Their findings indicated the superiority of positive instances is due to the small proportion of positive instances in the universe. Bourne and Guy (1968) also indicate when the task is to identify the two relevant features of a concept, subjects perform best when instances are chosen from the smaller of the two classes, positive or negative. For conjunctive concepts, the smaller class is composed of positive instances.

Research has also shown subjects to be unable or, perhaps, unwilling to use negative instances (Bruner, et al., 1956; Donaldson, 1959). However, Friebergs and Tulving (1961) used a feature identification task of a conjunctive concept and found positive instances were used more efficiently than negative instances "by subjects who are relatively unsophisticated with respect to concept identification tasks," (pp.105-6). They found subjects became very good at using sequences of negative instances when sufficient practice was allowed. Others have found similar results (e.g., Heim and Scholnick, 1972).

However, the findings from experimental research of conjunctive concept learning overwhelmingly support the use of sequences of positive instances over sequences of positive and negative instances (Bourne and Cominowski, 1972; Erickson and Jones, 1978). Clark (1971) reviewed over 250 experimental studies and found 20 out of 25 studies in conjunctive concept learning produced results favoring sequences of all positive instances.
In summary, the evidence clearly indicates sequences of positive instances are to be favored over any other sequences in conjunctive concept learning. However, one must keep in mind concepts used in these laboratory studies had finite universes and features (both relevant and irrelevant) which could all be known. Concepts of this sort, as Clark (1971) indicates, are not like the concepts studied by students in the classroom.

**Instructional Research.** A distinction exists between laboratory and instructional research in concept learning. In all the investigations cited from experimental psychology, the concepts studied were defined over a finite universe. In instructional (classroom type) research, concepts are defined over an infinite universe and the number of irrelevant features cannot be exhausted.

Educational psychologists have advocated the use of negative instances when teaching concepts (Markle and Tiemann, 1970; Tennyson, et al., 1972). When teaching concepts whose instances might have a large or infinite number of irrelevant dimensions, educational psychologists indicate the best presentation of instances is in a rational, or concept, set (Markle and Tiemann, 1969; Tennyson and Park, 1980). A rational set consists of two divergent positive instances each matched to a negative instance with prompting of the irrelevant dimensions. The rational set helps the learner focus attention on features which define the concept class
and aids in concept learning (Tennyson and Boutwell, 1973). Several studies have been done using such a scheme for teaching a concept. Swanson (1972) found students could better classify previously unencountered instances of environmental concepts when instructed using positive and negative instances in a rational set. In a similar study, Feldman (1972) found providing a definition along with a rational set was more facilitative in learning geometrical concepts than rational sets alone. Others have obtained similar results (e.g., Frayer, 1970; Sheppard, 1972).

Mechner (1965) defined correct classification behavior as the ability of the learner to generalize within a concept class while discriminating between classes. Woolley and Tennyson (1972), having developed a conceptual model of classification behavior, stated matching positive instances to negative instances helps the learner focus on the relevant features and produces an ability to discriminate among classes, providing divergent positive instances help reduce acceptance of irrelevant features as relevant, and divergency gives the learner an idea of the range of possibilities within a concept class.

Tennyson, Woolley, and Merrill (1972) researched what was necessary in order for a subject to learn a concept when dealing with an infinite concept class (i.e., one in which all irrelevant features cannot be exhausted). They constructed instructional sequences of positive and negative
instances by manipulating the difficulty of an instance, the
matching of positive and negative instances, and the diver­
gency of positive instances which caused specified behaviors
in their subjects (i.e., correct classification, overgen­
eralization, undergeneralization, and misconception).
Divergent positive instances varied in as many irrelevant
dimensions as possible. Matched positive and negative
instances had irrelevant features as similar as possible.
Results indicated the best sequence for teaching concepts
from infinite classes is one where positive and negative
instances vary in difficulty and positive instances are
presented in divergent pairs, each member of which is
matched to a negative instance.

Tennyson (1973) successfully replicated the Tennyson,
et al., (1972) study. In addition, he found negative
instances to be an integral part of concept learning since
subjects responded randomly to posttest items after having
received a treatment without negative instances.

Cohen and Carpenter (1980) studied the effects of
negative instances on learning a conjunctive geometrical
concept task. Positive and negative instances presented in
a rational set or a sequence of four positive instances
followed by four negative instances were favored over a
sequence of only positive instances.
Shumway (1971) investigated the effect of negative instances on the formation of mathematical concepts in a classroom setting. He found subjects trained with a mixed sequence of positive and negative instances could classify new instances of the concept more accurately than subjects trained with only positive instances. Shumway and Lester (1974) studied the effect of negative instances on the learning of two mathematical concepts: associativity and commutativity. Results supported the use of negative instances. In other research done using conjunctive concepts from mathematics, support is consistently found for using negative instances (Shumway 1974, 1977).

Thus, it appears research done in classroom-type settings with conjunctive concepts defined over an infinite universe clearly supports the use of negative instances.

Background Research for This Study. Shumway and White (1977) analyzed concepts from mathematics and science and hypothesized prevalent levels of irrelevant dimensions to be the confounding factor in conjunctive concept research. The frequencies of levels of irrelevant dimensions were manipulated across positive and negative instances. Letter-strings with five dimensions of two levels each were used. The concept was a two letter conjunctive concept. The frequency of the levels of irrelevant dimensions were either all .5 (i.e., each level appeared about half the time) or
one level had a frequency of .9 (i.e., it appeared about 90% of the time) and all others were .5. Their results supported the classical experimental psychological research findings (i.e., favored all positive instances) when frequency of levels was .5; however, with a .9 frequency, results supported the use of both positive and negative instances. Thus, a potentially powerful variable for explaining the differences in laboratory and instructional conjunctive concept research results was found. In a refinement of their study, Shumway, White, and Wilson (1981) have replicated their results.

Stout (1981) attempted to integrate the results of Shumway, et al., (1981) and Tennyson, et al., (1972). The conjunctive concept "D and K" of Shumway, et al., (1981) was used. Letter strings of seven dimensions (two relevant, five irrelevant) were used. The frequency, or prevalency, of two irrelevant features was held constant at 1. That is, one level of each of two irrelevant dimensions was present in all instances all the time. All other irrelevant features had .5 prevalency. Instances were arranged in a modified rational set. That is, positive instances were divergent and negative instances were matched to the positive instances (Tennyson, et al., 1972). No prompting of irrelevant dimensions was used. Three focusing strategies were employed: three-dimension, five-dimension, and seven-dimension. The three- and five-dimension focus strategies limited the number of irrelevant features subjects had to consider. In both
the three- and five-dimension strategies, subjects did not have to deal with the prevalent irrelevant features whose frequency was 1. The seven-dimension strategy required subjects to consider all irrelevant dimensions. Results supported the laboratory research results for conjunctive concepts when either a three- or a five-dimension strategy was used. However, when subjects had to consider all seven dimensions, results strongly favored the use of positive and negative instances.

In summary, a potentially powerful variable, namely prevalency of irrelevant features, has been shown to account for much of the discrepancy between laboratory and instructional conjunctive concept research results. It is the intention to translate the letter strings used by Shumway, et al., (1981) and Stout (1981) into a conjunctive mathematical concept. If replication of the results is achieved, a piece of the discrepancy puzzle will have been found.
CHAPTER III

METHOD AND PROCEDURES

Methods and procedures will parallel those used by Stout (1981). The Stout (1981) study can be found in Appendix A.

Subjects. The subjects were college students enrolled in beginning algebra courses during the Spring or Summer terms, 1982. Treatments were administered under the supervision of the author during regular class time. Participation was voluntary. Assignment to treatments was random.

Procedure. Two experiments, Experiment I and II, were performed with the same sample. Experiment I investigated the concept "difference of squares." Experiment II investigated the concept "equilateral quadralateral." The stimulus population of each concept consisted of sixteen instances, each with seven dimensions. Eight instances were positive and eight were negative. Five of the seven dimensions varied along two features.

All instances of the mathematical concepts were obtained by translating the letter strings used by Stout (1981). The translation from the letter strings of Stout (1981) to the instances of the mathematical concepts was accomplished via the correspondence shown in Table 1.
<table>
<thead>
<tr>
<th>Letter</th>
<th>Algebra concept</th>
<th>Geometric concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>simple terms</td>
<td>dashed border</td>
</tr>
<tr>
<td>B</td>
<td>complex terms</td>
<td>solid border</td>
</tr>
<tr>
<td>D</td>
<td>subtraction</td>
<td>equal sides</td>
</tr>
<tr>
<td>F</td>
<td>addition</td>
<td>sides not equal</td>
</tr>
<tr>
<td>G</td>
<td>coefficients of terms all 1's</td>
<td>number of diagonals drawn = 2</td>
</tr>
<tr>
<td>H</td>
<td>coefficients of terms not all 1's</td>
<td>number of diagonals drawn ≠ 2</td>
</tr>
<tr>
<td>K</td>
<td>terms (incl. coeff.) are all squares</td>
<td>4 sides</td>
</tr>
<tr>
<td>J</td>
<td>terms (incl. coeff.) are not all squares</td>
<td>not 4 sides</td>
</tr>
<tr>
<td>L</td>
<td>exponents on terms are all powers of two</td>
<td>angles are =</td>
</tr>
<tr>
<td>M</td>
<td>exponents on terms are not all powers of two</td>
<td>angles are not =</td>
</tr>
<tr>
<td>P</td>
<td>variables are in both terms</td>
<td>figure is a polygon</td>
</tr>
<tr>
<td>Q</td>
<td>variables are not in both terms</td>
<td>figure is not a polygon</td>
</tr>
<tr>
<td>R</td>
<td>constants are all integers</td>
<td>base // to bottom of page</td>
</tr>
<tr>
<td>S</td>
<td>constants are not all integers</td>
<td>base not // to bottom of page</td>
</tr>
</tbody>
</table>
For Experiment I, the names and features of each of the seven dimensions were: Terms are: simple or complex; Basic Operation is: subtraction or addition; Coefficients of Terms are: all 1's or not all 1's; Terms (incl. coeff.) are: all squares or not all squares; Exponents on Terms are: all powers of 2 or not all powers of 2; Variables are: in both terms or not in both terms; and Constants are: all integers or not all integers.

The names and features of each of the seven dimensions in Experiment II were: Border: — or — — —; Length of Sides: = or ≠; Number of Diagonals: = 2 or ≠ 2; Number of Sides: = 4 or ≠ 4; Angles: = or ≠; Figure is: a polygon or not a polygon; and Base Orientation: // to bottom of page or ≠ to bottom of page.

Each stimulus for each concept contained all seven dimensions. Consider, for example, the letter string C D G K L P R. In Stout (1981) this was a positive instance of the concept "D and K." For the present study, the corresponding positive instance of the concept "difference of squares" was \( b^2 - a^2 \). That is, the terms are simple, the basic operation is subtraction, coefficients of terms are all 1's, terms (incl. coeff.) are all squares, exponents on terms are all powers of 2, variables are in both terms, and constants are all integers. The corresponding positive instance of "equilateral quadrilateral" was
That is, the border is dashed, sides are equal in length, the number of diagonals is two, the number of sides is four, angles are equal, the figure is a polygon, and the base of the figure is parallel to the bottom of the page.

Next, consider the letter string B F G K M P R. This letter string was a negative instance of the concept "D and K." The corresponding negative instance for "difference of squares" was \((b + 3a)^6 + (2b - 5a + 8)^4\). That is, the terms are complex, the basic operation is addition, the coefficients of terms are all ones, terms (incl. coeff.) are all squares, exponents on terms are not all powers of two, variables are in both terms, and constants are all integers. The corresponding negative instance of "equilateral-
That is, the border is solid, sides are not equal in length, the number of diagonals is two, the number of sides is four, angles are not equal, the figure is a polygon, and the base of the figure is parallel to the bottom of the page. Each letter string of Stout (1981) was translated in like manner.

Prior to the actual experiments subjects were drilled in the meaning of each of the seven dimensions. Data from those subjects who showed a lack of understanding of the dimensions was not used.

Subjects received two packets of materials—a packet of instances and a packet of focus sheets. Subjects were told the concept consisted of two characteristics which described the mathematical stimulus and which were both present in any positive instance. Furthermore, subjects were told to consider as possible choices for describing the concept only pairs of characteristics present on their respective focus sheets. All answers were recorded on focus sheets.

Prior to the actual experiments the two letter string practice tasks and one problem task used by Stout (1981) were employed. All subjects were given the same two practice tasks, one with three dimensions and one with four dimensions. Each dimension had two features. The three-dimension task had a focus strategy of two dimensions. The four-dimension task had a focus strategy of four dimensions. The author carefully worked through the practice tasks with the subjects. During and after the practice tasks, questions
such as how to use negative instances or how to record answers were entertained. At the conclusion of this question and answer period, the problem task was given. The problem had five dimensions and a focus strategy of four dimensions. Subjects worked through the task without help. The purpose of the problem task was to determine whether subjects understood the protocol.

On each page of the training problem and actual mathematical task activities, there were four instances. Each of the accompanying focus strategies also contained four response opportunities per page. Subjects used cards to cover subsequent instances as they worked their way down each page, at their own rate. Subjects were able to view as many as three previous instances and responses. Subjects were told not to look back at previous pages. Subjects circled the features of the dimensions of the instances which appeared on their focus sheets. Subjects then uncovered the correct feedback presented as a + or - under each instance. Subjects recorded this correct feedback on their focus sheets. Subjects were then asked to write the two characteristics which they thought described the concept. All subjects viewed 40 instances (repetition of the eight positive and eight negative instances occurred).

The criterion variable was the number of trials until solution was obtained. Each time a subject wrote the two characteristics which he/she thought described the concept was considered a trial. Thus, the number of trials until
solution was the same as the number of instances a subject had to view until he/she continually recorded the two correct characteristics.

Three subjects from each treatment group were randomly selected for an interview following the experiments. Each interviewee was asked to describe his/her reasons for selecting his/her solution. The same procedure was used for both experiments.

In Experiment I, positive instances were presented as follows: \( a^{16} - b^8 \). Negative instances were presented as follows: 

\[ (a^2 + b^2 - a + b)^3 - (2 - a)^{24} \]. In Experiment II, positive instances were presented as follows:

\[ (a^2 + b^2 - a + b)^3 - (2 - a)^{24} \]. Negative instances were presented as follows:
Each subject received one of three focus strategies: three, five, or seven dimensions. Each of the strategies was developed by translating the focus strategies used by Stout (1981). Appendixes H, I, J, K, L, and M contain the focus strategies used. Samples of completed student response sheets can be found in Appendix 0.

**Design.** The design was a $2 \times 3$ factorial with two sequence conditions (all positive or mixed positive and negative) and three focus strategies (3, 5, and 7 dimensions). The stimulus population consisted of sixteen instances varying in five of seven dimensions of two features each.

Table 2 gives the sample distribution and design matrix for Experiment I. Table 3 gives the sample distribution and design matrix for Experiment II. The same design was used in both experiments.

All subjects participated in both experiments. Subjects were first randomly assigned to treatments for Experiment I. At the conclusion of Experiment I, subjects were then
randomly assigned to treatments for Experiment II. No record of a subject's participation in a treatment group for Experiment I versus Experiment II was kept. Subjects in each row were given the same sequence.

**TABLE 2. SAMPLE DISTRIBUTION**

**EXPERIMENT I**

Focus Strategy

<table>
<thead>
<tr>
<th>Sequence</th>
<th>3-focus</th>
<th>5-focus</th>
<th>7-focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>n= 17</td>
<td>n= 17</td>
<td>n= 17</td>
</tr>
<tr>
<td>Condition</td>
<td>+</td>
<td>n= 17</td>
<td>n= 17</td>
</tr>
</tbody>
</table>

**TABLE 3. SAMPLE DISTRIBUTION**

**EXPERIMENT II**

Focus Strategy

<table>
<thead>
<tr>
<th>Sequence</th>
<th>3-focus</th>
<th>5-focus</th>
<th>7-focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>n= 17</td>
<td>n= 17</td>
<td>n= 17</td>
</tr>
<tr>
<td>Condition</td>
<td>+</td>
<td>n= 17</td>
<td>n= 17</td>
</tr>
</tbody>
</table>

**Sequence Condition:**

**Experiment I.** In the all positive sequence (+), eight (8) positive instances were presented in divergent pairs. Each pair was repeated on a page. Instances were considered divergent when all but the two irrelevant features "variables are in both terms" and "constants are all integers" changed. For example, the following four instances were the first
instances seen by each subject in the all positive sequence condition:

\[ a^2 - 4b^2 \]

\[ + \]

\[ (b + 3a)^6 - (2b - 5a + 8)^4 \]

\[ + \]

\[ a^2 - 4b^2 \]

\[ + \]

\[ (b + 3a)^6 - (2b - 5a + 8)^4 \]

For \( a^2 - 4b^2 \), the terms are simple, coefficients of terms are not all ones, and exponents on terms are all powers of two. However, for \((b + 3a)^6 - (2b - 5a + 8)^4\), the terms are complex, coefficients of terms are all ones, and exponents on terms are not all powers of two. Thus, \( a^2 - 4b^2 \) and \((b + 3a)^6 - (2b - 5a + 8)^4\) are considered divergent.

The mixed sequence \((\pm)\) contained the same eight positive instances used in the all positive sequence. Each positive instance, however, was matched to a negative instance. Two such pairs were presented on each page. That is, two divergent positive instances were each matched to a negative instance. Matching occurred when all irrelevant features remained unchanged, but one relevant feature changed in order to create a negative instance. For example, the following four instances comprised the first
page of instances seen by each subject in the mixed sequence condition:

\[ a^2 - 4b^2 \]
\[ + \]
\[ a^2 - 3b^2 \]
\[ - \]
\[ (b + 3a)^6 - (2b - 5a + 8)^4 \]
\[ + \]
\[ (b + 3a)^6 + (2b - 5a + 8)^4 \]

Notice the two positive instances are the same instances seen by subjects in the all positive sequence condition. 

\[ a^2 - 4b^2 \] and \[ a^2 - 3b^2 \] are matched since they only differ on the relevant dimension "terms (incl. coeff.) are: all squares or not all squares." Similarly, 

\[ (b + 3a)^6 - (2b - 5a + 8)^4 \] and \[ (b + 3a)^6 + (2b - 5a + 8)^4 \] are matched since they only differ on the relevant dimension "basic operation is: subtraction or addition."

**Experiment II.** In the all positive sequence (+), eight (8) positive instances were presented in divergent pairs. Each pair was repeated on a page. Instances were considered divergent when all but two irrelevant features, "figure is a polygon" and "base orientation: // to bottom of page," changed. For example, the following instances comprised the first page of instances seen by subjects in the all positive condition:
In the first and third instances, the border is dashed, number of diagonals is two, and angles are equal. However,
for the second and fourth instances, the border is solid, number of diagonals is not two, and angles are not equal. Thus, the instances are considered divergent.

The mixed sequence (+) contained the same eight positive instances used in the all positive sequence. Each positive instance, however, was matched to a negative instance. Two such pairs were presented per page. Matching occurred when all irrelevant features remained the same, but one relevant feature changed in order to create a negative instance. For example, the following four instances, two divergent positive instances each matched to a negative instance, comprised the first page of instances viewed by subjects in the mixed sequence condition:

![Diagram of instances](image-url)
Notice the two positive instances are the same instances seen by subjects in the all positive sequence condition.

The first and second instances are matched since they differ only on the relevant dimension "number of sides: = 4 or \( \neq 4 \)." The third and fourth instances are matched since they differ only on the relevant dimension "length of sides: = or \( \neq \)."

Focus Strategy:

Each subject received one of three focus strategies: three, five, or seven dimensions. The seven-dimension focus strategy was, in reality, a no-focus strategy since subjects receiving this strategy had to consider all seven dimensions.
### Experiment I.

#### Three-focus strategy:

<table>
<thead>
<tr>
<th>Terms Are:</th>
<th>Operation Is:</th>
<th>Terms (incl. coeff.) Are:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>Subtraction</td>
<td>All squares</td>
</tr>
<tr>
<td>Complex</td>
<td>Addition</td>
<td>Not all squares</td>
</tr>
</tbody>
</table>

This instance is: ♦ OR -

Guess: + OR -

I think the two characteristics which describe the concept are:

___ AND ___

#### Five-focus strategy:

<table>
<thead>
<tr>
<th>Terms Are:</th>
<th>Operation Is:</th>
<th>Coefficients of terms Are:</th>
<th>Terms (incl. coeff.) Are:</th>
<th>Exponents on terms Are:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>Subtraction</td>
<td>All 1's</td>
<td>All squares</td>
<td>All powers of 2</td>
</tr>
<tr>
<td>Complex</td>
<td>Addition</td>
<td>Not all 1's</td>
<td>Not all squares</td>
<td>Not all powers of 2</td>
</tr>
</tbody>
</table>

This instance is: ♦ OR -

Guess: + OR -

I think the two characteristics which describe the concept are:

___ AND ___

#### Seven-focus strategy:

<table>
<thead>
<tr>
<th>Terms Are:</th>
<th>Operation Is:</th>
<th>Coefficients of terms Are:</th>
<th>Terms (incl. coeff.) Are:</th>
<th>Exponents on terms Are:</th>
<th>Variables Are:</th>
<th>Constants Are:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>Subtraction</td>
<td>All 1's</td>
<td>All squares</td>
<td>All powers of 2</td>
<td>In both terms</td>
<td>All integers</td>
</tr>
<tr>
<td>Complex</td>
<td>Addition</td>
<td>Not all 1's</td>
<td>Not all squares</td>
<td>Not all powers of 2</td>
<td>Not in both terms</td>
<td>Not all integers</td>
</tr>
</tbody>
</table>

This instance is: ♦ OR -

Guess: + OR -

I think the two characteristics which describe the concept are:

___ AND ___
Experiment II. Three-focus strategy:

<table>
<thead>
<tr>
<th>BORDER:</th>
<th>LENGTH OF SIDES:</th>
<th>NUMBER OF SIDES:</th>
<th>GUESS: + OR -</th>
<th>THIS INSTANCE IS: + OR -</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>=</td>
<td>= 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>≠</td>
<td>≠ 4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I think the two characteristics which describe the concept are: ____________________________ and ____________________________.

Five-focus strategy:

<table>
<thead>
<tr>
<th>BORDER:</th>
<th>LENGTH OF SIDES:</th>
<th>NUMBERS OF DIAGONALS:</th>
<th>NUMBER OF SIDES:</th>
<th>ANGLES:</th>
<th>GUESS: + OR -</th>
<th>THIS INSTANCE IS: + OR -</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>=</td>
<td>= 2</td>
<td>= 4</td>
<td>=</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>≠</td>
<td>≠ 2</td>
<td>≠ 4</td>
<td>≠</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I think the two characteristics which describe the concept are: ____________________________ and ____________________________.

Seven-focus strategy:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>=</td>
<td>= 2</td>
<td>= 4</td>
<td>=</td>
<td>A POLYGON</td>
<td>// TO BOTTOM OF PAGE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>≠</td>
<td>≠ 2</td>
<td>≠ 4</td>
<td>≠</td>
<td>NOT A POLYGON</td>
<td>// TO BOTTOM OF PAGE</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I think the two characteristics which describe the concept are: ____________________________ and ____________________________.
Treatments. The all positive sequence and the mixed sequence were each crossed with the three focus strategies to produce six treatments (in both experiments):

(1) The positive sequence with a three-dimension focus strategy;
(2) The positive sequence with a five-dimension focus strategy;
(3) The positive sequence with a seven-dimension focus strategy;
(4) The mixed sequence with a three-dimension focus strategy;
(5) The mixed sequence with a five-dimension focus strategy; and
(6) The mixed sequence with a seven-dimension focus strategy.

Data Collection. Data was collected during regular class meetings by the author. Experiment I was completed by all subjects before starting Experiment II. At the end of Experiment II, subjects were allowed to leave. The total class time (about fifty minutes) was usually required to complete both experiments.

Statistical Analysis. The data was analyzed using analysis of variance. Post hoc analysis was accomplished with Tukey's test.

Instrumentation. Appendix B contains the stimulus population for Experiment I. Appendix C contains the
stimulus population for Experiment II. Appendix D contains materials used by subjects in the all positive sequence condition of Experiment I. Appendix E contains the materials used by subjects in the mixed sequence condition of Experiment I. Appendix F contains materials used by subjects in the all positive sequence condition of Experiment II. Appendix G contains materials used by subjects in the mixed sequence condition of Experiment II. Appendix H contains the three-dimension focus sheets used by subjects in Experiment I. Appendix I contains the five-dimension focus sheets used by subjects in Experiment I. Appendix J contains the seven-dimension focus sheets used by subjects in Experiment I. Appendix K contains the three-dimension focus sheets used by subjects in Experiment II. Appendix L contains the five-dimension focus sheets used by subjects in Experiment II. Appendix M contains the seven-dimension focus sheets used by subjects in Experiment II.
CHAPTER IV
DATA ANALYSIS

Results:

Experiment I. Before subjecting the data to analysis, the data from two subjects in the all positive and five-dimension focus strategy treatment group were randomly discarded to allow for equal cell sizes of \( n = 17 \) each. This allowed a conventional analysis of variance to be performed with the robustness usually associated with equal \( n \) designs and avoided problems associated with computations and interpretation of unequal \( n \) designs.

Table 4 gives the means and standard deviations for trials to solution for each cell on each of the treatment tasks of Experiment I. Table 5 gives the frequency distribution for trials to solution for each cell.

TABLE 4. MEANS AND STANDARD DEVIATIONS
OF TRIALS TO SOLUTION

<table>
<thead>
<tr>
<th></th>
<th>3-focus</th>
<th>5-focus</th>
<th>7-focus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( x = 2.18 )</td>
<td>( x = 2.47 )</td>
<td>( x = 33.47 )</td>
</tr>
<tr>
<td></td>
<td>( sd = 0.33 )</td>
<td>( sd = 1.33 )</td>
<td>( sd = 14.56 )</td>
</tr>
<tr>
<td></td>
<td>( n = 17 )</td>
<td>( n = 17 )</td>
<td>( n = 17 )</td>
</tr>
<tr>
<td>( \pm )</td>
<td>( x = 2.29 )</td>
<td>( x = 3.47 )</td>
<td>( x = 6.18 )</td>
</tr>
<tr>
<td></td>
<td>( sd = 1.36 )</td>
<td>( sd = 1.55 )</td>
<td>( sd = 4.64 )</td>
</tr>
<tr>
<td></td>
<td>( n = 17 )</td>
<td>( n = 17 )</td>
<td>( n = 17 )</td>
</tr>
</tbody>
</table>
TABLE 5. FREQUENCY DISTRIBUTION OF TRIALS (n) TO SOLUTION—EXPERIMENT I *

Positive Sequence Condition:

<table>
<thead>
<tr>
<th>Three-Focus</th>
<th>Five-Focus</th>
<th>Seven-Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/f</td>
<td>n/f</td>
<td>n/f</td>
</tr>
<tr>
<td>1/3</td>
<td>1/2</td>
<td>1/1</td>
</tr>
<tr>
<td>2/10</td>
<td>2/11</td>
<td>3/1</td>
</tr>
<tr>
<td>3/2</td>
<td>3/1</td>
<td>5/1</td>
</tr>
<tr>
<td>4/2</td>
<td>4/1</td>
<td>40/14</td>
</tr>
<tr>
<td></td>
<td>5/1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6/1</td>
<td></td>
</tr>
</tbody>
</table>

Mixed Sequence Condition:

<table>
<thead>
<tr>
<th>Three-Focus</th>
<th>Five-Focus</th>
<th>Seven-Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/f</td>
<td>n/f</td>
<td>n/f</td>
</tr>
<tr>
<td>1/7</td>
<td>1/1</td>
<td>1/2</td>
</tr>
<tr>
<td>2/1</td>
<td>2/1</td>
<td>2/2</td>
</tr>
<tr>
<td>3/8</td>
<td>3/11</td>
<td>3/4</td>
</tr>
<tr>
<td>6/1</td>
<td>4/1</td>
<td>4/1</td>
</tr>
<tr>
<td></td>
<td>5/1</td>
<td>7/1</td>
</tr>
<tr>
<td></td>
<td>7/2</td>
<td>8/3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11/1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12/1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13/1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16/1</td>
</tr>
</tbody>
</table>

* n denotes the number of trials to solution, f denotes the frequency of occurrence of n
An examination of Table 4 suggests subjects receiving all positive or mixed instances and a focus strategy of three dimensions appear to have obtained the lowest mean trials to solution followed by subjects who received all positive instances and a focus strategy of five dimensions. The largest mean score appears to be obtained by subjects receiving all positive instances and a focus strategy of seven dimensions. An examination of the table also suggests interaction of the variables may be occurring.

An analysis of variance was performed. The results are summarized in Table 6.

**TABLE 6. ANOVA OF SEQUENCE CONDITION BY FOCUS STRATEGY**

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(SEQUENCE CONDITION)</td>
<td>1</td>
<td>1941.43</td>
<td>48.50</td>
<td>.001</td>
</tr>
<tr>
<td>B (FOCUS STRATEGY)</td>
<td>2</td>
<td>3355.43</td>
<td>84.07</td>
<td>.001</td>
</tr>
<tr>
<td>AB</td>
<td>2</td>
<td>2199.71</td>
<td>54.95</td>
<td>.001</td>
</tr>
<tr>
<td>S/AB</td>
<td>96</td>
<td>40.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>101</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A first-order interaction between sequence condition and focus strategy proved to be significant, $F(2,96) = 54.95, p < .001$, so post hoc pairwise comparisons were performed using Tukey's test. Figure 1 is a plot of the interaction.
FIGURE 1. PLOT OF FIRST ORDER INTERACTION

EXPERIMENT I
Of the fifteen pairwise comparisons tested at the $p < .01$ level of significance, five proved to be statistically significant. Specifically, subjects receiving the seven-dimension focus strategy and all positive instances achieved a statistically higher mean score ($33.47$) than any other treatment group ($p < .01$). Those subjects receiving a mixed sequence and the seven-dimension focus strategy had a mean score of $6.18$. This group was the only one where a treatment involving mixed instances produced a statistically significant lower mean score than the corresponding group receiving all positive instances. Thus, negative instances greatly enhanced the ability of subjects to acquire the concept when a seven-focus strategy was used and irrelevant features were prevalent. When only positive instances were used in conjunction with the seven-dimension focus strategy, subjects practically failed to acquire the concept, as is evident by the mean score $33.47$.

The mean scores obtained by the subjects when sequence condition was crossed with the three-dimension and five-dimension focus strategies (even though no statistically significant differences existed) tend to support the classical psychological results for conjunctive concepts. That is, the scores obtained in the all positive sequence conditions for both the three-dimension and five-dimension focus strategies appeared to be lower. However, the reversal of this result when subjects were given the seven-dimension focus
strategy lends support to results obtained by Shumway, et al., (1981) and Stout (1981). Clearly, the value of negative instances was greatly enhanced by the presence of highly frequent, or prevalent, features of irrelevant dimensions in the seven-dimension focus strategy.

Experiment II. Before subjecting the data to analysis, the data from two subjects in the all positive and three-dimension focus strategy treatment group, two subjects in the all positive and five-dimension focus strategy treatment group, three subjects in the all positive and seven-dimension focus strategy treatment group, and one subject in the mixed and seven-dimension focus strategy treatment group were randomly discarded to allow for equal cell sizes of \( n = 17 \) each. This allowed a conventional analysis of variance to be performed with the robustness usually associated with equal \( n \) designs and avoided problems associated with computations and interpretation of unequal \( n \) designs.

Table 7 gives the means and standard deviations for trials to solution for each cell on each of the treatment tasks of Experiment II. Table 8 gives the frequency distribution for trials to solution for each cell.
TABLE 7. MEANS AND STANDARD DEVIATIONS OF TRIALS TO SOLUTION BY CELL

<table>
<thead>
<tr>
<th></th>
<th>3-focus</th>
<th>5-focus</th>
<th>7-focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>µ=2.00</td>
<td>µ=2.13</td>
<td>µ=24.29</td>
</tr>
<tr>
<td></td>
<td>σ=1.00</td>
<td>σ=1.01</td>
<td>σ=19.35</td>
</tr>
<tr>
<td>n</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>µ=2.24</td>
<td>µ=2.32</td>
<td>µ=0.33</td>
</tr>
<tr>
<td></td>
<td>σ=1.56</td>
<td>σ=2.51</td>
<td>σ=0.77</td>
</tr>
<tr>
<td>n</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
</tbody>
</table>

An examination of Table 7 suggests subjects receiving all positive instances and either a three- or five-dimension focus strategy appear to have obtained the lowest mean trials to solution. However, subjects receiving all positive instances and a seven-dimension focus strategy appear to have the highest mean score of all treatment groups. Interaction of the variables is also suggested by an examination of Table 7.
TABLE 8. FREQUENCY DISTRIBUTION OF TRIALS(n) TO SOLUTION—EXPERIMENT II *

Positive Sequence Condition:

<table>
<thead>
<tr>
<th></th>
<th>Three-Focus</th>
<th>Five-Focus</th>
<th>Seven-Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/f</td>
<td>n/f</td>
<td>n/f</td>
<td></td>
</tr>
<tr>
<td>1/5</td>
<td>1/3</td>
<td>1/1</td>
<td></td>
</tr>
<tr>
<td>2/9</td>
<td>2/11</td>
<td>2/6</td>
<td></td>
</tr>
<tr>
<td>3/2</td>
<td>3/1</td>
<td>40/10</td>
<td></td>
</tr>
<tr>
<td>5/1</td>
<td>4/1</td>
<td>5/1</td>
<td></td>
</tr>
</tbody>
</table>

Mixed Sequence Condition:

<table>
<thead>
<tr>
<th></th>
<th>Three-Focus</th>
<th>Five-Focus</th>
<th>Seven-Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/f</td>
<td>n/f</td>
<td>n/f</td>
<td></td>
</tr>
<tr>
<td>1/9</td>
<td>1/7</td>
<td>1/1</td>
<td></td>
</tr>
<tr>
<td>3/6</td>
<td>2/1</td>
<td>2/1</td>
<td></td>
</tr>
<tr>
<td>5/1</td>
<td>3/7</td>
<td>3/5</td>
<td></td>
</tr>
<tr>
<td>6/1</td>
<td>9/2</td>
<td>4/2</td>
<td></td>
</tr>
<tr>
<td>7/1</td>
<td>6/1</td>
<td>7/1</td>
<td></td>
</tr>
<tr>
<td>11/1</td>
<td>11/1</td>
<td>11/1</td>
<td></td>
</tr>
<tr>
<td>12/2</td>
<td>12/2</td>
<td>12/2</td>
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</tr>
<tr>
<td>22/1</td>
<td>22/1</td>
<td>22/1</td>
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<td>27/1</td>
<td>27/1</td>
<td>27/1</td>
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</tr>
<tr>
<td>28/1</td>
<td>28/1</td>
<td>28/1</td>
<td></td>
</tr>
</tbody>
</table>

* n denotes the number of trials to solution, f denotes the frequency of occurrence of n
An analysis of variance was performed. The results are summarized in Table 9.

**TABLE 9. ANOVA OF SEQUENCE CONDITION BY FOCUS STRATEGY**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>F</th>
<th>p &lt;</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A (SEQUENCE CONDITION)</td>
<td>1</td>
<td>593.15</td>
<td>7.75</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>B (FOCUS STRATEGY)</td>
<td>2</td>
<td>2312.13</td>
<td>30.01</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>2</td>
<td>712.42</td>
<td>9.25</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>AB/AB</td>
<td>96</td>
<td></td>
<td>77.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>101</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A first-order interaction proved to be statistically significant, $F(2,96) = 9.25$, $p < 0.001$, so Tukey's test was performed as a post hoc analysis. Figure 2 is a plot of the interaction.
FIGURE 2. PLOT OF FIRST ORDER INTERACTION

EXPERIMENT II
Of the fifteen possible pairwise comparisons, five proved to be statistically significant \((p < .01)\). Specifically, those subjects receiving all positive instances and a seven-dimension focus strategy obtained the highest mean score \((24.29)\) of all treatment groups \((p < .01)\). Subjects receiving a mixed sequence and the seven-dimension focus strategy had a mean score of 8.38. This group was the only one where a treatment involving mixed instances produced a statistically significant lower mean score than the corresponding group receiving all positive instances. Thus, it again appears that negative instances greatly enhanced the ability of subjects to acquire the concept when a seven-focus strategy was used and irrelevant features were prevalent. The high mean score of 33.47 obtained by subjects receiving all positive instances and the seven-dimension focus strategy indicates subjects practically failed to acquire the concept.

The mean scores obtained by subjects when sequence condition was crossed with the three- and five-dimension focus strategies (even though no statistically significant differences existed) tend to support classical psychological results for conjunctive concepts. The reversal of this was again found when subjects were given the seven-dimension focus strategy crossed with sequence condition. This reversal of results lends support to results obtained by Shumway, et al., (1981), and Stout (1981). It seems clear
that the value of negative instances was greatly enhanced by the presence of prevalent levels of irrelevant dimensions in the seven-dimension focus strategy.

**Discussion.** No statistically significant differences were obtained between the all positive sequence and the mixed sequence in the three-dimension and five-dimension focus strategies (where irrelevant features had a .5 prevalency) for both experiments.

A possible explanation for this lack of statistical significance is that the minimum number of trials logically needed to obtain the solution in both experiments for the three- and five-dimension focus strategies crossed with the all positive condition was two, whereas the logical minimum for the corresponding mixed sequence condition was three. Thus, if subjects used all the information available in each instance to obtain the solution, the difference in mean trials to solution would be one trial. However, a trend of favoring all positive instances over sequences of positive and negative instances appears to be present. Usual findings in psychological research involving conjunctive concepts favor all positive sequences over mixed sequences. The reversal of this result (favoring a mixed sequence over all all positive sequence) for both experiments in the seven-dimension focus strategy is highly significant.
The reason for this change of results seems clear.
In Experiment I, "all constants are integers" and "variables are in both terms" appeared in every instance. In the positive condition crossed with the seven-dimension focus strategy, these two characteristics were always a possible choice for the concept. However, since both "all constants are integers" and "variables are in both terms" also appeared in every negative instance, they would be eliminated from consideration as the concept after one negative instance was encountered. Similarly, in Experiment II, the features "figure is a polygon" and "base is parallel to bottom of page" appeared in every instance. Again, these two characteristics were always a possible choice for the concept in the positive condition crossed with the seven-dimension focus strategy. Since both characteristics also appeared in every negative instance, they would be eliminated from consideration as the concept after a single negative instance.

The significance of the seven-dimension focus strategy was that it represented the situation where levels of irrelevant dimensions which were always prevalent had to be considered. Such a situation is common among mathematical concepts (e.g., using a's, b's, and c's, in $c^2 = a^2 + b^2$, using $x$ as the unknown, the position of exercises in a text, etc.).
Three subjects were randomly selected from each treatment group in each experiment to be interviewed. Each interviewee was asked to describe how his/her solution was obtained. In both experiments, interviewees randomly selected from the seven-dimension focus strategy and all positive instances treatment group consistently remarked they felt somewhat confused since they had found more than one solution to the task. Even interviewees who had selected the correct two characteristics were not sure they had found the solution. However, interviewees who had received the seven-dimension focus strategy and mixed instances were able to eliminate those incorrect choices of the characteristics by using the negative instances.

Interviewees from the other treatment groups in both experiments had little or no difficulty in arriving at the correct solution. Some subjects remarked they could have arrived at the solution earlier.

The implications of these results seem to be: 1) focusing the learner's attention on a narrow range of possible attractive irrelevant dimensions reduces the value of negative instances; 2) not knowing about or perhaps not being able to manipulate some prevalent irrelevant features of a concept dictates the need to use negative instances of the concept; 3) using negative instances as in this study, reduces the need to focus the learner's attention.
The present study evolved from the Shumway, et al., (1981) and Stout (1981) research. The Shumway, et al., (1981) study discovered a potentially important variable which could account for the discrepancy between the results of laboratory-type and instructional-type conjunctive concept learning research. This potentially important variable is the prevalency of irrelevant features of a conjunctive concept.

Stout (1981) extended the results of Shumway, et al., (1981), Tennyson, et al., (1972), and used focusing strategies the author believed depicted classroom practice to further validate the importance of the prevalency variable and to try to determine when negative instances should be used. Results indicated negative instances should be used when irrelevant features were difficult, impossible, or inconvenient to manipulate.

The present study extends the results of Stout (1981). Letter strings were translated into classroom concepts. The results were obtained using two mathematical concepts: difference of squares and equilateral quadrilateral. Results parallel those of Stout (1981) and support the findings of Shumway, et al., (1981). That is, a mixed
sequence was favored over an all positive sequence when crossed with the seven-dimension focus strategy in both experiments. Also, when irrelevant features had a prevalency of .5, mean trials to solution obtained by subjects in the all positive sequence crossed with a three- or five-dimension focus strategy always appeared to be lower than corresponding means in the mixed sequence condition in both experiments. However, no statistically significant difference was found. A possible explanation for the lack of statistical significance lies in the minimum number of trials logically needed to obtain a solution.

Conclusions which might be drawn from the present study are: (1) prevalent irrelevant features of a conjunctive concept which are difficult, impossible, or inconvenient to manipulate can be effectively eliminated from a learner's consideration by providing matched negative instances which contain those prevalent irrelevant features; (2) focusing the learner's attention away from prevalent irrelevant features of a conjunctive concept appears to reduce the need to use matched negative instances; and, (3) not providing negative instances coupled with a lack of effectively focusing the learner's attention can result in the failure of the learner to acquire the conjunctive concept.

Implications of this study for teaching conjunctive mathematical concepts are inherent in the above conclusions.
However, taking the results of this study coupled with the author's own teaching experience, a major implication seems to be: use negative instances in the mathematics classroom. Many of the irrelevant features of mathematical concepts go unheeded by mathematics teachers but are attractive distractors for students. Using matched negative instances can help students acquire the concept being taught.

Future research efforts could involve the use of a focusing strategy versus a mixed learning sequence in a teaching episode. The results from such a study would provide further information about the effectiveness of focusing versus the effectiveness of a mixed learning sequence in a classroom setting.

Other directions could include looking at the pattern of responses of subjects in a clinical-type setting where no focusing is provided. Subjects would generate their own list of (what they believed to be) relevant features as they viewed individual instances in the learning sequence. This could provide valuable information concerning what subjects perceived as relevant and lead, perhaps, to better focusing strategies.
BIBLIOGRAPHY


Feldman, K.V. The effects of number of positive and negative instances, concept definition, and emphasis of relevant attributes in the attainment of mathematical concepts. Technical Report No. 1, Wisconsin University, Madison Research and Development Center, 1972.

Friebergs, V., & Tulving, E. The effect of practice on utilization of information from positive and negative


Shumway, R.J. Negative instances in mathematical concept acquisition: Transfer effects between the concepts of commutativity and associativity. *Journal for Research in Mathematics Education*, 1974, 2, 197-211.


Shumway, R.J. and White, A.L. Prevalent levels of irrelevant attributes: A potentially important variable in mathematical concept learning. Unpublished manuscript, Ohio State University, 1977.


Swanson, J.E. The effects of number of positive and negative instances, concept definition, and emphasis of relevant attributes on the attainment of three environmental concepts by sixth grade children. Technical Report No. 244, Wisconsin University, Madison Research and Development Center for Cognitive Learning, 1972.


Tennyson, R.D., Woolley, F.R., & Merrill, M.D. Exemplar and nonexemplar variables which produce correct concept classification behavior and specified classification errors, Journal of Educational Psychology, 1972, vol. 63, no. 2, 144-152.

APPENDIX A

Negative Instances and Focusing Strategies in Concept Learning

David L. Stout

The Ohio State University
1981
Introduction. Recent research by educators (e.g., Shumway, White, and Wilson, 1981) has explored the seemingly contradictory evidence regarding the use of nonexamples in conjunctive feature identification tasks. Psychologists have done considerable amounts of careful experimental research on feature identification tasks. Reviews of this research consistently support the use of learning sequences consisting of only positive instances for conjunctive concepts (Bourne and Dominowski, 1972; Erickson and Jones, 1978). However, research done using conjunctive concepts from mathematics consistently supports the use of learning sequences containing a mixture of positive and negative instances (Shumway, 1971, 1974, 1977). In an attempt to explain this discrepancy, Shumway and White (1977) analyzed concepts from mathematics and science and then hypothesized prevalent levels of irrelevant dimensions to be the confounding factor. Paralleling the work of Bourne, et al., (1976), on frequency analysis, Shumway and White (1977) manipulated the frequencies of levels of irrelevant dimensions across positive and negative instances of a conjunctive concept. Their results supported the classical psychological research findings (i.e., favored all positive instances) when the frequency of levels of irrelevant dimensions was .5; however, when the frequency was .9, the results supported the use of both positive and negative instances. In a refinement of their experiment, Shumway, et al., (1981) achieved the same results.
When teaching concepts which have a large or an infinite number of irrelevant dimensions, educational psychologists have suggested the most efficient organization of positive and negative instances is in a concept set. A concept set consists of two divergent positive instances each matched to a negative instance with prompting of the irrelevant dimensions. This helps the learner focus attention on features which define the concept class and aids in concept learning. (Tennyson and Boutwell, 1973; Tennyson, et al., 1972).

The author's pilot study was an attempt to use the results of Shumway, et al., (1981), and Tennyson, et al., (1972), to explore the effectiveness of three focusing strategies. After analyzing concepts from mathematics, it becomes apparent that many irrelevant dimensions are either never or, at best, rarely manipulated. Thus, the learner can, and frequently does, erroneously ascribe relevancy to an irrelevant dimension due, perhaps, to its prevalence. However, if we as teachers were able to focus the attention of our students on the relevant dimensions or limit the number of irrelevant dimensions which they must consider, then giving only positive instances of the concept would seem to be adequate. Teachers must also be aware of irrelevant dimensions which exist and which are attractive to learners but which are impossible or inconvenient to manipulate (e.g., the position of exercises in a text). It is in this area where negative instances
would seem to be most powerful since the frequency of levels of irrelevant dimensions approaches 1.0. That is, a level of an irrelevant dimension is present in both positive and negative instances practically all the time. Thus, a non-example (negative instance) of the concept would help the learner to correctly judge the irrelevant dimension as irrelevant. Because of this apparent interaction of the use of positive and negative instances with the use of a focusing strategy, I hypothesized that as the intensity of focusing decreased, the value of negative instances would increase. I chose a two-letter conjunctive concept, "D and K", to test my hypothesis.

Sixteen seven-letter strings constituted the stimulus population. The seven letters in each string represent the dimensions. Each dimension had two features, for a total of fourteen features. The features for dimension one were B or C; for two, D or F; for three, G or H; for four, J or K; for five, L or M; for six, P or Q; and for seven, R or S. Eight of the strings were positive instances and eight were negative instances of the concept. Two learning sequences were constructed. One sequence consisted entirely of positive instances and the other mixed (positive and negative) instances. In each sequence, the positive instances were divergent. On each page there were two identical pairs. Thus, there were four instances per page.
For this experiment divergent means all irrelevant features, except two, changed from one positive instance to the next positive instance. The two irrelevant features left unchanged were P and R, representing those irrelevant features which we are not able to manipulate or perhaps those about which we are unaware. Thus, the prevalency for one level of each of the two unchanged irrelevant features was 1, while all others had .5 prevalency.

The instances in the mixed sequence were arranged in a modified concept set. Each page contained two positive instances and two negative instances. The two positive instances were divergent and each was matched to a negative instance. However, no prompting of the irrelevant dimensions was given.

Three focus strategies were also developed: three-focus, five-focus, and seven-focus. The three-focus strategy required the subject to limit attention to three dimensions each consisting of two features. D and K represented two of the six features. In the five-focus strategy, the subject's attention was focused on five dimensions of two features each. Again two of the features were D and K. The seven-focus strategy in reality was a no-focus strategy since the subject was required to view all seven dimensions with two features each. D and K represented two of the fourteen features.
Method: Subjects. The subjects were 72 Northwest Florida junior college students enrolled in mathematics courses taught by the author during the Fall term, 1981. Treatments were administered by the author during class. Participation was voluntary and all students in each class chose to participate.

Procedure. The stimulus population consisted of 16 letter strings each with seven dimensions. Eight strings were positive and eight were negative. Five of the seven dimensions varied along two levels. The levels of each dimension were B or C, D or F, G or H, J or K, L or M, P or Q, and R or S. Each stimulus was a string of seven letters. Each subject received two packets of materials -- a packet of instances and a packet of focus sheets. Subjects were told the concept consisted of two letters which were both present in any positive instance. Furthermore, subjects were told to consider as possible choices for the concept only pairs of letters present on their respective focus sheets. All answers were recorded on the focus sheets. The concept was "D and K". All subjects were given the same two training tasks, one with three dimensions and one with four dimensions. Each dimension had two levels. The three dimension task had a focus strategy of two dimensions. The four dimension task had a focus strategy of four dimensions. The author carefully worked through the training tasks with the subjects. During and after the training tasks, questions
concerning negative instances, how to record answers, etc., were entertained. At the conclusion of this question and answer period, a problem task was given. This problem task had five dimensions and a focus strategy of four dimensions. Subjects worked through this task without help. The purpose of the problem task was to determine whether subjects understood the essence of the experiment. On each page of the training, problem, and actual task activities, there were four instances. Each of the accompanying focus strategies also contained four response opportunities per page. Subjects used cards to cover subsequent instances and accompanying response opportunities as they worked their way down each page, at their own rate. Subjects were able to view as many as three previous instances and responses. Subjects were told not to look back at previous pages. For each instance subjects were asked to judge it as positive or negative. They recorded their guesses on their focus sheets. Subjects then uncovered the correct feedback presented as a + or - under each instance. Subjects recorded this correct feedback on their focus sheets. Subjects also circled the levels of the dimensions of the instances which appeared on their focus sheets. Subjects were then asked to write the letters which they thought were the concept. All subjects viewed 40 instances (repetition of the eight positive and eight negative instances was thus necessary). The criterion
variable was the number of trials until solution was obtained. Instances were presented as follows:

\[ \text{D D H K L P R} \]

Each subject received one of three focus strategies: three, five, or seven dimensions. The three, five, and seven dimensions were presented as follows:

- **C D J**
  - Guess: + or -
  - I think the two letters are: **+ or - ____ and ____**.

- **C D G K L**
  - Guess: + or -
  - I think the two letters are: **+ or - ____ and ____**.

- **C D G K L P R**
  - Guess: + or -
  - I think the two letters are: **+ or - ____ and ____**.

**Design.** The design was a 2 x 3 factorial with two sequence conditions (all positive or mixed positive and negative) and three focus strategies (3, 5, and 7 dimensions). The stimulus population consisted of sixteen seven-letter strings varying in five of the seven dimensions of two levels each. Table 10 gives the sample distribution and design matrix.
The 36 subjects in each row were given the same sequence. In the all positive sequence (+), 8 positive instances were presented in divergent pairs. Each pair was repeated on a page. Instances were considered divergent when all but the last two irrelevant features changed. The mixed sequence (-) contained the same eight positive instances used in the all positive sequence. Each positive instance, however, was matched to a negative instance. Two such pairs were presented on each page. Matching occurred when all irrelevant features remained unchanged but one relevant feature changed in order to make a negative instance. Both sequences were crossed with the three focus strategies for six possible treatments.

Results. The following table gives the means and standard deviations for trials to solution for each cell on each of the treatment tasks.

Insert Table 11 About Here

An examination of Table 11 suggests that subjects receiving all positive instances and a focus strategy of either 3 or 5 dimensions had the lowest mean trials to solution. The highest mean score was obtained by subjects receiving all positive instances and a focus strategy of 7 dimensions. However, the third lowest mean score was obtained by subjects receiving themixed sequence of instances and a focus strategy
of 7 dimensions. Thus, an examination of Table also suggests that interaction of the variables is occurring.

An analysis of variance was performed to subject these tentative findings to formal test. The results are summarized in Table 12.

| Insert Table 12 About Here |

A first order interaction between sequence condition and focus strategy proved to be significant, \( F(2,66) = 7.79, p < .001 \), so post hoc pairwise comparisons were performed using Tukey's test. Figure 3 is a plot of this disordinal interaction. Of the 15 pairwise comparisons tested at the \( p < .05 \) level of significance, five proved to be statistically significant. Specifically, subjects receiving the seven-dimension focus strategy with all positive instances achieved a statistically higher mean score (30.5) than any other treatment group (\( p < .05 \)). Those subjects receiving a mixed sequence and the seven-dimension focus strategy had a mean score of 9.58. This group was the only one where a treatment involving mixed instances produced a lower mean score than the corresponding group receiving all positive instances. Thus, negative instances greatly enhanced the ability of subjects to acquire the concept when irrelevant features were prevalent. When only positive instances were
used in conjunction with the seven-dimension focus strategy, subjects practically failed to acquire the concept as is evident by the mean score 30.5.

The mean scores obtained by the subjects when sequence condition was crossed with the three-dimension and five-dimension focus strategies (even though no statistically significant differences existed) tend to support the classical psychological results for conjunctive concepts. That is, lower mean scores were obtained in the all positive sequence condition for both the three-dimension and five-dimension focus strategies. However, the reversal of this result when subjects were given the seven-dimension focus strategy lends support to results obtained by Shumway, et al., 1981. Clearly, the value of negative instances was greatly enhanced by the presence of highly frequent, or prevalent, levels of irrelevant dimensions in the seven-dimension focus strategy.

Discussion. The favoring of all positive instances over sequences of positive and negative instances in the three-dimension and five-dimension focus strategies supports the usual findings in psychological research involving conjunctive concepts. The reversal of this result (favoring a mixed sequence over an all positive sequence) in the seven-dimension focus strategy is highly significant. The reason for this reversal of results seems clear. The letters P and R appeared in every instance. In the positive condition
crossed with the seven-dimension focus strategy, P and R was always a possible choice for the concept. However, since both P and R also appeared in every negative instance they would be eliminated immediately from consideration as the concept after one negative instance was encountered.

The significance of the seven-dimension focus strategy was that it represented the situation where levels of irrelevant dimensions were very prevalent. Such a situation is common among mathematical concepts (e.g., using a's, b's, and c's in \( c^2 = a^2 + b^2 \), using x as the unknown, etc.).

The implications of these results seem to be: 1) focusing the learner's attention on a narrow range of possible irrelevant dimensions reduces the value of negative instances; 2) not knowing about, or perhaps not being able to manipulate, some prevalent irrelevant features of a concept dictates the need to use negative instances of the concept; and 3) using negative instances as in this study, reduces the need to focus the learner's attention.

Future directions of the author's research will be in the area of using conjunctive mathematical concepts in the manner done here with letter strings.
### TABLE 10

Sample Distribution

<table>
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<th>3-focus</th>
<th>5-focus</th>
<th>7-focus</th>
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<tr>
<td>+ Condition</td>
<td>n=12</td>
<td>n=12</td>
<td>n=12</td>
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<tr>
<td>− Condition</td>
<td>n=12</td>
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<td>n=12</td>
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### TABLE 11

Means and Standard Deviations of Trials to Solution by Cell

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<th>3-focus</th>
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<th>7-focus</th>
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<tbody>
<tr>
<td>+ Condition</td>
<td>m=4.75</td>
<td>m=5.83</td>
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<td></td>
<td>sd=11.11</td>
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<td>sd=14.41</td>
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TABLE 12

ANOVA of Sequence Condition by Focus Strategy

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<td>A(Sequence Condition)</td>
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<td>B(Focus Strategy)</td>
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<td>AB</td>
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<td>7.79</td>
<td>.001</td>
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<tr>
<td>S/AB</td>
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<td>71</td>
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FIGURE 3. PLOT OF FIRST ORDER INTERACTION
References


Shumway, R. J. Negative instances and mathematical concept formation:


Shumway, R. J. Negative instances in mathematical concept acquisition:

Transfer effects between the concepts of commutativity and associativity. Journal for Research in Mathematics Education, 1974, 5, 197-211.


Shumway, R. J. & White, A. L. Prevalent levels of irrelevant attributes:

A potentially important variable in mathematical concept learning.

Unpublished manuscript, Ohio State University, 1977.

Shumway, R. J., White, A. L., and Wilson, Patricia. Frequency of Levels of Irrelevant Dimensions in Feature Identification.

Unpublished manuscript, Ohio State University, 1981.

### APPENDIX B

**STIMULUS POPULATION: EXPERIMENT I**

#### POSITIVE INSTANCES:

- $a^2 - 4b^2$
- $(b + 3a)^6 - (2b - 5a + b)^4$
- $a^{16} - b^8$
- $9(5 + b)^4 - 16(a^3 + b)^6^2$
- $16b^4 - 9a^2$
- $(a^2 + b^2 - a + b)^2 - (2 - a)^{24}$
- $b^2 - a^2$
- $36(a^2 - b^2)^6 - (3a + 1)^4$

#### NEGATIVE INSTANCES:

- $a^2 - 3b^2$
- $(b + 3a)^6 + (2b - 5a + b)^4$
- $a^{15} - b^8$
- $9(5 + b)^4 + 16(a^3 + b)^6^2$
- $16b^4 + 9a^2$
- $(a^2 + b^2 - a + b)^3 - (2 - a)^{24}$
- $b^2 + a^2$
- $35(a^2 - b^2)^6 - (3a + 1)^4$

---

78
APPENDIX C. STIMULUS POPULATION: EXPERIMENT II

+ 

- 

[Diagrams of various geometric shapes, including squares, rectangles, and pentagons, with dashed and solid lines.]
APPENDIX D

POSITIVE SEQUENCE CONDITION: EXPERIMENT I
1. \(a^2 - 4b^2\)

2. \((b + 3a)^6 - (2b - 5a + 8)^4\)

3. \(a^2 - 4b^2\)

4. \((b + 3a)^6 - (2b - 5a + 8)^4\)
5. \(16b^4 - 9a^2\)

6. \((a^2 + b^2 - a + b)^2 - (2 - a)^2\)

7. \(16b^4 - 9a^2\)

8. \((a^2 + b^2 - a + b)^2 - (2 - a)^2\)
9. \( a^{16} - b^8 \) + \\
10. \( 9(5 + b)^4 - 16(a^3 + 8)^6 \) + \\
11. \( a^{16} - b^8 \) + \\
12. \( 9(5 + b)^4 - 16(a^3 + 8)^6 \) +
13. \( b^2 - a^2 \)

14. \( 36(a^2 - b^2)^6 - (3a + 1)^4 \)

15. \( b^2 - a^2 \)

16. \( 36(a^2 - b^2)^6 - (3a + 1)^4 \)
17. \( a^2 - 4b^2 \)

+ 

18. \( (b + 3a)^6 - (2b - 5a + 8)^4 \)

+ 

19. \( a^2 - 4b^2 \)

+ 

20. \( (b + 3a)^6 - (2b - 5a + 8)^4 \)

+ 

21. \(16b^4 - 9a^2\)

+ 

22. \((a^2 + b^2 - a + b)^2 - (2 - a)^2\)

+ 

23. \(16b^4 - 9a^2\)

+ 

24. \((a^2 + b^2 - a + b)^2 - (2 - a)^2\)

+
25. \[ a^{16} - b^8 \] + \\
26. \[ 9(5 + b)^4 - 16(a^3 + 8)^6 \] + \\
27. \[ a^{16} - b^8 \] + \\
28. \[ 9(5 + b)^4 - 16(a^3 + 8)^6 \] +
29. \( b^2 - a^2 \) 

+ 

30. \( 36(a^2 - b^2)^6 - (3a + 1)^4 \) 

+ 

31. \( b^2 - a^2 \) 

+ 

32. \( 36(a^2 - b^2)^6 - (3a + 1)^4 \) 

+
33. \(a^2 - 4b^2\) 

34. \((b + 3a)^6 - (2b - 5a + 8)^4\) 

35. \(a^2 - 4b^2\) 

36. \((b + 3a)^6 - (2b - 5a + 8)^4\)
37. \(16b^4 - 9a^2\)

38. \((a^2 + b^2 - a + b)^2 - (2 - a)^2\)

39. \(16b^4 - 9a^2\)

40. \((a^2 + b^2 - a + b)^2 - (2 - a)^2\)
APPENDIX E

MIXED SEQUENCE CONDITION: EXPERIMENT I
1. \( a^2 - 4b^2 \)

2. \( a^2 - 3b^2 \)

3. \( (b + 3a)^6 - (2b - 5a + 8)^4 \)

4. \( (b + 3a)^6 + (2b - 5a + 8)^4 \)
5. \(16b^4 - 9a^2\)

6. \(16b^4 + 9a^2\)

7. \((a^2 + b^2 - a + b)^2 - (2 - a)^2\)

8. \((a^2 + b^2 - a + b)^3 - (2 - a)^2\)
9. \( a^{16} - b^8 \)

10. \( a^{15} - b^8 \)

11. \( 9(5 + b)^4 - 16(a^3 + 8)^6 \)

12. \( 9(5 + b)^4 + 16(a^3 + 8)^6 \)
13. \( b^2 - a^2 \)

+ 

14. \( b^2 + a^2 \)

- 

15. \( 36(a^2 - b^2)^6 - (3a + 1)^4 \) 

+ 

16. \( 35(a^2 - b^2)^6 - (3a + 1)^4 \) 

-
17. \( a^2 - 4b^2 \)

18. \( a^2 - 3b^2 \)

19. \( (b + 3a)^6 - (2b - 5a + 8)^4 \)

20. \( (b + 3a)^6 + (2b - 5a + 8)^4 \)
21. \(16b^4 - 9a^2\)

22. \(16b^4 + 9a^2\)

23. \((a^2 + b^2 - a + b)^2 - (2 - a)^{24}\)

24. \((a^2 + b^2 - a + b)^3 - (2 - a)^{24}\)
25. $a^{16} - b^8$

26. $a^{15} - b^8$

27. $9(5 + b)^4 - 16(a^3 + 8)^6$

28. $9(5 + b)^4 + 16(a^3 + 8)^6$
29. \( b^2 - a^2 \)

30. \( b^2 + a^2 \)

31. \( 36(a^2 - b^2)^6 - (3a + 1)^4 \)

32. \( 35(a^2 - b^2)^6 - (3a + 1)^4 \)
33. \( a^2 - 4b^2 \)

34. \( a^2 - 3b^2 \)

35. \((b + 3a)^6 - (2b - 5a + 8)^4\)

36. \((b + 3a)^6 + (2b - 5a + 8)^4\)
37. \(16b^4 - 9a^2\)

38. \(16b^4 + 9a^2\)

39. \((a^2 + b^2 - a + b)^2 - (2 - a)^{24}\)

40. \((a^2 + b^2 - a + b)^3 - (2 - a)^{24}\)
APPENDIX F

POSITIVE SEQUENCE CONDITION: EXPERIMENT II
APPENDIX G

MIXED SEQUENCE CONDITION: EXPERIMENT II
APPENDIX H

THREE-DIMENSION FOCUS SHEETS: EXPERIMENT I
**BASIC**  
TERMS ARE:  
SIMPLE  
COMPLEX  
OPERATION IS:  
SUBTRACTION  
ADDITION  
THIS INSTANCE IS:  + OR -  

TERMS(INCL. COEFF.) ARE:  
ALL SQUARES  
NOT ALL SQUARES  
GUESS:  + OR -  

I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE:  

**BASIC**  
TERMS ARE:  
SIMPLE  
COMPLEX  
OPERATION IS:  
SUBTRACTION  
ADDITION  
THIS INSTANCE IS:  + OR -  

TERMS(INCL. COEFF.) ARE:  
ALL SQUARES  
NOT ALL SQUARES  
GUESS:  + OR -  

I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE:  

**BASIC**  
TERMS ARE:  
SIMPLE  
COMPLEX  
OPERATION IS:  
SUBTRACTION  
ADDITION  
THIS INSTANCE IS:  + OR -  

TERMS(INCL. COEFF.) ARE:  
ALL SQUARES  
NOT ALL SQUARES  
GUESS:  + OR -  

I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE:  

**BASIC**  
TERMS ARE:  
SIMPLE  
COMPLEX  
OPERATION IS:  
SUBTRACTION  
ADDITION  
THIS INSTANCE IS:  + OR -  

TERMS(INCL. COEFF.) ARE:  
ALL SQUARES  
NOT ALL SQUARES  
GUESS:  + OR -  

I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE:  

126
APPENDIX I

FIVE-DIMENSION FOCUS SHEETS: EXPERIMENT I
<table>
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<th>Terms Are:</th>
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<th>Operation Is:</th>
<th>Terms (Incl. Coeff.)</th>
<th>Exponents</th>
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<td>Simple</td>
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<td>All 1's</td>
<td>All squares</td>
<td>All powers of 2</td>
<td></td>
</tr>
<tr>
<td>Complex</td>
<td>Addition</td>
<td>Not all 1's</td>
<td>Not all squares</td>
<td>Not all powers of 2</td>
<td></td>
</tr>
</tbody>
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This instance is: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

This instance is: + or -

Guess: + or -

This instance is: + or -

Guess: + or -

This instance is: + or -

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)

Guess: + or -

I think the two characteristics which describe the concept are: (blank line)
### Terms are:

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<td>Addition</td>
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### Coefficients of terms are:

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### Basic operation is:

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### GUESS:

- OR

I think the two characteristics which describe the concept are:

### AND

I think the two characteristics which describe the concept are:

### AND

I think the two characteristics which describe the concept are:

### AND
APPENDIX K

THREE-DIMENSION FOCUS SHEETS: EXPERIMENT II
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<td>AND __________________________.</td>
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<td>AND __________________________.</td>
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APPENDIX L

FIVE-DIMENSION FOCUS SHEETS: EXPERIMENT II

133
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GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___

GUESS: + OR -
I THINK THE TWO CHARACTERISTICS WHICH DESCRIBE THE CONCEPT ARE: ___ AND ___
APPENDIX M

SEVEN-DIMENSION FOCUS SHEETS: EXPERIMENT II
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<td>-</td>
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<tr>
<td>NOT A POLYGON</td>
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THIS INSTANCE IS: + OR -

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THIS INSTANCE IS: + OR -

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<th>FIGURE IS</th>
<th>BASE ORIENTATION</th>
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<td>1</td>
<td>4</td>
<td>-</td>
<td>A POLYGON</td>
<td>// TO BOTTOM OF PAGE</td>
</tr>
<tr>
<td>NOT A POLYGON</td>
<td># TO BOTTOM OF PAGE</td>
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THIS INSTANCE IS: + OR -
APPENDIX N

PRACTICE AND PROBLEM TASKS
Practice Task A

1. U X Y
   +

2. U W Y
   -

3. V X Y
   -

4. U X Z
   +
5. V W Z

6. U X Y

(STOP)
Practice Task B

1. A N R T

2. A N R S

3. E N R T

4. E P R T
5. ENRT

6. EPRS

7. ANQT

8. APRT
9. A N Q T

10. A P Q T

11. E N Q T

12. A P R T

(STOP)
PROBLEM 1

1. A P Q S U

2. A P Q T U

3. A N Q S U

4. E P Q S U
5. A P Q S U

6. A P Q T U

7. A P Q S V

8. E P Q S U
9. A P Q S U

+ 

10. A P Q T U

- 

11. A P R S U

+ 

12. A P Q T U

-
APPENDIX O

SAMPLE STUDENT RESPONSE SHEETS
1. \( a^2 - 4b^2 \)

2. \( (b + 3a)^6 - (2b - 5a + 8)^4 \)

3. \( a^2 - 4b^2 \)

4. \( (b + 3a)^6 - (2b - 5a + 8)^4 \)
<table>
<thead>
<tr>
<th>TERMS ARE:</th>
<th>OPERATION IS:</th>
<th>TERMS (INCL. COEFF.) ARE:</th>
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<tbody>
<tr>
<td>SIMPLE</td>
<td>SUBTRACTION</td>
<td>ALL SQUARES</td>
</tr>
<tr>
<td>COMPLEX</td>
<td>ADDITION</td>
<td>NOT ALL SQUARES</td>
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</table>

This instance is: \[+\] or \[\] .

**GUESS:** \[+\] or \[-\]

I think the two characteristics which describe the concept are:

- **SIMPLE**
- **SUBTRACTION**

**GUESS:** \[+\] or \[-\]

I think the two characteristics which describe the concept are:

- **SUBTRACTION**
- **All squares**

**GUESS:** \[+\] or \[-\]

I think the two characteristics which describe the concept are:

- **SUBTRACTION**
- **All squares**

**GUESS:** \[+\] or \[-\]

I think the two characteristics which describe the concept are:

- **SUBTRACTION**
- **All squares**

**GUESS:** \[+\] or \[-\]

I think the two characteristics which describe the concept are:

- **SUBTRACTION**
- **All squares**
I think the two characteristics which describe the concept are:

\[
\begin{align*}
\text{Length} &= \frac{\text{Diagonals}}{2} \\
\text{Number of Sides} &= \frac{\text{Number of Sides}}{4}
\end{align*}
\]

And

\[
\text{Number of Sides} = 4
\]
1. \( a^2 - 4b^2 \)

2. \( a^2 - 3b^2 \)

3. \( (b + 3a)^6 - (2b - 5a + 8)^4 \)

4. \( (b + 3a)^6 + (2b - 5a + 8)^4 \)
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**Guess:** (C) or -

I think the two characteristics which describe the concept are:

- Subtraction and
- Not all squares.

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